Effect of an absorbing medium on particle oscillations

V. I. Nazaruk*
Institute for Nuclear Research of RAS, 60th October Anniversary Prospect 7a, 117312 Moscow, Russia

(Received 24 September 2018; revised manuscript received 16 April 2019; published 11 July 2019)

Particle oscillations in absorbing matter are considered. The approach based on the optical potential is shown to be inapplicable in the strong absorption region. Models with Hermitian Hamiltonian are analyzed. They give an increase of the process width in comparison with the model based on the optical potential.

DOI: 10.1103/PhysRevC.100.015202

I. INTRODUCTION

In-particle oscillations in the medium absorption can play an important role, for example, in $K^0\bar{K}^0$ [1–4] and $n\bar{n}$ [5–8] oscillations. In this paper I consider $n\bar{n}$ transitions in the medium followed by annihilation,

$$n \to \bar{n} \to M. \tag{1}$$

Here M are the annihilation mesons. The reason for considering this process is that the absorption (annihilation) of $\bar{n}$ is extremely strong.

In the standard approach (later referred to as a *potential model*) the $\bar{n}$-medium interaction is described by antineutron optical potential $U_{\bar{n}}$. We have objections to this model (Sec. II). In Sec. III the alternative models based on the field-theoretical approach are considered. For these models two possibilities exist: a model with bare (Sec. III A) and dressed (Sec. III C) propagators. (In the latter case I come to the S-matrix problem formulation.) In the models with bare and dressed propagators I directly calculate the off-diagonal matrix element without using the optical potential.

The results are compared in Sec. IV. The potential model contains double counting. This has been proved in the standard S-matrix approach. This fact in particular should be emphasized.

In Sec. V the results are summarized. The problems of the models based on the S-matrix approach are pointed out as well. The restriction on the free-space $n\bar{n}$ oscillation time τ critically depends on the description of absorption. In this regard, the main goal of this paper is to consider the absorption model itself.

*nazaruk@inr.ru

II. POTENTIAL MODEL

We consider process (1). In the standard approach [5–7] the $n\bar{n}$ transitions in the medium are described by Schrodinger equations:

$$\begin{aligned}(i\partial_t - H_0)n(x) &= \epsilon_{n\bar{n}}\bar{n}(x),\\ (i\partial_t - H_0 - V)\bar{n}(x) &= \epsilon_{n\bar{n}}n(x),\\ H_0 &= -\nabla^2/2m + U_n,\\ V = U_{\bar{n}} - U_n &= \mathrm{Re}U_{\bar{n}} + i\mathrm{Im}U_{\bar{n}} - U_n,\end{aligned} \tag{2}$$

$\mathrm{Im}U_{\bar{n}} = -\Gamma/2$, $\bar{n}(0,\mathbf{x}) = 0$. Here U_n and $U_{\bar{n}}$ are the potential of n and the optical potential of $\bar{n}$, respectively; $\epsilon_{n\bar{n}}$ is a small parameter with $\epsilon_{n\bar{n}} = 1/\tau$, where τ is the free-space $n\bar{n}$ oscillation time, Γ being the annihilation width of $\bar{n}$.

In the lowest order in $\epsilon_{n\bar{n}}$ the process width is [5–7]

$$\Gamma_{\rm pot} = \epsilon_{n\bar{n}}^2 \frac{1}{(\mathrm{Re}V)^2 + (\Gamma/2)^2}\Gamma. \tag{3}$$

$U_{\bar{n}}$ is the basic element of the model. In this connection the following problems arise:

(1) The optical model was developed for the Schrödinger-type equations. The physical meaning of $\mathrm{Im}U_{\bar{n}}$ follows from the corresponding continuity equation. Coupled Eqs. (2) give rise to the following equation:

$$\left[\partial_t^2 + i\partial_t(V + 2H_0) - H_0^2 - H_0V + \epsilon_{n\bar{n}}^2\right]n(x) = 0. \tag{4}$$

The continuity equation cannot be derived from (4).

(2) To get $\Gamma_{\rm pot}$, the optical theorem or condition of probability conservation are used. However, the S matrix is essentially nonunitary.

(3) The structure and Γ dependence of (3) provoke some objections. Due to this, an alternative model should be considered.

