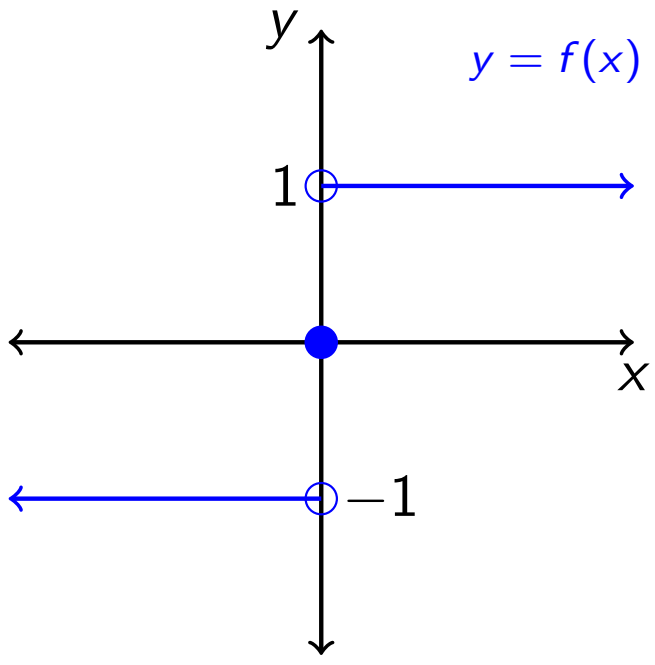
Limits and Continuity

June 12, 2014

A Strange Function

Let $f(x) = \begin{cases} \dfrac{|x|}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0. \end{cases}$



Limits

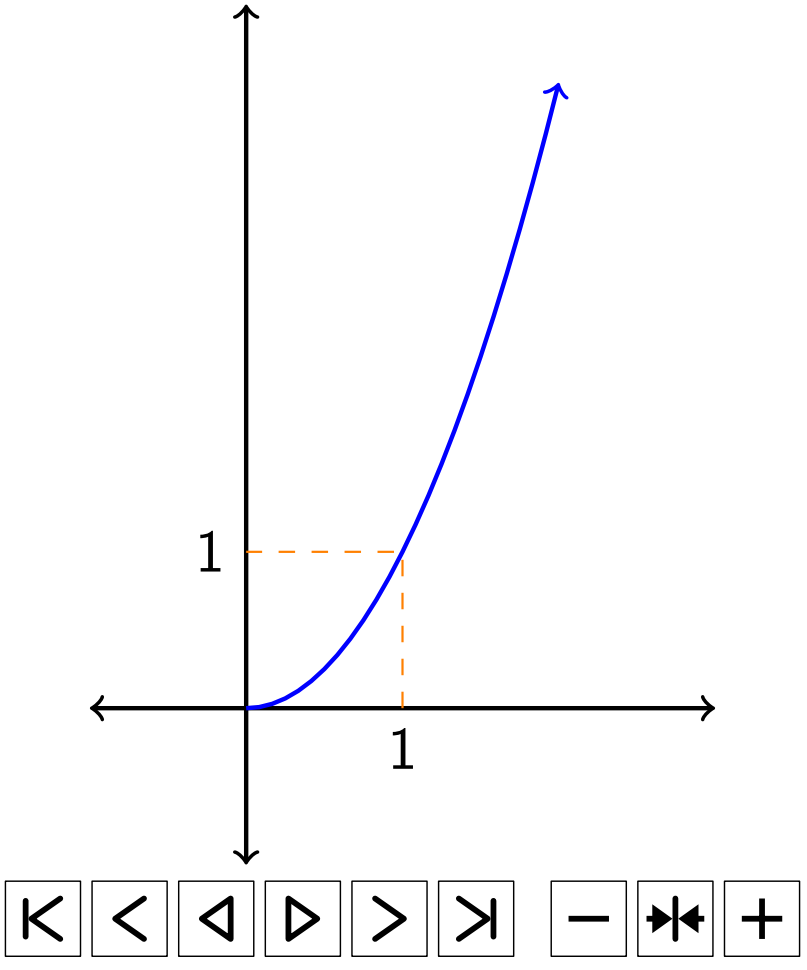
The limit of $f(x)$ as x tends to a is written as $\lim_{x \to a} f(x)$.

