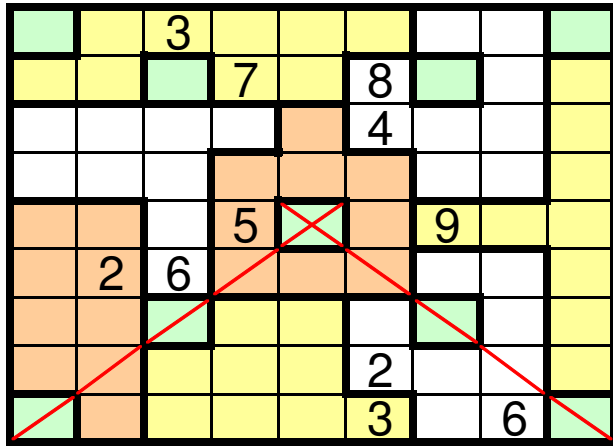


Proof of the Unique Solution of the V Pandemonion submitted to Wikipedia

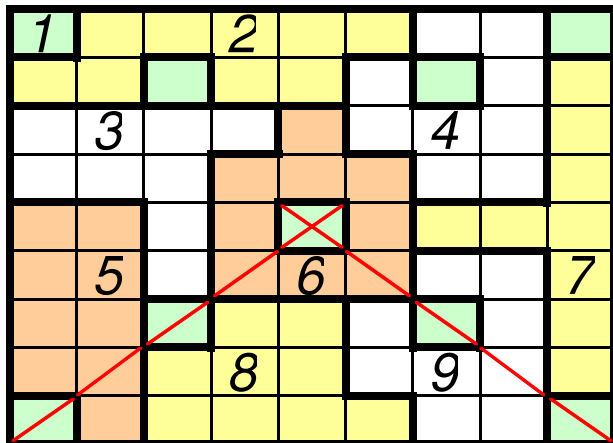
The Puzzle



The Rules

- 1 Every row, column and cluster must contain the numbers 1 to 9.
- 2 The green cluster is in nine parts. It must also contain the numbers 1 to 9.
- 3 The numbers 1 to 9 must appear on the red, V, line.

Identifying the clusters



Giving each cluster a number will make the task of explanation much easier.

The Proof

Entry Number 1

	A	B	C	D	E	F	G	H	I
1			3						
2				7		8			
3						4			
4									
5				5	8		9		
6		2	6						
7									
8						2			
9						3		6	

Note that the numbers located at cells F2 and F3 in cluster 4 and at F7 and F8 (cluster 9) must be replicated in some order in the green (cluster 1) cells at I1, G2, G7 and I9. If not, it would be impossible to place any of these numbers in all of columns G, H and I. So, 3 at F9 blocks not only green cell A9 but also cells I1, G2, G7 and I9. 3 at C1 also blocks green cells A1, C2 and C7, which leaves only cluster 1 cell E5 to 3.

Entry Number 2

	A	B	C	D	E	F	G	H	I
1			3				7		
2				7		8			
3						4	1		
4							6		
5				5	8		9		
6		2	6						
7									
8		8				2			
9						3		6	

6 at H9 forces a 6 in cluster 4 into one of cells G1, G3 or G4 which means that 6 cannot occupy G7 on the V line. 6 at H9 also blocks A9 and I9 on the V line. In addition, 6 at C6 blocks C7, D6, and F6. So, the only cell available to 6 on the V line is B8.

Entry Number 3

	A	B	C	D	E	F	G	H	I
1	6		3						
2				7		8			
3						4			
4									
5				5	8		9		
6		2	6						
7									
8		6				2			
9						3		6	

As noted at entry number 1, the numbers located at F2, F3, F7 and F8 must be replicated in some order at cluster-1 cells I1, G2, G7 and I9. Therefore, 6 at H9 excludes all those cluster-1 cells as well as A9. Moreover, 6 at C6 excludes cluster-1 cells C2 and C7. So, the only green (cluster-1) cell available to 6 is A1.

Entry Number 4

	A	B	C	D	E	F	G	H	I
1	6		3						
2				7	6	8			
3						4			
4									
5				5	8		9		
6		2	6						
7									
8		6				2			
9						3		6	

The only cell available to 6 in cluster 2 is E2.

