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# THESIS



PYROTECHNIC DEVICE RELIABILITY

by

Altan Özkil

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Thesis Advisor: Lyn R. Whitaker

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The Naval Weapons Support Center is planning to implement a bonus system to improve the reliability of pyrotechnic devices. The measure of effectiveness that they wish to use to determine how to award bonuses is the reliability of pyrotechnic devices. The data available to estimate this reliability is based on the current sampling inspection plan in which devices are tested in different environments. The models which include both dependence and independence assumptions between the outcomes of these tests are implemented and estimates of overall reliability along with 95 % lower confidence bound are obtained. The 95 % lower confidence bounds are found by bootstrapping. Using these estimates, models for making the decision to award bonuses are discussed and studied using Monte Carlo simulation .


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# Pyrotechnic Device Reliability

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## ABSTRACT

The Naval Weapons Support Center is planning to implement a bonus system to improve the reliability of pyrotechnic devices. The measure of effectiveness that they wish to use to determine how to award bonuses is the reliability of pyrotechnic devices. The data available to estimate this reliability is based on the current sampling inspection plan in which devices are tested in different environments. The models which include both dependence and independence assumptions between the outcomes of these tests are implemented and estimates of overall reliability along with 95 % lower confidence bound are obtained. The 95 % lower confidence bounds are found by bootstrapping. Using these estimates, models for making the decision to award bonuses are discussed and studied using Monte Carlo simulation .

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# I. INTRODUCTION

## A. BACKGROUND

Nations spend a lot of money to establish a strong defense network. It is essential that nations buy reliable weapons and ammunition from the contractors. To ensure reliability, contracts must include a lot acceptance sampling plan that specifies the minimal acceptable quality.

The Naval Weapons Support Center purchases pyrotechnic devices. Unless otherwise specified in the contract, the supplier is responsible to see that his devices meet all inspection requirements as specified. The inspection requirements are particular to characteristics of each type of device and are specified on the reference drawings and supplemental quality assurance provisions of the contract. Any testing that needs to be done on these devices is explained in this contract.

When nations buy weapons and ammunition from the same contractor, they would like the quality to improve over time. As contracts are now written, contractors need only to satisfy the requirements of the sampling inspection plan for lot acceptance. Under such contracts, contractors have no incentive to improve the quality of items they provide. For this reason, to improve quality, The Naval Weapons Support Center has decided to implement a bonus system. The contractor will be awarded a bonus if the result of the sampling inspection exceeds the minimum requirements for lot acceptance.

The Naval Weapons Support Center will begin to implement a bonus system for pyrotechnic devices in FY91. The data available to make the decision whether to award a bonus is based on the current sampling inspection plan. This plan is a series of destructive tests in different environments. The purpose of this thesis is to provide the

Naval Weapons Support Center with guidance for implementing a bonus system for pyrotechnic devices. Once implemented, the pyrotechnic bonus system will serve as a prototype for bonus systems for other devices.

## B. PYROTECHNIC DEVICE RELIABILITY

A pyrotechnic device is a chemical and grenade ammunition. There are three categories of pyrotechnic devices. The three categories are: **Aerial** display, **Surface** display and **Grenades** [Ref. 1: pp. 1-2]. Samples of the Pyrotechnic devices are exposed to various environments and then activated. The criteria for successful activation depends on the type of device :

1. Aerial display (Ground signals, flares, airburst simulators, sub signals, signal kits, etc.)

**"Successful activation means that the item will, after simulating user environment, successfully ....."**

- launch,
- have proper separation / signal ignition,
- reach desired altitude at correct angle,
- have proper parachute deployment,
- have proper display color,
- have proper display time,
- have no subsequent interference with next item.

2. Surface (Ground or Water) display (flare, hand-held signals, smoke / illume grenades, simulators, etc.)

**"Successful activation means that the item will, after simulating user environment, successfully ...."**

- have signal ignition with proper display,
- have proper display,
- have proper display time,

3. Grenades (fragmentation, defensive, white phosphors, etc.)

**"Successful activation means that the item will, after simulating user environment, successfully ...."**

- function (high order detonation following delay),
- have proper dissipation of payload,
- have completely consumed payload.

## C. PYROTECHNIC DEVICE RELIABILITY PROBLEM

Samples from any large lot of pyrotechnic devices submitted by a manufacturer must activate after exposure to different environments. These are;

1. Manufacturer Environment,
2. Temperature and Humidity Environment,
3. Vibration Environment,
4. Altitude Environment.

All items tested are subjected to the manufacturer environment. However items are only subjected to one of the three remaining environments : Temperature and Humidity, Vibration or Altitude.

The sampling plan consists of using four distinct samples from a lot that can be assumed (approximately) statistically independent. The items tested in each sample are also assumed to be independent. According to the sampling plan;

- **20** items are subjected to the Manufacturer Test,

- **20** items are subjected to both the Temperature and Humidity Test and Manufacturer Test,
- **32** items are subjected to both the Vibration Test and Manufacturer Test,
- **20** items are subjected to both Altitude Test and Manufacturer Test.

A total of **92** items are tested.

Acceptance criteria for each test are :

1. **Manufacturer Test** : Of the 20 items; if no more than 1 fails to activate, the lot passes.

2. **Joint Temperature and Humidity and Manufacturer Test** : Of the 20 items; if no more than 1 fails to activate, the lot passes.

3. **Joint Vibration and Manufacturer Test** : Of the 32 items; if no more than 2 fails to activate, the lot passes.

4. **Joint Altitude and Manufacturer Test** : Of the 20 items; if no more than 1 fails to activate, the lot passes.

The number of failures for these tests will be summarized by the vector :

$$( FOM, FOTH, FOV, FOA ) \qquad (1.2)$$

where

- **FOM** represents the number of failures after **the manufacturer** test,
- **FOTH** represents the number of failures after both **the temperature and humidity** and **manufacturer** tests,
- **FOV** represents the number of failures after both **the vibration** and **manufacturer** tests,
- **FOA** represents the number of failures after **the altitude** and **manufacturer** tests.

For example,

$$( 1, 1, 2, 1 ). \tag{1.2}$$

represents, 1 failure in manufacturer test, 1 failure in the joint temperature and humidity and manufacturer test, 2 failures in the joint vibration and manufacturer test, and 1 failure in the joint altitude and manufacturer test. It is also the maximum number of failures in each test that still leads to lot acceptance.

The marginal distributions of the number of failures in each of the above tests are modeled by **binomial** distributions. There are 24 possible realizations of the sampling inspection; ranging from the best case with no failures to activate, to the worst case with 1, 1, 2 and 1 devices failing to activate in these tests manufacturer, temperature and humidity, vibration and altitude respectively. These cases are tabulated in Table 1.

To award bonuses we need one measure of effectiveness for pyrotechnic devices that can be estimated from the available data. Ideally, this measure is the reliability of the device. However, because each of the tests are destructive, there is no one natural definition of reliability for these devices. To be on the conservative side, we define the reliability of a device to be the probability that the device will activate after exposure to all of the environments. It is not easy to estimate this reliability from the sampling plan data. This data is incomplete in the sense that we have limited information about the joint probability of activation after exposure to more than one environment. To try to compensate for this lack in the data, we will use models for the joint distribution of ( FOM, FOTH, FOV, FOA ) that specify particular types of dependence between the events that a device activates after exposure to different environments.

Table 1. POSSIBLE CASES

| CASE #'S | FOM | FOTH | FOV | FOA |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 1 |
| 3 | 0 | 0 | 1 | 0 |
| 4 | 0 | 1 | 0 | 0 |
| 5 | 1 | 0 | 0 | 0 |
| 6 | 0 | 0 | 1 | 1 |
| 7 | 0 | 1 | 0 | 1 |
| 8 | 0 | 1 | 1 | 0 |
| 9 | 1 | 0 | 0 | 1 |
| 10 | 1 | 0 | 1 | 0 |
| 11 | 1 | 1 | 0 | 0 |
| 12 | 0 | 0 | 2 | 0 |
| 13 | 0 | 1 | 1 | 1 |
| 14 | 1 | 0 | 1 | 1 |
| 15 | 1 | 1 | 0 | 1 |
| 16 | 1 | 1 | 1 | 0 |
| 17 | 0 | 0 | 2 | 1 |
| 18 | 0 | 1 | 2 | 0 |
| 19 | 1 | 0 | 2 | 0 |
| 20 | 1 | 1 | 1 | 1 |
| 21 | 0 | 1 | 2 | 1 |
| 22 | 1 | 0 | 2 | 1 |
| 23 | 1 | 1 | 2 | 0 |
| 24 | 1 | 1 | 2 | 1 |

Using these models and based on sampling plan data, estimates of the overall reliability along with lower confidence bounds are obtained. These will be used to implement the bonus system for pyrotechnic devices. We compute the maximum likelihood estimator ( **MLE** ) of the reliability by maximizing the appropriate likelihood; lower confidence bounds ( **LCB** ) are found by bootstrapping. The MLE's are computed under both independence and dependence assumptions. The estimation procedures assuming independence are described in Chapter II. In Chapter III we incorporate dependence by fitting a Log Linear Model to our data. The MLE's from Chapter II and Chapter III lead to inappropriate results for this for this problem; thus in Chapter IV we consider alternate and very conservative estimates of reliability. Using the estimates of Chapter IV, we investigate a sequential scheme for making the decision to award bonuses in Chapter V. The results of simulations are presented in Chapter VI. Finally, conclusions and recommendations are given in Chapter VII.

# II. THE ESTIMATION OF THE MAXIMUM LIKELIHOOD ESTIMATOR (MLE) OF THE RELIABILITY WITH INDEPENDENCE ASSUMPTION

## A. DEFINITIONS

We will say that a device **survives environment E**, if it is still potentially capable of activation after exposure to environment E. Let,

- $E_1$ be the device activates after exposure to Manufacturer environment,
- $E_2$ be the device activates after exposure to Temperature and Humidity environment,
- $E_3$ be the device activates after exposure to Vibration environment,
- $E_4$ be the device activates after exposure to Altitude environment.

We define the reliability of device as below,

$$R = P(E_1 \cap E_2 \cap E_3 \cap E_4). \tag{2.1}$$

In this formula, R means the probability that a device activates after exposure to four environments. We will estimate R for each of the 24 cases which lead to lot acceptance.

Let $Q_i = P(E_i)$ be the probability that device activates after exposure to environment i, for i = 1, 2, 3, 4. In the acceptance sampling plan several of the items must activate after exposure to a joint manufacturer and another environment. To avoid confusion we will denote tests 1 through 4 as the manufacturer test, the joint temperature humidity and manufacturer test, the joint vibration and manufacturer test and joint altitude and manufacturer test respectively.

Let

$$R_1 = P(E_1) = Q_1 \tag{2.2}$$

and let

$$R_i = P(E_1 \cap E_i) \,. \tag{2.3}$$

Here $R_i$ is the probability that a device survives test i for i = 1, 2, 3, 4. The simplest model is to assume that $E_1$, $E_2$, $E_3$, $E_4$ are independent. If we assume that $E_1, \ldots, E_4$ are independent then

$$R_i = Q_1 Q_i \tag{2.4}$$

for i = 2, 3, 4 and the reliability of device is,

$$R = Q_1 Q_2 Q_3 Q_4 \,. \tag{2.5}$$

## B. THE LIKELIHOOD EQUATION

Let

- $X_i$ be the number of devices that activate after test i,
- $n_i$ be the number of items given test i.

Then $X_i$ is binomial with parameters $R_i$ and $n_i$ for i=1, 2, 3, 4. Under the assumption of independence the joint likelihood function of observing $X_1 = x_1, \ldots, X_4 = x_4$ is

$$L(x_1, x_2, x_3, x_4 \mid R_1, R_2, R_3, R_4) = \prod_{i=1}^{4} \binom{n_i}{x_i} R_i^{x_i} (1 - R_i)^{n_i - x_i} \tag{2.6}$$

with constraints :

$$0 \leq R_1 \leq 1,$$

$$0 \leq R_2 \leq R_1,$$

$$0 \leq R_3 \leq R_1,$$

$$0 \leq R_4 \leq R_1.$$

Our aim is to maximize this likelihood function subject to the constraints that $(R_1, R_2, R_3, R_4) \in S$ where

$$S = \{(R_1, R_2, R_3, R_4): 0 \leq R_1 \leq 1,\ 0 \leq R_i \leq R_1 \quad i = 2, 3, 4\}.$$

From the equation (2.6), we see that maximizing L is equivalent to maximizing

$$l = \sum_{i=1}^{4} \{ (x_i \ln R_i) + (n_i - x_i) \ln(1 - R_i) \}, \tag{2.7}$$

where the constant multipliers $\binom{n_i}{x_i}$ for i = 1, 2, 3, 4 have been dropped ( because they do not effect the maximization procedure ) and the natural logarithm of L is taken.

We first show that $l$ is a concave function. To show that $l$ is a concave function, we can show - $l$ is a convex function. According to Theorem 3.3.6 [Ref. 2: p. 92], by looking at its Hessian matrix, we can learn whether function is convex or not. If its Hessian matrix is positive semi-definite at each point S then function $l$ is convex. To create the Hessian matrix, we must calculate partial derivatives of the function - $l$,

$$-\frac{\partial l}{\partial R_i} = -\frac{x_i}{R_i} + \frac{n_i - x_i}{1 - R_i} \tag{2.8}$$

$$- \frac{\partial^2 l}{\partial R_i^2} = \frac{x_i}{R_i^2} + \frac{n_i - x_i}{1 - R_i^2} \tag{2.9}$$

$$- \frac{\partial^2 l}{\partial R_i R_j} = 0 \qquad i \neq j. \tag{2.10}$$

Then, the determinant of the Hessian is:

$$| H | = \prod_{i=1}^{4} \left( \frac{x_i}{R_i^2} + \frac{n_i - x_i}{1 - R_i^2} \right). \tag{2.11}$$

Clearly we can see that for $0 < R_i < 1$ i = 1, 2, 3, 4, $| H |$ is always positive. Because - $l$ is continuous, this implies that - $l$ is a convex function on S. As a result of this, $l$ is a concave function.

We note that with the constraints on the probabilities $R_1, \ldots, R_4$, there does not in general exist a closed form solution to MLE. However, with only 24 realizations of $x_1, \ldots, x_4$ of interest, the estimated reliabilities for these 24 cases can be found with some rather tedious but straight-forward computations.

## C. COMPUTING THE MAXIMUM LIKELIHOOD ESTIMATORS AND LOWER CONFIDENCE BOUNDS

Because $l$ is concave over the convex set S, if the maximum occurs in the interior of S, it is a unique maximum and is given by

$$\hat{R}_i = \frac{x_i}{n_i} \tag{2.12}$$

for i = 1, 2, 3, 4. where

$$0 \le \hat{R}_1 \le 1 \quad , \; 0 \le \hat{R}_i \le \hat{R}_1$$

for i = 2, 3, 4.

In this case the MLE's for $Q_i$ i = 1, 2, 3, 4 are

$$\hat{Q}_1 = \frac{x_1}{n_1} \tag{2.13}$$

$$\hat{Q}_i = \frac{\frac{x_i}{n_i}}{\frac{x_1}{n_1}} \tag{2.14}$$

for i = 2, 3, 4. Finally we can estimate the reliability of the device as

$$\hat{R} = \hat{Q}_1 \hat{Q}_2 \hat{Q}_3 \hat{Q}_4 \, . \tag{2.15}$$

Table 2 summarizes the cases for which the MLE's can be found using (2.12) - (2.15). When the maximum falls on the boundary of S, there is no explicit expression for the MLE of ( $R_1$ , . . . , $R_4$ ) . These are the cases with the exception of ( 1 1 2 1 ) which have one failed item in the manufacturer test. This implies that $\hat{R}_1$ will be less than 1.0 . To find the MLE, we find the maximum of ( $R_1$ , . . , $R_4$ ) on each of the boundaries, compute $l$ for each of these and let the MLE be the one with the largest value of $l$ . It is clear that in most cases several of the boundaries can be eliminated from consideration, simplifying computation considerably.

Table 2. EASY AND HARD CASES

| EASY CASES | HARD CASES |
|---|---|
| ( 0 0 0 0 ) | ( 1 0 0 0 ) |
| ( 0 0 0 1 ) | ( 1 0 0 1 ) |
| ( 0 0 1 0 ) | ( 1 0 1 0 ) |
| ( 0 1 0 0 ) | ( 1 1 0 0 ) |
| ( 0 0 1 1 ) | ( 1 0 1 1 ) |
| ( 0 1 0 1 ) | ( 1 1 0 1 ) |
| ( 0 1 1 0 ) | ( 1 1 1 0 ) |
| ( 0 0 2 0 ) | ( 1 0 2 0 ) |
| ( 0 1 1 1 ) | ( 1 1 1 1 ) |
| ( 0 0 2 1 ) | ( 1 0 2 1 ) |
| ( 0 1 2 0 ) | ( 1 1 2 0 ) |
| ( 0 1 2 1 ) | |
| ( 1 1 2 1 ) | |

## D. EXAMPLES

### 1. Easy Case

In this example our failure vector is,

( 0 1 2 1 ).

According to this failure vector, we can write our likelihood equation by using equation (2.6) to get

$$L = R_1^{20}(1 - R_1)^0 R_2^{19}(1 - R_2)^1 R_3^{30}(1 - R_3)^2 R_4^{19}(1 - R_4)^1 \quad (2.16)$$

$$0 \leq R_1 \leq 1 \ , \quad 0 \leq R_i \leq R_1 \ ,$$

for i = 2, 3, 4. From the likelihood above;

$$\hat{R}_1 = \frac{x_1}{n_1} = \frac{20}{20} = 1.0000$$

$$\hat{R}_2 = \frac{x_2}{n_2} = \frac{19}{20} = 0.9500$$

$$\hat{R}_3 = \frac{x_3}{n_3} = \frac{30}{32} = 0.9375$$

$$\hat{R}_4 = \frac{x_4}{n_4} = \frac{19}{20} = 0.9500 \,. \quad (2.17)$$

Results imply that all $\hat{R}_i$'s for i = 2, 3, 4 are between 0.0 and $\hat{R}_1$, which means that constraints are met. Now we can estimate $Q_i$'s using equations (2.13) and (2.14).

$$\begin{aligned} Q_1 &= 1.0000 \\ Q_2 &= \frac{0.9500}{1} = 0.9500 \\ Q_3 &= \frac{0.9375}{1} = 0.9375 \\ Q_4 &= \frac{0.9500}{1} = 0.9500 \,. \end{aligned} \quad (2.18)$$

And finally, reliability of the device can be estimated by using equation (2.15)

$$R = (1.0000)(0.9500)(0.9375)(0.9500) = 0.84609375 . \quad (2.19)$$

### 2. Hard Case

In this example our failure vector is

$$( \; 1 \quad 0 \quad 2 \quad 1 \; )$$

According to this failure vector, we can write our likelihood equation by using equation (2.6) as

$$L = R_1^{19}(1 - R_1)^1 R_2^{20}(1 - R_2)^0 R_3^{30}(1 - R_3)^2 R_4^{19}(1 - R_4)^1 \quad (2.20)$$

$$0 \leq R_1 \leq 1 \;, \quad 0 \leq R_i \leq R_1$$

for i = 2, 3, 4. From the likelihood above;

$$\frac{x_1}{n_1} = \frac{19}{20} = 0.9500$$

$$\frac{x_2}{n_2} = \frac{20}{20} = 1.0000$$

$$\frac{x_3}{n_3} = \frac{30}{32} = 0.9375$$

$$\frac{x_4}{n_4} = \frac{19}{20} = 0.9500. \quad (2.21)$$

As you see from the above $\frac{x_2}{n_2} > \frac{x_1}{n_1}$ thus, the MLE does not lie in the interior of S. We begin by considering the boundary

$$R_1 = R_2 = R_3 = R_4 .$$

The likelihood equation on this boundary is,

$$L = R_1^{88} ( 1 - R_1 )^4 \tag{2.22}$$

$$0 \leq R_1 \leq 1 .$$

From the likelihood equation,

$$\hat{R}_1 = \hat{R}_2 = \hat{R}_3 = \hat{R}_4 = \frac{88}{92} = 0.9565 .$$

Then value of the likelihood with these estimated $\hat{R}_i$'s is

$$L = ( \frac{88}{92} )^{88} ( \frac{4}{92} )^4 = 0.714 \times 10^{-7} . \tag{2.23}$$

For the boundary

$$R_1 = R_2 = R_3 , \; R_4 \leq R_1 ,$$

the likelihood equation is,

$$L = R_1^{69} ( 1 - R_1 )^3 R_4^{19} ( 1 - R_4 )^1 \tag{2.24}$$

$$0 \leq R_1 \leq 1 , \quad 0 \leq R_4 \leq R_1 .$$

From the likelihood equation,

$$\hat{R}_1 = \hat{R}_2 = \hat{R}_3 = \frac{69}{72} = 0.9583 \qquad \hat{R}_4 = \frac{19}{20} = 0.9500 ,$$

and the value of the likelihood with these estimated $\hat{R}_i$'s is

$$L = \left( \frac{69}{72} \right)^{69} \left( \frac{3}{72} \right)^{3} \left( \frac{19}{20} \right)^{19} \left( \frac{1}{20} \right)^{1} = 0.723 \times 10^{-7} . \tag{2.25}$$

For the boundary

$$R_1 = R_2 = R_4 \ , \ R_3 \leq R_1$$

the likelihood equation is,

$$L = R_1^{58} ( 1 - R_1 )^2 R_3^{30} ( 1 - R_3 )^2 \tag{2.26}$$

$$0 \leq R_1 \leq 1 \qquad 0 \leq R_3 \leq R_1 .$$

From the likelihood equation,

$$\hat{R}_1 = \hat{R}_2 = \hat{R}_4 = \frac{58}{60} = 0.9666$$

$$\hat{R}_3 = \frac{30}{32} = 0.9375 .$$

Then value of the likelihood with these estimated $\hat{R}_i$'s is

$$L = \left( \frac{58}{60} \right)^{58} \left( \frac{2}{60} \right)^{2} \left( \frac{30}{32} \right)^{30} \left( \frac{2}{32} \right)^{2} = 0.876 \times 10^{-7} . \tag{2.27}$$

For the boundary

$$R_1 = R_3 = R_4 \ , \ R_2 \leq R_1 \ ,$$

the likelihood equation is,

$$L = R_1^{68}(1 - R_1)^4 R_2^{20} \tag{2.28}$$

$$0 \le R_1 \le 1 \quad 0 \le R_2 \le R_1 .$$

Clearly L is maximized on the boundary of the constraints (2.28), i.e. on another of the boundaries of S thus we can eleminate this case from consideration. For the boundary

$$R_1 = R_2 , \; R_3 \le R_1 , \; R_4 \le R_1 ,$$

the likelihood equation is,

$$L = R_1^{39}(1 - R_1)^1 R_3^{30}(1 - R_3)^2 R_4^{19}(1 - R_4)^1 \tag{2.29}$$

$$0 \le R_i \le 1 , 0 \le R_3 \le R_1 , 0 \le R_4 \le R_1 .$$

From the likelihood equation,

$$\hat{R}_1 = \hat{R}_2 = \frac{39}{40} = 0.9750 \quad \hat{R}_3 = \frac{30}{32} = 0.9375 \quad \hat{R}_4 = \frac{19}{20} = 0.9500 .$$

Then value of the likelihood with these estimated $\hat{R}_i$'s is

$$L = \left(\frac{39}{40}\right)^{39}\left(\frac{1}{40}\right)^1\left(\frac{30}{32}\right)^{30}\left(\frac{2}{32}\right)^2\left(\frac{19}{20}\right)^{19}\left(\frac{2}{20}\right)^1 = 0.990 \times 10^{-7} \tag{2.30}$$

Quick inspection reveals that the remaining boundaries can be eliminated from consideration.

After boundary analysis, we can see that equation (2.30) gives us the maximum likelihood value. Thus the MLE's for $R_1$, $R_2$, $R_3$, $R_4$ are

$$\hat{R}_1 = 0.9750$$

$$\hat{R}_2 = 0.9750$$

$$\hat{R}_3 = 0.9375$$

$$\hat{R}_4 = 0.9500 .$$

Now we can estimate $Q_i$'s with equations (2.13) and (2.14)

$$\begin{aligned} Q_1 &= 0.9750 \\ Q_2 &= \frac{0.9750}{0.9750} = 1.0000 \\ Q_3 &= \frac{0.9375}{0.9750} = 0.9615 \\ Q_4 &= \frac{0.9500}{0.9750} = 0.9744 . \end{aligned} \tag{2.31}$$

And finally reliability of the device can be estimated by using equation (2.15)

$$R = (0.9750)(1.0000)(0.9615)(0.9744) = 0.91346100 . \tag{2.32}$$

We computed **MLE's** with a FORTRAN program MLEA given in Appendix A. After 24 replications of this program, we can estimate $\hat{R}_i$'s for $i = 1, 2, 3, 4$ for each of the 24 outcomes of the sampling plan that lead to lot acceptance. These are given in Table 3, $\hat{R}$ is given in Table 4.

We compute lower confidence bounds for each possible case by bootstrapping. The bootstrap can be used to produce approximate confidence intervals in an automatic way. There are several ways to set approximate confidence intervals with bootstrapping. These are the percentile method, the standard method, the bias-corrected percentile method and the nonparametric method [Ref. 3 : pp. 67-70]. Let $\theta$ be an unknown parameter with estimator $\hat{\theta}$. To bootstrap, samples are generated using $\hat{\theta}$ in place of the unknown parameter $\theta$. For each of these samples an estimator $\hat{\theta}^{*}$ is computed. We define $\hat{G}(s)$ to be the parametric bootstrap cumulative distribution function of $\hat{\theta}^{*}$, i.e. $\hat{G}(s)$ is the empirical distribution of the $\hat{\theta}^{*}$'s. All methods mentioned above use percentiles of $\hat{G}$ to define the confidence interval. They differ in which percentiles are used. The percentile method was used in our calculations.

If we use the notation $\theta(\alpha)$ for the level $\alpha$ endpoint of one sided lower confidence interval for $\theta$, then

$$P[\,\theta(\alpha) \leq \theta\,] = \alpha\,. \tag{2.33}$$

An estimate of $\theta(\alpha)$ from the bootstrap cumulative distribution is given by

$$\hat{\theta}(\alpha) \equiv \hat{G}^{-1}(\alpha)\,. \tag{2.34}$$

To get LCB's for R, for each of the 24 realizations of the sampling plan , we generate bootstrap samples of random failure vectors, in which failures come from independent binomial distribution with parameters ( $n_i$ , $\hat{R}_i$ ) for i = 1, 2, 3, 4. This is done using the FORTRAN program RANVEC [Ref. 4] in Appendix B. We generate 5000 failure vectors for each case. And then we estimate $\hat{R}$ 's from each of the 5000 failure vectors by the means of the program MLEA. The next step is to compute the order statistics of $\hat{R}$ 's from $\hat{R}_1$ to $\hat{R}_{5000}$. We get the parametric bootstrap cumulative distribution function ( i.e. the emprical distribution of $\hat{R}_1, \ldots, \hat{R}_{5000}$ ) with this computation. This is done using the FORTRAN program SORT in Appendix C.

Finally we can get the 95 % lower confidence bound using equation (2.34) from this routine. Reliabilities and 95 % lower confidence bounds are listed in Table 4. Results in Table 4 are given in descending order and $\hat{R}$'s and LCB's are not ordered as we expected them to be. For example, the failure vector that has the maximum number of failure in each test is in the middle of the table with respect to $\hat{R}$. It has also a bigger 95 % LCB than the failure vector that has a total of 3 failed items in each of the joint tests. These results are counter-intuitive because we expect that more failures indicate a lower overall reliability. This is reasonable because it is likely that a device that is poorly constructed is more likely to fail in any of the four environments. From the results in Table 4, it is clear that an attempt to model dependence must be made in order to get believable estimates of R.

Table 3. TEST PROBABILITIES ( A )

| CASE | $\hat{R}_1$ | $\hat{R}_2$ | $\hat{R}_3$ | $\hat{R}_4$ |
|---|---|---|---|---|
| ( 0 0 0 0 ) | 1.0000000 | 1.0000000 | 1.0000000 | 1.0000000 |
| ( 0 0 0 1 ) | 1.0000000 | 1.0000000 | 1.0000000 | 0.9500000 |
| ( 0 0 1 0 ) | 1.0000000 | 1.0000000 | 0.9687500 | 0.1.00000 |
| ( 0 1 0 0 ) | 1.0000000 | 0.9500000 | 1.0000000 | 1.0000000 |
| ( 1 0 0 0 ) | 0.9891304 | 0.9891304 | 0.9891304 | 0.9891304 |
| ( 0 0 1 1 ) | 1.0000000 | 1.0000000 | 0.9687500 | 0.9500000 |
| ( 0 1 0 1 ) | 1.0000000 | 0.9500000 | 1.0000000 | 0.9500000 |
| ( 0 1 1 0 ) | 1.0000000 | 0.9500000 | 1.9687500 | 1.0000000 |
| ( 1 0 0 1 ) | 0.9861111 | 0.9861111 | 0.9861111 | 0.9500000 |
| ( 1 0 1 0 ) | 0.9833333 | 0.9833333 | 0.9687500 | 0.9833333 |
| ( 1 1 0 0 ) | 0.9861111 | 0.9500000 | 0.9861111 | 0.9861111 |
| ( 0 0 2 0 ) | 1.0000000 | 1.0000000 | 0.9375000 | 1.0000000 |
| ( 0 1 1 1 ) | 1.0000000 | 0.9500000 | 0.9687500 | 0.9500000 |
| ( 1 0 1 1 ) | 0.9750000 | 0.9750000 | 0.9687500 | 0.9500000 |
| ( 1 1 0 1 ) | 0.9807692 | 0.9500000 | 0.9807692 | 0.9500000 |
| ( 1 1 1 0 ) | 0.9750000 | 0.9500000 | 0.9687500 | 0.9750000 |
| ( 0 0 2 1 ) | 1.0000000 | 1.0000000 | 0.9375000 | 0.9500000 |
| ( 0 1 2 0 ) | 1.0000000 | 0.9500000 | 0.9375000 | 1.0000000 |
| ( 1 0 2 0 ) | 0.9833333 | 0.9833333 | 0.9375000 | 0.9833333 |
| ( 1 1 1 1 ) | 0.9615384 | 0.9500000 | 0.9615384 | 0.9500000 |
| ( 0 1 2 1 ) | 1.0000000 | 0.9500000 | 0.9375000 | 0.9500000 |
| ( 1 0 2 1 ) | 0.9750000 | 0.9750000 | 0.9375000 | 0.9500000 |
| ( 1 1 2 0 ) | 0.9750000 | 0.9500000 | 0.9375000 | 0.9750000 |
| ( 1 1 2 1 ) | 0.9500000 | 0.9500000 | 0.9375000 | 0.9500000 |

Table 4. RELIABILITIES AND 95 % LOWER CONFIDENCE BOUNDS ( A )

| FAILURE VECTOR | $\hat{R}$ | FAILURE VECTOR | 95 % LCB |
|---|---|---|---|
| ( 0 0 0 0 ) | 1.0000000 | ( 0 0 0 0 ) | 1.0000000 |
| ( 1 0 0 0 ) | 0.9891304 | ( 0 0 1 0 ) | 0.9062500 |
| ( 0 0 1 0 ) | 0.9687500 | ( 1 0 0 0 ) | 0.9000000 |
| ( 1 0 1 0 ) | 0.9687499 | ( 0 0 2 0 ) | 0.8750000 |
| ( 0 0 0 1 ) | 0.9500000 | ( 1 0 1 0 ) | 0.8550000 |
| ( 0 1 0 0 ) | 0.9500000 | ( 0 0 0 1 ) | 0.8500000 |
| ( 1 0 0 1 ) | 0.9499999 | ( 0 1 0 0 ) | 0.8500000 |
| ( 1 1 0 0 ) | 0.9499999 | ( 1 0 0 1 ) | 0.8282812 |
| ( 1 0 1 1 ) | 0.9439102 | ( 1 1 0 0 ) | 0.8258822 |
| ( 1 1 1 0 ) | 0.9439102 | ( 0 1 1 0 ) | 0.8234775 |
| ( 1 1 1 1 ) | 0.9385999 | ( 0 0 1 1 ) | 0.8234375 |
| ( 0 0 2 0 ) | 0.9375000 | ( 1 0 2 0 ) | 0.8125000 |
| ( 1 0 2 0 ) | 0.9374999 | ( 1 0 1 1 ) | 0.8015624 |
| ( 1 1 2 1 ) | 0.937499 | ( 0 1 0 1 ) | 0.8000000 |
| ( 0 0 1 1 ) | 0.9203125 | ( 1 1 1 0 ) | 0.8000000 |
| ( 0 1 1 0 ) | 0.9203125 | ( 0 1 2 0 ) | 0.7875000 |
| ( 1 1 0 1 ) | 0.9201959 | ( 0 0 2 1 ) | 0.7749999 |
| ( 1 0 2 1 ) | 0.9134614 | ( 1 1 0 1 ) | 0.7749018 |
| ( 1 1 2 0 ) | 0.9134614 | ( 1 1 1 1 ) | 0.7649999 |
| ( 0 1 0 1 ) | 0.9025000 | ( 1 0 2 1 ) | 0.7647058 |
| ( 0 0 2 1 ) | 0.8906249 | ( 1 1 2 0 ) | 0.7614843 |
| ( 0 1 2 0 ) | 0.8906249 | ( 1 1 2 1 ) | 0.7505192 |
| ( 0 1 1 1 ) | 0.8742968 | ( 0 1 1 1 ) | 0.7499999 |
| ( 0 1 2 1 ) | 0.8460937 | ( 0 1 2 1 ) | 0.7171874 |

# III. LOG LINEAR MODEL WITH DEPENDENCE ASSUMPTION

## A. BACKGROUND

Modeling the outcomes of the various environmental tests as independent is clearly inappropriate. However, the nature of the acceptance sampling plan makes it impossible to estimate the reliability of the device ( i.e. the probability that it would activate after exposure to all four environments ) without some assumptions about the dependence between outcomes of various tests. Thus, our second approach is to model the pyrotechnic device reliability with a log linear model.

