ASTRONOMISCHE NACHRICHTEN.

Nº. 46.

On Mr. Babbage's new machine for calculating and printing mathematical and astronomical tables.

From F. Baily.

London 1823. Nov. 28.

This invention of Mr. Babbage's is one of the most curious and important in modern times; whether we regard the ingenuity and skill displayed in the arrangement of the parts, or the great utility and importance of the results. Its probable effect on those particular branches of science which it is most adapted to promote, can only be compared with those rapid improvements in the arts which have followed the introduction of the steam engine: and which are too notorious to be here mentioned.

The object which Mr. Babbage has in view, in constructing his machine, is the formation and printing of mathematical tables of all kinds, totally free from error in each individual copy: and, from what I have seen of the mechanism of the instrument, I have not the least doubt but that his efforts will be crowned with success. It would be impossible to give you a correct idea of the form and arrangement of this machine, or of its mode of operation, without the help of various plates, and a more minute description than is consistent with the nature of your journal. But, it will be sufficient to say that it is extremely simple in its construction, and performs all its operations with the assistance of a very triffling mechanical power. Its plan may be divided into two parts, the mechanical and the mathematical.

The mechanical part has already been attained by the actual construction of a machine of this kind: a machine for computing numbers with two orders of differences only, but which I have seen perform all that it was intended to do, not only with perfect accuracy, but also with much greater expedition than I could myself perform the same operations with the pen. From the simplicity of the mechanism employed, the same principles may be applied in forming a much larger machine for computing tables depending on any order of differences, without any probability of failure from the multitude of

wheels employed. The liberality of our government (always disposed to encourage works of true science and real merit) has induced and enabled Mr. Babbage to construct a machine of this kind, capable of computing members with four orders of differences; and which will shortly be completed. To this machine will be attached an apparatus that shall receive, on a soft substance, the impression of the figures computed by the machine: which may be afterwards stereotyped or subjected to some other process, in order to ensure their permanency. By this means, each individual impression will be perfect.

The mathematical part depends on the method of differences to which I have above alluded: a principle well known to be, at once, simple and correct in its nature, and of very extensive use in the formation of tables, from the almost unlimited variety of its applications. It has been already successfully applied in the computation of the large tables of logarithms in France; and is equally applicable in the construction not only of astronomical tables of every kind, but likewise of most of the mathematical tables now in use. —

But, the full and complete application of this, and indeed of every other principle in the formation of tables, has been hitherto very much impeded by the impossibility of confining the attention of the computers to the dull and tedious repetition of many thousand consecutive additions and subtractions, or other adequate numerical operations. The substitution however of the unvarying action of machinery for this laborious yet uncertain operation of the mind, confers an extent of practical power and utility on the method of differences, unrivalled by any thing which it has hitherto produced: and which will in various ways tend to the promotion of science.

The great object of all tables is to save time and labour, and to prevent the occurrence of error in various

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computations. The best proof of their utility and convenience is the immense variety that has been produced since the origin of printing; and the diversity of those which are annually issuing from the press.

The general tables, formed for the purpose of assisting us in our computations, may be divided into two classes: 10 those consisting of natural numbers: 20 those consisting of logarithms. Of the former kind are the tables of the products and powers of numbers, of the reciprocals of numbers, of the natural sines, cosines etc. etc. Of the latter kind are not only the usual logarithmic tables, whose utility and importance are so well known and duly appreciated, but also various other tables for facilitating the several calculations which are constantly required in mathematical and physical investigations. I shall allude to each of these in their order.

