A COMPUTERIZED ALGORITHM FOR THE DETERMINATION OF THE OPTIMAL ALLOCATION OF TWO WEAPONS SYSTEMS AGAINST N TARGETS

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THESIS

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ABSTRACT

The problem of allocating munitions from M weapons systems to N target complexes is studied and a review of pertinent literature is presented. An algorithm for the solution of the problem in the special case of two weapons systems against N targets is developed and programmed for computerized solution. The results of an example problem are shown and tested. Discussion of the algorithm's extension to more than two weapons systems is included as are alternative solution techniques.

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I. INTRODUCTION

Defense planners are vitally concerned with the problem of assigning weapons systems to strategic targets. Information from intelligence sources is obtained, from which decisions are made regarding the military value of such targets. Judgements as to the effectiveness of weapons systems when used against target types can be made on the basis of test data gathered during Manufacturers' or Armed Services' observed evaluation firings. Constraints on the numbers of units of particular systems are dictated by Defense Department budget or resource availability considerations.

All of the above factors are known to be subject to change and can only be estimated to a varying degree of accuracy at any given time. Because of the uncertain nature of these factors, an optimum targeting policy often calls for a mixture of weapons to be assigned to any single target complex. Problems of this nature may be formulated as nonlinear-programming problems.

The problem of optimal allocation of units of two weapons systems against N targets or a complex of N targets is the subject of this thesis. An algorithm for solution of this problem is presented and a program is given for rapid solution by computer.

II. STUDY OF PERTINENT LITERATURE

The problem considered by this paper is concerned with the allocation of X and Y munitions of two particular weapons systems among N target complexes. Koopman [1] pointed out that the problem of optimum distribution of search effort and the allocation of munitions projected against area targets are analogous. Charnes and Cooper [4] provided a link between search theory and linear programming. F. A. Miercort and R. M. Soland [11] show an integer nonlinear programming formulation of the problem of allocation from a single weapons system against area and point defense. They develop a branch and bound algorithm for its solution.

John Danskin formulated a nonlinear model of M weapons systems' munitions assignment to N targets which he labelled a "simple maximum" problem. The iterative method of solution which he developed is the basis for the algorithm presented in this paper [2].

Lemus and David [8] use an analytic technique similiar to Danskin's to solve the problem and suggest that this should be considered a general solution in the case where the attacker has more than one type of weapon available for assignment to an undefended or virtually undefended target complex. They point out that whenever the number of weapons is large compared with the number of targets that analytic solution methods offer a great savings in computer time for little loss of accuracy. Earlier writings dealing with solutions to problems

of this type, such as those by Manne [6] and by DenBroeder, Ellison and Emerling [7], reformulate the allocation problem as a linear program. The resulting linear program will ordinarily provide a close enough approximation to the original problem to achieve satisfactory results.

III. DEVELOPMENT OF THE SOLUTION ALGORITHM

A. CHARACTERIZATION OF THE OPTIMAL ALLOCATION

The allocation of rounds of two distinct weapons systems among N targets is a nonlinear programming problem, stated mathematically as

$$\text{Maximize} \quad \sum_{i=1}^{N} V_i \left\{ 1 - \exp\left(-\left[\mu_i x_i + \nu_i y_i\right]\right) \right\} ,$$

$$\text{Subject to:} \quad \sum_{i=1}^{N} x_i \le X$$

$$\sum_{i=1}^{N} y_i \le Y$$

$$x_i, y_i \ge 0 \text{ for all } i ,$$

where V_i represents the military worth of target i, μ_i is the effectiveness of system X against target i and ν_i is the effectiveness of system Y against target i. The linear constraints imply that the well-known Kuhn-Tucker constraint qualification holds at all points belonging to the constraint region. Since the objective function is concave as well, our problem becomes one of concave programming and the denoted Kuhn-Tucker conditions are necessary and sufficient for an optimal allocation. Let us denote the optimal allocation as

$$x^* = (x_1^*, x_2^*, \ldots, x_N^*) \quad , \quad y^* = (y_1^*, y_2^*, \ldots, y_N^*)$$

The Lagrangian function is given by

$$L(x, y, \lambda) = \sum_{i=1}^{N} V_i \left\{ 1 - \exp\left(-\left[\mu_i x_i + \nu_i y_i\right]\right) \right\} + \lambda_1 \left(X - \sum_{i=1}^{N} x_i \right) + \lambda_2 \left(Y - \sum_{i=1}^{N} y_i \right),$$

such that

$$\frac{\partial L}{\partial x_i} = \mu_i V_i \exp(-[\mu_i x_i + \nu_i y_i]) - \lambda_1$$

and

$$\frac{\partial L}{\partial y_i} = \nu_i V_i \exp(-[\mu_i x_i + \nu_i y_i]) - \lambda_2 \quad .$$

