CASSELL'S TECHNICAL MANUALS.

DRAWING FOR METAL PLATE

CASSELL, PETTER, GALPIN & CO.'S

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***** This Set of Models has been added to the List of Examples towards the purchase of which the Science and Art Department allows, in the case of National and similar Schools, a grant of 50 per cent. of the cost, thus enabling Schools to possess these Models for the sum of £1.**

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AUTHOR OF "LINEAR DRAWING" AND "PROJECTION," "DRAWING FOR MACHINISTS," "BUILDING CONSTRUCTION," DRAWING FOR CARPENTERS AND JOINERS," "DRAWING FOR BRICKLAYERS," "DRAWING FOR STONEMASONS," ETC., ETC.

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CONTENTS.

INTRODUCTION.

The general principles on which these manuals have been designed have already been fully set forth in other volumes of the series, some of which, it is hoped, have reached the hands of the readers of the present one.

Although each book may be devoted to the drawing specially required in any one branch of industry, it must not be supposed that the elementary principles which are common to all can be repeated in every volume; this would leave only small space for the subject immediately under consideration, and would interfere with the size and portability of the books—matters of no small importance to working men.

The metal plate worker should in the first place make himself thoroughly acquainted with Practical Geometry, so as to be able to strike the various geometrical figures with ease, correctness, and rapidity.

It is painful to see the attempts made by some of our working men, who have not had an opportunity of being instructed, to draw even simple polygons, and the increased difficulty under which they labour when the figure is to be of a certain size. The general spread of drawing as a subject of instruction in our primary schools is doing much towards altering such a state of things; and it is with the view of assisting men to whom a knowledge of how to work better and quicker is practically an addition to their wages, that the lessons in this course are put forth.

The reader is therefore advised, when he has mastered the mere combinations of lines and the construction of polygons, ellipses, &c., to follow up the method of converting geometrical figures into others of equal area. He should in every case work by fixed measurements. Thus the moment he can construct (say) a pentagon, he should draw the figure on a given side, or in a given circle. The author speaks from experience when he states that want of accuracy is a great fault with many of our artisans, whose work is otherwise excellent.

As a test of accuracy, the following exercise is suggested :—Draw any polygon, say a nonagon, on a strong piece of cardboard, and cut it out, holding the penknife upright, so that the edges of the aperture in the cardboard may be as upright and as cleanly cut as possible. Next, construct several nonagons, of the same size; cut them out, and test whether they fit exactly into the aperture in the gauge-plate, turning them round so that the angles *a*, *b*, *c*, &c., of the polygon may fit into *d*, *e*, *f*, &c., in the plate; for of course, if both have been correctly constructed, the figure and aperture should agree in every position.

As the object of this is to attain the power of accurate measurement and construction, the compasses should be altered after each of the polygons has been constructed, and the measurements should each time be taken direct from the rule.

In addition to the figures contained in "Linear Drawing," others, involving applications of them and demanding increased accuracy, are given in the present course; and thus prepared, the student will be enabled to take up "Projection" with interest and intelligence.

In order that the reader may not be absolutely compelled to provide himself with the manual on projection, and that each volume may be as far as possible complete in itself, the elements of the subject are here given, and thus he is led on in an uninterrupted course to studies immediately concerning the work of the metal plate worker. The extent of the subject is, however, such as wholly to preclude anything but a very brief glance at projection as a whole being given, and the student is therefore urged to follow up his lessons from the manual devoted to this important branch.

How much altering, hacking, cutting, and filing would be avoided; how much labour and time, and consequently how much money would be spared, if workmen understood the principles on which their work should be done. Fig. 3 in Plate XX. is an instance in which it will be seen that by means of two correctly drawn curves the piece of metal may be cut so as to form a double elbow-pipe without any waste whatever; and this will become more and more evident as the lessons advance.

Workmen are often heard to say they "don't know how it is, but the work *would not come* right." Thanks to the now rapidly spreading system of instruction, they are learning that work *must* come right *if rightly done*, and that the form, however complex, *must* be the result of correct and accurate construction.

It is in order to carry on this instruction that so much stress is in the present volume laid on "Development;" but drawing on paper is one thing, practical application is another; and the student is earnestly advised not to rest satisfied with the drawing only, but to cut out the developments, and by folding them, really make the objects, the form of which he has been studying.

It is very important that this should be done, for some of the developments are, when flat, so very unlike the objects they are intended to form, that unless they are made up the student is often left unconvinced as to their correctness; but "seeing is believing," and by adopting the plan suggested, it would at once be proved that the tapering piece or elbow-pipe fits exactly when correctly designed, or the faulty point would be discovered.

These developments, too, when joined at the seams by a little gum or glue, become most useful objects for drawing by hand, and the lessons in "Model Drawing" will prepare the student for this practice. The whole subject of model drawing, with developments of the various models, is given in "Object Drawing," one of the Technical Series, and an elementary course of lessons is given in the present manual.

In the studies designed for the instruction of the pupil in this branch, however, only very few rules can be given. Object drawing is a freehand application of the principles of perspective, and thus the sooner any one who would really draw well applies himself to that study the better.

One branch only now remains to be touched on—Freehand Drawing—and on this subject a few elementary lessons are given in the course. But the student must not be under the impression that by the term "Freehand" ornamental drawing *only* is meant.

The power of drawing by freehand, combined with a knowledge of the principles of object drawing, should enable a man to delineate with rapidity and correctness any of the objects around, and not only the objects as he sees them, but as he knows they would appear even if not immediately before him.

There is, therefore, no necessity to wait or to seek for "copies" for freehand and object drawing, since numberless studies present themselves in whichever direction we turn.

With these few hints, then, the present volume is dedicated to working men, in the hope that they will find in it some useful instruction which may assist them in their work, and which may elevate them in the intellectual scale.

ELLIS A. DAVIDSON.

London, 1872.

DRAWING FOR METAL PLATE WORKERS.

PRACTICAL GEOMETRY.

In order that the student may be able to draw the various forms required in "development"—that is, the covering of surfaces—which forms such an important portion of the trade of the metal plate worker, it is necessary that he should acquire a sound knowledge of the geometrical methods by which the various figures may be constructed with correctness and accuracy.

In "Linear Drawing," the first volume of the Technical Series, an important system of lessons is worked out. To this volume, therefore, the student is referred for all elementary constructions, and the following figures are added as being specially useful in the branch of trade to which the present manual is devoted.

Fig. 16, then, in "Linear Drawing" shows the method of drawing a circle through any three points, and Fig. 33 teaches how to find the centre of a circle. In Fig. 1 in the following plate the instruction is carried further.

PLATE I.

FIG. 1.—**To draw a curve which shall be a portion of a circle when the centre is not available.**

Let A B be the *chord* of the arc, and D C its rise.

From A and B as centres, with the radius A B, describe the arcs A E and B F.

From A draw a line through C, cutting the arc B F in G.

From B draw a line through C, cutting the arc A E in H.

Divide A H and B G into any number of equal parts, as 1, 2, 3, 4, 5, and set off a number of these parts from G and H, as I, II, III, IV, V.

Draw lines from A to 1, 2, 3, 4, 5, and from B to I, II, III, IV, V.

Then it will be seen that the first line above H, viz. I., intersects the first line below G, viz. 1, in the point X.

Again, the line 2 will intersect II, line 3 will intersect III, and lines 4 and 5 will cut IV and V.

Proceed in the same manner on the opposite side, and through the intersections trace the curve by hand.

For inking, a "templet" may be made, and as this plan will be recommended in several other cases, the mode of making this useful article is given.

Draw the figure accurately on a smooth piece of veneer; if of a light colour so much the better, or a small quantity of veneer may be kept by you with thin white paper glued over it. Cut out the form *near* the curve required, and bring it exactly up to the mark by means of a fine file—a half-round one is best for this, as it enables you to finish concave as well as convex surfaces.

The final smoothing is then to be done with fine glass-paper, and in this process the edges should be very slightly bevilled off, in order to prevent the ink dragging on the paper.

Sets of curves of different radii, and "French curves" of various forms, may be purchased, and although these will be found very useful in their way, the above hints are given as it is deemed advisable to promote self-help as much as possible.

The student will remember that no portion of a true ellipse is a part of a circle, and the curve cannot be drawn with compasses so as to be mathematically correct; but there are many ways in which figures, nearly approximating to ellipses may be drawn by arcs of circles, which are very useful for general practical purposes.

In mechanical drawing, therefore, such figures are used, and have the advantage that they can be drawn by means of compasses instead of by hand.

The following method is given, in addition to Figs. 86, 87, and 88 in "Linear Drawing."

FIG. 2.—**To construct an elliptical figure by means of arcs of circles.**

Place the two diameters A B and C D at right angles, and intersecting each other at their middle point E.

From B on the line A B set off B F equal to B C.

From E on E C set off E G equal to F.

Draw G F and bisect it in I.

From F set off F J, equal to F I.

Draw J K parallel to G F.

From E set off E L and E M equal to E J.

Complete the square J K L M and produce the sides beyond J and L. The angles of the square are the centres from which the elliptical figure may be drawn.

From K and M, with radius K D or M C, describe arcs cutting the produced sides of the square in N O and P Q.

From J and L, with radius L A or J B, describe arcs joining N P and O Q, which will complete the figure.

FIG. 3.—**To bisect the space contained between two lines, A and B, inclined to each other, when the point at which they would meet is inaccessible.**

NOTE—Fig. 15*a* in "Linear Drawing" shows how to bisect the angle which would be formed if the lines met, and the application of that figure and the above is shown in Fig. 120 of the same volume.

At any part of each line erect equal perpendiculars, as C E and D F, and from their extremities draw lines parallel to A and B, intersecting in G.

Bisect the angle E G F, and the line G H will bisect the space contained between the lines A and B.

FIG. 4.—**To describe a circle touching two given circles, A and B, and one of them in a given point of contact, C.**

Join the centres D and E.

Draw a line from C passing through D, and produce it.

At E draw E F parallel to D C.

Draw C F parallel to E D, and produce it to G.

Draw G E and produce it until it intersects C D produced in H.

From H, with radius H C, describe the required circle, which will touch both the circles A and B, and one of them in the given point C.

PLATE II.

FIG. 1.—**To divide a circle into any number of equal parts.**

The following constructions, which require the compasses only, are best made with the steel dividers, and if

two or three pairs can be employed, the distances (such as the radius of the circle) often required, can be kept unaltered.

With the given radius describe the circle, and divide it into **six** parts, in **B, C, D, E, F, G.** B E is a diameter, and therefore divides it into two equal parts. B D is a chord of ⅔ or ⅓, and the circle is divided in B, D, E into **three** equal parts.

From each end of the diameter B E with the chord of B D, or C E, describe arcs intersecting in X.* Then the distance A X being set off from B and E, the circumference will be divided into **four** parts, H, B, I, E.

The arcs described with the radius A B from X X as centres will cut the circumference in **K, L, M, and N,** which points bisect the quadrants B H, H E, E I, and I B, and thus divide the circle into **eight** equal parts.

The radius A B, set off from H I to **O P Q R,** bisects the arcs B C, D E, E F, and G B, which completes the trisection of each quadrant, and therefore divides the circle into **twelve** equal parts.

The radius A B set off from K, L, M, and N, both ways, from each point, will bisect the two arcs on each side of the extremities of the diameters B E, I H, in **S, Y, U, Y, T, W, Z,** &c., and thus complete the division of the circle into **twenty-four** equal parts. Any further subdivision may either be done by bisecting the arcs already formed, or by trial. Thus each of the twenty-four parts being bisected, the circle will be divided into **forty-eight** parts.

All the foregoing instructions by which one circum-

* In all these constructions, in order to ensure greater accuracy, the arc should be described on both sides of the line joining the centres, thus the point x should be found on both sides of the diameter A B E.

ference is divided into twenty-four parts, are performed, it will be seen, by **three distances only**—the radius A B, the chord D B, and A X—consequently, if these be kept unaltered in separate pairs of dividers, the operations are performed with the greatest accuracy.

FIG. 2.—In order to avoid confusion, the continuation of this problem is given in a separate figure. With the distance A X as a radius, describe from O, P, Q, R (these points having been found as in the last figure) arcs intersecting in Y. Then the distance B Y or E Y will divide the circumference into **five** equal parts in **B, C, D, F,** and **G,** the distance A Y will bisect the arcs B C, C D, D F, &c., in **H, I, E, K, L,** and thus divide the circle into **ten** parts.

The distance B Y set off from S T (the extremities of the diameter) perpendicular to B E will bisect the arcs D E, B H, E F, B L, in the points **U, V, W, Z,** and will thus give one-twentieth of the circumference. The same distance being set off from these points will bisect the other arcs of the decagon. The division into forty parts may be effected by bisecting the arcs last found. These methods, when once clearly understood, are extremely useful in the rapid construction of regular polygons. Of course, no more of the figure need be worked than is necessary for the immediate purpose.

