# INDUSTRIAL DRAWING AND GEOMETRY

HENRY J. SPOONER, C.E.

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AN INTRODUCTION TO VARIOUS BRANCHES OF TECHNICAL DRAWING

BY

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"NOTES ON, AND DRAWINGS OF, A FOUR-CYLINDER PETROL ENGINE," ETC., ETC.

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# PREFACE

Many years have passed since the Rev. Canon Moseley, the great Educationist, in reporting to the Government on the importance of *Geometrical Drawing as a Branch of National Education,* wrote that "the use of scale and compasses in Drawing is a useful acquirement for workmen in almost every handicraft—to the blacksmith, the carpenter, and the mason, for instance, in making plans and sections of their work; and to the gardener or agricultural labourer, in laying out a plot of ground to a scale, or planning a shed or a cottage, or the drainage of a field. It is, besides, a useful expedient of practical Education, to associate the conception of form with accurate linear dimensions, as is done in Geometrical Drawing. *This kind of drawing might easily be taught to the first classes in Boys' Schools.* The first step in it would probably be to make them copy drawings of the plans and sections of familiar objects; then to practise them in making, to a scale, drawings of such plans and sections from the objects themselves; lastly, the inventive faculty might (even in the case of such youthful scholars) be exercised by requiring them to design, and to draw to a scale, the plans and sections of simple and useful objects, such as are made by the carpenter, the mason, the blacksmith, or the plumber.

"Familiarized with the drawings of such plans and sections, they would hereafter be enabled to work from the latter with greater ease and correctness; and *that habit of reasoning and understanding about what they are working upon, which is the education of the workman, would be encouraged and promoted.*

"Model Drawing, which your Lordships have so much and so beneficially encouraged, is, for the practical man, a subsequent step to the Geometrical Drawing of which I speak; and however important and valuable in itself—if it be estimated by the extent to which it is used or, indeed, useful in the mechanical arts—it must give place to it."

It is quite believable that the above sagacious Report (the italics in which are mine) has had not a little to do with the very general inclusion of Geometrical and Mechanical Drawing in the curricula of our Elementary and Secondary Schools. The educational value of the subject is now well established; for in addition to being a universal language and a means of expression, it is closely allied and associated with pure Geometry and calculation, and is a powerful means of cultivating the inventive faculty and habits of thought and reflection; but these points are now well understood, and need not be further pursued.

Now there are many books published on practical Geometry, and many on various kinds of Technical Drawing, but I do not know of one that comprehensively embraces the two in a form suitable for beginners, so, in writing this little book I have attempted to produce an up-to-date and introductory work for beginners, employing modern expedients in giving effect to Moseley's recommendations; and in doing so have kept in view how necessary it is for the pupil to make a good start by first giving attention to the selection and workmanlike manipulation of the simple instruments and materials used for such work; and for the guidance of those working without a teacher I have included a set of six half-tone prints, made from photographs showing the actual operations, which expedient I believe was first used in my advanced work, "Machine Design and Drawing."

I have explained how fairly hard pencils, sharpened in the right way, should be used to practise drawing different kinds of lines of good quality in various directions on the paper; how circles and arcs can be neatly drawn so as to make proper contact with one another and with straight lines, to ensure neatness and precision in execution. These simple operations should be performed again and again before more difficult work is taken in hand; as slovenly habits of drawing once acquired are extremely difficult to correct. A glance over the page of contents will show that a comprehensive, and in some chapters an unusual but useful selection of matter has been made for treatment in what is well-nigh an inexhaustible subject. Many of the drawings relate to the work of the architect, bricklayer, carpenter, engineer, industrial artist, mason, and metal-plate worker; but apart from these, attention is called in suitable places to the application of geometry to a wide range of industrial work, including engraving, gardening, land surveying, lithography, optical work, printing, stereotyping, etc., etc.

Most teachers, after giving a lecture, like to test the knowledge of their pupils by asking them pertinent questions; bearing this in mind, I have given at the end of most chapters a few suggestive or typical oral questions, followed by sketching and drawing exercises.

The order in which I have arranged the chapters seemed to me to give, on the whole, the best sequence, but of course teachers can vary this very considerably in accordance with their own views.

To make the book as attractive as possible to those who may use it without the help of a teacher, I have, when convenient, and whenever the matter treated seemed to lend itself to it, adopted a conversational style.

I am hoping that the little work will meet the requirements of many who are teaching the subject in Elementary, Secondary, and Trade Schools, and that it will conveniently lead up to works on Machine Construction and Drawing (such as my "Machine Drawing and Design for Beginners"), on Building Construction, and other branches of Technical Drawing.

I may add that the London University Matriculation Syllabus in Geometrical and Mechanical Drawing is nearly covered by the contents of the book.

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# CONTENTS

# DETAILS OF CONTENTS

## CHAPTER I

### DRAWING INSTRUMENTS, MATERIALS, ETC., AND THEIR USE



## CHAPTER II

### HOW TO DRAW STRAIGHT LINES AND SIMPLE FIGURES



## CHAPTER III

### MEASUREMENT AND CONSTRUCTION OF ANGLES, ETC.



## CHAPTER IV

### CONSTRUCTION OF TRIANGLES



## CHAPTER V

### SCALES, THEIR CONSTRUCTION AND USE



## CHAPTER VI

### PROPORTION, ETC.

## CHAPTER VII

### CIRCLES, ARCS, AND LINES



## CHAPTER VIII

### USE OF SQUARED OR SECTIONAL PAPER



## CHAPTER IX

### AREAS AND THEIR MEASUREMENTS



## CHAPTER X

### REDUCING AND ENLARGING FIGURES, ETC.

## CHAPTER XI

### SYMMETRY AND SYMMETRICAL FIGURES



## CHAPTER XII

### THE ELLIPSE



## CHAPTER XIII

### THE PARABOLA



## CHAPTER XIV

### THE HYPERBOLA



## CHAPTER XV

### SPIRALS AND MISCELLANEOUS CURVES



## CHAPTER XVI

### THE APPLICATION OF GEOMETRY TO ORNAMENTAL AND DECORATIVE DESIGN

## CHAPTER XVII

### PLAN AND ELEVATION—HOW TO MAKE A WORKING DRAWING OF A SOLID BODY



## CHAPTER XVIII

### PROJECTIONS AND SECTIONS OF SOME TYPICAL SOLIDS



## CHAPTER XIX

### FURTHER STUDIES IN PROJECTION



## CHAPTER XX

### ISOMETRICAL PROJECTIONS AND DRAWINGS



## CHAPTER XXI

### OBLIQUE OR PICTORIAL DRAWING



## CHAPTER XXII

### SIMPLE FASTENINGS USED IN METAL WORK, AND HOW TO DRAW THEM



## CHAPTER XXIII

### MISCELLANEOUS DRAWING EXERCISES IN WOODWORK, BRICKWORK, AND MASONRY

## CHAPTER XXIV

### VARIOUS MACHINE-DRAWING EXERCISES



## CHAPTER XXV

### INTERSECTIONS AND DEVELOPMENTS OF SIMPLE BODIES



## CHAPTER XXVI

### PRINTING, SHADING, TRACING, ETC.



## CHAPTER XXVII

### MISCELLANEOUS DRAWING EXERCISES



### DEFINITIONS, ETC.



### TABLES OF BRITISH AND METRICAL EQUIVALENTS

# INDUSTRIAL DRAWING AND GEOMETRY

## CHAPTER I

### DRAWING INSTRUMENTS, MATERIALS, ETC., AND THEIR USE

**Introduction.**—A few of the instruments and things described in this chapter you may not elect to use for some time to come; indeed, if you are doing only pencil-work, you will not require them; but even so, you will probably like to know something about these things, and it will not be a waste of time to quietly look through the chapter at your leisure.

1. **Compasses, Dividers, Pens, etc.**—A few shillings will now buy a small set of well-made drawing instruments of the English type, with which much useful work can be done. Usually these sets contain a pair of compasses with pen and pencil points, a pair of dividers, a bow-pencil, a bow-pen, and a drawing or ruling pen. Of course these instruments are not to be compared with the heavier and better made ones turned out by the best English makers, which you will be anxious to provide yourself with later.

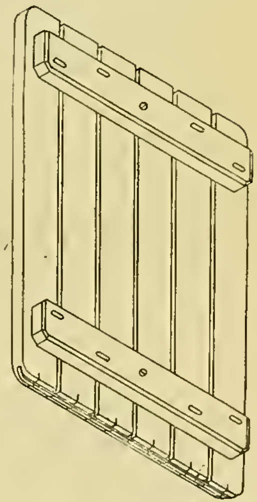


FIG. 1.—Battened drawing board.

2. **Drawing Board.**—This instrument is used for holding and supporting a sheet of paper flat, whilst a drawing is being made upon it. Care should be exercised in its selection, or trouble may be occasioned by its becoming twisted and out of truth, after very little use. There are many kinds of drawing boards, but the "**Battened**" form, shown in Fig. 1, is the best. The size most suitable for exercise work is about 24″ × 17″,[1] which takes the half of an "imperial" sheet of paper, or a "medium" sheet.

3. **Working Position of Drawing Board.**—To enable you to get a good view of your work without leaning over too much, the drawing board when in use should be tilted to an angle of about 15°,[2] either by using it on a sloping desk, or with the aid of *wooden blocks*.

4. **T-Square.**—This instrument is used for drawing long lines perpendicular to an edge of the drawing board; and Fig. 2 shows the "**English shape**," which is best for general purposes. It is made of well-seasoned pearwood, maple, or mahogany. Those of pearwood are the cheapest and answer very well for

[1] The mark or suffix (″) signifies *inch* or inches: thus $\frac{1}{16}$″ reads—one-sixteenth of an inch. A single dash (′) signifies foot or feet: thus 16′ reads—16 feet. The sign × coming between the two dimensions 24 and 17 signifies *multiplication* or the word *by*. Thus the above would read, 24 inches by 17 inches, one dimension being multiplied by the other giving the **area of the board**.

[2] The suffix (°) signifies *degrees*: thus 15° reads—15 degrees. Degrees are divided for measuring purposes into 60 equal parts, called **minutes**, represented by the suffix (′); and minutes into 60 parts called **seconds**, represented by (″). Thus 15°—30′—40″ would read fifteen degrees, thirty minutes, forty seconds.



B

rough use in a school, but the mahogany ones, with the working edges of ebony, are generally used for office work, and should always be used by those who can afford them. An enlarged section of the ruling edge is shown at A on Fig. 2.

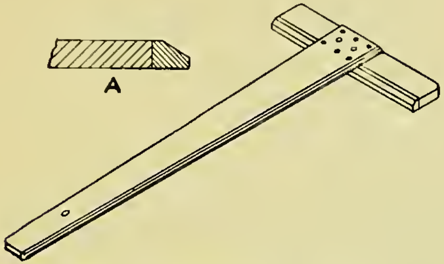



FIG. 2.—English shape T-square.

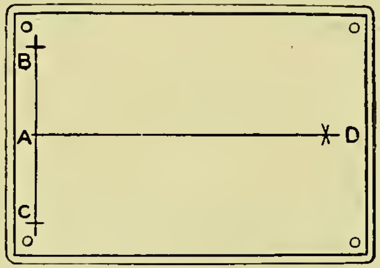



FIG. 3.—Testing T-square.

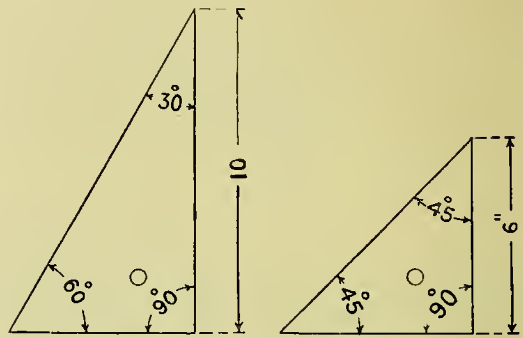



FIG. 4.—The two set-squares.

**5. Set-Squares, or Triangles.**[1]—These are right-angled triangles. They are made of various materials—such as pearwood, mahogany, and other woods, vulcanite, and transparent celluloid; and are used for drawing short lines perpendicular to a straight edge, T-square, or another set-square. They are also used for drawing angles of 30°, 45°, and 60°.

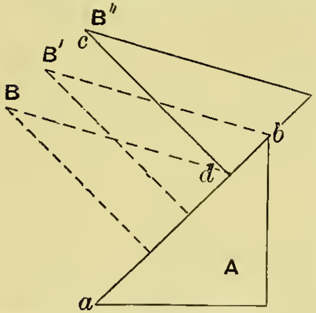



FIG. 5.—Using set-squares for parallel lines.

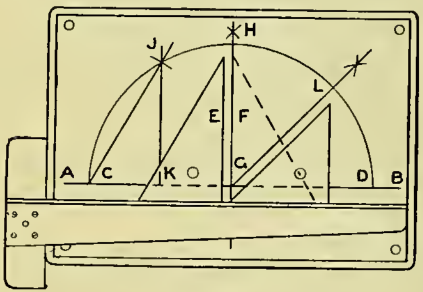



FIG. 6.—Testing set-squares.

Two set-squares are generally used, the angles and most useful sizes for which are shown on Fig. 4.

Set-squares of pearwood are cheap and useful for school use, but they are easily soiled, and often warp and become untrue. They are not to be compared with those made of transparent celluloid, which on the whole should be preferred.

**6. Using Set-Squares for Parallel Lines.**—Parallel lines that cannot be drawn by using the T- and set-squares in the ordinary way may be drawn by sliding the set-squares on one another, as shown in Fig. 5, where the set-square A is held firmly on the paper, and the other B is slid along the edge *ab*. Three positions, B, B′, B″, of the set-square B are shown; the line *cd* being parallel to the corresponding lines drawn through B and B′, and perpendicular to the line *ab*. Thus by manipulating the squares in this way, lines parallel or perpendicular to one another can be drawn on any part of the paper.

[1] The name given to these instruments in America.

7. **Drawing Paper.**—Two kinds of paper are in general use for drawing purposes: viz. "*Cartridge*" *paper* and "*Drawing*" *paper*. "**Cartridge**" (or "*Machine-made*") **drawing paper** is used for ordinary school drawing purposes. It is much cheaper than Whatman's "drawing paper" (which is used by engineers and architects), and can be obtained either in sheets or in rolls up to 62″ wide and 60 yds. long, rendering it extremely useful for diagrams, etc. Cartridge paper has two surfaces, a rough and a smooth one; the smooth surface is the **proper side to draw upon**, and is usually the front side when the water-mark [1] can be read correctly on holding the sheet between the eyes and the light.

Cartridge paper **does not** usually **take tints of colour evenly**, but with good paper, and care, a very fair effect can be obtained in light tints. But this paper is **most suitable for line drawings**.

8. **Whatman's Papers.**—For drawings that are to be finished in ink, without colour, the "Hand-made" drawing paper known as Whatman's "Hot-pressed," H.P., "*Smooth*" *or* "*Rolled*" *surface*, is most suitable. This paper should also be used for drawings when very fine lines are a necessity, and but little colour is required. For drawings which are to be **coloured or shaded**, or are to stand **frequent erasing of lines, Whatman's N.H.P. Paper** (*not hot-pressed*) or *rough surface* is to be preferred; its surface will take a fairly fine line, and tints can be laid very evenly upon it.

9. **Pencils—Different Kinds and Qualities.**—You should, if possible, only use blacklead pencils of a good quality, such as Stanley's, Faber's, or Hardtmuth's *prepared lead*, or Cohen's *Cumberland lead;* inferior makes are very unsatisfactory for drawing purposes. The following are the **requirements of a good pencil** for mechanical drawing: It should be **moderately hard**, of **even colour throughout**, and **durable** enough to retain a working point for a long time. It should not be liable to roll off the board and injure its point, and the **lines** drawn by it should be **easily rubbed out.** The ordinary round cedar-covered blacklead pencil, shown at A, Fig. 7, of good quality, is a serviceable pencil, but it easily rolls off the board. To retard the rolling action, some pencils are made hexagonal (Fig. 7, B), whilst Messrs. Stanley & Co. sell a pencil of specially prepared lead, the wooden cover of which is made elliptical, as shown at C. **Degrees of Hardness, etc.**—Pencils are made in various degrees of hardness, varying from BBBB (the softest) to HHHHHH (the hardest), and Nos. 1 to 6 in the solid lead, and in some makes, such as Stanley's pencils.

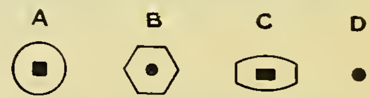



Fig. 7.—Sections of blacklead pencils.

Usually No. 1 = BB. No. 2 = HB. No. 3 = H. No. 4 = HH, or 2H. No. 5 = HHH, or 3H. No. 6 = HHHH, or 4H.

Nos. 3 and 4 will be found most useful for ordinary school work. **The hardest pencils** are only useful when of the very highest quality; they are expensive, and are used for very fine work to a small scale.

10. **How to Sharpen the Pencil.**—For ordinary line drawing, the pencil should be sharpened to a flat or chisel point, as shown in Fig. 8; this gives a strong point, which retains its sharpness longer than a round one, and it can be worked closer up to the squares, and is more easily sharpened; with the added advantage that the lines are more equal in quality. Needless to say, it is used with its flat side laid against the edge of the T- or set-square. To make a flat or chisel point to a wood-covered pencil, the wood is first cut away; and the best way to do this is to hold the pencil, as shown in Fig. 9, between the thumb and first finger of the left hand, and to rest it upon the second finger, which should be turned upwards, while

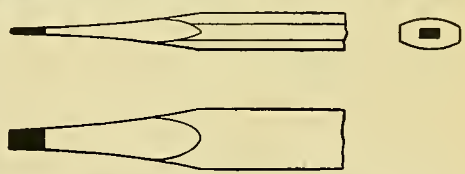


Fig. 8.—Chisel-pointed pencil.

[1] The best qualities only are water-marked.

the penknife (which should be sharp) is held in the four fingers of the right hand, which should be turned downwards, the thumb of this hand being placed under the pencil to steady it, as shown. A little practice will enable you to cut a good point with

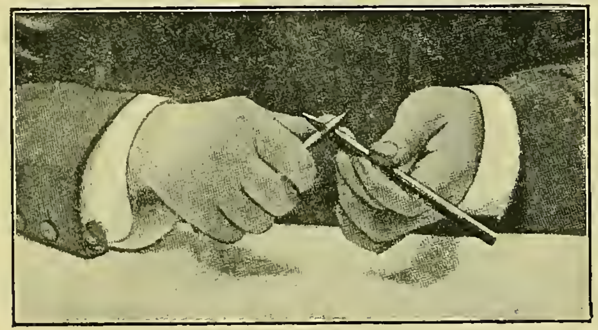


FIG. 9.—Knifing pencil point.

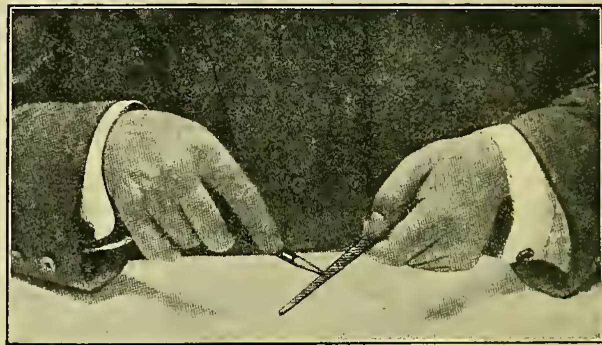


FIG. 10.—Filing pencil point.

precision and facility, as you have perfect control over the knife, which, should it slip, moves away from your hand. The lead part is best sharpened by rubbing it upon a smooth file,[1] as shown in Fig. 10, after which a stroke or two upon a piece of smooth paper gives it a good finish.

**11. Compass Pencils.**—The points of compass pencils should be made narrower than for straight-line purposes, and must be carefully adjusted so as not to draw a thick line; indeed, the beginner is more likely to do better work with a **conical-pointed** lead in his compasses. It is not enough to start with a good point, its **sharpness must be maintained**, and this requires constant attention.

**12. The Conical-pointed Pencil.**—For the making of freehand sketches, dimensioning, or descriptive writing upon a pencil drawing, it is desirable to use a softer pencil than that used for line drawing (such as a No. 2, or 3, or HB or H), and to sharpen it to a long conical point, as shown in Fig. 11. The **point should on no account be moistened** when used, as marks made by it in that condition are very difficult to erase.

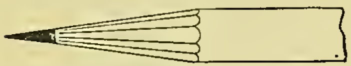


FIG. 11.—Conical-pointed pencil.

FIG. 12.—Drawing pin.

**13. Drawing Pins.**—To secure the paper to the drawing board drawing pins are used. For common school use small stamped pins answer very well; but a better, although more expensive, form is shown in Fig. 12.

[1] A 4″ smooth file, or a 4″ **triangular or three-square saw file**, should be preferred. If a file is not available, a piece of fine emery paper or cloth, "F" or "FF," or glass paper, "O," fastened to a strip of hard wood about 6″ long, 1″ wide, and a ¼″ thick, is a good substitute, or small blocks, containing about 16 surfaces of glass paper, specially made for pencil sharpening, can be obtained.

14. **The Rule,**[1] **and taking Measurements from it.**—A 12-inch steel rule, divided to 64ths and 100ths of an inch, should be preferred. Dimensions can very conveniently be taken off it by the dividers, as in Fig. 13, and pricked off on the drawing. In doing this care should be taken not to place the points of the dividers upon the rule in a *normal* or upright direction, or they will be injured. Fig. 13 shows how, by inclining the dividers to the surface of the rule, the sides of the points may be made to rest in the cuts or divisions without injuring the points or the divisions of the rule, if the latter be made of a soft material. The figure also shows how the dividers and rule should be held if the right hand is to have complete command over the former in adjusting the points to take off any required dimension. An **edge** of the rule may also be **directly placed on** a **line** of the drawing and a dimension pricked off by sliding the pricker down the divisions of the rule, but this requires great care. The **accuracy of the steel rule** and its durability make it superior to any other at the command of the draughtsman. As you may be frequently called upon to set out work with metric measurements, the back of the rule should be divided into centimetres and millimetres. You will also find a **pair of calipers** and a 60″ **measuring tape** useful, as you will see later.

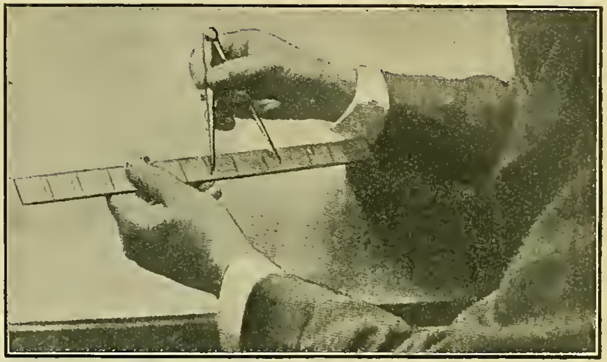


FIG. 13.—Showing application of dividers to rule.

15. **Drawing or Ruling Pens** are used in inking in drawings. The best type of these pens is **jointed**, so that when the screw is taken out one of the nibs can be moved away from the other about its hinge or joint for cleaning purposes.

16. **Indian Ink.**—It is well known that ordinary **writing ink is unsuitable** for use on drawings, as, although it is more or less indelible,[2] it has not the **blackness and body** that are considered necessary, to say nothing of the **corrosive action** of such inks on steel, which alone would preclude its use in the ordinary *drawing pen*. In addition to these objections ordinary writing ink runs too freely from the pen and blurs when touched by a brush in colouring. The only ink that satisfies all the draughtsman's requirements is known as *Indian ink;*[3] this ink, which may be produced by grinding or rubbing down an ink stick, when properly used, produces a clean, dense, jet-black line, and, being free from acid, it does not corrode the instruments; it can also be obtained in a liquid form.

17. **Colours, etc.**, for tinting drawings, may be obtained in cakes, or small pans, and very few suffice to begin with. If cake colours are used, they are ground up with water in a saucer until of the required depth of tint. **Moist water colours** in **pans** are to be preferred for school use. They are to be had in tin trays or cases in sets of five or six. **The most useful colours** and the

[1] **Rule** *v.* **Ruler.** A **rule** is an instrument with straight edges divided into inches and fractions of an inch (or into metric measurements). It is an **instrument used for making linear measurements.** The *regula* (ruler) of the ancient Romans was thus divided. A **ruler or straight-edge** is an instrument with straight edges (usually bevelled) for guiding a pencil, pen, or scriber in drawing straight lines. Thus, although a rule can be used as a ruler, **to call the former a ruler would be a misnomer**, one often used by non-technical writers. **Round desk-rulers** are very convenient for drawing parallel lines for ledger and such like purposes by those accustomed to their use.

[2] Not to be blotted out or effaced.

[3] The quality of Indian ink differs very much, but if good the stick will have a brownish glazed appearance at the end after being used.

materials they are used to represent are given below. The first four will suffice if only ordinary metals are to be indicated; the remaining ones are required when the other materials of construction, etc., are to be shown in colours.

| | | | |
|---|---|---|---|
| Colours for ordinary metals | **Prussian blue** | to represent | **wrought iron (and for dimension lines)** |
| | **Payne's grey** | ,, | **cast iron** |
| | **Crimson lake** | ,, | **centre and datum lines (and for dimension lines on tracings for blue prints)** |
| | **Gamboge, or Indian yellow** | ,, | **brass and gunmetal** |
| | **Prussian blue and crimson lake** | ,, | **steel** |
| Colours for other materials of construction, etc. | **Yellow ochre** | ,, | **stone** |
| | **Burnt sienna** | ,, | **wood** |
| | **Sepia** | ,, | **leather** |
| | **Light red** | ,, | **brickwork** |
| | **Indigo lead** | ,, | **lead** |
| | **Burnt umber** | ,, | **packing** |
| | **French ultramarine** | ,, | **water** |

**18. Saucers for mixing Colours.**—The lid of the tin case of colours that are sold for school use is stamped in the form of saucers for mixing the colours in; but the most useful saucers are the cabinet nests of white china, which are sold in sets of five and a cover.

**19. Brushes.**—For colouring drawings you will require at least two brushes, the most suitable being a "**middle swan**" and a "**small goose.**" These may be of camel hair,[1] but preferably of red or brown sable hair. You will also require a camel-hair

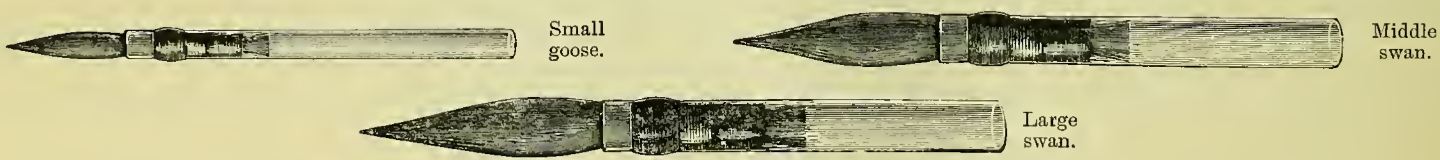



Fig. 14.—Brushes—sizes recommended for school use.

water brush of about "**large swan**" size, for transferring water to the saucers, etc. These three brushes are shown full size in Fig. 14.

[1] A **camel-hair brush** will not keep its point, nor spring back as well as a **sable hair** one does. But the former are much cheaper than sable, and answer well for rough work, but, not being so well made, the hairs often work out and adhere to the tinted surfaces.

# CHAPTER II

## HOW TO DRAW STRAIGHT LINES AND SIMPLE FIGURES

20. **Introduction**.—It is a waste of valuable time for the beginner to attempt to draw even the simplest forms and objects without some previous practice in drawing, in a workmanlike way, different kinds of lines, and a few representative symmetrical figures bounded by straight lines. You should carefully practise drawing the following progressive exercises, and after a few hours' work you should be able to draw simple plane figures neatly and with accuracy. Such operations seem so simple, but you cannot too soon find out that theory will not give precision in execution; it is practice, guided by theory, with never-ceasing efforts to work with accuracy, that has gained for us as a nation supremacy in manufactures, and will gain for you the reputation of being a good draughtsman.

21. EXAMPLE.—**Straight Lines drawn with the Assistance of the T-Square.**—You should patiently practise with your pencil and T-square in the following way:—

Commence by **pinning the paper flat on the drawing board**; this can best be done by first pinning one corner until the under side of the pin-head is in close contact with the paper. Then press upon the paper near this pin, and move your hand diagonally across the sheet to the opposite corner, drawing the paper taut by the friction exerted. Hold this corner down by the thumb and fingers of your other hand, and insert a drawing pin as before described. Smooth the paper by hand from the centre to the other corners and pin them, and the sheet will be as flat as it is possible to have it by using pins only. The T-square can now be placed in position and held firmly by the left hand in such a way as to keep the stock in contact with the edge of the board, and the blade tight on the paper, as shown in Fig. 15. The pencil should be held between the first two fingers and thumb of the right hand, and kept in contact with the edge of the T-square, resting the third and fourth fingers on the square as the stroke is made.

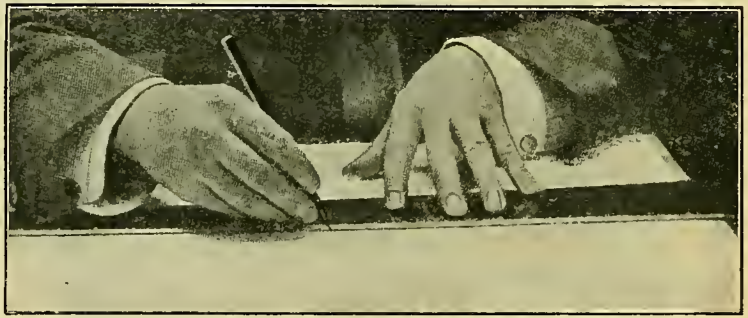


FIG. 15.—Showing how the T-square and pencil should be held.

You should now aim at producing lines equal in thickness throughout their length, and, as the thickness and quality of a line depend upon the sharpness of the pencil and amount of unvarying pressure exerted upon it, you will understand that only practice will enable you to draw them with certainty and facility. Each line should be drawn the full length of the T-square, and several of each kind should be drawn; in fact, they should be drawn again and again till they can be freely produced at least equal in quality to those shown in the following figure (16), where it will be seen that A is a very fine line, suitable for centre and construction lines. This should be drawn with a very sharp chisel-pointed pencil, and should be so fine that a light touch of the indiarubber will clean it out. At B is a line sensibly thicker than the previous one, and suitable for the finished lines of a very small drawing. C is thicker, and suitable for ordinary drawing purposes. D is more suitable for working drawings of simple objects, drawn to a large scale; and E is a suitable line for shade lines on drawings; this line is best drawn with three strokes of the pencil, as the pressure necessary with a point thick enough to produce it with one stroke, would in most cases break the lead. The two outside ones should be sharp and distinct, and the distance between them decided by thickness[1] of the required line. In making the third stroke, the pencil should be turned sideways, so as to fill the space between the outer lines.

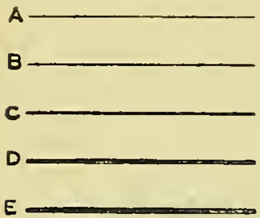



FIG. 16.—Thickness of lines.

The thickness and blackness of a line very much depend upon the pressure exerted on the pencil.

21A. **Defects in Lines.**—The main defects in lines which should be avoided are: **Varying thickness**, caused by varying the amount of pressure exerted upon the pencil. **Want of sharpness**, the sides of the lines having a blurred appearance, caused by softness of lead or want of sharpness in the pencil. **Uneven colour**, due to unequal quality of the lead or paper, or uneven pressure upon the pencil.

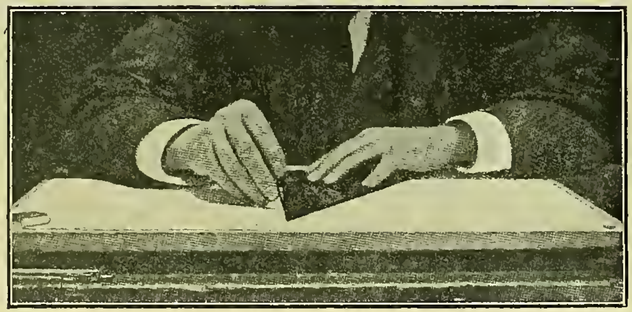


FIG. 17.—Using set-square with downward stroke of pencil as at A, Fig. 18.

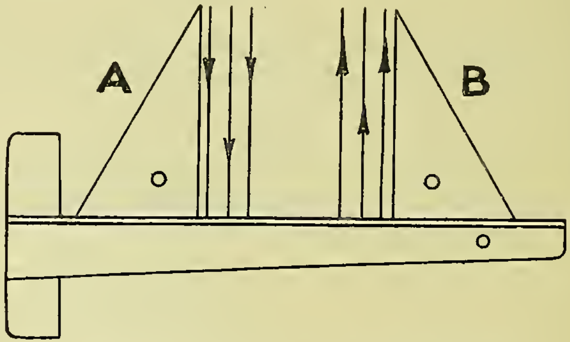



FIG. 18.—Diagram showing use of set-squares.

22. EXAMPLE.—**Straight Lines, drawn with the Assistance of a Set-Square.**—You should remember the instructions given for the

[1] The **ideal line** of the geometrician has length only, without breadth; but all lines drawn by the draughtsman and executed in the arts have breadth as well as length. The term "line" is often used in referring to things whose breadth or diameter is small compared with the length. Thus, ropemakers call **cord**, if small in diameter, **line**. And in **writing** and **printing**, the height of the letters being small compared to the length of the line, the term **line** is applied to the series of words running across the page. Again, in **elevations** or **excavations** of considerable length compared with their height or depth, the term **line** is often applied, as it is to the **trenches** and **earthworks** thrown up by the besiegers or besieged in military operations, also to rails upon which vehicles, etc., run.

previous example, and should now practise drawing similar lines with the assistance of one of your set-squares. The larger one had better be used, and the lines drawn its full length, at first to the right-hand side of the square as shown in Fig. 17 (and at A, Fig. 18), and afterwards to the left as shown in Fig. 19 (and at B Fig. 18) in the direction indicated by the arrows. It will be seen that the left hand in each case is firmly holding the set-square and T-square together and on to the board in such a way that the stock of the T-square is kept closely in contact with the edge of the board. The remarks upon the previous exercise respecting the quality of the lines apply equally to this one, and the necessity of practising the drawing of these lines from both sides of the set-square will be understood after your first attempts, as you will find that to steadily move your hand about with ease, in the required ways, needs considerable practice.

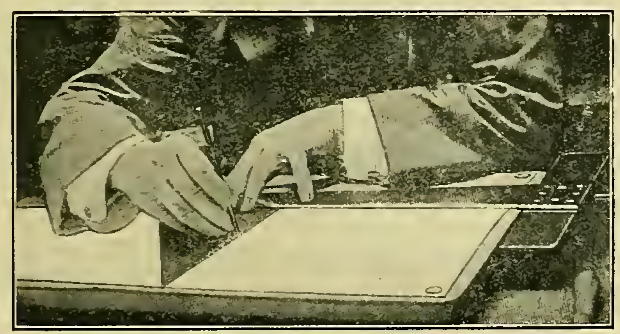


FIG. 19.—Using set-square with upward stroke of pencil as at B, Fig. 18.

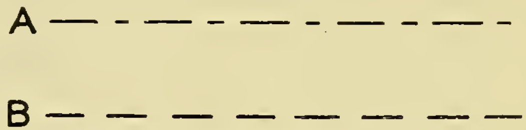



FIG. 20.—Dotted lines: different forms.


FIG. 21.—Examples of dotted lines: different thicknesses.

**23. Dotted Lines.**—Dotted lines are used on drawings either to indicate the line upon which a section has been taken or to mark the position of any existing part which is unseen; for the former, **dot-and-dash** lines, as at A (Fig. 20), are used, whilst for the latter **chain-dotted lines**, B, should be used. In the former case, A, they look best when the dots are equally spaced, and the short lines or dashes are equal in length, and about four or five times the lengths of the spaces; and in the latter case, B, when of equal length and equally spaced, the lines being made three or four times the length of the spaces, as shown. Obviously, if the dashes are made shorter, they take a longer time to draw. The thickness of the lines, and the lengths of the spaces and dashes, should be regulated by the size of the drawing. A glance at some of the following lines, A to E (Fig. 21), will give you some idea of what is considered good proportion, showing how they should vary in form with the thickness; and you should patiently practise drawing such lines until you can space them with a fair amount of neatness and facility.

**24. Rectangles.**—You should now be in a position to draw some simple figures. Having practised on lines drawn in the direction of the T-square, and at right angles to it, figures whose sides are made up of such lines should be easily drawn. So, by carefully working the following progressive exercises, which are very fully described, you should make an important step in the practice of mechanical drawing.

**25.** Example.—**To draw a Rectangle whose Length (2″) and Breadth ($1\frac{1}{2}''$) are given.**—Draw, with the aid of the **T**-square, a very fine indefinite line, AB, about $2\frac{1}{2}''$ long (Fig. 22). With the aid of a rule and a pair of dividers prick off (Art. 14) the length CD equal to 2″, and between these two points draw a good finished line as shown. Then, with the aid of a set-square, draw from C and D very fine distinct lines perpendicular to C and D, a little longer than the given breath ($1\frac{1}{2}''$).[1] Now, prick off as before the point E (Fig. 23) from C, making CE equal to $1\frac{1}{2}''$, the given breadth, and with the aid of the **T**-square, draw the finished line EF parallel[2] to CD.

The rectangle is completed by re-drawing CE and DF (Fig. 24) with the aid of the set-square (being careful to regulate the thickness of the lines, so that they are the same throughout the figure), and removing with indiarubber the ends of the construction lines AC and DB, and those above E and F, leaving the rectangle completed as shown, care being taken not to remove the sharp corners formed by the intersection of the lines.

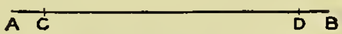


Fig. 22.—Construction of a rectangle. First step.

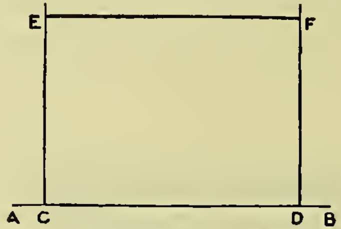


Fig. 23.—Construction of a rectangle. Second step.

Fig. 24.—Construction of a rectangle. The complete figure.

Note.—You should always aim at constructing a figure by drawing the least number of lines possible; in other words, **a line should not be gone over twice, if once will suffice.** As an illustration of this advice, with reference to the rectangle just drawn, many students would first have drawn the complete figure in fine lines, and then pencilled over each line to make it of the required thickness. Such a practice usually produces a poor result, as it is **difficult to exactly cover the the previous lines, and, further, it takes a longer time.**

**26. Exercises upon the Use of Centre Lines.**[3]—*First Case.* **Figure Symmetrical about a Single Centre Line.**—Whenever a figure has more than one line each side of its centre, and is symmetrical about that centre, it is best drawn by commencing with the centre line. To illustrate this, let us proceed to draw the figure shown complete in the dimensioned drawing (Fig. 25).

[1] The student, after a little practice, will be able to estimate these distances and lengths to within a quarter of an inch, so that such lines need not be drawn much longer than their required length **to minimize rubbing out,** but in no case should they be drawn too short at first, as any attempt at joining a length on is usually noticeable, and should be avoided.

[2] **Parallel lines, at equal distances,** are of frequent occurrence in the arts. When an **engraver** wishes to give us an idea of level and uniform surfaces, he represents the parts of them which are more or less in the shade by stronger or weaker lines, which are always parallel and at equal distances from one another. The **ploughman** forms his furrows in parallel lines; and in **music** parallel lines at equal distances are used, the five lines are sometimes drawn at the same time, by means of a ruling pen with five points, at equal distances from one another. In a similar way the lines on drawings representing the **wheel-tracks,** or two rails, of **rail-roads** or **tramways,** are drawn. It has been remarked that the celebrated **Cashmere shawls** made by the Indians, which are remarkable for their fineness and beauty, cannot be compared to those made in Europe for **uniformity of texture,** as the Indians have not the same accurate instruments for preserving the **parallelism and equal distances of the threads** that the Europeans have. Thus the latter, by an approach to the precision of ideal geometry in the parallelism of straight lines, have obtained a superiority in an art practised for centuries and carried to great perfection in India.

[3] **Centre lines** used in setting out work should be very fine continuous ones, undotted, as at A, Fig. 16; then any part of them can be used to measure to or from.

Commence by drawing a very fine line AB (Fig. 26), with the aid of the T-square; then with dividers prick off upon it two points C and D, 2″ apart. Through these points, with the aid of a set-square, draw two fine indefinite lines EG and FH. Then, with the dividers, prick off on one of these lines, say from C, the points J and K (Fig. 27), the opening of the dividers being $\frac{5}{8}''$, equal to a half of the breadth ($1\frac{1}{4}''$) of the given figure, and with the aid of the T-square draw through these points the finished full lines KM and JL. In a similar way mark off N and P from C, with the dividers open to $\frac{3}{8}''$, and through these points draw, in a similar way, the lines NO and PQ.

The figure should now be completed by going over the lines KJ and LM with the pencil, taking care to give the lines the same thickness and finish as the others, and the figure will be now complete as in Fig. 25.

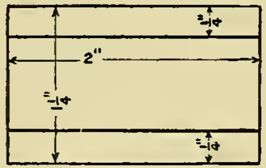



Fig. 25.—Rectangular figure, symmetrical about a centre line. The complete figure.

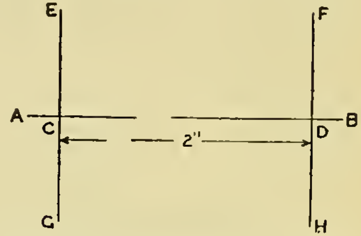



Fig. 26.—Rectangular figure. First step.

The projecting parts of the construction lines, should now be rubbed out with indiarubber, as in the previous exercise, the centre line AB being left projecting about $\frac{1}{4}''$ beyond the figure on each side.

Note.—The appearance and finish of the figure depend upon the lines being perfectly uniform in thickness and colour, and the student should constantly bear in mind the instructions previously given respecting the production of such lines.

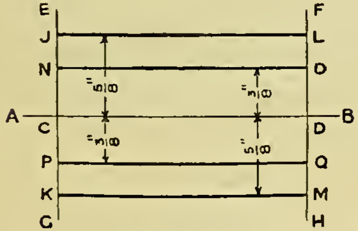



Fig. 27.—Rectangular figure. Second step.

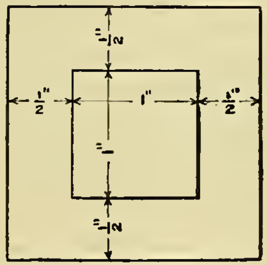



Fig. 28.—Square figure. Use of two centre lines. The complete figure.

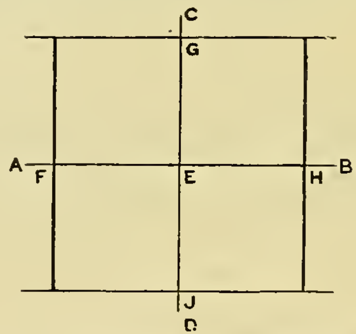



Fig. 29.—Square figure. Construction lines.

27. *Second Case.* **Figure Symmetrical about Two Centre Lines.**—The complete figure, No. 28, consists of two concentric squares which are symmetrical about two centre lines, at right angles to each other. So, first draw any two indefinite centre lines AB and

CD, perpendicular to one another (Fig. 29), and intersecting at E; then, with rule and dividers, prick off from E, along the centre lines distances EF, EG, EH, and EJ, equal to half the side of the outer square, viz. 1", and complete the square as in the previous case. The inner square should be drawn in the same way, the construction lines removed, and the required figure completed as shown in Fig. 28.

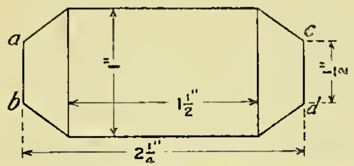



Fig. 30.—Complete figure symmetrical about two centre lines.

Fig. 31.—First step.

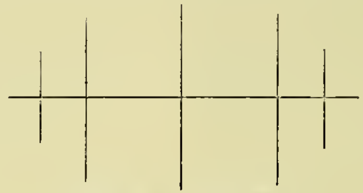


Fig. 32.—Second step.

Fig. 33.—Third step.

**28. Another Case of a Figure Symmetrical about Two Centre Lines.**—The figure to be drawn in this exercise consists of a rectangle, with a trapezoid at each end (Fig. 30). It will not be necessary to explain every step in the construction of this figure, as you should by this time be familiar with the method of working from centre lines, and might now attempt to draw the figure in what appears to you the best way, with a hint that the small ends *ab* and *cd* of the trapezoids should be drawn before the sloping sides. The Figures 31, 32, and 33 show the steps in the construction. These should speak for themselves now. Of course Fig. 30 shows the complete figure. But you should not trouble about writing dimensions on your drawings yet.

## DRAWING EXERCISES.

All the following exercises should be drawn full size, care being taken to make the lines sharp and distinct, and the dimensions as accurate as possible; these should be checked, where two or more occur in the same line, by scaling off the overall dimensions. Your teacher in awarding marks will take these points into consideration.

1. Draw a straight line, using your T-square, and mark off from one end successive lengths of $2\frac{1}{8}$", $3\frac{1}{16}$", and $2\frac{13}{16}$". Then measure the line formed by these three parts, compare this length with the calculated sum of the three quantities, and write down any error you may detect due to the faulty manipulation of your instruments.

2. Draw 3 lines, A, B, C, any length, write on them their apparent or estimated lengths, then measure them with your rule, and tabulate the results as in table below. Note.—By this kind of practice you will educate your eye for measurements.

| | Estimated L. | Measured L. | Error. |
|---|---|---|---|
| A | | | |
| B | | | |
| C | | | |

3. Draw Fig. 34, commencing by drawing the bottom or base line.

4. Two keel blocks are represented by Fig. 35: commence by drawing the base line and vertical centre line.

5. Fig. 36 shows the end view of a block of steps, whose risers and treads are represented on the drawing by 3 cm. (3 centimetres). Commence by drawing the base and back lines.

6. Commence Fig. 37 by drawing the horizontal centre line.

7. The dovetail key, Fig. 38, is symmetrical about two centre lines: draw these first.

8. Commence the concentric squares by drawing the two centre lines about which they are concentric.

9. The Maltese cross, Fig. 40, is symmetrical about two centre lines: draw these first.

10. The panelled door represented by Fig. 41 is symmetrical about a vertical centre line, which may be drawn first.

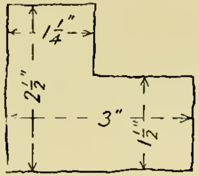



FIG. 34.

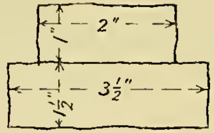



FIG. 35.—Keel blocks.

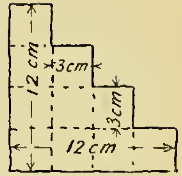



FIG. 36.—Block of steps.

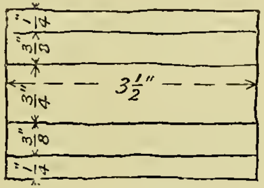


FIG. 37.

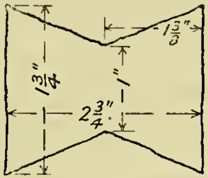


FIG. 38.—Dovetail key.

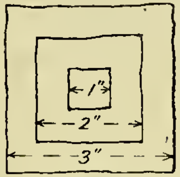



FIG. 39.—Concentric squares.

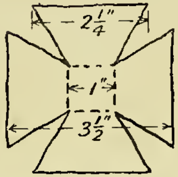



FIG. 40.—Maltese cross.

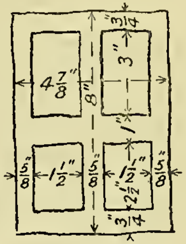



FIG. 41.—Panelled door.

# CHAPTER III

## MEASUREMENT AND CONSTRUCTION OF ANGLES, ETC.

**29. Introduction.**—Before working any problems relating to angles you should have clear ideas as to how angles are measured. An angle is selected as the **unit**, and the measure of any other angle is the number of units which it contains. Any angle might be taken for the unit—as, for example, a *right angle*—but it is obvious that a smaller angle than a right angle would be more convenient. Accordingly a right angle is divided into 90 equal parts called degrees, the circle containing 360;[1] and any angle may be estimated by ascertaining the number of **degrees (or standard angles)** contained by it. Thus, if CD (Fig. 42) be at right angles to AB, the angle BCD will contain 90°, and the arc AD will be a quarter of a circle; that is, CD will be inclined to AB 90°. Now, the angle ACD is equal to the angle BCD; therefore the angle ACD equals

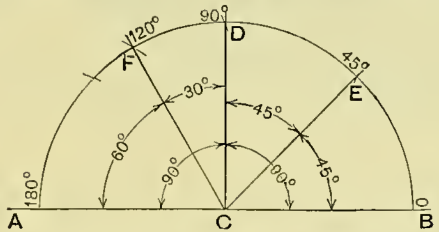



Fig. 42.—Construction of angles.

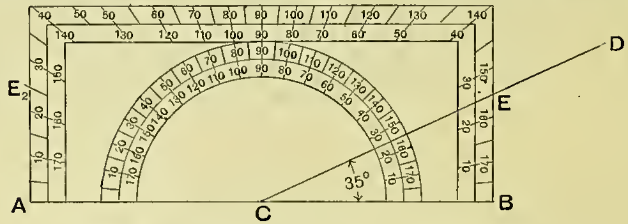



Fig. 43.—Setting out an angle with the protractor.

90°, and the angle between the arms of the straight line ACB will be 180°. Obviously, if we bisect the angle BCD in E, we get CE inclined 45° to CB.

Other angles can be constructed with equal facility. Thus, if we take the angle ACE we get 90° + 45°, or an angle of 135°; and by taking A as centre, AC as radius, and describing arc ACF, we divide the angle ACD in such a way that the angle ACF equals two-thirds of the angle ACD, that is, equals 60°; and of course the remaining angle DCF must equal 30°.

An angle of 15° is obtained by bisecting 30°, and 75° by adding 15° to 60°.

**30. To draw, with the assistance of a Protractor, a Line making a given Angle (say 25°) with a given Line, and passing through a fixed Point in that Line.**—Let AB (Fig. 43) be the given line, and C the fixed point in it. Then place the bottom edge of the protractor[2]

[1] Thales, 640 B.C., first applied the circle to the measurement of angles. We still adhere to the practice of the ancient Egyptian astronomers, which was to divide the circle into 360 parts called degrees. Each of these degrees was divided into 60 parts called minutes; these, again, into 60 parts called seconds, as we have seen. (We divide these seconds decimally, not into 60 parts called thirds, as the ancients did.)

[2] These instruments are made in two forms as shown on the figure. The rectangular one being generally made of boxwood or ivory, whilst the semicircular one is invariably made either in metal, horn, or celluloid.

on AB, and move the instrument along it until the star or centre point, from which all the lines or degrees radiate is directly over the point C. Then mark the point, E, from the edge of the instrument, which corresponds to the reading 25° (of course there is such a point the other side of the protactor at $E_2$), and through C and E draw the line CD, which will make an angle of 25° with AB as required.

**31. Use of the Set-Squares in Setting out Angles.**—In ordinary mechanical drawing, geometrical constructions and the protractor are never used for setting out angles if an easy manipulation of the set-squares will give them, and Figs. 44 to 49, which speak for themselves, show how such useful angles as 15°, 30°, 45°, 60°, 75°, 105°, 120°, 135°, and 150° can be drawn.

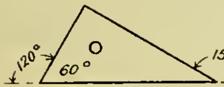



FIG. 44.

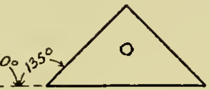



FIG. 45.

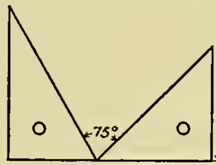



FIG. 46.

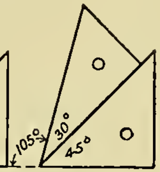



FIG. 47.

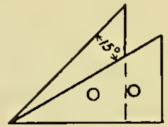



FIG. 48.

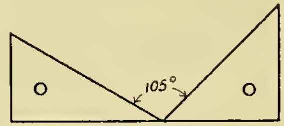



FIG. 49.

**32. Use of the Set-Squares in Trisecting and Bisecting Angles.**—By using the 60° set-square, as shown in Fig. 50, a right angle, ABC, can be readily trisected. Whilst to bisect an angle, BAC, Fig. 51, with any suitable radius, and A as centre, cut the lines in B and C. Then apply either of the set-squares, as shown, and draw the lines CD and BD, intersecting in D. Join D to A, and this line bisects the angle.

**33. Copying Angles.**—In copying an angle we may use either (*a*) a bevel, (*b*) an adjustable protractor, or (*c*) geometrical means.

(*a*) A **bevel** is an instrument used by engineers, carpenters, etc., for drawing and transferring all kinds of angles. It is shown in Fig. 52, applied to an angle, BAC, and it consists of two parts or rulers, ED and EF, attached to and hinging on the same pin or pivot, P, in such a manner that angles

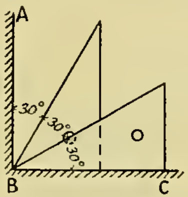



FIG. 50.—Trisecting a right angle.

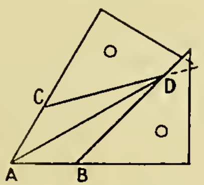



FIG. 51.—Bisecting an angle.

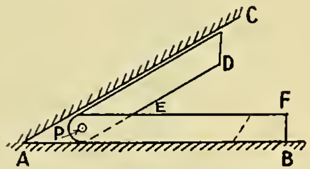



FIG. 52.—Bevel applied to angle.

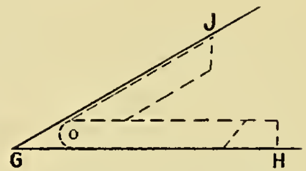



FIG. 53.—Copy of an angle by using a bevel.

of every size may be formed or copied by them; the pin joint is a fairly stiff one, so that some little exertion is required to open or close the instrument for different angles; then, once it has been set, as in Fig. 52, to the angle BAC, it can be used on a line GH, Fig. 53, and GJ can be drawn, making the angle HGJ the same as the angle BAC. Or it may be used to ascertain whether the two angles are of the same magnitude either on a drawing or on a piece of work.

(*b*) **Angles may be copied by using** some form of **adjustable protractor**, such as the useful little instrument invented by Prof. Low, and shown in Fig. 54. It consists of two parts, A and B, the former tongued on its circular edge, DEF, to snugly fit a corresponding groove in B (as shown in the detail sketch at C, which is a section at F). As will be seen, the circular edge of the part A is divided, so that the angle that MN makes with GH can be read off at the division E. Thus this handy instrument combines the functions of a bevel and protractor for drawing purposes, and may in many cases be used as explained in connection with (*a*).

(*c*) **Angles may also be copied by geometrical means.** A simple, very old expedient is to proceed as follows in copying the angle BAC (Fig. 55). With any convenient radius, and centre A, describe an arc, DE, as shown. Then draw a straight line GH (Fig. 56), and with centre G and the same radius draw an arc KF. Next open the compasses to measure DE with the points, and with centre K draw an arc to cut FK in F. Then, obviously, KF equals ED. In fact, if the two chords of the arcs were drawn, AED and GKF would be two equal and similar isosceles triangles, with their angles at A and G equal, of course.

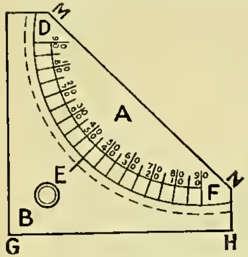



FIG. 54.—Low's adjustable protractor.

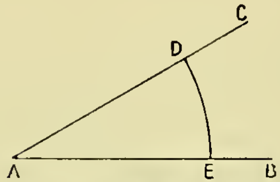



FIG. 55.—A given angle BAC.

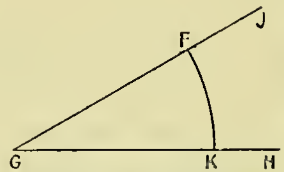



FIG. 56.—Copy of the angle BAC in Fig. 55.

**34. The Fitter's Square.**—The fitter's, or carpenter's square,[1] Fig. 57, is the instrument which is extensively used in the industrial arts in working on and arranging edges and planes which are to be at right angles (or square) to one another. The two working positions of the square are shown in the figure (57) at A and B.

**35. The Plumb[2] Rule,** Fig. 58, is an instrument largely used in a number of trades, particularly in those relating to the construction of buildings, for testing the uprightness or verticality of walls, etc. A lead or brass bob is suspended from the upper part of a wood or steel straight-edge, as shown

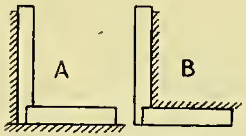



FIG. 57.—The two ways of using a fitter's square.

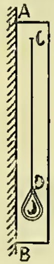



FIG. 58.—Plumb line.

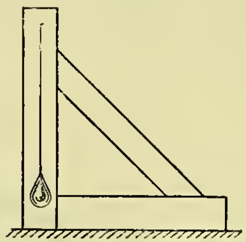


FIG. 59.—Plumb line for horizontal work.

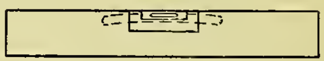


FIG. 60.—Spirit level.

(with a centre line CD marked on it parallel to AB), by a piece of string, and its edge AB is offered to the wall; if this is vertical the string or plumb line and CD will coincide, as the former always hangs in a vertical position when at rest.

Fig. 59 shows a plumb line fitted to a large wood square, which allows of its being used for horizontal or level work.

**36. The Spirit Level** is another important instrument used for testing whether a plane is horizontal. It often consists of a block of hard wood of square section with a slightly curved glass tube, full of alcohol or some **limpid spirit** (except a small bubble of air), fitted to its upper part, as shown

[1] Those used by engineers are wholly made of steel, whilst usually the carpenter's square has a steel blade fitted to a wood back or stock.
[2] The name is derived from the Latin word *plumbum*, lead.

in Fig. 60. The tube is carefully embedded in plaster of Paris in such a way that when the bottom of the block is resting on a horizontal plane, the bubble of air settles in the middle of the tube. Should the body upon which the instrument rests be out of the horizontal, the bubble moves along the tube towards the higher end, and the more the tube is curved the shorter the distance the bubble moves, in other words, the less sensitive it is.

## EXERCISES.

1. Set out the following angles, using your set-squares only: 75°, 105°, 120°, and 150°.
2. Construct a triangle with sides 4″, 5″, and 6″, and measure with your protractor its three angles. You, of course, will remember that in all triangles the sum of the angles equals 180°. So add the three angles together, and check your work. Repeat this by drawing other triangles of any shape; you will soon find that great care must be taken in using the protractor to ensure accuracy.
3. Set out a triangle with its sides 3″, 4″, and 5″, and measure its angles. If you are acquainted with the 47th problem in the first book of Euclid, you will be able to satisfy yourself that one of the angles must be a right angle.
4. Set out an angle of 30° with one of your set-squares, and then copy in the way explained in Art. 33 (*c*), checking the copy by applying your set-squares to it.
5. Set out at random any three angles, A, B, C, and write on each your estimate of its magnitude in degrees. Then measure them, and tabulate thus:—

| Angle. | Estimated degrees. | Measured degrees. | Error. |
|---|---|---|---|
| A | | | |
| B | | | |
| C | | | |

Note.—Occasionally practise this and you will cultivate an eye for angles. In recent years teachers have found this type of experimental question of great educational value.

# CHAPTER IV

## CONSTRUCTION OF TRIANGLES

**37. Definitions, etc.** **Also refer to the definitions, etc., at the end of the book** (p. 160).

A triangle is a closed figure having three sides and three angles. The sum of any two of its sides must be greater than the third (Euc. I. 20).

**The sum of the three angles of any triangle** is 180° (two right angles) (Euc. I. 32).

**The perimeter** of a triangle is the sum of its three sides.

**Similar triangles** are those having equal angles, not necessarily equal sides.

**Apex or Vertex.**—The angular point opposite the base of a triangle is called the *apex* or *vertex*.

**Median.**—A line drawn from a vertex to the middle point of the opposite side is called a *median*.

The greater side of every triangle has the greater angle opposite it (Euc. I. 18).

If a line which bisects the vertical angle A of any triangle ABC cut the base BC in D, the ratio of BD to DC is the same as the ratio of BA to AC (Euc. VI. 3).

**Congruent Triangles.**—If the three sides of any triangle be bisected, and the points of bisection be joined, the four triangles formed will be similar and equal in all respects; they are therefore called congruent triangles.

We may now work a few representative problems on triangles.

**38. To construct a Triangle with Sides any Given Lengths, say, 2·5″, 2·2″, and 1·75″.**—First draw one of the sides, AB 2·5″ (Fig. 61), as base.[1] Then with A as centre and radius of 2·2″, describe an arc. With B as centre and radius of 1·75″, describe another arc, cutting the former one in C. Draw AC and BC, completing the required triangle.

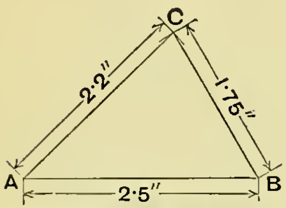



Fig. 61.—Scalene triangle.

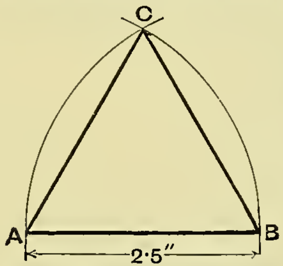



Fig. 62.—Equilateral triangle.

**39. To construct an Equilateral Triangle with, say, 2·5″ Sides.**—The construction is similar to that in the previous problem. Draw AB 2·5″ long (Fig. 62). With A as centre and AB as radius, describe an arc. With B as centre and with same radius, describe another arc cutting the former one in C. Complete the triangle by joining AC and BC (Euc. I. 1).

Note.—An equilateral triangle has three equal sides, and therefore three equal angles. Then, as the three angles of any triangle together equal 180°, each angle will equal one-third of 180°, equal 60°. So you may **use your 60° set-square** to draw the two sides.

**40. To construct a Triangle upon a given Base (say $2\frac{1}{2}$″), the Angles being in a given Proportion (say 1 : 2 : 3).**—Draw the base AB $2\frac{1}{2}$″ long (Fig. 63), and with A as centre (any radius) describe a semicircle. Divide the semicircle with the dividers into six (1 + 2 + 3) equal parts, and number them as shown in the figure. Join 2 and 5 to the centre A, and from B draw a line parallel to A5 till it intersects A2 in C. ABC is the required triangle.

[1] Note.—The line on which the triangle stands is usually called the **base**, but for geometrical purposes any side may be considered as such.

NOTE.—The number of parts into which the semicircle is divided is always the sum of the terms of the proportion. You will see that the angle CAB = 2A0, CBA = 5A6, and therefore ACB must equal 5A2, as the sum of the two angles 5A6 and CA0 in the semicircle are equal to the sum of the two angles CAB and ABC in the triangle, and the sum of the three angles in each figure is equal to 180°.

**41. To construct a Triangle when the Length of a Side (say 2″), an Angle at that Side (say 65°), and the Perimeter[1] (say $4\frac{1}{2}''$) are given.**—Draw the side AB 2″ long (Fig. 64). At either end A set off AC at the given angle 65° and $2\frac{1}{2}''$ long ($4\frac{1}{2} - 2$); as the sum of the base and AC must equal the perimeter. Connect BC, and bisect it at D by a perpendicular cutting AC in E. Join BE, and ABE is the required triangle.

NOTE.—It should be noticed that EB equals in length EC; therefore EB added to AE equals AC (Euc. I. 4).

**41A. To construct a Triangle when the Base (say $1\frac{1}{2}''$), Vertical Angle (say 35°), and Altitude (say $1\frac{1}{4}''$) are given.**

The working of this problem depends upon the fact that all angles in the same segment of a circle are equal.[2]

Thus in the semicircle Fig. 65 the angles ABE and ADE are equal, and are right angles, as are all angles in a semicircle (Euc. III. 31).

Again, in the segment Fig. 66 the angles FGI and FHI are equal, and, being in a segment greater than a semicircle, they are less than right angles (Euc. III. 31).

The segment Fig. 67 is less than a semicircle, and the equal angles KLN and KMN are therefore greater than right angles (Euc. III. 31).

To proceed with the problem. You will understand from the preceding remarks that the first thing to do is to draw a segment with a chord $1\frac{1}{2}''$ long, such that an angle at the circumference will equal 35°. This may be done thus. Draw AB $1\frac{1}{2}''$ long (Fig. 68), and at its middle point F erect a perpendicular. At either end of AB draw a line inclined[3] 90° − 35° = 55, intersecting the perpendicular at E. With radius EA and centre E, draw the arc ADCB. Draw DC parallel to AB and $1\frac{1}{4}''$ (the altitude) from it. Then join CA and CB, and ACB is the required triangle (Euc. III. 33).

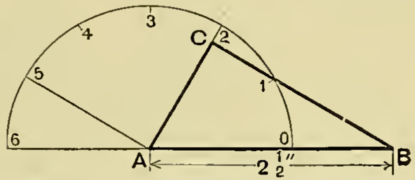



FIG. 63.—Triangle, with angles in given proportion.

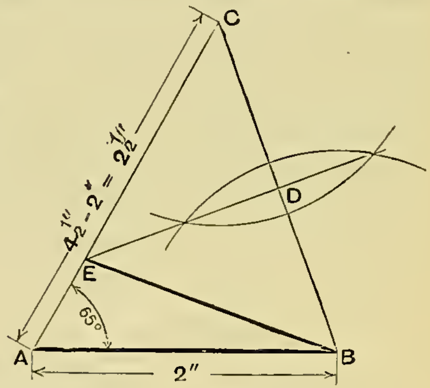



FIG. 64.

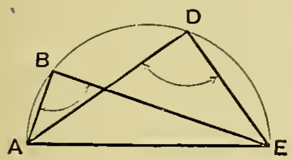



FIG. 65.

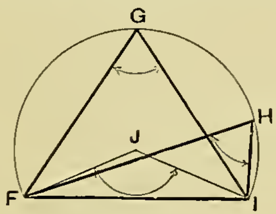



FIG. 66.

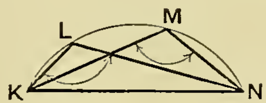



FIG. 67.

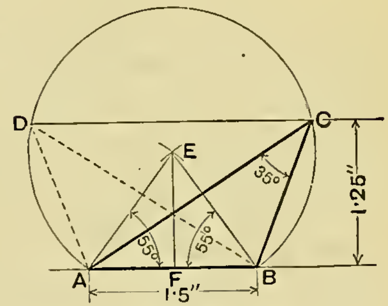



FIG. 68.

[1] Sum of the three sides.

[2] The angle at the centre is always twice the angle at the circumference; thus FJI (Fig. 66) is twice FGI (Euc. III. 20).

[3] This angle is always the complement of the vertical angle (Euc. III. 20).

NOTES.—1. The dotted triangle ADB is the same shape and size as ACB, so that two triangles can be drawn to satisfy the problem. Therefore this is called an **ambiguous case.**

2. Of course the data could have been varied by giving an angle at the base instead of the altitude.

3. You will remember that the **altitude** of a triangle is the perpendicular distance of vertex (corner opposite base) from base or base produced.

**42. To construct a Right-angled Triangle, having given the Hypotenuse (say $2\frac{1}{2}''$) and One of the Acute Angles (say 25°).**—This problem is best worked by putting the triangle in a semicircle whose diameter is equal to the hypotenuse. Proceed thus. Draw AB $2\frac{1}{2}''$ long (Fig. 69), and describe a semicircle upon it. At either end B draw a line, making an angle of 25° with it. This line cuts the semicircle in the point C. Join CA, and the triangle ABC is the required one, for as the angle ABC is in the semicircle, it is a right angle (Euc. III. 31). Refer to Fig. 65.

NOTE.—If the **hypotenuse and a side** had been **given,** a similar construction would obviously work the problem.

**43. To construct a Triangle whose Perimeter**

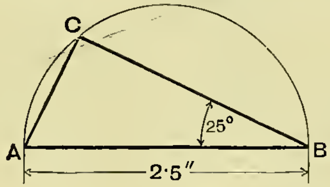


FIG. 69.

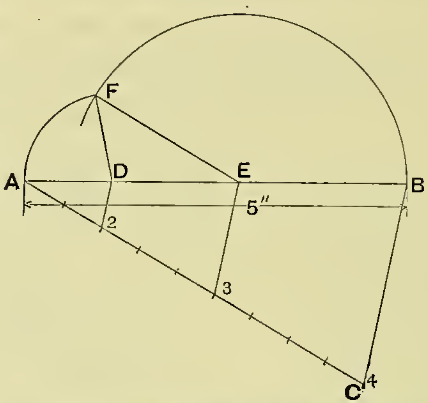


FIG. 70.—Triangle, sides in given proportion.

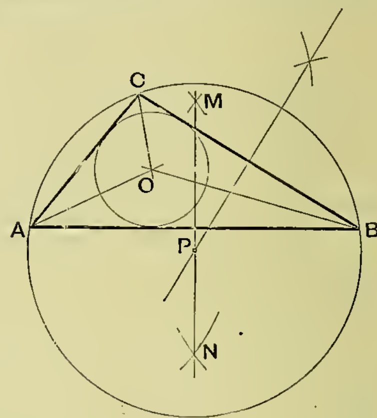


FIG. 71.