For the above problem, the necessary and sufficient Kuhn-Tucker conditions are;

1) $\frac{\partial L}{\partial x_i}, \frac{\partial L}{\partial y_i} \leq 0$

2) $x_i^* \frac{\partial L}{\partial x_i} = 0, \; y_i^* \frac{\partial L}{\partial y_i} = 0$

3) $x_i^*, \; y_i^* \geq 0$

4) $\lambda_1^*, \; \lambda_2^* \geq 0$

5) $\lambda_1^* (X - \sum_{i=1}^{N} x_i^*) = 0, \; \lambda_2^* (Y - \sum_{i=1}^{N} y_i^*) = 0$

6) $X - \sum_{i=1}^{N} x_i^* \leq 0, \quad Y - \sum_{i=1}^{N} y_i^* \leq 0$

Consideration of $\frac{\partial L}{\partial x_i} \leq 0$ implies

$$\mu_i V_i \exp(-[\mu_i x_i^* + \nu_i y_i^*]) \leq \lambda_1^* .$$

$x_i^* \frac{\partial L}{\partial x_i} = 0$ implies;

$x_i^* = 0$ if and only if $\mu_i V_i \exp(-[\nu_i y_i^*]) \leq \lambda_1^*$

$x_i^* > 0$ whenever $\mu_i V_i \exp(-[\mu_i x_i^* + \nu_i y_i^*]) = \lambda_1^*$.

It is easily shown that $\lambda_i^* > 0$. For example, consider λ_1^*, which we know by 4) satisfied

$$\lambda_1^* \geq 0 ,$$

if $\lambda_1^* = 0$, then $\mu_i V_i > 0 = \lambda_1^*$ for all i.

Hence

$$x_i^* = \lim_{\lambda_1 \to 0} \left\{ \frac{1}{\mu_i} \ln. \left[\frac{\mu_i x_i^* + \nu_i y_i^*}{\lambda_1^*} \right] \right\} = + \infty$$

and the constraint is violated.

In like manner, $y_i^* = 0$ if and only if

$$\nu_i V_i \exp(-[\mu_i x_i^*]) \leq \lambda_2^* ,$$

$y_i^* > 0$ whenever $\nu_i V_i \exp(-[\mu_i x_i^* + \nu_i y_i^*]) = \lambda_2^*$

and $\lambda_2^* > 0$.

The above considerations yield that there are four possibilities for optimal allocation against a target;

1) $x_i^* = 0$, $y_i^* = 0$

then $\mu_i V_i \leq \lambda_1^*$, $\nu_i V_i \leq \lambda_2^*$

2) $x_i^* > 0$, $y_i^* = 0$

then $\mu_i V_i \exp(-\mu_i x_i^*) = \lambda_1^*$,

$$\nu_i V_i \exp(-\mu_i x_i^*) \leq \lambda_2^*$$

and $x_i^* = \frac{1}{\mu_i} \ln. \left(\frac{\mu_i V_i}{\lambda_1^*} \right)$

3) $y_i^* > 0$, $x_i^* = 0$

then, similiarly,

$$y_i^* = \frac{1}{\nu_i} \ln. \left(\frac{\nu_i V_i}{\lambda_2^*} \right)$$

4) The case of a shared target, $x_i^*, y_i^* > 0$.

then $\mu_i V_i \exp(-[\mu_i x_i^* + \nu_i y_i^*]) = \lambda_1^*$,

$\nu_i V_i \exp(-[\mu_i x_i^* + \nu_i y_i^*]) = \lambda_2^*$

and $\frac{\mu_i}{\nu_i} = \frac{\lambda_1^*}{\lambda_2^*}$ so that;

$$x_i^* = \frac{\ln\left(\frac{\mu_i V_i}{\lambda_1^*}\right) - \nu_i y_i^*}{\mu_i}$$

$$y_i^* = \frac{\ln\left(\frac{\nu_i V_i}{\lambda_2^*}\right) - \mu_i x_i^*}{\nu_i} .$$

This paper assumes that for all i and j

$$\frac{\mu_i}{\nu_i} \neq \frac{\mu_j}{\nu_j} ,$$

and thus there can exist at most one shared target. Two cases are then possible for analysis, that of one shared target and that of no shared targets.

