Colton Conjecture -(n-1)

When evaluating integers from 1 to infinity using the following Collatz Conjecture "3n+1" function recursively until the result equals 1, the sum of recursive iteration result differences always results in a greater decreasing subtotal value than increasing subtotal value, and the total sum equals the evaluated number minus 1 multiplied by -1, or -(n-1).

Collatz Conjecture (3n+1):

$$f(n) = \begin{cases} n/2 & \text{if } n \equiv 0 \pmod{2} \\ 3n+1 & \text{if } n \equiv 1 \pmod{2} \end{cases}$$

$$a_i = \begin{cases} n & \text{for } i = 0 \\ f(a_{i-1}) & \text{for } i > 0 \end{cases}$$

Example: $n = 17$

17 mod 2 > 0
3(17) + 1 = 52
52 mod 2 = 0
(52) / 2 = 26
26 mod 2 = 0
(26) / 2 = 13
13 mod 2 > 0
3(13) + 1 = 40
40 mod 2 = 0
(40) / 2 = 20
20 mod 2 = 0
(20) / 2 = 10
10 mod 2 = 0
(10) / 2 = 5
5 mod 2 > 0
3(5) + 1 = 16
16 mod 2 = 0
(16) / 2 = 8
8 mod 2 = 0
(8) / 2 = 4
4 mod 2 = 0
(4) / 2 = 2
2 mod 2 = 0
(2) / 2 = 1
1 mod 2 > 0
3(1) + 1 = 4, 2, 1...loop

Recursive iteration results for 17: 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1

Difference (increase/decrease) between recursive iteration results:

$$\sum a_i = (a_1 - a_i) + (a_2 - a_1)\ldots(a_n - a_{n-1}) = -1(n-1)$$

$a_i = 17$
$a_1 = 52$
$a_1 - a_i = 35$
...
$a_n - a_{n-1} = -1$

52 - 17 = 35
26 - 52 = -26
13 - 26 = -13
40 - 13 = 27
20 - 40 = -20
10 - 20 = -10
5 - 10 = -5
16 - 5 = 11
8 - 16 = -8
4 - 8 = -4
2 - 4 = -2
1 - 2 = -1

Sum of differences (increases/decreases) between recursive iteration results:

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \ldots + a_n = 35-26-13+27-20-10-5+11-8-4-2-1 = -16$$

Increased values subtotal: 35 + 27 + 11 = 73

Decreased values subtotal: -26 + -13 + -20 + -10 + -5 + -8 + -4 + -2 + -1 = -89

Total sum: -89 + 73 = -16 or -1(17 - 1) = -16

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