THE ELEMENTS

OF

MOLECULAR MECHANICS

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PREFACE.

ABOUT three years ago I had the honour to lay before the Royal Society a paper on Molecular Mechanics, containing the outlines of the work which I now present to the public. The subject was considered one of great interest, but at the same time of such difficulty, that a scientific man of high reputation expressed his doubt of the possibility of carrying out the scheme in the present state of science. He was not then aware that I had already half carried out the scheme before I presented it to the Royal Society. As for myself, taught by experience that the greatest difficulties sooner or later yield to labour, and countenanced by the warm encouragements of distinguished men, who had taken a favourable view of the matter, I continued my work patiently, and succeeded at last in completing my ELEMENTS OF MOLECULAR MECHANICS.

What these Elements are may be learned in detail by a glance at the table of contents, or at the Introduction, which explains the nature, aim, and use of the work.

Here it suffices to say in general that my intention has been to open a path into a region of science hitherto deemed inaccessible, to the exploration of which scientific men of the present day are anxiously looking forward.

May I entertain the hope that my readers, considering the difficulty of the task before me, will receive this volume with indulgence, and be disposed to judge of it by the amount of useful matter it actually contains rather than by any deficiency in the development of the subject.

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J. BAYMA, S. J.

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THE ELEMENTS

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INTRODUCTION.

THE theory of universal gravitation, though so immense a discovery and so decisive a step towards a correct view of material agencies, has hitherto failed to give an explanation of molecular phenomena. Not, indeed, that such phenomena are not brought about by some kind of gravitation (for we know the contrary), but simply because molecular attraction has refused up to this day to reveal the secret of its nature, and the laws of its manifold causation. This I considered to be an unfortunate fact, and often put myself the question: Must we for ever remain ignorant of the laws of molecular action, and their relations, and the practical inferences from them? If not, may we begin at once, and try to find out at least a part of the secret? or must we wait till we are furnished with a greater supply of scientific materials?—To this I thought there was one answer only, that is, *We may try.* Accordingly, *I tried:* and found a fresh verification of the truth of the saying, that difficulties are not to be looked upon as a source of discouragement, but as a stimulus to exertion.

Thus this work is the result of a first endeavour towards ascertaining the laws of molecular action: an endeavour which, from the nature of the subject, might well have proved fruitless, but which in fact has resulted, if I am not mistaken, in the establishment of a body of principles, which may form the groundwork of a new branch of Science.

The first thing the reader will be prepared to look for in

opening this volume is, I imagine, *the fundamental hypothesis,* from which this new branch of science has sprung up. I think, I can truly answer with Newton: *Hypotheses non fingo. Quidquid enim ex phænomenis non deducitur hypothesis vocanda est: et hypotheses seu metaphysicæ, seu physicæ, seu qualitatum occultarum, seu mechanicæ, in philosophia experimentali locum non habent**. Reasoning based on assumption may have its advantages in many abstruse questions of natural philosophy: still there is no doubt that assumptions are made use of only for want of something better. On the other hand, it is evident that a work of such a nature as the present, which was intended to lay down the very first foundation of a molecular theory, could not possibly rest on hypothetical ground. Hence the reader will find no *fundamental hypotheses:* he will only find what I call 'principles' in the strict meaning of the word, i.e. truths about which I think that no reasonable doubt can be raised, and which accordingly may be safely employed as premisses for the demonstration of other less tangible truths. Of course, the 'principles' of natural philosophy, those at least which are not self-evident, are proved from the laws of nature, as the laws of nature are inferred from natural facts. Hence I have embodied in this work, as principles of molecular mechanics, those propositions only which are evident, or which I thought I had the power of rigorously demonstrating from known laws of nature.

So far, then, I have tried to follow Newton's steps. Still, as the line of thought which the mind follows in the investigation of scientific truth is not always the best to follow in its exposition, the reader will find in these pages scarcely a trace of the analytic process by which I was enabled to discover the truth of many of my propositions. For, upon careful examination, I acquired the conviction that the process which would best suit the purpose of showing clearly, briefly, and logically, what I had to show, was exactly the reverse of that which I had followed in my earlier investigations. Thus the form of exposition which I have adopted gives to the work that character of rigour which we are accustomed to find, almost exclusively, in mathematical treatises, though we should be glad to find it also in other scien-

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* *Principia,* Lib. III. Scholium generale.

tific writings. The work is divided into Books; and the Books into Propositions, Theorems, Problems, &c., according to the old style of geometrical writers. Thus each point of discussion comes distinctly before the reader, who is enabled to see at once what he has to deal with, and to concentrate his attention upon each separate proposition on which he wishes to form a judgment. This, in a book which gives the elements of a new branch of knowledge, was so great an advantage, that, to secure it, I did not hesitate to expose myself to the adverse criticism of those modern thinkers who despise the deductive method as a useless relic of the past. For myself, I am of quite a different opinion: I believe that deductive reasoning possesses among other advantages this one most especially, of being the proper test, or touchstone, of the inductive process, which then only I consider to be conclusive and unobjectionable, when its results, by an inversion of the process, can be transformed into legitimate deductions. But, whatever be the relative merits of the two processes, it cannot be denied, that a method of exposition, the difficulty of which discourages its adoption on the part of so many scientific writers, and which has been kept up only among those, who, like mathematicians, profess to prove rigorously what they assert, is a method calculated to give the reader more satisfaction and relish than might be drawn from simple induction, as it gives both the results of induction and the process by which those results are verified.

Of the twelve Books into which the present treatise is divided, the first and second give the demonstration of the principles which bear directly on the constitution and the properties of matter. The next three Books contain a series of theorems and of problems on the laws of motion of elementary substances. In the sixth and seventh the mechanical constitution of molecules is investigated and determined: and by it the general properties of bodies are explained. The eighth Book treats of luminiferous æther. The ninth explains some special properties of bodies. The tenth and eleventh contain a radical and lengthy investigation of chemical principles and relations, which may lead to practical results of high importance. The twelfth and last Book treats of molecular masses, distances, and powers.

Such is, in short, the substance of the whole treatise. It may be reduced to four principal points: I. Leading principles; II. Mathematical application of them; III. Their application to the constitution and physical properties of bodies; IV. Their application to chemical properties.

Among the leading principles of which I give the demonstration, the following may here deserve a special mention.

1. Bodies do not act by mathematical contact, however much our prejudices incline us to think the contrary.

2. There is no such thing existing as matter materially and mathematically continuous, that is to say, such that its parts touch each other with true and perfect contact; and therefore all bodies ultimately consist of simple, i.e. *unextended* elements, the sum of which constitutes the absolute mass of the body.

3. No other powers exist in the elements of matter, except locomotive or mechanical powers; so that we need have no anxiety about the *vires occultæ* of the ancients, nor need we make search after any other kind of *primitive* powers besides such as are mechanical. Hence chemical, electric, magnetic, calorific, and other such actions are all to be reduced to mechanical actions, complex indeed, but all following certain definite laws, and capable of being expressed by mathematical formulæ, as in general mechanics.

4. There are not only attractive, but also repulsive elements; and this is the reason why the molecules of bodies, as being made up of both sorts, can at certain distances attract, and at others repel each other.

5. Simple elements* cannot be at once attractive at greater and repulsive at less distances. If then a given element is attractive at any distance, it will be so at all distances: and if it be repulsive at any distance, it will be repulsive at all distances.

6. Simple elements must not be confounded with the 'atoms' of the chemists, nor with the molecules of which bodies are composed. Molecules are, according to their name, small extended masses, i.e. they imply volume. Elements are indivisible points

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* Throughout the work the word *simple* is used in the mathematical sense of *unextended*, not in any metaphysical sense.

without material extension. Again, molecules of every kind, even those of primitive bodies, are so many systems resulting from elements acting on each other; consequently, elements differ from molecules as parts differ from the whole; so that much may be said about separate elements which cannot be said of separate molecules or chemical 'atoms,' and *vice versa*. *Element, molecule, body,* have the same relation to each other in the physical order, that *individual, family, state,* bear to each other in the social order; for a body results from molecules, and molecules from elements holding together mechanically in a similar way to that in which a state results from families, and families from individuals bound together by social ties.

7. Simple elements have a sphere of activity, and throughout the whole sphere, even at molecular distances, act according to the inverse ratio of the square of the distances. This proposition, which is true of *elementary*, not of *molecular*, actions, not only does not contradict certain known laws, e.g. of cohesion, molecular reaction, chemical affinity, &c., but supplies the only means of accounting for these and other molecular relations. This truth is, as all must see, of the utmost importance, since it is the foundation of molecular analysis, of which it would be impossible to treat at all, unless the law of elementary actions at infinitesimal distances were known. This theorem universalizes Newton's law of celestial attraction by extending it to all *elementary* actions whether attractive or repulsive: and makes it applicable not only to telescopic but also to microscopic distances.

These and such like conclusions of capital importance I was bound to prove most irrefragably, lest my molecular mechanics should be raised on an insecure foundation. I therefore was not satisfied with direct and, as I think, peremptory physical proofs, but added a great number of *Scholia* calculated to dispel old popular and philosophical prejudices. It is not, indeed, the custom of scientific writers to answer metaphysical objections: but there are cases in which the opposite course seems to be preferable; and it is when the objection presents itself under the aspect of undeniable truth. In such cases, I think, we must solve the objection if we can. Accordingly, as far as the nature of this work permitted it, I have examined and solved a great number of

philosophical difficulties, which otherwise might have embarrassed some of my readers. On the other hand, it was necessary to take care that these polemical digressions should not interfere with the rest of the work: and for this reason I gave room to them in separate scholia to be printed in a smaller type, lest the reader should mistake them for the substance of the work, and engage unawares in the awful mysteries of philosophical speculation.

With regard to the mathematical part of the theory, I have little to say. After establishing in the third Book the law of motion for the most rudimentary systems of material elements, I begin to treat of those regular systems which may be assumed to play a part in the constitution of molecules. The molecules of *primitive* bodies, such as hydrogen, nitrogen, &c., cannot be supposed to be irregular in form: a conclusion which I prove in another place (Book VI. Prop. VI.). Consequently, while treating of primitive systems, I was entitled to confine myself to the consideration of regular polyhedrons.

Hence I divided these regular systems into different *classes*, according to their geometrical figure, as tetrahedric, octahedric, hexahedric, &c.

The several parts, of which any regular compound system of elements can consist, are reduced by me to a *centre*, *nuclei* to any number, and an external *envelope*. Thus I obtained not only a method of nomenclature for the different systems (a most important point), but also a method of exhibiting each system under brief and intelligible symbols. The reader by a glance at the table of *contents* (Book V.) will see what these symbols are.

I then subdivided the above classes of systems into different *species*, which I called *uninuclear, binuclear, trinuclear ... polynuclear*.

Lastly, besides classes and species, I pointed out the distinct *varieties* which may lie under each species. And by this means I completed the classification of primitive molecules.

As to the determination of the dynamical conditions of such elementary systems, I have nothing peculiar to say: only I am afraid, many will think that this part of the work is too extensive, though others might equally say that it is too limited. My own impression is that the subject deserved indeed a more

extensive mathematical development : but, on the one hand, it
was not, nor could it be, my intention to come forward with a
full and exhaustive treatise on a matter which now for the first
time was to be reduced to a scientific form : on the other hand,
I considered that a great number of students and other scientific
readers, who are less acquainted with mathematics, and to whom
these elements of molecular mechanics may be of some use in
their practical, no less than speculative investigations, would not
thank me for purposely spreading thorns and stumbling-blocks on
their path with too profuse a liberality. Hence, instead of making
additions, I suppressed many mathematical theorems and problems
which I had already prepared, and, moreover, strove to develope
the matters contained in the remaining Books as independently of
mathematical processes as I could. Thus it seems to me that
both those who are fond of mathematical working, and those who
are not, may feel satisfied : the first will find the mine fairly
opened and ready to be worked to whatever depth they choose :
the others will remain free to turn aside, or jump over to the
sixth Book directly and without any great inconvenience ; since,
even so, they will be able to understand enough and turn into use
everything that follows.

The third thing I had in view was to determine the constitu-
tion of molecules and the physical properties of bodies in general.
From what I show in the sixth Book, the definition of a molecule
of a primitive body, as hydrogen, would be this : *A molecule is
a system of simple elements, or material points, constituted by a
centre, a number of regular concentric polyhedric nuclei, and a
regular polyhedric repulsive envelope, all indissolubly bound with
one another by dynamical ties, and subject to a kind of palpitatory
motion by which they constantly contract and dilate with a sur-
prising rapidity.* All the parts of this definition are carefully
demonstrated in so many separate propositions.

The general constitution and properties of a molecule being
known, only a few additional considerations were wanted to lay
open the general constitution and properties of the body formed
of molecules. Accordingly, the conclusions presented in Book VII.
are scarcely anything more than corollaries of pre-established facts.
Yet these corollaries are of the greatest importance, as they give

for the first time the radical explanation of many points of physics which have always been treated with a remarkable incompleteness and want of accuracy. The reader will be amazed to hear, e.g. that *the action of a body A on a body B is not necessarily equal to the action of the body B on the body A*. He will wonder how the proposition affirming that *two heavenly bodies attract one another proportionally to their masses* is ambiguous, and may be as false in one sense as true in another. So also with regard to the force of inertia, of which men (from Newton inclusively down to us) are wont to speak very inaccurately, and which is not a new causality, but only a mode of exertion of the ordinary powers of matter, dependent on inertia not as a cause, but as a condition *sine qua non.* These and such like interesting topics are discussed in Book VII.

As to the special properties of bodies, they should have come immediately after the general, had it not been for the necessity of premising some considerations on luminiferous æther as a preparation for what I intended to say on the colour of bodies. The whole Book VIII. is on æther. I establish therein, that æther is a special substance, wholly attractive, unresisting, and elastic, but whose elasticity differs in kind from that of known ponderable fluids as negative from positive.

The explanation, which next comes, of the special properties of bodies, is made to depend on the curve of molecular actions. The idea, of course, is not original; but I may be allowed to say that, with the knowledge we have now gained of the constitution of molecules, the curve of actions seems to have more meaning and to be more suggestive than ever before. The reader will see also how I account for the change of liquids into vapours, and *vice versa*. I show that the obvious explanation of such a change is to be found in a *transposition* of ,molecular nuclei. The law of the calorific capacities of primitive bodies I have traced to its origin, and discovered that for the said bodies, and for equal weights of them, *the calorific capacities are directly proportional to the numbers of discrete molecules that are heated.* Thus the enunciation of the law takes a form exceedingly simple and intelligible, and the law itself leads us more directly to other consequences of great importance: it, namely, enables us to under-

stand very easily why in compound substances the law of calorific capacities ceases to be uniform, and why it so widely differs from that observed in primitive bodies.

The last point I had in view was to ascertain what use could be made of the principles and conclusions already established for the promotion and improvement of chemical science. I had, then, to find out in what the difference between physical and chemical action consists: to determine the causes of affinity: to show how intimate a relation exists between the geometrical figure of molecules and the number of combining equivalents: to examine in detail the numbers of equivalents that can combine with molecules of given forms: and lastly, to draw from this investigation general rules for binary combinations. This I have done in Book x., which, accordingly, contains the rules and principles by which chemists will be able to account for a number of facts hitherto wrapt up in a dark impenetrable veil of mystery. If I am not mistaken, the results of this investigation, which cost me a very great deal of labour, constitute a great step towards the foundation and establishment of a *rational* chemistry, which will ere long supply or suggest new means of fostering experimental discoveries. " A right method," says Dr Mayer, " is the most important condition for the successful prosecution of scientific inquiry;" and a right method in chemistry is the first fruit of a rational view of chemical relations.

What follows in Book xi. is a first application of the general rules to concrete examples. Of course, the evidence, by which the results of such an application are supported, is not sufficient of itself to exclude all possibility of doubt to the contrary. Not that its ground is hypothetical, but because we may suspect that the view we have taken of the subject is, as yet, too incomplete to become the *only* foundation of peremptory conclusions. But, though these conclusions must be considered as simply provisional until they are confirmed by some other kind of proof, I hope the reader will not fail to be struck by the fact that they afford a most natural and unexpected explanation of many chemical compounds, the complexity of which would seem hitherto to have defied the utmost efforts of scientific ingenuity. But let this be as it may : my object has been not so much to resolve these great

problems at once, as to call the attention of chemists to them : and I shall be happy, whatever be the result of their future investigations, to have contributed even by a simple suggestion to the development of so useful a branch of knowledge.

To conclude, a sound theory of molecular mechanics is, in a manner, a scientific necessity of the day. Competent men have ere now begun to feel this necessity: and though the difficulty of the task has prevented them from carrying out the arduous undertaking, yet some of them at least have not ceased to work in that direction, well knowing that the time is at hand when toil, skill, and perseverance cannot fail to meet with their due reward. Professor Tyndall's interesting *contributions to molecular physics* would suffice to bear me out in this assertion. I hope, then, that this work, in spite of its unavoidable imperfection, will be welcomed by all who take an interest in the promotion of science. The subject of which I have treated was of such a nature as to require *on my part* a great deal of mathematical and metaphysical labour: still such labour will be unnecessary on the part of my readers, who, accordingly, are not required to possess any great knowledge of either mathematics or metaphysics. The young student and the 'amateur,' no less than the natural philosopher and the accomplished professor, will, I am sure, understand and master the subject sufficiently to be able to form a correct idea of the soundness, importance, and practical use of molecular science.

As the employment of the geometrical method may have given to the work an air of dogmatism in questions regarding which there are great differences of opinion among philosophers, I beg to say once for all that I have merely stated my own views without pretending to render further discussion unnecessary.

BOOK I.

FUNDAMENTAL NOTIONS ON MATERIAL SUBSTANCE.

THE first basis of scientific inquiry is fact. Facts duly ascertained, well analyzed, carefully discussed and compared with one another, reveal the laws of nature, and unfold the divers properties of natural things. Such properties, though so numerous, complex, and various in kind, yet, as far as material substance is concerned, may ultimately be reduced to three only, viz. *motive power, mobility,* and *inertia.* This, of course, we must prove, if we wish to give the reader an exact idea of material substance. For, as the only means of determining the constitution of any given substance is supplied by a full consideration of its properties, we cannot give a correct and reliable view of material substance in general, unless we satisfactorily determine the properties without which no material substance can be conceived. This is the object to which the following propositions are devoted.

PROPOSITION I.

Every material substance is endowed with active power, passivity, and inertia, for causing, receiving and conserving local motion.

DEMONSTRATION. That which is wholly destitute of active power cannot make any impression on our senses, nor show its own existence and properties; and, consequently, no one can know what it is, or even whether it exists at all. But we all (without even excepting the idealist, who by his practice refutes his own theory) know that material substance exists, and, to a certain extent at least, we know what it is. Therefore material substance is endowed with active power. As to the fact, that the exertion of such a power is naturally connected with local motion, it is too

well known from experience and observation, to need any special proof.

Again, that which is without passivity is not capable of receiving motion; for *passivity* means capability of being acted upon and of being determined to motion. Now, all matter can be acted upon and be determined to motion; since we see that everything in this material world can receive local motion. Therefore, material substance has a passivity, on account of which it can receive local motion.

Thirdly, inertia is the incapability of changing its own state: so that an inert substance, if it be determined to motion of any intensity and direction, must move with that intensity and in that direction until it receive some other determination from without; in the same manner, if it be at rest, it must remain at rest until it receive a determination to motion from without. Now, this is the case with material substance, as a universal and constant experience proves. Therefore, material substance is inert. Q. E. D.

COROLLARY I. Therefore there is something in material substance, on account of which it possesses the power of imparting motion to other material substances. This is ordinarily called the " principle of activity" (*virtus activa*): but inasmuch as it is a constituent part of substance, it is called by philosophers the *substantial act* or the *substantial form*.

COROLLARY II. Therefore there is something in material substance, on account of which it can receive motion of any intensity and direction: and this is wont to be called the " principle of passivity;" but inasmuch as it is a constituent part of substance, is called by philosophers *the matter*, or the *potentia passiva*.

COROLLARY III. Therefore, in material substance, the form and the matter are so related to each other, that the matter, although capable of receiving the action of any exterior cause, cannot receive any action which proceeds from its own intrinsic principle of activity. For, if it could receive it, matter would move of itself, and cease to be inert.

SCHOLIUM. Ordinarily the word "matter" signifies material substance: but amongst philosophers material substance is that in which

one of the constituents is *the* matter. Physicists too take the word "matter" in the sense of one of the constituents of material substance, whenever they distinguish *the* matter from the active power of matter. It is very desirable that this distinction, which is of great importance for guarding against the sophisms of the ignorant, should be clearly pointed out by all writers on this subject. Then all would style that *the matter* on account of which material substance can receive the determination to motion, and would style that *the power* on account of which material substance can give the determination to motion; finally, they would call that *material substance* which can both give and receive the determination to motion. This method of speaking is very reasonable, because it distinguishes between those things which cannot be confounded without danger, and does away with ambiguous expressions: in addition to this it opens the way for understanding those primary principles of the ancients : *Omne agens agit in quantum est in actu: Omne patiens patitur in quantum est in potentia : Quo aliquid est, eo agit : Forma est id quo agens agit,* etc. All these principles are very philosophical and absolutely true, as is clear from the above corollaries; although the ancients, limited as they were in experimental knowledge, had the misfortune of being often deceived in the application of these same principles to particular cases.

PROPOSITION II.

No natural cause can communicate a finite velocity to a body in an infinitesimal unit of time.

DEMONSTRATION. If any natural cause could communicate a finite velocity to a body in an infinitesimal unit of time dt, it evidently could communicate to it an infinite velocity in any finite time. For, let v be the intensity of the action of that cause for a finite unit of time ; the intensity of the action in an infinitesimal unit dt will be vdt. If, therefore, vdt were of a finite intensity in order to produce finite velocity, then v itself must be of an infinite intensity. But it is clear that no natural cause exerts in a finite unit of time an action of an infinite intensity ; for we never see, nay we cannot even conceive a motion of infinite velocity as possible. Therefore, every action v of any natural cause whatever is of finite intensity : and consequently every action vdt is of an infinitesimal intensity. But an action of infinitesimal intensity

cannot communicate to a body a finite velocity. Therefore, no natural cause can communicate finite velocity to a body in an infinitesimal unit of time.   Q. E. D.

COROLLARY.   Therefore, rigorously speaking, there is no *force* in nature, which causes motion instantaneously : and if sometimes a force is said to produce motion instantaneously, it is only because the finite time, in which motion is produced, is so short that it cannot be determined by observation.

SCHOLIUM.   That which physicists call *Force* is strictly nothing else than the intensity of the action measured by the intensity or the quantity of the motion which in the given circumstances it is capable of communicating.   For this reason the more accurate physicists do not say that *forces* are continuous or instantaneous, but that the *action* is continuous or instantaneous, according as it has a duration mensurable or not.   As for the active power itself, it is always continuous, not only because it lasts as long as the substance of which it is a constituent, but also because it has a natural and intrinsic determination to act, and cannot suspend its action so long as there is a moveable body within its reach.

PROPOSITION III.

In the impact of bodies no communication of finite motion can be made by means of a true and immediate contact of matter with matter.

DEMONSTRATION.   The true and immediate contact of matter with matter would take place in the indivisible instant in which the distance between the points which are coming into contact becomes $= 0$.   But finite velocity cannot be communicated in an indivisible instant, as we have seen (Prop. II.).   Therefore, there can be no communication of finite velocity by means of a true and immediate contact of matter with matter.   Q. E. D.

Nor can the demonstration be evaded by having recourse to the *multitude* of points among which the contact would be supposed to take place.   For, since each individual contact does not contain the causality of each partial effect, it is evident that the multitude of such contacts cannot contain the causality of the total effect.   And, in fact, if each individual point of matter only acquires an infinitesimal velocity vdt, the whole multitude will

acquire only an infinitesimal velocity: that is, there will be no motion caused at all.

Nor can it be said that the motion is communicated by means of a *prolonged* contact. A prolonged contact is impossible, unless the velocities have become equal at the very commencement of the contact. Therefore, if velocity were communicated by the contact of matter with matter, it would have to be communicated in the very first instant of the contact, not in its prolongation.

Nor can we have recourse to the *elasticity* of bodies; because elasticity is the power of reacting after compression, and cannot be exercised till after the action, which works a change on the body, has taken place: and, therefore, if the body is not changed by an instantaneous contact, there can be no reaction owing to elasticity.

Nor can we even, with some writers, have recourse to the action of an ætherial substance interposed between the agent and the object acted upon; both because there is nothing interposed between two things which are supposed to touch each other truly and immediately by material contact, as also because our argument, which is altogether universal, applies to æther no less than to other material agents: hence, even if there were æther placed between, and the contact made through it, still any communication of finite velocity by means of an immediate contact of matter with matter would be absolutely impossible.

COROLLARY I. Therefore, distance is a necessary condition of the action of matter upon matter.

COROLLARY II. Therefore, the contact between the agent and the object acted upon is not material but *virtual*, inasmuch as it is by its active power (*virtus*), not by its matter, that the agent reaches the matter of the object acted upon.

COROLLARY III. Therefore, a material substance which is anywhere by reason of its matter, has within itself a power prepared to act where the substance itself is not present by its matter. What *the* matter is, as distinguished from the power of acting, has been explained above (Prop. I. Cor. II.), and will be more clearly understood from what we shall say hereafter.

SCHOLIUM. Boscovich proves this same proposition by the law of continuity in his theory of natural philosophy, part I. n. 18. *Concipiantur,* he says, *duo corpora æqualia, quæ moveantur in directum versus eamdem plagam : et id, quod præcedit, habeat gradus velocitatis* 6, *id vero, quod ipsum persequitur, gradus* 12. *Si hoc posterius cum sua illa velocitate illæsa deveniat ad immediatum contactum cum illo priore, oportebit utique, ut ipso momento temporis, quo ad contactum devenerint, illud posterius minuat velocitatem suam, et illud prius suam augeat, utrumque per saltum, abeunte hoc a* 12 *ad* 9, *illo a* 6 *ad* 9, *sine ullo transitu per gradus intermedios.* And hence he justly concludes that no communication of motion can take place, unless there be actually a distance between the bodies impinging and impinged upon.

To those who somehow or other retain the prejudices of infancy, our proposition with its corollaries may appear absurd indeed. But this comes only from want of reflection : and, if they admit attraction and repulsion, it is very easy to prove that they are not consistent with themselves, unless they admit also our proposition with its corollaries. And, with regard to attraction; when a stone is thrown up into the air, we see that its velocity is gradually destroyed, and replaced by a contrary and increasing velocity causing the fall of the body : and the same thing takes place in an oscillating pendulum. Now, no cause can be conceived which, by means of an immediate contact of matter with matter, produces such an *increasing* velocity. Moreover, attraction tends to bring bodies nearer to one another; and no body which touches immediately and materially another body can be brought nearer to it. We must, therefore, conclude that attraction admits of no material contact, and proves the existence of a natural power residing in matter, which is ready to act there where the substance of the agent is not present with its own matter. Now, with regard to repulsion. When a jar is full of gas, suppose hydrogen, an outward pressure is exerted against the sides of the jar, whether the jar be large or small, as compared with the mass of the fluid. This too cannot be explained by any immediate contact of the molecules of the fluid; for these molecules can become nearer than they are for greater and greater pressures. Consequently, the action by which the molecules repel one another is not exerted by means of an immediate contact of matter with matter, but only, as the ancients would say, *in contactu virtutis,* with intervening distance. These two examples show how really true it is that the power, with which matter is endowed, acts independently of any material contact, whatever vulgar prejudices may urge to the contrary.

With regard to our third corollary, Prof. Faraday expresses it in the following terms : "Each atom extends, so to say, throughout the whole of the solar system (*at least*), yet always retaining its own centre of force*." Many other illustrious physicists have held this same opinion, and many more hold it in our own time. Metaphysicians, however, object, that *No cause can act where it is not.* We answer, that this objection is based on a false supposition, which metaphysicians ought to be able to get rid of. Material substance has indeed a *formal* ubication in space by reason of its matter, from which it directs its exertion, but not by reason of its active power. This is evident by the fact that distances are always measured from the matter to the matter, and never from the matter to the active power, nor *vice versâ.* The matter alone marks out a point in space, and from this point we can take the direction of the action. But the power, as distinguished from the matter, is not a point in space, and does not mark out a point in space. Hence it is that, in speaking of material substance, it may indeed be said, that it has a formal ubication *from which* it directs its action, but it is absurd to seek the formal ubication *in which* the active power is situated. Such a power is not capable of situation, and is not confined to what we call place, but is altogether above place : and for this very reason also it cannot be placed under geometric dimensions, nor does it come under any kind of figure whatever, as every one knows. Hence also it is, that, in the movement of a material thing, the direction indeed, which extends from the matter to the matter, can be traced in space, while nothing similar can be said of the intensity, which proceeds from the active power of matter. We cannot in this place explain all these things at greater length ; for we do not write especially for metaphysicians, but for physicists : still what we have said is sufficient, we trust, to show that the aforesaid objection is only grounded on a false supposition. A further answer will be found below, Book II. Prop. VIII.

PROPOSITION IV.

An increase or decrease of intensity in motion is always due to a real production or extinction of velocity.

DEMONSTRATION. If the velocity acquired by a body is not a velocity preexisting in other bodies, then motion is in a strict

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* *A Speculation touching Electric Conduction and the Nature of Matter.* Phil. Mag. 1844, Vol. XXIV. p. 136.

sense produced; and if the velocity lost by a body totally ceases to exist in the world, then motion is in a strict sense extinguished. Now, this is always the case whenever a body receives an increase or a decrease of velocity. And, in fact, since motion cannot be communicated by means of a true and immediate contact of matter with matter (Prop. III.), the velocity of a body A will not pass into another body B either totally or partially, unless it be possible for it to leave the body A, to which it belongs, and traverse an interval of space between A and B. Now this process is utterly impossible and absurd. Velocity is a mode of being, and a mode of being cannot leave the subject of which it is a mode: *a fortiori* it cannot travel out of any subject whatever. Consequently, the velocity of the body A cannot be identically transmitted to the body B. Therefore, the velocity acquired by the body B is not the velocity pre-existing in the body A, but a velocity really produced by A acting upon B.

To prove the real extinction of velocity, a few more words will suffice. A decrease in motion does not take place of itself, but only by the exertion of a power which tends to communicate motion in an opposite direction. Now, the exertion of power produces velocity, as stated: and, if the velocity produced by a body A happens to be opposite to the velocity existing in another body B, the motion of B must totally or partially cease to be. This inference is evident; for, the same body B cannot move at one and the same time in two opposite directions, and, therefore, the two opposite velocities must actually destroy each other, it being impossible to have two actual velocities without two actual motions. Therefore, an increase or a decrease of intensity in motion is always due to a real production or extinction of velocity. Q. E. D.

COROLLARY I. Therefore, every acceleration, retardation and extinction of motion is the effect of actual production of velocity, not the effect of a velocity preexisting in any body whatever.

COROLLARY II. Therefore motion is not indestructible.

COROLLARY III. Therefore material substances act as efficient causes not by exerting their velocities (an expression unheard of,

though unfortunately the opinion which it expresses is by no means uncommon), but by exerting their active powers. Velocity itself is not the efficient cause or the causality, but simply the variable condition on the existence of which the various modes of application of the active power depend. And, accordingly, a body is in no need of velocity to be able to attract or to repel; to do this, it is sufficiently enabled by its natural powers; and, therefore, simple attraction or repulsion does not imply velocity as a necessary condition: but an immense number of other phenomena, those especially which imply vibrations, and those which can be reduced to the theory of impact, depend upon velocity as an essential condition, as we shall explain further on.

SCHOLIUM. The above proved proposition might be illustrated and confirmed by other reasons, which I omit for the sake of brevity. Still, I am obliged to dwell a little more on this same subject, to show what may be answered to those philosophers who have asserted that *Motion is the product of motion, and of motion alone.* These philosophers think that they are able to explain natural facts, even though all active powers, properly so called, be taken away from material substance. Their arguments may be reduced to the following allegations.

I. There is an old axiom *Nihil movet nisi motum*, meaning that a body which is at rest has no power of imparting motion. This axiom seems to be confirmed by observation and science; for, that which experience hitherto has shown as *force* in inorganic matter, is *mass animated by velocity:* and a force, which is not reducible to such terms, seems to be a product of mere·imagination.

This first argument has no weight at all. The assertion *Nihil movet nisi motum*, as a general proposition, lost the dignity of an axiom the very moment the fact of universal attraction was satisfactorily established. There have been philosophers, as far back as two centuries ago (Suarez, for example, in his great metaphysical work), who already rejected such an axiom: and to refute it peremptorily a scientific glance at the oscillation of a pendulum would now suffice. For, where is there a body to be found that by its motion communicates motion to the pendulum? Old philosophers, who, as we have said, were ignorant of universal attraction, in speaking of the cause of motion referred only to that causation of motion which was observed to take place in the impact of bodies; they had by no means the intention of applying the

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same theory to the falling of bodies; since they considered gravity as a quality proceeding from the natural constitution of the body itself; nor had they the least intention of applying the same theory to the motion of heavenly bodies; since for these they had provided already by assuming that astronomical revolutions were the work of intellectual beings. So, then, the axiom in question was necessarily limited by them to the causality of motion through impact. What was, then, the meaning of the axiom? That no impact is possible without motion: and that motion is, for this very reason, a necessary condition for every communication of motion due to impact. The old saying, therefore, holds good only as far as impact is concerned: and yet we are not entitled to conclude that motion, even in the case of impact, is the "efficient cause" of motion.

Still less can we conclude that all power of matter is "mass animated by velocity." This conclusion is wrong not only in the theory of gravitation, but even in that of impact, from which it has been deduced without sufficient consideration. Let us take a mass m animated by a velocity $2u$ and impinging directly on a body having an equal mass m, which is *at rest.* After the impact, both will move with a velocity u. Now, where is here the "mass animated by velocity" that diminishes the velocity of the impinging body? For, it must be remembered that, just as velocity cannot be increased, so also it cannot be diminished, except by exertion of power. Now, the mass which causes a diminution of velocity in the impinging body is at rest. It is clear, therefore, that a mass can act, even if it be "not animated by velocity." As for those philosophers who teach the contrary, it is evident that they mistook the quantity of motion (which is the adopted measure of the intensity of the action) for the action itself. Such a mistake, as remarked by M. Hirn*, implies nothing less than the assumption of the absolute identity of the effect with its cause: an assumption which is too openly untrue to need further refutation.

II. A second argument, in which many a modern philosopher has been entangled, tends to prove that material substance cannot be active. The argument is this: Activity is not consistent with inertia. Matter is inert. Therefore matter is not active.

The answer is very simple. The inertia, as admitted by all physicists, does not exclude all active power, but only such an active power as would make matter capable of imparting motion to itself. As long,

* *Exposition Analytique et Expérimentale de la Théorie de la Chaleur.* Paris, 1862.

then, as a particle of matter is incapable of imparting motion to itself, it is inert, whatever be its power of imparting motion to other particles.

III. A third argument is drawn from the principle of conservation of *vis viva*. This principle, according to some philosophers, would imply that there is always in the world the same amount of motion. If, then, the quantity of motion is constant, motion is neither extinguished nor produced. Whence they further infer, that motion must pass identically from one body to another, almost, we would say, as wine passes from the bottle to the glass.

We think that nothing like these last inferences really follows from the principle of conservation of *vis viva*. The *vis viva* of a system of bodies is not an arithmetical, but an algebraical sum of the *vires vivæ* of those bodies : and, since this algebraical sum can remain unaltered whether motion can be extinguished or not, no argument can possibly be drawn from the conservation of *vis viva* to prove that motion is neither extinguished nor produced. Let us conceive two bodies having equal masses m, and equal, but opposite, velocities u and $-u$: the sum of the *vires vivæ* both before and after the impact will be

$$mu^2 - mu^2 = 0 \; ;$$

and yet before the impact they had a velocity of translation, which after the impact has disappeared. Is it not obvious, then, that the *vis viva* is not the measure of the motion actually existing in a system of bodies ?

Some will say, that the translatory motion has not disappeared, but has only been transformed into vibratory, and is to be found, even after the impact, in the very molecules of the two bodies. We reply, that, true though it is that after the impact there must be a greater amount of vibratory motion in the molecules, nevertheless this increase of motion is by no means the result of any distribution or appropriation of the preexisting velocities, but an effect of mutual action, by which the molecules strive to restore their equilibrium which has been disturbed. This is a matter of fact. And indeed, in vibratory motion each vibration has a limited amplitude, and for each vibrating particle the velocity at the limit of the excursion changes sign, and consequently becomes $= 0$. Therefore, the velocity of the vibrating particles is a velocity which is alternately extinguished and reproduced : or, in other words, is not the preexisting velocity, as the objection assumes. Hence it sufficiently appears that the principle of conservation of *vis viva* affords no ground to prove the indestructibility of motion.

IV.   A fourth argument, extensively circulated among speculative philosophers, is drawn from the supposed impossibility of *actio in distans.*

We have proved (Prop. III. Corol. II.) that *actio in distans,* far from being an impossibility, is the only action of which material substance is capable.   Those who wish to prove the impossibility of such an action, start from a false supposition which radically vitiates their argument ; viz. that the active power is one of the two termini between which distance is to be measured. This we have shown above (Prop. III. Schol.).

V.   The fifth reason consists in the remark that no one can tell what active power is.

This reason is most futile.   Let us suppose for a moment that no one can tell what active power is; would it be lawful to deny active power?   We do not know what the causality is, on which a given effect depends ; are we bound to conclude that such a causality has no existence ?   The writer, who makes the present objection, admits the existence of matter : and yet is he able to say "what matter is" ?   Most certainly not ; for, since he conceives material substance as totally deprived of power, he takes away, in fact, that by which alone matter can act on his senses and show its existence and properties.   If, then, his argument had any strength, it would avail only against him as against every one who, like him, suppresses active power.   As for us, we dare to say we know what active power is ; and consequently we know also what *the* matter is, since the one cannot be conceived without reference to the other.   Philosophers have taught for centuries that the active power of a substance is nothing else but its substantial act, viz. one of its own essential constituents : *quo res est, eo agit.* This doctrine, for all who know the first principles of real philosophy, is a tangible truth : it can be ignored, but has never been shaken.   It is not exact, then, that "no one can tell what active power is."   We do not intend, of course, to write a dissertation on the notion in question.   We only state that those who assert mankind to be ignorant of what power is, wrongly argue from the singular to the universal.

VI.   A sixth reason, alleged by some writers, consists in affirming that active forces are mere abstractions, to which we give an undue reality.

It is astonishing how easily scientific men have allowed themselves to be subdued by this little sophism.   Surely, to judge after this test alone, philosophy must have sunk very low in our own times.   It is a

fact that this utterance *There are no abstract forces,* has become quite fashionable: and yet no one knows why our scientific men should insist on this point. No one is so ignorant as to deny that abstractions, as such, do not exist, except in the intellect. But many speak as if they were totally ignorant that what we conceive abstractedly exists concretely in the object from which it is abstracted. *Humanity* is an abstraction; and yet is to be found in every living man: *man* also is an abstraction; and yet is to be found in all of our contemporaries: *quantity* is an abstraction; and nevertheless is to be found not only in abstract geometry, as might have been anticipated, but in all real masses and volumes: *velocity* is an abstraction; and yet accompanies real motion. Hence abstract things and abstract forces do not cease to be real in nature, though they are abstract in our conception. These are commonplace remarks. It is deplorable that we should be obliged to make them. We boast of our extensive knowledge: but posterity will laugh at our inexplicable ignorance. Yet the most striking feature of the case is the open self-contradiction of these new philosophers. The very same men, who banish forces from this real world on account of their being abstractions, are so blind as to build their system of mechanics on motion and velocity: as if velocity and motion were not abstractions no less to be relegated, according to their principle, out of the real world. And these remarks will suffice to show the degree of philosophical accuracy employed by many a modern celebrity in writing scientific works.

VII. A seventh objection is, that active powers are neither spirit nor matter; whence it is concluded that they are nothing at all.

We could say as well: Velocity, motion, time, etc. are neither spirit nor matter: if, however, they were nothing at all, mechanical treatises would be theories based on nothing and treating of nothing: for, in fact, they are based on the consideration of nothing but velocity, time and motion, which would be nothing. The truth is, that substance alone, and not its attributes and properties, is capable of being divided into spirit and matter; and yet there are things which are spiritual, without being spirits (as the faculty of reasoning and willing), and things which are material, without being matter, as the activity and the passivity of material substance. And so, we are satisfied that a power is to be called spiritual when it resides in a spirit, material when it belongs to matter: and that a power may consequently be a reality either spiritual or material without being either spirit or matter, provided it be a real constituent either of the one or of the other.

VIII.   The eighth and last reason which is adduced to prove the inadmissibility of active powers capable of producing and extinguishing motion is drawn from the absurdity inherent in an uninterrupted creation and annihilation of motion.   Motion must have been created with matter, and be indestructible: motion is the only active principle of matter.

This reasoning is grounded on the assumption that the production of motion is a *creation*, and the extinction of motion an *annihilation*. Now, no supposition can be more false than this.   No accident is, or can be, created, because it is the property of substance alone to come out of nothing.   Motion is an accident.   Therefore motion is not created, and cannot be created.   And the same reasoning holds good with regard to annihilation.   It is most true, then, that an uninterrupted creation and annihilation of motion is absurd, as the objection says: but it by no means follows that production and extinction of motion is also absurd.   This last assertion would be not only gratuitous, but in flagrant opposition to the evidence afforded by all natural facts.   We conclude that the proposition "Motion must have been created with matter" is not a suggestion of reason, but the consequence of an abuse of words: also that the assertion "Motion is indestructible" cannot be admitted in natural philosophy, as being contrary to fact: lastly, that the assumption "Motion is the only active principle of matter" is condemned by reason, inasmuch as reason shows unmistakeably that activity and motion are two most distinct and opposite things.

The Author who makes use of this last argument seems to have taken his notion of the principle of activity and its indestructibility from the *Philosophical Magazine*, Vol. xxv. p. 378, and Vol. xxvii. pp. 81, 82, where we find the passages, which he quotes, of the German Dr. Mayer, and of the English Mr. Graham, Master of the Mint, who both hold motion to be indestructible.   We dare to say that these two men are no authority.   Surely, they make wondrous assertions; but the more wondrous the assertions, the more we are entitled to be favoured with reasons in their support.   Now, Mr. Graham asserts simply what he cannot prove; and, in fact, he does not pretend to have proved his assumption: whilst, on the other hand, Dr. Mayer makes indeed a great show of reasoning, which, however, when carefully analysed, has but one merit, viz. that of holding the inattentive reader under the false impression that the Doctor has demonstrated what he has only reasserted in other words, without the least exhibition of real proof.   Of course, Dr. Mayer being a German, we might not im-

prudently suspect his reasoning to be of the same kind with that of many of his fellow-countrymen, who became pantheists. A sprinkling of pantheistic doctrine, for the time we live in, is a kind of recommendation and entitles to celebrity. "Forces," says he, "are different forms, under which *one and the same object* makes its appearance*." And a little before he had reduced all being to *Deity* and *phenomena*, by a thorough exclusion or elimination of all created causes. "The exact sciences," says he, "are concerned with phenomena and measurable quantities. The first cause of things is Deity, a being ever inscrutable by the intellect of man; while higher causes, supersensuous forces, and the rest, with all their consequences, belong to the delusive middle region of naturalistic philosophy and mysticism †." To refute these philosophical errors would require a long digression which would scarcely harmonize with the character of the present work: the important thing for us is to state only that these errors are at the bottom of every dynamical system which denies the existence of active power, properly so called, in material things.

PROPOSITION V.

Material substance acts, cœteris paribus, with different intensity on bodies placed at different distances.

DEMONSTRATION. The observation of natural facts, physical, chemical, astronomical, electrical, &c. affords a permanent proof of this proposition, which is found true whether the distances in question be great or small, whether astronomical or molecular. It is superfluous to dwell any longer on this point; for among men of science there is not the least doubt on the subject.

PROPOSITION VI.

Material substance is not prevented by intervening bodies from acting on other bodies placed at a greater distance‡.

DEMONSTRATION. This proposition asserts a matter of fact. For example, a mass m is equally attracted by the earth whether

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* *Phil. Mag.* Vol. XXV. p. 515.   † *Ibid.* p. 511.

‡ This proposition may appear somewhat ludicrous to those who are wont to consider the intervening matter as a medium for transmitting action. On this question, see what we say Book II. Prop. IX.

it be alone or hidden in the middle of a solid mass M. In like manner, a mass m which has a weight mg undergoes no diminution of weight from any interposition, between it and the earth, of another mass m'; though this second mass is also attracted by the earth, as is evident by its weight $m'g$. The earth, accordingly, multiplies its action exactly as the material particles are multiplied in the objects all around it : which truth is commonly expressed by saying that the action of the earth on an object placed at a given distance, however much or little of matter be interposed, is always proportional to the mass acted upon. Hence no material object placed nearer the earth can prevent its active power from exerting itself on other material objects placed at a greater distance : and, consequently, intervening matter does not interfere with the exertion of power on more distant matter, or, in other words, the active power of material substance is by no means obstructed by intervening matter. Q. E. D.

COROLLARY I. Therefore the active power of material substance is not subject to diminution, debilitation or exhaustion, however much it be exerted.

COROLLARY II. Therefore a power, which is actually applied to a given moveable body, is still applicable to any other, even at the same instant.

COROLLARY III. Therefore the multiplicity of the effects does not necessarily imply a multitude of powers, but only a multiplicity of applications of one permanent power. So that if, in this very instant, two or three or ten new planets were created, the sun would not be in need of receiving new powers to be able to attract them all; nay, the very same power already possessed by the sun would be applied to these new bodies without any loss whatever of gravitation being sustained by the now existing planets. In like manner, if a hundred millions of drops of rain were now created, the earth would apply its attractive power to each and all of them without any loss of acceleration for the other millions of drops which may be actually falling.

SCHOLIUM. No cause is weakened precisely on account of its natural exertion and production of any number of effects, but only by

reason of the circumstance, that the cause, whilst acting, is exposed to reaction, and reaction can be such as will alter and impair the nature of the substance. Now a substance can suffer alteration and corruption then only, when it is a result of material composition: and a cause can be impaired then only, when its causality lies in a complex power resultant from the coexistence and concurrence of a multitude of simple powers. In this case, if the composition is changed, the resultant will be changed, though the simple powers remain unchanged. We shall prove presently that bodies and their molecules are made up of simple elements, or simple substances having a simple power: this power cannot be weakened or impaired. In combustion, for instance, wood loses its organic structure, and the substance of wood is resolved into carbon, hydrogen, etc.: but the first elements lose nothing at all, and still remain in carbon, hydrogen, etc. what they were in wood, without the least degree of substantial alteration. Wood is burnt, its elements are not burnt. So then, primitive or elementary substance is never weakened. This fact might be confirmed by the remark, that nothing is impaired but by its contrary. Now, the active power of primitive substance has no contrary in nature. For the only thing in nature, which might appear contrary to active power, would be an opposite power. Now, powers do not act on one another, but only on *the matter* of each respectively; which is evident from reason as well as from fact. From reason, because *the matter* alone, as such, is passive: from fact, because the exertion of power gives rise to motion alone: and every one knows that motion is not destruction of active power. Consequently, though matter subjected to opposite actions can remain at rest, i. e. though two opposite actions received in the same subject can be such as to neutralize each other, nevertheless the active powers have not the same opposition to each other, cannot be exhausted by any amount of exertion, and, though finite, are applicable indefinitely to the production of motion. This much I wished to state in order to show the ambiguity inherent in the terminology of those physicists, who use the words *action, power, force* as synonymous. Actions can and do destroy one another; powers never do, since they never can; forces can, or cannot, according as they are meant to express measures of action or of power.

Proposition VII.

The hypothesis that bodies are made up of particles materially continuous, viz. intrinsically extended and filling with their matter

all the space occupied by them, leads to an absolute impossibility of communication of motion.

DEMONSTRATION. Let us imagine two such particles or globules of continuous matter, and suppose that the one impinges on the other. Whatever the circumstances and the conditions of the impact, one thing is sure, viz. motion will not be communicated by means of a true and immediate contact of matter with matter (Prop. III.). It remains, then, that motion be communicated by virtual contact, i. e. by the contact of power with matter, as explained above, according to some law dependent on the distance of the two particles. Now, the matter of these particles being extended, by the hypothesis, it is evident that different points of one particle must be at different distances from any given point of the other; hence such different points will be acted upon in a different manner and acquire different velocities (Prop. V.).

Now, this being the case, since it is certain that one and the same piece of matter cannot move onward with different velocities, let us inquire whether there be a possibility of reducing all the different velocities, which are imparted to different points, to a mean one, which would be the common velocity of the mass acted upon. A little reflexion will enable us to understand that such a reduction is by no means possible. In a piece of continuous matter, any point which can be designated is so *invariably* united with the other points, that no impact and no mutual reaction is conceivable : the obvious consequence of which is that no work can be done within the continuous particle in order to equalize the unequal velocities impressed from without. Moreover, in our case the reduction ought to be rigorously instantaneous : which is another impossibility. In fact, if distinct points of a continuous piece of matter were for any short duration of time animated by different velocities, the continuum would evidently undergo immediate and unavoidable resolution; which is against the hypothesis. Since, then, the said reduction cannot be made instantaneously, as we have proved above (Prop. II.), nor indeed in any other way, and on the other hand our continuous particle cannot move onward before the different velocities are reduced to one of mean intensity, it is quite evident that the same continuous particle will never be capable of moving, whatever be the con-

ditions of the impact. And since what is true of one particle on account of its supposed continuity, is true also of each of the other particles equally continuous, we must conclude that bodies made up of particles materially continuous are totally incapable of receiving any communication of motion*. Q. E. D.

COROLLARY I. Therefore, all bodies being moveable, no body is made up of particles materially continuous, however small they might be conceived.

COROLLARY II. Therefore, the primitive material particles of bodies are unextended elements.

COROLLARY III. Therefore, what is usually called an " atom" in chemistry, is a molecule, or a system of unextended elements, or material points, mechanically connected with one another: as is shown also by the difference of the atomic weight for different substances. For the atomic weights must be proportional to the absolute numbers of elements constituting the respective " atoms" or molecules.

SCHOLIUM. We have proved that no actual continuity of matter is to be found in nature: still a time-honoured and deeply-rooted prejudice in favour of an opposite view makes it difficult for many to receive our conclusion with perfect confidence. To help this class of men to dispel all doubts on this point, we intend to prove here that, even by sup- posing that motion could be communicated by material contact, it yet would not be possible to impart motion to continuous matter.

And, indeed, since contact takes place only at some points of the surface, the velocity, which by hypothesis would be communicated to continuous matter, would be immediately imparted to those superficial points, and from these points would have to be communicated to all the other inner points, which are of course innumerable, and in fact infinite; for, continuum cannot be exhausted but by endless division. The velocity, then, of those exterior points ought to be communicated or distributed without any new impact (which is impossible between the parts of a perfect continuum) amongst an infinite multitude of

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* I have not seen this point examined by those philosophers who hold a different opinion. I should be glad if this proposition with its corollaries were to be the cause of the question being ventilated by them. They will see, I hope, that my conclusion is by no means gratuitous.

distinct points.   Now, this is simply absurd : firstly, on account of the absence of any new impact : secondly, because a finite quantity of motion distributed amongst an infinite multitude of distinct material points would become infinitesimal for each of them : and so the continuous matter would in any length of time go through but infinitesimal space ; that is, would not move at all.   And, therefore, there is no gain in supposing the possibility of imparting motion by means of material contact.

To give to the same class of men still greater satisfaction, we will now try to show that matter not only is not, but cannot be continuous. Out of many conclusive philosophical reasons, which might be alleged, I choose the one which is most intelligible, even to ordinary people. A compound which has no first components is a sheer impossibility. Continuous matter, if admitted, would be a compound which has no first components.   Therefore continuous matter is a sheer impossibility. In this argument the first proposition is self-evident ; for the components are the material constituents of the compound ; and, therefore, a compound which has no first components is a thing which is constituted without its first constituents, or a pure contradiction.   The second proposition is also undeniable.   And, first, there can be no doubt that continuous matter would be *a compound ;* for, continuous matter would be extended, and would have, accordingly, parts distinct from parts : which is the exclusive property of compounds.   Now, that this compound would be *without first components,* can be proved as follows.   If continuous matter has any first components, these components will either be extended or unextended.   If they are supposed to be *extended,* then they are by no means the *first* components ; since it is clear that in this case they have distinct parts, and therefore are themselves made up of other components.   If they are supposed to be *unextended,* then they are by no means the *components* of continuum ; since all know and admit that no continuum can be made up of unextended points.   And, indeed, unextended points have no parts, and therefore cannot touch one another partially ; whence it follows that either they touch each other totally, or they do not touch at all.   If they do not touch at all, they do not make a continuum, as is evident.   If they touch totally, the one will occupy exactly the same place which is occupied by the other, and no material extension will arise.   And for this reason geometrical writers consider that a mathematical line cannot be conceived as made up of points, but only as the track of a single point in motion. We see, then, that a material continuum is a compound, of which the

first components cannot be extended and cannot be unextended. And since it is impossible to think of a third sort of first components which would be neither extended nor unextended, we must needs conclude that continuous matter is a compound which has no first components. And, therefore, continuous matter is a mere absurdity.

PROPOSITION VIII.

In a material element, the matter *is a point, from which the action of the element is directed towards other points in space, and to which the actions of other material points in space are directed.*

DEMONSTRATION. An action productive of local motion must have both intensity and direction. Now, direction can only be taken from a point to a point in space; and *the matter* alone marks out a point in space, as is evident from our preceding considerations. Therefore, locomotive action must be directed from the matter of one element to the matter of some other. Q.E.D.

SCHOLIUM. From the fact that a single direction can only be taken from a mathematical point to a mathematical point, and that a single action has a single direction, we might prove again that in a primitive substance the matter cannot be extended, but must be a mathematical point, as already shown in Prop. VII. For, if the matter in the primitive substance were extended, the action of such a substance upon any point in space would have many directions: which means, that such an action would be a multitude of distinct actions. Now, a multitude of distinct actions directed towards the same point, and at the same time, from different points in space, would evidently require distinct principles of activity: that is to say, distinct substantial acts (Prop. I. Coroll. I.), and, therefore, distinct substances. Therefore, a primitive substance cannot be materially extended.

PROPOSITION IX.

Each material element acts in every direction with equal intensity for equal distances.

DEMONSTRATION. It is known that terrestrial gravitation causes equal accelerations, all around the earth, in the motion of bodies equally distant from the centre. It is likewise known that the

action of the sun is felt by planets, whatever be their relative position in space, and that such an action tends to impart to them a degree of centripetal velocity, which depends on their distance from the sun, so that a planet, though in distinct positions, will always be equally attracted, provided its distance from the sun remain unaltered. And the same is to be said of satellites with regard to their planets. These facts are certain. Nor can there be any doubt that such actions as we attribute to the earth, sun, and planets respectively, proceed really from these very bodies. For, since the cause of motion is the source both of the velocity and of the direction imparted to the body moved, locomotive actions thence proceed, whence both the direction and the intensity of motion proceed. Now, the direction of the motion follows a line, which can be drawn from the centre of the earth to the falling body, v. gr. to the drop of rain, from the centre of the sun to the planet, from the centre of the planet to the satellite : and, in like manner, the intensity of the motion varies only for a change of distance between the earth and the drop, between the sun and the planet, between the planet and the satellite. We are certain, therefore, that those actions which we attribute to these bodies really proceed from them : a truth on which men of science agree. Now, it is manifest that, if a great body, like the earth, or the sun, acts in every direction and with equal intensity for equal distances, every single element of matter contained in such a body must also act in every direction and with equal intensity for equal distances. For, it is evident that the total action of the earth, sun, etc. is a resultant, and must follow the nature of its components ; hence the action of the whole mass cannot be equal on every side for equal distances, unless the component actions, viz. the actions of simple elements, be also equal on every side for equal distances. Q. E. D.

COROLLARY I. Therefore, material substance has a sphere of activity : and to admit such a sphere is not to indulge in a metaphor or to speak from fancy, but to express in proper and exact terms an important reality.

COROLLARY II. Therefore polarity is not a property of simple elements, but of molecules only. Crystals, for instance, have

different cohesion for different planes; and this only proves that the crystalline molecules are not simple elements, but compound systems so constituted and shaped that they can approach nearer to each other by some faces than by others.

SCHOLIUM. The proposition which we have proved is very generally admitted. Still speculative philosophers, who, in matters like this, become easily perplexed, and are always difficult to content, might be glad perhaps to find some other proof, or at least illustration of it, drawn from the arsenal of metaphysics. We shall try to show, then, how far metaphysical reasoning can support what we have already proved by facts. Philosophers very generally agree that the constituent principles of substance, viz. the *act* and the *potentia*, or, in scientific language, the principle of activity and that of passivity, must have with one another a perfect correspondence; otherwise the substance would not have a perfect unity of essence. Hence, whatever we can say in the *active* voice of the substance by reason of its principle of *activity*, can be said in the *passive* of the same substance by reason of its principle of *passivity*: and *vice versa*. Now, of the substance, by reason of its principle of passivity, we can say in the passive that it can *be acted upon* from any point around it, and is ready to *receive* motion in any direction whatever. Therefore, of the same substance, by reason of its principle of activity, we ought to say in the active that it can *act* on any of the points around it, and is ready to *give* motion in any direction whatever. This means, in other words, that material substance acts in a sphere. This argument, which is drawn from the very essence of substance, might be strengthened again by a reason which metaphysicians will not contest.

Things having been created by God, clearly not without an intention, and decidedly for the manifestation of His own perfections, it was necessary that every created substance should have an intrinsic capability of attaining such an end *per se*, and, therefore, independently of any accidental condition*. And, since manifestation is action, the substance is *per se* capable of attaining its end so far only as it is endowed with an active power independent (for its applicability) of any

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* It does not follow, of course, that the intensity of the actions will also be inde-pendent of any accidental condition. The action and its intensity are accidents, and therefore can and must depend on accidental conditions, viz. on distances : but the active power far from being an accident, is an essential attribute of the substance ; and thus the case is quite different.

accidental condition. Now, if matter were not able to act in every direction, its active power would not be independent (for its applicability) of any accidental condition; for it would be necessary that the object to be acted on should first take an accidental position in that part of space to which such power would exclusively be applicable. Therefore, if matter cannot act in every direction, it cannot attain its end *per se*, but only *per accidens*. And, since this would imply imperfection in the Creator himself, it is to be concluded that matter must have the power of acting in any direction whatever.

This point being established, it remains to show that the actions ought to be equal for equal distances. Actions must be equal when the agent and the object acted on are to one another in the same relation. Now, they will be to each other in the same relation whenever their distance is the same, whatever we may say with regard to direction. And, indeed, the direction between two points is said to change only in this sense, that, if we take a third point in space as a term of comparison, the line joining the first two points may change position with regard to the third point : but, if no third point existed in space, it would not be possible to conceive of any change of direction between two points only, whatever the absolute position of the points in space. Hence, no change of direction can be admitted between two points as simply related to each other. If then the distance between the two points remains the same, all things remain the same, as far as the same points are concerned ; and, therefore, the same points, whenever they are equally distant, will act with equal intensity.

The speculation just made shows the agreement of metaphysical with physical philosophy with regard to the necessity of admitting a sphere of activity around each existing material point. Of course, in any matter of fact, the proper arguments are those which arise from the consideration of facts themselves : the others, which are drawn from higher sources, are useful only inasmuch as they prove the consistency of principles with facts.