Case 1): No shared targets;

If no shared targets exists, either x_i^* or $y_i^* = 0$, such that $x_i^* = 0$ whenever $y_i^* > 0$ and $y_i^* = 0$ whenever $x_i^* > 0$.

Letting $x_i^* > 0$,

then $x_i^* = \frac{1}{\mu_i} \ln\left(\frac{\mu_i V_i}{\lambda_1^*}\right)$

and $\sum_{\forall x_i^* > 0} x_i^* = X \Rightarrow \sum_{\forall x_i^* > 0} x_i^* = \sum_{\forall x_i^* > 0} \frac{1}{\mu_i} \ln(\lambda_1^*)$

or,

$$\lambda_1^* = \exp\left\{ \frac{\left[\sum_{\forall x_i^* > 0} \frac{1}{\mu_i} \ln(\mu_i V_i)\right] - X}{\sum_{\forall x_i^* > 0} \frac{1}{\mu_i}} \right\}$$

and likewise,

$$\lambda_2^* = \exp\left\{ \frac{\left[\sum_{\forall y_i^* > 0} \frac{1}{\nu_i} \ln(\nu_i V_i)\right] - Y}{\sum_{\forall y_i^* > 0} \frac{1}{\nu_i}} \right\} .$$

Case 2): One shared target;

If target j is shared, then

$$\frac{\mu_j}{v_j} = \frac{\lambda_1^*}{\lambda_2^*} \Rightarrow \lambda_2^* = \frac{\lambda_1^* v_j}{\mu_j} \quad . \text{(a)}$$

It is known that,

$$\sum_{\substack{\forall x_i^* > 0 \\ i \neq j}} \frac{1}{\mu_i} \ln.\left(\frac{\mu_i V_i}{\lambda_1^*}\right) + x_j^* = X$$

and therefore,

$$x_j^* = X - \sum_{\substack{\forall x_i^* > 0 \\ i \neq j}} \frac{1}{\mu_i} \ln.\left(\frac{\mu_i V_i}{\lambda_1^*}\right). \quad \text{(b)}$$

Likewise,

$$y_j^* = Y - \sum_{\substack{\forall y_i^* > 0 \\ i \neq j}} \frac{1}{v_i} \ln.\left(\frac{v_i V_i}{\lambda_2^*}\right).$$

Substituting from (a) then,

$$y_j^* = Y - \sum_{\substack{\forall y_i^* > 0 \\ i \neq j}} \frac{1}{v_i} \ln\left(\frac{v_i V_i \mu_j}{\lambda_1^* v_j}\right). \quad \text{(c)}$$

From allocation rule 4), the shared target case;

$$\mu_j x_j^* + v_j y_j^* = \ln.\left(\frac{\mu_j V_j}{\lambda_1^*}\right) = \ln.\left(\frac{v_j V_j}{\lambda_2^*}\right). \quad \text{(d)}$$

Substituting (b) and (c) in (d) leads to the result:

$$Z = \lambda_1^* = \exp\left\{\frac{\left[\sum_{\forall x_i^* > 0} \mu_j/\mu_i \ln.(\mu_i V_i) + \sum_{\substack{\forall y_i^* > 0 \\ i \neq j}} v_j/v_i \ln.\left(\frac{\mu_j V_i}{v_j/v_i}\right) - \mu_j \quad - v_j\right]}{\left[1 + \sum_{\substack{\forall x_i^* > 0 \\ i \neq j}} \mu_j/\mu_i + \sum_{\substack{\forall y_i^* > 0 \\ i \neq j}} v_j/v_i\right]}\right\}$$

and from

$$\lambda_2^* = \lambda_1^* \left(\frac{v_j}{\mu_j}\right),$$

then

$$\lambda_2^* = Z\left(\frac{v_j}{\mu_j}\right).$$

B. AN ALGORITHM FOR COMPUTER SOLUTION

The non-linear program solved in part A. is identical to the "simple maximum" problem studied by John Danskin [2]. Danskin developed an iterative procedure for solution of this problem. the ideas of this algorithm were employed to outline a step by step solution suitable for computerization. Programming and testing this procedure forms the basis of this report. The program, as is the case in Danskin's original algorithm, yields allocations of rounds of weapons systems one and two for each target. Figure (1) presents a graphical flow-chart of this algorithm.