If it exists, it is the number that $f(x)$ gets arbitrarily close to as x approaches a.

Importantly, it is completely independent of $f(a)$, since it only involves values of x which are extremely close to a: $\lim_{x \to a} f(x)$ can exist even if a is not in the domain of f!

Example

What is $\lim_{x \to 1} x^2$?



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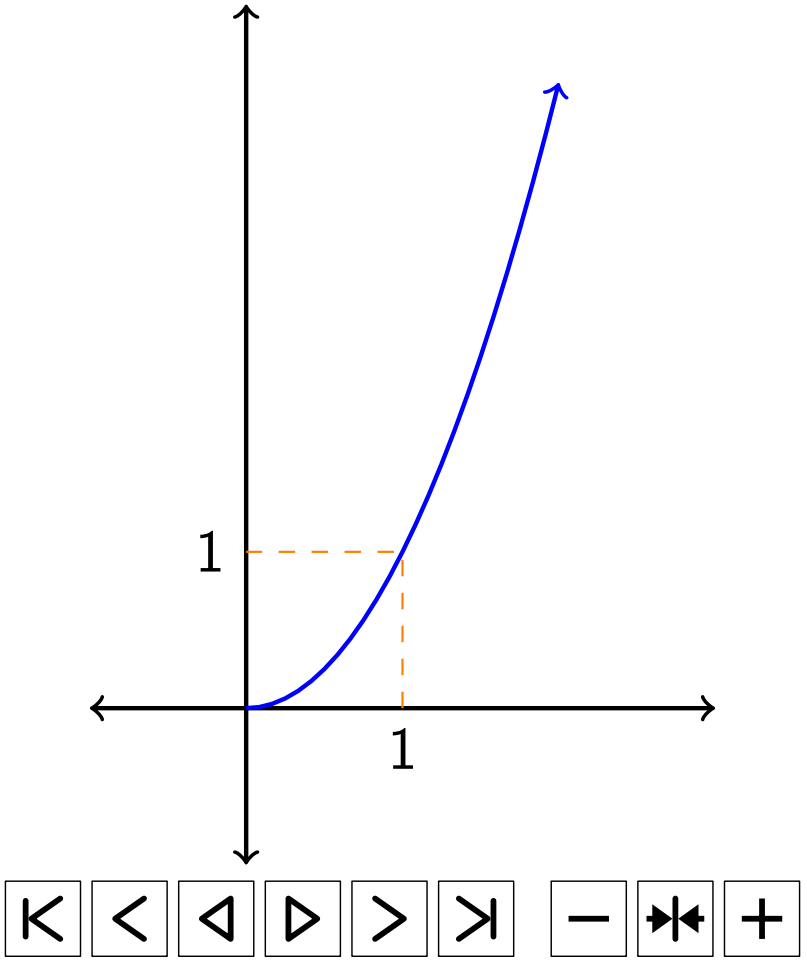

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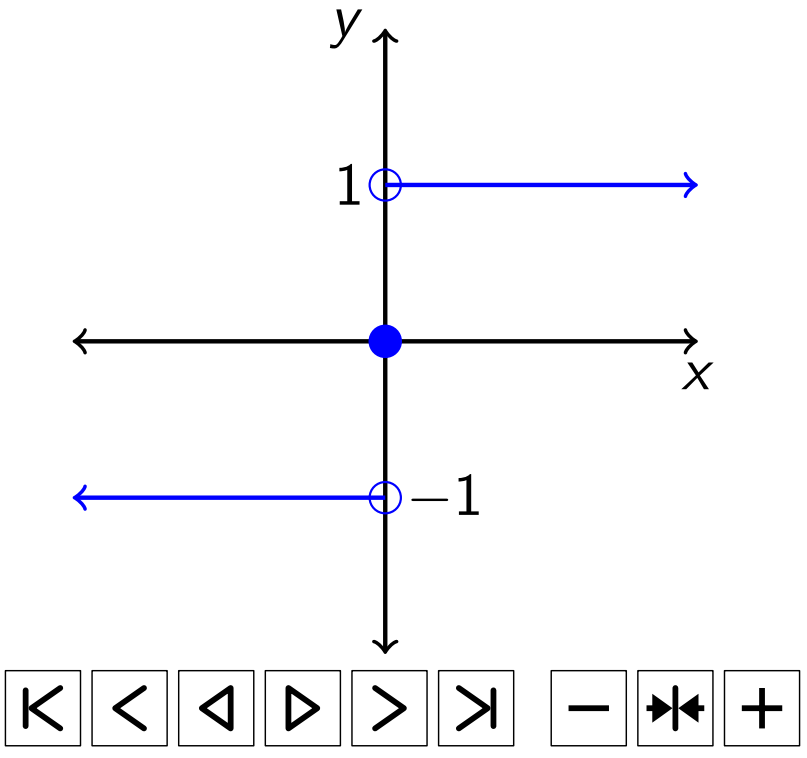
A function f is continuous if it is continuous at every point in the domain.

