# FOR CRYSTAL DRAWING

MARGARET REEKS

# HINTS FOR CRYSTAL DRAWING

# HINTS

FOR

# CRYSTAL DRAWING


BY

MARGARET REEKS


*WITH A PREFACE BY*


JOHN W. EVANS, D.Sc., LL.B., F.G.S.

MINERALOGIST TO THE SCIENTIFIC AND TECHNICAL DEPARTMENT OF THE IMPERIAL INSTITUTE


*WITH ILLUSTRATIONS BY THE AUTHOR*




LONGMANS, GREEN, AND CO.

39 PATERNOSTER ROW, LONDON

NEW YORK, BOMBAY, AND CALCUTTA

1908

R64

TO

PROFESSOR J. W. JUDD, C.B., LL.D., F.R.S.,

LATE DEAN AND PROFESSOR OF GEOLOGY AT THE ROYAL COLLEGE OF SCIENCE

WITH SINCERE RESPECT

# PREFACE.

THE accurate representation of geometrical relations in three dimensions is always a matter of difficulty, and this is especially the case with the forms of crystals. The most obvious course is to resort to models, but these are troublesome to construct, and occupy considerable space, so that for all practical purposes some method of representation in two dimensions, in other words a projection on a plane surface, such as a sheet of paper, must be employed.

The stereographic, gnomonic and linear projections, though admirable as exact records of the disposition in space of the faces, are unsuited to the elementary student who requires a projection that will enable him to realise at once the general form of the crystal. Ordinary perspective is, on the other hand, objectionable because the parallelism of lines is not preserved, and the scale of representation varies according to the distance from the point of view. It therefore becomes necessary to substitute parallel lines of sight or projection for those converging to the observer's eye, or to use a convenient, if somewhat self-contradictory locution, to represent the object as seen from infinity.

There still, however, remains a considerable choice as to the procedure to be adopted. We may, if we like, construct a plan and front and side elevations—in other words, representations of the object on a horizontal plane, as seen from above, and on vertical planes, as seen from a horizontal direction. In these, the lines of projection are at right angles to the plane of the drawing or projection, so that this method of representation is a special case of "orthographic projection". It is of great value for both educational and purely scientific purposes, but has the drawback that, as a rule, less than half of the faces are seen as such in the same view, others being represented only by their edges, which form the whole or part of the outline of the figure. A better idea of the crystal is therefore obtained if it be viewed obliquely from above and one side.

It remains to consider the position of the plane of projection, the canvas of our picture. Formerly it was usually placed at right angles to the lines of sight—another example of orthographic projection. It is now, however, considered preferable to follow the usages of pictorial art and, while placing the plane of representation or projection facing the supposed direction of the observer, to keep it in an upright position. Every photographer knows the importance of keeping the plate-holder upright if a natural picture is to be obtained. The descending lines of sight will then meet the plane at an oblique angle, and the projection is therefore referred to as inclined or clinographic. The crystal will accordingly stand with its vertical axis parallel to the plane of projection, but its principal horizontal directions will be oblique to the observer, and therefore to the plane of projection; so that the crystal may be considered as rotated on its vertical axis relatively to both.

As a result of these relations vertical directions retain their original dimensions; but distances from front to back or right to left are diminished in certain fixed proportions dependent upon the angle of rotation and the angular elevation of the line of sight.

In the present work the author, who has had a long and varied experience in mathematical drawing of every description, has devoted herself with her usual energy to the drawing of crystals in clinographic projection. While following in many respects the method of the late Professor Penfield, she has been able to give the student the advantage of numerous practical expedients which will greatly facilitate his work, and in some cases she has made distinct contributions to mathematical draftsmanship. Her method of determining the position of the axes in the triclinic system is, in fact, an ingenious graphic solution of a spherical triangle when three sides are given. Her procedure in drawing twin crystals is also marked by simplicity and originality.

If the student who is commencing the study of the geometry of crystals will work carefully through at least the simpler of the author's constructions, and follow this up by drawing other examples by the same methods, he will obtain a clearer grasp of crystal forms than he could possibly do by means of mere book work. Even the value of models for purposes of instruction is seriously discounted without the constructive training which is afforded by a course of crystal drawing.

JOHN W. EVANS.

IMPERIAL INSTITUTE,
*8th April*, 1908.

# AUTHOR'S PREFACE.

By way of preface, I would ask one boon of the teacher of Crystallography on behalf of the student, already bewildered by the difficulties of the subject—that he should not be confronted by a needless complexity of nomenclature and notation, but that one scheme should be selected and authorised by use.

The classification and symbols employed in this book have been very generally adopted. They are used in the classes at the "Imperial College of Science and Technology," and in many of the standard text-books.

So admirably has our great national collection of minerals, at the Natural History Museum, South Kensington, been arranged, that the student who has no opportunity of attending classes, may there see in the cases the entire science of mineralogy, as it were written in minerals themselves.

I have only to add, with regard to the solutions given in this book, that it is quite possible, that preferable methods may present themselves to the individual student for any particular point of construction—*tant mieux*, let him follow his own road

—nevertheless it is hoped that the hints supplied will aid him when struggling with the irritating elusiveness of crystal form.

It is believed that all axial ratios and other data given are accurate, but should any error have crept in, it will not affect the demonstration.

