ANALYSIS OF DATA DESCRIBING CONGRESSIONAL RESPONSES TO DOD BUDGET REQUESTS

James Robert Capra

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Analysis of Data Describing Congressional Responses to DOD Budget Requests		5. TYPE OF REPORT & PERIOD COVERED Ph.D. Thesis; June 1974
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) James Robert Capra		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		12. REPORT DATE June 1974
		13. NUMBER OF PAGES 300
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Naval Postgraduate School Monterey, California 93940		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Congressional Appropriations
Defense Appropriations
Robust Estimation
Jackknifing

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This thesis represents an attempt to characterize Congressional appropriations for Defense procurement and research and development between 1953 and 1973. The approach used involves formulating alternative models of appropriations as a percentage of requests and deriving point and interval estimates for parameters using robust statistical procedures. The results of the analysis lend support to the hypothesis that the response

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
(Page 1) S/N 0102-014-6601

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

(20. ABSTRACT continued)

of the Congress to Defense procurement and research and development budget requests has changed considerably, starting in fiscal 1969 and that the Air Force received uniquely favorable action on its research and development budget requests in the 1957-1969 time period .

The first chapter explores the institutional setting of Defense budgeting, from the budget prepreparation stage through the audit stage. In Chapter II, alternative models are formulated for appropriations as a percentage of requests, parameters are estimated using least squares, and residuals are analyzed. The following two chapters explore the advantages of robust alternatives to least squares and include robust point and interval estimates for the parameters of the models of Chapter II. The final chapter includes (i) an examination, using Monte Carlo techniques, of the operating characteristics of the statistical procedures employed in earlier chapters and (ii) a discussion of the results of dividing the data by groups of years and by service.

Analysis of Data Describing
Congressional Responses
to DOD Budget Requests

by

James Robert Capra
Lieutenant, United States Navy
B.A., Georgetown University, 1968
M.S., Naval Postgraduate School, 1970

Submitted in partial fulfillment of the
requirements for the degree of

DOCTOR OF PHILOSOPHY

from the

NAVAL POSTGRADUATE SCHOOL
June 1974

ABSTRACT

This thesis represents an attempt to characterize Congressional appropriations for Defense procurement and research and development between 1953 and 1973. The approach used involves formulating alternative models of appropriations as a percentage of requests and deriving point and interval estimates for parameters using robust statistical procedures. The results of the analysis lend support to the hypothesis that the response of the Congress to Defense procurement and research and development budget requests has changed considerably, starting in fiscal 1969 and that the Air Force received uniquely favorable action on its research and development budget requests in the 1957-1969 time period.

The first chapter explores the institutional setting of Defense budgeting, from the budget prepreparation stage through the audit stage. In Chapter II, alternative models are formulated for appropriations as a percentage of requests, parameters are estimated using least squares, and residuals are analyzed. The following two chapters explore the advantages of robust alternatives to least squares and include robust point and interval estimates for the parameters of the models of Chapter II. The final chapter includes (i) an examination, using Monte Carlo techniques, of the operating characteristics of the statistical procedures employed in earlier chapters and (ii) a discussion of the results of dividing the data by groups of years and by service.

TABLE OF CONTENTS

ACKNOWLEDGMENT

I would like to gratefully acknowledge, first and foremost, he day in and day out assistance of my advisor, Professor Donald . Gaver. Without his guidance this dissertation would certainly ot have been possible. I would also like to thank my dissertation ommittee who devoted valuable time to reading and commenting on ough drafts, especially Professors Carl R. Jones and Donald Barr.

This dissertation is dedicated to my wife Ellen. Not only id she type all the rough drafts, but also never failed to ncourage me to pursue the analysis to completion.

"The budget process. . . is primarily a system of communications, regularized and cyclical. Its purposes fall into two logical categories: first, the bringing of information to the proper level for the making of decisions — a category in governmental policies, programs and objectives which we may roughly classify as policy; and, second, the providing of information both upward and downward so that those decisions will be carried out — a category we may roughly classify as administrative."*

Frederick Mosher

The topic of this thesis is the regularized and cyclical system of communications which forms the Department of Defense budget process. At various points in the budget cycle, decisions or budget outcomes are recorded for both policy and administrative reasons. Some of the data in these records will be analyzed in order to draw inferences about the process of decision-making on the defense budgets. Because budget data is basically quantitative data, the analyses that will be undertaken will use quantitative techniques to formulate and test hypotheses concerning the defense budget process. In addition to attempting to

---

*Frederick Mosher, Program Budgeting: Theory and Practice, Chicago, Public Administration Service, 1954, p. 5.

characterize decision-making in defense budget making, this thesis will try to show the potentialities of quantitative analysis of public budgeting systems.

Most analyses of federal budgeting have focused on one or two parts of the federal budget cycle. The program budget literature has mainly addressed the budget preparation stage, while either ignoring or treating in a cursory fashion the Congressional review stage [Hitch,1965, Hitch and McKean, 1960, Novick,1965]. Quantitative analysis of budget preparation has been confined to the formulation and testing of hypotheses based on information about this stage alone. The literature on Congressional review of budget requests has focused on interactions within the Congress and between Congressional Committees and agencies [Fenno,1966, Wildavsky,1964, Horn,1970]. The budget preparation stage and budget execution stage have not been considered in this literature, except to a certain extent by [Wildavsky,1964]. Quantitative analysis of Congressional budget behavior has been dominated by those who have tried to operationalize some of the Fenno and Wildavsky statements concerning Committee-agency interactions. [Davis, Dempster and Wildavsky, 1966,1967,1971, Johnson,1972, Davis and Guillot,1967].

The analysis contained in succeeding chapters focuses on the Congressional budget review stage of the budget process. More specifically, it deals with Congressional review of Department of Defense budget requests for aircraft, missiles and research and development. In contrast to the work of Davis, Dempster, Wildavsky and others, this study concentrates

on just one agency rather than many, and uses recently developed statistical techniques to analyze budget data for the years 1953-1973.

The somewhat different focus of this study is the result of a number of developments that have taken place following the publication of the first papers of Davis, et al. First, a recent study by James Jernberg [Jernberg,1972] shows that different Congressional appropriations subcommittees have different interests and different approaches to budget review. Some are interested in budget components such as personnel and travel expenditures, while others are interested in outputs or program objectives. Instead of undertaking the ambitious task of characterizing all Congressional budget review activity, this analysis concentrates solely on such activity within the Department of Defense. Second, a recent and noteworthy development is the objection by Johnson [1967] , Kanter [1972] , and others [Natchez and Bupp,1973] , to the effect that the studies by Davis et al. have been done at too high a level of aggregation. The models Davis et al. propose are for total agency budget requests and total agency appropriations. For Defense budgets, Kanter and others [Stromberg,1970] have suggested conducting analysis at a lower level of aggregation such as Navy RDTE (Research, Development, Test and Evaluation). It is at this lower level that our study is conducted. Finally, a methodological development that has gathered momentum since the publication of the first budgeting articles by Davis et al. is that of the theory and techniques of robust estimation [Andrews et al, 1973]. These are

particularly well-suited to the objectives of our analysis of defense budget data, which is based on the belief that Congressional behavior since the early Fifties demonstrates the characteristics of a succession of regular, stable processes mixed with a few somewhat unusual events. The data analysis problem is to separate the two, and to prevent the unusual events from contaminating or distorting the estimates of the characteristics of the regular process.

The analysis to be presented omits one aspect of the budget process which forms a part of the Davis, Dempster, Wildavsky studies. Little attempt is made to structure detailed mechanistic models of both the process of agency budget request formulation, or of Congressional review. One reason for this omission is that in the Department of Defense, the process of formulation of budgets has changed considerably since the Fifties with the implementation of planning, programming and budgeting (PPB) and various revisions to the PPB system. The avoidance of the detailed process of budget request formulation could conceivably result in simultaneous equation problems [Johnston, Ch. 13 and 14, 1963]. However for the simple models used in this analysis, single equation methods have been shown to be acceptable [Wold, 1953, Johnston, 1963, pp 377-380]. Although the theory of robust estimation has not yet been extended to simultaneous equation estimation, we do fairly extensive Monte Carlo investigations of efficiency and mis-specification problems. Our results reflect favorably upon the operating characteristics of the robust statistical estimation procedures put to use.

The chapters which follow combine (a) an analysis of substantive questions which arise from trying to characterize Congressional activity in the area of defense budgeting with (b) a detailed consideration of methodological questions which arise when trying to apply various statistical estimation procedures to data like that on defense appropriations. Chapter I is an overview of the defense budget process from budget prepreparation to budget execution. The purpose of the chapter is to place in perspective the part of the process which is being studied in this analysis and for the first time to gather into one package a narrative description of the processes and institutions of defense budgeting as it has been and is currently conducted. Chapter II reviews the Davis, Dempster, Wildavsky work and proposes some alternative approaches and models for Congressional appropriations. Estimates for coefficients or parameters in these models are studied. In Chapter III the methodological literature on robust estimation is reviewed, and new estimates for the models are derived. Confidence intervals for the estimates are presented in detail in Chapter IV, together with a discussion of the methods for constructing the estimates. Chapter V employs Monte Carlo techniques in order to gain insight into the validity of the techniques used in earlier chapters. The result of the Monte Carlo studies in Chapter V also point to some important substantive conclusions about which of the models best characterizes what will be termed the <u>regular part</u> of the defense budget process, and for which years these models are most appropriate.

CHAPTER I

Descriptions of the federal budget process in the United States can focus either on a sequence of events or on the interactions among the major institutions in the political system. One can first view the process as an annual sequence of events which includes budget prepreparation, budget preparation, budget submission and Office of Management and Budget (OMB) review, budget submission and Congressional review, budget execution, audit. Each of these steps or events, however, represents activities and possibly interactions of three major political instituions: agency, President and Congress. When studying the process, the analyst can either focus on the events and attempt to examine the respective roles of the institutions at each step, or he can focus on the institutions and their interactions, with the budgetary sequence of events providing the setting. The discussion which follows is concerned with budgeting in the defense sector. It will be organized, as depicted in Figure I.1, from the sequential point of view, describing in succession budget prepreparation, budget preparation, OMB review, Congressional review, budget execution and audit. However, the reader should bear in mind

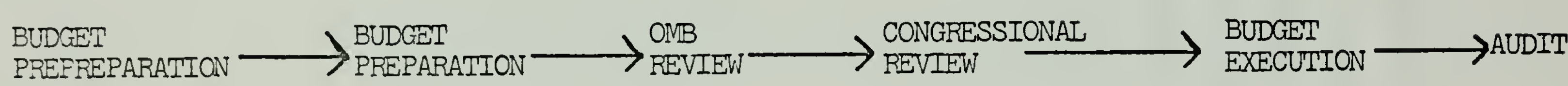


FIGURE I.1. Budgetary sequence of events for any given fiscal year's budget.

that at each of these stages all of the three major actors or institutions have some direct or indirect impact on the activities which take place.

As was stated in the introduction, the purpose of this chapter is to convey a picture of the defense budget process as a whole so that when a selected portion of the process is analyzed in detail in succeeding chapters, the reader will have some idea of how the analysis fits into a broad view of the process. A secondary purpose of this chapter is to bring together in one place current information and literature on defense budgeting.

I.1. Budget Prepreparation

Program Budgeting

In the Department of Defense, budget prepreparation occurs during the 18-month Planning, Programming and Budgeting cycle. Planning, programming and budgeting is an attempt to tie planned resource usage to objectives or goals [Schick, p. 33]. In simple terms, it is an attempt to budget in terms of outputs instead of inputs. For example, program budget decision-makers who are considering a health-care budget will be concerned with the budget division between health care for the elderly and child health care rather than the budget division between travel expenses (for all programs), rents, and maintenance. Budget review in terms of travel, rents, maintenance, etc. is generally considered to be budgeting in terms of objects of expenditure [Schick,1972,pp.20-21, Johnson,1972,

p. 2]. According to Allen Schick, program budgets are associated with a planning orientation, that is, the budget is a forward planning device, while object budgets have a control orientation, with the budget used as a method of insuring honesty and integrity [Schick,1972, pp. 20-23 and pp. 30-36].

When Robert McNamara became Secretary of Defense in 1961, he and the new Comptroller, Charles Hitch, committed the Department of Defense (DOD) to the basic principles of program budgeting: multiyear budgets cast in terms of outputs [Hitch, 1965,p.27, Ruefli,1971,pp. 166-169]. The Defense budget was divided among nine major programs or missions:

Strategic Retaliatory Forces
Continental Defense
General Purpose Forces
Airlift/Sealift
Reserve and National Guard
Research and Development
General Support
Retired Pay
Military Assistance

The force levels and dollars allocated to the above programs for the next five years were called the Five Year Defense Program. Within each program or mission were program elements which were supposed to contribute to the objectives or output of the program (See Table I.1). In theory, the program elements are supposed to be substitutes for each other, such as B-52's, land based missiles, Polaris submarine-launched missiles within the Strategic Retaliatory Forces program. Other considerations involved in the decision of where to place program elements are discussed by Smithies [Smithies,1965,

TABLE I.1

FIVE YEAR DEFENSE PROGRAM

Outline of Program Structure
(with selected examples of program elements)

1. STRATEGIC FORCES
   - 11XXX. Offensive Forces
     - 11113F: B-52 Squadrons
     - 11213F: Minuteman Squadrons
     - 11221N: Fleet Ballistic Missile System
   - 12XXX. Defensive Forces
     - 12114F: F-106 Squadrons
     - 12214A: Sentinal System
     - 12427N: SPASUR
   - 13XXX. Civil Defense
     - 13111C: Shelter Survey

2. GENERAL PURPOSE FORCES
   - 21XXX. Unified Commands
     - 211140: PACOM
   - 22XXX. Forces (Army)
     - 22113A: Infantry Divisions
     - 22122A: Mechanized Brigades
     - 22222A: Helicopter Companies (Medium)
     - 22233A: Hawk Battalions
     - 22313A: Construction Engineering Battalions
   - 23XXX. Other Support (Army)
     - 23196A: Base Operations (Europe)
     - 23296A: Base Operations (Pacific)
     - 23613A: Redeye (Operational Systems Development)
   - 24XXX. Forces (Navy)
     - 24114N: A-6 Squadrons
     - 24141N: Attack Carriers
     - 24221N: P-3 Squadrons
     - 24231N: ASW Carriers
     - 24311N: Submarines
     - 24411N: Fleet Escort (Major)
     - 24514N: Coastal/River Patrol and Assault Forces
     - 24611N: Mine Countermeasures Ships
     - 24711N: Underway Replenishment Ships

25XXX. Fleet Support
- 25196N: Base Operations
- 25613N: Phoenix Missile System (Operational Systems Devel.)

26XXX. Fleet Marine Forces
- 26122M: CH-53 Squadrons
- 26211M: Divisions
- 26311M: Force Troops (Combat Support)
- 26496M: Base Operations
- 26612M: Helicopter Avionics (Operational Systems Devel.)

27XXX. Forces (Air Force)
- 27129F: F-111 Squadrons
- 27213F: RF-4 Squadrons
- 27241F: Special Air Warfare Forces (SAWF)
- 27311F: MACE
- 27412F: Tactical Air Control System
- 27596F: Base Operations

28XXX. Other
- 280110: JCS Directed and Coordinated Exercises
- 28015N: Deepfreeze

INTELLIGENCE AND COMMUNICATIONS

31XXX. Intelligence and Security
- 310110: Cryptologic Activities
- 310130: Defense Attaché System

32XXX. National Military Command System
- 32011F: National Military Command Center

33XXX. Communications
- 33111A: STARCOM
- 33115K: DCA Satellite Project

34XXX. Special Activities
- 342110: National Activities

35XXX. Activities (Other)
- 35110F: Satellite Control Facility
- 35112N: Oceanography
- 35120A: Combat Development Activities

pp. 34-43]. The system of categories, including major programs and program elements, is called a program structure.

Ideally, a program structure is supposed to assign activities (program elements) to mutually exclusive programs. For optimal allocation of resources within a program (presuming an absence of problems like indivisibilities), it is necessary that the marginal productivity of a resource allocated to a program element equal the marginal productivity of that resource with respect to every other program element within the program. For optimal resource allocation between programs which yield different outputs the marginal utility of a resource allocated to one program must equal the marginal utility of the resource with respect to every other program.

Program Budget Cycle

Currently, the program budget cycle is an 18-month cycle conducted in three phases and resulting in the President's annual budget submission to the Congress in January. The first phase involves military threat and requirement evaluation. In the second phase, multiyear programs to meet the threat are derived. These programs and the dollars allocated to them are listed in the Five Year Defense Program. The third phase of the cycle is the development of the annual (next year's) budget.*

---

*The rationale behind the three phases is discussed by Hitch in [Hitch, pp. 28-39, 1965].

Since its inception, each phase of the program budget cycle has included extensive discussion and negotiation between the Office of the Secretary of Defense (OSD) and the military services. The general approach in each phase has been the issuance of some kind of guidance or constraints by OSD, such as fiscal or strategy guidance; next the services respond with proposals; OSD approves, disapproves or modifies the proposals; finally, some kind of agreement is reached before the next phase is entered [Kanter and Anger,1973]. In the McNamara era (1961-1969), OSD appeared to be controlling the process [Enke,1963, Crecine,1971]. Under Melvin Laird and David Packard the role of the military services has apparently been emphasized and broadened* [Kanter and Anger,1973,pp. 6-10].

In order to pass from the second phase of the program budget cycle, programming, into the third phase, budgeting, or in terms of this analysis to pass from budget prepreparation to budget preparation, it is necessary to translate the program budget into a different system of accounts. The Congress does not appropriate funds specifically for the program elements in the Five Year Defense Program. Rather, they consider the inputs or resources which produce the program element force levels. The budget submitted to the

*It is difficult to yet assess the differences between PPB under Laird and PPB under Secretary James Schlesinger.

Congress is an input-oriented budget (although it is not called an object budget). The general categories or <u>line items</u> are:*

Military Personnel (MPN)
Operations and Maintenance (OMN)
Procurement (PAMN, OPN)
Research and Development (RDTE)
Military Construction (MILCON)

These categories are further subdivided into budget activities such as Permanent Change of Station Travel (Navy).

Since the budget enacted by the Congress is in terms of dollar allocations for resources or inputs and the program budget is in terms of dollar allocations for outputs, it is necessary each year to translate or <u>crosswalk</u> the program budget into a resource budget.

<u>Crosswalking</u>

Crosswalking is a term applied to the transformation of the program budget into a line item resource budget, or vice versa. Translation from a program budget into a line item budget should be rather straightforward, as the following discussion will demonstrate.

A program element within the Navy Tactical Air Program or Mission is program element 2-41-22-N, F-14 squadrons. The approved budgets (hypothetical) for F-14 squadrons for FY 72 through FY 76, as contained in the FYDP, are (in

---

*The actual budget submission to the Congress contains estimates for Army, Navy, Air Force, Marine Corps and Defense Agencies in most of these accounts and the Procurement account is further divided among aircraft, missiles, ships, torpedoes, combat tracked vehicles and other procurement.

millions of dollars): 100, 150, 444.2, 398.027, 240.172. The following table shows how these dollars are used in terms of the line item categories:

	72	73	74	75	76
MPN				.872	4.290
OMN				6.326	14.079
PAMN*			413.5	310.00	217.69
OPN*				.717	4.113
RDTE	100	150	30.7	150.072	
Total:	100	150	444.2	467.987	240.172

*PAMN stands for Procurement Air and Missile Systems Navy,
OPN stands for Other Procurement Navy,
RDTE stands for Research, Development, Test and Evaluation,
OMN stands for Operations and Maintenance, Navy
MPN stands for Military Personnel, Navy

**The above numbers are hypothetical numbers only.

TABLE I.1a

These hypothetical budgets will supposedly yield eleven squadrons in FY75 and forty-four in FY76. The information contained in the table is called Program Element Summary Data. To obtain the Navy line item budget in RDTE for FY72 it is only necessary to sum over all the program elements the RDTE dollars they require. The same is true for the other line item categories.

Crosswalking from line item categories to program categories is not quite as straightforward, since it requires allocation of an overall budget figure for an input such as

Military Personnel or RDTE among all program elements. The problems involved in crosswalking from line item categories to program categories have been addressed in [Crecine and Fischer, 1971, Ruefli, 1971].

If we aggregate dollars for program elements, such as the F-14, into dollars for the major programs of which these elements form a part, then a crosswalked budget for the defense department might look something like Table I.2.

I.2. Budget Preparation

The Process

Phase III of the program budgeting cycle involves the preparation of the annual budget. As discussed in the previous section, the budget which will be sent to the Congress is in terms of line items rather than programs. Budget estimates are submitted by the services to the Secretary of Defense on or about October 1 of each year. These estimates are then sent to the Assistant Secretary of Defense (Comptroller), ASD(C), who in turn divides the budget review among directorates for military personnel, operations and maintenance, procurement, military construction, research and development [Crecine and Fischer, 1971, pp. 46-50]. The Department of Defense holds its own budget hearings during the budget preparation stage [Navy Programming Manual, pp. IV-5]. As will be pointed out later, since 1961 OMB (or until 1969 the Bureau of the Budget) review of defense budgets has consisted of participation in these hearings, but their influence has been limited [Halperin, 1972, p. 317].

	FORCES	FORCES	COMMS	SEALIFT	FORCES	DEVELOPMENT	MAINT	ACTIVITIES	ACTIVITIES	TO OTHER NATIONS	UNDISTRIB ADJUST	TOTAL
MILITARY PERSONNEL-ARMY	86,844	4,125,755	401,513	6,072	46,713	65,777	113,039	3,429,773	129,353	302,311		8,586,953
MILITARY PERSONNEL-NAVY	134,289	2,362,416	231,750	5,153	115,953	83,758	105,424	1,550,943	94,874	39,257		4,711,318
MILITARY PERSONNEL-MARINE CORPS	762	800,745	13,935		30,316	579	19,193	534,661	44,330	16,622		1,461,643
MILITARY PERSONNEL-AIR FORCE	1,396,306	1,795,822	776,366	403,421	19,569	201,022	66,960	1,621,558	207,127	13,774		6,524,017
RESERVE PERSONNEL-ARMY					344,462			25,164				369,626
RESERVE PERSONNEL-NAVY					146,932			9,051				155,983
RESERVE PERSONNEL-MARINE CORPS					51,850			1,822				53,672
RESERVE PERSONNEL-AIR FORCE					67,700			8,951				95,651
NATIONAL GUARD PERSONNEL-ARMY					442,634							442,634
NATIONAL GUARD PERSONNEL-AIR FORCE					122,586							122,586
TOTAL MILITARY PERSONNEL	1,600,261	9,084,738	1,423,566	414,646	1,408,215	330,136	329,415	7,189,953	476,184	371,964		22,525,079
RETIRED PAY-DEFENSE								3,388,999				3,388,999
OPERATION AND MAINTENANCE-ARMY	90,548	1,957,517	385,015		141,109		2,174,742	1,245,838	337,250	470,009		6,824,028
OPERATION AND MAINTENANCE-NAVY	254,150	1,571,406	277,648		109,906		1,912,174	530,002	204,332	85,936		4,945,264
OPERATION AND MAINTENANCE-MAR CORPS	6	199,167	786		6,601		119,823	42,336	26,234	6,485		403,329
OPERATION AND MAINTENANCE-AIR FORCE	1,017,458	1,064,488	519,091	193,639	155,872		2,343,706	780,057	187,134	147,017		6,325,434
OPERATION AND MAINTENANCE-DEF AGCYS		1,599	411,819				663,483	11,684	103,518			1,191,223
OPERATION AND MAINTENANCE-ARNG					314,956							314,955
OPERATION AND MAINTENANCE-ANG					368,689							368,689
NATL BRD FOR PROMOTION RIFLE PRAC									82			82
CLAIMS-DEFENSE									51,607			51,637
CONTINGENCIES-DEFENSE									297			297
COURT OF MILITARY APPEALS-DEFENSE									768			720
TOTAL OPERATION AND MAINTENANCE	1,362,172	4,794,169	1,593,759	193,639	1,099,133		7,215,848	2,529,887	911,714	709,367		20,409,688
PROC OF EQUIPMENT AND MISSILES-ARMY	647,592	1,449,555	66,196	752	60,185		276,842	71,968	1,556	918,957		3,473,613
PROC OF AIRCRAFT AND MISSILES-NAVY	515,298	2,597,605					29,496	199,298	1,134	33		3,341,054
SHIPBUILDING AND CONVERSION-NAVY	322,800	1,987,870	11,049				18,070					2,339,759
OTHER PROCUREMENT-NAVY	163,275	1,087,220	69,470		863		224,811	69,282	27,812	19,389		1,552,122
PROCUREMENT-MARINE CORPS		196,680	25		28,023		19,207	3,615	2	2,091		241,843
AIRCRAFT PROCUREMENT-AIR FORCE	346,429	1,542,059	632,280	639,283	52,766		31,374	93,826	22,256	172,400		3,532,673
M[illegible] F PROCUREMENT-AIR FORCE	885,100	39,560	533,240				4,808					1,462,700
[illegible] PROCUREMENT-AIR FORCE	217,662	628,732	443,074	12,245	1,211		120,010	-5,008		187,552		1,543,494
[illegible]MENT-DEFENSE AGENCIES			37,798				1,716	111	108			39,727
TOTAL PROCUREMENT	3,098,156	9,529,481	1,793,132	652,280	135,048		734,336	443,100	52,560	1,220,432		17,655,825
R, D, T AND E-ARMY	364,000	214,896	19,735			1,056,434			583			1,655,649
R, D, T AND E-NAVY	139,722	494,537	104,585			1,459,945						2,195,789
R, D, T AND E-AIR FORCE	447,845	67,059	303,602	11,600		1,976,333				10,708		2,817,339
R, D, T AND E-DEFENSE AGENCIES		13,820	98,949			336,740						451,509
TOTAL R, D, T AND E	951,567	790,312	527,071	11,600		4,831,452			583	10,700		7,123,255
MILITARY CONSTRUCTION-ARMY	258,200	51,055	9,742			28,595	15,624	113,356	41,702	76,811		595,035
MILITARY CONSTRUCTION-NAVY		92,918	7,757			16,973	66,718	72,207	37,940			294,513
MILITARY CONSTRUCTION-AIR FORCE	43,346	35,762	6,444	31,144		14,028	31,963	64,333	32,887			259,937
MILITARY CONSTRUCTION-DEFENSE AGCYS			2,607				6,102		1,420			13,129
MILITARY CONSTRUCTION-ARMY RESERVE					10,000							13,030
MILITARY CONSTRUCTION-NAVY RESERVE					5,000							5,030
MILITARY CONSTRUCTION-AIR FORCE RES					5,501							5,531
MILITARY CONSTRUCTION-ARMY NATL GRD					16,005							16,035
MILITARY CONSTRUCTION-AIR NATL GRD					18,539							18,539
TOTAL MILITARY CONSTRUCTION	301,546	179,735	26,558	31,144	47,045	59,596	120,407	249,896	113,949	76,811		1,235,679
FAMILY HOUSING-DEFENSE								714,319				714,319
HOMEOWNERS ASSISTANCE FUND - DEFENSE								8,684				8,684
RESEARCH, SHELTER SURVEY, MARKING	22,000											22,000
OPERATION AND MAINTENANCE-CIVIL DEF	50,776											50,776
CONSTRUCTION OF FACILITIES CIV DEF	498											498
SPECIAL FOREIGN CURRENCY									7,888			7,885
TOTAL MILITARY FUNCTIONS	7,386,976	24,378,435	5,364,078	1,303,309	2,689,441	5,221,184	8,396,006	14,524,838	1,563,178	2,389,274		73,215,719
MILITARY ASSISTANCE										786,012		785,012
FOREIGN MILITARY CREDIT SALES										199,362		199,362
FOREIGN MILITARY CREDIT SALES-ISRAEL										500,000		500,000
TOTAL MILITARY ASSISTANCE PROGRAM										1,467,374		1,467,374
GRAND TOTAL	7,386,974	24,378,435	5,364,078	1,303,309	2,687,441	5,221,184	8,396,006	14,524,838	1,563,178	3,876,648		74,704,093
[illegible]RTMENT OF THE ARMY	1,427,164	7,798,778	882,201	6,824	1,373,764	1,150,336	2,583,[illegible]	4,886,099	510,526	1,768,098		22,385,627
[illegible]MENT OF THE NAVY	1,530,312	11,390,764	717,205	5,153	469,444	1,540,755	2,514,[illegible]	3,821,187	437,559	169,733		21,517,124
[illegible]RTMENT OF THE AIR FORCE	4,358,206	5,173,474	3,214,299	1,291,332	524,433	2,191,333	2,628,[illegible]	3,493,755	449,404	431,443		23,372,550
DEFENSE AGENCIES, OSD AND JCS		15,419	550,373			338,740	871,[illegible]	11,795	105,336			1,892,589
DEFENSE-WIDE								4,112,002	60,552			4,172,550
DEFENSE CIVIL PREPAREDNESS AGENCY	73,274											73,274
MILITARY ASSISTANCE PROGRAM										1,467,374		1,467,374

In theory, budget preparation should be a relatively simple process which merely requires crosswalking the costs of approved programs for the next year into line item categories. In reality, the budget preparation stage has not proved to be that simple. Especially during the McNamara era, many decisions were made during this stage [Crecine and Fischer,1971].

<u>Budget Preparation Under McNamara</u>

The discussion in this section emphasizes the fact that budget preparation in the McNamara era (FY1962 - FY1970) was not merely a straightforward crosswalking of approved programs into line items. It is largely based on the analysis of John Crecine and Gregory Fischer in their paper "On the Resource Allocation Process in the Department of Defense." [Crecine and Fischer,1971]

McNamara program budgeting was based on a requirements approach. The idea was that requirements were to be set and then programs developed which satisfied the requirements at the minimum cost. As Secretary McNamara himself said

> "The President's charge to me was a two-pronged one — to determine what forces were required and to procure and support them as economically as possible." [Tucker, 1966, p. 14]

> "We start with the political objective, the formulation of which is presented to us by the Secretary of State and upon which the President indicates his desires that we develop a military program that will support the political objective. As you know, the President has stated the defense budget is to be established without regard to arbitrary budget ceilings. We determine the force levels which we believe are necessary to support the political objective and then act to

fulfill the President's second direction to us. He has indicated that we are to attain the specific force levels necessary to support the political objective at the lowest cost." [Tucker, 1966,p. 27]

Because of this philosophy, the services were never given explicit fiscal or budgetary constraints in the McNamara era. However, regardless of budget philosophy, the defense budget plan must satisfy the following identity, called the "Great Identity" by Crecine and Fischer:

$$\begin{array}{c}\text{Defense Expenditures} \\ + \\ \text{Non-Defense Expenditures}\end{array} = \text{Tax Revenues} + \text{Deficit}$$

If none of the other three terms are controlled by the Secretary of Defense, then, like it or not, the Defense Department must operate under some kind of budget constraint. The research of Crecine and Fischer is largely based on this observation.

As Crecine and Fischer point out, tax revenues (estimated) depend on the state of the economy, as estimated by the Treasury Department and the Council of Economic Advisors, and on the tax laws, changes to which must be approved by the Congress in general and the House Ways and Means Committee in particular. Non-defense expenditures are, of course, proposed by the agencies involved and reviewed by OMB. Another factor related to non-defense expenditures is their uncontrollability. Many government expenditures, such as Social Security and welfare payments, are made according to

fixed formulas and require basic changes in legislation if they are to be changed. Others, like interest on the national debt, must be paid, regardless of what else is in the budget. According to the Tax Foundation, Inc., in 1969 over $118 billion in outlays were relatively uncontrollable. Of this total, $90 billion were outlays in non-defense areas [Tax Foundation, 1973, p. 10]. The other term in the equation is the deficit. The President, together with the Council of Economic Advisors, OMB and the Treasury Department yearly set a deficit target which they believe will achieve the appropriate balance between government expenditure requirements and stability of the economy — low unemployment and low inflation. Violation of this target will generally be viewed as involving economic penalties.

During the McNamara era, the approved set of programs in the FYDP always implied expenditures higher than would be feasible under the "Great Identity" [Crecine and Fischer, 1971, p.37, Anger, 1973, p.6]. Consequently, after receiving the service budget requests, the directorates in the office of ASD(C) would submit planning estimates to McNamara which stated how much they believed could be cut from the budget requests. By this time (mid-October), McNamara had a reasonably good idea what the other terms of the Great Identity were going to be and could tell the directorates either to cut more or less than the planning estimates. In the meantime, he negotiated for deficit increases and for non-defense expenditure decreases.

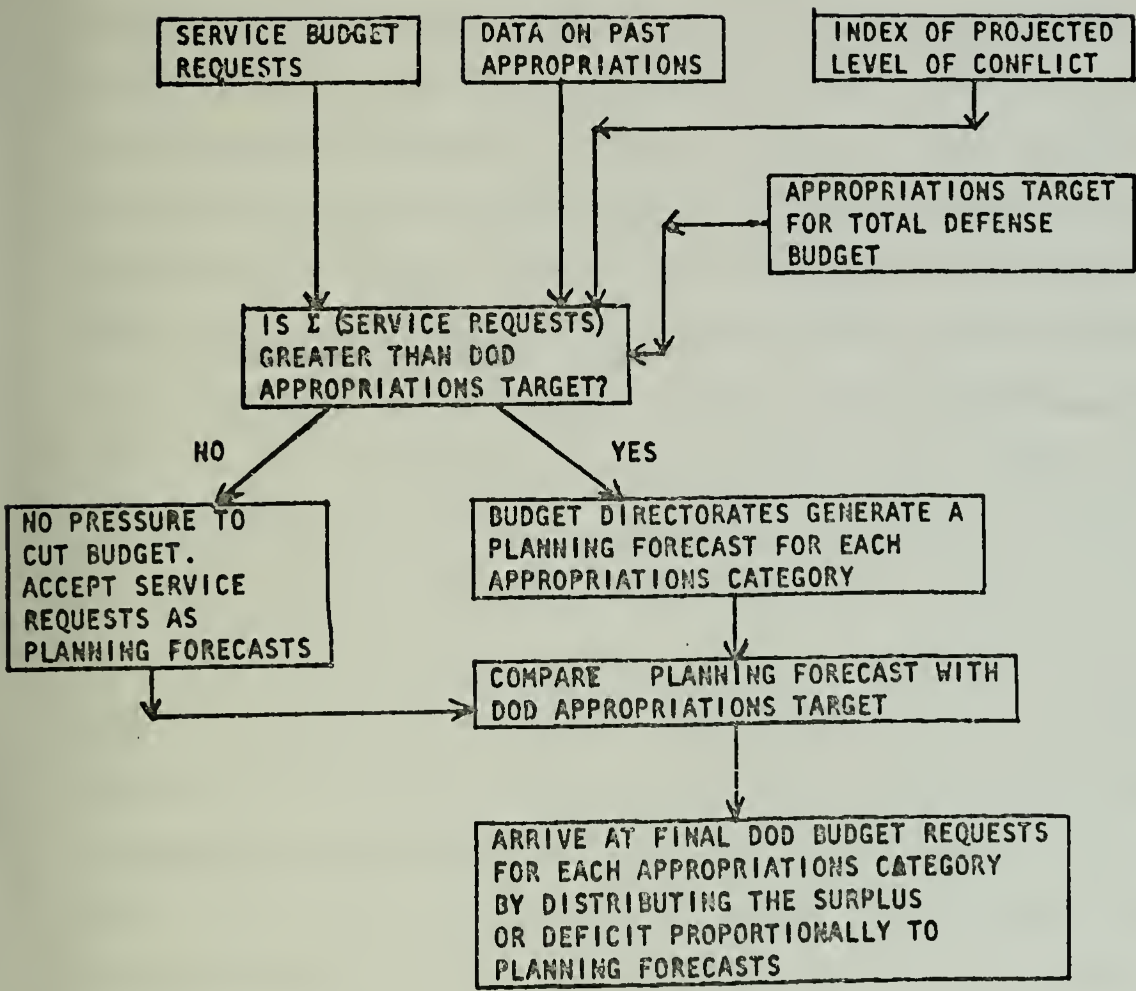
One wonders how programs requiring resources in more than one category were pieced back together after the directorates completed their review. According to Crecine and Fischer, analysts from the following offices participated in putting the programs back together in November and December: Assistant Secretary of Defense for Systems Analysis, Assistant Secretary of Defense for Installations and Logistics, Assistant Secretary of Defense, Comptroller [Crecine and Fischer,1971,p.56]. Figure I.2 is a copy of Crecine and Fischer's flow diagram of this process.

The record of budget cuts at the Secretary of Defense level in the McNamara era is consistent with the view that significant changes were made in the budget at this point in the process. The following table gives the percentage reduction in service budget estimates prior to submission to Congress for 1962-1970:

1962	8.07
1963	9.78
1964	19.70
1965	28.09
1966	20.10
1967	21.59
1968	22.91
1969	24.09
1970	24.69

TABLE I.3

Total O.S.D. Reductions in Service Budget Estimates
Source: Office of Assistant Secretary of Defense (Comptroller).



:.2 MODEL OF OSD (COMPTROLLER'S) BUDGET REVIEW

Crecine and Fischer suspected that because the approved program (FYDP) would not necessarily be included in the budget, and because the services had been submitting budgets in line item categories for years, simple linear models might explain or predict service budget requests in each of the line item categories. These models included explanatory variables like the previous year's appropriation, the difference in the last two years' appropriations, the difference between previous year's request and appropriation, and dummy variables for administration in power and whether or not the nation was at war. In most cases the models explained well over 95% of the variance in service budget requests from 1969 back to the Forties. Although analysis of this kind should be applied and used with care,* their results appear to indicate that regardless of budget philosophy at the Secretary of Defense level, the service budget requests have a remarkable stability, being related to some rather straightforward and uncomplicated variables. Crecine and Fischer also formulated simple linear models for directorate cuts. The results were similar to those for service requests.

In summary, the total analysis of Crecine and Fischer leads us to suspect that budget preparation in the McNamara era was something other than a simple crosswalking of approved programs into line items.

---

*Methodological problems with regression analysis of this type for Congressional appropriations will be discussed in Chapter II.

Budget Preparation Under Laird

When Melvin Laird became Secretary of Defense, he and his Deputy Secretary, David Packard, retained a three phase program budget cycle. However, they made some basic changes in emphasis. The Laird system has largely changed the approach from one which focuses on requirements. A five year budget constraint is set early in the programming phase, the second phase of the cycle [Kanter,1973,p.8]. In addition, the military services were given more of a say in the allocation of resources to programs [Anger,1973,p.10]. Laird and his successors have communicated the 5-year budget constraint, by year, to the services via either a document called the Fiscal Guidance Memorandum (issued in the Spring of 1970) or Defense Planning and Programming Guidance Memorandum (issued in the late winter or early spring of 1971, 1972, and 1973). The first budget submission which reflected this new system was the FY 1972 budget. Table I.3 on the next page gives an example of the budget or fiscal constraint given to the services. Note that the dollar constraints are given by program or mission, such as Land Forces or Tactical Air Forces; the service then puts together a proposal specifying how its Land Forces' dollars will be divided among the program elements under Land Forces. The set of service proposals is called the Program Objectives Memorandum (POM). The OSD responses or decisions on issues raised in the POM's are called the Program Decision Memoranda (PDM).

TABLE I.3

FISCAL GUIDANCE

Fiscal Guidance categories are divided into three groups. Group I, called "Major Mission," is made up of the FYDP program elements containing DoD combat forces and certain special support activities such as Research and Development (FYDP Program 6), Communications, Intelligence, and Support to Other Nations. Group II, called "General Support", contains elements from all Defense programs which provide mission and central support to Group I programs. Group III contains miscellaneous costs which are a function of management policies not directly related to the forces or their support.

*****

Fiscal Guidance Categories and Services Concerned

Categories:	Program Elements in: Army	Navy	Marine Corps	Air Force
I. Major Mission				
A. Force Mission				
1. Strategic Forces:				
a. Offensive		X		X
b. Defensive	X	X		X
c. Control & Surveillance	X	X		X
2. Land Forces	X		X	
3. Tactical Air Forces		X	X	X
4. Naval Forces:				
a. ASW & Fleet Air Defense		X		
b. Amphibious		X		
c. Support		X		
5. Mobility Forces	X	X	X	X
B. Other Mission				
1. Intelligence & Security	X	X		X
2. Communications	X	X		X
3. R&D (program 6)	X	X	X	X
4. Support to Other Nations:				
a&b. Military Assist.	X	X	X	X
c. Allies W.R. Stocks	X	X		X
II. General Support				
A. Base & Individual Support	X	X	X	X
B. Training	X	X	X	X
C. Command	X	X	X	X
D. Logistics	X	X	X	X
III. Miscellaneous Costs (retired pay, family housing, Military construction)	X	X	X	X

Miscellaneous costs are defined by particular Resource Identification Codes (RICs) drawn from certain program elements. The TOA identified by these RICs are deleted from the Program Elements in which originally appearing and recategorized by RIC.

Under the Laird system, by the time the services submit their budgets in October, they have been specifying programs subject to a budget constraint for over seven months. Consequently, one might suspect that OSD cuts of the service budget requests, following the October submission, would be minimal. This turns out to be the case. In FY1972, the cut was .13 per cent and in FY1973 it was 2.06 per cent. By comparison, the average cut in the FY1962-FY1971 time period was 18.8% with the lowest figure being 8% in FY1962. These figures show that under the Laird system, major budget reductions have not been made during the budget preparation stage. Also, major program decisions have not been delayed until November and December; rather, most decisions have been communicated to the services by August. These two factors, the small budget cuts and the issuance of PDM's in August, appear to indicate that budget preparation under Laird has for the most part become the crosswalking of the budget from program categories into line item accounts discussed earlier.

Having completed the discussion of the program budget cycle, and the budget prepreparation and preparation stages, it might be useful to summarize the chronological sequence of events. The 18 month program budget cycle for FY1974 began in June, 1971 with threat analysis and long range planning. The programming phase began in February of 1972 and continued through the spring and summer of that year. On October 1, the Services submitted their budget estimates to OSD. This budgeting phase (i.e., budget preparation) continued through

December of 1972. Finally, in January of 1973 the President transmitted his FY1974 budget estimate to the Congress. Figure I.3 depicts this sequence of events in general and Figure I.4 depicts the events in detail.

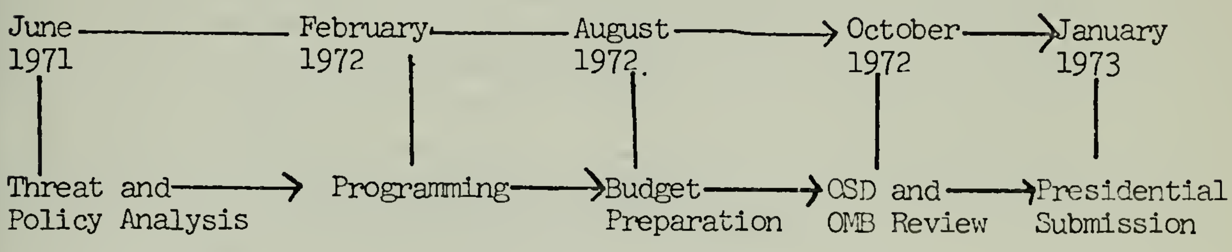


FIGURE I.3

I.3. OMB Review

The Budget and Accounting Act of 1921 created the Bureau of the Budget (later re-named the Office of Management and Budget by President Richard Nixon) and made the executive budget part of the federal budgetary process. The Act provided for Presidential review of the budget and Presidential submission of the budget proposal to the Congress. One of the functions of the Bureau of the Budget was to assist the President in his annual review of agency budget requests. The Bureau's role as a mechanism for Presidential control became more clear when Franklin Roosevelt moved it from the Treasury Department into the Executive Office of the President in 1939 [Redford et al., 1965, p. 325].

FIGURE I.4

PROGRAM/BUDGET REVIEW SCHEDULE
CALENDAR YEAR 1972

ITEM	ACTION	AGENCY	ACTION DATE
1	Submit JSOP-Vol I (74-81) Strategy and Force Planning	J	Jun 23, 1971
2	Provide comments on JSDP-Vol 1 to JCS	O	Aug 4, 1971
3	Issue Defense Policy and Planning Guidance	O	Oct 23, 1971
4	Identify and issue Selected Analysis Topics	O	Nov 1, 1971
5	Issue Materiel Support Planning Guidance	O	Dec 7, 1971
6	Submit comments on Defense Policy and Planning Guidance	JC	Dec 7, 1971
7	Submit JSOP-Vol 11 (FY 74-81) Analysis and Force Tabulations	J	Dec 23, 1971
8	Submit comments on Materiel Support Planning Guidance	JC	Dec 30, 1971
9	Update Five Year Defense Program through FY 1973	C	Jan 10, 1972
10	Update Five Year Defense Program for FY 1974-1977	C	Jan 21, 1972
11	Issue PDM Guidance	O	Jan 31, 1972
12	Issue Planning and Programming Guidance Memorandum Note: Includes (1) Force Planning (2) Fiscal Levels (3) SEA Assumptions and (4) Materiel Support Planning Guidance	D	Feb 22, 1972
13	Submit Joint Research and Development Objectives Document (JRDOD)	J	Feb 22, 1972
14	Provide selected analysis	JC	Prior to Mar 31, 1972
15	Submit Joint Force Memorandum (JFM)	J	May 8, 1972
16	Submit Program Objectives Memorandum (POM)	C	May 22, 1972
17	Submit JSOP-Vol 1 (75-82) Strategy and Force Planning (CY 1973 cycle)	J	Jun 1, 1972
18	Issue initial Budget Guidance for preparation of FY 1974 budget estimates	D	Jun 15, 1972
19	Issue first "Issue Paper" (IP)	O	Jun 19, 1972
20	Transmit first "Issue Paper" (IP) to SecDef	O	Jun 23, 1972
21	Issue last "Issue Paper" (IP)	O	Jul 17, 1972
22	Transmit last "Issue Paper" (IP) to SecDef	D	Jul 21, 1972
23.	Issue Program Decision Memorandum (PDM)	O	Aug 4, 1972
24	Submit reclamas to PDMs	C	Aug 14, 1972
25	Issue reclama decisions on PDMs	D	Aug 24, 1972
26	Issue Defense Policy and Planning Guidance (CY 1973 cycle)	D	Sep 1, 1972
27	Identify and issue Selected Analysis Topics (CY 1973 cycle)	D	Sep 1, 1972
28	Issue Materiel Support Planning Guidance (CY 1973 cycle)	O	Sep 5, 1972
29	Submit annual Budget Estimates and backup information	C	Oct 2, 1972
30	Start Budget Hearings	O	Oct 9, 1972
31	Submit comments of Defense Policy and Planning Guidance	JC	Oct 13, 1972
32	Update Five Year Defense Program	C	Oct 13, 1972
33	Submit comments on Materiel Support Planning Guidance	JC	Oct 20, 1972
34	Start issue of Program/Budget Decisions (PBDs)	D	Nov 6, 1972
35	Provide comments (reclamas) on PBDs	C	Nov 13, 1972
36	Issue revised PBDs based on reclama comments	O	Dec 1, 1972 to Dec 18, 1972
37	Conduct joint meetings with JCS and Service Secretaries to discuss major unresolved budget issues	D	Dec 15, 1972
38	Submit JSDP-Vol 11 (FY 75-81) Analysis and Force Tabulations (CY 1973 cycle)	J	Dec 22, 1972

LEGEND:
D - SecDef
J - JCS
C - Military Departments and Defense Agencies
JC - JCS, Military Departments, Defense Agencies

Enclosure

Between the end of the Second World War and the advent of the Kennedy Administration, defense budget requests were treated much like other agency requests by the Bureau of the Budget [Korb,1969, Ch. 3]; that is, BoB conducted an independent review of the budget and recommended adjustments (usually cuts) to the President prior to Presidential submission to the Congress. The mechanics of this review are discussed in [Ott and Ott,1969,Ch.2]. Table I.4 contains BoB cuts from 1949 through 1960.

Fiscal	Army		Navy		Air Force	
Year	Amount	Percentage	Amount	Percentage	Amount	Percentage
1950	5.82	57	5.19	49	4.11	47
1951	0.17	4	0.22	5	0.80	17
1954	0.90	9	1.28	12	3.70	25
1955	1.77	17	1.65	14	1.32	10
1956	1.30	14	1.28	12	1.32	8
1957	0.44	5	1.68	15	2.26	13
1958	2.14	20	0.96	8	2.98	15
1959	0.58	6	0.06	1	1.32	7
1960	2.47	22	2.46	19	1.96	10
1961	0.73	7	0.23	2	2.64	14
(1950-1961)	16.52		15.01		22.11	
	(1.65)	(16.1)	(1.50)	(13.7)	(2.21)	(16.6)

TABLE I.4

Since the beginning of the McNamara years during the Kennedy Administration, BoB and OMB have not conducted an independent review of defense budget estimates.

"It should be noted that the budget of the Department of Defense is handled somewhat differently from those of other agencies. The Bureau of the Budget participates with the financial officers of the Defense Department in a review of the requests of the various services for budgetary allowances, but its role here is not quite the same as with other agencies. It acts more as an advisor to the Secretary of Defense than as an arbiter." [Ott and Ott, 1969, p. 23].

The current situation is discussed in the Navy Programming Manual,

"The analysts of OSD and OMB normally make a joint review of the budgets submitted by the military departments. However, OMB analysts have authority to submit separate decisions on the markups." [p. IV-5]

These "separate decisions" mentioned in the above paragraph apparently are not really decisions, since according to Morton Halperin,

"Under Eisenhower, Kennedy and Johnson it became a matter of tradition that the Budget Director would have to appeal Secretarial decisions on the Defense budget to the President, the reverse of the situation in all other departments." [Halperin,1972,p. 317]

Other literature conflicts with Halperin's view of the power of the Secretary of Defense's decisions under Eisenhower but is in agreement with his view of the Kennedy and Johnson years [Korb,1969, Ch. 3]. Currently, the Director of OMB sits on the Defense Program Review Committee, a committee of the National Security Council "whose purpose is to keep the annual defense budget in line with foreign policy objectives" [Leacocos,1971,p.7]. The influence of this committee is more important in the fiscal guidance stage of the PPB cycle than after budget preparation.

In summary, OMB review of defense budget estimates currently appears to be limited. The Director may play a part in setting the overall budget ceiling for the Department of Defense through his membership on the Defense Program Review Committee. However, this is quite different from the role which OMB plays in the budget review of non-defense agencies.

I.4. Congressional Review of Budget Estimates

Introduction

In most Western democratic countries, the power of the purse is held by the legislature. (One exception is West Germany) [Macridis and Ward,1963,p.352]. The Congressional appropriations process in the United States is probably more complex than the legislative appropriations process in any other country in the world [Macridis and Ward,1963]. The process involves numerous interactions in committees of both houses, floor debate in each house, and negotiation between the houses. Also, for most agencies, Congressional scrutiny occurs not only when appropriations are being considered, but also when authorizations are being examined. The purpose of this section is to discuss the basics of Congressional consideration of Defense budget requests.

Before discussing Congressional budget review in detail, the importance of the Congress in the overall defense budget process should probably be discussed briefly. Research on

Congressional appropriations for defense has been sparse. Recent notable exceptions are [Korb,1973, Kanter,1972, Goss,1970]. The classic studies of Fenno and of Wildavsky have explicitly been confined to non-defense areas as were the quantitative models of Davis, Dempster and Wildavsky. One of the reasons for this paucity of research might be the record of Congressional changes in defense budget requests between FY1954 and FY1968. The following table gives the percentage and magnitude of changes, cuts or increases, made by the Congress in that period.

Fiscal Year	%	Millions of $
1954	-3.9	-1089
1955	-3.6	-1063
1956	-1.1	- 350
1957	+1.5	+ 493
1958	-1.8	- 639
1959	+2.0	+ 933
1960	-0.1	- 24
1961	+1.7	+ 660
1962	+0.58	+ 268
1963	+0.48	+ 230
1964	-3.66	-1797
1965	-1.51	- 717
1966	-0.18	- 81
1967	+0.70	+ 403
1968	-2.30	-1638

TABLE I.5

Source: Office of Assistant Secretary of Defense (Comptroller)

By comparison, in the 1962-1968 time frame, the Secretary of Defense cut service budget requests by an average of 18.6% with the lowest cut being 8% in 1962. Note from the previous

table that Congressional cuts of Presidential budget requests in 1962-1968 never exceeded 3.66%.

One of the reasons for the increasing amount of research on Congress and defense appropriations is the record of Congressional cuts since FY 1969.

Fiscal Year	%	Millions of $
1969	-6.75	-5201
1970	-7.49	-5638
1971	-3.13	-2147
1972	-4.02	-2951
1973	-6.56	-5221

TABLE I.6

Source: Office of Assistant Secretary of Defense (Comptroller)

Richard Fenno in his analysis of non-defense appropriations has argued that Congressional changes of less than 5 percent are marginal or insignificant [Fenno,1966,p.353]. Since 1969, changes to the defense budget request were greater than 5 percent three times and in the other two years they were three and four percent respectively. It should also be noted at this point that in the 1962-1973 time frame, Congressional changes to procurement and RDTE requests were higher than the figures in Tables I.5 and I.6 [Korb,1973,p.16].

Important Terms

Three important terms that arise in any discussion of budgeting are authorization, appropriation, and outlay or expenditure. An authorization by the Congress gives approval of functions or activities of an agency. In other words, the Congress passes legislation authorizing or approving activities

such as defense research and development. For many agencies, the authorizing legislation specifies a maximum amount that can be appropriated [Ott and Ott,1972,pp.51-52] , and in essence says the agency can now seek appropriations for its approved programs. Theoretically, substantive issues such as busing or anti-busing provisions, program cancellations, and so forth should be settled in the authorization bill since the appropriations bill is supposed to "appropriate not legislate."* In reality, the line between appropriation and legislation is somewhat unclear, and consequently substantive issues are often addressed in appropriations bills [Harris,1964,p.87].

An appropriation is the authority to obligate or commit the government to certain expenditures. Appropriations are generally defined as new obligational authority (NOA). Obligational authority may be granted for one, two or some specified number of years, that is, the agency may have one, two or some specified number of years to obligate the appropriations. In some cases, an agency may have no year accounts, which are available for obligation until the purpose of the spending is accomplished. An agency's total obligational authority (TOA) includes not only NOA but also unobligated balances from prior years' appropriations. In the Department of Defense the following time limits are in effect for the

---

*Authorizations can in effect appropriate if they provide funds through "backdoor financing" [Fenno, 1966,p. 114]. However, this is definitely the exception rather than the rule and is almost unheard of in defense areas.

six major accounts:

MP	1 year
OM	1 year
Procurement (except SCN)	3 years
SCN**	5 years
RDTE	2 years
MILCON	2 years

**SCN refers to Shipbuilding and Conversion, Navy

TABLE I.7

Expenditures or outlays are defined as the payment of liabilities incurred by the government, or the actual cash flow.* Expenditures in a particular year for the Defense Department may be different from appropriations. For example, NOA in 1970 was $69.4 billion while outlays were $76.3 billion [Korb,1973,p.27]. The reason for this is that an obligation might be incurred when a contract is signed, but payment may not be completed until the item is delivered. Agencies generally have two years after the expiration of obligational authority to complete the financial transactions associated with that authority [31 U.S. Code, 701-706]. If an agency has obligated funds but not expended them, these funds are referred to as unspent obligations. The term unexpended balances refers to

---

*The Navy Programming Manual defines outlays as "the amount of funds that must be drawn from the Treasury for goods and services received within the fiscal year under review." (p. L-1)

the sum of unobligated balances and unspent obligations. In 1971, unexpended balances for the Department of Defense totalled $35.2 billion of which $11.5 billion was unobligated balances [House Report 92-1389]. Figure I.5 summarizes these relationships.

TOA = NOA + Unobligated balances

Unexpended Balances = Unobligated balances + unspent obligations.

Figure I.5

Although Congressional appropriations are defined as NOA, unobligated and unexpended balances can have indirect effects on Congressional budget review. For example, if an agency has unobligated balances for which the obligational authority is about to expire, the Congress may extend the obligational authority for one or more years but cut the NOA request by an amount equal to the amount of the extended obligational authority [House Report 92-666, p. 84]. Every year, the House Appropriations Committee includes the dollar figures for unexpended and unobligated balances in their report and occasionally comments on the magnitude. An extensive discussion of unobligated balances for RDTE is contained in the House Appropriations Committee report on the FY1969 Appropriations bill [House Report 90-349, pp. 51-52].

Because of uncertainty about just exactly when funds will be obligated, especially in procurement accounts, and when checks will be drawn, it is impossible to say for certain when

a certain amount of obligational authority will be expended. For planning purposes, the Assistant Secretary of Defense (Comptroller) has published the following table for relating Department of the Navy outlays to TOA during the 1972-1976 time span. The table is based on data for outlays and TOA for the past ten years. Suppose one wanted to compare the expenditure rate for military personnel with the expenditure rate for shipbuilding and conversion in the years 1972 and 1973. Based on the table, it was estimated that 99% of the 1972 TOA would be spent in 1972 while .90% would be spent in 1973. This picture is quite different from the picture in the slow expenditure account shipbuilding and conversion where it was estimated that 6% of the TOA would be spent in 1972 while 21% would be spent in 1973.

(As percent of estimated total payments)

Appropriation	In First Year	In Second Year	In Third Year	In Fourth Year	In Fifth Year
Military Personnel, Navy	99.00	.90	.10		
Military Personnel, Marine Corps	96.50	3.30	.20		
Reserve Personnel, Navy	85.00	14.50	.50		
Reserve Personnel, Marine Corps	92.00	7.00	1.00		
Operations & Maintenance, Navy	82.50	16.00	1.50		
Operations & Maintenance, Marine Corps	81.00	17.50	1.50		
Procurement of Aircraft & Missiles, Navy	16.00	53.00	22.00	7.00	2.00
Procurement, Marine Corps	10.00	40.00	30.00	13.00	7.00
Shipbuilding & Conversion, Navy	6.00	21.00	23.00	27.00	23.00
Other Procurement, Navy	26.00	47.00	17.00	6.00	4.00
RDT&E, Navy	54.00	38.00	6.00	1.50	.50
Military Construction, Navy (Dir. Prog. Only)	7.00	40.00	30.00	13.00	10.00
Military Construction, Naval Reserve	6.00	34.00	30.00	10.00	20.00

TABLE I.7

Note: The above rates relate to gross payments within a fiscal year program; they are not net outlay rates which may vary substantially because of changes in the volume of reimbursable transactions. The above rates are representative and are subject to change.