**shall be a given Length (say 5″) and Sides in a given Proportion (say 2 : 3 : 4).**—Draw the line AB 5″ long (Fig. 70). At A draw AC, making any angle with AB, and, with a distance between the legs of the compasses equal to about $\left(\frac{1}{2+3+4}\text{ of AB}\right)$, or about one-ninth of AB, as near as can be judged by the eye, set off nine of these distances along AC. Join the ninth (at C) to B, and draw through the second from A a line 2D, parallel to BC, and through the fifth from A a line 3E also parallel to BC. Then AD, DE, and EB are the three sides of the required triangle, which may be constructed by drawing arcs from centres D and E, and radii DA and EB, respectively, intersecting in F. Join FD and FE, and the triangle is complete.

**44. To draw the Inscribed and Circumscribing Circles of any given Triangle.**—Let ABC (Fig. 71) be the given triangle. **To inscribe** the triangle with a circle, bisect any two of its angles (see Problem 32), such as ACB and CBA, by CO and BO, intersecting in O. Then with O as centre, and radius equal to the perpendicular distance of O from either side, describe the circle. The **centre of the circumscribing circle** is found by perpendicularly bisecting two sides of the triangle with the compasses, as shown; thus MN bisects AB; drawing a similar line across CB, they intersect at P, which is equidistant from the corners A, B, and C, and is the centre of the required circle.

## EXERCISES.

### Oral Questions.

1. What is a triangle?
2. What is a scalene triangle?
3. What is the sum of the angles of any triangle in degrees?
4. Take your 60° set-square and measure its sides. What do you find? What is the ratio of the longest to the shortest side? What is the name of the longest side?
5. Two angles of a triangle are found to measure 80° and 40°. What must the third angle measure, and why?
6. The angle at the apex of an isosceles triangle is found to be 49°. What are the angles at the base?
7. What is an obtuse-angled triangle?
8. Why are certain triangles called equilateral?
9. When are two triangles said to be congruent?
10. Your 45° set-square is the same shape as another boy's, although yours is smaller. Are the two figures they represent congruent? if not, why?

### Drawing Exercises.

11. Construct a triangle with sides 2·75″, 2″, and $1\frac{1}{2}$″.
12. Construct an equilateral triangle with $5\frac{3}{4}$″ perimeter.
13. Construct an isosceles triangle, base 2″, and an angle at the base of 40°.
14. Construct an isosceles triangle, 2″ base, and vertical angle equal 40°.
15. Draw a triangle whose perimeter equals $6\frac{1}{2}$″, the angles to be in the proportion 5 : 6 : 7.
16. Construct a triangle with a perimeter of 7″, the base being $2\frac{1}{2}$″, and an angle at the base 45°.
17. Construct a triangle, perimeter 6″, base 2″. The length of the other two sides to be in the proportion of 2 : 1.
18. Construct a triangle, its altitude $2\frac{1}{4}$″, an angle at the base 60°, and its perimeter 7·5″.
19. Construct a triangle, altitude 1·75″, vertical angle 35°, base 1·5″, and draw the circumscribing and inscribed circles.
20. Draw a right-angled triangle, the vertical angle 35° and hypotenuse 3″.
21. The perimeter of a triangle whose sides are in the proportion of 9 : 7 : 4 is 7·5″. Draw the triangle.
22. Construct a triangle with a base of 2″, one angle at the base of 40°, and a perimeter of 6″.
23. Construct a triangle whose sides are 10′ 6″, 10′ 0″, 16′ 3″. Scale $\frac{1}{8}$″ = 1′ 0″.
24. Construct a triangle with a base $2\frac{1}{2}$″, one angle at the base 50°, and the angle opposite the base 55°.
25. Construct a triangle ABC, making the base AB = $2\frac{1}{2}$″, the side BC = 2″, and the angle BAC = 45°. Note.—You will find that two triangles can be drawn to satisfy the conditions of this problem. This is called the **ambiguous** case.
26. Draw five different-shaped triangles. Carefully measure the three angles ABC of each one, and tabulate these measurements as follows:—

| No. of triangle. | Angle A. | Angle B. | Angle C. | A° + B° + C°. | Total error. |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |
| Etc. | | | | | |

In filling up the last column you will have to satisfy yourself what the sum of the angles should be, and compare that with what your measurements make it in column 5.

# CHAPTER V

## SCALES, THEIR CONSTRUCTION AND USE

**45. Introduction.**—If we wish to draw the elevation of a machine whose height[1] is, say, 5′, and length 12′, upon a sheet of paper whose surface does not exceed two or three square feet in area, it is evident it would be impossible to make this drawing of the machine full size. Now, suppose we make a line 3″ in length on the drawing represent a foot on the machine, then a line 5″ × 3″ = 15″ long would represent the height of the machine, and one 12″ × 3″, or 36″ long its length; and we should speak of the scale as being one of 3″ to the foot, and the *fraction of the scale,* as it is called (or representative fraction as it is sometimes called), would be—

$$\frac{3 \text{ inches}}{1 \text{ foot}} = \frac{3}{12} = \frac{1}{4}$$

In the same way: If $\frac{1}{4}$ inch represented 1 foot, the scale would be $\frac{1}{48}$

„ $\frac{1}{2}$ „ „ „ „ $\frac{1}{24}$

„ 1 „ „ „ „ $\frac{1}{12}$

„ $1\frac{1}{2}$ „ „ „ „ $\frac{1}{8}$

„ 4 „ „ „ „ $\frac{1}{3}$

„ $4\frac{1}{2}$ „ „ „ „ $\frac{3}{8}$

„ 6 „ „ „ „ $\frac{1}{2}$

And if 1 inch represented 1 yard, the scale would be $\frac{1}{1 \times 12 \times 3} = \frac{1}{36}$

„ 1 „ 1 chain „ „ $\frac{1}{12 \times 66} = \frac{1}{792}$

„ 1 millimetre represent 1 centimetre, scale would be $\frac{1}{10}$

„ 1 „ „ 1 decimetre „ „ $\frac{1}{100}$

„ 1 „ „ 1 metre „ „ $\frac{1}{1000}$

[1] The **dimensions of machines**, details, etc., are usually written in feet and inches; the former, as we have seen, being indicated by the suffix ′, and the latter by the suffix ″. Thus, 5′ reads 5 feet, and 5′ $3\frac{5}{8}$″ reads 5 feet $3\frac{5}{8}$ inches. Further, 0·783″ reads decimal (or point) seven eight three of an inch, equal to $\frac{783}{1000}$ of an inch. When **metric measurements** are used, the following abbreviations, *m.*, *dm.*, *cm.*, *mm.* respectively represent *metres*, *decimetres*, *centimetres*, and *millimetres*.

**Angles** are measured in *degrees*, *minutes*, and *seconds*. Thus 45° reads 45 degrees, and 20°, 40′, 50″ reads, twenty degrees, forty minutes, fifty seconds.

**46. Drawing to Scale.**—Of course, whenever practicable, the drawing is made the same size as the thing to be drawn; the drawing is then spoken of as being **full size**. If the size of the object will not admit of its being drawn full size, then as large a scale as is practicable should be selected. This applies more particularly to **detail drawings**, where every minute feature must be clearly shown. The great size of some work necessitates its being set out in detail on large specially prepared boards, whilst, on the other hand, the **details of watches, clocks**, and small instruments can only be satisfactorily shown when **drawn larger than their true size**. In every case, whatever scale is decided upon, care must be taken to draw all parts of the object to the same scale, and thus get an exact, although a reduced or enlarged, representation of it. **Scales** should always be constructed and drawn **on important drawings**, that are not fully dimensioned; so that the various parts may, with the aid of a pair of dividers, be scaled off, and so that any alteration in size, due to the shrinking of the paper, will affect both scale and drawing alike. These scales must be constructed and divided with great care and accuracy.

**47. Engineer's Scales.**—Although most of the drawings made by the beginner will be **full-size** or **half-size**, for which any ordinary rule can be used, yet after some practice he may be called upon to make them to a smaller scale, such as $\frac{1}{4}$ or $\frac{1}{8}$ full size, or even less, so that he will require an instrument with these scales marked on it. Such instruments are called **Scales**, or **Drawing Scales**, and they can be had made of various materials, such as cardboard, vulcanite, boxwood, ivory, and steel. The ordinary length is 12″, and they are made with thin edges to enable a distance to be marked off from the scale to the drawing direct with pencil or pricker; but a more accurate method is to take the distance off with dividers, as explained in Art. 14.

**Vulcanite scales** should be avoided, as they expand and contract greatly with changes of temperature. On the whole, the best materials for them are **boxwood** and **ivory**. The scales are 3″, $1\frac{1}{2}$″, 1″, $\frac{3}{4}$″, $\frac{1}{2}$″, $\frac{3}{8}$″, $\frac{1}{4}$″, and $\frac{1}{8}$″ to the foot, or $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{12}$, $\frac{1}{16}$, $\frac{1}{24}$, $\frac{1}{32}$, $\frac{1}{48}$, and $\frac{1}{96}$ full size respectively. Two scales on each edge of the instrument.

**48. Construction of Scales.**—Although you can never hope to make a scale with the accuracy that is possible when a **dividing machine** is used, that is to say, with the truth of a good ivory scale or steel rule, or even a boxwood scale, there is no reason why you should not, with care and a little practice, make scales accurate enough for some purposes; indeed, you have seen that scales have to sometimes be set out on a drawing, and you should therefore make an effort to understand how this is done. You will see that the construction of scales is based on the equal division of lines, executed on the principle of similar triangles; and the following example or two, worked in the form of problems, should put you in the way of making them yourself from suitable data, and better understanding their use.

**49. To divide a Line (say 3·5″ long) into any number (say thirteen) of Equal Parts.**—Let AB (Fig. 72) be the line 3·5″ long. At one of its ends, A, draw AC at any angle (an angle of 30° is a convenient one); now open the compasses so that the distance between the points is about one-thirteenth part of AB (by guessing, not by trial), and step along the line AC, making points H, I, J, etc., to C, the thirteenth point from A. Join CB, and through D, F, J, I, H, etc., draw lines parallel to CB, cutting AB in points in E, G, K, L, M, etc. These lines will divide AB into thirteen equal parts.

Note.—The truth of this and similar constructions can be proved thus. The triangle ADE is *similar* to the triangle ACB, because their angles are respectively equal. Therefore the side AD will be in the same proportion to AE as the side AC is to AB, and DC will be to EB in this same proportion; but DC is one-thirteenth of AC, therefore EB must be one-thirteenth of AB. And so on for the other divisions (Euc. I. 26 and VI. 2, 4, and 10).

**50. To draw a Scale of $\frac{1}{16}$, to read Feet and Inches, and to make it long enough to measure 4′.**—A length of 1′ will be represented by $\frac{1}{16}$ of a foot, or by $\frac{12}{16}$″ = $\frac{3}{4}$ of an inch on the scale, and the whole length of the scale will be 4 × $\frac{3}{4}$″ = 3″. Draw a line AB (Fig. 73) 3″ long, and carefully divide it into four equal parts (Problem 49). Then each of these parts will represent 1′; divide the first division AC into twelve equal parts, and these parts will represent inches. The scale may be finished by drawing the lines shown in the figure, and if they are figured in the way shown,

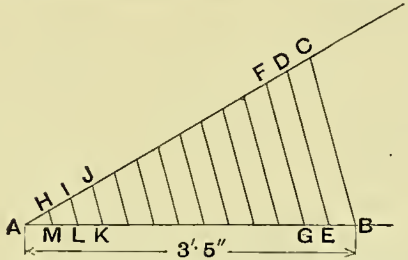



FIG. 72.—Line divided into equal parts.

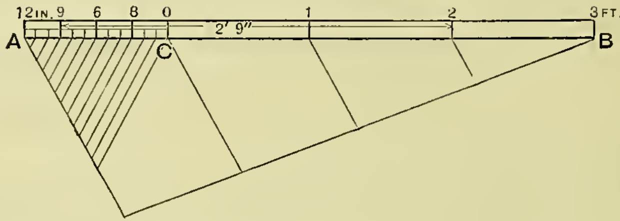



FIG. 73.—Construction of a scale of $\frac{1}{16}$.

which is the correct way, dimensions can be readily taken off with the dividers, by placing the points on the feet and inches in the required positions. Thus, to take off 2′ 9″, place one leg of the dividers on point 2′, and the other on 9″. The distance between the legs will represent 2′ 9″.

**51. To draw a Scale of $1\frac{1}{8}''$ to 1 yard, the Scale to show Feet, and be long enough to measure 3 yards.**—The representative fraction of this scale is $\frac{1\frac{1}{8}}{3 \times 12} = \frac{9}{8 \times 3 \times 12} = \frac{1}{32}$, and the length of the scale will be $3 \times 1\frac{1}{8}'' = 3\frac{3}{8}''$. Proceed as in the previous problem; draw a line AB (Fig. 74) this length, and divide it into three equal parts; then each of these parts will represent 1 yard. Divide AC, the first of these divisions, into three equal parts, and each of these parts will equal 1′. The scale should be figured and completed as explained in the previous problem.

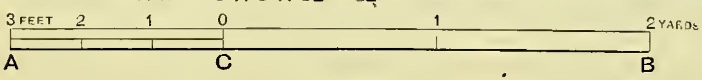



FIG. 74.

**52. To draw a Centimetre Scale or Rule, making it long enough to measure 13 Centimetres.**—Referring to the Metric Tables at the end of the book, we find that a centimetre = 0·394″. Therefore 13 × 0·394″ = 5·122″ = $5\frac{1}{8}''$ very nearly. So draw a line AB $5\frac{1}{8}''$ long (Fig. 75), and divide it into 13 equal parts, as in Problem 49, and complete the scale by figuring, and drawing the parallels to AB, and the divisions. Of course, a length AC (10 centimetres) equals 1 decimetre.

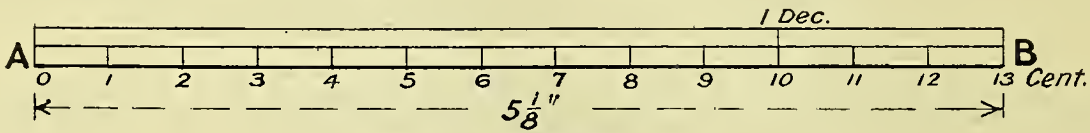



FIG. 75.—Centimetre scale or rule.

## Diagonal Scales.

**53.** Diagonal scales are used when the divisions on an ordinary scale would become very minute.

The principle of the scale can be explained by referring to Fig. 76. Let the problem be, to divide a distance DC by a diagonal line into any number (say four) of equal parts. Draw lines CB and DA from the extremities of the given line DC, and perpendicular to it, making them any length; with the dividers prick off any four equal distances, CG, GF, FE and EB, along AB, then through GFE and B draw lines parallel to DC; complete the figure by drawing the diagonal DB, cutting the lines in HIJ.

Now, the triangles CBD and FBI are similar, therefore $\frac{DC}{IF} = \frac{CB}{FB} = \frac{1}{\frac{1}{2}}$. That is to say, the distance FI is half the distance CD. And again, $\frac{IF}{JE} = \frac{FB}{EB} = \frac{\frac{1}{2}}{\frac{1}{4}}$. Therefore JE is half IF and a quarter DC. This simple expedient is equivalent to dividing the given distance or line into four equal parts. The value of this principle can be realized when we notice that DC may be as small as we like. Thus make it $\frac{1}{10}''$, then EJ will equal $\frac{1}{4} \times \frac{1}{10}'' = \frac{1}{40}''$. We may now proceed to construct a proper scale on these lines.

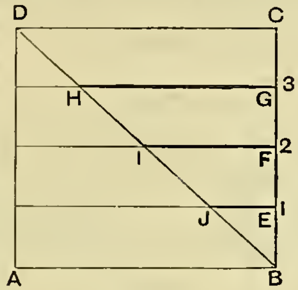



FIG. 76.

**54. To construct a Diagonal Scale to show Eighths and Sixty-fourths of an Inch.**—Draw a line AB (Fig. 77) any number of inches in length (say 3), and divide it into inches as at E and F. Divide AE into eight equal parts; each of these will be an eighth of an inch in length. Then at A and E draw perpendiculars AD and E0, and from A set off, along AD, $\frac{64}{8}$ = 8 equal divisions (any convenient size), and through each of these divisions draw a line parallel to AB. Then join 8 on AD to 7 on AE, and through 6, 5, 4, 3, 2, 1 and E on AB draw lines parallel to 8, 7, and complete the scale as shown in the figure.

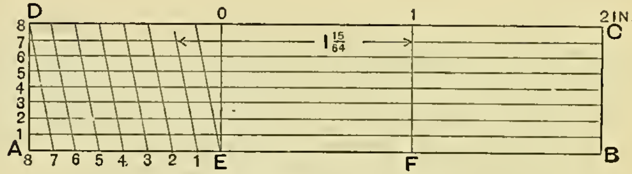



FIG. 77.—Diagonal scale.

The student will understand, after studying the previous figure, No. 76, that the divisions between the lines AD and D7 are $\frac{1}{8} \times \frac{1}{8} = \frac{1}{64}$, $\frac{2}{8} \times \frac{1}{8} = \frac{2}{64}$, etc. To take off any distance with the dividers, say $1\frac{15}{64}''$ (this will equal $1\frac{1}{8} + \frac{7}{64}$), place one leg of the dividers on F1″, where the horizontal line 7 cuts it, and move the other leg till it is on the diagonal 1. Then the distance between the legs will be the required one, namely $1\frac{15}{64}''$.

It will be noticed that the product of the divisions in AE and AD (8 × 8 = 64) equals the number of parts into which the distance AE has been divided by the scale.

It follows that, if the divisions had been 10 and 10, the line would have been divided into 10 × 10 = 100 parts, so that if a diagonal scale of yards is to be arranged to show readings of feet and inches, the divisions on the respective lines would be 3 and 12.

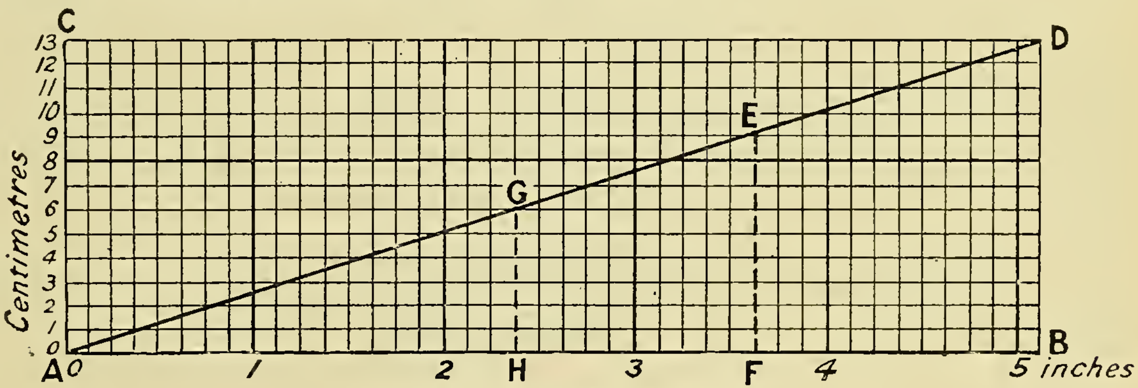



FIG. 78.—British and metric conversion scale.

**55. To draw a British and Metric Conversion Scale.**—In Problem 52 we found that 130 millimetres or 13 centimetres equal $5\frac{1}{8}''$. So take a piece of squared paper with $\frac{1}{8}''$ squares, and mark off a line AB $5\frac{1}{8}''$ long (Fig. 78). Draw AC perpendicular to AB, and 13 squares in height, and through C draw a parallel to AB, cutting a perpendicular at B in D, join AD, and figure the lines AB and AC as shown. Complete by drawing the perpendiculars through the inch divisions on AB.

**Use of Scale.**—Example.—(*a*) Suppose you use a foot rule and find that the dimension of a body you are measuring is $3\frac{5}{8}''$. Refer to your scale, and AF is this length, run your eye up the line FE, and you find that it cuts the diagonal AD in E, a point $\frac{1}{5}$ of a square above the 9 centimetres line, therefore your measurement is 9·2 centimetres.

(*b*) Suppose that you are making a drawing in British measurements from a sketch with the dimensions in millimetres, and that you come to a dimension 60 mm., this is 6 centimetres; running your eye along the horizontal line through 6 on AC, you find it cuts the diagonal at G, which is on a vertical GH whose foot is $2\frac{3}{8}''$ from A, therefore your equivalent length in inches is $2\frac{3}{8}''$.

## EXERCISES.

1. Draw a scale of 1 inch to the foot, making it long enough to measure 6 feet. Note.—You will draw two parallel lines about $\frac{1}{8}''$ apart, making the upper one thin and the bottom one thick, as in Fig. 74. Set off the length 6″, and divide it into six equal parts, and divide the first inch from the left-hand end into twelve equal parts, as in Fig. 72, and be careful to mark the divisions as shown.
2. Make a scale of 1 inch to the yard, making it long enough to measure 6 yards. State what the representative fraction of this scale is.
3. Draw a scale of 2 inches to the mile, and make it long enough to measure 4 miles, dividing the first mile into furlongs.
4. Construct a scale of 1 inch to the chain, make it long enough to measure 8 chains, and divide the first chain into poles. Note.—The land chain is 66 feet in length, and there are four poles to the chain.
5. Draw a scale of 2 inches to the pole, showing yards.
6. Draw a diagonal scale to show tenths and hundredths of an inch, making it long enough to measure 6 inches.
7. Draw a British and metric conversion scale.

# CHAPTER VI

## PROPORTION, ETC.

### Simple Problems in Proportion

**56. Introduction.**—When four quantities—A, B, C, D—are proportionals, it is correct to say that A is to B as C is to D, and to write them thus, A : B : : C : D ; or thus, A : B = C : D.

Thus it is easily seen that B bears the same proportion to A as D does to C, and therefore the ratio (when two quantities are compared with each other a ratio is formed) in each case is the same. And A and D multiplied together will equal B and C multiplied together. This is called multiplying extremes and means. Now, applying this to lines, we have A = 2″, B = 3″, C = 4″, and D = 6″; and using these values for our proportion, we get 2 : 3 : : 4 : 6, and the extremes and means multiplied give us $2 \times 6 = 3 \times 4$.

In geometry this is equivalent to saying that the rectangle made up of sides 2″ and 6″ is equal to the rectangle with sides 3″ and 4″. (See Chapter IX. on **Areas.**)

Now, when the third term of a proportion is in the same ratio to the second as the second is to the first—thus, 3 : 6 : : 12 : 24 —the quantities are said to be in **continued proportion.** It is easily seen that 3 bears the same ratio to 6 that 6 does to 12 and 12 to 24; and taking the continued proportion 3 : 6 : : 12 : 24, we have $3 \times 12 = 6 \times 6$; that is, the first and third terms multiplied together equal the second term multiplied by itself (squared); and, further, $6 \times 24 = 12 \times 12$—that is, the second term multiplied by the fourth term equals the third term multiplied by itself. This is equivalent to saying that *any pair of alternate terms multiplied together will equal the intermediate term squared*, or, in other words, the rectangle made up of sides equal to the alternate terms will be equal to the square with sides equal to the intermediate term. The intermediate term is called a mean proportional between the other two.

You would do well to thoroughly master the few simple principles of proportion, if you have not already done so, as many **problems in mechanics and practical mathematics** are easily solved by a simple geometrical application of these principles.

In the previous chapter we have seen how **problems on the division of lines** can easily be worked on the principle of **similar triangles,**[1] and you had better again examine that before working the following interesting variation of it, which will conveniently lead up to the useful geometrical problems in proportion which follow it.

**57. To divide any given Line (say 2·75″ long) into Three Parts so that the Parts will be in a given Proportion (say of 3 : 4 : 5).**—Let AB (Fig. 79) be the line 2·75″ long. At one of its ends, A, draw AC at any angle, and from A step along AC twelve

[1] This is also the principle which governs the working of the **proportional compasses,** used for reducing the size of drawings, etc.

(3 + 4 + 5 = 12) equal parts. Join C (the twelfth part) to B, and from E, five parts from C, draw EF parallel to CB, and from G, four parts from E, draw GH parallel to CB. Then the given line AB is divided at H and F, so that AH : HF : FB as 3 : 4 : 5.

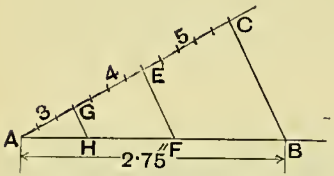


FIG. 79.—Line divided in given proportion.

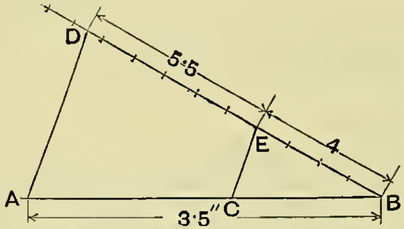


FIG. 80.—Line divided in given ratio.

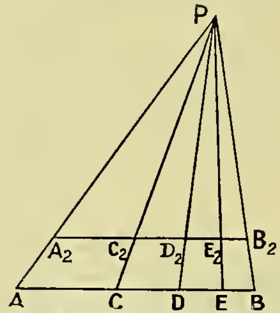


FIG. 81.—Division of a line.

### Direct Proportion

**58. To divide a Line AB (say 3·5″ long) in a Point C, so that BC : AC : : 4 : 5·5.** Draw the line AB (Fig. 80), and at one of the ends, say B, draw BD at any angle, and from B step along BD nine and a half equal parts (5·5 + 4 = 9·5). Mark this point D, and join AD, and from E, five and a half parts from D, draw EC parallel to AD. Then C is the point required. And BC will be in the same proportion to AC as 4 is to 5·5.

**59. To divide a line $A_2B_2$ Proportionally to any given Divided Line AB.**—Draw the two lines parallel to one another as in Fig. 81. Join their ends $AA_2$ and $BB_2$, and produce the lines to meet in a point P. Through P draw lines to CDE, cutting $A_2B_2$ in $C_2$, $D_2$, and $E_2$. You will then have divided the line $A_2B_2$ in a similar way, or proportionally to the given line AB.

NOTE.—When the given lines are nearly equal in length, you must place them as near together as possible to keep the point P within the edges of your drawing paper.

**60. To divide the Space contained between Two Parallel Lines AB and CD into Equal Parts (say 6) by Lines Parallel to them.**—Draw CK (Fig. 82) any perpendicular to them, and set off 1, 2, 3, 4, 5, 6 equal spaces; then with centre C, and C6 radius describe an arc 6J, cutting AB in J. Join C to J, and with centre C describe the arcs 1E, 2F, 3G, etc., and through the points E, F, G, etc., draw lines parallel to AB. These divide the space as required.

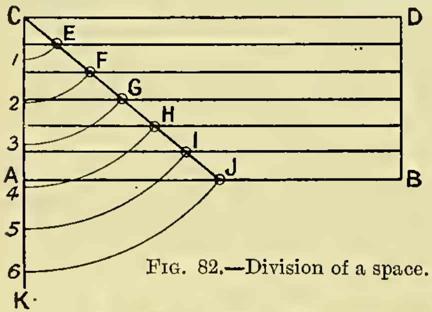


FIG. 82.—Division of a space.

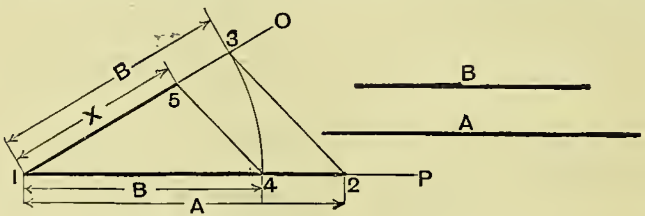


FIG. 83.—A third proportional to two given lines.

NOTE.—Suppose AC represented the height of a wall of six courses of brickwork, you have a means of finding the divisions representing the courses, or layers of bricks; but of course the result is only satisfactory when the drawing is accurate.

**61. To find a Third Proportional to Two given Lines, A and B.**—Let A and B (Fig. 83) be the given lines. Then a line

X is required, such that A : B : : B : X. Draw any indefinite line, 1P. At 1 draw the line 1O, making any angle with 1P. From 1 on the line 1P cut off 1 2 equal to A, and from 1 on the line 1O cut off 1 3 equal to B. Then, using 1 as centre, describe an arc 3 4, cutting 1P in 4. From 4 draw a line 4 5 parallel to 2 3. Then the line 1 5 is the required *third proportional* X.

NOTE.—If we state the relationship of the four quantities in the form of an equation, namely, AX = [illegible]B, we have the product of A and X equal to $B^2$. That is to say, the rectangle made up of sides A and X equals the square on B. Further, B is the mean proportional of A and X. Refer to Problem 64.

**62. To find a Fourth Proportional to Three given Lines, A, B, and C.**—In this problem the fourth term, X, in the proportion A : B : : C : X, is to be found.

Take, as in the previous problem, any indefinite line, 1P (Fig. 84), and at 1 draw 1O at any angle with 1P. From 1 on 1P cut off a length 1 2 equal to A, and on 1O from 1 cut off 1 3 equal to B. Then from 1 on 1P cut off 1 4 equal to the third line C. Connect 2 and 3, and through 4 draw 4 5 parallel to 2 3. Then 1 5 is X, the required *fourth proportional.*

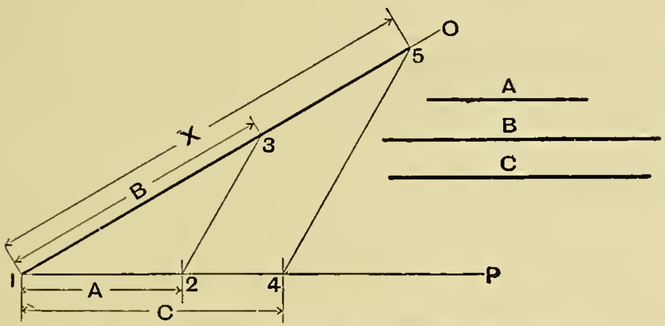



FIG. 84.—Fourth proportional to three given lines.

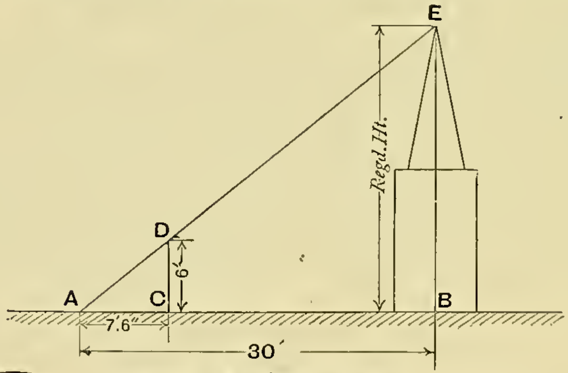



FIG. 85.—Finding the height of a tower.

**63. The Shadow cast on the Ground by a Vertical Rod 6′ high is 7′ 6″ in length. Find the Height of a Tower whose Shadow is 30′ long. Scale $\frac{1}{10}''$ = 1 foot.**—Obviously, the required height can be found by proportion. Again making use of similar triangles, draw a straight line AB (Fig. 85) to represent the shadow of the tower. This will be, to the given scale, 30 × $\frac{1}{10}''$ or 3″. Along AB from A mark off AC equal to the 7′ 6″, the length of the rod's shadow, and at C erect a perpendicular to AB, making its length CD equal 6′, the height of the rod. Next, carefully draw through A and D a line cutting a perpendicular to AB at B in E. Then the length of BE will represent the height of the tower.

NOTE.—Of course the figure represents a side view of the rod and tower. It will at once be seen that the solution of this problem is an application of the well-known property of similar triangles, namely, that their similar sides are proportional (Euc. I. 26 and VI. 4), as explained in Problem 43. Thus $\frac{AB}{AC} = \frac{BE}{CD}$, or the required height is a fourth proportional to the three other quantities.

**64. To find the Mean Proportional C of Two given Lines A and B.**—Draw any indefinite line OP (Fig. 86), and along it mark off the distances O1 equal to A, and 1 2 equal to B. Bisect the line O2. Let 4 be the point of bisection; then, with 4 as centre and radius 4O, describe a semicircle, and at 1 raise a perpendicular to cut the semicircle in 3. The line 1 3 is the required mean proportional C (Euc. VI. 13).

NOTES.—1. The **mean proportional squared** is equal to the product of the two given lines; that is, $A \times B = C^2$, or $A : C :: C : B$. As you will remember, the product of the extremes (first and fourth terms) of any proportion is equal to the product of the means (second and third terms).

2. The **mean proportional** C is called the **geometrical mean** of A and B. The **area of a square** on C is equal to the **area of the rectangle** whose sides are A and B, as we have seen, and as shown in Fig. 87.

This must not be confused with the **arithmetrical mean** of A and B, which is half their sum. That is, arithmetrical mean $= \frac{A + B}{2}$.

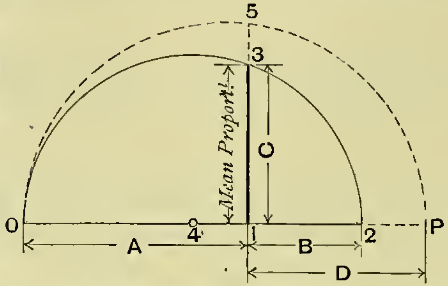


FIG. 86.—Mean proportional.

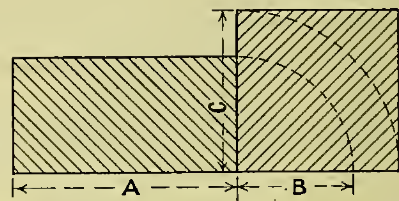


FIG. 87.—Square and rectangle equal in area.

3. You should study the relation the above problem bears to Problem 61.

4. When B equals unity, C will equal the $\sqrt{A}$, and the length of the line A represents the area of the rectangle formed by A and B.

5. The line 1 5 is the mean proportional of O1 and 1P, or of A and D, and **the square of the two mean proportionals** 1 3, and 1 5, are in the same ratio as B is to D,

$$\text{For } \frac{(1\,3)^2}{(1\,5)^2} = \frac{A \times B}{A \times D} = \frac{B}{D}$$

You will easily understand that if 1 3 and 1 5 be two **corresponding sides of similar figures**, the lines B and D will represent the ratio of their areas. Therefore this construction is very useful in handling such problems as No. 115, when the ratio of the areas is given.

## EXERCISES.

### ORAL EXERCISES.

1. If four quantities are proportionals, what relationship must exist between the first and second, and third and fourth?
2. When are four quantities said to be in continued proportion?
3. What relationship must exist between three terms of a proportion if the intermediate one is a mean proportional of the other two?

### DRAWING EXERCISES.

4. A line $6\frac{1}{2}''$ in length is the perimeter of a triangle whose sides are in the proportion of 2 : 3 : 4. Divide the line to find the lengths of the sides of the figure. Measure these lengths with your rule and write them down.

5. Two boys walk towards each other from the ends of a straight road a mile in length. One walks at the rate of $2\frac{1}{2}$ miles per hour, and the other at $3\frac{1}{2}$ miles per hour. Draw a line 6″ in length to represent the mile, and geometrically find the point in the line where the boys would meet. Measure its distances from the ends.

6. A line 5·4″ in length represents, to a scale of one-twelfth, the height of a bookcase, and the positions of the shelves are indicated by divisions 1·7″, 3·2″, and

4·4″ from one end of the line. Another line, 4·5″ in length, represents the height of a second bookcase whose shelves are to be arranged at heights proportional to the first one. Geometrically find the positions of the shelves of the second bookcase, and carefully measure and write down the distances between them.

7. One side of a rectangle whose area is equal to that of a square of $1\frac{1}{2}''$ side is 1″. Find by the method of proportion the length of the other side.

8. The sides of a rectangle are 2·75″ and 3·6″, and the long side is lengthened by 1″. Find how much the other side must be lengthened if the sides are to remain in the same proportion.

9. The shadow cast on the ground by a lamp-post, whose height is 9′, is 10′, and the shadow cast by a telegraph pole is 22′ 6″. Geometrically find the height of the pole.

10. Find the mean proportional of two lines whose lengths are 2·8″ and 1·6″, and measure its length.

# CHAPTER VII

## CIRCLES, ARCS, AND LINES

65. **Introduction.**—As ordinary mechanical drawings mainly consist of combinations of circles, arcs, and lines, the art of correctly and neatly drawing a few of them in various positions in relation one to the others, representing typical cases, should be cultivated by the beginner; for if such lines are faulty in form and finish, or do not satisfy the geometrical conditions of proper contact, they spoil the appearance and detract from the value of any drawing upon which they appear.

A few of the more important definitions and problems relating to circles and arcs are given here to help you, but for more complete information on these matters refer to definitions, etc., at end of the book; you may also refer to the author's "Geometrical Drawing," p. 61, etc.

66. **Definitions.**—**The radius of a circle** is a straight line drawn from the centre to its circumference.

**A diameter of a circle** is a straight line passing through its centre, and terminated at both ends by the circumference.

**An arc of a circle** is any part of the circumference.

**A chord** is a straight line joining the extremities of an arc.

**A segment** is any part of a circle bounded by an arc and its chord.

**A semicircle** is half a circle. or a segment cut off by a diameter.

**A sector** is any part of a circle bounded by an arc and two radii drawn to its extremities.

**A quadrant**, or quarter of a circle, is a sector having a quarter of a circumference for its arc, and the two radii perpendicular to each other.

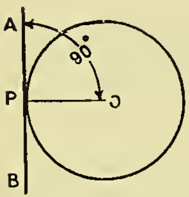



FIG. 88.—Circle and tangent.

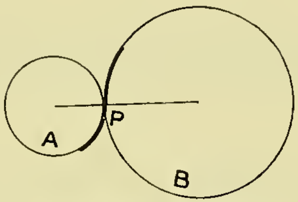



FIG. 89.—Point of contact of two circles.

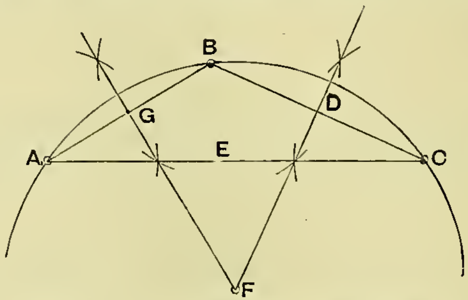



FIG. 90.—An arc described through three points.

**A sextant**, or sixth of a circle, is a sector having a sixth of the circumference for its arc, and the two radii making an angle of 60° with each other.

**An octant**, or eighth of a circle, is a sector having an eighth of the circumference for its arc, and the two radii making an angle of 45° with each other.

**A tangent** is any line perpendicular to a radius at its extremity in the circle. A tangent touches the circle in a point, as at P, Fig. 88 (which is called the point of contact), where the line AB touches the circle, and it is perpendicular to the radius OP.

**Point of Contact.**—Where two circles touch one another, they do so in a point only, called the *point of contact*, and the straight line which joins their centres passes through this point. Thus, Fig. 89 shows two circles, A and B, touching one another in the point P, which is the point of contact. It is only when this condition is satisfied that a part of one circle can be made to flow into a part of the other; the thick line in the figure shows how this condition must be satisfied.

To enable you to correctly treat cases where circles are in contact with one another and with straight lines, you should carefully study and work the following problems before attempting the exercises at the end of the chapter.

## Some Important Cases worked as Problems.

**67. To describe a Circular Arc through Three given Points.**—Let A, B, C (Fig. 90) be the given points. Join AB and BC, and draw the perpendicular bisectors GF, DF, intersecting in F. Then, with F as centre and radius FB, describe the required arc ABC.

**68. To find the Centre and Radius of a given Arc or Circle.**—Let A, B, and C (Fig. 90), be any points in the given arc or circle. Then join AB and BC, and perpendicularly bisect the lines AB and BC by the lines GF and DF, which intersect in F the required centre.

In the case of a circle, if one of the bisectors GF or DF be produced both ways it cuts the circle in the extremities of a diameter, and by bisecting the diameter the centre can be found.

**69. To draw a Tangent to a Circle through a fixed Point in its Circumference.**—Let B (Fig. 91) be the fixed point in the circle. Join B to the centre A, and through B draw CD perpendicular to AB. Then CD is the tangent required.

**70. To draw a Tangent to a Circle through a fixed Point without it.**—Let the circle in Fig. 91 be the given one, and P the point. Join the centre A with P, the fixed point without the circle, and bisect AP in E. With E as centre, radius EA, describe the semicircle AFP, cutting the given circle in F. Join PF. Then PF is the required tangent.

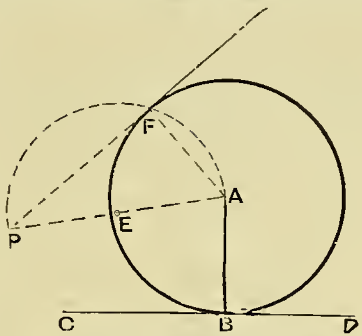


Fig. 91.—Tangents to a circle.

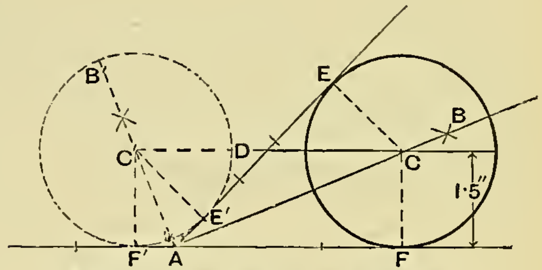


Fig. 92.—Circle touching two given lines.

It is evident that in a similar way a tangent the other side of PA could have been drawn.

Notes.—1. You will notice that if F be joined with A, the angle PFA will be a right angle, being an angle in a semicircle. (Euc. III. 31.) And FA will be a normal to the tangent at F.

2. The Euclidean geometry does not allow a tangent from a fixed point to a given circle to be drawn without first finding the point of contact as above, and the same remarks apply to the case of a common tangent to two circles; but for practical drawing purposes a tangent may be drawn from an external point to a circle, or a common tangent to two circles directly by carefully adjusting the straight-edge; and should the actual point of contact be required, a perpendicular to the tangent from the centre fixes it.

**70a. To inscribe in a given Angle a Circle of given Radius (say 1·5″).**—Let EAF (Fig. 92) be the given angle. Bisect the angle by the line AB, and draw CD parallel to AF and 1·5″ from it, intersecting AB in C. With C as centre, radius 1·5″, draw the circle touching the sides of the angle in E and F. The exact points of contact can be found by drawing from C the lines CE and CF perpendicular to AE and AF respectively.[1]

The dotted lines refer to a case when the angle is obtuse, and the same letters apply.

Notes.—1. This is a problem often met with in mechanical drawing, when two lines are to be connected by an arc of a circle of given radius.

[1] AE and AF are two tangents to the circle from A, and they are equal to one another (Euc. III. 17).

2. For most practical purposes a common tangent to two given circles (such as E′E, Fig. 92) can be drawn with a sufficient degree of accuracy by offering the edge of a square to the two circles and drawing a line to touch them, the points of contact being found by drawing perpendiculars from the centres to the tangent as shown.

70B. **To describe a Circle of given Radius to touch a given Line and a given Circle.**—From C (Fig. 93), the centre of the given circle, draw any line CE, cutting the circle in F; from F mark off FE, equal to the given radius, and with centre C, radius CE, describe the arc ED. At any point H in AB draw GH equal to the given radius EF, and perpendicular to AB. Through G draw GD parallel to AB, and cutting the arc DE in D. Then, with D as centre, radius EF, describe the required circle.

NOTE.—D is equidistant from line and circle.

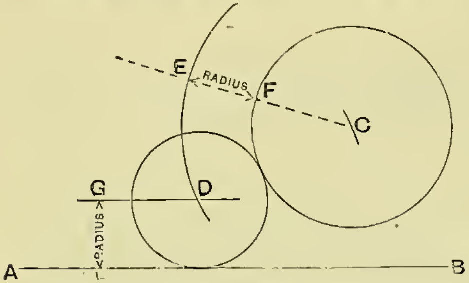



FIG. 93.—Circle of given size touching a fixed line and circle.

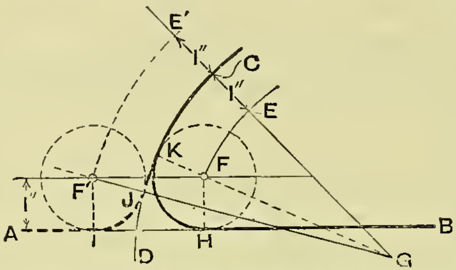



FIG. 94.—Arc touching straight line and arc.

71. **To describe an Arc of a Circle of given Radius (say 1″) to touch a given Arc and a given Straight Line.**—This is an obvious variant of the preceding problem. AB (Fig. 94) is the given line, and G the centre of the given arc CD. Draw GC, any line passing through the centre of the arc G, and cutting it in C. Mark off CE equal to the given radius of 1″, and with centre G, radius GE, describe the arc EF, and draw F′F parallel to AB and 1″ from it, intersecting EF in F, which is the centre of the required arc. With F as centre, radius FH, a perpendicular to AB, describe the required arc. Draw through G and F the line GK, cutting the circle in K. Then the points of contact are H and K.

If the arc were to touch the given circle externally, F′ would be its centre, and I and J its points of contact. The working is similar, and can be easily followed on the figure.

NOTE.—This problem occurs when a wheel with arms is drawn. HB is then the side of an arm, and the arc CK a part of the rim.

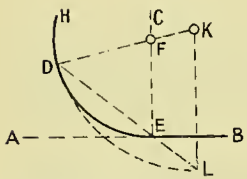



FIG. 95.—Arc touching a given arc and a line in a fixed point.

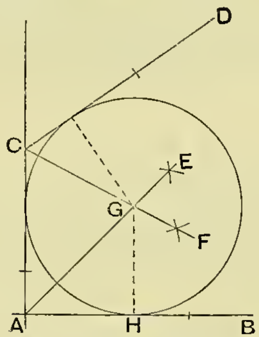



FIG. 96.—Circle touching three straight lines.

72. **To describe an Arc of a Circle to touch a given Line in a fixed Point and also a given Arc.**—Let AB (Fig. 95) be the given line, E the fixed point in it, and HDL the given arc, whose centre is K. Draw from E a line CE perpendicular to AB, and through K draw KL parallel to CE, and cutting the given arc in L. Join LE and produce it to cut the given arc in D. Join D to K, cutting CE in F. Then with centre F and radius FD describe the required arc DE, which satisfies the conditions of the problem.

73. **To draw a Circle to touch Three given Straight Lines.**—Let the given lines be AB, AC, and CD (Fig. 96), intersecting in A and C. Bisect the angle BAC by the line AE. The centre of the required circle must be somewhere in this line. Bisect the angle ACD by the line CF; the centre must also be somewhere in CF. Therefore it is in G, the intersection of AE and CF. From G draw GH perpendicular to AB and cutting it in H. With centre G, radius GH, describe the required circle or arc.[1] Then perpendiculars from G, such as GH, give the points of contact.

NOTE.—This problem sometimes occurs when a small bevel wheel is drawn, and AB is part of the boss or hub, and CD is the back of the rim.

[1] Three other circles can be drawn to touch the given lines, and one of them will obviously be contained by the triangle made by producing DC and BA till they meet.

**74. To describe Two Arcs to meet each other in the Line of their Centres and to touch Two given Lines at Fixed Points in them, the Radius of the Lesser Arc being also given.**—Let AB and CD (Fig. 97) be the given lines, and E and F the fixed points. Draw EG perpendicular to AB and equal to the radius of the lesser arc, and from F draw FH perpendicular to CD and equal to EG. Join G to H and bisect it by the perpendicular KN cutting FH produced in N. Join NG and produce it. Then with N as centre, and radius NF, describe an arc meeting NG produced in L. With G as centre, radius GL or GE, describe the arc EL. Then the arcs FL and LE are tangent to the given lines CD and AB respectively in the points F and E, as required, and the two arcs meet at L in the line NG of their centres.

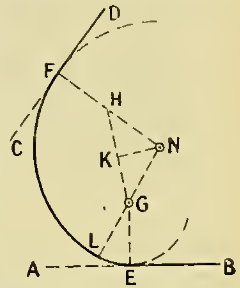



Fig. 97.

**75. Interesting cases of Arcs flowing into one another.**—The few examples of proper tangential contact of line and circle, and of circles of different curvature, shown in Figs. 98 to 104, should now speak for themselves, and you should be able to draw any of these in such a way that the geometrical conditions of proper meeting or contact are conformed to. You will not fail to notice that in such cases as Fig. 99 the centres A, B, and C of the three arcs are all on the same line MN. And that the points D, E, and F (Fig. 101), each common to two arcs, are in the lines of centres BC, CA, and AB respectively.

If you are fond of devising geometrical patterns for decorative work you will soon begin to find out endless arrangements of circles and lines that will have a pleasing effect if they are properly drawn.

**76. Hints on working the Exercises.**—Having studied the preceding problems, the student should be able to work the following exercises without further help. They should be carefully constructed from the dimensions shown, and not merely copied. Having pinned down a sheet of paper, the T and set-squares should be carefully dusted, and the pencils and lead of the pencil bows to be used sharpened, and the latter adjusted so that the pencil and steel point are of equal length; the exercises can then be proceeded with.

## IMPORTANT TANGENTIAL ARCS.

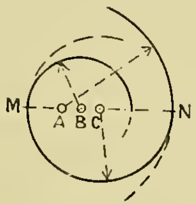



Fig. 99.

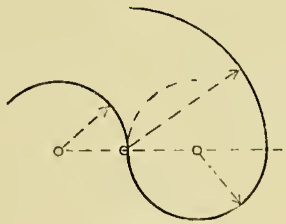


Fig. 100.

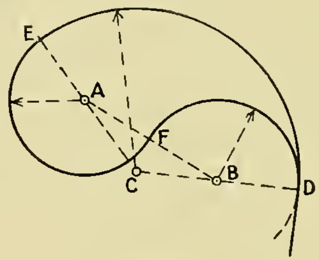



Fig. 101.

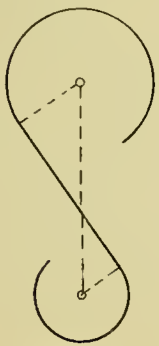


Fig. 98.

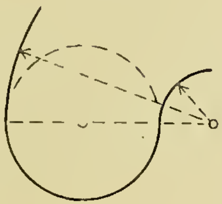


Fig. 102.

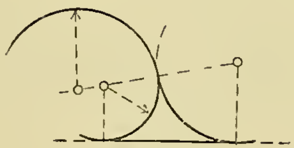


Fig. 103.

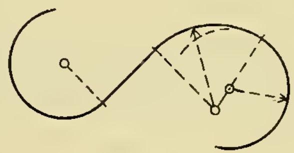


Fig. 104.

## EXERCISES.

The diagram relating to each of the following problems should be drawn full size before attempting the solution.

1. Assume any point P in the given circle (Fig. 105), and draw a tangent at the point.
2. Through the fixed point P (Fig. 106), draw a tangent to the given circle.
3. In the angles ABD and CBD (Fig. 107) inscribe arcs of $1\frac{1}{4}''$ radius.
4. Describe a circle of $1\frac{3}{4}''$ diameter to touch both the line AB (Fig. 108) and the given circle.
5. Describe a circle touching the three given lines, BA, AC, and CD (Fig. 109), and mark the points of contact.
6. Describe a $2\frac{1}{2}''$ circle to touch both the given circles (Fig. 110).
7. Describe an arc to touch the line AB (Fig. 111) in the point P and to flow into the given arc DE whose radius is 2″ and whose centre is C.
8. Describe two arcs to meet each other in the line of their centres. One of them to touch the line AB (Fig. 112) at E, its centre being P, and the other to touch the line AC in the point D.

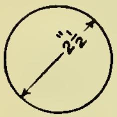


Fig. 105.

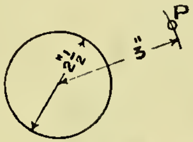



Fig. 106.

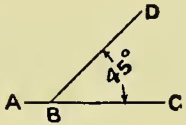



Fig. 107.

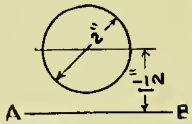



Fig. 108.

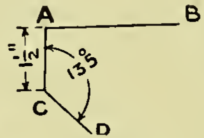



Fig. 109.

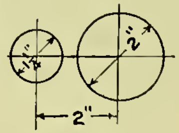



Fig. 110.

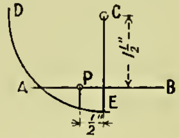



Fig. 111.

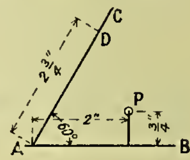



Fig. 112.

# CHAPTER VIII

## USE OF SQUARED OR SECTIONAL PAPER

**77. Introduction.**—Before commencing to study the interesting problems in this chapter, you should read the introduction to the chapter on Areas: you will then see how squared or sectional paper may be used in connection with areas of plane figures, and after you have examined the applications we shall deal with directly, you will understand what great use can be made of it in many branches of practical work; further, if you are fond of thinking things out for yourself, you will soon be using squared paper in a number of interesting ways.

**78. Different Kinds of Squared Paper.**—Squared paper is made with a great variety of rulings, and you should remember that the extra quality papers have the lines on them **lithographed**; they are then more mathematically accurate than papers that are merely **ruled.** Fig. 113 shows a sample of squared paper with $\frac{1}{8}''$ Ruling, and the inch lines thick. Among the other rulings commonly in use may be mentioned: $\frac{1}{8}''$ Ruling, $\frac{1}{2}''$ lines thick; $\frac{1}{10}''$ Ruling, $1''$ lines thick; $\frac{1}{10}''$ Ruling, $\frac{1}{2}''$ lines thick; 2 mm. Ruling, cm. lines thick; Millimetre Ruling, cm. lines thick; and $\frac{1}{4}''$ Ruling.

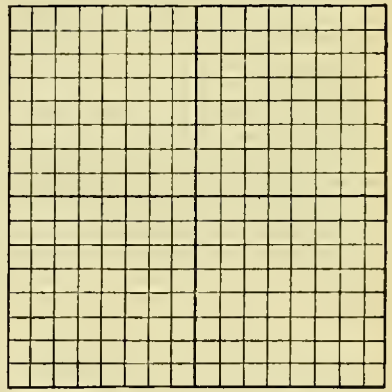


Fig. 113.—Squared paper $\frac{1}{8}''$ ruling, $1''$ lines thick.

The sheets can be had either ruled on both sides or on one side only, and they are commonly made $23'' \times 18''$ and $16\frac{1}{4}'' \times 13\frac{1}{4}''$. Squared paper can also be had in *rolls* of 10 or 50 yards by $26''$ in breadth. **Square tracing papers** can also be had with the same rulings.

**79. Use of Squared Paper.**—The following few articles will give you a good idea of the usefulness of squared paper in working simple problems in co-ordinate geometry, and in graphically solving algebraic equations.

**80. Position of a Point in a Plane.**—In fixing the position of a point in a plane, say the plane of the paper, it is convenient to use two lines, OY and OX, Fig. 114, intersecting at right angles as **axes or lines of reference**; the point O where they intersect or meet is called the **origin.** Let us suppose that P is a point 12 units from[1] OY, and 6 units from OX, the unit in this case being $\frac{1}{8}''$. Then the position of the point is referred to as "the point $(x, y)$," or as "the point (12, 6)." And the lines P6 and P12 are called the **rectangular co-ordinates** of the point P.

**81. Position of a Straight Line in a Plane.**—Obviously, the position of a straight line in a plane can be fixed by stating the positions of its ends,

[1] It is usual to state the distance of a point from OY first.

or, indeed, the positions of any two points in the line, because only one straight line can be drawn through two points. In Fig. 115, the line $PP_2$ passes through the origin, and if you draw through P, lines PN and PM (called co-ordinates) perpendicular to the axes, and draw similar lines through $P_2$, as shown,[1] you will find that $\frac{y}{x} = \frac{y_2}{x_2} \therefore y = \left(\frac{y_2}{x_2}\right)x$. But, of course, for the same straight line the ratio $\frac{y_2}{x_2}$ is constant, and may be written $k$. We may then write the above equation $y = kx$.

And in this form it is known as the **equation of a straight line**, which passes through the origin. Further, a straight line passing through the origin is referred to as a **graph** of the equation $y = kx$.

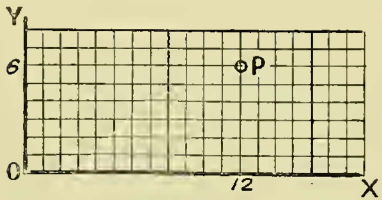



Fig. 114.—Position of a point in a plane.

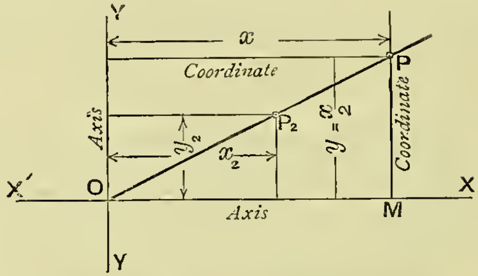



Fig. 115.—Position of line in a plane.

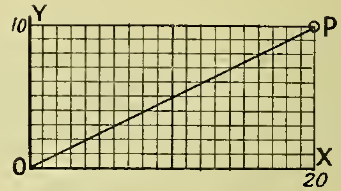



Fig. 116.—Graph of $y = \frac{1}{2}x$.

**82. To draw a Graph.**—Let $y = \frac{1}{2}x$. This being of the form $y = kx$, you now know that the graph is a straight line which must pass through the origin. So the first thing to do is to substitute some convenient value for $x$, say 20 (as large as your paper will take), and calculate the value of $y$, namely, $y = \frac{1}{2} \times 20 = 10$. With these values as co-ordinates, a second point in the graph on your squared paper is found, which, joined to the first, the origin, gives the required graph, as shown in Fig. 116.

**83. Straight Lines that do not pass through the Origin, and their Equations.**—Let us assume that we are to draw the graph of the equation $x = 3y - 20$. This is called an **equation of the first degree**; that is to say, no factor in the equation is a squared quantity or higher order. You can easily prove, as in Article 81, that all equations of the first degree represent straight lines, and, conversely, that all straight lines can be represented by equations of the first degree. The graph is easily drawn if you first find two or more values[2] of $x$ for corresponding values of $y$, namely, 0, 10, 20, 30, and 40.

| | | | |
|---|---|---|---|
| Thus, let | $y = 0$, | then $x = (3 \times 0) - 20 =$ | $-20$ |
| ,, | $y = 10$ | ,, $x = (3 \times 10) - 20 =$ | 10 |
| ,, | $y = 20$ | ,, $x = (3 \times 20) - 20 =$ | 40 |
| ,, | $y = 30$ | ,, $x = (3 \times 30) - 20 =$ | 70 |
| ,, | $y = 40$ | ,, $x = (3 \times 40) - 20 =$ | 100 |

and so on.

[1] You can do this by actual measurement, but of course you will remember that all such triangles as OPM on the figure will be similar, and therefore their bases and perpendiculars will be in the same ratio.

[2] Of course, points in the curves can be determined by giving arbitrary values to either $x$ or $y$. The curve in this case being a **straight line**, of course **two points** only in it **will fix its position** in relation to the axes OY, OX, as you have seen; but it will serve as an interesting and useful exercise to find two or three additional ones.

We are here treating $x$ as an **independent variable**, as it is called, $y$ being the **dependent variable**; but you need not trouble about these terms at present, unless they have been previously explained to you.

Now, to plot the line, you may commence by taking the first value of $y$. You find it is 0, which means that it must be somewhere on the line OX (Fig. 117), but the corresponding value of $x$ is $-20$, which means that it must also be twenty units from the axis OY, but **to the left of O, as it is a minus quantity.** (The **positive values** of $x$ and $y$ are measured to the right and upwards, while the **negative values** are measured to the left and downwards respectively.) The point P satisfies these two conditions, and obviously the points $P_2$, $P_3$, $P_4$, and $P_5$ equally satisfy the above values of their co-ordinates $x$ and $y$, and all these points, you will find, are on the straight line $PP_5$.

**84. Position of a Rectilinear Figure in a Plane.**[1]—If you understand how the position of a point is fixed in a plane in relation to the two axes or lines of reference, as explained in Article 80, you will see that the three points, A, B, and C (Fig. 118) (the corners of a triangle), are fixed by their **co-ordinates,** or each one by two numbers. Thus, A is fixed by the numbers (2, 1), B by (16, 5), and C by (7, 15). So, if you were told to mark on a sheet of squared paper the points (2, 1), (16, 5), (7, 15), and consider them to be the corners of a triangle, your drawing would be like Fig. 118.

**85. An Irregular Polygon (Hexagon) fixed in a Plane by its Co-ordinates.**—You will by now see how exceedingly convenient this method of fixing the position of points in a plane by means of their co-ordinates is, and you will now be able to solve some problems for yourself. For instance, if you were given a sheet of squared paper, and told to plot an irregular polygon (in this case a hexagon) whose corners had the following co-ordinates (24, 1), (20, 16·5), (13, 13), (10, 16), (3, 10), you would produce a figure like that shown in Fig. 119.

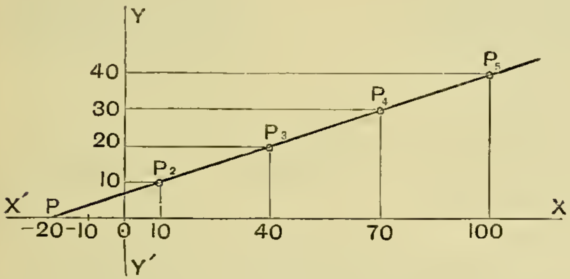



FIG. 117.—Graph of $x = 3y - 20$.

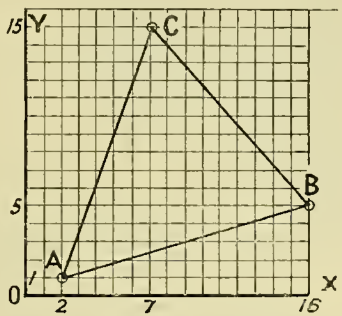



FIG. 118.—Position of a triangle.

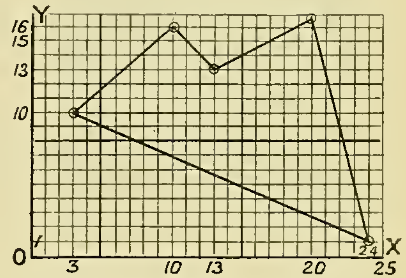



FIG. 119.—Position of an irregular polygon.

**86. Plotting Experimental Results on Squared Paper.**—If you have not already done some work in a mechanics or physics laboratory, you probably soon will, then you will find that the results of experiments in mechanical or physical science are invariably, when practicable, plotted on squared paper, giving what are called **experimental curves,** and showing at a glance the relations between simultaneously varying quantities which are mutually dependent. And you will soon realize that no symbols or figures can possibly convey to your mind so clear an idea of these relations as a simple figure plotted from the experimental data. As an illustration of the method, we will proceed to draw a curve to show how the mean **velocity of water passing through a 10-feet circular sewer or culvert** varied in a particular case with the depth of the water in it. It was experimentally found that, for the fractional parts of the full depth represented in Columns A, the relative velocities were represented by the quantities in Columns B.

[1] In the **ordnance** and other extensive **topographical surveys** the area of which a map is to be drawn is divided into squares, which are surveyed by different persons. All the surveys (squares) being afterwards collected and placed together to form the whole area.

| Table A. Proportion of depth of flow. | Table B. Relative velocity. | Table A. Proportion of depth of flow. | Table B. Relative velocity. | Table A. Proportion of depth of flow. | Table B. Relative velocity. | Table A. Proportion of depth of flow. | Table B. Relative velocity. | Table A. Proportion of depth of flow. | Table B. Relative velocity. | Table A. Proportion of depth of flow. | Table B. Relative velocity. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0·00 | 0·000 | 0·20 | 0·696 | 0·40 | 0·929 | 0·60 | 1·053 | 0·80 | 1·100 | 1·00 | 1·000 |
| 0·05 | 0·367 | 0·25 | 0·765 | 0·45 | 0·968 | 0·65 | 1·069 | 0·85 | 1·097 | | |
| 0·10 | 0·506 | 0·30 | 0·828 | 0·50 | 1·000 | 0·70 | 1·085 | 0·90 | 1·088 | | |
| 0·15 | 0·613 | 0·35 | 0·884 | 0·55 | 1·028 | 0·75 | 1·094 | 0·95 | 1·066 | | |

To set out the curve, take a sheet of squared paper, and from A (Fig. 120), plot the quantities representing the proportions of depth of flow along the axis OX (marked AB), as shown in the figure. Then along the axis OY (marked AC), at right angles to OX, mark off quantities to represent the relative velocities, letting them increase by 0·1.

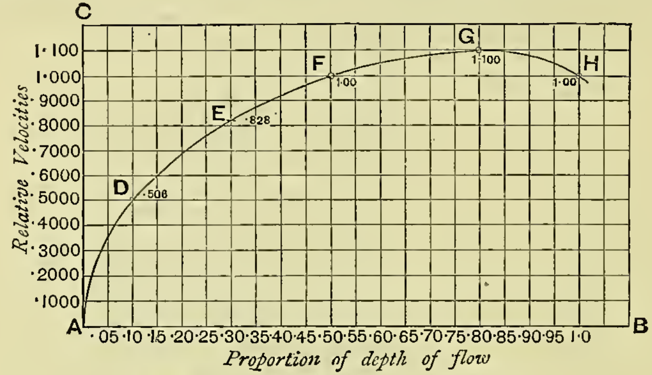



Fig. 120.—Plotting experimental results.

The height of a point to represent any actual relative velocity from the table can then be easily estimated and marked on its proper line; for instance, the highest point will be G = 1·100 for a depth of 0·80, so that the intersection of the line through 0·80 and 1·100 will give this point, and the other points having been similarly found, a fair flowing line through the points will be the required curve.[1]

In some cases only a few points, such as D, E, F, G, H, need be fixed to get the run of the curve. This graphic method of studying mechanical and physical matters has a great additional advantage over any other system, as an error in the determination of any of the quantities is immediately detected owing to its point coming noticeably outside the curve.

[1] In examining the curve it will be noticed that it attains its greatest value at 0·80 of the full depth, this value being considerably greater than at full depth. This clearly shows that when the sewer is running full, it is not discharging to its full capacity. This is mainly due to the increase in the value of the wetted perimeter being much more rapid towards the top than that of the area.

## EXERCISES.

### Suggestions for Oral Questions.

1. If in doubt about the accuracy of the ruling of your squared paper, how would you test it?
2. In fixing the position of a point by its co-ordinates, what is the name of the lines you measure from?
3. What name is given to the point where the axes or lines of reference meet or intersect?
4. State the form the general equation of a straight line which passes through the origin takes?
5. What form does the graph of the equation $y = kx$ take?

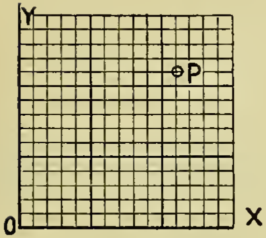



Fig. 121.

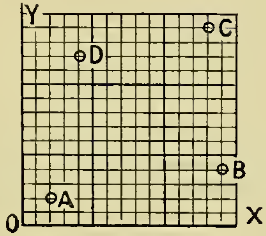



Fig. 122.

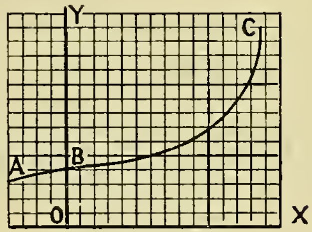



Fig. 123.

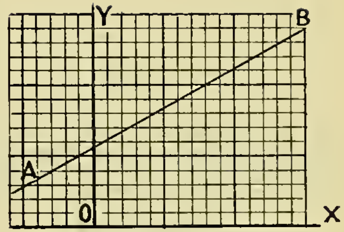



Fig. 124.

6. The large squares on a sheet of squared paper are 1″ and the small ones $\frac{1}{10}$″: how many of the small ones are there enclosed in each large one?
7. What are the co-ordinates of the point P in the given figure? (Fig. 121, which may be drawn on the blackboard). What number represents $x$, and what $y$?

### Drawing Exercises, etc.

8. Write down the co-ordinates of the points A, B, C, and D in Fig. 122.
9. Plot (*i.e.* mark on squared paper) the following points: (6, 13), (2, 15), (12, 9), (4, 17), (10, 18).
10. Plot the points (0, 0), (5, 3), (7, 5), (6, 10), (0, 14), (−5, 13), (−11, 11), (−15, 5), and draw a fair line or curve through them.
11. Write down the co-ordinates of a point P which is 2″ from (15, 0) and (0, 5).
12. The centre of a circle 2″ diameter has co-ordinates (8, 5). Find the co-ordinates of the points on the axes cut by the circle.
13. Plot the line whose equation is $y = 0{\cdot}3x$.
14. Graph the equation $y = 0{\cdot}45x$, and write down the co-ordinates of the point in the line which is 1″ from the $x$-axis.
15. Graph the line whose equation is $x = 2y - 15$.
16. Draw the triangle whose corners A, B, and C have the following co-ordinates: A (3, 1), B (22, 6), C (8, 12). And find the cc-ordinate of a point inside the triangle equidistant from its three sides.
17. Write down a few co-ordinates of the curve ABC, Fig. 123.
18. A line passes through the origin and is inclined 30° to the $x$-axis; find, if you can, the equation of the line.
19. Find, if you can, the equation of the line AB, Fig. 124.