In conclusion, I would gratefully thank Dr. Gilbert A. Cullis, of the "Imperial College of Science and Technology," and Dr. J. W. Evans, of the Imperial Institute (to whom I am indebted for the preface), for the kind encouragement and advice they have given me whilst my little book "was in the making".

MARGARET REEKS.

IMPERIAL COLLEGE OF SCIENCE AND TECHNOLOGY,
*May*, 1908.

# CONTENTS.

# ILLUSTRATIONS.

# CHAPTER I.

## INTRODUCTORY.

THE sole object of this little book is to assist the student of Mineralogy in making drawings of the crystal forms and combinations with which he has to deal. Nothing beyond this is aimed at. No idea is entertained of competing with the many standard works on crystallography, to which the student is referred for information. Merely solution of graphical problems is attempted here, with the hope that the attempt will not be entirely without value.

In order to draw crystal forms successfully it is absolutely essential to have some slight knowledge of geometrical projection, to know how to make the simple plan and elevation of a solid, in orthographic projection.

It must be assumed therefore that the student has such knowledge, also that he has thoroughly mastered—from the text-books on the subject—the several characteristics of the six systems under which crystals are classed, with the different methods of notation most in use.

Throughout the problems here given, Miller's symbols will be employed, Bravais-Miller in the hexagonal system. The examples are all typical.

The subject of geometrical projection will only be followed so far as it is helpful for crystal projection. Trigonometry will hardly appear except in the guise of geometry.

Of the many projections used by the geometrician, that known as clinographic has been mainly selected by the mineralogist as suitable for crystal projection. Modern text-books almost invariably adopt it for their illustration—therefore clinographic projection will be chiefly considered.

Crystal projection is very dependent on parallel lines. For drawing such lines set squares are most convenient. All acquainted with mechanical drawing know the value of the 60° and 45° squares, yet at the risk of appearing tedious we will venture a few hints on their correct use.

Fig. 1 will illustrate it. A and B represent the position of the right and left hands whilst the squares are being set for drawing parallel lines.

The fourth, third, middle fingers and thumb of the left hand are holding the 60° set square firmly pressed against the paper on which the series of parallel lines is to be drawn. The first finger is merely touching the 45° square lightly, acting as a guide, so that the square, the hypotenuse of which rests against the hypotenuse of the 60° square, can slide backwards and forwards as required.

The right hand effects the shifting movements, holding the ruling pen ready to assume quickly the second position B′, whilst the line is being drawn.

The first finger of the left hand is pressed firmly during the ruling, releasing the pressure again to enable the 45° square to be shifted for a second line.

Should it be desired to draw lines at right angles to those first drawn, the 45° square is shifted backwards, the edge at right angles to that first used coming into play.

Clinographic projection has probably been so generally chosen for crystal representation, because it in some measure combines the elevation and plan

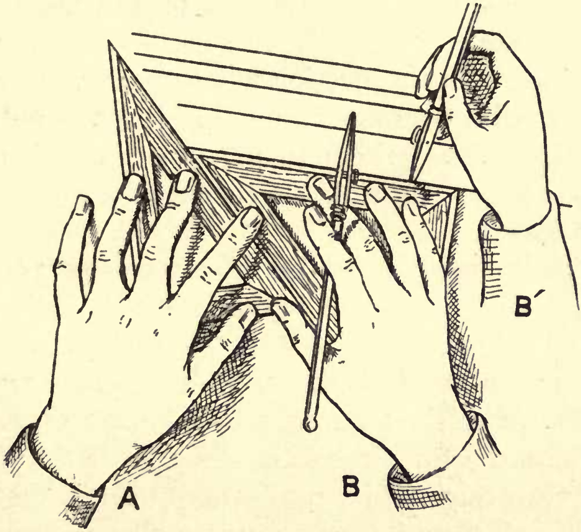



Fig. 1.

of orthographic projection, thus making one figure instead of two, suffice for the portrayal of one solid.

It has been assumed that the student is fairly well acquainted with orthographic projection, that he knows the relations of the vertical and horizontal planes on which the elevation and plan are projected, that he knows the relation of the projectors to each of these planes and to the object they are projecting; he knows that the projectors in the case of the plan fall vertically on every point and line of the object, as it were transpierce it, and convey the impress to the horizontal plane below; he knows that in the case of the elevation, the projectors transpierce the object horizontally, and carry its lines and points to a vertical plane behind it, which plane is at right angles to the horizontal projectors.

Clinographic projection much resembles orthographic, with some variation. It is made only on a vertical plane, which plane, it is convenient to imagine passes through the centre of the object to be projected.

The clinographic projectors are neither vertical nor horizontal, they are *inclined;* but though themselves inclined, they are, and *this is important, all contained in vertical planes*, which *planes* are at *right angles* to the plane of projection.

The projectors just as in the case of the elevation of orthographic projection, transpierce the object and carry its points to the plane of projection, but since the plane is taken through the centre of the object, all points in *front* of the plane will necessarily be carried by the projectors *downwards* and *backwards* to it; all points *behind* the plane will be carried to it, *forwards* and *upwards.*

These relations of projectors to planes of projection both in orthographic and clinographic projection will be clear from Fig. 2, which is a profile or side view of the planes of projection—A orthographic, B clinographic. The figure *a*, *b*, *c*, *d* is the side view of a cube which is being projected by the projectors—shown by dashed lines—to the respective planes of projection, as seen in profile represented by lines.

