國立浙江大學
工程季刊

第二卷 第三四期

ENGINEERING QUARTERLY
UNIVERSITY OF CHEKIANG

Vol. 2 No. 3&4 Dec. 1942

目錄

中華民國三十一年十二月出版

Economical problems in Surface Condenser Design — A Mathematical Analysis

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(adapted and condensed from a paper presented at the annual meeting of The Chinese Institute of Engineers in October 1941.)

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INTRODUCTION

This paper is an attempt at a systematic study of the economical problems involved in the design and selection of a surface condenser in steam power plant. It aims to bring out the relevant factors in every problem, and by means of mathematical approximations, to reach some generalized conclusions.

It does not claim to contribute anything towards the mechanical perfection or satisfactory working of a condenser. — In these respects a modern condenser is a very efficient apparatus. But the best conditions of its application are not always realized. The condenser in a power plant is usually ordered as a stock article its specifications following the requirements of the steam turbine. This practice is economically unsound because no other equipments in the power plant is so profoundly affected by the local conditions not essentially thermodynamical in nature.

Perhaps the would-be theoreticians are deterred from an exhaustive analysis by the enormous complication of the condensing system. It comprises first the turbine exhaust and the condenser proper, with the various attached pumps. The cooling-water supply and the external pumping installation is an independent problem, And lastly, there may be a choice of artificial cooling systems (tower, pond, etc) with the necessary pumping and air draft. Each of these components offers an alternative choice and poses an economical problem. It may indeed seem doubtful whether such a conglomeration of diverse apparatus can be logically treated in a single paper. The task seems hopeless until it is realized that all these are only successive steps of an integral problem, to be solved in systematic order.

Thermodynamically, the fundamental phenomenon of a heat-power plant is the temperature drop. In a steam plant the boiler steam represents the upper limit and the atmospheric air the lower limit of temperature. This total temperature drop is to be subdivided among various equipments: By far the largest part is usefully consumed for expansion work in turbine or engine. The first question is, how much temperature difference (θ_c) should be left for heat rejection to atmosphere, i.e. (1) the problem of economical back-pressure to turbine, or economical vacuum in condenser. In a direct cooling system with natural water supply, this temperature difference (θ_c) is available inside the condenser between the exhaust steam and the incoming cooling water. The question here is to determine the temperature of outgoing water which subdivides θ_c into two parts: the temperature rise of water and the temperature difference for heat transfer. This is (2) the problem of economical water-circulation. In an indirect cooling system, the water passes back and forth between the condenser and the cooling tower or pond. And θ_c is to be divided between these two apparatus. Thus there is also (3) the problem of economical temperature of inlet water. All these divisions are determined by economical criteria

Leakage air is a minor point. But the question of air-coolar vs. air-pump is determined by the maximum degree of undercooling allowable at the pump suction, or the minimum temperature difference in air cooler (θ_a). This offers (4) an economical temperature division at the other end inside the condenser.

The only major problem not connecteo with temperature division is (5) that of water velocity in condenser tubes, which is a fundamental economical quantity by its own merit.

These 5 problems cover all the important aspects in condenser design and operation. How they can be separately treated and yet build up to a unifield whole will be shown below.

COSTS AND SYMBOLS

The condensing system comprises all quipments connected to the condenser and helpful to the rejection of heat. All of these require investment and expenditure of power.

Let K_1 = yearly cost of condensing surface (fixed charge and maintenqnce)

K_2 = yearly cost of internal puming (in tubes and water boxes)

K_3 = yearly cot of external puming (intake and conduit, pumping, etc.)

K_4 = yearly cost of water consumption (purchase, treatment, etc.)

K_5 = yearly cost of artificial cooling (ground, structure, pumping, draft.)

K_6 = yearly cost of oil removed

ΣK = total yearly cost of condensing system.
$= K_1 + K_2 + K_3 + K_4 + K_5 + K_6$

$_0K$ = minimum yearly cost of any combination.

The following symbols will be used as defined

S = cooling surface (tubes) in sq ft.

l = total tube length in series; in ft.

N = no. of tubes per pass

N_p = no. of water passes

d_o, d_i = O.D. and I.D. of tube, in inches

v_t, v_b = water velocities in tube and water box, in ft per sec

W_w = total flow of cooling water, in lbs. per hr.

W_s = total flow of exhaust steam, in lbs. per hr.

t_s = exhaust steam temperature

t_1, t_2 = inlet and outlet temperatures of cooling water

$t_w = t_2 - t_1$

θ_1, θ_2 = nominal temperature differences at water inlet and outlet

θ_m = logarithmic mean temperature difference

U = overall coefficient of heat transfer (average)

q = heat rejected per lb. of exhaust steam
= 900 to 950 B.T.U. per lb.

Q = total heat flow, in B.T.U. per hr. $= W_s q$
$= W_w t_w = US\theta_m$

Then, $K = a_1 S$

$$K_2 = (b_1 \bar{v}_t^{n} \frac{Q}{a_{10}} + b_2 \bar{v}_t^{n} \Delta p) \bar{W}_w$$

$$K_3 = c \bar{W}_w^{m} L$$

$$K_4 = c' \bar{W}_w$$

$$K_5 = j \frac{Q}{\Delta t}$$

The meaning of most of these coefficients are obvious. Other symbols will be explained later.

ECONOMICAL BALANCES

The ultimate criterion of economy is minimum total cost. That is to say if ΣK = total yearly cost of condensing system and F = yearly fuel cost for turbine steam, then

$$\Sigma K + F = \text{minimum}$$

To attain this result, the whole problem may be broken up into 5 steps, to be considered in the following order:

(1) Economical water velocity -------- $K_1 + K_2 = \text{min.}$
(2) Economy of air removal ------------- $K_6 = \text{min.}$
(3) Economical division of temperature drop $K_{12} + K_{34} - \text{min}$
(4) Economy of artificial cooling ----- $K_{1234} + K_5 = \text{min.}$
(5) Economical condenser vacuum----- $\Sigma K + F = \text{min.}$

(1) Economical water velocity in tubes.

A higher water velocity causes more effective heat transmission and reduces the surface required, but it means also increasing pumping lost inside the condenser. The conditions on the steam side and the temperature of water are taken as constant for the time being oar

Let $\overline{W}_s$, t_s t_1 t_2 = constant

Then θ_1 θ_2 θ_m and $\Delta t_w \doteq$ constant

$$Q = \text{constant} \quad \overline{W}_W = \frac{Q}{\Delta t_W} = \text{constant}$$

K_3, K_4, K_5, are independent of V_t and taken as constant; and K_6 is a part of K_1 but a relatively negligible one

$$\therefore K_1 + K_2 = \min.$$

In this; $K_1 = aS = a\left(\frac{Q}{U\theta_m}\right)$

$$K_2 = \overline{W}_W \left(b_1 V_t^n \frac{\ell}{d_1^{1+a}} + b_2 V_t^{n'} N_p\right)$$

$$\text{where } N = \frac{\overline{W}_W}{3600 \times 62.4 \times V_t \times \left(\frac{\pi}{4}\frac{d_1^2}{144}\right)}$$

$$\text{and } \ell = \frac{S}{N\left(\pi \frac{d_0}{12}\right)} = \frac{S}{\overline{W}_W} V_t \left(1680 \frac{d_1^2}{d_0}\right)$$

N_p is determined by a suitable proportion of condenser diameter to length

$\frac{\ell}{N} : \sqrt{N_p N} \times \frac{d_0}{12} = 3$ to 4.2 for most large condensers

= 7 for smaller condensers ($\leqq$ 5000 ft²) to avoid too many passes

or $N_p \doteq \sqrt[3]{\text{const.} \times \frac{\ell}{N d_0^2}} = \text{constant} \times \frac{S^{\frac{2}{3}}}{\overline{W}_W} V_t \times \left(\frac{d_1}{d^{1.33}}\right)$

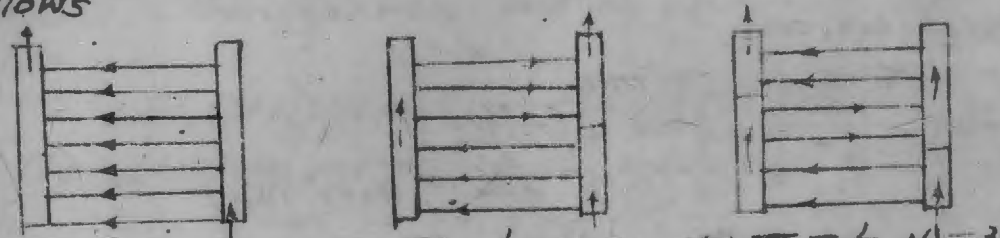
$$K_2 = \overline{W}_W \left(b_1 V_t^n \frac{\ell}{d_1^{1+a}} + b_2 V_t^n N_p\right)$$

$$= b_1 S V_t^{n+1} \frac{d_1^{1-a}}{d_0} + b_2 S^{\frac{2}{3}} V_t^{n+1} \frac{d_1^2}{d_0^{1.33}} = b_0 S V_t^{nw}$$

(Appendix II)

(This brings out the remarkable fact that at a given velocity the internal pumping-loss increases with the total surface and not with the ...

quantity of water. This will sound ess paradoxical if we remember the provision that the no. of tubes per pass should b change to keep V_t constant and the no. of passes should be changed to keep a correct dimensional proportion as follows



(a) $\overline{W}_W = 1$, $N_p = 1$ (b) $\overline{W}_W = \frac{1}{2}$ $N_p = 2$ (c) $\overline{W}_W = \frac{1}{3}$ $N_p' = 3$

same V_t, same surface, same power loss)

$$K_1 + K_2 = aS + b_0 S V_t^{n+1} = \min$$

$$\therefore \frac{\partial}{\partial V_t}(K_1 + K_2) = 0 : \quad {}_0V_t^{n+1} = \frac{1}{2n+1}\frac{a}{b_0}$$

$$\text{and } \left(\frac{K_1}{K_2}\right)_0 = 2n + 1, \text{ or } {}_0K_{12} = \left(1 + \frac{1}{2n+1}\right){}_0K_1$$

$$(\text{when } V_t = V_t)$$

Thus, the most economical water velocity in design (${}_0V_0$) is determined entirely by the cost data; and is independent of all characteristics of the particular condenser (as its size, tube arrangement, vacuum loading, etc.) For same wall thickness, tube size has only a negligible effect. Tube cleanliness has same effect owing to the change in frictional coefficient (b_1); but the correct way to provide for fouling of tubes is to allow a lower water velocity and use a bigculation by excess pump-capacity as in commonly practiced.

The result can be put into a more concrete form if we accept some experimental data for frictional loss in condenser:

If we take frictional head in tubes $= 0.008 \frac{V^{1.8}}{d^{1.2}} l$ and add 20% to 30% for water-box losses (Appendix II) we obtain the following results:

$$\text{Water h.p} = \frac{W_w(\Delta h)}{83000 \times 60} = \frac{S}{52850} \frac{d_i^{0.75}}{d_o} V^{2.75} \times (1.25 \pm)$$

Let b = yearly power cost per water h.p of pumping capacity.

Then $K_L = b \times (\text{internal water h.p.}) = b_o S V_o^{a+1}$

where $b_o = \frac{b}{52850} \times \frac{d_i^{0.7}}{d_o} \times (1.25 \pm)$

and $a = 1.75$

$$\therefore V_o^{2.75} \doteq 9400 \frac{d_o}{d_i^{0.75}} \left(\frac{a}{b_o}\right) \doteq 10^4 \frac{a}{b_o} \quad (\pm 5\% \text{ for usual dimentions})$$

For dirty tubes and increased friction, V_t should be reduced.

(2) Economy of Air Removal (Larger air cooler vs. larger air pump.)

The primary fact about air removal is that all the noncondensible gases must be pumped out. As for the water vapor accompanying the air there is the option of removing it either by condensation in the air cooler or by pumping with larger air pump.

For air cooler $\Delta S_a = \frac{\Delta Y}{U_a \theta_a} = \frac{(\Delta \overline{W}_r) q}{U_a \theta_a}$

For air ejector (steam-jet), let the increase in steam consumption $= \Delta \overline{W}_s = k_{\mu} \Delta \overline{W}_r$

Then, at economical balance ($K_6 = \text{min.}$), the cost of increment surface just equals the increment of fuel costs, or

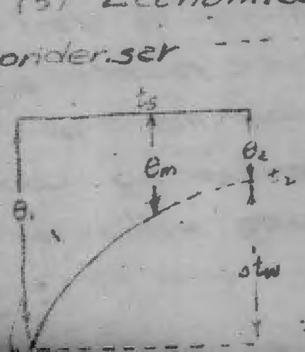
$$a(\Delta S_a) = f(\Delta \overline{W}_s)$$

∴ at pump suction, $\theta_a = \frac{q\, i}{U_a K_F}$

where $k_p = \left(\frac{\dot{W}_s}{\dot{W}_r}\right)$ for air pump

The air cooler is of course a continuation of the main part of condensing surface, and is differentiated only by a progressive increase in air concentration. It should be included in the economical balance in (1). The lowered value of U_a in air cooler however lowers the average condenser performance and is affected by V_t not to the same extent. This fact has been ignored in (1) because the effect is negligible except during the last percent of condensation.

(3) Economical division of temperature drop in condenser --- economical rate of water circulation



At a given vacuum and a given temperature of cold water, the total temperature drop (θ_1) is fixed. This θ_1 can be maintained either by circulating more water ($\Delta t_w \propto \frac{1}{W_w}$) or by having a larger surface ($\theta_m \propto \frac{1}{S}$)

$K_{12} + K_{34} = min.$

The combined costs of tube surface and internal pumping (K_{12}) increase directly with the surface (see n) and when the economical water velocity is used, the two components bear a definite ratio:

$$_oK_{12} = \left(1 + \frac{1}{2n+1}\right)_o K_1 = \left(1 + \frac{1}{2n+1}\right) aS_o = a'S_o$$

The cost of purchase or treatment of water increases proportionately to the quantity circulated (K_4 : $c'\overline{W}_w$). It is a negligible item in most plants.

The cost of external pumping increases less rapidly than the water circulation ($K_3 \propto \overline{W}_W^{0.5}$ to $\overline{W}_W^{0.6}$ — Appendix II) and consists largely of the cost of hydraulic structures (80% orencee). It also vary widely from one plant to another, being largely governed by the topographical features of the site (the distance of available water supply, the necessary lift, the extent of intake work, etc.). This aspect of the plant can be characterized by a factor L which is roughly an equivalent distance.

$$\therefore K_3 = c\overline{W}_W^{\,m} L \quad \text{where } m = 0.5 \text{ to } 0.6.$$

The combined cost of cooling water supply (delivered to the condenser and subsequently removed) = K_{34}. We can assume $K_{34} = C_o \overline{W}_W$, where C_o is a characteristic of the plant and m = 0.5 to 0.9, depending upon whether K_3 or K_4 is predominating

$$\therefore K_{12} + K_{34} = a'S_o + C_o \overline{W}_W^{\,m} = a'\left(\frac{Q}{U_o \theta_m}\right) + C_o\left(\frac{Q}{\sigma t_W}\right)^m$$

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$$= a'\left(\frac{Q}{U_0\theta_1}\right)\left(\frac{\theta_1}{\theta_m}\right) + C_0\left(\frac{Q}{\theta_1}\right)^m\left(\frac{\theta_1}{\Delta t_w}\right)^m = A\left(\frac{\theta_1}{\theta_m}\right) + C\left(\frac{\theta_1}{\Delta t_w}\right)^m$$

Let $\frac{\Delta t_w}{\theta_1} = x$, $\frac{\theta_2}{\theta_1} = 1 - x$ and $\frac{\theta_m}{\theta_1} = \frac{-x}{\log(1-x)}$

$\therefore \frac{\partial}{\partial x}(K_{1234}) = 0$ and solve for x

$$\frac{x^m}{1-x} - \frac{1}{x^{1-m}}\log\frac{1}{1-x} = m\frac{C}{A} = \text{constant}$$

and $\left(\frac{K_{34}}{K_{12}}\right)_0 \doteq \frac{1}{m}\left(\frac{\theta_m}{\theta_2} - 1\right)$ when the temperature is correctly divided.

Let $\left(2m\frac{C}{A}\right)^{\frac{1}{1+m}} = M$, and $1.8m - 0.42 = k$

As an approximation, $\left(\frac{\Delta t_w}{\theta_2}\right) = \frac{x}{1-x} \doteq M\left(1+\frac{M}{4}\right)^k$

($\pm 0.5\%$ for all $\theta_2 > \frac{1}{10}\theta_1$)

and $\left(\frac{K_{34}}{K_{12}}\right)_0 \doteq \frac{M}{2m}(1+\Delta M)$ where $\Delta = 0.173(m+1)(m-0.52)$

($\pm 1\%$ for $m \geqq 0.6$)

$$\therefore K_{1234} \doteq \left(1+\frac{1+\Delta M}{2m}M\right)\left(\frac{\theta_1}{\theta_m}\right)_0 a'\left(\frac{Q}{U_0\theta_1}\right) = a_0\frac{Q}{\theta_1} \propto \frac{Q}{\theta_1}$$

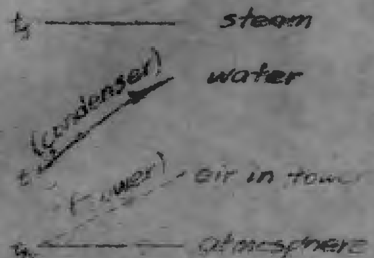
It is to be noted that the economical division of temperature drop inside a condenser is not determined by the condenser itself, but is entirely fixed by external circumstances (costs and topography). Once when these circumstances are specified, the best ratios of $\theta_1 : \theta_2 : \Delta t_w$ are fixed.

It can also be seen that $\overline{W}_w$ should be reduced with increasing water cost or external resistance (C_0) but not so rapidly.

$$\overline{W}_w \propto \frac{1}{\Delta t_w} \propto \frac{1}{M} \propto \frac{1}{(C)^{\frac{1}{1+m}}}$$

(4) Economy of Artificial Cooling — — Economical division of temperature drop (continued)

With artificial cooling of circulating water, the inlet water temperature to condenser is not fixed but controllable. The total available temperature drop is now between the steam temperature (t_s), and the wet-bulb temperature of the atmospheric air (t_b). This can be divided in various ratios between the condenser and the cooling tower or pond.



i.e. $(t_s - t_b) = (t_s - t_1) + (t_1 - t_b)$

or $\theta_c = \theta_1 + \theta_{t}$

Now $\Sigma K = K_{1234} + K_5$

As previously discussed, $K_{1234} = a_0 \frac{Q}{\theta_1} \propto \frac{1}{\theta_1} = \frac{1}{t_s - t_1}$

The best type of artificial-cooling installation depends mainly upon the ground area available, and its cost (K_5) varies with local conditions. A generalized expression, however, can be assumed by analogy: $K_5 = j \frac{Q}{Z_1 - Z_b}$ where j is a characteristic of the particular type of cooling installation, and $(Z_1 - Z_b)$ is a measure of the "cooling head" (Appendix IV)

$$\Sigma K = a_0 \left(\frac{Q}{\theta_1}\right) + j \frac{Q}{\Delta Z_{1b}} = a_0 \frac{Q}{t_s - t_1} + j \frac{Q}{Z_1 - Z_b}$$

For $\Sigma K = \min$, $\frac{\partial}{\partial t_1}(\Sigma K) = 0$ ($t_s, t_b = \text{constant}$)

$$\frac{a_0}{j} = \left(\frac{t_s - t_1}{Z_1 - Z_b}\right)^2 \left(\frac{dZ}{dt}\right)_1$$

To solve this use the approximation:

$$\left(\frac{Z_1 - Z_b}{t_1 - t_b}\right) \approx \text{average} \left(\frac{dZ}{dt}\right)_{1b} \doteq \sqrt{\left(\frac{dZ}{dt}\right)_1 \times \left(\frac{dZ}{dt}\right)_b} \text{ within } 2\%$$

For

Then $\frac{a_0}{J} = \left(\frac{t_s - t_1}{t_1 - t_b}\right)^2 \frac{1}{\left(\frac{dz}{dt}\right)_0}$

or $\left(\frac{t_s - t_1}{t_1 - t_b}\right)_0 = \sqrt{\frac{a_0}{J}\left(\frac{dz}{dt}\right)_0}$, where $\left(\frac{dz}{dt}\right)_0$ is a constant at a given t_b

Also $\left(\frac{K_{1z}}{K_s}\right)_0 = \frac{t_s - t_1}{z_1 - z_b}\left(\frac{dz}{dt}\right)_1 \doteq \sqrt{\frac{a_0}{J}\left(\frac{dz}{dt}\right)_1}$

where t_1 is calculated from above

and $\Sigma K_0 \doteq \left(1 + \sqrt{\frac{J}{a}\left(\frac{dt}{dz}\right)_1}\right) a_0 \frac{Q}{\theta_1}$

$\doteq \left(1 + \sqrt{\frac{J}{a}\left(\frac{dt}{dz}\right)_1}\right)\left(1 + \sqrt{\frac{J}{a_0}\left(\frac{dt}{dz}\right)_b}\right) a_0 \frac{Q}{(t_s - t_b)}$

$= a_0' \frac{Q}{t_s - t_b}$

a_0' is not exactly constant because of the nonlinear variation of the cooling head (i.e. $\frac{dz}{dt} \neq$ constant) But the error is small for small deviations.

(5) Economical vacuum.

So far the condenser vacuum is arbitrarily fixed and therefore all other changes in the condenser do not affect the operation of the turbine. If the vacuum is now allowed to vary, the turbine thermal efficiency will rise with increasing vacuum but the total cost of condensing system will also be increased.

$\Sigma K_0 = a_0 \frac{Q}{t_s - t_1}$ without artificial cooling,

$= a_0' \frac{Q}{t_s - t_b}$ with artificial cooling,

$= a_0 \frac{Q}{\theta_c}$ or $a_0' \frac{Q}{\theta_c}$, where θ_c = total available temp drop

yearly fuel cost $= F = f\,W_s$ where W_s = normal flow of exhaust steam

$\therefore F + \Sigma K = f\,W_s + a_0' \frac{W_s q}{\theta_c} = W_s\left(f + a_0' \frac{q}{\theta_c}\right)$

Let the throttle steam condition (H_1) be independently ... output of the turbine

Then $\overline{W}_s \neq$ constant, but may vary inversely with the expansion work done per lb. of steam, i.e.

$$\overline{W}_s = \frac{(\text{B.t.u work to shaft})}{H_1 - H_2} = \frac{M.E.}{H_1 - H_2} \propto \frac{1}{H_1 - H_2}$$

$$\therefore F + \Sigma K = \frac{M.E}{H_1 - H_2}\left(f + a_0 \frac{g}{\theta_c}\right)$$

For best economy, $\frac{d}{dt_s}(F + \Sigma K) = 0$

where $M.E$, H_1, g (see Appendix VI), f, and $a_0' =$ constant

$$\therefore \left(f + a_0 \frac{g}{\theta_c}\right) \frac{1}{H_1 - H_2} \cdot \frac{dH_2}{dt} + a_0' g \frac{d}{dt}\left(\frac{1}{\theta_c}\right) = 0$$

Let $(H_1 - H_2) / \left(\frac{dH_2}{dt_s}\right) = \Delta T =$ equivalent temperature drop in turbine.

Also $\frac{d}{dt_s}\left(\frac{1}{\theta_c}\right) = \frac{d}{dt_s}\left(\frac{1}{t_s - t_i}\right)$ or $\frac{d}{dt_s}\left(\frac{1}{t_s - t_o}\right) = -\frac{1}{\theta_c^2}$

$$\therefore \theta_c^2 = \frac{a_0' g}{f + \frac{a_0' g}{\theta_c}} \cdot \Delta T = \frac{a_0'}{f(1+\alpha)} \cdot g \Delta T$$

where $(g \Delta T) = f(P_1, T_1, P_2)$ only. (Appendix V)

Though θ_c and H_2 are both unknown on the right-hand side, the above can be solved by successive approximation — we may assume $\alpha = \frac{\Sigma K}{F} = 0.1$ to start with.

It is to be noted that even this last step may also be considered as a question of economical division of temperature drop. The total temperature difference now is between the steam at turbine throttle and the atmosphere, and the problem is, where in this temperature scale to place the condenser. The significant steam temperature here is not the throttle temperature, but rather the saturation temperature corresponding ...

will be found that ΔT as defind above is ro[illegible]
equal to the difference between the saturation
perature at throttle and at exhaust (Appendix V)
Other things being the same, it seems that a [illegible]
pressure high expansion plant allow a higher [illegible]
pressure or a lower vacuum ($\theta_c \propto \sqrt{\Delta T}$). But per[illegible]
the unit costs of such a pla[illegible] on not the same [illegible]
those of a low-presu[illegible]

Summary of

	Costs involved	Variable deter.	Cost Ratio
(1)	$K_1 + K_2 = min$	${}_0V_t^{2n+1} = \frac{1}{2n+1}\frac{a}{b_0}$	$\left(\frac{K_1}{K_2}\right)_0 = 2n+1$
(2)	$K_6 \quad = min.$	${}_0\theta_a = \frac{aq}{U_0 f k_p}$	
(3)	$K_{12} + K_{34} = min$	$\left(\frac{\Delta t_w}{\theta_2}\right) = (2m\frac{c}{A})^{\frac{1}{m+1}}(1+\frac{M}{4})^{[illegible]}$	$\left(\frac{K_{34}}{K_{12}}\right)_0 = \frac{1}{m}\left(\frac{[illegible]\theta_2}{{}_0\theta_1} - 1\right)$
(4)	$K_{1234} + K_5 = min.$	$\left(\frac{t_s - t_i}{t_i - t_0}\right)_0 = \sqrt{\frac{a_0}{j}\left(\frac{dz}{dt}\right)_0}$	$\left(\frac{K_{1234}}{K_5}\right)_0 = \sqrt{\frac{a_0}{j}\left(\frac{dz}{dt}\right)}$
(5)	$\Sigma K + F = min$	${}_0\theta_c = \frac{a_i q}{f(1+a)}\Delta T$	$\left(\frac{F}{\Sigma K}\right)_0 = \frac{\Delta T}{{}_0\theta_c} - 1$

General Conclusions

(1) The unit costs and the characteristic of [illegible]
local cooling water supply are the only relevan[illegible]
data for economical determinations in condenser
design; the water velocity in tubes and the [illegible]
[illegible] condenser are the f[illegible]

veriables to be determined. All these quantities, including the size of the unit, and the absolute pressure and temperatures, enter only indirectly in o-consideration and the thermodynamic requirements of the system denot at all interfere with the aconomizal consideration.

(2) In an economical condensing system, the different components of costs tear simple and definite ratio to each other. i.e. If external circumstances (not controlled by the designer) should cause the cost of one element to increase independently of the others, an attempt should be made to restore the economical ratio by using more of the other elements

(3) Some typical numerical calculations show that:

(a) The current practice in condenser design generally errs in too small tube surface (including air cooler), with consequent excessive tube velocity and excessive power consumptions by circulating and air pumps.

(b) The current tendency towards high vacuum (Say 29" to 28.5" Hg.) cannot be justified by fuel-saving alone.

Appendix I: Film Coefficient of Heat Transfer in Condenser

In 1916, G.A. Orrock gave the following empirical formula from his experiments on new single-tubes

condensing air-free steam: $U_0 = 325\,V^{+\,0.6}/e_m^{\frac{1}{\theta}}$
Earlier condensers shew average performance very far short of this, because of crowding of tubes and air-blanketing (1), but recent tests (1934-1936) on condensers of modern design show considerable improvements in this respect (2, 3), and substantiate the curves adopted by the leading condenser manufacturers (Power, Sept. 1932) which may be summarized by the formula $U = 264\,\sqrt{V}\,(1.42 - \frac{\theta}{\lambda_1})$ for commercially clean condensers. This is about 85% of Orrock's value for the same water velocity and perhaps takes into account some contamination of tubes on both sides.

Formerly, excessive fouling, reducing U to 50% or lower, was only prevented by frequent mechanical cleanings. The adoption of chlorination treatment for cooling water can keep the tubes almost continuously clean, maintaining a value of U at say 80% of Orrock's value.

The effect of air concentration is given by Orrock as $U = U_0 \left(\frac{P_v}{P_v + P_a}\right)^2$, but for higher air concentration as in air cooler, we may assume the total resistance increase linearly with air concentration (as suggested by some experimental results.) and this increase is of course independent of the changes on the water side. Therefore

$$\frac{1}{U_0} \text{ in air cooler} \doteq \frac{1}{U} + \frac{2}{(\alpha U_0)} \frac{P_a}{P_v} \doteq \frac{1}{U} + \frac{1}{350} \frac{P_a}{P_v}$$

According to this formula, U_0 will be fairly high ($\doteq 200\pm$) even for $P_a \doteq P_v$. This is probably true in the air-cooler section, in spite of some experimental observations which indicate extreme low values of U inside some tube banks (3). The latter condition is undoubtedly due to stagnation which usually cannot occur in the air-cooling section.

Appendix II: Hydraulic Losses in Condenser

The curve adopted by the leading condenser makers of U.S.A. for the frictional head losses in tubes and water boxes (commercially clean) can be represented by a formula

$$\Delta h = 0.00823 \frac{V^{1.83}}{d^{1.13}} \ell + 0.06 V^{1.55} N_p$$

Guy and Winstanley of Metropolitan Vickers, England give (in modified form) the following empirical coefficients (for new tubes) to a rational formula (3):

$$\Delta h = 0.0626 \left(\frac{70°F}{\text{mean } t_w}\right)^{\frac{1}{4}} \frac{V^{1.75}}{d^{1.25}} \ell + 1.5 \frac{V^2}{2g} N_p - \frac{V_d^2}{2g} \text{ (at discharge)}$$

and stated that for commercially clean tubes, the tube friction may be increased by 20% and for dirty tubes it may rise to two times this value. It can be seen that the increase in friction is of the same order as the decrease in heat transfer.

For condenser conditions in general use, these two formulae (or curves) give practically identical results. Therefore for a condenser in good conditions and normal operation, we may use

$$\Delta h = \left(0.008 \frac{\ell}{d^{1.13}} + 0.05 N_p\right) V^{1.75}.$$

and $\frac{\text{waterbox loss}}{\text{tube loss}} = \frac{0.05 d_i^{1.13}}{0.008} \left(\frac{N_p}{\ell}\right) = 20\% \text{ to } 35\%$ for $\frac{7}{8}$ tube O.D. 18 - 24 [illegible]

Appendix II. Yearly Cost of Pumping

Suppose water flows at the same velocity through pipes of different sizes. The flow will be proportional to D^2 but both piping cost and pumping power will not increase so rapidly.

$$\text{Pumpcost} \propto \overline{W}_w h \propto \overline{W}_w \frac{V^{1.86}}{D^{1.16}}\left(\frac{L}{D}\right) \propto D^{0.81} \propto \overline{W}_w^{0.43}$$

Piping cost ∝ weight per unit length $\propto D^{1.5}$ or $\overline{W}_w^{0.7}$ for smaller pipes (< 10") and $\propto D \propto \overline{W}_w^{0.5}$ for larger pipes.

