ALTERNATING-CURRENT MACHINES

BEING THE SECOND VOLUME OF

DYNAMO ELECTRIC MACHINERY

ITS CONSTRUCTION, DESIGN,
AND OPERATION

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PREFACE TO FIRST EDITION.

THIS book, like its companion volume on Direct Current Machines, is primarily intended as a text-book for use in technical educational institutions. It is hoped and believed that it will also be of use to those electrical, civil, mechanical, and hydraulic engineers who are not perfectly familiar with the subject of Alternating Currents, but whose work leads them into this field. It is furthermore intended for use by those who are earnestly studying the subject by themselves, and who have previously acquired some proficiency in mathematics.

There are several methods of treatment of alternating-current problems. Any point is susceptible of demonstration by each of the methods. The use of all methods in connection with every point leads to complexity, and is undesirable in a book of this character. In each case that method has been chosen which was deemed clearest and most concise. No use has been made of the method of complex imaginary numbers.

A thorough understanding of what takes place in an alternating-current circuit is not to be easily acquired. It is believed, however, that one who has mastered the first four chapters of this book will be able to solve any practical problem concerning the relations which exist between power, electro-motive forces, currents, and their phases in series or multiple alternating-current circuits containing resistance, capacity, and inductance.

The next four chapters are devoted to the construction, principle of operation, and behavior of the various types of alternating-current machines. Only American machines have been considered.

A large amount of alternating-current apparatus is used in connection with plants for the long-distance transmission of power. This subject is treated in the ninth chapter. The last chapter gives directions for making a variety of tests on alternating-current circuits and apparatus.

No apology is necessary for the introduction of cuts and material supplied by the various manufacturing companies. The information and ability of their engineers, and the taste and skill of their artists, are unsurpassed, and the information supplied by them is not available from other sources. For their courteous favors thanks is hereby given.

PREFACE TO THE SEVENTH EDITION.

THE extensive adoption of this volume as a text-book for the use of students on other than electrical courses and the growing tendency, in many Institutions, to require more thorough and extended work in electrical subjects from such students, have determined the scope of the present revision. In those cases where insufficient time is available for covering all the ground contained herein, it will be found that portions, which the instructor will probably desire to omit, are so treated that the remainder will constitute a coordinated treatment. It is also believed that, in the majority of Institutions, the book as a whole will be found adapted for the use of students on electrical courses. The manner of presentation is in many parts different from that which would be employed in a book written for engineers, but an extended experience in teaching young men of average attainments has proved it to be effective. As a student seldom gets a thorough understanding of a subject of this character without making numerical computations, problems have been introduced at the conclusion of each chapter.

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ALTERNATING-CURRENT MACHINES.

CHAPTER I.

PROPERTIES OF ALTERNATING CURRENTS.

1. **Definition of an Alternating Current.** — An alternating current of electricity is a current which changes its direction of flow at regularly recurring intervals. Between these intervals the value of the current may vary in any way. In usual practice, the value varies with some regularity from zero to a maximum, and decreases with the same regularity to zero, then to an equal maximum in the other direction, and finally to zero again. In practice, too, the intervals of current flow are very short, ranging from $\frac{1}{50}$ to $\frac{1}{240}$ second.

2. **Frequency.** — When, as stated above, a current has passed from zero to a maximum in one direction, to zero, to a maximum in the other direction, and finally to zero again, it is said to have completed one *cycle.* That is to say, it has returned to the condition in which it was first considered, both as to value and as to direction, and is prepared to repeat the process described, making a second cycle. It should be noted that it takes two *alternations* to make one *cycle.* The tilde (~) is frequently used to denote cycles.

The term *frequency* is applied to the number of cycles completed in a unit time, i.e., in one second. Occasionally the word *alternations* is used, in which case, unless otherwise specified, the number of alternations per minute is meant. Thus the same current is spoken of as having a frequency of 25, or as having 3000 alternations. The use of the word alternations is condemned by good practice. In algebraic notation the letter f usually stands for the frequency.

The frequency of a commercial alternating current depends upon the work expected of it. For power a low frequency is desirable, particularly for converters. The great Niagara power plant uses a frequency of 25. Lamps, however, are operated satisfactorily only on frequencies of 50 or more. Early machines had higher frequencies, — 125 and 133 (16,000 alternations) being usual, — but these are almost entirely abandoned because of their increased losses and their unadaptability to the operation of motors and similar apparatus.

In the Report of the Committee on Standardization of the American Institute of Electrical Engineers is the following: "In alternating-current circuits, the following frequencies are standard:

$$\left.\begin{matrix} 25 \sim \\ 60 \sim \end{matrix}\right\}$$

"These frequencies are already in extensive use, and it is deemed advisable to adhere to them as closely as possible."

The frequency of an alternating current is always that of the *E.M.F.* producing it. To find the frequency of the pressure or the current produced by any alternating-current generator, if V be the number of revolutions per minute, and p be the number of pairs of poles, then

$$f = p\frac{V}{60}.$$

3. **Wave-shape.** — If, in an alternating current, the instantaneous values of current be taken as ordinates, and time be the abscissæ, a curve, as in Fig. 1, may be developed. The length of the abscissa for one complete cycle is $\frac{1}{f}$ seconds.

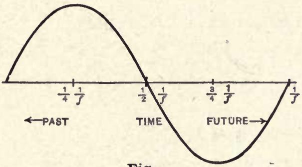


Fig. 1.

Imagine a small cylinder, Fig. 2, carried on one end of a wire, and rotated uniformly about the other end in a vertical plane. Imagine a horizontal beam of parallel rays of light to be parallel to the plane of rotation, and to cast a shadow of the cylinder on

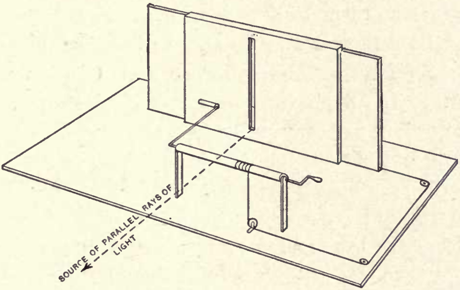


Fig. 2.

a plane screen perpendicular to the rays. The shadow will move up and down, passing from the top of its travel to the bottom in a half revolution, and from the bottom back to the top in another half revolution with a perfect harmonic motion. Now imagine the screen to be moved horizontally in its own plane with a uniform motion, and the positions of the shadow suitably recorded on it, — as on sensitized paper or on a photographic film, a slotted screen protecting all but the desired portion from exposure. Then the trace of the shadow will be as in Fig. 3. The abscissæ of this curve may be taken as time, as in the preceding curve, the abscissa of one complete cycle being the time in seconds of one revolution. Or, with equal relevancy, the abscissæ may be expressed in degrees. Consider the cylinder to be in a zero position when the radius to which it is attached is horizontal. Then the abscissa of any point is the angle which must be turned through in order that the cylinder may cast its shadow at that point. In this case the abscissa of a complete cycle will be 360°, or 2π. Consideration of the manner in which the curve has been formed shows that the ordinate of any point is proportional to the sine of the abscissa of that point, expressed in degrees. Hence this is called a *sinusoid* or *sine curve.*

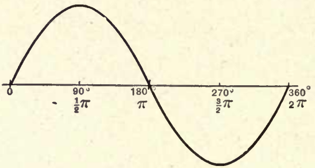


Fig. 3.

If the maximum ordinate of this curve, which corresponds to the length of the moving radius, or OA in Fig. 4, represents E_m, then the instantaneous value of the voltage, E', at t seconds after the beginning of any cycle, will be AB, or $E_m \sin \theta$. But, since OA traverses 2π radians during one complete revolution, it will sweep over $2\pi f$ radians per second, and, as angular velocity, represented by ω, is the angle turned through in unit time, it follows that the angular velocity of OA is $2\pi f$. The angular velocity may also be expressed as θ/t, or $\theta = \omega t = 2\pi ft$.

Hence
$$E' = E_m \sin \omega t$$
$$= E_m \sin 2\pi ft,$$

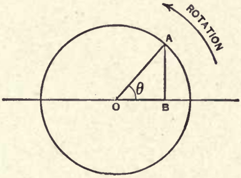


Fig. 4.

which is equivalent to neglecting all those intervals of time corresponding to whole cycles, and considering only the time elapsed since the end of the last completed cycle. In Fig. 4, OA is termed the radius vector, and θ, the vectorial angle or displacement. Graphic solutions of alternating-current problems may be effected by the use of vectors.

4. **Distortion.** — The ideal pressure curve from an alternator is sinusoidal. Commercial alternators, however, do not generate true sinusoidal pressures. But the sine curve can be treated with relative simplicity, and the curves of practice approximate so closely to the sine form, that mathematical deductions based on sine curves can with propriety be applied to those of practice. Two of these actual curves are shown in Fig. 5.

The shape of the pressure curve is affected by irregular distribution of the magnetic flux. Also uneven angular velocity of the generator will distort the wave-shape, making it, relative to the true curve, lower in the slow spots and higher in the fast ones. Again, the magnetic reluctance of the armature may vary in different angular positions, particularly if the inductors are laid in a few large slots. This would cause a periodic variation in the reluctance of the whole magnetic circuit and a corresponding pulsation of the total magnetic flux. All these influences operate at open circuit as well as under load.

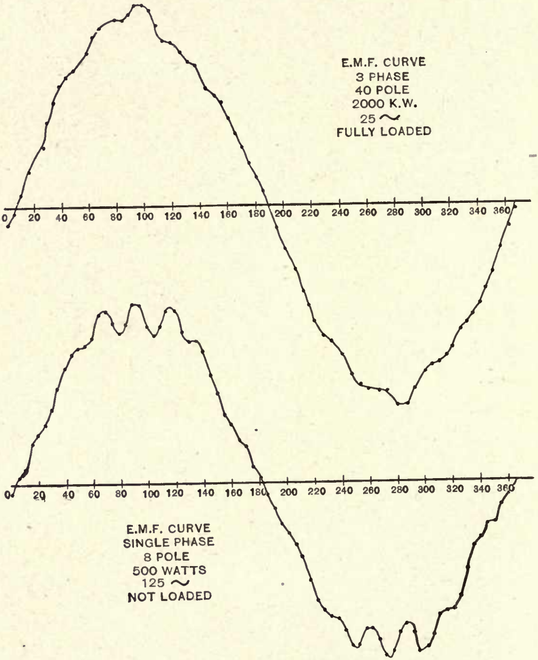


Fig. 5.

There are two other causes which act to distort the wave-shape only when under load. For any separately excited generator, a change in the resistance or apparent resistance of the external circuit will cause a change in the terminal voltage of the machine. As is explained later, the apparent resistance (impedance) of a circuit to alternating currents depends upon the permeability of the iron adjacent to the circuit. Permeability changes with magnetization. Now, because an alternating current is flowing, the magnetization changes with the changing values of current. This, by varying the permeability, sets up a pulsation in the impedance and affects the terminal voltage of the machine, periodically distorting the wave of pressure from the true sine.

There are cases of synchronously pulsating resistances. The most common is that of the alternating arc. With the same arc the apparent resistance of the arc varies inversely as the current. So when operated by alternating currents, the resistance of a circuit of arc lamps varies synchronously, and distorts the pressure wave-shape in a manner analogous to the above.

Summing up, the wave-shape of pressure may be distorted: *At open circuit as well as under load;* by lack of uniformity of magnetic distribution, by pulsating of magnetic field, by variation in angular velocity of armature; and *under load only;* by pulsation of impedance, by pulsation of resistance. And the effects of any or all may be superimposed.

5. Effective Values of E.M.F. and of Current. — One ampere of alternating current is a current of such instantaneous values as to have the same heating effect in a conductor as one ampere of direct current. This somewhat arbitrary definition probably arose from the fact that alternating currents were first commercially employed in lighting circuits, where their utility was measured by the heat they produced in the filaments; and further from the fact that the only means then at hand of measuring alternating currents were the hot-wire instruments and the electro-dynamometer, either of which gives the same indication for an ampere of direct current or for what is now called an ampere of alternating current.

The heat produced in a conductor carrying a current is proportional to the square of the current. In an alternating current, whose instantaneous current values vary, the instantaneous rate of heating is not proportional to the instantaneous value, nor yet to the square of the average of the current values, but to the square of the instantaneous current value. And so the average heating effect is proportional to the mean of the squares of the instantaneous currents.

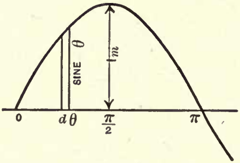


Fig. 6.

The *average* current of a sinusoidal wave of alternating current, whose maximum value is I_m, is equal to the area of one lobe of the curve, Fig. 6, divided by its base line π. Thus

$$I_{av} = \frac{\int_0^\pi I_m \sin\theta d\theta}{\pi} = \frac{I_m}{\pi}[-\cos\theta]_0^\pi = \frac{2}{\pi} I_m.$$

But the heating value of such a current varies, as

$$I^2 = \frac{\int_0^\pi I_m^2 \sin^2\theta d\theta}{\pi} = \frac{I_m^2}{\pi}\left[\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right]_0^\pi = \frac{1}{2} I_m^2.$$

The square root of this quantity is called the *effective* value of the current, $I = \frac{I_m}{\sqrt{2}}$. This has the same heating effect as a direct current I, and the effective values are always referred to unless expressly stated otherwise. Alternating-current ammeters are designed to read in effective amperes.

Since current is dependent upon the pressure, the resistance or apparent resistance of a circuit remaining constant, it is obvious that if $I = \frac{I_m}{\sqrt{2}}$ then does also $E = \frac{E_m}{\sqrt{2}}$. Likewise if average $I = \frac{2}{\pi} I_m$ then does also average $E = \frac{2}{\pi} E_m$. Or these may be demonstrated in a manner analogous to the above.

The maximum value of pressure is frequently referred to in designing alternator armatures, and in calculating dielectric strength of insulation. There have arisen various ways of indicating that *effective* values are meant, for instance, the expressions, sq. root of mean sq., $\sqrt{\bar{e}^2}$, $\sqrt{\text{mean square}}$. In England the initials R.M.S. are frequently used for root mean square.

The ratio $\frac{\text{Effective } E.M.F.}{\text{Average } E.M.F.}$ is called the *form-factor*, since its value depends upon the shape of the pressure wave. For the curve Fig. 7, the form-factor is unity. As a curve becomes more peaked, its form-factor increases, due to the superior weight of the squares of the longer ordinates.

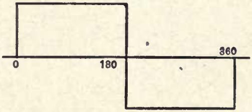


Fig. 7.

In the sinusoid the values found above give

$$\text{Form-factor} = \frac{\frac{1}{\sqrt{2}} E_m}{\frac{2}{\pi} E_m} = 1.11.$$

6. Form Factor of Non-Sine Curves. — For the determination of the form factor, three methods may be used, according to the character of the wave shape. First, if the equation of the curve is known, the analytical method may be employed. For example, take the ellipse, Fig. 8. Its equation is $y = \frac{b}{a}\sqrt{2\,ax - x^2}$.

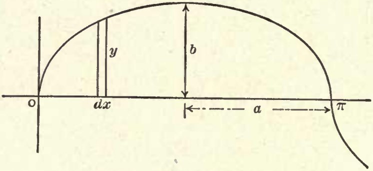


Fig. 8.

The average ordinate is

$$\frac{\int_0^{2a} y\,dx}{\pi} = \frac{\frac{b}{a}\int_0^{2a}\sqrt{2\,ax - x^2}\,dx}{\pi}$$

$$= \frac{\frac{b}{a}\left[\frac{x-a}{2}\sqrt{2\,ax - x^2} + \frac{a^2}{2}\operatorname{vers}^{-1}\frac{x}{a}\right]_0^{2a}}{\pi} = \frac{\frac{b}{a}\left(\frac{a^2}{2}\pi\right)}{\pi} = \frac{ab}{2}$$

and since $a = \frac{\pi}{2}$ this becomes $\frac{\pi b}{4}$. The square of the mean ordinate is

$$\frac{\int_0^{2a} y^2\,dx}{\pi} = \frac{\frac{b^2}{a^2}\int_0^{2a}(2\,ax - x^2)\,dx}{\pi} = \frac{\frac{b^2}{a^2}\left[ax^2 - \frac{x^3}{3}\right]_0^{2a}}{\pi}$$

$$= \frac{\frac{b^2}{a^2}\left(4\,a^3 - \frac{8\,a^3}{3}\right)}{\pi} = \frac{4\,ab^2}{3\,\pi},$$

but $a = \frac{\pi}{2}$, hence this becomes $\frac{2\,b^2}{3}$ and it follows that the effective value is $\sqrt{\frac{2}{3}}\,b$.

Therefore the form factor $= \dfrac{\sqrt{\frac{2}{3}}\; b}{\dfrac{\pi b}{4}} = \dfrac{4\sqrt{2}}{\pi\sqrt{3}} = 1.04.$

Second, the geometrical method may be used in calculating the form factor of simple wave shapes, as for example, Fig. 9.

The average ordinate $= \dfrac{\text{area of Fig. 9}}{\text{base}} = \dfrac{b\,[a + 2\,a]}{4\,a} = \frac{3}{4}\,b.$

The effective value $= \sqrt{\dfrac{\text{volume of Fig. 10}}{\text{base line}}}.$

The volume of Fig. 10 is

$$2\,ab^2 + 2\,.\,\tfrac{1}{3}\,ab^2 = b^2\,(2\;a + \tfrac{2}{3}\,a) = \tfrac{8}{3}\,ab^2.$$

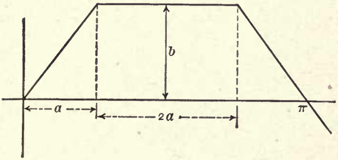


Fig. 9.

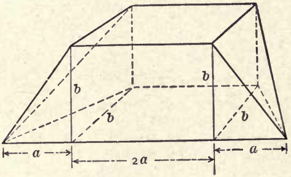


Fig. 10.

Hence the effective value is

$\sqrt{\dfrac{\frac{8}{3}\,ab^2}{4\,a}} = b\,\sqrt{\tfrac{2}{3}}$ and the form factor is $\dfrac{b\,\sqrt{\frac{2}{3}}}{\frac{3}{4}\,b} = \dfrac{4\sqrt{2}}{3\sqrt{3}} = 1.09.$

And third, the form factor of irregular curves, as for example the lower *E.M.F.* curve of Fig. 5, may be determined by the use of a planimeter. The average value $= \dfrac{\text{area of Fig. 5}}{\text{base}} = .60\,E_m.$ To obtain the effective value, a curve of squared ordinates must be plotted. The area of this curve divided by its base is the mean ordinate and the square root of this mean square is the effective value of the voltage, which for the curve in question is $.685\,E_m$. Hence the form factor $= \dfrac{.685}{.60} = 1.14.$

Probably no alternators give sine waves, but they approach it so nearly that the value 1.11 can be used in most calculations without sensible error.

7. **Phase.** — The curves of the pressure and the current in a circuit can be plotted together, with their respective ordinates and common abscissæ, as in Fig. 11. In some cases the zero and the maximum values of the current curve will occur at the same abscissæ as do those values of the pressure curve, as in Fig. 11. In such a case the current is said to be *in phase* with the pressure. In other cases the current will reach a maximum or a zero value at a time later than the corresponding values of the pressure, and since the abscissæ are indifferently time or degrees, the condition is represented in Fig. 12. In such a case, the current is said to be *out of phase* with, and to *lag* behind the pressure. In still other cases the curves are placed as in Fig. 13, and the current and pressure are again *out of phase*, but the current is said to *lead* the pressure. The distance between the zero ordinate of one sine curve and the corresponding zero ordinate of another, may be measured in degrees, and is called the *angular displacement* or *phase difference*. This *angle* of *lag* or of *lead* is usually represented by ϕ. When one

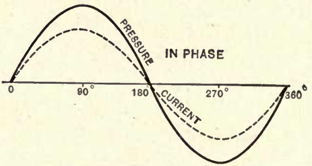


Fig. 11.

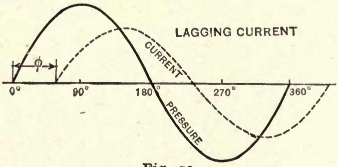


Fig. 12.

curve has its zero ordinate coincident with the maximum ordinate of the other, as in Fig. 14, there is a displacement of a quarter cycle ($\phi = 90°$), and the curves are said to be at right angles. This term owes its origin to the fact that the radii whose projections will trace these curves, as in § 3, are at right angles to each other.

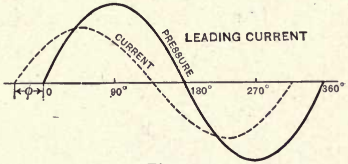


Fig. 13.

If the zero ordinates of the two curves coincide, but the positive maximum of one coincides with the negative maximum of the other, as in Fig. 15, then $\phi = 180°$, and the curves are in *opposite phase*.

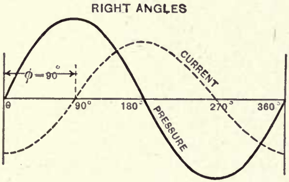


Fig. 14.

An alternator arranged to give a single pressure wave to a two-wire circuit is said to be a *single phaser*, and the current in the circuit a *single-phase current*. Some machines are arranged to give pressure to two distinct circuits — each of which, considered alone, is a single-phase circuit — but the time of maximum pressure in one is the time of zero pressure in the other, so that simultaneous pressure curves from the two circuits take the form of Fig. 16. Such is said to be a *two-phase* or *quarter-phase*

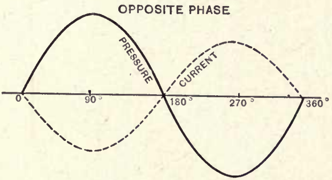
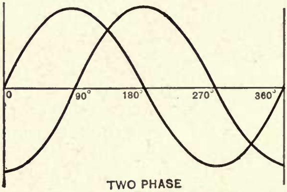


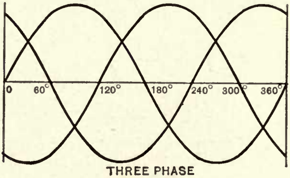
Fig. 15.

system, and the generator is a *two-phaser.* A *three-phase* system theoretically has three circuits of two wires each. The maximum positive pressure on any circuit is displaced from that of either of the other circuits by 120°. As the algebraic sum of the currents in all these circuits (if balanced) is at every instant equal to zero, the three return wires, one on each circuit, may be dispensed with, leaving but three wires. The three simultaneous curves of *E.M.F.* are shown in Fig. 17. The term *polyphase* applies to any system of two or more phases. An *n*-phase system has *n* circuits and *n* pressures with successive phase differences of $\frac{360}{n}$ degrees.



TWO PHASE
Fig. 16.

8. Power in Alternating-Current Circuits. — With a direct-current circuit, the power in the circuit is equal to the product of the pressure in volts by the current strength in amperes. In an alternating-current circuit, the *instantaneous* power is the product of the instantaneous values of current strength and pressure. If the current and pressure are out of phase there will be some instants when the pressure will have a positive value and the current a negative value or *vice versa.* At such times the instantaneous power will be a negative quantity, i.e.,



THREE PHASE
Fig. 17.

power is being returned to the generator by the disappearing magnetic field which had been previously produced by the current. This condition is shown in Fig. 18, where the power curve has for its ordinates the product of the corresponding ordinates of pressure and current. These are reduced by multiplying by a constant so as to make them of convenient size. The circuit, therefore, receives power from the generator and gives power back again in alternating pulsations having twice the frequency of the generator. It is clear that the relative magnitudes of the negative and positive lobes of the power curve will vary for different values of ϕ, even though the original curves maintain the same size and shape. So it follows that the power in an alternating-current circuit is not merely a function of E and I, as in direct-current circuits, but is a function of E, I, and ϕ, and the relation is deduced as follows: —

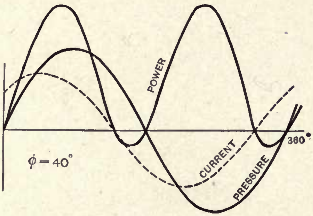


Fig. 18.

Let the accent (′) denote instantaneous values. If the current lag by the angle ϕ, then from § 3,

$$E' = E_m \sin \alpha,$$

where, for convenience,

$$\alpha = 2\,\pi f t,$$

and

$$I' = I_m \sin (\alpha - \phi).$$

Remembering that

$E = \dfrac{E_m}{\sqrt{2}}$, and $I = \dfrac{I_m}{\sqrt{2}}$ (§ 5) the instantaneous power,

$$P' = E'I' = 2EI \sin \alpha \sin (\alpha - \phi).$$

But $\sin(a - \phi) = \sin a \cos \phi - \cos a \sin \phi,$

so $P' = 2\,EI(\sin^2 a \cos \phi - \sin a \cos a \sin \phi).$

Remembering that ϕ is a constant, the average power over 180°,

$$P = \frac{2\,EI\cos\phi}{\pi}\int_0^\pi \sin^2 a\,da - \frac{2EI\sin\phi}{\pi}\int_0^\pi \sin a \cos a\,da$$

$$= \frac{2\,EI\cos\phi}{\pi}\left[\frac{1}{2}a - \frac{1}{4}\sin 2a\right]_0^\pi - \frac{2\,EI\sin\phi}{\pi}\left[\frac{1}{2}\sin^2 a\right]_0^\pi.$$

$$P = EI\cos\phi.$$

Should the current *lead* the pressure by $\phi°$, then the leading equation would be

$$P' = 2\,EI\sin a \sin(a + \phi),$$

which gives the same expression,

$$P = EI\cos\phi,$$

which is the general expression for power in an alternating-current circuit.

The above may also be shown by the use of vectors. Let *OA* and *OB* of Fig. 19 represent the effective values of *E.M.F.* and current respectively, taking the former as the datum line and assuming the latter to lag ϕ degrees behind the *E.M.F.* The line, *OB*, may be resolved into two components, one along *OA* and the other at right angles to it. These components, *OP* and *OQ*, are termed respectively the power and wattless components of the current. The actual power expended in the circuit is $OA \times OP = EI\cos\phi$

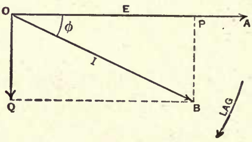


Fig. 19.

and the wattless power, or that alternately supplied to and received from the circuit, is $OA \times OQ = EI \sin \phi$.

Since, to get the true power in the circuit, the apparent power, or volt-amperes, must be multiplied by $\cos \phi$, this quantity is called the *power factor* of the circuit. If the pressure and current are in phase, $\phi = 0°$, and the power factor is unity.

It is important, at this point, to consider the graphical method of addition or subtraction of vector quantities, a process which is frequently employed in the treatment of alternating-current circuits. Let A and B, Fig. 20, be two lines whose lengths and whose directions respectively represent the magnitudes and time or space locations of two vector

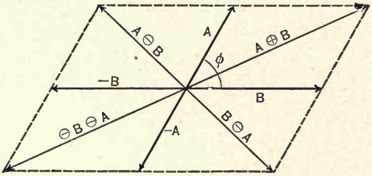


Fig. 20.

quantities. These may be $E.M.F.$'s, or currents as the case may be. The sum of two vectors is given in magnitude and in direction by the concurrent diagonal of a parallelogram the adjacent sides of which represent the vectors in size and direction. To substract one quantity from another vectorially, it is but necessary to change its sign and add it to the other. Representing vectorial addition by the symbol ⊕, and vectorial subtraction by ⊖, the results of the various additions and subtractions of A and B become evident from the figure.

9. Non-Sine Waves. — As sine waves of *E.M.F.* or current are seldom obtained in practice, it is convenient, in accurate calculations, to refer to their equivalent sine waves. An equivalent sine wave is one having the same frequency and the same mean effective value as the given wave. Consider two non-sine waves, one of *E.M.F.* and the other of current, their zero or maximum values being displaced by an angle ϕ_n. The phase difference of these two non-sine waves cannot be considered as the angle ϕ_n, but is that phase displacement of their equivalent sine waves which would give the same average of the instantaneous power values as the

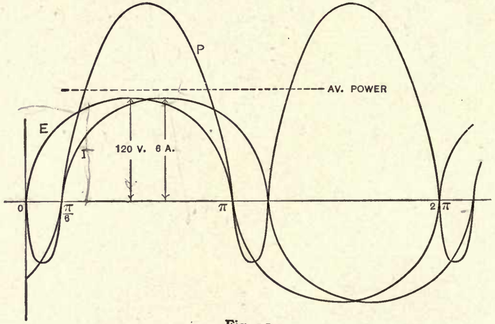


Fig. 21.

non-sine waves. Therefore, to find the phase displacement of two non-sine waves, it is necessary, first, to plot a power curve and determine its average ordinate, P_{av}; second, to calculate the mean effective values of the curves, represented respectively by E and I; and third, to substitute these values in the equation $P_{av} = EI \cos \phi$ from which ϕ can be obtained.

In general, it can be said, that when the form factors of both waves are less than that of the sine curve, their phase difference is greater than the displacement of their zero or maximum values; and likewise, if their form factors exceed the value 1.11, then the phase displacement is less than the displacement of corresponding values of the curves.

As a numerical example: Find the phase displacement of two semi-circular waves, having their zero values one-twelfth of a cycle or 30° apart. Let one be a pressure curve, whose maximum value is 120 volts, and the other, a current curve, whose maximum value is 6 amperes. These are shown in Fig. 21.

The average ordinate of the power curve, determined by subtracting the negative area from the positive and then dividing by the base, is found to be 383 watts. The effective value of each curve is $\sqrt{\frac{\int_0^\pi y^2\,dx}{\pi}}$, y^2 for the circle being $2\,bx - x^2$ where b is the maximum ordinate.

$$\frac{2\,b\int_0^\pi x\,dx - \int_0^\pi x^2\,dx}{\pi} = \frac{b\pi^2 - \frac{\pi^3}{3}}{\pi} = b\pi - \frac{\pi^2}{3}.$$

Since $b = \frac{\pi}{2}$, this becomes $\frac{\pi^2}{6}$, and hence the effective value is $\frac{\pi}{\sqrt{6}}$. From the relations

$$\frac{\pi}{2} : \frac{\pi}{\sqrt{6}} = 6 : I \quad \text{and} \quad \frac{\pi}{2} : \frac{\pi}{\sqrt{6}} = 120 : E,$$

it follows that the effective values of current and voltage are respectively 4.9 amperes and 98 volts. Then $383 = 98 \times 4.9 \cos\phi$, which gives as the phase displacement, $\phi = 37°$, instead of 30°.

10. E.M.F.'s in Series. — Alternating *E.M.F.*'s that may be put in series may differ in magnitude, in frequency, in phase relation, and in form or shape of wave.

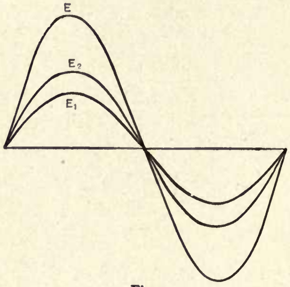


Fig. 22.

If two harmonic *E.M.F.*'s of the same frequency and phase be in series, the resulting *E.M.F.* is merely the sum of the separate *E.M.F.*'s. This condition is shown in Fig. 22, in which the two *E.M.F.*'s are plotted together, and the resulting *E.M.F.* plotted by making its instantaneous values equal to the sum of the corresponding instantaneous values of the component *E.M.F.*'s. The maximum of the resultant *E.M.F.* is evidently

$$E_m = E_{1m} + E_{2m},$$

and since $$E = \frac{E_m}{\sqrt{2}}, \quad E_1 = \frac{E_{1m}}{\sqrt{2}},$$

and $$E_2 = \frac{E_{2m}}{\sqrt{2}}, \quad E = E_1 + E_2,$$

as was stated.

If two *E.M.F.*'s of the same frequency, but exactly opposite in phase, be placed in series, it may be similarly shown that the resultant *E.M.F.* is the numerical difference of the component *E.M.F.*'s. This case may occur in the operation of motors.

The most general case that occurs is that of a number of alternating *E.M.F.*'s of the same frequency, but of different magnitudes and phase displacements. The changes in magnitude and phase and the phase relation of the resulting curve of *E.M.F.* are shown in Fig. 23, where recourse is had once again to the harmonic shadowgraph. But two components, E_1 and E_2, are treated, whose phase displacement is ϕ_1. The radii vectors E_{1m} and E_{2m} are laid off from o with the proper angle ϕ_1 between them, and the shadows traced by their extremities are shown in the dotted curves. The instantaneous value of the resultant *E.M.F.* is the algebraic sum of the corresponding in-

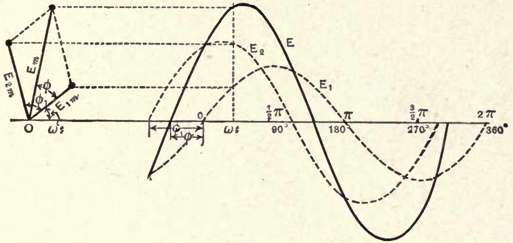


Fig. 23.

stantaneous values of the component *E.M.F.*'s, and the resultant curve of *E.M.F.* is traced in the figure by the solid line. But this solid curve is also the trace of the extremity of the line E_m, which is the vector sum (the resultant of the force polygon) of the component pressures, E_{1m} and E_{2m}. This is evident from the fact that any instantaneous value of the resultant pressure curve is the sum of the corresponding instantaneous values of the component curves, or (§ 3)

$$E' = E_{1m} \sin \omega t + E_{2m} \sin (\omega t + \phi_1).$$

Again from the force polygon

$$E_m \sin (\omega t + \phi) = E_{1m} \sin \omega t + E_{2m} \sin (\omega t + \phi_1).$$

Hence at any instant

$$E' = E_m \sin(\omega t + \phi),$$

wherefore the extremity of the line E_m traces the curve of resultant pressure, ϕ being its angular displacement from E_1. If a third component *E.M.F.* is to be added in series, it may be combined with the resultant of the first two in an exactly similar manner.

So it may be stated as a general proposition, that if any number of harmonic *E.M.F.*'s, of the same frequency, but of various magnitudes and phase displacements, be connected in series, the resulting harmonic *E.M.F.* will be given in magnitude and phase by the vector sum of the component *E.M.F.*'s. The analytic expressions for E and ϕ may be derived by inspection of the diagram, and are

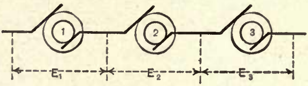


Fig. 24.

$$E = \sqrt{[E_1 \sin\phi_1 + E_2 \sin\phi_2 + \cdots]^2 + [E_1 \cos\phi_1 + E_2 \cos\phi_2 + \cdots]^2},$$

and

$$\tan\phi = \frac{E_1 \sin\phi_1 + E_2 \sin\phi_2 + \cdots}{E_1 \cos\phi_1 + E_2 \cos\phi_2 + \cdots}.$$

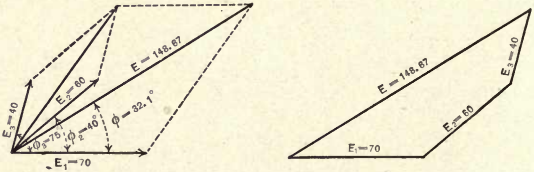


Fig. 25.

As a numerical example, suppose three alternators, Fig. 24, to be connected in series. Suppose these to give sine waves of pressure of values $E_1 = 70$, $E_2 = 60$, and $E_3 = 40$

volts respectively. Considering the phase of E_1 to be the datum phase, let the phase displacements be $\phi_1 = 0°$, $\phi_2 = 40°$, and $\phi_3 = 75°$, respectively. It is required to find E and ϕ. Completing the parallelograms or completing the force polygon as shown in Fig. 25, it is found that $E = 148.7$ volts and $\phi = 32.1°$.

Alternating *E.M.F.*'s of different frequencies in series will give, in general, an irregular wave form. In practice, the frequencies of some *E.M.F.*'s are multiples of the frequency of another, called the fundamental *E.M.F.*, or first harmonic. The pressure curve having twice this frequency is termed the second harmonic; another having three times this frequency, the third harmonic, and so on. The resultant instantaneous *E.M.F.* is obtained by adding the pressure

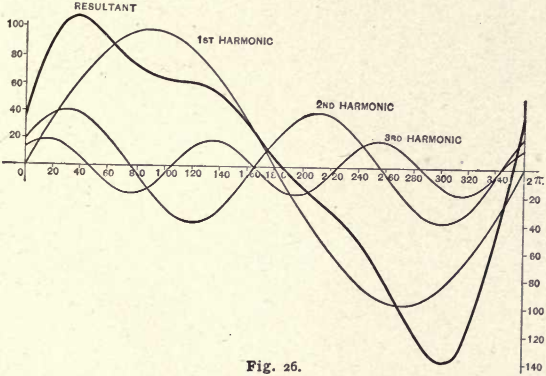


Fig. 26.

values of all the components at that instant. It is expressed as

$$E' = E_{1m} \sin \omega t + E_{2m} \sin (2\omega t + \phi_1) + E_{3m} \sin (3\omega t + \phi_2) + \dots + E_{nm} \sin (n\omega t + \phi_{n-1})$$

where $\phi_1, \phi_2, \ldots, \phi_{n-1}$, are the phase differences between E_{1m} and E_{2m}, E_{1m} and $E_{3m}, \ldots, E_{1m}$ and E_{nm} respectively when $\sin \omega t = 0$.

When both odd and even harmonics are present, the resulting curve will have unlike lobes, but when only odd harmonics occur, as is usual in electrical machinery, the lobe above the horizontal axis and the other below it will be similar. Fig. 26 shows the resulting *E.M.F.* of three harmonic components for the values, $E_{1m} = 100$ volts, $E_{2m} = 40$ volts, $E_{3m} = 20$ volts, $\phi_1 = 30°$ and $\phi_2 = 45°$.

Let it be required to find E', $2\frac{1}{3}$ seconds after the beginning of a cycle, the frequency of E_1 being 25 ~.

$$\omega t = 2\pi\, 25 \cdot 2\tfrac{1}{3} = 116\tfrac{2}{3}\,\pi \text{ or } \frac{2\pi}{3}.$$

$$2\,\omega t = \frac{4\pi}{3} \text{ and } (2\,\omega t + \phi_1) = \frac{8\pi}{6} + \frac{\pi}{6} = \frac{3\pi}{2};$$

$$3\,\omega t = 3 \cdot \frac{2\pi}{3} = 2\pi \text{ or } 0, \text{ then } (3\,\omega t + \phi_2) = \frac{\pi}{4}.$$

$$\text{Hence}\quad E' = 100 \sin\frac{2\pi}{3} + 40 \sin\frac{3\pi}{2} + 20 \sin\frac{\pi}{4}$$
$$= 86.6 - 40 + 14.1 = 60.7$$

volts, which agrees with the value of the ordinate at 120°, in the figure.

When the resulting pressure curve is given, it is possible, by graphical and analytical methods, to determine which harmonics are present, their maximum values, and phase displacements.

PROBLEMS.

1. What must be the speed of a 12-pole alternator to yield an *E.M.F.* of 60 cycles?

2. Find the instantaneous current value in a circuit, in which a 25 ∼ alternating current of 70.7 amperes flows, 6.0066 seconds after the completion of a cycle.

3. How many amperes flow in a circuit, when the instantaneous value of the current is 5 amperes, 30° after the beginning of a cycle?

4. What is the frequency of an *E.M.F.* which assumes its effective value every .01 second?

5. Find the average and effective values of a semi-circular wave shape, the maximum value being $\frac{\pi}{2}$. Determine form factor.

6. Find the form factor of a triangular wave shape.

7. Determine the form factor of the upper curve of Fig. 5.

8. Find the instantaneous voltage produced by a 50 ∼ alternator, generating parabolic waves of 120 volts maximum value, $1\frac{2}{3}$ seconds after the beginning of a cycle.

9. What is the phase displacement between E and I, respectively of 100 volts and 10 amperes maximum value, when the power in the circuit is 424 watts?

10. Find the phase difference of two non-sine waves of voltage and current, whose zero values are 45° apart. Let the wave shape be as shown in Fig. 9 and let $E_m = 100$ volts and $I_m = 10$ amperes.

11. Four 60 ∼ alternators, generating respectively 100, 80, 90 and 50 volts, are connected to a circuit. What will be the value of the resulting pressure, and what will be its phase with respect to that of the 100 volts, if the phase difference between successive components is 45°?

12. If the three *E.M.F.*'s, shown in Fig. 26, are impressed upon a circuit, what will be the resulting instantaneous voltage 5.015 seconds after the beginning of a cycle?

CHAPTER II.

SELF-INDUCTION.

11. Self-Inductance. — The subject of inductance was briefly treated of in § 15, vol. i., of this work; but, since it is an essential part of alternating-current phenomena, it will be discussed more fully in this chapter. When lines of force are cut by a conductor an *E.M.F.* is generated in that conductor (§ 13, vol. i.). A conductor carrying current is encircled by lines of force. When the current is first started in such a conductor, these lines of force must be established. In establishing itself, each line is considered as having cut the conductor, or, what is equivalent, been cut by the conductor. This notion of lines of force is a convenient fiction, designed to render an understanding of the subject more easy. To account for the *E.M.F.* of self-induction, the encircling lines must be considered as cutting the conductor which carries the current that establishes them, during their establishment. It may be considered that they start from the axis of the conductor at the moment of starting the current in the circuit; that they grow in diameter while the current is increasing; that they shrink in diameter when the current is decreasing; and that all their diameters reduce to zero upon stopping the current. At any given current strength the conductor is surrounded by many circular lines, the circles having various diameters. Upon decreasing the strength those of smaller diameter cut the conductor and disappear into a point on the axis of the conductor previous to the cutting by those of larger diameter. The number of lines accompanying a large current is greater than the number accompanying a smaller current.

The *E.M.F.* of self-induction is always a *counter E.M.F.* By this is meant that its direction is such as to tend to prevent the change of current which causes it. When the current is started the self-induced pressure tends to oppose the flow of the current and prevents its reaching its full value immediately. When the circuit is interrupted the *E.M.F.* of self-induction tends to keep the current flowing in the same direction that it had originally.

12. Unit of Self-Inductance. — The *self-inductance*, or the *coefficient of self-induction* of a circuit generally represented by L or l, is that constant by which the time rate of change of the current in a circuit must be multiplied in order to give the *E.M.F.* induced in that circuit. Its absolute value is numerically equal to the number of lines of force linked with the circuit, per absolute unit of current in the circuit, as is shown below. By linkages, or number of lines linked with a circuit, is meant the sum of the number of lines surrounding each portion of the circuit. For instance, a coil of wire consisting of ten turns, and threaded completely through by twelve lines of force, is said to have 120 linkages.

The absolute unit of self-inductance is too small for ordinary purposes, and a practical unit, the *henry*, is used. This is 10^9 times as large as the C. G. S. or absolute unit.

The Paris electrical congress of 1900 adopted as the unit of magnetic flux the maxwell, and of flux density the gauss. A maxwell is one line of force. A gauss is one line of force per square centimeter. If a core of an electromagnet has a transverse cross-section of 30 sq. cm., and is uniformly permeated with 60,000 lines of force, such a core may be said to have a flux of 60,000 maxwells and a flux density of 2000 gausses.

In § 13, vol. I., it has been shown that the pressure generated in a coil of wire when it is cut by lines of force is

$$e = -\frac{n\,d\Phi}{dt},$$

where n is the number of turns in a coil, and where e is measured in C. G. S. units, Φ in maxwells, and t in seconds. In a simple case of self-induction the maxwells set up are due solely to the current in the conductor. Now let K be a constant, dependent upon the permeability of the magnetic circuit, such that it represents the number of maxwells set up per unit current in the electric circuit; then, indicating instantaneous values by prime accents,

$$\Phi' = Ki',$$

and $$d\Phi = Kdi.$$

The *E.M.F.* of self-induction may then be written

$$e_s = -Kn\frac{di}{dt}.$$

By the definition of the coefficient of self-induction, whose C. G. S. value is represented by l,

$$e_s = -l\frac{di}{dt}.$$

From the last two equations, it is seen that $l = Kn$. Kn is evidently the number of linkages per absolute unit current. The negative sign indicates that the pressure is counter *E.M.F.*

In practical units,

$$E_s = - L\frac{dI}{dt}.$$

A circuit having an inductance of one henry will have a pressure of one volt induced in it by a uniform change of current of one ampere per second.

13. Practical Values of Inductances. — To give the student an idea of the values of self-inductance met with in practice, a number of examples are here cited.

A pair of copper line wires, say a telephone pole line, will have from two to four milhenrys (.002 to .004 henrys) per mile, according to the distance between them, the larger value being for the greater distance.

The secondary of an induction coil giving a 2″ spark has a resistance of about 6000 ohms and 50 henrys.

The secondary of a much larger coil has 30,000 ohms and about 2000 henrys.

A telephone call bell with about 75 ohms has 1.5 henrys.

A coil found very useful in illustrative and quantitive experiments in the alternating-current laboratory is of the following dimensions. It is wound on a pasteboard cylinder with wooden ends, making a spool 8.5 inches long and 2 inches internal diameter. This is wound to a depth of 1.5 inch with No. 16 B. and S. double cotton-covered copper wire, there being about 3000 turns in all. A bundle of iron wires, 16 inches long, fits loosely in the hole of the spool. The resistance of the coil is 10 ohms, and its inductance without the core is 0.2 henry. With the iron core in place and a current of about 0.2 ampere, the inductance is about 1.75 henrys. This coil is referred to again in § 16.

The inductance of a spool on the field frame of a generator is numerically

$$L = \frac{\Phi n}{10^8 I_f},$$

where Φ is the total flux from one pole, n the number of turns per spool, and I_f the field current of the machine. It is evident that the value of L may vary through a wide range with different machines.

14. Things Which Infuence the Magnitude of L. — If all the conditions remain constant, save those under consideration, then the self-inductance of a coil will vary: directly as the square of the number of turns; directly as the linear dimension if the coil changes its size without changing its shape; and inversely as the reluctance of the magnetic circuit.

Any of the above relations is apparent from the following equations. The numerical value of the self-inductance is

$$l = n\frac{\Phi}{i}.$$

As shown in Chapter 2, vol. i.,

$$\Phi = \frac{M.M.F.}{\text{reluctance}} = \frac{4\pi ni}{\frac{c}{\mu A}},$$

where c is the mean length in centimeters of the magnetic circuit, A its mean cross-sectional area in square centimeters, and μ is permeability.

Then, if $\mathcal{R}$ stand for the reluctance,

$$l = \frac{n}{i}\cdot\frac{4\pi ni}{\frac{c}{\mu A}} = 4\pi n^2\mu\frac{A}{c} = \frac{4\pi n^2}{\mathcal{R}},$$

which is independent of i.

If, as is generally the case, there is iron in the magnetic circuit, it is practically impossible to keep μ constant if any of the conditions are altered ; and it is to be particularly noted, that with iron in the magnetic circuit, L is by no means independent of I.

15. Formulæ for Calculating Inductances. — *Circle:* For a cylindrical conductor of radius r cm. and length l cm., bent into a circle and surrounded with a medium of unit permeability the self-inductance in henries is

$$L = 10^{-9}\, 2\, l \left\{ \log_\epsilon \frac{l}{r} - 1.508 \right\}.$$

This is accurate to within 0.2 % when the radius of the circle is greater than ten times that of the cylindrical conductor.

Straight Wire: For a straight cylindrical conductor of radius r cm. and length l cm. in a medium of unit permeability the self-inductance in henries is

$$L = 10^{-9}\, 2\, l \left[\log_\epsilon \frac{2\, l}{r} - 0.75 \right].$$

Parallel Wires: For a return circuit of two parallel cylindrical wires of radius r cm., d cm. apart from center to center, each of permeability μ and of l cm. length, the self-inductance in henries is

$$L = 10^{-9}\, 4\, l \left[\log_\epsilon \frac{d}{r} + \frac{\mu}{4} \right].$$

Solenoids: The formula given below for the self-inductance of a solenoid of any number of layers will give results accurate to within one-half of a per cent even for short solenoids, where the length is only twice the diameter, the accuracy increasing as the length increases.

$$L = 4\,\pi^2 n^2 m \left\{\frac{2\,a_0^4 + a_0^2 l^2}{\sqrt{4\,a_0^2 + l^2}} - \frac{8\,a_0^3}{3\,\pi}\right\} + 8\,\pi^2 n^2 \times$$

$$\Big\{[(m-1)\,a_1^2 + (m-2)\,a_2^2 + \dots][\sqrt{a_1^2 + l^2} - \tfrac{7}{8}\,a_1]$$

$$+ \tfrac{1}{2}[m(m-1)a_1^2 + (m-1)(m-2)a_2^2 + (m-2)(m-3)a_3^2 + \dots]$$

$$\left[\frac{a_1\,\delta a}{\sqrt{a_1^2 + l^2}} - \delta a\right]\Big\}.$$

Where

m is the number of layers,

a_0 is the mean radius of the solenoid,

$a_1, a_2, a_3, \dots a_m$ are the mean radii of the various layers,

l is the length of the solenoid,

δa is the radial distance between two consecutive layers,

n is the number of turns per unit length.

A simple and convenient formula for the calculation of the self-inductance of a single-layer solenoid is as follows:

$$L = 4\,\pi^2 n^2 \left\{\frac{2\,a^4 + a^2 l^2}{\sqrt{4\,a^2 + l^2}} - \frac{8}{3}\,\frac{a^3}{\pi}\right\}$$

where a is the mean radius and l is the length of the solenoid.

The natural logarithms used in preceding formulæ can be obtained by multiplying the common logarithm of the number, the mantissa and characteristic being included, by 2.3026.

The inductance of all circuits is somewhat less for extremely high frequencies than for low ones.

16. Growth of Current in an Inductive Circuit. — If a constant *E.M.F.* be applied to the terminals of a circuit having both resistance and inductance, the current does not instantly assume its full ultimate value, but logarithmically increases to that value.

At the instant of closing the circuit there is no current flowing. Let time be reckoned from this instant. At any subsequent instant, t seconds later, the impressed *E.M.F.* may be considered as the sum of two parts, E_1 and E_r. The first, E_1, is that part which is opposed to, and just neutralizes, the *E.M.F.* of self-induction, so that $E_1 = -E_s$;

but
$$E_s = -L\frac{dI}{dt},$$

so
$$E_1 = L\frac{dI}{dt}.$$

The second part, E_r, is that which is necessary to send current through the resistance of the circuit, according to Ohm's Law, so that

$$E_r = RI.$$

If the impressed *E.M.F.*

$$E = E_r + E_1 = RI + L\frac{dI}{dt},$$

then
$$(E - RI)\,dt = LdI,$$

and
$$dt = \frac{L}{E - RI}\,dI = -\frac{L}{R}\cdot\frac{-RdI}{E - RI}.$$

Integrating from the initial conditions $t=0$, $I=0$ to any conditions $t=t$, $I=I'$,

$$t = -\frac{L}{R}[\log(E - RI') - \log E]$$

$$-\frac{Rt}{L} = \log\left(\frac{E - RI'}{E}\right),$$

and $$I' = \frac{E}{R} - \frac{E}{R}\epsilon^{-\frac{R}{L}t} = \frac{E}{R}\left(1 - \epsilon^{-\frac{R}{L}t}\right),$$

where ϵ is the base of the natural system of logarithms.

This equation shows that the rise of current in such a circuit is along a logarithmic curve, as stated, and that when t is of sufficient magnitude to render the term $\epsilon^{-\frac{R}{L}t}$ negligible the current will follow Ohm's Law, a condition that agrees with observed facts.

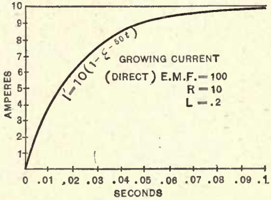


Fig. 27.

Fig. 27 shows the curve of growth of current in the coil referred to in §13. The curve is calculated by the above formula for the conditions noted.

The ratio $\frac{L}{R}$ is called the *time constant* of the circuit, for the greater this ratio is, the longer it takes the current to obtain its full ultimate value.

17. Decay of Current in an Inductive Circuit. — If a current be flowing in a circuit containing inductance and resistance, and the supply of *E.M.F.* be discontinued, without, however, interrupting the continuity of the circuit, the current will not cease instantly, but the *E.M.F.* of self-induction will keep it flowing for a time, with values decreasing according to a logarithmic law.

An expression for the value of this current at any time, t seconds after cutting off the source of impressed *E.M.F.*, may be obtained as in the preceding section. Let time be reckoned from the instant of interruption of the impressed *E.M.F.* The current at this instant may be represented by $\frac{E}{R}$, and is due solely to the *E.M.F.* of self-induction.

Therefore
$$E = RI + L\frac{dI}{dt} = 0,$$
or
$$RI = -L\frac{dI}{dt}.$$
$$\therefore dt = -\frac{L}{R}\frac{dI}{I}.$$

Integrating from the initial conditions $t = 0$, $I = \frac{E}{R}$, to the conditions, $t = t$, $I = I'$,

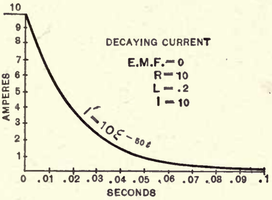


Fig. 28.

$$\int_0^t dt = -\frac{L}{R}\int_{\frac{E}{R}}^{I'}\frac{dI}{I}$$
$$t = -\frac{L}{R}\log\frac{I'}{\frac{E}{R}},$$
and
$$I' = \frac{E}{R}\epsilon^{-\frac{R}{L}t},$$

which is seen to be the term that had to be subtracted in the formula for growth of current. This shows clearly that while self-induction prevents the instantaneous attainment of the normal value of current, there is eventually no loss of energy, since what is subtracted from the growing current is given back to the decaying current.

Fig. 28 is the curve of decay of current in the same circuit as was considered in Fig. 27. The ordinates of the one figure are seen to be complementary to those of the other.

18. Magnetic Energy of a Started Current. — If a current I is flowing under the pressure of E volts, the power expenditure is EI watts, and the work performed in the interval of time dt is

$$dW = EI\,dt.$$

During the time required to establish a steady flow of current after closing the circuit, the impressed E may be considered as made up of two parts, one, E_r, required to send I' through the resistance of the circuit, and the other, E_s, which opposes the *E.M.F.* of self-induction. $E_rI\,dt$ appears as heat, while $E_sI\,dt$ is stored in the magnetic field. Since

$$E_s = -L\frac{dI}{dt},$$

$$dW = -LI\,dI.$$

Integrating through the full range, from 0 to W and from 0 to I,

$$\int_0^W dW = -L\int_0^I I\,dI.$$

$$\therefore W = -\tfrac{1}{2}LI^2,$$

which is an expression for the work done upon the magnetic field in starting the current. When the current is stopped the work is done by the field, and the energy is returned to the circuit.

The formula assumes the value of L to be constant during the rise and fall of the current, but this is not the case with an iron magnetic circuit. If L is taken as the average of the instantaneous values of self-inductance between the limiting values of the current, then the formula for the energy stored in the field still holds true.

Since iron has always a hysteretic loss, some of the energy is consumed, and the work given back at the disappearance of the field is less than that used to establish the field by the amount consumed in hysteresis.

19. Current Produced by a Harmonic E.M.F. in a Circuit Having Resistance and Inductance. — Given a circuit of resistance R and inductance L upon which is impressed a harmonic *E.M.F.* E of frequency f, to find the current I in that circuit.

Represent by ω the quantity $2\pi f$.

At any instant of time, t, let the instantaneous value of the current be I'.

To maintain this current requires an *E.M.F.* whose value at this instant is $I'R$. Represent this by E'_r.

From § 3, in a harmonic current,

$$I' = I_m \sin \omega t,$$

hence,

$$E_r' = RI_m \sin \omega t.$$

Evidently E_r' has its maximum value $RI_m = E_{rm}$ at $\omega t = 90°$ or $270°$, and its effective value is $E_r = RI$.

The counter *E.M.F.* of self-induction at the same instant of time, t, is

$$E_s' = -L \frac{dI'}{dt}.$$

But as before,

$$I' = I_m \sin \omega t,$$

so

$$dI' = \omega I_m \cos \omega t \, dt,$$

and

$$E_s' = -\omega L I_m \cos \omega t.$$

Evidently E_s' has a maximum value of $-\omega L I_m = E_{sm}$ at $\omega t = 0°$ or $180°$, and its effective value

$$E_s = -\omega L I.$$

It is clear that the impressed *E.M.F.* must be of such a value as to neutralize E_s and also supply E_r. But these two pressures cannot be simply added, since the maximum value of one occurs at the zero value of the other; that is, they are at right angles to each other, as defined in § 7. Reference to Fig. 29 will make it clear that combining these at right angles will give as a resultant the pressure $\sqrt{E_r^2 + E_s^2}$; and it is this pressure that the impressed *E.M.F.* E must equal and oppose. So

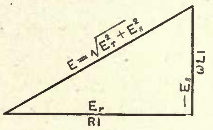


Fig. 29.

$$E = \sqrt{(IR)^2 + (\omega L I)^2},$$

from which

$$I = \frac{E}{\sqrt{R^2 + \omega^2 L^2}}.$$

This is a formula which must be used in place of Ohm's Law when treating inductive circuits carrying harmonic currents. It is evident that, if the inductance or the frequency be negligibly small (direct current has $f = 0$), the formula reduces to Ohm's Law; but for any sensible values of ω and L the current in the circuit will be less than that called for by Ohm's Law.

The expression $\sqrt{R^2 + \omega^2 L^2}$ is called the *impedance* of the circuit, and also the *apparent resistance.* The term R is of course called *resistance,* while the term ωL, which is $2\pi f L$, is called the *reactance.* Both are measured in ohms.

The effective value of the counter *E.M.F.* of self-induction can be determined as follows, without employing the calculus; that it must be combined at right angles with RI is not directly evident. Disregarding the direction of flow, an alternating current i reaches a maximum value i_m, $2f$ times per second. The maximum number of lines of force linked with the circuit on each of these occasions is li_m. The interval of time, from when the current is zero with no linkages, to when the current is a maximum with li_m linkages, is $\frac{1}{4f}$ second. The average rate of cutting lines, then, is $\frac{li_m}{\frac{1}{4f}}$, and is equal to the average $E.M.F.$ of self-induction during the interval. It has the same value during succeeding equal intervals; i.e.,

$$e_{s\,av} = -\frac{li_m}{\frac{1}{4f}} = -4fli_m.$$

The effective value is (§ 5) therefore,

$$e_s = -2\pi fli = \omega li,$$

and in practical units,

$$E_s = -2\pi fLI.$$

Since the squares of the quantities R, L, and ω enter into the expression for the impedance, if one, say R, is moderately small when compared with L or ω, its square will be negligibly small when compared with L^2 or ω^2. The frequency, because it is a part of ω, may be a considerable factor in determining the impedance of a circuit.

