Elliptic Flow of Heavy Quarkonia in pA Collisions

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Using the dilute-dense factorization in the color glass condensate framework, we investigate the azimuthal angular correlation between a heavy quarkonium and a charged light hadron in proton-nucleus collisions. We extract the second harmonic v_2, commonly known as the elliptic flow, with the light hadron as the reference. This particular azimuthal angular correlation between a heavy meson and a light hadron was first measured at the LHC recently. The experimental results indicate that the elliptic flows for heavy flavor mesons (J/ψ and D^0) are almost as large as those for light hadrons. Our calculation demonstrates that this result can be naturally interpreted as an initial state effect due to the interaction between the incoming partons from the proton and the dense gluons inside the target nucleus. Since the heavy quarkonium v_2 exhibits a weak mass dependence according to our calculation, we predict that the heavy quarkonium Υ should have a similar elliptic flow as compared to that of the J/ψ, which can be tested in future measurements.

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Introduction.—Plenty of evidence for strong collectivity phenomenon in small collisional systems, such as pp and pPb collisions at the LHC and dAu collisions at the Relativistic Heavy Ion Collider (RHIC), has been reported [1–8] in the last few years. The collectivity in small systems is measured and computed in terms of particle azimuthal correlations in high multiplicity pp and pA collisions and has become one of the most interesting and important topics in heavy ion physics. In these high multiplicity events, the azimuthal angular distributions of a measured particle can be decomposed into Fourier harmonics with the corresponding coefficients $v_n \equiv \langle \cos n\Delta\phi \rangle$, where $\Delta\phi$ is the azimuthal angle difference between the measured particle and the reference particle or the reaction plane.

In addition, recently there has been significant direct evidence that charm quarks also have a sizable collectivity in small collisional systems. Both the ALICE [9] and CMS [10,11] collaborations have reported large values of elliptic flow v_2 for J/ψ mesons and for D^0 mesons in pPb collisions at the LHC, although they are slightly less than the v_2 values of light hadrons.

One of the most successful explanations of the collectivity phenomenon in small collisional systems comes from the relativistic hydrodynamics approach. In this approach, the quark gluon plasma created in the collisions with high multiplicity are treated as relativistic fluids, and the flow harmonics can be viewed as the final state effect due to hydrodynamic evolution of small collisional systems with a certain amount of initial anisotropy. Excellent agreement [12–23] has been found between the hydrodynamics approach and the measured flow harmonics of light hadrons at both the RHIC and LHC. On the other hand, it is difficult for hydrodynamics to generate large collectivity for heavy mesons, since heavy quark in general does not flow as much as the light quark or gluon due to the large quark mass. Furthermore, recent calculation [24] also indicates that the observed v_2 from the ALICE and CMS collaborations cannot come from final state interactions alone, since the final state interactions can only provide a small fraction of the observed v_2 for J/ψ mesons. In addition, besides the hydrodynamics approach, there could be other possible mechanisms, as suggested in Refs. [25–28].

Meanwhile, the color glass condensate (CGC) framework or the saturation formalism shows that correlations between partons originating from the projectile proton and dense gluons inside the target nucleus, which can be written in terms of Wilson lines, can also provide a significant amount of collectivity [29–58] for light hadrons. Usually this is regarded as the initial state effect prior to the onset of hydrodynamic evolution. The CGC framework has been quite useful in understanding the heavy quarkonium productions [59–61] in pp and pPb collisions in the low transverse momentum region. However, calculations on the $J/\psi v_2$ in the CGC framework are still lacking.

The objective of this Letter is to study the elliptic flow harmonic v_2 of J/Ψ mesons within a simplified model based on the color evaporation model (CEM) and the dilute-dense factorization [62,63] in the CGC framework and to demonstrate that a significant amount of v_2 can be generated due to the nontrivial QCD dynamics of the interaction between the partons from the proton projectile and dense gluons in the nuclear target. This calculation is a further extension of the two-particle azimuthal correlation CGC calculation developed in Refs. [41,43,47,48,58]. Besides, we need to consider the splitting of the $c\bar{c}$ pair from a gluon ($g \to c\bar{c}$) in order to produce a J/ψ meson in the final state. Similar to the measurements carried out at the LHC, we compute $v_2 \equiv \langle \cos 2\Delta\phi \rangle$ based on the production of a J/ψ meson in the CEM accompanied by another reference quark, which eventually fragments into a charged hadron; i.e., we disregard the $q \to qg \to qJ/\psi$ jetlike contributions.