III. FIELD-THEORETICAL APPROACH

The interaction Hamiltonian of process (1) is given by

$$\begin{aligned}\mathcal{H}_I &= \mathcal{H}_{n\bar{n}} + \mathcal{H},\\ \mathcal{H}_{n\bar{n}} &= \epsilon_{n\bar{n}}\bar{\Psi}_{\bar{n}}\Psi_n + \mathrm{H.c.},\end{aligned} \tag{5}$$

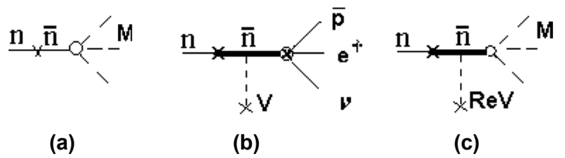

FIG. 1. (a) $n\bar{n}$ transition in the medium followed by annihilation. The annihilation is shown by a circle. The propagator is bare (b) $n\bar{n}$ transition in the medium followed by decay (c). The same as in (a) but the antineutron propagator is dressed (see text).

where $\mathcal{H}_{n\bar{n}}$ and $\mathcal{H}$ are the Hamiltonians of $n\bar{n}$ conversion and the $\bar{n}$-medium interaction, respectively. The background neutron potential is included in the neutron wave function:

$$n(x) = \Omega^{-1/2} \exp(-ipx), \tag{6}$$

$p = (\epsilon, \mathbf{p})$, $\epsilon = \mathbf{p}^2/2m + U_n$.

A. Model with a bare propagator

The $n\bar{n}$ conversion comes from the exchange of Higs bosons with $m_H > 10^5$ GeV. The $\bar{n}$ annihilates in a time $\tau_a \sim 1/\Gamma$. We deal with a two-step process with a characteristic time τ_a.

The general definition of the antineutron annihilation amplitude M_a is given by

$$\langle M0|T \exp\left[-i\int dx\mathcal{H}(x)\right] - 1|0\bar{n}_p\rangle = N(2\pi)^4\delta^4(p_f - p_i)M_a. \tag{7}$$

Here $|0\bar{n}_p\rangle$ is the state of the medium containing the $\bar{n}$ with the four-momentum $p = (\epsilon, \mathbf{p})$; $\langle M|$ denotes the annihilation mesons and N includes the normalization factors of the wave functions. The antineutron annihilation width Γ is expressed through M_a:

$$\Gamma = N_1 \int d\Phi |M_a|^2, \tag{8}$$

where N_1 is the normalization factor.

The amplitude of process (1) M_1 is given by

$$\langle M0|T \exp\left\{-i\int dx[\mathcal{H}_{n\bar{n}}(x) + \mathcal{H}(x)]\right\} - 1|0n_p\rangle = N(2\pi)^4\delta^4(p_f - p_i)M_1. \tag{9}$$

In the lowest order in $\mathcal{H}_{n\bar{n}}$ for the process amplitude M_1 one obtains [see Fig. 1(a)]

$$M_1 = \epsilon_{n\bar{n}} G_0 M_a, \tag{10}$$

$$G_0 = \frac{1}{\epsilon_{\bar{n}} - \mathbf{p}_{\bar{n}}^2/2m - U_n + i0}, \tag{11}$$

where G_0 is the antineutron propagator. Since $\mathbf{p}_{\bar{n}} = \mathbf{p}$, $\epsilon_{\bar{n}} = \epsilon$, then $G_0 \sim 1/0$. M_a contains all the $\bar{n}$-medium interactions followed by annihilation, including antineutron rescattering in the initial state. So in this case the antineutron propagator is bare.

When dealing with infrared singularity, for solving the problem a field-theoretical approach with a finite-time interval has been proposed [9]. The process (1) probability was found to be [10]

$$W(t) \approx W_f(t) = \epsilon_{n\bar{n}}^2 t^2, \tag{12}$$

where W_f is the free-space $n\bar{n}$ transition probability. Equation (12) leads to a very strong restriction on the free-space $n\bar{n}$ oscillation time: $\tau = 10^{16}$ yr.

