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Fig. 1.

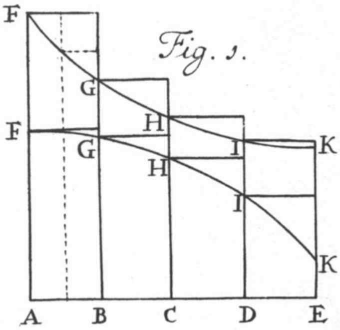


Fig. 2.

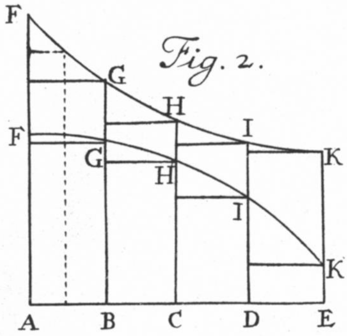


Fig. 3.

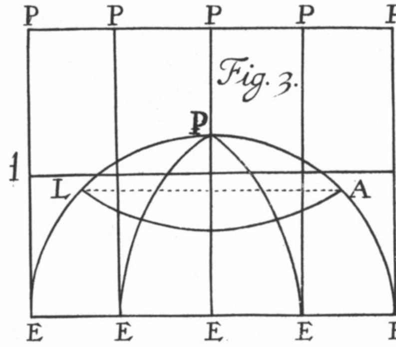


Fig. 5.

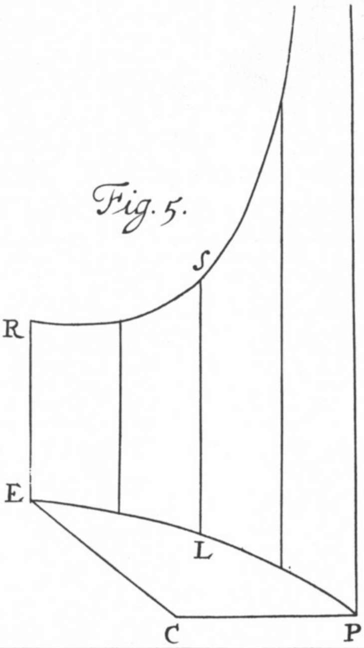


Fig. 6.

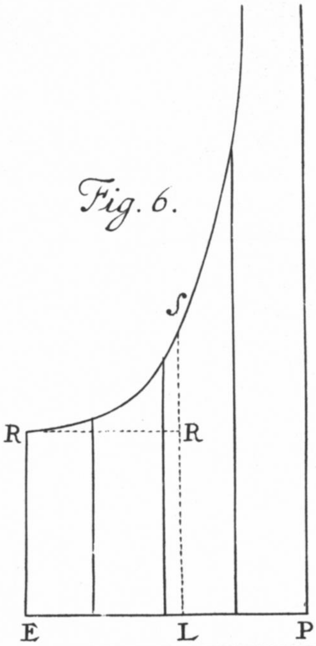


Fig. 7.

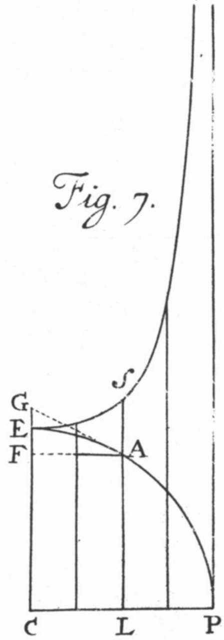


Fig. 8.

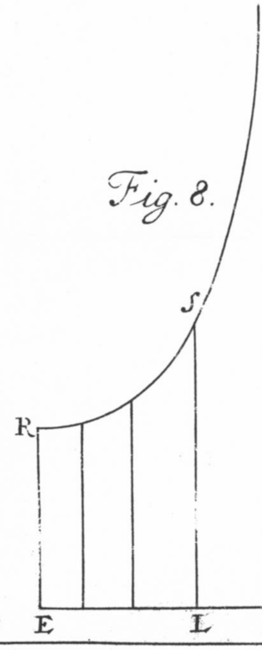


Fig. 10.

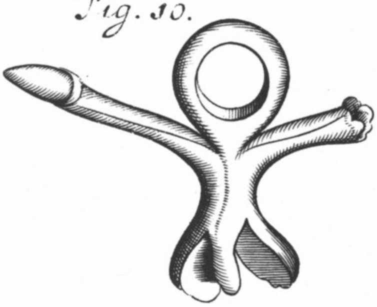


Fig. 11.