FIG. 3.—**To join two lines, A B, inclined to each other, by an arc of a circle.**

Produce A and B until they meet in C.

Bisect the angle A C B.

At D, the extremity of one of the lines, erect a perpendicular cutting the bisector in E.

From E, with radius E D, describe the arc which will meet A in F.

If the point C is not accessible, the angle must be bisected by the method shown in Plate I., Fig 3.

It will be seen that the above is an application of Fig. 118 in "Linear Drawing."

FIG. 4.—**To draw a circle touching another circle in a given point, and passing through a given point lying without the circle.**

Let A be the point of contact in the given circle, and B the point lying without it.

The centre of the required circle will evidently lie on the radius O A produced, and on a perpendicular at the middle of a line joining A B, which line will be a chord of the required circle.

Therefore, produce O A, draw a line from A to B, and bisect it in C.

Produce the bisecting line until it cuts O A produced in D, which is the centre of the required circle, D A being the radius.

PLATE III.

The methods of inscribing circles in triangles and squares has been given in "Linear Drawing," and those constructions are now supplemented by the following figures.

FIG. 1.—**To inscribe within a given equilateral triangle three equal semicircles, having their diameters adjacent and equal.**

Let A B C be the equilateral triangle.

Bisect the angles of the triangle by lines A *a*, B *b*, and C *c*.

Join A *b*, and on this line describe a semicircle touching the sides of the triangle.

To avoid confusion of lines, this semicircle is omitted in the figure, the point only (*d*) where it would cut *b a* being shown.

From *d* draw a line parallel to *c* A, cutting *b* B in *e*; from *e* draw a line parallel to *b a*, cutting A *a* in *f*.

Draw *f g* parallel to C A.

Join *g e* by a line parallel to B C.

Then *e f*, *f g*, and *g e* will be the adjacent diameters of three semicircles, the curves of which will touch the triangle A B C.

FIG. 2.—**To inscribe in a given circle three equal semicircles having their diameters adjacent.**

Find the centre of the circle A.

Draw the diameter B C, and from B and C set off the radius of the circle, thus dividing it into six equal parts in E G, F D.

Draw E F and G D.

Draw the radius A H at right angles to B C.

From F set off F I equal to F H, thus trisecting the quadrant H B in F I.

From I draw a line to G cutting E F in J.

From A set off A K and A L equal to A J.

Join J K, K L, and L J, which will give the adjacent diameters of the three required semicircles, the centres of which will be at M, N, O.

PLATE IV.

To inscribe within a circle four equal semicircles, having their diameters adjacent.

FIG. 1.—Draw two diameters, A B and C D, at right angles to each other, intersecting in O.

Bisect the quadrants in E F, G H.

Bisect O B in I.

Draw a line through I parallel to C D, cutting E F and G H in J and K.

Draw K L and J M parallel to A B, and join M L. Then the sides of the square J, K, L, M will be the adjacent diameters of the required semicircles, of which I, N, P, Q will be the centres, and I B the radius.

FIG. 2.—**Within a given square to inscribe four equal semicircles, each touching one side of the square and having their diameters adjacent.**

Let A B C D be the given square.

Draw the diagonals A C and B D intersecting in O.

Through O draw E F parallel to B C, and G H parallel to A B, thus dividing A B C D into four smaller squares, each having one diagonal.

Draw the second diagonal in each of these smaller squares, cutting A O, B O, C O, and D O, in I, J, K, and L.

Draw I J, J K, K L and L I, thus constructing the square, the sides of which will be the four adjacent diameters of the semicircles, which may then be drawn from M, N, P, Q, with radius M F.

PLATE V.

FIG. 1.—**Within a given square to inscribe four equal semicircles, each touching TWO sides of the square, and having their diameters adjacent.**

Let A B C D be the given square.

Draw the diagonals A D and B C, intersecting in O.

Through O draw E F and G H parallel to A B and A C.

Bisect E B in I, and D H in J.

Draw I J, cutting G H in L.

From O, set off on the diagonals O M, O N, and O P, equal to O L.

Join L M, M N, N P, P L, and a square will be constructed, the sides of which will be parallel to the diagonals of the original square, which will intersect the sides of the square L M N P in Q R S T. These points will be the centres for the required semicircles, the radius of which will be Q L, &c.

FIG. 2.—**To describe a regular polygon about a given circle.**

The required polygon is in this case a Hexagon.

Divide the circle into six equal parts (this number will of course vary with the required number of sides of the polygon) and draw radii A, B, C, D, E, F, producing them beyond the circle.

Bisect the angles thus formed in G, H, I, J, K, L.

Draw tangents to each of these radii, which, meeting in the produced radii A, B, C, D, E, and F, will form the required polygon about the circle.

The method of drawing a tangent at a given point is shown in "Linear Drawing," Fig. 49. It is only necessary to go through this operation once in the present figure.

Thus, the tangent drawn at G will cut the two radii O A and O B in M and N.

From O, with radius O M or O N, describe a circle cutting the radii O C, O D, O E, and O F in P, Q, R, S. Join these points, and a hexagon will be constructed about the circle.

PLATE VI.

FIG. 1.—The subject of this study is a section of a brass sash-bar of a very simple form, given as an application of geometrical drawing.

Draw the perpendicular A and the horizontal B C.

Next draw the horizontals D E and F G, and terminate them by the perpendiculars D F and E G.

Draw the horizontal line H I, setting off H I at equal distances from the central perpendicular.

From the centre draw the outer circle meeting the horizontal in H I.

From F and H and G and I, with radius F J, describe arcs cutting each other in J and K.

From J and K draw the arcs H F and I G, and from A set off on B C the distances L and M.

Draw D L and E M.

Draw the horizontal N O, and the arcs L N and O M, which will complete the external form.

The inner line is parallel to the outer one, and is drawn in the same manner.

The section lines are, of course, drawn by the aid of the set-square of 45°.

FIG. 2.—This is another exercise of accurate drawing similar in character to the last.

Set out the horizontal lines as in the previous figure, and find the points A, B, C, D.

Draw A B and C D, and bisect these lines in E and F.

From A and E describe arcs cutting each other in G, and from E and B describe arcs cutting each other in H.

From H describe the arc E B, and from G describe the arc E A, carefully observing that these arcs must meet with the utmost accuracy without overlapping. This practice has been shown in the construction of the Cyma Recta moulding in "Drawing for Stonemasons."

PROJECTION.

WE now proceed to consider the methods of representing solid objects—viz., such as possess length, breadth, and thickness. By the term "solid," however, we mean such as are *apparently* so. Thus, by a cube, we do not necessarily mean a block of stone or wood, but possibly an object of the same form made of sheet metal, and therefore *appearing* solid.

Whether an object is solid or hollow, and if the latter, what may be the thickness of the material of which it is made, would be shown in a section or cutting.

PLATE VII.

FIG. 1.—If we place two planes or surfaces at right angles to each other, to form as it were a floor and a wall, the floor, A B, would, in projection, be called the horizontal and the wall, C D, the vertical plane.

The Projection of Lines.

No. 1.—Let us take a piece of wire and fix it in an upright position, *a b*, then the point on which the wire rests is called the horizontal projection, or *plan*, and if we were to carry lines directly back from its extremities until they cut the vertical plane in *c* and *d*, the line C D would be the vertical projection or elevation of the wire.

No. 2.—If a wire, *e f*, be fixed at right angles to the vertical plane, the point *f* in which it is fixed is the *elevation*, being the view which would be obtained if the model were placed on an exact level with the eye, the point *e* being immediately opposite the spectator, so that the end only of the wire could be seen. If now perpendiculars are dropped from *e* and *f*, until they meet the horizontal plane in *g* and *h*, the line uniting *g* and *h* will be the *plan* of the wire, or the view obtained by looking down on it. Further, if we suppose a line, *i j*, No. 3, to be suspended in space, perpendiculars dropped from its extremities to cut the horizontal plane, will give the *plan*, *k l*. Then, if the lines be drawn from *k* and *l* to meet the vertical plane in *m* and *n*, and perpendiculars be raised from these points, intersected by lines drawn from the ends of the wire parallel to *k m* and *l n*, the points *o* and *p* will be obtained, and the line joining these will be the *elcvation* of the wire, *i j*.

In the model used for illustrating this lesson, the vertical and horizontal planes are connected by hinges and, are kept at right angles to each other by means of a brass loop. If now the wires be removed and the pin *r* be withdrawn so as to allow the plane C D to fall backward, the two planes of projection will form one surface separated only by the line I L, Fig. 2, and the plans and elevations will be seen in their respective positions on the two planes.

The line separating the two planes is called the *intersecting line*, and is therefore lettered I L in these lessons.

It must be pointed out that the "plan" of an object does not mean merely the piece of ground it stands upon, but the space it overhangs as well. Thus, the piece of ground on which the small lodge (Fig. 1) in the next plate would stand is represented by the dotted square in the plan, whilst the true space which the building *covers* or *overhangs* is represented by the outer square.

It will be seen that in all the figures shown in Plate VII. the lengths of the plans and the heights of the elevations are the same as the lengths and heights of the objects they represent—thus *c d* is the same length as *a b*, and *k l* and *o p* are the same length as *i j*.

But plans are not always the size, nor are elevations always the full height of the object, both being dependent on the position or angle in which the subject to be drawn is placed.

Before proceeding to treat of the changes which lines undergo by alteration of position, it is necessary that the terms used to define such positions should be understood, and for this purpose we again refer to the plate.

Here we have the line *a b* standing *upright* on the floor of the model, and as its distance from the wall is the same throughout its entire length, it is said to be at right angles to the horizontal and parallel to the vertical plane.

In No. 2 the line *e f* is said to be at right angles to the vertical and parallel to the horizontal plane; and it is evident that the line *i j* (No. 3) is parallel to both planes.

It will be seen that whilst the plan of a line when standing upright is a mere point (*a*, Fig. 2), the plan of the same line when placed horizontally—as *k l*—is the full length of the original. The figures in the next plate will account for this difference, and will show how the length of a line is dependent on the angle at which the line is inclined.

PLATE VIII.

FIG. 2.—Let the position of the wire be perfectly upright. Then its plan will be the point *a*, and its elevation the line *b c*.

Now, if this wire be made to work on a hinge joint at *b*, and if the end *c* be moved from left to right, as from *c* to *d*, the end *d* being kept the same distance from the wall of the model, the wire will still be parallel to the vertical, but *inclined* to the horizontal plane. In the present example it is inclined at 60°, but it may, of course, be inclined at any angle.

To find the plan of this wire, draw a line from *a* parallel to I L. From *d* drop a perpendicular to cut this line, then *a e* is the plan of *b d* in the position in which it is now placed—viz., parallel to the vertical and inclined at 60° to the horizontal plane; and if the movement of the wire were continued until it reached *f*, it would then be parallel to both planes. The plan would be the same line extended to *g*.

The line *b d* is said to be placed at a *simple* angle, because it is inclined to one plane, but remains parallel to the other.

Let us now suppose the wire fixed in this slanting position as far as its inclination to the horizontal plane is concerned; but if the whole hinge is made to rotate on a pivot, so that without altering the slant the end *d* may be turned forward, the line will then be at a *compound* angle—that is, it will be inclined or slanting to both planes.

Now it will be remembered that although we have turned the wire round, we have not altered its slant to the horizontal plane; it will therefore overhang a piece of ground exactly the same shape and size as it did in Fig. 2; but the *position of that space will be changed.* Let us now assume that in addition to the wire being inclined at 60° to the horizontal, it is required to slant at 45° to the vertical plane. Place the plan *h i*, Fig. 3 (equal to *a e*, Fig. 2), at 45° to the intersecting line, and draw perpendiculars from its extremities. The line *h* will cut the intersecting line in *j*, and will give the base of the line. To find its height we must remember that, although we turned the wire round, we did not alter its slant, and therefore the height of the end *d* remains the same as it was; so that a horizontal line being drawn from *d* (Fig. 2) to meet the perpendicular drawn from *i*, in the point *k*, the line *j k* will be the *projection* of the wire inclined to both planes at the required angles, and it will be seen that in this case both plan and elevation are shorter than the line itself.

To find the real length of a line when it is inclined to both planes, and its plan, *h i*, and the height of the end are given.

Draw a line, *i l*, at right angles to *h i*, and make it equal to the given height. Then a line drawn from *h* to *l will be the real length.* For the plan, the original line and a perpendicular dropped from its extremity form a right-angled triangle, and this triangle, instead of standing upright, as in Fig. 2, is shown horizontally in Fig. 3, and the line *h l* will thus be found to be of the same length as *b d.* This may be illustrated by holding a set-square vertically, and rotating it on its edge until it lies flat on the horizontal plane. The real length of the long edge

(or hypothenuse), which, when the set-square was vertical, was represented by its plan, *h i*, will then become visible.