The results of our test series create a ( 2 x 2 x 2 x 2 ) contingency table. Let **p** represent passing a test and **f** represent failing a test.

Table 5. CONTINGENCY TABLE STRUCTURE FOR TEST SERIES

| | | Passed Manufacturer Test | | Failed Manufacturer Test | |
|---|---|---|---|---|---|
| | | Passed Tem.&Hum. Test | Failed Tem.&Hum. Test | Passed Tem.&Hum. Test | Failed Tem.&Hum. Test |
| Passed Vibration Test | Passed Altitude Test | $M_{pppp}$ | $M_{pfpp}$ | $M_{fppp}$ | $M_{ffpp}$ |
| | Failed Altitude Test | $M_{pppf}$ | $M_{pfpf}$ | $M_{fppf}$ | $M_{ffpf}$ |
| Failed Vibration Test | Passed Altitude Test | $M_{ppfp}$ | $M_{pffp}$ | $M_{fpfp}$ | $M_{fffp}$ |
| | Failed Altitude Test | $M_{ppff}$ | $M_{pfff}$ | $M_{fpff}$ | $M_{ffff}$ |

The results of our tests can be thought of as censored data from a hypothetical ( 2 X 2 X 2 X 2 ) contingency table (See Table 1) . The frequency in each cell of this table is

$$M_{ijkl}, \quad (i, j, k, l) \in \{p, f\}^4,$$

the number of devices out of 92 which would have result i in environment 1 (manufacturer), result j in environment 2 (temperature and humidity alone), result k in environment 3 (vibration alone), result $l$ in environment 4 (altitude alone).

This is a hypothetical table, because if a device was exposed to all four environments and then failed to activate, there would be no way to discern which combination of the four environments caused failure. The data from the acceptance sampling plan can be thought of as censored data from such a ( 2 X 2 X 2 X 2 ) contingency table. As an example, a device that is given just the manufacturer test belongs in one of the cells ( p, j, k, $l$ ) where ( j, k, $l$ ) $\in \{p, f\}^3$, because it is not clear what would have happened to it had it been exposed to the other three environments.

Using a log linear model under independence, the expected value of each cell frequency is

$$E[\, M_{ijkl} \,] = e^{\mu + \lambda_i^1 + \lambda_j^2 + \lambda_k^3 + \lambda_l^4} \tag{3.1}$$

where i, j, k, $l$ = p, f and

$$\lambda_p^i = -\lambda_f^i \tag{3.2}$$

for i = 1, 2, 3, 4. Here the parameter $\mu$ represents the overall effect and the parameters $\lambda_p^i$ represent the effect of passing environment i. It is simple to show that the log linear model (3.1) is equivalent to the independence of the environments [Ref. 5: pp. 25-46].

On a logarithmic scale, the independence relation is equivalent to the additive relationship. As an example,

$$\log M_{pppp} = \mu + \lambda_p^1 + \lambda_p^2 + \lambda_p^3 + \lambda_p^4 . \tag{3.3}$$

Log linear models, which take into account dependence include extra interaction terms. Because of the extreme amount of censoring, we will only consider models with two way interaction terms of the type.

$$E[\, M_{ijkl} \,] = e^{\mu + \lambda_i^1 + \lambda_j^2 + \lambda_k^3 + \lambda_l^4 + \lambda_{ij}^{12} + \lambda_{ik}^{13} + \lambda_{il}^{14} + \lambda_{jk}^{23} + \lambda_{jl}^{24} + \lambda_{kl}^{34}} \tag{3.4}$$

where i, j, k, $l$ = p, f,

$$\lambda_p^i = -\lambda_f^i \tag{3.5}$$

for i = 1, 2, 3, 4 and

$$\lambda_{pp}^{ij} = \lambda_{ff}^{ij} = -\lambda_{pf}^{ij} = -\lambda_{fp}^{ij} \tag{3.6}$$

for i, j, = 1, 2, 3, 4. As an example;

$$E[\, M_{pppp} \,] = e^{\mu + \lambda_p^1 + \lambda_p^2 + \lambda_p^3 + \lambda_p^4 + \lambda_{pp}^{12} + \lambda_{pp}^{13} + \lambda_{pp}^{14} + \lambda_{pp}^{23} + \lambda_{pp}^{24} + \lambda_{pp}^{34}} . \tag{3.7}$$

The censored data does not tell us much about partial association between pairs of tests. For this reason, we assume that partial associations between any pair of tests are all the same. Let $\theta$ be the partial association between tests. Then we can reformulate equation (3.7) as follows:

$$E[\, M_{pppp} \,] = e^{\mu + \lambda_p^1 + \lambda_p^2 + \lambda_p^3 + \lambda_p^4 + 6\theta} . \tag{3.8}$$

The MLE's for the expected number of devices in each cell ( or equivalently the MLE's for the parameters ) can not be found explicitly. The EM Algorithm ( Expectation - Maximization ) will be used to approximate the MLE's of the expected number of devices in each cell.

## B. EXPECTATION MAXIMIZATION ( EM ) ALGORITHM

A general method of maximum likelihood estimation from incomplete data is the EM Algorithm. The expectation maximization algorithm is an iterative procedure where each stage consists of:

- an expectation step ( E ) followed by
- a maximization step ( M ).

This algorithm is generally used to compute maximum likelihood estimators in incomplete data problems. In the application of the EM algorithm we:

- replace missing values by their estimated expected values given the incomplete data
- estimate parameters
- reestimate the missing values assuming the new parameter estimates are correct
- reestimate parameters

and so forth, iterating until convergence [Ref. 6: pp. 127-141]. This iterative algorithm works as follows in pyrotechnic device problem.

### 1. Initialization

The algorithm requires initial guesses for the parameters of the log linear model cell frequency in contingency table. It uses initial guesses and calculates initial cell probabilities. We will get our initial guesses by first fitting a log linear model under independence.

### 2. Iterations

- **Expectation Step**

EM Algorithm estimates expected cell frequencies given the data by using the most current estimates of the parameters. It compares these estimated expected cell frequencies with the previous estimates. If the differences between these two estimates are small enough, then the EM Algorithm has converged. We then accept the final estimates of cell frequencies as MLE, so we can easily calculate cell probabilities.

- **Maximization Step**

If the algorithm has not converged, then it starts to estimate new parameters for the log linear model using expected cell frequencies from the E step as if there were actual data available. Estimation is done by maximizing the likelihood using an iterative Newton-Raphson method. The estimated parameters from this step are than used in the next E step of the EM algorithm.

## C. CALCULATIONS

We apply the EM algorithm for each realization of the failure vector ( FOM, FOTH, FOV, FOA ). During the maximization step of the EM Algorithm, we use the Newton-Raphson procedure which is described by SAS, [Ref. 7: pp. 190-212]. We first describe the M step using as an example the following failure vector.

$$( 1 \quad 1 \quad 2 \quad 0 )$$

- **Maximization Step**

We start with initial guesses (IG) for $M_{ijkl}$'s. These are the conditional expected values provided by the previous E step. For example, initial guesses for the failure vector above, are shown below.

$$IG = \begin{bmatrix} M_{pppp} \\ M_{pppf} \\ M_{ppfp} \\ M_{ppff} \\ M_{pfpp} \\ M_{pfpf} \\ M_{pffp} \\ M_{pfff} \\ M_{fppp} \\ M_{fppf} \\ M_{fpfp} \\ M_{fpff} \\ M_{ffpp} \\ M_{ffpf} \\ M_{fffp} \\ M_{ffff} \end{bmatrix} \qquad IG = \begin{bmatrix} 83.08 \\ 0.1 \\ 3.36 \\ 0.1 \\ 2.21 \\ 0.1 \\ 0.1 \\ 0.1 \\ 2.15 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \end{bmatrix} . \tag{3.9}$$

From these, we can compute the proportion of observations which fall into each cell as:

$$\hat{P} = \begin{bmatrix} \frac{M_{pppp}}{92} \\ \frac{M_{pppf}}{92} \\ \frac{M_{ppfp}}{92} \\ \vdots \\ \frac{M_{ffff}}{92} \end{bmatrix} = \begin{bmatrix} 0.9030 \\ 0.0011 \\ 0.0365 \\ \vdots \\ 0.0011 \end{bmatrix} . \tag{3.10}$$

We will use $\hat{P}$ to get MLE's for the parameters of the log linear model. Our log linear model which includes two way interactions can be written as

$$\begin{bmatrix} \log P_{pppp} \\ \log P_{pppf} \\ \log P_{ppfp} \\ \vdots \\ \log P_{ffff} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 6 \\ 1 & 1 & 1 & 1 & -1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & -1 & -1 & -1 & -1 & 6 \end{bmatrix} \gamma - \rho \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \tag{3.11}$$

where $\gamma^T = (\mu, \lambda_p^1, \lambda_p^2, \lambda_p^3, \lambda_p^4)$ is the parameter vector and $\rho$ is normalizing constant required by the restriction that the probabilities sum to 1 and where $P_{ijkl}$ is the probability corresponding to the $(i\,j\,k\,l)^{th}$ cell in the hypothetical contingency table. We can derive from equation (3.11)

$$\begin{bmatrix} \log P_{pppp} \\ \log P_{pppf} \\ \log P_{ppfp} \\ \vdots \\ \log P_{ffff} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 6 \\ 1 & 1 & 1 & -1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -1 & -1 & -1 & -1 & 6 \end{bmatrix} \beta - \delta \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \tag{3.12}$$

where $\beta^T = (\lambda_p^1, \lambda_p^2, \lambda_p^3, \lambda_p^4)$ is the parameter vector and $\delta$ is normalizing constant required by the restriction that the probabilities sum to 1. Then, to use the SAS procedure, we rewrite $\log P_{pppp}$, $\log P_{pppf}$, . . . , $\log P_{ffff}$ as the 15 logits, $F_1 = (\log P_{pppp} / \log P_{ffff})$, . . . , $F_{15} = (\log P_{fffp} / \log P_{ffff})$ so that

$$F(P) = \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_{15} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \log P_{pppp} \\ \log P_{pppf} \\ \log P_{ppfp} \\ \vdots \\ \log P_{ffff} \end{bmatrix} \tag{3.13}$$

where $P = (P_{pppp}, P_{pppf}, \ldots, P_{ffff})$. Using equations above the following result is obtained

$$F(P) = \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_{15} \end{bmatrix} = \begin{bmatrix} 2 & 2 & 2 & 2 & 0 \\ 2 & 2 & 2 & 0 & -6 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 2 & -6 \end{bmatrix} \beta . \qquad (3.14)$$

The design matrix (X) of this problem, from the equation (3.14) is as follows:

$$X = \begin{bmatrix} 2 & 2 & 2 & 2 & 0 \\ 2 & 2 & 2 & 0 & -6 \\ 2 & 2 & 0 & 2 & -6 \\ 2 & 2 & 0 & 0 & -8 \\ 2 & 0 & 2 & 2 & -6 \\ 2 & 0 & 2 & 0 & -8 \\ 2 & 0 & 0 & 2 & -8 \\ 2 & 0 & 0 & 0 & -6 \\ 0 & 2 & 2 & 2 & -6 \\ 0 & 2 & 2 & 0 & -8 \\ 0 & 2 & 0 & 2 & -8 \\ 0 & 2 & 0 & 0 & -6 \\ 0 & 0 & 2 & 2 & -8 \\ 0 & 0 & 2 & 0 & -6 \\ 0 & 0 & 0 & 2 & -6 \end{bmatrix} . \qquad (3.15)$$

In the application of Newton-Raphson Method, we use the variance and covariance matrix S of $F(\hat{P})$. It's inverse is given by

$$S^{-1}(P) = \begin{bmatrix} P_{pppp} - P_{pppp}^2 & -P_{pppp} \times P_{pppf} & \cdots & -P_{pppp} \times P_{fffp} \\ -P_{pppp} \times P_{pppf} & P_{pppf} - P_{pppf}^2 & \cdots & -P_{pppf} \times P_{fffp} \\ \vdots & \vdots & \vdots & \vdots \\ -P_{pppp} \times P_{fffp} & -P_{pppf} \times P_{fffp} & \cdots & P_{fffp} - P_{fffp}^2 \end{bmatrix} . \qquad (3.16)$$

The first estimates of parameters for the Newton-Raphson procedure are calculated as follows:

$$b_0 = [\, X^T S^{-1} ( \hat{P} ) X \,]^{-1} [\, X^T S^{-1} ( \hat{P} ) F ( \hat{P} ) \,]. \tag{3.17}$$

where $S^{-1}$ and F are estimated using the proportions in the vector $\hat{P}$. We estimate the reduced logit response functions using the equation

$$\hat{F}_0 = X b_0 . \tag{3.18}$$

From $F_0$ we can compute the updated estimates of $P_{pppp}$, $P_{pppf}$, . . . , $P_{ffff}$ Let $\Pi = ( \Pi_1 ( 1 ), \Pi_2 ( 1 ), \ldots, \Pi_{16} ( 1 ) )$ be the vector, which contains these estimates of the cell probabilities. The value of the log likelihood evaluated at $\Pi$ is

$$LHE = \sum_{i=1}^{16} x_i \log \Pi ( i ) \tag{3.19}$$

where $x_i$ is the number of items in cell i with respect to probability in $\Pi$.

We estimate parameters iteratively until the difference between last estimate and previous is small enough. At each iteration we update the inverse of variance and covariance matrix with probabilities of $\Pi$ from the previous iteration and we do following matrix computations.

$$\begin{aligned} C &= X^T S^{-1} ( \Pi ) X \\ G &= X^T [\, 92.0 \times ( \hat{P} - \Pi ) \,] \end{aligned} \tag{3.20}$$

Let $b_i$ be the next estimate of parameters $\beta$ in the i th iteration. Then $b_i$ as follows:

$$b_i = b_{i-1} - \delta\, C^{-1} G \tag{3.21}$$

where $\delta \leq 1$ is a constant supplied by the user.

We get first cell expectations from initialization. The failure vector, which is defined at page 5, has initial guesses for cells which are shown in equation (3.9). After the first application of Newton -Raphson Algorithm, we have following initial cell expectations ( CE ).

$$CE = \begin{bmatrix} 82.4278259 \\ 0.7278117 \\ 3.1047230 \\ 0.1208856 \\ 2.0790358 \\ 0.0809494 \\ 0.3453168 \\ 0.0592893 \\ 2.0268116 \\ 0.0789160 \\ 0.3366428 \\ 0.0578000 \\ 0.2254283 \\ 0.0387050 \\ 0.1651089 \\ 0.1250073 \end{bmatrix} . \tag{3.22}$$

- **Expectation Step**

The conditional expected frequency for each cell expectations are calculated using the estimated parameters of the log linear model from the previous M step. Some of the cell expectations formulated below

$$\begin{aligned} E[M_{pppp} \mid (FOM, FOTH, FOV, FOA)] = {} & (20 - FOM) \times P(pppp \mid p\ldots) \\ & + (20 - FOTH) \times P(pppp \mid pp..) \\ & + (32 - FOV) \times P(pppp \mid p.p.) \\ & + (20 - FOA) \times P(pppp \mid p..p) \end{aligned} \tag{3.23}$$

$$E[M_{pfpf} \mid (FOM, FOTH, FOV, FOA)] = (20 - FOM) \times P(pfpf \mid p\ldots) + (FOTH) \times P(pfpf \mid pf..) + (32 - FOV) \times P(pfpf \mid p.f.) + (FOA) \times P(pfpf \mid p..f) \tag{3.24}$$

where " . " represents either a pass or fail. For example,

$P(pppp \mid p\ldots)$ is the probability that device passes all environments given that it passed manufacturer test.

$P(pppp \mid pp..)$ is the probability that a device passes all environments given that it passed manufacturer test and temperature-humidity environment.

The conditional probabilities are computed from the estimated cell probabilities from the previous M step. As an example, we have following failure vector,

$$(\ 1 \quad 1 \quad 2 \quad 0\ )$$

Then $E[\,M_{pppp} \mid (1,\ 1,\ 2,\ 0)\,]$ is calculated as follows:

$$P(pppp \mid p\ldots) = \frac{82.4278259}{88.9458375} = 0.9267193$$

$$P(pppp \mid pp..) = \frac{82.4278259}{86.3812462} = 0.9542328$$

$$P(pppp \mid p.p.) = \frac{82.4278259}{85.3156228} = 0.9661516$$

$$P(pppp \mid p..p) = \frac{82.4278259}{87.9569015} = 0.9371388,$$

where the numbers come from (Ꝫ.22) the estimated cell expected values after from the previous M step. Thus,

$$E[\, M_{pppp} \,|\, (\, 1,\ 1,\ 2,\ 0\,)\,] = 83.4654163$$

The remaining 16 conditional expectations are calculated similarly. Then we compare these expectations with previous expectations. If the difference between compared expectations are small enough then EM Algorithm is assumed to have converged.

When EM Algorithm converges, we can estimate reliability of device ( R ) after exposure all four environments as

$$\hat{R} = \frac{E[\, \hat{M}_{pppp} \,]}{92.0} \,. \tag{3.25}$$

For example, for the failure vector above, after 27 iterations, we obtain

$$E[\, \hat{M}_{pppp} \,] = 84.412714 \,,$$

and

$$\hat{R} = \frac{84.412714}{92.0} = 0.9175259.$$

## D. INITIAL GUESS PROBLEM

The EM Algorithm uses an initial guess vector as in equation (3.9). We use the **independence** assumption as described in equations (3.1), (3.2) and (3.3). The following procedures are used during the application of EM Algorithm to the log linear model with the independence assumption.

We have still an initial guesses problem for parameters in the log linear model with the independence assumption. For this reason, we do some calculations to find initial guesses for the parameters. Moreover, one can choose random initial guesses for the parameters too.

Assume that we have the following failure vector,

$$( \ 1 \ \ 0 \ \ 1 \ \ 0 \ ).$$

In this procedure we divide the number of devices which pass the manufacturer test by the number of cells which include items that have passed the manufacturer test. In this example, we can calculate $M_{ppff}$ as follows:

- $[ \frac{19.0}{8.0} ] = 2.375$ items for manufacturer test
- $[ \frac{20.0}{8.0} ] = 2.500$ items for temperature and humidity test
- $[ \frac{1.0}{8.0} ] = 0.125$ item for vibration test
- $[ \frac{0.0}{8.0} ] = 0.000$ item for altitude test

finally the expected number of device for this cell is,

$$M_{ppff} = 5.000 .$$

After computing $M_{ijkl}$ for each cell using the same procedure above, we can group these cells to estimate $M_{pp00}$ , $M_{pf00}$ , $M_{fp00}$ , $M_{ff00}$ as follows:

- $\sum M_{ppkl} = M_{pp00}$
- $\sum M_{pfkl} = M_{pf00}$
- $\sum M_{fpkl} = M_{fp00}$
- $\sum M_{ffkl} = M_{ff00}$

where k, $l$ = p, f.

In this example,

Table 6. GRUOPED DATA FOR INITAL GUESS CALCULATION

| TERMS | ESTIMATED VALUES |
|---|---|
| $M_{pp00}$ | 32.500 |
| $M_{pf00}$ | 22.500 |
| $M_{fp00}$ | 23.500 |
| $M_{ff00}$ | 13.500 |

By combining the parameters $\mu$ and $\lambda_p^1$, we have four equations and four unknown parameters. We can easily solve for the parameters from the initial guesses.

Table 7. ESTIMATED PARAMETERS FOR INITIAL GUESS

| TERMS | ESTIMATED VALUES |
|---|---|
| $\mu + \lambda_p^1$ | -1.97 |
| $\lambda_p^2$ | 1.08 |
| $\lambda_p^3$ | 1.08 |
| $\lambda_p^4$ | 0.56 |

Finally, we can split the sum of the $\mu$ and $\lambda_p^1$ into two parts. One possibility is to divide the sum by two assigning half to $\mu$ and half to $\lambda_p^1$. Results using a random initial guess and the above procedure for an initial guess are very close. Calculation of initial guess parameters are done by the FORTRAN program INITIAL in Appendix D and the FORTRAN program PARAM in Appendix E.

## E. RESULTS

Reliabilities of the pyrotechnic device are calculated by a FORTRAN program LLMDEP in Appendix F. They are given in Table 8. Results from the log linear model design with dependence assumption are similar to the first model. We can easily see from the previous table that the ordering of the estimates of R is counter-intuitive. We expect ( 1 1 2 1 ) to yield the smallest $\hat{R}$ rather than an $\hat{R} = 0.934$ which is larger than $\hat{R}$ for about half of the 24 realizations of the failure vector.

It is clear from these results that this log linear model is inappropriate for modeling the outcomes of the sampling inspection plan. More realistic models would include three-way and four-way interaction terms. However, due to the extreme censoring in the data, we can not estimate R for these models. Moreover, reliabilities of several cases for even this model were not calculated because of computational limitations. Taking the account of dependence with log linear models is clearly not a reasonable approach for estimating reliabilities from this data.

Table 8. RELIABILITIES WITH LOGLINEAR MODEL

| FAILURE VECTOR | $\hat{R}$ |
|---|---|
| ( 0 0 0 0 ) | 1.0000000 |
| ( 1 0 0 0 ) | 0.9887247 |
| ( 1 0 1 0 ) | 0.9693505 |
| ( 1 0 0 1 ) | 0.9518477 |
| ( 1 1 0 0 ) | 0.9503552 |
| ( 1 1 1 0 ) | 0.9457148 |
| ( 1 0 1 1 ) | 0.9456805 |
| ( 1 1 1 1 ) | 0.9399773 |
| ( 1 0 2 0 ) | 0.9374950 |
| ( 1 1 2 1 ) | 0.9336501 |
| ( 0 0 1 1 ) | 0.9255090 |
| ( 0 1 1 0 ) | 0.9255087 |
| ( 1 1 0 1 ) | 0.9246104 |
| ( 1 0 2 1 ) | 0.9175486 |
| ( 1 1 2 0 ) | 0.9175295 |
| ( 0 1 0 1 ) | 0.9089977 |
| ( 0 0 2 1 ) | 0.8978485 |
| ( 0 1 2 0 ) | 0.8978109 |
| ( 0 1 1 1 ) | 0.8825021 |
| ( 0 1 2 1 ) | 0.8584540 |

# IV. WORST CASE SCENARIO

## A. ASSOCIATION ANALYSIS

In the inspection sampling plan, failures are not assigned to an individual failure mechanism except for the manufacturer test. For example, from the following failure vector.

$$( \; 1 \quad 1 \quad 2 \quad 1 \; )$$

We know that there is at least one failure due to the manufacturing test. There is one failure from the joint temperature and humidity and manufacturer test, but we do not know which failure mechanism generated this failure. Items can fail due to a manufacturing related failure mechanism or other failure mechanism or both. The worst case is to assume that the cause of failure is due to all of the failure mechanisms that it was exposed to.

Let MT represents manufacturer test, TAHT represents temperature and humidity test, VT represents vibration test, AT represents altitude test.

Because all 92 items are exposed to manufacturer test, the worst case of the failure analysis of the failure vector above is shown by the following table.

**Table 9. FAILURE ANALYSIS**

| | MT | TAHT | VT | AT |
|---|---|---|---|---|
| # OF TESTED ITEMS | 92 | 20 | 32 | 20 |
| # OF SUCCESFUL ITEMS | 87 | 19 | 30 | 19 |

This " worst case " scenario should give us lower bounds for the estimates $\hat{R}$ of R.

If we calculate the reliability of series system assuming failure mechanisms in test are independent when in fact they are associated but not independent, then we underestimate system reliability. It is reasonable to expect that there is positive dependence between four tests. If one item fails in manufacturer test, it is more likely to fail in other tests. One way of modelling positive dependance between tests is to assume that tests are positively quadrant dependent.

Given random variables $T_1, \ldots, T_n$. They are said to be positively quadrant dependent (PQD) [Ref. 8: p. 33], if

$$P(T_1 \leq t_1, \ldots, T_n \leq t_n) \geq \prod_{i=1}^{n} P(T_i \leq t_i) \tag{4.1}$$

for all $(t_1, t_2, \ldots, t_n) \in R^n$. An equivalent formulation of positive quadrant dependence is $T_1, T_2, \ldots, T^n$ are PQD iff

$$P(T_1 > t_1, \ldots, T_n > t_n) \geq \prod_{i=1}^{n} P(T_i > t_i) \tag{4.2}$$

for all $(t_1, t_2, \ldots, t_n) \in R^n$. The proof that (4.1) and (4.2) are equivalent as given in [Ref. 8: pp. 32-33].

We may take account positive dependence between tests with the PQD assumption. Let

$$T_i = \begin{cases} 1, & \text{if an item passes environment i} \\ 0, & \text{if an item fails environment i} \end{cases} , \tag{4.3}$$

for i = 1, 2, 3, 4. Assume that $T_1$, $T_2$, $T_3$, $T_4$ are positively quadrant independent, i.e.

$$P(T_1 \leq t_1, T_2 \leq t_2, T_3 \leq t_3, T_4 \leq t_4) \geq \prod_{i=1}^{4} P(T_i \leq t_i) \quad (4.4)$$

or equivalently that

$$P(T_1 > t_1, T_2 > t_2 T_3 > t_3, T_4 > t_4) \geq \prod_{i=1}^{4} P(T_i > t_i). \quad (4.5)$$

Let R be the probability that an item activates after exposure to all 4 environments. Then

$$R = P(T_1 = 1, T_2 = 1, T_3 = 1, T_4 = 1). \quad (4.6)$$

Using equations (4.4) and (4.5)

$$R \geq P(T_1 = 1) P(T_2 = 1) P(T_3 = 1) P(T_4 = 1). \quad (4.7)$$

Using the notation from the previous section

$$R \geq Q_1 Q_2 Q_3 Q_4, \quad (4.8)$$

where $Q_i$ is the probability that an item passes environment i. With the censored data, we can't estimate $Q_1$, $Q_2$, $Q_3$ and $Q_4$ without building a more structured model for $T_1$, $T_2$, $T_3$ and $T_4$. One alternative is get a lower bound for estimates of $Q_1$, $Q_2$, $Q_3$, $Q_4$. This is **worst case scenario** .

## B. CALCULATIONS WITH EXAMPLE

Let

- $\tilde{R}$ be the MLE for the reliability of the device.
- $\tilde{Q}_i$ be the MLE for the probability that an item passes environment i.

- $\hat{R}$ be a lower bound of $\tilde{R}$ for reliability of the device.
- $\hat{Q}_i$ be a lower bound for the probability that an item passes environment i.

Then according to equation (4.8)

$$\tilde{R} \geq \prod_{i=1}^{4} \tilde{Q}_i \tag{4.9}$$

Further we will construct estimates $\hat{Q}_i$ of $Q_i$ such that

$$\hat{Q}_i \leq \tilde{Q}_i \tag{4.10}$$

and then define

$$\hat{R} = \prod_{i=1}^{4} \hat{Q}_i . \tag{4.11}$$

Thus

$$\hat{R} \leq \tilde{R}. \tag{4.12}$$

From Chapter II, the likelihood of observing $X_1 = x_1,\ X_2 = x_2,\ X_3 = x_3,\ X_4 = x_4$ is

$$
\begin{aligned}
L(x_1, x_2, x_3, x_4 \mid R_1, R_2, R_3, R_4) &= \prod_{i=1}^{4} \binom{n_i}{x_i} R_i^{x_i} (1 - R_i)^{n_i - x_i} \\
&= \binom{n_1}{x_1} P(T_1 = 1)^{x_1} P(T_1 = 0)^{n_1 - x_1} \\
&\times \binom{n_2}{x_2} P(T_1 = 1, T_2 = 1)^{x_2} \\
&\times (1 - P(T_1 = 1, T_2 = 1))^{n_2 - x_2} \qquad (4.13) \\
&\times \binom{n_3}{x_3} P(T_1 = 1, T_3 = 1)^{x_3} \\
&\times (1 - P(T_1 = 1, T_3 = 1))^{n_3 - x_3} \\
&\times \binom{n_4}{x_4} P(T_1 = 1, T_4 = 1)^{x_4} \\
&\times (1 - P(T_1 = 1, T_4 = 1))^{n_4 - x_4}
\end{aligned}
$$

where $x_i$ is the number of devices out of $n_i$ that activate after test i. If we know why the device failed for the tests i = 2, 3, 4 which include manufacturer test along with exposure to environment i then our likelihood could be written as

$$
\begin{aligned}
L(x_1, x_2, x_3, x_4 \mid R_1, R_2, R_3, R_4) = & \binom{n_1}{x_1} P(T_1 = 1)^{x_1} P(T_1 = 0)^{n_1 - x_1} \\
& \times \binom{n_2}{x_2, x_{21}, x_{22}, x_{23}} \\
& \times P(T_1 = 1, T_2 = 1)^{x_2} P(T_1 = 1, T_2 = 0)^{x_{21}} \\
& \times P(T_1 = 0, T_2 = 1)^{x_{22}} P(T_1 = 0, T_2 = 0)^{x_{23}} \\
& \times \binom{n_3}{x_3, x_{31}, x_{32}, x_{33}} \\
& \times P(T_1 = 1, T_3 = 1)^{x_3} P(T_1 = 1, T_3 = 0)^{x_{31}} \\
& \times P(T_1 = 0, T_3 = 1)^{x_{32}} P(T_1 = 0, T_3 = 0)^{x_{33}} \\
& \times \binom{n_4}{x_4, x_{41}, x_{42}, x_{43}} \\
& \times P(T_1 = 1, T_4 = 1)^{x_4} P(T_1 = 1, T_4 = 0)^{x_{41}} \\
& \times P(T_1 = 0, T_4 = 1)^{x_{42}} P(T_1 = 0, T_4 = 0)^{x_{43}}
\end{aligned}
\tag{4.14}
$$

where $x_{i1}$ is the number given test i which failed due to environment i but passed manufacturing, $x_{i2}$ is the number given test i that passed environment i but failed manufacturing and $x_{i3}$ is the number given test i that failed both manufacturing and environment i for i = 2, 3, 4. Note that $n_i - x_i = x_{i1} + x_{i2} + x_{i3}$ for i = 2, 3, 4.