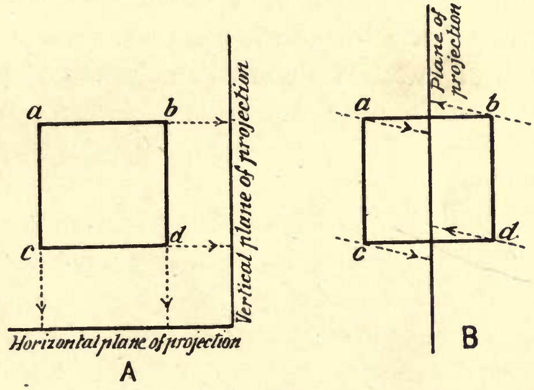



FIG. 2.

It should be apparent from these figures that in clinographic projection the top face of the cube will be visible, in orthographic projection it will be represented as a line.

The student, being acquainted with orthographic projection, knows that the projectors are at right angles to the plane of projection, because the eye is assumed to be opposite each point projected ; in clinographic projection, the eye or point of vision is supposed raised above each point a certain number of degrees, for which reason the projectors are represented by inclined lines.

In orthographic projection the object may be inclined to either plane ; in clinographic it is always placed with one axis in a vertical position and is then rotated round this axis towards the left, through some selected angle.

The result of this rotation is, that side faces become visible, just as, on account of the elevation of the eye, the upper faces are visible.

Fig. 3. *A* is an orthographic elevation of an octahedron. *B* is a clinographic projection of the same form. In both it has been rotated in the direction of the movement of the hands of a clock,

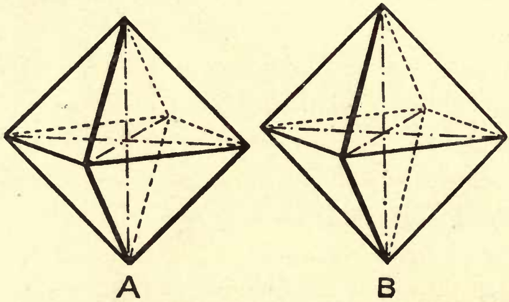



Fig. 3.

on its upright axis. In the orthographic projection that axis has been inclined a certain number of degrees to the horizontal. In the clinographic projection the axis is vertical but the point of vision has been elevated.

The figures are almost identical; there is one difference: in the clinographic drawing the object has its actual height, in the orthographic the height is slightly foreshortened.

In clinographic projection there are two most important angles—the angle of rotation of the object towards the left and the angle of the elevation of the point of vision.

Both these angles are optional—the draughtsman will vary them according to the kind of view he wishes to obtain—he may wish to show more of the side or of the top of any particular crystal; he will then choose wider angles.

Nevertheless, though this is justifiable—in some cases expedient—certain angles have been selected as suitable for crystal representation, and because of their suitability and almost invariable acceptance it is well to adhere to them.

These conventional angles are for the rotation 18° 26′, for the elevation of the eye 9° 28′.

As they are of such importance they must be constructed with most careful accuracy. To achieve this accuracy a certain trigonometrical ratio of the angles comes most opportunely to our aid. This ratio is the tangent.

The tangent of the angle 18° 26′ = 1 : 3.

The tangent of the angle 9° 28′ = 1 : 6.

By which is simply meant, stated in language not trigonometrical, that, if a perpendicular be drawn at any point on either of the lines enclosing the angle, and be produced to cut the other enclosing line, the proportion of the perpendicular to the line on which it is raised, measured from its base to the apex of the angle, will be in the case of angle 18° 26′ as 1 : 3; in the case of angle 9° 28′ as 1 : 6.

Knowledge of these ratios greatly aids in the construction of the angles.

It is applied thus: a line is drawn at some point; on it a perpendicular is raised, the proportion of the perpendicular to the line is made as 1 to 3 or as 1 to 6, the top of the perpendicular is joined to the end of the line to form the angle. Figs. 4 and 5 should demonstrate this sufficiently.

Since the words orthographic and clinographic are somewhat lengthy for constant use, they will often be represented by *C* and *O*. Since also the terms horizontal and vertical will frequently recur, they will be replaced by *H* and *V*.

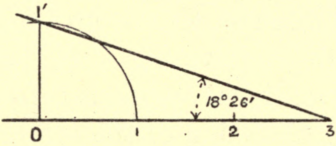



FIG. 4.

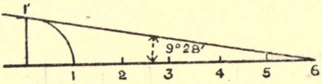



FIG. 5.

The student is reminded that with reference to projection, lines parallel to the side edges of the paper are called vertical; those parallel to the upper and lower edges are called horizontal.

## CHAPTER II.

### ISOMETRIC SYSTEM—PROJECTION OF THE AXES.

### PLATE I.

AXES all at right angles, all equal, all lettered, $a$, $a_1$, $a_2$, $a_3$. $a_1$ and $a_2$ horizontal; $a_3$ vertical.

The first step towards the construction of any crystal form is the construction of the axes.

The construction of the isometric axes in $C$ projection shall be the first problem (see Plate I.).

In $C$ projection, the vertical axis will be a vertical line of its actual length. We can at once draw it, placing it towards the bottom of the paper so as to leave plenty of space above, making it the desired length and lettering its + and − ends, $+ a_3$, $- a_3$, as shown in the figure.