Having recourse once again to the harmonic shadowgraph described in § 3, the phase relation between impressed $E.M.F.$ and current may be made plain. It has already been shown that E_r and E_s are at right angles to each other. Since the pressure E_r is the part of the impressed *E.M.F.* which sends the current, the current must be in phase with it. Therefore there is always a phase displacement of 90° between I and E_s. This relation is also evident from a consideration of the fact that when I reaches its maximum value it has, for the instant, no rate of change; hence the flux, which is in phase with the current, is not changing, and consequently the *E.M.F.* of self-induction must be, for the instant, zero. That is, I is maximum when E_s is zero, which means a displacement of 90°.

In Fig. 30 the triangle of *E.M.F.*'s of Fig. 29 is altered

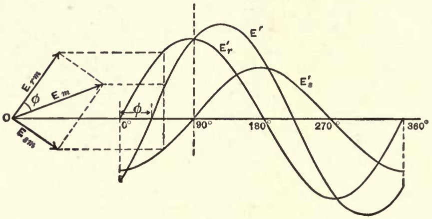


Fig. 30.

to the corresponding parallelogram of *E.M.F.*'s, and the maximum values substituted for the effective. If now the parallelogram revolve about the center o, the traces of the harmonic shadows of the extremities of E_m, E_{rm} and E_{sm} will develop as shown. It is evident that the curve E_r' — and so also the curve of current — leads the curve E' by the angle ϕ. It is clear that the magnitude of ϕ depends upon the relative values of L and R in the circuit, the exact relation being derived from the triangle of forces.

$$\tan \phi = \frac{E_s}{E_r} = \frac{\omega LI}{RI} = \frac{\omega L}{R} = \frac{2\pi fL}{R}.$$

Furthermore

$$\cos \phi = \frac{E_r}{E},$$

that is, the cosine of the angle of lag is equal to the ratio of the volts actually engaged in sending current to the volts impressed on the circuit, and this ratio is again equal to the power-factor as stated in § 8.

20. Instantaneous Current produced by a Harmonic E.M.F. in a Circuit having Resistance and Inductance. — The *E.M.F.* at any instant *t*, impressed upon a circuit containing resistance and inductance, must be of such magnitude as to send the instantaneous current I' through the resistance, and also to neutralize the *E.M.F.* of self-induction. That is

$$E' = RI' + L\frac{dI'}{dt}.$$

But $$E' = E_m \sin \omega t. \qquad \text{(Art. 3.)}$$

Hence $$E_m \sin \omega t = RI' + L\frac{dI'}{dt},$$

$$\frac{dI'}{dt} + \frac{R}{L}I' = \frac{E_m}{L}\sin \omega t.$$

Multiply by integrating factor $e^{\int \frac{R}{L} dt}$

$$\frac{dI'}{dt}\, e^{\int \frac{R}{L} dt} + \frac{R}{L} I' e^{\int \frac{R}{L} dt} = \frac{E_m}{L} \sin \omega t \,.\, e^{\int \frac{R}{L} dt}.$$

But $\int \frac{R}{L}\, dt = \frac{Rt}{L}$ and writing in differential form,

$$dI' e^{\frac{Rt}{L}} + I' e^{\frac{Rt}{L}} \frac{R}{L} dt = \frac{E_m}{L} \sin \omega t \,.\, e^{\frac{Rt}{L}} dt.$$

The second term is in the form $da^x = a^x \log_e a\, dx$.

Hence $$dI'e^{\frac{Rt}{L}} + I'd\left(e^{\frac{Rt}{L}}\right) = \frac{E_m}{L} \sin \omega t \,.\, e^{\frac{Rt}{L}}\, dt.$$

Since the first member is in the form $d(xy) = y\,dx + x\,dy$, it may be replaced by $d\left(I'e^{\frac{Rt}{L}}\right)$. Integrating,

$$I'e^{\frac{Rt}{L}} = \frac{E_m}{L}\int e^{\frac{Rt}{L}} \sin \omega t\, dt + C.$$

Hence $$I' = e^{-\frac{Rt}{L}}\frac{E_m}{L}\int e^{\frac{Rt}{L}} \sin \omega t\, dt + Ce^{-\frac{Rt}{L}}. \qquad (1)$$

To determine value of the integral, use the formula

$$\int u\, dv = uv - \int v\, du.$$

Let $u = \sin \omega t$, then $du = \omega \cos \omega t\, dt$, and let

$$dv = e^{\frac{Rt}{L}}\, dt = \frac{L}{R} e^{\frac{Rt}{L}} \frac{R}{L}\, dt.$$

Hence $$v = \frac{L}{R} e^{\frac{Rt}{L}}.$$

Then the integral becomes

$$\int e^{\frac{Rt}{L}} \sin \omega t\, dt = \frac{L}{R} e^{\frac{Rt}{L}} \sin \omega t - \frac{\omega L}{R}\int e^{\frac{Rt}{L}} \cos \omega t\, dt.$$

Use the same formula again for this second integral, but here $u = \cos \omega t$, hence $du = -\omega \sin \omega t\, dt$; and where

$$dv = e^{\frac{Rt}{L}}\, dt.$$

Hence $$v = \frac{L}{R} e^{\frac{Rt}{L}}.$$

Then $$\int e^{\frac{Rt}{L}} \sin \omega t \, dt = \frac{L}{R} e^{\frac{Rt}{L}} \sin \omega t - \frac{\omega L^2}{R^2} e^{\frac{Rt}{L}} \cos \omega t - \frac{\omega^2 L^2}{R^2} \int e^{\frac{Rt}{L}} \sin \omega t \, dt.$$

Or $$\left(1 + \frac{\omega^2 L^2}{R^2}\right) \int e^{\frac{Rt}{L}} \sin \omega t \, dt = \frac{L}{R} e^{\frac{Rt}{L}} \sin \omega t - \frac{\omega L^2}{R^2} e^{\frac{Rt}{L}} \cos \omega t.$$

Substitute value of integral in (1), then

$$I' = e^{-\frac{Rt}{L}} \frac{E_m}{L} \left[\frac{\frac{L}{R} e^{\frac{Rt}{L}} \sin \omega t - \frac{\omega L^2}{R^2} e^{\frac{Rt}{L}} \cos \omega t}{\frac{R^2 + \omega^2 L^2}{R^2}} \right] + C e^{-\frac{Rt}{L}},$$

$$I' = \frac{E_m}{L} \left[\frac{\frac{R}{L} \sin \omega - \omega \cos \omega t}{\frac{R^2}{L^2} + \omega^2} \right] + C e^{-\frac{Rt}{L}}. \qquad (2)$$

Let the angle ϕ be chosen so that $\tan \phi = \frac{\omega L}{R}$, thus representing the angle of lag of the current behind the *E.M.F.* Therefore

$$\cos \phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}},$$

and $$\sin \phi = \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}}.$$

Hence $R = \sqrt{R^2 + \omega^2 L^2} \cos \phi$ and $\omega = \frac{\sqrt{R^2 + \omega^2 L^2} \sin \phi}{L}$.

Substitute these values in the numerator of (2). Then

$$I' = \frac{E_m}{L^2} \times$$

$$\left[\frac{\sqrt{R^2+\omega^2L^2}\cos\phi\sin\omega t - \sqrt{R^2+\omega^2L^2}\sin\phi\cos\omega t}{\dfrac{R^2+\omega^2L^2}{L^2}} \right] + Ce^{-\frac{Rt}{L}}.$$

Or $$I' = \frac{E_m}{\sqrt{R^2+\omega^2L^2}}(\cos\phi\sin\omega t - \sin\phi\cos\omega t) + Ce^{-\frac{Rt}{L}}.$$

And $$I' = \frac{E_m}{\sqrt{R^2+\omega^2L^2}}\sin(\omega t - \phi) + Ce^{-\frac{Rt}{L}}. \qquad (3)$$

The term $Ce^{-\frac{Rt}{L}}$ shows the natural rise of the current when the voltage is first impressed upon the circuit. After a few cycles have been completed this term may be neglected.

Then $$I' = \frac{E_m}{\sqrt{R^2+\omega^2L^2}}\sin(\omega t - \phi). \qquad (4)$$

This expression gives the instantaneous current in a circuit having resistance and inductance at any instant, when a harmonic *E.M.F.* is impressed upon that circuit. When $L = 0$ and $\phi = 0$, the equation reduces to

$$I' = I_m \sin\omega t \text{ as in Article 3.}$$

21. Choke Coils.—The term choke coil is applied to any device designed to utilize counter electromotive force of self-induction to cut down the flow of current in an alternating-current circuit. Disregarding losses by hysteresis, a choke coil does not absorb any power, except that which is due to the current passing through its resistance. It can therefore be more economically used than a rheostat which would perform the same functions.

These coils are often used on alternating-current circuits in such places as resistances are used on direct-current circuits. For instance, in the starting devices employed in connection with alternating-current motors, the counter *E.M.F.* of inductance is made to cut down the pressure applied at the motor terminals. The starter for direct-current motors employs resistance.

It is often desirable to adjust the reactance of choke coils, and for this purpose several simple arrangements may be utilized. The coil may have a sliding iron core, or its winding may have several taps. Choke coils having U-shaped magnetic circuits are sometimes provided with movable polepieces, which serve to change the length of the air gap.

Since a lightning discharge is oscillatory in character and of enormous frequency, a coil which would offer a negligible impedance to an ordinary alternating current will offer a high impedance to a lightning discharge. This fact is recognized in the construction of lightning arresters. A choke coil of but few turns will offer so great an impedance to a lightning discharge that the high-tension, high-frequency current will find an easier path to the ground through an air gap suitably provided than through the machinery, and the latter is thus protected.

A choke coil for this purpose has no iron core, and consists of a few turns of wire, insulated from one another, wound in spiral or helical form. A lightning arrester choke coil used in railway service, for station use, is shown in Fig. 31.

The choking effect is not alone due to the high impedance offered to an oscillatory discharge, but also due to the "skin effect" of the wire. By this is meant, the tendency of the alternating current to have a greater density near the surface than along the axis of the conductor, thus increasing the resistance. To illustrate, a $1\frac{1}{2}''$ round copper conductor offers a true resistance, twice as great as its ohmic resistance, to a 130 ~ alternating current. Even in small wires, the true resistance presented to currents of very high frequency,

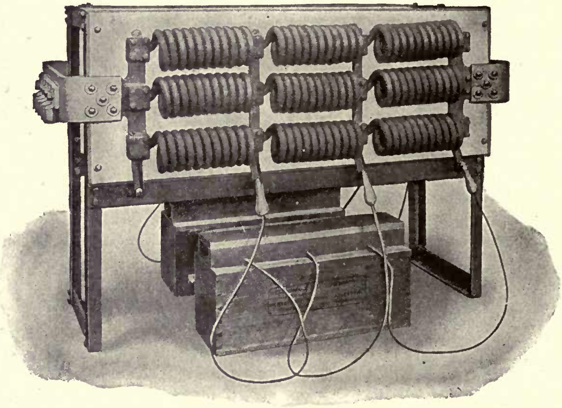


Fig. 31.

such as those produced by wireless telegraph transmitters, greatly exceeds the ohmic resistance, and therefore conductors are required possessing a large surface compared to the cross-section.

Choke coils are also used in connection with alternating-current incandescent lamps, to vary the current passing through them, and in consequence to vary the brilliancy.

PROBLEMS.

1. What is the field winding inductance of a bi-polar generator, having 7500 ampere turns per spool and a total flux of 2.4 mega-maxwells, when the exciting current is 2 amperes?

2. Find the inductance of a cast steel test ring coil of 300 turns when carrying 6 amperes, the test ring being 6″ outside and 5″ inside diameter and 2½″ in axial depth.

3. Determine the inductance of a pole-line 10 miles long and consisting of a pair of No. 1 copper wires separated by a distance between centers of 24 inches.

4. Determine the self-inductance of a solenoid consisting of 10 layers of No. 16 double-cotton covered wire, 100 turns per layer, wound upon a cylindrical wooden core 2 inches in diameter.

5. Find the value of the current in a circuit having 5 ohms resistance and an inductance of 0.15 henry, .03 seconds after impressing 110 volts upon that circuit.

6. What is the time constant of a circuit in which the current reaches half of its ultimate value .0018 second after connection with a source of *E.M.F.*?

7. What would be the current .02 second after suppressing the *E.M.F.* in the circuit of problem 5, a constant flow having been previously established?

8. Determine the energy stored in the magnetic field of the generator of problem 1, assuming L to be constant during the rise or fall of the current.

9. Find the current produced by a 60 ~ alternating *E.M.F.* of 120 volts in a circuit having 10 ohms resistance and an inductance of .04 henry. What is the power factor of the circuit?

10. What should the inductance of the circuit of problem 9 be, to attain a power factor of 85%?

11. Derive an expression for the current in a circuit whose resistance and reactance are equal. What will be the power factor?

12. Find the instantaneous value of a 25 ~ alternating current, 2.342 seconds after impressing a harmonic *E.M.F.* of 125 volts maximum upon a circuit which has a resistance of 8 ohms and an inductance of 0.04 henry.

CHAPTER III.

CAPACITY.

22. **Condensers.** — Any two conductors separated by a dielectric constitute a *condenser*. In practice the word is generally applied to a collection of thin sheets of metal separated by thin sheets of dielectric, every alternate metal plate being connected to one terminal and the intervening plates to the other terminal. The Leyden jar is also a common form of condenser.

The function of a condenser is to store electrical energy by utilizing the principle of electrostatic induction. Whenever a difference of potential is impressed upon the condenser terminals, stresses are set up in the dielectric which exhibit themselves electrically as a counter electromotive force, opposing and neutralizing the impressed *E.M.F.* During the period of establishment of the stresses a current flows through the dielectric, and it is known as a displacement current. This, however, ceases to flow as soon as the counter electromotive force of dielectric polarization is equal in magnitude to the impressed *E.M.F.* The condenser is then said to be charged. It should be remembered that the charge resides in the dielectric as the result of the stresses produced in it by the impressed *E.M.F.*

The nature of the stresses in the dielectric can be more readily understood by considering the conductors to be surrounded by an electrostatic field. This field may be considered as composed of electrostatic lines of force, shown in Fig. 32, which indicate by their directions the directions of the stresses, and by their nearness to each other the magnitude of the stresses. The greater the impressed *E.M.F.*, the greater will be the number of these lines and

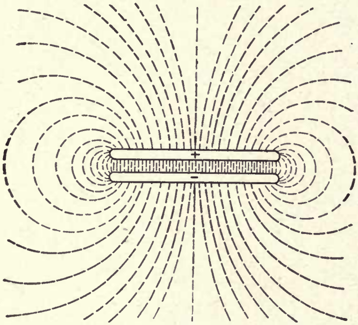


Fig. 32.

the greater will be the charge. The property of a dielectric which opposes the passage of this dielectric flux may be termed its *obstructance*, and it is similar in this respect to reluctance in opposing the passage of magnetic flux, and to resistance in opposing the flow of current. The obstructivity of a dielectric is three hundred times the reciprocal of its specific inductive capacity or its dielectric constant, which is the ratio of the electric strain to the stresses produced by it in the dielectric.

No dielectric is capable of supplying more than a definite maximum amount of counter electromotive force per unit of length measured along a line of force. If the impressed potential difference exceeds this maximum counter *E.M.F.*, which is the measure of its dielectric strength, the dielectric is ruptured and breaks down mechanically. Of course, if the dielectric is a liquid or a gas, it will be restored to its original state when the impressed *E.M.F.* is diminished. At the point of rupture, a current in the form of a spark or an arc passes from one conductor to the other, the tendency being to lessen the potential difference. Such rupture, followed by an arc, is a frequent source of trouble in electric machinery.

The dielectric strength of any dielectric depends upon its thickness, the form of the opposed conducting surfaces, and the manner in which the *E.M.F.* is applied, whether gradually, suddenly, or periodically varying. It has been stated that the dielectric strength approximately varies inversely as the cube root of the thickness, showing that a thin sheet is relatively stronger than a thick one of the same material. For example, the dielectric strength of crystal glass when 5 mm. thick is 183 kilovolts per centimeter, but when 1 mm. thick it is 285 kilovolts per centimeter. In the following table giving the dielectric strengths of various materials, the particular thicknesses for which the values are given are stated:

Material.	Thickness in mm.	Dielectric Strength in Kilovolts per cm.
Air	10	29.8
Air	1	43.6
Glass	5	183
Mica	1	610
Mica	0.1	1150
Micanite	1	400
Linseed Oil	6	84
Vaseline Oil	6	60
Lubricating Oil	6	48
Ebonite	2	430

The capacity of a condenser is numerically equal to the quantity of electricity with which it must be charged in order to raise the potential difference between its terminals from zero to unity.

If the quantity and potential be measured in C. G. S. units, the capacity, c, will be in C. G. S. units. If practical units be employed, the capacity, c, is expressed in *farads*. The farad is the practical unit of capacity. A condenser whose potential is raised one volt by a charge of one coulomb has one farad capacity. The farad is 10^{-9} times the absolute unit, and even then is too large to conveniently express the magnitudes encountered in practice. The term microfarad ($\frac{1}{1000000}$ farad) is in most general use.

In electrostatics, both air and glass are used as dielectrics in condensers; but the mechanical difficulties of construction necessitate a low capacity per unit volume, and therefore render these substances impracticable in electrodynamic engineering. Mica, although it is expensive and difficult of manipulation, is generally used as the dielectric in standard condensers and in those which are intended to withstand high voltages. Many commercial condensers are made from sheets of tinfoil, alternating with slightly larger sheets of paraffined paper. Though not so good as mica, paraffin will make a good dielectric if properly treated. It is essential that all the moisture be expelled from the paraffin when employed in a condenser. If it is not, the water particles are alternately attracted and repelled by the changes of potential on the contiguous plates, till, by a purely mechanical action, a hole is worn completely through the dielectric, and the whole condenser rendered useless by short-circuit. Ordinary paper almost invariably contains small particles of metal, which become detached from the calendar rolls used in manufacture.

These occasion short-circuits even when the paper is doubled.

The capacity of a condenser is proportional directly to the area and inversely to the thickness of the dielectric. It is also directly proportional to the dielectric constant of the insulating material, which, in addition to the definition already cited, may be defined as the number expressing the ratio of increase of the capacity of an air condenser, when the air is entirely replaced by that dielectric. This constant, usually represented by *K*, decreases with the temperature and with the time of charge. For these reasons the values of *K* given by different observers differ considerably, but some accepted values are given in the following table:

DIELECTRIC CONSTANTS AT 15° C.

Flint Glass (dense)	10.1	Quartz	4.55
Flint Glass (light)	6.57	Sulphur	2.9 to 4.0
Crown Glass (hard)	6.96	Shellac	2.7 to 3.0
Mica	6.64	Ebonite	2.05 to 3.15
Tourmaline	6.05	Paraffin Wax	2.0 to 2.3

The resistance of a condenser is not infinite, but a measurable quantity, and is usually expressed in megohms per microfarad, or, when referring to cables, in megohms per mile. Hence there is always a leakage from one charged plate to the other, both through the dielectric and over its surface. Poor insulation may occasion a considerable loss of energy appearing in the form of heat, and is therefore to be avoided.

Analogous to magnetic hysteresis in iron, is dielectric hysteresis in condensers, but, contrary to the former, it decreases as the frequency increases. Thus, at a frequency of the order of 10 million cycles, dielectric hysteresis is entirely absent. A dielectric having a high hysteretic constant, such as glass — 6.1, may consume a considerable amount of energy on low frequency circuits, this loss also appearing as heat.

23. Capacity Formulæ. — The following formulæ, in which r is the radius of the conductor and l its length, both in centimeters, give the capacity in microfarads of conductors with respect to the earth:

Sphere in free space,

$$C = \frac{r}{900,000}.$$

Circular disk in free space,

$$C = \frac{r}{1,413,720}.$$

One cylindrical wire in free space,

$$C = \frac{l}{4,144,680 \log_{10} \frac{l}{r}}.$$

One cylindrical wire h cm. from the earth,

$$C = \frac{l}{4,144,680 \log_{10} \frac{2h}{r}}.$$

In the following formulæ, giving the capacity of condensers of various forms, only that portion of the dielectric flux which passes perpendicularly between the conducting surfaces is considered; that is, the end flux shown by the curved dotted lines in Fig. 32 is neglected. Under this consideration, the following expressions may only be used when the thickness of the dielectric is very small compared to the conductor area.

Two concentric spheres,

$$C = \frac{r_1 r_2 K}{900{,}000\ (r_2 - r_1)} \text{ where } r_2 > r_1.$$

Two concentric cylinders,

$$C = \frac{lK}{4{,}144{,}680\ \log_{10} \frac{r_2}{r_1}} \text{ where } r_2 > r_1.$$

Two cylindrical wires *d* cm. apart,

$$C = \frac{lK}{8{,}289{,}360\ \log_{10} \frac{d}{r}}.$$

Two circular plates, *d* cm. apart,

$$C = \frac{r^2 K}{3{,}600{,}000\ d}.$$

From this last formula, another may be readily derived for the calculation of the capacity of a condenser having n dielectric sheets, and having its symbols expressed in inches. The capacity is

$$C = .000225\ \frac{An}{t} K,$$

where A is the area of each sheet in square inches, and t is its thickness in mils.

The following data of a condenser, used in duplex telegraphy, give an idea of capacity and dielectric resistance.

The condenser consists of tinfoil and paper sheets, the former being brought out alternately to one terminal and then to the other. There are 92 sheets of beeswaxed paper, 7 × 5 inches and two mils thick, which constitute the dielectric. The capacity of the condenser is 1.47 microfarads, and its dielectric resistance is 160 megohms.

24. Connection of Condensers in Parallel and in Series. — Condensers may be connected in parallel as in Fig. 33. If the capacities of the individual condensers be respectively C_1, C_2, C_3, etc., the capacity C of the combination will be

$$C = C_1 + C_2 + C_3 + \dots.$$

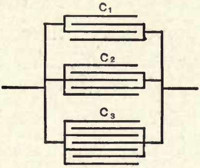


Fig. 33.

For the potential difference on each condenser is the same, and equal to the impressed *E.M.F.*, and the total charge is equal to the sum of the individual charges, or

$$E = E_1 = E_2 = E_3 = \dots.$$

and $$Q = Q_1 + Q_2 + Q_3 + \dots.$$

Then by division $\dfrac{Q}{E} = \dfrac{Q_1}{E_1} + \dfrac{Q_2}{E_2} + \dfrac{Q_3}{E_3} + \dots.$

But by definition $\dfrac{Q}{E} = C$, $\dfrac{Q_1}{E_1} = C_1$ and so on,

therefore $$C = C_1 + C_2 + C_3 + \dots.$$

The parallel arrangement of several condensers is equivalent to increasing the number of plates in one condenser. An increase in the number of plates results in an increase in the quantity of electricity necessary to raise the potential difference between the terminals of the condenser one volt; that is, an increase in the capacity results.

If the condensers be connected in series, as in Fig. 34, the capacity of the combination will be

$$C = \frac{1}{\dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} + \dots}.$$

For, if a quantity of positive electricity, Q, flow into the left side of C_1, it will induce and keep bound an equal negative quantity on the right side of C_1, and will repel an equal positive quantity. This last quantity will constitute the charge for the left side of C_2. The operation is repeated in the case of each of the condensers. It is thus clear that the quantity of charge in each condenser is Q. The impressed *E.M.F.* must consist of the sum of the potential differences on the separate condensers. Let these differences be respectively E_1, E_2, E_3, etc. Then the impressed *E.M.F.*

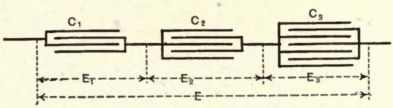


Fig. 34.

$$E = E_1 + E_2 + E_3 + \cdots$$

But $$E_1 = \frac{Q}{C_1}, \quad E_2 = \frac{Q}{C_2}, \quad E_3 = \frac{Q}{C_3}, \text{ etc.,}$$

and also, $$E = \frac{Q}{C},$$

therefore $$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} = \frac{Q}{C_3} + \cdots$$

or $$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots}.$$

As an example, consider three condensers of respective capacities of 1, 2, and 5 microfarads. Since the factor to reduce to farads will appear on both sides of the equations, it may here be omitted. With the three in multiple (Fig. 33), the capacity of the combination will be

$$C = 1 + 2 + 5 = 8 \text{ mf.}$$

With the three in series (Fig. 34),

$$C = \frac{1}{\frac{1}{1} + \frac{1}{2} + \frac{1}{5}} = .588 \text{ mf.}$$

With the two smaller in parallel and in series with the larger (Fig. 35),

$$C = \frac{1}{\frac{1}{1+2} + \frac{1}{5}} = 1.875 \text{ mf.}$$

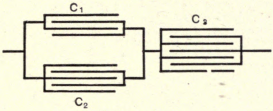


Fig. 35.

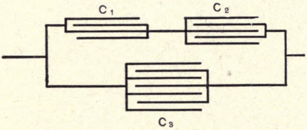


Fig. 36

With the two smaller in series and in parallel with the larger (Fig. 36),

$$C = \frac{1}{\frac{1}{1} + \frac{1}{2}} + 5 = 5.666 \text{ mf.}$$

If with any condensers

$$C_1 = C_2 = C_3 = \cdot \cdot \cdot \cdot = C_n,$$

then, with n in multiple,

$$C = nC_1,$$

and with n in series,

$$C = \frac{1}{n} C_1.$$

It is interesting to note that the formulas for *capacities* in parallel and in series respectively are just the reverse of those for *resistances* in parallel and in series respectively.

25. Decay of Current in a Condensive Circuit. — The opposition to a flow of current which is caused by a condenser is quite different from that which is caused by a resistance. To be sure, there is some resistance in the leads and condenser plates, but this is generally so small as to be negligible. The practically infinite resistance of the condenser dielectric does not obstruct the current as an ordinary resistance is generally considered to do. The dielectric is the seat of a polarization *E.M.F.* which is developed by the condenser charge and which grows with it. It is a counter *E.M.F.*; and when it reaches a value equal to that of the impressed voltage, the charging current is forced to cease.

To find the current at any instant of time, t, in a circuit (Fig. 37) containing a resistance R and a capacity C, the constant impressed pressure E must be considered as consisting of two variable parts, one E_r, being active in sending current through the resistance, and the other part, E_c, being required to balance the potential of the condenser. Then at all times

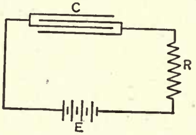


Fig. 37.

$$E' = E_r' + E_c'.$$

Let time be reckoned from the instant the pressure E is applied; when, therefore, $t = 0$ and $I_0 = \frac{E}{R}$. Consider the current at any instant of time to be I'. Then if it flow for dt seconds it will cause dQ coulombs to traverse the circuit, and

$$I' = \frac{dQ'}{dt} \text{ or } dQ' = I'dt,$$

from which

$$Q' = \int I'dt.$$

By definition,

$$C = \frac{Q'}{E_c'},$$

therefore,

$$E_c' = \frac{Q'}{C} = \frac{\int I'dt}{C}.$$

And by Ohm's Law,

$$E_r' = I'R,$$

so at this instant of time

$$E' = E_r' + E_c' = I'R + \frac{\int I'dt}{C},$$

whence

$$E'C = RCI' + \int I'dt,$$

which upon differentiating, becomes

$$0 = RCdI' + I'dt.$$

Integrating

$$\int_0^t dt = -RC \int_{I_0}^{I'} \frac{dI'}{I'},$$

$$t = -RC\left[\log I' - \log \frac{E}{R}\right].$$

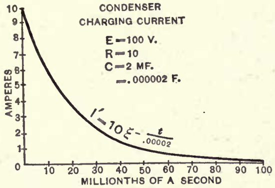


Fig. 38.

Solving for I',

$$I' = \frac{E}{R}\epsilon^{-\frac{t}{RC}},$$

which is the expression sought. Like the corresponding expression for an inductive circuit, it is logarithmic.

Fig. 38 is a curve showing the decay of current in a condenser for the conditions indicated. The product *RC* is the time constant of a condensive circuit and is similar to the ratio $\frac{L}{R}$ in an inductive circuit.

26. Energy Stored in Dielectric. — A current I flowing in a condensive circuit against a dielectric polarization of E volts, represents a power of EI watts. The work performed in an interval of time dt is

$$dW = EI\,dt$$

and represents the elementary work done in establishing the stresses in the dielectric. Since

$$E = \frac{Q}{C} \text{ and } I\,dt = dQ,$$

there results by substitution

$$dW = \frac{Q\,dQ}{C},$$

which when integrated through the full range, that is

$$\int_0^W dW = \frac{1}{C}\int_0^Q Q\,d\,Q,$$

becomes $$W = \frac{Q^2}{2\,C} \text{ joules.}$$

This is the expression for the energy required to establish the dielectric stresses when the current is first applied, and also the expression for the energy returned to the circuit by the dielectric when the impressed *E.M.F.* is withdrawn.

27. Condensers in Alternating-Current Circuits — Hydraulic Analogy. — Imagine a circuit consisting of a pipe through which water is made to flow, first one way, then the other, by a piston oscillated pump-like in one section of it. The pipe circuit corresponds to an electric circuit, the pump to a generator of alternating *E.M.F.*, and the flow of water to a flow of alternating current. Further imagine one section of the pipe to be enlarged, and in it placed a transverse elastic diaphragm. This section corresponds to a condenser. Its capacity with a unit pressure of water on one side depends upon the area of the diaphragm, its thinness, and the elastic coefficients of the material of which it is made. In a condenser the capacity depends upon the area of the dielectric under strain, its thinness, and the specific inductive capacity of the dielectric employed. As the water surges to and fro in the pipe, some work must be done upon the diaphragm, since it is not perfectly elastic. This loss corresponds to the loss in a condenser by dielectric hysteresis. The fact that the diaphragm is not absolutely impervious to water corresponds to the fact that a dielectric is not an absolute electric insulator. As the diaphragm may be burst by too great a hydrostatic pressure, so may the dielectric be ruptured by too great an electric pressure.

28. Phase Relations. — To understand the relation between pressure and current in a condensive circuit, consider the above analogy. Imagine the diaphragm in its medial position, with equal volumes of water on either side of it, and the piston in the middle of its travel. This middle point corresponds to zero pressure. When the piston is completely depressed there is a maximum negative pressure, when completely elevated, a maximum positive pressure, if pressure and flow upward be considered in the positive direction. If the piston oscillate in its path with a regular motion, it is clear that the water will flow upward from the extreme lowest to the extreme highest position of the piston. That is, there will be flow in the positive direction from the maximum negative to the maximum positive values of pressure. The direction of flow is seen to remain unchanged while the piston passes through its middle position or the point of zero pressure. These facts are indicated in Fig. 39, which shows that portion of the pipe having the piston and the diaphragm.

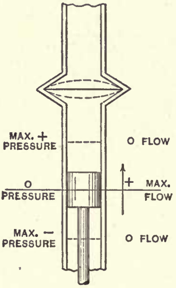


Fig. 39.

Returning to electric phenomena, if a harmonic *E.M.F.* be impressed upon any circuit, a harmonic current will flow in it. So in a circuit containing a condenser and subject to a sinusoidal *E.M.F.*, the current flow will be sinusoidal. This flow will be in the positive direction from the negative maximum to the positive maximum of pressure, and in a negative direction from the positive maximum to the negative maximum, as described above. This necessitates that the zero values of current occur at the maximum values of pressure; and since the curves are both sinusoids, their relation may be plotted as in Fig. 40. It is immediately seen that these curves are at right angles, as described in § 7, and that the current leads the pressure by 90°.

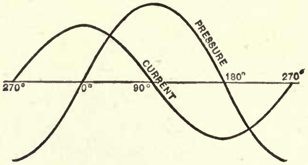


Fig. 40.

Reference again to the hydraulic analogy will show that the condenser is completely charged at the instant of maximum positive pressure, discharged at the instant of zero pressure, charged in the opposite direction at the instant of maximum negative pressure, and finally discharged at the instant of the next zero pressure. Thus the charge is zero at the maximum current flow, and at a maximum at zero current, that is, when the current turns and starts to flow out. These points are marked in Fig. 41.

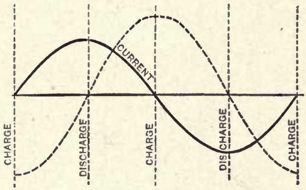


Fig. 41.

29. Current and Voltage Relations. — If a sinusoidal pressure E of frequency f be impressed upon a condenser, the latter is charged in $\frac{1}{4}\cdot\frac{1}{f}$ seconds, discharged in the next $\frac{1}{4f}$ seconds, and charged and discharged in the opposite direction in the equal succeeding intervals. The maximum voltage $E_m = \sqrt{2}\,E$ (§ 5), hence the quantity at full charge is

$$Q_m = \sqrt{2}\,EC.$$

The quantity flowing through the circuit per second is

$$4\,fQ_m = 4\,f\sqrt{2}\,EC.$$

This number therefore represents the average current, or

$$I_{av} = 4\sqrt{2}\,fEC.$$

From § 5, the effective current

$$I = \frac{\pi}{2\sqrt{2}}\,I_{av},$$

whence $$I = 2\pi f C E,$$

and $$E = \frac{1}{2\pi f C} I.$$

The last is an expression for the volts necessary to send the capacity current through a circuit. The expression $1/2\pi f C$ is called the *capacity reactance* of the circuit. It is analogous to $2\pi f L$, the inductive reactance of an inductive circuit.

If the circuit contain both a resistance R and a capacity C, the voltage E impressed upon it must be considered as made up of two parts, E_r, which sends current through the resistance and is therefore in phase with the current, and E_c, which balances the counter pressure of the condenser and is therefore 90° behind the current in phase.

By Ohm's Law

$$E_r = RI,$$

and from above

$$E_c = \frac{1}{2\pi f C} I.$$

The impressed E must overcome the resultant of these two $E.M.F.$'s; and since they are at right angles

$$E = \sqrt{E_r^2 + E_c^2},$$

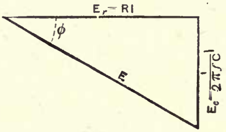


Fig. 42.

or

$$I = \frac{E}{\sqrt{R^2 + \left(\frac{1}{2\pi f C}\right)^2}}.$$

The relation of the $E.M.F.$'s is shown graphically in Fig. 42, where the current, which is in phase with the pressure E_r, is seen to lead the impressed pressure by the angle ϕ.

30. Instantaneous Current in a Circuit Having Capacity and Resistance. — The value of the *E.M.F.*, impressed upon a circuit containing capacity and resistance at any instant t, must be sufficient to send the instantaneous current I' through the resistance and also neutralize the *E.M.F.* of dielectric polarization. Hence

$$E' = E_r' + E_c'.$$

But
$$E' = E_m \sin \omega t, \qquad (\S\ 3)$$

$$E_r' = I'R, \qquad (\S\ 25)$$

and
$$E_c' = \frac{\int I'dt}{C}; \qquad (\S\ 25)$$

therefore
$$E_m \sin \omega t = I'R + \frac{\int I'dt}{C}.$$

Differentiating $\omega E_m \cos \omega t\, dt = R\, dI' + \dfrac{I'dt}{C}$.

Multiplying by integrating factor $e^{\int \frac{dt}{RC}}$ or $e^{\frac{t}{RC}}$ and dividing by R, this becomes

$$dI'e^{\frac{t}{RC}} + I'e^{\frac{t}{RC}} \frac{dt}{RC} = \frac{\omega E_m}{R} e^{\frac{t}{RC}} \cos \omega t\, dt.$$

The second term is in the form $da^x = a^x \log_e a\, dx$, hence this is

$$dI'e^{\frac{t}{RC}} + I'd\left(e^{\frac{t}{RC}}\right) = \frac{\omega E_m}{R} e^{\frac{t}{RC}} \cos \omega t\, dt.$$

Since the first member is in the form $d(xy) = y\,dx + x\,dy$ it equals $d\left(I'e^{\frac{t}{RC}}\right)$. Substitute and integrate, then

$$I'e^{\frac{t}{RC}} = \frac{\omega E_m}{R} \int e^{\frac{t}{RC}} \cos \omega t\, dt + C,$$

or $$I' = e^{-\frac{t}{RC}} \frac{\omega E_m}{R} \int e^{\frac{t}{RC}} \cos \omega t \, dt + Ce^{-\frac{t}{RC}}. \qquad (1)$$

To determine value of the integral, use the formula $\int u\,dv = uv - \int v\,du$, where $u = \cos \omega t$, hence $du = -\omega \sin \omega t \, dt$; and where $dv = e^{\frac{t}{RC}} dt = RCe^{\frac{t}{RC}} \frac{dt}{RC}$, hence $v = RCe^{\frac{t}{RC}}$. Then

$$\int e^{\frac{t}{RC}} \cos \omega t \, dt = RCe^{\frac{t}{RC}} \cos \omega t + RC\omega \int e^{\frac{t}{RC}} \sin \omega t \, dt.$$

The second integral is in the same form, but here $u = \sin \omega t$, hence $du = \omega \cos \omega t \, dt$, dv and v remain the same. Then

$$\int e^{\frac{t}{RC}} \cos \omega t \, dt = RCe^{\frac{t}{RC}} \cos \omega t + R^2C^2\omega e^{\frac{t}{RC}} \sin \omega t - R^2C^2\omega^2 \int e^{\frac{t}{RC}} \cos \omega t \, dt.$$

$$\left(1 + R^2C^2\omega^2\right) \int e^{\frac{t}{RC}} \cos \omega t \, dt = RCe^{\frac{t}{RC}} \cos \omega t + R^2C^2\omega e^{\frac{t}{RC}} \sin \omega t.$$

Substitute value of integral in (1), there results

$$I' = e^{-\frac{t}{RC}} \frac{\omega E_m}{R} \left[\frac{RCe^{\frac{t}{RC}} \cos \omega t + R^2C^2\omega e^{\frac{t}{RC}} \sin \omega t}{1 + R^2C^2\omega^2} \right] + Ce^{-\frac{t}{RC}}.$$

$$I' = \omega C E_m \left[\frac{\cos \omega t + RC\omega \sin \omega t}{1 + R^2C^2\omega^2} \right] + Ce^{-\frac{t}{RC}},$$

or $$I' = E_m \left[\frac{\frac{1}{\omega C} \cos \omega t + R \sin \omega t}{\frac{1}{\omega^2 C^2} + R^2} \right] + Ce^{-\frac{t}{RC}}. \qquad (2)$$

Let the angle ϕ be chosen so that

$$\tan\phi = \frac{\frac{1}{\omega C}}{R} = \frac{1}{\omega CR},$$

thus representing the angle by which the current leads the *E.M.F.* Therefore

$$R = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}\cos\phi,$$

and
$$\frac{1}{\omega C} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}\sin\phi.$$

By substitution in (2), there results,

$$I' = E_m \frac{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}{R^2 + \frac{1}{\omega^2 C^2}}[\sin\phi\cos\omega t + \cos\phi\sin\omega t] + Ce^{-\frac{t}{RC}},$$

or
$$I' = \frac{E_m}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}\sin(\omega t + \phi) + Ce^{-\frac{t}{RC}} \qquad (3)$$

where the exponential term shows the natural current decay in a condensive circuit when the *E.M.F.* is first applied. Neglecting this term, (3) reduces to

$$I' = \frac{E_m}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}\sin(\omega t + \phi), \qquad (4)$$

giving an expression for the instantaneous current in a circuit, having resistance and capacity, at any instant when a harmonic *E.M.F.* is impressed upon that circuit. When the capacity of a circuit is an infinitesimal, such as is the case when its two terminals are slightly separated, then in the formula, $C = 0$ and the current is also zero, which is evidently true for an open circuit. When the circuit contains no capacity relative to itself, and only resistance, then the term $\left(\frac{1}{\omega C}\right)^2$ should not enter the equation, which will then reduce to

$$I' = I_m \sin(\omega t + \phi), \qquad \text{as in § 3.}$$

PROBLEMS.

1. Determine the capacity of a pair of No. 000 line wires, two feet apart, and three miles long.

2. Calculate the dielectric constant of the condenser mentioned in § 23. What is its insulation resistance expressed in megohms per microfarad?

3. Derive the formula $C = .000225 \frac{An}{t} K$ of § 23.

4. Find the equivalent capacity of the group of condensers shown in

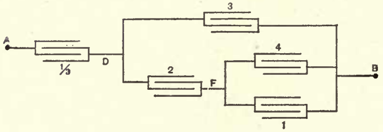


Fig. 43.

Fig. 43, the number adjacent to each condenser representing its capacity in microfarads.

5. If a constant *E.M.F.* of 150 volts is applied to the terminals *A* and *B* of the group of condensers shown in Fig. 43, what will be the voltage across the terminals of each condenser?

6. If a circuit having a resistance of 10 ohms and a capacity of 20 microfarads has a constant *E.M.F.* of 100 volts impressed upon it, how long will it take for the current to sink to half its initial value?

7. Determine the energy which can be electrically stored in a cubic inch of mica dielectric when the applied potential is 450 volts per mil thickness.

8. Find the current produced by a 25~ alternating *E.M.F.* of 100 volts in a circuit having 25 ohms resistance and a capacity of 30 microfarads. What is the power factor of the circuit?

9. It is desired to construct a condenser of crown glass plates 10 × 12 inches so that the power factor of its circuit having 12.5 ohms resistance shall be 90% for an oscillatory current of 80,000 cycles. How many plates will be required if the thickness of each is .15 inch?

10. Determine the instantaneous value of a 60~ alternating current 5.71 seconds after impressing a harmonic *E.M.F.* of 220 volts (effective) upon a circuit having a resistance of 100 ohms and a capacity of 25 microfarads.

CHAPTER IV.

ALTERNATING-CURRENT CIRCUITS.

31. Resistance, Inductance and Capacity in an Alternating-Current Circuit. — In general, alternating-current circuits have resistance, inductance and capacity. An expression for the current flow in such a circuit may be derived mathematically, as in § 34, or the current may be found graphically by combining results already obtained. In § 19 it was shown that the counter *E.M.F.* due to the inductive reactance of a circuit is $2\pi fLI$ and leads the current by 90°, and in § 29 it was shown that the *E.M.F.*

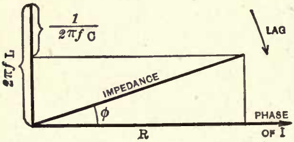


Fig. 44.

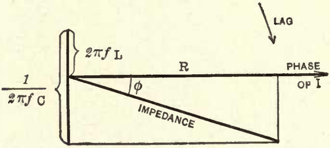


Fig. 45.

of dielectric polarization due to the capacity reactance of a circuit is $\frac{I}{2\pi fC}$ and lags behind the current by 90°; hence these two *E.M.F.*'s are opposite in phase, or 180° apart. These relations are shown in Fig. 44, where the inductive reactance is greater than that due to capacity, and in Fig. 45, where the latter exceeds the former, the resistance being the same in both cases. The common factor *I* is omitted in these diagrams, as is very often done for convenience, but it should be remembered that neither resistance, reactance nor impedance is a vector quantity. Clearly the impedance resulting from the three factors, R, L and C, is represented in direction and in magnitude by the hypothenuse as shown, and the impressed pressure is I times this quantity.

The general expression for the flow of an alternating current through any kind of a circuit is therefore

$$I = \frac{E}{\sqrt{R^2 + \left[2\,\pi f L - \frac{1}{2\,\pi f C}\right]^2}},$$

the quantity within the brackets indicating an angle of lag of current when positive. and an angle of lead when negative.

32. **Definitions of Terms.** — In considering the flow of alternating currents through series circuits and through parallel circuits, continual use must be made of various expressions, some of which have been defined during the development of the previous chapters. For convenience the names of all the expressions connected with the general equation

$$I = \frac{E}{\sqrt{R^2 + \left(2\,\pi f L - \frac{1}{2\pi f C}\right)^2}}$$

will be given and defined.

I is the *current* flowing in the circuit. It is expressed in amperes, and lags behind or leads the pressure, by an angle whose value is

$$\phi = \tan^{-1} \frac{2\,\pi f L - \frac{1}{2\,\pi f C}}{R}\,.$$

E is the harmonic *pressure*, of maximum value $\sqrt{2}\,E$, which is applied to the circuit, and has a frequency f. It is expressed in volts.

R is the *resistance* of the circuit, and is expressed in ohms. It is numerically equal to the product of the impedance by the cosine of ϕ.

L is the *inductance* of the circuit, and is expressed in henrys.

C is the localized *capacity* of the circuit, and is expressed in farads.

$2\,\pi f L$ is the *inductive reactance* of the circuit, and is expressed in ohms.

$$\frac{1}{2\,\pi f C}$$

is the *capacity reactance*, or *capacitance*, of the circuit, and is expressed in ohms.

$$\left(2\,\pi f L - \frac{1}{2\,\pi f C}\right)$$

is the *reactance* of the circuit, and is expressed in ohms and usually represented by X. It is numerically equal to the product of the impedance by the sine of ϕ.

$$\sqrt{R^2 + \left[2\,\pi f L - \frac{1}{2\,\pi f C}\right]^2} \quad \text{or} \quad \sqrt{R^2 + X^2}$$

is the *impedance* or apparent resistance of a circuit, and is expressed in ohms and usually represented by Z.

$$\frac{1}{\sqrt{R^2 + \left[2\,\pi f L - \frac{1}{2\,\pi f C}\right]^2}} \quad \text{or} \quad \frac{1}{Z},$$

the reciprocal of the impedance, is the *admittance* of the circuit, and is represented by Y. It is expressed in terms of a unit that has never been officially named, but which has sometimes been called the mho. There are two components of the admittance, as shown in Fig. 46.

The *conductance* of a circuit, usually represented by g, is that quantity by which E must be multiplied to give the

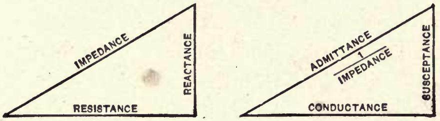


Fig. 46.

component of I parallel to E. It is expressed in the same units as the admittance, and is numerically equal to

$$\frac{\cos\phi}{Z} \quad \text{or} \quad Y\cos\phi,$$

but $$\cos\phi = \frac{R}{Z}, \text{ hence } g = \frac{R}{Z^2}.$$

The *susceptance* of a circuit, represented by b, is that quantity by which E must be multiplied to give the component of I perpendicular to E. It is measured in the same units as the admittance, and is numerically equal to

$$\frac{\sin\phi}{Z} \quad \text{or} \quad Y\sin\phi,$$

but $$\sin\phi = \frac{X}{Z}, \text{ hence } b = \frac{X}{Z^2}.$$

Admittance may then be expressed as

$$Y = \sqrt{g^2 + b^2}.$$

It should be noticed that while admittance is the reciprocal of impedance, conductance is not the reciprocal of resistance, nor is susceptance the reciprocal of reactance. This becomes evident, upon considering numerical values in connection with the impedance right-angled triangle, *e.g.* 3, 4 and 5 for the sides.

33. Representation of Impedance and Admittance by Complex Numbers. — The problem of determining current, voltage and phase relations in alternating-current circuits may be solved graphically, by means of vector diagrams, or trigonometrically. To facilitate the solution of particular problems by the latter method, use is made of complex numbers.

In Fig. 47, let I be the current produced in a circuit by the harmonic *E.M.F.*, E, the current lagging behind the electromotive force by the angle ϕ. Taking the rectangular reference axes x and y as shown, both E and I may be resolved into components along them. Let the symbol j be placed before the y-components, thus distinguishing them from the x-components. Then

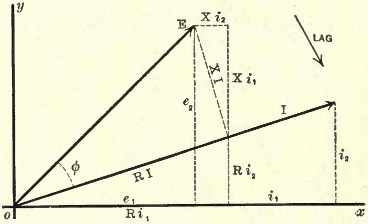


Fig. 47.

$$E = e_1 + je_2$$

and
$$I = i_1 + ji_2,$$

the plus sign indicating vector addition at right angles of the x and y components respectively. But E may also be resolved into a component in phase with the current and into another at right angles thereto, that is, it may be expressed as

$$E = RI + jXI,$$

and substituting the values of E and I, there results

$$e_1 + je_2 = Ri_1 + jRi_2 + jXi_1 + j^2Xi_2.$$

Both RI and XI may be resolved into components along the axes of reference as indicated, and hence it follows that

$$e_1 = Ri_1 - Xi_2$$

and $$e_2 = Ri_2 + Xi_1.$$

Then $Ri_1 - Xi_2 + jRi_2 + jXi_1 = Ri_1 + jRi_2 + jXi_1 + j^2Xi_2.$

Therefore $$-Xi_2 = j^2Xi_2$$

or $$j^2 = -1$$

and $$j = \sqrt{-1},$$

which is therefore the interpretation of the symbol j, as already defined.

From the foregoing,

$$I = \frac{E}{R + jX},$$

but $$I = \frac{E}{Z}, \qquad (§ 32)$$

hence the impedance Z may be properly represented by $R + jX$.

Admittance, being the reciprocal of impedance, may then be represented by $\frac{1}{R + jX}$, and multiplying both numerator and denominator by $R - jX$, there results,

$$\frac{R - jX}{(R + jX)(R - jX)} = \frac{R - jX}{R^2 - j^2X^2} = \frac{R - jX}{R^2 + X^2}.$$

Separating, $Y = \frac{R}{R^2 + X^2} - j\frac{X}{R^2 + X^2} = \frac{R}{Z^2} - j\frac{X}{Z^2}.$

But $\frac{R}{Z^2} = g$ and $\frac{X}{Z^2} = b.$ (§ 32)

Hence the admittance Y is to be represented by $g - jb$.

34. Instantaneous Current in a Circuit Having Inductance, Capacity and Resistance. — In § 20 an expression was derived for the instantaneous current produced by a harmonic *E.M.F.* in a circuit having inductance and resistance, and in § 30 a similar expression was derived for a circuit having capacity and resistance. Proceeding along the same lines, a general expression could be obtained for the instantaneous current produced by a harmonic *E.M.F.* in any alternating-current circuit, that is, in one having inductance, capacity and resistance. This method, however, is rather cumbersome, and a simpler one is given as follows:

The harmonic *E.M.F.* is represented by $E_m \varepsilon^{j\omega t}$, an expression which results from the use of Maclaurin's Series, that is,

$$f(x) = [f(x)]_{x=0} + x[f'(x)]_{x=0} + \frac{x^2}{\lfloor 2}[f''(x)]_{x=0} + \frac{x^3}{\lfloor 3}[f'''(x)]_{x=0} + \ldots$$

where $f'(x)$, $f''(x)$, $f'''(x)$, ... are the respective derivatives of $f(x)$.

When the function is $\sin\theta$,

$$\sin\theta = \theta - \frac{\theta^3}{\lfloor 3} + \frac{\theta^5}{\lfloor 5} - \frac{\theta^7}{\lfloor 7} + \ldots$$

and when the function is $\cos\theta$,

$$\cos\theta = 1 - \frac{\theta^2}{\lfloor 2} + \frac{\theta^4}{\lfloor 4} - \frac{\theta^6}{\lfloor 6} + \ldots$$

When, however, the function is $\varepsilon^{j\theta}$, then

$$\varepsilon^{j\theta} = 1 + \frac{j\theta}{\underline{|1}} + \frac{(j\theta)^2}{\underline{|2}} + \frac{(j\theta)^3}{\underline{|3}} + \frac{(j\theta)^4}{\underline{|4}} + \frac{(j\theta)^5}{\underline{|5}} + \frac{(j\theta)^6}{\underline{|6}} + \frac{(j\theta)^7}{\underline{|7}} + \ldots$$

Remembering that $j^2 = -1$, $j^4 = 1$, $j^6 = -1, \ldots$ this becomes

$$\varepsilon^{j\theta} = 1 - \frac{\theta^2}{\underline{|2}} + \frac{\theta^4}{\underline{|4}} - \frac{\theta^6}{\underline{|6}} + \ldots + j\left[\theta - \frac{\theta^3}{\underline{|3}} + \frac{\theta^5}{\underline{|5}} - \frac{\theta^7}{\underline{|7}} + \ldots\right]$$

Hence $$\varepsilon^{j\theta} = \cos\theta + j\sin\theta.$$

Multiply through by E_m and replace θ by ωt, then

$$E_m\varepsilon^{j\omega t} = E_m(\cos\omega t + j\sin\omega t),$$

which is evidently a proper expression for a harmonic *E.M.F.*

Consider a circuit having a resistance R, a capacity C and an inductance L. The *E.M.F.* impressed upon this circuit must be of such magnitude as to neutralize both the counter *E.M.F.* of self-induction and the *E.M.F.* of dielectric polarization, and also send the instantaneous current I' through the resistance. Therefore

$$E' = E_1' + E_c' + E_r'$$

or $$E_m\varepsilon^{j\omega t} = L\frac{dI'}{dt} + \frac{1}{C}\int I'dt + I'R. \qquad (1)$$

Since the current is of the same character as the impressed *E.M.F.*, it may be represented by $B\varepsilon^{j\omega t}$, where B is a constant to be determined. Then

$$I' = B\varepsilon^{j\omega t}$$

and $$\frac{dI'}{dt} = jB\omega\varepsilon^{j\omega t},$$

and $$\int I'dt = B\int\varepsilon^{j\omega t}dt = \frac{B}{j\omega}\varepsilon^{j\omega t} = \frac{jB}{j^2\omega}\varepsilon^{j\omega t} = -j\frac{B}{\omega}\varepsilon^{j\omega t}.$$

Substituting these values in (1), there results

$$E_m \varepsilon^{j\omega t} = j\omega LB\varepsilon^{j\omega t} - j\frac{B}{\omega C}\varepsilon^{j\omega t} + RB\varepsilon^{j\omega t},$$

or

$$E_m = B\left[R + j\left(\omega L - \frac{1}{\omega C}\right)\right].$$

Hence

$$B = \frac{E_m}{R + j\left(\omega L - \frac{1}{\omega C}\right)} \cdot \frac{R - j\left(\omega L - \frac{1}{\omega C}\right)}{R - j\left(\omega L - \frac{1}{\omega C}\right)},$$

or

$$B = \frac{E_m\left[R - j\left(\omega L - \frac{1}{\omega C}\right)\right]}{R^2 - j^2\left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= \frac{E_m}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}\left[R - j\left(\omega L - \frac{1}{\omega C}\right)\right].$$

But

$$I' = B\varepsilon^{j\omega t} = B(\cos\omega t + j\sin\omega t),$$

and substituting value of B, there results

$$I' = \frac{E_m}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}\left[R - j\left(\omega L - \frac{1}{\omega C}\right)\right]$$

$$[\cos\omega t + j\sin\omega t],$$

or

$$I' = \frac{E_m}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}\left[\left(R\cos\omega t + \left[\omega L - \frac{1}{\omega C}\right]\sin\omega t\right)\right.$$

$$\left. + j\left(R\sin\omega t - \left[\omega L - \frac{1}{\omega C}\right]\cos\omega t\right)\right].$$

Assuming the impressed harmonic *E.M.F.* as a simple sine function, then only the second part within the bracket of this expression need be taken; hence

$$I' = \frac{E_m}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}\left[R \sin \omega t - \left(\omega L - \frac{1}{\omega C}\right)\cos \omega t\right]. \quad (2)$$

Now let an angle ϕ be chosen so that

$$\tan \phi = \frac{\omega L - \frac{1}{\omega C}}{R};$$

then $$\omega L - \frac{1}{\omega C} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \sin \phi$$

and $$R = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \cos \phi.$$

Substituting these in (2),

$$I' = \frac{E_m \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} [\sin \omega t \cos \phi - \cos \omega t \sin \phi]$$

or $$I' = \frac{E_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \sin (\omega t - \phi), \quad (3)$$

which is the required expression for the instantaneous current produced by a harmonic *E.M.F.* in a circuit containing inductance, capacity and resistance.

When $L = 0$, the expression reduces to the form given in § 30; and when the circuit has no capacity with respect to itself, the term $\frac{1}{\omega C}$ drops out, and the expression reduces to the form given in § 20. It follows, then, that equation (3) may be applied to any alternating-current circuit.

35. Resonance. — An electrical circuit is said to be *resonant*, or in *resonance* with an impressed *E.M.F.*, when the natural period of that circuit and the period of the *E.M.F.* are the same. The natural period of the circuit is the reciprocal of that frequency at which the current is a maximum. By reference to the formula

$$I = \frac{E}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}}$$

it becomes evident that the maximum current is $\frac{E}{R}$, which occurs when

$$2\pi fL - \frac{1}{2\pi fC} = 0,$$

that is, when the capacity and the inductance are so proportioned that their reactances are equal. From this relation, it follows that the critical frequency at which resonance occurs is

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}},$$

and that the natural period of the circuit is $2\pi\sqrt{LC}$.

To show the current values for different frequencies, a curve as in Fig. 48 may be drawn. It is plotted for a series circuit, having 5 ohms resistance and an inductance of 0.422 henry and a capacity of 6 microfarads; upon which circuit is impressed a harmonic *E.M.F.* of 100 volts. The frequency of the impressed *E.M.F.* required for resonance is seen to be 100 ∼. At this frequency the potential difference between the terminals of the condenser $= \frac{I}{2\pi fC}$ $= 5300$ volts, and that across the inductance coil is $2\pi fLI$ $= 5300$ volts also, whereas only 100 volts is impressed upon the circuit. Hence when resonance occurs in a circuit in which the capacity and the inductance are in series, the potential difference across either may rise to such a value as to puncture the insulation of the apparatus. If the capacity and inductance be in parallel, enormous currents may flow between the two. This is because the two are balanced, and the one is at any time ready to receive the energy given up by the other; and a surging once started between them receives periodical increments of energy from the line. This is analogous to the well-known mechanical phenomena that a number of gentle, but well-timed, mechanical impulses can set a very heavy suspended body into violent motion. The frequency of these impulses must correspond exactly to the natural period of oscillation of the body. In this parallel arrangement, serious damage is likely to result from resonance in overloading and burning out the conductors between the inductance and capacity.

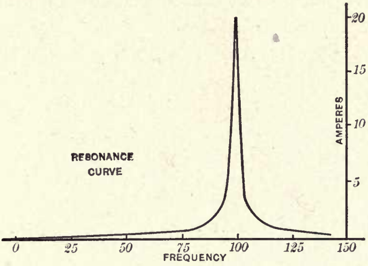


Fig. 48.

The protection of transformers and other station apparatus against high-potential surges coming from transmission lines is effected by the use of choke coils interposed between the lines and the station wiring. It is essential for proper protection, that the electrostatic capacity of this wiring be as small as possible and that the choke coil have as low an inductance as will allow the lightning arrester to take up the discharge, so that the frequency of resonance will be raised, thus decreasing the liability of picking up destructive voltages from line impulses or lightning discharges.

36. Damped Oscillations. — When a condenser is discharged through a circuit having resistance and inductance, an oscillatory current flows, the maximum values of which decrease logarithmically. The ratio of two successive maximum values can be shown equal to $\varepsilon^{\frac{\alpha}{2f}}$, where α is the damping factor and is equal to $\frac{R}{2L}$, R being the high-frequency resistance of the circuit. The entire exponent, $\frac{\alpha}{2f}$, is called the *logarithmic decrement*, and is represented by δ.