Now that the terms authorization, appropriation and outlays have been defined, the role of the Congress in the consideration of defense budget requests can be discussed. In the following two sections, authorizations and appropriations will be discussed respectively. The direct role of the Congress in examining and/or limiting outlays has been minimal and will not be discussed specifically. Control of outlays has been only in the form of setting an expenditure ceiling on the entire federal government through the Revenue and Expenditure Control Act of 1968. Aside from that one instance, the Congress has not legislated on the matter [Maxon, 1972].

The Authorization Process

> "No funds may be appropriated after December 31, 1960 to or for the use of any armed force of the United States for the procurement of aircraft, missiles or naval vessels unless the appropriation of such funds has been authorized by legislation enacted after such date."
>
> Section 412(b)
> Military Construction Act of 1960
> (Public Law 86-149)

Prior to the enactment of P.L. 86-149, only Military Construction required authorization as well as appropriation by the Congress. With the enactment of that law, procurement of aircraft, missiles and naval vessels came under authorization as well as appropriation scrutiny. Later, other procurement accounts and RDTE also were required to have authorizations

prior to appropriations action.* Currently, only operations and maintenance, military personnel and some small procurement accounts** may receive appropriations without prior authorization.

As with non-defense agencies, authorizations for the Defense Department are first examined by the substantive committees, in this case the House and Senate Armed Services Committees. Prior to 1961, the House Armed Services Committee was characterized as a "real estate committee" [Dexter,1963] since its annual review of the Defense Department was confined to the Military Construction Authorization. Today, the Armed Services Committees annually authorize aircraft, missiles, naval vessels, tracked combat vehicles, other weapons, RDTE and military construction. An actual authorization contains upper limits on appropriations for each of the services in the relevant categories, together with any special legislative provisions that may be deemed necessary — such as a ban on shipbuilding in foreign shipyards. Authorization acts do not contain detailed breakdowns of the subcategories under broad categories of aircraft, missiles, and so forth which are listed above.*** However, if a committee wants funds to be

*RDTE in fiscal 1963, Tracked Combat Vehicles in fiscal 1968, Other Weapons in fiscal 1970, Torpedoes in fiscal 1971.

**Other Procurement Army, Other Procurement Navy and Other Procurement Air Force and the Army's ammunition account are the procurement accounts not requiring authorization.

***Occasionally, the statute will contain a statement about a specific program. This has been the case with the Safeguard anti-ballistic missile system. However, such specific language is seldom found in the statute itself.

cut from a specific program, such as the CH 47 cargo transport helicopter, it will generally say so in its committee report, which forms part of the legislative history of the authorization. The legislative history is used later to determine the intent of the law and actually forms as important a part of the authorization as the actual statute [Harris,1964,pp. 90-95].

The authorization process begins with the House and the Senate Armed Services Committees' hearings on the defense budget requests in the procurement and RDTE areas. The actual authorization bill has generally been introduced in each house by the chairman of the Armed Services Committee for that house. After a committee's hearings are concluded, it marks-up (modifies) the bill in executive or closed session and writes a report. The committee's bill is then reported to the Floor where it is debated and sometimes amended prior to passage. If the House and Senate versions of an authorization statute are in disagreement, a Conference Committee composed of members of the Armed Services Committees meets and works out a compromise. There are only two votes in Conference, the House vote and the Senate vote. A compromise requires the approval of a majority of the House members and a majority of the Senate members. The Conference Report summarizes the compromises that have been reached by the Conference Committee. Once the report is approved by each house the compromise version of the bill is sent to the President for his signature. Occasionally one house sends its conference representatives to Conference with instructions, that is it instructs the representatives

not to back down on a particular provision. Such action is rare, however, since it makes it more difficult to reach a compromise [Redford, 1965,pp. 419-420].

Some authors have argued that the authorization role of the Armed Services Committees is related to an increasing concern on the part of the Congress over national security policy. Raymond Dawson, in the early 1960's, expressed the belief that Section 412(b) opened a new era in Congressional oversight of foreign and military policy [Dawson,1963]. A decade later, Arnold Kanter argued that Congressional behavior in the 1960-1970 period was "policy oriented" [Kanter,1972]. Interviews of Armed Services Committee staff members indicate that they believe that they have developed expertise in certain military areas such as tactical air warfare [Berry and Peckham,1973]*.

On the other hand, other authors have noted very little change in the approach of the Congress toward the defense issues [Kolodzei,1963, Goss,1970]. Recently, Leslie Korb has argued that even recent large cuts in Defense budgets do not reflect a policy orientation on the part of the Congress [Korb,1973].

Appropriations

Appropriations bills originate in the House of Representatives. The Constitution specifies that revenue bills must originate in the House, and this has been extended by tradition

---

*This information is based on interviews by Berry and Peckham which were not discussed in their thesis.

to appropriations bills, probably because appropriations and revenues were originally considered simultaneously by the Congress [Harris,1964,p. 52]. In the defense area, military personnel, operations and maintenance, procurement and RDTE are all funded by one appropriations bill, the Defense Appropriations Bill. Military Construction is funded separately.*

After its introduction in the House, the Defense Appropriations Bill is referred to the House Appropriations Committee which in turn refers it to the Defense Appropriations Subcommittee. The Subcommittee holds hearings, marks-up or modifies the bill and reports it to the full committee where it is usually reported to the full House without further changes [Fenno,1966,p.134]. As with Armed Services Committee reports, Appropriations Committee reports form an integral part of the legislative history of the appropriation and are used to determine the intent of the Congress. Changes to appropriations bills on the House Floor are rare. Richard Fenno has discussed in considerable detail the reasons for this phenomenon [Fenno,1966, pp. 433-434]. When they do occur, they are usually minor. Of 591 non-defense bills in the early Sixties examined by Fenno, 517 were passed without amendment [Fenno,1966,p.450]. Between 1961 and 1972, 7 defense bills were

*Military Construction, which comprises a very small proportion of the defense budget, will not be discussed specifically in this analysis.

passed without amendment, while three were passed with just one amendment. Two of those three included amendments introduced by George Mahon, Chairman of the Defense Appropriations Subcommittee and were supported by the Subcommittee. The other two years, the defense bill was passed after two amendments were approved [Congressional Quarterly Almanacs, 1961 through 1972]. Of the seven amendments approved in this time frame, two amendments prohibited the construction of ships in foreign shipyards, two amendments prohibited any use of appropriated funds for a domestic peace corps, one amendment reduced funds for the Army by $10 million since a price change had occurred since the Subcommittee had considered the budget request, one amendment included funds for a new destroyer which had not been included in the Subcommittee's version of the bill since Senate authorization of the destroyer had not been completed, and one included increases in operations and maintenance funds for Army, Navy and Air Force of $50 million each, an amendment introduced by Mahon.

After House passage, the Defense Appropriations bill is brought to the Senate where it is referred to the Senate Appropriations Committee and subsequently to the Defense Appropriations Subcommittee. It should be noted, though, that the Subcommittee has usually been holding hearings on the original House bill for quite some time, even though the bill has not yet been formally referred to the Senate. The following table gives the average number of days between the

opening of Senate hearings and the passage of the Defense Appropriations Bill in the House for the 1946-1949, 1952-1955 and 1962-1965 time periods

1946-1949	1952-1955	1962-1965
+26.2	-48.0	-90.8

[Note: "+" means bill passed before hearing started.]

TABLE I.9

In recent years, the Senate Defense Appropriations Subcommittee and Senate Armed Services Committee have been holding joint hearings on procurement and RDTE.

After hearings are concluded and the House passes its bill, the Subcommittee marks up the final House bill and writes its report. The full Appropriations Committee then refers the Subcommittee bill to the Senate Floor. Since 1961, twenty amendments to defense appropriations bills have been adopted on the Senate Floor. Some of these amendments, especially in recent years, have involved significant changes.* Even these totals, however, probably do not adequately capture the role of Floor consideration in the Senate. For example in 1970 only one amendment was offered and it was adopted,

---

*For example, in FY1971, a floor amendment provided $500 million to Israel of which $250 million was for the purchase of F-4 phantom jets [Congressional Quarterly Almanac, 1971, p. 341].

but Congressional Quarterly states, "Many members had prepared floor amendments to trim the bill if the committee had not done so." [Congressional Quarterly Almanac, 1970, p. 419]

After Senate passage, a Conference Committee composed of some or all of the Defense Appropriations Subcommittees' members meets to iron out differences between the House and Senate versions of the bill. As with authorization bills, the Conference reaches a compromise agreement and writes a Conference Report. After House and Senate approval of the Conference Report, the compromise bill is sent to the President for his signature.

Like authorizations , appropriations acts set dollar limits for broad categories like Navy RDTE, and Army Aircraft procurement. Specific programs are seldom, if ever, mentioned in the actual legislation. The Congress and the Defense Department know how the cuts in budget requests are to be allocated (unless the cut is an undistributed or nonspecific reduction) by the legislative history of the appropriations act. Most important in this legislative history are the Committee reports. Study of these reports yields significant information about the reasons for budget reductions and the intent of the Congress. For example, since budget prepreparation took place over 18 months prior to Congressional review, the Department of Defense itself will often recommend reductions because of factors such as

schedule slippages.* In the budget execution stage Committee reports are important sources of information for determining the legality of switching dollars between different accounts.

I.5 Budget Execution

The passage of an appropriations act by no means marks the end of the budget process. In fact the budget execution stage, which begins with the passage of the act, is one of the most important of all the budget stages. During budget execution, funds are apportioned, allocated and allotted by OMB, OSD, and the Services to those levels of the Defense establishment which will obligate and spend the appropriated funds. Also, during this stage the Services frequently seek and obtain authority from the Secretary of Defense and the Congress for transferring appropriations from one account to another — a process called reprogramming. The budget execution stage as discussed here includes all the activity between the signing of the appropriations act and the actual expenditure (outlay) of appropriated funds. Activity in this stage is affected by myriad rules and regulations; consequently, only the selected portions or elements will be discussed.

History

A traditional budgetary problem has been the problem of deficiencies; that is, overobligation by an agency and then

---

*For example, REDEYE missile in 1964. [House Report 1329, p. 35, April 17, 1964]

presentation to Congress of a "fait accompli." As one member puts it, the departments "can make these deficiencies and Congress can refuse to allow them; but after they are made it is hard to refuse to allow them." In order to address this problem, Congress passed the Anti-deficiency Act of 1903 which provided for agency apportionment or allotment of obligational authority, generally on a quarterly basis. It also provided for penalties for exceeding the apportionments or allotments [U.S. Code 665(a)-(i)]. Later, in 1921 the Budget and Accounting Act vested apportionment power in the Bureau of the Budget (later OMB). To this day, "OMB reflects Presidential control and can restrict the rate or purpose of obligations as provided for by law." [Navy Programming Manual, p. IV-7]

In the Department of Defense, the Office of the Secretary of Defense becomes very much involved in the apportionment process. Apportionment decisions have actually been the joint responsibility of the OMB and OSD. During the 1960's impoundments (refusal to apportion Congressionally approved obligational authority) were the result of OSD decisions.*

The Apportionment Process

The apportionment process resembles the budget preparation stage of the budget process. As the Navy RDTE manual states

---

*The classic example is Secretary McNamara's refusal to apportion funds appropriated for the B-70 bomber.

"The bureaus, systems commands and offices conduct their reviews of apportionment programs in much the same manner as budget estimates are reviewed. The review and apportionment program to OSD is the same as for budget estimates." [Navy RDTE Management Guide, p. 5-2].

Preparation of apportionment proposals or requests begins well before the passage of the appropriations act. The Service Comptroller offices generally obtain all the apportionment requests within one week after the passage of the appropriations act while the final request is submitted to OMB within fifteen days after the passage of the act.* Hearings are held on the apportionment request by the Services, OMB and OSD [Navy RDTE Management Guide, pp. 5-2 to 5-3].

The reason for all this paperwork is two-fold. First of all, by the time the apportionment request is acted upon approximately two years have elapsed since the budget pre-preparation stage for the fiscal year in question. During that time the threat may have changed, priorities may have changed, projects may have fallen behind schedule, costs may have changed. A second reason is that the Congress has generally cut the appropriations request (in line item categories) and those cuts must be applied to individual programs in the absence of specific Congressional instructions.**

---

*The apportionment request is submitted on for DD1105. Final authorization is approved on approved form DD1105 and on form SD-440.

*For example, in 1968, 1969 and 1970 the Congress made undistributed reductions in the RDTE budgets.

Reprogramming

As the fiscal year progresses, needs may change, some projects may fall behind schedule, new projects may arise or any number of unforeseen eventualities may surface. The device for handling this problem during the budget execution stage is called reprogramming. Reprogramming is the shifting of funds from specific uses originally planned to others where these funds can theoretically make a greater contribution to organizational or military effectiveness. In other words, it is using money for reasons other than those for which it was appropriated. Reprogramming in the Defense Department plays a very important part in the budget process. However, it is not always looked upon favorably by the Congress. For example, in 1971 Representative Whitten of the House Defense Appropriations Subcommittee stated in a hearing on defense reprogramming, "We tend to give money for a high priority project and then you...come in and say you want to use it for a low priority project." [House Appropriations Committee Hearings on Defense Appropriations, 1971, Part II, p. 603]

Reprogramming procedures have been well-defined in the Defense Department since the early Sixties [Parker,1973,Ch. 2]. Following the passage of the appropriations act, the Services submit to the Congress, via OSD, their Base for Reprogramming Decisions. This document (DD1414) "identifies the purposes in terms of budget subactivities of the...appropriation and the amounts for which funds have been authorized and appropriated." [Navy RDTE Guide, pp. 5-6] It also reflects

the application of cuts made by the Congress. For example, the Congress may make an across-the-board 3-percent cut in operations and maintenance. The Base for Reprogramming decisions will show how this reduction was applied to specific programs.

Suppose the Services desire to reprogram. Specific approval is required by SECDEF and the Armed Services and Appropriations Committees if the reprogramming involves any programs or functions specifically reduced by Congressional action or items which the Committees have expressed an interest in.* The latter can be determined by a review of the legislative history of the authorization and appropriations bills: hearings, committee reports, floor debate. Approval of SECDEF and notification of the Congressional Committees is required if the reprogramming action involves an increase of $2 million or more in any budget subactivity or the addition of a new subactivity line item the cumulative cost of which is estimated to be $10 million or more over a three year period. Because of the way the procedures are set up, the Services obtain maximum flexibility the more programs they fund under each line item (rather than increasing the number of line items) since reprogramming is based on "fences" drawn by the line items.

On the other hand, the Congress increases its control by drawing more "fences" or requiring more specific breakdowns of budget estimates.

---

*A reprogramming request together with its justification is transmitted on a form DD1415.

Twice each year the Services transmit to the Congress a "Report on Programs" which summarizes all reprogramming approved during a 6 month period, including those actions which did not require SECDEF or Congressional approval. The Report, called a DD1416, includes columns reflecting the original program (appropriations) as approved by Congress that was earlier transmitted on the DD1414, all approved reprogramming actions since that time, all reprogramming actions that were taken which did not require reprogramming and the current program (as modified by reprogramming).

Most reprogramming requests are handled by the chairmen, ranking minority members and senior members of the Defense Appropriations Subcommittees and the Armed Services Committees. As Stephen Horn, a former Senate Appropriations staff member has said, "Most junior members of Senate Appropriations are unaware of the vast reprogramming responsibilities held by their senior colleagues. . .the requests and the disposition of them almost never become known to others on the subcommittees or full committee." [Horn,1970,p.194] Occasionally, a reprogramming request will stir up controversy, such as a reprogramming request seeking funds for the STEP program in the Sixties.* Members who are worried about potential reprogramming without Congressional oversight will often seek an amendment to the appropriations or authorization bills or at least will try to

---

*STEP was a program to train individuals who failed to pass armed services qualification tests. [Horn,1970,p. 195]

generate floor debate which will cover a particular item. A good example of this are the amendments to Defense Appropriations bills introduced by H. R. Gross and passed by the House in the mid-Sixties which prohibited the Department of Defense from using appropriated funds to establish a domestic peace corps. With the passage of these amendments any attempt to reprogram funds (no matter how small the amount) into a domestic peace corps program would have required Congressional Committee approval. However, if Congressman Gross's amendment were opposed by Committee members, a reprogramming request which might subvert the purpose of the amendment could have been approved without being brought to the Floor of the House or the Senate.

An important question that comes to mind when discussing reprogramming concerns its magnitude. How much money is reprogrammed yearly? Statistics for DoD are scarce since some of the reprogramming documents are classified. During the four fiscal years 1961 through 1964, over $8.8 billion in Defense reprogramming was undertaken for missiles, ships, and RDTE alone.[Report of House Armed Services Committee, July 8, 1965]. The approved procurement and RDTE budgets for these years totalled approximately $83 billion.

A recent study by Commander John T. Parker [Parker,1973,pp. 90-91 and 97-98], includes the following figures for reprogramming. Unfortunately some of these figures do not reflect what is termed "below threshold reprogramming" —

that is reprogramming for which official notification of SECDEF and/or the Congress is not required.

FY	Reprogrammed	DOD Budget
61*	3.79	40.28
62	1.91	46.49
63	1.77	48.35
64	2.86	49.81
65	2.91	49.25
66	3.60	61.73
67	5.00	71.83
68**	7.04	72.03
69	4.44	73.63
70	2.43	71.29
71	3.26	68.63
72	1.93	72.56

TABLE I.10 (in billions)

Note: *1961-1963 data is only RDTE and Aircraft-Missile reprogramming.

**1968-1972 data is only above threshold programming

I.6. Audit

The final step of the budget process is the audit. The General Accounting Office undertakes audits for the Congress while various components within the Defense Department and the Services undertake them for the executive branch.

The legislative audit undertaken by the General Accounting Office (GAO), includes not only an examination of accounts for accuracy and adequacy but also includes scrutiny of the legal

basis for expenditures in order to ascertain whether or not they were made in accordance with the letter and intent of the law. As might be suspected, the intent of the law is usually determined by examining the legislative history of the appropriation against which the obligation was charged. A comprehensive discussion of GAO, its history and its functions is contained in [Harris,1964,Chapter 6].

GAO was established in 1921 by the Budget and Accounting Act. Until 1921 auditing of governmental accounts was done by auditors in the Treasury Department. The Budget and Accounting Act declared GAO to be independent of the executive department. The Comptroller General is appointed by the President to a fifteen year non-renewable term and can be removed only by a joint resolution of Congress. During the first three decades of its existence, GAO was attacked by executive agencies as being unduly restrictive in its accounting and financial management requirements. Finally, the Budget and Accounting Act of 1950 called for reform in accounting and auditing methods. Cooperation between GAO, the Treasury, and the Budget Bureau in the area of accounting and auditing followed this legislation [Harris, 1964,p. 135].

GAO has the power of disallowance. Consequently, an executive officer may be subject to penalties if GAO does not agree that an expenditure was within the letter and intent of the law. To protect themselves, many executive officers seek advance GAO rulings on questionable expenditures

[Harris,1964,p. 145]. Also, at times the Congress has granted exemptions to agencies which state that decisions on the legality of expenditures by executive officers are final, which means that GAO cannot disallow these expenditures. Occasionally certain Defense activities have been exempted, especially during wartime [See Harris,1964,p. 147, for an example].

In addition to auditing agency books, the General Accounting Office has in recent years assumed the role of investigator for the Congress. Every year numerous reports on the financial management systems of all or part of various federal agencies are submitted to the Congress.* Every agency is not audited every year, but GAO can decide to audit or investigate it at any time. The audit or investigation may be authorized by GAO on its own authority, by a Congressional Committee or by an individual Congressman.

The National Security Act Amendments of 1949 established the internal audit of defense accounts as a function to be performed by the offices of the Comptrollers of the Services and the Comptroller of the Department of Defense. Since GAO does not audit all DoD accounts every year, internal audits are both protective and constructive, attempting to detect items that might be of interest to GAO and to provide management with information related to economy and efficiency.

The audit stage is seldom included in discussions of Defense budgeting. Nevertheless, it is important to realize

---

*Recent examples include studies of the DD963 and the C5A.

that the audit function is performed and that it places some limits on Service activities; for example, unreported reprogramming is likely to be caught by GAO. This in turn means that the Services have some incentive to try to insure that appropriations are granted for the items they believe to be important. In other words, the Services can in general not take an appropriations estimate "off a license plate" and expect to be able to reprogram with impunity later, given the previously mentioned reprogramming reporting requirements and the possibility of GAO audit and disallowance.

I.7. Summary

In this chapter, the defense budget process has been outlined, beginning with the budget prepreparation stage, and terminating at the audit stage. This description is a necessary preliminary to the more technical analysis contained in the later chapters. These chapters will focus on the Congressional review stage and will attempt to characterize the response of the Congress as a whole to the President's budget request for Defense procurement and research and development. The discussion of this chapter hopefully will have impressed the reader with the numerous factors behind and the background of the budget request of the President and likewise the numerous elements which affect final Congressional action on that request. Although any given year's budget process may be viewed as a sequence of events, budget execution for year t-2 may affect the budget preparation for the budget

of year t. Thus any attempt to formulate deterministic models of even part of the process does not appear to have much chance of success unless the models contain an exceptionally large number of variables. The analysis to be undertaken here abstracts from many of the potential variables that might affect Congressional action in any one year in order to formulate, and estimate parameters for simple, probabilistic models. These models are designed to characterize Congressional activity on the average and to identify departures from this routine or usual behavior.

CHAPTER II

II.1. Background

The literature on Congressional budgeting is extensive, with major contributions dating back to Arther MacMahon's classic discussion of Congressional oversight [MacMahon, 1943]. Other works include those of Huzar, Carroll, Smithies, Wallace, Harris. Since 1960, the two most frequently cited studies of Congressional budgeting have been those of Wildavsky [Wildavsky,1964] and Fenno [Fenno,1966]. These two works are similar in many ways. Both are narrative discussions of Congressional budgeting based on interviews with Congressmen, Senators and staff members. Both conclude that, faced with complex and detailed budget requests, the Congress (appropriations subcommittees in particular) adopts simple approaches to budget review. These approaches are best characterized as <u>incrementalism</u> and <u>sampling</u> [Wildavsky,1964,pp. 14-15, and Fenno,1966, pp. 332-340]. By <u>incrementalism</u> it is meant that the Congress often makes marginal, percentage-like decisions on budget requests. [For example, an incremental decision rule might, as a rule, grant 90 percent of the budget request.] By <u>sampling</u> it is meant that the Congress will select some item,

often a small, insignificant and familiar one, and examine it in great detail [Fenno,1966,p. 336]. If the item is justified, other items will be approved. If not, the item will be cut, possibly eliminated, and another item will be examined.

As a sequel to Wildavsky's work, Davis, Dempster and Wildavsky [Davis,et al.,1966,1967,1971] formulated a number of simple linear models of budget requests and Congressional action on budget requests in the non-defense sector. The analysis to be presented here was partially stimulated by examination of these models. The Davis, Dempster, Wildavsky (DDW) models choose as their level of analysis the total agency budget, that is the budget of agencies or bureaus such as the Bureau of Land Management in the Department of the Interior. The models and data considered here refer to a lower level of aggregation.

Recently it has been suggested [Kanter,1972] that the Fenno and Wildavsky analyses and the DDW models do not apply to Defense appropriations. The argument has been made that Congressional action in this area is motivated by concern over questions of national security [Kanter,1972,p. 138]. Another author has argued that Congressional action has been motivated by fiscal or economic concerns [Korb,1973]. On the other hand, a 1970 Rand study [Stromberg,1970] attempts, with some success, to apply DDW models to Defense budget data from the years 1953 to 1968.

The purpose of the analysis done here will not be to reconcile the differences between DDW, Kanter, Korb and

Stromberg. However, it should be pointed out that those studies are not necessarily totally inconsistent. For example, it is possible for the Congress in general and the House and Senate defense appropriations subcommittees in particular to be making specific cuts in specific procurement programs such as the F-14 aircraft for fiscal or policy reasons. Yet, at the same time, it is possible that these subcommittees have implicit (or possibly explicit) target figures for the appropriations in the general categories like procurement aircraft and missiles Navy (PAMN).

In this chapter and in subsequent chapters, appropriations at the level of PAMN will be analyzed using several simple models. The data is comprised of requests and appropriations for aircraft and missile procurement for the Army, Navy and Air Force, and research and development (RDTE) requests and appropriations for the Army, Navy and Air Force. The analysis was limited to these categories since they are the ones in which Congressional cuts (or additions) have been most frequent and of the greatest size [Korb,1973,p. 16].

The purpose of the analysis will be to characterize the average or usual results of Congressional appropriations activity (at the levels of aggregation reflected in the data), and to identify as vividly as possible departures from the usual or average results. A number of behavioral explanations exist for the characterizations presented in the following chapters, including incrementalism, sampling, and policy and fiscal concerns at the program level. However, the focus of this

analysis is not on the behavioral explanations of the budget process but rather on the results of the process.*

The analysis in this chapter begins with a discussion of the question of how one might construct a model of appropriations as a percentage of budget requests. Next the methodology employed by DDW and Stromberg in their studies is reviewed and the methodology adopted in the data analysis reported later in the chapter is presented. The data; its sources and organization, will be explained and results of the analysis of the data, using a variety of analytical techniques, will then be reviewed. The final section is a summary of the major conclusions of the chapter.

II.2. Modelling Appropriations as a Percentage of Request.

In any one year, after the Congress appropriates funds, one can look back on the request and note that the appropriations are some percentage of the request — or

$$y_t = \beta_t x_t$$

where y_t is the appropriation in year t

x_t is the request in year t

β_t is the percentage or average, of the request that is appropriated.

If one is attempting to model the regular or usual results of Congressional action, a potential approach would be to say

---

*This is similar to the focus of a recent study of Atomic Energy Commission budgets by Natchez and Bupp [Natchez and Bupp, 1973].

that the percentage is a constant, depending neither on the request or the year. In other words, in order to sort out usual budget outcomes from unusual ones, one might contend that on the average, the appropriations are a fixed percentage of the request. However, since such a simplified approach leaves out numerous intricacies of the budget process which in any one year may affect the final budget outcome, a reasonable modification of the above might include a stochastic error component which in some sense symbolizes other factors not a part of the simple percentage statement. These other factors, or errors, can be modelled in several ways. For example, they can be viewed as being unrelated to the percentage, β, and unrelated to the request. This would lead to a model of the form:

$$y_t = \beta x_t + \varepsilon_t$$ where ε_t is a random disturbance.

On the other hand, one might view the percentage in any given year as random, with it being the result of the interaction of a fixed and a random component. This might be modelled by an equation of the form:

$$\frac{y_t}{x_t} = \beta e^{u_t}$$

where u_t is a random disturbance.*

---

*The exponential form will enable us to use standard estimating procedures after taking logarithms.

The percentage in a given year might also be viewed as random but the result of the additive effect of a random disturbance on a fixed component:

$$\frac{y_t}{x_t} = \beta + \varepsilon_t$$

Note the difference between $y_t/x_t = \beta + \varepsilon_t$ and $y_t/x_t = \beta e^{u_t}$. The latter equals $\beta + \beta u_t + \beta u_t^2/2! + \beta u_t^3/3! + \ldots$. Thus, the random component (the sum of all the terms after β) is related to the size of β.

One point which has not been addressed is that another way of looking at appropriations as being a percentage of requests that this percentage as having some regular or usual component to it is to view the percentage as being dependent on the size of the request; for example:

$$y_t = \beta x_t^{\alpha}$$

or

$$y_t = [\beta x_t^{(\alpha-1)}]\, x_t$$

where $[\beta x_t^{\alpha-1}]$ represents the percentage of the request appropriated. This representation resembles a graduated income tax scheme (if $0 < \alpha < 1$) in that as x_t gets larger the percentage of the request appropriated gets smaller. More specifically, since the elasticity of $[\beta x_t^{\alpha-1}]$ equals $(\alpha-1)$ the above equation implies that if the request changes by z percent, the percentage of the request appropriated will change by $(\alpha-1)z$ percent.* Again, other factors discussed earlier may cause the percentage not to be exactly $\beta x_t^{(\alpha-1)}$. Thus, a random component might be included in the model making it look like:

$$\frac{y_t}{x_t} = \beta x_t^{(\alpha-1)} e^{u_t} \qquad \text{where } u_t \text{ is a random disturbance}$$

In the above paragraphs, four somewhat different ways of characterizing appropriations outcomes have been discussed. These were formulated into the following equations:

---

*Elasticity of $x_t^{(\alpha-1)}$ is equal to

$$\frac{\dfrac{\partial \beta x_t^{(\alpha-1)}}{\partial x_t}}{\dfrac{\beta x_t^{(\alpha-1)}}{x_t}}$$

(2.1) $$y_t = \beta x_t + \varepsilon_t$$

(2.2) $$y_t = \beta x_t e^{u_t}$$

(2.3) $$y_t = \beta x_t + \varepsilon_t x_t$$

(2.4) $$y_t = \beta x_t^{\alpha} e^{u_t}$$

A final characteristic which is attractive for estimation purposes is

(2.5) $$y_t = \beta x_t^{\alpha} e^{u_t \ln x_t}$$

The equations are the result of different ways of thinking about the routine, percentage aspect of Congressional appropriations. As discussed earlier, they do not imply that Congressional decision-makers always, or even ever, think in precise percentage terms. (Although they might.) Rather, they are different ways of characterizing the usual end-product or overall results of Congressional action and may provide insight into what outcomes are unusual or irregular.

Up to this point, nothing has been said about the statistical questions involved in estimating β (and α) from sets of data. It turns out that the equations just formulated also appear to be alternatives one might consider when faced with the problem of estimation and the assumptions that necessarily underlie the computation of estimates. The discussion that will follow will concentrate mainly on the statistical aspects of the problem of characterizing Congressional appropriations as a percentage of requests. The methodology of DDW and Stromberg will be discussed first. Then alternative approaches will be suggested and the results of the application of these approaches discussed.

II.3. Methodology

Prior analysis of budget data by Davis, Dempster, and Wildavsky and by Stromberg have generally commenced with the assumption of a simple model for Congressional budget behavior of the following form [Stromberg, 1970, p. 61, DDW, 1971, p. 534]:

$$y_t = \beta x_t + \varepsilon_t$$

where y_t = appropriations in year t

x_t = request in year t

ε_t is a stochastic error or disturbance term assumed to be distributed $N(0,\sigma^2)$ with the sequence $\{\varepsilon_t\}$ being one of independent and identically distributed (i.i.d.) random variables.

The constant coefficient β is then estimated using least squares and some criteria like R^2, the "coefficient of determination," is used to judge the adequacy of the "fit" of the model to the data [DDW, 1971, p. 536, Stromberg, 1970, pp. 21-25].* In most cases [DDW, 1971, p. 537] the R^2 was very high and thus more complex models employing additional variables seemed unnecessary. Both DDW and Stromberg attempt to determine breakpoints — years in which the coefficient β shifts — by performing a hypothesis test originally suggested by Chow [Chow, 1960].

---

*"The principal selection criterion (among alternative models) is the criterion of the maximum adjusted correlation coefficient, $\overline{R}$" [DDW, 1971, p. 536].

II.3.1. Methodological Problems

There are several problems with this methodology. The difficulties can be classified under three subject headings labelled problems with (i) the model, (ii) fitting techniques, centered around the use of least squares, and (iii) goodness of fit criteria, exemplified by the use of R^2 as a criterion of closeness of fit.

(i) Model Specification Problems

One of the first problems is that the model specified in (2.1) assumes no interdependence between either β and x or between ε and x. As suggested earlier, if Congress were using a percentage appropriation or percentage cut decision rule, it may conceivably be the case that the percentage changes with the size of the request. The problem would not be noticeable if all requests for a particular agency (in the case of DDW) or for a particular functional category (in the case of Stromberg) were of approximately the same magnitude. In the Stromberg study, however, requests for Procurement Aircraft and Missiles, Navy, for example, ranged between $380 million in 1953 and $3.06 billion in 1963. Certainly it will be desirable to let available data speak for itself on this issue of interdependence between size of request and magnitude of the percentage appropriated, and on others to be discussed.

Another type of interdependence is that between ε and x. As will be discussed later and as is well known [Scheffe,1959], the optimal properties of least squares depend on the sequence

$\{\varepsilon_t\}$ being uncorrelated, identically distributed, and having $var(\varepsilon_t) = \sigma^2$, a constant. Under such conditions the Gauss-Markov theorem is applicable, guaranteeing efficiency of least squares estimates among the class of estimators linear in the observations. Were this condition to be true, and considering a model of the form of (2.1), then as the request gets larger the spread of actual data points or variance around the regression line remains unchanged. Both DDW and Stromberg view the error term, ε_t, as a random shock or variation from the usual percentage associated with "special events and circumstances relevant to particular years." [DDW,1971,p. 534, and Stromberg,1970,p. 8]. However, it is likely to be the case that the "special circumstances" that affect a $380 million request are neither the same nor create the same distribution of external affects, as the circumstances that affect a $3.06 billion request.

(ii) Least Squares Problems

The Gauss-Markov theorem insures that if the ε_t are i.i.d. then least squares estimators are best (minimum variance) linear unbiased estimators. If (2.1) were the true underlying model, then provided that an

estimate that is linear in the data is desired, the least squares estimate would appear to be a logical choice.

However. as mentioned previously, one obvious form of lack of homogeneity of variance of error terms — σ^2 changing with x_t — is certainly possible. In fact, since the regression line passes through the origin in equation (2.1) and negative appropriations are not possible, there is a lack of homogeneity of the error variance built into the model.

Another problem is the presence of cutting or spending moods on the part of Congress. Perhaps the residuals in the nineteen fifties decade are predominantly positive, and those in the sixties negative. One way DDW try to take this explicitly into account is by using a classical autoregressive model:

$$y_t = \beta x_t + \varepsilon_t$$

where $\varepsilon_t = \rho\varepsilon_{t-1} + v_t$

and the sequence $\{v_t\}$ is one of independent identically distributed random variables.

However, in most cases they were only dealing with 8 observations (at the most 16 and at the fewest 3).

Both DDW and Stromberg attempted to detect changes in the coefficient β, changes which could be associated with a cutting or spending mood on the part of the Congress and the public. DDW ran an F test for each agency on the residuals from the following regressions [Chow,1960]:

$$y_{1t} = \beta_1 x_{1t} + \varepsilon_{1t}$$

$$y_{2t} = \beta_2 x_{2t} + \varepsilon_{2t}$$

$$y_t = \beta x_t + \varepsilon_t \qquad \text{[DDW,1967,p. 278]}$$

where the first equation is for the first n_1 years of data, the second equation is for the second n_2 years of data and the third equation is for the pooled ($n_1+n_2=n$ years) data. The value of n_1 was allowed to vary between 3 and n-3 so that all possible break points were tried. In addition to the problems with using this approach on data sets numbering 8 or fewer, there is the added difficulty that the test assumes the error terms are normally distributed. Simpler approaches, such as examination of residuals, may be more informative.

Stromberg attempted to detect changes in β by analyzing the model

$$y_t = \beta_1 x_{1t} + \beta_2 x_{2t} + \varepsilon_t \qquad \text{where}$$

$x_{1t} = x_t$ (the request) for the first n_1 years of data and zero for the remaining years and $x_{2t} = x_t$ (the request) for the

last n_2 years of data and zero otherwise. He then ran a t-test for the equality of $\beta_1 = \beta_2$. In his formulation the $\{\epsilon_t\}$ are assumed to have the same distribution before and after the shift in the parameter β.

In many cases least squares may be more of a hindrance than a help to inference and achieving an understanding of the data. One grossly outlying observation may seriously affect the least squares estimate. As an illustration, consider the situation portrayed in Figure II.1, when the regression line is forced through the origin.

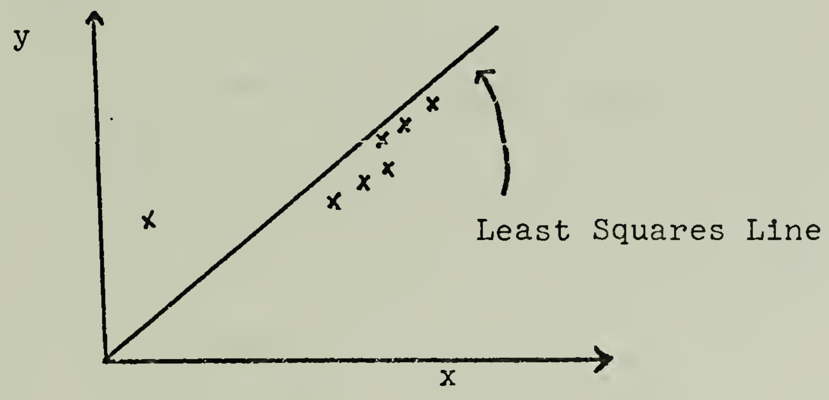


FIGURE II.1

The upper point in Figure II.1 may be a genuine outlier. For example, the President may have sent to Congress an urgent request for more funds. This request, occasioned by an international crisis, may have been made in March after the budget was submitted and consequently was not reflected in the budget request data analyzed. Because least squares

minimizes the squared deviations from the regression line, the resulting estimate of β in certain situations is apt to be a meaningless compromise. Least squares will produce a line with a slope larger than the true β in order to avoid the one extremely large deviation.

Another situation that might arise is depicted in Figure II.2, where the regression is not forced through the origin.

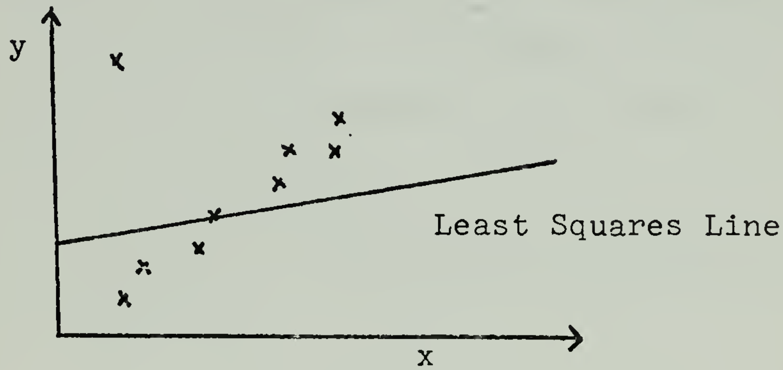


FIGURE II.2

Even if the previously discussed problems are not present and the error terms are i.i.d., the Gauss-Markov theorem only insures that least squares estimators are the best from the class of estimators that is linear in the observations. Anscombe [1967] and Huber [1973] have proposed so-called robust* estimators which are modifications of the least squares estimators but which are not linear in the observations. These estimators appear to be especially appealing when

assumptions of equality of the variance of the disturbances are not satisfied and when long tailed error distributions are suspected to be in operation.* Robust estimators will be discussed in more detail in later chapters.

(iii) <u>The Use and Interpretation of R^2</u>

Davis, Dempster and Wildavsky propose the use of $\bar{R}^2$, the coefficient of determination, adjusted for degrees of freedom, as the criterion for the closeness of fit of the data to their models. There are two major problems with this criterion. One was pointed out by Stromberg; the other has apparently not been discussed before in the context of the subject matter being investigated.

The usual computational formula for the estimate of R^2 obtained from a sample is:

$$(2.6) \qquad 1 - \frac{\sum_{t=1}^{T} e_t^2}{\sum_{t=1}^{T} (y_t - \bar{y})^2}$$

where e_t is the t^{th} residual from the fitted function, y_t is the t^{th} observation on the dependent variable, and $\bar{y}$ is the sample mean of the dependent variable. Dividing the numerator and the denominator of equation (2.6) by T, it is

*Anscombe [1967] and Huber [1973] contend that in practice it is difficult to distinguish between the effects of heteroscedasticity and long-tailed error distributions.

to see why R^2 is usually proposed as a criterion and given an interpretation of 1 minus the unexplained variance as a percentage of total variance of the dependent variable, or the explained variance as a percentage of total variance. Stromberg [1967, pp. 21-24] has pointed out that when performing regression with an intercept forced to be equal to zero as in equation (2.1) the interpretation of $\frac{\Sigma e_t^2}{T}$ as the (sample) unexplained variance is not correct. Regression with an unconstrained term insures that the $\bar{e}$, the mean residual, will be identically zero since when the partial derivative of the function to be minimized (Σe_t^2) with respect to the intercept is set equal to zero, the resulting "normal" equation automatically sets $\bar{e}$ equal to zero. However when a zero intercept is assumed, this normal equation is not a result of the least squares minimization and $\bar{e}$ either may or may not equal zero. Injecting $\bar{e}$ into equation (2.5) won't help since then one could theoretically obtain a large R^2 when the average error around the regression line was large but the spread around the average small.

Stromberg [1967,p. 24] and the BIOMED statistical package [BIOMED,1965,p. 233] have addressed the problem by computing a somewhat different number than that in equation (2.5). They have computed, what Stromberg called W^2

(2.7) $$W^2 = 1 - \frac{\sum_{t=1}^{T} e_t^2}{\sum_{t=1}^{T} y_t^2}$$

The interpretation given is that W^2 gives 1 minus the unexplained variation about zero as a percentage of the total variation of the dependent variable about zero. The problem here is that when performing regression under the assumption of a zero intercept, zero appears to be chosen as the point around which variation is computed more for convenience than for any other reason. Also, suppose that $\bar{e}$ happens to equal zero or be near zero (which will be the case if regression with an unconstrained intercept would have yielded a zero or near zero intercept) then with positive y_t's (which will always be the case with budget data) W^2 will be larger than the corresponding R^2 value and could be somewhat misleading to someone used to thinking in terms of R^2.

Another problem with R^2 is associated with the question of its usefulness as a tool in analyzing budget models similar to (2.1). The model (2.1) states that appropriations equal a percentage of request — plus some random error term. There should be no difference between this statement and the statement — on the average Congress cuts a certain percentage of the request — or

$$(2.8) \qquad (x_t - y_t) = \gamma x_t + \xi_t$$

In fact, if (2.1) is a correct model, then β should equal $(1-\gamma)$ and ε_t should equal $-\xi_t$ since

$$(x_t - y_t) = \gamma x_t + \xi_t$$

$$y_t = (1-\gamma)x_t - \xi_t \quad .$$

DDW and Stromberg have always tested equations similar to equation (2.1) and in general reported that the models were appropriate since the R^2 (or W^2) values were .98 or .99.

A simple test of equation (2.8) on data which yields a .99 R^2 for equation (2.1) will reveal a much reduced R^2. (For example, data which yielded a .98 R^2 for equation (2.1) achieved only a .26 R^2 for equation (2.8).)

The point of the discussion is that the value for the measure of fit, R^2, is going to be sensitive to the way the model for appropriations is formulated because changes in location of the dependent variable alter the distribution of R^2. As a result, one should probably not use R^2 as an absolute measure of fit and should instead consider it in a probabilistic context.

II.3.2 A Description of the Methodology Employed

Methodological problems with analysis of appropriations behavior were divided into model specification, least squares, and the use of R^2 as a criterion of closeness of fit of the data to the model. The methodology of the analysis of this chapter was designed to address these problems.

Plotting

The first approach to the data involved constructing scatterplots of appropriations and requests. Scatterplots

graphically indicate the range of the data and the character of the relationship between the variables, whether it be linear or curvilinear. Scatterplots also reveal potential outliers. Stem and leaf plots* of the appropriations and the requests were also constructed to display the range and variation of the data. In addition to being revealing, scatterplots and stem and leaf plots are relatively easy to construct, for example using the SNAP/IEDA computer package from Princeton University.

Plots of the logarithms of the data were also analyzed. These plots were useful for consideration of the models 2.3 through 2.5.

Alternative Models and Estimation Procedures

Five alternative models were chosen as potential representations of appropriations as a percentage of requests. These were the models (2.1) through (2.5) discussed in section II.3. It should be noted that the models, except for (2.1), are proposed in order to take into account systematic interdependence between the request, x_t, and the percentage appropriated, the error component, or both. They are not designed to take into account cutting or spending moods, or eras in the mood of Congress.

*Stem and leaf plots are discussed in Appendix A.

Our attitude towards the models is that (a) they cannot be universally or perpetually valid — one can specify representations of more complex behavior on the part of Congress —, but (b) such simple models do aid in understanding the phenomena underlying the data. In particular, estimation of model coefficients by various techniques, and subsequent examination of the residuals around the fitted relationship is a procedure that brings to attention certain historical periods and possibly certain programs that differ from the average or "usual." The fact that such periods (or eras) and programs exist should give pause to those who would attempt to mechanistically predict future appropriations behavior on the basis of simple models fitted to historical data. On the other hand, simple fitted models can provide a starting point for future discussions of amounts to be appropriated [Senate Report 93-688].

The simplest model of appropriations behavior, that given by equation (2.1), was estimated using ordinary least squares. The residuals were analyzed using scatterplots, and stem and leaf plots of both the residuals themselves and their absolute value.

The model specified in equation (2.2) was analyzed in two ways. The least squares estimator for β is the mean value for the ratio y_t/x_t. However, the median y_t/x_t ratio was also utilized in order to provide a robust estimator of β [Andrews, et al.,1972], and to allow outliers to reveal themselves more clearly. The residuals for both estimates

were compared on the basis of scatterplots and stem and leaf plots, both of the residuals themselves and of their absolute value.

Equation (2.3) specified a percentage type model, in some ways similar to equation (2.1), except that the error term is multiplicative. It is also similar to equation (2.4) except that the percentage of the request that is appropriated does not change with the size of the request. The coefficient β was estimated in two ways. The first method, which yielded the least squares estimate, was to use the mean difference of $\ln y_t - \ln x_t$ as the estimate of $\ln \beta$. The second approach was to use the median difference. Residuals for both estimates were compared.

The model specified in equation (2.4) is linear in the logs. Consequently logarithms were taken of both sides of (2.4) and the coefficients α and $\ln \beta$ were estimated using least squares. Residuals were analyzed as in the case of the other models.

If one takes logarithms of both sides of (2.5) the resulting equation is:

$$\text{(2.9)} \qquad \ln y_t = \ln \beta + \alpha \ln x_t + u_t \ln x_t \quad .$$

As suggested in [Johnston,1963,p. 211], estimates of α and $\ln \beta$ were computed by applying least squares after dividing both sides of equation (2.9) by $\ln x_t$.*

*Since none of the values for the x_t's were equal to 1.0, division by $\ln x_t$ was always possible.

In the cases of (2.3) - (2.5) it should be obvious that β was not estimated directly; rather, estimates of $\ln \beta$ were derived. The exponential of this estimate was used as the estimate of β.

It should be pointed out that the goals of estimation at this point in the analysis are exploratory and tentative. One question to be answered was: which methods tolerate and reveal these outliers more effectively — the log transformed data or the basic numbers, mean estimators or median estimators. Another question to be answered was whether or not any of the approaches (analysis of the logs, use of medians or means) revealed cutting or spending moods.

II.4. Data

Two basic batches of data were analyzed. The first were procurement requests and appropriations for the years 1953-1973. The following categories were analyzed as a group:

Procurement Equipment and Missiles, Army	(PEMA)
Procurement Aircraft and Missiles, Navy	(PAMN)
Procurement Aircraft, Air Force	(AF A/C)
Procurement Missiles, Air Force	(AF Missiles)

Twenty one years of data and 4 observations per year should yield 84 total observations. However, from 1955-1958 either PEMA requests or appropriations or both were zero and consequently were not included in the data. Thus, to start with, 80 observations were analyzed. The above four categories

constitute roughly three quarters of annual Defense Department procurement requests and appropriations. The only major category not included which is comparable in size to these is Shipbuilding and Conversion, Navy. The high and low appropriations figures for each category were (in billions):

Category	High	Year	Low	Year
PEMA	$5.462	1968	.971	1960
PAMN	3.955	1972	.113	1953
APAF	8.048	1953	2.072	1955
MPAF	2.903	1953	.796	1966

TABLE II.1

HIGH AND LOW APPROPRIATIONS FOR PROCUREMENT APPROPRIATIONS IN THE PERIOD 1953-1973

The data from the years 1953-1968 were taken from Stromberg [Appendices A and B]. Stromberg took the appropriations categories of 1968 as given; he then traced all category changes back to 1953 (there were few in the procurement area) and thus reconciled the data with the 1968 categories. The 1969-1973 data were reconciled with the 1968 categories (there were several changes in the Army procurement categories in 1972) by the author. The basic data was obtained from [U.S. Senate, Budget Estimates and Appropriations, 1969-1973].

The second batch of data analyzed was the Research and Development (RDTE) requests and appropriations for 1953-1973 for the Army, Navy and Air Force. In all there were 63

observations. The high and low appropriations figures for each category were (in billions):

Category	High	Year	Low	Year
RDTE Army	$1.839	1972	.333	1956
RDTE Navy	2.545	1973	.059	1954
RDTE Air Force	3.632	1963	.418	1955

TABLE II.2

HIGH AND LOW APPROPRIATIONS FOR RDTE IN THE YEARS 1953-1973

The data sources for RDTE were the same as for the Procurement data. A listing of the data can be found in Appendix D of this chapter.

II.5. Results

Analysis of the procurement and RDTE data using the approaches discussed earlier yielded some notable differences in results depending on which approach was used. These differences can be summarized under the categories of differences in coefficients, residuals and outliers, and closeness of "estimated" appropriations to actual appropriations, i.e. predictability. Each of these topics will be taken up in turn.

II.5.1. Coefficients

The three models (2.1) - (2.3) all say that on the average, appropriations are a constant percentage of the request. The models differ in the way the random

component enters their specification. The estimated coefficients using the different approaches for procurement and RDTE data are presented in Table II.3.

Model		Procurement	RDTE
(2.1)		.959	.982
(2.2)	median	.974	.989
	mean	.993	1.090
(2.3)	median	.977	.990
	mean	.973	1.030

TABLE II.3
ESTIMATED PERCENTAGE OF REQUEST APPROPRIATED USING (2.1) - (2.3)

Although it is difficult to assess the differences among the various "estimated" percentages without some measures of stability or variability of the estimates or without examining how close the estimated appropriations are to actual appropriations both for the set of data analyzed and for an independent set, some differences stand out immediately upon examination of Table II.3. The least squares estimates for the percentages in (2.1) and (2.2), that is .959, and .993, for procurement are different from the other estimated percentages which are about .975. Confidence intervals computed in Chapter IV will allow us to assess the importance of such differences.

A second interesting aspect of the estimated coefficients is that for the RDTE data the least squares estimates for (2.2) and (2.3) are greater than 1.0, being 1.09 and 1.03

respectively. In other words, according to these results the Congress has increased the appropriations over and above the request by some percentage figure. This result is somewhat at odds with the current view of Congressional activity in the RDTE area, that is, the view that the Congress is carefully scrutinizing and cutting RDTE budget requests. These and other points will be pursued further in the discussion of outliers and closeness of fit.

The two models (2.4) and (2.5) both say that the percentage of the request which is granted depends on the size of the request. The difference between the two is in the specification of the error component. The estimated coefficients for both (2.4) and (2.5) are presented in Table II.4.

Model	Procurement		RDTE	
	$\hat{\alpha}$	$\ln \hat{\beta}$	$\hat{\alpha}$	$\ln \hat{\beta}$
(2.4)	.948	.384	.903	.702
(2.5)	.969	.215	.852	1.056

TABLE II.4

ESTIMATED COEFFICIENTS FOR (2.4) AND (2.5)

The coefficients themselves are not immediately informative. More informative are the percentages they imply. For example, what percentage of the request do the estimated coefficients imply would have been granted for a request of 3 billion dollars. This can be assessed by computing $\hat{\beta} x_t^{(\hat{\alpha}-1)}$, where $\hat{\beta}$ and $\hat{\alpha}$ are the estimated coefficients and x_t is, in this

case, 3.0. The following table gives the average, maximum and minimum percentage of the request that the estimated coefficients imply would have been granted for the procurement and RDTE data that was analyzed. This provides a method of comparison of the estimated percentages from the approaches (2.1) - (2.3) with the results of the estimation of coefficients for (2.4) and (2.5).

Model	Procurement			RDTE		
	High	Average	Low	High	Average	Low
(2.4)	1.14	.981	.91	1.35	1.03	.91
(2.5)	1.06	.975	.93	1.56	1.03	.85

TABLE II.5
ESTIMATED PERCENTAGE OF REQUEST GRANTED FOR DATA ANALYZED

Appendix C to this chapter contains stem and leaf plots for the percentages for procurement and RDTE using the approaches of (2.4) and (2.5).

An interesting question is: How does the "estimated" percentage of the request granted change with changes in the size of the request? As the request gets larger does the percentage granted increase or decrease? (Of course the approaches in (2.1) - (2.3) assume the percentage remains the same.) This sensitivity of the percentage granted to the size of the request can be investigated using the elasticity of the percentage with respect to the request or

$\frac{\partial \beta x^{\alpha-1}}{\partial x} \Big/ \frac{\beta x^{\alpha-1}}{x}$. The following table gives the "estimated" elasticities for (2.4) and (2.5). Note that these elasticities can be obtained by computing $(\hat{\alpha} - 1)$.

Model	Procurement	RDTE
(2.4)	-.052	-.097
(2.5)	-.031	-.148

TABLE II.6

ELASTICITY OF PERCENTAGE OF REQUEST GRANTED WITH RESPECT TO SIZE OF REQUEST

What these elasticities say is that for a 100 percent increase in the request there is an estimated 5.2 percent decrease in the percentage of the request granted (in the case of procurement, with Model (2.4)).

II.5.2. Residuals

As discussed in the methodology section, for each approach residuals were calculated and these residuals were then studied. An initial study of the residuals was undertaken (i) by making stem and leaf plots, and (ii) by making scatterplots of the residuals versus the request for (2.1)-(2.2) or scatterplots of the residuals and logs of requests for (2.3)-(2.5).

The residuals studied were the results of the following arithmetic computations for each approach.

(2.1') $y_t - \hat{\beta} x_t$

(2.2') $\frac{y_t}{x_t} - \hat{\beta}$

(2.3') $\ln y_t - \ln x_t - \ln \hat{\beta}$

(2.4') $\ln \bar{y}_t - \hat{\alpha} \ln x_t - \ln \hat{\beta}$

(2.5') $\frac{\ln \bar{y}_t}{\ln \bar{x}_t} - \hat{\alpha} - \ln \hat{\beta} \; (\frac{1}{\ln x_t})$

The stem and leaf plots and scatter plots are contained in Appendix B of this chapter. The results for the procurement data will be discussed first, followed by a discussion of the results for the RDTE data.

Discussion of the Procurement Analysis: Residuals and Outliers

For each model, as applied to the procurement data, the following items or data points produced residuals whose absolute values were more than twice the mean absolute residual for that model. This provides a preliminary indication of outliers.

Negative	1954	AF A/C
	1954	AF Missiles
	1954	PAMN
	1953	PEMA
Positive	1954	PEMA
	1959	PEMA
	1962	PEMA
	1960	AF Missiles
	1963	PAMN

TABLE II.7

ITEMS PRODUCING LARGE RESIDUALS

These results reveal some interesting aspects of the phenomena giving rise to the data. First, 1954 probably is not part of the same population as the rest of the data. Appropriations for fiscal 1954 were approved shortly after the signing of the Korean truce and appear to be affected by this event. (The high positive residual for 1954 PEMA – representing a Congressional appropriation of over $2 billion after the Army requested only $1 billion – either represents a desire to replenish Army stocks at the close of the war <u>or</u> possibly is related to the very large cut of the 1953 PEMA request – a cut of almost $700 million). Until 1960, PEMA is a very unstable category. In 1957 and 1958 during the era when "massive retaliation" and "brinksmanship" were the popular foreign policy doctrines [Crecine and Fischer,1971,pp. 25-26] there were neither requests nor appropriations for PEMA. The large positive residuals for fiscal 1962 PEMA and fiscal 1963 PAMN reflect the increasing emphasis on conventional weaponry in the early 1960's and may very well be

related to Congressional concern over the unsuccessful Bay of Pigs invasion of April 1961 and the growth of the Military Assistance Command, Vietnam.

A second interesting aspect of analysis of the residuals is that almost all post-1969 items yield negative residuals; also, these are of approximately equal magnitude. In other words, the various simple model approaches imply that recent appropriations have been higher than they actually were. For example, using the approach of (2.1) on procurement data, thirteen of the twenty most negative residuals were for post-1969 items. This grouping of the post-1969 items suggests that decision rules changed in 1969 and the Congress began cutting a larger percentage of the request. In other words, there is some evidence that an era terminated and the mood of Congress changed. Since the data analyzed represent total Congressional activity, this analysis does not pinpoint whether the change was associated with any special Congressional Committee. Other research [Laurance,1973] indicates that the Senate Armed Services Committee began to closely scrutinize defense authorization requests at this time. The only peculiar aspect of the fiscal 1969 budget picture was that since it was acted on in calendar 1968, it was an election year budget. Also, fiscal 1968 represents the low point in President Johnson's popularity and a low point in public support for the Vietnam war [Gallup,1968].

One notable difference in the residuals for the different approaches is associated with the pre-1960 items of less than $2 billion. The analysis of the logarithms, the approaches of

(2.3) through (2.5), grouped together a number of pre-1960 items that were under $2 billion, several being under $1 billion, as the most negative residuals. The other approaches, which used the original untransformed numbers did not group these residuals together. For example, under the approach of (2.4) 1953 PAMN with a request of $124.5 million and appropriations of $113 million produced the 5th most negative residual. Under the approach in (2.1), the residual was slightly negative and ranked 33rd (of 80) among residuals ordered from most negative to most positive. A similar result occurred for 1956 PAMN with a request of $945 million and appropriations of $804 million.

The particular phenomenon observed here resembles one discussed by Daniel and Wood[1971,p. 25] as part of a general discussion of the distribution of data points. The plot of requests versus appropriations looks something like Figure II.3.

