NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

COMPUTER SIMULATION OF A CRUISE MISSILE USING BRUSHLESS DC MOTOR FIN CONTROL

by

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March 1985

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REPORT DOCUMENTATION PAGE

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1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (*and Subtitle*) Computer Simulation of a Cruise Missile Using Brushless DC Motor Fin Control		5. TYPE OF REPORT & PERIOD COVERED Master's Thesis; March 1985
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(*s*) Gene C. Franklin		8. CONTRACT OR GRANT NUMBER(*s*)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93943		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93943		12. REPORT DATE March 1985
		13. NUMBER OF PAGES 60
14. MONITORING AGENCY NAME & ADDRESS(*if different from Controlling Office*)		15. SECURITY CLASS. (*of this report*) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (*of this Report*)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (*of the abstract entered in Block 20, if different from Report*)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (*Continue on reverse side if necessary and identify by block number*)

Cruise Missile Simulation
Brushless DC Motor
Altitude Hold Controller
Computer Missile Simulation

20. ABSTRACT (*Continue on reverse side if necessary and identify by block number*)

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A proportional integral control scheme in conjunction with tachometer feedback provides the position control for the missile

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The load on the fin is developed from the dynamic fluid environment that the missile will be operating in and is proportional to such factors as fin size and air density.

The program written in CSMP language is suitable for parametric studies including motor and torque loan characteristics, and missile and control system parameters.

Computer Simulation of a Cruise Missile using Brushless DC Motor Fin Control

by

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
March 1985

ABSTRACT

A computer simulation was developed in order to provide a method of establishing the potential of brushless DC motors for applications to tactical cruise missile control surface positioning. In particular an altitude hold controller has been developed that provides an operational load test condition for the evaluation of the electromechanical actuator.

A proportional integral control scheme in conjunction with tachometer feedback provides the position control for the missile tailfin surfaces. The fin control system is further imbedded in a cruise missile model to allow altitude control of the missile.

The load on the fin is developed from the dynamic fluid environment that the missile will be operating in and is proportional to such factors as fin size and air density.

The program written in CSMP language is suitable for parametric studies including motor and torque load characteristics, and missile and control system parameters.

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1. INTRODUCTION

Hydraulic actuators have been the most effective means to control the fins of cruise missiles for many years. High torque loads are often felt on the control surfaces and heretofore servo motor control has not often been attempted because of size constraints and the necessary torques required to dynamically control the missile fins.

The advent of brushless DC motors possessing very strong magnetic fields presents an opportunity to investigate missile control using these motors rather than the more conventional hydraulic actuators. A simplified model of a brushless DC motor is used as a basis for a fin positioning controller. This controller is then utilized in a simplified generic missile model to develop an altitude hold controller. The characteristics of the DC motor can be adjusted to determine the ability of that particular motor to dynamically control the fins of the missile and ultimately to dynamically control a missile in flight.

Four phases of development are necessary to develop the computer program for missile flight simulation. The first phase entails the development of the DC motor model and the necessary assumptions. The second phase places the motor in a proportional - integral (PI) controlling system with tachometer feedback supplementing the natural back electromotive force (BEMF) exerted by the motor itself. Next the missile model is developed. The final phase is the development of an altitude hold controller which incorporates the missile model and the fin positioning controller.

To verify the CSMP simulation, the equations of motion were also programmed in the FORTRAN language. These routines were used to compare the test runs described in section V., for accuracy of the model simulation.

The computer program provides to the user, the ability to modify the motor parameters, missile parameters, and to adjust the dynamic characteristics of the fin controller and altitude hold controller. This program provides an effective tool in the study of brushless DC motor actuators used in dynamic missile systems.

II. DC MOTOR MODEL DEVELOPMENT

The initial brushless DC motor model developed by Steve Thomas [Ref. 1]. has been used as a starting point for this work. Program number 2 of [Ref. 1] incorporated a commutator switching scheme to allow modeling of brushless operation, see Fig. 2.1. The need to create a model which will allow the study of motor performance under dynamic load conditions as well as static load conditions seems to be a natural follow on to the work performed in [Ref. 1].

It is well known that the electrical time constant of a DC motor is small compared to the mechanical time constant of the motor. This allows the assumption that the armature inductance is small enough to neglect. Using this knowledge, a continuous model of the DC motor with the same physical parameters as in [Ref. 1] has been selected, see Fig. 2.2. In computer simulation of navigation type controllers, only the slower time constants are of interest. Whenever fast time constants are also included, computational difficulties are encountered because of the small integration time interval requirements.

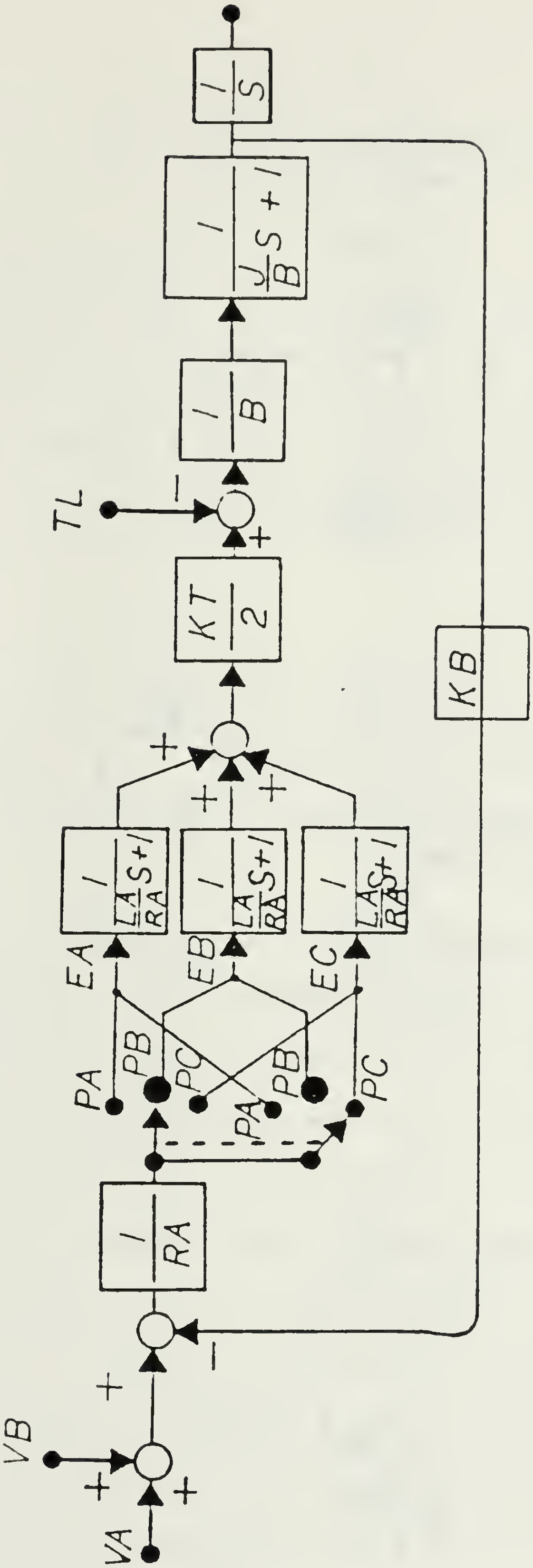

Figure 2.1 Basic Brushless DC Motor Model

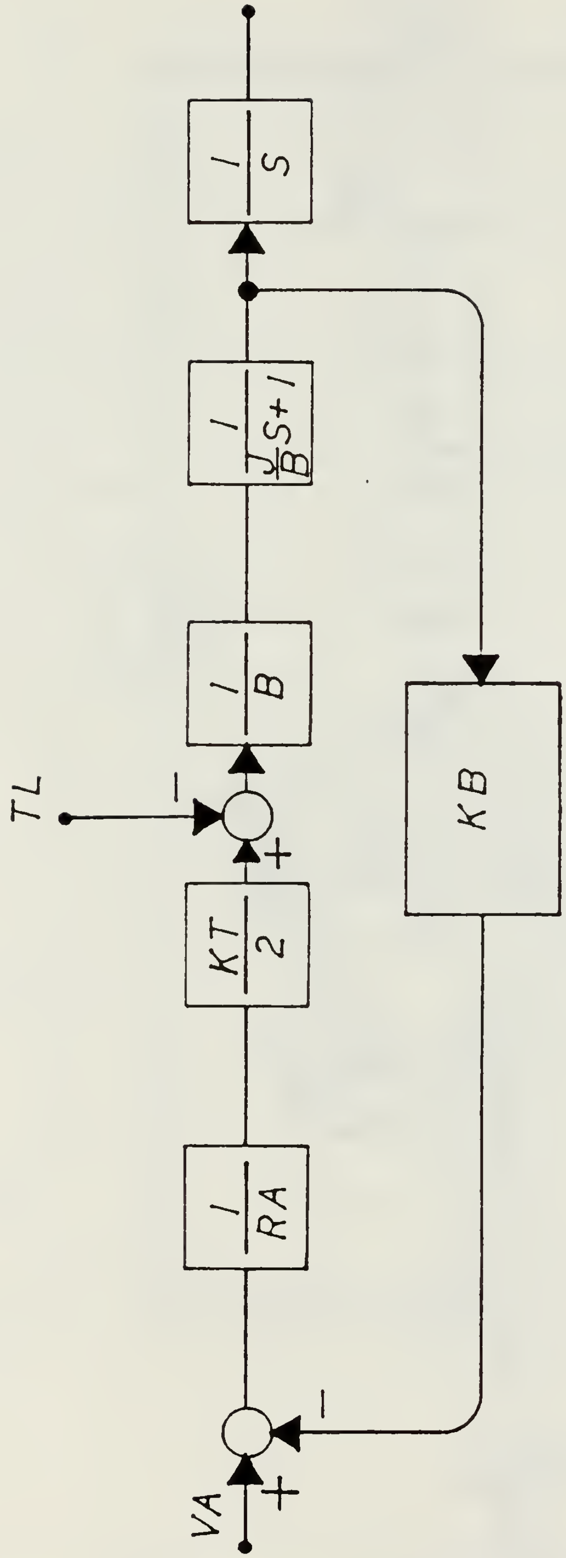

Figure 2.2 Modified DC Motor Model

III. FIN POSITION CONTROLLER

A. TORQUE GENERATION MODEL

One of the first steps in the design of a fin position controller is to design a dynamic input to the load torque of the DC motor model. This is accomplished using the dynamic lift force relationship established in [Ref. 3], as follows:

Lift Force = (Cl x air density x area x velocity^2) / 2

where Cl is the force coefficient

Cl = velocity x characteristic length / kinematic viscosity

The force coefficient is usually determined experimentally from airfoil tests. The lift force is defined to be perpendicular to the velocity vector of the fin. The total force vector is the sum of the lift force and the drag force vectors.

Some simplifying assumptions have been made. The total force vector is assumed to be approximately equal to the lift force vector for small fin deflections ie. less than 15 degrees deflection above or below the velocity vector. The coefficient of drag is less than one tenth the coefficient of lift for these angles, see Fig. 3.1.

In the computer simulation, Cl is modeled as a linear ramp that saturates at plus or minus 13 degrees fin deflection, and is approximately equal to 1/10 fin deflection angle. The lift force is converted to a torque by multiplying by the moment-arm. The moment-arm is the distance from the rotational axis of the fin to it's center of pressure. Although in a dynamic system the center of pressure changes slightly, it has been assumed that it is constant. Fig. 3.2 shows the torque load block diagram. System generated Motor torque and Load torque for maximum size step inputs is shown in Fig. 3.3 and Fig 3.4 respectively.

B. TACHOMETER FEEDBACK

Inherent back emf of the motor provides some speed regulation, but is not sufficient to adequately control the motor speed. The back emf constant of the motor is supplemented by an additive constant that would be provided by a tachometer. This rate feedback allows the desired speed regulation and quick response to commanded inputs.

C. PROPORTIONAL INTEGRAL CONTROL

1. General

A series proportional-integral control scheme is used for precise position control of the fin. The position error is generated by closing the loop with a constant gain amplifier as shown in Fig. 3.5.

2. Transfer Function Analysis

The uncompensated motor transfer function is easily obtained for various system parameters using Program 1 written in Basic language in appendix C. A sample output is provided in appendix B. For the parameters used in this simulation the uncompensated motor transfer function Gl(s) becomes:

$$Gl(s) = \frac{15.9}{S^2(.03014) + S(176.8125) + (125.5584)}$$

The open loop transfer function is linearized by restricting the operational range so as to not include the saturation region of the coefficient of lift equation, (this is not necessary for program operation, but only for linear analysis. The forward loop transfer function G(s) = Gc(s)Gl(s) becomes:

$$G(s) = \frac{S(3180) + (1590)}{S^3(.03) + S^2(176.8) + S(125.5)}$$

where Gc(s) is the transfer function for the PI controller. Using the coefficients derived from this forward transfer function and including the feedback constant (Knvrt), which has been set to 35, the compensated open loop and closed loop Bode plots can be seen in Fig. 3.6 and Fig. 3.7

respectively. System response is quite fast with a response time of 3 msec. for full scale fin deflection accurate to within .01 degrees of desired fin angle.

For substitution of the Fin controller into the missile plant the overall fin controller transfer function can also be evaluated using Program 1 in appendix C. This transfer function

$$Geq(s) = G(s)/1 + G(s)H(s)$$

for the parameters used in this simulation is as follows:

$$Geq(s) = \frac{S(3180) + (1590)}{S^3(.03) + S^2(176.8) + S(1113126) + (556500)}$$

3. Bode Analysis

Bode plot analysis was used to design the fin position controller. The motor transfer function including velocity feedback and torque load generator is reduced to the equivalent transfer function

$$Gl(s) = \frac{K_t}{S^2(D_2) + S(D_1) + D_0}$$

where all transfer function coefficients are detailed in Program 1 appendix C. and can be obtained for specific parameters.

This is a type zero system which for the constants chosen (see appendix C for chosen parameters) yields poles at -.71 and -5865. The permanent magnet motor itself is a type 1 system. Derivation of the torque felt from the fin allows reduction of the system to a single input single output system, however it uses a feedback loop which causes the effective transfer function of the motor system to become a type 0 system. The motor system also includes velocity feedback which was derived from the original back emf constant of the system plus an additional amplifier to boost the rate feedback stabilization effect. Without boosting this feedback which is easily derived with the use of a tachometer the system oscillates and does not damp out well.