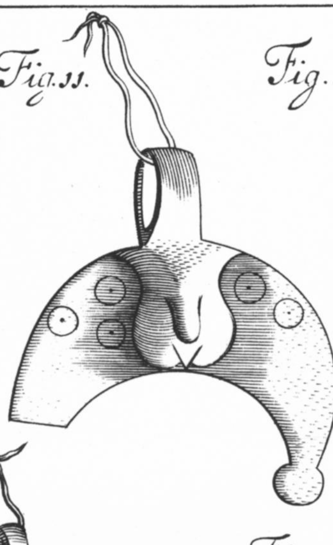


Fig. 12.

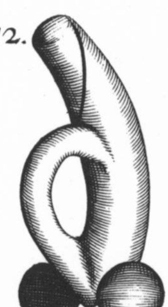


Fig. 13.

Fig. 26.

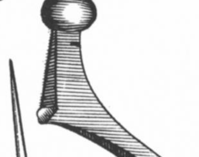
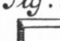


Fig. 18.

Fig. 14.

Fig. 20.

Fig. 19.



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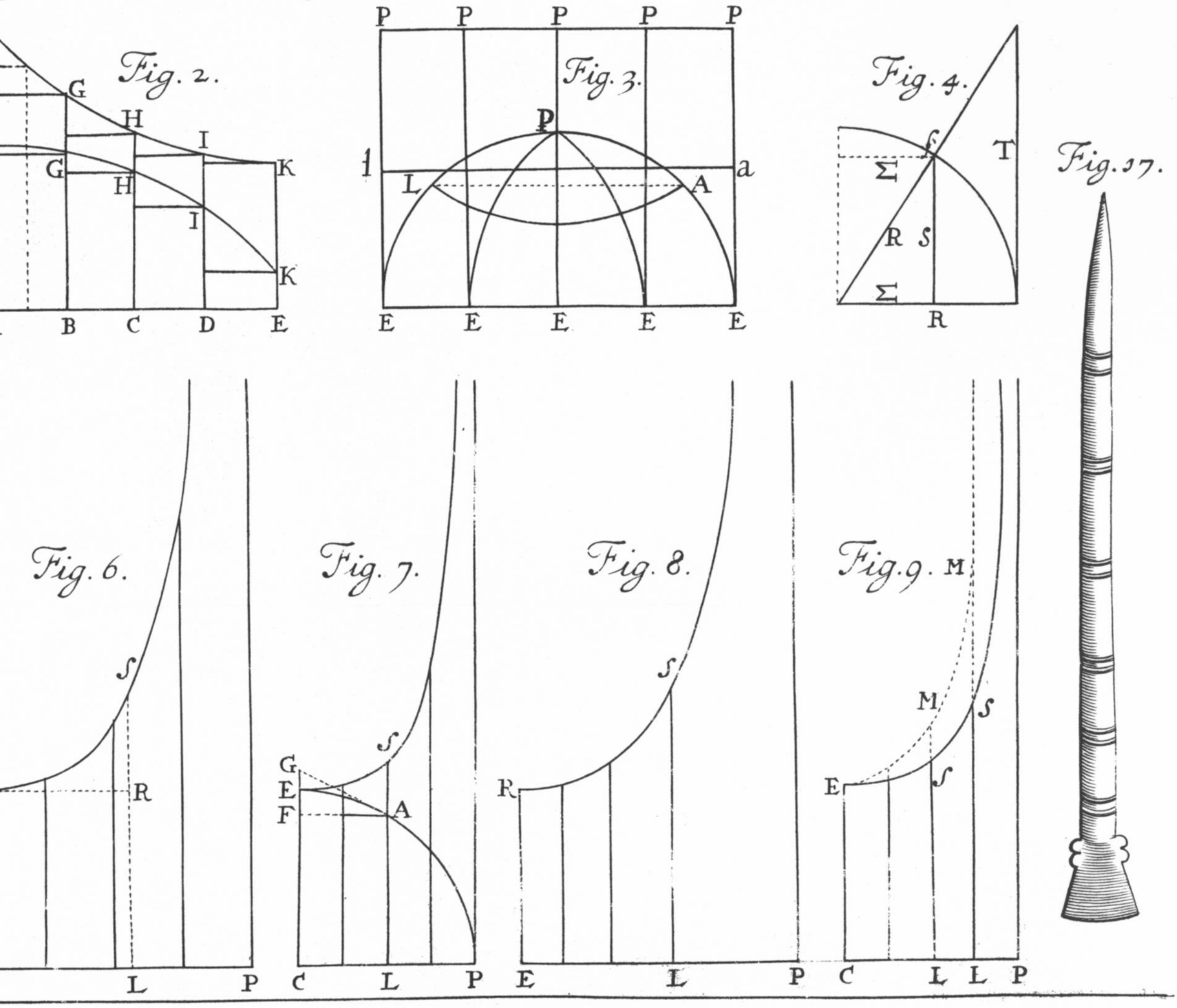