10. Tables of the products of numbers. The numerous tables of this species which have been published at various time, and in different countries, sufficiently attest their utility and importance: and there can be no doubt that, if their accuracy were undeniable, their employment would be much more frequent. One of the first tables of this class was published in "Dodson's Calculator"; and contains a table of the first nine multiples of all numbers from 1 to 1000. In 1775 this table was much extended, and printed in an octavo size: it comprehended the first nine multiples of all numbers from 1 to 10000. Notwithstanding these and other tables of the same kind, the Board of Longitude considered that still more extended tables might be useful to science, and employed the late Dr. Hutton to form a multiplication table of all numbers from 1 to 1000, multiplied by all numbers less than 100. These were printed by their directions; and it is to be presumed that no expense was spared to render them accurate: yet in one page only of those tables (page 20) no less than forty errors occur, not one of which is noticed in the printed list of the errata. The French government, likewise, sensible of the utility of such tables, ordered the construction of a still more extensive set for the use of These are comprised in one several of its departments. volume quarto, and extend from the multiplication of 1 by 1 to 500 by 500: and in the year 1812, they caused a second edition of those tables to be printed. But, the most convenient tables of this kind which have yet appeared were recently published at Berlin, by M. Crelle; and comprise, in one octavo volume, double the quantity of the French tables. Another volume, of the same size, which is announced by the same author, will render these by far the most valuable of their kind, provided their accuracy can be relied on. The quantity of mental labour saved, in the construction of such tables, by the help of the machine, is literally infinite: for, in fact, no previous calculation is at all requisite; and it will be necessary merely to put into the machine, at the end of every two pages, the number whose multiples are required. This number will be successivily 1, 2, 3 etc. . . to 500.

20. Tables of Square Numbers. The squares of all numbers, as far as 1000, were a long time ago published on the continent by M. Lambert. These have been since extended as far as the square of 10000 by Mr. Barlow of the Royal Military Academy at Woolwich. The Board of Longitude employed the late Dr. Hutton to calculate a similar table as far as the square of 25400. In computing a table of this kind by the machine, even if extended to the most remote point that could be desired, the whole of the mental labour would be saved: and when the numbers 1, 1, 2 are once placed in it, it will continue to produce all the square numbers in succession without interruption. This is, in fact, one of those tables which the engine already made is capable of computing, as far as its limited number of wheels will admit.

3°. Tables of Cube Numbers. Tables of this kind have likewise been already computed by Mr. Lambert and Mr. Barlow; and also by the late Dr. Hutton, by order of the Board of Longitude. In computing such a table by the machine, the whole of the mental labour would be in this case also saved: since it would be merely necessary to place in the machine the numbers 1, 7, 6, 6; and it would then produce in succession all the cube numbers.

40 Tables of the higher Powers of Numbers. The Board of Longitude employed Dr. Hutton also to construct a limited table of this kind; which should contain the first ten powers of all numbers from 1 to 100. And Mr. Barlow has published, in his collection, a table of the fourth and fifth powers of numbers between 100 and 1000. Should it be thought desireable to re-compute or extend these tables, the whole labour may be performed by the help of the machine, except the few figures required to be first placed in it; and which might perhaps occupy the computer about ten minutes for each power. In fact the computation of these few fundamental figures would not occupy so much time, nor be so liable to error, as the calculation of one of the tabular numbers, according to the usual method.

50. Tables of the Square Roots and Cube Roots of Numbers. A table of the first kind has been given by Mr. Lambert: and a more extended one by Mr. Barlow, in his Collection. The latter writer constructed his table by means of differences; ane advantage which may be applied with greater effect to the table of Cube Roots, on account of the greater convergency of this order of differences.

60. Tables of the Reciprocals of Numbers. These are amongst the most simple but most useful of arithmetical tables; and are peculiarly valuable in converting various series into numbers, - thus facilitating the calculation of differences for the more ready construction of other tables. In order, however, to be employed in such operations, it is absolutely necessary that they should be infallible. Several tables of this kind have been printed: the most recent and extensive of which are those of Mr. Barlow, and Mr. Goodwin.

7 · Tables of Natural Sines, Cosines, Tangents etc. The utility of tables of this kind is evident from the variety of forms in which they have been, from time to time, printed: and it is needless to insist on their importance at the present day, since no seaman dare venture out of sight of land without a knowledge of their use. In ordre to be of any real service however, they should be accurate; et diligently revised from time to time: otherwise they may be worse than useless. The labour of computing tables of this kind will vary according to the number of figures contained in the result. It appears desirable that the larger tables of this sort should be printed with their several orders of differences to a much greater extend than formerly, for the purpose of making other tables, and for executing several mathematical operations beneficial to science. It would be difficult to state precisely the quantity of mental labour saved by the machine, in constructing tables of this kind; but, I believe, it may be facily reduced to the two thousandth part of the whole.