Entry Number 5

	A	B	C	D	E	F	G	H	I
1	6		3						
2				7	6	8			
3						4			
4									
5				5	8		9		
6		2	6						
7						9			
8		6				2			
9						3		6	

Although we don't know at this stage we can deduce with 100% certainty that a 9 must occupy either I1 or I9. So, the numbers that will eventually occupy G2, G7, I1 and I9 (in whatever order) are 2, 4, 8 and 9. This means that, in accordance with the terms of entry number 1, 9 must occupy the last remaining cell of the cells F2, F3, F7 and F8, i.e. F8.

Entry Number 6

	A	B	C	D	E	F	G	H	I
1	6		3			5			
2				7	6	8			
3						4			
4									
5				5	8		9		
6		2	6						
7						9			
8		6				2			
9						3		6	

5 at D5 blocks F4, F5 and F6 which means that the only cell available to 5 in column F is F1.

Three important observations

	A	B	C	D	E	F	G	H	I
1	6		3			5			
2			15	7	6	8			
3						4			
4						167			
5				5	8	167	9		
6		2	6			7			
7			157			9			
8		6				2			
9	157					3		6	

First, the numbers 1, 6 and 7 must occupy F4, F5 and F6 in some order, thus excluding them from the rest of cluster 6. Second, the numbers left available to cluster-1 cells C2, C7 and A9 are 1, 5 and 7. Third, the numbers 1, 5 and 7 must occupy the V line in some order in cells F6, C7 and A9, thus excluding them from the other four cells on the V line. This means that 2, 4, 8 and 9 must occupy cells D6, G7, H8 and I9.

Entry Number 7

	A	B	C	D	E	F	G	H	I
1	6	8	3	8	8	5			
2				7	6	8			
3						4			
4									
5				5	8		9		
6		2	6						
7						9	8		
8		6				2		4	
9						3		6	8

2 and 9 cannot occupy cell H8 as they already occupy the same cluster. As can be readily-seen 8 cannot occupy G8; neither can 8 occupy I1 as in doing so it would exclude 8 from cluster 2 where it must sit on any of cells B1, D1 or E1. So, on the V line 8 must occupy either G7 or I9 which leaves on 4 to occupy H8.

Entry Number 8

	A	B	C	D	E	F	G	H	I
1	6		3			5			
2				7	6	8			
3						4			
4									
5				5	8		9		
6		2	6	9					
7						9			
8		6				2		4	
9						3		6	

Apart from 1,5 and 7 which have already been located on the V line in some order in cells F6, C7 and A9, only 2, 8 and are available to the V line. As established at entry number 7, 8 must occupy G7 or I9 thus excluding it from D6. Moreover, 2 at B6 blocks D6, leaving only 9 to occupy D6.

Entry Number 9

	A	B	C	D	E	F	G	H	I
1	6		3			5			9
2				7	6	8	4		
3						4			
4									
5				5	8		9		
6		2	6	9					
7						9			
8		6				2		4	
9						3		6	

As 2 and 8 must occupy G7 and I9 in some order 4 and 9 must occupy I1 and G2. 9 at G5 blocks G2. So, 9 must occupy I1, leaving 4 to occupy G2.

Entry Number 10

	A	B	C	D	E	F	G	H	I
1	6		3			5			9
2				7	6	8	4		
3					28	4			
4				3	248167				
5				5	8	167	9		
6		2	6	9	4817				
7						9			
8		6				2		4	
9						3		6	

3s at E5 and F9 force 3 into cell D4 in cluster 6. (This means that 2, 4 and 8 are the only numbers still available to the column-F cells in cluster 6.)

Entry Number 11

	A	B	C	D	E	F	G	H	I
1	6		3		1	5			9
2				7	6	8	4		
3					2 8	4			
4				3	2 4 8				
5				5	8		9		
6		2	6	9	4 8				
7					5 7	9			
8		6			5 7 9	2		4	
9					5 7 9	3		6	

1, 5, 7 and 9 are the numbers still available in column (other than 2, 4 and 8 in cluster 6). As 5 at F1, 7 at D2 and 9 at I1 block E1, 1 is the only number available to this cell. This means that 5, 7 and 9 must occupy cells E7, E8 and E9 in cluster 8.