From this likelihood, the MLE's of $Q_i = P(T_i = 1)$ are given by

$$
\tilde{Q}_1 = \frac{x_1 + (x_2 + x_{21}) + (x_3 + x_{31}) + (x_4 + x_{41})}{n_1 + n_2 + n_3 + n_4}, \tag{4.15}
$$

$$\tilde{Q}_2 = \frac{x_2 + x_{22}}{n_2}, \tag{4.16}$$

$$\tilde{Q}_3 = \frac{x_3 + x_{32}}{n_3}, \tag{4.17}$$

$$\tilde{Q}_4 = \frac{x_4 + x_{42}}{n_4}. \tag{4.18}$$

However since we do not know the $x_{ij}$'s, we see that

$$\tilde{Q}_1 \geq \frac{x_1 + x_2 + x_3 + x_4}{n} \tag{4.19}$$

and

$$\tilde{Q}_i \geq \frac{x_i}{n_i}, \tag{4.20}$$

for i = 2, 3, 4. Let

$$\hat{Q}_1 = \frac{x_1 + x_2 + x_3 + x_4}{n} \tag{4.21}$$

and

$$\hat{Q}_i = \frac{x_i}{n_i} \tag{4.22}$$

for i = 2, 3, 4. Then $\hat{Q}_i$'s are lower bounds for the true MLE's $\tilde{Q}_i$, and

$$\hat{R} = \hat{Q}_1 \hat{Q}_2 \hat{Q}_3 \hat{Q}_4 \tag{4.23}$$

It is clear that $\hat{R}$ is a lower bound for the MLE $\tilde{R}$.

In Worst Case Scenario, we assume that the item, which fails in test i, fails because of both manufacturing related failure mechanism and other failure mechanism which is induced by the i th test environment. With this assumption, the reliability of of device is given as follows

$$\hat{R} = \hat{Q}_1 \hat{Q}_2 \hat{Q}_3 \hat{Q}_4 \tag{4.24}$$

where

$$\begin{aligned} \hat{Q}_1 &= \frac{x_1 + x_2 + x_3 + x_4}{92} \\ \hat{Q}_2 &= \frac{x_2}{20} \\ \hat{Q}_3 &= \frac{x_3}{32} \\ \hat{Q}_4 &= \frac{x_4}{20} \end{aligned} \tag{4.25}$$

Here is an example, in this example our failure vector is,

$$( \; 0 \quad 1 \quad 2 \quad 1 \; )$$

$$\begin{aligned} \hat{Q}_1 &= \frac{20 + 19 + 30 + 19}{92} = 0.9456521 \\ \hat{Q}_2 &= \frac{19}{20} = 0.9500000 \\ \hat{Q}_3 &= \frac{30}{32} = 0.9375000 \\ \hat{Q}_4 &= \frac{19}{20} = 0.9500000 \end{aligned} \tag{4.26}$$

And finally reliability of the device can be estimated by using equation (4.24)

$$\hat{R} = (0.9456521)(0.9500000)(0.9375000)(0.9500000) = 0.8093069 \tag{4.27}$$

## C. RESULTS

We compute MLE's with a FORTRAN program MLEB in Appendix G. These are given Tables 10 and 11.

To get an approximate lower confidence bounds for R using the worst case data, we bootstrap using the procedure described in the previous chapter. The FORTRAN program RANVEC in Appendix B is used to generate the 5000 bootstrap samples for each case. We then estimate $\hat{R}$ 's for 5000 failure vectors by the means of MLEB for given case. The next step is to compute the order statistics of $\hat{R}$ 's from $\hat{R}_1$ to $\hat{R}_{5000}$. This is done by FORTRAN program SORT in Appendix C. Finally, we obtain the 95 % lower confidence bound from this routine. Reliabilities and 95 % lower confidence bounds are tabulated in the following pages. Results of reliabilities and 95 % lower confidence bounds are given in descending order.

These estimates and LCB's for R are decreasing with the total number of the failures out of 92 items tested. This, at least, is consistent with how we believe pyrotechnic devices behave. It should be noted that these estimates are in fact conservative lower bounds for the time MLE's under the very weak assumption of PQD. How conservative these estimates are cannot be determined without more extensive data that allows us to estimate the degree of dependence between tests.

Table 10. TEST PROBABILITIES ( B )

| CASE | $\hat{R}_1$ | $\hat{R}_2$ | $\hat{R}_3$ | $\hat{R}_4$ |
|---|---|---|---|---|
| ( 0 0 0 0 ) | 1.0000000 | 1.0000000 | 1.0000000 | 1.0000000 |
| ( 1 0 0 0 ) | 0.9891340 | 1.0000000 | 1.0000000 | 1.0000000 |
| ( 0 0 1 0 ) | 0.9891340 | 1.0000000 | 0.9687500 | 0.1.00000 |
| ( 1 0 1 0 ) | 0.9782608 | 1.0000000 | 0.9687500 | 1.0000000 |
| ( 0 0 0 1 ) | 0.9891304 | 1.0000000 | 1.0000000 | 0.9500000 |
| ( 0 1 0 0 ) | 0.9891340 | 0.9500000 | 1.0000000 | 1.0000000 |
| ( 1 0 0 1 ) | 0.9782608 | 1.0000000 | 1.0000000 | 0.9500000 |
| ( 1 1 0 0 ) | 0.9782608 | 0.9500000 | 1.0000000 | 1.0000000 |
| ( 0 0 2 0 ) | 0.9782608 | 1.0000000 | 0.9375000 | 1.0000000 |
| ( 1 0 2 0 ) | 0.9673913 | 1.0000000 | 0.9375000 | 1.0000000 |
| ( 0 0 1 1 ) | 0.9782608 | 1.0000000 | 0.9687500 | 0.9500000 |
| ( 0 1 1 0 ) | 0.9782608 | 0.9500000 | 0.9687500 | 1.0000000 |
| ( 1 0 1 1 ) | 0.9673913 | 1.0000000 | 0.9687500 | 0.9500000 |
| ( 1 1 1 0 ) | 0.9673913 | 0.9500000 | 0.9687500 | 1.0000000 |
| ( 0 1 0 1 ) | 0.9782608 | 0.9500000 | 1.0000000 | 0.9500000 |
| ( 1 1 0 1 ) | 0.9673913 | 0.9500000 | 1.0000000 | 0.9500000 |
| ( 0 0 2 1 ) | 0.9673913 | 1.0000000 | 0.9375000 | 0.9500000 |
| ( 0 1 2 0 ) | 0.9673913 | 0.9500000 | 0.9375000 | 1.0000000 |
| ( 1 0 2 1 ) | 0.9565217 | 1.0000000 | 0.9375000 | 0.9500000 |
| ( 1 1 2 0 ) | 0.9565217 | 0.9500000 | 0.9375000 | 1.0000000 |
| ( 0 1 1 1 ) | 0.9673913 | 0.9500000 | 0.9687500 | 0.9500000 |
| ( 1 1 1 1 ) | 0.9565217 | 0.9500000 | 0.9687500 | 0.9500000 |
| ( 0 1 2 1 ) | 0.9565217 | 0.9500000 | 0.9375000 | 0.9500000 |
| ( 1 1 2 1 ) | 0.9456521 | 0.9500000 | 0.9375000 | 0.9500000 |

Table 11. RELAIBILITIES AND 95 % LOWER CONFIDENCE BOUNDS ( B )

| FAILURE VECTOR | $\hat{R}$ | FAILURE VECTOR | 95 % LCB |
|---|---|---|---|
| ( 0 0 0 0 ) | 1.0000000 | ( 0 0 0 0 ) | 1.0000000 |
| ( 1 0 0 0 ) | 0.9891304 | ( 1 0 0 0 ) | 0.9891304 |
| ( 0 0 1 0 ) | 0.9582200 | ( 0 0 1 0 ) | 0.8766983 |
| ( 1 0 1 0 ) | 0.9476901 | ( 1 0 1 0 ) | 0.8766983 |
| ( 0 0 0 1 ) | 0.9396738 | ( 0 0 0 1 ) | 0.8222825 |
| ( 0 1 0 0 ) | 0.9396738 | ( 0 0 0 1 ) | 0.8222825 |
| ( 1 0 0 1 ) | 0.9293478 | ( 0 1 0 0 ) | 0.8222825 |
| ( 1 1 0 0 ) | 0.9293478 | ( 1 1 0 0 ) | 0.8222825 |
| ( 0 0 2 0 ) | 0.9171195 | ( 0 0 2 0 ) | 0.8179346 |
| ( 1 0 2 0 ) | 0.9069293 | ( 1 0 2 0 ) | 0.8084239 |
| ( 0 0 1 1 ) | 0.9003056 | ( 0 0 1 1 ) | 0.7712974 |
| ( 0 1 1 0 ) | 0.9003056 | ( 0 1 1 0 ) | 0.7712974 |
| ( 1 0 1 1 ) | 0.8903022 | ( 1 0 1 1 ) | 0.7712974 |
| ( 1 1 1 0 ) | 0.8903022 | ( 0 1 0 1 ) | 0.7712974 |
| ( 0 1 0 1 ) | 0.8828803 | ( 0 1 0 1 ) | 0.7548369 |
| ( 1 1 0 1 ) | 0.8730705 | ( 1 1 0 1 ) | 0.7548369 |
| ( 0 1 2 0 ) | 0.8615828 | ( 0 1 2 0 ) | 0.7218070 |
| ( 0 0 2 1 ) | 0.8615828 | ( 0 0 2 1 ) | 0.7200747 |
| ( 1 0 2 1 ) | 0.8519021 | ( 1 0 2 1 ) | 0.7200747 |
| ( 1 1 2 0 ) | 0.8519021 | ( 1 1 2 0 ) | 0.7200747 |
| ( 0 1 1 1 ) | 0.8457870 | ( 0 1 1 1 ) | 0.6929346 |
| ( 1 1 1 1 ) | 0.8362838 | ( 1 1 1 1 ) | 0.6927614 |
| ( 0 1 2 1 ) | 0.8093069 | ( 0 1 2 1 ) | 0.6508338 |
| ( 1 1 2 1 ) | 0.8001103 | ( 1 1 2 1 ) | 0.6505433 |

# V. BONUS SYSTEM APPROACHES

## A. BACKGROUND

**Quality** is described as " especially high degree of goodness or worth " [Ref. 9 : p. 685]. In industry, a quality product is one that fulfills customer expectations. There are two general aspects of quality:

- Quality of Design
- Quality of Conformance.

All goods and services are produced in various grades or levels of quality. These variations in grades or levels of quality are intentional; therefore the appropriate technical term is quality of design. The quality of conformance is how well the product conforms to the specifications and tolerances required by the design. Quality of conformance is influenced by a number of the following factors:

- the choice of manufacturing process
- the training and supervision of the workforce
- type of quality assurance system ( process controls, tests, inspections, etc. ) used
- the extent to which these quality-assurance procedures are followed
- the motivation of workforce to achieve quality.

Quality Control is the engineering and management activity by which we measure the quality characteristics of a product, comparing them with specifications or requirements and taking appropriate remedial action whenever there is a difference between the actual performance and the standard [Ref. 10 : pp. 1-3].

As contracts are now written, contractors need only to satisfy the requirments of the sampling inspection plan for lot acceptance. Contractors have no incentive to improve the quality of the items they provide, although they are in a position to do so. As mentioned before, the quality of conformance is influenced by the motivation. Therefore we can motivate manufacturers by giving a **bonus** for improved quality. To improve quality, The Naval Weapons Support Center has decided to implement a bonus system for pyrotechnic devices.

## B. BONUS PLANS

In this chapter, we design a bonus system to improve the quality of pyrotechnic devices. A good bonus system encourages reliability growth, because firms try to reach a high quality to get a bonus. An effective bonus system must detect small differences among the offered lots. In the previous chapters, we estimated the reliability of a pyrotechnic device in several ways. We assumed independence in the first model, and we assumed dependence between tests in the last two models. The estimated reliabilities in the first two models are close to each other, but they exhibit different structures in order. One can easily see that the failure vector of the worst case ( 1 1 2 1 ) has bigger reliability value than ten of the possible cases, from Table 5 and Table 15. In addition, the order of 95 % LCB's of cases does not match to the order of MLE's in independent models. But in the worst case scenario, which assumes dependence between tests, we get the same order for both 95 % LCB's and MLE's. Thus, we will use **the worst case scenario** model in further calculations for the bonus system. With the assumptions above, we can apply three sampling plans for giving bonuses to the manufacturers. These are:

- Single Sampling Bonus System
- Double Sampling Bonus System
- Multi-Sampling Bonus System.

Manufacturers have to meet pyrotechnic device acceptance criteria first, before having a chance to get a bonus.

### 1. Single Sampling Bonus System

In this bonus system, we will decide two things at the end of inspection. First we will decide whether the offered lot is acceptable or not, and then for lots which are accepted we will decide whether to give a bonus to the manufacturer.

The Single sampling bonus system is designated by three numbers. These are,

$$n_1\ ,\ LCBFB\ ,\ LCB$$

where

- $n_1$ is the sample size for inspection ( here $n_1$ = 92 ).
- LCBFB is the cut off value applied to the lower confidence bound for awarding a bonus.
- LCB means estimated lower confidence bound after inspection. (If LCB is greater than or equal to LCBFB, then a bonus is awarded.)

The following algorithm shows us how single sampling bonus system works for pyrotechnic devices.

```
STEP # 1  :  DETERMINE CUT OFF VALUE FOR AWARDING THE BONUS.
STEP # 2  :  TAKE A SAMPLE SIZE OF 92 FROM OFFERED LOT.
STEP # 3  :  APPLY MANUFACTURER AND THREE ENVIRONMENT TESTS.
STEP # 4  :  COMPARE RESULTS OF TESTS WITH ACCEPTANCE CRITERIA.
STEP # 5  :  ESTIMATE ITS LOWER CONFIDENCE BOUND ( LCB ),
             IF THEY MET ACCEPTANCE CRITERIA.
STEP # 6  :  COMPARE LCB OF LOT WITH LCBFB
STEP # 7  :  IF THE LCB IS GREATER THAN OR EQUAL TO LCBFB,
             GIVE BONUS TO THE FIRM.
```

According to the algorithm described above, the following are possible events for the firms.

- Firm may not satisfy our acceptance criteria. This means that firm gets a failure vector worse than ( 1 1 2 1 ).

- Firm satisfies acceptance criteria, but its LCB may be less than LCBFB. This means that firm does not get bonus, but the lot is accepted.

- Firm satisfies acceptance criteria, and its LCB may be greater than or equal to LCBFB. This means that firm gets the bonus.

### 2. Double Sampling Bonus System

A double sampling plan has an advantage over a single sampling plan. Because a double sampling plan involves a larger sample size, it reduces the chance that a manufacturer who deserves a bonus will not get one. Double Sampling Bonus System permits the taking of two samples on which to make a decision [Ref. 11 : pp. 184-185].

In this system, we have two inspection stages. If the firm does not get a bonus after the first inspection, then the firm is given a second chance with a second inspection. A double sampling bonus system is designated by five numbers.

$$n_1 \, , \; n_2 \, , \; LCBFB \, , \; LCB1 \, , \; LCB2$$

where

- $n_1$ is the sample size for first inspection, ( here $n_1 = 92$ )

- $n_2$ is the sample size for second inspection, ( here $n_2 = 92$ )

- LCBFB is the cut off value applied to lower confidence bounds for awarding bonuses,

- LCB1 is the estimated lower confidence bound after first inspection,

- LCB2 is the updated estimate of lower confidence bound after second inspection.

In the double sampling bonus system, if the firm does not get a bonus after the first inspection but does meet the acceptance criteria for the first sample, then a second sample is taken. We already have LCB's of possible cases after the first inspection in Table 11. After the second inspection, we calculate the LCB using an aggregated failure vector which includes failures from both samples. When we compute LCB's using aggregated failure vector after the second inspection, we have a total sample size of 184; there are 135 different failure vectors for which the lot meets the acceptance criteria for both samples. After tabulating these possible 135 cases and the estimates of R, we used the bootstrap procedure to find LCB's for each case. We created 5000 random failure vectors for each of them by using case success probabilities in tests. After this, we estimated MLE's of 5000 failure vectors for each possible case. Finally, we estimated 95 % LCB of each case. These calculations were done by using programs RANVEC, MLEB, SORT in appendix B, G and C respectively. The results are tabulated at Appendix I. The following algorithm shows us how double sampling bonus system works for pyrotechnic device.

```
STEP #  1 :  DETERMINE LCBFB.
STEP #  2 :  TAKE A SAMPLE SIZE OF 92 FROM OFFERED LOT.
STEP #  3 :  APPLY MANUFACTURER AND THREE ENVIRONMENT TESTS.
STEP #  4 :  COMPARE RESULTS OF TESTS WITH ACCEPTANCE CRITERIA.
STEP #  5 :  ESTIMATE ITS LOWER CONFIDENCE BOUND ( LCB1 ),
             IF THEY MET ACCEPTANCE CRITERIA.
STEP #  6 :  COMPARE LCB1 OF LOT WITH LCBFB
STEP #  7 :  IF THE LCB1 IS GREATER THAN OR EQUAL TO LCBFB,
             GIVE BONUS TO THE FIRM.
STEP #  8 :  IF THE LCB1 IS LESS THAN LCBFB,
```

GIVE A SECOND CHANCE TO THE FIRM FOR BONUS.

STEP # 9 : TAKE A NEW SAMPLE SIZE OF 92 FROM OFFERED LOT.

STEP # 10 : APPLY MANUFACTURER AND THREE ENVIRONMENT TESTS.

STEP # 11 : AFTER GETTING THE NEW FAILURE VECTOR,
ADD THIS ONE TO THE FIRST FAILURE VECTOR.

STEP # 12 : ESTIMATE ITS LOWER CONFIDENCE BOUND ( LCB2 ),
WITH AGGREGATED FAILURE VECTOR.

STEP # 13 : COMPARE LCB2 OF LOT WITH LCBFB

STEP # 14 : IF THE LCB2 IS GREATER THAN OR EQUAL TO LCBFB,
GIVE BONUS TO THE FIRM.

According to the algorithm described above, the following are possible events for the firms.

- The firm may not satisfy our acceptance criteria after first inspection. This means that firm gets a failure vector worse than ( 1 1 2 1 ). The lot is not accepted and second sample is not taken.

- The firm satisfies acceptance criteria, and its LCB1 may be greater than or equal to LCBFB. This means that it gets bonus after first inspection, and a second sample is not needed.

- The firm satisfies acceptance criteria, but its LCB1 may be less than LCBFB. This means that it does not get the bonus after first inspection, but still has a chance to get a bonus if it submits a second sample.

Of those which submit a second sample

- LCB2 may be less than LCBFB. It means that firm does not get the bonus, but the lot is accepted.

▪ The aggregated failure vector meets acceptance criteria. But LCB2 may be grater than or equal to LCBFB. It means that firm gets bonus.

### 3. Multi-Sampling Bonus System

In this system approach, we may take a sample of size 92 from offered lot three times, four times or more. But if we use the three sampling bonus system, after the third inspection, we may get ( 4 X 4 X 7 x 4 = 448 ) 448 different failure vectors. Multi-Sampling Bonus System is going to be more computationally intensive with respect to double sampling bonus system. This can be done later using with the same reasoning in double sampling bonus system.

## C. EXAMPLES

### 1. Single Sampling Bonus System

Let us assume that we decided that lower confidence bound for bonus will be 0.8500. We are going to use this LCBFB in these examples.

▪ Case # 1

At the end of inspection, firm has following failure vector

$$( 2 \quad 0 \quad 0 \quad 0 ).$$

The firm did not meet the acceptance criteria. We immediately reject the lot and no bonus is given.

▪ Case # 2

At the end of inspection, firm has following failure vector

$$( 1 \quad 0 \quad 0 \quad 1 ).$$

The firm meets the acceptance criteria. We estimate its 95 % lower confidence bound to be

$$LCB = 0.8222825\,.$$

Because $LCB < LCBFB$ , we do not give a bonus to the firm.

- Case # 3

At the end of inspection, firm has following failure vector

$$(\ 0 \quad 0 \quad 1 \quad 0\ ).$$

The firm meets the acceptance criteria. We estimate its 95 % lower confidence bound to be

$$LCB = 0.8766983 \quad .$$

Because $LCB > LCBFB$ , we give a bonus to the firm.

### 2. Double Sampling Bonus System

- Case # 1

At the end of first inspection, firm has following failure vector.

$$(\ 1 \quad 0 \quad 0 \quad 1\ ).$$

The firm meets the acceptance criteria. We estimate its 95 % lower confidence bound to be

$$LCB1 = 0.8222825 \quad .$$

Because $LCB1 < LCBFB$ , we give a second chance to the firm. Here is the result of second inspection

$$(\ 0 \quad 3 \quad 2 \quad 1\ ).$$

The aggregated failure vector will be

$$( \ 1 \quad 3 \quad 2 \quad 2 \ ).$$

It is clear that LCB2 < LCB1 for this data, thus we do not give a bonus to the firm.

- Case # 2

At the end of first inspection, firm has following failure vector

$$( \ 1 \quad 0 \quad 0 \quad 1 \ ).$$

The firm meets the acceptance criteria. We estimate its 95 % lower confidence bound to be

$$LCB1 = 0.8222825 \ .$$

Here, $LCB < LCBFB$ thus we will give a second chance to the firm. Here is the result of second inspection

$$( \ 0 \quad 0 \quad 1 \quad 1 \ ).$$

The aggregated failure vector will be

$$( \ 1 \quad 0 \quad 1 \quad 2 \ ),$$

and we estimate its 95 % lower confidence bound to be

$$LCB1 = 0.8369564 \ .$$

Because $LCB < LCBFB$ , we do not give a bonus to the firm.

- Case # 3

At the end of first inspection, firm has following failure vector

$$( \ 1 \quad 0 \quad 0 \quad 1 \ ).$$

The firm meets the acceptance criteria. We estimate its 95 % lower confidence bound to be

$$LCB1 = 0.8222825 \quad .$$

Because $LCB1 < LCBFB$ , we will give a second chance to the firm. Here is the result of second inspection

$$( \; 0 \quad 0 \quad 0 \quad 1 \; ).$$

The aggregated failure vector will be

$$( \; 1 \quad 0 \quad 0 \quad 2 \; ),$$

and we estimate its 95 % lower confidence bound to be

$$LCB1 = 0.8706521 \; .$$

Because $LCB2 > LCBFB$ , we will give a bonus to the firm.

# VI. SIMULATION RESULTS OF BONUS SYSTEMS

## A. BACK GROUND

We simulated the bonus system to get an idea of how well the bonus system proposed in the previous chapter works. This provides the user with a means of setting the cut off criteria for awarding a bonus. To generate random failure vectors, we need to know the probabilities of being successful in each test for the firm. First, we assume that the firm has an equal probability of being successful in each test, with the following values of $P_1 = P_2 = P_3 = P_4$

- 0.9375
- 0.9500
- 0.9750
- 0.9900
- 0.9950.

In a second set of simulations, we assume that the manufacturer test has a bigger probability of being successful than the other environment tests. The following probabilities of being successful in manufacturer test were used.

- 0.9990
- 0.9950
- 0.9750
- 0.9500.

In these cases, we used equal probabilities of being successful in other environment tests with values between 0.9375 and manufacturer test probability for that given case. Finally, we assume that the manufacturer test should have smaller probability of being successful. When we study these cases with this assumption, we used probabilities of being successful in other environment tests between manufacturer test probability and 0.9990 for that given case. We assume that minimum value of the manufacturer test probability will be 0.9500 because of worst case in last two assumptions.

We generated 2000 random failure vectors for each possible combination of probabilities by using program RANVEC in Appendix B. They were used for 1000 replications of each bonus system, because the Double Sampling Bonus System ( DSBS ) potentially uses two failure vectors per replication.

After getting the failure vectors, the next step is to decide lower confidence bound for giving bonus ( LCBFB ). We chose 0.800, 0.825, 0.850, 0.875, 0.900, 0.950, 0.999 as LCBFB during our simulations. We simulated bonus systems by using program BONUS in Appendix G. Program BONUS counts how many times firm gets bonus during 1000 replications. And then it calculates the bonus percentage dividing counted number by 1000. For each scenario this bonus percent is an estimate of the probability of getting a bonus.

## B. INITIAL COMPARISON OF SYSTEMS

In this part, with equal probabilities of being successful in tests, we tried to see the difference between the Single and the Double Sampling Bonus Systems. For this reason, we simulated Single Sampling Bonus System. Results were tabulated and plotted in next pages.

Table 12. SINGLE SAMPLING BONUS SYSTEM (EQUAL PROBABILITIES)

| PROB.'S | CHOSEN LOWER CONFIDENCE BOUNDS FOR BONUS | | | | | | |
|---|---|---|---|---|---|---|---|
| | 0.800 | 0.825 | 0.850 | 0.875 | 0.900 | 0.950 | 0.999 |
| 0.9200 | 0.025 | 0.007 | 0.007 | 0.007 | 0.002 | 0.002 | 0.002 |
| 0.9375 | 0.060 | 0.023 | 0.023 | 0.023 | 0.010 | 0.010 | 0.007 |
| 0.9450 | 0.092 | 0.044 | 0.044 | 0.044 | 0.018 | 0.018 | 0.010 |
| 0.9500 | 0.117 | 0.053 | 0.053 | 0.053 | 0.022 | 0.022 | 0.011 |
| 0.9600 | 0.222 | 0.112 | 0.112 | 0.112 | 0.045 | 0.045 | 0.028 |
| 0.9700 | 0.380 | 0.198 | 0.198 | 0.198 | 0.107 | 0.107 | 0.067 |
| 0.9750 | 0.482 | 0.281 | 0.281 | 0.281 | 0.163 | 0.163 | 0.067 |
| 0.9800 | 0.609 | 0.381 | 0.381 | 0.381 | 0.237 | 0.237 | 0.106 |
| 0.9850 | 0.740 | 0.511 | 0.511 | 0.511 | 0.362 | 0.362 | 0.166 |
| 0.9900 | 0.868 | 0.646 | 0.646 | 0.646 | 0.502 | 0.502 | 0.274 |
| 0.9950 | 0.955 | 0.805 | 0.805 | 0.805 | 0.502 | 0.502 | 0.407 |

It is obvious that there is no difference between some lower confidence bounds for bonus from the table above. The firm gets the same bonus percentage when we use 0.825, 0.850, 0.875 as lower confidence bound for bonus. The same thing occurs when we use 0.900 and 0.950 as LCBFB. For this reason, we are going to see four curves in Figure 1. Figure 2 shows Double Sampling Bonus System with different LCBFB's.

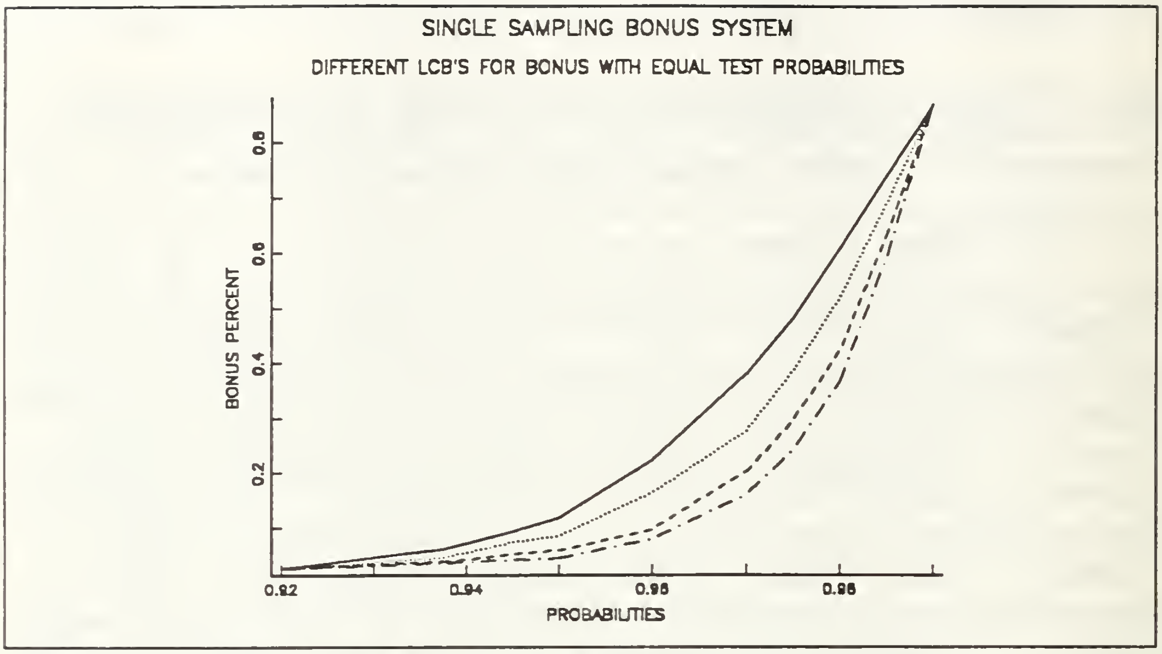


Figure 1. Single Sampling Bonus System With Different LCBFB's

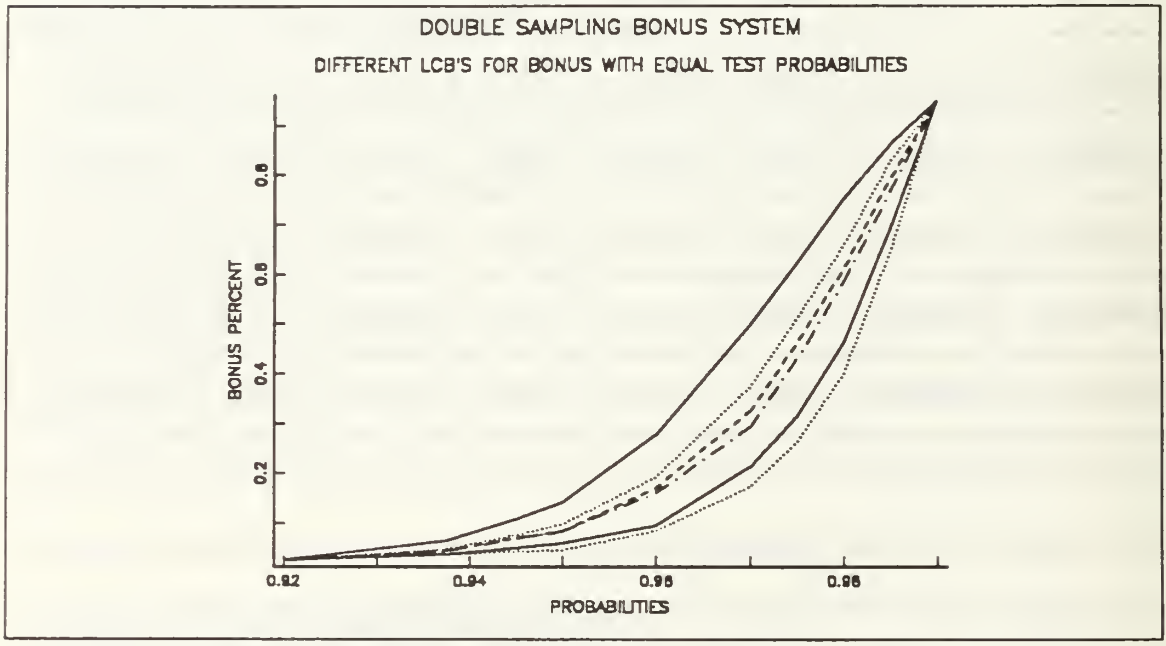


Figure 2. Double Sampling Bonus System With Different LCBFB's

As you see in figure 2, we have six different curves for Double Sampling Bonus System. Because in this system, we can see from the table that LCBFB's 0.900 and 0.950 have approximately the same bonus percentage. For this reason we did not plot for 0.950.