PROPOSITION X.

All material elements are equally inert.

DEMONSTRATION. When we say that matter is inert, we simply express the fact that material substance is absolutely incapable of imparting motion to itself. Now, absolute incapability is perfect incapability, and does not admit of degrees. There-

fore, inertia does not admit of degrees. And, accordingly, no element can have more or less inertia: or, in other words, all elements are equally inert. Q. E. D.

COROLLARY I. Therefore, all bodies also are equally inert. A mass weighing a hundred pounds is not more inert than a mass weighing only one pound. The two weights contain, of course, different quantities of matter, but their matter is equally incapable of imparting motion to itself. This truth is very exactly formulated by Professor Challis. " There may be," he says, " more or less of inert matter, but not more or less of inertia*."

COROLLARY II. Therefore, the *vis inertiæ* is not the same as the inertia of matter. And, in fact, the *vis inertiæ* (to say nothing else) admits of degrees; which is not the case with inertia.

SCHOLIUM. The inertia of matter may be admitted simply as a fact: and so we did at the beginning of this Book, Prop. I. But, since inertia is also admitted to be an essential property of matter, it will not be useless to illustrate this truth by showing in few words how it is that inertia necessarily results from the constitution of an element of matter.

Let there be a motive power which is ready of its own nature to exert itself in an indefinite sphere; which is the case with matter. A power of this sort must necessarily have a definite centre, from which it directs its action on every side indefinitely. Now, amongst all the points that can be marked out in indefinite space, none can be designated upon which this power cannot exert itself, but *that one* from which it directs its exertion on all others. And the reason is on the one hand, because no direction can exist from a point to itself, and on the other, that no motive action can exist without direction. The power, therefore, that we are considering, is ready to exert itself upon all other imaginable points, but not upon the centre of its own direction; and from this it follows that a material element, in spite of all the power it possesses of moving any other material element, cannot use it to move itself. No one will fail to see, how simple, natural, and complete is this explanation, and how intimately and necessarily connected with the simplicity of elements. For it seems that if elements of matter had more than one point, they would not be

* *Phil. Mag.* 1863, p. 283.

necessarily inert; because between point and point there would be a definite direction, and then the action of a material substance upon itself would be possible. Hence, what we have remarked regarding the source of inertia confirms the simplicity of material elements. That inertia is an essential property of matter can be established also by the following consideration. Matter is inert not only when it is at rest, but also when it is in motion, nay even when acting upon other matter. But that which is found always in a substance, and depends in no way upon accidental changes, but is to be presupposed in the substance itself before we can conceive any accidental change, is essential to such a substance. And, indeed, all unanimously acknowledge that inertia is an essential property of matter : though some speculative philosophers who did not know the definition of inertia, have entertained doubts about this subject. Had they known that the inertia is a mere incapability of imparting motion to itself, no doubt on their part would have been possible.

BOOK II.

ON THE NATURE OF ELEMENTARY POWERS.

THE propositions, which we have proved in the preceding Book, contain the more fundamental principles respecting the mechanical constitution of matter: those, which we shall explain presently, will help us to form an adequate idea of the same constitution by calling our attention to the nature of the powers with which matter is endowed. What we wish to inquire in this Book is, first, how many kinds of active powers ought to be admitted in material substance; and, secondly, whether there is a general law ruling the exertion of those powers, and what it is.

PROPOSITION I.

Both attractive and repulsive powers must be admitted as existing in this material world.

DEMONSTRATION. If the elements of matter were all exclusively attractive, no natural body would be found to possess expansivity, elasticity, impenetrability. Air and gases would not be expansive, if their molecules were not actually repelling each other: a stone would not resist pressure or react against an impinging body, had it not repulsive powers. The reason of this is, that to resist is to act, and action supposes activity; and, therefore, in the case of impact no resistance can be admitted, unless a power be granted which by its exertion produces a quantity of motion contrary to that of the impinging body; this being necessary in order to resist, *i.e.* in order to extinguish or diminish the motion of the impinging body. Now, since the motion of the impinging body causes it to approach the body impinged upon, it is clear that the quantity of motion which this second

body communicates to the first in the opposite direction must
be of such a nature as would cause an increase of distance. Now,
such an action is called *repulsion:* and the power, of which it is
the exertion, is called repulsive power.

On the other hand, if all the elements of matter were exclu-
sively repulsive, solid and liquid bodies would not exist; because
all matter would expand indefinitely through the immensity of
space, and no compound would be possible. We conclude that
the very existence of solid, liquid, and expansive bodies is a proof
of attraction and repulsion being real actions of matter. There-
fore, attractive and repulsive powers are real powers in nature.
Q. E. D.

SCHOLIUM. To resist is to act. This proposition is so evident that
it would seem superfluous to say a word in its support. Still in matters
of this sort we cannot make a step in our way without encountering
prejudice. Many a modern philosopher assumes that that, by which a
particle of matter is enabled to resist the motion of another particle, is
the inert matter itself, which stands in the way, and by its materiality
and impenetrability offers an invincible obstacle and a passive resistance
to the continuation of motion. This is assumed, of course, not proved;
and men believe that it is a principle too evident to deserve proof.
We beg pardon of these men of science, if we venture to remind them
that to act, and to be acted upon, are not the same thing. In order to
act, activity is necessary: to be acted on, passivity. That, therefore,
by means of which a substance can act, is not that on account of which
the same substance can be acted on. Now, *the matter* (see Book I. Prop.
I. Corol. II.) is not that by means of which a substance can act, but that
on account of which the substance can be acted on. Therefore, if re-
sistance is action, resistance is owing to the active power of material
substance, not to its matter. But resistance is action; for nothing but
action produces a quantity of motion contrary to that of the impinging
body. Therefore, resistance is not passive, and is not owing to the
inert matter standing in the way of the moving body, but is active and
owing to the active power of which the inert matter is the centre.

True it is, that the word *matter* is commonly used as synonymous
with material substance: and in this sense matter involves both activity
and passivity; hence we can say "Matter acts," and again "Matter is
acted on." But the philosophers of whom we are speaking, attribute
resistance to something especially material which stands in the way

and obstructs the passage by its materiality and inertia: and mean to say that the very matter as contradistinguished from its active power is the obstacle. Now, this view is not calculated to promote science, but to bring it back to its infancy. From the remotest times down to our own all great philosophers knew and taught with universal approbation and without the slightest opposition that *Corpus agit in quantum est in actu in aliud corpus in quantum est in potentia:* it would not do for our scientific reputation to ignore this very first principle of natural science. The mistake of those who attribute resistance to the inert matter originated probably in the consideration that inertia is a necessary *condition* of resistance: so necessary, that physicists call resistance itself *vis inertiæ*. But we must be aware that the power of resisting is called *vis inertiæ*, not on account of inertia being a force or a power, but only because the exertion of power in the impact of bodies depends on the fact that the body impinged upon, *on account of its inertia,* cannot retire till it has received a suitable velocity from without; and, therefore, during all the time which is necessary for the reception of such a velocity remains exposed to the action of the impinging body. For the same reason, the impinging body also is all along exposed to the action of the body impinged upon: and there results mutual compression and mutual communication of motion. Hence it is evident that the inertia is a necessary condition of the collision: but it is equally evident that the quantity of motion acquired by the conflicting bodies is efficiently produced by the powers, not by the inertia. This is what all physicists know; though we must confess that they often use words which seem scarcely proper to convey to others a correct idea of the doctrine. It is not astonishing, then, if the doctrine is sometimes misunderstood. On inertia we shall have more to say in Book VII. Prop. VII.

PROPOSITION II.

Attractive and repulsive powers cannot be replaced by molecular vortices.

DEMONSTRATION. A writer, who has recourse to molecular vortices to get rid of attractive and repulsive powers, imagines those vortices to be caused by the revolution of a ponderable atom around its axis. The atom by its centrifugal force is supposed to dilate the æther by which it is immediately surrounded: this first stratum of æther is supposed to acquire a centrifugal

tendency, which is communicated to the following stratum, hence
to a third, a fourth, and so on, with an intensity inversely pro-
portional to the squares of the distances from the centre. A
vortex, therefore,. is a sphere of concentric strata of æther hav-
ing a density directly proportional to the square of their distance
from a central revolving atom. In the theory which adopts this
sort of molecular vortices, expansion is considered as being a con-
sequence of centrifugal force communicated to the successive
strata of æther : and so repulsive powers are superseded by cen-
trifugal force. On the other hand, attraction is considered as
being the result of a difference of pressures exercised by the
æther on the opposite sides of a body situated within the vortex.
For, since the greater the distance of the strata from the cen-
tral atom, the greater, by the hypothesis, is their density, the
pressures exerted on different sides of the body cannot be equal :
and those parts of the body which are more remote must bear
a greater pressure. The body must, accordingly, approach that
centre with a quantity of motion proportional to the excess of
the one pressure over the other : and thus, attractive powers also
would be excluded.

This theory, as every one will see, assumes the existence of
a central atom materially extended ; for, an unextended point
has no axis, and cannot describe a circumference on itself. And
this first assumption is false, as we have demonstrated in the
preceding Book, Prop. VII. The same theory assumes also that
motion is communicated by means of material contact : and this
second assumption also is false, as we have proved above, Prop.
III. The same theory is in direct opposition to the law of com-
munication of motion in the impact of bodies, which requires
that the quantity of motion gained by one body be equal to
that lost by the other. And, in fact, if this law is observed,
the central atom by communicating motion to the surrounding
æther must lose its velocity of rotation, and come to a state of rest,
which entails the cessation of the vortex. Therefore the assumed
conservation of the vortex is a third false supposition. Since
these reasons suffice for our purpose, we may stop here, though
many other remarks might be added.

According to the inventor of this theory, the motion of the

ultimate atoms of matter necessarily takes place in an absolute vacuum : hence they find no resistance and suffer no diminution of *vis viva*. This is true so long as an atom has nothing to do with a medium : yet, if the surrounding medium comes into material contact with the central atom, as it is the case with this new theory, it matters very little whether motion takes place in an absolute vacuum, or not. For the central atom will always be deprived of its velocity by the surrounding medium, to which it is supposed to impart motion : and this would suffice to destroy both the vortices and the theory. Whilst, if the surrounding medium does not come into material contact with the central atom, then all the theory is again reduced to nothing, since there will be *actio in distans*, which requires those active powers which the theory is intended to get rid of. Hence it appears, how difficult it is even for acute men to avoid contradictions, when they build their theories, not on principles, but on arbitrary assumptions : and, at the same time, it remains demonstrated that attractive and repulsive powers cannot be replaced by these molecular vortices. Q. E. D.

SCHOLIUM. Mr T. Graham proposes another theory of molecular vortices. " Let us imagine," he says, "one kind of substance only to exist, ponderable matter : and further, that matter is divisible into ultimate atoms, uniform in size and weight. We shall have one substance, and a common atom. With the atom at rest, the uniformity of matter would be perfect. But the atom possesses always more or less motion, due, it must be assumed, to a primordial impulse. This motion gives rise to volume. The more rapid the movement, the greater the space occupied by the atom, somewhat as the orbit of a planet widens with the degree of projectile velocity. Matter is thus made to differ only in being lighter or denser matter. The specific motion of an atom being inalienable, light matter is no longer convertible into heavy matter[*]," &c. I do not see, by what mechanical principles Mr Graham would be able to account for the motion of his atoms. Is their motion progressive or vibratory ? Is it curvilinear or rectilinear ? If simply rectilinear, how can it give rise to volume ? and how is it compared with the orbits of planets ? If curvilinear, what and where is the cause of its curvature ? Are we to suppose that

* *Phil. Mag.* 1864, p. 61.

curvilinear motion can be the effect of a simple "primordial impulse" which is essentially rectilinear? We must therefore assume that there is an agent which urges each atom to abandon its rectilinear path. Mr Graham has neglected to inform us what this agent is: and wisely too. For the motion of his atoms being "inalienable" and "inconvertible," it would have been improper to suppose that there is a cause capable of working that change. If, lastly, the motion is assumed to be vibratory, the velocity of each atom will become $= 0$ at the limit of each vibration. Now is there anything that checks a motion "inalienable" and "inconvertible," and reduces it to zero? And, when motion has been reduced to zero, how is it restored? We think it evident, that Mr Graham's view cannot be reconciled with the known principles of mechanics; and, therefore, any ulterior remark, on our part, is quite superfluous.

Another kind of molecular vortices has been invented by Mr Samuel Brown for chemical purposes, and applied by others to the explanation of expansivity and elasticity. These vortices are supposed to be formed of material points revolving around a material centre, each vortex with its centre constituting a molecule: so that molecules, according to this view, would be so many planetary systems under microscopic dimensions. Whether a molecule can or cannot be conceived such as this theory assumes, we shall examine later. What we are here concerned with, is the use which has been made of this theory to the end of suppressing repulsive powers. It has been said, that the material points, of which the vortex is formed, by revolving around the centre must acquire a centrifugal force, and consequently a tendency towards expansion. This centrifugal tendency has been considered as being the cause of expansivity and resistance: and the phenomena ordinarily ascribed to repulsive powers have been considered as having their origin in a centrifugal force, which is not a power of matter, but a consequence of curvilinear motion. This theory, we must say, is a blunder from the beginning to the end. Its origin is a mistake: its evolution a sophism: its bearing an impossibility. Physicists, indeed, admit of something which is styled *centrifugal force:* but scientific men are supposed to know, that, in spite of the name, centrifugal force is by no means an efficient power, but only a quantity of motion, i.e. the one of the two components into which the actual quantity of motion animating a body at any point of its orbit is usually resolved. Hence, the centrifugal force of a point in motion is called *force* only because the velocity of the point in motion is the measure of the effort which

would have been necessary to produce it, or which would be neces-
sary to destroy it. Now, if a molecule is supposed to consist of ele-
ments revolving around a centre, the first consequence will be, that
such a molecule cannot possibly be expansive. Those who consider
centrifugal force as a source of expansivity, did not take into account
the fact that the centre of attraction must communicate to a body
moving in an orbit such a centripetal velocity as is exactly necessary
to retain it in its orbit, in spite of centrifugal force. So centrifugal
force cannot prevail ; and, therefore, the system cannot be expansive.
To make it expansive, one of two things would be indispensable : either
to impress on the system an increasing velocity of revolution, or to
weaken the intensity of central attraction. The first condition would
require an amount of exertion of extrinsic power : which is not the
case with expansive substances; for, they expand of themselves by
spontaneous work : the second condition cannot be accepted, because
the active power of a primitive substance is as unchangeable as the
substance of which it is the actual constituent. Nor is it less obvious,
that centrifugal force cannot be a source of reaction. Let us take a
bladder full of air, and reduce it by pressure to half its volume. What
can the supposed centrifugal force of the molecular systems do towards
reacting? The orbits, in each system, are reduced by pressure to a
smaller dimension ; and it is clear that they cannot dilate anew, unless
each point, which is now describing a smaller orbit, can fly off in the
direction of the tangent of the same orbit. Now, this is impossible ;
for, after the orbit has been reduced to smaller dimensions, the centri-
fugal force must have become such as is suitable for the new orbit,
viz. equal and opposite to the centripetal force : and, this being the
case, it is evident, that centrifugal force after the compression has
ceased, cannot draw the element out of its new orbit, in order to
dilate again the molecular system. Therefore, in this theory gases
would indeed be liable to indefinite compression, but would never
expand again. In other words, centrifugal force is wrongly assumed as
a substitute for repulsive powers.

I shall end by a remark which, in my opinion, is most important.
Scientific men would lose nothing, and gain not a little, by speaking
more correctly than they often do. In any branch whatever of science,
a terminology, which of its own nature leaves room for ambiguity, can
only multiply the chances of error. The word *force* (vis) both in its
vulgar and philosophical sense means either the efficient cause, or the
power of acting, or, lastly, and frequently, the intensity of the action.

Now, it is certain that the quantity of motion of a moving body is not the efficient cause of motion, nor the active power, nor the intensity of the action, but only the intensity of the effect. Why, then, should we give the name of *force* to motion? Of course, there is always a reason (good or bad) for everything: and one might say that we are not the inventors of this expression. But it is strange that, while we enjoy the advantage of a more exact scientific knowledge than our forefathers had, we should not try to improve their terminology by giving up those expressions in which the vagueness of the terms is a perpetual cause of blunders. When we find, for instance, that a man like M. Moigno has been so far misled by incorrect expressions as to say that "centrifugal force is a force of reaction*," we may well be sure that men of less scientific ability will be exposed to mistakes of still greater moment. It would be easy to make a catalogue of errors of this sort, which interfere more or less with a clear conception of things : but this is not the object of the present work. I shall only add that the correct expression of mechanical facts requires a well-marked distinction between *cause* and *power*, between *power* and its *exertion* and the *conditions* of its exertion, between *action* and *force*, between *force* and *motion* or *quantity of motion*.

The *cause* of motion is the substance itself, which by exertion of power produces motion. The *power* is that by which the cause is able to act: it is its activity and its causality. Such a power is simple in a simple element, whilst in a compound system of elements it is a result of simple powers, and depends on the disposition of the components and on the constitution of the compound. The *exertion* of power is the formal application of the activity of the agent to the passivity of the object acted upon. The *force* is not the action, but the intensity of the action : and is measured, either by its dynamical effect, i. e. by the space which a mass is obliged to go through in a given time, if no obstacle intervenes ; or by its statical effect, i. e. by the pressure exercised on another mass. Lastly, the *quantity of motion* is not a force, though it is so called, but only a representative or a measure of force, viz. the result of an action of a given intensity, and the test by which we judge of the intensity of the action. And the same is to be said of pressure,

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* I give his own words : " Lorsqu' une cause quelconque en agissant sur un corps l'oblige à changer de direction ou de vitesse, le corps en vertu de son inertie réagit contre cette cause avec une énergie égale et contraire dans une direction déterminée : cette force de réaction, dont la force centrifuge est un cas particulier, a reçu d'Ampère le nom de force épiménique." *Manuel de la Science,* an. I. p. 234.

as is evident. But as a body in motion and a body at rest are in a different condition for acting, and consequently a body in motion and a body at rest exert their powers in a different way, and give rise to different phenomena, so we are exposed to the temptation of considering motion as a kind of power modifying the ordinary powers of the body, whilst it modifies only their actual application. Thus scientific men and philosophers in general are wont to say that "Heat is force," though they have learned also that heat is only "a mode of motion." Now, these two propositions are irreconcilable, as Professor Challis observes[*]. The truth is that heat is really a mode of motion only, and not a force; and, hence, the argument of Professor Challis against Professor Tyndall is based on a false assumption. The *force*, in the case of heat, is the intensity of calorific action; now, this action with its intensity comes not efficiently from the heat, but from the active powers of the hot body: and the hot body has these powers, not because it is hot, but because it is constituted of molecules, each of which has power, whatever be their state and temperature; though for different states and temperatures those powers may have a very different mode of application: on which circumstance it depends that the action of a body, when heated, differs very sensibly from the action of the same, when cool. Since motion entails a change of distances, and distance is one of the conditions on which the intensity of actions depends, it is obvious that molecules in motion will exert their permanent powers with different intensity according to the nature and mode of their motion.

To conclude, I think that, when a mass m has the velocity v, the product mv ought not to be called a *force*, much less a *power*, but only a *quantity of motion*, or an *impetus* or a dynamic momentum. So also, when a mass m is subjected to an action capable of developing a velocity v in the unit of time, but is prevented from moving, the product mv ought to be called a *pressure*, or a static momentum, and not a *force*. Centripetal force is not the centripetal action, but only its intensity or its quantity: centrifugal *force* is neither action, nor the intensity of action, but only the quantity and intensity of centrifugal tendency. *Dead* force and *living* force (as distinguished from *vis viva* according to Dr Mayer) are names which we can totally suppress, since they are only metaphors, and say nothing more than pressure and impetus. *Vis viva* itself is another useless word, although it has so great a sway in mechanics. *Vis viva* is not a force; for no force can be measured by the product of the mass into the square of its velocity. *Work*, indeed,

* *Phil. Mag.* 1863, p. 290.

is measured by half the product of the mass into the square of its velocity; but work, either done, or accumulated, is not a force; for it can be easily proved that a mass m having the velocity v, in doing its work $\frac{1}{2}mv^2$, exerts an action, the intensity of which is only mv; whence it follows that the force employed in the work is measured by the impetus mv, and not by the work $\frac{1}{2}mv^2$. But let us stop here.

PROPOSITION III.

Attractive and repulsive powers are the only powers of matter: so that we need not look for any other occult agency.

DEMONSTRATION. An occult power is to be admitted then only, when a phenomenon occurs which cannot proceed from the powers already known. This is evident; for, when phenomena can be accounted for by known powers, there is no ground for any inquiry about occult causes; in other words, to look for occult causes without data, or indications, on which to ground the induction, is to propose to one's self a problem without conditions: which no man in his senses would do. Now no phenomenon has been observed anywhere in material things, which cannot proceed from the known powers of attraction and repulsion: nay, it is positively certain that all phenomena proceed from the same powers. For each material point, when acted on, can only change its place; and, therefore, the effect of the action of matter upon matter is only local motion, one element approaching to, or retiring from the other. And this is precisely what attractive and repulsive powers are especially competent to do.

The proposition, that a material element, when acted upon, can only receive a determination to local motion, can be proved very clearly. In a simple element of matter, the passivity, or *the matter*, is limited to a simple mathematical point, as we have already shown. Now, a mathematical point cannot suffer any intrinsic change, since it has nothing with regard to which it could be possibly modified. Whence it follows, that the same point is only capable of being determined to an extrinsic change, which can be only a change of extrinsic relations. Now this change of extrinsic relations is brought about by local motion. Therefore, the element of matter can be subjected to nothing

but local motion. Hence we further infer, that, if there were in matter any secret non-locomotive power, this power would be absolutely useless, as absolutely inapplicable to any other material thing, and would remain in this absurd condition for ever. Therefore, no phenomenon exists, or can exist, in nature, which shows the existence of any occult power in matter.   Q. E. D.

SCHOLIUM. The ancient philosophers admitted occult qualities and òccult powers : some admit them even now. And, indeed, many molecular phenomena depending on chemical affinity, cohesion, capillarity, electricity, and magnetism, may easily lead men to suspect that there is something in matter besides mechanical powers. Moreover, the very distinction admitted between chemical and physical phenomena, bétween universal and molecular attraction, &c. seems to show that even scientific men are still of opinion, that there are powers in matter, which are not simply mechanical. But the fact is, that physicists, in accounting for phenomena, do not trace them as yet to their primitive causes, i.e. to the simple elements, but only to their proximate causes, which are complex, and, as such, follow different laws of causality corresponding to the different mode of their constitution. The reason of this is, because no one, as yet, has gone beyond the chemical analysis of bodies, whilst, before we are able (if we ever shall be) to trace phenomena to their simple and primitive causes, it would be necessary to make also a mechanical analysis of chemical atoms, in order to ascertain and determine the intrinsic constitution of primitive molecules. Now, since this has not been done, it is obvious that scientific men ought to retain the distinctions above mentioned, not indeed in order to insinuate the existence of occult powers in the primitive substance, but to keep up the necessary distinction between complex powers of different kinds, and exerted according to different laws. This distinction of names was absolutely necessary when it was first introduced; because it was then a received opinion that phenomena of so very different kinds could not but be owing to agencies radically different in species. In our days, chemical actions have been acknowledged to be mechanical actions: yet no one would think it practicable to suppress the distinction made between chemical and mechanical forces. Chemical phenomena present themselves to our appreciation under a form too remote from that which characterises the ordinary mechanical phenomena, to be possibly confounded with them under one and the same denomination.

This much with regard to the different names, by which forces of different kinds are designated. As to the thing itself, it is known that the distinction between universal and molecular attraction is not calculated to point out any secret non-mechanical power. And, in fact, both molecular and non-molecular actions give rise to nothing but local motion; and local motion is a mechanical effect. The only suspicion entertained by philosophers bore on the radical difference of these actions with regard to the law of their intensities; and the doubt originated from the assumption that the law which holds for astronomical distances, did not hold good for molecular. We shall show further on that the assumption has no ground whatever. The Newtonian law holds as well for molecular distances as for astronomical: and, if philosophers have been unable to make the application of that law to molecular‑ distances, it is only because they have neglected some necessary considerations concerning the intrinsic constitution of molecules. When these considerations are presented to the reader, he will find that molecular action not only proceeds from the same kind of powers, to which the astronomical phenomena are due, but also is a resultant of elementary actions, which follow the Newtonian law, though the resultant itself (the molecular action) follows another law dependent on the molecular constitution.

PROPOSITION IV.

One and the same element A *cannot attract the element* B *and repel another element* C, *when* B *and* C *are equally distant from* A.

DEMONSTRATION. Attraction and repulsion are actions of an opposite nature, which, accordingly, cannot proceed from one and the same simple principle of activity. This, I think, is self-evident, at least for the case which we are now considering, viz. when the elements B and C are equidistant from A*. Therefore the element A cannot have two contrary determinations, the one to attract B, the other to repel C.

On the other hand, A cannot be determined by any element B to act on it by attraction, nor by any element C to act on it by repulsion. And for this we may allege many reasons. First, because the nature of the element A, as also of any other substance

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* Though the reader will not fail to see clearly the truth of this principle as far as simple and primitive substance is concerned, we will add some explanation of it in the next proposition and scholium.

whatever, is intrinsic to it; since nature is nothing but the substance itself as capable of acting. Accordingly, the nature of A cannot depend on anything which is extrinsic and accidental, such as the presence or the action of B or C. Secondly, because the actions of B and C are received in A only inasmuch as A is passive, viz. capable of being determined to motion; therefore, the actions of B and C can indeed impart to A a motion of a certain intensity, but not give to A the property of attracting rather than that of repelling; or *vice versa*. Thirdly, because, if the element B had the power of determining A to act on it by attraction, and the element C had the power of determining A to act on it by repulsion, then matter would not be perfectly inert, viz. indifferent to be acted upon by any power whatever, whether attractive or repulsive. Therefore, inert elements cannot be the cause of their being attracted or repelled. Therefore the element B cannot determine A to act on it by attraction, nor can the element C determine A to act on it by repulsion. And, since the elements B and C are also supposed to be equally distant from A, it is evident that the element A neither by an intrinsic determination, nor by reason of any exterior circumstance can at the same time attract an element B and repel another element C.

PROPOSITION V.

One and the same element of matter cannot be attractive for one distance and repulsive for another.

DEMONSTRATION. Actions of opposite nature, such as attraction and repulsion, cannot proceed from one simple principle of activity. Therefore, one simple element either is only attractive for all distances, or is only repulsive for all distances. Opposite powers can evidently be admitted in compound substances, which, on account of their composition, may involve distinct and opposite principles. In this case the action is a resultant: and may be attractive or repulsive according to circumstances. But, with regard to a simple element, no opposite principles are conceivable; because one simple element is one simple and primitive nature, and consequently one simple principle of activity. Therefore one and the same element of matter is either wholly attractive or wholly repulsive, whether the distances are great or small.   Q.E.D.

ANOTHER DEMONSTRATION. If ever the elementary action from attractive becomes repulsive, this change must depend (according to the hypothesis) on a change of distance. Now, this cannot be. A change of distance can lead, and leads really, to a change of *intensity* in the action : but the *nature* of the action must always remain unchanged. The intensity of the action is an accidental thing, and, accordingly, can vary for accidental changes, the nature of the agent still remaining the same. But the nature of the actions depends only on the nature of the substance which acts : and it is absurd to suppose that the nature of a primitive substance can depend on any accident whatever, much more if this accident is only an extrinsic relation, such as distance is. For, if the determination of an element A to attract, or to repel, comes from without, viz. from an extrinsic circumstance, then such a determination is accidental and adventitious. What would then be the nature of this element, which, of itself, is neither of an attractive nature, nor of a repulsive ? This manifest absurdity, i.e. a substance having no definite nature of its own, shows the impossibility of the hypothesis. Q.E.D.

A THIRD DEMONSTRATION. Let us suppose that an element of matter is repulsive to a small distance from its centre, and that for all greater distances it is attractive. Let d be the distance for which the change of repulsion into attraction takes place. Then the element, at the distance d from its centre, has no powers; d being the limit, where repulsion ends and attraction begins, i.e. where the intensity of its action becomes $= 0$. Now, since from the centre to the distance d there is no attraction, let us ask whence does that power of attracting emanate, which is to be felt at all greater distances? Does it emanate from the spherical surface virtually described with the radius d? But how is this possible, since the element, as we have just said, at this surface has no power whatever ? The attractive power must, therefore, emanate from the centre itself. But, then, it is clear that attraction must prevail all throughout even to the centre. Hence an element which is attractive at any distance is such for all distances: and, for the same reasons, an element which is repulsive at any distance is such for all distances. Q.E.D.

SCHOLIUM. Boscovich, whose opinion we have refuted in this proposition, contends that attraction and repulsion are not actions of a different nature. *Utraque vis,* says he, *ad eamdem pertinet speciem, quum altera respectu alterius negativa sit, et negativa a positivis specie non differant. Alteram negativam esse respectu alterius patet inde quod tantummodo differant in directione......Quod autem negativum et positivum ad eamdem pertineant speciem, id sane patet vel ex eo principio:* Magis et minus non differunt specie. *Nam a positivo per continuam subtractionem, nimirum diminutionem, habentur prius minora positiva, tum zero, ac demum negativa, continuando subtractionem eamdem.* This way of reasoning is not calculated to cause conviction; since it is not lawful to predicate of actions, as Boscovich does, whatever can be predicated of motion. Two points A and B being given, there is only one direction from A to B, whether A be attractive or repulsive. Hence the action of A on B, whether attractive or repulsive has always one and the same direction from A to B; and, therefore, the diversity of direction is not predicable of the action of A, but only of the motion of B; for, it is B itself that will go towards A, if attracted, or recede from A, if repelled: and these two motions will evidently have opposite directions. Consequently, the fact that the action of A causes B to take this or that direction does not show that A acts in this or that direction, but only that A exerts, in one and the same direction, a power which is of this or that nature, according as it attracts or repels.

True it is, that writers on mechanics speak indifferently of direction of forces, direction of actions, direction of motion, as if these expressions were equivalent to one another. They speak so, because men are only too much accustomed to attribute to the cause that reality which is to be found only in the effect. But we must take notice, that mechanical writers, in admitting a positive and a negative direction in motion, are supported by a special reason, which does not hold for actions as such. This reason is, that motion can be extinguished by degrees, and become $= 0$, and then gradually reappear with an opposite tendency, though the action be not subjected to the same changes. Thus, when a pendulum reaches the highest point of its course, its velocity at that point becomes $= 0$, and, since zero is the limit of positive and negative quantities, if the velocity of ascent has been taken as negative, the velocity of descent must be taken as positive. Still, this does not imply that the action of the earth, which first produced retardation,

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* *Theoria Phil. Nat.* Part. I. n. 108.

has changed its nature or direction when it began to produce acceleration; for, it is evident, that the action remains attractive as before, and directed vertically as before. Hence it appears, that opposite directions are possible in motion, which can pass through zero, though not in powers, which can meanwhile remain constant in intensity as well as direction. We may add, that, if Boscovich's assertion were true, that negative and positive do not differ in kind, but only in quantity, then also to be creditor for £1000, or debtor for £1000, would differ in quantity, and not in kind: a conclusion, which might be admitted indeed by a debtor, but not by his creditor. Again, it is evident, that attraction and repulsion are both positive exertions; they cannot, accordingly, differ from one another as positive and negative. Lastly, every direction is taken from a point to a point; and, consequently, whatever the power be, the direction must remain the same, if the points remain at the same place; and, therefore, it is evidently impossible to admit that a difference of power constitutes a difference of direction.

Some might ask, what reasons can have determined Boscovich to admit in one and the same element both attractive and repulsive power. Boscovich's only reason was that, if bodies were made up of attractive elements remaining attractive even to their centre, then bodies would not be impenetrable. Now, this reason may be solved by a very simple remark. For, true though it is that the impenetrability of bodies demonstrates the existence of repulsive powers, it nevertheless does not demonstrate, nor even suggest, that the repulsive power ought to coexist in one and the same simple element together with the attractive. And this is what Boscovich supposed without the shadow of a proof.

PROPOSITION VI.

The existence of repulsive elements is not opposed to the fact of universal attraction.

DEMONSTRATION. If all matter were repulsive, we should have evidently universal repulsion: but, if bodies are composed of elements, some of which are attractive and others repulsive, then we shall have either universal repulsion or universal attraction, according as the number and the intensity of the repulsive or of the attractive powers prevails. And, therefore, from the fact that it is attraction that prevails all throughout the world, it only follows that in the universe the number or intensity of the attrac-

tive powers is greater than that of the repulsive. Therefore, the existence of repulsive elements is not in opposition with the fact of universal attraction. Nor is it opposed to the law of attraction. The attraction which is known to prevail between celestial bodies, varies according to the law of the inverse square of the distance: now, this same law remains true, even though the heavenly bodies contain a great number of repulsive elements. And, indeed, the distances, for which the law of universal attraction is applicable, are such that the total action of one body on another may be considered as the algebraical sum of a multitude of parallel actions. Hence, if each elementary action is inversely proportional to the square of the distance, so also will be the total action. Therefore, the existence of repulsive elements does not interfere with universal attraction. Q. E. D.

PROPOSITION VII.

Simple elements act on one another with an intensity inversely proportional to the square of the distance, whether the distance be telescopic or microscopic, astronomical or molecular.

REMARK. Let the reader observe that this proposition regards the action of simple *elements,* not the action of *molecules.* That molecular actions are not inversely proportional to the square of the distance, is a known fact. Molecular cohesion, for instance, is immensely greater than it could possibly be by the Newtonian law: so also molecular repulsion. Yet, since a molecule, though often called an atom, is a system of elements, and elements acting according to the Newtonian law can give rise to molecular systems, which, at very small distances, will act according to any other law that may be indicated by molecular phenomena, it follows that simple elements may really act on one another, at molecular distances, according to the Newtonian law, however different be the law of action of the molecules for the same distances.

DEMONSTRATION. Celestial bodies, as is shown by astronomers, act on one another according to the Newtonian law, i.e. with intensities inversely proportional to the square of the distances. And, since the action of one body on another is the algebraical sum of the actions proceeding from each element of the

body, we must admit that each element, at least when acting from a great distance, obeys the Newtonian law. Now, this being admitted, it can be proved that each element will always obey the same law, even though the distance has become molecular. For, the natural determination to act follows the essential constitution of the agent, so that no agent which has no more than one essential principle of activity can have more than one natural determination to act. Now, a simple element has but one essential principle of activity; since a simple being has a simple essential act. Therefore, a simple element has only one natural determination to act, and this, of course, essential to it, and consequently immutable. Now, it is obvious, that a substance which has but one determination to act, cannot obey in its actions two different laws. For, the law which rules the actions is a result of an intrinsic determination of the agent; and, therefore, two different laws of acting are two formal results, which imply two formal principles and two natural determinations to act. And, since a simple element has but one formal principle, and but one natural determination to act, as already shown, it is evident that an element cannot possibly obey two different laws of action. Hence the law which elements obey when acting from a certain distance, must be obeyed by the same elements when acting from any other distance whatever. But for certain distances they obey the Newtonian law; therefore, for all distances, however molecular, they obey the Newtonian law.   Q. E. D.

ANOTHER DEMONSTRATION. Since scientific men naturally prefer mathematical to metaphysical reasoning, I shall give here a second demonstration drawn from mathematical considerations. Matter, as we have already proved, exerts its power in a sphere, i. e. with equal intensity for equal distances. Let us, then, imagine a spherical surface, of which the centre is a given material point, and the radius r. Let vdt be the action, in the instant dt, of the material centre on a material point situated on that surface, and Vdt the action of the same centre, in the same instant, on the whole of that surface when this is conceived to be thoroughly filled with material points. The mass formed by all these material points is $4\pi r^2 k$, k being its density, which here is to be con-

sidered as infinite, because in the unit of surface (when it is supposed to be thoroughly filled with matter) an infinite multitude of points is designable. Therefore, the total action of the central element, in the instant dt, at the distance r will be

$$V dt = v dt \,.\, 4\pi r^2 \,.\, k.$$

If we imagine another spherical surface, having the same centre, and the radius r', and call $v' dt$ the action of the central element on a given material point situated on this surface, $V' dt$ being the action of the same element on the whole of the same surface, we shall have also

$$V' dt = v' dt \,.\, 4\pi r'^2 \,.\, k.$$

From these two equations we obtain

$$V : V' :: v r^2 : v' r'^2 ;$$

but, for those distances, for which the Newtonian law is known to hold good, we have

$$v r^2 = v' r'^2, \text{ and consequently } V = V';$$

therefore, the whole exertion of a central element on a spherical surface is *constant;* at least when the radii of the spheres have a certain magnitude. This restriction, however, if we only understand what we are saying, must be rejected as useless and absurd. And, indeed, if the whole possible exertion is the same for two great distances, it will be the same for all imaginable distances, however small. For, that total exertion, being the total effort of which an element is capable at a given distance, is the measure of the whole activity of the agent as related to such a distance. And since the agent is a simple element, and has but one simple activity all throughout, and this as unchangeable as the simple substance itself, it is quite impossible to admit that the same activity, for some sphere of a smaller radius, can be different from itself. Therefore the equation

$$V = V'$$

will be true for the smallest imaginable distances, as well as for the greatest; and consequently also the equation

$$v r^2 = v' r'^2,$$

which expresses the Newtonian law, will be true for all distances, however small. Q.E.D.

A THIRD DEMONSTRATION, *ex absurdo*. The only ground for repudiating the Newtonian law, in the case of molecular distances, would be this, viz. because in the expression of the law of molecular actions the squares of the distances ought to be replaced by some higher power of the same distances. This was, in fact, the course followed hypothetically by Newton, Boscovich, and others, with regard to molecular actions. Now, it cannot be assumed without absurdity, that the law of elementary actions, in the case of molecular distances, requires such higher powers. And, in fact, let us take the equation

$$Vdt = vdt \cdot 4\pi r^2 \cdot k,$$

which, independently of any hypothesis whatever, expresses the total elementary action exerted, in the duration dt, on a given spherical surface. The hypothesis of actions proportional to a power of the distances higher than the square would lead us to the proportion

$$v : a :: 1 : r^{2+n},$$

v being the action at the distance r, a the action at the unit of distance, and n a positive number, either an integer or a fraction. Hence we should have

$$v = \frac{a}{r^{2+n}};$$

and this value of v being substituted in the preceding equation, gives

$$Vdt = \frac{adt}{r^n} \cdot 4\pi \cdot k.$$

Now, this result (in which $k = \infty$) is inadmissible; because, by taking r infinitesimal, it gives $Vdt = \infty$: in other words, because the element would possess a power capable of exerting itself with infinite intensity in a single instant of time. This would imply that the element possesses an activity of infinite actual intensity. Since, then, this is altogether absurd, and natural facts themselves show that material activity is not infinite, we con-

clude that the law of actions inversely proportional to a power of the distances higher than the square, cannot be true with regard to simple elements.   Q. E. D.

A FOURTH DEMONSTRATION. No new law of action is to be admitted without a sufficient reason.   Therefore, if the Newtonian law, which manifestly is a law existing in nature, is adequately sufficient to account for all phenomena, even molecular, without exception, no other law is to be admitted.   Now, the Newtonian law is adequately sufficient to account for all phenomena, even molecular, without exception.   For, the property of acting according to the Newtonian law, as possessed by simple elements, is perfectly consistent with the existence of molecules acting according to any other law, which may be required in the case of molecular phenomena, as we have already observed, and will be most evident from what we shall say hereafter on the properties of molecular systems.   Therefore, there is no sufficient reason for admitting, or even suspecting, that elements, when acting at molecular distances, follow any new law different from the Newtonian law.   Q. E. D.

PROPOSITION VIII.

The sphere of action of material elements extends beyond any assignable limit.

DEMONSTRATION.   Let v be the intensity of the action at the unit of distance, v' that of the action at the distance r. The law of action gives the relation

$$v' = \frac{v}{r^2} \, ;$$

in which v' cannot become $= 0$, unless r becomes $= \infty$.   Q. E. D.

SCHOLIUM.   Those, who are reluctant to admit *action at a distance*, will be the more so with regard to this proposition.   Still, many things are true, which are difficult to be understood: and it would be against reason to deny truths sufficiently inferred from facts, only on account of the difficulty which we experience in giving an intelligible explanation of them.   Those, who, to avoid objections of a serious appearance, deny this action at a distance, expose themselves to real difficulties,

which admit of no possible solution: if they reject action at a distance, because its explanation appears to be difficult, they are also bound to reject even more decidedly action by material contact, which indeed admits of no explanation whatever. But let us try to discover the source of such difficulties. To understand and explain how material elements can act at any distance whatever, is difficult for this one radical reason, that our intellectual work is never purely intellectual, but always accompanied by the working of that other power, which we call imagination, and that, when we are considering something that transcends imagination, and of which no sensible image can be formed, our intellect finds itself under the necessity of working without the assistance of suitable sensible representations. Nay, since fancy cannot remain inactive, and strives continually to supply the intellect with new images (which unhappily are not calculated to afford any exact representation of a thing which is not sensibly representable), from this it happens, that the intellect instead of receiving help from imagination, is rather led astray by it, with the chance of following a wrong direction. Such is the case with regard to the difficulty of understanding that which transcends imagination. On the other hand, if we suppose that this difficulty has been successfully overcome, and the truth clearly understood, there remains another great difficulty, that of properly expressing what we have clearly understood. The words, which we are obliged generally to make use of in speaking of intellectual objects, are more or less immediately drawn from sensible things, and have still, even in their figurative or derivative sense, a peculiar connection with the sensible images of which their roots are the representatives. With such words, our explanations must, of course, be metaphorical in a high degree: and there are circumstances, in which they will fail to express unobjectionably our most unobjectionable thoughts. Had we succeeded in expressing most exactly what we have in our minds, there would remain another formidable chance of failure. For, the greater our scrupulousness and exactness is, the more strange and absurd our style will appear to those (and they are very many), who know of no other language than that of their senses, their imagination, and popular prejudice.

With regard to our subject, the whole difficulty seems to arise from the assertion that *a cause cannot act where it is not*. This proposition, though it is to be ranked among those which popular prejudice, incompleteness of conception and imperfection of language cause to be received as axiomatic, has its origin in a false supposition, as we have already

shown (Book I. Prop. III. Schol.). But, as the objection is presented in popular terms, and our answer has no such advantage, the chance is that the objection will keep its ground. For this reason, Professor Faraday, without denying (as he could) the pretended axiomatic assertion, preferred to say that the atom of matter " is everywhere present," and, therefore, could act everywhere. By this answer the learned writer trying to avoid Scylla fell into Charybdis. If the element of matter is everywhere present, then Westminster Abbey, for example, is everywhere present. Yet we are certain that, in the opinion of Professor Faraday, Westminster Abbey is neither present in California nor in the moon, but in London exclusively. In fact, the word " present" cannot be used conveniently when we speak of active power. We are accustomed to say, that a body is present, not in that place in which its action is felt, but in that from which the direction of the action proceeds; and since such direction proceeds from the centres of power, to these centres alone we refer, when we point out the place occupied by a body. Professor Faraday, on the contrary, refers to the active powers, when he says that matter is everywhere present; and this way of speaking is irreconcilable with the notions we have of determinate places, distances, volumes, &c., and creates a chaotic confusion in all our ideas of material things. The learned professor speaks more correctly when stating that " each atom (element) extends, so to say, throughout the whole of the solar system, yet always retaining its own centre of force." Here the words *so to say* tell us clearly that the author, having found no proper terms to express himself, makes use of a metaphor, and attributes *extension* to material elements in a sense which is not commonly adopted. He doubtless wishes to say that " each element extends *virtually* throughout space, though it *materially* occupies only the central point from which its action is directed." This answer is very good. But people are not likely to fully realise its meaning. For, in speaking of material substance men often confound that which belongs to it by reason of its *matter* with that which belongs to it by reason of its *form*. If the substance had no matter, it would not mark out a point in space; it is, therefore, on account of its matter that a substance is ubicated. As for the form (which is the same thing as active power), although it is said to have a kind of ubication on account of the matter by which it is terminated, nevertheless, of itself, it has no capability of formal ubication, as we have already shown in another place. Hence the assertion "No cause can act where it is not" cannot have any reasonable meaning, unless it be limited as follows: "No cause can act where it is

not virtually by its power;" which implies that "A cause can act where it is not formally ubicated with its own matter."

Some will say that we may be right in making this distinction, but that we ought not to assume that the active power can be there where there is no matter to support it. We answer that this objection implies an hypothesis which is not true. The matter, in fact, does not *support* its form in the same manner in which the substance supports the accidents. The accidental act requires a subject already existing: the substantial act requires only a terminus to which it gives existence. This is evident; because if the substantial act ought to be supported by a real subject, this real subject would be a substance before receiving the same substantial act: which is a contradiction in terms. Therefore, the form is not really supported by the matter, but only *terminated* by it. To understand this more clearly, take a sheet of paper and make a hole in it. Such a hole has a form, say circular. Now, is it not clear that this circular form causes to exist a geometrical centre, by which it is terminated, though not supported? Is not the circular area altogether out of this centre, though essentially terminated by the same centre? In the same way the form is terminated, though not supported by its matter; and, therefore, there is no necessity of merging, so to say, the form in the matter in order to have it supported, as the objection supposes.

The image, to which we have had recourse to explain this point, is not a parity, but an imperfect likeness. Its defect lies chiefly in this, that the circular form, though not supported by the hole nor by its centre, is nevertheless supported by the paper in which it has been made: and this must be so, because a circumference is not a substantial form, and, therefore, is in need of being supported. The best means of conceiving the actual relation of the substantial form to its matter is furnished by the consideration of an indefinite sphere. I call an "indefinite sphere" that in which the density decreases continually in the same ratio as the squared distances from a given point increase, so that its indefinite sphericity is constituted by the density itself decreasing gradually and equally all around the given point or centre. But such a sphere, of course, is not of matter, but of power: and what we have called its *density* ought to mean *intensity*, because it has to be measured not by the frequency of material points, but only by the intensity of the exertions of which it is capable at any given distance from its centre. If such a sphere were of matter, no one would say that he cannot conceive it. Now, a sphere of power is a thing which

can be conceived with the same facility, if not greater. For, continuous decreasing intensity is at least as easily conceivable as continuous decreasing density. Here, then, we have a centre, the existence of which essentially depends on the existence of a power of a decreasing intensity and constituting an indefinite virtual sphere. Take away this power, this spherical form; the centre will be no more. On the contrary, let such a spherical form be created; the centre will be immediately called into existence, as the essential and intrinsic terminus of sphericity; it being impossible to have a real sphericity not traced to a real centre. Now, this sphericity, although absolutely inseparable from its centre, is not in the centre, but all around it: is not *supported* by the centre, but actuates it and gives to it an existence: and although this same spherical form possesses an intensity decreasing as the squared distances increase, still it has at all distances the same property of giving existence to its centre, since it has at all distances a spherical character essentially connected with a central point as its terminus.

That this is the case with a material element, appears from all our preceding demonstrations, especially from Prop. IX. Book I. We are entitled to conclude, therefore, that the power of a material element constitutes a virtual indefinite sphere : that *the* matter is the geometrical centre of this sphere : that it is not the matter that supports the power, but the power itself, or the form, that gives existence to what we call *the* matter : that the matter is a point in space ; and accordingly an element is ubicated only by reason of its matter : that the power is all virtually without the matter, though it is intrinsically terminated by it, and cannot be apart from it : that there is no insuperable difficulty in conceiving how a virtual sphericity may be all virtually out of its centre, in the same way as we conceive a material sphericity to be all materially out of its centre : that the active power, of itself, has no formal ubication, but is reduced to this predicament only on account of the ubication of its matter, which is the formal ubication of the substance, and the centre from which its action is directed. Hence, it is obvious that the proposed objection, viz. that "the power would virtually be there where there is no matter to support it" is not an objection, but a simple statement of a fact, which admits of a very natural explanation grounded on the intellectual analysis of the substance itself : though our opponents will certainly fail to understand it so long as they, in such an intellectual process, trust imagination more than intellect.

A last objection remains. If a material element has an indefinite

sphere of activity, its active power will possess a virtual ubiquity. Now, virtual ubiquity is an infinite perfection, an attribute which belongs to God alone, on account of His immensity. To this we answer, that virtual ubiquity may have two meanings. First it may mean that which is the primary source of all possibility of ubications, i. e. that which eminently contains in itself all ubications that can be : and in this sense God alone, on account of His immensity, has an absolute and substantial ubiquity. But virtual ubiquity may mean also something which is not the primary source of all possible ubications, but only the property of a being that reaches by its action an object having any ubication whatever : and in this sense the power of a material element may be said to have virtual ubiquity. By this alone everyone will see, that there is no danger of confounding the virtual ubiquity of created power with the immensity of God. Divine immensity has been ingeniously defined by a philosopher " A sphere of which the centre is everywhere :" the power of a material element is "a sphere of which the centre is in a single point." This remark might suffice to show the insignificance of the objection. We might add that, whilst Divine Substance is to be found whole and infinite in every possible ubication, without any imaginable loss, gradual diminution, or successive attenuation, a material element on the contrary, besides being present substantially only in one point, has a power which loses continually in intensity as its virtual ubications are increased, till millions of millions of them are required to produce the least sensible effect. In other words, the virtuality or intensity of such a power tends continually towards zero as its limit, although it never reaches it. And, as a decreasing series, though implying an infinity of terms, may have a finite value, so the virtuality of material powers, though extending after its own manner beyond any finite limit, represents only a finite property of a finite being.

The objection could be answered also by means of another consideration. The virtual ubiquity of material power, as compared with relative space (which is conceived through sensible representations) seems quite an incredible thing in a creature : but, when compared with absolute space as it is apprehended by the intellect, loses so much of its mysterious aspect, that it seems almost to sink into a nonentity. I shall not develope this consideration, because it would lead us into a series of abstractions of too metaphysical a nature.

Proposition IX.

A material element attracts or repels any distant element independently of any material medium of communication.

DEMONSTRATION. If material substance acts at a distance without its power or its action being transmitted from place to place, then the action of a material element does not depend on any medium of communication. But material substance acts at a distance without its power or its action being transmitted from place to place. Therefore, a material element acts independently of any material medium. The major of this argument is self-evident. The minor can be proved in two different ways. First, because the power and the action of an element are, of their own nature, intransmissible; secondly, because, were they transmissible, no medium of transmission is to be found. As to the first, we know that whatever is transmitted from place to place, is transmitted by local motion: but power and action cannot receive local motion, as is evident; for, capability of receiving local motion is the exclusive property of *the* matter. Therefore, neither power nor action can travel, and neither power nor action can be transmitted from place to place or from matter to matter. With regard to the second reason: if a medium were to be found for transmitting power or the exertion of power, it would be a material one, as is admitted by our opponents; otherwise it would not move from place to place, as is necessary for communicating the exertion. But matter is not a fit medium for such a transmission. For, that which is taken as a medium for transmitting power or action must acquire a transient activity; but matter is not capable of receiving activity as such, but only its accidental act, or the momentum of motion, which constitutes the quantity of its motion; and, therefore, matter is not a fit medium for transmitting action or power. Therefore the action of matter is independent of any material medium of communication. Q. E. D.

COROLLARY I. Therefore, all bodies act, by attracting or repelling, with equal promptitude and without loss of time, whether the distance of the object acted upon be great or small.

And, in fact, time follows motion ; now, the active power of bodies and their action does not reach the distant body through motion, i.e. by successive transmission ; on the contrary, each element is of its own nature prepared to affect *immediately* every other material element existing in its indefinite sphere of activity. Hence a body will indeed act with greater intensity at a less distance, but not sooner than at a greater distance.

COROLLARY II. Therefore, the opinion of those who assume that solar attraction wants time for reaching the earth or any other planet, is wholly without foundation, nay, quite inadmissible.

COROLLARY III. Therefore, though we are wont to say that actions are transmitted, yet this is not true of the actions themselves, but only expresses the fact of a progressive development in the series of effects resulting from those actions. In the same way, we say often that actions are conveyed through a material medium : but the meaning of this expression is simply this, that a material medium is indispensable for the progressive development of the aforesaid series of effects. Thus, if, a mass of air being at rest, a string is stretched in order to elicit sound, the vibrations of the string will be communicated to the neighbouring molecules of air by the action (not by the motion) of the string itself : these first molecules being thrust out of their position of equilibrium will by their action (viz. by the exertion of a power residing in each of their own elements, not of a power coming from the string, nor by their motion, nor by transmitted action) put in motion a following set of molecules ; and so on indefinitely : so that in the whole series of molecular vibrations each preceding molecule causes the motion of the following, and causes it by the exertion of its own powers, not of any power transmitted. It is evident, that the string cannot give activity to the molecules of air. These molecules, whether the string vibrates or not, have already their own activity and their own mutual action : only their actions balance each other so long as the mass of air is at rest. But, when the string begins to vibrate, the equilibrium being broken near it, those molecules of air which first cease to be in equilibrium begin to act on the following

molecules with a different intensity due to the change of distances. What we say of air and sound, is to be said of any other such medium, as v. gr. æther and its vibrations either luminous or calorific. The molecules of æther have their own powers and exert them, whether there exists a flame determining a series of vibrations or not : but with the flame, the first molecules of æther which are displaced from their position of equilibrium will acquire a new local relation with regard to the following, and their actions will be of a new intensity, sufficient to cause the displacement of the next set of molecules, and so on. The flame, then, causes the displacement of the first set of molecules : the first set displaced causes the displacement of the second : the second displaced causes the displacement of the third, &c.; each set producing its own effect by its own powers, not by the exertion of any power communicated to them by the flame, and their displacement being not a cause, but only a condition, on which the intensity of their exertion depends. Hence it appears that in this sort of phenomena it is not the action, much less the power, that is transmitted, but only the motion or the perturbation of equilibrium : and even motion is not properly *transmitted*, but only *propagated;* because the motion of each following molecule is not the identical motion of each preceding one, but is a motion really produced in the very impact of the one on the other, as we have proved above (Book I. Prop. IV). And, therefore, one motion succeeds another indefinitely, the one being a condition for the existence of the other : which constitutes propagation, not properly transmission.

BOOK III.

ON THE MOTION OF SIMPLE ELEMENTS.

WE have established that bodies are ultimately composed of simple elements or material unextended points: that elements are all equally inert: that these elements are of two kinds only, i.e. either attractive or repulsive: that an attractive element attracts always and at all distances, as also a repulsive element always and at all distances repels: that all the elements act with intensities inversely proportional to the squared distances, whether such distances be astronomical or molecular. We know, then, what is material substance as such, and what its dynamical constitution. We may now proceed to a first application of these general principles, by trying to determine the laws of motion and equilibrium of a system of material points. This we shall do in a series of theorems and problems. The simplest systems of elements consist of two material points: from these we shall begin.

THEOREM I.

Two attractive elements A *and* B (fig. 1) *of equal power, being originally at rest, will, in consequence of mutual action, make vibrations of an equal and constant amplitude* AB *through the point* C *taken in the middle of* AB.

DEMONSTRATION. The action of *A* upon *B* being equal, by the hypothesis, to that of *B* upon *A*, the two elements must approach each other with equal velocities till they meet at the · point *C*, which will be in the middle of *AB*. Now, when the two elements meet in *C*, the velocity, which they have acquired in the whole time employed by them in approaching, is not extinguished. For, no velocity can be extinguished except by an exertion of power

producing an equal and opposite velocity : but, in our case, there is no possibility of any such exertion; first, because the encountering of the two elements takes place in a single instant of time; a circumstance which excludes all possibility of producing finite velocity (Book I. Prop. II); then, because distance is an indispensable condition of the action of matter upon matter (Book I. Prop. III): now, in our case, the elements A and B have already reached the point C, and therefore no distance remains between them. Since, then, the velocity with which A and B meet is by no means destroyed in their meeting, and two elements having their velocity perfectly intact cannot possibly remain at rest, we must needs conclude, that A and B will, after their meeting, continue their course in the direction of their actual velocities, till the same velocities become extinct. Such an extinction will take place by degrees, in the same way as also the velocities have been imparted, but in an inverse order. And, indeed, the mutual attraction, which before the meeting of A with B produced mutual acceleration, after their meeting must produce a mutual retardation, the action being now directly opposite to the actual velocities. Hence the element A will continue its course till it reaches the place B, where its velocity becomes $= 0$: and in the same manner the element B will continue its course till it reaches the place A, where its velocity also will be exhausted. After this excursion, the elements A and B will be, with regard to one another, in the same condition as they were at the beginning. Their mutual attraction will cause a new approach, a new meeting in C, a new retardation, &c.; and so there will be a second excursion, then a third, a fourth, and so on : the amplitude of each excursion being always AB. Q.E.D.

SCHOLIUM. In the demonstration of this theorem we have stated that in the meeting of two attractive elements their velocities cannot be destroyed. Those who are wont to believe the contrary ought to remember, that *the matter* of the element A cannot resist the motion of the element B, whatever our prejudices may suggest to the contrary. Every resistance is action: every element acts by reason of its power, not of its matter, which is only a geometric centre of activity, or a terminus of direction, having no other reality than that of the power of which it is the centre. See what we have said on this subject in the

preceding books. Again, the power, by which the motion of the two elements could be stopped, ought to be repulsive; otherwise it would not be able to extinguish the velocities with which they meet. Now, in our case, no repulsive powers exist, since A and B are both attractive. Therefore, the velocities of A and B cannot possibly be extinguished in the meeting of the two elements.

Perhaps, some will say, that equal and opposite velocities must destroy each other. But this is not the case. Equal and opposite velocities, when impressed on one and the same material point, must undoubtedly neutralise each other, for this reason, that a single point cannot move at the same time in two opposite directions. But our theorem regards two distinct material points having their own distinct velocities, and perfectly capable of following two distinct directions. We might add, that, properly speaking, it is not even true that two equal and opposite *velocities* do destroy each other, but only that two *actions*, which either simultaneously or successively communicate two equal and opposite velocities to the same material point, neutralise the effect of each other, by the impossibility in which the material point is of moving in two directions. Velocity does not act, as we have proved (Book I. Prop. IV); and, therefore, one velocity cannot efficiently destroy another, though the one cannot coexist with the other.

THEOREM II.

Two attractive elements A *and* B *(fig. 2) of unequal power, being originally at rest, will, in consequence of mutual action, make vibrations of constant, but unequal, amplitude through a point* C *taken between* A *and* B, *but nearer to* A *or to* B *according as* A *or* B *is the more powerful of the two.*

DEMONSTRATION. If the element A has a greater power than the element B, then B will be more attracted by A than A can be attracted by B. Accordingly, B will acquire a greater velocity than A. Hence the two elements will not meet at the middle of the distance AB, but at a point C, which will be nearer to A in proportion as the power of A is greater than that of B. When they meet, their velocities are not altered, as we have just shown ; and so they will continue to move in their respective directions, but with retarded motion on account of their mutual action being

now opposed to their actual velocities. Hence the element A will go on from C towards A', and the element B from C towards B'; so that, when they have respectively measured the spaces $CA' = CA$ and $CB' = CB$, their velocities will be exhausted. After this first excursion, the elements A and B will be, with regard to each other, in the same condition as they were at the beginning. Their mutual attraction will produce a new approach, a new encounter in C, and then a new retardation, &c. The amplitude of the excursions will be constantly AA' for the element A, and BB' for the element B. Q. E. D.