Hence
$$\delta = \frac{R}{4Lf},$$

and replacing f by
$$\frac{1}{2\pi}\sqrt{\frac{1}{LC}}, \qquad (\S\ 35)$$

there results
$$\delta = \frac{\pi R}{2}\sqrt{\frac{C}{L}}.$$

The effective current value of a train of damped oscilla-

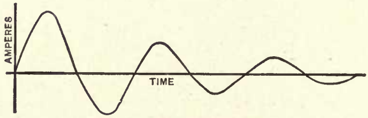


Fig. 49.

tions, one of which is shown in Fig. 49, can be deduced by considering the energy, stored in the condenser at each charge and discharged n times per second, to be consumed in heating the conductors. Then from § 26,

$$I^2R = n\frac{Q^2}{2C},$$

and substituting the value of R in the expression for δ, there is obtained

$$\delta = \frac{\pi n Q^2}{4CI^2}\sqrt{\frac{C}{L}};$$

and since $$Q = E_m C,$$

$$I^2 = \frac{n\pi E_m{}^2 C}{4\delta}\sqrt{\frac{C}{L}}.$$

Hence the effective value of the current is

$$I = \frac{E_m}{2}\sqrt{\frac{n\pi C}{\delta}\sqrt{\frac{C}{L}}}.$$

The natural frequency of a circuit in which a decaying oscillatory current flows is dependent only upon R, C and L, and may be obtained from the formula

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \alpha^2} \qquad \text{where } \frac{1}{LC} > \alpha^2.$$

When $\alpha^2 > \frac{1}{LC}$, the current is unidirectional and decreases as in Fig. 38.

37. Polygon of Impedances. — Consider a circuit having a number of pieces of apparatus in series, each of which may or may not possess resistance, inductance, and capacity. There can be but one current in that circuit when a pressure is applied, and that current must have the same phase throughout the circuit. The pressure at the terminals of the various pieces of apparatus, necessary to maintain through them this current, may, of course, be of different magnitude and in the same or different phases, being dependent upon the values of R, L, and C. Therefore to determine the pressure necessary to send a certain alter-

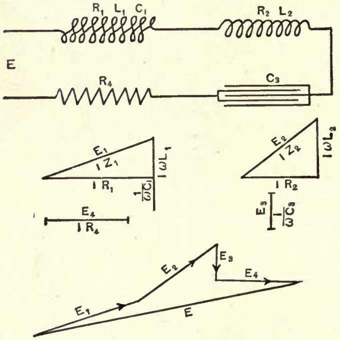


Fig. 50.

nating current through such a series circuit, it is but necessary to add vectorially the pressures needed to send such a current through the separate parts of the circuit. This is readily done graphically although in many cases the various quantities may be of such widely different magnitudes that it will be found more convenient to make use of trigonometrical expressions and methods.

Fig. 50 shows the pressures (according to § 31) necessary to send the current I through several pieces of apparatus, and the combination of these pressures into a polygon giving the resultant pressure E necessary to send the current I through the several pieces in series. In these diagrams, impedance is represented by the letter Z. C_1 and C_3 are localized, not distributed capacities.

For practical purposes, the quantity I, which is common to each side of the triangle, may be omitted; and merely the impedances may be added vectorially in a "polygon of impedances," giving an equivalent impedance, which, when multiplied by I, gives E.

Inspection of the figure shows that the analytical expression for the required E is

$$E = I\sqrt{(R_1+R_2+\cdots)^2+\left[\omega(L_1+L_2+\cdots)-\left(\frac{1}{\omega C_1}+\frac{1}{\omega C_2}+\cdots\right)\right]^2}.$$

The pressure at the terminals of any single part of the circuit is

$$E_1 = I\sqrt{R_1^2+\left[\omega L_1 - \frac{1}{\omega C_1}\right]^2},$$

$$E_2 = I\sqrt{R_2^2+\left[\omega L_2 - \frac{1}{\omega C_2}\right]^2},$$

$$E_3 = \cdot\ \cdot\ \cdot\ \cdot$$

It is evident that

$$E_1 + E_2 + \cdot\ \cdot\ \cdot\ \cdot > E,$$

and it is found by experiment that the sum of the potential differences, as measured by a voltmeter, in the various parts of the circuit, is greater than the impressed pressure.

38. A Numerical Example Applying to the Arrangement Shown in Fig. 50.—Suppose the pieces of apparatus to have the following constants:

$R_1 =$ 85 ohms,	$L_1 = .25$ henry,	$C_1 = .000018$ farad (18 mf.)
$R_2 =$ 40 ohms,	$L_2 = .3$ henry,	. . . .
. . . .	. . . .	$C_3 = .000025$ farad,
$R_4 =$ 100 ohms.	. . . .	. . . .

With a frequency of 60 cycles — whence $\omega = 377$ — it is required to find the pressure necessary to be applied to the circuit to send 10 amperes through it.

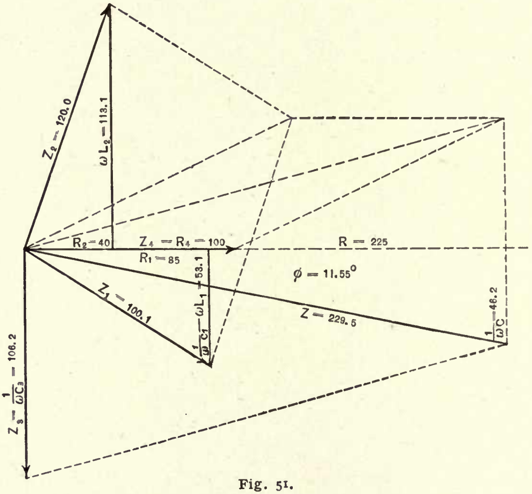


Fig. 51.

The completion of the successive parallelograms in Fig. 51, is equivalent to completing the impedance polygon, and the parts are so marked as to require no explanation. The solution shows that the equivalent impedance, $Z = 229.5$ ohms, that the equivalent resistance (= actual resistance in series), $R = 225$ ohms, that the equivalent reactance is condensive and equals 46.2 ohms, and that $\phi =$

11.55° of lead. Hence the pressure required to send 10 amperes through the circuit is

$$E = 10 \times 229.5 = 2295 \text{ volts.}$$

To obtain the same results analytically

$$E = 10\sqrt{[85+40+100]^2 + [(94.2+113.1)-(147.3+106.2)]^2}$$

$$E = 2295 \text{ volts.}$$

The voltages at the terminals of the various pieces of apparatus are:

$$\begin{aligned}
E_1 &= 10\sqrt{85^2+(94.2-147.3)^2} &&= 1001 \text{ volts,}\\
E_2 &= 10\sqrt{40^2+113.1^2} &&= 1200 \quad \text{"}\\
E_3 &= 10\sqrt{0^2+106.2^2} &&= 1062 \quad \text{"}\\
E_4 &= 10\sqrt{100^2+0^2} &&= 1000 \quad \text{"}\\
E_1+E_2+E_3+E_4 & &&= 4263 \quad \text{"}
\end{aligned}$$

which is greater than $E = 2195$ volts, showing that the *numerical* sum of the pressures is greater than the impressed pressure; while the vectorial sum of the separate pressures is equal to the impressed pressure.

39. **Polygon of Admittances.** — If a group of several impedances, Z_1, Z_2, etc., be connected in parallel to a common source of harmonic *E.M.F.* of E volts, their equivalent impedance is most easily determined by considering their admittances Y_1, Y_2, etc. The currents in these circuits would be

$$I_1 = EY_1,$$
$$I_2 = EY_2.$$

The total current, supplied by the source, would be the vector sum of these currents, due consideration being given to their phase relations. Calling this current I, the equation $I = EY$ can be written, where Y is the equivalent admittance of the group. To determine Y, a geometrical addition of Y_1, Y_2, etc., must be made, the angular relations being the same as the phase relations of I_1, I_2, etc., respectively. The value of the equivalent admittance may therefore be represented by the closing side of a polygon, whose other sides are represented in magnitude by the several admittances Y_1, Y_2, etc., and whose directions are determined by the phase angles of the currents I_1, I_2, etc., flowing through the admittances respectively.

Fig. 52 is a polygon of admittances showing the method of obtaining the equivalent admittance graphically for a

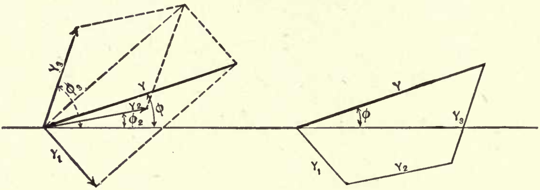


Fig. 52.

number of admittances in parallel. The equivalent admittance may also be determined analytically, since

$$Y = \sqrt{g^2 + b^2},$$

and hence it follows that the current

$$I = E\sqrt{(g_1 + g_2 + \ldots)^2 + (b_1 + b_2 + \ldots)^2},$$

where g_1, g_2, ... and b_1, b_2, ... are the respective conductances and susceptances of the various pieces of apparatus.

The instantaneous value of the current in the main circuit is equal to the sum of the instantaneous current values in the branch circuits, but since their maximum values occur at different times, the sum of the effective values of current in the branches generally exceeds the effective current value in the mains.

As a numerical example on the foregoing, consider the same apparatus as was used in the preceding example (§ 38), to be arranged in parallel, as in Fig. 53. It is required to find the current that will flow through the mains when a 60 ~ alternating *E.M.F.* of ten volts is impressed upon the circuit.

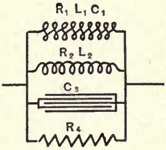


Fig. 53.

Remembering that $g = \frac{R}{Z^2}$ and that $b = \frac{X}{Z^2}$, and referring to § 38 for the numerical values, the conductances and susceptances of the branch circuits are

$$g_1 = \frac{85}{100.1^2} = .00848 \qquad b_1 = -\frac{53.1}{100.1^2} = -.0053$$

$$g_2 = \frac{40}{120^2} = .00278 \qquad b_2 = \frac{113.1}{120^2} = .00786$$

$$g_3 = 0 \qquad b_3 = -\frac{106.2}{106.2^2} = -.00942$$

$$g_4 = \frac{100}{100^2} = .01 \qquad b_4 = 0$$

Adding algebraically,

$$g = .02126 \qquad b = -.00686$$

Then $Y = \sqrt{(.02126)^2 + (-.00686)^2} = .0224.$

Hence $I = EY = 10 \times .0224 = .224$ amperes.

The phase of I is given by

$$\phi = \tan^{-1} \frac{b}{g} = \tan^{-1} \frac{-.00686}{.02126} = -17^\circ\ 53',$$

the negative sign indicating that the current leads the *E.M.F.* The admittance of each branch circuit and the value and phase of the current therein, may be calculated by proceeding in a similar manner.

40. Impedances in Series and in Parallel. — If a circuit have some impedances in series and some in parallel, or in any series parallel combination, the equivalent impedance can always be found by determining the equivalent impedances of the several groups, and then combining these resulting impedances to get the total equivalent impedance sought. To illustrate, a problem will be worked out in detail.

Let it be required to determine the values and phases of the currents in the main and in each of the four branch

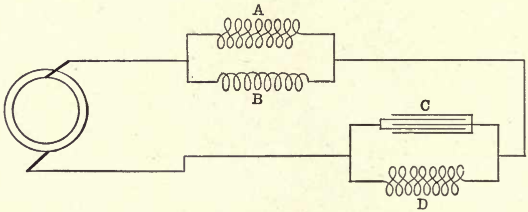


Fig. 54.

circuits, A, B, C and D of the combination shown in Fig. 54, when the main terminals are connected to a 200-volt 25-cycle alternator.

The constants of the apparatus and the results of the various steps in the calculation are given below in tabulated form, and require no explanation.

	A	*B*	*C*	*D*	
R	50	20	0	150	ohms
L	1	.5	0	6	henrys
C	.00002	0	.00001	.000008	farads
ωL	157.08	78.54	0	942.48	
$\frac{1}{\omega C}$	318.31	0	636.62	795.78	
X	− 161.23	78.54	− 636.62	146.70	
Z	168.80	81.05	636.62	209.80	
ϕ	− 72° 46′	75° 43′	− 90°	44° 38′	
Y	.00593	.01234	.001571	.00477	
g	.001758	.00305	0	.00339	
b	− .00566	.01195	− .001571	.00335	

g_{A+B}	= .00481	g_{C+D}	= .00339
b_{A+B}	= .00629	b_{C+D}	= .00178
Y_{AB}	= .00792	Y_{CD}	= .00383
ϕ_{AB}	= 52° 36′	ϕ_{CD}	= 27° 42′
Z_{AB}	= 126.3	Z_{CD}	= 261.2
R_{AB}	= 76.74	R_{CD}	= 231.3
X_{AB}	= 100.3	X_{CD}	= 121.4

R_t = 308.0 = total equiv. resistance

X_t = 221.7 = total equiv. reactance

Z_t = 379.5 = total equiv. impedance

I_t = .529 = current in mains

ϕ_t = 35° 45′ = phase of I_t.

$E_{AB} = Z_{AB} I_t = 66.81$ $\qquad$ $E_{CD} = Z_{CD} I_t = 138.17$

$I_A = .396$ $\qquad$ $I_C = .217$

$I_B = .824$ $\qquad$ $I_D = .649$

It is evident that the sum of the potential differences across the two groups is greater than the impressed *E.M.F.*, and that the arithmetical sum of the currents in the branch circuits exceeds the total current flowing in the main circuit. The relative magnitudes and the phases of the

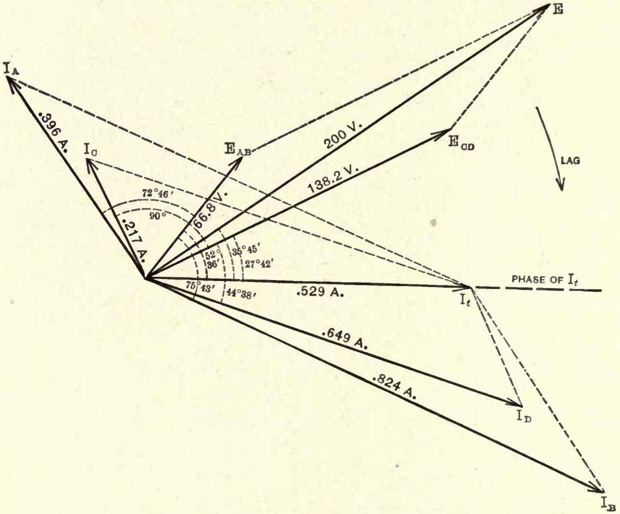


Fig. 55.

various currents and *E.M.F.*'s in the different circuits are represented in Fig. 55, which also serves as a rough check upon the calculations.

PROBLEMS.

1. A 60 ~ alternating *E.M.F.* of 200 volts maximum is impressed upon a circuit having 120 ohms resistance, an inductance of 1 henry and a capacity of 25 microfarads. Determine the value and phase of the current flowing in the circuit.

2. What are the values of X, Z, Y, b, and g in the preceding problem?

3. If a harmonic *E.M.F.* of 220 volts (effective) is impressed upon a circuit, producing a current of 20 amperes lagging 30°, what will be the resistance and reactance of the circuit?

4. Find the instantaneous current value in the circuit of Problem 1, $1\frac{3}{7}$ seconds after impressing the harmonic *E.M.F.* upon it.

5. What is the resonant frequency of a circuit having 10 microfarads capacity and an inductance of .352 henry? What will be the drop across the condenser at resonance, when 10 amperes flow through it?

6. A condenser of .003 mf. capacity is charged 20 times per second to a potential of 1000 volts. What is the mean effective current value of the discharge in a circuit having an inductance of 2 millihenrys, if the decrement of the oscillations is .2?

7. Determine the pressure required to send 10 amperes through the circuit shown in Fig. 50, if the frequency is 25 $\sim$ per second, the values of the resistances, inductances and capacities being the same as in § 38. Give also a graphic solution.

8. In the arrangement shown in Fig. 53, using the same impedances as in the preceding problem, what current will traverse the mains, if a 25-cycle alternating *E.M.F.* of 10 volts is impressed thereon?

9. If the 200-volt 25 $\sim$ alternator of § 40 were replaced by another of the same voltage but of 60 $\sim$ frequency, what would be the values and phases of the currents in the main and in the branch circuits?

10. Let the impedance *D* of the preceding problem be disconnected from the circuit. Then determine the values and phases of the currents in the main and in the branch circuits *A* and *B*. Construct a vector diagram showing the voltage and current relations.

CHAPTER V.

ALTERNATORS.

41. **Alternators.** — An alternator is a machine used for the conversion of mechanical energy into electrical energy, which is delivered as alternating current, either single-phase or polyphase. Alternators, like direct-current generators, have field-magnets and an armature; but the commutator of the direct-current machine is replaced in alternators by slip-rings, which deliver alternating current to brushes rubbing upon them when the armature rotates, or receive direct currents for exciting the field-magnets when they rotate. A polyphase alternator produces two or more single-phase alternating *E.M.F.*'s, which in operation send currents in circuits which may or may not be in electrical connection with each other. The only relation between these *E.M.F.*'s is that of time, that is, they differ in phase. These phase differences depend upon the relative positions of the armature windings and may be anything from 0° to 360°, but it is customary to place them so as to produce *E.M.F.*'s differing symmetrically in phase. In two-phase or four-phase alternators, the *E.M.F.*'s are 90° apart, in three-phase alternators 120° apart, and in six-phase alternators 60° apart.

As it is the relative motion of the armature and field-magnets which is essential in the generation of *E.M.F.*, it is quite as common to have the field-magnets of an alternator revolve inside the armature as to have the armature revolve; in fact, nearly all large alternators are of the revolving-field type, the revolving-armature type being now generally restricted to smaller units. The chief advantage of the revolving-field type alternator is that it avoids the collection of high-tension currents through brushes, since the armature connections are fixed, and only low-tension direct current need be fed through the rings to the field-coils. Other advantages are increased room for armature insulation, and, in polyphasers, the avoidance of more than two slip-rings. Revolving-field alternators have been constructed to generate 25,000 volts, whereas the *E.M.F.* produced in the revolving-armature type is practically limited to 5000 volts. In a few instances, notably at Niagara Falls, the field-magnets revolve outside the armature.

Besides the two types mentioned, there is the inductor type of alternator, which has both its armature and its field windings stationary and has an iron rotating member termed an inductor. In this type there are neither brushes, collector rings, nor moving electrical circuits.

It is necessary that all but the very smallest alternators should be multipolar to fit them to commercial requirements. For alternators must have in general a frequency between 25 and 125 cycles per second; the armature must be large enough to dissipate the heat generated at full load without its temperature rising high enough to injure the insulation; and finally, the peripheral speed of the armature cannot safely be made to exceed greatly a mile a minute. With these restrictions in mind, and knowing that a point on the armature must pass under two poles for each cycle, it becomes evident that alternators of anything but the smallest capacity must be multipolar.

42. Electromotive Force Generated. — In § 13, vol. i., it was shown that the pressure generated in an armature is

$$E_{av} = 2\,p\Phi S\,\frac{V}{60}\,10^{-8},$$

where p = number of pairs of poles,
Φ = maxwells of flux per pole,
V = revolutions per minute,
and S = number of inductors.

In an alternating circuit $E = k_1 E_{av}$, where k_1 is the form-factor, i.e., the ratio of the effective to the average *E.M.F.* Hence in an alternator yielding a sine wave *E.M.F.*,

$$E = 2.22\,p\Phi S\,\frac{V}{60}\,10^{-8}.$$

Inasmuch as $p\,\dfrac{V}{60}$ represents the frequency, f,

$$E = 2.22\,\Phi S f\,10^{-8}.$$

An alternator armature winding may be either concentrated or distributed. If, considering but a single phase, there is but one slot per pole, and all the inductors that are intended to be under one pole are laid in one slot, then the winding is said to be concentrated, and if the inductors are all in series the above formula for E is applicable. Nearly all engine-driven alternators have six slots per pole although twelve slots per pole are used when the output per pole is large and a long armature is undesirable. If now the inductors be not all laid in one slot, but be distributed in n more or less closely adjacent slots, the *E.M.F.* generated in the inductors of any one slot will be $\frac{1}{n}$ of that generated in the first case, and the pressures in the different slots will differ slightly in phase from each other, since they come under the center of a given pole at different times. The phase difference between the *E.M.F.* generated in two conductors which are placed in two successive armature slots, depends upon the ratio of the peripheral distance between the centers of the slots to the peripheral distance between two successive north poles considered as 360°. This phase difference angle

$$\phi = \frac{\frac{\text{width slot} + \text{width tooth}}{\text{circumference armature}}}{\text{no. pairs poles}} 360.$$

If the inductors of four adjacent slots be in series, and if the angle of phase difference between the pressures generated in the successive ones be ϕ, then letting E_1, E_2, E_3, and E_4 represent the respective pressures, which are

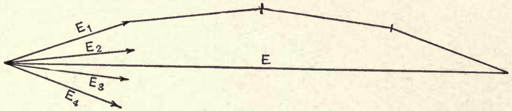


Fig. 56.

supposed to be harmonic, the total pressure, E, generated in them is equal to the closing side of the polygon as shown in Fig. 56. Obviously $E < E_1 + E_2 + E_3 + E_4$. If the winding had been concentrated, with all the inductors in one slot, the total pressure generated would have been equal to the algebraic sum.

The ratio of the vector sum to the algebraic sum of the pressures generated per pole and per phase is called the *distribution constant.* Not only may the number of slots under the pole vary, but they may be spaced so as to occupy the whole surface of the armature between successive pole centers (the peripheral distance between two poles is termed the pole distance), or they may be crowded together so as to occupy only one-half, one-fourth, or any other fraction of this space. Both the number of slots and the fractional part of the pole distance which they occupy affect the value of the distribution constant. A set of curves, Fig. 57, has been drawn, showing the values of this constant for various conditions. Curves are drawn for one slot (concentrated winding), 2, 3, 4 slots in a group, and many slots (i.e., smooth core with wires in close contact on the surface). The ordinates are the distribution constants, and the abscissæ the fractional part of the pole distance occupied by the slots.

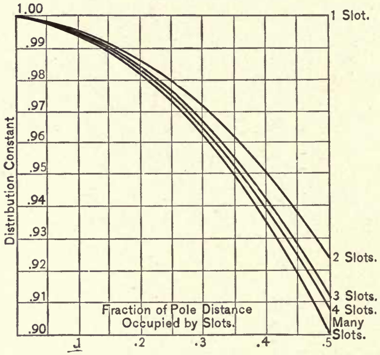


Fig. 57.

The distribution constant, k_2, must be introduced into the formula for the *E.M.F.* giving

$$E = 2\, k_1 k_2 p \Phi S \frac{V}{60}\, 10^{-8},$$

or, for sine waves,

$$E = 2.22\, k_2 \Phi S f\, 10^{-8}.$$

43. Armature Windings. — There are separate and distinct windings on the armature core of a polyphase alternator for each phase, and these may each be separately connected to an outside circuit through a pair of terminals, or they may be connected together in the armature according to some scheme whereby one terminal will be common to two phases. Some simple diagrams of the armature windings of multipolar alternators are given in the accompanying figures. Fig. 58 shows a single-phase concentrated winding, with the winding necessary to render it two-phase indicated by dotted lines. If the two windings be electrically connected where they cross at the point P, the machine becomes a star-connected four-phase or quarter-phase alternator.

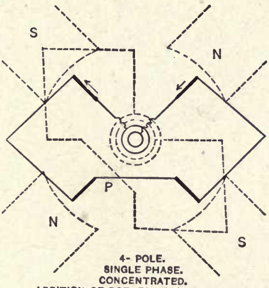


Fig. 58.

Three-phase alternators might be provided with six slip-rings or terminals, thus supplying three distinct circuits with single-phase alternating current, or with four slip-rings or terminals, one of which should be connected to a common return wire for the three currents. These are uncommon, however, since the usual practice is to provide only three slip-rings or terminals, each connected wire acting as a return path for the currents flowing in the other two. It follows, then, that the current in one wire of a three-phase system at any instant is equal and opposite to the sum of the currents in the other two wires at that instant. This is shown in Fig. 59, where the dotted curve, representing the sum of the two current curves, is exactly equal and opposite to the third current curve.

There are two methods of connecting the armature windings of three-phase alternators which are called respectively *Y*-and Δ-connections. In the first, one end of each winding

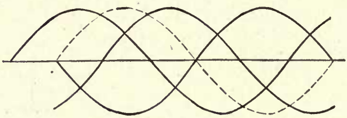


Fig. 59.

is connected to a slip-ring or terminal; the other ends being joined together form a neutral connection, which sometimes

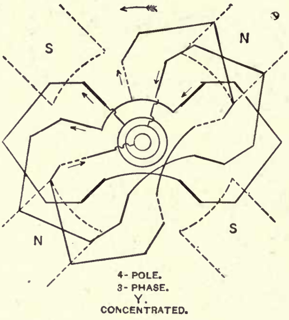


Fig. 60.

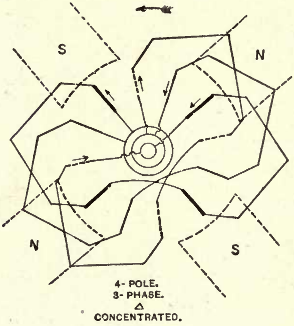


Fig. 61.

is connected with a fourth slip-ring or terminal adapting the alternator for use with a three-phase, four-wire system. In armatures having a Δ-connection, the three windings are connected together in series to form a closed circuit, each junction being connected to a slip-ring or terminal post. A three-phase, *Y*-connected, concentrated armature winding is shown in Fig. 60, and the same when Δ-connected is shown in Fig. 61.

In these winding diagrams the radial lines represent the inductors, and the other lines the connecting wires; the inductors of different phases being drawn differently for clearness. Where but one inductor is shown, in practice there would be a number wound into a coil and placed in one slot. For simplicity all the inductors of one phase are shown in series. Alternator armatures with distributed windings can also be represented diagrammatically similar to the foregoing, but the diagrams become very complex when there are many slots per pole per phase. For sim-

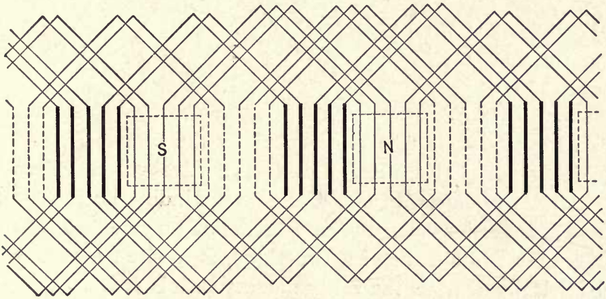


Fig. 62.

plicity, a rectified diagram is given, Fig. 62 representing the armature winding of a three-phase alternator. There are five slots per pole per phase.

44. Voltage and Current Relations in Two-Phase Systems. — A two-phase alternator may be considered as two separate single-phase alternators of the same size, the *E.M.F.*'s of which are maintained at a phase difference of 90°. The maintenance of the phase relation might be accomplished by mounting the two armatures on the same shaft and then placing the coils in the same relative position with the two field-magnets displaced from each other by ninety degrees, or with the field-magnets in the same relative position and the armature coils displaced by ninety degrees. Let these two alternators be denoted by 1 and 2, Fig. 63, and assume that the *E.M.F.* of alternator No. 2 lags 90° behind that generated in alternator No. 1. Then the time variations

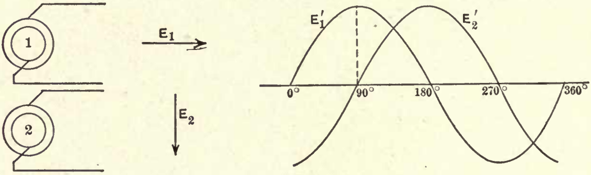


Fig. 63.

of their *E.M.F.*'s may be graphically represented as shown. This condition is also represented by the vectors E_1 and E_2, lag being clockwise. Let the effective values of the *E.M.F.*'s produced in either armature winding be E volts and the effective current value therein be I amperes. Then, since the two circuits of a two-phase, four-wire system are electrically distinct, the voltage across each is E volts, their

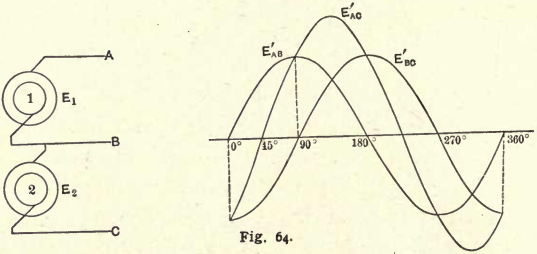


Fig. 64.

phase relations being shown by the vectors, and the current in each line wire is I amperes.

Now consider these two alternators to be connected as shown in Fig. 64, thus forming a two-phase, three-wire system. The other conditions remaining unaltered, the *E.M.F.*'s across AB and BC will be the same as before or E volts, and the current flowing in A or C will similarly be I amperes. The voltage across AC is due to the *E.M.F.*'s produced in both alternators, and its instantaneous value is equal to the algebraic sum of their simultaneous *E.M.F.*'s. The curves showing the time variation of these instantaneous values are annexed, and E_{AC} is seen to lag 45° behind E_{AB}. These conditions may also be represented by vectors as in Fig. 65, and therefrom

$$E_{AC} = E_{AB} \oplus E_{BC} = \sqrt{2}\, E,$$

which is therefore the voltage across the lines A and C, and it lags 45° behind E_{AB}.

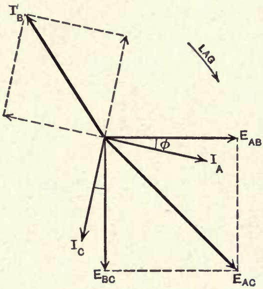


Fig. 65.

It should be noted that if E_2 leads E_1 by 90°, then E_{AC} will lead E_{AB} by 45°; and further, if the terminals of the receiving apparatus be reversed, the phases of the *E.M.F.*'s sending current through them will be reversed. Assuming load to be applied between A and B, and B and C only, and further that the circuits are balanced, the current in line C will then lag 90° behind the current in A, as shown in Fig. 65. Knowing that the instantaneous value of the current in line B is equal and opposite to the sum of the instantaneous current values in lines A and C, its value and phase may be determined by adding $-I_A$ and $-I_C$ vectorially as shown. Thus I_B is seen to be equal to $\sqrt{2}\,I$ and to lag $225° + \phi$ behind E_{AB}.

45. Voltage and Current Relations in Three-Phase Systems. — Consider a three-phase, Y-connected alternator to consist of three single-phase generators whose *E.M.F.*'s are maintained at a successive phase displacement of 120° (§ 44), their external connections being as shown in Fig. 66. Let the directions of the *E.M.F.*'s in the three armature coils as their axes successively pass a given fixed point, be positive, and let these conditions be indicated by the small arrows. Then, the phase relations of the armature electromotive forces will be represented as in Fig. 67, in which E_3 lags behind E_2, and E_2 lags behind E_1. The potential differences across the various line wires may then be determined by vectorial addition and subtraction; for example, the *E.M.F.* across AB is equal to the vectorial difference of E_1 and E_2, since they are oppositely directed. Taking the momentary positive flow as directed towards A, then

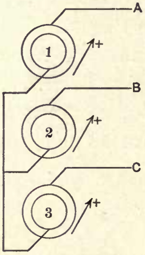


Fig. 66.

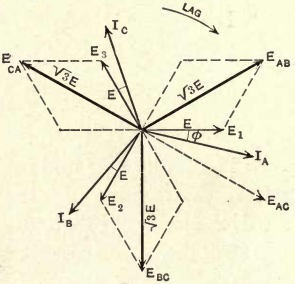


Fig. 67.

$$E_{AB} = E_1 \ominus E_2 \text{ and leads } E_1 \text{ by } 30°.$$

Similarly

$$E_{BC} = E_2 \ominus E_3 \text{ and lags } 90° \text{ behind } E_1,$$
$$E_{CA} = E_3 \ominus E_1 \text{ and lags } 210° \text{ behind } E_1.$$

Calling the *E.M.F.* generated in each armature E volts as before, the magnitudes of E_{AB}, E_{BC}, and E_{CA} will each be $\sqrt{3}\,E$, as may readily be proven by geometry. As the current flowing in each line wire is the same as that in each armature, it will be I amperes, and if the circuits are balanced, i.e. if three loads, each having the same resistance and the same reactance, are connected respectively between A and B, B and C, and C and A, the phases of the currents in them will be 120° apart, as shown. Therefore, in a three-phase, Y-connected system, the voltage between any two line wires is $\sqrt{3}\,E$ volts, and the current in each line is I amperes.

Now let these three alternators be connected as in Fig. 68, thus forming a three-phase mesh- or Δ-connected

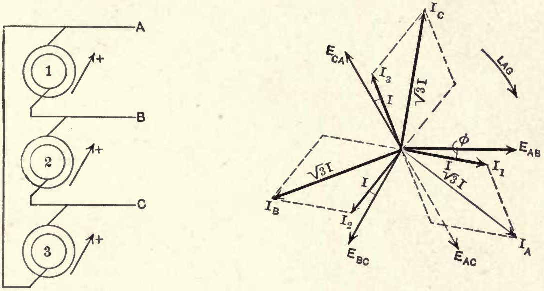


Fig. 68.

system. The *E.M.F.* across two line wires is produced by one alternator only and is therefore E volts, and if all the other conditions remain unchanged, E_{BC} will lag 120° behind E_{AB}, and E_{CA} will lag 120° behind E_{BC}. Assuming the three phases to be equally loaded, and representing positive current flow in the coils as their axes successively pass a fixed point by the small arrows, the magnitudes of the currents in the lines may be determined vectorially as in Fig. 68, where ϕ is the angle of lag. Hence

$$I_A = I_1 \ominus I_3 \text{ and lags } 30° + \phi \text{ behind } E_1,$$
$$I_B = I_2 \ominus I_1 \text{ and lags } 150° + \phi \text{ behind } E_1,$$
and $$I_C = I_3 \ominus I_2 \text{ and leads } E_1 \text{ by } 90° - \phi,$$

the magnitude of each being $\sqrt{3}\,I$. Then, to sum up, the voltage between any two lines in a balanced three-phase, Δ-connected system is E volts, and the current in each line wire is $\sqrt{3}\,I$ amperes.

The power delivered by a three-phase alternator is independent of the manner of connection, for in one case each leg is supplied with I amperes at $\sqrt{3}\,E$ volts, and in the other case with $\sqrt{3}\,I$ amperes at E volts.

46. Voltage and Current Relations in Four-Phase Systems. — To obtain the current and voltage relations in four-

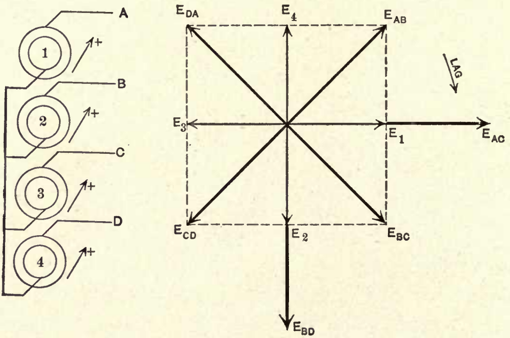


Fig. 69.

phase systems, consider the four-phase alternator to consist of four single-phase alternators whose $E.M.F.$'s are maintained ninety degrees apart successively. When these alternators are star-connected as in Fig. 69, it will become evident from an inspection of the vector diagram that the voltages between line wires are as follows, the order of the subscripts denoting momentary positive direction:

$$E_{AB} = E_1 \ominus E_2 = \sqrt{2}\,E \text{ and leads } E_1 \text{ by } 45^\circ,$$
$$E_{BC} = E_2 \ominus E_3 = \sqrt{2}\,E \text{ and lags } 45^\circ \text{ behind } E_1,$$
$$E_{CD} = E_3 \ominus E_4 = \sqrt{2}\,E \text{ and lags } 135^\circ \text{ behind } E_1,$$
$$E_{DA} = E_4 \ominus E_1 = \sqrt{2}\,E \text{ and lags } 225^\circ \text{ behind } E_1,$$
$$E_{AC} = E_1 \ominus E_3 = 2\,E \text{ and is in phase with } E_1,$$
$$E_{BD} = E_2 \ominus E_4 = 2\,E \text{ and lags } 90^\circ \text{ behind } E_1.$$

If the circuits are balanced, the current in each line wire is the same as that flowing in an armature winding, or I amperes.

When the four single-phase alternators are ring-connected as in Fig. 70, the voltage across adjacent line wires is E volts, and across alternate line wires is $\sqrt{2}\,E$ volts. The current in each line wire is $\sqrt{2}\,I$ amperes, and the phases of these currents are represented in Fig. 71.

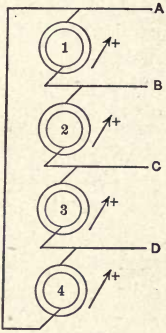


Fig. 70.

The relations of the voltages and currents in the armature windings of a six-phase alternator to the voltages across the line wires and to the currents therein may similarly be determined.

47. Measurement of Power. — The power delivered to the receiving circuits of a two-phase, four-wire system can be measured by two wattmeters, one connected in each phase. The sum of their readings is the total power supplied. If the load is balanced, one of the wattmeters may be dispensed with, and the total power is then double the reading of the other.

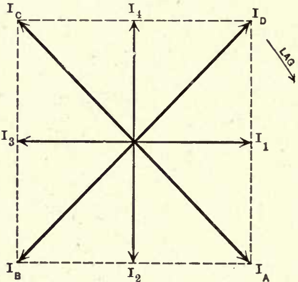


Fig. 71.

In any two-phase, three-wire system the power can be measured by two wattmeters connected as in Fig. 72. The sum of the instrument readings is the whole power. In a two-phase, three-wire system, where all the load is connected between the outside wires and the common wire, and none between the outside wires themselves, and where the load is balanced, then one wattmeter can be used to measure the whole power by connecting its current coil in the common wire and its pressure-coil between the common wire and one outside wire first, then shifting this connection to the other outside wire, as indicated in Fig. 73. The sum of the instrument readings in the two positions is the whole power. A wattmeter made with two pressure-coils could have one connected each way, and the instrument would automatically add the readings, giving the whole power directly. Or, again, a high non-reactive resistance could be placed between the two outside wires and the pressure-coil of the wattmeter connected between

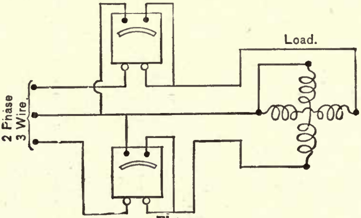


Fig. 72.

the common wire and the center point of this resistance. This requires that the wattmeter be recalibrated with half of this high resistance in series with its pressure-coil.

With the exception of the two-phase systems, the power in any balanced polyphase system may be measured by one wattmeter whose current coil is placed in one wire, and whose pressure-coil is connected between that wire and the neutral point. The instrument reading multiplied by the number of phases gives the whole power. The neutral point may be on an extra wire, as in a three-phase, four-wire system; or may be artificially constructed by connecting the ends of equal non-reactive resistances together, and connecting the free ends one to each of the phase wires.

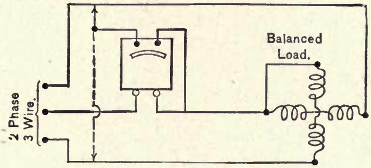


Fig. 73.

With the exception of the two-phase systems, the power in any n-phase, n-wire system, irrespective of balance, may be determined by the use of $n - 1$ wattmeters. The current coils are connected, one each, in $n - 1$ of the wires, and the pressure-coils have one of their ends connected to the respective phase wires, and their free ends all connected to the nth wire. The *algebraic* sum of the readings is the power in the whole circuit. Depending upon the power factor of the circuit, some of the wattmeters will read negatively, hence care must be taken that all connections are made in the same sense; then those instruments which require that their connections be changed, to make them deflect properly, are the ones to whose readings a negative sign must be affixed.

Some specific connections for indicating wattmeters in three-phase circuits are shown in the following figures. Fig. 74 shows the connection of three wattmeters to measure the power in an unbalanced three-phase system. All

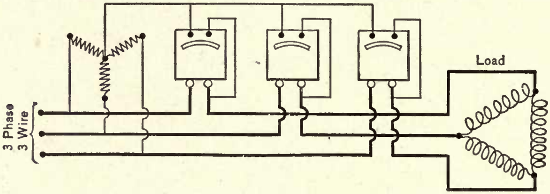


Fig. 74.

the readings will be in the positive direction, and their sum is the total power. If a fourth, or neutral wire be present, it should be used, instead of creating an artificial neutral, as shown. The magnitude of the equal non-reactive resistances, used to secure this neutral point, must be so chosen that the resistances of the pressure-coils of the wattmeters will be so large, compared thereto, as not to disturb the potential of the artificial neutral point.

Fig. 75 shows the connection of one wattmeter, so as to read one-third of the whole power in a balanced, three-phase, four-wire system. If the system be three-wire, a neutral point may be created as in Fig. 74.

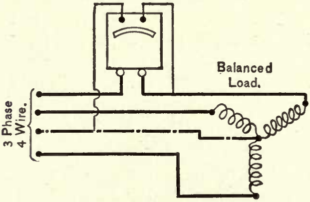


Fig. 75.

Another method of measuring power in a balanced three-phase system, either Δ-connected or Y-connected, is based upon the assumption that both pressures and currents vary harmonically. No neutral point is required, and the connections are shown in Fig. 76. The free end of the pressure-coil is connected first to one of the wires other than that in which the current coil is connected, and then to the other. The angular displacements between the current in any line wire and the *E.M.F.*'s between

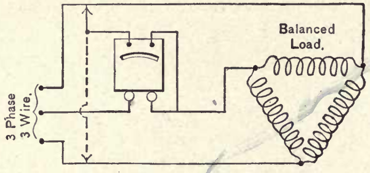


Fig. 76.

it and the other line wires are $30° + \phi$ and $30° - \phi$, as will become evident from an inspection of Fig. 67 and Fig. 68. The readings of the wattmeters are then

$$P_1 = \sqrt{3}\, EI \cos (30° + \phi)$$

and

$$P_2 = \sqrt{3}\, EI \cos (30° - \phi)$$

where E is the *E.M.F.* generated and I is the current flowing in each armature coil. The algebraic sum of the readings is

$$\begin{aligned} P_1 + P_2 &= \sqrt{3}\, EI [\cos (30° + \phi) + \cos (30° - \phi)] \\ &= \sqrt{3}\, EI [\tfrac{1}{2}\sqrt{3} \cos \phi - \tfrac{1}{2} \sin \phi + \tfrac{1}{2}\sqrt{3} \cos \phi + \tfrac{1}{2} \sin \phi] \\ &= 3\, EI \cos \phi \end{aligned}$$

which is the total power delivered. When ϕ is greater than 60°, P_1 becomes negative, hence care is required to avoid confusion of signs at low power factors. Both readings will be positive if the power factor is greater than .5, but one of them will be negative if it is less than this value.

The algebraic difference of the two wattmeter readings is

$$\begin{aligned} P_2 - P_1 &= \sqrt{3}\, EI [\cos (30° - \phi) - \cos (30° + \phi)] \\ &= \sqrt{3}\, EI [\tfrac{1}{2}\sqrt{3} \cos \phi + \tfrac{1}{2} \sin \phi - \tfrac{1}{2}\sqrt{3} \cos \phi + \tfrac{1}{2} \sin \phi] \\ &= \sqrt{3}\, EI \sin \phi. \end{aligned}$$

It is more convenient, however, to consider line voltages and line currents instead of those in the alternator armature windings or in the load of each phase. Therefore, representing the *E.M.F.* between any two line wires by E_l, and the current in each line by I_l, then, since either $E_l = \sqrt{3}\,E$ (Y-connection) or $I_l = \sqrt{3}\,I$ (Δ-connection), by dividing the previous results by $\sqrt{3}$, there is obtained

$$P_1 + P_2 = \sqrt{3}\,E_l I_l \cos\phi,$$
$$P_2 - P_1 = E_l I_l \sin\phi.$$

In the balanced three-phase system under consideration, it is possible to determine the power factor of the similar receiving circuits by the use of a single wattmeter connected as in Fig. 76. The readings of the wattmeter in the two positions are the only observations required. The power factor is clearly

$$\cos\phi = \cos\tan^{-1}\left[\sqrt{3}\,\frac{P_2 - P_1}{P_1 + P_2}\right],$$

which is derived from the two preceding equations.

An accurate method for the determination of the power in unbalanced three-phase systems, avoiding the necessity of a neutral point, involves the use of two wattmeters connected as in Fig. 77. The algebraic sum of the instrument indications is the total power supplied. It is possible to obtain negative readings, but since the currents lag behind their respective *E.M.F.*'s by different amounts in an unbal-

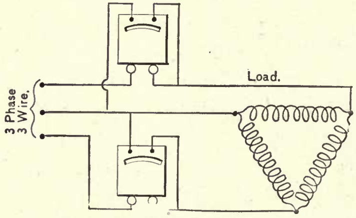


Fig. 77.

anced system, it cannot be said that when the power factor is less than 0.5 one instrument reads negatively, for the term power factor here has no definite significance. To determine, then, the correct signs of the wattmeter readings, the given load may be replaced by a non-inductive balanced load of lamps, and if the terminals of the potential coil of one instrument must now be reversed to deflect properly, it shows that the negative sign must be affixed to its reading on the load to be measured.

48. Saturation. — The electromotive force produced in an alternator at no-load is dependent upon the peripheral speed of the rotating member and upon the field excitation. The relation of the open circuit voltage to the field current when the alternator is driven at constant speed may be represented by a curve called the no-load saturation curve. For a certain 65 K.W. two-phase 2400-volt 60-cycle inductor alternator, running at 900 revolutions per minute (air-gap of 85 mils), the no-load saturation curve has been experimentally determined, and is shown in Fig. 78, curve *A*. It indicates, for example, that the field current necessary to produce the rated voltage on open circuit when the machine runs at its proper speed is 4.45 amperes. It is seen that this curve is almost straight for small exciting currents. At small excitation, the reluctance of the air-gap is very high and that of the iron very low, and therefore the former may be considered as constituting the entire reluctance of the magnetic circuit. Since the reluctivity of air is constant regardless of the flux density, at small excitations the flux will be proportional to the magnetomotive force, and therefore the open-circuit voltage is proportional to the field current, hence the curve is straight. As the field becomes stronger, however, the proportion of the air-gap reluctance to the entire reluctance decreases, for the permeability of iron decreases with increased flux-density, and therefore the *E.M.F.* increases less rapidly with increased excitation.

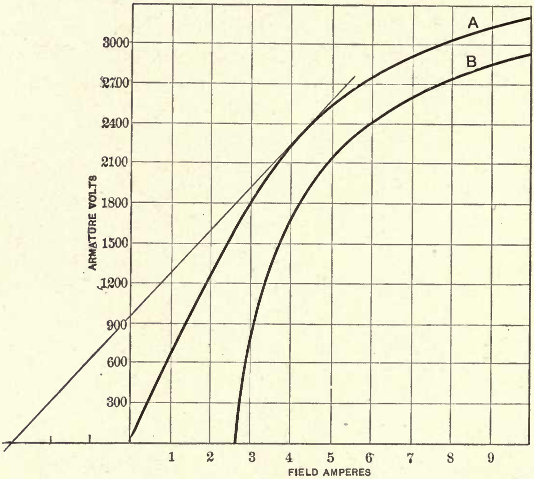


Fig. 78.

The percentage of saturation of an alternator at any excitation may be found from its saturation curve by drawing to it a tangent at the assigned excitation and determining its intercept on the axis of ordinates. The ratio of this intercept to the ordinate of the curve at the assigned excitation, expressed as a percentage, is the percentage of saturation. For example, the percentage of saturation of the alternator mentioned, when the field current is 4.45 amperes, is

$$\frac{950}{2400} \times 100 = 39.6\%.$$

The ratio of a small percentage increment of field excitation in an alternator to the corresponding percentage increment of terminal voltage produced thereby, is called the saturation factor. Unless otherwise specified, it refers to the excitation existing at normal rated speed and voltage and on open circuit. The saturation factor is a criterion of the degree of saturation and may be expressed as

$$k = \frac{1}{1 - m}$$

where m is the percentage of saturation. Thus, the saturation factor of the alternator whose saturation curve is shown in Fig. 78, is

$$\frac{1}{1 - .396} = 1.66.$$

The relation of the terminal voltage to the field current when the alternator is driven at its rated speed and delivering its rated current is given by the full-load saturation curve, which is somewhat similar in shape to the no-load saturation curve. It may be determined experimentally by employing variable non-inductive resistances for maintaining the constant full-load current on each phase, and noting the terminal voltage corresponding to various excitations. The full-load saturation curve for the 65 K.W. inductor alternator at unity power factor is shown as curve B in Fig. 78. As this curve takes into account all of the diverse causes of decrease in terminal voltage resulting from the application of a load to the machine, it is important in the calculation of regulation. In alternators of large capacity, it is a difficult matter to determine the full-load saturation curve by test, and consequently other methods are usually employed.

If the alternator is normally excited to above the knee of the saturation curve, it will require a considerable increase of field current to maintain the terminal voltage when the load is thrown on, while, if normally excited below the knee, a slight increase of excitation will suffice.

49. Regulation. — The regulation of an alternator is the ratio of the maximum difference of terminal voltage from the rated load value, occurring within the range from open circuit to rated load, to the rated load terminal voltage, the speed and field current remaining constant. As the maximum deviation during this range generally occurs at the rated load, it is customary to define regulation as the ratio of the rise in terminal voltage, that occurs when full load at unity power factor is thrown off, to the terminal voltage. Or, expressing it in the form of an equation,

$$\text{Regulation} = \frac{\text{No-load Voltage} - \text{Full-load Voltage}}{\text{Full-load Voltage}}.$$

An alternator having perfect regulation is one which shows no increase in terminal voltage upon opening its load circuit, that is, the regulation is zero.

With small machines, the regulation can be easily determined by test, provided artificial loads are available. It is simply necessary to plot the no-load and full-load saturation curves, and from them the regulation at any load can be found. Referring to Fig. 78, for example, the regulation of the alternator at full load with unity power factor is $\frac{2750 - 2400}{2400} = .146$ or 14.6 per cent. With large machines, however, artificial loads are not usually available, and the determination of regulation in this case is more difficult and less accurate.

The factors affecting the voltage drop in an alternator upon the application of load thereto, are, the armature resistance, armature reactance, and magnetization or demagnetization, occurring especially at low power factors. These factors are sometimes grouped together and dealt with collectively by the use of a quantity called the synchronous impedance. It is that impedance, which, if connected in series with the outside circuit and to an impressed voltage of the same value as the open-circuit voltage at the given speed and excitation, would permit a current of the same value to flow as does flow.

The armature resistance drop, seldom exceeding three per cent of the terminal voltage, is, for each phase, equal to the product of the resistance of the armature winding of that phase and the current flowing through it. In calculating regulation, the hot resistance (at 75° C.) of the windings is always taken.

The armature conductors of an alternator cut the magnetic flux due to the field current, and this flux may be considered as sinusoidally distributed at no-load. An *E.M.F.* will thereby be produced, which will cause a current to flow through the armature windings and through the load circuit. The armature ampere-turns set up a magnetic flux which is superimposed upon the field flux. The magnitude and phase of the terminal electromotive force will depend upon this resulting flux, and, if that due to the field excitation be constant, then the terminal *E.M.F.* will vary in a manner depending upon the flux due to the armature current, which, for brevity, will be called the armature flux. The armature self-induction, being proportional to the armature flux, varies and depends upon the relative positions of the armature and the field and upon the magnitude and phase of the currents in the armature windings. This variation is shown in Fig. 79, which gives the inductance corresponding to different angular positions of the

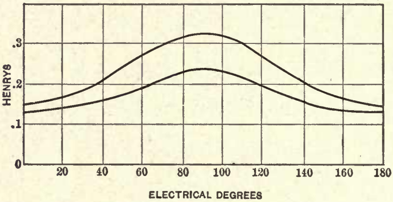


Fig. 79.

armature, zero degrees representing coincidence of pole and coil group center lines. These curves refer to the 65 K.W. two-phase inductor alternator (§ 48), with a current of 9 amperes flowing through the winding of one phase only. The upper curve embodies results taken with the field coil open-circuited, and the lower one with the field coil short-circuited. Thus, in single-phase alternators, the armature flux varies in time and in space.

Consider a polyphase alternator having a revolving armature, a distributed armature winding, a magnetic circuit yielding a uniform magnetic reluctance as regards the flux due to the current in any armature conductor, and a balanced load. The armature flux will be approximately sinusoidally distributed in space and stationary as regards the field winding, for it would revolve backward as fast as the armature revolves forward. The axis of the armature flux, when the current through the conductors is in phase with the *E.M.F.*, is at right angles to that of the field flux, as shown in Fig. 80 by the dotted line *sn*. When the load on each phase is inductive, the axis of the armature flux is displaced in the direction of rotation; and when the current supplied to the load leads the *E.M.F.* of the alternator, the axis of the armature flux is displaced in the direction opposite rotation. These conditions are represented by the dotted lines $s'n'$ and $s''n''$ respectively. From an inspection of the figure, it becomes evident, that with a non-reactive load the armature flux neither assists nor opposes the field flux; with an inductive load, the armature flux has a component which is opposite to the field flux; and with a capacity load the armature flux has a component which is in the same direction as the field flux. The magnetomotive force causing the armature flux may then be considered as composed of two components, a transverse component, which is a measure of the armature inductance, and the magnetizing or demagnetizing component, which acts either with or against the field magnetomotive force, depending upon the nature of the load.

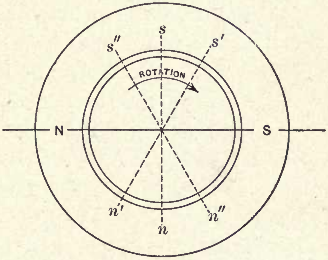


Fig. 80.

Commercial alternators do not have a uniform magnetic reluctance, a perfectly distributed winding, nor a sinusoidal flux-distribution; and therefore an exact theoretical treatment of alternator regulation is impossible.

50. E.M.F. and M.M.F. Methods of Calculating Regulation. — Two methods of calculating the regulation of alternators from the results of other than full-load tests have been widely employed, but the results are only approximate. The first, called the *E.M.F.* method, generally gives a larger regulation, and the second, called the *M.M.F.* or *A.I.E.E.* method, generally gives a smaller regulation than what is obtained by test.

The *E.M.F.* method may be stated as follows: To determine the regulation of an alternator when supplying a given current to a receiving circuit of unity power factor, add the armature resistance drop to the rated terminal voltage, and add the sum vectorially at right angles to the armature impedance voltage, that is, the open-circuit voltage corresponding to the given short-circuit current value. This result minus the rated voltage gives the voltage rise at the required load, and dividing this by the rated voltage, the regulation at that load will be obtained.

Consider these factors in detail. Instead of expressing the armature resistance drop in terms of the resistance of each winding and the current therein, it is desirable in practice to express it in terms of the line current and the resistance between any two armature terminals. For example, take a three-phase alternator and assume it to be connected to a balanced load. Representing the line voltages and line currents respectively by E and I, and the resistance of each armature winding by r, then, in a Y-connected alternator, the total copper loss is $3\,I^2r$, and in a Δ-connected machine the total copper loss is $3\left(\frac{I}{\sqrt{3}}\right)^2 r = I^2r$.

If R is the armature resistance between terminals, then $R = 2\,r$ in a Y-connected alternator, and $R = \dfrac{1}{\dfrac{1}{r} + \dfrac{1}{2r}} = \dfrac{2}{3}\,r$ in a Δ-connected alternator. Hence the total copper loss, whether the machine is Y- or Δ-connected, is $\frac{3}{2}\,I^2R$ or $3\,I^2\frac{R}{2}$.

To reduce this result to an equivalent single-phase circuit with the same voltage between line wires and representing the same power, P, consider that the rated current per terminal in a single-phaser is $\frac{P}{E}$, in a two-phaser is $\frac{P}{2E}$, and in a three-phaser is $\frac{P}{\sqrt{3}E}$. Denoting the equivalent single-phase current by I_{eq}, it follows that in a single-phase circuit $I_{eq} = I$, in a two-phase circuit $I_{eq} = 2I$, and in a three-phase circuit $I_{eq} = \sqrt{3}I$. The equivalent single-phase copper loss in a three-phase alternator is $\frac{1}{2}I^2_{eq}R$. Dividing by I_{eq}, there results the equivalent single-phase armature resistance drop of a three-phase alternator, which is $\frac{1}{2}I_{eq}R$. This result is also true for two-phase star- or ring-connected alternators, as may readily be proven. Thus, the armature resistance drop in a polyphase alternator is the product of the equivalent single-phase current and half the armature resistance as measured between terminals.

The armature impedance voltage is obtained from the short-circuit current and the no-load saturation curves. The short-circuit current curve represents the field excitations required to send various currents through the short-circuited armature windings, and may be obtained by direct test without requiring large power expenditures. The short-circuit current curve for the alternator considered in the two preceding articles is shown in Fig. 81.

As a numerical example, let it be required to determine the regulation of this 65 K.W. two-phase 2400-volt alternator at full load with unity power factor, the armature resistance between terminals being 5 ohms at 25° C.

The rated current per terminal of the alternator is $\frac{P}{2E} = \frac{65{,}000}{2 \times 2400} = 13.5$ amperes, and the equivalent single-phase current is $\frac{P}{E} = 27$ amperes. Half of the armature resistance measured between terminals at 75° C. is $\frac{5}{2} + 50 \times 2.5 \times .004 = 3$ ohms. Hence the armature *IR* drop is $27 \times 3 = 81$ volts and is in phase with the terminal voltage, since the power factor of the load is assumed to be unity. The sum of the armature resistance drop and the

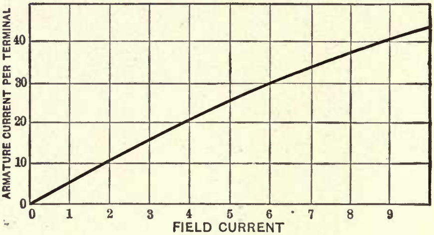


Fig. 81.

terminal voltage is 2481 volts. The excitation required to produce the rated current (13.5 amperes) is 2.55 amperes, as obtained from Fig. 81. From the no-load saturation curve of Fig. 78 is found the impedance voltage corresponding to this excitation, and is 1550 volts. Adding the 2481 volts and the 1550 volts at right angles, there results the voltage that is considered from the standpoint of this method to be actually generated in the alternator,

$$\sqrt{(2481)^2 + (1550)^2} = 2924 \text{ volts.}$$

Hence the regulation at full load with unity power factor is

$$\frac{2924 - 2400}{2400} = .218 \text{ or } 21.8\%.$$

The result for the same conditions obtained from the no-load and the full-load saturation curves is 14.6%, thus showing that the *E.M.F.* method gives a poorer regulation than is obtained by test.