The Letter is organized as follows. In the next section, we briefly introduce the framework employed in our calculation of v_2 for heavy quarkonia, including the CGC correlators and the dilute-dense formalism for particle production as well as the CEM. Then, we show the comparison between our numerical results and the LHC data, with some further comments. The phenomenological implications of our model calculation are discussed in the Conclusion.

Elliptic flow of heavy quarkonia in pA collisions.—Let us now briefly mention the essential ingredients of the calculation that lead to the elliptic flow of heavy quarkonia in pA collisions with a charged light hadron as the reference particle. In correspondence to high multiplicity events in pA collisions, we assume that there are multiple active partons from the proton projectile participating the interaction with the target nucleus. To measure the J/Ψ elliptic flow, a charged hadron is used as a reference particle in the LHC experiment. Similarly, to simplify the calculation, we pick a gluon and a quark from the proton with the quark serving as the reference with the gluon splitting into a pair of heavy quarks $\mathcal{Q}\bar{\mathcal{Q}}$ and compute their interactions with the target nucleus. (We have checked that the numerical result with a gluon as the reference particle yields a similar v_2 coefficient.) We take into account all the possible correlations between the gluon (or $\mathcal{Q}\bar{\mathcal{Q}}$) and the reference quark generated by those interactions, up to $1/N_c^2$ order and neglect higher order corrections. As we show below, nontrivial color correlation starts to appear at the $1/N_c^2$ order, which generates sizable elliptic flow for heavy quarkonia. Based on our numerical calculation of the light hadron v_2, we expect that the contributions from the $1/N_c^4$ order are negligible in the region of interest where the transverse momentum is small. In the large $k_\perp$ region, other effects such as higher order N_c corrections may become important. Our model is akin to the CGC model calculations [41,43,47,48,58] with the additional $g \to \mathcal{Q}\bar{\mathcal{Q}}$ splitting in order to produce heavy quarkonia in the final state.

Accompanied by a reference quark, the incoming gluon splits into a pair of heavy quarks $\mathcal{Q}\bar{\mathcal{Q}}$ before or after they traverse the dense nuclear target. Therefore, the differential spectrum of the production of $\mathcal{Q}\bar{\mathcal{Q}}$ and another light quark in the large N_c limit can be written as [64]

$$\frac{dN^{gqA\to\mathcal{Q}\bar{\mathcal{Q}}qX}}{d^2\boldsymbol{k}_1 d^2\Delta\boldsymbol{k}_1 d^2\boldsymbol{k}_2} = \mathcal{N}\int\frac{d^2\boldsymbol{r}d^2\boldsymbol{r}'}{(2\pi)^2}e^{-i\Delta\boldsymbol{k}_1\cdot(\boldsymbol{r}-\boldsymbol{r}')}\prod_{i=1}^{2}\int\frac{d^2\boldsymbol{b}_i d^2\boldsymbol{r}_i}{(2\pi)^2}e^{-i\boldsymbol{k}_i\cdot\boldsymbol{r}_i}\langle DDD\rangle\psi(\boldsymbol{r})\psi^*(\boldsymbol{r}'), \tag{1}$$

with the normalization factor $\mathcal{N}$, which cancels out when we compute v_2, and

$$DDD \equiv [D(\boldsymbol{x}_\mathcal{Q},\boldsymbol{x}'_\mathcal{Q})D(\boldsymbol{x}'_{\bar{\mathcal{Q}}},\boldsymbol{x}_{\bar{\mathcal{Q}}}) + D(\boldsymbol{x}_g,\boldsymbol{x}'_g)D(\boldsymbol{x}'_g,\boldsymbol{x}_g) - D(\boldsymbol{x}_\mathcal{Q},\boldsymbol{x}'_g)D(\boldsymbol{x}'_g,\boldsymbol{x}_{\bar{\mathcal{Q}}}) - D(\boldsymbol{x}'_{\bar{\mathcal{Q}}},\boldsymbol{x}_g)D(\boldsymbol{x}_g,\boldsymbol{x}'_\mathcal{Q})]D(\boldsymbol{x}_q,\boldsymbol{x}'_q), \tag{2}$$