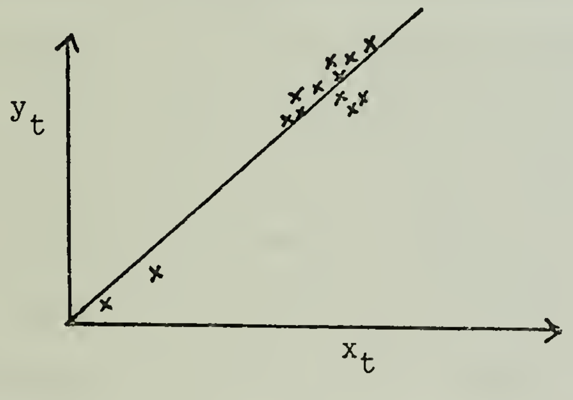


FIGURE II.3

As Daniel and Wood point out, if least squares is used on data similar to that in Figure II.3, the cluster at the right acts as essentially one point in determining the slope. The slope is, thus, highly dependent on the extreme values at $100 and $800 million. Thus, the residuals for those items will tend to be small. If the approach of (2.3) is used and the median difference between ln y_t and ln x_t is employed as an estimator for ln β, then all of the points have equivalent importance in determining the estimate.

In the discussion of coefficients it was pointed out that the .959 coefficient obtained by using least squares on (2.1) was somewhat different from the coefficients produced by the other approaches. In the stem and leaf plot of the residuals (p. B-4) note the large number of residuals in the 0-300 range. There are thirty-three negative residuals and forty-seven positive ones. The other approaches yield distributions of residuals which are somewhat more symmetrical around zero. For example note the stem and leaf plot (p. B-15) for the approach of (2.4). There are forty-two negative and thirty-eight positive residuals. The pattern of the residuals and the estimated coefficients indicate that when least squares was applied to (2.1) a few large cuts, that is appropriations significantly smaller than requests, pulled

the estimated line down slightly; hence the smaller slope and different pattern of residuals. In later chapters we will explore estimation procedures which are not as sensitive as least squares to a few extreme observations.

Discussion of the RDTE Analysis' Residuals and Outliers

As was true in the case of procurement, the residuals produced by applying each of the various approaches to RDTE data were examined in detail. The items which produced the largest negative residuals under each of the approaches were 1954 Army, Navy and Air Force. It should be noted that for the approach of equation (2.2), regardless of whether the mean or median of the ratio of appropriations to request were used, the result failed to yield a negative residual that was larger in absolute value than twice the mean of the absolute value of residuals.

The largest positive residuals under each of the approaches were produced by 1955 Navy, 1962 Air Force and 1960 Air Force.

The procurement data showed a marked tendency for post-1969 items to have negative residuals for each model-fitting approach. However, the RDTE data do not yield such definitive

results for all approaches. The approaches of (2.1) - (2.3), especially (2.1), show this tendency; in each case, after the residuals were ordered from most negative to most positive, 13 of 15 post-1969 items yielded residuals in the lower half of the ordered set. However, this was not true for the variable percentage approaches of (2.4) and (2.5). Under these approaches the most negative residuals were recorded for the pre-1960 items, where the request and cut were both small in absolute terms but the cut was large when considered relative to the request. For example, under the approach of (2.5) the seventeen most negative residuals were for items less than $1 billion, prior to 1960.

Insight can be gained into this pattern for the approaches of (2.4) and (2.5) by recalling the elasticities presented in Table II.6. Under the approach of (2.5) the estimated elasticity of the percentage of the request granted with respect to the size of the request was -.148. Consequently, the percentage of a 500 million dollar request that is appropriated was estimated to be .148 larger than the percentage of a 1 billion dollar request that is appropriated. The RDTE data varied between requests of $54 million and $3 billion. Any items for which the request was less than $1 billion and which did not show appropriations greater than the request yielded large negative residuals when the approaches of (2.4) and (2.5) were used.

One feature of the analysis of RDTE residuals which did not appear in the analysis of the procurement data was the

almost exclusive domination of the extreme positive residuals by the Air Force. No matter which approach was used, the residuals for the Air Force were among the most positive. Re-examination of the data reveals that except for small reductions in 1966 and 1968, between the years 1957 and 1969 the Air Force either received almost exactly what it requested or, more often, received more than it requested. This pattern is not present for the Army and Navy. This leads one to suspect that the Air Force data for the 1957-1969 period may be part of a population different from the Army and Navy data of the same period. On the other hand, the 1970-1973 data reveal no notable differences between Congressional action on Air Force RDTE requests and Congressional action on Army and Navy requests.

A final point of interest is associated with the distribution of residuals under the various approaches. The stem and leaf plots in Appendix B show two interesting features. First, once again the approach of (2.1) yields many small positive residuals. Note the large number of residuals in the 0 to 150 range. This is very similar to the pattern that appeared in the procurement data. The second interesting feature is associated with the difference in the distribution of residuals for the mean and median when used as estimators for the coefficients in the approaches of (2. 2) and (2.3). Recall, that the mean and median yielded different estimated coefficients (1.09 to .99 for (2. 2) and 1.03 to .99 for (2.3)). Next, note that when the mean is used

as the estimator almost all of the residuals are negative — small in absolute value but negative nevertheless. In fact, when the mean is used as the estimator for the coefficient for the approach of (2. 2), 57 of 63 residuals are negative. When the median is used exactly half are negative and half are positive, with one residual being zero.*

Closer examination of the data indicates that at least one apparent outlier, 1955 Navy, where the request was $61 million and appropriations were $419 million, is significantly affecting the least squares estimate — the mean — both for the approach of (2. 2) and the approach of (2.3).

The scatterplots of the residuals versus the predicted values and of the dependent variable versus the independent variable (see Appendix B) for approaches (2.4) and (2. 5) are also revealing. In both cases, for small predicted values the residuals tend to be negative while for large predicted values they tend to be positive.

A careful examination of the log x_t versus log y_t scatterplot for the approach of (2.4) on page B-33, however, reveals that there is one extremely large positive residual associated with a very small value for the independent variable. That is, the scatterplot and the estimated regression line look something like Figure II.4.

---

*For the approach of (2.3) 48 of 63 residuals are negative when the mean is used as estimator while half are negative and half are positive when the median is used.

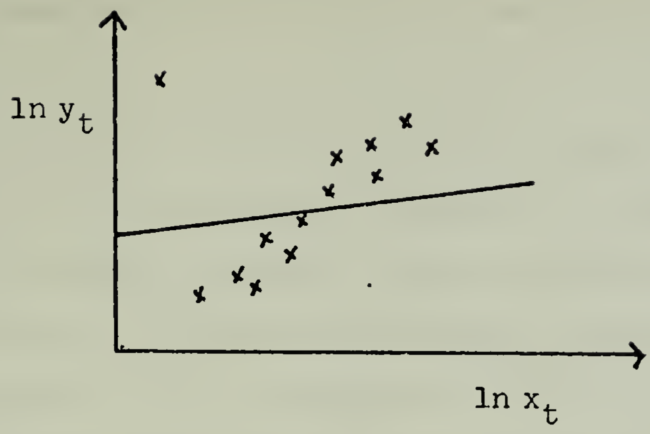


FIGURE II.4

This data point was, again, 1955 Navy RDTE for which the request was $61 million and appropriations were $419 million. It appears that this outlier, which is affecting the least squares estimates for (2.2) and (2.3), is also affecting the least squares results for the approach of (2.4).

II.5.3. Predictability

The numerical values of the residuals under each of the approaches are, of course, not directly comparable. For example, the residuals from the approach of (2.1) are found by computing, for each t,

$$y_t - \hat{\beta}x_t$$

while the residuals obtained from the approach of (2.2) are found by computing

$$\frac{y_t}{x_t} - \hat{\beta}$$

where the β's were estimated using the techniques discussed earlier.

In the past section the distributions of the residuals under each of the approaches were compared, as were the items associated with extreme residuals under each approach. But the numerical values of the residuals were not compared.

Some assessment of the proximity of what might be called the predicted appropriation under each approach, $\hat{\beta}x_t$, for (2.1) - (2.3) and $\hat{\beta}x_t^{\hat{\alpha}}$ for (2.4) - (2.5) and the actual appropriation (i.e. y_t) is useful and informative. Consequently, the difference $y_t - \hat{\beta}x_t$ (or $y_t - \hat{\beta}x_t^{\hat{\alpha}}$) was computed for each item under each approach, both for the procurement and RDTE data. Then both the mean and median of the absolute value of this difference were computed. (The mean of positive square roots of the squared differences could have been used, but the mean of the absolute differences was considered to be less sensitive to one or two large values. The median of the absolute differences was of course even less sensitive to extreme values.) Tables II.8 and II.9 contain the results of these computations.

For the procurement data, all of the approaches yield a comparable mean absolute difference with the median as estimator for equations (2.2) and (2.3) yielding the smallest values by a narrow margin.* The values for the median of absolute differences, $|y_t - \hat{\beta}x_t|$, and examination of the stem and leaf plot for the absolute differences indicate that when the

*Since absolute differences are being used, it is to be expected that the median will fare better than the mean under the approaches of (2.2) and (2.3).

Approach	Mean Absolute Difference	Median Absolute Difference
(2.1) $y_t = \beta x_t + \varepsilon_t$	253.2	112.78
(2.2) $y_t = \beta x_t + \varepsilon_t x_t$		
Median as estimator	249.7	93.9
Mean as estimator	262.8	119.7
(2.3) $y_t = \beta x_t e^{u_t}$		
Median as estimator	249.7	93.9
Mean as estimator	250.2	97.8
(2.4) $y_t = \beta x_t^{\alpha} e^{u_t}$	254.9	136.8
(2.5) $y_t = \beta x_t^{\alpha} e^{u_t \ln x_t}$	251.6	131.6

TABLE II. 8

Procurement
(Numbers are in Millions)

Approach	Mean Absolute Difference	Median Absolute Difference
(2.1) $y_t = \beta x_t + \varepsilon_t$	89.3	36.3
(2.2) $y_t = \beta x_t + \varepsilon_t x_t$		
Median as estimator	90.3	32.2
Mean as estimator	196.9	152.9
(2.3) $y_t = \beta x_t e^{u_t}$		
Median as estimator	90.3	32.2
Mean as estimator	115.2	77.8
(2.4) $y_t = \beta x_t^{\alpha} e^{u_t}$	111.4	56.7
(2.5) $y_t = \beta x_t^{\alpha} e^{u_t \ln x_t}$	142.9	71.9

TABLE II. 9
RDTE
(Numbers are in Millions)

median was used as estimator for (2. 2) and (2.3) most of the absolute differences were small while an obvious few were quite large. It is worth noting that the mean absolute differences are very close for the approach of equation (2.1) and those of equations (2. 2) and (2.3) (with the median as estimator) but the median absolute differences are considerably different.

Examination of Table II.9 for the RDTE data reveals that the mean and the median absolute difference are very large for the approach of equation (2. 2) when the mean of the appropriations/request ratio is used as the estimator for $\hat{\beta}$. This is consistent with the discussion in the last section of the problems with this estimator for the RDTE data. The median used as an estimator under the approaches of equations (2.2) and (2.3) and the approach of equation (2.1) all yield comparable absolute differences. This is to be expected, of course, since the estimated coefficients under each approach were nearly identical. Earlier, problems with the application of least squares to equations (2.4) and (2. 5) for the RDTE data were discussed. The results in Table II.9 are consistent with that discussion. The mean and median absolute differences are considerably larger for (2.4) and (2.5) than for equations (2.1) and for (2.2) - (2.3) (with the median as estimator).

II.5.4. Conclusions

Each approach was used on both the procurement and RDTE data, and was evaluated by examining (i) the estimated coefficient or percentage of the request which was appropriated, (ii) the pattern of the residuals, and (iii) the closeness of the "predicted" appropriations to actual appropriations. For each approach the results will be summarized.

(2.1) $y_t = \beta x_t + \varepsilon_t$

The least squares solution, i.e. estimate of β for (2.1), produced an estimated coefficient for procurement which was different from those produced by other approaches. A more complete assessment of such differences will be possible after confidence intervals are constructed in Chapter IV. Examination of residuals for this approach reveals rather disturbing patterns. For example many of the residuals are positive, which indicates that the classic least squares assumptions possibly are not being satisfied. Also, the distribution of data points is such that it appears that least squares forced the line directly through the low value data points. The closeness of fit of the predicted appropriations to actual appropriations for this approach was comparable to that of approaches (2.3) and (2.2) when the median was used as the estimator for these approaches, closeness of fit being measured by the mean $|y_t - \hat{\beta}x_t|$. The median $|y_t - \hat{\beta}x_t|$ was larger for the approach of (2.1). In summary, by the three standards used for evaluating the estimates, while the least

squares approach of (2.1) did not produce the worst estimates, neither did it produce the best. This is especially true for the procurement data.

$(2.4)\quad y_t = \beta x_t^{\alpha} e^{u_t}$

This is a "variable percentage" approach where the percentage of the request that is granted varies with the size of the request. Estimates of the coefficients were derived by applying least squares to the equation in the logs. The estimated coefficients for the procurement data were reasonable; they implied percentages that went from .91 to 1.14 depending on the size of the request, with an average of .98. However, the estimated coefficients for the RDTE data implied percentages that went from .91 to 1.35 with an average of 1.03. It is difficult to imagine that for a request of a certain size the Congress would consistently vote an increase of 35 percent. This approach yielded larger residuals for requests that were small in dollar magnitude than any other approach except for that of (2. 5). However, for the RDTE data, the residuals had a disturbing pattern, being negative for small requests and positive for large requests. The least squares line appears to have been unduly affected by some outliers. Insofar as closeness of fit is concerned, for the procurement data, the average $|y_t - \hat{\beta} x_t^{\hat{\alpha}}|$ difference when this approach was used was comparable to that yielded by other approaches. However the median absolute difference was larger.

For the RDTE data the closeness of fit was not as good as that for (2.2) and (2.3) – when the median was used – nor as that for (2.1). However it was better than that for (2.2) and (2.3) when the mean was used as an estimator.

(2.2) $y_t = \beta x_t + \varepsilon_t x_t$

For this approach first the median and then the mean of the ratio y_t/x_t were used as estimators of β. For the procurement data the estimates yielded by the mean and the median were comparable. However when the mean was used on the RDTE data the estimated β was 1.09. Examination of the data showed that this estimate, a least squares estimate, was also unduly affected by an outlier. For the procurement data, the residuals' pattern was reasonable and similar for both mean and median as estimator – separating out some early 1950's items – and pointing to a possible change in Congressional attitude after 1969, since many post-1969 items had negative residuals. For the RDTE data the least squares estimator, the mean, revealed almost all negative residuals with a few large positive ones. The residuals when the median was used as an estimator appeared reasonably distributed. For example there were as many positive as negative residuals. Insofar as closeness of fit is concerned, for the procurement data, the median showed the smallest mean (and median) absolute error or $|y_t - \hat{\beta} x_t|$ of all the approaches, except for the median using the approach of (2.3). The mean also showed a small absolute error. However, for the RDTE data, while the

median again showed small mean (and median) absolute errors the mean showed the largest. As mentioned earlier it appears that a few outliers seriously affected the estimates when the mean was used for the approaches of (2.2) and (2.3).

(2.5) $y_t = \beta x_t^{\alpha} e^{u_t \ln x_t}$

This approach is very similar to that of (2.4). Coefficients were estimated by taking the logs of both sides, dividing the equation by log x_t and applying least squares. The results are comparable to those for the approach of (2.4). Coefficients imply reasonable percentages for procurement data but not for RDTE data. Residuals show an undesirable pattern, from the standpoint of least squares, for RDTE data. The closeness of fit is not particularly impressive, especially for RDTE data.

(2.3) $y_t = \beta x_t e^{u_t}$

The coefficient was estimated by using the median and the mean of $\ln y_t - \ln x_t$ as an alternative estimate for $\ln \beta$. The estimated coefficients are comparable to those of the approach of (2.2) for both procurement and RDTE data. In other words, both the median and mean yield reasonable estimates for the procurement data but the mean estimate for the RDTE data is affected by outlier values, although not as much as the mean for the approach of (2.2).* Closeness of fit results are also comparable to those of (2.2).

---

*$\hat{\beta}$ using the mean as estimator for (2.2) is 1.09, while $\hat{\beta}$ when using the mean as estimator for (2.3) is 1.03.

II.6. Summary

This chapter represents an effort to analyze historical data under the general working hypothesis that Congressional action on budget requests usually results in percentage adjustments at the appropriations level of aggregation. In the first section the problem of modelling appropriations as a percentage of requests was discussed. The second section was devoted to methodology. The prior work of Davis, Dempster and Wildavsky and of Stromberg was reviewed and problems with their approaches were pointed out. The methodology used in this study was then presented. Alternative formulations of, or approaches to, the percentage adjustment hypothesis were analyzed by examining estimated coefficients, residuals, and closeness of fit of the "predicted appropriations" to the actual appropriations. Each approach produced somewhat different results. The most consistent results, on the basis of various criteria discussed in section II.5, were produced by the following approaches when the median (of the appropriations request ratio or the logarithm of the ratio) was used as the estimator for β in:

(2.2) $$y_t = \beta x_t + \varepsilon_t x_t$$

(2.3) $$y_t = \beta x_t e^{u_t}$$

The other approaches, which all involve least squares estimators in one form or another, produced at one time or another results which were either unreasonable or inferior in some way to the results of the approaches (2.2) and (2.3) when medians were used as estimators.

The analysis of the residuals combined with an examination of scatterplots for the approaches if (2.2) and (2.3) — and to a certain extent for the other approaches for that matter — produced three very interesting substantive results. First, the 1954 data, coming as it did at the end of the Korean conflict, may not be part of the same population as the other data. Second, the post-1969 items, especially for procurement, show a definite pattern of negative residuals which indicates that the percentage adjustment very likely changed in 1969. Finally, in the RDTE data, the 1957-1969 Air Force RDTE items dominate the positive residuals which indicates that decision-making on Air Force items may have been different from decision-making on Army and Navy items during that period.

The chapters which follow will be devoted to analysis of the same set of data using an alternative estimation procedure. The results of using this alternative procedure will be compared with the least squares results.

APPENDIX A

Stem and leaf plots are a method of quickly summarizing a large set of numbers. They are discussed extensively by John Tukey in his forthcoming book, <u>Exploratory Data Analysis</u> [Tukey]. The basic technique is best described by means of an example. Suppose one wants to summarize the following set of numbers which might be percentages of requests appropriated using (2.4) for various sizes of the request:

.981	.965	.921	.972	.991	.985
.995	.998	.943	.946	.993	.986
.990	.943	.954	.988	.964	.981
.982	.986	.955	.985	.966	.942
.976	.987	.968	.973	.987	.990
.995	.997	.994	.998	.991	.993

TABLE A.1

A stem and leaf plot of these numbers with the stem being the first two digits and the leaves being the third digit is given in Figure A.1.

```
.92  1
.93
.94  2,3,6
.95  4,5
.96  4,5,6,8
.97  2,3,6,8
.98  1,1,2,5,5,6,7,8
.99  0,0,0,1,1,3,3,4,5,7,8
```

FIGURE A.1

The plot reveals that the distribution is somewhat skewed left and also emphasizes a potential outlier, .921.

APPENDIX B

This appendix contains scatterplots and stem and leaf plots for residuals for each of the equations which were subjected to analysis in this chapter. It should be noted that in this appendix the term "residuals" refers to the following:

(2.1') $y_t - \hat{\beta} x_t$

(2.2') $\frac{y_t}{x_t} - \hat{\beta}$

(2.3') $\ln y_t - \ln x_t - \ln \hat{\beta}$

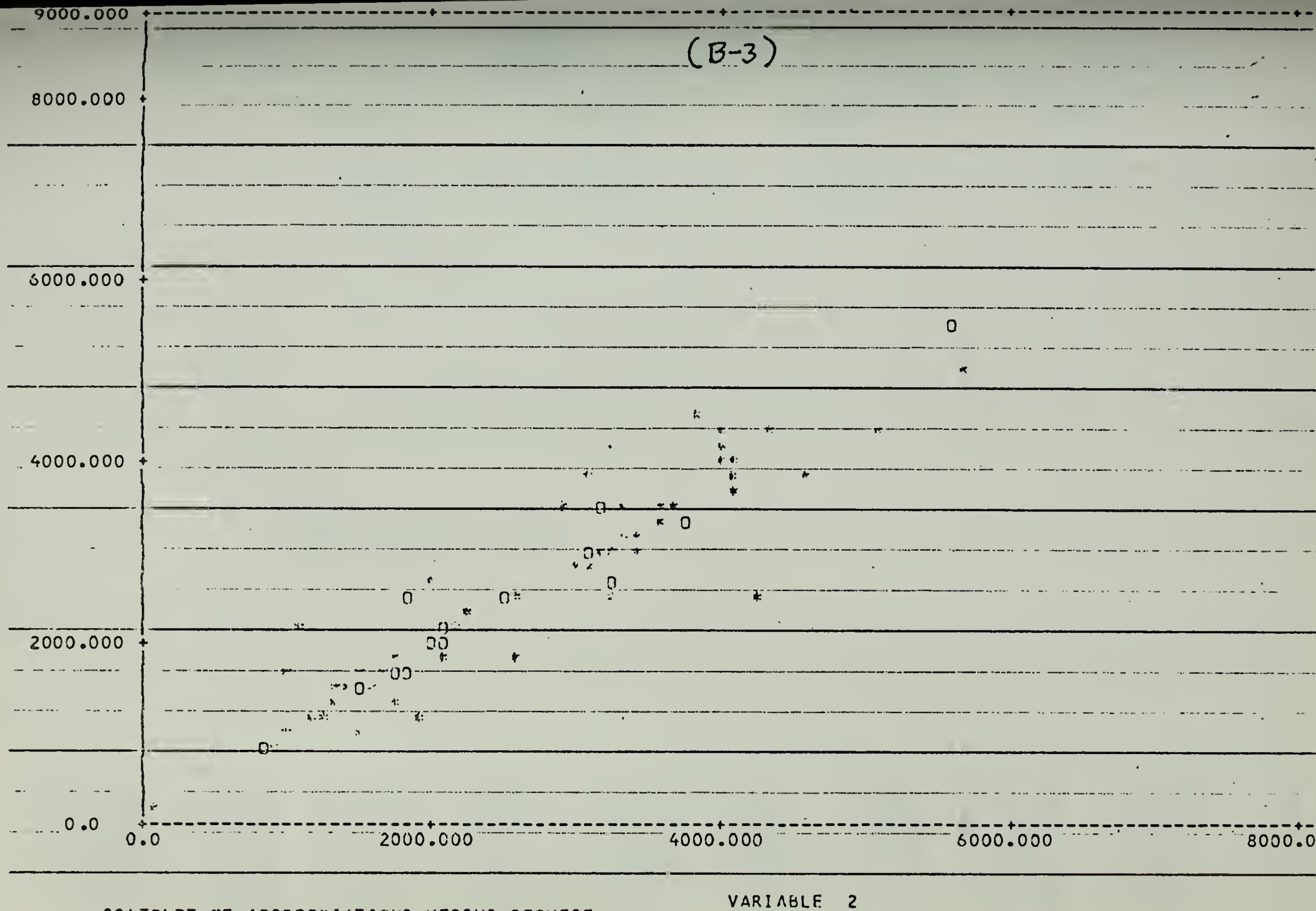
(2.4') $\ln y_t - \hat{\alpha} \ln x_t - \ln \hat{\beta}$

(2.5') $\frac{\ln y_t}{\ln x_t} - \hat{\alpha} - \ln \hat{\beta} \left(\frac{1}{\ln x_t}\right)$

Since in many cases the plots are not identified explicitly on the page on which they appear, the following list identifies each plot and gives its page number. For the $\ln x_t$ versus $\ln y_t$ scatterplot for (2.4) and the $\frac{1}{\ln x_t}$ versus $\frac{\ln y_t}{\ln x_t}$ scatterplot for (2.5) the least squares lines are included. These are indicated by the symbol "YY". The pages containing the stem and leaf plots of the residuals contain a complete listing of the residuals in addition to various measures such as mean, median and so forth.

*The small circles on the scatterplots indicate that more than one observation has been plotted in approximately the same place.

	Procurement Page	R&D Page
(2.5) $y_t = \beta x_t^{\alpha} e^{u_t \ln x_t}$		
Scatterplot of $\frac{1}{\ln x_t}$ versus $\frac{\ln y_t}{\ln x_t}$	B-18	B-37
Stem and leaf of residuals	B-19	B-38
Scatterplot of $(\frac{\ln y_t}{\ln x_t})$ "predicted" versus residuals	B-20	B-39
Scatterplot of $\ln x_t$ versus residuals	B-21	B-40

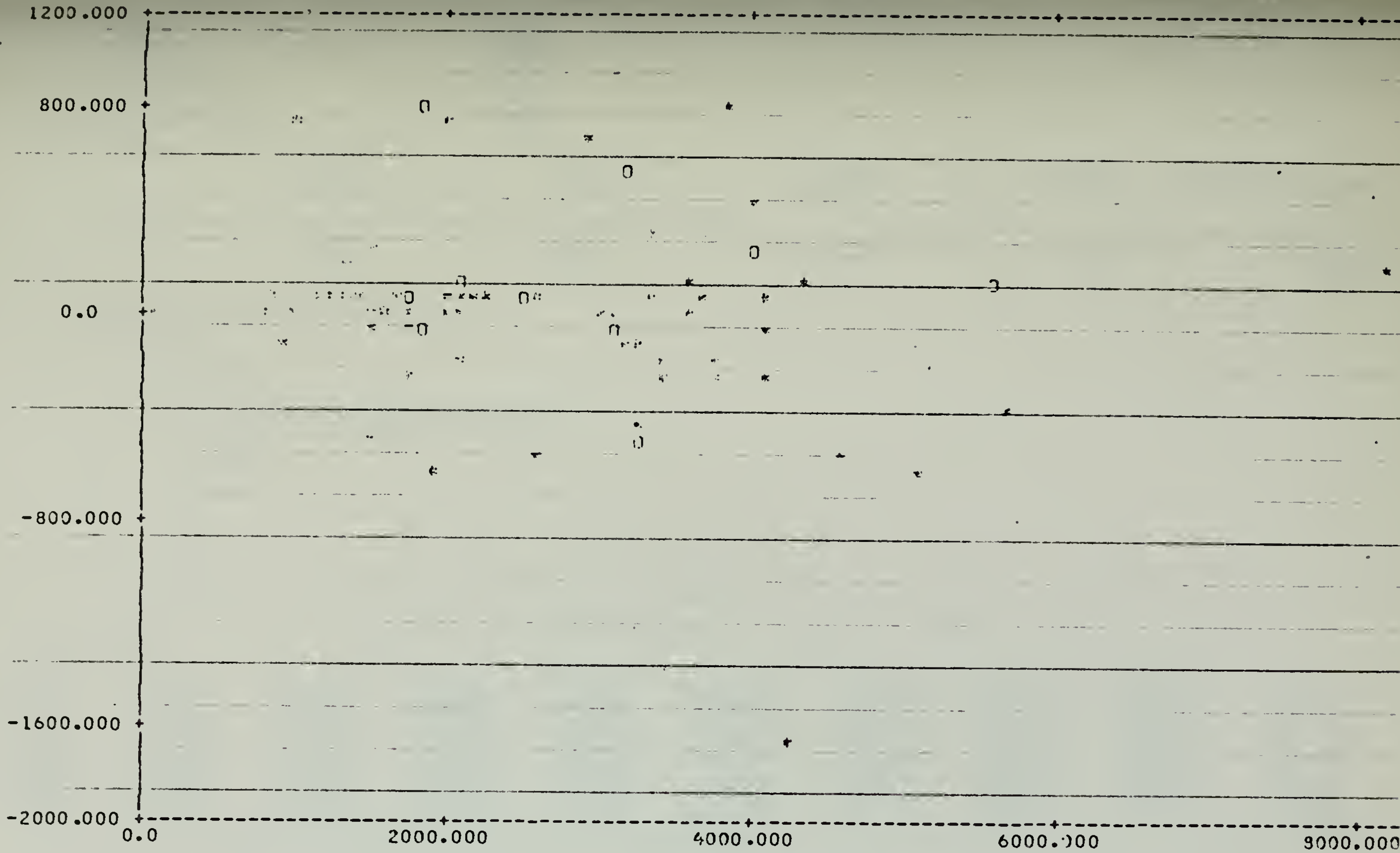


SCATPLOT OF APPROPRIATIONS VERSUS REQUEST

(B-4)

```
TRIMEAN                         31.389
LOWER HINGE                    -90.852
UPPER HINGE                    122.325
LOWEST VALUE                 -1655.839
HIGHEST VALUE                 1199.263

THE FOLLOWING VALUES WERE FOUND TO BE LESS THAN        -304.030 WHICH IS THE LOWER SIDE POINT.
         -1655.839            -623.470            -609.401            -565.214            -552.1
          -517.209            -511.465            -484.462            -441.544            -366.74

THE FOLLOWING VALUES WERE FOUND TO BE GREATER THAN       335.504 WHICH IS THE UPPER SIDE POINT.
           438.118             528.016             554.368             681.931             738.4
           761.900             782.600             802.622             829.526             893.8
          1199.263

STEM-AND-LEAF PLOT FOR X( 5).

   -16 | 6
   -15 |
   -14 |
   -13 |
   -12 |
   -11 |
   -10 |
    -9 |
    -8 |
    -7 |
    -6 | 21
    -5 | 7521
    -4 | 84
    -3 | 7
    -2 | 76620
    -1 | 76440
    -0 | 8555544431110
     0 | 12222233345555566677789
     1 | 1113447
     2 | 1267
     3 | 1
     4 | 4
     5 | 35
     6 | 8
     7 | 468
     8 | 039
     9 |
    10 |
    11 |
    12 | 0

INFO PR(5)=T

 -552.1518  738.4891  129.7980   -52.7270    72.4910    16.4630    60.2500    48.1748   38.6535   X( 5)
  174.7268  261.0555  438.1184    33.4956  -136.8470    21.3992   681.9311   528.0161  110.5831   X( 5)
  216.4327 -201.2416 -161.4643  -170.9782   -41.4107  1199.2634    61.7877  -511.4653  -50.5505   X( 5)
   82.3899   32.2421 -255.4967    84.6048   271.6980   212.4485   -29.2605  -274.8164   50.6986   X( 5)
 -216.5044  -38.2196   -3.8526  -102.4194   -52.3818   -41.0225  -258.1549    31.2356   16.1211   X( 5)
  114.8535  893.8325  136.8734  -609.4014  -517.2090    -5.9577 -1655.8385   829.5259   13.6901   X( 5)
  306.2996   49.0515  137.4710   -79.2853  -623.4699    58.5014    16.1231   802.6215  107.5305   X( 5)
   85.0319  554.3675  -47.9817  -441.5442   -11.4996  -141.2130    53.0685  -356.7470  -14.0626   X( 5)
   68.4775  761.9000  782.6001    70.0650    51.3915  -484.4622  -565.2140    23.8040              X( 5)
STEM AND LEAF OF RESIDUALS FROM LEAST SQUARES FIT
```

SCATTER PLOT

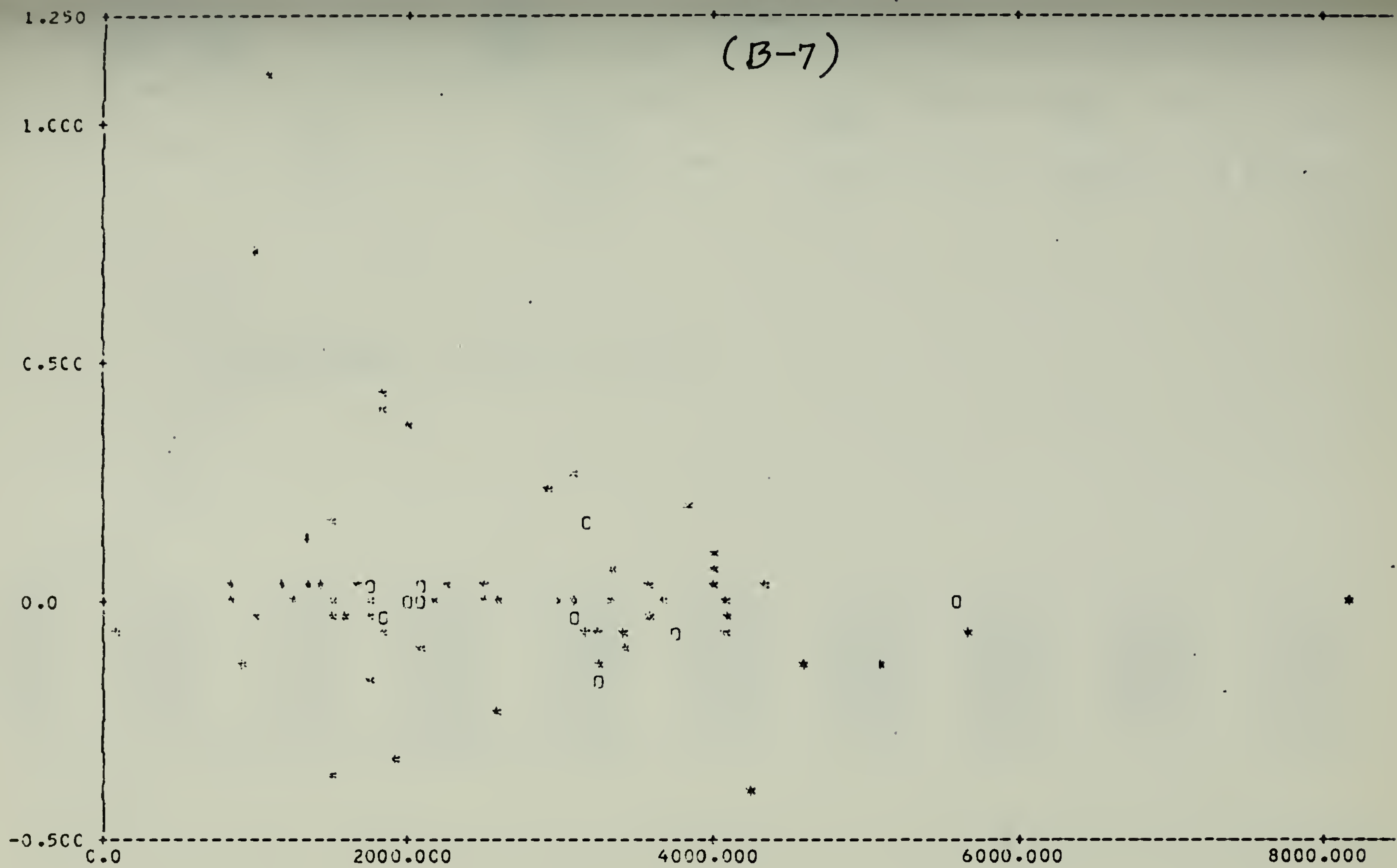


SCAT PLOT OF RESIDUAS VERSUS X FROM LEAST SQUARES FIT

INPT NOBS=63. NVIN=2. SINGLE=T

```
PROPERTIES OF THE  90 OBSERVATIONS OF VARIABLE   4:

          MEAN                                0.020
          MEDIAN                              0.000
          MIDMEAN                            -0.003
          TRIMEAN                            -0.004
          LOWER HINGE                        -0.057
          UPPER HINGE                         0.026
          LOWEST VALUE                       -0.401
          HIGHEST VALUE                       1.106

          THE FOLLOWING VALUES WERE FOUND TO BE LESS THAN           -0.141 WHICH IS THE LOWER SIDE POINT:
                           -0.401               -0.353         -0.338               -0.231               -0.175
                           -0.164               -0.164         -0.150

          THE FOLLOWING VALUES WERE FOUND TO BE GREATER THAN           0.110 WHICH IS THE UPPER SIDE POINT.
                            0.145                0.154          0.163               0.169                0.201
                            0.218                0.278          0.367               0.413                0.431
                            0.747                1.106

          STEM-AND-LEAF PLOT FOR X( 4).

                -4 | 0
                -3 | 54
                -2 | 3
                -1 | 76654321
                -0 | 9887666666444433322221110000
                 0 | 001111111112222222333334445 58
                 1 | 04567
                 2 | 028
                 3 | 7
                 4 | 13
                 5 |
                 6 |
                 7 | 5
                 8 |
                 9 |
                10 |
                11 | 1

       INFO PR(4)=T

 -0.2211   1.1060   0.7472   0.4311  -0.0620  -0.3381   0.3669   0.0072  -0.4007     X( 4)
  0.1543  -0.0096  -0.3529  -0.0253   0.1448   0.0127  -0.0582  -0.0425   0.0114     X( 4)
  0.0784   0.0052  -0.0793  -0.1343  -0.0565  -0.0575  -0.0940  -0.0160  -0.1225     X( 4)
  0.0222  -0.0424   0.0482  -0.0065   0.0402   0.2775  -0.0313   0.0187   0.0232     X( 4)
  0.0264  -0.0087  -0.1746  -0.1638  -0.0640  -0.0016  -0.0761   0.0138   0.0507     X( 4)
  0.2003  -0.0240   0.0951   0.0176   0.2183   0.1627  -0.0223  -0.0007   0.0170     X( 4)
  0.0405   0.0105  -0.1367  -0.0714  -0.0024  -0.0295  -0.1497   0.0053   0.0443     X( 4)
  0.1690  -0.0230  -0.1640   0.4131  -0.1088   0.0029   0.0100   0.0103   0.0264     X( 4)
  0.0264   0.0264   0.0242  -0.0006   0.0006  -0.0412  -0.0572  -0.0351              X( 4)
   THESE ARE THE RESIDUALS WITH THE MEDIAN AS THE ESTIMATOR

       INFO FU(4)=T

       INFO SCATX(2)=4
```

SCATTER PLOT



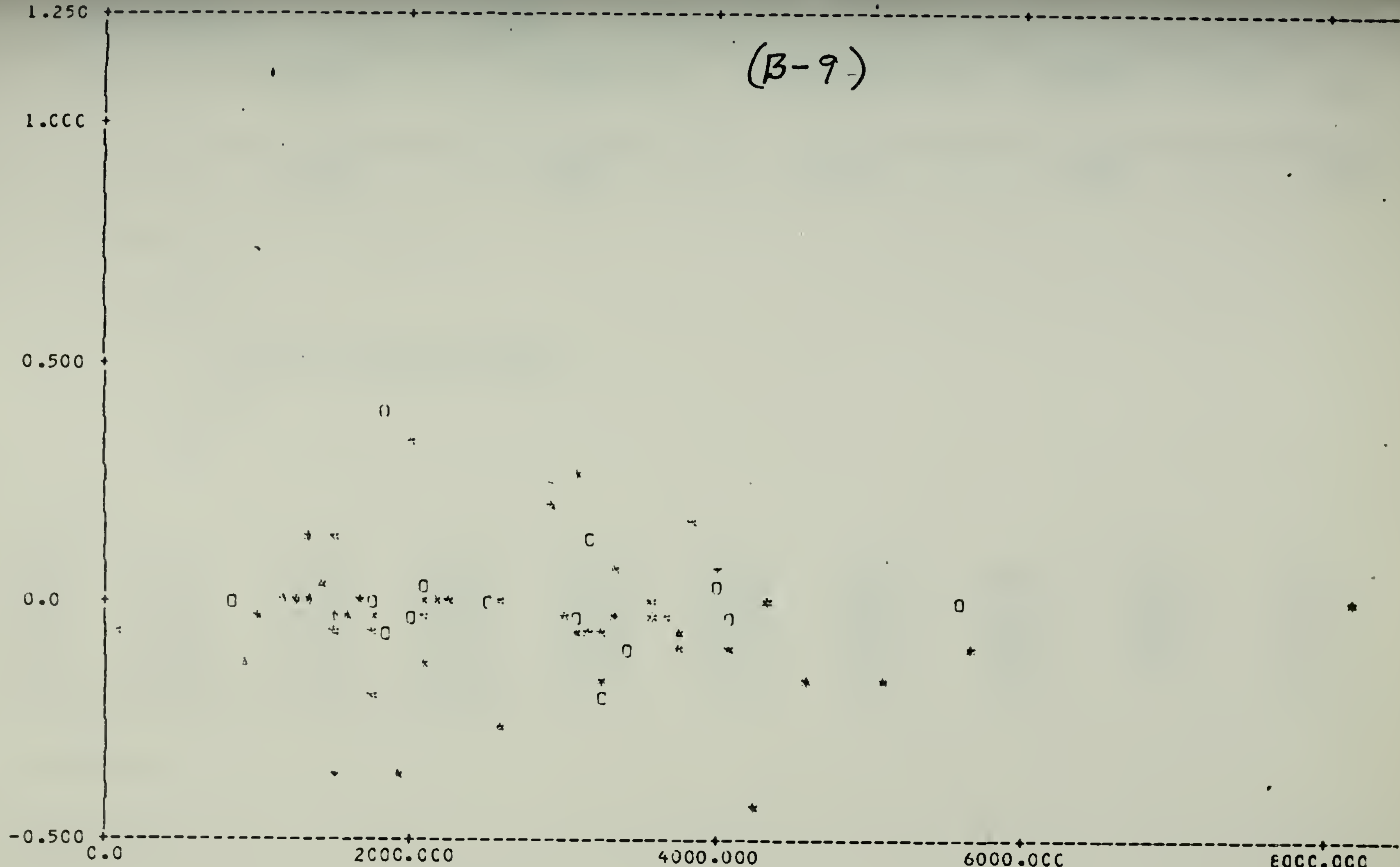
INFC FX(4)=Y

(B-8)

```
MEDIAN                                  -0.020
MIDMEAN                                 -0.022
TRIMEAN                                 -0.024
LOWER HINGE                             -0.077
UPPER HINGE                              0.007
LOWEST VALUE                            -0.421
HIGHEST VALUE                            1.086

THE FOLLOWING VALUES WERE FOUND TO BE LESS THAN          -0.161 WHICH IS THE LOWER SIDE POINT.
             -0.421             -0.373            -0.358             -0.251            -0.195
             -0.184             -0.184            -0.170

THE FOLLOWING VALUES WERE FOUND TO BE GREATER THAN        0.090 WHICH IS THE UPPER SIDE POINT.
              0.125              0.134             0.143              0.149             0.181
              0.198              0.258             0.347              0.393             0.411
              0.727              1.086

STEM-AND-LEAF PLOT FOR X( 4).

    -4 | 2
    -3 | 76
    -2 | 5
    -1 | 98876543100
    -0 | 9888888666655544443332222221111111111000
     0 | 00011112223368
     1 | 23458
     2 | 06
     3 | 59
     4 | 1
     5 |
     6 |
     7 | 3
     8 |
     9 |
    10 | 9

INFO FR(4)=T

-0.2510   1.0861   0.7273   0.4112  -0.0818  -0.3580   0.3470  -0.0127  -0.4206   X( 4)
 0.1344  -0.0294  -0.3728  -0.0452   0.1249  -0.0072  -0.0781  -0.0624  -0.0084   X( 4)
 0.0585  -0.0147  -0.0992  -0.1542  -0.0764  -0.0774  -0.1139  -0.0359  -0.1423   X( 4)
 0.0023  -0.0623   0.0283  -0.0264   0.0203   0.2576  -0.0512  -0.0012   0.0033   X( 4)
 0.0065  -0.0286  -0.1945  -0.1837  -0.0839  -0.0215  -0.0960  -0.0061   0.0308   X( 4)
 0.1809  -0.0439   0.0752  -0.0023   0.1984   0.1428  -0.0422  -0.0206  -0.0028   X( 4)
 0.0207  -0.0094  -0.1565  -0.0913  -0.0223  -0.0494  -0.1696  -0.0146   0.0244   X( 4)
 0.1491  -0.0429  -0.1839   0.3932  -0.1287  -0.0170  -0.0099  -0.0096   0.0065   X( 4)
 0.0065   0.0065   0.0043  -0.0205  -0.0193  -0.0610  -0.0771  -0.0550            X( 4)
 THESE ARE THE RESIDUALS WITH THE MEAN AS ESTIMATOR

INFO PU(4)=T

INFO SCATX(2)=4
```

SCATTER PLOT

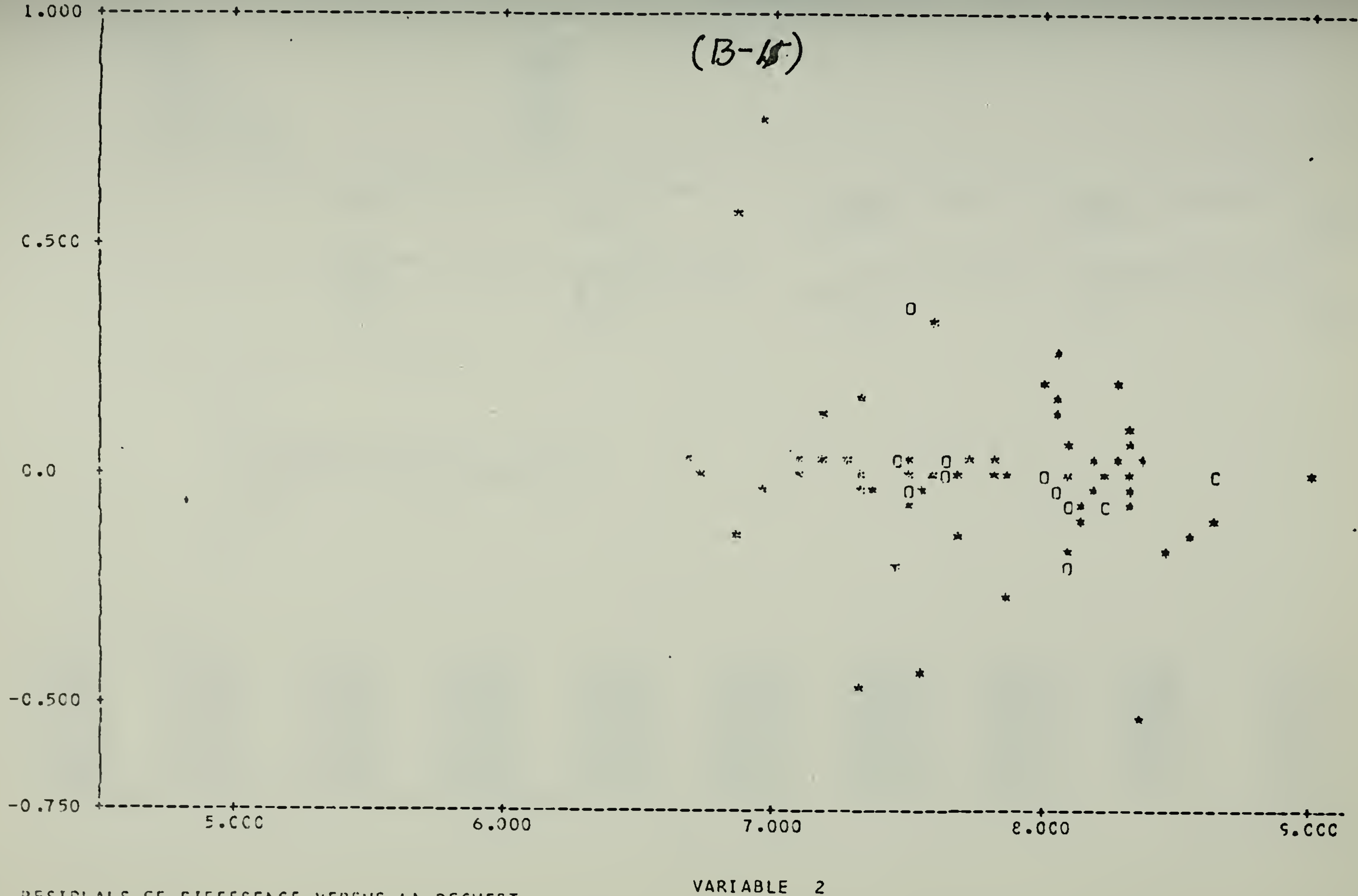
(B-9)



INFO FX(4)=1

```
LOWER HINGE                    -0.061
UPPER HINGE                     0.027
LOWEST VALUE                   -0.530
HIGHEST VALUE                   0.759

THE FOLLOWING VALUES WERE FOUND TO BE LESS THAN        -0.148 WHICH IS THE LOWER SIDE POINT.
          -0.530              -0.450          -0.427              -0.271              -0.198
          -0.184              -0.184          -0.167              -0.151              -0.148

THE FOLLOWING VALUES WERE FOUND TO BE GREATER THAN       0.114 WHICH IS THE UPPER SIDE POINT.
           0.139               0.147           0.155               0.160               0.188
           0.202               0.251           0.320               0.354               0.367
           0.570               0.759

STEM-AND-LEAF PLOT FOR X( 4).

   -5 | 3
   -4 | 53
   -3 |
   -2 | 70
   -1 | 88755320
   -0 | 888776666444433333222111000
    0 | 0011111111122222223333444455589
    1 | 45569
    2 | 05
    3 | 257
    4 |
    5 | 7
    6 |
    7 | 6

INFO PR(4)=T

-0.2710   0.7589   0.5695   0.3666  -0.0657  -0.4266   0.3198   0.0074  -0.5303     X( 4)
 0.1471  -0.0099  -0.4502  -0.0264   0.1387   0.0130  -0.0616  -0.0446   0.0117     X( 4)
 0.0775   0.0052  -0.0849  -0.1485  -0.0598  -0.0609  -0.1015  -0.0166  -0.1344     X( 4)
 0.0225  -0.0445   0.0483  -0.0067   0.0405   0.2508  -0.0327   0.0190   0.0235     X( 4)
 0.0268  -0.0090  -0.1977  -0.1843  -0.0680  -0.0017  -0.0814   0.0141   0.0507     X( 4)
 0.1875  -0.0250   0.0932   0.0179   0.2023   0.1546  -0.0232  -0.0007   0.0174     X( 4)
 0.0409   0.0108  -0.1512  -0.0762  -0.0025  -0.0308  -0.1670   0.0055   0.0445     X( 4)
 0.1601  -0.0239  -0.1845   0.3537  -0.1185   0.0029   0.0102   0.0105   0.0268     X( 4)
 0.0263   0.0268   0.0245  -0.0007   0.0007  -0.0432  -0.0606  -0.0368              X( 4)
RESIDUALS FOR DIFF WITH MED AS ESTIMATOR

INFO PU(4)=T

INFO PX(4)=T
```

SCATTER PLOT

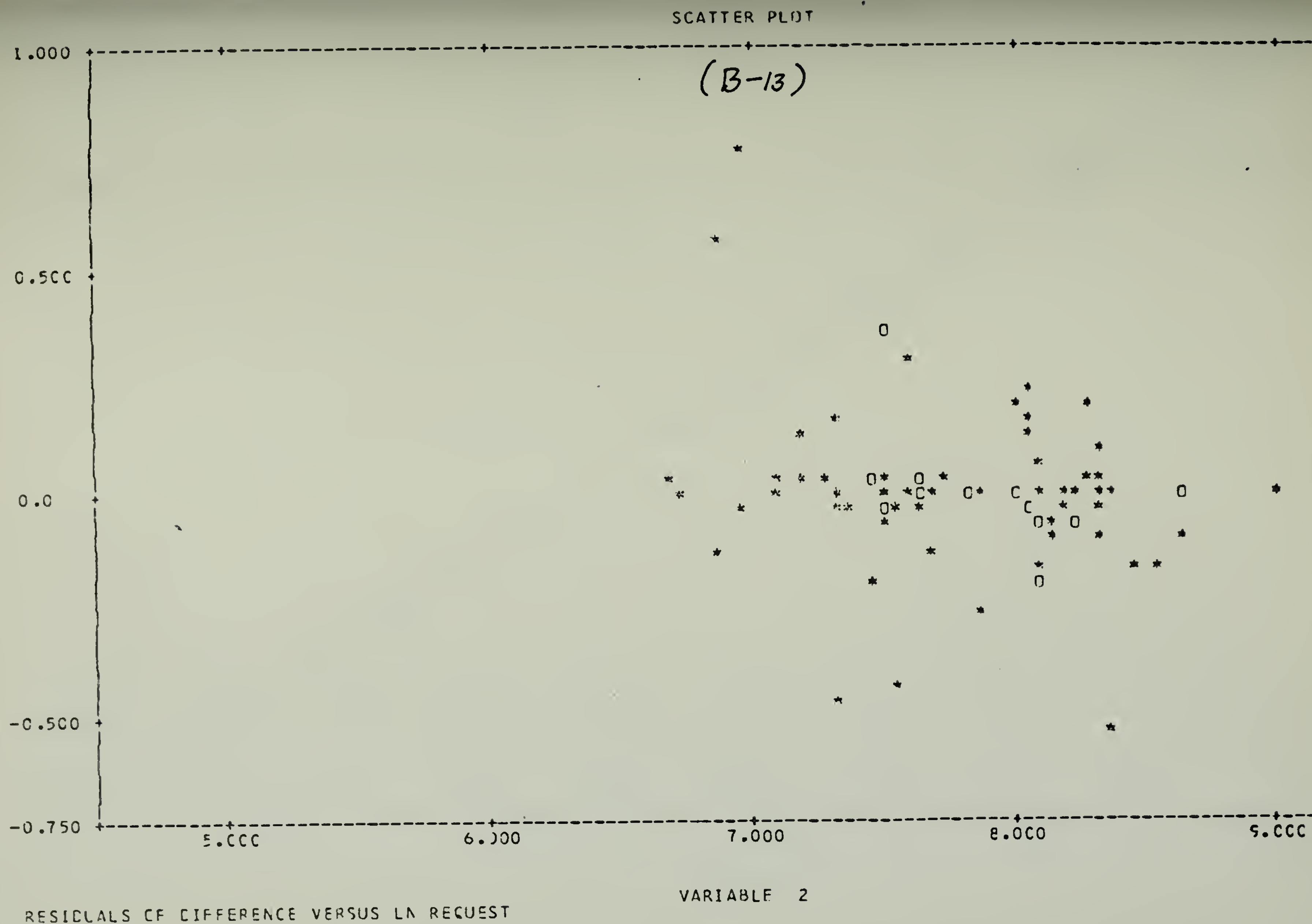


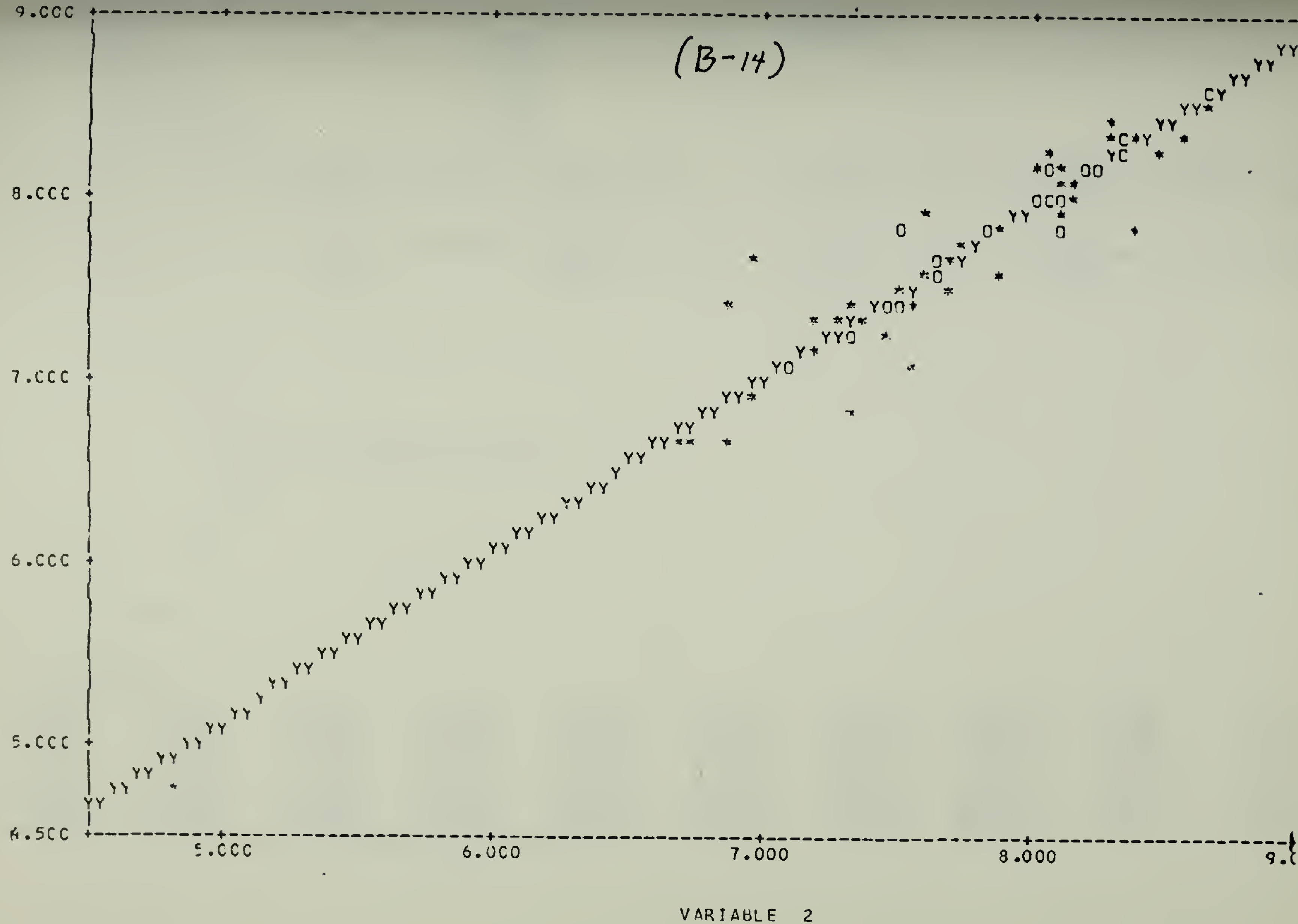
RESIDUALS OF DIFFERENCE VERSUS LN REQUEST