FIG. 4.—Again, it has been shown that if a wire were fixed at O, at right angles to the vertical, and parallel to the horizontal plane, its plan would be *m n* and its elevation the point *o*, and if it were rotated on the point *n* until it became parallel to I L, its plan would be *n p* and its elevation *o q*; but on the principle shown in Figs. 2 and 3 it will be evident that, if the wire be rotated only as far as *r*, the elevation will be the line *o s*.

PLATE IX.

The Projection of Planes and Surfaces.

The same laws which govern the projection of single lines, govern also the delineation of planes, which are flat surfaces bounded by lines. Let *a*, *b*, *c*, *d* (Fig. 1) be a metal plate, the surface of which is parallel to the vertical and perpendicular to the horizontal plane. Its *plan* will then be the line $a' b'$.

If, now, this plane be turned, so as to be at right angles to both planes, its plan—that is, the line on which it would stand—will be $a' b'$ (Fig. 2), and its elevation the line $a'' c''$, or the view obtained when looking straight at the long edge.

Now, let this plane rotate on the line $a'' c''$ as a door on its hinges, until the plan reaches b'', then a perpendicular drawn from b' will give the rectangle $a'' c'' b''' d'''$, which will be the projection of the plane, when perpendicular to the horizontal and inclined to the vertical plane, the height remaining unaltered. The other rectangles show the projections of the plane when further rotated.

FIG. 3.—In this figure the plane again rests on *a b*, its edge, *b d*, only being visible in the elevation ; but this edge hides the opposite one, which is parallel to it, and therefore the points *a* and *c''* are immediately at the back of, or "beyond," *b d*. Let us now rotate the plane on *a b*, as in closing a box-lid or trap-door. Then the plan of the plane will be the rectangle *a b c'' d''*, and the more the plane is lowered the longer the plan will become, as shown at *e* and *f*.

Notwithstanding the slanting direction which the plane has assumed in relation to the horizontal plane, it still remains at right angles to the horizontal plane. This is shown in the plan, where the lines *a b* and *c'' d''*, which represent the upper and lower edges of the plane, are perpendicular to I L. Let us now place the plane at a compound angle. This will be done by rotating the plan, *carefully lettered* as in Fig. 3. Then perpendiculars drawn from each of the points intersected by horizontal lines from the corresponding points in the elevation will give the required projection.

The process is so plainly shown in the elevation that further explanation is deemed unnecessary. The student is urgently recommended not to be content with simply copying the diagrams herein given, these being merely intended to illustrate principles, and unless these principles be applied nothing will be gained. He is therefore advised to vary the form of the plane and to project it at various angles.

PLATE X.

FIG. 1.—A B C D is the plan of a cube, and E F G H is the elevation, which in the present position of the cube

is the same shape as the plan. For it will be evident that since the cube consists entirely of squares at right angles to each other, the piece of ground the object stands upon must be a square, and also that the side standing on A B is parallel to the vertical plane, and hence its elevation is the square E F G H.

FIG. 2.—In this study the object has been rotated so that one angle, A, faces the spectator, and thus in the elevation two sides are visible, neither of them, however, appearing of its real width, whilst the height remains unchanged.

Fig. 3 is the plan of a triangular prism, the one face of which is turned towards the spectator, and thus as it is parallel to the vertical plane, the elevation is simply a rectangle.

Fig. 4 shows the same prism when turned round, so that the face parallel to the vertical plane is at the back, and one edge facing the spectator; the elevation is thus two rectangles, the boundary line, however, being the outline of the rectangle, which is parallel to the vertical plane.

PLATE XI.

It has already been said that an object apparently solid is not necessarily so, and it is a most important part of the work of the metal plate worker to know the exact shapes which metal is to be cut, so that when riveted, bent, or otherwise united, the object may be formed.

This portion of the subject is called development of surfaces.

We will in the first instance show the method of developing a cube.

Construct the square *a b c d*, representing the base or any one side of the cube. Produce the sides of this square to *e*, *f*, *g*, *h*, *i*, *j*, *k*, *l*, and construct the squares *a e f b*, *b g h d*, *d l k c*, and *c j i a*.

At *k l* (or any other of the external lines) construct the square *k l m n*, which will complete the development, for it will be seen that if the squares *a e f b*, *b g h d*, *d l k c*, and *c j i a* were bent upwards, they would form four walls of a box, of which *a b c d* would be the bottom, and that the square *k l m n* would fold over and form the top or lid, parallel to *a b c d*.

FIG. 2.—In this figure is given the method of developing a triangular prism.

It is clear that this object consists of three rectangular faces and two triangular ends. Therefore—

Draw the three rectangles A B C D, C D E F, and E G H F, and at the ends of one of them draw the equilateral triangles A C J and B D I, which will complete the required figure.

PLATE XII.

Fig. 1 is the plan and elevation of a square prism, projected as shown in Fig. 2, Plate X.

It is required to find the true shape of a section or cutting caused by a plane passing through the prism in the direction of the line *a d*.

This plane of section would cut through the diagonal *a c* of the top and the angle *d* of the bottom.

Draw the dotted lines *a c* and *d′ d′* at right angles to the line of section, and at any part draw *d′ e* parallel to *a d*.

Now it will be evident that this will be the greatest

length of the section, and that the width will be *somewhere* on each side of *e*; but where? How *wide* will the section be? These are questions which must of course arise when the work has proceeded thus far, and these points will, it is hoped, be rendered clear by the following explanation :—

The section line, in passing through *a c*, cuts the object in the widest part. Therefore if the eye be carried down from *a* in the elevation to *a c* in the plan, it will be seen that the real width on each side of the centre *c* is *e a* and *e c*. Therefore, if these lengths be set off on each side of *e* in the section line, and the points joined to *d'*, then *a c d'* will be the true section.

In Fig. 2 the section plane passes from one angle of the top to the opposite angle of the bottom, cutting through the middle of the two edges. The length will of course be equal to that of the section line, and the width across the middle will be equal to the diagonal of the square, as at *a c* in Fig. 1 ; and from this width the figure narrows to a point both at top and bottom.

FIG. 3.—Here the section plane passes from a line connecting the middle points of the two adjacent edges of the top to a similar line on the two opposite edges of the base. The width of the section at its middle will be equal to the diagonal *a c*, and at the top it will be equal to *g h*.

It is usual to cover sections with lines at 45° to their central line.

PLATE XIII.

In Fig. 1 of this plate is given the development of the square prism which forms the subject of the last lesson.

The mere development will of course be easily understood, since it is precisely the same as that of a cube, four of the faces being elongated instead of being square.

The object of the present lesson, however, is to show the section line on the development, thus enabling the workman to cut the metal out whilst flat, so that when bent, not only the general shape of the object may be formed, but the aperture caused by the section, and the covering of that aperture (which will be the true section), may be the result.

It will be remembered that the section plane enters at the top, on the diagonal *a c*.

Draw the diagonal *a c* in the square *a b c d* representing the top of the present development.

Now it will be clear that *a* is one of the points from which the section line is to be drawn, and also that when the development is folded into its proper form, *c′* would reach *c*; *c′* is thus another of the points of the section line, and *a′* will be the third.

Draw *a a′ c′*, which will be the line of section.

The covering for this aperture will then be an isosceles triangle, of which *a c* is the base, and *a a′* and *c′ a′* the sides.

In a material which may be bent, it may be economical to make the whole development in one piece, and therefore the present method is shown.

From *a′*, with radius *a′ c′*, describe an arc, and from *c′*, with radius *a c*, describe an arc cutting the previous arc in *e*.

Join *e a′*, *a′ c′*, and *c′ e*. Then the triangle *c′ e a′* is the required covering of the aperture.

Fig. 2 is another development of the same square prism,

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In a material which may be bent, it may be economical to make the whole development in one piece, and therefore the present method is shown.

From *a′*, with radius *a′ c′*, describe an arc, and from *c′*, with radius *a c*, describe an arc cutting the previous arc in *e*.

Join *e a′*, *a′ c′*, and *c′ e*. Then the triangle *c′ e a′* is the required covering of the aperture.

Fig. 2 is another development of the same square prism,

showing the section line caused by a plane passing from the middle points of two adjacent sides of the top to a line joining two corresponding points in the opposite edges of the base. The projection of this object has already been given in Fig. 3, Plate XII.

Having drawn the general form of the development, draw the diagonal *a b*, and the line *c d* parallel to it, from points in the middle of the sides of the square.

Draw a corresponding line at the opposite angle of the square representing the other end of the prism, viz., *e f.*

From *a'* set off *a' c'* equal to *a' d*, viz., the middle of the width of the face, and find the corresponding points, *g h*, on the edges of the base.

Draw *c' g* passing through *i*, the middle point of the perpendicular, and draw *d h* passing through *j*, and these will be the section lines.

Fig. 3 is the true section as shown in Fig. 3, Plate XII., from which the height must be obtained. The width at top and bottom are of course equal to *c d*, and the width across the middle is equal to the diagonal of the square.

PLATE XIV.

Fig. 1 shows the plan and elevation of a piece of square metal piping, when the plane of the section passes from one side to the other at 45°, the section will thus be a rectangle the length of which is equal to the section line, whilst the width corresponds with that of the side of the pipe.

Now if when such a pipe has been cut through at 45° the upper portion is turned round, rotating as it were on a pin fixed in the centre of the section, the two parts meeting on the line *c d* will form an elbow joint.

In the present example, the joint forms a right angle, because the section line was made at 45° (that is, *half* a right angle) but an elbow of any angle may be formed by the same method, it being understood that the angle of the section line must always be half the angle required to be formed by the joint.

Fig. 2 is a projection of the object when placed at an angle to the vertical plane.

To draw this figure, place the plan at the required angle to the intersecting line, and draw perpendiculars from all its angles.

It has already been pointed out that when an object is rotated horizontally without its inclination being altered, its height remains unchanged. This may be frequently seen in the objects around us; for instance, a door, however much it may be moved on the line of the hinges, remains the same height; the uppermost extremity of a crane, when it is rotated so as to bring a cask or bale which has been raised from a vessel, over the wagon waiting to receive it on shore, remains the same height, describing, in fact, as it moves round, a horizontal circle in the air.

Thus, then, this elbow joint, so rotated, is the same height as when its side was parallel to the vertical plane, as in Fig. 1. It is only necessary, therefore, to draw horizontals from Fig. 1 to cut the perpendiculars drawn from the angles of Fig. 2 to complete the projection.

Fig. 3 is the development with the shape of the section attached.

Set out, in the first place, the four sides of the prism, with the ends, one only of which is shown in the figure.

On this development, draw across one side a horizontal line from *c* in Fig. 1.

From *d* draw a horizontal line, which will give the height of the right-hand face. Join these points by oblique lines, on one of which construct the rectangle representing the true section, and this would form the lid, if one of the parts were formed into a common coal-scuttle, as shown in Fig. 4.

In Fig. 3, it will be seen, there would be no waste of metal. The section line would so divide the plate that each portion would form a part of the pipe, the only difference being that in the part shown in fine lines, the joint or seam will be in one of the edges of the *back*, whilst in the other it will be in the *front*.

PLATE XV.

This plate gives the method of obtaining the development and section line of a square pipe cut so as to form a double elbow, in continuation of the lesson which was illustrated by the last plate.

A B C D is the plan, and E F G H is the elevation of the pipe.

Draw I J and K L, the section lines as seen in the side elevation.

It will be clear that if the middle part, I J K L, were removed, the upper part, G H K L, could be brought down upon E F I J, and form a straight pipe as in the last lesson, and thus that the change is merely caused by rotating the middle piece, I J K L, as on a pin in the centre of each section, the upper and lower parts remaining vertical during the operation. This will be easily understood if a similar model be made of three pieces of wood.

When therefore this middle piece is rotated in the manner described, the lines M and N of Fig. 1 will take the positions of M′ and N′, in Fig. 2, whilst the lower portion, E F I J, will occupy precisely the same position as in Fig. 1, as will also the upper part, K′ L′ G′ H′, excepting that it is moved towards the right side and lowered.

Fig. 3 is the development showing the section line, which being constructed in precisely the same method as Fig. 3 in the last plate, will not require any further explanation.

PLATE XVI.

The principles shown in the two preceding lessons are further developed in the present plate. Here Fig. 1 is the plan and elevation of a hexagonal prism, to be cut off at an angle of 45°.

The method of projecting the elevation from the plan will not require any explanation in this place, and we therefore at once proceed to show the method of obtaining the true form of the section.

Let A B be the section line cutting the edges of the prism in C D.

It will be clear that A B in the elevation is represented by *a b* in the plan, and that C and D represent not only the two points so lettered, but two lying immediately behind them ; thus, C D in the elevation correspond with *c d* in the plan, and it will be seen that *c′* and *d′* lie directly at the back of these.

Thus, then, whilst the length of the section will be equal to A B, its width will be *c c′* or *d d′*.

These points being clearly understood, the execution becomes easy.