So the total cost of a pumping installation will probably increase as $\overline{W}_w^{0.5}$ to $\overline{W}_w^{0.6}$ and the economical water velocity in pipe will increase only slightly with the size of pipe or with total flow.

The costs of intake-work, etc., are not amenable to mathematical generalizations, but usually do not increase so rapidly as the capacity, except when extensive construction has to be undertaken to obtain a new supply of water after the existing reserve is exhausted.

Appendix IV. Cooling Head

When water is cooled in open-air, evaporation usually plays a more important part than conduction. Therefore the cooling effect is not proportional to the temperature difference only, but rather to the difference in "total heat" of the two air-vapor mixtures, called the "sigma function" by Carrier. This has been shown to be a function of wet-bulb temperature and atmospheric pressure, or assuming standard barometer of wet-bulb tempture only.

Instead of as B.t.u heat content, we can express the cooling head as a proportional temperature difference Z (the rapidity of evaporation being assuming to be due to an augmented temperature difference)

$$Sigma = 0.24 t_b + \frac{W_v}{W_a} L \quad (L = \text{Latent heat at } t_b)$$

$$= 0.24 t_b + \frac{18}{29}\frac{P_s}{P_b} L = 0.24(t_b + 180 P_s) \text{ for ordinary temperature and pressure}$$

$$\therefore Z \doteq t_b + 183 P_s, \text{ where } P_s = \text{Saturation pressure or } 5\% \text{ in p.s.i}$$

If we use the approximate relation

$$P_s \doteq \frac{1}{3.62}\left(\frac{t_b+150}{200°}\right)^{10} \text{ from } 40 \text{ to } 160°F,$$

$$\text{then } z = t_b + 32.5\left(\frac{t_b+150°}{200°}\right)^{7.5},$$

$$\text{and } \frac{dz}{dt} = 1 + 1.22\left(\frac{t_b+150°}{200°}\right)^{6.6}.$$

Appendix V: Equivalent Temperature Drop in Turbine

The term $(H_1 - H_2) \times \frac{1}{\frac{dH_1}{dt_s}} \doteq \Delta T$ is a quantity in the dimension of a temperature but proportional to the work done by expansion per lb of steam in turbine. The effect of stage efficiencies will somehow cancel out if we make some reasonable assumptions.

We may assume the stage efficiency is constant for all expansion of <u>superheated</u> steam $(e = e_1)$, and further it drops off for <u>wet</u> steam proportionately to the % moisture $(e_2 \doteq e_1 x_2$, where x = quality of steam), so

$$\frac{dH_1}{dt_s} = \left(\frac{dH_1}{dt_s}\right)_\phi e_1 \qquad \frac{dH_2}{dt_s} \doteq \left(\frac{dH_2}{dt_s}\right)_\phi e_1 x_2 \text{ (probably too high)}$$

$$\text{and average}\left(\frac{dH}{dt_s}\right)_{12} = \frac{H_1 - H_2}{t_{s1} - t_{s2}} \doteq \sqrt{\left(\frac{dH_1}{dt_s}\right)\left(\frac{dH_2}{dt_s}\right)}, \text{ about 1 to 2\% too high}$$

$$\Delta T = (H_1 - H_2) \times \frac{1}{\left(\frac{dH_1}{dt_s}\right)} \doteq (t_{s1} - t_{s2})\sqrt{\frac{\left(\frac{dH_2}{dt_s}\right)}{\left(\frac{dH_1}{dt_s}\right)}} \doteq (t_{s1} - t_{s2})\sqrt{\left(\frac{dH_2}{dt_s}\right)_\phi x_2 \Big/ \left(\frac{dH_1}{dt_s}\right)_\phi}$$

Now $\left(\frac{dH}{dt_s}\right)_\phi$ is a definite function of steam condi-
tion only and is independent of the actual processes.
So this reduces ΔT to a function of initial and final
steam conditions only.

In particular for wet steam, $\left(\frac{dH_1}{dt_s}\right)_\phi = x_2 \frac{L_2}{T_2}$ if the
pump work for liquid is small.

For superheated steam, no exact formula is avail-
able but we may write $\left(\frac{dh_1}{dt_1}\right)_\phi = \frac{L_1}{T_{s1}}(1+\delta_1)$, where $\delta_1 \doteq$
$\frac{D}{10^3}(1.5 + 0.[illegible] \frac{[illegible]}{10^4}) + \frac{[illegible]}{10^{[illegible]}}$ with P in p.s.i., and D degree
superheat.

Therefore, $\Delta T \doteq (t_{s1} - t_{s2})_n \sqrt{\left(\frac{dH}{dt_s}\right)_\phi \frac{T_2}{L_2} \frac{1}{x_2^2}} = \frac{t_{s1} - [illegible]}{x_2} [illegible]$

As $q = x_2 L_2$, $q \Delta T = (t_{s1} - t_{s2})_n \sqrt{\left(\frac{dH_1}{dt_1}\right)_\phi T_2 L_2} = f(P_1, T_1, P_2 \text{ only})$

It should be noted the % moisture drops out of the last
equation if we assume the stage efficiency decreases
proportionately to the quality of steam.

Appendix VI: "Constancy of "q""

We have already seen that the heat to be absorbed
from each pound of exhaust steam ranges from 900
to 950 B.t.u. But it may well be asked why, in the
balance of fuel cost ($\propto \frac{1}{H_1 - H_2}$) against condensing
cost ($\propto \frac{q}{\Theta_1}$), $q = (H_2 - h_e)$ is taken as constant while
$(H_1 - H_2)$ is a variable. The answer rests on a rather
fortuitous circumstance.

In Appendix V. we have stated that $\frac{dH}{dt_s} = e_2 (x_2 \frac{L_2}{T_2}) \doteq$
1.1 – 1.2. For typical values of condenser conditions
(1" – 2" Hg, 10% moisture) and last stage efficiency (65–
[illegible] $\frac{dh_e}{[illegible]}$ [illegible]

$\frac{dq}{dt_s} = \frac{dH_2}{dt_s} - \frac{dh_e}{dt_s} = 0.1$ to 0.2, whereas q itself ≈ 900 to 950

Thus, with every degree change in exhaust temperature, θ may be changed by 3 to 4%, $(H_1 - H_2)$ by about [illegible]% — the change in steam consumption, however small, cannot be neglected in this case, — and q by not much more than 0.01%, which is insignificant as compared to the uncertainty in value of θ. Bibliography

(1) J.H. Harlow and R.A. Bowman: Condenser Performance with Reduced Cooling Surface. A.S.M.E. Transactions, Feb. 1939.

(2) J.H. Landis and E.A. Tucker: Performance of two 10,000 sq.ft. Surface condensers. A.S.M.E. Transactions, vol. 56 (1934)

C.H. Van Hengel: Tests of a 30,000 sq.ft. Surface Condenser at Widely Varying Temperatures, Velocities of Inlet Water, and Loads. A.S.M.E. Transactions, Nov. 1936.

(3) H.L. Guy and E.V. Winstanley: Some Factors in the Design of Surface Condensing Plant. I.M.E. Proceedings, vol. 134 (1934).

(4) E.F. Church, Jr.: Steam Turbine, 2nd ed. p. 2[illegible]

遵義白土活性試驗

李壽恒 魏[illegible]

一、引言

化學工業中[illegible]以各種[illegible]

[illegible]

天然泥土之首被採用者為英國之漂布土(Fuller's earth)其後美之Florida earth，德之Silitonite，以及日本之酸性白土，相繼問世，每年應用於石油與油脂工業為量頗巨。繼後德日諸國，更利用化學與物理之方法，處理白土，使之活性化(Activated)以增強脫色與吸附能力，由此而得之白土，名活性白土，用於代替[illegible]，為世界所樂用。

[illegible]之[illegible]

州(1)[illegible]

[illegible]

研究，[illegible]

關於漂白土之脫色理論(2)(3)(4)，各專家化學家之研究，斷為一種物理的吸附現象，而活性化之作用(5)(6)，在於酸孔隙，使之疏鬆，[illegible]大為增加。

白土活性化之方法可大別為物理法(7)與化學法(8)(9)(10)(11)(12)兩種，前者係利用電氣，後者為利用化學藥品處理白土，工業上採用者，大多為化學法。

常用之化學藥品為鹽酸與硫酸，前者以無機類類溶液(氫氧化鈉(13)重碳酸鈉等(14))有機酸溶液(15)為酸(16))等為活化劑者，採用之較少而已。

[illegible](17)(18)(19)[illegible]

[illegible]

(Activation)時[illegible]

二、實驗

A 原土[illegible]

[illegible]

時屄皂代用品，售價每元十數塊，每塊長度約16—18糎，濶5—6糎，厚1.5—2.0糎。

茲將該土之物理性質分述如下：

色澤　略帶黃色。

斷面　用小刀劃土塊，察其斷面尚稱光滑。

崩潰　取白土一小塊投入水中，土即吸收水分而崩潰。

比重　2.68

反應　取乾燥白土粉末少許，分置於紅色與藍色石蕊試紙上，滴水潤土，詳察試紙有無變化，所得結果兩者均不變色。

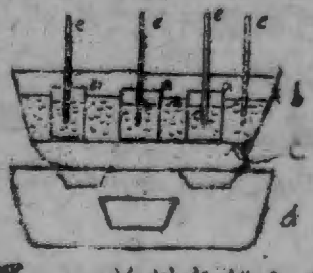
又在白土之表面分別滴以 Phenolphthalein 與 Methyl orange 二指示劑，結果亦不變色，故知該土之酸性與鹽基性均不甚顯著。

對魚肝油之顯色反應　如將日本酸性白土加入魚肝油中，則白土呈美藍色而沈澱，今以同法處理遵義白土，未見有顯色反應。

脫色　取原土粉末0.5克加入 Methylene Blue 試劑100c.c.中，震盪十五分鐘，過濾之，盛濾液於納氏比色管中，與比色標準液相比較，察其脫色能力，得 Methylene Blue Number 為40（試驗與比色標準液之配取，以及比色所用之方法，詳見(c)試驗方法項下）

(B)儀器裝置

加熱裝置　原土活性化時須加高溫度，所用熱媒，如溫度在100°C以上，可用植物油如菜油，倘溫度低於此數，可改用熱水，其裝置可參見左圖



a　玻杯（600—800c.c.）
b　油鍋或水浴
c　鐵綫網
d　炭爐
e　溫度計（0～200°C）

乾燥裝置　活性化後之白土經水洗後，須使之乾燥，所用儀器裝置與加熱所用者同。

研磨與篩粉用儀器　乾燥活性白土之研磨，因每次分量有限，故不用磨機，祇在研缽中研細之，所得粉末，均用每吋百孔之美國標準篩（U.S.

Standard Sieve）　篩過以法除粗粒

脫色試驗用儀器　活性白土脫色效率之測定係用比色法儀器用納氏比色管（Nessler tube）容積為50 cc

C 試驗方法

活性白土之製造　取原土100克和以各種分量與濃度之鹽酸在一定溫度下熱之使起活性化作用經適當時間以後去熱使冷加入少量稀氫氧化鈉溶液中和一部分殘酸澄清後傾去廢液再依傾瀉法（Decantation Method）用清水洗滌至洗液不含氯游子為止在100—120°C下烘乾研細篩過所得粉末貯瓶中備作脫色試驗之用

脫色試驗　本試驗方法係參考丁B. Hill, L.W.Nichois 與 H.C. Cowles 三氏所創之法並根據目前情形酌量更改而成其手續如下：

(1)試劑之製備　取 Methylene Blue 染料約0.4克溶解於0.1N左右之硫酸(比重1.84之硫酸3cc加水稀釋至1000cc)一研中製成藍色溶液取此藍色液一份用蒸餾水九份稀釋之盛納氏管中與另一氯化鈷標準溶液相比較如兩者顏色深淺不同時可於未稀釋之染液中再溶入適量之染料以增濃或加水以減淡逐漸調整務使此染液用九倍容積之水稀釋後所得之顏色適與標準鈷鹽液之深度相同而後已

以鈷鹽為顏色深淺之標準較為可靠因各廠出品之 Methylene Blue 純度不同故同量染料溶於同容積之水中所得溶液其顏色之深淺亦隨染料製造廠家而不同因之同一白土其脫色效率可因染料純度不同而不相一致倘用鈷鹽溶液作為試劑深淺之標準可免此弊

此標準氯化鈷溶液之配製係將適量固體氯化鈷結晶（$CoCl_2 \cdot 2H_2O$）溶於96%濃度之酒精內加少量鹽酸中使製成之溶液每1000 cc中含有34%鹽酸2.5cc鈷0.47克（即 $CoCl_2 \cdot 2H_2O$ 1.322克）

(2)比色標準液之製備　取不同容量之染料試劑分別盛於納氏管中各用蒸餾水稀釋至50cc作為比色標準液并以稀釋液中所含染料試劑之容積百分數標示之名曰 Methylene Blue Number

常用之標準液有如下表

第一表

Methylene Blue Number	溶液50cc中應含染料試劑之容積	Methylene Blue Number	溶液50cc中應含染料試劑之容積
1	0.5cc	16	80cc
2	10	19	95
3	10	22	110
5	26	25	12.5
7	33	30	150
10	50	35	17.5
13	65	40	1[illegible]0

(2)脫色效率之測定　　納乾燥活性土〇〇克入玻塞錐形瓶中加入藍色染料試劑100cc閉塞震盪一刻鐘即行過濾取濾液50cc盛納氏管中與比色標準液相比較記錄其與濾液深淺相同之標準液所標示之 **Methylene Blue Number** 以表示該土之脫色效率

三　實　驗　結　果

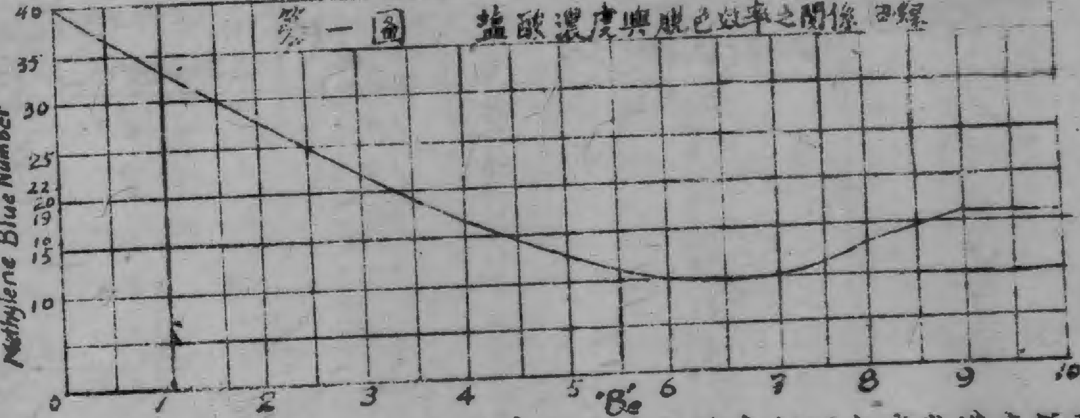
(A)活性化時鹽酸濃度對於脫色效率之影響

在同一狀况之下用各種比重不同之鹽酸處理原土所得結果見第二表

第二表

NO.	活性化情形							Methylene Blue Number	備註
	原土分量	鹽酸分量		鹽酸濃度		加熱溫度	加熱時間		
		無水氯化氫	定用酸量	°Bé	比重				
1	100克	—	—	--	—	--	—	4	未經處理之原土
2	〃	30克	90cc	3	1.011	100±5°	約5小時	22	
3	〃	〃	〃	4	1.026	〃	〃	19	
4	〃	〃	〃	5	1.034	〃	〃	13	
5	〃	〃	〃	6	1.043	〃	〃	10	
6	〃	〃	〃	7	1.021	〃	〃	10	
7	〃	〃	〃	8	1.008	〃	〃	13	
8	〃	〃	〃	9	1.065	〃	〃	15	
9	〃	〃	〃	10	1.072	〃	〃	10	

今以第一圖表示之如下



由此可知活性土之脫色效率與鹽酸之濃度有關在適當濃度範圍以內(6-7°Bé)鹽酸愈濃則製成之活性土脫色效率愈高過此濃度則適與前相反但如濃度增至9°Bé以上活性土之脫色效率受濃度之影響甚微.

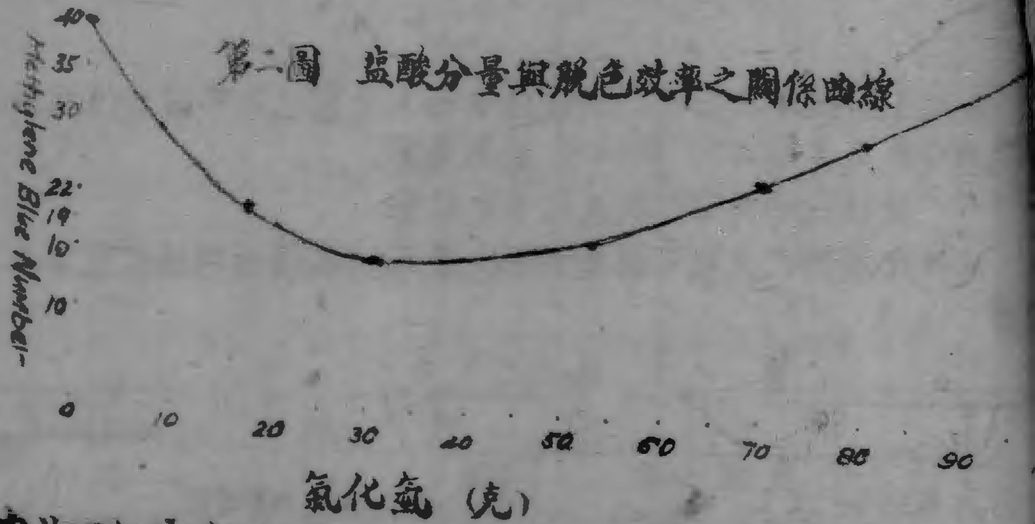
(b)活性化時鹽酸之分量對於脫色效率之影響

將一定濃度之鹽酸在同溫度下等時間內處理定量原土惟因鹽酸之分量不同故製成之漂白土脫色效率亦異.

鹽酸之分量以酸中所含氯化氫為計算標準茲將試驗情形與結果列表并圖示於次

第三表

NO.	活性化情形						Methylene Blue Number	備註
	原土分量	鹽酸之量 氯化氫	實用酸量	鹽酸濃度	加熱溫度	加熱時間		
1	100克	—	—	—	—	—	40	未經處理之原土
2	〃	20克	63.5	10°Bé	100-105℃	25小時	19	
3	〃	30	95.0	〃	〃	〃	16	
4	〃	50	158.0	〃	〃	〃	22	
5	〃	70	222.0	〃	〃	〃	25	
6	〃	80	253.5	〃	〃	〃	30	
7	〃	100	317.0	〃	〃	〃	40	



第二圖 鹽酸分量與脫色效率之關係曲線

由此可知鹽酸之分量不宜過多如超出一定限量則製成之活性土脫色效率反將降低。

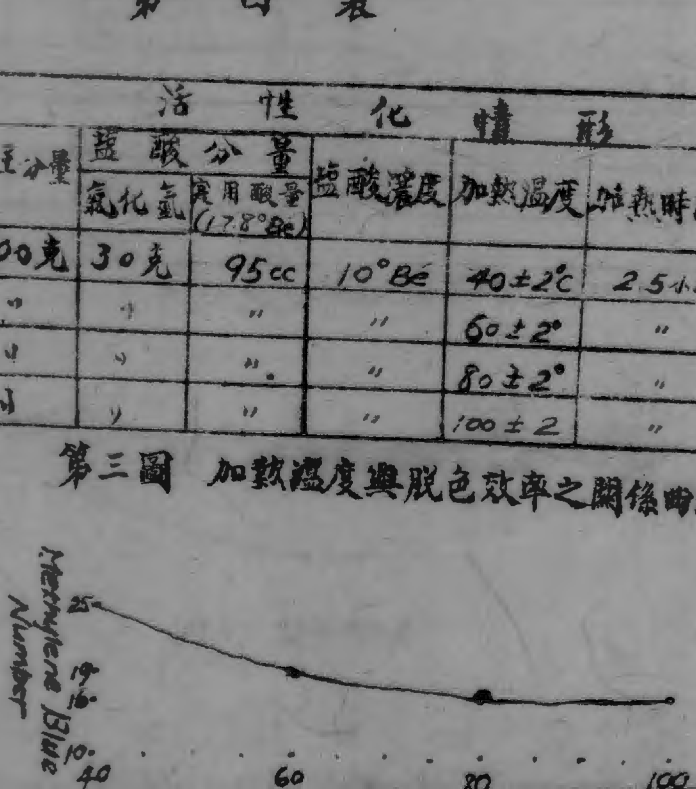
(a)活性化溫度對於脫色效率之影響

在同一條件之下（酸量溫度與加熱時間）以不同之溫度處理原土其結果如下

第 四 表

No.	活性化情形						Methylene Blue Number
	原土分量	鹽酸分量		鹽酸濃度	加熱溫度	加熱時間	
		氯化氫	實用酸量 (17.8°Bé)				
1	100克	30克	95cc	10°Bé	40±2°C	2.5小時	25
2	"	"	"	"	60±2°	"	19
3	"	"	"	"	80±2°	"	16
4	"	"	"	"	100±2	"	16

第三圖 加熱溫度與脫色效率之關係曲線



由此可知溫度在100°C以內活性白土之脫色效率因活性化溫度之不同而有差異其間以80--100°C最為適宜。

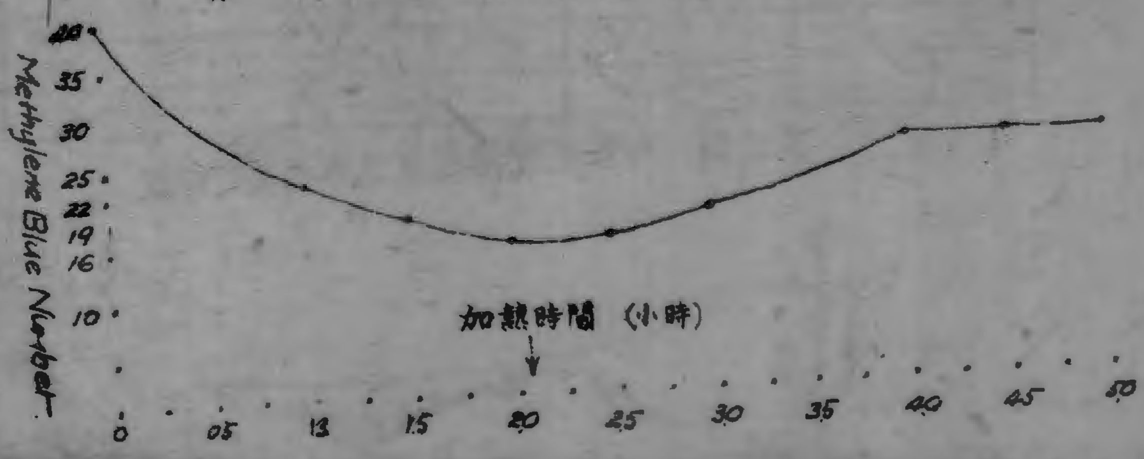
活性化時間對於脫色效率之影響

活性化時間之久暫可影響於脫色效率試驗情形與結果見第五表與附圖

第五表

No.	活性化情形						Methylene Blue Number	備註
	原土分量	鹽酸分量 氯化氫	定用酸量 17.8°/Bé	鹽酸濃度	加熱溫度	加熱時間		
1	100克	—	—	—	—	—	40	未經處理原土
2	〃	30克	95cc	10°Bé	100±5℃	0.5小時	25	
3	〃	〃	〃	〃	〃	1.0	22	
4	〃	〃	〃	〃	〃	1.5	19	
5	〃	〃	〃	〃	〃	2.0	16	
6	〃	〃	〃	〃	〃	2.5	16	
7	〃	〃	〃	〃	〃	3.0	19	
8	〃	〃	〃	〃	〃	4.0	25	
9	〃	〃	〃	〃	〃	4.5	25	
10	〃	〃	〃	〃	〃	5.0	25	

第四圖 加熱時間與脫色效率



據此實驗結果可見加熱時間二三小時已足若再增長則活性白土之脫色效率反有減弱如時間已達四小時而仍繼續加熱其於活性土之脫色效率不復有若何顯著之影響.

五 結 論

綜上所述吾人可知遵義白土在適當情形之下可用鹽酸法之活性化,而活性白土之脫色效率與所用鹽酸之分量濃度以及加熱溫度加熱時間均有關係.

酸之分量過多濃度過濃或處理時間過長均能使白土脫色效率低減此種現象認即係白土骨骼 Skeleton 被毀表面積減少之故.

六 摘 要

(1)遵義白土如用鹽酸處理可以活性化.
(2)鹽酸之濃度有一定限制過濃過淡均能使白土脫色效率減低
(3)鹽酸之分量不宜過多
(4)加熱之溫度在攝氏一百度以內愈高愈佳
(5)加熱之時間兩三小時已足時間過長對脫色效率并無幫助

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Structural Members in Torsion and Rigid Frames in Space

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Synopsis

The purpose of this paper is to present a general method for analyzing the stresses and deformations in a rigid frame by considering it as a three-dimensional structure. Elastic equations for a member, prismatical or haunched, are formed by expressing the flexural and torsional moments of both ends in terms of the twistings of the joints and the rotations of the members, which are the elastic unknowns, analogous to the two dimensional slope-deflection equations. These elastic equations are substituted into the equilibrium equations of the joints and the members, which are then solved for the elastic unknowns. The end moments are in turn obtained from the elastic equations. If the sidesways are absent or negligible, the tedious work of solving simultaneous equations can be replaced by moment distribution, performed in three mutually perpendicular directions independently. A few numerical examples are given at the end of the paper for demonstration of the method presented.

Introduction

All rigid frames are three dimensional in nature, but few are analyzed as such. In many cases, especially if direct torsional loads are avoided due to careful design, the interaction between various planes of the frame is small, and the stresses and deformations obtained by applying two-dimensional theories to separate planes of the frame is satisfactory. In general, a load in any plane of the frame will cause equally important stresses and deformations in members lying in other planes as those in the members coplanar with the load, and the bending and torsional moments in all members are interdependent. In analyzing such structures a three-dimensional theory is necessary.

Structural beams in torsion were treated theoretically and checked experimentally by Inge Lyse[1] and Bruce G. Johnston, in a view of strength of materials, not extended to haunched members nor intended for direct use in structural design. The application of the method of moment distribution to space frame-work has been demonstrated by Hardy Cross[2] and N.D. Morgan and later presented in detail by Paul Andersen[3] with the effect of sidesway entirely neglected.

(1) Proceedings, Am. Soc. C.E., April, 1935

(2) "Continuous Frames of Reinforced Concrete" by Hardy Cross and N.D. Morgan, John Wiley & Sons, N.Y., U.S.A.; 1932.

(3) Proceedings, Am. Soc. C.E., October, 1937

In this paper, it is demostrated that the flexual and torsional stresses and deformations in any rigid frame in space can be calculated with fair accuracy whether the haunching of members and side way of joints are involved or absent. The writer does not have the opinion that every rigid frame should be analyzed by three-dimensional theories, yet he does not realyze that two-dimensional theories can be applied only when the assumptions made in the latter theories are realyzed in the design of the frame and the error thus introduced is known to be negligible.

TORSIONAL MEMENTS & DEFORMATION

The essential feature of three-dimensional theory of rigid frames lies in the consideration of the torsional moments and deformations as well as the flexual moments and deformations. The problem of pure torsion as applied to prismatical shafts with non-circular section was first treated correctly by Saint Venant, and his general solution is applicable to any cross-section. Imoortant contributions to the problem have been made by Prandtl, Timoshenko, Griffith, Sonntag, and many others. The results of the authorities stated are condensed and turned usable forms in the following paragraphs.

The relative twisting deflection of two ends of a member (θ) caused by two equal and opposite toksional moments applied at the two ends are directly proportiona moment applied (M) directly proportional to the length of the member (L), and inversely proportional to the modulus of rigidity (G), or

$$\theta = \frac{ML}{JG} \qquad (1)$$

where J is a constant of proportionality, called the torsional constant, which depends on the size and shape of the cross-sectional area of the member, and is in the unit like the moment of inertia of areas, is in the unit of 4th power of linear dimensions.

The torsional stiffness of a prismatical member (... is the torsional moment required to produce a unit re lative twisting deflection between the two ends of th member, or

$$K = \frac{M}{\theta} = \frac{GJ}{L} \qquad (2)$$

For a circular section with radius r, the torsional constant is $J = \frac{1}{2} r^4$, same as the polar moment of inertia of the section an identity which does not hold true for other sections.

For a rectangular section with length b and breath n, the tor sional constant is given by an infinite series,

$$J = \frac{1}{3} bn^3 - 2qn^4 \qquad \text{where } q = \left(\frac{2}{\pi}\right)^5 \sum_{i=1,3,5\cdots}^{\infty} \left(\frac{1}{i}\right)^5 \tanh \frac{i\pi b}{2n}$$

the term qn^4 is the correction for each end of the rectangle. For $b/n = 1.000$, $q = 0.0964$ and for $b/n = 4.000$, $q = 0.1055$. For ordinary rectangular sections used in rigid-frame members, b/n varies from 1.200 to 1.800, q is practically 0.1. For extremely na rrow reatangles, the correction term can be neglected.