where the dipole correlators $D(\boldsymbol{x},\boldsymbol{y}) \equiv (1/N_c)\mathrm{Tr}U(\boldsymbol{x})U(\boldsymbol{y})^\dagger$. We denote $\boldsymbol{k}_1$ as the transverse momentum of the $\mathcal{Q}\bar{\mathcal{Q}}$ pair and $\Delta\boldsymbol{k}_1$ as the relative transverse momentum of $\mathcal{Q}\bar{\mathcal{Q}}$. $\boldsymbol{k}_2$ stands for the transverse momentum of the reference light quark. In the above expression, $D(\boldsymbol{x}_q,\boldsymbol{x}'_q)$ corresponds to the reference quark production, while the other two color dipoles come from the heavy quark pair or the incoming gluon in the large N_c limit. The full expression of the Wilson correlators for the $\mathcal{Q}\bar{\mathcal{Q}}$ production can be found in Ref. [64] without taking the large N_c limit. It is easy to see that the terms that we neglected above do not provide any correlation between the $\mathcal{Q}\bar{\mathcal{Q}}$ pair and the light reference quark. The transverse coordinates $\boldsymbol{x}_{\mathcal{Q},\bar{\mathcal{Q}},g,q}$ ($\mathcal{Q}$, $\bar{\mathcal{Q}}$, g and the

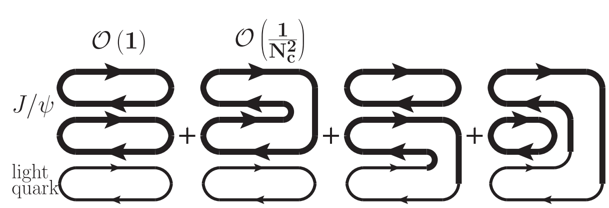

FIG. 1. Illustration of the expectation value of three dipole correlators in the gluon background fields of the target nucleus. These four diagrams also show the origins of the each terms in Eq. (3). It is clear that only the last two diagrams contain azimuthal angular correlations between the produced J/ψ and the reference light quark.

reference quark) inside the above dipole correlators can also be written as $\boldsymbol{x}_Q(\boldsymbol{x}_{\bar{Q}}) \equiv \boldsymbol{x}_g \pm (\boldsymbol{r}/2)$, $\boldsymbol{x}'_Q(\boldsymbol{x}'_{\bar{Q}}) \equiv \boldsymbol{x}'_g \pm (\boldsymbol{r}'/2)$, $\boldsymbol{x}_g(\boldsymbol{x}'_g) \equiv \boldsymbol{b}_1 \pm (\boldsymbol{r}_1/2)$, and $\boldsymbol{x}_q(\boldsymbol{x}'_q) \equiv \boldsymbol{b}_2 \pm (\boldsymbol{r}_2/2)$. Here the average $\langle DDD \rangle$ indicates the color averaging of three color dipoles in terms of the corresponding fundamental Wilson lines in the gluon background fields of target nuclei. The above four target averages can be computed in the McLerran-Venugopalan model [65,66], and nontrivial color correlations can appear when two color singlet dipoles are disconnected in order to form a singlet quadrupole, due to inelastic exchanges with the target gluon fields, as shown in Fig. 1. In the large N_c limit, a general three-dipole correlator can be cast into the following form up to the $1/N_c^2$ order [58]:

$$\langle D(\boldsymbol{x}_1,\boldsymbol{x}'_1)D(\boldsymbol{x}_2,\boldsymbol{x}'_2)D(\boldsymbol{x}_3,\boldsymbol{x}'_3)\rangle = e^{-(Q_s^2/4)[(\boldsymbol{x}_1-\boldsymbol{x}'_1)^2+(\boldsymbol{x}_2-\boldsymbol{x}'_2)^2+(\boldsymbol{x}_3-\boldsymbol{x}'_3)^2]}$$
$$\times [1 + F(\boldsymbol{x}_1,\boldsymbol{x}'_1;\boldsymbol{x}_2,\boldsymbol{x}'_2) + F(\boldsymbol{x}_3,\boldsymbol{x}'_3;\boldsymbol{x}_2,\boldsymbol{x}'_2) + F(\boldsymbol{x}_1,\boldsymbol{x}'_1;\boldsymbol{x}_3,\boldsymbol{x}'_3)],$$
$$\text{with} \quad F(\boldsymbol{x}_1,\boldsymbol{x}'_1;\boldsymbol{x}_2,\boldsymbol{x}'_2) = \frac{[Q_s^2(\boldsymbol{x}_1-\boldsymbol{x}'_1)\cdot(\boldsymbol{x}_2-\boldsymbol{x}'_2)]^2}{4N_c^2}\int_0^1 d\xi \int_0^\xi d\eta e^{(\eta Q_s^2/2)(\boldsymbol{x}_1-\boldsymbol{x}_2)\cdot(\boldsymbol{x}'_2-\boldsymbol{x}'_1)}. \tag{3}$$