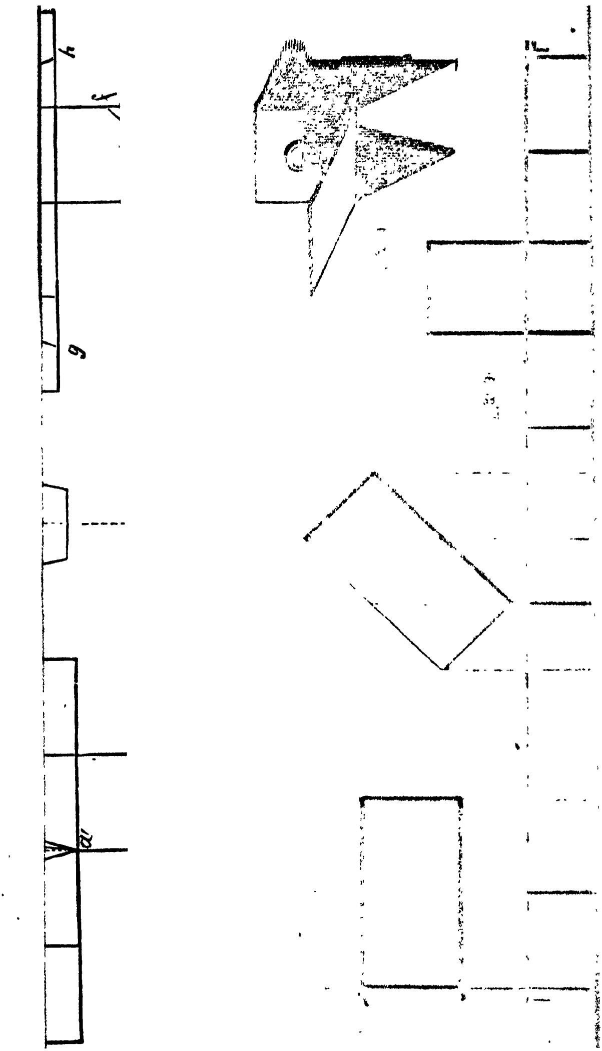
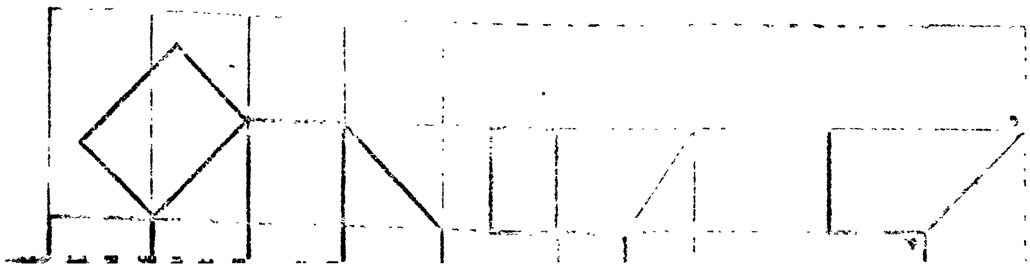
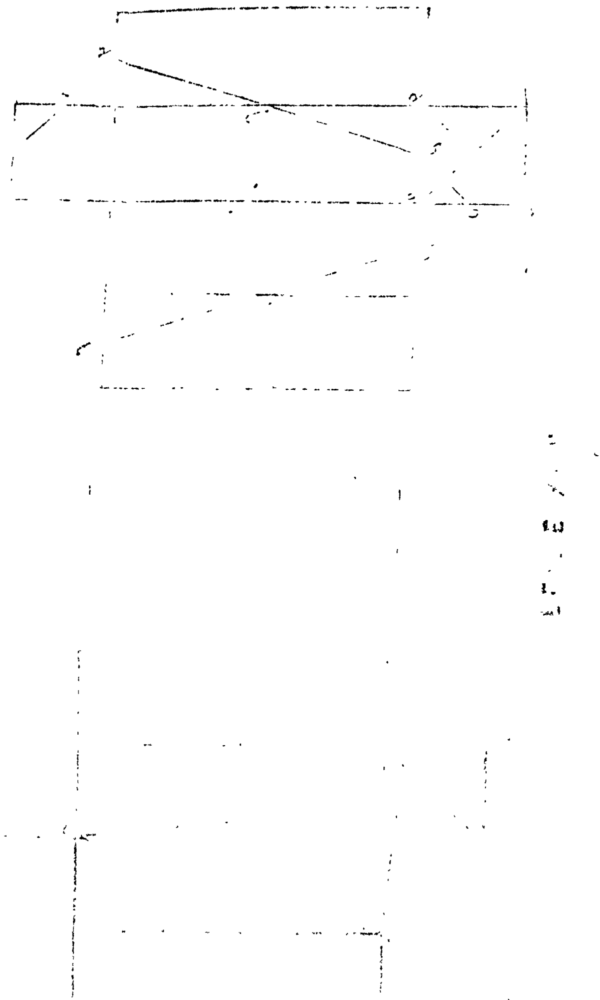


PLATE XIV.





From A and B draw lines at right angles to A B, and draw A′ B parallel to A B. Now, from C and D draw lines at right angles to A B cutting A′ B′ in 1 and 2.

On these lines set off from 1 and 2 the widths corresponding with 3 *c c′* or 4 *d d′* in the plan—viz., 1 *e f* and 2 *g h*.

Join A, *g*, *e*, B, *f*, *h*, A′, and the figure thus completed will be the true section.

It will be evident that if the upper part of the prism were rotated on the lower, as in Plate XIV., the two parts would form an elbow as already shown.

FIG. 2.—In this figure the prism is represented as rotated on its axis, so that one edge faces the spectator, whilst in Fig. 1 the side *c d* was parallel to the vertical plane of projection.

Horizontal lines drawn from A, B, C, D, will give the view of the section in this figure; but it must be understood that this is not the *true* section, the form being apparently shorter, owing to the position of the section which, of course, slants backward from the lower point.

Fig. 3 is the development of the prism, and on this the section line is shown. This is, as before, obtained by drawing horizontal lines from the points A, B, C, D in the elevation, to cut corresponding perpendiculars in the development.

The parts thus produced will, when folded into shape, be exactly equal.

PLATE XVII.

The lessons given in the previous plate will, it is hoped, be sufficient to enable the student to adapt the system to other prisms, and we therefore devote the present lesson to the study of pyramids.

Let it be required to project a pyramid the base and sides of which are equilateral triangles. Such a solid is called a "Tetrahedron."

FIG. I.—A B C is the plan and *c a* the width of the elevation. The point requiring consideration is, How high will this pyramid be?

Now, although the real shape and size of the side lettered A D C in the plan is similar to *a' c' b'* in the elevation of one side, shown in the equilateral in the left-hand corner of the plate, still the line *e' d'* would only be the height here shown *if the three triangular sides stood upright* on their edges on A B, B C, C A.

But they slant inward until they all meet in a point, which, as all the sides are equal, will be directly over the centre of the plan (point D).

This knowledge enables us to find the exact position of the apex.

Draw a perpendicular from D and another at *a*.

Mark on the perpendicular *a* the real height of *e d*, one side of the pyramid standing upright—viz., *a d'*.

This perpendicular will be the edge elevation of the side.

Then from *a*, with the radius *a a'*, describe an arc cutting the perpendicular drawn from D in *d''*, which will be the apex.

Join *a d''* and *c d''*, which will complete the elevation.

It will, perhaps, at first seem strange that although we *know* the apex to be over the middle of the solid, yet it does not appear so in this elevation. The reason of this is, that the length of *a d''* is the projection of the height

(or altitude) of the triangle, whilst the line *c d* is the projection of the *edge c d*, which it will be seen is longer and slants more than *e d*.

This appears plainly in the plan, where C D is longer than D *a*; yet D is in the centre.

The student is now advised to turn the plan, so that the edge A B may be parallel to the intersecting line, and in the elevation projected from this figure the apex will lie over the middle of the base.

Fig. 2 is the plan and elevation of a square pyramid, when the two edges A B and C D of the base are parallel to the vertical plane.

It will be seen that the position of the apex is found in the same manner as in the last figure, by marking on a perpendicular the real altitude of the side, and then inclining this elevation of the side until it cuts the axis in *e*.

Fig. 3 shows (in fine lines) the plan and elevation of the same pyramid, when one of the diagonals of the plan is parallel to the vertical plane.

Let it be required to draw the plan of the pyramid when resting on one angle, D, the plane of the base being at an angle of 30° with the horizontal plane.

Place the elevation, *a d e*, at the required angle, and project the square which forms the base of the pyramid.

In doing this it will be seen that the square base rests on D, whilst A is raised; thus, the diagonal *d a* of the elevation is in the plan reduced in length to D A′, whilst the other diagonal, *b′ c′*, retains its proper width.

Now produce the diagonal A D, and drop a perpendicular from the apex (*e*) to cut this line in *e.*′

This will give the plan of the apex. Join this point to the points of the plan of the base, and the plan of the solid will be completed.

To find the true shape of the section on the line *g h*, draw lines from *g*, *h*, *i*, at right angles to *g h*, and draw *i k* parallel to that line: *j k* will be the length of the section.

From *g*, *i*, *h*, draw perpendiculars cutting the plan in *l*, *m*, *n*, *o*. Join these points, and a *plan* of the section will be obtained.

On each side of *p'* in the upper figure set off the length *p m* or *p l* of the plan—viz., *p q* and *p r*. Join *j q*, *j r*, *q k*, and *r k*, and the true section will be completed.

To draw the development of this pyramid.

FIG. 4.—With radius equal to one of the *edges* of the pyramid describe an arc.

Draw the radius *e a*. From *a* mark on the arc the lengths *a b*, *b a'*, *a' c* and *c a''*, equal to the sides of the base.

Join these points, and from each of them draw lines to *e*.

On either of the lines, such as *c d*, construct a square for the base of the pyramid, and this will complete the development.

It remains, however, to mark on this development the line of section—that is, the line in which the material is to be cut, so that, when folded, the frustum (that is, the portion remaining of the pyramid when the upper part is cut off) may be formed.

To find this line, mark on *a' e* the length *a g* of the elevation. From *i* (Fig. 3) draw *i i'* parallel to *a d*. From *c* and *b* (Fig. 4) set off on *c e* and *b e* the length *d i'*—viz., *c i''* and *b'' i''*.

On *a″ e* and *a e* (Fig. 4), mark off the length *d″ h*, taken from Fig. 3. Join *h i″*, *i″ g*, *g i″*, and *i′ h*, which will give the line of section required.

PLATE XVIII.

FIG. 1.—A B C D E F is the plan and Fig. 2 is the elevation of a hexagonal pyramid, placed so that one of the edges faces the spectator, two of the sides of the base (B C and E F) being at right angles to the vertical plane of projection.

Fig. 2 is cut by a line parallel to the base, and it is required to find the true section of the pyramid when cut by a plane of which the line G H is the elevation.

Now it will be clear that in this position each of the lines in the elevation represents two edges rising from points similarly lettered in the plan, since in the plan the three distant angles of the hexagon are immediately behind the other three, and thus the lines *b c* O and *f e* O in the elevation represent not only two edges each, but the triangular side of the hexagonal pyramid contained between them—viz., B O C and F O C in the plan.

All sections of this pyramid, parallel to the base, are regular hexagons; therefore a perpendicular drawn from I to cut the plans of the edges E O and F O in *i* and *i′* will give *i i′*, the one side of the true section; and lines drawn from *i* and *i′* parallel to E D and F A will be two more, and thus the whole plan may be completed.

Similarly, Figs. 3 and 4 are the plan and elevation of the same pyramid when rotated on its axis so that two faces are parallel to the vertical plane.

Here the section line is seen cutting the elevation of the *edge* of the pyramid in A, and a perpendicular dropped to cut the plan of the edge in A′ gives the radius of a circle which will contain the true section; from the centre therefore, with radius O A′, describe this circle, and this, cutting the other five radii of the hexagon, will give the hexagon which is the true section, as in the last figure.

We now proceed to draw the development of this pyramid, and here it will be evident that the exact size could not be obtained from the elevation, Fig. 2, for, the length of *a o* is not the true length, because it slants backward; nor is the length *f o* the right length, because the base on which it stands, F O in the plan, is not parallel to the vertical plane, and thus *f o* is in a degree foreshortened.

In the elevation, Fig. 4, however, the line B O is the true length of the edges of the pyramid; therefore, from any point as O′, with radius B O, describe the arc, Fig. 5, and set off on it six distances equal to the side of the hexagon forming the plan. From each of the points draw a line to O′, which will complete the development.

Now on each of these lines set off the length B A taken from Fig. 4. Join the points, and this will give the section line, thus completing the figure.

PLATE XIX.

To draw an ellipse which shall be the true section of a cone on a given line.

If a cone be cut across so that the plane of section may pass through the axis at an angle and cut the slanting surface of the cone on the opposite sides, the section is called an Ellipse.

An ellipse differs from an oval in being the same at both ends; whilst in an oval the one end is more pointed than the other. See "Linear Drawing and Geometry," in which are given various methods for describing the conic sections as plane figures; whilst in "Projection" will be found the methods of obtaining the parabola and hyperbola by means of projection from given sections in cones.