B. Absorption in the intermediate state

Starting from (5) and (6) I have drawn the singular amplitude M_1. To gain a better understanding of the problem, I consider the $n\bar{n}$ transitions in the medium followed by β^+ decay:

$$n \to \bar{n} \to \bar{p}e^+\nu. \tag{13}$$

The neutron wave function is given by (6). The interaction Hamiltonian is

$$\mathcal{H}_I = \mathcal{H}_{n\bar{n}} + \mathcal{H}_W + V\bar{\Psi}_{\bar{n}}\Psi_{\bar{n}}, \tag{14}$$

where V is defined by (2) and $\mathcal{H}_W$ is the Hamiltonian of the decay $\bar{n} \to \bar{p}e^+\nu$. The process amplitude is nonsingular [see (15) below] and I use the S-matrix approach. In the lowest order in $\mathcal{H}_{n\bar{n}}$ the amplitude M_2 [see Fig. 1(b)] is given by

$$\begin{aligned} M_2 &= \epsilon_{n\bar{n}} G M_d, \\ G &= \frac{1}{\epsilon_{\bar{n}} - \mathbf{p}_{\bar{n}}^2/2m - U_{\bar{n}} + i0} \\ &= \frac{1}{\epsilon - \mathbf{p}^2/2m - (U_n + V) + i0} = -\frac{1}{V}, \end{aligned} \tag{15}$$

where M_d is the amplitude of the β^+-decay and G is the antineutron propagator.

The process width Γ_2 is

$$\Gamma_2 = \frac{\epsilon_{n\bar{n}}^2}{|V|^2}\Gamma_d, \tag{16}$$

where Γ_d is the width of the β^+ decay. The propagator is dressed due to the additional field V. There are no questions connected with $U_{\bar{n}}$ since G is the propagator of Schrodinger equation.

C. Model with a dressed propagator

We return to process (1). I compose a model with a dressed propagator. By analogy with (14) in the Hamiltonian $\mathcal{H}$ [see (5)] I separate out the scalar field V_1:

$$\mathcal{H} = V_1\bar{\Psi}_{\bar{n}}\Psi_{\bar{n}} + \mathcal{H}_a, \tag{17}$$

where $\mathcal{H}_a$ is the annihilation Hamiltonian. Now the antineutron annihilation amplitude M_{an} is defined through $\mathcal{H}_a$:

$$\langle M0|T \exp\left[-i\int dx\mathcal{H}_a(x)\right] - 1|0\bar{n}_p\rangle = N(2\pi)^4\delta^4(p_f - p_i)M_{\text{an}}. \tag{18}$$

The interaction Hamiltonian is given by

$$\mathcal{H}_I = \mathcal{H}_{n\bar{n}} + V_1\bar{\Psi}_{\bar{n}}\Psi_{\bar{n}} + \mathcal{H}_a. \tag{19}$$

In the lowest order in $\mathcal{H}_{n\bar{n}}$ the amplitude of process (1) is

$$
\begin{aligned}
M_s &= \epsilon_{n\bar{n}} G_d M_{\text{an}},\\
G_d &= G_0 + G_0 V_1 G_0 + \cdots\\
&= \frac{1}{(1/G_0) - V_1 + i0} = -\frac{1}{V_1}.
\end{aligned} \tag{20}
$$

The antineutron propagator G_d is dressed. V_1 plays the role of antineutron self-energy Σ. M_s corresponds to the first order in $\mathcal{H}_{n\bar{n}}$ and all the orders in V_1 and $\mathcal{H}_a$. Compared to (7), M_{an} is calculated through the reduced Hamiltonian $\mathcal{H}_a$ instead of $\mathcal{H}$; otherwise, $V_1 = 0$ and I arrive at the amplitude (10).