INFO BX(4)=T

(B-12)

```
PROPERTIES OF THE  90 OBSERVATIONS OF VARIABLE  4:

     MEAN                          0.000
     MEDIAN                       -0.003
     MIDMEAN                      -0.005
     TRIMEAN                      -0.007
     LOWER HINGE                  -0.064
     UPPER HINGE                   0.023
     LOWEST VALUE                 -0.534
     HIGHEST VALUE                 0.756

     THE FOLLOWING VALUES WERE FOUND TO BE LESS THAN        -0.152 WHICH IS THE LOWER SIDE POINT.
                   -0.534              -0.454             -0.430              -0.274              -0.201
                   -0.188              -0.188             -0.170              -0.155              -0.152

     THE FOLLOWING VALUES WERE FOUND TO BE GREATER THAN        0.111 WHICH IS THE UPPER SIDE POINT.
                    0.135               0.144              0.151               0.157               0.184
                    0.199               0.247              0.316               0.350               0.363
                    0.566               0.756

     STEM-AND-LEAF PLOT FOR X( 4).

          -5 | 3
          -4 | 53
          -3 |
          -2 | 70
          -1 | 99755420
          -0 | 9897766665554433333221110000
           0 | 00011111112222222224444579
           1 | 44563
           2 | 05
           3 | 256
           4 |
           5 | 7
           6 |
           7 | 6

INFO FR(4)=T

-0.2743   0.7556   0.5662   0.3632  -0.0691  -0.4299   0.3164   0.0040  -0.5336   X( 4)
 0.1438  -0.0132  -0.4535  -0.0297   0.1353   0.0096  -0.0650  -0.0480   0.0084   X( 4)
 0.0741   0.0020  -0.0883  -0.1518  -0.0631  -0.0642  -0.1049  -0.0199  -0.1377   X( 4)
 0.0192  -0.0478   0.0450  -0.0100   0.0372   0.2475  -0.0360   0.0157   0.0202   X( 4)
 0.0234  -0.0123  -0.2010  -0.1876  -0.0713  -0.0050  -0.0847   0.0108   0.0474   X( 4)
 0.1842  -0.0283   0.0899   0.0146   0.1990   0.1512  -0.0265  -0.0040   0.0140   X( 4)
 0.0275   0.0074  -0.1546  -0.0795  -0.0058  -0.0341  -0.1703   0.0021   0.0412   X( 4)
 0.1567  -0.0272  -0.1878   0.3503  -0.1218  -0.0004   0.0069   0.0072   0.0234   X( 4)
 0.0234   0.0234   0.0212  -0.0040  -0.0027  -0.0465  -0.0639  -0.0401            X( 4)
 RESIDUALS FOR DIFF WITH MEAN AS ESTIMATOR

INFO PU(4)=T

INFO HX(4)=T
```

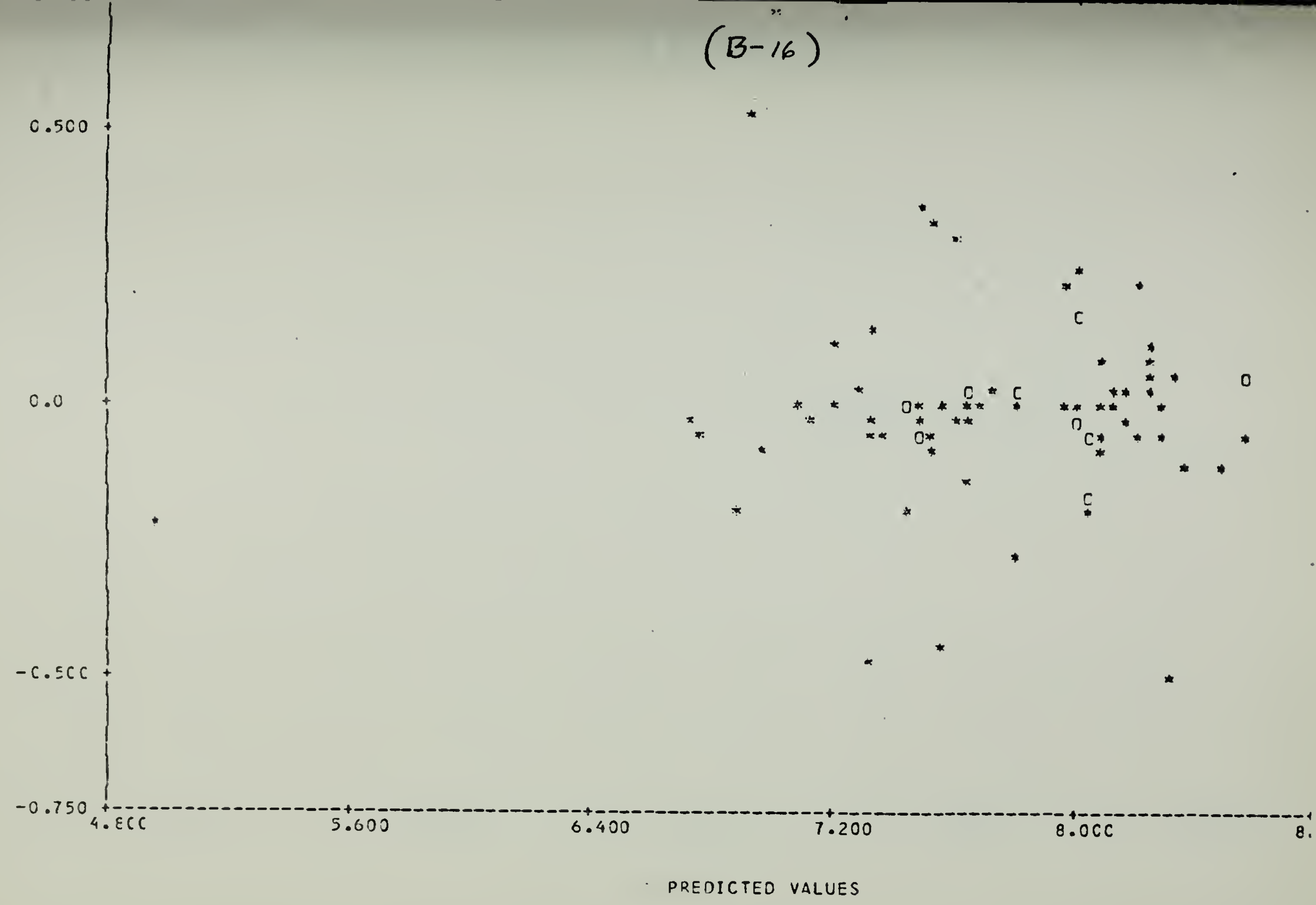

SCATTER PLOT
(B-13)
1.000
0.5CC
0.0
-C.5C0
-0.750
5.CCC
6.)00
7.000
8.0C0
9.CCC
VARIABLE 2
RESICLALS CF CIFFERENCE VERSUS LN RECUEST


(B-14)
9.CCC
8.CCC
7.CCC
6.CCC
5.CCC
4.5CC
5.CCC
6.000
7.000
8.000
VARIABLE 2

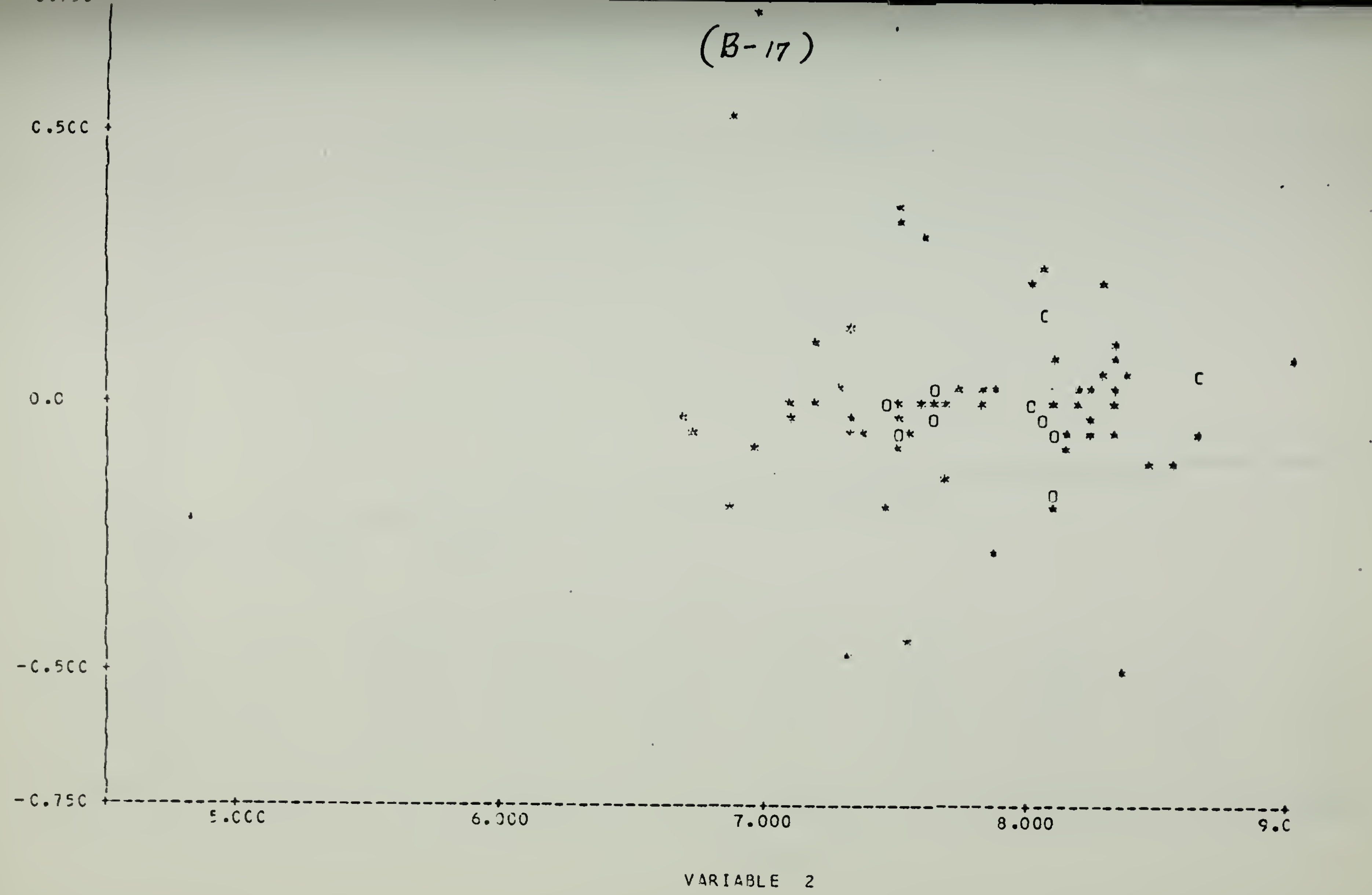
(B-15)

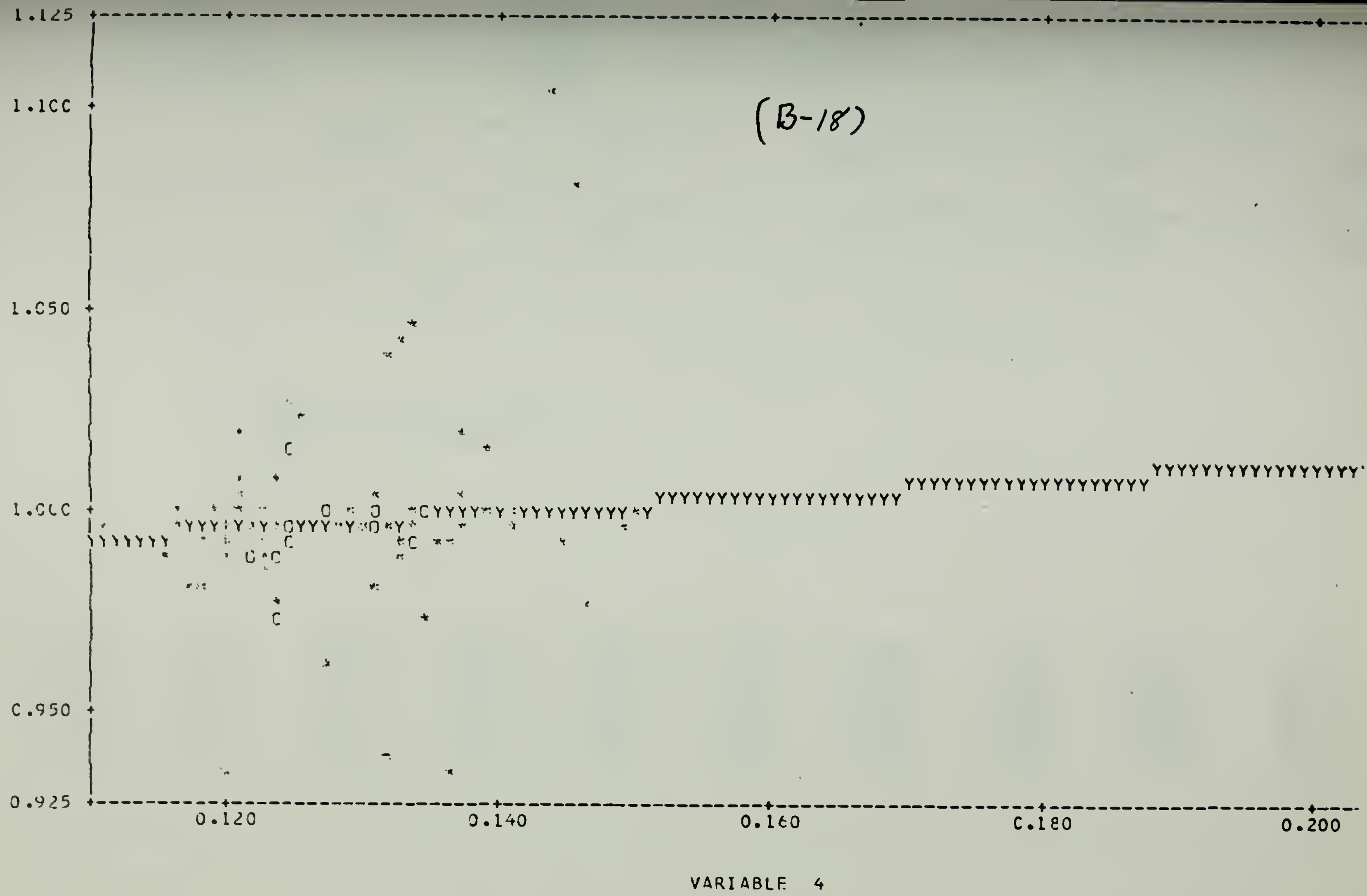
```
PROPERTIES OF THE  90 OBSERVATIONS OF VARIABLE  3:

   MEAN                          0.000
   MEDIAN                       -0.006
   MIDMEAN                      -0.011
   TRIMEAN                      -0.012
   LOWER HINGE                  -0.055
   UPPER HINGE                   0.043
   LOWEST VALUE                 -0.502
   HIGHEST VALUE                 0.714

   THE FOLLOWING VALUES WERE FOUND TO BE LESS THAN        -0.153 WHICH IS THE LOWER SIDE POINT.
               -0.502            -0.477             -0.440            -0.270            -0.223
               -0.204            -0.185             -0.184            -0.171            -0.153

   THE FOLLOWING VALUES WERE FOUND TO BE GREATER THAN      0.140 WHICH IS THE UPPER SIDE POINT.
                0.159             0.166              0.210             0.211             0.261
                0.308             0.337              0.349             0.520             0.714


   STEM-AND-LEAF PLOT FOR X( 3).

         -5 | 0
         -4 | 84
         -3 |
         -2 | 720
         -1 | 9875321
         -0 | 8877666555555443333222211100
          0 | 0000011111222223445556789
          1 | 12367
          2 | 116
          3 | 145
          4 |
          5 | 2
          6 |
          7 | 1

INFO PU(3)=T


INFO PR(3)=T

-0.2701    0.7144    0.5198    0.3494   -0.2231   -0.4404    0.3080    0.0697   -0.5021      X( 3)
 0.1589   -0.0001   -0.4767   -0.0732    0.1058    0.0141   -0.0487   -0.0625   -0.0259      X( 3)
 0.0922    0.0474   -0.0424   -0.1114   -0.0465   -0.0401   -0.0849   -0.0275   -0.1855      X( 3)
 0.0024   -0.0603    0.0387   -0.0155    0.0317    0.2614   -0.0220    0.0193    0.0187      X( 3)
 0.0092    0.0014   -0.1844   -0.1708   -0.0515    0.0238   -0.0553    0.0049    0.0757      X( 3)
 0.2103    0.0012    0.1180    0.0466    0.2107    0.1664   -0.0047    0.0193    0.0357      X( 3)
 0.0649    0.0529   -0.1192   -0.0546    0.0123   -0.0192   -0.1532   -0.0524    0.0158      X( 3)
 0.1327   -0.0481   -0.2041    0.3373   -0.1271   -0.0095    0.0102    0.0032    0.0074      X( 3)
-0.0333   -0.0122   -0.0081   -0.0189   -0.0267   -0.0690   -0.0768   -0.0535                X( 3)
 THESE ARE THE RESIDUALS FOR THE LN REGRESSION


 INFO HX(3)=T
```

(B-16)


0.500
0.0
-0.500
-0.750
4.800
5.600
6.400
7.200
8.000
8.
PREDICTED VALUES

(B-17)

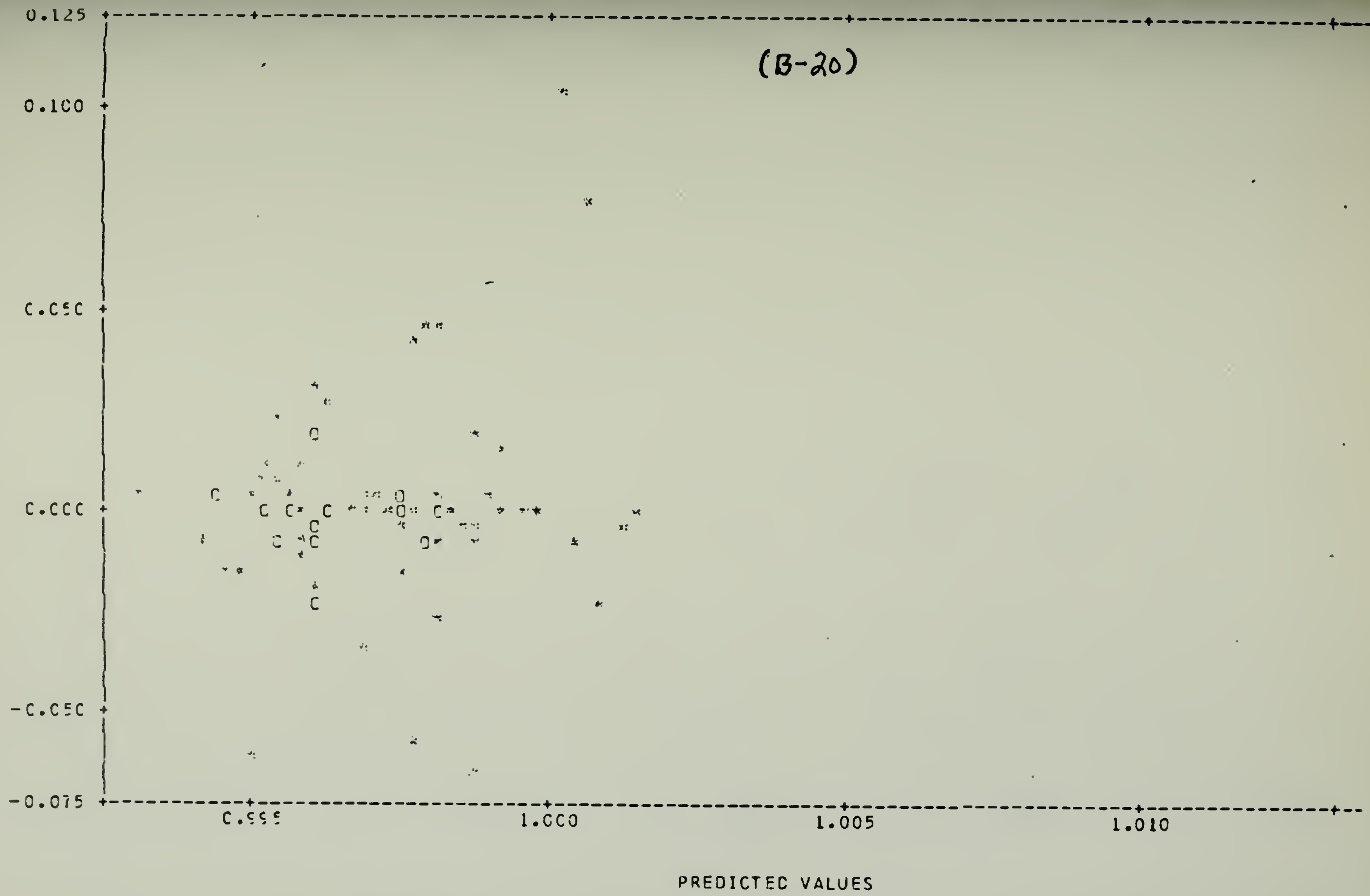



(B-18)
1.125
1.1CC
1.C50
1.CCC
C.950
0.925
0.120
0.140
0.160
C.180
0.200
VARIABLE 4

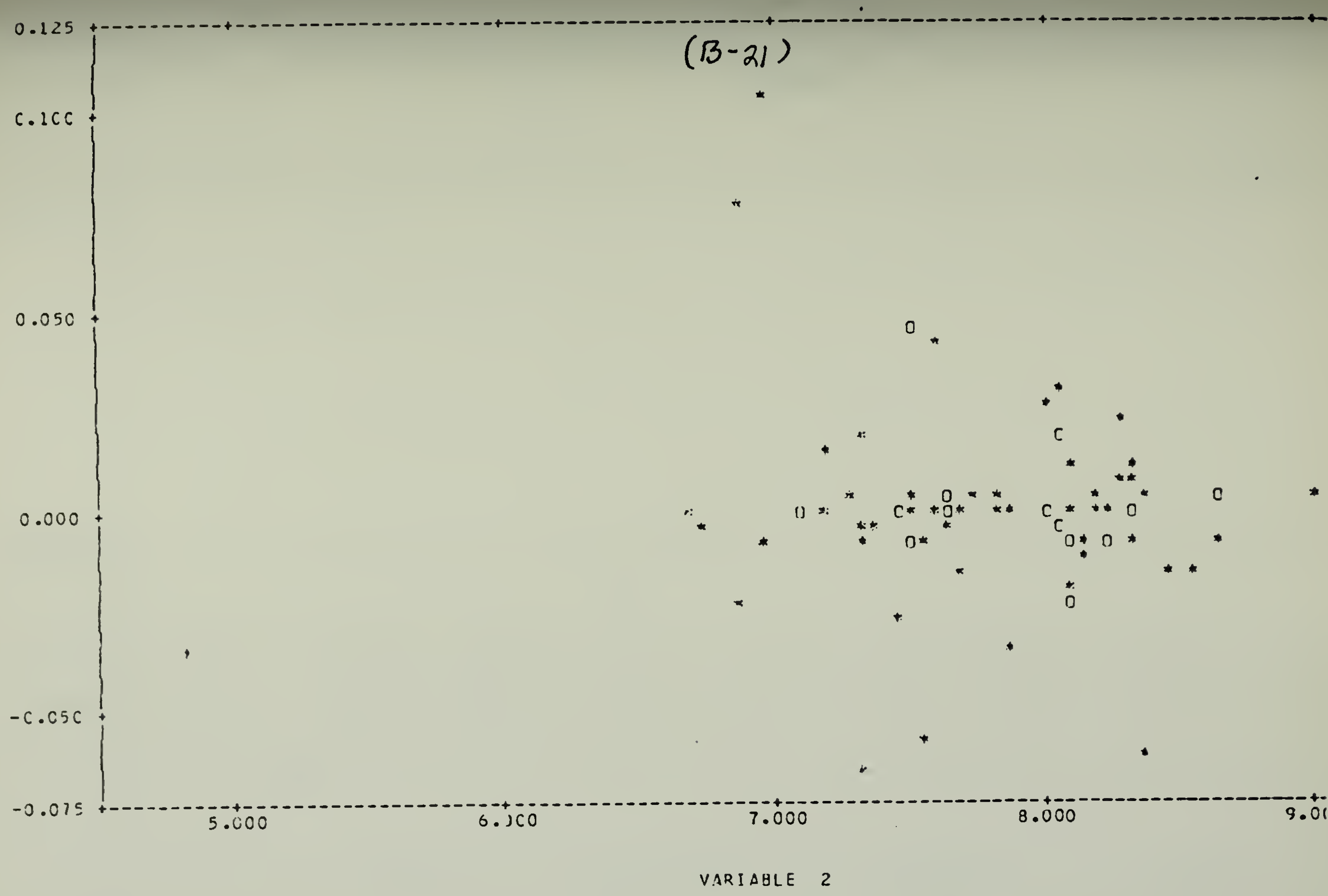
(B-19)

```
MEDIAN                           -0.001
MIDMEAN                          -0.001
TRIMEAN                          -0.002
LOWER HINGE                      -0.009
UPPER HINGE                       0.004
LOWEST VALUE                     -0.064
HIGHEST VALUE                     0.105

THE FOLLOWING VALUES WERE FOUND TO BE LESS THAN          -0.019 WHICH IS THE LOWER SIDE POINT.
              -0.064            -0.062          -0.058            -0.035            -0.033
              -0.027            -0.024          -0.024            -0.022            -0.020

THE FOLLOWING VALUES WERE FOUND TO BE GREATER THAN        0.015 WHICH IS THE UPPER SIDE POINT.
               0.016             0.019           0.019             0.020             0.024
               0.026             0.032           0.041             0.046             0.047
               0.078             0.105

STEM-AND-LEAF PLOT FOR X( 5).

     -6 | 42
     -5 | 8
     -4 |
     -3 | 53
     -2 | 74420
     -1 | 66520
     -0 | 8888777776655433222222221111
      0 | 001111111112222334444445568
      1 | 03699
      2 | 046
      3 | 2
      4 | 167
      5 |
      6 |
      7 | 8
      8 |
      9 |
     10 | 5

-0.0347   0.1048   0.0783   0.0473  -0.0331  -0.0577   0.0409   0.0046  -0.0617   X( 5)
 0.0189  -0.0008  -0.0639  -0.0030   0.0163   0.0015  -0.0069  -0.0076  -0.0017   X( 5)
 0.0104   0.0032  -0.0072  -0.0151  -0.0067  -0.0062  -0.0115  -0.0033  -0.0243   X( 5)
 0.0012  -0.0074   0.0053  -0.0013   0.0044   0.0318  -0.0035   0.0022   0.0024   X( 5)
 0.0019  -0.0006  -0.0238  -0.0221  -0.0074   0.0013  -0.0082   0.0009   0.0076   X( 5)
 0.0241  -0.0014   0.0127   0.0039   0.0257   0.0198  -0.0018   0.0011   0.0032   X( 5)
 0.0064   0.0039  -0.0159  -0.0030   0.0005  -0.0032  -0.0199  -0.0045   0.0035   X( 5)
 0.0195  -0.0054  -0.0266   0.0455  -0.0164  -0.0008   0.0010   0.0006   0.0018   X( 5)
-0.0015   0.0003   0.0005  -0.0018  -0.0024  -0.0082  -0.0096  -0.0065            X( 5)
   RESIDUALS FROM LN RATIO REGRE

INFO PU(5)=T

INFO BX(5)=T
```

(B-20)

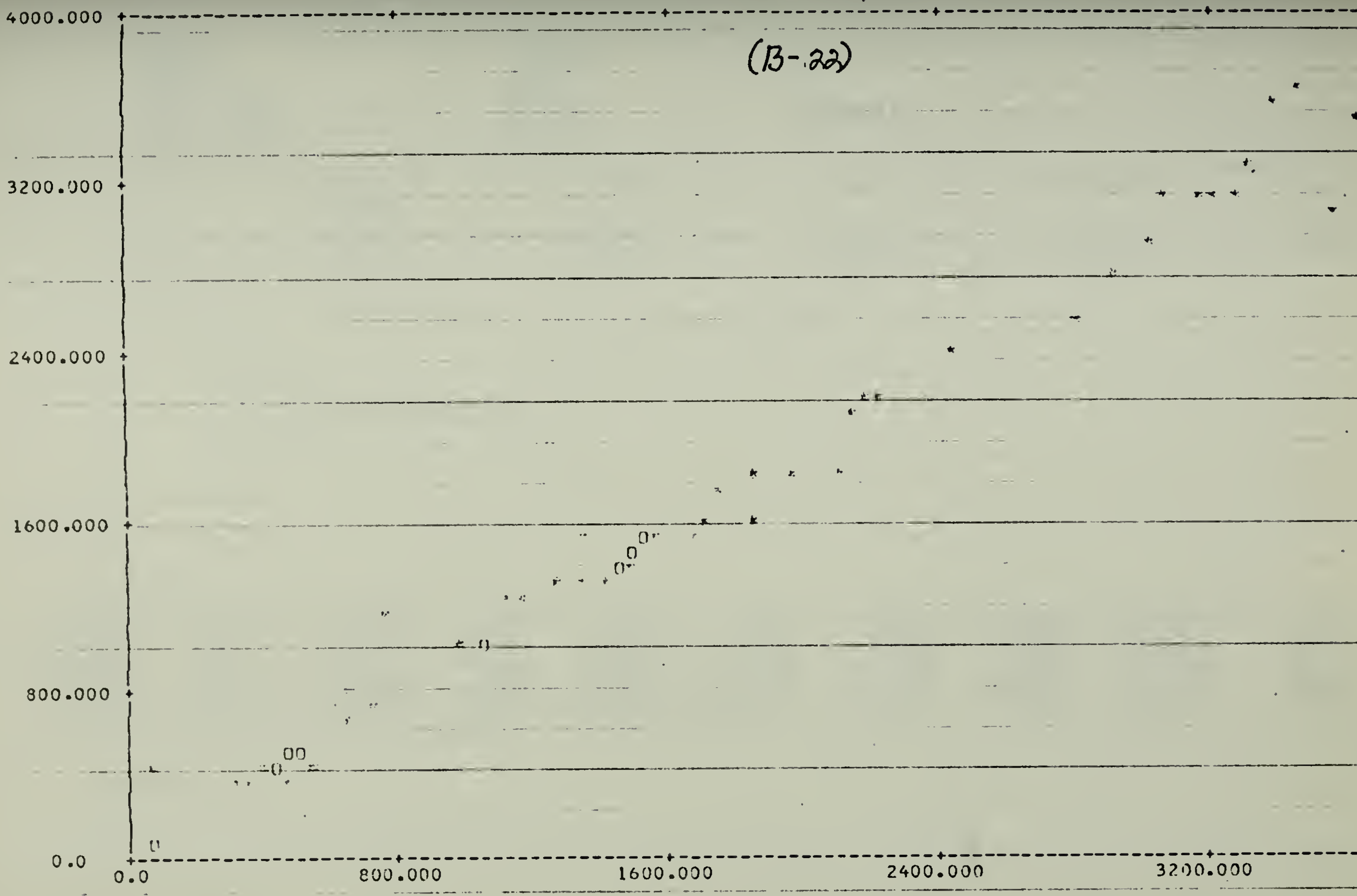


0.125
0.100
0.050
0.000
-0.050
-0.075
0.995
1.000
1.005
1.010
PREDICTED VALUES


(B-21)
0.125
0.100
0.050
0.000
-0.050
-0.075
5.000
6.000
7.000
8.000
9.00
VARIABLE 2

INFC FR(5)=T

SCATTER PLOT



SCATPLOT OF APPROPRIATIONS VERSUS REQUEST

(B-23)

MEAN	20.239
MEDIAN	7.791
MIDMEAN	5.390
TRIMEAN	5.947
LOWER HINGE	-35.162
UPPER HINGE	37.086
LOWEST VALUE	-435.786
HIGHEST VALUE	795.993

THE FOLLOWING VALUES WERE FOUND TO BE LESS THAN -107.409 WHICH IS THE LOWER SIDE POINT.

-435.786	-255.067	-219.039	-217.289	-121.355
-109.053				

THE FOLLOWING VALUES WERE FOUND TO BE GREATER THAN 109.333 WHICH IS THE UPPER SIDE POINT.

111.102	114.357	191.866	243.179	255.690
266.838	360.010	423.550	795.993	

STEM-AND-LEAF PLOT FOR X(5).

```
-4 | 4
-3 |
-2 | 622
-1 | 210
-0 | 99887655432111110000O
 0 | 11111111111122223344446679
 1 | 119
 2 | 467
 3 | 6
 4 | 2
 5 |
 6 |
 7 |
 8 | 0
```

INFO PR(5)=T

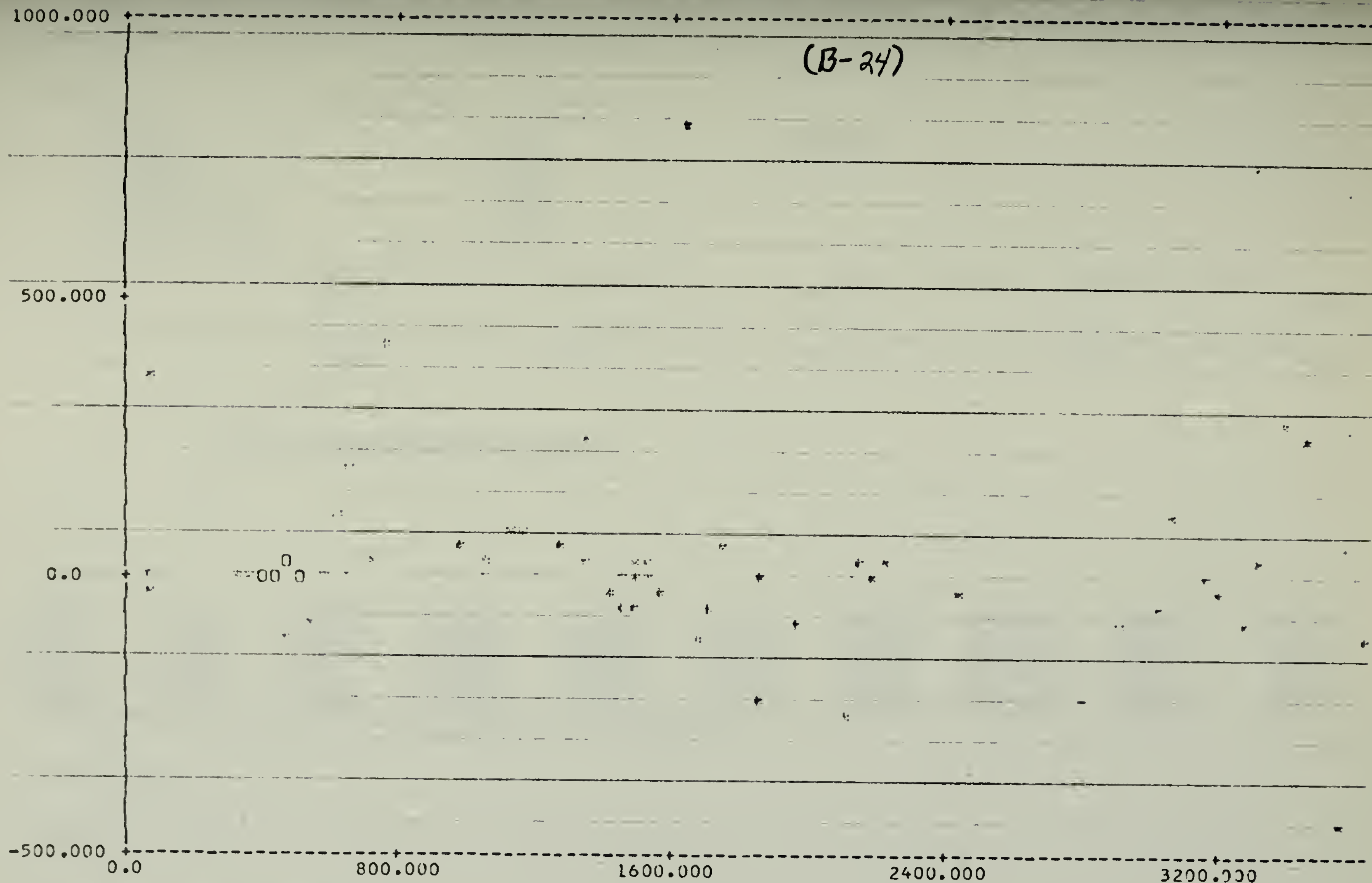
-1.8100	-121.3550	-3.5390	6.0606	7.4620	7.2800	36.2722	8.2463	18.4583	X(5)
93.3733	14.6878	-57.5623	-31.8927	-5.8429	37.4040	-1.6992	-109.0534	-219.0391	X(5)
-68.4342	-76.4827	-255.0669	-4.3223	-14.9368	360.0102	7.9934	23.6814	9.1910	X(5)
191.8662	62.6704	70.8758	57.5594	28.7268	-19.1731	-52.2953	-6.9986	41.7955	X(5)
-3.4770	33.9645	15.1493	7.7909	-14.8485	-217.2888	9.5550	-87.2266	-5.0558	X(5)
10.3740	111.1020	12.0302	37.0858	423.5500	243.1788	795.9934	255.6898	-103.1722	X(5)
-35.1616	12.9010	114.3574	17.5435	256.8375	-435.7862	-94.6435	-49.1906	-80.3280	X(5)

STEM AND LEAF OF RESIDUALS FROM LEAST SQUARES FIT

INFO HX(5)=T

SCATTER PLOT

(B-24)

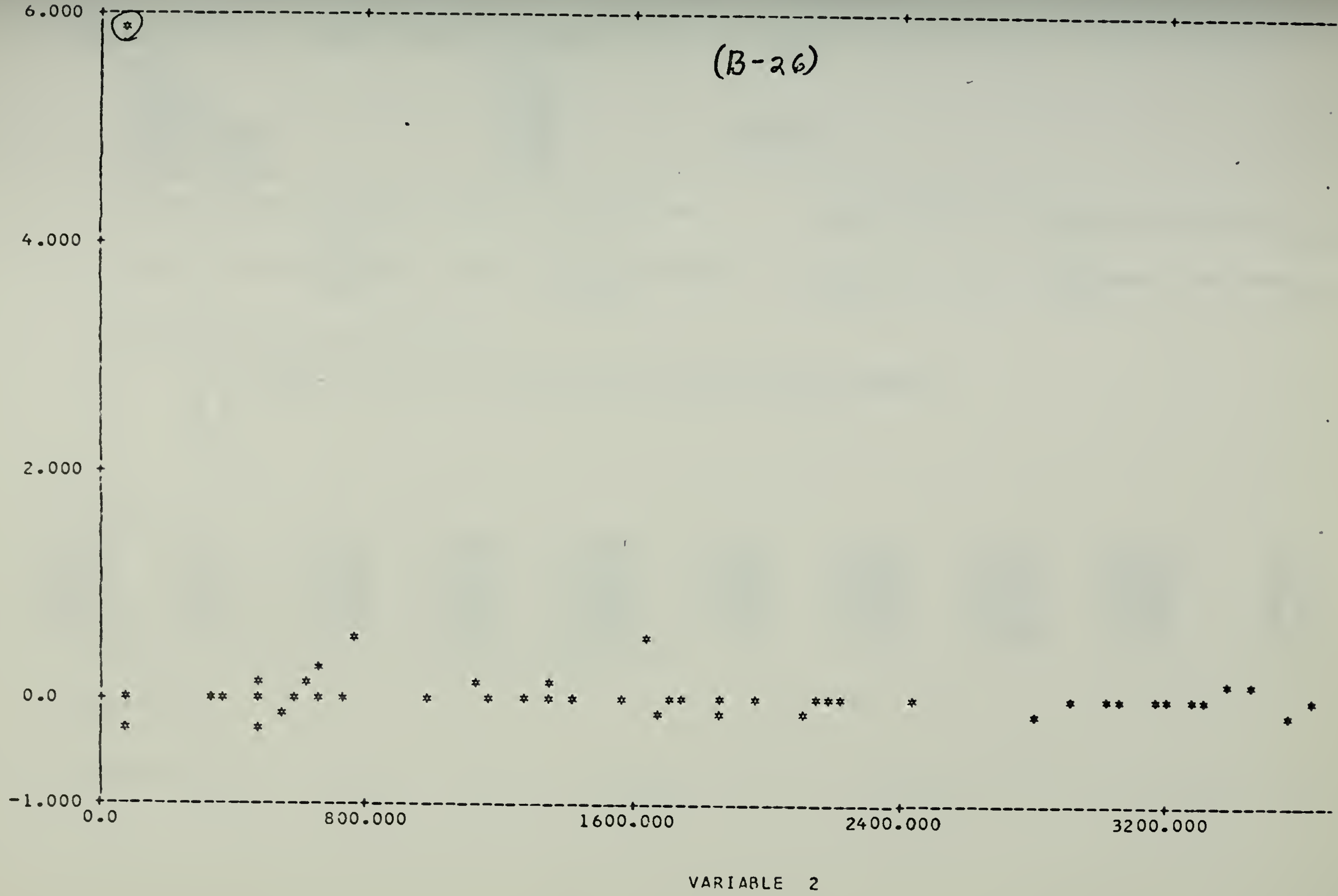


SCAT PLOT OF RESIDUAS VERSUS X FROM LEAST SQUARES FIT VARIABLE 2

```
INFO BX(4)=T

PROPERTIES OF THE  63 OBSERVATIONS OF VARIABLE  4:
     MEAN                          0.104
     MEDIAN                        0.0
     MIDMEAN                      -0.003
     TRIMEAN                      -0.003
     LOWER HINGE                  -0.031
     UPPER HINGE                   0.031
     LOWEST VALUE                 -0.262
     HIGHEST VALUE                 5.895

     THE FOLLOWING VALUES WERE FOUND TO BE LESS THAN       -0.093 WHICH IS THE LOWER SIDE POINT.
               -0.262             -0.206              -0.169            -0.129           -0.127
               -0.125

     THE FOLLOWING VALUES WERE FOUND TO BE GREATER THAN     0.093 WHICH IS THE UPPER SIDE POINT.
                0.175              0.175               0.292             0.479            0.558
                5.895

     STEM-AND-LEAF PLOT FOR X( 4).

          -0 | 3221111110000000000000000000000
           0 | 2000000000000000000011111122356
           1 |
           2 |
           3 |
           4 |
           5 | 9

INFO PR(4)=T

-0.0109  -0.2623  -0.0168   0.0113   0.0113   0.0113   0.0702   0.0010   0.0109   X( 4)
 0.0758   0.0042  -0.0459  -0.0296  -0.0109   0.0177  -0.0080  -0.0725  -0.1253   X( 4)
-0.0467  -0.0460  -0.1270  -0.0639  -0.2063   5.8950   0.0113   0.0428   0.0113   X( 4)
 0.2925   0.0577   0.0538   0.0386   0.0126  -0.0190  -0.0428  -0.0116   0.0170   X( 4)
-0.0087   0.0090   0.0      -0.0033  -0.0130  -0.0841   0.0113  -0.1693  -0.0186   X( 4)
 0.0113   0.1753   0.0113   0.0447   0.5579   0.1754   0.4794   0.0675  -0.0353   X( 4)
-0.0178  -0.0028   0.0305  -0.0015   0.0725  -0.1292  -0.0394  -0.0232  -0.0315   X( 4)
   THESE ARE THE RESIDUALS WITH THE MEDIAN AS THE ESTIMATOR

INFO PU(4)=T

INFO SCATX(2)=4
```

(B-25)

SCATTER PLOT '

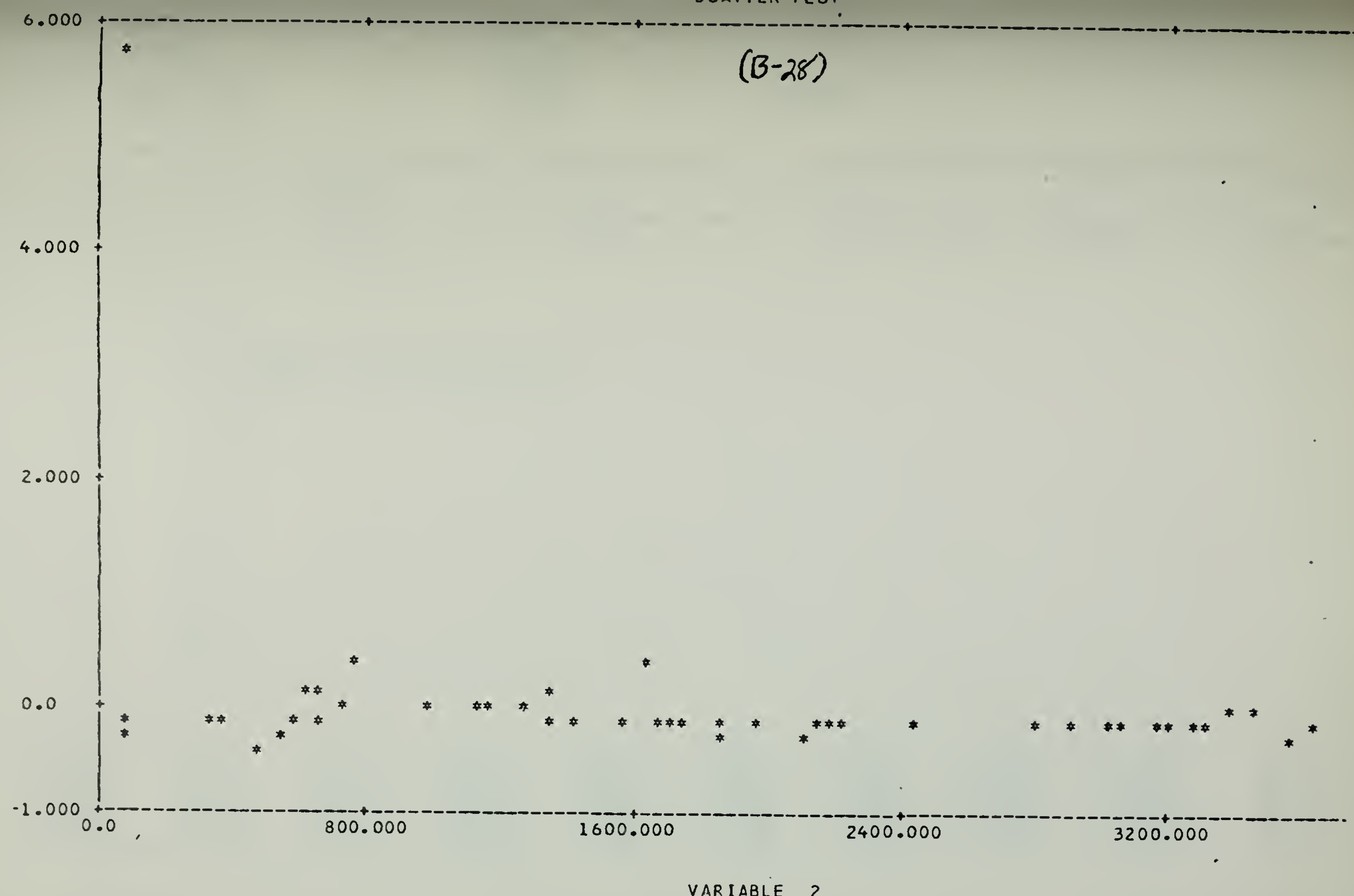


INFO HX(4)=T

```
PROPERTIES OF THE  63 OBSERVATIONS OF VARIABLE  4:

     MEAN                                 0.000
     MEDIAN                              -0.104
     MIDMEAN                             -0.108
     TRIMEAN                             -0.108
     LOWER HINGE                         -0.136
     UPPER HINGE                         -0.074
     LOWEST VALUE                        -0.367
     HIGHEST VALUE                        5.791
```

(B-27)

```
     THE FOLLOWING VALUES WERE FOUND TO BE LESS THAN          -0.198 WHICH IS THE LOWER SIDE POINT.
                   -0.367              -0.311              -0.274              -0.234              -0.231
                   -0.230

     THE FOLLOWING VALUES WERE FOUND TO BE GREATER THAN       -0.012 WHICH IS THE UPPER SIDE POINT.
                    0.071               0.071               0.188               0.375               0.454
                    5.791

     STEM-AND-LEAF PLOT FOR X( 4).

          -0 | 4332222222221111111111111111111111111111111111111100000
           0 | 11245
           1 |
           2 |
           3 |
           4 |
           5 | 8

   INFO PR(4)=T

-0.1152   -0.3666   -0.1211   -0.0929   -0.0929   -0.0929   -0.0341   -0.1033   -0.0934      X( 4)
-0.0285   -0.1001   -0.1502   -0.1339   -0.1152   -0.0866   -0.1122   -0.1768   -0.2296      X( 4)
-0.1510   -0.1503   -0.2313   -0.1682   -0.3106    5.7907   -0.0929   -0.0615   -0.0929      X( 4)
 0.1882   -0.0466   -0.0505   -0.0657   -0.0916   -0.1233   -0.1470   -0.1159   -0.0873      X( 4)
-0.1130   -0.0953   -0.1043   -0.1076   -0.1172   -0.1884   -0.0929   -0.2736   -0.1229      X( 4)
-0.0929    0.0710   -0.0929   -0.0596    0.4536    0.0712    0.3751   -0.0368   -0.1396      X( 4)
-0.1221   -0.1070   -0.0737   -0.1058   -0.0318   -0.2335   -0.1437   -0.1274   -0.1358      X( 4)
   THESE ARE THE RESIDUALS WITH THE MEAN AS ESTIMATOR

   INFO PU(4)=T

   INFO SCATX(2)=4
```



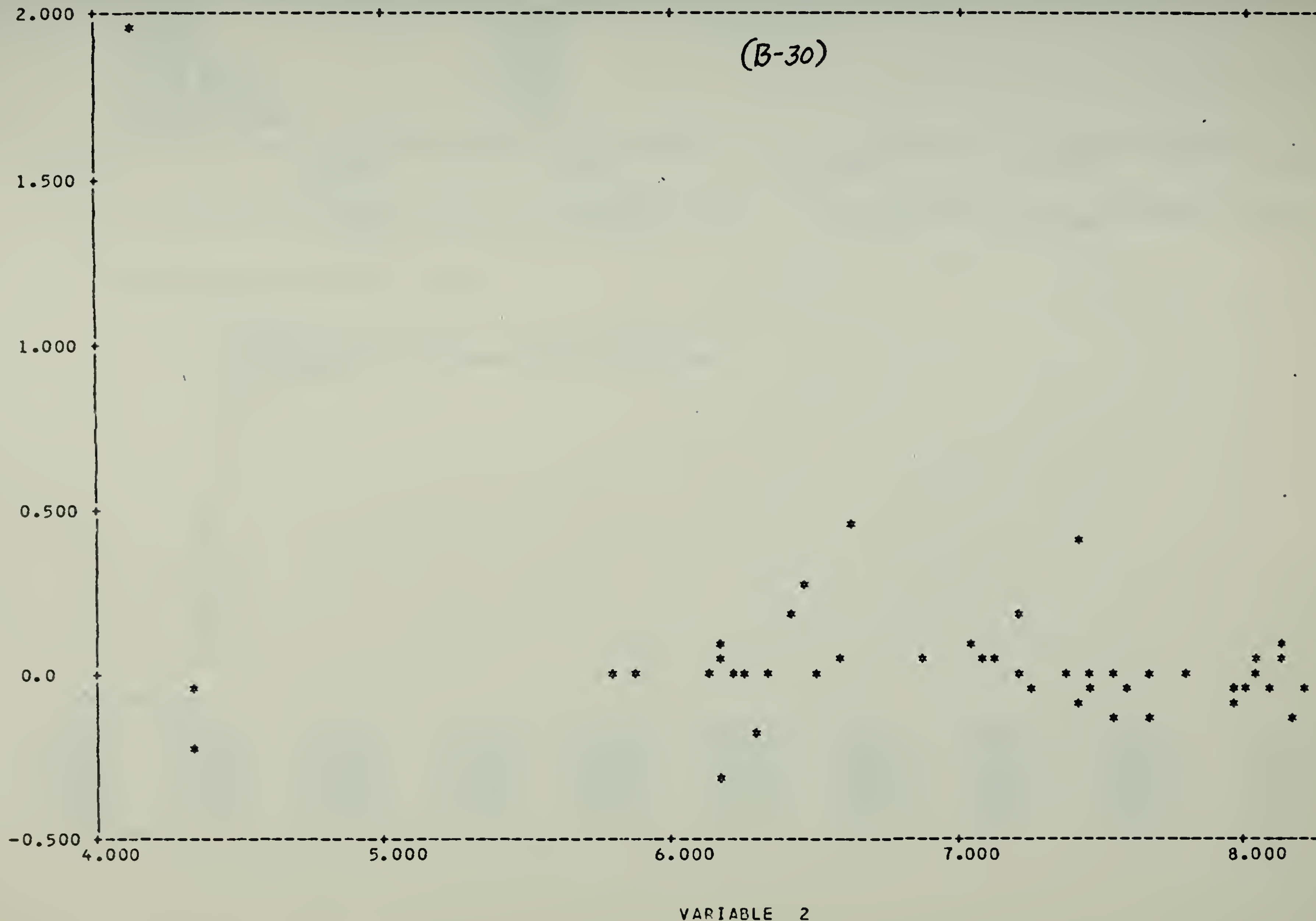
(B-28)
6.000
4.000
2.000
0.0
-1.000
0.0
800.000
1600.000
2400.000
3200.000
VARIABLE 2

INFO HX(4)=T

(B-29)

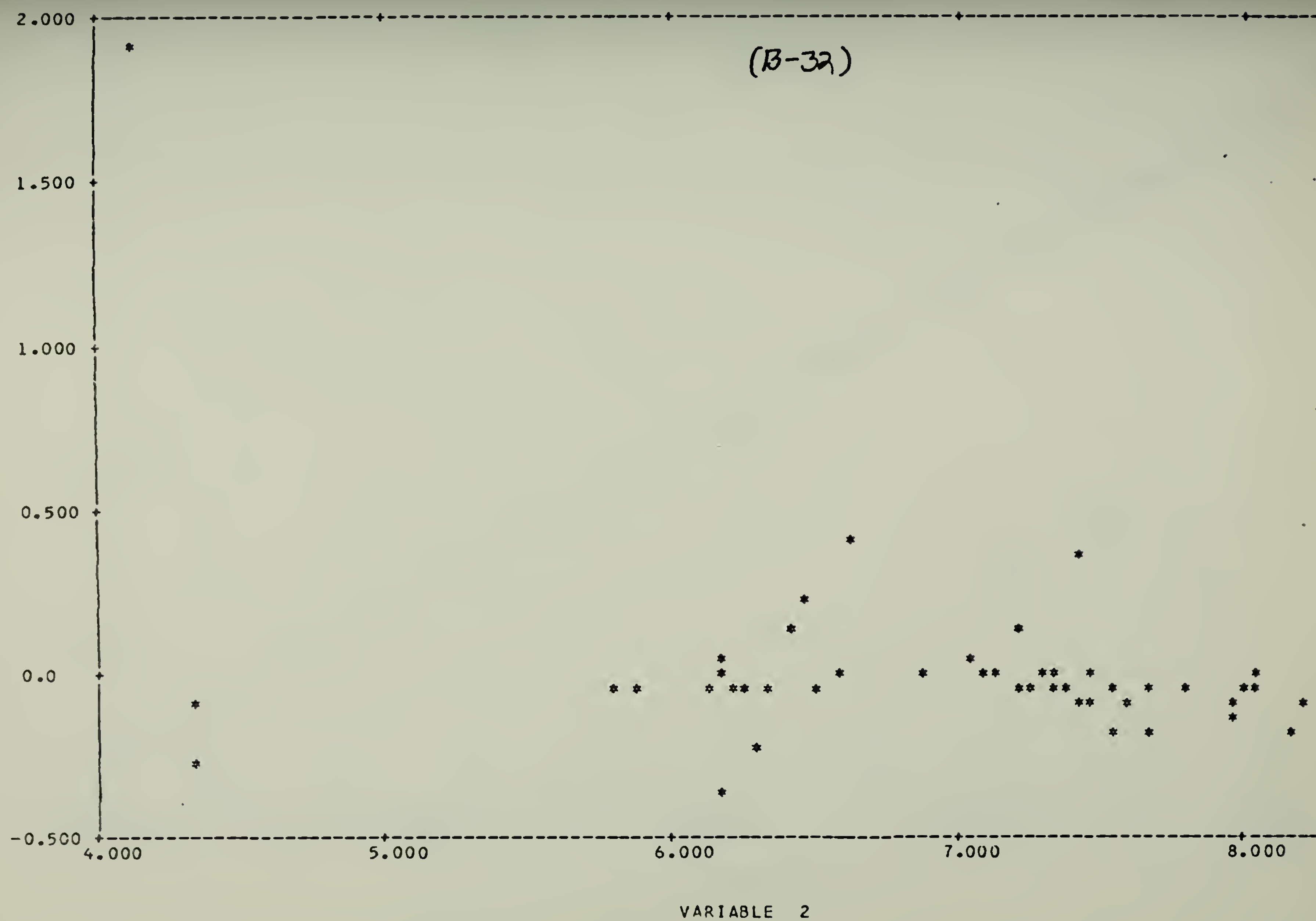
```
MEAN                     0.035
MEDIAN                   0.0
MIDMEAN                 -0.003
TRIMEAN                 -0.003
LOWER HINGE             -0.032
UPPER HINGE              0.030
LOWEST VALUE            -0.308
HIGHEST VALUE            1.941

THE FOLLOWING VALUES WERE FOUND TO BE LESS THAN           -0.095 WHICH IS THE LOWER SIDE POINT.
          -0.308                 -0.234                  -0.188             -0.140              -0.138
          -0.135

THE FOLLOWING VALUES WERE FOUND TO BE GREATER THAN         0.093 WHICH IS THE UPPER SIDE POINT.
           0.163                  0.163                   0.259             0.395               0.447
           1.941


STEM-AND-LEAF PLOT FOR X( 4).

      -3 | 1
      -2 | 3
      -1 | 9444
      -0 | 98755544433222221111111000
       0 | Z001111111111223444567777
       1 | 66
       2 | 6
       3 |
       4 | 05
       5 |
       6 |
       7 |
       8 |
       9 |
      10 |
      11 |
      12 |
      13 |
      14 |
      15 |
      16 |
      17 |
      18 |
      19 | 4

  INFO PR(4)=T

-0.0111   -0.3084   -0.0172    0.0114    0.0114    0.0114    0.0686    0.0010    0.0109    X( 4)
 0.0738    0.0042   -0.0475   -0.0304   -0.0111    0.0178   -0.0081   -0.0761   -0.1355    X( 4)
-0.0484   -0.0477   -0.1375   -0.0669   -0.2340    1.9406    0.0114    0.0424    0.0114    X( 4)
 0.2592    0.0567    0.0530    0.0383    0.0127   -0.0194   -0.0442   -0.0118    0.0170    X( 4)
-0.0089    0.0090    0.0       -0.0033   -0.0132   -0.0889    0.0114   -0.1878   -0.0190    X( 4)
 0.0114    0.1632    0.0114    0.0442    0.4474    0.1634    0.3954    0.0660   -0.0363    X( 4)
-0.0182   -0.0028    0.0304   -0.0015    0.0707   -0.1401   -0.0406   -0.0237   -0.0324    X( 4)
  RESIDUALS FOR DIFF WITH MED AS ESTIMATOR


  INFO PU(4)=T


  INFO HX(4)=T
```