Let Fig. 1, Plate XIX., be the plan and elevation of the cone, and A B the line of section.

Divide the circumference of the plan into any number of equal parts, C, D, E, F, G, H, I, and D′, E′, F′, G′, H′, I, and draw radii.

Project these points on to the base of the elevation of the cone, and from C″, D″, &c., draw lines to the apex J′.

The diagram up to this point represents a cone on the slanting surface of which straight lines have been drawn, which on looking down on the apex would appear as radii of the circle forming the plan.

The line E″ J″ in the elevation is therefore represented by E J in the plan, and thus the plan of any point marked on E″ J″ must fall somewhere on the radius E J.

Now the section line, A B, cuts through all the lines drawn to the apex of the cone in the points *d*, *e*, *f*, *g*, *h*, and it will be remembered that although in the elevation the section is represented by a single line A B, it will assume a different form in the plan.

From points A and B draw perpendiculars cutting the diameter C D in *a* and *b*, and from *d*, *e*, *g*, *h*, in the elevation, draw perpendiculars cutting the radii of the plan which are similarly lettered.

Draw the curve which will unite *e*, *d a*, *a*, *d*, *e*, and also the curve uniting *g*, *h b*, *b*, *h*, *g*. It will at once be seen that these two curves form the ends of an ellipse which is to be the plan of the section, but that points are wanted on F F′ in order to complete the figure.

But we cannot draw a perpendicular from the point *f* in the elevation to cut the radius F in the plan, as we have done in the other lines, *because the radii* F, F′, *are but portions of the same perpendicular on which the point f is situated*, and therefore no intersection could be obtained as before.

Now let us remember that the line F″ J″, though appearing perpendicular to I L when looked at in its present position, would, if looked at from K in the direction of the arrow, be seen to be as much a portion of the slanting surface of the cone as I J′, and therefore the line F J′ would be seen to make the same angle with the horizontal plane as I J′.

If, then, we rotate the cone on its axis, the point *f* will move to *f f*, and a perpendicular drawn from *f f* will give us *j f* in the plan. If now we turn back the cone to its original position (which will be represented by drawing a quadrant from the centre of the plan with radius J *f f*), the quadrant will cut the radius F in *f′* and F′ in *f″*. Join *e* and *g*, and *e′* and *g′* by curves passing through *f* and *f′*, which will complete the plan of the section.

This is not, however, the *true section*, but the view when looking down upon it, and as it is slanting, its length from *a* to *b* will seem shorter than it really is.

It will be evident that the true length of the section is the line A B. From these points then, and also from *d*, *e*, *f*, *g*, *h*, draw lines at right angles to the section line, and A′ B′ parallel to it. On each side of the points *d*, *e*, *f*, *g*, *h*, in the line A′ B, set off the distances which the points similarly lettered are from C I in the plan, and these will give the points through which the true section may be drawn.

To metal plate workers it will be a matter of importance to know the exact curve at which the metal must be cut, so that when formed into a cone it may be truncated—that is, cut off at the required angle.

We will therefore, in the first place, develop the entire cone, and then mark on it the section line.

FIG. 2.—From any point, as J″, draw a line equal to J″ C in the elevation, viz., J′ C″, and with J C″ as radius describe an arc. On each side of C″ set off on this arc the equal distances C, D, E, &c., of the plan (Fig. 1).

Join I I to J, and the sector thus formed will be the development of the cone on which it is now required to trace the line of section.

To do this, draw lines to J″ from the points D D′, E E′, F F′, &c., already marked on the arc I I′.

From the points *d*, *e*, *f*, *g*, *h*, and B in the section line A B, draw lines parallel to the base of the cone, cutting the line C″ J″ in *d*′, *e*′, *f*′, *g*′, *h*′, B′.

Now, from C″, Fig. 2, set off the length C′ A taken from Fig. 1, viz., C″ A′.

On the lines D′ D, Fig. 2, set off the length C *e*′ taken from Fig. 1, viz., E′ *e*.

Proceed thus for the other points, transferring the lengths on C″ J″, Fig. 1, to the lines similarly lettered in Fig. 2, and through the points thus obtained draw the curve, the line in which the material would be cut, so that when I B and I B are brought together, a truncated cone may be formed, the section of which on the line A B will be the ellipse A″ B′.

PLATE XX.

On referring to Plate XIV. the student will be reminded that if a solid be cut across, the parts will, when rotated on a centre, form an elbow—that is, they may be joined so as to turn a corner, or the parts may be turned obliquely to each other, forming an elbow of any required angle.

This principle holds equally good in relation to cylinders.

Fig. 1 is the half-plan, and Fig. 2, A, B, C, D, is the elevation of a cylinder which it is required to cut so that when the parts are joined a double right-angled elbow pipe may be formed. The zinc worker will at once see the usefulness of this study.

It is necessary that the student should again be reminded, that whatever may be the required angle, the *section must be made at half that angle with the axis.*

Thus, if a pipe is to follow two walls which meet at an angle of 120°, each part must be cut at 60°. Therefore, in the present instance, draw the section lines E F and G H at 45° to the axis.

If now, supposing for the time the cylinder to be solid, the middle part be rotated on the pins I and J placed in the centres of the sections, and at right angles to E F and G H, the double elbow will be formed. The point E of the middle portion will move to F, and the point G will move to G′.

The point F of the middle portion will move to E of the lower, and H will move to H′.

In this operation it will be seen that the lower part is not moved at all, and that the upper part merely moves

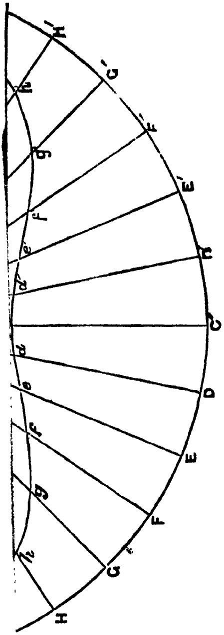


Fig. 2.

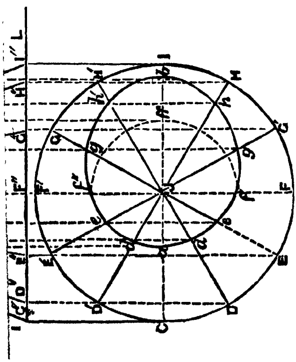


Fig. 1.

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vertically and towards the right hand, but that onlv the middle portion is rotated.

The student is advised to get a cylinder such as this made of wood, cut and pinned as in the example.

With such a model he will easily realise the description ere given, for if the lower portion be held with the left and whilst the right hand grasps the upper, moving it in he required directions, it will be seen that only the middle portion will be rotated.

FIG. 3.—To develop this cylinder, divide the plan into any number of equal parts, as shown at A, *b*, *c*, *d*, *e*, *f*, D in the half-plan, Fig. 1.

Draw a horizontal line, and erect on it the perpendicular A′.

On each side of A′ set off the divisions *b*′, *c*′, *d*′, *e*′, *f*′, D′, and *b*″, *c*″, *d*″, *f*″, D″, taken from Fig. 1.

At D′ and D″ erect perpendiculars, and from B in the elevation draw a horizontal. The rectangle D′ B″ B‴ D″ will then be the development of the entire cylinder.

To trace on this development the lines in which the metal is to be cut so that the three parts may be accurately formed without any waste whatever—

From *b*, *c*, *d*, *e*, *f* in the plan draw perpendiculars cutting the section lines in *b*′, *c*′, *d*′, *e*′, *f*′.

From the points *b*′, *c*′, *d*′, *e*′, *f*′, in Fig. 3, erect perpendiculars, and from the intersections *b*′, *c*′, *d*′, *e*′, *f*′, and E F, in Fig. 2, draw horizontal lines cutting the perpendiculars in Fig. 3 in F′, E′, F″, *b*″, *c*″, *d*″, *e*″, *f*″.

The line drawn through these points will be the curve required.

The upper curve is obtained in precisely the same manner, the horizontals being drawn from the points in the line G H, of Fig. 2.

To find the true section on either of the two lines, draw lines through the points *b*, *c*, *d*, *e*, *f*, at right angles to G H.

On each side of G H set off on these lines lengths equal to those drawn from *b*, *c*, *d*, *e*, *f*, to A D in the half-plan, Fig. 1.

Join these points, and the ellipse thus drawn will be the true section.

PLATE XXI.

To describe the form of a "tapering piece" of piping, to join two pieces of piping, which are both vertical, but not in the same axis, and which are of different diameters.

Let A B C D be a portion of the one pipe, and E F G H the other.

Join B E and C F, and produce the lines until they meet in O; then if O C be produced until it is equal to O B, viz., to I, and I B be joined, it will be evident that O I B is the elevation of a cone placed obliquely on the lower cylinder, and which is cut off at B C and E F.

Now draw any diameter to the cylinder, as J K, and on it describe a semicircle, representing half of the section of the cylinder.

Divide this semicircle into any number of equal parts, viz., L, M, N, P, Q.

Through these points draw perpendiculars cutting the line B C in *l*, *m*, *n*, *p*, *q*, and from *l*, *m*, *n*, *p*, *q* draw lines to O.

Now from O, with radius O *n*, describe an arc, N′ N″, and on this arc set off the lengths into which the semicircle is divided.

From O draw radii through all these points, producing them beyond the arc N′ N″.

From O as a centre, and with O B, O *l*, O *m*, O *p*, O *q*, and O C as radii, describe arcs cutting the radii in Fig. 2 in C′, *q*′, *p*′, *n*′, *m*′, *l*′, and B, &c., and the curve being drawn through these points will give the bottom of the tapering piece.

The upper curve is to be drawn in the same manner, and will be understood from the diagram.

Of Roofs and Domes,

AND THE METHOD OF COVERING THEM.

Although the whole subject of the development of prisms has been treated of in "Projection," it is deemed desirable to enlarge on the subject here in order to show the application of these studies to the zinc worker and plumber.

PLATE XXII.

FIG. 1.—*a b c d* in this plate is the plan of a building to be covered by a hipped roof.

To draw the plan of the roof.

Bisect the angles of the parallelogram, and the bisectors meeting in *e* and *f* will form the plans of the hip-lines, and the line joining *e* and *f* will be the plan of the ridge.

Let it now be required to project the elevation from this plan.

Draw any horizontal, as A B, Fig. 2, and the perpendiculars from *c*, *e*, *f*, *d*, cutting A B in *g*, *h*, *i*, *j*, and produce *h* and *i* indefinitely.

Produce the perpendicular at *e* until it reaches *l*; then it will be clear that *k l* is the width of the roof trusses (at *k l* and *m n*) which would be at right angles to *a b* and *c d*.

Draw *k′ l′*, Fig. 3, equal to *k l* in Fig. 1, and at the middle point, o, draw the perpendicular *o p* equal to the real height of the truss, which is of course a matter dependent on the taste or defined purpose of the architect. This triangle will then be the shape of the truss at this point, and is the section *across* the roof.

Make *h q* and *i r* in Fig. 2 equal to *o p* in Fig. 3. Draw *g q*, *q r*, and *r j*, which will complete the elevation, and this will also be the longitudinal section through the ridge.

We now have to find the real length of the hip. To do this, draw *f s*, Fig. 1, equal to *o p*, Fig. 3, and at right angles to *f d*.

Join *d s*, then the right-angled triangle, *d f s*, is the true shape of the hip truss. This will be understood by cutting a piece of cardboard of the shape described, and placing it on its edge, *d f*. Then it will be seen that *d s* will be the length of the hip.

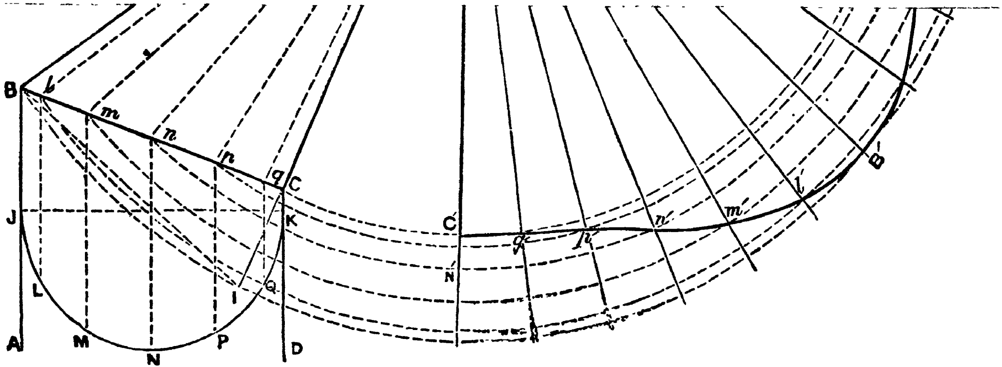
To develop the covering of this roof—a matter of the greatest importance to the plumber and zink worker.