C. COMPUTER PROGRAM FOR SOLUTION:

Computer Program A solves the Weapons Systems Allocation problem for two systems against N targets. The problem is written in FORTRAN for solution on an IBM system 360 computer. Computer Program A shows the data required to solve the example problem of Section IV in the initialization statements. Data required to solve any problem are as follows:

N = number of targets

X = rounds available from system 1

y = rounds available from system 2

Box MU (I) = effectiveness of system 1 versus target I

Box ETA (I) = effectiveness of system 2 versus target I

BOX VEE (I) = military value of target I

Box K (I) = target number of target I

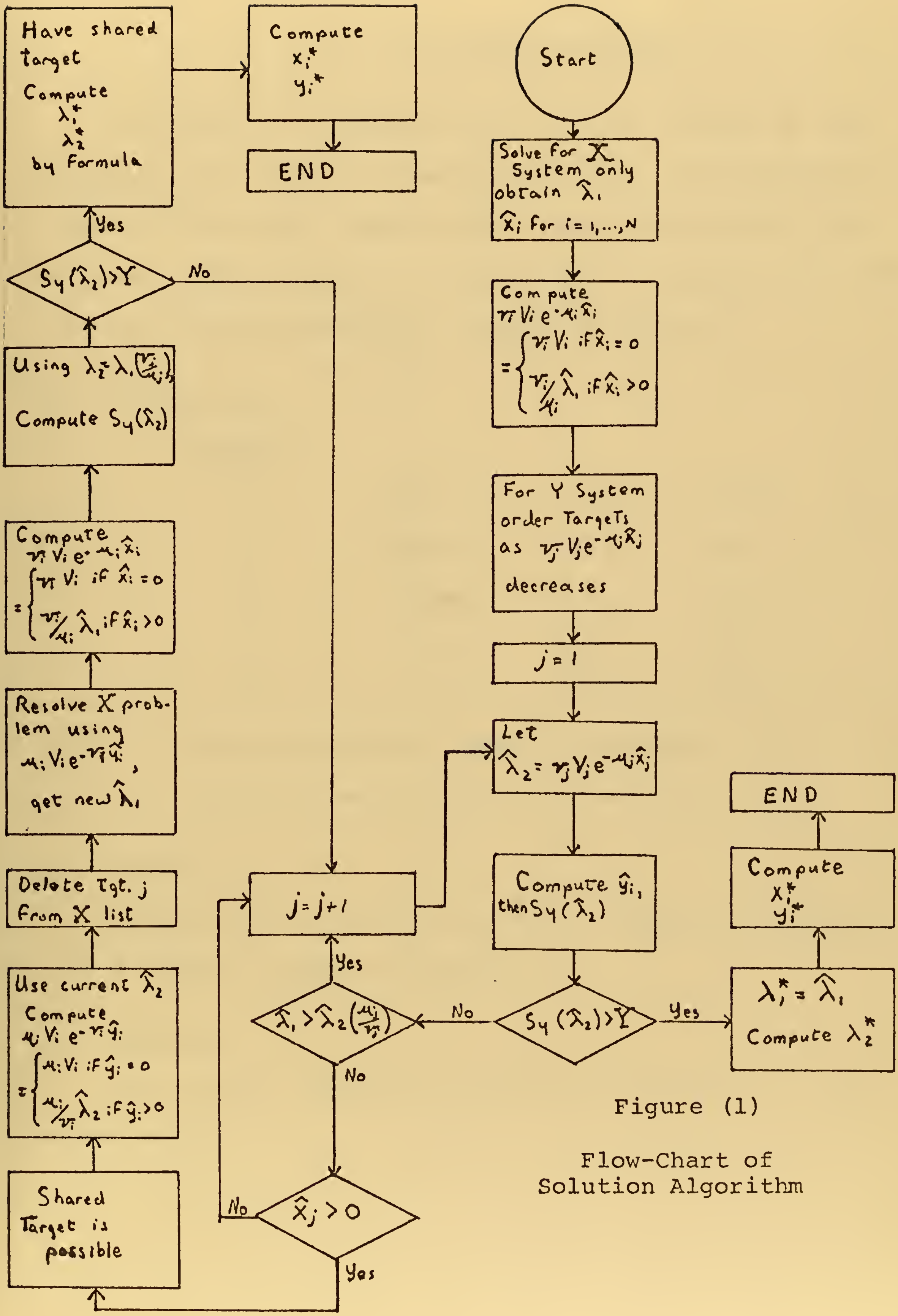

Figure (1)

Flow-Chart of Solution Algorithm

IV. AN EXAMPLE PROBLEM

An example is presented here to show the use of the solution algorithm and computer program. It represents a sample of the data required for a problem involving two weapons systems against a complex of five targets and gives the results as computed by Computer Program A.