SCHOLIUM. One might ask: Is there any material element possessing a greater power than any other? This question cannot be answered in the present state of science. Still, it would be rashness, on our part, to assume, without necessity or indication of any kind, that all elements have equal power: and consequently, in a general treatise like this, we must admit as possible a difference in the degree of power for different elements. If, in the application of the general formulas of dynamics to the molecular phenomena, we were to find out that all elements have powers of equal intensity, such a discovery (which I deem highly improbable) would indeed simplify our mechanical results: but we have no right meanwhile to assume what is not demonstrated, and probably never will be.

THEOREM III.

In any system of two attractive elements of equal power, the vibrations due to mutual actions are such, that the cube of their amplitude is directly as the square of the time employed, and inversely as the power.

DEMONSTRATION. Let t be the time employed by A in measuring the space AD (fig. 1), and by B in measuring the space BE. Let $AC = BC = a$, $AD = BE = x$. Let v be the action of each element at the unit of distance and for the unit of time. The mutual acceleration, after the time t, will be

$$\frac{d^2x}{dt} = \frac{vdt}{DE^2} = \frac{vdt}{4(a-x)^2}.$$

This equation, being first multiplied by $\dfrac{dx}{dt}$, then integrated, gives

$$\frac{1}{2}\left(\frac{dx}{dt}\right)^2 = \frac{v}{4\,(a-x)} + C.$$

If the time t is reckoned from the beginning of motion, then for $t = 0$ we shall have $x = 0$, and $\dfrac{dx}{dt} = 0$. Therefore

$$C = -\frac{v}{4a}.$$

Hence the preceding equation will give, after reduction,

$$\frac{dx}{dt} = \sqrt{\frac{vx}{2a\,(a-x)}}.$$

The second member is taken as positive, because both x and t increase together, and therefore dx and dt are of the same sign.

From the last equation we obtain

$$dt = dx\sqrt{\frac{2a}{v}}\sqrt{\frac{a-x}{x}};$$

and this, by making $\sqrt{\dfrac{a-x}{x}} = \dfrac{1}{z}$, will be transformed into

$$dt = \frac{2a\,dz}{(1+z^2)^2}\sqrt{\frac{2a}{v}},$$

which, being integrated, gives

$$t = a\sqrt{\frac{2a}{v}}\left\{\frac{z}{1+z^2} + \tan^{-1} z + C\right\},$$

or

$$t = \sqrt{\frac{2a}{v}}\left\{\sqrt{x\,(a-x)} + a\,.\tan^{-1}\sqrt{\frac{x}{a-x}} + C\right\}.$$

In order to determine C, we may remark that for $t = 0$, we have $x = 0$; and therefore

$$0 = a\,.\tan^{-1} 0 + C;$$

now, the arcs, whose tangent is $= 0$, are without number, viz.

$$0, \ \pm \pi, \ \pm 2\pi, \ \pm 3\pi, \ \ldots\ldots$$

both positive and negative. The negative, if introduced into our equation, would tend to diminish, and the positive to augment, the value of t by constant quantities. The negative imply the hypothesis that the time t begins after a number of excursions of the elements: the positive imply the hypothesis that the time t extends to a number of excursions made before that one, which is actually under consideration, and of which x is a portion. Since, then, we take the beginning of our excursion as the beginning of the time t, we must neither diminish, nor augment the time t by those constant quantities. In other words, we must take $C = 0$. We have then

$$t = \sqrt{\frac{2a}{v}} \left\{ \sqrt{x\,(a-x)} + a \cdot \tan^{-1} \sqrt{\frac{x}{a-x}} \right\}.$$

When the elements meet in C, we have $x = a$. Let t' be what t becomes at that instant; it will be

$$t' = \sqrt{\frac{2a}{v}} \cdot a \cdot \tan^{-1} \infty = \frac{a\pi}{2} \sqrt{\frac{2a}{v}};$$

hence, if T be the time employed in the whole excursion from A to B, we shall have

$$T = \pi \sqrt{\frac{2a^3}{v}}.$$

For another couple of oscillating elements, we shall have evidently

$$T' = \pi \sqrt{\frac{2a'^3}{v'}};$$

whence, in general,

$$T^2 \ : \ T'^2 \ :: \ \frac{a^3}{v} \ : \ \frac{a'^3}{v'}; \quad \text{Q.E.D.}$$

COROLLARY I. If $v = v'$, then $T^2 \ : \ T'^2 \ :: \ a^3 \ : \ a'^3$; i.e. for two couples of elements of equal powers, the cubes of the amplitudes are directly as the squares of the times employed.

COROLLARY II. If $T = T'$, then $a^3 : a'^3 :: v : v'$; i.e. for two couples making isochronous oscillations, the intensities of the powers are directly as the cubes of the amplitudes.

COROLLARY III. If $a = a'$, then $T^2 : T'^2 :: v' : v$; i.e. for two couples which oscillate through equal amplitudes, the intensities of the powers are inversely as the squares of the times employed.

THEOREM IV.

Two attractive elements of unequal power, in consequence of mutual action, make oscillations, the amplitudes of which are directly as the powers by which they are determined.

DEMONSTRATION. Let A and B (fig. 2) be the two elements, v and w their respective powers, or rather their measures, i.e. their actions at the unit of distance and for the unit of time. Let $AC = a$, $BC = b$, and thus $AB = a + b$; and let x and y be the spaces measured by A and B respectively in the time t. The equations of motion will be

$$\frac{d^2x}{dt} = \frac{wdt}{(a+b-x-y)^2}, \quad \frac{d^2y}{dt} = \frac{vdt}{(a+b-x-y)^2};$$

hence

$$\frac{\dfrac{d^2x}{dt}}{\dfrac{d^2y}{dt}} = \frac{w}{v}, \text{ or } v \cdot d\left(\frac{dx}{dt}\right) = w \cdot d\left(\frac{dy}{dt}\right);$$

whence, by integrating, we shall obtain

$$\frac{vdx}{dt} = \frac{wdy}{dt} + C, \text{ and } vx = wy + Ct + C'.$$

But, since for $x = 0$ it is also $y = 0$, $\dfrac{dx}{dt} = 0$, $\dfrac{dy}{dt} = 0$, we shall have $C = 0$, $C' = 0$; and consequently

$$x : y :: w : v.$$

And as a is a particular value of x, and b a particular value of y, we shall have also

$$a : b :: w : v; \quad \text{Q. E. D.}$$

REMARK.   From the two last proportions we obtain

$$x \,:\, y \,::\, a \,:\, b, \quad x = \frac{ay}{b}, \quad y = \frac{bx}{a};$$

hence, by substituting these values of x and of y in the equations of motion respectively, we shall have, after reduction,

$$\frac{d^2x}{dt} = \frac{a^2}{(a+b)^2} \cdot \frac{w\,dt}{(a-x)^2}, \quad \frac{d^2y}{dt} = \frac{b^2}{(a+b)^2} \cdot \frac{v\,dt}{(b-y)^2}.$$

These equations give

$$\frac{dx}{dt} = \frac{a}{a+b} \sqrt{\frac{2w}{a} \cdot \frac{x}{a-x}}, \quad \frac{dy}{dt} = \frac{b}{a+b} \sqrt{\frac{2v}{b} \cdot \frac{y}{b-y}},$$

whence, by the same method of integration made use of in Theorem III., we obtain

$$t = \frac{a+b}{a} \sqrt{\frac{a}{2w}} \left\{ \sqrt{x\,(a-x)} + a \cdot \tan^{-1} \sqrt{\frac{x}{a-x}} \right\},$$

$$t = \frac{a+b}{b} \sqrt{\frac{b}{2v}} \left\{ \sqrt{y\,(b-y)} + b \cdot \tan^{-1} \sqrt{\frac{y}{b-y}} \right\}.$$

Let t' be what t becomes when the elements meet in C; in that instant we have $x = a$, $y = b$; hence

$$t' = (a+b)\frac{\pi}{2} \sqrt{\frac{a}{2w}}, \quad \text{and} \quad t' = (a+b)\frac{\pi}{2} \sqrt{\frac{b}{2v}};$$

and therefore the time T of a whole excursion from A to A' and from B to B' will be

$$T = (a+b)\,\pi \sqrt{\frac{a}{2w}} = (a+b)\,\pi \sqrt{\frac{b}{2v}}.$$

THEOREM V.

Two elements A *and* R *(fig. 3) of an equal degree of power, but the first attractive, the second repulsive, will, under mutual action, move with uniformly accelerated velocity on the side of the attractive element* A, *and remain, in the whole of their course, equally distant from one another.*

DEMONSTRATION. Let v be the intensity of the action of A at the unit of distance and for the unit of time. The action of A upon R in the instant dt will be

$$\frac{d^2x}{dt} = \frac{vdt}{AR^2}.$$

So also the action of R upon A will be

$$\frac{d^2x}{dt} = \frac{vdt}{AR^2};$$

and consequently R is as much urged to approach A, as A to recede from R. Therefore the distance AR will remain constant. Let $AR = a$; we shall have

$$\frac{d^2x}{dt} = \frac{vdt}{a^2}, \quad \text{whence} \quad \frac{1}{2}\left(\frac{dx}{dt}\right)^2 = \frac{vx}{a^2};$$

and then

$$dt = \frac{adx}{\sqrt{2vx}}, \quad \text{and} \quad t = 2a\sqrt{\frac{x}{2v}};$$

and consequently

$$x = \frac{v}{a^2} \cdot \frac{t^2}{2},$$

which expresses the law of uniformly accelerated motion. Q.E.D.

THEOREM VI.

An attractive element A *and a repulsive element* R (fig. 4), *if* A *is the more powerful of the two, will oscillate through one another by constant oscillations of unequal amplitude.*

DEMONSTRATION. Since attraction is here greater than repulsion, by the hypothesis, the element R will approach A more than A can recede from R. Hence the two elements will become nearer and nearer, and at last meet at a point C, with different velocities, since R has been acted upon more intensely than A. The element R, therefore, when passing through the point C, will have a greater velocity than the element A at the same moment. After passing through C, the attractive action of A tends to retard the advance of R towards R', and in like manner the re-

pulsive action of R tends to retard the advance of A towards A.'
And, since these retardations follow the same law as the preceding
accelerations, the space measured by R from the point C to the
point R', and the space measured by A from the point C to the
point A', will be respectively equal to the spaces RC and AC
measured by the same elements before meeting. When the ele-
ments R and A reach the places R' and A' respectively, their
velocities will be exhausted. Hence, they will begin a second
excursion, recross C, and return to R and A respectively, whence
they will again and again make other excursions of the same
amplitudes as before. Q. E. D.

REMARK. Let $AC = a$, $RC = b$, and consequently $AR = b - a$;
let x and y be the spaces gone over by A and R respectively
in the time t, and v and w the actions of A and R respectively at
the unit of distance, and for the unit of time. We shall have

$$\frac{d^2x}{dt} = \frac{wdt}{(b-a+x-y)^2}, \qquad \frac{d^2y}{dt} = \frac{vdt}{(b-a+x-y)^2}.$$

These equations, by the method employed in Theorem III., may
be transformed into

$$\frac{d^2x}{dt} = \frac{a^2}{(b-a)^2} \cdot \frac{wdt}{(a-x)^2}, \qquad \frac{d^2y}{dt} = \frac{b^2}{(b-a)^2} \cdot \frac{vdt}{(b-y)^2},$$

which by two successive integrations will give

$$t = \frac{b-a}{a} \sqrt{\frac{a}{2w}} \left\{ \sqrt{x(a-x)} + a \cdot \tan^{-1}\sqrt{\frac{x}{a-x}} \right\},$$

$$t = \frac{b-a}{b} \sqrt{\frac{b}{2v}} \left\{ \sqrt{y(b-y)} + b \cdot \tan^{-1}\sqrt{\frac{y}{b-y}} \right\}.$$

Let t' be what t becomes when the elements meet in C. In
that instant, we have $x = a$, $y = b$; hence

$$t' = \frac{b-a}{2} \pi \sqrt{\frac{a}{2w}} = \frac{b-a}{2} \pi \sqrt{\frac{b}{2v}};$$

and, therefore, the time T of a whole excursion will be for both
the elements

$$T = (b-a) \pi \sqrt{\frac{a}{2w}} = (b-a) \pi \sqrt{\frac{b}{2v}}.$$

THEOREM VII.

Two attractive elements A *and* B *having an equal power* v *will oscillate through a repulsive element* R *having a power* w, *placed between them at the middle of* AB (fig. 5), *whenever the inequality* v > 4w *subsists; and the oscillations will have a constant amplitude* AB.

DEMONSTRATION. The element A is attracted by B, as B is attracted by A; but both are repelled by R. Therefore, the acceleration will be, both for A and for B, the difference of the two actions. Hence, if we make $AR = a$, $AE = x$, the acceleration, after the time t, will be

$$\frac{d^2x}{dt} = \frac{vdt}{4\,(a-x)^2} - \frac{wdt}{(a-x)^2}.$$

From this equation we obtain

$$\frac{dx}{dt} = \sqrt{\frac{x\,(v-4w)}{2a\,(a-x)}},$$

which, being resolved and integrated, gives

$$t = \sqrt{\frac{4a}{v-4w}} \left\{ \sqrt{x\,(a-x)} + a \cdot \tan^{-1} \sqrt{\frac{x}{a-x}} \right\}.$$

Let t' be what t becomes when the elements meet in R. In that instant we have $x = a$; and consequently

$$t' = \frac{a\pi}{2} \sqrt{\frac{4a}{v-4w}};$$

and, consequently, the time T of the whole excursion will be

$$T = \pi \sqrt{\frac{4a^3}{v-4w}}.$$

Now, when $v = 4w$, then $t' = \infty$; because, in fact, the elements A and B would not move at all. When $v < 4w$, then t' becomes imaginary; because the elements A and B, instead of meeting, would recede in opposite directions with an increasing velocity. But when the inequality $v > 4w$ subsists, then the value of t' is real and finite; and therefore, when this inequality subsists, the

two elements oscillate through the element R with oscillations of a constant amplitude.   Q.E.D.

REMARK.   If the element R were attractive instead of repulsive, the law of motion of the elements A and B would be obtained by simply changing the sign of w in the preceding equations.

THEOREM VIII.

Two repulsive elements R *and* R′ (fig. 6) *having an equal power* w *will oscillate through an attractive element* A *having the power* v, *and placed between them at the middle of* RR′, *whenever the inequality* 4v > w *subsists; and the oscillations will have a constant amplitude* RR′.

DEMONSTRATION.   In this example, which is the reverse of the preceding, making $AR = a$, $RE = x$, the acceleration, after the time t, will be expressed by

$$\frac{d^2x}{dt} = \frac{vdt}{(a-x)^2} - \frac{wdt}{4(a-x)^2};$$

whence

$$\frac{dx}{dt} = \sqrt{\frac{x(4v-w)}{2a(a-x)}}.$$

This equation, being integrated, gives

$$t = \sqrt{\frac{4a}{4v-w}} \left\{ \sqrt{x(a-x)} + a \cdot \tan^{-1}\sqrt{\frac{x}{a-x}} \right\}.$$

Let t' be what t becomes when the elements meet in A, or when $x = a$.   We shall have

$$t' = \frac{a\pi}{2}\sqrt{\frac{4a}{4v-w}}.$$

The time T of the whole excursion from R to R' will be

$$T = \pi\sqrt{\frac{4a^3}{4v-w}}.$$

It is evident, that when $4v > w$, then t' is real and finite. Therefore, whenever this inequality subsists, the elements R and

R' will meet in A, and consequently oscillate through A with oscillations of a constant amplitude.   Q.E.D.

THEOREM IX.

If two repulsive elements R *and* R' *having an equal power* v, *are moving towards one another directly and with equal and opposite velocities, they will at length deprive one another of their velocities, and then retrace their way back, with velocities which at every point of their regressive course will be equal and opposite to the velocities which they possessed at the same points when they were approaching.*

DEMONSTRATION.   Let $2a$ be the distance of the two elements at the moment in which the time t begins, and let u be their velocity at that moment.   If x is the space measured by each element during the time t, the action of one element upon the other after the time t will be

$$\frac{d^2x}{dt} = -\frac{vdt}{4\left(a-x\right)^2};$$

whence

$$\frac{1}{2}\left(\frac{dx}{dt}\right)^2 = -\frac{v}{4\left(a-x\right)} + C.$$

The constant C will be determined from the conditions

$$x = 0, \quad \frac{dx}{dt} = u;$$

from which we obtain

$$C = \frac{1}{2}u^2 + \frac{v}{4a},$$

a..d consequently

$$\frac{dx}{dt} = \sqrt{\frac{2a^2u^2 - \left(v + 2au^2\right)x}{2a\left(a-x\right)}} \quad \text{.............. (1),}$$

where we take the positive sign before the root, because dx and dt are both of the same sign.

When $\dfrac{dx}{dt} = 0$, the preceding equation gives a particular value of x, which represents the whole space gone over by one of the

elements in approaching the other; for, the last instant of their approaching is that, for which their velocity becomes $= 0$. If this peculiar value of x be called b, we shall find

$$b = \frac{2a^2u^2}{v + 2au^2} \quad\dots\dots\dots\dots\dots\dots\dots\dots (2).$$

Hence, the expression (1) may be put under the form

$$\frac{dx}{dt} = \sqrt{\frac{\frac{2a^2u^2}{v + 2au^2} - x}{\frac{2a^2u^2}{v + 2au^2} \times \frac{a - x}{au^2}}} = \sqrt{\frac{b - x}{\frac{b(a - x)}{au^2}}} = u\sqrt{\frac{a}{b}}\sqrt{\frac{b - x}{a - x}};$$

and consequently we shall have

$$dt = \frac{dx}{u}\sqrt{\frac{b}{a}}\sqrt{\frac{a - x}{b - x}},$$

which gives

$$t = -\frac{a - b}{u}\sqrt{\frac{b}{a}}\left\{\sqrt{\frac{(a - x)(b - x)}{a - b}} + \tfrac{1}{2}\log\left(\frac{1 + \sqrt{\frac{b - x}{a - x}}}{1 - \sqrt{\frac{b - x}{a - x}}}\right) + C\right\}.$$

The constant C will be determined by the condition $x = 0$, which entails $t = 0$, and

$$C = -\sqrt{\frac{ab}{a - b}} - \tfrac{1}{2}\log\left(\frac{1 + \sqrt{\frac{b}{a}}}{1 - \sqrt{\frac{b}{a}}}\right).$$

Hence, we shall find

$$t = \frac{a - b}{2u}\sqrt{\frac{b}{a}}\left\{\frac{\sqrt{ab} - \sqrt{(a - x)(b - x)}}{\sqrt{a - b}}\right.$$
$$\left. + \tfrac{1}{2}\log\left(\frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}}\right)\left(\frac{\sqrt{a - x} - \sqrt{b - x}}{\sqrt{a - x} + \sqrt{b - x}}\right)\right\}.$$

Let t' be the whole time employed in approaching; its value will be obtained by making $x = b$ in the preceding equation; so that it will be

$$t' = \frac{a - b}{2u}\sqrt{\frac{b}{a}}\left\{\frac{\sqrt{ab}}{\sqrt{a - b}} + \tfrac{1}{2}\log\frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}}\right\} \quad\dots\dots\dots (3).$$

Let us consider now the motion of the two elements after the total extinction of their primitive velocities. From what we have just shown, it is evident, that, when u becomes $= 0$, the distance between the two elements is $= 2(a-b)$. Then repulsion begins to prevail, and, accordingly, the elements begin to recede from one another. Let us reckon the time t from this instant. If x is the space measured by one element in the time t, we shall have the equation

$$\frac{d^2x}{dt} = \frac{vdt}{4(a-b+x)^2},$$

which gives

$$\frac{1}{2}\left(\frac{dx}{dt}\right)^2 = -\frac{v}{4(a-b+x)} + C.$$

And, since for $x = 0$ we have also $\dfrac{dx}{dt} = 0$, we shall find

$$C = \frac{v}{4(a-b)};$$

hence, by substituting this value, we shall obtain

$$\frac{dx}{dt} = \sqrt{\frac{vx}{2(a-b)(a-b+x)}};$$

and consequently

$$dt = dx\sqrt{\frac{2(a-b)}{v}}\sqrt{\frac{a-b+x}{x}},$$

and

$$t = \frac{a-b}{2}\sqrt{\frac{2(a-b)}{v}}\left\{\sqrt{\frac{(a-b+x)x}{a-b}} + \tfrac{1}{2}\log\frac{\sqrt{a-b+x}+\sqrt{x}}{\sqrt{a-b+x}-\sqrt{x}} + C\right\}.$$

Now, since for $t = 0$ we have also $x = 0$, we shall have

$$C = \tfrac{1}{2}\log 1 = 0;$$

therefore

$$t = \frac{a-b}{2}\sqrt{\frac{2(a-b)}{v}}\left\{\sqrt{\frac{(a-b+x)x}{a-b}} + \tfrac{1}{2}\log\frac{\sqrt{a-b+x}+\sqrt{x}}{\sqrt{a-b+x}-\sqrt{x}}\right\}.$$

Let t' be the time employed by one of the elements in measuring the space b. By making $x = b$, we shall obtain

$$t' = \frac{a-b}{2} \sqrt{\frac{2(a-b)}{v}} \left\{ \sqrt{\frac{ab}{a-b}} + \tfrac{1}{2} \log \frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}} \right\} \dots \dots (4).$$

This value of t' is equal to that obtained above in the equation (3). And, in fact, the equation (2) gives us

$$\frac{1}{u} \sqrt{\frac{b}{a}} = \sqrt{\frac{2(a-b)}{v}} \; ;$$

and this relation makes the equations (3) and (4) identical. Hence also the velocity

$$\frac{dx}{dt} = \sqrt{\frac{vx}{2(a-b)(a-b+x)}},$$

which, in the case of $x = b$, becomes

$$\frac{dx}{dt} = \sqrt{\frac{b}{a}} \sqrt{\frac{v}{2(a-b)}},$$

will, on account of the same relation, be reduced to

$$\frac{dx}{dt} = \sqrt{\frac{b}{a}} \times u \sqrt{\frac{a}{b}} = u.$$

Therefore the two elements, after having come back through a space b, will have again (but in an opposite direction) the velocity u, which they possessed at the same point when they were approaching. Q.E.D.

THEOREM X.

When two repulsive elements R and R' having an equal power v are driven against one another directly and with a velocity u, as in the above theorem, the total amount of action, by which that initial velocity is destroyed, is precisely equal to the same initial velocity.

DEMONSTRATION. The total amount of action of the element R upon the element R' may be obtained by integrating the expression

$$\frac{vdt}{4(a-x)^2}$$

between the limits $x = 0$ and $x = b$, as is evident from what we have premised in the preceding theorem. Now, we have obtained there

$$dt = \frac{dx}{u} \sqrt{\frac{b}{a}} \sqrt{\frac{a-x}{b-x}};$$

therefore

$$\int \frac{vdt}{4(a-x)^2} = \frac{v}{4u} \sqrt{\frac{b}{a}} \int \frac{dx}{\sqrt{(b-x)(a-x)^3}}.$$

Let $a - x = \dfrac{1}{z^2}$, whence

$$x = \frac{az^2 - 1}{z^2}, \quad dx = \frac{2dz}{z^3};$$

we shall have

$$\int \frac{vdt}{4(a-x)^2} = \frac{v}{4u} \sqrt{\frac{b}{a}} \int \frac{2zdz}{\sqrt{1-(a-b)z^2}}.$$

Let the second member of this equation be multiplied and divided by $a - b$; then, by making our integration, we obtain

$$\int \frac{vdt}{4(a-x)^2} = -\frac{v}{2u} \sqrt{\frac{b}{a}} \left\{ \frac{\sqrt{1-(a-b)z^2}}{a-b} + C \right\};$$

and, replacing z^2 by $\dfrac{1}{a-x}$,

$$\int \frac{vdt}{4(a-x)^2} = -\frac{v}{2u(a-b)} \sqrt{\frac{b}{a}} \left\{ \sqrt{1-\frac{a-b}{a-x}} + C \right\}.$$

When $x = 0$, then also $t = 0$, and the first member of the equation becomes also $= 0$. Hence we obtain

$$C = -\sqrt{\frac{b}{a}};$$

and consequently

$$\int_0^x \frac{vdt}{4(a-x)^2} = -\frac{v}{2u(a-b)} \sqrt{\frac{b}{a}} \left\{ \sqrt{1-\frac{a-b}{a-x}} - \sqrt{\frac{b}{a}} \right\}.$$

If we make now $x = b$, our integral will become

$$\int_0^b \frac{vdt}{4(a-x)^2} = \frac{1}{u} \cdot \frac{bv}{2a(a-b)};$$

but the relation (2) gives

$$\frac{bv}{2a(a-b)} = u^2;$$

therefore

$$\int_0^b \frac{vdt}{4(a-x)^2} = \frac{u^2}{u} = u.$$

Therefore the total amount of exertion is equal to the initial velocity. Q.E.D.

COROLLARY. Therefore the total amount of *work*, which can be done by a material point advancing against a continuous resistance, is by no means to be confounded with the total amount of *action* by which it can destroy a velocity equal to its own, or by which the resistance exhausts its velocity. In fact, the quantity of *work*, as admitted in mechanical treatises, is a product of three factors, viz. of a moving mass m, of a resistance R, and of the space x measured by the mass under such a resistance. Hence, the differential expression of the *work* W would be in general

$$dW = mRdx.$$

In our case,

$$m = 1, \quad Rdt = -\frac{vdt}{4(a-x)^2},$$

whence

$$R = -\frac{v}{4(a-x)^2};$$

and, therefore,

$$dW = -\frac{vdx}{4(a-x)^2},$$

and consequently

$$W = -\frac{v}{4(a-x)} + C.$$

Now, when $x = 0$, then also $W = 0$; and so we obtain

$$C = \frac{v}{4a};$$

whence

$$W = -\frac{v}{4\,(a-x)^2} + \frac{v}{4a} = -\frac{1}{4} \cdot \frac{vx}{a\,(a-x)};$$

which expression, when $x = b$, on account of the relation (2), becomes

$$W = -\frac{1}{2}u^2.$$

BOOK IV.

DYNAMICAL CONSTITUTION OF
PRIMITIVE POLYHEDRIC SYSTEMS OF ELEMENTS.

FROM the examples which we ·have given in the preceding Book we may have seen how material elements, by acting on one another, can constitute a system of points connected by dynamical ties. The systems, which we have hitherto considered, consisted of two or three elements only; we shall now make a step further. We intend to resolve in this Book a series of problems regarding. the dynamical relations of any number of elements constituting a regular polyhedric system. The solution of such problems will facilitate that of others more complicated which will follow: and thus the reader, we hope, will be enabled to understand more clearly what we are to establish hereafter about the molecular constitution of bodies.

PROBLEM I.

Four repulsive elements A, B, C, D (fig. 7) *having equal powers* w *are so arranged as to form a regular tetrahedron around an attractive centre* O *which has a power* v. *Find the dynamical formula of this system.*

SOLUTION. Let us remark first, that any proposed regular system may be represented by what I shall call a *material for-mula*, as well as by a dynamical one. We can represent ma-terially our present system by the equation $m = 1 + 4$, or rather the equation

$$m = A + 4R,$$

in which the letter A is meant to indicate *attractivity*, and the letter R *repulsivity*. This formula is to be read as follows: The

mass m of the system consists of *one* attractive element, and of *four* repulsive. This kind of notation we shall employ with great advantage in the progress of this work to express briefly and neatly the material composition of any regular system. Let us come now to our problem.

Each of the elements surrounding the centre O is exposed to four actions. The element A, e.g. is exposed to one attractive action from O, and three repulsive from B, C, D. Let us take those actions as positive, which tend to augment the distance OA, or the radius of the system; and those as negative, which tend to diminish that distance: and let $OA = r$. Then, the attractive action of O upon A will be expressed by

$$- \frac{v}{r^2}.$$

The three repulsive actions of B, C, D on A, as being equal, give the resultant

$$+ \frac{3w}{AB^2} \cos BAO.$$

Let us take a point I at the middle of CD, and draw BI. Let us mark the point k, where the prolongation of AO meets BI, and let us draw the radius BO. We shall find

$$\cos BAO = \frac{Ak}{AB} = \frac{r + \frac{1}{3} r}{\sqrt{Ak^2 + Bk^2}} = \frac{\frac{4r}{3}}{\sqrt{\left(\frac{4r}{3}\right)^2 + r^2 - \left(\frac{r}{3}\right)^2}} = \sqrt{\frac{2}{3}};$$

hence the said resultant will be

$$+ \frac{3w}{AB^2} \sqrt{\frac{2}{3}}, \quad \text{or} \quad + \frac{3w}{4r^2} \sqrt{\frac{3}{2}}.$$

Therefore the element A is subject to an acceleration

$$\frac{d^2r}{dt^2} = -\frac{1}{r^2}\left(v - \frac{3w}{4}\sqrt{\frac{3}{2}}\right),$$

or

$$\frac{d^2r}{dt^2} = -\frac{1}{r^2}\left(v - w.0\cdot91856\right).$$

This equation contains the solution of the problem.

PROBLEM II.

Six repulsive elements A, B, C, D, E, F (fig. 8) *having equal powers* w *are so arranged as to form a regular octahedron around an attractive centre* O *having a power* v. *Find the dynamical formula of the system.*

SOLUTION. Each of the elements surrounding the centre O is exposed to six actions, one attractive from O, and five repulsive from the other material points. Let $OA = r$. The attractive action of O upon A will be expressed by

$$-\frac{v}{r^2};$$

the four repulsive actions proceeding from C, D, E, F, being equal, will have a resultant expressed by

$$+\frac{4w}{AC^2}\cos CAO = \frac{4w}{2r^2}\cos 45^0 = \frac{w}{r^2}\sqrt{2};$$

and lastly, the fifth repulsive action proceeding from B is

$$+\frac{w}{4r^2}.$$

Therefore

$$\frac{d^2r}{dt^2} = -\frac{1}{r^2}\left\{v - w\left(\frac{1}{4} + \sqrt{2}\right)\right\} = -\frac{1}{r^2}(v - w \cdot 1\!\cdot\!66425);$$

which equation contains the solution of the problem. The material formula of this system is $m = A + 6R$, according to what we have said in the preceding problem.

PROBLEM III.

Eight repulsive elements A, B, C, D, E, F, G, H (fig. 9) *having equal powers* w *are so arranged as to form a regular hexahedron around an attractive centre* O *having a power* v. *Find the formula of the system.*

SOLUTION. Let $OA = r$, as usual. Each repulsive element,

v. gr. A, is exposed to eight actions. The first is attractive, from O, and is expressed by

$$-\frac{v}{r^2};$$

the three repulsive from B, C, D, being equal, will have for their resultant

$$+\frac{3w}{AB^2}\cos BAO = \frac{3w}{AB^2} \cdot \frac{AE}{AB} = \frac{3w}{4r^2}\sqrt{3};$$

the three repulsive from F, G, H, being equal, will give

$$+\frac{3w}{AF^2}\cos FAE = \frac{3w}{AF^2} \cdot \frac{AF}{AE} = \frac{3w}{4r^2}\sqrt{\frac{3}{2}};$$

and, lastly, the action from E will be expressed by

$$+\frac{w}{4r^2}.$$

Therefore

$$\frac{d^2r}{dt^2} = -\frac{1}{r^2}\left\{v - w\left(\frac{3\sqrt{3}}{4} + \frac{3}{4}\sqrt{\frac{3}{2}} + \frac{1}{4}\right)\right\},$$

or

$$\frac{d^2r}{dt^2} = -\frac{1}{r^2}(v - w \cdot 2\cdot46759).$$

Such is the solution of the problem. The material formula of this system is $m = A + 8R$.

PROBLEM IV.

Twelve repulsive elements (fig. 10) *having equal powers* w *are so arranged as to form an octo-hexahedron, of which the centre is an attractive element* O *having the power* v. *Find the formula of the system.*

SOLUTION. Let $OA = r$, as before. Each repulsive element is exposed to twelve actions. The action of O upon A is attractive, and will be expressed by

$$-\frac{v}{r^2};$$

the four repulsive from $G,\ I,\ C,\ K,$ are equal, and their resultant is

$$+\frac{4w}{AG^2}\cos GAO=\frac{4w}{\frac{1}{2}AB^2}\cos 60^0=\frac{4w}{\frac{1}{2}(r\sqrt{2})^2}\times\frac{1}{2}=\frac{2w}{r^2}\ ;$$

the four from $D,\ H,\ L,\ M,$ are equal, and their resultant is

$$+\frac{4w}{AM^2}\cos MAF.$$

Let us draw MO; the triangle MOF will be equilateral; hence the line Mh drawn to the middle of OF will be perpendicular to this radius. Therefore

$$\cos MAF=\frac{Ah}{AM}=\frac{r+\frac{1}{2}r}{\sqrt{2r^2+r^2}}=\frac{3}{2\sqrt{3}}=\frac{1}{2}\sqrt{3};$$

and thus, the aforesaid resultant will be

$$+\frac{4w}{AM^2}\cdot\frac{1}{2}\sqrt{3}=\frac{2w\sqrt{3}}{3r^2}.$$

The two actions from B and E are equal, and their resultant is

$$+\frac{2w}{AE^2}\cos EAO=\frac{2w}{AE^2}\cos 45^0=\frac{2w}{AE^2}\sqrt{\frac{1}{2}}=\frac{w\sqrt{2}}{2r^2}.$$

Lastly, the action from F is

$$+\frac{w}{4r^2}.$$

Therefore

$$\frac{d^2r}{dt^2}=-\frac{1}{r^2}\left\{v-w\left(2+\frac{2\sqrt{3}}{3}+\frac{\sqrt{2}}{2}+\frac{1}{4}\right)\right\},$$

or

$$\frac{d^2r}{dt^2}=-\frac{1}{r^2}(v-w\,.\,4\cdot11170).$$

Such is the solution of the problem. The material formula of this system may be written thus, $m=A+(12)\,R$. We do not write simply $12R$, because this second way of writing will be presently employed in the formula of the regular icosahedron.

PROBLEM V.

Twelve repulsive elements (fig. 11) *of an equal power* w *are so arranged as to form a regular icosahedron, of which the centre* O *is attractive and has a power* v. *Find the formula of this system.*

SOLUTION. Let us determine first the edge of our polyhedron in terms of the radius $OA = r$. Drawing AA', AF', $A'F'$, we shall have

$$AA'^2 = 4r^2 = AF'^2 + A'F'^2.$$

Now, AF' is a diagonal of the pentagon $ABE'F'D$, and the triangle BAF' is isosceles; therefore

$$AF' \times 2 \cos BAF' = AB = A'F'; \text{ whence}$$

$$AF' = \frac{A'F'}{2 \cos BAF'} = \frac{A'F'}{2 \cos 72^0},$$

and consequently

$$4r^2 = A'F'^2 \left(1 + \frac{1}{4 \cos^2 72^0}\right);$$

therefore

$$A'F' = \frac{2r}{\sqrt{1 + \dfrac{1}{4 \cos^2 72^0}}} = \frac{2r}{\sqrt{3\cdot61803}} = \frac{2r}{1\cdot90210} = r\cdot1\cdot05145.$$

Such is the value of the edge of a dodecahedron in terms of r.

Let us come now to the actions. The element A is exposed to twelve actions. The first from O is attractive, and will be expressed by

$$-\frac{v}{r^2}.$$

The five actions from B, C, D, E, F are repulsive and equal, and their resultant is

$$+\frac{5w}{AD^2} \cos DAO.$$

But in the triangle ADO we have

$$AO = r, \quad DO = r, \quad AD = r\cdot1\cdot05145;$$

hence, if we put $AOD = \alpha$, $DAO = \beta$, we shall obtain

$$AO : AD :: \sin \beta : \sin \alpha :: 1 : 1{\cdot}05145,$$

whence $\quad\quad\quad \sin (180^0 - 2\beta) = \sin \beta \cdot 1{\cdot}05145,$

or $\quad\quad\quad \sin 2\beta = \sin \beta \cdot 1{\cdot}05145 = 2 \sin \beta \cos \beta,$

and consequently

$$\cos \beta = \frac{1{\cdot}05145}{2} = 0{\cdot}52572.$$

Therefore the resultant of those five actions will be

$$\frac{5w \cdot 1{\cdot}05145}{2r^2 (1{\cdot}05145)^2} = \frac{5w}{r^2} \cdot 0{\cdot}475533.$$

Five other actions from B', C', D', E', F' repulsive and equal give a resultant

$$\frac{5w}{AF'^2} \cos F'AO.$$

But, since $A'F'A$ is a right angle, we shall have

$$AF'^2 = AA'^2 - F'A'^2 = 4r^2 - r^2 (1{\cdot}05145)^2,$$

as also

$$\cos F'AO = \frac{AF'}{AA'} = \frac{A'F'}{2r \cdot 2 \cos 72^0} = \frac{r \cdot 1{\cdot}05145}{4r \cdot 0{\cdot}30902} = 0{\cdot}85065.$$

Whence the resultant will be

$$+ \frac{5w}{r^2 \{4 - (1{\cdot}05145)^2\}} \times 0{\cdot}85065 = \frac{5w}{r^2} \cdot 0{\cdot}293883.$$

Lastly, the action from A' will be expressed by

$$+ \frac{w}{4r^2}.$$

Therefore

$$\frac{d^2r}{dt^2} = - \frac{1}{r^2} \{v - w (2{\cdot}37766 + 1{\cdot}46941 + 0{\cdot}25)\},$$

or $\quad\quad\quad \frac{d^2r}{dt^2} = - \frac{1}{r^2} (v - w \cdot 4{\cdot}09707).$

This is the formula of the system. The material formula of this same system will be $m = A + 12R$.

PROBLEM VI.

Twenty repulsive elements of an equal power w *are so arranged as to form a regular pentagonal dodecahedron, of which the centre is occupied by an attractive element* O (fig. 12) *having the power* v. *Find the formula of this system.*

SOLUTION. Let $HO = r$. We shall observe first, that the lines *FE, EI, IL, LF, FU, UV,* ... are the edges of a cube, of which the diameter is $2r$; and, therefore,

$$4r^2 = 3FE^2, \text{ and } FE = 2r\sqrt{\frac{1}{3}}.$$

Again, the side EH of a pentagon can be expressed in terms of r; for, since $\frac{1}{2}HE = EF \cos FEH$, we shall have

$$EH = 4r \cos 72^0 \sqrt{\frac{1}{3}} = 4r \cdot 0\dot{5}773 \times 0\dot{3}0902 = r \cdot 0\dot{7}1364.$$

Now, the element H is exposed to twenty actions. The first from O is

$$-\frac{v}{r^2};$$

the three from E, G, I, give as their resultant

$$+\frac{3w}{GH^2} \cos GHO.$$

Now, as the line HO is perpendicular to the plane of the triangle GIE, which is the basis of the pyramid $GIEH$ having the edge

$$= 2r\sqrt{\frac{1}{3}},$$

it follows that the line Ii drawn from the point I to the middle of the opposite edge EG will be cut by HO into two parts, of which the one is to the other as $1 : 2$. Hence

$$\sin IHO = \frac{2}{3} \cdot \frac{Ii}{HI} = \frac{2Ii}{3r \cdot 0\dot{7}1364} = \sin GHO;$$

but

$$Ii = GI\sqrt{\frac{3}{4}} = 2r\sqrt{\frac{1}{3}}\sqrt{\frac{3}{4}} = r;$$

and consequently

$$\sin GHO = \frac{2}{2 \cdot 14092} = 0 \cdot 93417.$$

Therefore

$$\cos GHO = \sqrt{1 - \sin^2 GHO} = 0 \cdot 35679.$$

Therefore the resultant of those three actions will be

$$\frac{3w}{r^2 (0 \cdot 71364)^2} \times 0 \cdot 35679 = \frac{w}{r^2} . 2 \cdot 10173.$$

Then the six actions from F, L, K, R, D, A, all equal, will give

$$+ \frac{6w}{HL^2} \cos LHO.$$

And, since FL is the edge of a cube, of which the diameter is CL, and moreover

$$\cos OLF = \frac{LF}{CL} = \frac{2r\sqrt{\frac{1}{3}}}{2r} = \sqrt{\frac{1}{3}},$$

it follows that the resultant of those six actions will be

$$\frac{6w}{HL^2} \sqrt{\frac{1}{3}} = \frac{6w}{\frac{4r^2}{3}} \sqrt{\frac{1}{3}} = \frac{9w}{2r^2} \sqrt{\frac{1}{3}}.$$

Then we have six other equal actions from M, N, P, Q, B, C. The resultant of these is

$$+ \frac{6w}{NH^2} \cos NHO.$$

Now, since $NKCH$ is a regular tetrahedron, TH will make with the edges NH, KH, &c. angles that are all equal, and of which the cosine is expressed by the height of the tetrahedron divided by its edge. But the edge is HN, and the height is $HN\sqrt{\frac{2}{3}}$; therefore $\cos NHO = \sqrt{\frac{2}{3}}$. And, since HN is the diagonal of the square $NLHA$, we shall have for our resultant

$$\frac{6w}{NH^2} \sqrt{\frac{2}{3}} = \frac{6w}{2\frac{4r^2}{3}} \sqrt{\frac{2}{3}} = \frac{9w}{4r^2} \sqrt{\frac{2}{3}}.$$

We have then three other equal actions from U, S, V, of which the resultant is

$$+ \frac{3w}{UH^2} \cos UHO.$$

Now, as the height of the pyramid $UVST$ is equal to the height of the pyramid $GEIH$, and this second is expressed by

$$\sqrt{HI^2 - \frac{4}{9} Ii^2} = \sqrt{r^2 (0\text{·}71364)^2 - \frac{4}{9} r^2} = r \text{.} 0\text{·}25463,$$

we shall have

$$UH \cos UHO = 2r - r \text{.} 0\text{·}25463 = r \text{.} 1\text{·}74536 ;$$

whence

$$\cos UHO = \frac{r}{UH} \text{.} 1\text{·}74536.$$

But, the angle HUT is right ; and therefore

$$UH^2 = 4r^2 - r^2 (0\text{·}71364)^2 = r^2 \text{.} 3\text{·}49071 ;$$

consequently, the resultant will be

$$\frac{3w}{r^2 \text{.} 3\text{·}49071} \text{.} \frac{1\text{·}74536}{\sqrt{3\text{·}49071}} = \frac{3w}{r^2} \text{.} 0\text{·}267618.$$

We have, lastly, one more action from T, and its expression is

$$+ \frac{w}{4r^2}.$$

Therefore

$$\frac{d^2 r}{dt^2} = - \frac{1}{r^2} \left\{ v - w \left(2\text{·}10173 + \frac{9}{2} \sqrt{\frac{1}{3}} + \frac{9}{4} \sqrt{\frac{2}{3}} + 0\text{·}80285 + \frac{1}{4} \right) \right\},$$

$$\text{or} \quad \frac{d^2 r}{dt^2} = - \frac{1}{r^2} (v - w \text{.} 7\text{·}58978).$$

This is the dynamical formula of our present system. Its material formula is, of course, $m = A + 20R$.

PROBLEM VII.

A number of attractive elements of equal power w *are so arranged as to form a regular polyhedron, of which the centre is occupied by a material element having a repulsive power* v. *Find the formula of the system.*

SOLUTION. It is evident, that the solution of this general problem may be obtained by simply changing the signs of v and w in the above found equations. Since, then, the solution of all the preceding problems can be represented by the general equation

$$\frac{d^2r}{dt^2} = -\frac{1}{r^2}(v - Mw) \dots\dots\dots\dots\dots (1),$$

in which M is a constant number depending only on the form of the polyhedron, we shall have, for our present problem, the equation

$$\frac{d^2r}{dt^2} = +\frac{1}{r^2}(v - Mw) \dots\dots\dots\dots\dots (2),$$

in which we shall take for M the number which corresponds to the form of our polyhedron.

PROBLEM VIII.

A number of attractive elements of an equal power w *are so arranged as to form a regular polyhedron, of which the centre has an attractive power* v. *Find the formula of this system.*

SOLUTION. Evidently, it will suffice to change the sign of w in the equation (1). We shall have, accordingly,

$$\frac{d^2r}{dt^2} = -\frac{1}{r^2}(v + Mw) \dots\dots\dots\dots\dots (3),$$

where we shall take for M the number which corresponds to the form of the given system.

PROBLEM IX.

A number of repulsive elements of an equal power w *are so arranged as to form a regular polyhedron, of which the centre is occupied by another repulsive element having the power* v. *Find the formula of this system.*

SOLUTION. Evidently, it will suffice to change the sign of v in the equation (1). Therefore

$$\frac{d^2r}{dt^2} = +\frac{1}{r^2}(v + Mw) \dots\dots\dots\dots\dots (4),$$

where we shall take for M the number which suits our system.

PROBLEM X.

A number of elements having a power w *form a regular poly-hedron, of which the centre is occupied by another element having the power* v. *Find the dynamical conditions of the system.*

SOLUTION. The general equation of motion for a regular polyhedric system comprises the four cases implied in the four equations (1), (2), (3) (4) ; and, accordingly, we shall have

$$\frac{d^2r}{dt^2} = \pm \frac{1}{r^2} (v \pm Mw),$$

where different signs are to be taken according as the powers are of a different nature.   Let us come to some details.

1st.   If the powers are such as to give to this general equation the signs of the formula (1), we can make three hypotheses with regard to the powers; viz. either $v = Mw$, or $v < Mw$, or $v > Mw$.

When $v = Mw$, it is evident that the system will be, of itself, in equilibrium, whatever the radius r.   In this case the system will be indifferent to all dimensions, and resist neither traction nor pressure.

When $v < Mw$, then the elements forming the polyhedron will have a tendency to recede from the centre : and the system cannot keep together without exterior pressure.

When $v > Mw$, then the elements forming the polyhedron will move actually towards the centre, and vibrate through it by constant vibrations.   When these vibrations take place, the system contracts and dilates alternately, so that it might be called a *palpitating* system, and for each vibration we would point out a *palpitation.*   In this case the equation

$$\frac{d^2r}{dt^2} = - \frac{1}{r^2} (v - Mw)$$

integrated between the limits R and r (R being the greatest value of the variable radius r) gives

$$\frac{dr}{dt} = - \sqrt{\frac{2 (v - Mw) (R - r)}{Rr}},$$

where we take the root with the negative sign, because dr and dt are of different signs. Hence

$$t = \sqrt{\frac{R}{2\,(v - Mw)}} \left\{ \sqrt{r\,(R - r)} + R \cdot \tan^{-1} \sqrt{\frac{R - r}{r}} \right\}.$$

Let t' be what t becomes when the elements meet in O; since at this moment $r = 0$, it will be

$$t' = \frac{\pi R}{2} \sqrt{\frac{R}{2\,(v - Mw)}};$$

and the time T of a whole excursion, or of a *palpitation*, will be

$$T = \pi \sqrt{\frac{R^3}{2\,(v - Mw)}}.$$

If, instead of $v - Mw$, we substitute its value $-r^2 \dfrac{d^2x}{dt^2}$, this equation will give

$$\frac{d^2r}{dt^2} = -\frac{R}{2r^2} \left(\frac{\pi R}{T} \right)^2;$$

and this being multiplied by dr, and then integrated, gives

$$\frac{dr}{dt} = -\frac{\pi R}{T} \sqrt{\frac{R - r}{r}}.$$

This equation does not contain M; and, since M is a factor dependent on the special polyhedric form of the system, the equation is applicable to systems of any regular form whatever.

2nd. If the powers are such as to give to the general equation the signs of the formula (2), then, if $v = Mw$, the system will be in equilibrium for any value whatever of r. If $v > Mw$, the elements which form the polyhedron will tend to recede from the centre, and the system, of itself, will not hold together. If $v < Mw$, then the equation may be written thus

$$\frac{d^2r}{dt^2} = -\frac{1}{r^2}\,(Mw - v);$$

and this, being integrated, will give

$$t = \sqrt{\frac{R}{2(Mw - v)}} \left\{ \sqrt{r(R - r)} + R \cdot \tan^{-1} \sqrt{\frac{R - r}{r}} \right\};$$

whence

$$T = \pi \sqrt{\frac{R^3}{2(Mw - v)}}.$$

Substituting $-r^2 \dfrac{d^2r}{dt^2}$ for $Mw - v$, and working as in the preceding case, we shall find the equation

$$\frac{dr}{dt} = -\frac{\pi R}{T} \sqrt{\frac{R - r}{r}},$$

which gives the velocity of motion after the time t independently of M, when T is known.

3rd.   If the powers are such as to give to the general equation the form (3), then the system is essentially vibratory, whatever the relation between v and Mw.   In this case, we shall find

$$t = \sqrt{\frac{R}{2(v + Mw)}} \left\{ \sqrt{r(R - r)} + R \cdot \tan^{-1} \sqrt{\frac{R - r}{r}} \right\},$$

as also

$$T = \pi \sqrt{\frac{R^3}{2(v + Mw)}},$$

and lastly,

$$\frac{dr}{dt} = -\frac{\pi R}{T} \sqrt{\frac{R - r}{r}}.$$

4th.   If the powers are such as to give to the general equation the form (4), then the system essentially tends to its own dissolution.   The equation (4) gives, in this case,

$$\frac{1}{2} \left(\frac{dr}{dt} \right)^2 = -\frac{1}{r}(v + Mw) + C.$$

To determine C, let us suppose that, when $r = R$, then the velocity of the elements receding from the centre of the system is $= u$.   Then

$$C = \frac{1}{2} u^2 + \frac{1}{R}(v + Mw),$$

and consequently

$$\frac{dr}{dt} = \sqrt{2\,(v + Mw)\left(\frac{r - R}{Rr}\right) + u^2},$$

and

$$dt = \frac{dr}{\sqrt{u^2 + \dfrac{2\,(v + Mw)}{R} - \dfrac{2\,(v + Mw)}{r}}}.$$

Let us put, for brevity's sake,

$$u^2 + \frac{2\,(v + Mw)}{R} = a, \quad 2\,(v + Mw) = b\,;$$

it will be

$$dt = \frac{dr}{\sqrt{a - \dfrac{b}{r}}},$$

whence

$$t = \sqrt{r\,(ar - b)} + \frac{b}{2\sqrt{a}} \log \frac{\sqrt{ar} - \sqrt{ar - b}}{\sqrt{ar} + \sqrt{ar - b}} + C.$$

Let us suppose that the time t begins at the moment in which the velocity of the elements is u. The constant C will then be determined by the conditions $t = 0$, $r = R$; and we shall find, after some reductions,

$$C = -uR - \frac{b}{2\sqrt{a}} \log \frac{\sqrt{a} - u}{\sqrt{a} + u}\,;$$

and consequently

$$t = \sqrt{r\,(ar - b)} - uR + \frac{b}{2\sqrt{a}} \log \frac{\sqrt{ar} - \sqrt{ar - b}}{\sqrt{ar} + \sqrt{ar - b}} \cdot \frac{\sqrt{a} + u}{\sqrt{a} - u}.$$

And these considerations may suffice with regard to primitive polyhedric systems.

BOOK V.

DYNAMICAL CONSTITUTION OF
COMBINED POLYHEDRIC SYSTEMS OF ELEMENTS.

THE molecules of bodies, as we shall see later, cannot consist of
a single polyhedric system of elements connected with a central
point: they involve in their constitution a number of such poly-
hedric systems; so that the law of motion of any molecule must
be the result of the actions of all such combined systems. Let us,
then, apply to the motion and equilibrium of compound polyhedric
systems what we have already deduced for the case of simple poly-
hedrons. A compound regular system consists of any number of
polyhedric systems, having one common centre, and connected
with one another by mutual action.

A compound system may involve different polyhedric forms,
for which the number M of the above-found equations has a dif-
ferent value. This number M may be considered as a kind of
dynamical *modulus*, varying only with the form of the systems.
In order to distinguish the modulus of one polyhedric form from
that of any other, we shall add to the letter M a *suffix* destined
to show the number of elements contained in the polyhedric forms,
for which the values of M are calculated. So, we shall write

$$\text{for a tetrahedron} \dots\dots\dots\dots\dots M_4 = 0{\cdot}91866,$$
$$\text{for an octahedron} \dots\dots\dots\dots M_6 = 1{\cdot}66425,$$
$$\text{for a hexahedron} \dots\dots\dots\dots M_8 = 2{\cdot}46759,$$
$$\text{for an octo-hexahedron} \dots\dots M_{(12)} = 4{\cdot}11170,$$
$$\text{for an icosahedron} \dots\dots\dots M_{12} = 4{\cdot}09707,$$
$$\text{for a dodecahedron} \dots\dots\dots M_{20} = 7{\cdot}58978.$$

Amongst all the polyhedrons, of which the compound system is
made up, that which is the most remote from the centre we shall

call the *envelope* of the compound system.   The other polyhedrons, which lie between the centre and the envelope, we shall call *nuclei*. The whole compound system may be styled *uninuclear, binuclear, trinuclear*, ... according as it possesses one, two, three, ... nuclei.

<div style="text-align:center">PROBLEM I.</div>

To find the dynamical formulas of the tetrahedric uninuclear system $m = R + 4A + 4R'$, *in which the central point O (fig. 13) is repulsive, the nucleus ABCD attractive, and the envelope* $RR'R''R'''$ *repulsive*.*

SOLUTION.   Let v be the action of the centre, v' that of any element of the nucleus, w that of any element of the envelope: and let $OA = r$, $OR = \rho$. Let us consider first the action of the envelope upon any element A of the nucleus.

The action of R upon A tending to augment the distance of A from the centre, is evidently

$$+ \frac{w}{(\rho + r)^2}.$$

The actions from R', R'', R''' on A, which tend to diminish the distance of A from the centre, are equal, and their resultant is

$$- \frac{3w}{AR'^2} \cos RAp = - \frac{3w}{AR'^2} \cdot \frac{Ap}{AR'} = - \frac{3w}{AR'^2} \cdot \frac{Op - AO}{AR'}.$$

But, as the centre of a tetrahedron is the point, in which its height is divided in the ratio $3 : 1$, if p is the centre of the triangle $R'R''R'''$, we shall have $Op = \frac{1}{3}\rho$. Again, if i is the centre of the triangle BCD, we shall have

$$AR'^2 = RD^2 = Ri^2 + iD^2 = \left(\rho - \frac{1}{3}r\right)^2 + \frac{8r^2}{9}.$$

---

* The meaning of the notation $m = R + 4A + 4R'$ has been explained above (Book IV. Probl. I.). To avoid confusion, the terms of the second member, in all such formulas, must follow one another in this way : the centre being designated by the first term, the nucleus which is nearer to the centre must be written the second, and so on, following the order of distance. The envelope, therefore, is always indicated by the last term.

Hence our resultant will be

$$- \frac{3w\left(\frac{1}{3}\rho - r\right)}{\sqrt{\left\{\left(\rho - \frac{1}{3}r\right)^2 + \frac{8r^2}{9}\right\}^3}}.$$

Let P be the total action of the envelope on any element of the nucleus. We shall have

$$P = w\left\{\frac{1}{(\rho + r)^2} - \frac{3\left(\frac{1}{3}\rho - r\right)}{\sqrt{\left(\rho^2 + r^2 - \frac{2\rho r}{3}\right)^3}}\right\}.$$

This value of P is always positive; for, if we reduce the second member to a common denominator, the numerator will be

$$\sqrt{\left(\rho^2 + r^2 - \frac{2\rho r}{3}\right)^3} + 3r(\rho + r)^2 - \rho(\rho + r)^2,$$

which is always positive. And, therefore, the action of the repulsive envelope tends to expand the nucleus.

The action of the centre O and of the other elements B, C, D, of the nucleus upon the same point A, is expressed, according to what we have said in the preceding Book (Probl. VII.), by

$$+ \frac{1}{r^2}(v - M_4 v').$$

Hence the action of the whole system upon any point A of the nucleus is

$$\frac{d^2 r}{dt^2} = \frac{v - M_4 v'}{r^2} + w\left\{\frac{1}{(\rho + r)^2} - \frac{3\left(\frac{1}{3}\rho - r\right)}{\sqrt{\left(\rho^2 + r^2 - \frac{2\rho r}{3}\right)^3}}\right\} \quad \ldots\ldots (5).$$

Let us investigate now, what is the action of the whole system upon any element R of the envelope. The action of O and of the other elements R', R'', R''' of the envelope upon R, is expressed, according to Problem IX. of the preceding Book, by

$$+ \frac{1}{\rho^2}(v + M_4 w).$$

The action of the nucleus upon R is a resultant of three equal actions from B, C, D, and of a fourth from A. The first three give as their resultant

$$- \frac{3v'}{DR^2} \cos DRi = - \frac{3v'}{DR^2} \cdot \frac{Ri}{DR} = - \frac{3v'\left(\rho - \frac{1}{3}r\right)}{\sqrt{\left(\rho^2 + r^2 - \frac{2\rho r}{3}\right)^3}} \; ;$$

the fourth, from A, is expressed by

$$- \frac{v'}{(\rho + r)^2} ;$$

and all these actions are taken as negative, since they tend to diminish the distance OR. Hence the action of the whole system on any element R of the envelope is

$$\frac{d^2\rho}{dt^2} = \frac{v + M_4 w}{\rho^2} - v'\left\{ \frac{1}{(\rho + r)^2} + \frac{3\left(\rho - \frac{1}{3}r\right)}{\sqrt{\left(\rho^2 + r^2 - \frac{2\rho r}{3}\right)^3}} \right\} \ldots\ldots(6).$$

The equations (5) and (6) contain the solution of the problem.

The conditions of equilibrium are evidently

$$\frac{d^2r}{dt^2} = 0, \qquad \frac{d^2\rho}{dt^2} = 0,$$

$$\text{or} \quad \frac{M_4 v' - v}{r^2} = v\left\{ \frac{1}{(\rho + r)^2} - \frac{3\left(\frac{1}{3}\rho - r\right)}{\sqrt{\left(\rho^2 + r^2 - \frac{2\rho r}{3}\right)^3}} \right\}$$

$$\left.\vphantom{\begin{array}{c}1\\1\\1\\1\end{array}}\right\} \ldots\ldots\ldots(7).$$

$$\frac{M_4 w + v}{\rho^2} = v'\left\{ \frac{1}{(\rho + r)^2} + \frac{3\left(\rho - \frac{1}{3}r\right)}{\sqrt{\left(\rho^2 + r^2 - \frac{2\rho r}{3}\right)^3}} \right\}$$

EXAMPLES. Our equations being rather complex, we cannot determine directly the values of the radii r and ρ, for which the system of the powers v, v', w will be in equilibrium. But we may proceed by an inverse method. We can make any hypothesis

with regard to r and ρ, and find out what the powers ought to be in order to secure the equilibrium for that hypothesis.