Let it be required to calculate the regulation of the same alternator at $\frac{3}{4}$ full-load by the *E.M.F.* method, when the power factor of the receiving circuits is 80%.

The rated $\frac{3}{4}$ full-load current is 10.125 amperes and the equivalent single-phase current is 20.25 amperes, hence the armature *IR* drop is $20.25 \times 3 = 60.75$ volts. The terminal *E.M.F.* can be resolved into two components, one in phase with and the other at right angles to the current. These components are respectively $2400 \times .8$ or 1920 volts, and $2400 \times \sin \cos^{-1} .8 = 2400 \times .6 = 1440$ volts. The impedance voltage as obtained from the curves of Figs. 81 and 78 is found to be 1220 volts. The result of adding these *E.M.F.*'s in their proper phases is the voltage actually generated in the alternator, namely

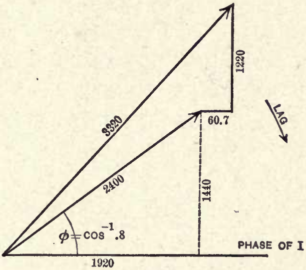


Fig. 82.

$$\sqrt{(1920 + 60.75)^2 + (1440 + 1220)^2} = 3320 \text{ volts,}$$

as shown diagrammatically in Fig. 82. The regulation, then, at $\frac{3}{4}$ full-load and 80% power factor is

$$\frac{3320 - 2400}{2400} = 38.3\%.$$

The *M.M.F.* method of calculating alternator regulation at unity power factor may be stated as follows: — The exciting ampere-turns corresponding to the terminal voltage plus the armature resistance drop, and the ampere-turns corresponding to the impedance voltage, are combined vectorially to obtain the resultant ampere turns, and the corresponding internal *E.M.F.* is obtained from the no-load saturation curve. The difference between this *E.M.F.*

and the rated voltage is divided by the rated voltage to obtain the regulation.

As a numerical example, let it be required to calculate the regulation of the same alternator at full load with unity power factor by the *M.M.F.* method.

The field current corresponding to 2400 + 81 volts is 4.7 amperes, as obtained from the no-load saturation curve of Fig. 78. The field current corresponding to the impedance voltage (1550 volts) is found from the same curve and is 2.55 amperes. This value can also be obtained directly from Fig. 81; it corresponds to the rated current (13.5 amperes). Then, adding 4.7 amperes and 2.55 amperes at right angles, there results $\sqrt{4.7^2 + 2.55^2}$ or 5.35 amperes. The no-load voltage corresponding to this excitation is 2620 volts Therefore the regulation is

$$\frac{2620 - 2400}{2400} = .0916 \text{ or } 9.16\%.$$

This result is much smaller than that obtained by test (14.6%), that is, the *M.M.F.* method for these conditions gives a better regulation than it should. The mean value of the results obtained by the *E.M.F.* and *M.M.F.* methods is 15.5% and agrees fairly well with the experimental result; but this is not always true.

51. Regulation for Constant Potential. — Alternators feeding light circuits must be closely regulated to give satisfactory service. The pressure can be maintained constant in a circuit by a series boosting transformer, but it is generally considered better to regulate the alternator by suitable alteration of the field strength.

The simplest method of regulating the potential is to have a hand-operated rheostat in the field circuit of the alternator, when the latter is to be excited from a common source of direct current, or in the field circuit of the exciter, if the alternator is provided with one. The latter method is generally employed in large machines, since the exciter field current is small, while the alternator field current may be of considerable magnitude, and would give a large I^2R loss if passed through a rheostat.

A second method of regulation employs a composite winding analogous to the compound windings of direct-

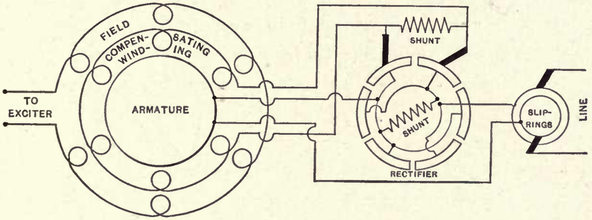


Fig. 83.

current generators. This consists of a set of coils, one on each pole. These are connected in series, and carry a portion of the armature current which has been rectified. The rectifier consists of a commutator, having as many segments as there are field poles. The alternate segments are connected together, forming two groups. The groups are connected respectively with the two ends of a resistance forming part of the armature circuit. Brushes, bearing upon the commutator, connect with the terminals of the composite winding. The magnetomotive force of the composite winding is used for regulation only, the main excitation being supplied by an ordinary separately excited field winding. The rectified current in the composite coils is a pulsating unidirectional current that increases the magnetizing force in the fields as the current in the armature increases. The rate of increase is determined by the resistance of a shunt placed across the brushes. By increasing the resistance of this shunt, the amount of compounding can be increased. With such an arrangement an alternator can be over-compounded to compensate for any percentage of potential drop in the distributing lines. The method here outlined is used by the General Electric Company in their single-phase stationary field alternators. The connections are shown in Fig. 83.

A third method of regulation is employed by the Westinghouse Company on their revolving armature alternators, one of which, a 75 K.W., 60~, single-phase machine, is shown in Fig. 84. A composite winding is employed, and the compensating coils are excited by current from a series transformer placed on the spokes of the armature spider. The primary of this transformer consists of but a few turns, and the whole armature current is conducted through it before reaching the collector rings. The secondary of this transformer is suitably connected to a simple commutator on the extreme end of the shaft. Upon this rest the brushes which are attached to the ends of the compensating coil. This commutator is subjected to only moderate currents and low voltages. The current in the secondary of the transformer, and hence that in the compensating coil, is proportional to the main armature current. The machine is wound for the maximum desirable over-compounding, and any less compensation can be secured by slightly shifting the commutator brushes. For there are only as many segments as poles; and if the brushes span the insulation just when the wave of current in the transformer secondary is passing through zero, then the pulsating direct current in the compounding coil is equal to the effective value of the alternating current; but if the brushes are at some other position, the current will flow in the field coil in one direction for a portion of the half cycle, and in the other direction for the remaining

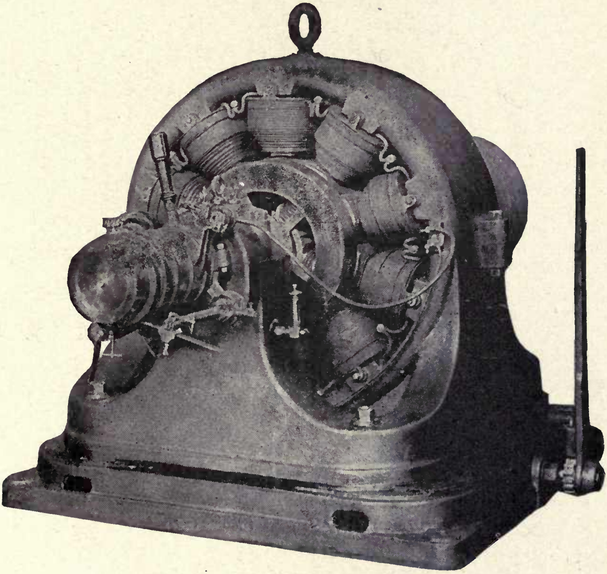


Fig. 84.

portion. A differential action, therefore, ensues, and the effective value of the compensating current is less than it was before.

In order to produce a constant potential on circuits having a variable inductance as well as a variable resistance, the General Electric Co. has designed its compensated revolving field generators, which are constructed for two- or three-phase circuits. The machine, Fig. 85, is of the

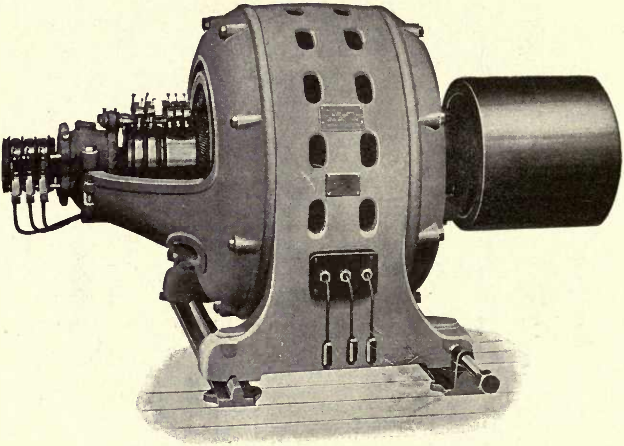


Fig. 85.

revolving field type, the field being wound with but one simple set of coils. On the same shaft as the field, and close beside it, is the armature of the exciter, as shown in Fig. 86. The outer casting contains the alternator armature windings, and close beside them the field of the exciter. This latter has as many poles as has the field of the alternator. Alternator and exciter, therefore, operate in a synchronous relation. The armature of the exciter is fitted with a regular commutator, which delivers direct current both to the exciter field and, through two slip-rings, to the alternator field. On the end of the shaft, outside of the bearings, is a set of slip-rings, four for a quarter-phaser, three for a three-phaser, through which the exciter armature receives alternating current from one or several series transformers inserted in the mains which lead from the alternator. This alternating current is passed through the exciter armature in such a manner as to cause an armature reaction, as described in § 49, that increases the magnetic flux. This raises the exciter voltage and hence increases the main field current. The

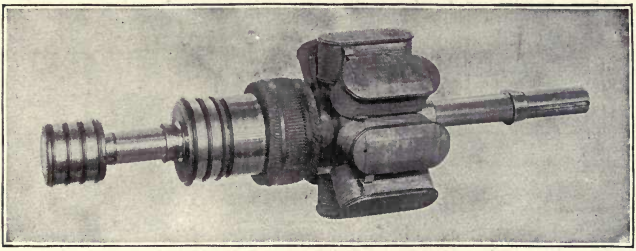


Fig. 86.

reactive *magnetization* produced in the exciter field is proportional to the magnitude and phase of the alternating current in the exciter armature. The reactive *demagnetization* of the alternator field is proportional to the magnitude and phase of the current in the alternator armature. And these currents have the fixed relations of current strength and phase, which are determined by the series transformers. Hence the exciter voltage varies so as to compensate for any drop in the terminal voltage. Neither the commutator nor any of the slip-rings carry pressures of over 75 volts. The amount of over-compounding is determined by the ratio in the series transformers. The normal voltage of the alternator may be regulated by a small rheostat in the field circuit of the exciter. The various connections of this type of compensated alternator are shown in Fig. 87.

The regulation of voltages by means of composite wind-

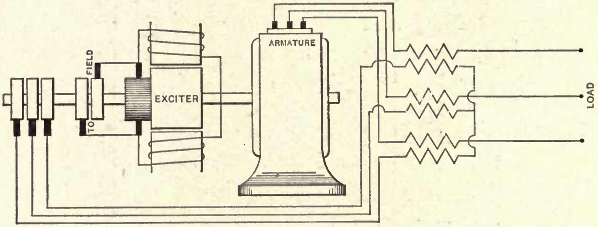


Fig. 87.

ings finds application on alternators up to about 250 K.W. output. The Tirrill Regulator, for use with large or small generators, is made by the General Electric Company, and shown in Fig. 88. This device operates by rapidly opening and closing a shunt circuit connected across the exciter field rheostat, the operation being accomplished by means of a differentially wound relay, which is connected to the exciter bus-bars. There are two control magnets, one for direct current and the other for alternating current. The current for the first is taken from the exciter bus-bars, and the current for the latter is taken from a potential transformer connected in the circuit to be regulated. Upon the same spool as this potential winding is an adjustable compensating winding which is connected to the secondary of a current transformer inserted in the principal lighting circuit. The cores of these control magnets are attached to pivoted levers provided with contacts at their other ends.

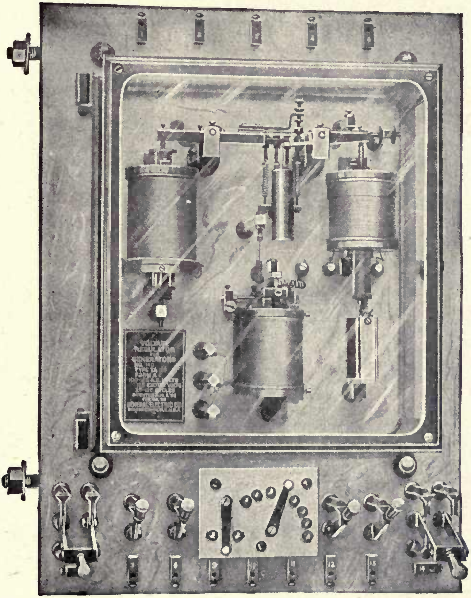


Fig. 88.

When a load is thrown on the alternator, the voltage will tend to drop and the alternating-current magnet will weaken, thus causing the main contacts to close. This also causes the relay contacts to close and short-circuit the exciter field rheostat, thereby increasing the potential

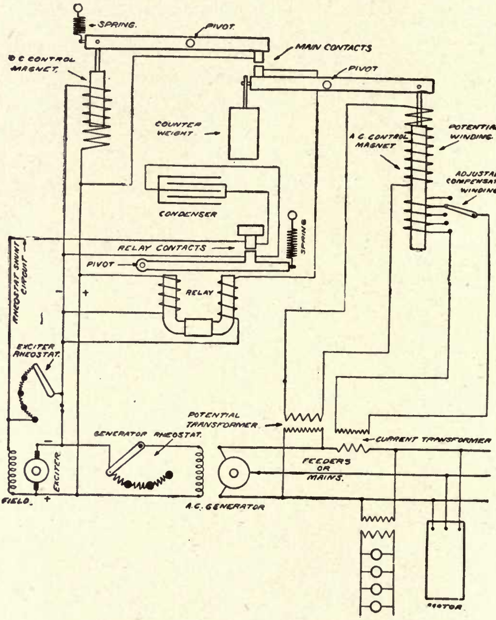


Fig. 89.

supplied to the alternator field. The general scheme and connections of this regulator for a single generator and exciter are shown in Fig. 89.

52. **Efficiency.** — The following is abstracted from the Report of the Committee on Standardization of the American Institute of Electrical Engineers. Only those portions are given which bear upon the efficiency of alternators. They will, however, apply equally well to synchronous motors.

The "efficiency" of an apparatus is the ratio of its net power output to its gross power input.

Electric power should be measured at the terminals of the apparatus.

In determining the efficiency of alternating-current apparatus, the electric power should be measured when the current is in phase with the *E.M.F.* unless otherwise specified, except when a definite phase difference is inherent in the apparatus, as in induction motors, etc.

Where a machine has auxiliary apparatus, such as an exciter, the power lost in the auxiliary apparatus should not be charged to the machine, but to the plant consisting of the machine and auxiliary apparatus taken together. The plant efficiency in such cases should be distinguished from the machine efficiency.

The efficiency may be determined by measuring all the losses individually, and adding their sum to the output to derive the input, or subtracting their sum from the input to derive the output. All losses should be measured at, or reduced to, the temperature assumed in continuous operation, or in operation under conditions specified.

In synchronous machines the output or input should be measured with the current in phase with the terminal *E.M.F.* except when otherwise expressly specified.

Owing to the uncertainty necessarily involved in the approximation of load losses, it is preferable, whenever possible, to determine the efficiency of synchronous machines by input and output tests.

The losses in synchronous machines are:

a. Bearing friction and windage.

b. Molecular magnetic friction and eddy currents in iron, copper, and other metallic parts. These losses should be determined at open circuit of the machine at the rated speed and at the rated voltage, $+ IR$ in a synchronous generator, $- IR$ in a synchronous motor, where $I =$ current in armature, $R =$ armature resistance. It is undesirable to compute these losses from observations made at other speeds or voltages.

These losses may be determined by either driving the machine by a motor, or by running it as a synchronous motor, and adjusting its fields so as to get minimum current input, and measuring the input by wattmeter. The former is the preferable method, and in polyphase machines the latter method is liable to give erroneous results in consequence of unequal distribution of currents in the different circuits caused by inequalities of the impedance of connecting leads, etc.

c. Armature-resistance loss, which may be expressed by $p\,I^2R$; where $R =$ resistance of one armature circuit or branch, $I =$ the current in such armature circuit or branch, and $p =$ the number of armature circuits or branches.

d. Load losses. While these losses cannot well be determined individually, they may be considerable, and, therefore, their joint influence should be determined by observation. This can be done by operating the machine on short circuit and at full-load current, that is, by determining what may be called the "short-circuit core loss."

With the low field intensity and great lag of current existing in this case, the load losses are usually greatly exaggerated.

One-third of the short-circuit core loss may, as an approximation, and in the absence of more accurate information, be assumed as the load loss.

e. Collector-ring friction and contact resistance. These are generally negligible, except in machines of extremely low voltage.

f. Field excitation. In separately excited machines, the I^2R of the field coils proper should be used. In self-exciting machines, however, the loss in the field rheostat should be included.

The efficiency curve of an alternator may be plotted when the losses at different loads have been determined. The efficiency curve of a 5000 K. W. 11,000 volt alternator, and that of a 1000 K. W. 500 volt alternator are shown in Fig. 90.

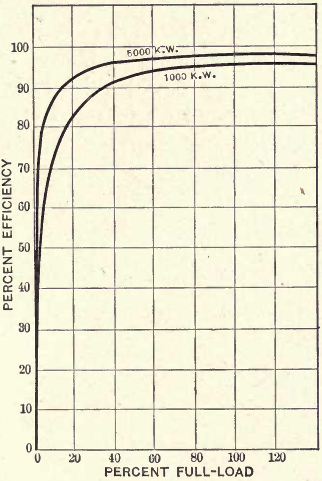


Fig. 90.

53. Rating. — Alternators are rated by their electrical output, either in kilowatts or in kilovolt-amperes. By rating is meant the power that the machine can deliver to the load without an excessive rise in temperature. This temperature rise is due to the losses in the alternator; these include the practically constant iron losses and the copper losses, variable with load. Under a fixed current output the temperature of the armature will rise until the rate of escape of heat from it is equal to the rate of its development. This ultimate temperature should not exceed 80° C. in any case. Under a given voltage the current output is limited by the rise of temperature and the power output is further limited by the power factor of the load circuit. Hence the power supplied by an alternator to a reactive load is less than that supplied to a non-reactive load for the same temperature rise in the machine. It is advisable to rate alternators in kilovolt-amperes and to specify the power factor on which this rating is based. Thus, a 6600 volt alternator whose rated current is 500 amperes, called a 3300 kilovolt-ampere alternator, could deliver 3300 kilowatts to a non-reactive load, but, for the same temperature rise it could only deliver 2640 kilowatts to a load of 80% power factor.

An alternator should be able to carry a 25% overload for two hours without serious injury because of heating, electrical or mechanical stresses, and with an additional temperature rise not exceeding 15° C. above that specified for rated load, the overload being applied after the machine has acquired the temperature corresponding to continuous operation at rated load.

54. Inductor Alternators. — Generators in which both armature and field coils are stationary are called inductor alternators. Fig. 91 shows the principle of operation of these machines. A moving member, carrying no wire, has pairs of soft iron projections, which are called inductors. These projections are magnetized by the current flowing in the annular field coil as shown in figure. The surrounding frame has internal projections corresponding

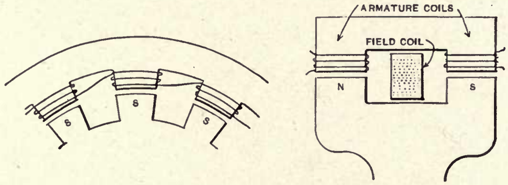


Fig. 91.

to the inductors in number and size. These latter projections constitute the cores of armature coils. When the faces of the inductors are directly opposite to the faces of the armature poles, the magnetic reluctance is a minimum, and the flux through the armature coil accordingly a maximum. For the opposite reason, when the inductors are in an intermediate position the flux linked with the armature coils is a minimum. As the inductors revolve, the linked flux changes from a maximum to a minimum, but it does not change in sign.

Absence of moving wire and the consequent liability to chafing of insulation, absence of collecting devices and their attendant brush friction, and increased facilities for insulation are claimed as advantages for this type of machine. By suitably disposing of the coils, inductor alternators may be wound for single- or polyphase currents.

The Stanley Electric Manufacturing Company manufactured two-phase inductor alternators. A view of one of their machines is given in Fig. 92, with the frame separated for inspection of the windings. In this picture the field coil is hanging loosely between the pairs of inductors. The theoretical operation of this machine is essentially that

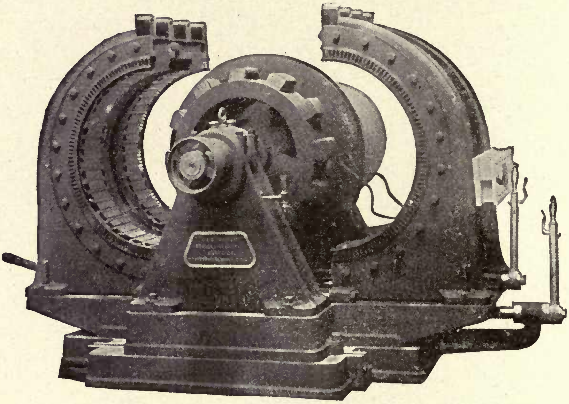


Fig. 92.

described above. All iron parts, both stationary and revolving, that are subjected to pulsations of magnetic flux, are made up of laminated iron. The large field coil is wound on a copper spool. Ordinarily when the field circuit of a large generator is broken, the *E.M.F.* of self-induction may rise to so high a value as to pierce the

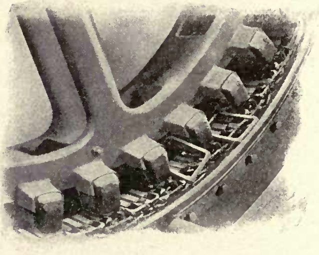


Fig. 93

insulation. With this construction the copper spool acts as a short circuit around the decaying flux, and prevents high

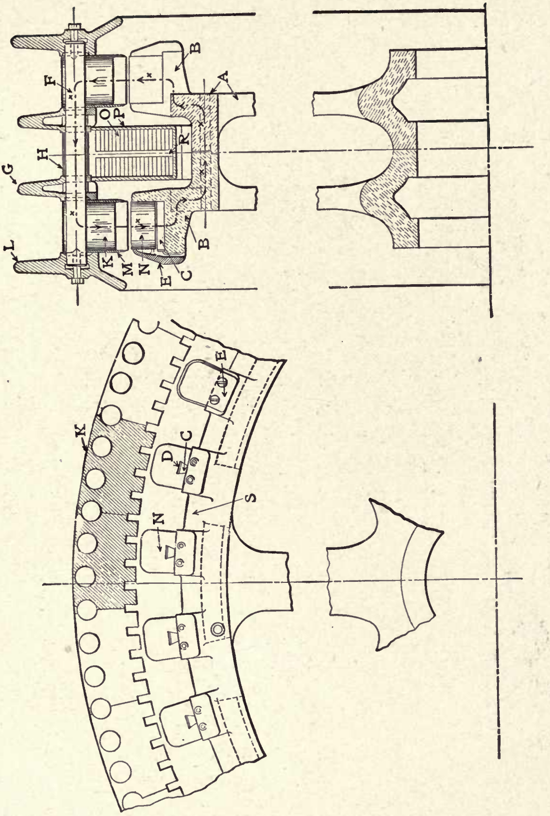


Fig. 94.

E.M.F.'s of self-induction. Figs. 93 and 94 show details of construction of larger machines of this type.

55. Revolving Field Alternators. — In this type of alternator, the armature windings are placed on the inside of the surrounding frame, and the field poles project radially from the rotating member. As was stated before this

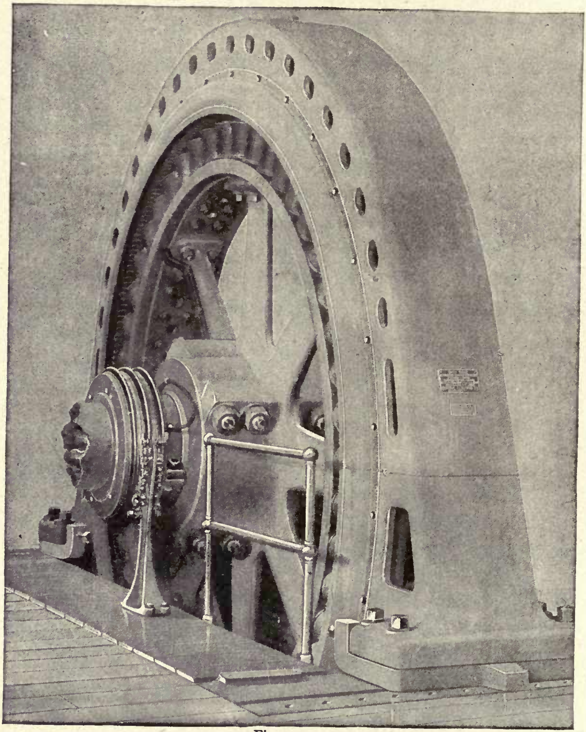


Fig. 95.

type of construction is to be recommended in the case of large machines which are required to give either high voltages or large currents. With the same peripheral velocity, there is more space for the armature coils; the coils can be better ventilated, air being forced through ducts by the rotating field; stationary coils can be more perfectly insulated than moving ones; and the only currents to be collected by brushes and collector rings are those necessary to excite the fields.

Fig. 95 shows a General Electric 750 K. W. revolving field generator. The two collector rings for the field current are shown, and in Fig. 96 the edgewise method of

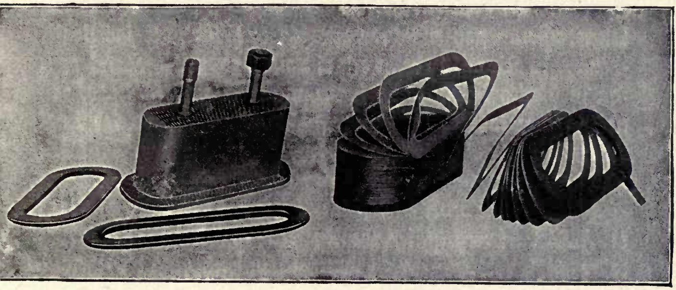


Fig. 96.

winding the field coils is shown. The collector rings are of cast iron and the brushes are of carbon. Fig. 97 shows the details of construction of a 5000 K.W. three-phase 6600-volt machine of this type as constructed for the Metropolitan Street Railway Co. of New York. This machine has 40 poles, runs at 75 R.P.M. at a peripheral velocity of 3900 feet per minute. This gives a frequency of 25. The air gap varies from five-sixteenths at the pole center to eleven-sixteenths at the tips. The short-circuit current at full-load excitation is less than 800 amperes per leg. The rated full-load current is slightly over 300 amperes.

Fig. 98 shows the method of assembling the armature

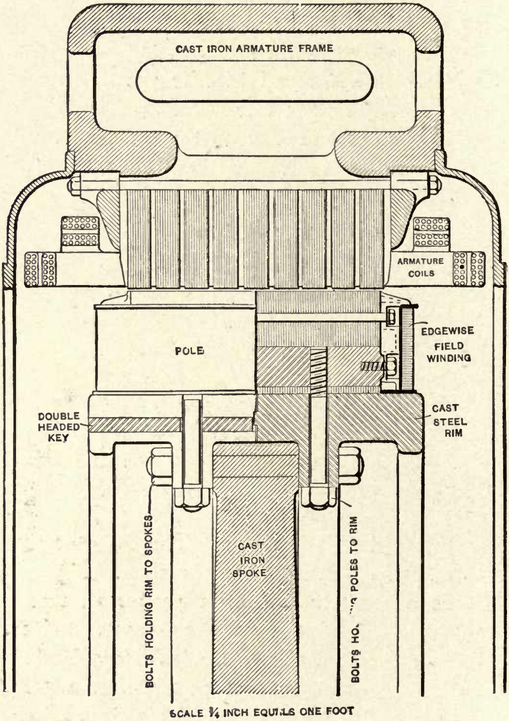


Fig. 97.

coils in the slots of the stationary core. In this machine there is a three-phase winding distributed so as to utilize

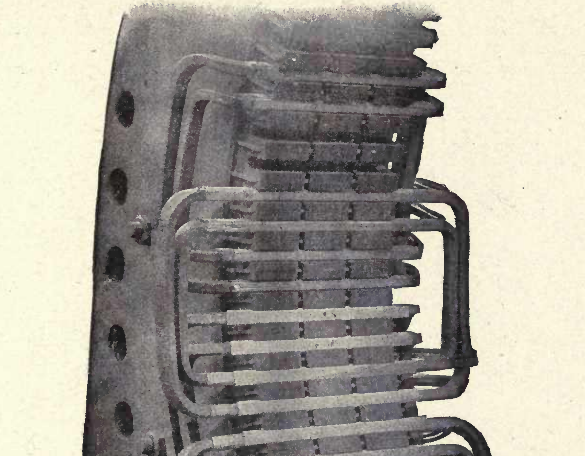


Fig. 98.

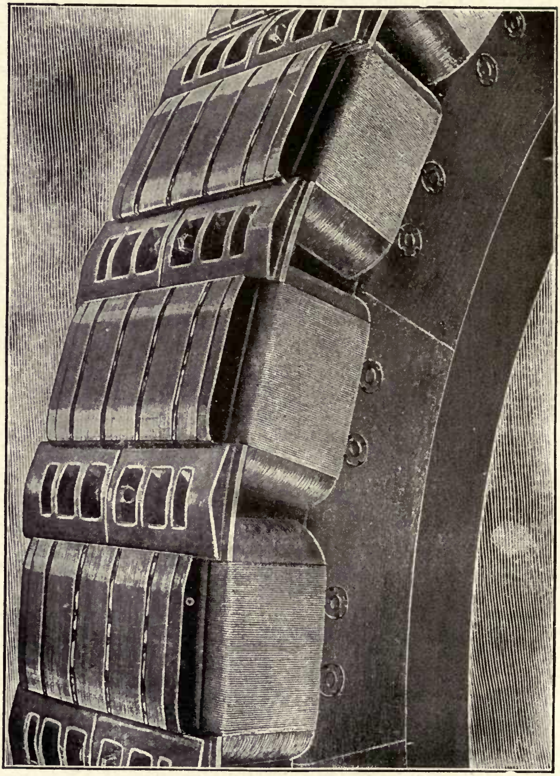


Fig. 99.

two slots per pole per phase. Fig. 99 shows the construction of a rotating field which consists of a steel rim mounted upon a cast-iron spider. Into dovetailed slots in the rim are fitted laminated plates with staggered joints. These plates are bolted together. The laminations are supplied at intervals with ventilating ducts. The coils are kept in place by retaining wedges of non-magnetic material.

56. Self-Exciting Alternators. — An alternator of a different type from those previously considered is the Alexanderson self-exciting alternator which is manufactured by the General Electric Company. The armature and field windings differ in no respect from the usual type used in alternating-current generators. The field current is

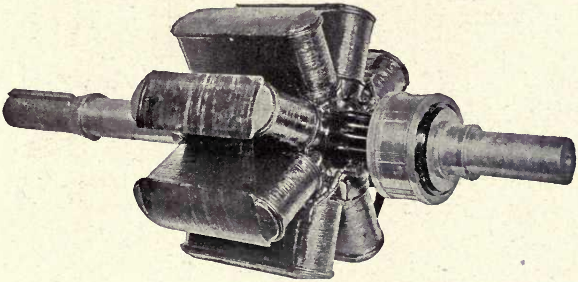


Fig. 100.

derived from an auxiliary winding placed in the same slots as the main armature winding, and is rectified by means of a segmental commutator with one active segment per pole. The revolving field of a 100 K.W. three-phase self-exciting alternator with its commutator is shown in Fig. 100. Alternate segments of this commutator are connected to two steel rings surrounding the segments, and these rings are connected to the field winding. A terminal of each auxiliary winding is connected to one of three brushes bearing on this commutator, and their other terminals are attached to a three-phase rheostat, as shown in Fig. 101.

Automatic compounding is effected by series transformers connected as indicated in the figure. The amount of boosting in the field circuit will depend upon the values

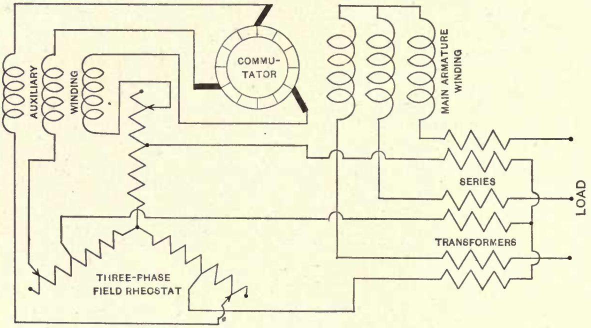


Fig. 101.

of the currents in the secondaries of the series transformers as well as upon the power factor of the load.

PROBLEMS.

1. The armature of a 25 cycle, eight pole single-phase alternator has three slots per pole with ten conductors in each slot, the slots occupying one-half of the pole distance. If the flux from each pole is 1,200,000 maxwells, what will be the effective *E.M.F.* generated in the armature, assuming this *E.M.F.* to be sinusoidal?

2. Determine the voltage across the outside terminals of a two-phase three-wire 100 volt alternator, when the windings on its armature are set 85° apart instead of 90° apart.

3. The *E.M.F.* generated in each armature winding of a three-phase alternator is 125 volts, and the current in each is 5 amperes when the alternator is connected to a certain load. Determine the voltages between the lines and the current flowing in each line wire, when the machine is *Y*-connected and when it is Δ-connected.

4. Find the magnitudes and phases of the various voltages across the

line wires of a four-phase star-connected system, the voltage generated in each armature winding being 75 volts.

5. A three-phase alternator is connected to a balanced non-reactive receiving circuit. It is required to determine the magnitude of the power supplied by the alternator, when the voltage across the line wires is 150 volts and the current in each line is 20 amperes.

6. A three-phase alternator is connected to a balanced inductive load and the power is measured according to the method of Fig. 76. What is the power factor of the receiving circuits if the observed indications on the wattmeter are 2,200 and 2,900 watts respectively?

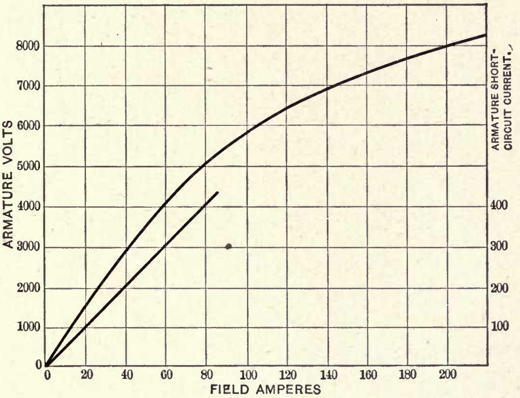


Fig. 102.

7. In the alternator of Fig. 78, determine the saturation factor when the exciting current is six amperes.

8. Calculate the regulation of the alternator of the preceding problem when the field excitation is 7 amperes, and when the power factor of the load is unity.

9. The no-load saturation and the armature short-circuit current curves of a 3500 K. W. three-phase 6,600 volt revolving field alternator are shown in Fig. 102. Calculate the regulation at full load with unity

power factor by the *E.M.F.* method. The armature resistance between terminals is .093 ohms at 25° C.

10. Determine the regulation of the alternator of the preceding problem on an inductive load of 80% power factor, by the *E.M.F.* method.

11. If the load of the preceding problem were replaced by a capacity load of the same power factor, what would be the regulation at full load, as calculated by the *E.M.F.* method.

12. Determine the regulation of the alternator of Fig. 102 for each of the conditions of the three preceding problems, applying the *M.M.F.* method.

CHAPTER VI.

THE TRANSFORMER.

57. **Definitions.** — The alternating-current transformer consists of one magnetic circuit interlinked with two electric circuits, of which one, the *primary*, receives electrical energy, and the other, the *secondary*, delivers electrical energy. If the electric circuits surround the magnetic circuit, as in Fig. 103, the transformer is said to be of the *core type*. If the reverse is true, as in Fig. 104, the transformer is of the *shell type*. The practical utility of the transformer lies in the fact that, when suitably designed, its primary can take electric energy at one potential, and its secondary deliver the same energy at some other potential; the ratio of the current in the primary to that in the secondary being approximately inversely as the ratio of the pressure on the primary to that on the secondary.

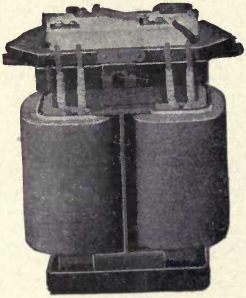


Fig. 103.

The *ratio of transformation* of a transformer is represented by τ, and is the ratio of the number of turns in the secondary coils to the number of turns in the primary coil. This would also be the ratio of the secondary voltage to

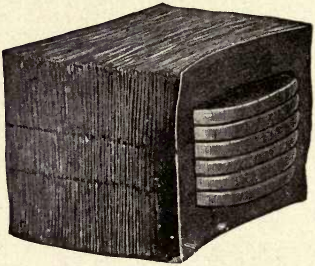


Fig. 104.

the primary voltage if there were no losses in the transformer. A transformer in which this ratio is greater than unity is called a "step-up" transformer, since it delivers electrical energy at a higher pressure than that at which it is received. When the ratio is less than unity it is called a "step-down" transformer. Step-up transformers find their chief use in generating plants, where because of the practical limitations of alternators, the alternating current generated is not of as high a potential as is demanded for economical transmission. Step-down transformers find their greatest use at or near the points of consumption of energy, where the pressure is reduced to a degree suitable for the service it must perform. The conventional representation of a transformer is given in Fig. 105. In general, little or no effort is made to indicate the ratio of transformation by the relative number of angles or loops shown, though the low-tension side is sometimes distinguished from the high-tension side by this means.

When using the same or part of the same electric circuit for both primary and secondary, the device is called an auto-transformer. These are sometimes used in the

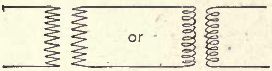


Fig. 105.

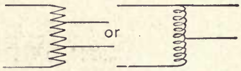


Fig. 106.

starting devices for induction motors, and sometimes connected in series in an alternating-current circuit, and arranged to vary the *E.M.F.* in that circuit. Fig. 106 is the conventional representation of an auto-transformer.

58. The Ideal Transformer. — The term ideal transformer may be applied to one possessing neither hysteresis and eddy current losses in the core nor ohmic resistance in the windings, and all the flux set up by one coil links with the other coil also. Actual transformers, however, do not satisfy these conditions, yet their behavior approximates closely to that of an ideal transformer.

When the secondary coil of a transformer is open-circuited it is perfectly idle, having no influence on the rest of the apparatus, and the primary becomes then merely a choke coil or reactor. The reactance of a commercial transformer is very large and its resistance very small, consequently the impedance is high and almost wholly reactive. In the ideal transformer the current that will flow in the primary when the secondary is open-circuited is very small and lags 90° behind the *E.M.F.* which sends it. This current is called the *exciting current*, and will be sinusoidal in the ideal transformer when the impressed electromotive force is sinusoidal. A flux will be set up in the iron of the transformer, which is sinusoidal and in phase with the exciting current. This flux induces a sinusoidal *E.M.F.* in the primary coil which is 90° behind the flux in phase because the induced *E.M.F.* is greatest when the time rate of flux change is greatest, and this flux change is greatest when passing through the zero value. This induced electromotive force is 90° behind the flux, which in turn is 90° behind the impressed *E.M.F.*; therefore the induced *E.M.F.* is 180° behind the impressed electromotive force, or is a *counter E.M.F.* In the ideal transformer under consideration, the counter pressure is exactly equal to the *E.M.F.* impressed upon the primary. The phase relations of the pressures, the exciting current, and the flux in an ideal transformer are shown in Fig. 107. It should be noted that the exciting current, being at right angles to the pressure for this transformer, does not represent a loss in power, for the energy is alternately received from and supplied to the circuit. This may be shown graphically as in Fig. 108, where the lobes of negative and positive power are equal.

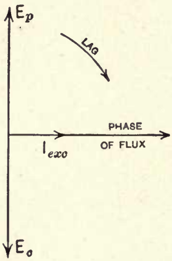


Fig. 107.

When the secondary winding of an ideal transformer is closed through an outside impedance, the variations in the flux, which is linked with the secondary as well as the primary, produce in the secondary an *E.M.F.* τ times as great as the counter *E.M.F.* in the primary, since there are τ times as many turns in the secondary coil as there are in the primary, or

$$E_s = \tau E_p;$$

and a current I_s will flow through the external circuit. The ampere-turns of the secondary, $n_s I_s$, will be opposed to the ampere-turns of the primary, and will thus tend to demagnetize the core. This tendency is opposed by a readjustment of the conditions in the primary circuit. Any demagnetization tends to lessen the counter *E.M.F.* in the primary coil, which immediately allows more current to flow in the primary, and thus restores the magnetization to a value but slightly less than the value on open-circuited secondary. Thus the core flux remains practically constant whether

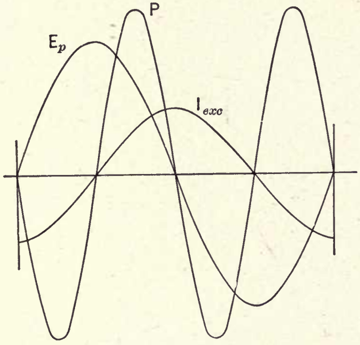


Fig. 108.

the secondary be loaded or not, the ampere turns of the secondary being opposed by a but slightly greater number of ampere turns in the primary. So

$$n_s I_s = n_p I_p, \text{ very nearly,}$$

and
$$I_s = \frac{n_p}{n_s} I_p = \frac{1}{\tau} I_p.$$

The counter *E.M.F.* in the primary of a transformer accommodates itself to variations of load on the secondary in a manner similar to the variation of the counter *E.M.F.* of a shunt wound motor under varying mechanical loads.

The vector diagram of the ideal transformer, when the secondary is closed through a circuit having a reactance X_2, and a resistance R_2, is shown in Fig. 109. It represents a step-down transformer where $\tau = \frac{1}{2}$. The secondary current I_s lags behind E_s by the angle $\phi = \tan^{-1}\frac{X_2}{R_2}$. The primary ampere-turns are composed of two components — one necessary to balance the secondary ampere-turns and the other necessary to magnetize the core, as shown.

When the transformer delivers little power, the magnetizing component of the magnetomotive force is comparable in magnitude with the active component, consequently the secondary current lags behind the primary current by less than 180°. When the transformer is fully loaded, however, the magnetizing current is comparatively small and therefore the directions of I_p and I_s are very nearly opposite. This is also true of commercial transformers, in which the exciting current is less than 90° behind E_p.

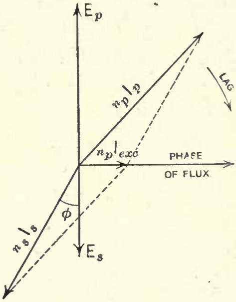


Fig. 109.

59. **Core Flux.** — The relation between the magnetic flux in a transformer core and the primary impressed *E.M.F.* can be determined by considering that the flux varies harmonically, and that its maximum value is Φ_m; then the flux at any time, t, is $\Phi_m \cos \omega t$, and the counter

E.M.F., which is equal and opposed to the impressed primary pressure E_p, may be written (§ 13, vol. i.)

$$E_p{}' = \frac{n_p}{10^8} \frac{d(\Phi_m \cos \omega t)}{dt};$$

and since Φ_m and ω are constant

$$E_p{}' = 10^{-8}\, n_p \omega \Phi_m \sin \omega t,$$

from which

$$E_{pm} = 10^{-8}\, n_p \omega \Phi_m,$$

and

$$\Phi_m = \frac{10^8\, E_{pm}}{n_p \omega} = \frac{10^8 \sqrt{2}\, E_p}{n_p \omega}.$$

This equation is used in designing transformers and choke coils. The values of Φ_m for 60 cycle transformers of different capacities, as determined by experiment and use, are shown in the curve, Fig. 110. It is usual in such

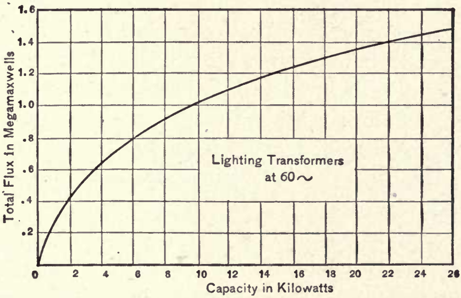


Fig. 110.

designs to assume a maximum flux density, $\mathcal{B}_m$. Transformer cores are worked at low flux densities, and, while the value assumed differs considerably with the various manufacturers, it is safe to say that for 25 cycles $\mathcal{B}_m$ varies between 6 and 8 kilogausses; for 60 cycles between 5 and 6 kilogausses; and for 125 cycles between 3 and 4 kilogausses. The necessary cross-section, A, of iron in square centimeters is found from the relation

$$\Phi_m = \mathcal{B}_m A.$$

60. Transformer Losses. — The transformer as thus far discussed would have 100 % efficiency, no power whatever being consumed in the apparatus. The efficiencies of loaded commercial transformers are very high, being generally above 95 % and frequently above 98 %. The losses in the apparatus are due to the resistance of the electric circuits, hysteresis, and eddy currents. These losses may be divided into core losses and copper losses, according as to whether they occur in the iron or the wire of the transformer.

61. Core Losses. — (*a*) *Eddy current loss.* If the core of a transformer were made of solid iron, strong eddy currents would be induced in it. These currents would not only cause excessive heating of the core, but would tend to demagnetize it, and would require excessive currents to flow in the primary winding in order to set up sufficient counter *E.M.F.*

To a great extent these troubles are prevented by making the core of laminated iron, the laminæ being transverse to the direction of flow of the eddy currents but longitudinal with the magnetic flux. Each lamina is more or less thoroughly insulated from its neighbors by the natural oxide on the surface or by Japan lacquer. The eddy current loss is practically independent of the load.

An empirical formula for the calculation of the watts lost in the transformer core due to eddy currents, based upon the assumption that the laminations are perfectly insulated from one another, is

$$P_e = kvf^2l^2\mathcal{B}_m^{\ 2},$$

where

k = a constant depending upon the resistivity of the iron,
v = volume of iron in cm.^{3},
l = thickness of one lamina in cm.,
f = frequency,

and

$\mathcal{B}_m$ = maximum flux density (Φ_m per cm.^{2}).

In practice k has a value of about 1.6×10^{-11}.

The values of P_e in watts per cubic inch and per pound in terms of flux density for 25 cycle and 60 cycle transformers may be taken directly from the curves of Fig. 111. These

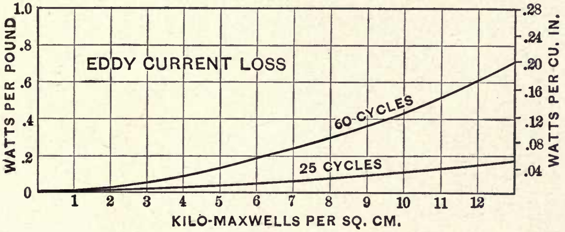


Fig. 111.

curves are plotted from values calculated by means of the formula and refer to laminations usually employed, these being 0.014 inch thick.

(*b*) *Hysteresis loss.* A certain amount of power, P_h, due to the presence of hysteresis, is required to carry the iron through its cyclic changes. The value of P_h can be calculated from the formula expressing Steinmetz's Law,

$$P_h = 10^{-7} v f \eta \mathcal{B}_m^{1.6},$$

where

v = volume of iron in cm.^{3},

f = frequency,

$\mathcal{B}_m$ = the maximum flux density,

and η = the hysteretic constant.

A fair value of η for transformer sheets is .0021. Curves of the hysteresis loss in 25 cycle and 60 cycle transformers based upon this value of η are shown in Fig. 112. In the better grades of transformers, however, the hysteresis loss

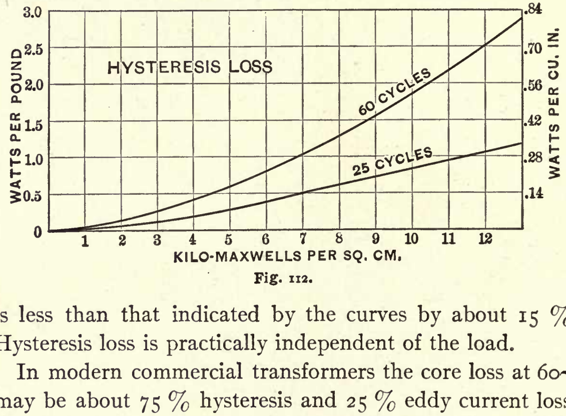


Fig. 112.

is less than that indicated by the curves by about 15 %. Hysteresis loss is practically independent of the load.

In modern commercial transformers the core loss at 60~ may be about 75 % hysteresis and 25 % eddy current loss. At 125 ~ it may be about 60 % hysteresis and 40 % eddy current loss. This might be expected, since it was shown that the first power of f enters into the formula for hysteresis loss, while the second power of f enters into the formula for eddy current loss.

The core loss is also dependent upon the wave-form of the impressed *E.M.F.*, a peaked wave giving a somewhat lower core loss than a flat wave. It is not uncommon to find alternators giving waves so peaked that transformers tested by current from them show from 5 % to 10 % less core loss than they would if tested by a true sine wave. On the other hand generators sometimes give waves so flat that the core loss will be greater than that obtained by the use of the sine wave.

The magnitude of the core loss depends also upon the temperature of the iron. Both the hysteresis and eddy current losses decrease slightly as the temperature of the iron increases. In commercial transformers, a rise in temperature of 40° C. will decrease the core loss from 5 % to 10 %. An accurate statement of the core loss thus requires that the conditions of temperature and wave-shape be specified.

62. Exciting Current. — In commercial transformers the exciting current lags less than 90° behind the primary impressed *E.M.F.*, because of the iron losses. The exciting current may therefore be resolved into two components, one in phase with the primary *E.M.F.*, and the other at right angles to it. The former is that current necessary to overcome the core losses and is called the *power component* of the exciting current. It is expressed as

$$I_{e+h} = \frac{P_e + P_h}{E_p},$$

the values of P_e and P_h being calculated from the formulæ of § 61.

The other component, being 90° behind E_p, is termed the *wattless component* of the exciting current, or the *magnetizing* *current* of a transformer. It is that current which sets up the magnetic flux in the core, and is denoted by the symbol I_{mag}.

Representing the reluctance of the core by $\mathcal{R}$, and the magnetomotive force necessary to produce the flux Φ_m by $\mathcal{H}$, from §§ 21 and 25, vol. i.,

$$\Phi = \frac{\mathcal{H}}{\mathcal{R}} = \frac{4\pi n_p \frac{I_{mag}}{10}}{\mathcal{R}},$$

whence
$$I_{mag} = \frac{10\,\mathcal{R}\Phi}{4\pi n_p} = \frac{10\,\mathcal{R}\Phi_m}{4\sqrt{2}\,\pi n_p}.$$

The value of $\mathcal{R}$ is calculated (§ 24, vol. i.) from

$$\mathcal{R} = \frac{l}{A}\rho,$$

where l is the length of magnetic circuit, A its cross-section and ρ the reluctivity of the iron

$$\left(\rho = \frac{1}{\mu} = \frac{1}{\text{permeability}}\right).$$

The phase relations of the power and wattless components of the exciting current are shown in Fig. 113. The angle between I_{exc} and I_{mag} is called the *angle of hysteretic advance* and is denoted by α. This angle is determined from the relation

$$\alpha = \tan^{-1}\frac{I_{e+h}}{I_{mag}}.$$

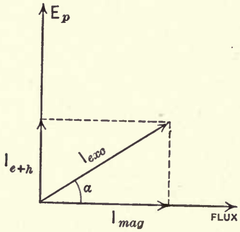


Fig. 113.

It should be noted that the use of the term hysteretic in this connection is somewhat misleading, for the value of α depends upon the eddy current loss as well as upon the hysteresis loss. The exciting current is

$$I_{exc} = \sqrt{I^2_{mag} + I^2_{e+h}}$$

and lags behind the primary impressed *E.M.F.* by an angle $90° - \alpha$.

The magnitude and position of the exciting current of a transformer can be determined experimentally by the use of a wattmeter, a voltmeter, and an ammeter connected in the primary circuit, the secondary, of course, being open-circuited. The ammeter reading gives the value of I_{exc} and its position is given by the equation

$$\alpha = \sin^{-1} \frac{P}{EI_{exc}},$$

P being the wattmeter reading minus the copper loss due to the exciting current in the primary winding.

If the impressed *E.M.F.* be harmonic the flux will also be harmonic and consequently the magnetizing current cannot be harmonic, because of the variation in the reluctance of the core. Besides a decrease in permeability with increasing flux density, the permeability on rising flux is smaller than on falling, under a given magnetomotive force, due to hysteresis. Therefore the magnetizing current wave will be peaked and will have a hump on the rising side. This magnetizing current wave can be plotted when the hysteresis loop of the core is given over the range of flux density produced by the primary *E.M.F.* Since $\mathcal{H}$ is directly proportional to the current, and $\mathcal{B}$ is proportional to Φ, if proper units are chosen, $\mathcal{H}_m$ may be taken equal to $I_{mag\ m}$, and $\mathcal{B}_m$ may be taken equal to Φ_m. Then the hysteresis loop and the sinusoidal flux curve may be drawn as in Fig. 114. The value of the magnetizing current corresponding to a given value of the flux is obtained by taking the abscissa corresponding to this flux value from the hysteresis loop and laying it off as an ordinate at the point on the time axis corresponding to the flux value taken. This process is indicated in the figure, and the entire current curve has been constructed by proceeding in this manner.

This distorted curve of magnetizing current may be resolved into true sine components (§ 10), a fundamental with higher harmonics, the third harmonic being the most pronounced. The exciting current, being composed of

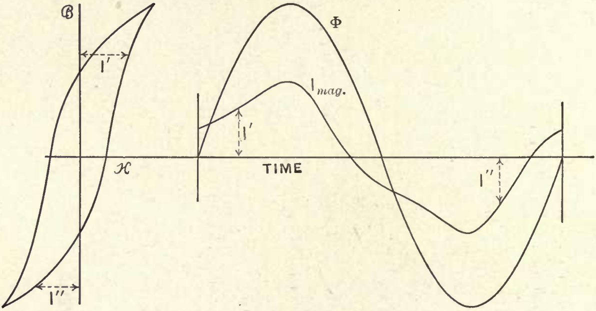


Fig. 114.

two components, one of which is non-sinusoidal, will also be non-sinusoidal, but since it is usually very small compared with the load current, no appreciable error will be introduced by considering I_{exc} as harmonic.

The exciting current varies in magnitude with the design of the transformer. In general it will not exceed 5 % of the full load current, and in standard lighting transformers it may be as low as 1 %. In transformers designed with joints in the magnetic circuit the magnitude of the exciting current is largely influenced by the character of the joints, being large if the joints are poorly constructed.

63. Equivalent Resistance and Reactance of a Transformer. — If a current of definite magnitude and lag be taken from the secondary of a transformer, a current of the same lag and τ times that magnitude will flow in the primary, neglecting resistance, reluctance, and hysteresis. An impedance which, placed across the primary mains, would allow an exactly similar current to flow as this primary current, is called an *equivalent impedance*, and its components are called *equivalent resistance* and *equivalent reactance.*

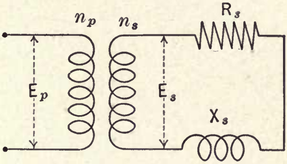


Fig. 115.

If the secondary winding of the transformer have a resistance R_s and a reactance X_s, and if the load have a resistance R_2 and a reactance X_2, then the current that will flow in the secondary circuit is

$$I_s = \frac{E_s}{\sqrt{(R_s + R_2)^2 + (X_s + X_2)^2}},$$

where E_s is the secondary induced pressure when E_p is the primary impressed $E.M.F.$ The secondary current lags behind E_s by an angle ϕ whose tangent is

$$\frac{X_s + X_2}{R_s + R_2}.$$

For convenience, X_2 and R_2 will be taken equal to zero, Fig. 115, and the expressions which will result will be the equivalent resistance and reactance of the secondary winding of the transformer.

Therefore $$\sqrt{R_s^2 + X_s^2} = \frac{E_s}{I_s}.$$

If the equivalent impedance have a resistance R and a reactance X then the ratios $\frac{X}{R}$ and $\frac{X_s}{R_s}$ must be equal, since the angle of current lag is the same in both primary and secondary. And since the current in the equivalent impedance has the same magnitude as that in the primary

$$I_p = \frac{E_p}{\sqrt{R^2 + X^2}},$$

and $$\sqrt{R^2 + X^2} = \frac{E_p}{I_p}.$$

But $$I_p = \tau I_s,$$

and $$E_p = \frac{1}{\tau} E_s,$$

therefore, $$\sqrt{R^2 + X^2} = \frac{\frac{E_s}{\tau}}{\tau I_s} = \frac{1}{\tau^2}\frac{E_s}{I_s} = \frac{1}{\tau^2}\sqrt{R_s^2 + X_s^2}.$$

But $$\frac{R}{X} = \frac{R_s}{X_s}.$$

Solving $$R = \frac{1}{\tau^2} R_s,$$

$$X = \frac{1}{\tau^2} X_s,$$

which are the values of the equivalent resistance and reactance of the secondary winding respectively. Similarly the equivalent load resistance and reactance are respectively

$$R = \frac{1}{\tau^2} R_2$$

and

$$X = \frac{1}{\tau^2} X_2.$$

64. Copper Losses. — The copper losses in a transformer are almost solely due to the regular current flowing through the coils. Eddy currents in the conductor are either negligible or considered together with the eddy currents in the core.

When the transformer has its secondary open-circuited the copper loss is merely that due to the exciting current in the primary coil, $I^2_{exc}R_p$. This is very small, much smaller than the core loss, for both I_{exc} and R_p are small quantities. When the transformer is regularly loaded the copper loss in watts may be expressed

$$P_c = I_p^2 R_p + I_s^2 R_s,$$

where R_p and R_s are the resistances of the primary and secondary coils respectively. In an ideal transformer with zero reluctance, I_s is 180° behind I_p, and this is also approximately true for a commercial transformer under a considerable load. Therefore, for convenience, the secondary resistance may be reduced to the primary circuit and the copper loss may then be expressed as

$$P_c = I_p^2 (R_p + R) = I_p{}^2 (R_p + \frac{1}{\tau^2} R_s).$$

At full load this loss will considerably exceed the core loss. While the core loss is constant at all loads, the copper loss varies as the square of the load.

65. Efficiency. — Since the efficiency of induction apparatus depends upon the wave-shape of *E.M.F.*, it should be referred to a sine wave of *E.M.F.*, except where expressly specified otherwise. The efficiency should be measured with non-inductive load, and at rated frequency, except where expressly specified otherwise.