The projection of the two horizontal axes is less simple; they will not appear as mere horizontal lines; the $a_1$ will be inclined downwards towards the left, its + end will appear depressed.

The inclination towards the left is dependent on the angle of rotation, the apparent depression on the angle of elevation of the eye. We have to find the amounts of inclination and apparent depression.

To do this we call to our aid an $O$ plan of the axes.

This we construct above the $C$ projection of the

$a_3$ axis already drawn. Choose a point $O$ (see Plate I.) vertically above it. This point is the $O$ plan of the $a_3$ axis, which in accordance with the rules of $O$ projection will be represented simply by a point.

Through $O$ draw a vertical and also the line $OA_1$ making with the vertical the angle of rotation 18° 26′.

This angle should be constructed according to the directions given (see Fig. 4).

Make the line $OA_1$ equal the semi-axial length.

Through $O$ draw line $OA_2$ at right angles to $OA_1$, making it equal $OA_1$ in length.

These two lines $OA_1$ and $OA_2$ are the plans of the semi $a_1$ and $a_2$ axes in the required position in orthographic projection.

We want these axes in $C$ projection.

We already have the vertical $a_3$ axis in line $+a_3$, $-a_3$. Bisect this axis and through the centre draw the $H$ line $X'Y'$.

Drop vertical projectors from the + ends of the $H$ axes of the $O$ plan. Somewhere on these projectors, below the level of $X'Y'$ will lie the + ends of the horizontal axes in $C$ projection; the problem we have to solve is to find their exact position on these projectors.

The simplest method is as follows:—

Through point $O$ of the $O$ plan draw the $H$ line $XY$; produce the vertical through $A_1$ to cut this line in $O'$, divide the distance $OO'$ into half as shown, then by means of the dividers convey the $\frac{1}{2}$ distance to the $C$ projection, marking it downwards from the line $X'Y'$ on the vertical projector dropped

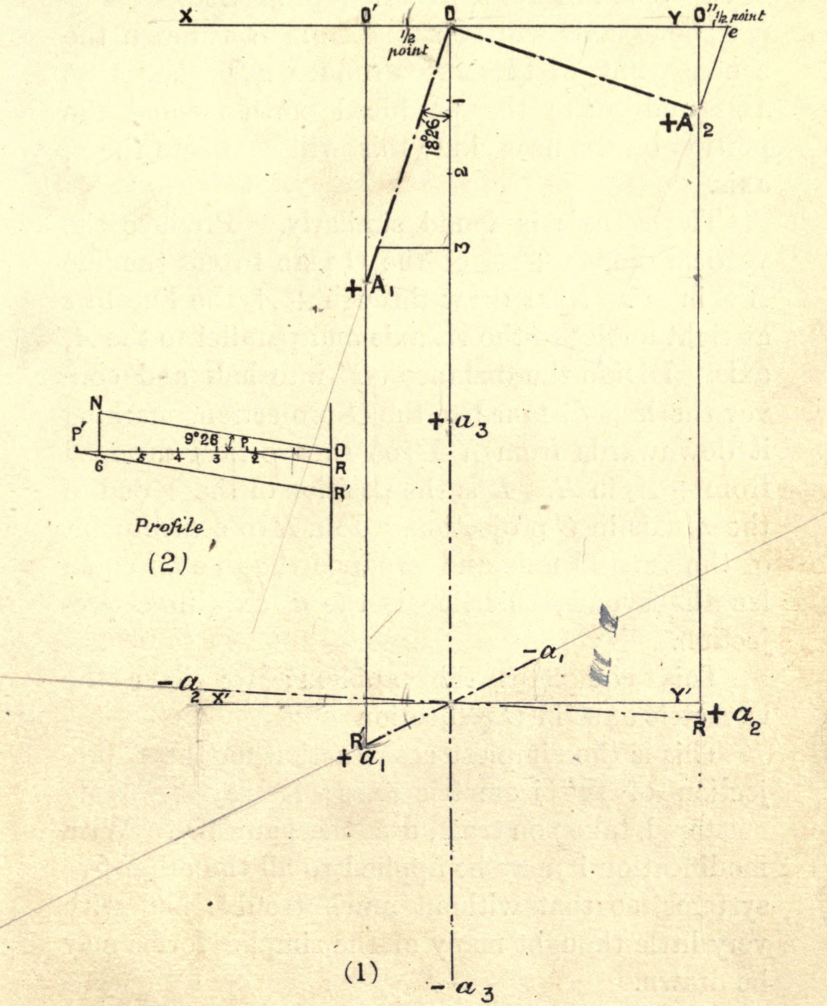



PLATE I.

ISOMETRIC SYSTEM.

Construction of Axes.

from $A_1$ in point $R'$. Point $R'$ will be the position of the + end of the $a_1$ axis in $C$ projection.

The $a_1$ axis will be a line from $R'$ through the centre point of the $a_3$. Produce it beyond that axis and make the produced portion equal the portion on the near side; this will complete the $a_1$ axis.