The obvious thing to do with the motor system transfer function G1(s) is to increase it to a type 1 system to allow zero steady state error to a step input. A proportional-integral (PI) controller is chosen with constants Kp and Ki such that Gc(s) = Kp + Ki/s. This PI controller is placed in cascade with the motor system transfer function to yield the forward system transfer function G(s) as follows:

$$G(s) = \frac{S(N_1) + N_0}{S^3(D_2) + S^2(D_1) + S(D_0)}$$

The effect of the PI controller is to introduce a pole at zero, and a zero at -Ki/Kp. Ki has been chosen to be 100 and Kp has been chosen to be 200. These values position the new system zero at minus .5. This zero is to the left of the dominant system root and allows very quick servo mechanism response, in a system which is stable for all positive values of system gain. A damping ratio approximately equal to .44 is achieved with the PI controller. From the Open loop Bode plot the phase margin is 44 degrees, gain margin is 7 DB, and band pass frequency is 4000 rad/sec. The peak frequency magnitude is 1.36 at 5000 rad/sec as seen on the closed loop Bode plot Fig. 3.7. From [Ref. 4] second order approximations show the natural frequency to be 2920 rad/sec, settling time is .003 sec, overshoot magnitude is 1.24, system time constant is .00086 sec., and transient oscillating frequency is 2676 rad/sec. Figure 3.8 shows the fin controller time response using the parameters described above.

The Bandwidth of the fin controller is strongly influenced by the feedback constant H(s). This feed back gain is set equal to 35 x N, where N is the gear reduction of the fin mechanism (N = 10 in this simulation). This large negative feedback constant greatly increases the bandwidth of the system, but is able to achieve quick fin response times without long settling times. Trials were conducted to attempt to reduce the system bandwidth by locating the PI zero further in the left half plane. It is possible to reduce the system bandwidth in this manner, however a tradeoff must be made in sacrificing response time. System bandwidth can be reduced to about 100 rad./sec. by placing the compensator zero at minus 50 by setting Ki = 100 and Kp = 2, see open and closed loop Bode plots Figures 3.9 and 3.10. Doing this allows a motor response time for full scale fin deflection of .05 sec., (see Figure 3.11) which would be adequate for many cruise missle applications.

Additional controller designs such as the use of derivative as well as proportional and integral control and lead-lag filter sections could be developed to provide low bandwidth and fast fin response times, however the optimization of the controller design as stated in Section II. was not the primary objective of this work.