A. PROBLEM DATA

Target	V		
1	1000.0	0.03	0.005
2	100.0	0.20	0.20
3	500.0	0.02	0.06
4	25.0	0.20	2.00
5	10.0	0.20	0.20

Units of Weapons System 1 available: 80.0

Units of Weapons System 2 available: 50.0

B. PROBLEM

Compute the optimum allocation of rounds of systems 1 and 2 against targets 1 through 5.

C. RESULTS

The solution obtained from Computer Program A is as shown below.

```
C       COMPUTER OUTPUT


        MUNITIONS AVAILABLE FROM WEAPONS SYSTEM 1 =80.0
C       MUNITIONS AVAILABLE FROM WEAPONS SYSTEM 2 =50.0


C       OPTIMAL LAGRANGE MULTIPLIER FOR SYSTEM 2 =2.7925
C       OPTIMAL LAGRANGE MULTIPLIER FOR SYSTEM 1 =2.7925


C       OPTIMUM ALLOCATIONS FOR SYSTEM 1
C              TGT          ROUNDS
C              1            79.1423
C              2             0.8577
C              3             0.0000
C              4             0.0000
C              5             0.0000

C       OPTIMUM ALLOCATIONS FOR SYSTEM 2
C              TGT           ROUNDS
C              1             0.0000
C              2             8.9863
C              3            39.5711
C              4             1.4425
C              5             0.0000

C       SHARED TARGET IS TARGET 2
```

These results were satisfactorily checked against allocation rules 1-4 as outlined in Section III. The algorithm was also hand computed for this problem and results were compatible. The problem of integer solution was not addressed, and so results must be rounded off to the nearest whole integer for satisfactory use. Program running time for this example problem was 8.38 seconds.

V. EXTENSION OF DANSKIN'S ALGORITHM TO M WEAPONS SYSTEMS

The characterization and existence of a solution to the non-linear programming problem representing allocation of munitions from two weapons systems among N targets were shown in Section III. In Danskin's work he proved that if a solution existed for the M-1 weapons systems case, then one iteration of his algorithm from the M-1 base case would result in the proper allocation in the M systems case [2]. It is possible, though tedious, to extend the computerized solution algorithm one step at a time from the two weapons systems case to the M weapons systems case.

VI. OTHER SOLUTION APPROACHES

Lemus and David [10] state that, in the case where the number of weapons is comparable with the number of targets, the allocation problem may be reduced to a formulation of the transportation problem and solved by linear programming. This method has the advantage of integer-valued solution but is disadvantageous since the optimal allocation is characterized by (except for alternate optima) all fire being concentrated on as many targets as there are constraints.

In his paper, R. H. Day [9] shows a method of solution which decomposes the general allocation problem into a set of targeting problems in the small and a targeting problem in the large. He solves the former problems using a Sequential Optimization Method and employs the results to obtain a solution to the targeting problem in the large by nonlinear programming. Mylander [12] discusses how the problem can be solved using the Sequential Unconstrained Minimization Technique, (SUMT).

Ury Passy [10] showed that the general assignment problem could be formulated as a geometric program and could be solved through its dual. He developed an algorithm for solution of weapons-assignment problems such as the one addressed in this thesis.

VII. CONCLUSIONS

The computerized solution of the weapons systems allocation problem in the special case of two weapons systems against N targets is shown. Extension of the solution procedure to the case of M systems is discussed but not attempted. Based on a survey of the literature, there appear to be several alternate means of solution which could possibly be profitably investigated.

The techniques studied should be useful in solving problems other than those of weapons allocation. Such problems as allocation of Research and Development funds and allocation of search effort are examples [15].