# CHAPTER IX

## AREAS AND THEIR MEASUREMENTS

(*The definitions relating to areas are given at the end of the book*)

**87. Introduction.**—By this time you are probably aware that the boundary-line or **perimeter of any closed figure encloses an amount of surface called its area.** The **unit of area** for ordinary measuring purposes may be a square foot, a square inch, or a square centimetre. A **square foot** is the area of a square drawn on a side one foot in length; a **square inch** (Fig. 125) is the area of a square drawn on a side one inch in length; and a **square centimetre** (Fig. 125) is a square drawn on a side one centimetre in length.

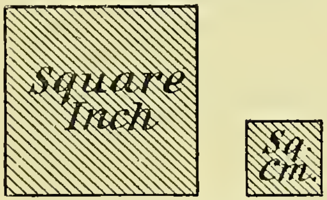



FIG. 125.—Comparison of square inch and square centimetre.

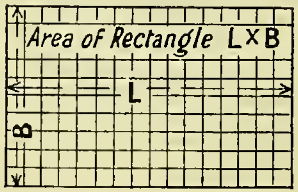



FIG. 126.—Area of rectangular floor.

If we had a rectangular floor laid with square tiles, each one having **sides of one foot**, you would speak of the **area of each tile** as being **one square foot.** Now supposing that on counting the number of tiles in one row you find there are 16, and that there are 10 rows, you would intuitively know, even if you had not been told, that the number of tiles on the whole floor can be found by multiplying the 16 by the 10, giving 160. That is to say, the area of the floor is 160 square feet.[1] Or, in other words, the area of the rectangular floor equals L × B, that is, its length multiplied by its breadth.

Now, supposing you were to cover your sheet of paper with inch squares, draught-board fashion, you would have before you a sheet of **squared paper**, as it is called, and you could mark off along one line of them 16, and along a line at right angles to it, 10. The rectangle so formed would represent the floor, as in Fig. 126, of course to a **scale of one inch to the foot**, or to one square inch to one square foot. The latter, of course, gives the **ratio of the areas**, but this gives you no direct idea of how many times larger the floor is than your piece of paper representing it. However, if you cut out a piece of tracing paper one foot square, and place it over your inch squares and count the number enclosed, you find that there are 12 × 12 = 144, as in Fig. 127; that is to say,

[1] You have doubtless in buying stamps often counted them in this way: you do not trouble to count each one, you count the number in a row and multiply it by the number of rows. Thus, if you were buying five shillings' worth of penny stamps, you would receive 60, and if there were 10 in a row, you would know at once that the number was right if you had 6 rows, as 6 × 10 = 60.

each tile has an area 144 times the area of one of your inch squares. And of course it follows that the scale of areas is 1 square inch to 144 square inches, or 1 to 144; that is to say, 144 pieces of paper containing 16 × 10 inch squares would cover the floor.

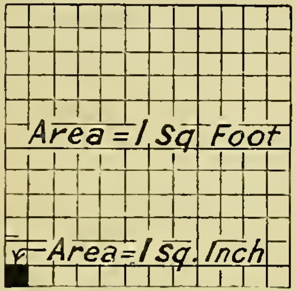



FIG. 127.—Square foot and square inch compared.

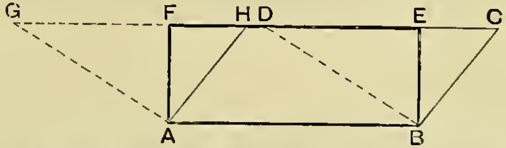



FIG. 128.—Equal parallelograms on the same base.

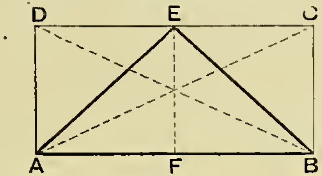



FIG. 129.

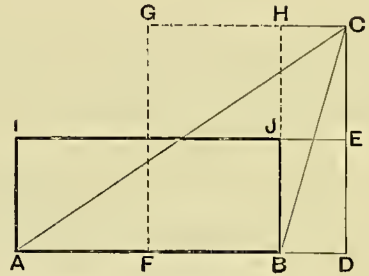



FIG. 130.—Rectangle equal in area to triangle.

It will now be helpful to examine a few typical problems relating to areas of simple figures, and you should experience no difficulty in understanding the following, and the bearing that one may have upon the others.

**88. To construct a Rectangle equal in Area to any given Parallelogram.**—The working of this problem depends upon the geometrical fact **that parallelograms upon the same or equal bases, and between the same parallels, are equal in area** (Euc. I. 35, 36). Thus, the parallelogram ABCH (Fig. 128) is on the base AB, and if on this base we construct a rectangle ABEF, the side EF being on the side CH produced, we shall have the rectangle (which is also a parallelogram) equal in area to the given parallelogram, as required, because the before-mentioned conditions are complied with.

NOTES.—1 It will be seen that the parallelogram ABDG is also on the same base and between the same parallels, and is therefore equal to ABCH and ABEF. The **area of a rectangle**, as you are aware, is expressed by multiplying two adjacent sides together. Thus, area of ABEF = AB × BE. It follows that the **area** of any **parallelogram** is equal to one of its sides multiplied by the perpendicular distance between that side and the opposite one; thus, the area of ABCH = AB × BE.

2. **Experiment.** Make a large drawing of the figure, and with your scissors cut out the triangle AFG. You will find that it exactly covers the triangle BED. Treating BEC in the same way, it will cover AFH. You will know what importance to attach to this **experimental proof**, which with a little thought can be applied to other cases, such as the next problem, for instance.

**89. To draw an Isosceles Triangle whose Area shall be half that of a given Rectangle.**—Let ABCD (Fig. 129) be the given rectangle. Bisect AB by the perpendicular FE, cutting CD in E. Join AE and BE; and AEB is the required triangle.

It will be at once obvious that the rectangle ABCD is cut into two equal parts by EF, and also that the diagonal AE cuts the rectangle AFED into two equal parts; and BE cuts FBCE into two equal parts. Therefore the area of the triangle AEB (made up of AEF and BEF) is equal to the sum of the areas AED and BEC; that is, it is equal to half the rectangle. And as AE and BE are equal, AEB is an isosceles triangle.

NOTE.—The diagonal AC cuts the rectangle into two equal parts, and so does BD. Therefore the triangles ADB and ABC are each equal to half the rectangle, and therefore must equal the triangle AEB. It will be noticed that **the three equal triangles are on the same base and between the same parallels.** Euclid (I. 37) proves that **all triangles on the same base and between the same parallels are equal in area.**

**90. To construct a Rectangle equal in Area to a given Triangle.**—If you understand the previous problem, the working of this one should be obvious.

Let ABC (Fig. 130) be the given triangle. At A and B erect AI and BJ, perpendiculars to AB. From C draw CD perpendicular to AB (or AB produced) and cutting it in D; bisect CD (the altitude) in E, and draw EJI parallel to AB, and cutting AI and BJ in I and J. Then ABJI is the required rectangle. (Based on Euc. I. 41.)

NOTE.—From this it will be seen that the **area of a triangle** may be expressed thus: Area = base × ½ altitude; or, obviously, the altitude multiplied by half base will also give the area. The dotted rectangle BFGH illustrates this.

**91. To construct a Rectangle equal in Area to a given Rectangle, One Side of the former being given.**—The solution is not so obvious as that of the preceding problems, and you need not attempt it if you are reading the chapter for the first time.

Let ABCD (Fig. 131) be the given rectangle, and AE the given side. Then the other side of the required rectangle is a fourth proportional to AE, AB, and AD. Set off along AB from A the given side AE; join E and D, and through B draw BF, cutting AD produced in F. Then AF is the fourth proportional and the required side. Complete the rectangle by drawing FG and EG parallel to AE and AF respectively.

PROOF.—By similar triangles, AB : AF :: AE : AD, ∴ AB × AD = AF × AE; that is, the two rectangles are equal.

NOTE.—This **problem is of great importance** when **moments** have to be manipulated for the geometrical determination of the **centre of gravity** of figures. It is also a simple way of finding **a line to represent a rectangle**, *i.e.* if AE = 1″, then AF would equal the area of ABCD in square inches.

**92. The Pythagorean Theorem.**—If you draw a right-angled triangle (Fig. 132) making its sides in the ratio of 3 : 4 : 5, as shown, and construct on each side a square, and divide each of the squares into smaller squares whose sides are the equal divisions of the lines, the square ACGH, you will find on counting, contains nine small squares, and ABFE sixteen squares, whilst CBJK, the largest one, contains 25. You will be struck with the fact, of course, that 25 = 9 + 16, that is to say, **the area of the square on the hypotenuse of the right-angled triangle equals the sum of the squares on the other two sides.** Indeed, this theorem, which was discovered by Pythagoras about 580 years B.C., and applies to all right-angle triangles, is proved in the 47th Prob. of Euclid's first book.

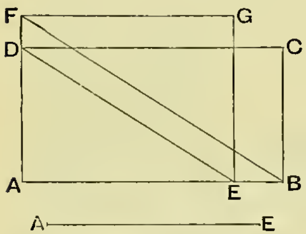


FIG. 131.—Rectangles of equal area.

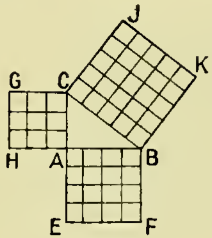


FIG. 132.

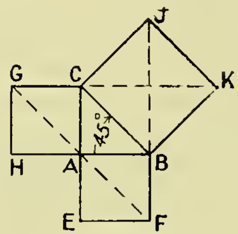


FIG. 133.

The isosceles triangles in Fig. 133 also shows this beautiful relationship between the areas of the squares; for obviously all the triangles into which the squares are divided by the diagonals are similar and equal, therefore the square on CB equals the sum of the squares on AB and AC, as there are four triangles in the former and two in each of the latter. You should draw this figure to a large scale, and cut out the triangles on the small squares, and place them together to cover the large square.

**93. To construct a Square equal in Area to the Sum of Two given Squares.**—You will now experience no trouble in working this and similar problems. Let the given squares be of $1\frac{3}{4}''$ and 1″ sides. Draw BC (Fig. 134) $1\frac{3}{4}''$ long, and AB 1″ long, perpendicular to BC. Join AC, and with AC as side construct a square ACDE, which is the one required.

NOTE.—This problem is of great practical importance, as it equally applies to all **similar figures**, and you should endeavour to understand that **the areas of similar figures are to one another as the squares upon their corresponding sides.** A glance at the dotted triangles and hexagons on the figure will help you to understand this truth, which is further illustrated by Prob. 96.

**94. Area of a Circle.**—If you divide a circle (Fig. 135) into, say, 24 equal parts or sectors as shown, and cut these parts out with a scissors, you can arrange them as in Fig. 136, and see that they form a figure which in shape is very nearly a rectangle. It will occur to you, of course, that the larger the number of divisions you make, the more the figure will approach an exact rectangle.

The area of the rectangle equals that of the circle, of course, and we know that the former = base × height = $\pi R \times R$, or $\pi R^2$ = the area of the circle.

This will conveniently lead up to the next problem.

**95. To construct a Rectangle equal in Area to a given Circle.**[1]—Let the circle (Fig. 137) be the given one. As you have seen, a rectangle whose short sides are equal to the circle's radius, and whose long sides are equal to $\pi R$ (or say $3\frac{1}{7}$ the radius), will equal in area the circle. Then, such a rectangle may be drawn by taking the radius AB as the short side, and at B setting off BC at right angles, and equal in length to $3\frac{1}{7}$ AB. Complete the rectangle by making AD and DC parallel to BC and AB respectively.

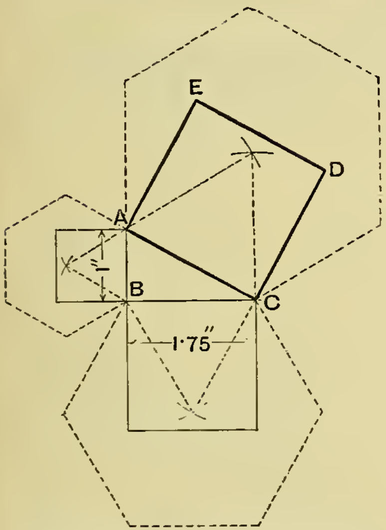



Fig. 134.—Figures equal in area to sum of two similar figures.

Notes.—1. A triangle equal in area to the circle may, of course, have a base equal to BC, and an altitude twice AB (equal to the circle's diameter), or it may have AB as altitude and base twice BC.

2. A **second method** of determining a line equal in length to half the circumference is shown by the dotted lines on the figure. AF is drawn making 30° with AE, and cutting a line through E parallel to BC in F. Mark a point G three times the radius of the circle from B, and the distance FG is a close approximation to $\pi R$.

**96. To describe a Circle equal in Area to the Sum of Two given Circles.**—Let the given circles be of 1″ and $1\frac{3}{4}$″ diameters. Draw BC (Fig. 138) $1\frac{3}{4}$″ long, and AB 1″ long, perpendicular to BC. Join AC, and with AC as a diameter, describe a circle, which is the one required.

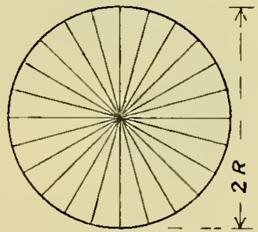



Fig. 135.

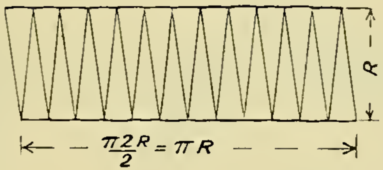



Fig. 136.—Figure equal in area to circle.

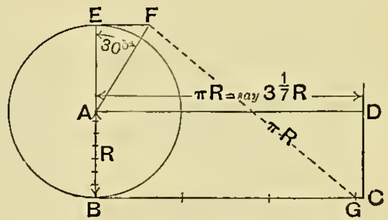



Fig. 137.—Rectangle equal in area to circle.

Notes.—1. **The areas of circles vary in the same proportion as the squares on their diameters.** Thus this problem is similar in principle to Prob. 93.

[1] A simple **practical test for the equality of the areas** (if you have been using a sensitive balance) is to carefully draw the figures (the circle and rectangle) upon a piece of millboard or thin sheet metal, and to cut them out and **weigh** them. The equality of the area of the circle to that of the rectangle ABCD can then be seen. If you draw the figures on squared paper you will not fail to notice that the rectangle is equal to $3\frac{1}{7}$ squares, whose sides are R, and that therefore the area of the circle = $3\frac{1}{7}R^2$. Of course $3\frac{1}{7}$ is only an approximate value of $\pi$, as we have seen.

2. The **areas of all similar figures** vary in a like way, as we have seen; thus, if pentagons were constructed on AB, BC, and AC, the one on AC would be equal in area to the sum of the other two.

3. This problem is of **practical use** when it is required **to find the diameter of a pipe** whose sectional **area** is **equal** to that of **two others**, *i.e.* one which will contain the same quantity of liquid as the two others together. An obvious variation of the above construction enables you to find a circle equal in area to the difference of two others.

4. A very elegant variation of this problem is **Hippocrates' theorem of the lunulæ** (or lunes). The semicircles being similar figures, by the I. 47, the area of the semicircle ACB (Fig. 139) is equal to the sum of the areas of the semicircles ADC and CFB; if from the equals we take away the crosshatched parts AEC and CGB, the remainders will be equal. That is, **the area of the triangle ACB will equal the sum of the areas of the lunes ADCE and CFBG.**

5. It follows from the preceding note, that if the triangle ACB (Fig. 139) be made isosceles, *i.e.* the angles BAC and ABC 45°, each of the lunes will have an area equal to that of the square on the radius of semicircles ADC or CFB. Draw such a figure, and satisfy yourself that it is so. In Fig. 140 we have half such a figure, CE being the axis of symmetry. Here we have the direct construction for making a lune AFEGA equal in area to that of a given square BCDA, where the radius of the semicircle is AD, the side of the square; and the radius of the arc AGE is CA, the diagonal of the square.

**97. To draw a Square whose Area shall equal, say, Five Square Inches.**—The direct method of working this problem is to construct a square upon a line whose length is equal to the square root of the given area. The length of this line will equal $\sqrt{5}$. Then draw a line, AB (Fig. 141), 2″ long, and at A erect the perpendicular AC, 1″ long. Join BC, and the length of this line will equal[1] $\sqrt{5}$. Upon this construct the required square.

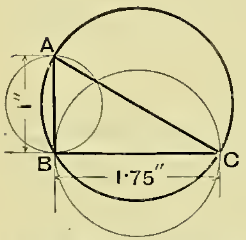



Fig. 138.—Circle equal in area to sum of two others.

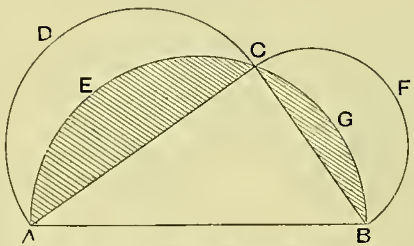



Fig. 139.—Area of the triangle equals sum of areas of the lunes.

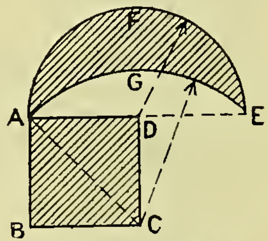



Fig. 140.—Lune equal in area to the square.

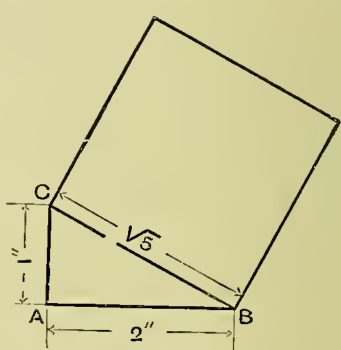



Fig. 141.

**98. To make a Square equal in Area to a given Rectangle.**—Let ABCD (Fig. 142) be the given rectangle. Produce BA to E, making AE equal to the side AD.

Find a mean proportional (AF) of AB and AE (Prob. 64. Also refer to Euc. II. 14, III. 35). Then upon AF construct the required square.

Note.—If the side AD of the rectangle be made equal to 1″, then, as we have seen, the **length** AB of the other side in inches **will represent the area of the rectangle**, also that of the square, and AF will equal the **square root** of the number representing the area.

---

[1] You will remember that the length of the side of any square is equal to the square root of the area of the square. For example, a square whose area is 4 square inches will have sides whose length is $\sqrt{4''} = 2''$, and one whose area is 1 square inch will have sides $= \sqrt{1''} = 1''$, and, again, one whose area is $\frac{1}{4}$ square inch will have sides $= \sqrt{\frac{1}{4}''} = \frac{1}{2}''$. Bearing in mind these facts, and making use of Prob. 97, you will be able to **draw a square of any given area**, or any line whose length is a root quantity, expressed thus $\sqrt{N}$, where N is known.

**99. To reduce any Irregular Figure to a Triangle of equal Area.**—Let ABCDE (Fig. 143) be the figure. Join AD, and through E draw a line parallel to AD, cutting AB in F; join DF. Similarly, join BD, and through C draw CG parallel to DB, cutting AB produced in G. Join DG, and the triangle FDG is equal in area to the given figure.

This will be readily understood when it is seen that the triangles BDC and BDG are upon the same base, BD, and between the same parallels, BD and CG, and are therefore equal in area (Prob. 89, and Euc. I. 37); that is, we have cut off the triangle BDC, and put on one, BDG, equal in area, and therefore have not altered the area of the figure; and, in the same way, at the other side of the figure, we have replaced the triangle FEA, on the base FE, by the triangle FED, on the same base and between the same parallels.

NOTE.—This principle once thoroughly understood, the most complicated rectilinear figure can be easily reduced to a triangle.

## Land Surveying

**100. Measurement of Land.**—Surveyors measure land with a chain invented by a Mr. Gunter, and therefore known by the name of *Gunter's Chain.* It is 22 yards, or 4 poles, in length,[1] and is divided into 100 equal parts, called links, each link being 7·92 inches.

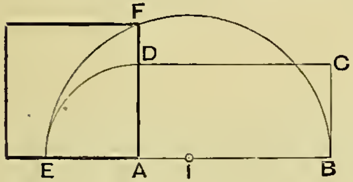



FIG. 142.

**100A. An Acre** of land is equal to 10 square chains; that is, a strip of land 1 chain in breadth and 10 in length (Fig. 144) has an area of 1 acre; and this equals $22 \times 220 = 4840$ square yards ($= 66 \times 660 = 43560$ sq. ft.); or $4 \times 40 = 160$ square rods, or perches; or $100 \times 1000 = 100{,}000$ square links.

It is usual to give the measurement in acres, roods, and perches: 4 roods being an acre, and 40 perches a rood.

A statute-pole is $16\frac{1}{2}$ feet long: but in different parts of the country there are by custom poles of different lengths—21, 18, 15 feet, etc.

**101. The Land Surveyor's Operations.**—The fundamental principles underlying the operations in any survey are the same. There are three separate operations:—

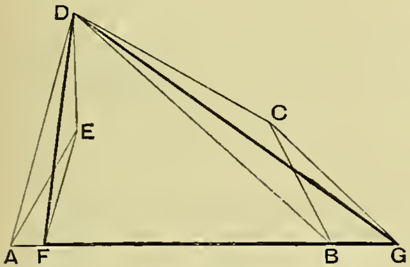



FIG. 143.—Triangle equal in area to irregular figure.

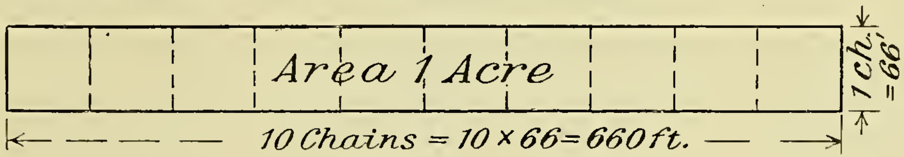



FIG. 144.

1st. The taking of the measurements[2] on the ground.

[1] The chain used by civil engineers is 100 feet in length, with 1 foot links.

[2] **The origin of the science of geometry** appears to be due to the efforts made in the early ages to measure land. This naturally suggested problems on the areas of triangles and other simple figures; indeed, the earliest solutions of such problems appear to be afforded by a Papyrus in the British Museum, giving rules for the calculation of triangles, trapezoids, and circles, which is believed to have been copied from a much older work about 1700 years before the Christian era.

2nd. Making the drawing or plan. In other words, plotting the measurements on paper.

3rd. From such plans or drawings measuring the areas, or arranging the work for which the survey was made.

Measurements in the field are taken either by linear or angle measuring instruments, or by a combination of both; but whatever the system of measurement may be it is a necessary condition of good practice to **check all measurements** (with the exception of **offsets** of a few links in length) by other linear or angular measurements.

Having made these passing remarks on a branch of work that has much to do with the geometrical side of our subject, we may pass on to the examination of a typical example or two.

**102. The Field-book.**—The usual method of entering the field-notes is to *begin at the bottom of the page* and write upwards. Each

FIELD-BOOK.

| | Links. | |
|---|---|---|
| | To B | |
| | 1456 | |
| C 722 | 882 | |
| From | A | go East |

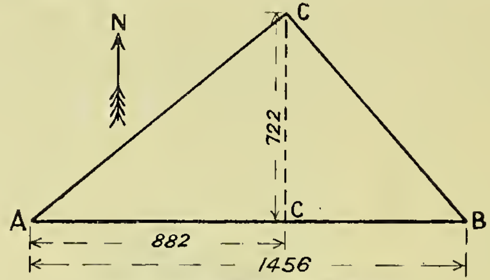



FIG. 145.—Field of three sides.

page of the field-book is divided into three columns. In the middle column is set down the distances on the chain line, at which any offsets, marks, or observations are made, and these offsets, etc., are entered in the right and left-hand column, as you will see.

**103. To measure a Field of Three Sides.**—Let ABC (Fig. 145) be the field. At each corner of the field place a station staff.[1] Then, in chaining from A to B (going east), measure till you find *c*, from which a perpendicular will rise[2] to the opposite corner C, and enter in the field-book the distance from the end A to this point, namely, 882 links, and then continue the measurement of the line, 1456 links; this length being the distance from A to B. The perpendicular (or **offset**) is now measured and found to be 722.

Centuries later the annual overflow of the Nile, and the consequent destruction of landmarks of the different proprietors who paid tribute to the king, led to the necessity of measurements being made each year to restore the marks, and gave an impetus to the study of simple geometrical problems.

The only geometry known to the Egyptian priests was that of surfaces, together with a sketch of that of solids, a geometry consisting of some simple quadratures and cubatures which they had apparently obtained empirically.

The ancient Greeks too, judging from the problems discussed by Hero of Alexandria, were led to study the science in connection with the surveys of their mines.

[1] A pole some 6 ft. in length, usually painted in red, black, and white, in broad alternate bands. Used with a red or white flag, when the poles are placed far apart.

[2] The exact position of c along the line AB is generally found by using a small instrument called an **optical square**.

**To Calculate its Area** proceed as follows:—

CALCULATION.

Triangle ABC.

```
      1456 Base AB
       772 Offset Cc
      ----
      2912
     10192
    10192
    ------
2 | 11,24032 Double area
    --------
    5,62016
          4
    -------
    2·48064
         40
    -------
    9·22560   Ans. Area = 5 a. 2 r. 9¼ p.
```

**104. To measure a Field with several Sides.**—If the field is somewhat elongated, as in Fig. 146, it is often convenient to choose the longest diagonal AE, as a **base line.** Then, after fixing station staffs or poles at the corners A, B, C, D, E, F, and G, to chain the line AE, measuring till you find a position from which a perpendicular will rise to the first corner B, and enter the measurement, as explained in Art. 102, and shown in the figure (146) which accompanies the field-book. Complete the survey by repetitions of this operation for the corners G, C, F, and D.

**105. Offsets, and how to measure them.**—It will be noticed that in Figs. 145 and 146 the straight lines which are measured in the field do not exactly coincide with the uneven boundary line of the field. This necessitates occasional **offsets** being taken, to the right or left as may be required, and these offsets are measured by what is called an **offset-staff**, a round wooden rod, usually 10 links in length. The offsets are entered in the field-book by taking a separate column for each side, AB, BC, etc., of the field, and entering the offsets as we did in connection with Fig. 146.

FIELD-BOOK.

| | Links. | |
|---|---|---|
| | To E | |
| | 1263 | |
| | 1096 | D 142 |
| F 156 | 989 | |
| | 863 | C 254 |
| G 280 | 581 | |
| | 456 | B 120 |
| From | A | go North |

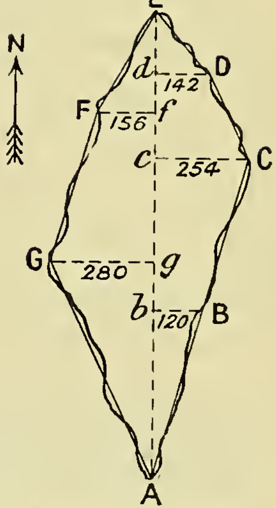



FIG. 146.—Plan of field.

**106. To find the Area of a Rectilinear Enclosure in Square Feet.**—Let Fig. 147 be the enclosure, with the measurements in feet,[1] tabulated in the field-book as in Art. 105. Then we have:—

FIELD-BOOK.

| | Feet. | |
|---|---|---|
| | To D | |
| | 140 | |
| | 120 | C 35 |
| E 40 | 90 | |
| | 30 | B 50 |
| From | A | |

CALCULATION OF AREA.

$$\triangle\, AbB = \tfrac{1}{2}Ab \times bB = \tfrac{1}{2} \times 30 \times 50 = 750$$
$$\text{Trapezium } bBCc = bc \times \tfrac{1}{2}(bB + cC) = 90 \times \tfrac{1}{2}(50 + 35) = 3825$$
$$\triangle\, cCD = \tfrac{1}{2}cD \times cC = \tfrac{1}{2} \times 20 \times 35 = 350$$
$$\triangle\, ADE = \tfrac{1}{2}AD \times eE = \tfrac{1}{2} \times 140 \times 40 = 2800$$

Area = 7725 sq. feet

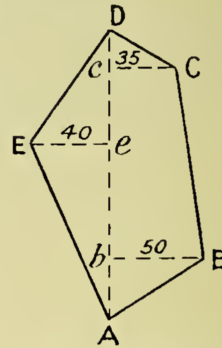



FIG. 147.—Rectilinear enclosure.

That is, the required area is 7725 square feet.

**107. To draw a Square or any Regular Figure equal in Area to a Closed Figure bounded by an Irregular Curved Line.**—The irregular figure shown in Fig. 148 may be supposed to be the plan of a field, which has been drawn from notes made in the field-book from the surveying operations. The object of the problem is to find a square or rectangular plot equal to it in area. You may take a piece of tracing paper and draw across it a fine straight line, and place it over the figure (Fig. 148) and prick off the positions of such lines as AB, BC, CD, and DA, being careful to place them so that the lines **give-and-take**; that is, cut off as much of the figure as they add to it. Most students have an eye true enough to enable them, with care, to draw in this way a straight-line figure (or polygon) closely approximating to the curved one. This figure can be divided into triangles, and these in their turn converted into rectangles and equivalent squares, etc., by the previous problems, as may be required.

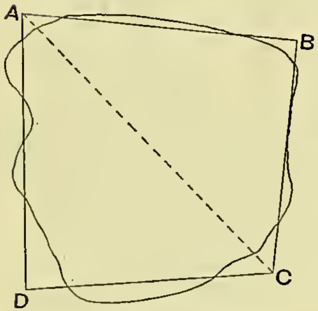



FIG. 148.—Irregular figure.

NOTE.—The area of the figure can be accurately measured by the use of an instrument called a planimeter, and its perimeter can be approximately measured by rolling along the boundary a coin or the lid of a pill-box, taking account of the diameter of the rolling circle and the number of revolutions.

**108. To find the Area of a Figure bounded by Three Rectangular Lines and an Irregular Curved Line.**[2]—FIRST METHOD.—Draw a fine line on a piece of tracing paper, and hold the paper over the curved line EC (Fig. 149) till the straight line EF becomes a give-and-take line; that is to say, place it so that it cuts off as much

[1] Measured by using an engineer's chain of 100 feet and a tape measure for the offsets.

[2] You will ere this have noticed that it is only in the case of a few regular figures that the area is connected by any simple relation with the linear dimensions, so that it can be calculated from those dimensions. In a great many difficult cases the following practical method of measuring an area can be adopted with advantage. Draw the figure on a sheet of cardboard, millboard, or thin sheet metal, cut out the figure and weigh it in a suitable balance. If you know by a separate experiment how many square units (centimetres or inches) weigh one gramme, you can thus find the area of the figure which has been cut out.

of the figure ABCD as it adds to it, as in the previous problem. A little careful judgment will enable you to do this with a degree of accuracy near enough for many practical purposes. Having fixed the line, prick off the points E and F where it cuts AD and BC. Then, obviously, $AB \times \frac{AE + BF}{2}$ will equal the area of the given figure.

**109.** Another Method of measuring Areas.—**Given a Closed Figure bounded by an Irregular Curved Line, representing a Field to a given scale (say, 3 chains to the inch), to determine its Approximate Area in Acres.**[1]—This required area could be determined by Prob. 107, by first finding an equivalent figure bounded by straight lines, and then by dividing the figure as found into triangles, or triangles and trapezia, whose areas could be readily measured by using a chain scale, and by adding the areas of the triangles, etc., to determine the whole area. But a very **simple, practical way** of determining the area is to set out a number of parallel lines on a **sheet of tracing-paper**, making the distance between them $= \frac{10}{(\text{No. of chains to the inch})^2} = \frac{10}{3^2} = \frac{10}{9} = 1\frac{1}{9}''$ for the scale of three chains to the inch, to which the figure is drawn. Now, if this sheet of tracing-paper is placed over the figure (as in Fig. 150), the lengths of **equivalent strips** of width $1\frac{1}{9}''$ into which the figure is divided can be measured (as shown) by a rule (or, better still, a strip of paper, ticking off the separate lengths), and each inch in length will be equal in area to one acre.

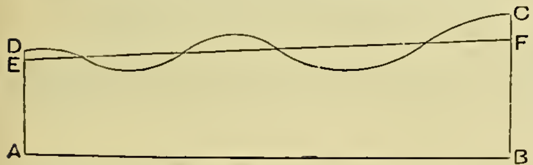


Fig. 149.—Example of a give-and-take line.

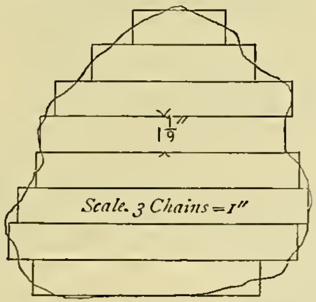


Fig. 150.—Measure of acreage.

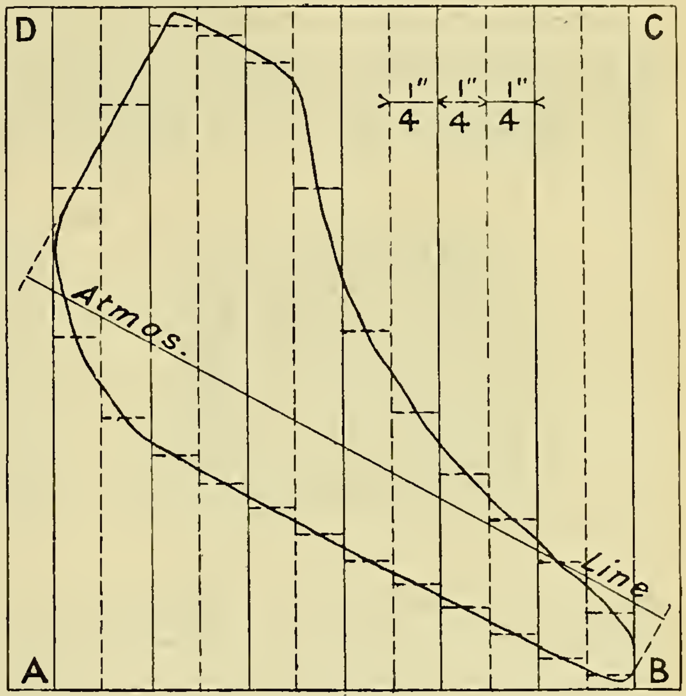


Fig. 151.—Area of an indicator diagram.

**110. Given a Closed Figure bounded by an Irregular Curved Line, representing an Indicator Diagram from a Steam Engine, to determine its Approximate Area in Square Inches.**—As will be seen from Fig. 151, the expedient employed in the previous problem may be used, making the distance between the parallel lines on the tracing paper (or celluloid), $\frac{1}{4}''$, then the sum of the lengths

[1] This ready way of determining such an area is sometimes practised by surveyors, when the figure represents a plot of land.

of the parallel strips divided by four will obviously give the area of the figure in square inches. As a matter of fact, the sum of the lengths in this case is 15·375″, as you will find, if you carefully measure the lengths of the strips. So that the area = 15·375 ÷ 4 = 3·844 square inches.

NOTE.—The error due to want of accuracy (the personal equation) should not exceed 5 per cent.

**111. Area of Figures on Squared Paper.**—The following typical cases should help you to measure areas of figures on squared paper. The figures may be either drawn on the squared paper, or **squared tracing paper** may be used to place over the figure or over part of a drawing.

(*a*) **When a Side or Diagonal of the Figure can be made to coincide with a Line of the Squared Paper.**—The area of any rectilinear figure ABCDE (Fig. 152) can be found with the aid of right-angled triangles and rectangles. The given figure, you will see, is

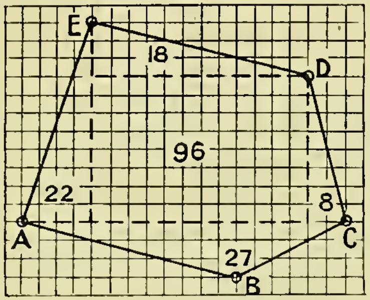



FIG. 152.—Area on squared paper.

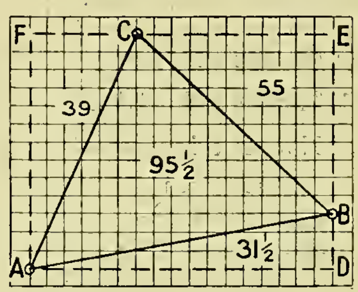



FIG. 153.—Area of triangle.

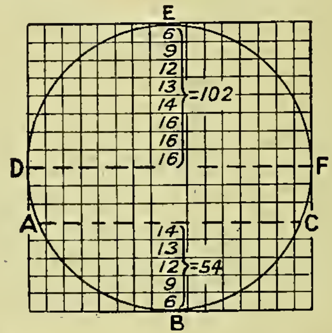



FIG. 154.—Approximate area of segments.

divided into four triangles and a rectangle, the numbers marked on the figures indicating the number of squares each contains. Thus the area of the whole figure equals 22 + 27 + 8 + 18 + 96 = 171 squares. Now, if the sides of the squares be 10 to the inch, obviously the area of the figure equals $\frac{171}{10^2}$ = 1·71 square inches.

(*b*) **When an Important Line of the Figure does not coincide with a Line of the Squared Paper.**—As in Fig. 153, it is convenient to draw rectangular lines outside the figure, making, if possible, a rectangle (ADEF) whose area can be readily determined (in this case 17 × 13 = 221), then by subtracting the right-angled triangles ADB, BEC, and CFA from the rectangle, the required area of ABC is found; that is, area of ABC = 221 − (39 + 55 + 31½) = 95½ squares.

(*c*) **When the Figure is Curvilinear.**—Although the areas of such figures cannot be found exactly by the **method of counting squares**, approximate values may be easily determined in the following way.

Let the figure be the segment of a circle ABCA (Fig. 154). In counting the squares covered by its area you will notice that the arc ABC *cuts through* several squares, and the question arises, how are you to deal with these broken squares? Of course you could estimate the fractional value of each one included in the boundary line or arc, but this would be a tedious proceeding; however, a *fairly correct value* can be found by *counting* 1 *if the broken square is more than half a complete square, and counting* 0 *if less than half a square.* Applying this rule to the figure, we find that its area is 54 squares.

Now, of course, if the segment had been a semicircle, we could more easily calculate its area, arithmetically, by first finding the area of the circle, whose diameter in this case would be represented by 16, the area being $R^2 \times \pi = 8^2 \times \pi = 64 \times 3{\cdot}14159 = 201{\cdot}062$, and half this, or 100·531 squares, is the area of the semicircle. But applying the method of counting squares as a test, as in the upper half, DFED, of the circle, we find that the area is 102, as shown, or roughly $1\frac{1}{2}$ per cent. over the true value.

## EXERCISES.

### Typical Oral Questions.

1. What is the name given to the length of the **boundary line** of a plane figure? And what name is given to the extent of its **surface**?
2. What do you understand by the term **area** of a plane figure?
3. Does a figure with a large perimeter necessarily have a large area?
4. What are the units by which areas are usually measured?
5. The area of a triangle is 4 square inches, and its altitude is 3″. What will the length of its base be?
6. What's the shape of the triangle which has the largest area for a given perimeter?
7. A square has an area equal to the sum of the areas of two other squares, one on a 3″ side, and the other on a 4″ side. What would be the area of the square, and the length of its side?
8. Enunciate the Pythagorean theorem.
9. The sides of a right-angled triangle are 2″ and 3″. Give the length of its hypotenuse as a root quantity.
10. The adjacent sides of a rectangle are 2″ and 8″. What is the length of its perimeter? What is the length of the side of a square equal to it in area?
11. What is the **unit of area** in measuring land?
12. Give the lengths of the sides of a rectangular piece of land whose area is one acre.
13. What is the length of Gunter's chain used for measuring land?
14. A strip of land 2 chains in breadth is 1 acre in area. What is its length?
15. Your cricket pitch is 22 yards long, and your average step 1 yard. You are allowed to use half an acre of land each side of the pitch. How many steps away from the pitch would give you the approximate boundary of the acre?

### Drawing Exercises.

Note.—In working the following exercises it will sometimes be convenient to prick off corners of the figures (or other suitable points in them) by placing a page of this book over your drawing paper.

16. Carefully measure the rectangle (Fig. 155), and give its area in square centimetres.
17. Draw a rectangle equal in area to the given parallelogram (Fig. 156).
18. Draw an isosceles triangle equal in area to the given parallelogram (Fig. 156). Note.—You are to first draw figures like this full size from the dimensions.
19. Draw a square equal in area to the sum of the two squares (Fig. 157).
20. Draw a square equal in area to the cross-hatched figure (Fig. 158).
21. Draw a rectangle approximately equal to the given figure (Fig. 159) when drawn full size.
22. The radii of an annulus are 2″ and 1″. Draw a circle equal in area to the surface between the two circles.

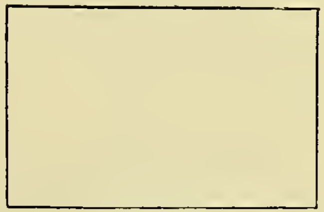


Fig. 155.

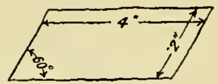



Fig. 156.

23. Draw a square equal in area to the rectangle (Fig. 155).
24. Draw a triangle equal in area to the quadrilateral (Fig. 160), making A its apex.
25. The base of a triangle is 4″ and the other sides make angles of 35° and 45° with it: determine the area of the figure, and check by calculation.

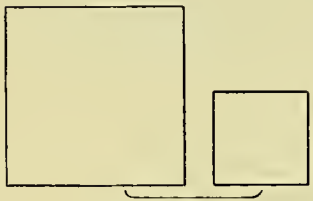


Fig. 157.

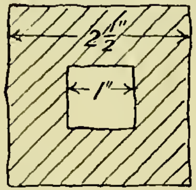



Fig. 158.

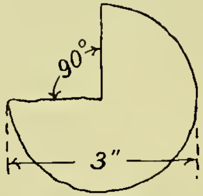



Fig. 159.

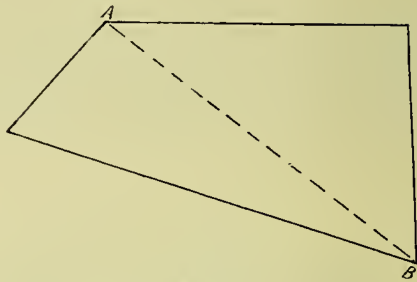



Fig. 160.

26. Two sides of a triangle are 3″ and 4″, and the included angle 40°: determine the area of the figure, and check by calculation.
27. Three sides of a triangle are 3″, 4″, and 2½″: determine the area of the figure.
28. Draw a rectangle equal in area to the given figure (Fig. 161), making AB its base. Note.—Refer to Problems 99 and 90.
29. Convert the irregular hexagon (Fig. 162) into a rectangular figure.
30. Carefully make the following measurements of the triangle ABC (Fig. 163): (*a*) the angle $\phi$; (*b*) the side CB; (*c*) the area in square centimetres.
31. You are to plot the triangular field from the given note below from a field-book (refer to Art. 102), and measure its area in acres, roods, and perches. Scale 1″ to the chain.

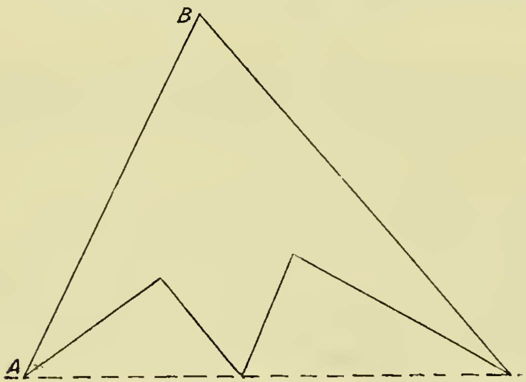



Fig. 161.

Fig. 162.

| | Links. | |
|---|---|---|
| | To B | |
| | 1264 | |
| | 782 | C 456 |
| From | A | go West |

Field-book note. Question 31.

32. The field notes of a six-sided field are given: plot (or draw) the field, and measure its area. Scale 1" = 1 chain.

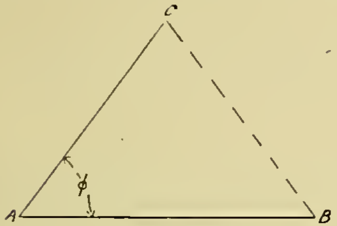



FIG. 163.

| | Links. | |
|---|---|---|
| | To D | |
| | 1070 | |
| | 828 | C 40 |
| E 50 | 762 | |
| | 436 | B 108 |
| F 84 | 315 | |
| From | A | go North |

Field-book note. Question 32.

FIG. 164.

33. The plan of a piece of land is given (Fig. 164), it is drawn to the scale of ½" to the chain. Measure as nearly as you can its area in acres, etc.
34. Carefully set out the given figure (Fig. 165), and measure its diagonal BD, and the area of the figure in square inches.
35. The plan of a piece of land is shown (Fig. 166). AD is a base line, and the angles at the base have been measured, also the lengths of two sides. BC is measured on the ground as a check. What should its length be, if all the measurements are correct? The scale that was used in plotting the figure is 1" = 1 chain.

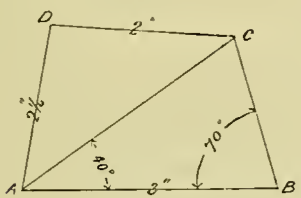



FIG. 165.

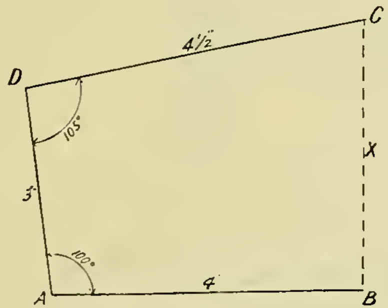



FIG. 166.

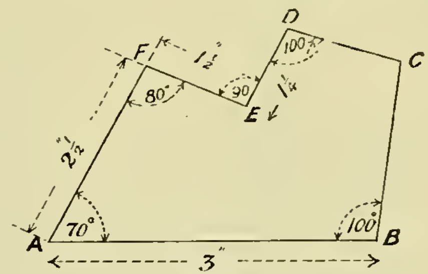



FIG. 167.

36. Carefully draw the figure (Fig. 167) from the dimensions given, and measure and write down the length of the diagonal AC. Also measure with your protractor the angle at C, and check it by calculation.

NOTE.—The sum of all the interior angles of a polygon is equal to twice as many right angles, less four, as the polygon has sides.

37. Set out the figure (Fig. 168) with great care, and measure the diagonal AC, the angle at C, and the distance of P from B.
38. Measure the area of the sector (Fig. 169) in square centimetres, after carefully reproducing it on your drawing paper.
39. Make a drawing of the section of a buttress wall (Fig. 170), scale half inch to the foot, and calculate the weight of the wall per foot run, assuming the weight of a cubic foot to be 140 lbs. NOTE.—You will be able to measure the area of the section in square feet. The product of this area and 140 gives the weight.

40. The section of a cast-iron bearer bar is given (Fig. 171). Draw it full size, and calculate the weight of the bar: its length is 36″, and the weight per cubic inch may be taken to be ¼ lb. Note.—AB is the bottom of the section.

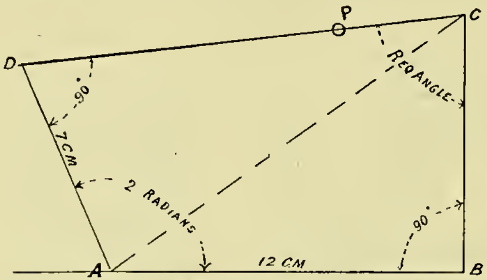



Fig. 168.

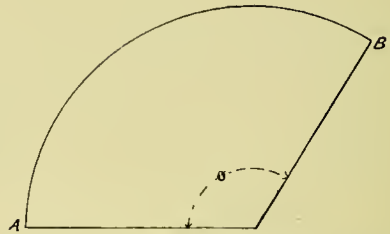



Fig. 169.

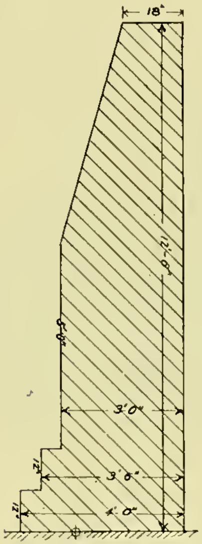



Fig. 170.—Buttress wall.

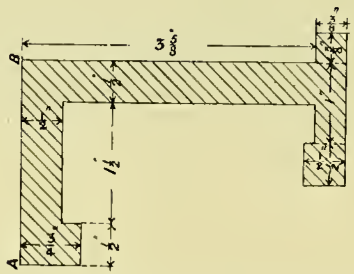



Fig. 171.—Section of cast-iron bearer.

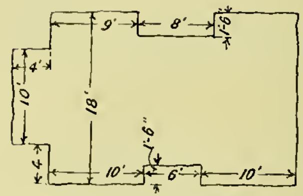



Fig. 172.—Plan of a floor.

41. The figured plan of a floor is given in Fig. 172. Make a drawing of it to a scale of ¼″ to 1 foot, and calculate the number of square yards of linoleum that would be required to cover the floor.

# CHAPTER X

## REDUCING AND ENLARGING FIGURES, ETC.

**112. Introduction.**—As the practical draughtsman is sometimes called upon to reduce or enlarge figures, we may with advantage give a little attention to the expedients usually employed in such operations. If you happen to have a good pair of **proportional compasses**, you can by means of this simple instrument reduce or enlarge drawings so that all the lines of the copy shall bear any required proportion to the lines of the original drawing. We will briefly describe the instrument, and work just one problem to show how it is used.

**113. Proportional Compasses.**—The ordinary form of this instrument is shown in Fig. 173. It can be either set to deal with **lines**,[1] **or** with **areas**. **To set the instrument** you must first accurately close it so that the two legs appear as one, the nut C of course being unscrewed; you next move the slider (attached to the nut) until the line across it coincides with any required division upon either of the scales; you now tighten the nut, and the compasses are ready for use. It should be mentioned that proportional compasses can also be used to inscribe regular polygons in circles, and extract the square roots and cube roots of numbers, but no one troubles about using this instrument for such purposes now. Further, it is only of use for any purpose when in perfect adjustment and in skilful hands.

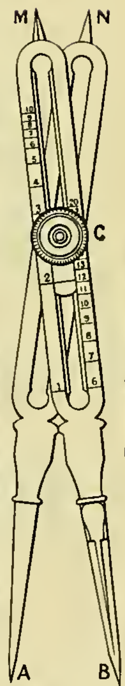


FIG. 173.

**114. To reduce or enlarge the Lines of a Figure, using Proportional Compasses.**—Let us suppose that the irregular polygon ABCDEF (Fig. 174) is the figure which is to be reduced to a similar one whose sides shall be, say, half those of the given one. From any corner F draw lines to corners B, C, and D, dividing the figure into triangles. Now set the compasses so that the line across the slider coincides with the division 2 on the scale of lines. The points A, B of the instrument (Fig. 173) will then open to double the distance between the points M, N (Euc. VI. 4). Next open the points A, B to the length of the side FA (Fig. 174), and prick off F*a* with the points M, N, making F*a* half FA. In the same way find the points *b*, *c*, *d*, and *e*, and join these points to form the reduced copy, *abcde*F, of the given figure. Obviously, if *abcde*F had been the given figure, and we had to enlarge it by doubling the lengths of its sides, we should in drawing the lines through the corner F produce them beyond the corners of the given figure, and apply the points M, N of the compasses to the given lines, the distance between the points A, B giving the lengths of the corresponding lines in the enlarged figure.

Of course you will understand that in enlarging figures any error made in measuring a line to be enlarged will be proportionally increased in the new figure. We may now work a few cases by ordinary geometrical methods, commencing with the one we have just worked by using the proportional compasses.

**115. To reduce a given Irregular Figure to a similar one whose Sides shall be, say, half of those of the given one.**—

[1] A simple form of proportional compasses is made called **wholes and halves**, because the longer legs are twice the length of the shorter ones. This instrument is also useful for **dividing lines by continual bisection.**

First Method.—Let ABCDEF (Fig. 174) be the given figure. First divide the figure into triangles by drawing lines FB, FC, and FD through any corner, F. Then from F along FE mark off F*e* equal to half FE, and through *e* draw *ed* parallel to ED and cutting FD in *d*, and complete the required figure by drawing *dc*, *cb*, and *ba* parallel to DC, CB, and BA, as shown in the figure.

Note.—Of course you will remember that **the areas of all similar figures** are as the squares on their corresponding sides (Prob. 93); therefore the area of the reduced figure equals one quarter the area of the given one, as shown by the squares EFGH and *efgh* in Fig. 175.

You should also remember that **the corresponding sides of similar figures or polygons are proportional.** Thus, *ab* : AB :: *a*F : AF, or the ratios of pairs of corresponding lines are all equal.

Second Method.—Let ABCDEF (Fig. 175) be the given figure. Then draw *ab* equal to half AB anywhere parallel to AB, and join A*a* and

## REDUCTION OF FIGURES.

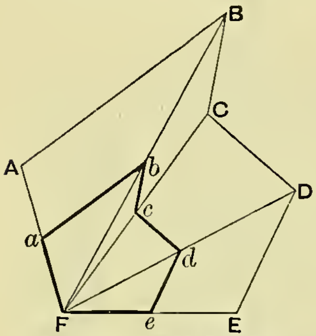


Fig. 174.—First method.

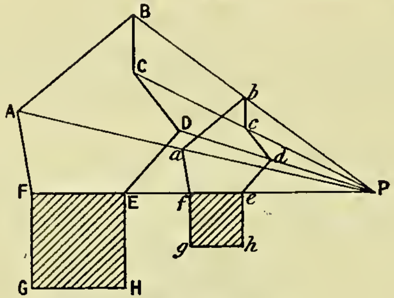


Fig. 175.—Second method.

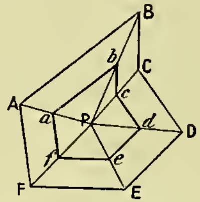


Fig. 176.—Variation of second method.

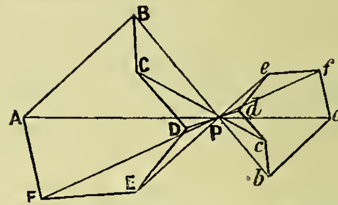


Fig. 177.—Method giving inverted figure.

B*b*, and produce the lines to meet in P. Through P draw lines to C, D, E, and F. Then through *b* draw *bc* parallel to BC, and cutting PC in *c*. And by repeating this operation complete the similar figure *abcdef* as shown.

**116. Variations of the Previous Methods.**—Obviously, if the point P (Fig. 175) be placed inside the figure, as in Fig. 176, the reduced figure *abcdef* can be drawn as shown.

Or, if the lines through P be produced as in Fig. 177, the reduced copy can be drawn the other side of P as shown. Of course the new figure then becomes inverted.

**117. To reduce a given Figure bounded by a Curved Line to a similar one of Fixed Size.**—The principle employed in working a problem like this can be readily understood by examining Fig. 178, where the figure ABCDEF is reduced to *abcde*F, by first drawing from F lines FD, FC, and FB, to any suitable points D, C, B in the curve, and from a point *a*, which is fixed by *a*A, the amount of reduction required, draw a parallel to AB, cutting FB in *b*, and from *b* draw *bc* parallel to BC, and cutting FC in *c*, and so on, to complete the figure *abcde*F, the curve being afterwards drawn through the points *a*, *b*, *c*, *d*, and *e*.

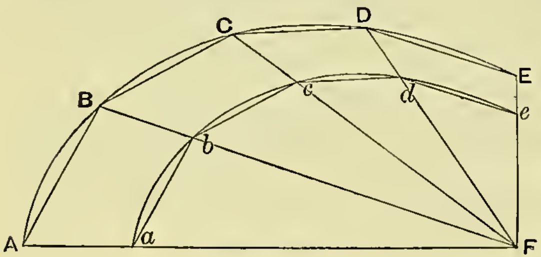


Fig. 178.

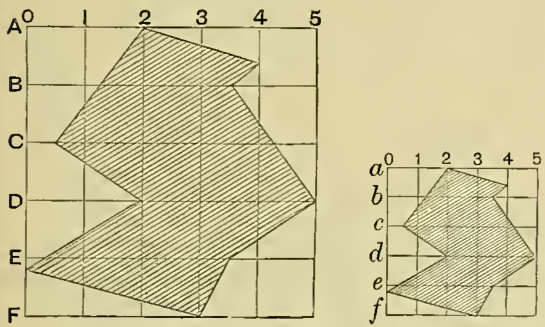
Note.—Of course the **curve only** might have been given **to be reduced.** The point F would then be any point taken at pleasure, and the same construction could be employed.

**118. Reducing and Enlarging Figures by the Use of Squares.**—Complicated figures can be readily reduced or enlarged by first drawing over the figure a number of squares, as shown in Fig. 179, or placing over it a sheet of squared tracing paper. It is then only necessary, if the figure is to be reduced, to draw another set of squares to the required scale (Fig. 180), and points in the squares corresponding to those on the other figure can be readily marked, and the required figure drawn. The figures shown explain themselves, and need no further remark.

NOTE.—Complicated figures are more easily reduced by using the pantograph, an instrument used by the land surveyor for reducing drawings. In every case of enlarging, the greatest accuracy is necessary both in drawing, by either of the methods explained, and in manipulating the pantograph, as it is obvious that original errors are magnified by enlarging, and new ones are often made.

**119. To construct a Figure similar to a given Figure but with Twice its Area.**—Let ABCDEFG (Fig. 181) be the given figure. By this time you are quite familiar with the fact that areas of similar figures are to each other as the squares on the corresponding sides of the figures. So, obviously, if you draw BH at right angles to AB, and equal to it in length, the square on the hypotenuse AH will have twice the area of the square on AB (Prob. 92). So with A as centre, and radius AH, the cut AB produced in *b*, then A*b*, will be the base of the new figure. The required figure A*bcdefg* can now be drawn as in Problem 115.

NOTE.—Problem 97 will help you to deal with figures whose areas are in any other ratio.



FIGS. 179, 180.—Reduction by use of squares.

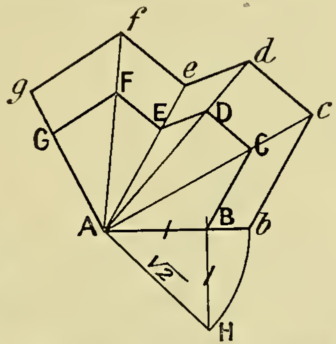


FIG. 181.—Doubling area of figure.

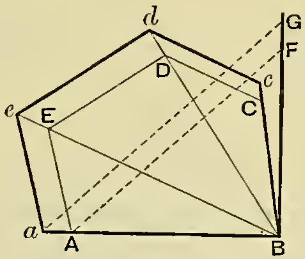


FIG. 182.

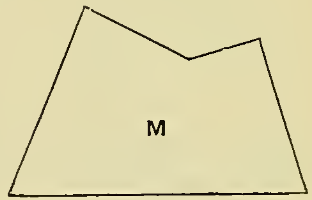


FIG. 183.

**120. To construct a Figure similar to a given Figure, ABCDE, Fig. 182, and having an Area equal to that of given Figure, M, Fig. 183.**—First reduce the given figures into triangles of equal area (Prob. 99), and then the triangles into squares of equal areas (Probs. 90 and 98). Then set off BF and BG, the sides of the squares, along a line BG through B, making any suitable angle with AB (this angle in the figure is 90°); the side of the square representing the figure ABCDE being BF, and that representing the figure M being BG. Join F to A, and through G draw G*a* parallel to FA, cutting BA produced in *a*. Then *a*B is the base of the required figure, which can be completed by drawing *ae*, *ed*, and *dc* parallel to AE, ED, and DC respectively, cutting the lines BE, BD, and BC produced in *e*, *d*, and *c*. (Refer to Prob. 115.)

$$\text{PROOF.—}\quad \frac{\text{Area } aBcde}{\text{area ABCDE}} = \frac{aB^2}{AB^2} \text{ (Prob. 93)}$$

$$\text{But } \frac{aB^2}{AB^2} = \frac{BG^2}{BF^2} = \frac{\text{area of M}}{\text{area of ABCDE}}$$

NOTE.—This represents a very important type of problem, and when you understand the expedients employed, you should experience no difficulty in working any variation of it.

**121. The Mass-centre, or Centre of Area or Gravity.**—The mass-centre of every straight line is its geometrical centre, and the mass-centre of any triangle is in a line bisecting it and its base, the distance of the mass-centre or c.g. (centre of gravity) being one-third the length of the bisector from the base.

This can be easily understood by reference to the figure ABC (Fig. 184). We may suppose that the triangle consists of a number of lines placed side by side parallel to AB. Then, as the mass-centre of each line is its geometrical centre, the line CD,[1] which passes through all these centres, will contain the c.g. of the whole figure.

But for the same reason BE, which bisects AC in E, contains the c.g., therefore the intersection of these two lines BE and CD is the mass-centre of

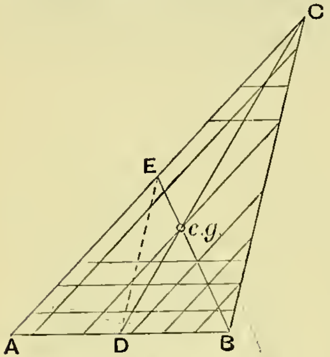



FIG. 184.—Mass-centre.

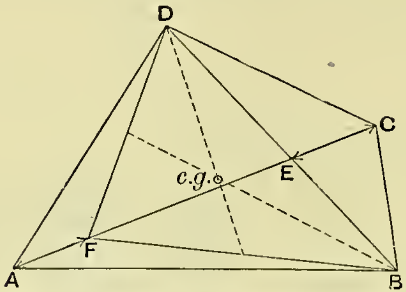



FIG. 185.—Mass-centre of quadrilateral figure.

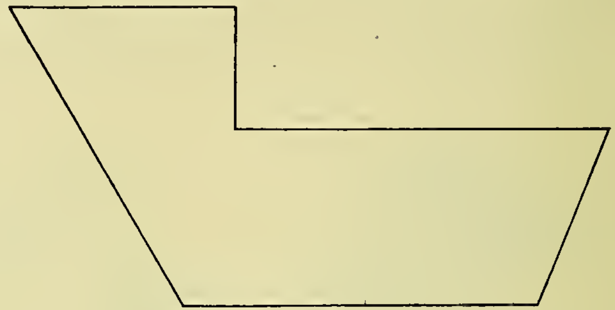


FIG. 186.

the figure. Proof—Join E to D, then, as the triangles ADE and ABC are obviously similar, ED will be parallel to CB, and the triangles ED(*cg*) and CB(*cg*) are also similar.

Therefore D(*cg*) : (*cg*)C : : ED : CB
But it will be seen that AD : AB : : DE : BC : : 1 : 2

Or D(*cg*) : (*cg*)C : : 1 : 2, that is (*cg*)D = $\frac{1}{3}$ CD, or the **mass-centre is $\frac{1}{3}$ the centre line from the base.**[2]

**122. To find the Mass-centre of any Quadrilateral Figure.**—Let ABCD (Fig. 185) be the given figure. Then join the opposite corners BD and AC. The lines intersect in E. Then set off AF from A along AC equal to CE. Draw BF and DF, and by the previous case find the c.g. of the triangle BDF, and it can be shown that this point is the c.g. of the given figure.[3]

**Alternative Method.**—Each diagonal AC and BD divides the figure ABCD into two triangles; if the c.g.'s of these two pairs of opposite triangles be joined, the intersection of the two lines will also contain the c.g. or mass-centre of the figure.

[1] This line is called the **axis of skew symmetry.**

[2] If over the area of a figure we have a distribution of small equal areas with equal masses (or weights), such that for every bit of the area, however small, the mass is proportional to the area, then there is a **uniform distribution of mass**, and the area is said to be **evenly covered or loaded.**

[3] You will be assisted in this reflection by satisfying yourself that the area of the quadrilateral BADF is equal to the area of the triangle BDC.

## DRAWING EXERCISES.

NOTE.—In working the following exercises it will sometimes be convenient to prick off corners of the figures (or suitable points in them) by placing the page of the book over your drawing paper.

1. Reduce the irregular polygon (Fig. 186) to a similar one, all the linear dimensions being halved. Work this exercise in two different ways, and compare the reduced copies.

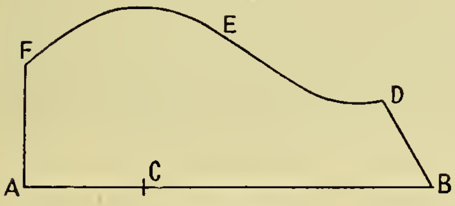



FIG. 187.

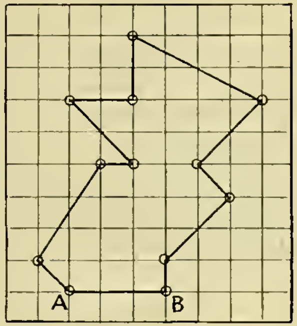



FIG. 188.

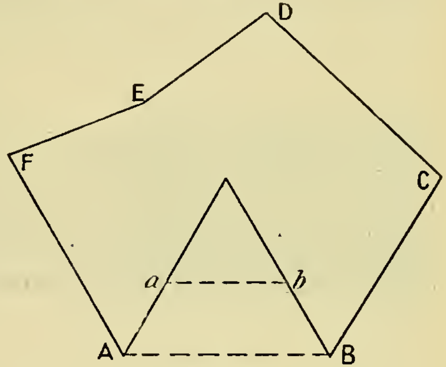



FIG. 189.

2. Draw a figure on base CB (Fig. 187), similar to the one on base AB.
3. Carefully copy the figure given in Fig. 188, and by using squares, as in the figure, draw an enlarged copy, doubling all the linear dimensions.
4. On *ab* (Fig. 189) as base, construct an irregular heptagon similar to the one ABCDEF on base AB. And draw two squares whose areas are in the same proportion as the areas of the polygons. Further, from these estimate what the ratio of the areas of the polygons is.
5. Set out the polygon (Fig. 190) to the dimensions given, then draw a similar one but with double its area.

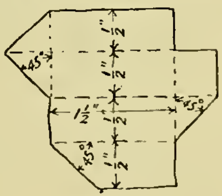



FIG. 190.

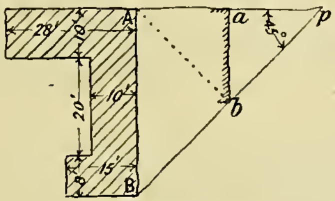



FIG. 191.

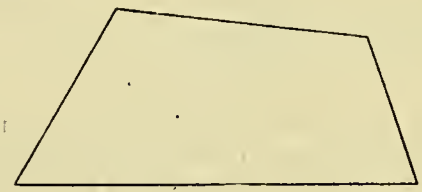


FIG. 192.

6. Draw a figure similar in shape to the polygon in Fig. 186, but with an area equal to that of ABCDEF, Fig. 189.
7. AB (Fig. 191) is the base of the plot of ground shown. Set out the plot, scale $\frac{1}{10}$″ to the foot, and draw a reduced copy of the plot on base *ab* which is half AB.
8. Draw a scalene triangle with sides, 2″, 4″, and 4½″, and determine the position of its centre of gravity, or mass-centre.
9. In two different ways find the mass-centre of the quadrilateral figure (Fig. 192).

# CHAPTER XI

## SYMMETRY AND SYMMETRICAL FIGURES

**123. Introduction.**—In Chapter II. you will remember dealing with some simple symmetrical figures. For instance, Fig. 27 shows a rectangular figure symmetrical about a centre line AB, and this line is called the **axis of symmetry**, whilst in Fig. 28 we have a square figure symmetrical about two axes of symmetry at right angles. You may have often seen pieces of paper folded, and cut with the scissors, which when opened out present very pleasing symmetrical figures. Fig. 193 shows a piece of foolscap paper, and MN is the folded edge. The folded sheet is cut along the lines MCN to form a triangle, and when the paper is opened out the **kite**, $MCNC_2$, is formed, and MN is the axis of symmetry. If you fold a sheet of, say foolscap, paper twice, so that the folds or creases are at right

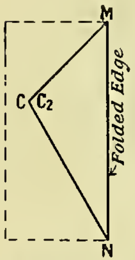



FIG. 193.

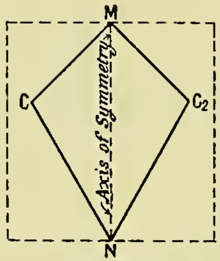



FIG. 194.—Axis of symmetry.

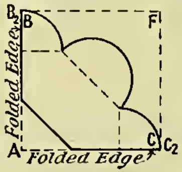



FIG. 195.

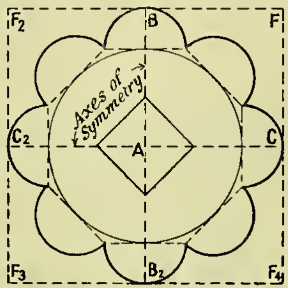



FIG. 196.—Axes of symmetry.

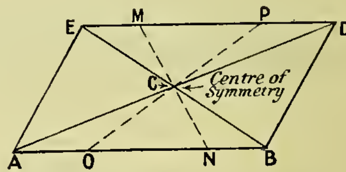



FIG. 197.—Centre of symmetry.

angles, and cut it along the original edge of the paper to form a square, ABFC, Fig. 195, then any pattern may be cut, such as shown in the figure, and when unfolded you will have made a figure with two axes of symmetry, as in Fig. 196. If the corner at A had been clipped off as shown, then a square hole would appear in the unfolded figure. When once you have seen how easy it is to produce symmetrical figures in this way, you will often be tempted to experiment with scissors and paper.