80. Tables of the Logarithms of Numbers. Tables of this kind are in the hands of every person engaged in numerical investigations: and it is needless to dwell on their utility et importance. The logarithms of numbers from 1 to 108000 have been 'already computed, with a greater or less number of figures; but this has been the work of various authors, and of several successive years: the labour is so immense that no human being has ventured to undertake the whole. The tables which now exist are

chiefly copies from those original and partial computations. By the help of the machine, however, this immense labour vanishes, and new tables may be readily computed and re-computed as often as may be required by the public. It is probable that the present tables if extended from 108000 to 1000000 would be of greater utility, than an extension of the present tables to a larger number of figures. The quantity of mental labour saved by the machine may be estimated in the following manner. Suppose a machine constructed, capable of computing with five orders of differences; it would be necessary to calculate those differences for every thousandth logarithm only: consequently, if the table extended from 10000 to 100000, there would be but ninety sets of differences to compute. Any one of these sets being placed in the machine, with its first five differences, it will deliver the 500 preceding logarithms and also the 500 succeding ones; thus producing a thousand logarithms: at the end of which term, another set of differences must be substituted. With five orders of differences, a table of logarithms may be computed to eight places of figures, which shall be true to the last figures, and it would not require more than half an hour to compute each set of differences; particularly as the higher numbers require very little labour, two or three terms of the series being quite sufficient.

90. Tables of Logarithmic Sines, Cosines, Tangents et Cotangents. The remarks which have been made in the preceding article, will apply with nearly equal propriety to the tables here alluded to. The mental labour required for their construction by the machine is reduced to a very insignificant quantity, when compared with the prodigious labour employed in the usual way.

10°. Tables of Hyperbolic Logarithms. Some small tables of this kind have been printed in several works, and are useful in various integrations: but the most comprehensive set was computed by Mr. Barlow, which contains the hyperbolic logarithms of all numbers from 1 to 10000. The labour of computing them is very great, which is the cause of their not being more extended. From a slight examination of the subject, it would appear that the mental labour may, in this case, be reduced by the machine to about a two hundredth part of what was formerly necessary.

110 Tables for finding the Logarithms of the sum or difference of two quantities, each of whose logarithms given. This table, which was first suggested, by Mr. Gauss, has been printed in at least three different

It is extremely convenient when many similar operations are required: the whole of it was computed by the method of differences; and consequently nearly the whole of the labour may be saved by the help of the machine.

120. Other general tables might also be here mentioned, which have been of great service in various mathematical investigations, and have been computed and printed, by different authors: such as tables of the powers of .01, .02, .03, etc.: tables of the squares of the natural sines, cosines, tangents etc.: tables of figurate numbers, and of polygonal numbers: tables of the length of circular arcs: tables for determining the irreducible case of cubic equations: tables of hyperbolic functions, viz hyperbolic sines, cosines etc., and logarithmic hyperbolic sines, cosines etc. These and various other tables which it is needless here to mention, may be computed by the machine, with very little mental labour, and with the greatest accuracy.

Besides the general tables above alluded to, there are many others which are applicable to particular subjects only: the most important of which are those connected with Astronomy and Navigation. - When we contemplate the ease and expedition with which the seaman determines the position of his vessel, and with what confidence he directs it to the most distant quarter of the globe, we are not perhaps aware of the immense variety of tables which have been formed almost solely for his use: and without the aid of which he dare not venture on the boundless ocean. Not only must the general tables of the sun and moon be first computed, together with the various equations for determining their apparent places; but those places also for every day in the year are prepared solely for his use; and even for different hours in the same day. The places of certain stars must likewise be given: and, as these depend on precession, aberration and nutation, tables of this kind also must be formed for each star. Then come the lunar distances, which are computed for every third hour in the day; and which depend likewise on a variety of other complicated tables. After these come the Requisite Tables, published by order of the Board of Longitude, and the usual Logarithmic Tables for facilitating the computations, both of which are dependent on other tables from which they have been deduced or copied. Now, when it is considered that an error, in any one of these multifarious tables, will affect the last result, and thereby render the navigator liable to be led into difficulties, if not danger, it must be acknow-

ledged that it is of very essential importance that all such tables should be computed and printed in so perfect a manner that they may in all cases be depended upon. This however, in the present mode of constructing them. is scarcely possible. I have myself discovered above five hundred errors in the work containing the Tables of the Sun et Moon, from which (still lately) the annual volumes of the Nautical Almanac have been computed: and a respectable author has asserted that, in the first edition of the Requisite Tables, published by order of the Board of Longitude, there were above a thousand errors. Many of the subsidiary tables, above alluded to, have not been computed since they were first printed: for, the mental and even manual labour of calculating them has been so great that the world has been obliged to remain contented with those original computations: and they are consequently subject to all the errors arising from subsequent editions and copies.