The $a_2$ axis is found similarly. Produce the vertical though $+ A_2$ of the $O$ plan to cut the line $XY$ in $O''$. Next draw through $+ A_2$ the line to $e$ at right angles to the $A_2$ axis and parallel to the $A_1$ axis. Divide the distance $eO''$ into half and convey the half distance to the $C$ projection, marking it downwards from $X'Y'$ on the vertical dropped from $+ A_2$ in $R$. $R$ is the position of the + end of the $a_2$ axis in $C$ projection. Join $R$ to centre point of the vertical axis and produce the line an equal length beyond; this line is the $a_2$ axis in $C$ projection.

This completes the problem; we have the isometric axes in $C$ projection.

This is the simplest construction for the $C$ projection of the isometric axes; it may be easily mastered, taken on trust, used mechanically. With modification it may be applied to all the other five systems, so that without much trouble and with very little thought many of the simpler forms may be drawn.

Nevertheless explanation of the several steps will be demanded.

It will at once be seen how the $O$ plan aids the $C$ projection much in the same way as it aids the orthographic elevation.

From the $O$ plan we get the amount of foreshortening consequent on the rotation to the left. The vertical projectors dropped from the + ends of the $A_1$ and $A_2$ axes convey this to the $C$ projection below.

But the $O$ plan does not indicate directly the degree of foreshortening of the axes consequent on the elevation of the point of sight.

We obtain at once from the $O$ plan how much to the right or left of the centre the + ends of the $H$ axes will lie. We do not at once know the distance below the centre they will appear to fall. We know they will lie somewhere on the vertical projectors dropped from the $O$ plan. We do not know the exact position on these projectors.

Let us first consider what the projection of the horizontal axes, the $a_1$ and $a_2$ would be supposing the point of vision had not been raised through $9° \ 28'$. They would appear in $C$ projection as a horizontal line, passing through the centre of the vertical axis. The $a_1$ axis would apparently coincide with the $a_2$ for a portion of the length of the latter, but though in reality of the same length it would appear shorter. Its + end would be where the $V$ projector from the $A_1$ axis of the $O$ plan cuts the $H$ line $X'Y'$ of the $C$ projection. The + end of the $a_2$ would be where the $V$ projector from $A_2$ cuts the same line.

But the elevation of the point of vision causes a proportional apparent depression of the + ends of the two horizontal axes.

Suppose the plane of projection and the projectors could be viewed from the side or in profile

(Fig. 2, Plate I. shows such a view). The plane of projection is represented simply by the vertical line through $ORR'$. The projectors appear as inclined lines. These inclined lines make the angle of elevation 9° 28′ with any horizontal line drawn at right angles to the plane of projection.

Draw the line $NO$, making the angle 9° 28′ with $OP'$ (for construction of the angle see Fig. 5); all the projectors will be parallel to $NO$.

Now from the $O$ plan we take the distance $O'$, $+ A_1$, which is the distance the + end of the $A_1$ axis is in front of the plane of projection. We mark this distance on the profile in $OP'$; draw $P'R'$ parallel to $NO$. $R'$ gives us the distance the end of the axis will appear to fall below the level of the centre $O$. $OR'$ marked on the vertical dropped from $+ A_1$ of the $O$ plan to the $C$ projection will give the exact position for the + end of the $a_1$ axis.

The + end of the $a_2$ is found similarly. The distance $+ A_2\ O''$ is marked on the profile in $OP$; the projector is drawn parallel to $NO$ to cut the plane of projection in $R$. $OR$ is the amount of apparent depression of the + end of the $a_2$ axis to be marked on the vertical projector dropped from $+ A_2$ of the $O$ plan in $R$. $R$ is the + end of the $a_2$ axis in $C$ projection.

But the profile is not necessary for construction and all unnecessary work is best dispensed with. We can after all obtain the apparent depression from the $O$ plan.

For as a consequence of the tangential relations of the angles selected for rotation and elevation, because the tangent of $9^{\circ}\ 28$ is $\frac{1}{6}$, whilst that of $18^{\circ}\ 26'$ is $\frac{1}{3}$, we can get the amount of depression for any point in a very simple way: we have only to take one-sixth part of a line passing through the point to line $XY$ at right angles to that line; this sixth part will be the amount of apparent depression to be marked downwards on the vertical projector below the line $X'Y'$ of the $C$ projection.

But since to get the exact sixth we must either set proportional compasses or use some geometrical method, and since such compasses may not be at hand, and the geometrical method or the more primitive trials by the dividers method, both take time, we can find the amount in the case of the + end of the $a_1$ axis by dividing the distance $OO'$ into half and taking the half.

Any point whose apparent depression we wish to obtain may be made the end of a line parallel to the $A_1$ axis. The + end of the $A_2$ is the end of the line to $e$, which is parallel to the $A_1$ axis; $\frac{1}{2}\ eO''$ will give the amount of apparent depression for the + end of the $a_2$ axis in $C$ projection. It is not necessary to draw the line to $e$, we need only mark the point.

An easy test for the accuracy of the construction of the axes is to draw verticals through the ends of the $a_1$ and $a_2$ axes and horizontals through the ends of the $a_1$. The space between the $H$ lines taken with the dividers should measure twice into the space between the verticals through the ends of the $a_1$ axis; and the space between the verticals through the ends of the $a_1$ axis, taken with the dividers, should measure three times into the space between the verticals through the ends of the $a_2$ axis.