**124. Centre of Symmetry.**—If you draw a parallelogram, Fig. 197, and through C, the intersection of its diagonals, you draw a number of straight lines, such as MN and OP, across the figure, you will find that they are all bisected at the centre C, which is then said to be the **centre of symmetry**; and the parallelogram is said to be **symmetrical about the point C.**

**125. Use of Squared Paper in Drawing Symmetrical Figures.**—Squared paper lends itself to the easy construction of symmetrical figures. We have an example in Fig. 198. Suppose the curved figure at the left of the centre line MN be first drawn, the right-hand half could obviously be easily drawn by using the corners of the squares as centres of the arcs. Indeed, the positions of any points corresponding to others on the other half of the figure can easily be located in this way.

**126. Practical Applications of Symmetrical Figures—Engraving, Lithography, and Printing.**—Obviously, the figures E and Ǝ, Fig. 199, are symmetrical about the **axis of symmetry**, MN. Now, suppose that the E is drawn in ink on a sheet of paper, CDEF, and the ink remains wet; if the paper be folded along the line MN, the impression Ǝ on MNDE will be made; and whilst this is wet you could get an impression of the E on another sheet of paper, which would be of course the original E. Now, if you understand this simple operation, you know something about the principle made use of by engravers and lithographers. For the object of the arts practised by these workers is to form on the surface of a block or plate of some suitable substance, such as wood, metal, or stone, certain figures, of which the impression is printed or transferred exactly to some other surface. Now, let us suppose that the E on CNMF is the figure required to be printed. Then, obviously, the figure on the block must be drawn in a reversed position, as on NDEM. For this reason the **types** used by the compositors to print this page are **cast reversed**, and placed in the frame by proceeding from right to left, in order that, when applied to the paper, the letters they produce may be in their natural position, and be read from left to right.

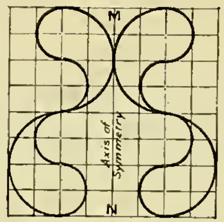



Fig. 198.—Axis of symmetry.

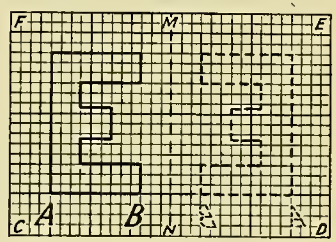



Fig. 199.—Symmetry by impression.

Thus mere impression does not produce copies equal to, or like the figure on the block or plate, but symmetrical reversed ones. Compare an **Indiarubber stamp** with the impression it makes, or some written matter with its impression on the blotting-pad, and you will better understand this.

**127. Stereotyping.**—In stereotyping, **matrices** are engraved, drawn or composed, and by means of them impressions are made on plates, which are again employed in the ordinary way to print drawings, music, or writing, etc. Of course, at the first impression, the figures pass from the left to the right (as in Fig. 199), and at the second repass from the right to the left. Therefore in stereotyping, the printed figures are identical on the primitive matrix and on the copies taken from the intermediate plate.

## EXERCISES.

### Oral Exercises.

1. When has a figure a line or axis of symmetry? Mention any geometrical figure you can think of that has an axis of symmetry.
2. Mention two geometrical figures each of which has two axes of symmetry at right angles to each other.
3. Give the name of any figure that has a centre of symmetry.
4. Has a scalene triangle an axis of symmetry?

### Drawing Exercises.

5. Draw the following figures and mark on them their axes of symmetry: an isosceles triangle, a rectangle, a pentagon, and a hexagon. How many axes of symmetry has the last-named?

6. Show by drawings the symmetrical figures that can be produced by cutting folded sheets into the following shapes: (*a*) a scalene triangle with its hypotenuse along the crease; (*b*) a sector with an angle of 60°, a side along the crease; (*c*) a rhombus with a side along the crease.

7. By using a piece of tracing paper, trace Fig. 187, and reproduce the figure on your drawing paper by pricking it off. Then draw the symmetrical figure produced by using AB as the line of symmetry.

8. Draw on a piece of squared paper the letter **L**, assume any axis of symmetry, as in Fig. 199, and then draw the shape the impression of this figure would make if it was drawn in ink, and you in blotting it made an impression on the blotting paper.

# CHAPTER XII

## THE ELLIPSE

**128. Introduction.**—Look at Fig. 203.[1] No doubt you have often heard figures that shape called **ovals**, although the proper geometrical name of the figure is **ellipse.** Strictly speaking, an **oval is broader at one end than at the other**—in fact, egg-like in form.[1] If you saw off a piece of broomstick, and the saw-cut is on the slant, as in Fig. 202, you know that the shape of the cut will not be circular, but elliptical, as in Fig. 203; and you no doubt also know that if a cone is cut, as in Figs. 200, 201, the cut or section is also an ellipse.[2] In fact, an ellipse may be defined as a curved figure formed by the intersection of a plane and a cone or of a plane

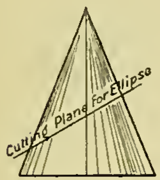



FIG. 200.—Cone cut by plane.

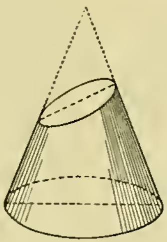


FIG. 201.—Elliptical section.


FIG. 202.—Cylinder cut by plane.

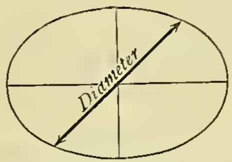



FIG. 203.—Showing axes and a diameter.

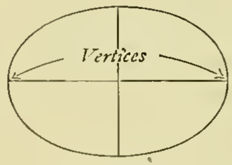



FIG. 204.

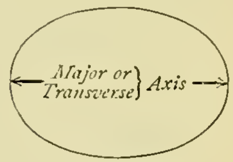



FIG. 205.

and a cylinder, when the plane passes obliquely through *opposite sides* of either of the solids.

An ellipse is **symmetrical about two** lines or **axes** which are at right angles to each other (Fig. 203). One, the **major** or transverse **axis** (Fig. 205), passes through the centre, and is the longest line that can be drawn on the figure, the other, the **minor** or conjugate **axis** (Fig. 206), bisects the major axis perpendicularly, and is the shortest line that can be drawn on the figure.

[1] Oval from *ovum* "an egg." Elliptical mirrors are commonly, but erroneously, referred to as being oval.

[2] The projection of a circle on a plane inclined to its surface is also an ellipse. It may interest you later to know that if a triangle move in such a way that two of its corners always lie in two fixed lines, the *locus* of the third corner will be an ellipse.

You may have been told that the paths in which the planets move round the sun are ellipses, the centre of the sun being one of their foci. Over 3000 years passed in studying astronomy and geometry before this beautiful truth was discovered.

**129. Some Definitions and Properties of the Curve.**—If you are reading this book for the first time, you need not trouble about the following definitions, etc. They appear here primarily for reference purposes.

**Diameter.**—Any line passing through the centre of an ellipse and terminated both ways by the curve is called a *diameter*. Hence it is manifest that the centre bisects all diameters (Fig. 203).

**Vertices.**—The extremities of a diameter are called *vertices* (Fig. 204).

**Transverse Axis.**—The diameter which passes through the *foci* is called the *transverse axis*. This is also called the *major axis* (Fig. 205), as we have seen.

**Conjugate Axis.**—The diameter which is perpendicular to the transverse axis is called the *conjugate axis*. This, as we have

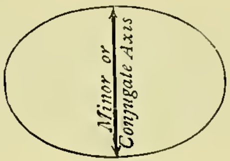



FIG. 206.

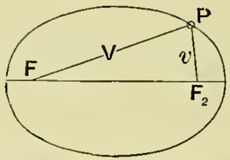



FIG. 207.—Showing the focal distances.

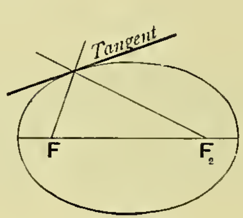



FIG. 208.

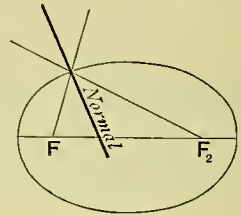



FIG. 209.

seen, is also called the *minor axis* (Fig. 206). Any two **diameters** are said to be **conjugate** when the tangents at the vertices of one diameter are parallel to the other diameter.

**The Focal Distance.**—The distance of a focus from the nearest vertex is called the *focal distance*, and the distances of the foci[1] from any point in the curve the *focal distances*[1] or **radii vectors**, such as V and $v$ in Fig. 207.

**Tangent.**—The line which passes through a point in an ellipse (Fig. 208) and bisects the exterior angle formed by the focal lines at that point, is called a *tangent*.

**Normal.**[2]—A line perpendicular to a tangent at the point of contact (Fig. 209) is called a normal to the curve. This line, therefore, bisects the focal angle.

**Conjugate Diameter.**—A diameter which is parallel to a tangent at a given point is said to be conjugate to the diameter which passes through this point (the sum of the squares on conjugate diameters is constant).

**The Area of an Ellipse** is found by multiplying the product of the major and minor axes by $\frac{\pi}{4}$ (= 0·7854, or, say, $\frac{11}{14}$ very nearly).

[1] **Foci,** plural of *focus*. **The sum of the focal distances is a constant quantity.** Due to this property of the curve, **elliptical wheels** can be geared to give a **slow forward and quick return motion,** each axis passing through a focus, and the distance between the axes or centres being equal to the major axis or the sum of the focal distances.

[2] Lat. *norma*, "a square or rule;" perpendicular.

## Simple Problems relating to Ellipses.

**130. To draw an Ellipse (First Method) as the Locus of a Point.**—The elliptical curve may be generated by a point moving in such a way that its distance from a fixed point (called a **focus**), Fig. 210, is in a constant ratio to its perpendicular distance from a fixed straight line (called a **directrix**), the generating point being nearer to the fixed point than to the line. Every complete ellipse has two *foci* and two directrices, as shown in the figure. Let A, E, D, and B be points in the curve; then the ratio referred to—

$$\frac{\text{focus to vertex}}{\text{vertex to directrix}} = \frac{CA}{AF} = \frac{CE}{EG} = \frac{CD}{DH} = \frac{CB}{BF} = \textbf{eccentricity.}$$

If this be understood, you will be able to easily determine the positions of any number of points in the curve when this ratio (or **eccentricity**, as it is called) is known. And a fair or flowing line through the points so determined will give the curve.

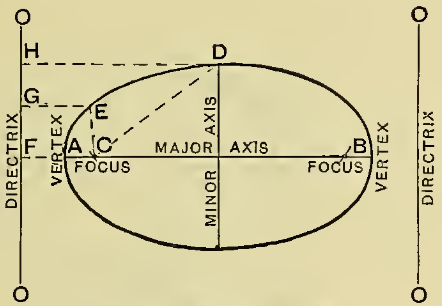



Fig. 210.—Ellipse, as the locus of a point.

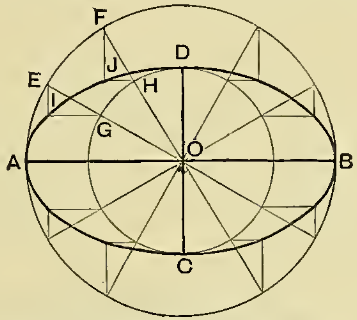



Fig. 211.—Ellipse, constructed by concentric circles.

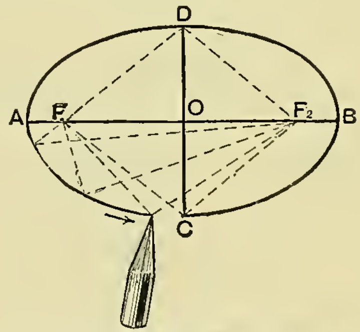



Fig. 212.—Ellipse, drawn by string method.

**131. To draw an Ellipse, having given the Major and Minor Axes—Second Method: By Concentric Circles.**—Let AB and CD, Fig. 211, be the two given axes. Place them at right angles to each other at their centres O. Then with O as centre, and radii OA and OD, describe two circles (the major and minor auxiliary circles), and from centre O draw lines (at any angles to AB) cutting the circles, as at GE and HF. From E and F draw lines EI and FJ parallel to CD (the minor axis), and through G and H draw parallels to AB (the major axis), cutting the parallels to CD in I and J, two points in the required curve; so draw a fair line through AIJD, and a quadrant of the ellipse is formed. Complete the curve, by finding other points in the same way, as shown in the figure, or by symmetry.

**132. To draw an Ellipse, having given the Major and Minor Axes—Third Method: Mechanically, by means of a Piece of String and Two Pins.**—Let AB and CD (Fig. 212) be the given axes. Place them at right angles to each other at their centres O. Then with radius AO (half major axis), and centre C or D, describe arcs cutting AB in F and $F_2$. Then these points are the foci of the ellipse. In each of these points stick a small pin; also place one in C (or D). Then pass a thread or string round the three pins, and tie the ends, making the string taut. The string now forms a triangle, $FCF_2$. Substitute a pencil for the pin at C, and move it along, keeping the string taut, and the pencil will trace a true ellipse.[1]

NOTE.—This method of drawing an ellipse is of great service to practical men, as the curve can be readily drawn on the work or material, but great care must be taken not to vary the **tension of the string**. **Gardeners** trace their **elliptical flower beds** in this way, stakes or poles being stuck in the ground at F and $F_2$, around which a cord is used.

**133. To draw an Ellipse, having given the Major and Minor Axes—Fourth Method: By Paper Trammel.**—Let AB and CD, Fig. 213, be the two given axes at right angles to each other at their centres O. Then if on a strip of paper, or the straight-edge of a card, the semi-axes be marked from any point, the strip can be used to find points in the curve in the following way. The strip *ac*, shown in the figure, is marked with *ac* equal to AO, the semi-major axis, and the distance *bc* equal to CO, the semi-minor axis. Then, if the points *a* and *b* on the straight-edge are kept on the axes, as

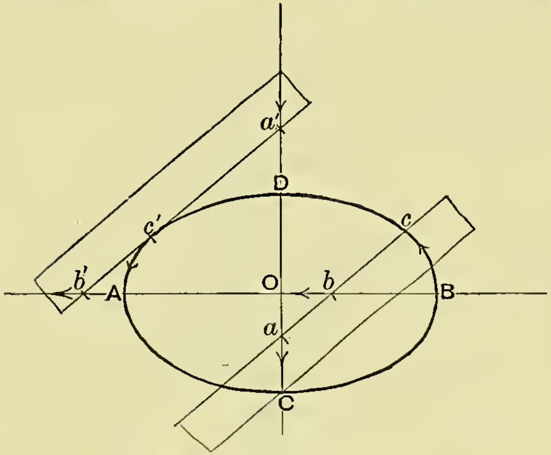



FIG. 213.—Ellipse, paper trammel method.

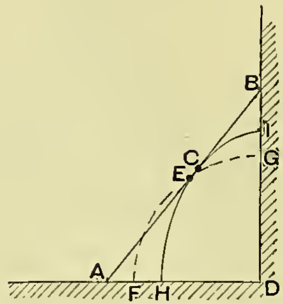



FIG. 214.—Quadrant of an ellipse as locus of a point in the line AB.

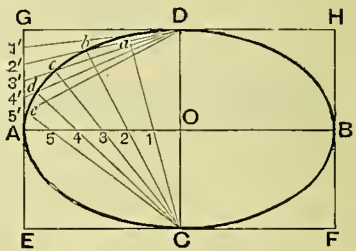



FIG. 215.—Rectangle and the inscribed ellipse.

shown, the point *c* will trace the curve. **When the two axes are nearly the same length,** *ac* and *bc* should be set off each side of *c*, as shown at *b'* and *a'* on the second strip. The curve is then traced by *c'*, whilst *a'* and *b'* are kept on the axes and the axes produced. **Tracing cloth or paper** can be used with advantage; any line on the cloth or paper can then be used instead of one of its edges, and the points pricked through *c* or *c'*.

NOTES.—1. This is a favourite expedient with draughtsmen to rapidly find a few points in an ellipse.

2. The **carpenter's elliptical trammel** is based upon this principle.

3. It directly follows from this problem that if we cause a line AB (Fig. 214) to move with its ends A and B on two lines AD and BD at right angles to each other, any point C on the line will trace the quadrant of an ellipse ICH. It is obvious that when the point is at **E, in the middle of AB,** the curve traced will be a **quadrant of a circle,** GEF.

[1] You will understand this expedient when you remember that **the sum of the focal distances of any point on the curve is always equal to the major axis.** Thus (Fig. 212) $FD + F_2D = AB$. This is important, and should not be forgotten.

**134. To draw an Ellipse, having given the Major and Minor Axes—Fifth Method: By Radial Lines from the Ends of the Minor Axis.**—Let AB and CD (Fig. 215) be the given axes at right angles to each other at their centres O. Then through AB and CD draw the rectangle EFHG, and divide AO and AG into any suitable number of equal parts (say six) in the points 1, 2, 3, 4, 5 and 1′, 2′, 3′, 4′, 5′ respectively. From D draw lines to pass through the points 1′, 2′, 3′, 4′, 5′, and from C draw lines passing through the points 1, 2, 3, 4, 5 to cut the other radial lines in the points *a*, *b*, *c*, *d*, *e*, which are points in a quadrant of the required ellipse. A repetition of this construction for the other quadrants completes the figure, or the figure may be completed by symmetry.

NOTE.—This construction is also applicable to cases where conjugate diameters do not intersect at right angles as they do above (see Prob. 138).

**135. To determine the Major and Minor Axes of a given Ellipse.**—Commence by drawing any two parallel lines across the given figure, such as EF and GH (Fig. 216). Bisect each of them at L and M respectively. Join L and M, and produce the line in both directions to cut the figure in I and J. Then IJ is a diameter,[1] and its centre O, which is found by bisection, is the centre of the ellipse. With this point O as centre, describe any arc cutting the curve in Q and P; and bisect the arc QP in R. Join OR, and produce the line both ways to cut the curve in A and B. Then AB is the required major axis, and the minor axis CD is found by drawing a perpendicular to AB at its centre O, cutting the curve in C and D.

**136. To draw a Tangent to a given Ellipse at a Fixed Point in the Curve.**—First find the major axis AB (Fig. 217), as in the previous

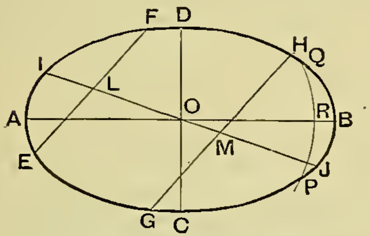



FIG. 216.—Construction giving the major and minor axes.

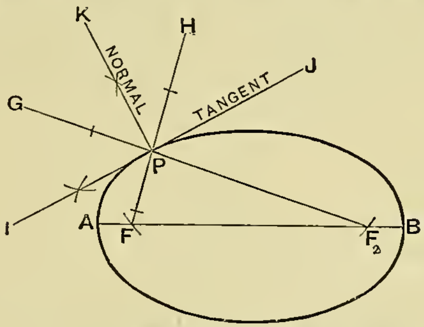



FIG. 217.—Tangent and normal at fixed point in the ellipse.

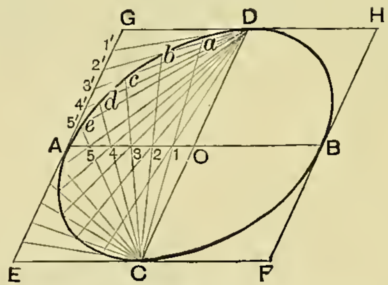



FIG. 218.—Ellipse, given conjugate diameters.

problem, and the foci F and $F_2$, as in Prob. 132. Then join the fixed point P to F and $F_2$. Produce $F_2P$ to G, and bisect the angle GPF by IJ, which is the required tangent.

A very important property of the ellipse is that the focal lines FP and $F_2P$ make the same angle at P with the curve (or with its tangent, IJ).

NOTE.—As you are probably aware, if a ray of light impinge on a mirror, or a wave of sound on a flat surface, the light or sound is reflected; and the angle of reflection is equal to the angle of incidence. The same law holds good for curved surfaces; thus, if we had a source of light at F (Fig. 217), and a ray impinged on an elliptical mirror APB at P, the angle of incidence would be FPI, and that of reflection $F_2$PJ. It follows from this fact that all the rays from F would be reflected to the other focus $F_2$, which would become a luminous point. The same would occur with sound: if the ellipse represent the plan of a building, a whisper at either focus F or $F_2$ would be heard at the other, although perhaps it could not be heard at any other part of the building. In some foreign prisons a cruel use has been made of this echoing property of the curve.

[1] It will be noticed that lines EF and GH are double ordinates of the diameter IJ, as the lines OD and OC are ordinates of the axis AB.

**137. To draw a Normal or Perpendicular to a given Ellipse at a Fixed Point in the Curve**[1] **(Fig. 217).**—First find the major axis AB (Prob. 135) and the foci F and $F_2$, as in Prob. 132. Then join the given point P to F and $F_2$. Produce $F_2P$ to G, and FP to H. Bisect the angle GPH by PK, which is the required normal or perpendicular.

**138. To draw an Ellipse, when two Conjugate Diameters, other than the Major and Minor Axes, are given intersecting each other at their Centres—First Method: By Radial Lines from the Ends of One of the Axes.**—Let AB and CD (Fig. 218) be the given conjugate axes. Then through the ends of AB and CD draw the parallelogram EFHG, with sides parallel to the given axes. Then the points *a, b, c, d, e* in the quadrant AD of the figure may be found in the same way as in Prob. 134, and the explanation given equally applies to this figure.

**139. Second Method: By using the Auxiliary Circle.**—Let AB and CD (Fig. 219) be the given conjugate diameters intersecting in

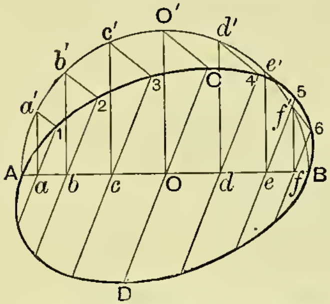



FIG. 219.—Ellipse, given conjugate diameters.

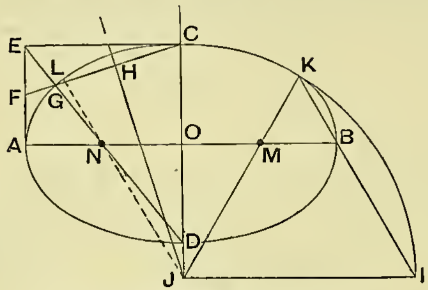



FIG. 220.—Ellipse, by circular arcs.

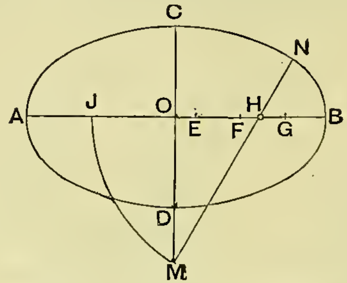



FIG. 221.—Ellipse, by circular arcs.

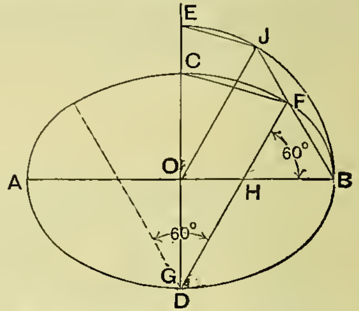



FIG. 222.—Ellipse, by circular arcs.

O. Then with O as centre describe the auxiliary semicircle AO′B, and divide AB into any number of parts in *a, b, c, d, e, f*, and from these points, and at O, erect perpendiculars to AB to cut the semicircle in *a′, b′, c′, d′, e′, f′* and O′. Next, join O′ to C, and through the points *a, b, c, d, e, f* draw lines parallel to the diameter CD, intersecting lines through *a′, b′, c′, d′, e′, f′* parallel to O′C in 1, 2, 3, 4, 5, and 6, which are points in the required ellipse. The other half of the curve can be drawn in the same way, or be put in by symmetry.

**140. Approximate Method of constructing Ellipses by Circular Arcs, having given the Number of Centres—First Method (By Three Centres): When the Major and Minor Axes are given.**—Let AB and CD (Fig. 220) be the major and minor axes perpendicular to each other at their centres O. Then through C and A draw parallels to AB and CD, intersecting in E. Bisect AE in F, and join CF and ED, intersecting in G, which is obviously a point on the true curve (Prob. 134). Next bisect CG in H by the perpendicular HJ, cutting CD produced in J, which is the **centre of curvature at C**. With J as centre, radius JC, describe the arc CKI, cutting a line

[1] This is a problem that sometimes occurs in connection with an **elliptical arch** when the **intrados and extrados are parallel**. When all the normals to an ellipse are intersected by a curve at equal distances, the **curve** is said to be **parallel to the ellipse**. This curve will not, as the student might suppose, be an ellipse, although in most cases it will much resemble one; it will have in every case a *different character*. A circle parallel to a circle is a concentric circle; with this exception, all parallel curves are different in character.

parallel to AB through J in I. Join I to B, and produce it to cut the arc in K; and join K to J, cutting AB in M; then with centre M, radius MK or MB, describe the arc KB. Then J and M are the centres of curvature of the ellipse at C and B, and the two arcs CK and KB form a quadrant of the required ellipse and give a close approximation to the true curve. Make AN = BM, and through J and N draw JL. Then the semi-ellipse ACB is formed of the three arcs AL, LK, and KB, described from the centres N, J, and M respectively. And the other half of the figure can be completed by symmetry.

**141. Second Method (Another Method by Three Centres): When the Major and Minor Axes are given.**—Let AB and CD (Fig. 221) be the given axes intersecting in O. Then make AE on AB equal to the minor axis CD, and divide BE into three equal parts in F and G. With O as centre, radius FB (two of the parts), mark off H and J. Use H as centre, radius HJ, and cut CD produced in M. Then with M as centre, radius MC, describe the arc CN, cutting a line through M and H in N. With H as centre, radius HN or HB, describe the arc NB, which completes the quadrant CNB of the required ellipse. The figure can be completed by symmetry.

**142. Third Method (By Three Centres): When the Minor Axis is not less than Two-thirds the Major Axis.**—Let AB and CD (Fig. 222) be the given axes intersecting in O. Then with O as centre, radius OB, describe an arc BE, cutting OC produced in E. Then on OB construct the equilateral triangle OBJ; join J to E, and through C draw CF parallel to EJ, cutting BJ in F. Through F draw FG parallel to OJ, cutting OD or OD produced in G and AB in H. Then G and H are the required centres, and the radii are HF and GF respectively. With these centres the arcs can be described as shown, and the ellipse completed by symmetry. It will be noticed that the angle subtended by each of the three arcs is 60°.

Note.—This construction gives a good curve for an arch, as it can easily be set out, is pleasing to the eye, and gives a very suitable waterway. When the minor axis is less than two-thirds and greater than half the major axis, five or more centres should be used, as three do not give a curve of agreeable form. (See Author's "Geometrical Drawing," p. 132.) These curves are sometimes called the basket-handle arches, or curves of many centres.

## EXERCISES.

### Typical Oral Exercises.

1. What is the difference between an ellipse and an oval?
2. How many axes of symmetry has an oval?
3. The end of your round pencil is a circle. If you cut the end in a slant direction, what is the shape of the cut surface or section?
4. What is the name of the longest line you can draw across an ellipse? And what the shortest line?
5. If you draw a line that just touches the curve of an ellipse without cutting it, what is the name of the line in relation to the curve?
6. What is a normal to an ellipse?
7. Mention any object or part of a structure that is elliptical in form.
8. In what shaped paths do the planets move round the sun?

### Drawing Exercises.

9. In a rectangle 3¼″ × 2″ inscribe an ellipse.
10. Construct an ellipse with axes 3″ and 1⅞″ by using a strip of stiff paper to find points in the curve.
11. The major and minor axes of an ellipse are 3½″ and 2″ long respectively. Draw the curve by three different methods, and find its foci.
12. After drawing the ellipse in the previous problem, show how you would determine its axes, supposing that their positions are unknown.
13. Draw a tangent to an ellipse whose axes are 3″ and 1½″, the point of contact to be 1¼″ from an extremity of either axis.
14. Draw a semi-ellipse (axes of 4″ and 2″), and set out a number of normals to the curve 0·75″ long (externally), and through their ends draw a curve parallel to the ellipse.
15. The distance between the foci of an ellipse is 2½″, and the major axis is 3¼″ long. Draw the ellipse.

16. Two conjugate diameters of an ellipse are 3″ and $2\frac{1}{2}$″ long, and they are inclined to one another at an angle of 70°. Set out the ellipse in two different ways.

17. The diagonals of a rhombus are 4″ and $2\frac{1}{4}$″. Draw the figure, and inscribe in it an ellipse.

18. A rectangle, 3″ × $1\frac{3}{4}$″, circumscribes an ellipse. What is the arithmetical difference between the areas of the ellipse and the rectangle ?

NOTE.—The area of an ellipse is equal to the product of the axes multiplied by $\frac{\pi}{4}$.

19. Draw a semi-ellipse (axes 5″ and $2\frac{1}{2}$″), making the major axis 5″ and the semi-minor axis $1\frac{1}{4}$″ ; then draw a parallel curve (inside) $\frac{1}{2}$″ from it.

NOTE.—This is a problem that sometimes occurs in connection with an **elliptical arch** when the intrados and the extrados are parallel. When a number of equal normals to an ellipse have their ends in a curve, the curve is said to be parallel to the ellipse, as we have seen.

20. Draw an ellipse with axes $2\frac{3}{4}$″ and $1\frac{3}{4}$″, and find the centres of curvature of the figure at the extremities of the major and minor axis ; also of one other point in the curve.

21. Draw in two different ways an approximate ellipse by means of circular arcs, using three centres and making the axes 3″ and 2″.

# CHAPTER XIII

## THE PARABOLA

**143. A Parabola**[1] may be defined as a curved figure formed by the intersection of a plane and a cone, when the **plane passes through the cone parallel to its side**,[2] as in Figs. 223, 224; or, in other words, the curve is a **conic section**, or **conic**, as it is sometimes called. In the language of co-ordinate geometry, a parabola is the **locus of a point** which moves so that its distance from a fixed point called the **focus** is equal to its distance from a fixed straight line called the **directrix**, as we shall see later.

The parabola is one of the most important curves the practical man has to deal with, as by its use many interesting problems can be graphically solved. The curve can be drawn from a great variety of data, but only a few simple cases come within the province of this work.

**144. Definitions and Properties, etc.**—If you are reading this chapter for the first time, don't trouble about these definitions, etc. They are here mainly for reference purposes.

**Diameter.**—A straight line perpendicular to the directrix (Fig. 225), terminated at one extremity by the parabola and produced indefinitely within, is called a *diameter.*

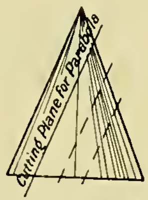


FIG. 223.—Cone cut by plane.

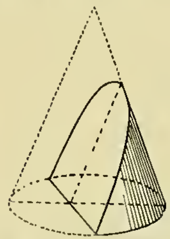


FIG. 224.—Parabolic section.

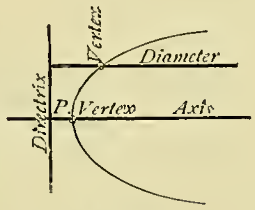



FIG. 225.

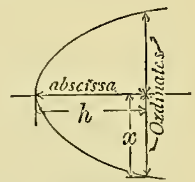



FIG. 226.

A diameter of a curve may be defined as the locus of the middle points of a series of parallel chords.

**Axis.**—The diameter of a parabola which passes through its focus is called its *axis* (see Figs. 225 and 235).

**Vertex.**—The point in which a diameter meets the parabola is called its vertex (see Fig. 225).

**Principal Vertex.**—The vertex of the axis is called the principal vertex (Fig. 225).

**Ordinate.**—A straight line bisected by a diameter and terminated both ways by the parabola (Fig. 226), is a *double ordinate of that diameter.* An ordinate $x$ is proportional to the square root of its abscissa $h$.

[1] The **path of a body** that is **thrown obliquely** in a vacuum is a parabola. A **shell** fired from a mortar moves approximately in a **parabolic** path. The **orbits** described by **comets** appear to be parabolas, the sun being at the focus. But they are known to be ellipses very much elongated.

[2] Or parallel to a single generator is, perhaps, a better way of defining it. The sides of a parabola come closer together as the cutting-plane approaches the side of the cone, so the limit of the figure is a straight line.

**Abscissa.**—The segment of a diameter between its vertex and an ordinate is called an *abscissa* (see Fig. 226).

**Chord.**—A straight line cutting the parabola in two points is called a chord.

**Tangent.**—The tangent at any point P in a parabola (Fig. 227) bisects the angle FPE between the focal distance FP and PE, a perpendicular on the directrix CE.

NOTE.—From any external point two tangents can be drawn to a parabola. **Tangents which meet in the directrix are at right angles to each other.**

**Sub-tangent.**—The distance JG (Fig. 227) between the intersections on the axis of the tangent at a point P and a perpendicular to the axis from P is called the **sub-tangent** at P.

NOTE.—The extremities J and G of the sub-tangent are at equal distances from the vertex. If this be remembered, a tangent at any point P can be directly drawn by first finding the sub-tangent.

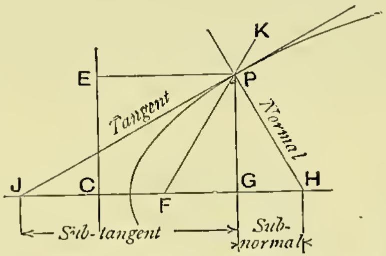


FIG. 227.—Tangents and normals to a parabola.

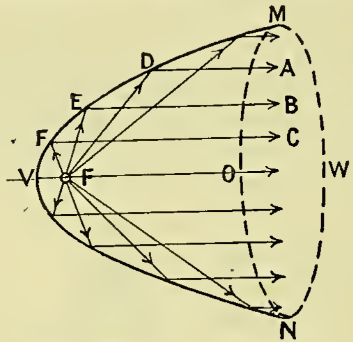


FIG. 228.—Parabolic reflector.

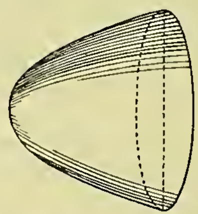


FIG. 229.—Paraboloid.

**Normal.**—The *normal* PH (Fig. 227) to a tangent is a line at right angles to the tangent at the point of contact. It bisects the exterior angle EPK, formed by the focal distance and a perpendicular on the directrix CE.

**Sub-normal.**—The distance GH (Fig. 227) between the intersections on the axis of the normal at a point in the curve and a perpendicular from the point to the axis is called the *sub-normal*. It is a property of the parabola that the sub-normal is constant. The sub-normal equals twice the distance of the focus from the vertex. That is to say, it equals the distance between the focus and directrix.

**145. Diameter and Focal Line.**—Any diameter, such as EP produced (Fig. 227), is inclined at the same angle[1] to the curve as the focal line FP.

[1] This is a **valuable property of the parabola**. For if we imagine the curve MEVN (Fig. 228) to be the plan of a parabolic wall capable of reflecting light, and that we have a source of light at the focus F, it means that parallel rays AD, BE, CF will be reflected; the angle of reflection being equal to the angle of incidence (refer to note on Prob. 136). Now, if we revolve the figure about its axis VW, a **surface of revolution** will be formed (in a geometrical sense), called a **paraboloid**. Indeed, we have in some **lighthouses** a plated copper **mirror**, having the form of the paraboloid (Fig. 229), and all the **rays** reflected from the internal surface when the source of light is at the focus F (Fig. 228) **form a beam of light** having the circular end for its base. You no doubt have often seen such a beam of light on the sea coast, or at sea. Sometimes, it is made to revolve, that it may be seen from all quarters of the compass.

**146. To draw a Parabola when its Axis and Base are given—First Method.**—Let AB (Fig. 230) be the base, and CD the axis or height. Then draw CD perpendicular to AB at its point of bisection, and through D draw EF parallel to AB, and intersecting the perpendiculars through A and B in E and F respectively, completing the parallelogram ABFE. Divide AC and CB into any number of equal parts (four are taken), and through these divisions draw parallels to CD. Next divide AE and BF into the same number of equal parts (four), and through these divisions draw lines to D. These lines will intersect the parallels to CD, giving points *a*, *b*, *c*, etc., in the required curve, as shown.

NOTES.—1. *This* **construction depends upon** *the fact that if a diameter be drawn through the centre point of any chord, the tangents at the ends of the chord intersect on the diameter, and the curve cuts the diameter at the centre point between the intersection of the tangents and the chord.* Thus, BD is a chord; and the

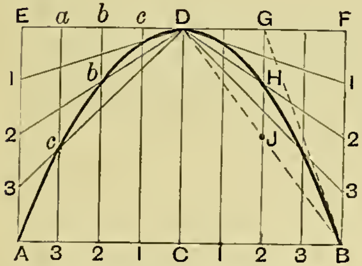



FIG. 230.—Parabola on base AB.

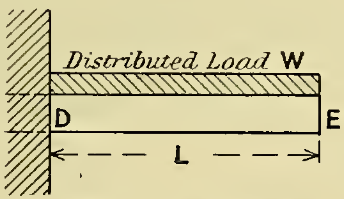



FIG. 231.—Loaded cantilever.

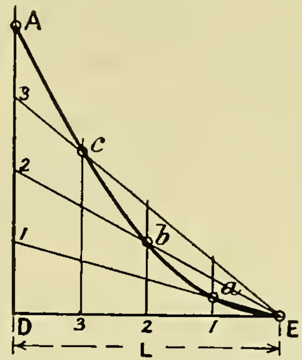



FIG. 232.—Bending moment diagram.

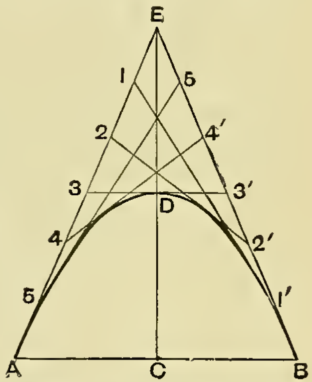



FIG. 233.—Parabola on base AB.

diameter through G will intersect it in its centre, J. DF is the tangent at D, and the tangent at B will also pass through G (refer to note on the Sub-tangent); and H, which bisects GJ, will be a point in the curve, since (by similar triangles) GJ : JH :: F2 : B2.

2. This is the curve (ADB) **which represents the variation of bending moment in a beam supporting a uniform load.** AB would represent the length of the beam, and the height CD the greatest bending moment $\frac{WL}{8}$, whilst the abscissæ from the divisions in AB measure the varying bending moment; for the distance of any point in this curve from EF varies as the square of its distance from CD, and all curves which satisfy this condition are parabolas.

In the case of the **cantilever** (Fig. 231), supporting a distributed load W, the **greatest bending moment** is, as you perhaps know, $\frac{WL}{2}$. And if DA (Fig 232), is made to represent this, the variation of the bending moment from the free end E to the fixed end D, is represented by the parabolic curve E*abc*A, the construction of which should now be obvious.

**147. To draw a Parabola when its Axis and Base are given—Second Method (by Tangents).**—Let AB (Fig. 233) be the base, and CD the axis, as in the first case. Then set up CD perpendicular to AB at its centre point C, and produce CD to E, making DE equal to CD. Join AE and BE, and these two lines will be tangents to the required curve (refer to note on the Sub-tangent). Now, if these lines be each divided into any number of equal parts (six in the figure), and the divisions be numbered and joined as shown, the lines 1 1′, 2 2′, etc., are also tangents, and it is only necessary to take a sufficient number of divisions for an almost perfect curve to be formed with very little touching up by hand.

**148. To describe a Parabola when its Base, Height, and Inclined Axis** (**which passes through the Centre of the Base**) **are given.**—In this case it is required to have the greatest height at some point not directly over the centre of the base. The position of the highest point, D (Fig. 234), in relation to the base fixes the inclination of an axis, CD, which bisects the base. If AB be the base, and CD this axis, the curve can at once be drawn by first drawing the circumscribing parallelogram, ABFE, and by finding points in its sides, as shown in the figure, and explained in Prob. 146. Of course, if desirable, points can be interpolated between any two points already found by subdividing the corresponding spaces on AE and DE.

NOTE.—It will be seen in the Fig. 234 that the tangent PH at P bisects GD in H.

**149. To describe a Parabola when the Directrix and Focus are given.**—Let AB (Fig. 235) be the given directrix, and F the focus.

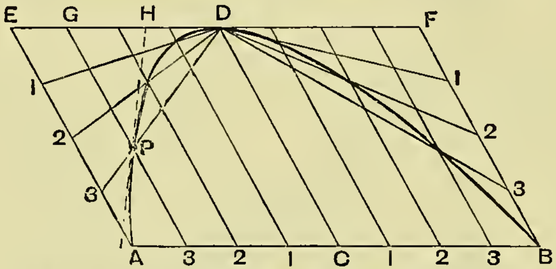



FIG. 234.—Parabola on base AB with axis inclined.

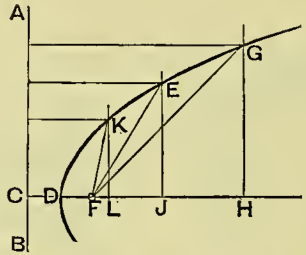



FIG. 235.—Parabola as locus of a point.

Then through F draw the axis CFH perpendicular to AB and cutting it in C. Bisect CF in D, and D, being a point on the axis equidistant from the focus and directrix, is the **vertex**. Next draw a number of indefinite lines parallel to the directrix, such as LK, JE, and HG, etc. Then, to find points in the curve contained in these lines, begin with line JE. Its distance from the directrix AB is CJ ; so with this radius and with centre F cut JE in E, and similarly with HG, take radius CH, centre F, and cut it in G. Then both E and G will be **points in the curve**, as they are **equidistant from the focus and directrix**. A third point, K, is shown in the figure, and as many more as may be required can be found in this way, and the curve drawn through them.

It will be seen that the **curve** is the **locus of a point** equidistant from a given point F, and a given line AB.

## EXERCISES.

### Typical Oral Exercises.

1. In what direction must a cone be cut to give a parabola?
2. A segment of a parabola has a base of 4″ and an axis perpendicular to it 3″ long. What is the area of the figure?
Note.—The area of a parabola is two-thirds that of the circumscribing rectangle.
3. Define a diameter of a parabola.
4. A projectile fired from a gun moves in a curved path till it comes to rest. What is the name of this curve?

### Drawing Exercises.

5. The base and height of a parabola are 3″ and 2″ respectively. Describe the curve in two different ways.
6. The axis of a parabola is 3″ in length, and makes an angle of 65° with its base, which is $2\frac{1}{2}$″ in length. Construct the curve.
7. Set out the parabola in Question 5, and mark a point on the curve 1″ from the vertex, and through the point draw a tangent to the curve by two different methods.
8. The base of a parabola is $3\frac{1}{2}$″, and its height $2\frac{1}{4}$″, the highest point above the base being vertically above a point in the base 1″ from the centre. Draw the curve.
9. The base of a parabola is 4″, and an axis of the curve passing through the centre of the base is inclined 70° to the base and is $1\frac{1}{2}$″ long. Construct the curve.
10. Set out the parabola in Exercise 5, and find its focus and directrix.
11. The focus of a parabola is $\frac{1}{2}$″ from the directrix. Describe the curve, making the axis 3″ long.
12. The distance between the vertex and focus of a parabola is $\frac{5}{8}$″. Set out a small part of the curve approximately by means of circular arcs.
13. The height of a parabola is $2\frac{1}{2}$″, and its base 2″. Describe the curve by the method of tangents.

# CHAPTER XIV

## THE HYBERBOLA

**150. Introduction.**—You will remember that it is explained in the preceding chapters that the *ellipse* and *parabola* may be defined as curved figures formed by the intersections of planes and a cone—in the former, when the plane *passes through opposite sides of the cone,* and in the latter, when the cutting plane is *parallel to the side* (or parallel to a single generator). In a similar way the **hyperbola**, which is the other important *conic,* may be defined as a curved figure formed by the intersection of a plane and a cone **when the plane is parallel to any two generators**, such as those whose elevations AB are shown in Fig. 236. All such planes cut both sheets of the conical surface, and the curves of such intersections have **two branches**,[1] as they are called, which are unlimited in extent.

As in the cases of the ellipse and parabola, this definition does not bring out the property of the curve which furnishes the most convenient method of constructing it. So we must again resort to the language of co-ordinate geometry, and define a *hyperbola* as the locus of a point which moves so that its distance from a fixed point called a *focus* bears a constant ratio to its distance from a fixed straight line called the *directrix,* the ratio being greater than unity.

So we have in the **parabola** a **ratio of equality**; in the **ellipse**, one **less than unity**; and in the **hyperbola**, one **greater than unity.**

The **eccentricity** of the curves is the numerical value of these ratios. The sides of the hyperbola become straighter as the cutting plane approaches the axis of the cone, so the limit of the figure is a pair of straight lines when the plane contains the axis.

The most important features of this interesting curve are shown in Fig. 237. AB is the transverse axis; EG the conjugate axis; $F_1$, $F_2$ the foci; and $F_3$, $F_4$ the foci of the conjugate to the curve.

**Asymptotes.**—The diagonals DH and SJ (Fig. 237) of the rectangle, formed by the tangents to the hyperbola and its conjugate at their vertices, are called the *asymptotes* of the hyperbola. They **continually approach the curves** without limit, but never meet them.

**The Rectangular Hyperbola.**—When the **axes** AB and EG (Fig. 237) of an hyperbola are **equal**, the curve is called equilateral or *rectangular,* and the angle between the asymptotes is a right angle, and the product of the abscissa QX and ordinate QY of any point Q in the curve is a constant.

For drawing purposes this is the most important case, and we will confine our attention to it.

[1] Obviously if the cone is generated by a line AC (Fig. 236) revolving about an axis AG, and if the line is produced beyond A to D, an **inverted cone**, DAE, sometimes called the **opposite cone**, is generated at the same time, with a common apex A. Our cutting plane cuts this other cone also and gives the other branch of the hyperbolic curve. Briefly, if a cone be cut into two parts by a plane, which, if continued, would meet the opposite cone, the section is called a hyperbola.

**151. Given an Ordinate and Abscissa, EF and ED** (Fig. 238), **of a Point in a Rectangular Hyperbolic Curve, and the Axes** (or **Asymptotes**) **AB and AD, to draw the Curve.**—Complete the parallelogram ABCD. Next divide EC into any number of parts (preferably equal parts) (say five) in 1, 2, 3, etc.; and through these divisions draw lines parallel to DA, terminating in DC and AB. Lines may now be drawn through the corner A to pass through the divisions 1, 2, 3, etc., on DC, as shown; then at their intersections 1′, 2′, 3′, etc., with EF draw lines 1′*a*, 2′*b*, 3′*c*, etc., to intersect the lines through 1 1, 2 2, 3 3, etc., in *a*, *b*, *c*, etc. (points in the required curve), as shown. If these points be now joined by a flowing line, any point *x* in the curve will be distant from the axes AD and

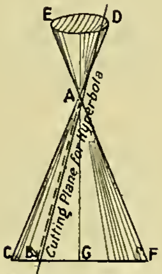



Fig. 236.—Cone cut by plane, giving hyperbolic section.

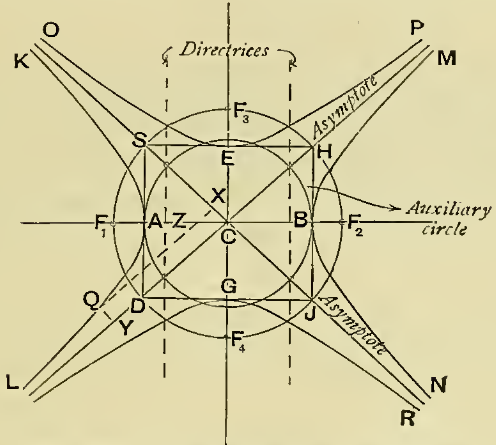



Fig. 237.—Hyperbola, with its most important features.

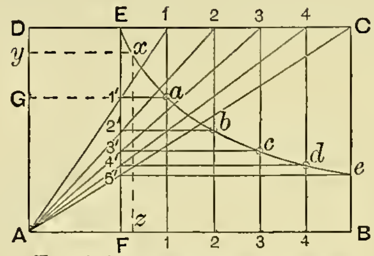



Fig. 238.—Rectangular hyperbola.

AB such that the product $xy \times xz = FE \times ED$; for by easy Euclid it can be proved that the rectangles AFED and A1*a*G, etc., are equal; and therefore the product of the rectangular distances of all points in the curve from the axes is a constant, as we have seen it must be.

Notes.—1. This curve is of great practical importance, as by it the varying pressure of an expanding gas can be shown in relation to its volume. In other words, it is the isothermal curve (curve of equal temperature) used by engineers to show the varying pressure of expanding gases, and the indicator diagrams taken from the cylinders of heat-engines approximate more or less to the form of the figure AB*eb*ED, where AD and FE would represent the initial absolute pressure, B*e* the terminal absolute pressure, and AB the stroke of the piston; the initial volume of the gas would be represented by AFED, and the terminal volume by ABCD.

2. **Area.**—The area of the figure ADE*e*B (Fig. 238) is often required, and is represented by the mean height of DE*be* above AB, times AB, or by $(AF \times FE) + \left(AF \times FE \times \log_e \frac{AB}{AF}\right)$.

The log is the hyperbolic one, which is the ordinary or common tabular log multiplied by the modulus 2·30258.

3. **The mean height of** DE*ae* **= the mean pressure.** If the area of the figure ADE*e*B be divided by the length AB, the quotient will equal the mean height.

Example.—Let the scale of pressures be 80 lbs. per sq. inch = 1″, and the area = $4\frac{1}{4}$″, and the length = 4″.

Then the mean height $= MH^t = \frac{\text{area}}{\text{length}} = 4\frac{1}{4}'' = 1\frac{1}{16}''$, and therefore mean pressure $= 80 \times MH^t = 80 \times 1\frac{1}{16}'' = 85$ lbs. per square inch.

## EXERCISES.

### Typical Oral Exercises.

1. Explain in what way a cone must be cut to give a hyperbolic section.
2. When is a hyperbolic curve called rectangular or equilateral?
3. Why do engineers sometimes call the rectangular hyperbola an isothermal curve?

### Drawing Exercises.

4. A point P moves in such a way that the product of its distances from two rectangular axes[1] is constant. Trace the point in the direction of one of the axes for a distance of 3″, placing it at the starting-point $1\frac{1}{2}$″ from that axis, and 1″ from the other. Note.—Refer to Problem 151.

5. The absolute pressure of a gas is 150 lbs. per sq. inch, and it is expanded till its absolute pressure is 15. Show by a diagram how the volume varies with the different pressures, assuming that the temperature remains constant. Note.—Refer to Problem 151.

6. A volume of 10 cubic feet of air at atmospheric pressure (15 lbs. per square inch) is compressed to a pressure of 75 lbs. per square inch isothermally. Show by a diagram how the volume varies with the increasing pressure. Note.—Refer to Problem 151.

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[1] The axes referred to here are the **asymptotes** of the rectangular hyperbola. When these are at right angles the curve is of course called rectangular, as we have seen.

# CHAPTER XV

## SPIRALS AND MISCELLANEOUS CURVES

**152. Introduction.**—If a line rotate in any plane about one of its extremities as a fixed point in that plane, and a point continuously travels along the line in the same direction according to some definite law, the curve traced by the point is called a **spiral**, and the fixed point is called its **pole**. A fixed line in the plane passing through the pole, from which the angle passed through by the moving line (called the **position or vectorial angle**) can be measured, is called the **initial line**, and any line joining a point in the curve to the pole is called a **radius vector**. (A spiral may also be defined as the path of a point whose radius vector is proportional in some way to its **vectorial angle**.)

As this radius vector increases in length as the position angle becomes larger, and the magnitude of the angle may increase without limit, it is obvious that spirals can extend to an infinite distance from the pole, and that the **number of convolutions** equals the position angle divided by 360°.

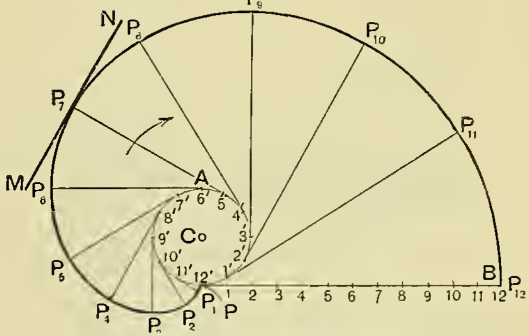

FIG. 239.—Involute of a circle.

**153. The Involute of the Circle.**—If a perfectly flexible line or inextensible thread be wound round any curve so as to coincide with it, and be kept taut as it is unwound again, a point in the line will trace another curve called the **involute** of the first curve, this first curve being called the **evolute**[1] relatively to the involute. Now, for example, if the curve around which the thread is wound be a circle, then the curve traced will be an **involute of the circle**. You can easily draw such a curve if you take a **reel of cotton** and press the circular end on the table, as in Fig. 239. A pencil attached to the free end of the thread will enable you to trace such a curve as $P_1$ $P_2$ . . . $P_{12}$ on a sheet of paper. The free part of the thread will be a tangent to the circle (or **evolute**) at the point of contact as it leaves it. This point of contact is the **instantaneous centre** of motion, and the **centre of curvature** at the tracing point.

**154. To describe the Involute of a Circle.**—Let ACP (Fig. 239) be a diameter of the given circle, and P a point in the required curve. Then draw $PP_{12}$ tangent to the circle at P, and make the distance P to $P_{12}$ equal to the circumference of the circle (= dia. × $\pi$). Next divide both the circle and

[1] The **evolute of a curve** is the **envelope of the normals** to the curve.

the line $PP_{12}$ into any number (say twelve [1]) of equal parts in 1′, 2′, 3′, etc., and 1, 2, 3, etc., respectively, and at the points 1′, 2′, 3′, etc., in the circle draw tangents. Now, it is obvious that if P be the fixed end of the thread, and $P_{12}$ the movable end, $P_{12}$ will cover P when the thread is wound round the circle, and that as it unwinds the point $P_1$ will be from 11′ a distance along the tangent at 11′ equal to P1 along PB, and that when P is at $P_2$ its distance from 10′ will be equal to P2 on PB, and so on till all the other points $P_3$, $P_4$, $P_5$, etc., have been fixed by taking the corresponding distances from P along PB, and marking them off along the tangents from the tangent points. A flowing curve through the points $P_1$, $P_2$, $P_3$, etc., will give one convolution of the required involute of the circle through the point P.

NOTE.—It is obvious that the moving string is tangent to the circle and normal to the curve at any point, and that a line at right angles to this line at the generating point is a *tangent* to the curve at that point. Thus $P_7A$ is a normal at $P_7$, and MN a tangent at that point.

**154A. Cams.**—The term "cam" is applied to a curved plate or curved groove used to communicate motion to another piece by the action of its curved part. We have such a plate, P, in Fig. 240 fixed to the shaft S and rotating with it, its edge in contact with the roller-end R of the oscillating lever HR of a **shearing machine.** When the end R of the lever is raised by the cam the other end H, to which one of the shear blades is fixed, descends on K, the piece of metal to be sheared. If you examine the cam P you will see that when it rotates in the direction of the arrow the roller R is gradually raised whilst in contact with the part CFD, during which the shearing work is done. Then, whilst in contact with the part of the cam BE, the heavy end R of the lever quickly falls (about the trunnions or axis A), giving to the upper shear blade at H a **slow downward** and a **quick upward motion.** The part EC of the cam is concentric with the shaft S, and therefore there is no motion of the lever whilst the roller R is in contact with this part; in fact, an interval of rest occurs, during which the piece to be sheared can be adjusted.

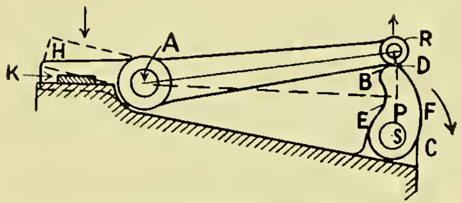


FIG. 240.—Cam used in shearing machine.

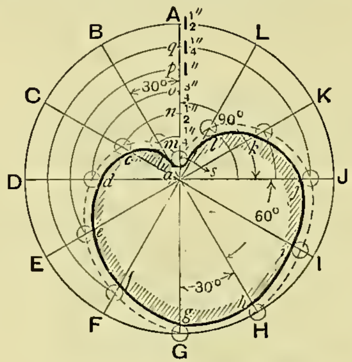


FIG. 241.—Cam, heart type.

**155. To set out a Cam so that a Point reciprocated by it in a Straight Line will move as follows: whilst the Cam is uniformly rotating 180°, uniformly upwards $1\frac{1}{2}''$, interval of Rest for 30°; uniformly down $\frac{1}{4}''$ for 60°; and the remaining $1\frac{1}{4}''$ for the 90°.**—Draw any straight line AG (Fig. 241), and mark off from *s*, a point in it, any distance *sa*; then mark off from *s* along *s*A the rise of $1\frac{1}{2}''$; divide this into any number of equal parts (say six) in the points *m*, *n*, *o*, *p*, and *q*. Next, with centre *a*, radius *b*A, describe a circle, and divide the semicircles ADG and AJG (each 180°) into the same number of equal parts (six) in B, C, D, etc. Now, with centre *a*, radius *am*, cut *a*B in *b*, and with same centre, radius *an*, describe an arc cutting *a*C in *c*, and so on for the points *d*, *e*, *f*, *g*, which are points in the cam to give the upward movement. The next part is to correspond to an interval of rest; so the arc GH of the circle will be that part. As the curve must now draw $\frac{1}{4}''$ nearer the centre for the next 60°, bisect A*q*, and describe arcs as shown, to give the points *i* and *j*. Next, with centre *a*, radius *as*, describe an arc cutting *a*J in *s*; divide *sj*, the remaining

[1] Obviously the larger the number of divisions the more accurate will the curve be. Of course, this curve could be easily described mechanically by means of templates, or by the movement of a lath on a cylinder.

distance the point is to fall through, into three (the numbers of the remaining divisions of the circle) equal parts in *v* and *w*. Then describe arcs through these points to give the points *k*, *l*. A flowing line through the points *a*, *b*, *c*, *d*, *e*, as shown dotted, will give the required curve.

Of course, the reciprocating point would only move in the line SA in the way explained when it had no magnitude. In the case that occurs in practice the point is replaced by a roller; then the curve has to be slightly modified, as shown in contact with the small circle above *a*, whose centre is S, and whose diameter equals that of the roller, the cross-hatched figure representing the cam.

NOTE.—As by a skilful arrangement of suitable cams almost any desired movement of a machine part can be arranged, the student would do well to work out a few examples.

**156. To set out a Simple Cam (another Example).**—Look at Figs. 242 to 244. You see three positions of a cam in relation

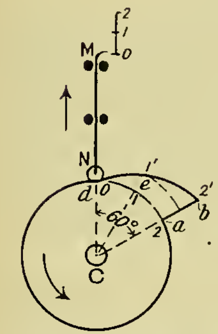


FIG. 242.—Bottom position of slider.

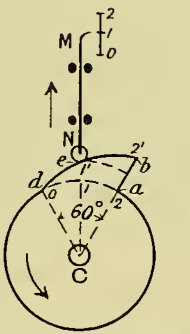


FIG. 243.—Intermediate position of slider.

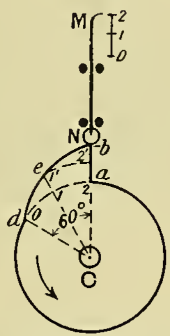


FIG. 244.—Top position of slider.

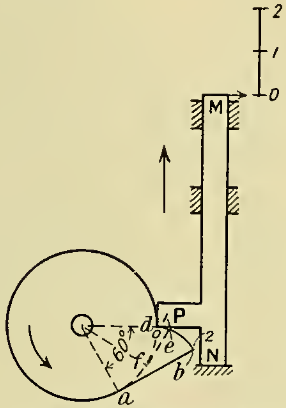


FIG. 245.—Bottom position of slider.

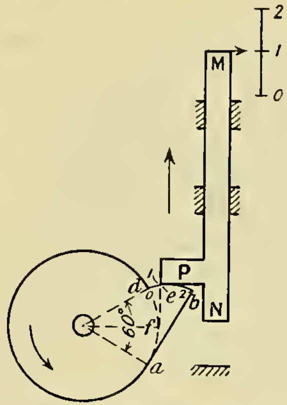


FIG. 246.—Intermediate position of slider.

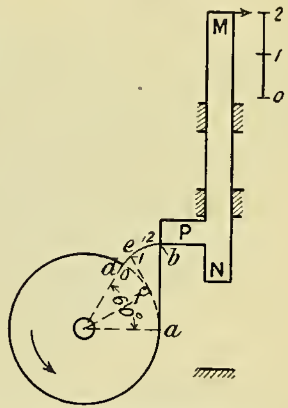


FIG. 247.—Top position of slider.

to a slider MN. As the angle *da* is 60°, it is obvious that as the cam rotates uniformly the slider is rising a distance from *a* to *b* whilst its roller is in contact from *d* to *b*. Three positions of the slider are shown in the Figs. 242 to 244, and these should speak for themselves. In setting out the cam, first mark off the angle of 60° and the lift *ab* on the radius produced through *a*, divide *ab* and *ad* into the same number of equal parts (in this case two), and with centre of the circle C, radius C to the point between *a* and *b*, describe an arc to cut the radius C1 produced in *e*. Through *d*, *e*, and *b* draw a flowing line to complete the cam. Of course, if the end N of the slider MN be fitted with a roller, the edge of the cam must be set back a distance equal to the radius of the roller, as in the previous problem. You will notice that with this arrangement the edge of the cam causes an oblique thrust to come on the slider, which then tends to bind it in the guides. Let us see how this can be avoided.

**157. Involute Cams.**—Look well at the Figs. 245 to 247, and keep before your mind that the object of the arrangement you see is to convert the rotary motion of the cam into an up-and-down sliding motion of the slider MN. This part is in its lowest position in Fig. 245, and in its highest one in Fig. 247, where it is on the point of disengagement. When this occurs, of course the slider falls by gravitation to its lowest position again, only to be again raised when the part *d* comes into position under the projecting piece P. The lower surface of this part is horizontal, and for all positions whilst in contact with the cam, this surface is tangential to the curve *deb*, as can be clearly seen in Fig. 246. If you have carefully read Problem 154, you will understand how to set out the involute curve *deb*, and will see that the normal to the surface of the curve at the point of contact is always vertical, and tangential to the circle; so that no matter how great the load, there is no tendency on the part of the cam to put a side thrust on the slider, as in the previous example.

**158. The Spiral of Archimedes.**—In this spiral consecutive points in the curve uniformly recede from the pole; that is to say, the length of the radius vector is directly proportional to its position or vectorial angle.[1] This will be understood by reading the following problem.

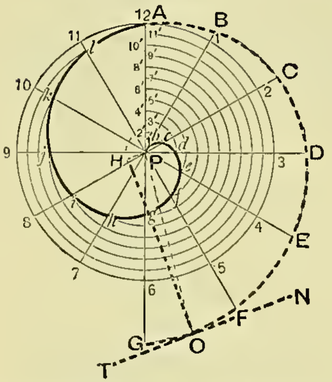

FIG. 248.—Archimedian spiral.

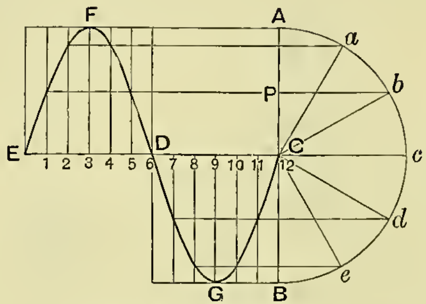

FIG. 249.—Curve of sines.

**159. To describe One Convolution of the Spiral of Archimedes, the Pole, the Initial Line, and the Length of the longest Radius Vector being given.**—Let P and PA (Fig. 248) be the given pole and initial line respectively. Then from P along PA mark off PA, equal to the largest radius vector. Then with P as centre, radius PA, describe a circle. Next divide the circle and the line PA into any number of equal parts (say 12), as shown in the figure. From the centre and pole P draw lines to the points 1, 2, 3, etc., dividing the circle. Then with centre P, radius P 11′, describe an arc cutting P 11 in *l*, and with the same centre, radius P 10′, describe an arc cutting P 10 in *k*, and so on for the other points, *j*, *i*, *h*, etc. A flowing line drawn through these points from A will terminate in P the pole.

NOTES.—1. It will be seen that the radii vectores, P*l*, P*k*, P*j*, etc., get shorter by the fixed distance of $\frac{1}{12}$ of PA for each angle of $\frac{1}{12}$ of a revolution passed through. This being the case, it is obvious that if a cam were arranged this shape, it would give a uniform reciprocating motion when uniformly revolving about an axis through the pole P. The part of the curve P*bcdefg*, if reproduced on the other side, making a figure symmetrical about the line P*g*, gives the form of a **cam** called the **Heart cam.**

2. The points BCDE, etc., are in a second convolution of the curve, and are found by marking off from *b*, *c*, *d*, *e*, etc., along the respective radii vectores, the distance PA. The curve can in this way be carried on to any extent, as the convolutions are parallel.

3. In the **centrifugal pump** the casing takes the form of this curve for a constant velocity, as the section must be proportional to the quantity passing through in a given time. See Goodman's "Mechanics Applied to Engineering," p. 546.

**160. The Curve of Sines, or Harmonic Curve.**—This is the curve in which a musical string vibrates. The ordinates in this curve are proportional to the sines of the angles, which are the same fractions of 360° as the corresponding abscissæ of the wave-length.

**161. To draw the Curve of Sines, the Amplitude and Length of a Vibration being given.**—Let AC (Fig. 249) be the given amplitude and EC the length. Then place AC at right angles to EC, as shown in the figure. With C as centre, radius AC, describe the semicircle AcB. Next divide the semicircle into any number (say six) of equal parts in *a*, *b*, *c*, etc.; also each half (ED and DC) of the length EC into the same number of parts in 1,

[1] When the radius vector is inversely proportional to the **vectorial angle**, the curve generated is called the **reciprocal**, or **hyperbolic spiral.**

2, 3, etc. Then through the points 1, 2, 3, etc., draw ordinates to cut abscissæ through $a, b, c$, etc., as shown. A flowing line drawn through the points of intersection is the required curve.

It will be noticed that the radius of curvature increases from the crests F and G to D, the **point of inflection**, where it becomes infinite. The distance ED is half a wave-length, which is repeated from D to C, the other side of EC.

NOTE.—If the point $b$ move uniformly in the circle, the point P, its projection on AB, will move in AB with a **simple harmonic motion**. The satellites of Jupiter as seen from the earth nearly have this motion.

**162. The Helix.**—If you cut out of a piece of paper a right-angled triangle ABC (Fig. 250), making its base AB equal in length to the circumference of a cylinder = $d\pi$, where $d$ is its diameter, and wrap the triangle round the cylinder, the hypotenuse BC will form a curve $BE_2C_2$ (winding round the cylinder) which is called a helix (or erroneously a spiral). On comparing Figs. 249 and 251, you will see the curve GDFE of 249 is exactly similar to the curve $BG_2E_2C_2$ of 251, but the former is a plane

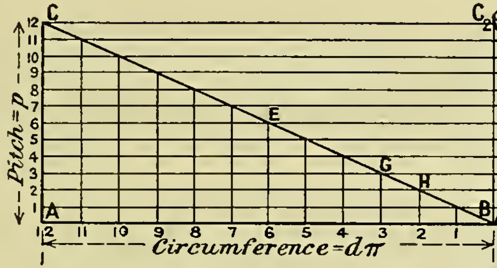


FIG. 250.—Development of a helix.

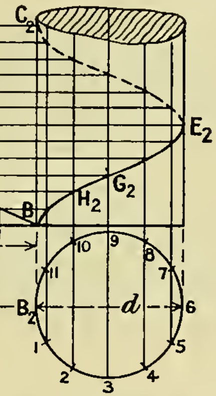


FIG. 251.—Projections of a helix.

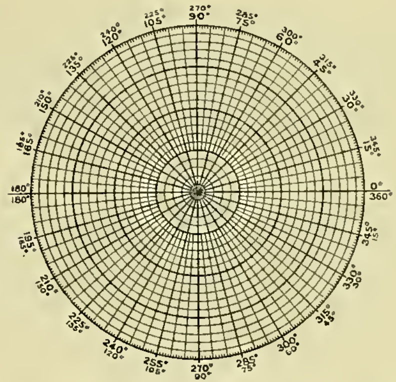

FIG. 252.—Plotting paper for polar co-ordinates.

curve, and its axis is in a horizontal position, whilst the latter is the elevation of a line winding round the cylinder, and its axis is in a vertical position.

The plans of the cylinder and helix are the circle in Fig. 251. $A_2B_2$, the plan of the triangle, in being wound round the cylinder, moves in the direction of the numbers 1, 2, 3, etc., in the circle, and the other features of the construction should now speak for themselves.

**163. Screw Threads are Helices.**—Thus, if the helix in Fig. 251 represented an edge of a screw thread, such a one as in Fig. 400 for instance, the distance $BC_2$ would be the **pitch of the thread or helix.**

**164. Plotting Paper for Polar Co-ordinates.**[1]—Fig. 252 shows to a reduced scale a circle divided by concentric circles and radial

[1] This paper can be bought at some instrument shops. Messrs. Bemrose & Sons make a speciality of it, selling it in pads of 40 sheets (diameter of outer circle $5\frac{5}{16}''$ or 135 mm.) for a shilling, with tables included.

lines; the latter divide the circles into angles of 5°, and if the concentric circles be equally spaced, say $\frac{1}{8}''$ apart, you can readily understand how useful such a diagram is in working problems on spiral and such curves as we have been dealing with in this chapter.