Entry Number 12

	A	B	C	D	E	F	G	H	I
1	6		3		1	5			9
2		9		7	6	8	4		
3						4			
4				3					
5				5	8		9		
6		2	6	9					
7						9			
8		6				2		4	
9						3		6	

Look around!, all numbers other than 9 block B2.

Entry Number 13

	A	B	C	D	E	F	G	H	I
1	6		3		1	5			9
2	2	9		7	6	8	4		
3						4			
4				3					
5				5	8		9		
6		2	6	9					
7						9			
8		6				2		4	
9						3		6	

Now all but 2 are blocked from A2.

Entry Number 14

	A	B	C	D	E	F	G	H	I
1	6		3		1	5			9
2	2	9		7	6	8	4		
3						4			
4				3					
5				5	8		9		
6		2	6	9					
7				6		9			
8		6				2		4	
9						3		6	

6s at E2, B8 and H9 force 6 in cluster 8 into cell D7.

Entry Number 15

	A	B	C	D	E	F	G	H	I
1	6		3		1	5			9
2	2	9		7	6	8	4		
3						4			
4				3					
5				5	8		9		
6		2	6	9					
7				6	57	9			
8		6	18	18	579	2		4	
9			24	24	579	3		6	8

Cluster 8 is an interesting cell. As established at entry number 11, 5, 7 and 9 must occupy cells E7, E8 and E9. This leaves 1, 2, 4 and 8 to the other four cells in cluster 8. 2 and 4 at F8 and H8 force 2 and 4 into C9 and D9, leaving 1 and 8 to complete cluster 8. As 2 must occupy either C9 or D9 it blocks I9 which until now was known to contain either 2 or 8. So, 2 is excluded from I9, leaving only 8.

Entry Number 16

	A	B	C	D	E	F	G	H	I
1	6		3		1	5			9
2	2	9		7	6	8	4		
3						4			
4				3					
5				5	8		9		
6		2	6	9					
7				6		9	2		
8		6				2		4	
9						3		6	8

In consequence of entry number 16, 2 must occupy cell G7.

Entry Number 17

	A	B	C	D	E	F	G	H	I
1	6		3		1	5			9
2	2	9		7	6	8	4		
3						4			
4				3					
5				5	8		9	8	
6		2	6	9					
7				6		9	2		
8		6				2		4	
9						3		6	8

H5 is the only cell available to 8 in cluster 7.

Entry Number 18

	A	B	C	D	E	F	G	H	I
1	6		3		1	5	7	2	9
2	2	9		7	6	8	4		
3						4			
4				3					
5				5	8		9	8	
6		2	6	9					
7				6		9	2		
8		6				2		4	
9						3		6	8

2 and 7 at A2 and D7 force 2 and 7 into cells G1 and H1. 2 at G7 forces 2 into H1, leaving 7 to G1.

Entry Number 19

	A	B	C	D	E	F	G	H	I
1	6		3		1	5	7	2	9
2	2	9		7	6	8	4		
3						4			
4				3					
5				5	8		9	8	
6		2	6	9					
7				6		9	2		
8		6				2		4	
9			24	24	9	3		6	8

At entry number 15 it was established that 2 and 4 must occupy cells C9 and D9 in some order. With those two cells already excluded 9 is blocked from all cells in row 9 other than E9.

Entry Number 20

	A	B	C	D	E	F	G	H	I
1	6		3		1	5	7	2	9
2	2	9		7	6	8	4		
3						4			
4				3					
5				5	8		9	8	
6		2	6	9					
7				6		9	2		
8	9	6				2		4	
9					9	3		6	8

A8 is the only cell available to 9 in cluster 5 and in row 8.

Entry Number 21

	A	B	C	D	E	F	G	H	I
1	6		3		1	5	7	2	9
2	2	9		7	6	8	4		
3						4			
4				3					
5				5	8		9	8	
6		2	6	9			8		
7				6		9	2		
8	9	6	18	18		2		4	
9					9	3		6	8

As established at entry number 15, 1 and 8 must occupy C8 and D8 in some order. So, the only cell available to 8 in cluster 9 is G6.