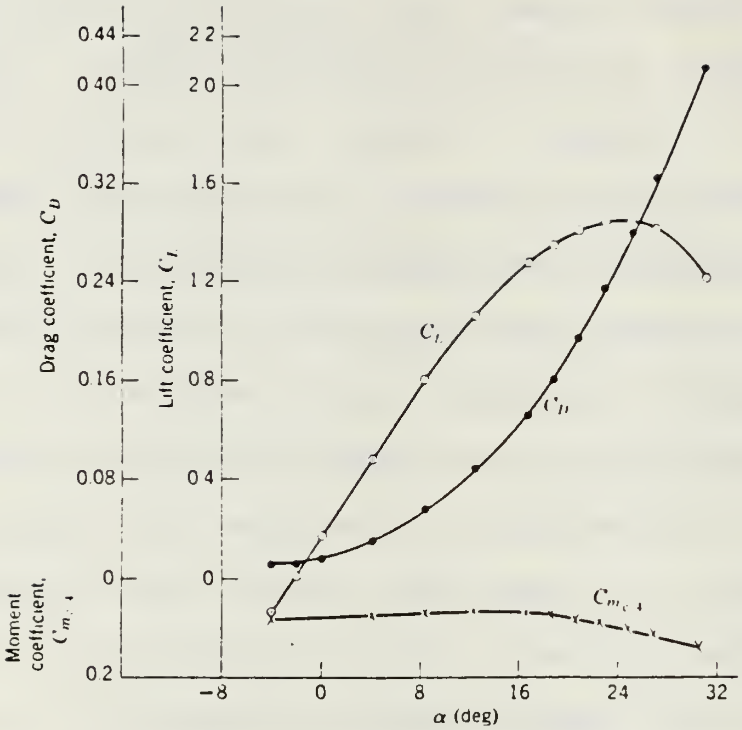

Figure 3.1 Lift Coefficient Graph

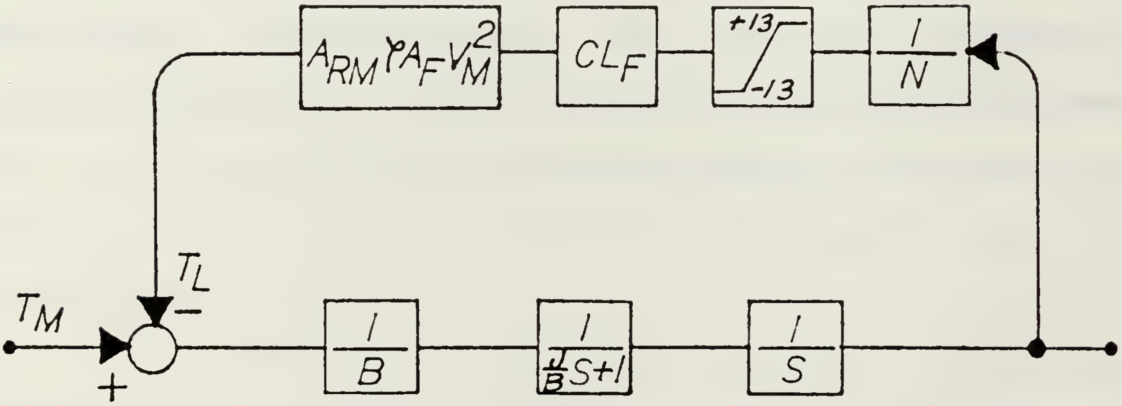

Figure 3.2 Torque Load Block Diagram

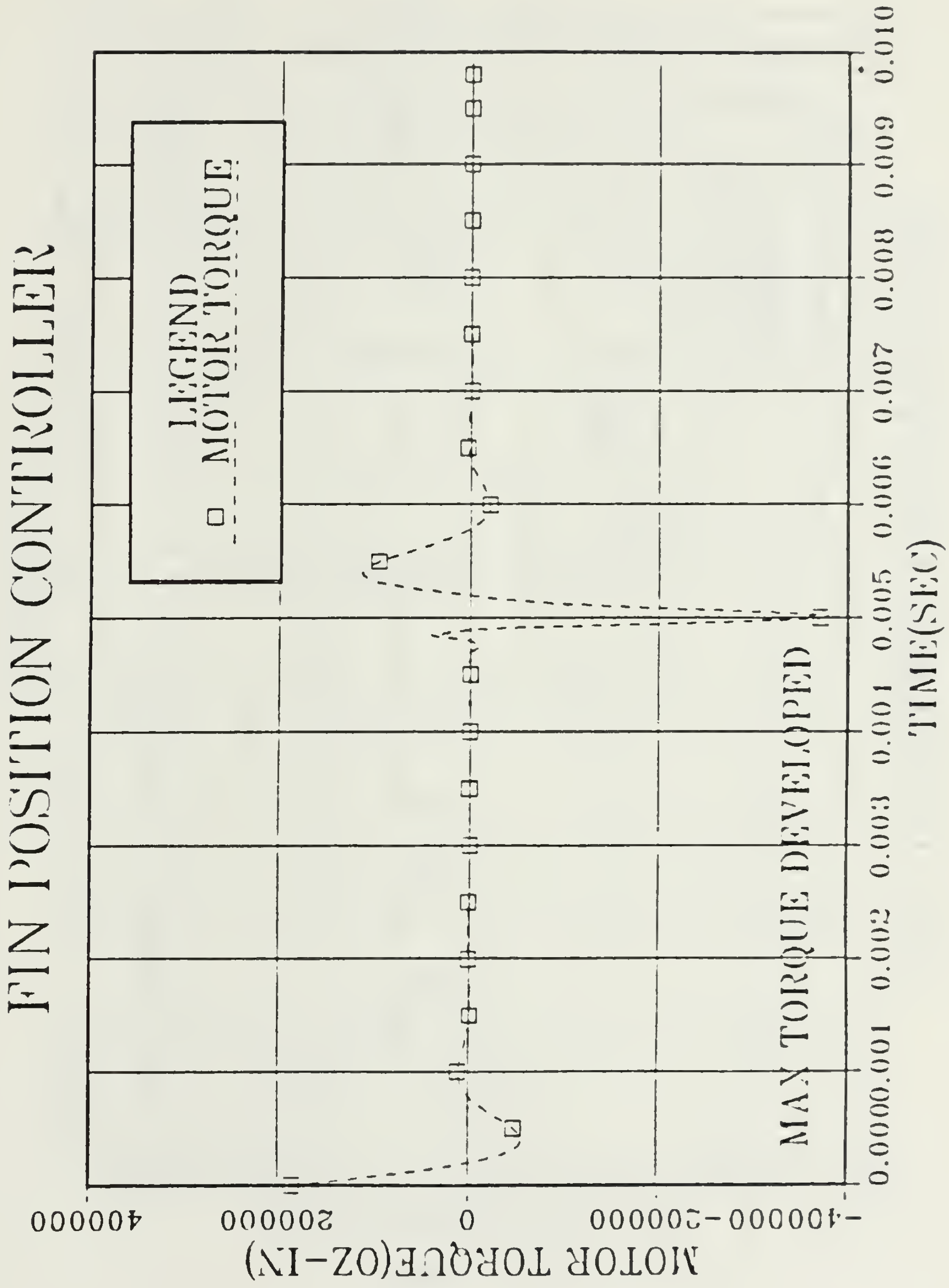

Figure 3.3 Motor Torque Response to Step Input

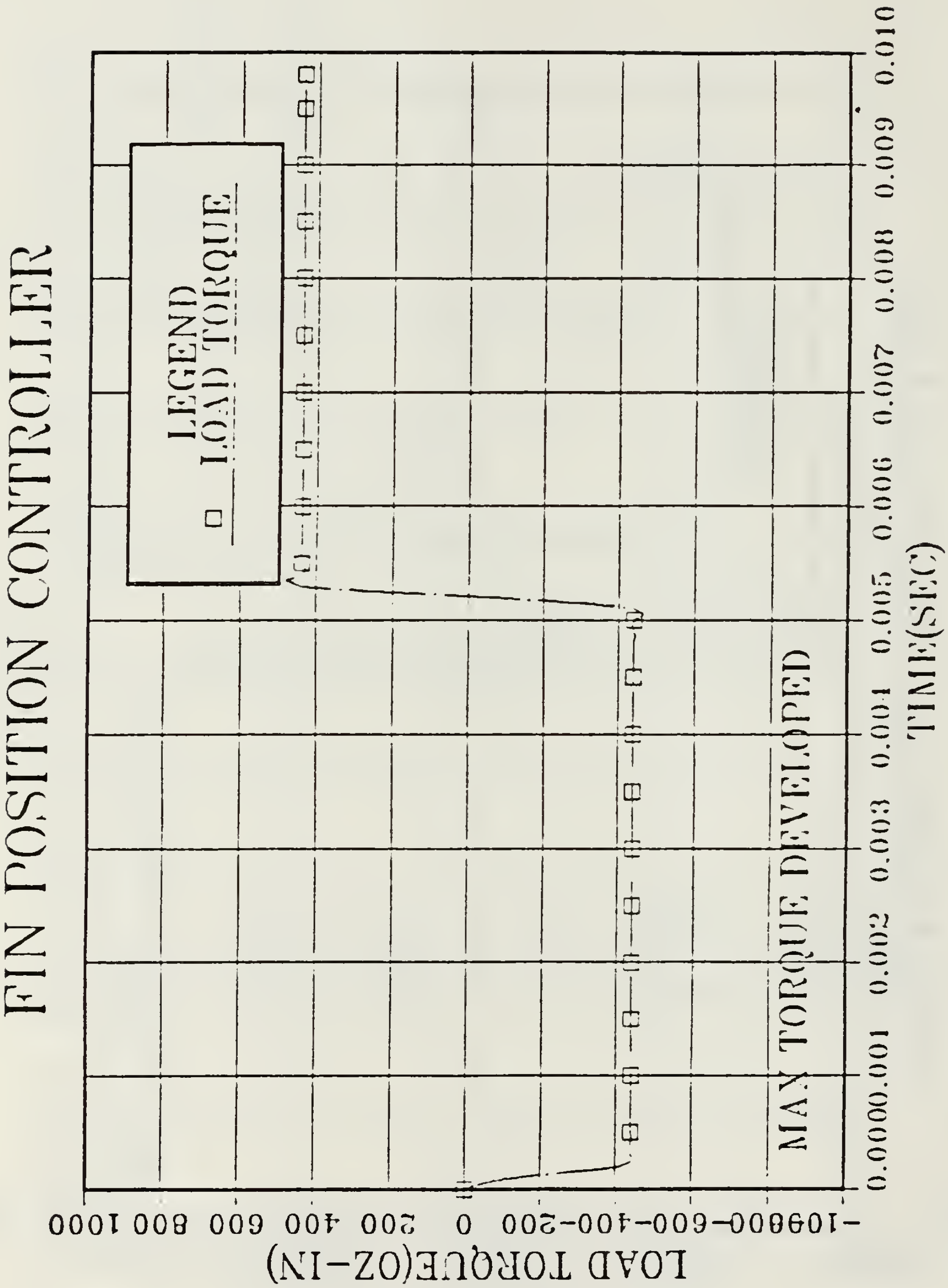

Figure 3.4 Load Torque Response for Step Input

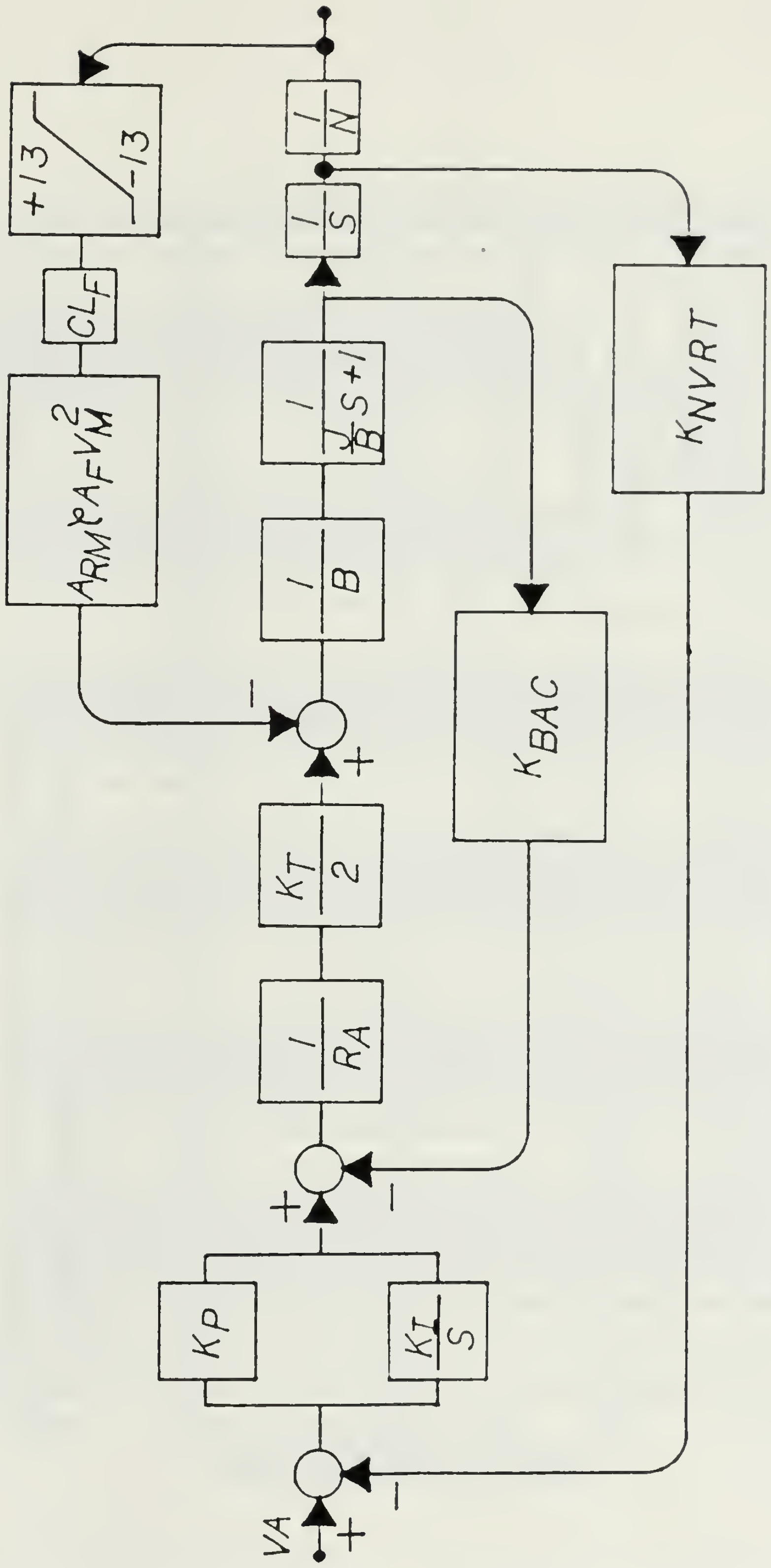

Figure 3.5 Fin Position P-I Controller

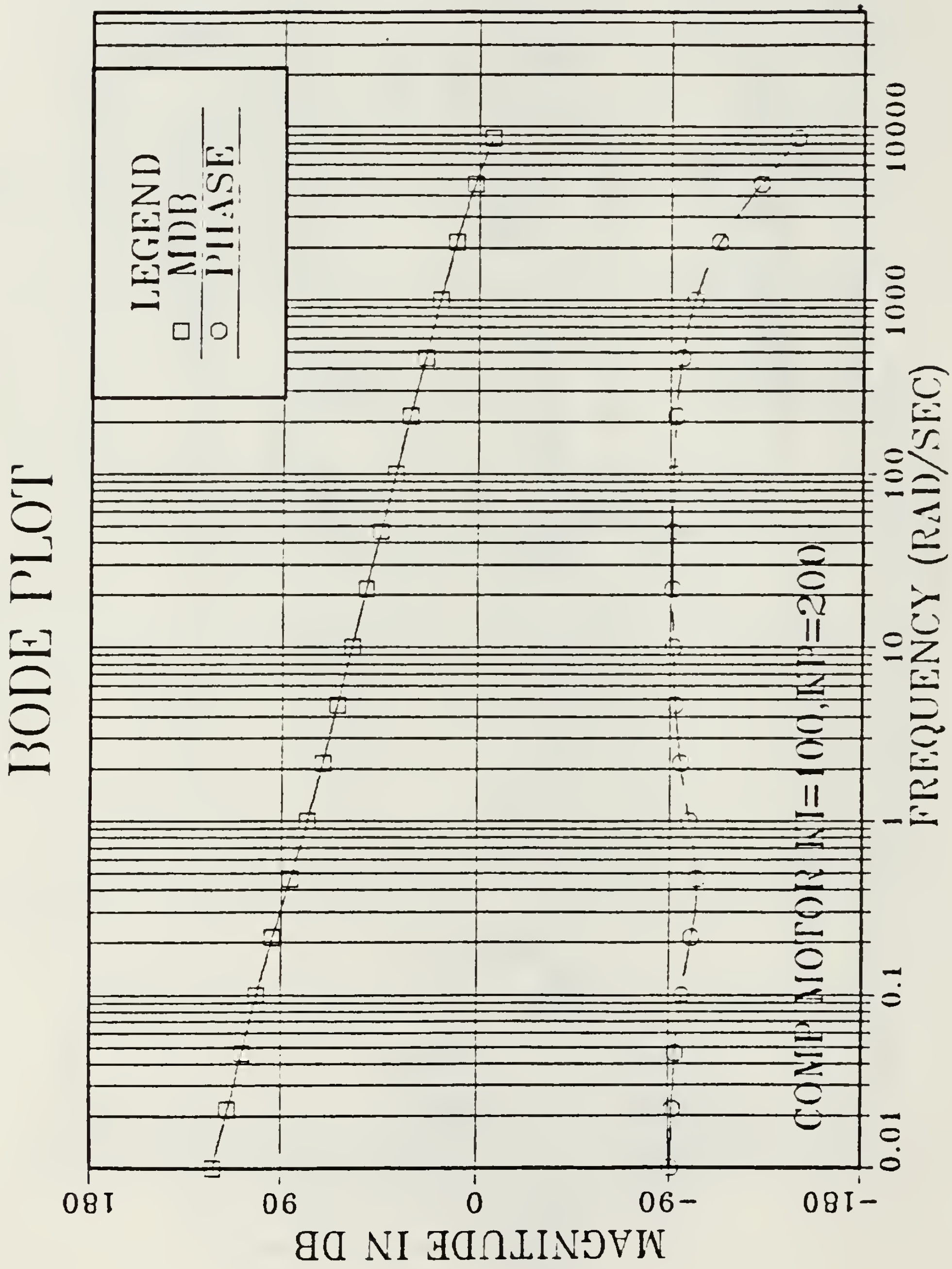

Figure 3.6 Open Loop Bode Plot Ki = 100, Kp = 200

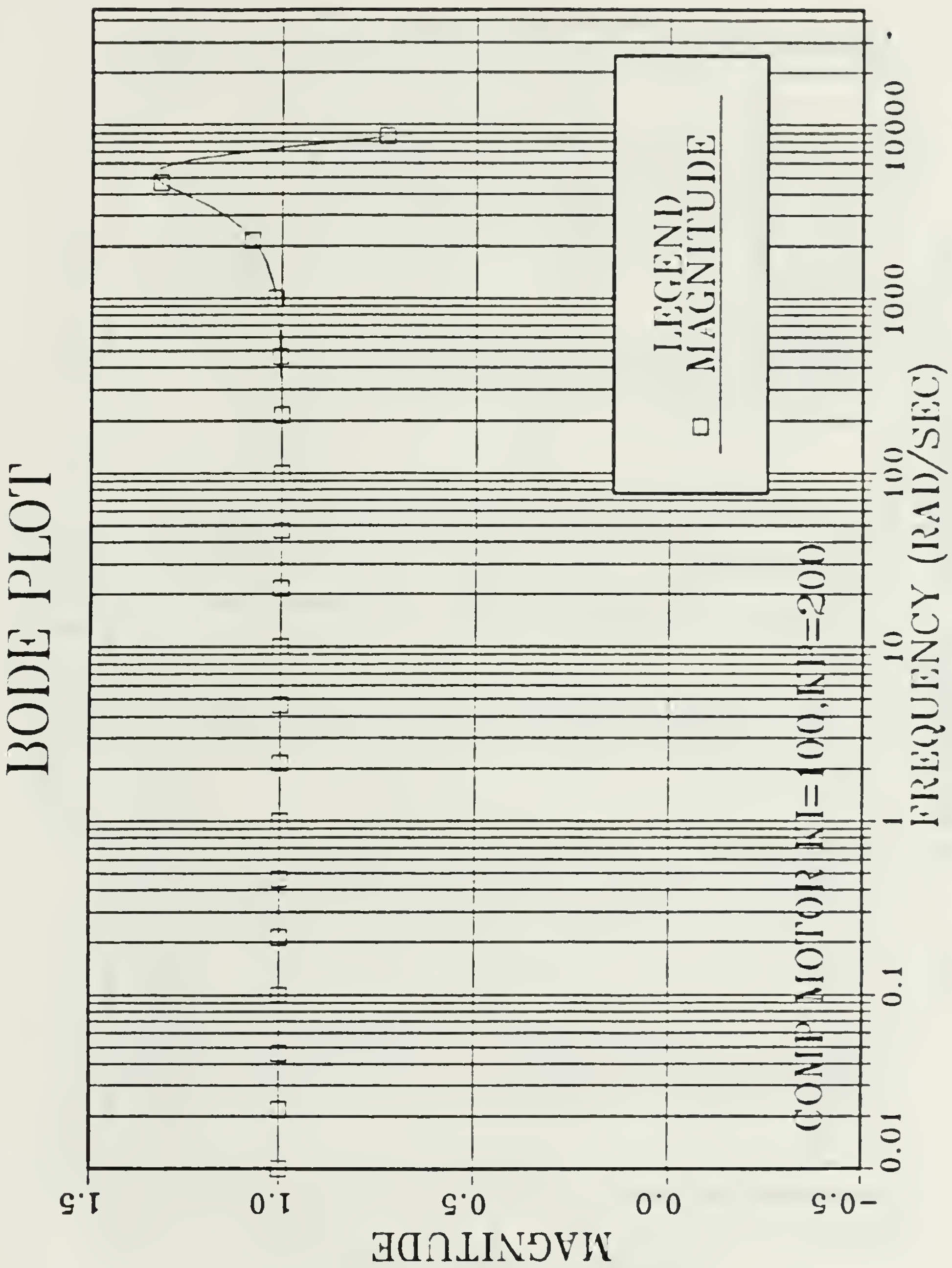

Figure 3.7 Closed Loop Bode Plot Ki = 100, Kp = 200

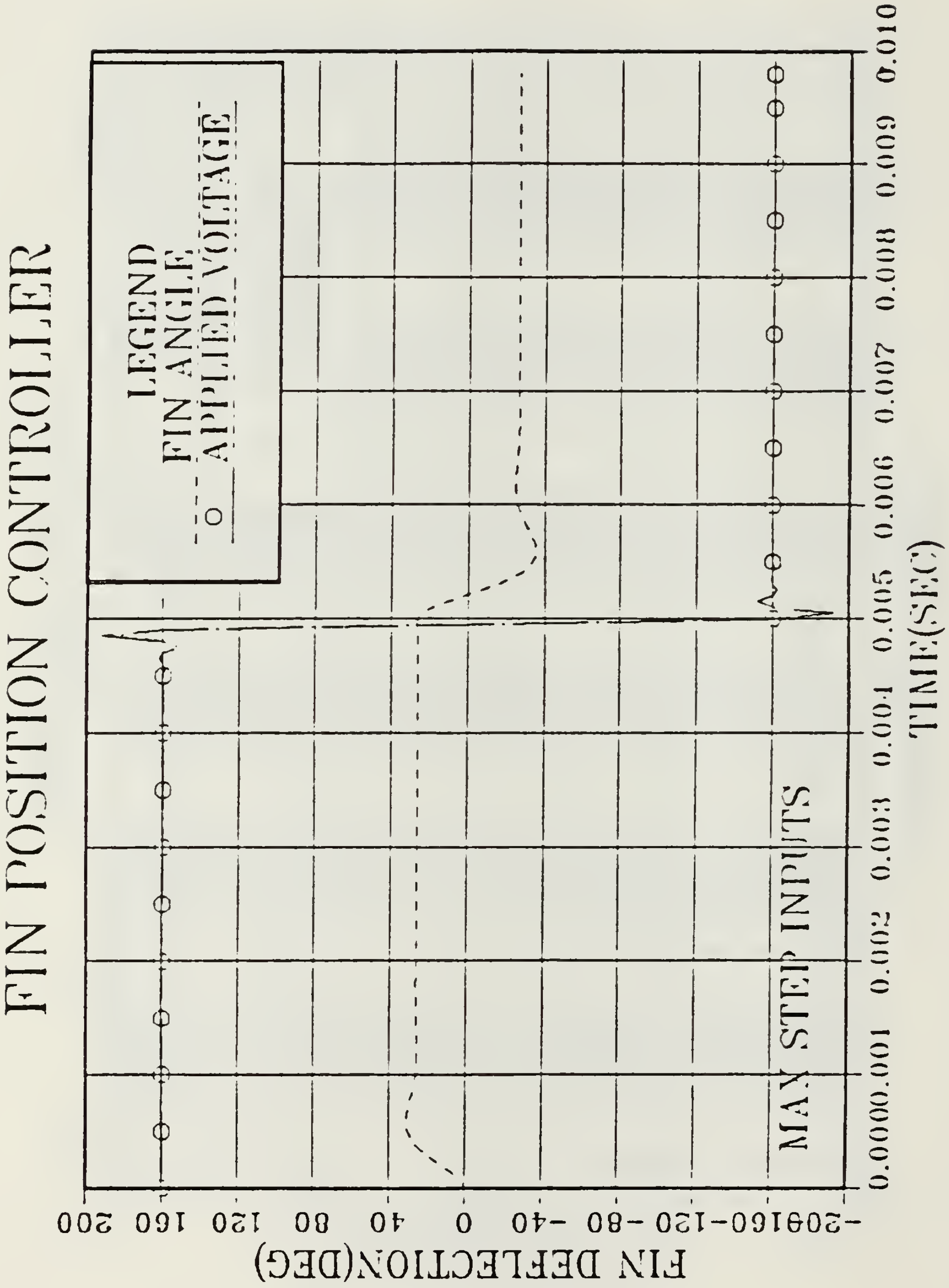

Figure 3.8 Fin Position Time Response

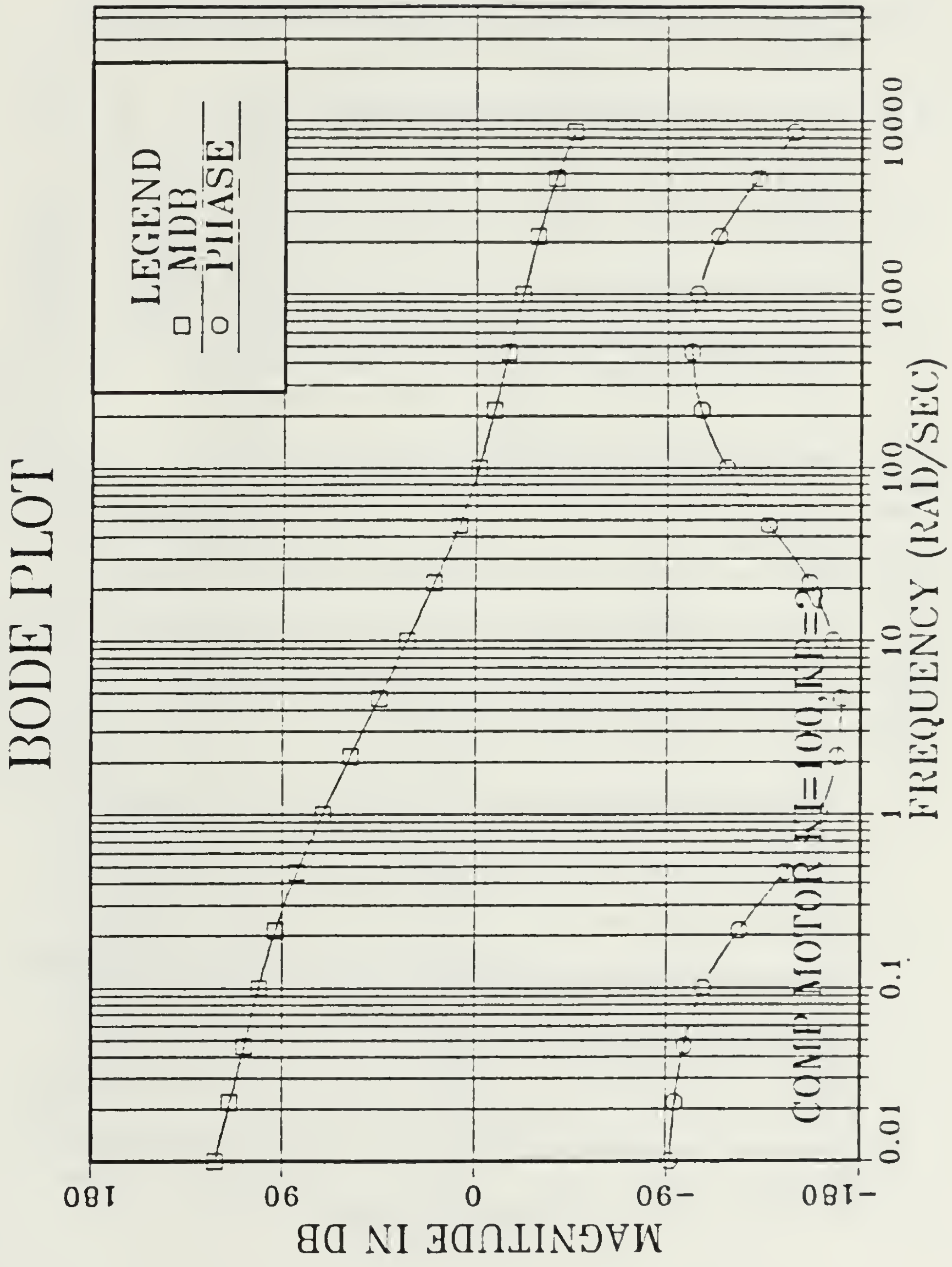

Figure 3.9 Open Loop Bode Ki = 100, Kp = 2

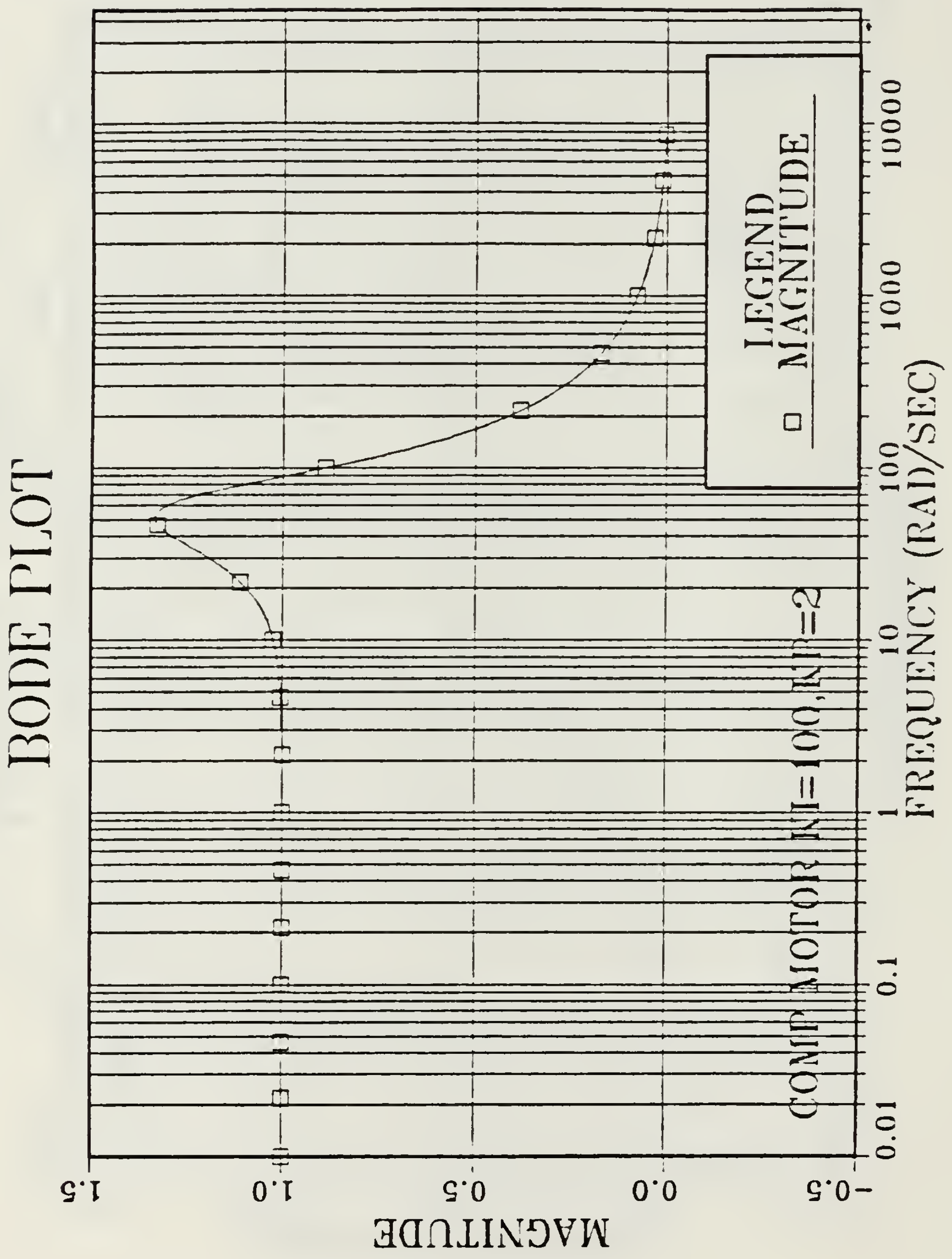

Figure 3.10 Closed Loop Bode Ki = 100, Kp = 2

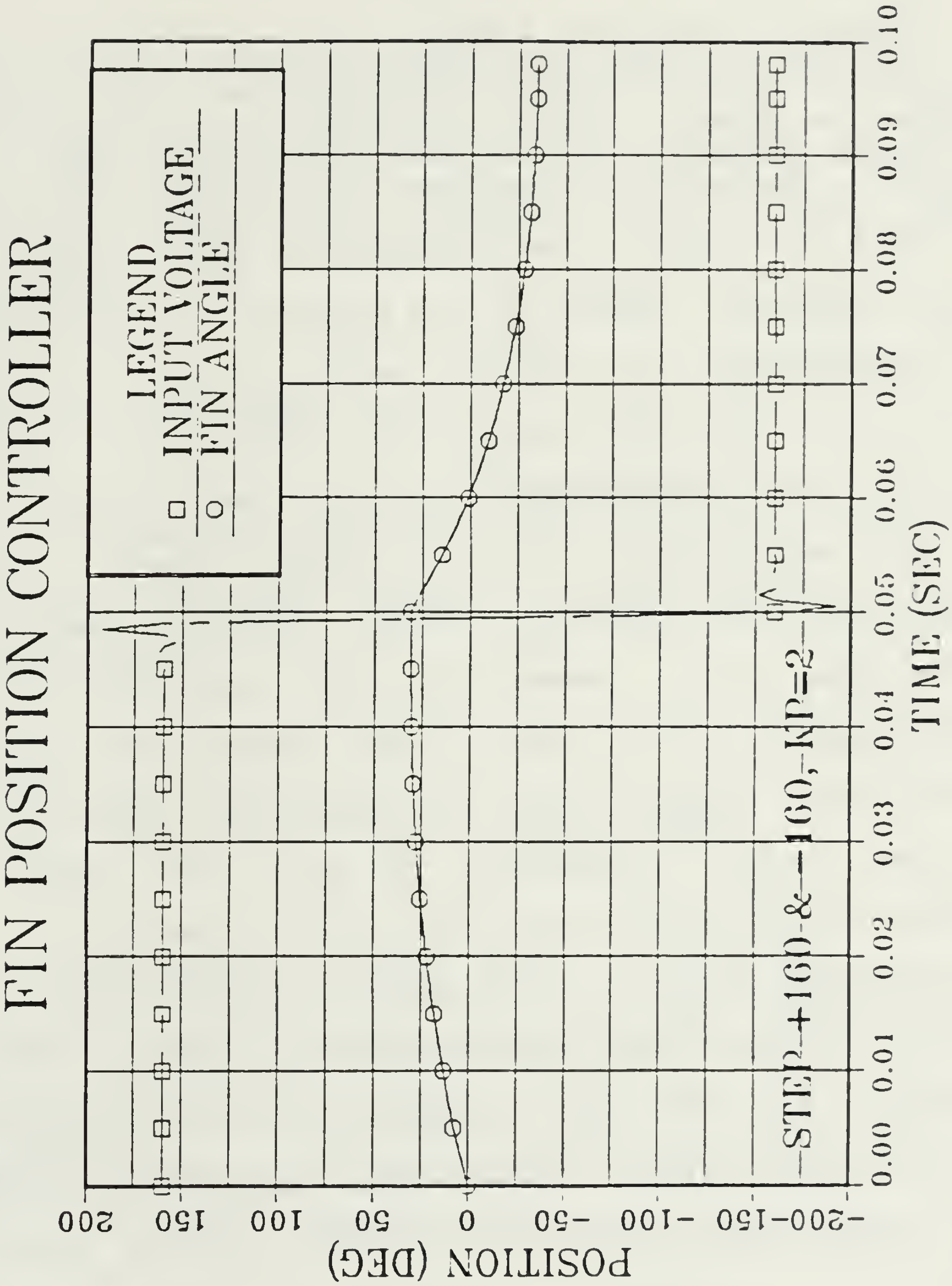

Figure 3.11 Fin Position Time Response

IV. MISSILE MODEL DEVELOPMENT

In order to study the effects of missile flight upon the motor, the missile system dynamics must first be adequately modeled. The missile equations are developed with reference to Fig. 4.1, which shows diagramatically the missile pitch plane dynamics. Control of the missile in the yaw plane can be accomplished in a similiar manner. The pitch plane was chosen so that the effects of gravity can easily be incorporated in the study of the motor characteristics.

A description of the missile plant, operating conditions, and specific system parameters used in the simulation are needed as ground work for a thorough understanding of the Pitch plane dynamic equations to be developed. Pitch control is effected by means of a rear mounted movable aerodynamic fin with total surface area equally distributed on each side of the missile for a total of two square feet. It is assumed that both fin sections operate together using one DC motor for control. In actual operation four fins each independently controlled by one DC motor is more likely. Missile roll stability is assumed to be provided by a similiar vertically oriented system, but certainly possessing independently controlled fins. Vertical missile orientation is therefore assured for this model and is assumed to be constant. The missile length is ten feet, effective surface area is twelve feet and the weight of the missile is assumed to be 1000 pounds. The missile is assumed to be a narrow rigid body for purposes of computation of the moment of inertia in the transverse plane. This assumption leads to the following moment of inertia (Iz) equation (see appendix A for equation symbol definitions):