For a trapezoidal section with altitude b, and bases n and n Fig. 1, the torsional constant can be obtained by integration and then correction terms

$$J = \int_0^b \frac{1}{3} y^3 dx - 0.1 m^4 - 0.1 n^4$$

$$= \int_0^b \frac{1}{3}\left(\frac{m-n}{b}x + n\right)^3 dx - 0.1 m^4 - 0.1 n^4$$

$$= \frac{b}{12}(m+n)(m^2+n^2) - 0.1(m^4+n^4)$$

For rolled I-beams of sloping or parallel flang surfaces, Fig. 2, the torsional constant can be obtained by summation of the torsional constants of the web and the flangs, neglecting the end correction.

$$J = 4 \times \frac{1}{12}\left(\frac{b-w}{2}\right)(m+n)(m^2+n^2) + \frac{1}{3} w^3 d$$

Approxmately

$$J = \frac{b}{6}(m+n)(m^2+n^2) + \frac{1}{3} w^3 d \qquad (5)$$

If the flanges are parallel, m=n, it becomes

$$J = \frac{2}{3} b n^3 + \frac{1}{3} w^3 d \qquad (5a)$$

For haunched members with rectangular cross sections Fig. 3, we can apply equations (1) and (3)

$$d\theta = \frac{M}{GJ} dx, \quad \text{and} \quad \theta = \frac{M}{G}\int_0^x \frac{dx}{J}$$

where $J = \frac{1}{3} b n^3 - 0.2 n^4 = \frac{b n^3}{3}\left(1 - 0.6\frac{n}{b}\right)$

For straight haunch, $\frac{x}{L} = \frac{b-b_1}{b_2-b_1}$, $dx = \frac{L}{b_2-b_1} db$

hence, $\theta = \frac{M}{G}\int_{b_1}^{b} \frac{\frac{L}{b_2-b_1} db}{\frac{bn^3}{3}\left(1-0.6\frac{n}{b}\right)} = \frac{3ML}{G(b_2-b_1)n^3} \log_e \frac{b_2-0.6n}{b_1-0.6n}$

therefore, $J = \frac{ML}{G\theta} = \frac{(b_2 - b_1)h^3}{3\log_e \frac{b_2 - a\,b\,h}{b_1 - a\,b\,h}}$ (6a)

and $K = \frac{GJ}{L} = \frac{G(b_2 - b_1)h^3}{3\log_e \frac{b_2 - a\,b\,h}{b_1 - a\,b\,h}}$ (6b)

If the member is made of several sections with different haunches, Fig. 4, then

$$\theta = \theta_1 + \theta_2 + \theta_3 + \theta_4 = \frac{ML_1}{GJ_1} + \frac{ML_2}{GJ_2} + \frac{ML_3}{GJ_3} + \frac{ML_4}{GJ_4} = \frac{M}{G}\Sigma\frac{L}{J}$$

So that, $K = G/\Sigma\frac{L}{J}$ (7)

where J for each section is obtained from equation (6a)

When a torsional moment M is applied at any point on the member, the fix end moments F_{ab} and F_{ba} can be obtained from two equation:

$$F_{ab} + F_{ba} + M = 0$$

and $\frac{-F_{ab}}{G}\left(\frac{L_1}{J_1} + \frac{L_2}{J_2}\right) = \frac{F_{ba}}{G}\left(\frac{L_3}{J_3} + \frac{L_4}{J_4}\right) = \theta$

hence, $F_{ab} = -M\frac{\frac{L_1}{J_1} + \frac{L_2}{J_2}}{\Sigma\frac{L}{J}}$ and $F_{ba} = -M\frac{\frac{L_3}{J_3} + \frac{L_4}{J_4}}{\Sigma\frac{L}{J}}$ (8)

in which the egative signs show that the fix-end moments and the applied moment are in opposite direction

If the member is prismatical, then,

$F_{ab} = -M\frac{L'}{L}$ and $F_{ba} = -M\frac{L''}{L}$ (8a)

where L' and L'' are the distances of the load from the ends a and b respectively.

THE GENERAL SLOPE DEFLECTION EQUATION

Let ab be any member in a rigid frame in space with elastically built-in ends, bent by loads in one of its principal planes (the y-z plane) as shown in Fig. 8. By ordinary beam theory, the slope-deflection equations can be derived in the form:

$$M_{abx} = K_{aax}\theta_{ax} + K_{abx}\theta_{bx} + (K_{aax} + K_{abx})\phi_{abx} + F_{abx} \quad (9a)$$

$$M_{bax} = K_{bbx}\theta_{bx} + K_{bax}\theta_{ax} + (K_{bbx} + K_{bax})\phi_{bax} + F_{bax} \quad (9b)$$

where $K_{abx} = K_{bax}$ and $\phi_{abx} = \phi_{bax}$

The moments, slopes and deflections are considered to be positive if they are in the clockwise direction on the member as viewed from the positive end of x-axis. Thus all those shown in Fig. 8 are positive.

Similarly if the same member is also bent in the x-z plane the slope-deflection equations will be

$$M_{aby} = K_{aay}\theta_{ay} + K_{aby}\theta_{by} - (K_{aay} + K_{aby})\phi_{aby} + F_{aby} \quad (10a)$$

$$M_{bay} = M_{bby}\theta_{by} + K_{bay}\theta_{ay} - (K_{bby} + K_{bay})\phi_{bay} + F_{bay} \quad (10b)$$

where $K_{aby} = K_{bay}$ and $\phi_{aby} = \phi_{bay}$

If the member is also twisted about the longitudinal axis (z-axis) by torsional loads and torsional end moments the elastic equations will be

$$M_{abz} = K_{aaz}\theta_{az} + K_{abz}\theta_{bz} + F_{abz} \quad (11a)$$

$$M_{baz} = K_{bbz}\theta_{bz} + K_{baz}\theta_{az} + F_{baz} \quad (11b)$$

where $K_{aaz} = K_{bbz} = K_{abz} = K_{baz} = \frac{GJ}{L}$

When the member is prismatical the elastic equations become

$$M_{abx} = K_x(\theta_{ax} + 0.5\theta_{bx} - 1.5\phi_{abx}) + F_{abx} \quad (12a)$$

$$M_{bax} = K_x(\theta_{bx} + 0.5\theta_{ax} - 1.5\psi_{abx}) + F_{bax} \quad (12b)$$

$$M_{aby} = K_y(\theta_{ay} + 0.5\theta_{by} - 1.5\psi_{aby}) + F_{aby} \quad (12c)$$

$$M_{bay} = K_y(\theta_{by} + 0.5\theta_{ay} - 1.5\psi_{aby}) + F_{bay} \quad (12d)$$

$$M_{abz} = K_z(\theta_{az} - \theta_{bz}) + F_{abz} \quad (12e)$$

$$M_{baz} = K_z(\theta_{bz} - \theta_{az}) + F_{baz} \quad (12f)$$

with $K_x = \frac{4EI_x}{L}$, $K_y = \frac{4EI_y}{L}$ and $K_z = \frac{GJ}{L}$

where I_x and I_y are the moments of inertia about x- [illegible] respectively and E is the modulus of elas- [illegible]

By inspecting equations (11) or (12e) and (12f), the following facts can be visualyzed: (i) F_{abz} and F_{baz} are the fixed end torsional moments and K_z is the torsional stiffness of the member. (ii) If there are no torsional loads on the member, $F_{abz} = F_{baz} = 0$, the torsional moments at the two ends are equal and opposite, proportional to the difference of the twisting angles of the two ends. (iii) The relative linear displacement of the two ends will not produce any torsional moments in the member. (iv) The load applied in each principal plane containing the longitudinal axis will not produce any torsional moments in the member.

The equation used to solved the elastic unknowns are of two types namely: the joint equations and the bent equations. The joint equations are obtained by considering the equilibrium of the moments acting on the joints, which are equal and opposite to the end moments of the member around the joints. Let ab, ad and ad' be the member around joint a. Fig 5 the equilibrium of the moments acting on the joint about x-axis, or $\Sigma M_a = 0$ gives:

$$M_{abx} + M_{acy} + M_{adx} = 0.$$

Similarly

$$M_{aby} + M_{acy} + M_{ady} = 0,$$

and $M_{abz} + M_{acz} + M_{adz} = 0,$

in which M_{adx}, M_{aby} and M_{acz} are torsional, other being flexual. The number of such equations is the same as number of θ's.

The bent equations are obtained by firstly considering the equilibrium of the moments acting on the member, due the end moments, the end shears and the loads on the member, and then solving for one of the end shears in the form of $V_{abx} = \frac{M_{abz} + M_{bax}}{L} - C_x$, (Fig. 7)

where C_x is the simple beam shear at a due to loads only. Several such end shears of members in one plane or in two perpendicular planes are added and set equal to zero loads in the direction of x-axis. The number of such equations is the same as the number of ψ's

By solving the joint equations and the bent equations simultaneously, the elastic unknowns (θ's and ψ's) are obtained. Substituting the known values of the elastic unknowns into the slope-deflection equation we obtain all the end moments

THE GENERALIZED METHOD OF MOMENT DISTRIBUTIO

The tedious work of solving simultaneous equations in
slope-deflection method can be replaced by moment distributi
If sidesways are involved, the end moments about the three ax
of the members are interrelated to each other, thus making t
moment distribution very complicate. If sidesways are abser
or negligible the end moments about one axis of the memb
not related to those about the other two axes since there
sa bent equations and all the terms of a joint equation are
moments about one of the axes. Thus the distribution of the
moments about the three axes can be performed independen
Due to θ_{ax} an rotation of the joint a about x-axis, th
produced an end moment at the near end of all joint the me
around the joint, proportional to their stiffnesses K_{aax}, flexu
torsional. Hence, the distribution factors are proportional t
stiffnesses. At the same time an end moment produces at th
end of every member For any one of the members, say ab, it
$K_{abx}\theta_{ax}$, with the same sign as the moment distributed at
near end. Hence the carry-over factor from a toward
K_{bax}/K_{aax}. For a prismatical member, this becomes ½ If
moment distribution is torsional this is always -1 no matter
ther the member is haunched or prismatical. The distribu
can be performed directly on the skeleton of the frame
a separate table (see example 2 and 3).

NUMERICAL EXAMPLES

Example 1. (Fig. 8a, 8b, and 8c)

Beam ab: 1' × 1.5' × 10'–0", fixed at end a

Column bc: 1' × 1' × 8'–0" fixed at end c.

$E = 2{,}000\ k/in.^2$, and $G = 800\ k/in.^2$

For beam ab: $K_x = \frac{4EI_x}{L} = \frac{4 \times 2{,}000 \times 144}{10}\,\frac{1 \times 1.5^3}{12} = 32{,}400$ ft.k.

$K_y = \frac{4EI_y}{L} = \frac{4 \times 2{,}000 \times 144}{10}\,\frac{1.5 \times 1^3}{12} = 14{,}400$ ft.k.

$K_z = \frac{GJ}{L} = \frac{800 \times 144}{10}\left(\frac{1.5 \times 1^3}{3} - 0.2 \times 1^4\right) = 3{,}450$ ft.k

$F_{abx} = -\frac{30 \times 5^3}{10^3} = -37.50$ ft.k $F_{bax} = +37.50$ ft.k.

$F_{abz} = F_{baz} = -30 \times \frac{5}{10} = -15.00$ ft.k.

For column bc: $K_x = K_z = \frac{4 \times 2{,}000 \times 144}{8} \times \frac{1^4}{12} = 12{,}000$ ft.k.

$K_y = \frac{800 \times 144}{8}\left(\frac{1 \times 1^3}{3} - 0.2 \times 1^4\right) = 1{,}920$ ft.k.

Elastic unknowns: θ_{bx}, θ_{by}, θ_{bz}, ϕ_{aby}, and ϕ_{bcz}.

Slope-deflection equations:

$M_{abx} = 32{,}400(0.5\theta_{bx}) - 37.50$, $M_{bax} = 32{,}400(\theta_{bx}) + 37.50$,

$M_{aby} = 14{,}400(0.5\theta_{by} - 1.5\phi_{aby})$, $M_{bay} = 14{,}400(\theta_{by} - 1.5\phi_{aby})$

$M_{abz} = 3{,}450(-\theta_{bz}) - 15.00$; $M_{baz} = 3{,}450(\theta_{bz}) - 15.00$,

$M_{bcx} = 12{,}000(\theta_{bx})$, $M_{cbx} = 12{,}000(0.5\theta_{bx})$,

$M_{bcy} = 1{,}920(\theta_{by})$, $M_{cby} = 1{,}920(-\theta_{by})$,

$M_{bcz} = 12{,}000(\theta_{bz} - 1.5\phi_{bcz})$, $M_{cbz} = 12{,}000(0.5\theta_{bz} - 1.5\phi_{bcz})$

$\Sigma M_{bx} = 0$ give $M_{bax} + M_{bcx} = 0$,

or $44{,}400\,\theta_{bx} + 37.50 = 0$

$\Sigma M_{by} = 0$ gives $M_{bay} + M_{bcy} = 0$ (13a)

or $16{,}320\,\theta_{by} - 21{,}600\,\phi_{aby} = 0$

$\Sigma M_{bz} = 0$ gives $M_{baz} + M_{bcz} = 0$ (13b) or $15{,}450\,\theta_{bz} - 18{,}000\,\phi_{bcz} - 15.00 = 0$

Isolating ab, $\Sigma M_{by} = 0$ gives $M_{aby} + M_{bay} + 10 V_{bay} = 0$. (Fig. 8

Isolating bc, $\Sigma M_{bz} = 0$ gives $M_{bcz} + M_{cbz} + 10 V_{cbz} = 0$ (Fig. 8

Isolating joint b, $\Sigma F_x = 0$ gives $V_{bay} - V_{bcz} = 0$

Hence, $M_{aby} + M_{bay} = M_{bcz} + M_{cbz}$

or, $21{,}600\,\theta_{by} - 18{,}000\,\theta_{bz} + 43{,}200\,\phi_{aby} + 36{,}000\,\phi_{bcz} = 0$.

By geometry, $\phi_{bcz} = -\phi_{aby}$.

Solving (13a), (13b), (13c), (13d), and (13e) simultaneously, we ob

$\theta_{bx} = -0.000844$, $\theta_{by} = -0.000780$, $\theta_{bz} = +0.001655$,

$\phi_{aby} = -0.000589$ and $\phi_{bcz} = +0.000589$.

Substituting these into the slope-deflection equations, we

$M_{abx} = -51.18$ ft.k., $M_{aby} = +7.11$ ft.k., $M_{abz} = -20.71$ ft.k.

$M_{bax} = +10.14$ ft.k., $M_{bay} = +1.50$ ft.k., $M_{baz} = -9.28$ ft.k.

$M_{bcx} = -10.14$ ft.k., $M_{bcy} = -1.50$ ft.k., $M_{bcz} = +9.28$ ft.k.

$M_{cbx} = -5.07$ ft.k., $M_{cby} = +1.50$ ft.k., $M_{cbz} = -0.67$ ft.k.

Example 2. The frame shown in Fig. 9 consists of members
flexural stiffnesses four times as great as the torsional stiffness
a load of 100 k. at 4' from b and 6' from c, the moments and
formations are all about the same axis (the y-axis), with bf, c
dh in torsion and others in bending.

$F_{bcy} = -400 \frac{4 \times 6^2}{10^2} = -576$ ft.k., and $F_{cby} = +400 \frac{6 \times 4^2}{10^2} = +384$ ft.k.

The end moments M_{bay} and M_{bly} are equal and have to bala
and M_{bfy}. Hence they are omitted in the distribution and only
final results are given. Similarly for M_{dey} and M_{dky} at jo
and also for M_{cjy} at joint c. The distribution factors

Joint	b		c			d	
Moments	Mbcy	Mcfy	Mcby	Mcdy	Mcgy	Mdcy	Mdhy
Distribution factor	0.308	0.076	0.308	0.076	0.308	0.076	0.076
F.E.M	-576		384				
1st. distribution	+177	+45	-118	-118	-29		
Carry-over	-59		+89			-59	
2nd distrib	+18	+4	-27	-27	-7	+18	+4
Carry-over	-14		+9	+9		-14	
3rd distribution	+4	+1	-9	-6	-1	+4	+1
Total	-450	+50	+331	-142	-37	-37	+5

$M_{bdy} = M_{bly} = +200$ $M_{aby} = M_{ipy} = +100$

$M_{cjy} = -152$, $M_{fcy} = -76$

$M_{dey} = M_{dkr} = +23$, $M_{edy} = +11 = M_{kdy}$

$M_{fby} = -50$ $M_{gcy} = 37$ and $M_{ody} = +3$

Example 3 Same frame as Example 2. Due to a torsional load of 500 ft.k applied at 4' from b 6 from c, and in a counter-clockwise direction as viewed from the right, we have by (8a).

$F_{bct} = +300$ ft k and $F_{cbt} = +200$ ft k

The moments in abcde are all torsional and those in the others are all flexural. The moment distribution is performed in the following table

Joint	b		c			d	
Moments	Mbax	Mbcx	Mcbx	Mcdx		Mdcx	Mdex
Distribution factor	0.1	0.1	0.1	0.1	—	0.1	0.01
Carry over factor		-1	-1	-1		-1	
F.E.M		+300.0	+200.0				
1st distribution	-30.0	-30.0	-20.0	-20.0	—		
Carry-over		+2.0	+30.0				
2nd distribution	-2.0	-2.0	-3.0	3.0	—	+20.0	
Carry-over		+3.0	+2.0	+2.0		-2.0	2.0
3rd distribution	-0.3	-0.3	-0.4	0.4	—	+3.0	
						-0.3	-0.3
Total	-32.3	+29.7	+208	2.7	—	9.7	2.2

$M_{bfx} = M_{oix} = +29.2$ $M_{og} = M_{nih} = -93.9$

$M_{dm} = M_{dkx} = -$ [illegible] $M_{fbx} = M_{lpx} = -23.6$

$M_{gex} = M_{icx} = -46.$ [illegible] $M_{hax} = M_{kax} = -44$

$M_{abx} = +32.3$, gmd $M_{gd} = 4.2$

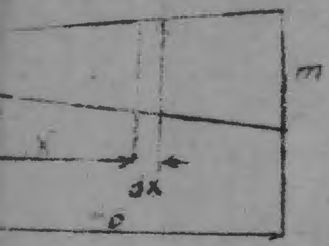


Fig. 1

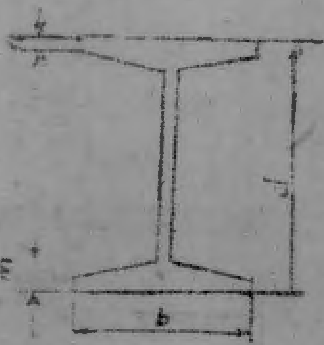


Fig. 2

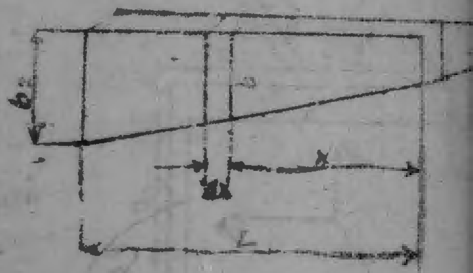


Fig. 3

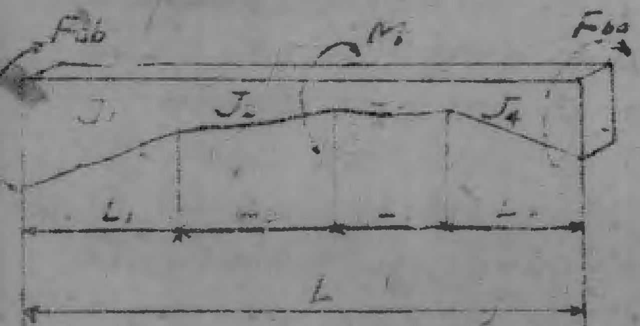


Fig 4

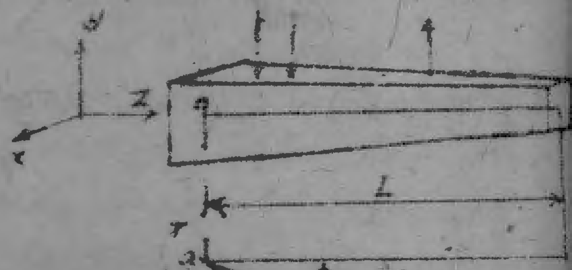
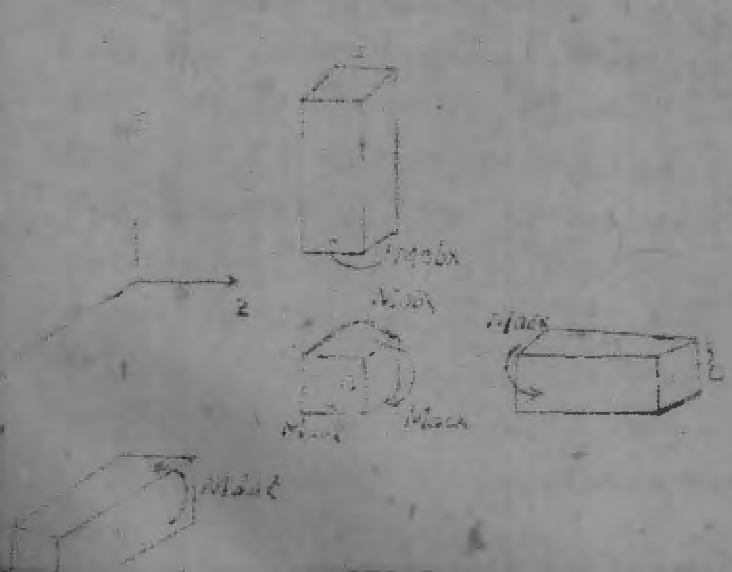
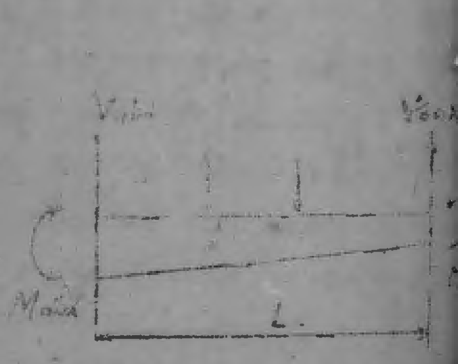


Fig 5





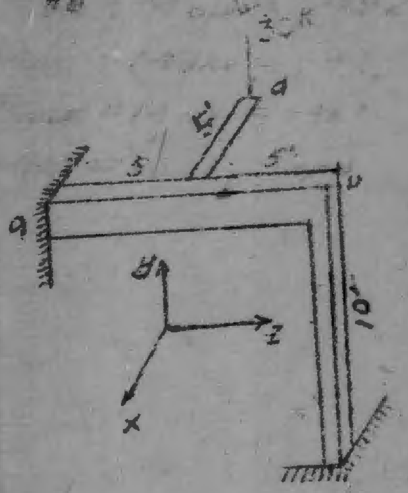


Fig. 8a

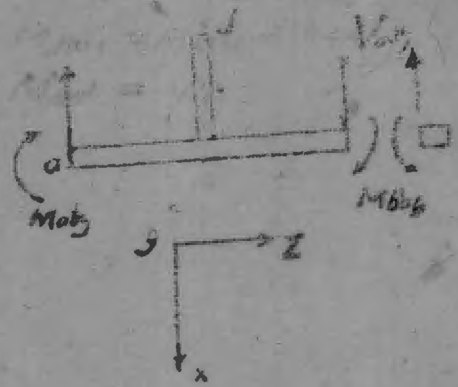


Fig. 8b

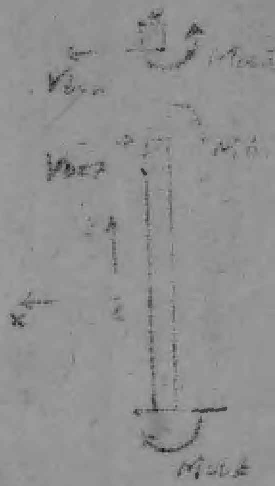


Fig. 8c

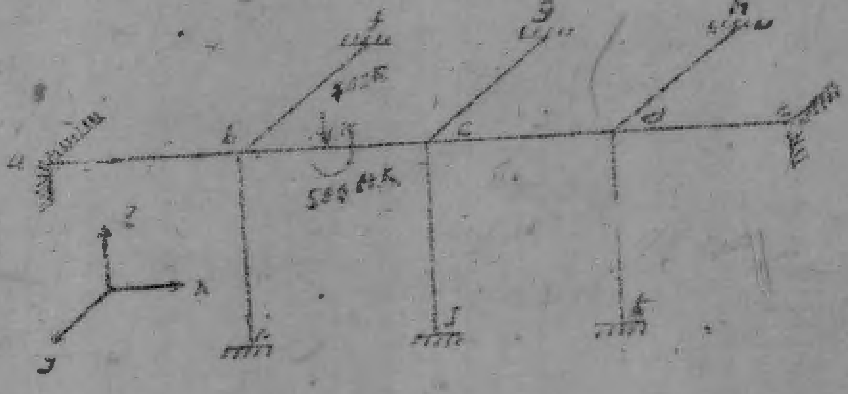


Fig. 9

USE OF INFLUENCE FUNCTION IN THE THEORY OF BENDING OF THIN PLATE

By HSU TSI FAN (范緒箕)

Introduction

Engineering mathematics is generally considered as a collection of mathematical mothods adapted for the solation of problems of different kind. Many of these problems require the evaluation of particular solations of linear differential equation which to satisfy certain boundary conditions the method of Green's function is often used in the theory of beams which is known to many engineers as influence lines or influence function. This paper is endeavoured [illegible] on the validity of this method as applied to the theory of bending of thin plates.

Application

If we have a rectangular thin plate subjecting to a bending load distribution $p(\eta,\xi)$, then the plate undergoes the vertical deflection w. Now let us define an function $\phi(x,y,\eta,\xi)$ as the deflection of the plate at point (x,y) due to application of unit force at point (η,ξ). This function is the so called influence function for the deflection. Hence if the load distribution $p(\eta,\xi)$ is given, in order to find the deflection curve we have first to find this influence function $\phi(x,y,\eta,\xi)$ and then the deflection w at any point (x,y) could be found by evaluating numerically or graphically the following integral

50 $$w(x,y) = \int_0^a \int_0^b P(\eta, \xi)\, \Phi(x, y, \eta, \xi)\, d\eta\, d\xi \quad \text{------ (1)}$$

where a is the length of the plate (along x axis)

b is the width of the plate (along y axis)

In order to do this let us consider the equation of bending of the plate.

$$\nabla^4 w = \frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p(x,y)}{N} \quad \text{-------- (2)}$$

where $N = \frac{Eh^3}{12(1-\nu^2)}$

h = thickness of the plate

E = modulus of elaslicity

ν = poissons ratio

$p(x,y)$ = normally applied load

If we take:

$$w(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn}\, \varphi_m(x)\, f_n(y) \quad \text{-------- (3)}$$

$$p(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{mn}\, \varphi_m(x)\, f_n(y) \quad \text{-------- (4)}$$

The deflection can be expressed as:

$$w(x,y) = -\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{B_{mn}}{N\lambda_{mn}}\, \varphi_m(x)\, f_n(y) \quad \text{----- (5)}$$

Where λs are the characteristic values associated with the problem, and are determined by the homogeneous differential equation

$$\nabla^4 w + \lambda_{mn} w = 0 \quad \text{-------------------- (6)}$$

This equation (6) must exist in the problem, otherwise equation (w) and (4) no longer hold.

According to equation (6), equation (2) can be written as:

$$-\lambda_{mn} w = \frac{p(x,y)}{N} \quad \text{--------------- (7)}$$

Substituting equations (3) and (4) into equation (7) we obtain:

$$-\lambda_{mn} A_{mn} = \frac{B_{mn}}{N}$$

and therefore:

$$A_{mn} = -\frac{B_{mn}}{\lambda_{mn} N}$$

Hence substituting this expression for A_{mn} into equation (3) we obtain equation (5).

Now if we have a load distribution:

$$P(\eta, \xi) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{mn} \varphi_m(\eta) f_n(\xi) \text{ ------- (8)}$$

where η, ξ lies on the same coordinate as x and y, but the point (η ξ) does not necessary coincide with point (x, y,). Then w(x, y) can be expressed in the following form:

$$W(x,y) = -\int_0^a \int_0^b \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{P(\eta, \xi)}{N \lambda_{mn}} \varphi_m(x) \varphi_m(\eta) f_n(y) f_n(\xi) d\eta d\xi$$

Now comparing equation (1) and (8) it follows that:

$$\Phi(x, y/\eta \xi) = -\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\varphi_m(x) f_n(y) \varphi_m(\eta) f_n(\xi)}{N \lambda_{mn}} \text{ ---- (10)}$$

The criterion for this method is that the functions $\varphi_1(x)$, φ ---- and $f_1(y)$, $f_2(y)$ ---- or functions $\varphi_1(\eta)$, $\varphi_2(\eta)$ ----- and $f_1(\xi)$, $f_2(\xi)$ ----- must be orthogonal, e.g. they must satisfy the relations:

$$\left.\begin{aligned} \int_0^a \varphi_m(\eta) \varphi_k(\eta) d\eta = 0 \\ \int_0^b f_n(\xi) f_l(\xi) d\xi = 0 \end{aligned}\right\} \text{ -------------- (11)}$$

where $m \neq k$, and $n \neq l$.

If we imply another restriction that:

$$\left.\begin{aligned} \int_0^a [\varphi_m(\eta)]^2 d\eta = 1 \\ \int_0^b [f_n(\xi)]^2 d\xi = 1 \end{aligned}\right\} \text{ ------------------- (12)}$$

$$- \lambda_{mn} A_{mn} = \frac{C_{mn}}{N}$$

and therefore:

$$A_{mn} = - \frac{C_{mn}}{\lambda_{mn} N}$$

Hence substituting this expression for A_{mn} into equation (3) we obtain equation (5).

Now if we have a load distribution:

$$P(\eta,\xi) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{mn} \varphi_m(\eta) f_n(\xi) \text{ -------}$$

where η, ξ lies on the same coordinate as x, y, but the point (η ξ) does not necessary coincide point (x, y,). Then w(x, y) can be expressed in the following form:

$$W(x,y) = - \int_0^a \int_0^b \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{P(\eta,\xi)}{N \lambda_{mn}} \varphi_m(x) \varphi_m(\eta) f_n(y) f_n(\xi)$$

Now comparing equation (1) and (8) it follows that:

$$g(x,y/\eta\xi) = - \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\varphi_m(x) f_n(y) \varphi_m(\eta) f_n(\xi)}{N \lambda_{mn}} \text{ ----}$$

The criterion for this method is that the functions φ, φ ---- and $f_1(\xi)$, $f_2(\xi)$---- or functions $\varphi_1(\eta)$, ----- and $f_1(\xi)$, $f_2(\xi)$ ----- must be orthogonal, e.. they must satisfy the relations:

$$\left.\begin{aligned} \int_0^a \varphi_m(\eta)\,\varphi_k(\eta)\,d\eta &= 0 \\ \int_0^b f_n(\xi)\,f_\ell(\xi)\,d\xi &= 0 \end{aligned}\right\} \text{ -------------}$$

where $m \neq K$. and $n \neq \ell$.

If we imply another restriction that:

$$\left.\begin{aligned} \int_0^a [\varphi_m(\eta)]^2 d\eta &= 1 \\ \int_0^b [f_n(\xi)]^2 d\xi &= 1 \end{aligned}\right\} \text{ ----------------}$$

o to normalize the functions $\varphi_m(\eta)$ and $f_n(\xi)$, then multiplying both sides of equation (8) by $\varphi_m(\eta) f_n(\xi)$ and integrate between the limits $\eta = 0$ and $\eta = a$. $\xi = c$ and $\xi = b$, we obtain

$$\int_0^a \int_0^b P(\eta,\xi)\,\varphi_m(\eta) f_n(\xi)\, d\eta\, d\xi = B_{mn} \int_0^a [\varphi_m(\eta)]^2 d\eta \int_0^b [f_n(\xi)]^2 d\xi$$

Hence: $b_{mn} = \int_0^a \int_0^b P(\eta,\xi)\,\varphi_m f_m(\xi)\, d\eta\, d\xi$

erefore substituting this into equation (5) equation (9) immediatly follows.