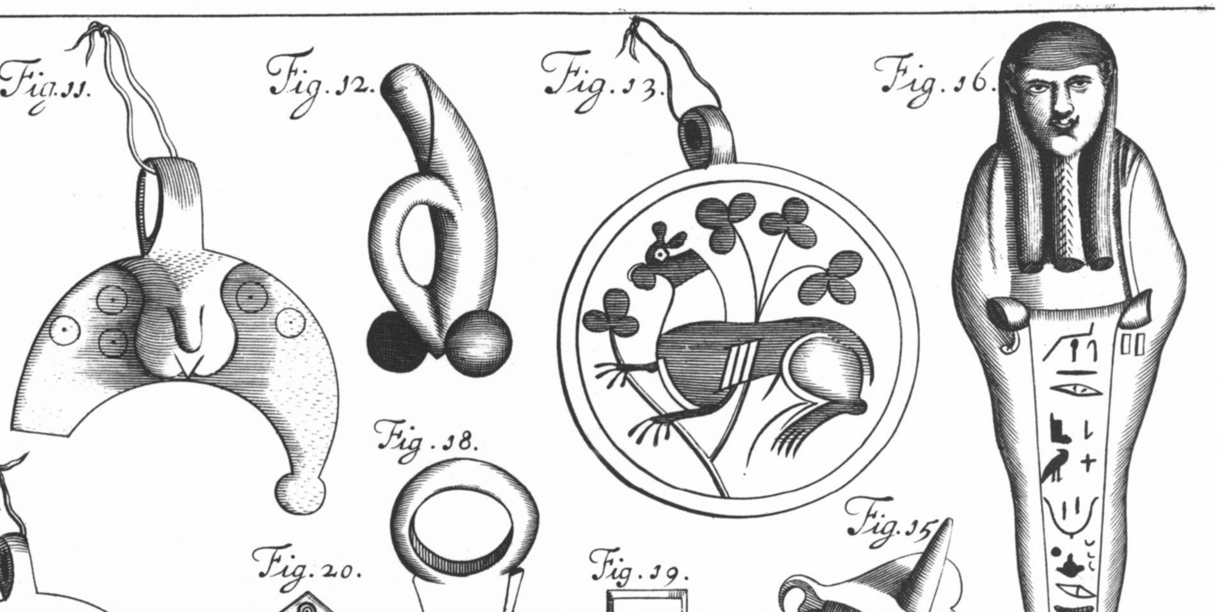

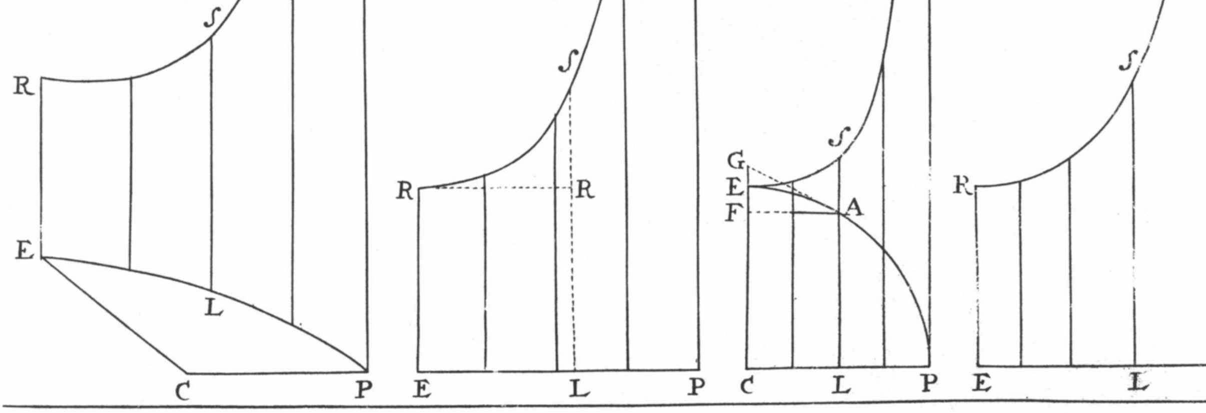
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A Letter from the Reverend Dr Wallis, Professor *of* Geometry *in the University of* Oxford, *and* Fellow *of the* Royal Society, *London, to Mr.* Richard Norris, *Concerning the Collection of* Secants; *and the true Division of the Meridians in the Sea-Chart.*