$$Iz = (1 / 12) \times MASS \times Lm^2$$

where Lm is the missile length. The formulation of a force balance equation with respect to the reference missile direction yields:

$$\frac{d^2Y}{dt^2} = [(K_1 \times \alpha) - (K_2 \times \gamma) - Wn] / MASS$$

where α is the missile attack angle, γ is the angle between the missile axis and the horizontal reference (pitch angle), and where $Wn = mg \cos(\phi)$ and $\phi = 0$ for a horizontally referenced system, i.e. the X direction is along the earths surface. A moment balance about the missile pitch axis gives:

$$\frac{d^2\gamma}{dt^2} = [(K_3 \times \delta) - (K_4 \times \alpha) - (K_5 \times \frac{d\gamma}{dt})] / Iz$$

The constants K1 through K5 in the missile equations of motion above are derived from the aerodynamic characteristics of the fin and the missile and give:

$$K_1 = (Clm) \times (Am) \times (Qk)$$

$$K_2 = (Cdm) \times (Am) \times (Qk)$$

$$K_3 = (Clf) \times (Af) \times (Qk) \times (L_2)$$

$$K_4 = (Clm) \times (Am) \times (Qk) \times (L_1)$$

$$K_5 = [(Cmq) \times (Am) \times (Lm)^2 \times (Qk)] / [2 \times (Vm)]$$

where Cmq (missile moment coefficient) is assumed to be .5 and Cl (coefficient of lift for both the fin and the missile) is set to .1 while the drag coefficient Cdm is set to .01 using the lift coefficient curves from wind tunnel data similiar to those of Fig. 3.1.

$$Qk = (\rho air / 2) \times (Vm)^2$$

ρair has been assumed to be constant at .002378 (slugs/ft^3) for near sea level operation and Vm, the missile velocity is 2000 ft/sec.. The missile surface area and fin surface area (Vm and Vf) have been set to 12 ft^2 and 2 ft^2 respectively. The missile center of pressure moment arm about the missile center of gravity (L1) has been set to 1 foot and the fin moment arm about the missile center of gravity (L2) has been set to 5 feet.

The pitch angle of the missile (γ) is equal to the sum of the missile attack angle (α) and (β), where β is the direction of the missile's velocity vector with respect to the horizontal reference . This assumes that there are no vertical crosswinds.

The vertical reference direction normal to the earths surface is referred to as the "Y" direction, "X" is the horizontal coordinate of the earth reference system and is the direction of missile flight. "Z" is the transverse coordinate. Propulsion of the missile along the "X" direction is assumed to be provided by a separate thrust control system such that the horizontal component of missile velocity remains constant.

