EFFECTS OF SKEH ANGLE ON SIMPLE SPAN BRIDGE DECKS UNDER SIML'LATED TRUCK LOADING

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## EFFECTS OF SKEW ANGLE ON SIMPLE SPAN BRIDGE

 DECKS UNDER SIMULATED TRUCK LOADING
## by

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# A REPORT PRESENTED TO THE GRADUATE COMMITTEE OF THE DEPARTMENT OF CIVIL ENGINEERING IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF ENGINEERING 

UNIVERSITY OF FLORIDA

Thesis
K96

## ACKNOWLEDGEMENTS

The author wishes to express his sincere appreciation to his advisor and committee chairman Dr. C. 0 . Hays for devoting much patience and guidance throughout the duration of this report. Thanks are also due to committee members Dr. F. E. Fagundo and Dr. B. H. Edwards for taking time out of their busy schedules to read and comment. Fine support is acknowledged from the Industrial and Systems Engineering Department, and in particular Mr. T. Kisko and Mr. R. Houston for allowing the use of GTSTRUDL on their VAX 11-750 computer.

The support and encouragement of family and friends over the last year have made many accomplishments possible. The attitude and dedication of fellow graduate students, particularly N. C. Kumar and K. L. Toye, has set high standard and fostered motivation key to success. Finally, special thanks are due to Ms. Cindy Swartz for providing splendid typing assistance with a fine attitude and pleasant demeanor.
ACKNOWLEDGEMENTS ..... ii
CHAPTER 1 INTRODUCTION ..... 1
1.1 Overview ..... 1
1.2 The Nature of Plate Bending ..... 2
1.3 Numerical Methods ..... 4
CHAPTER 2 MESH SIZE STUDY ..... 6
2.1 Introduction ..... 6
2.2 Determination of Element Size ..... 7
2.3 Effect of Aspect Ratio ..... 16
CHAPTER 3 PARAMETER STUDY ..... 22
3.1 Introduction ..... 22
3.2 Procedure ..... 24
3.3 Results ..... 39
3.4 Contour Plot Descriptions ..... 40
CHAPTER 4 REINFORCEMENT ..... 43
4.1 Introduction. ..... 43
4.2 Failure Analyis by Yield Lines ..... 44
4.3 Orthogonal Reinforcement in Skewed Decks ..... 51
4.4 Design for Skewed Steel ..... 52
CHAPTER 5 CONCLUSIONS ..... 67
5.1 Summary ..... 67
5.2 Further Study ..... 68
APPENDIX A PROGRAM SKEW LOADER ..... 69
A. 1 Introduction. ..... 70
A. 2 Flowchart ..... 72
A. 3 Program Description ..... 74
A. 4 Input Guide ..... 81
A. 5 Extension of the Program. ..... 83
APPENDIX B SAMPLE GTSTRUDL OUTPUT ..... 106
APPENDIX C CONTOUR PLOTS ..... 113
REFERENCES ..... 122


## CHAPTER 1

## INTRODUCTION

### 1.1 Overview

Modern bridge designs often include decks whose spans are at a skew angle to their supports for economic or aesthetic considerations. The behavior of these structures is largely dependent upon a variety of factors such as the angle of skew, direction of reinforcement, aspect ratio, orthotropic stiffeners and types of loading. The effects of these and other factors on the mechanics of deformation may be interdependent, thus requiring a study including variation of parameters. This report will examine the effects of skew angle on major principal moments of free spans under simulated truck loading. Orthogonal and skewed reinforcement will also be discussed.

The finite element approach has been selected as the method of analysis. A mesh size study is undertaken in Chapter 2 to examine the effects of singularities caused by concentrated loads. GTSTRUDL [5] software is used for the stiffness analysis on a VAX 11-750 digital computer. A service load simulator program is written in FORTRAN-77 (Appendix A) and is run semi-interactively with GTSTRUDL to incrementally adjust the truck position. The parameter studies are discussed in Chapter 3. Here the major principal moment at center span is plotted ayainst the location of the truck for various angles of deck skew. The moment reductions which are found for increased angles of skew do not necessarily allow for a reduction in reinforcing steel. The angles at

which the major principal moments intersect the steel plays an important role in the efficiency of resisting flexure. This relationship is discussed in Chapter 4.

### 1.2 The Nature of Plate Bending

An orthogonal isotropic plate subjected to transverse loading deflects according to the differential equation [18]

$$
\begin{equation*}
\frac{\partial^{4} w}{\partial x^{4}}+2 \frac{\partial^{4} w}{\partial x^{2} \partial y^{2}}+\frac{\partial^{4} w}{\partial y^{4}}=-q / D \tag{1}
\end{equation*}
$$

In the case of a simple span such as a bridge deck of Figure $1.1(a)$ the solution to (1) must also satisfy the boundary conditions
$w=0$
(Zero Deflection)
$\left(\frac{\partial^{2} W}{\partial x^{2}}+v \frac{\partial^{2} W}{\partial y^{2}}\right)=0$
(Zero Moment)

Along Simple Supports
and

$$
\left.\begin{array}{rl}
\left(\frac{\partial^{2} w}{\partial x^{2}}+v \frac{\partial^{2} w}{\partial y^{2}}\right) & =0 \quad \text { (Zero Moment) } \\
\left(\frac{\partial^{3} w}{\partial x^{3}}+(2-v) \frac{\partial^{3} w}{\partial x \partial y^{2}}\right)=0 \text { (Zero Shear) }
\end{array}\right\} \begin{aligned}
\text { where: } w & =\text { displacement } \\
x, y & =\text { cartesian coordinates } \\
v & =\text { poisson's ratio } \\
q & =\text { loading function } \\
D & =\text { isotropic stiffness }
\end{aligned}
$$

Along Free Edges



These equations can be solved for many loading conditions by the use of Fourier series. For skewed plate analysis, the introduction of an oblique coordinate system, such as that shown in Figure 1.1(b) is required for the analysis. After transformation of the Laplacian, equation (1) becomes [17]
$\frac{D}{\cos ^{4} \phi}\left\{\frac{\partial^{4} w}{\partial \bar{x}}+2\left(1+2 \sin ^{2} \phi\right) \frac{\partial^{4} w}{\partial \bar{x}^{2} \partial \bar{y}^{2}}+4 \sin \phi\left(\frac{\partial^{4} w}{\partial \bar{x}^{3} \partial \bar{y}}+\frac{\partial^{4} w}{\partial \bar{x}^{4} \bar{y}^{3}}\right)+\frac{\partial^{4} w}{\partial \bar{y}^{4}}\right\}=p(\bar{x}, \bar{y})$
where: $\bar{x}=x+y \tan \phi$
$\bar{y}=y \sec \phi$
$\phi=$ deck angle of skew
$p(\bar{x}, \bar{y})=$ transformed loading function
Solutions to (6) are difficult even for simple loading cases and are therefore not well suited to extensive parameter studies, although Kennedy [10] has been successful with series solutions on a computer. Analytical methods have also been applied by Krettner [12] and Lardy [13]. Energy methods have been used by Guzman and Luisoni [6].

### 1.3 Numerical Methods

Several researchers have been successful in analysis of skew slabs by the finite difference [8], [9] and finite element methods [1], [15]. The growing popularity of numerical methods for skew slab analysis may be attributed to both the inadequacy of analytical solutions and to the advances in today's computer technology. The latter, particularly, has allowed for extensive use of numerical methods in applied mechanics. Several package programs such as STRUDL [14] are available to the public and have become a widely accepted method for analysis, design and research.
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The Finite Element Method was selected for this study using GTSTRUDL [5] software on a VAX 11-750 computer. The finite element approach breaks up or "discretizes" the structure to be analyzed into a network of constituent elements. In plate bending each of these elements is usually allowed three types of displacements or "degrees of freedom" at each corner node (two orthogonal rotations and one transverse displacement). A mathematical function is assumed to describe the displacement variation between the nodes. Then, the stiffness properties of the individual elements can be developed in matrix form. If compatability between the adjacent elements is satisfied then the solution obtained should "converge" as the number of elements is increased. Here compatibility refers to the fact that the pieces must fit together and that all adjoining elements at similar nodes must have corresponding degrees of freedom. Strictly, compatibility is not completely satisfied for the Bending Plate Parallelogram (BPP) element which is used in this study. Zienkiewicz [20] shows it is not possible for a simple polynomial expression to ensure full compatibility when only one displacement and two rotations are prescribed at the nodes. However, experience [20] with the BPP element shows that it "converges" to a good engineering approximation in most practical cases.

Although many different types of elements may be used in discretizing a structure, the procedure is fundamentally the same. The material properties and boundary conditions are first defined for the problem. The element mesh is then selected and the structure stiffness matrix is formed from the known element stiffness matrix. Matrix algegra can then be used to solve the equation relating displacement, moment, strain and stress. Several texts (such as [3] and [20]) ara available which detail the finite element process and give many diverse applications.
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## CHAPTER 2

## MESH SIZE STUDY

### 2.1 Introduction

The first step in analyzing a structure by the finite element method is selecting a mesh size and pattern. The mesh is a very important feature of the study and must be selected with care. In this study the following aspects of mesh size, type and pattern were considered:

1. Accuracy of results
2. Economy of computer time

3:- Ease of comparison of results at varied angles of skew
4. Assurance of conforming to the criterion of non-distorted elements.

GTSTRUDL provides for a wide range of elements suitable for this study. The element type chosen is the Bending Plate Parallelogram (BPP) which is very well suited to skew plate analysis. The BPP uses a fourth order transverse displacement expansion and uses three degrees of freedom (one displacement and two rotations) at each corner node. At 90 degrees the BPP is equivalent to the more familiar Bending Plate Rectangle (BPR). The element is not considered distorted unless it is skewed to an interior acute angle of less than 30 degrees. Consequently, it can be used for deck skew angles of up to 60 degrees.

This study will model the deck as a free span between simple supports. The thickness is held constant at 18 inches and the moments $X$ and $Y$ are released at the joints along the supported edges. Before proceeding further it must be pointed out that there are two ways to geometrically "skew" a plate from a rectangle to a parallelogram, both of which are used extensively in the literature. The first way is to keep all edges the same length and allow the supports to move closer together as the angle of skew is increased. This is illustrated in Figure 2.1(b). The other widely used convention is to maintain a constant distance between the supports and allowing the free edge to "stretch" as a skew angle is increased. Figure 2.1(c) illustrates this convention. This study will adopt the first convention for the simple reason that a thin strip taken parallel to the inclined side will approximate the span of an equivalent simple beam, the length of which will be held constant as the deck skew angle is varied. Moreover, the reinforcement is generally laid parallel to the deck edges as well and holding these lengths constant may be a more realistic approach. Thus for all the examples in which the results are compared between the skewed deck and an orthogonal deck, the geometries shown in Figure 2.1(a) and 2.1(b) are implied.

### 2.2 Determination of Element Size

Before skew angle can be investigated, the proper element size must be determined by a convergence study of the solution of rectangular deck of dimensions similar to those used in practice. Hereafter, the term "span" will refer to the inclined distance (skewed dimension) between supports. Also the ratio of width to span will be abbreviated WSR where


FIGURE 2.1 TWO WAYS TO "SKEW" A PLATE. METHOD (b) WILL BE USEO IN THIS REPORT
the width is the distance along the supported edge. The WSR parameter will be varied throughout the study in much the same way as the aspect ratio is in orthogonal plate analysis. The span length chosen for the mesh size study was 30 feet and the supported edge length 45 feet $(W S R=1.50)$. Three mesh sizes were investigated based on these dimensions.

The first mesh was called "RIGHT" (Figure 2.2) and consisted of 24 elements ( 4 in the span direction and 6 along the supports) each 90 inches square. The idea of the study is to start with this coarse mesh and compare the results to increasingly more refined meshes. However, since the comparisons are made based upon a 10 kip concentrated central load, the finer meshes will not be approaching a finite value (because, as is well known from thin plate theory, a point load produces infinite stress).- Therefore it is anticipated that the true solution will lie somewhere between the first course mesh run and one of the finer modifications. A more accurate finite element analysis can be made using the load area equal to the tire contact (imprint) area, and comparisons can be made with the "point load" mesh runs to determine which will give the best approximation. The second mesh was titled "RTFINE" (Figure 2.3) and was given 8 span elements each 45" and 10 transverse elements each 54". For the third run these 80 elements were bisected bilaterally to give a 320 element mesh called "RTEXFINE" (Figure 2.4). As expected the values for the central span moment under the load increased in the two successive finer meshes (see Figure 2.5). The values went from 3.25 k -in/in (RIGHT) to 3.86 k -in/in (RTFINE) to $4.48 k$-in/in (RTEXFINE).




FIGURE 2.3 MESH "RTFINE"


FIGURE 2.5 TRANSVERSE MAJOR PRINCIPAL MOMENT ALONG MIDSPAN


### 2.2.1 Convergence

The problem now is to determine which (if any) of these results are close to the moment found under a more realistic tire load. The double wheel contact area of a water tanker truck was studied in Reference 4. The dimensions are approximated here to a rectangle of $10^{\prime \prime} \times 24$ ". Interior element loads (i.e., concentrated loads within the boundaries of an element) can be approximated by placing the force upon imaginary stringers parallel to the element edges and using the resultant reactions at the nodal points as a new collection of equivalent loads. This technique will be used later in the program SKEW LOADER as a method for finding statically equivalent nodal loads for any truck position on a skewed deck. This technique, however, is not applicable at this point as the objective here is to avoid the singularities caused by concentrated loads altogther. Therefore a more exact approach is required before we can obtain a solid basis for comparison in choosing the appropriate mesh size.

Since GTSTRUDL will accept uniformly distributed loads only over an entire element (i.e., no partial element loading), an extremely fine localized mesh has to be developed to surround this tire load. Extra care must be taken here to ensure that compatibility conditions are satisfied between element and that no elements are distorted. Distortion is defined in GTSTRUDL as having aspect ratios greater than 2.0 or acute angles less than 30 degrees. Rectangular elements cannot be used in this localized mesh as compatibility cannot be maintained without highly distorting element aspect ratios. Therefore, a triangular pattern was carefully assembled for the localized mesh (Figure 2.6) using GTSTRUDL element Cending Plate Hybrid Triangle

(BPHT). Due to the hiyh degree of correlation between the data obtained from three different meshes (RIGHT, RTFINE and RTEXFINE) for areas not in the vicinity of the load, the RTFINE mesh was chosen to house the localized triangular mesh. In other words, Figure 2.6 was inserted into the hatched portion of Figure 2.7. The central portion of the localized triangular mesh contains four rectangles each $5^{\prime \prime} \times 12$ " which are loaded with a uniform pressure of 0.04167 ksi . This modified system is therefore statically equivalent to a central 10 kip concentrated load on the plate. This mesh, called UTRTFINE, gave a central span moment of 4.107 k -in/in which falls slightly above the moment given by the RTFINE mesh (see Figure 2.8). Since the moment of the RTFINE mesh with a concentrated central 10 kip load is only $6.2 \%$ lower than this "exact" value, it is considered that the RTFINE mesh will be accurate enough for the purpose of investigating the variation in moment with deck skew angle. The cost in computer time for using the RTEXFINE mesh would be increased exponentially as the number of elements are quadrupled, and would as equally overestimate the moment as the RTFINE mesh would underestimate it.