In the calculation of Astronomical tables, the machine will be of very material assistance: not only because an immense variety of subsidiary tables are required to determine the place of the sun, moon, and planets and even of the fixed stars but likewise on account of the frequent change which it is found necessary to introduce in the elements from which those tables are deduced: and which vary from time to time according to the improvements in Physical astronomy and the progress of discovery.

Within the last twenty years it has been found necessary to revise almost all the tables connected with the solar system: and already many of these have been found inefficient for the refined purposes of modern astronomy. But the great expence of time and labour and money has been the principal obstacle to the advancement of this part of the science: since each revision has been attended with the introduction of new equations, which consequently require new tables. And, to this day, we have not been furnished with any tables whatever of three (out of the four) new planets that have been discovered since the commencement of the present centring: nor can the places of many thousands of the fixed stars be readily determined for want of the subsidiary tables necessary for that purpose.

It perhaps may be proper to state that all astronomical tables (with very few trifling exceptions) are deduced by the two following methods: 10. by the addition of certain constant quantities, whereby the mean motions of the body are determined; 20 by certain corrections (of those motions) which depend on the sine or cosine of

a given arc: and which are called equations of the mean motions. The mean motions of any of the celestial bodies may be computed by means of the machine, without any previous calculation: and those quantities depending of the sine and cosine may in all cases be computed by the machine with the help of two previous calculations of no great length or labour, and in most cases with the help of one only.

In the year 1804 Baron de Zach published his Tables of the Sun: and within two years of that date, Mr. Delambre published similar tables. In 1810 Mr. Carlini published his Tables of the Sun, on a new construction: so that within the space of six years it was considered necessary by these distinguished astronomers to publish these three interesting and highly useful works.

In the year 1806 Mr. Bürg published his very valuable Tables of the moon; a work which superseded the use of Mason's tables, and was rewarded with a double prize by the French government: It was received with gratitude by the scientific in every nation, and opened a new æra in the history of astronomy and navigation. These were followed by the Tables of Burckhardt in 1812; which are still more accurate than those of Bürg: and, at te present moment, the elements of some new tables have been deduced by Mr. Damviseau. But it is the elements only which have yet been deduced; since it is these alone which can be expected to engage the attention of the profound mathematician. Nevertheless laborious, yet useful, operation of computation cannot safely be left to inferior hands. The merit of each is however very unequally estimated by the world. Euler had three hundred pounds granted him by the English government for furnishing the elements, and Mayer three thousand for the actual computation of the tables of the Moon, which were published by the Board of Longitude in the year 1770.

The elements of Mr Damoiseau have been already two years before the public: but the time and labour necessary to compute the tables therefrom are so great that they have not yet appeared. In order to deduce the place of the moon from these elements, no less than 116 different equations are requisite, all depending on the sine or cosine of different arcs. The labour of computing each equation, with the pen, would be immense; and liable to innumerable errors: but, with the assistance of the machine, they are all deduced with equal facility and safety, and without much previous computation.

In the year 1808 Mr. Bouvard published his tables of Jupiter and Saturn: but in 1821, owing to the progress of discovery and the advancement of physical astronomy, it was found necessary to revise the elements; and an entire new set of tables was then published by this distinguished astronomer. In order to deduce the geocentric places of these planets, it is requisite to compute no less than 116 tables depending on the sine or cosine of certain arcs.

I shall not intende further on the time of your readers by alluding to the tables of the other planets, which are all liable to similar observations: but I shall take the liberty of calling their attention to those very useful tables which have, from time to time appeared for determining the apparent places of the fixed stars; and which generally assume the title of "Tables of Aberration et Nutation." Tables of this kind are of vast importance to the practical astronomer, since they save a great deal of time and labour in the reduction of observations: and it is believed that many valuable observations remain unreduced, for want of convenient tables of this sort.