Why not try and draw such a diagram, making the great circle 6″, say, spacing the circles $\frac{1}{8}''$ or $\frac{1}{4}''$ apart? You could then quite easily set out the Archimedian spiral shown in Fig. 248, and some interesting cam forms like those in Figs. 241 to 247.

## EXERCISES.

### Typical Oral Exercises.

1. If you unwind a thread from a reel of cotton and make the end of the thread guide a pencil on a piece of paper pressed against the end of the reel, what is the name of the curve that will be traced?

2. Whilst a man is walking at a uniform speed from the centre of a locomotive turntable to its outer edge, along a radius of the circle, the table makes a complete revolution. What is the name of the curve the man would be moving in in relation to the ground?

3. If you cut out a paper right-angled triangle and wrap it round a cylinder, ruler, or piece of broomstick, starting with the base at right angles to the axis of the cylinder, the hypotenuse will form a curve winding round the cylinder. What is the name of this curve? Can you call to mind any well-known bodies or machine details on which the curve is to be found?

### Drawing Exercises.

4. Draw the involute of a 2″ circle, and at a point in the curve 2″ from the centre of the circle draw a tangent.

5. The largest radius vector of an Archimedian spiral of one convolution is $1\frac{3}{4}''$. Draw the curve, and a tangent to it making an angle of 30° with the initial line.

6. Draw a cam so that a double uniform reciprocating motion through $1\frac{1}{2}''$ in a straight line is given to a point by the cam revolving once.

7. Set out a cam to give the following motion in a straight line: For the first 60° motion, uniform rise of 1″, interval of rest during next 45°, uniform fall of $\frac{1}{2}''$ during the next 45°, interval of rest during next 60°, and uniform rise of 1″ and fall of $1\frac{1}{2}''$ during the following 150°. Make the diameter of the roller $\frac{1}{2}''$, and the part of the cam nearest the centre or axis $\frac{1}{2}''$ from it.

8. Set out a curve of sines, making the amplitude $1\frac{1}{2}''$, and the length 3″.

# CHAPTER XVI

## THE APPLICATION OF GEOMETRY TO ORNAMENTAL AND DECORATIVE DESIGN

**165. Introductory Remarks.**—The beautiful artistic creations of the ancient Greeks appear to have been always controlled and perfected by applications of the laws and principles of geometry, and the decline of art in different ages can be traced to the neglect of these principles. In fact, it can be shown that whenever the geometrical spirit ceased to influence design, the decline of art was rapid. Few workers realize how universal are the applications of geometry to artistic design, and how the most beautiful forms, from a simple pattern to a most elaborate design giving the greatest charm, can be produced by the artist who has mastered the art of using geometry as an instrument to assist him to devise, arrange, and combine some of the simplest geometrical figures. Obviously, the possibilities in this direction are without limit. The artist who confines himself to the manipulation of geometrical figures, or to forms that have a geometrical formation or foundation, without entering the realms of pure art, gets many of his ideas from natural objects, such as shells, leaves, buds, and flowers, which he conventionalizes in such a way that they lend themselves to geometrical treatment. Without attempting to particularize the great variety of work that comes within his province, the following may be mentioned as being among the most important: **Geometrical patterns and simple tracery,**[1] for decorative purposes, such as tessellated mosaic work for floors, floor-cloths, parquet, wall-papers, carpets, rugs, marquetry, buhlwork, etc.; encaustic tiles for walls, floors, etc.; mural decorations; tracery of Gothic and other windows (which has given such characteristic beauty to the architecture of the fourteenth century); painted and sculptured patterns on vases, ivory, pottery, and porcelain; the ornamental treatment of glass and jewellery; the arches of bridges and ecclesiastical and other buildings; furniture, ironwork, such as gates, railings, grilles, and mediæval ironwork, including hinges of doors and church chests, ironwork of windows, etc.; works in bronze, and other metals, etc.

### Geometrical Patterns.

**166. Marquetry, or Buhl Work** (Fr. *marquetrie*), is inlaid work consisting of thin layers of coloured woods or ivory glued on to a backing of oak or fir, well dried and seasoned, which, to prevent warping, is composed of several thicknesses. The art was cultivated by the early Italian cabinetmakers, who represented by its means not only geometrical patterns, but landscapes and figures.

**167. Mosaic.**[2]—The filling up of a plane surface with small pieces of marble, opaque glass, coloured clays, or other substances, so as to form a pattern, was practised by the Greeks in the fourth century B.C., the best Hellenic examples of this kind of work being discovered during the excavations at Olympia about 1875. Some very fine specimens of mosaic are to be seen in the Chapel of the Confessor and in front of the high altar at Westminster.

When the design is formed of small cubes, generally of marble, it is called **tessellated**, and when formed of larger pieces of marble or glazed earthenware, shaped and cut or formed so as to fit one another accurately, **sextile.**[3]

NOTE.—In laying down a pavement of mosaic or inlaid work on which persons are to walk, too many summits (or corners) should not meet in any one point, as any considerable weight on that point may injure the texture and solidity of the work.

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[1] You must give a good deal of attention to the **study of some of the preceding chapters** before you can hope to draw tracery well.

[2] Greek for "small stone."

[3] When **pieces of opaque glass** are used to form complicated pictures for the ornamentation of walls and vaults, the design is **fictile or vermiculated.**

We may now proceed to examine a few examples of the application of geometry to ornamental and decorative design. Many of them have been selected from the author's work on "Geometrical Drawing," and in not a few cases designed to illustrate a variety of applications which will be referred to as we proceed. You should look upon many of them as examples that can be varied in an infinite number of ways, and you should, after drawing them as exercises (preferably to a larger scale), endeavour to devise suitable variations, realizing that in this direction there is boundless scope for the exercise of your ingenuity and taste ; for, as has been truly said, **geometry is the handmaid of ornament.**

Although in nearly each of the following figures an attempt has been made, by leaving a part unfinished and by the use of dotted lines, to show how it has been constructed, it is assumed that you have read the preceding chapters, which have a bearing on this part of our work, and you should therefore experience no difficulty in reproducing them.

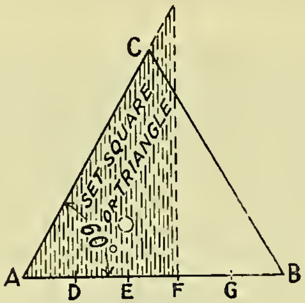

Fig. 253.—First step.

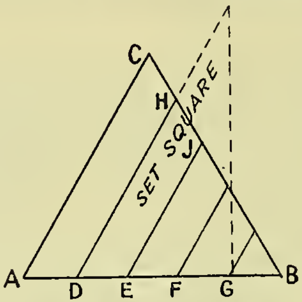

Fig. 254.—Second step.

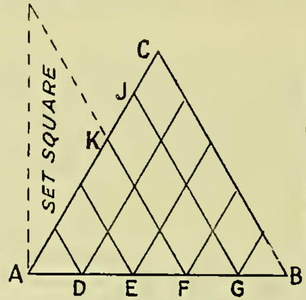

Fig. 255.—Third step.

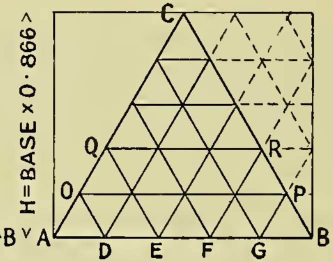

Fig. 256.—Complete pattern.

**168. Some Points relating to the Application of Simple Figures in Mosaic—Paving, Glazing, and generally to all Inlaid Work and Geometrical Patterns.**—These branches of industry are concerned in covering or filling a given area with figures terminated by straight lines.

If it is a condition that all the figures should be regular, and have the same number of sides, there are only three figures available, namely—

1st. **Equilateral triangles**, the summits or apices of which (six in number) meet at the same point, as in Fig. 256.

2nd. **Squares**, the summits of which meet, four and four, at the same point, as in Fig. 274 (the dotted squares).

3rd. **Hexagons**, the summits of which meet, three and three, at the same point, as in Fig. 282.

But, in addition to these regular figures, there are many more or less regular, which, when combined together, produce a pleasing and artistic appearance, as you will see.

**169.** Exercise—**Simple Patterns formed by Equilateral Triangles.**—Some interesting patterns can be drawn with the assistance of your 60° set-square and tee-square. Thus, if you prick off with your compasses a few equal divisions (say, ½″) along a line AB (Fig. 253), and with the set-square complete the equilateral triangle ABC. Then through these divisions, D, E, F, G, draw lines (Fig. 254) DH, EI, etc., with the 60° set-square. Reversing the square (Fig. 255) draw lines GJ, FK, etc., and (Fig. 256) through the apices of the small triangles draw lines OP, QR, etc., parallel to AB, to complete the figure or pattern. Or a rectangle can be filled with the small triangles (as shown dotted in the figure) if its height $= \frac{\sqrt{3}}{2}$AB $= 0{\cdot}866$ AB. If the sides of the small triangles (or the rhombuses, as the case may be) are produced till they cut the sides of the rectaugle, you may then shade or colour some of the triangles to get effect, or vary the pattern according to your taste and ingenuity.

**170. To draw a Diamond Chequered Pattern.**—You will experience no trouble now in making such drawings as are shown in

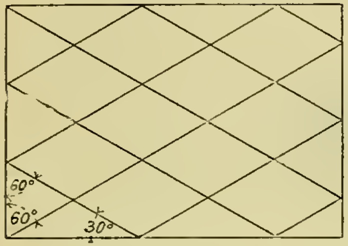


FIG. 257.—First step.

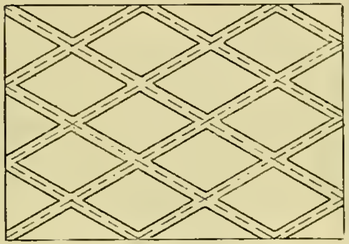

FIG. 258.—Second step.

FIG. 259.—Finished pattern of chequered plate.

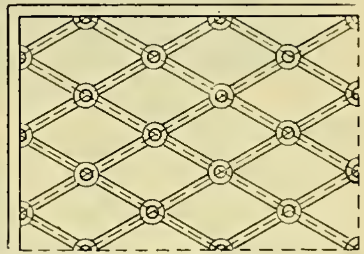

FIG. 260.—Design for cast-iron grating.

Figs. 257 to 259, which should speak for themselves. This particular design of a chequered plate is called the Admiralty pattern,[1] the rhombuses having acute angles of 60°.

**Cast-iron Grating.**—Fig. 260 shows a diamond chequered pattern that is much used for gratings. You will now experience no trouble in setting out this pattern.

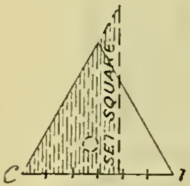


FIG. 261.

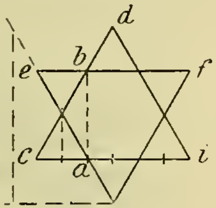


FIG. 262.

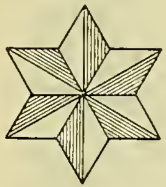

FIG. 263.

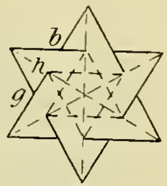


FIG. 264.

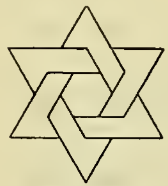

FIG. 265.

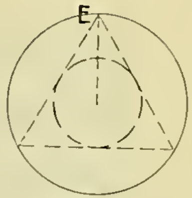


FIG. 266.

**171. Simple Star Forms** are readily drawn with the 60° set-square, as will be seen from an inspection of Figs. 261 to 265, which should now speak for themselves.

NOTE.—It will be noticed that the base *ci* of the triangle (Fig. 262) is divided into six equal parts, and that a line *ab*, the second from the end *c*, will enable you to draw through *b* the side *ef* of the second triangle. A circle with au inscribed equilateral triangle is shown in Fig. 266. If three other triangles be placed over this, as shown in Fig. 267, with apices at B, D, and E, either of the star forms (Figs. 268 or 269) with 12 points can be produced.

[1] As you will see from Fig. 259, the pattern represents a plate with raised strips; such plates are used for engine and boiler-room flooring and such purposes, to prevent the feet from slipping, and they are generally made of mild steel.

The **8-pointed stars** (Figs. 271 and 272) are commenced by first drawing the circumscribing circles, and using the 45° set-square, as shown in Fig. 270.

The **Rosette** (Fig. 273) has the diagonals and cross lines of the circumscribing square for guiding or construction lines.

**172. Patterns suitable for Tiles, Linoleum, etc.**, based on squares, can be made to have a very pleasing effect, such as Fig. 274, and a little ingenuity will enable you to draw a variety of these. A variation in form can be made by arranging the squares with their

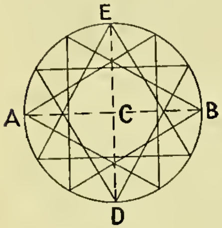

FIG. 267.

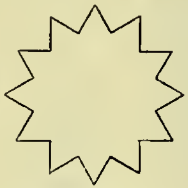
FIG. 268.

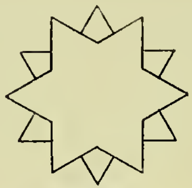
FIG. 269.

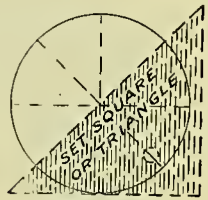

FIG. 270.

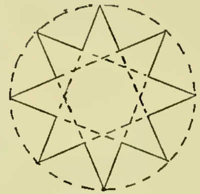
FIG. 271.


FIG. 272.

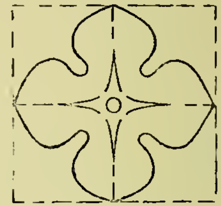
FIG. 273.

diagonals parallel to the sides of the enclosing rectangle, as in the trellis pattern, Fig. 279. Some very pleasing effects can be got by drawing suitable patterns on the squares as in Fig. 275. A **unit of the pattern** is formed by the squares ABCD, EFGH, or

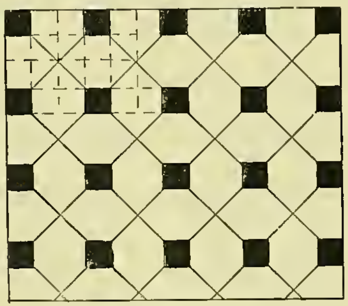
FIG. 274.

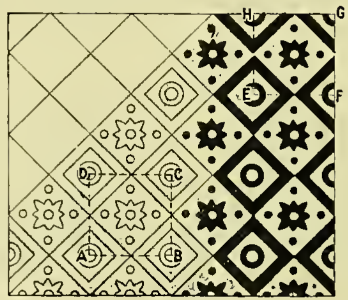

FIG. 275.

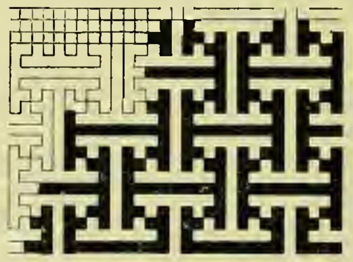
FIG. 276.

by those whose corners are at the centres of the stars. Another type of these patterns is shown in Fig. 276; it is obviously also based on squares as shown.

**173. Greek Frets.**—These are based on squares, which should be first drawn, then a variety of interesting fret patterns, such as those shown in Figs. 277 and 278, can easily be set out. You should get a sheet of squared paper, and practise drawing some.

**174. Trellis Patterns.**—The trellis pattern in Fig. 280 you will see, upon examination, is based on squares as arranged in Fig. 279. If these are drawn first the figure is easily completed.

FIG. 277.—Greek fret based on squares.

FIG. 278.—Greek fret, key pattern based on squares.

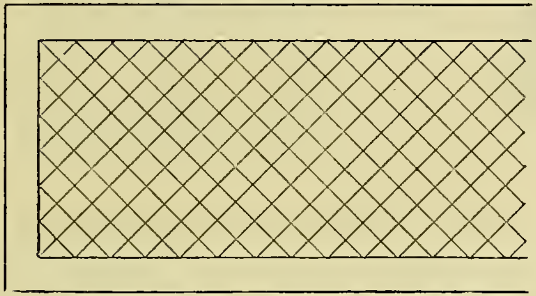

FIG. 279.—Arrangement of squares as base for trellis, etc.

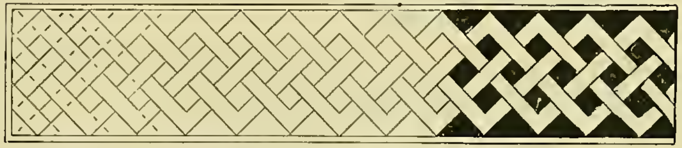

FIG. 280.—Trellis pattern.

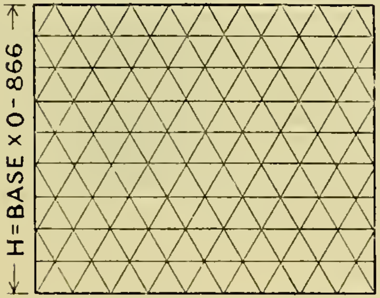


FIG. 281.—Base triangles, first step.

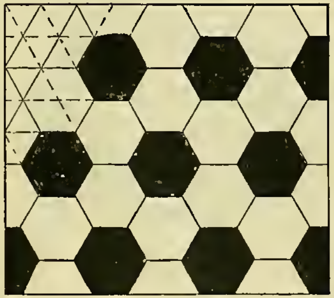

FIG. 282.—Complete pattern of tiles.

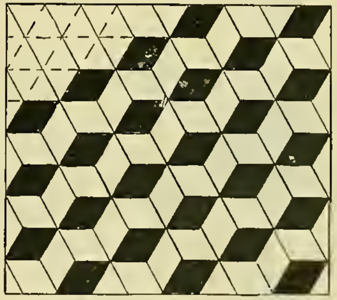

FIG. 283.—Pattern for tiles, etc.

**175. Hexagons in a Rectangle.**—Figs. 281 and 282 show two steps in drawing a pattern formed by hexagons, and based on equilateral triangles, as in Fig. 256. These should now speak for themselves. Two variations of these are shown in Figs. 283 and 284. A little scheming will enable you to devise many others.

**176. Use of Guiding Lines in Detail Work.**—You have had your attention called to the decorated squares in the pattern (Fig. 275).

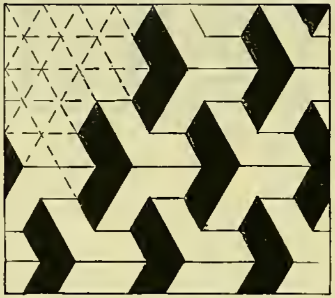

FIG. 284.—Pattern for tiles, etc.

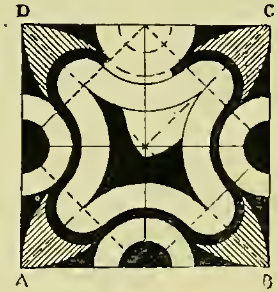


FIG. 285.—Unit of a pattern

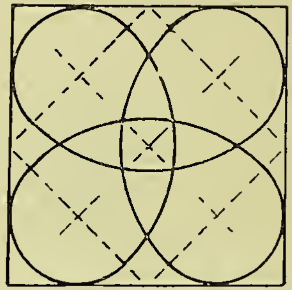

FIG. 286. —Showing use of construction lines.

In setting out such forms it is often helpful to make use of such lines as form the five squares in Fig. 285, or the diagonals and inscribed square in Fig. 286 for guiding purposes.

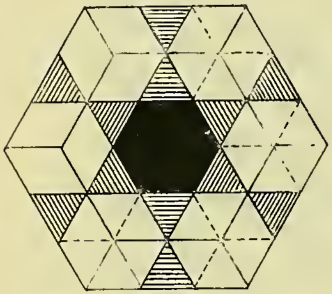

FIG. 287.—Hexagonal tile.

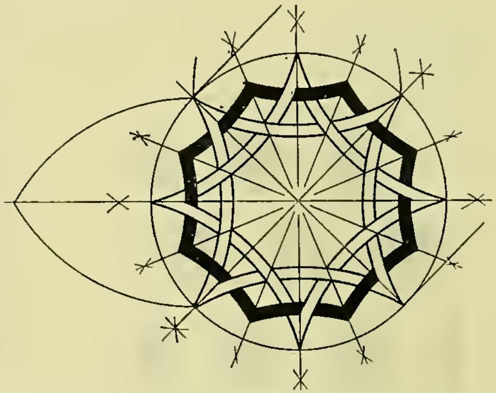

FIG. 288.

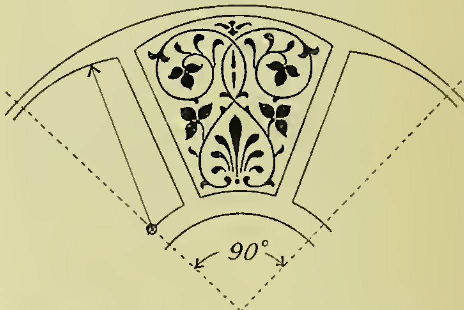


FIG. 289.—Decoration of a circular plaque.

Further, the hexagonal tiles in Fig. 282 may be decorated in a variety of ways: such as shown in Fig. 287, for example. The sides in this case being trisected, and lines drawn through the points parallel to the sides, shading the triangles, as shown, giving a central star.

Fig. 288 shows an elegant interlacing pattern suitable for the decoration of square or circle; and Fig. 289 one quarter of the decoration of a circular plaque.[1] Of course, all the panels would be decorated as the centre one is.

## EXERCISES.

### Typical Oral Exercises.

1. Small pieces of marble, opaque glass, and coloured clays are sometimes used for fancy paving purposes. What name is giving to such work?
2. What is the difference between tessellated and sextile pavements?
3. In laying down a pavement or inlaid work on which persons are to walk, what practical objection is there to making many summits or corners meet in the same point?
4. A certain kind of inlaid work consists of thin layers of coloured wood or ivory glued on a backing of wood. By what name is this work known?
5. If it is a condition that all the figures forming a geometrical pattern shall be regular and have the same number of sides, how many regular geometrical figures are available?

### Drawing Exercises.

6. Draw an equilateral triangle on a 6″ base and cover its surface with 1″ equilateral triangles. Count these and compare the areas of the whole figure to that of one of the elements.
7. Draw a rectangle with 5″ base and 4·33″ height, and cover it with ½″ equilateral triangles, as in Fig. 256. How many of these triangles, and how many half ones are there? Compare the area of the rectangle to that of one of the triangles, *i.e.* give the ratio of areas.
8. Make a drawing of a piece of diamond chequered plate (Fig. 259) 5″ × 4″, making the size of the equilateral triangles upon which it is based 1″. Note.—This is known as Admiralty pattern. What is the object of the ridges or raised strips on the surface of the plates?
9. Draw the three stars shown in Figs. 263 to 265, making the equilateral triangle, upon which they are based, with 3″ sides.
10. In 3″ circles draw the eight-pointed stars shown in Figs. 271 and 272.
11. Draw a 3¼″ square and in it set out the tile pattern shown in Fig. 274.
12. Draw the Greek fret key pattern (Fig. 278), making its breadth 2¾″ and its length about 8″ or 9″.
13. Draw the trellis pattern shown in Fig. 280, making its breadth 2″ and length about 6″.
14. Set out the hexagonal pattern shown in Fig. 282, enclose it in a square of about 3″ side, aud make the triangles, upon which it is based, with ⅜″ sides.
15. Draw a rectangle of 5″ base and 4·33″ in height, and set out the pattern in Fig. 283, using ½″ equilateral triangles.
16. Draw a rectaugle of 5″ base and 4·33″ in height, and set out the pattern in Fig. 284, using ½″ equilateral triangles.
17. The unit of a pattern is shown in Fig. 285. Set out the pattern showing at least 9 units, and making each 1½″ × 1½″.

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[1] Taken from a Board of Education Examination paper, by kind permission of H.M. Stationery Office.

# CHAPTER XVII

## PLAN AND ELEVATION—HOW TO MAKE A WORKING DRAWING OF A SOLID BODY.

**177. Introduction.**—We will assume that you have carefully read at least the first three chapters, particularly Chapter II., and that you are about to attempt a drawing of some simple object. Now, before you can do this intelligently, it is obvious that you should have a fair acquaintance with elementary projection, and as it is possible that you may not have received instruction in this useful branch of geometry, we will proceed to briefly explain how an object may be drawn in plan and elevation; for the shape and proportions of most simple solids can be completely shown by drawing two views only, namely—

**178. Plan and Elevation**, called their *projections*. The terms "plan" and "elevation," as applied to the representation of an object, are fairly well understood in a general way. Thus we speak of the elevation of a house, meaning the view we get by looking at its front, back, or sides. By such a view we see its height and breadth, and the height of everything shown is found on this *elevational* view. Again, we speak of the plan of a plot of ground. This view, of course, shows its length and breadth, and the distance it may be from some landmark. In the same way the **plan** of a house or any object is the **top view** we get by looking down on it from above. All this and much more can better be made clear by referring to an example; and as first steps cannot be made too easy, the subject frequently presenting considerable difficulties to beginners, you cannot do better than take a sheet of drawing paper and any rectangular solid, such as a box or a book, and work out the following simple exercise:—

Let BACD (Fig. 290) be the sheet of paper. Draw across it any line XY (this may be done in the ordinary way with the **T**-square), and place the bottom (EFGK) of your box on the paper, so that one of the long edges, EK, is resting on XY. Then bend the part of the paper BD about the line XY, as shown, until it touches the back of the box EKIJ. If, when the paper is in this position, a pencil point be drawn round the box, marking the lines EFGKIJE, we shall have on the horizontal plane (XYCA) a **plan EFGK** of the box, and on the vertical plane (XYD′B′) an **elevation EKIJ**. Now let us suppose that we are to draw the plan and elevation of the box in its present position, in the ordinary way. Begin by drawing XY (Fig. 291) with the aid of the **T**-square; then construct EKIJ (the elevation), a rectangle, making EK equal to the length of the box, and EJ equal to its thickness, remembering that EK must rest on the ground line (XY), as the box is resting on the ground (horizontal plane); and that as it is touching the vertical plane, the plan, which may now be projected (carried down) from the elevation, must be drawn showing the back EK of the box touching XY. Of course, all the lines on the plan and elevation are drawn with the assistance of the **T**-square and the set-square S. You will of course notice that in this case the plan might have been drawn first, and the elevation projected from it. That is to say, this is a case where either the plan or the elevation may be first drawn. (Cases will occur directly where this is not a matter of choice.) It will now be seen that in Fig. 291 we have represented the form and position of a body which possesses three dimensions (namely, length, breadth, and thickness) upon a plane having only two dimensions, namely, length and breadth. You should now bend the paper (Fig. 291) about its XY, so that the two parts are at right angles, as in Fig. 290, and then imagine that the box is in its place, as it is shown in that figure; for beginners frequently fail to make much progress owing to their inability to exercise their imagination in this way.

As a further exercise we may draw the plan and elevation of a rectangular block in such positions as shown in Figs. 292 and 293, where it will be seen that the two views in the latter figure are separated by the distance *aa′*, and to enable you to see what bearing this change of position has upon the previous case we will proceed to work a little problem which shall be a distinct step in advance of the previous study, but, nevertheless, one that ought to be readily understood. The problem may be stated thus:—

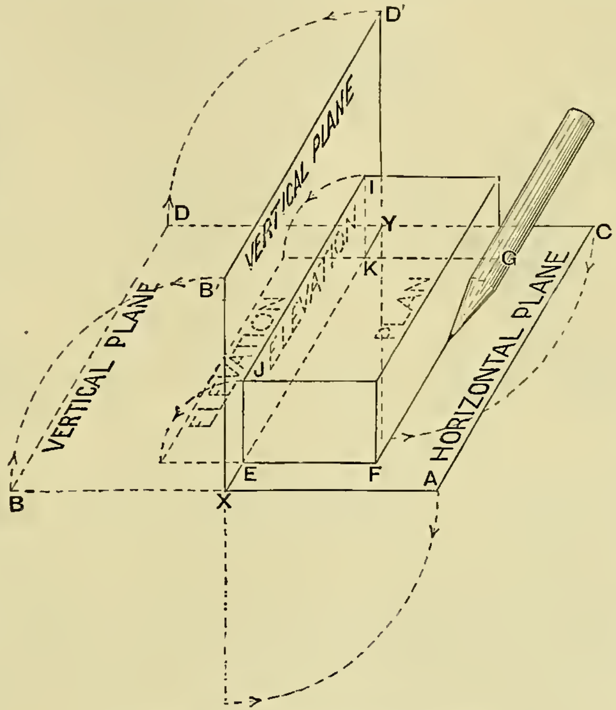


Fig. 290.—Relation of plan to elevation.

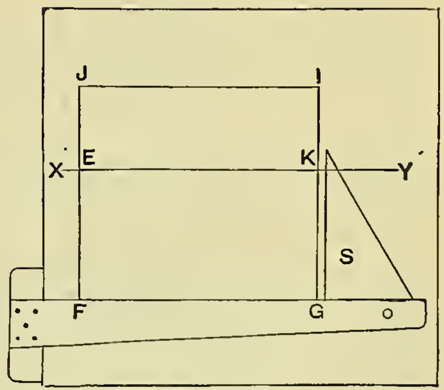


Fig. 291.—Projecting one view from the other.

**179. To draw the Plan and Elevation of a Rectangular Block 9″ long, 6″ wide, and 3″ thick, when a 9″ × 6″ Face is horizontal, and 1″ above the H.P. (or Ground) and One of its Sides is parallel to the Vertical Plane, and 2″ from it. (Scale half-size.)**—First draw across the paper a line, and mark it XY[1] (Fig. 292). Then fold or bend the paper about this line, as in the previous study, and as shown in the figure, and place the block on something 1″ thick; it will then be the right height above the

[1] This is the ground-line, as it is called; it is invariably marked XY in geometry.

ground, or horizontal plane. If we now move it till its back face is parallel to the vertical plane, and 2″ from it, the block will be in the required position. The figure clearly shows this position, and at this stage it will be instructive to compare this problem with the previous study (assuming that the box and the block are the same size). It will be noticed that the plan in Fig. 292 is the same shape as the plan in Fig. 291 (this must be so, as both solids are horizontal), but is 2″ distant from XY (that is, 2″ from the V.P.), and similarly with the elevations, they are the same shape. The one in Fig. 292, being 1″ above XY, shows that it is 1″ high. Of course it will be noticed that the lines (projectors) connecting the block with its elevation are perpendicular to the V.P., and also the lines connecting the block and the plan are perpendicular in the horizontal plane. The figure also shows by dotted lines the paper folded (constructed) back into its proper (normal) position, and the dotted elevation shown will be seen to be *in the same straight line with the plan, perpendicular to the ground-line* (XY). Thus, when the projections of an object are drawn, we always have the **plan and elevation in the same straight line perpendicular to the ground-line.**

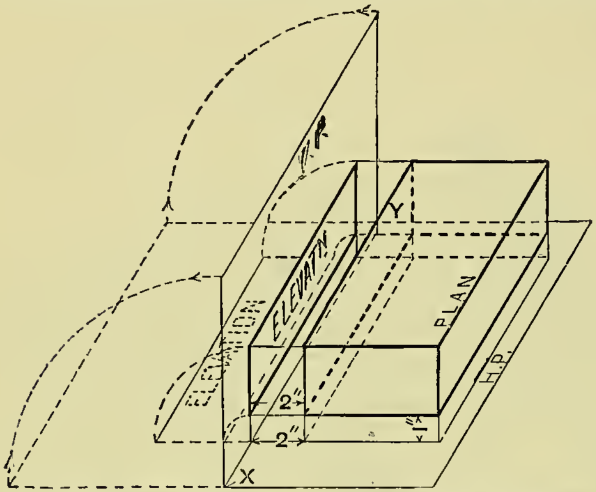


Fig. 292.—Block in position, between folded drawing paper.

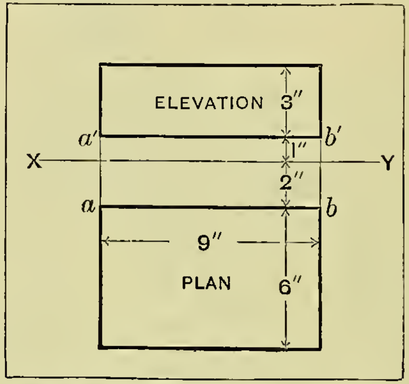


Fig. 293.—Projections of a rectangular block or cuboid.

To make this second study complete, let us suppose that we, knowing exactly how the views will appear in shape and position, wish **to draw in the ordinary way the projections of the block** to satisfy the problem. The first thing to do is to draw XY, the ground-line (Fig. 293). Then, as in this case we can first draw either projection, let us start on the plan. Remembering that the block is 2″ from the V.P., we draw a line *ab* parallel to XY and 2″ below it, and on this line we construct the plan, which of course is a rectangle, whose length is 9″ and breadth 6″. Then from each end of this plan draw a projector perpendicular to XY; between these projectors draw *a′b′* the bottom of the elevation, parallel to XY and 1″ above it, and on this line complete the rectangle, whose breadth is 3″ (the block's thickness), which forms the elevation. The projectors are best drawn undotted, but much thinner than the lines that form the projections.[1]

This completes the projections, and you would do well to repeat the operation explained in the previous study, and try to imagine that the solid itself is standing over the plan, and in front of the elevation, as shown in the figure.

Note.—Before leaving this study, we might notice that the line *a′b′* on the elevation represents the bottom of the block, a horizontal surface, a surface

[1] In an **ordinary mechanical drawing the projectors are not allowed to remain,** any that may have been drawn as a matter of necessity or convenience being rubbed out.

perpendicular to the vertical plane. You will directly better understand that the projections on a plane of all surfaces perpendicular to it are straight lines on that plane. Thus the line *ab* on the horizontal plane is the plan of a vertical side.

**180. End (or Side) Elevations, and Sections.**—Let us suppose we are looking at the rectangular block (Fig. 294) in the direction of the arrows A and B; the view we then get is called an *end elevation*, and it may be shown as at E, where the figure is obviously constructed with the assistance of the plan, the 3″ height being marked off with the dividers. It is generally more convenient to place this view by the side of the elevation, as shown at AF; the view is then projected from the elevation as shown, the 6″ breadth being marked off with the dividers or found by using the arcs *fm* and *hn*. If we were to cut through the solid with a vertical saw-cut along the line CD in plan, the true shape of the cut would be a vertical *section* (a section on the line CD as it is called) of the solid. This is shown at G in the position which is usually most convenient in relation to the elevation. It is drawn in the same way as the end elevation AF.

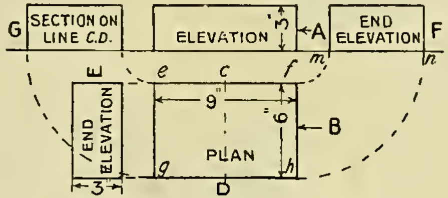


Fig. 294.—Projecting sections and end elevations.

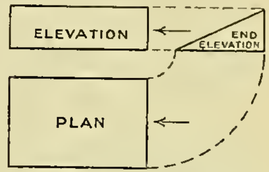


Fig. 295.—Projections of a triangular prism.

Usually an **end elevation becomes necessary because** some part or parts cannot be properly seen either in plan or elevation; or, taking the simplest case, it may be because one or more of the edges of the solid is not parallel to the H.P. or V.P., and therefore will not be seen in true length either in plan or elevation. You will better understand this if you look at Fig. 295. The end elevation, read with the other views, shows that the solid is a triangular prism, although its plan and elevation are the same shape as those of the rectangular block in Fig. 294.

We may now proceed to **review** the **salient points** as they would probably present themselves to you if you were about to commence a working drawing. You should begin by making up your mind as to how many views of the object you intend to show, bearing in mind that the drawings should clearly represent the object in such a way that its true dimensions and the form of every detail are shown. So long as this is satisfactorily accomplished, as few views as possible should be drawn. Two views at least are always required, and these may be an *Elevation* (which shows length and height), and a *Plan* (which shows length and breadth).

Or the front elevation and an end elevation may be used to obtain a similar result. But three views, namely a *Front elevation*, an *End elevation*, and a *Plan* are generally shown, with sufficient *sectional elevations* and *sectional plans* (part section and part elevation, and part section and part plan respectively) to make the external and internal form or construction of the object quite clear. The **use of dotted lines**, as in the end elevation at $MM_2$ $KK_2$ (Fig. 297), for indicating the position of unseen parts, should as a rule be **avoided as far as possible**; but a judicious use of a few of them may save the making of another view, provided always that they do not impair the clearness of the view upon which they are placed.

**Dotted lines** should not be used for unseen parts in highly finished **coloured drawings**, but only for working drawings. In cases where the object to be shown is symmetrical about the centre line, it is usual to show one **half of the view in elevation**, and the other **half in section**, as in the sectional elevation of the coupling, Fig. 476, Art. 248.

The section may extend slightly beyond the centre line, or may finish at it; in either case a black line is used to terminate the section. This saves the making of a separate sectional view.

Although it is obviously desirable to limit the number of views of an object, as previously explained, care must be taken not to carry this too far; as in the case of a complicated object, say a casting, much time is often spent by the pattern-maker and others in trying to read a drawing, where an additional view or section would have enabled the trained eye to see at a glance a mental picture of the required object.

It is usual in **English practice** to arrange **elevations above plans**, or sectional plans, when convenient. In **American practice** this is reversed, as we shall see in Chapter XIX.; but in all cases the views must be arranged so that the relation between two adjoining ones may be readily recognized, and so as to facilitate their being properly projected one from another.

Having decided upon the number of views to be shown, it is usual to take a spare piece of paper, and to roughly sketch upon it the views decided upon in their relative positions one to another, and to mark upon each the overall sizes, as in Fig. 297. **The scale to which the views can be drawn**, in dealing with large bodies, will depend upon the size of the sheet of drawing paper to be used.

All sheets of drawings forming one set should have equal outer margins, and as far as possible equal spaces between the views.

Having arranged the positions of the views upon the sheet, and the scale to which they are to be drawn, the next thing to be done is to draw the centre lines of the various views. The positions of these can be readily ascertained from a rough sketch used to adjust the spacings, and they should be carefully marked out; and after this has been done, the various views may be commenced. Of course **these remarks are for the guidance of the young draughtsman.** The beginner will always have plenty of paper to practise on, and need not trouble about the spacing out.

It is impossible to lay down any fixed **rule as to what view should be first completed**; in fact, it is usually the practice to work upon two or three views at the same time, drawing some part upon all views first, and then adding another part to these, and so on. But generally any known portion, such as the size of a shaft, stroke of a part, leading centres or outline is first drawn; and *always* the view from which the greatest number of parts of other views can be projected, or the greatest amount of information obtained (frequently a section) is then proceeded with. An axiom being to **put in outside sizes of work indefinitely first**, and to fill in all smaller details, as bolts, rivets, studs, nuts, keys, cotters, etc., afterwards. In the case where a part has a circular form the **circles should be drawn first**, and the other views projected from them, and when a number of similar parts, as rivets, bolts, and nuts, occur, it is best to put in the small circles of the entire number first, with one setting of the compasses, and then the similar lines of each. This will take less time than if each one is completed singly, and ensures a more uniform result.

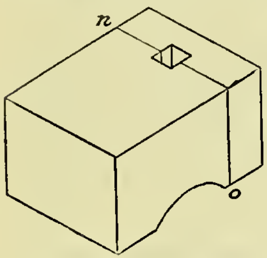

FIG. 296.—Isometric view of a bench block.

It is not usual to show upon working drawings, bolts, nuts, pins, rivets, studs, keys, cotters, rods, shafts, spindles, springs and levers in section when a section plane passes through their axes. The reason being that it is less trouble to show them in plan or elevation than in section, and it renders the drawing more clear. But all these matters can now be more conveniently dealt with as we proceed to explain how drawings of a few simple objects may be made, starting with a very easy example and selecting others so that they may gradually present to the student further features and expedients in a progressive way.

**181. Drawings of a Cast-iron Bench Block.**—The sketch, Fig. 296, shows the form often given to a bench block or anvil, such as is often used in an engineer's fitting shop. Cast Iron is used for the block in preference to Wrought Iron, as it is much cheaper in first cost, and, being harder, is not so easily injured by a blow. The flat surfaces may be planed, but in some cases the top only is machined, and in others it is used rough as cast. In this and the following exercises, the views and scale selected are so arranged as to enable the object to be drawn upon a half imperial sheet of paper, viz. 22″ × 15″.

As a drawing example, the four views of the block shown in Fig. 297, viz. a front elevation, a plan, an end elevation, and a section on the line *no* (see also Fig. 296) taken transversely through the centre of the hole and looking to the right (the left-hand portion being removed), are to be drawn full size.

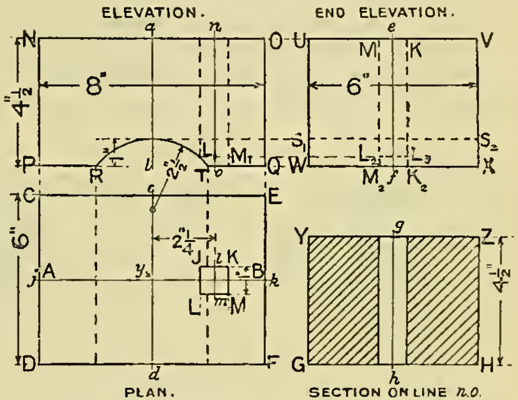


Fig. 297.—Four views of a cast-iron bench block.

So commence by placing a sheet of paper on the drawing board and pin it down taut and flat, as explained in Art. 21. This being a beginner's exercise, we need not trouble very much about spacing out the views of the block we wish to draw, as previously explained. If you have followed the previous exercises you will by this time be fairly able to manipulate your instruments correctly, and by the exercise of a little intelligence will easily draw the plan and elevation of the block; so, bearing in mind the hints previously given as to which view to draw first, it will be seen that this is a case where the plan should be first set out. Then start by drawing the centre lines *jk* and *cd*, intersecting in *y* (Fig. 297), in suitable positions. The length of the block should be first set out by pricking off *yj* and *yk* with a 4″ opening of the dividers, the scale being full size. The T-square is then drawn down to about 3¼″ below *jk*, and the 60° set-square is placed upon it and brought into position so that the pencil will be in line *k*. The line is then lightly drawn downward, nearly[1] to the T-square; and the set-square is then slid along the T-square, and a line drawn through *j* in a similar manner. Next prick off with the dividers *c* and *d*, 3″ on each side of *y*. The T-square is then raised to the lower mark *d*, and the finished line DF is drawn carefully, once and for all, between the two vertical lines previously drawn. The T-square is then raised to the upper mark *c* and a similar finished line, CE, drawn through it. Then rub out the extra portions of the lines at CD and EF and pencil in, completing the rectangle CEFD. Next draw the vertical centre line *lm* of the hole in the block, which will be 2¼″ from the centre of the block; and take the dividers and set them carefully to ½″, and prick off points in the sides of the square from its centre, and pencil in the sides JKML of the square in the same way as you did CEFD.

The elevation may now be proceeded with by first drawing an indefinite line PQ, a suitable distance from CE, and a similar line NO at the top, 4½″ from it. The side lines PN and OQ may now be projected from the plan and drawn their finished thickness. The arched opening RT may now be drawn; first mark up centre line *ba*, the height (1″) of the arch above the bottom of the block, and set the pencil compasses to an opening of 2½″ (the radius of the arch), and describe the arc RT as shown. Then from J and K in plan project the vertical finished dotted lines as shown, from bottom to top of the elevation, to indicate the position of the hole.

To commence the end elevation project two indefinite lines UV and WX from the top and bottom of the elevation respectively, and draw the centre line *ef* in a suitable position. Then mark off 3″ each side of this line and draw the finished sides UW and VX, completing the outline as before. To indicate the position of the square hole on this view set off *e*M and *e*K, ½″ each side of *e*, and draw the dotted lines $MM_2$ and $KK_2$. The dotted lines $S_1S_2$ and $L_2L_3$ are projected from the elevation, and indicate the position of top of the arch part and the intersection of the arch with the side of the square hole respectively. The section on line *no* is drawn in a

[1] As we do not know exactly where to stop, so we always draw it lightly and too long, and rub out what we do not require after its desired length has been obtained. This is much better than to draw a line too short, and to join a piece on to make it of the required length, as the joint always shows.

similar way about a centre line *gh*, the bottom GH being projected preferably from DF of the plan, and the sides YG and ZH from UW and VX respectively. Of course the height GY is 4½", the same as that of the elevations. As we are looking at the section from the left, we shall see the right-hand side of the section.

The **parts** actually **cut through** by the section plane should be **section-lined** as shown, and as described in Art. 23. And the section lines on both right- and left-hand side of the hole should be drawn sloping in one direction only, as it is one piece of metal.

The **section lines** used **to indicate cast iron** are **continuous ones** (Fig. 524); as shown, they are drawn with the 45° set-square, resting upon the T-square. The distance between them, or pitch of the lines, is a matter of taste, and should vary with the size of the part to be sectioned; in this case lines $\frac{1}{10}$th of an inch apart may be used. They can be drawn by judging the distances by the eye after a little practice, or a line can be drawn at right angles to the slope of the section lines, across the figure to be sectioned, and equal spaces set off upon it by ticking them off from a scale of equal parts, or by using a pair of dividers. To finish the drawing, carefully clean off any matter or lines not required, but the centre lines should be left, projecting about ¼" beyond the boundary of the view they are shown upon. The **dimensions** need not at present be shown on the drawing. The title of the drawing should be neatly written (printed) by hand, at the top of the drawing, making it clear and brief.

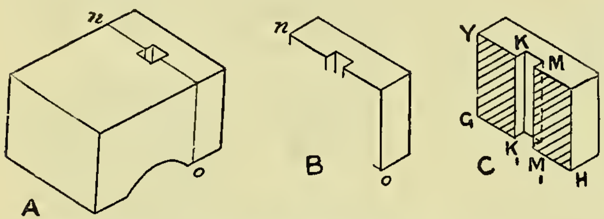


Fig. 298.—Illustrating the treatment of sections.

If you have any difficulty in realizing what the section on line *no* (or any other section) shows, you are strongly recommended to make a kind of **pictorial** or **isometric sketch** (refer to Chapter XX.) of the object, somewhat like that shown at "A" in Fig. 298, or better, if you will take the trouble to cut a model of the object out in yellow soap, or mould it in putty or modelling clay; it need not be to scale, but should be roughly proportionate in size; this model you can cut in the desired position to enable you to realize what shape the section would be. If you use a sketch, and have difficulty in deciding how the part cut by the section plane will appear, place the section line upon your sketch in the desired position as *no*; then rub out the forward portion (that to be removed) up to the section line, as shown at "B," and then try to complete the sketch "B," obtaining the data necessary to enable you to do so from the other views of the object. For instance, knowing the block to be rectangular with parallel sides, you can add to "B" the lines YG and HG, Fig. "C." Then you know from the elevations that the hole goes right through parallel to the sides, so you can draw the lines $KK_1$ and $MM_1$, indicating the cut hole. Of course this is only a sketch, but you should have no difficulty in identifying it with the section on line *no*, as given in Fig. 297.

## EXERCISES.

### Typical Oral Exercises.

1. What is the name you give to the view of the top of a house, as seen from a balloon, say?
2. You look at the windows of a house. What name would you give to the view you get?
3. What is the object of drawing an end elevation or side view of an object?
4. Why is it sometimes necessary to draw a sectional view of a body?

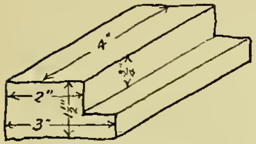

Fig. 299.

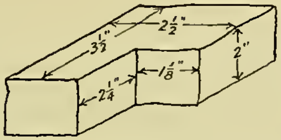

Fig. 300.

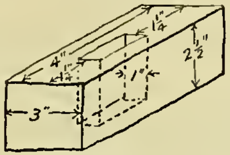

Fig. 301.

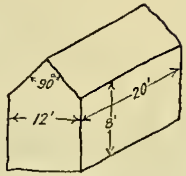

Fig. 302.

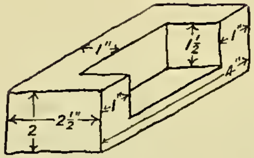

Fig. 303.

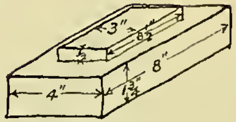

Fig. 304.

Figs. 299 to 304, illustrating drawing exercises numbers 5 to 10.

### Drawing Exercises.

5. In Fig. 299 a pictorial view of a solid is given. Draw its plan, elevation, and end elevation. Full size.
6. Draw plan and elevation of the solid shown in Fig. 300.
7. Draw plan, elevation, and a cross or transverse section through the cotter hole of the solid shown in Fig. 301.
8. A dimensioned sketch of a corrugated iron shed is given in Fig. 302. Draw three outline views of it. Scale $\frac{1}{4}''$ to the foot.
9. Draw a plan and elevation of the solid shown in Fig. 303. Is this a case where a third view could with advantage be drawn? If so, make the one that in your opinion gives the most information.
10. The pictorial view of a mounted oilstone is shown in Fig. 304. Draw two views of it. Scale $\frac{1}{2}$ size.

## CHAPTER XVIII

### PROJECTIONS AND SECTIONS OF SOME TYPICAL SOLIDS

182. We will now proceed to further study the science of projection, so far as solids are concerned, by working a few typical exercises that will bring out some of the operations in mechanical drawing which most frequently occur; and it will be convenient to present these in the form of the following problems:—

183. **To draw the Plan and Elevation of a Rectangular Block 3″ long, 2″ wide, and 1″ thick, placing it with its Base inclined 30° to the Horizontal, and one of its Sides parallel to the Vertical Plane and 1″ from it.**—After studying Art. 178, you will experience little trouble in working this problem. First draw across the paper the XY (Fig. 305), and bend the paper about this line in the way explained (this should be done with each problem until you can dispense with such help). Now place the block in the required position, and consider which view must be drawn first, plan or elevation. It will be noticed that as the sides are parallel to the vertical plane, the elevation will be the true shape of the side, and as the top and bottom are inclined to the ground (horizontal), the plan will give a foreshortened view of the top and the upper end, so that the elevation must be first drawn. To do this, at any point $a'$ on the XY, draw a line $a'd'$ inclined 30° to XY, and above it (this may be drawn with the set-square) construct the rectangular side, $a'b'c'd'$, of the block, which is the complete elevation. To construct the plan, draw (with the set-square) from the elevation the projectors $b'b$, $a'a$, $c'c$, and $d'd$, and then draw the line $b_2d_2$, 1″ below XY, and parallel to it; this is the plan of one side: next draw $bd$ parallel to $b_2d_2$, and 2″ (the width of the block) from it, for the other: pencil in $c_2c$, the plan of the top edge, of course this will be seen in plan, while $a_2a$ is the plan of the bottom edge, which is unseen, and is therefore shown dotted. Each corner of the elevation is, of course, the elevation of one of the horizontal edges.

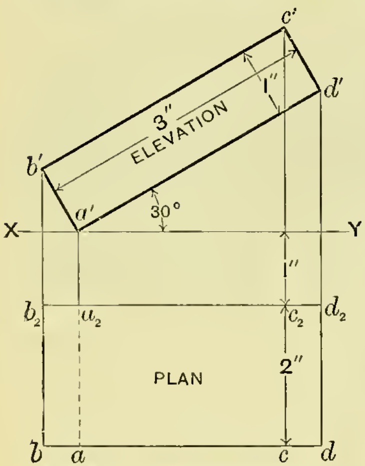

Fig. 305.—Rectangular block.

Note.—The line $b'b_2$ is a mere projector, and as such should be drawn very fine or dotted, but the other part, $b_2b$, of the line is a part of the plan, and should be a good bold well-defined line; the same remarks apply to the other similar lines in the figure.

184. **To draw the Plan and Elevation of a Hexagonal Pyramid (Axis 3″, edge of base $1\frac{1}{4}$″), when its Base is on the H.P., with an Edge of the Base inclined 35° to the V.P.**—Commence with the plan (Fig. 306), as the base will be seen in true shape. This will be a

hexagon with a side inclined 35° to the XY,[1] and the opposite corners joined give the plans of the slant edges of the solid. The line on the drawing inclined 35° to V.P. should be laid down with the protractor, and the $1\frac{1}{4}''$ edge marked on it. The hexagon can be constructed on this, and the plan completed. The elevations of the six corners of the base must be on the ground-line, as the base is on the H.P. The axis of the pyramid, 3″, must be set up from the point where the projector from the centre of the hexagon meets the ground-line. The whole elevation is completed by joining the elevation of the apex to the six points determined on the ground-line for the elevations of the six corners of the base, as shown. Obviously, two of the slant edges will be unseen in elevation, and will therefore be shown dotted.

**185. To draw the Projections of a Tetrahedron, whose Edges are $2\frac{1}{2}''$ long, when one of its Slant Edges is parallel to the V.P., and a Face is on the H.P.** —The tetrahedron is to stand with one of its four faces on the ground (Fig. 307), so the plan will be an equilateral triangle, with its three corners joined to its centre, and as a slant edge is to be parallel to the V.P., you should, in starting on the plan, construct the triangle with one of its edges perpendicular to the XY, as shown. Construct the elevation by drawing from the corners in the plan projectors, giving $c'$, $b'$, and $d'$ on the ground-line, as the elevations of the corners of the base. The elevation of the apex will be somewhere along the projector through $a$; its exact position is found by describing an arc about $c'$ as centre, with $2\frac{1}{2}''$ (true length of edges) radius, cutting the projector $aa'$ in $a'$, which is the required point. Complete the elevation by joining $a'c'$ and $a'b'$.

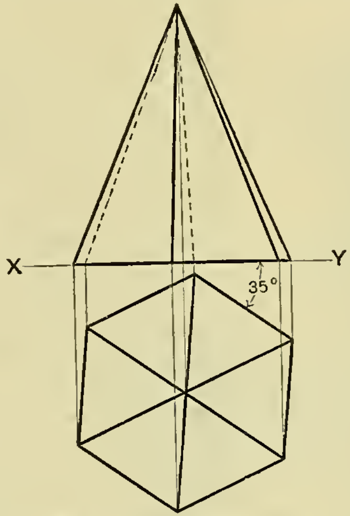


FIG. 306.—Hexagonal section.

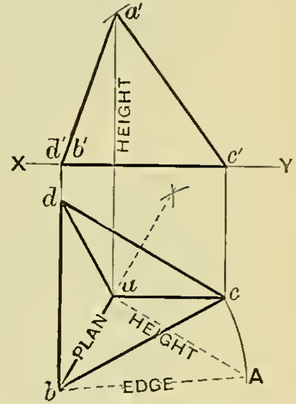


FIG. 307.—Tetrahedron.

It should be noticed that the edge AC is parallel to the V.P. (as required), and therefore the elevation $a'c'$ is the true length of the edge. Cases occur **when neither slant edge is parallel to the V.P.**; then the **height of the tetrahedron** must be determined before the elevation can be drawn. This height can always be found in the way just described by using an auxiliary plane parallel to a slant edge, or by a separate construction. If determined by a separate construction, it should be remembered that the height is found when a right-angled triangle, having the plan of a slant edge as its base, the true length of the edge as its hypotenuse, and the required height (or axis) as its perpendicular, is drawn (this triangle can always be readily drawn, as two sides and an angle are known). The triangle is often conveniently drawn on the plan, as shown at $ba$A, where $a$A is perpendicular to $ab$, and $ba$ is equal to $bc$, the true length of an edge.

NOTE.—You should carefully examine a **wire model** of this solid; beginners often blunder by assuming that the height is equal to the length of the edges.

**186. A Cylinder with its Base on the H.P. is cut by a Plane passing through the Top Left-hand Corner of its Elevation, and making an Angle of 60° with its Axis. To draw Plan and True Shape of Section (Axis $2\frac{1}{2}''$, Diameter 2″).**—Draw the projections of the

[1] The inclination of a line to a plane is the inclination of the line to its projection on that plane.

cylinder as shown (Fig. 308), and through the top left-hand corner of the elevation set off a line (representing the section plane), inclining it 60° to the axis (or sides). As the plans of all sections of a vertical cylinder are circles, and coincide with plan of the cylinder, we have only to cross-hatch the plan with dotted lines,[1] as shown, to represent the plan of the section. The true shape of the section may be found as shown in the figure. This projection of the section is drawn by first **dividing the plan into** any number of parts, say **twelve** (which is the most convenient number). This is easily done by manipulating the 60° set-square, or in the following way. Divide the circle into four parts by lines parallel and perpendicular to XY, and then with an opening of the compasses equal to the radius, mark off a point each side of each of these four points. The circle is then divided into twelve equal parts, *a*, *b*, *c*, etc.: from each of these points draw a projector to the top of the elevation. It will be noticed that with two exceptions one projector is only required for two points, as the points *b*, *c*, *d*, *e*, *f* come directly in front of the points *l*, *k*, *j*, *i*, *h*. To proceed with the projections, draw a line perpendicular to the cutting plane from each point made by a projector cutting this plane, and along this line **mark off a point, making it the same distance from the section plane as the plan of the projection is from the XY.** Thus the projector from *g* cuts the section plane in *g'*, and the line *g'*G is drawn perpendicular to the section plane, and the distance *g'*G is marked off equal to the distance of *g* from XY. Passing to the next projector, we have *f'h'* the common elevation of F and H, and therefore the two distances *f'*F and *h'*H on the same line must be made equal to the distance between *f* and XY, and *h* and XY respectively. In a similar way the points I, J, K, L, A, B, C, D, and E are found. Draw a fair line through these points, and the figure (an **ellipse**) will be the required **true shape** of the section.

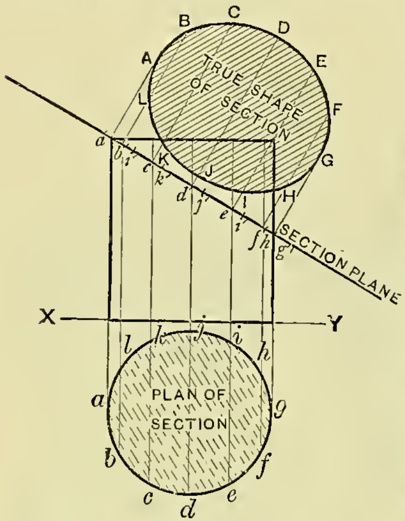


Fig. 308.—Section giving ellipse.

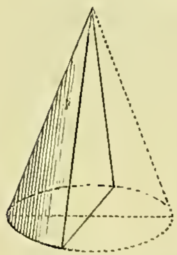

Fig. 309.—Section giving triangle.

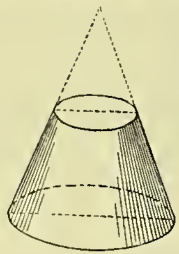

Fig. 310.—Section giving circle.

**187. Conic Sections.**—There are five different figures or sections of a cone (called **conic sections**) due to cutting planes, according to their different positions. The **section** will be a **triangle** if the cutting plane pass through the apex of the cone and any part of its base, as in Fig. 309. And if the cone be cut into two parts by a plane parallel to its base the **section** will be a **circle**, as in Fig. 310. The **other three sections** are the **hyperbola**, the **ellipse**, and the **parabola**, dealt with in Chapters XII. to XIV.; but we will now draw these curves as actual sections of a cone.

**188. Given a Cone (Diameter of Base $2\frac{1}{2}''$, Axis $3''$) with its Base on the H.P., to draw the True Shape of a Section made by a Cutting Plane.—First Case: By a Plane parallel to its Axis and $\frac{1}{2}''$ from it.**—Commence by drawing the plan and elevation of the cone, as shown in Fig. 311; then draw AB $\frac{1}{4}''$ from C, the centre of the plan, and parallel to XY, cutting the circular base in *m* and *n*. The elevations *m'*, *n'* of these points can be at once drawn; they must be on XY, as the base is resting on the ground, and they are points in the required section. Through C draw a perpendicular to AB, cutting it in *d*. This point will obviously be the plan of the **highest point in the section**; and **to**

[1] The **cross-hatching** is shown **dotted** in plan, as the cut surface is covered by the top part of pyramid.

**find its elevation**, with centre C, radius $Cd$, describe an arc cutting OP in $e$; then through $e$ draw a projector, cutting the side of the elevation in $e'$. Through $e'$ draw $e'f'$, parallel to XY and cutting the axis in $d'$, the required point. Next, with centre C, and any radii $Cg$ and $Ch$ (less than $Cm$), describe semicircles, cutting AB in J and K. These semicircles represent the plans of horizontal sections, whose elevations can be found by drawing projectors through $g$ and $h$ to cut the sides of the elevation of the cone $g'$ and $h'$, draw lines through these points parallel to XY; then $J'$ and $K'$, elevations of J and K on these lines, will be points in the required section, as they are on the cone, and must be contained by the cutting plane. Similar points can be found on the other side of the elevation, as shown in the figure, or **by symmetry**; and a fair line drawn through these points gives the conic section. This **section is a hyperbola**, for, as explained in Chapter XIV., any **cutting plane parallel to two generators** of the cone cuts the solid in a **hyperbolic section**.

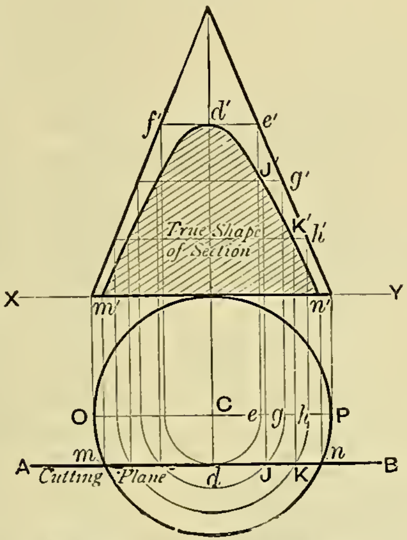


FIG. 311.—Section giving hyperbola.

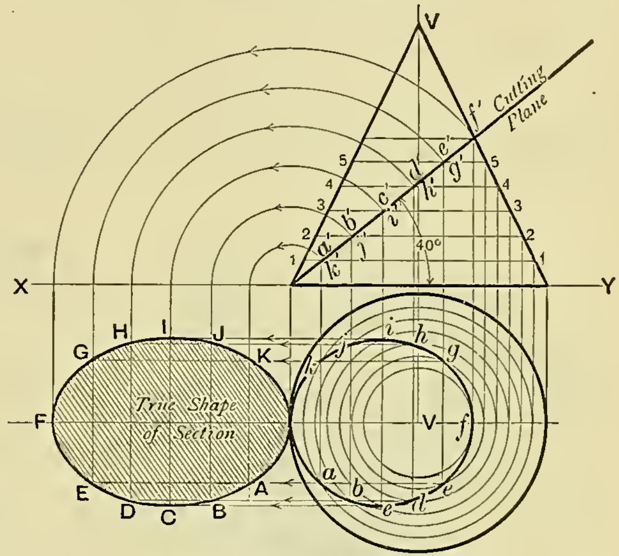


FIG. 312.—Section giving ellipse.

189. **Second Case: By a Plane inclined to the Axis at a given Angle (say 50°) and passing through a Point in the Base.**—Commence by drawing the projections of the cone (Fig. 312), and through a bottom corner of the elevation draw the section plane, making $90° - 50° = 40°$ with XY, cutting the opposite side in $f'$. Divide this line into any even number of parts (say six), and through these divisions, $k'a'$, $j'b'$, etc., draw projectors; also through these points draw lines across the elevation parallel to XY, cutting the sides in 1 1, 2 2, etc. These lines represent the elevations of circular sections of the cone whose circular plans can be projected, cutting projectors through $a'k'$, $b'j'$, etc., as shown in the figure. We may now think of the lines $ak$, $bj$ (the intercepts), etc., as the plans of lines contained by the section plane. It will then be obvious that the points $a$, $b$, $e$, etc., are the plans of **points both on the plane and on the cone**; that is to say, plans of points in the required section of the cone. If these points be joined by a flowing line, the figure formed will be the plan of the required section. But we require its true shape. To get this, we may construct or revolve the figure into the H.P., as shown in the figure, which should by now speak for itself.

NOTE.—Of course, **both the plan and true shape are ellipses**. It will occur to you that these figures could have been drawn direct from their major and minor axes, as explained in Chapter XII. And you might try to draw them in this way, but it will require a little care on your part to find the **true length of the minor axis**.

**190. Third Case: By a Plane parallel to its Side and $\frac{1}{4}''$ from it.**—Commence by drawing the projections, as before (Fig. 313). A line parallel to the side $n'V'$ and $\frac{1}{4}''$ from it will represent the plane, cutting the cone from $m'a'$ to $g'$ in elevation; and by dividing this line into any number of parts (say six) by lines parallel to XY, the points $a$, $b$, $c$, etc., in the plan of the section can be found, as in the previous case, and by constructing the section into the H.P. the **true shape** can be found. This, of course, will be a **parabola**, as explained in Chapter XIII.

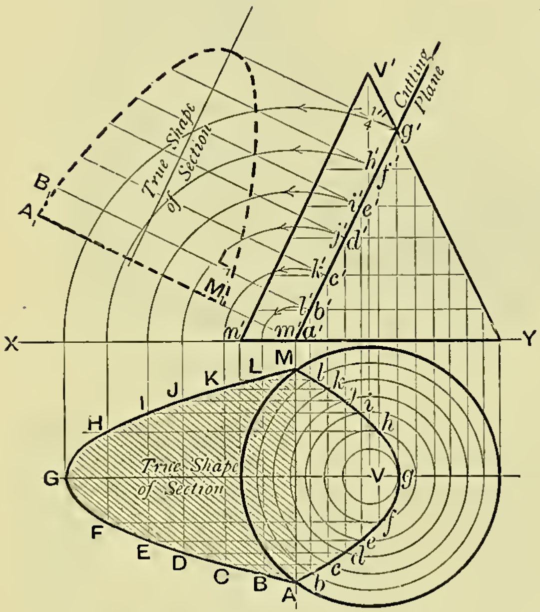


FIG. 313.—Section giving parabola.

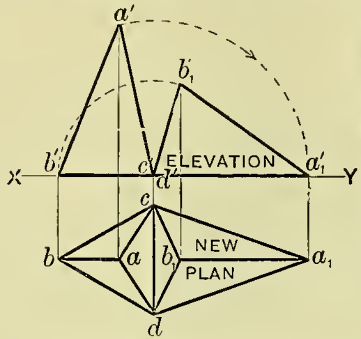


FIG. 314.—Triangular pyramid.

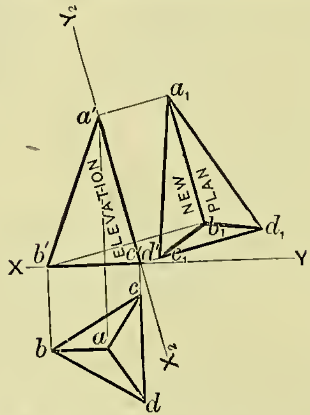


FIG. 315.—Use of preliminary plan.

NOTE.—**The dotted figure showing the true shape** is arrived at by the alternative method explained in Prob. 186, and, as will be seen, by using a fewer number of lines; but the author invariably finds that beginners get a better grasp of the geometry of the problems by working them as explained, to begin with.

Of course the distances $A_1M_1$, $B_1L_1$, etc., on the dotted figure from $n'V'$, the side of the cone, are made equal to the corresponding points in the plan from XY.

**191. Preliminary Projections.**—To construct the required plan and elevation, it is sometimes necessary to draw preliminary projections. For example, suppose our **problem is to draw the plan and elevation of a pyramid when resting on one of its faces.** This can be done, as in Fig. 314, by first drawing the plan and elevation $abcd$ and $a'b'c'd'$ of the pyramid when resting on its base; and then swinging the elevation over until the face ADC (represented on the elevation by the line $a'c'$) touches XY. As the pyramid is hinged on the edge $cd$, this line will be part of the new plan. The remaining corners, $b_1$, $a_1$, of this plan are found by dropping down projectors from $a'_1$ and $b'_1$ to intersect lines from $b$ and $a$, parallel to XY, in $a_1$ and $b_1$. Join these points, as shown, and the elevation and plan are completed.

Nothing more need be said about the construction of this figure, as a glance at it will explain the whole operation. So you may now compare it with the **alternative method** shown in Fig. 315. Here you will see that, instead of constructing (swinging) the face into the horizontal plane (XY), we have constructed the horizontal plane on to the face (in geometry this is, of course, easily done, as the H.P. is represented by a line), and found our new plan by running down projectors perpendicular to our new ground-line $X_2Y_2$; and by marking off $a_1$ the same distance from $X_2Y_2$ as $a$ is from XY, and doing the same with the other points—in fact, by making the new plan of each point the same distance from the new ground-line as the old plan is from the old ground-line—we have only to join these points to complete the plan.

NOTES.—1. By comparing the two methods, you will notice that in Fig. 315 there are four views of the pyramids against three in 316; and when you remember that in some cases the object projected is more complicated, you will, no doubt, conclude that the latter method, namely, moving the ground-line instead of the solid, is the best one to adopt.

2. If this little study is properly understood—and it should not be passed till it is—the following problem will be easily worked.