Table 13. DOUBLE SAMPLING BONUS SYSTEM (EQUAL PROBABILITIES)

| PROB.'S | CHOSEN LOWER CONFIDENCE BOUNDS FOR BONUS | | | | | | |
|---|---|---|---|---|---|---|---|
| | 0.800 | 0.825 | 0.850 | 0.875 | 0.900 | 0.950 | 0.999 |
| 0.9200 | 0.026 | 0.008 | 0.007 | 0.007 | 0.002 | 0.002 | 0.002 |
| 0.9375 | 0.064 | 0.030 | 0.025 | 0.023 | 0.010 | 0.010 | 0.007 |
| 0.9450 | 0.108 | 0.059 | 0.050 | 0.045 | 0.020 | 0.018 | 0.010 |
| 0.9500 | 0.142 | 0.079 | 0.063 | 0.057 | 0.027 | 0.022 | 0.011 |
| 0.9600 | 0.277 | 0.172 | 0.144 | 0.122 | 0.052 | 0.045 | 0.028 |
| 0.9700 | 0.498 | 0.351 | 0.293 | 0.236 | 0.138 | 0.107 | 0.067 |
| 0.9750 | 0.624 | 0.487 | 0.423 | 0.353 | 0.212 | 0.163 | 0.106 |
| 0.9800 | 0.752 | 0.632 | 0.565 | 0.487 | 0.319 | 0.237 | 0.166 |
| 0.9850 | 0.861 | 0.799 | 0.738 | 0.643 | 0.486 | 0.362 | 0.274 |
| 0.9900 | 0.948 | 0.920 | 0.887 | 0.799 | 0.671 | 0.502 | 0.407 |
| 0.9950 | 0.987 | 0.982 | 0.970 | 0.941 | 0.872 | 0.695 | 0.626 |

Figure 2 shows DSBS to be more sensitive in the sense that a higher percentage of the firms were awarded a bonus. For this reason, we decided to implement DSBS in all simulations.

## C. SIMULATION RESULTS WITH DIFFERENT LCB'S FOR BONUS

In this section, results for each chosen lower confidence bound for bonus will be presented as follows. We used values in Table 13 to draw plots with equal probabilities in each test. Polynomial approximation was used in curve fitting.

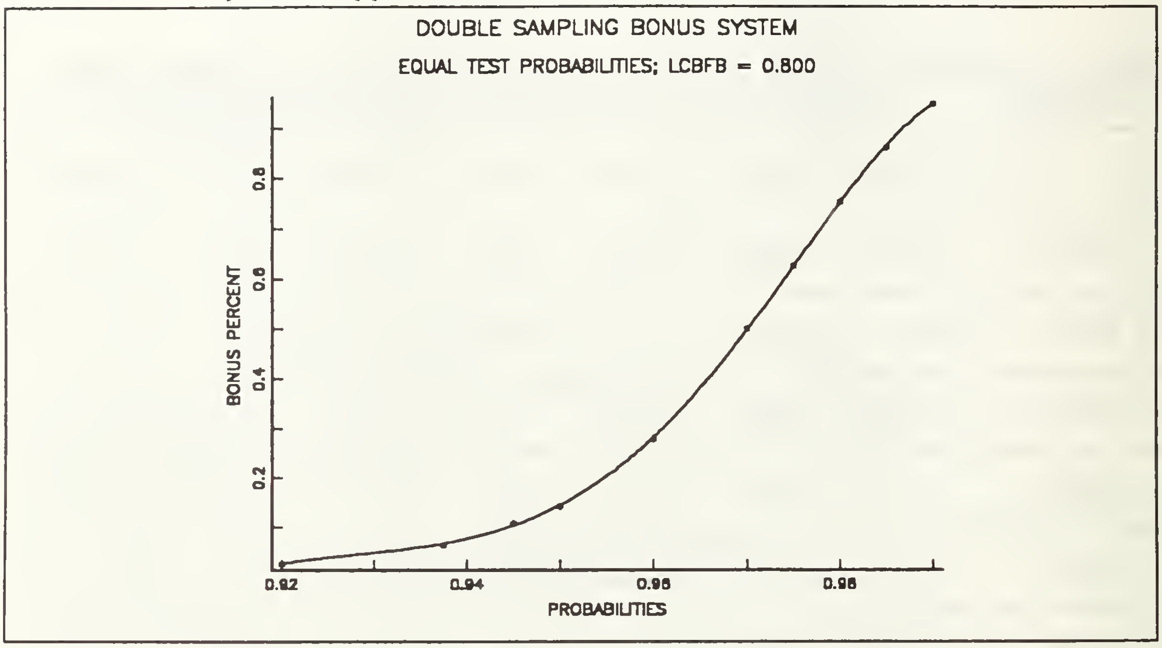


Figure 3. Double Sampling Bonus System With LCBFB = 0.800

Table 14. DSBS (EQUAL PROBABILITIES) LCBFB = 0.800

| LOWER CONFIDENCE BOUND FOR BONUS IS 0.800 | | | | | |
|---|---|---|---|---|---|
| PROB.'S | BONUS % | PROB.'S | BONUS % | PROB.'S | BONUS % |
| 0.9200 | 0.026 | 0.9375 | 0.064 | 0.9450 | 0.108 |
| 0.9500 | 0.142 | 0.9600 | 0.277 | 0.9700 | 0.498 |
| 0.9750 | 0.624 | 0.9800 | 0.752 | 0.9850 | 0.861 |
| 0.9900 | 0.948 | 0.9950 | 0.987 | | |

In Figure 3, when the probabilities of being successful in each test increase, then the bonus percent increases. For example, when the test probabilities is equal to 0.9200, then the bonus percent is 0.026. If the firm increases its probabilities of being succesful in each test to 0.9900, then the bonus percent becomes 0.948. The result of the double sampling bonus system with different lower confidence bounds are tabulated and plotted in Appendix J.

To see whether the probability of passing the manufacturer test effects the bonus percentage differently than the other probabilities, we assumed that the environment tests would have equal probabilities. But the firm will have a different probability of being successful in the manufacturer test. In the following table, the first row represents probability of being successful in the environment tests and the first column represents probability of being successful in the manufacturer test. The intersection of rows and columns gives us the bonus percentage of a firm with given probabilities of being successful in the tests. These procedures were done for each LCBFB value separately. The results are tabulated and plotted in Appendix K. Bonus percentages for LCBFB = 0.8000 are plotted on the following page. It is clear that the bonus percentage of the firm will be high if the firm has big probabilities of being succesful in both manufacturer test and other joint environment tests.

Bonus percentages are tabulated and ploted with different probabilities.

$$LCBFB = 0.800$$

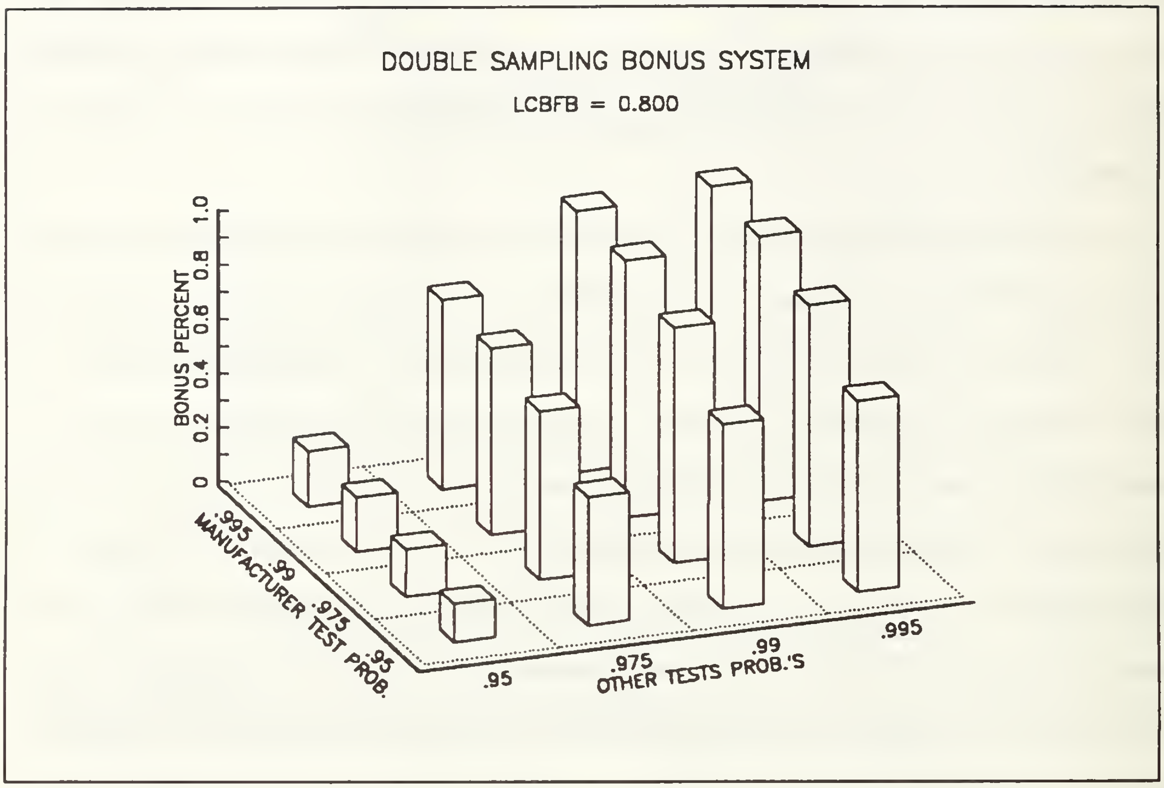


Figure 4. Double Sampling Bonus System With LCBFB = 0.800

Table 15. DSBS (DIFFERENT PROBABILITIES) LCBFB = 0.800

| | 0.950 | 0.975 | 0.990 | 0.995 |
|---|---|---|---|---|
| 0.950 | 0.142 | 0.476 | 0.683 | 0.700 |
| 0.975 | 0.176 | 0.624 | 0.869 | 0.891 |
| 0.990 | 0.202 | 0.688 | 0.948 | 0.973 |
| 0.995 | 0.206 | 0.698 | 0.962 | 0.987 |

## D. BONUS PERCENTAGE ( BPRCT ) FORMULATION

In this section we try to approximate probability of getting a bonus as a function of lower confidence bound for bonus ( LCBFB ), probability of passing manufacturer test ( $P_1$ ) , and probability of passing environment tests ( $P_2$ ). To do this, we use **regression analysis** with GRAFSTAT using the bonus percentages which are the simulated values of the probability of getting a bonus. If a reasonable relationship between the bonus percentage and LCBFB and the probabilities $P_1$ and $P_2$ can be found; it can be used to set LCBFB without resorting to simulation.

We want to formulate bonus percentages as a function of the following:

- Lower confidence bound for bonus ( LCBFB )
- Probability of passing manufacturer test $P_1$
- Probability of passing environment tests $P_2$.

After polynomial approximation, we can see that plotted graphs (Figures 3-10) look like logistic growth curves [Ref. 12: p. 383]. Then we can formulate bonus probabilities (BP) using the logistic growth function.

$$BP = \frac{1}{1 + e^{A}} \tag{7.4}$$

where

$$A = \beta_0 + \beta_1 P_1 + \beta_2 P_1^2 + \beta_3 P_2 + \beta_4 P_2^2 + \beta_5 P_1 P_2 + \beta_6 LCBFB \tag{7.5}$$

We can make a transformation described as below.

$$1 + e^{A} = \frac{1}{BP} \tag{7.6}$$

$$e^{A} = \frac{1 - BP}{BP} \tag{7.7}$$

$$A = \log \left[ \frac{1 - BP}{BP} \right]. \tag{7.8}$$

Now we have a linear equation as a function of LCBFB, $P_1$ and $P_2$. We may use simulation results and approximate parameters doing a linear regression.

- Let Y be the ( n x 1) column vector of observations on dependent variable.
- Let X be the ( n x $p'$ ) matrix consisting of a column of ones, which is labeled 1, followed by p column vectors of observations on independent variables.
- Let $\beta$ be the ( $p'$ x 1 ) vector of parameters to be estimated.
- Let $\varepsilon$ be the ( n x 1 ) vector of random errors. Then

$$Y = X\beta + \varepsilon \tag{7.9}$$

We can obtain observations on dependent variable as below,

$$Y = \log \left[ \frac{1 - BPRCT}{BPRCT} \right] \tag{7.10}$$

where BPRCT is the bonus percentage obtained from the simulation. As an example we have the following information from simulations.

- LCBFB = 0.875
- $P_1 = 0.9500$
- $P_1^2 = 0.9025$
- $P_2 = 0.9375$
- $P_2^2 = 0.8789$
- $P_1 P_2 = 0.8910$
- BPRCT = 0.029
- Y = 3.5110.

We can write following equation.

$$3.5110 = \beta_0 + \beta_1 0.95 + \beta_2 0.9025 + \beta_3 0.9375 + \beta_4 0.8789 + \beta_5 0.8910 + \beta_6 0.875 + \varepsilon .$$

We deliberately chose 30 random results from simulations. We formulate them as above. We can estimate unknown parameters on these equations doing linear regression. 30 equations can be written with matrix notation as follows:

$$Y = \begin{bmatrix} Y(1) \\ Y(2) \\ Y(3) \\ \vdots \\ Y(30) \end{bmatrix} . \tag{7.11}$$

And X matrix is designed as follows:

$$X = \begin{bmatrix} 1 & P_1(1) & P_1^2(1) & P_2(1) & P_2^2(1) & P_1\, P_2(1) \\ 1 & P_1(2) & P_1^2(2) & P_2(2) & P_2^2(2) & P_1\, P_2(2) \\ 1 & P_1(3) & P_1^2(3) & P_2(3) & P_2^2(3) & P_1\, P_2(3) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & P_1(30) & P_1^2(30) & P_2(30) & P_2^2(30) & P_1\, P_2(30) \end{bmatrix} . \tag{7.12}$$

And parameter vector will be as below.

$$\beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \end{bmatrix} . \tag{7.13}$$

We estimate parameters with linear regression using GRAFSTAT packages. And then we formulate the bonus percentage using the estimates of parameters. The estimated bonus probabilities are given by

$$\hat{BP} = \frac{1}{1 + e^{X\hat{\beta}}} . \tag{7.14}$$

The following estimates of the parameters were obtained from the regression.

Table 16. COEFFICIENT VECTORS OF REGRESSION ANALYSIS

| PARAMETERS | ESTIMATED VALUE |
|---|---|
| $\beta_0$ | -1042.3 |
| $\beta_1$ | 695.11 |
| $\beta_3$ | -86.661 |
| $\beta_4$ | 1560 |
| $\beta_5$ | -569.81 |
| $\beta_6$ | -570.62 |
| $\beta_7$ | 12.266 |

The standard error was 0.45471 after the linear regression. We can use these estimates of parameters in equation (7.5). And we can estimate the bonus probability of the firm when the probability of passing tests and lower confidence for bonus are known. Some examples are calculated with equation (7.5). Results of the calculations and comparison with simulation are summarized in following table.

Table 17. BONUS PERCENTAGES WITH REGRESSION ANALYSIS

| LCBFB | $P_1$ | $P_2$ | SIMU-LATION | EST.VALUE |
|---|---|---|---|---|
| 0.800 | 0.9750 | 0.9750 | 0.624 | 0.564 |
| 0.800 | 0.9990 | 0.9375 | 0.106 | 0.062 |
| 0.825 | 0.9750 | 0.9950 | 0.882 | 0.911 |
| 0.825 | 0.8500 | 0.9750 | 0.064 | 0.077 |
| 0.850 | 0.9500 | 0.9900 | 0.612 | 0.615 |
| 0.850 | 0.9900 | 0.9750 | 0.476 | 0.529 |
| 0.875 | 0.9000 | 0.9750 | 0.114 | 0.080 |
| 0.875 | 0.9700 | 0.9700 | 0.236 | 0.211 |
| 0.900 | 0.9950 | 0.9990 | 0.985 | 0.945 |
| 0.900 | 0.8750 | 0.9900 | 0.158 | 0.096 |
| 0.950 | 0.9450 | 0.9450 | 0.018 | 0.010 |
| 0.950 | 0.9750 | 0.9990 | 0.839 | 0.790 |
| 0.999 | 0.9850 | 0.9850 | 0.274 | 0.335 |
| 0.999 | 0.9900 | 0.9250 | 0.004 | 0.002 |

# VII. CONCLUSIONS AND RECOMMENDATIONS

We estimated the reliability of pyrotechnic device from the sampling plan using the following models:

- Maximum likelihood assuming independence of pyrotechnic device activation in different environments;
- Log linear model incorparating some dependence between pyrotechnic device activation in different environments;
- Worst case scenario which gives a lower bound for the estimated reliability for the general model where no assumptions are made about the form of dependence between pyrotechnic device activation in different environments.

Using these models and based on sampling plan data, estimates of overall reliability along with 95 % lower confidence bound are obtained. We computed the lower confidence bound for each possible case by bootstrapping.

Results from the first model are not consistent with the way pyrotechnic devices operate. In particular, the estimated reliabilities are not ordered as we expected them to be. Intiutively, we expect samples with fewer failures to give smaller $\hat{R}$ values and corresponding LCB's than samples with more failures. This is not the case for the first model. For example, the failure vector that has the maximum number of failures for each test ( a total of 5 failures ) is in the middle of the order with respect to $\hat{R}$ and the 95 % lower confidence bound. The failure vector ( 0 1 0 1 ) with a total of 2 failures has a much lower $\hat{R}$ and LCB. The discrepency between the results of this model and what we expected to see are probably due to the fact that there is a dependence between the events which a device activates under different environments.

The results of the log linear model design with two-way dependence assumptions are similar to the first model. Two-way interaction terms were used in this model. Moreover, reliabilities of several cases were not calculated because of computational limitations. Due to the extensive censoring in the sample data, it does not appear to be possible to estimate the reliability based on models which incorparate dependence. Thus, we turn to finding lower bounds for the estimated reliability based on models with dependence.

Finally, the worst case scenario model gives the most reasonable results for this problem. Both the lower bounds for estimates of reliabilities and 95 % lower confidence bounds for these lower bounds are ordered according to the total number of failures. Thus, the results from the worst case scenario were used to implement the bonus system for pyrotechnic devices.

After getting the estimate of reliability and 95 % lower confidence bound for each case, we tried to design a bonus system to improve the quality of pyrotechnic devices. We used 95 % lower confidence bounds instead of the estimated overall reliabilities to decide whether to give bonuses. Two sampling plans for giving the bonus to the manufacturers were considered:

- Single Sampling Bonus System
- Double Sampling Bonus System.

We simulated two sampling bonus systems to see the difference between them. We concluded that the double sampling system is more sensitive than single sampling bonus system.

To formulate an approximate bonus percentage as a function of lower confidence bound for bonus ( LCBFB ), the probability of passing manufacturer test $P_1$, and the probability of passing the environment tests $P_2$, we used regression analysis with GRAFSTAT.

Because the sampling plan results in so much censoring, the only reasonable estimates of overall reliability that we obtained were actually lower bounds. This makes the bonus system conservative in the sense that a bonus might not be awarded what it is deserved. Therefore, if a bonus system is to be implemented, a more comprehensive sampling plan needs to be devised which allows estimation of R. A simple solution to this problem can be to apply all environmental tests to the same sample which would give a measure of dependence between environmental tests.

# APPENDIX A. PROGRAM MLEA

```
*       ***************************************************************
        PROGRAM MLEA
*       ***************************************************************
*          THIS IS A FORTRAN PROGRAM TO CALCULATE THE RELIABILITY OF AN
*       ITEM AFTER EXPOSURE TO SEVERAL  ENVIRONMENTS WITH  INDEPENDENCE
*       ASSUMPTION WHICH IS  DESCRIBED IN CHAPTER I . THE PROGRAM READS
*       5000 SUCCESS VECTORS, WHICH ARE RANDOMLY GENERATED BY PROGRAM
*       RANVEC, FROM AN INPUT FILE CALLED SUCVECT ONE BY ONE. AFTER
*       CALCULATION IT THEN PROGRAM WRITES ESTIMATED RELIABILITIES TO
*       AN OUTPUT FILE CALLED RESULT.
*       ***************************************************************
*       VARIABLES
*       ***************************************************************
*       SOM   : NUMBER OF SUCCESSFUL ITEMS IN MANUFACTURER TEST.
*       SOTH  : NUMBER OF SUCCESFUL ITEMS IN TEMPERATURE AND HUMIDITY
*               TEST.
*       SOV   : NUMBER OF SUCCESSFUL ITEMS IN VIBRATION TEST.
*       SOA   : NUMBER OF SUCCESSFUL ITEMS IN ALTITUDE TEST.
*       R1H   : ESTIMATED PROBABILITY OF PASSING FROM MANUFACTURER TEST.
*       R2H   : ESTIMATED PROBABILITY OF PASSING FROM TEMPERATURE AND
*               HUMIDITY TEST.
*       R3H   : ESTIMATED PROBABILITY OF PASSING FROM VIBRATION TEST
*       R4H   : ESTIMATED PROBABILITY OF PASSING FROM ALTITUDE TEST
*       RHMLE : ESTIMATED RELIABILITY OF ITEM AFTER EXPOSURE TO
*               SEVERAL ENVIRONMENT TESTS.
*       N     : SAMPLE SIZES FOR EACH TEST
*       X     : NUMBER OF SUCCESSFUL ITEMS IN EACH TEST.
*       FLAG  : INDICATOR VARIABLE FOR DETERMINING EASY AND HARD CASE.
*       R1MAX : R1H VALUE WHICH MAXIMIZES LIKELIHOOD.
```

```
*     R2MAX : R2H VALUE WHICH MAXIMIZES LIKELIHOOD.
*     R3MAX : R3H VALUE WHICH MAXIMIZES LIKELIHOOD.
*     R4MAX : R4H VALUE WHICH MAXIMIZES LIKELIHOOD.
*     LMAX  : HIGHEST MAXIMUM LIKELIHOOD VALUE.
*     L     : LIKELIHOOD VALUES AT THE END OF EACH HARD CASE.
*     V     : TOTAL SAMPLE SIZES IN EACH HARD CASE.
*     Q1H   : REORGANIZED PROBABILITY OF MANUFACTURER TEST.
*     Q2H   : REORGANIZED PROBABILITY OF TEMP. AND HUMIDITY TEST.
*     Q3H   : REORGANIZED PROBABILITY OF VIBRATION TEST.
*     Q4H   : REORGANIZED PROBABILITY OF ALTITUDE TEST.
*     ****************************************************************
*     TYPE DECLARATION
      REAL SOM(5000),SOTH(5000),SOV(5000),SOA(5000),R1H,R2H,R3H,R4H,
     +RHMLE(5000),N(4),X(4),R1MAX,R2MAX,R3MAX,R4MAX,LMAX,V(7),Q1H,Q2H,
     +Q3H,Q4H,NTOT,L(7)
      INTEGER I,J
      LOGICAL FLAG(4)
*     ****************************************************************
*     READING SUCCESS VECTORS FROM SUSVECT FILE
      DO 60 I=1,5000
      READ(7,*) SOM(I),SOTH (I),SOV(I),SOA(I)
*     ****************************************************************
*     FILES FOR READING AND WRITING
      CALL EXCMS ('FILEDEF 7 DISK SUCVECT DATA A1')
      CALL EXCMS ('FILEDEF 16 DISK RESULT DATA A1')
*     ****************************************************************
*     INITIALIZATION OF SAMPLE SIZES
      N(1)= 20.0
      N(2)= 20.0
      N(3)= 32.0
      N(4)= 20.0
*     INITIALIZATION OF FLAG VARIABLES
      DO 10 J=2,4
         FLAG(J)= .FALSE.
```

```
   10 CONTINUE
*     NUMBER OF SUCCESSES IN EACH TEST
      X(1)= SOM(I)
      X(2)= SOTH(I)
      X(3)= SOV(I)
      X(4)= SOA(I)
*     ******************************************************************
*     CHECK OPERATION FOR EASY CASE
      IF ((X(1)/20.00).GE.(X(2)/20.00)) THEN
         FLAG(2)= .TRUE.
      END IF
      IF ((X(1)/20.00).GE.(X(3)/32.00)) THEN
         FLAG(3)= .TRUE.
      END IF
      IF ((X(1)/20.00).GE.(X(4)/20.00)) THEN
         FLAG(4)= .TRUE.
      END IF
      IF ( FLAG(2).AND.FLAG(3).AND.FLAG(4) ) THEN
*     ******************************************************************
*     CALCULATIONS IN EASY CASE
         R1H= X(1)/20.0
         R2H= X(2)/20.0
         R3H= X(3)/32.0
         R4H= X(4)/20.0
         Q1H= R1H
         Q2H= R2H/Q1H
         Q3H= R3H/Q1H
         Q4H= R4H/Q1H
         RHMLE(I)= Q1H*Q2H*Q3H*Q4H
         GO TO 50
      END IF
*     ******************************************************************
*     CALCULATIONS IN HARD CASES
*     INITIALIZATION
```

```
      DO 30 J=1,7
         V(J)= 0.0
         L(J)= 0.0
  30  CONTINUE
      LMAX= 0.0
      NTOT= 0.0
*     *****************************************************************
*     CASE 1  R1H.LT.1.0  AND  R1H = R2H = R3H = R4H
      DO 40 J=1,4
         V(1)= V(1)+X(J)
         NTOT= NTOT+N(J)
 40   CONTINUE
      R1H= V(1)/NTOT
      L(1)= ((R1H)**V(1))*((NTOT-V(1))/NTOT)**(NTOT-V(1))
      IF (L(1).GT.LMAX) THEN
         LMAX= L(1)
         R1MAX= R1H
         R2MAX= R1H
         R3MAX= R1H
         R4MAX= R1H
      END IF
*     *****************************************************************
*     CASE 2 R1H.LT.1.0, R1H = R2H = R3H , R4H IS BETWEEN 0.0 AND R1H
      IF ( X(4)/20.0.NE.1.0 ) THEN
         V(2)= X(1)+X(2)+X(3)
         NTOT= N(1)+N(2)+N(3)
         R1H= V(2)/NTOT
         IF((X(4)/20.00).LE.R1H) THEN
            L(2)= ((R1H)**V(2))*((NTOT-V(2))/NTOT)**(NTOT-V(2))*((X(4)
     +/20.0)**X(4))*(((20.0-X(4))/20.0)**(20.0-X(4)))
            IF (L(2).GT.LMAX) THEN
               LMAX= L(2)
               R1MAX= R1H
               R2MAX= R1H
```

```
            R3MAX= R1H
            R4MAX= X(4)/20.0
         END IF
      END IF
   END IF
*  ******************************************************************
*  CASE 3 R1H.LT.1.0, R1H = R2H = R4H , R3H IS BETWEEN 0.0 AND R1H
   IF ( X(3)/32.0.NE.1.0 ) THEN
      V(3)= X(1)+X(2)+X(4)
      NTOT= N(1)+N(2)+N(4)
      R1H= V(3)/NTOT
      IF ((X(3)/32.00).LE.R1H) THEN
        L(3)= ((R1H)**V(3))*((NTOT-V(3))/NTOT)**(NTOT-V(3))*((X(3)
     +/32.0)**X(3))*(((32.0-X(3))/32.0)**(32.0-X(3)))
         IF (L(3).GT.LMAX) THEN
            LMAX= L(3)
            R1MAX= R1H
            R2MAX= R1H
            R3MAX= X(3)/32.0
           R4MAX= R1H
         END IF
      END IF
   END IF
*  ******************************************************************
*  CASE 4 R1H.LT.1.0, R1H = R3H = R4H , R2H IS BETWEEN 0.0 AND R1H
   IF ( X(2)/20.0.NE.1.0 ) THEN
      V(4)= X(1)+X(3)+X(4)
      NTOT= N(1)+N(3)+N(4)
      R1H= V(4)/NTOT
      IF ((X(2)/20.00).LE.R1H) THEN
        L(4)= ((R1H)**V(4))*((NTOT-V(4))/NTOT)**(NTOT-V(4))*((X(2)
     +/20.0)**X(2))*(((20.0-X(2))/20.0)**(20.0-X(2)))
         IF (L(4).GT.LMAX) THEN
            LMAX= L(4)
```

```
            R1MAX= R1H
            R2MAX= X(2)/20.0
            R3MAX= R1H
            R4MAX= R1H
          END IF
        END IF
      END IF
*   ******************************************************************
*   CASE 5 R1H.LT.1.0, R1H = R2H  R3H , R4H ARE BETWEEN 0.0 AND R1H
      IF ((X(3)/32.0.NE.1.0).AND.(X(4)/20.0.NE.1.0))  THEN
        V(5)= X(1)+X(2)
        NTOT= N(1)+N(2)
        R1H= V(5)/NTOT
        IF((X(3)/32.00).LE.R1H.AND.(X(4)/20.00).LE.R1H) THEN
          L(5)= ((R1H)**V(5))*((NTOT-V(5))/NTOT)**(NTOT-V(5))*((X(3)
     +/32.0)**X(3))*((32.0-X(3))/32.0)**(32.0-X(3))*((X(4)/20.0)**
     +x(4))*((20.0-X(4))/20.0)**(20.0-X(4))
          IF (L(5).GT.LMAX) THEN
            LMAX= L(5)
            R1MAX= R1H
            R2MAX= R1H
            R3MAX= X(3)/32.0
           R4MAX= X(4)/20.0
          END IF
        END IF
      END IF
*      ***************************************************************
*      CASE 6 R1H.LT.1.0, R1H = R3H  R2H , R4H ARE BETWEEN 0.0 AND R1H
      IF ((X(2)/20.0.NE.1.0).AND.(X(4)/20.0.NE.1.0))  THEN
        V(6)= X(1)+X(3)
        NTOT= N(1)+N(3)
        R1H= V(6)/NTOT
        IF((X(2)/20.00).LE.R1H.AND.(X(4)/20.00).LE.R1H) THEN
         L(6)= ((R1H)**V(6))*((NTOT-V(6))/NTOT)**(NTOT-V(6))*((X(2)
```

```
     +/20.0)**X(2))*((20.0-X(2))/20.0)**(20.0-X(2))*((X(4)/20.0)**
     +x(4))*((20.0-X(4))/20.0)**(20.0-X(4))

          IF (L(6).GT.LMAX) THEN
               LMAX= L(6)
               R1MAX= R1H
               R2MAX= X(2)/20.0
               R3MAX= R1H
               R4MAX= X(4)/20.0
          END IF
         END IF
      END IF
*     ****************************************************************
*     CASE 7 R1H.LT.1.0, R1H = R4H   R2H , R3H ARE BETWEEN 0.0 AND R1H
      IF ((X(2)/20.0.NE.1.0).AND.(X(3)/32.0.NE.1.0))  THEN
         V(7)= X(1)+X(4)
         NTOT= N(1)+N(4)
         R1H= V(7)/NTOT
         IF((X(2)/20.00).LE.R1H.AND.(X(3)/32.00).LE.R1H) THEN
          L(7)= ((R1H)**V(7))*((NTOT-V(7))/NTOT)**(NTOT-V(7))*((X(3)
     +/32.0)**X(3))*((32.0-X(3))/32.0)**(32.0-X(3))*((X(2)/20.0)**X(2))
     +*((20.0-X(2))/20.0)**(20.0-X(2))

          IF (L(7).GT.LMAX) THEN
               LMAX= L(7)
               R1MAX= R1H
               R2MAX= X(2)/20.0
               R3MAX= X(3)/32.0
              R4MAX= R1H
          END IF
         END IF
      END IF
*     ****************************************************************
```

```
*     CALCULATION OF RHMLE VALUE WITH RESPECT TO CASE WHICH HAS LARGEST
*     MAXIMUM LIKELIHOOD IN HARD CASE
      R1H= R1MAX
      R2H= R2MAX
      R3H= R3MAX
      R4H= R4MAX
      Q1H= R1H
      Q2H= R2H/Q1H
      Q3H= R3H/Q1H
      Q4H= R4H/Q1H
      RHMLE(I)= Q1H*Q2H*Q3H*Q4H
*     ****************************************************************
*     WRITING AFTER EACH CALCULATION
   50 WRITE (16,*) RHMLE(I)
   60 CONTINUE
      STOP
      END
```