Let us suppose first $\rho > r$; v. gr. $\rho = 1$, $r = \dfrac{1}{3}$. We shall obtain

$$M_4 v' - v = \frac{w}{16}, \quad M_4 w + v = v'\left(\frac{9}{16} + \frac{9}{2\sqrt{2}}\right),$$

and, by eliminating v,

$$w = v' \cdot 3\cdot30138.$$

This value of w being placed in one of the preceding equations, we obtain

$$v' = v \cdot 1\cdot28973.$$

Hence, in order to secure the equilibrium with the radii $\rho = 1$, $r = \dfrac{1}{3}$, the powers of the system must fulfil the conditions

$$v = v, \quad v' = v \cdot 1\cdot28973, \quad w = v \cdot 4\cdot25757.$$

The hypothesis $\rho = 4$, $r = 1$, would lead to

$$v = v, \quad v' = v \cdot 1\cdot18036, \quad w = v \cdot 3\cdot88333.$$

Let us suppose now $\rho = r$. We shall find

$$M_4 v' - v = w \cdot 1\cdot5490, \quad M_4 w + v = v' \cdot 1\cdot5490,$$

whence

$$M_4 (v' + w) = (v' + w) \times 1\cdot5490,$$

and, therefore,

$$M_4 = 1\cdot5490.$$

Now this is impossible; for the value of M_4 is always $0\cdot91866$, as we have found it to be (Book IV. Probl. I.). Therefore, no equilibrium is possible in our system with $\rho = r$.

Let us suppose now $\rho < r$; v. gr. $\rho = \dfrac{1}{3}$, $r = 1$. We shall have

$$M_4 v' - v = w \cdot 7\cdot82561, \quad M_4 w + v = v' \cdot 5\cdot06250;$$

whence we obtain

$$+ w = - v' \cdot 0\cdot59996.$$

This result with opposite signs shows the impossibility of equilibrium with the radii $\rho = \dfrac{1}{3}$, $r = 1$, as long as the envelope is repulsive.

REMARK. The system tends to expand as long as $\dfrac{d^2\rho}{dt^2}$ is positive, viz. as long as the inequality

$$v + M_4w > \frac{v'}{\left(1 + \dfrac{r}{\rho}\right)^2} + \frac{3v'\left(1 - \dfrac{1}{3}\dfrac{r}{\rho}\right)}{\sqrt{\left\{1 + \left(\dfrac{r}{\rho}\right)^2 - \dfrac{2}{3}\dfrac{r}{\rho}\right\}^3}}$$

subsists. Hence, our system would be indefinitely expansive, if this inequality were to subsist for all greater and greater values of ρ. And, since it is not of the nature of the system, that the radius r should necessarily increase with ρ, the system will be indefinitely expansive whenever, r remaining finite, ρ becomes infinite. In this case the inequality is reduced to the simple expression

$$v + M_4w > 4v';$$

which may be called *the condition of indefinite expansivity* of the system*.

PROBLEM II.

To find the dynamical formulas of the octahedric uninuclear system $m = R + 8A + 6R'$, *in which the central point* O (fig. 14) *is repulsive, the nucleus is hexahedric and attractive, the envelope octahedric and repulsive.*

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* The uninuclear tetrahedric system admits of eight varieties, viz.

$$m = R + 4A + 4R',$$
$$m = R + 4R' + 4A,$$
$$m = A + 4A' + 4R,$$
$$m = A + 4R + 4A',$$
$$m = R + 4A + 4A',$$
$$m = R + 4R' + 4R'',$$
$$m = A + 4R + 4R',$$
$$m = A + 4A' + 4A''.$$

From what has been said on the first variety it is easy to see what may be said of all the others : they differ only from one another by the signs of v, v', w.

SOLUTION. Let v be the action of the central element, v' that of any element of the nucleus, w that of any element of the envelope; and let $OA = r$, $OR = \rho$. Let us consider first the action of the envelope on any point A of the nucleus. The triangle $RR'R''$ is equilateral, and the radius OA, when sufficiently prolonged, will pass through it perpendicularly at the point E, which is its centre. Let us draw RT to the middle of the side $R'R''$; we shall have $RE = 2\,TE$.

Now, the actions of the elements R, R', R'' on A are equal, and their resultant is

$$-\frac{3w}{AR^2}\cos RAE = -\frac{3w}{AR^2}\cdot\frac{AE}{AR}.$$

But

$$AE = OE - OA = \sqrt{\rho^2 - RE^2} - r,$$

and

$$RE^2 = \frac{4}{9}\,TR^2 = \frac{4}{9}\cdot\frac{3}{4}\,RR'^2 = \frac{1}{3}\,RR'^2 = \frac{2}{3}\,\rho^2\,;$$

therefore

$$AE = \sqrt{\rho^2 - \frac{2}{3}\rho^2} - r = \rho\,\sqrt{\frac{1}{3}} - r.$$

Hence our resultant will be

$$-\frac{3w\left(\rho\,\sqrt{\frac{1}{3}} - r\right)}{\sqrt{\left\{\left(\rho\,\sqrt{\frac{1}{3}} - r\right)^2 + \frac{2\rho^2}{3}\right\}^3}}.$$

This resultant, when $\rho\,\sqrt{\frac{1}{3}} - r > 0$, tends to push A towards the centre, and when $\rho\,\sqrt{\frac{1}{3}} - r < 0$, tends to draw it in the opposite direction. But, in both cases, it must be taken with the negative sign; for, in the second case, it will become positive of itself on account of $\rho\,\sqrt{\frac{1}{3}} < r$.

From the elements R''', R^{iv}, R^{v}, we have three other equal actions on A; their resultant is

$$+\frac{3w}{AR'''^2}\cos R'''AE' = \frac{3w}{AR'''^2}\cdot\frac{AE'}{AR'''} = \frac{3w\left(\rho\sqrt{\dfrac{1}{3}}+r\right)}{\sqrt{\left\{\left(\rho\sqrt{\dfrac{1}{3}}+r\right)^2+\dfrac{2\rho^2}{3}\right\}^3}},$$

as is evident from the remarks above made. This resultant is positive, because it always tends to augment the distance AO.

Let then P be the whole action of the envelope on the point A; we shall have

$$P = 3w\left\{\frac{r+\rho\sqrt{\dfrac{1}{3}}}{\sqrt{\left(\rho^2+r^2+\dfrac{2\rho r}{\sqrt{3}}\right)^3}}+\frac{r-\rho\sqrt{\dfrac{1}{3}}}{\sqrt{\left(\rho^2+r^2-\dfrac{2\rho r}{\sqrt{3}}\right)^3}}\right\}.$$

The value of P is always positive*; hence the action of the envelope on the nucleus tends always to expand the nucleus, as in the above problem.

* To prove this, let $P=0$; then

$$\left(\frac{\rho-r\sqrt{3}}{\rho+r\sqrt{3}}\right)^2 = \left\{\frac{(\rho^2+r^2)\sqrt{3}-2\rho r}{(\rho^2+r^2)\sqrt{3}+2\rho r}\right\}^3.$$

Let us suppose, for brevity's sake, $\rho=1$; the preceding equation will give

$$(1+3r^2)\{8r^3+18r(1+r^2)^2\} = 18r\{(1+r^2)^3+2r^2(1+r^2)\}.$$

Now, this equation can be divided by $2r$; and then it may be reduced to

$$9r^6+6r^4 = 7r^2.$$

This, again, can be divided by r^2, and reduced to

$$3r^2+1 = \pm2\sqrt{2};$$

whence $r = \pm\sqrt{-\dfrac{1}{3}+\dfrac{2}{3}\sqrt{2}}$, or $r = \pm\sqrt{-\dfrac{1}{3}-\dfrac{2}{3}\sqrt{2}}.$

The equation is thus fully resolved; for, we have here two real and finite roots, two others imaginary, and two others $=0$ on account of the common factor r which we found twice, and which, if supposed to be $=0$, could in two different manners satisfy the equation. The two imaginary roots cannot, of course, be adopted, as is evident.

The real finite roots $r = \pm\sqrt{-\dfrac{1}{3}+\dfrac{2}{3}\sqrt{2}} = \pm0.7803$ represent the radii OI and OI'; but, when the point A takes the place I or I', the condition $P=0$ is not fulfilled; and,

Lastly, the action of the centre and of the other elements of the nucleus upon A is, according to Problem VII. of the preceding Book,

$$+ \frac{1}{r^2}(v - M_8 v').$$

Hence, the action of the whole system on any element A of the nucleus will be

$$\frac{d^2 r}{dt^2} = \frac{v - M_8 v'}{r^2} + 3w \left\{ \frac{r + \rho \sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2 + r^2 + \frac{2\rho r}{\sqrt{3}}\right)^3}} + \frac{r - \rho \sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2 + r^2 - \frac{2\rho r}{\sqrt{3}}\right)^3}} \right\} \ldots(8).$$

Let us determine now the action of the whole system on any point R of the envelope. The action of the centre and of the other elements of the envelope upon the element R is, according to Problem IX. of the preceding Book,

$$+ \frac{1}{\rho^2}(v + M_6 w).$$

The action of the nucleus on R comprises, 1st, four equal actions proceeding from A, A', A'', A''', of which the resultant is

$$- \frac{4v'}{AR^2} \cos ARO = - \frac{4v'}{AR^2} \cdot \frac{BR}{AR} = - \frac{4v' \left(\rho - r\sqrt{\frac{1}{3}}\right)}{\sqrt{\left\{\left(\rho - r\sqrt{\frac{1}{3}}\right)^2 + \frac{2r^2}{3}\right\}^3}}.$$

2nd, four other equal actions from $A^{\text{IV}}, A^{\text{V}}, A^{\text{VI}}, A^{\text{VII}}$, of which the resultant is

$$- \frac{4v'}{A^{\text{V}}R^2} \cos A^{\text{V}}RO = - \frac{4v'}{A^{\text{V}}R^2} \cdot \frac{BR'''}{AR'''} = - \frac{4v' \left(\rho + r\sqrt{\frac{1}{3}}\right)}{\sqrt{\left\{\left(\rho + r\sqrt{\frac{1}{3}}\right)^2 + \frac{2r^2}{3}\right\}^3}}.$$

in fact, the actions of which P is the resultant, in this case, are all positive, i. e. tending to make r greater. Hence those two roots $r = \pm 0.7803$ have nothing to do with the condition $P = 0$; they have been introduced into our equation by raising it to the square, and by thus creating the possibility of taking the terms of the equation $P = 0$ with a double sign. From these remarks it follows, that the true root of the equation $P = 0$ is only $r = 0$; and consequently the action of the envelope on any element of the nucleus cannot become $= 0$, except when the nucleus disappears. Since, then, P cannot become negative unless it pass through zero, it follows that P remains positive for every finite value of r.

These two expressions are negative, because the actions represented by them tend to diminish the distance OR.

Therefore, the action of the whole system upon any element of the envelope will be

$$\frac{d^2\rho}{dt^2} = \frac{v + M_6 w}{\rho^2} - 4v'\left\{\frac{\rho + r\sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2 + r^2 + \frac{2\rho r}{\sqrt{3}}\right)^3}} + \frac{\rho - r\sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2 + r^2 - \frac{2\rho r}{\sqrt{3}}\right)^3}}\right\}\ldots(9).$$

The equations (8) and (9) contain the solution of the problem. In the case of equilibrium we have

$$\frac{d^2 r}{dt^2} = 0, \qquad \frac{d^2\rho}{dt^2} = 0,$$

or

$$\left.\begin{aligned}\frac{M_8 v' - v}{r^2} &= 3w\left\{\frac{r + \rho\sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2 + r^2 + \frac{2\rho r}{\sqrt{3}}\right)^3}} + \frac{r - \rho\sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2 + r^2 - \frac{2\rho r}{\sqrt{3}}\right)^3}}\right\}\\[2em]\frac{M_6 w + v}{\rho^2} &= 4v'\left\{\frac{\rho + r\sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2 + r^2 + \frac{2\rho r}{\sqrt{3}}\right)^3}} + \frac{\rho - r\sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2 + r^2 - \frac{2\rho r}{\sqrt{3}}\right)^3}}\right\}\end{aligned}\right\}\ldots(10).$$

EXAMPLES. The complexity of these equations prevents us from determining directly the radii r and ρ, for which the system is in equilibrium : but we may proceed by the inverse method, as in the preceding problem, and determine the ratio of the powers v, v', w, for which the system, under given radii, will be in equilibrium.

Let us take first $\rho = 1$, $r = \frac{1}{2}$. We shall have

$$M_8 v' - v = w \cdot 0\cdot3750, \qquad M_6 w + v = v' \cdot 7\cdot2448;$$

whence

$$w = v' \cdot 3\cdot70555, \qquad v = v' \cdot 1\cdot0780,$$

and the system will be in equilibrium for the said radii, when we have

$$v = v, \qquad v' = v \cdot 0\cdot92764, \qquad w = v \cdot 3\cdot43692.$$

Let us take now $\rho = 1$, $r = 0{\cdot}1$. We shall find

$$M_8 v' - v = w \,.\, 0{\cdot}0001, \quad M_6 w + v = v'\,.\,7{\cdot}9983 \,;$$

whence

$$w = v'\,.\,3{\cdot}3235, \quad v = v'\,.\,2{\cdot}4672 \,;$$

and the system will be in equilibrium for the said radii, when we have

$$v = v, \quad v' = 0{\cdot}4053, \quad w = v\,.\,1{\cdot}3460.$$

Let us take $\rho = 1$, $r = 0{\cdot}7803$; we shall find

$$M_8 v' - v = w \,.\, 1{\cdot}2499, \quad M_6 w + v = v'\,.\,5{\cdot}1484 \,;$$

whence

$$w = v'\,.\,6{\cdot}47067, \quad v = -\, v'\,.\,5{\cdot}6155.$$

This contrariety of signs shows that, with the said radii, the equilibrium is impossible, unless the centre from repulsive becomes attractive.

REMARK. The system tends to expand as long as $\dfrac{d^2\rho}{dt^2}$ is positive; viz. as long as the inequality

$$v + M_6 w > 4v' \left\{ \frac{1 + \dfrac{r}{\rho}\sqrt{\dfrac{1}{3}}}{\sqrt{\left\{1 + \left(\dfrac{r}{\rho}\right)^2 + \dfrac{2}{\sqrt{3}}\dfrac{r}{\rho}\right\}^3}} + \frac{1 - \dfrac{r}{\rho}\sqrt{\dfrac{1}{3}}}{\sqrt{\left\{1 + \left(\dfrac{r}{\rho}\right)^2 - \dfrac{2}{\sqrt{3}}\dfrac{r}{\rho}\right\}^3}} \right\}$$

subsists. Hence the system will be indefinitely expansive, if this inequality subsists for any greater and greater value of ρ, the radius r remaining finite. When $\rho = \infty$, the inequality is reduced to

$$v + M_6 w > 8v',$$

which is the condition of indefinite expansivity for the present system*.

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* We have in this system also eight varieties, as in the preceding problem. The conditions of motion and rest for all such varieties may be easily deduced from the equations (8), (9), (10), by changing the signs in a suitable manner before v, v', w.

PROBLEM III.

To find the dynamical formulas of the hexahedric uninuclear system $m = R + 6A + 8R'$, *in which the central element* O (fig. 15) *is repulsive, the nucleus is octahedric and attractive, the envelope hexahedric and repulsive.*

SOLUTION. Let v be the action of the centre, v' that of any point of the nucleus, w that of any point of the envelope: and let $OA = r$, $OR = \rho$.

The action of the centre O and of the other elements of the nucleus on A is, according to Problem VII. of the preceding Book,

$$+ \frac{1}{r^2}(v - M_6 v').$$

The action of the envelope on the point A will be found as follows. The elements $R, R', R'', \ldots$ are equidistant from O and symmetrically arranged with regard to the nucleus; so that each of them is situated on the prolongation of the lines drawn from the centre of the system to the centres of each surface of the nucleus. Let us draw, then, the radius OR and the straight line AT so as to divide AA'' into two equal parts. The radius OR will cut AT in E; and we shall have $AE = 2TE$. Now, the elements R, R', R', R''' exert themselves upon A with equal actions, of which the resultant is

$$- \frac{4w}{AR^2} \cos RAB = - \frac{4w}{AR^2} \cdot \frac{AB}{AR} ;$$

but

$$AB = OB - OA = \sqrt{\rho^2 - RB^2} - r,$$

and, on the other hand,

$$RB^2 = \frac{1}{2} RR'^2 = \frac{1}{2} \cdot \frac{4}{3} \rho^2 = \frac{2\rho^2}{3} ;$$

therefore

$$AB = \sqrt{\rho^2 - \frac{2\rho^2}{3}} - r = \rho \sqrt{\frac{1}{3}} - r.$$

Therefore the resultant of those four actions will be

$$-\frac{4w\left(\rho\sqrt{\frac{1}{3}}-r\right)}{\sqrt{\left\{\left(\rho\sqrt{\frac{1}{3}}-r\right)^2+\frac{2\rho^2}{3}\right\}^3}},$$

with the negative sign; since it tends to diminish the distance OA. The other four elements R^{iv}, R^{v}, R^{vi}, R^{vii} exert themselves on A with equal actions, of which the resultant is evidently

$$+\frac{4w}{AR^{\mathrm{iv}2}}\cos R^{\mathrm{iv}}AO=\frac{4w}{AR^{\mathrm{iv}2}}\cdot\frac{B'A}{AR^{\mathrm{iv}}}=\frac{4w\left(\rho\sqrt{\frac{1}{3}}+r\right)}{\sqrt{\left\{\left(\rho\sqrt{\frac{1}{3}}+r\right)^2+\frac{2\rho^2}{3}\right\}^3}},$$

with the positive sign; since it tends to augment the distance OA.

If, then, the total action of the envelope on any element of the nucleus be called P, we shall have

$$P=4w\left\{\frac{\rho\sqrt{\frac{1}{3}}+r}{\sqrt{\left(\rho^2+r^2+\frac{2\rho r}{\sqrt{3}}\right)^3}}-\frac{\rho\sqrt{\frac{1}{3}}-r}{\sqrt{\left(\rho^2+r^2-\frac{2\rho r}{\sqrt{3}}\right)^3}}\right\};$$

and this value of P will be always positive, as is evident from what we have said on the value of P in the preceding problem. And, therefore, the action of the envelope will tend, here also, to expand the nucleus.

The action of the whole system upon any element of the nucleus will be

$$\frac{d^2r}{dt^2}=\frac{v-M_6v'}{r^2}+4w\left\{\frac{r+\rho\sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2+r^2+\frac{2\rho r}{\sqrt{3}}\right)^3}}+\frac{r-\rho\sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2+r^2-\frac{2\rho r}{\sqrt{3}}\right)^3}}\right\}\dots(11).$$

Let us now determine the action of the whole system on any element R of the envelope. The action from O and from the other elements of the envelope upon R, is, according to Problem IX. of the preceding Book,

$$+\frac{1}{\rho^2}\left(v+M_8w\right);$$

The action of the nucleus upon R is a resultant 1st of the actions proceeding from A, A', A'', which give

$$- \frac{3v'}{AR^2} \cos ARE = - \frac{3v'}{AR^2} \cdot \frac{ER}{AR} = - \frac{3v'\left(\rho - r\sqrt{\frac{1}{3}}\right)}{\sqrt{\left\{\left(\rho - r\sqrt{\frac{1}{3}}\right)^2 + \frac{2r^2}{3}\right\}^3}},$$

and 2nd, of the actions proceeding from A''', A^{iv}, A^v, which give

$$- \frac{3v}{A'''R^2} \cos A'''RE' = - \frac{3v'}{A'''R^2} \cdot \frac{E'R}{A'''R} = - \frac{3v'\left(\rho + r\sqrt{\frac{1}{3}}\right)}{\sqrt{\left\{\left(\rho + r\sqrt{\frac{1}{3}}\right)^2 + \frac{2r^2}{3}\right\}^3}}.$$

These actions are negative, since they tend to diminish the distance OR. Hence the action of the whole system upon any element of the envelope will be

$$\frac{d^2\rho}{dt^2} = \frac{v + M_8 w}{\rho^2} - 3v'\left\{\frac{\rho + r\sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2 + r^2 + \frac{2\rho r}{\sqrt{3}}\right)^3}} + \frac{\rho - r\sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2 + r^2 - \frac{2\rho r}{\sqrt{3}}\right)^3}}\right\} \dots (12).$$

The equations (11) and (12) contain the solution of the problem. There will be equilibrium, when we have

$$\frac{d^2 r}{dt^2} = 0, \qquad \frac{d^2\rho}{dt^2} = 0,$$

or

$$\left. \begin{array}{l} \dfrac{M_6 v' - v}{r^2} = 4w\left\{\dfrac{r + \rho\sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2 + r^2 + \frac{2\rho r}{\sqrt{3}}\right)^3}} + \dfrac{r - \rho\sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2 + r^2 - \frac{2\rho r}{\sqrt{3}}\right)^3}}\right\} \\[4ex] \dfrac{M_8 w + v}{\rho^2} = 3v'\left\{\dfrac{\rho + r\sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2 + r^2 + \frac{2\rho r}{\sqrt{3}}\right)^3}} + \dfrac{\rho - r\sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2 + r^2 - \frac{2\rho r}{\sqrt{3}}\right)^3}}\right\} \end{array} \right\} \dots (13).$$

EXAMPLES.   Let us take first $\rho = 1$, $r = \dfrac{1}{2}$: from the equations (13) we obtain

$$M_6 v' - v = w \,.\, 0\!\cdot\!5000, \quad M_8 w + v = v' \,.\, 5\!\cdot\!4336,$$

whence

$$v' \,.\, 3\!\cdot\!7694 = w \,.\, 1\!\cdot\!9676\ ;$$

and the system will be in equilibrium for the said radii, when we have

$$v = v, \quad v' = v \,.\, 1\!\cdot\!41899, \quad w = v \,.\, 2\!\cdot\!71845.$$

Let us take now $\rho = 1$, $r = 0\!\cdot\!1$; we shall find

$$M_6 v' - v = w \,.\, 0\!\cdot\!00013, \quad M_8 w + v = v' \,.\, 7\!\cdot\!05240,$$

whence

$$v' \,.\, 5\!\cdot\!38819 = w \,.\, 2\!\cdot\!46746,$$

and the system will be in equilibrium for the said radii, if we have

$$v = v, \quad v' = v \,.\, 0\!\cdot\!60089, \quad w = v \,.\, 1\!\cdot\!31230.$$

Let us take $\rho = 1$, $r = 0\!\cdot\!7803$; we shall find

$$M_6 v' - v = w \,.\, 2\!\cdot\!24066, \quad M_8 w + v = v' \,.\, 1\!\cdot\!09430,$$

whence

$$+ v' \,.\, 0\!\cdot\!5699 = - w \,.\, 0\!\cdot\!22693\ ;$$

and this contrariety of signs shows the impossibility of equilibrium with the radii $\rho = 1$, $r = 0\!\cdot\!7803$ in the present system.

REMARK.   The system tends to expand as long as $\dfrac{d^2\rho}{dt^2}$ is positive.   Hence, by the method employed in the preceding problems, we shall find the inequality

$$M_8 w + v > 6v',$$

as the condition of indefinite expansivity for the present system. This same system admits eight varieties ; and their equations will be drawn from (11), (12), (13), by a suitable change of signs.

PROBLEM IV.

To find the dynamical formulas of the octahedric uninuclear system $\mathrm{m} = \mathrm{R} + (12)\,\mathrm{A} + 6\mathrm{R}'$, *in which the centre* O (*fig.* 16) *is re-*

pulsive, the nucleus is octo-hexahedric and attractive, the envelope octahedric and repulsive.

SOLUTION. Let v be the action of the centre, v' that of any element of the nucleus, w that of any element of the envelope; and let $AO = r$, $OR = \rho$. The action of the whole system on a point A of the nucleus will be determined as follows. The action of the centre and of the other elements of the nucleus is (Book IV. Prob. VII.)

$$+ \frac{1}{r^2} (v - M_{(12)} v').$$

The action of the envelope upon A comprises 1st, the actions from R and R', which are equal, and give

$$-\frac{2w}{RA_0{}^2} \cos \frac{1}{2} RA_0R' = -\frac{2w}{RA_0{}^2} \cdot \frac{A_0I}{RA_0} = -\frac{2w \left(\rho \sqrt{\dfrac{1}{2}} - r \right)}{\sqrt{\left\{ \left(\rho \sqrt{\dfrac{1}{2}} - r \right)^2 + \dfrac{\rho^2}{2} \right\}^3}};$$

for

$$A_0I = OI - OA_0 = \rho \sqrt{\frac{1}{2}} - r,$$

and

$$RA_0 = \sqrt{RI^2 + A_0I^2} = \sqrt{\frac{1}{2}\rho^2 + \left(\rho \sqrt{\frac{1}{2}} - r \right)^2}.$$

The resultant is taken as negative, as it tends to diminish r, at least when $OA < OI$; and when $OA > OI$, it will change into positive, of itself, on account of $\rho \sqrt{\dfrac{1}{2}} < r$.

2nd. The actions from R'' and R''', which are equal, and give

$$+\frac{2w}{R''A_0{}^2} \cos \frac{1}{2} R''A_0R''' = \frac{2w}{R''A_0{}^2} \cdot \frac{A_0I'}{R''A_0} = \frac{2w \left(\rho \sqrt{\dfrac{1}{2}} + r \right)}{\sqrt{\left\{ \left(\rho \sqrt{\dfrac{1}{2}} + r \right)^2 + \dfrac{\rho^2}{2} \right\}^3}};$$

for $A_0I' = OI' + OA_0 = \rho \sqrt{\dfrac{1}{2}} + r$. This resultant is positive, since it tends to augment r.

3rd. The actions from R^{iv} and R^{v}, which are equal, and give

$$+\frac{2w}{R^{\text{v}}A_0^2}\cos R'A_0O=\frac{2w}{R^{\text{v}}A_0^2}\frac{OA_0}{R^{\text{v}}A_0}=\frac{2wr}{\sqrt{(\rho^2+r^2)^3}};$$

and this resultant also is positive. Hence, if P be the total action of the envelope on any element of the nucleus, we shall have

$$P=2w\left\{\frac{r}{\sqrt{(\rho^2+r^2)^3}}+\frac{r+\rho\sqrt{\frac{1}{2}}}{\sqrt{\left(\rho^2+r^2+\frac{2\rho r}{\sqrt{2}}\right)^3}}+\frac{r-\rho\sqrt{\frac{1}{2}}}{\sqrt{\left(\rho^2+r^2-\frac{2\rho r}{\sqrt{2}}\right)^3}}\right\}.$$

This value of P is always positive; and therefore, in this system also, the action of the envelope tends always to expand the nucleus.

The action of the whole system on any point of the nucleus will be

$$\frac{d^2r}{dt^2}=\frac{v-M_{(12)}v'}{r^2}+2w\left\{\frac{r}{\sqrt{(\rho^2+r^2)^3}}+\frac{r+\rho\sqrt{\frac{1}{2}}}{\sqrt{\left(\rho^2+r^2+\frac{2\rho r}{\sqrt{2}}\right)^3}}\right.$$

$$\left.+\frac{r-\rho\sqrt{\frac{1}{2}}}{\sqrt{\left(\rho^2+r^2-\frac{2\rho r}{\sqrt{2}}\right)^3}}\right\}\ldots\ldots(14).$$

Let us now determine the action of the whole system on any element R of the envelope. The action of the centre O and of the other elements of the envelope is (Book IV. Prob. IX.)

$$+\frac{1}{\rho^2}(v+M_6w).$$

The action of the nucleus on R comprises 1st, four actions from the four elements of it, which are nearer to the point R, of which the resultant is

$$-\frac{4v'}{AR^2}\cos ARO=-\frac{4v'}{AR^2}\cdot\frac{\rho-r\sqrt{\frac{1}{2}}}{AR}=-\frac{4v'\left(\rho-r\sqrt{\frac{1}{2}}\right)}{\sqrt{\left(\rho^2+r^2-\frac{2\rho r}{\sqrt{2}}\right)}}.$$

2nd. Four actions from the four elements of it, which are most remote from R, of which the resultant is

$$-\frac{4v'}{A''R^2}\cos A''RO = -\frac{4v'}{A''R^2}\cdot\frac{\rho + r\sqrt{\frac{1}{2}}}{A''R} = -\frac{4v'\left(\rho + r\sqrt{\frac{1}{2}}\right)}{\sqrt{\left(\rho^2 + r^2 + \frac{2\rho r}{\sqrt{2}}\right)^3}}.$$

3rd. Four actions from the four intermediate elements, of which the resultant is

$$-\frac{4v'}{A'R^2}\cos A'RO = -\frac{4v'}{A'R^2}\cdot\frac{\rho}{A'R} = -\frac{4v'\rho}{\sqrt{(\rho^2 + r^2)^3}}.$$

These actions are all negative, as they all tend to diminish the radius ρ.

Therefore the action of the whole system upon any element of the envelope will be

$$\frac{d^2\rho}{dt^2} = \frac{v + M_6 w}{\rho^2} - 4v'\left\{\frac{\rho}{\sqrt{(\rho^2 + r^2)^3}} + \frac{\rho + r\sqrt{\frac{1}{2}}}{\sqrt{\left(\rho^2 + r^2 + \frac{2\rho r}{\sqrt{2}}\right)^3}}\right.$$
$$\left. + \frac{\rho - r\sqrt{\frac{1}{2}}}{\sqrt{\left(\rho^2 + r^2 - \frac{2\rho r}{\sqrt{2}}\right)^3}}\right\}\ldots\ldots (15).$$

The equations (14) and (15) contain the solution of the problem. The conditions of equilibrium are, of course,

$$\frac{d^2 r}{dt^2} = 0, \quad \frac{d^2\rho}{dt^2} = 0.$$

To give an example, let us take $\rho = 1$, $r = 0\cdot5$. The conditions of equilibrium will entail

$$v = v, \quad v' = v\,.\,0\cdot26485, \quad w = v\,.\,1\cdot25010.$$

The system cannot be indefinitely expansive, unless we have

$$M_6 w + v > 12v';$$

which inequality may be easily drawn from the equation (15) by the method followed in the preceding problems. The system

admits eight varieties, the formulas of which will be obtained from (14) and (15) by simple changes of signs.

PROBLEM V.

To find the dynamical formulas of the hexahedric uninuclear system $m = R + (12) A + 8R'$, *in which the centre* O (fig. 17) *is repulsive, the nucleus is octo-hexahedric and attractive, the envelope hexahedric and repulsive.*

SOLUTION. Let v be the action of the centre, v' that of any element of the nucleus, w that of any element of the envelope: and let $OA = r$, $OR = \rho$.

Let us find first the action of the whole system on any point A of the nucleus. The action of the centre and of the other elements of the nucleus upon A is (Book IV. Prob. VII.)

$$+ \frac{1}{r^2}(v - M_{(12)}v').$$

The action of the envelope upon A comprises 1st, the actions from R and R', which are equal, and give

$$-\frac{2w}{AR^2}\cos RAI = -\frac{2w}{AR^2}\cdot\frac{AI}{AR} = -\frac{2w\left(\rho\sqrt{\frac{2}{3}}-r\right)}{\sqrt{\left\{\left(\rho\sqrt{\frac{2}{3}}-r\right)^2+\frac{\rho^2}{3}\right\}^3}};$$

for, $$AI = OI - AO = \rho\sqrt{\frac{2}{3}} - r.$$

This resultant must be taken as negative, since it tends to diminish r, at least when $OA < OI$; and, if $OA > OI$, then it will change its sign of itself.

2nd. The actions from R^{iv} and R^v, which are equal, and give

$$+\frac{2w}{R^vA^2}\cos R^vAI' = \frac{2w}{R^vA^2}\cdot\frac{AI'}{R^vA} = \frac{2w\left(\rho\sqrt{\frac{2}{3}}+r\right)}{\sqrt{\left\{\left(\rho\sqrt{\frac{2}{3}}+r\right)^2+\frac{\rho^2}{3}\right\}^3}};$$

and this resultant is positive, as it tends to augment r.

3rd. The actions from R'', R''', R^{VI}, R^{VII}, which are equal, and give

$$+ \frac{4w}{R^{\text{VII}}A^2} \cos R^{\text{VII}} A I' = \frac{4w}{R^{\text{VII}}A^2} \cdot \frac{AO}{R^{\text{VII}}A} = \frac{4wr}{\sqrt{(\rho^2 + r^2)^3}}.$$

This resultant also is positive. Hence the total action P of the envelope on any element of the nucleus will be

$$P = 2w \left\{ \frac{2r}{\sqrt{(\rho^2 + r^2)^3}} + \frac{r + \rho\sqrt{\frac{2}{3}}}{\sqrt{\left(\rho^2 + r^2 + 2\rho r\sqrt{\frac{2}{3}}\right)^3}} + \frac{r - \rho\sqrt{\frac{2}{3}}}{\sqrt{\left(\rho^2 + r^2 - 2\rho r\sqrt{\frac{2}{3}}\right)^3}} \right\},$$

and it is always positive. Hence, in this case also, the action of the envelope tends to expand the nucleus.

The action of the whole system on any element of the nucleus will be

$$\frac{d^2 r}{dt^2} = \frac{v - M_{(12)}v'}{r^2} + 2w \left\{ \frac{2r}{\sqrt{(\rho^2 + r^2)^3}} + \frac{r + \rho\sqrt{\frac{2}{3}}}{\sqrt{\left(\rho^2 + r^2 + 2\rho r\sqrt{\frac{2}{3}}\right)^3}} + \frac{r - \rho\sqrt{\frac{2}{3}}}{\sqrt{\left(\rho^2 + r^2 - 2\rho r\sqrt{\frac{2}{3}}\right)^3}} \right\} \quad \ldots\ldots\ldots (16).$$

Let us now find the action of the whole system on any element R of the envelope. The action of the centre and of the other elements of the envelope on R is (Book IV. Prob. IX.)

$$+ \frac{1}{\rho^2} (v + M_8 w).$$

The action of the nucleus on R comprises 1st, three actions from the points A, A', A'' which are the nearest to R, of which the resultant is

$$- \frac{3v'}{A'R^2} \cos A'RO = - \frac{3v'}{A'R^2} \cdot \frac{RE}{A'R} = - \frac{3v'\left(\rho - r\sqrt{\frac{2}{3}}\right)}{\sqrt{\left\{\left(\rho - r\sqrt{\frac{2}{3}}\right)^2 + \frac{r^2}{3}\right\}^3}} \; ;$$

for, we have

$$ER = OR - OE = \rho - \sqrt{OA'^2 - A'E^2} = \rho - r\sqrt{\frac{2}{3}} \, .$$

2nd. Three other actions from the three points most remote from R; of which the resultant is drawn from the preceding, by changing the sign before $r\sqrt{\frac{2}{3}}$; and so we have

$$- \frac{3v'\left(\rho + r\sqrt{\frac{2}{3}}\right)}{\sqrt{\left\{\left(\rho + r\sqrt{\frac{2}{3}}\right)^2 + \frac{r^2}{3}\right\}^3}} \, .$$

3rd. Six actions from the six intermediate points; of which the resultant is

$$- \frac{6v'}{A'''R^2} \cos A'''RO = - \frac{6v'}{A'''R^2} \cdot \frac{OR}{A'''R} = - \frac{6v'\rho}{\sqrt{(\rho^2 + r^2)^3}} \, .$$

All these actions are negative, as they all tend to diminish ρ. The action of the whole system on any element of the envelope will be

$$\frac{d^2\rho}{dt^2} = \frac{v + M_8 w}{\rho^2} - 3v'\left\{ \frac{2\rho}{\sqrt{(\rho^2 + r^2)^3}} + \frac{\rho + r\sqrt{\frac{2}{3}}}{\sqrt{\left(\rho^2 + r^2 + 2\rho r\sqrt{\frac{2}{3}}\right)^3}} \right.$$

$$\left. + \frac{\rho - r\sqrt{\frac{2}{3}}}{\sqrt{\left(\rho^2 + r^2 - 2\rho r\sqrt{\frac{2}{3}}\right)^3}} \right\} \, \ldots\ldots\ldots(17).$$

The equations (16) and (17) contain the solution of the problem. The condition of indefinite expansivity will be here

$$M_8 w + v > 12 v'.$$

Problem VI.

To find the dynamical formulas of the octo-hexahedric uninuclear system $\mathrm{m} = \mathrm{R} + 6\mathrm{A} + (12)\,\mathrm{R}'$, *of which the centre* O (*fig.* 18) *is repulsive, the nucleus is octahedric and attractive, the envelope octo-hexahedric and repulsive.*

Solution. Let v be the action of the centre, v' that of any element of the nucleus, w that of any element of the envelope : and let $OA = r$, $OR = \rho$.

The action of the centre and of the other elements of the nucleus upon the point A is (Book IV. Prob. VII.)

$$+ \frac{1}{r^2} (v - M_6 v').$$

The action of the envelope upon A comprises 1st, four actions from the four elements of it, which are nearer to A, of which the resultant is

$$- \frac{4w}{AR^2} \cos \frac{1}{2} RAR = - \frac{4w}{AR^2} \cdot \frac{AI}{AR} = - \frac{4w \left(\rho \sqrt{\dfrac{1}{2}} - r \right)}{\sqrt{\left(\rho^2 + r^2 - 2\rho r \sqrt{\dfrac{1}{2}} \right)^3}},$$

for we have $AI = OI - OA = \rho \sqrt{\dfrac{1}{2}} - r$.

2nd. Four actions from the four elements of it, which are most remote from R; of which the resultant differs from the preceding only by the sign of r, and is

$$+ \frac{4w \left(\rho \sqrt{\dfrac{1}{2}} + r \right)}{\sqrt{\left(\rho^2 + r^2 + 2\rho r \sqrt{\dfrac{1}{2}} \right)^3}}.$$

3rd. Four actions from the four intermediate elements; of which the resultant is

$$+ \frac{4w}{AR'^2} \cos R'A\,O = \frac{4w}{AR'^2} \cdot \frac{OA}{AR'} = \frac{4wr}{\sqrt{(\rho^2 + r^2)}^3}.$$

Of these three expressions, the last two are positive, as tending to augment r. The action of the envelope on any point of the nucleus is, accordingly,

$$P = 4w \left\{ \frac{r}{\sqrt{(\rho^2 + r^2)}^3} + \frac{r + \rho\sqrt{\frac{1}{2}}}{\sqrt{\left(\rho^2 + r^2 + \frac{2\rho r}{\sqrt{2}}\right)}^3} + \frac{r - \rho\sqrt{\frac{1}{2}}}{\sqrt{\left(\rho^2 + r^2 - \frac{2\rho r}{\sqrt{2}}\right)}^3} \right\},$$

which is always positive, and tends, therefore, to expand the nucleus. And the action of the whole system on any point of the nucleus will be

$$\frac{d^2 r}{dt^2} = \frac{v - M_6 v'}{r^2} + 4w \left\{ \frac{r}{\sqrt{(\rho^2 + r^2)}^3} + \frac{r + \rho\sqrt{\frac{1}{2}}}{\sqrt{\left(\rho^2 + r^2 + \frac{2\rho r}{\sqrt{2}}\right)}^3} \right.$$

$$\left. + \frac{r - \rho\sqrt{\frac{1}{2}}}{\sqrt{\left(\rho^2 + r^2 - \frac{2\rho r}{\sqrt{2}}\right)}^3} \right\} \quad\ldots\ldots\ldots (18).$$

Let us find now the action of the whole system on any point R_1 of the envelope. The action of the centre and of the other elements of the envelope R_1 is (Book IV. Prob. IX.)

$$+ \frac{1}{\rho^2} (v + M_{(12)} w).$$

The action of the nucleus upon R_1 comprises 1st, two actions from A and A''', which give

$$- \frac{2v'}{AR_1^2} \cos \frac{1}{2} ARA''' = - \frac{2v'}{AR_1^2} \cdot \frac{\rho - r\sqrt{\frac{1}{2}}}{AR_1}$$

$$= - \frac{2v'\left(\rho - r\sqrt{\frac{1}{2}}\right)}{\sqrt{\left(\rho^2 + r^2 - \frac{2\rho r}{\sqrt{2}}\right)}^3}.$$

2nd. Two actions from A' and A'', which give

$$-\frac{2v'\left(\rho + r\sqrt{\dfrac{1}{2}}\right)}{\sqrt{\left(\rho^2 + r^2 + \dfrac{2\rho r}{\sqrt{2}}\right)^3}}.$$

3rd. Two actions from A^{iv} and A^{v}, which give

$$-\frac{2v'}{A^{\mathrm{v}}R_1^2}\cos\frac{1}{2}\,A^{\mathrm{iv}}R_1A^{\mathrm{v}} = -\frac{2v'}{A^{\mathrm{v}}R_1^2}\cdot\frac{R_1O}{A^{\mathrm{v}}R_1} = -\frac{2v'\rho}{\sqrt{(\rho^2 + r^2)^3}}.$$

These actions are all negative. Hence, the action of the whole system on any element of the envelope will be

$$\frac{d^2\rho}{dt^2} = \frac{v + M_{(12)}w}{\rho^2} - 2v'\left\{\frac{\rho}{\sqrt{(\rho^2 + r^2)^3}} + \frac{\rho + r\sqrt{\dfrac{1}{2}}}{\sqrt{\left(\rho^2 + r^2 + \dfrac{2\rho r}{\sqrt{2}}\right)^3}}\right.$$

$$\left. + \frac{\rho - r\sqrt{\dfrac{1}{2}}}{\sqrt{\left(\rho^2 + r^2 - \dfrac{2\rho r}{\sqrt{2}}\right)^3}}\right\} \quad\ldots\ldots\ldots (19).$$

The equations (18) and (19) contain the solution of the problem.

PROBLEM VII.

To find the dynamical formulas of the octo-hexahedric uninuclear system $\mathrm{m} = R + 8A + (12)\,R'$, *in which the centre* O (*fig.* 19) *is repulsive, the nucleus is hexahedric and attractive, the envelope octo-hexahedric and repulsive.*

SOLUTION. Let v be, as usual, the action of the centre, v' that of any element of the nucleus, w that of any element of the envelope: and let $OA = r$, $OR = \rho$.

The action of the centre and of the other elements of the nucleus upon A is (Book IV. Prob. VII.)

$$+\frac{1}{r^2}(v - M_8v').$$

The action of the envelope on A comprises 1st, three actions from the three points R; of which the resultant is

$$-\frac{3w}{AR^2}\cos RAI = -\frac{3w}{AR^2}\cdot\frac{AI}{AR} = -\frac{3w\left(\rho\sqrt{\dfrac{2}{3}} - r\right)}{\sqrt{\left(\rho^2 + r^2 - 2\rho r\sqrt{\dfrac{2}{3}}\right)^3}}.$$

2nd.   Three actions from the three points R'; of which the resultant is

$$+\frac{3w\left(\rho\sqrt{\dfrac{2}{3}} + r\right)}{\sqrt{\left(\rho^2 + r^2 + 2\rho r\sqrt{\dfrac{2}{3}}\right)^3}}.$$

3rd.   Six actions from the six points R''; of which the resultant is

$$+\frac{6w}{R''A^2}\cos R''AO = \frac{6w}{R''A^2}\cdot\frac{OA}{R''A} = \frac{6wr}{\sqrt{(\rho^2 + r^2)^3}}.$$

Hence, the total action of the envelope on any element of the nucleus will be

$$P = 3w\left\{\frac{2r}{\sqrt{(\rho^2 + r^2)^3}} + \frac{r + \rho\sqrt{\dfrac{2}{3}}}{\sqrt{\left(\rho^2 + r^2 + 2\rho r\sqrt{\dfrac{2}{3}}\right)^3}} + \frac{r - \rho\sqrt{\dfrac{2}{3}}}{\sqrt{\left(\rho^2 + r^2 - 2\rho r\sqrt{\dfrac{2}{3}}\right)^3}}\right\}.$$

This value of P is always positive, and, therefore, tends always to expand the nucleus.   The action of the whole system on any element of the nucleus is

$$\frac{d^2r}{dt^2} = \frac{v - M_8 v'}{r^2} + 3w\left\{\frac{2r}{\sqrt{(\rho^2 + r^2)^3}} + \frac{r + \rho\sqrt{\dfrac{2}{3}}}{\sqrt{\left(\rho^2 + r^2 + 2\rho r\sqrt{\dfrac{2}{3}}\right)^3}} + \frac{r - \rho\sqrt{\dfrac{2}{3}}}{\sqrt{\left(\rho^2 + r^2 - 2\rho r\sqrt{\dfrac{2}{3}}\right)^3}}\right\} \quad \ldots\ldots\ldots (20).$$

Let us now find the action of the whole system on any point R_1 of the envelope. The action of the centre and of the other elements of the envelope on R_1 is (Book IV. Prob. IX.)

$$+ \frac{1}{\rho^2}\left(v + M_{(12)}w\right).$$

The action of the nucleus on R_1 comprises 1st, two actions from A and A', which give

$$-\frac{2v'}{AR_1^2}\cos AR_1 O = -\frac{2v'}{AR_1^2}\cdot\frac{\rho - r\sqrt{\dfrac{2}{3}}}{AR_1} = -\frac{2v'\left(\rho - r\sqrt{\dfrac{2}{3}}\right)}{\sqrt{\left(\rho^2 + r^2 - 2\rho r\sqrt{\dfrac{2}{3}}\right)^3}}.$$

2nd. Two actions from A^{vi} and A^{vii}, which give

$$-\frac{2v'\left(\rho + r\sqrt{\dfrac{2}{3}}\right)}{\sqrt{\left(\rho^2 + r^2 + 2\rho r\sqrt{\dfrac{2}{3}}\right)^3}}.$$

3rd. Four actions from A'', A''', A^{iv}, A^{v}, which give

$$-\frac{4v'}{A''R_1^2}\cos A''R_1 O = -\frac{4v'}{A''R_1^2}\cdot\frac{OR_1}{A''R_1} = -\frac{4v'\rho}{\sqrt{(\rho^2 + r^2)^3}}.$$

These actions are all negative. The action of the whole system upon any element of the envelope will be

$$\frac{d^2\rho}{dt^2} = \frac{v + M_{(12)}w}{\rho^2} - 2v'\left\{\frac{2\rho}{\sqrt{(\rho^2 + r^2)^3}} + \frac{\rho + r\sqrt{\dfrac{2}{3}}}{\sqrt{\left(\rho^2 + r^2 + 2\rho r\sqrt{\dfrac{2}{3}}\right)^3}}\right.$$
$$\left. + \frac{\rho - r\sqrt{\dfrac{2}{3}}}{\sqrt{\left(\rho^2 + r^2 - 2\rho r\sqrt{\dfrac{2}{3}}\right)^3}}\right\}\ldots\ldots(21).$$

The equations (20) and (21) contain the solution of the problem.

PROBLEM VIII.

To find the dynamical formulas of the dodecahedric uninuclear system $m = R + 12A + 20R'$, *in which the centre* O (*fig.* 20) *is re-*

pulsive, the nucleus is icosahedric and attractive, the envelope dode-cahedric and repulsive.

SOLUTION.   Let v be, as usual, the action of the centre, v' that of any element of the nucleus, w that of any element of the envelope: and let $OA = r$, $OR = \rho$.

The action of the centre and of the other elements of the nucleus upon A is (Book IV. Prob. VII.)

$$+ \frac{1}{r^2} (v - M_{12} v').$$

The action of the envelope on A comprises 1st, five equal actions from the five points R which are nearest to A; of which the resultant is

$$- \frac{5w}{AR^2} \cos RAI = - \frac{5w}{AR^2} \cdot \frac{AI}{AR} = - \frac{5w}{AR^2} \cdot \frac{OI - OA}{AR}.$$

But

$$OI = \sqrt{\rho^2 - RI^2} = \sqrt{\rho^2 - a^2 \rho^2} = \rho \sqrt{1 - a^2},$$

a being a number to be determined hereafter.   So also

$$AR^2 = (\rho \sqrt{1 - a^2} - r)^2 + a^2 \rho^2 \, ;$$

hence our resultant will be

$$- \frac{5w (\rho \sqrt{1 - a^2} - r)}{\sqrt{\{(\rho \sqrt{1 - a^2} - r)^2 + a^2 \rho^2\}^3}}.$$

2nd.   Five equal actions from the five points R'; of which the resultant is

$$+ \frac{5w}{AR'^2} \cos R'AI' = \frac{5w}{AR'^2} \cdot \frac{AI'}{AR'} = \frac{5w (\rho \sqrt{1 - a^2} + r)}{\sqrt{\{(\rho \sqrt{1 - a^2} + r)^2 + a^2 \rho^2\}^3}}.$$

3rd.   Five equal actions from the five points R'', of which the resultant is

$$+ \frac{5w}{AR''^2} \cos R''AO = \frac{5w}{AR''^2} \cdot \frac{Ai}{AR''} = \frac{5w}{AR''^2} \cdot \frac{AO - Oi}{AR''}.$$

But

$$Oi = \sqrt{\rho^2 - R''i^2} = \sqrt{\rho^2 - b^2 \rho^2} = \rho \sqrt{1 - b^2},$$

b being a number to be determined hereafter. Hence this resultant will be expressed by

$$+ \frac{5w\,(r - \rho\,\sqrt{1 - b^2})}{\sqrt{\{(r - \rho\,\sqrt{1 - b^2})^2 + b^2\rho^2\}^3}}\,.$$

4th. Five equal actions from the five points R''', of which the resultant is

$$+ \frac{5w}{AR'''^2}\cos R'''A\,O = \frac{5w}{AR'''^2}\cdot\frac{Ai''}{AR''} = \frac{5w\,(r + \rho\,\sqrt{1 - b^2})}{\sqrt{\{(r + \rho\,\sqrt{1 - b^2})^2 + b^2\rho^2\}^3}}\,.$$

Hence, the action of the whole system on any point of the nucleus will be

$$\frac{d^2r}{dt^2} = \frac{v - M_{12}v'}{r^2} + 5w\left\{\frac{r + \rho\,\sqrt{1 - a^2}}{\sqrt{(\rho^2 + r^2 + 2\rho r\,\sqrt{1 - a^2})^3}} + \frac{r - \rho\,\sqrt{1 - a^2}}{\sqrt{(\rho^2 + r^2 - 2\rho r\,\sqrt{1 - a^2})^3}}\right.$$

$$\left. + \frac{r + \rho\,\sqrt{1 - b^2}}{\sqrt{(\rho^2 + r^2 + 2\rho r\,\sqrt{1 - b^2})^3}} + \frac{r - \rho\,\sqrt{1 - b^2}}{\sqrt{(\rho^2 + r^2 - 2\rho r\,\sqrt{1 - b^2})^3}}\right\} \dots (22).$$

Let us now determine the action of the whole system upon any point of the envelope. The action of the centre O' (fig. 21) and of the other elements of the envelope on R is (Book IV. Prob. IX.)

$$+ \frac{1}{\rho^2}\,(v + M_{20}w).$$

The action of the nucleus on R comprises 1st, three equal actions from the three elements A which are nearer to R; of which the resultant is

$$-\frac{3v'}{AR^2}\cos ARI = -\frac{3v'}{AR^2}\cdot\frac{RI}{AR} = -\frac{\rho - O'I}{AR}\cdot\frac{3v'}{AR^2}\,.$$

But

$$O'I^2 = r^2 - AI^2 = r^2 - \frac{4}{9}\cdot\frac{3}{4}\,AA^2 = r^2 - \frac{1}{3}\,AA^2 = r^2 - \frac{1}{3}\,r^2\,(1{\cdot}05145)^2,$$

as is evident from the preceding Book, Problem V. where we

determined the edge of an icosahedron in terms of its radius. Therefore

$$O'I = r \sqrt{1 - \frac{1}{3}(1\cdot05145)^2} = r\sqrt{1 - c^2};$$

and our resultant becomes

$$- \frac{3v'(\rho - r\sqrt{1-c^2})}{\sqrt{\{(\rho - r\sqrt{1-c^2})^2 + r^2c^2\}^3}}.$$

2nd. Three equal actions from the three elements A', which are the most remote from R; of which the resultant may be obtained from the last expression by changing the sign of $O'I = r\sqrt{1-c^2}$; whence

$$- \frac{3v'(\rho + r\sqrt{1-c^2})}{\sqrt{\{(\rho + r\sqrt{1-c^2})^2 + c^2r^2\}^3}}.$$

3rd. Three equal actions from the three elements A''; of which the resultant is

$$-\frac{3v'}{A''R^2}\cos A''RO = -\frac{3v'}{A''R^2} \cdot \frac{Ri}{A''R} = -\frac{3v'}{A''R^2} \cdot \frac{\rho - O'i}{A''R}.$$

But

$$O'i^2 = r^2 - A''i^2 = r^2 - \frac{4}{9}A''A''^2\frac{3}{4} = r^2 - \frac{1}{3}A''A''^2.$$

Now, since $A''A''$ is a diagonal of a pentagon, of which the side is expressed by $r \times 1\cdot05145$, we shall have

$$A''A'' = \frac{r\cdot1\cdot05145}{2\cos 72^\circ} = \frac{r\cdot1\cdot05145}{0\cdot618034} = r\cdot1\cdot701103;$$

therefore

$$O'i = r\sqrt{1 - \frac{1}{3}(1\cdot70110)^2} = r\sqrt{1 - d^2}.$$

Hence our resultant will be

$$- \frac{3v'(\rho - r\sqrt{1-d^2})}{\sqrt{\{(\rho - r\sqrt{1-d^2})^2 + d^2r^2\}^3}}.$$

4th. Three equal actions from the three elements A''', of which the resultant is

$$- \frac{3v' \left(\rho + r\sqrt{1-d^2}\right)}{\sqrt{\{(\rho + r\sqrt{1-d^2})^2 + d^2 r^2\}^3}},$$

and differs from the preceding by the sign of $r\sqrt{1-d^2}$. These actions are all negative, since they all tend to diminish the radius ρ. Therefore, the action of the whole system upon any point of the envelope will be

$$\frac{d^2\rho}{dt^2} = \frac{v + M_{20}w}{\rho^2} - 3v' \left\{ \frac{\rho + r\sqrt{1-c^2}}{\sqrt{(\rho^2 + r^2 + 2\rho r\sqrt{1-c^2})^3}} + \frac{\rho - r\sqrt{1-c^2}}{\sqrt{(\rho^2 + r^2 - 2\rho r\sqrt{1-c^2})^3}} \right.$$
$$\left. + \frac{\rho + r\sqrt{1-d^2}}{\sqrt{(\rho^2 + r^2 + 2\rho r\sqrt{1-d^2})^3}} + \frac{\rho - r\sqrt{1-d^2}}{\sqrt{(\rho^2 + r^2 - 2\rho r\sqrt{1-d^2})^3}} \right\} \ldots (23).$$

The equations (22) and (23) contain the solution of the problem. In the equation (23) we have

$$d^2 = 0{\cdot}96458, \quad c^2 = 0{\cdot}36851.$$

As for the numbers a^2 and b^2 of the equation (22), they may be determined as follows. We have (fig. 20)

$$RR = 2RI \sin 72^0, \text{ and } R''R'' = 2R''i \sin 72^0;$$

whence

$$RI = a\rho = \frac{RR}{2\sin 72^0}, \quad R''i = b\rho = \frac{R''R''}{2\sin 72^0}.$$

But RR is the edge of a dodecahedron; and therefore (Book IV. Prob. VI.)

$$RR = 4\rho \sqrt{\frac{1}{3}} \cos 72^0 = \rho \cdot 0{\cdot}71364,$$

as also

$$R''R'' = \frac{R''R}{\cos 72^0} = \frac{4\rho \sqrt{\frac{1}{3}} \cos 72^0}{2\cos 72^0} = 2\rho \sqrt{\frac{1}{3}} = \rho \cdot 1{\cdot}15460;$$

therefore,

$$a = \frac{0{\cdot}71364}{2\sin 72^0} = \frac{0{\cdot}71364}{1{\cdot}902113} = 0{\cdot}37518,$$

$$b = \frac{1{\cdot}15460}{2\sin 72^0} = \frac{1{\cdot}15460}{1{\cdot}902113} = 0{\cdot}60701;$$

whence

$$a^2 = 0\cdot 13976, \quad b^2 = 0\cdot 36851.$$

Hence we conclude that we have also $b^2 = c^2$.

PROBLEM IX.

To find the dynamical formulas of the icosahedric uninuclear system $\mathrm{m} = \mathrm{R} + 20\mathrm{A} + 12\mathrm{R}'$, *in which the centre* O (fig. 22) *is repulsive, the nucleus dodecahedric and attractive, the envelope icosahedric and repulsive.*

SOLUTION.   Let v be the action of the centre, v' that of any element of the nucleus, w that of any element of the envelope : and let $OA = r$, $OR = \rho$.