Computer Program A

This program determines the allocation of X munitions from system 1 and Y munitions from system 2 among N targets.

```
      DIMENSION BOXX(5),EXSTAP(5),STAOP(5),STAOR(5),WYSTAP(5),EXHAT(5)
      DIMENSION BOXI(5),BOXJ(5),BOXK(5),BOXMU(5),BOXVEE(5),BOXETA(5)
      DIMENSION BOXL(5),YOU(5),YOUR(5),ELBOX(5),VEE(5)
      DATA BOXK/1.0,2.0,3.0,4.0,5.0/,BOXMU/0.03,0.2,0.02,0.2,0.2/
      DATA BOXVEE/1000.0,100.0,500.0,25.0,10.0/,BOXETA/0.005,0.2,0.06,
     12.0,0.2/
      N=5
      X=80.0
      Y=50.0
      ENLAM=0.0
      J=1
      DO 300 I=1,N
      A=BOXMU(I)*BOXVEE(I)
      BOXI(I)=A
      B=BOXETA(I)*BOXVEE(I)
      BOXJ(I)=B
  300 CONTINUE
      M=N
      MA=N
      NA=N
      NB=N
      NG=N
      WAD=ENLAM
      ESWHY=ENLAM
      ROLAM=ENLAM
      WTM=ENLAM
      WART=ENLAM
      ESS=ENLAM
      BPAIN=ENLAM
      BRAWN=ENLAM
      TROUT=ENLAM
      TRIPE=ENLAM
      ROUND=ENLAM
      N=N-1
 1000 DO 3000 I=1,N
      IF (BOXI(I).LT.BOXI(I+1)) GO TO 2000
      GO TO 3000
 2000 P=BOXI(I)
      BOXI(I)=BOXI(I+1)
      BOXI(I+1)=P
```

```
      Q=BOXMU(I)
      BOXMU(I)=BOXMU(I+1)
      BOXMU(I+1)=Q
      R=BOXVEE(I)
      BOXVEE(I)=BOXVEE(I+1)
      BOXVEE(I+1)=R
      S=BOXETA(I)
      BOXETA(I)=BOXETA(I+1)
      BOXETA(I+1)=S
      T=BOXJ(I)
      BOXJ(I)=BOXJ(I+1)
      BOXJ(I+1)=T
 3000 CONTINUE
      N=N-1
      IF (N.GE.1) GO TO 1000
      CONTINUE
      DO 6000 I=1,M
      DALLAM=BOXI(I)
      DO 600 K=1,M
      IF (BOXI(K).LT.DALLAM) GO TO 600
      ESSLAM=1.0/BOXMU(K)
      ELLAM=(BOXI(K)/DALLAM)
      EXLAM=ESSLAM*ALOG(ELLAM)
      ENLAM=ENLAM+EXLAM
  600 CONTINUE
      BOXX(I)=ENLAM
      ENLAM=0.0
      IF (BOXX(I).GT.X) GO TO 9000
 6000 CONTINUE
      IF (I.GE.M) GO TO 90
 9000 I=I-1
   90 WAA=BOXI(I)
      WAB=X-BOXX(I)
      DO 60 L=1,I
      WAC=1/BOXMU(L)
      WAD=WAD+WAC
   60 CONTINUE
      WAE=WAB/WAD
      WAF=EXP(-WAE)
      STALAM=WAA*WAF
      DO 30 L=1,I
      WBA=1/BOXMU(L)
      WBB=BOXI(L)/STALAM
      WBC=WBA*ALOG(WBB)
      EXSTAR(L)=WBC
   30 CONTINUE
      I=I+1
      DO 20 L=I,M
```

```
      EXSTAR(L)=0.0
   20 CONTINUE
      DO 99 I=1,M
      IF (EXSTAR(I).GT.0.0) GO TO 10
      STAOP(I)=BOXVEE(I)*BOXETA(I)
      STAOR(I)=STAOP(I)
      GO TO 99
   10 STA=BOXETA(I)/BOXMU(I)
      STAOP(I)=STA*STALAM
      STAOR(I)=STAOP(I)
   99 CONTINUE
      M=M-1
   96 DO 98 I=1,M
      IF (STAOP(I).LT.STAOP(I+1)) GO TO 97
      GO TO 98
   97 U=STAOP(I)
      STAOP(I)=STAOP(I+1)
      STAOP(I+1)=U
   98 CONTINUE
      M=M-1
      IF (M.GE.1) GO TO 96
   68 CONTINUE
      TWOLAM=STAOP(J)
      DO 89 I=1,MA
      IF (STAOP(J).EQ.STAOR(I)) GO TO 87
   89 CONTINUE
   87 NR=I
      DO 86 I=1,MA
      RUN =1/BOXETA(I)
      ROW=(BOXJ(I)/TWOLAM)
      ESWAY=RUN* ALOG(ROW)
      ESWHY =ESWHY +ESWAY