### 2.3 Effect of Aspect Ratio

Before the RTFINE mesh could be selected as the proper mesh size for the study, one more check had to be made. If the results showed substantial changes in accuracy when the aspect ratio of the plate was varied, then this would indicate that the mesh size used in the study would have to vary as the WSR is changed. A similar run was therefore made to compare the output of the RTFINE mesh with a more exact value where the aspect ratio has been reduced from 1.50 to 0.60 . The

FIGURE 2.7 MESH "UTRTFINE"
respective meshes were renamed RTFINE060 and UTWSRO60 (Figure 2.9). The mesh of Figure 2.6 was renumbered to be housed in the mesh of Figure 2.9 as shown and comparisons were again made between the central span moments given by the two outputs. The results showed no substantial change in behavior at the new aspect ratio. The UTWSR060 (more exact) gave a central span moment of 5.69 k -in/in while the RTFINEO60 gave 5.44 k-in/in, or $4.3 \%$ less (see Figure 2.10). The RTFINE mesh is therefore considered a satisfactory approximation to the true deck behavior and will be used as the basis for investigating the effect of skew.

(8 @ 45")



FIGURE 2.10 COMPARISON OF RTFINE AND UTWSR060

## CHAPTER 3

## PARAMETER STUDIES

### 3.1 Introduction

Now that the mesh RTFINE has been selected as a guide for the element size, the effect of skew angle can be studied. In order to obtain a realistic evaluation of the variation in maximum moments, the loading used must correspond to the dimensions and axle weights of an actual truck. Furthermore, a consistent convention must be used to establish the truck position for varying angles of deck skew. The principal moment at key points of the slab will be evaluated by GTSTRUDL finite element analysis for each position of the truck as it moves incrementally across the span. Then the corresponding results for different angles of skew can be compared when plotted on the same graph. Since the WSR may affect the moment variation as the skew angle is changed, it too will be treated as a parameter.

Figure 3.1 shows the models to be studied. The principal moments will be computed for WSR's of $1.50,1.00$ and 0.75 while the skew angle is changed from 0 to 40 degrees. For the case of $W S R=1.00$ (rhombus), skew of 20 degrees will also be investigated. In all cases the truck will move along the inclined centerline as measured from support to support. In the case of the rhombus, the effects of edge loading will also be examined as the truck is moved across the span inset at 48" from the left free edge. In all cases the deck is simply supported at the top and bottom edges and free along the left and right edges. This will be the conventional orientation of the deck throughout this report.


The truck chosen for the investigation was the 70 kip FDOT-SU4 (Figure 3.2). The span dimension of the deck was selected for convenience to be 32 feet in order that 8 span elements of $48^{\prime \prime}$ (Figure 3.3) may be used as in the RTFINE deck. The RTFINE actually used 8 span elements of $45^{\prime \prime}$, though the element study showed the difference should be of negligible order. The transverse dimensions were varied from 48 to 32 to 24 feet to allow the WSR to change from 1.50 to 1.00 to 0.75 , respectively. Transverse element dimension was chosen as $48^{\prime \prime}$ for convenience. Again, these parallelogram element dimensions remain constant as the skew angle of the deck is varied (Figure 3.4(a)). Note the reference position of the skew angle in Figure $3.4(a)$ as the complement is sometimes used in the literature.

### 3.2 Procedure

For the span chosen for the study, the maximum centerline moment in an orthogonal deck would occur under axle \#3 (wheels \#5 and \#6 in Figure 3.2). Wheel \#5 was therefore chosen as the reference from which the position of the truck will be measured. The truck will always be positioned as if it were moving parallel to the free edges of the deck. Since the program SKEW LOADER (see Appendix A) allows for input in terms of skewed coordinates, the position of wheel \#5 will be given in such a coordinate system. The $X$ input will be as measured from the left edge of the deck and the $Y$ input will be the span (inclined) distance from the base support. For center loading, the position of wheel \#5 will be input such that the center of axle \#3 is over the transverse centerline of the deck (see Figure 3.4(b) for an example). The truck will be "moved" alony the span direction while maintaining

FIGURE 3.2 FDOT SU-4 TRUCK

FIGURE 3.3 ORTHOGONAL DECK WSR $=1.50$ USED IN PARAMETER STUDIES


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this centered position. In this way the advantage of symmetry can be maintained on the right deck and axisymmetric loading can be maintained on the skewed decks.

The following discussion applies to the analysis procedure for load cases $A$ and $B$ (see Figure 3.1) where the WSR is 1.50 and the skew angle is 0 and 40 degrees, respectively. The remaining cases are handled in an analogous manner except for the edge loading cases which will be discussed separately. The first position of the truck is located at the transverse center and 96" (i.e., 2 elements) up from the base support. A finite element analysis is then run with the truck in this position and the principal moments at joint 59 (deck center) are obtained. The truck is then moved up 12" (i.e., one quarter element) in the span direction to a new position 108" from the base support, finite element analysis is again run with the truck in the new position and the principal moments at joint 59 again noted. This procedure is repeated at increments of 12 " until the truck is $96^{\prime \prime}$ (i.e., 2 elements) away from the far support. The largest principal moment at joint 59 is plotted against the truck position in Figure 3.5 where the letter label of the graph corresponds to the respective case in Figure 3.1.

The above procedure is repeated for each of the remaining cases and the results are plotted for varied angles of skew at each WSR to obtain Figures 3.5, 3.7 and 3.9. In this way the effects of skew can be inspected separately for each WSR. Now for each of these graphs the transverse variation in moment must be studied on neighboring nodes in order to be sure that the moment at the transverse center is actually at or near the maximum in the deck. This is due to the fact that as deck skew angle is increased, the element edges are moved closer to the

wheels of the truck which may allow for greater localized moments at joints off-center. Thus each of the Figures 3.5, 3.7 and 3.9 showing moment variation with truck position is followed by a Figure 3.6, 3.8, and 3.10 , respectively, showing the section moment variation across midspan in the vicinity of the peak moment. This section diagram examines the moment variation at midspan across two elements to the left and right of the peak and therefore plots for a total of 5 joints. For example, the joint 59 moment-position graph of case $A(W S R=1.50$, skew $=0.0$ ) peaks when wheel \#5 is positioned 192" from the base support. Therefore to investigate the transverse variation in moment for this position of the truck, Figure 3.6 plots the moments at joints $41,50,59,68$ and 77. This graph is labeled "A-192-RT" giving the load case, truck position and deck skew angle, respectively.

For the case of edge loading a similar procedure is followed, though edge loading is considered only for the cases with a WSR of 1.00. The truck is again placed $96^{\prime \prime}$ from the base support but now $48^{\prime \prime}$ (i.e., one element) in from the left free edge. A finite element analysis is run with the truck in this position and the moment.at joint 5 is noted. The truck is then moved across the span in increments of 12 " with a finite element analysis run for each successive position. In each case the moment at joint 5 is noted, and as before the truck is stopped at 96 " from the far support. The moment at joint 5 (the edge node at midspan) is plotted against each position of the truck and is shown in Figure 3.11 for skew angles of 0,20 and 40 degrees. In a similar procedure as for the center loading cases, the transverse moment variation is plotted in Figure 3.12 which examines the moments across 4 elements starting from the left edge. Fur this parameter study,
$14$









FIGURE 3.12 MOMENT VARIATION ACROSS MIDSPAN
including both the center and edge loading cases, a total of 170 finite element runs were made.

### 3.3 Results

In many of the skewed decks, the maximum moment does not occur at the same joint as in the corresponding orthogonal deck. However, it is seen that the increase in moment from the corresponding joint never exceeds $3 \%$. Therefore the moment at the same joint will be used for comparison. It must be noted however, that the moments generally do not peak for the same position of the truck. For example, Figure 3.9 (WSR = 0.75 , center load) shows that the right deck attains maximum moment at joint 32 when the truck position (i.e., wheel \#5) is at 192" from the base support. The 40 degree deck however peaks earlier when the truck position is at $144^{\prime \prime}$. This is the general trend in all center load cases studied here. The right deck always peaks at $192^{\prime \prime}$ while the 40 degree deck peaks about 48" (one element) sooner. As might be expected, the 20 degree deck peaks about 24 " earlier.

The fact that the moments peak earlier for increased angles of skew can be attributed to the effect of wheel \#6 (also a critical wheel) reaching the deck center sooner. This allows the moment at the deck center to peak earlier and also accounts for a portion of the reduction in maximum principal moment from the right deck because both wheels 5 and 6 are not lying along the span center at the same time. The right deck peaking at $192^{\prime \prime}$ is to be expected as this corresponds to the position where the critical wheels are at span center. For the case of the edge loading however, it is noted that the joint 5 moment peaks occur when the truck is positioned at 192" regard?ess of the angle of

skew. Figure 3.12 shows that the principal span moment decreases from the left edge of the deck at low angles of skew, though it increases slightly at joint 14 at 40 degrees. This would seem to indicate that the localized effects of the free edge are less pronounced at greater angles of skew.

Table 3.1 summarizes the key data for the parameter study. It is interesting to note that the greater WSR's allow for the greater percentage in principal moment reduction for increased angles of skew. For example, it is seen that the WSR of 1.50 has a moment reduction of $24.3 \%$ in moving from the right deck to 40 degree skew deck. This can be compared to only a $15.6 \%$ reduction for the WSR of 0.75 . The rate of moment reduction is also non-linear as evidenced by the table entries of WSR $=1.00$ (center loading). Here the moment reduction for the first 20 degrees (i.e., 0 to 20) is only $5.4 \%$ while for the second 20 degrees (i.e., 20 to 40 ) is $13.5 \%$. Therefore the rate of moment reduction is increased with an increase in skew angle. In the case of edge loading, the behavior is similar except that the percentages are somewhat increased. It is important to note that these decreases in maximum principal moment do not necessarily allow for commensurate reduction in reinforcing steel. The angle at which the reinforcement is laid and the direction of these principal moments play an important role in the amount of steel required. These effects will be discussed in greater detail in Chapter 4.

### 3.4 Contour Plot Description

Appendix $C$ shows the contours of major principal moment and $Z$ (transverse) displacement for skew angles of 0 and 40 degrees. These

TABLE 3.1 PARAMETER STUDIES SUMMARY


A INDICATES CASE "A"
plots were obtained from GTSTRUDL graphics and plotted on a Tektronix print device from the scope environment. ${ }^{1}$ The loading conditions selected for comparison were a central 10 kip load and the peak SU-4 central load. The WSR is 1.50 throughout.

Figures C.1 and C. 2 show the effect of skew angle on the major principal moments under a central 10 kip load. Note that the contour gradient remains essentially perpendicular to the supports. This same effect can be seen on Figures C. 3 and C. 4 (SU-4 peak loading case) where the path of "steepest descent" is nearly along the shortest line to the supports. The concentration of contours near the obtuse corner show that the major principal moment increases more quickly (along a line towards the center) than at the acute corner. Figures C. 3 and C. 4 can be compared to the transverse moment variation graphs for load cases A and $B$ as shown in Figure 3.6.

Figures C. 5 through C. 8 illustrate the variation in deflected shape between skew angles of 0 and 40 degrees. Figures C. 4 and C. 5 refer to the central 10 kip load while C. 7 and C. 8 refer to the SU-4 peak load. It is seen that the behavior is similar for both loading cases. As would be expected physically, the displacement gradient is perpendicular to the supports and the direction of "minimum descent" is along the span centerline.

[^0]
## CHAPTER 4

## RE INFORCEMENT

### 4.1 Introduction

The safe and economic proportioning of reinforcement is critical in bridge deck design. Ideally, the re-bars should be laid orthogonal to the principal design moments. However since the principal angles vary from point to point throughout the deck for even a single loading case, the reinforcement cannot be placed ideally in a practical sense. Furthermore since design is often based on a series of different loading cases, the principal angle will often vary at the point as well. Therefore the concept of ideal reinforcement for a bridge deck is a trivial one.

Certain general directions for the reinforcement are a yood deal more economical (in terms of required steel quantity only) than others. Cope [2] has studied orthogonal reinforcement for skewed decks and compared the experimental results of placement parallel to the supports versus parallel to the skewed edge. The results of this study will be discussed in more detail in section 4.3. Orthogonal reinforcement in skewed decks however, has severe limitations in practice. Since a large number of re-bars of different lengths are required for this design, a great deal of extra labor is required for cutting and placing the reinforcement. Therefore, though it is generally desireable to have reinforcing steel running perpendicular to the supports (to resist the span moment), such an arrangement is usually not practical.

Discussions with FDOT engineers indicate that most reinforcement in skewed decks today is not orthogonal. The primary reinforcement is generally laid parallel to the deck edges and the transverse reinforcement is laid parallel to the supports. This is certainly the easiest from a construction point of view, but for largely skewed bridges it may require a considerable amount of extra reinforcement. Morely [16] has developed a design process for non-orthogonal reinforcement in skewed decks. Though the method may be tedious for practical design problems witout the aid of computer, it serves well to illustrate the effects of skewed reinforcement. This design will be discussed in section 4.4 and some examples worked which are relevant to the parameter study of Chapter 3.

Even though the STRUDL analyses are purely elastic, it is relevant to first examine the failure mechanisms of skewed decks by the theory of yield lines before discussing reinforcement in detail. For a more complete discussion on yield line theory and applications, several texts are available such as [7], [19].

### 4.2 Failure Analysis by Yield Lines

Analysis of plate capacity can be determined theoretically by the principle of virtual work and an assumption of a yield line pattern. The work done externally by the applied forces acting through the plate deflection is equated to the internal work done by the rotation of the moments acting on the yield lines. The collapse load can then be solved for in terms of the ultimate unit moment capacity of the plate. Often the main problem in the yield line approach is determining the correct (or critical) failure mechanism to be analyzed. Yield line patterns can
be assumed from some experience or from some generally established guidelines. Hughes [7] gives some such guidelines, although generally the critical pattern is not easily found even for some rather simple geometries.

For example, Hughes [7] shows three possible failure mechanisms for a simply supported skewed span as shown in Figure 4.1. The deck is given a central point load $W$ and the orthotropic reinforcement is oriented parallel to the deck edges as is generally used in practice (see Figure 4.1). The figure and equations have been modified to agree with the conventions used in this report thusfar. It is seen that the elliptical fan pattern represents the least allowable failure load. Since bridge decks are generally not subjected to a single central concentrated load, this case may be of purely academic interest. However, in deriving the critical load for the elliptical failure mechanism, Hughes outlines (and proves) a very useful principle for obtaining an affine isotropic right slab from an orthotropic skewed one. This procedure is discussed below.