The efficiency of a transformer is expressed by the ratio of the net power output to the gross power input or by the ratio of the power output to the power output plus all the losses. The efficiency, ε, may then be written,

$$\varepsilon = \frac{V_s I_s}{V_s I_s + P_h + P_e + P},$$

where V_s is the difference of potential at the secondary terminals.

The losses and efficiencies of a line of 2200 volt, 60 cycle transformers of the shell type are given in the following table: —

Rated Output in Kilowatts.	Core Loss in Watts.	Full-load Copper Loss in Watts.	Per cent Efficiency at Full-load.
1	30	32	94.1
2	50	56	94.9
3	66	78	95.4
5	90	105	96.3
7.5	116	135	96.8
10	135	170	97.0
15	169	233	97.4
20	200	304	97.5
25	225	375	97.65
30	250	444	97.8
40	300	586	97.9
50	350	725	98.0

The efficiencies of a certain 10 K.W. transformer at various loads are shown by the curve of Fig. 116.

If the transformer be artificially cooled, as many of the larger ones are, then to this denominator must be added the power required by the cooling device, as power con-

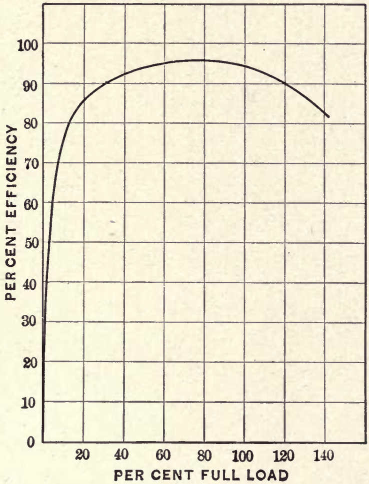


Fig. 116.

sumed by the blower in air-blast transformers, and power consumed by the motor-driven pumps in oil or water cooled transformers. Where the same cooling apparatus supplies a number of transformers or is installed to supply future additions, allowance should be made therefor.

Inasmuch as the losses in a transformer are affected by the temperature, the efficiency can be accurately specified only by reference to some definite temperature, such as 75° C.

The *all-day efficiency* of a transformer is the ratio of energy output to the energy input during the twenty-four hours. The usual conditions of practice will be met if the calculation is based on the assumption of five hours full-load and nineteen hours no-load in transformers used for ordinary lighting service. With a given limit to the first cost, the losses should be so adjusted as to give a maximum all-day efficiency. For instance, a transformer supplying a private residence with light will be loaded but a few hours each night. It should have relatively much copper and little iron. This will make the core losses, which continue through the twenty-four hours, small, and the copper losses, which last but a few hours, comparatively large. Too much copper in a transformer, however, results in bad regulation. In the case of a transformer working all the time under load, there should be a greater proportion of iron, thus requiring less copper and giving less copper loss. This is desirable in that a loaded transformer has usually a much greater copper loss than core loss, and a halving of the former is profitably purchased even at the expense of doubling the latter.

66. Calculation of Equivalent Leakage Inductance. — The magnetic leakage in a transformer is that flux which links with one winding and not with the other. Its magnitude depends upon the size and form of the coils and the manner of their arrangement. This magnetic leakage may be considered equivalent to an inductance connected in the primary circuit and to an inductance connected in the secondary circuit. After the leakage flux has been determined, the inductances L_p and L_s are found from the relation

$$l = n\frac{\Phi}{i}, \qquad \S\ 12$$

and then the reactances are obtained from $X_p = \omega L_p$ and $X_s = \omega L_s$.

To calculate the leakage flux, consider a shell type transformer having one primary and one secondary coil with many turns of wire in each. The paths of the leakage flux in this type of transformer are indicated in Fig. 117. Let the

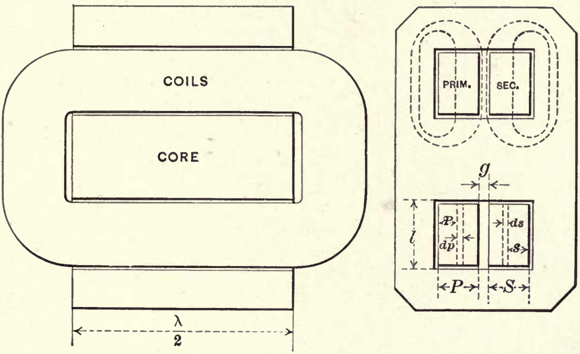


Fig. 117.

dimensions shown on the sections be expressed in centimeters.

It is convenient to consider the leakage flux as the sum of three portions, the part passing through the primary space, the part passing through the secondary space, and the part passing through the gap, g. The magnetomotive force tending to send flux through the elementary portion dp and back through the iron is $\frac{p}{P}$ of the whole $M.M.F.$ of the primary, so for any element

$$M.M.F. = 4\pi n_p i_p \frac{p}{P}$$

where i_p is expressed in absolute units. Since the permeability of iron is roughly 1000 times that of air, no appreciable error is introduced by considering the whole reluctance of the circuit of the leakage flux to be in the air portion of that circuit. The cross-section area of this air portion of the magnetic circuit for any element is

$$\frac{\lambda}{2} 2\, dp = \lambda dp,$$

and its length is l, therefore the reluctance is $\frac{l}{\lambda dp}$. The elementary primary leakage flux, $d\Phi_p$, is then

$$d\Phi_p = \frac{M.M.F.}{\mathcal{R}} = \frac{4\,\pi n_p i_p}{\frac{l}{\lambda dp}} \cdot \frac{p}{P} = \frac{4\,\pi n_p i_p \lambda p dp}{lP}.$$

Inductance, as defined in § 12, is numerically equal to the number of linkages per absolute unit of current, or $l = n\frac{\Phi}{i}$. The number of turns linked with the elementary flux $d\Phi_p$ is $\frac{p}{P}$ of the total number of primary turns, therefore the elementary leakage inductance dl_p is

$$dl_p = \frac{p}{P} n_p \frac{d\Phi_p}{i_p} = \frac{p}{P} n_p \frac{4\,\pi n_p i_p \lambda p dp}{lPi_p} = \frac{4\,\pi n_p^2 \lambda}{lP^2} p^2 dp.$$

Integrating over the full width of the primary coil from 0 to P, there is obtained

$$l_p = \frac{4\,\pi n_p^2 \lambda}{lP^2} \int_0^P p^2 dp = \frac{4\,\pi n_p^2 \lambda}{l} \cdot \frac{P}{3}.$$

Proceeding in like manner, the value of l_s is found to be

$$l_s = \frac{4\,\pi n_s^2 \lambda}{l} \cdot \frac{S}{3}.$$

The leakage flux passing through the gap between the coils is set up by the entire magnetomotive forces of either the primary or the secondary. Consider the flux passing through the right-hand half of the gap to be set up by the secondary *M.M.F.*, and that through the other half of the gap to be set up by the primary *M.M.F.* Then, proceeding as before, the leakage inductances equivalent to these portions of the leakage flux are respectively

$$l_{gp} = \frac{4\,\pi n_p^2 \lambda}{l} \cdot \frac{g}{2}$$

and

$$l_{gs} = \frac{4\,\pi n_s^2 \lambda}{l} \cdot \frac{g}{2}.$$

Adding; the leakage inductances due to the primary and secondary *M.M.F.*'s are respectively,

$$l_p + l_{gp} = \frac{4\,\pi n_p^2 \lambda}{l} \left(\frac{P}{3} + \frac{g}{2} \right)$$

and

$$l_s + l_{gs} = \frac{4\,\pi n_s^2 \lambda}{l} \left(\frac{S}{3} + \frac{g}{2} \right).$$

Reducing to practical units, and multiplying by ω, the primary and secondary leakage reactances are respectively

$$X_p = \frac{4\,\pi n_p^2 \lambda \omega}{l} \left(\frac{P}{3} + \frac{g}{2} \right) 10^{-9}, \qquad (1)$$

and

$$X_s = \frac{4\,\pi n_s^2 \lambda \omega}{l} \left(\frac{S}{3} + \frac{g}{2} \right) 10^{-9}, \qquad (2)$$

all the dimensions being in centimeters.

The secondary leakage reactance may be reduced to the primary circuit by dividing by τ^2 (§ 63), but it should be remembered that this is only permissible when the transformer is under considerable load or when the exciting current is entirely ignored, as in most practical calculations. The total equivalent leakage reactance in the primary circuit is then

$$X_{\text{total}} = \frac{4\,\pi n_p^2 \omega\lambda}{l}\left(\frac{P}{3} + \frac{S}{3} + g\right)10^{-9}. \qquad (3)$$

As some of the leakage flux passes through and between the coils where they project beyond the core, it is usual to take for λ the mean length of a turn of a coil diminished by $\frac{2}{3}$ of the length extending beyond the iron.

The minimum leakage reactance would result if each secondary turn were immediately adjacent to a primary turn, but obviously this ideal condition cannot be attained in practice. Still it may be approximated by interleaving the secondary and primary coils. When one coil is placed between the two halves of the other, as in Fig. 141, the leakage reactance is approximately one fourth of that expressed by the foregoing formulæ. The values assigned to the symbols for this case are indicated in Fig. 118. Thus,

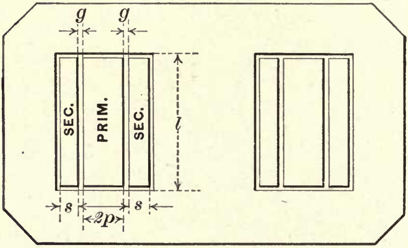


Fig. 118.

by having many coils and by alternating primary and secondary coils, the leakage reactance may be greatly reduced.

The formulæ for the calculation of leakage reactance may also be applied to the core type of transformers, but the notation will be slightly different. With this type, P and S are the radial depths of the primary and secondary coils respectively, g is the radial width of the gap, l is the axial length of the coils, λ is the mean length of a turn of the windings, and n_p and n_s are the number of primary and secondary turns respectively on both sections.

67. Regulation. — The definition of the regulation of a transformer as recommended by the American Institute of Electrical Engineers is as follows: "In constant-potential transformers, the regulation is the ratio of the rise of secondary terminal voltage from rated non-inductive load to no-load (at constant primary impressed terminal voltage) to the secondary terminal voltage at rated load." Further conditions are that the frequency be kept constant, and that the wave of impressed *E.M.F.* be sinusoidal.

Not the whole of the primary impressed *E.M.F.* is operative in producing secondary pressure, for I_pR_p volts are expended in overcoming the resistance of the primary coil, and I_sR_s volts are expended in overcoming the resistance of the secondary coil. In addition to these, a part of the impressed *E.M.F.* is lost in overcoming the primary and secondary reactances due to the leakage flux, the magnitudes of these decrements being I_pX_p and I_sX_s. To consider these various losses in voltage, imagine the transformer itself to be an ideal one, but to have a resistance R_s, equal to the resistance of the secondary coil, and a reactance X_s, equal to the secondary leakage reactance of the actual transformer, connected in the secondary circuit in series with the load resistance R_2 and reactance

X_2. And further, let there be a resistance R_p, equal to the resistance of the primary coil of the actual transformer, and a reactance X_p, equal to the primary leakage reactance thereof, connected in the primary circuit of the ideal transformer, as shown in Fig. 119.

The complete vector diagram of *E.M.F.*'s and currents in

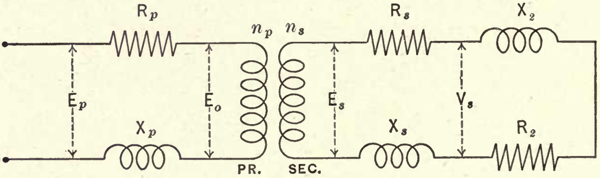


Fig. 119.

a transformer corresponding to the arrangement of Fig. 119 is represented in Fig. 120, where

V_s = the difference of potential at the secondary terminals,
E_s = *E.M.F.* induced in secondary winding,
E_p = impressed primary pressure,
E_o = operative part of E_p,
I_p and I_s = primary and secondary currents respectively.

For clearness a 1 to 1 ratio has been portrayed, and the various drops are greatly exaggerated. The diagram will be discussed in detail.

The exciting current, I_{exc}, has two components, namely I_{mag} in phase with the flux, and I_{e+h} in phase with E_0. The magnetizing current is determined from the expression

$$I_{mag} = \frac{10\, l\Phi_m}{4\sqrt{2}\,\pi A \mu n_p}, \qquad \S\ 62$$

and the power component of the exciting current is obtained from

$$I_{e+h} = \frac{vf(kfl^2\mathfrak{B}_m{}^2 + \eta\mathfrak{B}_m{}^{1.6}10^{-7})}{E_p}. \qquad \S\ 61$$

The current flowing in the secondary circuit is

$$I_s = \frac{E_s}{\sqrt{(X_s + X_2)^2 + (R_s + R_2)^2}}, \qquad \S\ 62$$

and lags or leads the secondary induced *E.M.F.* by an angle ϕ whose tangent is

$$\frac{X_s + X_2}{R_s + R_2},$$

the secondary induced electromotive force being 90° behind the flux. The primary current, I_p, is equal to the vectorial sum of $-\tau$ times the secondary current and the exciting current as shown. When a small current is taken from the secondary of the transformer, the directions of I_p and I_s are considerably less than 180° apart, but when the secondary current is large, the directions of I_p and I_s are approximately opposite.

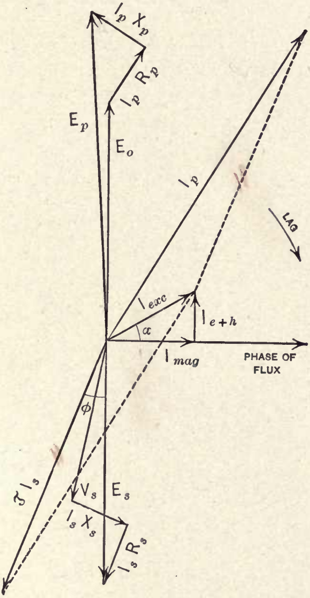


Fig. 120.

The secondary induced *E.M.F.* is not all utilizable at the terminals. There is a resistance drop of I_sR_s volts which is in phase with I_s, and a reactance drop of I_sX_s volts due to the leakage flux, this being at right angles to the phase of the secondary current. The result of subtracting $I_s R_s$ and $I_s X_s$ from E_s vectorially is V_s, which is the difference of potential at the secondary terminals.

The operative part, E_0, of the primary impressed electromotive force which is necessary to produce the secondary induced pressure E_s, leads the latter by 180° and its magnitude is $\frac{E_s}{\tau}$. There is a primary resistance drop of $I_p R_p$ volts in phase with I_p and a reactive drop due to leakage of $I_p X_p$ volts at right angles to I_p. Therefore the *E.M.F.* impressed upon the primary terminals necessary to produce E_0 is the vectorial sum of E_0, $I_p R_p$ and $I_p X_p$, and is denoted by E_p.

Both R_s and R_p become known quantities as soon as the size of the secondary and primary conductors is known. The values of X_s and X_p are calculated from the formulæ derived in Art. 66. Thus all the quantities entering into the calculation of the vectors shown in Fig. 120 are known.

Then, when I_s is the full-load current, the regulation of the transformer at power factor $= \cos \phi$ is

$$\text{Regulation} = \frac{E_p - \frac{V_s}{\tau}}{\frac{V_s}{\tau}} = \frac{\tau E_p - V_s}{V_s},$$

which, when multiplied by 100, gives the percentage regulation.

A circuit approximately equal to that of Fig. 119 is shown as Fig. 121, where the secondary resistances and reactances are reduced to the primary circuit, and where the exciting current is considered as flowing through a separate impedance, thus eliminating all transformer action.

A transformer diagram of practical importance is dependent upon the consideration that the exciting current may be neglected when the apparatus carries a large load. It

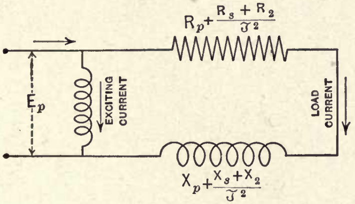


Fig. 121.

follows that $I_p = \tau I_s$, and that I_p is exactly opposite I_s. The primary and secondary resistance drops, being in phase respectively with I_p and I_s, are parallel, and the latter may be reduced to the primary circuit and added algebraically to I_pR_p. Then the total equivalent resistance drop of the transformer is $I_p\left(R_p + \frac{R_s}{\tau^2}\right)$. Similarly the total equivalent reactance drop of the transformer is $I_p\left(X_p + \frac{X_s}{\tau^2}\right)$ and is at right angles to I_p or I_s. The impressed primary *E.M.F.*, E_p, is equal to the vectorial sum of E_0, $I_p\left(R_p + \frac{R_s}{\tau^2}\right)$, and $I_p\left(X_p + \frac{X_s}{\tau^2}\right)$, as shown in Fig. 122. Hence the regulation is expressed by

$$\text{Regulation} = \frac{E_p - \frac{E_s}{\tau}}{\frac{E_s}{\tau}} = \frac{\tau E_p - E_s}{E_s}.$$

In practice it will be found impossible to complete the solution of these diagrams graphically because of the extreme flatness of the triangles. The better way is to draw an exaggerated but clear diagram, and obtain the true values of the sides by the methods of trigonometry and geometry.

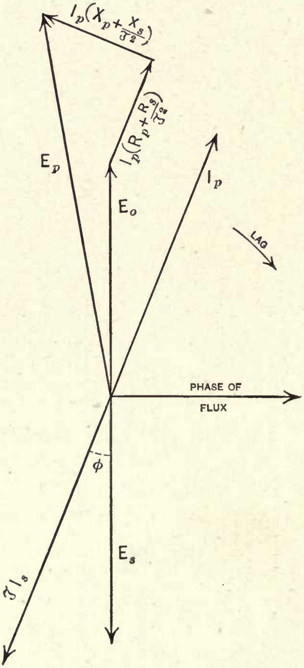


Fig. 122.

The regulation of a transformer at any load and power factor can be computed when the equivalent resistance and the equivalent reactance are known. The equivalent resistance can be determined experimentally by measuring the primary and secondary resistances using direct current, and then reducing the latter to the primary circuit by dividing by τ^2. The equivalent reactance can be determined by short-circuiting one winding and impressing a sufficient *E.M.F.* upon the other to permit full-load current to flow. This current value multiplied by the total equivalent resistance gives the resistance drop which must be subtracted from the impressed *E.M.F.* at the proper phase angle to obtain the total equivalent reactance drop. Dividing this by the current value there obtains the total equivalent reactance. The regulation of the transformer at any load and power factor may thereafter be calculated.

This method is more reliable than the load test, in which the no-load and full-load voltages are directly measured, because of the magnitudes of these quantities. A slight error in these measurements would introduce a considerable error in the regulation value, for taking the difference between these large quantities exaggerates the error of measurement.

68. Circle Diagram. — The magnitude and phase of the current produced by a constant impressed primary electromotive force, E_p in Fig. 121, depends upon the resistance and reactance of the circuit. If the load be non-inductive, the current supplied to it is dependent upon the resistance of the load. Neglecting the effect of shunt exciting circuit, the impressed *E.M.F.* has two components, that necessary to overcome the reactive drop due to the leakage flux in the transformer itself, and that necessary to overcome the resistance drop due to the resistance of the entire circuit. These are at right angles to each other and may be represented respectively by $I_p\left(X_p + \frac{X_s}{\tau^2}\right)$ and $I_p\left(R_p + \frac{R_s + R_2}{\tau^2}\right)$ as in Fig. 123. If the resistance of the load be altered, the current will change and the point A

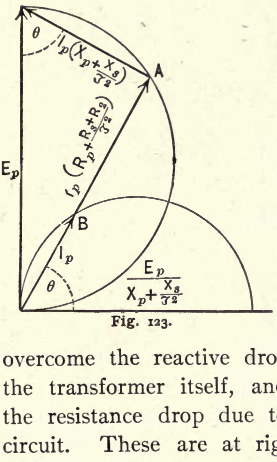


Fig. 123.

will be in a different position, since $X_p + \frac{X_s}{\tau^2}$ is constant. However, the impressed *E.M.F.* is always equal and opposite to the resultant of the reactance drop and the resistance drop, and to satisfy this condition the locus of the point *A* must be a semicircle. As I_p is proportional to the reactive drop, and since the two angles marked θ are equal, it follows that the locus of the point *B* is also a semicircle. The diameter of this semicircle is

$$\frac{E_p}{X_p + \frac{X_s}{\tau^2}} \text{ amperes,}$$

which is the condition corresponding to zero resistance. To sum up, then, the locus of the load current for various resistances, when the load is non-inductive, is a semicircle whose diameter is the ratio of the primary impressed *E.M.F.* to the total equivalent reactance of the transformer, and whose diameter is at right angles to E_p.

The total current produced by E_p of Fig. 121, when the load is non-inductive ($X_2 = 0$), is the vectorial sum of I_p and I_{exc}, as shown in Fig. 124. The resulting primary current lags behind E_p by an angle ϕ_p, and the power factor of the complete circuit is the ratio of *OM* to *ON*, or $\cos \phi_p$. The power supplied to the transformer is the product of E_p and *OM*. Knowing the copper and core losses, the out-

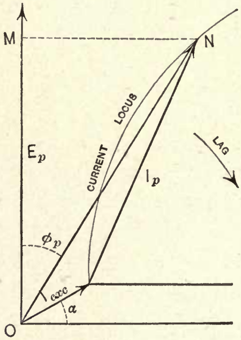


Fig. 124.

put P may be computed, and the efficiency of the transformer determined. The regulation is then obtained from

$$\text{Regulation} = \frac{\tau E_p - \frac{P}{I_s}}{\frac{P}{I_s}} = \frac{\tau E_p I_s - P}{P}.$$

69. **Methods of Connecting Transformers.** — There are numerous methods of connecting transformers to distributing circuits. The simplest case is that of a single transformer in a single-phase circuit. Fig. 125 shows such an arrangement. This and the succeeding figures have the pressure and current values of the different parts marked on them, assuming in each case a 1-K.W., 1 to 10 step-down transformer.

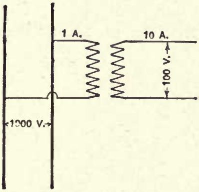


Fig. 125.

As in Fig. 126, two or more transformers may have their primaries in parallel on the same circuit, and have their secondaries independent. If the two secondaries of this case are connected properly in series a secondary system of double the potential will result, or by adding a third wire to the point of juncture, as shown by the dotted line of Fig. 127, a three-wire system of distribution can be secured.

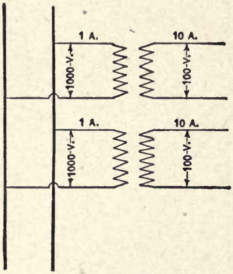


Fig. 126.

The secondaries must be connected cumulatively; that is, their instantaneous $E.M.F.$'s must be in the same direction. If connected differentially, there would be no pressure between the two outside secondary wires, the instantaneous pressures of the two coils being equal and opposed throughout the cycle. Again, with the same condition of primaries, the secondaries can be connected in multiple as in Fig. 128. Here the connections must be such that at any instant the *E.M.F.*'s of the secondaries are toward the same distributing wire. The connection of more than two secondaries in series is not common, but where a complex network of secondary distributing mains is fed at various points from a high-tension system, secondaries are necessarily put in multiple.

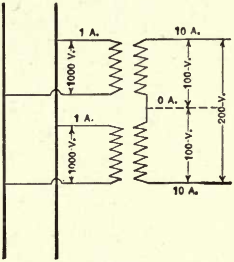


Fig. 127.

In many types of modern transformers it is usual to wind the secondaries (low-tension) in two separate and similar coils, all four ends being brought outside of the case. This allows of connections to two-wire systems of either of two pressures, or for a three-wire system according to Figs. 127 and 128, to be made with the one transformer, this being more economical than using two transformers of half the size, both in first cost and in cost of operation. In many transformers the primary coils are also wound in two parts. In these, however, the four terminals are not always brought outside, but in some

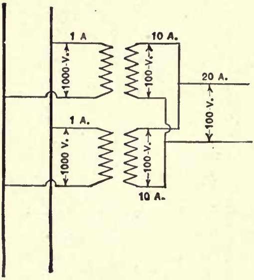


Fig. 128.

cases are led to a porcelain block on which are four screw-connectors and a pair of brass links, allowing the coils to be arranged in series or in multiple according to the pressure of the line to which they are to be connected. From this block two wires run through suitably bushed holes outside the case.

A two-phase four-wire system can be considered as two independent single-phase systems, transformation being accomplished by putting similar single-phase transformers in the circuit, one on each phase. If it is desired to tap a two-phase circuit to supply a two-phase three-wire circuit, the arrangement of Fig. 129 is employed. By the reverse connections two-phase three-wire can be transformed to two-phase four-wire. An interesting transformer connection is that devised by Scott, which permits of transformation from two-phase four-wire to three-phase three-wire. Fig. 130 shows the connections of the two transformers. If one

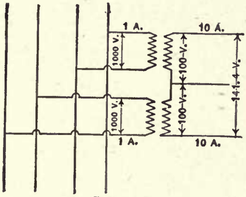


Fig. 129.

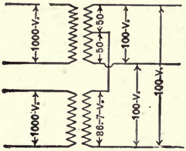


Fig. 130.

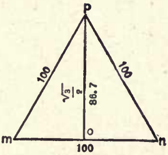


Fig. 131.

of the transformers has a ratio of 10 to 1 with a tap at the middle point of its secondary coil, the other must have a ratio of 10 to .867 $\left(10 \text{ to } \frac{\sqrt{3}}{2}\right)$. One terminal of the secondary of the latter is connected to the middle of the former, the remaining three free terminals being connected respectively to the three-phase wires. In Fig. 131, considering the secondary coils only, let *mn* represent the pressure generated in the first transformer. The pressure in the second transformer is at right angles (§ 7) to that in the first, and because of the manner of connection, proceeds from the center of *mn*. Therefore the line *op* represents in position, direction, and magnitude the pressure generated in the second. From the geometric conditions *mnp* is an equilateral triangle, and the pressures represented by the three sides are equal and at 60° with the others. This is suitable for supplying a three-phase system. In power transmission plants it is not uncommon to find the generators wound two-phase, and the step-up transformers arranged to feed a three-phase line.

In America it is common to use one transformer for each phase of a three-phase circuit. The three transformers may be connected either Y or Δ. They may be Y on the primary and Δ on the secondary, or *vice versa*. Fig. 132 shows both primary and secondary connected Δ. The pressure on each primary is 1000 volts, and as a 1-K.W. transformer was assumed, i.e., 1 K.W. per phase, there will be one ampere in each, calling for 1.7 ($\sqrt{3}$) amperes in each primary main (§ 45). This arrangement is most desirable where continuity of service is requisite, for one of the trans-

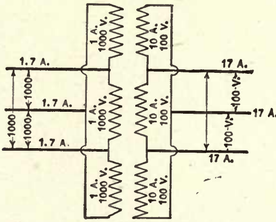


Fig. 132.

formers may be cut out and the system still be operative, the remaining transformers each taking up the difference between $\frac{1}{3}$ and $\frac{1}{2}$ the full load; that is, if the system was running at full load, and one transformer was cut out, the other two would be overloaded $16\frac{2}{3}$ per cent. Even if two of them were cut out, service over the remaining phase could be maintained. It is not uncommon to regularly supply motors from three-phase mains by two somewhat larger transformers rather than by three smaller ones.

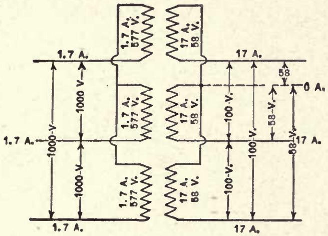


Fig. 133.

Fig. 133 shows the connections for both primaries and secondaries in Y. If in this arrangement one transformer be cut out, one wire of the system becomes idle, and only a reduced pressure can be maintained on the remaining phase. The advantage of the star connection lies in the fact that each transformer need be wound for only 57.7 per cent of the line voltage. In high-tension transmission this admits of building the transformers much smaller than would be necessary if they were Δ connected. Fig. 134 shows the connections for primaries in Δ, secondaries in Y; and Fig. 135 those for primaries in Y and secondaries in Δ. By taking advantage of these last two arrange-

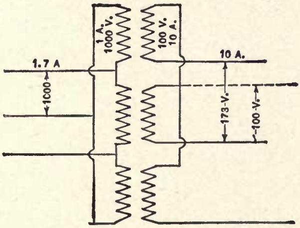


Fig. 134.

ments, it is possible to raise or lower the voltage with 1 to 1 transformers. With three 1 to 1 transformers, arranged as in Fig. 134, 100 volts can be transformed to 173 volts; while if connected as in Fig. 135, 100 volts can be transformed down to 58 volts.

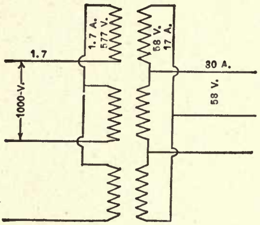


Fig. 135.

Fig. 136 shows a transformer and another one connected as an autotransformer doing the same work. Since the required ratio of transformation is 1 to 2, the autotransformer does the work of the regular transformer with one-half the first cost, one-half the losses, and one-half the drop in potential (regulation). The only objection to this method of transformation is that the primary and secondary circuits are not separate. With the circuits grounded at certain points, there is danger that the insulation of the low-tension circuit may be subjected to

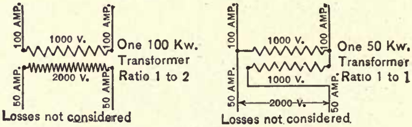


Fig. 136.

the voltage of the high-tension circuit. One coil of an autotransformer must be wound for the lower voltage, and the other coil for the difference between the two voltages of transformation. The capacity of an autotransformer is found by multiplying the high-tension current by the difference between the two operative voltages. Autotransformers are often called compensators. Compensators are advantageously used where it is desired to raise the potential by a small amount, as in boosting pressure for very long feeders. Fig. 137 shows three 1 to 2 transformers

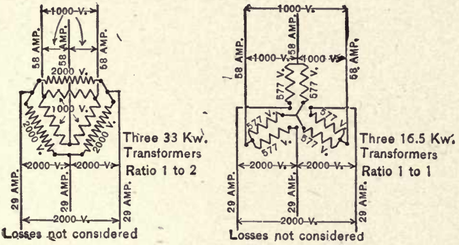


Fig. 137.

connected in Δ on a three-phase system, and three 1 to 1 compensators connected in Y to do the same work.

From a two-phase circuit, a single-phase *E.M.F.* of any desired magnitude and any desired phase-angle may be secured by means of suitable transformers, as shown in Fig. 138. Suppose the two phases *X* and *Y* of a two-phase system be of 100 volts pressure, and it is desired to obtain a single-phase *E.M.F.* of 1000 volts and leading the phase *X* by 30°. As in Fig. 139, draw a line representing the

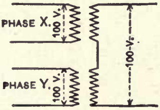


Fig. 138.

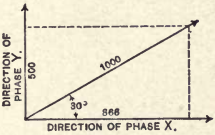


Fig. 139.

direction of phase *X*. At right angles thereto, draw a line representing the direction of phase *Y*. From their intersection draw a line 1000 units long, making an angle of

30° with X. It represents in direction and in length the phase and the pressure of the required $E.M.F.$ Resolve this line into components along X and Y, and it becomes evident that the secondary of the transformer connected to X must supply the secondary circuit with 866 volts, and that the secondary of the other must supply 500 volts. Therefore the transformer connected to X must step-up 1 to 8.66 and that connected to Y must step-up 1 to 5. If 10 amperes be the full load on the secondary circuit, the first transformer must have a capacity of 8.66 K.W., and the second a capacity of 5 K.W. The load on X and Y is not balanced.

70. Lighting Transformers. — Because of the extensive use of transformers on distributing systems for electric lighting, the various manufacturers have to a great extent standardized their lines of lighting transformers. Some of these will be briefly described.

The Wagner Electric Mfg. Co.'s "type M" transformer is illustrated in Fig. 140. It is of the shell type of construction, makers of this type claiming for it superiority of regulation and cool running. In the shell type the iron is cooler than the rest of the transformer, in the core type it is hotter. As the "ageing" of the iron, or the increase of hysteretic coefficient with time, is believed to be aggravated by heat, this is claimed as a point of superiority of the shell type. However, the prime object in keeping a transformer cool is not to save the iron, but to protect the insulation; and as the core type has less iron and generally less iron loss, the advantages do not seem to be remarkably in favor of either. In the Wagner "type M" transformers the usual practice of having two sets of primaries and sec-

Fig. 140.

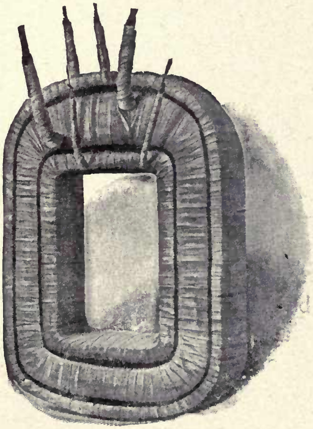


Fig. 141.

ondaries is followed. Fig. 141 shows the three coils composing one set. A low-tension coil is situated between two high-tension coils, this arrangement being conducive to a good regulation. The ideal method would be to have the coils still more subdivided and interspersed, but practical reasons prohibit this. The space between the coils and the iron is left to facilitate the circulation of the oil in which they are submerged. The laminæ for the shell are stamped each in two parts and assembled with joints staggered. As can be seen from the first cut, all the terminals of the two primary and the two secondary coils are brought outside the case. The smaller sizes of this line of transformers, those under 1.5 K.W., have sufficient area to allow their running without oil, so the manufacturers are enabled to fill the retaining case with an insulating compound which hardens on cooling.

The General Electric Co.'s "H" transformers are of the core type. In Fig. 142 is shown a sectional view giving a good idea of the arrangement of parts in this type. Fig. 103 is also one of this line of transformers. In it is shown the tablet board of porcelain on which the connections of the two high-tension coils may be changed from series to parallel or *vice versa*, so that only two high-tension wires are brought through the case. Fig. 143 shows the arrangement of the various parts in the assembled apparatus. The makers

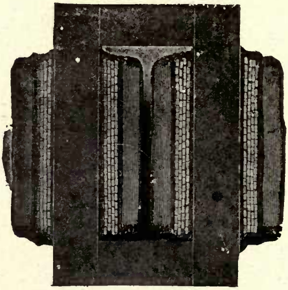


Fig. 142.

claim for this type that the coils run cooler because of their being more thoroughly surrounded with oil than those of the shell type. Another point brought forward is that copper is a better conductor of heat than iron; the heat from the inner portions of the apparatus is more readily dissipated than in the shell type. The core has the advantage of being made up of simple rectangular punchings, and the disadvantage of having four instead of two joints in the magnetic circuit. A particular advantage of the "type H" transformer is the ease and certainty with which the primary windings can be separated from the secondary windings. A properly formed seamless cylinder of fiber can be slipped over the inner winding and the outer one wound over it. This is much more secure than tape or other material that has to be wound on the coils.

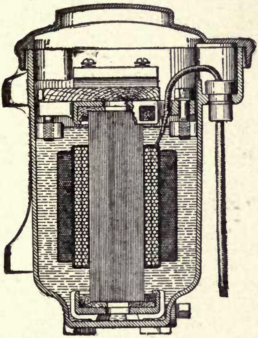


Fig. 143.

Fig. 144 shows a 2-K.W. O. D. transformer without the case. A tablet board is used for the terminals of the high-tension coils, but the low-tension wires are all run out of the case. Fig. 145 shows one of the coils. "Type O. D." transformers are built from $\frac{1}{4}$ to 25 K.W. for lighting and to 50 K.W. for power. Those of 10 K.W. or less are in cast-iron cases, those above 10 K.W. in corrugated iron cases with cast tops and bottoms. The corrugations quite materially increase the radiating surface. The windings are submerged in oil.

An example of the Stanley Electric Manufacturing Co.'s standard line of "type A. O." transformers is given in

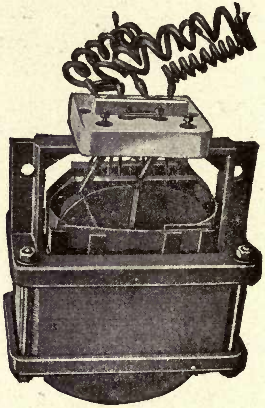


Fig. 144.

Fig. 146. These are also of the shell type, with divided primaries and secondaries.

71. Cooling of Transformers. — The use of oil to assist in the dissipation of the heat produced during the operations of transformers is almost universal in sizes of less than about 100 K.W., especially if designed for outdoor use. Some small transformers are designed to be self-ventilating, taking air in at the bottom, which goes out at top as a result of being heated. They are not well protected from the weather, and are liable to have the natural draft cut off by the building of insects' nests. Larger

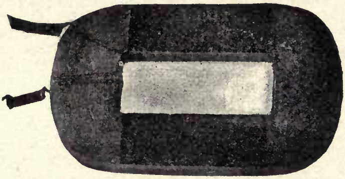


Fig. 145.

transformers that are air cooled and that supply their own draft are used to some extent in central stations and other

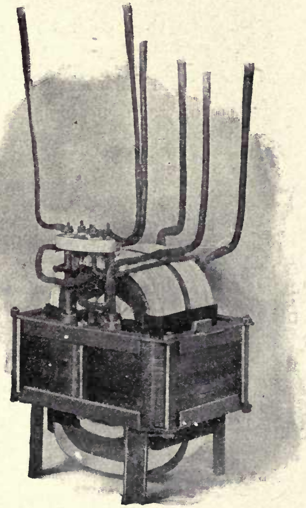


Fig. 146.

places where they can be properly protected and attended to. A forced draft is, however, the more common. Where such transformers are employed, there are usually a number of them; and they are all set up over a large chamber into which air is forced by a blower, as indicated in Fig. 147.

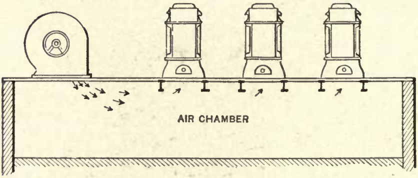


Fig. 147.

Dampers regulate the flow of air through the transformers. They can be adjusted so that each transformer gets its proper share.

Fig. 148 shows a General Electric Company's air-blast transformer in process of construction. The iron core is built up with spaces between the laminæ at intervals; and the coils, which are wound very thin, are assembled in small intermixed groups with air spaces maintained by pieces of insulation between them. The assembled structure is subjected to heavy pressure, and is bound together to prevent the possibility of vibration in the coils due to the periodic tendency to repulsion between the primary and the secondary. These transformers are made in sizes from 100 K.W. to 1000 K.W. and for pressures up to 35,000 volts.

Another method of cooling a large oil transformer is to circulate the oil by means of a pump, passing it through a radiator where it can dissipate its heat. Again cold water is forced through coils of pipe in the transformer case, and it takes up the heat from the oil. There is the slight danger in this method that the pipes may leak and the water may injure the insulation. Water-cooled transformers have been built up to 2000 K.W. capacity.

In those cases where the transformer requires some outside power for the operation of a blower or a pump, the power thus used must be charged against the trans-

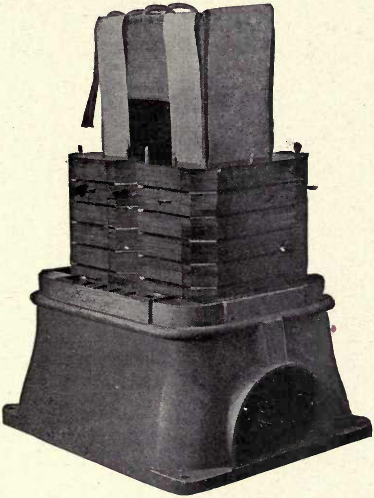


Fig. 148.

former when calculating its efficiency. In general this power will be considerably less than 1% of the transformer capacity.

72. Constant-Current Transformers. — For operating series arc-light circuits from constant potential alternating-current mains, a device called a constant-current transformer is frequently employed. A sketch showing the principle of operation is given in Fig. 149. A primary coil is fixed relative to the core, while a secondary coil is allowed room to move from a close contact with the primary to a considerable distance from it. This secondary coil is nearly but not entirely counter-balanced. If no current is taken off the secondary that coil rests upon the primary. When, however, a current flows in the two coils there is a repulsion between them. The counterpoise is so adjusted that there is an equilibrium when the current is at the proper value. If the current rises above this value the coil moves farther away, and there is an increased amount of leakage flux. This lowers the *E.M.F.* induced

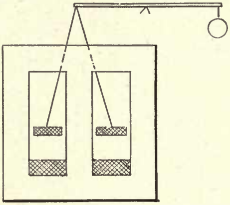


Fig. 149.

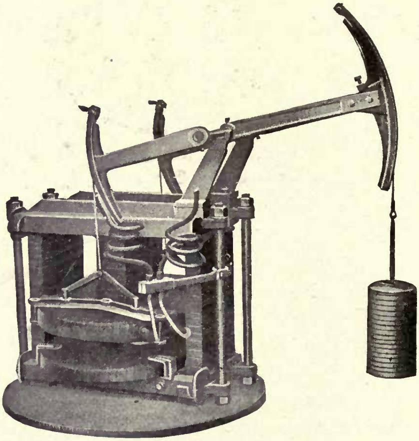


Fig. 150

in the secondary, and the current falls to its normal value. Thus the transformer automatically delivers a constant current from its secondary when a constant potential is impressed on its primary.

Fig. 150 shows the mechanism of such an apparatus as made by the General Electric Company. The cut is self-explanatory. Care is taken to have the leads to the mov-

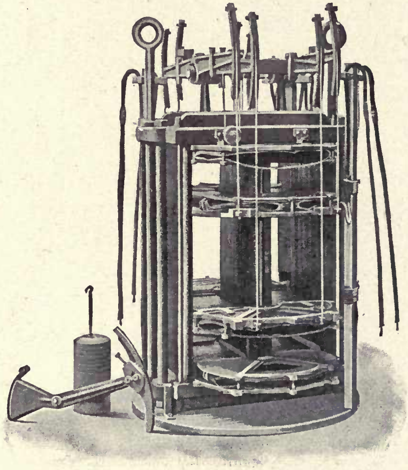


Fig. 151.

ing coil very flexible. Transformers for 50 lamps or more are made with two sets of coils, one primary coil being at the bottom, the other at the top. The moving coils are balanced one against the other, avoiding the necessity of a very heavy counterweight. Fig. 151 shows a 50-light constant-current transformer without its case. Fig. 152 shows a complete 25-lamp apparatus. The tank is filled with oil, the same as an ordinary transformer. Great care must be taken to keep these transformers level, and to assist in this the larger sizes have spirit-levels built

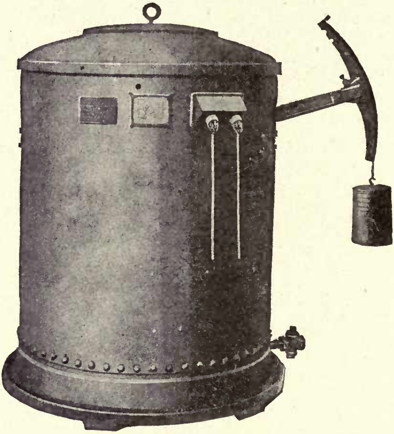


Fig. 152.

into the case. A pair of these transformers can be specially wound and connected to supply a series arc-light circuit from a three-phase line, keeping a balanced load on the latter.

73. Polyphase Transformers. — In transforming from one n-phase system to another n-phase system, instead of using n single-phase transformers, one n-phase transformer may be employed. A polyphase transformer consists of several single-phase transformers having portions of their magnetic circuits in common. As these common portions of the magnetic circuits carry fluxes differing in phase, an economy of material results due to the fact that the resultant flux is less than the arithmetical sum of the component fluxes. A further saving is effected due to the

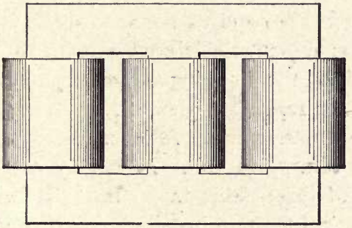


Fig. 153.

necessity of only one instead of several containing cases, but this disappears, however, when the single-phase transformers are all mounted in one case.

Three-phase transformers are used extensively in Europe

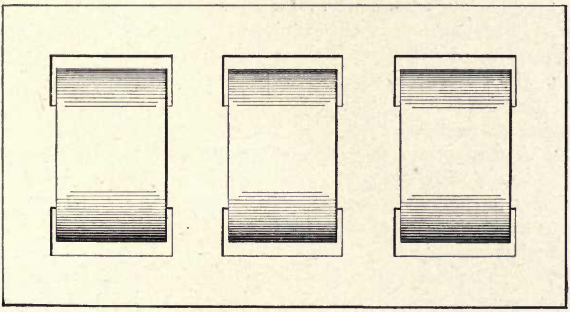


Fig. 154.

and the tendency toward their use in America is constantly increasing. The magnetic circuits of the two most common types of three-phase transformers are diagrammatically shown in Fig. 153 and Fig. 154. The first is known as the core type, and the other as the shell type.

The advantages claimed for the three-phase transformer over three single-phase units are: 1, a saving of about 15 % in first cost; 2, the required floor space is smaller and the weight is less; 3, greater efficiency. In the case of a break-down, however, the resulting derangement of the service and the cost of repair are greater for three-phase than for single-phase transformers. Another disadvantage is the greater cost of a spare unit. In large power stations an installation of three-phase transformers is believed to be more economical than an installation of single phase units.

PROBLEMS.

1. Determine the total flux of a 60 cycle lighting transformer having 0.8 primary turn per volt of primary impressed *E.M.F.*

2. Calculate the eddy current and hysteresis losses in the iron of a 125 cycle core-type transformer for which Φ_m is 0.2 megamaxwell. The mean length of the magnetic circuit is 35 inches and the cross-section is 9 square inches.

3. If the *E.M.F.* impressed upon the primary of the transformer of the preceding problem be 2200 volts, compute the value of the exciting current and its phase.

4. A transformer has 2000 turns of No. 16 B. & S. copper wire on the secondary winding, and 100 turns of No. 4 B. & S. copper wire on the primary winding. The mean lengths of the secondary and primary turns are respectively 17 and 28 inches. Determine the total equivalent primary resistance.

5. What is the copper loss in the transformer of the preceding problem when the primary current is 25 amperes, the exciting current being neglected?

6. Assuming that the transformer of problems 2 and 3 is a 5 K.W., 20 : 1 step-down transformer, and that the primary and secondary resistances are 16.6 and .041 ohms respectively; determine the efficiency at full load.

7. Find the all-day efficiency of the transformer of the preceding problem, basing the calculation upon 5 hours full-load and 19 hours no-load operation.

8. Calculate the equivalent primary and the equivalent secondary leakage reactances of a 60 ~ shell type transformer having one primary and one secondary coil. The constants indicated in Fig. 117 are:

Mean length of secondary turn		28	inches
Mean length of primary turn		37.5	inches
$n_p = 396$	$P = S =$	1.25	inches
$n_s = 18$	$g =$	.25	inch
$\lambda = 15.5$ inches	$l =$	6.5	inches.

What is the total equivalent primary leakage reactance, the transformer assumed to be under considerable load?

9. The resistances of the primary and secondary windings of a 1 : 2, 60 cycle, step-up transformer are respectively 0.1 and .34 ohms. The equivalent leakage reactances of the primary and of the secondary are .14 and .5 ohms respectively, and the secondary induced electromotive force, E_s, is 220 volts. Determine the *E.M.F.* to be impressed upon the primary terminals, the load on the secondary consisting of a 6 ohm resistance and an inductance of .01 henry. The exciting current is .85 amperes and lags 70° behind E_0.

10. Calculate the regulation of the transformer of the preceding problem.

11. It is desired to transform from 2200 volts two-phase to 500 volts three-phase by means of a Scott transformer. Allowing one volt per turn on the windings, find the number of turns on each primary and on each secondary coil.

12. A 100–1000 volt step-up transformer is connected to the *A*-phase of a two-phase, four-wire system, and a 100–2000 volt step-up transformer is connected to the other phase. Determine the magnitude and phase of the secondary electromotive force when the secondary coils are in series.

CHAPTER VII.

MOTORS.

INDUCTION MOTORS.

74. Rotating Field. — Suppose an iron frame, as in Fig. 155, to be provided with inwardly projecting poles, and that these be divided into three groups, arranged as in the diagram, poles of the same group being marked by the same letter. If the poles of each group be alternately wound in opposite directions, and be connected to a single source of *E.M.F.*, then the resulting current would magnetize the interior faces alternately north and south. If the impressed *E.M.F.* were alternating, then the polarity of each pole would change with each half cycle. If the three groups of windings be connected respectively with the three terminals of a three-phase supply circuit, any three successive poles will assume successively a maximum polarity of the same sign, the interval required to pass from one pole to its neighbor being one-third of the duration of a half cycle. The maximum intensity of either polarity is therefore passed from one pole to the next, and the result is a *rotating field.* If the frequency of the supply *E.M.F.* be f, and

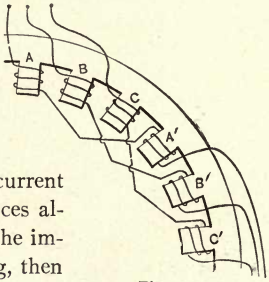


Fig. 155.

if there be p pairs of poles per phase, then the field will make one complete revolution in $\frac{p}{f}$ seconds. It will therefore make $\frac{f}{p} = \frac{V}{60}$ complete revolutions per second. A rotating field can be obtained from any polyphase supply-circuit by making use of appropriate windings.

75. The Induction Motor. — If a suitably mounted hollow conducting cylinder be placed inside a rotating field, it will have currents induced in it, due to the relative motion between it and the field whose flux cuts the surface of the cylinder. The currents in combination with the flux will react, and produce a rotation of the cylinder. As the current is not restrained as to the direction of its path, all of the force exerted between it and the field will not be in a tangential direction so as to be useful in producing rotation. This difficulty can be overcome by slotting the cylinder in a direction parallel with the axis of revolution. Nor will the torque exerted be as great as it would be if the cylinder were mounted upon a laminated iron core. Such a core would furnish a path of low reluctance for the flux between poles of opposite sign. The flux for a given magnetomotive force would thereby be greater, and the torque would be increased.

Induction motors operate according to these principles. The stationary part of an induction motor is called the *stator*, and the moving part is called the *rotor*. It is common practice to produce the rotating field by impressing *E.M.F.* upon the windings of the stator. There are, however, motors whose rotating fields are produced by the currents in the rotor windings.

The construction of a line of induction motors manufactured by the General Electric Company is shown in Fig. 156. In this type the outer edges of the stator laminations are directly exposed to the air, thus improving ventilation. The stator core and windings of a Westing-

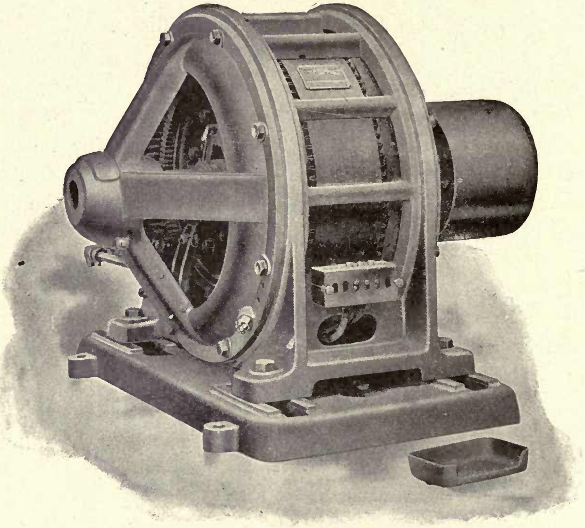


Fig. 156.

house induction motor are shown in Fig. 157. Each projection of the core does not necessarily mean a pole; for it is customary to employ a distributed winding, there being several slots per pole per phase. The stator windings are similar to the armature windings of polyphase alternators. The winding for each phase consists of a group of coils, one group for each pole. The individual coils of each group are laid in separate slots. The stator windings of a three-phase induction motor are shown in Fig. 158, where each loop represents the group of coils for one pole.

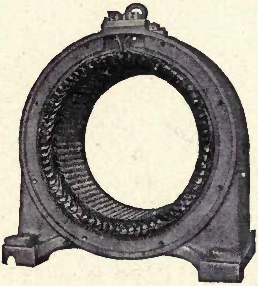


Fig. 157.

One type of rotor is shown in Fig. 159. The inductors are copper bars embedded in slots in the laminated steel core. They are all connected, in parallel, to copper collars or short-circuiting rings, one at each end of the rotor. They offer but a very small resistance, and the currents induced in them are forced to flow in a direction parallel with the axis. The reaction against the field flux is there-

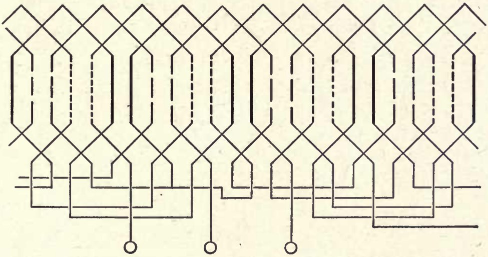


Fig. 158.

fore in a proper direction to be most efficient in producing rotation. A rotor or armature of this type is called a *squirrel cage*.

Another type of rotor frequently used, especially in large induction motors, has polar windings which are similar to the windings on the stator. Fig. 160 shows a

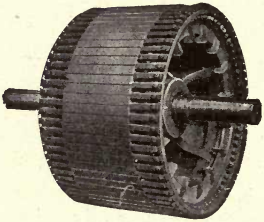


Fig. 159.

rotor of this type made by the General Electric Company. The windings are short-circuited through an adjustable resistance carried on the rotor spider. When starting the

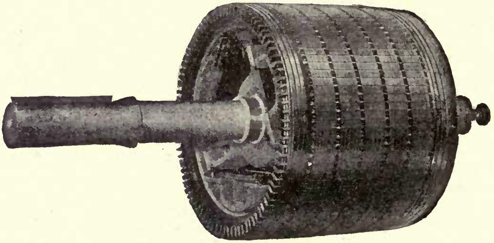


Fig. 160.

motor, all the resistance is in circuit, and after the proper speed has been attained, the resistance may be cut out by pushing a knob on the end of the shaft, as shown in the figure. This arrangement permits of a small starting current under load and a large torque, § 77. Fig. 161

shows another rotor of this type made by the same company; the windings are identical with those on the other, except that their terminals are brought out to three slip-

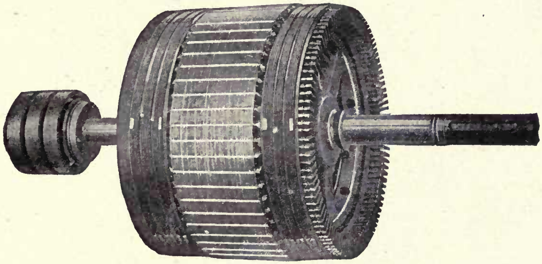


Fig. 161.

rings. A starting resistance can be placed away from the motor and be connected with the rotor windings by means of brushes rubbing upon the slip-rings.

76. Starting of Squirrel-Cage Motors. — To avoid the excessive rush of current which would result from connection of a loaded squirrel-cage motor to a supply circuit, use is made by both the Westinghouse Company and the General Electric Company of starting compensators. These are auto-transformers which are connected between the supply mains, and which, through taps, furnish to the motor circuits currents at a lower voltage than that of the supply mains. After the rotor has attained the speed appropriate to the higher voltage, the motor connections are transferred to the mains, and the compensator is thrown out of circuit. The connections are shown in Fig. 162, and the appearance of the General Electric Company compensator is shown in Fig. 163. The change of connections is accomplished by moving the handle shown at the right of the figure. While the compensator is supplied with various taps, only that one which is most suitable for

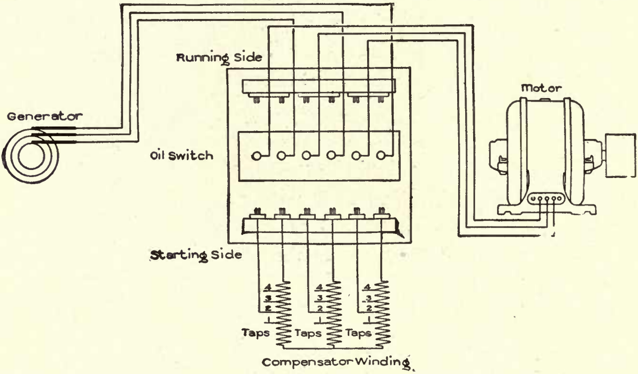


Fig. 162.

the work is used when once installed. The Westinghouse starter for squirrel-cage induction motors is shown in Fig. 164. It consists of auto-transformers and a multi-point drum-type switch, the latter being oil immersed so as to eliminate sparking at the points of contact. An important feature of this design is that the handle is moved in but one direction in passing from the off, through the starting, to the running position, thus making it impossible to connect the motor directly to the full line voltage.

Where special step-down transformers are used for individual motors, or where several motors are located close to and operated from a bank of transformers, it is sometimes practical to bring out taps from the secondary winding, and use a double-throw motor switch, thereby making provision

Fig. 163.

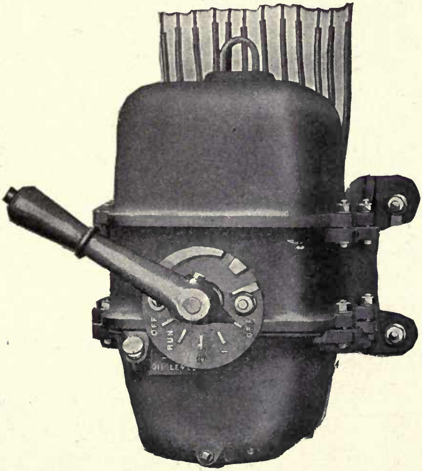


Fig. 164.

for starting the motor at low voltage, while avoiding the necessity for a starting compensator.

The General Electric Company make small squirrel-cage motors, with centrifugal friction clutch pulleys; so that although a load may be belted to the motor, it is not applied to the rotor until the latter has reached a certain speed. The starting current is therefore a no-load starting current.