The above result can be obtained by using the technique developed in many early works [67–72]. The saturation momentum Q_s^2, which is proportional to $A^{1/3}$, with A as the number of nucleons, characterizes the density of target nuclei and it increases with the collisional energy.

To reduce the number of integrations, we fix all the rapidities and set the rapidity of Q and $\bar{Q}$ to be approximately equal. As usual, the $g \to Q\bar{Q}$ splitting function $\psi(\boldsymbol{r})\psi^*(\boldsymbol{r}') \equiv \sum_{\lambda\alpha\beta}\psi^{T\lambda}_{\alpha\beta}(\boldsymbol{r})\psi^{T\lambda *}_{\alpha\beta}(\boldsymbol{r}') = (8\pi^2 m_Q^2/p_g^+)[\frac{1}{2}K_1(m_Q r)K_1(m_Q r')(\boldsymbol{r}\cdot\boldsymbol{r}'/rr') + K_0(m_Q r)K_0(m_Q r')]$, with p_g^+ as the longitudinal momentum of the incoming gluon. To make further simplification, we set the longitudinal momentum fraction of Q and $\bar{Q}$ with respect to the incoming gluon to be $\frac{1}{2}$ in the splitting function.

In addition, we assume the momentum and coordinate distribution of the incoming gluon and quark inside the proton as the Gaussian-type Wigner function $W(\boldsymbol{b},\boldsymbol{p}) = (1/\pi^2)e^{-\boldsymbol{b}^2/B_p - \boldsymbol{p}^2/\Delta^2}$, where the parameters B_p and Δ^2 are the variances of the impact parameter $\boldsymbol{b}$ and the transverse momentum $\boldsymbol{p}$, respectively. This parametrization of incoming quark and gluon distributions can help us to perform some of the impact parameters and dipole size integrations analytically and allows us to carry out the rest of the integrations numerically.

In the CEM, since the invariant mass of the heavy quark pair is integrated from the bare quark pair mass ($2m_Q$) to the mass of the open heavy meson pair ($2M_H$), we should convolute the factor $\theta(\sqrt{M_H^2 - m_Q^2} - \Delta k_1)F_{Q\bar{Q}\to J/\psi}$ and integrate over the relative momentum $\Delta\boldsymbol{k}_1$ to convert the produced $Q\bar{Q}$ into the corresponding heavy quarkonium with the probability $F_{Q\bar{Q}\to J/\psi}$. To further simplify the calculation, since the dominant contribution of the integration over Δk_1 comes from the region where $\Delta k_1 \sim m_Q$ [73], we assume that the threshold $\sqrt{M_H^2 - m_Q^2}$ is large enough (1.4 GeV for J/ψ) so we can integrate over Δk_1 up to infinity and obtain the delta function $(2\pi)^2\delta^{(2)}(\boldsymbol{r}-\boldsymbol{r}')$. We use such a crude approximation as a first step estimate. At last, the transverse momentum-dependent production spectrum of the heavy quarkonium accompanied by a light quark in pA collisions reads

$$\frac{dN^{pA\to J/\psi qX}}{d^2\boldsymbol{k}_1 d^2\boldsymbol{k}_2} = \prod_{i=1}^{2} W(\boldsymbol{b}_i,\boldsymbol{p}_i) \otimes \frac{dN^{gqA\to Q\bar{Q}qX}}{d^2\boldsymbol{k}_1 d^2\boldsymbol{k}_2} F_{Q\bar{Q}\to J/\psi}$$
$$= \mathcal{N}\int d^2\boldsymbol{r} \prod_{i=1}^{2}\int \frac{d^2\boldsymbol{b}_i d^2\boldsymbol{r}_i}{(2\pi)^2} d^2\boldsymbol{p}_i W(\boldsymbol{b}_i,\boldsymbol{p}_i) e^{-i(\boldsymbol{k}_i-\boldsymbol{p}_i)\cdot\boldsymbol{r}_i}\langle DDD|_{\boldsymbol{r}=\boldsymbol{r}'}\rangle |\psi(r)|^2 F_{Q\bar{Q}\to J/\psi}. \tag{4}$$