AN old enquiry,(about the sum or Aggregate of *Secants*,) having been of late moved a-new; I have thought fit to trace it from its Original: with such solution as seems proper to it: Beginning first with the general Preparation; and then applying it to the Particular Case.

General Preparation.

1. Because Curve lines are not so easily managed as Streight lines: the Ancients, when they were to consider of Figures terminated (at left on one side) by a Curve line (Convex or Concave,) as *AFKE*, Fig. 1. 2. did oft make use of some such expedient as this following, (but diversly varied as occasion required.) Namely,

2. By Parallel Streight lines, as *AF*, *BG*, *CH*, &c, (at equal or unequal distances as there was occasion,) they parted it into so many segments as they thought fit; (or supposed it to be so parted.)

3. These segments were *so many wanting one*, as was the number of those Parallels.

4. To each of these Parallels, wanting one; they fitted Parallelograms, of such breadths as were the Intervalls (equal or unequal) between each of them (respectively) and the next following. Which formed an Adscribed Figure made up of those Parallelograms.

 5. And

5. And, if they began with the Greateſt (and therefore neglected the leſt,) ſuch Figure was Circumſcribed (as Fig. 1.) and therefore Bigger than the Curvilinear propoſed.

6. If with the Leſt (neglecting the greateſt;) the Figure was Inſcribed (as Fig. 2.) and therefore Leſs than that propoſed.

7. But, as the number of Segments was increaſed, (and thereby their breadths diminiſhed;) the difference of the Circumſcribed from the Inſcribed (and therefore of either from that propoſed) did continually decreaſe, ſo as at laſt to be leſs than any aſſigned.

8. On which they grounded their Method of exhauſtions.

9. In caſes wherein the Breadth of the Parallelograms, or Intervalls of the Parallels, is not to be conſidered, but their length only; (or, which is much the ſame, where the Intervalls are all the ſame, and each repated = 1:) *Archimedes* (inſtead of Inſcribed and Circumſcribed Figures) uſed to ſay, *All except the Greateſt*, and *All except the Leſt*. As Prop. 11. Lin. Spiral.

Particular Caſe.

10. Though it be well known, that, in the Terreſtrial Globe, all the Meridians meet at the Pole, (as *EP*, *EP*, Fig. 3.) whereby the Parallels to the Equator, as they be nearer to the Pole, do continually decreaſe:

11. And hereby a degree of Longitude in ſuch Parallels, is leſs than a degree of Longitude in the Equator, or a degree of Latitude:

12. And that, in ſuch proportion, as is the Co-Sine of Latitude (which is the ſemidiameter of ſuch Parallel) to the Radius of the Globe, or of the Equator:

13. Yet hath it been thought fit (for ſome reaſons) to repreſent theſe Meridians, in the Sea Chart, by Parallel ſtreight lines; as *Ep*, *Ep*.

14. Where-

14. Whereby, each Parallel to the Equator (as *LA*) was repreſented in the Sea-Chart, (as *la*,) as equal to the Equator *EE*: and a degree of Longitude therein, as large as in the Equator.

15. By this means, each degree of Longitude in ſuch Parallels, was increaſed, beyond its juſt proportion, at ſuch rate as the Equator (or its Radius) is greater than ſuch Parallel, (or the Radius thereof.) Longitude

16. But, in the Old Sea-Charts, the degrees of Latitude were yet repreſented (as they are in themſelves) equal to each other; and, to thoſe of the Equator.

17. Hereby, amongſt many other Inconveniences, (as Mr. *Edward Wright* obſerves, in his *Correction of Errors in Navigation*, firſt publiſhed in the year 1599,) the repreſentation of places remote from the Equator, was ſo diſtorted in thoſe Charts, as that (for inſtance) an *Iſland* in the Latitude of 60 degrees, (where the Radius of the Parallel is but half ſo great as that of the Equator,) would have its Length (from Eaſt to Weſt) in compariſon of its Breadth (from North to South) repreſented in a double proportion of what indeed it is.