It will, of course, be understood that the surface will consist of four planes, which will meet at the hip lines.

Now it has already been shown that the ends are triangles, of which *a e c* and *b f d* are the plans; the length of lines *a c* and *b d* remains unaltered, but the real length of *c e*, *a e*, *b f*, *d f*, has been proved to be *d s*.

Therefore on *d b* and *a c* construct isosceles triangles, having *d s* for the two remaining sides; these triangles, then, *a t c* and *b n d*, are *the true shape of the coverings of the ends of the roof.*

Now from *c* and *d*, with radius *c t*, describe arcs cutting the perpendiculars *k* and *m* in *v* and *w*. Join *d w*, *v c*,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and *w v*. Then the trapezoid *c v w d* is the development of one of the planes forming the side of the roof covering.

The same length set off on the perpendiculars *l n* will give the points *x y*, which will complete the fourth plane.

We will now proceed to find the form of the hip when the roof is a groined one.

It will be clear that if a spectator stands on the platform of a railway at the side of a semicircular arch by which a road is carried over it, he will then see that, whilst the face or elevation of the arch, where it crosses the railway at right angles is semicircular, its span being of course the diameter of the circle, of which it is half; the length from the springing near which he is standing, to the most distant springing (that is, the one on the opposite of the line at the other end of the arch), will be much longer; yet the arch there is not any higher, although its span, thus taken crosswise, is longer, because the diagonal of a square or other rectangle is longer than any one of its sides.

The principle on which to find the curve which would reach to the springing at which the student is standing, to the one referred to, is also shown in Fig. 4.

On *a b* describe a semicircle, and from the points 1, 2, 3, 4 erect perpendiculars cutting the semicircle in 1′, 2′, 3′, 4′, or mark off *any* divisions in the diagonal, and from them draw perpendiculars to *a b*. Now from the points where the lines 1′, 2′, 3′, 4′, &c., cut *a c* draw lines perpendicular to *a c*; make each of these equal in height to those correspondingly lettered in the semicircle, and the curve drawn through their extremities will be the form required.

This study has already been fully worked out at Fig. 85 in "Building Construction," to which the student is referred.

It is, however, desirable for the continuity of the present branch of our subject, that we should in this place repeat Fig. 88 and its elucidation, desiring as we do that each volume of the Technical Series should as far as possible be complete in itself; and as the business of the zinc worker and the plumber is so intimately associated with the building trades, the lesson herein given forms a necessary step in the present course.

FIG. 5.—Here A B C D is the plan of a building to be covered by a groined roof. The arch, the springing of which is A B and C D, is a semicylinder. The arch, which has its springing in A C and B D, being of the same height but of wider span, is a semicylindroid.

A *cylindroid* is a solid body of the character of a cylinder; but whilst in a cylinder all sections taken at right angles to the axis are circles, in the cylindroid all such sections are ellipses. It is, in fact, a *flattened cylinder*. The curve of the groin then is generated by the penetration of a cylindroid and cylinder.

On A B describe the semicircle which represents the face of the arch, at the ends A B and C D, and divide it into any number of equal parts, *a*, *b*, *c*, &c.

It is only necessary to use the *quadrant*, as throughout the working the measurements are the same on each side.

Draw the diagonals A D and B C.

From *a*, *b*, *c*, *d*, *e*, *f* draw lines perpendicular to A B, cutting the diagonal A D in a', b', c', d', e', f', and set off the same distances on the other half of the diagonal.

From these points draw lines at right angles to A C, and passing through it in points 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11. Mark off on the perpendicular 6 the height 6 *f*, equal to the height of the semicircle *f*, and on the perpendiculars 5, 4, 4, 3, 2, 1 mark off in succession the heights of the perpendiculars *e*, *a*, *f*, *c*, *d*, *a*, as contained between the semicircle and its diameter A B.

Set off the same heights on the corresponding perpendiculars, on the other side of 6 *f*, and the curve traced through these points will be a semiellipse, which is the section of the semicylindroid forming the arch, of which A C and B D are the springings.

We now proceed to find the curve of the groin; and it will be evident that although the span is still further increased in length, the heights of the different points in the curve will be the same as in both the previous elevations. The span then of the arch at the groin is the diagonal A D (or B C) to which the divisions *a*, *b*, *c*, *d*, *e*, *f* have already been transferred from the semicircle, and from these the lines were carried at right angles to A C, on which the height of the points in the curve were set off.

These points (viz., a', b', c', d', e' f') in the diagonal, then, will be seen to be common to both arches, since they are the plans of the points in the roof where the cylindrical and cylindroidal bodies penetrate each other. At these points, therefore, draw lines perpendicular to the *diagonal*, and mark off on these the heights of the perpendiculars in the semicircle from which the points on which they stand were deduced. These extremities being connected, the curve so traced is the groin curve, and will give the shape for the centering of the groin, as the semicircle and semiellipse will for those used in the elevations of the arches.

It now only remains to develop the surfaces of these arches, that is, to find the shape of the zinc or lead which would cover the roof of a railway station, or other building, when formed as here described.

The student is advised to work this study on a large scale on cardboard, and then to cut out the separate parts, which he can afterwards join at their edges, thus constructing an accurate model of the roof required.

He is urged not to be content with *reading* this book, but to work out each study to different scales, and to cut out all the developments in cardboard or paper. This plan will not only instruct him as to the lessons herein given, but will lead him to think of applications of them to the daily work in which he may employed.

FIG. 6.—Draw any straight line, and commencing at A, set off on it the distances into which the curve A C is divided—(measuring on the *curve*, *not* on the springing line)—namely, the distances A, a b c, &c.

At the points on the straight line thus marked, draw perpendiculars ; make the middle one equal to 6 f ; those on e e equal to 5 e ; those on d d equal to 4 d ; those on c c equal to 3 c ; those on b b, to 2 b ; and those on a a equal to 1 a.

Join the extremities of these perpendiculars, and the two curves meeting in a point, and joined by the original straight line, will form the development of the covering of the cylindroidal arch.

Fig. 7 is the development of the semicylindrical arch. As this is worked in precisely the same manner as the last, but taking the measurements from the semicircle, no further instructions are deemed necessary.

The term **DOME** is applied to the covering of the whole or part of a building. The Germans call it ***dom***, and give the name to the cathedral or principal church in a city, although the building may not have a spherical or polygonal dome. From this and other circumstances it is inferred that the term is derived from the Latin *domus*, a house.

The following definition is given by Mr. Peter Nicholson, to whose excellent works we are also indebted for the illustrations of this branch of our subject.

"A dome is an outer or vaulted roof, springing from a polygonal, circular, or elliptic plan, presenting a convex surface on the outside, or a concavity within, so that every horizontal section may be of a similar figure, and have a common vertical axis. According to the plan from which they spring, domes are either circular, elliptical, or polygonal. Of these, the circular may be spherical, spheroidal, ellipsoidal, hyperboloidal, paraboloidal.

"The word *dome* is applied to the external part of the spherical or polygonal roof, and *cupola* to the internal part. Cupola is derived from the Italian *cupo*, deep, whence also our word "cup;" but "cupola" and "dome" are often used synonymously, although perhaps incorrectly. Such as rise higher than the radius of the base, are denominated *surmounted* domes; those that are of a less height than the radius are called *diminished* or *surbased;* and such as have circular bases are termed cupolas."

The forms named above, "hyperboloidal" and "paraboloidal," will be understood on referring to Figs. 93 and 94 in "Linear Drawing," which would be the vertical sections taken through the axes of the domes referred to, and these domes would be generated by each of these sections rotating on its axis.

Thus, if a piece of cardboard be cut of the shape of the parabola (Fig. 93, "Linear Drawing"), and a wire or thread inserted from C to D, then if the student, holding this string or wire a little distance from C and D, blows against the edge of the piece of card, he will, as it whirls round, see the paraboloid or solid parabola generated.

PLATE XXIII.

FIG. 1.—**Given the plan of a square dome, and one of the axal ribs, at right angles to the sides, to find the curve of the angle rib and the covering.**

The axis of a square dome is the vertical line where the diagonal planes would intersect each other.

Let A B C D be the plan of the dome, and A C and B D the intersections of the diagonal planes. E F is the base and E K the height of the given rib; and the curve line K I H G F the section of the upper surface which comes in contact with the boarding.

The student will realise this if he imagines the point K raised, so that it is over E, the quadrant E F K thus becoming vertical.

Produce E F to K, divide the curve line K F into any number of equal parts, and it will be understood as the operation progresses, that the more points taken, the more correct will be the result.

Set off the parts F G, G H, H I, I K upon the straight line F *k*. The first from F to *g*, the second from *g* to *h*, the third from *h* to *i*, and the fourth from *i* to *k*. Through *g*, *h*, and *i* draw lines parallel to A B. From the points G, H, I, in the curve of the given rib, draw G G′ N, H H′ O,

I I′ P, parallel to A B, cutting the base of the rib E F at the points G′, H′, I′, and the half diagonal D E at the points N, O, P.

Take the intercepted parts, G N, H O, I P, between E F and E D, apply them successively to the lines parallel to A D on each side of F *k*.

This is best done by drawing perpendiculars from I, H, G, cutting the lines drawn through *g*, *h*, *i* in *n*, *o*, *p*.

Draw through these points the curve *a n o p k*.

Measure on the other portion of the lines from *g*, *h*, *i*, the widths *g n*, *h o*, *i p*, viz., *i p′*, *h o′*, *g n′*, draw *k p′ o′ n′* D.

Then the space contained between A K D will be the form of the lead or zinc required to cover the one quarter, A E D, of the dome.

To find the hip line of the angle rib, the base of which is E D.

From the points N, O, P, E draw N Q, O R, P S, and E T perpendicular to E D. Make these lines successively equal to G′ G, H′ H, I′ I, and E K, and through the points D, Q, R, S, T draw a curve which will be the hip line.

To draw the angle rib and covering of an octagonal dome.

Fig. 2 is the plan of the octagonal dome, showing also the given rib.

Let A B C D E F G H be the plan, and let L O K be the given rib—that is, the rib passing to the apex, at right angles to the side A H.

Divide the arc L K into any number of equal parts, as *a*, *b*, *c*, and draw the radius A O.

From *a*, *b*, *c* draw lines parallel to O K, and cutting A O in *a′*, *b′*, *c′*.

From F draw the curves through these points, thus completing the development.

Now, to make the form of the whole of this still more clear to the learner.

Let us suppose the figure carefully drawn to a much larger scale on a piece of stiff drawing paper or card-board.

Cut with a penknife through the curve A B, through the lines B G and G A, and place the quadrant upright on its edge A G, securing it by gumming a piece of paper under the corner piece I.

Now cut through the curve A E, and at the back cut half through the thickness of the line G E, and bend the piece A E upwards until it is perpendicular to the surface of the drawing.

In the same way cut through the curve L G and the line G G′, and half through the line L G, and bend up the figure L G G′ until it is vertical.

Then it will be seen that the points A of the one rib, B of the other, and A of the third, will meet at one point, the three lines G A, G B, and G G′ uniting in one perpendicular, and these would then be the forms of the respective roof trusses.

Now cut out the curves J D L and L F K, and it will be seen that these pieces will exactly cover the ribs in the form required, D meeting F on the point G (in which A and B are also united).

As by this method, the curve L G′ cuts into the covering L, D, J, and there are other little inconveniences arising from making such a model out of one piece, it is advisable to make the three ribs out of separate pieces, and to gum them on the plan, using strips of linen by way of hinges; they can then be raised or laid down flat at pleasure.

FIG. 2.—This figure gives the plan and elevation of a pyramidical roof, the plan of which is square, and the surfaces of which curve inwardly.

A B C D is the plan and E F G the section through G′, and this would be the form of the truss or rib parallel to the sides.

The angle rib would, of course, be obtained as in the last figure.

Now to find the form of the covering.

Divide F G into any number of equal parts, as *a*, *b*, *c*, *d*, *e*, and from these points draw perpendiculars cutting the semidiagonals G′ C and G D in *a′ a′*, *b′ b′*, *c′ c′*, *d′ d′*, *e′ e′*, and draw the line G′ H.

Now draw any perpendicular, I, and a horizontal at its lower extremity, and on this set off on each side of I the lengths I C′ and I D′ equal to H C and H D in the plan.

On I set off the lengths *e*, *d*, *c*, *b*, *a* taken from those on the curve G F, Fig. 2, and through these points draw horizontal lines.

On these, on each side of the perpendicular, set off the widths of the lines on each side of the line G H in the plan (*a′*, *b′*, *c′*, *d′*, *e′*), and through the points thus obtained *a″*, *b″*, *c″*, *d″*, *e″*, draw the curves G″ C and G″ E, thus completing the covering.

PLATE XXV.

To find the shape of the metal plates forming a cylinder, penetrated by a smaller one.

Before the shapes of the plates can be ascertained it is necessary that the whole object should be projected ; the method of accomplishing this is, therefore, given in Plate XXV.