The action of the centre and of the other elements of the nucleus on the element A is (Book IV. Prob. VII.)

$$+ \frac{1}{r^2} (v - M_{20} v').$$

The action of the envelope upon A comprises, 1st, three equal actions from the three elements R which are nearer to A; of which the resultant is

$$- \frac{3w}{AR^2} \cos RAE = - \frac{3w}{AR^2} \cdot \frac{AE}{AR} = - \frac{3w}{AR^2} \cdot \frac{OE - OA}{AR} \cdot$$

But

$$OA = r, \quad OE = \rho \sqrt{1 - c^2},$$

as we have seen in the last problem.   And, again,

$$AR^2 = AE^2 + ER^2 = (\rho \sqrt{1 - c^2} - r)^2 + \frac{4}{9} RR^2 \cdot \frac{3}{4}$$

$$= (\rho \sqrt{1 - c^2} - r)^2 + \frac{1}{3} \rho^2 (1\cdot 05145)^2 = (\rho \sqrt{1 - c^2} - r)^2 + c^2 \rho^2.$$

Therefore, the said resultant will be expressed by

$$- \frac{3w (\rho \sqrt{1 - c^2} - r)}{\sqrt{\{(\rho \sqrt{1 - c^2} - r)^2 + c^2 \rho^2\}^3}} \cdot$$

2nd, three equal actions from the three elements R' which are the most remote from A, of which the resultant will be

$$+ \frac{3w (\rho \sqrt{1 - c^2} + r)}{\sqrt{\{(\rho \sqrt{1 - c^2} + r)^2 + c^2 \rho^2\}^3}} .$$

3rd, three equal actions from the three elements R'', of which the resultant is

$$+ \frac{3w}{R''A^2} \cos R''AO = \frac{3w}{R''A^2} \cdot \frac{Ai}{R''A} = \frac{3w}{R''A^2} \cdot \frac{AO - Oi}{R''A} .$$

But

$$Oi^2 = \rho^2 - R''i^2 = \rho^2 - \frac{4}{9} R''I^2 = \rho^2 - \frac{4}{9} \cdot \frac{3}{4} R''R''^2 = \rho^2 - \frac{1}{3} R''R''^2.$$

Now, $R''R''$ is the diagonal of a pentagon, of which the side is $\rho \times 1 \cdot 05145$; therefore

$$R''R'' = \frac{\rho \cdot 1 \cdot 05145}{2 \cos 72^0} = \rho \cdot 1 \cdot 701103,$$

and

$$Oi = \rho \sqrt{1 - \frac{1}{3} (1 \cdot 70113)^2} = \rho \sqrt{1 - d^2} ;$$

and so our resultant will be

$$+ \frac{3w (r - \rho \sqrt{1 - d^2})}{\sqrt{\{(r - \rho \sqrt{1 - d^2})^2 + d^2 \rho^2\}^3}} .$$

4th, three equal actions from the three elements R''', of which the resultant is

$$+ \frac{3w (r + \rho \sqrt{1 - d^2})}{\sqrt{\{(r + \rho \sqrt{1 - d^2})^2 + d^2 \rho^2\}^3}} .$$

Therefore, the action of the whole system upon any point of the nucleus will be

$$\frac{d^2 r}{dt^2} = \frac{v - M_{20} v'}{r^2} + 3w \left\{ \frac{r - \rho \sqrt{1 - c^2}}{\sqrt{(\rho^2 + r^2 - 2\rho r \sqrt{1 - c^2})^3}} + \frac{r + \rho \sqrt{1 - c^2}}{\sqrt{(\rho^2 + r^2 + 2\rho r \sqrt{1 - c^2})^3}} \right.$$
$$\left. + \frac{r - \rho \sqrt{1 - d^2}}{\sqrt{(\rho^2 + r^2 - 2\rho r \sqrt{1 - d^2})^3}} + \frac{r + \rho \sqrt{1 - d^2}}{\sqrt{(\rho^2 + r^2 - 2\rho r \sqrt{1 - d^2})^3}} \right\} \ldots (24).$$

Let us now find the action of the whole system on any element of the envelope. The action of the centre and of the other elements of the envelope upon R_1 is (Book IV. Prob. IX.)

$$+ \frac{1}{\rho^2} (v + M_{12} w).$$

The action of the nucleus on R_1 comprises, 1st, five equal actions from the points A, A', A'', A''', A^{IV}, which are nearest to R_1, of which the resultant is

$$- \frac{5v'}{AR_1^2} \cos AR_1\, O = - \frac{5v'}{AR_1^2} \cdot \frac{R_1 h}{AR_1} = - \frac{5v'}{AR_1^2} \cdot \frac{\rho - Oh}{AR_1}.$$

But $Oh = \sqrt{r^2 - Ah^2}$, Ah being a straight line drawn from A to the centre of the pentagon $AA'A''A'''A^{IV}$; and, therefore, $Ah = ar$, a having here the same value as in the preceding problem. Hence our resultant will be

$$- \frac{5v' (\rho - r\sqrt{1 - a^2})}{\sqrt{\{(\rho - r\sqrt{1-a^2})^2 + a^2 r^2\}^3}}.$$

2nd, five equal actions from the elements a, a', a'', a''', a^{IV}, which are the most remote from R_1, of which the resultant will be

$$- \frac{5v' (\rho + r\sqrt{1 - a^2})}{\sqrt{\{(\rho + r\sqrt{1-a^2})^2 + a^2 r^2\}^3}}.$$

3rd, five equal actions from the five elements m; of which the resultant is

$$- \frac{5v'}{R_1 m^2} \cos OR_1 m = - \frac{5v'}{R_1 m^2} \cdot \frac{R_1 k}{R_1 m} = - \frac{5v'}{R_1 m^2} \cdot \frac{\rho - Ok}{R_1 m}.$$

But $Ok = \sqrt{r^2 - mk^2}$, mk being a straight line drawn from m to the centre of the pentagon $mmmmm$; and, therefore, $mk = br$, b having here the same value as in the preceding problem. Hence our resultant will be

$$- \frac{5v' (\rho - r\sqrt{1 - b^2})}{\sqrt{\{(\rho - r\sqrt{1-b^2})^2 + b^2 r^2\}^3}}.$$

4th, five equal actions from the five elements n; of which the resultant will be

$$- \frac{5v'\left(\rho + r\sqrt{1-b^2}\right)}{\sqrt{\{(\rho + r\sqrt{1-b^2})^2 + b^2 r^2\}^3}}.$$

Therefore, the action of the whole system on any element of the envelope will be

$$\frac{d^2\rho}{dt^2} = \frac{v + M_{12}w}{\rho^2} - 5v'\left\{ \frac{\rho + r\sqrt{1-a^2}}{\sqrt{(\rho^2 + r^2 + 2\rho r\sqrt{1-a^2})^3}} + \frac{\rho - r\sqrt{1-a^2}}{\sqrt{(\rho^2 + r^2 - 2\rho r\sqrt{1-a^2})^3}} \right.$$
$$\left. + \frac{\rho + r\sqrt{1-b^2}}{\sqrt{(\rho^2 + r^2 + 2\rho r\sqrt{1-b^2})^3}} + \frac{\rho - r\sqrt{1-b^2}}{\sqrt{(\rho^2 + r^2 - 2\rho r\sqrt{1-b^2})^3}} \right\} \dots \dots (25).$$

The equations (24) and (25) contain the solution of the problem.

PROBLEM X.

To find the dynamical formulas of the tetrahedric binuclear system $\mathrm{m} = \mathrm{A} + 4\mathrm{R} + 4\mathrm{A}' + 4\mathrm{R}'$, *in which the centre* O (fig. 23) *is attractive, the first nucleus is repulsive, the second nucleus attractive, and the envelope repulsive.*

SOLUTION. Let v be the action of the centre, v' that of any element of the first nucleus, v''' that of any element of the second nucleus, w that of the envelope: and let $OR = r'$, $OA = r''$, $OR' = \rho$.

First, the action of the whole system upon any element R_1 of the first nucleus comprises, 1st, the action of the centre and of the other elements of the first nucleus, which, according to Problem I. of the preceding Book, is expressed by

$$- \frac{1}{r'^2}(v - M_4 v').$$

2nd, the action of the second nucleus on R, which, according to Problem I. of the present Book, is expressed by

$$- v''\left\{ \frac{1}{(r' + r'')^2} + \frac{3\left(r' - \frac{1}{3}r''\right)}{\sqrt{\left(r'^2 + r''^2 - \frac{2r'r''}{3}\right)^3}} \right\}.$$

3rd, the action of the envelope, which consists of the action of R_1' upon R, which is expressed by

$$- \frac{w}{(\rho - r')^2},$$

and of the three actions from the other three elements, giving a resultant

$$+ \frac{3w}{R_1 R'^2} \cos R' R_1 O = \frac{3w}{R_1 R'^2} \cdot \frac{R_1 I}{R_1 R'} = \frac{3w \left(r' + \frac{1}{3} \rho \right)}{\sqrt{\left(\rho^2 + r'^2 + \frac{2\rho r'}{3} \right)^3}}.$$

Hence, if the total action of the envelope on R is called S, we shall have

$$S = - \frac{w}{(\rho - r')^2} + \frac{3w \left(r' + \frac{1}{3} \rho \right)}{\sqrt{\left(\rho^2 + r'^2 + \frac{2\rho r'}{3} \right)^3}}.$$

And the action of the whole system on any element of the first nucleus will be

$$\frac{d^2 r'}{dt^2} = \frac{M_4 v' - v}{r'^2} - v'' \left\{ \frac{1}{(r' + r'')^2} + \frac{3 \left(r' - \frac{1}{3} r'' \right)}{\sqrt{\left(r'^2 + r''^2 - \frac{2 r' r''}{3} \right)^3}} \right\}$$

$$- w \left\{ \frac{1}{(\rho - r')^2} - \frac{3 \left(r' + \frac{1}{3} \rho \right)}{\sqrt{\left(\rho^2 + r'^2 + \frac{2\rho r'}{3} \right)^3}} \right\} \ldots \ldots (26).$$

Secondly, the action of the whole system on any point A of the second nucleus comprises, 1st, the action of the centre and of the other elements of the second nucleus, which is (Book IV. Prob. VIII.)

$$- \frac{1}{r''^2} (v + M_4 v'').$$

2nd, the action of the first nucleus on A, which is (Book V. Prob. I.)

$$+ v' \left\{ \frac{1}{(r' + r'')^2} + \frac{3\left(r'' - \frac{1}{3}r'\right)}{\sqrt{\left(\left(r'^2 + r''^2 - \frac{2r'r''}{3}\right)\right)^3}} \right\}.$$

3rd, the action of the envelope on A, which is ($ib.$)

$$+ w \left\{ \frac{1}{(\rho + r'')^2} + \frac{3\left(r'' - \frac{1}{3}\rho\right)}{\sqrt{\left(\rho^2 + r''^2 - \frac{2\rho r''}{3}\right)^3}} \right\}.$$

Therefore, the action of the whole system on any point of the second nucleus will be

$$\frac{d^2 r''}{dt^2} = - \frac{M_4 v'' + v}{r''^2} + v' \left\{ \frac{1}{(r' + r'')^2} + \frac{3\left(r'' - \frac{1}{3}r'\right)}{\sqrt{\left(r'^2 + r''^2 - \frac{2r'r''}{3}\right)^3}} \right\}$$

$$+ w \left\{ \frac{1}{(\rho + r'')^2} + \frac{3\left(r'' - \frac{1}{3}\rho\right)}{\sqrt{\left(\rho^2 + r''^2 - \frac{2\rho r''}{3}\right)^3}} \right\} \ldots \ldots (27).$$

Thirdly, the action of the whole system on any element R_1' of the envelope comprises, 1st, the action of the centre and of the other elements of the envelope, which is (Book IV. Prob. I.)

$$- \frac{1}{\rho^2} (v - M_4 w).$$

2nd, the action of the first nucleus upon R_1', which may be obtained in the same way as the expression of S, and is expressed by

$$+ v' \left\{ \frac{1}{(\rho - r')^2} + \frac{3\left(\rho + \frac{1}{3}r'\right)}{\sqrt{\left(\rho^2 + r'^2 + \frac{2\rho r'}{3}\right)^3}} \right\}.$$

3rd, the action of the second nucleus upon R_1', which is (Book V. Prob. I.)

$$- v'' \left\{ \frac{1}{(r''+\rho)^2} + \frac{3\left(\rho - \frac{1}{3}r''\right)}{\sqrt{\left(\rho^2 + r''^2 - \frac{2\rho r''}{3}\right)^3}} \right\}.$$

Therefore, the action of the whole system on any element of the envelope will be

$$\frac{d^2\rho}{dt^2} = \frac{M_4 w - v}{\rho^2} + v'\left\{ \frac{1}{(\rho - r')^2} + \frac{3\left(\rho + \frac{1}{3}r'\right)}{\sqrt{\left(\rho^2 + r'^2 + \frac{2\rho r'}{3}\right)^3}} \right\}$$

$$- v''\left\{ \frac{1}{(r''+\rho)^2} + \frac{3\left(\rho - \frac{1}{3}r''\right)}{\sqrt{\left(\rho^2 + r''^2 - \frac{2\rho r''}{3}\right)^3}} \right\} \dots\dots (28).$$

The equations (26), (27), (28) contain the solution of the problem.

PROBLEM XI.

To find the dynamical formulas of the octahedric binuclear system m = A + 6R + 8A' + 6R', *in which the centre* O (fig. 24) *is attractive, the first nucleus is octahedric and repulsive, the second nucleus hexahedric and attractive, the envelope octahedric and repulsive.*

SOLUTION. Let v be the action of the centre, v' that of any element of the first nucleus, v'' that of any element of the second nucleus, w that of any element of the envelope: and let $OR = r'$, $OA = r''$, $OR' = \rho$.

First, the action of the whole system upon any element R of the first nucleus comprises, 1st, the action of the centre and of the other elements of the first nucleus, which is (Book IV. Prob. II.)

$$-\frac{1}{r'^2}(v - M_6 v').$$

2nd, the action of the second nucleus on R, which is (Book V. Prob. III.)

$$-4v'' \left\{ \frac{r' + r'' \sqrt{\frac{1}{3}}}{\sqrt{\left(r'^2 + r''^2 + \frac{2r'r''}{\sqrt{3}}\right)^3}} + \frac{r' - r'' \sqrt{\frac{1}{3}}}{\sqrt{\left(r'^2 + r''^2 - \frac{2r'r''}{\sqrt{3}}\right)^3}} \right\}.$$

3rd, the action of the envelope on R, which consists of the action of R' expressed by

$$- \frac{w}{(\rho - r')^2},$$

of the action of the element $R_{,,,}$ expressed by

$$+ \frac{w}{(\rho + r')^2},$$

and of the other four actions from the elements $R_{,}, R_{,,}, R'', R'''$, which give

$$+ \frac{4w}{R''R^2} \cos R''RO = \frac{4w}{R''R^2} \cdot \frac{OR}{R''R} = \frac{4wr'}{\sqrt{(\rho^2 + r'^2)^3}}.$$

Hence the action of the envelope on any element of the first nucleus is

$$S = \frac{4wr'}{\sqrt{(\rho^2 + r'^2)^3}} + \frac{w}{(\rho + r')^2} - \frac{w}{(\rho - r')^2};$$

and the action of the whole system on any element of the first nucleus will be

$$\frac{d^2r'}{dt^2} = \frac{M_6 v' - v}{r'^2} - 4v'' \left\{ \frac{r' + r'' \sqrt{\frac{1}{3}}}{\sqrt{\left(r'^2 + r''^2 + \frac{2r'r''}{\sqrt{3}}\right)^3}} + \frac{r' - r'' \sqrt{\frac{1}{3}}}{\sqrt{\left(r'^2 + r''^2 - \frac{2r'r''}{\sqrt{3}}\right)^3}} \right\}$$

$$+ w \left\{ \frac{4r'}{\sqrt{(\rho^2 + r'^2)^3}} + \frac{1}{(\rho + r')^2} - \frac{1}{(\rho - r')^2} \right\} \ldots\ldots\ldots(29).$$

Secondly, the action of the whole system on any element A of the second nucleus comprises, 1st, the action of the centre and of the other elements of the second nucleus, which is (Book IV. Prob. VIII.)

$$- \frac{1}{r''^2} (v + M_8 v'').$$

2nd, the action of the first nucleus on A, which is (Book V. Prob. III.)

$$+3v'\left\{\frac{r''+r'\sqrt{\frac{1}{3}}}{\sqrt{\left(r'^2+r''^2+\frac{2r'r''}{\sqrt{3}}\right)^3}}+\frac{r''-r'\sqrt{\frac{1}{3}}}{\sqrt{\left(r'^2+r''^2-\frac{2r'r''}{\sqrt{3}}\right)^3}}\right\}.$$

3rd, the action of the envelope on A, which is of the same form, viz.

$$+3w\left\{\frac{r''+\rho\sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2+r''^2+\frac{2\rho r''}{\sqrt{3}}\right)^3}}+\frac{r''-\rho\sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2+r''^2-\frac{2\rho r''}{\sqrt{3}}\right)^3}}\right\}.$$

Therefore, the action of the whole system upon any element of the second nucleus will be

$$\frac{d^2r''}{dt^2}=-\frac{v+M_8v''}{r''^2}+3v'\left\{\frac{r''+r'\sqrt{\frac{1}{3}}}{\sqrt{\left(r'^2+r''^2+\frac{2r'r''}{\sqrt{3}}\right)^3}}+\frac{r''-r'\sqrt{\frac{1}{3}}}{\sqrt{\left(r'^2+r''^2-\frac{2r'r''}{\sqrt{3}}\right)^3}}\right\}$$

$$+3w\left\{\frac{r''+\rho\sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2+r''^2+\frac{2\rho r''}{\sqrt{3}}\right)^3}}+\frac{r''-\rho\sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2+r''^2-\frac{2\rho r''}{\sqrt{3}}\right)^3}}\right\}\dots(30).$$

Thirdly, the action of the whole system on any element R' of the envelope comprises, 1st, the action of the centre and of the other elements of the envelope on R', which is (Book IV. Prob. I.)

$$-\frac{1}{\rho^2}(v-M_6w).$$

2nd, the action of the first nucleus on R', which is

$$+v'\left\{\frac{1}{(\rho-r')^2}+\frac{1}{(\rho+r')^2}+\frac{4\rho}{\sqrt{(\rho^2+r'^2)^3}}\right\}.$$

3rd, the action of the second nucleus on R', which is

$$- 4v'' \left\{ \frac{\rho + r'' \sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2 + r''^2 + \frac{2\rho r''}{\sqrt{3}}\right)^3}} + \frac{\rho - r'' \sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2 + r''^2 - \frac{2\rho r''}{\sqrt{3}}\right)^3}} \right\}.$$

Therefore the action of the whole system on any element of the envelope will be

$$\frac{d^2\rho}{dt^2} = \frac{M_6 w - v}{\rho^2} + v' \left\{ \frac{4\rho}{\sqrt{(\rho^2 + r'^2)^3}} + \frac{1}{(\rho + r')^2} + \frac{1}{(\rho - r')^2} \right\}$$

$$- 4v'' \left\{ \frac{\rho + r'' \sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2 + r''^2 + \frac{2\rho r''}{\sqrt{3}}\right)^3}} + \frac{\rho - r'' \sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2 + r''^2 - \frac{2\rho r''}{\sqrt{3}}\right)^3}} \right\} \dots (31).$$

The equations (29), (30), (31) contain the solution of the problem.

REMARK.   The action of the repulsive envelope on the first nucleus tends to diminish its radius r.   And, indeed, it may easily be proved, that the expression of S is always negative, i.e. that we have

$$\frac{1}{(\rho - r')^2} > \frac{1}{(\rho + r')^2} + \frac{4r'}{\sqrt{(\rho^2 + r'^2)^3}},$$

or

$$\frac{1}{\left(1 - \frac{r'}{\rho}\right)^2} > \frac{1}{\left(1 + \frac{r'}{\rho}\right)^2} + \frac{4\frac{r'}{\rho}}{\sqrt{\left\{1 + \left(\frac{r'}{\rho}\right)^2\right\}^3}}.$$

And, in fact, since $\frac{r'}{\rho} > 0$ and < 1, by making $\frac{r'}{\rho} = x$, the inequality becomes

$$\frac{1}{(1 - x)^2} > \frac{1}{(1 + x)^2} + \frac{4x}{\sqrt{(1 + x^2)^3}},$$

whence

$$1 > \left(\frac{1-x}{1+x}\right)^2 + \frac{4\,(1-x)^2\,x}{(1+x^2)\,\sqrt{1+x^2}}\,.$$

Now, this inequality always subsists; for its second member becomes $= 1$ only when $x = 0$, viz. when the nucleus disappears. The same remark could be made with regard to the expression of S in the preceding problem.

PROBLEM XII.

To find the dynamical formulas of the dodecahedro-rhombic bi-nuclear system m $= R + 6A + 8R' + 6R''$, *in which the centre* O *(fig. 25) is repulsive, the first nucleus is octahedric and attractive, the second nucleus hexahedric and repulsive, the envelope octahedric and repulsive.*

SOLUTION. We may observe that a rhombic dodecahedron arises from a cube, when each of the six faces of the cube is surmounted by a square pyramid of the angle $45°$. Then the two faces of two pyramids, which end on the same edge of the cube, will be in one and the same plane, and constitute a rhombus; and, as the edges of the cube are twelve, the polyhedron will be a rhombic dodecahedron. Hence the system

$$m = R + 6A + 8R' + 6R''$$

will be a rhombic dodecahedron only when the length of the radius ρ of the envelope has a peculiar relation with the length of the radius r'' of the second nucleus. This relation will be found to be

$$\rho = r''\sqrt{\frac{4}{3}}\,.$$

Now, it is evident, that the equations of this system can be deduced from those of the preceding problem, by changing the signs before v, v', v''.

Hence, for the action of the whole system on the first nucleus, we shall have

$$\frac{d^2r'}{dt^2} = \frac{M_6v'-v}{r'^2} + 4v''\left\{\frac{r'+r''\sqrt{\frac{1}{3}}}{\sqrt{\left(r'^2+r''^2+\frac{2r'r''}{\sqrt{3}}\right)^3}}+\frac{r'-r''\sqrt{\frac{1}{3}}}{\sqrt{\left(r'^2+r''^2-\frac{2r'r''}{\sqrt{3}}\right)^3}}\right\}$$

$$+ w\left\{\frac{4r'}{\sqrt{(\rho^2+r'^2)^3}}+\frac{1}{(\rho+r')^2}-\frac{1}{(\rho-r')^2}\right\}.$$

The action of the whole system on the second nucleus will be expressed by

$$\frac{d^2r''}{dt^2} = \frac{v+M_8v''}{r''^2} - 3v'\left\{\frac{r''+r'\sqrt{\frac{1}{3}}}{\sqrt{\left(r'^2+r''^2+\frac{2r'r''}{\sqrt{3}}\right)^3}}+\frac{r''-r'\sqrt{\frac{1}{3}}}{\sqrt{\left(r'^2+r''^2-\frac{2r'r''}{\sqrt{3}}\right)^3}}\right\}$$

$$+ 3w\left\{\frac{r''+\rho\sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2+r''^2+\frac{2\rho r''}{\sqrt{3}}\right)^3}}+\frac{r''-\rho\sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2+r''^2-\frac{2\rho r''}{\sqrt{3}}\right)^3}}\right\}.$$

And the action of the whole system on the envelope will be

$$\frac{d^2\rho}{dt^2} = \frac{M_6w+v}{\rho^2} - v'\left\{\frac{4\rho}{\sqrt{(\rho^2+r'^2)^3}}+\frac{1}{(\rho+r')^2}+\frac{1}{(\rho-r')^2}\right\}$$

$$+ 4v''\left\{\frac{\rho+r''\sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2+r''^2+\frac{2\rho r''}{\sqrt{3}}\right)^3}}+\frac{\rho-r''\sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2+r''^2-\frac{2\rho r''}{\sqrt{3}}\right)^3}}\right\}.$$

By introducing into these equations the relation $\rho = r''\sqrt{\frac{4}{3}}$, we shall have the formulas of the problem.

BOOK VI.

ON THE MOLECULES OF BODIES IN GENERAL.

A GREAT number of problems might be added to those which we have hitherto resolved: but, as their solution, after what has been said, is only a question of material work, and on the other hand the equations, to which such problems lead, cannot be integrated on account of their ever-increasing complexity, so we think it better to dispense with further inquiries of this kind. Let us come to a first application of the doctrine contained in the preceding Books, and investigate the conditions, to which the molecules of solid, liquid, or expansive bodies are subjected, inasmuch as they are made up of attractive and repulsive elements connected by mutual action.

PROPOSITION I.

The chemical, or specific, atoms of any ponderable substance cannot consist of single separated elements.

DEMONSTRATION. A chemical 'atom' is the least quantity of matter under which the specific nature of a substance can exist. Thus, the specific 'atom' of water is the least possible quantity of water; so that nothing can be taken away without the nature of water being destroyed. Hence, the parts of a specific atom are not only *integrant* with regard to the quantity of the mass, but at the same time *constituent* with regard to the specific substance to which they belong. Now, we know from chemistry that the atomic weights may be represented by proportional numbers in the following manner, viz. if the atomic weight of hydrogen be chosen as the unit of weight, then the atomic weights of all other

primitive bodies will be represented by a number of such units: e.g. nitrogen by 14, sulphur by 16, iron by 28, mercury by 100, and so on. And, since the weights are proportional to the quantities of matter weighed, it is obvious that the quantity of matter existing in one atom of nitrogen, of sulphur, of iron, of mercury, etc. is 14, 16, 28, 100,...times greater respectively than the quantity of matter existing in one atom of hydrogen. Hence, evidently, none of these atoms can consist of one simple element: and, if any primitive atom were to consist of one simple element, it would be only the atom of hydrogen itself, which is the lightest, and therefore the most simple of all atoms. But, the hypothesis that the atom of hydrogen is a simple element cannot be upheld. For, this element would be repulsive, since hydrogen is an expanding fluid; hence, hydrogen would be purely and perfectly repulsive in all circumstances, and at all distances; and, therefore, it could not possess affinity for any other substance; which conclusion is contradicted by facts. And again (to omit other considerations) hydrogen observes Mariotte's law, i.e. a given mass of hydrogen exercises pressures, which are inversely proportional to the volumes which it occupies. Now, this law would not be kept if the atom of hydrogen were a single repulsive element. This is easily proved. Let v be the action of a repulsive element at the unit of distance, d the actual distance from element to element, l the edge of a cube full of such a repulsive substance, p the pressure (or the sum of the actions) exercised on the surface l^2 of the cube. If n is the number of elements filling the length l, we shall have $l = nd$, and

$$p = \frac{v}{d^2} \cdot n^2.$$

When the same mass, or number of elements, are subjected to another pressure p', then let l' be the side of the new cube occupied by it, and d' the new distance of the elements; we shall have $l' = nd'$, and

$$p' = \frac{v}{d'^2} \cdot n^2;$$

whence

$$p : p' :: \frac{1}{d^2} : \frac{1}{d'^2}.$$

But the relations $l = nd$, $l' = nd'$ give $l^2 : l'^2 :: d^2 : d'^2$; therefore

$$p : p' :: \frac{1}{l^2} : \frac{1}{l'^2};$$

and, consequently, the hypothesis of atoms being simple elements leads us to a law, according to which pressures would be inversely proportional to the surfaces on which they are exercised. And, since hydrogen follows another law, as we have already stated, we must conclude that the specific atoms of hydrogen cannot be simple elements. Q. E. D.

COROLLARY I. Therefore all bodies, whether solid or liquid or expansive, are made up of specific atoms having *composition*, i. e. quantity of mass. The absolute atomic mass is nothing else than the number of simple elements of which the specific atom is constituted.

COROLLARY II. Therefore each atom has also *bulk*, i. e. quantity of volume. And, indeed, simple elements cannot act on one another unless they are distant; and, therefore, no mechanical composition of elements is possible without intervening space. The atomic volume is the quantity of space comprised between the geometrical surfaces of the atom; these surfaces being determined by the extreme elements of the atom itself.

COROLLARY III. Therefore specific atoms are very justly called *molecules* (i. e. *parvæ moles*), since it is a fact that they imply volume.

PROPOSITION II.

A primitive molecule is a system of simple elements permanently connected by mutual actions.

DEMONSTRATION. As no continuous matter exists (Book I. Prop. VII.), it is evident that every bulk comes from material points related to one another through space. Now, since the molecules of primitive bodies are most refractory with regard to any change of nature, it follows that the material points, of which they are compounded, must be bound together by such ties as are sufficient to prevent permanently any change of specific composition. The

mutual connexion of these elements evidently arises from the active powers with which such elements are endowed, and which are permanently exerted according to the Newtonian law (Book II. Prop. VII.). These exertions, then, as far as primitive molecules are concerned, must be of such an intensity as to be sufficient without the aid of exterior causes to keep the system together. And, in fact, the molecules of primitive bodies, when acted upon by any exterior cause, undergo accidental modifications, but are never resolved, and cling invariably to their specific constitution. Now, molecules would not have this property if their specific constitution were dependent on external action; for, evidently, a change of external action would then lead to a change of specific composition. Hence it is evident that each primitive molecule consists of elements whose action is amply sufficient for the permanent constitution of an invariable specific system. Q. E. D.

COROLLARY I. Therefore primitive fluids, though tending to expand, are not made up of molecules indefinitely expansive. And, indeed, if a molecule were indefinitely expansive, it would tend, of itself, to its own dissolution, and consequently the fluid composed of such molecules would not be one of the permanent specific substances.

COROLLARY II. Therefore the molecules of primitive fluids, when they are not altered by pressure, have their natural bulk, beyond which they cannot expand. Thus a molecule of oxygen or of nitrogen situated at the outer surface of the atmosphere remains there under a certain volume, without any further tendency to expand. True it is that this result may be partially due to terrestrial attraction: but, even without this attraction, those molecules would have a limit of dilatation; since permanent substances cannot tend of themselves to their own dissolution.

These two corollaries are true, of course, of *primitive* molecules only; for it is known that molecules arising from chemical combination may be in need of exterior pressure to keep together without resolving, as is the case when two substances have very little affinity.

Proposition III.

A primitive molecule cannot consist of a centre and one envelope only, but must have also at least one nucleus.

Demonstration. A system consisting of a centre and an envelope contains four varieties, viz.

$$m = A + nA, \quad m = R + nR, \quad m = R + nA, \quad m = A + nR.$$

Now, no body can be made up of molecules of any of these varieties. The first, being purely attractive, possesses no resisting powers; and, therefore, a body made up of such molecules would be liable to indefinite compression without reacting; which is against the nature of all known ponderable bodies. The second must be excluded, because it represents a system which has no ties, and tends essentially to its own dissolution, and, therefore, cannot be found in natural substances (preceding Prop.). The third also must be excluded. For, if we have (Book IV. Prob. X.) $v - Mw > 0$, it will be impossible to make up a mass of such molecules, without the molecules themselves being resolved. And, indeed, the attractive envelopes will necessarily mingle with one another (such being the consequence of mutual attraction), and change the nature of the system. And, moreover, with the condition $v - Mw > 0$ the variety $m = R + nA$ would be indefinitely expansive, a reason sufficient, of itself, to exclude such a variety. If we have $v - Mw = 0$, or $v - Mw < 0$, then the body made up of such molecules would be unable to resist compression; which is not the case with known ponderable substances. The fourth variety also must be excluded; because, if we have $v - Mw > 0$ or $v - Mw = 0$, the system will not react against pressure, and, therefore, cannot belong to any known ponderable substance: and if we have $v - Mw < 0$, the system will have an insufficient constitution, and will tend to its own dissolution; and, therefore, cannot be admitted as existing in primitive substances, which are all permanent. Therefore no primitive substance is formed of molecules having only a centre and an envelope. Q. E. D.

PROPOSITION IV.

The envelope of a primitive molecule is always repulsive.

DEMONSTRATION. The hypothesis of molecules having attractive envelopes leads to consequences contrary to fact. And, first, a system of elements, if its envelope be attractive, cannot possibly preserve its specific constitution, when placed in proximity with other systems of the same nature. For, the attractive elements of the neighbouring envelopes will evidently approach one another and constitute a number of permanent vibratory systems totally distinct from the original systems and independent of them. Now, all primitive bodies consist of molecules which preserve invariably their specific constitution when uniting together in one mass. Therefore none of the primitive bodies are made up of systems having attractive envelopes. This proof might be developed by a detailed examination of the different systems uninuclear, binuclear, &c., of which we have treated in the preceding Books; yet I shall omit such a development, because otherwise I should fill whole pages without any very great advantage; the general argument being in its conciseness quite satisfactory.

Secondly, a system of elements, which represents a molecule of any primitive substance, must have an intrinsic constitution which guarantees the stability of its specific nature (Book VI. Prop. II.), and, therefore, must be endowed with powers calculated to resist any attempt at its dissolution. Hence, it is indispensable that in such a system the attractive powers, under all critical circumstances, should be able to check the repulsive. Now, suppose that a number of systems are placed near one another; if their envelopes be attractive, we shall see, that, whatever may be the geometrical figure of such systems, they will so arrange themselves with respect to each other as to form a new additional envelope around any system intercepted by them. This additional envelope will consist of the elements of the envelopes of the neighbouring systems, and will be attractive, since we suppose that the envelopes are attractive. The result will be that each system will acquire a new additional attractivity. And, as each system pos-

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sesses already, of its own, the attractivity which is amply sufficient to hold it together under a determinate volume, the additional envelope will double, triple, or in general multiply the amount of attraction in each system : so that each of them will be exposed to contract its bulk, at least under external pressure, without any possibility of resistance. And, as this would be true of all the systems which are actually in the mass (those only excepted which lie at its surface), it follows that a body formed of such systems would be utterly incapable of resisting compression. And, there-fore, ponderable bodies, which all resist compression, cannot be formed of molecules having an attractive envelope.

Thirdly, the molecules of expansive bodies repel each other, although attractivity must be predominant in their constitution (Book VI. Prop. II.) ; therefore their mutual repulsion cannot arise but from the greater vicinity of the repulsive elements. And since the elements of the envelopes are those that come nearer to one another, it follows that the envelopes are repulsive. The same must be said of the molecules of the liquid and solid bodies ; for, such molecules, if pressed against one another, refuse to ap-proach, and react with a great intensity : and, since attractivity must be absolutely predominant in their constitution, their reluct-ance to approach nearer to one another cannot arise but from the circumstance that the elements, which ought to approach more immediately, are repulsive. Now, such elements are those of which the envelopes are constituted. And consequently all the molecules of all ponderable bodies have a repulsive envelope. Q. E. D.

PROPOSITION V.

The molecules of primitive bodies are not quasi-planetary systems.

DEMONSTRATION. A quasi-planetary system (according to those who imagined the molecules to be so constituted) consists of a number of material points revolving around a material centre, and describing orbits, as planets do, though with a much greater amount of perturbation. If a molecule were made up of an attrac-

tive centre and a number of other elements incapable of constituting a regular polyhedric envelope, then, certainly, a molecule would be like a microscopic planetary system. Thus, if the elements of the molecular envelope were five, or seven, or eleven, each of them would be acted on with a different intensity on different sides, and each would be compelled, in its attempt at finding a position of equilibrium which is never to be found, to describe a sort of orbit. But it is easy to show that such systems cannot be admitted as having any part in the molecular constitution of any known substance.

And, first, the molecules of primitive bodies have a determinate nature, which remains always the same and retains the same specific properties; whilst a quasi-planetary system has not a permanent mode of being, but is perpetually changing; so that it must have in turn very different properties.

Secondly, the molecules of bodies have a determinate mass, which remains always the same, the atomic weights being constant: whilst in a quasi-planetary system, in which the elements of the envelope are constantly running about in all directions, after the fashion of planets or satellites, no reason can be found why new satellites should be prevented from joining the system, or some of the actual satellites should be prevented from withdrawing and joining another system. Should Venus approach sufficiently near the moon, what would prevent the moon from abandoning its orbit and becoming a satellite of this planet? Should Jupiter find in its way any number of moons, what would prevent them from becoming so many satellites of Jupiter? On the other hand, the molecules of bodies, in spite of all physical and chemical operations, cannot be made to change their atomic weights; and, therefore, molecules are not quasi-planetary systems.

Thirdly, the molecules of natural substances have their determinate figure, as is evident from the constancy of crystalline forms, and from other properties, especially chemical (as we shall see later on), of the same substances: whilst a quasi-planetary system has all sorts of figures in succession, and none of them regular.

Fourthly, the molecules of bodies are in possession of a nucleus

besides the centre and the envelope (Book VI. Prop. III.): now, the existence of a nucleus is inconceivable in a quasi-planetary system. For, in such a system the elements of the nucleus and of the envelope, while describing their orbits with mutual perturbation, cannot but mingle with one another in a perpetual confusion, which makes the constitution of the system altogether fortuitous and precarious. Hence, it is evident that the molecules of primitive bodies cannot be assumed to be planetary systems. Q.E.D.

COROLLARY. Therefore quasi-planetary systems are not a means of accounting for the combination of different substances. For, the reasons above alleged to prove that such systems do not exist in primitive substances, prove as well that they do not exist in compound substances.

PROPOSITION VI.

The molecules of primitive substances have, in general, a regular shape, i.e. they are regular polyhedrons.

DEMONSTRATION. Regular shape (tetrahedric, for instance, hexahedric, octahedric, &c.) is one of the conditions without which a molecular system cannot have stability. In fact, that which prevents a system of elements from having a regular shape, is either an inequality of power in the elements surrounding the centre and constituting the envelope or nucleus of the molecule, or else a number of elements not calculated to meet the geometrical exigency of regular figures. Now, both hypotheses are irreconcileable with stability of molecular constitution. Therefore, as the molecules of primitive bodies possess a very great stability of constitution (Book VI. Prop. II.), we conclude that these molecules are regular systems. First, then, if the elements of any nucleus or of the envelope possess a different amount of power, the system will evidently be in some parts less solidly built than in others; and consequently will offer less resistance to resolution in some parts than in others, and will be liable to greater alteration in some parts than in others. Hence, such a system, if acted on by very strong chemical agencies, will be exposed to resolution: which

is not the case with primitive substances. Secondly, if the elements surrounding the centre are not in sufficient number to form a regular polyhedron, then, either we fall into the same inconvenience as before, viz. a molecular constitution weaker in one part than in others, or we shall have a quasi-planetary system, which is inadmissible.

Moreover, we have sufficient indications of the regularity of primitive molecules in the shape of their crystals. Many such crystals are regular : and these cannot come from irregular molecules. Those crystals also, which are called irregular, often proceed very evidently from regular forms either immediately or mediately. A few primitive crystals, which cannot be traced to regular polyhedric forms, seem to constitute an exception; yet they may come from regular molecules which, in the act of crystallization, have suffered an accidental change of form. Doubtless, crystals having an invariable regular polyhedric form, like those of silver, gold, copper, diamond, lead, mercury, &c., cannot be conceived as resulting from molecules of irregular forms; for irregularity cannot be the source of constant regularity : and yet, since regular molecules, when meeting to form a crystal, may be accidentally modified, on one side only, by their mutual action, it is quite possible that molecules in the act of crystallization should be flattened or elongated, and that irregular crystals should be made up of molecules, which in their free state, viz. before crystallizing, were regular polyhedrons.

We shall not insist on this point. That a body, which is considered as a primitive substance, should be made up of molecules not perfectly regular, is not absolutely impossible : still, such an irregularity could not be proved from the irregularity of crystalline forms, and, were it demonstrated, we should then be entitled to suspect that that substance is not primitive, and a time may come when chemistry will be able to show its composition. However this may be, our proposition does not formally exclude all exceptions : and, therefore, even if a few substances, as sulphur* or

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* We think that the molecules of sulphur are regular tetrahedrons, though the two crystalline forms of this substance are very remote from the tetrahedric. See what we say in Book XI. numb. VII. and X.

tin, had irregular molecules, which cannot be proved, we are right
in saying that the molecules of primitive bodies have *in general*
a regular polyhedric form.   Q. E. D.

COROLLARY I.   Therefore, not only the envelope, but the
nucleus, or the nuclei, of primitive molecules are in general re-
gular polyhedrons ; for, the regularity of the one depends evidently
on the regularity of the others.

COROLLARY II.   Therefore all the elements of the same mole-
cular stratum, i. e. of the same nucleus or envelope, possess equal
power.   Were it otherwise, they would be unequally distant from
the centre of the system, and from themselves ; which would
destroy regularity of form.

COROLLARY III.   Therefore we can safely apply to primitive
molecules in general those considerations, which we have presented
in the preceding Books, about regular systems of elements.

PROPOSITION VII.

*The molecules of primitive bodies at certain distances attract
each other.*

DEMONSTRATION.   We assume as a known fact, that, as the
earth attracts bodies, so also bodies attract one another.   And,
since bodies are made up either of primitive molecules, or of
molecules resulting from their combination, it follows, that, though
repulsive action may prevail, in particular cases, between mole-
cules on account of the greater proximity of their repulsive ele-
ments, yet attractivity is absolutely greater than repulsivity in
each molecular system.   Consequently, every molecule, at least for
great distances in comparison with which the molecular radii may
be considered as infinitesimal quantities, and for which its total
action is equal to the algebraical sum of the actions of all its
elements, is attractive.   Q. E. D.

PROPOSITION VIII.

The molecules of primitive bodies are systems naturally endowed with a palpitatory motion of contraction and dilatation.

DEMONSTRATION. We have already shown that all molecules must have one nucleus, or many, and a repulsive envelope. Now, in the case of uninuclear molecules, the only system which can represent them is that whose material formula is

$$m = R + nA + n'R';$$

for, the other varieties, $m = R + nR' + n'R''$, $m = A + nA' + n'R$ and $m = A + nR + n'R'$, are not to be found in bodies, either on account of their incapability of reacting against pressure, or of their repulsivity at any distance however great. Now, that a molecule may keep permanently its own nature, it must retain its envelope firmly connected with the other constituent parts of the system. This connexion is obtained, of course, through the attraction exercised by the nucleus. Nor is it sufficient that this attraction should equilibrate the repulsive efforts of the centre and of the envelope upon the nucleus; for, if it were so, then the molecule would be repulsive at all great distances, as is evident from the expressions of the conditions of equilibrium which we have found (Book v. Prob. I. II. &c.), and from the numerical examples there given. Hence, we must admit that the attractive power of the nucleus is greater than that which would be necessary for the equilibrium of the nucleus: in other words, we must have

$$nv' > v + n'w,$$

according to our usual notation. And, consequently, the elements of the nucleus, on account of their prevailing attractivity, will approach the centre, and make regular vibrations through it, after the manner explained in Book IV. Prob. X. Meanwhile the envelope, being acted upon with successively different intensities by the nucleus, will itself contract and dilate, more or less, according to the conditions of the case, each element moving to and fro in

the direction of the radius, though it will never reach the centre nor vibrate through it, on account of mutual repulsivity.

In the case of binuclear molecules, the three varieties

$$m = R + nR' + n'A + n''R'',$$
$$m = A + nR + n'A' + n''R',$$
$$m = A + nR + n'R' + n''R'',$$

cannot be admitted as existing in natural bodies. The first and second cannot be admitted, because either they would be repulsive at any great distance, or, if they were attractive, the second nucleus would take the place of the first, and change the system. The third also cannot be admitted, because either it is repulsive at any great distance, or else the attractivity of its centre must be so great that the system would not be capable of reacting against compression. The only varieties that deserve consideration are the following,

$$m = R + n A + n'A' + n''R'',$$
$$m = A + nA' + n'R + n''R',$$
$$m = R + nA + n'R' + n''R'',$$

and they are all endowed with palpitatory motion. And, indeed, they are admissible only when they give rise respectively to the inequalities

$$nv' + n'v'' > v + n''w,$$
$$v + nv' > n'v'' + n''w,$$
$$nv' > v + n'v'' + n''w,$$

Now, by comparing these conditions with the dynamical formulas of their respective systems, we shall see that equilibrium is impossible with them, on account of the prevalence of attractivity. Hence, the system will be vibratory: and the envelopes themselves will have a motion of contraction and dilatation, as in the preceding case.

Hence it is clear, that the same motion of contraction and dilatation will be a property of all other molecules which are trinuclear, quadrinuclear, or, in general, polynuclear systems; for, in all such molecules equilibrium is impossible when their attractivity

absolutely prevails on their repulsivity, as is the case in natural bodies. We conclude, then, that all molecules, by reason of their own constitution*, are endowed with a movement of contraction and dilatation. Q. E. D.

PROPOSITION IX.

Molecules of the same constitution and figure at certain distances repel each other.

DEMONSTRATION. When a mass is formed of molecules having the same constitution and figure, these molecules have their envelopes directly opposite to each other; hence, when the distance of the repulsive elements which constitute the envelopes becomes much smaller than the distance of the attractive elements which constitute the nucleus immediately following, the repulsive actions become much more intense than the attractive: which is a corollary of the Newtonian law of actions. This proposition is evident also from the fact that liquid, solid, and fluid bodies resist compression; for, to resist mutual approach is evidently due to an exertion of power of a repulsive nature.

PROPOSITION X.

Molecules of the same constitution and figure can approach each other to a certain distance only, where they remain in equilibrium of position.

DEMONSTRATION. I call equilibrium of position the permanence of two molecules at the same distance, such distance being measured from the centre of the one to the centre of the other. This sort of equilibrium is only *relative*, and may coexist with the palpitation of the molecules, i. e. with the vibrations of the nuclei

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* I say "by reason of their constitution", viz. considering each molecule of itself; for, when a number of molecules are united in one mass, the action of surrounding molecules may perhaps cause a suspension of such a palpitatory motion; since the elements of the surrounding molecules form around the intercepted molecules a repulsive additional envelope, the action of which may be sufficient to establish equilibrium in the intercepted molecules.

through their centre, and the consequent oscillations of the en-
velopes.   Now, to prove our proposition, it suffices to remark that
molecules of the same constitution and figure at less distances
repel each other (Book VI. Prop. IX.), and at greater distances
attract each other (ib. Prop. VII.).   The consequence is, that at
some intermediate distance, where the action from attractive be-
comes repulsive, the expression of the action must change its
sign, and, therefore, becomes $= 0$.   At such distance, then, which
may be called *the distance of relative equilibrium,* attraction and
repulsion totally neutralize each other: and this is the only
distance at which any two molecules of the same constitution and
figure can and will remain in equilibrium of position.   Q.E.D.

BOOK VII.

ON THE GENERAL PROPERTIES OF BODIES.

FROM molecules constituted as we have explained the primitive bodies arise, whose properties we discover by means of experiment and observation. Bodies, accordingly, must have different properties, inasmuch as the molecules of which they are composed possess a different constitution; and from the special properties which are found to be inherent in different bodies, a special constitution of the molecules thereof may be inferred. But there are properties which are common to all ponderable bodies, on which something is to be said before we come to other special considerations. They result from the constitution and mutual relation of molecules considered in a general point of view, and they are styled "the general properties of ponderable bodies." Such general properties may be arranged in the following order:

1st. Quantity of mass and of volume.
2nd. Divisibility.
3rd. Porosity.
4th. Compressibility.
5th. Reactivity.
6th. Impenetrability.
7th. Capability of displaying the so called *force of inertia*.
8th. Vibrativity.

We do not rank among the general properties of bodies *activity, passivity* and *inertia*, because they are the properties of matter rather than of bodies; as they do not belong to bodies on account of their composition or bodily constitution, but only inas-

much as the elements themselves of which the molecules are made
up are essentially active, passive and inert, as we have seen in
Book I.

PROPERTY I.

The *quantity of mass* means, as all know, the quantity of
matter which exists in a body.   It is evident that the *absolute*
quantity of matter contained in a body must be expressed by the
absolute number of elements contained in the same body.   If,
then, we were to discover that the number of elements constitut-
ing one molecule of a homogeneous body is m, and that the body
contains k such molecules, we might know the absolute mass M of
the same body by the equation

$$M = mk.$$

But, as we cannot, in the present state of science, determine
exactly the factors of the absolute mass, so we prefer to measure
the quantity of mass by comparing it to a certain other quantity,
which we arbitrarily assume as the unit of mass.

The *volume* is the space comprised between the material points
that form the extreme boundary of the body.   As space is the
region of continuous motion, so we conceive space as continuous :
and, since in a continuous quantity no natural unit of measure is
to be found, we cannot measure space, unless we adopt an arbi-
trary unit of extension.   Hence quantity of volume is measured
only by another quantity of volume, which we consider as the
unit of volume : in the same way as we do with regard to the
quantity of mass.   There is, however, a difference between the
two ; for, the quantity of mass, though it is measured, for our con-
venience, by an arbitrary unit, is made up of simple elements,
each of which is a natural unit of measure ; whilst the quantity of
volume contains no such natural units, as we have remarked.

The masses of two or more bodies are exactly proportional to
their weights.   This principle (which we have proved in Book I.
Prop. VI.) gives us the means of determining the quantity of
mass.

The ratio of the mass to the volume is called the *density.* If M, V, δ are the mass, the volume, and the density of a given body, we shall have

$$\delta = \frac{M}{V};$$

and the unit of density will be the density of the body which in the unit of volume contains that quantity of mass which is taken as the unit of mass. From these common notions it is clear that each body must have mass and volume, and, therefore, density also.

PROPERTY II.

Divisibility is the capability of division or separation of parts. The absolute divisibility of bodies is exactly as great as the quantity of matter contained in the bodies. The relative divisibility is as great as the amount of power which is applicable to execute the division. These propositions are self-evident. It is equally evident, that the old question regarding the intrinsic possibility of dividing matter by an endless division, does not deserve, on our part, the least attention; since we know, that bodies are composed of a finite number of molecules, and these of a finite number of elements; and, these elements being attained, no division can proceed further. With regard to the infinite divisibility of the quantity of volume, physicists have nothing to say, as this is a question of geometry or rather of metaphysics, not of physics. We may only observe, that volume and space, like time, being continuous, must be capable of endless division: but, on the other hand, such division is only virtual, inasmuch as it does not *separate* one part of space or of time from another, but merely marks out the *distinction* of the one from the other. The reason of this is, because space and time have only virtual parts; whilst the quantity of matter which has material parts materially and formally distinct, though actually connected by mutual actions, is strictly and properly divisible, inasmuch as the dynamical connexion of those material parts can be destroyed by an external action forcing these parts asunder.

Property III.

Porosity may be defined the capability of containing a greater quantity of matter without an increase of volume. Porosity is either absolute or relative. The absolute porosity of a body is the whole space within the body unoccupied by matter. Our conception of absolute porosity will be true, if we consider the molecules of a body and each element of them as isolated in vacuo. The absolute pores of a body will then be those portions of space, which are actually determined by the relative position of a number of elements being in immediate vicinity : and the sum of the absolute pores will be equal to the volume itself of the body. This last inference is evident ; for, simple elements, though occupying space, cannot fill space, as they have no bulk ; and, therefore, the whole volume of the body is only *virtually* divided into smaller volumes, without any part of it being filled with matter. Those, who are wont to confound a space *occupied* with a space *filled* with matter, do not reflect that nothing but continuous matter could possibly fill any part of space ; whilst no continuous matter is required to occupy space; since occupation does not involve extension, and material unextended points are calculated to occupy space, as they mark out a point in space, which is their centre of action.

The relative porosity of a body is a space between its molecules more or less accessible to the molecules of another body, e.g. air, oil, water, &c. The body whose molecular constitution and arrangement permits the free access and permeation of such molecules, is *porous* with regard to the same; and, consequently, relative porosity means relative permeability. Relative porosity sometimes is quite sensible, as in the case of wood, sponge, &c.; but in many instances it is not sensible, as in steel, marble, &c.

That porosity is a general property of all bodies is an immediate corollary of their being composed of molecules, and the molecules of elements. So we have nothing more to say about this property.

PROPERTY IV.

Compressibility is the capability of undergoing a diminution of volume. From what we have said on the constitution of molecules it is clear that bodies must be absolutely compressible; for the distance between molecules, absolutely speaking, can be diminished, although we do not always possess a power sufficient to overcome the resistance we encounter, so as to produce a sensible diminution of bulk. Compressibility does not imply that the reduction of the bulk is durable; and, therefore, compressibility does not exclude the power of reacting, under compression, so as to restore the primitive bulk.

PROPERTY V.

Reactivity in general is the power of resisting an external action that tends to modify the state of the body: but very often we call reactivity more especially the power of struggling against compression. From what has been said above on the constitution of molecules it is evident that reactivity is a general property of all ponderable bodies; for, as the molecules must naturally maintain in each body their position of equilibrium (Book VI. Prop. X.), and cannot approach nearer to one another without their action becoming repulsive, the consequence is, that pressure, by diminishing the distance, puts those molecules in such a condition as is required for mutual repulsion. Such a repulsion, as opposite to the action by which it is occasioned, is called *reaction,* and the readiness of the body to react is called *reactivity.*

A body will react as long as its molecules remain at a distance less than that of relative equilibrium. There is only one case in which an exception to this general law is to be found; and this is when pressure alters the molecular constitution of the body so as to change its specific nature. In such a case the body is constrained to change its chemical constitution, or at least its physical state, as when vapours are condensed into liquids, or when two gases in consequence of compression combine with each other

and diminish in bulk. If, then, we except this one case (in which, however, molecular reaction has a chemical or quasi-chemical effect in promoting the constitution of molecules of a new kind, which require a new distance of relative equilibrium) the reaction is always, not only proportional, but perfectly equal to the action by which the body is compressed. This proposition may be proved thus.

Let us suppose, that a molecule whose centre is O (fig. 26) is projected with a certain velocity against another molecule whose centre is O'. At the instant for which the distance of the two molecules becomes less than that of the equilibrium of position, the repulsive powers of the molecule O' impinged upon tend to retard the advance of the impinging molecule O, whilst the repulsive powers of the impinging molecule O tend to repel the molecule O' impinged on. These two opposite efforts are felt more intensely by the elements a and by the elements a', which are nearer, than by the elements b or b', which are further apart. And, therefore, since at the beginning of the impact the elements a are more retarded than the elements b, and the elements a' receive a greater velocity than the elements b', the elements a and b will become nearer, and so also the elements a' and b' : in other words, there will be mutual compression. Now the molecules, when so compressed, are in an abnormal state, and tend, by reason of their own constitution, to reassume their natural shape by expansion : and, as they cannot expand on the side at which they are actually pressing one another, they will strive to dilate by extending in the opposite directions. Accordingly, the effect of mutual repulsion will be, that the molecule O' impinged on, while striving to free itself from unnatural compression, acquires, by interior working, a certain velocity in the direction $O'R$: and the impinging molecule O, while striving likewise to free itself from compression, acquires, by interior working, a certain velocity in the direction OP contrary to the direction of the impulse.

Now, the maximum of compression of these two molecules being an effect of the mutual repulsion of the elements a and a', as we have stated, such a maximum must be absolutely equal on both sides ; for, were it greater on one side, its excess would

induce a further compression, i. e. we should have a greater compression than the maximum of compression; which is impossible. Consequently, the maximum of compression being equal on both sides, the efforts towards restoring their own shape must be equal in both molecules; and, therefore, the molecule O' must acquire (by its interior work of reaction) the same quantity of motion in the direction $O'R$, as the molecule O acquires (by its interior work) in the direction OP. This fact, which is true in the case not only of two molecules, but of any two bodies whatever, as we shall see hereafter, is commonly expressed by saying that *action and reaction are always equal*, or that *the quantity of motion acquired by the mass impinged upon is equal to that lost by the impinging body*.

In the example which we have given, the two molecules are not necessarily of the same nature. If they were so, then it is obvious, that their compressions as produced by the exertion of equal powers on equally constituted systems, would be in every point of view, i. e. both materially and formally, equal. But, when two molecules are of a different nature, then what we have said of the equality of compression must be understood to mean only that *the amount* of compression is the same, though its material effect may be different, according to the different constitution of the compressed systems. That the amount of compression must be the same, we have already proved : that its material effect may be different, is evident from the difference of constitution, which implies a different disposition to yield or to resist. So that, as in the impact of bodies the quantities of motion lost and acquired are equal, though the velocities lost and acquired be not equal, so the amount of formal compression is equal in two molecules, even though their material compression, or change of bulk, is unequal, on account of their having a different mass, or, in general, a different nature.

The words *action* and *reaction* cannot be used in the case of mutual actions, through which the structure of bodies is not shaken, and of which the one is not provoked by the other. Thus, the action of the sun upon the earth, and the action of the earth upon the sun, are not action and reaction; for, the earth acts upon the sun not because the sun provokes its action, but

because the earth possesses active powers of its own : and, in the same manner, the sun acts on the earth, not because the earth provokes such action, but because the sun itself is endowed with activity, and is determined to act.   Therefore, the sun and the earth do not *act* and *react* on each other, but simply *act*.   The idea of reaction implies in the reacting body an exertion of power which has been awakened by violence, viz. by another exertion causing a disturbance in the body, and putting it in an unnatural state from which it strives to recover.   Now, this occurs only when traction or pressure or any other analogous exertion of power constrains the molecules of a body to alter their natural size or their distances of relative equilibrium: this is the state of things by which reaction is called into existence, and kept up until by it the molecules reduce themselves again to their normal size and distance.

Those physicists who, in speaking of the mutual action of the earth on a stone, and of the stone on the earth, use the words action and reaction, confound·the absolute principle of *the equality of action and reaction in the impact of bodies* with another principle which is less absolute, viz. that *the quantity of action of a body* A *on another body* B *is equal to the quantity of action of the body* B *on the body* A.   These two principles, we say, must not be confounded.   In the second, in fact, the question is one not of action and reaction, but of mutual action only : and, moreover, whilst the first principle is absolute and general, the second is true only with a restriction, viz. when the bodies A and B are of the same nature*.   Many a physicist has overlooked the neces-

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* This restriction is indispensable. Let M and M' be two masses of the same nature: each molecule a of the mass M will act on each molecule a' of the mass M' with an intensity equal to that with which each molecule a' acts on each molecule a. For it is clear, that in masses of the same nature the molecules have the same powers : and, since any two molecules act on one another from a common distance, the exertion of those powers will also be equal on both sides. Now, this conclusion would not be true, if the molecules a and a' were of a different nature, if *e.g.* they were constituted of a different proportion of attractive and repulsive elements.

When the two masses are of the same nature, we may conclude that the molecule a of the mass M acts on all the molecules of the mass M' by actions equal to those of all these molecules on a itself : so also any other molecule b of the mass M acts on all the molecules of the mass M' as b itself is acted upon by them : and so on. Hence,

sity of this restriction, from assuming (of course, without proof) that the particles of matter, at least at non-molecular distances, were all *equally* attractive. This, I repeat, they assumed; for, there is no fact in nature and no argument in scientific reasoning, that can prove such an assertion. The only apparent proof which can be brought in, would be based on the hypothesis that all matter is homogeneous: and this hypothesis is not only gratuitous, but ruinous and false, as is evident from the above given proofs of the existence of attractive and repulsive elements. And, indeed, if molecules are made up of attractive and repulsive elements, and different substances contain a different proportion of them, as is evident from their having so widely different physical and chemical properties, it is impossible not to see that bodies of different natures are made up of molecules which are not homogeneous, and that their actions are not necessarily equal. And consequently we cannot admit the principle of equality of mutual actions with regard to substances of a different species, whilst, on the other hand, we must admit the equality of action and reaction for all ponderable substances, however heterogeneous.

Let us add here a few remarks on another principle which has a certain connexion with the same subject, and which has been admitted without a sufficient limitation: viz. that *the heavenly bodies attract one another proportionally to their masses.* To show the ambiguous bearing of this principle, let *A, B,* and *C* be three heavenly bodies. Then the principle may mean, first, that the attraction of *A* upon *B* and *C* is proportional to the masses of the bodies *B* and *C, which are acted on.* It might mean also, that the attractions of *A* and of *B* upon *C* are proportional to the masses of the bodies *A* and *B, which act.* Now the principle is absolutely true in the first sense only: it cannot be true in the second, unless we make the restriction that the bodies *A*

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by making the sum of all the actions of all the molecules contained in *M*, we shall be satisfied that the total action of *M* upon *M'* is equal to the total action of *M'* upon *M*. Therefore, for masses of the same nature, the quantities of mutual action are equal: but with regard to masses of a different nature, the conclusion cannot be true, because molecules of a different kind may possess different powers.

and B are of the same specific constitution, and differ only in quantity of mass. Let us illustrate this by an example. The mass of the earth attracts an element of matter, which is placed at its surface, with an action of the intensity g. If, then, two bodies have the masses M and M' respectively, the action of the earth upon them will be respectively Mg and $M'g$: and we shall express this fact by saying that the weights are proportional to the masses weighed. So far the principle is self-evident; for weight is nothing but the sum of the earth's actions received in the body, and by which the body is pressed against the obstacle that prevents it from falling: and it is obvious that the number of such actions is equal to the number of the elements that are acted on. But, what if the mass of the earth were to become greater? would g also become greater? It is evident, that the mass of the earth might be augmented by addition of repulsive as well as of attractive elements. In the second case g would become greater, of course; but, in the first, it would become smaller. Again, the mass of the earth could be diminished by a subtraction of repulsive elements: and in this case the diminution of the mass would entail an increase of intensity in terrestrial attraction. In other words, gravity is the result of actions of different signs; and, therefore, it is not represented by the arithmetical, but by the algebraical, sum of the actions proceeding from each element of matter contained in the earth: whilst, on the contrary, the mass of the earth is always represented by an arithmetical, not by an algebraical, sum of its elements. We conclude, therefore, that the weights are proportional to the masses of the bodies which are weighed, but they are not necessarily proportional to the masses by which the bodies weighed are attracted. And thus, if it were possible to transfer a body from the earth to the moon, its weight would not necessarily change in the ratio of the earth's mass to the mass of the moon, unless we make the hypothesis that the moon and the earth contain the same proportion of attractive and repulsive elements having the same quantity of power respectively: but this hypothesis would have no foundation at all. Planets, likewise, are attracted by the sun proportionally to their own absolute masses; and yet they do not necessarily attract the sun with attractions

proportional to their own masses, but proportional to the excess of their attractivity on their repulsivity. And, indeed, to allow themselves to be attracted is the property of material elements on account of their matter or passivity, which is of the same kind in each and all of them; and therefore, each and all are, or can be attracted. On the contrary, to attract is a property of elements by reason of their form or activity, which is not of the same kind in all the elements, since some are attractive, and others repulsive. Therefore, if a planet contains a number m of attractive and a number m' of repulsive elements, each of equal intensity, it will indeed be attracted proportionally to the whole mass $m + m'$, but will attract only by an action proportional to the difference $m - m'$.

PROPERTY VI.