**192. To draw Plan and Elevation of a Cylindrical Roller, when a Face is inclined 55° to the Vertical Plane (size 3", diameter 1" thick).**—First draw plan and elevation of the roller, with a face parallel to the V.P., as shown in Fig. 316. Then draw a new (auxiliary) ground-line $X_2Y_2$, making 55° with the face in plan, and divide the elevation into a number of equal parts (Prob. 186), $a'$, $b'$, $c'$, $d'$, etc., and find their plans, $a$, $b$, $c$, $d$, etc. Then from these plans project the new elevations $a''$, $b''$, $c''$, $d''$, etc., making their heights the same above $X_2Y_2$ as they are above XY. Draw through these points $a''$, $b''$, $c''$, etc., a fair line, and the figure is the new elevation of the front face. In the same way project the elevation of the back face, and complete the projection by joining the top and bottom points, and dotting the part of the back that is unseen, as shown.

NOTE.—The figure $a''b''c''d''$, etc. (part of the new elevation), is an ellipse, and is perfectly symmetrical about centre lines parallel and perpendicular to $X_2Y_2$. Of course, when a circle is inclined to the plane of projection, its projection is always an ellipse.

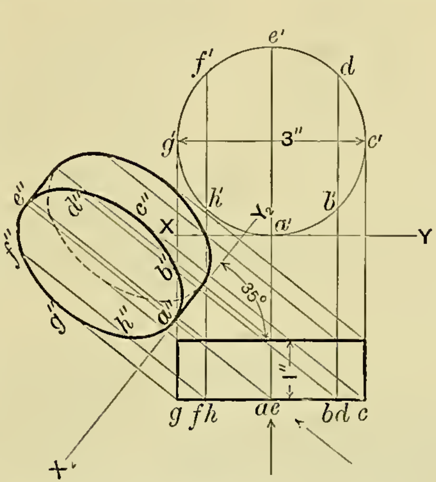


FIG. 316.—Use of preliminary or auxiliary elevation.

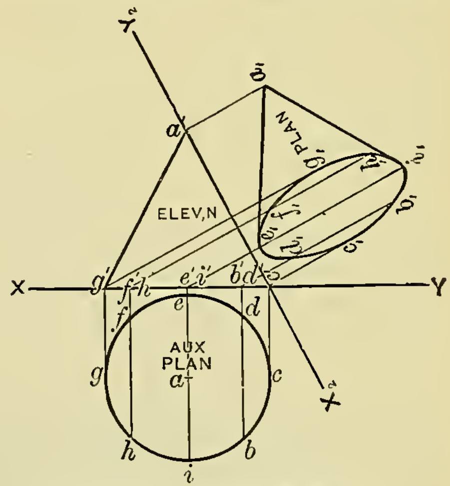


FIG. 317.—Cone lying on its side.

**193. To draw the Plan and Elevation of a Cone when lying on its Side.**—First draw the plan and elevation of the cone when standing on its base (Fig. 317). Then draw the new ground-line $X_2Y_2$, through the side $a'e'$, and project the plan of the circular base (as in the previous problem), which will be an ellipse, and having found $a$, the plan of the apex, draw from it tangents to the ellipse, and the plan is complete.

**194. A Horizontal Line parallel to the Vertical Plane is given by its Plan and Elevation, to determine the Distance from the XY.**—Let $ab$, $a'b'$ (Fig. 318) be the given plan and elevation of the line. Draw a second ground-line, $X_2Y_2$, at right angles to XY, and from $a$ and $b$ run up projectors perpendicular to it, and mark off $a''b''$ along these projectors, making their height above $X_2Y_2$ the same as their height above the old ground-line XY. The dotted lines in the figure show one way of marking off the height of the new elevation. Clearly the slant distance between the points XY and $a''b''$ is the required distance, as these two points are the end elevations of XY and the given line.

NOTE.—This expedient of assuming a new plane perpendicular to the V.P. so that an end view of the planes is obtained is most useful when the distance between the XY and a plane parallel to it is required.

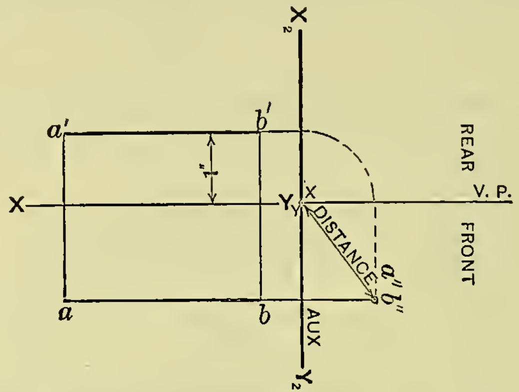


FIG. 318.—New elevation, showing distance of line AB from XY.

## DRAWING EXERCISES.

In selecting problems from the following it should be realized that this course in industrial drawing will probably in most cases extend over a period of some two or three years. The more difficult exercises are near the end of them, and a good deal of experience in working problems in the projection of solids is necessary before these can be satisfactorily tackled. Models of the solids should be freely used when available.

1. In Figs. 319 to 325 you have the projections of various solids shown, drawn to a scale of $\frac{1}{4}$ full size. Write particulars of the shape, name of solid, dimensions, and position of each one. As an example of what is required, the following particulars relating to Fig. 319 may be considered typical. **Name of solid.** Square prism. **Dimensions.** Edge of base $1\frac{11}{16}''$; length of solid $2\frac{11}{16}''$. **Position.** A long edge on the H.P. and perpendicular to V.P.; a side inclined 30° to H.P.

2. The length, breadth, and thickness of a rectangular solid are 3″, $1\frac{1}{2}''$, and 1″ respectively. Place it in any position in relation to the horizontal and vertical planes, draw its plan and elevation, and write particulars of its position in relation to those planes.

NOTE.—This is a type of problem of great educational value. The teacher will be easily able to vary it.

3. The axis of a square prism is inclined 30° to the horizontal plane, and one of its sides is in contact with the vertical plane. Draw projections of the solid, axis 3″, and edge of base 2″.

4. Draw plan and elevation of a brick 9″ × $4\frac{1}{2}''$ × 3″, when it is resting on an end, and a 9 × $4\frac{1}{2}$ surface is inclined 30° to the vertical plane. Scale, quarter full size.

5. Draw plan and elevation of a square prism (height 3″, edge of base $1\frac{1}{2}''$), when a long edge is on the H.P., and a face is inclined 60°.[1]

[1] It should be remembered that when the name of neither plane is mentioned, *inclined* means inclined to the horizontal plane.

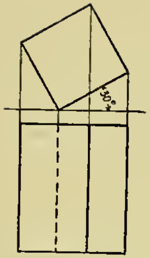


FIG. 319.

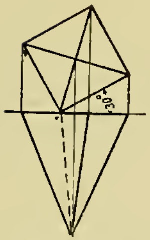


FIG. 320.

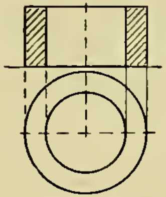

FIG. 321.

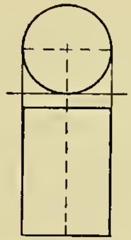

FIG. 322.

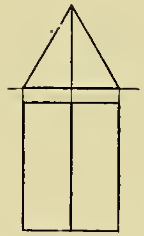

FIG. 323.

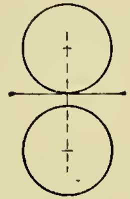

FIG. 324.

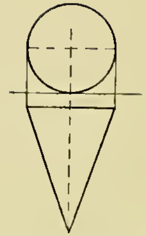

FIG. 325.

NOTE.—These solids are drawn to a scale of quarter full size.

6. Draw the projections of a square prism (height 3″, edge of base $1\frac{1}{2}$″), when a diagonal of a face is horizontal, and it is resting on one of its short edges.

NOTE.—You may draw the elevation first, and then place under it the XY, unless you can see a way of working it direct.

7. Draw plan and elevation of a cube of 2″ edge, when a diagonal of a face is inclined 30°.

8. Draw projections of a hexagonal prism (height 3″, $1\frac{1}{4}$″ edge), when resting on a face, and an end is 1″ from the V.P.

9. Two prisms (length 3″, edge of base 1″) are arranged one on the other, so as to form the letter T. Draw the plan and elevation when the front of the letter is inclined 25° to the V.P.

10. An inch square hole, 1″ deep, is made in the centre of a square slab (thickness $1\frac{1}{2}$″, diameter $2\frac{1}{2}$″), and a square prism (length $2\frac{1}{2}$″, edge of base 1″) is placed in the hole. The sides of the hole are parallel to the sides of the slab. Draw plan and elevation when a diagonal of the base of the slab is parallel to the V.P.

11. Show, by its projections a square pyramid (height 3″, edge of base 2″), when its base is on the V.P., and an edge of the base is inclined 35°.

12. Two square prisms (height $2\frac{1}{2}$″, diameter 1″) are placed one on the other, so that they form the letter T. They tilt over a little, the base of the vertical one being inclined 15°. Draw their projections.

13. Draw the plan and elevation of a hollow cylinder (height 3″, diameter 2″, diameter of hole 1″). Place it with its axis horizontal.

14. A cylinder, whose height is 2″, and diameter 1″, supports a square slab, whose thickness is 1″ and diameter 2″. Draw their projections when the axes are both vertical and in the same straight line.

15. A hollow cylinder (height 3″, diameter 2″, diameter of hole 1″) stands on the H.P., and supports a sphere of $2\frac{1}{2}$″ diameter. Draw projections.

NOTE.—A small circle of the sphere will rest on the top edge of the hole.

16. A hollow sphere (diameter 3″, diameter of hole 2″) rests on the ground, and is cut by a horizontal plane 2″ high. Draw the true shape of the section.

17. A vertical hexagonal prism is pierced by a $1\frac{1}{2}$″ cylindrical hole in the direction of its length, and is cut by a plane bisecting it, and making an angle at 45° with its axis. Draw the true shape of the section. Height of prism $3\frac{1}{2}$″, edge of base $1\frac{1}{4}$″.

18. Draw plan and elevation of a tetrahedron of 2″ edge, when an edge of a face is inclined 45° to the V.P.

19. A hexagonal prism (height 1″, edge of base $1\frac{1}{2}$″) supports a tetrahedron, three of whose corners rest on three of the top corners of the prism. Draw plan and elevation.

20. Draw the projections of a tetrahedron of 2″ edge, when a face is in contact with the V.P., and the *elevation* of one of the edges not on the V.P. is inclined 45°.

21. An octagonal slab, whose thickness is 1″, supports a square prism (height 2″, edge of base $1\frac{1}{2}$″), each bottom corner of the prism resting on a top corner of the slab. Draw plan and elevation when a diagonal of the prism is parallel to the V.P. Also show the true shape of a section made by a plane cutting through the top left-hand corner of the prism and the bottom right-hand corner of the slab.

22. Six square prisms (length 3″, edge of base 1″) are arranged horizontally in such a way that three of them, whose sides are in contact, support two also

in contact, and the remaining one is placed on top of the two, so that the six together form three equal steps. Draw their plan, and an elevation on a V.P., making 30° with the long horizontal edges.[1]

23. Draw plan and elevation of a tetrahedron of 2″ edge, when two of its faces are vertical.

Note.—If two or more faces or planes meet in an edge or line, they will be vertical if the edge or line be vertical.

24. Draw the projections of two spheres, diameters 2¼″ and 1″, placing them in contact with one another. Show on the projections where they touch one another.

25. A 2″ cylinder rests on the ground with its 2¼″ axis vertical and 1¾″ from the V.P. Show an 1¼″ sphere touching it and the plane of projection.

26. A cone, base 2″ diameter, axis 3″, rests with its base on the H.P., and its axis 1½″ from the V.P. An 1⅜″ sphere resting on the ground touches it. Draw their projections.

27. A cone with its base on the H.P. is cut by a vertical plane, the distances of the plane from the axis being ½″, diameter of base 2¼″, axis 3″. Draw the true shape of the section, and give its geometrical name.

28. A cone, base 2½″, axis 3″, stands with its axis vertical. Draw the plan and true shape of a section made by a plane bisecting the axis, and inclined 60° to it. What is the geometrical name of this section?

29. A cone, base 2½″, axis 3″, is cut by a plane in a direction parallel to its side, and ½″ from that part. Draw the true shape of the section, and state the geometrical name of the curve.

30. The shape of a slab is a kite[2] whose diagonals are 3″ and 1¾″, an angle between two of its sides being 30°, and its thickness ¾″. Draw its projections when one of its sides is resting on the H.P., and its plane is 1″ from the V.P.

31. Draw the plan and elevation of a square prism when its faces are inclined 45° to the H.P., and its ends make 45° with the V.P. Edge of base 1½″, axis 2½″.

32. Draw the projections of a square pyramid when its axis is horizontal and inclined 40° to the V.P. Edge of base 2″, axis 2½″.

33. Draw plan and elevation of a square pyramid when its base its vertical, and one of its faces is parallel to the vertical plane. Edge of base 2″, axis 2½″.

34. A face of a tetrahedron of 3″ edge is in the V.P., and an edge of that face is vertical. Draw its projections, and show two auxiliary elevations, made on planes parallel to the plan of a face and the plan of an edge.

35. Draw plan and elevation of a cylinder whose axis is horizontal, and draw a new elevation on a plane whose XY makes 30° with the plan of the axis. Diameter of base 2″, length 3½″.

36. Draw projections of a cone when its axis is horizontal and inclined 30° to the V.P. Diameter of base 2½″, height 4″.

37. A square prism, resting on one of its short edges, has its axis inclined 60°. Draw plan and elevation, and make a new elevation on a plane inclined 30° to one of its sides. Edge of base 2″, height 3″.

38. A square slab, with a round hole through its centre, is so placed that its square faces are inclined 30° to the V.P., and all its long edges are inclined 45°. Draw its projections. Edge of square 3″, diameter of hole 1½″, thickness 1″.

39. Draw plan and elevation of a square pyramid with one of its faces resting on the H.P. Edge of base 2″, height 3″.

40. Draw projections of hexagonal pyramid when one of its slant edges is vertical. Edge of base 1·5″, axis 3″.

41. A triangular pyramid rests with its apex on the ground and a face vertical. Draw projections. Base equilateral triangle of 2½″ edge, height 4″.

42. Draw the plan and elevation of a square prism, base 2″, axis 4″, when it is suspended from a point on a long edge 1″ from a corner.

Note.—The line passing through the given point and the centre of gravity of the prism (which in this solid is at the centre of its axis) must be vertical.

43. Draw the projections of a rectangular block 3″ × 2″ × 1″, when one of its diagonals is vertical.

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[1] If their long edges be inclined 30° to the V.P., the elevation will be drawn on a V.P. making 30° with those edges.

[2] A quadrilateral which has a diagonal as an axis of symmetry has been called by Professor Sylvester a *kite*. Refer to Fig. 194.

# CHAPTER XIX

## FURTHER STUDIES IN PROJECTION

**195. First-angle** *versus* **Third-angle Projection, or English** *versus* **American Practice.**—The expedient we employed in Fig. 290 in getting clear ideas about the relation of plan to elevation may be now carried a step further. The first glance at that figure might suggest that the horizontal and vertical planes meet or intersect in the XY and go no further; but in Fig. 326 we have a pictorial view of a model[1] which clearly shows that the **planes cross one another**, indeed, we always suppose that they **extend**

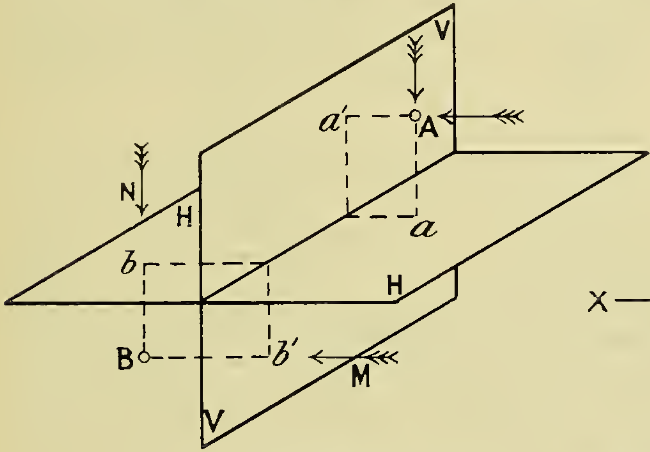



Fig. 326.—Pictorial view of planes of projection.

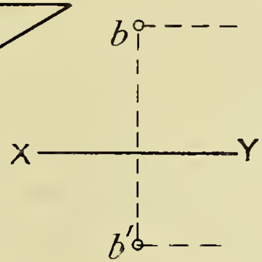



Fig. 328.—Third-angle projections of point B.

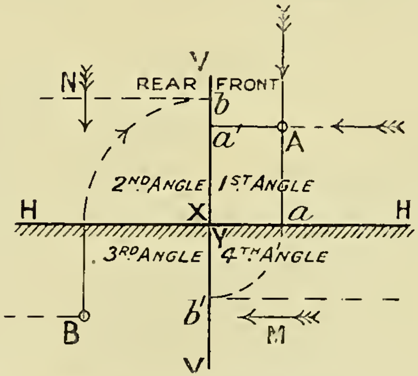



Fig. 327.—End view of planes of projection.

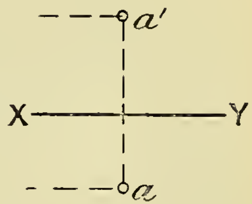



Fig. 329.—First-angle projection of point A.

**indefinitely in each direction.** Now, Fig. 327 shows an end view of these planes, and we see that **their intersection** in XY **forms four angles.** That above the H.P. and in front of the V.P. being called the first angle, and thus we have—

[1] This model can be readily made by cutting two cards, each through half its length, and halving them together.

The first Angle. Above H.P. and in front of V.P.
The second Angle. Above H.P. and behind V.P.
The third Angle. Below H.P. and behind V.P.
The fourth Angle. Below H.P. and in front of V.P.

The point A (Figs. 326 and 327), we see, is in the first angle, and its ordinary plan and elevation is shown in Fig. 329, with its **plan** *a* **(or view from top) below the elevation.** And this represents the practice usually followed by English draughtsmen.

Now, in Figs. 326 and 327, the **point B** is seen to be **in the third angle**, and looking in the direction of the arrow M (Fig. 326), its elevation will appear at *b*′, below the ground, and therefore below the ground-line or XY in Fig. 328. Looking down on B (Fig 326) in the direction of arrow N for its plan, we see this will appear at *b*, behind the vertical plane, and therefore above the XY, as in Fig. 328. Thus, when the **body** is supposed to be **placed in the third angle** we have the **plan above the elevation** on our paper, and this represents **the universal practice of American draughtsmen.** And all Americans are extremely sensitive when the convenience of this practice is questioned. This being so, it is perfectly certain that they are never likely to fall into line with the first-angle projection, which we have seen is generally adopted in this country. On the other hand, as a large amount of American machinery is used in this country, and drawings relating to it, made in third-angle projection, are often being received, and, further, as we have many American engineering books used in England in which, of course, things are shown projected in accordance with American practice, it is well to become familiar with the system. So, mainly for the above reasons, occasional examples in third-angle projection are given in the following chapters. Therefore, when you find the plan of a body is shown above its elevation, instead of below, as is usual with us, you will know it is American practice, even if it is not marked "Amer. Proj."

**196. End Elevations or Side Views.**—Look at the three views of a flanged taper pipe shown in Figs. 330 to 332, and those shown in Figs. 333 to 335. Which arrangement strikes you as being the most convenient? You will say at once, the former. For, otherwise, obviously, if the pipe happened to be very long, stretching across the paper, the two views, say of the square flange, would be at opposite ends (as in Figs. 334, 335), a considerable distance apart, and could not be easily compared. To avoid this, **every end elevation** or side view **should be shown as the near side of the adjacent view from which it is projected,**[1] as in the upper drawings.

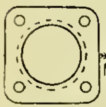


Fig. 330.

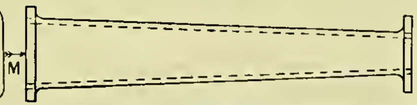


Fig. 331.—Arrangement of end views recommended.

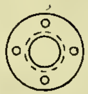


Fig. 332.

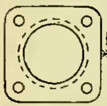


Fig. 333.

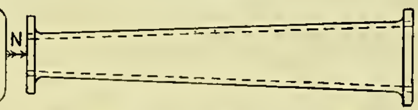


Fig. 334.—Inconvenient arrangement of end views often seen.

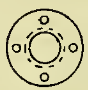


Fig. 335.

To get clear ideas about this, ask yourself how you would arrange the end views of a **long bolt** with a **square head** and a **hexagonal nut.** You would, I think, instinctively place the end elevation of the square head close to its elevation, and that of the nut near its elevation.

Before proceeding to draw projections of some bodies arranged with the object of being progressive, you may learn something more about the expedients employed in mechanical drawing, by setting out such a simple figure as Fig. 336, which is formed by a combination of lines

[1] Unfortunately, there is a want of uniformity of practice in this connection in this country, and the author must plead guilty to having set a bad example in some of his early work in this respect. The reasons for so doing need not be discussed here, but rather should efforts be made to secure uniformity of practice.

and arcs, and may be considered an advance on the figures dealt with in Arts. 26 to 28. We will state the case in the form of an exercise. Thus—

**197. To draw a Section of a Wrought-iron Beam or Joist.**—Fig. 336 is a finished drawing of the section of the beam, drawn in a conventional way, and fully dimensioned. After studying the previous exercises, each step the student should take in making this simple drawing should be obvious; indeed, all that he should require is a hint or two to enable him to go about it in a workmanlike way. Now, this section being symmetrical about a centre line, this line, *ab* (Fig. 337), should obviously be drawn first, and the rectangular outline of the section, drawn as shown in the figure, forms the first step. It will be noticed that the only lines in this figure that can be drawn in a finished state right off are AB and CD. The second step is to describe the arcs,[1] having previously found their centres as indicated at *c* and *d*, Fig. 338 (these centres can, with ordinary care and a little practice, be found by trial). All that now remains to be done is to carefully join the arcs and complete the outline with lines of uniform thick-

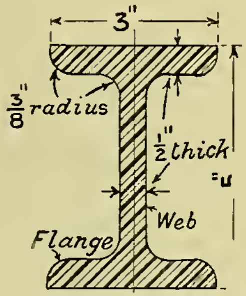


FIG. 336.—W. I. beam. Finished section.

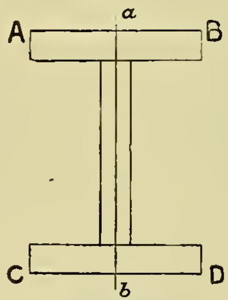


FIG. 337.—Section of beam. First step.

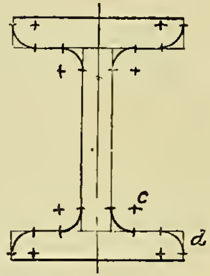


FIG. 338.—Section of beam. Second step.

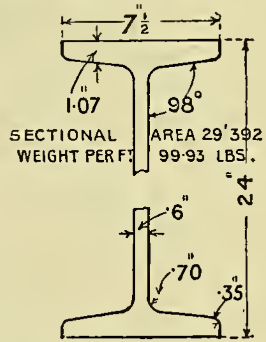


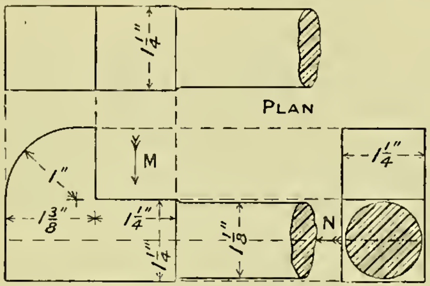
FIG. 339.—Section of maximum size standard beam.

ness throughout. The figure may now be **cross-hatched** or **section-lined.** The conventional lines in this case (as the material is wrought iron) are alternately thick and thin as shown in Fig. 524; but these beams are now usually made of steel.

**198. British Standard Beam Sections.**—The form given to the beam section in Fig. 336 is conventional, it being a convenient one for drawing purposes. Formerly there was a great want of uniformity in the relative thickness of flanges and web, and also of the radii of the fillets and edges, to say nothing of the amount of taper given in the flanges; but in 1904 the Engineering Standards Committee published their report on the Properties of British Standard Sections, in which all the sections commonly used by ship and bridge builders, etc., are standardized. Fig. 339 gives the standard dimensions for the largest beam section made, which is shown here as an example of a standardized section: and it may be used as an instructive drawing exercise.

[1] It will be noticed that the radius of the arcs is **three-fourths the thickness** of the metal. But it should be explained that these sections are now standardized and the actual radii fixed for all sizes, the flanges being made slightly taper in thickness (as shown in Fig. 339); but for some drawing purposes the above proportions may be used, and the flanges made of uniform thickness.

**199.—To draw Three Views of a Hook Bolt.**—This simple piece, the head part of a hook bolt, has been selected as a suitable exercise at this stage in projecting one view from another. In starting such a drawing, it is best, as a general rule, to draw the circles first, if there are any. So set out the end elevation (Fig. 342). You will now experience no difficulty in doing this. Next draw the elevation (Fig. 341), being careful to make the arc of 1″ radius flow into the straight lines, and complete the exercise by drawing the plan (Fig. 340), which in this case is shown above the elevation ("Amer. Proj."), but if you prefer it draw it below. You will notice that the end view (Fig. 342) is got by looking at the bolt in the direction of the arrow N, and the plan by looking iu the direction of the arrow M.




FIGS. 340, 341.—Hook bolt ("Amer. Proj."), plan over elevation.

FIG. 342.—End elevation.

**200. To draw a Stuffing Box Gland. Scale full size.**—Figs. 345 and 344 show, in plan and elevation, a gun-metal stuffing box gland (fully dimensioned) for a $2\frac{1}{2}''$ piston rod or valve spindle,[1] used to hold the packing in the box to keep the joint steam tight.

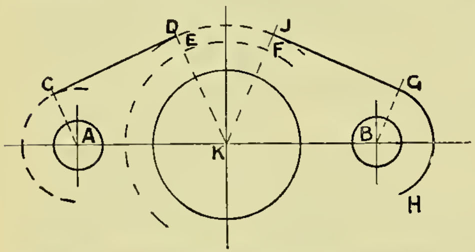



FIG. 343.—Auxiliary plan to show method of drawing.

In commencing a drawing of these views, you will first set out the centre lines *ab* and *cd*, as the object is symmetrical about these lines. Now, as matter of practice, as has been previously explained, whenever one of two views of a body or part of a body is circular in form, that view should be drawn first. So mark out centre lines for the holes A and B (as in Fig. 343), and describe the four circles in plan to the dimensions shown, giving the lines their finished thickness. Then, with 1″ radius, arcs may be drawn about the centres of the stud holes A and B with a light line, also arc DJ, of $2\frac{1}{16}''$ radius, about centre K; then tangents such as CD can be drawn, and the plan completed (as in Fig. 344) by going over the arcs EF and GH, etc., making them uniform in thickness with the other lines. The elevation presents no difficulty, and should be easily drawn now.

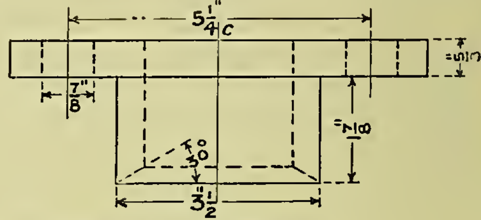



FIG. 344.—Elevation of stuffing box gland.

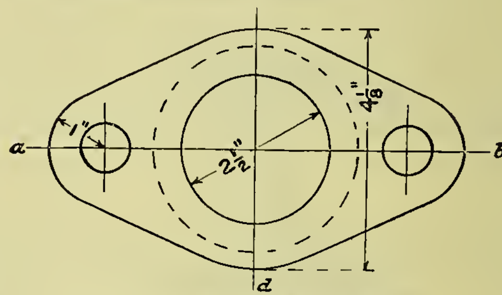



FIG. 345.—Plan of stuffing box gland.

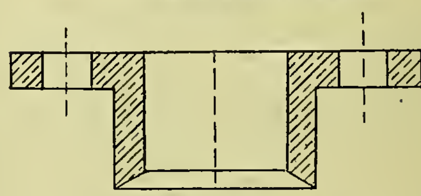


FIG. 346.—Section on line *ab*.

[1] For particulars relating to stuffing boxes, etc., see Arts. from 63, Author's "Machine Design, Construction, and Drawing for Beginners."

At this stage, a good exercise on the above would be *to* **draw a section of the gland** *made by a plane cutting it in halves through the line ab* (Fig. 345). Obviously, its outline would be similar to the elevation, Fig. 344, and Fig. 346 shows the finished section. Such a gland would be made of brass, and the conventional section-lining (as in Fig. 524) for this metal has been used.

**201. To draw the Plan and Elevation of a Steel Crank, also a Section on the line AB** (Figs. 347 to 349).—Have a good look at the projections of the crank, and you will agree that the elevation (which, as there are circles on it, you will draw first) must be drawn with great care, if the **tangential arcs** forming the outline are to meet properly. If you are in any doubt as to how the centres of the two arcs of 500 mm. radius are to be found, turn back to Art. 75, and Fig. 101 will help you. For, obviously, the centres of the arcs forming the sides, the radius of which is 500 mm., will be (500 – 135) mm. from C, and (500 – 75) mm. from $C_3$. The outline being drawn, the remaining part of the elevation should present no difficulty, and the plan is readily projected from the elevation.

Coming to the **section**: if the part ADB be cut away by the section plane, then, looking in the direction of the arrow M, we get the view shown in Fig. 349. **Section-lining** the parts that would be cut through by the plane, with dotted lines, as in Fig. 524, to represent the material, steel.

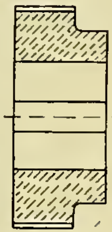
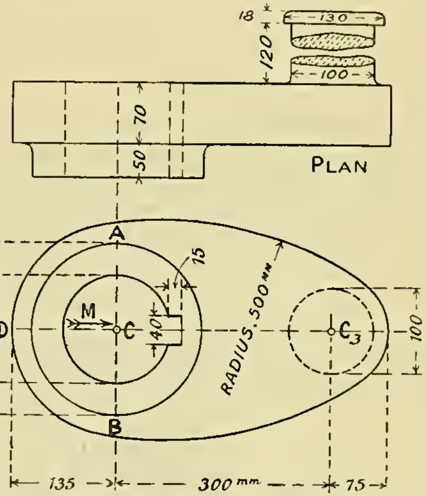


Fig. 349.—Section on line AB.




Figs. 347, 348.—Elevation and plan of steel crank ("Amer. Proj."), plan above elevation.

**202. Use of Pictorial Sketches and Drawings.**—Look at Fig. 292, which is a pictorial drawing or sketch. You see at a glance the shape of the block of wood represented, also its position in relation to the planes of projection, as the drawing has the advantage of conveying in one view ideas of the three dimensions. The two views of the same block shown in Fig. 293 do not so readily convey to your mind the same information. Also have a look at Figs. 299 to 304, the pictorial sketches shown are dimensioned. If you were sent somewhere to take particulars of one of these pieces, it would be convenient to make such a freehand sketch, and after carefully measuring the piece with your rule, write on it the dimensions; you could then, on your return, make a proper drawing in plan and elevation of the piece. You will also notice that the front face of each of the pieces shown in the figures is seen in **true shape**. But if you scrutinize Fig. 296 you will notice that no face is shown in true shape; as a matter of fact, it is an **isometric**[1] **drawing**, a view of the block tipped up into such a position that **all its edges are equally inclined** to the ground, and therefore **equally foreshortened** in plan. To enable you to make such drawings and sketches with facility, we will devote the next two chapters to their consideration.

[1] Gr. from *isos*, equal, and *metron*, a measure. This system of drawing was introduced by Professor Sir George Stokes of Cambridge.

# CHAPTER XX

## ISOMETRICAL PROJECTIONS AND DRAWINGS

**203. Introduction.**—You have read the previous article, and will understand that if the cube shown in Fig. 350 be cut by a plane passing through the corners *b, f, d,* the three face diagonals *bf, fd,* and *db,* being the same length, form an equilateral triangle, to the plane of which the three equal edges *cb, cd, cf* are equally inclined.

In Fig. 351 the **orthographic** views P and N, the plan and elevation (third-angle projections, Art. 195) of the cube are shown, and you will notice that the solid is so placed that the face diagonal *a'c'* is paralled to the V.P. Therefore *h'c'* (view N) is a **diagonal of the cube** seen in **true length.** Further, the cutting plane we have referred to in connection with Fig. 350 is shown here by the line *b'f'*, passing through the three corners *b', d', f'*, and perpendicular to the diagonal *c'h'*. The **new plan** R, due to looking at the cube in the direction of this diagonal (or due to the diagonal being vertical) is shown, and you will experience no trouble in drawing it, after having studied Fig. 316, Art. 192. You won't fail to notice that the boundary line is a regular hexagon, that $c_1$ is the centre of the triangle $b_1d_1f_1$, which is equilateral, therefore the three angles at $c_1$ are each equal to 120°; and that every other edge of the cube is equal and parallel to one of those meeting in the centre $c_1$, and the plan of each face a rhombus. The plan R is called an **Isometric Projection** because all the edges of the cube are foreshortened the same amount, and therefore all lines parallel to them can be measured with the same scale.

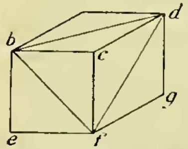


FIG. 350.—Pictorial drawing of cube.

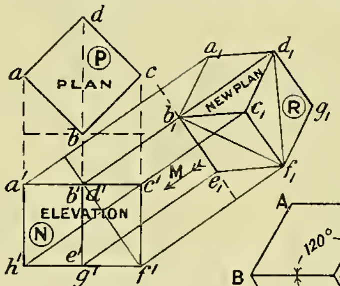


FIG. 351.—Isometric projection of cube.

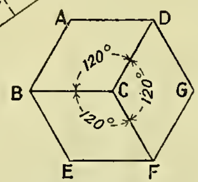


FIG. 352.—Isometric drawing of cube.

The three edges $c_1b_1$, $c_1d_1$, $c_1f_1$, are called the **Isometric Axes**, and the planes which they determine (that is, there are three such planes, each containing two of the axes), and all planes parallel to them, are called **Isometric Planes**, whilst all lines parallel to the axes are called **Isometric Lines**.

**204. The Difference between Isometric Projection and Isometric Drawing.**—Look at view N, Fig. 351. You will see that *c'f'* is the **true length** of the cube's edge, whilst in the new plan R the line $c_1f_1$ is its **apparent length.**[1] Now, obviously, if *c'f'* is made 1 inch in length, $c_1f_1$ will be what we may call an **isometric inch.** And if these lines are produced, inches can be laid off

[1] You ought to be able to satisfy yourself that the **ratio of the apparent length to the true length** is $\sqrt{2} : \sqrt{3}$.

from $fc'$ produced, and their projections on $f_1c_1$ produced will give an **isometric scale**, which can be used in constructing any **isometric projection.** Now, this question of the scale is purely an academical one, as it has no practical value, for since the isometric lines are all equally foreshortened, there is no reason why they should represented as foreshortened at all. Therefore in practice an **isometric drawing** (Fig. 352) of our cube would ordinarily be made by **drawing each edge its true length**, dimensions being set off from an ordinary rule or scale. Obviously, drawings made in this way have the great advantage that the ordinary rule or scales can be applied to them by a workman to measure any part.

Of course an **isometric drawing** of a body will be **larger than** its **isometric projection** in the proportion of Fig. 352 to the projection R (plan) in Fig. 351. And it can be shown that the corresponding lines are in the ratio of $\sqrt{3}$ to $\sqrt{2}$.

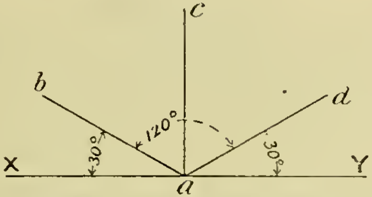


FIG. 353.—Isometric axes, *ab* and *ad*.

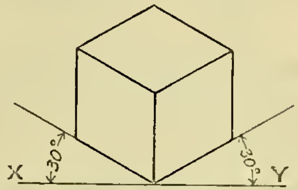


FIG. 354.—Isometric drawing of a cube.

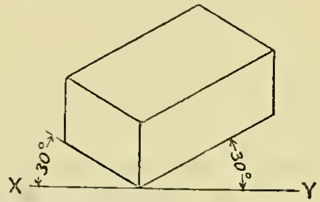
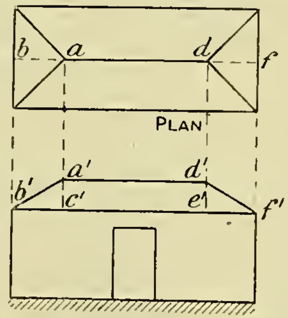


FIG. 355.—Isometric drawing of a brick.

**205. Some Typical Isometric Drawings.**—Of course, to make an isometric drawing of a rectangular body, such as the cube, direct, all we have to do is to draw lines *ab* and *ad*, from any point *a*, Fig. 353, in a line XY, inclined 30° in opposite directions and measure the length and breadth of the body along them, and the thickness along a line *ac* perpendicular to XY; the view of the solid is then easily completed by drawing parallels to these lines, as shown in Fig. 354. The block or brick shown in Fig. 290 is drawn in this way in Fig. 355, and such drawings should now speak for themselves.

In Figs. 356, 357 you have the orthographic projections ("Amer. Proj.") of an **iron shed** shown, and in Fig. 358 an isometric drawing of it. The only point about the latter that will trouble you when you attempt to make the drawing is the positions of the points A and D, where the slant hip-rafters meet. But you will, no doubt, see that the offsets BC and CA, in Fig. 358, must be made equal to $b'c'$ and $c'a'$ in Fig. 357 respectively.



FIGS. 356, 357.—Elevation.

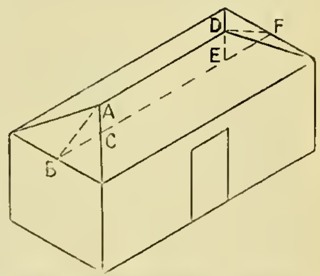


FIG. 358.—Isometric drawing.

**206. Isometric Drawing of a Circle.**—If a circle be circumscribed by a square, as in Fig. 359, and the diagonals drawn, these cut the circle in *a, c, c, g*, through which lines parallel to the sides enable you, in making the isometric drawing (Fig. 360), to find similar points in the ellipse, whose axes coincide with the diagonals, as shown in the figure. The curve can be carefully drawn through the eight points so determined; or additional points can be used (and transferred) as shown at the intersection of the dotted lines (Fig 359). Another method of finding points in the ellipse is also shown in Fig. 360. On the diameter *ae* of the circle describe the semicircle *a*BD*e*, and by setting off angles at 45° and 30° from the centre, the former gives

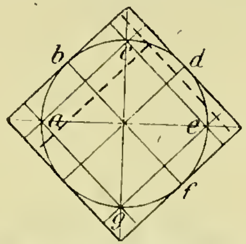


FIG. 359.—Circle circumscribed by square.

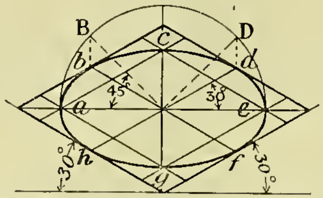


FIG. 360.—Isometric drawing of circle with the circumscribing isometric squares.

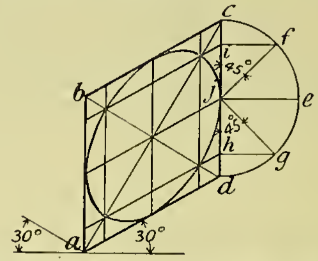


FIG. 361.—Isometric drawing of a circle. Another method.

B and D, from which perpendiculars to *ae* cut the latter in *b* and *d*; whilst the angles of 30° intersect in *e*; and the remaining points *g*, *h*, *f* in the ellipse may be found by symmetry. A variation of this method, shown in Fig. 361, should almost speak for itself. A semicircle *ced* is described on one of the sides of the rhombus, and divided into four equal parts by *f*, *e*, and *g*. The perpendiculars *fi*, *ej*, and *gh* to the side *cd* give the points *i*, *j*, and *h*, through which the parallels are drawn, the cross ones being found in the same way or by symmetry; the intersections giving four points in the ellipse, and the centres of the sides of the rhombus the other four.

**207. Setting off Angles to the Sides of the Isometric Cube.**—Let Fig. 363 be the isometric cube, then draw a square, whose side *ab* (Fig. 362) is equal to the edge of the cube. With one of its corners, say *a*, as centre, describe the quadrant *bd*, divide it into the required angles, and produce the radial lines through the points of division to cut the sides of the square, as in *e*, *f*, *e*, *g*, forming a **scale of tangents**. As an illustration of the use of this scale: with the compasses prick off *b′e′*, Fig. 363, making it equal to *be* (Fig. 362); join *a′* to *e′*, and this is the isometric projection of a line which makes an angle $\theta$ with the edge *ab* of the cube (Fig. 362). You will see that two other angles have also been set off on Fig. 363 by making *a′f″* and *d‴g‴* respectively equal to *bf* and *dg* on Fig. 362.

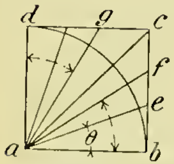


FIG. 362.—Scale of tangents.

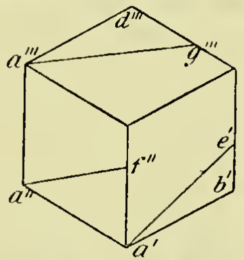


FIG. 363.—Setting off angles by using the scale of tangents.

Other expedients could be shown you, some of which you may discover for yourself, if you take any real interest in this system of projection; but, as we have seen, the oblique or pictorial system of projection, which enables us to show one face of a body in true shape, is so much more used for practical purposes that your time will be better spent in becoming more familiar with it, as you will be if you carefully follow the next chapter. However, before doing so, you might with advantage work the following exercises.

## EXERCISES.

### Typical Oral Exercises.

1. What is the meaning of the term "isometric" when applied to a projection?
2. What is the difference between *isometric projections* and *isometric drawings?*
3. How many isometric axes has the isometric projection of a cube?

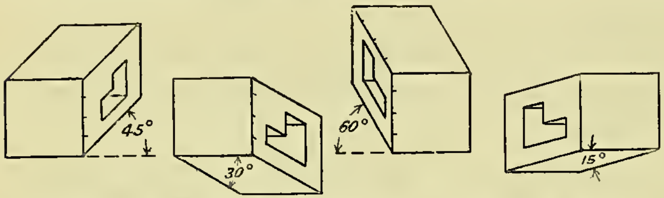
### Drawing Exercises.

4. Make isometric drawings of the solids shown in Figs. 300, 301, and 302.
5. Make an isometric drawing of a square pyramid, edge of base 1¾″, axis 2½″.
6. Make an isometric drawing of a 3″ circle.
7. A cylinder is 2″ in diameter and 3″ in length. Make an isometric drawing of it.
8. Make an isometric drawing of your instrument box, with the lid upright.
9. A square slab 2½″ across and 1″ thick has a 2″ hole bored through its centre. Make an isometric drawing of it.
10. Ask your teacher to lend you the chalk-box. Carefully measure it, and make an isometric drawing of it when the lid is in some inclined position, say, making an angle of 45° with the top of the box.
11. A 2″ cube is cut by a plane passing through one of its edges and making an angle of 30° with a face. Make isometric drawings of the two parts when separated.
12. Make an isometric drawing of the shed shown in Fig. 302. Scale ¼″ to the foot.
13. Make isometric drawings of the blocks shown in Figs. 303 and 304.

# CHAPTER XXI

## OBLIQUE OR PICTORIAL DRAWING

**208. Introduction.**—We have already made use of a system of drawing that lends itself to the rapid representation of a body showing its three dimensions on one view, and have referred to it in Art. 202, which you might again glance through. By this system, known as oblique projection, all lines lying in planes parallel to the paper are shown in their true forms, lengths, and relations. Further, all lines perpendicular to the paper are shown in true length or in some suitable fraction of that length. It is a system well adapted to some of the requirements of many trades, particularly those of the carpenter and joiner, where difficult joints and fitted pieces have to be shown.




Figs. 364 to 367.—Four pictorial views of a perforated prism.

The four different drawings of the same perforated prism, in Figs. 364 to 367, are arranged to show that the edges, which are **perpendicular to the paper**, may be **represented by parallel lines having any suitable direction**; this enables you to show, not only the front face, but either top or bottom, or the right or left side of a body, and either of these faces can be made more noticeable than the other by care in the arrangement of the angle. Of course, it is a convenience

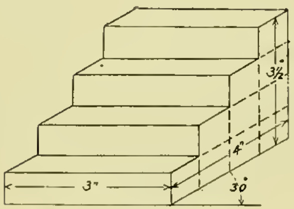
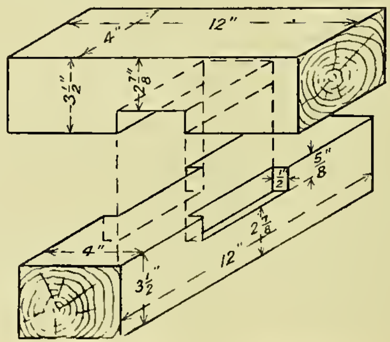
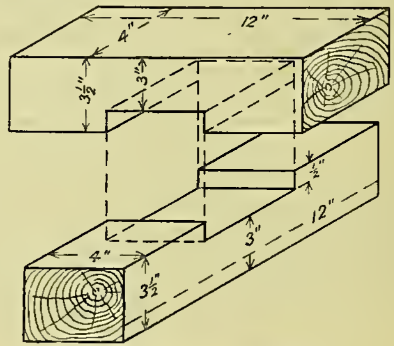



Fig. 368.—Block of steps.

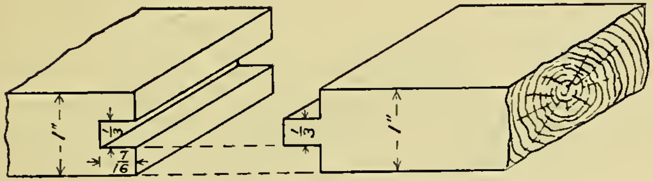



Figs. 369, 370.—Notched joint.

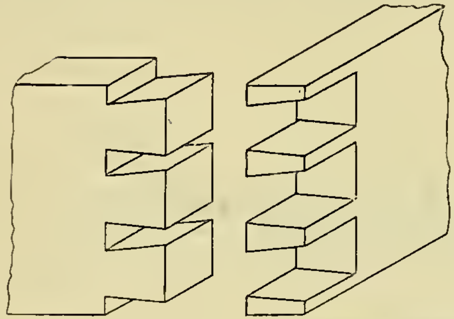


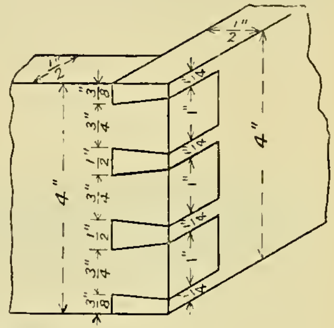

Figs. 371, 372.—Cogged joint.

to select one of the set-square angles. Thus it will be seen that 45° has been used in Fig. 364, 30° in Fig. 365, and 60° in Fig. 366, whilst in Fig. 367 it is 15°. Circles on planes perpendicular to the paper project as ellipses, and the rules apply as explained in Art. 206.




FIGS. 373, 374.—Grooved and tongued joint.

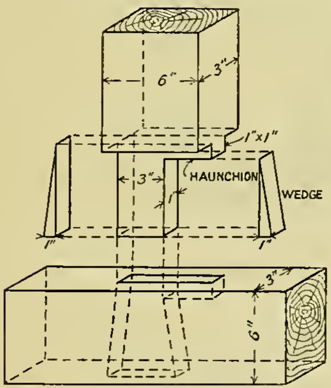





Common dovetail joint.

FIGS. 375, 376.—Showing the two parts separated.

FIG. 377.—Showing the complete joint.




FIGS. 378, 379.—Mortise and tenon joint.

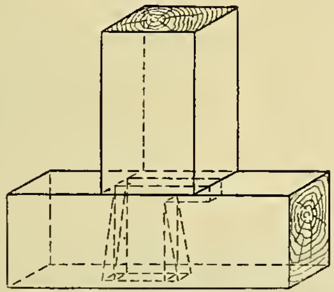


FIG. 380.—Mortise and tenon joint put together.

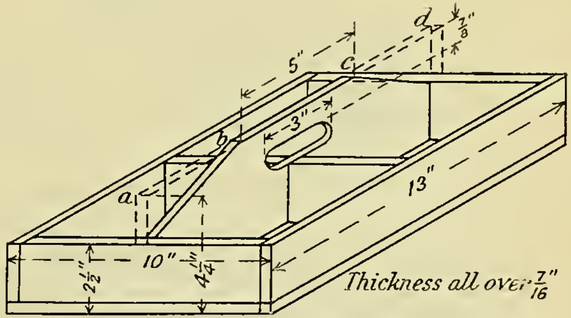



FIG. 381.—Housemaid's blacklead box.

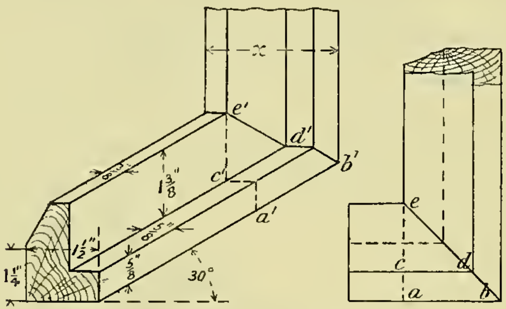
**209. Some Simple Bodies drawn in Oblique Projection.**—Most of the examples shown in Figs. 368 to 387 will practically speak for themselves. Some of the **joints** most commonly **used in woodwork** are shown. The picture-frame mitred joint (Figs. 382, 383) is an example which shows that there are certain limitations both in this and in isometric projections beyond which the systems should not be pushed.[1] It will be noticed that the apparent breadth $x$ of the side of the frame comes out in the drawing a good deal wider than the real breadth; indeed, this is an example which would have come out better in an isometric drawing. Of course, the method of using offsets employed in connection with Fig. 358 is applicable here for the mitre $c'd'$ in Fig. 382, being equal to $cd$ in Fig. 383, also $a'b'$ is made equal to $ab$. Fig. 384 is an oblique projection of the cast-iron bearing block shown in Figs. 471, 472.

## DRAWING EXERCISES.

1. Draw a pictorial view of the stationery case (Fig. 385). Scale ½ size.
2. Make drawing in oblique projection of the sign-post in Figs. 386, 387. Scale ¼ size.
3. Make a drawing in oblique projection of the hook-bolt in Figs. 340 to 342.

Draw in oblique or pictorial projection the following:—

4. The notched joint (Figs. 369, 370). First draw the parts separated, as shown, then draw the complete joint. Scale ¼ full size.




FIGS. 382, 383.—Picture-frame mitred joint.

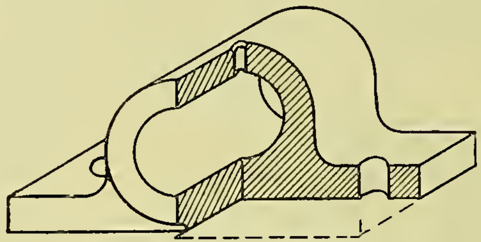


FIG. 384.—Cast-iron beariug block.
Showing parts in section.

5. The cogged joint (Figs. 371, 372). First draw the parts separated, as shown, then draw the complete joint. Scale ¼ full size.
6. The grooved and tongued joint (Figs. 373, 374). First draw the parts separated, as shown, then draw the complete joint. Full size.
7. The common dovetail joint (Figs. 375 to 377). First draw the parts separated, as shown, then draw the complete joint. Full size.
8. The mortise and tenon joint (Figs. 378 to 380). First draw the parts separated, as shown, then draw the complete joint. Scale ¼ full size.

---

[1] Many attempts have been made to draw a general plan of a complicated machine in isometric or oblique projection, but the contortions, which are hardly noticed in the case of small details, beceme so pronounced as to make the system quite useless for such purposes.

9. The housemaid's blacklead box (Fig. 381). Scale ½ full size.
10. The picture-frame mitred joint (Figs. 382, 383). Full size.
11. The bearing block, sectional view, as in Fig. 384. Scale double the size shown.
12. A 3″ square slab, ¾″ thick, supporting at its centre an 1½″ sphere.
13. A cone, base 2″, axis 3″, resting on a 3″ square slab, ¾″ thick.

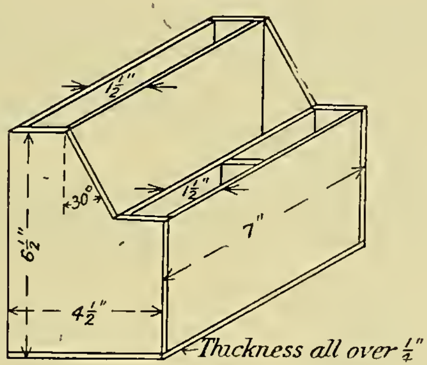
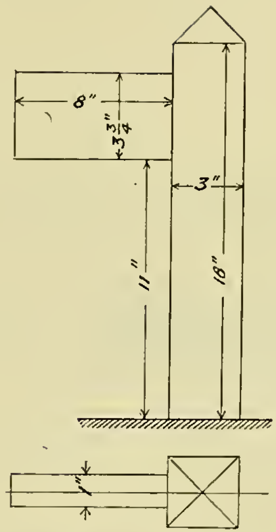



FIG. 385.—Stationery case.




FIGS. 386, 387.—Sign-post.

# CHAPTER XXII

## SIMPLE FASTENINGS USED IN METAL WORK, AND HOW TO DRAW THEM

### Riveted Joints

**210.** One of the most simple and efficient fastenings, which has been extensively used for a great variety of purposes from very ancient times, is the rivet. As a fastening, it somewhat **resembles a bolt**, but differs from it in two important respects; for a **bolt can be used as a temporary fastening**, and can be withdrawn by unscrewing the nut; but a **rivet is a permanent fastening**, and the parts held together by it can only be separated by chipping off a head. Further, a bolt is used satisfactorily when the straining force acts in the direction of its axis, giving it a tensional load, but it is not considered advisable to load a rivet in this way, its **proper function** being **to resist shearing** in a direction normal to its axis.

**Rivets** are made in special machines (from special round iron or steel bar), with heads either **cup-shaped**, as in Fig. 388, or **pan-shaped**, as in Fig. 389; the heads are formed while red hot by dies of these shapes, and their finished forms before and after use are shown in the figures; the dotted lines showing the length of the stub end required to form the second head.

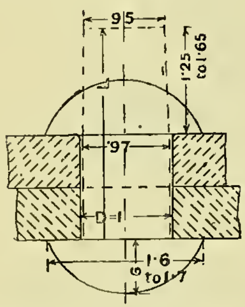



Fig. 388.—Head and second head cup-shaped.

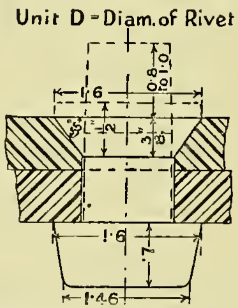



Fig. 389.—Pan-shaped head, second head fully countersunk.

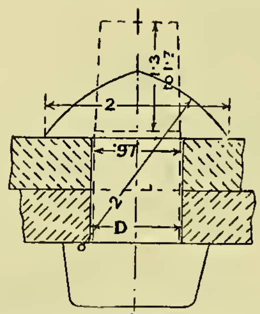



Fig. 390.—Conoidal second head, hammer-finished.

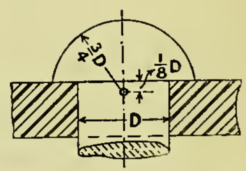
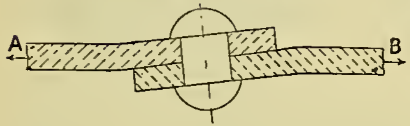


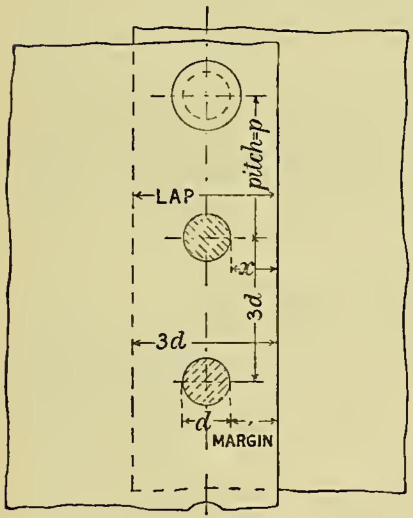

Fig. 391.—Proportions for drawing purposes.

In riveting plates, whenever practicable **riveting machines** are used, the rivet is made red hot, passed through the plates and pressed between two dies, by hydraulic or steam pressure. The heads are then usually made **cup or spherical shaped**, as in Fig. 388, and are said to be **machine** riveted. When machines are not available the rivets are **hand riveted**. For this job a full gang consists of three men and a boy, the latter to heat the rivet and bring it from the furnace to the holder up, who inserts it into the rivet hole and presses against the rivet with a tool called a **dolly**, cupped to receive the head of the rivet, while the other two men on the opposite side hammer the stub end down with riveting hammers and finish it off by a blow or two from a sledge hammer, a **snap-headed** tool being interposed to give the head the cup shape in Fig. 388. In confined positions where it is not possible to snap the heads, they are finished by hammering to the conical or **conoidal** form shown in Fig. 390, which has usually not quite the strength of the cup head. In many classes of work, such as the plating of ships, the seatings of girders, etc., the heads must not project; the plates are then **countersunk**, as shown in Fig. 389 (which shows a *full* counter-sunk head), and the heads finished off flush with the plate, or with a slight fulness or projection, as shown dotted.

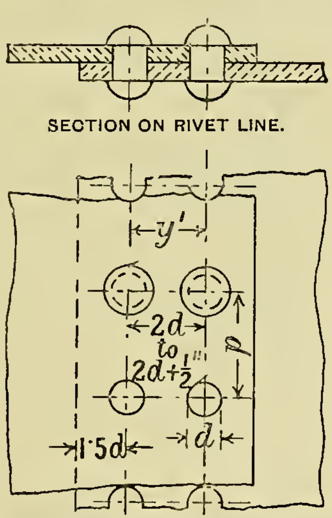
**For drawing purposes** an approximation to the ordinary cup head is easily made by using a radius of $\frac{3}{4}$ the diameter, as shown in Fig. 391, and striking the head from a point on the centre line $\frac{1}{8}$D from the shoulder.



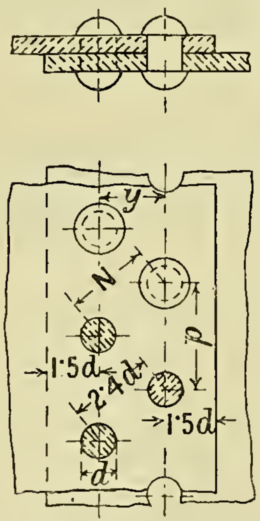

**211. Proportions of Rivet Heads, etc.**—These proportions vary somewhat in practice, as they have not yet been standardized; but those shown in Figs. 388 to 390 may be taken to be average ones, they are in terms of D, the diameter of the hole. The dotted **lengths for forming the heads** should be taken to be approximate. They vary from 1·25 to 1·7 times the diameter,



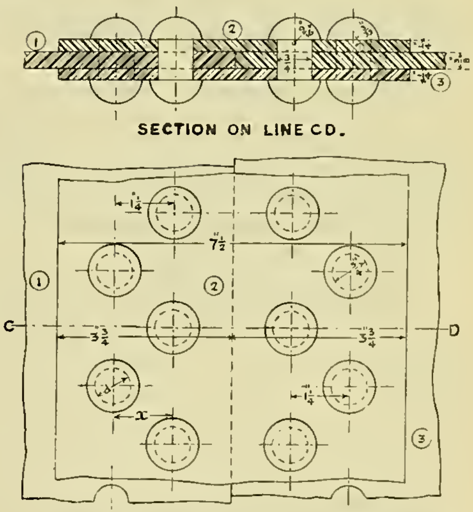

FIGS. 392, 393.—Single-riveted lap joint.




FIGS. 394, 395.—Double-riveted lap joint (chain).




FIGS. 396, 397.—Double-riveted lap joint (zigzag).




FIGS. 398, 399.—Butt joint with double butt straps or cover plates (zigzag).

the actual length required depending upon the completeness with which the rivet fills the hole, and upon whether the head is formed by hand or by machine, the former requires about $\frac{1}{8}$D less length than the latter, as the machine compresses and swells the rivet till it completely fills the hole, thus making a very perfect job.

**212. Forms of Joints.**—The simplest form of riveted joint is the **lap joint**, with a single row of rivets, shown in Figs. 392, 393; this joint, although largely used for many purposes, has, when subjected to great straining actions (as it is in boiler work), an obvious fault, for a couple acts about the rivets, tending to bend the joint (as shown in Fig. 392), owing to the plates A and B not being in the same plane. The **butt joints**, to be directly described, make a much more satisfactory but more expensive job.

**213. Double-riveted Lap Joints** are shown in Figs. 394 to 397. In Figs. 396, 397 we have **rivets** arranged **zigzag**, and in Figs. 394, 395 they are opposite to one another, or the joint is said to be **chain** riveted.

**214. Proportions of Joints.**—The usual practice is to make the distance $x$ (Fig. 392) between the side of a rivet and edge of the plate (called the **margin**) at least equal to the rivet diameter, thus making the **minimum lap** equal to $3d$, as shown, but in cases where the edges of the plates are more or less rough, a $\frac{1}{4}''$ is added to this.

**215. Diameters of Rivets.**—These are shown on the drawings, but the diameters of the rivets for other thicknesses of plate may be found by using the empirical formula dia. $d = 1{\cdot}2\sqrt{t}$, where $t$ is the thickness of the plates.

**216. Pitch of the Rivets.**—The **minimum** pitch in any given case may be determined by the rough formula, pitch $= d + 1\frac{1}{8}''$. In the example, Figs. 392, 393, the pitch, it will be noticed, is $3d$. The smaller pitches are used when the joint is to be kept steam-tight. When the strength of the joint is to be the greatest possible, the pitch is determined as explained in the author's "Machine Design."

**217. Butt Joints with Double Straps.**—Figs. 398, 399 show a **double-riveted** butt joint with *double butt straps*. In this joint, the plates to be joined are in the same plane, and they butt one on the other, top and bottom cover plates or straps being placed over and under as shown. It has been found by experiments that when the straps are made half the thickness of the plates (as it would appear they should be) the **straps** are then the weakest part. This has led to the practice of making their **thickness from** $\frac{5}{8}t$ **to** $t$ (thickness of plate). The dimensions on the figures are suitable for $\frac{5}{8}''$ plates; other proportions are shown on the figure.

The **distance between rivet lines** for chain riveting is given on Fig. 395. The distance $y$ (Fig. 397) for zigzag riveting may be found by the rough rule $y = 1{\cdot}7d$.

For further information about riveted joints, refer to the author's "Machine Drawing and Design for Beginners."

**218. Hints on Making the Drawings.**—The sections in all the joints shown should be drawn first, commencing with the centre lines through the rivets, the thickness of the plates can then be marked off on this line, and the plate lines drawn. The rivet should be next drawn. Referring to Fig. 391 for its details, you will see that the **radius of the head**, for ordinary drawing purposes, may be $\frac{3}{4}$ of D; then, as D $= \frac{3}{4}''$, the radius is $\frac{3}{4} \times \frac{3}{4}'' = \frac{9}{16}''$, and the centre is $\frac{1}{8} \times \frac{3}{4}'' = \frac{3}{32}''$ below the shoulder, so, with radius of $\frac{9}{16}''$, and this position of the centre, form the heads, and complete by section-lining, as shown, the upper view; the plan can then be readily projected from the section. Scale for all the views, full size.

**219. Screws, Bolts, etc.**—It will now be convenient to give some attention to the **pair of elements** forming the fastening, which in the science of kinematics[1] is called a **screw-pair**, the simplest form of which is the **common bolt** and nut shown in Fig. 407. A fundamental feature of bolts and screws is that parts connected by them can be easily disconnected when required, and, when it is realized what a great variety of work these interesting fastenings are used for, some idea can be formed of the multiplicity of forms

[1] Science of pure motion.

and kinds that are in actual use; but for our purpose we shall only give attention to two or three of the most important ones. Now, to completely specify some special form of bolt or screw it may be necessary to mention eight features, namely, (*a*) shape or form of the thread, (*b*) pitch or number of threads to the inch, (*c*) shape of head, (*d*) outline of body, barrel or stem, (*e*) size or diameter, (*f*) direction of threads (as *right-hand* or *left-hand*), (*g*) length, (*h*) material, as *iron*, *brass*, etc.

**220. Forms of Screw Threads.**—Figs. 400 to 403 show the threads most commonly used by the engineer. Figs. 400, 401, a vee

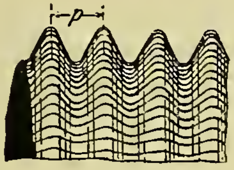



Fig. 400.—Whitworth's.

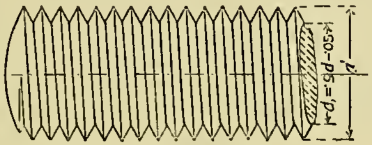



Fig. 401.—Whitworth screw.

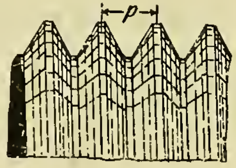



Fig. 402.—Seller's screw.

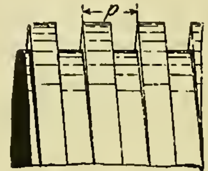



Fig. 403.—Square thread.

thread, slightly rounded at the top and bottom, is **Whitworth's, the standard British thread.** Fig. 402 is also a **vee**, with the top and bottom slightly flat, it is **Seller's** and **the standard thread of America**[1] Fig. 403 is the **square thread**, which you have no doubt seen on the letter-copying press; unlike vee threads, this screw does not subject the nut to a bursting strain.

**221. Proportions of Screw Threads.**—Fig. 404 shows the shape of our **Whitworth vee thread.** The angle between the threads being 55°, and $\frac{1}{6}$th of the full depth of the triangle *abc* being **rounded** off at the **top and bottom** (to a radius of $0{\cdot}137329p$), as shown. But the **full depth** of the triangle is 0·96 the pitch, so that the actual **depth of the thread** is $\frac{4}{6} \times 0{\cdot}96p = 0{\cdot}64p$, or, to be exact, $0{\cdot}640327p$. And if $d$ = diameter of the screw at top of the threads, Fig. 401, and $d_1$ = diameter at bottom of the threads (the **net or core** diameter),

WHITWORTH'S THREAD

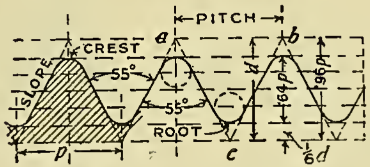



Fig. 404.—Detail of the British Standard Whitworth thread.

SELLER'S THREAD

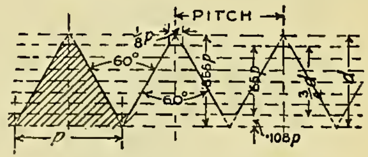



Fig. 405.—Detail of the American Standard Seller's thread.

Then **the core diameter** $$d_1 = 0{\cdot}9d - 0{\cdot}05, \text{ nearly} \qquad (1)$$

And if $n$ = number of threads per inch, and $p$ = the pitch of the threads,

Then **the pitch** $$p = \frac{1}{n} = 0{\cdot}08d + 0{\cdot}04, \text{ nearly} \qquad (2)$$

In Fig. 405 is shown **Seller's thread**, which we have explained is the standard thread adopted by America. The **triangle** in this case is **equilateral**, the angle therefore being 60°, $\frac{1}{8}$ the **full depth** of the triangle being **cut off top and bottom**, as shown, to

[1] It is claimed that the dies and taps used to produce these threads can be used longer before becoming blunt than ours; but, strangely enough, our Whitworth threads are used in the American Navy work.

form flats parallel with axis. So that the **actual depth** of the thread $d' = \frac{3}{4}d$, or $d' = \frac{3}{4} \times 0{\cdot}866p = 0{\cdot}65p$. The **proportions of the square thread** (Fig. 403) are shown in Fig. 406; the **pitch** for standard screws being **twice that for vee threads** the same diameter,

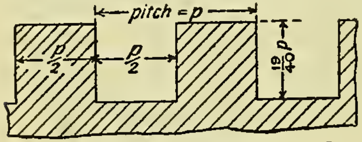
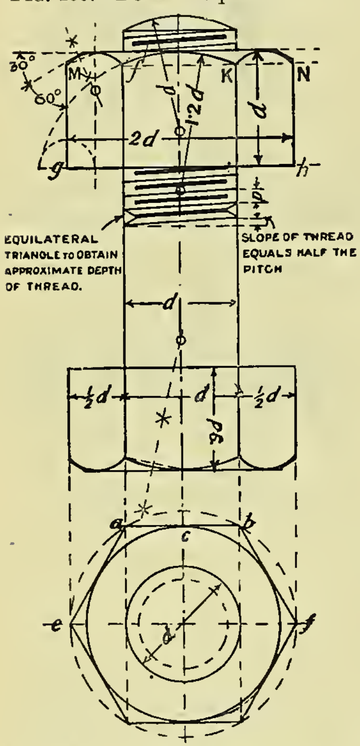



FIG. 406.—Detail of square threads.

or, the **pitch for square threads** $p = \frac{1}{n} = 0{\cdot}16d + 0{\cdot}08$, nearly . . . . . (3)

And, if $d_1$ = **diameter at bottom of threads** (the core diameter), as in other cases,

Then the **core diameter for square threads** $d_1 = 0{\cdot}85d - 0{\cdot}075$ . . . . . . . . (4)




FIGS. 407, 408.—Proportions of hexagonal bolts for drawing purposes.