Here the circle represents the plan of the larger, and the parallelogram D D′ E E′, that of the smaller cylinder.

From this figure project the mere cross which forms the elevation, by drawing perpendiculars from A B to be terminated at the length above the picture line corresponding with the height of the larger cylinder.

Next draw the elevation of the smaller cylinder crossing the larger one horizontally.

The great object we have in view at this stage is to find the curves generated where the penetrations take place, so that from these the exact form of the aperture may be deduced.

The student is here reminded that as the *plan* is the view of the object *when looking down upon* it, the line C A B C, which is the top of the smaller cylinder in the elevation is the *middle* line in the plan, and thus the line D E, which is the front or most prominent line of the cylinder in the plan, is represented by D E, the middle line in the elevation.

From C′ in the plan, with radius C′ E, describe a semicircle, which represents half the plane of the end of the cylinder. This plane, although laid down flat, is supposed to stand upright on the line E E′ at right angles to the plan.

Divide the semicircle into any number of equal parts, and from these divisions draw lines meeting E E at right angles in F and G.

Set off the lengths of these perpendiculars on each side

of the line D E in the elevation, viz., F F and G G, and draw lines from these points across the whole length of the elevation of the smaller cylinder.

Draw similar lines parallel to C C′ from the corresponding points in the plan, viz., F F′, G G′, which lines will be seen to pass, not only through the smaller, but also through the larger cylinder, representing as they do planes common to both solids.

From the points A and B, *f g e*, draw perpendiculars to meet the horizontals drawn from the points similarly lettered in the elevation, and the intersections *e*, *f f*, *g g*, will give the points through which the curves of penetration are to be drawn.

PLATE XXVI.

Having thus prepared the necessary drawings, we proceed to develop the surfaces of the larger cylinder, with the view of accurately delineating the form of the aperture through which the smaller cylinder is to pass, and it must be borne in mind that this aperture, notwithstanding that it is to contain a cylinder, will not be a circle when the surface through which it is pierced is laid out flat.

To develop the cylinder, divide the circle (its plan) into any number of equal parts, set these off along a straight line, B B″, erect perpendiculars at B and B″ of the required height, and join them by a horizontal which will complete the general development.

Now in this development draw a centre line A° representing A in the plan. On each side of A° set off the lengths *g*, *f*, *e*, and erect perpendiculars, then the heights of the points correspondingly lettered in the elevation marked off on these perpendiculars, will give the points through which the form of the aperture may be traced.

As this development meets at the middle of the other aperture, half of the form must be drawn at B′ and half at B″. Since the difference between the chord from *e* to *e′* and the curve is considerable, the distance should be again divided into a number of parts and set off on B′ B″, as shown at X X X, by this means the inaccuracy may be considerably lessened.

It now only remains to develop the form of one of the ends of the penetrating or smaller cylinder.

To do this, draw a horizontal line, and erect a perpendicular, E. On each side of this point set off the distances, *f g*, *c g*, *f* E, into which the end of the smaller cylinder is divided, and from these points erect perpendiculars.

On these, set off the lengths of the lines between E E′ and the plan of the larger cylinder, viz., E *e*, F *f*, G *g*, C′ B, &c.

The curve uniting the extremities of these perpendiculars will give the form in which the piece of metal is to be cut, so that when rolled and joined at its outer edges it may form a part of a cylinder of the required size, which will exactly fit to the aperture in the larger cylinder already described.

FREEHAND DRAWING.

IN several of the studies which have been given in the preceding pages the student has been called upon to draw curves by hand through points obtained by measurement,

or by intersection of ruled lines. As this requires some practice, a brief course of freehand drawing is here added, and the student is advised to copy the examples at intervals whilst proceeding with the linear drawing; he will thus acquire the power of drawing the curves required.

In designing any piece of work, or in copying drawings, curved lines of the most varied forms are of constant occurrence, and although some of them which are arcs of circles can be done with compasses, and others may be inked by means of "French curves," there will still be many which are best executed by freehand, and there will occur curved lines continuous with straight ones which can always be more neatly joined by hand than by instruments, or which a certain amount of practice will enable the draughtsman to execute with his pen or pencil in less time than it would take him to find the centres. But this is not all. The study and practice of freehand drawing gives accuracy to the eye and refines the perceptive faculties; it enables a man to raise his ideas beyond mere straight lines, to cultivate his taste, and in many ways to add beauty to utility.

PLATE XXVII.

The annexed plate affords a series of elementary studies in freehand drawing. They should be copied at least twice the size of the examples, and if the first attempt should not be satisfactory, the student must remember that it is only by repeated and earnest efforts that success is attained, and that "try again" is at the foundation of all progress and of every great work.

In order to economise space, Fig. 1 may be used by the learner as three separate studies, of which the easiest may be taken first.

Draw the vertical line A B and the oblique line C D on the *right* side, giving the general inclination and extent of the curve to be drawn. Now set off between C and D the points E and F, and draw lines at right angles to C D. Make these equal to E H and F G, and trace the curve through D, G, and H, to C.

It must be pointed out that the curve ought not to spring abruptly out of the perpendicular line, as shown in Fig. 2, but must merge so gradually into it that it shall at last become a portion of the straight line itself. This will require some little practice, but as it is one of the most important points in ornamental drawing, attention is at once called to it. The two lower curves on the same side may now be drawn, each a little larger than the one preceding it. The straight lines used in the first curve are, however, merely intended as leading-strings, and therefore as soon as the smallest amount of power has been attained they should be dispensed with, and the curve drawn without such extraneous aid.

Proceeding now to a further stage of the study, draw another vertical line, and draw the curves on the *left* side, which in the present study will be found more difficult to draw than those on the other. The hand should be placed rather higher up on the paper, and the pencil should be held rather longer than in the previous practice.

It may here be well to mention that in freehand drawing generally the pencil should be held as long, and the eye should be kept as far from the paper, as may be convenient. By these means freedom of hand and a just view of the form is attained.

In sketching, these curves should at first be very slightly traced; they should not be made up of repeated scratches touching or crossing each other, as in Fig. 3, but of small pieces which in themselves form portions of the line to be drawn (Fig. 4).

Having thus practised the method of drawing curves inclined towards the right and left sides, the example may be copied as presented in the figure, the great object to be kept in view being that the curves must balance each other; that is, that the one on the right must be the exact counterpart of that on the left.

Draw the vertical line A B, and a horizontal, D H, crossing it; set off on this the distance to which the curves are to extend—viz., D H, and on the perpendicular mark the point C. Now, whenever lines are to be balanced, the one on the left side should be the first drawn, for it will be clear that if the curve on the right side were sketched first, the hand would cover it whilst drawing the other, and this would add much to the difficulty of balancing the different parts. Horizontal lines, in order to regulate the heights of the curves, may be used in elementary practice; but after a while a few touches rapidly sketched across will be found sufficient, and all squaring of freehand forms should be discarded.

FIG. 5.—The lines which form the subject of this study differ from those in the previous one, being each composed of two distinct curves.

It will be seen that the curve starting from *a* proceeds as far as *b*, and then turns in another direction. This peculiar bend requires much care, for the exact position of the point at which the change takes place materially alters the form.

In order to afford some guidance to the student, the line *a c* is drawn, and the change of direction in the curve takes place at the point *b* of this line.

As in the last subject, it is intended that the single curve should be practised before the balancing is attempted.

Draw a vertical line and a horizontal at its extremity; draw *a c* at the required inclination, and mark on it the point *b*.

Now divide *a b* and *b c* into equal parts, and draw the lines *d e* and *f g* at right angles to *a c*. Make *d e* and *f g* equal to the depth of the curve; then, commencing at *a*, trace the curve through *e*, *b*, *f* to *c*. Care must be taken to avoid a sudden bend at the juncture, so that the curves may flow gradually and imperceptibly into each other.

Assuming, then, that the student has had some practice in drawing the separate curves as they appear on each side of the vertical line, he may next proceed to draw the complete subject, aided at first by the guide-lines, but subsequently rejecting them.

In the lower portion of the example the practice is advanced to the delineation of a curve growing out of another. This, too, is an important point in ornamental drawing, and has been referred to in Fig. 1. It will be seen that although the branch *i* springs from *h*, it does not project suddenly from the curve *g i*, but merges gracefully out of it, so that if the part of the curve *g h* were removed, the branch *j h i* would still be complete, and if the branch *h i* were taken away, the remaining curve, *g h j*, would not be interrupted.

FIG. 6.—In this study the practice afforded in both the previous lessons is applied, the simple and compound

curves being employed and the branch carried round so as to form a scroll. This is only the curve forming the lower portion of the last figure continued, and the remarks in relation to the curve growing out of the other apply equally to this exercise.

In every case the example should be lightly sketched at first, and when it has been examined and corrected, it should be partially rubbed out, and the lines should then be repeated with a rather harder pencil, great attention being paid to their smoothness and uniform thickness. In the sketching an HB. pencil should be used, and for "lining in" an H. will be found the best.

PLATE XXVIII.

This example is a border or ornamental moulding, and gives the pattern and half of the repeat.

Having drawn the upper and lower horizontal lines, draw A B, C D, E F, and G H, the distance between them being equal. Then it will be seen that C D and G H are the centre lines of the heart, and that A B and E F are the centre lines of the tongue or leaf between the hearts.

Now draw the curve J, and balance it by the curve I K.

It will, of course, be understood that although the instructions and lettering refer to the complete figure, it is intended that the corresponding lines in the repeat are to be drawn at the same time; in fact, whatever length of the moulding is to be drawn, these divisions or compartments should be first set out, and the single curve drawn in each before proceeding any further. On no account should one portion be completed before the others have been sketched, for as each set of curves is drawn the drawing becomes more complex, and the difficulty of accurate balancing is increased. When these curves have been completed, the interior ones, which depend *upon*, but are not parallel *to* them, are to follow. In drawing these the greatest care is necessary, so that the curves may run gracefully downwards, the space between the inner and outer curves becoming gradually narrower.

The centre part at C is now to be drawn, following the plan already laid down, viz., to draw first the left and then the right side of the figure; and after this the leaves between the hearts are to be drawn in the same manner.

PLATE XXIX.

This is a pattern for a running scroll, arranged so as to repeat; *a* will therefore join on at *b*, and thus the design may be continued.

It will be seen that in order to equalise the spaces so as to carry out this arrangement, the whole is divided into squares, and the central flower is placed on the intersection of the diagonals.

In commencing this design, the general form is to be sketched of each scroll rising out of the previous one. At this stage no notice should be taken of the husks or foliage, *c*, *d*, &c., but the scrolls should be sketched as if consisting of the flowing stem only, and the husks should then be drawn outside the original form.

Great care must be exercised to ensure the smooth spiral character of the curves. There must be no angular breaks, but the eye must be carried onward towards the centre of each scroll, and the husks must appear as additions, but not as excrescences. In order to test the

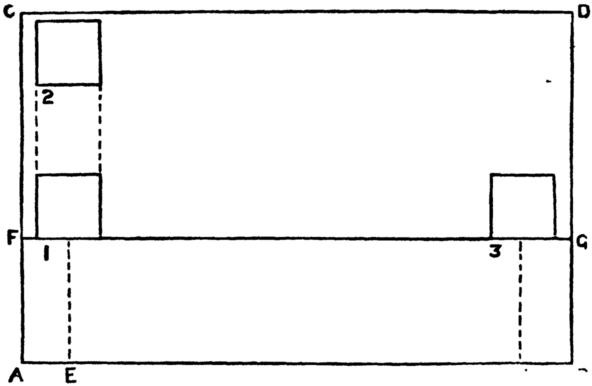
correctness of the forms, turn the sketch upside down, place it vertically, or in any other direction, and if the design has been correctly sketched, the scrolls should be equally perfect in whatever position they may be viewed. This should be repeatedly done during the progress of the work, so that any part which may be too full or too flat may be improved before the husks, flowers, foliage, or other details are added.

DRAWING FROM SOLID OBJECTS.

IN all the examples hitherto given, *one surface* only has been shown of each of the subjects under consideration. Now for absolutely working drawings this is all that is necessary, such drawings being required to show the exact position of every wall, cornice, sill, or step, so that the work may be executed according to the exact measurement of the design.

But if it be required to draw a house or any other object in such a manner that an impression of solidity may be given, further study and practice become necessary, and for this purpose the present section is introduced. It is not intended, however, to burden the mind of the student with a number of rules, such being given in the volume of this series devoted to the study of Perspective. A few simple principles only will therefore be given in order that the beginner may be enabled to sketch from solid objects in a correct manner.

Let it be assumed that the student has provided three cubes, or rectangular blocks, of any proportion, and that he has placed them on a table of which A B C D is the surface, in the manner indicated in the adjoining diagram, and that he is seated at E. It will be seen that the figure A B C D is thus the *plan* of the table with the blocks on it.



Now to make a sketch of the models as they appear to the spectator placed as follows.

PLATE XXX.