In general, in order to solve a problem we have to find functions $\varphi_m(x)$ $f_n(y)$ and λ_{mn} by mathematical methods. e methods most frequently used are the interation method ich is known to many e gineers as the Vianelo - Stodola thod, the direct methods of the calculus of variations which known as the energy method, Rayleigh - Ritz method, Galer- method etc.

But in examining the equation (10) we see that it is the me expression as for Fourier's coefficient. Hence the validity of Fourier expansion in problems of this kind is evident. Expressing the load into an Fourier Series

$$P = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} P_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$

Comparing this equation with equation (4) and with the normalizing condition (12) in mind we see that:

$$\varphi_m(x) = \sqrt{\frac{2}{a}} \sin\frac{m\pi x}{a}$$

$$f_n(y) = \sqrt{\frac{2}{b}} \sin\frac{n\pi y}{b}$$

原书缺页

原书缺页

可控整流器之臨界電感

沈尚賢

摘要——本文應用Fourier級數求得一簡單之公式,用以計算可控整流器之臨界電感,並附算例以示應用.

(一) 緒言

整流器中電流開始傳導之相角,可藉適當方法,任意控制者,稱為可控整流器(Controllable rectifier),例如電閘管(Thyratron)電燃管(Ignitron)及磁變管(Permatron),均屬此類.整流器所接之輸入過流圈(Input choke)不宜過小,否則整流器之輸出電流,瞬值將有時中斷,對於其運用情形,有不利之影響,應設法避免.適足維持電流連續不斷時所需過流圈之電感,謂為臨界電感(Critical inductance).實際應用時,輸入過流圈之電感,須大於臨界電感,庶不致電流時斷時續.

普通不控制之整流器之臨界電感,可照Termon氏(1)及Scott氏(2)所示之公式,約略計算.至於可控整流器之臨界電感,則Overbeck氏(3)亦曾為文加以討論,惟該文所述有數點尚待商酌.本文則用Stout氏(4)之方法,以分析此問題,乃與Overbeck氏所用者,完全不同.

Stout氏法係先將整流器之輸出電壓波形,依Fourier級數,分成直流及各項交流部分,再就負載(Load)及電路之阻抗,分別計算各直交流部分之電流,然後綜合成實際之電流.此法可不用微分方程式,故頗淺易.

(1) F.E.Termon: "Radio Engineering" 第180頁,第二版(1937)

(2) R.J.Scott: "Rectifier Filter Design" Electronics, 第2[illegible]頁 1938年第六期

(3) W.R.Overbeck: "Critical Inductance and Controlled Rectifiers" Proc. I.R.E. 第27卷第10期 第655-659頁(1939)

(4) M.B Stout: "Analysis of Rectifier Filter Circuit" Elec. Eng. 第54卷第9期第977-984頁.(1935)

(17)

$R_0 = R$ (18)

$Z_n = n\omega L - \frac{1}{n\omega C}$

若所用之電感L為臨界電感L_c則(16)式中n次諧波部份電流之幅值適等於其直流部分即

$$\frac{E_0}{R} = \frac{E_n}{n\omega L_c - \frac{1}{n\omega C}} \quad (20)$$

因之得 $L_c = \frac{K_n R}{n\omega K_0 (1 - E_t/E_0')} + \frac{1}{n^2\omega^2 C}$ 若E_t較E_0為甚少

此即為所求之臨界電感之公式。式中 $E_0' = K_0 E = E_0 + E_t$

而C又極大則復可化簡為

$$L_c = \frac{K_n R}{n\omega K_0}$$

(三)算例

(例甲)有一三相整流器每相之實效電壓為220伏特頻率為50週波所接之負載電阻R為1000歐姆電容C為8兆分法拉特整流管之電壓降E_t為10伏特所控制之延遲傳導相角為30度求其直流輸出電壓及所需之臨界電感。

$E = \sqrt{2} \times 220 = 311$ 伏特

$n\omega = 3 \times 2\pi \times 50 = 942$

自第二圖可得在$\phi = 30°$時

$K_0 = 0.716$ $K_n = 0.358$

故直流輸出電壓E_0為 $K_0 E - E_t = 0.716 \times 311 - 10 = 213$ 伏特

$E_0 = 223$ 伏特

$$L_c = \frac{K_n R}{n\omega K_0 (1 - E_t/E_0')} + \frac{1}{(n\omega)^2 C}$$

$$= \frac{0.358 \times 1000}{942.5 \times 0.716 (1 - \frac{10}{223})} + \frac{1}{(942.5)^2 \times 8 \times 10^{-6}}$$

$= 0.695$ 亨利 (約0.7亨利)

(例乙)一單相全波可控整流器其整流管於傳導時之電壓降為10伏特而於衝燃[5] Firing 至少須40伏特電容C為20兆分法拉特電源頻率為50週波整流器之直流輸出電壓可任意控制使在100至200伏特之間其最小直流輸出為0.5安培求所需電源電壓及濾流器之臨界電感。

$n\omega = 2 \times 2\pi \times 50 = 628$

在200伏特輸出時相角 必為最小適足以獲得所需之衝燃電壓以開始

(四)結論

上列分析中有下列各項假定：

(1)在傳導時整流管之電壓降為一常數 E_t.

(2) n 次諧波以上之電流可以忽視.

(3)電阻 R 與電容阻抗 $\frac{1}{n\omega C}$ 相較為甚大，故 R 與 C 並聯時 R 可以忽視。

(4)過流圈並無電阻，電源變壓器並無電阻及磁漏電感。

第(1)項假定在充氣式整流管中與實際情形尚稱符合，第(2)項假定亦尚與通常情形符合，例如在三相整流器中在 ϕ 為零時 $\frac{E_3}{E_1}$ 及 $\frac{E_2}{E}$ 之比為 0.288 與 0.100，而 $\frac{E_1}{E_3}$ 及 $\frac{E_1}{E_2}$ 約為 3 與 9，故 $\frac{E_3}{E_1}$ 及 $\frac{E_2}{E}$ 之比僅為 0.076 與 0.011，由於此項假定故實際所需之臨界電感當較由(22)式計算所得者須略大。第(3)項假定通常亦易於適合，設不加忽視則所需之臨界電感較(20)式所示者略小，故由於第(2)(3)兩項假定所生之誤差可相互抵消一部分。第(4)項假定實際上亦可容許，如須計算較為準確，可將電阻包括於 R 內，而將變壓器磁漏電感認為過流圈電感之一部。

由本文分析所得之公式不適用於單相半波整流器，對於單相全波整流器則可以 n 等於 2 代入即得，而對於三相全波整流器或三相半波雙 Y 整流器之情形則與六相半波整流器相似，以 n 等於 6 代入各公式即可。

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(4) M. B. Stout: "Analysis of Rectifier Filter Circuit" Elec. Eng., 第 54 卷第 9 期第 977-984 頁, (1935).

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10

USE OF MOLDS IN INDUSTRIAL FERMENTATIONS

S.C. Pan (潘尚貞) and C.Y. Yu (俞其塑)

The term molds as is popularly used, refers to a group of lower fungi, growing in the form of mycelium and reproducing commonly by means of asexual spores. Their metabolic activities are of wide diversity, attacking all kinds of organic compound — fats, proteins and carbohydrates including cellulose. Fulmer and Werkman (1) list 32 non-nitrogenous compounds that are fermented by A. niger, and more than 40 compounds have been isolated among their metabolic products from glucose alone. (2) (3) In general, their metabolic changes either involve a simple hydrolysis such as the saccharification of starch or a partial aerobic oxidation such as the production of gluconic acid from glucose, both not been mistakenly called fermentations. The utilization of the former activity in industry has been practiced since ancient times such as the hydrolysis of rice starch for 'wine' fermentation in oriental contries or cheese ripening in the occidental, while the utilization of the latter activity is of recent origin which has led to the development of new industries. In the present discussion, the industries based on the latter activity will be taken up first

Citric Acid

The production of citric and oxalic acids by molds has been known since the classical study of Wehmer. He believed that the citric acid formation was characteristic for the new genus Citromyces (Penicillium like) that he introduced and the formation of oxalic acid was characteristic for the Aspergilli. Later researches showed that a number of Penicillium and Aspergillus species were capable of producing the two acids and Wehmer's idea was consequently abandoned. Thom (4) in his book "The Penicillia" offers no special group for Wehmer's Citromyces.

Although the production of oxalic acid by molds has never been applied commercially due to the cheaper chemical method, the production of citric acid has given rise to a new industry. Wehmer's Citromyces was discarded for commercial uses due to their weak fermentation power and Zahorski (5) first introduced A. niger as the suitable organism for citric fermentation. This was confirmed by Thom and Church (6) who pointed out that "Aspergillus niger does not designate a definite strain or species" but is used to designate "a whole group of black Aspergilli with fundamental characteristics in common". The fermentation power varies widely among the strains.

The favorable conditions for citric acid formation was first studied in detail by Currie (7). His result forms the basis of the process now practiced in the Charles

Although the details of the
Pfizer Co. of Brooklyn, U.S.A
process are kept secret, the general features are
known as fellows. Sucrose is employed as the substrate
as it introduces the least amount of impurities and is ferment-
ed the fastest. Aeration, temperature, and the length of
the fermentation periods play significant roles in the eff-
iciency of the fermentations while the two most important
factors seem to be the inital reaction and the amount
of nitrogenous nutrients. By acidifying the medium with
hydrochloric acid to pH 3.5 or lower, the citric acid fermen-
tation goes at a high rate while the formation of oxalic acid
is greatly suppressed. Citric acid accumulates without
causing any trouble and no calcium carbonate should
therefore be added. Although there is no definite agreement
about the most suitable natrogen source, the nutrient
concentration seems to exert important influence upon
the efficiency of the fermentation as it has long been
known that well norrished heavy mold mat produces
less acid than a thin growth. Of the mineral elements,
potassium, phosphorous, magnesium and sulfur are found
essential. Currie's medium therefore includes: sucrose
12.5—15%, NH_4NO_3 0.2—0.25% KH_2PO_4 0.075—0.10% and
$MgSO_4$ 0.02—0.025%. Recent work of Doelger and
Prescott (8) agreed upon these general features and
it also shows that there are other factors which tend
to fluctuate the consistency of the process.

As the mold fermentation is a strict aerobic
process, the accessibity of oxygen is apparently an

important factor. Furthermore, the reaction takes place only in the film of the mold growth lying upon the surface of the medium specially designed equipment is apparently necessary to meet these two requirements. As has been reported by Doelger and Prescott and also practiced commercially (9), shallow pans with ventilation [illegible] to be the most suitable. Shallow alluminum pan [illegible] purity have been adopted as they avoids corros[ion] and the harmful effect of other metals such copper, iron etc. Usually, 15–20% sucrose solution with the nutrient salts is seeded with the spores in the pans at 25°–35°. Good growth on the surface occurs in 2 days and the fermentation is well underway in 4 days. The fermentation is usually complete in 7–10 days and the acid can be crystallized out directly from the medium, after a yeast fermentation of the residual sugar, or can be separated as calcium citrate on the addition of $CaCO_3$. The yield is about 60% of the sugar used. This is however far too low in comparison with the theoretical value as indicated by the following equation $2C_6H_{12}O_6 + 3O_2 \longrightarrow 2C_6H_8O_7 + 4H_2O$
Further researches are therefore needed for improving the process.

Gluconic Acid

Although gluconic acid is also a common metabolic product of molds, it was not known until its first isolation, by Molleard in 1922 (10), from the fermentation by A. niger. It is formed by a partial deprivation of the inorganic nutrients. The detailed studies of Bernhau[er] er(11) and other authors indicate that the essential factors for gluconic acid production are high initial pH and low nitrogen content of the medium, which suppress the formation of citric and oxalic acids. Addition of $CaCO_3$ is therefore necessary for this fermentation. Glucose, sucrose and maltose prove to be the suitable substrates. Other conditions, of course, are the same as in the citric fermentation.

The application of this type of fermentation to industry was developed mainly by the extensive researches of May, Herrick, and their associates of the Bureau of Chemistry and Soils, U.S.D.A. They first worked with Pencillium species and found that by carrying out the fermentation in aerated shallow aluminum pans, the fermentation went at a very high rate, completing in 3-5 days. With an initial glucose concentration of as high as 20%, a yield of 80% of the theoretical was yet obtainable. (12) Later they found that the fermentation period could be further shortened if the fermentation was carried out in a glass vessel aerated through a sintered glass false bottom where submerged growth of the mold took place.(13) On 30 principle, they designed for gluconic acid fermentation a revolving drum equipped with internal buckets and baffles and a means of introduction and removed of air under pressure.(14) A. niger was adopted in place of the Penicillia due to the abundant and uniform sporulating power of the former thus insuring a consistency in yield and rate of fermentation. The spores from the growth in a specified medium is first germin[ated] in a peptone-glucose-salts medium, and then inoculate[d] into the main fermentation midium containing: 15.0% gl[u]cose, 0.0188% KH_2PO_4, 0.0156% $MgSO_4 \cdot 7H_2O$, 0.0388% (NH_4) PO_4 and $CaCO_3$ in an amount slightly lower than to neu[-] tralize the gluconic acid theoretically obtainable (15) [...] 3.21. of the medium with 75 gm. of $CaCO_3$ per drum. Under the optimum conditions that they worked out (16) [...] 13 r.p.m. for agitation, 1.0—1.21 per min. for air flow and 30 lb. per sq. in. for air pressure, the fermentation starts after 2—3 hours lag and goes on at such a rapidity that it is complete in 18 hours. During fermentation the pH of the medium drops from 5.5—6.0 at the begining to 3.5 at the end. the yield of gluconic acid is higher than 84% based on glucose available or 97% based on glucose consumed. they also found that glucose concentration of 25—30% can be fermented with equally high efficiency and propose to dispense with the sterilization as no appreciable contamanation will occur in such a short time interval.

Use of Molds in Alcoholic fermentation of Grain.

Many molds are known to be rich in diastase. this has long been utilized in the oriental countries for hydrolysis of starchy material in making rice wine and distilled liquors. the general procedure involves first the preparation of the starter which is called "chu" (麯) in chinese, sometimes also known as Chinese yeast, or "Koji" in Japanese, and the hydrolysis proper. The starter is essentially a nutrient substarte upon which abundant mold growth is to be raised. Usually, a mexture of wheat, barley, becans, and peas, are made into cakes, cooked and seeded with the desired mold, or in old practice they are left to "spontaneous growth" from spores carried in air. Takamine selected wheat bran in place of the above mentioned material for Takadiastase production because it is a cheap, bulky, fibrous nutrient base (6). Asp. oryzae (rather members of A. flavus-oryzae group) and mucor rouxii are the most important organisms occuring naturally or artificially employed. The meterial is incubated at a temperature slightly above the room temperature and at a proper humidity. Growth is interrupted at the time of maximum enzyme activity, usually 3-5 day. It is then dried and ready for market or they can be extracted with water and the diastase precipitated with alcohol or ammonium sulfate followed by dialysis (17). The preparations are known as Taka diastase, polysime or ptotozyme etc.

For alcoholic fermentation of grains, the cooked raw material is mixed with the starter and incubated. Inoculum of yeast is added after the saccharification of the starch becomes well underway. Modifications in the details of the process are practiced in different regions giving different types of products.

The same method can be applied to making industrial alcohol. Recent results of Underkofler et al (23) (24) indicate that saccharification of a corn with moldy bran— growth of Asp. orysae on wheat br— gives far better yield of alcohol than with barley m

Amylo process of Alcoholic Fermentation

The Utilization of molds for saccharifying th starchy material for industrial alcohol production has been practiced in European countries under the namo amploprocess (18)(19). It has the advantages over the malt amylase in the fact that saccharification takes place under sterile conditions, thus giving higher yield and greater purity to the product. This process was developed by Calmette and Boidin who first isolated the Mucor rouxii or Amyloyces rouxii as wa called by him, from the "Chinese yeast". This mold is capable of giving a submerged growth in an aerated liqu culture in the form of detached, oval-shaped yeast lik cells of extremely high amylolytic power.

The industrial process involves: 1. soaking of grains, e.g. crushed corn with a little hydrochloric acid, and steamed to a gelatinized mash, 2 inoculating with 1/10 — 1/6 of its volume of a pure culture of the Mucor species in the form of submerged growth; and 3. incubating and aerating for 24 hours when saccharification is practically complete and the mash is ready to be incolated with yeast. This process has been repeatedly improved since its first discovery, through the introduction of stronger mold strains. With proper choice of the mold and yeast (Sacch. anamensis), the two can be inoculated at the same time and the whole process is run at 35°—38°C. The fermentation completes itsself in 48 hours with a yield of alcohol as high as 97.5% of the theoretical

Soy Fermentation

Another use of the mold in the fermentation industry is based upon the action of their proteolytic enzymes. The soybean sauce of the oriental countries is one of the prominent examples. The process (20) also involves first the preparation of the starter ("chu or "Koji"). this is made essentially in the same way as the starter for alcoholic fermentation, except soybean and wheat constitute the raw materials and strains ASP. oryzae of high protedlytic power and low amylase activity is prefered. The matured starter is subjected to a ripening process. It is immersed in a brine of high salt content (20° to 22° Bé) in which a very slow digestion process is allowed to develop in a period of several months to 2-3 years, at room temperature or slightly higher. Special types of yeasts (Zygosacchar-omyces species) are believed to play some part in this digesition process. The final product is a dark heavy salty liquid with flavors attributed to various factors principally to the metabolic products of the molds. The process is finished by filter pressing and pasturization of the sauce. Slight modifications in the details of the process leads to different types of the product.

Ripening of Cheese

The proteolytic enzymes of the penicillia is al

Utilized in the ripening of Roquefort and Camenbert cheeses (21). The Camenbert cheese is a kind of soft cheese. The curd is first made into thin cakes salted and inoculated with a pure culture of Penicillium camenberti or with spores carried in the air in the incubation room. The incubation temperature is usually kept at 13°–14°C. and a floccose white covering of the mold develops on the surface in about a week. Proteolysis becomes apparent as indicated by the softening of the surface which is in direct contact with the mold growth. It extends from all sides toward the center while the acidity gradually disappears. In about four weeks the whole mass becomes softened and assumes a characteristic flavor and a buttery texture. Oidium lactis is also believed to play a part in the development of flavor and aroma.

In contrast to Camenbert Cheese, Roquefort cheese is a variety of hard cheese and it is ripened by the development of P. roqueforti throughout the entire interior of the curd. The curd is so prepared as to leave cracks, channels, and openings between the particles or it can be punched so that air can penetrate through the holes. P. roqueforti grows in these air spaces and its green color makes this type of cheese also known as blue veined cheese. The characteristic flavor is due to the splitting of the butter fat by the organism resulting in the formation of a definite mixture of fatty acids. The success of this process is said to depend upon the tolerance of P. roqueforti to low oxygen tension and high salt content. (21) Golding (22), however, recently reported that CO_2 instead of O_2 is the controling factor for the growth of this mold within the cheese.

Gallic Acid

Enzyme activities of molds also find applications in the production of gallic acid from tannins. Tannins are glucosides of gallic acid complex or its related compounds. They are present in the barks and leaves of a number of trees and in the gall nuts produced by varous species of oak when stimulated by insects. Van Tieghem in 1867 first noticed that Asp. niger and P. glaucum when growing in a tannin liquor hydrolyzed the tannins into gallic acid. This hydrolysis was later shown to be due to the tannase activity of the molds. The present day mycological process (25) of making gallic acid consists essentially of growing a pure culture of Asp niger in a sterile tannin extract in accordance with Calmette's patent (26) or some modification of it. In China, Fong and his associates have recently made extensive studies upon the gallic acid fermentation (27). They are making gallic acid with more or less the same process as mentioned above — growing Asp niger on a tannin extract.

Molds in Tanning and Other Industries

In tanning industry, mold protease preparations are sometimes applied for piring and bating, i.e. to soften the collagen for absorbing the tannin liquor (18). On the other hand, certain molds (e.g. Asp. niger) are capable of resisring high tannin content and therefore cause spoilage of the tannin liquor (6).

Molds are also used in other industries (18) like tobacco and coffee fermentations, the desizing in the textile industry, the degumming of silk etc. They are of minor importance and are not discussed in detall here.

Conclusion

This brief discussion shows that molds are of exceedingly diverse metabolic activities. Many of them have been utilized in industries since ancient times, while recent resarches have opened up many new fields of application. The development of these industries is yet in its infancy. Future investigations are needed for their improvement.

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編輯餘瀋

本刊復刊之首期原定於七月間出版，因遭印刷事業落後，加以抗戰期間人力物力均感缺[illegible]印刷所耗時間倍於平時，以致出版略為延期。首期刊作二卷二期，係銜接最在杭州出版至二[illegible]一期，八一三事變後，浙大奉令遷移，本刊不得已[illegible]中輟，今為仍舊貫起見，故復刊後卷序期數與早[illegible]聯接。

此期篇幅較上期增多，插圖亦較叢積，故印刷[illegible]時乃意中之事。本刊未能按預定日期與社會人士相[illegible]見，誠屬萬分抱憾，實因非常時期出版困難，以致有[illegible]讀者之盼望，此本刊所不得不[illegible]諒於諸君子者也。

考印刷與羅盤火藥為吾國最早三大發明，蔡倫[illegible]造紙遠在漢代，徒以故步自封，反落人後，近年來印刷[illegible]機與材料紙張等之供給多賴舶來，在平時固予取予[illegible]求，供應不虞匱乏，在戰時則來源不繼，未免感到困難。仰給舶來之各項工業製品，莫不如是，若印刷者僅其[illegible]端而已。雖然，前事不忘，後事之師也；從艱難抗戰中[illegible]獲得之經驗彌足珍貴，苟能本此經驗以研究將來工[illegible]業建設之計劃，以及與該計劃有關之各種工程問[illegible]則其意義與價值豈不重且大耶？

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第三卷 第一二期

ENGINEERING QUARTERLY
UNIVERSITY OF CHEKIANG

Vol. 3 No. 1 & 2 June 1943

目錄

中華民國三十二年六月出版

電抗調頻器之理論與設計

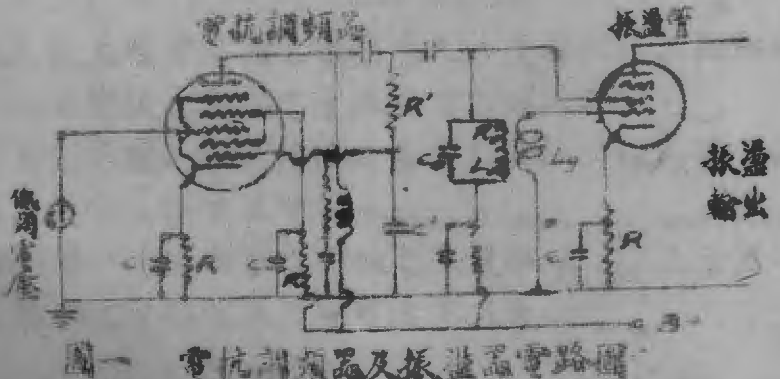
(The Theory & Design of a Reactance Modulator)

馬師亮 毛樣琮 顧論訓

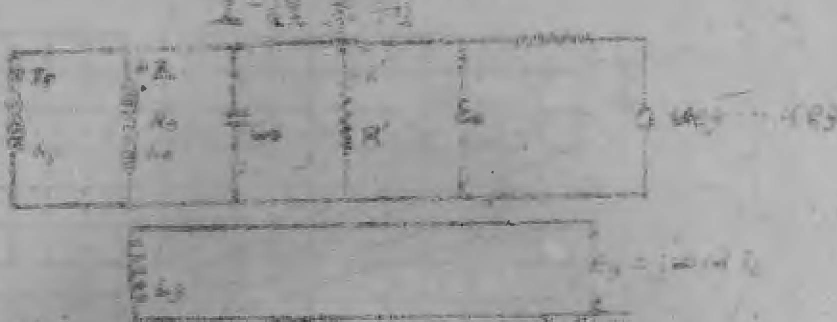
近數年來調頻(Frequency Modulation)作用之完成多用"五柵管"(pentagrid tube)並聯於振盪器(oscillator)之並聯電路(tank circuit)上五柵管所生之板流(plate current)其頻率與振盪器相同若將五柵管第一柵極所加電壓之相位予以適當之調節則板流之相位可較在並聯電路兩端間電壓之相位差九十度此五柵管之性質猶如一電抗(reactance)其電抗量之大小可由加於第一柵極間之電壓控制之由於電抗量之變更振盪器之頻率亦隨之變動因此五柵管名之曰電抗調頻器(Reactance Modulator)

(1)電抗調頻器之理論

電抗調頻器之簡單電路如圖一所示圖二為其相當(equivalent)電路



圖一 電抗調頻器及振盪器電路圖



圖二 電抗調制器及振盪器之耦合電路

振盪器並聯電路內含有 R_0、L_0 及 C_0，又 R 及 C 組成一串聯電路與並聯電路平行之。通常 R 之值遠較諸電容 C 之電抗值為大，故串聯電路內之電流與並聯電路間之電壓幾同相。設並聯電路間之電壓為

$$e_0 = \hat{E}_0 \sin\omega t$$

則串聯電路內之電流約為

$$i = \frac{\hat{E}_0}{R - j\frac{1}{\omega C}} \sin\omega t \quad (2)$$

因 $\frac{1}{\omega C}$ 之值較 R 甚小，故亦可寫為 $i = \frac{\hat{E}_0}{R}$

而C電容兩端間之電壓將為

$$e_g = -\frac{\hat{E}_0}{\omega C R}\cos\omega t \quad (3)$$

(3)式表示加入五極管第一柵極之電壓與並聯電路間之電壓差九十度。由於第三柵極音頻電壓之變化，則板流將為

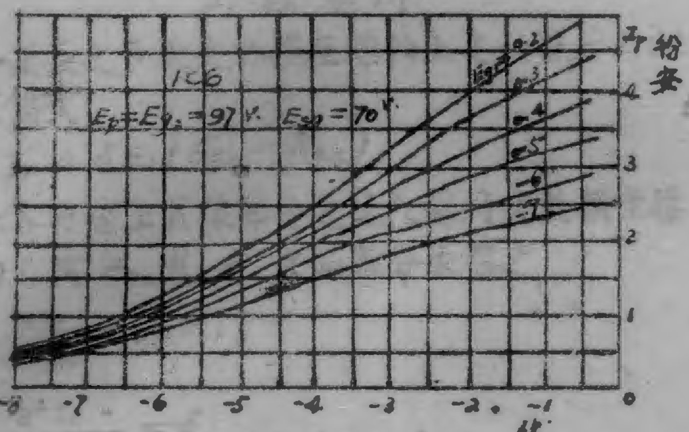
$$i_p = -\hat{I}_p(1 + m\cos pt)\cos\omega t \quad (4)$$

(4)式所示之板流較並聯電路間之電壓遲九十度，因此則五極管即相當於一電導力電抗。(4)式中板流求得之方法及其證明詳於下：

若五極管其他各極之電位均保持不變，則其板流將為第一柵極及控制極之函數。若用五極變頻管(Pentagrid converter tube)代替之，其板流將為第一柵極與第四柵極之函數。令以 E_g 表示第一柵之電壓，E_{g4} 表示第四柵之電壓，則板流即可用下式表示之

$$i_p = f(E_g, E_{g4})$$

在各種不同 E_g 值時之 i_p 與 E_{g4} 之關係如圖三所示



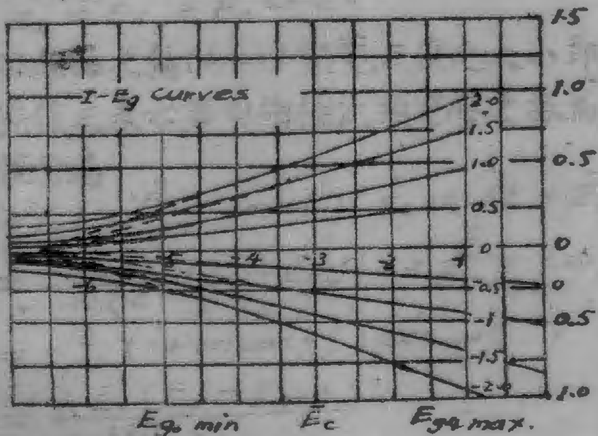
圖三 $I_p - E_{g4}$

依第三圖所示之結果最堪注意若將圖中曲線變換座標取適當之E_{g1}如E_{g1}為假定零電位(zero potential)，並以此曲線為橫座標軸以任何值E_{g1}時之板流與在E_{g1}等於$\bar{E}_{g1}$時之板流之差爲新縱座標此種變換可由下列兩式求得之

$$E_1 = E_{g1} + \bar{E}_{g1} \quad (6)$$

$$I_r = I_{pa} + (-\bar{I}_{pa}) \quad (7)$$

(6)(7)兩式中 E_1表示變換後E_{g1}之值 I_{pa}表示任何E_{g1}值之板流，$\bar{I}_{pa}$表示當E_{g1}等於$\bar{E}_{g1}$時之板流，I_r表示變換後之板流經過變換後各種不同E_1值之$I_r - E_{g4}$曲線如圖四所示



圖四 $I_r - E_{g4}$ 曲線

其斜度(slope)與未經變換前顯然不同變換後之曲線大部分幾爲直線，

若將直線部份延長之多數交於橫座標軸上之一點若 I_r 以 E_1 及 E_{g4} 表示,即可得下列簡式

$$I_r = k E_1 (E_{g4} - \bar{E}_{g4}) \qquad (8)$$

式中 $\bar{E}_{g4}$ 表示各曲線上直線部份之延長線同交於橫坐標 E_{g4} 之值,k 一常數可由 I_r 對 E_{g4} 及 E_1 之偏微分求得之

$$\frac{\partial I_r}{\partial E_{g4}} = k E_1 \qquad (9)$$

$$\frac{\partial^2 I_r}{\partial E_{g4} \partial E_1} = k \qquad (10)$$

$$I_r = \frac{\partial^2 I_r}{\partial E_{g4} \partial E_1} E_1 (E_{g4} - \bar{E}_{g4}) \qquad (8)$$

以電位數伏(volt)間,k 為一常數.