Impenetrability is the result of molecular reactivity, and consists in this, that one molecule does not allow another molecule to occupy the place actually occupied by itself. Impenetrability, as well as reactivity, arises evidently from the constitution of molecular systems. Hence, it is without reason that Boscovich, as we have already observed (Book II. Prop. V.), from the impenetrability of bodies inferred the impenetrability of each simple element of matter. And, indeed, if molecules possess a reactive constitution, on account of which the one is relatively impermeable to the other, bodies will be impenetrable, whatever we may say of simple elements in general.

Molecules, in order to be impenetrable, must have the power of retaining a certain bulk, whatever the pressure to which they are subjected. I say a *certain* bulk; because the very same body which is impenetrable is also compressible; and, therefore, the volume of the molecules is not absolutely invariable.

Impenetrability is not essentially interfered with by chemical affinity, although the combination of two substances, that have mutual affinity, gives rise to a compound, whose volume is less than that of its components. In fact, the impenetrability of bodies is not an absolute, but a relative property, physical, not chemical; and, therefore, bodies are impenetrable as viewed in their own

permanent constitution, whatever may be their relative permeability when they are actually undergoing a change of constitution. Still, the molecules of different substances, when actually combining, do not strictly compenetrate one another, though the one unites with the other so as to form a new and unique molecular system. And, after all, even though a true penetration were unavoidable, we might turn the difficulty by answering, that impenetrability, being a physical property, regards only a set of molecules either of the same nature or such as have not, or are not actually exercising, molecular affinity; and, therefore, no case of chemical combination can be alleged against the impenetrability of bodies.

PROPERTY VII.

The capability of displaying the so-called *force of inertia* is, like impenetrability, nothing more than a particular case of reactivity. A body by reason of its reactivity resists the passage of another body: and by reason of the same reactivity a body checks or diminishes the motion of another body that impinges upon it: and it is when it produces such a diminution of motion that it is said to display its force of inertia. Hence, the *vis inertiæ* is nothing else than the intensity (Book II. Prop. II. Schol.) of a reaction displayed by a body when its structure is altered by exterior violence: so that, whenever a body is free from exterior violence, viz. whenever its molecules remain at the distance of relative equilibrium, any display of *vis inertiæ* is impossible.

The *vis inertiæ* might be defined as the quantity of effort by which a body, when enduring violence from without, strives to restore its molecules to their natural size and to their position of relative equilibrium. From what we have already observed, it is evident that this effort proceeds from the powers residing in the molecules. If the exterior action, to which they are subjected, tends to produce compression, the resistance of the body will consist of an exertion of repulsive powers: if the exterior action tends to dilate the body, the resistance of the body will consist of exertion of attractive powers (Book VI. Prop. VII. and IX.).

Now let it be observed, that a body A, whether pushing or drawing another body B, acts always with a much greater intensity on those molecules of the body B which are closer to it, than on those others which are more distant. The consequence is, that the body B will be incapable of yielding its place to the pushing body, or of following the direction of the drawing body, till the molecules of B by mutual compression in the first case, and by mutual traction in the second, have all acquired a suitable velocity. Accordingly, so long as by such interior actions and reactions the molecules of B have not acquired a common velocity, the body A cannot freely pursue its course, but suffers either compression or traction according as it is itself acting by compression or traction: and in both cases the same body A, through the internal reaction of its own molecules, acquires a quantity of motion which is contrary to its original tendency. Such is the fact, to account for which the *vis inertiæ* has been had recourse to. The reason, then, why reaction received the name of *vis inertiæ* is, because the resistance of the body acted on is developed in that lapse of time, during which the inert body, *inasmuch as inert*, i.e. incapable of leaving its place without its mass having acquired a common velocity, stands still in need of acquiring it, and therefore delays to quit its place. This is the only reason why the body B, if acted on by compression, will, for a time, cause a similar compression in the impinging body, and, if acted on by traction, will, for a time, cause a similar traction or tension in the body by which it is drawn: so that the compression in the first case, and the traction in the second, is common both to the acting body and to the body acted upon.

Hence we see, that *inertia* is not the cause, but only the condition *sine qua non*, of the resistance: that it is not anything like an active power, or an exertion of power, but only an incapability of moving, on the part of the resisting body, without having acquired a suitable velocity. And consequently, the *vis inertiæ* also, far from being a new causality, is only another name for *quantity of reaction*, drawn from the consideration of the mode of being and the dynamical state of the inert body.

In the impact of two bodies A and B, it is evident that both are compressed; hence the so-called *vis inertiæ* is awakened

in both, and equally too, according to what we have already
established about the equality of action and reaction amongst
molecules (Book VII. Property V.). But this last consequence, in
the case of two bodies, especially if they are of a different nature,
deserves a further explanation, which will be here in its proper
place. The compression of the bodies impinging and impinged
upon is common* and cannot be greater in A than in B, or *vice
versâ*, as the one is the condition of the existence of the other,
whatever the nature of the bodies A and B. And as such com-
pressions have opposite tendencies, they must give rise to oppo-
site results. Now, when A begins to press B, we have *mutual
action*, as is evident: but, when compression has reached its maxi-
mum (and there cannot be a maximum, unless the two bodies
are under an equal degree of intensive compression, as we have
already intimated), then *reaction* begins: by which both A and
B free themselves from their unnatural compression. This reaction
does not take place from A to B, nor from B to A, but is
separately exercised by the molecules of A among themselves, and
by the molecules of B among themselves. Wherefore the action
which produces compression is indeed mutual, but the reaction,
though it takes place both in A and in B, is not mutual, as it
is not exercised from A to B, nor from B to A. Now let it be
remarked, that the reaction, by which the body B frees itself
from compression, must undo what has been done by the action
of the body A, by which the compression was brought about:
and, in the same manner, the reaction, by which the body A frees
itself from compression, must undo what has been done by the
action of the body B which has been the cause of such a com-
pression. If, then, the actions of A and of B have been equal,
the reactions also will be equal. But these actions are always
equal; since they are measured by the maximum of compression.
Therefore the reactions are also equal; and, consequently, the
quantity of motion acquired by the body A in freeing itself from

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* What we say here on compression may easily be applied to traction. When a
body is drawn with a rope, compression is replaced by tension : and tension, like com-
pression, will be common and require equal efforts on both sides ; and consequently,
here also, action and reaction will be equal.

compression equals that acquired by the body B in the same circumstance.

We have said that the so-called *vis inertiæ* is brought into existence when the equilibrium among the molecules of a body ceases to exist. Accordingly, no *vis inertiæ* is called into existence when an agent acts with equal intensity on each molecule of a body; for in this case their equilibrium will not be disturbed, and the whole of the body will simultaneously receive a common velocity and move in.the direction of it without undergoing change of form. Thus, when the earth acts on a body which is free to fall, the *vis inertiæ* has no part to play, and the body cannot react, since its interior equilibrium is not disturbed. Thus also no sensible reaction is called forth by the action of the sun upon the earth, because the action of the sun on each part of the earth is sensibly of one and the same intensity. On the contrary, the action of the moon on the earth is calculated to give rise to a reaction on the part of the earth, because the action of the moon on the less distant parts of the earth is approximately to its action on the most distant parts, as 16 : 15; and this difference is sensible. Hence the moon tends to cause a swelling in that part of the earth to which it is nearest; and the effect of its action, though inappreciable with regard to those parts which are solid, is most sensible in the ebb and flow of the tides.

PROPERTY VIII.

Vibrativity is the capability of vibratory motion. We have already proved, that the molecules of primitive bodies have, by reason of their constitution, a movement of contraction and dilatation : but vibrativity, as a general property of bodies, implies the capability of making vibrations determined by an extrinsic agent. Vibrativity may be different in different bodies. Peculiar kinds of vibrativity correspond to peculiar classes of phenomena brought about by vibrations, as in heat, light, sound, and other modes of motion of periodic recurrence, to which an innumerable multitude of natural phenomena are to be traced, and for which dif-

ferent bodies have a different disposition depending on their molecular constitution. Hence, when we see that two bodies are differently affected by light, heat, &c., we are entitled to infer that those bodies differ in their molecular constitution. These general remarks may here suffice : for further details see Book IX. on the special properties of bodies.

BOOK VIII.

ON LUMINIFEROUS ÆTHER*.

To give a correct notion of some of the special properties of
bodies it is indispensable to know the nature of the agency on
which the luminous and calorific phenomena depend. This obliges
us to say a few words on luminiferous æther, before we proceed
further. And since there are scientific men who deny the exist-
ence of a special substance deserving the name of luminiferous
æther, it is necessary for us to show its reality, before we determine
the nature and properties of the fluid itself.

I. ÆTHER, A SPECIAL SUBSTANCE.

The question is not precisely whether we must admit a medium
for the transmission of light, but whether this medium is a special
substance distinct from all substances coming under the name
of ponderable matter. Mr Grove, in his valuable book *on the
correlation of physical forces,* shows a great tendency to replace
luminiferous æther by common matter, or gross matter, as it is
often called. "The difference," says he, "between the view which
I am advocating and that of the ætherial theory as generally
enunciated is, that the matter which in the interplanetary spaces
serves as the means of transmitting by its undulations light and
heat, I should regard as possessing the qualities of ordinary or,
as it has sometimes been called, gross matter, and particularly
weight; though from its extreme rarefaction it would manifest
these properties in an infinitely small degree; whilst, on the
surface of the earth, that matter attains a density cognisable by
our means of experiment, and the matter is itself, in great part,
the conveyer of the undulations in which these agents consist"
(page 149). As for the ætherial theory, he thinks it to be a

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* I write *æther,* to make a distinction between the medium of transmission of
light, and those substances which are called *ethers* in chemistry, as nitric ether,
sulphuric ether, &c.

gratuitous assumption: in this theory "a specific matter without weight must be assumed, of the existence of which there is no evidence but in the phenomena for the explanation of which its existence is supposed. To account for the phenomena, the æther is assumed, and to prove the existence of the æther the phenomena are cited" (page 150).

We shall remark first, that the ætherial theory assumes by no means the existence of "a specific matter without weight." Æther is called an *imponderable*, not to express that it is without weight, but to state the fact, that we cannot weigh it. Æther, like all other material things is essentially subject to gravitation : this follows from what has been demonstrated in the first and second Books of this work. And, since weight is nothing but the resultant of attractions applied to a mass, a mass of æther cannot be under attraction without having weight. Yet weight in general is not necessarily the resultant of terrestrial actions alone. The earth itself has weight as related to the sun, and the satellites have weight as related to their planets. In other words, the notion of weight extends to all gravitating matter throughout the universe. To say, then, that since we cannot ascertain the amount of terrestrial action on æther, æther must be without weight, would mean, either that we admit an absurd kind of matter which cannot be acted upon, or that we consider the earth as being the only, or at least the most active, centre of gravitation in the world.

We may remark, moreover, that the ætherial theory does not assume the existence of a new "specific matter," as Mr Grove thinks. We have shown in another place that the analysis of bodies must ultimately lead to simple elements, some attractive, and others repulsive. A matter specifically different from attractive and repulsive elements would be a matter destitute of that which essentially constitutes what we call matter : it would be a sheer impossibility. Thus far, then, we agree with Mr Grove : we, like him, reject a new specific matter. Still, æther might be, after all, a substance specifically different from all other known substances. Thus, hydrogen and nitrogen, although made up of elements of common matter, are substances of a different species, as every one must allow, inasmuch as they have a different specific constitution from which their different specific properties result.

If, then, luminiferous æther possesses properties which are incompatible with the constitution of common ponderable bodies, we are compelled to say that æther is endowed with a peculiar specific constitution. In such a case, which is a real one, as we shall presently see, luminiferous æther will be a special substance, though by no means formed of a new specific matter.

A third remark may be, that the gross matter which "from its extreme rarefaction would manifest its properties only in an indefinitely small degree" cannot be a proper medium for transmitting light. The rate of velocity, at which light travels, is so enormous that it is quite impossible to account for it by a medium consisting of a highly rarefied fluid. The rapidity of the transmission of undulations undoubtedly depends upon the intensity of the mutual action of the particles constituting the transmitting medium : and this intensity again depends on the vicinity of the same particles, according to the general law which we have proved in a preceding Book. Now, is it possible to admit that common matter in a state of high rarefaction is capable of transmitting light with so prodigious a rapidity ? Air is common matter : and we know from acoustics that the waves of air have in their propagation a velocity of about 340 metres. Now, let air be extremely expanded and highly rarefied ; is it conceivable that it will acquire just then the power of transmitting waves of any kind with the amazing velocity of 298,000,000 metres ? If a man can conceive this, he must be ready to admit also that in proportion as the material particles of a fluid become more distant from one another, their mutual action must become intenser. We trust that this new theorem will never be adopted by mechanical writers.

This last consideration, to which others might be added, sufficiently shows that common matter extremely rarefied cannot be the medium which transmits light. Accordingly, luminiferous æther is a special substance altogether distinct from any of the substances ranked amongst primitive ponderable bodies and their compounds. What we shall say hereafter concerning the properties of æther will amply confirm our present conclusion*.

* We think that Mr Grove is quite wrong, when he points to a vicious circle on the part of those who hold luminiferous æther to be a special substance. "Of the

II. ÆTHER, AN UNRESISTING MEDIUM.

Though there are scientific men, who do not admit luminiferous æther to be an unresisting medium, yet their opinion is not generally adopted. The greater number, compelled by the examination of facts, adhere to Newton's saying, that throughout celestial spaces no resisting medium is to be found, for the reason that *non solum solida planetarum et cometarum corpora, sed etiam rarissimi caudarum vapores, motus suos velocissimos liberrime peragunt, et diutissime conservant.** This reason is so good and conclusive, that, had it been sufficiently considered by those who adhere to the opposite view, it would have brought them to a total change of opinion. If planets move through a resisting medium, how is it, that from centuries they have not suffered any sensible retardation in their orbits? The patrons of the resisting medium have said that the masses of planets being enormously great as compared with the mass of the medium through which they move, the resistance of the medium must be very small, and cause only an imperceptible retardation. This answer, plausible though it may have appeared to some philosophers, is utterly vain. Before we assume that the density of the medium is incomparably smaller than that of the celestial bodies, we must examine whether this assumption is consistent with the nature of the light-transmitting medium. Let us see the consequences of such an assumption.

If the ætherial particles are so far apart from one another, that through their united actions no sensible retardation can be

existence of æther," says he, "there is no evidence, but in the phenomena for the explanation of which its existence is supposed. To account for the phenomena the æther is assumed, and to prove the existence of the æther the phenomena are cited." Now, by the same way of reasoning we might show that the existence of a book does not prove the existence of a writer. We might say with him : To account for the book the writer is assumed, and to prove the existence of the writer the book is cited. Is this a vicious circle? I think not. The vicious circle would be to assume the existence of a book in order to prove the existence of the writer, and then from the existence of the writer to infer that of the book. " There is no evidence, in the case of æther, but in the phenomena." This, of course, is true; still, this evidence if real is quite sufficient: the phenomena are the book : nothing more is required. It must have its writer.

  * *Principia*, Vol. III. Prop. 41, Prob. 21.

produced in the motion of planets, what will become of that pro-
digious elasticity, by which æther transmits the luminous vibra-
tions at the rate of 298,000,000 metres per second? This fact
can be accounted for in two ways only : i. e. either by saying that
the particles of the transmitting medium are much nearer to one
another than the particles of air, hydrogen, or any other known
fluid, or by saying that those particles are indeed far apart, but
possess repulsive powers of such an incredible intensity that the
diminution of elastic reaction, which would arise from their sup-
posed enormous distance, is fully compensated. Now, the first
answer would give us the idea of a very dense medium ; so
dense, that the planets, after a short time, would infallibly stop
their course : the second answer would make æther very rare :
but, rare as it would be, it would nevertheless extinguish the
motion of the planets as infallibly as in the first case ; since its
rarity would indeed diminish the number of the resisting parti-
cles in the ratio 700,000 : 1, according to Newton's calculation :
but the intensity of the resistance proceeding from each particle
would increase in the ratio
$$490,000,000,000 : 1.$$
This leads us to an evident conclusion. The light-transmitting
medium, whether dense or rare, will always, and in a short time
too, stop the course of the planets, if its elasticity be owing to
any resisting powers having part in its constitution. Therefore,
since the planets have continued to move for centuries without
the least sensible retardation, the medium through which they
move cannot but be destitute of resisting powers.

Our argument rests exclusively on the undiminished motion of
planets. Newton adds the consideration of comets and cometic
tails, which may seem to lead even more easily to the same con-
clusion, on account of their masses which are relatively smaller
than those of planets, and, therefore, would be more sensibly
retarded in their revolution by a resisting medium. But, since
the orbits of comets, in spite of all astronomical calculations, are
not exactly known, I thought it better to rest the argument on
the more positive ground of planetary motion only : and, for the
same reason, I omit to argue from the unresisted motion of the
cometic tails. The nature of such tails is not sufficiently known

to base an argument upon. It may be true, that those tails are a kind of vapour, *caudarum vapores,* as Newton and many others have believed : and, if this were the case, the argument based on cometic tails would be very good indeed. But, the assumption is not proved, and might be false. One might say that the tails are not made up of a peculiar vaporous matter especially attached to the body of the comets : we could say as well that a cometic tail is a pure modification of the æther itself, i.e. *a phenomenon of light* accompanying some substantial or accidental alteration of the cometic body, and depending most probably on the high degree of heat which is experienced by comets in getting nearer to the sun. This is a view, which in the present state of astronomy might be fairly advocated, without giving rise to positive objections of any weight, nay with the support of some probable reasons. The very possibility of this last hypothesis would have impaired a demonstration based on the motion of cometic tails.

A difficulty has been often raised. Encke's comet has suffered a change in its velocity : how can this change be accounted for, unless we admit that the comet moves through a resisting medium ? My answer is, that this is no difficulty at all : it is only a sample of rash reasoning on the part of those who propound it. How do we know that Encke's comet cannot possibly have suffered a change in its orbit, unless it moves through a resisting medium ? Have we any evidence, or even any ground whatever for a probable conjecture, that there is absolutely nothing in interplanetary spaces, except the medium, and those bodies which we have hitherto observed ? To discourage such a supposition it would suffice to mention Le Verrier's discovery. I do not say that we shall hereafter discover any new celestial body by whose action to account for the modification of the orbit of Encke's comet : there are perhaps thousands of bodies in the solar system of which we have no notion, and never shall have, on account of their being unobservable. I say only, that with the scanty knowledge we have of the bodies that move in heavenly spaces, and after the discovery of so many new planets, the existence of which had never been suspected before, it would be too rash on the part of a man of science to pronounce that

no other cause exists in the heavens to which we can trace the
change of Encke's comet's course, except a resisting medium.
The more so, since Encke's comet shows a regular *acceleration*
of its motion, instead of a retardation. Now acceleration does
not proclaim, but refute, the theory of a resisting medium ; and
therefore Encke's comet and all the other celestial bodies with
one loud voice proclaim, and witness in fact, the absolute non-
existence of a resisting medium.

III. ÆTHER WHOLLY ATTRACTIVE.

It follows from its incapability of resisting the motion
of planets, that æther is exclusively made up of attractive ele-
ments, and that its elasticity has a different source and differs in
kind from the elasticity of the known ponderable bodies. The
first part of this proposition is a simple and direct corollary of
the non-resistance of æther. For every one knows that the loss
of velocity sustained by an impinging body must be attributed
to the repulsive elements contained in the body on which it
impinges. And consequently the loss of velocity sustained by a
body on account of its moving through a medium must be at-
tributed to the action of repulsive elements contained in the
medium. Hence it is that a medium involving repulsive ele-
ments in its constitution must be a resisting medium : and, there-
fore, an unresisting medium excludes repulsive elements from its
own constitution.

As for the second part of our proposition, it may seem dif-
ficult to conceive any kind of elasticity in a fluid which excludes
repulsive elements from its constitution. This difficulty will soon
disappear. We will only remark here, that *elasticity* is not ex-
actly *expansivity*, although expansivity is never to be found
without elasticity. Both elasticity and expansivity, when they
coexist, arise from molecular repulsion : and such is the case
with regard to ponderable substances : but, if a substance be
elastic without being expansive, its elasticity will be owing to
molecular attraction alone. That this is the case with luminiferous
æther, it may be very easily proved. For, if the elasticity of
æther were due to repulsive, or expansive powers, the amount

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of repulsivity would be proportional to the amount of elasticity : and thus an immense elasticity would be the result of an immense repulsivity. But such an immense repulsivity would have shown its existence ere now by stopping, or at least modifying very sensibly, the course of planets : which is not the case. Therefore the elasticity of æther is not due to repulsive elements.

This same truth might be confirmed by another consideration. If the elasticity of æther were due to its containing repulsive elements, a beam of light admitted through a small hole into a dark room would not occupy a conical space exclusively, but would dilate in every direction. This is evident; for, when the elasticity of a fluid is owing to mutual molecular repulsion, the pressure which each molecule exercises on each of its neighbouring molecules must be felt all around, and be transmitted in all directions, as it is the case with a sound emitted from a pipe, which, though directed towards a given spot, is nevertheless heard directly and distinctly from other places which are out of the cone determined by the direction of the pipe. The transmission of sound is, then, due to a medium whose elasticity finds its explanation in molecular repulsivity : but the transmission of light is owing to a medium the elasticity of which cannot be explained by any hypothesis of molecular repulsion.

The consequences of this statement are very numerous, and affect very materially the whole theory of light. Still we have no intention of coming to particulars on this subject. Details would lead us too far : and we must not abandon the line which we have proposed to follow in the present work.

IV.   THE ÆTHER'S ELASTICITY OF TRANSMISSION.

Let us inquire now, whether it is possible or not for a fluid to be elastic without being repulsive. We may anticipate that it is possible; for we have already seen that the immense velocity of transmission of light demonstrates the immense elasticity of the transmitting medium, whilst the total absence of retardation with regard to the motion of planets demonstrates an absence of repulsivity in the medium of motion. It is therefore a fact, and

not a mere possible conjecture, that elasticity does not always imply repulsivity.

Now, to give an idea of this new kind of elasticity, let us imagine an indefinite series of attractive elements placed at equal distances and forming a straight line. Let the elements a, b, c, d, ... (fig. 27) possess equal attractive powers, and be in equilibrium under the equal attractions exercised on them from each side of the indefinite line. If any cause whatever compels a to move towards b, all the other elements on each side of the line will immediately begin to vibrate, two and two together, viz. b with a, c with d, e with f, &c.; and this kind of vibratory motion will be propagated to any great distance with a surprising rapidity and an undiminished energy. And, in fact, when a begins to approach b, the attraction to which b is subjected becomes greater on the side of a than on the side of c; and accordingly b must immediately move towards a. But when b begins to move towards a, then c becomes less attracted by b than by d; and accordingly c must immediately begin to move towards d; and thus d also is obliged to move towards c. When d is going towards c, then e must yield to the prevailing action of f; and so on indefinitely through all the line, and on each side of the line*.

It is evident, that the property of so transmitting or propagating motion implies elasticity, though not the common elasticity. For this reason we call it *a new kind of elasticity*, or, if preferable, *elasticity of transmission*. We might call it also *negative elasticity*, since it arises from attractive powers whose exertion tends to diminish distances : then the common elasticity, as arising from repulsivity and tending to augment the distances, should be called *positive*.

Such a negative elasticity is the only one that æther must possess in a high degree, to be a suitable medium of quick transmission : and is the only one that will not interfere with the motion of celestial bodies. Of course, we do not intend to say, that æther consists of separate points or elements of matter, nor that luminiferous waves are propagated precisely according to the

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* By examining this kind of propagation of motion in a system of parallel lines, it will be easy to prove, that their vibratory motions are also propagated perpendicularly to the direction of their transmission.

example presented in our figure. Our intention has been only to point out a new principle of elasticity, of which we find no example in the ponderable substances, and which explains the possibility of a quick propagation of motion through a medium which is destitute of resisting power. If æther is nothing but separate attractive elements, then our figure gives a true idea of the real mode of propagation of light: and, if æther is made up of attractive elements grouped into molecules, then the real mode of propagation of light will imply also molecular palpitation, according to Book IV. Prob. X.

V. TWO POPULAR OBJECTIONS.

The existence of a fluid exclusively attractive and occupying all the celestial regions gives rise to a plausible objection. The particles of this fluid will gravitate towards the centre of the whole mass, and finally meet in that common centre. But, if this be true, then our interstellar medium ought to have soon disappeared from space by collecting itself in a dense mass around a point.

To show that this objection contains no real difficulty, it might suffice to remind the reader of what has been proved in Book III. Theor. I. about the vibrations of two attractive elements. What has been said there of two elements, may be said here of any great multitude. In fact, let us conceive a very large sphere full of attractive particles of equal power, and equally distributed throughout it, with exclusion of any other substance. It is obvious that the particles will all gravitate towards the centre of figure. But any particle whatever, when moving towards such a centre, acquires a velocity, which is ·a maximum when the same particle reaches the centre, and cannot be exhausted until the same particle, in continuing its course, has been subjected to a retardation equal to the previous acceleration which has produced such maximum of velocity. This shows very plainly, that each particle will always recede from the centre on the one side as much as it has approached to it from the other: and thus it is evident, that, although the density of the fluid is greater and greater in proportion as we come nearer to the centre of figure, still there is

no chance of a general and permanent condensation, which would leave a vacuum in any part of the primitive sphere.

It must be observed, however, that if the sphere, besides being filled with the fluid under consideration, contains other bodies of a different nature, then the action of such bodies must be felt by the fluid. Hence the centre of figure will be no more the point in which the density of the fluid reaches its maximum. If those bodies were to possess a very great power of attraction, the motion of the fluid might be so affected as to take another direction altogether, and each body might become a separate centre of ætherial condensation. In such a case, the fluid would move towards each body in streams of greater or less thickness according to circumstances. Of course, if a body were endowed with a great power of repulsion, the motion of the fluid might also be turned in an opposite direction: but this is not the case; for every solid body owes its solidity to the absolute prevalence of attractivity in its constitution: and consequently all the solid bodies existing in the heavenly spaces must be attractive.

Another objection remains. How can the celestial bodies preserve their relative distances, if they are continually attracted towards a point, viz. towards the centre of the universe? This objection was very popular, when the doctrine of universal attraction was first brought to play a part in astronomical discussions. The answers then given were different, according to the systems adopted by philosophers. The Cartesians, by admitting that the world has no limit, evaded the objection: other philosophers had recourse to angelic intervention: the true scientific answer was drawn from the theory of central, i. e. centripetal and centrifugal forces, and assumed only that, as the moon revolves around the earth, and the earth and other planets around the sun, so the sun moves around another centre of attraction, and this centre around another, till we come to a first centre, which is the centre of the universe. Such an assumption was very reasonable, as suggested by analogy, and containing, not a new arbitrary invention, but the mere application to the universe of the laws observed in the solar system: accordingly the answer has been held satisfactory. Now, with regard to the attractive medium and the point towards which the heavenly bodies would have to converge, the very same answer

is applicable. If celestial bodies, while describing their orbits around their immediate centres, revolve at the same time with a suitable velocity around the centre of ætherial attraction, nothing more is required to prevent them from meeting at that centre. And, indeed, our case is exactly identical with the preceding one : only, instead of considering the centre of the universe as occupied by a particular celestial body surrounded by æther, we consider æther itself with its immense bulk enveloping all the heavenly spaces as that very body the centre of which is to be regarded as the centre of the universe. We have, of course, no means of thoroughly ascertaining the fact : He alone knows all about it, to whose infinite power and wisdom the universe owes its existence and harmony. But those who make the objection must be satisfied that our answer is quite sufficient ; since we have shown the possibility of what they consider as impossible.

VI. DENSITY OF LUMINIFEROUS ÆTHER.

Newton conjectured that the density of æther is to that of the air as 1 to 700,000 ; and all physical philosophers have been wont to say that æther is extremely rare. Their reason was, because a dense æther would have resisted the planetary motion. We have already proved, that, if æther were repulsive, even by making it extremely rare, the planets would not be freed from sensible retardation. On the other hand, if the medium is all attractive, be it extremely rare or extremely dense, the motion of planets remains unaltered. Hence, the proper test for judging of the density of luminiferous æther is to be sought for, not in the motion of planets, but in the motion of æther itself.

Luminiferous æther vibrates very freely through atmospheric air : let us see whether from this fact we can draw any inference about its density at the surface of the earth. Taking for granted that air is the result of a mixture, not of a combination, of oxygen and nitrogen, the number of molecules which are necessary to fill up, at sea level, the length of a single millimetre is not less, and possibly greater, than 281,740, as we shall see further on. And, since a beam of red light in going over the same length of a millimetre makes 1550 vibrations, we may conclude, that the length of

one vibration of red light extends through a line occupied by at least 181 molecules of air. Now it seems but reasonable to think that the vibratory motion of æther, which is actually advancing through such a number of obstacles without a sensible loss of intensity, would not be able to triumph over those obstacles again and again, were it not for the mass of æther being much more considerable than that against which it impinges.

This conclusion receives an additional confirmation, if we observe that a ray of solar light, when coming in an almost horizontal direction, is not sensibly altered by traversing some 1000 metres more through atmospheric air. Now, in 1000 metres the ray finds on its path not less than 281,740,000,000 molecules of air, against which it has to struggle without intermission. Yet it passes undisturbed. I do not see how such a fact can be accounted for if æther is not immensely denser than atmospheric air*.

I abstain from further considerations on this point. Every one sees that a very great density perfectly agrees with the notion of a mass which is all attractive: but the positive determination of that density at the surface of the earth is extremely difficult, since it regards a problem of which the data are not fully known, and of which the solution depends on the conditions of mutual actions of ponderable and imponderable masses: conditions whose determination requires a perfect knowledge of the phenomena of light, heat, electricity and magnetism. Modern science has yet much to learn before any very conclusive statement about the constitution of æther can be arrived at. It is obvious, however, by what we have said up to this point, that we are not of the opinion of Mr Grove when he says: "æther is a most convenient medium for hypothesis: thus, if to account for a given phenomenon the hypothesis requires that æther be more elastic, it is said to be more elastic: if more dense, it is said to be more dense: if it be required by hypothesis to be less elastic, it is pronounced to be

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* With this great density æther possesses also a very great subtlety. The one does not exclude the other. A great density comes from a great proximity of molecules or particles: subtlety comes from the small number of elements contained in each molecule. If æther were a mass of simple elements not tied up into molecules, it would reach the highest possible degree of subtlety, however great its density.

less elastic ; and so on. The advocates of the ætherial hypothesis certainly have this advantage, that the æther, being hypothetical, can have its characters modified or changed without any possibility of disproof either of its existence or modifications*." Our answer to Mr Grove is, 1st, that the existence of a medium for the transmission of light is not an hypothesis, but a fact: 2nd, that the distinction of such a medium from any ponderable substance is not an hypothesis, but a necessary inference drawn from observed facts: 3rd, that to admit in such a medium now a greater, now a less density or elasticity, may be perfectly scientific, not less indeed than to admit that steam or gases may be, under different conditions, more or less dense, and react with more or less of energy. Hence, in our opinion, the advantage which the advocates of the ætherial medium certainly have, does not consist in that which the writer imagines, but in this, that they are able to prove that the ætherial medium is not a dream.

VII.　ÆTHER, AND PONDERABLE MATTER.

A great number of scientific men, to give an explanation of calorific, electric, and luminous phenomena, assume that æther pervades all ponderable bodies: whence many of them have come to the conclusion, that every molecule of a body is surrounded by an ætherial atmosphere, the action of which is considered to be the source of those phenomena. Professor W. A. Norton, in a series of interesting articles published in Silliman's American Journal, gives a theory of molecular physics, of which the fundamental principle is, that each molecule is formed by an atom of ponderable matter surrounded by two ætherial atmospheres of a different kind. I give his words:

" The established truths and generally received ideas, which form the basis of the theory, are as follows :

" 1st. All the phenomena of material nature result from the action of force upon matter.

" 2nd. All the forces in operation in nature are traceable to two primary forces, viz. attraction and repulsion.

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* *Correlation of Physical Forces*, p. 142.

" 3rd. All bodies of matter consist of separate indivisible parts, called atoms, each of which is conceived to be spherical in form.

" 4th. Matter exists in three different forms essentially different from each other. These are 1st, ordinary or gross matter, of which all bodies of matter directly detected by our senses either wholly or chiefly consist. 2nd. A subtile fluid, or æther, associated with ordinary matter, by the intervention of which all electrical phenomena originate or are produced. This *electric æther*, as it may be termed, is attracted by ordinary matter, while its individual atoms repel each other. 3rd. A still more subtile form of æther, which pervades all space and the insterstices between the atoms of bodies. This is the medium by which light is propagated, and is called the *luminiferous æther*, or the *universal æther*. The atoms, or 'atomettes,' of this æther mutually repel each other; and it is attracted by ordinary matter, and is consequently more dense in the interior of bodies, than in free space.

" 5th. Heat in all its recognized actions upon matter manifests itself as a force of repulsion.

" The corner stone of a physical theory of molecular phenomena must consist in the conception that is formed of the essential constitution of a single molecule, understanding by a molecule an atom of ordinary matter endued with the properties and invested with the arrangements which enable it to exert forces of attraction and repulsion upon other molecules. In seeking for this, the most philosophical course that can be pursued is to follow out to their legitimate conclusions the general principles already laid down......The conception here formed of a molecule involves the idea of the operation of the two forces of attraction and repulsion : a force of attraction is exerted by the atom upon each of the two atmospheres surrounding it : and a force of mutual repulsion between the atoms of each atmosphere. These we regard as the *primary forces* of nature, from which all known forces are derived*."

These are the capital points of Prof. Norton's ingenious theory.

* *Phil. Mag.* September, 1864, p. 193.

But we think that such a theory contains a great deal of arbitrary assumption. And indeed, on what evidence are we to grant that matter exists in three different forms essentially different from each other? Then, how can we know the existence of atoms of gross matter having a spherical form, and therefore extended, though indivisible? Why should we admit two ætherial fluids, which are both repulsive and only differ in subtlety? All this the learned Professor assumes without proof, apparently because it consists of "established truths and generally received ideas." But we say that no one has up to this day established the truth of such propositions. As for "received ideas," every one knows how often questionable notions have been, and are received without serious examination, especially when expressed by Professors in a very dogmatic style. Are not a thousand hypotheses received? and do they cease to be hypotheses, although he who makes use of them for building a theory adorns them with the high name of principles?

But let us come to a second remark. The atoms of gross matter being "indivisible" cannot be extended, and cannot be "conceived to be spherical in form;" for if they were extended and indivisible, they would be so many pieces of continuous matter, which we have already proved (Book I. Prop. VII.) to be impossible. Again, ætherial substance, according to the author, is repulsive : now this is inconsistent with astronomical facts, as we have sufficiently shown. Moreover, the writer, after having assumed that the electric and the luminiferous æthers are both made up of atoms that repel each other, assumes also that electric æther attracts luminiferous æther; for he admits that a molecule is formed of an atom of gross matter with two atmospheres, of which the first consisting of condensed luminiferous æther is attracted by the other which consists of electric æther. Now, if the atoms of electric æther are repulsive, how can they attract? So then we must conclude that Prof. Norton's theory as presented by him, in spite of the talent and learning of its author, cannot be adopted in science.

As for the examples, by which he illustrates the theory, they consist of a series of phenomena of different kinds, the explanation of which does not show that the theory is not at fault. For it

must be remarked, that those explanations do not imply the existence of *extended atoms* or of *two distinct ætherial substances;* and, therefore, the theory assumes more than is necessary for, or guaranteed by, the explanation of phenomena. What we have said (Book VI. Prop. IV.) on the constitution of molecules demonstrates indeed the necessity of granting to each molecule of ponderable matter a repulsive atmosphere, which we have called the *molecular envelope.* But this envelope is not of æther, since æther is not repulsive. Had Prof. Norton known the impossibility of continuous matter, he would have found out that what he calls an "atom of gross matter" comprises already not only the central element of a molecule, but its nuclei and its envelope: and consequently is already " endued with the properties and invested with the arrangements which enable it to exert forces of attraction and repulsion upon other molecules," without requiring any new and special atmosphere of electric or luminiferous æther.

To come now to a more general point of view, it may be asked whether ponderable bodies are at all permeated by the ætherial fluid. Mr Grove, by denying the existence of any special ætherial substance, evidently answers in the negative: on the other hand, the great majority answer in the affirmative, though they give no sufficient proof of their opinion, at least with regard to solid bodies, as iron, diamond, &c. And indeed their affirmation is based only on the phenomena of heat, light, electricity and magnetism. Now, these phenomena can be explained in the opposite hypothesis. And in fact, electricity and magnetism play only at the surface of bodies, and nothing compels us to say that an electric fluid has its habitual residence between the molecules of iron, zinc, or copper. As to calorific vibrations, they may be assumed to be the vibrations of the molecules of a hot body : and the contrary opinion may be accounted for by the fact that in former days heat was considered as a peculiar substance, and men have been long accustomed to think that a body could not become hot without something substantial being received in it. This something was an imponderable fluid. In our own time this theory has been abandoned, but the old expressions "latent heat," "accumulated heat," and others of the same kind, remain in use, and it is difficult to get rid of them. Hence it is that

scientific men are still inclined to say that bodies become hot, if not because a new substance finds its way into it, at least because a substance which is permanently residing between its molecules is compelled to make vibrations of a certain period. Even Prof. Tyndall himself, whose lectures on heat are so instructive and interesting, has been unable to resist the temptation, as he assumes all throughout that heat is the motion of a fluid residing between the molecules of bodies. What we say of heat, might be said of light, especially by those who assume æther to be a very rare and very repulsive fluid. And, indeed, a transparent body, e.g. a crystal, transmits the ray of light not only in the direction in which the molecular interstices leave the way open in a straight line, but also in all other directions. Now, though we were willing to admit that æther can move freely in the direction of the interstices, yet it would be difficult to understand how it could move as freely in the other oblique directions, where it finds obstacles upon obstacles. Diamond, for instance, offers 2700 obstacles (molecules) at least in the length of a single undulation of red light. Is it possible to conceive that the pencil of rays which passes through so many obstacles will not differ sensibly from another pencil that is supposed to pass without obstacle in the direction of the interstices? This, I say, would be evidently inadmissible, were æther a substance extremely rare, as the common theory supposes. And, therefore, those who follow the common theory cannot explain how the transmission of light through a crystal can be due to the undulations of æther residing in the crystal: they would be more consistent by saying that the motion, which the luminous ray impresses on the molecules first encountered at the surface of the crystal, is propagated to the opposite surface through the vibrations of the molecules themselves, and that this last surface again communicates its motion to the free neighbouring æther and determines in it those undulations which constitute the so-called *emergent* ray.

If, however, we admit that æther is all attractive, as we have already shown, and that, instead of being extremely rare, is extremely dense, and at the same time extremely subtile, then the existence of æther between the molecules of solid bodies may perhaps be considered as a tolerably scientific hypothesis. Still,

there is no argument, I think, even in this case, by which to prove that the hypothesis is the enunciation of a fact: and in this I agree with Mr Grove, who sees no necessity of admitting a new substance within the bodies. But, as for the transmission of light through transparent bodies, I am convinced that it cannot be owing to æther existing between the molecules of the bodies. Water is transparent: snow is not transparent. Now, if the transparency of water is owing to the undulations of luminiferous æther residing between the molecules of water, why is not snow equally transparent? The molecules of snow are doubtless more distant from one another than the molecules of water; and therefore, if æther was free to make its undulations between the molecules of water, it ought to be at least equally free to make its undulations between the molecules of snow. Again, glass is transparent: but if we reduce it to a fine powder, this powder will not be transparent. Here then we have the same result as in water and snow. The molecules of the glass are certainly nearer to each other when the glass is unbroken, than when it has been reduced to powder. If, then, æther had been the agent which transmitted the ray of light through the unbroken glass, it is evident that it would transmit it through the powder with still greater facility. We might ask moreover, why should not a sponge be transparent? It is evident that the æther can move more freely through the sponge than through the glass.

An objection against this reasoning may be that transparency requires a regular arrangement of molecules: such an arrangement exists in the glass and not in its powder; and therefore it is that the powder, and so also the sponge, does not transmit the luminous ray. To this we answer, that the regular arrangement of the molecules is necessary, because they cannot vibrate in regular periods if they are not regularly arranged; whence we infer that light is transmitted by the motion of these same molecules: whilst, in the hypothesis that the ray is transmitted by the motion of the æther intercepted between the molecules, the regular arrangement would not explain the fact of the transmission of a ray in any other direction than that determined by molecular interstices in a straight line. Now the ray is in fact transmitted in all other directions. Accordingly, we maintain

that the transparent bodies transmit the rays of light by the motion of their own molecules, not by the motion of intercepted æther: a conclusion which might be confirmed by other facts and reasons, if necessary. See Grove, *Correlation of Physical Forces* (pp. 129—142).

BOOK IX.

ON SOME SPECIAL PROPERTIES OF BODIES.

THE special properties of bodies are those which are not common to all bodies. A few of such properties connote the actual mode of being of the bodies with regard to themselves: the far greater number connote various dispositions of bodies with regard to other bodies.

The properties which connote the mode of being of a body constitute its *intrinsic state,* as solidity, liquidity, expansivity, or its *extrinsic shape,* as the crystalline form. The properties, which connote a peculiar disposition of a body with regard to another body, are of different kinds, according as they are calculated to give rise to phenomena of a different class, e.g. mechanical, chemical, &c.

In the present Book we shall limit ourselves to a few remarks on solidity, liquidity, expansivity, elasticity, hardness, softness, capability of changing state, calorific capacity, and colour of bodies. Chemical affinity and other conditions of chemical combination will form the subject of the following Book.

I. THE THREE STATES OF BODIES.

We have shown (Book VI. Prop. X.) that in any ponderable body whatever the action between molecules is either attractive or repulsive, according as the actual distance of the molecules is greater or less than the distance of relative equilibrium. Hence the action between two neighbouring molecules is a function of their distance. If we take the distances on a line OX (fig. 28) and represent the intensity of the corresponding actions by the lengths of so many ordinates drawn perpendicularly to OX from the end of each distance, then the series of all the actions, by which one molecule can successively affect the other, will be

represented by the series of the ordinates which determine the curve *PNAM.* In this curve the branch *ANP,* of which the ordinates are negative, corresponds to the distances at which the molecules attract each other; and the branch *AM,* of which the ordinates are positive, corresponds to the distances at which the molecules repel each other. This manner of representing opposite actions agrees with the law which we have adopted in Book IV. and V, where we took repulsion as positive and attraction as negative, on account of the tendency of the first to augment, and of the second to lessen the distances.

The curve *MANP* is asymptotic on both sides. With regard to the negative branch *ANP* the proposition is evident; since the molecular attraction extends, though with an ever decreasing intensity, to all great distances. As for the positive branch *AM,* the proposition is made manifest by the fact that molecules of regular shape and of the same kind (such as are here supposed, since we treat of the molecules of one and the same body) must arrange themselves regularly in such a manner as to have their repulsive envelopes directly opposite to each other; so that the more they approach, the greater will be their mutual repulsion; and consequently the branch *AM* also is asymptotic*.

If *BN* is the greatest of the negative ordinates, *OB* will be the distance at which the two molecules exert the greatest attraction. On the contrary, the distance *OA* for which the ordinate is $= 0$, is the distance of relative equilibrium or of the equilibrium of position (Book VI. Prop. X.).

After these remarks, it is easy to understand that a body will be solid, when, its molecules being in the position of relative equilibrium, from a small increase of their distance an attractive action results which does not allow of the molecules being easily separated or arranged in a different order around one another.

A body will be liquid when, its molecules being in the position of relative equilibrium, from a small increase of their distance a weak attractive action results, which allows of the molecules being easily separated or easily arranged in a different order around one another.

* An exception, however, may occur in the case of tetrahedric molecules.

A body will be expansive and fluid, when its molecules are at a distance sensibly less than that of relative equilibrium, and therefore repel each other, and are in need of exterior pressure to be kept at such a distance.

Hence it is clear that the state of the body depends on the magnitude of the ordinate BN, i.e. of the maximum of attraction. The intrinsic difference between a liquid and an expansive fluid consists only in a different degree of compressibility. The liquid is composed of molecules which, when placed at the distance of relative equilibrium, resist compression very powerfully: the fluid is composed of molecules which allow themselves to be urged to a distance much less than that of relative equilibrium, without showing any very great resistance. If hydrogen, or any other gas, were freed from pressure, it would expand as much as its constitution requires, viz. to a certain determinate bulk only; for there must be a limit of expansivity for all permanent substances (Book VI. Prop. II.): then it would remain at rest in the same manner as if it were a liquid. Its molecules would then be at the distance of relative equilibrium, and its difference from known liquids would only be that, whilst other liquids do not allow themselves to be sensibly reduced in bulk, hydrogen would allow of a very sensible reduction.

It is evident, that the greater or less compressibility of a body depends on the angle α at which the curve of the actions cuts the axis OX. If the angle α is very nearly $= 90°$, the ordinates will become enormously great for a very small change in the distance of relative equilibrium; and so, a small compression may develope an enormous resistance. This is the case with liquids. If on the contrary the angle α is small, the ordinates will not become very great except for a great change in the distance of relative equilibrium; and then, a great compression may be required to develope a great resistance. This is the case with gases.

Hence, as the difference between a solid and a liquid is mainly drawn from the value of the greatest negative ordinate BN, so the difference between a liquid and an expansive fluid is drawn from the value of the angle α at which the curve cuts the axis of the abscissas. Of course, the smaller the angle, the greater, as a rule, will be the distance of relative equilibrium.

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II. Elasticity.

Elasticity is the power of reacting in order to restore the relative state of equilibrium between the molecules, when it has been sensibly altered by mechanical action. If the body reacts after allowing itself to be sensibly altered, its elasticity is called *sensible* or relative, and constitutes a special property of the body. If a body reacts before it can be sensibly altered, its elasticity is called *absolute*, and is nothing more than its reactivity, which is not a special property, as it is found in all ponderable bodies. Accordingly, sensible or relative elasticity implies 1st, that the body which is called elastic can undergo a sensible change through mechanical action: and 2nd, that the body can recover itself more or less quickly and completely from the effects of such an action, and restore itself to its primitive state.

A body may be altered by mechanical action in four distinct ways, viz. by *compression, traction, flexion,* and *torsion.* Hence four kinds of elasticity can be admitted.

The *elasticity of compression,* which is a property of expansive fluids and of solid bodies, is to be found in those bodies, in which the distance of relative molecular equilibrium OA (fig. 28) is sufficiently great, and for which the curve of the molecular actions cuts the axis at a small angle. The compressibility of a body will be greater and greater in proportion as the angle α is smaller and the distance AO greater; for, that the body should allow itself to be appreciably compressed, it is necessary that, when the molecular distance becomes $< OA$, the molecules should not immediately develope a great repulsion on each other. In the liquids, where a very great repulsion is immediately developed, the exertion of pressure does not cause any sensible diminution of volume; and it is for this reason that liquids are not ranked amongst elastic bodies, although their absolute elasticity is very great.

The *elasticity of traction* is to be found in those bodies, for which the curve of the molecular actions cuts the axis at a small angle, and in which the maximum of attraction BN corresponds to a point B which is at a considerable distance from the position of equilibrium A, and indeed, the elasticity of traction requires that, when the distance between the molecules is augmented, attraction should be weak at the beginning in order to allow the body to

yield sensibly to traction, and that the same attraction should become and remain sufficiently great when the distance continues to increase, that it may prevent the molecules from total separation, and restore the body, after the traction has ceased, to its primitive shape. This kind of elasticity is to be found in liquids and solids ; not in gases, as is evident.

The *elasticity of flexion* is a result of both elasticity of compression and elasticity of traction. For, the body that allows itself to be inflected, suffers compression on the one side, and traction on the other ; and therefore, in reacting for the restoration of its primitive figure, it must exert repulsion on the side which has been compressed, and attraction on the side which has been elongated. In other words, the body reacts on the one side by its elasticity of compression, on the other by its elasticity of traction. It is evident, that this kind of elasticity cannot be found in gases, in which there is no elasticity of traction, nor in liquids, in which there is no elasticity of compression. Still, as it is not absolutely necessary for the elasticity of flexion that the amount of compression on the one side of the body should be as great as the amount of traction on the other, we may conceive an imperfect elasticity of flexion, by which a body, without allowing itself to be sensibly compressed on the one side, should allow itself to be sensibly elongated on the other. Such a body would be liable to inflexion : and in this sense some liquids may be said to possess a certain elasticity of flexion.

The *elasticity of torsion* consists in this, that the molecules of a body, when obliged to deflect laterally from their position of equilibrium, are able to restore themselves to their normal position. This they will be able to do, provided the increase of distance occasioned by their deflection be followed by a development of attractive actions of an intensity sufficient both to prevent any break of the structure, and restore the body, after the torsion has ceased, to its normal state. This kind of elasticity is to be found in solid bodies, and, to some extent, also in some gluish liquids.

III. HARDNESS AND SOFTNESS.

Solid bodies, which do not allow their form to be sensibly altered by pressure, are called *hard* bodies. If they resist inflexion

and torsion, they are called *rigid* bodies. If they resist traction, they are styled *tenacious*. Hence hardness is the opposite of elasticity of compression, tenacity the opposite of elasticity of traction, rigidity the opposite of elasticity of flexion and of torsion. Of course, no body is perfectly hard, or rigid, or tenacious: they are so called only in a relative sense, inasmuch as they possess in a high degree the power of resisting pressure, traction, flexion, and torsion.

Hardness requires that the curve of the molecular actions should cut the axis OX (fig. 28) at an angle approaching 90^0. For rigidity the angle must again approach 90^0, and moreover the distance AB must be very small, i. e. the maximum of attraction must follow close upon a small increase of the distance of relative equilibrium. Tenacity requires something more, viz. that the absolute value of the attraction, which is developed for a small increase of the distance, should have a considerable magnitude.

Hardness admits of three contraries besides elasticity: and first *softness*. A body is called *soft* which allows itself to be compressed without reacting very sensibly for the restoration of its form. A soft body may possess slight elasticity, as the sponge, or scarcely any, as lead. Softness requires that the angle, at which the curve of the actions cuts the axis, be small. A soft body may also be clammy if, in the curve of the actions, AB is great as compared with OA.

Secondly, *friability*. A body is said to be *friable*, which allows of its particles being easily separated by friction. Friability requires that the maximum of attraction be sufficiently small, though not so small as in liquids. Another condition of friability is that the maximum of attraction should come immediately after a little increase of the molecular distance.

Thirdly, *fragility* or *brittleness*. A body is said to be fragile or brittle, which allows of its particles being easily separated by percussion or flexion. Brittleness differs from friability in this, that the molecules of a brittle body cling to one another more firmly than those of a friable body; and for this reason, percussion or flexion is necessary to produce the fracture.

When a body has been broken, it would seem that its parts, if drawn sufficiently near to each other, ought to display anew

their cohesive power and unite again as firmly as before. And yet this is not the case. When the body is being broken, those of its molecules which lie in the fracture undergo a change in their mode of being, since they become free to adapt themselves one way or another, as the circumstances require, to the new conditions of equilibrium which are brought about. Hence the curve of the actions, for these molecules, will be changed : and consequently the molecules lying all along the fracture will not be able to display the same amount of power after the fracture as before. It is to be observed, moreover, that at the surface of the fracture the molecules are free to expand, and the elements of their envelopes may arrange themselves in such a manner as to fill the intervals which, before the fracture, were occupied by the other, now separated, molecules. Hence, when the two parts of the body are drawn near again and pressed, there is no chance of restoring them to their primitive place : and, accordingly, they will not be able to exert the same cohesive power as before the fracture. Still there are cases, in which the conditions necessary for the exertion of cohesive power can be restored by calorific action. The motion resulting from calorific action disturbs the new state of the molecules at the surface of the fracture, and changes their mode of being : and, if the two parts of the body are meanwhile held together or pressed on each other, the molecules will be obliged to partake of the same kind of calorific motion, and there will be a chance of their finding their way into a fitting place and of their becoming connected again with one another as firmly as before.

A hard body is said to be *malleable*, when it allows its shape to be changed under the action of the hammer, without breaking. Hence malleability is the reverse of friability and brittleness, and implies softness to a certain degree. Malleability is to be found only in those bodies, which under the action of the hammer allow the intrusion of a molecule through other molecules, and are at the same time tenacious enough not to cleave. For such bodies, then, the curve of the actions must give both BN and α sufficiently great.

A hard body is said to be *ductile*, when it allows its shape to be changed by united compression and traction, so as to receive, without cleaving, the form of a lamina or of a wire. Ductility

requires almost the same conditions as malleability. What in the one case is done by the strokes of a hammer, in the other is done *in a different way* by compression : and for this reason the degree of ductility is not necessarily equal to the degree of malleability.

The properties hitherto explained, as far as they are connected with the curve of the molecular actions, depend on the magnitude of the quantities α, BN, AB, and OA, as is shown in the following table.

Table of some properties of bodies as connected with the curve of molecular actions.

Properties.	Angle α	Maximum BN	Length AB	Distance OA
Solidity	great	mean	any	any
Liquidity	great	small	mean	any
Expansivity	small	any	any	very great
Elasticity of compression	small	great	any	any
,, of traction......	small	great	great	any
,, of flexion	small	great	great	any
,, of torsion	any	great	great	any
Hardness	great	great	any	small
Rigidity.....................	great	great	small	small
Tenacity	great	very great	small	small
Softness	small	mean	mean	great
Friability	great	mean	small	any
Brittleness.................	great	great	small	any
Malleability	great	great	mean	any
Ductility	great	great	great	any

IV. CHANGEABLENESS OF STATE.

Solid bodies often can be changed into liquid and even into expansive fluids : expansive fluids can be changed into liquids and solids : so also liquids can be reduced to solid bodies or to elastic fluids. Hence we infer that the molecules of a large class of bodies admit of a considerable, though accidental, change in their mode of existence, and consequently in their mode of acting on the neighbouring molecules.

The ordinary means employed in changing the state of a body

is heat, or calorific motion. When a body is raised, or reduced, to a certain temperature, its molecules are constrained to make vibrations of a certain extent: and as the temperature increases or decreases, the molecular envelopes gradually expand or contract. Now, the expansion of the envelopes tends to lessen molecular attraction; and therefore a solid body by being raised to a higher temperature, may possibly be changed into a liquid. Again, the contraction of the envelopes tends to augment molecular attraction; and therefore a liquid body by being reduced to a lower temperature may possibly be changed into a solid.

The change from the liquid to the expansive state may be accounted for in an analogous manner. If a liquid is subjected to an increase of temperature, or is freed from the pressure under which it naturally maintains its liquidity, the molecular envelopes will gradually dilate: and thus the molecular attraction of the liquid will gradually disappear and be replaced by repulsion; i. e. the liquid will become an expansive fluid. This change, however, requires more than a simple dilatation of the molecular envelopes; for it implies as a necessary condition *a change of order in the nuclei*, or what we shall call *transposition of nuclei*. Let, for instance, the molecule of the liquid be represented by the formula

$$m = A + nA' + n'R + n''R'.$$

As long as the body remains liquid, the attractive powers of the centre and of the first nucleus will not allow the second nucleus and the envelope to expand freely. But when the molecules by a diminution of pressure or by an increase of temperature are allowed, or obliged, to make vibrations of a greater amplitude, the agitation of the system may become such as to actually bring the oscillating elements of the second nucleus $n'R$ nearer to the centre than the elements of the first nucleus nA'. And, this being the case, the elements of the nucleus nA' by the repulsive exertions of the invading enemy may be compelled to recede from the centre, and cease to form the first nucleus by taking the place of the second. Then the system will be changed into

$$m = A + n'R + nA' + n''R'.$$

After this transposition, the molecules will necessarily tend to

dilate; for the attractive elements nA' are in a less favourable condition for counteracting the repulsive elements $n'R$ and $n''R'$ after the transposition, than before it: and, on the other hand, the repulsive elements $n'R$, after the transposition, are in a more favourable condition for nullifying the efforts of the attractive elements A and nA', than they were before it. And thus it is easy to conceive how a liquid can be changed into an expansive fluid. Vice versa, an expansive fluid may be reduced to the liquid state by causing the transposition to cease: and this can be obtained either by a reduction of temperature or by an increase of pressure.

It follows from this explanation, that the change of a liquid into an expansive fluid is accompanied by a *specific* change of molecular constitution. And, indeed, the dynamical formulas of the system after the transposition (which leads to a partial change of signs) are not of the same form as before; and consequently the elastic vapours, of water, e.g. are, in a dynamical point of view, a substance *specifically* different from liquid water, though *materially* they are identical. We might assimilate such a change of species to that which is brought about in the ellipse by changing only the sign of one term in its equation. Every one knows that this change transforms the equation of the ellipse into that of an hyperbola: and whilst the ellipse, like the liquid, holds a determinate place, the hyperbola, like the expansive fluid, is of such a nature as to demand indefinite extension.

When a solid is melted, no specific change is required; for no transposition is necessary. And indeed, in the melting of a solid, the law of the actions is modified only so as to diminish cohesion to a certain degree, without however destroying it totally, and *a fortiori* without fostering expansivity. Yet, if the molecules of a body are so constituted as to allow the transposition of a nucleus without becoming expansive, the transposition might be admitted as possible in liquefaction also; and, in such a case, the liquid would be a substance specifically different from the solid with which it is materially identical.

In the melting of a solid, the expansion of the molecular envelopes causes an increase of molecular bulk. Hence it is that a body in the liquid state ordinarily fills a greater volume than

in the solid. Still, it is not absolutely necessary that the volume of the liquid should be greater than that of the solid. For, even though each molecule has become greater in bulk, the volume of the body cannot become greater unless the distance of the molecules, measured from their respective centres, is also greater. Now the molecules of a body may be so constituted, that the curve of the actions will cut the axis of the abscissas in the same point A (fig. 28), whether the molecular envelope has been expanded or not. So the system

$$m = A + nA' + n'R + n''R'$$

is so constituted that, when the envelope $n''R'$ expands, the nucleus $n'R$ must contract, and at the same time the attractive nucleus nA' expands. Accordingly, the increase of repulsion due to the dilatation of the envelopes can be compensated by the decrease of repulsion due to the contraction of the nucleus $n'R$, and by the increase of attraction due to the dilatation of the nucleus nA'. More than this; it may happen that, for a given molecular system, the curve of the actions in the case of liquefaction should cut the axis of the abscissas in a point A less distant from O than that which corresponds to the state of solidity; and in such a case, the distance of relative equilibrium being less, the liquid will have less volume than the solid. Of this an instance is to be found in the melting of ice; since the water so obtained occupies a less volume than the ice from which it comes. Hence also, when a liquid solidifies, it is not necessary that its volume should diminish; though, in fact, the far greater number of known substances diminish in bulk when solidifying.

Different bodies require a different degree of heat, that they may melt into liquids or expand into vapours. This is an obvious consequence of a different molecular constitution.

V. Calorific capacity.

Bodies are said to have a greater or less calorific capacity, according as they require a greater or less quantity of calorific action to be raised from a given temperature to another higher

temperature. Physicists have proved that heat is not a substance, but a mode of motion ; and thus the capacity for heat corresponds to the quantity of action by which a certain quantity of motion is to be communicated to the body. If the quantity of calorific action, which is necessary to raise a mass of water from $0^{\circ} C$ to $1^{\circ} C*$ be taken as the unit of action, then the quantity of action which is necessary to raise an equal mass of another substance from $0^{\circ} C$ to $1^{\circ} C$ can be expressed by a number which will represent the calorific capacity of this second substance as compared to that of water. The calorific capacity is often called *specific heat,* and can be determined for equal volumes as well as for equal masses.