A skewed deck such as that shown in Figure $4.2(a)$ can be transformed into an equivalent orthogonal deck for the purposes of analysis. The equivalent right deck (or so called "affine deck") shown in Figure $4.2(b)$ is of course much easier to analyze and can be obtained by the following rules:
(a) Deflections are identical at corresponding locations in both decks.
(b) Given that $m$ and $\mu m$ are the ultimate resisting moments in the reinforcing direction of the actual deck, then the affine deck has ultimate resisting isotropic moment m.

$\begin{aligned} & \\ & E \| \\ & \text { E }\end{aligned}$

MECHANISM 2 (PARALLALOGRAM)
$W=8 \sqrt{\mu}(m \cos \phi)$
(IN BETWEEN)


E E


FIGURE 4.1 THREE FAILURE MECHANISMS FOR CENTRALLY LOADED SKEWED SPANS (FROM HUGHES [7])


FIGURE 4.2 AFFINE DECK TRANSFORMATION
(c) Given that the $Y$ coordinate axis is along the $m$ reinforcement direction in the actual deck, then all distances measured in the $Y$ direction are the same for both decks.
(d) Given that the $X$ coordinate axis is along the $\mu m$ reinforcement direction in the actual deck, then it is taken at a right angle to the $Y$ axis for the affine deck.
(e) In order to obtain a dimension in the $X$ direction of the affine deck, divide the corresponding length on the actual deck by $\sqrt{\mu}$.
(f) Divide the loads on the actual deck by $\sqrt{\mu} \cos \phi$ to obtain the corresponding loads on the affine deck.

Using the above procedure, an interesting result is obtained if the deck in Figure 4.2(a) is analyzed assuming a yield line occurs across midspän- (see Figure 4.3). This assumption for the failure mechanism may be valid for the multiple load cases arising on bridge spans. The load $P$ is a sum of the concentrated wheel loads acting across midspan and should cause failure to occur in beam action. If $m$ represents the unit moment resisting capacity of the affine deck in Figure 4.3(b), then by statics

$$
\left(\frac{L}{4}\right)\left(\frac{p}{\sqrt{H} \cos \phi}\right)=m\left(\frac{b}{\sqrt{H}}\right)
$$

or

$$
m=\frac{P L}{4 b \cos \phi}
$$

Therefore the deck of Figure $4.3(\mathrm{a})$ is equivalent to an orthogonal deck of span length $L$ and base of a length diminished by the cosine of the angle of skew. This illustrated in Figure 4.4.

## $=$





### 4.3 Orthogonal Reinforcement in Skewed Decks

An experimental investigation was conducted by Cope [2] on the effects of orientation of orthogonal reinforcement in 45 degree skewed decks. Although orthogonal reinforcement is generally not used in skewed bridge design, the study finds some interesting results which may lead to a better understanding of flexural behavior. The test procedure was fundamentally a comparison between two 45 deyree skew slabs, each with orthogonal reinforcement. Slab A had secondary reinforcement parallel to the supports. Slab $B$ had primary reinforcement parallel to the skewed edges. The propagation of cracking and deflections in the two slabs were examined for increasing loads in various locations.

The tests showed considerable behavioral differences in the two slabs in the areas of deflections, cracking and failure mechanisms. Sagging cracks were first initialized on slab A running towards the free edges and parallel to the supports, and under heavier load hogging cracks appeared at the obtuse corners. Initial sagging cracks in slab B occured earlier (at about $80 \%$ of the load for $A$ ) as did the hogging cracks at the obtuse corners (at about $67 \%$ of the load for A). The cracking patterns were similar in direction and spacing, though somewhat straighter and more continuous in slab B. The modes fo failure of the two decks were different as well. Slab A initially failed in shear on the free edge at the obtuse corner. At about $40 \%$ higher load, slab B developed a top surface crack between the two obtuse corner supports which was wide enough to produce a discontinuity in slope. Both slabs were able to carry considerably more load after their initial
failures. Ultimate failures were punching shear at the obtuse corner for slab $A$ and excessive deflections for slab $B$.

The report concluded that the behavior of skewed decks is strongly influenced by the direction of the reinforcenent. When the reinforcing is placed orthogonally and parallel to the supports, the slab is stiff and behaves well under service loads. However, heavy concentrated loads are not distributed well across the slab and a large reactive force develops at the obtuse corner. This led to local failure in the test. For slabs with orthogonal reinforcement parallel to the free edyes, a more flexible slab results which better distributes moments due to concentrated loads. Greater deflections can occur and ultimate failure load is increased, though again hogging cracks at the obtuse corner may limit servicibility.

The results of this study are consistent with a finding by Kennedy [11] which experimentally investigated the stress near corners of simply spanned-skewed plates. Here too it was found that stresses near the obtuse corners are significant and increase with increasing angles of skew. Furthermore, the stress at the obtuse corners may exceed the maximum stress at center span. Kennedy then recommends that the obtuse corners of concrete skewed decks be heavily reinforced top and bottom in directions parallel and perpendicular to the supports as well as parallel to the free edge. The accute corners should be heavily reinforced at the bottom and nominally reinforced at the top. The direction of reinforcement near the acute corners should be perpendicular to a line between them and parallel to the free edge.

### 4.4 Design for Skewed Steel

A design procedure was worked out by Morely [16] on proportioning skewed reinforcement which is laid parallel to the edges of the deck.

Since this is generally the orientation used in construction, this method could prove useful in design practice. Equations are developed based on the optimum proportionment of steel to resist a moment triad at a point in the deck. Therefore the method begins with the assumption that elastic analysis has been performed for a load case on the deck and that a set of three moments (two bending and one torsional) are known at the point considered. The design procedure is then carried out using a series of charts, tables and equations given in the reference.

Morley's design for skew reinforcement to resist a single moment triad consists of the following steps:
(a) Orient the sign convention of the known moment triad in accordance with Figure 4.5
(b) Compute $\frac{M_{x}}{\mid M_{x y}} \left\lvert\, \frac{M_{y}}{\mid M_{x y} T}\right.$ and $k=\left(\sigma_{y}{ }^{\prime}{ }_{a} /\left|M_{x y}\right|\right)$ using a rough =... approximation for $l_{a}$. ( $k=0$ if no minimum steel is specified)
(c) Enter the charts with the nearest values of $\phi$ and $k$ and locate the appropriate region
(d) Evaluate $a_{1 x}, a_{1 \phi}, a_{2 x}, a_{2 \phi}, \theta_{s}$ and $\theta_{h}$ from the appropriate equations in the tables
(e) Compute $l_{a}$ by an iteration equation of section equilibrium
(f). Compute $A_{1 x}, A_{1 \phi}, A_{2 x}, A_{2 \phi}$, from $\left.a_{1 x}=\sigma_{y}\right\rceil_{a} A_{1 x}(x \not t \phi)$ where these variables and others used in the design examples are defined as follows:

| $A_{\min }$ | specified minimum steel are per unit slab width |
| :--- | :--- |
| $a_{l x}, a_{l \phi}$ | 'area functions' with dimensions of moment |
| $d_{e}, d_{0}$ | depths for bottom and top steel, respectively |
| $k$ | failure factor connecting $A_{m i n}$ and applied |
|  | twisiny moment $M_{x y}$ |




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为
FIGURE 4.5 MORLEY'S [16] SIGN CONVENTION FOR NON ORTHOGONAL REINFORCEMENT DESIGN EXAMPLES

$6$

condition, since only the moment at the geometric center is considered in each case. The examples are intended only to illustrate the effects of non-orthogonal reinforcement at a point for a few different load conditions. Morely emphasizes the effects of the sign of the torsional moment by working two similar examples of a 20 degree skew deck. The first has a point subjected to $M_{y}=+50 k-i n / i n$, $M_{x}=+35 k-i n / i n$ and $M_{x y}=+20$ $k-i n / i n$ and the total required steel is found to be $3.96 \mathrm{in}^{2} / \mathrm{ft}$. In the second example the sign of $M_{x y}$ is changed to $-20 k-i n / i n$ and the total amount of steel becomes $8.20 \mathrm{in}^{2} / \mathrm{ft}$, or roughly twice the amount. This increase occurs because the direction of the major principal moment is shifted, due to the sign change of $M_{x y}$, from within the acute angle between the reinforcement to the obtuse angle. This reduces the unit effective reistance of the steel, and hence a greater amount is required.

For the examples discussed below, reference is made to Figure 4.6 and the following data is applicable to each:

$$
\begin{aligned}
\text { Deck thickness } & =18.0 \mathrm{in} \\
E & =3324 \mathrm{ksi} \\
\mathrm{f}_{c}^{\prime} & =3401 \mathrm{psi} \\
\sigma_{c} & =2891 \mathrm{psi} \\
W S R & =1.00 \\
\sigma_{y} & =50 \mathrm{ksi}
\end{aligned}
$$

In addition, the following equations from Morley [16] are common to all examples:

$$
\begin{aligned}
& \text { I. } T_{1}=\frac{1}{T_{a}}\left[a_{1 x} \cos ^{2} \theta+a_{1 \phi} \sin ^{2}(\theta-\phi)\right] \\
& \text { II. } \quad T_{a}=d_{e}+\left(\frac{T_{2}}{T_{1}}\right) d_{0}-\frac{\left(T_{1}+T_{2}\right)^{2}}{2 \sigma_{c} T_{1}}
\end{aligned}
$$

III. $\quad a_{1 x}=\sigma_{y}{ }^{1} A_{1 x}(x \leftrightarrow \phi)$


FIGURE 4.6 SKEWED REINFORCEMENT EXAMPLES

Example 1. Right deck 100 kip central load

Step (a) GTSTRUDL gives the following moments in the appropriate sign convention
$M_{x}=+28.49 k-i n / i n$
$M_{y}=+43.03 \mathrm{k}$-infin
$M_{x y}=0.0 k-i n / i n$
(b) $\frac{M_{x}}{T M_{x y} T^{-}}=\frac{M_{y}}{\left|M_{x y}\right|} \rightarrow \infty$
(c) Using Figure 2 [16] the appropriate region is $A$
(d) From Table 1 [16]
$a_{2 x}=a_{2 y}=0 \rightarrow$ No Top Steel Required
$=a_{1 x}=M_{x}+\left|M_{x y}\right|=28.49$
$a_{1 y}=M_{y}+\left|M_{x y}\right|=43.03$
$\theta_{S}=-45^{\circ}$
(e) From equation $I, \quad T_{1}=\frac{1}{T_{a}}\left[28.49 \cos ^{2}(-45)+43.03 \sin ^{2}(-45)\right]$ $T_{1}=35.76 / I_{a}, T_{2}=0$

From equation II, $\mathrm{T}_{\mathrm{a}}=16-\frac{\mathrm{T}_{1}}{2(2.891)}$
or

$$
1_{a}^{2}-16 \mathrm{r}_{\mathrm{a}}+6.18=0 \rightarrow \mathrm{r}_{\mathrm{a}}=15.60
$$

(f) From equation III, $12(28.49)=50(15.60) A_{1 x}$

$$
\begin{aligned}
A_{l x} & =0.44 \mathrm{in}^{2} / \mathrm{ft} \\
12(43.03) & =50(15.60) \mathrm{A}_{1 y} \\
A_{l y} & =0.66 \mathrm{in}^{2} / \mathrm{ft}
\end{aligned}
$$

Example 2. 20 deg skew deck using moment triad of example 1

Step (a) $M_{X}=+28.49 \mathrm{k}-\mathrm{in} / \mathrm{in}$
$M_{y}=+43.03 k-i n / i n$
$M_{x y}=0.0 k-i n / i n$
(b)

(c) Using Figure 3 [16] the appropriate region is $A 2$
(d) From Table 1 [16]

$$
\begin{aligned}
a_{2 x} & =a_{2 \phi}=0 \rightarrow \text { No Top Steel Required } \\
a_{1 x} & =M_{x}=M_{y} \tan ^{2} \phi+\frac{(1+2 \sin \phi)}{\cos \phi}\left[M_{y} \tan \phi-M_{x y} \approx\right. \\
a_{1 x} & =28.49-43.03(\tan 20)^{2}+\frac{(1+2 \sin 20)}{\cos 20}[43.03 \tan 20-0 \approx \\
=a_{1 x} & =50.85 \\
a_{1 \phi} & =\frac{M_{y}}{\cos ^{2} \phi}+\frac{1}{\cos \phi}\left|M_{y} \tan \phi-M_{x y}\right| \\
a_{1 \phi} & =\frac{43.03}{(\cos 20)^{2}}+\frac{1}{\cos 20}\left|43.03 \tan 20^{\circ}-0\right| \\
a_{1 \phi} & =65.36 \\
\theta_{s} & =\frac{1}{2}(90+\phi) \\
\theta_{s} & =+55^{\circ}
\end{aligned}
$$

(e) From equation $I, \quad T_{1}=\frac{1}{T_{a}}\left[50.85(\cos 20)^{2}+65.36(\sin 35)^{2} \approx\right.$

$$
T_{1}=66.4 / 1_{\mathrm{a}}, T_{2}=0
$$

From equation II, $1_{a}=16-\frac{T_{1}}{2(2.891)}$
or

$$
\begin{aligned}
& 1_{a}^{2}-161_{a}+11.48=0 \\
& 1_{a}=15.25^{\prime \prime}
\end{aligned}
$$

(f) From equation III, $12(50.85)=50(15.25) A_{1 x}$

$$
\begin{aligned}
A_{1 x} & =0.80 \mathrm{in}^{2} / \mathrm{ft} \\
12(65.36) & =50(15.25) \mathrm{A}_{1 \phi} \\
A_{1 \phi} & =1.03 \mathrm{in}^{2} / \mathrm{ft}
\end{aligned}
$$

Comparison of this result with Example 1 illustrates the effects of 20 deg skew reinforcement minimized for the same moment triad. The total steel per unit width becomes $(0.80+1.03) \cos 20=1.72 \mathrm{in}^{2} / \mathrm{ft}$ or $56 \%$ more than Example 1.