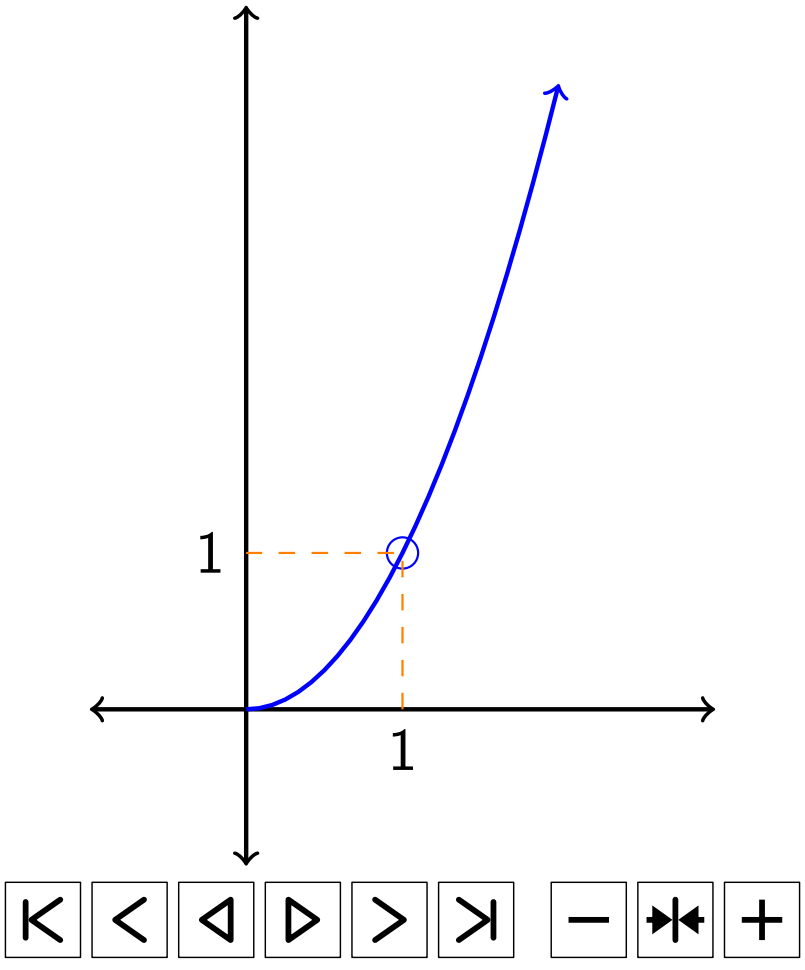
Non-example

What is $\lim_{x \to 0} \frac{|x|}{x}$?



Example

What is $\lim_{x \to 1} \dfrac{x^3 - x^2}{x - 1}$?



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- If a is not in the domain of f, at this stage it's usually because you're dividing by 0. Try factorising and simplifying to make it work: graphically, all that's happening is that there is a 'hole' in the domain (remember the previous example).
- Substitute something very close to a into the equation with your calculator. (Only use this if you have to, since it's extremely barbaric.)
    - If you get a sensible answer doing this, go back and try to get the same answer another way.