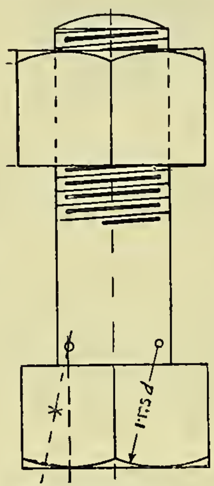


FIG. 409.—Side elevation of Fig. 407.

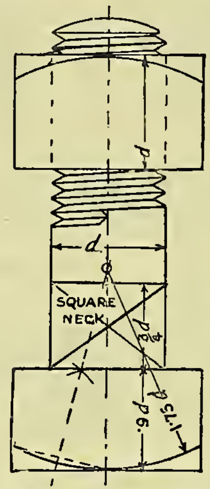



FIG. 410.—Bolt with square head and nut.

With this thread the thrust is very nearly parallel to the axis of the screw, and therefore there is **no bursting strain** on the nut, as we have seen, which is an important advantage. But the thread is more costly to produce than the vee thread, more particularly as it cannot be satisfactorily cut with dies. The figure (406) shows the usual proportions of the thickness and depth of the threads.

**222. Drawing Exercise. To draw an 1″ Whitworth Bolt and Nut.**—From a drawing point of view by far the most important detail you will have to deal with is the bolt and nut, as any want of accuracy in presenting it mars the appearance of what otherwise might be a very good drawing, and offends the trained eye. Further, as the detail so often occurs on drawings, a real effort should be made to set it out in the usual *conventional way* shown in Figs. 407, 408, and 409.

Commence with the *Plan*,[1] Fig. 408, by drawing the circumscribing circle (with a radius[2] equal to $d$, the diameter of the bolt = 1″), and the bolt circle (radius ½″), and from the latter draw projectors, cutting the former in $a$ and $b$, join $ab$, and describe the chamfer circle, touching $ab$ in $c$. The hexagon is then completed with the 60° set-square, making each of the other sides just touch the chamfer circle. Projectors from the corners $e$, $f$ can now be drawn, and these, with projectors from $a$ and $b$, give the indefinite elevation of the bolt body, and edges of nut and head. The **thickness of the nut** (= $d$) can now be set off, and with radius $1{\cdot}2d$, and centre on centre line, the arc $f$K can be drawn, and a line through these points gives M and N, which are used, as shown, to draw the arcs on the side

[1] As you have seen, if you were drawing this in accordance with **American practice**, you would place the **plan above the elevation.**

[2] As we have explained, for drawing purposes (for 1″ bolts and under) it is convenient to make the diameter across the angles = $2d$.

faces,[1] the elevation of the nut is then completed (if you wish to make a very exact drawing) by drawing the chamfers at 30°, to just touch the arcs; but you **need not trouble about this chamfer** for your purpose. The **head** is drawn in the same way, making its **thickness equal to 0·9**$d$, whilst the point or end of the bolt is usually rounded with a radius = $d$. The **screw threads** are easily drawn in the conventional way shown, the **slope** being fixed by marking up ½ the pitch; the **thick lines**, of course, represent the bottom of the threads, and their length may be found by making the small equilateral triangle shown, of side equal to the pitch, which gives the **approximate depth of the thread.** The use of the dotted lines on the nut will be apparent.

Fig. 410 shows a **bolt with square head and nut**, and **square neck** to prevent the bolt rotating whilst screwing up, the bolt hole being square; the proportions given in the table below, with the exception of the diameter across the angles, also apply to these bolts.

**223. Standard Bolts and Screws.**—We may now give some attention to the **proportions** of **bolts** and **bolt heads** in general use. Figs. 407, 408, and 409 show, as we have seen, the form of the common hexagonal bolt and nut; their proportions are now **standardized**,[2] they are practically those given in the table below, which are in common use. The practice of some manufacturers in the past has been to make **bright** nuts and heads somewhat smaller in diameter than **black** ones, but this is very inconvenient, as, if for no other reason, it necessitates the use of two spanners for the same size bolt. However, as now standardized, both the bright and black have the same *maximum* dimensions, the *minimum* dimensions fixed for the latter giving a larger *allowance*, as it is called.

PROPORTIONS OF STANDARD WHITWORTH BOLTS AND SCREWS.

| Diam. of Bolt or Screw. Inches. | No. of Threads per inch = $n$. | Diam. across Flats = D. | Diam. across angles = 1·155 D. | Diam. of Bolt or Screw. Inches. | No. of Threads per inch = $n$. | Diam. across Flats = D. | Diam. across angles = 1·155 D. |
|---|---|---|---|---|---|---|---|
| ¼ | 20 | 0·525 | 0·6062 | 1 | 8 | 1·6701 | 1·9284 |
| ⅜ | 16 | 0·7014 | 0·8191 | 1¼ | 7 | 2·0483 | 2·3651 |
| ½ | 12 | 0·9200 | 1·0612 | 1½ | 6 | 2·4134 | 2·7867 |
| ⅝ | 11 | 1·101 | 1·2713 | 1¾ | 5 | 2·7578 | 3·1844 |
| ¾ | 10 | 1·3012 | 1·5024 | 2 | 4·5 | 3·1491 | 3·6362 |
| ⅞ | 9 | 1·4788 | 1·7075 | | | | |

**224. Locking Nuts and Arrangements.**—No matter how perfect the fit of a nut on its bolt may be, when it is subjected to vibration, or to the jarring tremulous motion of machinery, the nut gradually works loose or tends to do so, and may, if there is nothing to stop it, work off the bolt. Now, one of the best known expedients to prevent this, and the one usually employed when pieces subject to rapid motion are connected by bolts, is the **Lock Nut**, which is an extra nut screwed tightly down on to the ordinary one, as in Fig. 411, to **jamb or lock it on the bolt** in such a way that it will not work loose. This lock nut is sometimes made half the ordinary thickness of a nut, on the assumption that it is only to jamb the other nut and take only a small part (if any)

[1] A little practice will enable you to draw these with considerable accuracy and facility by feeling for the centre and radius, assuming tentative radii and positions of the centre till the true centre is found.

[2] Refer to Reports on British Standard Screw Threads, published by Crosby Lockwood & Son, for further information, if required.

of the load, but a little consideration will satisfy you that it is **the top nut which practically takes the whole load,** whatever its thickness may be, and, therefore, of course the **thick nut** should be there (as in Fig. 411), as the **true lock nut,**[1] but **spanners** (or wrenches) are rarely thin enough to take the lock nut when it is thin and is placed at the bottom, and this has led to the growth of the faulty practice shown in Fig. 412. An obvious way out of the difficulty would be to make both nuts the full thickness, but there is not always room for this, and when there is it offends the eye, so the compromise of keeping the total thickness the same and making them both the same thickness, namely $\frac{2}{3}$ to $\frac{3}{4}d$, as in Fig. 413, is one that is often met with. However, the **standard arrangement,** shown in Fig. 411, should always be used when convenient. Fig. 413 also shows how the end of the bolt is sometimes turned down to allow the nut to be easily screwed on, and to more conveniently allow of a **split pin** to be used, where the bolt is subject to much vibration, **to prevent the nuts working off,** should they get loose.

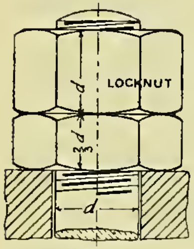



FIG. 411.—Standard practice.

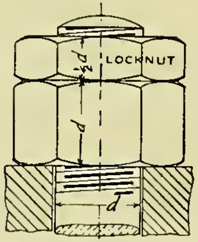



FIG. 412.—Practically convenient, theoretically faulty.

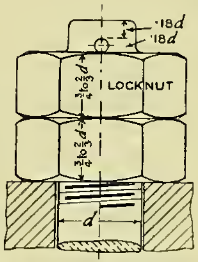
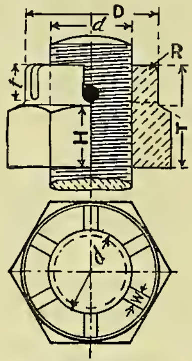



FIG. 413.—Compromise, sometimes convenient.




FIGS. 414, 415.—Capstan nut.

**225.** The **Capstan nut** or **Castle nut** (Figs. 414, 415) is largely used for locking purposes in **motor-car work,** and on jobs generally that are subjected to sudden shocks and much vibration. It consists of an hexagonal nut, with a portion turned off making a circular collar, through which rectangular slots are made, and into which, after the nut has been adjusted, a round or rectangular **cotter with split ends** is fitted through both nut and bolt. The **standard proportions** are, D = width across flats $-\frac{1}{16}''$, T $= 1{\cdot}25d$, H $= 0{\cdot}75d$, $t = 0{\cdot}4375d$, W $= 0{\cdot}25d$, and the radius R may $= \frac{d}{8}$.

## EXERCISES.

### TYPICAL ORAL EXERCISES.

1. What advantage has a bolt over a rivet?
2. In structures such as girders and boilers the parts are riveted together, as the handle of a frying-pan is riveted to the pan. On the other hand, a bolt is used to attach the bell to your bicycle handle. Why should not bolts be used for the former cases, and rivets for the latter?
3. Why does a butt joint in riveted plates subjected to great tension, make a better mechanical job than a lap joint?
4. What advantage has a square-threaded screw over a vee-threaded one?
5. How are nuts prevented from working loose or coming off when bolts are subjected to considerable vibration?
6. What is the difference between the screw threads used in this country and those used in America?

[1] It is the practice of some engineers to arrange the nuts in this way, and to make the thickness of the bottom one equal to $d$ and the top one equal to $\frac{7}{8}d$.

## SKETCHING EXERCISES.

7. Make good bold freehand sketches of the following:—Rivet heads. (*a*) A cup or snap head. (*b*) A countersunk head. (*c*) A hammer-finished head.

8. Make a freehand sketch in good proportion of (*a*) a single-riveted lap joint; (*b*) a double-riveted zigzag butt joint.

9. Make effective freehand sketches of (*a*) a Whitworth screw thread; (*b*) a Seller's screw thread; (*c*) a square screw thread.

10. Show by a freehand sketch in good proportion a standard lock nut arrangement.

11. Make a careful freehand sketch of a castle lock nut. You had better use an actual nut to work from if you can borrow one from your teacher.

## DRAWING EXERCISES.

12. Make a neat drawing of the lap joiut shown in Figs. 392, 393. Full size. Making the thickness of the plates $\frac{3}{8}''$, and the diameter of the rivets $\frac{3}{4}''$.

13. Draw plan and section of the butt joint shown in Figs. 398, 399. Full size.

14. Set out a single-riveted lap joint for $\frac{1}{2}''$ plates, making the rivets a suitable diameter, and pitching them as close together as you can, consistent with good practice. Full size.

15. Draw three views of a $\frac{7}{8}''$ hexagonal-headed bolt. You may make it long enough to hold together two plates whose total thickness is 3".

16. Set out a pair of lock nuts for a $1\frac{1}{4}''$ bolt.

17. Make full-size drawings of a 1" castle nut.

18. Practise drawing different-sized bolts, using the table for dimensions, when required.

19. Set out a double-riveted lap joint (chain riveting), Figs. 394, 395, making the thickness of the plates $\frac{1}{2}''$, diameter of rivets $\frac{13}{16}''$, and the distance between rivet lines $1\frac{3}{4}''$.

# CHAPTER XXIII

## MISCELLANEOUS DRAWING EXERCISES IN WOODWORK, BRICKWORK, AND MASONRY

**226. Introduction.**—In Figs. 369 to 387 you have some examples of joints, etc., in woodwork shown, which you, no doubt, have studied and drawn. The following pieces of woodwork may now be set out. You will notice that to avoid apparent complication no attempt has been made to show in detail the joints. After you have drawn these, or, for the matter of that, whilst you have them in hand, no doubt you will be carefully examining any such pieces of woodwork you may come across, and trying to understand how they are pieced together.

**227. Wooden Stand for a Machine.**—The framed stand (Fig. 416) is typical of the kind of support largely used by machinists for light machines. The figure shows a pictorial sketch of a stand that has been measured for the purpose of setting it out. By this time you will experience no difficulty in making such a sketch from the actual thing and running your rule over it for the dimensions. The elevation and end elevation of the stand (Figs. 417, 418) present no difficulty, so draw them to a scale of 3″ = 1 ft.

**228. Kitchen Table.**—The sketch (Fig. 419) shows the table upside down, for convenience of

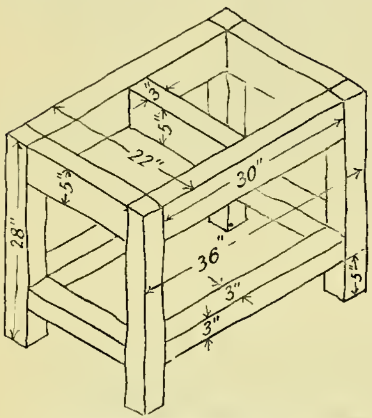
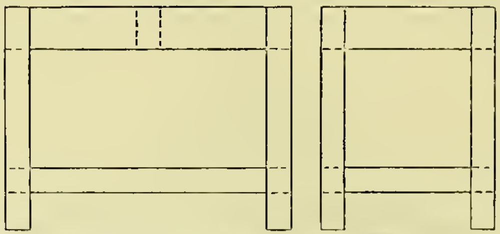



FIG. 416.—Isometric sketch of wooden stand for a machine.



FIGS. 417, 418.—Front and end elevations of stand.

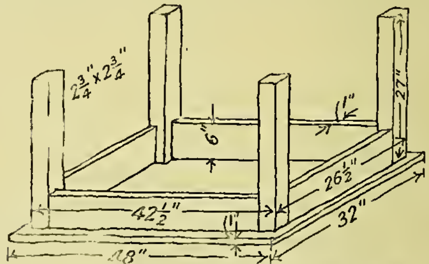
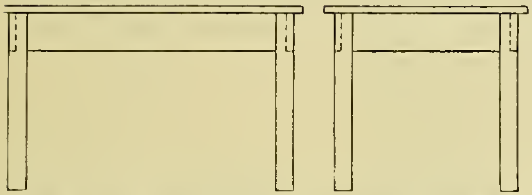



FIG. 419.—Pictorial sketch of kitchen table, inverted.

better showing the way the legs are arranged. Draw the two views (Figs. 420, 421) and add a plan, dotting in the legs, etc., to a scale of $1\frac{1}{2}''$ = 1 ft.

**229. Dining-room Sideboard.**—This simple piece of furniture has been measured, as shown in the pictorial sketch, Fig. 422, and two views (Figs. 423, 424) have been drawn in orthographic projection. Set these out, scale $1\frac{1}{2}'' = 1$ ft.



FIGS. 420, 421.—Front and end elevations of kitchen table.

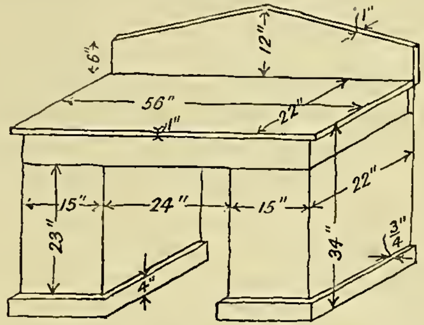
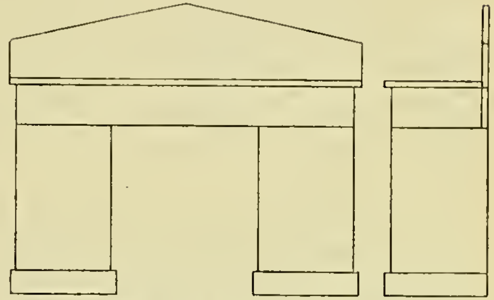



FIG. 422.—Pictorial sketch of sideboard.



FIGS. 423, 424.—Front and end elevations of sideboard.

## Brickwork and Masonry.

**230.** You cannot walk very far, whether it be in the streets of London or in some large village, without coming across a bricklayer at work. You may have noticed that in building a wall he is very careful to place the bricks in accordance with some


FIG. 425. *Plan of Header Course*

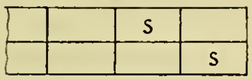
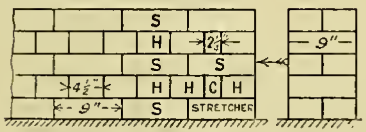



FIG 426. *Plan of Stretcher Course*




FIGS. 427, 428.—Front and end elevations of 9" wall. English bond.

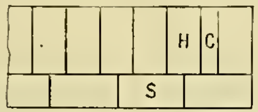



FIG. 429 *Plan of Stretcher Course*

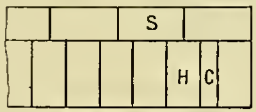
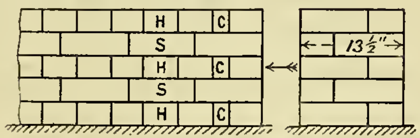



FIG. 430. *Plan of Header Course*




FIGS. 431, 432.—Front and end elevations of 14" wall. English bond.

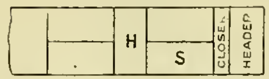
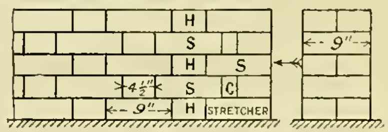



FIG. 433. *Plan of Second Course*


FIG. 434. *Plan of Bottom Course*




FIGS. 435, 436.—Front and end elevations of 9" wall. Flemish bond.

set rules. It would take him a very long time to tell you about these. And if he did, you would probably not understand very much, or, at least, remember it if you did. However, there is no reason why you should not try to understand a few simple matters relating to this work, and this can be best done by making a drawing or two, which should tend to cultivate your powers of observation, and lead you to take a greater interest in such work. We will commence with—

**231. A 9″ Brick Wall.**—Look at Fig. 427, and then at Fig. 435. These elevations differ in the way in which the bricks are arranged in alternate **courses** or layers. The former is known as **English bond**, and the latter as **Flemish bond.** The **term bond** being given to any arrangement of bricks in which no vertical joint of one course is exactly over or above one in an adjacent course. In the English bond (Fig. 427) you will notice that the bottom course consists of bricks laid in the direction of their length, and therefore called **stretchers**, whilst in the courses H they are laid across the wall so that their ends are only seen in elevation. These ends are said to be **headers.** Now look at the plans of these courses, Figs. 426 and 425 (also the end elevation, Fig. 428). You will see that when one course is laid over the other, some of the joints cross one another, whilst others that are parallel are separated by a distance of half the breadth of a brick. Of course, to make this arrangement possible half bricks, C, called **closers**, have to be used near the ends of alternate courses, as shown. Comparing the English and Flemish bonds you will see that the former has more headers

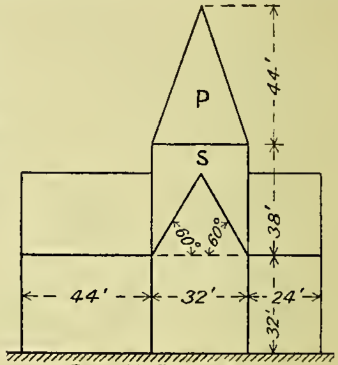



FIG. 443. ELEVATION.

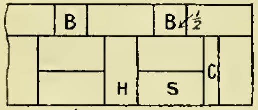



FIG. 437. *Plan of Second Course*

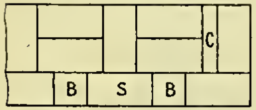
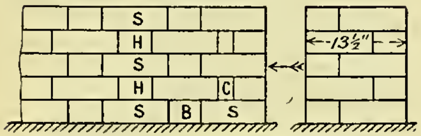
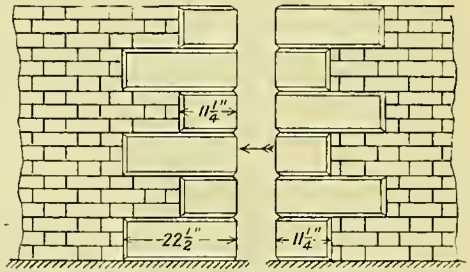



FIG. 438 *Plan of Bottom Course*




FIGS. 439, 440.—Front and end elevations of 14″ wall. Flemish bond.




FIGS. 441, 442.—Front and end elevations of angle quoins with bevelled edges.

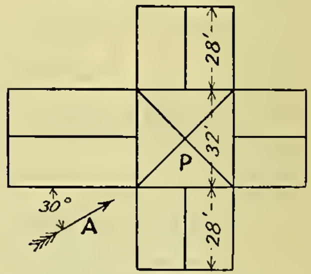



FIG. 444.—Plan of small church.

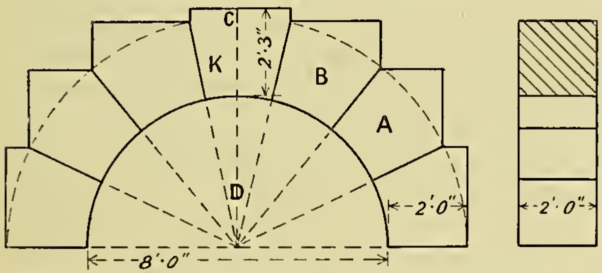
for a given height and length of the wall than the latter, and is therefore better bound or held together, and forms a stronger structure; for this reason civil engineers prefer this bond for their works; whilst, on the other hand, where appearance is an important factor, as in architectural work, the Flemish bond is generally used.

**Make Drawings** of the two 9″ walls as shown, scale $1\frac{1}{2}''$ to the foot. Note the average size of our *stock* bricks is $8\frac{3}{4}'' \times 4\frac{1}{4}'' \times 2\frac{3}{4}''$ or, including one thickness of **mortar**, $9'' \times 4\frac{1}{2}'' \times 3''$. Thus a so-called 9″ wall is really an $8\frac{3}{4}''$, and a so-called 14″ wall $13\frac{1}{4}''$.

**232. A 14″ Brick Wall.**—After giving attention to the previous article, a careful inspection of Figs. 429 to 432, and 437 to 440 will enable you to understand how the bricks must be arranged in the alternate courses to be properly bonded for the two different bonds in common use. You will notice that to secure the proper break of joint in the Flemish bond $\frac{1}{2}$ bricks, B, called **bats** (Figs. 437, 438), have to be used.

Draw the views shown of both walls to a scale of $1\frac{1}{2}''$ to the foot.

FIG 445. PLAN




FIGS. 446, 447.—Front elevation and section on CD of stone semicircular arch.

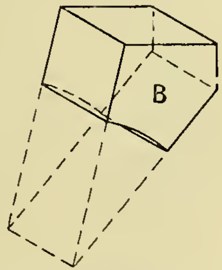



FIG. 448.—Pictorial view of voussoir A.

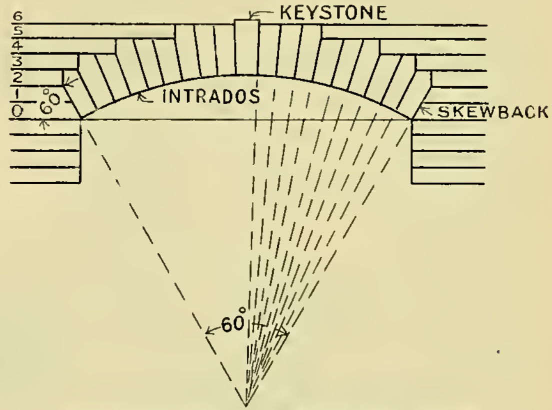



FIG. 449.—Segmental arch of a stone bridge.

**233. A Small Church** in outline is shown in Figs. 443, 444. You will see that the steeple is represented by a square pyramid, P, resting on a square prism, S, whilst the top pieces of the other parts are equivalent triangular prisms, and these are supported by rectangular prisms.

Make drawings of this structure as shown. Scale $\frac{1}{4}''$ to 4 ft.

Also an elevation looking in the direction of the arrow A. This view you should be now able to draw, with the exercise of a little ingenuity, and the assistance of Problem 192.

**234. Angle Quoins with Bevelled Edges.**—Look at Figs. 441, 442, you will see represented a combination of stone and brickwork you have often come across. The stone quoins are always made equal in thickness to a multiple of the thickness of the bricks,

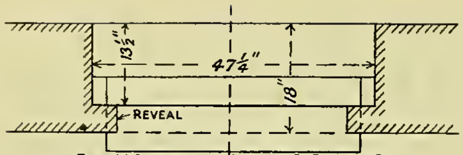



FIG. 450. *Section through line* C D.

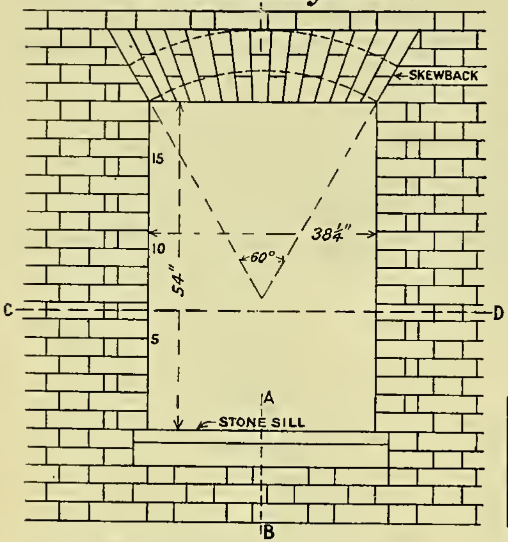



FIG. 451.—Elevation of square-headed window with flat-gauged arch.

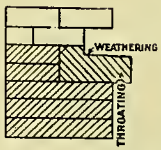



FIG. 452.—Section of window sill, etc., on line AB.

therefore in this case the thickness of quoin equal 9″, due to 3 bricks, and the length and breadth of the quoins are usually multiples of $2\frac{1}{4}$″, the quarter of a brick's length.

**235. Stone Semicircular Arch.**—The three views (Figs. 445 to 447) of this arch should now speak for themselves. The section (Fig. 447) is taken through the keystone, K, and the joints of the voussoirs on the intrados[1] projected from the elevation (Fig. 446). One of the voussoirs, B, is shown in oblique projection in Fig. 448. Draw the views shown to a scale of $\frac{3}{4}$″ to the foot, and the voussoir B to a scale $1\frac{1}{2}$″ to the foot.

**236. Segmental Arch of a Stone Bridge.**—The view shown (Fig. 449) is not a working drawing, but it gives you a good idea, particularly after working the previous exercise, of a type of structure that requires considerable skill to design on a large scale.

**237. Square-headed Window with Flat-gauged Arch.**—If you use your eyes when you have a chance of going over a house in course of construction there are many parts, such as the window shown in Figs. 450 to 452, that will be of interest. You will not be able to concentrate your attention on many of these until you have tried to make such drawings as those shown, and endeavoured to understand them. It should be explained that the dotted arch takes the load over the window opening, and that the joints shown on the gauged arch are not always the actual joints, but ones that are **pointed** on by the bricklayer in finishing the work. The top of the stone sill, Fig. 452, slopes slightly downwards to allow the water to run off, as you will see, and is therefore referred to as the **weathering**. To prevent the water flowing on to the brickwork below, the under edge of the sill is channelled, and this groove is called the **throating**.

The figures are dimensioned, and no doubt you would like to make drawings of the window. A suitable scale would be $\frac{1}{8}$″ to the foot.

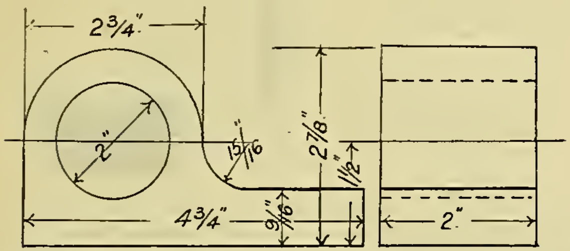
[1] The under surface of the arch, as shown in Fig. 449.

# CHAPTER XXIV

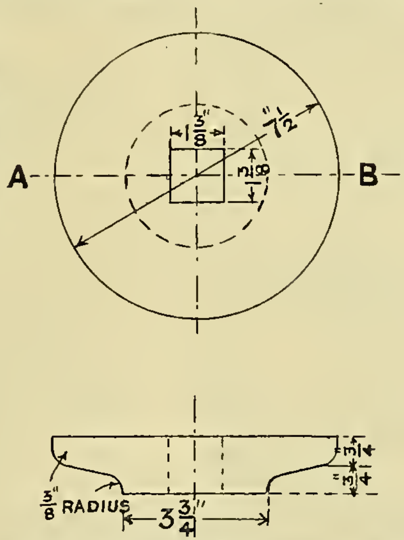
## VARIOUS MACHINE-DRAWING EXERCISES

**238.** In Figs. 453, 454 you have a dimensioned sketch (elevation and end view) of a **cast-iron bracket** used to support a handrail. It is secured by a $\frac{5}{8}''$ bolt. Make a working drawing of it, showing the two views given, and a plan. Also showing the bolt hole, which you are to put in a suitable position. Scale full size. Draw the circular parts first, and the plan before you decide upon the position of the hole.

**239.** The Figs. 455, 456 show two dimensioned views of a **cast-iron foundation washer**: one form of the washers used in connection with the bolts that hold down heavy machinery to concrete foundations, or with bolts that are sometimes used to keep opposite walls of a building from bulging out. The washers distribute the pressure (due to




Figs. 453, 454.—Dimensioned sketches of a cast-iron bracket.




Figs. 455, 456.—Cast-iron foundation washer.

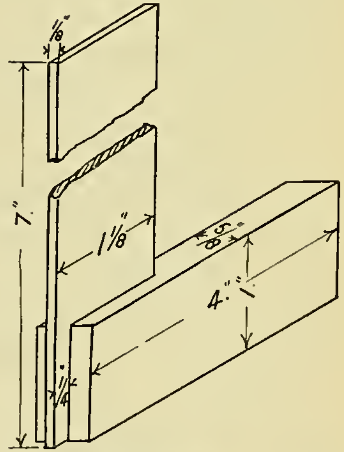



Fig. 457.—Engine fitter's square.

the bolts) over a large area. Draw the two views shown and add a section through the line AB, also a plan of the under side. Scale half size. Draw the circular view first.

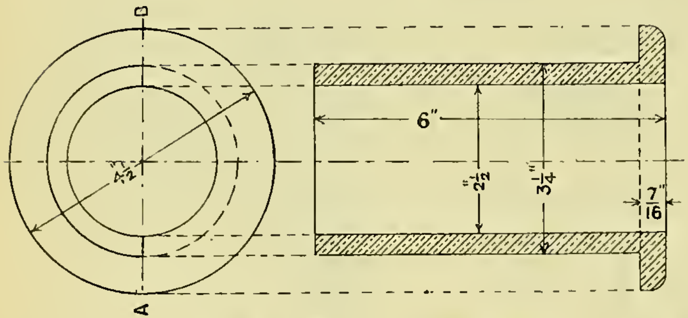
**240.** A dimensioned sketch of an **engine fitter's square** is given in Fig. 457. Draw a plan and elevation of it. Scale full size.

**241.** Two views of a **gun-metal flanged bush** are shown in Figs. 458, 458A. It is **used to bush** what is called a **loose belt pulley,** that is, a sort of idle pulley which is free to rotate upon a shaft, so that when the belt is not required to drive the **fixed pulley** (placed close to it, side by side), which is keyed to the shaft and rotates with it, it is made to run on the loose pulley; so that the hole gets worn in time; or, rather, the bush does. The latter is made a driving fit (as it is called) in the hub (or boss) of the wheel. That is to say, the fit is so tight that the bush is driven in by a hammer. When the bush has worn slack, the wheel can be rebushed, and made to run as true as ever.

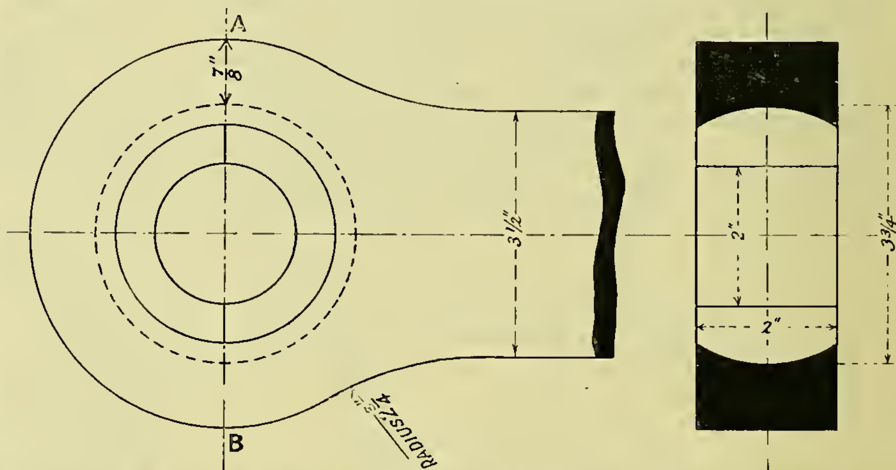
Draw the two views, but make Fig. 458A half elevation and half section through the axis. Scale full size. Commence on the end elevation, and remember that the diameter of the shaft on which it runs is $2\frac{1}{2}''$.

**242. Link End with Spherical Seat.**—In Figs. 459, 459A are shown the elevation and section (on line AB) of a link end with spherical seating. You will notice that the spherical bush or seat is split, to enable you to get it into and out of position. If one half of the seating be moved round about the axis of the rod a quarter of a turn, it can be withdrawn, after which the other part can be manipulated in the same way. With such an arrangement the link or rod can have a small movement about its axis without the pin (which fits the hole) binding in the hole. Draw the two views, full size, commencing on the circles in the elevation.

**243. Joint Pins.**—In Figs. 460 to 462A three pins for pin joints are shown, with different methods of



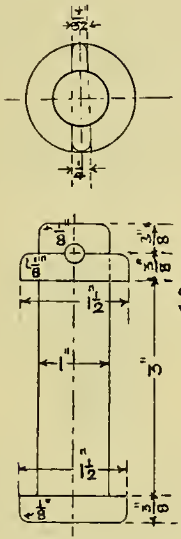

Figs. 458, 458A.—Gun-metal bush.



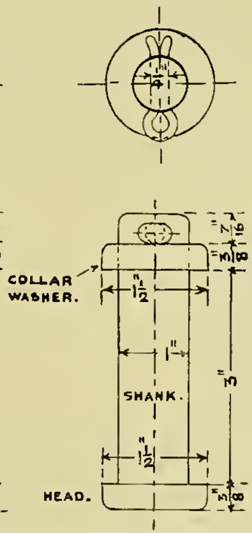

Figs. 459, 459A.—Front elevation and section on line AB, of a link end with spherical seated bush.

securing the collar. In 460, 460A a **taper pin** passes through the pin end and is seated in the groove in the collar. In Figs. 461, 461A a split pin (a detail of which is shown in Fig. 463) passes through the pin end, and its tail is spread open, as shown, to prevent it coming out. Figs. 462, 462A show a taper pin passing through the collar; the hole is made taper by using a taper reamer after the hole is drilled. These taper pins usually have a taper of $\frac{1}{4}''$ to the foot (or are made to Morse taper), so that if they are driven home they hold tight by the wedge action; but when they are likely to be subjected to much vibration they are made with a split end, being either forged with a split, or sawn, as in Figs. 464, 464A. The proportions shown are those in common use. Exercise.—Draw the three pins full size, and the details twice full size.

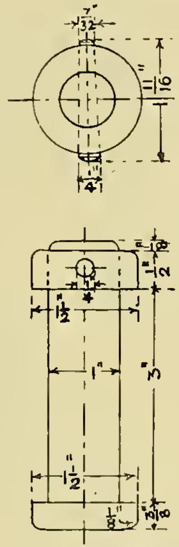
JOINT PINS.



Figs. 460, 460a.—Arranged with taper pin.




Figs. 461, 461a.—Arranged with split pin.



Figs. 462, 462a.—Arranged with taper pin through collar.

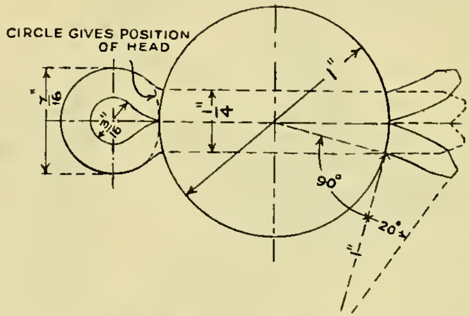
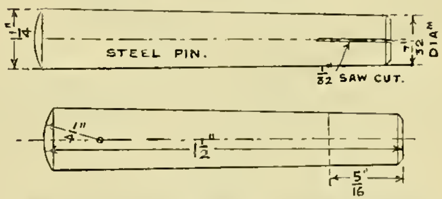



Fig. 463.—Detail of split pin.



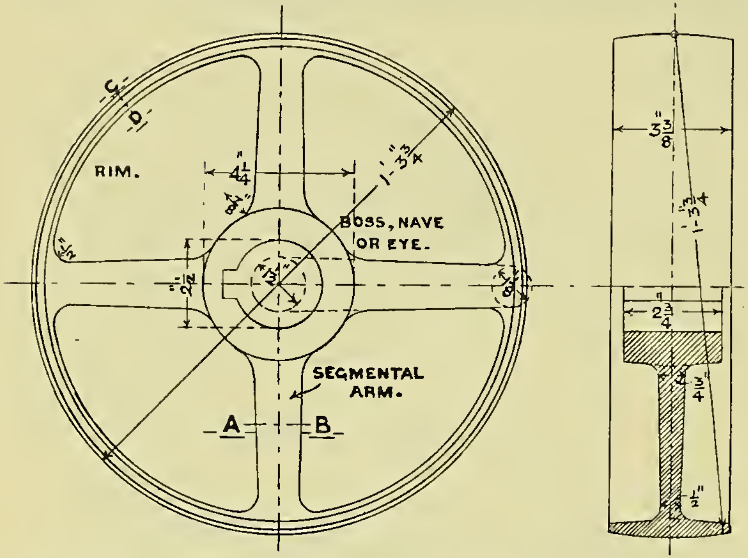

Figs. 464, 464a.—Detail of taper split pin.

**244. Belt Pulley.**—Figs. 465, 466 are two views of a heavy pulley for a double 3″ belt, the arms are segmental in section, as in Fig. 467, or elliptical, as in Fig. 468. Draw to scale of half full size. Note that the breadths of the arms are set off at the centre of the hub or boss, and at the rim. Commence with the circular view. These drawings should present no difficulty now.

**245. Locomotive Crank.**—A very simple form of locomotive crank, from a drawing point of view, is shown in Figs. 469, 470. It is a pattern introduced by Mr. Wordsell, the famous locomotive engineer, some years ago. To make a drawing of it, use a scale of $1\frac{1}{2}''$ to the foot. Commence on the end view, Fig. 469.

**246. Cast-iron Bearing Block.**—A very simple and inexpensive form of bearing is shown in Figs. 471, 472. The drawings will speak for themselves now. You should draw this piece full size, and commence on the sectional elevation. Also draw a plan. Be careful to find the proper position of the centre C, so that the lines flow into one another. A pictorial view of the block is shown in Fig. 384.

**247. Gun-metal Tail-guide for Valve Rod.**—Figs. 473, 474 show two views of a guide you may have noticed projecting from the end of the slide-valve jacket of a steam engine; its function is to steady and keep in its place the end of the valve rod, so that the valve may not leave the face upon which it works. In making the drawings (scale full size), commence on the end view, Fig. 473, and project from that the elevation, from which project a section on the line CD. You will have to be very careful in doing




Figs. 465, 466.—Elevation and sectional elevation of a straight arm belt-pulley.

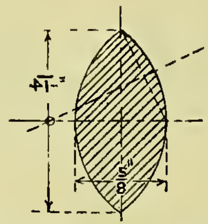


Fig. 467.—Section of segmental arm.

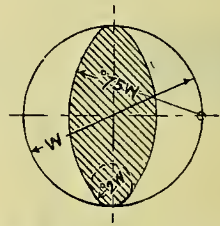
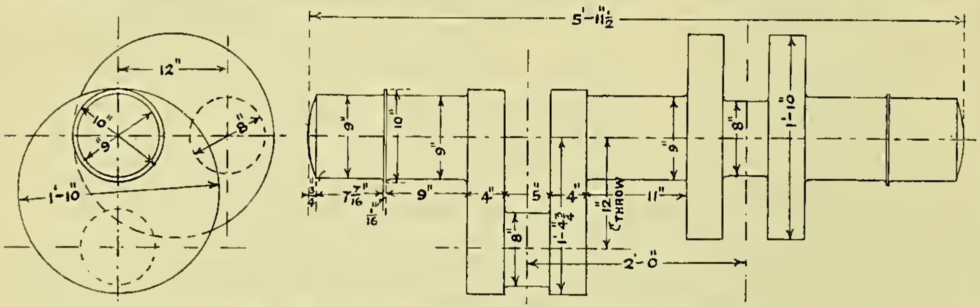


Fig. 468.—Section of elliptical arm.

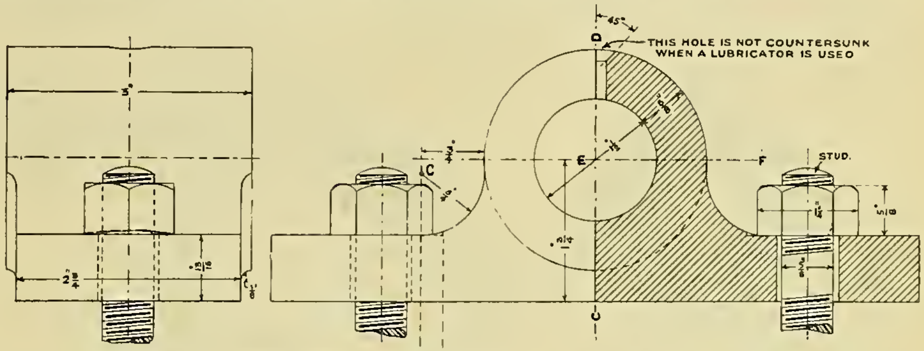
this to think out which lines will be seen after the body has been cut along the section line. By now you should be able to do this very well.




Figs. 469, 470.—Locomotive crank, Wordsell's pattern.

**248. Couplings.** — Whenever a **line of shafting** exceeds some 20′ in length it is made up of two or more lengths, connected together by what are technically called **couplings**, many forms of which are in use. One of the simplest of these is the **butt-muff coupling**, three views of which are shown in Figs. 475, 476, and 477. They are arranged to form a drawing exercise, in continuation of the previous ones, and it will be convenient to touch upon the principal features of the arrangement as we describe how it may be drawn.

**249. Drawing Exercise.—To draw a Butt-Muff Coupling. Scale half full size.**—From an inspection of the figures [1] it will be seen that the **sleeve, muff,** or **box,** B, is slid over the ends M and N of the two pieces of shafting that butt, and are required to be coupled together, and a taper key, K, is used, as shown, to fix the box to the shafting so that one length may transmit a torque, or twisting action, to the other. Now, remembering what we have said about commencing a drawing of an object that has a circular part, it will be seen that this is a case where the end views, Figs. 475 and 477 (or as much of them as possible), should be drawn first; so, having drawn the circles, the **section of the key** (taken through the centre of the coupling), Fig. 475, can be set out. As this is an important detail, it is shown in Fig. 478 to a larger scale. The point A, on the centre line and circle, is the centre of the section, and the **thickness of the key** here should be **half its breadth.** Now, the rule for **breadth** W may be, $\frac{d}{4} + \frac{1}{8}''$, then in this case, $W = \frac{3\frac{1}{2}}{4} + \frac{1}{8}'' = 1''$, and therefore BC = $\frac{1}{2}''$, so *the* **depth,** AC, **of the keyway** (which is uniform throughout the length) [2] becomes $\frac{1}{4}''$, the full taper of $\frac{1}{8}''$ to the foot being given to the keyway in the box. Fig. 479 shows the key in pictorial projection. With these hints, you should now experience no difficulty in drawing the three views shown, and in setting out a complete plan of the coupling. You will notice that you are instructed to make the drawings to a **scale of half full size,** that is to say, you are to draw the object one-half its real size, but you will not *dimension* the drawing with figures one-half of the original ones, as the **dimensions on a drawing indicate the real size,** and are independent of the scale to which the drawing may be made. All horizontal dimensions




FIGS. 471, 472.—End elevation and sectional elevation of cast-iron bearing block.

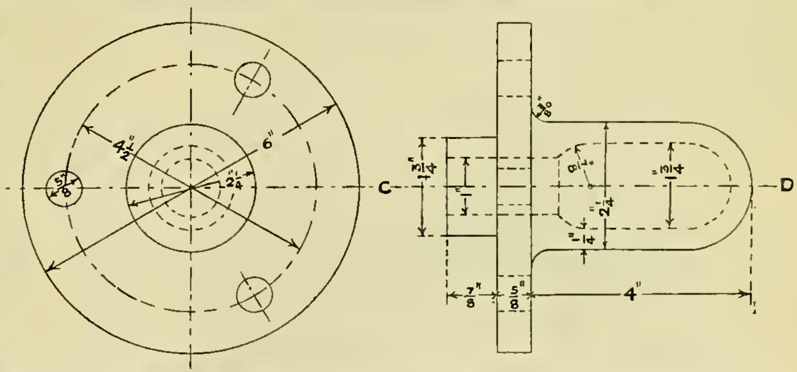



FIG. 473, 474.—End elevation and elevation of a gun-metal tail guide for valve rod.

[1] It will be seen that the proportional parts in terms of the **unit** used ($d + \frac{1}{2}''$) in designing it are given, but it has been also dimensioned for a 3″ shaft as a drawing exercise, and you only need pay attention to the actual dimensions given.

[2] The taper is always made on the coupling or boss, which is fitted to the shaft, excepting when the **key is fixed,** and the boss moves along the shaft a short distance; the key (which is then called a **feather**) is then parallel.

are placed to read horizontally in the spaces left for them between the dimension lines, and all vertical dimensions read from bottom to top of drawing when looking from its right-hand edge. The points of the arrow-heads must touch the lines between which the dimension is taken.

Every important part should be dimensioned on at least one of the views, and in cases where a body consists of two or

BUTT-MUFF COUPLING.

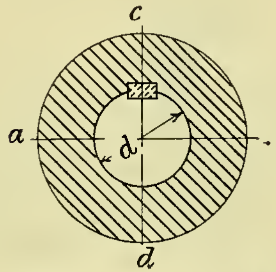



Fig. 475.—Section on line *ef*.

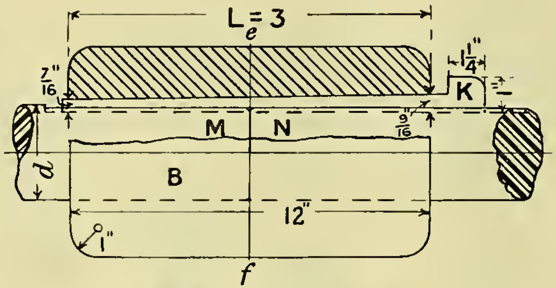



Fig. 476.—Sectional elevation.

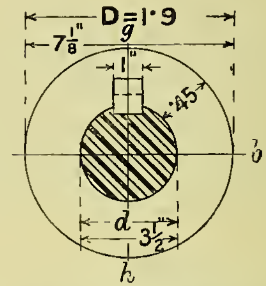



Fig. 477.—End elevation.

more divisions of its length, breadth, or thickness, the **overall** (sum of its parts) **dimensions** should be shown; indeed, in some cases it saves time in reading a drawing (when it gets into the works) if important dimensions are occasionally repeated on different views.

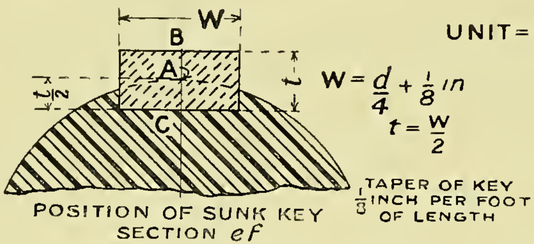



Fig. 478.—Proportions and position of sunk key.

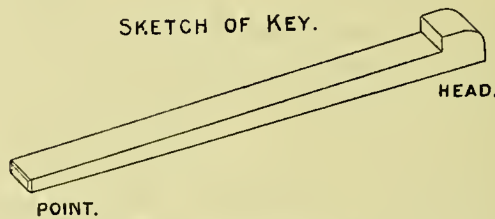


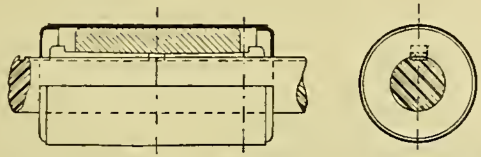

Fig. 479.—Sketch of key.

It should be explained that although the muff-coupling is the simplest one in general use, it requires to be very carefully fitted if it is to be a first-rate job, for, obviously, unless the depth of the keyway in each of the shafts to be coupled be exactly the same and the diameters be the same, the key will be bedded on one shaft whilst the other will be loose. To prevent this happening, some engineers make the **key in two lengths**, and drive them both in from the same end, one for each shaft. Or they may be driven from opposite ends, as shown in Fig. 480. This figure and Fig. 481 also show how the **coupling** is **cased** to protect the clothes of workers from coming into contact with the key-heads.

**Proportions for other Sizes.**—Taking the *unit* as $d + \frac{1}{2}''$, the usual proportions are shown on the figures in terms of the unit for other sizes of the shaft. As a **further exercise**, you might make drawings of such a coupling for a 2″ shaft. Full size.

**Materials.**—The **box** is made of **cast iron**; the **shafts**, usually of mild steel or wrought iron; and **keys**, of mild steel.

**249A. Drum or Barrel of Hoisting Machine.**—A section, Fig. 482, and end elevation, Fig. 483, of a drum or barrel of a hoisting machine, such as a crane or crab, are shown. The axle is supported by bearings at its **journals** J, and the spur driving wheel is fixed on the end A. It is fully dimensioned, and the two views can be drawn half full size. *Note.*—The **keys** which secure the barrel and wheel to the shaft are shown on the end view.



FIGS. 480, 481.—Sectional elevation and end elevation of cased butt coupling.

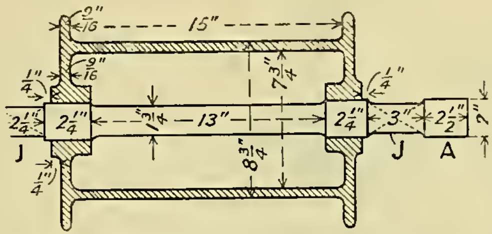



FIG. 482.—Section of drum or barrel of hoisting machine.

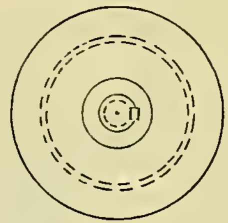


FIG. 483.—End elevation.

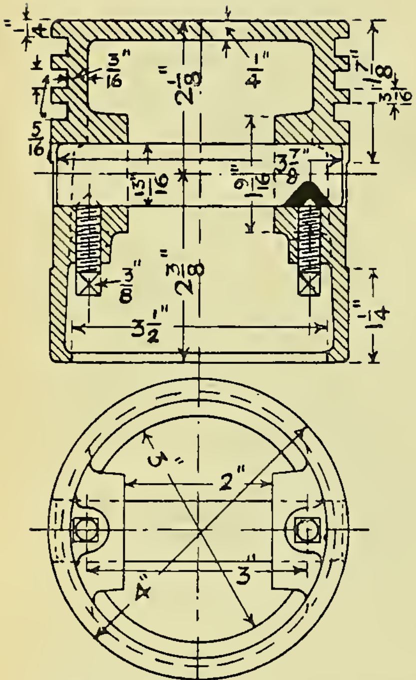
**250. How to measure the Diameter of a Large Cylindrical Body.**—If you had a pair of calipers large enough, the diameter of a body such as the rough drum, Figs. 482, 483, could be accurately measured, but your small pocket calipers would not be large enough for the purpose. However, you may take a piece of fine string, or, better still a tape, and measure the circumference of the drum, from which you will be able to find the diameter thus: The circumference in this case measures 27·48″, but diam. $\times \frac{22}{7}$ = circumference very nearly, or the diam. $= \frac{7 \times \text{circum.}}{22} = \frac{7 \times 27{\cdot}48}{22} = 8{\cdot}75''$. An advantage of this method is that the measurement is truer when the body is not quite round, than if it had been made with the calipers, as it gives the *mean* diameter.

**251.** A section and the plan of the under part of a **petrol engine piston** are shown in Figs. 484, 485. It is of the ordinary trunk type, fitted with three rings, $\frac{5}{16}'' \times \frac{1}{8}''$ section made of good grade cast iron. The steel gudgeon-pin is held in position by two $\frac{3}{8}''$ set-screws. Draw the views to a scale of full size, commencing with the plan. Such pistons for high-speed engines are made lighter.

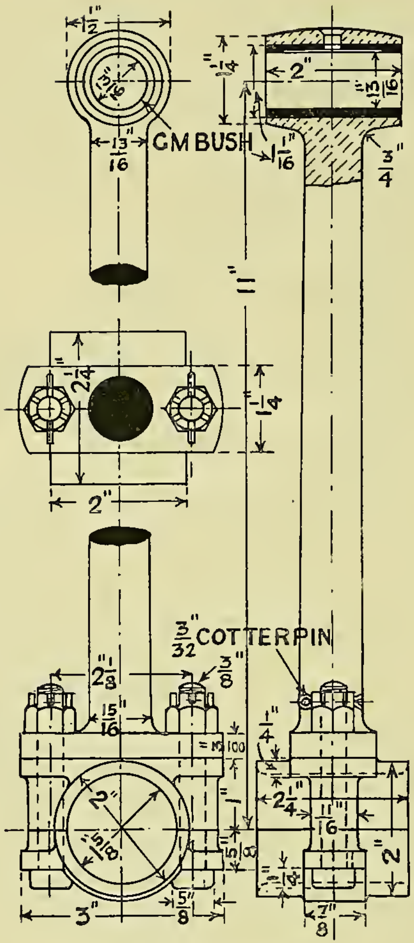
**252. Petrol Engine Connecting Rod.**—The views of the connecting rod (for the piston in Art. 251) shown in Figs. 486, 487 are fully dimensioned, and should now speak for themselves. Make separate drawings of each part of the rod. Scale full size.

**253. Cast-iron Bracket with Pin.**—The pictorial views[1] (Figs. 488, 489) of the bracket and pin are fully dimensioned, and you may draw an elevation looking on the face AA, a plan, and a sectional elevation taken through the axis BB, projected from the first view. Scale full size.

[1] Taken from the 1909 Board of Education paper in Machine Construction and Drawing, by kind permission of H.M. Sationery Office.



FIGS. 484, 485.—Petrol engine piston.




FIGS. 486, 487.—Petrol engine connecting rod.

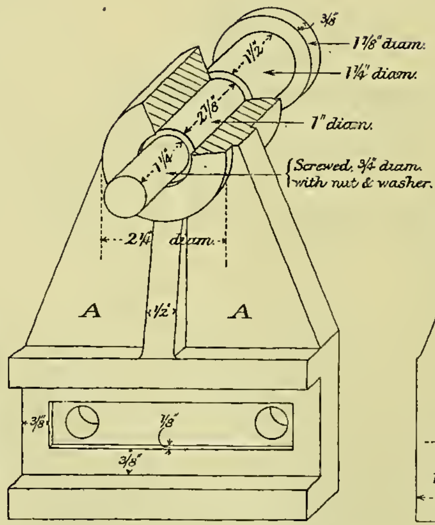



FIG. 488.—Front elevation of cast-iron bracket with pin.

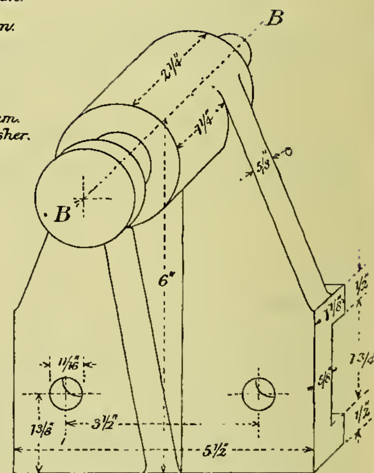



FIG. 489.—Back elevation of bracket.

# CHAPTER XXV

## INTERSECTIONS AND DEVELOPMENTS OF SIMPLE SOLIDS

### Intersections.

**254. Intersections.**—The determination of the lines of interpenetration due to the intersection of two solids is a branch of advanced geometry beyond the scope of this work, but there are a few very simple but important cases which you will understand. Some of these are based upon the fact that if two cylinders, or two cones, or a cone and cylinder, envelop a common sphere (that is to say, both solids enclose the same sphere), then the elliptical joint or section on the one enveloping body is exactly the same in shape and size as that on the other, and the projection of the section or joint on a plane parallel to the axes is a straight line. The following examples will make this clear.

**255. Intersection of Two Equal Cylinders.**—Fig. 490 shows two equal cylinders with axes *ac* and *bc* intersecting in *c* (the centre of the sphere), to form an elbow; the line *de* being the elevation of the **line of intersection**; the true shape of the intersection, of course, being an ellipse. Another case, that of **two equal cylinders**, is shown in Figs. 494, 495; the two parts T and R forming a **tee piece**, as it is called.

**256. Intersection of Cylinder and Cone.**—The two axes (Figs. 491, 492) intersect in *c*, the centre of the enveloped sphere, and the intersection of the surfaces at the sides gives *de*, the **section** (in each case), whose **true shape** you will know by this time must be an **ellipse**. The two parts of Fig. 491 may form a **conical ventilator**, and those in Fig. 492 a **conical nozzle**.

**257. Intersection of Two Cones and a Cylinder.**—The three intersecting pieces (Fig. 493), if in the form of pipes, would make what is called an irregular **breeches-piece**. It will be seen that they all envelop the sphere whose centre is *c*; the intersection of the cylinder and cone *cv* is obviously on the line *cf*, and of the two cones on the line *dg*; these lines intersect in *i*, so join this point to *k*, and the intersections *id*, *ie*, and *ik*, of the three solids are complete.

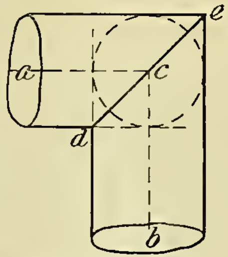


Fig. 490.—Intersection of two equal cylinders.

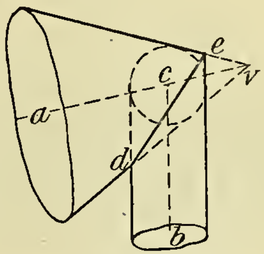


Fig. 491.—Intersection of cylinder and cone.

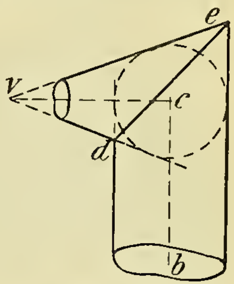


Fig. 492.—Intersection of cylinder and cone. Second case.

**258. Intersection of Two Unequal Cylinders.**—A very interesting and important case is shown in Figs. 494, 495, where we have the axes of the two unequal cylinders R and S not in the same plane. The points *ee'* and *ff'* in the intersection cau be at once found, as shown, and by using an auxiliary elevation any additional number of points can be found, such as *dd''*; dividing the semicircle of the auxiliary elevation into a suitable number of parts (in this case 6), and measuring the distances, such as *b'd'*, and marking them off above and below the axis *m'm'*, as at *b''d''*. The Figures should now speak for themselves.

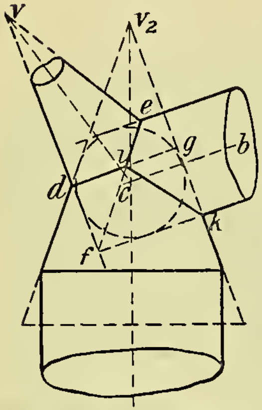
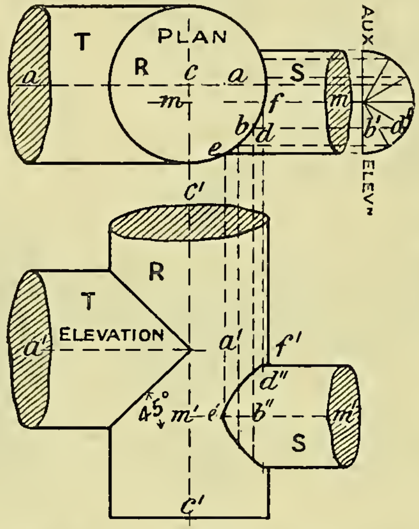


FIG. 493.—Intersection of two cones and a cylinder.




FIGS. 494, 495.—Intersection of cylinders. Two cases.

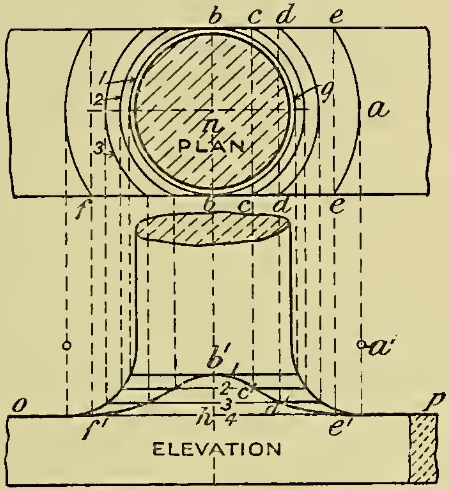
**259. Intersection of a Circular Fillet and Plane Surface.**—Look well at the tee-end of the connecting rod in Figs. 496, 497. You will see that the intersection of the fillet and the sides of the tee-head is the curve *b'c'd'e'*. The question arises how this curve is to be properly projected so as to truly represent the edge or line of intersection on the actual piece. A careful examination of Figs. 496, 497, should make this clear. The plan of the highest point in the curve will be *b*, and the elevation of the circular section containing it gives the position *b'*. Divide *b'h* into, say, three parts, and draw the lines 2 and 3 parallel to *op*, then these will give the diameters of the circles 2 and 3 in the plan, which circles cut the sides of the head in the points *c* and *d*, projectors from these points giving the points *c' d'* in the required curve in elevation. The point *e'*, the lowest one in the curve, is found by drawing a projector through the centre *a'* of the fillet, to *a*, in the plan and with centre *n*, radius *na*, stiiking the arc *eae*, then a projector from *e* cuts the line *op* in the elevation in the required point *e'*. The points on the left side of the curve are found in the same way, or by symmetry.

**260. Intersection of Circular Fillet with Cylindrical Surface.**—Look at your bicycle cranks, and notice where the arm merges into the boss or hub there is a curve, something like the one *b'e'* (Figs. 498, 499) in the elevation. Obviously the arc *bcd* in plan is on a cylindrical surface which intersects the circular fillet between the arm and boss. To find *b'*, the highest point in the curve, joiu the centres *m* and *n* (in plan), cutting the circle in *b*, through which drop a projector, *bb'*, cutting a horizontal through *a* (in elevation), the centre of the fillet arc, in *b'*. The other points are found as shown in the figures, and explained in the previous case.

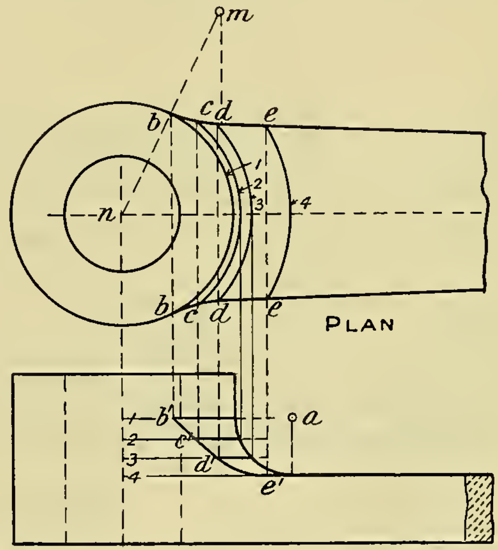
## Developments.

**261. Introduction.**—Take a model of a cylinder, and cut a piece of paper so that when it is wound round the body of the solid it exactly fits the cylindrical surface; the shape of this sheet will be a rectangle whose breadth equals the length of the axis of the cylinder, and whose length equals the circumference of its base. Now, this sheet of paper may be referred to as the **development or (lay-out)**

of the cylindrical surface. And of course you could place the sheet on the drawing board and roll the cylinder over it, keeping it in contact with the paper throughout the whole of its length during a complete revol ution.




FIGS. 496, 497.—Intersection of circular fillet and plane surface.




FIGS. 498, 499.—Intersection of circular fillet with cylindrical surface.

**262. Development of the Five Regular Solids.**—You can now understand in what sense Fig. 500 is referred to as the development of a **tetrahedron** (refer to Fig. 307). You will see that if you set out this figure, consisting of four equilateral triangles, each representing a face of the solid, you could fold it into a model of the solid. Do this with some stiff paper or millboard, and fasten the edges where they meet with sealing wax, or leave a little margin on the edges (as shown dotted), and gum or glue them together. You will now better see that Fig. 501 is the development of a **cube**; Fig. 502 that of an **octahedron** (refer to Fig. 591), consisting of eight equilateral triangles; Fig. 503 the development of a **dodecahedron** (Fig. 592), consisting of twelve pentagons; whilst 504 is the development of the **icosahedron** (refer to Fig. 593), consisting of twenty equilateral triangles.

If you are fond of such work, and have patience, you might make models of these interesting solids, and of others.

**263. Development of a Square Elbow.**—When two equal pipes, with ends bevelled at 45°, are brought together (as shown in Fig. 490), they form what is called a square elbow. Now, suppose you wish to make one out of metal plate, you will have to cut each plate to the exact shape, so that when it is bent round into the form of a cylinder one end will be bevelled, as in Fig. 505; in other words, you would want to find its development; and Figs. 505, 506 show how this is done. Divide the semicircle in Fig. 505 into, say, six equal parts, and draw the lines shown from these divisions, making the base line *dd*, in Fig. 506, $\pi D$ in length, or $\frac{22}{7}D$ (where D is the diameter of the pipe); or you may step off the divisions along *dd*, making them equal to those in the semi-


FIG. 500.—Development of tetrahedron.

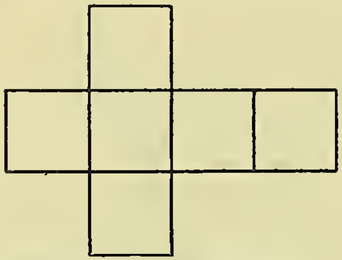


FIG. 501.—Development of cube.

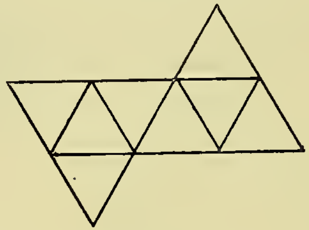


FIG. 502.—Development of octahedron.

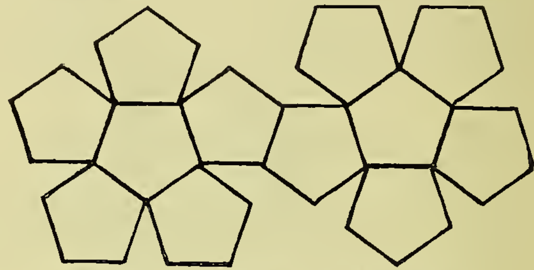


FIG. 503.—Development of dodecahedron.

circle; but, of course, that is not so exact. Number the divisions, as shown, and draw the vertical lines from the divisions to intersect horizontal lines from the points in the joint line *cd* (Fig. 505) in points in the required curve *dcd* (Fig. 506), which can be neatly drawn freehand through the intersecting points. Or this development may be made by the following direct method.

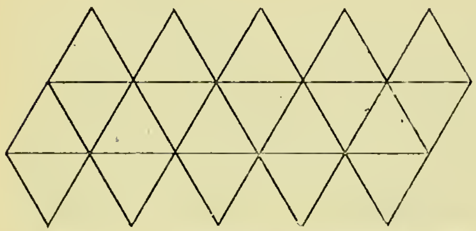
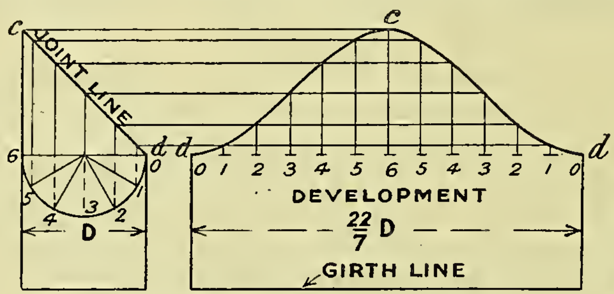


FIG. 504.—Development of icosahedron.




FIGS. 505, 506.—Development of square elbow.

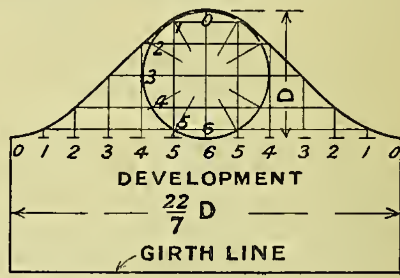



FIG. 507.—Development of square elbow. Direct method.

**264. Direct Method.**—Fig. 507 shows a direct method of setting out the curve, which should speak for itself when compared with the other figures.