五柵管第一柵極間所加電壓之瞬時值(Instantaneous value)為

$$e_1 = -\frac{\hat{E}_o}{\omega R'C'} \cos \omega t$$

第四柵極間之瞬時值為

$$e_4 = \bar{E}_c + \hat{E}_a \cos pt$$

式中 $\bar{E}_c$ 表示第四柵極之偏(bios)電壓,$\bar{E}_c$ 值之大小即為 $I_r - E_{g4}$ 曲線直線部份之中點值,$\hat{E}_a$ 表示第四柵極低頻電壓之振幅,若將 e_1 及 e_4 代入式則 I_r 之瞬時值當為

$$I_r = k\left(-\frac{\hat{E}_o}{\omega R'C'} \cos \omega t\right)\left(\hat{E}_a \cos pt + \bar{E}_c - \bar{E}_{g4}\right)$$

$$= -\frac{k \hat{E}_o}{\omega R'C'} (\bar{E}_c - \bar{E}_{g4}) \left[1 + \frac{\hat{E}_a}{\bar{E}_c - \bar{E}_{g4}} \cos pt\right] \cos \omega t$$

$$= -\hat{I}_r (1 + m \cos pt) \cos \omega t \qquad (11)$$

式中

$$\hat{I}_r = \frac{k \hat{E}_o (\bar{E}_c - \bar{E}_{g4})}{\omega R'C'} \qquad (12)$$

$$m = \frac{\hat{E}_a}{\bar{E}_c - \bar{E}_{g4}} \qquad (13)$$

(11)式中兩常數當由(12)及(13)兩式表示之,以下當需求頻偏(Deviation

frequency與 $\hat{E}'_0$, $\hat{E}_a$ 及電路常數之關係

從耦當電路中知

$$I_r=\frac{\hat{I}_r}{\sqrt{2}}(1+m\cos\omega t)$$

$$I'=\frac{\hat{E}_0}{\sqrt{2}R}$$

$$E_g=j\omega M I_L$$

由振蕩電路內求得下列四式

$$j\omega M I_L=I_p r_p+I_L(R_0+j\omega L_0) \quad (14)$$

$$-j\frac{I_c}{\omega C_0}=I_L(R_0+j\omega L_0)$$

或

$$I_c=j\omega I_L C_0(R_0+j\omega L_0) \quad (15)$$

$$I'R'=I_L(R_0+j\omega L_0)$$

$$I'=\frac{I_L}{R'}(R_0+j\omega L_0) \quad (16)$$

或

$$jI_r\omega L_r=I_L(R_0+j\omega L_0)$$

$$I_r=-j\frac{I_L}{\omega L_r}(R_0+j\omega L_0) \quad (17)$$

其總流為

$$I_p=I_L+I_r+I_c+I_c'$$

將(15)(16)(17)及(18)式代入(14)式則

$$j\omega M I_L=r_p\left[I_L+j\omega C_0(R_0+j\omega L_0)I_L+\frac{1}{R'}(R_0+j\omega L_0)I_L-j\frac{I_L}{\omega L_r}(R_0+j\omega L_0)\right]+I_L(R_0+j\omega L_0)$$

振盪器之頻率可將上式中之實數(real value)部份等於零求之

$$r_p+r_p\omega^2L_0C_0+\frac{R_0}{R'}r_p+r_p\frac{\omega L_0}{\omega L_r}+R_0=0$$

或

$$-\omega^2L_0C_0+\frac{\omega L_0}{\omega L_r}+\left\{1+\frac{R_0}{r_p}+\frac{R_0}{R'}\right\}=0. \quad (19)$$

L_r 並非常數但 $\frac{1}{\omega L_r}$ 值可用下式表式之

$$\frac{1}{\omega L_r}=\frac{\hat{I}_r(1+m\cos pt)}{\hat{E}_0}=\frac{k\hat{E}_0(\bar{E}_c-\bar{E}_{ga})}{\omega R'C'\hat{E}_0}(1+m\cos pt)$$

$$=k\frac{\bar{E}_c-\bar{E}_{ga}}{\omega C'R'}(1+m\cos pt)=\frac{G}{\omega}(1+m\cos pt) \quad (20)$$

上式中 $G=\frac{k(\bar{E}_c-\bar{E}_{ga})}{R'C'}$ 將G代入(19)式，ω之值將為

$$\omega=\sqrt{\frac{1}{L_0C_0}\left\{1+\frac{R_0}{R'}+\frac{R_0}{r_p}+L_0G(1+m\cos pt)\right\}}$$

$$\omega = \omega_0 \sqrt{\alpha + \beta \cos pt} \qquad (21)$$

(21)式中 $\omega_0 = \frac{1}{\sqrt{L_0 C_0}}$，$\alpha = 1 + \frac{R_0}{R_i} + \frac{R_0}{R_p} + L_0 G$，$\beta = L_0 G m$ 但(21)式亦可用二項定理(Binomial theorem)展開如下

$$\omega = \omega_0 \left\{ \alpha^{1/2} + \tfrac{1}{2}\alpha^{-\frac{1}{2}}\beta \cos pt - \tfrac{1}{8}\alpha^{-3/2}\beta^2 \cos^2 pt + \tfrac{1}{16}\alpha^{-5/2}\beta^3 \cos^3 pt + \cdots \right\} \qquad (22)$$

顯然可知頻偏與第四柵極電壓之關係不為一直線若β值很小仍可有直線性質此點與實際情形頗為相符合。

$$\omega = \omega_0 \left\{ \alpha^{1/2} + \tfrac{1}{2}\alpha^{-\frac{1}{2}}\beta \cos pt \right\} \qquad (23)$$

$$\begin{aligned} f &= f_0 \left\{ \alpha^{1/2} + \tfrac{1}{2}\alpha^{-\frac{1}{2}}\beta \cos pt \right\} \\ &= \alpha^{1/2} f_0 \left\{ 1 + \frac{\beta}{2\alpha} \cos pt \right\} \\ &= f_c \left\{ 1 + mf \cos pt \right\} \end{aligned} \qquad (24)$$

(24)式中 f_c 表示載波頻率(carrier frequency)亦有稱為平均頻率(mean frequency)因頻差係以此為準常數 mf 可稱頻偏係數(coefficient of frequency deviation)若五柵管之性質已知在電路常數決定後(21)式中之α將為一固定常數β亦可寫為

$$\beta = L_0 G \frac{\hat{E}_0}{(E_c - E_{g4})}$$

$$mf = \frac{L_0 G}{2\alpha} \frac{\hat{E}_0}{(E_c - E_{g4})}$$

從此可知頻偏係數與第四柵極之低頻電壓成正比若α為常數β較α為小則直線調幅(Linear Modulation)當可期實際上若在E及E_g數伏間直線調幅決無問題。

又振盪管內板流之交流部分將為

$$\begin{aligned} i_p &= A \sin 2\pi \int_0^t f_c (1 + mf \cos pt)\, dt \\ &= A \sin \left\{ \omega_c t + \frac{\omega_c mf}{p} \sin pt \right\} \end{aligned} \qquad (25)$$

(25)式為調頻電流之標準式。

(14) 電抗調頻器及調頻振盪器之設計

於設計中最先需决定者為載波頻率及最大頻差，當數然後選擇五柵及决定電路常數從已决定之五柵管由實驗可以求得 $\hat{E}_{a\,max}$, $\hat{E}_{c\,max}$, $\bar{E}_{g4}$, $\bar{E}_{g1}$ 及 k 之值設計中需用之公式列表如下：

(A) $k = \frac{\partial^2 I_r}{2\bar{E}_{g4}\partial E_1}$ (10)

(B) $m = \frac{\hat{E}_a}{\bar{E}_c - \bar{E}_{g4}}$ (13)

(C) $G = \frac{k(\bar{E}_c - \bar{E}_{g4})}{RC'}$

(D) $\omega_0 = \frac{1}{\sqrt{L_0C_0}}$ (21)

(E) $\alpha = 1 + \frac{R_0}{R_1} + \frac{R_0}{r_p} + L_0G$ (21)

若所用者為去載波調頻 (Suppressed carrier modulation) 法則

$$\alpha = 1 + \frac{R_0}{R'} + \frac{R_0}{r_p}$$

(F) $\beta = L_0Gm$ (21)

(G) $f_c = f_0\alpha^{1/2}$ (24)

(H) $mf = \frac{\beta}{2\alpha}$ (24)

調頻振盪器之設計除加 $\frac{R_0}{R'}$ 及 L_0G 兩項外其餘與通常之振盪器完全相同若將 R' 值加大則 R'-C' 串聯電路內高頻功率之損失可以減小即 $\frac{R_0}{R'}$ 值小 L_0G 亦可用 f_0, f_c 及 f_d $(=f_c\,mf)$ 表示之

$$L_0G = \frac{2f_cf_d}{f_0^2m} = \frac{2f_cf_d}{f_0^2}\left(\frac{\bar{E}_c - \bar{E}_{g4}}{\hat{E}_a}\right)$$

若用去載波調法則(21)式中無 L_0G 項因此振盪器之設計亦很簡單

若欲免除電抗調頻器內之畸變 (Distortion) 則下列三條件必須適合.

$$\hat{E}_a \leq \hat{E}_{a\,max}$$
$$\hat{E}_c \leq \hat{E}_{c\,max}$$
$$\beta \ll \alpha$$

$\hat{E}_{a\,max}$ 表示加於第四柵極低頻電壓之最大振幅即

$\hat{E}_{a\,max} = \frac{E_{g4\,max} - E_{g4\,min}}{2}$ $\hat{E}_{c\,max}$ 表示加於第一柵極 $\hat{E}_c$ 之最大

振幅在 $E_{g4\,max.}$ 及 $E_{g4\,min.}$ 間，$Z_r - E_{g4}$ 曲線須仍為直線。

$$\hat{E}_c = \frac{\hat{E}}{\omega R'C'}$$

$$\hat{E}_{c\,max.} = \frac{\hat{E}_o}{\omega R'C'_{min.}}$$

$$C'_{min.} = \frac{\hat{E}_o}{\hat{E}_{c\,max.}}\left(\frac{1}{\omega_c R'}\right) \qquad (25)$$

若欲無畸變發生，$R-C$ 電路中 C 不能小於(25)式所求之值 C 之值可從(21)及(24)式中求得之

$$C' = \frac{kL_o\hat{E}_a f_o^2}{2f_c f_d R'} = \frac{kL_o\hat{E}_a}{2\pi f R'}\left(\frac{f_o}{f_c}\right)^2 \qquad (26)$$

$$\frac{C'}{C'_{min.}} = \frac{\omega_c\hat{E}_a\hat{E}_{c\,max.}kL_o f}{2\hat{E}_o f_c f_d} = \frac{\omega_c\hat{E}_a\hat{E}_{c\,max.}kL_o}{2\pi f\,\hat{E}_c}\left(\frac{f_o}{f_c}\right)^2 \qquad (27)$$

此比值 $\frac{C'}{C'_{min.}}$ 必須大於一始能避免畸變，若 R 之值已選定 C 即可從(26)式中求得。

(III)電抗調頻器設計之實例

今以兩隻五柵管 106 為電抗調頻管，其線路圖與去載波法相同，將高頻電壓加於五柵管之第一柵極，而低頻電壓加於第四柵極，從實驗所求得之常數如下：

(1) $K = 87.3 \times 10^{-6}$

(2) $\hat{E}_{a\,max.} = 2.0$ 伏

(3) $\hat{E}_{g\,max.} = 2.0$ 伏

(4) $\bar{E}_c = -3.0$ 伏

(5) $\bar{E}_g = -6.5$ 伏

(A)已知之條件

$R_a = 50\ \Omega$　　　　$r_p = 20,000\ \Omega$

$R' = 50,000\ \Omega$　　　　$g_m = 5.0\times10^{-5}$ ʊ/特

$f_a = 5000$ 週/秒　　　　$\hat{E}_s = 100$ 伏

$\hat{E}_a = 50$ 伏（主載波調幅）

(B)依計算所求得之常數

(1)　$m = \frac{E_A}{E_0 - E_{34}} = \frac{2}{3.5} = 57.2\%$

(2)　$\alpha = 1 + \frac{R_a}{R'} + \frac{R_a}{r_p} = 1.0015$

(3)　$f_0 = 1.0015 f_0' \doteq f_0$

(4)　$\beta = 2\alpha m g = 2\times\frac{5\times10^{3}}{5\times10^{6}} = 2\times10^{-3}$

(5)　$f_0 = \frac{1}{2\pi\sqrt{L_0 C_0}}$

若 $C_0 = 100$ 微微法拉特

$$L_0 = \frac{1}{(2\pi\times5\times10^{6})^2\times C_0} = 10.1 \text{ 微亨利}$$

$$B = L_0 G_m = \frac{L_0 K \hat{E}_a}{R' C} = \frac{L_0 K \hat{E}_a}{C C_0 R'}$$

$$C = \frac{10.1\times10^{-6}\times 330000^{-5}}{2\times10^{-3}\times10^{5}\times0.5} = 17.6 \text{ 微微法拉特}$$

二

$$L_m = \frac{\hat{E}_0/2}{\hat{E}_{l\,max}\,\omega R'} = \frac{100}{2\times2\pi\times5\times10^{6}\times10^{5}} = 15.92 \text{ 微微法拉特}$$

結論

由(1)(2)兩節所得之結果知電抗調頻器類較阿氏(Armstrong)之調頻法為優因阿氏之發射機需用多級之倍頻(frequency multipliers)級，(22)式中所示若β值遠較α為小直線調幅可以期望若需較大之最大頻偏用電抗調頻器所成發射機亦需用二三級之倍頻級。

特性曲線之變換可使電抗調頻管之振流式較為簡單　(8)式明顯指示電抗調頻管之特性此式可使設計中之計算較為簡單耳

申謝

本文承蒙經濟部中央無線電器材廠技術室主任葉金灣先生詳讀原稿多所指示甚為感謝 國立浙江大學電機系主任王國松博士及電機系講師□□□先生對本文亦多建議特表謝忱

附註一

圖三所示曲線皆為靜曲線(Static curve)但與動曲線(Dynamic curves)幾完全相同因若 de_p 很小則 $\frac{\partial e_p}{\partial e_g}$ 亦很小又板電壓較第二及第四柵極(Screen Grid)電壓為高故此真空管可作為定量電流發電機。

附註二

若其他各極之電壓保持一定則板流將為 e_g 及 E_{gs} 之函數如

若照泰來氏級數(Taylor's series)展開則

$$i_p = f(e_g, E_{gs}) + \frac{\partial i_p}{\partial e_g} de_g + \frac{\partial i_p}{\partial E_{gs}} dE_{gs} + \frac{1}{2!}\frac{\partial^2 i_p}{\partial e_g^2} de_g^2 + \frac{\partial^2 i_p}{\partial e_g \partial E_{gs}} de_g dE_{gs} + \cdots$$

故所得板流式內之調幅項當為 $i_{pm} = \frac{\partial^2 i_p}{\partial e_g \partial E_{gs}} de_g dE_{gs}$ 此與(K)式完全相同

(完)

石油工業之趨勢

劉毓英

緒論：近年來因飛機汽車質與量之進步，一日千里，故其原動力之供給亦不能不使人注意，蓋此等內燃機之燃料皆取諸石油，而世界石油之蘊藏量有限，據1913年國際地質調查所之統計，全世界之石油蘊藏量，照當時消耗情形計算，三十年內即將告罄，如不設法節省，於今兩三年，飛機汽車無幾油可用矣，但事實上數十年來飛機汽車之增加正是日甚一日，而今日石油之供給尚未達告罄時期，蓋因由石油中取汽油之方法亦在逐漸改進，使汽油之產量增加，石油之消耗減少，同時改進汽油之品質，以節省汽油。由石油取得汽油，最初係用直接蒸餾法，其產量僅百分之二十，改用裂解法後，汽油產量增至百分之六十，自聚化法發明以後，與裂解並用，汽油之產量可增至百分之一百零八[1]，但用此二法于作用後，有一部分石油破裂成無用之氣體，仍覺不經濟，最近趨勢將利用此種氣體，以各種方法使之合成為有用之液體燃料，如是則石油之利用可謂盡其極矣。

汽油品質之改良可增加機器之效率，即可節省燃料，據美人 Gustav Egloff 之研究[2]，如將汽油之辛烷值(Octane-Va.)由八十七增至一百時，可使飛行一千四百英哩之飛機少帶一千二百磅燃料，換言之，即可節省如許汽油，而可使該飛機多帶一千二百磅之貨物或炸彈。由此可知汽油質之改良亦甚重要。欲研究汽油性質之優劣，即須明瞭其化學成分及其化學構造，因汽油辛烷值之高低與化學構造有關，故目前之趨勢為使汽油化學性質之研究與工業上之發展同時並進。

汽油之分子組織與制震性之關係：制震為內燃機燃料最重要之性質，其高低以辛烷值表示，蓋燃料效率之大小及所生能力之多寡皆依壓縮比(Compression Ratio)而定，在某一範圍之中，壓縮比與燃料之效力即所生之能力成正比，但壓縮比之最高限度以燃料之辛烷值而定，因壓縮如超過這限度，引擎即發生機震，一部分能力即變生機震，燃料之效率因之減低，而制震性之高低與燃料之分子構造有關，以各種不同之屬碳氫化合

油系經一增[illegible]之[illegible]可得百分之九十總產量對抗爆性[illegible]之關係[illegible]將各種碳氫化合物分述於下：

(一)直鏈烷屬化合物：此類化合物分子中之直鏈愈長其抗爆性愈低，如增加支鏈[illegible]集中，抗爆性會增高，增加支鏈可使抗爆性改善。

(二)烯屬碳氫化合物：烯族屬碳氫化合物與抗爆性之關係與烷屬碳氫化合物同，抗爆性與分子之大小無關，但與直鏈之長短則顯相關連。

(三)環烷化合物：環烷化合物之抗爆性較同數碳原子之鏈族烷屬化合物高，至其烷基支鏈之插入，則其影響與直鏈烷屬碳氫物相似，倘增加支鏈可增加抗爆性。

(四)含一個苯環之芳香族化合物：此類碳氫物若在環上加直鏈可使抗爆性增高，但至直鏈上含三個碳原子為限，再加支鏈再繼續增長則反使抗爆性減低。

抗爆性與分子構造固然有關，但抗爆性低之碳氫化合物加以適宜之抗爆劑亦可使抗爆性提高，但一種抗爆劑並不能使各種燃料之抗爆性皆能提高，應用之處亦有限度，如四乙鉛能使烷屬及飽和環族碳氫化合物之抗爆提高，而對於烯屬、雙烯屬及芳香族碳氫化合物則毫無影響，對於不飽和之環族碳氫化合物則得相反之結果，故欲加入抗爆劑，尤須明瞭抗爆劑及汽油之性質。

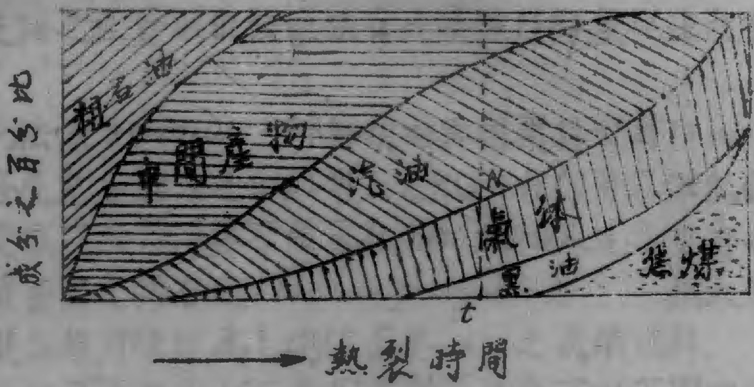
液體燃料製造之沿革：液體燃料提取之進步可分下列四步驟，即直接蒸餾、熱裂、氫化、綜合是也，茲分述之如下：

(甲)直接蒸餾：在1918年以前世界上百分之八十以上之汽油皆以直接蒸餾法從石油中取得，但由直餾法所得之汽油，其[illegible]甚多，其產量有限，第一、產量有限，至多僅百分之二十五；第二、產物之品質無法控制，至三十年前最好蒸餾之汽油其品質僅及今三四等之出品。

(乙)熱裂法 (Thermal Cracking)：因直餾法產量之限制，故有熱裂法之發明，熱裂法可使汽油之產量由百分之二十五增至百分之四十，今日美國半數以上之汽油皆由熱裂法製造，熱裂法與直餾法之不同在於

直餾法僅將石油中原有之沸點在汽油範圍中之部分提取而出，而熱裂法則更使沸點較高之部分加熱分解，而增加沸點在汽油部分之產物。熱裂法不但能提高產量，亦能提高品質，因其對產物之化學成分亦畧能控制。產物質與量之控制，皆由于熱裂時之溫度、壓力及作用時間而定，其關係可以下圖示之(1)：

圖一：熱裂作用與時間之關係



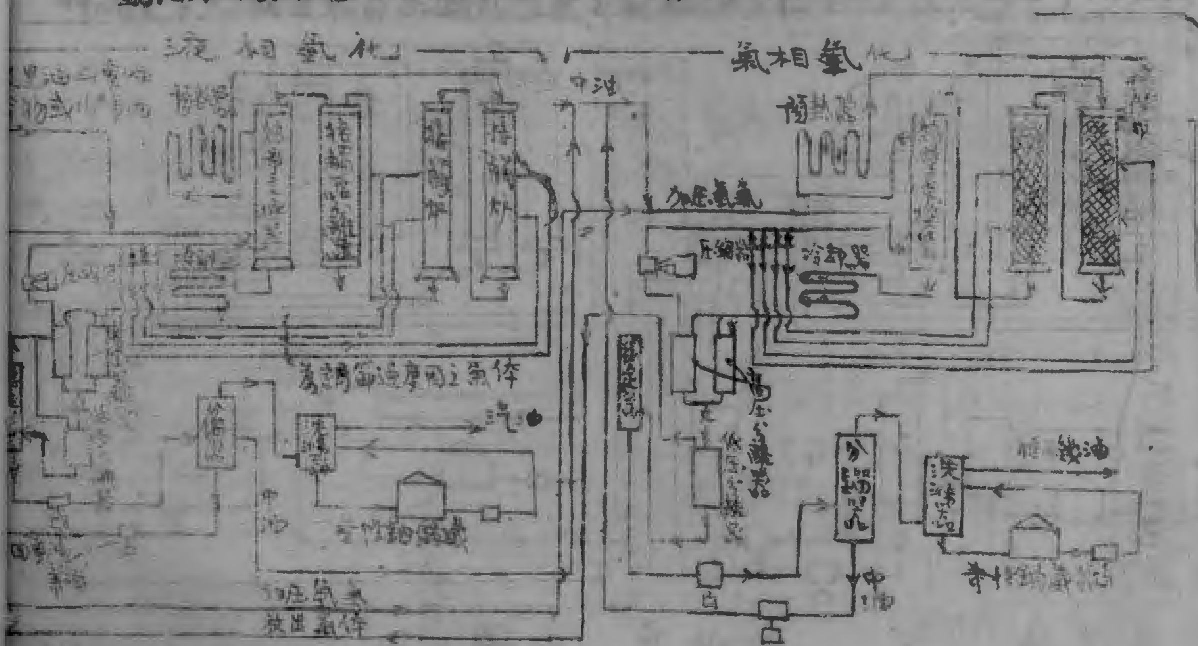
如圖表示在某一組特定之溫度及壓力下作用時間與各種產物之關係，由此圖解吾等可推知下列數點：(一)在任何情況（即溫度及壓力）之下分解作用及重疊作用每為同時進行，換言之，即當熱裂時一部分石油經加熱分解成沸點較低之汽油，但同時另一部分石油經加熱起重疊作用而反成沸點較高之重油。(二)熱裂時間佔非常重要地位，時間過短，石油未起作用，待時間漸次增長，石油即分解成中間產物，由中間產物再分裂成有用之汽油，但時間過長，汽油即繼續分解而成無用之氣體及焦碳。(三)在每一組溫度及壓力之配合下，總可找到一適當之熱裂時間能使汽油之產量為最高，如圖中所示，當作用時間為 t 時汽油之產量達最高率 MN。如吾等在各種不同之溫度及壓力之配合下，求得許多 MN，然後在其中選得一最高之值，即可決定汽油最高率之溫度壓力及熱裂時間。

熱裂法除能增加汽油之產量外，尚有一優點，即能任支配產物中所含高沸點部分及低沸點部分之對比，以適當特殊需要，例如冬季天冷，需要沸點較低之汽油，則工廠可調節其產物，使多含低沸點部分。熱裂法較直餾固優，但仍不免有缺點：(一)汽油產量仍屬有限，不能超過最高點 MN。(二)產物之

質與量不能同時控制，因產量最高之時，產物品質不定，佳蓋各種產物之成分亦依熱裂時間之長短而改變。(三)產物中常夾雜不飽和之碳氫化合物，因熱裂法係將長鏈碳氫化合物分裂而成短鏈，凡分裂之處至少必需有二個氫原子以補足每碳四價之值，故氫之缺乏乃意中事，若不以外界加入氫氣，則產物定帶不飽和性，此種不飽和之碳氫化合物雖沸點適合於普通汽油，但易於氧化及重疊，使汽油變質不適於用。(四)產物中同時有氣體烷屬碳氫化合物之生成，為使分裂所得之低級不飽和產物飽和而成為烷屬氣體，必須有一部分原料供給氫原子，而自己則重疊成為高沸點之重油，因此損失一部份原料，即使汽油之產量減低。熱裂法有此缺點，乃有氫化法之發明以補救之。

(四)氫化法：此法係 Bergius 所發明，由德國顏料公司(I. G. Farben Industrie)繼續發展完善。此法包含熱裂及氫化二種作用，不但能使石油重油煤脂(Tar)變成汽油，即煤亦可經氫化而成輕油，人造之重疊物亦可由氫化而得辛烷值甚高之摻合劑(blending agent)，與天然汽油或熱裂所得汽油混合後，可使成為辛烷值高於一百之飛機燃料。

氫化分液相氫化及氣相氫化兩步驟，其製造程序可以下圖明之。[56]



圖二 氫化法

當液相氫化時如所用原料係重油石油或煤則將粉狀之接觸劑漂浮于油中如所原料係煤則將煤粉與重油或煤脂加少量粉狀接觸劑混成可以幫浦運輸于管中之稀漿原料製備停當後乃與壓氫氣混合經過熱量交換器利用反應熱以提高原料之溫度乃再經一預熱器使之熱至作用溫度然後入接觸爐在爐內起氫化及熱裂作用由接觸爐出來之物先經一沉澱器使接觸劑煤中之灰份以及一部份之重油在此分出所分出之漿狀物加熱在離心器中使固體及液體分離液體部分可仍用作調煤固體部分經洗去灰粉後接觸器可循環再用。

經沉澱器後未凝聚部分乃經熱量交換器及冷却器使降低溫度在常溫時為液體之部分在此凝聚經高壓分離器後使液體與氣體分離氣體經壓縮器後作循環器氣分別通入于熱量交換器及兩接觸爐中以調節溫度液體部分再經抵壓分離器及穩定器(Stabilizer)放出低沸點之烷屬碳氫化合物所剩液體乃經分餾器分成汽油中油及重油汽油以鹼液洗後即可應用中油作為氣相氫化之原料再經氫化以得汽油重油則回至液相氫化中與原料混合再用

氣相氫化與液相氫化相似其唯一不同之點在于接觸爐因中油在接觸溫度下已成氣體故接觸劑係裝於接觸爐內且於通過接觸爐後亦無需沉澱器其餘路線與液相氫化無異可由圖中明之。

此法所用之溫度及壓力為460° 及220-300氣壓氣相氫化及液相氫化可用同一溫度及壓力亦可不同如液相氫化在460°C舉行氣相氫化則在513°C(7)由溫度壓力之變更可改變產物之成分。

此法所需氫氣大多取諸沼氣及蒸氣(8)將沼氣及水蒸氣對摻通過一排裝有接觸劑之平行管此管由二個同心管套成兩同心管皆係含百分之二十五鉻及百分之二十五鎳之鋼做成兩管長均為五呎外管之外徑為七吋內管之外徑為五吋兩管之間裝以鎳或鈷製之接觸劑(將鎳或鈷沉澱於硅藻土(Kieselguhr 上)管內之溫度為1050°C氣體通過接觸劑之速度為每小時二百立方呎氣體先通入內管使熱至作用溫度然後出至兩管之間與接觸劑相遇乃起下列作用:

$$CH_4 + H_2O \rightarrow CO + 3H_2$$
$$CH_4 + 2H_2O \rightarrow CO_2 + 4H_2$$
$$CH_4 + CO_2 \rightarrow 2CO + 2H_2$$

由接觸器出來之氣體含百分之七十五氫百分二十一之一氧化碳百分之一之二氧化碳及百分之一未作用之沼氣欲除去 CO 及 CO_2 必須先將 CO 變成 CO_2 然後以水或鹼溶液洗去之故將此混合氣體再加入水蒸氣通過一裝有銅鈷接觸劑之垂直管溫度為300—350°C則起下列作用：

$$CO + H_2O \rightarrow CO_2 + H_2$$

由接觸器出來之氣體乃經冷卻器以去除剩餘之水蒸汽然後經一壓縮器壓至二十五氣壓以水洗去 CO_2 洗過之氣體大部分為氫此外尚含極少量之 CO 及低沸點之碳氫化合物乃將此氣經壓縮器壓至200—300氣壓以備應用

氫化法所用之接觸劑須具下列條件[5]：(一)能使沸點高於普通汽油之一切石油產物變成低沸點之環屬或支鏈碳氫化合物此類碳氫化合物能使汽油之辛烷值提高(二)表面須粗糙不平(三)須不為硫所毒(四)可繼續使用一年以上經長時間研究結果以鉬及鎢最為適宜最近亦有應用氧化鉬(NiO)硫化鈷(CoS)及硫化錫(SnS)者[7]

氫化法之產率為85—95%每經一次接觸爐可望產生50—75%之燒油

由氫化法所得之產物具下列特性[5]：(一)可於適當之環境下儲藏相當時期而不變其性質其穩定度可與直餾汽油完全一樣(二)其揮發度適合于各國規定(三)其辛烷值較直餾汽油略高約為76.5與71.2之比(四)每斗加3 c.c.之四乙烷鉛即可使其辛烷值增至87—90(五)含烯量甚少(六)硫之含量極少且完全不會膠化由上述性質觀之可知由氫化法所得之汽油遠勝于由直餾法所得者尤以其辛烷值及與四乙烷鉛之混合性而言.

由氫化法所得之產物經分餾後知其正烷相當份量之環族及支鏈烷屬碳氫物(此中大部份為甲烷基環戊烷及少量之環 烷)及6—7%之芳香族碳氫化合物.

(丁)綜合法：綜合法以利用其他燃料工業所產生之副產物以合成有

用之液體燃料爲目的茲將其法分述於下：

(子)由低級烷屬碳氫化合物製造上等液體燃料：由直餾法熱裂法及氫化法製造液體燃料皆有氣體副產物產生此等副產物乃由有用之液體燃料經過度熱裂所成此類產物愈多即有用之液體燃料之產率愈低如能將此副產物設法再造成液體燃料則原料之消耗自可減低不少由上述各法所得之氣體副產品大部份爲低級烷屬碳氫化合物欲由低級烷屬碳氫化合物使成液體燃料必須使其分子量增高而烷屬碳氫物爲比較穩定之物質不易起重叠及加成作用(Polymerization and Addition)

故欲增加其活動力必須先將烷屬碳氫物減氫(Dehydrogenation)

使成爲烯屬碳氫物然後更經重疊或加成作用而得高分子量之液體碳氫化合物.