# APPENDIX B. PROGRAM RANVEC

```
      ******************************************************************
      PROGRAM RANVEC
      ******************************************************************
*         THIS IS THE PROGRAM TO GENARETE RANDOM NUMBERS FROM BINOMIAL
*     DISTRIBUTION.THE PROGRAM READS PROBABILITIES OF BEING SUCCESSFUL
*     IN FOUR TESTS INTERACTIVELY. IT GENERATES UNIFORMLY DISTRIBUTED
*     RANDOM NUMBERS  WITH THESE PROBABILITIES ACORDING TO SAMPLE SIZE
*     OF EACH TEST. FOR EACH TEST PROGRAM COUNTS UNIFORMLY DISTRIBUTED
*     RANDOM  NUMBERS, WHICH  HAVE  GREATER THAN  OR EQUAL PROBABILITY
*     WITH RESPECT TO THE GIVEN PROBABILITY FOR THAT TEST.TOTAL COUNTS
*     GIVE US  SUCCESSFUL ITEM NUMBERS  FOR EACH TEST. PROGRAM UPDATES
*     SEEDS AND CALLS SUBROUTINE RANNUM IN EACH ITERATION. THE PROGRAM
*     GENERATES 5000 SUCCESS VECTOR AND WRITES THEM TO AN OUTPUT FILE
*     CALLED SUSVECT.
*     ******************************************************************
*     VARIABLES .
*     ******************************************************************
*     PSIM   : PROBABILITY OF SUCCESS IN MANUFACTURER TEST.
*     PSITH  : PROBABILITY OF SUCCESS IN TEMP. AND HUMIDITY TEST.
*     PSIV   : PROBABILITY OF SUCCESS IN VIBRATION TEST
*     PSIA   : PROBABILITY OF SUCCESS IN ALTITUDE TEST
*     NUM1   : COUNTER FOR MANUFACTURER TEST
*     NUM2   : COUNTER FOR TEMP. AND HUMIDITY TEST
*     NUM3   : COUNTER FOR VIBRATION TEST
*     NUM4   : COUNTER FOR ALTIUTDE TEST
*     X      : BINOMIALLY DISTRIBUTED RANDOM NUMBER FOR MANUFACTURER
*              TEST
*     V      : BINOMIALLY DISTRIBUTED RANDOM NUMBER FOR TEMP.AND
*              HUMUDITY TEST
```

```
*     Y      : BINOMIALLY DISTRIBUTED RANDOM NUMBER FOR VIBRATION TEST
*     Z      : BINOMIALLY DISTRIBUTED RANDOM NUMBER FOR ALTITUDE TEST
*     A      : UNIFORMLY DISTRIBUTED RANDOM NUMBER FOR MANUFACTURER
*              TEST
*     B      : UNIFORMLY DISTRIBUTED RANDOM NUMBER FOR TEMP.AND
*              HUMIDITY TEST
*     C      : UNIFORMLY DISTRIBUTED RANDOM NUMBER FOR VIBRATION TEST
*     D      : UNIFORMLY DISTRIBUTED RANDOM NUMBER FOR ALTITUDE TEST
*     ISEED  : SEED NUMBER FOR MANUFACTURER TEST
*     KSEED  : SEED NUMBER FOR TEMP. AND HUMIDITY TEST
*     LSEED  : SEED NUMBER FOR VIBRATION TEST
*     MSEED  : SEED NUMBER FOR ALTITUDE TEST
*     ****************************************************************
*     TYPE DECLARATION
      REAL PSIM,PSITH,PSIV,PSIA,NUM1,NUM2,NUM3,NUM4,X(5000),V(5000)
     +,Y(5000),Z(5000),A,B,C,D
      INTEGER ISEED,KSEED,LSEED,MSEED
*     *****************************************************************
*     INITIALIZATION
      ISEED = 45267
      KSEED = 113234
      LSEED = 435
      MSEED = 1
*     *****************************************************************
*     READING TEST PROBABILITIES OF BEING SUCCESSFUL IN EACH TEST
      WRITE(*,*)'PLEASE WRITE PROBABILITY OF BEING SUCCESSFUL IN
     + MANUFACTURER TEST'
      READ (*,*) PSIM
      WRITE(*,*)'PLEASE WRITE PROBABILITY OF BEING SUCCESSFUL IN
     + TEMPERATURE AND HUMIDITY TEST'
      READ (*,*) PSITH
      WRITE(*,*)'PLEASE WRITE PROBABILITY OF BEING SUCCESSFUL IN
     + VIBRATION TEST'
      READ (*,*) PSIV
```

```
      WRITE(*,*)'PLEASE WRITE PROBABILITY OF BEING SUCCESSFUL IN
     + ALTITIDE TEST'
      READ (*,*) PSIA
*     ******************************************************************
*     OPENING AN OUTPUT FILE TO WRITE THE RESULTS
      CALL EXCMS ('FILEDEF 16 DISK SUSVECT DATA A1')
*     ******************************************************************
*     GENERATION
      DO 50 J=1,5000,1
*     ******************************************************************
*     INITIALIZATION IN EACH ITERATION
          NUM1= 0.0
          NUM2= 0.0
          NUM3= 0.0
          NUM4= 0.0
*     ******************************************************************
*     SEEDS UPDATATION IN EACH ITERATION
          ISEED=ISEED+17
          KSEED=KSEED+1356
          LSEED=LSEED+1
          MSEED=MSEED+789
*     ******************************************************************
*     GENERATION OF SUCCESSFUL NUMBER OF ITEMS IN MANUFACTURER TEST
          DO 10 I=1,20,1
              CALL RANNUM (1,ISEED,0.0,1.0,0.0,A)
              IF ( A.LT.PSIM ) THEN
                  NUM1=NUM1+1
              END IF
              X(J) = NUM1
  10      CONTINUE
*     ******************************************************************
*     GENERATION OF SUCCESSFUL NUMBER OF ITEMS IN TEMP. AND HUM. TEST
          DO 20 K=1,20,1
              CALL RANNUM (1,KSEED,0.0,1.0,0.0,B)
```

```
            IF ( B.LT.PSITH ) THEN
                 NUM2=NUM2+1
            END IF
            V(J) = NUM2
 20       CONTINUE
*    *****************************************************************
*    GENERATION OF SUCCESSFUL NUMBER OF ITEMS IN VIBRATION TEST
          DO 30 L=1,32,1
            CALL RANNUM (1,LSEED,0.0,1.0,0.0,C)
            IF ( C.LT.PSIV ) THEN
                 NUM3=NUM3+1
            END IF
            Y(J) = NUM3
 30       CONTINUE
*    *****************************************************************
*    GENERATION OF SUCCESSFUL NUMBER OF ITEMS IN ALTITUDE TEST
          DO 40 M=1,20,1
            CALL RANNUM (1,MSEED,0.0,1.0,0.0,D)
            IF ( D.LT.PSIA ) THEN
                 NUM4=NUM4+1
            END IF
            Z(J) = NUM4
 40       CONTINUE
*    *****************************************************************
*    WRITING THE RESULTS TO AN OUTPUT FILE AS 4 TUPLE
          WRITE (16,1) X(J),V(J),Y(J),Z(J)
  1       FORMAT (1X,F12.7,4X,F12.7,4X,F12.7,4X,F12.7,4X)
 50  CONTINUE
     STOP
     END
*    ******************************************************************
     SUBROUTINE RANNUM(DISTN, SEED, RPARM1, RPARM2, IPARM, X)
*    ******************************************************************
*        THIS SUBROUTINE IS A PART OF SIMUTIL FORTRAN WHICH IS WRITTEN
```

```
*     BY DR. M. P. BAILEY.  THIS SUBROUTINE PROVIDES AN INTERFACE WITH
*     THE  LLRANDOMII  ROUTINES PROVIDED IN  THE NONIMSL  LIBRARY.  THE
*     PARAMETER REQUIRMENTS  AND CALLING PROCEDURES ARE AS FOLLOWS:
*     DISTN = DISTRIBUTION TYPE YOU WANT TO SELECT AN INTEGER BETWEEN 1
*             AND 7.
*     SEED = THE RANDOM NUMBER SEED YOU WISH TO USE.
*     RPARM1, RPARM2, AND IPARM ARE REAL AND INTEGER PARAMETERS PASSED
*     TO THE ROUTINE WITH MEANINGS WHICH VARY WITH THE TYPE OF DISTRI_
*     BUTION YOU DESIRE.
*     X = THE RETURNED RANDOM NUMBER, IT IS ALWAYS REAL.
*     DISTRIBUTION NUMBERS AND THE ASSOCIATED PARM DEFINITIONS
*     1--UNIFORM ON THE INTERVAL RPARM1 TO RPARM2.
*     2--NORMAL WITH MEAN RPARM1 AND VARIANCE RPARM2.
*     3--EXPONENTIAL WITH RATE RPARM1.
*     4--COUCHY WITH A = RPARM1 AND B = RPARM2.
*     5--GAMMA WITH SHAPE RPARM2 AND RATE RPARM1.
*     6--POISSON WITH RATE RPARM1.
*     7--GEOMETRIC WITH P = RPARM1.
*     *****************************************************************
*     TYPE DECLARATION
      REAL RPARM1,RPARM2,X,TEMP,VARIAT(1)
      INTEGER DISTN, SEED, IPARM, N
*     *****************************************************************
      IF (DISTN.LE.0.OR.DISTN.GT.8) THEN
         WRITE(10, *) 'ILLEGAL CALL TO RANNUM, BAD DISTN'
         STOP
      ENDIF
      GOTO (10, 20, 30, 40, 50, 60, 70), DISTN
*     *****************************************************************
*     GENERATE A UNIFORM BETWEEN RPARM1 AND RPARM2
 10   CONTINUE
      IF (RPARM1 - RPARM2.EQ.0) THEN
         WRITE(10, *) 'ILLEGAL EQUAL RPARMS IN RANNUM'
         STOP
```

```
      ENDIF
      IF (RPARM1.GT.RPARM2) THEN
         TEMP = RPARM1
         RPARM1 = RPARM2
         RPARM2 = TEMP
      ENDIF
      CALL LRND(SEED, VARIAT, 1, 1, 0)
      VARIAT(1) = RPARM1 + (RPARM2 - RPARM1) * VARIAT(1)
      GOTO 80
*     ****************************************************************
*     GENERATE A NORMAL WITH MEAN RPARM1 AND STDDEV RPARM2
 20   CALL LNORM(SEED, VARIAT, 1, 1, 0)
      VARIAT(1) = (VARIAT(1) * RPARM2) + RPARM1
      GOTO 80
*     ****************************************************************
*     GENERATE AN EXPONENTIAL WITH RATE (1/MEAN) RPARM1
 30   CONTINUE
      IF (RPARM1.EQ.0) THEN
         WRITE(10, *) 'ILLEGAL ZERO RATE IN RANNUM'
         STOP
      ENDIF
      CALL LEXPN(SEED, VARIAT, 1, 1, 0)
      VARIAT(1) = VARIAT(1) / RPARM1
      GOTO 80
*     ****************************************************************
*     GENERATE A COUCHY WITH A = RPARM1 AND B = RPARM2
 40   CONTINUE
      IF (RPARM2.LE.0) THEN
         WRITE(10, *) 'ILLEGAL COUCHY SPREAD IN RANNUM, B = ',RPARM2
         STOP
      ENDIF
      CALL LCCHY(SEED, VARIAT, 1, 1, 0)
      VARIAT(1) = (VARIAT(1) * RPARM2) + RPARM1
```

```
      GOTO 80
*     *****************************************************************
*     GENERATE GAMMA WITH SHAPE RPARM2 AND RATE RPRAM1
 50   CONTINUE
      IF (RPARM1.LE.0) THEN
         WRITE(10, *) 'ILLEGAL NONPOSITIVE GAMMA RATE IN RANNUM'
         STOP
      ENDIF
      IF (RPARM2.LE.0) THEN
         WRITE(10, *) 'ILLEGAL SHAPE PARAMETER IN RANNUM'
         STOP
      ENDIF
      CALL LGAMA(SEED, VARIAT, 1, 1, 0, RPARM2)
      VARIAT(1) = VARIAT(1) * (1.0 / RPARM1)
      GOTO 80
*     *****************************************************************
*     GENERATE POISSON WITH RATE RPRAM1
 60   CONTINUE
      IF (RPARM1.LE.0) THEN
         WRITE(10, *) 'ILLEGAL POISSON RATE IN RANNUM'
         STOP
      ENDIF
      CALL LPOIS(SEED, VARIAT, 1, 1, 0, RPARM1)
      GOTO 80
*     *****************************************************************
*     GENERATE GEOMETRIC WITH P = RPRAM1
 70   CONTINUE
      IF (RPARM1.LE.0) THEN
         WRITE(10, *) 'ILLEGAL GEOM PROB IN RANNUM'
         STOP
      ENDIF
      CALL LGEOM(SEED, VARIAT, 1, 1, 0, RPARM1)
      GOTO 80
 80   CONTINUE
```

```
X = VARIAT(1)
END
```

# APPENDIX C. PROGRAM SORT

```
*     **********************************************************************
      PROGRAM SORT
*     **********************************************************************
*     THIS IS THE SORTING PROGRAM. PROGRAM USES BUBBLE SORT ALGORITHM.
*     PROGRAM READS ESTIMATED RELIABILITIES FROM AN INPUT FILE CALLED
*     RESULT. IT SORTS FROM  SMALLEST TO LARGEST, AND WRITES IN TO AN
*     OUTPUT FILE CALLED FRESULT WITH 95 % LOWER CONFIDENCE BOUND.
*     **********************************************************************
*     VARIABLES
*     **********************************************************************
*     A     : ESTIMATED RELIABILITY
*     FLAG  : INDICATOR VARIABLE TAKES VALUE ' OF ' AND ' OFF '
*     **********************************************************************
*     TYPE DECLARATION
      CHARACTER FLAG*3
      REAL A(5000)
      INTEGER I,N,J
*     **********************************************************************
*     OPENING AN INPUT AND AN OUTPUT FILE
      CALL EXCMS ('FILEDEF 9 DISK RESULT DATA A1')
      CALL EXCMS ('FILEDEF 15 DISK FRESULT DATA A1')
*     **********************************************************************
*     READING ESTIMATED RELIABILITIES
      DO 10 I=1,5000
         READ(9,*) A(I)
 10   CONTINUE
*     **********************************************************************
*     SORTING OPERATION
      N=I-1
```

```
      DO 30 I=N,2,-1
         FLAG='OFF'
         DO 20 J=1,I-1
            IF (A(J).GT.A(J+1)) THEN
               TEMP=A(J)
               A(J)=A(J+1)
               A(J+1)=TEMP
               FLAG='ON'
            END IF
 20      CONTINUE
         IF (FLAG.EQ.'OFF') THEN
            GO TO 40
         END IF
 30   CONTINUE
 40   CONTINUE
*     ****************************************************************
*     WRITING THE RESULTS IN ASCENDING ORDER TO OUTPUT FILE
      DO 50 I=1,5000
         WRITE (15,*) A(I)
 50   CONTINUE
      WRITE (15,1) A(250)
 1    FORMAT (///,15X,'95 % LOWER CONFIDENCE BOUND IS',1X,F12.7)
      STOP
      END
```

# APPENDIX D. PROGRAM INITIAL

```
*       ****************************************************************
        PROGRAM INITIAL
*       ****************************************************************
*           THIS PROGRAM, CALCULATES  INITIAL  GUESSES FOR PARAMETERS IN
*       LOGLINEAR MODEL BY MEANS OF PROGRAM PARAM WHICH IS IN APPENDIX E.
*       PROGRAM SUPPLIES PARTIAL SUMS OF EXPECTATION, TO SOLVE EQUATIONS
*       IN  PROGRAM  PARAM. IT READS INTERACTIVELY NUMBER OF FAILURES IN
*       TESTS. PROGRAM WRITES RESULTS TO AN OUTPUT FILE CALLED EXPECT.
*       ****************************************************************
*       VARIABLES
*       ****************************************************************
*       FOM   : NUMBER OF FAILURES IN MANUFACTURER TEST
*       FOTH  : NUMBER OF FAILURES IN TEMPERATURE AND HUMIDITY TEST
*       FOV   : NUMBER OF FAILURES IN VIBRATION TEST
*       FOA   : NUMBER OF FAILURES IN ALTITUDE TEST
*       P1    : SUCCESS RATIO FOR MANUFACTURER TEST
*       P2    : SUCCESS RATIO FOR TEMPERATURE AND HUMIDITY TEST
*       P3    : SUCCESS RATIO FOR VIBRATION TEST
*       P4    : SUCCESS RATIO FOR ALTITUDE TEST
*       Q1    : FAILURE RATIO FOR MANUFACTURER TEST
*       Q2    : FAILURE RATIO FOR TEMPERATURE AND HUMIDITY TEST
*       Q3    : FAILURE RATIO FOR VIBRATION TEST
*       Q4    : FAILURE RATIO FOR ALTITUDE TEST
*       X1    : PARTIAL SUM OF EXPECTED NUMBERS IN CELLS FOR EQUATION 1
*       X2    : PARTIAL SUM OF EXPECTED NUMBERS IN CELLS FOR EQUATION 2
*       X3    : PARTIAL SUM OF EXPECTED NUMBERS IN CELLS FOR EQUATION 3
*       X4    : PARTIAL SUM OF EXPECTED NUMBERS IN CELLS FOR EQUATION 4
*       ****************************************************************
*       TYPE DECLARATION
```

```
      REAL FOM,FOTH,FOV,FOA,P1,P2,P3,P4,Q1,Q2,Q3,Q4,PROD,X1,X2,X3,X4
*     ****************************************************************
*     OPENING AN OUTPUT FILE
      CALL EXCMS ('FILEDEF 13 DISK EXPECT DATA A1')
*     ****************************************************************
*     READING NUMBER OF FAILURES IN EACH TEST INTERACTIVELY
      WRITE(*,*)'PLEASE WRITE NUMBER OF FAILURES IN MANUFACTURER TEST'
      READ(*,*) FOM
      WRITE(*,*)'PLEASE WRITE NUMBER OF FAILURES IN TEMP.AND HUM.TEST'
      READ(*,*) FOTH
      WRITE(*,*)'PLEASE WRITE NUMBER OF FAILURES IN VIBRATION TEST'
      READ(*,*) FOV
      WRITE(*,*)'PLEASE WRITE NUMBER OF FAILURES IN ALTITUDE TEST'
      READ(*,*) FOA
*     ****************************************************************
*     CALACULATION OF SUCCESS RATIOS
      P1=(20.0-FOM)/8.00
      P2=(20.0-FOTH)/8.00
      P3=(32.0-FOV)/8.00
      P4=(20.0-FOA)/8.00
*     ****************************************************************
*     CALCULATION OF FAILURE RATIOS
      Q1=FOM/8.00
      Q2=FOTH/8.00
      Q3=FOV/8.00
      Q4=FOA/8.00
*     ****************************************************************
*     CALCULATION PARTIAL SUMS OF EXPECTATION FOR PROGRAM PARAM
      PROD=2.0*P3+2.0*Q3+2.0*P4+2.0*Q4
      X1=4.0*P1+4.0*P2+PROD
      X2=4.0*P1+4.0*Q2+PROD
      X3=4.0*Q1+4.0*P2+PROD
```

```
      X4=4.0*Q1+4.0*Q2+PROD
*     *****************************************************************
*     WRITING THE RESULTS TO AN OUTPUT FILE
      WRITE (13,*) X1,X2,X3,X4
      STOP
      END
```

# APPENDIX E. PROGRAM PARAM

```
*       *****************************************************************
        PROGRAM PARAM
*       *****************************************************************
*          THIS IS THE PROGRAM TO CALCULATE PARAMETERS OF LOGLINEAR MODEL
*       WITH IMSL SUBROUTINE. IT TAKES PARTIAL SUMS FROM PROGRAM INITIAL
*       AND SOLVES  FOUR  NONLINEAR EQUATIONS, WHICH HAVE FOUR UNKNOWNS.
*       THE PROGRAM USES AN IMSL SUBROUTINE CALLED DNEQNF TO SOLVE THIS
*       EQUATION.IT WRITES SOLUTIONS OF EQUATIONS TO OUTPUT FILE CALLED
*       PARAM DATA.
*       *****************************************************************
*       VARIABLES
*       *****************************************************************
*       ITMAX   : MAXIMUM ITERATION NUMBER.
*       N       : PARAMETER
*       XGUESS  : INITIAL GUESS FOR FOUR NONLINEAR EQUATIONS.
*       F       : NONLINEAR EQUATIONS.
*       *****************************************************************
*       TYPE DECLARATION
*       *****************************************************************
        PARAMETER (N=4)
        REAL*8 ERRREL
        INTEGER ITMAX,N

        INTEGER K
        REAL*8  FNORM,X(N),XGUESS(N)
        EXTERNAL ACN
*       *****************************************************************
*       OPENING A FILE FOR WRITING RESULTS
```

```
      CALL EXCMS ('FILEDEF 9 DISK PARAM DATA A')
*     ******************************************************************
*     INITIAL GUESS
      DATA XGUESS/3.5D0,3.5D0,3.5D0,3.5D0/
*     ******************************************************************
*     INITIALIZATION
      ERRREL = 0.0001D0
      ITMAX = 10000
*     ******************************************************************
*     CALLING OF IMSL SUBROUTINE
      CALL DNEQNF ( ACN,ERRREL,N,ITMAX,XGUESS,X,FNORM)
*     ******************************************************************
*     RESULTS
      WRITE (9,1) (X(K),K=1,N),FNORM
    1 FORMAT('THE SOLUTION TO THE SYSTEM IS',/,'X=(',4F8.2,')',/,'WITH
     +FNORM=',F8.2,//)
      END
*     ******************************************************************
      SUBROUTINE ACN (X,F,N)
*     ******************************************************************
*     VARIABLES
*     X   : INITIAL GUESS
*     F   : NONLINEAR EQUATIONS
*     ******************************************************************
*     TYPE DECLARATION
      REAL*8 X(N),F(N)
      INTEGER N
*     ******************************************************************
*     1 ST EQUATION
      F(1)=DEXP(X(1))*(DEXP(X(2)+X(3)+X(4))+
     +     (1/DEXP(X(2)+X(3)+X(4)))+
     +     (DEXP(X(2)+X(3))/DEXP(X(4)))+
     +     (DEXP(X(2)+X(4))/DEXP(X(3)))+(DEXP(X(3)
     +     +X(4))/DEXP(X(2)))+(DEXP(X(2))/DEXP(X(3)+X(4)))+
```

```
     +     (DEXP(X(3))/DEXP(X(2)
     +      +X(3)))+(DEXP(X(4))/DEXP(X(2)+X(3))))-LOG(32.00)
*      *****************************************************************
*      2 ST EQUATION
       F(2)=DEXP(X(1)+X(2))*(DEXP(X(3)+X(4))+
     +     (1/DEXP(X(3)+X(4)))+(DEXP(X(3))/
     +     DEXP(X(4)))+(DEXP(X(4))/DEXP(X(3))))-LOG(23.00)
*      *****************************************************************
*      3 ST EQUATION
       F(3)=DEXP(X(1)+X(3))*(DEXP(X(2)+X(4))+
     +     (1/DEXP(X(2)+X(4)))+(DEXP(X(2))/
     +     DEXP(X(4)))+(DEXP(X(4))/DEXP(X(2))))-LOG(23.00)
*      *****************************************************************
*      4 ST EQUATION
       F(4)=DEXP(X(1)+X(4))*(DEXP(X(2)+X(3))+
     +      (1/DEXP(X(2)+X(3)))+(DEXP(X(2))/
     +      DEXP(X(3)))+(DEXP(X(3))/DEXP(X(2))))-LOG(14.00)
       RETURN
       END
```