Now, let it be observed that the quantity of calorific action which is necessary to raise the temperature of a body in a given ratio, is exactly proportional to the amount of resistance offered by the body to the causation of the calorific vibrations that constitute its new temperature. Such a resistance increases, *cæteris paribus,* with the number of molecules contained in the body : and, even *cæteris imparibus,* though with a little less exactness, as we shall presently see.

It has been remarked long ago by physicists, that the calorific capacities of the primitive bodies are inversely proportional to the chemical equivalents of the same bodies. This law was believed to admit of a few exceptions : but the exceptions themselves, in the present state of science, have almost totally disappeared, for reasons which we shall explain further on (Book X. § II.) ; so that the aforesaid law may now be considered as general. Let us then call c and c' the calorific capacities of two equal masses of different substances, whose atomic (molecular) weights are p and p' respectively. The law which we have enunciated will give

$$c : c' :: p' : p.$$

But, if n and n' be the numbers of molecules contained in the two masses respectively, the weight of the first mass will

* I adopt the French measures for heat, as well as for weight, &c., because the French system is becoming daily more common. Scientific men, even in England, seem to have a strong tendency to take it up. I think that such a tendency will lead to beneficial results.

be np, and the weight of the second $n'p'$; and since equal masses must have an equal weight, we shall have $np = n'p'$, or

$$n : n' :: p' : p;$$

and consequently

$$c : c' :: n : n';$$

or *the calorific capacities are, for equal masses, directly proportional to the numbers of molecules.*

This general result is remarkable for its simplicity; yet it is only sensibly, not rigorously, true, as we have already intimated. But it is very easy to account for the little discrepancy existing between the theoretical and the experimental results; nay, when we consider how greatly one substance differs from another, we might well be surprised at the fact, that substances of such opposite natures do not give rise to discrepancies capable of making the law of calorific capacities quite unrecognisable.

The discrepancies just mentioned arise evidently from the different molecular constitution of different substances. The calorific motion is communicated from molecule to molecule mainly through their respective envelopes, which are in the best condition for strongly influencing one another. The rest of the molecular masses, i.e. the nuclei, move in consequence of the motion to which the respective envelopes have been subjected, according to the nature of the molecular constitution; and therefore the difficulty of communicating calorific motion to a body does not depend, except in a very secondary degree, on the inner part of the molecules. Hence, whether the mass lying under the envelope be greater or smaller, the calorific capacity must remain sensibly proportional to the number of the molecules whose envelopes are to be put in motion. That this is the case, is proved, as already observed, by the small discrepancy of the real from the theoretical capacities. Yet the different mass of the nuclei, and still more the different number of elements that constitute the molecular envelopes of different substances, do really alter to a small extent the law of calorific capacities: and moreover, as heat ordinarily expands the bodies, and augments also the molecular distances, the calorific action is partially employed in mechanical work, which constitutes a loss with regard to calorific motion. Now this work is not equal when the mole-

cules are unequal. For these reasons it ought to be expected
that the law of calorific capacities would not be rigorously true
for bodies of a different nature.

The difference which is found to exist between the theo-
retical and the experimental calorific capacities might become a
means, or instrument, of discovery. For two substances being
given, that, for which the theoretical law is altered by defect,
must have molecules possessing envelopes more ready to receive
calorific motion : and that, for which the theoretical law is altered
by excess, must have molecules possessing envelopes less ready
to receive calorific motion. This leads us to admit, that in the
first case the envelopes are lighter or more simple or more in-
dependent, in their motion, of the rest of the molecular mass,
and in the second heavier or more complex or more dependent,
in their motion, on the rest of the molecular mass. Thus, the
equivalent of zinc being 32·53, and the equivalent of iron 28,
and the real calorific capacity of zinc 0·09555, the theoretical
capacity x of iron will be drawn from the proportion

$$28 \,:\, 32·53 \,::\, 0·09555 \,:\, x = 0·111008.$$

Now the real capacity of iron is $x = 0·11379$; therefore, the
law is altered by excess when iron is compared with zinc; and,
accordingly, in iron the molecular envelope is heavier or more
complex or more dependent on the rest of the molecular mass
than in zinc. Again, the equivalent of oxygen being 16 (as we
shall see hereafter), that of hydrogen 1, and the real capacity of
oxygen 0·2182, the theoretical capacity x of ·hydrogen will be
drawn from the proportion

$$1 \,:\, 16 \,::\, 0·2182 \,:\, x = 3·4912.$$

Now the real capacity of hydrogen is $x = 3·4046$; therefore
the law is altered by defect when hydrogen is compared with
oxygen; and, accordingly, in hydrogen the molecular envelope
is either less complex or less dependent, in its motion, on the
rest of the molecular mass than in oxygen.

The calorific capacities being, for equal weights, proportional
to the numbers of molecules, it follows that the diversity of the
molecular masses exercises a very small influence ·on calorific

vibrations. Whence we must conclude that the molecules of primitive bodies are so constituted that, when their envelopes are compelled to vibrate, the nuclei and the centre of the molecules *do not oppose a resistance proportional to their masses.* Of this we can give a good reason; for the inner part of the molecules is neither totally attractive nor totally repulsive; and, therefore, the resultant of the actions of the centre and of the nuclei is not proportional to the mass; and consequently a considerable diversity of masses does not entail a considerable diversity of action.

What we have said up to this concerns the primitive bodies. In compound bodies the calorific capacities follow a more complex law, dependent on their mode of combination. And, in fact, chemical combination alters more or less the state of the combining molecules, and such an alteration must be ordinarily greater with regard to the molecular envelopes, on which, as we have stated, the calorific capacities mainly depend. Of the different kinds of combination we shall speak further on (Book x. § III. &c.).

Bodies are good or bad conductors of heat, according as they transmit calorific motion quickly or slowly. It would seem that conductivity ought to be inversely proportional to calorific capacity; for, the greater the calorific capacity, the greater is the difficulty of the communication of motion from molecule to molecule: and the greater this difficulty, the less the conductivity. But the reverse is true in many instances. Thus, the calorific capacities of silver and gold are 0·05701 and 0·03244 respectively: and their conductivities are 100 and 53 respectively. To account for this fact, that the conductivity is not inversely proportional to the capacity for heat, it must be observed that the capacities for heat, as above considered, are reckoned for equal masses, whilst the conductivities are reckoned for equal volumes: and, again, radiation influences conductivity, inasmuch as the heat which is radiated cannot be conducted; whilst the calorific capacities are independent of radiated heat. We may also remark that bodies of different molecular constitution have a different calorific motion, though they be at the same temperature as measured by the thermometer. The flute and the trumpet, though playing the same note, and consequently causing the same number of vibrations in the same time, nevertheless

do not emit the same kind of sound : so two bodies, though raised to the same temperature, have a different kind of heat. Now, one kind of heat may consist of a vibratory motion easily transmissible, whatever be its intensity, and another kind may consist of a motion which cannot be easily transmitted unless when it has reached a given intensity. From these remarks it sufficiently appears that conductivity and calorific capacity do not follow necessarily one and the same law.

VI. Colour.

Bodies, with regard to light, are divided into luminous, transparent, and opaque. A *luminous* body, as a flame, is that whose molecules by making spontaneous vibrations of a certain period are able to communicate to the neighbouring æther a motion of the same period, calculated to affect our eye. A *transparent* body, as water, is that whose molecules are so constituted and arranged that the luminous vibrations of æther can be taken up by them and continued through the mass of the body. See in Book VIII. § VII, the principle on which this explanation, and others which follow, are grounded. An *opaque* body, as iron, is that whose molecules are so constituted or arranged as to be unable to take up the luminous vibrations of æther, or to propagate an analogous motion through the mass of the body. Opaque bodies reflect or absorb, instead of transmitting light : and, hence, they have *colour*, as we shall presently explain.

From the velocity of propagation of light, which, according to M. Foucault, is of 298,000,000,000 millimetres in a second, and from other known optical data first determined by Newton, we can find the numbers of vibrations made in a second of time by different rays of the solar spectrum. They are as in the following table :

Extreme red...	vibrations	462,015,500,000,000,
Red orange ...	„	510,806,190,000,000,
Yellow	„	540,834,840,000,000,
Green	„	582,031,200,000,000,
Light blue ...	„	649,237,300,000,000,
Deep blue......	„	663,690,000,000,000,
Extreme violet	„	733,990,100,000,000.

The length of one undulation may be obtained for each coloured ray by dividing the whole line travelled over by light in a second by the number of undulations made by each coloured ray in the same time. Hence the length of the undulations in each ray of the spectrum will be as follows:

Extreme red	fraction of millimetre	0·0006450,
Red orange	„	0·0005807,
Yellow	„	0·0005510,
Green	„	0·0005120,
Light blue	„	0·0004590,
Deep blue	„	0·0004490,
Extreme violet	„	0·0004060.

And, consequently, the numbers of undulations contained in the length of a millimetre will be the following:

Extreme red	number of vibrations	1550,
Red orange	„	1722,
Yellow	„	1814,
Green	„	1953,
Light blue	„	2178,
Deep blue	„	2227,
Extreme violet	„	2463.

These numbers show how great a velocity is that which animates molecular vibrations. As for luminous bodies, it is evident that they cannot excite luminous undulations of a certain period in the surrounding æther, unless they themselves move at the same period and make as many undulations. With regard to non-luminous bodies, they either transmit or reflect light; and therefore although they are incapable of setting themselves into spontaneous vibrations suitable to make a sensible impression on our organ of vision, yet they are prepared, when acted upon by impinging rays, to take up the same kind of vibratory motion, at their surface at least, if they are opaque.

An opaque body is a body that does not allow light to pass

through it.   Perfect opacity would consist in the molecules of the
body being so constituted and united with one another that they
are not prepared to propagate among themselves any kind of
luminous undulations.   A body perfectly opaque would, then,
either reflect or absorb all the rays impinging on its surface.   But
bodies have very often only an imperfect opacity: which arises
from their molecules being so constituted and connected with each
other, that they are prepared to propagate amongst themselves the
vibratory motions which correspond to some kind of light, though
not those corresponding to other kinds.   Thus, the reason why
a piece of green glass is opaque for all the rays, except the green,
is, that in the green glass the molecules are so constituted and
connected with one another, that the length of their undulations
must be $= 0^{.mm}0005120$, which is the length of the undulation of
a green ray: and so the green ray is transmitted whilst the
others are necessarily reflected or absorbed at the surface of in-
cidence*.

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* Professor Tyndall says: "All ordinary transparent and colourless substances
owe their transparency to the discord which exists between the oscillating periods of
their molecules and those of the waves of the whole visible spectrum.   The general
discord of the vibrating periods of the molecules of compound bodies with the light-
giving waves of the spectrum may be inferred from the prevalence of the property of
transparency in compounds, solid, liquid, and gaseous; while their greater harmony
with extra-red periods is to be inferred from their opacity to the extra-red rays" (*Pro-
ceedings of the Royal Society*, 1864, p. 160).   I am very much inclined to think that
the reverse is true.   A body is opaque with regard to light or heat, when the period of
the molecular vibrations cannot coincide with the period of the luminous or calorific
waves: and a body is transparent with regard to light or heat, when the period of the
molecular vibrations coincides with the period of the luminous or calorific waves.   To
prove this, it would suffice to observe that discord implies opposition: now, opposition
is calculated to quench, not to propagate, motion.   How then can waves of any period
be conveyed or propagated by, or through, molecules which cannot vibrate in those
periods ?   On the other hand, if the molecules move in periods harmonizing with the
periods of the luminous or of the calorific waves, how can this motion interfere with
the propagation or transmission of those waves ?   My firm conviction is, that mole-
cular vibrations of a given period constitute the actual propagation of light or heat of
the same period through the transparent solid body.   Mr Tyndall, of course, admits
with most others, that aetherial, not molecular, vibrations constitute the actual propa-
gation of light and heat through the transparent bodies: but, even in this hypothesis,
how could we say, that the aether intercepted between the molecules can easily con-
tinue its vibrations when it finds resistance, and cannot when no resistance is to be
found ?   Moreover, from Prof. Tyndall's theory it would follow, as he explicitly states,

It is obvious, that a body imperfectly opaque is also imperfectly transparent; and therefore its opacity as well as its transparency is only relative.

A body is white, when its molecules are so constituted that they can move in any period of luminous vibrations, but are so arranged that such a kind of motion cannot be regularly communicated from molecule to molecule, and consequently is not propagated throughout the body. In such a case, the molecules which lie at the surface of the body, and those perhaps which come immediately after them, will vibrate in periods like those of the incident rays (which are white, of course, if they consist of solar light); and those molecules, when once set in motion, will communicate the same kind of motion to the neighbouring æther, according to the laws of impact of elastic bodies. In other words, the beams of white light will be either reflected or dispersed (according as the surface of the body is polished or not) from the surface of the body into the eyes of the spectators.

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that the flame of hydrogen burning in oxygen would consist of vibrations of an extra-red period. Now, surely, that flame has a visible hue; and therefore its period is one of those contained in the visible spectrum; and consequently is not extra-red. Hence, we cannot adopt Prof. Tyndall's view on this subject. The more so, because the author, who is wont to prove fairly all his assertions, has not thought proper to give us the proof of this one, nor of the existence of æther between the molecules of solid or liquid transparent bodies. He says, indeed, that æther "surrounds the very atoms of solid and liquid substances;" that "transparent bodies are such, because the æther and their atoms (molecules) are so related to each other, that the waves which excite light can pass through them, without transferring the motion to the atoms" (*Heat as a mode of motion*, Lect. IX. p. 293). But no reason is given for the assumption. In a lecture (Jan. 1866) at the Royal Institution, Prof. Tyndall observed that, if light communicated the motion of its undulations to the ice, the ice would be melted; whence he concluded that ice possesses periods of vibrations which are discordant from the periods of light. But every one will see that this is not a proof. If light communicated the *calorific* motion to the ice, the ice would be melted; but if light communicates the *luminous* motion to the ice, it does not follow that ice will be melted. The only conclusion which can be drawn is, that luminous rays and calorific rays are distinct, and that, therefore, the motion of light through ice is not competent to produce the heat necessary for the melting of ice. It appears, then, that the assumption of æther vibrating in the transparent bodies, and being the instrument of transmission, is not proved. This would probably show that the view advocated by Professor Tyndall is admitted by him only on account of its having been admitted by others before him. If this be the case, the learned Professor cannot fail to see the propriety of treating anew this important point of science.

An instance of this is to be found in pounded glass and snow, which are both white. Their molecules are undoubtedly prepared to make luminous vibrations of any period; since glass and water are transparent bodies: but the same molecules, in pounded glass and in snow, have ceased to have the regular and homogeneous arrangement indispensable for the continuation of the regular periods of vibrations throughout the mass; and therefore it is, that those periods are broken, and the motion is not propagated through the body, or, if any motion be propagated, it is not luminous.

A body is black, when its molecules are so constituted, that they cannot vibrate according to any of the periods of the luminous rays of the solar spectrum. In such a case, the luminous ray is *absorbed*, that is, the action of the impinging luminiferous æther exhausts itself in communicating to those molecules a kind of motion which does not consist of periods calculated to make any appreciable impression on our organ of vision. This is evident; for the action of a luminous ray of a certain period would oblige the molecules of the body to vibrate at the same period, if they were able to take up such a kind of vibrations; and, accordingly, the extinction of the luminous ray must be the consequence of an incompatibility of its periods with the nature of the molecules acted upon. A molecule is, in this respect, like a pendulum, the vibrations of which are of a certain rapidity according to given dynamical relations: and it is as impossible for the molecules of lamp-black to synchronize with the spectral luminous vibrations, as it is impossible for a pendulum of one foot in length to synchronize with a pendulum of an inch.

Some will say, that if this be true, then a black body would be perfectly invisible. This, I think, would only prove that no visible body is perfectly black. Still, the molecules which lie at the surface of a black body, while acted upon by the luminous rays, must react on the impinging æther; hence the black body gives back to the surrounding æther a certain amount of motion of which a portion may still preserve in some degree the properties of visible ray.

A body is red, or yellow, or green, &c., when its molecules are so constituted that they can vibrate according to the periods

of the luminous rays known under the names of these colours ; but they are so arranged and united, that the respective vibrations cannot be propagated throughout the body. We speak of opaque bodies, of course ; for, when the body is transparent for a certain ray, then its molecules must be so arranged and united that the interior propagation of the corresponding periods of vibrations should be possible.

A body may present a colour different from any of those of the solar spectrum. This happens in two cases : first, when the molecules of the body are so constituted that they can vibrate in periods of some kinds without being able to vibrate in the intermediate periods, as when the molecules can move in the periods of blue and in the periods of red, but cannot move in the periods of orange, of yellow and of green. Secondly, when the body is composed of molecules of a different nature. If, e.g. some molecules can vibrate in the periods of red, and others in the periods of green, the body will have a mixed colour, which is not to be seen in the solar spectrum.

There are bodies which are opaque in the solid, and transparent in the liquid state, as wax, butter, oil, sugar, &c. The fact is accounted for by observing that, when a body is melting, its molecules change their mutual relations, and modify their intrinsic constitution.

We will here close these remarks, or rather hints, on colours. What we have said suffices to show that the colours of bodies are intimately connected with their molecular constitution and arrangement ; and, therefore, the phenomena of light are each and all to be looked upon as a very valuable and indispensable means for ascertaining the true nature of the same bodies.

BOOK X.

ON CHEMICAL PROPERTIES IN GENERAL.

CHEMISTRY is considered only as an experimental science : and chemists in general, when they have succeeded in determining experimentally the laws, conditions, and results of combination and resolution, are satisfied that they have done the whole of their duty. Yet natural philosophers are well aware, that a great deal of work remains to be done, if we wish to raise chemistry to its proper perfection. Laws have been established; but the principles on which such laws repose have not been established at all : and a science cannot but be very imperfect, which has yet to find out what are its own higher principles. Hence, scientific men, whilst continuing to develope material knowledge, as they do, by experimental research, ought not to forget that chemistry is expected to rise, sooner or later, to a higher region, whence the principles are to be seen by which the laws experimentally known are to be accounted for. In a word, they might well consider, that besides the experimental, we want a rational science of chemistry. The first elements of a rational chemistry constitute a part of the science of which I am treating under the name of *Molecular Mechanics*. Unhappily, every science has its infancy ; so we must for the present content ourselves with a few generalities and still fewer details. We shall treat in this Book of chemical affinity, of chemical equivalents, and of the disposition of bodies to combine in different definite proportions.

I. CHEMICAL AFFINITY.

Two substances are said to be endowed with chemical affinity, when the molecules of the one can unite with the molecules of the other so as to constitute a new molecular system. Thus, when nitrogen and hydrogen unite to form ammoniacal gas, the molecules of the two substances not only *mix*, but *combine* with one another, i.e. affect and modify each other intrinsically as much as is necessary to form a single molecular system.

Chemical affinity is not to be found between molecules of the same kind. For molecules of the same nature, on account of the direct opposition of their repulsive envelopes, cannot have any tendency to fuse themselves into a single molecular system*. Thus, in a mass of hexahedric molecules of the same nature, any two molecules O and O' (fig. 29) will repel each other at all distances less than the distance of relative equilibrium, on account of the direct opposition of the elements R to the elements R'.

On the contrary, chemical affinity is to be found between molecules of a different kind. And, in fact, in molecules of a different nature, the repulsive envelopes are not necessarily situated in direct opposition; and therefore it is not necessary that repulsion should prevail between them at all distances less than the distance of relative equilibrium. Thus, if an hexahedric molecule is surrounded by tetrahedric molecules, there will be no direct opposition of the envelopes. A tetrahedric molecule will present its vertex R' (fig. 30) to the surface $RRRR$ of the hexahedric one : and it is evident, that if the distance of the two molecules is sufficiently lessened, the attraction exercised by the element A upon R' can prevail over the repulsions exercised by the elements R upon the same R'. For, the nearer R' approaches, the greater will become the attraction of A upon R', whilst the resultant of the four repulsions exercised from the points R evidently is lessened on account of the increasing obliquity of their direction. Hence, when two molecules of a different nature are constrained to approach to a distance less than that of relative equilibrium, the repulsion which is immediately developed may, after it has reached a maximum, by a further decrease of the distance become weaker and weaker till it vanishes, and is replaced again by attraction. This new attraction may be called the *chemical attraction;* for it is in fact the immediate causality of combination.

From the preceding remarks it follows, that the curve of molecular actions, which we have found above for molecules of the same nature, does not hold for molecules of a different nature having chemical affinity. In this last·case, the positive branch $A M$ (fig. 31) of the curve, which represents the repulsive actions

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* Tetrahedric molecules may, in some cases, be excepted, as the opposition of their envelopes is not direct.

corresponding to distances less than OA, i.e. less than the distance of relative equilibrium, is not asymptotic, but, after having reached its maximum in M, cuts again the axis of the abscissas in A', and then describes a new arc $A'N'A''$, whose ordinates correspond to the chemical attractions causing the molecule A to combine with the molecule O. The distance OA'', at which the molecules will remain after their combination, might be called the *distance of chemical equilibrium:* and then the distance OA would have to be called the *distance of physical equilibrium.*

Here, then, we have a very neat representation of the radical distinction existing between chemical and physical actions. The physical are those by which the generally called *physical* phenomena are produced; and are represented by the branches PNA and AMA' of the curve: the chemical are those on which the affinity and the combination depend; and are represented by the branches $A'N'A''$ and $A''M'$. The point A' marks the end of the physical, and the beginning of the chemical actions; so that, whilst two molecules can act on each other by physical action at all distances from OA' to infinity, their chemical action cannot extend farther than OA'.

The mere inspection of the curve will enable us to understand, 1st, that a mixture of two substances is not the same as their combination, but quite a different thing; for, when two substances are mixed, their molecules rest at the distance OA, and when they combine, their molecules rest at the distance OA'. Mixture means *juxtaposition,* whilst combination implies partial *introsusception* or something of that kind. 2nd, that, when for two substances the repulsion represented by the ordinate of the point M is not very great, a mean pressure will be sufficient of itself, if continued, to determine the combination. In this case, the two substances may be said to have a considerable affinity. But, if the said repulsion is very great, their combination will require a very great exertion, and the two substances might be said to have little affinity. Still, pressure is not the only, nor the principal standard by which we judge of the degree of affinity existing between two substances. 3rd, that molecules, which have little or no affinity in one state and temperature, may acquire or intensify it, as the case may be, by a change of state or of temperature.

For, it is evident from the reasons above given (Book IX, § IV.), that a change of state or of temperature causes a change in the molecular radii, and alters very materially the form of the curve of molecular actions. We will not lose time in giving instances of this fact; every one knows that this is the most common case in chemistry. 4th, that the degree of affinity is the degree of facility with which two substances combine; so that affinity increases in the inverse ratio of the ordinate of the point M. But a substance A may have in certain circumstances a greater affinity for a second substance B than for a third C, whilst by a change of circumstances, or conditions, it may acquire a greater affinity for the substance C than for the substance B. This also is one of the most ordinary facts in chemistry, and the key to all chemical analyses. 5th, that the greater the ordinate of the point N' in the curve of actions, the greater will be the firmness of the combination and the stability of the compound. 6th, that when two substances combine, the volume of the compound will very often be smaller than the united volumes of the components. This is evident from the difference existing between the distance of physical and of chemical equilibrium. Thus, one volume of nitrogen and two of oxygen are reduced by combination to two volumes of hyponitric acid: so also one volume of nitrogen and three of hydrogen contract into two volumes of ammoniacal gas. But there are numerous exceptions. A volume of carbon, e.g. and a volume of oxygen give two volumes of oxide of carbon without sensible reduction: one volume of hydrogen and one of chlorine give two volumes of hydrochloric acid without reduction, &c. These exceptions may be easily accounted for by two obvious remarks. The first is, that the difference between the distance of physical equilibrium and that of chemical equilibrium, in the case of two given substances, may be small; hence the reduction of the volume, on this ground, might be small. The second is, that when molecules of a different nature actually combine, the molecules of one kind cannot reach their position of chemical equilibrium without causing an amount of expansion in the envelopes of the molecules of the other kind; and, in fact, when the vertex R' (fig. 30) is about to reach the plane $RRRR$, it is evident that the elements R are removed farther apart, and the volume of the molecule O

augmented.   Such an increase of volume may be sufficient to
compensate the diminution due to the greater proximity of the
combining molecules ; and consequently, the result of combination
is not necessarily a reduction of volume.

II.   Chemical equivalents.

Substances of a different kind combine only in definite propor-
tions.   A substance being given, and the least mass of it that is
capable of entering into combination being taken as a unit, the
least masses of other substances that are capable of entering into
combination will be represented by numbers which are called
Proportionals. The masses represented by such numbers are called
Chemical equivalents or *Chemical atoms.*   This last name, as well
as that of *atomic weight, atomic doctrine,* &c. is extensively em-
ployed in chemical works, in England at least ; but we shall ex-
clusively retain the name of chemical equivalents.   " The word
atom," says Prof. Faraday, " which can never be used without
involving much that is purely hypothetical, is often *intended* to
be used to express a simple fact : but good as the *intention* is, I
have not yet found a mind that did habitually separate it from
its accompanying temptations : and there can be no doubt that
the words *Definite proportions, Equivalents, Primes,* &c. which
did and do express fully all the *facts* of what is usually called the
' atomic' theory in chemistry, were dismissed because they were
not expressive enough, and did not say all that was in the mind
of him who used the word 'atom' in their stead : they did not
express the hypothesis as well as the fact *."

The chemical equivalents of different substances, if properly
determined, must be proportional to their molecular masses respec-
tively ; but the absolute molecular masses cannot be known until
the common factor is found by which the numbers representing
the equivalents should be multiplied.   The determination of this
common factor would require the determination of the absolute
mass of one at least of those substances.

I have said that the equivalents are proportional to the mole-
cular masses *if they have been properly determined.*   The reason of

* *A speculation touching electric conduction and the nature of matter.—Phil. Mag.*
1844, Vol. xxiv.

this reserve is, that about the equivalents of some substances, as they are generally given in elementary works, a suspicion has arisen that they are faulty. The suspicion originated chiefly from the fact, that, whilst for the far greater number of primitive substances the equivalents are inversely proportional to their calorific capacities, a few primitive bodies, on the contrary, had equivalents which could not tolerably agree with the same law, unless they were either divided or multiplied by 2.

I shall give here the equivalents of some substances from M. Regnault's Chemistry, adding in separate columns the calorific capacities of the same, and the products of those capacities into the equivalents.

Substance.	Symbol.	Equivalent.	Capacity.	Product.
Hydrogen	H	1·00	3·4046	3·4046
Oxygen	O	8·00	0·2182	1·6376
Carbon*	C	6·00	0·1469	0·8914
Nitrogen	N	14·00	0·2440	3·4160
Chlorine...........	Cl	35·42	0·1214	4·3000
Bromine (vapour)	Br	78·26	0·0552	4·4608
Sulphur	S	16·12	0·2026	3·2759
Iodine	I	125·33	0·0541	6·7803
Phosphorus	P	32·00	0·1895	6·0640
Arsenic	As	75·00	0·0814	6·1050
Iron	F	28·00	0·1138	3·1864
Cobalt	Co	29·52	0·1069	3·1556
Nickel	Ni	29·57	0·1086	3·2113
Cadmium	Cd	55·74	0·0567	3·1604
Zinc	Zn	32·53	0·0955	3·2042
Tin	Sn	58·82	0·0562	3·3056
Lead	Pb	103·56	0·0314	3·2499
Bismuth...........	Bi	106·40	0·0308	3·2771
Antimony	Sb	64·52	0·0508	3·2776
Copper	Cu	31·65	0·0951	3·0099
Mercury...........	Hg	100·00	0·0333	3·3320
Silver.......	Ag	108·00	0·0570	6·1560
Palladium	Pd	53·22	0·0593	3·1559
Gold	Au	98·22	0·0324	3·1823
Platinum	Pt	98·56	0·0324	3·1913

* The calorific capacity of carbon is different in diamond, graphite, charcoal, &c. We here give to carbon the capacity of diamond, though we think, for reasons that will be adduced further on, that carbon vapour has twice the same capacity.

From the last column of this table we see that the products of the equivalents into the calorific capacities are sensibly constant, with a few exceptions. Silver, arsenic, phosphorus, iodine, if their equivalents were divided by 2, would cease to be exceptional. In like manner, oxygen would be reduced to the general law by multiplying its equivalent by 2. And so on.

Now, are these corrections to be admitted? Of course, it would not have been prudent to adopt them, when they rested on this ground only: but in our own days eminent chemists have shown that there are other reasons, besides the law of calorific capacities, which make such corrections indispensable. M. Regnault proves the necessity of dividing by 2 the equivalents of silver, sodium, and potassium, from the known law of isomorphism*. Mr Rose establishes that "the atomic weights of the five alkaline metals

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* Here is one of his proofs: "Si nous adoptons ces équivalents (thermiques), les composés que nous formulions KO, NaO, AgO, KCl, $NaCl$, $AgCl$,......prendront les formules K^2O, Na^2O, Ag^2O, K^2Cl, Na^2Cl^2, Ag^2Cl^2,......Nous allons chercher à montrer, que ce sont en effet ces dernières formules qu'il faut adopter, si l'on veut satisfaire non seulement aux lois des combinaisons, que nous avons développées, mais encore à la loi de l'isomorphisme. Nous connaissons deux métaux, le cuivre et le mercure, qui forment chacun deux oxydes basiques, dont les formules Cu^2O, CuO, Hg^2O, HgO, sont vérifiées par la constitution des sels neutres, qu'ils forment avec des acides puissants. Comparons les sels, que forment ces oxydes, ou les composés binaires qui leur correspondent, avec les sels analogues, ou aux composés binaires correspondents, formés par le potassium, le sodium, et l'argent. On trouve dans la nature, à l'état crystallisé, le sulfure de cuivre Cu^2S et le sulfure d'argent: ces deux minéraux présentent exactement la même forme crystalline. On y rencontre, en outre, des minéraux présentant la même forme crystalline, et renfermant à la fois le sulfure de cuivre Cu^2S et le sulfure d'argent en des quantités relatives variables à l'infini. De sorte, que l'on est conduit à admettre que ces corps peuvent se remplacer en proportions quelconques, sans changer la forme crystalline du composé. Le sulfure d'argent et le sulfure de cuivre Cu^2S présentent donc tous les charactères de l'isomorphisme. On est en droit d'en conclure que le sulfure d'argent doit avoir le même mode de constitution que le sulfure de cuivre Cu^2S; et que la formule du sulfure d'argent doit être Ag^2S. Mais, si la formule du sulfure d'argent est Ag^2S, celle de l'oxyde d'argent doit être Ag^2O.

"Maintenant, l'observation démontre que le sulfate d'argent est isomorphe avec le sulfate de soude anhydre. Pour satisfaire à la loi de l'isomorphisme il faudra donc écrire la formule du sulfate de soude $Na^2O . SO^3$, si l'on écrit celle du sulfate d'argent $Ag^2O . SO^3$. Mais les composés de potassium sont isomorphes avec ceux de sodium; on ne peut donc pas écrire leur formules de deux manières différentes; la formule de la potasse doit donc être écrite K^2O."—Regnault, *Chimie*, Vol. III. § 1240. See also Vol. III. pp. 350, 351.

(as well as that of thallium) must be divided by two[*]." That the equivalent of oxygen must be doubled, is already the prevalent opinion among scientific men. Hence Prof. Odling decidedly asserts that the molecule of water "contains not, as was formerly held, the same amount of hydrogen, but double the amount of hydrogen contained in the molecule of muriatic acid (HCl). Or, in other words, the molecular weight of water, muriatic acid, and hydrogen, are to one another as 18 : 36·5 : 2, or as 9 : 18·25 : 1[†]."

As for chlorine and iodine, we have also special reasons for dividing their equivalents by 2. The compounds ordinarily expressed by the formulas ClO^7, IO^7 would now (on account of $O = 16$) be expressed by Cl^2O^7, I^2O^7: but we shall see (Book XI. § III.) that these last formulas cannot be accounted for, as two molecules of one substance and seven molecules of another cannot arrange themselves, as is necessary for combination. On the contrary, if we take $Cl = 17·71$, and $I = 62·77$, those formulas will become Cl^4O^7, I^4O^7, which admit of a very natural explanation. And moreover, as the equivalent of sodium is now admitted to be $Na = 11·49$ instead of $Na = 22·98$, sulphate of soda will be expressed by $Na^4O.S^2O^3$. Now, assuming with all chemists that chloride of sodium contains one equivalent of sodium and one of chlorine, we know that four equivalents of chloride of sodium with one equivalent of water and one of sulphuric acid give rise to the following reaction:

Chloride of sodium 4 equivalents {Chlorine $4Cl$, Sodium $4Na$}
Water 1 equivalent {Hydrogen H^2, Oxygen O}
Sulphuric acid...................... S^2O^3

$\to$ {$2Cl^2H$ hydrochloric acid.
$Na^4O.S^2O^3$ sulphate of soda.
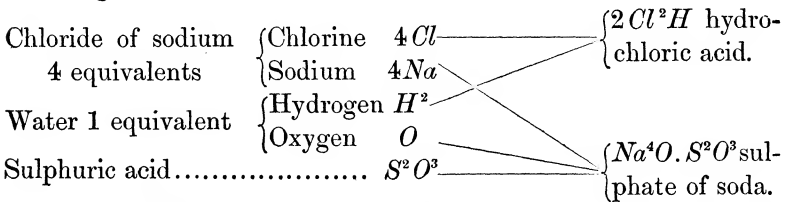
And therefore, hydrochloric acid contains not one, but two equivalents of chlorine with one of hydrogen: and the common formula ClH must become Cl^2H. In other words chlorine has for its equivalent 17·71.

The equivalent of phosphorus must also be divided by 2. Though this is very clearly indicated by its calorific capacity, we

* *Phil. Mag.* 1863, Vol. XXVI. p. 366.
† *Phil. Mag.* 1863, Vol. XXVI. p. 382.

can prove it from the relation of phosphorus to chlorine. Thus, we know that chloride of phosphorus dissolves in water and yields phosphorous acid and hydrochloric acid. The reaction is as follows:

Chloride of phosphorus $\begin{cases} \text{Chlorine} & 4\,Cl^3 \\ \text{Phosphorus} & 4P \end{cases}$ ——————— $6HCl^2$,
4 equivalents

Water 3 equivalents $\begin{cases} \text{Hydrogen} & 3H^2 \\ \text{Oxygen} & 3\,O \end{cases}$ ——————— P^4O^3;
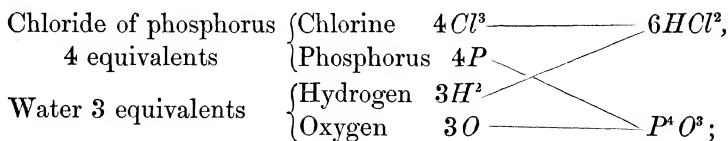
and, since the formula of hydrochloric acid is really HCl^2, as we have seen, it follows that the formula of phosphorous acid is really P^4O^3, not P^2O^3; and therefore the equivalent of phosphorus is not 32 but 16.

So also perchloride of phosphorus dissolves in water, and gives the reaction

Perchloride of phosphorus $\begin{cases} \text{Chlorine} & 4\,Cl^5 \\ \text{Phosphorus} & 4P \end{cases}$ ——————— $10HCl^2$,
4 equivalents

Water 5 equivalents $\begin{cases} \text{Hydrogen} & 5H^2 \\ \text{Oxygen} & 5\,O \end{cases}$ ——————— P^4O^5;
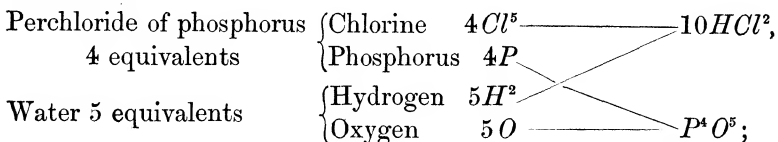
the last product P^4O^5 representing phosphoric acid; whence we come again to the same conclusion $P = 16$.

With regard to arsenic, we shall also divide its equivalent by 2, as is suggested by its calorific capacity, and required by its relation to phosphorus. For, arsenic combines with metallic and non-metallic substances in the same manner as phosphorus, which it resembles in many respects. Thus, we have the following compounds of phosphorus:

	Usual formula.	New formula.
Phosphorous acid	PO^3	P^4O^3,
Phosphoric acid	PO^5	P^4O^5,
Phosphuretted hydrogen	PH^3	P^2H^3,
Chloride of phosphorus	PCl^3	PCl^3,
Iodide of phosphorus	PI^3	PI^3;

and we have in the same manner

Arsenious acid	AsO^3	As^4O^3,
Arsenic acid	AsO^5	As^4O^5,
Arseniuretted hydrogen	AsH^3	As^2H^3,
Chloride of arsenic	$AsCl^3$	$AsCl^3$,
Iodide of arsenic	AsI^3	AsI^3.

These examples of parallelism between arsenic and phosphorus show that, the equivalent of phosphorus having been divided by 2, that of arsenic also must be divided by 2.

With regard to carbon, as its equivalent has been determined from that of oxygen, the change adopted for oxygen obliges us to take .12 instead of 6 for the equivalent of carbon. But the calorific capacity of diamond might tempt us to go further, and to take 24 for equivalent. The more so, because the determination of the equivalent of carbon has been made under a great disadvantage, as it has never been possible to obtain the vapour of carbon in its pure state, and to weigh it separately. An equivalent thus determined might well be erroneous, in this sense at least, that it might require to be multiplied by 2, as the law of calorific capacities seems to suggest. But carbon in different states shows different calorific capacities. In diamond its capacity is only 0·1469; but in plumbago it is 0·2020, and in charcoal 0·2420; and therefore the capacity increases for a decrease of density. Hence we may well hold that carbon in its gaseous state will have a still greater capacity than in charcoal. And, since the capacity 0·2420 of charcoal multiplied by the number 12 gives 2·9040, which nearly agrees with the general law, a still greater capacity will give a product even more satisfactory. And, therefore, the law of calorific capacities is quite safe with carbon = 12 : and, as we find no other reasons for a change, we shall retain 12 for the equivalent of carbon.

Bromine resembles iodine and chlorine in many. respects. Hydrobromic acid bears the strongest resemblance in every particular to hydriodic acid : it has the same constitution by volume, very nearly the same properties, and may be prepared by means exactly similar, substituting the one body for the other. So also bromic acid closely resembles chloric acid. Hence, the equivalents of chlorine and iodine having been divided by 2, that of bromine also must be divided by 2.

We may, then, safely conclude that the corrections more or less clearly suggested by the law of calorific capacities are to be admitted : and consequently we shall adopt the following equivalents.

Oxygen .... 16·00   Carbon .... 12·00
Phosphorus ...... 16·00   Arsenic .... 37·50

Iodine .... 62·77   Sodium .... 11·49
Chlorine .... 17·71   Lithium............. 3·22
Bromine .... 39·13   Potassium .... 19·60
Silver .... 54·00

III. RELATIVE NUMBERS OF COMBINING EQUIVALENTS AS DEPENDENT ON THE GEOMETRICAL FORM OF MOLECULES.

Chemical combination is the annexation of one or many molecules of a certain substance to one or many of another substance, giving rise to a new molecular system.

When a given number of molecules of one substance A are annexed to, or bound up with, one of another substance B, the compound will consist of a number of compounded molecules equal to that contained in the substance B. For example, if the substance B contains n tetrahedric molecules, each of which unites to itself, at its four faces, four molecules of the substance A, the compound will evidently consist of n new molecules. In this case, the constitution of the compound can be *discrete;* and, therefore, its equivalent will represent one of its real molecules in the same manner as the equivalent of each component represents one of its molecules.

When a molecule of a given substance A is annexed to, or bound up with, a number of molecules of another substance B, the compound may consist of a number of *discrete* molecules equal to that contained in the substance A. Thus, if the substance A contains n tetrahedric molecules, each of which is laid hold of by four molecules of the substance B, n groups will be formed, and the constitution of the compound will be *discrete,* as in the preceding example. The only difference between the two cases lies in this, that in the first the molecule situated in the centre of the group is considered as that which *actively* annexes the four others to itself, and in the second is considered as being itself *passively* annexed to the four others. It is evident, that the equivalent of the compound will again correspond to one of its molecules.

But, whether a molecule actively annexes many others to itself, or is passively annexed to them, the compound will not be

discrete, but *continuous*, whenever the combining molecules are on both sides connected with one another. In such a case, it is obvious that the chemical equivalent of the compound will not represent any distinct molecule existing in it, but only the smallest possible mass, under which the compound can be conceived, viz. a *virtual* or *intentional* molecule. The reader will find below many examples of this kind of continuous combination.

The molecules of primitive bodies being regular polyhedric systems of elements (Book VI. Prop. VI.), and having faces, edges, and vertices, the combination may be conceived to be possible inasmuch as molecules can be annexed to molecules either by their faces, or by their edges, or by their vertices. This means, that one molecule will draw to itself a molecule (or many) of another kind, and bind it either to one of its own faces or to one of its own edges : the drawn molecule (or molecules) at the same time offering itself to that annexation by one of its own vertices, or of its own edges, or of its own faces, as the case may be. It is evident, however, that two molecules cannot directly unite through their vertices, the vertices being always repulsive (Book VI. Prop. IV.).

After these general statements, we must now inquire how many molecules of any given regular polyhedric form can be chemically united to a molecule, or to a number of molecules, of a different nature, and of any regular polyhedric form whatever. The general principle, on which this inquiry will be based, is, that the combination cannot take place except between those numbers of molecules for which there is a possibility of a geometrical arrangement; for, it is evident, that *molecules cannot combine, if they cannot approach to one another.* Hence any geometrical exigency which prevents the approaching of molecules in a given number makes the combination impossible for such a number. Let us come to particulars.

IV. NUMBERS OF MOLECULES THAT CAN COMBINE WITH TETRAHEDRIC MOLECULES.

First, then, a *tetrahedric* molecule, by reason of its four faces and of the attractive elements which, in the neighbouring nucleus, may correspond to each of them, is geometrically prepared

to admit around itself *four* molecules of another substance : and, if its power be sufficient, to annex them to itself by the action of the said attractive elements. In this case the numbers of the combining molecules will be in the ratio 1 : 4, and the molecules of the compound will be *discrete* and *tetrahedric*.

Again, a tetrahedric molecule has six edges, and, therefore, is geometrically disposed to admit around itself *six* molecules of another substance, and to unite them to itself by the action of the elements lying under its own envelope. In this case the number of the combining molecules will be in the ratio 1 : 6, and the molecules of the compound will be regular *octahedrons.*

Consequently, a tetrahedric molecule can also admit around itself *ten* molecules of another substance, viz. *four* on its faces and *six* on its edges, and unite them all to itself, as before, at least when the volume of these molecules is considerably less than that of the said tetrahedric molecule. In this case, the molecules of the compound will be *tetrahedrons* or *octahedrons,* according as the molecules which unite through the faces, or those which unite through the edges, are most prominent.

In these three cases the compound consists of *discrete* molecules.

Let us now conceive $4n$ tetrahedric molecules of the same kind, which are free to arrange themselves in space according to their own nature and exigency. They will arrange themselves in the following manner: n out of them will occupy the places $a, a, a, \ldots$ (fig. 32), so disposed as to form the common vertices of a number $2n$ of tetrahedrons and of a number n of octahedrons, regularly distributed : n other molecules will occupy the centres O' of those octahedrons, and the remaining $2n$ the centres O of the tetrahedrons.

Now, let us suppose that, instead of having $4n$ molecules of the same kind, we have $3n$ molecules of one substance, and n of another. Either these n molecules will occupy the vertices a, and the remaining $3n$ the centres of the octahedrons and of the tetrahedrons, or the n molecules will occupy the centres O' of the octahedrons, and the remaining $3n$ the vertices a and the centres O of the tetrahedrons. These arrangements, by allowing each of the $3n$ molecules of one substance to approach to the

n molecules of the other substance, favour the combination of them ; and, therefore, tetrahedric molecules (if no other obstacle is to be found) can combine in numbers that are in the ratio 1 : 3. In this case the compound will not consist of discrete compounded molecules, as is evident, and its constitution may be called *continuous*. When the n molecules of the one substance have a predominant power of annexation, and occupy the places $a, a, a, \ldots$, then the form of the compound will be tetrahedric, or traceable to the tetrahedron : but, if those n molecules occupy the centres of the octahedrons, then the form of the compound will be octahedric or traceable to the octahedron.

The above ratio 1 : 3 does not require that the molecules be all tetrahedric. If $3n$ molecules were tetrahedric, and the remaining n hexahedric, or octahedric, or octohexahedric, or rhombododecahedric, the geometrical arrangement would still be favourable to the combination. The n molecules would, in fact, occupy the centres of the octahedrons above mentioned, and could annex to themselves, or be annexed to, the $3n$ molecules of the other substance. But, in this case, those amongst the $3n$ molecules which occupy the places $a, a, a, \ldots$ would possibly be more intimately united with the centres of the octahedrons, than those others which occupy the centres of the tetrahedrons : nay, these last, by reason of their greater distance from the molecules of the other substance, may be materially included in the compound without any chemical union.

Let us now suppose, that among the $4n$ tetrahedric molecules $2n$ are of one kind and $2n$ of another. The combination will be possible again; for the $2n$ of the one kind will occupy the centres of the above-mentioned tetrahedrons, whilst the $2n$ of the other kind will occupy the centres and vertices of the octahedrons. The numbers of the combining molecules will be in the ratio 2 : 2 or 1 : 1. This ratio requires that $2n$ molecules at least should be tetrahedric: the other $2n$ may be hexahedric, octahedric or octohexahedric. The constitution of the compound will be *continuous*, and its form will be traceable to the tetrahedron or to the octahedron, according as the molecules which possess a greater power of annexation occupy the centres of the tetrahedrons or the centres and vertices of the octahedrons.

15—2

Let us suppress the n molecules, which occupy the centres of the octahedrons. There will remain $2n$ molecules of one kind in the centres O of the tetrahedrons, and n of the other kind in the vertices $a, a, a, \ldots$ They will be suitably arranged for combination. Hence, we must admit that tetrahedric molecules can combine in the ratio 2 : 1. The compound will be *continuous*, and its form will be tetrahedric, or traceable to the tetrahedron.

The ratio 2 : 1 is also admissible when, $2n$ molecules being tetrahedric, the others n are hexahedric or octahedric. With n hexahedric, each vertex of a hexahedric molecule will be connected with the neighbouring face of a tetrahedric one. With n octahedric, each vertex of a tetrahedric molecule would be connected with the neighbouring face of an octahedric one. The compound will be *continuous*, as is evident, and its form will be traceable to the octahedron or to the tetrahedron.

These are the only definite ratios which I find to be admissible for tetrahedric molecules.

V. NUMBER OF MOLECULES WHICH CAN COMBINE WITH OCTAHEDRIC MOLECULES.

An octahedric molecule, by reason of its eight faces, and of the attractive elements corresponding, in the neighbouring nucleus, to each of them, has a special capability of combination with *eight* surrounding molecules of another substance : and, by reason of its twelve edges, with *twelve* such molecules : and consequently, by reason both of edges and faces, with *twenty* such molecules; under the same conditions and limitations as we have mentioned in the case of tetrahedric molecules. The compounds, in these three cases, will consist of *discrete* molecules.

Again, n octahedric molecules can combine with $2n$ tetrahedric, as we have already shown ; and the compound will be *continuous*.

Moreover, n octahedric molecules can unite with $4n$ of another kind having two vertices diametrically opposite, whether hexahedric, octahedric, octohexahedric, icosahedric, or rhombo-dodecahedric. In this case, each of the $4n$ molecules will turn one vertex towards the neighbouring face of one, and the opposite vertex towards the neighbouring face of another of the n octa-

hedric molecules, between which it lies: and thus the $4n$ molecules can be connected, each by two vertices, with the n others. In the compound the n octahedric molecules will constitute the vertices and the centres of a continuous system of cubical spaces, and the $4n$ will find their place between each vertex and centre, as in figure 33.

By adding n other molecules of the second kind (for which there is a suitable space between each four molecules of the first substance) we shall obtain also the ratio $1 : 5$.

Moreover, n octahedric molecules can be arranged alternately in straight parallel lines with n others hexahedric or octohexahedric. Then, the six vertices of each octahedric molecule will be turned towards the neighbouring faces of six hexahedric or octohexahedric molecules, and can be united with them. If the n octahedric molecules were arranged alternately with n others not hexahedric nor octohexahedric, but having two opposite vertices, then either these last would turn their opposite vertices to the faces of two neighbouring octahedric molecules, or each of the octahedric molecules would turn its opposite vertices to the faces of the two neighbouring molecules of the other substance. In each case the combination is possible: and the compound may acquire a *laminar* structure.

These are the only definite ratios which I could find to be admissible for octahedric molecules.

VI. NUMBER OF MOLECULES WHICH CAN COMBINE WITH HEXAHEDRIC MOLECULES.

A hexahedric molecule, by reason of its six faces, and of the attractive elements corresponding, in the neighbouring nucleus, to each of them, is capable of combination with *six* molecules of another substance: and, by reason of its twelve edges, with *twelve* such molecules : and consequently, by reason both of edges and faces, with *eighteen* such molecules, whenever the other necessary conditions are fulfilled. The compounds in these three cases consist of *discrete* molecules.

Again, n hexahedric molecules can unite with $3n$ of another kind having two vertices diametrically opposite, whether octahedric, hexahedric, octohexahedric, icosahedric, or rhombo-dodeca-

hedric. In this case, each of the $3n$ molecules will have two opposite vertices turned to the faces of the two hexahedric molecules between which it lies: and thus the $3n$ molecules can be united, each by two vertices, with the n others.

This arrangement leaves a cubic space unoccupied between the vertices of every eight hexahedric molecules: and so there will remain room for n other octahedric or octohexahedric molecules. And, therefore, n hexahedric molecules can possibly combine with $4n$ others, if these are octahedric or octohexahedric.

Moreover, n hexahedric molecules can unite with n molecules of another kind having two vertices diametrically opposite. In this case, they will be arranged alternately in straight parallel lines. If the molecules of the second kind are octahedric, the compound will be equally connected in all directions, and its form will be traceable to the cube or the octahedron : and if they are not octahedric, the compound will be more firm in one direction than in others, and may have a *laminar* structure.

These are the only new ratios, which, in addition to the others already pointed out in the two preceding numbers, I find to be admissible for hexahedric molecules.

VII. NUMBER OF MOLECULES WHICH CAN COMBINE WITH OCTOHEXAHEDRIC MOLECULES.

An octohexahedric molecule, on account of its six square faces, has a special geometrical disposition to combine with *six* surrounding molecules of another substance, and, on account of its eight triangular faces, to combine with *eight* such molecules : and consequently, on account of all its faces, to combine with *fourteen* such molecules. It might also unite with *twelve* by its twelve edges : as also with *eighteen* by its edges and square faces : and, lastly, with *twenty-six* by its twelve edges and fourteen faces, if its volume is considerably greater than that of the molecules of the other substance. All these compounds would consist of *discrete* molecules.

We might think, that n octohexahedric molecules can unite with $3n$ of another kind having two opposite vertices, as we have just said with regard to hexahedric molecules : but we must remark, that by the natural arrangement of the octohexahedric

molecules their square faces will be parallel and opposite to each other, and n places regularly disposed will be formed between their triangular faces, of such a size that they will necessarily be occupied either by some of the octohexahedric molecules themselves, or by molecules of the other substance admitted into combination. Hence, if we add n new molecules of the first substance, which is octohexahedric, we shall have $2n$ molecules ready to combine with $3n$ others: if, on the contrary, we add n molecules of the second substance, we shall have n octohexahedric molecules ready to combine with $4n$ others. The compound will be *continuous*, and its form will be traceable to the cube or the octahedron.

Again, n octohexahedric molecules can unite with n others alternately arranged, and having at least two vertices diametrically opposite, and, consequently, not tetrahedric. They could be arranged also in parallel lines, so that one line of octohexahedric molecules would alternate with a line of molecules of another kind. In this case, the octohexahedric would have their square faces immediately opposite in all directions, and the molecules of the other substance would each find their place between eight triangular faces of eight neighbouring octohexahedric molecules.

If n octohexahedric molecules are so arranged as to form the centre and vertices of a continuous series of cubic spaces, then there will be room for $4n$ other molecules between each centre and vertex, as we said with regard to octahedric molecules; and, moreover, there will be room for $3n$ others between each two vertices, as we said with regard to cubic molecules; and thus we shall have n octohexahedric molecules ready to unite with $7n$ others having at least two vertices diametrically opposite.

These are the only ratios, besides those above noticed, which I find to be admissible with regard to octohexahedric molecules.

VIII. Number of Molecules which can combine with Icosahedric Molecules.

An icosahedric molecule can unite with *twenty* molecules of another kind by reason of its twenty faces: and with *thirty* on account of its thirty edges, if its volume be considerably greater than theirs: a hypothesis which seems scarcely admissible with regard to primitive molecules. The compound will be *discrete*.

Again, n icosahedric molecules can unite with $5n$ others of any kind. And, indeed, n icosahedrons can be arranged regularly so as to present to one another their vertices; they will then leave between them $5n$ octahedric spaces, which can be occupied by $5n$ molecules. The compound will be *continuous*.

These are the only definite ratios, besides those already noticed, which I find to be admissible for icosahedric molecules.

IX. NUMBER OF MOLECULES WHICH CAN COMBINE WITH DODECAHEDRO-PENTAGONAL MOLECULES.

A dodecahedro-pentagonal molecule can unite with *twelve* molecules of another kind by reason of its twelve faces, and with *thirty* by reason of its thirty edges, if its volume be considerably greater than theirs (which is scarcely probable with primitive molecules); and, lastly, with *forty-two* by reason both of the faces and of the edges. These compounds will be *discrete*.

Again, n dodecahedro-pentagonal molecules can unite with $6n$ molecules of another kind having two opposite vertices, and lying each between two parallel faces of two neighbouring dodecahedrons, by which they may be drawn into combination. The compound will be *continuous*.

I cannot find any other ratio besides these and the others already noticed, for dodecahedro-pentagonal molecules.

X. NUMBER OF MOLECULES WHICH CAN COMBINE WITH DODECAHEDRO-RHOMBIC MOLECULES.

A dodecahedro-rhombic molecule can unite with *twelve* of another kind by reason of its twelve faces, and with *twenty-four* by reason of its twenty-four edges: and, consequently, with *thirty-six* by reason both of its faces and of its edges. These compounds will be *discrete*.

Again, n dodecahedro-rhombic molecules can unite with $6n$ of another kind placed each between two parallel rhombic faces of two neighbouring dodecahedrons, to which they can be annexed by their opposite vertices. The compound will be *continuous*, and its form will be either tetrahedric, or octahedric, or rhombohedric (angle $60°$), with six equal faces, or any modification of these forms.

Moreover, n dodecahedro-rhombic molecules can unite with n others, each of these being intercepted between eight rhombic faces. In this case, the dodecahedrons are situated face to face in straight parallel lines, and the other molecules fill up the octahedric places which by the nature of the arrangement will remain between those lines. The molecules of the second kind must be either hexahedric, or octohexahedric, or tetrahedric: they cannot be octahedric; for, six vertices cannot meet eight surfaces.

These are the only definite ratios which I find to be admissible for dodecahedro-rhombic molecules.

XI. GENERAL RESULT.

It may be useful to give here a table of the results which we have obtained in the preceding pages. Such a table, in addition to other advantages, has that of showing how far the number of molecules, which can enter into combination, depends on their geometrical figures. The reader, however, will remember, that our inquiry has been limited to the combination of *two* substances, and those *primitive*. The combination of two substances which are not primitive, or that of three or more substances, depends on a greater number of conditions; and so, in this last case, the proportional numbers of molecules which are apt to combine cannot be determined generally without a great deal of additional work. Nevertheless, the results arrived at in the preceding pages are sufficient to give a first idea of the kind of reasoning which could be employed in making further inquiries, and offer the first ground for a fuller investigation of the subject. As to the correctness of the same results I have no doubt whatever: I have verified them, as far as I could, by tangible methods: nay, for this special purpose I constructed a host of solid regular polyhedrons of every kind, which, by being arranged successively in different ways, gave me the most sure means of ascertaining the reality of each case. Still, I am quite willing to admit that there may be other ratios of combining molecules which I have not been able to find out. And, therefore, the following table ought to be looked upon as provisional.

Table of the relative numbers in which molecules of two primitive substances admit of a possible combination.

Number and shape of Molecules of a first substance.	Number and shape of Molecules of a second substance.	Constitution of the compound.
n tetrahedric ... with	n, $2n$, $3n$ tetrahedric, or octohexahedric, or hexahedric, or octahedric	continuous.
$2n$, $3n$ id. ... „	n octahedric, or hexahedric, or dodecahedro-rhombic, or octohexahedric	continuous.
n, $2n$, $3n$ id. ... „	n tetrahedric or hexahedric............	continuous.
n id. ... „	$4n$, $6n$, $10n$ of any form	discrete.
n octahedric ... with	$2n$, $3n$ tetrahedric	continuous.
n id. ... „	n, $4n$, $5n$ octahedric, or hexahedric, or octohexahedric, or icosahedric, or rhombo-dodecahedric	continuous.
n id. ... „	$8n$, $12n$, $20n$ of any form.	discrete.
n hexahedric ... with	n, $2n$, $3n$ tetrahedric	continuous.
n id. ... „	$3n$, $4n$ octahedric, or octohexahedric	continuous.
n id. ... „	$3n$ hexahedric, or icosahedric, or dodecahedro-rhombic	continuous.
n id. ... „	$6n$, $12n$, $18n$ of any form..............	discrete.
n octohexahedric with	$2n$, $3n$ tetrahedric, or hexahedric ...	continuous.
$2n$ id. „	$3n$ octahedric, or octohexahedric, or icosahedric, or dodecahedro-rhombic.......................................	continuous.
n id. „	$4n$, $7n$ hexahedric, or octahedric, or octohexahedric, or icosahedric, or dodecahedro-rhombic	continuous.
n id. „	n of any form...	continuous.
n id. „	$6n$, $8n$, $12n$, $14n$, $18n$, $26n$ of any form	discrete.
n icosahedric ... with	$5n$ tetrahedric, hexahedric, &c.	continuous.
n id. ... „	$20n$, $30n$ of any form	discrete.
n dodecahedro-pentag. with	$6n$ of any form having two vertices diametrically opposite	continuous.
n id. „	$12n$, $30n$, $42n$ of any form.............	discrete.
n dodecahedro-rhombic with	n tetrahedric, or hexahedric, or octohexahedric	continuous.
n id. „	$6n$ of any form having two vertices diametrically opposite	continuous.
n id. „	$12n$, $24n$, $36n$ of any form	discrete.

BOOK XI.

ON THE SHAPE AND ARRANGEMENT
OF MOLECULES.