18. For rectifying this in ſome meaſure (and of ſome other inconveniences,) Mr. *Wright* adviſeth; that (the Meridians remaining Parallel, as before) the degrees of Latitude, remote from the Equator, ſhould at each Parallel, be protracted in like proportion with thoſe of Longitude.

19. That is; As the Co-Sine of Latitude (which is the Semidiameter of the Parallel) to the Radius of the Globe, (which is that of the Equator:) ſo ſhould be a degree of Latitude (which is every where equal to a degree of Longitude in the Equator,) to ſuch degree of Latitude ſo protracted (at ſuch diſtance from the Equator;) and ſo to be repreſented in the Chart.

20. That is; every where, in ſuch proportion as is

the respective Secant (for such Latitude) to the Radius. For, As the Co-Sine, to the Radius; so is the Radius, to the Secant (of the same Arch or Angle;) as Fig. 4. Σ. R :: R. ſ.

21. So that (by this means) the position of each Parallel in the Chart, should be at such distance from the Equator, compared with so many *Equinoctial* Degrees or Minutes, (as are those of Latitude,) as are all the Secants (taken at equal distances in the Arch) to so many times the Radius.

22. Which is equivalent (as Mr. *Wright* there notes) to a Projection of the *Spherical* surface (supposing the Ey at the Center) on the concave surface of a Cylinder erected at right Angles to the Plain of the Equator.

23. And the division of Meridians, represented by the surface of a Cylinder erected (on the Arch of Latitude) at right Angles to the Plain of the Meridian (or a portion thereof.) The Altitude of such Projection (or portion of such Cylindrick surface) being, (at each point of such Circular base) equal to the secant (of Latitude) answering to such point. As Fig. 5.

24. This Projection (or portion of the Cylindrick surface) if expanded into a Plain, will be the same with a Plain Figure, who's base is equal to a Quadrantal Arch extended (or a portion thereof) on which (as ordinates) are erected Perpendiculars equal to the Secants, answering to the respective points of the Arch so extended: The least of which (answering to the *Equinoctial*) is equal to the Radius; and the rest continually increasing, till (at the Pole) it be Infinite. As at Fig. 6.

25. So that, as *ERſL*. (a Figure of Secants erected at right Angles on *EL*, the Arch of Latitude extended,) to *ERRL*, (a rectangle on the same base, who's altitude *ER* is equal to the Radius;) so is *EL* (an Arch of the Equator equal to that of Latitude,) to the di-

distance of such Parallel, (in the Chart) from the Equator.

26. For finding this distance, answering to each degree and Minute of Latitude; Mr. *Wright* (as the most obvious way) Adds all the Secants (as they are found calculated in the Trigonometrical Canon) from the beginning, to the degree or Minute of Latitude proposed.

27. The sum of all which except the Greatest, (answering to the Figure Inscribed) is too Little: The sum of all except the Least, (answering to the Circumscribed,) is too Great, (which is that he follows:) And it would be nearer to the truth than either, if (omitting all these) we take the intermediates; for Min. $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, &c. or (the doubles of these) Min. 1, 3, 5, 7, &c. Which yet (because on the Convex side of the Curve) would be somewhat too Little.

28. But any of these ways are exact enough for the use intended, as creating no sensible difference in the Chart.

29. If we would be more exact; Mr. *Oughtred* directs (and so had Mr. *Wright* done before him) to divide the Arch into parts yet smaller than Minutes, and calculate Secants suting thereunto.

30. Since the Arithmetick of Infinites introduced, and (in pursuance thereof) the doctrine of Infinite series (for such cases as would not, without them, come to a determinate proportion;) Methods have been found for squaring some such Figures; and (particularly) the Exterior Hyperbola (in a way of continual approach) by the help of an Infinite series. As, in the *Philosophical Transactions*, Numb. 38, (for the Month of *August* 1668,) And my Book *De motu*, Cap 5. prop. 31.

31. In Imitation whereof, it hath been desired (I find) by some, that a like Quadrature for this Figure of Secants (by an Infinite series fitted thereunto) might be found.