77. Principle of Operation of the Induction Motor. — If the speed of rotation of the field be V R.P.M. and that of the rotor be V' R.P.M., then the relative speed between a given inductor on the rotor and the rotating field will be $V - V'$ R.P.M. The ratio of this speed to that of the field, viz., $\frac{V - V'}{V} = s$, is termed the *slip*, and is generally expressed as a per cent of the synchronous speed. If the flux from a single north pole of the stator be Φ maxwells, then the effective *E.M.F.* induced in a single rotor inductor is $2.22\ p\Phi\, s \frac{V}{60}\, 10^{-8}$, where p represents the number of pairs of stator poles. The frequency of this induced *E.M.F.* is different from that of the *E.M.F.* impressed upon the stator. It is s times the latter frequency. The frequency would be zero if the rotor revolved in synchronism with the field, and would be that of the field current if the rotor were stationary. As the slip of modern machines is but a few per cent (2 % to 15 %), the frequency of the *E.M.F.* in the rotor inductors, under operative conditions, is quite low. The current which will flow in a given inductor of a squirrel-cage rotor is difficult to determine. All the inductors have *E.M.F.*'s in them, which at any instant are of different values, and in some of them the current may flow in opposition to the *E.M.F.* It can be seen, however, that the rotor impedance is very small. As the impedance is dependent upon the frequency, it will be larger when the rotor is at rest than when revolving. It will reduce to the simple resistance when the rotor is revolving in synchronism. Suppose a rotor to be running light without load. It will revolve but slightly slower than the revolving field, so that just enough *E.M.F.* is generated to produce such a current in the rotor inductors that the electrical power is equal to the losses due to friction, windage, and the core and copper losses of the rotor. If now a mechanical load be applied to the pulley of the rotor, the speed will drop, i.e., the slip will increase. The *E.M.F.* and current in the rotor will increase also, and the rotor will receive additional electrical power, equivalent to the increase in load. The induction motor operates in this respect like a shunt motor on a constant potential direct-current circuit. If the strength of the rotating field, which cuts the rotor inductors, were maintained constant, the slip, the rotor *E.M.F.*, and the rotor current would vary directly as the mechanical torque exerted. If the rotor resistance were increased, the same torque would require an increase of slip to produce the increased *E.M.F.* necessary to send the same current, but the strict proportionality would be maintained. The rotating magnetism, which cuts the rotor inductors, does not, however, remain constant under varying loads. As the slip increases, more and more of the stator flux passes between the stator and rotor windings, without linking them. This increase of magnetic leakage is due to the cross magnetizing action of the increased rotor currents. The decrease of linked field flux not only lessens the torque for the same rotor current, but also makes a greater slip necessary to produce the same current. The relation which exists between torque and slip for various rotor resistances is shown in Fig. 165, where the full lines represent torque, and the dotted lines current. An inspection of the curves shows that the maximum

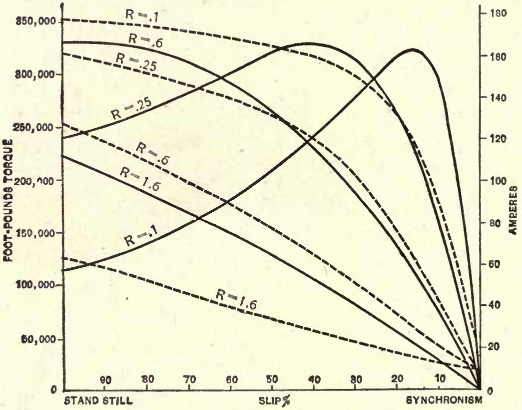


Fig. 165.

torque which a motor can give is the same for different rotor resistances. The speed of the rotor, however, when the motor is exerting this maximum torque, is different for different resistances. This fact is made use of in starting induction motors having wound rotors so that the starting current may not be excessive. The rotor resistance is designed to give full-load torque at starting with full load current. When the motor reaches its proper speed, this resistance is gradually cut out so that a large torque is secured within the operative range.

78. Relation between Speed and Efficiency. — A portion of the total power supplied to the stator of an induction motor is consumed in the resistance of the stator windings, another portion is consumed in overcoming the stator iron losses, and the remainder is supplied to the rotor. Expressing this statement in the form of an equation, the total power supplied to the stator is

$$P_1 = P_{l_1} + P_2,$$

where P_{l_1} is the sum of the stator copper and iron losses. Similarly, a portion of the power delivered to the rotor is consumed in heating the rotor windings, and another portion, very small and usually negligible, is consumed in overcoming the rotor iron losses, the rest being available as mechanical power. A small amount of the latter is wasted in bearing friction and windage, but this loss will be ignored. Then the power input to rotor is

$$P_2 = P_{c2} + P_0,$$

where P_{c2} is the power expended in heating the rotor windings, and where P_0 is the mechanical power developed in the rotor. Combining these expressions

$$P_1 = P_{l_1} + P_{c2} + P_0.$$

To obtain an approximate relation between efficiency and speed, it is convenient to neglect the stator losses; that is, the total power taken by the motor is considered as effective in producing the revolving field. Then the power input to the motor is

$$P_1 = P_2 = P_0 + P_{c2}.$$

If the torque exerted by the rotating field upon the rotor be T lb.-ft., the power required to rotate this field at V

revolutions per minute against the reaction of the rotor will be $2\pi VT$ ft.-lbs. per minute. The speed of the rotor being V' revolutions per minute, the power developed in the rotor will therefore be $2\pi V'T$ ft.-lbs. per minute. The power expended in heating the rotor windings is $2\pi VT - 2\pi V'T$, or $2\pi T(V - V')$ ft.-lbs. per minute. Neglecting the stator losses and the rotor iron losses, the following proportion results:

$$P_1 : P_0 : P_{c2} = 2\pi VT : 2\pi V'T : 2\pi T(V - V'),$$

or $$P_1 : P_0 : P_{c2} = V : V' : V - V' = 1 : 1 - s : s;$$

from which

$$\text{Efficiency} = \frac{P_0}{P_1} = \frac{V'}{V},$$

$$V' = V(1 - s), \quad P_0 = P_2(1 - s), \quad \text{and } P_{c2} = P_1 s.$$

Thus the efficiency of an induction motor is approximately the ratio of the rotor speed to the synchronous speed, and the power expended in the rotor windings is approximately proportional to the slip. Therefore, to secure a high efficiency, the slip should be small so that V' more nearly approaches V; and to have a small slip requires a rotor having windings of low resistance.

79. Determination of Torque. — The torque exerted by an induction motor may be expressed in terms of the stator input, stator losses, and synchronous speed. If P_0 be the motor output in watts, then the torque in lbs. (one foot radius) is

$$T = \frac{33000\, P_0}{2\pi V'\, 746}.$$

But $V' = V(1 - s)$, and $P_0 = P_2(1 - s)$. (§ 78)
When P_0 is expressed in watts, the term P_2 is the rotor

input in *synchronous watts*, so-called. Therefore the torque, neglecting as usual the rotor iron losses, becomes

$$T = \frac{33000\, P_2\, (1 - s)}{2\,\pi V\,(1 - s)\, 746} = 7.04\,\frac{P_2}{V}.$$

The power which is delivered to the rotor is the difference between the motor power intake and the stator losses, that is, $P_2 = P_1 - P_{l_1}$; and, since $V = \frac{60 f}{p}$, the expression for torque becomes

$$T = 0.1174\, p\,\frac{P_1 - P_{l_1}}{f},$$

which is independent of rotor speed and mechanical output.

80. The Transformer Method of Treatment. — It is customary in theoretical discussions to consider the induction motor as a transformer. Evidently when the rotor is stationary the machine is nothing but a transformer, with a magnetic circuit so constructed as to have considerable magnetic leakage. When the rotor is moving, the machine does not act exactly like the ordinary transformer, but its action can be more conveniently and accurately determined by reference to transformer action. When no load is put upon the rotor of an induction motor, the currents supplied to the stator are called the exciting currents, just as is the current in the primary of a transformer when its secondary winding is open-circuited. The counter *E.M.F.*'s induced in the stator windings by the revolving flux is less than the impressed *E.M.F.*'s by amounts sufficient to allow the exciting currents to flow, and these overcome the eddy current and hysteresis losses of the stator iron, and set up the *M.M.F.*'s necessary to establish the rotating field.

When the induction motor operates under load, the slip, which before was practically zero, is increased, and *E.M.F.*'s are induced in the rotor windings due to the relative motion of the rotating field and the rotor. The demagnetizing effects of the rotor currents produced thereby are neutralized, as in the transformer with loaded secondary, by an increase of current in the stator windings, this being possible because of the diminished counter-electromotive forces. On account of the similarity of the actions of the induction motor and the transformer, the stator of machines as ordinarily constructed is also called the *primary*, and the rotor the *secondary* of an induction motor.

When an induction motor is running at a certain slip, s, the frequency of the electromotive forces induced in the rotor windings is s-times the frequency of the supply voltage. Because of this fact, quantities in the secondary circuit cannot be directly added to quantities in the primary circuit, but the reactions of the rotor currents and magnetic fluxes upon the primary are of the same frequency as the primary *E.M.F.*'s; for the flux produced by the secondary currents revolves relative to the rotor with a speed equal to the frequency of the induced secondary *E.M.F.*'s, so that the speed of this flux plus the speed of the rotor is the same as the speed of the revolving field. Thus, the secondary flux of an induction motor reacts upon the primary flux with the same frequency, exactly as in the transformer.

81. Leakage Reactance of Induction Motors. — Not all of the flux set up by the stator currents traverses the air gap between the rotor and stator iron, nor does all of this air gap flux link with the rotor turns, and, similarly, not all of the flux set up by the rotor currents links with the stator turns. The flux which links with one winding and not with the other is called the leakage flux. This magnetic leakage will be considered under the following heads: slot leakage, tooth-tip or "zig-zag" leakage, coil-end leakage, and belt leakage.

Slot Leakage. The flux which passes across the slots and the slot openings is termed the slot leakage flux, the

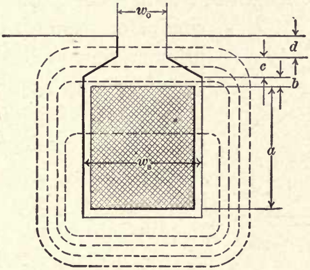


Fig. 166.

various paths thereof being shown by the dotted lines in Figs. 166 and 167. The magnitude of the slot leakage flux is dependent upon the form of the slot, and may be determined when the dimensions shown in the figures are known. Neglecting the reluctance of the iron portion of the magnetic circuit, the permeance of the path of the primary slot leakage flux per inch of slot length of the slot shown in Fig. 166 is

$$\mathcal{P}_1 = 2.54\left[\frac{a_1}{3\,w_{s_1}} + \frac{b_1}{w_{s_1}} + \frac{2c_1}{w_{s_1} + w_{01}} + \frac{d_1}{w_{01}}\right],$$

where the subscripts 1 designate a primary slot. The method of derivation of this formula is identical with that given in § 66.

The slot leakage flux per ampere inch of primary slot is

$$\Phi_{s_1} = 0.4\,\pi \mathcal{P}_1 n_1,$$

and hence the reactance of the primary slot leakage flux per phase in ohms is

$$X_{s_1} = 2\,\pi f l_s\, N_1 n_1\, \Phi_{s_1}\, 10^{-8},$$

where l_s is the length of slot in inches, n_1 is the number of conductors connected in series per primary slot, and N_1 is the number of primary slots per phase.

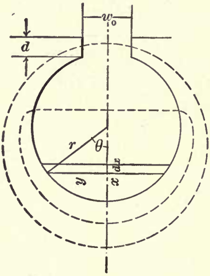


Fig. 167.

In motors having full-pitch windings, all the conductors in one slot belong to the same phase winding, but in motors having fractional-pitch windings some or all of the slots contain conductors belonging to different phase windings, and therefore the conductors in these slots carry currents differing in phase. To take this influence into account, the pitch factor, C, must be inserted in the expression for slot leakage reactance. The value of C is plotted in terms of $\frac{\textit{coil span}}{\textit{pole pitch}}$ in Fig. 168. Then the expression for the equivalent reactance of the primary slot leakage flux per phase in ohms is

$$X_{s_1} = 2\,C f l_s N_1 n_1^2 \left[\frac{a_1}{3\,w_{s_1}} + \frac{b_1}{w_{s_1}} + \frac{2\,c_1}{w_{s_1} + w_{01}} + \frac{d_1}{w_{01}}\right] 10^{-7}. \quad (1)$$

Similarly, the reactance of the secondary slot leakage flux per phase in ohms is

$$X_{s2} = 2\,\pi C f l_s N_2 n_2 \Phi_{s2} 10^{-8},$$

which, when multiplied by $\left(\frac{n_1 N_1}{n_2 N_2}\right)^2 \frac{p_1'}{p_2'}$, where p_1' and p_2' are the number of primary and secondary phases respectively, reduces the secondary slot leakage reactance to the primary circuit, or

$$X_{s2} = 2\, C f l_s n_1^2 \frac{N_1^2 p_1'}{N_2 p_2'}\left[\frac{a_2}{3\, w_{s2}} + \frac{b_2}{w_{s2}} + \frac{2\, c_2}{w_{s2}+w_{02}} + \frac{d_2}{w_{02}}\right] 10^{-7}. \quad (2)$$

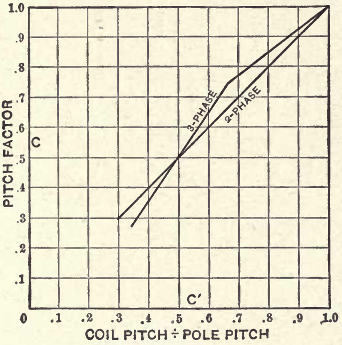


Fig. 168.

If the slots are of the form shown in Fig. 167, the permeance of the elementary path dx per inch of slot length is

$$2.54\,\frac{dx}{2y} = 2.54\,\frac{d\,(r - r\cos\theta)}{2\,r\sin\theta} = 1.27\,d\theta.$$

The leakage flux through this element per ampere inch of slot is

$$d\Phi_s = 0.4\,\pi\,(1.27\,d\theta)\left[n\,\frac{\theta r^2 - r\cos\theta\,.\,r\sin\theta}{\pi r^2}\right].$$

Hence the inductance per phase (considering only the circular portions of the slots) is

$$L_s = \frac{0.508}{\pi\ 10^8}\ n^2 N l_s \int_0^{\pi} (\theta - \cos\theta \sin\theta)^2\, d\theta,$$

and consequently the reactance of this portion of the slot leakage per phase in ohms is

$$X_s = 12.5\ f n^2 N l_s 10^{-8} = 2\ f l_s N n^2\,(0.625)\ 10^{-7},$$

which is similar to the preceding equations for slot leakage. Hence equations (1) and (2) may be employed for calculating the slot leakage reactance of round slots, when the

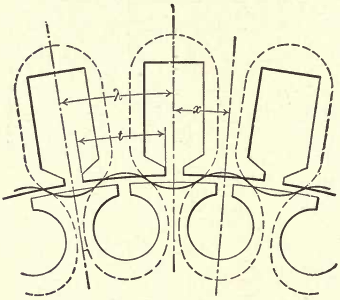


Fig. 169.

first three terms within the brackets of these expressions are replaced by the constant 0.625.

Tooth-Tip Leakage. Tooth-tip leakage is that flux which passes through a portion of the tooth tip opposite a slot. The path of this leakage flux is shown in Fig. 169. For convenience in the following discussion, the number of rotor and stator slots will be assumed equal and their openings extremely small. The permeance of the path of this leakage flux is variable, being zero when a rotor slot is opposite a stator slot, and a maximum when a rotor slot is midway between two stator slots. The maximum permeance per inch of slot length is

$$\mathcal{P}_m = 2.54 \frac{\frac{\lambda}{2}}{2\Delta},$$

where λ is the average or common tooth pitch, in inches, and Δ is the radial length of the air gap, in inches. The permeance of the path, when the rotor and stator are in an intermediate position, is $2.54 \frac{x}{\lambda} \cdot \frac{\lambda - x}{\Delta}$, and it follows that the average permeance per inch of slot length is

$$\mathcal{P} = \frac{2}{\lambda} \cdot \frac{2.54}{\Delta\lambda} \int_0^{\frac{\lambda}{2}} (\lambda x - x^2)\, dx = 0.423 \frac{\lambda}{\Delta}.$$

The tooth-tip leakage flux per ampere inch of slot for both stator and rotor is

$$\Phi_t = 0.4\,\pi \mathcal{P} n_1 = 0.532 \frac{\lambda n_1}{\Delta},$$

and hence the equivalent reactance of the tooth-tip leakage per phase (stator and rotor) is

$$X_t = 2\,\pi f l_s N_1 n_1\, \Phi_t\, 10^{-8}.$$

When the number of slots in the rotor and in the stator is not the same, and when the slot openings are appreciable, the value of Φ_t must be multiplied by $\left(\frac{t_1}{\lambda_1} + \frac{t_2}{\lambda_2} - 1\right)^2$, where t_1 and t_2 are the equivalent stator and rotor tooth-tip widths respectively, and where λ_1 and λ_2 are the stator and rotor tooth pitches respectively. The equivalent tooth tips are determined by adding 2 $\Delta f'$ to the actual tooth-tip width, where f' is the flux-fringing constant. The value of this constant is given in Fig. 170, where f' is plotted in terms of $\frac{w_0}{2\Delta}$. Then introducing the pitch factor, C, the expression for the equivalent reactance (stator and rotor) of the tooth-tip leakage flux per phase in ohms is

$$X_t = 3.35\, Cfl_sN_1n_1^2\frac{\lambda}{\Delta}\left(\frac{t_1}{\lambda_1} + \frac{t_2}{\lambda_2} - 1\right)^2 10^{-8}. \qquad (3)$$

For motors having squirrel-cage rotors, the value of X_t obtained from (3) should be reduced by 20 %.

Coil-End Leakage. The flux which passes around the

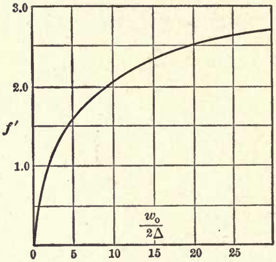


Fig. 170.

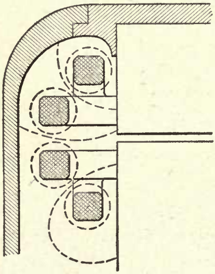


Fig. 171.

ends of the coils where they project beyond the slots is called the coil-end leakage flux, the path of this flux being entirely or partly in air, as shown in Fig. 171. It is almost impossible to calculate accurately the coil-end leakage flux because of the proximity of the motor end plates and the influence of the mutual flux of the different phases. For a full-pitch three-phase winding, it is usual to assume this flux as one maxwell per ampere inch of exposed conductor. This assumption is experimentally justified. Then the flux in maxwells for all the conductors per pole per phase (i.e. per phase-belt) per ampere is

$$\Phi_c = l_c \frac{n_1 N_1}{2\,p},$$

where l_c is the length of the end connections per primary turn, and p is the number of pairs of poles.

The value of Φ_c depends upon the ratio of the pole pitch to the diagonal of the section of the coil end, and is approximately proportional to the logarithm of this ratio. But the diagonal of the section of the coil end is approximately equal to $\frac{\text{pole pitch}}{\text{number of phases}} = \frac{\lambda_p}{p'}$, hence Φ_c is proportional to log p'. For fractional pitch windings, the ratio $\frac{\text{coil span}}{\text{pole pitch}} = C'$ must be introduced. Therefore the value of Φ_c is proportional to log $C'p'$. The value of log $C'p'$ for a full-pitch three-phase winding being 0.477, the inductance per phase-belt for any winding is therefore

$$L_c = \frac{\log C'p'}{0.477} \left(\frac{n_1 N_1}{2\,p}\right)^2 l_c \, 10^{-8}.$$

If the length of the rotor coil-ends be considered 80 % of the stator coil-ends, then the total coil-end inductance per phase is

$$L_c = 3.78 \left(\frac{n_1 N_1}{2\,p}\right)^2 p l_c \log C'p' \,.\, 10^{-8},$$

and the coil-end leakage reactance per phase (stator and rotor) in ohms is

$$X_c = 5.95\, f \frac{n_1^2 N_1^2}{p} l_c \log C'p' \,.\, 10^{-8}. \qquad (4)$$

Belt Leakage. Neglecting the exciting current of the induction motor, and considering an instant when a primary phase-belt completely overlaps a secondary phase-belt, the magnetomotive forces due to the currents in these belts of conductors will be in opposition, and there will be no belt leakage. But, if the belts of conductors be in

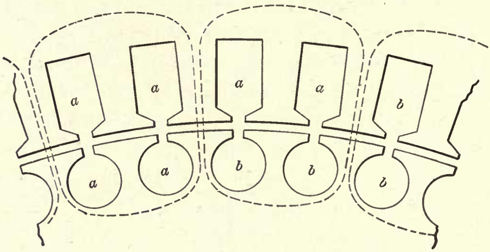


Fig. 172.

any other position, a secondary phase-belt overlaps two primary phase-belts, and their magnetomotive forces are no longer in opposition. The resultant *M.M.F.* will be effective in producing a leakage flux, termed belt leakage, which links with primary and secondary conductors, thus resulting in a leakage reactance. The relative position of stator and rotor for which this reactance is a maximum, is shown in Fig. 172, the paths of the flux being indicated by the dotted lines, and the slots for conductors of different phases being lettered differently.

An expression for the average belt leakage inductance per phase, similar to that given by Adams, is

$$L_b = k' l_s K K_1 K_2 C S_{12}^{\,2} \frac{D}{\Delta} 10^{-10},$$

where S_{12} is the number of series conductors per phase per pole (primary and secondary), D is the rotor diameter in inches, k' is 3.32 for two-phase and 1.005 for three-phase motors, K is the slot contraction factor, or $\frac{t_1 t_2}{\lambda_1 \lambda_2}$, K_1 is a constant depending upon the number of slots per pole as obtained from Fig. 173, K_2 is a constant taking into account the ampere turns for the iron portions of the

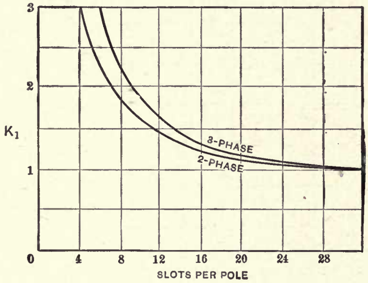


Fig. 173.

belt leakage paths and may be taken as 0.85, and C is the pitch factor as determined from Fig. 168. The belt leakage reactance per phase (primary and secondary), in ohms, is $2\,\pi f L_b$, or

$$X_b = k'' K_1 C f l_s S_{12}^{\;2}\, \frac{D}{\Delta} \cdot \frac{t_1 t_2}{\lambda_1 \lambda_2}\, 10^{-10}, \qquad (5)$$

where k'' is 17.8 for two-phase and 5.36 for three-phase motors. This expression applies to induction motors having phase-wound rotors; for motors having squirrel-cage rotors the value of k'' should be reduced by about 65 %.

Total Leakage Reactance. The total leakage reactance per phase of an induction motor is the sum of the various leakage reactances for which expressions have just been derived. That is, the total reactance per phase, X_T, is equal to the sum of equations (1), (2), (3), (4), and (5).

To secure a high starting torque and efficiency it is

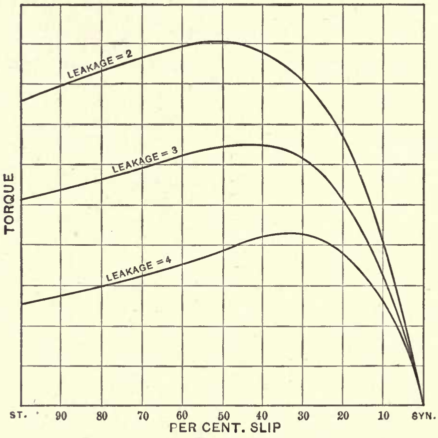


Fig. 174.

necessary to keep the magnetic leakage as small as possible. The relation of torque to speed with various arbitrary values of magnetic leakage is shown in Fig. 174. It is seen that the maximum torque increases directly as the leakage decreases.

The leakage reactance of induction motors may be decreased by employing fractional-pitch windings, and by increasing the reluctance of the path of the leakage flux. The reluctance of the path of the useful flux, however, should be kept as low as possible, and it is usual to make the air gap just as small as is consistent with good mechanical clearance. Concentricity of rotor and stator is to be obtained by making the bearings in the form of end plates fastened to the stator frame. Some makers send wedge gap-gauges with their machines so that a customer may test for eccentricity due to wear of the bearings. A small air gap, besides lowering the leakage and raising the power factor, increases the efficiency and capacity of the motor.

A convenient expression giving the proper radial depth of the air gap in inches, in terms of the horsepower rating of the motor, is

$$\Delta = \frac{\sqrt[3]{\text{hp.}}}{80} .$$

82. Calculation of Exciting Current. — The exciting current per phase of an induction motor has two components; one, the power component, which overcomes the eddy current and the hysteresis losses of the iron, the small stator copper loss due to the exciting current being neglected; and the other, the wattless component of the exciting current, which sets up the magnetomotive force necessary to overcome the reluctance of the magnetic circuit. The iron losses of the rotor under normal conditions are extremely small because of the very low frequency of rotor flux; therefore only the stator iron losses need be considered. In the following discussion, star-connected windings are assumed.

The power component of the exciting current per phase is

$$I_{e+h} = \frac{P_e + P_h}{Ep'}, \qquad \S\ 62.$$

where P_e is the total eddy current loss, P_h is the total hysteresis loss, E is the impressed electromotive force per stator winding, and p' is the number of phases. The values of P_e and P_h must be calculated separately for the stator teeth and for the stator yoke, because the flux densities in these parts of the magnetic circuit are different. The maximum flux density in the teeth will first be determined.

The counter *E.M.F.* induced in a full-pitch distributed stator winding (one phase) by the rotating magnetic field is

$$2\, k_1 k_2 p \Phi S \frac{V'}{60} 10^{-8}, \qquad \S\, 42$$

where k_1 is the form factor or the ratio of the effective to the average *E.M.F.*, k_2 is the distribution constant as obtained from Fig. 57, p is the number of pairs of stator poles, Φ is the flux per pole and is assumed to be sinusoidally distributed, S is the number of conductors connected in series per stator phase, and V' is the rotor speed in revolutions per minute. In single-phase motors, the winding is usually distributed over two-thirds of the pole distance. When the rotor revolves at synchronous speed, then $p \frac{V'}{60} = f$. If the ohmic drop due to the exciting current in the stator winding be neglected, the counter *E.M.F.* and the impressed *E.M.F.* are practically equal. Finally, introducing the pitch factor, C, as obtained from Fig. 168, to take care of fractional-pitch windings, the value of the impressed electromotive force becomes

$$E = 2\, k_1 k_2 C \Phi S f\, 10^{-8},$$

whence

$$\Phi = \frac{E\, 10^8}{2\, k_1 k_2 C S f}.$$

Representing the core length in inches by l_s, and the pole pitch in inches by λ_p, the average flux density (maxwells per square inch) in the air section becomes

$$\mathfrak{B} = \frac{\Phi}{l_s \lambda_p},$$

and the maximum flux density

$$\mathfrak{B}_m = \frac{\pi}{2} \mathfrak{B} = \frac{\pi \Phi}{2\, l_s \lambda_p}.$$

This equation assumes a continuous surface on both sides of the air gap over the polar region, but this does not occur in practice because of the presence of rotor and stator slots. In the following, the rotor slots are assumed extremely small so that their influence on flux distribution is inappreciable. If t_1 be the equivalent stator tooth tip width in inches, and λ_1 be the stator tooth pitch in inches, then the maximum flux density (maxwells per square inch) in the air gap as well as in the tooth, which at that instant is at the center of the polar region, is

$$\mathfrak{B}_{mt} = \frac{\lambda_1}{t_1} \mathfrak{B}_m = \frac{\pi \lambda_1 E\, 10^8}{4\, k_1 k_2 C l_s \lambda_p t_1 S f}, \qquad (1)$$

which is the value to be taken for the maximum flux density in calculating the eddy current and hysteresis losses in the stator teeth.

The maximum flux density in the stator yoke is half of the maximum flux density in the air gap where continuous surfaces on both rotor and stator were assumed. Therefore the maximum flux density to be employed in calculating the eddy current and hysteresis losses in the stator yoke is

$$\mathfrak{B}_{my} = \frac{\pi E\, 10^8}{8\, k_1 k_2 C l_s \lambda_p S f}. \qquad (2)$$

The values of P_e and P_h are calculated as in § 61.

The wattless component of the exciting current, or the magnetizing current, of an induction motor supplies the *M.M.F.* necessary to overcome the air-gap reluctance, the reluctance of the iron being neglected. The magnetomotive force required is

$$M.M.F. = \mathcal{R}\,\Phi = \frac{2\,\Delta \mathcal{B}_{mt}}{2.54} = \frac{\pi\lambda_1 \Delta E\, 10^8}{5.08\, k_1 k_2 C l_s \lambda_p t_1 S f}, \quad (3)$$

where Δ is the radial length of the air gap in inches.

In a two-phase machine, the magnetizing currents in both phases together set up this magnetomotive force. When the magnetizing current in one winding is at maximum value ($\sqrt{2}\, I_{mag}$), at that instant the magnetizing current in the other winding is zero. Therefore the total *M.M.F.* set up per pole per phase will be

$$M.M.F. = \frac{4\,\pi S \sqrt{2}\, I_{mag}}{10 \,.\, 2\,p}. \quad (4)$$

Equating (3) and (4) and solving, the magnetizing current per phase for a two-phase induction motor is

$$I_{mag} = \frac{5\,p\lambda_1 \Delta E\, 10^8}{5.08\,\sqrt{2}\; k_1 k_2 C l_s \lambda_p t_1 S^2 f}. \quad (5)$$

In a three-phase machine, at the instant when the magnetizing current in one winding is at maximum value ($\sqrt{2}\, I_{mag}$), the magnetizing current in each of the other two windings is at half maximum value $\left(\frac{\sqrt{2}}{2} I_{mag}\right)$, and hence the current which sets up the *M.M.F.* is $\sqrt{2}\, I_{mag} + 2\left(\frac{\sqrt{2}}{2} I_{mag}\right) = 2\sqrt{2}\, I_{mag}$, and the magnetomotive force supplied thereby, per pole per phase, is

$$M.M.F. = \frac{4\pi S\, 2\sqrt{2}\, I_{mag}}{10 \cdot 2p}. \qquad (6)$$

Equating (3) and (6) and solving, the magnetizing current per phase for a three-phase induction motor is found to be one-half that for a two-phase motor, or half that given by equation (5). It should be noted that in the foregoing, E is the *E.M.F.* impressed upon each stator winding, and not the *E.M.F.* across motor terminals.

After the two components of the exciting current have been calculated, the magnitude of I_{exc} per phase may be obtained from the relation

$$I_{exc} = \sqrt{I_{e+h}^2 + I_{mag}^2}, \qquad (7)$$

and its angle of lag behind the impressed *E.M.F.* is given by

$$\phi_e = \cot^{-1} \frac{I_{e+h}}{I_{mag}}. \qquad \S\ 62$$

In induction motors the size of I_{e+h} is small compared to I_{mag}, the exciting current differing from the magnetizing current by but a few per cent.

83. Circle Diagram by Calculation. — The similarity between an induction motor operating under a mechanical load and a transformer operating under a resistance load has already been pointed out; from whence it follows that the transformer circle diagram, § 68, may be applied to the induction motor. The circle diagram may be constructed when the magnitude and phase of the exciting current are known, and when the leakage reactance of the motor has been calculated. The magnitude and position of the exciting current per phase may be computed from the expressions derived in the preceding article. This value is laid off as in Fig. 175 and the point D is thus located, which is one of the points on the circular current locus. The diameter of this circle is the ratio of the impressed *E.M.F.* per stator winding to the total leakage reactance per phase. That is, $DC = \frac{E}{X_T}$, where X_T is the sum of equations (1), (2), (3), (4), and (5) of § 81. Thus the circle diagram is completely determined.

The current in a stator winding of an induction motor

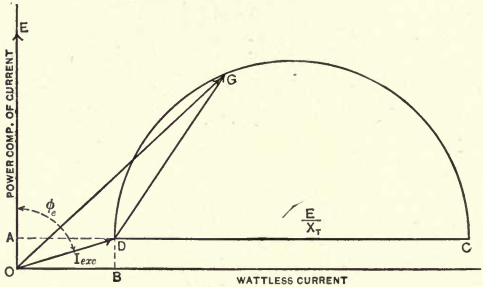


Fig. 175.

may be considered as the resultant of two components: one, the primary exciting current per phase, and the other, the effective current which supplies the magnetomotive force necessary to counterbalance the *M.M.F.* per phase due to the rotor currents. Thus in Fig. 175, OG is the stator current per phase, and is the resultant of OD, the exciting current per phase, and DG, the effective current per phase.

The power factor of an induction motor depends upon the value of the stator current, as may be seen from the figure. The maximum power factor is attained when OG is tangent to the semicircle. Neglecting the power component of the exciting current, DB, the maximum power factor may be expressed as

$$\cos \phi_m = \frac{\frac{DC}{2}}{AD + \frac{DC}{2}} = \frac{\frac{DC}{2}}{\sigma DC + \frac{DC}{2}} = \frac{1}{2\,\sigma + 1},$$

where $\sigma = \frac{AD}{DC}$ = leakage coefficient. The determination of the performance curves of an induction motor from the circle diagram will be considered later.

84. Circle Diagram by Test. — The circle diagram of an induction motor is completely determined and may be constructed when the magnitude and position of the exciting current per phase and when the magnitude and corresponding position of any other current value per stator phase are known.

The exciting current per stator phase can be determined when the primary amperes, the primary voltage between line wires, and the watts input have been obtained while the motor operates at no-load with full voltage. Thus, for a three-phase induction motor having its stator windings Y-connected, Fig. 176, the magnitude of I_{exc} per phase is the ammeter reading. The power input is the sum of the wattmeter readings in the two positions, that is,

$$P_1 + P_2 = \sqrt{3}\, E_l I_{exc} \cos \phi_e. \qquad \S\ 47$$

Therefore the angle by which I_{exc} lags behind the impressed $E.M.F.$ is

$$\phi_e = \cos^{-1} \frac{P_1 + P_2}{\sqrt{3}\, E_l I_{exc}},$$

where E_l is the voltmeter reading. The exciting current per phase, OD, Fig. 175, is now completely determined and may be drawn to a convenient scale. The point D constitutes one point on the circular current locus.

If current, voltage, and power measurements be made when the motor is operating under load, or when the rotor is locked, another point, G, on the current locus may be

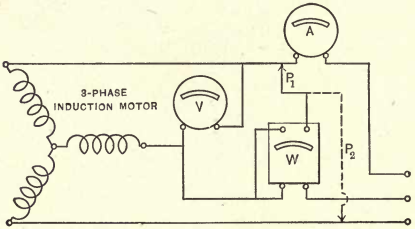


Fig 176.

similarly located. The latter measurement is to be preferred because it determines the extreme value of the stator current, and intermediate points on the diagram are less likely to be in error. The impressed *E.M.F.*, for the measurement with rotor locked, should be reduced to such a value as will send a current which will produce the same heating in the windings as does continuous full-load operation. The current and power may then be calculated for full voltage by increasing the former in direct proportion to the voltage, and by increasing the latter in proportion to the square of the voltage. The corrected current value is then laid off on the diagram to the same scale. The semicircle drawn through the points D and

G, having a center on a line through D perpendicular to the $E.M.F.$ vector, is the required current locus.

Having constructed the circle diagram, the total leakage reactance per phase of the induction motor may be readily obtained, since the diameter of the semicircle is the ratio of the volts per stator phase to the total reactance per phase. The volts per stator phase in the three-phase induction motor with Y-connected windings is $\frac{E_l}{\sqrt{3}}$.

Before proceeding to the calculation of the performance curves of an induction motor, it is necessary to know the resistance per phase of the stator winding and the *equivalent* rotor resistance per stator phase. The resistance, R_1, between terminals of the stator winding may be directly measured by direct-current methods; thus for a three-phase motor, the total stator copper loss (§ 50) is $\frac{3}{2} I^2R_1$, where I is the current per line; hence the stator copper loss per phase is $\frac{I^2R_1}{2}$, and the resistance per phase is $r_1 = \frac{R_1}{2}$.

The equivalent rotor resistance per stator phase, r_2, of a three-phase coil-wound rotor with a winding identical with that of the stator, is half the rotor resistance measured between two slip-rings. Therefore the copper loss of this coil-wound rotor per stator phase is I^2r_2.

The equivalent resistance of a squirrel-cage rotor per stator phase cannot be determined directly, but may be calculated from the observations taken during the motor test with the rotor locked. As already explained, the impressed $E.M.F.$ for this test is reduced, and the current and power input must thereafter be computed for full voltage operation. Representing the stator current per phase at lock by I_L, the power per phase at lock by P_L, and the constant iron and friction loss per phase, as obtained at no-load running, by P_{fi}, then the copper loss in the squirrel-cage rotor of an induction motor per stator phase is

$$P_{c_2} = P_L - P_{fi} - I_L^2 r_1,$$

and the equivalent resistance of the squirrel-cage rotor per stator phase is

$$r_2 = \frac{P_{c2}}{I_L^2}.$$

85. Performance Curves from Circle Diagram. — Having constructed the circle diagram as described in the two

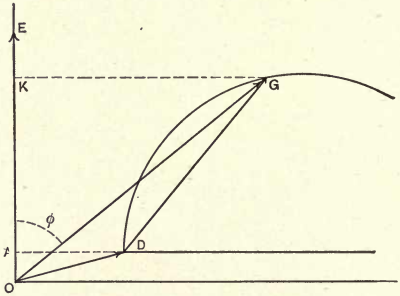


Fig. 177.

foregoing articles, the performance curves of the induction motor may be determined therefrom by calculating the following quantities for a number of positions of the point G on the current locus, Fig. 177.

$$\text{Stator Current per phase} = OG.$$

$$\text{Input to Motor} = E \,.\, OK \,.\, p'.$$

$$\text{Power Factor of Motor} = \cos\phi = \frac{OK}{OG}$$

$$\text{Stator Copper Loss} = r_1 \,.\, \overline{OG}^2 \,.\, p'.$$

$$\text{Input to Rotor} = E \,.\, AK \,.\, p' - r_1 \,.\, \overline{OG}^2 \,.\, p'.$$

$$\text{Rotor Copper Losses} = r_2 \,.\, \overline{DG}^2 \,.\, p'.$$

$$\text{Motor Mechanical Output} = p' \left[E \,.\, AK - r_1 \,.\, \overline{OG}^2 - r_2 \,.\, \overline{DG}^2\right].$$

$$\text{Motor Efficiency} = \frac{\text{Mechanical Output}}{E \,.\, OK \,.\, p'}.$$

$$\text{Slip} = \frac{r_2 \,.\, \overline{DG}^2}{E \,.\, AK - r_1 \,.\, \overline{OG}^2}. \qquad \S\ 78$$

$$\text{Torque} = \text{c.}1174\, p\, \frac{E \,.\, AK \,.\, p' - r_1 \,.\, \overline{OG}^2 \,.\, p'}{f}. \qquad \S\ 79$$

In the foregoing, E is the impressed *E.M.F.* per phase, p' is the number of phases, r_1 is the stator resistance per phase, r_2 is the equivalent rotor resistance per stator phase, p is the number of pairs of poles, and f is the frequency.

The results obtained may then be embodied in a set of curves as in Fig. 178, where abscissæ represent per cent full-load power output, and ordinates represent per cents.

If the voltage impressed upon an induction motor be increased, there will result a proportional increase in the flux linked with the rotor, and in consequence a proportional increase in the rotor current. As the torque depends upon the product of the flux and the rotor ampere turns, it follows that the torque varies as the square of the impressed voltage. The capacity of a motor is therefore changed when it is operated on circuits of different voltages.

Owing to the low power factor of induction motors, transformers intended to supply current for their operation should have a higher rated capacity than that of the

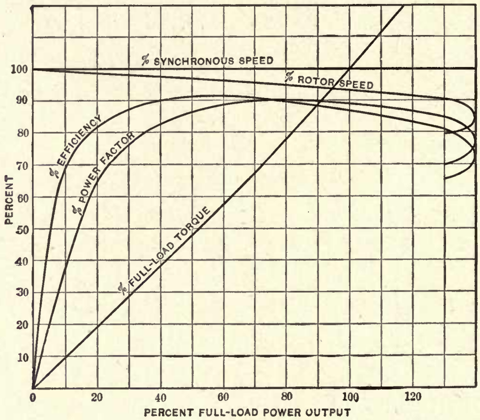


Fig. 178.

motors. It is customary to have the kilowatt capacity of the transformer equal to the horsepower capacity of the motor.

The direction of rotation of a three-phase motor can be changed by transposing the supply connections to any two terminals of the motor. In the case of a two-phase, four-wire motor, the connections to either one of the phases may be transposed.

86. Method of Test with Load. — The complete performance of two-phase or three-phase induction motors when operated from balanced two-phase or three-phase circuits may be calculated when the values of power input, as measured by the two-wattmeter method, § 47, have been determined by test for various mechanical loads upon the rotor. The instruments required are a voltmeter and a wattmeter, three observations being necessary at each load, namely, P_1, P_2, and the line voltage, E_l. The primary resistance measured between terminals and the equivalent secondary resistance per stator phase must be known. An outline of the method employed for a three-phase induction motor follows.

By reference to § 47, it is seen that

$$\text{Total primary input} = P_1 + P_2 = \sqrt{3}\, E_l I \cos\phi, \tag{1}$$

$$P_2 - P_1 = E_l I \sin\phi, \text{ and} \tag{2}$$

$$\text{Primary power factor} = \cos\phi = \cos\tan^{-1}\left[\sqrt{3}\,\frac{P_2 - P_1}{P_1 + P_2}\right]. \tag{3}$$

The primary current per terminal, as obtained from (2), is

$$I = \frac{P_2 - P_1}{E_l \sin\phi}, \tag{4}$$

where

$$\sin\phi = \sin\tan^{-1}\left[\sqrt{3}\,\frac{P_2 - P_1}{P_1 + P_2}\right]. \tag{5}$$

The equivalent single-phase current is $\sqrt{3}\, I$, §50. The power and wattless components of the equivalent single-phase current are respectively

$$\sqrt{3}\, I\cos\phi = \frac{P_1 + P_2}{E_l}, \tag{6}$$

and

$$\sqrt{3}\, I\sin\phi = \sqrt{3}\,\frac{P_2 - P_1}{E_l}. \tag{7}$$

The primary copper loss, § 50, is $\frac{3}{2} I^2 R_1$, (8) where R_1 is the stator resistance measured between terminals. The value of I is given by (4) and (5).

The iron and friction loss is obtained by subtracting the primary copper loss at no-load from the total primary input at no-load, that is,

$$P_{fi} = P_1^0 + P_2^0 - \tfrac{3}{2} I_0^2 R_1, \qquad (9)$$

where P_1^0, P_2^0, and I_0 are the no-load values. I_0 is obtained from (4) and (5) by taking P_2^0 for P_2, and P_1^0 for P_1.

The total primary losses including friction are therefore

$$P_{fi} + \tfrac{3}{2} I^2 R_1. \qquad (10)$$

The secondary input in synchronous watts is the difference between the total primary input and the total primary losses, or

$$P_r = P_1 + P_2 - P_{fi} - \tfrac{3}{2} I^2 R_1. \qquad (11)$$

The external rotor torque, § 79, is $0.1174 \dfrac{P_r p}{f}$, (12)

where p is the number of pairs of poles and f is the frequency of the impressed *E.M.F.*

The power and wattless components of the equivalent secondary current are respectively

$$I_2 \cos \phi_2 = (6) - (6 \text{ at no-load}) = \frac{P_1 + P_2 - P_1^0 - P_2^0}{E_l}, \qquad (13)$$

and

$$I_2 \sin \phi_2 = (7) - (7 \text{ at no-load}) = \frac{\sqrt{3}}{E_l}[P_2 - P_1 - P_2^0 + P_1^0], \qquad (14)$$

hence the equivalent single-phase secondary current is

$$I_2 = \sqrt{(I_2 \cos \phi_2)^2 + (I_2 \sin \phi_2)^2}. \qquad (15)$$

The secondary copper loss $= r_2 I_2^2$, where r_2 is the equivalent secondary resistance.

The percentage rotor slip is the ratio of the secondary copper loss to the secondary input in synchronous watts, or

$$s = \frac{100\, r_2 I_2^2}{P_r}. \tag{16}$$

The output of the motor in horse power is

$$\frac{P_r - r_2 I_2^2}{746}. \tag{17}$$

The efficiency of the induction motor, being the ratio of the watts output to the watts input, is

$$Eff. = \frac{P_r - r_2 I_2^2}{P_1 + P_2}, \tag{18}$$

which, when multiplied by 100, gives the percentage efficiency.

When numerous values of P_1 and P_2 have been experimentally determined and when the foregoing computations have been made for each set of readings, curves of the various factors may be plotted in terms of the motor output, as in the preceding article.

87. Phase Splitters. — In order to operate polyphase induction motors upon single-phase circuits, use is made of inductances in series with one motor circuit to produce a lagging current, or of condensers to produce a leading current, or of both — one in each of two legs. The General Electric Company, in its condenser compensator, for use with small motors, as shown in Fig. 179, employs an autotransformer and condenser, connected as in diagram, Fig. 180.

The autotransformer is used to step-up the voltage, which is impressed upon the condenser, to 500 volts.

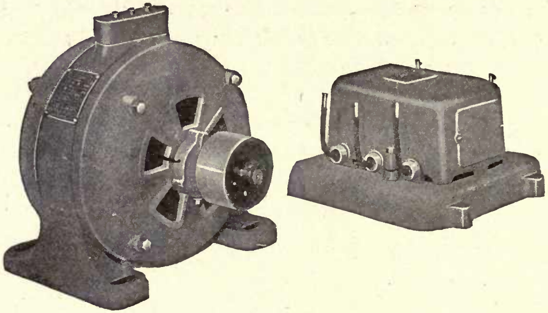


Fig. 179.

The necessary size of the condenser is thereby reduced. The equivalent impedance of the autotransformer and condenser, as connected, is such as to produce a leading current in the one-phase sufficient to give a satisfactory starting torque, and it brings the power factor practically up to unity at all loads.

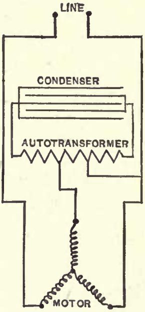


Fig. 180.

88. Single-Phase Induction Motors. — The difference between the single-phase and the polyphase induction motor lies principally in the character of their magnetic fields. In the polyphase motor, the revolving field is practically sinuscidally distributed in space and constant in value. This is also true of the single-phase induction motor when running at synchronous speed, but at any other speed the field is not constant in value, nor is it sinusoidally distributed in space. If an alternating *E.M.F.* be impressed upon the stator winding of a single-phase induction motor an alternating flux will be set up which passes through the rotor. This pulsating flux lags approximately 90° behind the impressed *E.M.F.* When the rotor is in motion, its conductors cut this flux and *E.M.F.*'s are set up in them which are in time phase with the flux. The value of these generated *E.M.F.*'s depends upon the magnitude of the stator flux and upon the speed of the rotor. They set up currents in the rotor conductors, the magnitudes of which are directly proportional to the electromotive forces generated therein. These currents set up a rotor flux which lags 90° in time behind the rotor *E.M.F.*'s, and is displaced 90 electrical degrees in space. Thus the pulsating flux through the rotor due to the impressed alternating *E.M.F.* is at right angles, in a bipolar machine, to the rotor flux due to the motion of the rotor. When the stator flux is a maximum there will be no rotor flux, and when the stator flux is zero the rotor flux will be a maximum. In this way the resultant magnetic flux in the gap changes its position and revolves in the direction of rotation. At synchronous speed, the maximum values of the stator flux and the rotor flux are equal; thus a true rotating field of uniform intensity is produced. At a lower speed, the maximum values are unequal and consequently the rotating field will be of variable intensity. At standstill no revolving field exists and no torque is developed.

The inability of the single-phase induction motor to exert a torque at standstill has led to the introduction of numerous starting devices, but these are usually only applicable to small-sized motors. Three general methods are employed to render the motor self-starting under load. First, the motor can be started as a repulsion motor (§ 103), and when normal speed is attained, a centrifugal device automatically short-circuits the commutator and simultaneously lifts off the brushes, thus changing the machine to a single-phase induction motor. Second, an auxiliary stator winding may be connected to the line through a phase-splitting device, as in § 87. Either a squirrel-cage or a coil-wound rotor may be used. Third, an auxiliary winding on the stator is connected through a non-inductive resistance and switching device to the line. An automatic clutch is employed, thus permitting the motor to approach normal speed before taking up its load.

89. **The Monocyclic System.** — This is a system advocated by the General Electric Company for the use of plants whose load is chiefly lights, but which contains some motors. The monocyclic generator is a modified single-phase alternator. In addition to its regular winding, it has a so-called teaser winding, made of wire of suitable cross-section to carry the motor load, and with enough turns to produce a voltage one-fourth that of the regular winding, and lagging 90° in phase behind it. One end of the teaser winding is connected to the middle of the regular winding, and the other end is connected through a slip-ring to a third line wire.

A three-terminal induction motor is used, the terminals being connected to the line wires either directly or through transformers.

90. **Frequency Changers.** — These are machines which are used to transform alternating currents of one frequency

into those of another frequency. They are commonly used to transform from a low frequency (say from 25 or 40) to a higher one. They depend for their operation upon the variation with slip of the frequency of the rotor *E.M.F.*'s of an induction motor. The common practice for raising the frequency is to have a synchronous motor turn the rotor of an induction motor in a direction opposite to the direction of rotation of the latter's field. The synchronous motor and the stator windings of the induction motor are connected to the low-frequency supply mains. Slip-rings connected to the rotor windings of the induction motor supply current at the higher frequency. The size of the synchronous motor necessary to drive the frequency changer is the same percentage of the total output as the rise of frequency is to the higher frequency.

91. **Speed Regulation of Induction Motors.** — The speed of an induction motor can be varied by altering the voltage impressed upon the stator, by altering the resistance of the rotor circuit, or by commutating the stator windings so as to alter the multipolarity. The first two methods depend for their operation upon the fact that, inasmuch as the motor torque is proportional to the product of the stator flux and the rotor current, for a given torque the product must be constant. Lessening the voltage impressed upon the stator lessens the flux, and also the rotor current, if the same speed be maintained. The speed, therefore, drops until enough *E.M.F.* is developed to send sufficient current to produce, in combination with the reduced flux, the equivalent torque. Increasing the resistance of the rotor circuit decreases the rotor current, and requires a drop in speed to restore its value. Both of these methods result

in inefficient operation. If the impressed voltage be reduced, the capacity of the motor is reduced. In fact, the capacity varies as the square of the impressed voltage. Changes in the multipolarity of the stator require complicated commutating devices.

92. The Induction Wattmeter. — The operation of the induction wattmeter, like that of the induction motor, is based

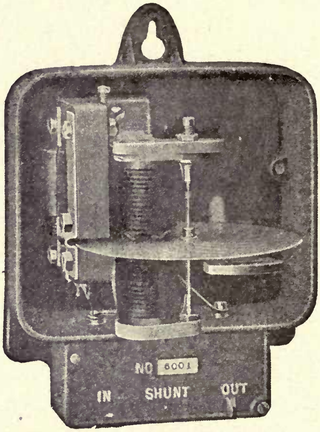


Fig. 181.

upon the action of a revolving or shifting magnetic field upon a metallic body capable of rotation. The rotating field is developed because of the difference in phase of the magnetic fields produced by the currents in the series and shunt coils of the wattmeter. The coils and the rotating member of an induction wattmeter are shown assembled in Fig. 181. The disk or armature is carried on a short shaft which is mounted in the usual way and is provided with a worm gear at its upper end for actuating the dial train.

The series coil has no iron core and consists of a few turns of heavy wire, thus it possesses very little self-induction. If the power factor of the load circuit be unity, then the current flowing through this winding will be practically in phase with the impressed *E.M.F.*, and, as the flux is in phase with the current producing it, this also will be approximately in phase with the impressed electromotive force. The shunt coil consists of a large number of turns of small wire wound on a laminated iron core. This winding has considerable self induction, hence the current flowing through it is almost at right angles to the impressed *E.M.F.* This angle of lag is slightly less than 90° owing to the iron and copper losses of the shunt circuit. The vector diagram corresponding to these conditions is shown in Fig. 182.

The alternating magnetic fluxes due to the series and shunt coils pass through the disk and develop eddy currents therein, which react on the fluxes and produce torque. As the torque is dependent upon both the flux of the series coil and that of the shunt coil, it is proportional to the energy which is to be measured. To render the angular velocity of the disk proportional to the torque, a permanent brake magnet is employed, and it is so mounted as to allow the disk to revolve between its poles. The permanent magnet may be moved radially with respect to the disk, and its position is adjusted to obtain the proper retarding force.

It is necessary to have the series and shunt fluxes in time quadrature on non-inductive load in order that the wattmeter may indicate correctly on inductive load. To accomplish this, a copper band with a small gap in it, called a *shading coil,* is placed around the limb of the laminated iron core. This gap is closed by means of a resistance wire of such length and size that the *E.M.F.*

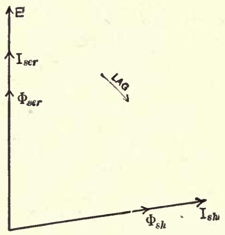


Fig. 182.

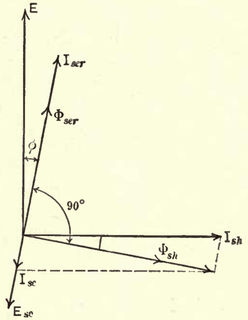


Fig. 183.

induced in this band by the alternating shunt flux will send a current through it of such value that, when combined vectorially with the current in the shunt coil, a flux at right angles to the flux in the series coil will result. This is shown in Fig. 183, where ϕ represents the angle by which the current in the series coil lags behind the impressed *E.M.F.* The vectors E_{sc} and I_{sc} represent respectively the electromotive force and the current in the shading coil. The resistance of the shading coil must be decreased when using the meter on circuits of lower frequency.

Series transformers are used with induction wattmeters of more than 50 amperes capacity, and potential transformers are employed where the pressure exceeds 300 volts. Polyphase induction wattmeters consist of separate single-phase elements assembled in the same case. The disks are mounted on a common shaft, and each revolves in its own field.

SYNCHRONOUS MOTORS.

93. Synchronous Motors. — Any excited single-phase or polyphase alternator, if brought up to speed, and if connected with a source of alternating *E.M.F.* of the same frequency and approximately the same pressure, will operate as a motor. The speed of the rotor in revolutions per second will be the quotient of the frequency by the number of pairs of poles. This is called the synchronous speed; and the rotor, when it has this speed, is said to be running in synchronism. This exact speed will be maintained throughout wide ranges of load upon the motor up to several times full-load capacity.

To understand the action of the synchronous motor, suppose it to be supplied with current from a single generator. The following discussion refers to a single-phase motor, but may equally well be applied to the polyphase synchronous motor. In the latter case each phase is to be considered as a single-phase circuit.

Let E_1 = *E.M.F.* of the generator,

E_2 = *E.M.F.* of the motor at the time of connection with the generator,

θ = Phase angle between E_1 and E_2,

R = Resistance of generator armature, plus that of the connecting wires and of the motor armature, and

ωL = Reactance of the above.

The resultant *E.M.F.*, E, which is operative in sending current through the complete circuit, is found by combining E_1 and E_2 with each other at a phase difference θ, as in Fig. 184.

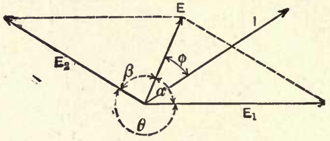


Fig. 184.

Representing the angle between E_1 and E and E_2 and E by α and β respectively, it follows that

$$E = E_1 \cos \alpha + E_2 \cos \beta.$$

This resulting *E.M.F.* sends through the circuit a current whose value is

$$I = \frac{E}{\sqrt{R^2 + \omega^2 L^2}},$$

and it lags behind E by an angle ϕ, such that $\tan \phi = \frac{\omega L}{R}$.

The power P_1 which the generator gives to the circuit is

$$P_1 = E_1 I \cos (\alpha - \phi)$$

and the power P_2 which the motor gives to the circuit is

$$P_2 = E_2 I \cos (\beta + \phi).$$

Now, if in either of the above expressions for power, the cosine has any other value than unity, then the power will consist of energy pulsations, there being four pulsations per cycle. The energy is alternately given to and received from the circuit by the machine. If the cosine be positive, the amount of energy in one pulsation, which is given to the circuit, will exceed the amount in one of the received pulsations. The machine is then acting as a generator. If the cosine be negative the opposite takes place, and the machine operates as a motor. As α

and β are but functions of E_1, E_2, and θ, and as these latter are the quantities to be considered in operation, it is desirable to eliminate the former. From the foregoing

$$P_1 = \frac{E_1 E}{\sqrt{R^2 + \omega^2 L^2}} \cos (\alpha - \phi),$$

or

$$P_1 = \frac{E_1^2 \cos \alpha + E_1 E_2 \cos \beta}{Z} [\cos \alpha \cos \phi + \sin \alpha \sin \phi].$$

Expanding, this becomes

$$P_1 = \frac{E_1^2}{Z} (\cos^2 \alpha \cos \phi + \sin \alpha \cos \alpha \sin \phi)$$
$$+ \frac{E_1 E_2}{Z} (\cos \alpha \cos \beta \cos \phi + \sin \alpha \cos \beta \sin \phi).$$

But $$2\pi - \theta = \alpha + \beta;$$

hence $$\cos \alpha \cos \beta = \cos \theta + \sin \alpha \sin \beta,$$

and $$\sin \alpha \cos \beta = - \sin \theta - \cos \alpha \sin \beta;$$

also $$\cos^2 \alpha = 1 - \sin^2 \alpha.$$

Therefore

$$P_1 = \frac{E_1^2}{Z} (\cos \phi - \sin^2 \alpha \cos \phi + \sin \alpha \cos \alpha \sin \phi)$$
$$+ \frac{E_1 E_2}{Z} (\cos \theta \cos \phi + \sin \alpha \sin \beta \cos \phi$$
$$- \sin \theta \sin \phi - \cos \alpha \sin \beta \sin \phi),$$

or

$$P_1 = \left[\frac{E_1 E_2}{Z} \cos (\theta + \phi) + \frac{E_1^2}{Z} \cos \phi\right]$$
$$+ \frac{E_1}{Z} [E_2 (\sin \alpha \sin \beta \cos \phi - \cos \alpha \sin \beta \sin \phi)$$
$$- E_1 (\sin^2 \alpha \cos \phi - \sin \alpha \cos \alpha \sin \phi)].$$

But, since

$$\frac{E_1}{E_2} = \frac{\sin \beta}{\sin \alpha},$$

the second term reduces to zero, and therefore

$$P_1 = \frac{E_1E_2}{\sqrt{R^2 + \omega^2L^2}} \cos(\theta + \phi) + \frac{E_1^2}{\sqrt{R^2 + \omega^2L^2}} \cos \phi.$$

Similarly, the power supplied to the circuit by the motor is

$$P_2 = \frac{E_1E_2}{\sqrt{R^2 + \omega^2L^2}} \cos(\theta - \phi) + \frac{E_2^2}{\sqrt{R^2 + \omega^2L^2}} \cos \phi.$$

If there were no losses due to resistance, etc., P_1 would be numerically exactly equal to P_2. Neglecting any losses in the machines, except that due to resistance, the algebraic sum of P_1 and P_2 is equal to RI^2. In order to determine the behavior of a synchronous motor when on a given circuit, use is made of the above formula for power, and each case must be considered by itself. The method of procedure is shown in the next article.