The process width Γ_s is

$$
\begin{aligned}
\Gamma_s &= N_1 \int d\Phi |M_s|^2 = \frac{\epsilon_{n\bar{n}}^2}{|V_1|^2}\Gamma_{\text{an}},\\
\Gamma_{\text{an}} &= N_1 \int d\Phi |M_{\text{an}}|^2.
\end{aligned} \tag{21}
$$

The amplitude M_s is nonsingular because the propagator is dressed. The antineutron self-energy $\Sigma = V_1$ appears due to separation of the field V_1. This procedure seems to be artificial and unjustified as well as definition of the M_{an}. There are no similar problems for process (13) since the self-energy and decay of $\bar{n}$ are generated by different fields $\mathcal{H}_W$ and V. This point should be given particular emphasis. In any case $\Gamma_{\text{an}} \sim \Gamma$, and so

$$\Gamma_s \sim \Gamma_{\text{an}} \sim \Gamma. \tag{22}$$

IV. COMPARISON WITH POTENTIAL MODEL

A. Double counting in the potential model

First, I compare the potential model with the model with a dressed propagator. In (21) I have to take the same parameters as in the potential model: $V_1 = V$ and $\Gamma_{\text{an}} = \Gamma$. Then I get

$$\Gamma_s = \epsilon_{n\bar{n}}^2 \frac{1}{(\text{Re}V)^2 + (\Gamma/2)^2}\Gamma. \tag{23}$$

Equation (23) coincides with (3): $\Gamma_s = \Gamma_{\text{pot}}$. By means of the model with a dressed propagator I have obtained Γ_{pot}. The antineutron annihilation width Γ is involved in the propagator [see (20), where $V_1 = V$] as well as vertex function which means double counting.

The same conclusion has been done in Ref. [8]. It was shown that double counting leads to full cancellation of the leading terms. However, in Ref. [8] the consideration was qualitative and performed on the finite-time interval. Equation (23) reproduces (3) exactly.

B. Model with Hermitian Hamiltonian

As proved earlier, the model with dressed propagator is unjustified. Nevertheless, the correction of the type (17) cannot be excluded. As an alternative to the model with bare propagator I consider the model with dressed propagator [see Fig. 1(c)]. The model is simple: U_n and $U_{\bar{n}}$ are the real potentials of n and $\bar{n}$, respectively; annihilation included in the vertex function only; energy gap ReV leads to the process suppression. As with model with bare propagator, the Hamiltonin is Hermitian.

In (21) I take $V_1 = \text{Re}V$ (in this case the Hamiltonin is Hermitian), and $\Gamma_{\text{an}} = \Gamma$. The process width Γ_s is

$$\Gamma_s = \frac{\epsilon_{n\bar{n}}^2}{(\text{Re}V)^2}\Gamma. \tag{24}$$

The model described above is the most realistic variant of the model with dressed propagator.

Therefore, $\Gamma_s \sim \Gamma$. For the $K^0\bar{K}^0$ transitions in the medium followed by decay and regeneration of the K_S^0 component an identical Γ dependence takes place [11,12]. In the potential model $\Gamma_{\text{pot}} \sim \Gamma$ only at light absorption. Indeed, if $\Gamma/2 \ll |\text{Re}V|$, then

$$\Gamma_{\text{pot}} = \frac{\epsilon_{n\bar{n}}^2}{(\text{Re}V)^2}\Gamma\left[1 - \left(\frac{\Gamma}{2\text{Re}V}\right)^2\right]. \tag{25}$$

In the first approximation (25) coincides with (24). This agreement was expected since the dominant role was played by Re$U_{\bar{n}}$.

If $\Gamma/2 \gg |\text{Re}V|$, then

$$\Gamma_{\text{pot}} = \frac{4\epsilon_{n\bar{n}}^2}{\Gamma}. \tag{26}$$

$\Gamma_{\text{pot}} \sim 1/\Gamma$, whereas $\Gamma_s \sim \Gamma$.

The difference in the results is seen from the ratio

$$r = \frac{\Gamma_s}{\Gamma_{\text{pot}}} = 1 + \left(\frac{\Gamma}{2\text{Re}V}\right)^2. \tag{27}$$

If $|\text{Re}V| = \Gamma/2$, then $r = 2$. If $|\text{Re}V| = \Gamma/4$, then $r = 5$. When $|\text{Re}V|$ decreases, Γ_s and r increase.

To conclude: (1) The smaller the $|\text{Re}V|$ (antineutron self-energy), the greater the difference in the results. It is a maximum for the model with a bare propagator. (2) In the strong absorption region $\Gamma_{\text{pot}} \sim 1/\Gamma$, whereas $\Gamma_s \sim \Gamma$. (3) The potential model contains double counting. These conclusions are also true for the model with bare propagator since it is the limiting case $V_1 \to 0$. These conclusions do not depend on the specific models of the blocks M_a and M_{an}.