Example 3. 20 deg skew deck 100 kip central load

Step (a) GTSTRUDL gives the following moments in the appropriate sign convention

$$
\begin{aligned}
\therefore M_{x} & =+24.36 k-i n / i n \\
M_{y} & =+40.77 k-i n / i n \\
M_{x y} & =+0.646 k-i n / i n
\end{aligned}
$$

(b) $\frac{M_{x}}{T_{x y} T}=\frac{24.36}{0.646}=37.71 \frac{M_{y}}{T M_{x y} T}=\frac{40.77}{0.646}=63.11$
(c) Using Figure 3 [16] the appropriate region is A2
(d) From Table 1 [16]

$$
\begin{aligned}
& a_{2 x}=a_{2 \phi}=0 \rightarrow \text { No top steel required } \\
& a_{1 x}=M_{x}-M_{y} \tan ^{2} \phi+\frac{(1+2 \sin \phi)}{\cos \phi}\left[M_{y} \tan \phi-M_{x y}\right] \\
& a_{1 x}=24.36-40.77(\tan 20)^{2}+\left(\frac{1+2 \sin 20}{\cos 20}\right)[40.77 \tan 20-0.646] \\
& a_{1 x}=44.39
\end{aligned}
$$

$$
a_{1 \phi}=\frac{M_{y}}{\cos ^{2}}+\frac{1}{\cos \phi}\left|M_{y} \tan \phi-M_{x y}\right|
$$

$$
\begin{aligned}
& a_{1 \phi}=\frac{40.77}{(\cos 20)^{2}}+\frac{1}{\cos 20}|40.77 \tan 20-0.646| \\
& a_{1 \phi}=61.24 \\
& \theta_{s}=+\frac{1}{2}(90+\phi) \\
& \theta_{s}=+55^{\circ}
\end{aligned}
$$

(e) From equation $I, \quad T_{1}=\frac{1}{T_{a}}\left[44.39(\cos 55)^{2}+61.24(\sin 35)^{2}\right]$

$$
T_{1}=34.7 / 1_{a}, \quad T_{2}=0
$$

$$
\text { From equation II, } \begin{aligned}
& 1_{a}=16-\frac{T_{1}}{2(2.891)} \\
& \text { or } \\
& 1_{\mathrm{a}}{ }^{2}=-161_{\mathrm{a}}+6.0=0 \\
& 1_{\mathrm{a}}=15.62^{\prime \prime}
\end{aligned}
$$

(f) From equation III, 12(44.39) $=50(15.62) A_{1 x}$

$$
\begin{aligned}
A_{1 x} & =0.68 \mathrm{in}^{2} / \mathrm{ft} \\
12(61.24) & =50(15.62) A_{1 \phi} \\
A_{1 \phi} & =0.94 \mathrm{in}^{2} / \mathrm{ft}
\end{aligned}
$$

Comparison of this result with Example 1 illustrates the additional steel required at the geometric center for the 20 deg skew deck vs. the right deck, both under a central 100 kip load. The total steel per unit width becomes $(0.68+0.94) \cos 20=1.52 \mathrm{in}^{2} / \mathrm{ft}$ or $38 \%$ more than Example 1. This increase is due to the major principal moment lying within the obtuse angle of the reinforcement as shown in Figure 4.7.


(a) EXAMPLE 1 STEEL COINCIDES
FIGURE 4.7 REINFORCEMENT EXAMPLES
100 KIP CENTRAL LOAD

Example 4. Right deck peak SU-4 loading

Step (a) GTSTRUDL gives the following moments in the appropriate sign convention

$$
\begin{aligned}
& M_{x}=+5.07 k-i n / i n \\
& M_{y}=+13.90 k-i n / i n \\
& M_{x y}=0.0 k-i n / i n
\end{aligned}
$$

(b) $\frac{M_{x}}{T M_{x y} T}=\frac{M_{y}}{T M_{x y}^{-} T} \rightarrow \infty$
(c) Using Figure 2 [16] the appropriate region is A
(d) From Table 1 [16]

$$
\begin{aligned}
a_{2 x} & =a_{2 y}=0 \rightarrow \text { No Top Steel Required } \\
\therefore a_{1 x} & =M_{x}+\left|M_{x y}\right|=5.07 \\
a_{1 y} & =M_{y}+\left|M_{x y}\right|=13.90 \\
\theta_{s} & =-45^{\circ}
\end{aligned}
$$

(e) From equation $I, \quad T_{1}=9.49 / 1_{a}, \quad T_{2}=0$

$$
\begin{array}{ll}
\text { From equation II, } & \mathrm{T}_{\mathrm{a}}=16-\frac{\mathrm{T}_{1}}{2(2.891)} \\
& \text { or } \\
& 1_{\mathrm{a}}{ }^{2}=-161_{a}=1.64=0 \\
1_{\mathrm{a}}=15.90^{\prime \prime}
\end{array}
$$

(f) From equation III, $12(5.07)=50(15.90) A_{1 x}$

$$
\begin{aligned}
A_{1 x} & =0.08 \mathrm{in}^{2} / \mathrm{ft} \\
12(13.90) & =50(15.90) A_{1 y} \\
A_{1 y} & =0.21 \mathrm{in}^{2} / \mathrm{ft}
\end{aligned}
$$

(a)

Example 5. 20 deg skew deck peak SU-4 loading

Step (a) GTSTRUDL gives the following moments in the appropriate sign convention

$$
\begin{aligned}
& M_{x}=+4.66 k-i n / i n \\
& M_{y}=+13.13 k-i n / i n \\
& M_{x y}=-0.36 k-i n / i n
\end{aligned}
$$

(b) $\frac{M_{x}}{M_{x y} T}=\frac{4.66}{0.36}=12.9 \quad \frac{M_{y}}{T M_{x y} T^{\prime}}=\frac{13.13}{0.36}=36.5$
(c) Using Figure 3 [16] and noting that $M_{x y}<0$

$$
M_{x} \rightarrow-4.66, M_{y} \rightarrow-13.13
$$

and the appropriate region is $B$
(d) From Table 1 [16] with suffixes 1, 2, s, h interchanged

$$
a_{2 x}=a_{2 \phi}=0 \rightarrow \text { No Top Steel Required }
$$

$$
a_{1 x}=-M_{x}+M_{y} \tan ^{2} \phi-\frac{(1+2 \sin \phi)}{\cos \phi}\left[M_{y} \tan \phi-M_{x y}\right]
$$

$$
a_{1 x}=4.66-13.13(\tan 20)^{2}-\frac{(1+2 \sin 20)}{\cos 20}[-13.13 \tan 20+0.36]
$$

$$
{ }^{a} 1 x=10.84
$$

$$
{ }^{a_{1 \phi}}=\frac{M_{y}}{\cos ^{2} \phi}+\frac{1}{\cos \phi}\left|M_{y} \tan \phi-M_{x y}\right|
$$

$$
a_{1 \phi}=\frac{13.13}{(\cos 20)^{2}}+\frac{1}{\cos 20}|-13.13 \tan 20+0.36|
$$

$$
a_{1 \phi}=19.57
$$

$$
\theta_{s}=+\frac{1}{2}(90+\phi)=55^{0}
$$

(e) From equation $I, \quad T_{1}=\frac{1}{T_{a}}\left[10.84(\cos 20)^{2}+19.57(\sin 35)^{2}\right]$

$$
T_{1}=16.0 / 1_{\mathrm{a}}, T_{2}=0
$$

From equation II, $1_{a}=16-\frac{T_{1}}{2(2.891)}$
or

$$
1_{a}^{2}-161_{a}+2.77=0 \rightarrow 1_{a}=15.82
$$

(f) From equation III, $12(10.84)=50(15.82) A_{1 x}$

$$
\begin{aligned}
A_{1 x} & =0.16 \mathrm{in}^{2} / \mathrm{ft} \\
12(19.57) & =50(15.82) A_{1 \phi} \\
A_{1 \phi} & =0.30 \mathrm{in}^{2} / \mathrm{ft}
\end{aligned}
$$

Comparison of this result with Example 4 illustrates the additional steel required at the geometric center for the 20 deg skew deck vs. the right deck, both under peak SU-4 loading. The total steel per unit width becomes $(0.16+0.30) \cos 20=0.43 \mathrm{in}^{2} / \mathrm{ft}$ or $49 \%$ more than Example 4. This increase is again due to the major principal moment lying in the obtuse corner of the reinforcement as shown in Figure 4.8.

(a) EXAMPLE 4 STEEL COINCIDES
WITH PRINCIPAL MOMENTS
FIGURE 4.8 REINFORCEMENT EXAMPLES

## CHAPTER 5

## CONCLUSIONS

### 5.1 Summary

The finite element method was used to examine the effects of skew angle on the major principal moments of simple spans. A mesh size of eight span elements in 30 feet was found to give results within about $6 \%$ of those found from a more intricate localized mesh surrounding a more realistic tire load. These results varied little with change in aspect ratio. This mesh size was therefore chosen as a guide for the parameter studies which followed.

Skew angles of 0,20 and 40 degrees were investigated for various width to span ratios (WSR's) as shown in Figure 3.1. The major principal moments at key points in the deck were examined as an FDOT SU-4 type truck was moved across the span. The service load simulator program of Appendix $A$ was used to calculate the equivalent nodal forces for the truck in any position on the skewed deck. The results showed that for center loading, peak moment reductions of up to $24 \%$ were found for 40 degree skew decks over orthogonal decks. The percent reductions were lower for smaller WSR's. In general, the major principal moments peaked earlier (i.e., at lesser advanced positions of the truck along the span) for increased angles of skew, This can be attributed to the fact that the wheels of the axles do not reach span center simultaneously.

The direction of reinforcement plays an important role in the flexural behavior of the deck [2]. Orthogonal reinforcement laid parallel to the supports provides for a stiffer slab which behaves well under service loads. Orthogonal reinforcement laid parallel to the free edges provides a more flexible slab. However, skewed reinforcement is the preferred method from the construction point of view. The affine deck method discussed in section 4.2 seems to be the most practical for use in the design office. Morely's [16] method for minimum steel at a point would seem to have limitations in practice, although it serves well to illustrate the considerable effect that the direction of the major principal moment has on the quantity of skewed steel required.

### 5.2 Further Study

More extensive parameter studies could be used with different truck types and combination loading. Program SKEW LOADER (Appendix A) could be easily modified for this application. In addition, the effects of girders, deck thickness and material properties could also be studied. The reinforcement design method developed by Morely [16] is well suited to computer programming. This could be an area of further study along with complete design examples and comparison with the affine deck method.

The stress concentration effect at the obtuse corners could be investigated by the finite element method. This could be of particular importance as experiments have shown that this is often the area of initial failure. An extremely fine local mesh could be assembled (such as along the line of Figure 2.6) in the area of the obtuse corner and the moments investigated with variation in deck angle of skew.

## APPENDIX A

PROGRAM SKEW LOADER

# APPENDIX A <br> PROGRAM SKEW LOADER 

## A. 1 Introduction

Since the parameter study calls for moving a truck incrementally across a skewed deck and evaluating the moments for each truck position, a method had to be developed for loading the deck. GTSTRUDL allows for concentrated loads to be placed only at the nodal points of the mesh. Thus the weight of each truck wheel within an element must be broken into its statically equivalent nodal forces before it can be input to GTSTRUDL for stiffness analysis. To do this by hand would be extremely tedious as 4 reactions would have to be calculated for each of 8 wheels for 170 different load cases. Furthermore, the positions of the wheels in relation to the mesh nodes are geometrically complex making large numbers of calculations prohibitive. This type of problem is therefore well suited to computer programming.

Program SKEW LOADER was developed to output the equivalent nodal forces for a truck positioned on a skewed deck. The program will generate any mesh size on a deck of any dimensions at any angle of skew. The data for the truck FDOT SU-4 is stored internally in the program. Although any type of truck may be used if the truck width, axle spacing and axle weights are known. The user also has the option of specifying the truck position in rectangular or skewed coordinates. The latter is recommended for most applications and in general greatly simplifies the input procedure. The position of any wheel may be input to establish the truck position, which is always assumed to lie parallel to the left
and right edges of the deck. The program uses the stringer method with each loaded element to solve for the equivalent nodal forces.

A set command and regeneration prograns were also written to expedite the study. These programs were used to operate on the first GTSTRUDL program by rewriting the joint load input portion based on the output of SKEW LOADER. In this way the truck could be repositioned automatically and sent back to GTSTRUDL quasi-interactively. This provided for an orderly and systematic approach to data acquisition. This program set was written in command language on a VAX 11-750 computer. The program SKEW LOADER is written in FORTRAN-77.

The rest of Appendix A gives a flowchart, program description and input guide for SKEW LOADER. Copies of the program are also included along with two examples and sample outputs.

$$
\because . \ddot{z}
$$




## A. 3 Program Description

## A.3.1 Main

The program SKEW LOADER is structured as a series of subroutine calls from the main. The initial matrix sizes are dimensioned arbitrarily in the opening statement allowing for a maximum mesh size of 30 nodes each way and a maximum truck size of 18 wheels. These numbers are only set for convenience so that variably dimensioned subroutines can be used later. Therefore the maximums above can be increased easily by adjusting the sizes in the opening dimension statement (no other program changes are required). The input is read from LOADER.DAT and the output goes to OUTPUT.LIS. Only the key subroutines will be discussed below as the functions of the others are self evident. The subroutine descriptions below contain a complete listing of the variables used within. Variables which are not listed are defined within the subroutine in terms of the ones described here.

## A.3.2 Subroutine INITL

This is where most of the user supplied data is read in, stored and echo printed. The following variables are used here:

VARIABLE
DESCRIPTION
AXLSP Axle spacing matrix
AXLWT Axle weight matrix
DDK
Deck dimension along $X$-axis
DDYSK
Deck dimension along $Y$ (skew)-axis
ERR
Error switch
KEY
Mesh print request switch

LCN
NA
NAXLSP
NH
NXCROS
NXSTEP
PHI
TRTYPE
WIDTH
WJX
WJY
WNUM
WT
XINC
XSKINC
XSHIFT
YSHIFT

Load case number
Number of axles
Number of axle spaces
Number of wheels
Number of crossings (mesh lines) $X$ direction
Number of steps (elements) $X$ direction
Angle of deck skew
Truck type (SU-4 or custom)
Width of truck
$X$ coordinate of user's input wheel
Y coordinate of user's input wheel
Input wheel number
Wheel weight number
Increment (element size) X-axis
Increment (element size) $Y$ (skew)-axis
Shift coordinate switch $X$-axis
Shift coordinate switch $Y$-axis

## A.3.3 Subroutine GMESH

The mesh is generated here based on the user inputs specified in INITL. The nodes are generated first in the span direction and then along the $X$-axis. The coordinates of each node are stored in column vectors XCOR and YCOR. The following variables are used here:

VARIABLE
ERR
KEY
NXCROS
NXSTEP
NYCROS
NYSTEP
PHI
XCOR
XINC
YCOR
YSKINC

Error switch
Mesh print request switch
Number of crossings (mesh lines) $X$ direction Number of steps (elements) $X$ direction Number of crossings (mesh lines) Y direction Number of steps (elements) Y direction Angle of deck skew Nodal $X$ coordinate storage matrix Increment (element size) $X$-axis Nodal $Y$ coordinate storage matrix Increment (element size) $Y$ (skew)-axis

## A.3.4: Subroutine LOCATE

Now that the position of one of the truck wheels is known (user's input) the rest can be determined assuming that the truck is lying along the span direction (i.e., parallel to the deck's free edges). From trigonometry the position of the left front wheel can be found from the known position of the input wheel. Then a standard procedure can be followed to locate all wheels relative to this position by geometry. It may often occur that one or more of the truck wheels are lying outside the boundary of the deck. In this case a warning is printed to show the user that the total weight of the truck is not on the deck. This must be taken into account in the statics check at the end of the program.