The first general tables of this kind were published by Mr. Mezger at Maunheim in 1778, and contained the corrections of 352 stars. In 1807 Mr. Cagnoli extended these tables to the corrections of 501 stars: and in the same year Baron de Zach published at Gotha his "Tabulae speciales Aberrationis et Nutationis" which contained the corrections for 494 zodiacal stars. But, already these tables have nearly ontlived their utility. Independent of their very limited extent, the elements from which they were deduced, have been superseded by others more agreable to actual observation; which, together with their exclusion of the solar nutation, and other minute quantities which cannot safely be neglected in the present state of astronomy, renders these tables of doubtful utility to the practical astronomer.

The number of zodiacal stars (without including the very minute ones) is considerably above a thousand: each of which may, in the course of a revolution of the nodes, suffer an occultation by the moon. These occultations are ascertained to be visible at sea, even from the unsteady deck of a vessel under sail: and afford the surest means of determining the longitude, provided the position of the star could be well ascertained. In order to furnish the corrections for each star, ten equations are requisite, depending on the sine et cosine of given ares; so that it would require the computation of upwards of ten thousand

subsidiary tables in order to produce the necessary corrections; a labour so gigantic as to preclude all hope of seeing it accomplished by the pen. By the help of the machine, however, the manual labour vanishes, and the mental labour is reduced to a very insignificant quantity. For, as. I have already stated, astronomical tables of every kind are reducible to the same general mode of computation: viz. by the continual addition of certain constant quantities, whereby the mean motions of the body may be determined ad infinitum; and by the numerial computation of certain circular functions for the correction of The quantities depending on these circular the same. functions, let them arise from whatever source they may, or let them be dependent on any given law whatever, are deducible with equal ease, expedition and accuracy by the help of the machine. So that in fact there is no limit to the application of it, in the computation of astronomical tables of every kind.

I might now direct your attention to those other subjects of a particular nature, to which the machine is applicable; such as tables of Interest, Annuities etc. etc.: all of which are reducible to the same general principles, and will be found to be capable of being computed by the machine with equal facility and safety. But, I trust that enough has been said to show the utility and importance of the invention; an invention, inferior to none, of the present day: and which, when followed up by the construction of a machine of larger dimensions now in progress (by which alone its powers and merit can be duly appreciated) will tend considerably to the advancement of science, and add to the reputation of its distinguished inventor.

I have omitted to state that this machine computes, in all cases, to the nearest figure, whatever it may be. That is, after the required number of figures are computed, if the next following figure should be a 5 or upwards, the last figure is increased by unity, without any attention on the part of the operator.

But, it is not in these mechanical contrivances alone, that the beauty and utility of the machine consist. Mr. Babbage, who stands deservedly high in the mathematical world, considers these but of a secondary kind, and has met with many curious and interesting results, which may ultimately lead to the advancement of the science. The machine which he is at present constructing will tabulate the equation $\Delta^4 u_3 = c$: consequently there must be a means of representing the given constant c, and also

There are five axes in the machine, in each of which one of these may be placed. It is evident that the arbitrary constant must be given numerically, although the members may be any whatever. The multiplication is not like that of all other machines with which I am acquainted, viz. a repeated addition—but is an actual multiplication: and the multiplier as well as the multiplicand may be decimal. A machine possessing five axes (similar to the one now constructing) would tabulate, according to the peculiar arrangement, any of the following equations

$$\Delta^{3}u_{3+1} = a\Delta u_{3} + \Delta^{3}u_{3} \qquad \Delta^{3}u_{3} = au_{3}$$
If there were but two axes, we might tabulate

$$\triangle^2 u_3 = a u_{3+1}$$

These equations appear to be restricted; and so they certainly are. But, since they can be computed and printed by machinery, of no very great complication, and since it is not necessary (after setting the machine at the beginning) to do any thing more than twin the handle of the instrument, it becomes a matter of some consequence to reduce the mode of calculating our tables to such forms as those above alluded to.

A table of logarithms may be computed by the equation $\triangle^4 u_3 = c$: but in this case, the intervals must not be greater than a few hundred terms. Now, it may be possible to find some equation, similar to those above mentioned, which shall represent a much more extensive portion of such tables, — possibly many thousand terms: and the importance that would result from such an equation renders it worthy the attention of mathematicians in general.