94. **Special Case.** — Suppose a single-phase synchronous motor, excited so as to generate 2100 volts, to be connected to a generator giving 2200 volts, the total resistance of the circuit being 2 ohms and the reactance 1 ohm. Then the angle ϕ of current lag behind the resultant *E.M.F.* has a value $\tan \phi = \frac{\omega L}{R} = 0.5$, whence $\phi = 26° \, 34'$.

Calculations of P_1 and P_2 for values of θ between 0° and 360° have been made using the formulæ of the preceding article, the results being embodied in the form of curves in Fig. 185. Phase differences, θ, are represented as abscissæ and P_1 and P_2, in kilowatts, are represented as ordinates. An enlargement of the lower portion of Fig. 185 is shown in Fig. 186. The ratio of P_2 to P_1, when the former is negative and the latter positive, and when all losses excepting the copper losses are neglected, is the motor efficiency. From an inspection of these curves, and

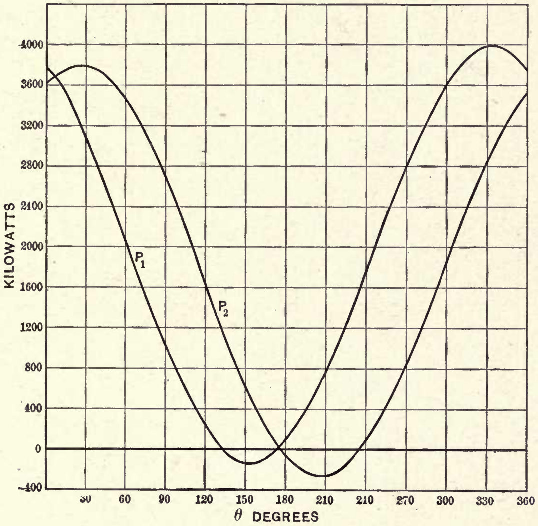


Fig. 185.

a consideration of the equations from which the curves are derived, the following conclusions may be drawn: —

(*a*) The motor will operate as such for values of θ between 175° and 238°. The difference between these angles may be termed the *operative range.*

(*b*) The generator would operate as a motor for values of θ between 133° and 174°, providing the motor were mechanically driven so as to supply the current and power; i.e., what was previously the motor must now operate as a generator.

(*c*) The motor, within its operative range, can absorb any amount of power between zero and a certain maximum. To vary the amount of received power, the motor

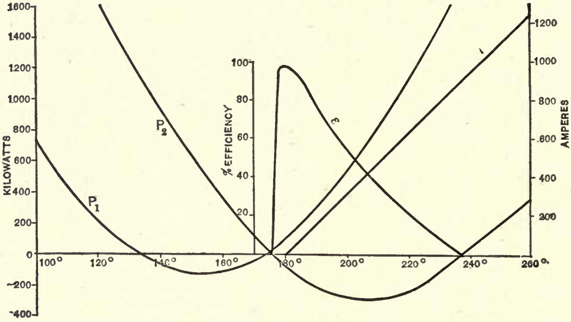


Fig. 186.

has but to slightly shift the phase of its *E.M.F.* in respect to the impressed *E.M.F.*, and then to resume running in synchronism. The sudden shift of phase under change of load is the fundamental means of power adjustment in the synchronous motor. It corresponds to change of slip in the induction motor, to change of speed in the shunt motor, and to change of magnetomotive force in the transformer.

(*d*) For all values of the received power, except the maximum, there are two values of phase difference θ. At one of these phase differences more current is required for the same power than at the other. The value of the current in either case can be calculated as follows: —

Since $$P_1 + P_2 = RI^2,$$

$$I = \sqrt{\frac{P_1 + P_2}{R}}.$$

The values of I are plotted in the diagram. The efficiency of transmission $\epsilon = \frac{P_2}{P_1}$ is also different for the two values of θ. It is also represented by a curve.

If the phase alteration, produced by an added mechanical load on the motor, results in an increase of power received by the motor, the running is said to be *stable*. If, on the other hand, the increase of load produces a decrease of absorbed power, the running is unstable.

(*e*) If for any reason the phase difference θ, between the *E.M.F.*'s of the motor and generator, be changed to a value without the operative range for the motor, the motor will cease to receive as much energy from the circuit as it gives back, and it will, therefore, fall out of step. Among the causes which may produce this result are sudden variations in the frequency of the generator, variations in the angular velocity of the generator, or excessive mechanical load applied to the motor. In slowing down, all possible values of θ will be successively assumed; and it may happen that the motor armature may receive sufficient energy at some value of θ to check its fall in speed, and restore it to synchronism, or it may come to a standstill.

(*f*) Under varying loads the inertia of the motor armature plays an important part. The shifting from one value of θ to another, which corresponds to a new mechanical load, does not take place instantly. The new value is overreached, and there is an oscillation on both sides of its mean value. This oscillation about the synchronous speed is termed *hunting*. If the armature required no energy to accelerate or retard it, this would not take place.

(g) The maximum negative value of P_2 — that is, the maximum load that the motor can carry — is evidently when $\cos(\theta - \phi) = -1$ or when $\theta - \phi = 180°$. The formula for the power absorbed by the motor then reduces to

$$P_{2m} = \frac{E_2^2 \cos\phi - E_1 E_2}{\sqrt{R^2 + \omega^2 L^2}} = 302 \text{ K.W.}$$

(h) The operative range of the motor can be determined by making P_2 equal to zero. By transformation the formula then becomes

$$\cos(\theta - \phi) = -\frac{E_2}{E_1}\cos\phi.$$

Two values of $(\theta - \phi)$ result, one on each side of 180°. In the case under consideration $\cos(\theta - \phi) = -.8537$, and $\theta - \phi = 211° 23'$ or $148° 37'$. Since $\phi = 26° 34'$, $\theta = 237° 57'$ or $175° 11'$.

95. The Motor E.M.F. — To determine what value of E_2 will give the maximum value of power to be absorbed by a motor, consider E_2 as a variable in the equation given in (g) above.

Differentiating

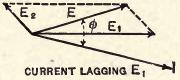
$$\frac{dP_{2m}}{dE_2} = \frac{2 E_2 \cos\phi - E_1}{\sqrt{R^2 + \omega^2 L^2}},$$

and setting this equal to zero and solving,

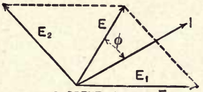
$$E_2 = \frac{E_1}{2\cos\phi} = 1230 \text{ volts.}$$

At this voltage the maximum possible intake of the motor is 605 K.W. If the voltage of the motor be above this or below it, its maximum intake will be smaller.

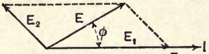
Remembering that the current lags behind the resultant pressure of the generator and motor pressures by an angle ϕ, which is solely dependent upon ω, L, and R, it will be easily seen, from an inspection of Figs. 187, 188, and 189, that the current may be made to lag behind, lead, or be in phase with E_1, by simply altering the value of E_2. This may be done by varying the motor's field excitation. A proper excitation can produce a unit power factor in the transmitting line. The over-excited synchronous motor, therefore, acts like a condenser in producing a leading current, and can be made to neutralize the effect of inductance. The current which is consumed by the motor for a given load accordingly varies with the excitation. The relations between motor voltage and absorbed current for various loads are shown in Fig. 190.



CURRENT LAGGING E_1
Fig. 187.



CURRENT LEADING E_1
Fig. 188.



CURRENT IN PHASE WITH E_1
Fig. 189.

Synchronous motors are sometimes used for the purpose of regulating the phase relations of transmission lines. The excitation is varied to suit the conditions, and the motor is run without load. Under such circumstances the machines are termed *synchronous compensators.*

The capacity of a synchronous motor is limited by its heating. If it is made to take a leading current in order to adjust the phase of a line current, it cannot carry its full motor load in addition without excessive heating.

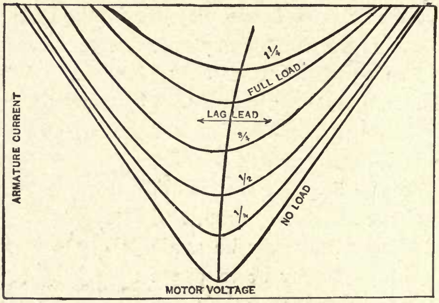


Fig. 190.

96. Starting Synchronous Motors. — Synchronous motors do not have sufficient torque at starting to satisfactorily come up to speed under load. They are, therefore, preferably brought up to synchronous speed by some auxiliary source of power. In the case of polyphase systems an induction motor is very satisfactory. Its capacity need be but $\frac{1}{10}$ that of the large motor. Fig. 191 shows a 750-K.W. quarter-phase General Electric motor with a small induction motor geared to the shaft for this purpose. This motor may be mechanically disconnected after synchronism is reached. Before connection of the synchronous motor to the mains it is necessary that the motor should not only be in synchronism, but should have its electromotive force at a difference of phase of about 180° with the impressed pressure. To determine both these points a simple device, known as a *synchronizer*, is employed. The simplest of these is the connection of incandescent lamps across a switch in the circuit of the generator and motor, as shown in Fig. 192. When the phase difference between the generator and motor *E.M.F.*'s is zero, the lamps will be brightest, and when the phase difference is 180°, the lamps will be dark. As the motor comes up to synchronous speed, the lamps become alternately bright and dark. As synchronism is approached, these alternations grow slower and finally become so slow as to permit closing of the main switch at an instant when the lamps are dark.

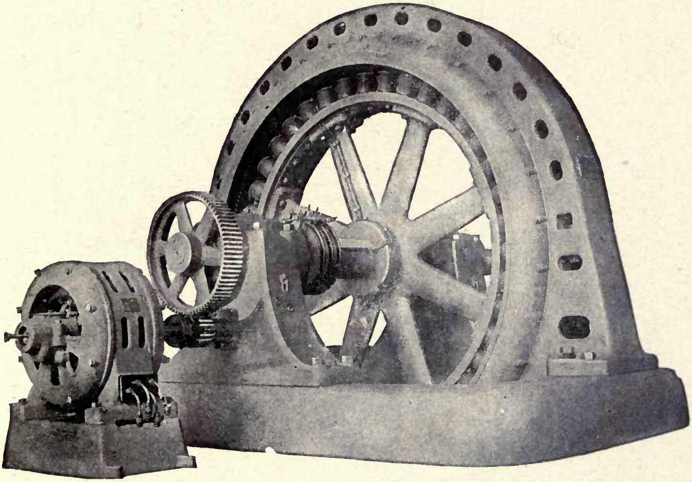


Fig. 191.

Instead of connecting the synchronizing device directly in the main circuit, it may be connected in series with the secondaries of two transformers, whose primaries are connected respectively across the generator and motor terminals, as shown in Fig. 193. With this arrangement, maximum brightness of the lamps may indicate that the generator and motor *E.M.F.*'s are either in phase or in opposition, according to the manner in which the transformer connections are made.

Another synchronizing device which is now extensively used is known as the synchroscope, and is shown in

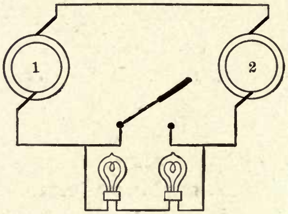


Fig. 192.

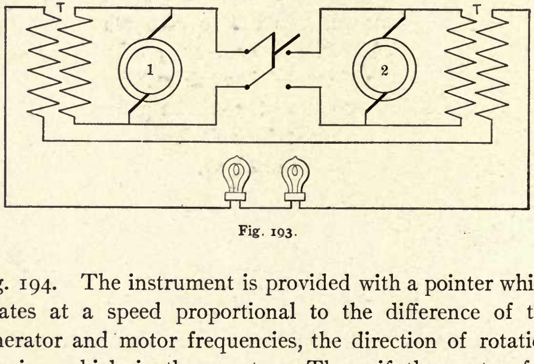


Fig. 193.

Fig. 194. The instrument is provided with a pointer which rotates at a speed proportional to the difference of the generator and motor frequencies, the direction of rotation showing which is the greater. Thus, if the motor frequency is too high, the pointer will rotate anti-clockwise. When the frequencies are identical, the pointer assumes some position on the scale, and when this position coincides with the index at the top of the scale, the main

switch may be closed, thus connecting the two machines together.

Synchronous motors may be brought up to speed without any auxiliary source of power. The field circuits are left open, and the armature is connected either to the full pressure of the supply, or to this pressure reduced by means of a starting compensator, such as was described in §76. The magnetizing effect of the armature ampere

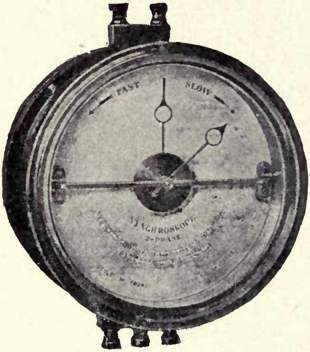


Fig. 194.

turns sets up a flux in the poles sufficient to supply a small starting torque. When synchronism is nearly attained, the fields may be excited and the motor will come into step. The load is afterwards applied to the motor through friction clutches or other devices. There is great danger of perforating the insulation of the field coils when starting in this manner. This is because of the high voltage produced in them by the varying flux. In such cases each field spool is customarily open-circuited on starting. Switches which are designed to accomplish this purpose are called break-up switches.

97. Parallel Running of Alternators. — Any two alternators adjusted to have the same *E.M.F.* and the same frequency may be synchronized and run in parallel. Machines of low armature reactance have large synchronizing power, but may give rise to heavy cross currents, if thrown out of step by accident. The contrary is true of machines having large armature reactance. Cross currents due to differences of wave-form are also reduced by large armature reactance. The electrical load is distributed between the two machines according to the power which is being furnished by the prime movers. This is accomplished, as in the case of the synchronous motor, by a slight shift of phase between the *E.M.F.*'s of the two machines. The difficulties which have been experienced in the parallel running of alternators have almost invariably been due to bad regulation of the speed of the prime mover. Trouble may arise from the electrical side if the alternators are designed with a large number of poles. Composite wound alternators should have their series compounding coils connected to equalizing bus bars, the same as compound wound direct-current generators.

SINGLE-PHASE COMMUTATOR MOTORS.

98. Single-Phase Commutator Motors. — If the current in both field winding and armature of any direct-current motor be periodically reversed, the direction of rotation of the armature will remain unchanged. Therefore direct-current motors might be operated on alternating-current circuits. Shunt motors cannot be operated satisfactorily when fed with alternating current, because the reversals of current do not take place simultaneously in armature and field windings owing to the high inductance of the latter winding. This would cause momentary currents in the armature in a reversed direction and would tend to produce a counter-torque, thus considerably decreasing the effective torque.

When direct-current series motors are supplied with alternating current, the instantaneous current value is

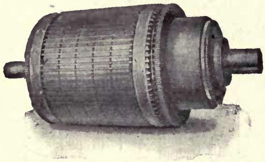


Fig. 195.

necessarily the same in both armature and field winding, and therefore no counter-torque is developed. The direct-current series motor with various modifications may be operated on alternating-current circuits, and when so used is termed the *single-phase series motor*, or the *single-phase commutator motor*. It is essential that the entire magnetic circuits of motors of this type be laminated in order to decrease the otherwise excessive hysteresis and eddy current losses. Series motors, when operated on alternating current, produce a pulsating torque varying from zero to a certain maximum value.

The armature of a single-phase series motor is similar to that of the direct-current motor. The armature of a 150 horse-power single-phase alternating-current railway motor is represented in Fig. 195.

99. Plain Series Motor. — Consider a direct-current armature mounted within a single-phase alternating magnetic field, as in Fig. 196. When the armature is stationary an electromotive force will be induced in the armature turns, due to the alternating flux which passes between the

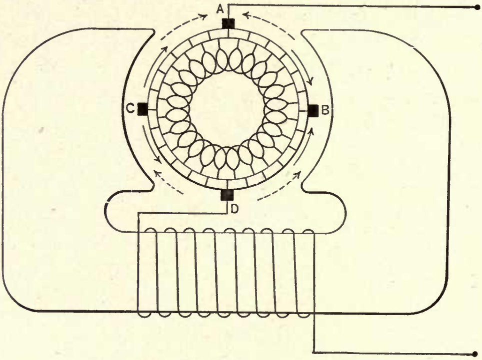


Fig. 196.

field poles. The greatest *E.M.F.*'s will be induced in the turns perpendicular to the field axis, since these turns link with the greatest number of lines of force; and no *E.M.F.*'s will be induced in the turns in line with the field axis. The directions of the *E.M.F.*'s induced in the armature turns by the change in field flux are indicated in the figure by the full arrows, and it is seen that the maximum value of this *E.M.F.* is across *BC*. As in transformers, the effective value of this electromotive force (§ 59) is

$$E_T = \frac{2\pi f \Phi_m N}{\sqrt{2}\ 10^8}, \qquad (1)$$

where Φ_m is the maximum value of the flux entering the armature and N is the equivalent number of armature turns.

The maximum number of lines of force linked with a single turn depends upon the position of this turn in the magnetic field, and is proportional to the greatest value of Φ_m times the cosine of the angle of displacement of the turn from the position *AD*. Assuming the turns to be evenly distributed over the periphery of the armature, the average value of the maximum flux linked with the armature turns will be $\frac{2}{\pi} \Phi_m$. If there are N_a conductors on the armature, the number of turns connected in continuous series will be $\frac{N_a}{2}$. The electromotive forces induced in the upper and lower groups of armature turns are added in parallel, consequently the effective number of turns in series is $\frac{1}{2} \cdot \frac{N_a}{2} = \frac{N_a}{4}$. Therefore the equivalent number of armature turns may be expressed as

$$N = \frac{2}{\pi} \cdot \frac{N_a}{4} = \frac{N_a}{2\pi}. \tag{2}$$

Substituting this value of N in equation (1), the *E.M.F.* induced in the armature winding by the change in value of the field flux is

$$E_T = \frac{f \Phi_m N_a}{\sqrt{2}\ 10^8}, \tag{3}$$

and it lags 90° behind field flux in time.

If the brushes of the motor, *A* and *D*, are placed at the points shown in Fig. 196, this electromotive force will not manifest itself externally, since it consists of two equal and opposite components directed toward these brushes. This *E.M.F.* appears, however, in the coils short-circuited by the brushes, as will be shown later. The current, which enters the armature by way of the brush and which traverses the two halves of its windings in parallel, produces an armature flux of maximum value Φ_{am}. This sets up a reactance *E.M.F.* in the armature which in the case of uniform gap reluctance can be similarly expressed as

$$E_a = \frac{f\Phi_{am}N_a}{\sqrt{2}\,10^8}. \qquad (4)$$

This lags 90° behind the current.

When the armature revolves, there are, in addition, electromotive forces induced in the armature conductors as a result of their cutting the field flux. The directions of these *E.M.F.*'s are indicated by the dotted arrows, and it is seen that these *E.M.F.*'s, generated by the rotation of the armature, add to each other and appear on the commutator as a maximum across *AD*.

The average value of the electromotive force due to the rotation of the armature is

$$E_{rot\ av} = \Phi_f N_a \frac{V}{60} 10^{-8}, \qquad \S\ 42$$

where V is the armature speed in rev. per min. in a bipolar field, and Φ_f is the field flux; and the effective value of this *E.M.F.* is

$$E_{rot} = \frac{\Phi_{fm}N_a}{\sqrt{2}\,10^8} \cdot \frac{V}{60}, \qquad (5)$$

and is in time phase with the field flux, but appears as a counter *E.M.F.* at the brushes *AD*.

When an alternating current is passed through the field coils, the alternating field flux is set up, and this flux produces a reactive *E.M.F.* in the field winding lagging 90° behind the flux in phase, exactly as in a choke coil. The magnitude of this *E.M.F.* is

$$E_f = \frac{2\pi f \Phi_{fm} N_f}{\sqrt{2}\ 10^8}, \qquad (6)$$

where Φ_{fm} is the maximum value of the field flux, and N_f is the number of field turns.

The electromotive force, *E*, which is impressed upon the motor terminals, is equal and opposite to the vectorial

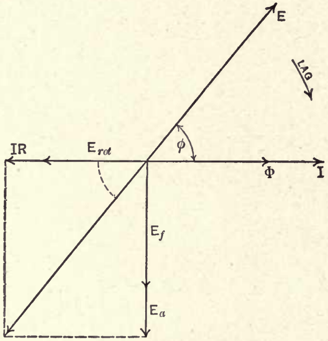


Fig. 197.

sum of E_a, E_{rot}, E_f, and the *IR* drop of the armature and field windings, as shown in Fig. 197, where *I* is the current flowing through the field and armature, and Φ represents the phase of the flux. In this diagram, eddy current and hysteresis losses are ignored. The impressed electromotive force is therefore

$$E = \sqrt{(E_{rot} + IR)^2 + (E_a + E_f)^2}. \qquad (7)$$

100. Characteristics of the Plain Series Motor. — In the series motor, the same current passes through field and armature windings, and, if uniform reluctance around the air gap be assumed, then the armature and field fluxes will be proportional to the equivalent armature turns and field turns respectively. Therefore

$$\Phi_{am} : \Phi_{fm} = N : N_f = \frac{N_a}{2\pi} : N_f. \qquad (1)$$

Representing the ratio of the field turns to the effective armature turns by τ, then $\Phi_{fm} = \tau\Phi_{am}$, and $N_f = \tau \frac{N_a}{2\pi}$. Therefore expressions (4) and (6) of § 99 become respectively

$$E_a = \frac{\Phi_{fm} N_a}{\sqrt{2}\, 10^8} \cdot \frac{f}{\tau},$$

and
$$E_f = \frac{\Phi_{fm} N_a}{\sqrt{2}\, 10^8} \cdot f\tau.$$

Equation (5) of § 99 is

$$E_{rot} = \frac{\Phi_{fm} N_a}{\sqrt{2}\, 10^8} \cdot \frac{V}{60},$$

which then reduces to

$$E_{rot} = \frac{\tau}{f} E_a \frac{V}{60}, \text{ and } E_{rot} = \frac{1}{f\tau} E_f \frac{V}{60}.$$

Therefore
$$E_f = \tau^2 E_a.$$

Neglecting the armature and field resistance drop, the impressed *E.M.F.* becomes

$$E = E_a \sqrt{\left(\frac{\tau V}{60 f}\right)^2 + (\tau^2 + 1)^2}, \qquad (2)$$

which is the fundamental $E.M.F.$ equation of the plain series motor.

The power factor of the motor is

$$\cos\phi = \frac{E_{rot}}{E} = \frac{\tau V}{60f\sqrt{\left(\frac{\tau V}{60f}\right)^2 + (\tau^2 + 1)^2}}, \qquad (3)$$

and the current supplied to the motor is

$$I = \frac{E}{Z} = \frac{E}{X_a\sqrt{\left(\frac{\tau V}{60f}\right)^2 + (\tau^2 + 1)^2}}, \qquad (4)$$

still neglecting the motor resistance.

When $V = 60f$, the motor is said to run at synchronous speed (bipolar field). The power factor of a plain series motor, having $\tau = 1$, when running at this speed, is $\frac{1}{\sqrt{5}}$, or 0.446, and for values of τ other than unity the power factor is less than 0.446. It is true that if the resistance of the motor be considered, the power factor will exceed this value, but nevertheless it remains extremely low.

The current intake under these same conditions is $\frac{E}{\sqrt{5}X_a}$. When the motor is at standstill, $V = 0$, and the power factor is zero. The current intake at standstill is $\frac{E}{2X_a}$. Hence the ratio of the current at synchronism to the current at standstill is $\frac{1}{\sqrt{5}} \div \frac{1}{2} = 0.894$. The ratio of the torque at synchronous speed to the torque at standstill, since it varies as the square of the current, is $\left(\frac{1}{\sqrt{5}}\right)^2 \div$

$\left(\frac{1}{2}\right)^2 = 0.80$, which shows that the starting torque is but little greater than the torque at synchronous speed. For railway service motors are required having large starting torque and whose torque rapidly decreases as the speed of the motor increases. It is seen, therefore, that independent of its low power factor, the plain series motor, having uniform magnetic reluctance around the air gap, is unsuitable for traction and for similar purposes.

If, however, the reluctance of the air gap in the direction *AD*, Fig. 196, is increased, the power factor and speed-torque characteristics will be improved, and these will depend largely upon the ratio of field turns to effective armature turns, as will be seen by considering the construction of the motor to be such that the proportion, equation (1), must be modified by the introduction of a constant considerably greater than unity, into its antecedents. A motor of this kind, with few field turns compared to armature turns, might be suitable for traction, but more important improvements have been made, which will now be discussed.

101. Compensated Series Motors.—From an inspection of Fig. 197 it is seen that the power factor of series motors may be increased by increasing IR and E_{rot}, or by decreasing E_f and E_a. It is obvious that increasing IR signifies an increase in losses, thus resulting in a lower efficiency. E_{rot} can be increased by increasing the number of armature turns. Both E_f and E_a can be decreased by lowering the frequency without affecting E_{rot}, hence low frequencies are desirable. To decrease the reactive electromotive force of the field, it is necessary that the reluctance of the magnetic circuit be low, i.e., small air gap and low flux densities in the iron, in order that the required flux can be produced by a minimum number of ampere turns. The armature reactive *E.M.F.*, E_a, is not essential to the operation of the motor, and can be neutralized by the use of compensating windings, and this feature of alternating-current series motors is a very important one.

The compensating winding is embedded in slots in the pole faces, as shown in Fig. 198, which represents a West-

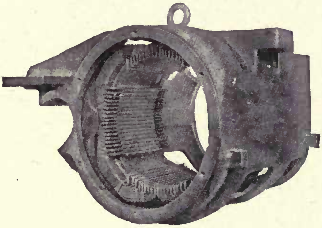


Fig. 198.

inghouse four-pole compensated single-phase railway motor with its armature and field windings removed. The number of turns of the compensating winding is adjusted so as to set up a magnetomotive force equal and opposite to that due to the current in the armature coils. The compensating winding may be energized either by the main current, by placing this winding in series with field and armature, or by an induced current, which is obtained by short-circuiting the compensating winding upon itself, thus utilizing the principle of the transformer in that the main and induced currents are opposite in phase. The former method of neutralizing E_a is known as *conductive* or *forced compensation*, and may be used with both alternating and direct currents, and the latter method is known as *inductive compensation*, and may be used only with alternating current.

Figs. 199 and 200 show schematically the connections of

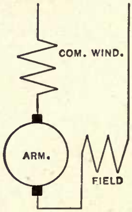


Fig. 199.

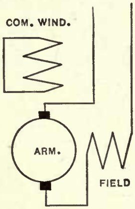


Fig. 200.

the conductively and inductively compensated alternating-current series motors respectively. The compensating

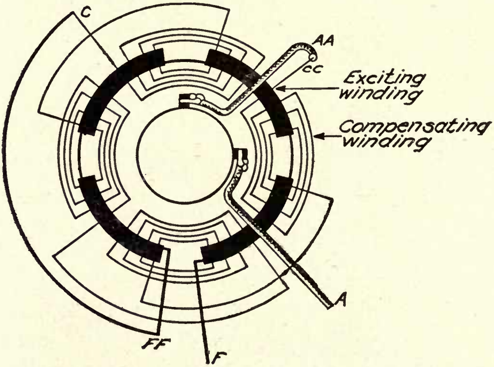


Fig. 201.

winding is preferably distributed so that the armature reactance is neutralized as completely as possible. The current flows in the same direction in all of the conductors of the compensating winding embedded in one field pole, and flows in the opposite direction in the conductors embedded in the adjacent poles. Fig. 201 illustrates a distributed conductive-compensating winding of a four-pole machine.

When the compensating winding completely neutralizes the armature reactance, the impressed electromotive force (Eq. 7, §99) is

$$E = \sqrt{(E_{rot} + IR)^2 + E_f^2},$$

where R is the resistance of the motor including that of the compensating winding. If the resistance, R, be neglected, then, since

$$E_{rot} = \frac{V}{60 f \tau} E_f, \qquad \text{§ 100}$$

the impressed electromotive force becomes

$$E = E_f \sqrt{\left(\frac{V}{60 f \tau}\right)^2 + 1},$$

and therefore the power factor is

$$\cos \phi = \frac{E_{rot}}{E} = \frac{V}{\sqrt{V^2 + (60 f \tau)^2}}.$$

The motor current is

$$I = \frac{E}{X_f \sqrt{\left(\frac{V}{60 f \tau}\right)^2 + 1}}.$$

At synchronous speed $V = 60 f$, and therefore the power factor at this speed becomes $\frac{1}{\sqrt{1 + \tau^2}}$.

Still neglecting the motor resistance, the current intake at synchronous speed is $\frac{E\tau}{X_f\sqrt{1+\tau^2}}$, and at standstill it is $\frac{E}{X_f}$, consequently the ratio of the current at synchronous speed to the current at standstill is $\frac{\tau}{\sqrt{1+\tau^2}}\cdot$ Since torque varies as the square of the current, the ratio of the torque at synchronous speed to the starting torque is $\frac{\tau^2}{1+\tau^2}\cdot$ Hence it follows that the speed-torque characteristics of a compensated series motor may be adjusted to the required conditions by properly proportioning the number of armature and field turns.

Compensated series motors are well suited for traction. The performance curves of the 250-horse-power 25-cycle Westinghouse conductively compensated single-phase series motor used on the New York, New Haven and Hartford Railroad are shown in Fig. 202. Each locomotive is equipped with four of these motors operating at 225 volts, which is procured by step-down transformation from overhead 11,000-volt trolley. On parts of the road the motors operate on direct current, the current being supplied directly to the motors.

102. Sparking in Series Motors. — The principal difficulty encountered in the operation of single-phase series motors is the sparking at the brushes. This is caused by the local currents produced by the *E.M.F.* generated in the armature turns short-circuited by the brushes, due to the periodic reversals of the field flux. With the brushes located in the neutral position with respect to the *E.M.F.*

of rotation, the short-circuited turns are perpendicular to the axis of the field flux, and therefore the flux linked with these turns is a maximum. The electromotive force generated in a short-circuited armature section is

$$E_s = \frac{2\pi f \Phi_m N_s}{\sqrt{2}\ 10^8},$$

where N_s is the equivalent number of armature turns per section which is short-circuited by a brush. If the resist-

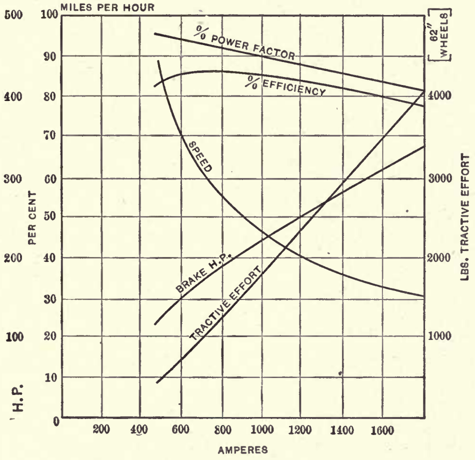


Fig. 202.

ance of the section be small, an enormous current will flow, and will cause excessive heating of the brush, commutator segments, and armature conductors.

In order to decrease this local current, the *E.M.F.*

induced in each section may be decreased and the resistance thereof may be increased. From an inspection of the preceding formula it is seen that E_s may be decreased by reducing the number of armature turns per section, by lowering the maximum value of the flux, and by lowering the frequency. Thus single-phase series motors are usually provided with more than two poles and with many commutator segments, and are designed to operate on low frequency circuits. A simple way to increase the

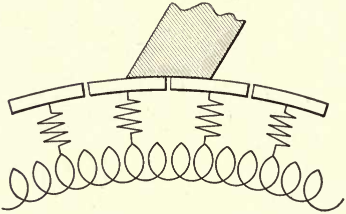


Fig. 203.

resistance of the armature sections involves the use of *preventive* or *resistance leads*, which are connected between armature conductors and commutator segments, as illustrated in Fig. 203. It has been shown by experiment that the losses are a minimum when the resistance of the preventive leads is so proportioned that the short-circuit currents and normal currents are equal. The resistance leads are usually of German silver and have a large current-carrying capacity. They are placed in the same slots as the armature conductors, and usually at the bottom thereof. Only a few of these leads are in circuit at any instant, and, when the armature rotates, all of the preventive leads carry current in turn, hence the average loss of power per lead is small. As the heating effect of the short-circuit current depends upon the duration of the short circuit, it is essential that the brushes be made quite narrow.

103. Repulsion Motors. — The repulsion motor consists of a field resembling the stator of the single-phase induction motor, and an armature which is similar to the armatures of direct-current and alternating-current series motors. The armature winding always remains short-

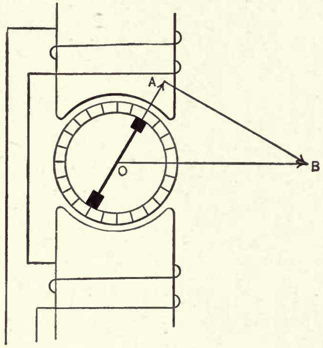


Fig. 204.

circuited in a line inclined at a definite angle with the field axis, this being accomplished by means of brushes, bearing on the commutator, which are joined together by a conductor of low resistance. The field winding is supplied with single-phase alternating current. The fact that the armature and field windings are electrically distinct makes it possible to operate the motor on high voltage systems, the armature winding being so adjusted that the currents therein can be commutated satisfactorily.

The pulsating flux through the armature, produced by the alternating current in the field winding, may be considered as the resultant of two components, one in the direction of the brush axis, and the other perpendicular thereto; these being represented in Fig. 204 respectively by *OA* and *AB*. The component *OA* produces an *E.M.F.* in the armature conductors and causes a current to flow

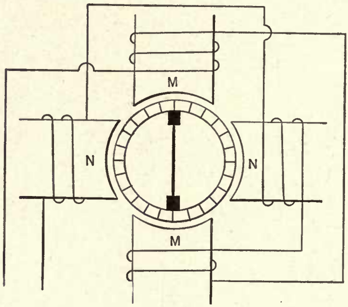


Fig. 205.

through them. The other component, *AB*, reacts upon this armature current, thereby developing torque.

To represent the action of a repulsion motor more clearly, the field winding may be considered as composed of two parts placed at right angles to each other, as at *M* and *N*, Fig. 205, and the brushes may be located in line with one of them. When the rotor is stationary, the pulsating flux from poles *MM* causes the flow of current in the short-circuited armature, the effect of which is the production of a flux opposite and nearly equal to that which caused the current flow. The flux from poles *MM* is thereby reduced, thus resulting in increased stator current and flux from poles NN. Neglecting iron losses, this flux will be in phase with the line current, whereas the phase of the armature current is opposite to that of the current in coils MM. Hence the flux from poles NN is in phase with the armature current, and their product, torque,

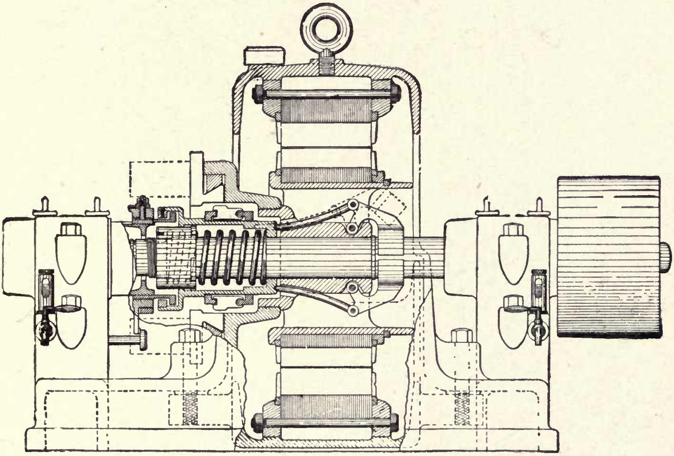


Fig. 206.

retains its sign as both reverse in direction together. The repulsion motor exerts its maximum torque at starting, and this torque decreases with decreasing current and with increased speed, and consequently this type of commutator motor is well adapted for single-phase traction. The power factor of repulsion motors is low at starting and rises rapidly as the speed increases. Repulsion motors may be operated on 25 ~ or even on 60 ~ supply circuits.

The fact that the repulsion motor may be converted readily into a single-phase induction motor, by simply short-circuiting the entire commutator, thus changing the armature to a squirrel-cage rotor, has led to the design of single-phase induction motors which start and come up to speed as repulsion motors. A motor of this type, manu-

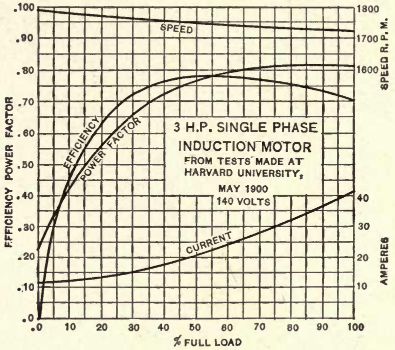


Fig. 207.

factured by the Wagner Electric Company, is shown in Fig. 206. Upon reaching normal speed, a centrifugal device, shown in the figure, causes the commutator bars to be short-circuited, and the brushes are simultaneously lifted from the commutator. The results of tests made upon this type of motor are represented in the curves of Fig. 207.

104. Series-Repulsion Motor. — A single-phase railway motor which embodies many of the best features of the repulsion motor and of the compensated series motor, and therefore called the series-repulsion motor, has recently been developed by the General Electric Company.

The windings resemble those of a series motor, and the armature has a fractional-pitch winding such as is used on direct-current motors. The connections of the motor circuits for the starting and running positions are shown in Fig. 208.

The motor starts as a repulsion motor and possesses all the characteristics thereof when in this position. The

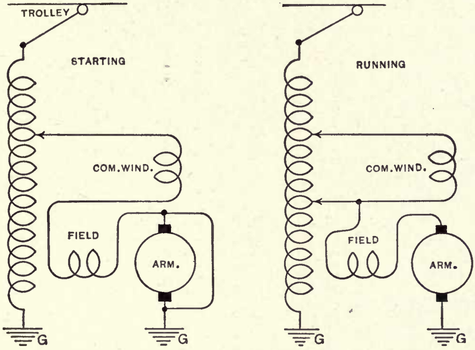


Fig. 208.

compensating coils differ from those on series motors in that they have twice as many turns as there are on the armature, and therefore the armature current at starting is twice as large as the current flowing through the field and compensating windings. Thus the starting torque is twice as great as would be obtained for the same current with a compensated series motor. When in the running position, the motor characteristics are similar to those of the compensated series motor, but it possesses an advantage over the latter in regard to sparking.

At starting there is very little sparking at the brushes, but this increases up to a certain value of voltage induced in the short-circuited armature sections. This value practically corresponds to that which gives good commutation in the running position. Better inherent commutation is the chief advantage of the series-repulsion motor.

PROBLEMS.

1. Determine approximately the full-load efficiency of a certain 15-horse-power three-phase six-pole 50-cycle induction motor which makes 950 revolutions per minute when carrying full load.

2. What torque does the motor of the preceding problem exert when operating under full load?

3. Calculate the leakage reactance per phase of a 5000-volt three-phase 25-cycle 30-pole induction motor having a three-phase wound

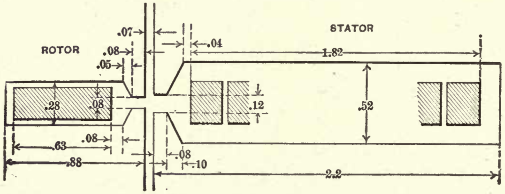


Fig. 209.

rotor (both stator and rotor have Y-connected full-pitch windings). The dimensions of the slots in inches are indicated in Fig. 209, and the constants of the motor are:

Rotor diameter	118	in.
Total number of primary slots	450	
Total number of secondary slots	720	
Series conductors per primary slot	8	
Series conductors per secondary slot	$\frac{1}{2}$	
Length of slots	29.5	in.
Length of end connection per primary turn	48.5	in.
Depth of laminations back of slots	2.36	in.

4. Calculate the exciting current per phase of the induction motor of the preceding problem.

5. The stator resistance per phase of the induction motor of problem 3 is 1.7 ohms, and the rotor resistance per phase is .02 ohm, which, when reduced to the stator circuit, is 2 ohms. Determine the motor performance curves from its circle diagram, this being based upon the results of the two preceding problems.

6. Plot curves of P_1 and P_2 for a single-phase synchronous motor, excited so as to generate 1000 volts, connected to a single-phase alternator having an *E.M.F.* of 1200 volts, the total resistance of the circuit being 1.75 ohms and the total impedance 2 ohms.

CHAPTER VIII.

CONVERTERS.

105. The Converter. — The converter is a machine having one field, and one armature, the latter being supplied with both a direct-current commutator and alternating-current slip-rings. When brushes, which rub upon the slip-rings, are connected with a source of alternating current of proper voltage, the armature will rotate synchronously, acting the same as the armature of a synchronous motor. While so revolving, direct current can be taken from brushes rubbing upon the commutator. The intake of current from the alternating-current mains is sufficient to supply the direct-current circuit, and to overcome the losses due to resistance, friction, windage, hysteresis, and eddy currents. The windings of a converter armature are closed, and simply those of a direct-current dynamo armature with properly located taps leading to the slip-rings. Each ring must be connected to the armature winding by as many taps as there are pairs of poles in the field. These taps are equidistant from each other.

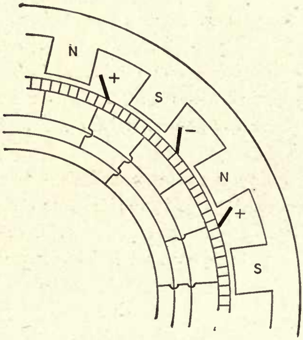


Fig. 210.

There may be any number of rings greater than one. A converter having n rings is called an n-ring converter. The taps to successive rings are $\frac{1}{n}$th of the distance between the centers of two successive north poles from each other. Fig. 210 shows the points of tapping for a 3-ring multipolar converter.

A converter may also be supplied with direct current

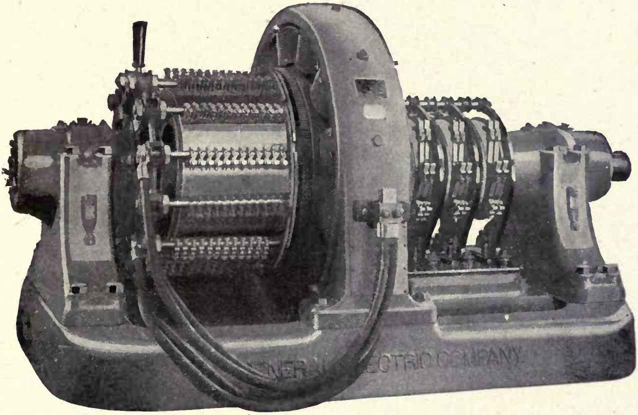


Fig. 211.

through its commutator, while alternating current is taken from the slip-rings. Under these circumstances the machine is termed an *inverted converter.* Converters are much used in lighting and in power plants, sometimes receiving alternating current, and at other times direct current. In large city distributing systems they are often used in connection with storage batteries to charge them from alternating-current mains during periods of light load, and to give back the energy during the heavy load. They are also used in transforming alternating into direct currents for electrolytic purposes. A three-phase machine for this purpose is shown in Fig. 211.

A converter is sometimes called a rotary converter or simply a rotary.

106. E.M.F. Relations. — In order to determine the relations which exist between the pressures available at the various brushes of a converter,

Let E_d = the voltage between successive direct-current brushes.

E_n = the effective voltage between successive rings of an n-ring converter.

a = the maximum *E.M.F.* in volts generated in a single armature inductor. This will exist when the conductor is under the center of a pole.

b = the number of armature inductors in a unit electrical angle of the periphery. The electrical angle subtended by the centers of two successive poles of the same polarity is considered as 2π

The *E.M.F.* generated in a conductor may be considered as varying as the cosine of the angle of its position relative to a point directly under the center of any north pole, the angles being measured in electrical degrees. At an angle β, Fig. 212, the *E.M.F.* generated in a single inductor G is $a \cos \beta$ volts. In an element $d\beta$ of the periphery of the armature there are $bd\beta$ inductors, each with this *E.M.F.* If connected in series they will yield an *E.M.F.*

of $ab \cos \beta \, d\beta$ volts. The value of ab can be determined if an expression for the *E.M.F.* between two successive direct-current brushes be determined by integration, and be set equal to this value E_d as follows:

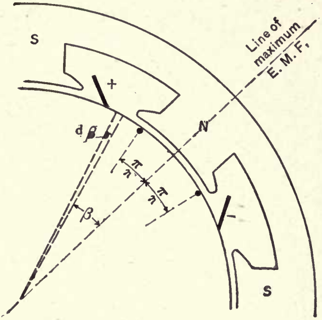


Fig. 212.

$$E_d = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} ab \cos \beta d\beta = 2\, ab.$$

$$\therefore \quad ab = \frac{E_d}{2}.$$

In an n-ring converter, the electrical angular distance between the taps for two successive rings is $\frac{2\pi}{n}$. The maximum *E.M.F.* will be generated in the coils between the two taps for the successive rings, when the taps are at an equal angular distance from the center of a pole, one on each side of it, as shown in the figure. This maximum *E.M.F.* is

$$\sqrt{2}\, E_n = \int_{-\frac{\pi}{n}}^{+\frac{\pi}{n}} ab \cos \beta d\beta = 2\, ab \sin \frac{\pi}{n}$$

$$= E_d \sin \frac{\pi}{n}.$$

The effective voltage between the successive rings is therefore

$$E_n = \frac{E_d}{\sqrt{2}} \sin \frac{\pi}{n}.$$

By substituting numerical values in this formula, it is found that the coefficient by which the voltage between the direct-current brushes must be multiplied in order to get the effective voltage between successive rings is for

2 rings	0.707
3 rings	0.612
4 rings	0.500
6 rings	0.354

In practice there is a slight variation from these co-efficients due to the fact that the air-gap flux is not sinusoidally distributed.

107. Current Relations.— In the following discussion it is assumed that a converter has its field excited so as to cause the alternating currents in the armature inductors to lag 180° behind the alternating *E.M.F.* generated in them.

The armature coils carry currents which vary cyclically with the same frequency as that of the alternating-current supply. They differ widely in wave-form from sine curves. This is because they consist of two currents superposed upon each other. Consider a coil *B*, Fig. 213. It carries a direct current whose value $\frac{I_d}{2}$ is half that carried by one direct-current brush, and it reverses its direction every time that the coil passes under a brush. The coil, as well as all others between two taps for successive slip-rings, also carries an alternating current. This current has its zero

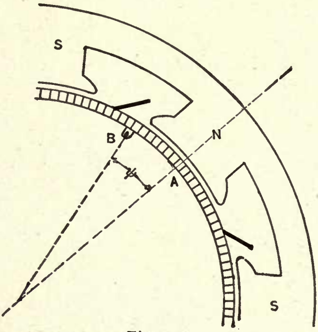


Fig. 213.

value when the point A, which is midway between the successive taps, passes under the brush. The coil being ψ electrical degrees ahead of the point A, the alternating current will pass through zero $\frac{\psi}{2\pi}$ of a cycle later than the direct current. The time relations of the two currents are shown in Fig. 214.

To determine the maximum value of the alternating current consider that, after subtracting the machine losses,

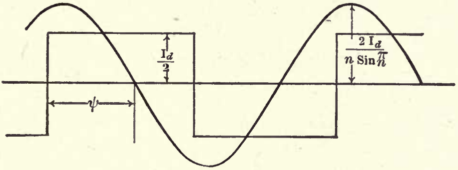


Fig. 214.

the alternating-current power intake is equal to the direct-current power output. Neglecting these losses for the present, if E_n represents the pressure and I_n the effective alternating current in the armature coils between the successive slip-rings, then for the parts of the armature windings covered by each pair of poles

$$E_d I_d = n E_n I_n$$

$$= n \frac{E_d}{\sqrt{2}} \sin \frac{\pi}{n} I_n.$$

Therefore, the maximum value of the alternating current is

$$\sqrt{2}\, I_n = \frac{2 I_d}{n \sin \frac{\pi}{n}}.$$

The time variation of current in the particular coil B is obtained by taking the algebraic sum of the ordinates of the two curves. This yields the curve shown in Fig. 215. Each inductor has its own wave-shape of current, depending upon its angular distance ψ from the point A. Converter coils, therefore, alternately functionate as motor and as generator coils.

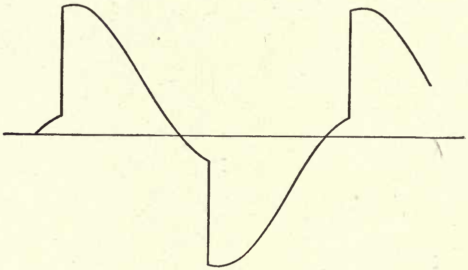


Fig. 215.

108. Heating of the Armature Coils. — The heating effect in an armature coil due to a current of such peculiar wave-shape as that shown in Fig. 215 can be determined either graphically or analytically. The graphic determination requires that a new curve be plotted, whose ordinates shall be equal to the squares of the corresponding current values. The area contained between this new curve and the time axis is then determined by means of a planimeter. The area of one lobe is proportional to the heating value of the current. This value may be determined for each of the coils between two successive taps. An average of these values will give the average heating effect of the currents in all the armature coils. The heating is different in the different coils. It is a maximum for coils at the points of tap to the slip-rings and is a minimum for coils midway between the taps.

109. Capacity of a Converter. — As the result of a rather involved analysis it is found that a machine has different capacities, based upon the same temperature rise, according to the number of slip-rings, as shown in the following table. The armature is supposed to have a closed-coil winding.

CONVERTER CAPACITIES.

Used as a	Kilowatt Capacity
Direct-current generator	100
Single-phase converter	85
Three-phase converter	134
Four-phase converter	164
Six-phase converter	196
Twelve-phase converter	227

The overload capacity of a converter is limited by commutator performance and not by heating. As there is but small armature reaction, the limit is much higher than is the case with a direct-current generator.

110. Starting a Converter. — Converters may be started and be brought up to synchronism by the same methods which are employed in the case of synchronous motors. It is preferable, however, that they be started from the direct-current side by the use of storage batteries or other sources of direct current. They may be brought to a little above synchronous speed by means of a starting resistance as in the case of a direct-current shunt motor, and then, after disconnecting and after opening the field circuit, the connections with the alternating-current mains may be made. This will bring it into step.

111. Armature Reaction. — The converter armature currents give rise to reactions which consist of direct-current generator armature reactions superposed upon synchronous motor armature reactions. It proves best in practice to set the direct-current brushes so as to commutate the current in coils when they are midway between two succes-

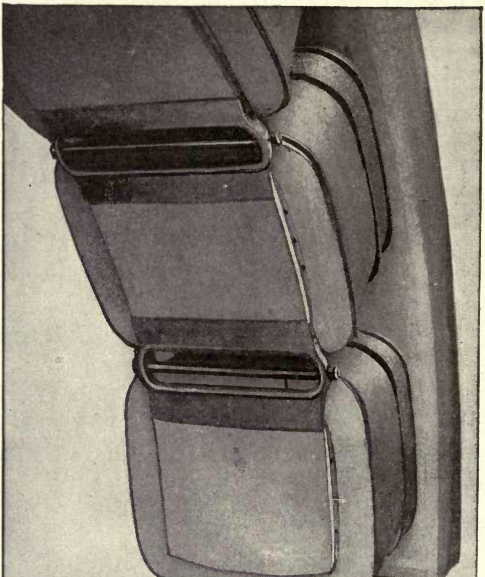


Fig. 216.

sive poles. The direct-current armature reaction, then, consists in a cross-magnetization which tends to twist the field flux in the direction of rotation. When the alternating currents are in phase with the impressed *E.M.F.* they also exert a cross-magnetizing effect which tends to twist the field flux in the opposite direction. The result of this neutralization is a fairly constant distribution of flux at all loads. Within limits even an unbalanced polyphase converter operates satisfactorily. There is no change of field excitation necessary with changes of load.

The converter is subject to hunting the same as the synchronous motor. As its speed oscillates above and below synchronism, the phase of the armature current, in reference to the impressed *E.M.F.*, changes. This results in a distortion of the field flux, of varying magnitude. This hunting is much reduced by placing heavy copper circuits near the pole horns so as to be cut by the oscillating flux from the two horns of the pole. The shifting of flux induces heavy currents in these circuits which oppose the shifting. Fig. 216 shows copper bridges placed between the poles of a converter for this purpose.

When running as an inverted converter from a direct-current circuit, anything which tends to cause a lag of the alternating current behind its *E.M.F.* is to be avoided. The demagnetization of the field by the lagging current causes the armature to race the same as in the case of an unloaded shunt motor with weakened fields. Converters have been raced to destruction because of the enormous lagging currents due to a short circuit on the alternating-current system.

112. Regulation of Converters. — The field current of a converter is generally taken from the direct-current brushes. By varying this current the power factor of the alternating-current system may be changed. This may vary, through a limited range, the voltage impressed between the slip-rings. As the direct-current voltage bears to the latter a constant ratio it may also be varied. This is, however, an uneconomical method of regulation. Converters are usually fed through step-down transformers. In such cases there are two common methods of regulation, which vary the voltage supplied to the converter's slip-rings. The first is the method of Stillwell, which is shown in the diagram, Fig. 217.

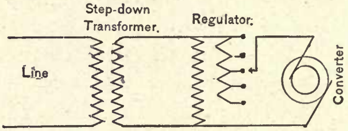


Fig. 217.

The regulator consists of a transformer with a sectional

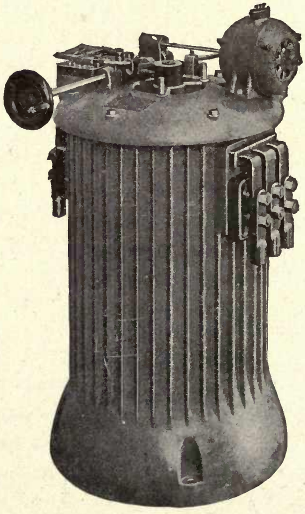


Fig. 218.

secondary. Its ratio of transformation can be altered by moving a contact-arm over blocks connected with the various sections, as shown in the diagram. The primary of the regulator is connected with the secondary terminals of the step-down transformer. The sections of the secondary, which are in use, are connected in series with the step-down secondary and the converter windings.

The second method of regulation is that employed by the General Electric Co. The ratio of transformation of a regulating transformer, which is connected in circuit in the same manner as the Stillwell regulator, is altered by shifting the axes of the primary and secondary coils in respect to each other. Fig. 218 shows such a transformer, the shifting being accomplished by means of a small, direct-current motor mounted upon the regulator. The primary windings are placed in slots on the interior of a laminated iron frame, which has the appearance of the stator of an induction motor. The secondary windings are placed in what corresponds to the slots of the rotor core. The winding is polar; and if the secondary core be rotated by an angle corresponding to the distance between two successive poles, the action of the regulator will change from that of *booster* to that of *crusher*.

Another method of converter regulation, sometimes used in railway work, makes use of reactance coils, connected between the step-down transformer coil terminals and the slip-rings of the converter, as well as of an ordinary series compounding coil on the field-cores of the converter. The series and shunt field coils are so adjusted that the converter takes a lagging current at no load and a leading current at full load. The step-down transformer voltage being assumed as constant, the voltage impressed upon the slip-rings will be the remainder resulting from the vector subtraction of the reactance drop from the constant voltage. On heavy loads and leading currents this remainder is greater than the constant voltage. There is therefore a constantly increasing voltage impressed upon the slip-rings as the load increases. Too large a reactance, however, is liable to introduce pulsation troubles.

In Europe some use is made of a small auxiliary alternator mounted upon the shaft of the converter and operating synchronously with it. By varying and reversing the field excitation of this alternator, whose armature phases are connected between the transformer terminals and the slip-rings of the converter, it may be caused to act as a booster or as a crusher.

Recently converters have been constructed in a manner that permits of altering their ratios of voltage conversion by changing the distribution of flux in the air gap. Non-sine waves of *E.M.F.* are then induced in the armature inductors. The ratios of voltage conversion hitherto deduced upon the assumption of sine wave-forms do not then hold. The change of flux distribution is accomplished by splitting each pole into sections along axial planes. The sections are then subjected to different magnetomotive forces which may be independently varied during operation.

113. Mercury Vapor Converter. — A mercury vapor converter, which is suitable for use in charging storage batteries from a single-phase circuit, is shown with its connections in Fig. 219. It consists of a very highly exhausted glass bulb equipped with four electrodes, of which two are positive, one negative, and the other an auxiliary which is used only in starting. The two latter electrodes are of mercury. The two external terminals of an auto-transformer are connected with the two positive electrodes, while the internal terminals are connected to the single-phase supply circuit. The operation of this converter is based upon the facts that (*a*) to start a current between two electrodes in a vacuum bulb of this character there must be impressed upon these electrodes a very high voltage (25,000 volts), most of which is consumed in overcoming a transition resistance at the negative electrode, and (*b*) once started this cathode transition voltage drops to a very small value (4 volts). In operation, and after starting, therefore, current flows during one-half of a cycle from the left-hand terminal of the transformer to the left-hand positive electrode through the vapor to the main negative electrode and thence through the battery to the center of the transformer coil, and during the following half cycle flows from the right-hand terminal to the right-hand positive electrode through the tube and battery as before. The positive electrodes permit current to flow

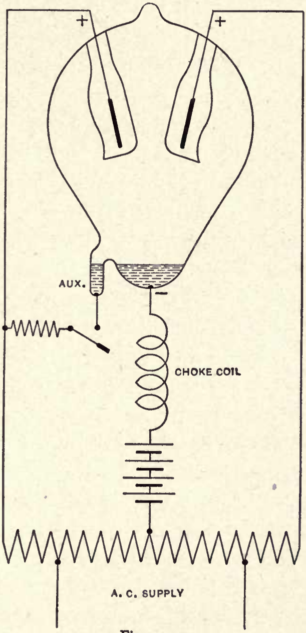


Fig. 219.

from them into the tube but never in the reverse direction. They are therefore each idle during alternate half cycles. The transition resistance of the negative main electrode when once broken down remains so as long as current enters it from the vapor.