      IF(ESWAY.GT.0.0) GO TO 81
      ESWAY=0.0
   81 BOXL(I) =ESWAY
   86 CONTINUE
      IF (ESWHY.GT.Y) GO TO 85
      ESWHY=0.0
      SAM=BOXMU(NR)/BOXETA(NR)
      RAS=TWOLAM*SAM
      IF (RAS.LT.STALAM) GO TO 69
      IF (EXSTAR(NR).GT.0.0) GO TO 65
   69 J=J+1
      IF (J.GT.MA) GO TO 85
      GO TO 68
   65 DO 67 I=1,MA
      IF (BOXL(I).GT.0.0) GO TO 66
      SEAT =BOXMU(I)*BOXVEE(I)
```

```
      YOU(I)=SEAT
      GO TO 67
   66 TEA=BOXMU(I)/BOXETA(I)
      SEAT=TEA*TWOLAM
      YOU(I) =SEAT
   67 CONTINUE
      BOXK(NR)=0.0
      DO 58 I=1,NA
      YOUR(I)=YOU(I)
   58 CONTINUE
      NA=NA-1
   56 DO 59 I=1,NA
      IF (YOU(I).LT.YOU(I+1)) GO TO 57
      GO TO 59
   57 S=YOU(I)
      YOU(I)=YOU(I+1)
      YOU (I+1)=S
   59 CONTINUE
      NA=NA-1
      IF (NA.GE.1) GO TO 56
      DO 49 I=1,NB
      IF (BOXK(I).EQ.0.0) GO TO 49
      DALLAM=YOU(I)
      DO 48 K=1,NB
      IF (BOXK(K).EQ.0.0) GO TO 48
      ESSLAM=(1.0/BOXMU(K))
      ELLAM=(BOXI(K)/DALLAM)
      EXLAM=ESSLAM*ALOG(ELLAM)
      ROLAM=ROLAM+EXLAM
   48 CONTINUE
      ELBOX(I)=ROLAM
      IF(ROLAM.GT.X) GO TO 47
      ROLAM =0.0
   49 CONTINUE
   47 I=I-1
      IF(BOXK(I).EQ.0.0) GO TO 47
      DO 43 K=1,NB
      IF (YOU(I).EQ.YOUR(K)) GO TO 42
   43 CONTINUE
   42 NC=K
      DO 45 I=1,NC
      IF(BOXK(I).EQ.0.0) GO TO 45
      WEM=1/BOXMU(I)
      WTM=WTM+WEM
      WUM=ALOG(BOXI(I))
      WAR =WEM*WUM
      WART=WART+WAR
   45 CONTINUE
```

```
      WARN=WART-X
      WARP=WARN/WTM
      WUNLAM=EXP(WARP)
      WTM=0.0
      WART=0.0
      DO 44 I=1,NB
      IF(BOXK(I).EQ.0.0) GO TO 40
      WOP=1/BOXMU(I)
      WEP =BOXI(I)/WUNLAM
      WIP= WOP*ALOG(WEP)
      EXHAT(I)=WIP
      GO TO 44
   40 EXHAT(I)=0.0
   44 CONTINUE
      DO 41 I=1,NB
      IF (EXHAT(I).LE.0.0) GO TO 39
      GO TO 41
   39 EXHAT(I)=0.0
   41 CONTINUE
      NC=NC+1
      DO 29 I=NC,NB
      EXHAT(I)=0.0
   29 CONTINUE
      DO 38 I=1,NB
      IF(EXHAT(I).EQ.0.0) GO TO 37
      MARS=BOXETA(I)/BOXMU(I)
      VEEP=MARS*WUNLAM
      GO TO 36
   37 VEEP=BOXJ(I)
   36 VEE(I)=VEEP
   38 CONTINUE
      DO 26 I=1,NG
      IF(BOXK(I).EQ.0.0) GO TO 28
      GO TO 26
   28 SHIP=BOXETA(I)/BOXMU(I)
      HIP=WUNLAM*SHIP
      TWOLAM=HIP
   26 CONTINUE
      DO 27 I=1,NG
      WY=1/BOXETA(I)
      WYY=VEE(I)/TWOLAM
      WYE=ALOG(WYY)
      WYEY=WY*WYE
      IF(WYEY.LE.0.0) GO TO 22
      ESS=ESS+WYEY
      WYSTAR(I)=WYEY
      GO TO 27
   22 WYSTAR(I)=0.0
```

```
   27 CONTINUE
      IF(ESS.LE.Y) GO TO 25
      ESS=0.0
      GO TO 24
   25 R=NR
      BOXK(NR)=R
      GO TO 69
   24 CONTINUE
      DO 532 I=1,NG
      IF(BOXK(I).EQ.0.0) GO TO 533
      GO TO 532
  533 ZED=BOXMU(I)*X
      ZEP=BOXETA(I)*Y
      NS=I
      ZEE=ZED+ZEP
  532 CONTINUE
      DO 632 I=1,NG
      IF(EXHAT(I).GT.0.0) GO TO 631
      IF(I.EQ.NS)GO TO 631