The nth Fourier harmonics of the transverse momentum-dependent differential spectrum is defined as [74]

$$\frac{d\kappa_n}{dk_1} \equiv k_1 \int d\phi_1 d^2\boldsymbol{k}_2 e^{in(\phi_1-\phi_2)}\frac{dN^{pA\to J/\psi qX}}{d^2\boldsymbol{k}_1 d^2\boldsymbol{k}_2}, \tag{5}$$

The $k_\perp$-dependent elliptic flow (the second Fourier harmonic) of the produced heavy quarkonium then can be obtained as follows [10]:

$$v_2(k_\perp) \equiv \frac{\frac{d\kappa_2}{dk_\perp}}{\frac{d\kappa_0}{dk_\perp}}\frac{1}{v_2[\mathrm{ref}]}, \tag{6}$$

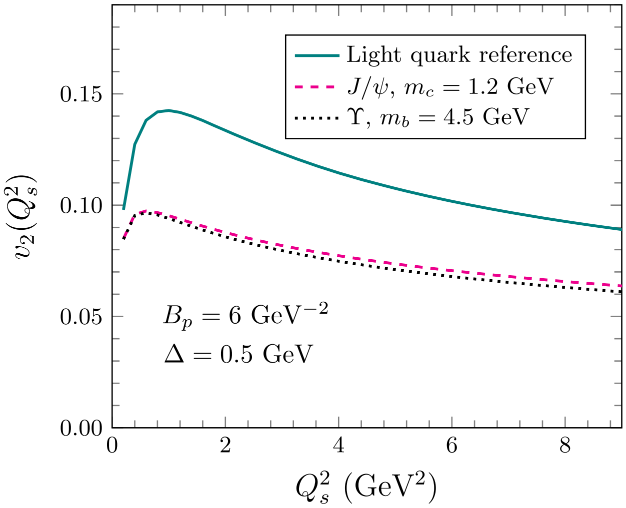

FIG. 2. The integrated v_2 of J/ψ and Υ compared with the v_2 of the reference light quark as function of the saturation momentum Q_s^2.

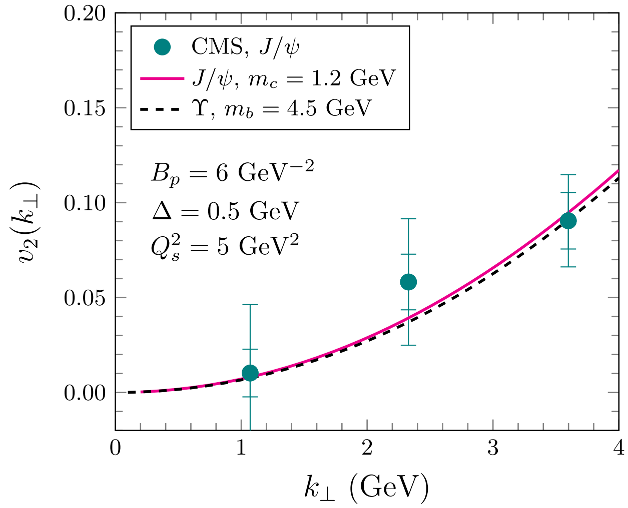

FIG. 3. The $k_\perp$-dependent elliptic flow v_2 of J/ψ as function of its transverse momentum $k_\perp$ compared with the CMS data [10] where both systematic (inner ones) and statistic (outer ones) error bars are shown. Our result is also consistent with the ALICE [9] data. In addition, as a prediction, the v_2 of Υ is also plotted in this figure.

where $v_2[\mathrm{ref}] = \sqrt{\kappa_2[\mathrm{ref}]/\kappa_0[\mathrm{ref}]}$ is the transverse momentum integrated elliptic flow of the reference light quark, which has been computed in Ref. [58]. Similarly, the integrated v_2 for heavy quarkonia can be written as $v_2 \equiv (\kappa_2/\kappa_0)/v_2[\mathrm{ref}]$.