It will be noticed that only half the $O$ plan of the axes has been drawn, for since half supplies all necessary points, no more is needed, the fewer lines used for construction the better; for construction lines are, as it were, the scaffolding necessary whilst a building is in progress, to be removed when it is complete.

In all figures given the back edges will be shown dotted, in order that the entire form or combination of forms may be perfectly realised. The front edges will be shown by thick lines, the construction lines will be kept fine.

The axes will always be indicated by chain lines; when the figure embraces several forms the axial ratio, the proportion of the axes, will always be shown in the centre of the figure.

There is no occasion to repeat the construction for the axes each time; a method, which we will call "parallelism," may be used for reproducing them.

Let the axes be constructed once with the utmost care and accuracy and of a fair length on a piece of thin paper; the vertical axis must be absolutely parallel to the right-hand edge.

Draw on the card or paper on which the projection is to be made a perfectly vertical line. Place the right-hand edge of the pattern sheet accurately against this line, holding it firmly in position, whilst a set square is adjusted in turn to each of the horizontal axes and lines drawn by the parallel method from a point selected for the centre of the figure to be projected. These lines will give the directions of the horizontal axes. This done the pattern sheet can be removed and the axes completed.

Should it be desired that the axes of the drawing be of different lengths from those of the pattern this may be attained by a further application of "parallelism".

Before removing the pattern sheet adjust a set square to touch the apex of the pattern vertical axis and the + end of either of the *H* axes, then having marked the desired length of the new vertical axis, shift the square without changing its inclination, by keeping it well against the edge of the guiding square until it touches the apex of the new vertical axis; it will cut off the correct proportional length on the new *H* axis. The length of the other *H* axis may be obtained in a similar way; reference to Fig. 1 should make this method clear (see also Plate XIV.).

This method if carried out with great care is possibly more exact than that of pricking through, because the latter, by constant use, wears the paper, causing inaccuracy.

Several methods may be employed for the solution of the problems offered by crystal drawing.

These methods we will call :—

The parallel method or parallelism.

The method of symmetry or repetition.

The method of plane intersection.

The profile method.

The word "repetition" has been substituted for symmetry to obviate confusion between geometrical and crystallographic symmetry and will be used throughout the explanations.

## CHAPTER III.

### ISOMETRIC SYSTEM (*CONT.*)—OCTAHEDRON—CUBE—RHOMBDODECAHEDRON—COMBINATIONS.

### PLATES II. TO IV.

BEFORE attempting to project any crystal it is absolutely necessary to *realise* it, that is to say, to have a clear idea of its proportions and their interrelation before the mental vision.

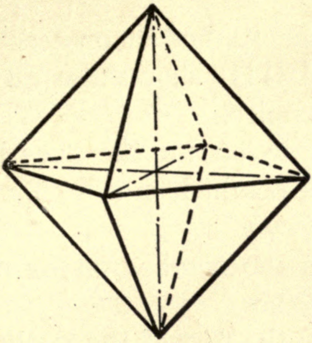
To attain such an idea it is useful to have a model, but if such a thing is not at hand the practised draughtsman can construct a mental image from certain angles and ratios found in the text-books; to do this is an exercise for the reasoning and imaginative faculties. The power to realise a form not seen, is a power well worth acquiring.

To get such a clear vision of a crystal form, it is seldom necessary to have exact measurements of every edge and angle. The repetition of symmetry in most of the systems obviates this.

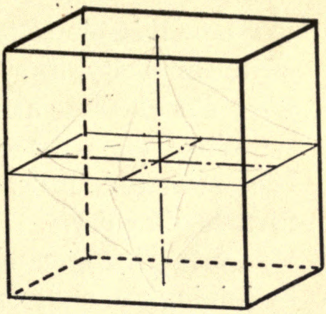
In the isometric system very few data will generally be required, practice alone will enable the student to select such as will best aid him to realise a form or combination of forms.

It will be assumed that the student knows the several forms by name or possesses a text-book of mineralogy in which he may find them described.

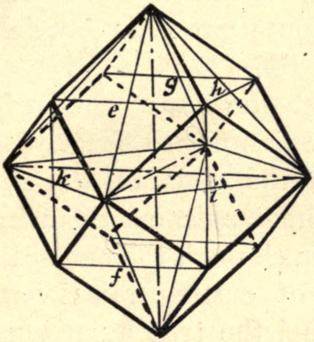
Though the cube is figured first in the books, it



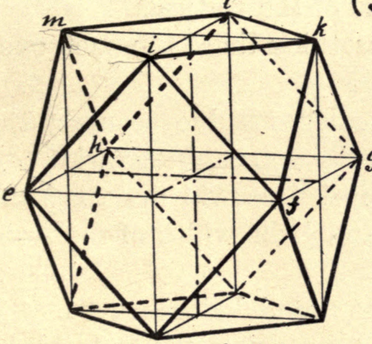
(1)



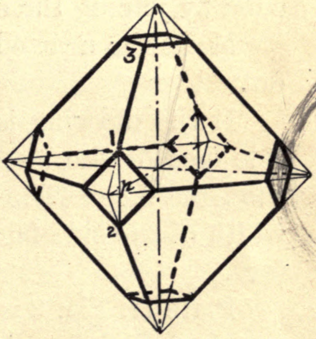
(2)




(3)




(4)




(5)

## PLATE II.

ISOMETRIC SYSTEM.