32. In

32. In order to which; Put we, for the Radius of a Circle, R; the right Sine of an Arch or Angle, S; the Versed Sine, V; the Co-Sine (or Sine of the Complement) $\Sigma = R - V = \surd : Rq - Sq :$ the Secant, ſ; the Tangent, T. Fig. 4.

33. Then is, $\Sigma . R :: R . ſ$. That is, $\Sigma) R^2 (S = \frac{R^2}{\Sigma}$; the Secant.

34. And $\Sigma . S :: R . T$. That is, $\Sigma) SR (T = \frac{SR}{\Sigma}$; the Tangent.

35. Now, if we ſuppoſe the Radius CP, Fig. 7. divided into equal parts, (and each of them $= \frac{1}{\infty} R$;) and, on theſe, to be erected the Co-Sines of Latitude LA:

36. Then are the Sines of Latitude in *Arithmetick* Progreſſion.

37. And the Secants anſwering thereunto, $Lſ = \frac{R^2}{\Sigma}$.

38. But theſe Secants, (anſwering to right Sines in *Arithmetical* progreſſion,) are not thoſe that ſtand at equal diſtances on the Quadrantal Arch extended, Fig. 6.

39. But, ſtanding at unequal diſtances (on the ſame extended Arch;) Namely, on thoſe points thereof, who's right Sines (whilſt it was a Curve) are in *Arithmetical* Progreſſion. As Fig. 8.

40. To find therefore the magnitude of *RELſ*, Fig. 6. Which is the ſame with that of Fig. 8. (ſuppoſing *EL* of the ſame length in both; however the number of Secants therein may be unequal:) we are to conſider the Secants, though at unequal diſtances, Fig. 8. to be the ſame with thoſe at equal diſtances in Fig. 7. anſwering to Sines in *Arithmetical* Progreſſion.

41. Now theſe Intervalls (or portions of the baſe) in Fig. 8. are the ſame with the intercepted Arches (or portions of the Arch) in Fig. 7. For this Baſe is but that Arch extended.

42. And theſe Arches (in parts infinitely ſmall) are to be reputed equivalent to the portions of their reſpective Tangents intercepted between the ſame ordinates. As in Fig. 7. 9.

43. That

43. That is, Equivalent to the portions of the Tangents of Latitude.

44. And these portions of Tangents are, to the Equal intervalls in the base, as the Tangent (of Latitude) to its Sine.

45. To find therefore the true Magnitude of the Parallelograms (or segments of the Figure;) we must either protract the equal segments of the base Fig. 7. (in such proportion as is the respective Tangent to the Sine) to make them equal to those of Fig. 8.

46. Or else (which is equivalent) retaining the equal intervalls of Fig. 7. protract the Secants in the same proportion. (For, either way, the Intercepted Rectangles or Parallelograms will be equally increased) As *LM.* Fig. 9.

47. Namely; As the Sine (of Latitude) to its Tangent; so is the Secant, to a Fourth; which is to stand (on the Radius equally divided) instead of that Secant.

$$S.\ \frac{SR}{\Sigma}\ (::\Sigma.\ R)\ ::\ \frac{R2}{\Sigma}.\ \frac{R3}{\Sigma2 = R2 - S2} = LM,\ \text{Fig. } 9.$$

48. Which therefore are as the Ordinates in (what I call *Arith. Infin. Prop.* 104.) *Reciproca Secundanorum:* supposing Σ^2 to be squares in the order of Secundanes.

$$
\begin{array}{l}
R^2 - S^2)\ R^3\ (R,\ +\frac{S2}{R},\ +\frac{S4}{R3},\ + \\
\quad R3 - S2R \\
\quad \overline{+S2R} \\
\quad +S^2R - \frac{S4}{R} \\
\quad \overline{} \\
\quad\quad +\frac{S4}{R} \\
\quad\quad +\frac{S4}{R} - \frac{S6}{R3} \\
\quad\quad \overline{} \\
\quad\quad\quad +\frac{S6}{R3}
\end{array}
$$

49. This (because of $\Sigma^2 = R^2 - S^2$; and the Sines S, in Arithmetical Progression) is reduced (by division) into this Infinite Series

$$R + \frac{S2}{R} + \frac{S4}{R3} + \frac{S6}{R5}, \&c.$$

50. That is, (putting $R = 1.$)

$$1 + S^2 + S^4 + S^6, \&c.$$

51. Then (according to the *Arithmetick* of Infinites)

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we are to interpret S, ſucceſſively, by 1 *S*, 2 *S*, 3 *S*, &c. till we come to S, the greateſt. Which therefore repreſents the number of All.