(一)減氫(Dehydrogenation)：　(a) 自由低級烯屬碳氫物製造液體燃料成功以後減氫作用成爲工業上重要問題最近丁烷及其異性體丙烷以及乙烷之減氫極有進展茲畧述其法於後：

減氫法之原理甚爲簡單係將預熱之烷屬碳氫物通過一適當之接觸劑出來之氣體即含有烯屬碳氫物氫氣及未作用之烷屬碳氫物烯屬碳氫物用以製造液體燃料氫氣經分離器分去後可用於氫化法未作用之氣體重入接觸爐工廠之設備亦甚簡單僅需氣體加熱爐一接觸爐數座及氫氣分離器一已足矣接觸器內之接觸劑須時工刷新故數座接觸爐乃輪流使用刷新時通入適量之空氣或燃燒氣或空氣及水蒸汽之混合物使附着于接觸劑上之碳在適宜之溫度(低于900°C)下燒去.

此處所用適宜之接觸劑須具下列條件：該接觸劑須僅能促進減氫作用而不能促進碳與碳間之分裂此爲一必具而難能之條件因由能量上觀之碳與碳間之分裂較碳與氫間之分裂爲易例如

	卡/克分子	伏特數
碳與氫間之連繫	87,300	3.79
碳與碳間之連繫	58,600	2.54

減氫作用須在較高溫度進行(500－700) 因低溫度有利於反對方向之

作用且在低溫時作用進行之速率極低(二)接觸劑須易於刷新(三)其壽命至少須有數十天之久(四)價須賤以合經濟條件以具備上述四條件為原則向週期表上去選擇則以第VI族(若鉻鉬)第V族(若釩)及第四屬(若鈦鈰)等金屬之氧化物最為適宜將此種氧化物附着於無甚接觸作用之他種金屬氧化物上(如鋁或鎂之氧化物)即成此種接觸劑如處理適當可使減氫作用如下列方程式進行：$C_nH_{2n+2} \rightarrow C_nH_{2n} + H_2$
可全無甲烷或其他低級碳氫物生成即可不使碳與碳間之連繫破裂以鉻之接觸劑為例其製法如後將氧化鉻硝酸鉻鉻酸銨或重鉻酸鉀溶于水中然後以氧化鋁浸入此溶液內取出烘乾後加熱使鉻化合物分解成氧化鉻而附着於氧化鋁上上述鉻化合物溶液之濃度須依氧化鉻在氧化鋁上之濃度而變而溶液之分量則使溶液之鉻適盡為氧化鋁吸收為度接觸劑須時加刷新可使其壽命延長同時接觸劑之壽命與合作時之溫度壓力及氣體經過接觸劑之速度亦均有關如處理得當可用一千小時以上而不減其活動力。

減氫作用如於適當之情況下進行產率可達 90－95% 且有等容積之純粹氫氣為副產品可供氫化法之所需。

減氫作用所需之溫度約為500°C 高于500°C 即增加反對方面之作用速率壓力約為一氣壓接觸時間則與產物之成分甚有關係如接觸時間過短作用未及完全如接觸時間過長則除減氫作用外更有其他作用開始如 CH_4, C_2H_4, C_2H_6 均為可能產物甚至可將氫與碳完全分裂適宜時間約為2.65 秒左右如所用原料為丙烷則經接觸後所得產物為氫20.5%丙烯 19.5%由於其他作用之發生有少量甲烷(0.5－1%)與碳(附着於接觸劑上)產生其餘60%為未作用之丙烷使之循環再用則可得總產率 95%

(二)由烯屬碳氫物合成液體燃料：以烯屬碳氫化合物合成液體燃料可有三種方法即接觸法加熱法及重疊氫化法是也前兩者係將烯屬碳氫物與烷屬碳氫物起加成作用而得分子較大之異性烷屬碳氫物其沸點適合于汽油且其辛烷值甚高後者係使烯屬碳氫物加接觸劑使之重疊成分

子較大之烴屬碳氫物再經氫化後乃得沸點適宜辛烷值甚高之液體燃料茲將此二法分述於下：

(1)接觸法：所謂接觸法即加接觸劑使烯與烷化合(Addition)而得分子較大之烷所用接觸劑以硫酸為最適宜原料以採用丁烯及異丁烷為最適宜其特殊作用為：

$$(CH_3)_2CH-CH_3 + CH_3CH_2CH{:}CH_2 \longrightarrow CH_3CH_2-\underset{}{\overset{CH_3}{\overset{|}{C}H}}-\underset{\underset{CH_3}{|}}{\overset{CH_3}{\overset{|}{C}}}-CH_3$$

異丁烷　　　丁烯　　　異辛烷

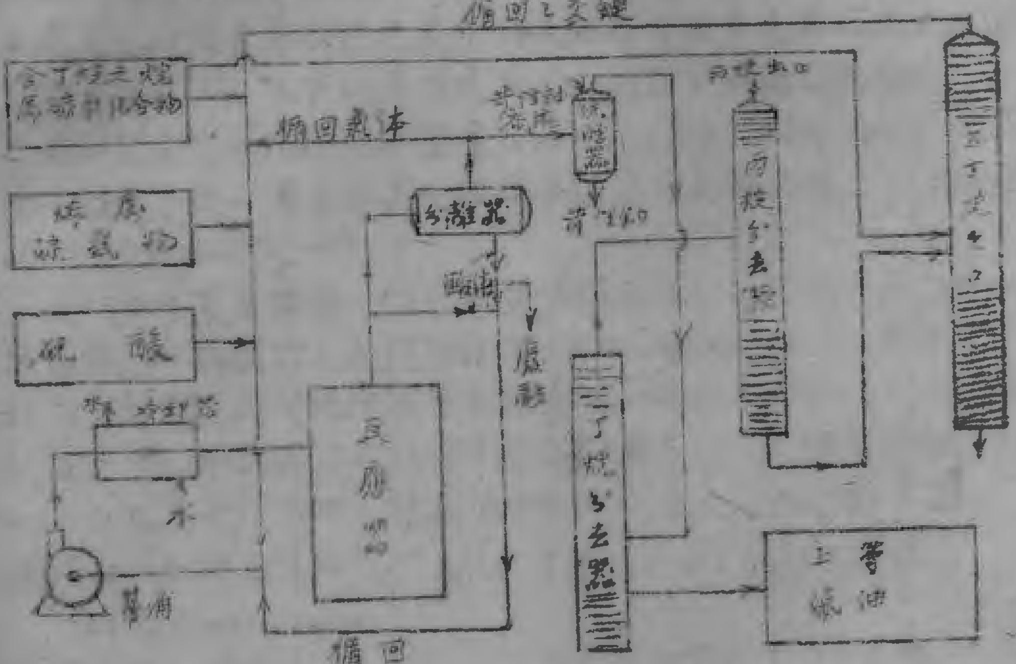
但事實上作用並不如是簡單除加成作用外尚有重疊作用、異構作用(Isomerization)及分裂作用發出且第一次產物亦可能與原料再起作用甚至起第三次作用故經作用後所得產物之沸點範圍甚廣大部分為自21°C至185°C其辛烷值在90以上此法能用原料除丁烯及異丁烯外烯屬碳氫物如丙烯異丁烯1及2丁烯三甲烷基乙烯異丁烯及其兩分子或三分子之重疊物以及正丁烯與異丁烯之相互重疊物(Copolymer)。

均可應用烷屬碳氫物如異丁烷異戊烷及異己烷亦均可用唯正丁烷及異辛烷不起如是作用烷屬碳氫物之直鏈愈長所得產物之辛烷值亦愈低

產物之成分及產率之多寡須視作用時之環境而定作用時原料最好在液體狀態之下普通係將烯漸漸加入於酸與烷之混合液中用力攪動之反應率與攪動之有力與否酸與烷及烯之比例酸之濃度作用時之溫度及壓力及烯加入之速度均有關係攪動必須有力否則酸與碳氫物混合不勻使作用遲滯因而增加烯之重疊作用使產物中多含高沸點成分酸與烷之比例須高因過低亦使烯起重疊作用但其確數須視所用器具而變酸之濃度以95-97%為最適宜過濃即有二氧化硫發生但當丙烯與異丁烷作用時為要增加產率及減少丙烷之硫酸酯起見酸之濃度須增至101%烷與烯之比最好較一分子對一分子略高如是可減少烯之重疊作用可使產物之沸點完全在普通汽油沸點範圍之內加烯之速度亦須視所用器具而異據S.F. Birch及其合作之實驗加入時間為九十分鐘如烯過快亦能增

加重叠作用過慢則使所成產物爲酸分裂而成氣體作用時溫度之範圍爲-10°C至+30°C最適宜溫度爲20°C當溫度接近-10°C時重叠作用增加故其產物不但有不飽和性（溴值甚高）其沸點之範圍亦大大增加當溫度接近30°C時氧化作用增加酸液爲氧化物所污不能復用酸之損失因之增大作用時之壓力除用異丁烷時須加每方吋45—50磅之壓力以助其液化外其餘均可在普通壓力下進行操縱其製造程序以圖三示之

圖三. 由丁烯及異丁烷製造汽油



所用反應器係有一夾層之熱壓器（Autoclave）內裝攪動器夾層可通冷卻劑（如冷水或液體二氧化碳冷卻之煤油（kerosene））以維持所需溫度蓋此乃放熱作用開始時先將熱壓器以熱空氣烘乾乃加入所需之硫酸然後以壓力加入烷以攪動器攪動之使成均勻之乳狀液（Emulsion）乃以冷卻劑通入夾層中使器內之物冷至所需溫度然後徐徐加入烯液並繼續攪動待烯液加完後繼續攪動半小時乃通入更冷之冷卻劑使溫度降至-10°C蓋在低溫度處理可減少產物之損失也然後將熱壓器之內容物放出取得可循環再用產物乃入一分離器分去氣體（大部

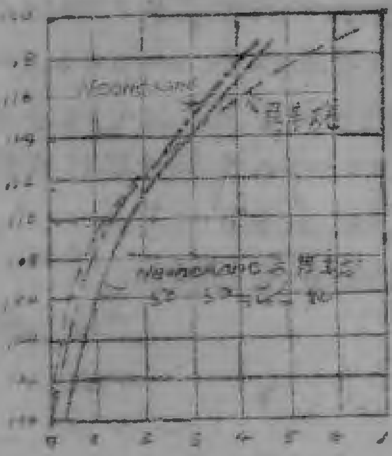
分餾加壓力用之汽氣)此氣體可循回至[illegible]處部而有殘液分出此液如能利用可再用如已不需則去之上層之液體再入洗滌器中以氫氧化鈉中和之洗後入一分餾器分去丁烷所剩液體即為上等汽油丁烷另再入一分餾器分去丙烷剩下之丁烷再得循回入第一分餾器使異丁烷及正丁烷分開異丁烷後經冷卻器入反應器中再用

如所用原料係丁烯及異丁烷則所得產物大部為2,2,4三甲烷基戊烷及辛烷之各種同素異性體之混合物此外尚有少量之異戊烷2,3二甲烷基丁烷2-3-及2,4二甲烷基戊烷及2,2,5-三甲烷基己烷存在

(二)加熱法[1]:此法係用以製造Neohexane 所謂Neo者謂分子中有一碳原子其四價皆與其他碳原子連結故此物又名2,2二甲烷基丁烷 ($CH_3-\overset{CH_3}{\underset{CH_3}{C}}-CH_2-CH_3$)

1939年夏美國有一謠言謂斐立浦石油公司發明一種新汽油辛烷值高于100遇後謂此種汽油名曰(Neohexane)不久斐立浦公司自己宣佈謂此謠傳確係事實該確發明Neohexane。謂此物之辛烷值僅94,如與適宜之抗爆劑混合則其辛烷值可高至115°,Neohexane及異辛烷與四乙烷鉛混合後辛烷值之增加示如下圖:

圖[illegible]:辛烷值與四乙烷鉛之關係



加入四乙烷鉛之體積(cc)

此圖明示如加四乙烷鉛以上Neohexane辛烷值之增加卻大于異辛烷以前辛烷值高之汽油其沸點皆高于普通飛機原料故必需與沸點較低之燃料混合如異辛烷之沸點為87°至115°而飛機燃料所需之沸點為52.5-100°之間Neohexane未發明之前辛烷值與沸點往往不

能同時滿意今則此問題可迎刃而解矣因 Neohexane 之沸點爲50℃故若與異辛烷或其他碳氫混合可得沸點適宜而辛烷甚高之飛機燃料

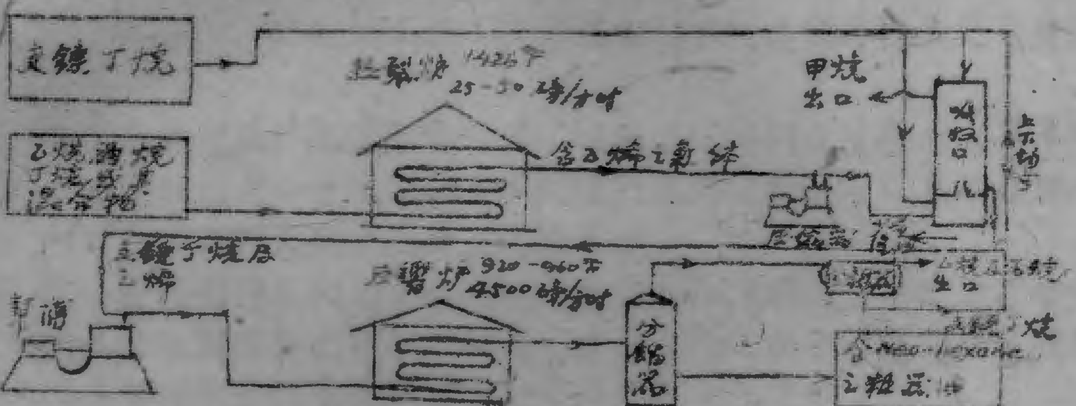
Neohexane 之製造係加熱使異丁烷及乙烯作用其主要反應如下

$$(CH_3)_2 \cdot CH \cdot CH_3 + CH_2{:}CH_2 \rightarrow CH_3-\underset{CH_3}{\overset{CH_3}{\underset{|}{\overset{|}{C}}}}-CH_2-CH_3$$

異丁烷　　乙烯　　Neohexane

工業上製造程序如下圖：

圖五：Neohexane 之製造



工業上所用原料自非純粹乙烯及異丁烷乙烯來自烷屬碳氫化合物之混合氣體其中含甲烷乙烷丙烷及丁烷將此混合氣體以每方吋25—50磅壓力通過一熱至1750°F之管形爐由爐中出來之氣體即含有乙烯使之冷卻後乃吸收于異丁烷中乙烯與異丁烷之混合物乃通入反應爐此爐亦爲管形乙烯及異丁烷分數路通入同時只有異丁烷分別通入爐中之溫度爲910°F壓力爲每方吋3000—5000磅所得產物便經一分餾器分去丙烷乙烷及其他低沸點之碳氫物分出之異丁烷可循環再用Neohexane及較重部分再經一次分餾即可得純粹之Neohexane及其他碳氫化合物茲將Neohexane　及其他可能產物之沸點錄於下表可知欲得純粹之Neohexane　並不困難因其沸點與其他產物相差甚遠

表一：碳氫化合物之沸點

異戊烷	28°C
正戊烷	36°C

2,3二甲烷基丁烷	58°C
2甲烷基戊烷	60°C
3甲烷基戊烷	63°C
正己烷	68°C

Neohexane

加熱法之設備較接觸法貴因加熱法須用高壓耐熱之器具如以後亦能以接觸法在常溫常壓下製造則其發展之希望更大

(3) 重疊氫化法 此法係先將異丁烯與正丁烯相互重疊而成異辛烯此物之沸點雖已適合于汽油但其辛烷值較低僅82至84且不易與四乙烷鉛混合故必須再經氫化使成為異辛烷

當一種或兩種不同之烯屬碳氫物起重疊作用時其重疊之方式有一定規則故由所用原料之分子結構即可推知產物之分子結構因分子結構與辛烷值有關故必須明瞭茲將重疊作用之規則略述如下:

凡一種或數種之烯屬碳氫物當其起重疊作用時普通形式均可以下式表示:

被飽和之分子曰接受者或簡稱受者未飽和之分子為授予者或簡稱授者如二分子為同一化合物重疊時僅有一種可能性故可為受者或授者但如二分子為兩種不同之化合物則究竟孰為受者孰為授者必須有所規定據由各種重疊觀察觀察結果含有較活潑之双鍵者應為受者双鍵活動力之比較可以下列規則決定之

① 烯屬碳氫物自乙烯起分子愈大愈易重疊直至戊烯達最高點分子如再增大則其重疊性又漸漸減低

② 同素異性體之分子結構不同亦可影響双鍵之活動力一般說法第三烯(Tertiary olefine)如2甲烷基1丙烯及2甲烷基2丁烯較其含同數碳原子之第二同素異性體(secondary isomer)如2丁烯及2戊烯易起重疊作用

低級之烯屬碳氫物起重疊作用之難易根據上述高低規則可排列如下：乙烯<丙烯<第二丁烯<乙甲烷基1丙烯及乙甲烷基+及乙丁烯<第二戊烯<高級烯屬碳氫物.

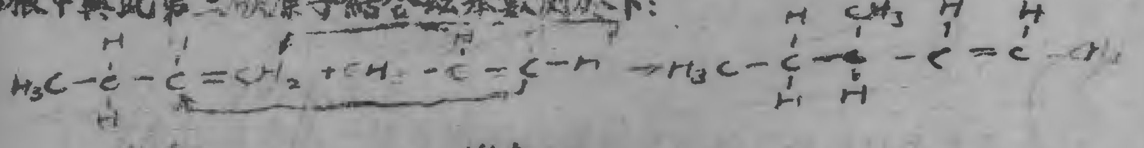
(3)二分子及三分子低級烯之重疊產物較未經重疊之分子不易重疊例如乙甲烷基+丙烯($CH_3C(CH_3)=CH_2$)之變成辛烯較辛烯及双辛烯之再重疊快得多、同樣二分子之重疊物較三分子之重疊物易起重疊作用.

分子之受者及授者雖定但分子中之那一個碳原子與另一分子之那一個碳原子或氫原子作用亦須明瞭故下列二條規定:

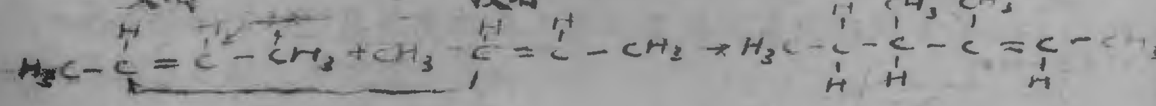
(1)依授者與受者之定義觀之,知受者之双鍵為授者之氫原子及烯根所飽和,而授者之活潑氫原子常附着於双鍵發點之碳原子上或双鍵中接着含最少碳原子之烷根之碳原子上,茲舉數例以明之,下列公式中作記號之氫點為活潑之氫原子

$CH_3-CH=CH-H^*$, $CH_3-\underset{CH_3}{C}=CH^*-CH_3$, $CH_3-CH_2-CH=CH^*-CH_3$

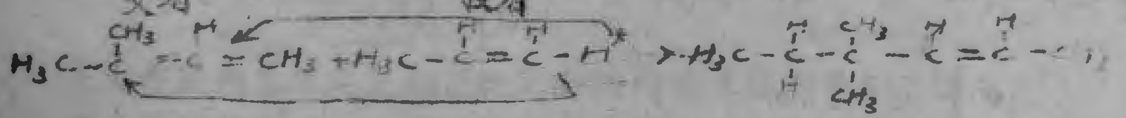
(2)授者之氫原子常與受者双鍵中之最終碳原子或双鍵中與含最少碳原子之烷根接着之碳原子連接,而烯根則與双鍵中之另一碳原子連接,但如受者之双鍵中含有一第三碳原子(tertiary carbon)則授者之烯根與此第三碳原子結合,茲舉數例於下:



受者　　　　授者



受者　　　　授者



受者　　　　授者

由重疊所得之產物或由去氫後所得之烯屬碳氫物,其双鍵之位置常有變更,可能變之成為較穩定之化合物,促進重疊作用之條件往往亦能促

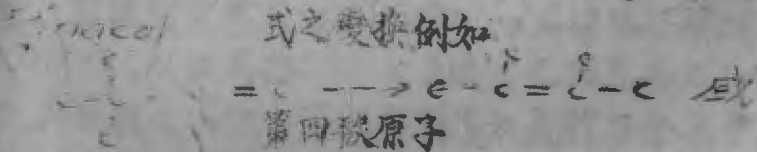
[illegible]双鍵異構作用故欲推知最後產物應注意双鍵位置之變更其改變亦依一定規則總之如下：

① C…C-C=C 之化合物往往較 C…C-C=C-C 不穩定故前者[illegible]溫度[illegible]劑以其他之影響而變為後者

② 第二烯 (secondary olefine) 甚易變為第三烯 (tertiary olefine) 如第二碳原子 (secondary carbon) 位於双鍵之α地位時例如

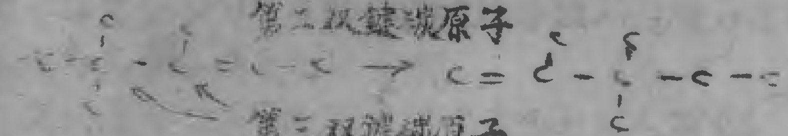
3甲烷基1丁烯　　　　3甲烷基2丁烯

③ 如化合物中有一第四碳原子 (Quarternary carbon) 在第二 (Secondary) 或第三 (tertiary) 双鍵碳原子之α位置時則 Pinacol 式之變換例如



第四碳原子

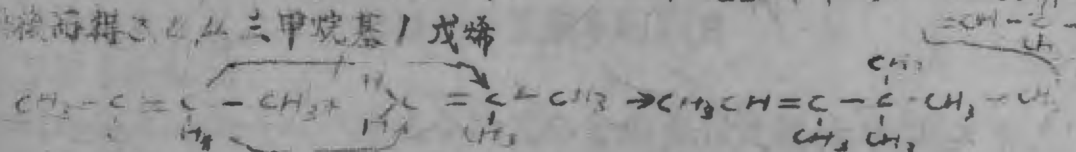
第二双鍵碳原子



第三双鍵碳原子

第四碳原子

[illegible]作用常在一裝有磷酸接觸劑之接觸器中進行溫度為150℃，壓力每方吋600磅，其次產物為3,4,4三甲烷基戊烯經 Pinacol 變換而得2,3,4三甲烷基1戊烯



氫化方法與上述之氣相氫化完全相同唯其接觸器有不同而已。

由此法所得之產物之辛烷值可高至97-100，且與四乙烷鉛之混合性亦甚佳，但其揮發性較低，因其沸點不在普通汽油沸點範圍之內，故不能單獨為燃機之燃料，僅能用以與其他液體燃料混合以提高辛烷值而

已加氫汽油之辛烷值高于100

凡氣體烷屬碳氫物除甲烷外皆可用以合成上等液體燃料此等氣體產量甚多如天然氣及一切石油工業及煉焦工業之副產物故由低沸點之烷屬碳氫化合物製造液體燃料近年來正在設法推進中。

(四)由一氧化碳及氫合成汽油：此法係由 Franz Fischer 及 Hans Fropsch 發明其原理為將 CO 及 $H_2(1:2)$ 之混合氣體通過一適當之接觸劑而合成液體碳氫化合物所起作用視所用之接觸劑而異如用鈷或鎳為接觸劑其作用為：

$$xCO+2xH_2\longrightarrow(CH_2)_x+xH_2O$$

如以鐵為接觸劑則其反應為：

$$2xCO+xH_2\rightarrow(CH_2)_x+xCO_2$$

同時反應溫度亦依接觸劑而變以鈷或鎳為接觸劑最適宜之溫度為180-200°C，以鐵為接觸劑則最適宜之溫度為230-250°C且溫度之高低與氣體之成分亦有相當關係如氣體內含氫量高則溫度須較低溫度之控制為此法中之最困難問題因產物之成分亦依溫度而變溫度如稍上下即可得完全不同之產物產物之成分與所用壓力亦有關增高壓力可增加石蠟即高分子碳氫物之生成。

關於適用之接觸劑近年來研究者不乏其人有較 原來所 者更佳此種接觸劑必須有下列各種條件：

(1)活動力須大使氣體以相當之空間速率 經過一次接觸後即起大量作用。

(2)活動力須持久至少能維持數星期多則數月

(3)須易於翻新。

欲適合上述條件以一種混合物或合金為最佳是類接觸劑中含三類物質第一類為主要接觸劑如鎳鈷鐵等自身有促進反應之力量第二類為加速劑 如銅錳釷鈿等其本身無促進反應之力量但如加入於主要接觸劑中後不但可幫助接作用且可使作用溫度減低或生物成分改變第三類為支撐物

加矽藻土(Kieselguhr) 用以增加接觸 Fischer 自己所用之接觸劑以鎳為主要接觸劑以錳及鋁之氧化物為加速劑以矽藻土為支撐劑但據英國 Aito 博士之研究謂鈾及釷之氧化物為最好之加速劑矽藻土為最佳之支撐劑彼曾做一接觸劑含80%鎳10%銅2.5%氧化釷0.5%氧化鈾其餘為矽藻土用此接觸劑可由一立方尺之混合氣體製成1.44cc汽油後來又做一接觸劑含四十份重量之鈷五十份鎳十五份錳三份氧化釷五份氧化鈾及一百二十五份矽藻土以氫在400°C下還原四小時而成用此接觸劑于190°C下使之作用每立方公尺氣體可產155cc汽油 Tsutsumi 亦曾將所有可能之接觸劑作有系統之研究結果謂鎳之接觸劑含20%錳8%氧化鈾及4%氧化釷者為最佳用此接觸劑在200°C時作用如氣體通過接觸劑之空間速率為每小時六卅則每立公尺氣體可產汽油168cc 此為最高記錄。

凡此法所能用之接觸劑均為硫所毒故氣體中之硫化氫及有機硫化物均需在事前完全除去去硫方法須求手續簡單費用便宜且 H_2 及 CO 體積之比須不因去硫而有所改變硫化氫係由氫氧化鐵去除有機硫化物之去除最初係用鎳粉為接觸劑現在則用特製之鹼性氧化鐵在176-300°C之間能使有機硫化物分解成硫化氫此物即留于器中另有一法為以鎳為接觸劑將硫化物氧化成二氧化硫然後去除二氧化硫採用現有方法已能將有機硫化物減低至每立方公尺僅含0.2克硫雖如此但接觸劑之活動力仍難免漸次減低一則因硫漸漸積多二則因其毒性之擴大三則因接觸劑上為生成之固體石蠟所蓋住有時附着于接觸劑上之石蠟重量可數倍於接觸劑本身故其活動力自然減少此外如作用溫度過高亦易使接觸劑之活動力減低因此接觸劑須常加刷新之法係將溶劑溶去固體石蠟乃以鹽溶解接觸劑過濾將過濾之溶液重新沉澱于支撐物上所得石蠟溶液將溶劑蒸去後可得人造石蠟。

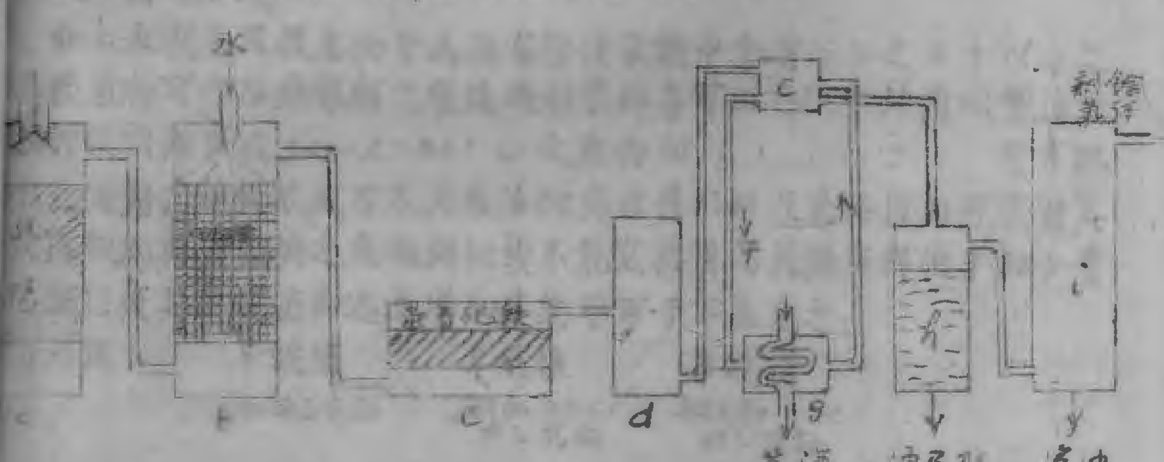
此法可用原料之範圍甚廣凡含有碳而能製成 CO 及 H_2 之物質均可應用如煤炭天然氣及工礦中之廢氣等如以焦煤為原料在水煤氣發生爐中通入加倍之蒸氣使一部份生成之 CO 再與蒸氣化合而成 CO_2 及 H_2 如

CO 與 H_2 之比適為 1:2 同時生成一份 CO_2 此氣體應無甚…不
使 CO 及 H_2 之濃度易為沖淡而已如以天然氣或由焦爐爐出來之煤氣
等計因此等氣體內含 CH_4 故若將此等氣體與過熱蒸氣或二氧化碳
通過附着於活性礬土或氧化鋁或碳酸鎂等之鎳鈷銅鋼或鎢之接觸劑
得 CO 及 H_2 之混合物作用溫度為 840—920°C 其所起反應如
下

$$CH_4 + H_2O \rightarrow CO + 3H_2$$
$$CH_4 + 2H_2O \rightarrow CO_2 + 2H_3$$
$$CH_4 + CO_2 \rightarrow 2CO + 2H_2$$

由一氧化碳合成汽油之製造程序示如圖六[14]

圖六 Fishner 及 Tropsch 法製造人造汽油



a 為水煤氣發生爐 b 為洗滌器 C 為去硫器內裝氫氧化鐵 d 為有機硫化物去除器內裝特製之鹼性氧化鐵或…濾清器 e 為反應爐其兩端與鍋爐連接使反應所生之熱為循環於反應爐空隙間之油移去至鍋爐而發生水蒸氣以供作水煤氣之用由反應爐出來之氣體含有 25% 未起作用乃將此氣通過一冷凝器以直接或間接法冷至常溫而沸點之油井產物及水即在此凝為液體未凝固之輕油及氣體乃入洗滌器或吸收器 輕油在此用油洗去或以活性碳吸收剩餘氣體可作原料或可作為 CO 及 H 之原料因剩餘氣中含 CO_2 使之與 CH_4 及 H_2O 化合可得 CO 及 H_2 其作用

$CH_4 + CO_2 + 2H_2O \rightarrow 4CO + 8H_2$　　但如剩餘氣中 CO_2 之含量少，則不能利用。由分離器及冷卻器中所得之液體，再分別蒸餾，可得五種物，其產物及含烯量可于下表知之：

	沸點	產率之百分數	烯之含量（容積百分數）
氣體產物	低于30°C	4%	50%
汽油（Kogasin I）	30－200°C	62%	30%
油狀物（Kogasin II）	高于200°C	23%	10%
由油狀物取得之石蠟	熔點 50°C	7%	—
由接觸器取得之石蠟	70～80°C	4%	—

由上表觀之，五種產物皆為極有價值。氣體中含有百分之五十以上之烯屬碳氫物，可作各種醇類之製造絕好原料，亦可以已述方法用以製造上等液體燃料。沸點在30－2000°C之產物即 Kogasin I 可直接用作汽油，因其絕對無硫，故可不必經任何處理，僅以鹼液洗滌後即可應用。其辛烷值較低，與直餾法之產物相似，故不能直接用為飛機燃料，但于加少量四乙烷鉛後，其辛烷值即迅速增加，其效率可于下表見之：

沸點	辛烷值（未加四乙烷鉛）	辛烷值（每錏加0.5cc.四乙烷鉛）	辛烷值（每錏加1cc.四乙烷鉛）
30－180°C	47.0	67.0	71.5
30－150°C	57.0	72.5	76.0
30－125°C	62.0	76.0	80.0

于加四乙烷鉛後，其辛烷值之增加甚速，故亦可作飛機燃料之用。其第三種產物 Kogasin II 可再經熱裂而變成汽油，或可用以製造滑潤油。其製法有三：(1) 將 Kogasin II 氯化，使平均每分子含有一或二之氯原子，然後以活性鋁處理之；(2) 直接以氯化鋁處理；(3) 以 Friedel Craft 反應使與芳香族碳氫物凝聚。Kogasin II. 雖不能直接作為滑潤油，但由上述各法製得之滑潤油之性質較普通滑潤油為佳。

德國因缺乏石油燒政府竭力獎勵人造物之發明,自此物發明後即積極利設廠,已於1936年出貨,目前雖產量尚不多,然對此次戰爭亦不無小補。

結論:汽油製造上之沿革,即如上述,由直餾而熱裂而氫化乃至於綜合方法之採用,須依環境之需要而異,如美國蘇聯產石油多之國家,應 法應用使石油原料之利用達於最高限度,至產石油少之國家如中國,若欲解決汽油問題,自根本着想,應國家出資本,大量建設人造汽油廠,方法自以採用 Bergius 之煤之氫化法及 Fischer 法為最適宜,由此二法所得之副產物(氣體)亦可以接觸法或加熱法用以製造液體燃料,以目前情形論,國人應對 Fischer 法加以研究,因此法所需設備較為簡單,或可在國內自己設法製造,用氫化法則必須高壓設備,在目前形勢,自較難以辦到.