FIG. 1. Octahedron {111}.
FIG. 2. Cube {100}.
FIG. 3. Rhombdodecahedron {101}.
FIG. 4. Cube and Octahedron in Equilibrium.
FIG. 5. Cube and Octahedron, latter predominant.

will be advisable to take the octahedron on account of its simplicity, its symbol {111} shows that all its faces cut all three axes at unity.

When once the construction for the axes has been mastered, drawing the octahedron will offer no further difficulty.

Plate II. Fig. 1 shows the octahedron formed by joining the ends of the axes.

The cube is drawn by the first of the methods mentioned on page 17, "parallelism" (see Plate II. Fig 2).

Construct the axes or transfer them, by the method given on page 16.

Through the ends of the $a_1$ axis draw lines parallel to the $a_2$ axis, cut them by lines parallel to the $a_1$, drawn through the ends of the $a_2$. Through the points of cutting raise verticals, make the verticals equal the true axial length, complete the cube by joining the ends of the verticals.

Make the near edges thick lines, the back ones dotted.

If the drawing is made with due accuracy the side face of the cube will appear $\frac{1}{3}$ the width of the front face. The upper face will appear $\frac{1}{2}$ the width of the side face and $\frac{1}{6}$ the width of the front face.

Plate II. Fig. 3 is the rhombdodecahedron.

Having obtained the axes, join the ends of the $a_1$ and $a_2$. Join them also to the ends of the $a_3$ axis; in other words, complete the octahedron lightly, divide each edge into exactly half in the points *e*, *f*, *g*, *h*, *i*; through these points draw lines parallel respectively to the three axes, *i.e.*, lines parallel to the $a_2$ through $e, f, g$; parallel to the $a_1$ through $h$; to the $a_3$ through $i, k$.

Where these lines intersect will be the solid angles of the form.

The figure is completed by the second method (see p. 17) by "repetition". The draughtsman knowing the points which are vertically below or on the same level with those found, that is to say, knowing thoroughly the symmetry of the rhombdodecahedron, completes it by marking off similar lengths on selected parallel lines. He will test the accuracy of his work by trying with the set squares whether the several points lie on lines parallel to the axes; he will not need actually to draw the lines, as he becomes skilful he will more and more dispense with auxiliary lines, merely marking the required points of intersection by a sharply defined pencil dot.

Fig. 4 shows a combination of forms—the cube and the octahedron; they are in so-called equilibrium, the faces of the octahedron meet at the middle points of the edges of the cube.

The construction is at once apparent. Lines are drawn through the ends of each axis parallel to the other two axes; they intersect in points $e, f, g, h, i, k, l, m$, etc.; join these points for the edges of the combination.

Fig. 5 shows the same combination, but in this case the octahedral faces greatly predominate. The cube cuts the $a_1$ axis in point $p$. Draw a short vertical through $p$ to cut the two edges of the octahedron at points 1 and 2. From these points the intersections of the two forms can be drawn;

they are each parallel to one edge of the octahedron.

For the upper cubic face point 3 can be found by taking the distance from the end of the $a_1$ axis to point 1 and marking it from the vertex of the $a_3$ axis in point 3. The edges of intersection can again be drawn from 3 parallel to octahedral edges.

All the other cubic faces may be found similarly.

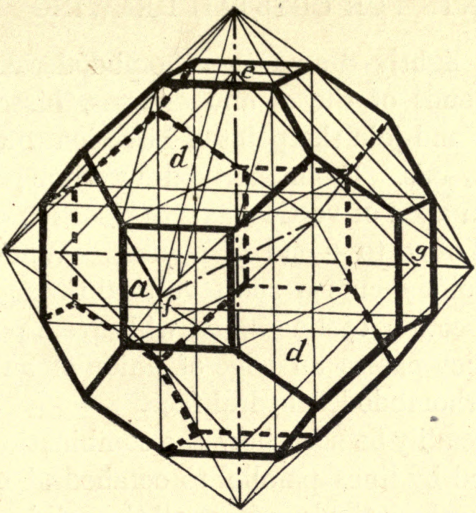
Plate III. Fig. 1. The rhombdodecahedron cut by cubic faces gives an extreme example of the use of "parallelism".

Draw first the rhombdodecahedron quite lightly.

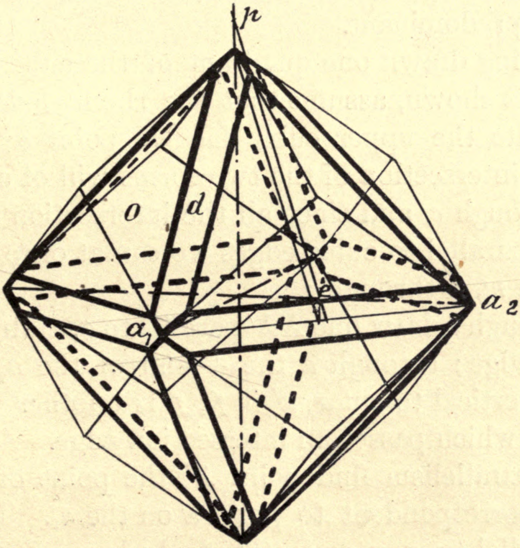
It is assumed that the cubic faces cut the axes in points $e, f, g$. These points can be marked from the ends of the respective axes. A line through point $e$ parallel to the $a_2$ axis, and a line through point $f$ parallel to the vertical axis will cut lines drawn through the centres of the rhombdodecahedral faces and by cutting them will give points through which the intersection edges of the two forms can be drawn parallel to the several axes; careful attention to the figure will make this clear.