Entry Number 22

	A	B	C	D	E	F	G	H	I
1	6		3		1	5	7	2	9
2	2	9		7	6	8	4		
3						4			
4				3					
5				5	8		9	8	
6		2	6	9	4		8		
7				6		9	2		
8	9	6				2		4	
9					9	3		6	8

As established at entry number 11, 2, 4 and 8 must occupy E3, E4 and E6 in some order. 2 at C6 and 8 at G6 block E6, leaving the cell available only to 4.

Entry Number 23

	A	B	C	D	E	F	G	H	I
1	6		3		1	5	7	2	9
2	2	9		7	6	8	4	3	
3	3	3				4			
4				3					
5				5	8		9	8	
6		2	6	9	4		8		
7				6		9	2		
8	9	6				2		4	
9					9	3		6	8

3s at C1 and D3 force 3 in cluster 3 into either A3 or B3. Whichever cell 3 occupies doesn't matter for the moment; suffice to know that it and 3 at D4 force 3 in cluster 4 into cell H2.

Entry Number 24

	A	B	C	D	E	F	G	H	I
1	6		3		1	5	7	2	9
2	2	9		7	6	8	4	3	
3						4			
4				3					
5				5	8		9	8	
6		2	6	9	4		8		
7				6		9	2		
8	9	6				2	3	4	
9					9	3		6	8

G8 is the only cell available to 3 in cluster 9 and in column G.

Entry Number 25

	A	B	C	D	E	F	G	H	I
1	6		3		1	5	7	2	9
2	2	9		7	6	8	4	3	
3						4			
4				3					
5				5	8		9	8	
6		2	6	9	4	1	8		
7				6		9	2		
8	9	6				2	3	4	
9					9	3		6	8

We have already established that 1, 5 and 7 must occupy cluster-1 cells C2, C7 and A9 in some order. At present the combinations are: C2 (1 or 5), C7 (1, 5 or 7), A9 (1, 5 or 7). We have also established that cell F6 on the V line must contain either 1 or 7. Now, if we place a 7 in F6 there would be no cell available to 7 among the remaining green, cluster-1, cells. Therefore, the only number that can occupy F6 is 1.

Entry Number 26

	A	B	C	D	E	F	G	H	I
1	6		3		1	5	7	2	9
2	2	9	1	7	6	8	4	3	
3						4			
4				3					
5				5	8		9	8	
6		2	6	9	4	1	8		
7				6		9	2		
8	9	6				2	3	4	
9					9	3		6	8

A result of entry number 25 is that 5 and 7 must share cluster-1 cells C7 and A9, leaving 1 to cell C2.

Entry Number 27

	A	B	C	D	E	F	G	H	I
1	6		3		1	5	7	2	9
2	2	9	1	7	6	8	4	3	5
3						4			
4				3					
5				5	8		9	8	
6		2	6	9	4	1	8		
7				6		9	2		
8	9	6				2	3	4	
9					9	3		6	8

5 must now complete row 2 at I2.

Entry Number 28

	A	B	C	D	E	F	G	H	I
1	6		3		1	5	7	2	9
2	2	9	1	7	6	8	4	3	5
3						4			
4				3					
5				5	8		9	8	
6		2	6	9	4	1	8		
7				6		9	2		
8	9	6	8	1		2	3	4	
9					9	3		6	8

As established at entry 15, 1 and 8 must occupy cells C8 and D8. 1 at C2 blocks C8, forcing 1 into D8, leaving 8 to C8.

Entry Number 29

	A	B	C	D	E	F	G	H	I
1	6		3		1	5	7	2	9
2	2	9	1	7	6	8	4	3	5
3						4			
4				3					
5				5	8		9	8	
6		2	6	9	4	1	8		
7				6		9	2		
8	9	6	8	1	5	2	3	4	7
9					9	3		6	8

The only cell available to 5 in row 8 is E8, leaving 7 at I8, to complete row 8.

Entry Number 30

	A	B	C	D	E	F	G	H	I
1	6		3		1	5	7	2	9
2	2	9	1	7	6	8	4	3	5
3						4			
4				3					
5				5	8		9	8	
6		2	6	9	4	1	8		
7				6	7	9	2		
8	9	6	8	1	5	2	3	4	7
9					9	3		6	8

We have already established that 2 and 4 must occupy C9 and D9. So, the only cell available to 7 in cluster 8 is E7.