# APPENDIX F. PROGRAM LLMDEP

```
*       ************************************************************************
        PROGRAM LLMDEP
*       ************************************************************************
*              THIS IS THE  FORTRAN PROGRAM TO CALCULATE THE RELIABILITY
*       OF  PYROTECHNIC DEVICE.  IT  ASSUMES  THAT THERE IS A DEPENDENCE
*       BETWEEN MANUFACTURER AND ENVIRONMENT TESTS. EXPECTATION-MAXIMIZA_
*       TION ALGORITHM IS USED IN THIS MODULE.   ALGORITHM  STARTS WITH
*       INITIAL GUESSES FOR PARAMETERS  AND ESTIMATES EXPECTATIONS. IT
*       RECALCULATES CELL PROBABILITIES AND UPDATES EXPECTATIONS UNTIL
*       IT CONVERGES.  AN ITERATIVE NEWTON AND RAPHSON PROCEDURE IS USED
*       DURING UPDATATION OF CELL PROBABILITIES.  THIS PROCEDURE IS DONE
*       BY A SUBROUTINE NAMED UCPROB.
*       ***********************************************************************
*       VARIABLES
*       ***********************************************************************
*       FOM    : NUMBER OF FAILURES IN MANUFACTURER TEST.
*       FOTH   : NUMBER OF FAILURES IN TEMP. AND HUMIDITY TEST.
*       FOV    : NUMBER OF FAILURES IN VIBRATION TEST.
*       FOA    :  NUMBER OF FAILURES IN ALTITUDE TEST.
*       MU     : OVERALL MEAN.
*       LP1    : MEAN EFFECT OF MANUFACTURER TEST.
*       LP2    : MEAN EFFECT OF MANUFACTURER TEST.
*       LP3    : MEAN EFFECT OF MANUFACTURER TEST.
*       LP4    : MEAN EFFECT OF MANUFACTURER TEST.
*       TETHA  : TWO WAY INTERACTION TERMS
*       RHMLE  : RELIABILITY OF DEVICE
*       MPPPP  : CELL FREQUENCY WITH RESPECT TO TESTS RESULTS.
*       MPOPO  : SUM OF CELL FREQUENCIES WHICH HAVE PASSED DEVICES
*                MANUFACTURER AND VIBRATION TEST.
```

```
*     MP000  : SUM OF CELL FREQUENCIES WHICH HAVE PASSED DEVICES
*              MANUFACTURER TEST.
*     IP     : INITIAL PROBABILITY VECTOR
*     FP     : UPDATED (FINAL) PROBABILITY VECTOR
*     Y      : CELL EXPECTATION VECTOR
*     A,B,C  : SOME TERMS TO MAKE THE CALCULATIONS EASY.
*     EXPPPP : EXPECTED NUMBER OF DEVICES IN CELL WHICH
*              HAS A RESULTANT VECTOR ( P P P P ) IN MANUFACTURER,
*              TEMPERATURE,  VIBRATION AND ALTITUDE TEST RESPECTIVELY
*     FLAG   : INDICATOR VARIABLE OF CONVERGENCE FOR PARAMETERS.
*     RHMLE  : ESTIMATED RELIABILITY OF PYROTECHNIC DEVICE.
*     ***************************************************************
*     TYPE DECLERATION
      PARAMETER (K=10000)
      LOGICAL FLAG(16)
      INTEGER I
      REAL FOM, FOTH, FOV, FOA, LP1, LP2, LP3, LP4, TETHA
      REAL MPPPP,MPPPF,MPPFP,MPPFF,
     +     MPFPP,MPFPF,MPFFP,MPFFF,
     +     MFPPP,MFPPF,MFPFP,MFPFF,
     +     MFFPP,MFFPF,MFFFP,MFFFF
      REAL EXPPPP(K),EXPPPF(K),EXPPFP(K),EXPPFF(K),
     +     EXPFPP(K),EXPFPF(K),EXPFFP(K),EXPFFF(K),
     +     EXFPPP(K),EXFPPF(K),EXFPFP(K),EXFPFF(K),
     +     EXFFPP(K),EXFFPF(K),EXFFFP(K),EXFFFF(K)
      REAL MP000, MF000, MPP00, MP0P0, MP00P, A, B, C
      REAL IP(16,1),FP(16,1),X(16)
      REAL Y(16)
      COMMON / PROB / Y
*     ***************************************************************
      CALL EXCMS ('FILEDEF 15 DISK END DATA A1 ' )
*     ***************************************************************
*     INITIALIZATION
      RHMLE = 0.0
```

```
      EPS = 0.001
      DO 10 L=1,16
         FLAG(L)=.FALSE.
 10   CONTINUE
*     ******************************************************************
*     READING THE NUMBER OF FAILURES IN EACH TEST INTERACTIVELY
      WRITE(*,*)'PLEASE ENTER THE # OF FAILURES IN MANUFACTURER TEST'
      READ(*,*) FOM
      WRITE(*,*)'PLEASE ENTER THE # OF FAILURES IN TEMP. AND HUM. TEST'
      READ(*,*) FOTH
      WRITE(*,*)'PLEASE ENTER THE # OF FAILURES IN VIBRATION TEST'
      READ(*,*) FOV
      WRITE(*,*)'PLEASE ENTER THE # OF FAILURES IN ALTITUDE TEST'
      READ(*,*) FOA
*     ******************************************************************
*     READING THE INITIAL GUESS FOR EACH CELL IN HYPOTHETICAL
*     CONTINGENCY TABLE
      WRITE (*,*)'PLEASE ENTER INITIAL GUESS FOR MPPPP'
      READ(*,*) MPPPP
      WRITE (*,*)'PLEASE ENTER INITIAL GUESS FOR MPPPF'
      READ(*,*) MPPPF
      WRITE (*,*)'PLEASE ENTER INITIAL GUESS FOR MPPFP'
      READ(*,*) MPPFP
      WRITE (*,*)'PLEASE ENTER INITIAL GUESS FOR MPPFF'
      READ(*,*) MPPFF
      WRITE (*,*)'PLEASE ENTER INITIAL GUESS FOR MPFPP'
      READ(*,*) MPFPP
      WRITE (*,*)'PLEASE ENTER INITIAL GUESS FOR MPFPF'
      READ(*,*) MPFPF
      WRITE (*,*)'PLEASE ENTER INITIAL GUESS FOR MPFFP'
      READ(*,*) MPFFP
      WRITE (*,*)'PLEASE ENTER INITIAL GUESS FOR MPFFF'
      READ(*,*) MPFFF
      WRITE (*,*)'PLEASE ENTER INITIAL GUESS FOR MFPPP'
```

```
      READ(*,*) MFPPP
      WRITE (*,*)'PLEASE ENTER INITIAL GUESS FOR MFPPF'
      READ(*,*) MFPPF
      WRITE (*,*)'PLEASE ENTER INITIAL GUESS FOR MFPFP'
      READ(*,*) MFPFP
      WRITE (*,*)'PLEASE ENTER INITIAL GUESS FOR MFPFF'
      READ(*,*) MFPFF
      WRITE (*,*)'PLEASE ENTER INITIAL GUESS FOR MFFPP'
      READ(*,*) MFFPP
      WRITE (*,*)'PLEASE ENTER INITIAL GUESS FOR MFFPF'
      READ(*,*) MFFPF
      WRITE (*,*)'PLEASE ENTER INITIAL GUESS FOR MFFFP'
      READ(*,*) MFFFP
      WRITE (*,*)'PLEASE ENTER INITIAL GUESS FOR MFFFF'
      READ(*,*) MFFFF
*     ****************************************************************
*     CALCULATION OF INITIAL PROBABALITIES USING CELL FREQUENCIES
      IP(1,1) = MPPPP/92.00
      IP(2,1) = MPPPF/92.00
      IP(3,1) = MPPFP/92.00
      IP(4,1) = MPPFF/92.00
      IP(5,1) = MPFPP/92.00
      IP(6,1) = MPFPF/92.00
      IP(7,1) = MPFFP/92.00
      IP(8,1) = MPFFF/92.00
      IP(9,1) = MFPPP/92.00
      IP(10,1)= MFPPF/92.00
      IP(11,1)= MFPFP/92.00
      IP(12,1)= MFPFF/92.00
      IP(13,1)= MFFPP/92.00
      IP(14,1)= MFFPF/92.00
      IP(15,1)= MFFFP/92.00
```

```
      IP(16,1)= MFFFF/92.00
*     ***********************************************************************
*     DETERMINATION OF INITIAL FREQUENCIES FOR LIKELIHOOD ESTIMATION
      Y(1) = MPPPP
      Y(2) = MPPPF
      Y(3) = MPPFP
      Y(4) = MPPFF
      Y(5) = MPFPP
      Y(6) = MPFPF
      Y(7) = MPFFP
      Y(8) = MPFFF
      Y(9) = MFPPP
      Y(10)= MFPPF
      Y(11)= MFPFP
      Y(12)= MFPFF
      Y(13)= MFFPP
      Y(14)= MFFPF
      Y(15)= MFFFP
      Y(16)= MFFFF
*     ***********************************************************************
*     CALL A SUBROUTINE WHICH UPDATES CELL PROBABILITIES USING NEWTON
*     AND RAPHSON PROCEDURE
      CALL UCPROB( IP,FP )
*     ***********************************************************************
*     UPDATATION OF CELL FREQUENCIES
      MPPPP=FP(1,1)*92.00
      MPPPF=FP(2,1)*92.00
      MPPFP=FP(3,1)*92.00
      MPPFF=FP(4,1)*92.00
      MPFPP=FP(5,1)*92.00
      MPFPF=FP(6,1)*92.00
      MPFFP=FP(7,1)*92.00
      MPFFF=FP(8,1)*92.00
      MFPPP=FP(9,1)*92.00
```

```
      MFPPF=FP(10,1)*92.00
      MFPFP=FP(11,1)*92.00
      MFPFF=FP(12,1)*92.00
      MFFPP=FP(13,1)*92.00
      MFFPF=FP(14,1)*92.00
      MFFFP=FP(15,1)*92.00
      MFFFF=FP(16,1)*92.00
*     *****************************************************************
*     INITIAL EXPECTATIONS
      EXPPPP(1)=MPPPP
      EXPPPF(1)=MPPPF
      EXPPFP(1)=MPPFP
      EXPPFF(1)=MPPFF
      EXPFPP(1)=MPFPP
      EXPFPF(1)=MPFPF
      EXPFFP(1)=MPFFP
      EXPFFF(1)=MPFFF
      EXFPPP(1)=MFPPP
      EXFPPF(1)=MFPPF
      EXFPFP(1)=MFPFP
      EXFPFF(1)=MFPFF
      EXFFPP(1)=MFFPP
      EXFFPF(1)=MFFPF
      EXFFFP(1)=MFFFP
      EXFFFF(1)=MFFFF
*     *****************************************************************
      MP000=MPPPP+MPPPF+MPPFP+MPPFF+MPFPP+MPFPF+MPFFP+MPFFF
      MF000=MFPPP+MFPPF+MFPFP+MFPFF+MFFPP+MFFPF+MFFFP+MFFFF
      MPP00=MPPPP+MPPPF+MPPFP+MPPFF
      MP0P0=MPPPP+MPPPF+MPFPP+MPFPF
      MP00P=MPPPP+MPPFP+MPFPP+MPFFP
      A=92.00-MPP00
      B=92.00-MP0P0
```

```
      C=92.00-MP00P
*     ******************************************************************
*     NEXT EXPECTATIONS
      DO 20 I=2,K
        EXPPPP(I)=(20.00-FOM)*(MPPPP/MP000)+(20.00-FOTH)*(MPPPP/MPP00)+
     +  (32.00-FOV)*(MPPPP/MP0P0)+(20.00-FOA)*(MPPPP/MP00P)
        IF(ABS(EXPPPP(I)-EXPPPP(I-1)).LE.EPS) THEN
          FLAG(1)=.TRUE.
        END IF
        EXPPPF(I)=(20.00-FOM)*(MPPPF/MP000)+(20.00-FOTH)*(MPPPF/MPP00)+
     +  (32.00-FOV)*(MPPPF/MP0P0)+FOA*(MPPPF/C)
        IF(ABS(EXPPPF(I)-EXPPPF(I-1)).LE.EPS) THEN
          FLAG(2)=.TRUE.
        END IF
        EXPPFP(I)=(20.00-FOM)*(MPPFP/MP000)+(20.00-FOTH)*(MPPFP/MPP00)+
     +  FOV*(MPPFP/B)+(20.00-FOA)*(MPPFP/MP00P)
        IF(ABS(EXPPFP(I)-EXPPFP(I-1)).LE.EPS) THEN
           FLAG(3)=.TRUE.
        END IF
        EXPPFF(I)=(20.00-FOM)*(MPPFF/MP000)+(20.00-FOTH)*(MPPFF/MPP00)+
     +  FOV*(MPPFF/B)+FOA*(MPPFF/C)
        IF(ABS(EXPPFF(I)-EXPPFF(I-1)).LE.EPS) THEN
           FLAG(4)=.TRUE.
        END IF
        EXPFPP(I)=(20.00-FOM)*(MPFPP/MP000)+FOTH*(MPFPP/A)+(32.00-FOV)*
     +  (MPFPP/MP0P0)+(20.00-FOA)*(MPFPP/MP00P)
        IF(ABS(EXPFPP(I)-EXPFPP(I-1)).LE.EPS) THEN
           FLAG(5)=.TRUE.
        END IF
        EXPFPF(I)=(20.00-FOM)*(MPFPF/MP000)+FOTH*(MPFPF/A)+(32.00-FOV)*
     +  (MPFPF/MP0P0)+FOA*(MPFPF/C)
        IF(ABS(EXPFPF(I)-EXPFPF(I-1)).LE.EPS) THEN
           FLAG(6)=.TRUE.
        END IF
```

```
      EXPFFP(I)=(20.00-FOM)*(MPFFP/MP000)+FOTH*(MPFFP/A)+FOV*(MPFFP/B)+
     + (20.00-FOA)*(MPFFP/MP00P)
      IF(ABS(EXPFFP(I)-EXPFFP(I-1)).LE.EPS) THEN
         FLAG(7)=.TRUE.
      END IF
      EXPFFF(I)=(20.00-FOM)*(MPFFF/MP000)+FOTH*(MPFFF/A)+FOV*(MPFFF/B)+
     + FOA*(MPFFF/C)
      IF(ABS(EXPFFF(I)-EXPFFF(I-1)).LE.EPS) THEN
         FLAG(8)=.TRUE.
      END IF
      EXFPPP(I)=FOM*(MFPPP/MF000)+FOTH*(MFPPP/A)+FOV*(MFPPP/B)+FOA*
     + (MFPPP/C)
      IF(ABS(EXFPPP(I)-EXFPPP(I-1)).LE.EPS) THEN
         FLAG(9)=.TRUE.
      END IF
      EXFPPF(I)=FOM*(MFPPF/MF000)+FOTH*(MFPPF/A)+FOV*(MFPPF/B)+FOA*
     + (MFPPF/C)
      IF(ABS(EXFPPF(I)-EXFPPF(I-1)).LE.EPS) THEN
         FLAG(10)=.TRUE.
      END IF
      EXFPFP(I)=FOM*(MFPFP/MF000)+FOTH*(MFPFP/A)+FOV*(MFPFP/B)+FOA*
     + (MFPFP/C)
      IF(ABS(EXFPFP(I)-EXFPFP(I-1)).LE.EPS) THEN
         FLAG(11)=.TRUE.
      END IF
      EXFPFF(I)=FOM*(MFPFF/MF000)+FOTH*(MFPFF/A)+FOV*(MFPFF/B)+FOA*
     + (MFPFF/C)
      IF(ABS(EXFPFF(I)-EXFPFF(I-1)).LE.EPS) THEN
         FLAG(12)=.TRUE.
      END IF
      EXFFPP(I)=FOM*(MFFPP/MF000)+FOTH*(MFFPP/A)+FOV*(MFFPP/B)+FOA*
     + (MFFPP/C)
      IF(ABS(EXFFPP(I)-EXFFPP(I-1)).LE.EPS) THEN
         FLAG(13)=.TRUE.
```

```
      END IF
      EXFFPF(I)=FOM*(MFFPF/MF000)+FOTH*(MFFPF/A)+FOV*(MFFPF/B)+FOA*
     + (MFFPF/C)
      IF(ABS(EXFFPF(I)-EXFFPF(I-1)).LE.EPS) THEN
         FLAG(14)=.TRUE.
      END IF
      EXFFFP(I)=FOM*(MFFFP/MF000)+FOTH*(MFFFP/A)+FOV*(MFFFP/B)+FOA*
     + (MFFFP/C)
      IF(ABS(EXFFFP(I)-EXFFFP(I-1)).LE.EPS) THEN
         FLAG(15)=.TRUE.
      END IF
      EXFFFF(I)=FOM*(MFFFF/MF000)+FOTH*(MFFFF/A)+FOV*(MFFFF/B)+FOA*
     + (MFFFF/C)
      IF(ABS(EXFFFF(I)-EXFFFF(I-1)).LE.EPS) THEN
         FLAG(16)=.TRUE.
      END IF
*     ****************************************************************
      MPPPP = EXPPPP(I)
      MPPPF = EXPPPF(I)
      MPPFP = EXPPFP(I)
      MPPFF = EXPPFF(I)
      MPFPP = EXPFPP(I)
      MPFPF = EXPFPF(I)
      MPFFP = EXPFFP(I)
      MPFFF = EXPFFF(I)
      MFPPP = EXFPPP(I)
      MFPPF = EXFPPF(I)
      MFPFP = EXFPFP(I)
      MFPFF = EXFPFF(I)
      MFFPP = EXFFPP(I)
      MFFPF = EXFFPF(I)
      MFFFP = EXFFFP(I)
```

```
      MFFFF = EXFFFF(I)
*     ********************************************************************
*     CHECK FOR THE STOPING CONDITION
      IF( FLAG(1).AND.FLAG(2).AND.FLAG(3).AND.FLAG(4).AND.FLAG(5)
     + .AND.FLAG(6).AND.FLAG(7).AND.FLAG(8).AND.FLAG(9).AND.FLAG(10)
     + .AND.FLAG(11).AND.FLAG(12).AND.FLAG(13).AND.FLAG(14).AND.
     + FLAG(15).AND.FLAG(16)) THEN
*     ******************************************************************
*     CALCULATION OF FINAL EXPPPP (STOPING CONDITION IS MET)
        RHMLE=EXPPPP(I)/92.00
        GO TO 30
      END IF
*     ******************************************************************
*     STOPING CONDITION IS NOT MET.  PROBABILITIES FOR THE NEXT
*     NEWTON AND RAPHSON PROCEDURE
      IP(1,1) = MPPPP/92.00
      IP(2,1) = MPPPF/92.00
      IP(3,1) = MPPFP/92.00
      IP(4,1) = MPPFF/92.00
      IP(5,1) = MPFPP/92.00
      IP(6,1) = MPFPF/92.00
      IP(7,1) = MPFFP/92.00
      IP(8,1) = MPFFF/92.00
      IP(9,1) = MFPPP/92.00
      IP(10,1)= MFPPF/92.00
      IP(11,1)= MFPFP/92.00
      IP(12,1)= MFPFF/92.00
      IP(13,1)= MFFPP/92.00
      IP(14,1)= MFFPF/92.00
      IP(15,1)= MFFFP/92.00
      IP(16,1)= MFFFF/92.00
*     ******************************************************************
      CALL UCPROB(IP,FP)
*     ******************************************************************
```

```
      MPPPP=FP(1,1)*92.00
      MPPPF=FP(2,1)*92.00
      MPPFP=FP(3,1)*92.00
      MPPFF=FP(4,1)*92.00
      MPFPP=FP(5,1)*92.00
      MPFPF=FP(6,1)*92.00
      MPFFP=FP(7,1)*92.00
      MPFFF=FP(8,1)*92.00
      MFPPP=FP(9,1)*92.00
      MFPPF=FP(10,1)*92.00
      MFPFP=FP(11,1)*92.00
      MFPFF=FP(12,1)*92.00
      MFFPP=FP(13,1)*92.00
      MFFPF=FP(14,1)*92.00
      MFFFP=FP(15,1)*92.00
      MFFFF=FP(16,1)*92.00
*     ***************************************************************
      MP000=MPPPP+MPPPF+MPPFP+MPPFF+MPFPP+MPFPF+MPFFP+MPFFF
      MF000=MFPPP+MFPPF+MFPFP+MFPFF+MFFPP+MFFPF+MFFFP+MFFFF
      MPP00=MPPPP+MPPPF+MPPFP+MPPFF
      MP0P0=MPPPP+MPPPF+MPFPP+MPFPF
      MP00P=MPPPP+MPPFP+MPFPP+MPFFP
      A=92.00-MPP00
      B=92.00-MP0P0
      C=92.00-MP00P
*     ***************************************************************
   20 CONTINUE
*     ***************************************************************
   30 WRITE(15,40)FOM,FOTH,FOV,FOA,RHMLE
   40 FORMAT(/,5X,'CASE',4X,F5.2,2X,F5.2,2X,F5.2,2X,F5.2,/,15X,
     +'MLE = ',F12.7)
*     ***************************************************************
      STOP
```

```
      END
*     ******************************************************************
      SUBROUTINE UCPROB (IP,FP)
*     ******************************************************************
*         THIS SUBROUTINE UPDATES CELL PROBABILITIES USING NEWTON AND
*     RAPHSON PROCEDURE WHICH IS DESCRIBED IN SAS .
*     ******************************************************************
*     VARIABLES
*     ******************************************************************
*     FOM    : NUMBER OF FAILURES IN MANUFACTURER TEST.
*     FOTH   : NUMBER OF FAILURES IN TEMP. AND HUMIDITY TEST.
*     FOV    : NUMBER OF FAILURES IN VIBRATION TEST.
*     FOA    :  NUMBER OF FAILURES IN ALTITUDE TEST.
*     MU     : OVERALL MEAN.
*     LP1    : MEAN EFFECT OF MANUFACTURER TEST.
*     LP2    : MEAN EFFECT OF MANUFACTURER TEST.
*     LP3    : MEAN EFFECT OF MANUFACTURER TEST.
*     LP4    : MEAN EFFECT OF MANUFACTURER TEST.
*     TETHA  : TWO WAY INTERACTION TERMS
*     RHMLE  : RELAIBILITY OF DEVICE
*     MPPPP  : CELL FREQUENCY WITH RESPECT TO TESTS RESULTS.
*     MPOPO  : SUM OF CELL FREQUENCIES WHICH HAVE PASSED DEVICES
*              MANUFACTURER AND VIBRATION TEST.
*     MPOOO  : SUM OF CELL FREQUENCIES WHICH HAVE PASSED DEVICES
*              MANUFACTURER TEST.
*     IP     : INITIAL PROBABILITY VECTOR
*     FP     : UPDATED (FINAL) PROBABILITY VECTOR
*     Y      : CELL EXPECTATION VECTOR
*     A,B,C  : SOME TERMS TO MAKE THE CALCULATIONS EASY.
*     EXPPPP : EXPECTED NUMBER OF DEVICES IN CELL WHICH
*              HAS A RESULTANT VECTOR ( P P P P ) IN MANUFACTURER,
*              TEMPERATURE,  VIBRATION AND ALTITUDE TEST RESPECTIVELY
*     SIGN   : INDICATOR VARIABLE OF CONVERGENCE FOR PARAMETERS.
*     RHMLE  : ESTIMATED RELIABILITY OF PYROTECHNIC DEVICE.
```

```
*     ****************************************************************
*     TYPE DECLERATION
      LOGICAL SIGN(5)
      REAL IP(16,1),FP(16,1)
      REAL F0(15,1),F1(15,1),S(15,15),X(15,5),B0(5,1),
     +B1(5,1),PI0(15,1),PI1(15,1),C(5,5),G(5,1),SINV(15,15),F(15,1),
     +PR|(15,1),PR2(5,1),PR3(15,5),PR4(5,5),SUM,LAST,PI01(16,1),LHE,
     +LHEMAX,RIP(15,1),PR5(15,5),CINV(5,5),DIF(15,1),PR6(5,1),LAMBDA,
     +PI11(16,1),EPS,BLAST(5,1),PLAST(15,1),BNEW(5,1),FLAST(15,1)
     +,U,XT(5,15),PR4INV(5,5)
      REAL Y(16)
      COMMON / PROB / Y
      INTEGER I,J,K
*     ****************************************************************
*     INITIALIZATION

      DO 50 I=1,5
         SIGN(I)= .FALSE.
   50 CONTINUE
      DO 70 I=1,5
         DO 60 J=1,5
            S(I,J)=0.0
   60    CONTINUE
   70 CONTINUE
*     ****************************************************************
*     READING THE DESIGN MATRIX
      CALL EXCMS ('FILEDEF 9 DISK DESIGN INPUT  A1')
      DO 80 I=1,15
         READ(9,*) X(I,1),X(I,2),X(I,3),X(I,4),X(I,5)
   80 CONTINUE
      REWIND 9
*     ****************************************************************
*     INVERSE OF VARIANCE AND COVERIANCE MATRIX FOR INITIAL B0
      DO 100 I=1,15
```

```
        DO 90 J=1,15
           IF(I.EQ.J) THEN
              SINV(I,I)=(IP(I,1)-(IP(I,1)**2.0))*92.00
           END IF
           IF(I.NE.J) THEN
              SINV(I,J)=(-IP(J,1))*IP(I,1)*92.00
           END IF
 90     CONTINUE
100   CONTINUE
*     *****************************************************************
*     LOGIT RESPONSE FUNCTIONS
      DO 110 I=1,15
          F(I,1) =ALOG(IP(I,1)/IP(16,1))
110   CONTINUE
*     *****************************************************************
*     THE TRANSPOZE OF THE DESIGN MATRIX
      CALL TRNRR (15,5,X,15,5,15,XT,5)
*     *****************************************************************
*     MATRIX MULTIPLICATION PR1=(SINV*F)
      CALL MRRRR (15,15,SINV,15,15,1,F,15,15,1,PR1,15)
*     *****************************************************************
*     MATRIX MULTIPLICATION PR2=(XT*PR1)
      CALL MRRRR (5,15,XT,5,15,1,PR1,15,5,1,PR2,5)
*     *****************************************************************
*     MATRIX MULTIPLICATION PR3=(SINV*X)
      CALL MRRRR (15,15,SINV,15,15,5,X,15,15,5,PR3,15)
*     *****************************************************************
*     MATRIX MULTIPLICATION PR4=(XT*PR3)
      CALL MRRRR (5,15,XT,5,15,5,PR3,15,5,5,PR4,5)
*     *****************************************************************
*     INVERSE OF THE MATRIX MULTIPLICATION PR4=PR4INV
      CALL LINRG (5,PR4,5,PR4INV,5)
*     *****************************************************************
```

```
*     INITIAL ESTIMATION OF PARAMETERS B0
*     MATRIX MULTIPLICATION B0=(PR4INV*PR2)
      CALL MRRRR (5,5,PR4INV,5,5,1,PR2,5,5,1,B0,5)
*     ****************************************************************
*     F0= X * B0
      CALL MRRRR (15,5,X,15,5,1,B0,5,15,1,F0,15)
*     ****************************************************************
*     INITIAL PROBABILITIES PI0=(EXP(F0))
      DO 120 I=1,15
           PI0(I,1)=EXP(F0(I,1))
 120  CONTINUE
      SUM=0.0
      DO 130 I=1,15
           SUM=SUM+PI0(I,1)
 130  CONTINUE
      LAST=1.0/(1.0+SUM)
*     ****************************************************************
*     PROBABILITY MATRIX WHICH INCLUDES 16 VALUES  PI01
*     FOR THE INITIAL ESTIMATE OF LIKELIHOOD ESTIMATION
      DO 140 I=1,15
         PI01(I,1)=PI0(I,1)*LAST
 140  CONTINUE
      PI01(16,1)=LAST
*     ****************************************************************
*     INITIAL LIKELIHOOD FOR NEXT ITERATION AT STEP B0
      LHE=0.0
      DO 150 I=1,16
         LHE=LHE+Y(I)*ALOG(PI01(I,1))
 150  CONTINUE
      LHEMAX=LHE
*     ****************************************************************
*     REORGANIZED INITIAL PROBABILITIES FOR UPDATATION RIP
      DO 160 I=1,15
         RIP(I,1)=IP(I,1)
```

```
 160  CONTINUE
*     *******************************************************************
*     FIRST ITERATION IN NEWTON AND RAPHSON METHOD
      DO 170 I=1,15
         DO 180 J=1,15
            IF(I.EQ.J) THEN
               SINV(I,I)=(PIO(I,1)-(PIO(I,1)**2.0))*92.00
            END IF
            IF(I.NE.J) THEN
               SINV(I,J)=(-PIO(J,1))*PIO(I,1)*92.00
            END IF
 170     CONTINUE
 180  CONTINUE
*     *******************************************************************
*     MATRIX MULTIPLICATION PR5=(SINV*X)
      CALL MRRRR (15,15,SINV,15,15,5,X,15,15,5,PR5,15)
*     *******************************************************************
*     MATRIX MULTIPLICATION C=(XT*PR5)
      CALL MRRRR (5,15,XT,5,15,5,PR5,15,5,5,C,5)
*     *******************************************************************
*     INVERSE OF THE MATRIX C=CINV
      CALL LINRG (5,C,5,CINV,5)
*     *******************************************************************
      DO 190 I=1,15
          DIF(I,1)= 92.00*(RIP(I,1)-PIO(I,1))
 190  CONTINUE
*     *******************************************************************
*     MATRIX MULTIPLICATION FOR G=(XT*DIF)
      CALL MRRRR (5,15,XT,5,15,1,DIF,15,5,1,G,5)
*     *******************************************************************
*     MATRIX MULTIPLICATION (PR6= CINV*G)
      CALL MRRRR (5,5,CINV,5,5,1,G,5,5,1,PR6,5 )
*     *******************************************************************
      LAMBDA=1.0
```

```
 200  DO 210 I=1,5
          PR6(I,1)=PR6(I,1)*LAMBDA
 210  CONTINUE
*     *****************************************************************
*     INITIAL VALUE FOR B1
      DO 220 I=1,5
          B1(I,1)=B0(I,1)-PR6(I,1)
 220  CONTINUE
*     *****************************************************************
*     MATRIX MULTIPLICATION  F1=(X*B1)
      CALL MRRRR (15,5,X,15,5,1,B1,5,15,1,F1,15)
*     *****************************************************************
*     CALCULATION OF PROBABILITIES FOR B1 PI1=EFP(F1)
      DO 230 I=1,15
           PI1(I,1)=EXP(F1(I,1))
 230  CONTINUE
*     *****************************************************************
*     CALCULATION OF THE 16 TH PROBABILITY VALUE
      SUM=0.0
      DO 240 I=1,15
           SUM=SUM+PI1(I,1)
 240  CONTINUE
      LAST=1.0/(1.0+SUM)
*     *****************************************************************
      DO 250 I=1,15
         PI11(I,1)=PI1(I,1)*LAST
 250  CONTINUE
      PI11(16,1)=LAST
*     *****************************************************************
*     INITIAL LIKELIHOOD ESTIMATION
      LHE=0.0
      DO 260 I=1,16
         LHE=LHE+Y(I)*ALOG(PI11(I,1))
```

```
 260  CONTINUE
*     *****************************************************************
      K=0
      EPS=0.001
      IF(LHE.LT.LHEMAX) THEN
          K=K+1
          IF(K.GT.10) THEN
             DO 270 I=1,5
                 BLAST(I,1)=B0(I,1)
 270         CONTINUE
             DO 280 I=1,15
                 PLAST(I,1)=PI01(I,1)
 280         CONTINUE
             GO TO 490
          END IF
          LAMBDA=LAMBDA/2.0
          GO TO 200
      END IF
      LHEMAX=LHE
*     *****************************************************************
      DO 290 I=1,5
         IF(ABS(B0(I,1)-B1(I,1)).LE.EPS) THEN
             SIGN(I)=.TRUE.
         END IF
 290  CONTINUE
*     *****************************************************************
*     CHECKING CRITERIAS
      IF(SIGN(1).AND.SIGN(2).AND.SIGN(3).AND.SIGN(4).AND.SIGN(5)) THEN
          DO 300 I=1,5
              BLAST(I,1)=B1(I,1)
 300      CONTINUE
          DO 310 I=1,15
              PLAST(I,1)=PI11(I,1)
 310      CONTINUE
```

```
         GO TO 490
      END IF
*     ****************************************************************
*     CRITERIAS ARE NOT MET THEN NEW ITERATIONS
      DO 320 I=1,5
         BLAST(I,1)=B1(I,1)
 320  CONTINUE
      DO 330 I=1,15
         PLAST(I,1)=PI1(I,1)
 330  CONTINUE
*     ****************************************************************
 340  DO 360 I=1,15
         DO 350 J=1,15
            IF(I.EQ.J) THEN
               SINV(I,I)=(PLAST(I,1)-(PLAST(I,1)**2.0)) *92.00
            END IF
            IF(I.NE.J) THEN
               SINV(I,J)=(-PLAST(J,1))*PLAST(I,1)*92.00
            END IF
 350     CONTINUE
 360  CONTINUE
*     ****************************************************************
*     MATRIX MULTIPLICATION PR3=(SINV*X)
      CALL MRRRR (15,15,SINV,15,15,5,X,15,15,5,PR3,15)
*     ****************************************************************
*     MATRIX MULTIPLICATION C=(XT*PR3)
      CALL MRRRR (5,15,XT,5,15,5,PR3,15,5,5,C,5)
*     ****************************************************************
      CALL LINRG ( 5,C,5,CINV,5 )
*     ****************************************************************
      DO 370 I=1,15
          DIF(I,1)= 92.00*(RIP(I,1)-PLAST(I,1))
```

```
 370  CONTINUE
*     ************************************************************
*     MATRIX MULTIPLICATION G=(XT*DIF)
      CALL MRRRR (5,15,XT,5,15,1,DIF,15,5,1,G,5)
*     ************************************************************
*     MATRIX MULTIPLICATION PR6=(CINV*G)
      CALL MRRRR (5,5,CINV,5,5,1,G,5,5,1,PR6,5)
*     ************************************************************
      LAMBDA=1.0
*     ************************************************************
*     NEW PARAMETER ESTIMATES
 380  DO 390 I=1,5
         BNEW(I,1)=BLAST(I,1)-(LAMBDA*PR6(I,1))
 390  CONTINUE
*     ************************************************************
*     MATRIX MULTIPLICATION F1=(X*BNEW)
      CALL MRRRR (15,5,X,15,5,1,BNEW,5,15,1,F1,15)
      DO 400 I=1,15
          PI1(I,1)=EXP(F1(I,1))
 400  CONTINUE
*     ************************************************************
      SUM=0.0
      DO 410 I=1,15
           SUM=SUM+PI1(I,1)
 410  CONTINUE
      LAST=1.0/(1.0+SUM)
*     ************************************************************
      DO 420 I=1,15
         PI11(I,1)=PI1(I,1)*LAST
 420  CONTINUE
      PI11(16,1)=LAST
*     ************************************************************
*     LIKELIHOOD ESTIMATION
      LHE=0.0
```

```
      DO 430 I=1,16
         LHE=LHE+Y(I)*ALOG(PI11(I,1))
 430  CONTINUE
*     ****************************************************************
      K=0
      EPS=0.001
      IF(LHE.LT.LHEMAX) THEN
          K=K+1
          IF(K.GT.10) THEN
             GO TO 490
          END IF
          LAMBDA=LAMBDA/2.0
          GO TO 380
      END IF
      LHEMAX =LHE
*     ****************************************************************
      DO 440 I=1,5
         IF(ABS(B1(I,1)-BLAST(I,1)).LE.EPS) THEN
             SIGN(I)=.TRUE.
         END IF
 440  CONTINUE
*     ****************************************************************
      IF(SIGN(1).AND.SIGN(2).AND.SIGN(3).AND.SIGN(4).AND.SIGN(5)) THEN
          DO 450 I=1,5
              BLAST(I,1)=BNEW(I,1)
 450      CONTINUE
          DO 460 I=1,15
              PLAST(I,1)=PI11(I,1)
 460      CONTINUE
          GO TO 490
      END IF
*     ****************************************************************
*     CRITERIAS ARE NOT MET THEN NEW ITERATIONS
      DO 470 I=1,5
```

```
         BLAST(I,1)=B1(I,1)
  470  CONTINUE
       DO 480 I=1,15
         PLAST(I,1)=PI11(I,1)
  480  CONTINUE
       GO TO 340
*      ******************************************************************
  490  CONTINUE
*      ******************************************************************
*      MATRIX MULTIPLICATION FLAST=(X*BLAST)
       CALL MRRRR (15,5,X,15,5,1,BLAST,5,15,1,FLAST,15)
*      ******************************************************************
*      CALCULATION OF FINAL PROBABILITIES
       DO 500 I=1,15
         PLAST(I,1)=EXP(FLAST(I,1))
  500  CONTINUE
*      ******************************************************************
       SUM=0.0
       DO 510 I=1,15
            SUM=SUM+PLAST(I,1)
  510  CONTINUE
*      ******************************************************************
       LAST=1.0/(1.0+SUM)
       DO 520 I=1,15
         FP(I,1)=PLAST(I,1)*LAST
  520  CONTINUE
       FP(16,1)=LAST
       RETURN
       END
```