SCATTER PLOT

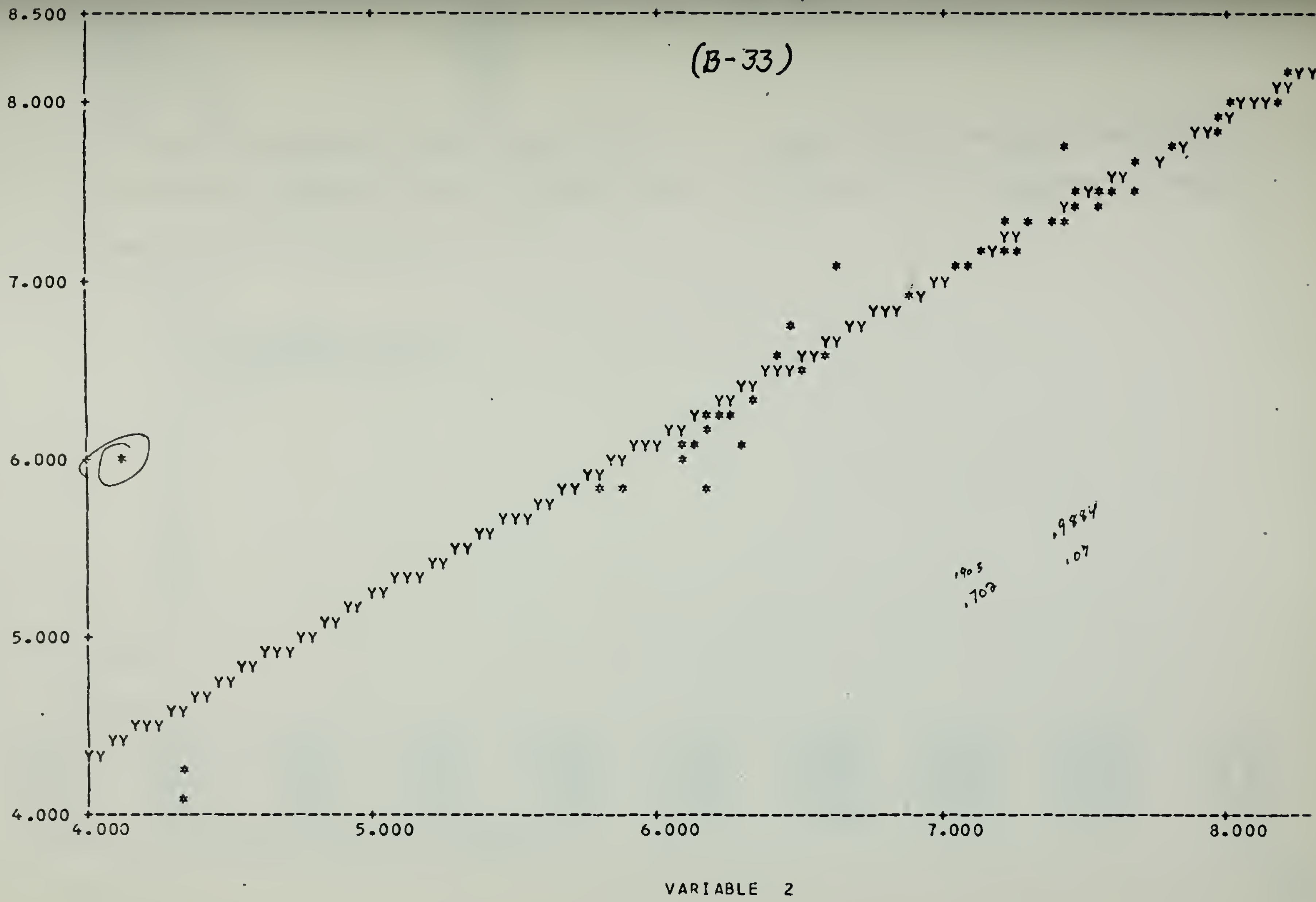


```
PROPERTIES OF THE  63 OBSERVATIONS OF VARIABLE  4:

     MEAN                          0.000
     MEDIAN                       -0.035
     MIDMEAN                      -0.038
     TRIMEAN                      -0.038
     LOWER HINGE                  -0.067
     UPPER HINGE                  -0.005
     LOWEST VALUE                 -0.343
     HIGHEST VALUE                 1.906

     THE FOLLOWING VALUES WERE FOUND TO BE LESS THAN           -0.130 WHICH IS THE LOWER SIDE POINT.
               -0.343                -0.269              -0.223               -0.175              -0.173
               -0.171

     THE FOLLOWING VALUES WERE FOUND TO BE GREATER THAN          0.058 WHICH IS THE UPPER SIDE POINT.
                0.128                 0.128               0.224                0.360               0.412
                1.906


     STEM-AND-LEAF PLOT FOR X( 4).

          -3 | 4
          -2 | 72
          -1 | 877210
          -0 | 88888777655555555444444333222222222220
           0 | 011223344
           1 | 33
           2 | 2
           3 | 6
           4 | 1
           5 |
           6 |
           7 |
           8 |
           9 |
          10 |
          11 |
          12 |
          13 |
          14 |
          15 |
          16 |
          17 |
          18 |
          19 | 1

 INFO PR(4)=T

-0.0461  -0.3434  -0.0522  -0.0236  -0.0236  -0.0236   0.0336  -0.0340  -0.0241    X( 4)
 0.0388  -0.0308  -0.0825  -0.0654  -0.0461  -0.0172  -0.0431  -0.1111  -0.1705    X( 4)
-0.0834  -0.0827  -0.1725  -0.1019  -0.2690   1.9055  -0.0236   0.0074  -0.0236    X( 4)
 0.2241   0.0217   0.0180   0.0033  -0.0223  -0.0544  -0.0792  -0.0468  -0.0180    X( 4)
-0.0439  -0.0260  -0.0350  -0.0384  -0.0432  -0.1239  -0.0236  -0.2228  -0.0540    X( 4)
-0.0236   0.1282  -0.0236   0.0092   0.4124   0.1283   0.3603   0.0310  -0.0714    X( 4)
-0.0532  -0.0378  -0.0046  -0.0366   0.0357  -0.1751  -0.0757  -0.0587  -0.0674    X( 4)
 RESIDUALS FOR DIFF WITH MEAN AS ESTIMATOR
```

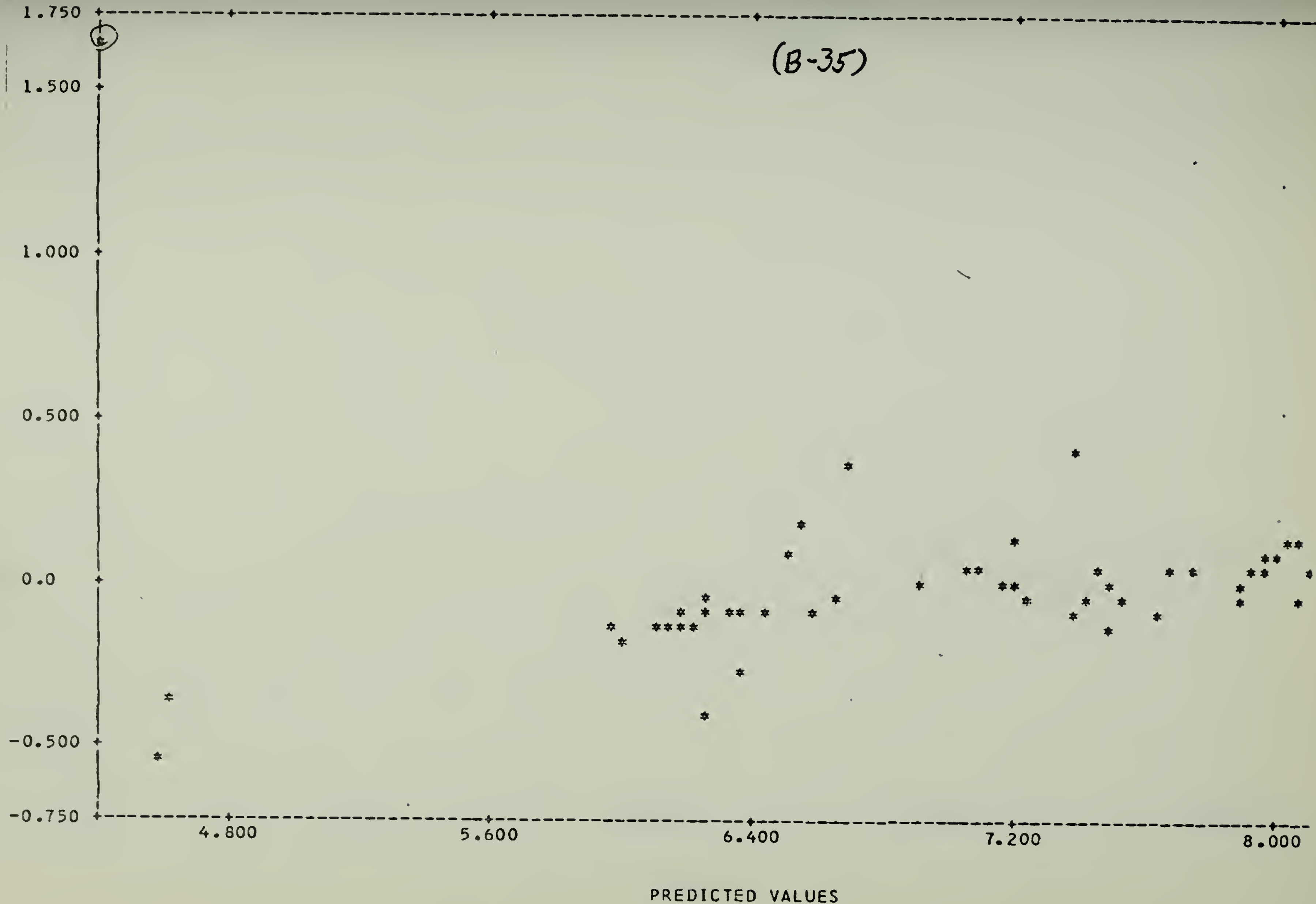
SCATTER PLOT .

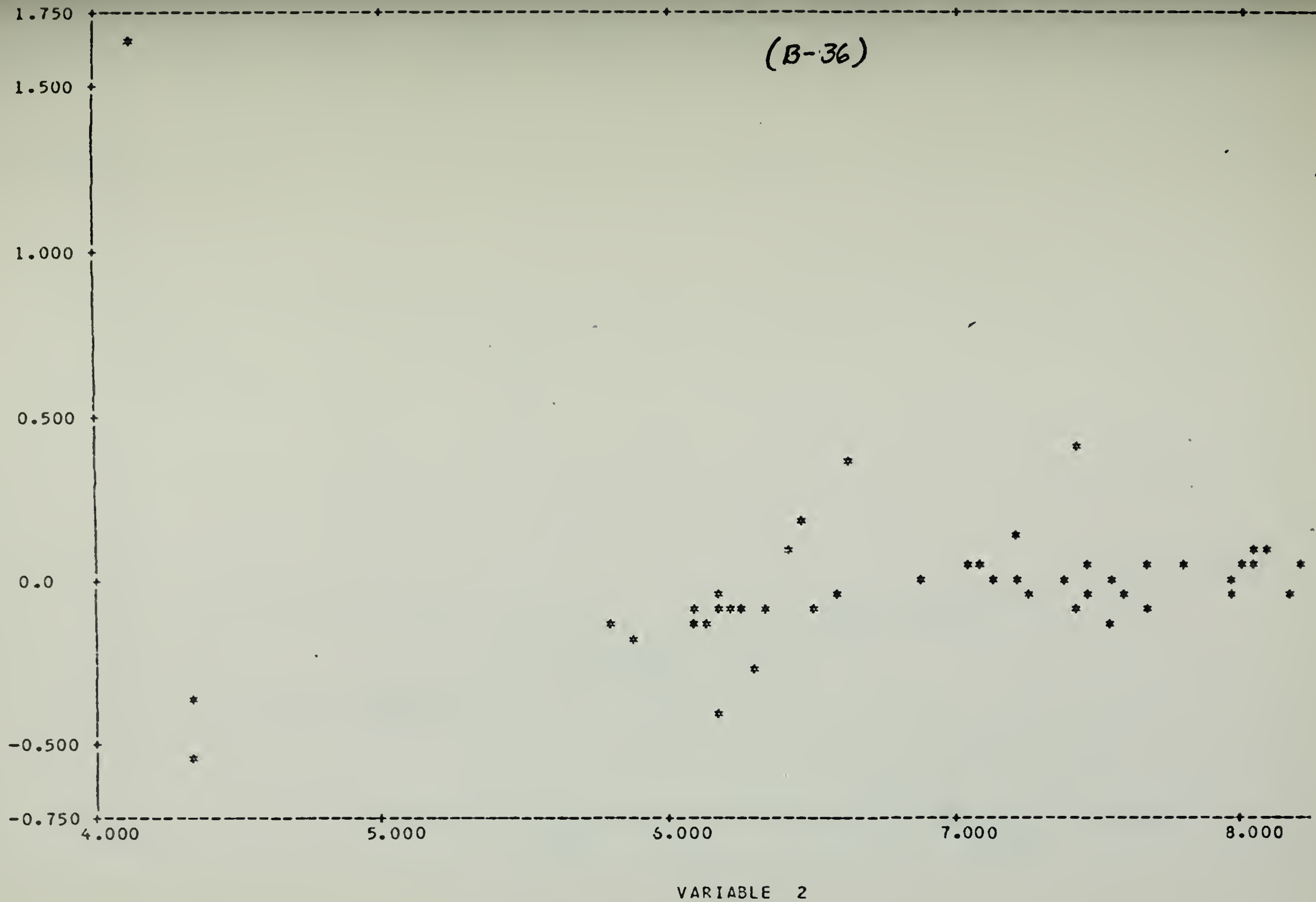


RESIDUALS OF DIFFERENCE VERSUS LN REQUEST

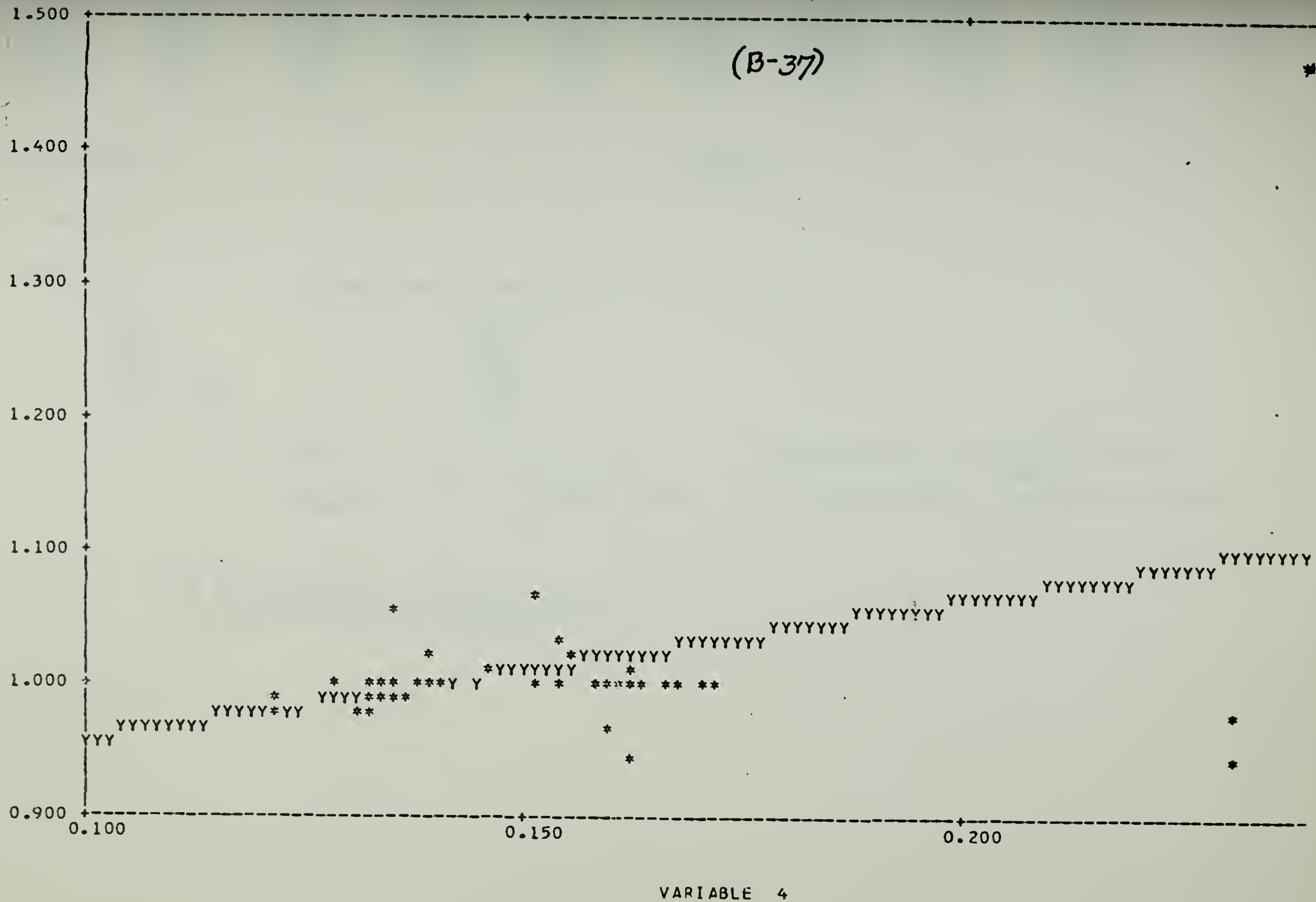

(B-33)
8.500
8.000
7.000
6.000
5.000
4.000
4.000
5.000
6.000
7.000
8.000
VARIABLE 2

```
PROPERTIES OF THE 63 OBSERVATIONS OF VARIABLE 3.                (B-34)

     MEAN                          0.000
     MEDIAN                       -0.019
     MIDMEAN                      -0.021
     TRIMEAN                      -0.022
     LOWER HINGE                  -0.087
     UPPER HINGE                   0.039
     LOWEST VALUE                 -0.529
     HIGHEST VALUE                 1.626

     THE FOLLOWING VALUES WERE FOUND TO BE LESS THAN        -0.213 WHICH IS THE LOWER SIDE POINT.
               -0.529                  -0.424              -0.361                 -0.292

     THE FOLLOWING VALUES WERE FOUND TO BE GREATER THAN      0.166 WHICH IS THE UPPER SIDE POINT.
                0.172                   0.376               0.399                  1.626


     STEM-AND-LEAF PLOT FOR X( 3).

          -5 | 3
          -4 | 2
          -3 | 6
          -2 | 9
          -1 | 6443222110
          -0 | 9977765554443333322211
           0 | 1111222333344444445677
           1 | 04457
           2 |
           3 | 8
           4 | 0
           5 |
           6 |
           7 |
           8 |
           9 |
          10 |
          11 |
          12 |
          13 |
          14 |
          15 |
          16 | 3

   INFO PU(3)=T


   INFO PR(3)=T
 -0.1322  -0.4242  -0.1613  -0.1389  -0.1187  -0.1211  -0.0481  -0.0382  -0.0288    X( 3)
  0.0420  -0.0118  -0.0535  -0.0413  -0.0192   0.0148  -0.0096  -0.0705  -0.1195    X( 3)
 -0.0396  -0.0265  -0.1082  -0.3608  -0.5290   1.6257  -0.1120  -0.0731  -0.0985    X( 3)
  0.1724   0.0102   0.0244   0.0176   0.0067  -0.0188  -0.0514  -0.0176   0.0278    X( 3)
  0.0079   0.0394   0.0333   0.0293   0.0293  -0.0322  -0.0947  -0.2918  -0.1443    X( 3)
 -0.0868   0.0716  -0.0724  -0.0314   0.3759   0.1476   0.3995   0.1422   0.0450    X( 3)
  0.0513   0.0649   0.0952   0.0704   0.1447  -0.0606   0.0193   0.0397   0.0386    X( 3)
   THESE ARE THE RESIDUALS FOR THE LN REGRESSION


   INFO HX(3)=T
```

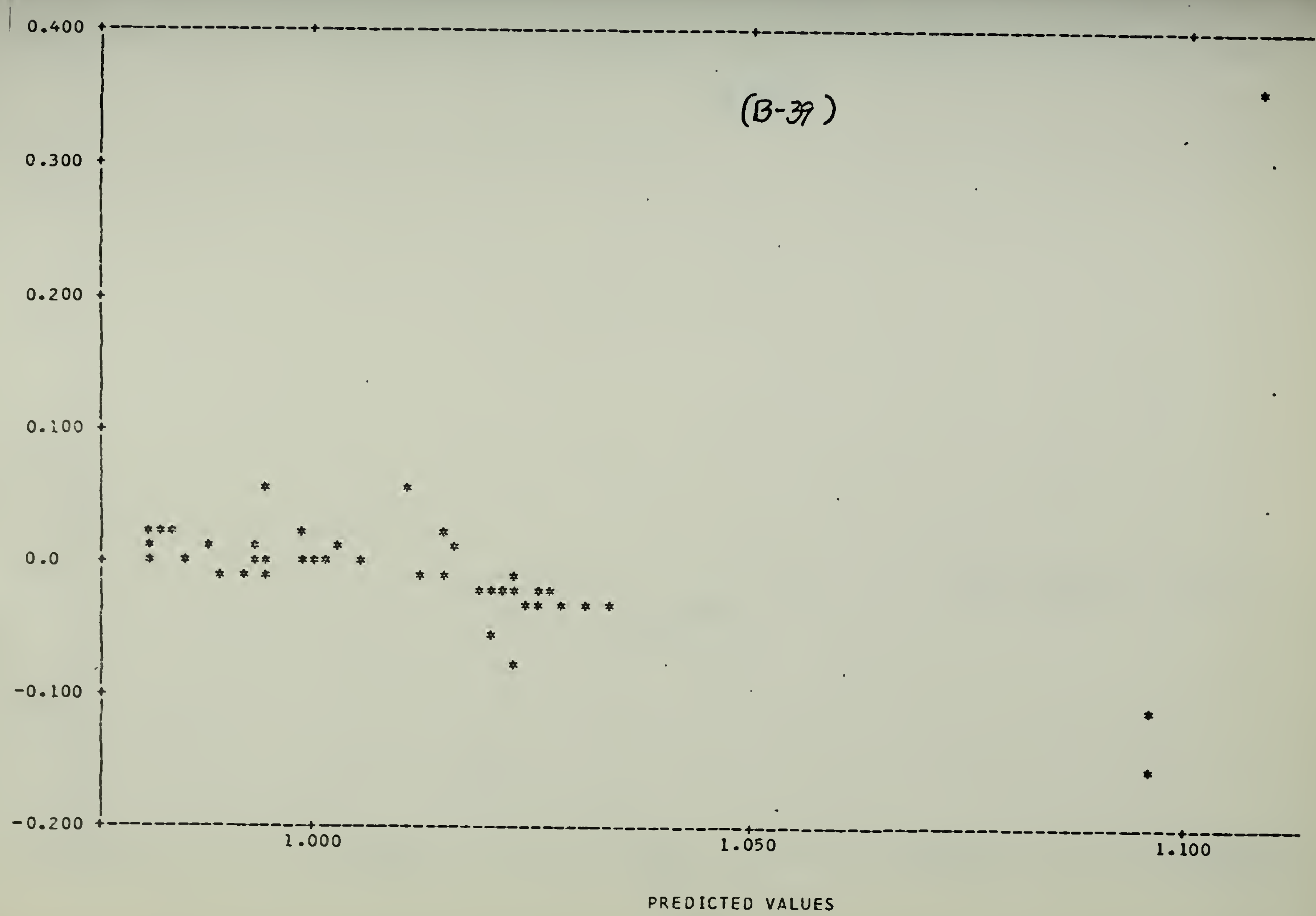

(B-35)
1.750
1.500
1.000
0.500
0.0
-0.500
-0.750
4.800
5.600
6.400
7.200
8.000
PREDICTED VALUES

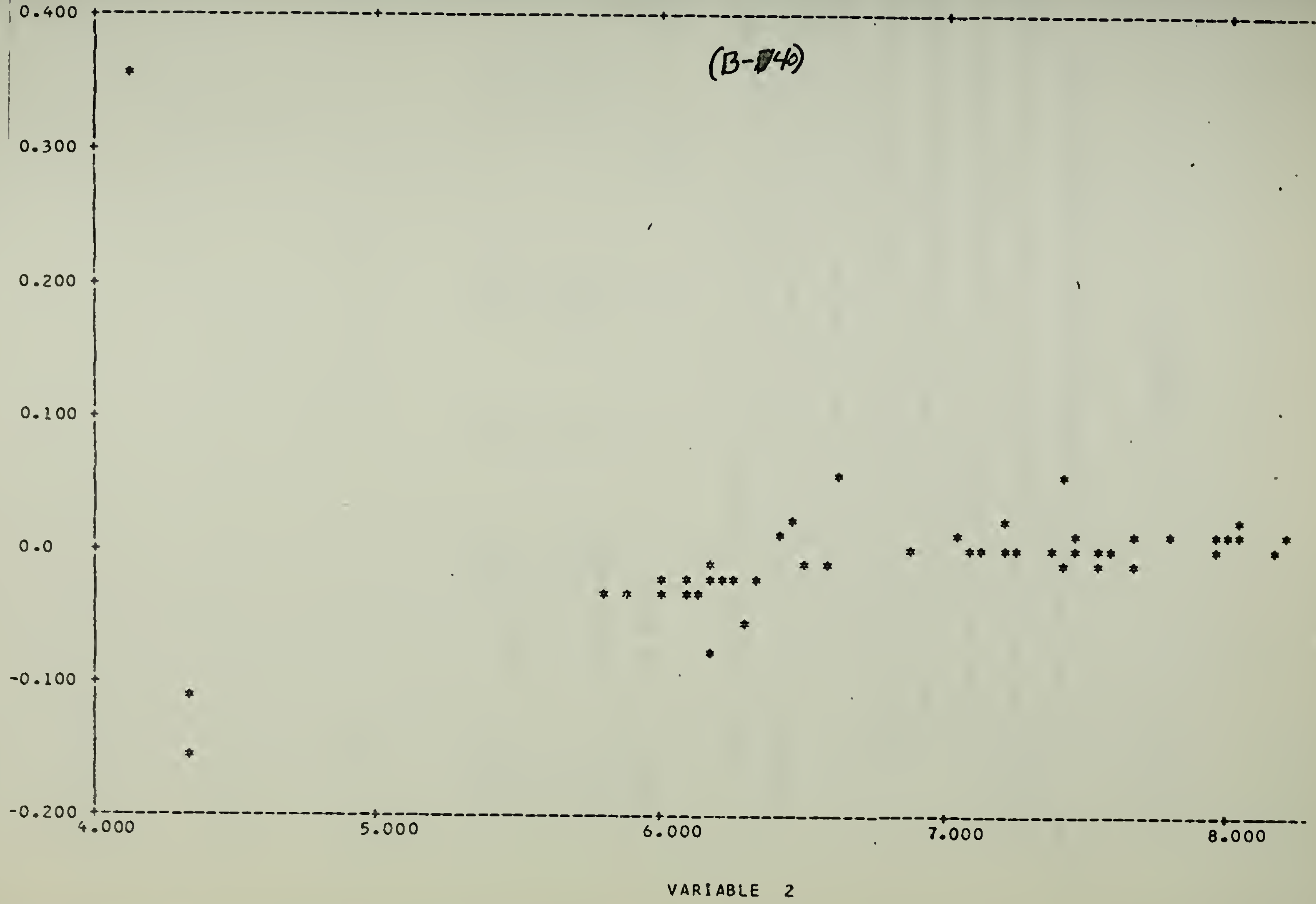

(B-36)
1.750
1.500
1.000
0.500
0.0
-0.500
-0.750
4.000
5.000
6.000
7.000
8.000
VARIABLE 2

(B-37)



```
-0.0284   -0.0751   -0.0366   -0.0337   -0.0274   -0.0281   -0.0141   -0.0052   -0.0039     X( 5)
 0.0068    0.0003   -0.0047   -0.0034   -0.0001    0.0049    0.0017   -0.0061   -0.0118     X( 5)
-0.0017    0.0010   -0.0092   -0.1140   -0.1534    0.3605   -0.0254   -0.0181   -0.0215     X( 5)
 0.0231    0.0012    0.0045    0.0041    0.0036    0.0006   -0.0045    0.0003    0.0075     X( 5)
 0.0052    0.0102    0.0095    0.0090    0.0095    0.0025   -0.0205   -0.0515   -0.0310     X( 5)
-0.0183    0.0072   -0.0145   -0.0074    0.0545    0.0225    0.0573    0.0252    0.0135     X( 5)
 0.0137    0.0153    0.0189    0.0162    0.0254    0.0005    0.0092    0.0119    0.0122     X( 5)
  RESIDUALS FROM LN RATIO REGRE

  INFO PU(5)=T                                 (B-38)

  INFO BX(5)=T

  PROPERTIES OF THE  63 OBSERVATIONS OF VARIABLE  5:

       MEAN                          0.000
       MEDIAN                        0.001
       MIDMEAN                       0.003
       TRIMEAN                       0.003
       LOWER HINGE                  -0.014
       UPPER HINGE                   0.010
       LOWEST VALUE                 -0.153
       HIGHEST VALUE                 0.361

       THE FOLLOWING VALUES WERE FOUND TO BE LESS THAN         -0.038 WHICH IS THE LOWER SIDE POINT.
                  -0.153              -0.114              -0.075              -0.052

       THE FOLLOWING VALUES WERE FOUND TO BE GREATER THAN        0.034 WHICH IS THE UPPER SIDE POINT.
                   0.054               0.057               0.361

       STEM-AND-LEAF PLOT FOR X( 5).

            -1 | 51
            -0 | 854333333222211111111000000
             0 | 00000000000011111111111111222223356
             1 |
             2 |
             3 | 6

  INFO HX(5)=T
```


(B-39)
0.400
0.300
0.200
0.100
0.0
-0.100
-0.200
1.000
1.050
1.100
PREDICTED VALUES


(B-[redacted]40)
0.400
0.300
0.200
0.100
0.0
-0.100
-0.200
4.000
5.000
6.000
7.000
8.000
VARIABLE 2

APPENDIX C

This appendix contains summaries (stem and leaf plots, means, medians and so forth, for "so-called" predicted percentages of requests which would be appropriated when the requests are of the size of the data analyzed in the chapter. These predicted percentages are obtained by computing $\hat{\beta} x_t^{(\hat{\alpha}-1)}$ where $\hat{\beta}$ and $\hat{\alpha}$ were estimated using (2.4) and (2.5). The following list states which values for $\hat{\beta}$ and $\hat{\alpha}$ were used and identifies the pages on which the summaries can be found:

Equation Number	$\hat{\beta}$	$\hat{\alpha}$	x_t data set	Page
(2.4)	1.473	.948	Procurement	C-2
(2.4)	2.028	.903	RDTE	C-3
(2.5)	1.242	.969	Procurement	C-4
(2.5)	2.877	.852	RDTE	C-5

```
INFO BX(3)=T

PROPERTIES OF THE  80 OBSERVATIONS OF VARIABLE  3:          (C-2)

        MEAN                              0.981
        MEDIAN                            0.977
        MIDMEAN                           0.977
        TRIMEAN                           0.977
        LOWER HINGE                       0.961
        UPPER HINGE                       0.996
        LOWEST VALUE                      0.919
        HIGHEST VALUE                     1.142

        THE FOLLOWING VALUES WERE FOUND TO BE LESS THAN          0.925 WHICH IS THE LOWER SIDE POINT.
                    0.919

        THE FOLLOWING VALUES WERE FOUND TO BE GREATER THAN          1.031 WHICH IS THE UPPER SIDE POINT.
                    1.035                1.037                1.142

        STEM-AND-LEAF PLOT FOR X( 3).

             91 | 9
             92 |
             93 | 777
             94 | 27
             95 | 002233446778
             96 | 0011334455566677789
             97 | 6777
             98 | 2466667899
             99 | 1133445555677
            100 | 133446
            101 | 0046
            102 | 1478
            103 | 57
            104 |
            105 |
            106 |
            107 |
            108 |
            109 |
            110 |
            111 |
            112 |
            113 |
            114 | 2

   INFO PR(3)=T

 0.9189   0.9765   1.0215   1.0267   1.0238   1.0097   0.9942   0.9763   0.9649   X( 3
 0.9949   1.0144   0.9632   0.9374   0.9370   0.9421   0.9645   0.9574   0.9613   X( 3
 1.1424   0.9908   0.9830   1.0281   0.9971   0.9928   0.9867   0.9860   0.9859   X( 3
 0.9883   0.9671   0.9671   0.9771   0.9821   0.9945   0.9674   0.9646   0.9644   X( 3
 0.9615   0.9530   0.9524   0.9504   0.9863   0.9534   0.9556   0.9523   0.9537   X( 3
 0.9500   0.9693   0.9660   0.9660   0.9596   0.9582   0.9597   0.9543   0.9374   X( 3
 0.9463   0.9567   0.9632   0.9663   0.9641   0.9680   1.0034   1.0351   1.0055   X( 3
 1.0043   1.0011   0.9965   0.9933   0.9857   0.9895   0.9774   0.9845   0.9963   X( 3
 1.0373   1.0155   1.0095   0.9952   1.0042   1.0027   0.9932   0.9938            X( 3

   INFO BX(1)=T
```

(C-3)

```
FILL CONST(3)=-.C97

ARIT PROD(3,1)=3

ARIT SUM(3,2)=3

REEX EXP(1)=1

REEX EXP(3)=3

INFO BX(3)=T

PROPERTIES OF THE  63 OBSERVATIONS OF VARIABLE  3:
        MEAN                          1.023
        MEDIAN                        0.996
        MIDMEAN                       1.004
        TRIMEAN                       1.007
        LOWER HINGE                   0.959
        UPPER HINGE                   1.097
        LOWEST VALUE                  0.911
        HIGHEST VALUE                 1.354

        THE FOLLOWING VALUES WERE FOUND TO BE GREATER THAN       1.235 WHICH IS THE UPPER SIDE POINT.
                    1.326               1.328               1.354

        STEM-AND-LEAF PLOT FOR X( 3).

             9 | 1122222233335666677788889999999
            10 | CC0001223346677889
            11 | CC01112223345
            12 |
            13 | 335

INFO PR(3)=T

1.1156  1.1098  1.1416  1.1487  1.1257  1.1284  1.1107  1.0279  1.0284    X( 3)
1.0203  1.C044  0.9943  0.9992  0.9964  0.9912  0.9899  0.9828  0.9727    X( 3)
C.9797  C.9676  0.9598  1.3262  1.3275  1.3542  1.1182  1.1093  1.1032    X( 3)
1.0780  1.0354  1.0169  1.0090  C.9943  0.9878  0.9955  0.9941  0.9778    X( 3)
C.9719  C.9587  C.9560  0.9566  0.9472  0.9339  1.0991  1.0966  1.1203    X( 3)
1.0903  1.C832  1.C748  1.0660  1.0617  1.0040  0.9843  0.9159  0.9111    X( 3)
0.9220  0.9236  0.9264  0.9197  0.9178  0.9128  0.9309  0.9276  C.9206    X( 3)

INFO BX(1)=T
```

(C-4)

```
MEAN                                 0.975
MEDIAN                               0.973
MIDMEAN                              0.973
TRIMEAN                              0.973
LOWER HINGE                          0.963
UPPER HINGE                          0.984
LOWEST VALUE                         0.938
HIGHEST VALUE                        1.068

THE FOLLOWING VALUES WERE FOUND TO BE LESS THAN            0.942 WHICH IS THE LOWER SIDE POINT.
          0.938

THE FOLLOWING VALUES WERE FOUND TO BE GREATER THAN         1.005 WHICH IS THE UPPER SIDE POINT.
          1.007            1.008            1.068

STEM-AND-LEAF PLOT FOR X( 3).

        93 | 8
        94 | 999
        95 | 2567888999
        96 | 001122334455555666677778
        97 | 223367888889
        98 | 001222333344478899 9
        99 | 22559
       100 | 02378
       101 |
       102 |
       103 |
       104 |
       105 |
       106 | 8

  INFO PR(3)=T

0.9376   0.9723   0.9987   1.0018   1.0001   0.9919   0.9827   0.9722   0.9654   X( 3
0.9831   0.9946   0.9644   0.9489   0.9487   0.9517   0.9652   0.9609   0.9633   X( 3
1.0676   0.9808   0.9791   1.0026   0.9845   0.9819   0.9783   0.9779   0.9779   X( 3
0.9796   0.9667   0.9667   0.9726   0.9756   0.9830   0.9669   0.9652   0.9651   X( 3
0.9634   0.9583   0.9579   0.9567   0.9781   0.9585   0.9598   0.9579   0.9587   X( 3
0.9565   0.9680   0.9660   0.9660   0.9622   0.9614   0.9623   0.9590   0.9489   X( 3
0.9545   0.9605   0.9644   0.9662   0.9649   0.9672   0.9882   1.0067   0.9894   X( 3
0.9887   0.9868   0.9841   0.9822   0.9777   0.9800   0.9728   0.9770   0.9840   X( 3
1.0080   0.9955   0.9918   0.9833   0.9886   0.9877   0.9822   0.9825            X( 3

  INFO BX(1)=T

  PROPERTIES OF THE 80 OBSERVATIONS OF VARIABLE 1:

MEAN                              2705.429
MEDIAN                            2507.900
MIDMEAN                           2449.150
TRIMEAN                           2374.475
LOWER HINGE                       1749.000
UPPER HINGE                       3494.650
LOWEST VALUE                       124.500
HIGHEST VALUE                     8205.600

THE FOLLOWING VALUES WERE FOUND TO BE GREATER THAN         5240.300 WHICH IS THE UPPER SIDE POINT.
```

```
PROPERTIES OF THE  63 OBSERVATIONS OF VARIABLE  3:

        MEAN                                 1.031          (C-5)
        MEDIAN                               0.980
        MIDMEAN                              0.992
        TRIMEAN                              0.936
        LOWER HINGE                          0.924
        UPPER HINGE                          1.134
        LOWEST VALUE                         0.855
        HIGHEST VALUE                        1.565

        THE FOLLOWING VALUES WERE FOUND TO BE GREATER THAN       1.344 WHICH IS THE UPPER SIDE POINT.
                   1.515               1.518               1.565

        STEM-AND-LEAF PLOT FOR X( 3).

                 8 | 566677778889
                 9 | 12223444556677788888899
                10 | 01233489
                11 | 001234455667788
                12 | 12
                13 |
                14 |
                15 | 226

     INFC PR(3)=T

 1.1640    1.1547    1.2055    1.2170    1.1801    1.1844    1.1561    1.0273    1.0280    X( 3
 1.0156    0.9916    0.9765    0.9838    0.9796    0.9719    0.9698    0.9593    0.9443    X( 3
 0.9546    0.9368    0.9252    1.5153    1.5177    1.5645    1.1682    1.1540    1.1443    X( 3
 1.1046    1.0388    1.0106    0.9986    0.9765    0.9667    0.9783    0.9761    0.9518    X( 3
 0.9432    0.9237    0.9196    0.9205    0.9068    0.8874    1.1377    1.1339    1.1714    X( 3
 1.1239    1.1127    1.0996    1.0360    1.0792    0.9910    0.9615    0.8614    0.8546    X( 3
 0.8702    0.8725    0.8765    0.8670    0.8642    0.8570    0.8830    0.8783    0.8682    X( 3

     INFC BX(1)=T

     PROPERTIES OF THE  63 OBSERVATIONS OF VARIABLE  1:

        MEAN                             1506.137
        MEDIAN                           1442.700
        MIDMEAN                          1328.006
        TRIMEAN                          1392.200
        LOWER HINGE                       537.000
        UPPER HINGE                      2146.400
        LOWEST VALUE                       61.000
        HIGHEST VALUE                    3627.900

        STEM-AND-LEAF PLOT FOR X( 1).

                 0 | 111344444455555566677
                 1 | 0001223344555556677889
                 2 | 01122439
                 3 | 0122334466
```

APPENDIX D

DATA

APPROPRIATIONS	REQUEST	
1889.2000	2544.4000	1953PEMA
2226.6000	1070.7000	1954PEMA
1669.3000	970.1000	1959PEMA
971.7000	1024.7000	1960PEMA
1495.3000	1337.0000	1961PEMA
2532.6000	1803.0000	1962PEMA
2520.0000	2555.0000	1963PEMA
2931.1000	3202.0000	1964PEMA
1656.4000	1779.0000	1965PEMA
1204.8000	1223.1000	1966PEMA
3483.3000	3311.1000	1967PEMA
5462.5000	5581.0000	1968PEMA
5031.4000	5626.0000	1969PEMA
4254.4000	5069.1000	1970PEMA
2958.5000	3226.0000	1971PEMAREV
3407.3000	3719.4000	1972PEMAREV
3025.0000	3439.1000	1973PEMAREV
113.5000	124.5000	1953PAMN
1222.8000	1924.2000	1954PAMN
1944.7000	2030.0000	1955PAMN
804.5000	945.2000	1956PAMN
1696.2000	1703.4000	1957PAMN
1724.9000	1852.3000	1958PAMN
2129.3000	2083.9000	1959PAMN
2044.6000	2114.1000	1960PAMN
2144.1000	2114.9000	1961PAMN
2680.9000	2000.0000	1962PAMN
3834.7000	3065.0000	1963PAMN
2889.1000	3066.0000	1964PAMN
2496.3000	2515.8000	1965PAMN
2272.5000	2279.8000	1966PAMN
1789.9000	1789.9000	1967PAMN
2939.1000	3046.0000	1968PAMN
2574.3000	3222.0000	1969PAMN
2620.0000	3235.5000	1970PAMN
3117.9000	3427.7000	1971PAMNREV
3955.0000	4069.1000	1972PAMNREV
3596.3000	4118.6000	1973PAMNREV
2453.7000	4233.0000	1954AFAIRCR
2072.4000	2098.8000	1955AFAIRCR
4128.8000	4031.0000	1956AFAIRCR
4533.1000	3859.9000	1957AFAIRCR
3914.9000	4122.9000	1958AFAIRCR
4288.4000	4012.8000	1959AFAIRCR
4284.6000	4322.8000	1960AFAIRCR
3497.2000	2934.1000	1961AFAIRCR
3537.2000	3136.2000	1962AFAIRCR
3562.4000	3135.0000	1963AFAIRCR
3385.6000	3559.0000	1964AFAIRCR
3563.7000	3663.0000	1965AFAIRCR
3517.0000	3550.2000	1966AFAIRCR
4017.3000	3961.3000	1967AFAIRCR
5493.4000	5532.0000	1968AFAIRCR
3860.0000	4612.0000	1969AFAIRCR
3405.8000	3775.2000	1970AFAIRCR
3219.3000	3314.9000	1971AFAIRCR
2942.3000	3116.5000	1972AFAIRCR
2682.3000	3255.7000	1973AFAIRCR
2903.8000	3012.1000	1953AFMISSI

APPROPRIATIONS	REQUEST	
936.9000	1509.5000	1954AFMISSI
812.7000	830.2000	1955AFMISSI
1475.4000	1449.5000	1956AFMISSI
1695.5000	1483.9000	1957AFMISSI
1500.7000	1578.7000	1958AFMISSI
1394.2000	1722.1000	1959AFMISSI
2540.5000	1832.1000	1960AFMISSI
1837.6000	2124.9000	1961AFMISSI
1928.7000	1975.2000	1962AFMISSI
2459.0000	2500.0000	1963AFMISSI
2141.9000	2177.0000	1964AFMISSI
1730.0000	1730.0000	1965AFMISSI
796.1000	796.1000	1966AFMISSI
1189.5000	1189.5000	1967AFMISSI
1340.0000	1343.0000	1968AFMISSI
1720.2000	1768.0000	1969AFMISSI
1448.1000	1486.4000	1970AFMISSI
1427.2000	1530.6000	1971AFMISSR
1683.7000	1837.4000	1972AFMISSR
1705.0000	1816.8000	1973AFMISSR

APPROPRIATIONS	REQUEST	
440.0000	450.0000	1953 ARDTE
345.0000	475.0000	1954 ARDTE
345.0000	355.0000	1955 ARDTE
333.0000	333.0000	1956 ARDTE
410.0000	410.0000	1957 ARDTE
400.0000	400.0000	1958 ARDTE
498.7000	471.0000	1959 ARDTE
1035.7000	1046.5000	1960 ARDTE
1041.2000	1041.7000	1961 ARDTE
1203.2000	1130.4000	1962 ARDTE
1319.5000	1329.0000	1963 ARDTE
1390.2000	1474.6000	1964 ARDTE
1344.1000	1401.5000	1965 ARDTE
1410.6000	1442.7000	1966 ARDTE
1531.9000	1522.2000	1967 ARDTE
1514.2000	1544.0000	1968 ARDTE
1522.6000	1661.9000	1969RDTEARM
1596.8000	1849.5000	1970RDTEARM
1618.2000	1717.9000	1971RDTEARM
1839.5000	1951.5000	1972RDTEARM
1829.0000	2122.7000	1973RDTEARM
70.0000	75.7000	1953 NRDTE
58.6000	74.9000	1954 NRDTE
419.9000	61.0000	1955 NRDTE
439.2000	439.2000	1956 NRDTE
492.0000	477.0000	1957 NRDTE
505.0000	505.0000	1958 NRDTE
821.2000	641.0000	1959 NRDTE
1015.9000	970.9000	1960 NRDTE
1218.6000	1169.0000	1961 NRDTE
1301.5000	1267.0000	1962 NRDTE
1475.9000	1474.0000	1963 NRDTE
1530.5000	1578.4000	1964 NRDTE
1377.5000	1456.3000	1965 NRTDE
1444.2000	1478.1000	1966 NRTDE
1762.4000	1752.5000	1967 NRTDE
1826.5000	1863.9000	1968 NRTDE
2141.3000	2146.4000	1969RDTENAV
2186.4000	2211.5000	1970RDTENAV
2165.1000	2197.3000	1971RDTENAV
2372.3000	2431.4000	1972RDTENAV
2545.3000	2813.8000	1973RDTENAV
525.0000	525.0000	1953 AFRDTE
440.0000	537.0000	1954 AFRDTE
418.1000	431.0000	1955 AFRDTE
570.0000	570.0000	1956 AFRDTE
710.0000	610.0000	1957 AFRDTE
661.0000	661.0000	1958 AFRDTE
743.0000	719.0000	1959 AFRDTE
1159.9000	750.0000	1960 AFRDTE
1552.9000	1334.0000	1961 AFRDTE
2403.2000	1637.0000	1962 AFRDTE
3632.1000	3439.0000	1963 AFRDTE
3458.7000	3627.9000	1964 AFRDTE
3117.3000	3210.9000	1965 AFRDTE
3109.4000	3153.9000	1966 AFRDTE
3116.8000	3058.1000	1967 AFRDTE
3251.2000	3293.6000	1968 AFRDTE
3570.3000	3364.7000	1969RDTEAF
3060.6000	3561.2000	1970RDTEAF
2762.1000	2909.7000	1971RDTEAF
2912.9000	3017.0000	1972RDTEAF
3122.5000	3262.2000	1973RDTEAF

CHAPTER III

Introduction

In the previous chapters, the institutional background of our analysis of Congressional defense budget activity was discussed, alternative approaches to models for characterizing this activity were presented, coefficients for characteristic equations were estimated and residuals analyzed. For example, it was pointed out that if the relationship between request in year t, x_t, and appropriation, y_t, is modeled as

$$(2.2) \qquad y_t = \beta x_t + \varepsilon_t x_t$$

then the median of the ratio y_t/x_t is a useful estimator for β; that is it is superior under certain circumstances to a least squares estimate because of the median's de-emphasis of extreme values.

Following the work of Andrews, Tukey, and others [Andrews, et al., 1972], this chapter explores the usefulness of an alternative robust estimator for the five different approaches discussed in the previous chapter. The estimator is the Huber "M" estimator [Huber, 1973]. This estimator was originally proposed for use in the estimation of a location

parameter [Huber, 1964]. However, it has recently been extended to and asymptotic properites have been derived for the regression problem [Huber,1973]. The format of this chapter will be the following: first, the basic problem of concern in robust regression will be discussed, with examples taken from the analysis of the previous chapter; next, the technique and rationale of Huber "M" estimates will be outlined; estimates for defense budget data for each of our five basic equations will be presented, discussed and compared with non-robust, least squares estimates. Finally, residuals and outliers will be analyzed.

III.1 The Problem

It may be useful at this point to re-state the five basic equations that have been used in the analysis of defense budget data in the previous chapter

(2.1) $$y_t = \beta x_t + \varepsilon_t$$

(2.2) $$y_t = \beta x_t + \varepsilon_t x_t$$

(2.3) $$y_t = \beta x_t e^{u_t}$$

(2.4) $$y_t = \beta x_t^{\alpha} e^{u_t}$$

(2.5) $$y_t = \beta x_t^{\alpha} e^{u_t \ln x_t}$$

where y_t stands for appropriations in year t, x_t stands for the agency budget request in year t, $\{\varepsilon_t\}$ and $\{u_t\}$ are stochastic

disturbances which are assumed to be independent and identically distributed.

In the previous chapter, results (especially for the RDTE data) indicated that for (2.2) and (2.3) the median y_t/x_t ratio and $\ln(y_t/x_t)$ ratio were more useful estimators for β and $\ln \beta$ than were the least squares estimators. This is because (i) for both (2.2) and (2.3), the mean gave coefficient estimates which were basically suspect. They indicated that the percentage of the request appropriated is greater than 1.0, a result which even a casual glance at plotted data did not support, and (ii) stem and leaf plots of the residuals and scatterplots of y_t versus x_t and $\ln y_t$ versus $\ln x_t$ showed that a few extreme observations were having a disproportionate impact on the estimates. The robustness of the median as opposed to the mean is not particularly surprising since (2.2) and (2.3) reduce (when appropriate transformations are made) to a location parameter problem of the form:

$$y = \theta + \varepsilon$$

where ε is a random variable with a given distribution and θ is a fixed constant

Andrews, Tukey et al. found the median to be more robust than the mean for the location parameter problem [Andrews, et al., 1972]. Also, see [Mosteller,1973, pp. 248-250]. The problem with the mean is that it is particularly

susceptible to extreme observations or outliers. For example suppose ε is drawn from a distribution of the form

$$f(x) = p \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} + q \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{x^2}{\sigma^2}}$$

where $p+q = 1$. If one compares the variance of the mean, $\bar{y}$, as an estimator for θ, to the variance of $\tilde{y}$, the median, also an estimator of θ, he finds that as σ^2 gets large, greater than 9 in the case of $p = .9$ and $q = .1$, then the variance of $\tilde{y}$ becomes smaller than the variance of $\bar{y}$.* Thus for this model of random error the median is more efficient than the mean.

Regression problems, e.g. analysis of data in terms of relations like (2.1), (2.4) and (2.5), also involve outlier difficulties. However the analogue to the median in regression problems is not as easy to calculate.** Also, even in the location parameter case, although the median is not as susceptible to "wild shot" or extreme observations as the mean, it is only 64 percent efficient if the distribution of the error component is normal. What is desired for each of the equations (2.1) - (2.5) is an estimator with robustness properties similar to the median but one which is relatively efficient under normality assumptions.

---

*This is not a contradiction of the Gauss-Markov theorem since the median is not linear in the observations. For a more detailed discussion of this so-called "wild shot" problem, see Appendix I.

*The estimator is that which results from minimizing $\Sigma|y-\hat{y}|$. This minimization involves solving a linear programming problem.

III.2. The Technique and Rationale of Huber "M" Estimates

Huber [1964] proposed a robust alternative to the mean and the median in the location parameter problem. The estimate is obtained by solving the equation

$$\sum_{t=1}^{T} \phi(y_t - \hat{\theta}) = 0 \qquad \text{for } \hat{\theta} \tag{3.1}$$

$$\text{where} \quad \phi(z) = \begin{cases} z & \text{for } |z| < c \\ c & \text{for } z \geq c \\ -c & \text{for } z \leq -c \end{cases} \tag{3.2}$$

and c is a constant.

This estimate is not linear in the observations unless c is very large, in which case it reduces to the least squares estimate. Huber [1964] derived the asymptotic properties of the estimator as $T \to \infty$, and in tests run by Andrews et al. [1972] it performed well in terms of robustness compared to the sample mean. It was more efficient than the sample mean in the case of long-tailed error distributions and almost as efficient when the error distribution was normal. The reason for the term "M" estimate is that, in general, maximum likelihood estimates are the result of solving an equation like (3.1), where $\phi(z) = \frac{f'(z)}{f(z)}$ and f(z) is a density function. For example, for $\phi(z) = z$ ($c = \infty$) the solution to (3.1) will yield a maximum likelihood estimate where ε is N(0,1). The specification of $\phi(z)$ in (3.2) corresponds to a distribution with fatter tails than the normal distribution. In other

words, between -c and +c the assumed distribution is normal except for multiplication by a constant; however, beyond ±c the assumed distribution resembles an exponential, with tails fatter than the normal.* An error model of this type means that a "wild shot" observation is more likely than in the strict normal distribution case; consequently it is not given as much weight in the calculation of the estimated location parameter as in least squares, which is maximum likelihood if the distribution is normal.

Other functional forms for ϕ have been proposed by Andrews, Hampel and Tukey [Andrews, et al.,1972] which allow the function ϕ to go to zero as z becomes large. For example the ϕ suggested by Hampel is

$$\begin{aligned} \phi(z) &= \sin(z/c) \quad \text{for} \quad |z| \leq \pi c \\ &= 0 \quad \text{for} \quad |z| > \pi c \end{aligned}$$

while the ϕ suggested by Tukey is

$$\begin{aligned} \phi(z) &= z(1 - (z/c)^2) \quad |z| \leq c \\ &= 0 \quad |z| > c \end{aligned}$$

Note, that when a normal distribution is assumed and the corresponding ϕ used, each observation has an effect directly proportional to $(y_t - \theta)$ on the maximum likelihood estimator.

---

*See Appendix B for a derivation of the distribution corresponding to the ϕ proposed by Huber.

For Huber's estimator each observation has an effect proportional to $(y_t - \theta)$ for values of $|y_t - \theta| < c$ and an effect proportional to c for values of $|y_t - \theta| \geq c$. For Andrews, Hampel and Tukey estimators, values $|y_t - \theta|$ which are greater than c have no effect on the estimate. The choice of c for these estimators involves a trade off between Gaussian efficiency (if f is really normal) and robustness (if the true distribution is really a longer tailed distribution than the normal).* The Andrews, Hampel and Tukey estimators will not be discussed further in this chapter. Rather, attention will now focus on the application of the Huber "M" estimate to the regression problem. In particular, the application will be to the DOD budget data previously discussed.

Huber [1973] proposed an analogue to the M estimator for a location parameter as a robust estimator of regression coefficients: for a regression problem of the form

$$y_t = \sum_{i=1}^{p} \beta_i x_{it} + \varepsilon_t$$

the proposal was to find $\hat{\beta} = (\hat{\beta}_1, \ldots, \hat{\beta}_i, \ldots, \hat{\beta}_p)$ that is the solution to the equations

---

*See Andrews [1973] for a short discussion.

$$(3.3) \qquad \sum_{t=1}^{T} x_{1t}\, \phi(y_t - \sum_{i=1}^{p} \hat{\beta}_i x_{it}) = 0$$

$$\vdots$$

$$\sum_{t=1}^{T} x_{pt}\, \phi(y_t - \sum_{i=1}^{p} \hat{\beta}_i x_{it}) = 0$$

where
$$(3.4) \qquad \begin{aligned} \phi(z) &= z \quad \text{for } |z| < c \\ &= c \quad \text{for } z \geq c \\ &= -c \quad \text{for } z \leq -c \end{aligned}$$

The estimates will not in general be linear in the observations. Note that for very large c, the estimator becomes, in effect, the least squares (or maximum likelihood if normal residuals are assumed) estimator. However for smaller c, the estimator reduces the impact of extreme observations. Huber regression estimates can also be viewed as the result of solving the following minimization problem:

$$(3.5) \qquad \min \sum_{t=1}^{T} \rho(y_t - \sum_{i=1}^{p} \hat{\beta}_i x_{it})$$

where
$$\begin{aligned} \rho(z) &= \tfrac{1}{2}z^2 \quad && \text{for } |z| < c \\ &= cz - \tfrac{1}{2}c^2 && \text{for } z \geq c \\ &= -cz - \tfrac{1}{2}c^2 && \text{for } z \leq -c \end{aligned}$$

Note as c gets large the minimization in (3.5) becomes simply a least squares problem.

Huber [1973] derives the asymptotic properties for estimators yielded by solving the system (3.3) under quite general conditions for defining ϕ. In other words, his asymptotic results are not confined to a ϕ of the form of (3.4).

There are two computational difficulties with Huber estimates. First, they are not one step estimates. In order to define the function ϕ, some value for $\hat{\beta}$ is needed. Once the function is defined a new value for $\hat{\beta}$ can be found, and so forth. Huber [1973] has suggested an algorithm for solving the equations.* It is not immediately clear that when the algorithm stops a global minimum to (3.5) or a global maximum to the likelihood function has been achieved. In fact the optimization properties are not discussed by Huber. Fortunately, the algorithm is relatively easy to implement and given some rather weak conditions on ϕ (and consequently on ρ) the stopping conditions specified by Huber are necessary and sufficient for a global minimum to (3.5). This is discussed in Appendix C to this chapter. The second problem is that usually in a regression problem, it is not desirable to assume that ε_t is distributed like a N(0,1) random variable between +c and -c. Rather, the standard assumption is that

---

*This problem and the algorithm are discussed in Appendix C.