**265. Development of a Square Tee Piece.**—Fig. 495 shows the axes *a'a'* and *c'c'* of two cylinders, which form a square tee piece;

Fig. 508 shows the end view of such a piece. To draw the development, Fig. 510, first draw the semicircle in Fig. 508, and divide it, as shown; through the three divisions erect perpendiculars cutting the upper circle in 1′, 2′, 3′, 0′; horizontals through these points cut the ordinates from the base or girth line (Fig. 510), and give points in the required curved boundary line. The development of the hole in the pipe *mn* is shown in Fig. 509; it is symmetrical about the line $a_1a_1$, and the distance $3_10$ is found by stepping off distances $3_12_1$ and $2_11_1$, also $1_10$ equal to 3′2′ and 2′1′, also 1′0 in Fig. 508. For the widths: reading the two views together, $a_13_1$ is equal to $a3$; $b_12_1$ equal to $b2$, and $c_11_1$ equal to $c1$.

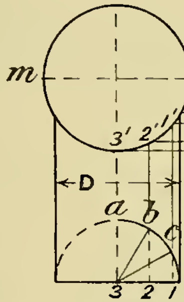


Fig. 508.—Square tee piece.

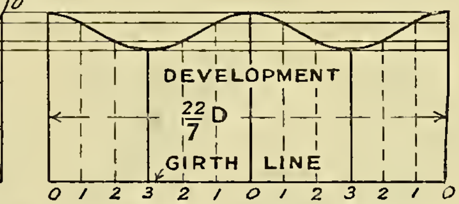


Fig. 510.—Development of pipe for square tee piece.

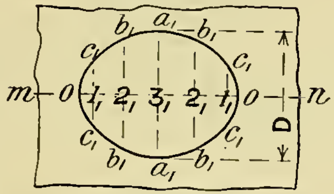


Fig. 509.—Development of the hole in pipe *mn*.

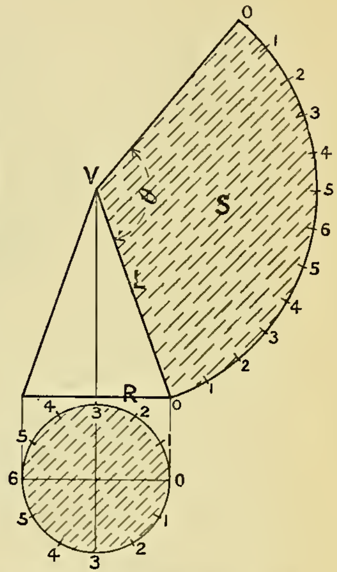


Fig. 511.—Development of cone.

**266. Development of a Cone.**—If the plan and elevation be drawn as in Fig. 511, and each semicircle be divided into, say, six equal parts, then the arc 0.6.0 can be drawn with V, the vertex, as centre, and L, the length of the slant side, as radius; take the length of the chord 01 (one of the divisions) in the semicircles as an opening of your compasses, and step off twelve divisions, along the arc as shown, to determine its length, then join the last one 0 to V, and the sector S is the required development. Note: the sector is only the **true development** when the length of its arc is equal to the circumference of the cone's base. Obviously the equal chords are measured on circles of different curvature, so the lengths of the arcs they represent cannot be quite the same. Therefore, to get the **true development**, make the angle $\theta = \dfrac{360 \times R}{L}$. Because it can be proved that $\theta : 360° :: R : L$, where R is the radius of the cone's base. In Fig. 511 $L = 3R$, $\therefore\ \theta = \frac{360}{3} = 120°$.

**267. Developments of Pyramids.**—If you have read Problem 191, and examined or drawn Fig. 315, you will see at once that *Acd* in Fig. 512 is the true shape of a side of the **triangular pyramid**, and, therefore, a part of its development; the base *cdb* is also in true shape, and it forms another part. It only remains to deal with the other two sides, and this is conveniently done by taking A as centre, radius *Ad*, and describing the arc $cdc_1$. It only then remains to mark off the points $b_1$ and $c_1$ with an opening of the dividers equal to the edge *cd*. Join $Ac_1$ and $Ab_1$; then the figure $cAc_1b_1db$ is the development of the solid. And on the same lines you will now be able to treat a **square pyramid** as in Fig. 513, or a frustum of it, as in Fig. 514. To complete the development in Fig. 514 you must find the true shape of the section of the pyramid made by the cutting plane. This you can do as in Fig. 312, Problem 189. But the construction shown in Fig. 514 should now be easily followed.

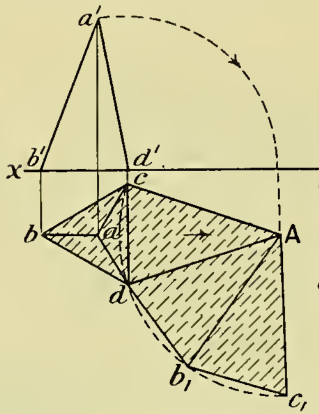



Fig. 512.—Development of a triangular pyramid.

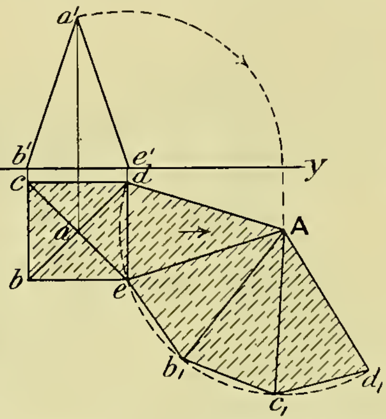



Fig. 513.—Development of a square pyramid.

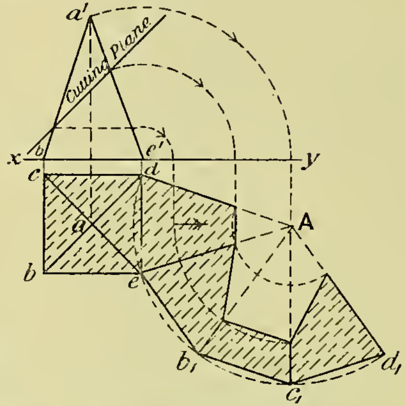



Fig. 514.—Development of the frustum of a square pyramid.

## EXERCISES.

1. A square elbow is formed by two 3″ pipes intersecting, as in Fig. 490. Draw the arrangement to a scale of half-size.

2. A conical ventilator is formed by a 3″ pipe intersecting a sheet-metal cone, as in Fig. 491. Draw the ventilator, making the angle of the conical part 60° at its vertex, and the diameter of the mouth part 8″. Scale half-size.

3. A conical nozzle is formed by a 4″ pipe and a conical part, as in Fig. 492. Draw the nozzle, making the angle at the vertex of the mouth-piece 40°, and the diameter of the nozzle 2″. Scale half-size.

4. A tee-piece is formed by two 3″ pipes intersecting at right angles, as in Figs. 494, 495 (the left-hand part). Draw two views of the arrangement. Scale half-size.

5. A tee-piece is formed by the intersection of a 3″ pipe and a 4″ one, their axes being in the same plane and at right angles. Draw two views of the arrangement, showing the line of intersection. Scale half-size. Note.—The right-hand part of Figs. 494, 495 shows how the line of intersection is obtained.

6. Figs. 496, 497 show the curve formed by the intersection of the tee-end and fillet of part of an engine connecting rod. Assuming that the diameter of the rod is 2″, the radius of the fillet 1″, and the breadth of the tee-head is 2″, draw the curve of intersection. Scale full size.

7. Figs. 498, 499 show the curve formed on a crank boss due to the intersection of the fillet with the curved part of the arm. Assuming that the boss has a diameter of $2\frac{1}{2}''$, that the radius *mb* is $1\frac{1}{4}''$, and the radius of the fillet $\frac{7}{8}''$, draw the curve as shown. Scale full size.

8. Draw on stiff paper the developments of the following regular solids, cut them out, and fold them to form models of the solids; if you make them with

jointings, as in Fig. 500, you can gum the edges together where they meet. (*a*) A tetrahedron of 2″ edge. (*b*) A cube of 2″ edge. (*c*) An octahedron of 2″ edge. (*d*) An icosahedron of $1\frac{1}{4}$″ edge.

9. Draw the development of an $1\frac{1}{2}$″ diameter square elbow, as in Figs. 505, 506. Scale full size.
10. Draw the development of a square tee-piece, diameter $1\frac{1}{2}$″. Scale full size. (Refer to Figs. 508–510.)
11. A cone has a 2″ base and 3″ axis. Draw its development.
12. The cone in the previous exercise is cut by a plane bisecting its axis and inclined 30° to its base. Draw the development of its lower part or frustum.
13. Draw an equilateral triangular pyramid of 2″ edge of base and 3″ axis, and show its development.
14. Draw a square pyramid, edge of base 2″, axis 3″, and show its development.
15. The square pyramid in the previous exercise is cut by a plane bisecting its axis and iuclined 30° to its base. Draw the development of its frustum.

Note.—If the developments in Exercises 11 to 15 be drawn with jointing, as in Fig. 500, models of the solids can be made by cutting out the figures, folding and gumming the edges.

# CHAPTER XXVI

## PRINTING, SHADING, TRACING, ETC.

**268. Printing, etc.**—The following style of lettering is most suitable for notes or remarks on a drawing. The letters, etc., should be neatly written with a fine pointed writing pen of the ordinary type: probably the best for this kind of work is Perry's No. 120 EF.

*a b c d e f g h i j k l m n o p q r s t u v w x y z*

In drawing office practice it is usual to stencil[1] headings and titles, etc., in plain letters, such as the following, the size varying from $\frac{1}{8}''$ to $\frac{3}{4}''$, according to the size of the drawing; for example, the heading or title on *medium* or *royal* size sheets would be in good proportion if made with $\frac{1}{4}''$ letters for *half imperial sheets*, $22'' \times 15''$, $\frac{3}{8}''$ for *imperial*, $30'' \times 22''$, with $\frac{1}{2}''$ or $\frac{5}{8}''$ letters for *double elephant*, $40'' \times 27''$, and $\frac{5}{8}''$ or $\frac{3}{4}''$ for *antiquarian*, $53'' \times 31''$; and such sub-titles as **plan, elevation**, etc., with $\frac{1}{8}''$ or $\frac{3}{16}''$ letters:—

**ABCDEFGHIJKLMNOPQRSTUVWXYZ**
**1234567890**

Although most of this printing is done by stencilling,[2] you should endeavour by practice to do it neatly by freehand, to enable you to do finished work, or proceed when stencil plates are not available. The quality of printing and writing upon a drawing greatly adds to or detracts from its appearance.

**269. Working Drawings** of machinery are made in such a way that the form and size of every detail are clearly shown for the guidance of those in the works. The rule is to make them to as large a scale as possible, generally full size for all small details, and $\frac{1}{2}$ and $\frac{1}{4}$ full size for larger ones. Such drawings are first carefully and completely set out in pencil,[3] and then inked in if required. All parts cut by section planes being hatched with sectional lines indicating the materials they are made of, in accordance with the shading shown in Fig. 524, or alternately, they are coloured[4] to indicate the materials, as explained in Art. 17. The edges of surfaces that are to be machined are usually coloured with a narrow band of a deeper tint,

[1] A good deal of practice is necessary to enable the beginner to do this neatly. He usually commences by making the stencil brush too wet, which causes the ink to flow between the stencil plate and paper. The best expedient is to recess a piece of Indian ink in a thin block of wood, and, after wetting the brush, rub it over the ink and wood till it is dry enough to use on the plate.

[2] It is best to start from the middle of a title when stencilling, so as to get it quite symmetrical with the drawing. This can easily be done by counting the letters which come each side of the centre, allowing one for each interval between two words.

[3] Many students and draughtsmen leave a large proportion of the details in their minds for inking, instead of completely pencilling them on the drawing. Nothing can be said to defend this objectionable habit, which puts an unnecessary tax on the mind. It has the forther disadvantage that the pencil-drawing cannot be passed on to another for inking in. To make a finished pencil-drawing quickly, neatly, and properly, you must patiently practise; for it represents an art, and is seldom a gift.

[4] Drawings from which tracings are to be made for reproduction by photographic printing to give blue prints, are of course always section-lined, and not coloured.

or hatched. The next step is to ink in with red ink[1] the centre lines,[2] and the dimension lines with Prussian blue.[3] The arrowheads and the dimensions should be now neatly written with a writing or mapping pen of the kind explained above, care being taken to make the dimensions bold and neat, so that they can be easily read from the drawing. The value of a drawing for workshop purposes greatly depends upon the clearness and accuracy of the figures or dimensions and the skilful way in which they have been arranged. Often an occasional duplication of a dimension on different views will save much time in the works.

In cases where original drawings are not likely to be much used, it is the practice of many engineers not to ink them in. This, of course, necessitates more careful finishing in pencil. Indeed, the beginner should not be encouraged to do any inking-in work until he has become fairly proficient in the somewhat difficult art of making a good pencil drawing.

The particulars as to the scale to which the drawing is made *must* always be clearly shown upon the drawing, not in order to enable workmen to "scale it," as sufficient dimensions should always be given to entirely obviate this. If a drawing is not completely dimensioned and there is any probability of it being sent abroad where a different system of measurement is used, or to where it will be exposed to variations of temperature, the scale should always be drawn upon the drawing. Sometimes the scale of a working drawing has to be reduced to make it suitable for attachment to a specification, or some such purpose; in such a case, proportional compasses may be advantageously employed; the best practice being to locate the centres of the circles and curves and to ink the latter in direct, and then to proceed with the straight lines, avoiding the use of pencils as much as possible.

## Shade Lines and Line Shading.

**270. Shade Lines.**—The appearance of finished drawings (which are usually made to a small scale) is improved, and the true form of parts made more intelligible in a single view, by the use of shade or dark lines, which give an appearance of relief, to the various parts.

Shade lines indicate the intersection of two surfaces, one of which is in the shade and the other illuminated. In arranging the shade lines, the parallel rays of light are conventionally assumed to come from the left and from behind (over the left shoulder) towards the object, their plans and elevations making angles of 45° with the vertical and horizontal planes respectively, their real inclination to the ground being 35°·15 nearly.[4] Thus, applying these rules to the body shown in Fig. 515, we have the back and right-hand edges *ab* and *bc*, also *ef* and *fg* of the projecting piece of the plan as shade lines; whilst the rules applied to the elevation give us the bottom and right-hand edges, *hi* and *ic'*, as shade lines. But, it should be explained, the line *hi* would not be a shade line if the body was actually resting on a horizontal surface, as the two surfaces would be in contact, and the upper not projecting beyond the lower. For these reasons *jk* is not a shade line but *g'k* is. These rules applied to a case where there is a recess or hole, as in Fig. 516, give us the front and left-hand edges, *bc* and *ab*, as shade lines; the upper surface being in the light or illuminated, and the front and left-hand sides of the hole in the shade.

In dealing with curved surfaces shade lines are never used to denote their contour or outlines. Thus, in Fig. 517 the only shade line on the elevation of the vertical cylinder is *de*, the line representing the solid's base, *fe*, being a boundary line of a curved surface, is not a shade line.

Now, the plan of the cylinder has a curved outline, and the rule relating to such cases is to make the shade line begin at the points *a* and *b*, at which the projections of the rays touch this outline, and let it gradually increase in thickness till its full strength is reached at *c*. Similarly, for the hole, the shade line increases in thickness from *m* and *n* to *g*.

The rules we have given relating to the rays of light we shall see are also concerned in the art of **Shading**; but, strangely enough, although generally followed by artists, many English engineers prefer to take the rays of light as shown in Fig. 523, where the rays *in plan* are *parallel* to those in *elevation*. This makes no difference to the elevation, but in plan the shade lines come in front, as shown, instead of at the back.

**271. Shading by Lines.**—By shading a projection of an object its true form can often be rendered intelligible in a single view. For example, the shaded view of a cylinder explains itself. But, on account of the time and labour involved, shading by tinting is only rarely used, even in finished

[1] This may be prepared by rubbing down a little colour from the cake of crimson lake. The practice in some offices is to use blue ink for centre lines, and red for dimension lines.

[2] The dimension lines on tracings prepared for blue prints may be drawn in red ink, and centre lines in Indian ink; the latter are formed by alternating dashes (of $\frac{1}{8}''$ and $\frac{3}{8}''$ lengths) not too close together and evenly spaced. Only short dashes are used on short centre lines.

[3] This may also be made by rubbing down a cake of the colour required, but most draughtsmen have the use of bottles of specially prepared red and blue inks.

[4] The cosine of the angle being obviously the $\sqrt{2} \div \sqrt{3}$.

## EXAMPLES OF SHADE LINES AND LINE SHADING.

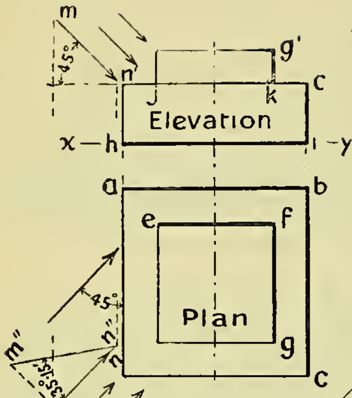


FIG. 515.

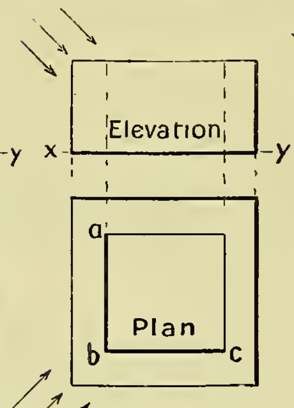


FIG. 516.

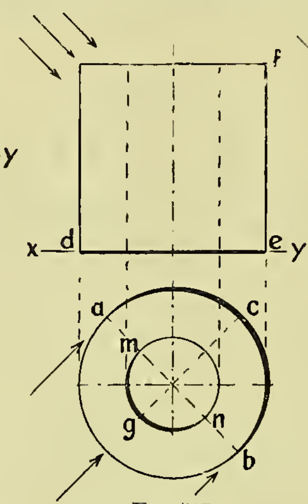


FIG. 517.

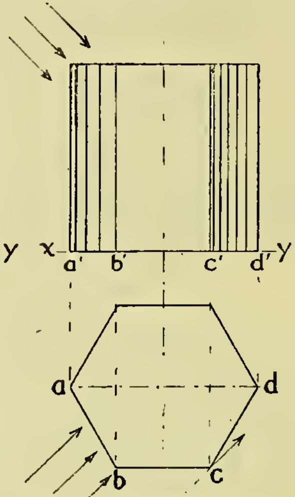


FIG. 518.

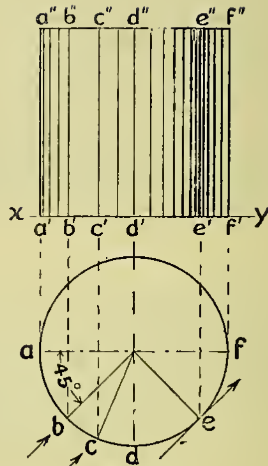


FIG. 519.

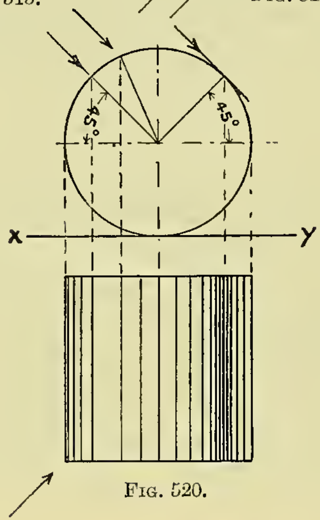


FIG. 520.

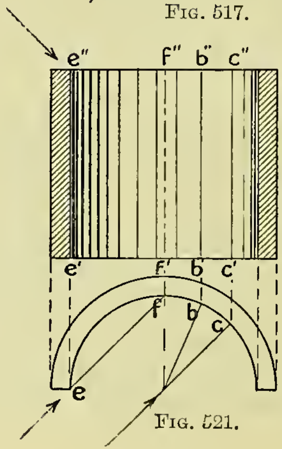


FIG. 521.

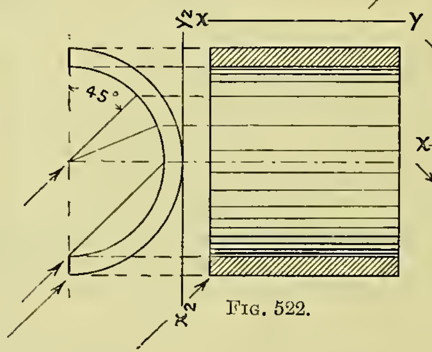


FIG. 522.

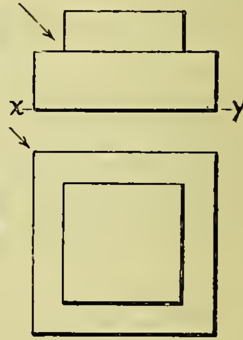


FIG. 523.

machine drawings. However, a similar effect can be easily produced by a few shading lines,[1] which are or should be drawn in accordance with the rules followed in shading proper. To commence with a simple example that very often in many forms appears on machine drawings, we have in Fig. 518 a vertical hexagonal prism, with its front face in the light or illuminated. Such surfaces parallel to the vertical plane would receive flat tints, and the nearer the surface is to the eye the lighter such tints would be, and the shading lines would be equally spaced (between $b'$ and $c'$), the spacing being increased on the lighter surfaces parallel to the plane of projection, and the surfaces in the shade also receive flat tints, but the nearer such surfaces are to the eye the darker such tints are, or the closer the shade lines. Thus—

Surfaces in the light inclined to the plane of projection have given them graduated tints (represented by graduated lines, as shown between $a'$, $b'$, Fig. 518), and as such surfaces recede from the eye the tints are made darker, or the lines closer together, as shown.

Surfaces in the shade inclined to the plane of projection, also have given them graduated tints (or lines), and as such surfaces recede from the eye they are made lighter, or the lines further apart, as between $c'$ and $d'$, Fig. 518.

When two such surfaces are unequally inclined, the one upon which the rays impinge most directly is made lightest.

Curved Surfaces.—The above rules in the main are followed in shading curved surfaces. Thus in Fig. 519 we have the plan and elevation of a vertical cylinder upon which the light falls from $a'a''$ to $e'e''$, but most directly at the generator whose plan is $b$; this, therefore, as we have seen, should be the lightest part, but succeeding generators from $b'b''$ to $d'd''$ approach the eye, and according to what we have seen should therefore be increasingly lighter. So, in order to meet both these considerations, it is the practice to bisect $bd$ in $c$, and make the surface between $b'b''$ and $c'c''$ the lightest; in fact, it is usually untinted, and remains white. Obviously, the darkest part of the cylinder is at $e'e''$, so that the shade and shading increase in depth from $c'c''$ to $e'e''$, and diminish from $e'e''$ to $f'f''$.

A horizontal cylinder, with its axis perpendicular to the vertical plane is shown in Fig. 520, and the student will see that similar lines are used in arranging the shading. The case of a vertical hollow semi-cylinder is shown in Fig. 521, and, for reasons we have explained, the lightest part of the cylindrical surface is between the generators $b'b''$ and $c'c''$, and the darkest at the generator $e'e''$; the part between $e'e''$ and $f'f''$ being in the shade. Fig. 522 is a hollow horizontal semi-cylinder whose axis is parallel to the vertical plane, and the shading shown should now speak for itself.

NOTE.—Figs. 515 to 523 are first-angle projections. If they had been third-angle projections, the explanations given would equally apply.

**272. Copying Workshop Drawings.**—The original drawings are kept in the drawing office for reference purposes, and copies only, produced in various ways, are used in the workshops. The most direct way of copying a drawing is to trace it on a sheet of tracing paper or tracing cloth, and, if more than one copy is required, the tracing is used to produce blue prints by sun-printing.

There are several photo-copying processes used for reproducing copies, or blue prints, by heliography, or sun-printing as it is called, in which the tracing is placed in front of, and in close contact with, a sensitized sheet of paper, both being clamped in a glass frame and exposed to the actinic[2] rays of light which, falling upon the tracing, pass through the transparent portions, decomposing the sensitized paper below, leaving the opaque lines upon the tracing undecomposed and transferred to the sensitized sheet. This sheet is then removed from the frame, and washed in water or certain solutions to remove the sensitizing matter and thereby develop the lines.

The sun-printing process has the drawback of being somewhat slow, since it is mainly dependent upon the character of the natural light, and as this varies a great deal, so does the time taken to make the prints; but since the invention some years ago of the electrical photo-copying apparatus, in which electricity is used to produce the requisite light, engineers have had at their command a simple, handy apparatus which makes them independent of the weather, and in which prints may be made in two or three minutes. Perhaps the best-known apparatus of this kind is the one invented by Messrs. Shaw and Halden, and manufactured by Messrs. J. Halden of Manchester.

In using the apparatus the tracing and sensitized paper are laid upon a vertical semi-cylindrical glass plate, and a cover or jacket is then laid over the back of the sensitized sheet and firmly clamped by engaging with a rod. The cylinder is then turned into position, and the arc lamp lowered and raised gradually up and down its interior, the speed being regulated to suit the exposure required of various sensitized papers. The lime-light apparatus of a lecture lantern can also be used with excellent results for this purpose with a little scheming, but of course it is not so expeditious.

**273. Tracing.**—No small amount of skill is required to expeditiously make a good tracing. The beginner cannot do better than commence by drawing a number of straight lines and arcs of different thicknesses on tracing paper and cloth with his drawing pen and bow pen respectively. The ink

[1] These are only used in connection with rounded surfaces on machine drawings.
[2] The action, as in photography, of the sun's rays in their chemical, as distinct from their illuminating and heating, effects.

may be introduced between the nibs of the pen by a pointed quill, or by a writing pen, care being taken to wipe the outside of the nibs, to prevent any ink from them touching the edge of the square, straight edge, or set-square, for should the ink get in contact with these instruments it runs on to the paper and spoils the tracing. Care must be taken to preserve **uniformity of thickness** in the lines where required, and to make **arcs and curves flow into straight lines** without any apparent break,—in other words, to satisfy the geometrical condition for tangential contact. If the **ink does not freely run** on the tracing paper or cloth, a little powdered chalk may be rubbed over the sheet, or a drop or two of ox-gall may be added to the ink.

In commencing a tracing, be careful to pin the tracing paper over the drawing and on to the drawing board in such a way that the sheets are taut, and the principal line of the drawing is square with the working edge of the board. As a general rule it is best to draw all the lines that are in the direction of the length of the board first. In working down from top to bottom in doing this, many lines will probably be missed, but they will be picked up by working down a second, and even a third time if necessary. The transverse lines can then be drawn in the same way, and then any connecting arcs drawn, and the circles, if any, described.

Usually the first tracing made by a beginner is not much of a success, but by persevering in the way indicated he should soon become proficient. Great care, of course, has to be taken in writing the dimensions to ensure absolute accuracy.

**274. Tracing Exercises.**—You will find that many of the larger diagrams in the figures given in this book are suitable to commence on. After a little practice on these, you will be able to attempt the tracing of most of the drawing exercises given in the book, of course working on the simpler ones first.

**275. Sectional Shading or Lining for Various Materials.**—Fig. 524 shows the sectional shading that is very generally used to indicate the materials used in engineering work. They speak for themselves.

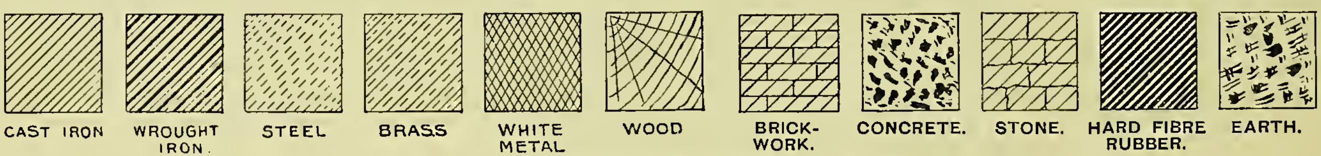
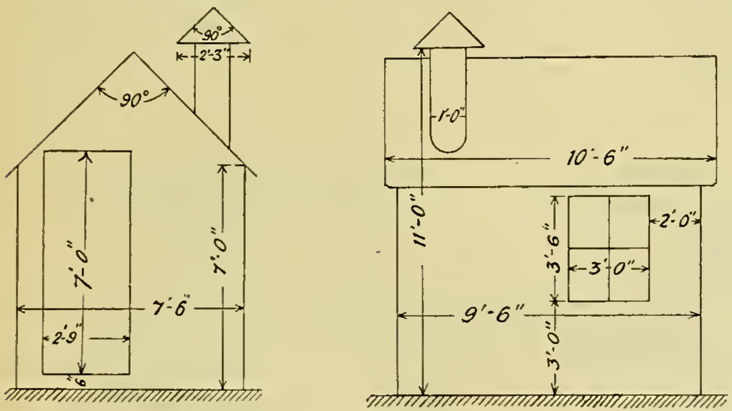



FIG. 524.—Conventional sectional lining for various materials.

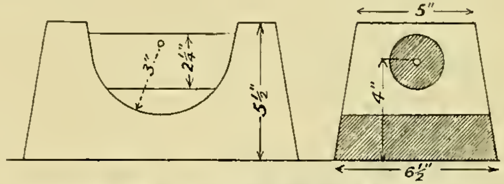
# CHAPTER XXVII

## MISCELLANEOUS DRAWING EXERCISES

1. A **hut** is shown in Figs. 525, 526, dimensioned, but not to scale. Draw these two views, and add a plan. Scale ½″ = 1 foot.
2. The Figs. 527 to 529 show a **cast-iron weight**, the plan being incomplete. Draw the complete three views. Scale half full size.
3. A pictorial view of a **box** is shown in Fig. 530. Draw a plan, elevation, and end elevation of it. Full size.
4. An isometric sketch of a **gun-metal block** (forming part of a thrust bearing) is shown in Fig. 535. Draw three views of it, making the radius of the arcs 1½″. Scale full size.
5. Draw a pictorial view of the **dovetailed joint** shown in Figs. 536, 537.




FIGS. 525, 526.—Elevations of a hut.




FIGS. 527, 528.—Cast-iron weight. Elevation and section.

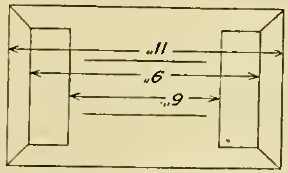



FIG. 529.—Incomplete plan.

6. The end views (Figs. 540, 541) of two valve cams for petrol motor cam shaft, with their common front elevation (Fig. 542), are shown. Carefully set them out. Full size.
7. A valve rod gland (gun-metal) is shown in plan and elevation in Figs. 543, 544. Draw the two views. Full size.
8. Make full-size drawings of the cam shaft bearing shown in Figs. 550, 551. Full size.

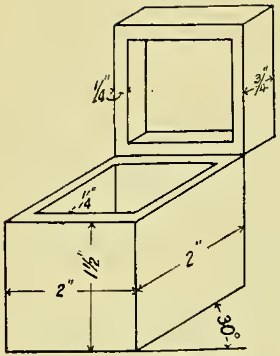


FIG. 530.—Wooden box.

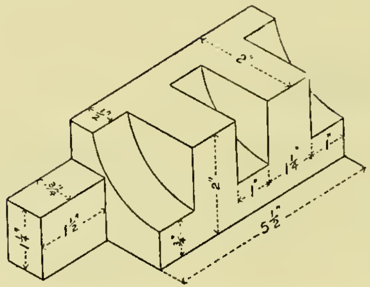
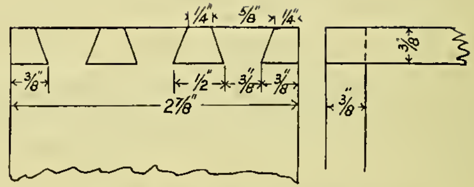
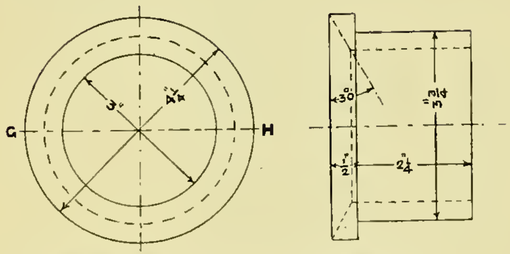


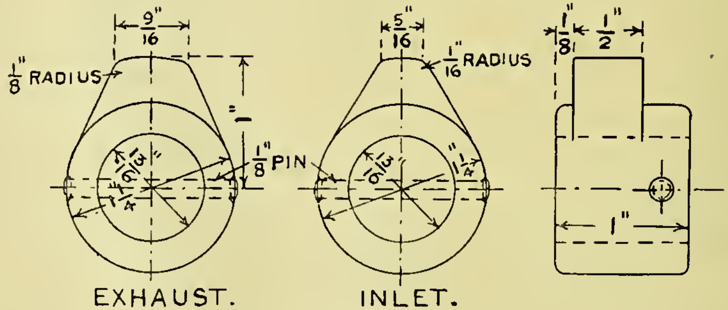
FIG. 535.—Part of a thrust bearing.



FIGS. 536, 537.—Dovetailed joint.




FIGS. 538, 539.—Neck bush for stuffing box.




FIGS. 540–542.—Valve cams.

9. A cast-iron tank flange joint is shown in section (Fig. 547). Assume that the pitch of the bolts is 3", and set out in two views a length of joint that will show three bolts. Full size.

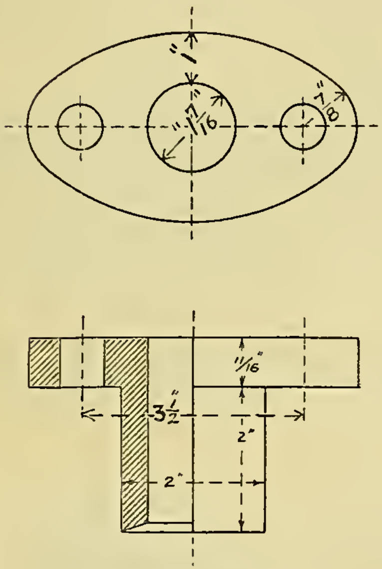
10. Draw, full size, the valve cover for a petrol engine cylinder (Figs. 548, 549). The hole is threaded to suit the sparking plug.

11. A petrol engine cylinder cover is shown in Figs. 550, 551. Carefully make the two drawings. Scale full size.

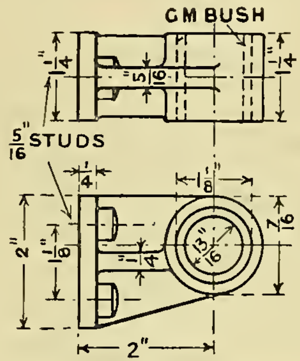
12. Make a pictorial view of the hut shown in Exercise No. 1.

13. Show the cast-iron weight (Figs. 527 to 529) in pictorial projection. Half-size.

14. Draw two views of the gun-metal stuffing box bush (Figs. 538, 539), full size, and add a sectional plan.



FIGS. 543, 544.—Gun-metal gland.




FIGS. 545, 546.—Bracket bearing for cam shaft.

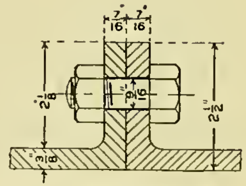
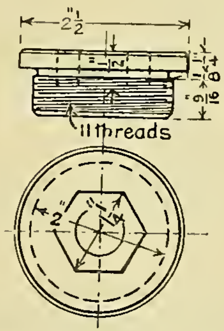
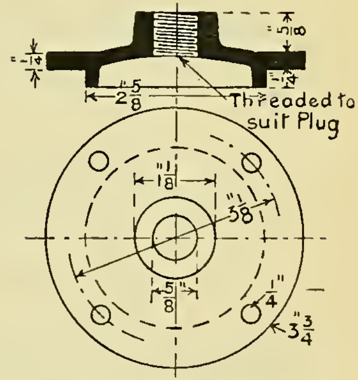


FIG. 547.—Cast-iron flange joint.




FIGS. 548, 549.—Valve cover for petrol engine cylinder.




FIGS. 550, 551.—Petrol engine cylinder cover plate.

# DEFINITIONS, ETC.

You are not expected to laboriously study the following definitions, etc.; they have been arranged, and find a place here, mainly for reference purposes. But notwithstanding this intention, you will probably read them, and in so doing no doubt learn some things that you will remember, and find useful in executing work in this subject. Some of the definitions will be sure to strike you as being self-evident, ones that you instinctively understand, and would take for granted. However, it will do you no harm to see them in print.

**A Point**[1] (*punctum*, "a small hole") is that which has position, but not magnitude. It is generally indicated by a dot, thus (·), sometimes enclosed in a small circle, but can be more accurately represented by two short cross-lines.

**A Line** (*linea*, "a linen thread") is that which has only length. It has really no thickness or breadth in pure geometry.

**A Straight Line** is the shortest distance between its extremities, or is such that, if any two points be taken in it, the part which lies between them is the shortest line which can be drawn between those points. For drawing and practical purposes straight lines are drawn or produced with a **straight edge** to guide the pencil, marking or cutting tool. A thin cord stretched by the two ends takes the form of a straight line in plan, and when such a line has been rubbed with chalk, and pulled tight over a surface on which a straight line is to be drawn, it can be lifted up at the middle, so as to make it, when let go, strike the surface with a little force, and form what is technically called a **chalk line**.

**Parallel Lines** are lines the same distance apart throughout (as in Fig. 552), and which, if produced ever so far both ways, never meet.

Fig. 552.—Parallel lines.

**An Angle** (*angulus*, "a corner") is the inclination to each other of two straight lines which meet in a point. The point is called the vertex, and the lines meeting in it, sides or legs of the angle. The size of the angle does not depend on the length of the lines, but on their inclination to one another.

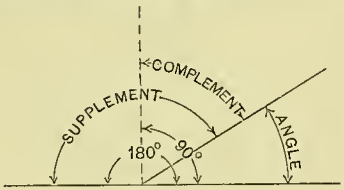



Fig. 553.—Complement and supplement of an angle.

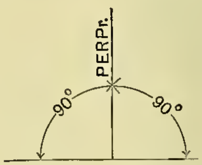



Fig. 554.—Right angle.

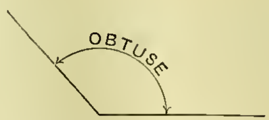



Fig. 555.—Obtuse angle.

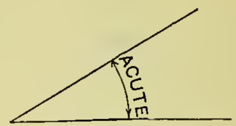



Fig. 556.—Acute angle.

The **Complement of an Angle** (Fig. 553) is the angle it requires to complete a right angle.

The **Supplement of an Angle** (Fig. 553) is the angle it requires to complete two right angles.

[1] For practical optical purposes, the nearest approach to a geometrical point is the point made by the intersection of two threads of a spider's web.

**Right Angle.**—When a straight line standing on another straight line makes the adjacent angles (those on each side of it) equal to one another, each of them is called a *right angle:* and the lines are mutually perpendicular, and are inclined to one another at an angle of 90° (Fig. 554).

**An Obtuse Angle** (*obtusus*, "blunt") is an angle greater than a right angle (Fig. 555).

**An Acute Angle** (*actus*, "sharp") is an angle less than a right angle (Fig. 556).

**A Circle**[1] (*circulus*, "a ring," "a hoop") (Fig. 557), is a plane figure contained by one curved line called the circumference, every point of which is equally distant from a point within it called the *centre.*[2] The curved line itself forming the **circumference** or **periphery** is also called a circle (or ring).

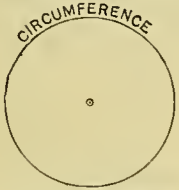



FIG. 557.—Circumference of a circle.

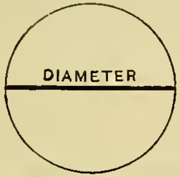



FIG. 558.—Diameter of a circle.

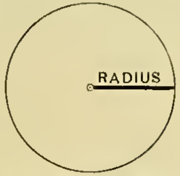



FIG. 559.—Radius of a circle.


FIG. 560.—Arc of a circle.

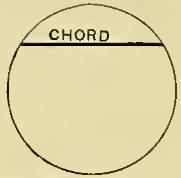



FIG. 561.—Chord of a circle.

The circumference (*circumferens*,[3] "carrying round") = $\pi$ × the diameter. The area of a circle = its diameter$^2 \times \frac{\pi}{4}$ nearly; or = the radius$^2 \times \pi$ = 3·14159, or $\frac{22}{7}$ nearly; therefore $\frac{\pi}{4}$ = 0·7854, or $\frac{11}{14}$ nearly.

**A Diameter of a Circle** (*diametros*, "a measure through") is a straight line which passes through the centre, and is terminated both ways by the circumference (Fig. 558).

**A Radius of a Circle** (*radius*, "spoke of a wheel") is a straight line drawn from the centre to the circumference, and is half a diameter (Fig. 559).

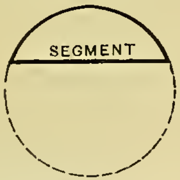



FIG. 562.—Segment of a circle.

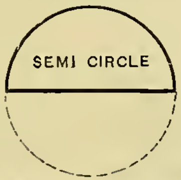



FIG. 563.—Semicircle.

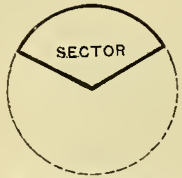



FIG. 564.—Sector of a circle.

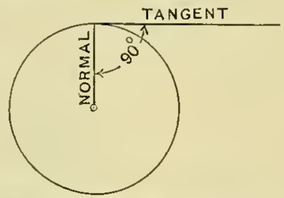



FIG. 565.—Tangent and normal to a circle.

**An Arc of a Circle** (*arcus*, "a bow") (Fig. 560) is any part of its circumference.

**A Chord** (*chorde*, "a harp-string") (Fig. 561) is the straight line which joins the extremities of an arc.

**A Segment of a Circle** (*segmentum*, "a cutting," "a slice") (Fig. 562) is a figure contained by an arc and its chord.

**A Semicircle** is half a circle (Fig. 563).

[1] Pythagoras (550 B.C.) discovered that of all figures having the same boundary, the circle among plane figures and the sphere among solids are the most capacious.

[2] *Kĕntrŏn*, "a goad," "a point."

[3] Archimedes (287 B.C.) discovered the relation between circumference and diameter. Refer to the author's "Origin, Rise, and Progress of the Science of Geometry," etc., p. 23.

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A **Sector of a Circle** (*sector*, "a cutter") (Fig. 564) is the figure contained by an arc and the two radii drawn to its extremities. If the radii be at right angles to each other, it is called a *quadrant.*

**Concyclic.**—Points which lie on the same circle are said to be concyclic.

**Area of sector** = length of arc × $\frac{1}{2}$ radius.

A **Tangent** (Lat. *tango*, "to touch") is a straight line which touches a circle in a point, but which, when produced, does not cut it (Fig. 565).

A **Triangle** (Lat. *tri*, "three;" and *angulus*, "a corner") is a figure which has three sides. (It has been called a *trigon.*)

Area of a triangle = base × $\frac{1}{2}$ altitude. Centre of *gravity* is on a centre line from its apex, and $\frac{1}{3}$ of its altitude from the base.

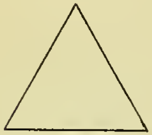

FIG. 566.—Equilateral triangle.

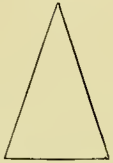

FIG. 567.—Isosceles triangle.

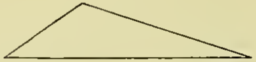

FIG. 568.—Scalene triangle.

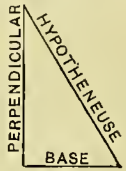


FIG. 569.

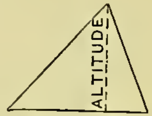


FIG. 570.—Showing altitude.

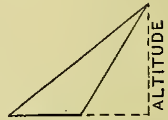


FIG. 571.—Showing altitude.

An **Equilateral Triangle** (Lat. *æquus*, "equal;" and *latus, lateris*, "a side") has three equal sides (Fig. 566).

An **Isosceles Triangle** (Gr. *isos*, "equal;" and *skelos*, "a leg") has two equal sides. The side which is not one of the equal sides is called the base (Fig. 567).

A **Scalene Triangle** (Gr. *scalenos*, "uneven") has none of its sides equal (Fig. 568).

A **Right-Angled Triangle** contains a right angle. The side opposite the right angle is called the *hypotenuse* (Gr. *hypo*, "under" or "beneath;" and *teino*, "to stretch"), one of the other sides is called the *base*, and the remaining side the *perpendicular* or *side*, these being interchangeable according to the position of the triangle (Fig. 569).

The **Vertical Angle** of a triangle is the one which is opposite the base.


FIG. 572.


FIG. 573.

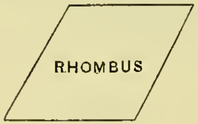


FIG. 574.


FIG. 575.

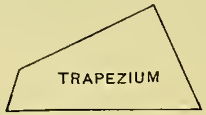


FIG. 576.


FIG. 577.

The **Altitude** or height of a triangle is the perpendicular drawn from the vertical angle to the base or the base produced (Figs. 570, 571).

**Orthocentre.**—The intersection of the perpendiculars from corners of a triangle to the opposite sides.

A **Quadrilateral or Quadrangular Figure** (Lat. *quatuor*, "four;" *latus, lateris*, "a side") is contained by any four straight lines which form a closed figure. It is sometimes called a *tetragon.*

A **Parallelogram** (Gr. *parallel*; and *gramma*, "a figure") is a figure which has the opposite sides equal and parallel to each other.

A **Square** is a quadrilateral figure having all its sides equal to each other, and its angles right angles (Fig. 572).

A **Rectangle** (Lat. *rectus*, "right," and *angle*) (Fig. 573) or oblong has only its opposite sides equal to each other, and all its angles right angles.[1]

[1] The simplest way to test whether the figure is a true rectangle or not, when the opposite sides are equal, is to measure the diagonals, which should be equal.

A **Rhombus** (Gr. *rhombos*) is a quadrilateral figure which has all its sides equal to each other, but has no right angles (Fig. 574).

A **Rhomboid** (Gr. *rhombos:* and *eidos,* "like") is a quadrilateral figure which has only its opposite sides equal to each other, and has no right angles (Fig. 575).

A **Trapezium** (Lat. *trapeza,* "a table," from *tetra,* "four;" and *peza,* "foot") is a quadrilateral figure, the opposite sides of which are neither parallel nor equal (Fig. 576).

A **Trapezoid** (Gr. *trapezion ;* and *eidos,* "like") is a quadrilateral figure, two of its sides being parallel, but none equal (Fig. 577).

A **Diagonal** (Gr. *dia,* "through;" and *gonia,* "an angle") of a straight-lined figure is a straight line joining opposite angular points.

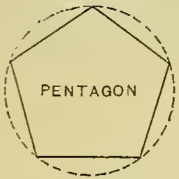

FIG. 578.

FIG. 579.

FIG. 580.

FIG. 581.

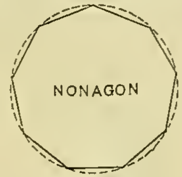

FIG. 582.

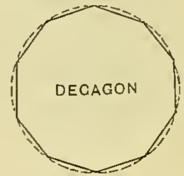

FIG. 583.

A **Polygon** (Gr. *poly,* "many;" and *gonia,* "angle") is a figure which has more than four sides. When all its sides are equal, a polygon is said to be *equilateral.* A polygon is equiangular when all its angles are equal. It is sometimes referred to as a *multilateral figure.*

A **Regular Polygon** is both equilateral and equiangular.

An **Irregular Polygon** has its sides and angles unequal. It may have such a form that all its external angles are prominent or **salient** (Lat. *salio,* "to leap"), or one or more external angle may re-enter or be **re-entrant**. Thus, in the irregular polygon (Fig. 129), the exterior angles at A, B, D, E, F, are salient, but the angle at C is re-entrant.

Special names are given to polygons to indicate the number of sides. For example:—

A **Pentagon** (Gr. *pente,* "five;" and *gonia,* "an angle") is a figure of five sides (Fig. 578).

A **Hexagon** (Gr. *hex,* "six;" and *gonia,* "an angle") is a figure of six sides (Fig. 579).

FIG. 584.

FIG. 585.

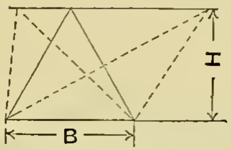

FIG. 586.—Triangles of equal area.

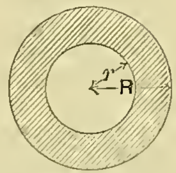

FIG. 587.—Annulus.

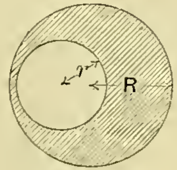

FIG. 588.—Eccentric annulus.

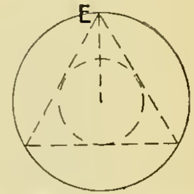

FIG. 588A.—Circumscribed and inscribed circles.

A **Heptagon** (Gr. *hepta,* "seven;" and *gonia,* "an angle") is a figure of seven sides (Fig. 580).

An **Octagon** (Gr. *octo,* "eight;" and *gonia,* "an angle") is a figure of eight sides (Fig. 581).

A **Nonagon** (Lat. *nonus,* "ninth;" and *gonia,* "an angle") is a figure of nine sides (Fig. 582).

A **Decagon** (Gr. *deka,* "ten;" and *gonia,* "an angle") is a figure of ten sides (Fig. 583).

An **Undecagon** (Lat. *undecim,* "eleven;" and Gr. *gonia,* "an angle") is a figure of eleven sides (Fig. 584).

A **Duodecagon** (Lat. *duo,* "two;" Gr. *deka,* "ten;" and *gonia,* "an angle") is a figure of twelve sides (Fig. 585).

**Circumscribed and Inscribed Circles.**—Fig. 588A shows an equilateral triangle circumscribed and inscribed.

The **Perimeter** (Gr. *peri*, "around;" and *metron*, "that which measures") of a plane figure is the sum of all its sides, or its boundary.

A **Proposition** (Gr. *pro*, "before;" and *pono*, "to place") is that which is offered or proposed for adoption or consideration.

**Propositions** (for geometrical purposes) are of two kinds, viz. *problems* and *theorems*.

A **Problem** (Gr. *pro*, "before;" and *ballo*, "to throw") is a proposal to do a thing, such as to solve a question, or to draw a figure.

A **Theorem** (Gr. *theorema*, "something which can be seen," literally, "a sight;" *median*, "traversing the middle, lengthwise") is a proposition to be proved by a certain chain of reasoning. In a *theorem* some new principle is asserted to be true, the truth of which is almost self-evident.

A **Corollary** is usually defined as a statement the truth of which follows readily from an established proposition; it is therefore appended to the proposition as an inference or deduction, which usually requires no further proof.

**Congruent Figures.**—Figures which are equal in all respects are said to be congruent (Lat. *congruo*, "to agree").

**Superposition.**—If two figures when applied to or laid over one another can be made to fit exactly, or **coincide**, they must be equal in all respects, and this method of testing equality is known as the **method of superposition**.

**Equivalent Figures.**—Figures which are **equal in area** (but not necessarily congruent) are said to be **equivalent**.

The **Apothem** of a regular polygon is the perpendicular from the centre to the side.

## Definitions, and Summary of some Useful Particulars relating to Areas.

A knowledge of the arithmetical measure of an area may often enable you to solve with facility a problem on areas; therefore the *measures of areas of various figures* which follow should be found useful for reference.

**Area. Definition.**—The boundary-line or perimeter of any closed figure encloses an amount of surface called its *area*.

| | |
|---|---|
| **Area of Rectangle** | = length × breadth. |
| ,, **parallelogram** | = length of side × distance between sides. |
| ,, **triangle** | = base B × $\frac{1}{2}$ perpendicular height H. Fig. 586. (Euc. I. 377.) |
| ,, **any regular polygon** | = radius of inscribed circle × number of sides × $\frac{1}{2}$ length of one side. |
| ,, **circle** | = radius$^2$ × $\pi$, or diameter$^2$ × $\frac{\pi}{4}$ = $d^2$ 0·7854 nearly, and the ratio $\frac{\text{area of a circle}}{\text{area of circumscribing square}} = \frac{3\frac{1}{7}}{4} = \frac{22}{28} = \frac{11}{14}$. |
| ,, **sector of circle** | = radius × $\frac{1}{2}$ length of arc or |
| | = $\frac{\text{no. of degrees in arc}}{360}$ × area of the circle. |
| ,, **segment of circle** | = area of the sector − $\frac{1}{2}$ chord × (radius − versin).[1] |
| ,, **the ring, or annulus** | = $\pi(R^2 - r^2)$. (Figs. 587, 588.) |
| ,, **ellipse** | = major axis × minor axis × 0·7854. |
| ,, **parabola** | = base × $\frac{2}{3}$ height. |
| ,, **surface of sphere** | = diameter × $\pi$ = ($\frac{2}{3}$ surface of circumscribing cylinder). |
| ,, **cylinder** | = (length × circumference) + area of both ends. |
| ,, **cone** | = (circumference of base × $\frac{1}{2}$ slant height) + area of base. |
| ,, **frustum of cone** | = (sum of circumferences at both ends × $\frac{1}{2}$ slant height) + area of both ends. |

## Definitions, etc., relating to Solids and their Projection.

These are given not necessarily for systematic study, but rather for reference purposes. Many of them are in common use in the science of projection.

**Solids** are all bodies that have the three dimensions, length, breadth, and thickness; they have an infinite variety of shape, some being bounded

[1] The versin is the perpendicular distance between the chord and arc.

by curve surfaces, and some by plane surfaces, whilst others are bounded by a combination of such surfaces. The following particulars of some of the principal solids will give examples of each kind. In geometrical language those that are terminated or bounded by regular and equal similar planes are called regular solids, such as the tetrahedron, cube, octahedron, dodecahedron, and icosahedron. (These are called the **five regular solids.**) All the regular solids can be inscribed in or made to circumscribe a sphere.

**The Tetrahedron** (Gr. *tetra*, "four;" and *hedra*, "a side") (Fig. 589), one of the five regular solids, is a triangular pyramid bounded by four equal equilateral triangles.

**The Cube** (Gr. *kybos*, "a die") (Fig. 590) is one of the five regular solids, consisting of or bounded by six equal square bases or sides, and its angles are all right angles.

**The Octahedron** (Gr. *okto*, "eight;" and *hedra*, "a side") (Fig. 591) is one of the five regular solids, bounded by eight equal equilateral triangles.

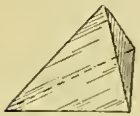
FIG. 589.—Tetrahedron.

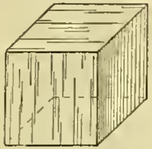
FIG. 590.—Cube.

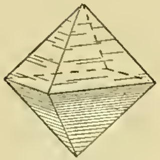
FIG. 591.—Octahedron.

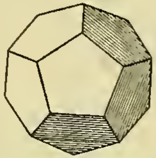
FIG. 592.—Dodecahedron.

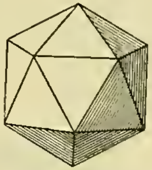
FIG. 593.—Icosahedron.


FIG. 594.—Sphere.

**The Dodecahedron** (Gr. *dodeka*, "twelve;" and *hedra*, "a base or side") (Fig. 592) is one of the five regular solids; it is bounded by twelve regular pentagons as faces.

**The Icosahedron** (Gr. *eikosi*, "twenty;" and *hedra*, "a base or side") (Fig. 593) is one of the five regular solids; it is bounded by twenty equilateral triangles.

**A Polyhedron** is any solid bounded by plane figures.

## SURFACES AND VOLUMES OF THE FIVE REGULAR SOLIDS (EDGES = 1).

| Name of Solid. | Surface. | Volume. |
|---|---|---|
| Tetrahedron . . . . | 1·7320508 | 0·1178511 |
| Cube or Hexahedron . | 6·0000000 | 1·0000000 |
| Octahedron . . . . | 3·4641016 | 0·4714045 |
| Dodecahedron . . . | 20·6457288 | 7·6631189 |
| Icosahedron . . . . | 8·6602540 | 2·1816949 |

**The Sphere**[1] (Gr. *sphaira*, "a ball") (Fig. 594) is a solid contained within one uniform surface, every point of which is equally distant from a point within called the centre, and may be conceived to be generated by the revolution of a semicircle about its diameter, which is fixed. All sections

[1] Archimedes (287 B.C.) discovered that the *solidity* and *surface* of the sphere are $\frac{2}{3}$ of the circumscribing cylinder. Refer to the author's "Origin, Rise, and Progress of the Science of Geometry," etc., p. 23.

of a sphere are **circles.** If a sphere be cut by a plane passing through it, each part is called a **segment.** When the cutting plane passes through the centre, each part is a **hemisphere**; any part cut off between two planes is called a **zone.**

(**The area of a sphere's surface** = $\pi \times$ its diameter$^2$. **The capacity of a sphere** = 0·5236 × its diameter$^2$.)

**A Circular Spindle** (Fig. 595) is a solid that may be conceived to be formed by the revolution of a circular arc or segment ABC about its chord AB, which remains fixed.

**A spheroid** (or ellipsoid) (Fig. 596) is a solid that may be conceived to be formed or generated by the revolution of an ellipse about one of its axes. If the revolution be made about the major axis, the solid is called a **prolate spheroid**; but if about the minor axis, an **oblate spheroid**

**A Right**[1] **Cylinder** (Gr. *kylindros*, "a roller") (Fig. 597) is a solid bounded by one circular and two plane surfaces, and may be conceived to be formed or generated by the revolution of a rectangle about one of its sides, which is fixed, and is called the axis of the cylinder. Either of the two plane surfaces is called the base of the cylinder. When the axis is inclined to the bases, it is an **oblique cylinder.**

(**Area of a cylinder's surface** = area of both ends + length × circumference. **Capacity** = area of one end × length.)

**A Prism**[2] (Gr. *prisma*, from *prizo*, "to saw") is a solid whose two ends are any plane figures which are equal, similar,[3] and parallel, and its sides parallelograms. Its axis is the straight line joining the centres of its ends or bases.

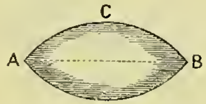


FIG. 595.—Circular spindle.

FIG 596.—Spheroid.

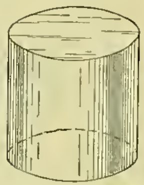

FIG. 597.—Cylinder.

FIG. 598.—Triangular prism.

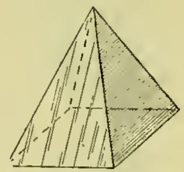

FIG. 599.—Square pyramid.

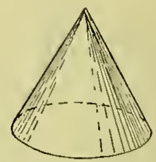

FIG. 600.—Cone.

**A Right**[4] **Prism** (Fig. 598) is one having its axis perpendicular to its ends. The one shown is a triangular prism. When the axis is inclined to its ends, it is an **oblique prism.**

**A Cuboid** is a cube-like solid, such as a square or rectangular slab (Fig. 601).

**A Pyramid** (Egyptian word) (Fig. 599) is a solid whose base is a polygon, and whose sides are triangles, their apices meeting in one point called the **apex or vertex** of the pyramid. When the axis is inclined to the base it is an **oblique pyramid.**

(**Capacity of a pyramid** = area of base × $\frac{1}{3}$ perpendicular height. Its *c.g.* (centre of gravity) is on the axis, and $\frac{1}{4}$ height from base.)

**A Right**[4] **Pyramid** is one having its axis perpendicular to its base. The one shown is a *square pyramid* (Fig. 599).

**A Cone** (Fig. 600) (Gr. *konos*) is a solid having a circular base, and its other extremity terminating in a single point called its **apex or vertex.** It may be conceived to be generated by the revolution of a right-angled triangle about one of its sides containing the right angle, which is fixed, and is called the axis of the cone. It is sometimes convenient to consider it as a pyramid with an infinite number of sides, and its base as a polygon with an infinite number of sides (that is to say, a circle). When its axis is inclined to the base, it is an **oblique cone.**

(**Area of surface of a Cone** = area of base + circumference of base × $\frac{1}{2}$ slant height. **Capacity** = area of × $\frac{1}{3}$ perpendicular height. Its *c.g.* (centre of gravity) is on the axis, $\frac{1}{4}$ height from base.)

[1] It is said to be a *right* cylinder when the axis is perpendicular to its bases or ends.

[2] If a solid be terminated by two dissimilar parallel planes as ends, and the remaining surfaces joining the ends be also planes, the solid is called a **prismoid.**

[3] As you have seen, figures that are similar and equal are called **congruent** or **identical.** For much interesting information relating to the properties of congruent figures, refer to Prof. Henrici's "Geometry of Congruent Figures."

[4] It is said to be *right* when its axis is perpendicular to its bases or ends.

**A Parallelopiped** (Fig. 601) (Gr. *parallel, epi,* "upon;" and *pedon,* "the ground") is a regular solid bounded by six parallelograms, the opposite ones of which are equal and parallel, or it is a prism whose base is a parallelogram. Thus a brick is a parallelopiped. These solids are sometimes called **cuboids.**

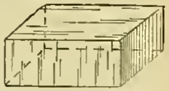
Fig. 601.—Cuboid.

**Frustum** (Lat. "a piece," "a bit").—The frustum of a cone, pyramid, or other solid is the part near the base formed by cutting off the top. It is said to be **truncated.**

**Volume of a Solid.**—As in the case of areas of plane figures, the number of solids whose volume can be calculated from their linear dimensions is very limited. But by **immersing a solid completely in a liquid, it** displaces a quantity of that liquid whose volume is the same as that of the solid. This gives us a simple means of finding such volumes, and obviously we can in this way also find the internal volume of a vessel of any shape, as the engineer sometimes does in measuring the **volume of the steam-passages** and clearance space of a steam-cylinder.

**Projections.**—The plan and elevation of an object are called its *projections.* The projection of a point on a plane is the extremity of the *projector* let fall from the point to the plane.

When a line, or the plane surface of a solid, is parallel to a plane, its projection on that plane is equal to the line or surface, and the greater the inclination the shorter becomes its projection, the limit occurring when the line or surface is perpendicular to the plane, when its projection becomes a point or line respectively. The **conventional way** to represent or **name the corners** of an object is by capital letters, and their plans by small italics, and their elevations by the same italics with dashes.

**Projectors** (Lat. *pro,* "forward;" and *jacio, jactum,* "to throw").—The lines connecting the projections of an object, or lines from points on an object to the corresponding points on the projection, are called *projectors.*

**Section** (Lat. *seco,* "to cut").—When a body is cut by a plane,[1] the surface or shape of the cut part or surface of separation is called a *section.*

**Sectional Elevation.**—If the section of a body be drawn in elevation, and the parts attached to it, but not cut by the section plane, be shown, the view is called a sectional elevation.

**Orthographic** (Gr. *orthos,* "straight;" and *grapho,* "to write").—Pertaining to orthography, which in geometry refers to the projections of objects showing all the parts thereof in their true proportions.

**Orthogonal** (Gr. *orthos,* and *gonia,* "an angle").—Right-angled, from *orthogon,* "a rectangular figure." Thus we speak of the planes of projection as being **orthogonal planes.** In the case of an **ordinary projection,** the projections are perpendicular to the planes of projection, and the system is said to be **orthographic**; but when the **projectors** are all **equally inclined** at any other angle to the planes, as in the case of **shadows,** the system is referred to as **orthogonal.**

**A Plane** (Lat. *planus,* "level," "flat") is a plain level surface. If any two points be taken in a true plane, the line joining the points will be wholly in the plane. The term "plane" is often used to express an imaginary surface.

**Co-ordinate Planes.**—The horizontal and vertical planes (H.P. and V.P. respectively) of projection are called the *co-ordinate planes of projection,* and their intersection is represented by the letters XY.

**The Traces of a Plane** are the lines made by the plane cutting the co-ordinate planes.

**Inclined Planes** are planes that are inclined to the *horizontal plane,* and are also perpendicular to the vertical plane.

**Oblique Planes** (Lat. *obliquus,* "slanting") are planes that are inclined to both the *horizontal and vertical planes.* They are, therefore, *inclined planes* which are not perpendicular to the vertical plane. Strictly speaking, all inclined planes are oblique planes.

**Tangent Planes.**—When a plane touches a sphere in a point on its surface, it is said to be tangent to it. Similarly, if a plane touches the surface of a cone in a line passing through its apex, the plane is called a tangent plane; and, again, if a plane touches the surface of a cylinder in a line parallel to its axis, it is tangent to the solid.

**Constructed.**—A plane is said to be constructed when it has been folded or revolved about its trace (as an axis) into a plane of projection, carrying with it all figures, lines, and points which are contained in it.

**Locus** (Lat. "a place").—When a plane is being *constructed,* say, about its horizontal trace, the successive plans of any point in the plane form a line perpendicular to the horizontal trace. This line is called the *locus* of the plan of the point. **Loci is the plural of locus.**

[1] This is a geometrical expression. Of course the body is not literally cut. A line or plane is used to represent the position of a cut.

**Horizontal** (Gr. *horos*, "a boundary").—A line or plane is said to be horizontal when it is parallel to the horizon, or to the surface of still water, or perpendicular to the direction of a plumb-line at any point.

**Horizontal Trace.**—The point in which any line or line produced pierces the horizontal plane is called its *horizontal trace;* similarly, the line in which any plane cuts the horizontal plane is called the *horizontal trace of the plane.*

**Vertical** (Lat. *verto*, "to turn").—A line or plane placed perpendicular to the horizontal plane is vertical (the plumb-line used by bricklayers and others always hangs in a vertical line).

**Vertical Trace.**—The point in which any line or line produced pierces the vertical plane is called its *vertical trace;* similarly, the line in which any plane cuts the vertical plane is called the *vertical trace of the plane.*

**Hidden Lines.**—The projections of hidden lines are drawn dotted.

**Contiguous.**—Two faces of a solid are said to be contiguous when they intersect or are adjacent.

**Dihedral Angle** (Gr. *di*, "two;" and *hedra*, "a side").—The true angle formed by the intersection of two planes is termed a *dihedral angle.*

**Trihedral Angle** (Gr. *tri*, "three;" *hedra*, "a side").—The solid angle of a solid formed by the intersection of three of its sides in a corner.

# TABLES OF BRITISH AND METRICAL EQUIVALENTS

## 1.—LENGTH

### English to Metrical

1 inch = 25·4 millimetres = 2·54 centimetres
1 foot = 30·4799 centimetres
1 yard = 0·914399 metre
1 chain = 66 ft. = 20·1168 metres
1 mile = 5280 ft. = 80 chains
0·62137 mile = 1 kilometre

### Metrical to English

1 millimetre = 0·03937 inches = $\frac{1}{25}$″ nearly, or $\frac{2\cdot5''}{64}$ nearly
1 centimetre = 10 mm. = 0·3937 inch = a full $\frac{3}{8}$″
1 metre = { 39·37 inches = $39\frac{3}{8}$″ nearly; 3·280843 feet; 1·093614 yards }
1 kilometre = 1000 metres = 3280·9 feet

## 2.—SURFACE AND AREA

### English to Metrical

1 sq. inch = 6·4516 sq. centimetres
1 sq. foot = 929·03 sq. centimetres
= 0·092903 sq. metre
1 sq. yard = 0·836126 sq. metre
1 acre = 0·40468 hectare
1 sq. mile = 259 hectares

### Metrical to English

1 sq. centimetre = 0·155 sq. inch
1 sq. metre = 10·7639 sq. feet
„ = 1·196 sq. yards
100 sq. metres = 1 are
1 hectare = 100 ares = 10,000 sq. metres = 2·4711 acres

The **Square** is used in measuring roofing and flooring. It equals 100 sq. feet. The **Rod** is used in measuring brickwork. It equals 272 super. feet $1\frac{1}{2}$ brick thick = $11\frac{1}{3}$ cu. yards = 306 cu. feet.

## 3.—LAND MEASURE.

| | |
|---|---|
| 7·92 inches | = 1 link |
| 100 links or 66 feet | = 1 chain (Gunter's) |
| 10 chains | = 1 furlong |
| 80 chains, or 8 furlongs | = 1 mile |
| 9 square feet | = 1 square yard |
| $30\frac{1}{4}$ square yards | = 1 square rod, pole, or perch |
| 16 perches | = 1 square chain |
| 40 perches | = 1 rood |
| 4 roods | = 1 acre |
| 640 acres | = 1 square mile |
| 30 acres | = 1 yard of land |
| 100 acres | = 1 hide of land |

The side of a square whose area is one acre is equal to 208·71 feet.

## 4.—VOLUME

### English to Metrical

1 cu. inch = 16·387 cu. centimetres
1 cu. foot = 0·028317 cu. metre
„ = 28·317 litres
1 cu. yard = 0·764553 cu. metres
„ = 764·553 litres
1 gallon = 4·545963 litres
„ = 0·1605 cu. feet
„ = 277·27 cu. inches

### Metrical to English

1 cu. centimetre = 0·061 cu. inch
1 cu. decimetre = 61·024 cu. inches
1 litre = 1000 cu. centimetres = 1·7598 pints
1 cu. metre = 35·3148 cu. feet
„ = 1·307954 cu. yards

1 U.S.A. gallon = 0·83254 Imperial gallon = 231 cu. inches.

# MISCELLANEOUS CONSTANTS

One Radian = 57·30 degrees
1 Gallon = 0·1604 cubic foot = 10 lbs. of water at 62° F.
1 Knot = 6080 feet per hour = 1 Nautical mile per hour

Weight of 1 lb. in London = 445,000 dynes
1 lb. avoirdupois = 7000 grains = 453·6 grammes
1 Cubic Foot of Water weighs 62·3 lbs.

THE END.

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