Entry Number 31

	A	B	C	D	E	F	G	H	I
1	6		3		1	5	7	2	9
2	2	9	1	7	6	8	4	3	5
3						4			
4				3					
5				5	8		9	8	
6		2	6	9	4	1	8		
7			5	6	7	9	2		
8	9	6	8	1	5	2	3	4	7
9	7				9	3		6	8

7 at E7 forces 7 to A9, leaving 5 to C7.

Entry Number 32

	A	B	C	D	E	F	G	H	I
1	6		3		1	5	7	2	9
2	2	9	1	7	6	8	4	3	5
3						4			
4				3					
5				5	8		9	8	
6		2	6	9	4	1	8	7	
7			5	6	7	9	2		
8	9	6	8	1	5	2	3	4	7
9	7				9	3		6	8

H6 is the only cell available to 7 in cluster 9.

Entry Number 33

	A	B	C	D	E	F	G	H	I
1	6		3		1	5	7	2	9
2	2	9	1	7	6	8	4	3	5
3						4			
4				3					
5				5	8		9	8	
6	5	2	6	9	4	1	8	7	3
7			5	6	7	9	2		
8	9	6	8	1	5	2	3	4	7
9	7				9	3		6	8

A6 is the only cell available to 5 in row 6, leaving 3 to complete the row at I9.

Entry Number 34

	A	B	C	D	E	F	G	H	I
1	6		3		1	5	7	2	9
2	2	9	1	7	6	8	4	3	5
3						4			
4				3					
5				5	8		9	8	
6	5	2	6	9	4	1	8	7	3
7			5	6	7	9	2	1	
8	9	6	8	1	5	2	3	4	7
9	7	1	24	24	9	3	5	6	8

G9 is the only cell available to 5 in row 9, leaving 1 to B9. (We have already established 2 and 4 at C9 and D9.) Then, 5 at G9 forces the only remaining number in cluster - 1 - into cell H7.

Entry Number 35

	A	B	C	D	E	F	G	H	I
1	6		3		1	5	7	2	9
2	2	9	1	7	6	8	4	3	5
3						4			
4				3					
5				5	8		9	8	
6	5	2	6	9	4	1	8	7	3
7	3	8	5	6	7	9	2	1	4
8	9	6	8	1	5	2	3	4	7
9	7	1			9	3	5	6	8

3 and 8 block cluster-5 cells A5 and B5, forcing them into A7 and B7. So, 4 is forced to occupy I7.

Entry Number 36

	A	B	C	D	E	F	G	H	I
1	6		3		1	5	7	2	9
2	2	9	1	7	6	8	4	3	5
3						4			
4				3					
5	4	7		5	8		9	8	
6	5	2	6	9	4	1	8	7	3
7	3	8	5	6	7	9	2	1	4
8	9	6	8	1	5	2	3	4	7
9	7	1			9	3	5	6	8

With 3 and 8 occupying A7 and B7, 7 at A9 forces 7 in cluster 5 into cell B5, leaving 4 to A5.

Entry Number 37

	A	B	C	D	E	F	G	H	I
1	6		3		1	5	7	2	9
2	2	9	1	7	6	8	4	3	5
3						4			
4				3		7			
5	4	7		5	8	6	9	8	
6	5	2	6	9	4	1	8	7	3
7			5	6	7	9	2	1	4
8	9	6	8	1	5	2	3	4	7
9	7	1			9	3	5	6	8

7 at B5 forces 7 in column F into F4, leaving 6 to F5.

Entry Number 38

	A	B	C	D	E	F	G	H	I
1	6		3		1	5	7	2	9
2	2	9	1	7	6	8	4	3	5
3						4			
4				3		7			
5	4	7	2	5	8	6	9	8	1
6	5	2	6	9	4	1	8	7	3
7			5	6	7	9	2	1	4
8	9	6	8	1	5	2	3	4	7
9	7	1			9	3	5	6	8

Entry Number 39

	A	B	C	D	E	F	G	H	I
1	6		3		1	5	7	2	9
2	2	9	1	7	6	8	4	3	5
3						4			
4				3		7			
5	4	7	2	5	8	6	9	8	1
6	5	2	6	9	4	1	8	7	3
7			5	6	7	9	2	1	4
8	9	6	8	1	5	2	3	4	7
9	7	1	4	2	9	3	5	6	8

2 at C5 forces 2 into cell D9, leaving 4 at C9, to complete cluster 8 and row 9.