It is interesting to notice that the four correlators inside $\langle DDD|_{r=r'}\rangle$ cancel completely if we set the coordinate separation of the $\mathcal{Q}\bar{\mathcal{Q}}$ pair r to 0. Therefore, if we perform the small r expansion, we can see the first nontrivial contribution comes at r^2 order and the mass dependence is associated with the r integration. The heavy quark mass dependences cancel completely between κ_2 and κ_0 when we only compute v_2 up to the r^2 order. Note that our numerical calculation is carried out without taking the small expansion in order to see the heavy quarkonia mass dependence.

As shown above, we have to evaluate a large number of integrations in order to numerically plot the elliptic flow of heavy quarkonia. Our strategy is to analytically integrate as many integrals as possible and numerically evaluate the remaining five dimensional integrations.

Numerical results and comments.—Using the aforementioned simplified model, we are able to compute the elliptic flow v_2 for heavy quarkonia, such as J/ψ and Υ mesons. In Fig. 2, we show the integrated v_2 of J/ψ and Υ comparing with the v_2 of light reference quark as functions of Q_s^2 in the CGC formalism. This plot shows that heavy quarkonia in the CGC formalism can typically have the $k_\perp$ integrated v_2 roughly between 5% and 10%, which is about 2/3 of that for light reference quarks. Similar curves for light quarks can also be found in Refs. [47,48]. It is important to note that, due to the splitting $g \to \mathcal{Q}\bar{\mathcal{Q}}$, the production mechanism of heavy quarkonia is generically different from that of light hadrons. We believe that this leads to a slightly smaller v_2 for heavy quarkonia. In the meantime, the quarkonium mass dependence is rather weak for the elliptic flow if one compares J/ψ and Υ. It will be very interesting to measure the v_2 of Υ in the near future. In Fig. 3, excellent agreement is found by comparing our calculation of $v_2(k_\perp)$ for J/ψ to the CMS data with the parameter consistent with Ref. [58]. Here we use a slightly larger $Q_s^2 = 5\ \mathrm{GeV}^2$ for the LHC instead of $Q_s^2 = 4\ \mathrm{GeV}^2$ for RHIC. We find that v_2 is not very sensitive to the choice of B_p and Δ values, under variations of $\pm 50\%$ or smaller.

Conclusion and outlook.—As a conclusion, let us make some further comments on the consequence of this Letter. (1) First of all, as we have demonstrated above, the heavy quarkonia can have significant elliptic flow due to color interactions and transitions that have little mass dependence. Intuitively, this can be understood as the cancellation of mass dependence between the anisotropic spectrum κ_2 and the isotropic spectrum κ_0; thus v_2 contains little mass dependence. This allows us to predict that the Υ meson should also have a similar size of elliptic flow at the RHIC and LHC, although it is much heavier than the J/ψ meson. This prediction could be tested in future measurements. (2) Furthermore, instead of integrating over the relative transverse momentum of heavy quark pairs, we can integrate over the momentum of $\bar{\mathcal{Q}}$ and measure the outgoing $\mathcal{Q}$. This allows us to generalize the above calculation and compute the elliptic flow for open charm particles, namely, the D^0 meson. The numerical evaluation may be more demanding, but we expect the corresponding v_2 for D mesons should lie in the similar range as that of J/ψ. (3) In addition, instead of using the CEM, one could also compute the elliptic flow for heavy quarkonia in more sophisticated models by separating the color singlet states from the color octet states. Nevertheless, since the elliptic flow is computed from the ratio of κ_2 and κ_0 where most of the detailed information of the hadronization from heavy quark pairs to physical quarkonia cancels, we expect that our prediction for v_2 should be robust. (4) Last but not least, the framework employed in this Letter is consistent with previous calculations on the spectra of J/ψ and Υ mesons in both pp and pPb collisions [61]. It is worth noting that one can describe both the elliptic flow v_2 and the nuclear modification factor $R_{p\mathrm{Pb}}$ for heavy quarkonia in the low transverse momentum region within this framework. A similar but more comprehensive description of J/ψ production in pp and pPb collisions can be also found in Refs. [59,60].

In this Letter, we have computed the elliptic flow for heavy quarkonia and found excellent agreement with the J/ψ data measured at the LHC. This suggests that the observed large v_2 for J/ψ at the LHC can be naturally explained as the initial state effect in the CGC formalism.

Because of the complexity of this problem, a number of approximations have been made in order to simplify the calculation before we can perform the numerical calculation. Nevertheless, we expect the main feature of our results will retain in a more complete calculation. We leave such study and the detailed derivation as well as the calculation of the v_2 of D^0 mesons to a future work.

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