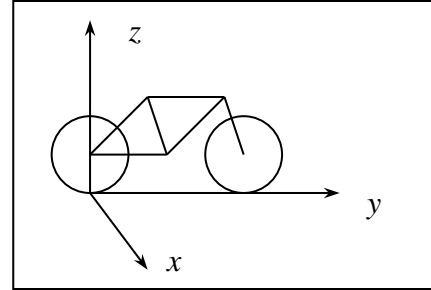


A simplified look at bike roll rate with Euler's Equations for 3D rigid body motion:

$$\begin{aligned}\sum M_x &= I_{xx} \dot{\omega}_x - (I_{yy} - I_{zz}) \omega_y \omega_z \\ \sum M_y &= I_{yy} \dot{\omega}_y - (I_{zz} - I_{xx}) \omega_z \omega_x \\ \sum M_z &= I_{zz} \dot{\omega}_z - (I_{xx} - I_{yy}) \omega_x \omega_y\end{aligned}$$



Use a right-hand coordinate system such that x and y axes define the ground plane with y in the direction the bicycle is traveling, and the z axis is vertical. Then some terms immediately cancel:

The friction of the wheels on the ground prevents rotation about the vertical axis so $\omega_z = 0$ and its rate of change $\dot{\omega}_z = 0$.

Further simplification can also be made, without loss of generality, by letting the wheel spin rate be constant so its rate of change $\dot{\omega}_x = 0$.

That leaves just:

$$\sum M_x = 0, \quad \sum M_y = I_{yy} \dot{\omega}_y, \quad \sum M_z = -(I_{xx} - I_{yy}) \omega_x \omega_y$$

Then let the center of mass be half way between the front and rear wheel and z_{cm} off the ground. The moment about the y -axis due to gravity on the center of mass will be $mgz_{cm} \sin \theta_y$ and the moment about the z -axis due to tire friction will be equal to or less than $\frac{1}{2} \mu mgy_{wheelbase}$. So finally:

$$\sum M_y = I_{yy} \dot{\omega}_y = mgz_{cm} \sin \theta_y, \quad \sum M_z = -(I_{xx} - I_{yy}) \omega_x \omega_y \leq \frac{1}{2} \mu mgy_{wheelbase}$$

The moment about the y -axis is due only to gravity, the lean angular rate, $mgz_{cm} \sin \theta_y / I_{yy}$, is independent of the wheel spin rates, and the bike simply tips over as if the wheels were not spinning at all.