A table of sines may, for a small portion of its course, be represented by the equation $\triangle^2 u_3 = c$: but it may be represented in its whole extent by the equation $\triangle^2 u_3 = a u_{3+1}$. Now, this is precisely one of the equations above quoted: and if a proper machine were made (and it need not be a large one) it would tabulate the expression A sin θ from one end of the quadrant to the other, at any interval (whether minutes or seconds) by only once setting it. It would not be very complicated to place three such machines by the side of each other,

and cause them to transfer their results to a common axis, with which the printing apparatus might be connected. Such a machine would, amongst other tables, compute one from the expression

 $A \sin \theta + B \sin 2\theta + C \sin 3\theta$

the utility of which, in astronomy, is well known. In fact Mr. Babbage is of opinion that it would not be impossible to form a machine which should tabulate almost any individual equation of differences.

Amongst the singular and curious powers produced by small additions to the machinery, may be reckoned the possibility of tabulating series expressed by the following equation:

 $\triangle^2 u_3$ = the units figure of u^3

 $\triangle^3 u_3 = 2 \times^3$ the figures found in the tens place of u_{3+1}

 $\triangle^3 u_3 = 4x$ the figures found in the units and tens place of μ_{3+1}

and many others similar thereto.

Again, let the machine be in the act of tabulating any series, a part may be attached by means of which, whenever any particular figure (a 6 for example) occurs in the units place, any other number (23 for instance) shall be added to that and all the succeeding terms: and when, in consequence of this, another figure 6 occurs in the units place, then 23 more will be added to that and all the succeeding terms. Or, if it be preferred, the number added shall be added to the term ending in 6 only, and not to all succeeding ones.

These views may appear to some persons more curious than useful. They lead however to speculations of a very singular and difficult nature in determining the laws which such series follow: and they are not altogether so remote from utility as may be imagined. I avoid alluding to many other curious properties which this machine is capable of exhibiting, as they will scarcely be intelligible till the machine itself is more known in the world. Indeed I fear I have already tired your patience with this long letter.

Francis Baily.

Auszug aus einem Schreiben des Herrn Professors und Ritters Bessel an den Herausgeber.

Königsberg 1823. Dec. 4.

Zu Rümkers Beobachtungen theile ich Ihnen folgende correspondirende hier gemachte mit:

AR. appar. 1822 Juni 29. Mond IR. ... 14 46 24,32 . 0,08 nach d. Culm. Aug 29. 4 Capricorni 20 7 37,86 XX. 76 10 14,22 833 Mayeri ... 14 51,81 Mond IR. 24 3,18 . 0,15 nach -Oct. 26. 3 Piscium 22 51 34,35 Mond IR. 59 17,60 . 0,33 nach XXIII. 17 23 5 1,15 96 Aquarii ... 10 14,27 19 Piscium ... 23 27 22,09 22 ----42 55,55 46 6,02 Mond IR. ... 50 5,64 0,38 nach 1823 März 20. VI. 168 6 27 30,15 & Geminorum 33 4,30

44 27,30

37 Geminor.

1823 Mürz 20. Mond IR. ... 6 41 58,04 . 0,40 vor d. Culm.

— 21. Mond IR. ... 7 54 27,02 . 0,37 vor

VIII. 42. 8 10 3,18

344 Mayeri ... 21 33,94

Bei der Mondsbeobachtung vom 20^{sten} März bemerke ich, daß sie zwar an sich gut, aber durch einen Fehler im Zählen der Secunden wahrscheinlich entstellt ist, worüber die Rechnung aufklären muß. Vielleicht sind diese Beobachtungen mit des Herrn Rümkers so zu vergleichen, daß man sowohl die hier, als die in Paramatta verglichenen Sterne aus Piazzi's Cataloge reducirt und bei den letzteren die constante Differenz anbringt, welche die ersteren wahrscheinlich geben werden. Am 21^{sten} März hat Hr. Rümker α und β Gemin. verglichen; hier wurden zwar diese beiden Sterne nicht observirt, allein die angegebene AR. ist, wegen der genau bekannten Oerter dieser und der übrigen Fundamentalsterne, als unmittelbar aus denselben abgeleitet, anzusehen.

Bessel.