A FULL knowledge of the molecular constitution of bodies requires the solution of the following problems:

1. How many elements are in a molecule of the given substance?

2. What is the figure of the molecule?

3. How many nuclei are in it?

4. Which of the nuclei are attractive, and which repulsive?

5. What is the relative intensity of the active powers in the centre, in the nuclei, and in the envelope?

6. What is the relative length of the respective radii?

7. What is the distance between two neighbouring molecules?

8. How are the molecules arranged?

Though there are data in great numbers, from which we might determine many of the conditions implied in these problems, yet, unfortunately, the means which are at our disposal are, as yet, most inadequate to the heavy task. The forms of crystals, the atomic weights, the calorific capacities, the transmission, conduction, absorption, radiation, refraction, polarization, of light and heat, the heat of fusion and of ebullition, the degree of hardness, or softness, or elasticity, or tenacity, the laws and conditions of combination, the different degrees of affinity, the relations between mechanical and chemical work, and a great number of other analogous matters, would, if well known, furnish precious data for answering the questions proposed: but our present knowledge of them (to say nothing of the overwhelming complexity of the conditions on which such problems depend) is evidently too incomplete to allow the least hope of any but very partial

success. And this is one of the reasons why I apprehend that
molecular mechanics will have a long infancy before it acquires
its natural development. Let us, then, limit ourselves to a few
hints concerning the easiest of those problems, viz. the deter-
mination of the figure of primitive molecules. Our method of
solution is grounded on the preceding table of the numbers of
molecules, which may, by reason of their figure, come into com-
bination. The use of that table seems to be very obvious : but
in certain cases it may require some precautions, as will be
evident from the examples given below. In general, the greater
the numbers of the molecules between which the combination
takes place, the greater is the chance of failure. And, accord-
ingly, it will be but prudent on our part to base our inquiry
on examples of the greatest possible simplicity.

I. The molecule of oxygen.

The form of the molecules of oxygen can be determined by
the consideration of the following compounds :

|		Common formula.	New formula.
Hypochlorous acid	....	OCl	....	OCl^4
Chlorous acid	....	O^3Cl	....	O^3Cl^4
Hypochloric acid	....	O^4Cl	....	O^4Cl^4
Chloric acid	....	O^5Cl	....	O^5Cl^4
Perchloric acid	....	O^7Cl	....	O^7Cl^4.

We see that in hypochlorous acid four molecules of chlorine
unite around one of oxygen : and the term Cl^4, which remains
constantly the same in all the following compounds, shows that
the four molecules of chlorine united with the one of oxygen
constitute a permanent group or form, around which other mole-
cules of oxygen can be united in definite numbers. Hence, as-
suming the molecule or group OCl^4 of hypochlorous acid to be
a component of the other four compounds, their formulas may
be written thus :

| Chlorous acid	....	$OCl^4. O^2$,
Hypochloric acid	....	$OCl^4. O^3$,
Chloric acid	....	$OCl^4. O^4$,
Perchloric acid	....	$OCl^4. O^6$.

Now, from our table, the ratios $1:2$, $1:3$, $1:4$, $1:6$ require

that the molecule OCl^4 should be *discrete;* and, since it is formed by the union of four molecules of chlorine around one of oxygen, it cannot but be tetrahedric. But OCl^4 cannot be discrete and tetrahedric, unless the molecule O also is tetrahedric. Hence we conclude that the molecule of oxygen is a tetrahedron.

The same conclusion might be arrived at by the consideration of many other compounds. Phosphorus and oxygen combine in the proportions represented by the formulas (new style)

$$P^4O, \quad P^4O^3, \quad P^4O^5,$$
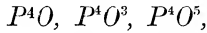
which may be written thus, $OP^4, \quad OP^4.O^2, \quad OP^4.O^3$, and explained as the above. Iodine and oxygen give the compounds (new style)

$$I^4O^5, \quad I^4O^7,$$
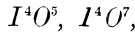
which may be written $OI^4.O^4, \quad OI^4.O^6$, and explained as above. These formulas, as well as the preceding, show clearly that oxygen is tetrahedric.

Let us remark, that, if we had taken for chlorine the usual equivalent 35·42, our formula of perchloric acid would have been Cl^2O^7, which cannot be explained by any reference to our general table. And the same would have been the case with periodic acid, had we not divided the equivalent of iodine by 2. On the contrary, the equivalents which we have adopted account most advantageously for the composition of the said acids, a composition that with the old equivalents would be quite unintelligible.

II. The molecule of nitrogen.

The form of the molecules of nitrogen can be determined by the consideration of the following compounds :

	Usual formula.	New formula.
Protoxide of nitrogen	NO	N^2O
Deutoxide of nitrogen	NO^2	N^2O^2
Nitric acid (anhydrous)	NO^5	N^2O^5
Iodide of nitrogen	NI^3	NI^6
Chloride of nitrogen	NCl^3	NCl^6
Ammoniacal gas	NH^3	NH^3
Sulphide of nitrogen	NS^3	NS^3
Cyanogen	NC	N^2C
Phosphide of nitrogen	N^2P	$N^2P^2.$

Here we have between nitrogen and other substances the ratios $2:2$, $2:1$, $1:3$, $1:6$; and our table shows that such ratios cannot coexist, unless the molecule of nitrogen is a tetrahedron. We shall admit, then, that nitrogen is tetrahedric.

The conclusion is confirmed by the formula of nitric acid, which gives between the molecules of nitrogen and of oxygen the ratio $2:5$. This ratio is not to be found in our table; but will be easily accounted for, if nitrogen is tetrahedric. For, the formula $N^2 O^5$ can be written $NO^4 . N . O$; the term NO^4 will then represent a molecule of nitrogen united by its four faces with four molecules of oxygen, and constituting with them one tetrahedric group, or form; which is again united with the two remaining molecules N and O in the ratio $1:2$. This explanation being, in my opinion, the only one which can account for the ratio $2:5$, would probably suffice of itself to prove that nitrogen is really tetrahedric.

III.   THE MOLECULE OF CARBON.

The form of the molecules of carbon may be determined by the consideration of the following compounds:

	Usual formula.	New formula.
Light carburetted hydrogen ...	CH^2	CH^4
Olefiant gas	$C^2 H^2$	CH^2
Oil of turpentine	$C^5 H^4$	$C^5 H^8$
Carbonic acid	CO^2	CO^2
Oxide of carbon	CO	CO
Cyanogen	CN	CN^2
Sulphide of carbon	CS^2	CS^4
Chloride of carbon	$C^4 Cl^2$	CCl^2
Bichloride of carbon	$C^4 Cl^4$	CCl^4
Perchloride of carbon	$C^4 Cl^6$	CCl^6
„ (another*)	$C^2 Cl^4$	CCl^8.

We have here between carbon and other substances the ratios $1:1$, $1:2$, $1:4$, $1:6$, $1:8$. Now, these ratios, according to our table, cannot be accounted for, unless carbon is octohexahedric. The ratio $5:8$ in the oil of turpentine may be very

* A volatile liquid found by M. Regnault.

easily explained, if we write the formula thus $CH^8 . C^4$; for, we shall have first a group of eight molecules of hydrogen annexed to one of carbon and constituting a cubic form; and then, since, according to our table, a cubic form can combine with four octo-hexahedric forms, we shall admit that the group CH^8 unites with C^4 or $4C$ in the ratio 1 : 4.

The octohexahedric form of carbon accounts very satisfactorily for the crystalline form of diamond, which is octahedric, sometimes modified by hexahedric faces, and for the crystalline form of graphite, which consists of hexagonal plates. In diamond the molecules turn their square faces to each other; whilst in graphite they turn to each other their triangular faces.

IV. The molecule of hydrogen.

The form of the molecules of hydrogen may be easily determined by the consideration of the following compounds:

	Usual formula.	New formula.
Ammoniacal gas	NH^3	NH^3
Marsh gas	CH^2	CH^4
Olefiant gas	C^2H^2	CH^2
Phosphuretted hydrogen	PH^3	P^2H^3
Arseniuretted hydrogen	AsH^3	As^2H^3
Hydrochloric acid	ClH	Cl^2H
Bisulphide of hydrogen	S^2H	S^2H.

Here we have between hydrogen and other substances the ratios 1 : 2, 2 : 1, 3 : 1, 3 : 2, 4 : 1. According to our table, the ratio 3 : 2 excludes the possibility of supposing hydrogen to be tetrahedric. Now, if hydrogen is not tetrahedric, the ratio 2 : 1 given by the formula CH^2 proves that hydrogen is hexahedric. And, in fact, carbon is octohexahedric, as we have shown: and an octohexahedric molecule, according to our table, cannot combine with two others, unless they are tetrahedric or hexahedric. Therefore, hydrogen, which cannot be supposed to be tetrahedric, must be assumed to be hexahedric.

V. The molecule of phosphorus.

The molecule of phosphorus may be known from the following compounds:

	Usual formula.	New formula.
Phosphide of nitrogen	PN^2	P^2N^2
Phosphorous acid	PO^3	P^4O^3
Phosphoric acid	PO^5	P^4O^5
Phosphuretted hydrogen	PH^3	P^2H^3
Chloride of phosphorus	PCl^3	PCl^3
Perchloride of phosphorus	PCl^5	PCl^5
Iodide of phosphorus	PI^2	PI^2
„ another	PI^3	PI^3.

We have here the ratios 1 : 1, 1 : 2, 1 : 3, 2 : 3, which by being compared with those of our table, sufficiently show that the molecule of phosphorus is octohexahedric.

But, how should we account for the ratio 1 : 5, which we find in perchloride of phosphorus, and which is not to be found amongst those given by our table for an octohexahedric molecule? This objection seems to contain a serious difficulty. Yet, since chlorine is tetrahedric, as we shall see hereafter, the difficulty can be got rid of. To understand the mode of arrangement of n molecules of phosphorus with $5n$ of chlorine, it is sufficient to consider that the formula of the compound may be written thus $PCl^3. Cl^2. P$; then we easily conceive, by reference to fig. 32, that the molecules $2Cl$ will occupy the centres O of the tetrahedrons, and the molecules P and the groups PCl^3 the vertices a and the centres O^1 of the octahedrons. This arrangement would tend to show that in the molecule of phosphorus the attractive nucleus which lies under the envelope is octahedric. But, as the difficulty arising from the formula PCl^5 might perhaps be solved by some other consideration, we shall not insist on this conclusion.

Phosphorus crystallizes in rhombic-dodecahedrons. This crystalline form is very easily accounted for by the union of fifteen octohexahedric molecules in one group. One of these molecules being in the centre of the group, eight other molecules would unite with it by its eight triangular faces, and constitute a cubic form, and the last six would meet this cubic form at its six faces. This arrangement would again suggest, that the above-mentioned nucleus of phosphorus is octahedric.

VI. The molecule of chlorine.

Let us consider the following compounds:

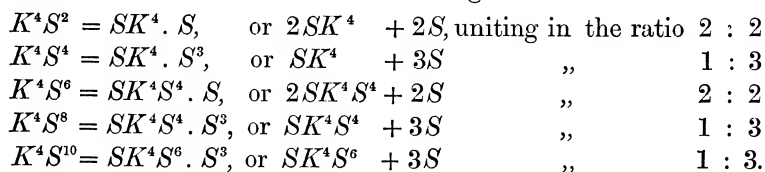|	Usual formula.	New formula.
Hydrochloric acid	ClH	Cl^2H
Chloride of sulphur	ClS^2	Cl^2S^2
„ another	ClS	Cl^2S
Chloride of potassium	ClK	Cl^2K^2
Chloride of sodium	$ClNa$	Cl^2Na^2
Chloride of copper	$ClCu^2$	Cl^2Cu^2
Chloride of mercury	$ClHg^2$	Cl^2Hg^2
Chloride of magnesium	$ClMg$	Cl^2Mg
Chloride of phosphorus	Cl^3P	Cl^3P
Chloride of arsenic	Cl^3As	Cl^3As.

Here we have between chlorine and other substances the ratios 2 : 1, 3 : 1, 2 : 2. From these ratios we may immediately infer, by a glance at our table, that chlorine is either tetrahedric or hexahedric. On the other hand, if chlorine were hexahedric, the formula Cl^2H would be totally inexplicable. For, as hydrogen is itself hexahedric, this formula would lead us to admit that $2n$ hexahedric molecules can combine with n others, also hexahedric. Now this cannot be admitted, as is evident from our table. Hence we must conclude that chlorine is not hexahedric. Accordingly, it must be tetrahedric.

VII. The molecule of sulphur.

The form of the molecules of sulphur can be determined by the consideration of the following compounds:

	Usual formula.	New formula.
Chloride of sulphur	S^2Cl	S^2Cl^2
Sulphide of nitrogen	S^3N	S^3N
Sulphide of copper	SCu^2	SCu^2
Sulphide of silver	SAg	SAg^2
Sulphide of carbon	S^2C	S^4C
Bisulphide of hydrogen	S^2H	S^2H
Monosulphide of potassium	K^2S^2	K^4S^2
Bisulphide of potassium	K^2S^4	K^4S^4
Tersulphide of potassium	K^2S^6	K^4S^6
Tetrasulphide of potassium	K^2S^8	K^4S^8
Pentasulphide of potassium	K^2S^{10}	K^4S^{10}.

In these compounds we have between sulphur and other substances the ratios 1 : 2, 2 : 1, 2 : 2, 3 : 1, 4 : 1, &c., from which we can conclude that sulphur is tetrahedric. And in fact the ratio 2 : 1 in the bisulphide of hydrogen (hydrogen being hexahedric) cannot be accounted for, unless sulphur is tetrahedric. The same conclusion can be drawn from the compounds of sulphur and potassium, sulphur and sodium, sulphur and oxygen, and others. Thus, in the sulphides of potassium the invariability of the term K^4 shows that the first group resulting from the mutual action of sulphur and potassium consists of four molecules of potassium annexed to one of sulphur, and that the formulas of those compounds are resolvable in the following manner:

$$K^4S^2 = SK^4 . S, \quad \text{or } 2SK^4 \quad + 2S, \text{uniting in the ratio } 2 : 2$$
$$K^4S^4 = SK^4 . S^3, \quad \text{or } SK^4 \quad + 3S \quad \text{,,} \qquad 1 : 3$$
$$K^4S^6 = SK^4S^4 . S, \quad \text{or } 2SK^4S^4 + 2S \quad \text{,,} \qquad 2 : 2$$
$$K^4S^8 = SK^4S^4 . S^3, \text{ or } SK^4S^4 \quad + 3S \quad \text{,,} \qquad 1 : 3$$
$$K^4S^{10}= SK^4S^6 . S^3, \text{ or } SK^4S^6 \quad + 3S \quad \text{,,} \qquad 1 : 3.$$

This interpretation of the proposed formulas is in perfect accordance with the results of our table, and is the only one which the same table allows us to admit. We conclude, accordingly, that, though the crystalline form of sulphur (of which we shall say something in the sequel) is not tetrahedric, the molecule of sulphur is a tetrahedron.

VIII. The molecule of arsenic.

Arsenic is of the same form as phosphorus. Many of its compounds with oxygen, hydrogen, chlorine, iodine, contain the same number of equivalents as the corresponding compounds of phosphorus. Let us consider the following compounds:

	Usual formula.	New formula.
Arsenic acid	AsO^5	As^4O^5,
Arsenious acid	AsO^3	As^4O^3,
Realgar	AsS^2	As^4S^4,
Orpiment	AsS^3	As^4S^6,
Sulpharsenic acid	AsS^5	As^4S^{10},
Arseniuretted hydrogen	AsH^3	As^2H^3,
Chloride of arsenic	$AsCl^3$	$AsCl^3$,
Iodide of arsenic	AsI^3	AsI^3.

The ratio $2 : 3$ in As^2H^3 suffices, according to our table, to show that the molecules of arsenic are octohexahedric. For, hydrogen being hexahedric, the formula As^2H^3 cannot be transformed into As^4H^6 so as to become similar to that of orpiment As^4S^6. Of this last, the tetrahedric form of sulphur affords us a natural interpretation, viz. $SAs^4S^4 . S$; whilst the expression As^4H^6, with hydrogen hexahedric, admits of no interpretation. Hence the ratio $2 : 3$ in arseniuretted hydrogen does not stand for $4 : 6$, but really represents two molecules of arsenic combining with three of hydrogen; and consequently it really shows that arsenic is octohexahedric.

IX. THE MOLECULE OF IODINE.

Let us consider the following compounds :

	Usual formula.	New formula.
Iodide of arsenic	I^3As	I^3As,
Iodide of phosphorus	I^3P	I^3P,
„ another	I^2P	I^2P,
Iodhydric acid	IH	I^2H.

These compounds suffice to determine the form of the molecules. For the ratio $3 : 1$ between iodine and arsenic, and iodine and phosphorus, shows that iodine is either tetrahedric or hexahedric. At the same time, the ratio $2 : 1$ between iodine and hydrogen shows that iodine is not hexahedric; for, two hexahedric molecules cannot combine with another hexahedric. And, therefore, we must conclude that the molecule of iodine is tetrahedric.

X. RELATION OF CRYSTALLINE FORMS TO THE SHAPE OF THE MOLECULES.

Many primitive bodies crystallize, and it seems quite natural to infer from the form of their crystal that of their molecules. Thus, we should say that the molecule of potassium is a hexahedron, that of mercury an octahedron, that of silver an octahedron or a hexahedron, since it crystallizes in both forms, that of iron a hexahedron, that of lead an octahedron, that of copper an octahedron, that of gold a hexahedron, and so on. Yet, what

has been said above on the molecule of phosphorus, viz. that, though the crystal is rhombo-dodecahedric, the molecule is octo-hexahedric, suggests the notion that it is not lawful, even in the case of primitive bodies, to assume that the form of the molecule is the form of the crystal. The molecule of sulphur, as we have proved by geometrico-chemical considerations, is tetrahedric; whilst the two crystalline forms of sulphur are very remote from the tetrahedron, the one being that of an octahedron with a rhombic base, the other that of an oblique prism with a rhombic base. In the case of phosphorus we found no difficulty; for it is evident that a rhombo-dodecahedric crystal can arise from the regular arrangement of fifteen octohexahedric molecules, and con-sequently also from other numbers of such molecules regularly arranged in an analogous way. But, in the case of sulphur, it seems very difficult to show how the passage can be made from the tetrahedric form of the molecules to the crystalline forms of the body.

Nevertheless, if we recall to our mind the natural mode of arrangement of a set of equal tetrahedric molecules, we shall find a solution to this difficulty. Tetrahedric molecules, as we have seen, arrange themselves in such a way as to form a series of tetrahe-drons alternately intercepting a series of octahedrons. Hence a body formed of tetrahedric molecules can crystallize in octahedrons, if the molecules which constitute the octahedrons are actually exerting on one another a greater cohesive power than those which constitute the tetrahedrons. The difficulty is thus half solved. Now, for the other half, i.e. with regard to the rhombic base, let us remark, that the molecule O' (fig. 32) which is in the centre of the octahedron ought to present its six edges to the respectively opposite edges of the six neighbouring molecules $a, a, a, ...$; and thus these seven molecules come closer to one another with their attractive elements than those which constitute the surrounding tetrahedrons, and consequently may not only cause the form of the crystal to be octahedric, but at the same time disturb the regularity of the crystalline form. The reason of this is, because the mutual action, and therefore the distance of relative equilibrium for these molecules, is not the same when they turn their edges to one another, as they do in the octa-

hedrons, and when the one turns its vertex to the face of the other, as in the tetrahedrons.

The same considerations show the possibility of oblique prismatic four-sided crystals being formed by regular tetrahedric molecules. Only, in the case of sulphur, it seems difficult to conceive how and why the principal axis of those prismatic crystals should make an angle $85^\circ 54'$ with the base, and why the obtuse angle of the base should be $90^\circ 32'$. These two angles are indeed widely different from those of the quadrilateral prisms resulting from a regular tetrahedric arrangement: but what we have just said of the difference between the actions of the molecules, according as they turn their edges to one another, or not, contains an implicit solution of this question also. For, the mass made up of such molecules must alter its regular form in proportion as the conjunction of the molecules in the octahedrons is more intimate than that of the molecules constituting the tetrahedrons.

These remarks may be applied to other crystals arising from tetrahedric molecules. For instance, iodine crystallizes in right rhombic prisms, which are the result of an alteration in the arrangement of the molecules dependent on the different degree of mutual conjunction according as they are related to the octahedric or to the tetrahedric groups. Of course, the molecules of iodine having a constitution different from that of sulphur, their arrangement is altered to a different extent, and gives rise to crystals of different form.

As for the dimorphism of sulphur, it evidently comes from the different state of the molecules of sulphur at different temperatures. For, according as the molecular envelopes are more or less expanded, the nuclei will be in a more or less favourable position for exerting their attractive powers; and consequently the intensity of the cohesive force (which chiefly depends on the relative position of the nuclei) will in one circumstance be sufficient to hold the tetrahedric groups united with the octahedric so as to form a prism, and in other circumstances will not be sufficient, and the crystal will then be octahedric.

We may here add a general remark about the crystals of compound bodies as compared with those of their components. From

the fact that two compounds assume similar or dissimilar crystalline forms, we are often tempted to infer that their components also have a similar or dissimilar crystalline form. Still this inference, to say the least, is not always true. We know of compounds having the same crystalline form, and yet resulting from components having dissimilar crystalline forms. Thus, sulphide of copper Cu^2S^2 and sulphide of mercury Hg^2S^2 yield different crystals, the first being a regular octahedron, and the second a rhombohedron of the angle 71^0. And yet mercury as well as copper crystallizes in regular octahedrons, and the two sulphides contain the same number of equivalents. Again, chloride of copper Cu^2Cl^2, chloride of silver Ag^2Cl^2, and chloride of potassium K^2Cl^2, crystallize, the first in tetrahedrons, the second in octahedrons, the third in cubes. Hence the second and third retain the form of silver and of potassium respectively, whilst the first, instead of retaining the form of copper, takes that of chlorine. This suffices to show that, even in the case of isomeric bodies, it is not safe to infer the form of the compounds from that of their components.

XI. REMARKS ON THE FORM AND ARRANGEMENT OF COMPOUND MOLECULES.

In the preceding pages we have considered the arrangement of the molecules in the most simple of compound substances. Let us now say a word on some of those compounds which involve in their constitution a very great number of equivalents of primitive substances. We give a few examples.

	Usual formula.	New formula.
Ether	C^4H^5O	$C^4H^{10}O$,
Alcohol	$C^4H^6O^2$	$C^4H^{12}O^2$,
Nitric ether	$C^4H^5O \cdot NO^5$	$C^4H^{10}ON^2O^5$,
Sulphurous ether	$C^4H^5O \cdot SO^2$	$C^4H^{10}OS^2O^2$,
Cane-sugar, crystallized	$C^{24}H^{22}O^{22}$	$C^{12}H^{22}O^{11}$,
Gum arabic	$C^{24}H^{22}O^{22}$	$C^{12}H^{22}O^{11}$,
Starch	$C^{24}H^{20}O^{20}$	$C^{12}H^{20}O^{10}$,
Albumen	$C^{400}H^{310}N^{50}O^{120}PS^2$	$C^{100}H^{155}N^{25}O^{30}PS.$

Though, of course, we do not presume to determine the real

form or molecular arrangement of any of these compounds, we can nevertheless throw a little light on the subject, by considering the peculiar aptitude inherent in every component for constituting compounds of a determinate form.

Oxygen being tetrahedric, the formula of *ether* may be explained by saying that a molecule of oxygen unites with four of carbon, and that the tetrahedric compound OC^4 unites to itself the ten molecules of hydrogen, the ratio 1 : 10 being one of those which we find in our table for tetrahedric molecules. And so ether would consist of *discrete* molecules OC^4H^{10}.

The formula of *alcohol* may be written thus $OC^4H^{10} + H + H + O$, and contains no apparent difficulty. The discrete molecules OC^4H^{10} (which are tetrahedric or octahedric, according as 4 out of the 10 molecules H are more or less prominent than the other 6) would occupy the centres O' (fig. 32) of the octahedric spaces, the molecules O the vertices $a, a, \ldots$ and the remaining molecules $2H$ the centres O of the tetrahedric spaces. This explanation is so natural and spontaneous, that we are tempted to say that it cannot but be true. The constitution of alcohol would then be *continuous*.

The formula of *nitric ether* may be written thus,

$$OC^4H^{10} + NO^4 + N + O.$$

This expression may be supposed to represent the arrangement of the different components. The groups NO^4 would occupy the vertices a (fig. 32), the groups OC^4H^{10} the centres of the octahedrons, and the remaining molecules O and N would occupy alternately the centres of the tetrahedrons.

The formula of *sulphurous ether* may be doubled and written in the following manner, $OC^4H^{10}S^4 + OC^4H^{10}O^4$. This expression would show that an equivalent of sulphurous ether consists of *two discrete molecules*, both tetrahedric.

The formula of *cane-sugar* $C^{12}H^{22}O^{11}$ is exactly identical with that of gum arabic. Hence two different arrangements of the same equivalents must be admitted as possible. The formula may be written thus, $OH^4C^4H^{10} + 2OC^4O^4H^4$; then the molecules of the first group would unite with those of the second in the ratio 1 : 2, the first being tetrahedric or octahedric, the second tetrahedric.

The same formula might also be written thus,

$$OH^4C^4H^{10} + 2OC^4H^4O^4,$$
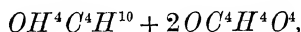
or thus, $OH^4C^4H^{10} + 2OH^4C^4O^4$; and the molecules of the first groups would again unite with those of the second in the ratio 1 : 2. These arrangements may be assumed to correspond to the two substances represented by the formula $C^{12}H^{22}O^{11}$.

The formula of *starch* may be written thus,

$$OC^4O^6 + 2OC^4H^{10} + O;$$
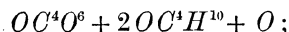
then the molecules of the first group would occupy the centres of the octahedrons (fig. 32), those of the second group the centres of the tetrahedrons, and the last O the vertices $a, a, \ldots$

The formula of albumen, in spite of its prodigious complexity, may be reduced to the following expression,

$$5(OC^4N^4H^{10} + OC^4O^4H^{10} + NC^4H^{10} + HC^8) + P + S;$$
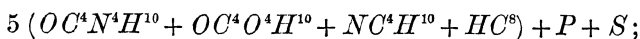
and thus the equivalent of albumen would be made up of two and twenty discrete groups or molecules, the arrangement of which we are not tempted to waste time about.

We shall end this Book by giving a table of the arrangements which, according to the preceding views, might best account for the constitution of some ordinary compounds. We give in the first column the name of the substance, in the following its formula, and in the three next the mode of arrangement of the groups or molecules in the compound. These last three columns are headed a, O, O' with reference to fig. 32, a indicating the vertices of the tetrahedric and of the octahedric spaces, O the centres of the tetrahedric, O' the centres of the octahedric.

Table of Molecular Arrangements accounting for the formation of compound substances.

Substance.	Formula.	Arrangement of molecules.		
		a	*O*	*O'*
Water	H^2O	O	$2H$	—
Binoxide of hydrogen ...	H^2O^2	O	$2H$	O
Carbonic oxide.............	C^2O^2	O	$2C$	O
Carbonic acid	CO^2	O	$2C$	—
Protoxide of nitrogen ...	N^2O	O	$2N$	—
Deutoxide of nitrogen ...	N^2O^2	O	$2N$	O
Nitric acid	N^2O^5	NO^4	$N+O$	—
Ammoniacal gas	NH^3	N	$2H$	H
Cyanogen	CN^2	C	$2N$	—
Hypochlorous acid	OCl^4	—	—	—
Chlorous acid	O^3Cl^4	OCl^4	$2O$	—
Chloric acid	$O\,Cl^4$	—	—	—
Perchloric acid............	O^7Cl^4	OCl^4O^4	$2O$	—
Hyposulphurous acid ...	S^2O	O	$2S$	—
Sulphurous acid	S^2O^2	O	$2S$	O
Hyposulphuric acid......	S^4O^5	—	—	—
„ sulphuretted	S^6O^5	OS^4O^4	$2S$	—
„ bisulphuretted	S^8O^5	—	—	—
„ trisulphuretted	$S^{10}O^5$	$OS^4O^4S^4$	$2S$	—
Sulphuric acid	S^2O^3	O	$2OS^4O^4$	O
Hydrosulphuric acid ...	S^2H^2	S	$2H$	S
Bisulphide of hydrogen ..	S^2H	—	$2S$	H
Arsenious acid	As^4O^3	OAs^4	$2O$	—
Arsenic acid...............	As^4O^5	—	—	—
Chloride of arsenic	$AsCl^3$	Cl	$2Cl$	As
Realgar....................	As^2S	S	$2As$	S
Sulpharsenic acid.........	As^2S^5	S	$2S$	SAs^4S^6
Hypophosphorous acid ..	OP^4	—	—	—
Phosphorous acid.........	O^3P^4	OP^4	$2O$	—
Phosphoric acid	O^5P^4	—	—	—
Phosphide of nitrogen ...	N^2P^2	N	$2P$	N
Chloride of phosphorus ..	PCl^3	Cl	$2Cl$	P
Sulphide of nitrogen......	NS^3	N	$2S$	S
Concentrated sulph. acid	$S^2O^3.H^2O$	$OS^4O^4H^4$	$2O$	O
Hypochloric acid	$Cl^4O^3+Cl^4O^5$	OCl^4	$2O$	OCl^4O^4
Crystallized iodic acid...	$I^4O^5+H^2O$	OI^4O^4	$2H$	O
Phosphoric acid monohy.	$P^4O^5+H^2O$	OP^4O^4	$2H$	O
„ bihydrated	$P^4O^5+2H^2O$	$OP^4O^4H^4$	$2O$	—
„ terhydrated	$P^4O^5+3H^2O$	—	$2O$	$OP^4O^4H^6$
Potash	OK	—	—	—
Peroxide of potassium...	O^3K^4	OK^4	$2O$	—
Monohydrated potash ...	$OK^4.H\,O$	OK^4	$2H$	O
Carbonate of potash......	$OK^4.CO^2$	OK^4	$2O$	C
Chlorate of potash	$OK^4.Cl^4O^5$	$OK^4O^4Cl^4$	$2O$	$OK^4O^4\,Cl^4$

In this table, hypochlorous acid, chloric acid, hyposulphuric acid, arsenic acid, hypophosphorous acid, phosphoric acid, potash, are shown to consist of *discrete* molecules OCl^4, OCl^4O^4, OS^4O^4, OAs^4O^4, OP^4, OP^4O^4, OK^4, and so also bisulphuretted hyposulphuric acid, whose formula gives the *discrete* molecule $OS^4O^4S^4$. And, for this reason, nothing is to be found in the last three columns regarding these substances.

I do not affirm, of course, that the results presented in this table contain the true interpretation of natural facts: I know that my conclusions are provisional as the table of the proportional numbers of combining molecules which I have given in Book x. My intention has been only to show how intimately the theory of combination is connected with that of the geometric form of molecules. This subject deserves a profound investigation, as it may lead to important practical results; and I am satisfied that the few hints above given will sooner or later be taken up by natural philosophers, and become in their hands a valuable instrument for the promotion of science.

BOOK XII.

ON THE MASSES, DISTANCES, AND POWERS OF MOLECULES.

THOUGH I am not prepared to solve those problems which I have mentioned at the beginning of the preceding Book, I think that it will not be improper to say a few words on the masses of molecules, their distances in different bodies, and their active powers. I shall say very little : and everyone who knows the present state of molecular science will excuse me for not saying more.

I. ABSOLUTE MASSES OF MOLECULES.

The absolute mass of a molecule is the number of simple elements of which the molecule consists. Chemical equivalents, as we have above observed, do not express absolute, but only relative molecular masses. Thus, for hydrogen and oxygen we have the proportion $H : O :: 1 : 16$; whence $O = 16H$; and therefore we cannot know the absolute mass O without knowing the absolute mass H.

Now, since hydrogen is hexahedric, and must have at least a centre R, a nucleus $6A$ and an envelope $8R'$, as we have proved (Book VI. Prop. III.), the least possible mass of the molecule of hydrogen is 15. Hence the least possible mass of the molecule of oxygen is $15 \times 16 = 240$. So also the least possible mass of carbon is $15 \times 12 = 180$, of iron $15 \times 28 = 420$, of mercury $15 \times 100 = 1500$; and so on.

And, if the molecule of hydrogen had more than one nucleus, if e.g. its formula were $m = A + 8R + 6A' + 8R'$, then, its absolute mass being $= 23$, the mass of oxygen would be 368, that of carbon 276, that of iron 644, that of mercury 2300; and so on.

All these numbers are so great, that, were it possible, we would endeavour to make them smaller: and yet, the real numbers may be even greater.

As the absolute mass of each molecule of a given substance

is the product of its equivalent by a constant factor, i.e. by the absolute mass of a hydrogen molecule, and as such a product represents a multitude of elements, and therefore cannot be fractional, one might think that the determination of the absolute mass of a molecule may be reduced to a problem of arithmetic, and can be easily effected by finding the factor which transforms all the equivalents into whole numbers. But, as the equivalents themselves are known but approximately, the thing is impracticable. On the other hand, were we able to find out the true number of elements contained in any molecule, so as to know with certainty that oxygen e.g. contains 240 and mercury 1500 of them, what could we do with such numbers? It is evident that a system of 240 material points transcends in complexity every means at our disposal for the determination of its dynamical conditions and properties. Here then we find an impassable barrier, and nothing remains for us, when we have come to these *real* pillars of Hercules, but to humbly admire the infinite wisdom of the Creator in the least of his works, and to meditate that inspired sentence: " He hath made all things good in their time, and hath delivered the world to their consideration, so that man cannot find out the work which God hath made from the beginning to the end*."

II.   MOLECULAR DISTANCES.

The data, from which we can form an idea of molecular distances, are much of the same value as those from which the astronomers infer the distance of the stars from the earth. They have grounds for saying that the distance of any star from us is at least 2000 times greater than that of the earth from the sun ; but they cannot guarantee that it is not immensely greater : and we, on our own side, can prove that the distance of two neighbouring molecules in a body is not greater than 0·0000036 of a millimetre, though we cannot assume that it is not much smaller.

Several metals, as nickel, cobalt, iron, are reduced from their oxides at a very low temperature by means of a current of hydrogen gas. Each particle of metal slowly evolving its oxygen forms a powder which may be considered as composed of primitive

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* Ecclesiastes iii. 11.

molecules, or ultimate atoms, as they are called. They are in every case less than $\dfrac{1}{100,000,000}$ of an inch in diameter*. In other words, we can state that 4,000,000 molecules of iron would fill up only the length of a millimetre, or even less. From this fact we can easily infer what is the greatest distance of two neighbouring molecules of any known body.

Let us conceive two equal volumes, the first of iron, the second of hydrogen existing under the ordinary barometric pressure and at the temperature $0^\circ C$. The numbers of the molecules will be proportional directly to the densities of the substances, and inversely to the masses of the molecules. For, let α and α' be the absolute masses of two molecules, the one of hydrogen, the other of iron, n^3 and n'^3 the numbers of molecules of the two substances filling equal cubic volumes V, and ρ and ρ' the densities of the fluid and of the metal. The mass of hydrogen contained in the volume V will be $V\rho = \alpha n^3$; and the mass of iron contained in an equal volume will be $V\rho' = \alpha'n'^3$. Therefore

$$n^3 \; : \; n'^3 \; :: \; \frac{\rho}{\rho'} \; : \; \frac{\alpha}{\alpha'},$$

whence

$$n = n' \sqrt[3]{\frac{\rho}{\rho'} \cdot \frac{\alpha'}{\alpha}}.$$

Now, since the absolute molecular masses of different substances are proportional to their chemical equivalents, we have here

$$\frac{\alpha'}{\alpha} = \frac{28}{1};$$

and on the other hand, if the density of hydrogen is chosen as a unit, we shall have also

$$\frac{\rho}{\rho'} = \frac{1}{80129};$$

hence, by substituting these numbers, and making $n' = 4,000,000$ according to the preceding statement, we shall have for the number of molecules of hydrogen in the linear millimetre

$$n = 4,000,000 \sqrt[3]{\frac{28}{80129}} = 281,740.$$

Since the ratio $\dfrac{\alpha'}{\alpha}$ in our preceding equation is the ratio of

* Golding Bird, *Elements of Natural Philosophy*, n. 5.

the chemical equivalents of the two substances that are compared, we can take $\alpha' = 1$ as the equivalent of hydrogen, and $\rho' = 1$ as its density, and substitute 281,740 for n'; and so we shall have

$$n = 281{,}740 \sqrt[3]{\frac{\rho}{\alpha}}$$

for the number of molecules of any substance which are required to fill up the length of a millimetre, α being the chemical equivalent, and ρ the density (related to hydrogen) of that substance.

The distance d between two adjacent molecules, the millimetre being taken as the unit of distance, will be

$$d = \frac{1}{n} = \frac{1}{281{,}740} \sqrt[3]{\frac{\alpha}{\rho}}.$$

To enable the reader to form a comparative judgment on the distances of molecules in different bodies, I have prepared the following table, in which I have added also the number of molecules of each substance contained in the length of a red luminous wave : this addition will perhaps be found of great use in many questions of optics, as we already observed in Book VIII. when speaking of the density of luminiferous æther.

Table of Molecular Distances, &c.

Substance.	Equi-valent.	Density.	Molecular dis-tances in fractions of millimetre.	Number of molecules in 1 millim.	in a red wave.
Hydrogen	1·00	1·	0·0000035494	281,740	181
Oxygen	16·00	16·	0·0000035494	281,740	181
Nitrogen	14·00	14·	0·0000035494	281,740	181
Carbon (vapour) ...	12·00	12·	0·0000035494	261,740	181
Chlorine	17·72	35·44	0·0000028171	354,970	229
Sulphur (vapour) ...	16·00	96·00	0·0000019533	511,956	330
Bromine (vapour)...	39·13	77·98	0·0000028171	354,970	229
Iodine (vapour)......	62·66	126·00	0 0000028171	354,970	229
Phosphorus (vapour)	16·00	64·	0·0000022359	447,233	288
Arsenic (vapour) ..	37·50	150·	0 0000023359	447,233	288
Mercury (vapour)...	100·00	100·	0·0000035399	282,490	182
Sulphur (solid)	16·00	22,603·00	0·0000003250	3,076,970	1985
Chlorine (liquid) ...	17·72	14,784·00	0·0000003770	2,652,530	1711
Bromine (liquid) ...	39·13	33,002·	0·0000003757	2,661,900	1717
Iodine	62·66	55,004·	0 0000003707	2,697,635	1740
Phosphorus	16·00	20,334·	0·0000002601	3,844,880	2481
Arsenic	37·50	64,449·	0·0000002963	3,374,760	2127

Substance.	Equiva-lent.	Density.	Molecular dis-tances in fractions of millimetre.	Number of molecules	
				in 1 millim.	in a red wave.
Diamond	12·00	39,258·	0·0000002396	4,182,447	2700
Potassium............	19.60	9,617·3	0·0000004500	2,222,190	1434
Sodium	11·49	10,778·5	0·0000003626	2,758,002	1779
Iron	28·00	80,129·	0·0000002500	4,000,000	2580
Cobalt	29·52	94,507·	0·0000002408	4,152,400	2679
Nickel	29·57	97,842·	0·0000002382	4,198,300	2708
Zinc	32·53	76,282·	0·0000002672	3,743,070	2415
Cadmium	55·74	96,732·	0·0000002953	3,385,715	2184
Tin	58·82	78,417·	0·0000003225	3,100,834	2000
Lead	103.56	126,214·	0·0000003323	3,009,454	1941
Manganese	27·57	88,948·	0·0000002402	4,163,090	2686
Bismuth	106·40	109,203·	0·0000003518	2,841,955	1833
Antimony............	64·52	74,625·	0 0000003381	2,957,420	1908
Copper	31·65	99,707·	0·0000002438	4,102,353	2647
Mercury	100·00	151,186·	0·0000003093	3,233,600	2086
Silver	54.00	116,452·	0·0000002747	3,639,990	2348
Gold	98 22	214,115·	0·0000002737	3,653,143	2357
Platinum	98.56	245,363·	0·0000002619	3,818,300	2462

III. MOLECULAR ACTIONS.

The intensity of mutual action between the molecules of some substances can be determined with sufficient approximation, when their distance is known.

Wires of different metals are capable of resisting different tractions, i.e. break under the traction of different weights. M. Regnault gives for wires of two millimetres in diameter the following data:

> Iron wire breaks for 250 kilogrammes,
> Copper   „   137   „
> Platinum   „   125   „
> Silver   „   85   „
> Gold   „   68   „
> Zinc   „   50   „
> Tin   „   16   „
> Lead   „   12   „

These wires having a diameter $= 2$ millim., the area of their circular section will be $\pi \times 1 = 3\cdot14159$. Hence, if the number of

molecules occupying one square millimetre is n^2, the number of
molecules occupying that circular section will be $x = n^2\pi$. If then
x molecules of the iron wire by their united actions are capable
of resisting traction and preventing the fracture of the same wire
till the weight has become $= 250$ kil., the greatest attractive or
cohesive action of each molecule on each neighbouring molecule
will be $\dfrac{250 \text{ kil.}}{n^2\pi}$; and the same may be said proportionately of
copper, &c.; so that, the weight for which the wire breaks being
called P, the greatest cohesive exertion of each molecule will be
in general

$$\frac{P}{n^2\pi}.$$

As for gases, we know that their tendency to expand is
measured by the pressure by which such a tendency is neutralized.
Hence, when a gas is in equilibrium under the barometric
pressure $0^{m.}76$, the total exertion of the molecules which press
the area of a square millimetre will be equal to the weight of
a mercurial column having 1 millim. of base and 760 millim. of
height.   And, since 1 cubic decimetre (1,000,000 cubic milli-
metres) of water weighs 1 kilogramme, and the density of
mercury is $13\cdot596$, hence 1,000,000 cubic millimetres of mercury
weigh $13^{\text{Kil.}}\cdot596$; and consequently 760 cubic millim. of mercury
will weigh

$$\frac{760 \times 13^{\text{Kil.}}\cdot596}{1,000,000} = 0^{\text{Kil.}}\cdot010333.$$

Let then n^2 be the number of gaseous molecules which are
contained in a square millimetre; their united effort against
pressure will be measured by the weight $0^{\text{Kil.}}\cdot010333$, by which
they are kept in equilibrium: and the action of each molecule
will be

$$\frac{0^{\text{Kil.}}\cdot010333}{n^2}.$$

If d is the distance of two neighbouring molecules, since we have
$n^2 d^2 = 1$, the preceding expression may be changed into

$$0^{\text{Kil.}}\cdot010333 \times d^2;$$

from which we infer that, *when two gases are under equal pressure, their molecular actions are directly proportional to the squares of the molecular distances.*

Taking the values of n from the preceding table, we find for the molecular actions of some gases the following results:

Substance.	Number of molecules in a square millimetre.	Amount of repulsion from molecule to molecule.
		Kil.
Hydrogen	79,337,403,000	0·00000000000013017
Oxygen	79,337,403,000	0·00000000000013017
Nitrogen	79,337,403,000	0·00000000000013017
Carbon (vapour)......	79,337,403,000	0·00000000000013017
Sulphur ,, ...	262,099,000,000	0·00000000000003942
Chlorine ,, ...	126,038,000,000	0·00000000000008200
Bromine ,, ...	126,038,000,000	0·00000000000008200
Iodine ,, ...	126,038,000,000	0·00000000000008200
Phosphorus ,, ...	200,017,300,000	0·00000000000005165
Arsenic ,, ...	200,017,300,000	0·00000000000005165
Mercury ,, ...	79,800,620,000	0·00000000000012947

It is to be observed that, although some substances, as hydrogen, oxygen, nitrogen, &c., exert equal repulsive powers from molecule to molecule for equal molecular distances, it by no means follows that they are endowed with equal absolute powers. This remark is important. For the distance of two molecules is measured from the centre of the one to the centre of the other: whilst, on the contrary, the action of the one upon the other depends chiefly on the distance of the elements which constitute the molecular envelopes. These elements can be more or less distant, according as the molecular radii in different substances are smaller or greater, the distance of the molecules themselves being the same. Thus, these equal actions proceed from different powers exerting themselves under different conditions.

With regard to solids, limiting ourselves to the molecular actions of the eight above-named metals, we shall find the following values:

Substance.	Number of molecules in the square millimetre.	Molecular resistance developed by traction.
		Kil.
Iron	16,000,000,000,000	0·00000000000497356
Copper......... ...	16,829,300,000,000	0·0000000000259121
Platinum	14,579,590,000,000	0·00000000000272910
Silver	13,249,533,000,000	0·00000000000204205
Gold...............	13,284,150,000,000	0·00000000000162931
Zinc	14,010,506,000,000	0·00000000000113595
Tin	9,615,182,000,000	0·00000000000052967
Lead...............	9,056,816,000,000	0·00000000000042175

I do not give any table of the molecular actions of liquids, because this would require a series of delicate experiments which I have not the means of making with sufficient accuracy.

IV. LAW OF MOLECULAR ACTIONS IN THE EXPANSION
OF FLUIDS.

In a given mass of fluid, which by diminution of pressure is supposed to expand, and of which the volumes are inversely proportional to the pressures, according to Mariotte's law, the molecular actions are inversely proportional to the cubes of the distances. This is easily shown.

Let V be a cubic recipient containing n^3 molecules of gas, let d be the distance of the neighbouring molecules, p the pressure sustained by one face of the cube, ϖ the action of one molecule. We shall have

$$V = n^3 d^3, \quad p = n^2 \varpi.$$

Let V' be another cubic recipient containing the same number n^3 of molecules, let d' be their distance, p' the pressure sustained by one face of the cube, ϖ' the action of one molecule: we shall have also

$$V' = n^3 d'^3, \quad p' = n^2 \varpi'.$$

By Mariotte's law we have

$$V : V' :: p' : p;$$

therefore

$$\varpi : \varpi' :: p : p' :: d'^3 : d^3;$$

and, consequently, *the actions of neighbouring molecules, within the limits in which Mariotte's law is applicable, are for the same substance inversely proportional to the cubes of the molecular distances.*

This theorem is remarkable both on account of its simplicity, and because it proves the reality of a law which many modern philosophers* have surmised and even endeavoured to substitute for the general law of action of matter at molecular distances. Their conjecture is now a demonstrated truth, if applied to the molecules of an expansive fluid dilating according to Mariotte's law: but this is not a reason for concluding that the Newtonian law ought to cease to be true at molecular distances; for as the law of the inverse cubic distances is true, in the case just considered, for the actions of *molecules*, so also the Newtonian law remains true, in this case as in others, for the actions of the *simple elements of matter*, of which the molecules consist.

From this theorem and from other general notions given in the preceding Books, it follows that the molecules of a gas, when their distance is changed, suffer a change in their radii. And, indeed, the action of one molecule on another is the resultant of the actions of its centre, nuclei, and envelope: and each of these component actions is a function of the distance and of the radii. Now, let ρ, $\alpha\rho$, $\beta\rho$, ... be the molecular radii (α, β, ... being numerical fractions), $d = N\rho$ the molecular distance, v, v', v'', ... w the actions of each element of the centre, nuclei, and envelope respectively; and let $f, f', f'', \ldots$ be any indices of functions. The aforesaid component actions will be capable of being expressed by terms of the form

$$\frac{f(v, \alpha, \beta, \ldots N)}{d^2}, \quad \frac{f'(v', \alpha, \beta, \ldots N)}{d^2}, \quad \frac{f''(v'', \alpha, \beta, \ldots N)}{d^2}, \quad \ldots$$

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* See in *Cambridge Philosophical Transactions*, Vol. VII. part 1, a paper, where it is proved that, if the particles of an elastic medium act on each other according to an inverse power of the distance, that power must be greater than 2. This, of course, is an obvious truth. But it is inferred that, since it is hardly accordant with the simplicity of nature to suppose the power fractional, the simplest and therefore the most likely power is 3. "My investigation," says Mr Earnshaw (*Phil. Mag.* Vol. XXVII. p. 99), "led me to the same result." Our readers will have seen, that this result, when true, rests on more certain grounds than those alleged by these writers

whence we conclude, that their resultant ϖ will be of the form

$$\varpi = \frac{F\,(v,\ v',\ v''\ \dots\ \alpha,\ \beta,\ \dots\ N)}{d^2}\,.$$

If the distance $d = N\rho$ becomes $d' = N'\rho'$, the resultant ϖ' will be of the form

$$\varpi' = \frac{F\,(v,\ v',\ v'',\ \dots\ \alpha',\ \beta',\ \dots\ N')}{d'^2}\,;$$

and since $\varpi\ :\ \varpi'\ ::\ d'^3\ :\ d^3$, we shall find

$$\frac{F\,(v,\ v',\ v'',\ \dots\ \alpha,\ \beta,\ \dots\ N)}{F\,(v,\ v',\ v'',\ \dots\ \alpha',\ \beta',\ \dots\ N')} = \frac{d'}{d}\,.$$

This relation requires that, when d is changed into d', the numbers $\alpha,\ \beta,\ \dots\ N$, be changed into $\alpha',\ \beta',\ \dots\ N'$; or, in other words, a change of distance is accompanied by a change of molecular radii.

V. Molecular Powers as compared with terrestrial Gravitation.

A kilogramme is the weight of a cubic decimetre of water at its maximum of density. If we were able to ascertain the number of simple elements contained in a cubic decimetre of water, we should immediately find also the weight of one element at the surface of the earth. Now, let us observe that the number of molecules contained in a cubic millimetre of hydrogen under the usual pressure $0^{\text{m}\cdot}76$ is $(281740)^3 = 22,363,769,000,000,000$ at least. And the same will be the number of the molecules of oxygen contained in an equal volume. But, on the one hand, water contains 2 volumes of hydrogen and 1 of oxygen; therefore, a cubic millimetre of the mixture of oxygen and hydrogen will contain in hydrogen $\frac{2}{3}$ of the said number of molecules, and in oxygen $\frac{1}{3}$ of the same number. On the other hand, if we assume that the molecule of hydrogen is made up of 15 simple elements (this being, as we have seen above, the least number possible), that of oxygen will be made up of 240 simple elements, as we have already stated. Hence the least number of elements con-

tained in a cubic millimetre of the said mixture will be found by multiplying the number $(281740)^3$, first by $\frac{2}{3} \times 15 = 10$, and then by $\frac{1}{3} \times 240 = 80$, and by adding the products together. The result will be

$$2{,}012{,}739{,}210{,}000{,}000{,}000.$$

Hence the absolute mass of a cubic decimetre of the same mixture will be

$$2{,}012{,}739{,}210{,}000{,}000{,}000{,}000{,}000.$$

Now the density of this mixture at the usual barometric pressure is to that of water as 6 : 11112. Hence the last number multiplied by $\dfrac{11112}{6}$ will give for the absolute mass of a cubic decimetre of water

$$3{,}727{,}593{,}016{,}920{,}000{,}000{,}000{,}000{,}000.$$

Such is the least possible number of simple elements contained in a cubic decimetre of water, and weighing 1 kilogramme. And, since the weight is expressed by the action of gravity g multiplied by the mass, we shall have

$$g \times 3{,}727{,}593{,}016{,}920{,}000{,}000{,}000{,}000{,}000 = \overset{\text{Kil.}}{1} ;$$

and the weight of a single element would accordingly be

$$g = \overset{\text{Kil}}{0}{\cdot}00000000000000000000000000000268269.$$

By means of the relation just found we can express molecular actions in terms of terrestrial gravitation. Thus, the action between the neighbouring molecules of iron being, according to one of our preceding tables,

$$\overset{\text{Kil.}}{0}{\cdot}00000000000497356,$$

we may substitute for it this other expression

$$g \times 37275930169 \times 497356,$$

or nearly

$$g \times 185{,}377{,}350{,}000{,}000 ;$$

and similar expressions may be obtained for the molecular actions of other substances.

VI. Variation of Molecular Actions with the Temperature.

Let us consider molecular actions as influenced by heat. A fluid expansive mass, which at the temperature $0^\circ C$ ($32^\circ F$) and under the ordinary pressure $0^{m}\cdot 76$ occupies a volume V, when brought to the temperature t° will occupy a volume V' determined by the equation

$$V' = V(1 + kt^0),$$

k being the coefficient of expansion, which is different for different substances.

Let us suppose that, when the fluid mass has reached the temperature t^0, we augment the pressure so as to reduce it again to its primitive volume V; if the ordinary pressure $0^{m}\cdot 76$ is represented by p, the new pressure p', which is necessary for reducing the heated mass from the volume V' to the volume V, will be, according to Mariotte's law,

$$p' = p\frac{V'}{V} = p(1 + kt^0).$$

By this equation we can easily calculate the changes which any given degree of heat causes in the molecular actions of a fluid, at least between those limits, within which the coefficient k remains constant.

Let us take for example a mass of hydrogen, for which we have

$$k = 0\cdot 0036678.$$

Let ϖ and ϖ' be the pressures sustained by a molecule of the gas at the temperatures 0° and t^0 respectively, the volume V remaining unaltered. As $n^2\varpi = p$, and $n^2\varpi' = p'$, we shall have

$$\varpi' = \varpi(1 + 0\cdot 0036678 \times t^0).$$

Now for hydrogen we have found already (Book XII. § III.)

$$\overset{\text{Kil.}}{\varpi} = 0\cdot 00000000000013017;$$

therefore, by making successively $t = 1$, $= 2$, $= 3$, ... we shall obtain the following series of values:

Kil.

for $t = 0^0$ $\varpi' = 0.00000000000013017,$
$t = 1^0$ $\varpi' = 0.00000000000013529,$
$t = 2^0$ $\varpi' = 0.00000000000014040,$
$t = 3^0$ $\varpi' = 0.00000000000014552,$
$t = 4^0$ $\varpi' = 0.00000000000015064,$
$t = 5^0$ $\varpi' = 0.00000000000015576,$
$t = 6^0$ $\varpi' = 0.00000000000016088,$
$t = 7^0$ $\varpi' = 0.00000000000016600,$
$t = 8^0$ $\varpi' = 0.00000000000017112,$
$t = 9^0$ $\varpi' = 0.00000000000017623,$
$t = 10^0$ $\varpi' = 0.00000000000018135,$
$t = 20^0$ $\varpi' = 0.00000000000023247,$
$t = 40^0$ $\varpi' = 0.00000000000033477,$
$t = 60^0$ $\varpi' = 0.00000000000043707,$
$t = 80^0$ $\varpi' = 0.00000000000053937,$
$t = 100^0$ $\varpi' = 0.00000000000064167.$

We see by these values that each molecule of hydrogen in passing from the temperature 0^0C to the temperature 100^0C becomes at 100^0, capable of exerting a power about five times greater than that which it displays at 0^0; and this, too, *without any change of molecular distances.*

Now it is obvious, that two molecules, whilst remaining at the same distance, cannot act upon one another with different intensities, unless some change intervenes in their mechanical conditions, i.e. in the relative magnitude of the molecular radii, or, which comes to the same, in the amplitude of molecular vibrations. Hence it follows, that *heat modifies the radii of the molecules* that are heated. The radius of the molecular envelope becomes greater; for the increase of molecular repulsion cannot be accounted for, except by admitting that the envelopes (which are always repulsive) have come nearer to one another, while the centres of the molecules remain at the primitive distance. On the other hand, the more the repulsive envelopes expand, the more the nuclei become free to move towards the centre, as it has been shown in another place. So then, heat, or rather the cause which communicates heat to a molecule, has the capability of changing the mutual relations of the molecular radii, or the

amplitude of molecular vibrations. The cause, which communicates heat to a molecule, is, of course, another molecule already hot, and therefore subject to a certain kind of vibratory motion, which it communicates through the exertion of its ordinary powers.

The first cause of calorific motion is to be found in the very constitution of molecules, as we have seen (Book VII. Prop. VIII. and Book VI. Prop. VIII.), where we have shown that the molecules of any body whatever are always subject to vibratory motion. And this is the reason why we find no substance *absolutely* cold, or which could not be colder than it is. Cold, then, is nothing but a quantity of heat less than the quantity from which we start in counting the degrees of heat. There is no absolute cold, because there is no absolute molecular rest: and, when we say that a molecule is heated, we cannot mean that calorific motion is then first imparted to it, but only that the preexisting calorific motion is increased : and when we say that a molecule becomes cold, we do not mean that calorific motion is extinguished, but only that it is diminished.

Heat, by altering the relations which exist between the radii of a molecule, can produce a characteristic change in the constitution of it, by causing a *transposition* of nuclei. This transposition is the necessary condition of vaporization and volatilization, as we have already shown (Book IX. § IV). In order to explain how a fluid, by cooling, or by being subjected to pressure, or both, resumes the liquid state, we have only to suppress the condition of expansivity, viz. we have to give back to the nuclei the place which they had before the transposition. In fact, pressure tends to reduce the volume of the molecular envelopes : so also cooling allows the envelopes to contract ; and in both cases these envelopes will act with a new vigour on the nuclei, and recall them, so to say, to their natural places, and thus cause the transposition to cease.

Since the act of vaporization and of volatilization consists precisely in the transposition of the nuclei, it follows that the facility or difficulty of changing a body into vapour is measured by the facility or difficulty of producing the transposition of the nuclei. This facility or difficulty depends evidently on the nature, number, proportion, and arrangement of the simple elements of which the molecule is constituted.

When a liquid is changed into vapour, heat, according to the common expression, becomes *latent:* and when the vapour is again reduced to the liquid state, heat is emitted and becomes *sensible.* So also heat becomes sensible when a liquid solidifies, and becomes latent when a solid melts into a liquid. These facts may be very easily accounted for. Vaporization and liquefaction alter very materially the molecular constitution. The mechanical conditions on which calorific vibrations depend, are changed in the act of vaporization; and other conditions are brought in by nuclear transposition, under which the molecules must lay out in tension an amount of energy which otherwise would have been spent in vibration. And so also with regard to the melting of solids : heat, in this case also, is spent in tension of a certain kind, i.e. in keeping up a state of molecular independence by counteracting the natural tendency of the molecules towards cohesion.

Since heat is communicated from molecule to molecule through the envelopes, as we have proved in treating of the law of calorific capacities (Book IX. § V), we may perhaps say that *heat becomes latent, when the mutual action of the envelopes ceases to increase their vibratory motion,* or, in other words, when the molecules are already in such a state, that *the vibrations of the envelopes are ruled and controlled by the vibrations of the inner nuclei.* On the contrary, heat would become sensible, when the inner nuclei are again ruled in their vibrations by the vibrations of the envelope.

CONCLUSION.

And here I stop. My object in writing these pages has been to give a definite scientific form to the first elements of *Molecular Mechanics;* a science, which, if properly developed, cannot fail to lead to practical results of great moment. I have established the principles of this new science, and endeavoured to show by a few applications what use can be made of them in scientific research. The whole work presents, as it were, the first map of a world little known : I have found its poles, drawn its meridians and parallels, and marked some coasts of unexplored land. Towns and villages, rivers and lakes, and the rest, remain in blank. Still I hope that this outline of molecular science, such as it is, will not be com-

pletely useless.   My reason for so thinking is simply that this
first step of mine is calculated to call the attention of more com-
petent men to this most important subject.   I may be allowed to
add, that the truths which I have demonstrated (especially in the
first Books) will help natural philosophers to discard vulgar pre-
judices, and to distrust, more than they do, those reasonings of
which the premisses have only the sanction of popular belief.
Would that the time, which often is wasted in building elaborate
theories on the sandy ground of gratuitous assumptions, were em-
ployed as much as possible in building upon the rock of de-
monstrated truth.

THE END.

Plate 1.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Fig. 11

Fig. 12

Fig. 13

Fig. 14

Fig. 15

Plate II.

Fig. 16

Fig. 17

Fig. 18

Fig. 19

Fig. 20

Fig. 21

Fig. 22

Fig. 24

Fig. 23

Plate III.

Fig. 25

Fig. 26

Fig. 27

Fig. 28

Fig. 31

Fig. 33.

Fig. 32

Fig. 29

Fig. 30.

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