52. And becauſe the firſt member doth repreſent a Series of Equals; the ſecond, of Secundans; the third, of Quartans, &c. Therefore the firſt member is to be multiplied, by S; the ſecond, by $\frac{1}{3}$S; the third, by $\frac{1}{5}$ S; the fourth, by $\frac{1}{7}$ S; &c.

53. Which makes the Aggregate,

$$S + \tfrac{1}{3}S^3 + \tfrac{1}{5}S^5 + \tfrac{1}{7}S^7 + \tfrac{1}{9}S^9, \text{ \&c.} = ECLM, \text{ Fig. } 9.$$

54. This (becauſe S is allways leſs than R = 1) may be ſo far continued, till ſome power of S become ſo ſmall as that it (and all which follow it) may be ſafely neglected.

55. Now (to fit this to the Sea-Chart, according to Mr. *Wrights* deſign:) Having the propoſed Parallel (of Latitude) given; we are to find (by the Trigonometrical Canon) the Sine of ſuch Latitude; and take, equal to it, CL = S. And, by this, find the magnitude of ECLM, Fig. 9; that is, of RELſ, Fig. 8. that is, of RELſ, Fig. 6. And then, As RRLE (or ſo many times the Radius,) to RELſ (the Aggregate of all the Secants;) ſo muſt be a like Arch of the Equator (equal to the Latitude propoſed,) to the diſtance of ſuch Parallel, (repreſenting the Latitude in the Chart) from the Equator. Which is the thing required.

56. The ſame may be obtained, in like manner, by taking the Verſed Sines in *Arithmetical* progreſſion. For if the right Sines (as here) beginning at the Equator, be in *Arithmetical* progreſſion, as 1, 2, 3, &c. Then will the Verſed Sines, beginning at the Pole, (as being their complements to the Radius) be ſo alſo.

The Collection of Tangents.

57. The ſame may be applyed in like manner, (though that be not the preſent buſineſs,) to the Aggregate of Tangents, (anſwering to the Arch divided into equal parts.)

58. For,

58. For, those answering to the Radius so divided, are $\frac{SR}{\Sigma}$; (taking S in *Arithmetical* progression.)

59. And then, inlarging the Base (as in Fig. 8.) or the Tangent (as in Fig. 9.) in the proportion of the Tangent to the Sine.

$$S.\frac{SR}{\Sigma}\,(::\Sigma.R)::\frac{SR}{\Sigma}.\ \frac{SR^2}{\Sigma^2}=\frac{SR^2}{R^2-S^2}.$$

60. We have (by division) this Series,

$$R^2-S^2)\ SR^2\ \left(S,\ +\frac{S^3}{R^2},\ +\frac{S^5}{R^4},\ +\right.$$

$$\frac{SR^2-S^3}{S^3}$$

$$+S^3-\frac{S^5}{R^2}$$

$$+\frac{S^5}{R^2}$$

$$+\frac{S^5}{R^2}-\frac{S^7}{R^4}$$

$$+\frac{S^7}{R^4}$$

$$S+\frac{S^3}{R^2}+\frac{S^5}{R^4}+\frac{S^7}{R^6}+\frac{S^9}{R^8}\ \&c.$$

61. That is (putting $R=1$)

$$S+S^3+S^5+S^7+S^9,\ \&c.$$

62. Which (multiplying the respective members by $\frac{1}{2}S$, $\frac{1}{4}S$, $\frac{1}{6}S$, $\frac{1}{8}S$, $\frac{1}{10}S$, &c) becomes

$$\tfrac{1}{2}S^2+\tfrac{1}{4}S^4+\tfrac{1}{6}S^6+\tfrac{1}{8}S^8+\tfrac{1}{10}S^{10},\ \&c.$$

Which is the Aggregate of Tangents to the Arch who's right Sine is S.

63. And this method may be a pattern for the like process in other cases of like nature.

An Explanation of the Figures of Several Antiquities, communicated by a Member *of the* Royal Society.

FIG. 10, 11, 12, 13, 14, 15. *Res Turpiculæ*, or *Priapi*, worn by Roman Children against Fascination.

16. An Ægyptian brass *Serapis*, or *Teraphim*.

17. A brass *Stilus Scriptorius*.

 18, 19. Old