The missile plant must be merged with the motor driven fin positioning controller. High speed DC motors using rare earth permanent magnets for field excitation permit relatively high voltage inputs on the order of 150-200 VDC to operate at maximum speed. A limitation of 160 volts has been established in this simulation and allows approximately + or - 26 degrees of fin deflection for missile control.

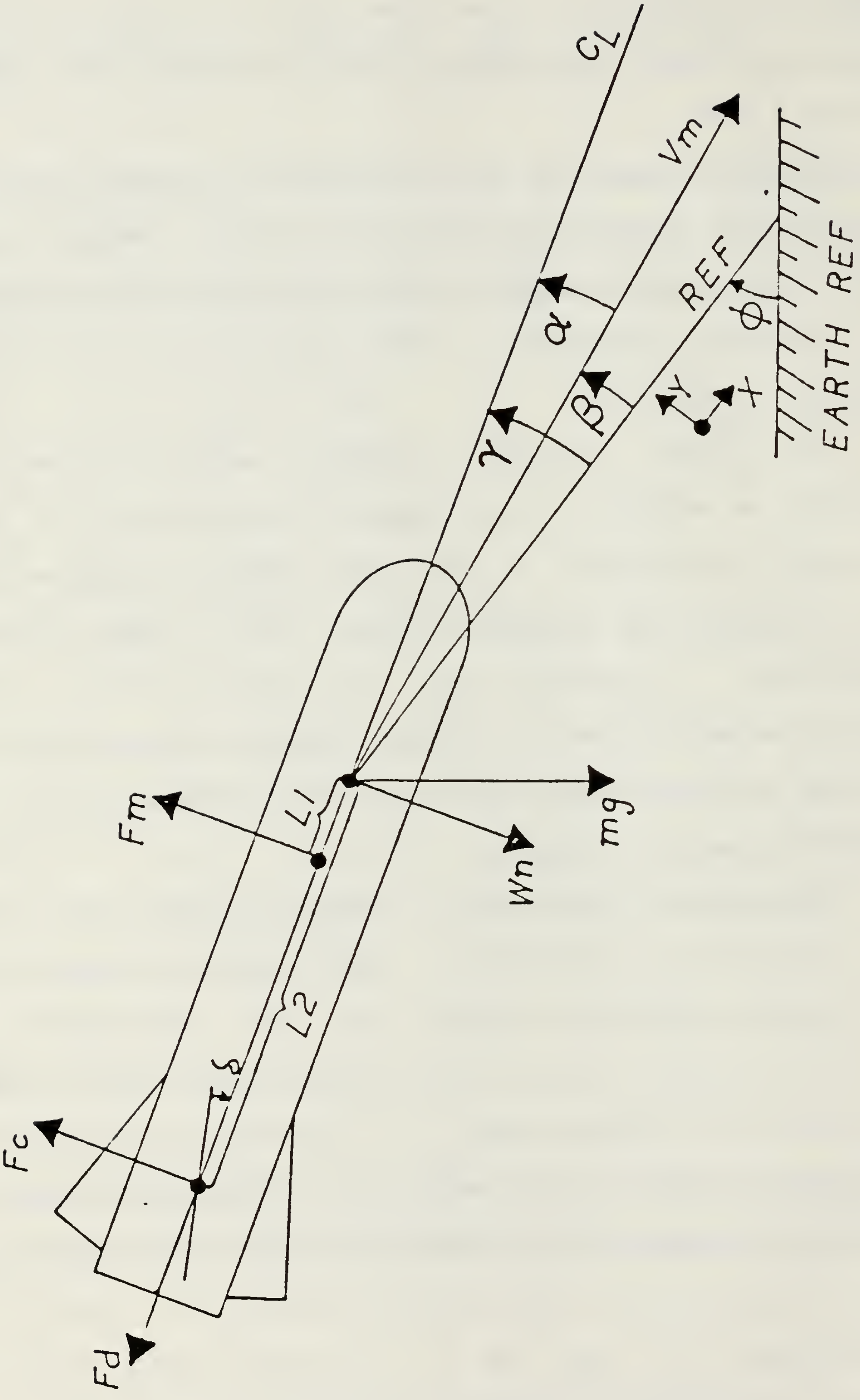

Figure 4.1 Missile Pitch Plane Dynamics

V. ALTITUDE HOLD CONTROLLER

The purpose of the design of an altitude controller in this simulation is to provide a means of accessing the real ability of a typical state of the art brushless DC motor to adequately control a typical tactical cruise missile in steady state flight. Parameters may be varied to model specific missile parameters including missile length, weight, effective surface area, distance from center of gravity to the center of pressure of the missile and to the center of rotation of the fin axis, and the control surface size. Control system parameters can be adjusted to within reasonable limits to allow system bandwidth to be above 100 radians per second if desired. The amount of position overshoot can also be minimized however a tradeoff in time required to reach the desired steady state altitude will occur.

It is desired in this simulation to design the controller so that for the typical cruise missile specifications chosen, that steady state oscillations in height are within + or - one foot from a desired height signal and that this condition is reached in no more than four missile time constants. The specific missile being simulated has a time constant approximately equal to one second, therefore a steady state condition should exist within four seconds from initial missile release. Although it is not specifically the goal of the controller designed in this simulation to respond quickly to step changes in height, this would be a desirable characteristic of operation and would increase the value of the model.

Further assumptions include the existence of a gyro on board the missile to provide the missile pitch angle (γ), an altimiter to provide missile height above the earth reference plane, and a clock timer and associated electronics to determine the vertical acceleration of the missile (d^2Y/dt^2). It has been assumed that no relative wind exists for simplicity. Using the approximation that the angle of the velocity vector above the horizontal reference plane (β) is

$$\beta = \tan^{-1}[dY/dt) / (dX/dt)]$$

it is evident that the missile attack angle (α) can be obtained as:

$$\alpha = \gamma - \beta$$

Thus the necessary information is provided to make use of the previously derived missile equations of motion to begin the altitude controller.

A. UNCOMPENSATED MISSILE PLANT

The missile dynamics are combined with the fin controller in Fig. 5.1. This system to be controlled has three inputs. The primary input signal provides voltage through a power amplifier to drive the fin controller which then positions the control fins to provide the fin angle (δ) input to the missile model. The Secondary input , but of great significance is the input that comes from the acceleration caused by the gravity acting on the mass of the missile itself. Finally a fin bias voltage (Vb) must be applied to help counteract the initial application of the missile weight to the system dynamic equations. The effect of the fin bias is to increase the initial system stabilization time dramatically. Without it the missile will fall a great deal before the system can catch up to this initial weight input.

To assist in controller design, in addition to calculation of the equivalent transfer function of the fin controller, Program 1 (appendix C) also calculates missile system parameters K_1 through K_5, and the overall uncompensated missile plant transfer function. All missile and fin parameters can be changed within this program. Sample output is provided in appendix B for the parameters used in this simulation.

It is seen from appendix B that the uncompensated missile plant yields a 7th order type one system with one zero. Several sets of missile parameters were evaluated. In general a mechanical resonance occurs in the open loop Bode plot at approximately one radian per second input frequency, see Fig. 5.2.

B. COMPENSATION OF MISSILE PLANT

The mechanical resonance is evidenced by the presence of two complex roots in the uncompensated missile transfer function near the resonance peak as seen on the Bode plot of the uncompensated plant.

The most common first approach to compensating for a mechanical resonance is to decrease the system gain such that the resonance peak falls well below (at least 6 dB) the 0 gain crossover point. The problem that occurs, however is that a minimum bandwidth of 6.28 radians per second should be achieved to insure adequate response for systems down to 1 second time constants. Adding some conservatism to this bandwidth requirement calls for a system bandwidth requirement of approximately 10 radians per second.

The above observations and previously established design requirements lead to the idea of the possible employment of a PID controller to compensate for the dominant complex roots of the uncompensated plant. In theory once this is accomplished it should be a relatively simple matter to use Lead-Lag compensation to adjust the open loop Bode response of the system transfer function for proper phase margin and bandwidth for a well damped quick responding plant. Recall that the uncompensated system is type one, and that a PID controller also adds a pole at 0 to the overall system. This should allow zero steady state error to ramp inputs, but also necessitates the use of two Lead compensator sections to provide the necesary system phase margin required.

The compensated system block diagram is shown in Fig. 5.3. Unless changes to motor or missile parameters are relatively small, the system will need to be recompensated. The first step to recompensate is enter the new parameters in the Basic program to determine the uncompensated missile transfer function then use this as the basis for the Bode plot. Next, because the zero of this transfer function is always nearly cancelled by one of the real roots of the function, the main consideration is the set of dominant complex roots. These roots can be multiplied to give the equivalent second order polynomial in coefficient form. The PID controller contributes the following transfer function to the forward transfer function of the missile plant:

$$V_{PID} = [(KK_D)S^2 + (KK_P)S + (KK_I)] / S$$

The complex roots of the system can therefore be cancelled by coosing the PID constants equal to the coefficients of the above derived second order system polynomial. The next step is to include this transfer function along with two lead sections in the forward system transfer function for the Bode analysis.

One method that works well for choosing the proper lead compensator sections to give a system bandwidth above 10 radians per second is to place a zero at .1 and a zero at 300 then use the second Lead section to fine tune the open loop Bode plot to give a phase margin of 45 to 55 degrees. For the parameters of the system as indicated previously placing the second zero at .8 and the pole at 15 satisfies the originally specified design requirements, see Fig. 5.6 for the output response to a zero reference signal input. Also see Fig. 5.7 and Fig 5.8 for response to step inputs to provide a missile climb of 100 feet and a missile drop in altitude of 10 feet respectively.