Entry Number 40

	A	B	C	D	E	F	G	H	I
1	6		3	4	1	5	7	2	9
2	2	9	1	7	6	8	4	3	5
3				8		4			
4				3		7			
5	4	7	2	5	8	6	9	8	1
6	5	2	6	9	4	1	8	7	3
7			5	6	7	9	2	1	4
8	9	6	8	1	5	2	3	4	7
9	7	1	4	2	9	3	5	6	8

4 at F3 forces 4 into cell D1 in column D, leaving 8 to complete column D at D3.

Entry Number 41

	A	B	C	D	E	F	G	H	I
1	6		3	4	1	5	7	2	9
2	2	9	1	7	6	8	4	3	5
3				8	2	4			
4				3	8	7			
5	4	7	2	5	8	6	9	8	1
6	5	2	6	9	4	1	8	7	3
7			5	6	7	9	2	1	4
8	9	6	8	1	5	2	3	4	7
9	7	1	4	2	9	3	5	6	8

Thanks to 8 at D3, 8 and 2 complete cluster 6 and column E in the order shown

Entry Number 42

	A	B	C	D	E	F	G	H	I
1	6		3	4	1	5	7	2	9
2	2	9	1	7	6	8	4	3	5
3			7	8	2	4			
4				3	8	7			
5	4	7	2	5	8	6	9	8	1
6	5	2	6	9	4	1	8	7	3
7			5	6	7	9	2	1	4
8	9	6	8	1	5	2	3	4	7
9	7	1	4	2	9	3	5	6	8

Look around!, the only cell available to 7 in cluster 3 is C3.

Entry Number 43

	A	B	C	D	E	F	G	H	I
1	6		3	4	1	5	7	2	9
2	2	9	1	7	6	8	4	3	5
3			7	8	2	4			6
4				3	8	7			2
5	4	7	2	5	8	6	9	8	1
6	5	2	6	9	4	1	8	7	3
7			5	6	7	9	2	1	4
8	9	6	8	1	5	2	3	4	7
9	7	1	4	2	9	3	5	6	8

2 at E3 forces 2 into cell I4, leaving 6 to complete cluster 7 and column I at I3.

Entry Number 44

	A	B	C	D	E	F	G	H	I
1	6	8	3	4	1	5	7	2	9
2	2	9	1	7	6	8	4	3	5
3			7	8	2	4			6
4				3	8	7			2
5	4	7	2	5	8	6	9	8	1
6	5	2	6	9	4	1	8	7	3
7			5	6	7	9	2	1	4
8	9	6	8	1	5	2	3	4	7
9	7	1	4	2	9	3	5	6	8

8 completes row 1 at B2.

Entry Number 45

	A	B	C	D	E	F	G	H	I
1	6	8	3	4	1	5	7	2	9
2	2	9	1	7	6	8	4	3	5
3	3	5	7	8	2	4	1	9	6
4	1	4	9	3	8	7	6	5	2
5	4	7	2	5	8	6	9	8	1
6	5	2	6	9	4	1	8	7	3
7	8	3	5	6	7	9	2	1	4
8	9	6	8	1	5	2	3	4	7
9	7	1	4	2	9	3	5	6	8

We now have more than enough entries to complete the puzzle without controversy.

The completed V Pandemonion

	A	B	C	D	E	F	G	H	I
1	6	8	3	4	1	5	7	2	9
2	2	9	1	7	6	8	4	3	5
3	3	5	7	8	2	4	1	9	6
4	1	4	9	3	8	7	6	5	2
5	4	7	2	5	8	6	9	8	1
6	5	2	6	9	4	1	8	7	3
7	8	3	5	6	7	9	2	1	4
8	9	6	8	1	5	2	3	4	7
9	7	1	4	2	9	3	5	6	8