Draw *a b c d*, representing the front elevation of the cube, which in this case must be a *square*, as it is in the model, since the face is parallel to the plane of the picture —that is, to a surface (as a sheet of glass) standing on the line F G in the previous figure.

Now it will be clear that if an object were placed lower than the eye of the spectator, he would see the top of it, as he would of a box standing on the floor, and that if the model were placed higher than his eye, the bottom would be visible, like that of a birdcage hung up against the wall.

This, then, is the first point which demands the attention of the student, and he will of course be able instantly to decide whether he sees the upper or under surface of the object. In the present instance it has been assumed that it is lower than the level of the eye. In order to test *how much* the eye is higher than the model, it should be raised until the top is no longer visible, and this height should be compared with that of the object.

Now, supposing it should appear that the height of the eye of the spectator is three times the real height of the model, draw a line, H L, at a distance above the square *a b c d*, which shall have the same proportion to the square as the height of the eye has to the size of the model. This line is called the horizontal line, and in reference to this the following brief definition will be found useful :—

The Horizontal Line represents the height of the eye of the spectator.

If a sheet of glass were placed upright between the eye of the spectator and the object, and this line were drawn on it, then, when looking straight forward, the point on the horizontal line immediately opposite the eye would be called the Point of Sight. From this we obtain the following definition :—

The Point of Sight is the point in the horizontal line which is immediately opposite to the eye of the spectator.

We now proceed to draw the objects, of which the plans are given on page 74.

It will easily be understood that since the cube, the plan of which is Fig. 1, stands so that its front and back are parallel to the picture-plane, these two faces will be absolute squares, not in any way altered in appearance by their position.

Having, therefore, drawn the square *a b c d*, representing the front of the cube, and having determined the height of the eye in relation to that of the cube, draw the horizontal line, H L, and mark on it the point of sight, P S, opposite to the spectator, whose position is at E (in the previous figure).

All lines which in the object are at right angles to the plane of the picture must be drawn to the Point of Sight.

Now on referring to the plan of cube No. 1, shown on page 74, it will be seen that as two of the faces are parallel to the picture, the other two must be at right angles to it; running, in fact, directly from the spectator to the distance.

Therefore, from *a b c d* draw lines to the point of sight, and between these draw the vertical line *e f*, representing the distant edge of the side of the cube; and from *f* draw *f g*, for the distant edge of the top.

In drawing from objects by the eye, no exact rule can be given as to the placing of the line *e f*. A little practice will, however, enable the student to judge of its position with tolerable correctness, and the precise method of representing dimensions in the distance is given in "Practical Perspective," to which the student is referred.

In order to account for the sides of the cube which are not visible in the drawing, draw a line from *a* to the point of sight, and a horizontal from *e* meeting it in *a'*; draw a perpendicular from *g* to *a'*. All the edges of the object will thus become visible, as if the whole were constructed of wires. It must be observed that the back surface of the cube is not altered in shape by being removed from the immediate foreground; it is a square like the front, but becomes smaller, owing to the convergence of the lines between which it is contained.

Reverting now to the plan of the table on page 74, it will be seen that the cube, of which Fig. 2 is the plan, is not in the immediate foreground, but has travelled directly backward, as if on a tramway, the track being shown by the dotted lines.

Now it will be clear that the sides of the plan, and the track in which they have travelled, are continuations of the sides of the cube No. 1, which are at right angles to the plane of the picture.

Again, it will be remembered that all such lines converge to the point of sight, and as the cube now to be drawn is supposed to be similar in size to the former one, the track in which it has travelled will be represented by the lines drawn to the point of sight from *a* and *b*. The line *a b* will, therefore, in Fig. 2 be represented by *h i*, which will be the base of the square forming the front of the cube.

Having thus drawn *h i*, draw perpendiculars from its extremities, then the lines from *c* and *d* to the point of sight will give the points *j* and *k*; the apparent height of the distant cube and a horizontal line joining *j k* will complete the front.

It will be evident that *i j* and *h k* could at once have been drawn equal to *h i*, but this would only be a correct method when it is known that the required figure is a square; but by the plan here shown the correct height would be obtained, whatever might be its proportion to the width. The distant side and top are now to be drawn as in Fig. 1.

FIG. 3.—This cube, as will be seen from the plan, is placed immediately in front of the spectator, so that only the front and top, but neither of the sides, would be visible. The front having been completed, draw lines from *n* and *o* to the point of sight; and between these draw the distant edge *p q* of the top, corresponding with the line *g f* in Fig. 1.

Lines drawn from *l* and *m* to the point of sight, and perpendiculars drawn from *p* and *q*, will give the points *r*, *s*, which being joined by a horizontal line, will complete the view of the object rendered as if transparent.

PLATE XXXI.

Fig. 1 is a cube on which rests a pyramid, the apex of which is not over the centre of the top of the cube. Having sketched the cube as in former lessons, draw the line A B representing the front edge of the base of the pyramid, Fig. 2. As the pyramid is to be represented as if moved forward, this line must of course be drawn in front of C D, the edge of the cube, and although it would in reality be the same length, it must be longer than C D, being, in fact, the most prominent line in the picture. You will understand this if you refer to the cut on page 74, which represents the line on which the picture-plane stands.

Now, if you place such a plane in front of the present subject, and gradually move it nearer and nearer to it, the first part at which it would touch would be the line A B.

From A and B draw lines to the point of sight; for although the pyramid has been moved sideways and forward, its edges have been kept parallel to those of the cube, and the lines A E and B F in the model are at right angles to the plane of the picture. Next draw the line E F, which completes the base of the pyramid. In the quadrilateral A B E F draw diagonals; at their intersection erect a perpendicular equal to the apparent height, G, of the object, and finally draw G A, G B, G E, and G F.

FIG. 3.—This is a cube resting on one of its edges, whilst another touches the cube, Fig. 1.

Now, it is necessary to bear in mind that, although this cube rests on one edge only, the plane of the side A′ B′ C′ D′ is parallel to the picture. To prove that this is so, place the cube in the first instance on one of its sides, the face A′ *b c d* being parallel to the picture, as in Fig. 1; raise the object at *b*, until the edge *d* rests against the side of the cube, Fig. 1, at D′. It will then be evident that the object has merely rotated on A′ from left to right, but that the surface remains parallel to the picture-plane as before.

The shape of the front therefore remains unaltered—a perfect square—but resting on the angle A′ instead of on the side A′ *b*.

To draw this object, draw the line A′ D′, carefully observing—(1) that it must be of the length of the side of the cube, Fig. 1, the cubes being equal; (2) that it slants in the degree required, the line A′ D forms the hypothenuse of the right-angled triangle A′ E′ D′, and by comparing the angles at A' and D in your drawing with those formed by the meeting of the two models, you will soon discover whether the cube, Fig. 3 is sufficiently inclined.

On A′ D′ draw the square A′ B′ C′ D′, which will give the face of the cube in the position required.

Reverting to the original cube drawn in dots, it will be seen that the edges *b b′* and *c c′*, being at right angles to the picture, converge to the point of sight, and this will still be the case, for though the edges of the cube have altered in *position*, they have not altered in *direction*, but still run directly from the foreground into the distance at right angles to the plane of the picture. Therefore from A′, B′, C′, D′ draw lines to the point of sight.

The line drawn from D′ will cut the distant perpendicular of Fig. 1 in E′, and as the cubes are equal, this will determine the depth of the distant sides; therefore from E′ draw a line parallel to D′ A′, cutting a line drawn from A′ to the point of sight in F. Also from E′ draw a line parallel to D C, cutting a line drawn from C to the point of sight in H.

From F′ draw a line parallel to A′ B′, cutting a line drawn from B′ to the point of sight in G; draw G H, which will complete the object.

PLATE XXXII.

In the previous lessons, the models have been so placed that their front and back surfaces have been parallel to the plane of the picture. Under such circumstances, the sides alluded to retain their original shape, however much

they may be diminished in size by being moved into the distance.

It now becomes necessary to consider the method of drawing objects when their sides are at various angles with the picture.

Let A B, Fig. 1, be the vertical edge of a cube which is nearest to the spectator.

The lines B C and A D represent the edges of the right side, which in the object are horizontal, but which in the present view converge towards the right-hand side, until if continued, they would meet on the horizontal line, *not* at the point of sight, to which it will be remembered "all lines at *right angles to the picture*" are drawn, but to a point called the "*vanishing point.*"

The lines B E and A F converge in a similar manner to a point on the left-hand side. Care must be taken that these lines are not drawn up too obliquely, which would make the sketch appear as if the object were tilted up from the back.

It must be understood that the vanishing points need not necessarily be on the paper, nor need the lines be drawn completely up to them, a little observation and practice will enable the student to judge of the amount of inclination required, and to sketch the object with tolerable correctness.

The rules for finding the exact position of the vanishing and other points do not fall within the range of this volume, but are fully treated of in another ("Practical Perspective," Technical Series).

The student must use his judgment too in determining, the positions of the distant perpendiculars D C and E F, and from the upper extremities of these lines are to be drawn representing the opposite edges of the top. Those in the object are parallel to the front edges previously drawn, and must therefore converge to the same point.

The lines C G and E G therefore should be so drawn that, if continued, they would meet the lines B E, A F, B C, and A D on the horizontal line.

The cube being thus completed, a block, the end of which is a square smaller than the end of the cube, and the sides of which are oblong, is to be placed upon it in such a manner as to leave a margin all round.

Draw diagonals in the upper surface of the cube B G E C.

Now it will be clear, that if a true square were drawn within another, the angles of the inner figure would be on the diagonals of the outer one, and that the sides of the two figures would be parallel.

Therefore, having determined that the nearest angle of the upper block shall be at H, draw lines from that point to the vanishing points on each side ; these will cut the diagonals in I and J.

Then the lines H I and H J will represent two of the edges of the base of the solid.

From I and J draw lines to the opposite vanishing points. These will cross each other in K on the diagonal B G.

Then H I K J will be the ground plan of the upper block traced perspectively on the cube.

Now erect the perpendicular H L, which must be slightly shorter than it would be if it were in the immediate foreground, for it must be observed, that this line stands back a short distance, represented by the length B H.

From L draw lines to the vanishing points, thus obtaining M and N; and lines drawn from these points to the opposite vanishing points will intersect in O, and thus complete the figure, which is now to be "lined in."

The use of rules and compasses in these studies is strictly forbidden, the object being to accustom the student to making sketches by hand, the importance of which cannot be overrated.

We now come to the consideration of circles, and Fig. 2 will assist the student in comprehending the lesson about to be given.

From this figure it will be seen that if a circle be enclosed in the square A B C D, it would touch the sides at E, F, G, H, and would pass through the diagonals in I, J, K, and L.

Here then are eight points in the circle, and it is now necessary that these should be found in the drawing.

When the square has been perspectively drawn, as in Fig. 3 *a b c d*, draw the diagonals, and the diameters passing through their intersections. One of these, *e g*, will of course be parallel to *a d*, whilst the other, being at right angles to the plane of the picture, will vanish in the point of sight.

Now through the points I L and I K, Fig. 2, draw lines meeting A D in M and N. Mark off on *a d*, Fig. 3, the length *a m* and *d n*, equal to A M in the previous figure.

From *m* and *n* draw lines to the point of sight, passing through the diagonals, which will give the points *i*, *j*, *k*, and *l*.

The square with the point through which the circle passes is now sketched perspectively, and the figure representing the circle is to be traced by hand through *h*, *k*, *g*, *j*, *f*, *i*, *e*, *l*, *h*.

Fig. 4 represents a cylinder so placed that its circular end is at an angle with the plane of the picture.

Now it has been shown that in order to put a circle into perspective, it must be enclosed in a square, and this will assist in drawing the cylinder, which by the same rule would be contained in a block, the ends of which are square, as they might be in a piece of timber out of which it had been turned.

Commence, therefore, by drawing the line A B equal to the diameter of the circle.

This is to be the nearest angle of the block; and from the extremities lines are to converge to the vanishing points, as shown in Fig. 1. In the present figure, however, the sides are of different relative sizes.

The figure representing the circle is then to be sketched within the perspective view of the square A B C D, as shown in the previous figure. Reference to the example will show the method of completing the view, the distant end being drawn as already described.

The knowledge of the above principles, together with a little practice, will soon enable the student to sketch circular and cylindrical forms without the guiding lines, and the sooner such aid is dispensed with the better.

In addition to the blocks here referred to, the student should accustom himself to draw from the common objects around him, many of which are based on the solids already mentioned. The very lessons on the development of surfaces too will assist in providing drawing

models, for if the studies be carefully worked out to a large scale on stiff paper or cardboard, and the figure when drawn be cut out and made up, a most useful and important set of objects will be obtained, and each lesson will thus have served two purposes, and an amount of practice will thus be obtained which will prove of the utmost value to those who would use drawing as a language.

THE END.

CASSELL, PETTER, & GALPIN, BELLE SAUVAGE WORKS, LONDON, E.C.

(*List continued on next page.*)