ε_t is distributed $N(0,\sigma^2)$ between $+c\sigma$ and $-c\sigma$. This complicates the problem since σ, a scale parameter must be estimated. Thus one additional equation which involves $\hat{\beta}$ must be solved and the system (3.3) is modified by replacing

$$(y_t - \sum_{i=1}^{p} \hat{\beta}_i x_{it}) \quad \text{with} \quad [(y_t - \sum_{i=1}^{p} \hat{\beta}_{it} x_{it})/\hat{\sigma}] \quad .^*$$

III.3. Comparison of Huber M Estimates and the Estimates of Chapter II

Huber "M" estimates were calculated for the procurement and RDTE data for equations (2.1) through (2.5). Estimates were calculated for c values of 2.0, 1.5, and 1.0. Only the results for c values of 2.0 and 1.0 are shown. The values for c = 1.5 lie between these. Tables III.1 and III.2 present the results and the corresponding results from Chapter II.

---

*See [Huber, 1972, Andrews et al., 1972 , Huber, 1973 and Huber, 1964] for discussions of the estimation of this scale factor.

	c=1.0		c=2.0		Least squares	
	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$
(2.1) $y_t = \beta x_t + \varepsilon_t$	.961	NA	.962	NA	.969	NA
(2.2) $y_t = \beta x_t + \varepsilon_t x_t$	.969	NA	.974	NA	.993 .974 (median)	NA
(2.3) $y_t = \beta x_t e^{u_t}$	.963	NA	.971	NA	.977 .973 (median)	NA
(2.4) $y_t = \beta x_t^{\alpha} e^{u_t}$	1.110	.982	1.181	.975	1.473	.948
(2.5) $y_t = \beta x_t^{\alpha} e^{u_t \ln x_t}$	.997	.996	1.041	.991	1.242	.969

TABLE III.1 (NA = not applicable)
Procurement Estimates

Estimates for $\hat{\beta}$ in (2.3) through (2.5) are derived by taking $\exp(\widehat{\ln \beta})$. For (2.3) through (2.5) actually $\ln \beta$ was estimated. The estimates for $\ln \beta$ are in Table III.1.1.

	c=1.0	c=2.0	Least squares
(2.3)	-.033	-.029	-.023 (-.027) (median)
(2.4)	.108	.169	.384
(2.5)	-.0025	.038	.215

TABLE III.1.1
Estimated $\ln \beta$ for Procurement Data

	c=1.0 $\hat{\beta}$	c=1.0 $\hat{\alpha}$	c=2.0 $\hat{\beta}$	c=2.0 $\hat{\alpha}$	Least squares $\hat{\beta}$	Least squares $\hat{\alpha}$
(2.1) $y_t = \beta x_t + \varepsilon_t$	.977	NA	.978	NA	.982	NA
(2.2) $y_t = \beta x_t + \varepsilon_t x_t$	.989	NA	.990	NA	1.093 .989 (median)	NA
(2.3) $y_t = \beta x_t e^{u_t}$	.989	NA	.989	NA	1.025 .989 (median)	NA
(2.4) $y_t = \beta x_t^{\alpha} e^{u_t}$	1.061	.990	1.076	.988	2.028	.903
(2.5) $y_t = \beta x_t^{\alpha} e^{u_t \ln x_t}$	1.029	.994	1.005	.998	2.877	.852

TABLE III.2 (NA = not applicable)
RDTE

Again, for (2.3) through (2.5) it was ln β, not β, that was estimated. The estimates for ln β are in Table III.2.1.

	c=1.0	c=2.0	Least squares
(.3)	-.012	-.011	.024 -.011 (median)
(.4)	.059	.072	.702
(.5)	.028	.005	1.056

TABLE III.2.1
Estimated ln β for RDTE Data

Some of the most interesting results are those for equations (2.4) and (2.5) both for procurement and RDTE data. Before addressing those results, salient features of the results for (2.1) through (2.3) will be pointed out. First, for the procurement data all of the techniques estimated coefficients – estimated percentages of the budget request that are granted – which were equivalent. The only exception to this statement is for equation (2.2) for which the least squares estimate was .993 while the median y_t/x_t estimate and the Huber (c=2.0) estimates were .974, and the Huber (c=1.0) estimate was .969. In other words, the least squares estimate implies that over 99% of the budget request is appropriated, while the others say about 97% of the request is appropriated. It is difficult to assess these differences without some measure of stability or variation of the estimates. This topic will be taken up in Chapter IV. Also of interest is the fact that for the procurement data the results were similar when c=1.0 and c=2.0.

The RDTE results for (2.1) through (2.3) are of even more interest. Recall that in the previous chapter it was observed that certain data points appeared to be affecting the least squares estimates for (2.2) and (2.3). For example, the least squares estimator for (2.2) implied that 109% of the budget request is appropriated. Using the median y_t/x_t value as an estimator in model (2.2), and the median $\ln (y_t/x_t)$ value as an estimator in (2.3) produced results which appeared more reasonable than the least squares results. The application

of the Huber "M" estimator to (2.2) and (2.3)yielded results that were almost identical to those achieved by the use of the median. The result was not particularly surprising since, as mentioned in section III.1 of this chapter, estimation for (2.2) and (2.3) was really a location parameter problem. Andrews, Tukey and others have already noted that both the median and the Huber "M" are useful estimators in location parameter problems when the error distribution is non-normal. Huber "M" is more efficient when near-normality of errors holds. It is interesting to note that the extra computational effort involved in computing the Huber "M" in (2.2) and (2.3) did not yield results different from the "easy to compute" median. As with the procurement data the importance of small differences such as that between the least squares estimate and the Huber "M" estimates for (2.1) using RDTE data must await the analysis of the stability of the estimates in the following chapter and analysis of residuals in the next section.

In the case of both the Procurement and RDTE data — but especially for the RDTE data — the results of applying least squares and the Huber technique to (2.4) and (2.5) appear to be different. For example, for the RDTE data, in (2.5) $\hat{\beta}$ is almost three times as large when least squares is used as when Huber "M" estimates are computed. Again the full impact of the differences must await an analysis of (i) the stability of the coefficients in the next chapter, (ii) examination of the residuals. However, it is possible to sharpen

our intuition about the differences between the results for (2.4) and (2.5) by viewing them in another way.

Since the percentage of a request granted for a request of size x_t is modelled in (2.4) and (2.5) as being equal to $\beta x_t^{(\alpha-1)}$, the actual requests for the procurement and RDTE data were taken and $\hat{\beta} x_t^{\hat{\alpha}-1}$ was computed (i) when ln β and α were estimated via least squares and (ii) when they were estimated using Huber's method. Thus, for the same x_t values it was possible to compare the percentages estimated to have been approved when least squares was used and the percentage estimated to have been approved when the Huber "M" estimate was computed. Tables III.3 and III.4 contain stem and leaf plots of the percentages.

Before discussing the tables it should be noted that since $\hat{\alpha} < 1.0$ under all estimation techniques each of the techniques estimates that as x_t, the request, increases the percentage of the request granted will decrease.

Examination of Table III.3 highlights the fact that the Huber "M" estimates the percentage of the request granted to be smaller than does least squares. For (2.4), the estimated β and α under least squares imply that for 17 of 80 values tried it is estimated that Congress would be appropriating greater than 100% of the request. On the other hand, when Huber "M" estimates are used (c=2.0) it is estimated that for only one of the 80 x_t values tried would Congress be appropriating greater than 100% of the request. (99% for c=1.0). For 80 x_t values tried, the estimated percentage of the request granted would be for the most part between .96 and .99. For

(2.4) $y_t = \beta x_t^{\alpha} e^{u_t}$

c=1.0

94	6
95	3335678889999
96	000111122222333333344777799
97	000001112222333333456666788
98	00234467
99	
100	
101	
102	1

Mean = .968

c=2.0

94	1
95	1113577888999
96	00112223334444455555566
97	000034444555667888889999
98	002233346689
99	134589
100	
101	
102	
103	
104	7

Mean = .972

Least squares

91	9
92	
93	77
94	27
95	002233446778
96	0011334455566677789
97	6777
98	2466667899
99	133344555677
100	133446
101	0046
102	1476
103	57
104	
105	
106	
107	
108	
109	
110	
111	
112	
113	
114	2

Mean = .981

(2.4) $y_t = \beta x_t^{\alpha} e^{u_t}$

TABLE III.3

Procurement: Stem and leaf plots for the percentages of the request "estimated" to be granted when x_t takes on the values found in the procurement data.

c=1.0

963	1
964	55559
965	2556677789
966	00112233555666677788889
967	45559
968	01222234457777888899
969	034455699
970	2379
971	1279
972	
973	
974	
975	
976	
977	
978	9

Mean = .969

c=2.0

96	04445566666667777788888888888999999
97	0000122222222223333333344445555566778899
98	00
99	6

Mean = .971

Least squares

93	8
94	999
95	2567888999
96	00112233445555566677778
97	223367888889
98	00122233333444788999
99	22559
100	02378
101	
102	
103	
104	
105	
106	8

Mean = .975

(2.5) $y_t = \beta x_t e^{u_t \ln x_t}$

TABLE III.3 (continued)

Procurement: Stem and leaf plots for the percentages of the request "estimated" to be granted when x_t takes on the values found in the procurement data.

(2.5) similar differences are noticeable. When the Huber "M" approach is used, these percentages are mostly between .96 and .98 (c=2.0) or .96 and .97 (c=1.0). The mean estimated percentages of the requests that would be granted for the eighty x_t values tried are given in Table III.3.1.

	c=1.0	c=2.0	Least squares
(2.4) $y_t = \beta x_t^{\alpha} e^{u_t}$	.968	.972	.981
(2.5) $y_t = \beta x_t^{\alpha} e^{u_t \ln x_t}$	.969	.971	.975

TABLE III.3.1

The differences between the estimated percentages of the request that will be granted when least squares as opposed to Huber "M" estimates for β and α are used are even more noticeable in Table III.4 for the RDTE data.

The results in Table III.4 probably reflect better than any other information presented so far the implications of the differences between the Huber "M" estimates for β and α for (2.4) and (2.5) and the least squares estimates. For equation (2.4), the least squares estimates for β and α imply that for a low enough request the estimated percentage of the request appropriated is greater than 1.10. The Huber estimates for β and α in (2.4) imply a maximum percentage of 1.025 for the x_t values tried. The differences are more marked for (2.5).

(2.4)

c=1.0

97	7788899999
98	0023333455556667777777888999
99	0011256778899
100	00111122345
101	
102	225

mean = .974

c=2.0

97	8888999999
98	002233344455555666666677777899
99	00134455667788888999
100	11
101	668

mean = .991

Least squares

9	112222223333566667778888999999
10	00000122334677889
11	0001112223345
12	
13	335

mean = 1.028

$$(2.4)\quad y_t = \beta x_t^{\alpha} e^{u_t}$$

(2.5)

c=1.0

982	5689
983	00124579
984	7
985	22349
986	125688
987	022444557
988	00379
989	337
990	
991	249
992	037
993	014778
994	01257
995	37
996	
997	
998	
999	
1000	
1001	
1002	
1003	00
1004	2

mean = .989

c=2.0

987	223344455667
988	022335667889
989	0001111223346799
990	06799
991	023446667789
992	024
993	
994	
995	66
996	1

mean = .998

Least squares

8	566677778889
9	12223444556677788888899
10	01233489
11	001234455667788
12	12
13	
14	
15	226

mean = 1.031

$$(2.5)\quad y_t = \beta x_t^{\alpha} e^{u_t \ln x_t}$$

TABLE III.4

RDTE: Stem and leaf plots for the percentage of the request "estimated" to be granted when x_t takes on the values found in the RDTE data.

The least squares estimates for β and α imply that, for three of the x_t values, as much as 150% of the request was granted. The largest value, for the x_t values tried, when the Huber "M" approach was used was 100.5%. Table III.4.1 contains the mean estimated percentage of the request granted, for the 63 RDTE x_t values tried.

	c=1.0	c=2.0	Least Squares
(2.4) $y_t = \beta x_t^{\alpha} e^{u_t}$	.979	.991	1.029
(2.5) $y_t = \beta x_t^{\alpha} e^{u_t \ln x_t}$	.989	.998	1.031

TABLE III.4.1

When trying to assess the reasons for the differences between the least squares and Huber "M" results which are reported in Tables III.3 and III.4, it is useful to recall that scatterplots of $\ln y_t$ versus $\ln x_t$ in Chapter II reveal that the least squares results were dominated (distorted) by a few extreme or outlier observations.

III.4. Residuals

One of the interesting features of the Huber "M" approach is that it provides an easy way to examine residuals. Recall that the approach solves the following:

$$\underset{\beta}{\mathrm{Min}} \sum_{t=1}^{T} \rho(y_t - \sum_{i=1}^{p} \hat{\beta}_i x_{it})$$

where

$$\begin{aligned} \rho(z) &= \tfrac{1}{2}z^2 && \text{for} \quad |z| < c\sigma \\ &= cz\sigma - \tfrac{1}{2}(c\sigma)^2 && \text{for} \quad z \geq c\sigma \\ &\ \ -cz\sigma - \tfrac{1}{2}(c\sigma)^2 && \text{for} \quad z \leq -c\sigma \end{aligned}$$

and σ is a scale parameter.

We can label the residuals for which $|y_t - \sum_{i=1}^{p} \beta_i x_{it}| < c\sigma$ as Group 1 residuals, residuals for which $y_t - \sum_{i=1}^{p} \beta_i x_{it} \geq c\sigma$ as Group 2 residuals and those for which $y_t - \sum_{i=1}^{p} \beta_i x_{it} \leq -c\sigma$ as Group 3 residuals.

III.4.1. Group 2 and Group 3 Residuals

The items or appropriations accounts which were in the Group 2 and Group 3 residuals were approximately the same for each of the different models (2.1) through (2.5). However, the rank order of these residuals varied between approaches. For example, 1955 Navy RDTE, where the request was $61 million and appropriations were $419 million, produced the largest positive or Group 2 residual for (2.2) through (2.5), and only the third largest positive residual for (2.1). However, the rank order of the Group 2 and Group 3 residuals is not nearly as important as the order of the residuals when ordinary

least squares is used. When least squares is used, each observation receives a weight proportional to $|y_t - \sum_{i=1}^{p} \hat{\beta}_i x_{it}|$ in the calculation of $\hat{\beta}$, while when the Huber "M" approach is used, Group 2 and Group 3 residuals receive a weight proportional to $c\sigma$.

The following table gives, for c=1.0, the items corresponding to Group 2 and Group 3 residuals common to all the models, (2.1) through (2.5), and the budget request and final appropriations for each: The tables reflect no particular rank order since this varied from model to model.

Item and Year	REQUEST	APPROPRIATION
1954 PEMA	1070.7	2226.6
1963 PAMN	3065.0	3834.7
1957 AF Aircraft	3859.9	4533.1
1962 PEMA	1803.0	2532.6
1960 AF Missiles	1832.1	2540.5
1962 PAMN	2000.0	2680.9
1959 PEMA	970.0	1669.3
1961 AF Aircraft	2934.1	3497.2
1963 AF Aircraft	3135.0	3562.4
1962 AF Aircraft	3136.2	3537.2

TABLE III.5

Procurement Group 2 Residual Items

There were minor differences between the Group 3 residuals for (2.1) and for the other equations. These involved two items, not in the table, which are different in the set of Group 3 residuals for (2.1).

Item and Year	REQUEST	APPROPRIATION
1954 AF Aircraft	4283.0	2453.7
1954 AF Missile	3222.0	2574.3
1954 PAMN	1924.2	1222.8
1953 PEMA	2544.4	1889.2 *
1959 AF Missile	1722.1	1394.2
1970 PAMN	3235.5	2620.0
1969 PAMN	3222.0	2574.3
1973 AF Aircraft	3255.7	2682.3 **
1970 PEMA	5069.1	4254.4 **
1969 PEMA	5626.0	5031.4 **
1969 AF Aircraft	4612.0	3860.0

*Not a Group 3 for (2.1).

**Because of the magnitude of the request and because the (2.4) and (2.5) results imply that the percentage of the request granted decreases as request increases, these items do not appear as Group 3 residuals for (2.4) and (2.5).

TABLE III.6

Procurement Group 3 Residuals

Note that the Group 2 residuals contain items from the early 1960's and late 1950's except for 1954 PEMA. Air Force items are particularly noticeable in the Group 2 residuals. On the other hand, the Group 3 residuals are dominated by items from the early 1950's and post-1969 items. This is consistent with the discussion in the last chapter of a possible change in Congressional decision rules starting in 1969.

Item	Request	Appropriations
1955 Navy	61.0	419.9
1960 AF	750.0	1159.9
1962 AF	1637.0	2403.2
1959 Navy	641.0	821.2
1957 AF	610.0	710.0

Group 2 Residuals RDTE

The Group 2 residuals for (2.1) also contained more Air Force items for 1961, 1963, and 1969.

Item	Request	Appropriations
1954 Army	475.0	345.0
1973 Army	2122.7	1829.0
1970 Army	7849.5	1596.8
1970 AF	3561.2	3060.6
1973 Navy	2813.8	2545.3 *
1954 Navy	74.9	58.6 **
1954 AF	537.0	440.0 **

Group 3 Residuals

*Not part of Group 3 for (2.4) and (2.5).

**Not part of Group for (2.1).

The Group 3 residuals for (2.4) and (2.5) also contained the item "1953 Navy", where the request was 75.7 and the appropriations 70.0. Despite the fact that this was only a 5 million dollar cut, (2.4) and (2.5) imply that a small request should generally be cut very little if at all. Thus 1953 Navy appears as a Group 2 residual for (2.4) and (2.5) indicating the appropriation was much less than expected.

The Group 2 and Group 3 residuals for RDTE demonstrate patterns similar to those noted for procurement. That is, the late 1950's and early 1960's items are noticeable in the Group 2 residuals, while the early 1950's and post-1969 items dominate the Group 3 residuals.

In summary, the items producing the largest residuals when the Huber "M" technique was used resemble those producing the largest least squares residuals. However, the magnitude and distribution of the least squares residuals differ. For the RDTE and procurement data analyzed, use of the Huber method allowed the residuals to stand out more dramatically. This is because, heuristically speaking, the Huber method does not try as hard as least squares to avoid having a few very large residuals.

III.4.2 Distribution of Residuals

Both the least squares and the Huber "M" techniques assume that the distribution of the stochastic disturbance is symmetric about zero. One way of examining the appropriateness of an estimator is to examine whether or not the residuals

which result when that estimator is used are symmetric or nearly symmetric about zero. There are certainly many other ways to probe whether or not the assumptions underlying the estimation technique are satisfied; such as, for example, constructing histograms, stem and leaf plots and making probability plots [Wilk and Gnanadesikon,1968]. However the simple technique of counting positive and negative residuals will quickly reveal some of the basic information being sought. The following tables III.6 and III.7, show the number of negative residuals as opposed to the total number of items for (2.1) through (2.5) when the Huber "M" approach, with c=1.0, and when least squares were used.

	Huber M, c=1.0	Least squares
(2.1) $y_t = \beta x_t + \varepsilon_t$	33/80	33/80
(2.2) $y_t = \beta x_t + \varepsilon_t x_t$	35/80	42/80
(2.3) $y_t = \beta x_t e^{u_t}$	36/80	42/80
(2.4) $y_t = \beta x_t^{\alpha} e^{u_t}$	37/80	46/80
(2.5) $y_t = \beta x_t^{\alpha} e^{u_t \ln x_t}$	36/80	54/80

TABLE III.5

Procurement — Number of negative residuals divided by total number of residuals

	Huber M, c=1.0	Least squares
(2.1) $y_t = \beta x_t + \varepsilon_t$	23/63	28/63
(2.2) $y_t = \beta x_t + \varepsilon_t x_t$	32/63	57/63
(2.3) $y_t = \beta x_t e^{u_t}$	30/63	48/63
(2.4) $y_t = \beta x_t^{\alpha} e^{u_t}$	30/63	35/63
(2.5) $y_t = \beta x_t^{\alpha} e^{u_t \ln x_t}$	31/63	28/63

TABLE III.6

RDTE — Number of negative residuals divided by total number of residuals

The results in Tables III.5 and III.6 reveal a few marked differences between overall residual patterns resulting from the application of least squares as opposed to the Huber "M" estimator. For the procurement data, least squares applied to (2.5) yields a large percentage of negative residuals — 14 more than one-half — while the Huber "M" yields a percentage slightly smaller than one-half. A similar but not quite so noticeable difference results for (2.4). On the other hand, for (2.1)-(2.5) using the Huber "M" the fraction of negative residuals is always less than one-half. For the RDTE data the major differences between the two approaches, in terms of positive-negative symmetry of the residuals, is for (2.2) and (2.3), the location parameter problems. Least squares applied to (2.2) yields an estimate for β which results in 57 of a possible 63 negative residuals as opposed to the

Huber "M" which yields 32 of a possible 63. For (2.2), least squares results in 48 negative residuals while the Huber "M" results in 30 negative residuals. The similarity between the numbers of positive and negative residuals when least squares and the Huber "M" are applied to (2.4) and (2.5) is somewhat deceiving. An inspection of the individual items which correspond to the positive and negative residuals reveal marked differences between the two approaches. For example, for (2.5) the residuals of a total of twenty-eight items had signs which were different when the Huber "M" approach was applied than the signs resulting when least squares was applied. In order to look into this matter a little more closely, recall first that for (2.4) and (2.5) the least squares residuals were mostly negative (except for a few large positive ones) for small values of $\log x_t$ and mostly positive for large values of $\log x_t$ see Figure III.2 for illustration.

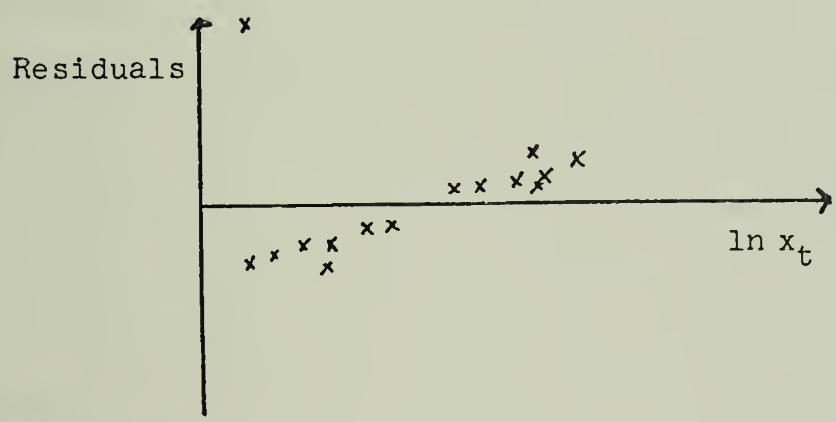


FIGURE III.2

Thus despite the fact that the numbers of positive and negative residuals were nearly equal, they bore an undesirable relationship to x_t. As reported in Table III.6, the residuals resulting from the application of the Huber "M" approach to (2.4) and (2.5) are also symmetric around zero but do not show a relationship to x_t like that in Figure III.2.

III.5 Summary

This chapter commenced with a discussion of the problem of robust estimation when a linear regression can not be reduced to a location parameter problem and when one is not prepared to pay for the lack of efficiency of the sample median if the error distribution is normal in the location parameter problem. The need for a robust estimation technique was especially evident in Chapter II upon examination of scatterplots, least squares coefficient estimates, residual patterns for the models (2.4) and (2.5), and, to a lesser extent for (2.1). The Huber "M" estimator has been proposed and utilized as a robust alternative to least squares. The logical basis for the estimator, and its relationship to maximum likelihood estimation, were discussed. Section III.3 contains a comparison of Huber "M" and least squares estimates for the coefficients in (2.1) through (2.5) for both procurement and RDTE data. The results

were noticeably different for (2.4) and (2.5) both for procurement and RDTE and for (2.2) and (2.3) for RDTE data. The final section, prior to this summary, contains a discussion of the Group 2 and Group 3 residuals produced by the Huber "M" approach and a comparison of the distribution of the residuals under least squares and Huber "M" estimation.

At this point a short assessment of what has been accomplished is in order. The Huber "M" approach yielded results, for (2.4) and (2.5) for procurement and RDTE and for (2.2) and (2.3) for RDTE, which appear to be more reasonable, substantively, and more consistent with the assumptions underlying the estimation technique than those produced by least squares. However, the implication for the defense budget question is possibly even broader. No claim has been made in this analysis that Congressional action on defense budgets is completely deterministic and predictable. Rather, the analysis has been based, more or less implicitly up to this point, on the view that embedded in the process of Congressional consideration are certain regular elements which resemble the outcomes which occur when simple rules of thumb are applied and certain irregular, and unusual elements which do not appear to be the product of the application of fixed decision rules or simple rules of thumb (or may be the result of the application of different decision rules). The robust regression approach, through the specification of Group 2 and Group 3 residuals, has allowed us to distinguish or separate

out of these two sets of elements. One might argue that analysis of least squares residuals would do the same thing. However, the robust regression approach has in some sense diminished the contamination of the estimation of what is going on in the fixed or regular area by the data generated by unusual or irregular occurrences. Such a contamination can be considerable when least squares is used.

Several questions still remain unanswered. For example, how stable are the coefficient estimates – both least squares and Huber "M" estimates? That is, can reasonable confidence limits be placed on estimated coefficients? How do (2.1) through (2.5) compare among themselves in terms of stability of estimates and prediction intervals? It may not be possible to answer these questions by only re-analyzing the data at hand, since the theory underlying interval estimation is not well-developed for the Huber "M". Thus, data might have to be constructed which provide a standard against which to compare results for the data at hand. One reasonable procedure is to synthesize such data, i.e. by sampling from a suitable disturbance distribution.

Several questions have been addressed and partially answered, but need further analysis. What can be said about what appear to be the "irregular" elements of the process. Are they completely inexplicable or are there as many different explanations as there are different budget items and years. Certain observations have already been made, such as the post-1969 "cutting" mood of the Congress and the generosity

of the Congress to the Air Force between 1957 and 1969. Possibly some models are more appropriate for parts of the data than others. These and other questions will be addressed in the next two chapters.

APPENDIX A

The Mean and Median as Location Parameter Estimators

A.1. The Wild Shot Problem

The classic location parameter problem is one with the following form:

$$(1) \qquad y_t = \theta + \varepsilon_t$$

where θ is a location parameter and the ε_t are independent and identically distributed, $E(\varepsilon_t)=0$ and $var(\varepsilon_t)=\sigma^2$. The Gauss-Markov theorem [Scheffé,1959] says that the least-squares estimator, in this case the sample mean, is the best (minimum variance) unbiased estimator which is also linear in the observations. If the ε_t are $N(0,\sigma^2)$, then the least squares estimator is a best (minimum variance) estimator.

Unfortunately the sample mean is susceptible to extreme observations, since the value of a least squares estimator, $\hat{\theta}$ is proportional to the absolute difference $|y_t-\hat{\theta}|$. The following example demonstrates this point.

If the ε_t are $N(0,1)$ then for a sample of 10 observations the variance of $\bar{y}$ is 0.1. However, suppose that the errors, ε_t , are of the following form:

(2) $$\varepsilon_t = \begin{cases} \xi_t \sim N(0,1) & \text{with probability } p \\ \eta_t \sim N(0,\sigma^2) & \text{with probability } q = 1-p \end{cases}$$

The density for ε_t then becomes

(3) $$f_{\varepsilon_t}(x) = \frac{pe^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} + \frac{qe^{-\frac{1}{2}(x/\sigma)^2}}{\sqrt{2\pi\sigma^2}}$$

In this case, $E(\varepsilon_t) = 0$, and

(4) $$Var(\varepsilon_t) = E(\varepsilon_t^2) = p + q\sigma^2$$

Suppose $p = 0.9$, and $q = 0.1$ and $\sigma^2 = 10$. Then, on the average, one observation out of 10 comes from a distribution with large variance. However, using (4), $var(\varepsilon_t)$ is equal to

$$E(\varepsilon_t^2) = 0.9 + (0.1)10$$
$$= 1.9$$

and the $var(\bar{y})$ equals 0.19. Note what has happened: when one out of ten observations comes, on the average, from a contaminating distribution, the variance of the estimator, $\bar{y}$, nearly doubles.

A.2 The Median as a Location Parameter Estimate: The Normal Case

The median is an alternative estimator to the mean in location parameter problems when disturbanc distributions are symmetric. It can be shown that as the sample size, n, gets large, then for a distribution that is symmetric around zero, the following statement can be made about $\tilde{y}$, the sample median [David,1970]:

$$E(\tilde{y}) \approx 0$$

$$\operatorname{var}(\tilde{y}) \approx \frac{1}{4n(f(o))^2}$$

where f is the density from which a sample is being taken. Thus if ε_t is N(0,1), then

$$\operatorname{var}(\tilde{y}) \approx \frac{2\pi}{4n} = \frac{1.57}{n} \quad \text{(as n gets large)}$$

$$\text{while } \operatorname{var}(\bar{y}) = \frac{1}{n} \quad .$$

Thus the median has an efficiency, with respect to the mean, of roughly $\frac{1}{1.57} = 0.64$. In other words, if the ε_t are N(0,1), then it will take 1.57 as many observations when using the sample median as opposed to the sample mean in order to achieve the same variance for the estimator.

A.3. The Mean Versus the Median: The Wild Shot Case

The relationship between the variance of the sample median and the variance of the sample mean reverses when the ε_t are not normally distributed but are distributed according to (2).

The variance of the sample median, when the ε_t are distributed according to (2) is approximately given by

$$\text{(6)} \qquad \text{var}(\tilde{y}) \approx \frac{2\pi}{4n(p + \frac{q}{\sigma})^2}$$

If $p = 0.9$, $q = 0.1$ and $\sigma^2 = 10$ then

$$\text{var}(\tilde{y}) \approx \frac{1.68}{n}$$

Thus for $n = 100$, $\text{var}(\tilde{y}) \approx .0168$ and $\text{var}(\bar{y}) = .019$.

This "robust" behavior of the median as compared to the mean is noticeable in the case of many long-tailed error distributions [Andrews, et al., 1972].

APPENDIX B

The Huber "M" Estimator as a Maximum Likelihood Estimator

B.1. Basic Concepts and the Normal Case as An Example

For the discussion that follows, assume that what is being sought is an estimate for β in the model

$$(1) \qquad y_t = \beta x_t + \varepsilon_t$$

where ε_t is a random variable and the ε_t's are independent and identically distributed. The following discussion can be generalized to the case in which β and x_t are vectors. Let the density for ε be $p(y - \beta x)$. If one observes $y_1, y_2, \ldots, y_T$ and $x_1, x_2, \ldots, x_T$ then the log likelihood of the sample is given by

$$(3) \qquad L = \sum_{t=1}^{T} \log p(y_t - \beta x_t)$$

A maximum likelihood estimate for β, $\hat{\beta}$, is an estimate which maximizes the likelihood, which is equivalent to maximizing L. The necessary condition for a maximum yields the equation

(4) $$\frac{\partial L}{\partial \beta} = \sum_{t=1}^{T} x_t \frac{p'(y_t - \beta x_t)}{p(y_t - \beta x_t)} = 0$$

or

$$\sum_{t=1}^{T} x_t \phi(y_t - \beta x_t) = 0 \ , \quad \text{where}$$

$$\phi = - \frac{p'}{p}$$

The maximum likelihood estimate, $\hat{\beta}$, satisfies equation (4). We recognize the possibility of multiple roots, but disregard the complication for the present.

If the ε_t are assumed to be N(0,1) then $\phi(z) = z$. This is easily seen since

$$p(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z)^2}$$

$$p'(z) = -z \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

$$\frac{p'(z)}{p(z)} = -z$$

and

$$\sum_{t=1}^{T} x_t(y_t - \beta x_t) = 0$$

or

$$\hat{\beta} = \frac{\Sigma\, x_t y_t}{\Sigma\, x_t^2}$$, which is also the least squares

estimator for β.

B.2. A Distribution That is Implied by the Huber "M" Estimator

The Huber "M" approach, replaces $\phi(z) = z$, by

(5) $$\phi(z) = \begin{cases} -c & \text{for } z < c \\ z & \text{for } |z| < c \\ +c & \text{for } z \geq +c \end{cases}$$

It is possible to find a distribution for which the likelihood is maximized when the following equation (6) is solved for β and when certain restrictions, such as continuity, are placed on the distribution.

(6) $$\sum_{t=1}^{T} x_t\ \phi(y_t - \beta x_t) = 0$$

where ϕ is as specified in (5).

The density function (and distribution) is defined over three disjoint regions of interest: $(-\infty, c]$, $(-c, +c)$, $[c, \infty)$. Call these Regions 1, 2, and 3 respectively. For

For these regions, the specification ϕ in (5) implies the following differential equations, where p_i refers to a density defined over region i.

$$\frac{p_1'(z)}{p_1(z)} = +c$$

$$\frac{p_2'(z)}{p_2(z)} = -z$$

$$\frac{p_3'(z)}{p_3(z)} = -c$$

The solutions to these equations are $p_1(z) = K_1 e^{+cz}$, $p_2(z) = He^{-\frac{1}{2}z^2}$ and $p_3(z) = K_2 e^{-cz}$. If one desires the distribution which is being derived to be symmetric around zero, then $K_1 = K_2$. Call this constant K. Also, to simplify matters, let $K = K^*c$, so that $p_1(z) = K^*ce^{cz}$ and $p_3(z) = K^*ce^{-cz}$. The constants H and K* must satisfy certain conditions. First,

(7) $$\int_{-\infty}^{-c} K^*ce^{cz}\,dz + \int_{-c}^{+c} He^{-\frac{1}{2}z^2}\,dz + \int_{c}^{\infty} K^*ce^{-cz}\,dz = 1.$$

Next, it is attractive (but not essential) that the density p, which is being derived be continuous. This will be the case if

(8) $$p_1(-c) = p_2(-c)$$

(9) $$p_2(c) = p_3(c) \quad .$$

Equations (8) and (9) are not independent. However, (7) and (8) are. Consequently they can be used to derive K* and H.

First, using equation (8) it follows that

$$K^* c e^{-c^2} = H e^{-\frac{1}{2}c^2}$$

and

$$K^* = \frac{H}{c} e^{+\frac{1}{2}c^2} \tag{10}$$

Substituting in equation (7) yields the following

$$\frac{H}{c} e^{-\frac{1}{2}c^2} + H \int_{-c}^{+c} e^{-\frac{1}{2}z^2} dz + \frac{H}{c} e^{-\frac{1}{2}c^2} = 1$$

$$\frac{2H}{c} e^{-\frac{1}{2}c^2} + H \int_{-c}^{+c} e^{-\frac{1}{2}z^2} dz = 1 . \tag{11}$$

The second term in equation (10) can not be put into closed form so that a closed form expression for H can be stated. However, it is easy to obtain an approximate value for $\int_{-c}^{+c} e^{-\frac{1}{2}z^2} dz$ by using the standard normal tables to get the area under the standard normal density between -c and +c and then multiply this value by $\sqrt{2\pi}$.

Let us call the area $\Phi(c) - \Phi(-c)$. Using this, H is found to be

$$H = \frac{c}{2e^{-\frac{1}{2}c^2} + \sqrt{2\pi}\, c(\ \Phi(c) - \Phi(-c)\ )} \tag{12}$$

Returning to the original problem, the Huber "M" approach yields a maximum likelihood estimator for β when the density of ε_t is of the form

$$(13) \qquad p(z) = \begin{cases} K\ e^{+cz} & z \le -c \\ H\ e^{-\frac{1}{2}z^2} & |z| < c \\ K\ e^{-cz} & z \ge c \end{cases}$$

where K = K*c and K* and H are found using (11) and (12). This solution assumes that any scale parameter is already implicit in c. If a scale parameter also needs to be estimated, the formulas change slightly, however the basic results remain the same: p(z) is like a normal density between (-c,+c) but has "fatter," exponential tails than a normal density beyond -c and +c.

Of course, the distribution defined in (13) is not the only distribution implied by the Huber "M" estimator. For example, if one wanted p(z) to behave exactly like a normal density between -c and +c, H could be made equal to $\frac{1}{\sqrt{2\pi}}$, in which case p(z) would not be continuous unless c had a value which allowed (12) to be satisfied.

APPENDIX C

Algorithm for Deriving Huber "M" Estimates and a Proof of Optimality

C.1. Iterative Algorithm for Deriving Huber "M" Estimates

In Appendix B it was shown that the Huber "M" estimator is a maximum likelihood estimator. It is also true that the Huber "M" estimator is the solution to the problem

$$(1) \qquad \min_{\beta} Z(\beta) = \sum_{t=1}^{T} \rho(y_t - \beta x_t)$$

$$\text{where } \rho(z) = \begin{cases} -cz-\tfrac{1}{2}c^2 & z \leq -c \\ \tfrac{1}{2}z^2 & |z| < c \\ cz-\tfrac{1}{2}c^2 & z \geq c \end{cases}$$

Notice that (1) is a modified least squares function. However, intuitively, the influence or effect on the estimate, $\hat{\beta}$, of observations for which $|y_t - \hat{\beta}x_t| \geq c$ is not proportional to $|y_t - \beta x_t|$ as in the least squares case.

The procedures for solving (1) which have been proposed are iterative procedures where an estimate of β at step i is used to obtain a new estimate at step i+1 [Huber,1973, Andrews,1973]. One of these algorithms [Huber,1973] will be discussed in detail in this appendix.

The algorithm requires the following steps.

(a) Start with an initial value of $\hat{\beta}$, for example the least squares estimate; call this $\hat{\beta}^{(1)}$. The i^{th} estimate is $\hat{\beta}^{(i)}$ $(i=1,2,...)$.

(b) Based on the current estimate of $\hat{\beta}$, partition the observations into three groups, $T_1^{(i)}$, $T_2^{(i)}$, $T_3^{(i)}$.

$$\begin{aligned} &\text{Group } T_1^{(i)} = \{t: \quad y_t - \hat{\beta}^{(i)}x_t \leq -c\} \\ (2) \qquad &\text{Group } T_2^{(i)} = \{t: \quad |y_t - \hat{\beta}^{(i)}x_t| < c\} \\ &\text{Group } T_3^{(i)} = \{t: \quad y_t - \hat{\beta}^{(i)}x_t \geq c\} . \end{aligned}$$

If the partition is identical to that achieved for $\hat{\beta}^{(i-1)}$ then stop. Use $\hat{\beta}^{(i)}$ as the estimate for β. Otherwise go to step (c).

(c) Minimize, if possible the function $Z_i^*(\beta)$ with respect to β where Z_i^* is defined like Z except that the partition $(T_1^{(i)},T_2^{(i)},T_3^{(i)})$ is held fixed or in other words is not allowed to change as β changes. Call the solution $\hat{\beta}^{(i+1)}$. Go to step (b).

C.2. Some Properties of the Huber Algorithm

Very little is said about the algorithm in [Huber,1973]. The literature on robust regression apparently does not contain statements concerning the characteristics of the

---

*The only problem that could arise is if a minimum of $Z^*(\beta)$ can not be determined, that is, if $T_2^{(i)}$ is empty in which case the solution to the minimization of Z^* is unbounded.

algorithm, except that it appears to work. Two questions which will be addressed here are: Is the stopping rule a true optimality condition? Does the algorithm achieve a global optimum or can it stop at a local optimum which may be far away from the global solution?

<u>Proposition</u>: The function Z is differentiable with respect to β, and

$$Z'(\beta) = - \sum_{t=1}^{T} x_t \rho'(y_t - \beta x_t) = \sum_{t=1}^{T} x_t \phi(y_t - \beta x_t)$$

where

$$\phi(y_t - \beta x_t) = \begin{cases} -c & \text{if } y_t - \beta x_t \leq -c \\ (y_t - \beta x_t) & \text{if } |y_t - \beta x_t| < c \\ +c & \text{if } y_t - x_t \geq c \end{cases}$$

Proof: This is an immediate consequence of the chain rule, since Z is a sum of composites of differentiable functions.

The next result shows that satisfaction of the algorithm's stopping condition automatically satisfies a necessary condition for a minimum to Z(β) .

<u>Proposition</u>: When the stopping condition is invoked, then $Z'(\beta) = 0$.

Proof: Let $\tau^{(i)} = (T_1^{(i)}, T_2^{(i)}, T_3^{(i)})$. The stopping condition says $\tau^{(i+1)} = \tau^{(i)}$. Recall that $\hat{\beta}^{(i+1)}$ is the β which minimizes Z*(β). Thus

$$(3) \quad \sum_{t \varepsilon T_1^{(i)}} x_t(y_t - \beta^{(i+1)} x_t) + \sum_{t \varepsilon T_2^{(i)}} - x_t c + \sum_{t \varepsilon T_3^{(i)}} x_t c = 0 .$$

If $\tau^{(i+1)} = \tau^{(i)}$ then

$$(4.a) \quad \sum_{t \varepsilon T_1^{(i+1)}} x_t(y_t - \hat{\beta}^{(i+1)}) + \sum_{t \varepsilon T_2^{(i+1)}} - x_t c$$

$$+ \sum_{t \varepsilon T_3^{(i+1)}} x_t c = 0 .$$

and

$$\text{for} \quad t \varepsilon T_1^{(i+1)} , \quad (y_t - \hat{\beta}^{(i+1)} x_t) \leq -c$$

$$(4.b) \quad t \varepsilon T_2^{(i+1)} , \quad |y_t - \hat{\beta}^{(i+1)} x_t| < c$$

$$t \varepsilon T_3^{(i+1)} , \quad y_t - \hat{\beta}^{(i+1)} x_t \geq c .$$

The left hand side of (4.a), together with the inequalities in (4.b) correspond to the definition of $Z'(\beta)$ while equation (4.a) itself says $Z'(\beta) = 0$.

Since $Z'(\beta) = 0$ when the stopping condition is invoked, the necessary condition for a minimum to $Z(\beta)$ is satisfied. If satisfaction of the algorithm's stopping condition

satisfies not only necessary but also sufficient conditions for a minimum of $Z(\beta)$, then it is not necessary to worry about potential local minima or stationary points when the stopping condition is satisfied. The approach to proving sufficiency is based on proof of an interesting characteristic of $Z(\beta)$. This characteristic and the proof to it not only are useful for the present analysis, but also indicate a type of function ρ for which necessary and sufficient conditions for a minimization like (1) can be stated for an algorithm like Huber's.

Proposition: $Z(\beta)$ is convex in β.

Proof: By the definition of $\rho(\dot{z})$, ρ is convex in $\dot{z}$. Thus $\rho(a+bz)$, where a and b are constants is also convex in z.

Theorem: When the stopping condition is satisfied necessary and sufficient conditions for a minimum are satisfied.

Proof: The theorem is true since $Z'(\beta) = 0$ is a necessary and sufficient condition for the minimization of a convex differentiable function.

CHAPTER IV

In the previous two chapters, defense budget data were analyzed using equations which were designed to describe or represent general tendencies and characteristics of Congressional appropriations action, as a function of requests. In addition, some attempt was made to identify situations in which these tendencies or characteristics are not found or present. Different techniques were employed in estimating the coefficients in these equations, some robust and others not so robust. In Chapters II and III, coefficient estimators and residuals were compared both for analysis using least squares and analysis using Huber "M" estimates. In Chapter V, the implications of the patterns of the residuals will be discussed further. However, before that discussion begins the discussion of estimation begun in the previous chapter will be completed.

Up to this point our attention has focused on obtaining point estimates. The purpose of this chapter is to expand on the work of earlier chapters by deriving some measures of variability or stability of the point estimates and by estimating confidence intervals for the coefficients in equations (2.1)

through (2.5). The first section of this chapter poses the problem of determining the variance of an estimator. In section two, the jackknife is proposed as a possible solution to this problem. Finally, results of jackknifing least squares and Huber "M" estimates are summarized and discussed. The appendices contain discussions of the problem of biasedness of estimators for β in equations like (2.3), and of the rationale behind probability plotting, together with probability plots for jackknifed estimates.

IV.1. The Problem

IV.1.1. Background

The basic questions that are addressed in this chapter are the result of the fact that estimators (least squares, Huber, etc.) are modelled as random variables.

Since estimators are random variables, important questions arise concerning their properties. What is the expected value, what is the variance, what is the distribution of a particular estimator? If the expectation of the estimator equals the true value of the parameter being estimated, the estimator is said to be unbiased [Kendall and Stuart,I, 1963, p. 222]. If as the sample size gets large the estimator converges in probability to a generate random variable which equals with probability one the value of the parameter being estimated, then the estimator is said to be consistent [Kendall and Stuart,II,1951,p. 3]. Different estimators of the same parameter are often compared by examining their

variance. If the estimators are unbiased, this is done by computing their <u>relative efficiency</u> [Kendall and Stuart,II, 1951,pp. 5-6], which is defined as the ratio of the variance of the estimators. Numerous other characteristics of estimators become important when the estimation problem is put into a decision theoretic context [Ferguson,1967].

Unfortunately, it is not always possible to obtain an unbiased, consistent, efficient estimator, and sometimes even when one is found, these properties are sensitive to certain assumptions made about the problem (such as the normality of ε_t). The wild shot problem of Appendix A of Chapter III gives an example of an estimator which suffers when certain assumptions are not met.

If there are competing estimators – and competing problem formulations, such as (2.1) through (2.5) – it is useful to ask and attempt to answer certain questions. Is the variance of the estimator large? Consequently, the question of how this variance should be estimated arises. Another question is associated with the distribution of the estimator. Is it normal? What about large sample behavior?

Not all of these questions will be answered fully in what follows. However, estimates will be derived for the standard error of the estimated coefficients, both when using the

Huber "M" approach and least squares. This will allow us to construct interval estimates for β (and α where applicable) in (2.1) through (2.5).

IV.1.2. The Problem in the Least Squares Case

The problem that is of interest at this point is twofold. First there is the question of the bias of the estimator; secondly, there is the question of how to estimate its stability, as measured by the variance.

Least squares applied to (2.1) yields an unbiased estimator for β. This is shown in most standard textbooks [Johnston,1963,p. 15]. Also, least squares applied to (2.2), after dividing through by x_t, yields an unbiased estimator for β. However, there are some problems when estimating comparable parameters in (2.3) through (2.5). In each case logarithms are taken and ln β is estimated. The least squares method applied to the transformed equations yields unbiased and maximum likelihood estimators (assuming independent normal residuals)for ln β and α. However, as shown in Appendix C, the estimator for β given by $\hat{\beta} = e^{\widehat{\ln \beta}}$ is not unbiased.

When the least squares method is applied to (2.1), the variance of the estimator for β, if it exists, given in most standard texts is:

(4.1) $$\sigma_{\hat{\beta}}^2 = \frac{\sigma^2}{\left(\sum_{t=1}^{T} x_t^2\right)}$$

where σ^2 is the variance of ε_t.

One method of estimating $\sigma_{\hat{\beta}}^2$ for (2.1) is to estimate σ^2 by

$$(4.2) \qquad \hat{\sigma}^2 = \frac{\sum_{t=1}^{T} e_t^2}{T-1}$$

where $e_t = y_t - \hat{\beta}x_t$, and substitute $\hat{\sigma}^2$ for σ^2 in (4.1).

Similar formulas can be derived for $var(\hat{\beta})$ in (2.2) and $var(\hat{\alpha})$ in (2.4) and (2.5).

It is possible to derive an expression or estimator for β which for large T is nearly unbiased and also to derive an approximate expression for the variance of the estimator as T, the sample size, gets large. The method is the Tukey-Quenouille-Miller jackknife, and it will be discussed in Section IV.2.

If the ε_t (or u_t) are assumed to be normal, then $\hat{\beta}$ in (2.1) and (2.2), $\ln \hat{\beta}$ for (2.3) through (2.5) and $\hat{\alpha}$ in (2.4) and (2.5) are also normal. However, if this assumption is not made, the estimators are in general not normally distributed. Asymptotic normality can be shown to hold for jackknifed estimators to be discussed in Section IV.2 [Miller,1974].

IV.1.3. The Problem in the Huber "M" Case

Small sample properties of the Huber "M" estimators have not been determined analytically. Most studies comparing the properties of these estimators with those of least squares estimators have employed Monte Carlo techniques [Huber,1972, Huber,1973, Andrews, et al.,1972].

Huber has derived asymptotic results which show that the "M" estimators are asymptotically normal under certain mathematical conditions. Approximate expressions for the variance of the estimator as the sample size, T, gets large have also been derived [Huber,1973].* Nevertheless, existing mathematical results give no precise guide to action in the finite sample case except to use a finite sample version of the asymptotic variance. An especially thorny problem is what to use as an estimate for $\mathrm{var}(\hat{\beta})$ in (2.3) through (2.5).

In the next section the jackknife will be proposed as a method of obtaining estimates of the variances of estimates for β and α. In the case of least squares the appropriateness of the jackknife has been established, however the same is not true for the Huber "M". This question of the appropriateness of jackknifing Huber "M" estimates will be addressed in Chapter V.

---

*See Appendix D.

IV.2. The Jackknife

The jackknife is a technique which was originally proposed by Quenouille [1949] for reducing bias in estimation problems. Tukey extended the use of the technique by pointing out that it could be used to obtain approximate confidence intervals in situations for which standard statistical procedures do not exist, or are difficult to apply. Miller [1964] analyzed the utility of the jackknife for obtaining confidence intervals and presented examples to illustrate occasions when such a use is not appropriate. He also showed the conditions under which some jackknife estimators (jackknifed transformations of the sample mean) are asymptotically normal. Recently, the properties of jackknifed least squares estimators have been analyzed [Miller, 1974].

The jackknife procedure is based on dividing the data into groups, obtaining estimates from combinations of the groups and then averaging the estimates. More specifically in its use in this analysis , a jackknifed estimator is formed in the following way. First, successively leave out one observation and compute

$$\hat{\beta}_{-t} = T\hat{\beta} - (T-1)\hat{\beta}_{T-t}$$

where $\hat{\beta}$ is the estimate when all the data are used, $\hat{\beta}_{T-t}$ is the estimate when all the data but the t^{th} observation are used. $\hat{\beta}_{-t}$ is an estimate called the t^{th} pseudo-value. Next compute the average of the pseudo-values, designated by

(4.8) $$\tilde{\beta} = \frac{\sum_{t=1}^{T} \hat{\beta}_{-t}}{T}$$

$\tilde{\beta}$ is the jackknifed estimator. Tukey has been reported as suggesting that under certain conditions the pseudo-values are approximately independent and that

(4.4) $$\sum_{t=1}^{T} \frac{(\tilde{\beta} - \hat{\beta}_{-t})^2}{T(T-1)}$$

would be an appropriate estimate for the variance of $\tilde{\beta}$ [Miller,1964].

A major part of the attractiveness of the jackknife in this analysis stems from the fact that Miller has derived results for the case where $\hat{\beta} = f(\hat{\theta})$ and $\hat{\theta}$ is a least squares estimate in a regression context [Miller,1974]. In particular, suppose $\hat{\beta} = e^{\widehat{\ln \beta}}$. Miller's results show that under suitable conditions, to be specified later, an approximate t-statistic and confidence interval can be constructed for $\tilde{\beta}$, using (4.3) and (4.4).

Although the properties of jackknifed estimators have not yet been proven for the Huber "M" and other robust estimators, their use in order to obtain an estimate of β and to obtain a useable estimate for $\sigma_{\tilde{\beta}}$ appears reasonable and is one of the few attractive options for computing variance estimates for "Huberized" estimators. Huber [1972, p. 1053] has pointed out that each pseudo-value is a finite sample version of the influence of a single observation on a complicated estimation [Andrews,et al.,1972]. He adds that "As a rule, jackknifing gives

a useable estimate for the variance of a robust estimate," and that this estimate of the variance can then be used to produce an approximate confidence interval.

In the following section results of the application of the jackknife both for least squares and Huber "M" estimates will be presented, together with estimates of $\sigma_{\tilde{\beta}}$ and $\sigma_{\tilde{\alpha}}$. Approximate confidence intervals will also be presented.

IV.3. Results

IV.3.1. Basic Results

Revised estimates of β and α were computed by jackknifing. This was done for (2.1) through (2.5) both when using least squares and when using the Huber "M" approach, with c=1.0. In addition, estimates for the variance of $\tilde{\beta}$ and $\tilde{\alpha}$ were computed, as suggested by Miller in [Miller,1974]. All computations were performed both on the procurement and the RDTE data.

Several important questions arise in the course of analyzing the results. First, since jackknifing was originally proposed as a technique for bias reduction, how do the jackknifed estimates for the coefficients compare with the "non-jackknifed" estimates, especially for $\hat{\beta}$ in (2.3) through (2.5)? Secondly, how do the variance estimates compare — both between equations and between estimation techniques?

First, comparison of jackknifed versus non-jackknifed estimates for β and α, using least squares and the Huber "M" approaches, is contained in Tables IV.1 (procurement) and IV.2 (RDTE).

As discussed earlier, least squares when applied to (2.3) through (2.5) in logarithmic form yields biased estimates for β. The jackknifed estimates for (2.3) do not appear to differ very much from the unjackknifed estimates. This is true for both least squares and Huber "M" estimates. On the other hand, there is a marked difference between jackknifed and unjackknifed estimates for β in (2.4) and (2.5). While jackknifed and unjackknifed estimates for β differ, the estimates for α appear to be equivalent.

The Huber "M" estimates demonstrate some interesting characteristics. For RDTE, the jackknifed and unjackknifed estimates for β in (2.4) and (2.5) are nearly the same. However, for the procurement data, the jackknifed estimates differ considerably from the unjackknifed estimate although not as much as is true for the least squares estimates. Reasons for these differences will be suggested later.

A final interesting point about Tables IV.1 and IV.2 deals with the estimate for β obtained by using least squares on (2.2) with the RDTE data. The unjackknifed $\hat{\beta}$ was 1.09 while the jackknifed $\tilde{\beta}$ was .908. One result estimates Congress on the average to grant 109% of the request while the other says they grant 91% of the request. This point will be pursued further after the estimates for $\sigma_{\tilde{\beta}}$ and $\sigma_{\tilde{\alpha}}$

Procurement

	Least Squares		Huber "M"	
	β	α	β	α
(2.1) $y_t = \beta x_t + \varepsilon_t$	.961 (.959)	NA	.948 (.961)	NA
(2.2) $y_t = \beta x_t + \varepsilon_t x_t$	.955 (.993)	NA	.962 (.969)	NA
(2.3) $y_t = \beta x_t e^{u_t}$	1.011 (.977)	NA	.950 (.963)	NA
(2.4) $y_t = \beta x_t^{\alpha} e^{u_t}$	1.272 (1.471)	.994 (.948)	.967 (1.111)	.995 (.982)
(2.5) $y_t = \beta x_t^{\alpha} e^{u_t \ln x_t}$	.761 (1.242)	.970 (.969)	.765 (.997)	1.021 (.996)

TABLE IV.1

Jackknifed Least Squares and Huber "M" Estimates (Top estimate is jackknifed estimate. Bottom estimate, in parentheses, is junjackknifed estimate.)

RDTE

	Least Squares		Huber "M"	
	β	α	β	α
(2.1) $y_t = \beta x_t + \varepsilon_t$	.982 (.982)	NA	.973 (.977)	NA
(2.2) $y_t = \beta x_t + \varepsilon_t x_t$	.908 (1.093)	NA	.984 (.989)	NA
(2.3) $y_t = \beta x_t e^{u_t}$	1.017 (1.025)	NA	.987 (.989)	NA
(2.4) $y_t = \beta x_t^{\alpha} e^{u_t}$	1.584 (2.028)	.893 (.903)	1.036 (1.061)	.990 (.990)
(2.5) $y_t = \beta x_t^{\alpha} e^{u_t \ln x_t}$	1.377 (2.877)	.837 (.852)	1.022 (1.029)	.990 (.994)

TABLE IV.2

Jackknifed Least Squares and Huber "M" Estimates

are presented since these estimates are relevant to the question of why there are some large differences between jackknifed and unjackknifed results.

Tables IV.3 and IV.4 contain the jackknifed estimates for β and α together with estimates for $\sigma_{\tilde{\beta}}$ and $\sigma_{\tilde{\alpha}}$. These estimates for the standard error were obtained by computing the square root of the sum of the squared differences of the pseudo-values from the jackknifed estimate and then divided by T(T-1).

The results in Tables IV.3 and IV.4 are consistent with the results in Tables IV.1 and IV.2 in the sense that both highlight estimation problems for equations (2.4) and (2.5). The estimates for $\sigma_{\tilde{\beta}}$ and $\sigma_{\tilde{\alpha}}$ in (2.4) and (2.5) are very large, both for RDTE and procurement. Thus, the confidence intervals will be very wide. For the procurement data the estimates for $\sigma_{\tilde{\beta}}$ and $\sigma_{\tilde{\alpha}}$ are large both when least squares and when Huber "M" estimation techniques are used, although the Huber "M" standard errors are approximately one half the size of those associated with least squares estimates. For the RDTE data, the Huber "M" standard errors (in (2.4) and (2.5)) are less than one-seventh the size of those of the least squares estimates.

Also worthy of note is the size of the estimated standard error for the least squares $\tilde{\beta}$ in (2.2) when the RDTE data was used. Recall, that the unjackknifed least squares estimate for β in (2.2) was 1.09 while the jackknifed estimate is .908.

Procurement

	Least Squares		Huber "M"	
	β	α	β	α
(2.1) $y_t = \beta x_t + \varepsilon$	.961 (.016)	NA	.948 (.012)	NA
(2.2) $y_t = \beta x_t + \varepsilon_t x_t$	.955 (.023)	NA	.962 (.012)	NA
(2.3) $y_t = \beta x_t e^{u_t}$	1.011 (.020)	NA	.950 (.011)	NA
(2.4) $y_t = \beta x_t^{\alpha} e^{u_t}$	1.272 (.593)	.994 (.048)	.967 (.259)	.995 (.028)
(2.5) $y_t = \beta x_t^{\alpha} e^{u_t \ln x_t}$	.761 (.794)	.970 (.066)	.765 (.337)	1.02 (.038)

TABLE IV.3

(Estimates of standard errors are in parentheses.)