對於汽油質之改良,近年來亦頗有進步,二十年來汽油之辛烷值平均升高10,一面由於加入制爆劑,一面則亦由於汽油本身成分之改良,汽油今後改進之途徑有二:①傾向於合成某種所希望之物質,或與此種物質,密切相似之物質,以適應各種特殊需要,②傾向於混合某幾種已知之單純物質,使適合數種所需條件,換言之,即今後之趨勢為製造具有各種特殊性質之單純物質,再將此種物質依所需條件混合,以適合各種不同之需求。

本文材料大部份係採自沈善炯莊自強徐鴻方吳滄華李盤生諸同學之雜誌報告,沈善炯莊自強二同學且已將報告之材料譯成中文,對本文幫助不少,特此在此誌謝.

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Extended Simpson's Rule For Area Moment

T. Wang (王仁東)

[Introduction] The calculation of the deflection of beams is usually a tedious except when the loading is extremely simple. In solving problems of beams with more complicate loading, the usual methods of double integration, area moment or the conjugate beam needs so many steps of involved calculations that mistakes are very liable to occur. Reliable results can only be obtained after very elaborate works. In the present engineering practice, the graphical method is usually resorted to. With good draftsmanship, results with errors not greater than a few percent are usually obtainable with comparative easiness. But more accurate results are impossible with the graphical method. It seems that a method combining the accuracy of the analytic method and the simplicity of the graphic method is yet wanting.

After months of searching, the author found that the conjugate beam method would be a powerful weapon of the area and the area moment of the moment curve can be evaluated easily. He also found that the idea of the Simpson's rule can be applied to such evaluations. The original Simpson's rule is for area only. He then derived an extended rule for area moment after Simpson's method. The combined use of the area rule and the area moment rule seems to be remarkably successful in handling problems of deflection of beams however complicate the loading may be. He hopes that the method will be found useful in the engineering practice.

[The Difficulties In The Deflection Calculation]

In the method of double integration one set of equation is required for each span of a particular kind of loading. Thus for the beam of example 3 below, five sets of equations are required, and ten integration constants are to be determined. The method of area moment and the method of conjugate beam are generally considered possible to make solutions simpler. The moment diagrams are constructed in the both methods and the area and the area moment are calculated accordingly.

[Note]: The theorems of area moment are as follows:
(1) The angle of deflection of beam between points a and b is equal to the area of moment diagram between a and b devided by EI. (2) The vertical distance between point b on the beam and the tangent to the beam at a is equal to the moment of area of the diagram between a and b about b, divided by EI. The theorems of conjugate beam are as follows: (1) The angle of deflection of a beam at a given section is equal to the shear of the conjugate beam at the same section. (2) The deflection of beam supported at ends is equal to the moment of its conjugate beam at the same section devided by EI.

In case the moment diagram is a simple mathematical curve, both the method of area moment and the method of conjugate beam offer simple and satisfactory solution. In case the moment diagram is complicated (i.e. made of a large number of triangles, parabolas, and cubic parabolas superposed upon each other) the calculation is usually very elaborate.

The determination of area under a complicate or an irregular curve can be carried out with the well known Simpson's one third rule or three eighth rule with fair degree of accuracy. By the same principle, a rule for area moment can be derived. This extended Simpson's rule gives equally accurate and reliable results in area-moment calculation as the original rule in area calculation. With the both rules in hand, the deflection of beam can be computed with reasonable promptness and fair accuracy.

[Derivation Of The Area Moment Rule]

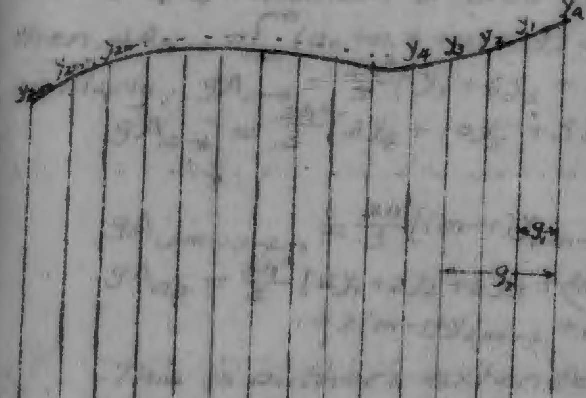


fig. 1

Referring to fig. 1, the area under the given curve from a to b is divided into 2m equal parts of width h each. A parabola of the form

$$y = a_0 + a_1 x + a_2 x^2$$

is passed through three successive points on the curve. Thus for the area from a to 2, the equation must satisfy the three points: $(y_a, 0)$, (y_1, h), $(y_2, 2h)$

$$y_a = a_0 \quad \cdots\cdots\cdots\cdots\cdots (1)$$

$$y_1 = a_0 + a_1 h + a_2 h^2 \quad \cdots\cdots\cdots\cdots (2)$$

$$y_2 = a_0 + 2a_1h + 4a_2h^2 \quad \text{-----------------} \quad (3)$$

by solving (1), (2), and (3), we get:

$$a_0 = y_a \qquad a_1 = \frac{4y_1 - y_2 - 3y_a}{2h} \qquad a_2 = \frac{y_2 - 2y_1 + y_a}{2h^2}$$

The area under curve from a to 2 is:

$$A_{a-2} = \int_0^{2h} (a_0 + a_1x + a_2x^2)dx = \frac{h}{3}(y_a + 4y_1 + y_2)$$

Similarly, from 2 to 4,

$$a_0 = 6y_2 - 8y_3 + 3y_4, \quad a_1 = \frac{2y_3 - 7y_2 - 5y_4}{2h}, \quad a_2 = \frac{y_4 - 2y_3 + y_2}{2h^2}$$

and $A_{2-4} = \frac{h}{3}(y_2 + 4y_3 + y_4)$

$$A_{4-6} = \frac{h}{3}(y_4 + 4y_5 + y_6)$$

. . . .

$$A_{a-b} = \frac{h}{3}(y_a + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + 2y_{2m-2} + 4y_{2m-1} + y_{2m})$$

This is Simpson's one third rule for area.

Similarly, we derived the rule for area moment.

Let gA_{a-2} = moment of area from a to 2 about 2,

gA_{2-4} = moment of area from 2 to 4 about a, etc.

then, $gA_{a-2} = \int_{2h}^{4h}(a_0 + a_1x + a_2x^2)x\,dx = \frac{2h^2}{3}(2y_1 + y_2)$

similarly, $gA_{2-4} = \frac{2h^2}{3}(y_2 + 6y_3 + 2y_4)$

$$gA_{4-6} = \frac{2h^2}{3}(2y_4 + 10y_5 + 3y_6)$$

. . . .

$$gA_{(2m-2)-2m} = \frac{2h^2}{3}[(m-1)y_{2m-2} + (4m-2)y_{2m-1} + my_{2m}]$$

$$gA_{ab} = \frac{2h^2}{3}[2y_1 + 2y_2 + 6y_3 + 4y_4 + 10y_5 + 6y_6 + \cdots + 2(m-1)y_{2m-2} + 2(2m-1)y_{2m-1} + my_{2m}.$$

This is auther's extended rule for area moment.

Note: The coefficients in the extended rule are such: for odd terms, the coefficients is equal to twice the number of term, thus for the fifth term the coefficient is 10.

for even terms, the coefficient is equal to the number

Illustrative Examples. In example 1 & 2 the accuracy of the method is illustrated. The examples 3 and 4 are intended to show the straightforwardness of the method. The calculations are simply arithmetics. The chance of mistakes seems to be slight.

Example I. a simple suported beam of length L is loaded at the center with the force P. Find the maximum deflection. (Referring to fig. 2)

Draw the moment diagram in the usual way and divide it into 8 equal parts.

The maximum deflection at the center is equal to the moment of the moment diagram from a to 4 about a. (by Wang's Rule)

$EIy_{max} = \Sigma A_{a-4} = \frac{2L^2}{3\times 64}$ ([illegible]) PL

$= PL^3/48$

Example II. A simple suported beam carries a uniform load of w lbs. per ft. The length of the beam is L. Find the maximum deflection. (Referring to fig. 3)

Draw the moment diagram, and divide it into 8 equal parts. The maximum deflection is at the center and is equal to the moment of the moment diagram from a to 4 about a. (by Wang's Rule)

$$EIy_{max} = \frac{2L^2}{3\times 64}(2\times 7 + 2\times 12 + 6\times 15 + 2\times 16)\frac{wL^2}{128}$$

$$= 5wL^4/384$$

Example III. A beam 16 ft. long and with I = 70 in.^{4} is loaded as shown in fig. 4. find the maximum deflection and its position.

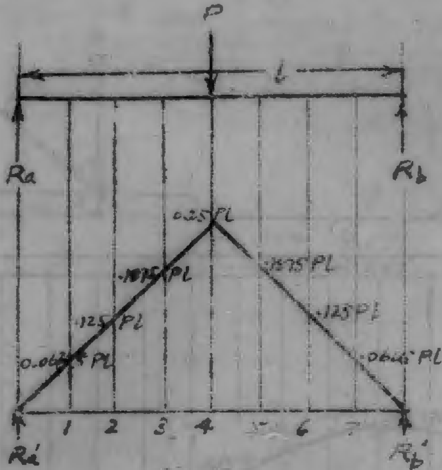


fig. 2

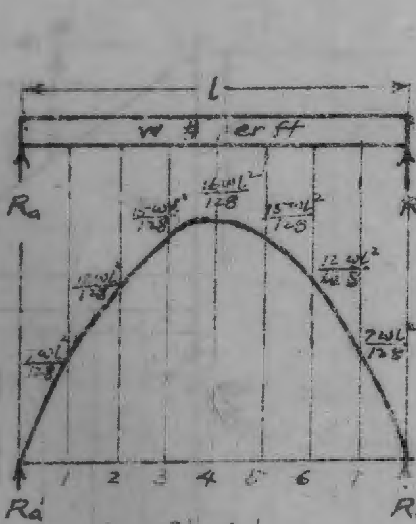


fig. 3

Draw the moment diagram and divide it into 8 equal parts (if the diagram is divided into 16 equal parts the accuracy will be improved by 0.2 %)

Find the moment of the complete diagram about the ordinate b (by Wang's Rule)

$$9A_{a-b} = \frac{2\times 4}{3}(2\times 10.36 + 2\times 15.6 + 6\times 17.19 + 4\times 17.27 + 10\times 16.15 + 6\times 13.79 + 14\times 8.82)\times 1000$$

$$= 1,579,000$$

$$R_a' = EI\theta_a = 1,579,000 \div 16 = 98,700$$

At the point of maximum deflection, the area of the moment diagram from a to the point should be equal to R_a', which is equal to 98.700. The point of maximum deflection is very near to the center of the beam for any type of loading. Hence a simple approximate method is possible with error probably not greater than $\frac{1}{2}$ to 1 % in the most cases.

The area of the diagram from a to 8 is :

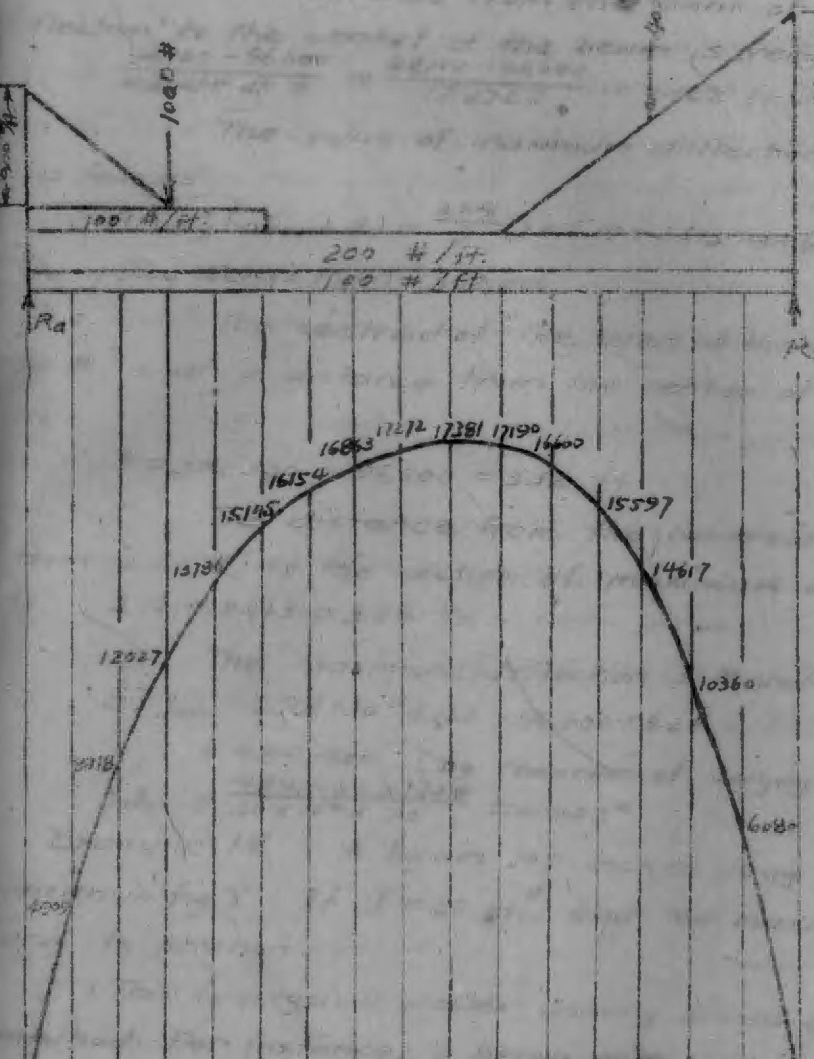


fig. 4

y Simpson's Rule)

$-8 = \frac{2}{3}(4 \times 8.82 + 2 \times 13.79 + 4 \times 16.15 + 17.27) \times 1000$

$= 96,600$

The distance from the point of maximum deflection to the center of the beam is very nearly

$$\frac{98{,}700-96{,}600}{\text{ordinate at } 8} = \frac{98{,}700-96{,}600}{17.272} = 0.122 \text{ ft.}$$

The value of maximum deflection can be found as follows:

$$gA_{a-8} (\text{about } 8) = \frac{2\times 4}{3}(2\times 16.15 + 2\times 13.79 + 6\times 8.82)\times 1000$$

(by Wang's Rule) $= 302{,}000$

The centroid of "The area of the diagram from a to 8" is at a distance from the center of the beam equal to

$$g = 302{,}000 \div 96{,}600 = 3.12 \text{ ft.}$$

The distance from the centroid of "the diagram from a to 8" to the section of maximum deflection is then:

$3.12 + 0.123 = 3.24$ ft.

The maximum deflection is therefore very nearly:

$$EI\,y_{max.} = 98{,}700\times 8.122 - 96{,}600\times 3.24 - 17.27\times\frac{(0.122)^2}{2}$$

$= 489{,}400$ (by theorem of conjugate beam)

$$y_{max} = \frac{489{,}400\times 1728}{30\times 10^6\times 70} = 0.403''$$

Example IV. A beam 100 inches long is loaded as shown in fig. 5. If $I = 20$ in.^{4} find the maximum deflection and its position.

(This is a typical problem usually solved by the graphic method. For instance, a beam with varying section can be divided into a number of equal parts, considering each as a concentrated load. An approximate solution is then obtained by the graphic method.)

Fig. 5 shows the complete solution by the graphic method on a reduced scale as the original drawing paper. The attempt give the result, $y_{max.} = 0.203''$.

Possibly, better results can sometimes be obtained. Nevertheless, the solution by Simpson's Rule and Wang's rule illustrated below enables one to get a comparison of the two methods with respect to conveniency and accuracy.

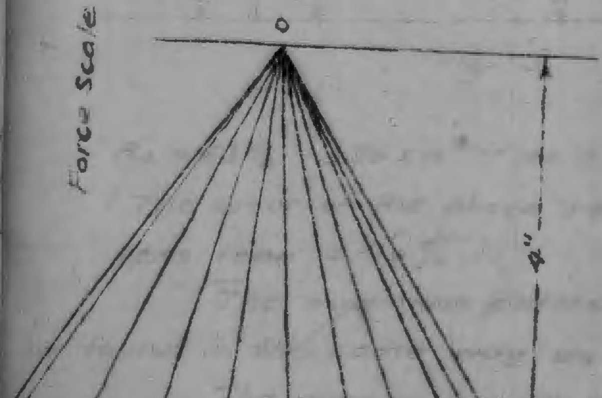
Draw the moment diagram as shown in fig. 6 and divide the area into 20 equal parts of width 5" each.

The moment of the complete diagram about b is:

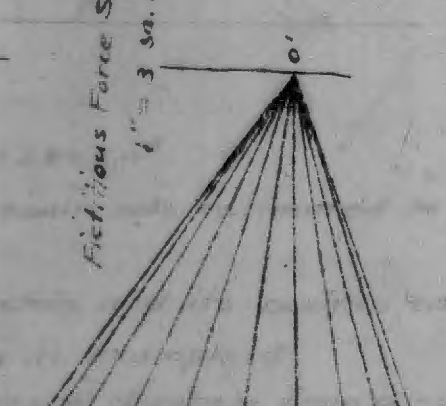
$$3A_{a-b} = \frac{2\times 25}{3}(2\times 17.75 + 2\times 34.5 + 6\times 51.25 + 4\times 65.5 + 10\times 79.75 + 6\times 90 + 14\times 100.25 + 8\times 106.5 + 18\times 112.75 + 10\times \text{[illegible]} + 22\times 115.25 + 12\times 111.5 + 26\times 107.75 + 14\times 99 + 30\times 90.25 + 16\times 76 + 34\times 61.75 + 18\times 41.5 + 38\times 21.25)\times 1000$$

$$= 3.84\times 10^{8}$$

Force Scale: 1" = 150 #



Fictitious Force Scale: 1" = 3 sq. in.



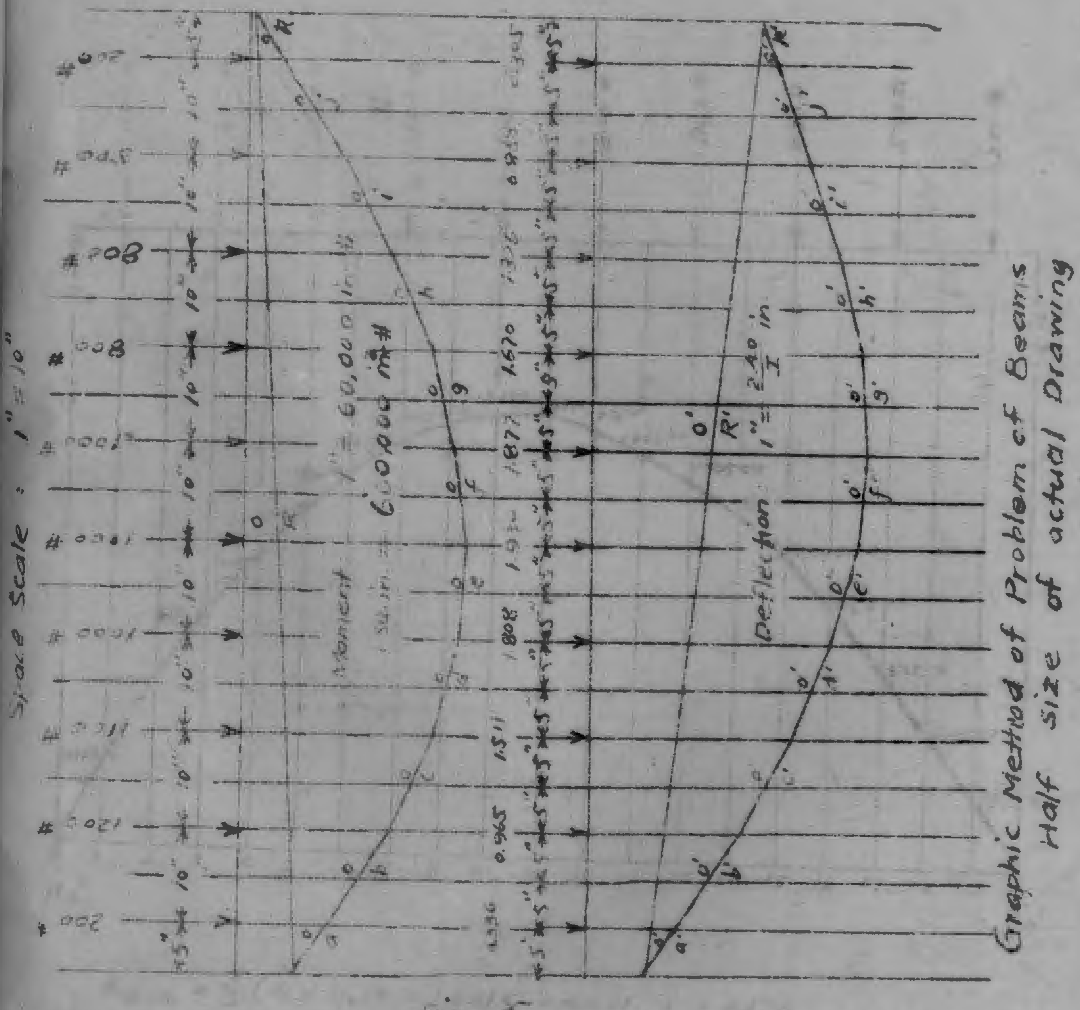


fig. 5

$R'_a = EI\theta_a = 3.84 \times 10^8 \div 100 = 3.84 \times 10^6$

(The error of the above result was estimated to be less than 0.40 %)

The maximum deflection and its position can be found in the same way as in example 3.

The area of the moment diagram from a to 10 is : (by Simpson's Rule)

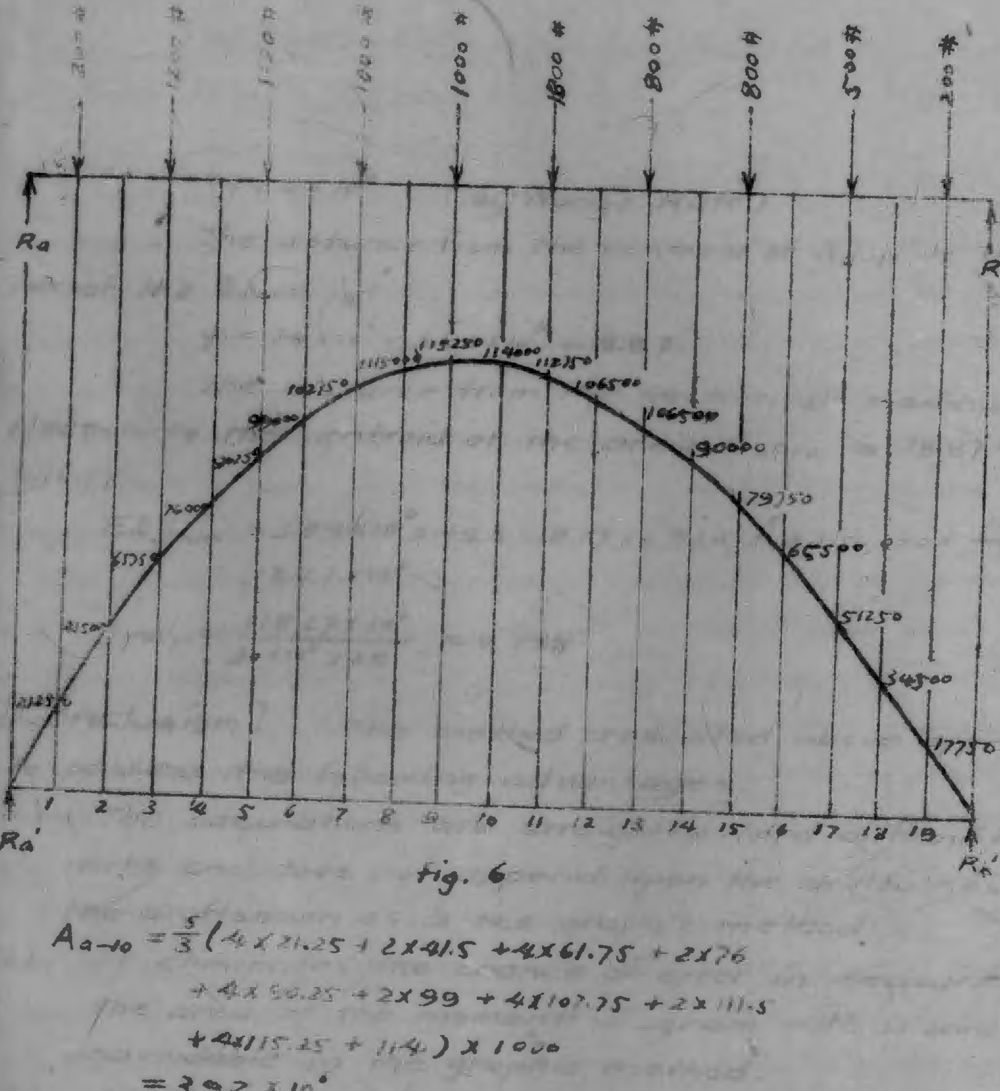


fig. 6

$$A_{a-10} = \frac{5}{3}(4 \times 21.25 + 2 \times 41.5 + 4 \times 61.75 + 2 \times 76 + 4 \times 90.25 + 2 \times 99 + 4 \times 107.75 + 2 \times 111.5 + 4 \times 115.25 + 114) \times 1000$$

$$= 3.92 \times 10^6$$

The distance from the section of maximum deflection to the center of the beam is:

$$\frac{(3.92 - 3.84) \times 10^6}{114,000} = 0.7''$$

To find the maximum deflection

$$gA_{a-10} = \frac{3 \times 25}{3}(2 \times 115.25 + 2 \times 111.5 + 6 \times 107.75 + 4 \times 99 + 10 \times 90.25 + 6 \times 76 + 14 \times 61.75 + 8 \times 41.5 + 18 \times 21.5) \times 1000$$

$= 74.0 \times 10^6$ (by Wang's Rule)

The distance from the centroid of A_{4-10} to the center of the beam is:

$\bar{y} = 74 \times 10^6 \div 3.92 \times 10^6 = 18.87''$

The distance from the section at maximum deflection to the centroid of the area A_{4-10} is $18.87 - 0.7 = 18.17''$

$EI y_{max.} = 3.84 \times 10^6 \times 49.3 - 18.17 \times 3.92 \times 10^6 - 114{,}000 \times \frac{(0.7)^2}{2}$
$= 118.27 \times 10^6$

$y_{max.} = \frac{118.27 \times 10^6}{30 \times 10^6 \times 20} = 0.198''$

[Conclusion] The method presented above appears to possess the following advantages:

(1) The calculations are straightforward arithmetic works and does not depend upon the skillfulness of the draftsman as is the graphic method.

(2) It eliminates the chance of error in measuring the area of the moment diagram with is sometimes unavoidable in the graphic method.

(3) After drawing the moment diagram, which is not very elaborate even for the most complicatety loaded beam, there will be no much further difficulty in finding the deflection.

(The End)

The Improvent of Highways In China

Wing. S. Shee (王師義)

Introduction

A highway is one kind of means of transportation. It [is] a step stone to the material civilization. The social, po[lit]ical, commerciaal, industrial, agricultural and educational [de]velopments all develop upon the development of commu[ni]cation lines. Its function is the same as the veins of blood [o]f a human body. In war time, a quick means of commu[ni]cation is very [u]rgent for the mobilization of troops and [th]e transportation of war supplies.

The development of communication may be by war [by] water, and by land. But, in free China there are [on]ly a few navigatible rivers. Its use is much limited [a]nd due to a strict blocade of the enemy, the construc[ti]on of new railroads is practically impossible at present [H]ence the possible development is only by air, and by the [hi]ghways. A big project of air transportation will be carri[e]d out in near future. The construction of highways has [m]ade a remarkable development, especially during this [gr]eat war. Due to the urgent need of the defensive [w]ar, the communication system in our country is greatly [i]mproved. There are many thousand miles of highways [to] be constructed in short time.

When the writer travels along some trunk lines, [h]e finds that there are many great improvements in both [th]e construction and maintenance of the high-ways, but [h]e thinks that the futher improvements are still possible nd imperative. The main object of this paper is to discuss the various methods of the improvements of highways in our country.

Methods of Improvements

The best requirement of engineering works is based ~~upo~~ upon the basic principles of economics, i.e., to secure the maximum profit with the minimum amount of investment. The sum of investment should include the first cost and the cost of maintenance of road. But it is not a real economy to make this sum a minimum only because the cost of operation and depreciation of cars has a great influence on the net profit. Hence the real economy is to secure all those costs a minimum sum. But the cost of construction and maintenance of roads belongs to the public government, while the cost of operation and depreciation of moving traffics belongs to the private owners. The more the former is saved, the more the latter is wasted. So their interests are just conflicting to each other. It is difficult to make a compromise between them. Logically the prime duty of the government is to render the service to the people. She should take care of their interests. It is worthwhile to build good roads for the benefit and convenience of drivers. I think they prefer to pay more tax to drive on the better roads. Hence the improvement of highways is really an economical cy.

