# CHAOS THEORY

"It has been said that something as small as a flutter of a butterfly is wing can ultimately cause a typhoon half way around the world. "

Chaos: When the present determines the future, but the approximate present does not approximately determine the future.

A commonly used definition says that, for a dynamical system to be classified as chaotic, it must have the following properties:

I. It must be sensitive to initial conditions;
II. It must be topologically mixing; and
III. Its periodic orbits must be dense.

Minimum complexity of a chaotic system

The Poincaré–Bendixson theorem states that a two dimensional differential equation has very regular behavior. The Lorenz attractor discussed above is generated by a system of three differential equations with a total of seven terms on the right hand side, five of which are linear terms and two of which are quadratic (and therefore nonlinear). Another well-known chaotic attractor is generated by the Rossler equations with seven terms on the right hand side, only one of which is (quadratic) nonlinear. Sprott found a three dimensional system with just five terms on the right hand side, and with just one quadratic nonlinearity, which exhibits chaos for certain parameter values. Zhang and Heidel showed that, at least for dissipative and conservative quadratic systems, three dimensional quadratic systems with only three or four terms on the right hand side cannot exhibit chaotic behavior. The reason is, simply put, that solutions to such systems are asymptotic to a two dimensional surface and therefore solutions are well behaved.

## Distinguishing random from chaotic data

It can be difficult to tell from data whether a physical or other observed process is random or chaotic, because in practice no time series consists of pure 'signal.' There will always be some form of corrupting noise, even if it is present as round-off or truncation error. Thus any real time series, even if mostly deterministic, will contain some randomness.

All methods for distinguishing deterministic and stochastic processes rely on the fact that a deterministic system always evolves in the same way from a given starting point.

I. Pick a test state;
II. Search the time series for a similar or 'nearby' state; and
III. Compare their respective time evolutions.

Essentially, all measures of determinism taken from time series rely upon finding the closest states to a given 'test' state (e.g., correlation dimension, Lyapunov exponents, etc.). To define the state of a system one typically relies on phase space embedding methods.

When a non-linear deterministic system is attended by external fluctuations, its trajectories present serious and permanent distortions. Furthermore, the noise is amplified due to the inherent non-linearity and reveals totally new dynamical properties. Statistical tests attempting to separate noise from the deterministic skeleton or inversely isolate the deterministic part risk failure. Things become worse when the deterministic component is a non-linear feedback system.