## The following variables are used in LOCATE:

VARIABLE
AXLSP
DDX
DDYSK
ERR
NA
NAXLSP
NW
NXCROS
NYCROS
OFFWT
PHI
PSD
REACT
USERX
USERY
WHEELX
WHEELY
WIDTH
WJX
WJY
WT
W1X
W1Y
XSHIFT
YSHIFT

DESCRIPTION

Axle spacing matrix
Deck dimension along $X$-axis
Deck dimension along $Y$ (skew)-axis
Error swtich
Number of axis
Number of axle spaces
Number of wheels
Number of crossing (mesh lines) $X$ direction
Number of crossing (mesh lines) $Y$ direction
Matrix containing weights of wheels off deck
Angle of deck skew
Perpendicular support distance
Matrix of nodal reactions
$X$ input (carried)
$Y$ input (carried)
$X$ coordinate of wheel under consideration
$Y$ coordinate of wheel under consideration Truck width
$X$ coordinate of user's input wheel
$Y$ coordinate of user's input wheel
Matrix of wheel weights
$X$ coordinate of wheel \#1
Y coordinate of wheel \#l
Shift coordinate switch $X$-axis
Shift coordiante switch $\gamma$-axis

### 3.5 Subroutine SEARCH

This subroutine is the heart of the program and certainly the most complex. The function here is to find the location of each wheel with respect to its surrounding nodes, then compute the nodal point reactions by the stringer method. For a right angle deck the search is a relatively
simple procedure as the nodal coordinates can be easily compared to the coordiantes of the wheel. The skewed deck however, provides for complications in locating the position of the wheel relative to its neighboring nodes as a series of offsets from the edge of the skewed element must be computed and compared. The idea is to first search the span direction for the first node with a $Y$ coordinate larger than the $Y$ coordinate of the wheel. Calling the $Y$ coordinate of this node and the one immediately below it D1 and D2, respectively, the lower "track" D2 can then be searched. Now the first node along this track with an $X$ coordinate larger than the $X$ coordinate of the wheel is noted. The $X$ coordinate of this node is called CHECK1. Now the $X$ offset distance of the wheel from the element edge is examined to see on which side of the line it falls (see Figure A.1). If XOFF1 is greater than HXOFF1 the wheel is in the element on the right. If it is less than WXOFF1 then it is in the element on the left. If the two distances are equal then it is on the element edge. A similar procedure is used to locate the neighboring nodes above the wheel (i.e., on upper track D1). Provisions are made for cases in which the wheel falls directly on a node or on an element edge.

Now that the coordinates of the four neighboring nodes are known, the distances to the wheel can be easily computed. An immaginary grid of stringers is laid on the element as follows:

1. One is positioned under the wheel and laid horizontally just reaching the element edges
2. Then two more are laid under the first along the skewed edges The reactions of the four corners from the weight of the wheel are then computed by statics and a running total of the results is kept in a storage matrix for the mesh called REACT. The following variahles are used in subroutine SEARCH:

I $, L, N$
K
LCOUNT
NA, NB, NE ,NW
NNE,NNW,NSE,NSW
NW
NXCROS
NXSTEP
NYCROS
NYSTEP
PHI
RA,RB,RL, RR
REACT
RNNE, RNNW,RNSE,RNSW
WHEELX
WHEELY
WT
XCOR
XINC
YCOR
YSKINC

## Node counters

Wheel counter
Element counter
Nodes above, below, east and west Node northeast, northwest, etc.
Number of wheels
Number of crossings (mesh lines) $X$ direction
Number of steps (elements) $X$ direction
Number of crossings (mesh lines) $Y$ direction
Number of steps (elements) Y direction
Angle of deck skew
Reaction above, below, left, right
Matrix of nodal reactions
Reactions node northeast, etc.
$X$ coordinate of wheel under consideration
$Y$ coordinate of wheel under consideration
Matrix of wheel weights
Nodal coordinate storage matrix
Increment (element size) $X$-axis
Nodal coordinate storage matrix
Increment (element size) $Y$ (skew)-axis

## A.3.5 Subroutine RESULT

The non-zero nodal reactions obtained from subroutine SEARCH are printed out here and summed to check statics. The weight of the truck is checked against the sum of the total load on the deck and the weight of the "off wheels". The user can verify this result to be the original weight of the truck. The following variables are used here:

## VARIABLE

DESCRIPTION

NCOUNT
NXCROS
NYCROS

Node counter
Number of crossing (mesh lines) $X$ direction
Number of crossing (mesh lines) Y direction

OFFWT
REACT
TOTAL
TRKWT
USERX
USERY
WJX
WJY
WNUM
XSHIFT
YSHIFT

Total weight of wheels off the deck
Matrix of nodal reactions
Running sum of reactions for statics check
Back summed weight of truck
$X$ input (carried)
$Y$ input (carried)
$X$ coordinate of user's input wheel
$Y$ coordinate of user's input wheel
Input wheel number (user's choice)
Shift coordinate switch X -axis
Shift coordinate switch $Y$-axis

## A. 4 Input Guide

## A.4.1 Data deck

The data file must be titled LOADER.DAT and be accessible from the main. The following data must be input in the order shown beginning in column \#1. The term "card" refers to one line of data in the file.

| CARD | FORMAT |
| :---: | :--- |
| 1 | Free |
| 2 | $1 I 1$ |
|  |  |
| 3 | $1 F 10.1$ |
| 4 | 112 |
| 5 | $1 I 2$ |
| 6 | $1 I 1$ |

DESCRIPTION

Alpha-numeric title of output
Enter truck type as follows
(a) 1 is FDOT SU-4
(b) 9 is custom*

Angle of deck skew (degrees)
Load case number (user's option)
Input wheel number
XSHIFT switch
(a) 1 is on
(b) 0 is off

1 I1

1F7. 2
(a) If YSHIFT is on, give the distance to the wheel as measured along the inclined span edge for the base support
(b) if YSHIFT is off, give the $\gamma$ coordinate of that wheel

Mesh print request switch
(a) 1 is yes
(b) 0 is no

Number of elements in the $X$ direction
Number of elements along the $Y$ (skewed)
direction
Element size $X$ direction
Element size $Y$ (skewed) direction
*Note: If the custom option is chosen, then the following data must be entered on the next data cards (i.e., after card \#2):
(a) Number of axles [format 1I1]
(b) Truck width [format 1F6.2]
(c) Spacing for each successive axle as measured from the previous axle (one per data card) [format 1F6.2]
(d) Axle weights (one per data card) [format 1F6.2]

## A.4.2 Examples

The above proceure is illustrated by two examples. The first is an input to SKEW LOADER for the SU-4 case, and the second is for the custom option. Figures A. 2 through A. 5 given the input data and resulting truck position diagrams for the two cases. Comments are provided on the sample inputs for clarity.

## A. 5 Extension of the Program

The program can be easily extended to include a number of standard truck types beyond the SU-4. The easiest way would be to write more subroutines in the form of TRTYPE1 to include data for any trucks. Then of course modify the switches in subroutine INITL. The program could also be easily extended to include more than one truck on the deck at a time. Finally, a more sophisticated mech generator could be included to allow for elements of different dimensions. This feature would be particularly useful in the analysis decks with stiffening girders.



FIGURE A. 3 SKEW LOADER EXAMPLE 1 (SU-4)

LOADER EXAMPLE 1 (SU-4)

IT CHECK
; K TYPE
E OF SKEW (DEG)
FDOT SU-A
) CASE NUMBER 40.0
$-1$
IT WHEEL NUMBER
IPUT (SHIFT ON)
192.00

SPUT (SHIFT ON)

+ PRINT REQUEST
240.00

NO

| Y-AXIS | Y-AXIS (SKEWED) |  |
| :--- | :---: | :---: |
| 3ER OF ELEMENTS | 12 | 8 |
| IENT SIZE | 48.00 | 48.00 |
| DIMENSION | 576.00 | 384.00 |




EEL \# WEIGHT IN ELEMENT \# AFFECTING NODES $x$ COORD Y COORD

| 9.35 | RT OF 31 | ONLY 4344 | 5.59 | 222.15 |
| :--- | :---: | ---: | ---: | ---: | ---: |
| 9.35 | 56 | 62637172 | 63.81 | 271.00 |
| 9.35 | ABOUE 29 | ONLY 42 | 37.73 | 183.85 |
| 9.35 | 55 | 61627071 | 95.95 | 232.70 |
| 9.35 | RT OF 28 | ONIY 4041 | 69.87 | 145.55 |
| 9.35 | 54 | 60616970 | 123.09 | 194.40 |

ELLTANT NODAL LOADS
r：\＃LOAD（KIPS）
0.39
8.76
9.35
8.96
0.39
6.22

8． 36
8.36
3.23
0.45
0.50
0.60
0.23

TICS CHECK
AL LOAD DN DECK $=56.10 \mathrm{KIPS}$
CK WEIGHT $=70.00 \mathrm{KIPS}$
＊NOTE莒芥れ
CE THE XSHIFT IS ON $X$ INPUT OF 192．00 GIVES $X$ COORD 37．73
＊NOTEれがが
CE THE YSHIFT IS ON Y INPUT OF 240.00 GIVES $Y$ COORD 183.85


(2)


LOADER EXAMPLE 2 (CUSTOM)

## CT CHECK

K TYPE
NUMBER OF AXLES
TRUCK AXLE WIDTH
AXLE SPACE \# 1
AXLE SPACE \# 2
AXLE WEIGHT \# 1
AXLE WEIGHT * 2
AXLE WEIGHT \# 3
E OF SKEW (DEG)
) CASE NUMBER
IT WHEEL NUMBER
IPUT (SHIFT DN)
IPUT (SHIFT ON)
PRINT REQUEST
72. 00
192.00
custam
3
72.00
168.00
180.00
8.00
32.00
32.00
30.0

1
3

NO

|  |  |  |
| :--- | :---: | :---: |
| DATA | 12 | 15 |
| DER OF ELEMENTS | 36.00 | 24.00 |
| DENT SIZE | 432.00 | 360.00 |




EL \# WEIGHT IN ELEMENT \# AFFECTING NODES $X$ COORD Y COORD
4.00 ONLY 48 $30 \quad 108.00 \quad 311.77$

| 3 | 16.00 | ABOVE 23 | ONLY 41 | -24.00 | 166.28 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 7 | 16.00 | 70 | 74759081 | 38.35 | 202.23 |
| $j$ | 16.00 | RT OF 16 | ONLY 3334 | 66.00 | 10.37 |
| 16.00 | 63 | 67638384 | 123.35 | 46.39 |  |

ILTANT NODAL LOADS
E \# LOAD (KIPS)

> 8.00
> 8.00
> 16.00
> 4.00
> 8.49
> 2.56
> 2.96
> 8.09
> 3.80
> 1.15
> 1.33
> 3.62

## TICS CHECK

AL LOAD ON DECK $=68.00 \mathrm{KIPS}$
CK WEIGHT $=72.00 \mathrm{KIPS}$


THIS PROGRAM CALCULATES THE SET OF EQUIVALENT NODAL LDADE FOR ANY TYPE OF TRUCK MOUING ACROSS A SKEWED (PARALLELOGRAM) DECK OF ANY DIMENSIONS WITH ANY MESH SIZE. THE MESH IS GENERATED INTERNALLY AND CAN BE OUTPUT OPTIONALLY. THE USER MUST INPUT HE FOLLOWING:
(1) 'ALPHA' GIVE ON THE FIRST DATA CARD THE ALPHA-NUMERIC TITLE TO BE PRINTED ON THE DUTPUT LISTING. E77 CHAPACTER MAXIMUMJ
(2) 'TRTYPE' ENTER HERE THE INTEGER CQRRESPONDING TG THE TRUCK TYPE
FOR WHICH THE ANALYSIS IS TO BE PERFORMED. THE OPTIONS
ARE LISTED BELOW
(A) 1 IS FDOT TYPE SU-4
(B) 9 IS CUSTOM
***NOTE*** IF THE CUSTOM OPTION IS CHOSEN, THEN THE FOLLDWING DATA MUST BE ENTERED ON THE NEXT DATA CARDS
(A) NUMBER OF AXLES. [INTEGER UP TO 9]
(B) TRUCK WIDTH (CDAXIAL WHEEL SPACING). [REAL]
(C) SPACING FOR EACH SUCCESSIVE AXLE AS MEASURED
(C) SPACING FRR EACH SUCCESSIVE AXLE AS MEASURED. NOTE THAT THE \# OF AXLE SPACES IS ONE LESS THAN THE NUMBER OF AXLES. [REAL]
(D) AXLE WEIGHTS (ONE PER DATA CARD IN KIPS). [REAL] ALSO NOTE THAT IF THE CUSTOM OPTION IS IN EFFECT THEN THE NUMBER OF DATA CARDS WILL VARY.
(3) 'PHI'. THE ANGLE OF SKEW (DEGREES). [REAL]
(4) 'LCN'. THE LOAD CASE NUMBER (ONLY FQR USER'S REFERENCE)
(5) 'WNUM' HERE THE USER GIVES AN INTEGER INDICATING WHICH ONE OF WHEELS HE WILL LOCATE. THE PROGRAM WILL AUTOMATICALLY LOCATE THE REST (SEE SKETCH FOR RELATIVE POSITIONS AND NUMBERING OF THE WHEELS). NOTE THAT THIS CARD COMPLETELY ESTABLISHES THE POSITION OF THE TRUCK ASSUMING THAT IT IS TRAVELLING PARALLEL TO THE CURB (SKEWED EDGE OF THE DECK). [INTEGER]
(6) 'XSHIFT' THIS CARD ALLOWS THE USER THE OPTION OF GIVING THE $X$ INPUT AS EITHER SHIFTED OR NORMAL. IF A SHIFTED X COORDINATE DESIRED THEN SPECIFY $A \cdot 1$ ' ON THIS CARD. IF NOT, GIVE A 'O' [INTEGER]
(7) 'YSHIFT' THIS CARD ALLOWS THE USER THE OPTION OF INPUTING EITHER THE Y COURDINATE OR INCLINED SPAN DISTANCE ALONG THE SKEWED EDGE. IF SKEWED Y COORDINATE IS DESIRED THEN SPECIFY A, 1 ON THIS CARD. IF NOT, GIVE A \%\% [INTEGER]
(E) IF' CARD \#5 IS 'O' THEN GIVE ON \#7: *

## [INTEGER]



| *み $\neq$ NOTE** THE OPTIONS OF SKEWED COORDINATES ARE TNDEPENDENTFOR THE X AND Y AXES. IT IS RECDMMENDED THAT THE SKENED OPTION BE USED WHEN PLACING A WHEEL ON ANY GRID LINE OR MESH NODE, ALTHOUGH IT IS OFTEN CONVENIENT AT OTHER TIMES AS WELL. |
| :---: |
|  |  |
|  |  | BE USED WHEN PLACING A WHEEL ON ANY GRID LINE OR NESH NODE, ALTHOUGH IT IS OFTEN CONVENIENT AT OTHER TIMES AS WELL.