      GO TO 632
  631 BRA=ALOG(BOXI(I))
      BRAT=BOXMU(NS)/BOXMU(I)
      BRAN=BRA*BRAT
      BRAIN=BRAIN+BRAN
  632 CONTINUE
      DO 432 I=1,NG
      IF(EXHAT(I).GT.0.0) GO TO 431
      GO TO 432
  431 IF(I.EQ.NS) GO TO 432
      BRAT=BOXMU(NS)/BOXMU(I)
      BRAWN=BRAWN+BRAT
  432 CONTINUE
      DO 732 I=1, NG
      IF(WYSTAR(I).GT.0.0)GO TO 731
      GO TO 732
  731 IF(I.EQ.NS) GO TO 732
      TRA=BOXETA(NS)/BOXETA(I)
      TRAIN=BOXMU(NS)*BOXVEE(I)
      TRAT=TRAIN/TRA
      TROT=ALOG(TRAT)
      TRIP=TRA*TROT
      TROUT=TROUT+TRIP
      TRIPE=TRIPE+TRA
  732 CONTINUE
      WIPE=BRAWN+TRIPE+1.0
      WORK=BRAIN+TROUT-ZEE
      WORN=WORK/WIPE
      WUNLAM=EXP(WORN)
```

```
      WARP=BOXETA(NS)/BOXMU(NS)
      TWOLAM=WUNLAM*WARP
      DO 832 I=1,NG
      IF(I.EQ.NS)GO TO 832
      IF (EXHAT(I).LE.0.0) GO TO 831
      RAUNCH=1/BOXMU(I)
      RANCH=BOXI(I)/WUNLAM
      RATT=ALOG(RANCH)
      RALPH=RAUNCH*RATT
      EXSTAR(I)=RALPH
      ROUND=ROUND+RALPH
      GO TO 832
  831 EXSTAR(I)=0.0
  832 CONTINUE
      ROTT=X-ROUND
      EXSTAR(NS)=ROTT
      ROUND=0.0
      DO 932 I=1,NG
      IF(I.EQ.NS) GO TO 932
      IF(WYSTAR(I).LE.0.0) GO TO 931
      RAUNCH =1/BOXETA(I)
      RANCH=BOXJ(I)/TWOLAM
      RATT=ALOG(RANCH)
      RALPH=RAUNCH*RATT
      WYSTAR(I)=RALPH
      ROUND=ROUND+RALPH
      GO TO 932
  931 WYSTAR(I)=0.0
  932 CONTINUE
      ROOT=Y-ROUND
      WYSTAR(NS)=ROOT
      GO TO 82
   85 WUNLAM=STALAM
      DO 84 I=1,NG
      DOG=1/BOXMU(I)
      CAT=BOXI(I)/WUNLAM
      WOLF=ALOG(CAT)
      EXSTAR(I)=DOG*WOLF
      IF(EXSTAR(I).GT.0.0) GO TO 83
      EXSTAR(I)=0.0
      WYSTAR(I)=1.0
      GO TO 84
   83 WYSTAR(I)=0.0
   84 CONTINUE
      DO 79 I=1,NG
      IF(WYSTAR(I).LE.0.0) GO TO 79
      ROPE=1/BOXETA(I)
      VERB=BOXJ(I)
```

```
      ROUND=ROUND+ROPE
      RICK=ALOG(VERB)
      DD=ROPE*RICK
      BRAIN=BRAIN+DD
   79 CONTINUE
      ROUT=BRAIN-Y
      CANDY=ROUT/ROMP
      TWOLAM=EXP(CANDY)
      DO 78 I=1,NG
      DOG=1/BOXETA(I)
      CAT=BOXJ(I)/TWOLAM
      WOLF=ALOG(CAT)
      WYSTAR(I)=DOG*WOLF
   78 CONTINUE
   82 CONTINUE
 9700 FORMAT(1X,F12.4)
      WRITE (6,9700) X,Y,WUNLAM,TWOLAM,EXSTAR,WYSTAR
      STOP
      END
//
```

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The problem of allocating munitions from M weapons systems to N target complexes is studied and a review of pertinent literature is presented. An algorithm for the solution of the problem in the special case of two weapons systems against N targets is developed and programmed for computerized solution. The results of an example problem are shown and tested. Discussion of the algorithm's extension to more than two weapons systems is included as are alternative solution techniques.

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