# APPENDIX G. PROGRAM MLEB

```
*       ******************************************************************
        PROGRAM MLEB
*       ******************************************************************
*             THIS IS THE PROGRAM TO CALCULATE RELIABILITY OF THE DEVICE
*       WITH DEPENDENCE ASSUMPTION.PROGRAM ASSUMES THAT FAILED ITEM FROM
*       ANY OF ENVIRONMENT TESTS FAILS  FROM MANUFACTURER TEST TOO. THIS
*       IS WORST CASE SCENARIO. IT READS NUMBER OF SUCCESSFUL ITEMS FROM
*       AN INPUT DATA CALLED SUSVECT. FINALLY THE PROGRAM WRITES RESULTS
*       TO AN OUTPUT FILE CALLED RESULT.
*       ******************************************************************
*       VARIABLES
*       ******************************************************************
*       SOM    : NUMBER OF SUCCESSFUL ITEMS IN MANUFACTURER TEST
*       SOTH   : NUMBER OF SUCCESSFUL ITEMS IN TEMP. AND HUMIDITY TEST
*       SOV    : NUMBER OF SUCCESSFUL ITEMS IN VIBRATION TEST
*       SOA    : NUMBER OF SUCCESSFUL ITEMS IN ALTITUDE TEST
*       R1H    : ESTIMATED PROBABILITY OF PASSING FROM MANUFACTURER TEST
*       R2H    : ESTIMATED PROBABILITY OF PASSING FROM TEMPRATURE AND
*                HUMIDITY TEST
*       R3H    : ESTIMATED PROBABILITY OF PASSING FROM VIBRATION TEST
*       R4H    : ESTIMATED PROBABILITY OF PASSING FROM ALTITUDE TEST
*       RHMLE  : ESTIMATED RELIABILITY OF ITEM AFTER EXPOSURE TO
*                SEVERAL ENVIRONMENT TESTS.
*       X      : DUMMY VARIABLE
*       ******************************************************************
*       TYPE DECLARATION
        REAL SOM(5000),SOTH(5000),SOV(5000),SOA(5000),X(4),R1H,R2H,R3H,
      + R4H,RHMLE(5000)
```

```
      INTEGER I
*     *****************************************************************
*     FILES FOR READING AND WRITING
      CALL EXCMS ('FILEDEF 9 DISK SUSVECT DATA A1')
      CALL EXCMS ('FILEDEF 17 DISK RESULT DATA A1')
*     *****************************************************************
*     READING NUMBER OF SUCCESS IN  EACH TEST
      DO 10 I=1,5000
          READ(9,*) SOM(I),SOTH(I),SOV(I),SOA(I)
*     *****************************************************************
*     NUMBER OF SUCCESSES IN EACH TEST
          X(1)= SOM(I)+SOTH(I)+SOV(I)+SOA(I)
          X(2)= SOTH(I)
          X(3)= SOV(I)
          X(4)= SOA(I)
*     *****************************************************************
*     CALCULATIONS SUCCESS PROBABILITIES IN EACH TEST
          R1H= X(1)/184.0
          R2H= X(2)/40.0
          R3H= X(3)/64.0
          R4H= X(4)/40.0
          RHMLE(I)= R1H * R2H * R3H * R4H
*     *****************************************************************
*     WRITING  RESULTS TO AN OUTPUT FILE CALLED RESULT
          WRITE (17,*) RHMLE(I)
10    CONTINUE
      STOP
      END
```

# APPENDIX H. PROGRAM BONUS

```
*       *****************************************************************
        PROGRAM  BONUS
*       *****************************************************************
*          THIS IS THE PROGRAM TO CALCULATE BONUS PERCENTAGE OF ANY FIRM
*       WHOSE LONG RUN SUCCESS  PROBABILITIES ARE KNOWN. IN THIS PROGRAM
*       IT  USES 2000 SUCCESS VECTORS, WHICH  ARE GENERATED BY RANVEC IN
*       APPENDIX  B. THEY ARE GENERATED  BY KNOWN LONG RUN PROBABILITIES
*       THE PROGRAM USES TWO DATA SETS.THEY ARE PRECALCULATED LCB'S SETS
*       .FIRST DATA  REPRESENTS FIRST INSPECTION, SECOND DATA REPRESENTS
*       SECOND INSPECTION LOWER CONFIDENCE BOUNDS.
*          SUCCESS VECTORS REPRESENT OFFERED LOTS.IT HAS A DETERMINISTIC
*       BONUS LINE.PROGRAM CALCULATES LCB OF OFFERED LOT,WITH FIRST DATA
*       AND COMPARES IT WITH LCB OF BONUS LINE.IF FIRM LCB IS GRATER THAN
*       FIRM GETS BONUS. OTHERWISE FIRM HAS A CHANCE TO ONE MORE TRY. IN
*       SECOND TRY, PROGRAM CUMULATES SUCCESS VECTORS AND IT USES SECOND
*       DATA TO FIND OUT LCB OF CUMULATED LOT. AFTER THIS CALCULATION IT
*       COMPARES AGAIN. FINALLY IT COUNTS  NUMBER OF TIMES THAT THE FIRM
*       GETS THE BONUS IN 1000 REPLICATIONS AND ESTIMATES BONUS PERCENT.
*       IT WRITES RESULTS TO AN OUTPUT FILE CALLED BONUS DATA.
*       *****************************************************************
*       VARIABLES
*       *****************************************************************
*       SOM      : NUMBER OF SUCCESS IN MANUFACTURER TEST.
*       SOTH     : NUMBER OF SUCCESS IN TEMPERATURE AND HUMIDITY TEST.
*       SOV      : NUMBER OF SUCCESS IN VIBRATION TEST.
*       SOA      : NUMBER OF SUCCESS IN ALTITUDE TEST.
*       BLINE    : LOWER CONFIDENCE BOUND OF BONUS LINE
*       LCB      : LOWER CONFIDENCE BOUND
*       FILCB    : LCB VALUES ARRAY IN FIRST INSPECTION
```

```
*     SELCB   : LCB VALUES ARRAY IN SECOND INSPECTION
*     A,B,C,D : DIMENSIONS FOR USE OF LCB DATAS
*     BFI     : NUMBER OF TIMES THAT FIRM GAT BONUS AFTER 1 ST INSP.
*     BSI     : NUMBER OF TIMES THAT FIRM GAT BONUS AFTER 2 ST INSP.
*     BTOT    : TOTAL NUMBER OF TIMES THAT FIRM GAT BONUS.
*     PRCT    : BONUS PERCENT.
*     COUNT   : COUNTER FOR 1000 REPLICATIONS.
*     SIGN    : INDICATOR OF ACCEPTANCE FOR FIRST INSPECTION
*     FLAG    : INDICATOR OF ACCEPTANCE FOR SECOND INSPECTION
*     ****************************************************************
*     TYPE DECLARATION
      LOGICAL SIGN(4),FLAG(4)
      REAL SOM(2000),SOTH(2000),SOV(2000),SOA(2000),BLINE,LCB,BFI,BSI,
     + BTOT,FILCB(2,2,3,2),SELCB(3,3,5,3),A,B,C,D,PRCT,COUNT
      INTEGER I,J,K,L
*     ****************************************************************
*     OPENING  FILES FOR READING AND WRITING
      CALL EXCMS ( ' FILEDEF 7 DISK SUCVECT DATA A1' )
      CALL EXCMS ( ' FILEDEF 8 DISK FIRST DATA A1' )
      CALL EXCMS ( ' FILEDEF 9 DISK SECOND DATA A1' )
      CALL EXCMS ( ' FILEDEF 15 DISK BONUS DATA A1' )
*     ****************************************************************
*     INITIALIZATION
      COUNT = 1.0
      BFI = 0.0
      BSI = 0.0
      BLINE = 0.9250000
      DO 10 I=1,4
          SIGN(I)=.TRUE.
          FLAG(I)=.TRUE.
 10   CONTINUE
*     ****************************************************************
*     READING FIRST INSPECTION LOWER CONFIDENCE BOUNDS FROM DATA FILE
      DO 50 I=1,2
```

```
          DO 40 J=1,2
              DO 30 K=1,3
                  DO 20 L=1,2
                      READ(8,*) FILCB(I,J,K,L)
 20               CONTINUE
 30           CONTINUE
 40       CONTINUE
 50   CONTINUE
*     ****************************************************************
*     READING SECOND INSPECTION LOWER CONFIDENCE BOUNDS FROM DATA FILE
      DO 90 I=1,3
          DO 80 J=1,3
              DO 70 K=1,5
                  DO 60 L=1,3
                      READ(9,*) SELCB(I,J,K,L)
 60               CONTINUE
 70           CONTINUE
 80       CONTINUE
 90   CONTINUE
*     ****************************************************************
*     READING SUCCESS PROBABILITY OF FIRM IN EACH TEST
      WRITE(*,*)'WRITE THE PROBABILITY OF SUCCESS IN MANUFACTURER TEST'
      READ (*,*) PSIM
      WRITE(*,*)'WRITE THE PROBABILITY OF SUCCESS IN TEMPERATURE AND
     +HUMIDITY TEST'
      READ (*,*) PSITH
      WRITE(*,*)'WRITE THE PROBABILITY OF SUCCESS IN VIBRATION TEST'
      READ (*,*) PSIV
      WRITE(*,*)'WRITE THE PROBABILITY OF SUCCESS IN ALTITUDE TEST'
      READ (*,*) PSIA
*     ******************************************************************
*     READING  NUMBER OF SUCCESFUL ITEMS
*     FOR FIRST INSPECTION AND SECOND INSPECTION
      DO 100 I=1,2000
```

```
         READ(7,*) SOM(I),SOTH(I),SOV(I),SOA(I)
100   CONTINUE
*     *****************************************************************
*     INSPECTIONS BEGIN
      DO 120 N=1,2000,2
*     *****************************************************************
*     SUBSCRIPT  DEFINITION  FOR FIRST INSPECTION
         A=INT(20.0-SOM(N))+1
         B=INT(20.0-SOTH(N))+1
         C=INT(32.0-SOV(N))+1
         D=INT(20.0-SOA(N))+1
*     *****************************************************************
*     CHECK FOR ACCEPTANCE OF FIRST OFFERED LOT
         IF (A.GT.2) THEN
             SIGN(1)=.FALSE.
         END IF
         IF (B.GT.2) THEN
             SIGN(2)=.FALSE.
         END IF
         IF (C.GT.3) THEN
             SIGN(3)=.FALSE.
         END IF
         IF (D.GT.2) THEN
             SIGN(4)=.FALSE.
         END IF
         IF(.NOT.(SIGN(1).AND.SIGN(2).AND.SIGN(3).AND.SIGN(4))) THEN
             SIGN(1)=.TRUE.
             SIGN(2)=.TRUE.
             SIGN(3)=.TRUE.
             SIGN(4)=.TRUE.
             GO TO  112
         END IF
*     *****************************************************************
*     AFTER FIRST INSPECTION  DETERMINATION OF LCB OF FIRM
```

```
      LCB=FILCB(A,B,C,D)
*   *******************************************************************
*   IS LOT WORTH WHILE  FOR  GETTING BONUS ?
      IF ( LCB.GT.BLINE ) THEN
         BFI=BFI+1.0
         GO TO  110
      END IF
*   *******************************************************************
*   SUBSCRIPT DETERMINATION  FOR SECOND INSPECTION
      A=A+INT(20.0-SOM(N+1))
      B=B+INT(20.0-SOTH(N+1))
      C=C+INT(32.0-SOV(N+1))
      D=D+INT(20.0-SOA(N+1))
*   *******************************************************************
*   CHECKING  RESULTS OF SECOND TEST SERIES ABOUT ACCEPTANCE
      IF (A.GT.3) THEN
          FLAG(1)=.FALSE.
      END IF
      IF (B.GT.3) THEN
          FLAG(2)=.FALSE.
      END IF
      IF (C.GT.5) THEN
          FLAG(3)=.FALSE.
      END IF
      IF (D.GT.3) THEN
          FLAG(4)=.FALSE.
      END IF
      IF(.NOT.(FLAG(1).AND.FLAG(2).AND.FLAG(3).AND.FLAG(4))) THEN
          FLAG(1)=.TRUE.
          FLAG(2)=.TRUE.
          FLAG(3)=.TRUE.
          FLAG(4)=.TRUE.
          GO TO  112
```

```
         END IF
*     ******************************************************************
*     AFTER SECOND INSPECTION  DETERMINATION OF LCB
         LCB=SELCB(A,B,C,D)
*     ******************************************************************
*      CHECKING FOR BONUS AFTER SECOND INSPECTION
         IF ( LCB.GT.BLINE ) THEN
            BSI=BSI+1
            GO TO  112
         END IF
*     ******************************************************************
*     COUNTING FOR CHECKING 1000 REPLICATIONS
 110     COUNT = COUNT + 1.0
         A=0.0
         B=0.0
         C=0.0
         D=0.0
*     ******************************************************************
*     CHECKING FOR 1000 REPLICATIONS
         IF (COUNT.GT.1000) THEN
             GO TO  130
         END IF
 120  CONTINUE
*     ******************************************************************
*     PERCENTAGE ESTIMATION OF GETING BONUS FOR FIRM A
 130  BTOT = BFI+BSI
      PRCT=BTOT/1000.0
*     ******************************************************************
*     WRITING RESULTS
      WRITE (15,1)
 1    FORMAT (//,16X,' BONUS PLAN SIMULATION FOR FIRM A ',2X)
      WRITE (15,2)
 2    FORMAT (16X,'**********************************',2X,//)
      WRITE (15,3)
```

```
3     FORMAT(4X,' FIRM A HAS FOLLOWING LONG RUN PROBABILITIES IN TESTS '
     +,2X)
      WRITE (15,4)
4     FORMAT(4X,'******************************************************'
     +,/,2X)
      WRITE (15,5) PSIM,PSITH,PSIV,PSIA
5     FORMAT(4X,'PROBABILITY OF SUCCESS IN MANUFACTURER TEST IS',2X,
     +F8.6,//,4X,'PROBABILITY OF SUCCESS IN TEMP. AND HUM. TEST IS',2X,
     +F8.6,//,4X,'PROBABILITY OF SUCCESS IN VIBRATION IS',2X,
     +F8.6,//,4X,'PROBABILITY OF SUCCESS IN ALTITUDE IS',F8.6,2X,//)
      WRITE (15,6) BFI
6     FORMAT(4X,'FIRM A GAT BONUS AFTER FIRST INSPECTION ',2X,F6.1,2X,
     +'TIMES',2X)
      WRITE (15,7)
7    FORMAT(4X,'**********************************************************
     +******',2X,/)
      WRITE (15,8) BSI
8     FORMAT(4X,'FIRM A GAT BONUS AFTER SECOND INSPECTION ',2X,F6.1,2X,
     +'TIMES',2X)
      WRITE (15,9)
9    FORMAT(4X,'**********************************************************
     +******',2X,/)
      WRITE (15,11) BTOT
11    FORMAT(4X,'TOTALLY FIRM A GAT BONUS IN 1000 REPLICATIONS ', 2X,
     +F5.1,2X,'TIMES',2X)
      WRITE (15,12)
12    FORMAT(4X,'***********************************************************
     +******',2X,/)
      WRITE (15,13) PRCT
13    FORMAT(4X,'GETTING BONUS PERCENTAGE OF FIRM A IS',1X,F6.3,2X)
      WRITE (15,14)
14    FORMAT(4X,'***********************************************************
     +******',2X,///)
      WRITE (15,15) BLINE
```

```
15    FORMAT(4X,' BONUS LINE FOR FIRMS IS',1X,F6.4,2X)
      WRITE (15,16)
16    FORMAT(4X,'********************************************************
     +******',2X,///)
      STOP
      END
```

# APPENDIX I. 95 % LCB'S FOR DSBS ( EQUAL PROBABILITIES )

Table 18. 95 % LCB'S FOR DOUBLE SAMPLING BONUS SYSTEM

| FAILURE VECTOR | 95 % LCB | FAILURE VECTOR | 95 % LCB |
|---|---|---|---|
| ( 0 0 0 0 ) | 1.0000000 | ( 0 0 0 1 ) | 0.9099184 |
| ( 0 0 0 2 ) | 0.8706521 | ( 0 0 1 0 ) | 0.9375849 |
| ( 0 0 1 1 ) | 0.8808635 | ( 0 0 1 2 ) | 0.8387057 |
| ( 0 0 2 0 ) | 0.8968240 | ( 0 0 2 1 ) | 0.8528913 |
| ( 0 0 2 2 ) | 0.8141473 | ( 0 0 3 0 ) | 0.8717731 |
| ( 0 0 3 1 ) | 0.8281843 | ( 0 0 3 2 ) | 0.7881665 |
| ( 0 0 4 0 ) | 0.8422214 | ( 0 0 4 1 ) | 0.8021229 |
| ( 0 0 4 2 ) | 0.7645337 | ( 0 1 0 0 ) | 0.9099184 |
| ( 0 1 0 1 ) | 0.8548708 | ( 0 1 0 2 ) | 0.8224728 |
| ( 0 1 1 0 ) | 0.8808635 | ( 0 1 1 1 ) | 0.8368122 |
| ( 0 1 1 2 ) | 0.8010584 | ( 0 1 2 0 ) | 0.8548636 |
| ( 0 1 2 1 ) | 0.8131180 | ( 0 1 2 2 ) | 0.7779465 |
| ( 0 1 3 0 ) | 0.8294836 | ( 0 1 3 1 ) | 0.7867751 |
| ( 0 1 3 2 ) | 0.7541937 | ( 0 1 4 0 ) | 0.8001103 |
| ( 0 1 4 1 ) | 0.7624319 | ( 0 1 4 2 ) | 0.7305543 |
| ( 0 2 0 0 ) | 0.8657608 | ( 0 2 0 1 ) | 0.8224728 |
| ( 0 2 0 2 ) | 0.7905909 | ( 0 2 1 0 ) | 0.8387058 |
| ( 0 2 1 1 ) | 0.7996263 | ( 0 2 1 2 ) | 0.7653871 |
| ( 0 2 2 0 ) | 0.8154084 | ( 0 2 2 1 ) | 0.7777317 |
| ( 0 2 2 2 ) | 0.7414687 | ( 0 2 3 0 ) | 0.7881665 |
| ( 0 2 3 1 ) | 0.7541937 | ( 0 2 3 2 ) | 0.7210189 |
| ( 0 2 4 0 ) | 0.7624319 | ( 0 2 4 1 ) | 0.7326533 |
| ( 0 2 4 2 ) | 0.6996729 | ( 1 0 0 0 ) | 0.9945652 |
| ( 1 0 0 1 ) | 0.9099184 | ( 1 0 0 2 ) | 0.8706521 |

Table 19. 95 % LCB'S FOR DOUBLE SAMPLING BONUS SYSTEM

| FAILURE VECTOR | 95 % LCB | FAILURE VECTOR | 95 % LCB |
|---|---|---|---|
| ( 1 0 1 0 ) | 0.9375849 | ( 1 0 1 1 ) | 0.8808635 |
| ( 1 0 1 2 ) | 0.8369564 | ( 1 0 2 0 ) | 0.8968240 |
| ( 1 0 2 1 ) | 0.8519021 | ( 1 0 2 2 ) | 0.8111921 |
| ( 1 0 3 0 ) | 0.8717731 | ( 1 0 3 1 ) | 0.8258852 |
| ( 1 0 3 2 ) | 0.7881665 | ( 1 0 4 0 ) | 0.8422214 |
| ( 1 0 4 1 ) | 0.8001103 | ( 1 0 4 2 ) | 0.7631623 |
| ( 1 1 0 0 ) | 0.9099184 | ( 1 1 0 1 ) | 0.8548708 |
| ( 1 1 0 2 ) | 0.8224728 | ( 1 1 1 0 ) | 0.8808635 |
| ( 1 1 1 1 ) | 0.8368122 | ( 1 1 1 2 ) | 0.8002886 |
| ( 1 1 2 0 ) | 0.8528913 | ( 1 1 2 1 ) | 0.8121204 |
| ( 1 1 2 2 ) | 0.7768002 | ( 1 1 3 0 ) | 0.8258852 |
| ( 1 1 3 1 ) | 0.7862302 | ( 1 1 3 2 ) | 0.7532073 |
| ( 1 1 4 0 ) | 0.8001103 | ( 1 1 4 1 ) | 0.7616678 |
| ( 1 1 4 2 ) | 0.7295684 | ( 1 2 0 0 ) | 0.8657608 |
| ( 1 2 0 1 ) | 0.8224728 | ( 1 2 0 2 ) | 0.7872553 |
| ( 1 2 1 0 ) | 0.8387058 | ( 1 2 1 1 ) | 0.7987179 |
| ( 1 2 1 2 ) | 0.7626535 | ( 1 2 2 0 ) | 0.8130434 |
| ( 1 2 2 1 ) | 0.7758973 | ( 1 2 2 2 ) | 0.7406143 |
| ( 1 2 3 0 ) | 0.7860773 | ( 1 2 3 1 ) | 0.7538874 |
| ( 1 2 3 2 ) | 0.7200024 | ( 1 2 4 0 ) | 0.7624319 |
| ( 1 2 4 1 ) | 0.7309083 | ( 1 2 4 2 ) | 0.6994067 |
| ( 2 0 0 0 ) | 0.9891304 | ( 2 0 0 1 ) | 0.9099184 |
| ( 2 0 0 2 ) | 0.8657608 | ( 2 0 1 0 ) | 0.9324048 |
| ( 2 0 1 1 ) | 0.8808550 | ( 2 0 1 2 ) | 0.8341966 |

Table 20. 95 % LCB'S FOR DOUBLE SAMPLING BONUS SYSTEM

| FAILURE VECTOR | 95 % LCB | FAILURE VECTOR | 95 % LCB |
|---|---|---|---|
| ( 2 0 2 0 ) | 0.8968240 | ( 2 0 2 1 ) | 0.8499787 |
| ( 2 0 2 2 ) | 0.8110244 | ( 2 0 3 0 ) | 0.8717731 |
| ( 2 0 3 1 ) | 0.8247706 | ( 2 0 3 2 ) | 0.7845937 |
| ( 2 0 4 0 ) | 0.8422214 | ( 2 0 4 1 ) | 0.8001103 |
| ( 2 0 4 2 ) | 0.7624320 | ( 2 1 0 0 ) | 0.9099184 |
| ( 2 1 0 1 ) | 0.8548708 | ( 2 1 0 2 ) | 0.8224728 |
| ( 2 1 1 0 ) | 0.8808550 | ( 2 1 1 1 ) | 0.8362838 |
| ( 2 1 1 2 ) | 0.8003395 | ( 2 1 2 0 ) | 0.8519021 |
| ( 2 1 2 1 ) | 0.8121204 | ( 2 1 2 2 ) | 0.7750615 |
| ( 2 1 3 0 ) | 0.8258852 | ( 2 1 3 1 ) | 0.7846695 |
| ( 2 1 3 2 ) | 0.7517914 | ( 2 1 4 0 ) | 0.8001103 |
| ( 2 1 4 1 ) | 0.7605843 | ( 2 1 4 2 ) | 0.7284731 |
| ( 2 2 0 0 ) | 0.8608695 | ( 2 2 0 1 ) | 0.8222825 |
| ( 2 2 0 2 ) | 0.7860733 | ( 2 2 1 0 ) | 0.8341965 |
| ( 2 2 1 1 ) | 0.7965909 | ( 2 2 1 2 ) | 0.7626535 |
| ( 2 2 2 0 ) | 0.8110244 | ( 2 2 2 1 ) | 0.7750614 |
| ( 2 2 2 2 ) | 0.7401020 | ( 2 2 3 0 ) | 0.7845957 |
| ( 2 2 3 1 ) | 0.7519901 | ( 2 2 3 2 ) | 0.7188347 |
| ( 2 2 4 0 ) | 0.7611454 | ( 2 2 4 1 ) | 0.7295684 |
| ( 2 2 4 2 ) | 0.6975686 | | |

# APPENDIX J. DOUBLE SAMPLING BONUS SYSTEM WITH EQUAL PROBABILITIES

Table 21. DSBS (EQUAL PROBABILITIES) LCBFB = 0.825

| LOWER CONFIDENCE BOUND FOR BONUS IS 0.825 | | | | | |
|---|---|---|---|---|---|
| PROB.'S | BONUS % | PROB.'S | BONUS % | PROB.'S | BONUS % |
| 0.9200 | 0.008 | 0.9375 | 0.030 | 0.9450 | 0.059 |
| 0.9500 | 0.079 | 0.9600 | 0.172 | 0.9700 | 0.351 |
| 0.9750 | 0.487 | 0.9800 | 0.632 | 0.9850 | 0.799 |
| 0.9900 | 0.920 | 0.9950 | 0.982 | | |

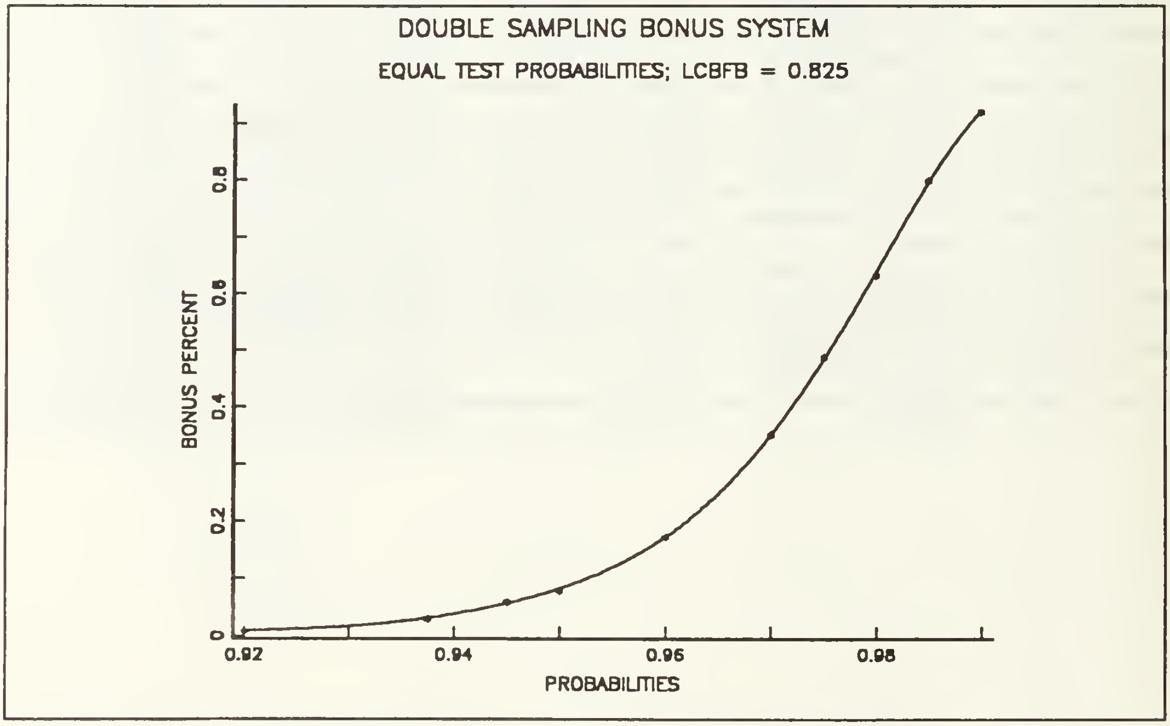


Figure 5. Double Sampling Bonus System With LCBFB = 0.825

Bonus percentages are tabulated and plotted below with LCBFB 0.850

Table 22. DSBS (EQUAL PROBABILITIES) LCBFB = 0.850

| LOWER CONFIDENCE BOUND FOR BONUS IS 0.850 | | | | | |
|---|---|---|---|---|---|
| PROB.'S | BONUS % | PROB.'S | BONUS % | PROB.'S | BONUS % |
| 0.9200 | 0.007 | 0.9375 | 0.025 | 0.9450 | 0.050 |
| 0.9500 | 0.063 | 0.9600 | 0.144 | 0.9700 | 0.293 |
| 0.9750 | 0.423 | 0.9800 | 0.565 | 0.9850 | 0.738 |
| 0.9900 | 0.887 | 0.9950 | 0.970 | | |

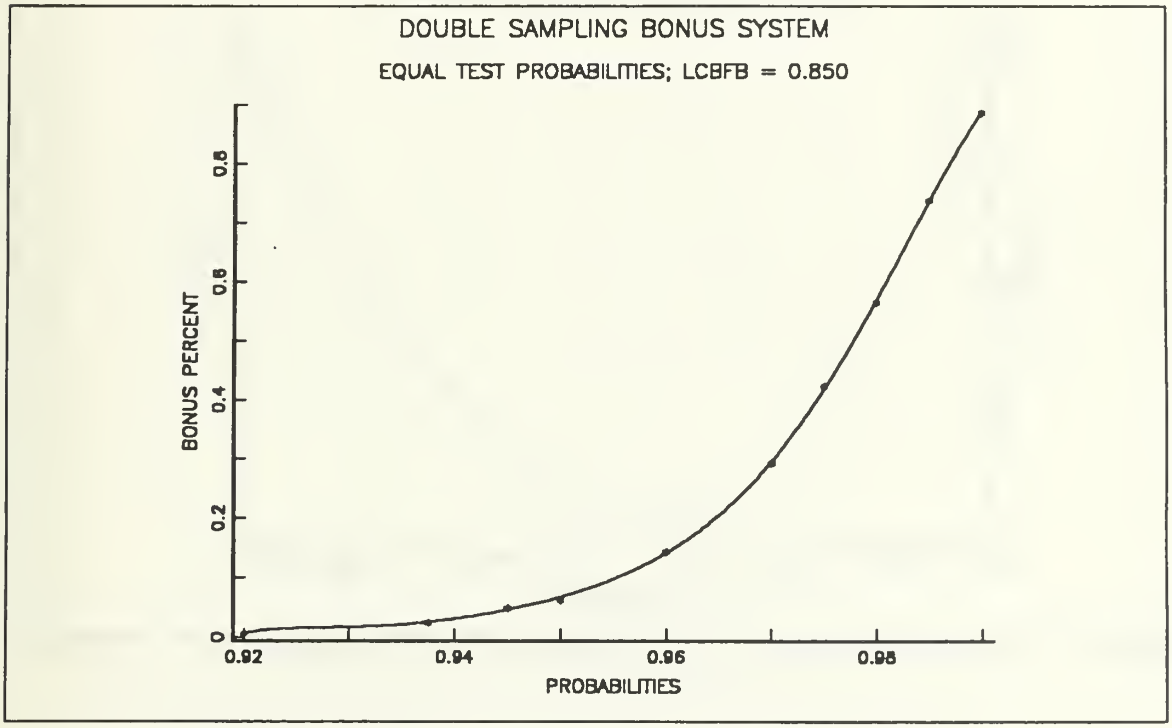


Figure 6. Double Sampling Bonus System With LCBFB = 0.850

Bonus percentages are tabulated and plotted below with LCBFB 0.875

Table 23. DSBS (EQUAL PROBABILITIES) LCBFB = 0.875

| LOWER CONFIDENCE BOUND FOR BONUS IS 0.875 | | | | | |
|---|---|---|---|---|---|
| PROB.'S | BONUS % | PROB.'S | BONUS % | PROB.'S | BONUS % |
| 0.9200 | 0.007 | 0.9375 | 0.023 | 0.9450 | 0.045 |
| 0.9500 | 0.057 | 0.9600 | 0.122 | 0.9700 | 0.236 |
| 0.9750 | 0.353 | 0.9800 | 0.487 | 0.9850 | 0.643 |
| 0.9900 | 0.799 | 0.9950 | 0.941 | | |