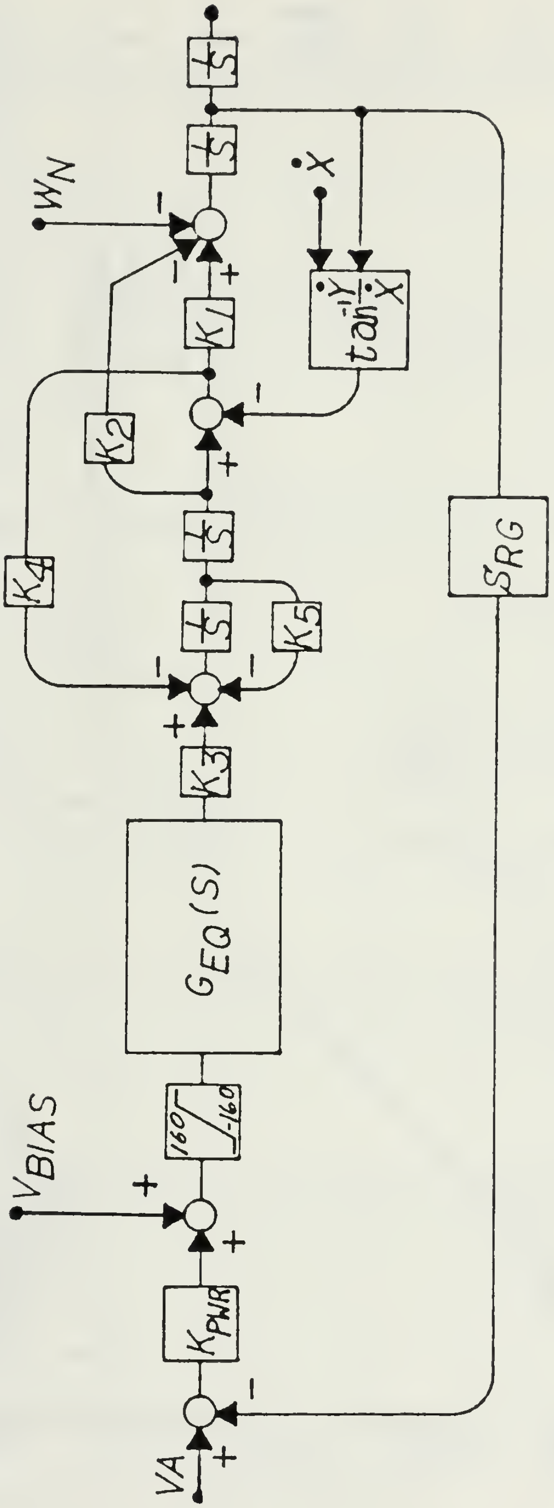

Figure 5.1 Uncompensated Missile Plant

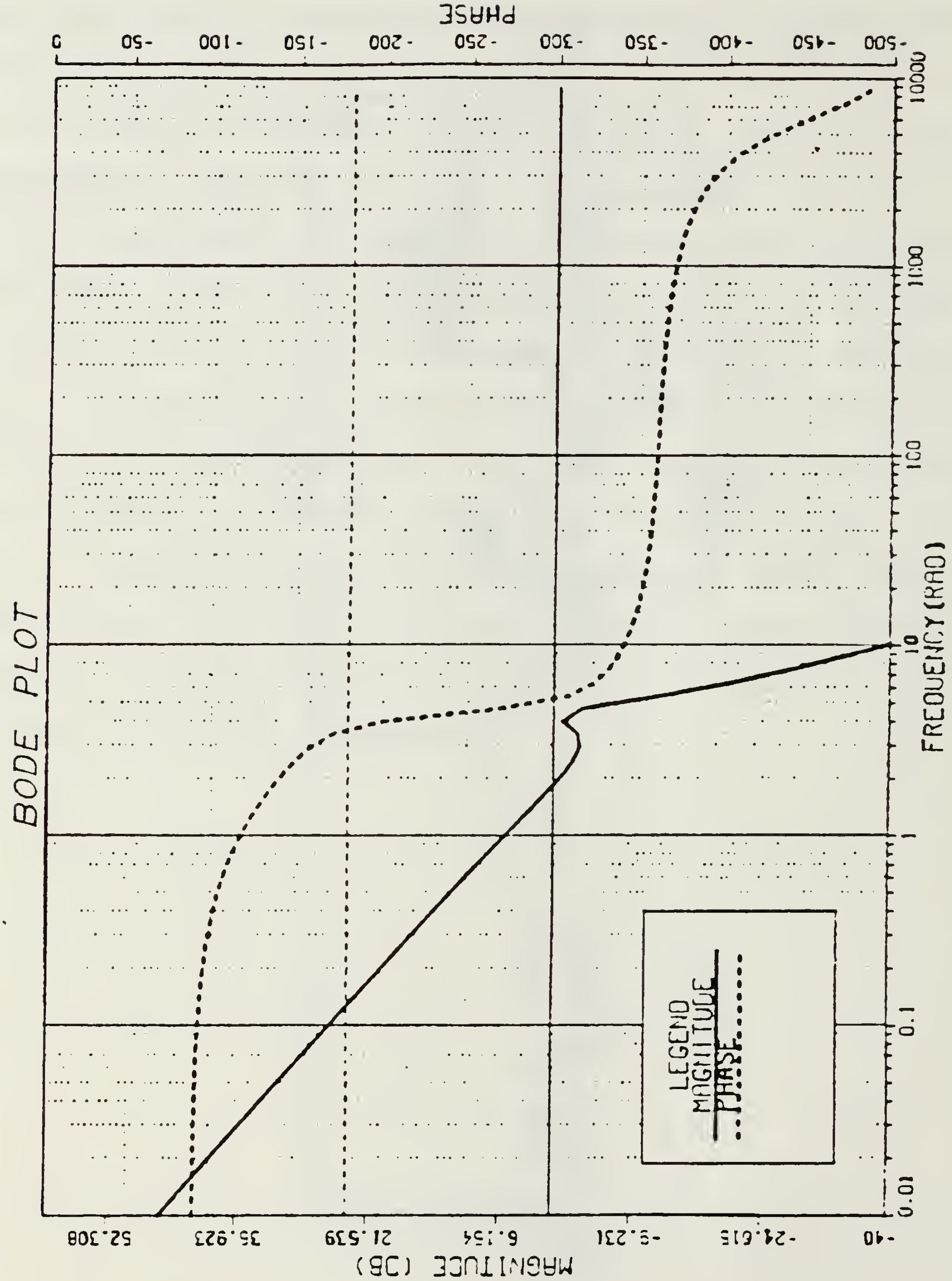

Figure 5.2 Open Loop Uncompensated Missile Plant

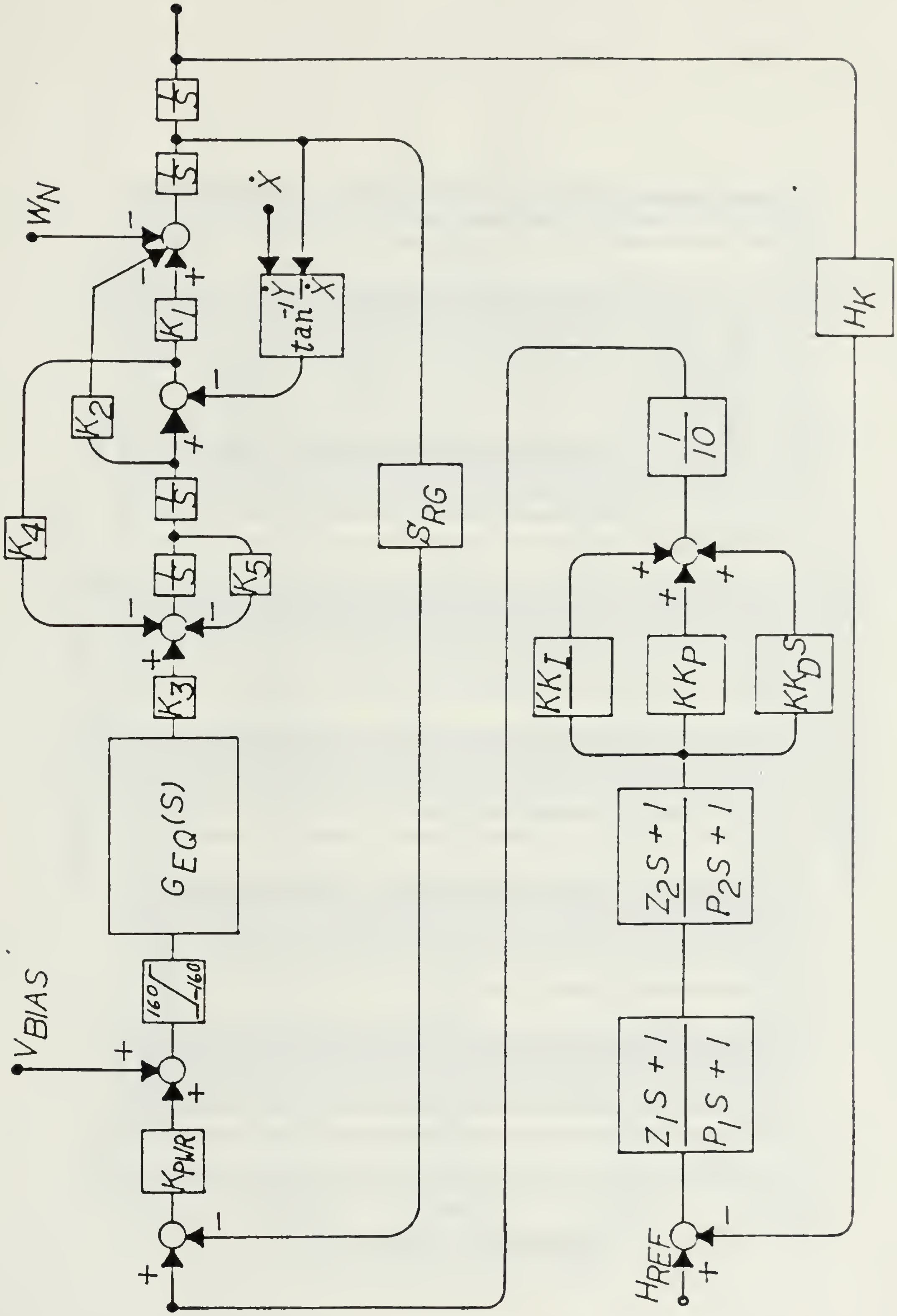

Figure 5.3 Compensated Missile Plant

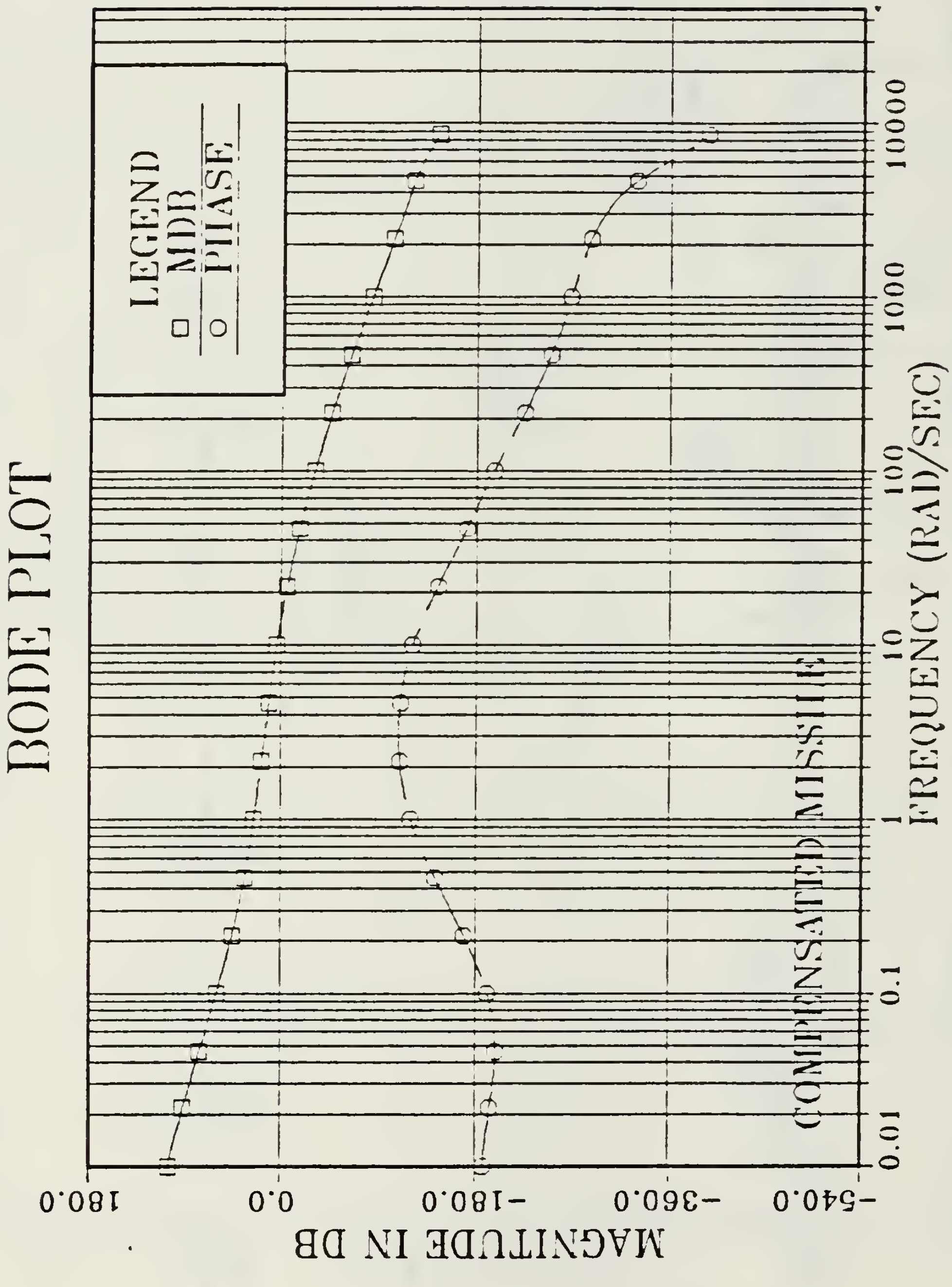

Figure 5.4 Open Loop Compensated Missile Plant

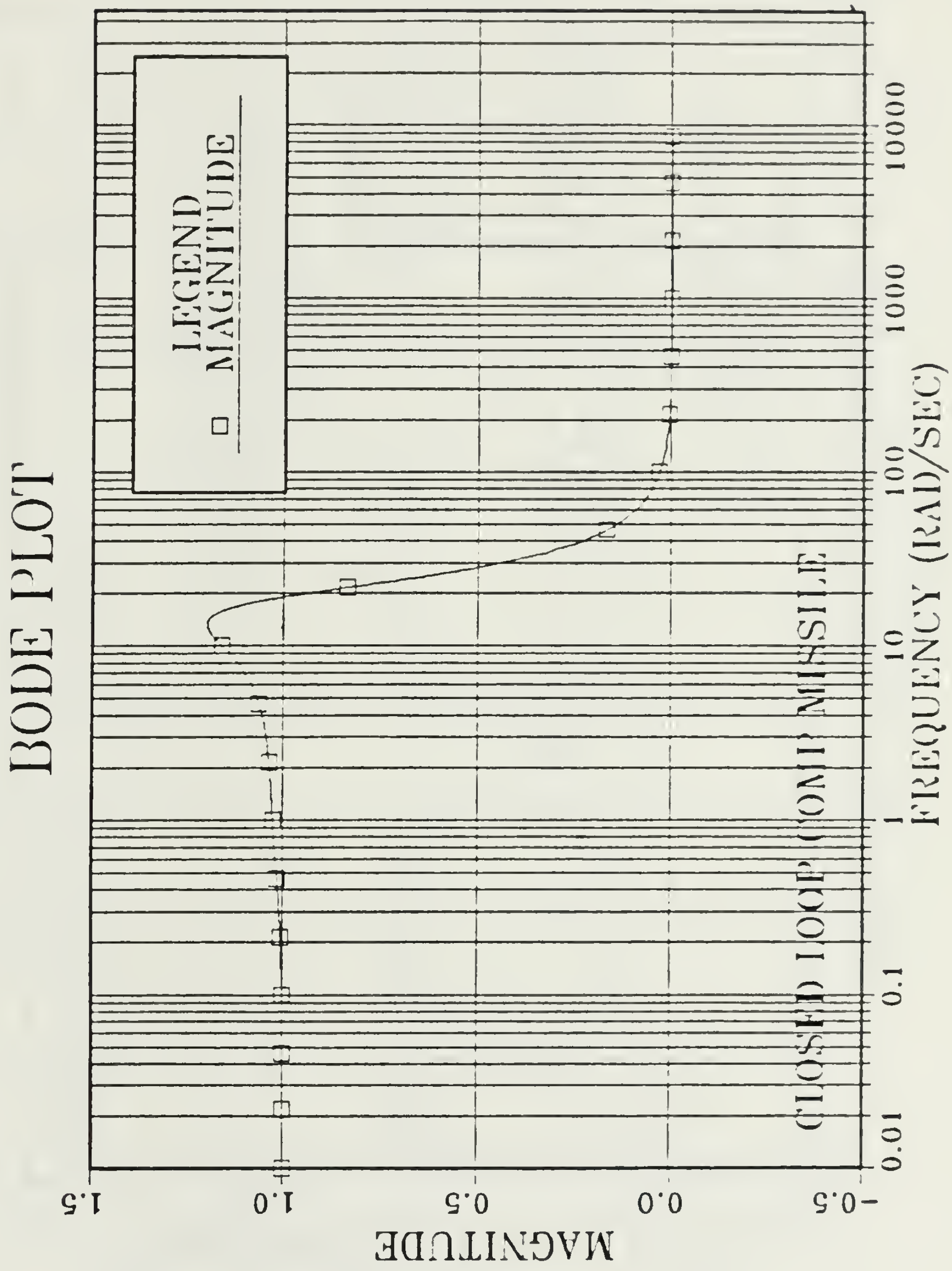

Figure 5.5 Closed Loop Compensated Missile Plant

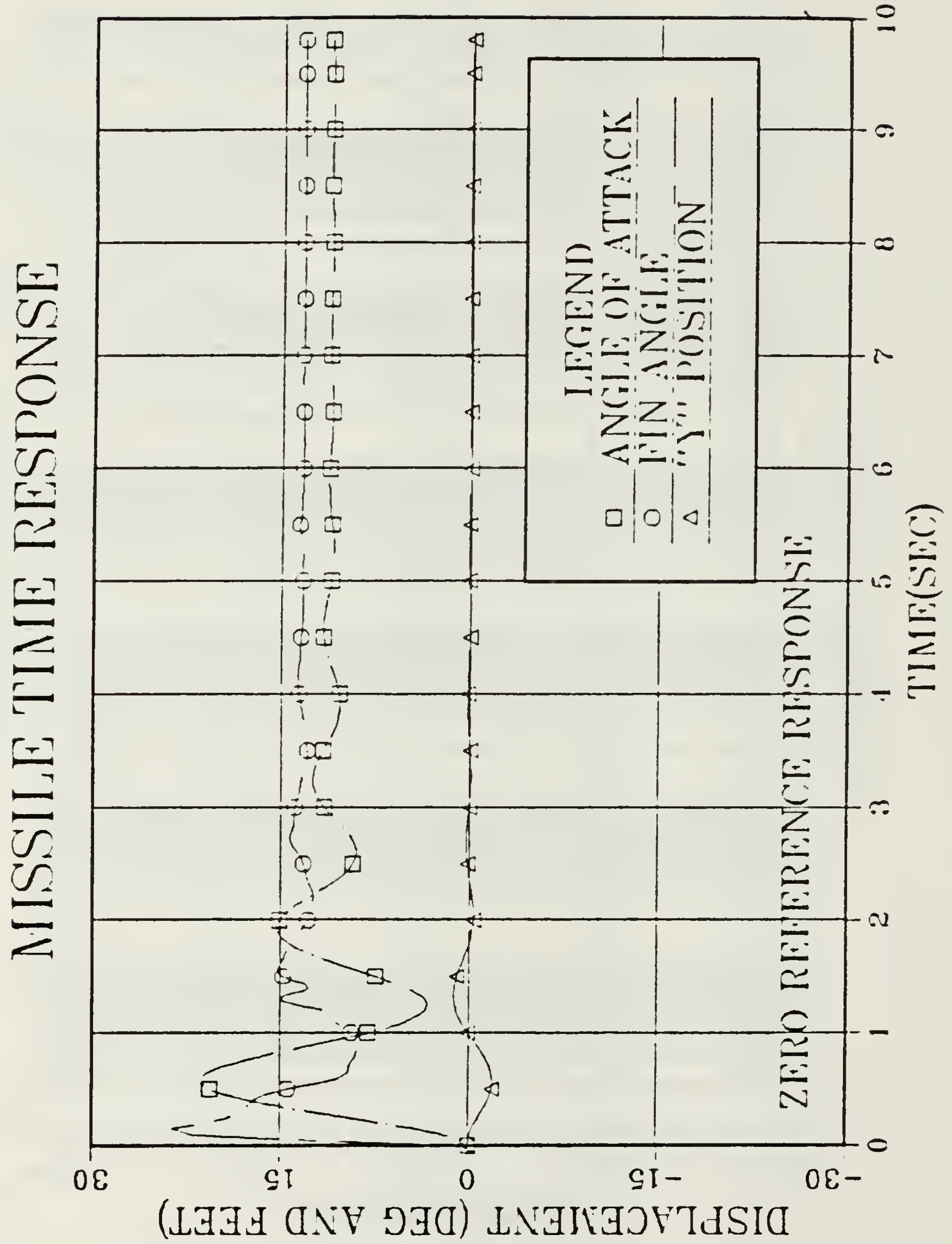

Figure 5.6 Zero Reference Time Response

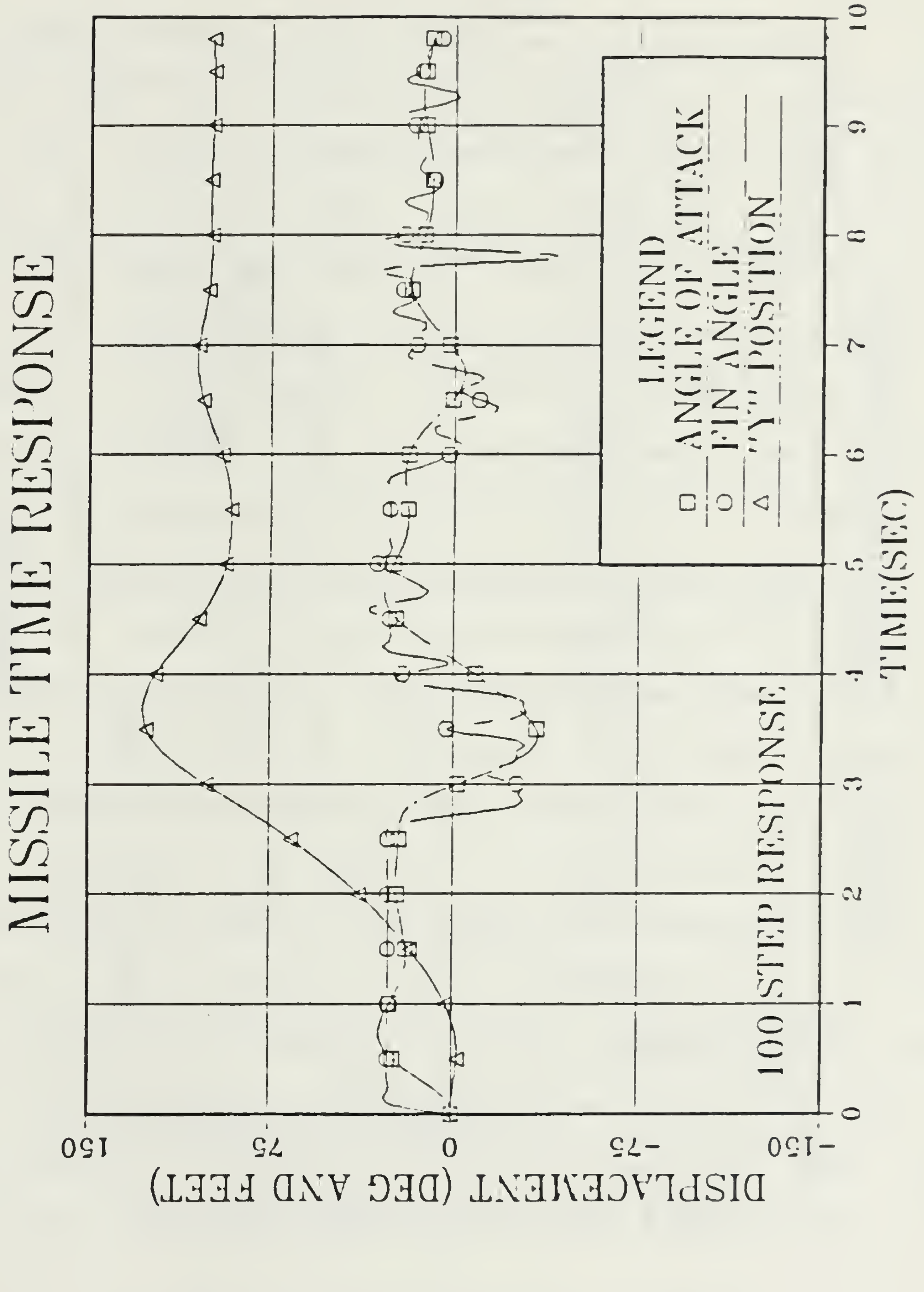

Figure 5.7 Step Response to +100

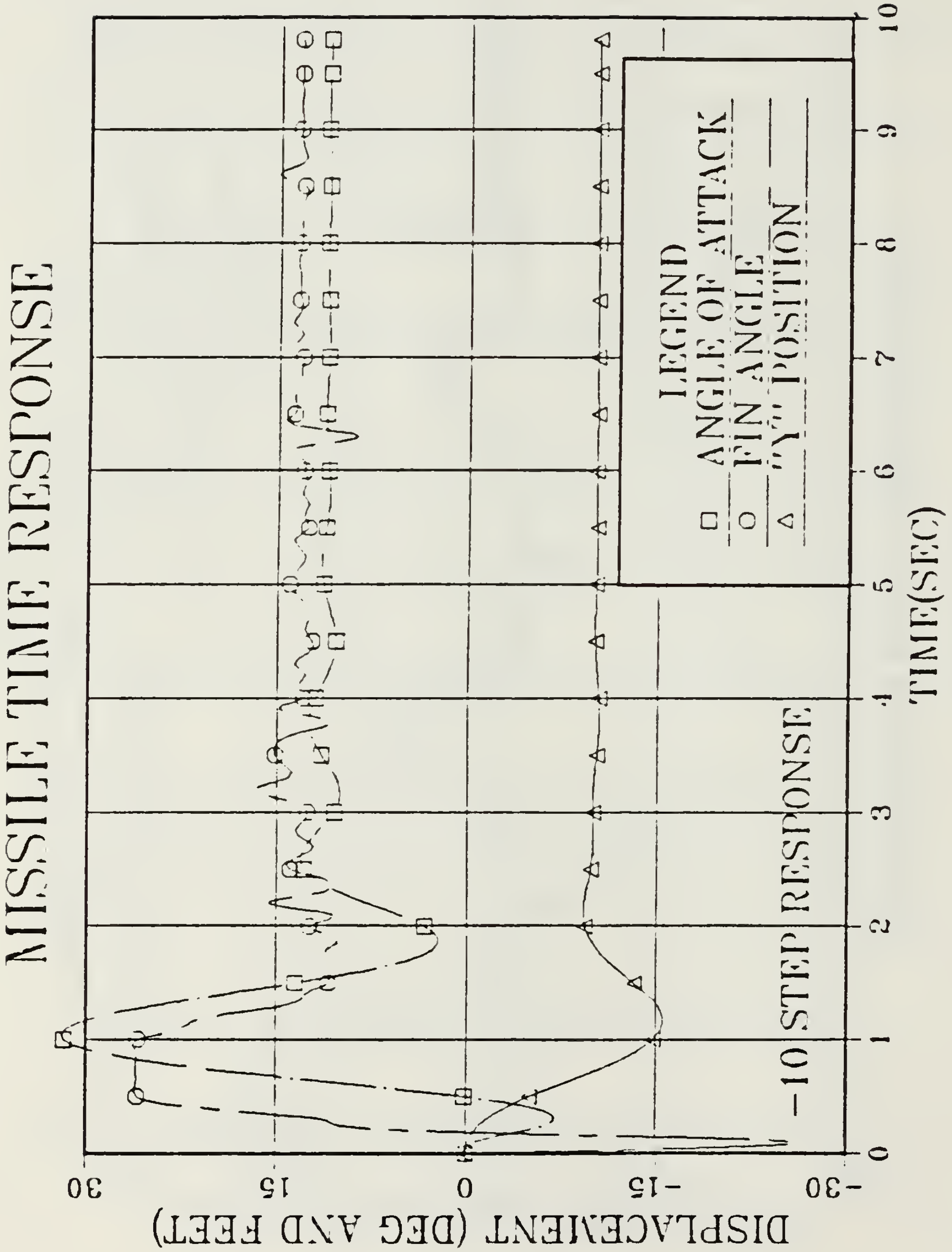

Figure 5.8 Step Response to -10

VI. CONCLUSIONS AND RECOMENDATIONS

A. CONCLUSIONS

The CSMP model results for simulations as described in section V. were confirmed to be within .001% of the FORTRAN language model for the cruise missile altitude hold controller under the same test conditions.

The missile system altitude hold controller meets and exceeds original design requirements established for the study of the effects of real load requirements on a DC motor having similiar parameters to state of the art brushless DC motors. Steady state oscillations in height were less than .1 ft. as compared to the design specification of less than or equal to one foot. Settling time was less than 4 sec. for all simulations. This meets or exceeds the design specification on settling time of less than or equal to 4 time constants (time constant of the simulated missile is approximately 1 sec.). The system gain had to be lowered by a factor of 10 to allow the compensator output to better interact with the vertical velocity feedback of the system. This factor along with saturation of the fin position controller during transient operation caused some steady state error to input reference signals. This effect can be counteracted by choosing reference signals appropriate to establish the desired output height.

The closed loop bandwidth of the fin controller is large for a servo mechanism. This is theoretically possible since large input voltages are used and because the load inertia felt on the motor is inversly proportional to the gear reduction ratio. The large bandwidth should be further investigated for actual values of load inertia from the fin mechanism.

B. RECOMMENDATIONS

For future work it is highly recommended to establish within the simulation program a means for adapting the controller to the desired changes in system parameters. Other means of control may be investigated, in particular incorporating feedback compensation other than proportional. In addition the steady state error problem requires further study to specifically identify and correct its sources.

APPENDIX A:

LIST OF SYMBOLS

Symbol name	Description/Units
Af	area of fin surface (ft^2)
Am	missile lift surface (ft^2)
Arm(L2)	fin moment arm (ft)
Atkang(α)	missile attack angle (radians)