(10) 'KEY' IF KEY=O THE COQRDINATE MESH WILL NOT BE PRINTED QUT. IF KEY $=1$ IT WILL. [INTEEER]
(11) NXSTEP, THE NUMBER OF ELEMENTS IN THE $x$ DIRECTION. [INTEGER]
(12) NYSTEP: THE NUMBER OF ELEMETS ALONG THE (SKEWED) Y DIRECTION. [INTEGER]
(13) XINC' THE X INCREMENT (INCHES) [REAL]
(14) YSKINC'.THE INCLINED Y INCREMENT (INCHES), [REAL]
(13) 'XINC' THE X INCREMENT (INCHES) [REAL]
(14) YSKINC'.THE INCLINED Y INCREMENT 〔INCHES). [REAL]
(A) THE X COORDINATE OF THE WHEEL NUMBER GIVEN ON CARD \# 4: [REAL]
IF CARD $\# 5$ IS 1 , THEN GIVE ON \#7:
(B) THE X DISTANCE OF THE WHEEL FROM THE LEFT SKEWED EDGE OF THE DECK [REAL]
(9) IF CARD \#G IS $0^{\circ}$ THEN GIVE ON \#B
(A) THE Y COORDINATE DF THE WHEEL NUMBEP GIVEN ON CAPD \# 4. [REAL]
IF CARD *S IS ' 1 ', THEN GIVE ON \#B:
(B) THE DISTANCE TO THE WHEEL AS MEASURED ALONG THE INCLINED
SPAN EDGE OF THE DECK. [REAL]

PLACE WHEEL \# 3 ON NODE $(5,4)$ OF A 30 DEG SKEWED DECK. THE DECK MEASURES 54O" ${ }^{\circ} 30^{\prime \prime}$ (X DIMENSION AND SKEWED SPAN, RESP) AND THE MESH SIZE IS 54" IN THE X DIRECTIDN AND 45" IN THE Y DIRECTION. THUS THERE ARE 10 INGREMENTS ALONG THE X AKIS AND 8 INCREMENTS ALONG THE SKEVED Y AXIS (PLEASE SEE SKETCH). SAY THE TRUCK TYPE IS FDOT SU-4. WE WOULD INPUT AS FOLLOWS:
CARD
INPUT
COMMENTS
1
2
3
4
4
FDOT SU-4 EXAMPLE 30.0

| 5 | 3 |
| :---: | :---: |
| 6 | 1 |
| 7 | 1 |
| 8 | 162.0 |
| 9 | 180.0 |


| 10 | 0 |
| :--- | :--- |
| 11 | 10 |
| 12 | 8 |
| 13 | 54.0 |
| 14 | 45.0 |


DIMENSION $X \operatorname{COR}(30,30), \operatorname{YCOR}(30,30), \operatorname{REACT}(30,30)$,
\$ WHEELX (18), hHEELY(18), WT (18), AXLSP (8), AXLWT (9)
INTEGER XSHIFT, YSHIFT, ERR, WNUM, TRTYPE
CHARACTER ALPHA*77
OPEN (UNIT = =, $\mathrm{FILE}=$ ' [. PAUL JLOADER. DAT', STATUS='OLD')
OPEN (UNIT $=6, F I L E=$ 'L. PAULJOUTPUT. LIS', STATUS = 'NEW')
CALL TITLEI (ALPHA)
CALL INITL (WX, WYY, PHI, NXSTEP, NXCROS, XINC, NYSTEP, NYCROS,
\$
YSK INC, LCN, KEY, XSHIFT, YSHIFT, WNUM, AXLSP, NA, NAXLSP, NW,


```
--_m
```



WRITE（6，382）WいX
FORMAT（ $1,7 \mathrm{~F},{ }^{\prime} \mathrm{X}$ INPUT（SHIFT ON）＇， $10 \mathrm{X}, \mathrm{F} 7.2$ ） GO TO 386
WRITE $(6,384)$ WJX
FORMAT（／，7X，＇X INPUT（SHIFT OFF）＇， $8 \mathrm{X}, \mathrm{F} 7.2$ ） IF（YSHIFT．EQ．O）GO TO 389
WRITE（6，388）WJY
FORMAT（ $/, 7 X,{ }^{\prime} Y$ INPUT（SHIFT ON）＇， $10 X, F 7.2$ ） GO TO 392
WRITE（ 6,390 ）WJY
FORMAT（ $/, 7 X,{ }^{\prime} Y$ INPUT（SHIFT OFF）＇， $8 X, F 7.2$ ）
READ（1，393）KEY
FORMAT（1I1）
？．F（KEY．EQ．O）GO TO 395
WR ITE $(6,394)$
FORMAT（ $/, 7 \mathrm{X}$ ，＇MESH PRINT REQUEST＇， 13 K, ＇YES＇）
GO TD 400
396
398
400
中
FORMAT（＇／， 7 X, ＇MESH PRINT REQUEST＇， 12 X ，＇NO＇）
WRITE $(6,405)$
FORMAT（／／／，7X，＇－－－－－－－－－－， 7 X，＇MESH DATA＇， $16 X$, ＇X－AXI＇S＇， GX，＇Y－AXIS（SKEWED）＇，$/, 7 X, \quad, \quad$ MESH DAIA，1甘X，＇X－AXIS＇，． READ（1，410）NXSTEP
410 FGRMAT（1I2）
READ（1，412）NYSTEP
412 FORMAT（1I2）
WRITE（ 6,414 ）NXSTEP，NYSTEP
414 FORMAT（ $/, 7 X$ ，＇NUMBER OF ELEMENTS＇， $9 \mathrm{X}, 12,10 \mathrm{X}, \mathrm{I2)}$ READ（ 1,420 ）XINC
420 FORMAT（ $1 F 6.2$ ）
READ（ 1,430 ）YSKINC
430 FQRMAT（1F6．2）
WRITE $(6,432)$ XINC，YSKINC
432 FORMAT（ $/ 7,7 \mathrm{X}$, ＇ELEMENT SIZE＇， $14 \mathrm{X}, \mathrm{FG} .2,6 \mathrm{~K}, \mathrm{FG} .2$ ）
DDX＝NXSTEPれXINC
DDYSK＝NYSTEP：YSKINC
WRITE（ 6,434 ）DDX，DDYSK
434 FORMAT（／，7X，＇DECK DIMENSION＇，11X，F7．2，5X，F7．2）
NKCROS＝NXSTEP +1
NYCROS＝NYSTEP +1
RETURN
END
SURROUTINE GMESH（PHI，NXSTEP，XINC，NXCROS，XCOR，NYSTEP，
\＄YSKINC，NYCROS，YCOR，KEY，ERR）

C SUB GMESH GENERATES THE SKEWED MESH AND ALLOWS FOR OPTIGNAL＊
C PRINTOUT FOR KEY＝1．


## DIMENSION XCOR（NYCROS，NXCROS），YCOR \｛NYCROS，NXCROS）

## INTEGER ERR

$\mathrm{PHI}=3.14159265359 \approx \mathrm{PHI} / 180$.
$\mathrm{X}=0.0$
$\mathrm{Y}=0.0$
$A=0$ ． 0
XSTEP＝0．0
NCOUNT $=0$
ERR＝0
DO $590 \quad J=1$ ，NXCROS
DO $580 \mathrm{I}=1$ ，NYCROS
NCOUNT $=$ NCOUNT +1
$\mathrm{YCOR}(I, J)=Y$
$\operatorname{XCOR}(I, J)=X$
IF（KEY．EQ．O）GO TO 570
WRITE（ 6,560 ）NCOUNT，$I, ~ J, X, Y$
560
 \＄$X=$ ，F7．2， $3 X, Y=$ ，F7．2）


```
\[
X 1=A \neq S I N(P H I)
\]
\[
Y 1=A \times C O S(P H I)
\]
\[
\begin{aligned}
& x=x \operatorname{STEP}-x 1
\end{aligned}
\]
\[
\hat{Y}=\hat{Y}
\]
CONT INUE
\(A=0.0\)
\(Y=0.0\)
XSTEP \(=\) XSTEP + XINC
\(X=\) XSTEP
590
CDNTINUE RETURN
END
940
SUBRDUTINE LDCATE (WJX, WJY, WIX, WIY, PHI, WMUM, ERR, DFFWT,
\$
XSHIFT, YSHIFT, USERXX, USERY, AXLSP, NA, MAXLEP, NH, WIDTH, WT,
NKCROS, NYCRIS, REACT, WHEELX, WHEELY, DDX, DDYSK
С \(C\) SUB LOCATE USES THE INPUT WHEEL CODRDINATES OF WJX AND WVY TO z C FIND THE COORDINATES OF THE LEFT FRONT (i=e.: DRIVER'S) WHEEL. K C THEN IT WILL LOCATE THE CDORDINATES OF THE REST BASED ON THIS C POSITION. IT ALSO CHECKS TO BE CERTAIN THAT EACH WHEEL FALLS C ON THE DECK ITSELF AND PRINTS A WARNING LIST FOR THOSE WHEELS C THAT ARE NOT WITHIN THE DECK BOUNDARIES. THE TOTAL WEIGHT OF C THE WHEELS THAT ARE OFF THE DECK WILL BE CDNSIDERED IN THE C STATICS CHECK AT THE END OF THE PROGRAM.
```



```
DIMENSIDN WT (NW), AXLSP (NAXLSP), WHEELX̉ (NW), WHEELY (NW),
```



```
    W1X=WUX-(A+B+C)*SIN(PHI)
    W1Y:=WJY+(A+B+C)*COS (PHI)
    GO TD }78
IF (WNUM.NE.8) GO TO 95S
    WIX=WJX-((A+B+C)*SIN(?HI)+Z1)
    W1Y=WUY+((A+B+C)*COS(PHI)-Z2)
    OO TO }98
    D=AXLSP(4)
    IF (WNUM.NE.9) GO TD 75马
    W1X=WJX- (A+B+C+D)*SIN(PHI)
    W1Y=WJY+(A+B+C+D)*C口S (PHI)
    GO TO 980
    IF (WNUM.NE.10) GO TO 960
    W1X=WUX-((A+B+C+D)*STN(PHI)+Z1)
    WIY=WWY+( (A+B+C+D)*CDS (PHI)-Z2)
    GO TO }98
    E=AXLSP(5)
    IF (WNUM.NE. 11) GO TO 96P
    W1X=W\X-(A+B+C+D+E)*SIN(PHI)
    WIY=WJY+(A+B+C+D+E) н气0S(PHI)
    GO TO 930
    IF (WNUM.NE. 12) GO TO 964
    W1X=WUX-((A+B+C+D+E)*SIN(PHI)+Z1)
    W1Y=WUY+((A+B+C+D+E)*CDS (PHI)-Z2)
    GD TO }98
    F=AXLSP(6)
    IF (WNUM.NE.13) GO TO 906
    W1X=WUX-(A+B+C+D+E+F)&SIN(PHI)
    W1Y=WJY+(A+B+C+D+E+F) &COS (PHI)
    GO TD }98
    IF (WNUM.NE. 14) GO TO 96S
    W1X=WJX-( (A+B+C+D+E+F)*SIN(PHI)+Z1)
    W1Y=WNY+((A+B+C+D+E+F)*CDS (PHI)-Z'S)
    G0 TD 980
    G=AXLSP(7)
    IF (WNUM.NE. 15) GO TO 970
    W1X=WJX-(A+B+C+D+E+F+C)*SIN(PHI)
    W1Y=WJY+(A+B+C+D+E+F+G)*CDS (PHI)
    CO TO 980
    IF (WNUM.NE.16) GO TG 972
    W1X=WJX-((A+B+C+D+E+F+G)*SIN(PHI)+ZI)
    W1Y=WJY+((A+B+C+D+E+F+G)*COS (PHI)-Z2)
    GO TD }98
    H=AXLSP (8)
    IF (WNUM.NE,17) ED TO }97
    W1X=WJX-(A+B+C+D+E+F+G+H)*SIN(PHI)
    W1Y=WJY+(A+B+C+D+E+F+G+H)*COS(PHI)
    GO TD 980
    IF (WNUM.NE. 18) GO TC 976
        W1X=WJX-((A+B+C+D+E+F+G+H)*SIN(PHI)+Z1)
        W1Y=WJY+((A+B+C+D+E+F+G+H)*CDS (PHI)-Z2)
        GO TD 980
```