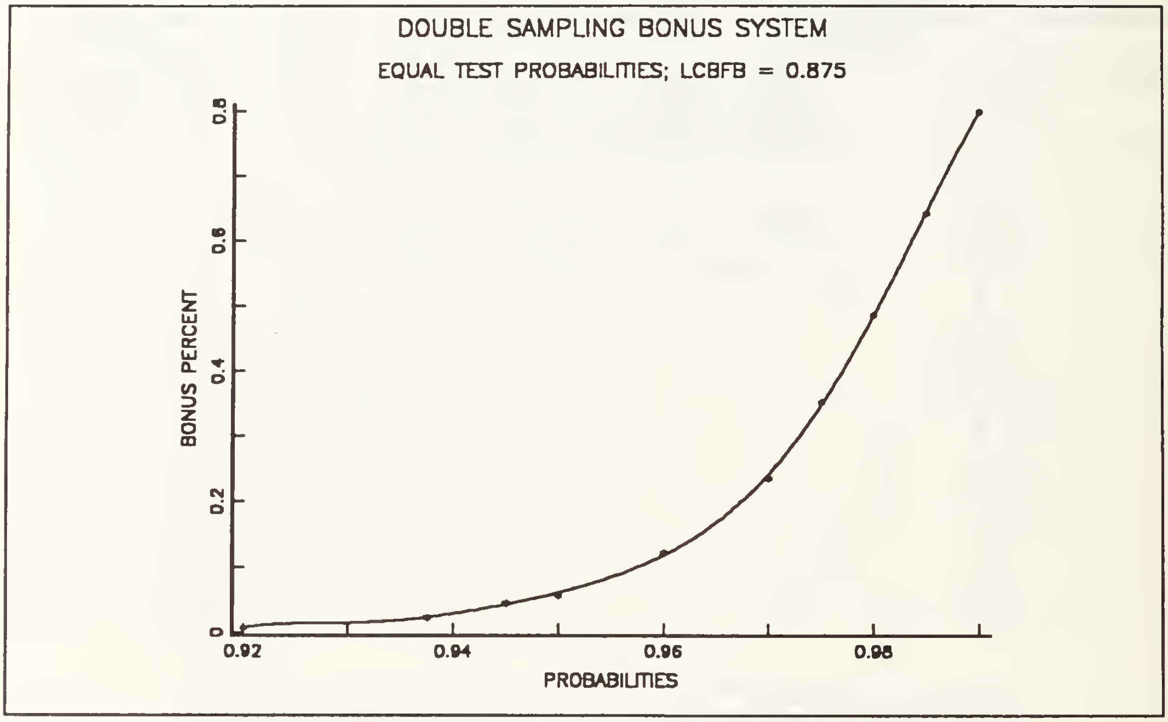


Figure 7. Double Sampling Bonus System With LCBFB = 0.875

Bonus percentages are tabulated and plotted below with LCBFB 0.900

Table 24. DSBS (EQUAL PROBABILITIES) LCBFB = 0.900

| LOWER CONFIDENCE BOUND FOR BONUS IS 0.900 | | | | | |
|---|---|---|---|---|---|
| PROB.'S | BONUS % | PROB.'S | BONUS % | PROB.'S | BONUS % |
| 0.9200 | 0.002 | 0.9375 | 0.010 | 0.9450 | 0.020 |
| 0.9500 | 0.027 | 0.9600 | 0.052 | 0.9700 | 0.138 |
| 0.9750 | 0.212 | 0.9800 | 0.319 | 0.9850 | 0.486 |
| 0.9900 | 0.671 | 0.9950 | 0.872 | | |

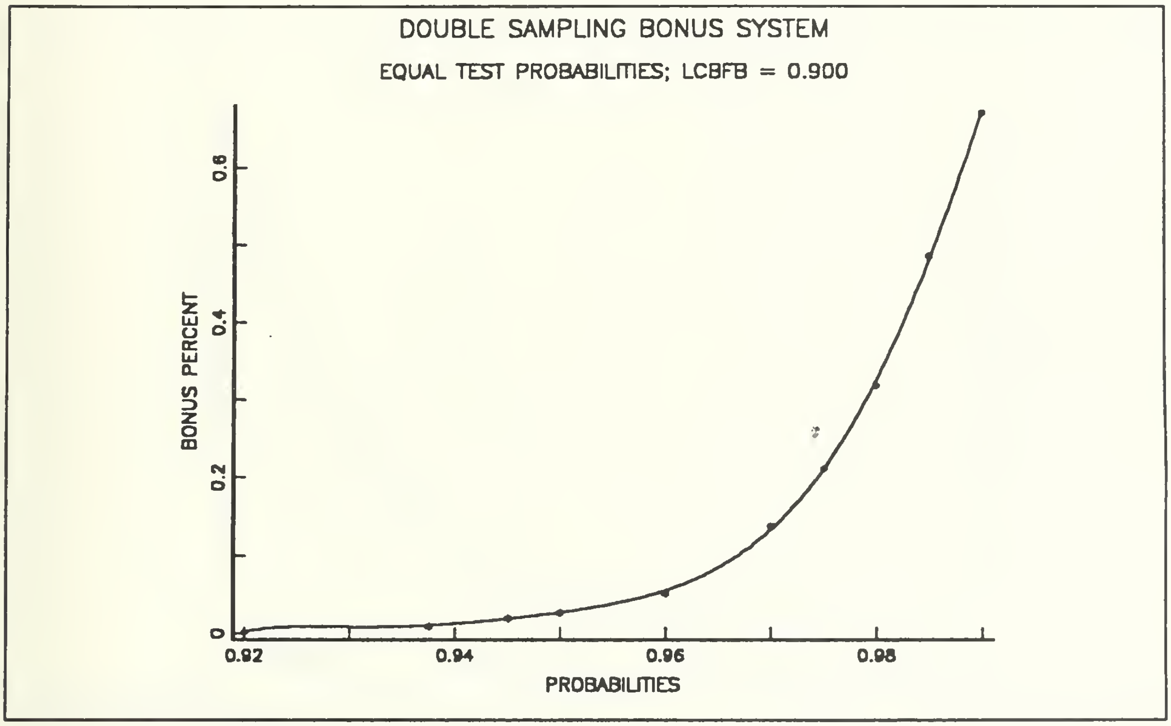


Figure 8. Double Sampling Bonus System With LCBFB = 0.900

Bonus percentages are tabulated and ploted below with LCBFB 0.950

Table 25. DSBS (EQUAL PROBABILITIES) LCBFB = 0.950

| LOWER CONFIDENCE BOUND FOR BONUS IS 0.950 | | | | | |
|---|---|---|---|---|---|
| PROB.'S | BONUS % | PROB.'S | BONUS % | PROB.'S | BONUS % |
| 0.9200 | 0.002 | 0.9375 | 0.010 | 0.9450 | 0.018 |
| 0.9500 | 0.022 | 0.9600 | 0.045 | 0.9700 | 0.107 |
| 0.9750 | 0.163 | 0.9800 | 0.237 | 0.9850 | 0.362 |
| 0.9900 | 0.502 | 0.9950 | 0.695 | | |

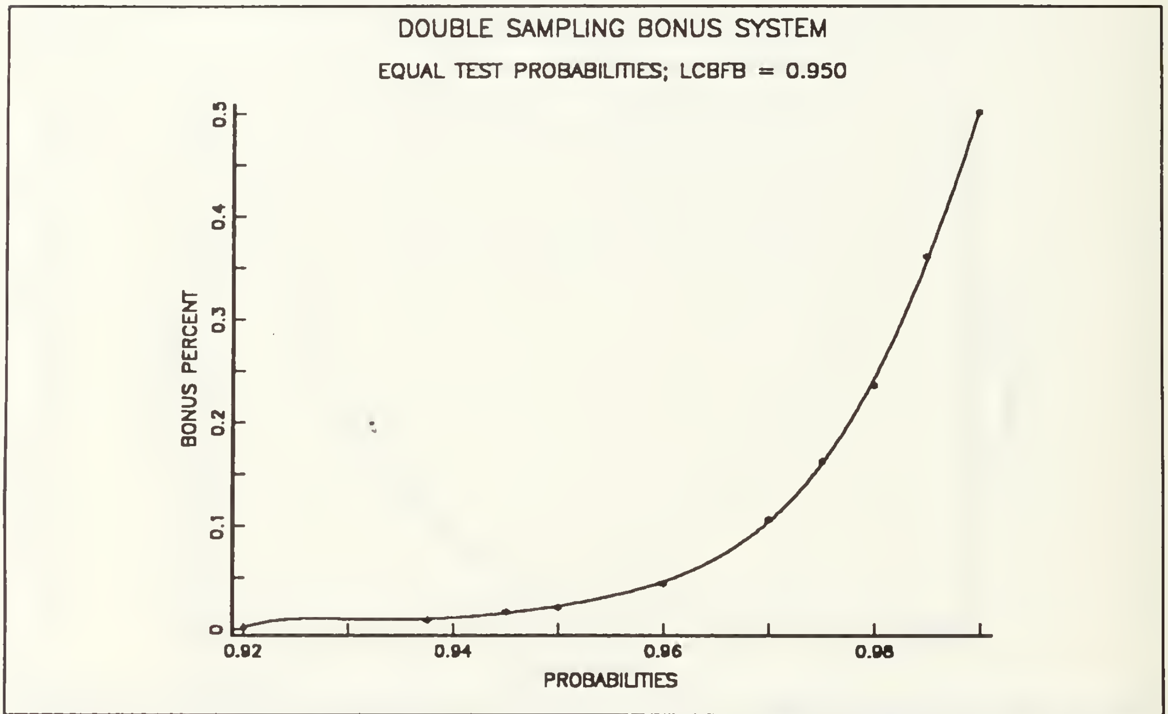


Figure 9. Double Sampling Bonus System With LCBFB = 0.950

Bonus percentages are tabulated and ploted below with LCBFB 0.999

Table 26. DSBS (EQUAL PROBABILITIES) LCBFB = 0.999

| LOWER CONFIDENCE BOUND FOR BONUS IS 0.999 | | | | | |
|---|---|---|---|---|---|
| PROB.'S | BONUS % | PROB.'S | BONUS % | PROB.'S | BONUS % |
| 0.9200 | 0.002 | 0.9375 | 0.007 | 0.9450 | 0.010 |
| 0.9500 | 0.011 | 0.9600 | 0.028 | 0.9700 | 0.067 |
| 0.9750 | 0.106 | 0.9800 | 0.166 | 0.9850 | 0.274 |
| 0.9900 | 0.407 | 0.9950 | 0.626 | | |

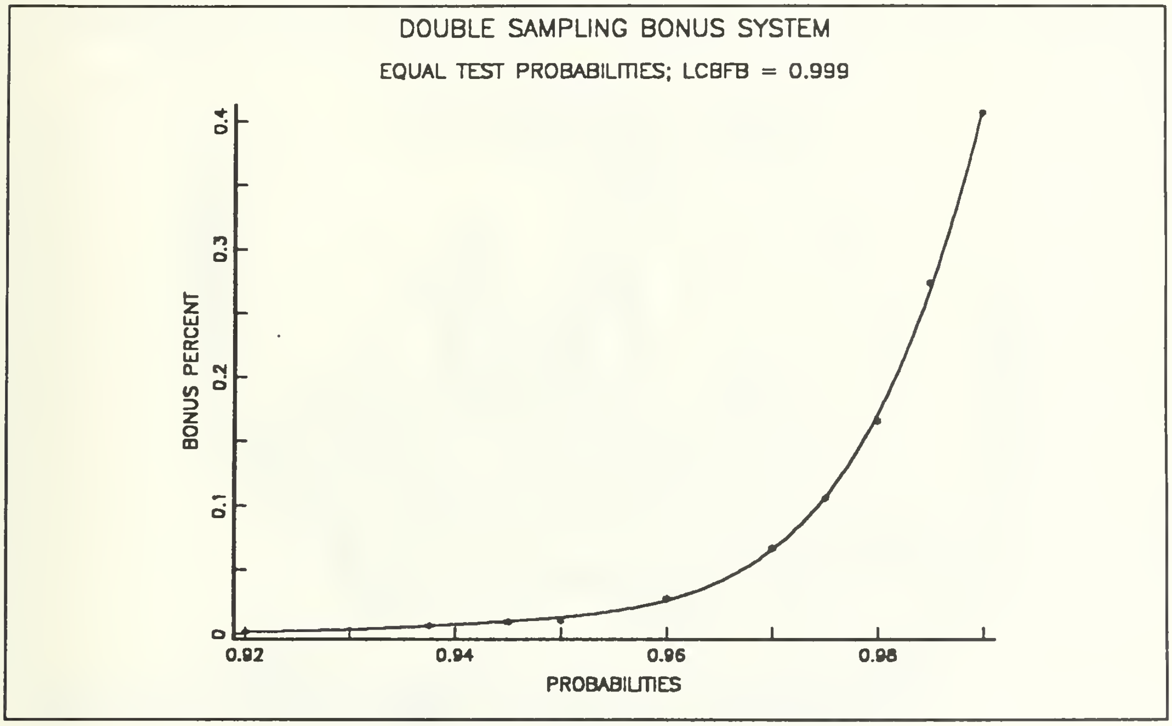


Figure 10. Double Sampling Bonus System With LCBFB = 0.999

# APPENDIX K. 95 % LCB'S FOR DSBS (DIFFERENT PROBABILITIES)

Bonus percentages are tabulated and ploted with different probabilities.

$$LCBFB = 0.825$$

Table 27. DSBS (DIFFERENT PROBABILITIES) LCBFB = 0.825

| | 0.950 | 0.975 | 0.990 | 0.995 |
|---|---|---|---|---|
| 0.950 | 0.079 | 0.364 | 0.632 | 0.689 |
| 0.975 | 0.099 | 0.487 | 0.831 | 0.882 |
| 0.990 | 0.109 | 0.550 | 0.920 | 0.968 |
| 0.995 | 0.110 | 0.558 | 0.934 | 0.982 |

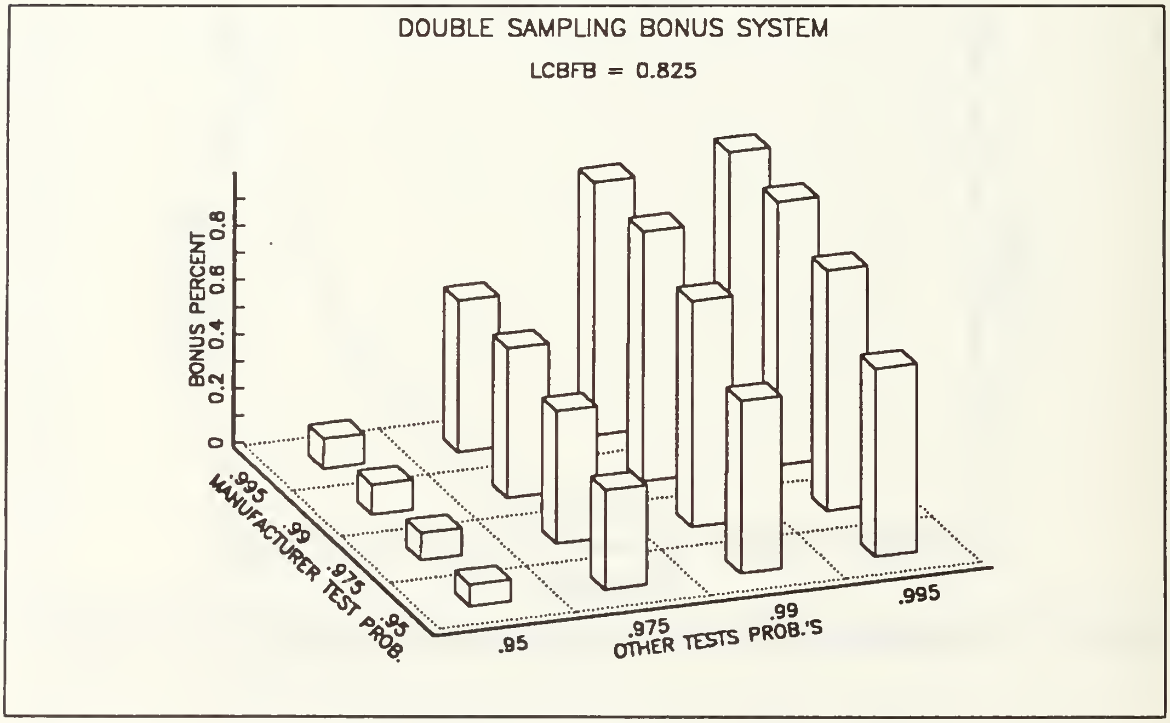


Figure 11. Double Sampling Bonus System With LCBFB = 0.825

Bonus percentages are tabulated and ploted with different probabilities.

$$LCBFB = 0.850$$

Table 28. DSBS (DIFFERENT PROBABILITIES) LCBFB = 0.850

| | 0.950 | 0.975 | 0.990 | 0.995 |
|---|---|---|---|---|
| 0.950 | 0.063 | 0.320 | 0.612 | 0.680 |
| 0.975 | 0.077 | 0.423 | 0.802 | 0.871 |
| 0.990 | 0.083 | 0.476 | 0.887 | 0.957 |
| 0.995 | 0.084 | 0.483 | 0.900 | 0.970 |

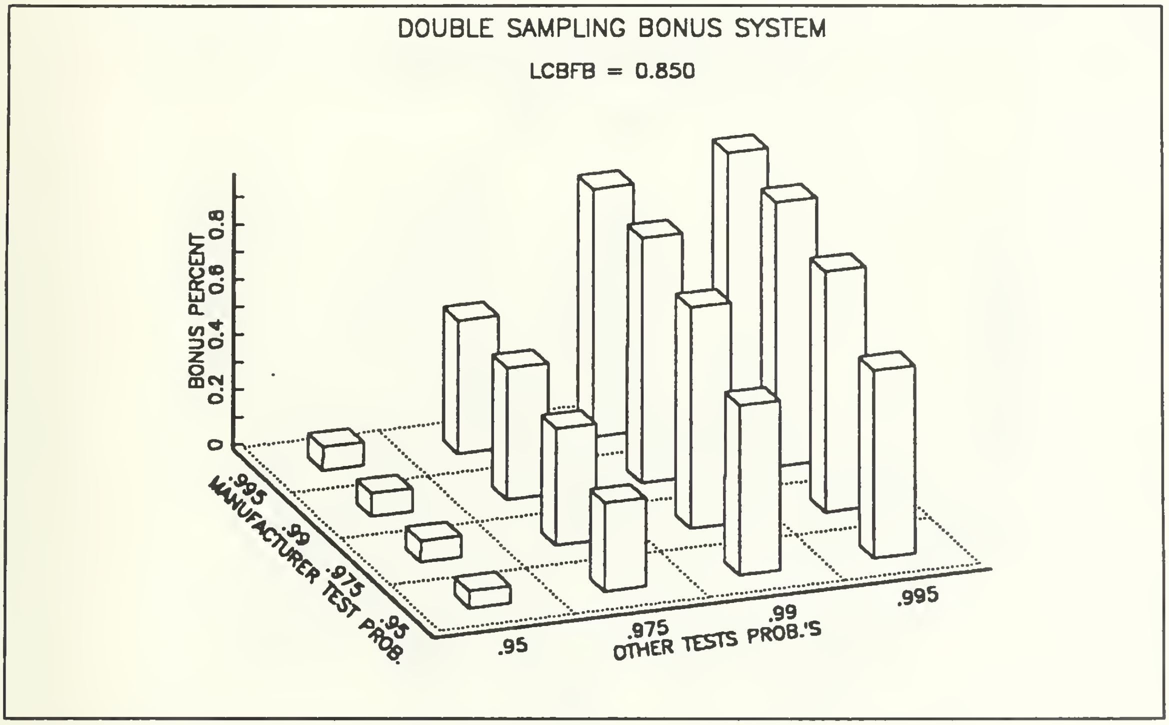


Figure 12. Double Sampling Bonus System With LCBFB = 0.850

Bonus percentages are tabulated and ploted with different probabilities.

$$LCBFB = 0.875$$

Table 29. DSBS (DIFFERENT PROBABILITIES) LCBFB = 0.875

| | 0.950 | 0.975 | 0.990 | 0.995 |
|---|---|---|---|---|
| 0.950 | 0.057 | 0.277 | 0.566 | 0.664 |
| 0.975 | 0.067 | 0.353 | 0.721 | 0.846 |
| 0.990 | 0.072 | 0.388 | 0.799 | 0.929 |
| 0.995 | 0.072 | 0.392 | 0.809 | 0.941 |

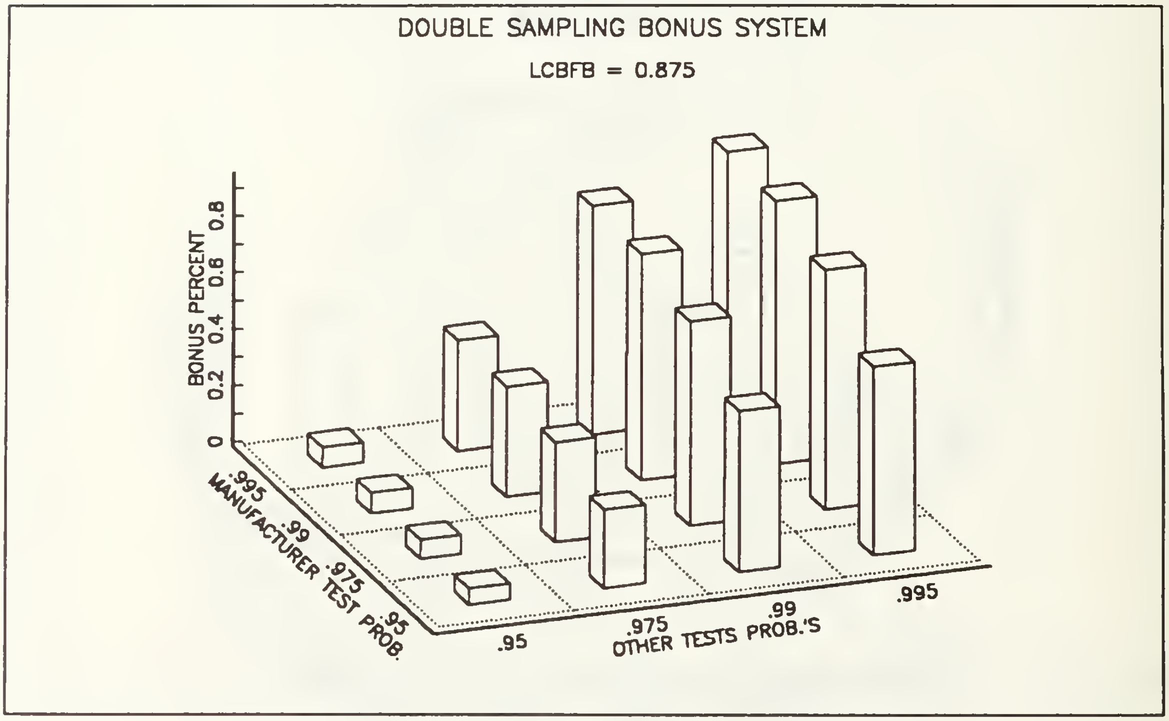


Figure 13. Double Sampling Bonus System With LCBFB = 0.875

Bonus percentages are tabulated and ploted with different probabilities.

$$LCBFB = 0.900$$

Table 30. DSBS (DIFFERENT PROBABILITIES) LCBFB = 0.900

| | 0.950 | 0.975 | 0.990 | 0.995 |
|---|---|---|---|---|
| 0.950 | 0.027 | 0.164 | 0.470 | 0.605 |
| 0.975 | 0.031 | 0.212 | 0.603 | 0.778 |
| 0.990 | 0.034 | 0.235 | 0.671 | 0.860 |
| 0.995 | 0.034 | 0.238 | 0.679 | 0.872 |

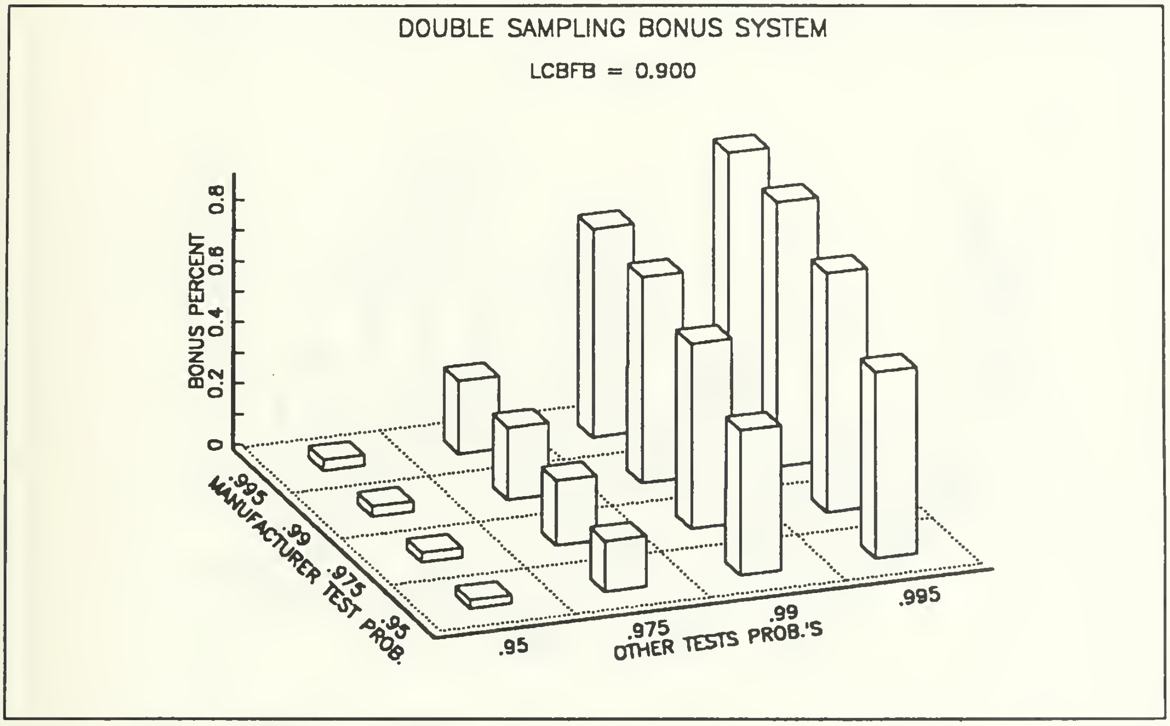


Figure 14. Double Sampling Bonus System With LCBFB = 0.900

Bonus percentages are tabulated and ploted with different probabilities.

$$LCBFB = 0.950$$

**Table 31. DSBS (DIFFERENT PROBABILITIES) LCBFB = 0.950**

| | 0.950 | 0.970 | 0.990 | 0.995 |
|---|---|---|---|---|
| 0.950 | 0.022 | 0.126 | 0.369 | 0.506 |
| 0.975 | 0.029 | 0.182 | 0.502 | 0.686 |
| 0.990 | 0.029 | 0.182 | 0.502 | 0.686 |
| 0.995 | 0.029 | 0.185 | 0.509 | 0.695 |

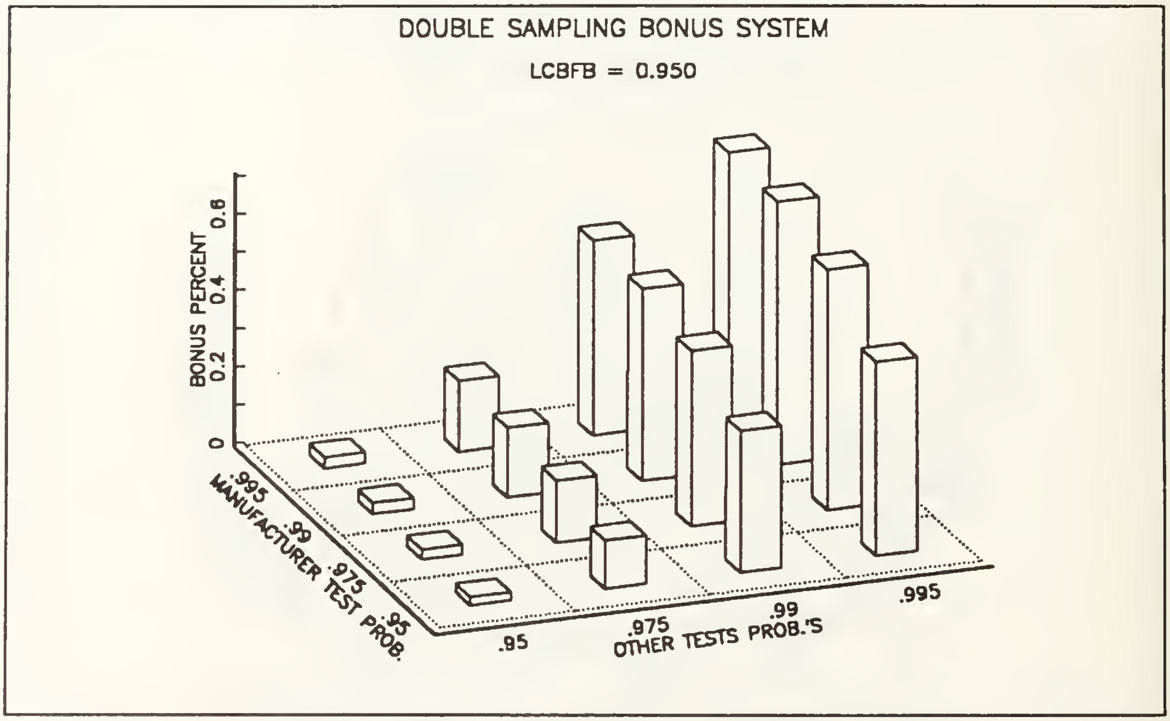


**Figure 15. Double Sampling Bonus System With LCBFB = 0.950**

Bonus percentages are tabulated and ploted with different probabilities.

$$LCBFB = 0.999$$

**Table 32. DSBS (DIFFERENT PROBABILITIES) LCBFB = 0.999**

| | 0.950 | 0.970 | 0.990 | 0.995 |
|---|---|---|---|---|
| 0.950 | 0.011 | 0.057 | 0.174 | 0.229 |
| 0.975 | 0.021 | 0.106 | 0.302 | 0.406 |
| 0.990 | 0.024 | 0.151 | 0.407 | 0.570 |
| 0.995 | 0.026 | 0.161 | 0.463 | 0.626 |

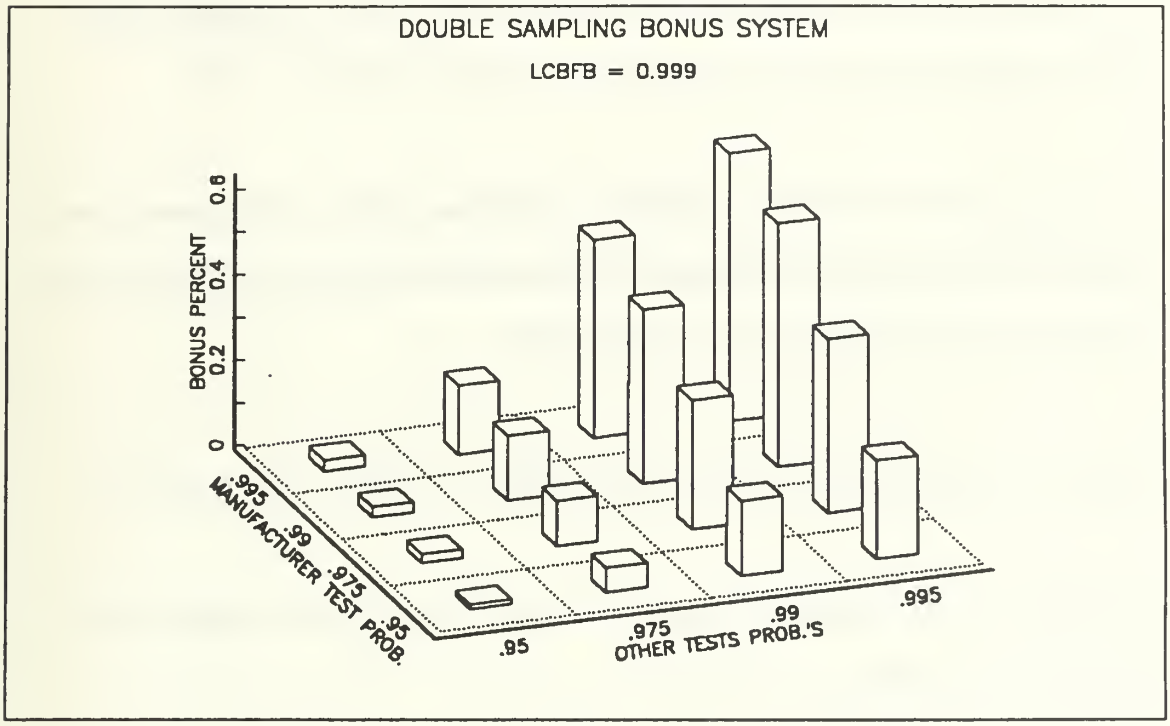


**Figure 16. Double Sampling Bonus System With LCBFB = 0.999**

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