Beta(β)	angle of Vm wrt horizontal (radians)
Bl	viscous load friction (oz-in/rad/s)
Bm	viscous motor friction (oz-in/rad/s)
Bt	total viscous friction (oz-in/rad/s)
Cd	coefficient of drag
Cg	missile center of mass
Clf	fin coefficient of lift
Cll	fin load torque generator lift coefficient
Clm	missile coefficient of lift
Cmq	moment constant of aerodynamic body
Cp	missile center of pressure
Cr	center of fin rotation
DDOT	second derivative wrt time of a variable
DOT	first derivative wrt time of a variable
Eint	motor integral control output
Epi	motor integ-prop control output
Eprop	motor prop control output
Epsh	missile position error
Errl	fin position error
Finang(δ)	fin position (rad)
Findeg	fin position (degrees)
Finsiz(Af)	lift area of fins (ft^2)
Gamma(γ)	missile axis angle wrt ref direction (rad)
Grav	Gravity constant 32 (ft/s^2)
Hk	missile position feedback constant
Href	missile commanded height (ft)
Im	motor current (amps)
Jl	fin system inertia (oz/in/s^2)
Jlp	fin system inertia through gear reduction
Jm	motor inertia (oz/in/s^2)
Jt	total motor system inertia (oz/in/s^2)
K1	missile constant
K2	missile constant
K3	missile constant
K4	missile constant
K5	missile constant
Kb	back emf constant (volts/rad/s)
Kbac	tack feedback plus motor back emf

Kh	missile position feedback constant
Ki	motor compensator integral constant
KKd	missile compensator derivative constant
KKi	missile compensator integral constant
KKp	missile compensator proportional constant
Knvrt	motor position feedback constant
Kp	motor compensator proportional constant
Kpwr	power amplifier gain
Kt	motor torque constant (oz-in/amp)
Ktac	tachometer feedback constant
L1	distance from Cg to Cp (ft)
L2	distance from Cg to Cr (ft)
LL1	missile lead filter 1
LL2	missile lead filter 2
Lm	missile length (ft)
Mass	mass of missile
Missiz(Am)	lift area of missile (ft^2)
N	gear reduction ratio
Nalph	fin aerodynamic parameter
Ndelt	missile aerodynamic parameter
P1	missile compensator pole
P2	missile compensator pole
Phi(ϕ)	missile ref direction angle to earth surface
Pwr	motor power (watts)
Qk	aerodynamic quality constant
Ra	motor armature resistance (ohms)
Roe(ρ)	air density ($slugs/ft^3$)
Srg	rate feedback constant of missile
Terr	motor-load torque error
Theta	motor shaft position (rad)
Tl	fin load torque (oz-in)
Tm	motor torque (oz-in)
Tmissu	uncompensated missile transfer function
Vafin	voltage applied to vin (volts)
Vb	fin bias voltage (volts)
Vbias	fin bias voltage (volts)
Vdir	missile compensator derivative sect. output
Velfed	missile velocity feedback signal
Vinpwr	input signal to power amp
Vint	missile compensator integral sect. output
Vprop	missile compensator integral sect. output
Wm	motor speed (rad/s)
Wn	normal force due to missile weight
Wt	missile weight (pounds)
X	missile ref. flight direction, horizontal
Y	vertical coordinate axis
Z	transverse coordinate axis
Z1	missile compensator zero
Z2	missile compensator zero

APPENDIX B:

PROGRAM 1 OUTPUT

```
************************************************************
UNCOMPENSATED MOTOR TRANSFER FUNCTION:

           15.9
G1(S) = ----------------------------------------------------
        SS(.03014) + S(176.8125) + (125.5584)

************************************************************
COMPENSATED MOTOR FORWARD TRANSFER FUNCTION G(S)=GC(S)G1(S):

           S(3180) + (1590)
G(S) = -----------------------------------------------------
       SSS(.03014) + SS(176.8125) + (125.5584)

************************************************************
COMPENSATED FIN CONTROL SYSTEM GEQ(S)=G(S)/(1+G(S)H(S)):

           S(3180) + (1590)
GEQ(S)=-----------------------------------------------------
       SSS(.03014) + SS(176.8125) + S(1113126) + (556500)

************************************************************
MISSILE EQUATION OF MOTION COEFFICIENTS:

YDDOT = (182.6304) * ATKANG - (18.26304) * GAMMA - WN

GDDOT = (18.26) * FINANG - (219156) * ATKANG - (2.739)* GDOT

************************************************************
UNCOMPENSATED MISSILE TRANSFER FUNCTION:

           S(9545876) + (4772938)
TMISSU = ---------------------------------------------------
         S^7(.03)+S^6(176.9)+S^5(1113611)+S^4(4166233)+

         ------------------------------------------------
         S^3(2.59E+07)+S^2(1.697E+07)+S(2386469)

************************************************************
SRG = 2  KPWR = 1  WT = 1000  VM = 2000
```

APPENDIX C:

PROGRAM 1 LISTING

```
REM THIS PROGRAM IS WRITTEN IN BASIC LANGUAGE AND WILL
REM RUN ON MOST PERSONAL COMPUTERS
REM THE PURPOSE OF THIS PROGRAM IS TO DETERMINE TRANSFER
REM FUNCTION COEFFICIENTS FOR MOTOR AND UNCOMPENSATED
REM MISSILE USING VARIABLE SYSTEM PARAMETERS
REM
REM NOTE: MOST BASIC INTERPRETERS REQUIRE LINE NUMBERS
REM TO PRECEED THE PROGRAM STATEMENTS. INCLUDE THESE
REM IN THE ACTUAL PROGRAM.
REM
KP = 200
KI = 100
KT = 15.9
KNVRT = 35
N = 10
BM = .00015
BL = .0015
BT = BM + (B./(N^2))
RA = 2.74
KTAC = 1
KB = .112
KBAC = KTAC + KB
JM = .001
JL = .01
JT = JM + (JL/(N^2))
CL = .1
ARM = .132
ROE = .002378
AF = 2
VM = 2000
REM
REM THIS PORTION DETERMINES THE UNCOMPENSATED MOTOR
REM TRANSFER FUNCTION
REM
PRINT "***************************************************"
KTL = (CL*ARM*ROE*AF*VM*VM)/2
D2 = JT*RA*N
D1 = (RA*BT*N)+(KT*KBAC*N)
D0 = KTL
PRINT "UNCOMPENSATED MOTOR TRANSFER FUNCTION:"
PRINT
PRINT "          ";KT
PRINT "G1(S) = -------------------------------------------"
```

```
PRINT "         SS(";D2;") + S(";D1;") + (";D0;")"
PRINT
PRINT "***************************************************"
REM
REM THIS PORTION OF THE PROGRAM DETERMINES THE COEFFICIENTS
REM OF THE FORWARD TRANSFER FUNCTION OF THE MOTOR
REM WHERE  G(S) = GC(S) * G1(S)
REM
N1 = KP * KT
N0 = KI * KT
PRINT "COMPENSATED MOTOR FORWARD TRANSFER FUNCTION"
PRINT "G(S) = GC(S)G1(S) :"
PRINT
PRINT "          S(";N1;") + (";N0;")"
PRINT "G(S) = -------------------------------------------"
PRINT "        SSS(";D2;") + SS(;D1;") + S(";D0;")"
PRINT
PRINT "****************************************************"
REM
REM THIS PORTION DETERMINES THE EQUIVALENT TRANSFER
REM FUNCTION OF THE COMPENSATED MOTOR SYSTEM INCLUDING
REM THE FEEDBACK LOOP.
REM
E1 = N1
E0 = N0
F3 = D2
F2 = D1
F1 = D0 + (N1*KNVRT*N)
F0 = N0 * KNVRT * N
PRINT "COMPENSATED FIN CONTROL SYSTEM"
PRINT "GEQ(S) = G(S)/(1+G(S)H(S)):"
PRINT
PRINT "           S(";E1;") + (";E0;")"
PRINT "GEQ(S) = -------------------------------------------"
PRINT "          SSS(";F3;")+SS(";F2;")+S(";F1;")+(";F0;")"
PRINT
PRINT "*****************************************************"
REM
REM THIS PORTION DETERMINES THE COEFFICIENTS FOR THE
REM UNCOMPENSATED MISSILE EQUATIONS OF MOTION.
REM
QK = (ROE*VM*VM)/2
AM = 12
CD = .01
L2 = 5
L1 = 1
LM = 10
CMQ = .5
GRAV = 32
WT = 1000
MASS = WT / GRAV
```

```
IZ = (1/12) * MASS * LM * LM
NDELT = CL * AF * QK
NALPH = CL * AM * QK
K1 = NALPH / MASS
K2 = (CD * AM * QK) / MASS
K3 = (NDELT * L2) / IZ
K4 = (NALPH * L1) / IZ
K5 = ((CMQ * AM * LM * LM * QK) / (2 * VM)) / IZ
PRINT "MISSILE EQUATION OF MOTION COEFFICIENTS :"
PRINT
PRINT "YDDOT = (";K1;") * ATKANG - (";K2;") * GAMMA - WN"
PRINT
PRINT "GDDOT= (";K3;")*FINANG-(";K4;")*ATKANG-(";K5;")*GDOT"
PRINT
PRINT "****************************************************"
REM
REM THIS SECTION INCORPORATES THE FIN CONTROLLER AND
REM MISSILE EQUATIONS OF MOTION ALONG WITH A VERTICAL
REM VELOCITY FEEDBACK LOOP AND POWER AMP TO FORM THE
REM BASIC MISSILE SYSTEM THAT IS TO BE CONTROLLED. THUS
REM WE HAVE TMISSU, (THE UNCOMPENSATED MISSILE TRANSFER
REM FUNCTION).
REM
SRG = 10
KPWR = 1
M1 = KPWR * K3 * (K1 - K2)
NN1 = E1 * M1
NN0 = E0 * M1
DD7 = F3
DD6 = (F2) + (F3 * K5)
DD5 = (F1) + (F2 * K5) + (F3 * K4)
DD4 = (F0 * SRG) + (F1 * K5) + (F2 * K4)
DD3 = (F0 * K5) + (F1 * K4)
DD2 = (F0 * K4) + (E1 * (M1/SRG))
DD1 = E0 * (M1/SRG)
PRINT "UNCOMPENSATED MISSILE TRANSFER FUNCTION:"
PRINT
PRINT "          S(";NN1;") + (";NN0;")"
PRINT "TMISSU = -------------------------------------------"
PRINT "          S^7(";DD7;")+S^6(";DD6;")+S^5(";DD5;")+ "
PRINT
PRINT "    ------------------------------------------------"
PRINT "    S^4(";DD4;")+S^3(";DD3;")+S^2(;DD2;")+S(";DD1;")"
PRINT
PRINT "****************************************************"
PRINT "SRG = ";SRG;" KPWR = ";KPWR;" WT = ";WT;" VM = ";VM;"
PRINT "LENGTH = ";LM
END
```

APPENDIX D:

PROGRAM 2 LISTING

```
//FRANKLIN JOB (2832,0116),'FRANKLIN',CLASS=C
//*MAIN ORG=NPGVM1.2832P
// EXEC CSMPXV
//X.COMPRINT DD DUMMY
//X.SYSPRINT DD DUMMY
//X.SYSIN DD *

INITIAL
CONSTANT  BM = 0.00015, BL = 0.0015, KT = 15.9
CONSTANT  JL = 0.01, PI = 3.14159265
CONSTANT  JM = 0.001, KB = 0.112
CONSTANT  LA = .0016, RA = 2.740, N = 10.0
PARAMETER THFSAT = 13.0, ARM = 0.132, ROE = 0.002378
PARAMETER MISSIZ = 12.0, FINSIZ = 2.0, AIRSPD = 2000.
PARAMETER WT = 1000.,L1 = 1.0, L2 = 5.0, LM = 10.
PARAMETER CLF = .1, CLM = .1
PARAMETER GRAV = 32.,CD = .01, CMQ = .5, LAMBDA = 0.
PARAMETER Z1 = 0.1, P1 = 200., Z2 = 5., P2 = 50.

* KT -- TORQUE CONSTANT (OZ-IN/AMP)
* KB -- BACK EMF CONSTANT (VOLT/RAD/S)
* RA -- RESISTANCE OF THE MOTOR (OHM)
* BM -- VISCOUS FRICTION COEFFICIENT OF THE MOTOR
*        (OZ-IN/RAD/S)
* BL -- VISCOUS FRICTION COEFFICIENT OF THE LOAD
* BLP -- VISCOUS FRICTION COEFFICIENT OF LOAD THROUGH
*         REDUCTION GEARS
* BT -- TOTAL VISCOUS FRICTION OF THE MOTOR SYSTEM
* JM -- INERTIA OF THE MOTOR (OZ-IN/S-S)
* JL -- INERTIA OF THE LOAD
* JLP -- INERTIA OF THE LOAD THRU REDUCTION GEARS
* JT -- TOTAL INERTIA OF THE MOTOR SYSTEM
* A1 = LA/RA  --  THE ELECTRICAL TIME CONSTANT OF THE
*                 MOTOR
* A2 = JT/BT    --  THE MECHANICAL TIME CONSTANT OF THE
*                 MOTOR

NOSORT
      BLP = BL/(N**2)
      JLP = JL/(N**2)
      JT  = JM + JLP
      BT  = BM + BLP
      A1  = LA / RA
      A2  = JT / BT
```

```
      MASS = WT / GRAV
      IZ = (1./12) * MASS * L * L
      XDDOT = 0.0
      GDDOT = 0.0

* CONTROL PARAMETERS
      VXINIT = AIRSPD
      VM = AIRSPD
      KI = 100.0
      KP = 200.0
      KTAC = 1.0
      KNVRT = 35.0
      KKI = 21.0
      KKP = 3.0
      KKD = 1.0
      INVZ1 = 1.0/Z1
      INVP1 = 1.0/P1
      INVZ2 = 1.0/Z2
      INVP2 = 1.0/P2
      KPWR = 1.
      VBIAS = 82.
      SRG = 2.
      HK = 1.0

DYNAMIC
* INPUT SECTION
      HREF = -10.

* BEGIN MISSLE CONTROL SECTION
      EPSH = HREF - (HK * Y)
      LL1 = LEDLAG(0.0001,INVZ1,INVP1,EPSH)
      LL2 = LEDLAG(0.0001,INVZ2,INVP2,LL1)
      VPROP = LL2 * KKP
      VINT = KKI * INTGRL(0.0,LL2)
      VDIR = KKD * DERIV(0.0,LL2)
      VPID = VPROP + VINT + VDIR
      VELFED = (SRG * YDOT)
      VINPWR = (VPID/10.) - VELFED
      VPWR = VINPWR * KPWR
      VA = VPWR + VBIAS
      VAFIN = LIMIT(-160.,160.,VA)

* BEGIN FIN CONTROL SECTION
      ERR1 = VAFIN - (KNVRT * THETA)
* PROPORTIONAL INTEGRAL CONTROL
      EPROP = ERR1 * KP
      EINT = KI * INTGRL(0.0,ERR1)
      EPI = EPROP + EINT
* TACHOMETER FEEDBACK PLUS BACK EMF
      KBAC = (KTAC + KB)
      VBACK = KBAC * WM
```

```
      ERR2 = EPI - VBACK

* MOTOR MODEL
      IM = ERR2 * (1.0/RA)
      TM = IM * KT
      TERR = TM + TL
      WM = REALPL(0.0,A2,TERR/BT)
      WMRPM = WM * (30./PI)
      THETA = INTGRL (0.0,WM)
* MOTOR SHAFT POSITION
      THDEG = THETA * (180./PI)
      WMFIN = WM / N
* FIN POSITION (POSITIVE IS UPWARD MOVEMENT OF AFT
* PORTION OF FIN)
      FINANG = THETA / N
      FINDEG = FINANG * (180./PI)
      PWR = WM * TM * .0070615

* FIN LOAD TORQUE RESULTING FROM DYNAMIC FIN DEFLECTION
      CLL = -(0.1 * LIMIT(-THFSAT,THFSAT,FINDEG))
      TL = ARM * ((ROE)/2) * FINSIZ * (AIRSPD**2)*CLL

* MISSILE MODEL
* FIN POSITION IS INPUT TO THE MISSILE MODEL HERE AS
* FINANG IN RADIANS
* AIRSPD USED HERE FOR SIMPLIFICATION AND (I.C.)
* RATHER THAN VM
      QK = (ROE * AIRSPD * AIRSPD) / 2.
      NDELT = CLF * FINSIZ * QK
      NALPH = CLM * MISSIZ * QK
      K1 = NALPH
      K2 = CD * MISSIZ * QK
* REFERENCE DIRECTION IS HORIZONTAL THEREFORE
* COS(LAMBDA)=1.
      WTN = MASS * GRAV * 1.
      K3 = NDELT * L2
      K4 = NALPH * L1
      K5 = (CMQ * MISSIZ * LM * LM * QK) / (2. * VM)
      GDDOT = (K3 * FINANG - K4 * ATKANG - K5 * GDOT)/IZ
      GDOT = INTGRL(0.0,GDDOT)
      G = INTGRL(0.0,GDOT)
      GDEG = G * (180./PI)
      YDDOT = ( K1 * ATKANG - K2 * G - WTN)/ MASS
      YDOT = INTGRL(0.0,YDDOT)
      Y = INTGRL(0.0,YDOT)
      XDDOT = 0.0
      XDOT = INTGRL(VXINIT,XDDOT)
      X = INTGRL(0.0,XDOT)
      VM = (XDOT **2 + YDOT **2)**0.5
      BETA = ATAN2(YDOT,XDOT)
      BETDEG = BETA * (180./PI)
```

```
      ATKANG = G - BETA
      ATKDEG = ATKANG * (180./PI)

TERMINAL
  TITLE MISSILE ALTITUDE HOLD CONTROLLER
  METHOD RKSFX
  TIMER FINTIM = 10.0,OUTDEL=.1,PRDEL=.1,DELT=.0001
  OUTPUT Y,BETDEG,GDEG,ATKDEG,FINDEG
  PRINT ATKDEG,FINDEG,Y
  LABEL MISSILE PARAMETERS
  PAGE MERGE
       END
       STOP
ENDJOB
/*
```

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