    WITE (6,978)
                            FORMAT (///, 7X, '****ERROR***为 ILLEEAL WHEEL NUMBER INPUT',
            ERR=1
            GO TO 1090
    I=0
        WHEELX(1)=W1 
        WHEELX(2)=W1X+Z1
        WHEELY(1)=W1Y
        WHEELY(2)=W11Y+Z2
        I= I+2
    IF (I.EQ.NW) GO TO 1010
    A=AXLSP (1)
    WHEELX(3)=W1X+(A)*SIN(PHI)
    WHEELX (4)=WHEELX (3)+Z1
    WHEEIY Y (3) =W1Y-A*COS (PHI)
    WHEELY}(4)=WHEELY(3)+Z
    ```
```

    I=I+2
    IF (I.EQ.NW) GQ TO 1010
    B=AX゙LSO
    WHEELX (5)=W1X+(A+B)*SIN(PHI)
    WHEELX(6)=WHEELX(5)+Z1
    WHEELY (5)=W1Y-(A+B) HCOS (PHI)
    WHEELY(6)=WHEELY(5)+Z2
    I=I+2
    IF (I.EQ.NW) GD TO 1010
    C=AXLSP (3)
    WHEELX(7)=W X + (A+B+C) *SIN(PHI)
    WHEELX (8)=WHEELX(7)+Z1
    WHEELY (7) =W1Y- (A+B+C)*COS (PHI)
    WHEELY (8)=WHEELY(7)+7?
    I=I+2
    IF (I.EQ.NW) GD TD 1010
    D=AXLSP(4)
    WHEELX(9)=WIX+(A+B+C+D) %SIN(PHI)
    WHEELX (10)=WHEELX(Я)+Z1
    WHEELY(9)=W1Y-(A+B+C+D)*CDS (PHI)
    WHEELY(10)=WHEELY(9)+Z2
    I=I+2
    IF (I.EQ.NW) GD TD 1010
    E=AXLSP(5)
    WHEELX(11)=W1X+(A+B+C+D+E)*SIN(PHI)
    WHEELX (12)=WHEELX(11)+Z1
    WHEELY (11) =W1Y-(A+B+C+D+E)*COS (PHX)
    WHEELY (12)=WHEELY(11)+Z2
    I=I+2
    IF (I.EQ.NW) GO TO 1010
    F=AXLSP(6)
    WHEELX(13)=W1X+(A+D+C+D+E+F)*SIN(PHI)
    WHEELX(14)=WHEELX(13)+Z1
    WHEELY}(13)=W1Y-(A+B+C+D+E+F)*COS (PHI
    WHEELY(14)=WHEELY(13)+Z2
    I=I+2
    IF (I.EQ.NW) G口 TO 1010
    C=AXLSP(7)
    WHEELX (15)=W1X+(A+E+C+D+E+F+G)*SIN(PHI)
    WHEELX(16)=WHEELX(15)+Z1
    WHEELY(15)=W1Y-(A+B+C+D+E+F+G)*COS (PHI)
    WHEELY (16)=WHEELY(15)+Z2
    I=I+2
    IF (I.EQ.NW) GD TO 1010
    H=AXLSP(8)
    WHEELX(17)=W1X+(A+B+C+D+E+F+G+H) %SIN(PHI)
    WHEELX(18)=WHEELX(17)+Z1
    WHEELY (17)=W1Y-(A+B+C+D+E+F+G+H) &COS(PHI)
    WHEELY(18)=WHEELY(17)+Z2
    I=I+2
    IF (I.EQ.NW) GD TD 1010
    WRITE (6,990), ILLEGAL NUMBER OF WHEELS INPUT')
    FORMAT (//,7X, 'ILLEGAL NUMBER GF WHEELS INPUT')
    GO TD 1090
    OFFWT=0.0
    D口 1080 I= 1,8
    A=WHEELY(I)
    IF (A.GE.O.O) GQ TO 1015
    OFFWT= OFFWT+WT(I)
    WT (I) =0.0
    WRITE (b,1013) I
    1013 FORMAT (/l/, 7X,'****WARNING甘***,WHEEL \# %,I2,
\$
IS OFF THE DECK TO THE SOUTH')
GO TO 1030
1015 PSD=DDYSK{COS(PHI)
IF (A.LE.PGD) GO TO 1025
GFFWT= OFFWT+WT (I)
WT (I)=0.0
WRITE (6,1020) I

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        XOFF1=(D-D2)*SIN(PHI)/SIN(1.570796327-PHI)
        CHECK1=XCOR(I2,L)
        WXDFF1=CHECK1-F
        CALL RNDOFF(WXOFF1, XDFF1)
        IF (WXOFF1.GT. XOFF1) GO TO }75
        IS=L+1
        IA=L
        LCOUNT=LCOUNT+NYSTEP
        NSE=NSE+NYCROS
        GO TD 760
    750 I3=L
        I4=L-1
    700 D3=XCOR(I2,I3)
        D4=XCOR(I2,I4)
        NSW=NSE-NYCRGS
        N=1
        G=XCOR (II,N)
        CALL RNDOFF (G,F)
        IF (G.GT.F) CO TD 78O
        N=N+1
        NNE=NNE+NYCROS
    GO TO 770
    780 XOFF2=(D1-D)*SIN(PHI)/SIN(1.57079.6327-PHI)
        CHECK2=XCOR (I 1,N)
        WXOFF2=F-(CHECK2-XINC)
        CALL RNDOFF(WXOFF2, KOFF2)
        IF (WXOFF2.GT. XOFF2) GO TO }79
        I 5=N-1
        I6=N-2
        NNE=NNE-NYCRDS
        LCDUNT=LCDUNT-NYSTEP
            GO TO }79
    I 5=N
    795 IG=N-1
    D5=XCOR(I1,I5)
        DG=XCOR(II,IG)
        NNW=NNE-NYCROS
        W=WT (K)
        A=WHEELX(K)
        B=WHEELY(K)
    WRITE (6,797) K,W,LCDUNT,NSW,NNW,NSE,NNE,A,B
    797 FORMAT (/, 9X, I1,6X,F5.2,7X,I工, 1OX,I2,1X,I2,1X,I2,
\$ 1X,I2,4X,F7.2, 2^,F7.2)
C

```

```

C HORIZONTALLY ACROSS THE ELEMENT AND FINDING THE EQUIVALENT *
C STATIC LDADS DN THE SKEWED EDGES. THEN PUT TWO IMAGINARY
C STRINGERS ALDNG THE SKEWED EDGES AND SOLVE FOR THE NODAL FOINT *
C LOADS. AS THE REACTIONS FRDM EACH WHEEL ARE FOUND, KEEP A
C RUNNING TOTAL OF EACH WHEEL'S CONTRIBUTIDN IN A STORAGE MATRIX *
C REACT (I, J).

```

```

            D7=WHEELX(K)-D4
            D8=(WHEELY(K)-D2)*SIN(PHI)/SIN(1.570796327-PHI)
            RL=WT(K)*((D3-D4)-(D7+D8))/(D3-D4)
            RR=WT (K)-RL
            RNNW=RL*(WHEELY(K)-D2)/(D1-D2)
            RNSW=RL*(D1-WHEELY(K))/(D1-D2)
            RNNE=RR*(WHEELY(K)-D2)/(D1-D2)
            RNSE=RR*(D1-WHEELY(K))/(D1-D2)
                    800
            REACT (I1, IG)=REACT (I1,IG)+RNNG
                        REACT (I2, I4)=REACT (I2,I4)+RNSW
                            REACT (I1,I5)=REACT (I1,I5)&RNNE
                        REACT(I2,I3)=REACT (I2,I3)+RNSE
                    CONTINUE
                        RETURN
                        END
    915 SUBRDUTINE RNDOFF (X,Y)

```
(

C SUBROUTINE RNDOFF ROUNDS OFF THE VALUES ASSIGNED TO \(X\) AND \(Y\) *
C TO THREE DECIMAL PLACES.

\(X=1000\). \(0 \pi X\)
\(Y=1000.0 * Y\)
\(X=\operatorname{ANINT}(X) / 1000.0\)
\(Y=\operatorname{ANINT}(Y) / 1000.0\)
RETURN
END
SUBROUTINE RESULT (NXCROS, NYCROS, REACT, OFFWT, XSHIFT,
\$ YSHIFT, WNUM, USERX, USERY, WJX, WJY)

C SUB RESULT QUTPUTS THE RESULTANT (NON-ZERO) NODAL LOADS AND \#
C CHECKS BY STATICS TO ASSURE THAT THE SUM OF THE WEIGHTS OF \#
C THE 'OFF-WHEELS' ADDS UP TO THE TRUCK WEIGHT.
 DIMENSION REACT (NYCROS, NXCROS)
INTEGER XSHIFT, YSHIFT, WNUM
TOTAL \(=0.0\)
NCOUNT=1
WRITE \((6,855)\)

855
中
 5X, 'LOAD (KIPS)', ノ)

DO \(880 \mathrm{~J}=1\), NXCROS
DO 870 I=1, NYCROS
\(R=R E A C T(I, ل)\)
IF (R.EQ.O.O) GD TO 860
TOTAL = TOTAL \(+R\)
WRITE ( 6,857 ) NCOUNT, R
FORMAT ( \(/, 8 \mathrm{X}, \mathrm{I} 2,9 \mathrm{X}, F 7 \mathrm{Z}\) 2)
NCOUNT = NCOUNT +1
CONTINUE
CONTINUE
TRKWT = TOTAL + OFFWT
WRITE ( 6,890 ) TOTAL, TRKWT
890 FORMAT (////,7X,'STATICS CHECK', //, 7X,
\$
\(\$\)

895
\$
900
905
\$
910
RETURN SUBROUTINE TRTYP1 (AXLSP, NA, NAXLSP, NW, WIDTH, WT)

C SUB TRTYPI HOLDS THE REQUIRED INPUT DATA FOR THE FDOT SU-4 *
C TYPE TRUCK
 DIMENSION AXLSP (3), WT (8)
\(\mathrm{NA}=4\)
NAXLSP \(=3\)
\(\mathrm{NW}=8\)
WIDTH \(=76.0\)
AXLSP (1) \(=110.0\)
\(\operatorname{AXLSP}(2)=50.0\)
AXLSP \((3)=50.0\)
WT (1) \(=6.95\)
\(W T(2)=6.95\)
DO \(950 \quad \mathrm{I}=3,8\)
WT (I) \(=9.35\)
950 CONTINUE
RETURN
END
SUBRDUTINE CUSTM1 (NA, NAXLSP, NW)

C SUB CUSTM1 READS IN THE NUMBER OF AXLES AND FINDS THE NUMBER *
C OF AXLE SPACES AND THE NUMBER OF WHEELS. THE INFORMATION IS \#
C REQUIRED FOR THE VARIABLE DIMENSIONS IN SUB CUSTM2.

READ \((1,705) \mathrm{NA}\)
705 FORMAT (111)
NAXLSP = NA-1
NW=2*NA
RETURN
END
SUBRDUTINE CUSTMZ (AXLSP, NA, NAXLSP, NW, WIDTH, WT, AKLWT)
 C SUB CUSTMZ READS IN THE REST DF THE REQUIRED DATA FOR ANY *
C TYPE DF TRUCK
 DIMENSIDN AXLSP (NAXLSP), WT (NW), AXLITT (NA)
READ (1,740) WIDTH
FORMAT (1FG.2)
DO 770 I=1, NAMLSP
READ (1,750) AXLSP (I)
FORMAT (1F6.2)
CONTINUE
DO \(780 \mathrm{I}=1, \mathrm{NA}\)
READ ( 1,775 ) AXLWT (I)
FORMAT (1FG.2)
CONTINUE
DO \(790 \mathrm{~K}=1\), NA
\(\mathrm{I}=2 \mathrm{z}\) K-1
\(J=2 \times k\)
WT (I) =AXLWT (K)/2
WT (J) =Wr (I)
CONTINUE
RETURN
END
SUBROUTINE TITLEI (ALPHA)
 C SUB TITLEI READS IN AND ECHO PRINTS THE IITLE CF THE OUTPUT *

\section*{C LISTING}

CHARACTER ALPHAK 77
READ (1,200) ALPHA
FQRMAT (A77) WRITE \((6,210)\) ALPHA
210 FORMAT (///, 7X, A77) RETURN END SUBROUTINE TITLE2 WRITE (6,700)



2X, 'WEIGHT',2X,'IN ELEMENT \#', 2x', 'AFFECTING NODES', \(2 x\). \$ 'x COORD', 2 X, ' \(Y\) COORD', /) RETURN END


\section*{GTSTR. COM}
\begin{tabular}{|c|c|c|}
\hline & GTSTRUDL [. E[. PAULIRUN & PI \\
\hline & GTSTRUDL [. PAULIPS20100. DAT; & \(\mathrm{PR}=[. \mathrm{PALL}] \mathrm{PS201002}\) \\
\hline & er.paul & \\
\hline & GTSTRUDL [.PAULJPS20100. DAT; & \(P R=[. P A U L] P 5201003 . L I S / N\) \\
\hline & Q[.PAULIRUN & PR=[.PAULJPS201003.LIS.N \\
\hline \[
\$
\] & GTSTRUDL [. PAULIPS20100. DAT; & \(\mathrm{PR}=\mathrm{L}\). \\
\hline & E[.PAULJ & \\
\hline \[
\$
\] & GTSTRUDL [. PAULJPS20100. DAT; & \(P R=[. P A U L] P S 201005 . L I S / N\) \\
\hline & & \\
\hline \$ & GTSTRUDL [.PAULJPS20100. DAT; & PR=[.PAUL]PS201006.LIS/H \\
\hline & E[.PAUL & \\
\hline \$ & GTSTRUDL [.P & \\
\hline & E[.PAULJRUN & \\
\hline \$ & GTSTRUDL [. PAUL]PS20100. & \(P R=[. P A U L] P S 201008 . L I S / N ~\) \\
\hline & e[. PAUL & \\
\hline \[
\$
\] & GTSTRUDL [ & \(P R=[. P A U L] P S 201009 . L I S / N\) \\
\hline & c & \\
\hline \$ & GTSTRUDL [ & PR=[.PAUL ]PS20 \\
\hline & E[.PAULIR & \\
\hline & GTSTRUDL [.PA & \(P R=[. P A U L] P S 2010\) \\
\hline & E[. PAULJR & \\
\hline & GTSTRUDL [.PAULJPS20100. & \(P R=[. P A U L J P S 2010012\). \\
\hline & - & \\
\hline \$ & GTSTRUDL [. P & \(P R=[. P A U L] P S 201\) \\
\hline \$ & E[.PAUL IR & \\
\hline \$ & GTSTRUDL & 520 \\
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\$
\] & ©[.PAULIRUN & \\
\hline & GTSTRUDL [.PAULJPS20100. & \(P R=[. P A U L] P S 3010\) \\
\hline \$ & E[-PAULIRUN & \\
\hline \[
\$
\] & GTSTRUDL [. PAULJPS20100. DAT; & \(P R=\). PAULJPS2010016. \\
\hline & E[.PAULJRU & \\
\hline & GTSTRUDL [.PAULJPS20100 & \\
\hline
\end{tabular}

\section*{RUN. COM}
```

\$ FOR [.PAUL]INC
\$ LINKK INC
FOR [.PAUL]LOADER
LINK LOADER
RUN LOADER
FOR [. PAUL]CREATE
\$ LINK CREATE
\$ RUN CREATE
\$ DEL INC.*;*
\$ DEL CREATE.\#; *
\$ DEL LOADER.*;*
\$ DEL [.PAULJPS20100.SAV湆

```


APPENDIX B
SAMPLE GTSTRUDL OUTPUT
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\]
101.2 нж Proprietary to the Ceorgia Tech Research Institute．
 1）IRET＇L．PAlll］PS20100．SAU＇
［－NEnS5，Me55age number 0805805］


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\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline \multicolumn{13}{|l|}{} \\
\hline
\end{tabular}


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๕る GEORGIA TECH RESEARCH INSTITUTE＊
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\section*{TE BENDING}

mter study is used ta inuestidate the effect af
Hele an bridge decks af variaus aspect ratids．
PS，IN，DEG
KERATE पFF
DEDIMATES
9 JロInTS ID \(1.1 \times 0.0-16.41698683\) Y 0.045 .1052458

\section*{THHES ID \(9 \times 48.0\)}

UPPIRT 1 TI 73 RY 9,9 TD 81 RY 9
IMCIDEMCES
8 ELEMENTS ID 1，1 FROH 1,1 TD 10,1 T口 11,1 TD 2,1 TIUES ID 8 FRDA 9 T日 9 T0 9 T0 9
LEASES
BY 9 HOAENT X，RMHENT Y