RDTE

	Least Squares		Huber "M"	
	β	α	β	α
(2.1) $y_t = \beta x_t + \varepsilon$	.982 (.013)	NA	.973 (.007)	NA
(2.2) $y_t = \beta x_t + \varepsilon_t x_t$	.908 (.095)	NA	.984 (.007)	NA
(2.3) $y_t = \beta x_t e^{u_t}$	1.017 (.034)	NA	.987 (.007)	NA
(2.4) $y_t = \beta x_t^{\alpha} e^{u_t}$	1.584 (1.238)	.893 (.110)	1.036 (.142)	.990 (.018)
(2.5) $y_t = \beta x_t^{\alpha} e^{u_t \ln x_t}$	1.377 (2.771)	.837 (.188)	1.022 (.178)	.990 (.024)

TABLE IV.4

IV.3.2. Variations or "Data Snooping"

In the results just reviewed two things caught our attention. First, the differences between the jackknifed and unjackknifed estimates for β in (2.4) and (2.5) were noticeable. This held for both least squares and Huber estimates when procurement data were analyzed, and held only for least squares estimates when the RDTE data were analyzed. Also, least squares applied to (2.2) yielded different jackknifed versus unjackknifed estimates for β when RDTE data was used. A second interesting result was the size of the estimated standard errors for $\tilde{\beta}$, again in (2.4) and (2.5). The estimates were large for both least squares and Huber "M" estimates for the procurement data. Least squares estimates yielded large estimated standard errors for $\tilde{\beta}$ in (2.2), (2.4) and (2.5) for the RDTE data.

The reasons for the results for (2.4) and (2.5) may have been the same both for procurement and RDTE data. However, the fact that the Huber "M" estimates demonstrated the same overall characteristics as the least squares estimates when procurement data was used, and different overall characteristics when RDTE data was used, leads one to suspect otherwise. In order to further explore the question, the pseudo-values, scatterplots and the original data were examined.

Examination of the pseudo-values for the procurement data revealed one which stood out. It was the pseudo-value corresponding to the item 1953 PAMN: the request was \$124 million, and appropriations were \$113 million.

These numbers are an order of magnitude smaller than the rest of the numbers in the data set, most of which are in the $2 to $3 billion range. In Table IV.5, the pseudo-values for this item both when least squares and when Huber "M" techniques were used are compared to the jackknifed estimate.

	Least Squares		Huber "M"	
	β	α	β	α
(2.4) pseudo-value	-35.18	-.670	-16.48	2.80
jackknifed estimate	1.27	.994	.967	.995
(2.5) pseudo-value	13.02	5.80	-25.14	3.88
jackknifed estimate	.76	.97	.764	1.02

Table IV.5
Pseudo-Values for 1953 PAMN

The figures in Table IV.5 indicated that it might be wise to compare $\tilde{\beta}$ (and $\tilde{\alpha}$) when the full sample was used to $\tilde{\beta}$ (and $\tilde{\alpha}$) when 1953 PAMN is omitted from the sample. This comparison is found in Table IV.6:

		Least Squares		Huber "M"	
		$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$
(2.4)	Full Data	1.47	.948	1.11	.982
	Data without 1954 PAMN	1.932	.913	1.337	.959
(2.5)	Full Data	1.24	.969	.997	.996
	Data without 1954 PAMN	2.007	.908	1.328	.960

TABLE IV.6

Tables IV.5 and IV.6 indicate the significant impact of one observation on the estimates. One of the puzzling facts is that this impact is strong not only when least squares is used but also when the supposedly robust Huber "M" procedure is used. This raises some questions about the Huber "M" procedure.

Examination of scatterplots of the data indicate that for the full data set both least squares and the Huber "M" procedure appear to be the victims of what might be called a "straggler-effect." The "straggler-effect" was discussed briefly in Chapter II. It is characterized by a scatterplot similar to that in Figure IV.1.

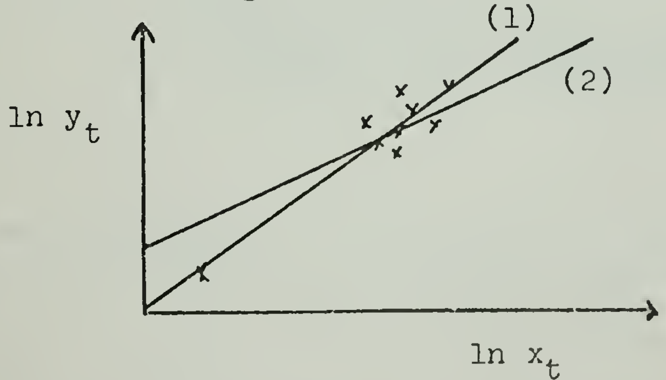


FIGURE IV.1. The"Straggler Effect"

The estimated line with 1953 PAMN as part of the data is line number (1). The estimated line with 1953 PAMN omitted is line number (2).

Application of the Huber "M" technique does not solve this problem. Recall that the technique acts like least squares as long as $|\ln y_t - (\ln\hat{\beta} + \hat{\alpha} \ln x_t)| \leq c$. If the absolute value of the difference is greater than c then a particular observation, t, does not receive as much weight as it would under least squares. This affords "protection" against outliers such as in the situation in Figure IV.2.

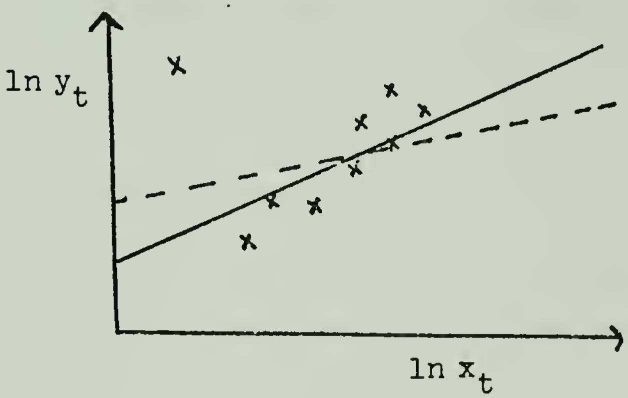


FIGURE IV.2

Effect of an Outlier

(broken line is least squares estimate, unbroken line is Huber "M" estimate.)

If the "straggler" is in the Group 1 residuals (i.e. it is one of the observations for which $|\ln y_t - (\ln\hat{\beta} + \hat{\alpha} \ln x_t| \leq c)$ then it has received as much (in fact more) weight under the Huber "M" approach as it would have under least squares.

To summarize at this stage, the preceding discussion has led to two useful insights. The first is that the Huber "M" procedure is not particularly robust in at least one situation, the case of the "straggler," if the "straggler" is not forced into the Group 2 or Group 3 residuals.* The second is that jackknifing and an examination of pseudo-values is useful for spotting this situation. It should be added that examination of the original data will alert the analyst to the possibility that a "straggler" effect problem may exist, however examination of the pseudo-values is helpful in determining how serious it is.

Table IV.7 contains the revised estimates of the coefficients and standard errors for equations (2.4) and (2.5) when the "straggler", 1953 PAMN, is purged from the data. Of course, elimination of this point, where both request and appropriation are very small will limit generalizations that can be drawn from the data to situations where procurement requests and appropriations are larger than $100 million. In fact it limits generalizations to situations where requests and appropriations are greater than $800 million. This is not a very serious limitation, however, given the size of procurement requests over the last fifteen years. Also included in the table are the unjackknifed estimates. Table IV.8 repeats the information in Tables IV.1 and IV.3; that is, it presents estimates when 1953 PAMN is not removed from the data in order to contrast these results with those in Table IV.

*Estimates based on functional forms for ϕ similar to those discussed in Chapter III (p. III-6) may be more robust.

	Procurement							
	Least Squares				Huber			
	Unjackknifed		Jackknifed		Unjackknifed		Jackknifed	
	β	α	β	α	β	α	β	α
(2.4) $y_t = \beta x_t^{\alpha} e^{u_t}$	1.93	.912	1.782 (.532)	.912 (.002)	1.337	.959	1.346 (.269)	.955 (.023)
(2.5) $y_t = \beta x_t^{\alpha} e^{u_t \ln x_t}$	2.01	.908	1.829 (.652)	.908 (.002)	1.328	.960	1.263 (.262)	.963 (.025)

TABLE IV.7

Results when 1953 PAMN is not used.
(Numbers in parentheses are estimated standard errors)

	Procurement							
	Least Squares				Huber			
	Unjackknifed		Jackknifed		Unjackknifed		Jackknifed	
	β	α	β	α	β	α	β	α
(2.4) $y_t = \beta x_t^{\alpha} e^{u_t}$	1.47	.948	1.272 (.593)	.994 (.048)	1.11	.982	.967 (.259)	.995 (.027)
(2.5) $y_t = \beta x_t^{\alpha} e^{u_t \ln x_t}$	.997	.996	.761 (.794)	.970 (.066)	1.24	.969	.765 (.337)	1.02 (.038)

TABLE IV.8

Results with 1953 PAMN included in data.
(Numbers in parentheses are estimated standard errors)

There are some interesting differences between the results in Table IV.7 and Table IV.8. Without 1953 PAMN included, the estimates for α in Table IV.7 indicate that the effect of the size of the request on the percentage of the request that was granted is more important than the results in Table IV.8 (where 1953 PAMN is included) indicate. For example, for equation (2.4) the jackknifed least squares approach estimates α to be .995, when 1953 PAMN is in the data set. An α of 1.0 says that the percentage of the request granted does not, on the average, change proportionately as the size of the request changes.* When 1953 PAMN is left out of the data, the estimated α is .955 for the jackknifed least squares estimate. When using the jackknifed Huber estimate, note that for equation (2.5) the estimated α is 1.02 in Table IV.8, which implies that the percentage of the request granted increases as the request increases. In Table IV.7, the comparable estimate for α is .908.

The differences between $\tilde{\beta}$ in Tables IV.7 and IV.8 are considerable also. Note for the jackknifed Huber the $\tilde{\beta}$ of 1.263 in Table IV.7 compared to a $\tilde{\beta}$ of .765 in Table IV.8.

Insofar as the estimated standard errors are concerned, in every case but one (the standard error for $\tilde{\beta}$ in equation (2.4) when a jackknifed Huber estimate is used), the standard errors are smaller when 1953 PAMN is not part of the data set.

---

*Recall that α may be interpreted as the elasticity of the percentage of the request granted with respect to the request.

The estimates for equations (2.4) and (2.5) for the procurement data raised questions both when least squares and when the Huber "M" approaches were used. However, the estimates for equations (2.2), (2.4) and (2.5) for the RDTE data raised questions only when the least squares method was used. Again, inspection of pseudo-values for theRDTE data and re-inspection of scatterplots helped to clarify these questions. It appears that one data point, 1955 RDTE Navy, for which the request was $61.2 million and appropriations were $419.9 million, was seriously affecting the estimates. The situation resembles that depicted in Figure IV.2.

As discussed in previous chapters, the effect of any one observation on the estimated line, when least squares is used, is directly proportional to the magnitude of the residual. However, the Huber "M" technique does not give this observation as much weight, and the result is as depicted in Figure IV.2.

Insofar as equation (2.2) is concerned, the estimated β, using least squares, is seriously affected by this one observation, 1955 Navy RDTE, because the estimator is the sample mean y_t/x_t ratio. The effect of extreme observations on the sample mean as a location parameter estimator has been well-documented in Andrews, et al. [1972].

Table IV.9 contains the revised estimates after 1955 Navy RDTE was eliminated from the data set. Also included are the unjackknifed estimates and estimates of the standard errors. The least squares method was used to obtain these estimates. Table IV.10 contains the same information as Table IV.9, except that 1955 Navy RDTE was not left out.

RDTE

Least Squares

	Unjackknifed		Jackknifed	
	β	α	β	α
(2.2) $y_t = \beta x_t + \varepsilon_t x_t$	.999	NA	.999 (.016)	NA
(2.4) $y_t = \beta x_t^{\alpha} e^{u_t}$	1.006	.945	.914 (.143)	1.01 (.02)
(2.5) $y_t = \beta x_t^{\alpha} e^{u_t \ln x_t}$	1.02	.828	.779 (.155)	1.03 (.02)

TABLE IV.9

1955 Navy RDTE not part of data

RDTE

Least Squares

	Unjackknifed		Jackknifed	
	β	α	β	α
(2.2) $y_t = \beta x_t + \varepsilon_t x_t$	1.093	NA	.908 (.095)	NA
(2.4) $y_t = \beta x_t^{\alpha} e^{u_t}$	2.028	.903	1.584 (1.238)	.893 (.110)
(2.5) $y_t = \beta x_t^{\alpha} e^{u_t \ln x_t}$	2.877	.852	1.377 (2.77)	.838 (.188)

TABLE IV.10

1955 Navy RDTE part of data

Tables IV.9 and IV.10 reveal some rather dramatic differences between the results when 1955 Navy RDTE is part of the data set (Table IV.9) and when it is not part of the data set (Table IV.10). On the average all of the estimated standard errors are smaller in Table IV.9 by about a factor of 10. The coefficients also differ. For example, in Table IV.10 both the estimates for α are less than one. In Table IV.9 the jackknifed α estimates are close to 1.0. The large discrepancy between the jackknifed and unjackknifed $\tilde{\alpha}$ (and $\tilde{\beta}$) for equation (2.5) in Table IV.9 appears to be due to a combination of "straggler effects" and outliers. Examination of the pseudo-values revealed three more items which appeared to be influential in causing the differences observed.

Item	Request	Appropriation
1954 Army	475.0	345.0
1953 Navy	75.7	70.0
1954 Navy	58.6	74.9
	(in millions)	

Table IV.9(a) contains revised estimates for α,β and the standard errors when the least squares method was applied (unjackknifed and jackknifed). Notice that the unjackknifed and jackknifed estimates are not as different from each other as in Tables IV.9 and IV.10.

	Unjackknifed		Jackknifed	
	β	α	β	α
(2.2) $y_t = \beta x_t + \varepsilon_t x_t$	1.009	NA	1.008 (.015)	NA
(2.4) $y_t = \beta x_t{}^{\alpha} e^{u_t}$	1.23	.971	1.22 (.14)	.971 (.015)
(2.5) $y_t = \beta x_t{}^{\alpha} e^{\dot{u}_t \ln x_t}$	1.19	.976	1.18 (.13)	.976 (.015)

TABLE IV.9(a)

When examining differences between the estimates for β in (2.2) and (2.4)-(2.5), it should be recalled that (2.2) and (2.4)-(2.5), when $\alpha \neq 1$, are different characterizations of appropriations behavior. In (4.2) appropriations are characterized as a constant percentage of the request while in (2.4)-(2.5) that percentage changes as the size of the request changes. In the next chapter, we will investigate whether the estimates for β would be different among (2.1) through (2.5) if the true model is (2.3).

Confidence regions:

The recent work of Miller [1974] has shown that under certain conditions, in large samples, when $f(\hat{\beta})$ is jackknifed ($\hat{\beta}$ being a least squares estimate) a t-statistic confidence interval can be constructed for $\widetilde{f(\beta)}$, using the jackknifed estimate of the standard error of $\widetilde{f(\beta)}$.

Two of the assumptions that must be met in order to use Miller's results are: (1) the fourth moment of ε_t (or u_t)

must be finite, (2) f must have bounded second derivatives in an open interval around β. These assumptions are satisfied if the ε_t are assumed $N(0,\sigma^2)$ and if $\tilde{f}(\beta)$ equals β, or e^{β}, as in the models used for this study. (3) The other assumption deals with whether $\frac{X^T X}{T} \rightarrow \Sigma$, a positive definite matrix as $T \rightarrow \infty$. If convergence takes place, then Miller's results hold, given the other two assumptions.

Although Miller's results have not been extended to jackknifed Huber estimates, following the lead of Huber concerning jackknifed estimates [Huber,1972,p. 1053] we decided to compute confidence intervals for the Huber estimates that are analogous to those computed for least squares.*

For both the least squares and Huber estimates the number of degrees of freedom was large enough to permit the use of the normal approximation for the t distribution. Confidence intervals based on a standard normal interval of (-2.0σ,+2.0σ) were constructed, yielding a nominal confidence level of .95. Based on the considerations discussed in the previous section, the straggler observation 1953 Navy PAMN was omitted from the data when computing the confidence intervals for $\tilde{\beta}$ and $\tilde{\alpha}$ in

---

*Huber [1973] suggests some possible estimates for $\sigma_{\hat{\beta}}$ in (2.1). The suggestion was expanded upon for the purposes of deriving confidence intervals for $\hat{\beta}$ in (2.1) through (2.5). Estimated intervals and how they were derived are discussed in Appendix D of this chapter. Apparently there has not been determined any way of finding out whether the jackknifed Huber or the approach followed in Appendix D produce the better interval without conducting some sampling experiments.

equations (2.4) and (2.5) for the procurement data. Also, the RDTE outliers 1955 Navy and 1954 Army and the stragglers 1953 Navy and 1954 Navy were omitted from the data when the least squares estimates were used in setting up the confidence intervals for the parameters in equations (2.2), (2.4) and (2.5).

Tables IV.11 and IV.12 contain the confidence intervals for β and α for procurement and RDTE data, both when least squares was used and when the Huber "M" approach was used.

Among the most noticeable aspects of Tables IV.11 and IV.12 is the fact that for the procurement data the intervals for all the Huber "M" intervals are smaller or the same size as the least squares intervals. For the RDTE data, the Huber "M" intervals are smaller for (2.1) - (2.3). For (2.4) - (2.5), the least squares intervals (with 4 points not included in the data set) are slightly smaller than the Huber "M" intervals (with all the data included). Since a theory of confidence intervals for jackknifed Huber "M" estimates has not yet been derived, one ought to be cautious in drawing inferences from the differing sizes of the confidence intervals for least squares and the Huber "M" estimates. The sampling experiments reported in the next chapter shed more light on this question. A second fact is that all of the intervals for the coefficients in (2.1) through (2.3) are considerably smaller than those for (2.4) and (2.5) and those for (2.4) are slightly smaller than those for (2.5). This point will again become of interest in Chapter V.

Procurement

	Least Squares β	Least Squares α	Huber β	Huber α
(2.1) $y_t = \beta x_t + \varepsilon_t$	(.93 , .99)	NA	(.92 , .97)	NA
(2.2) $y_t = \beta x_t + \varepsilon_t x_t$	(.91 ,1.00)	NA	(.94 , .99)	NA
(2.3) $y_t = \beta x_t e^{u_t}$	(.98 ,1.00)	NA	(.94 , .97)	NA
(2.4) $y_t = \beta x_t^{\alpha} e^{u_t}$	(.72 ,2.85)	(.91 ,.92)	(.81 ,1.88)	(.95 ,.96)
(2.5) $y_t = \beta x_t^{\alpha} e^{u_t \ln x_t}$	(.53 ,3.13)	(.90 ,.91)	(.74 ,1.79)	(.96 ,.97)

TABLE IV.11

RDTE

	Least Squares β	Least Squares α	Huber β	Huber α
(2.1) $y_t = \beta x_t + \varepsilon_t$	(.95 ,1.01)	NA	(.96 , .99)	NA
(2.2) $y_t = \beta x_t + \varepsilon_t x_t$	(.98 ,1.04)	NA	(.97 ,1.00)	NA
(2.3) $y_t = \beta x_t e^{u_t}$	(.95 ,1.05)	NA	(.97 ,1.00)	NA
(2.4) $y_t = \beta x_t^{\alpha} e^{u_t}$	(.94 ,1.50)	(.94 ,1.00)	(.75 ,1.32)	(.95 ,1.03)
(2.5) $y_t = \beta x_t^{\alpha} e^{u_t \ln x_t}$	(.92 ,1.44)	(.95 ,1.01)	(.67 ,1.38)	(.94 ,1.04)

TABLE IV.12

IV.4. Summary

This chapter commenced with a discussion of the problem of estimation of the variances and construction of confidence intervals for Huber "M" and least squares estimators. The jackknife was proposed as a technique for estimation of the variances. Next, jackknifed estimates were computed using the procurement and RDTE data. For the procurement data a "straggler," 1953 PAMN appeared to be having a disproportionate affect on the jackknifed Huber "M" and least squares estimates. For the RDTE data, four observations, especially 1955 Navy RDTE, were having a disproportionate affect on the least squares estimates. Confidence intervals were found for parameters in (2.1) - (2.5) both on the basis of jackknifed least squares and on the basis of jackknifed Huber "M" estimates. In Chapter V some of the sampling properties of Huber "M" and least squares estimates will be explored when the error distribution is Cauchy. The coverage of the jackknifed Huber "M" confidence intervals will also be addressed. Finally, questions raised in Chapters II and, to a certain extent, in Chapter III, concerning the possible existence of special eras of Congressional activity and special Congressional treatment of the Air Force in the RDTE area will be discussed.

APPENDIX A

In Section IV.3, it was stated that a straggler observation disturbed the jackknifed least squares and Huber "M" estimates for the procurement data and at least one outlier observation disturbed the jackknifed least squares estimate. It was noted at the time these statements were made that scatterplots, pseudo-values and the raw data were of assistance in identifying stragglers and outliers. Normal plots provided another data analytic aid.

As mentioned in the body of the chapter, it has been reported that Tukey has suggested that under certain conditions pseudo-values are approximately independent and normally distributed [Miller,1964]. If this were true, then

$$\hat{\beta}_{-(t)} = E(\hat{\beta}_{-(t)}) + \sigma_{\hat{\beta}_{-(t)}} \Phi^{-1}(Y_{(t)})$$

where $\hat{\beta}_{-(t)}$ is the t^{th} ordered pseudo-value and $Y_{(t)}$ is the t^{th} order statistic from a distribution which is U(0,1). Under these circumstances, it is reasonable to consider the regression of the t^{th} ordered pseudo-value versus $E(\Phi^{-1}(Y_{(t)}))$ where T is the total sample size [Wilk and Shapiro,1965]. However $E(\Phi^{-1}(Y_{(t)}))$ must be determined by numerical methods [Teichrow,1962]. In its place, what has been found to provide a good approximation is the standard normal inverse

of the expected value of the t^{th} order statistic of a sample of size T from a uniform, U(0,1) distribution, or $\Phi^{-1}(\frac{t}{T+1})$ [Blom,1958,pp. 70-71].

For jackknifed least squares and Huber "M" estimates, so-called "probability plots" were made of $\hat{\beta}_{-(t)}$ versus $\Phi^{-1}(\frac{t}{T+1})$. Although the plots did not (and were not meant to) provide a means of formally testing the normality of the pseudo-values, they were useful as a means of examining characteristics of their distribution.

Appendix B contains some of the typical probability plots. Most have a shape similar to that in Figure IV.A.1.

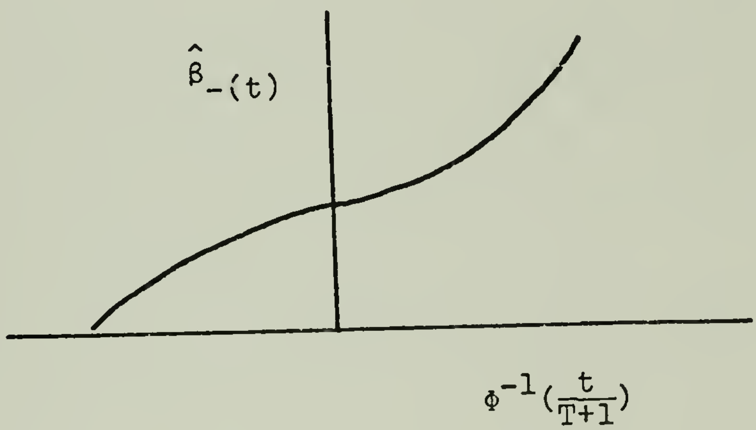


FIGURE IV.A.1

The changes in slope associated with very high and very low values for $\Phi^{-1}(\frac{t}{T+1})$ are more dramatic for some of the plots than for others. Plots like those in Figure IV.A.1 are frequently associated with distributions which, though symmetric, have thicker, fatter tails than the normal, like

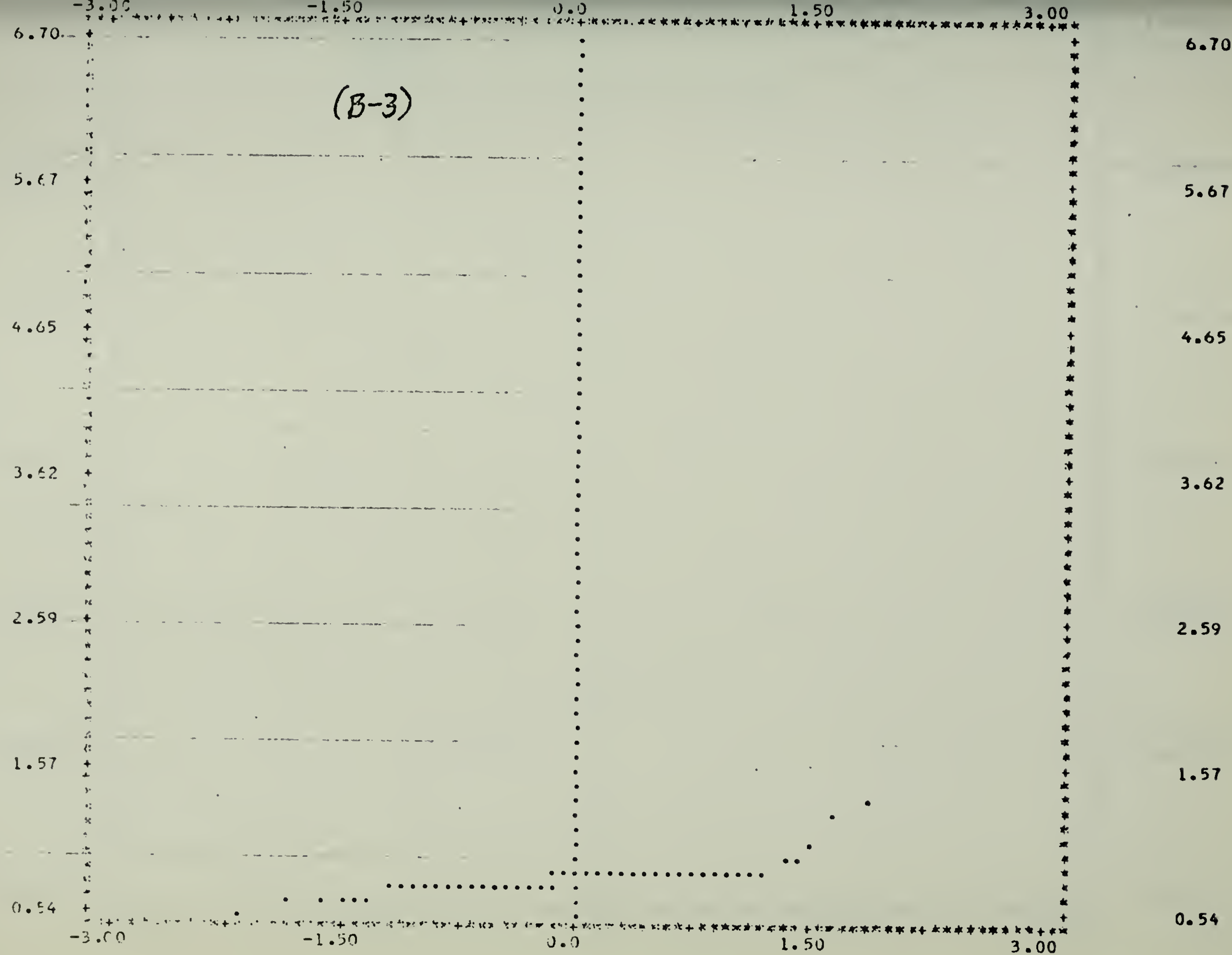
the double exponential and the Cauchy [Wilk and Gnanadesikan, 1968]. This is easily seen by noting that the increasing slope with increasing values of $|\Phi^{-1}(\frac{t}{T+1})|$ can be associated with the fact that for high values of $|\Phi^{-1}(\frac{t}{T+1})|$ the values for $\hat{\beta}_{-(t)}$ are larger than would be expected if the $\hat{\beta}$'s were normally distributed.

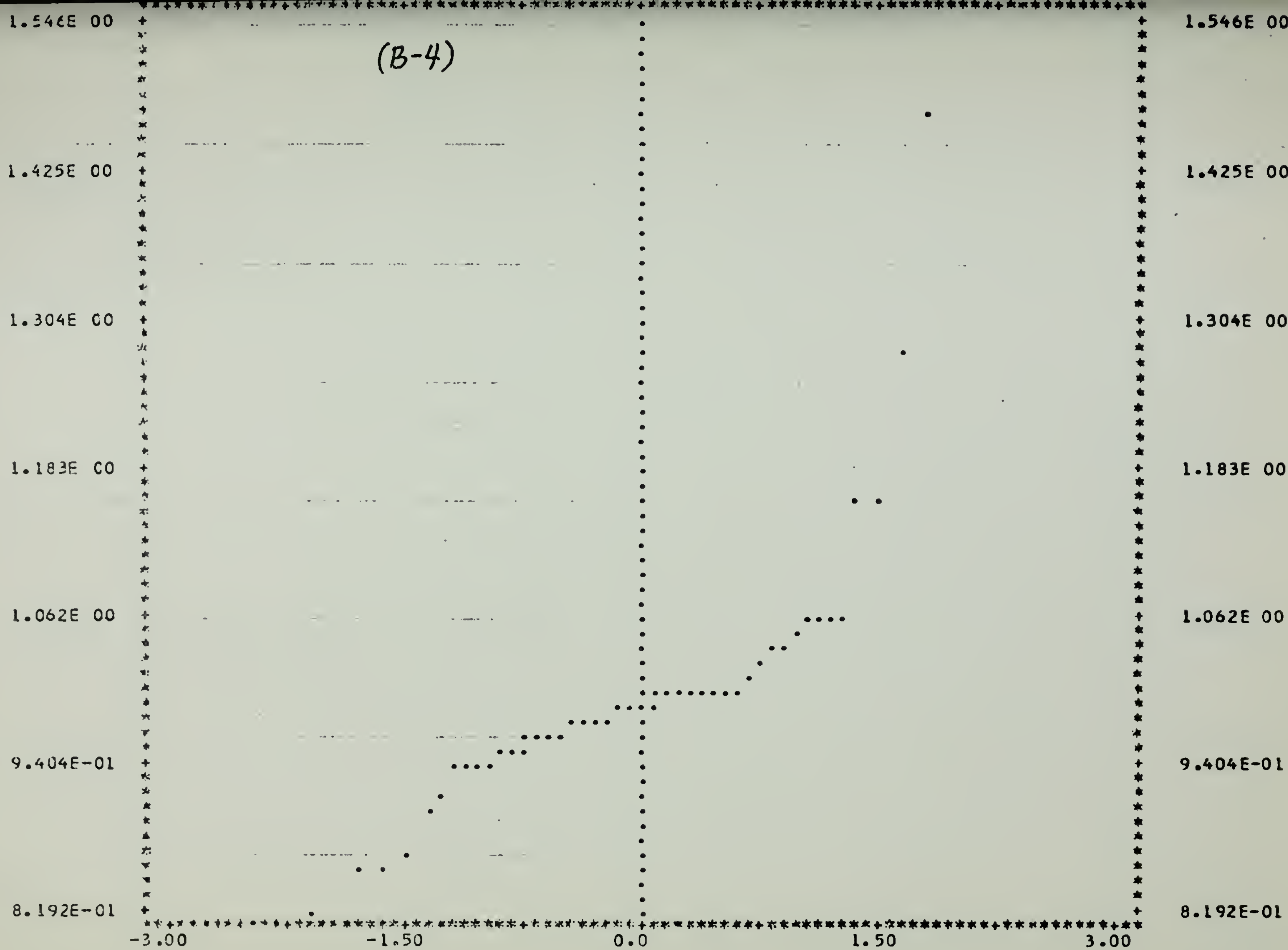
APPENDIX B

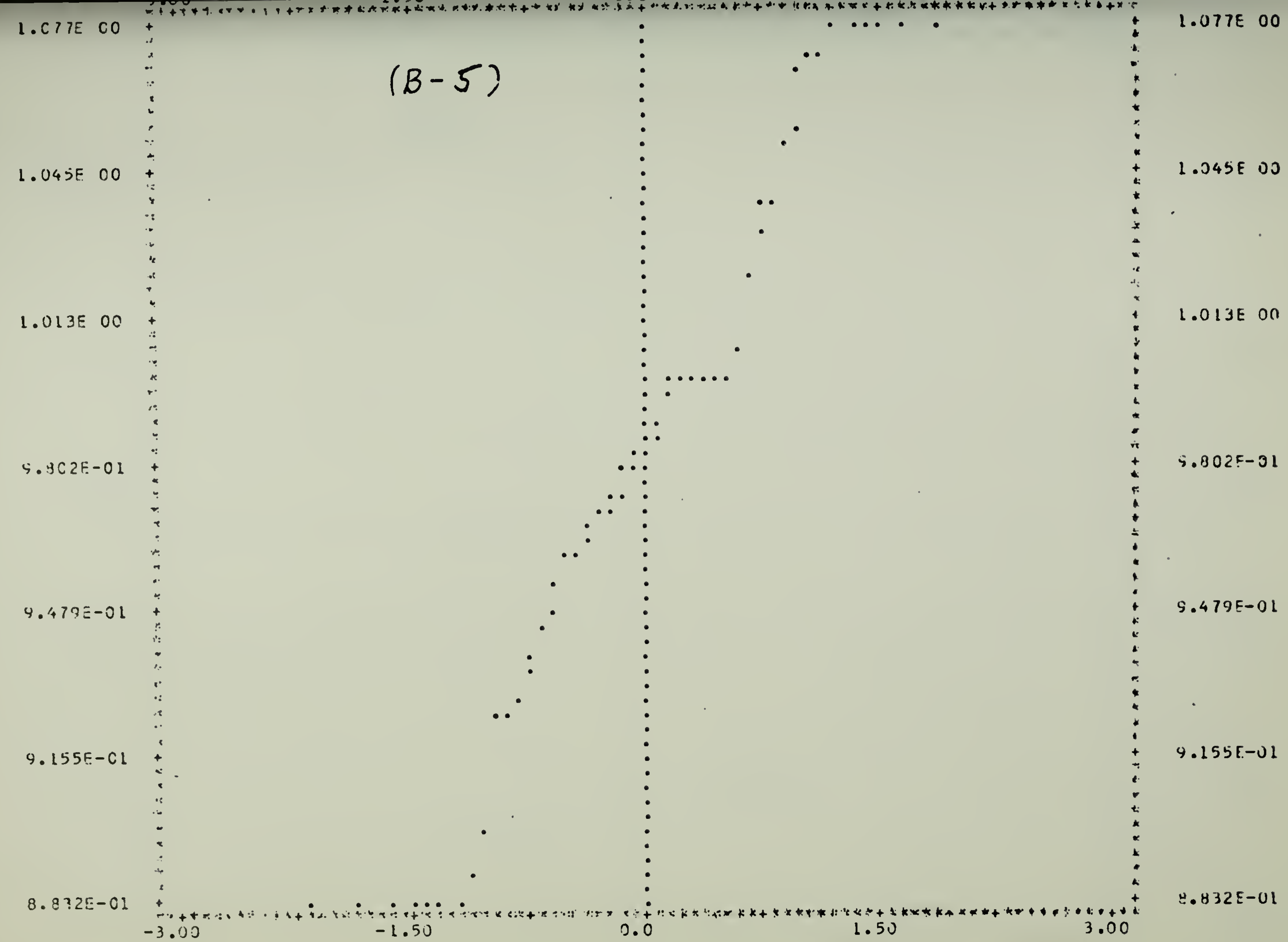
The plots contained in this appendix are a subset of the plots which were made. However they do indicate general characteristics of all the plots since they become more linear when (1) straggler and outlier observations are omitted (e.g., compare pages B-2 and B-3) and (2) the Huber "M" technique, as opposed to least squares is employed (e.g. compare pages B-2 and B-4; also compare pages B-5 and B-6).

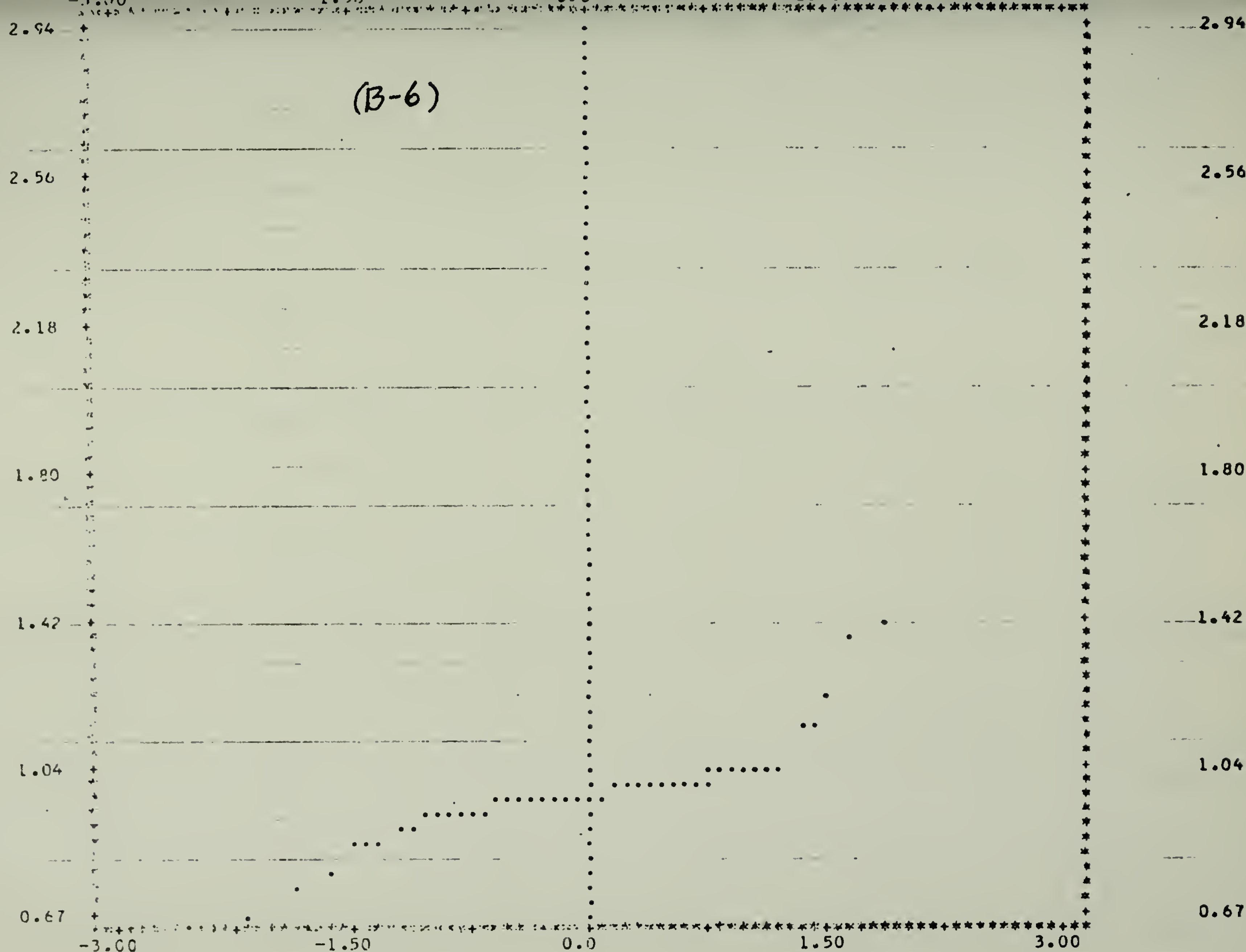
This appendix contains normal plots for the pseudo-values of jackknifed least squares and jackknifed Huber "M" estimates. The rationale behind these plots is discussed in Appendix A of this chapter. For all plots $\Phi^{-1}(\frac{t}{T+1})$ is plotted as the horizontal component while $\hat{\beta}_{-(t)}$, the t^{th} ordered pseudo-value, is plotted as the vertical component. The following is a listing which identifies the plots.

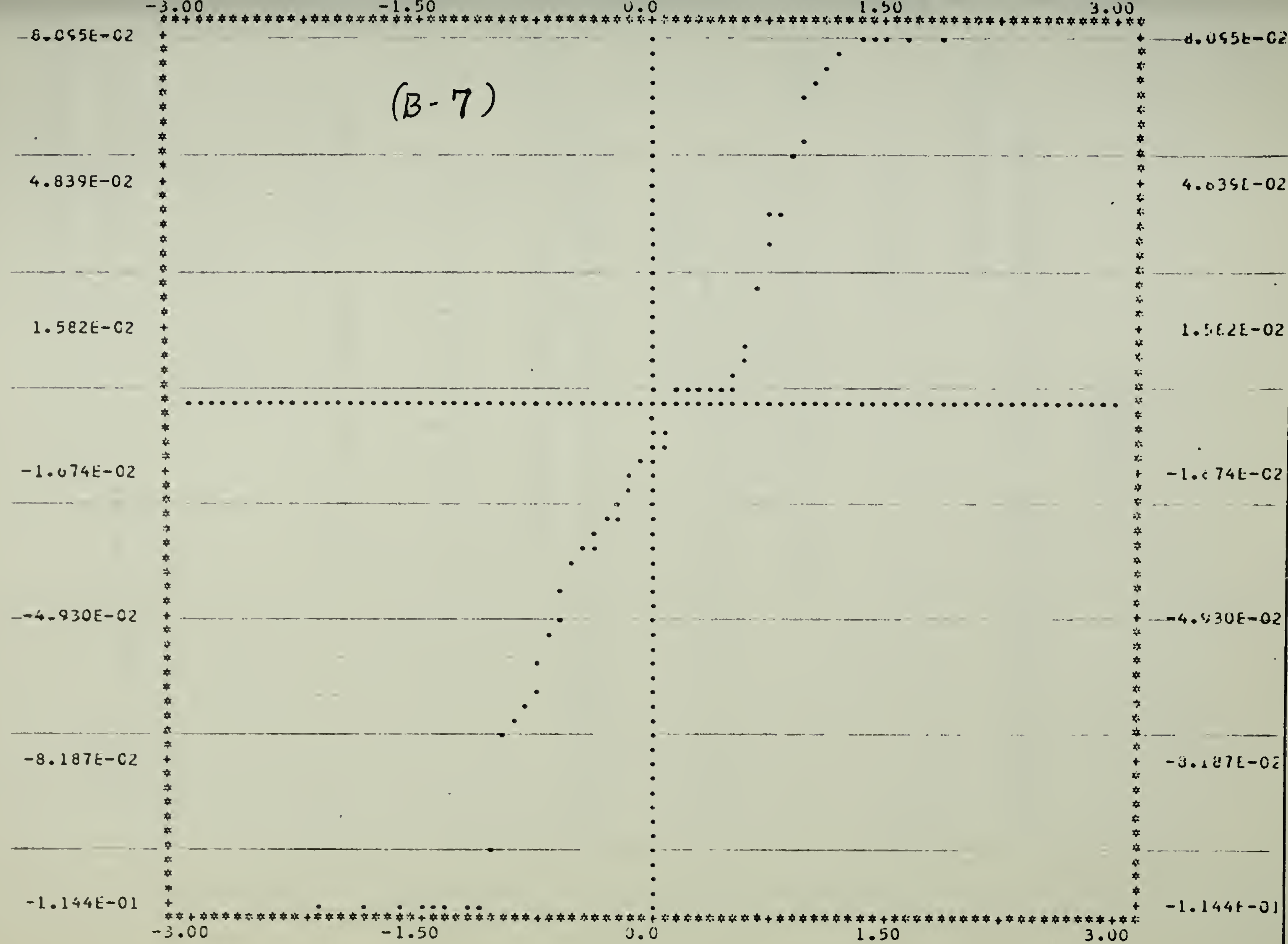
	Least Squares Page	Huber M Page
(2.2) $y_t = \beta x_t + \varepsilon_t x_t$		
RDTE	B-2	B-4
RDTE (59 observations)	B-3	
(2.3) $y_t = \beta x_t e^{u_t}$		
Procurement	B-5	B-6


(B-3)
-3.00
-1.50
0.0
1.50
3.00
6.70
5.67
4.65
3.62
2.59
1.57
0.54


(B-4)
1.546E 00
1.425E 00
1.304E 00
1.183E 00
1.062E 00
9.404E-01
8.192E-01
-3.00
-1.50
0.0
1.50
3.00


(B-5)
1.077E 00
1.045E 00
1.013E 00
9.802E-01
9.479E-01
9.155E-01
8.832E-01
-3.00
-1.50
0.0
1.50
3.00


(B-6)
2.94
2.56
2.18
1.80
1.42
1.04
0.67
-3.00
-1.50
0.0
1.50
3.00


(B-7)
-3.00
-1.50
0.0
1.50
3.00
8.055E-02
4.839E-02
1.582E-02
-1.674E-02
-4.930E-02
-8.187E-02
-1.144E-01

APPENDIX C

In this chapter, one of the models that was discussed was the following:

$$y_t = \beta x_t e^{u_t}$$

where y_t is the appropriations in year t

x_t is the request in year t

u_t is a random variable with expectation of 0 and variance of σ_t^2.

One method of estimation of β is to take logarithms of both sides of the equation to yield:

$$\ln y_t = \ln \beta + \ln x_t + u_t$$

The least squares estimator for ln β is

$$\ln \hat{\beta} = \sum_{t=1}^{T} \frac{\ln \left(\frac{y_t}{x_t}\right)}{T} .$$

If the estimator for β is taken to be $\hat{\beta} = e^{\ln \hat{\beta}}$, then expected value of $\hat{\beta}$ is given by

$$E(\hat{\beta}) = E[\ (\prod_{t=1}^{T}\ (\frac{y_t}{x_t})\ )^{1/T}\ ]\ .$$

$$= E[\ (\prod_{t=1}^{T}\ \frac{\beta x_t e^{u_t}}{x_t}\ )^{1/T}\ ]$$

$$= E[\ (\prod_{t=1}^{T}\ \beta e^{u_t}\ )^{1/T}\ ] \geq |\beta|[E\ (\prod_{t=1}^{T}\ e^{u_t})]^{1/T} = |\beta|E(e^{u_t})\ ,$$

if the u_t's are assumed to be independent and identically distributed. Thus, in general the estimator is biased since $E(e^{u_t})$ is not equal to 1.0 for any nondegenerate distribution of u_t for which $E(u_t) = 0$, for by Jensen's inequality [Feller, II,1958, p. 152] our assumptions imply

$$E[e^{u_t}] \geq e^{E[u_t]} = 1\ .$$

In fact, by the Taylor series expansion of the exponential

$$E[e^{u_t}] \geq 1 + \frac{\sigma^2}{2}\ .$$

Thus, $E(\hat{\beta}) > \beta$.

APPENDIX D

In the body of this chapter, confidence intervals were developed for the coefficients in (2.1) through (2.5) by jackknifing Huber "M" estimates. Huber [1973, pp. 814-815 and p. 818] has suggested using the following as an estimate of $\sigma_{\hat\beta}^2$ in an equation like (2.1), $y_t = \beta x_t + \varepsilon_t$:

$$\sigma_{\hat\beta}^2 = \left(1 + \frac{T - N(T_1)}{T \cdot N(T_1)}\right) \sum_{t \varepsilon T_1} \frac{(y_t - \hat\beta x_t)^2 + (cs)^2 (N(T_2) + N(T_3))}{N(T_1)}$$

$$\cdot \frac{1}{\sum_{t \varepsilon T_1} x_t^2} \quad \frac{T}{T-1}$$

where T is the total sample size, s is the estimated scale parameter, $\hat\beta$ is the estimated coefficient, $N(T_i)$ is the number of items in the Group i residuals. This estimate, is based on the theoretical value for the variance of the estimator derived by Huber [1973, pp. 812-813].

Confidence intervals were constructed for (2.1) through (2.5) based on Huber's suggestion. For those cases where $\hat{\beta} = e^{\ln \hat{\beta}}$, that is equations (2.3) and (2.5), confidence intervals were constructed for ln β and the upper and lower limits of these intervals were then transformed into upper and lower limits for β. In other words, if we let the upper limit of the confidence interval for ln β be designated as $(\ln \beta)^{\dagger}$ then the upper limit for β , which is designated as $\beta^{\dagger}$ was computed by taking $\beta^{\dagger} = e^{(\ln \beta)^{\dagger}}$.

The estimated confidence intervals are contained in Tables D.1 and D.2.

	Procurement β	α
(2.1) $y_t = \beta x_t + \varepsilon_t$	(.94 ,.98)	NA
(2.2) $y_t = \beta x_t + \varepsilon_t x_t$	(.94 ,.99)	NA
(2.3) $y_t = \beta x_t e^{u_t}$	(.94 ,.99)	NA
(2.4) $y_t = \beta x_t^{\alpha} e^{u_t}$	(.81 ,1.52) (.90 ,1.99)*	(.94 ,1.02) (.91 ,1.01)*
(2.5) $y_t = \beta x_t^{\alpha} e^{u_t \ln x_t}$	(.77 ,1.29) (.90 ,1.97)*	(.96 ,1.03) (.91 ,1.01)*

TABLE D.1

*Estimates computed without 1953 PAMN in data set.

RDTE

		β	α
(2.1)	$y_t = \beta x_t + \varepsilon_t$	(.96 ,.99)	NA
(2.2)	$y_t = \beta x_t + \varepsilon_t x_t$	(.97 ,1.00)	NA
(2.3)	$y_t = \beta x_t e^{u_t}$	(.97 ,1.00)	NA
(2.4)	$y_t = \beta x_t^{\alpha} e^{u_t}$	(.91 ,1.24)	(.97 ,1.01)
(2.5)	$y_t = \beta x_t^{\alpha} e^{u_t \ln x_t}$	(.89 ,1.18)	(.97 1.01)

TABLE D.2

The estimates in Tables D.1 and D.2 differ somewhat from those in Tables IV.11 and IV.12. This is especially true of the estimates for (2.4) and (2.5). However, the intervals are very similar for (2.1) through (2.3). The question of which approach, the jackknife or Huber's suggestion, produces a higher coverage rate, and on the average, smaller intervals will be addressed in the next chapter where artificial data are generated and confidence intervals constructed using both approaches.

CHAPTER V

In the previous two chapters least squares and Huber "M" estimates were computed and compared using five different equations which were designed to represent general tendencies on the part of the Congress in the budgeting of defense. In Chapter III point estimates were derived, while in Chapter IV confidence intervals were constructed. At this point in the analysis, there are some important unresolved questions. Which of the equations best represents the Congressional tendencies we are trying to characterize? How do Huber "M" and least squares estimates compare when the error distribution is known to be a long-tailed one, such as, for example, the Cauchy. How sensitive are least squares and Huber "M" to specification error? For example, suppose the "true" model were given by equation (4.2) but equation (4.4) was used for estimation purposes. Since there is as yet no formal theory which supports the jackknifing of Huber "M" estimates, how do jackknifed estimates, both point and interval estimates, behave when computed for artificially generated data? These are questions which will be addressed in this chapter.

This chapter is divided into four basic sections. The first section discusses the background and rationale of Monte Carlo studies which were designed to examine questions such as the ones just raised. In the next section, the specifics of the Monte Carlo techniques used are presented. Results of the studies are presented in the third section. The final section re-examines some questions raised in previous chapters concerning "eras" in Congressional behavior, and similarities and differences between action on the three services' requests. These questions are discussed in the light of results from the Monte Carlo studies and the results of analysis in previous chapters.

V.1. Background

V.1.1. Small Sample Properties

Asymptotic properties for Huber "M" estimates have been derived by Huber in [1973]. To date, however, there have apparently been no formal studies of the small sample properties of the estimates. Results of certain Monte Carlo studies were reported in [Huber,1973] and [Andrews, 1973]. Since the estimation procedure was applied to five different equations using RDTE and Procurement data, we have sought to investigate sampling properties of the estimates for relatively small samples of size 63 (the same as that for the RDTE data) for each of the five equations. Also since the RDTE data contained some unusual or extreme observations, representing Congressional

appropriations much larger (or smaller) than usual, the artificial data generated was based on samples from a long-tailed error distribution, the Cauchy.

Among the results sought from the Monte Carlo studies are answers to questions concerning the distribution of the estimates, and the expectation and variance of the estimates where these concepts are meaningful.

V.1.2. Specification Error

A major difficulty which arises when trying to estimate coefficients in regression equations is the problem of specification error. Among the problems generally referred to as specification error are incorrect functional form and non-independent error terms.

The problem of an incorrect functional form can best be explained using an example. Suppose tentatively that the "true" model is (2.1) — that is, $y_t = \beta x_t + \varepsilon_t$ — but equation (2.4), $y_t = \beta x_t^{\alpha} e^{u_t}$ is chosen by the analyst to depict the appropriations process. Can one count on the estimates for α and β for (2.4) to generally be near the values of 1.0 (for $\hat{\alpha}$) and the true β (for $\hat{\beta}$)? On the other hand, what if the "true" model were (2.4)? What would the estimates for β in (2.1) look like in this case; this obviously depends, at least partially, upon the "true" value for α. Although some analytical work has been devoted to examining the problem of incorrect functional form, especially in a least squares context [Box and Cox,1964, Zarembka,1968], we chose to address this

problem using Monte Carlo methods, by generating data using one equation, (2.3) for example, and estimating coefficients using the five different regression models. The behavior of least squares and, more importantly, Huber "M" estimates was compared by this process. Results of these studies are reported in section 3 of this chapter.

Another specification problem arises if the error terms in the equations are not independent. More specifically, suppose the ε_t in (2.1) are not independent but instead are related by first order autocorrelation. Huber has said that lack of independence of the error terms causes difficulties for Huber "M" estimates – at least for deriving asymptotic results [Huber, 1973 , p. 804]. Tests for autocorrelation in a least squares context and correction to the functional form based on the results of these tests have been used extensively by econometricians [Kane,1968,pp.364-373,Durbin,1951,Theil,1961]. However, often these tests are inconclusive, and such tests and adjustments are not well understood when Huber "M" estimates are computed. Consequently, in order to examine this problem and to ascertain its potential impact in the specific situation being studied here (that is, Congressional action on defense budgets), artificial data resembling the actual RDTE data was generated using error terms related through first-order autocorrelation. The effect on the distribution of the Huber "M" estimates was assessed; results will also be discussed in section 3.

V.1.3. Confidence Interval Estimates

In Chapter IV, confidence intervals were estimated for coefficients in equations (2.1) through (2.5) using the jackknife. Also, confidence intervals were constructed for the Huber "M" estimates using a finite sample approximation to the asymptotic variance of the Huber "M" estimate which was suggested by Huber [See Appendix D, Chapter IV]. Several questions were left unresolved. First, since no theory exists for jackknifing Huber "M" estimates, was the procedure followed in Chapter IV justified? Even if the procedure were justified, which method provides the better estimate in terms of coverage of the true value of the parameter and size, the estimate suggested by Huber or the jackknifed estimate?

Again, these questions were investigated using Monte Carlo techniques. Artifical data was constructed, using Cauchy errors. Because the data were constructed artifically, the true parameter value was known. Next, 95% confidence intervals were constructed for a number of samples of size 63, using the jackknife and using Huber's suggested estimate. The coverage and size of these confidence intervals were compared. The results follow in Section 3 of this chapter.

V.2. Techniques

The artificial data set which was analysed was constructed in the following way. The 63 RDTE observations provided the values for the x variable in equation (2.3).

The values for u_t were generated by sampling from a Cauchy distribution. A value for β equal to .989 (the value estimated for the real or non-artifical data using the Huber "M" technique) was used as the "true" value for the parameter. Using these three components (an assumed β, the x_t values from the original data, the Cauchy distributed u_t values) sample y_t values were generated.

The Cauchy random errors were assumed to have a value of zero for their location parameter. The value for the scale parameter which was used was the median absolute residual for the original RDTE data when equation (2.3) (in logarithmic form) was analyzed using the Huber "M" approach. In those cases when the "true" model was assumed to be (2.4) the values used for β and α were 1.061 and .99 respectively, the estimated values for the original data when the Huber "M" technique was applied to (2.4). The scale parameter for the distribution of u_t was set equal to the median absolute residual for (2.4) when the Huber "M" approach was applied.

The Cauchy errors were generated using a uniform random number generator devised and tested by Lewis and Learmonth [1973] and then applying an inverse transformation.

In some cases it was desired to introduce autocorrelated errors. In order to to this, a basic Cauchy error, v_1, was generated for observation one. However, for observation two the error was computed to be equal to u_2 where

$u_2 = .5\ v_1 + .5\ v_2$,

and v_2 is a Cauchy error independent from v_1.

In general, $u_t = .5\ v_{t-1} + .5\ v_t$. This produced a sequence of correlated Cauchy errors $\{u_1, u_2, \ldots, u_{63}\}$ with the same scale as the independent Cauchy errors, $\{v_1, v_2, \ldots, v_{63}\}$.*

V.3. Results

V.3.1. Huber "M" and Least Squares Estimates with Artificial Data: Monte Carlo Sampling Properties

In the previous section we described the manner in which artificial data were generated. A total

*This is true because the average of two independent identically distributed Cauchy random variables is a Cauchy random variable with the same distribution (i.e. scale and location parameters).

of 1000 samples or histories of size 63 were generated. For these samples, the coefficients in (2.1) through (2.5) were estimated by least squares and the Huber "M" technique. The empirical frequency distributions were examined and compared. Table V.1 gives the sample mean values for the coefficient(s) estimated for (2.1) through (2.5) using least squares and the Huber "M". Below these mean values are the sample standard deviations.

	Huber "M" (c=1.0)		Least Squares	
	β	α	β	α
(2.1) $y_t = \beta x_t + \varepsilon_t$	.990 (.009)	NA		NA
(2.2) $y_t = \beta x_t + \varepsilon_t x_t$	.990 (.008)	NA		NA
(2.3) $y_t = \beta x_t e^{u_t}$	.989 (.008)	NA	1.012	NA
(2.4) $y_t = \beta x_t^{\alpha} e^{u_t}$	.993 (.068)	1.000 (.010)	3626. (6x10^5)	.997 (.247)
(2.5) $y_t = \beta x_t^{\alpha} e^{u_t \ln x_t}$	.992 (.068)	1.000 (.010)	.998 (.187)	5.415 (907.0)

TABLE V.1

For (2.1) and (2.2), using least squares, the distribution of estimators was very skewed with approximately 3/4 of the observations lying between .92 and 1.05 and the remainder scattered among values greater than 1.05, some values being extremely large. The few large values made the computation of a mean seem

meaningless. Actually, when least squares is applied, the distribution of the estimated β has no finite expected value. Recall from Chapter IV, Appendix C, that for the expected value of the estimate for $\hat{\beta}$ in (2.3)

$$E(\hat{\beta}) \geq \beta E(e^{u_t}) \quad .$$

However, if u_t is Cauchy distributed, then $E(e^{u_t})$ does not exist. This explains the extremely erratic sample mean and variances for (2.1) and (2.2) and the unusual values found for (2.3) through (2.5).