To start the converter the bulb is tilted until there is a mercury connection between the main negative and auxiliary electrodes. This permits a current to flow from the storage battery through the mercury into the auxiliary electrode. If now the mercury bridge be broken, by restoring the bulb to its original position, vapor conduction will be established between the main negative and auxiliary electrodes. The transition resistance of the latter is thus broken down and the converter begins to operate, current flowing alternately from the two positive electrodes to the auxiliary electrode. If now the converter be again tilted and restored to its normal position the point of entrance of the vapor current into the mercury can be transferred to the main negative electrode. A second tilting is seldom necessary, the mercury generally making several makes and breaks of the circuit during the first tilt as a result of its fluidity. If for an instant (one millionth of a second) the current ceases to enter the mercury, the cathode transition resistance will reëstablish itself. An inductance inserted in the battery circuit causes a sufficient lag of current behind the voltage between a positive and the negative electrode to enable the voltage due to the other positive electrode to maintain the operation. The current in the battery circuit is unidirectional but pulsating.

PROBLEMS.

1. From what points on the armature winding should taps be taken for connection with the successive rings of a 5-ring 6-pole converter?

2. A 4-pole converter is supplied with six slip-rings so as to be adapted for use on single-, two-, or three-phase circuits. The rings used on single-phase are 1 and 4; on two-phase are 1 and 4, and 2 and 6; on three-phase 1, 3, and 5. Locate the points of attachment of taps from each ring to the armature winding.

3. A 12-ring converter delivers 600 volts to a direct-current railway circuit. What is the voltage between successive slip-rings?

4. A 20-pole 6-ring converter delivers 1000 amperes of direct current at full load. Neglecting armature resistance and other losses, determine the current wave-shape in a conductor 20 electrical degrees in advance of tap to a slip-ring.

5. During what portion of a revolution is the current in the conductor mentioned in problem 4 so directed as to exert a motor effort?

6. Compare the heating effect of full-load current in the conductor of problem 4 with that in a conductor midway between taps.

CHAPTER IX.

POWER TRANSMISSION.

114. Superiority of Alternating Currents. — In transmitting power electrically over long distances, it is necessary to employ high voltages, so that, with a reasonable line loss, the cost of the conductors will not be excessive. In the United States, power transmission at high voltages has been accomplished by means of alternating current only. In Europe, however, considerable attention has been given to the development of the Thury system of direct-current transmission. There are a number of plants successfully employing this system at the present time, but the highest voltages used are in the neighborhood of 20,000, and the amounts of power transmitted are comparatively small. If the line alone be considered, direct current is far superior to alternating current. The former has unity power factor, is free from inductive disturbances, such as surges, and it has no wattless charging current to reduce the effective output of the machines. As will be shown later, the amount of conductor material required in a direct-current line is less than that required in an alternating-current line with the same maximum voltage in the two cases.

A comparison of the station apparatus of both systems of power transmission shows that the direct-current system is at a great disadvantage. Three thousand volts is the maximum that can be successfully handled on a commutator, even with the special design of machine such as Thury has developed. Consequently, to obtain the required high line voltage, a number of generators must be connected in series, series-wound machines being used. When a machine is generating 3000 volts, the maximum current that can be commutated is about 100 amperes, so that the individual machines have small output. Each generator must be insulated from ground, and, as several machines are connected to the same prime mover, they must be insulated therefrom and from each other. The system is grounded at the middle point, so as to limit the amount of insulation required; that is, the insulation under each machine must be capable of withstanding the maximum difference of potential between its terminals and ground. The line current is maintained constant by several complicated auxiliary devices. These automatically regulate the speed of all the prime movers, so as to keep the line voltage proportional to the load; cut in or out of circuit one or more machines if there be large changes in load, and short-circuit any disabled machine.

In the substations a number of series-wound motors are connected in series across the line, the motors being arranged in groups, each group driving a generator. The generators, which may deliver either direct or alternating current, are connected in multiple for distribution. The motors and generators in the substation must be insulated from each other and from ground, just as are the machines in the generating station. The current taken by the motors is kept constant by an automatic shifting of the brushes. The Thury system is adapted only to undertakings where the power is to be transmitted over a long distance and the load is to be concentrated at few points, since at every tap a complete substation must be provided containing motors having an aggregate voltage equal to the line voltage. On the other hand, in an alternating-current system, a static transformer can be installed anywhere along the line and it will operate satisfactorily with practically no attention.

Considering the line, alone, the employment of direct current is better and more economical than that of alternating current. But when the whole plant, including generating station, line, and substations, is considered, the employment of the alternating-current system is held by many engineers to be the most advantageous. The alternating-current system is more reliable, more flexible, and, with the exception of special cases, is probably cheaper than the direct-current system, in spite of the greater cost of the line conductors.

115. Frequency. — According to the Standardization Rules of the A. I. E. E., there are two standard frequencies, namely, 60 cycles and 25 cycles. In early transmission plants the frequency employed was 60 cycles or higher. All recent transmissions, however, are at 25 cycles, and there is a strong tendency to lower this frequency to 15 or even to $12\frac{1}{2}$ for certain classes of work. Sixty-cycle generators and transformers are smaller and cheaper than are those of lower frequency. It was formerly thought that for lighting, a frequency higher than 25 cycles was necessary in order to prevent flickering of the lamps. But the success of 25-cycle lighting in Buffalo from the circuits of the Niagara Falls Power Company has proved that, if the form factor of the voltage wave is not greater than that for a sine wave, the higher frequency is unnecessary. The Niagara generators give a wave slightly flatter than a sine wave; and all modern generators of large output can be depended upon to give good wave forms.

The advantages of low frequency for transmission lines are as follows: (*a*) The inductive drop, $2\pi fLI$, is less, and consequently the regulation is better than for high frequencies.

(*b*) The capacity current, $2\pi fEC$, also increases with the frequency. Its effect is to reduce the energy output of the generators and transformers.

(*c*) The lower the frequency, the less difficult becomes the problem of operating generators and other synchronous apparatus in parallel. This is because the unavoidable variations in speed are smaller in proportion to the angular velocity, the lower the frequency.

(*d*) The power factor of an induction motor decreases as the frequency is raised. This is an extremely important reason for using a low frequency, since the power load generally constitutes a large part of the total load of a transmission system.

(*e*) A low frequency is also less liable to set up electrical oscillations as a result of the coincidence of the natural frequency of the line with that of an odd harmonic of the impressed *E.M.F.* If the distributed inductance and capacity of the line be L henrys and C farads respectively, then its natural frequency, as shown by Steinmetz, is to be expressed as

$$f = \frac{1}{4\sqrt{LC}}.$$

If the resistance be sufficiently low, as is often the case, oscillations at this frequency are liable to occur. A triple harmonic of some magnitude usually exists in the *E.M.F.* wave of each phase winding of an alternator. This does not appear at the terminals of a three-phase machine whether Y- or Δ-connected. It does appear, however, between the terminals and a grounded neutral. In the armature windings there is usually a triple harmonic component of current which sets up an armature reaction causing magnetic field distortion that results in fifth and seventh harmonic *E.M.F.*'s. Triple harmonics of *E.M.F.* or of current also result from the use of transformers. In three-phase work their influence upon the line may be overcome by the use of Δ connections.

With lines constructed in accordance with present practice, the natural frequency for a length of 150 miles is about three hundred. This is the same as that of the fifth harmonic on a 60-cycle system, whereas for a frequency of 25 the fifth harmonic frequency would be but 125. It would therefore be unwise to select a frequency of 60 cycles for such a line.

116. Number of Phases. — A comparison of the weights of line wire of a given material, necessary to be used in transmitting a given power, at a given loss, over the same distance, must be based upon equal maximum voltages between the wires. For the losses by leakage, the thickness and cost of insulation, and perhaps the risk of danger to life, are dependent upon the maximum value. A comparison upon this basis gives, according to Steinmetz, the following results: —

Relative weights of line wire to transmit equal power over the same distance at the same loss, with unit power factor.

2 Wires.	Single-phase	100.0
	Continuous current	50.0
3 Wires.	Three-phase	75.0
	Quarter-phase	145.7
4 Wires.	Quarter-phase	100.0

The continuous current does not receive the approval of American engineers, as previously stated. The single-phase and four-wire quarter-phase system each requires one-third more wire than the three-phase system.

By use of the Scott three-phase quarter-phase transformer, the transmission system may be three-phase, while the distribution and utilization system may be quarter-phase.

Each conductor of a three-phase line must be of the size required in a single-phase line transmitting half as much power, with the same percentage of loss, at the same voltage and distance between conductors.

117. Voltage. — If the frequency, the amount of transmitted power, and the percentage of power lost in the line, remain constant, the weight of line wire will vary inversely as the square of the voltage impressed upon the line. This depends upon the fact that the cross-section of the wire is not determined by the current density and the limit of temperature elevation, but by the permissible voltage drop. If the impressed voltage on a line be multiplied by n, the drop in the line may be increased n times without altering the line loss. For the line loss is to the total power given to the line as the drop in volts is to the impressed voltage. To transmit the same power, but $\frac{1}{n}$th the previous current is necessary; and this current, to produce n times the drop, must, therefore, traverse a resistance n^2 times as great as previously.

In a long transmission line the conductors constitute one of the largest items, if not the largest item, of investment of the entire plant. Consequently it is desirable to have the voltage as high as possible. But raising the voltage increases the investment for transformers, switching apparatus, lightning protection, and insulators; and the depreciation and repair charges on these items are much greater than the corresponding charges on the conductors. The economic voltage to be employed for transmitting a given amount of power over a certain distance is that voltage which will lead to the minimum annual cost for the entire plant. Theoretically, this economic voltage can be determined by expressing the several elements of cost as functions of the voltage and equating the differential of this expression to zero. This method is complicated, and it requires so many assumptions as to render it of little use.

118. Economic Drop. — A more practical way of determining the voltage is based upon the fact that there are certain standard voltages for high-tension transformers. Except for special cases, a standard voltage should be used. For a given voltage, the amount of conductor material varies inversely as the drop, whereas the line loss varies directly with the drop. If the economic drop, which fixes the cross-section of the conductor, be calculated for the several standard voltages, the best voltage to employ can readily be determined. For a given voltage at the generating station, the economic drop and cross-section of conductor for a single-phase circuit may be found as follows:

Let E = voltage at generating station,
P = power in kilowatts at generating station,
L_1 = length of line in miles, i.e., length of a single conductor,
R = total resistance of line in ohms,
x = loss in terms of impressed quantities,
S = section of conductor in circular mils,
c_1 = cost of energy in dollars per kilowatt-year at generating station,
c_2 = cost of conductor in dollars per pound,
p_2 = interest rate on cost of line conductors,
K_1 = resistance in ohms per mile of conductor having one circular mil cross-section, and
K_2 = weight in pounds per mile of conductor having one circular mil cross-section.

Then, line loss = Px.

Annual cost of line loss = c_1Px.
Weight of line conductors = $2\,K_2L_1S$.
Cost of line conductors = $2\,c_2K_2L_1S$.
Annual cost of line conductors = $2\,p_2c_2K_2L_1S$.

$$\text{Line resistance} = R = K_1\frac{2\,L_1}{S}.$$

$$\text{Line drop} = Ex = \frac{1000\,P}{E}R = \frac{2000\,PK_1L_1}{ES}.$$

$$\text{Section of conductor} = S = \frac{1000\,P}{E^2}K_1\frac{2\,L_1}{x}.$$

The total annual charge due to line loss plus interest on conductors is

$$c_1Px + 2\ p_2c_2K_2L_1S,$$

and per delivered kilowatt is

$$q = \frac{c_1Px + 2\ p_2c_2K_2L_1S}{P - Px}. \qquad (1)$$

Substituting the value of S, this becomes

$$q = \frac{c_1Px + 2\ p_2c_2K_2L_1 \dfrac{1000\ P}{E^2} K_1 \dfrac{2\ L_1}{x}}{P\ (1 - x)}, \qquad (2)$$

or $$q = \frac{c_1x}{1 - x} + \frac{p_2c_2K_1K_2\ 4\ L_1^2\ 1{,}000}{E^2x\ (1 - x)}. \qquad (3)$$

If K is substituted for $p_2c_2K_1K_2\ 4\ L_1^2\ 1000$, then

$$q = \frac{c_1x}{1 - x} + \frac{K}{E^2x\ (1 - x)}. \qquad (4)$$

To find the minimum value of q, its derivative is placed equal to zero, and there obtains

$$\frac{dq}{dx} = c_1x^2 + \frac{2\ K}{E^2}\ x - \frac{K}{E^2} = 0; \qquad (5)$$

whence $$x = -\ \frac{K}{E^2c_1} \pm \left[\left(\frac{K}{E^2c_1}\right)^2 + \frac{K}{E^2c_1}\right]^{\frac{1}{2}}.$$

But as x is positive

$$x = -\ \frac{K}{E^2c_1} + \frac{1}{E^2c_1}\sqrt{K^2 + E^2c_1K}. \qquad (6)$$

If the preceding were worked out for a constant or fixed delivered *E.M.F.*, instead of a fixed impressed *E.M.F.*, allowing the latter to become what it might, the expression for q would be the same, except that the denominator would be P instead of $P\ (1 - x)$, the quantities E, x, P, etc., being then delivered quantities instead of impressed quantities. If this be done, and the value of q be differentiated, there obtains

$$\frac{dq}{dx} = c_1 - \frac{K}{E^2x^2} = 0.$$

Hence

$$\frac{K}{E^2x} = c_1x, \text{ that is, the well-known relation}$$

$$\text{Interest} = \text{Loss.}$$

A three-phase line requires three-quarters as much conductor material as a single-phase line transmitting the same amount of power with the same loss. Each conductor of a three-phase transmission line has one-half the area of each conductor of the equivalent single-phase line. To find the economic drop for a three-phase line, multiply $2\ p_2c_2K_2L_1S$ in equation (1) by $\frac{3}{4}$. Then $\frac{3}{4}\ K$ will appear in equation (4) instead of K. Solving for the economic drop,

$$x = -\frac{3}{4} \cdot \frac{K}{E^2c_1} + \frac{1}{4\ E^2c_1}\sqrt{9\ K^2 + 12\ E^2c_1K}. \qquad (7)$$

The area of each conductor is

$$S = \frac{1}{2}\left(\frac{1000\ P}{E^2}\ K_1\ \frac{2\ L_1}{x}\right). \qquad (8)$$

119. Line Resistance. — The resistance of anything but very large lines is the same for alternating currents as for direct currents. In the larger sizes, however, the resistance is greater for the alternating currents. The reason for the increase is the fact that the current density is not uniform throughout a cross-section of the conductor, but is greater toward its outside. The lack of uniformity of density is due to counter electromotive forces set up, in

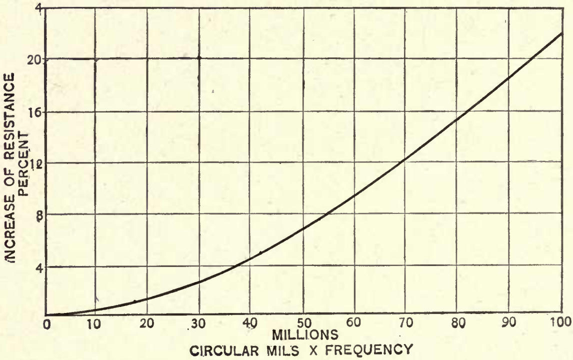


Fig. 220.

the interior of the wire, by the varying flux around the axis of the wire which accompanies the alternations of the current. This phenomenon is termed skin effect, §21. Its magnitude may be determined from the curve, Fig. 220.

120. Line Inductance. — The varying flux which is set up between the two line wires of a single-phase transmission circuit by the current flowing in them gives rise to a self-induced counter *E.M.F.* The inductance per unit length of single wire is numerically equal to the flux per unit current, which links a unit length of the line. To determine this value consider a single-phase line, with wires of R cms. radius, strung with d cms. between their centers, and carrying a current i. Let a cross-section of the line be represented in Fig. 221. The flux $d\Phi_1$, which passes through an element dr wide and of unit length, is

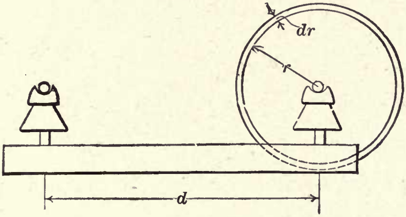


Fig. 221.

equal to the magnetomotive force divided by the reluctance, or

$$d\Phi_1 = \frac{4\,\pi i}{\frac{2\,\pi r}{dr}}\,.$$

Integrating for values of r between $d - R$ and R,

$$\Phi_1 = 2\,i \log\left(\frac{d-R}{R}\right),$$

and practically

$$= 2\,i \log\left(\frac{d}{R}\right).$$

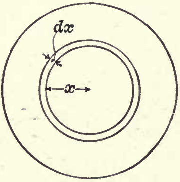


Fig. 222.

There is some flux which surrounds the axis of the right-hand wire, and which lies inside the metal. This is of appreciable magnitude owing to the greater flux density near the wire. Represent the wire by the circle in Fig. 222, and suppose that the current is uniformly distributed over the wire. Then the current inside the circle of radius x is $\frac{x^2}{R^2}i$, and the magnetomotive force which it produces is

$$4\pi\frac{x^2}{R^2}i.$$

The flux, however, which it produces links itself with but $\frac{x^2}{R^2}$ths of the wire. The flux through the element dx, which can be considered as linking the circuit, is therefore

$$d\Phi_2 = \frac{2\,x^3 i\,dx}{\mu R^4}.$$

Integrating for values of x between o and R,

$$\Phi_2 = \frac{2\,i}{\mu\,4}.$$

For copper or aluminum wires $\mu = 1$. Hence the total flux linked with the line is

$$\Phi_1 + \Phi_2 = 2\,i\left[\log_\epsilon\left(\frac{d}{R}\right) + \frac{1}{4}\right],$$

and the inductance, in absolute units, being the flux per unit current, is

$$l = 2\log_\epsilon\left(\frac{d}{R}\right) + \frac{1}{2}.$$

This gives by reduction the inductance in henrys per wire per mile as

$$L = \left[80.5 + 740\log\left(\frac{d}{R}\right)\right]10^{-6}.$$

The drop in volts due to the inductance, per mile of line (two conductors) per unit current, is therefore

$$E_L = .00405\, f \left[2.3 \log\left(\frac{d}{R}\right) + \frac{1}{4}\right],$$

where d and R must be in terms of the same unit.

It will be noted that the inductance depends upon the distance between conductors. This distance should increase with the voltage, but there is no definite relation between them. The following values represent average practice for bare overhead conductors:

Kilovolts.	Distance between Conductors in Inches.
2.3 to 6.6	28
10 to 20	40
20 to 30	48
30 to 50	60
50 to 60	72

121. Line Capacity. — The two conductors of a single-phase transmission line, together with the air between them, act as a condenser. The conductors correspond to the condenser plates, and the air corresponds to the dielectric. When the lines are long, or when the conductors are close together, the capacity is quite appreciable.

The capacity between two parallel cylindrical conductors may be determined as follows: Let A and B (Fig. 223) represent the two conductors of R cms. radius and d cms. apart between centers. Let A be charged with $+Q$ electrostatic units of electricity per centimeter of length, and B with $-Q$ units. If the charge on A be alone considered, there emanates from each unit length an electrostatic flux of $4\pi Q$ lines directed radially away from the axis of A. Similarly, a flux of $-4\pi Q$ emanates radially from each unit length of B, due to its charge. The negative sign indicates that the flux is directed towards the axis of B. The superposition of these two fluxes results in an electrostatic field such as would exist if a neutral conducting

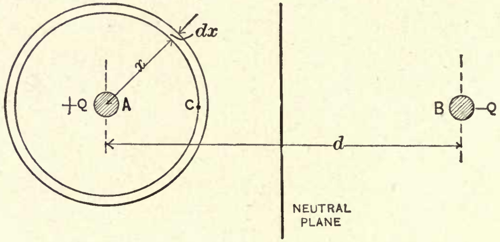


Fig. 223.

plane were introduced halfway between A and B and perpendicular to the plane of their axes, and the potential of the neutral conducting plane were maintained as much below that of A as it is above that of B. To determine the potential differences, consider that the difference of potential between two points is equal to the work that must be performed on a unit positive charge to move it from one point to the other. The intensity of the field at a point C, at a distance x from the axis of A, due to the charge on A, this intensity being the flux-density or force which would be exerted upon a unit positive charge, is

$$F_A = \frac{4\pi Q}{2\pi x} = \frac{2Q}{x},$$

and that due to the charge on B, noting that it is in the same direction as that due to A, is

$$F_B = \frac{4\pi Q}{2\pi(d-x)} = \frac{2Q}{d-x}.$$

Therefore the total intensity or force is

$$F = F_A + F_B = 2Q\left[\frac{1}{x} + \frac{1}{d - x}\right].$$

Hence the difference of potential between A and the neutral plane, which is the same as that between the neutral plane and B, is

$$E = \int_R^{\frac{d}{2}} 2Q\left[\frac{1}{x} + \frac{1}{d - x}\right] dx = 2Q \log_e \frac{d - R}{R}.$$

The capacity between either conductor and the neutral plane is therefore

$$C = \frac{Q}{E} = \frac{1}{2 \log_e \frac{d - R}{R}},$$

in electrostatic units per centimeter length of conductor. In transmission lines, R is usually so small compared with d as to be neglected. Reducing to miles, microfarads, and common logarithms, the capacity between neutral plane and either conductor is

$$C = \frac{0.0388}{\log\left(\frac{d}{R}\right)} \text{ microfarads per mile,}$$

and between conductors it is half as much.

Because of its capacity, a line takes a charging current when an alternating $E.M.F.$ is impressed upon it, even though it be not connected to a load. The value of this current is

$$I = 2\pi f E C \, 10^{-6} \text{ amperes,}$$

where E and C are the voltage and the capacity in microfarads respectively between conductors or between one conductor and the neutral plane.

To determine the charging current per conductor of a three-phase line, the above formula is used, C and E being the capacity and voltage respectively between conductor and neutral plane. The voltage is $\frac{1}{\sqrt{3}}$ of that between conductors.

The capacity between a conductor and ground may be derived by considering the ground as the neutral plane. In a circuit not employing a ground return, there is no charging current due to the capacity between the conductors and ground, but in the case of a single conductor with ground return there is a charging current. The value of this current is given by the preceding formula, in which case C is the capacity to ground and E is the voltage to ground.

122. Regulation. — The regulation of a transmission line is the ratio of the maximum voltage difference at the receiving end, between rated non-inductive load and no-load, to the rated-load voltage at the receiving end, constant voltage being impressed upon the sending end.

In short aerial transmission lines, the capacity and charging current may be considered as negligible. The voltage at the receiving end will then be the same as that at the sending end on no-load. On full-load the sending voltage is equal to the vectorial sum of the delivered voltage, that necessary to overcome the resistance drop, and that necessary to overcome the inductive drop at 90° ahead of the delivered voltage.

In long transmission lines the capacity cannot be neglected, and the following method due to Steinmetz may be employed: Consider the line to be made up of a number of sections, say ten, to each of which is apportioned one-tenth the total capacity and inductance of the line. The capacities may be considered as localized condensers at the sending end of the section and connected across the lines. The inductances may be considered as connected in series with the line. The connections of a few sections are shown in Fig. 224. The inductive and resistance drop of voltage in each section is

Fig. 224.

greater than in any other section more remote from the sending end, because, although the inductances and resistances are the same in all sections, the current is greater as a result of the extra charging current due to the capacity of intervening sections. The charging current is also different for each section because the voltage which occasions it increases as the sending end is approached.

The voltages and currents in each section can be determined with sufficient accuracy by making use of a large vector diagram, such as indicated in Fig. 225, where OE and OI represent the delivered voltage and current respectively. The power factor of the load being unity, the latter are in phase with each other. Let $E, E_1, E_2, \ldots$ be the voltages, and $I, I_1, I_2, \ldots$ be the currents delivered to the load and the successive sections respectively. Then, if R and X be the resistance and inductive reactance in ohms, and C be the capacity in farads, of each and every section,

$$E_1 = (E + RI) \oplus XI \text{ at } 90^\circ \text{ lead},$$
$$E_2 = (E_1 + RI_1) \oplus XI_1 \text{ at } 90^\circ \text{ lead, etc.},$$

and

$$I_1 = I \oplus \omega E_1 C \text{ at } 90^\circ \text{ lead},$$
$$I_2 = I_1 \oplus \omega E_2 C \text{ at } 90^\circ \text{ lead, etc.}$$

The various phase relations and magnitudes are seen in the figure, where the E's and I's with various subscripts

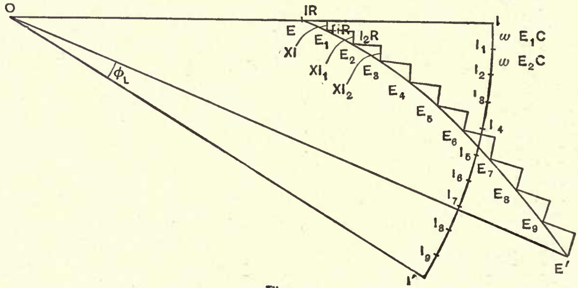


Fig. 225.

mark the terminals of the vectors from the origin, which are not drawn for the sake of clearness.

The cosine of ϕ_L, the angle between the current I' and the voltage E' at the sending end of the line, is the *power factor of the line.*

This method is strictly accurate only when the number of sections is infinite. Ten sections give sufficient accuracy for practical work.

123. Conductor Material. — The high permeability of iron prohibits its use as a conductor for transmission lines. There are but two other materials available, copper and aluminum. The physical constants of these metals are:

	Copper.	Aluminum.
Specific gravity	8.93	2.68
Conductivity, in terms of Matthiessen's Standard	.98	.61
Tensile strength, pounds per square inch	60,000	26,000
Elastic limit, pounds per square inch	40,000	14,000
Stretch modulus of elasticity, pounds per square inch	16,000,000	9,000,000
Coefficient of expansion per degree Fahrenheit	0.0000096	0.0000128

For the same conductivity, an aluminum conductor must have $\frac{.98}{.61}$ or 1.6 times the area of a copper conductor. But as aluminum is three-tenths as heavy as copper, the former weighs but 0.48 as much as the latter for equal conductivities. Therefore, if aluminum costs less than 2.08 times as much as copper per unit of weight, it is cheaper than the latter. The prices of both metals vary, but that of aluminum is usually much less than 2.08 times that of copper. If its lower tensile strength and greater coefficient of expansion are properly allowed for while the line is being strung, the use of aluminum for transmission line conductors is just about as satisfactory as that of copper. Because of the resultant saving, aluminum is being used very extensively.

124. Insulators. — There is no material from which insulators can be made that possesses all the mechanical and electrical qualities to be desired. Glass has been used to some extent, but it is easily broken, either in transit, or by stones and bullets after the insulators are installed. The two materials now used for insulators on high-tension transmission lines are porcelain and a substance known as electrose.

Porcelain is much tougher than glass and is therefore not so liable to be broken. Porcelain insulators are heavily glazed to prevent absorption of moisture, since

Fig. 226.

even the best porcelain is somewhat porous. A brown or gray tint is usually introduced into the glaze so that the insulators will not attract the attention of marksmen. A Thomas 33,000-volt porcelain pin-type insulator is shown in Fig. 226. It is $7\frac{1}{2}$ inches high and $8\frac{1}{2}$ inches in diameter.

Electrose possesses good insulating qualities, is very strong mechanically, and is free from cracks. It has a brown, smooth, polished surface and does not absorb moisture. Metal parts may be molded into it readily if so required. A 24,000-volt electrose pin-type insulator is shown in section in Fig. 227. It is 7 inches high and 12 inches in diameter.

In designing an insulator, the distance along the surface

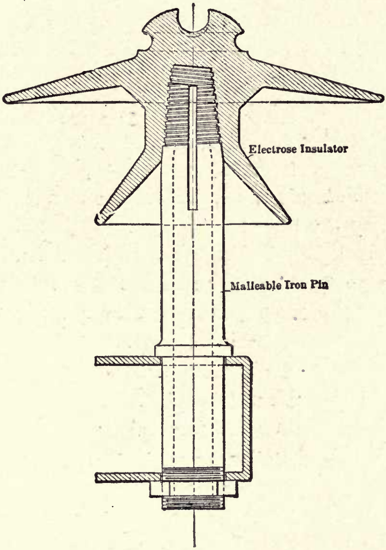


Fig. 227.

from the conductor to the point of support should be as long as possible in order to decrease the leakage current. The shortest distance through the air between these two points should be great enough to prevent the line voltage flashing over, even when the insulator is wet. Furthermore the distance through the dielectric should be great enough to prevent puncture. At the same time the size of the insulator and the quantity of material used should be kept as small as possible. In view of these requirements high-voltage insulators have taken the form of a series of umbrella-shaped petticoats. Manufacturing difficulties prevent the construction of a satisfactory large porcelain insulator of the usual type in a single piece. The petticoats therefore are made separately and are glazed or cemented together, this being usually done where the line is being erected. Electrose insulators may be molded in one piece, regardless of shape.

Metal pins are generally used for pin-type insulators, even on wooden poles, because of their strength and the fact that they are not burned by the leakage current. It was formerly thought that the additional insulation of a wooden pin was desirable, but it has been found that even the best treated pins will absorb moisture in time, especially in salt atmospheres. With steel towers and long spans, the large strains on the insulators necessitate the use of heavy iron or steel pins.

The tendency toward higher voltages for power transmission has led to the application of flexible suspension-type insulators. These consist of individual units securely coupled together, the number of units to be employed depending upon the line voltage. Some 110,000-volt insulators of this type are shown attached to a transmission tower in Fig. 230.

125. Sag of Conductors. — In stringing the conductors there are two things to be taken into consideration. First, the greatest possible tension in the conductor, which will occur at minimum temperature, must be less than the elastic limit of the conductor. At the minimum temperature there may be a coating of ice on the conductor. This not only results in increased weight, but also presents a greater area to the wind. Second, the clearance between conductors and ground at the maximum temperature must be great enough to prevent accidental contact or malicious interference. These questions are of greater importance when using aluminum than when using copper, because the tensile strength of aluminum is less than that of copper, its coefficient of expansion is greater, and, for a given conductivity, it presents a larger surface to the wind.

The conductors are of necessity strung under varying conditions as to temperature, and consequently they should be given an appropriate sag so that at the extreme temperatures the required conditions will be fulfilled. It is common to make use of curves, one for each span length, whose ordinates represent the appropriate sag and whose abscissæ represent temperatures between, say, $-40°$ and $110°$ F. There is considerable difference of opinion as to the proper assumptions to be made for sleet and wind pressure. In northern countries, conductors will frequently be covered with ice from one-half to one inch thick all around, and hence this should be allowed for. In regard to wind pressure, it should be noted that the wind velocities published by the United States Weather Bureau are observed and not actual velocities. An observed velocity of 100 miles per hour corresponds to an actual velocity of about 75 miles per hour. The wind pressure in pounds per square foot exerted upon a plane surface normal to the direction of the wind may be expressed as CV^2, where V is the actual wind velocity in miles per hour and C is a constant whose value may be taken as .005. Thus the wind pressure on a plane normal surface for an actual velocity of 75 miles per hour is approximately 30 pounds per square foot. The wind pressure on the conductors is usually taken as half of this value, or 15 pounds per square foot of projected conductor area. Except in the case of tornadoes, observed wind velocities in excess of 100 miles per hour are practically unknown. The assumption of a wind pressure of 15 pounds per square foot of projected area when the conductor is covered with one-half inch of ice all around is conservative.

The weight of the conductor and ice acts vertically downward, while the wind pressure at worst acts transversely to the direction of the line. The sag of the conductor is therefore in the direction of the resultant of these

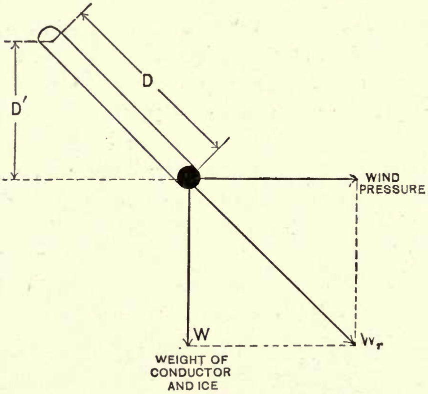


Fig. 228.

two forces, as shown in Fig. 228. The appropriate sag of a conductor at any given temperature such that the elastic limit of the metal shall not be exceeded at the minimum temperature may be found in the following manner:

Let S_1 = span in feet (Fig. 229),

D = sag in feet (Fig. 228),

W = weight of conductor and ice in pounds per foot,

W_r = resultant of W and wind pressure in pounds per foot,

T = maximum allowable tension in the conductor, usually taken as the elastic limit,

t = temperature in degrees Fahr. above the minimum (−40°),

k = temperature coefficient of linear expansion per degree F.

A = cross-section of conductor in square inches,

E_1 = stretch modulus of elasticity in pound-inch units,

L_s = length of single span of strung cable at the minimum temperature in feet, and

L_u = length of unstressed single span of cable at minimum temperature.

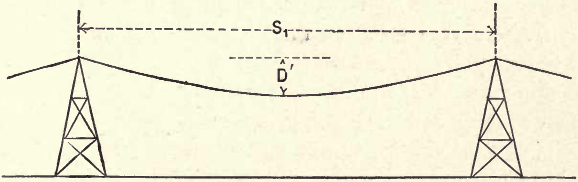


Fig. 229.

Then the following relations are sufficiently exact:

$$L_s = S_1 + \frac{S_1^3 W_r^2}{24\, T^2},$$

$$L_u = \frac{L_s}{1 + \frac{T}{AE_1}},$$

$$D^3 - \frac{3\,S_1}{8}\left[L_u\,(1 + kt) - S_1\right]D = \frac{3\,S_1^{\,3}L_uW_r}{64\,AE_1},$$

from which D, the sag in the direction indicated in Fig. 228, may be found. The vertical sag, D', is equal to $\frac{DW}{W_r}$.

126. Line Structure. — There are two types of line structure for transmission lines carrying large amounts of power at high voltages, namely, wooden poles and steel towers. If wooden poles are used the spans must be short, in order that the poles may withstand the forces to which they are subjected. With short spans, the expense for insulators will be large. On the other hand, while a single steel tower costs a great deal more than a wooden pole, the use of towers permits of longer spans and thereby reduces the number of structures and the cost of insulators. The depreciation and repair charges for steel towers are very small compared with the same items for wooden poles, especially if the towers be galvanized. Taking account of interest on the investment and the depreciation and repair charges on the line structures, including insulators, it will usually be found that towers are cheaper than wooden poles for lines using heavy conductors. Even if the cost of a proposed line with towers is a little more than with poles, towers would nevertheless be used in most cases because of their greater reliability. Towers can be designed to withstand the maximum forces which will be exerted upon them, allowing any desired

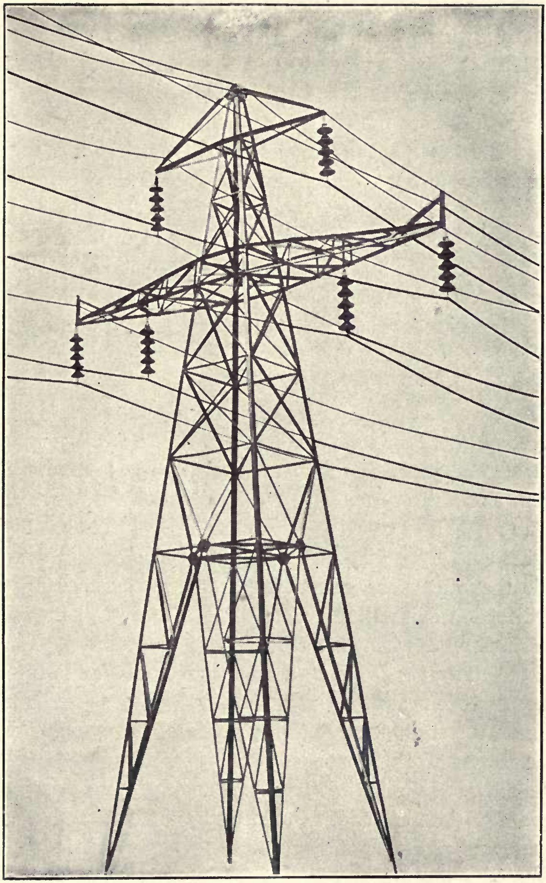


Fig. 230.

factor of safety. The strength of a tower will remain constant, whereas the strength of poles of a given size and kind of wood will vary, and the original strength will gradually diminish until the poles fail.

Fig. 230 shows the transmission line tower used by the Hydro-Electric Power Commission of Ontario, Canada, in transmitting power from Niagara to various points in that province at 110,000 volts and 25 cycles. The line consists of two three-phase circuits, the cables of each circuit being nine feet apart. The tower is 60 feet high, with a base 16 feet square, the span being 550 feet. Lightning protection is secured by the use of overhead ground wires fastened to the tower, as shown.

The forces acting on a structure in a straight portion of the line, where the spans are of equal length, are: (*a*) the weight of the conductors and ice; (*b*) the wind pressure on the conductors when covered with ice; (*c*) the wind pressure on the structure; (*d*) the weight of the structure.

The method of calculating the first three forces has been considered. The first and last act vertically downward, while the others are considered as acting transversely to the direction of the line. The value of the wind pressure is a maximum when the direction of the wind is transverse to that of the line, and zero when the wind is in the same direction as the line. Theoretically, there is no force on the structure due to the tension in the conductors, since the tensions in two adjacent spans counterbalance each other. In practice, however, such a force exists, on account of differences of wind pressure, of elevation of the towers, and of length of spans.

Should one or more of the conductors break, the full tension thereof will come on the structures on each side of the break. Such an accident is unusual, and the additional expense of building each structure to withstand it would not be justified. Consequently the conductors are secured to the insulators by means of loose ties. In the event of the breaking of a conductor the tension will then be taken up by several structures. At intervals of a few miles there are guyed towers capable of taking the full tension, and the conductors are firmly clamped to the insulators on such structures.

Knowing the magnitude of the transverse forces, the strength of the pole required to resist them can readily be calculated by means of the well-known formula for a beam fixed at one end. The resisting moment

$$M = \frac{SI}{C},$$

where S is the stress in the section, I is the moment of inertia of the section, and C is the distance from center to the fiber under maximum stress. From this formula, the most economical wooden pole is one whose vertical section is a parabola. The strength of the pole necessary to resist the other two forces can be determined by means of a formula for the compressive strength of a long column. However, if a structure is strong enough to resist the transverse forces, it will undoubtedly be strong enough to resist the vertical forces.

127. Spans and Layout. — For a given sized conductor, the necessary heights of the line structure, and the loads which the latter must sustain, increase as the span is lengthened. Consequently the cost of a single structure increases as the span is increased. At the same time, however, the number of structures and of insulators required is diminished. If the total cost of the structures complete with insulators be calculated for different spans, the most economical length of span can be determined. The foregoing applies to either wooden poles or steel towers, but of course poles can be used only for comparatively short spans because of strength limitations.

On account of irregularities in the ground and in order to clear obstacles, spans longer or shorter than the standard are frequently necessary. In such cases the forces acting on the structures are other than normal. These may sometimes be taken care of by using the regular structures with guys, but otherwise special structures are necessary.

In straight portions of the line ordinarily there are no forces acting on the structures due to the tension in the conductors, since the tension on one side balances that on the other. But when the line changes in direction this is not so. Figs. 231 and 232 show two methods of making a change in direction. In the first, the conductors are dead-ended on the two structures, and the tension is taken up by guys. In Fig. 232, for simplicity, but one conductor is shown. The effect of the tension in the conductors on the structures in changing the direction of the line by this method is shown in Fig. 233. For each conductor there is a force acting transversely on the structure equal to $2 \sin \frac{\alpha}{2}$ times the tension in the conductor. The

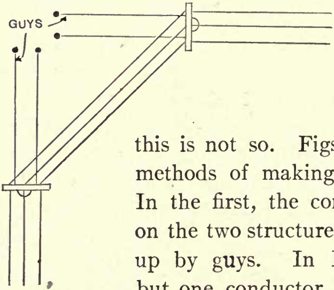


Fig. 231.

forces acting on structures at curves should be the same as on straight portions. Therefore the spans on curves are shortened by such an amount that the sum of the wind

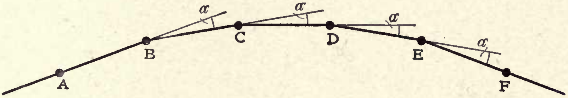


Fig. 232.

pressure on the conductors and the transverse component of the tension shall be equal to the wind pressure on the conductors on straight portions of the line. With shorter

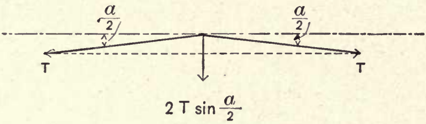


Fig. 233.

spans the tension in the conductors is less, and therefore the structures A and F, Fig. 232, must be guyed in order to equalize the tensions.

128. Example of Design of Transmission Line. — Let it be required to transmit 5000 kilowatts a distance of 100 miles over a three-phase circuit, using aluminum conductors, with a voltage of 66,000 at the generating station, the frequency being 25.

Economic Drop. The formulæ for the economic drop and cross-section of conductor for a three-phase line are given in § 118 as equations (7) and (8) respectively. Since the conductors are to be of aluminum,

$$K_1 = 85{,}000 \text{ ohms},$$
$$K_2 = 0.0048 \text{ pound},$$

as calculated from the constants given in § 123. Assuming

$$c_1 = \$15.00,$$
$$c_2 = \$\ 0.25,$$
$$p_2 = \ \ 0.05,$$

then $K = 4 \times .05 \times .25 \times 85{,}000 \times .0048 \times 10{,}000 \times 1000$
$= 204{,}000{,}000.$

Hence

$$x = -\frac{3 \times 204{,}000{,}000}{4\,(66{,}000)^2 \times 15} + \frac{[9\,(204{,}000{,}000)^2 + 12\,(66{,}000)^2 \times 15 \times 204{,}000{,}000]^{\frac{1}{2}}}{4\,(66{,}000)^2 \times 15} = .0461,$$

or the economic drop is 4.61 % of the impressed voltage. If 5000 K.W. are to be delivered, then $\frac{5000}{1 - .0461} = 5242$ K.W. must be supplied to the line. The size of the conductor required is

$$S = \frac{1}{2}\left[\frac{1000 \times 5242}{4{,}356{,}000{,}000}\,85{,}000\,\frac{200}{.0461}\right]$$
$$= 221{,}900 \text{ circular mils.}$$

An aluminum conductor of this size weighs $221{,}900 \times .0048$ or 1065 pounds per mile.

Proof that 4.61 % is the economic drop for the given conditions:

(*a*) Annual cost of lost power = 242 × \$15 = \$3630.

(*b*) Interest on cost of line conductors = 1065 × 300 × \$0.25 × .05 = \$3994.

(*c*) Cost of lost power per delivered kilowatt-year = $\frac{3630}{5000}$ = \$0.726.

(*d*) Interest on cost of line conductors per delivered kilowatt-year = $\frac{3994}{5000}$ = \$0.799.

Hence the sum of loss and interest per delivered kilowatt-year is \$1.525.

Now suppose $\frac{7}{8}$ as much conductor material were used.

$$\text{Line loss} = \tfrac{8}{7} \times 242 = 277 \text{ K.W.}$$
$$\text{Drop} = \tfrac{8}{7} \times 4.61 = 5.27\ \%.$$
$$\text{Delivered power} = 5242 - 277 = 4965 \text{ K.W.}$$

Then the new values of *a*, *b*, *c*, and *d* are:

$$a = 277 \times \$15 = \$4155,$$
$$b = \tfrac{7}{8} \times \$3994 = \$3498,$$
$$c = \tfrac{4155}{4965} = \$0.838,$$
$$d = \tfrac{3498}{4965} = \$0.704,$$

and hence the sum of loss and interest per delivered kilowatt-year is \$1.542. Therefore using $\frac{7}{8}$ as much conductor material increases the cost of delivering power. The cost of lost power and interest on the cost of the line conductors have been similarly calculated for $\frac{8}{7}$, $\frac{3}{4}$, and $\frac{5}{4}$ as much conductor material. The results, which are plotted in Fig. 234, prove that 4.61 % is the economic drop, since the cost per delivered kilowatt-year is a minimum at that value.

It has been assumed that the conductor is solid wire; but for a conductor of this size, cable is always used. The resistance of a cable is a few per cent higher than that of a solid conductor having the same cross-sectional area. A cable of 228,000 circular mils has approximately the same resistance as a solid wire of 221,900 circular mils. A cable of this size weighs 1100 pounds per mile, has a resistance of 0.396 ohm per mile, and is 0.55 inch in diameter.

INDUCTANCE. — Assuming the conductors to be spaced

72 inches apart, center to center, the inductance per mile of two conductors 0.55 inch in diameter, § 120, is

$$2\left[80.5 + 740 \log \frac{72}{.275}\right] 10^{-6} = 0.00374 \text{ henry},$$

and the inductance of the whole length of the line, for two conductors, is 0.374 henry.

CAPACITY. From § 121, the capacity between two conductors 0.55 inch in diameter is $\frac{0.0194}{2.418} = 0.00803$ microfarad per mile, and the capacity between two conductors for the whole length of line is 0.803 microfarad.

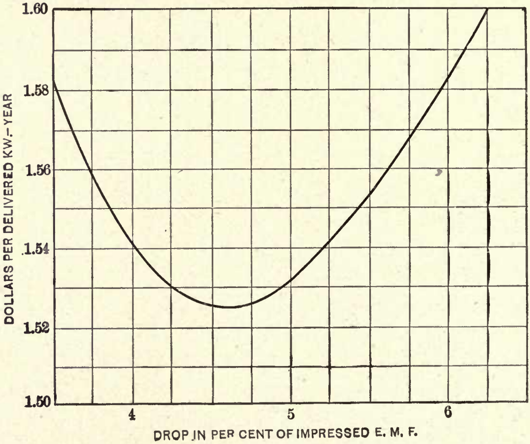


Fig. 234.

REGULATION. A convenient way of calculating the regulation of a three-phase circuit is based upon the fact that a three-phase circuit is equivalent to two single-phase circuits employing conductors of the same size. In other words, the regulation of a three-phase circuit is the same as that of a single-phase circuit carrying half as much power with the same percentage loss, at the same voltage and distance between conductors. The inductance and capacity of a single-phase line with the same sized conductors as the three-phase line under consideration have just been calculated. Dividing the line into ten equal sections, the constants of each are

$$L = .0374 \text{ henry.}$$

$$C = .0803 \text{ microfarad.}$$

$$R = .396 \times 20 = 7.92 \text{ ohms.}$$

Using the notation of § 122, and assuming the voltage at the receiving end of the line

$$E = 66{,}000\ (1 - .0461) = 62{,}957 \text{ volts.}$$

If 2500 kilowatts are delivered, the load current is

$$I = \frac{2{,}500{,}000}{62{,}957} = 39.71 \text{ amperes.}$$

The resistance and reactance drops of section 1 are respectively

$$IR = 39.71 \times 7.92 = 314 \text{ volts,}$$

$$2\,\pi f L I = 50\,\pi\ .0374 \times 39.71 = 233 \text{ volts.}$$

Hence

$$E_1 = \sqrt{(62{,}957 + 314)^2 + (233)^2} = 63{,}271.4 \text{ volts.}$$

The current in section 1 is

$$I_1 = \sqrt{(39.71)^2 + (50\,\pi\ 63{,}271.4 \times .0000000803)^2}$$

$$= 39.72 \text{ amperes.}$$

Similarly, the *E.M.F.* across section 2 is

$$E_2 = \sqrt{(63{,}271.4 + 39.72 \times 7.92)^2 + (50\,\pi\,.0374 \times 39.72)^2}$$
$$= 63{,}586.0 \text{ volts,}$$

and the current therein is

$$I_2 = \sqrt{(39.72)^2 + (50\,\pi\; 63{,}586 \times .0000000803)^2} = 39.73 \text{ amps.}$$

Proceeding in like manner, the values of the *E.M.F.*'s and currents in each section may be determined, and the regulation then calculated.

NATURAL FREQUENCY. The inductance of the line is 0.374 henry and the capacity is 0.000000803 farad. From § 115, the natural frequency is

$$\frac{1}{4\sqrt{0.374 \times .000000803}} = 456 \text{ cycles.}$$

There is therefore no probability of trouble from harmonics at the chosen frequency of 25.

SAG OF CONDUCTOR. The conductors are to be strung with such a sag that at the minimum temperature with one-half inch of ice all around the cable, and a wind pressure of 15 pounds per square foot of projected area, the tension in the cable shall not exceed the elastic limit of the material (14,000 pounds per square inch).

Weight per foot of cable is $\frac{1100}{5280} = 0.208$ pound.

Area of conductor is 0.179 square inch.

Outside diameter of cable is 0.55 inch.

Since ice weighs 57 pounds per cubic foot, the weight of an ice coating one-half inch thick is

$$12\,\pi\,\frac{\left(\frac{1.55}{2}\right)^2 - \left(\frac{.55}{2}\right)^2}{1728} \times 57 = 0.652 \text{ pound per foot of cable.}$$

The weight of cable and ice is therefore 0.86 pound per foot of cable.

The wind pressure on the ice-covered cable is

$$\frac{1.55 \times 12}{144} \times 15 = 1.94 \text{ pounds per foot of cable.}$$

The resultant of weight and wind pressure is

$$\sqrt{(0.86)^2 + (1.94)^2} = 2.122 \text{ pounds per foot of cable.}$$

Assuming a span of 400 feet, then in the notation of § 125,

$$S_1 = 400.$$
$$W = 0.86.$$
$$W_r = 2.122.$$
$$A = 0.179.$$
$$T = 14{,}000 \times 0.179 = 2510.$$
$$t = 0,\ 75,\ \text{and } 150.$$
$$k = 0.0000128.$$
$$E_1 = 9{,}000{,}000.$$

Hence

$$L_s = 400 + \frac{(400)^3 \times (2.122)^2}{24\ (2510)^2} = 401.91,$$

$$L_u = \frac{401.91}{1 + \dfrac{2510}{9{,}000{,}000 \times 0.179}} = 401.28.$$

If $t = 0$, then

$$D^3 - \frac{3 \times 400}{8}[401.28 - 400]\, D = \frac{3 \times (400)^3 \times 401.28 \times 2.122}{64 \times 9{,}000{,}000 \times 0.179},$$

or $$D^3 - 192\, D = 1585.7$$

Solving by estimation and trial, the sag, D, is found to be 16.9 feet.

The vertical sag $D' = \frac{0.86}{2.122} \times 16.9 = 6.84$ feet.

If $t = 75$, there results

$$D^3 - \frac{3 \times 400}{8}[401.28\,(1 + 0.00096) - 400]\,D = 1585.7,$$

from which $D = 18.35$ feet.

The vertical sag $= \frac{0.86}{2.122} \times 18.35 = 7.43$ feet.

If $t = 150$, then

$$D^3 - \frac{3 \times 400}{8}[401.28\,(1 + 0.00192) - 400]\,D = 1585.7;$$

$$\therefore D = 19.69 \text{ feet.}$$

Vertical sag $= \frac{0.86}{2.122} \times 19.69 = 7.98$ feet.

If the minimum temperature is taken as $-40°$ F., $75°$ above the minimum is $35°$ F., and $150°$ above the minimum is $110°$ F. In Fig. 235 the vertical sags for spans of 400 feet, 500 feet, 600 feet, and 700 feet have been plotted in terms of temperatures between $-40°$ and $110°$ F. In stringing the cables, the proper sag to be allowed should be obtained from these curves, its value depending upon the temperature at that time.

Length of Standard Span. From the curves of Fig. 235, the lower curve of Fig. 236 has been drawn, showing the vertical sag at $110°$ F. for different span lengths. If the minimum clearance of the cables from the ground is to be 20 feet, the point of support of the cables must be at a distance from the ground equal to 20 feet plus the maximum sag. The distance of the point of support of the lowest cable from the ground is called, for

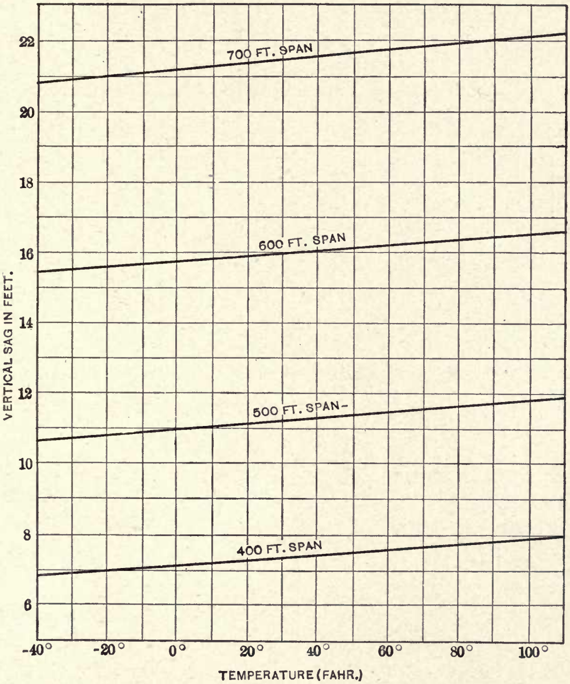


Fig. 235.

convenience, the height of the tower. The upper curve of Fig. 236 has been drawn with ordinates representing 20 feet more than those of the lower one, and therefore shows the heights of towers for different span lengths. Assume that 66,000-volt insulators cost $5.00 each erected, and that

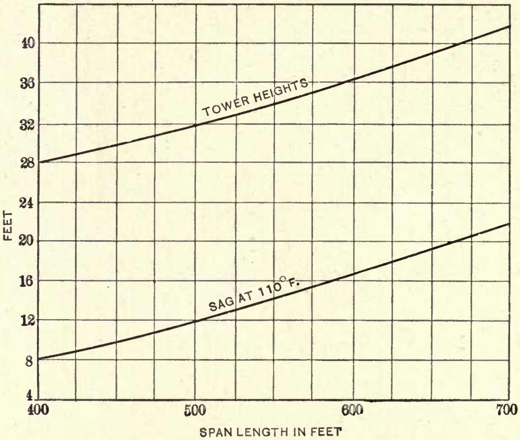


Fig. 236.

the costs of towers of various heights, erected, are as follows:

Tower Height in Feet.	Cost of Tower in Dollars.
30	95
32.5	100
35	110
37.5	125
40	145

From the upper curve of Fig. 236, it is seen that the greatest span length for which a 30-foot tower can be used under the given conditions is 450 feet. With this span length there will be required 11.73 towers per mile, and the cost of towers and insulators per mile of line will be

$$11.73 \times \$95 = \$1114.35 \text{ for towers.}$$
$$3 \times 11.73 \times \$5 = \underline{175.95} \text{ for insulators.}$$
$$\$1290.30 = \text{total cost per mile.}$$

For 32.5-foot towers the span is 515 feet, and 10.25 towers are required per mile. Then

$$10.25 \times \$100 = \$1025.00 \text{ for towers.}$$
$$3 \times 10.25 \times \$5 = \underline{153.75} \text{ for insulators.}$$
$$\$1178.75 = \text{total cost per mile.}$$

For 35-foot towers the span is 570 feet, and 9.26 towers are required per mile. Then

$$9.26 \times \$110 = \$1018.60 \text{ for towers.}$$
$$3 \times 9.26 \times \$5 = \underline{138.90} \text{ for insulators.}$$
$$\$1157.50 = \text{total cost per mile.}$$

For 37.5-foot towers the span is 615 feet, and 8.6 towers are required per mile. Then

$$8.6 \times \$125 = \$1075.00 \text{ for towers.}$$
$$3 \times 8.6 \times \$5 = \underline{129.00} \text{ for insulators.}$$
$$\$1204.00 = \text{total cost per mile.}$$

For 40-foot towers the span is 665 feet, and 7.94 towers are required per mile. Then

$$7.94 \times \$145 = \$1151.30 \text{ for towers.}$$
$$3 \times 7.94 \times \$5 = \underline{119.10} \text{ for insulators.}$$
$$\$1270.40 = \text{total cost per mile.}$$

It is evident from the foregoing calculations that the economic span is 570 feet, employing 35-foot towers.

FORCES ACTING ON THE TOWERS. For spans of 570 feet, the force acting on each tower due to the weight of line conductors when covered with ice will be

$$3 \times 570 \times 0.86 = 1470.6 \text{ pounds.}$$

The pressure due to the wind, being 15 pounds per square foot of projected cable area, when the cable is covered with one-half inch of ice all around, is

$$3 \times 570 \times 1.94 = 3317.4 \text{ pounds.}$$

The weights of towers vary considerably, depending upon their design. One ton may be taken as the average weight of a 35-foot tower.

A tower of the size under consideration will have the equivalent of about 25 square feet of normal surface exposed to the wind. Hence the wind pressure on the tower is $25 \times 30 = 750$ pounds. This acts at the center of gravity of the exposed surface, but for the purpose of calculation it is assumed that half this force, 375 pounds, acts at the top of the tower.

Therefore the tower must be strong enough to resist a force of $1470 + 2000 = 3470$ pounds acting vertically downward, and a force of $3317 + 375 = 3692$ pounds acting horizontally at the top of the tower.

LENGTH OF SPAN ON CURVES. Where the line is carried around a curve as shown in Fig. 232, the transverse force acting on the tower due to the tension in the cables should be allowed for by shortening the span length. If the angle α be 2°, the transverse force due to the tension in the cables (Fig. 233) is

$$3 \times 2 \times 2510 \times \sin 1° = 263.5 \text{ pounds.}$$

The transverse force due to wind pressure on the conductors is 3317.4 pounds in the standard span. Subtracting 263.5 therefrom leaves 3054 pounds as the desired wind pressure on the conductors per span on the curve. Hence the length of such spans should be

$$\frac{3054}{3317} \times 570 = 525 \text{ feet.}$$

If the angle α be 4°, then the transverse force due to the tension in the cables is 525.6 pounds. The span length for this value of α is

$$\frac{3317.4 - 525.6}{3317} \times 570 = 480 \text{ feet.}$$

Similarly, when

$\alpha =$ 6°, the span is 432 feet,
$\alpha =$ 8°, the span is 390 feet,
$\alpha =$ 10°, the span is 345 feet.

It is not advisable to have the angle α greater than 10°. If too many towers will then be required to make the necessary turn, it is better to make it as shown in Fig. 231, by dead-ending the line on two towers and having a short slack span between them, rather than by means of a curve.

PROBLEM.

Thirty thousand kilowatts are to be transmitted over a section of a transmission line 53 miles long, using a three-phase circuit of aluminum conductors, with 110,000 volts at the generating station. The various constants are:

Frequency = 25.
Cost of power per kilowatt-year at generating station = $12.00.
Cost of aluminum per pound = $0.25.
Interest rate thereon = 4 %.
Distance between cables = 9 feet.
(Fig. 230 shows the type of towers used.)

Determine the economic drop, cross-section of conductor, natural frequency of the line, and the charging current per conductor. Prepare curves showing the vertical sag at different temperatures for various span lengths.

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