For the realistic parameters $\Gamma = 100$ MeV and $|\text{Re}V| = 10$ MeV, the lower limit on the free-space $n\bar{n}$ oscillations time is $\tau = 1.2 \times 10^9$ s. When $V_1 = 0$, the model with a dressed propagator converts to the model with a bare propagator. It gives $\tau = 10^{16}$ yr. On the basis of this one can accept that the lower limit on the free-space $n\bar{n}$ oscillations time is in the range 10^{16} yr $> \tau > 1.2 \times 10^9$ s.

Finally, in the strong absorption region the model with an optical potential is inapplicable. In the models I calculate directly the off-diagonal matrix element. The optical potential is not used. (Note that in the case of Hermitian Hamiltonian the optical theorem is applicable.)

V. CONCLUSION

The model based on the optical potential compared with direct calculation of off-diagonal matrix element. The potential model is applicable only in the case of slight absorption.

If absorption is strong, then the potential model is inapplicable: (1) It contains double counting. (2) The Γ dependence of the result is inverse: $\Gamma_{\rm pot} \sim 1/\Gamma$, whereas $\Gamma_s \sim \Gamma$. (3) The physical meaning of Im$U_{\bar{n}}$ is uncertain. (4) The using of the optical theorem or condition of probability conservation contradicts the fact that the S matrix is essentially nonunitary.

The field-theoretical approach is free from drawback mentioned above. Two variants of the models have been considered: the model with bare and dressed propagators. (In the latter case I come to the S-matrix problem formulation.) If the scalar field $V_1 \to 0$ (the antineutron self-energy $\Sigma \to 0$), then the model with a dressed propagator converts to the model with a bare propagator and so the results are valid for the model with bare propagator as well. In both variants the optical potential is not used. The amplitudes of annihilation M_a and $M_{\rm an}$ are defined through Hermitian Hamiltonians.

The chief drawback in the model with a dressed propagator is that the procedure of separation of V_1 (or ReV) is artificial and unjustified. There are a lot of arguments in favor of the model with a bare propagator [10]. The only objection to this model is that it gives the result which essentially differs from the result of the potential model. The potential model has been considered above.

In my opinion the model with a bare propagator is preferable. The model with the dressed propagator has been considered for the study of the process since the problem is of a great importance. It also gives the conservative limit $\tau = 1.2 \times 10^9$ s.

In the oscillation of other particles the difference between Γ_s and $\Gamma_{\rm pot}$ is less; however, this difference can be essential for the problem under study. Specifically, for the K_S^0 regeneration, the model with Hermitian Hamiltonian [13] gives the reinforcement as well.

[1] K. M. Case, Phys. Rev. **103**, 1449 (1956).
[2] M. L. Good, Phys. Rev. **110**, 550 (1958).
[3] T. D. Lee and C. S. Wu, Annu. Rev. Nucl. Sci. **16**, 511 (1966).
[4] E. D. Commins and P. H. Bucksbaum, *Weak Interactions of Leptons and Quarks* (Cambridge University Press, Cambridge, 1983).
[5] K. G. Chetyrkin, M. V. Kazarnovsky, V. A. Kuzmin, and M. E. Shaposhnikov, Phys. Lett. B **99**, 358 (1981).
[6] P. G. H. Sandars, J. Phys. G **6**, L161 (1980).
[7] W. M. Alberico *et al.*, Nucl. Phys. A **523**, 488 (1991).
[8] V. I. Nazaruk, Phys. Rev. C **58**, R1884 (1998).
[9] V. I. Nazaruk, Phys. Lett. B **337**, 328 (1994).
[10] V. I. Nazaruk, Eur. Phys. J. C **53**, 573 (2008).
[11] V. I. Nazaruk, Int. J. Mod. Phys. E **26**, 1750007 (2017).
[12] V. I. Nazaruk, Chinese Physics C **42**, 023108 (2018).
[13] V. I. Nazaruk, arXiv:1906.00447v5.