PMPERTIES
TYFE＇EPP＇THICKMESS 18.0

\section*{fill}

Q． 15 ALL
＇FDOT SU－4 ROLLS ACRDSS DECK CEMTER＇





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2-7.55
2 -7.88
z-7.55
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2 -3.90
2-1.95
z-0.06
z-2.10
z-2.88
z -2.82
z-0.97
z-0.83
z -1.36
z-3.67
z-7.55
z-7.55
z-4.87
z-0.53
2-5.3]
ANALYSIS

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IFE BAMDIIDTH IS 8.889
MRD DEUIATIMN DF THE EAURATDTH IS 2.846
11.735

REDUCTIDA HAS FAILED TD PRDDUCE A PETTER KUMRERIMG. MMPERRING HILL PE USED.
\begin{tabular}{|c|c|}
\hline TEMCY CHECKS FIR 64 MEHARES & 6.38 SECDMDS \\
\hline RAMDUIDTH REDUCTITM & 5.95 SECDYDS \\
\hline UNERATE 64 ELEMEMT STIF. MATRICES & 12.54 SECDIDS \\
\hline ISSEMRLE THE STIFFHESS HATRIX & 12.51 SECDMDS \\
\hline Malcess 81 Jinits & 5.66 SECDMDS \\
\hline OLUE UITH 21 Piotitimes & 21.24 SECDMDS \\
\hline RIECESS 81 JIINT DISPLACEREMTS & 1.58 SECDMDS \\
\hline dRCESS 64 ELEMEHT REACTIDKS & 9.11 SECDMDS \\
\hline PLECESS 64 ELEMENT STRESSES & 9.14 SECDYDS \\
\hline STATICS CHECK & 2.46 SECDI \\
\hline
\end{tabular}
: AUERAGE PRINCIPAL EENDING RESLLTAMTS
CE SPECIFICATION HISSING - HIDDLE SURFNCE ASSUTED
OHT LIST MISSIMG - ALL ASSHMED

LTS DF LATEST AMALYSESK ฉะะหะ

LEM - SKEH=20. TIILE - RAME GIUEN

UE LNITS IMCH KIP DEG DEGF SEC

\section*{6-1 \\ FDOT SU-4 ROLLS ACRDSS DECK CENTER}

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\(0.1979715+02\)
\(0.7350025+01\)
\(0.7130045 \mathrm{E}+01\)
\(0.6207075+01\)
\(0.482972 \mathrm{E}+01\)
\(0.213989 \mathrm{E}+01\)
\(-0.2883335+01\)
\(-9.3413915+02\)
\(-0.295704 E+02\)
\(0.455526 E+02\)
\(0.129142 E+02\)
\(0.832295 \mathrm{E}+01\)
\(0.5724405+103\)
0. \(377513 E+01\)
\(-0.7161685+00\)


GTICES 1.2 13-JUL-1984 12:01:38.61 PASE
\begin{tabular}{|c|c|c|c|}
\hline \(0.289881 \mathrm{E}+00\) & \(-0.427647 E+91\) & \(0.228227 \mathrm{E}+01\) & \(-0.80050 .05+01\) \\
\hline \(0.495851 E+00\) & -0.228644E 01 & \(0.139105 E+01\) & \(-0.227182 \mathrm{E}+02\) \\
\hline \(0.974702 E+00\) & -0.122590 E (.) 1 & \(0.1099895+01\) & -0.474216E+02 \\
\hline \(0.159745 E+01\) & \(-0.171185 E+01\) & \(0.1699655+01\) & \(0.432510 \mathrm{E}+02\) \\
\hline \(-0.3954665+00\) & \(-0.6205625+91\) & \(0.290508 \mathrm{E}+01\) & 0. 1654235102 \\
\hline \(-0.9088752+00\) & \(-0.943323 E+01\) & \(0.4261685+0.1\) & \(0.798250 E+01\) \\
\hline \(-0.924320 E+00\) & \(-0.100962 E+02\) & \(0.458592 \mathrm{E}+01\) & 0. \(34402585+01\) \\
\hline \(-0.914574 E+00\) & \(-0.8987765+01\) & \(0.4026595+01\) & \(0.4798045+00\) \\
\hline \(-0.608510 E+00\) & \(-0.7487622^{2}+01\) & \(0.3439555+01\) &  \\
\hline -0.811583E-01 & -0.522623E+01 & \(0.2572345+01\) & -0.110802E+02 \\
\hline \(0.5884115+00\) & \(-0.2984345+01\) & \(0.1786385+01\) & -0.249124E+02 \\
\hline \(0.1501382+01\) & \(-0.135872 \mathrm{E}+01\) & \(0.1470055+0.1\) & -0. \(5449523 E+02\) \\
\hline \(0.1643135+00\) & -0.2485675+90 & \(0.2064405+00\) & 0.5257015ic +02 \\
\hline -0.29585]E+01 & -0.693211 E+01 & \(0.198574 E+01\) & 0. \(6889235+01\) \\
\hline \(-0.421925 E+01\) & \(-0.1058955+02\) & \(0.3180 .63 \mathrm{~F}+01\) & \(0.3672255+01\) \\
\hline \(-0.4102315+01\) & \(-0.1163855+02\) & \(0.3792835+01\) & - .228834E+00 \\
\hline \(-0.293585 E+01\) & -0.9542805 701 & \(0.3303775+01\) & - \(0.3302005+01\) \\
\hline \(-0.243978 E+01\) & \(-0.8708975+01\) & \(0.313460 \mathrm{E}+01\) & \(-0.595912 \mathrm{E}+01\) \\
\hline \(-0.126594 E+01\) & \(-0.6184075+01\) & \(0.2459065+01\) & -0.1.73459E+02 \\
\hline \(0.174997 E+00\) & \(-0.348868 \mathrm{E}+01\) & \(0.183184 \mathrm{E}+01\) & -0.270213E+02 \\
\hline \(0.162351 \mathrm{E}+01\) & \(-0.161569 \mathrm{E}+01\) & \(0.1619605+01\) & -0.452945E+02 \\
\hline \(0.1356065+01\) & -0.125626E+01 & \(0.1306165+01\) & -0.401249E+02 \\
\hline -0.201699E+01 & -0.676691E+01 & 0.217496 E +01 & -0.703950E+01 \\
\hline \(-0.371546 E+01\) & \(-0.105597 \mathrm{E}+02\) & \(0.342214 E+01\) & \(0.1369855+01\) \\
\hline \(-0.4228645+01\) & \(-6.1189345+02\) & \(0.3832365+01\) & \(0.2424145+01\) \\
\hline \(-0.376627 E+01\) & \(-0.107074 E+02\) & \(0.3270575+01\) & -0.695013E+00 \\
\hline -0.296351E+01 & \(-0.8647905+01\) & \(0.2842205+01\) & -0. \(388926 \mathrm{E}+01\) \\
\hline \(-0.1939555+01\) & \(-0.670112 \mathrm{c}+01\) & \(0.2380295+01\) & -0.815069E+01 \\
\hline \(-0.420614 E+00\) & \(-0.356196 E+01\) & \(0.1570675+01\) & \(-0.2444467 E+02\) \\
\hline
\end{tabular}

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\begin{tabular}{|c|c|c|c|}
\hline \(0.140617 \mathrm{E}+01\) & \(-0.141997 E+01\) & \(0.141307 \mathrm{E}+01\) & -0.453914E+02 \\
\hline 0.208901 E+11 & -1. \(198578 \mathrm{E}+01\) & \(0.203739 E+21\) & \(-1.1459332 \mathrm{E}+02\) \\
\hline \(-0.120304 E+01\) & \(-0.5027672+01\) & \(0.2412325+01\) & \(-7.294407 E+02\) \\
\hline \(-0.363555 \mathrm{E}+01\) & \(-0.101225 E+02\) & \(0.3243485+41\) & - \(0.555129 \mathrm{E}+01\) \\
\hline -0.461190E+01 & -1. 117610 E +02 & \(0.357455 \mathrm{~F}+01\) & 3. 8451935.00 \\
\hline \(-0.429291 E+01\) & \(\cdots\)-0.1090145 +12 & \(0.3304235+91\) & \(0.2412005+01\) \\
\hline \(-0.325693 \mathrm{E}+01\) & \(-0.849245 E+01\) & \(0.261776 \mathrm{E}+01\) & \(-2.1657085+00\) \\
\hline \(-0.270676 E+01\) & \(-0.726017 \mathrm{E}+01\) & \(0.227670 E+01\) & -0. \(339362 E+01\) \\
\hline -0.112403E+01 & \(-0.338322 E+01\) & \(0.112960 \mathrm{E}+01\) & \(-0.1601035+02\) \\
\hline \(0.848903 \mathrm{E}+00\) & \(-0.941884 E+00\) & \(0.8953935+00\) & \(-0.464664 E+02\) \\
\hline \(0.212414 E+01\) & \(-0.187912 \mathrm{t}+11\) & \(0.200163 \mathrm{E}+01\) & \(-0.455515 \mathrm{E}+02\) \\
\hline 0.4517665100 & \(-0.468235 \mathrm{E}+01\) & \(0.256706 \mathrm{E}+01\) & \(-0.189869 E+02\) \\
\hline -0.687199E+00 & -0.804400E +01 & 9.367040E +01 & \(-0.3637935+01\) \\
\hline \(-0.142582 E+01\) & -0.101885E+02 & \(0.4381345+01\) & \(0.4173065+01\) \\
\hline \(-0.174295 E+01\) & \(-0.103202 E+02\) & 0.428854501 & \(0.786717 \mathrm{E}+01\) \\
\hline -0.164245t +01 & \(-0.881564 E+01\) & 0.3585805101 & \(0.675601 \mathrm{t}+01\) \\
\hline \(-0.119035 E+01\) & -0.682792E+01 & \(0.281854 \mathrm{E}+0.1\) & \(0.434723 E+01\) \\
\hline \(-0.771359 E+00\) & -0.3478835201 & \(0.1353735+01\) & \(0.933909 E+00\) \\
\hline \(0.225273 E+00\) & -0.879910E-01 & \(0.156532 \mathrm{E}+08\) & \(-0.443486 E+02\) \\
\hline \(0.125557 \mathrm{E}+01\) & -0.127696E+01 & \(0.126527 E+01\) & \(-0.4532128+02\) \\
\hline \(0.504257 \mathrm{E}+00\) & \(-0.323648 E+01\) & \(0.187037 \mathrm{E}+01\) & \(-0.171203 E+02\) \\
\hline 0.217426 E +00 & \(-0.6301745+01\) & \(0.3259585+01\) & -0.195370E+01 \\
\hline 0.351920 E-01 & \(-0.880502 E+01\) & 0.442011 E 01 & \(0.498789 E+01\) \\
\hline -0.127372F+00 & \(-0.978867 E+01\) & \(0.483040 \mathrm{E}+01\) & \(0.812018 \mathrm{E}+01\) \\
\hline -0.207855E+00 & \(-0.913800 \mathrm{E}+01\) & \(0.446507 E+01\) & \(0.855166 \pm \div 01\) \\
\hline -0.104914E+00 & \(-0.716631 \mathrm{E}+01\) & \(0.353070 E+01\) & \(0.619545 \mathrm{E}+01\) \\
\hline -0.147884E-01 & \(-0.410275 E+01\) & \(0.2047985+01\) & \(0.10454845+01\) \\
\hline \(0.124473 E+00\) & \(-0.686508 E+00\) & \(0.555490 \mathrm{E}+00\) & -0.553066E+02 \\
\hline \(0.114590 E+01\) & -0.222828E+00 & \(0.684364 \mathrm{E}+00\) & -0.420617E+02 \\
\hline
\end{tabular}
(20)
\begin{tabular}{llll}
\(0.699577 E+00\) & \(-0.212557 E+01\) & \(0.141263 E+01\) & \(-0.967801 E+01\) \\
\(0.574027 E+00\) & \(-0.590493 E+01\) & \(0.278948 E+01\) & \(0.139119 E+01\) \\
\(0.445091 E+00\) & \(-0.773233 E+01\) & \(0.409246 E+01\) & \(0.616367 E+01\) \\
\(0.345125 E+00\) & \(-0.933681 E+01\) & \(0.484097 E+0.1\) & \(0.833424 E+01\) \\
\(0.327960 E+00\) & \(-0.935569 E+01\) & \(0.484683 E+01\) & \(0.851708 E+01\) \\
\(0.449907 E+00\) & \(-0.792587 E+01\) & \(0.4183395+01\) & \(0.655736 E+01\) \\
\(0.796842 E+00\) & \(-0.529800 E+01\) & \(0.304742 E+01\) & \(0.533925 E+00\) \\
\(0.172483 E+01\) & \(-0.403484 E+00\) & \(0.106418 E+01\) & \(-0.186555 E+02\)
\end{tabular}


APPENDIX C
CONTOUR PLOTS

ジロッ


15.0日18 HORIZONTAL IN UNITS PER IHCH
45.0018 UERTICAL IN UWITS PER IMCH
ROTATIONI \(2 \quad 0.0 \vee 0.0 \times 0.0\)
(1)
FIGure C. 1 CONTOUR: MAJOR PRINCIPAL MOMENT FOR CENTRAL 10 KIP LOAD, SKEW \(=0^{\circ}\)


\begin{tabular}{l}
61.3048 HORIZONYAL IN UNITS PER IHCH \\
\hline \(61.304 G\) UERTICAL IN UNITS PER IMCH
\end{tabular}

.

FIGURE C. 2 CONTOUR: MAJOR PRINCIPAL MOMENT FOR CENTRAL 10 KIP LOAD, SKEW \(=40^{\circ}\)


FIGURE C. 3 CONTOUR: MAJOR PRINCIPAL MOMENT FOR PEAK SU-4 CENTER LOAD, SKEW \(=0^{\circ}\)


61.3046 HORIZONTAL IN UNITB PER IMCH
61.3046 UERTICAL IN UNITS PER IMCH
ROTATIONI 2 O.O \(\vee>.0 \times\) INCH
\(x\)

figure c. 4 Contour: Major principal monent for peak su-4 center load, skew \(=40^{\circ}\)

FIGURE C. 5 CONTOUR: Z DISPLACEMENT FOR CENTRAL 10 KIP LOAD, SKEW \(=0^{\circ}\)


figure c. 6 contour: Z displacenent for central 10 Kip Load, sken \(=40^{\circ}\)

nctron

Figure c. 8 CONTOUR: \(z\) displacement for peak su-4 Center load, skew \(=40^{\circ}\)

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I I


\(+10-1=-2\)
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2```


[^0]:    1 The scope environment is a characteristic operating domain in GTSTRUDL which allows for interactive graphics at the terminal CRT.