To improve the highway, there are various methods, to imprcve the alinement, grade, curvature, and ement. The extent to which the improvement is le, depends upon the volume of traffics and the acity of the financial resoures. It can be determin only by good engineers. After the inspection trip of writer, he suggests the following practical methods improvements.

(A) Alinement. The first requirement of a good d is to secure the best location. The most ideal e is so chosen to be as straight and as level as sible. It is first to locate the principal cities d towns between the terminal points and then select the best belt to link them together, from h the best line can be finally determined. The lection of the best belt is a rather difficult task. re are many variable factors and some of them are flicting to each other. If the belt is not good, the st line can never be obtained. In case the best belt impossible to be located between some cities, some acent ones must be considered. A carefully recon-issance must be conducted by the well experinced gineers. The success or failure of the future load entirely dependent upon the skill of engineers to termine the best line in the reconnoisance survey.

The most striking example is the twenty-four -turns near An-nang (安南). It is artifficially castructed

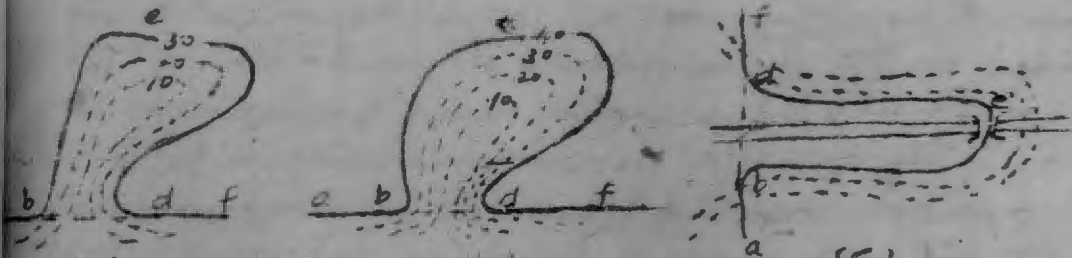
the very narrow and very steep slope of a high moun-
tain. On account of the worst condition of the belt, it
needs difficult to improve any better line. I don't
critisize the construction of such a road with a
heavy grade, sharp curvature, and short multiple re-
versed curve. To construct such a road with such a
result within such a worst condition is not bad at all.
I will say it is a marvelous construction in the sight
seeing district as to the Yellow Stone Park in the
States, or in the Ngow Mei Mountain (峨嵋山) in Sze-
chuen. I think all travellers are preferable to drive
on such a labyrinth and steeproad, and it will rather
take as a great enjoyment. But it is absolutely not
suitable for the international trunk line, because it
carries a great volume of freight traffics which are
usually very busy and very heavy, especially during
the war time. The writer has witnessed many times
that a heavy track is really very hard to be handled there,
so the chaffeur always feels in danger. The most dan-
gerous place is around every turn where the curvature
is too sharp, and the grade is too excessive. It is very
difficult to make a smooth riding. It frequently requires
the backward movement two or three times. If the brake
is out of control the backward movement, accelerated
by the force of gravitation, can not be stopped as
desired, so it still keeps on rolling straightly backward
and thus falls down to the bottom. But there is a
wonderful story that one truck just made a complete

overturing from the above road to the next below it without producing any damage, so it could proceed as usual. But such a fortunate accident is only a once out of several a million time. If the truck is stalled or overturned there, the subsequent trucks in both directions will cause some delay at the spot. So that portion of the road will become a bottle-necking. In order to secure a greater safety, and to speed up the movement of traffics, it is advisable to relocate it at once.

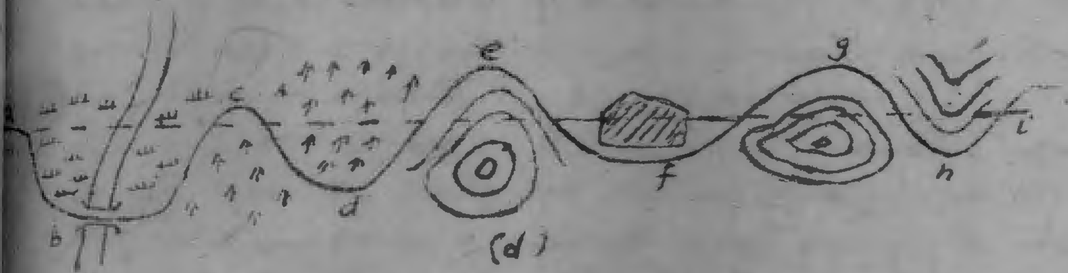
The next factor to improve the allinement is the selection of the bridge site and its approaches on both ends. For long and high bridges, the major important factor is to secure a sound foundation, preferable on the rock or on the hardpan. A relocation is worth while to choose the best site and easy approaches to the bridge. Some suspension bridges on the Kunming Burma Road do not provide with easy approaches. So delays and collisions are likely to happened. It seems to the writer that there is no strong reason to decide those bridge sites. It is advisable to make some improvements.

The third factor is the reduction of distance. The unnecessary distance of loops and zigzag lines are objectional features in alinement. The policy of ultilizing the existing road and avoiding the heavy cut and fill over a short section are both unwise.

Some typical examples will be illustrated as shown in fig. 2. The present line is constructed along the heavy line abcdf with the general idea



(a) (b) (c)



(d)

void the heavy cut or fill and to ultilize the old bridge. Its advantage is the only saving of earthwork or bridge, but the disadvantages are too numerous, as the long distance, sharp curvature, poor alinement, and more accidents. The common fallacy of young engineers is usually to consider the first cost too much but neglect the important factor of safety and the indefinite sum of the costs of maintenance, operation and depreciation. As a matter of fact, the first cost is a fixed amount spent once for all, or at least over a period of several years but the latter is accumulative, increasing with the time. The writer favous the better construction of these works of permanent nature. It is advisable to replace the heavy fill as in case (a), to construct a tunnel or heavy cut as in case (b), and to build a high trestle as in case (c). Similarly, the zigzag line abcdefgi may be much imprved. by a straight line ai. A little cut near the side hill, a little fill over a shallow pond, a little damage to the adjacent house means only a little to the total cost of the final construction. Although the cost of construction may be more expensive, the sum of other costs can be saved a great deal. So the total cost is still reduced to a minimum. A good engineer should have a broad view and consider everything in his mind in making his big projection.

(E) Grade. The most favorable grade is as level to facilitate the disposal of water from the road surface. A perfect level section is practically impossible, because the grade resistance is directly proportional to the percentage of grade. If the rate of grade is too large a great part of tractive force is used up to overcome the resistance, so it will stall on the heavy grade, and possibly move backward. If one truck is forced to stop there, it will cause a congestion and also a delay of traffics. Again, when the truck moves down the heavy grade, it will accelerate to a dangerous velocity due to the force of gravitation; so air brakes are necessary to be applied. Thus it wastes a great amount of useful energy. Both cases are not desirable. It is advisable not to exceed the ruling grade.

There are many typical cases to disobey this rule. Firstly, when a heavy fill is followed by a heavy cut over a short section, a loss of sta...

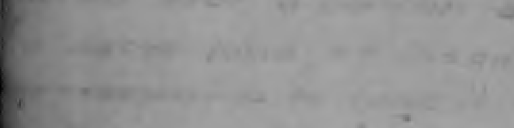
urvature is introduced to trace closely along the contour of side hill. A gentle slop is secured is at the expense of long distance. There is no saving of ill. The practical design, very simple and very economical, is to make the curvy cal and fill by the otted line as shown in (b) ... A smooth curve r grade can be easily obtained to balance the mount of cut and fill. The cost of earthwork n our country.

(a)

(b)

(c)

(d)



(e)

(f)

rather cheap. A heavy cut or fill, especially over short sections, is always more economical

Secondary, a short portion of excessive grade is constructed over a summit or depression of the rocky foundation. Sometimes, in oder to save the deep feeling over the bridge or culvert, an excessive grade is also employed on both approaches. Really, the cost of filling, of excavating, or even blasting is not expensive in comparison with the indefinite sum of running expenses. The expenditure of money to improve the grade is justificable. The saving of small cost of earth-work may cause a great trouble in riding and also increase the operating expenses.

Thirdly, a rolling grade in case (d), and a hump or a sag above or below the ruling grade as in case (e) are both undesirable, because the truck in high speed, especially on rough surface, will produce a sharp impact to cause damage to itself. A smooth grade in such cases is always preferable.

Fourthly, a roadbed is constructed below the adjacent land, so the surronding water will drain into the roadbed. The drainage system is rather difficult to be provided for and the pavement will be out of maintenance. Sometimes it is constructed over a portion of shallow ground. It has the same kind of disadvantage. The method of improvement is to raise it at least one foot above the adjacent ground or to fill to as high as the elevation of adjacent sections. A shallow filling over the long section is generally very economical. It is justificable to improve it as early as possible.

(C) Curvature. Curvature is an objectionable feature in alinement but can never be avoided, because it is impossible to construct a straight road between two terminals. Different kinds of curves are necessary to be employed to connect a series of straight lines. The method of improvement is usually to elliminate unnecessary curves or to replace the sharp curature with a flatter one. The common mistake to introduce sharp curvature is to avoid the heavy cut or a tunnel. There may be two reasons to commit this fault. Firstly, young engineers usually consider to cut down the first cost as an economical policy. Secondly, they may be too lazy to make a detailed reconnaisance for the best belt. Perhaps, they only examine those adjacent portions of the existing highway, or follow blindly the old alinement. So far the writer's opinion is concerned, many dangerous curvatures can be easily eliminated by making heavy cut or short tunnels. The first cost may be more expensive but the operating cost is much saved by shortening distance and reducing grade. The time of travel and the consumption of fuels are both reduced. The one more advantage is the security of safety.

It is a common sence that more accidents or collisions are happened on or near curvatures because of exececessive grade, improper superelevation, and short sight distance. Sinee a curve offers a greater resistance to the traffic, the rulling grade on the curve must be compensated for curvature, while the actual practice does not follow this rulé. A heavy truck, carrying a maximum load, will frequently stall there. Most chaffeurs are obliged to speed up the trucks before hand in order to climb up the excessive grade with the momentum effect. This is very dangerous because it will easily produce overturning, if the curvature is very sharp and the superelevation not provided properly for. The writer observed that most superelevations are not carefully adjusted. It is dangerous to drive on those places. The other dangerous element is not to provided a safe sight distance around sharp curvatures in deep excavation. A line of sight is usually interrupted by the intervening obstacles or high slopes of earth. If whistles are not frequently sounded, nose collisions of opposite traffics are likely to occur. The best way to eliminate such accidents is to avoid the sharp or reversed curve, especially in deep cut, or to widen the curve and divide the both lanes by conspicious marks of a center line. A sharp curve at the end of steep and long descending grade is also a dangerous element and should be eliminated, if possible.

(D) Pavement The principle of engineering economics is to design the work in such a way as to secure the costs of construction and operation a minimum sum. In general, if the first cost is cheap, the operating cost must be expensive. Hence, when the volume of traffics is small, it is economical to use cheap pavement, but when large, to change a better class of pavement, because the saving of operating expenses must be great enough to reconstruct the better road.

In our country, a great part of highway is constructed during the great war, and due to the financial embarrasement, there is no wearing surface on the water bound macadam road. At beginning, the number of traffics is small, so it may satisfy the local demand, while, at present, the imports of machines and war supplies are so greatly increased that the disadvantages of using poor pavement are very remarkable. The work of maintenance, is so difficult, the cost of operation so expensive, the rate of depreciation so high, the time of travel so prolonged, and the number of accidents so much increached that a poor road will evidently cause a great loss, both financially and materially, and also render a great hinderance, to the progress of the defensive war. The best way of improvement is to change the good pavement.

The best pavement used on modern roads may be either concreat or bituminous materials. If these materials are not easily obtained, native oil should be tried. The essential properties of the wearing surface are the binding power and waterproof. The writer believes that wood oil may satisfy these requirements under proper conditions. An exhaustive study is worthwhile to be carried out along the direction. A final suceess may be expected soon.

(E) Safety. "Safety first" is a golden proverb. Every body must remember it in his whole life. Engineers must also apply it to designing works. To design the highway according to the method, as mentioned above, is the best way to secure safety.

But there are other important factors as signaling, drainage, safe structure, and strategic points. At grade crossing and dangerous places, accidents are likely occured, so attractive signals, both manual and automatic, should be carefully provided. In our country there is only one kind of day-time signals. But experience tells us that, in order to avoid the air bombing, night riding is preferable, so night signals are necessary to be equipped.

It is a well-known fact that water is the worst enemy to the solidity of the roadbed. If the drainage system or drainage area is not properly designed, the washout of the wearing surface, bridges, culverts is ikely the result. To design the drainag system in the untainous regions, it is safer to provide an ample capacity. Or the washout, if once happened, may cause suspension of communication over a certain period, which will, in turn, result in a congestion of traffics at that point.

In war time, structures are sofe frequently mbed by the enemy planes or destroyed by the traitors and the fifth column. A carefully inspection and guard may reduce such accidents. Aling some busy nsportation lines, heavy trucks under overloading are frequently operated in high speed. so the impact force to the structure may be very exceessive. Hence, the design or repair of a safer structure is very important.

From the severe teaching of the aggressive war against our country, we suffer a great loss due to the fact that all communication lines are not well protected. So the enemy catch this weak point to wedge in our defense line by the mehanical units. Our lost rritories are, therefore, mostly along those lines. If former engineers located those lines with a military point of view, our brave soldiers may still hold them. Hence, locating engineers must acknowledge one more factor in reconnoisance, i.e., to utilize the stratagic points to protect the whole line and to defeat the whole enemy.

Our sages teaches us that the memory of the
st events may serve as a best guide for the future
ons. A big project of engineering works must be close-
co-omperated with the military strength. Otherwise,
soon as it is accomplished, it will immediately fall into
e foreign enemy's hand. The airdrome in Gyoh Shay
縣) and the Kunming - Burma Road are typical ex-
mples.

When the writer travels along different lines,
observes many cars and trucks are overturned by
e side of the road, partly due to the poor skill of
e drivers but chiefly due to the poor design by en-
ineers. Those locations if carefully ~~located~~ recorded
nd with photoes taken in field, can furnish very valua-
le informations for the investigation of the causes of
ccidents. A extensive study of those causes may
rmulate general rules for the improvement of better
linement. But it is very pitiful that no such a record
s yet been made and no further improvement is
ctually carried out, so future accidents can still be
epeated. It is the duty of engineers to investigate the
al causes of accidents and to make the design
ith safety first.

Conclusion

It is well recognized that China has made a re-
arkable development of highways during this war. But
ut such development and improvements are still far

from satisfaction. Engineers are advised to make improvement after improvements in order to accomplish the same result as in the western countries.

The design of engineering works is a specialized problem, which can be successfully solved only by the specialist. But many engineering projects in our country are determined not by civil engineers, but by civil officers; or even by engineers, but not by the best engineers. So all big jobs are not well designed or carried out according to the best methods. It is a wrong policy to design the big projects with the short sighted mined. To cut down the first cost is not a real economy. The economical point of view is to make the sum of all costs a minimum and not the first cost only. Those works of permanent nature must be designed with the best standard. The first cost, even if greater, is still economical, because of the accumulated costs of subsequent improvements may be so great as the first cost.

The construction of modern roads requires maintenance as soon as finished. Good maintenance can keep the road always in good conditions. The cost of maintenance should be considered as important as the cost of construction. If roads are properly maintained, the time of service can be much increased and the cost of operation and depreciation also much reduced. It is a general rule of real economy.

From the review of history, It is learned that the international war is inavoidable after a certain period. All communication lines, especially leading to the foreign country, must be well protected against any aggresive invention. Otherwise any line may be easily fallen into and renders convenient means to the enemy. In order to secure the best result, engineers should be also trained with a military knowledge. All strategic points should be well fortified to protect communication lines. The falling of great works, when just accomplished, is a great financial and material loss.

The concensus of opinion is that the transportation of freight by highways is not economical as by railroads. This comment is more true in our country. Since fuels and trucks are both imported, the construction of more highways requires a greater import of foreign goods and drains a greater amount of money into foreign countries. On the other hand, the construction of more railroads needs only the import of locomotives. The manufacture of rails and the construction of cars may be made in our country. The advantages of railroads are too numerous to be metioned. If the alinement and grades are so located so as to satisfy the requirements of railroads, those highways may be at once transfered to the road bed of future railroads. It is one stone to kill two birds.

在萬能銑床上如何可切任何齒數之斜齒輪(Helical gear)

鍾與銳

在高速度之機器中用正齒輪傳動，不如改用斜齒輪，可以減低噪聲，且可增加所傳能力，又在不平行不相交之軸，亦可應用斜齒輪傳動，故斜齒輪之應用頗為普遍也。製斜齒輪之機器可分為兩類，一普通工作機器，如萬能銑床，二專製齒輪機器，如 Gear generating machine, gear hobbing 等。我國工廠備有專製齒輪機器者殊屬少見，故欲修配或製造斜齒輪者，惟有利用萬能銑床。在銑床上切斜齒輪之要點有二，甲、切齒時齒輪坯需轉動，使成斜齒，乙、需將齒輪坯圓周分成與齒數相同之等分。欲完成此兩點，則有賴於萬能指示頭 (Universal index head)。萬能指示頭為裝于銑床工作台之左端，其內部構造見圖一。若轉動軸E，經斜齒輪、指示板、指針、手柄、手柄軸、螺齒杆、螺齒輪，而軸C旋動，與齒輪坯為同裝于軸C，故齒輪坯亦旋轉矣。又若使軸E不動，即指示板不動，則可將指針拔出，與指示板分離，轉動手柄軸，齒輪坯亦能旋轉矣。茲將其應用方法及計算公式列下：

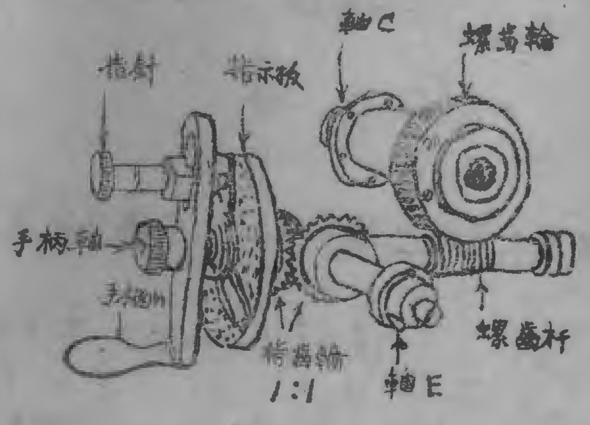


圖 一

甲 換齒切斜齒原理

于銑床工作台前進時，齒輪坯需同時旋轉，其間關係可令工作台之螺絲杆與指示頭之軸E以齒輪連動之，即成，見圖二。

設 L代表斜齒輪之前進(lead)

P 〃 螺絲杆 〃 〃

N代表旋轉數其右下角小字爲代表其部分之代表字母見圖一或圖二、

令 $\frac{N_S}{N_E} = e_1$

依斜齒原理則得

$\frac{N_S}{N_C} = \pm \frac{L}{D}$ (銑床之螺絲杆爲右手向螺絲)上式中之正號爲表示右手向斜齒負號爲表示左手向斜齒、

在一般之指示頭 $\frac{N_E}{N_C} = 40$ (兩軸均爲同方向旋轉故爲正號)

得 $e_1 = \pm \frac{L}{40P}$

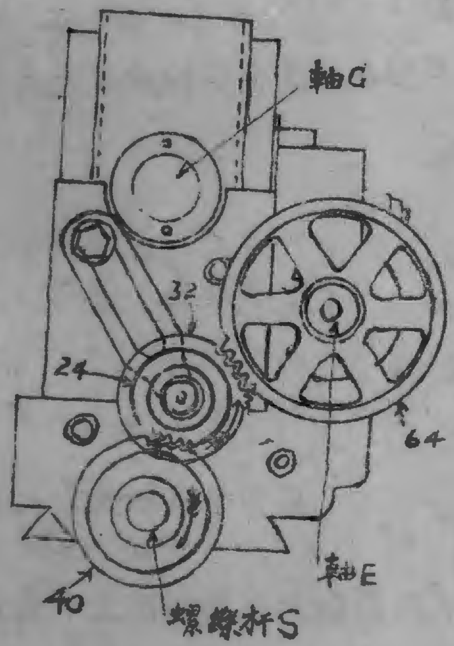


圖 二

上式 e_1 爲正數時表示軸C與軸E爲同方向旋轉若爲負數則反是

舉例 切右手向斜齒輪一隻其前進爲12吋在一般之銑床上螺絲杆之前進爲¼吋、

由上式 得 $e_1 = +\frac{12}{40 \times \frac{1}{4}} = +\frac{6}{5}$ 可改成 $\frac{24}{40} \times \frac{64}{32}$

是則由螺絲杆至軸E間之齒輪齒數爲40, 24, 32及64，斜齒爲右手向，e_1爲正數則軸C與軸E應爲同方向旋轉是則兩軸間用上述之四齒輪已符合同方向旋轉見圖二故不需另加惰輪(idler)矣.

乙 分齒輪坯圓週爲所需齒數之等分

一固定指示板即軸E不動可拔出指針轉動手柄而齒輪坯旋轉矣.齒輪坯旋轉之多少則視指針經過指示板上之小孔之多少而定在 Brown and Sharp 公司所造之指示頭有三塊指示板其上有小孔綴成大小

不同之同心圓每圓周上之小孔數目見下表、

第一板、 15, 16, 17, 18, 19, 20

第二板 21, 23, 27, 29, 31, 33

第三板 37, 39, 41, 43, 47, 49

指針應經過某圓週上若干孔數使齒輪坯旋轉所需齒數分之一圓週可由下式求之、

設 T代表所需齒數

H代表指示板上某圓週上之小孔數目

D代表指針所經過某圓週上之孔數（應成整數）

N代表旋轉數（與甲節同）

已知 $N_h/N_c = 40$

則得 $40H = DT$

舉例 設所需齒數爲25 求指針每次需旋轉若干孔數

擇 $H=15$ 由上式 $40 \times 15 = 25 \times D$

得 $D=24$ 是則指針對于圓週十五孔者應每次經過二十四孔或（$\frac{24}{15}$ =1轉）一轉又九孔。

二、照上所述方法因指示板上之小孔數目有限凡齒數在50以上之若干數目如51, 53, ……等不能等分之、其補救之術、惟有應用差動指示法（Differential indexing）即令軸C與軸E以齒輪連動之、是乃令指示板轉動一些、以補手柄轉動之不足也見圖三其計算之法述下

所有代表字母均與前同、可參看之、

令 $\frac{N_E}{N_C} = e$

得 $e = 40 - \frac{TD}{H}$

上式若得e爲正數乃表示軸C與軸E作同方向旋轉如得負數則反是.

舉例 設所需齒數爲96 齒、求軸C與軸E間需何齒輪連動.

如96齒時如不用差動指示法用 $40H = DT$ 公式將任何指示板上之小孔數目代入均不能令D成整數故惟有差動指示法方可也茲擇

$H=20 \quad D=8$

則 $C=40-\frac{96\times 8}{20}=\frac{8}{5}$

可改為 $\frac{64}{40}$

是則軸C上裝64齒齒輪軸E上裝40齒齒輪C之值為正數軸C與軸E需同方向旋轉故需于兩齒輪間加一惰輪方可見圖三

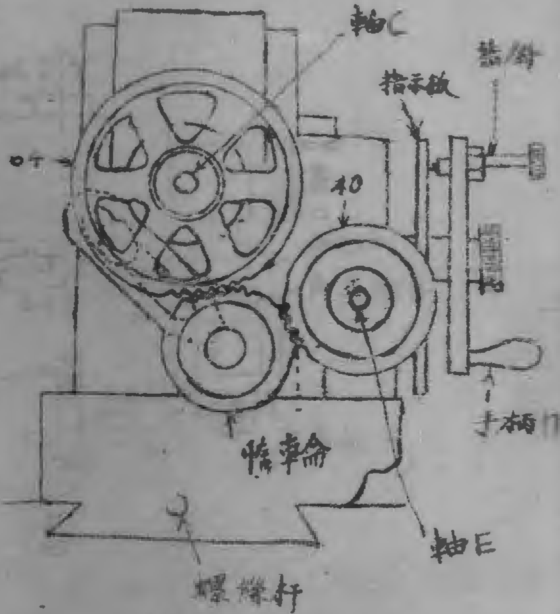


圖 三

用此方法在 Brown and Sharp 之萬能指示頭及規定之一組齒輪則可分二至三百八十二等分若干更多之等分也

(丙) 由前甲乙兩節對于製斜齒輪之要點明矣惟軸E需與工作台之螺絲杆而不能再與軸C連動矣由此之故凡斜齒輪齒數屬於節二後者不能在萬能銑床上製之矣茲思得一法即加一差動齒輪系 (Differential gear train) 于差動指示頭軸E之前端則任何齒數之斜齒輪皆能在萬能銑床上製之矣差動齒輪系構造見圖四乃由四個(或三個)角齒輪及十四軸組成之應用時十四軸 E_3 與指示頭軸E固接角齒輪 B_1 與軸C以齒輪連動之角齒輪 B_2 與螺絲杆以齒輪連動之見圖五

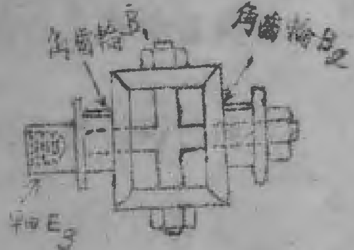


圖 四

差動齒輪系中軸E(即 E_3)乃角齒輪 B_1 與 B_2 之旋轉數可由

用公式計算之、

設 N代表旋轉數、其右下角小字為代表某部份

則 $2N_E = N_{B_1} + N_{B_2}$

應用上式時應注意旋轉方向以時針向為正數則反時針為負數于應用時、其間之關係討論之、

一、分齒輪坯圓週成所需齒數分 在轉動手柄等分齒坯時工作台不動則角齒輪不動在差動齒輪系中得 $2N_E = N_{B_1}$

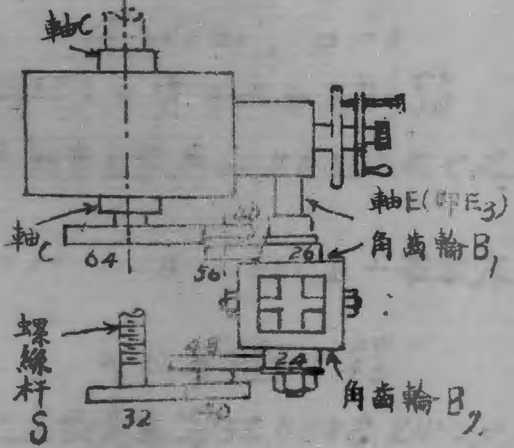


圖 五

設 $\frac{N_{B_1}}{N_C} = e_2$ 及所有各字母之代表均與前乙節用.

由乙節知 $e = \frac{N_E}{N_C}$ 將 e_2 代入得 $e = \frac{1}{2}e_2$

故 $e_2 = 2e = 2(40 - \frac{TD}{H})$

如若得 e_2 為正數時表示軸C與角齒輪 B_1 為同方向施轉、如為負數則反是、

二、切成斜齒 在切斜齒時工作台前進螺絲杆旋轉故角齒輪 B_2 旋轉又齒輪坯旋轉、故角齒輪 B_1 旋轉、故軸E之旋轉、乃由角齒輪 B_1 與 B_2 二者旋轉所組成由前知 $2N_E = N_{B_1} + N_{B_2}$

設 $\frac{N_S}{N_{B_2}} = e_3$ 又所有各字母之代表均與前甲乙兩節同

由前知 $N_E = 40N_C$ $\frac{N_{B_1}}{N_C} = e_2$

則 $2 \times 40 N_C = e_2 N_C + \frac{N_S}{E_3}$

故 $e_3(80 - E_2)N_C = N_S$

由前 $\frac{N_S}{N_C} = \pm\frac{L}{P}$, $e_2 = 2(40 - \frac{TD}{H})$

故 $e_3 = \pm\frac{LH}{2PTD}$

此式 e_3 之值為正數時表示螺絲S與角齒輪 B_2 之間應以若干齒數之齒

轉運動之。

先求 e_2　已知 $T=96$，　擇 $H=20$，$D=8$

$$e_2=2\left(40-\frac{96\times 8}{4}\right)=\frac{16}{5}\ \text{可改為}\ \frac{64}{40}\times\frac{56}{28}$$

則由軸 c 至角齒輪 B 間之齒輪其齒數為64、40、56、28，e_2之值為正數故不需加惰輪見圖五

次求 e_3，已知 $T=96$，$H=20$，$D=8$，$L=12$，在一般銑床上 $F=\frac{1}{4}$ 吋，斜齒為右手向用正號。

$$e_3=\frac{12\times 20}{2\times\frac{1}{4}\times 96\times 8}=\frac{5}{8}\ \text{可改為}\ \frac{40}{32}\times\frac{24}{48}$$

則由螺絲軸 S 至角齒輪 B 間之齒輪其齒為32、40、48、24，e_3之值為正數故其間不需加惰輪見圖五。

應用差動齒輪系加於差動指示頭上以切任何齒數之斜齒輪由舉例可知之甚明矣，故自配一副差動齒輪系于差動指示頭上，則可自製成修配任何齒數之斜齒輪矣，惟差動齒輪系中之角齒輪吾人知不易製之準確，恐有差誤，可以用正齒輪組成差動齒輪系，其構造與用角齒輪者不同，但其動作關係不變，茲將其構造簡圖（見圖六）列下以供參攷焉。

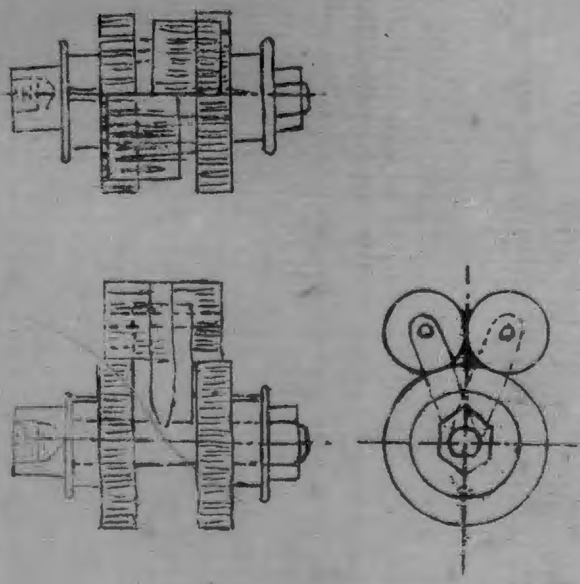


圖　六