On the other hand, it is not clear whether the expectation of the Huber "M" estimator exists or not. The sample means and variances derived were reasonable and in fact for (2.1) through (2.5) the sample means came very close to equalling the true parameter value, .989. It should be noted that the mean values for $\hat{\beta}$ and $\hat{\alpha}$ in (2.4) and (2.5) were .992 and 1.000. This is interesting since, in using (2.3) to generate the data, α was implicitly set equal to 1.0. However, note that the sample standard deviations for $\hat{\beta}$ in (2.4) through (2.5) were seven to eight times larger than the sample standard deviations for $\hat{\beta}$ in (2.1) through (2.3). These facts indicate that if data are generated by (2.3), on the average the estimated results will be the same no matter which equation is assumed to be the correct model for estimation purposes. However, it is not altogether unlikely on any one occasion that the estimate values may differ considerably, with the

estimate for $\hat{\beta}$ using (2.2) being near .998 and that for (2.4) near 1.06. These two values are only one sample standard deviation unit away from the sample means of the respective equations.

For each equation (2.1) through (2.5) normal plots of the 1000 estimates were constructed. Once again, the rationale behind plotting the ordered estimates in this way is that if the distribution of the estimates were normal, then the plots would be approximately linear. The plots for the Huber "M" estimates, found in Appendix A of this chapter, appear to be almost perfect straight lines. Sample skewness and kurtosis were also computed for each of the sets of 1000 estimates and goodness-of-fit tests run on each set [See Appendix A].

The least squares sample mean and variances were for most cases useless as indicators of the location and scale of the distribution of least squares estimates of β. Alternative indicators of the location and scale of the distribution are the median and semi-interquartile range. The results are contained in Table V.2.

If one compares the least squares results in Table V.2 with the Huber "M" results in Table V.1 it is seen that the central tendency (measured by the sample mean) of the Huber "M" results, for (2.1) through (2.5), is slightly closer to the true parameter value of .989 than the central tendency (measured by the sample median) of the least squares results.

Least Squares

	β	α
(2.1)	1.008 (.077)	NA
(2.2)	1.019 (.866)	NA
(2.3)	.988 (.032)	NA
(2.4)	.999 (.173)	.998 (.012)
(2.5)	.999 (.022)	.994 (.147)

TABLE V.2

Also the dispersion of each of the Huber "M" results (measured by the sample standard deviation) is smaller than the dispersion of the least squares results (measured by the semi-interquartile range).

V.3.2. The Huber "M" and Specification Error

The remainder of this section will be devoted to further discussion of Huber "M" estimates and the properties which they demonstrate in the Monte Carlo studies. First the question of specification error will be explored; then the coverage of confidence intervals constructed by jackknifing Huber "M" estimates will be compared to the coverage of intervals constructed using Huber's suggested estimate for the standard error of an estimated coefficient.

(i) Incorrect Functional Form

The first specification error problem is concerned with what happens to the estimates for β if the data are generated using (2.3) and coefficients are estimated using one of the other equations. Table V.1 contains some very interesting results.

On the average, it appeared to make very little difference which equation was used for estimation purposes. There was almost no perceptible difference between the results obtained when using equations (2.1), (2.2) or (2.3). These estimates, on the average, were equal to the true value of the parameter which generated the data. The distributions of estimates when using (2.4) or (2.5) were somewhat more widely dispersed than those for (2.1) through (2.3). (The sample standard deviation for $\hat{\beta}$ using (2.3) was .008 while the sample standard deviation for $\hat{\beta}$ using (2.4) was .068.) Also, the estimates for $\hat{\beta}$ using (4.4) - (4.5) on the average, were slightly greater than the true β value of .989. In a way this is a tautology. If estimates are equal, then there is little to choose from for the models (2.3) and (2.4) become equivalent.

In order to examine this question from another viewpoint, data were generated using (2.4). Table V.3 contains sample means and standard deviations of the distributions of 1000 estimates of β and α (where appropriate) using (2.1) through (2.5) when the data were generated using $y_t = 1.061\ x_t^{.99} e^{u_t}$.

Data generated using $y_t = 1.061\, x_t^{.99} e^{u_t}$

	β	α
(2.1)	.983 (.009)	NA
(2.2)	.990 (.008)	NA
(2.3)	.990 (.008)	NA
(2.4)	1.066 (.069)	.990 (.009)
(2.5)	1.066 (.071)	.990 (.009)

TABLE V.3

Means and standard deviations of 1000 estimates

Notice that on the average the estimates using (2.4) and (2.5) were nearly equal to the true values for β and α of 1.061 and .990, respectively. The average estimates for β using (2.1) through (2.3) were not equal to the average estimates using (2.4) and (2.5), which is to be expected since if $\alpha \neq 1.0$ the interpretation of β in (2.4) through (2.5) is different from that in (2.1) through (2.3). On the other hand the estimates for β using (2.2) and (2.3) were on the average equal to the values obtained from the real data when using the Huber "M". The average estimate for β using (2.1) was close to, but not equal to, the value obtained when the Huber "M" was applied to the real data. (That value was .977.)

Another aspect of the results is that although true model, the one which generated the data, was (2.4), the sample standard error for the distribution of $\hat{\beta}$ when (2.4) was used for estimation purposes was larger than that for β when (2.1) through (2.3) were used for estimation purposes. This highlights the fact that if $\alpha \neq 1$ in (2.4)-(2.5), the parameter β has a different interpretation from the parameter β in (2.1)-(2.3).

The results of the Monte Carlo studies of specification error when the data was generated uisng (2.3) or (2.4), and estimated using (2.1)-(2.5), help to clarify some of the issues involved in model selection. Assuming one limits his characterization of Congressional appropriations behavior to models (2.1)-(2.5), then if the estimates for β in (2.1)-(2.5) are the same, with $\hat{\alpha} = 1.0$ in (2.4)-(2.5), then one is left with little choice but to characterize the usual result of Congressional appropriations action as appropriations being a fixed percentage of requests. However if the results for (2.1)-(2.3) differ from those for (2.4)-(2.5), with $\hat{\alpha} \neq 1.0$ for the latter models, there are two possible explanations. The data could have been generated by a fixed percentage model and the high dispersion of estimates (using (2.4)-(2.5)) for β and α resulted in values for $\hat{\alpha}$ and $\hat{\beta}$ which did not characterize appropriations as being a fixed percentage of requests as estimated by (2.1)-(2.3). On the other hand, the Monte Carlo studies appear to indicate that if the data were

generated by a variable percentage model like (2.4), then the average estimates for (2.4)-(2.5) will differ from the average estimate for (2.1)-(2.3). The dispersion of the estimates for (2.1)-(2.3) may be of assistance in this case. Further studies, using different scale factors for the errors indicate that in many cases when the true model was (2.3) the estimates for β using (2.1)-(2.3) were much less dispersed than those for β using (2.4)-(2.5), but when the true model was (2.4), the estimates for β using (2.1)-(2.3) were less dispersed than those for β using (2.4)-(2.5) by a smaller margin. The problem of specification error is one of the areas, requiring more research.

(ii) Autocorrelated errors

Table V.4 contains the sample means and standard deviations for 1000 Huber "M" estimates of β (and α where applicable) when the artificial data was generated using (2.3), but with autocorrelated disturbances, and (2.1) through (2.5) were used for estimation purposes.

An interesting characteristic of Table V.4 is the small effect the autocorrelated disturbances have on the means or average estimates, affecting only (2.4) and (2.5) slightly. However, note that autocorrelation does appear to increase the dispersion of the estimates. The experiments which were run using a second scheme (where errors took the form $u_t = .5u_{t-1} + .5v_t$) further emphasized these conclusions. When this stronger autocorrelation scheme was used the mean estimates were still close to the true parameter

	Data generated by $y_t = .989\, x_t\, e^{u_t}$ with			
	u_t autocorrelated		u_t independent	
	β	α	β	α
(2.1)	.990 (.014)	NA	.990 (.009)	NA
(2.2)	.990 (.012)	NA	.990 (.008)	NA
(2.3)	.989 (.012)	NA	.989 (.008)	NA
(2.4)	.985 (.098)	1.001 (.014)	.993 (.068)	1.000 (.010)
(2.5)	.996 (.113)	1.000 (.015	.992 (.068)	1.000 (.010)

TABLE V.4

Affect of Autocorrelated Errors

value for (2.1) through (2.5). However, the sample standard deviations of the estimates all increased.*

Another point of interest is that while the normal plots, when the u_t's were independent, were very nearly straight lines, this did not hold as true when autocorrelation was introduced, especially for (2.4) through (2.5).

In summary, although the Monte Carlo studies of the effects of autocorrelation which were reported here were by no means exhaustive, they do indicate that the effects of autocorrelation will be mainly felt in the dispersion of the estimates rather than in their location or central tendency.

---

*The autocorrelation schemes used were actually quite weak. To examine the question in more detail, data were generated for which

$$u_1 = v_1$$

$$u_2 = .9u_1 + .1v_2$$

$$\vdots$$

$$u_t = .9u_{t-1} + .1v_t$$

Once again, results showed that the main impact of this strong first order autocorrelation was on the dispersion of the estimates.

V.3.3. Huber "M" Confidence Intervals

In Chapter IV confidence intervals were constructed by jackknifing Huber "M" estimates. In order to examine some of the properties of jackknifed robust estimates, sampling experiments were undertaken. Data was generated, as in the previous sections, by the equation $y_t = .989\, x_t\, e^{u_t}$, $\beta = 0.989$, where the $\{u_t\}$ were distributed as independent Cauchy random variables and the $\{x_t\}$ were the 63 appropriations requests found in the RDTE data. A total of 1000 samples were drawn, and jackknifed estimates for β (and α where applicable) were derived for (2.1) through (2.5). Confidence intervals were constructed, using the estimate of the variance of the jackknifed estimator employed in Chapter IV. In addition, for these same 1000 samples, confidence intervals were constructed using Huber's suggested estimate for the variance of a robust estimator, as discussed in Appendix D of Chapter IV.

When examining confidence intervals two questions are most important: coverage and length. For one thousand samples, the percentage of time the intervals covered or included the true parameter value was determined. Secondly, the mean and standard deviation of the size of the confidence intervals were used as measures of the length of the intervals. Of course, the ideal interval covers the true parameter value the fraction of time specified and is also short in length.

The confidence interval chosen was one for which coverage assuming normality, is .9544.

Table V.5 contains a comparison of the coverage and interval length for (a) confidence intervals constructed using the jackknife, and (b) confidence intervals constructed using Huber's suggested approach for computing an estimate of the variance of a robust estimator. Recall that if the estimator is normally distributed and if the estimate of its variance is correct, one would expect the confidence interval to cover the true parameter value approximately 95% of the time. Because of the size of our samples the normal approximation should be adequate.

Several features of Table V.5 are worthy of note. First, in many cases both confidence intervals, the jackknifed interval and Huber's suggestion appear to be conservative, covering more than 95% of the time. The coverage of the jackknifed interval is always greater than that of Huber's suggestion. However, the length of the jackknifed confidence intervals is slightly greater than the length of the intervals constructed using Huber's suggestion. In two cases, (2.1) and (2.5) the coverage of

	Jackknifed Interval				Huber's Suggestion			
	Coverage*		Mean Length (Standard Dev. of Length)		Coverage*		Mean Length (Standard Dev. of Length)	
	β	α	β	α	β	α	β	α
(2.1)	.959	NA	.038 (.012)	NA	.844	NA	.026 (.006)	NA
(2.2)	.973	NA	.033 (.007)	NA	.961	NA	.032 (.007)	NA
(2.3)	.974	NA	.033 (.007)	NA	.963	NA	.032 (.007)	NA
(2.4)	.967	.969	.272 (.111)	.039 (.014)	.934	.935	.249 (.057)	.035 (.008)
(2.5)**	.948	.950	.273 (.113)	.039 (.015)	.874	.882	.203 (.055)	.030 (.008)

TABLE V.5

(with approximately 95% confidence the true coverage is ±.013 of the sample coverage)

*Coverage refers to percentage of 1000 confidence intervals which covered true parameter value. For (2.4) and (2.5) true value for α was 1.0.

**Because of expense of computation, jackknifed values based on 614 confidence intervals.

the jackknifed intervals is considerably greater than that of Huber's suggested interval. However, in (2.5) the length of the jackknifed intervals is also considerably larger.

To sum up the results of Table V.5, the following points should be noted. The coverage of the intervals is very encouraging, especially when one realizes that the errors or disturbances were Cauchy distributed. Secondly, although the coverage of the jackknifed generated intervals is greater than Huber intervals, Huber's method produces shorter intervals. Considering the extra effort required to construct jackknifed confidence intervals the expense may not be justified. The exception to this is the case in which the data are generated by means of (2.3) and the coefficients estimated using equation (2.1): here the coverage of Huber's suggested interval is quite low.

V.4. Congressional Moods or Eras and the Air Force

V.4.1. Background

In Chapters II and III, analysis of residuals indicated that appropriations from fiscal 1969 to the present were smaller than the estimated equations (2.1) through (2.5) would indicate if one were to disregard the error or disturbance components in those equations. It was suggested that possibly budget outcomes after fiscal 1969 should be examined separate from budget outcomes or appropriations prior to that time. Also, recall that for some of the models, budget items

for years prior to 1960 appeared in the Group 2 and Group 3 residuals a disproportionate number of times. The tentative conclusion was that pre-1960 items should be modelled differently from post-1960 items.

Another question which arose in the analysis of Chapters II and III centered around the Air Force RDTE budgets between 1957 and 1969. During those years, the Air Force received appropriations which were either larger than the corresponding requests or nearly equal to them. This contrasted sharply with the experience of the Army and the Navy. As a result, Air Force RDTE items between 1957 and 1969 always yielded positive residuals after the coefficients in the various equations were estimated.

V.4.2. Breaking Up the Data by Years

The data were divided in several ways in order to examine the question of possible changes in the regular or routine pattern of budget outcomes in 1960 and in 1969. First, the data from FY60 through FY68 were segregated and analyzed using the Huber "M" technique. The remaining data were analyzed as a block. Application of the Huber "M" technique to the FY53-FY59 and FY69-FY73 data revealed that the Group 2 and Group 3 residuals were always composed of items from the Fifties. This led us to believe it may be useful for estimation purposes to divide the data into three parts or eras:

FY53-FY59 FY60-FY68 FY69-FY73

Procurement

Since the data contains requests by the three services by year, the number of data points for procurement in the three categories were 22*, 36 and 20 respectively. Table V.6 contains the results of the analysis of the three data sets for procurement. Table V.7 contains nominal 95% confidence intervals. These intervals were computed using Huber's method. Although the sample sizes are such that the normal approximation to the Student "t" is somewhat inaccurate, the over-coverage of the intervals reported earlier in the discussion of the Monte Carlo studies somewhat counterbalances this inaccuracy.

The point estimates in Table V.6 are very revealing. For the 1960-1968 time frame, the estimated β is about 1.01 (depending on the model used) while for 1969-1973 the estimated β for the fixed percentage models is approximately .90. This represents a decrease of approximately .10 in the percentage of requests which have been appropriated in the 1969-1973 time period. For the fixed percentage models, (2.1) through (2.3), the results for 1953-1959 lie between the 1960-1968 and 1969-1973 results. Also of interest in Table V.6 is the fact that for the variable percentage models (2.4) and (2.5) using 1960-1968 data estimated α is at or near 1.0. This says that the percentage

---

*The straggler 1953 PAMN was omitted from consideration. Also, 1953 Air Force Aircraft when appropriations totalled 8 billion was also eliminated from consideration.

	1953-1959		1960-1968		1969-1973		Full Data	
	β	α	β	α	β	α	β	α
(2.1)	.967	NA	.999	NA	.889	NA	.957	NA
(2.2)	.959	NA	1.010	NA	.903	NA	.970	NA
(2.3)	.952	NA	1.010	NA	.902	NA	.969	NA
(2.4)	2.474	.874	1.044	.996	1.607	.928	1.394	.953
(2.5)	2.579	.869	1.011	1.000	1.620	.927	1.373	.955

TABLE V.6

Procurement Point Estimates by Era

	1953-1959		1960-1968		1969-1973		Full Data	
	β	α	β	α	β	α	β	α
(2.1)	(.881,1.052)	NA	(.978,1.021)	NA	(.864,.912)	NA	(.964,.989)	NA
(2.2)	(.867,1.050)	NA	(.982,1.038)	NA	(.876,.930)	NA	(.973,1.004)	NA
(2.3)	(.865,1.049)	NA	(.982,1.038)	NA	(.875,.930)	NA	(.974,1.005)	NA
(2.4)	(.317,19.314)	(.606,1.142)	(.670,1.627)	(.939,1.052)	(.943,2.740)	(.862,.995)	(.909,1.239)	(.968,1.011)
(2.5)	(.419,15.870)	(.630,1.106)	(.662,1.542)	(.945,1.055)	(.972,2.702)	(.863,.991)	(.891,1.188)	(.974,1.014)

TABLE V.7

Confidence Intervals Procurement by Era

of the request appropriated is not sensitive to the size of the request. Recall that for the analysis of specification error the results using (2.1) through (2.5) were on the average the same when the data were generated by (2.3), a fixed percentage model. It appears that a fixed percentage model is most appropriate for the 1960-1968 period.

The interval estimates in Table V.7 contain three interesting results. (i) All of the confidence intervals for (2.1) through (2.5) in the 1953-1959 period are quite large. For example, the interval for β in (2.3) is (.865,1.049), the interval for (2.4) is (.317,19.314). Despite the fact that the Huber "M" technique was forcing items like 1954 AF A/C (request of $4283 million and appropriations of $2453 million) and 1959 PEMA (request of $970 million and appropriations of $1669 million) into the Group 2 and Group 3 residuals, the estimates for this period are very unstable. While in general our models are probably too simple to describe what is really happening, this is especially true for the 1953-1959 time period in the procurement area. There are a number of explanations for this. First, this period includes the end of the Korean War and the subsequent change in military policy to that of massive retaliation [Crecine and Fischer,1971,pp. 21-23] with less emphasis on conventional forces. A Congress unwilling

to completely adhere to such a policy might respond by adding dollars in the PEMA account and cutting some of the dollars from the Air Force accounts. On the other hand, by 1960 and certainly during the first Kennedy budgets, the administration policy provided for more dollars for all the Services [Crecine and Fischer,1971,pp. 4-5]. This fact together with the Congressional unwillingness to cut procurement during the Vietnam conflict account for the greater stability of the 1960-1968 estimates. The 1953-1959 data may possibly be better explained by a complex gaming model in which prior year cuts are taken into account, although there is no special reason why this should be the case. (ii) A second interesting aspect of Table V.7 is the length of the intervals for (2.4) and (2.5) for the 1969-1973 time frame. A comparison of these intervals to those for (2.1) through (2.3) suggests that a fixed percentage model may be most appropriate for this period, if only because of the instability of the estimates for (2.1) through (2.3). (iii) Finally, note that the intervals for the fixed percentage models,(2.1) through (2.3), for 1960-1968 do not overlap with the intervals for 1969-1973. Although this result does not represent the result of a classical statistical test, it does strongly support the contention that in the area of procurement the regular part of 1969-1973 budget outcomes in procurement follows a pattern different from that followed by the regular or routine part of 1960-1968 budget outcomes.

RDTE

Tables V.8 and V.9 contain point estimates and nominal 95% confidence interval estimates for the RDTE data when broken into the segments 1953-1959, 1960-1968, 1969-1973. The sample sizes were 21, 27 and 15 respectively. Again the normal approximation causes some inaccuracy in that the true coverage is not quite 95%, especially for the sample of size 15. However, the over-coverage of Huber's suggested technique for constructing confidence intervals which was reported in previous sections somewhat counter-balances this inaccuracy.

The results in Table V.8 show that the point estimates for the 1969-1973 time frame in the fixed percentage models are smaller by approximately 5 percentage points than those for 1960-1968 (and 1953-1959). The positive elasticities resulting from estimates of α in (2.4) and (2.5) of 1.05 in the 1969-1973 data are somewhat puzzling. However, the confidence intervals for α in Table V.9 are unusually wide, which indicates that the estimates in Table V.8 should probably not be taken too seriously. As with the procurement data, the confidence intervals for β in (2.1) through (2.3) for the 1960-1968 data do not overlap with the intervals for the 1969-1973 data. Finally the similarity between the estimates (point and interval) for 1960-1968 data and 1953-1959 data using (2.1) through (2.3), the fixed percentage models, is almost as interesting as the difference between the

	1953-1959		1960-1968		1969-1973		Full Data	
	β	α	β	α	β	α	β	α
(2.1)	1.009	NA	.995	NA	.946	NA	.977	NA
(2.2)	.998	NA	1.003	NA	.943	NA	.989	NA
(2.3)	.997	NA	1.003	NA	.942	NA	.989	NA
(2.4)	.740	1.049	1.287	.966	.634	1.051	1.061	.990
(2.5)	.766	1.044	1.322	.963	.628	1.052	1.029	.994

TABLE V.8

RDTE Point Estimates by Era

	1953-1959		1960-1968		1969-1973		Full Data	
	β	α	β	α	β	α	β	α
(2.1)	(.985,1.032)	NA	(.979,1.010)	NA	(.917,.975)	NA	(.934,.982)	NA
(2.2)	(.958,1.038)	NA	(.986,1.021)	NA	(.914,.972)	NA	(.944,.994)	NA
(2.3)	(.958,1.038)	NA	(.985,1.020)	NA	(.912,.972)	NA	(.944,.994)	NA
(2.4)	(.499,1.097)	(.984,1.114)	(.916,1.806)	(.921,1.012)	(.181,2.216)	(.889,1.212)	(.913,2.129)	(.899,1.007)
(2.5)	(.571,1.027)	(.994,1.093)	(.923,1.892)	(.914,1.011)	(.213,1.857)	(.912,1.192)	(.908,2.074)	(.902,1.008)

TABLE V.9

RDTE Confidence Intervals by Era

estimates using (2.4) and (2.5), the variable percentage models. A further refinement of the data, that is separating out 1957-1969 AF, will clarify the matter.

V.4.3. The Air Force and RDTE

Analysis of residuals in Chapters II and III has indicated that Air Force RDTE 1957-1969 should probably be separated from the rest of the data. In nine of these thirteen years the Congress appropriated more than was requested. Tables V.10 and V.11 contain point estimates and confidence intervals for the following RDTE data groups: Air Force (1957-1959), Army, and Navy (1960-1968), Army, Navy and Air Force (1953-1959, except 1957-1959 Air Force), Army, Navy and Air Force (1969-1973, except 1969 Air Force).

There are several interesting results in Tables V.10 and V.11 which need elaboration. (i) For the first time in the data analyzed thus far there is a noticeable difference in the results for (2.1) and those for (2.2) and (2.3). For the 1957-69 Air Force data, the estimate for β in (2.1) is 1.017 while that for (2.2) is 1.066. Examination of the data points indicates that six of the thirteen x_t values are smaller than 1.6 billion (4 being less than .750 billion) while the other seven x_t values are greater than 3.0 billion. This wide range of x_t values leads us to believe that for this data a model which allows some interdependence between x_t and the disturbance, like (2.2) or (2.3), may be preferable to one which doesn't. A similar observation is relevant when

	AF, 1957-1969		1960-1968 less AF		1953-1959 less AF 1957-1959		1969-1973 less AF 1969	
	β	α	β	α	β	α	β	α
(2.1)	1.017	NA	.989	NA	.998	NA	.938	NA
(2.2)	1.066	NA	.993	NA	.987	NA	.936	NA
(2.3)	1.063	NA	.993	NA	.986	NA	.935	NA
(2.4)	1.919	.922	2.042	.900	.789	1.038	.962	.996
(2.5)	1.782	.932	2.067	.899	.803	1.034	.923	1.002

TABLE V.10

RDTE Taking into Account AF Uniqueness

	AF, 1957-1969		1960-1968 less AF		1953-1959 less AF 1957-1959		1969-1973 less AF 1969	
	β	α	β	α	β	α	β	α
(2.1)	(.976,1.059)	NA	(.974,1.003)	NA	(.973,1.024)	NA	(.912,.964)	NA
(2.2)	(1.001,1.130)	NA	(.978,1.009)	NA	(.943,1.029)	NA	(.911,.962)	NA
(2.3)	(1.000,1.130)	NA	(.978,1.009)	NA	(.944,1.030)	NA	(.911,.962)	NA
(2.4)	(1.010,3.645)	(.838,1.006)	(1.142,3.655)	(.820,.981)	(.502,1.239)	(.962,1.113)	(.379,2.440)	(.876,1.116)
(2.5)	(.961,3.306)	(.849,1.015)	(1.164,3.671)	(.820,.978)	(.571,1.131)	(.976,1.093)	(.362,2.349)	(.881,1.122)

TABLE V.11

Confidence Intervals for RDTE
Taking Into Account AF Uniqueness

considering differences between the results for (2.4)-(2.5) and those for (2.2)-(2.3). The variable percentage models allow some interdependence between request size and the percentage of the request appropriated. In most of the data sets analyzed so far in this section, the request size has not varied enough to lend credence to any estimated interdependence. This is not true for the 1957-1969 Air Force data. However, the confidence intervals in Table V.11 for β and α in (2.4) and (2.5) are rather wide when compared to the intervals for β in (2.2) and (2.3). Although the Monte Carlo studies reported earlier indicate that even if the data were generated by (2.4) the value of the variance of $\hat{\beta}$ when using (2.4) for estimation is higher than the value of the variance of β when using (2.2) or (2.3) for estimation, the differences were not as great as those found in Table V.11.

(ii) A second interesting result is associated with the 1969-1973 data set. The elimination of one data point, 1969 Air Force, from the data set has a significant impact upon the estimates for α and β in (2.4) and (2.5), and a somewhat smaller impact on the estimates for β in the fixed percentage models (2.1) through (2.3). [Compare Table V.10 with Table V.8]. Of the fifteen data points between 1969-1973 all represented reductions by the Congress except for 1969 Air Force, where appropriations were $3.57 billion and the request was $3.36 billion. The Huber "M" technique made this item a Group 2 residual, thus reducing its impact on the estimate. However, as a comparison of Tables V.8 and V.10

indicate, this observation still had a significant effect on the estimates. This appears to be an instance where the more severe ϕ functions such as those suggested by Andrews and Hampel or by Tukey [Andrews et al. , 1972] might produce estimates which are preferable to the Huber estimates. With respect to Table V.10 it should also be noted that for the 1969-1973 data set (without 1969 Air Force) a fixed percentage model appears to be most appropriate since the estimates for α in (2.4) and (2.5) are approximately equal to 1.0.

(iii) Insofar as confidence intervals are concerned, the confidence intervals (using (2.1) through (2.3)) for the 1969-1973 data (without 1969 Air Force) do not overlap with those for the 1960-1968 data (without Air Force) nor with those from the 1957-1969 Air Force data. This reinforces the contention that in RDTE, as well as in procurement, the relationship of appropriations to requests changed in 1969 or 1970. Also, there is only a slight overlap between the confidence intervals (using (2.2)-(2.3)) for the 1957-1969 Air Force data and the confidence intervals for the 1960-1968 data (without Air Force). The same holds true for the 1957-1969 Air Force data and the 1953-1959 data (without 1957-1959 Air Force).

(iv) Finally, the estimates (point and interval estimates) for the 1953-1959 data (without 1957-1959 Air Force) and the 1960-1968 data (without Air Force) are very similar when using (2.1)

through (2.3). However, this is not true for (2.4) and (2.5), the variable percentage models. Closer inspection of the data shows that the values for the requests in each data set taken alone were all very similar. The requests in the pre-1960 data were all less than 1 billion while those for 1960-1968 were all greater than 1 billion. It appears that this small dispersion of the requests in each of these data sets resulted in unstable and unreliable estimates for α and β in (2.4) and (2.5). Table V.12 contains the results when the 1953-1968 data (without 1957-1968 Air Force) were analyzed as a unit. The small confidence intervals for all of the coefficients leads us to believe that this data should be treated as a unit. Of interest, also, is the fact that α in (2.4) and (2.5) is estimated to be approximately equal to 1.0 which means that even though the data contained rather widely dispersed x_t values, the percentage of the request appropriated did not appear to be sensitive to the size of the request.

Some tentative explanations for the results in the RDTE area are probably in order at this time. The three major results are that for the 1957-1969 Air Force data the estimated β was approximately 1.06. For the 1953-1968 data (without 1957-1968 Air Force) the estimated β was near 1.0, while for the 1969-1973 data (without 1969 Air Force) the estimated β was approximately .94. The first result, the β of 1.06 for the Air Force in the 1957-1969 period, is probably associated with the missile gap fears starting in the late 1950's when

		Point Estimates		Confidence Intervals	
		β	α	β	α
(2.1)	$y_t = \beta x_t + \varepsilon_t$	.990	NA	(.976,1.003)	NA
(2.2)	$y_t = \beta x_t + \varepsilon_t x_t$	.993	NA	(.977,1.008)	NA
(2.3)	$y_t = \beta x_t e^{u_t}$	.992	NA	(.977,1.008)	NA
(2.4)	$y_t = \beta x_t^{\alpha} e^{u_t}$	.944	1.007	(.814,1.094)	(.986,1.029)
(2.5)	$y_t = \beta x_t^{\alpha} e^{u_t \ln x_t}$	.900	1.014	(.799,1.015)	(.996,1.033)

TABLE V.12

1953-1968 RDTE less 1957-1968 AF

the Soviets orbitted the first Sputniks and with the dispute between Secretary McNamara and the Armed Services Committees over the B-70 bomber in the mid-1960's. The Secretary believed the bomber was unnecessary while the Committees disagreed. With respect to the apparent "rubber stamping" in the 1953-1968 period, except for the 1957-1969 Air Force, there are two possible explanations. The first is that prior to fiscal 1963 authorizations were not required for RDTE. Consequently the Armed Services Committees were not examining budget requests. Until recently, one of the characteristics of RDTE budget requests was that it was difficult to pinpoint and identify individual projects for which funding was being requested. Finally, prior to the 1960's, RDTE was a small percentage of the overall Defense budget request, especially Army and Navy RDTE. Thus, since the Defense Appropriations Subcommittees in the House and Senate were the only ones examining the requests prior to 1963, since the requests did not clearly identify projects to be funded and since the amount of money involved was not too great, it is reasonable to conclude that a subcommittee might essentially "rubber stamp" a request. The 1964-1968 approvals of RDTE requests are probably associated with the Congressional reluctance to cut the Defense budget during the Vietnam buildup.

The third major result of the analysis of RDTE data was the decrease in appropriations as a percentage of requests in the 1969-1973 time frame. By 1969, the Armed Services Committees had been reviewing authorization requests for

five years and had developed some expertise in this area. Also, by 1969 the budget backup data which stated what the funds were to be used for was organized into a number relatively visible and identifiable units [Armed Services Committee Print]. These factors, together with public dissatisfaction with military spending in general, appear to have contributed to the change in 1969 and its persistence up to the current time.

V.5. Summary and Conclusions

This chapter brings to a close our present exploratory analysis of Defense appropriations in procurement and RDTE as they depend upon requests. Alternative ways of specifying what has been termed the regular aspects of budget outcomes were discussed in Chapter II. The remainder of the analysis has consisted of attempts to probe more deeply into the questions of how to best specify these regular aspects and how best to estimate them. The Huber "M" technique was shown to be, in many instances, quite suitable for estimation purposes because of its designed insensitivity to unusual or irregular data points or occurrences. The question of confidence intervals was explored in Chapter IV, while small sample properties and specification error were explored in the first part of Chapter V. The discussion in IV.4 in general favored models which state that appropriations are a fixed percentage of requests, (2.1)-(2.3). The models (2.1)-(2.3) differ among

themselves in the way the random disturbance enters their specification. Tables V.13 and V.14 contain coefficient estimates that were chosen as the most appropriate models.

The estimates in Tables V.13 and V.14 in no way represent the final word on the topic of Congressional appropriations as a percentage of requests. Hopefully, future research will probe more deeply into questions such as the following. Why are the estimated percentages at, or near, the values in Tables V.13 and V.14? What caused the change in 1969 and can such changes be predicted? Why was the Air Force so successful in 1957-1969 in the RDTE area?

This analysis has made no attempt to include formulations which treat the percentage of the request appropriated in year t as being related to the percentage appropriated in year t-1. Formulations of this kind are a natural extension of this analysis. Another area of future research is related to individual committee action. Although this analysis has examined appropriations granted by the Congress as a whole, a similar analysis could be performed on Armed Services Committee and Defense Appropriations Subcommittee recommendations.

One very interesting question is concerned with the Kanter-DDW dispute and probably offers one of the more promising avenues for future research. Are the percentages in Tables V.13 and V.14 the result of a conscious effort on the part of the Congress to make percentage adjustments in budget requests, or are they the sum total of non-percentage type adjustments

	Best Model and Estimate	
1953-1959	None	
1960-1968*	(2.3) $y_t = \beta x_t e^{u_t}$	$\hat{\beta} = 1.010$
1969-1973*	(2.3) $y_t = \beta x_t e^{u_t}$	$\hat{\beta} = .901$

TABLE V.13

Procurement

*(2.2) and, to a certain extent (2.1), also provide adequate representations of the data.

1953-1968 (without 1957-1968 AF)*	(2.3) $y_t = \beta x_t e^{u_t}$	$\hat{\beta} = .993$
1957-1969 Air Force**	(2.4) $y_t = \beta x_t^{\alpha} e^{u_t}$	$\hat{\beta} = 1.919$, $\hat{\alpha} = .922$
	or (2.3) $y_t = \beta x_t e^{u_t}$	$\hat{\beta} = 1.063$
1969-1973 (without 1969 Air Force	(2.3) $y_t = \beta x_t e^{u_t}$	$\hat{\beta} = .935$

TABLE V.14

RDTE

*(2.2) and (2.1) also provide adequate representations of the data. The estimates for β using (2.2) and (2.1) were equivalent to those of (2.3).

**(2.2) but not (2.1) also provides an adequate representation of the data.

***The similarity between the results for (2.2) and (2.3) can be explained by recalling the discussion in Chapter II of these models. Recall that for (2.3)

$\frac{y}{x} = \beta + \beta u + \beta \frac{u^2}{2!} + \beta \frac{u^3}{3!} + \ldots$, while for (2.2) $\frac{y}{x} = \beta + \varepsilon$,

with u and ε being random. The two representations are very similar if β is near one, and u is small.

in budget requests, or are they the sum total of non-percentage type adjustments at a lower level of aggregation (i.e. cuts in specific weapon systems) by a Congress which has only limited time and resources to review budget requests?

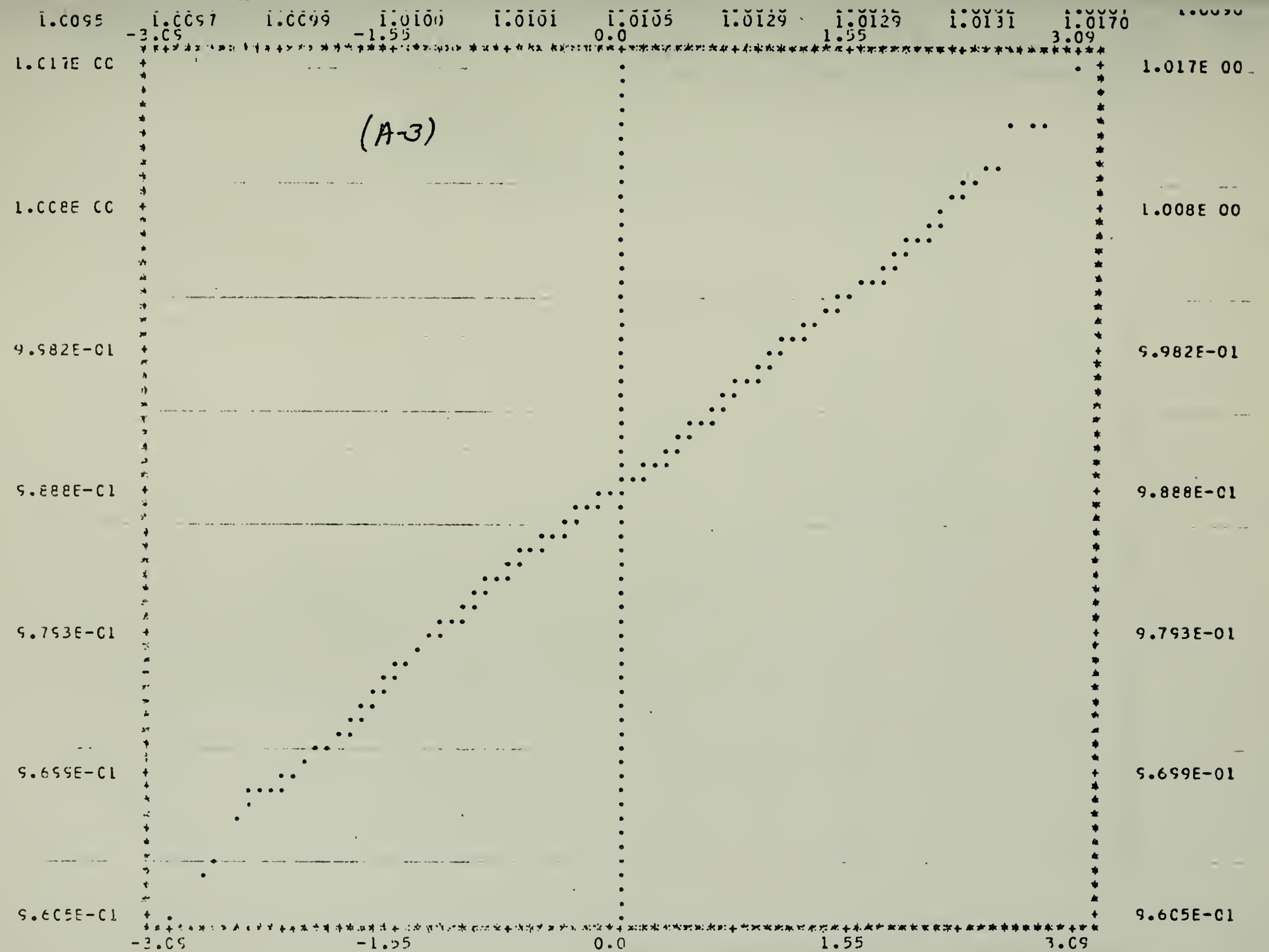
APPENDIX A

In Section V.3, data was generated by (i) model (2.3), (ii) model (2.4) and (iii) model (2.3) with autocorrelation. The disturbances, u_t, were Cauchy distributed in each case. Estimates of the coefficient β (and α where appropriate) were computed using (2.1)-(2.5). Since 1000 samples of size 63 were generated, it was possible to examine the distributions of the estimates for β (and α). Probability plots were constructed for the 1000 estimates of β (and α) when using (2.1)-(2.5) under conditions (i), (ii) and (iii). Also, the hypothesis tests, b_2 and $\sqrt{b_1}$ [Pearson and Hartley, 1966,pp. 67-68], were made in order to determine whether or not it was possible to reject a hypothesis of normality.

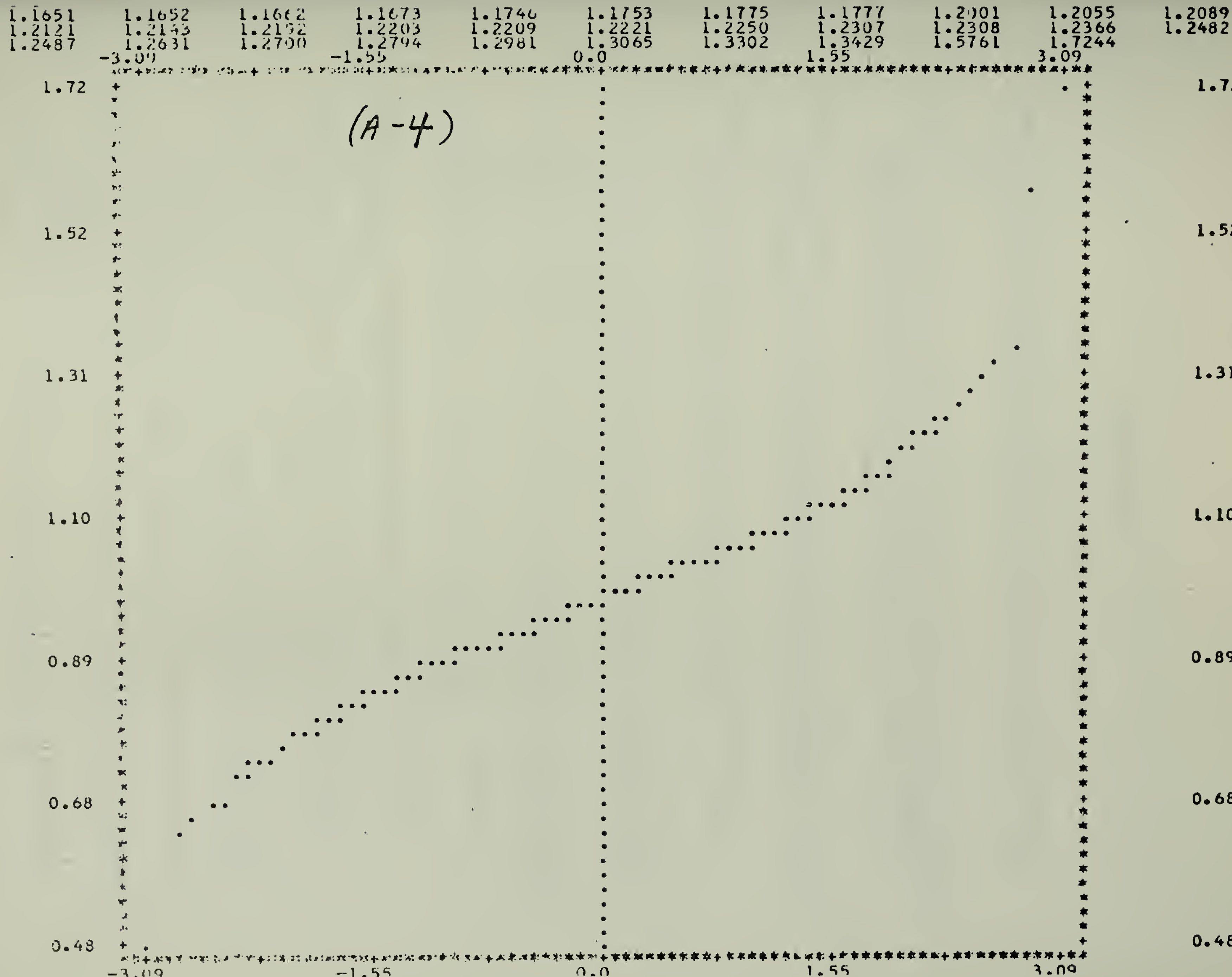
In an extensive Monte Carlo study made by Wilk, Shapiro and Chen [1968], $\sqrt{b_1}$ performed well in comparison to other standard goodness-of-fit tests for detecting non-normality under a wide variety of circumstances. The test which performed best was the W test [Wilk and Shapiro,1965]. However the W test is designed for samples of 50 or less. In no case was it possible to reject the null hypothesis, using b_2 or $\sqrt{b_1}$. For example, when the data were generated by means of (2.3) with autocorrelation and model (2.3) was used for estimation purposes, the value obtained for b_2-3, using the estimated coefficients from 1000 samples, was 7.1×10^{-8}. The upper

5% point, as tabulated in [Pearson and Hartley,1966,p. 208] is 2.6×10^{-1}. This was typical of the results obtained from the tests.

Examples of the probability plots are contained in this appendix. The plots on pages A-3 and A-4 are typical of the plots obtained. Almost all of the plots appeared linear, like that on page A-3, where the data was generated by (2.3) and estimates computed using (2.3). The plots, like that on page A-4 were not quite so linear when autocorrelation (data generated by (2.3) with autocorrelation) was introduced on top of specification error (estimates computed using (2.4)).



(A-3)
1.0095
1.0097
1.0099
1.0100
1.0101
1.0105
1.0129
1.0129
1.0131
1.0170
-3.09
-1.55
0.0
1.55
3.09
1.017E 00
1.008E 00
9.982E-01
9.888E-01
9.793E-01
9.699E-01
9.605E-01



1.1651 1.2121 1.2487
1.1652 1.2143 1.2631
1.1662 1.2132 1.2700
1.1673 1.2203 1.2794
1.1746 1.2209 1.2981
1.1753 1.2221 1.3065
1.1775 1.2250 1.3302
1.1777 1.2307 1.3429
1.2001 1.2308 1.5761
1.2055 1.2366 1.7244
1.2089 1.2482
-3.09
-1.55
0.0
1.55
3.09
1.72
1.52
1.31
1.10
0.89
0.68
0.48
(A-4)

REFERENCES

Armed Services Committee, Senate, Committee Print showing budget estimates and appropriations for specific items, 1973.

David F. Andrews, "Robust Regression: Practical and Computational Considerations," in Proceedings of Computer Science and Statistics, 7th Annual Symposium on the Interface, Iowa State University, 173-175, 1973.

D.F. Andrews, P.J. Bickel, F.R. Hampel, P.J. Huber, W.H. Rogers, J.W. Tukey, Robust Estimates of Location: Survey and Advances, Princeton University Press, 1972.

Thomas E. Anger, "A Critical Review of Defense Resource Planning and the Role of Analysis," Center for Naval Analysis Memorandum 864-73, March, 1973.

F.J. Anscombe, "Topics in the Investigation of Linear Relations Fitted by the Method of Least Squares," Journal of the Royal Statistical Society, B, 29, 1967.

R. Crist Berry and D.E. Peckham, "Interactions of Navy Program Managers with Congressional Committees and Their Staffs," Master's Thesis, Naval Postgraduate School, March, 1973.

BIOMED, University of California at Los Angeles, Write-up for BMD02R, Stepwise Regression, Revised 1965.

Gunnar Blom, Statistical Estimates of Transformed Beta Variables, John Wiley and Sons, 1958.

Peter Bloomfield and Geoffrey Watson, "The Inefficiency of Least Squares," Department of Statistics, Princeton University, Technical Report 53, Series 2, December 1973.

G.E.P. Box and D.R. Cox, "An Analysis of Transformations," Journal of the Royal Statistical Society, B, 26, 1964.

K.A. Brownlee, Statistical Theory and Methodology in Science and Engineering, John Wiley and Sons, 1960.

R.F. Byrne, A. Charnes, W.W. Cooper, O.A. Davis, D. Gilford, Studies in Budgeting, American Elsevier, 1971.

Holbert Carroll, The House of Representatives and Foreign Affairs, University of Pittsburgh Press, 1958.

Gregory Chow, "Tests of Equality Between Sets of Coefficients in Two Linear Regressions," Econometrica, 28, 591-605, 1960.

W.G. Cochran, "Errors of Measurement in Statistics," Technometrics, 10, 637-666, 1968.

Congressional Quarterly Almanac, Congressional Quarterly Service, 16-29, 1960-1973.

John P. Crecine, "Defense Budgeting: Organizational Adaptation to Environmental Constraints," in Studies in Budgeting, R.F. Byrne, A. Charnes et al., ed., American Elsevier, 1971.

John P. Crecine and Gregory Fischer, "On the Resource Allocation Process in the Department of Defense," Institute of Policy Studies, University of Michigan, 1971.

Cuthbert Daniel and Fred S. Wood, assisted by John W. Gorman, Fitting Equations to Data, Wiley Interscience, 1971.

H.A. David, Order Statistics, John Wiley and Sons, 1970.

Otto Davis, M.A.H. Dempster, Aaron Wildavsky, (a), "A Theory of the Budgetary Process," The American Political Science Review, LX, 3, 1966.

Otto Davis, M.A.H. Dempster, Aaron Wildavsky, (b), "On the Process of Budgeting," reprinted in The Planning-Programming-Budgeting System, Hearings before the Subcommittee on Economy in Government of the Joint Economic Committee, September 14, 19, 20, 21, 1967.

Otto Davis, M.A.H. Dempster, Aaron Wildavsky, (c), "On the Process of Budgeting, II," in Studies in Budgeting, R.F. Byrne, A. Charnes et al., ed., American Elsevier, 1971.

Otto Davis, Henry Gailliot, Gary Bowman, Alan Hess, "A Note on Supplemental Appropriations in the Federal Budgetary Process," reprinted in The Planning-Programming-Budgeting System, Hearings before the Subcommittee on Economy in Government of the Joint Economic Committee, September 14, 19, 20, 21, 1967.

Raymond H. Dawson, "Innovation and Intervention in Defense Policy," in New Perspectives in the House of Representatives, Robert Peabody and Nelson Polsby, ed., Rand McNally, 1963.

Peter J. Huber, "Robust Regression," Annals of Statistics, 1, 799-821, 1973.

Elias Huzar, The Purse and the Sword, Cornell University Press, 1950.

L.A. Jaeckel, "Estimating Regression Coefficients by Minimizing the Dispersion of the Residuals," Annals of Mathematical Statistics, 43, 1449-1458, 1972.

James E. Jernberg, "Information Change and Congressional Behavior," in Planning-Programming-Budgeting, 2nd edition, F.J. Lyden and E.G. Miller, ed., Markham, 1972.

Ronald W. Johnson, "A Model of Federal Agency Budgetary Behavior," paper prepared for the 1972 annual meeting of the Public Choice Society, May 3-6, 1972.

J. Johnston, Econometric Methods, 1st edition, McGraw Hill, 1963.

Edward J. Kane, Economic Statistics and Econometrics, Harper and Row, 1968.

A. Kanter, "Congress and the Defense Budget," American Political Science Review, LXVI, 129-142, March, 1972.

Herschel Kanter and Thomas Anger, "Navy Responses to Changes in the Defense Resource Planning Process," Center for Naval Analysis Memorandum 644-73, March, 1973.

M.G. Kendall and Alan Stuart, The Advanced Theory of Statistics, I , 2nd edition, Hafner Publishing Company, 1963.

M.G. Kendall and Alan Stuart, The Advanced Theory of Statistics, II, 3rd edition, Hafner Publishing Company, 1951.

Edward Kolodzei, The Uncommon Defense and the Congress, 1945-1963, Ohio State University Press, 1966.

Leslie Korb, (a), "Congressional Impact on Defense Spending, 1962-1973: The Programmatic and Fiscal Hypotheses," paper delivered at the American Political Science Association Meeting, September, 1973.

Leslie Korb, (b), The Role of the Joint Chiefs of Staff in the Defense Budget Process, Ph.D. Dissertation, Columbia University, 1969.

Edward Laurance, The Changing Role of Congress in Defense Policy-making, Ph.D. dissertation, University of Pennsylvania, 1973.

John P. Leacacos, "Kissinger's Apparat," Foreign Policy, I, 3-27, Winter, 1971.

P.A.W. Lewis and G.P. Learmonth, Random Number Generator Package: LLRANDOM, Research Report NPS55LW73061A, Naval Postgraduate School, Monterey, 1973.

P.A.W. Lewis and G.P. Learmonth, "Statistical Tests of Some Widely Used and Recently Proposed Uniform Random Number Generators," in Proceedings of Computer Science and Statistics, 7th Annual Symposium on the Interface, Iowa State University, 163-171, 1973.

F. Lyden and E. Miller, ed., Planning-Programming-Budgeting, 2nd edition, Markham, 1972.

Roy C. Macridis and Robert E. Ward, Modern Political Systems, Europe, 1st edition, Prentice Hall, 1963.

B.B. Maxon, "Expenditure Control," Unpublished paper, Naval Postgraduate School, June, 1972.

Rupert Miller, "A Trustworthy Jackknife," Annals of Mathematical Statistics, 1594-1605, 35, 1964.

Rupert Miller, "An Unbalanced Jackknife," Unpublished paper, Stanford University, 1974.

Frederick Mosher, Program Budgeting: Theory and Practice, Chicago, Public Administration Service, 1954.

Frederick Mosteller and Robert Rourke, Sturdy Statistics, Addison Wesley, 1973.

Arthur MacMahon, "Congressional Oversight of Administration: The Power of the Purse," Political Science Quarterly, 161-190, 380-414, March, 1943 and June 1943.

Peter B. Natchez and Irvin C. Bupp, "Policy and Priority in the Budgetary Process," American Political Science Review, LXVII, 951-963, 1973.

Navy Programming Manual, United States Navy, Department of the Navy Program Information Center, 1973.

Department of the Navy RDTE Management Guide, Part I: System Description, NAVSOP-2457 (Revised July, 1972).

David Novick, ed., Program Budgeting, Harvard University Press, 1965.

David Ott and Attiat Ott, (a), Federal Budget Policy, 2nd edition, The Brookings Institution, 1969.

David Ott and Attiat Ott, (b), "The Budget Process," in Planning-Programming-Budgeting, 2nd edition, F.J. Lyden and E.G. Miller, ed., Markham, 1972.

John T. Parker, Congressional Restraints on Reprogramming of Appropriated Funds in the Department of Defense, unpublished Student Research Report No. 83, Industrial College of the Armed Forces, 1973.

E.S. Pearson and H.O. Hartley, Biometrika Tables for Statisticians, Vol. I, Cambridge, 1966.

M.H. Quenouille, "Approximate Tests of Correlation in Time Series," Journal of the Royal Statistical Society, B, 11, 68-84, 1949.

M.H. Quenouille, "Notes on Bias Estimation," Biometrika, 43, 353-360, 1956.

E.S. Redford, D.B. Truman, A. Hacker, A.F. Westin, R.C. Wood, Politics and Government in the United States, Harcourt, Brace and World, 1965.

Timothy W. Ruefli, "PPBS - An Analytic Approach," in Studies in Budgeting, R.F. Byrne, A. Charnes et al., ed., American Elsevier, 1971.

A. Sarhan and B. Greenberg, ed., Contributions to Order Statistics, John Wiley, 1962.

H. Scheffé, The Analysis of Variance, John Wiley and Sons, 1959.

Allen Schick, "The Road to PPB: The Stages of Budget Reform," in Planning-Programming-Budgeting, 2nd edition, F.J. Lyden and E.G. Miller, ed., Markham, 1972.

W. Schilling, P. Hammond, G. Snyder, Strategy, Politics and Defense Budgets, Columbia University Press, 1962.

Arthur Smithies, "Conceptual Framework for the Program Budget," in Program Budgeting, David Novick, ed., Harvard University Press, 1965.

Arthur Smithies, (a), The Budgetary Process in the United States, McGraw Hill, 1965.

John L. Stromberg, The Internal Mechanisms of the Defense Budget Process, Fiscal 1953-1968, Rand Corporation Research Memorandum RM-6243-PR, September, 1970.

Prem P. Talwar, Robust Estimation of Regression Parameters, Unpublished Ph.D. Dissertation, Carnegie-Mellon Institute, 1974.

Tax Foundation, Incorporated, "Spending Control Issues and the U.S. Budget," Government Finance Brief No. 22, 1973.

David Teichroew, "Tables of Lower Moments of Order Statistics for Samples from the Normal Distribution," in Contributions to Order Statistics, A. Sarhan and B. Greenberg, ed., John Wiley and Sons, 1962.

H. Theil and A.L. Nagar, "Testing the Independence of Regression Disturbances," Journal of the American Statistical Association, 56, 793-806, 1961.

Samuel A. Tucker, ed., A Modern Design for Defense Decision, Industrial College of the Armed Forces, Washington, D.C., 1966.

John Tukey, Exploratory Data Analysis, to be published 1974.

John Tukey, (a), Chapter II, in Robust Estimates of Location, David Andrews et al., Princeton University Press, 1972.

John Tukey, (b), "A Survey of Sampling from Contaminated Distributions," in Contributions to Probability and Statistics, Stanford University Press, 1960.

John Tukey, (c), "Data Analysis and Behavioral Science," Unpublished manuscript.

John Tukey, (d), "Introduction to Today's Data Analysis," Princeton University, Department of Statistics, Technical Report 40, Series 2, 1973.

Earle Wallace, "The Politics of River Basin Appropriations: A Case Study of the Roanoke River Basin," Unpublished manuscript, University of North Carolina, 1959.

Aaron Wildavsky, The Politics of the Budgetary Process, Little, Brown, 1964.

M.B. Wilk and R. Gnanadesikan, "Probability Plotting Methods for the Analysis of Data," Biometrika, 55, 1-17, 1968.

M.B. Wilk, S.S. Shapiro and Mrs. H.J. Chen, "A Comparative Study of Various Tests for Normality," Journal of the American Statistical Association, 63, pp. 1343-1372.

M.B. Wilk and S.S. Shapiro, "An Analysis of Variance Test for Normality," Biometrika, 52, pp. 591-611.

H. Wold and L. Jureen, Demand Analysis, John Wiley and Sons, 1953.

P. Zarembka, "Functional Forms and the Demand for Money," Journal of the American Statistical Association, 502-511, June 1968.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Documentation Center Cameron Station Alexandria, Virginia 22314	2
2.	Library, Code 0212 Naval Postgraduate School Monterey, California 93940	2
3.	Dr. J.R. Borsting, Code 02 Academic Dean Naval Postgraduate School Monterey, California 93940	1
4.	Assoc. Prof. C.R. Jones, Code 55Js Department of Operations Research and Administrative Sciences Naval Postgraduate School Monterey, California 93940	1
5.	Prof. D.P. Gaver, Code 55Gv Department of Operations Research and Administrative Sciences Naval Postgraduate School Monterey, California 93940	13
6.	Assoc. Prof. R.S. Elster, Code 55Ea Department of Operations Research and Administrative Sciences Naval Postgraduate School Monterey, California 93940	1
7.	Assoc. Prof. D.R. Barr, Code 55Bn Department of Operations Research and Administrative Sciences Naval Postgraduate School Monterey, California 93940	1
8.	Assoc. Prof. C.O. Wilde, Code 53Wo Department of Mathematics Naval Postgraduate School Monterey, California 93940	1
9.	Prof. W. A. Mauer Army War College Carlisle, Pennsylvania	1

10. Library, Code 55 1
Department of Operations Research
and Administrative Sciences
Naval Postgraduate School
Monterey, California 93940

11. LT James Robert Capra, Code 55Zm 1
Department of Operations Research
and Administrative Sciences
Naval Postgraduate School
Monterey, California 93940

Thesis 153624
C1959 Capra
c.1
Analysis of data describing Congressional responses to DOD budget requests.