Many more lines than are necessary have been shown in this example to illustrate how "parallelism" may be used to test accuracy, but to the experienced worker such a network of lines is of course needless; by mere adjustment of the set squares he will check the parallelism.

Fig. 2 shows the rhombdodecahedron in combination with the octahedron. The rhombdodecahedral faces truncate the octahedral. Point $p$ is where the apex of the octahedron would be if it had not given place to the other form.




(1)




(2)

**PLATE III.**

ISOMETRIC SYSTEM.

FIG. 1. Rhombdodecahedron and Cube.

FIG. 2. Rhombdodecahedron and Octahedron.

*a* {100} *o* {111} *d* {101}.

Having lightly drawn the rhombdodecahedron join the + ends of the $a_1$ and $a_2$ axes, bisect the joining line and join the point of bisection to the + end of the $a_3$ axis. From point $p$ draw a line parallel to that from apex of vertical axis to bisection of line from + end of $a_1$ to + end of $a_2$ axis. Where the line from point $p$ cuts a short vertical through the point of bisection of line $a_1$, $a_2$, will give a point $e$, through which passes an edge of union of an octahedral and rhombdodecahedral face.

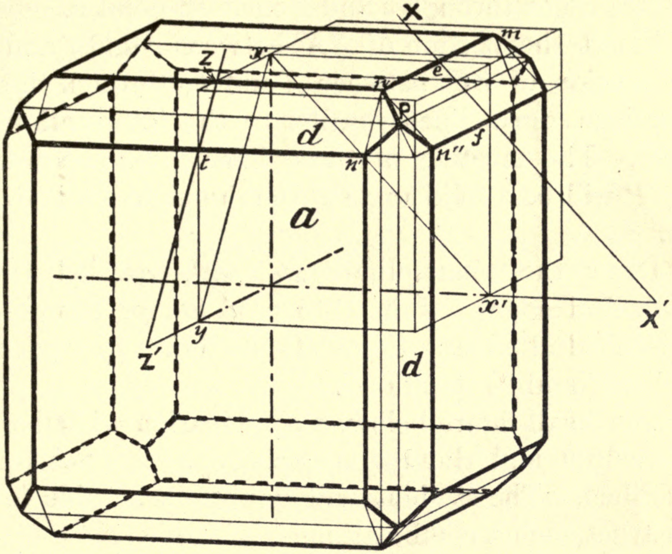
It will readily be seen how the combination can be completed by lines parallel to octahedral edges. Rhombdodecahedral edges form all the solid angles.

Plate IV. Fig. 1 shows the cube in combination with the rhombdodecahedron, the former being greatly predominant.

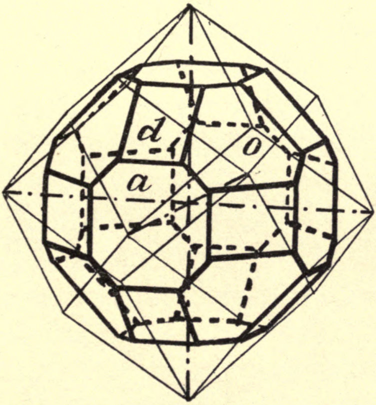
Having drawn one quadrant of the cube, quite lightly as shown, assume that the rhombdodecahedron cuts the upper cubic face in point $e$. One edge of intersection of the two forms will of course pass through $e$, and since all the intersection edges will be parallel to cubic edges we can at once draw the edge $m$, $n$ through $e$.

Through $e$ draw line $XX'$ parallel to $x$, $x'$ an octahedral edge; through $x$ the + end of the $a_2$ axis raise a vertical to cut $x$, $x'$ in $f$; $f$ is another point through which passes an intersection edge.

By parallelism find point $Z'$, the point on the $a_1$ axis correspondent to point $x$ on the $a_2$. Draw $Z'Z$ parallel to $x$, $y$, an octahedral edge. $Z'Z$ will meet the upper cubic face in $Z$ and the front one in $t$. Through $Z$ and $t$, intersection edges can be drawn.




(1)




(2)

PLATE IV.

ISOMETRIC SYSTEM.

FIG. 1. Cube and Rhombdodecahedron.
FIG. 2. Cube, Octahedron and Rhombdodecahedron.
*a* {100} *o* {111} *d* {101}.

The edges through *e* and *Z* meet in point *n*.

Bisect the portion of $XX'$ between *e* and *f* and the portion of $ZZ'$ between *Z* and *t*; through the bisections draw lines parallel to the intersection edges. The meeting of these lines gives a point *P*. *P* will be a solid angle of the rhombdodecahedron.

Draw the short lines $nn'$, $nn''$, parallel to octahedral edges. Join *P* to $n$, $n'$, $n''$ for rhombdodecahedral edges. Complete the figure by "parallelism" and "repetition".

The small figure below, a combination of cube, octahedron and rhombdodecahedron, need not be described. The student will doubtless be able to construct such a combination.

## CHAPTER IV.

### ISOMETRIC SYSTEM (*cont.*)—TRAPEZOHEDRON—COMBINATION WITH RHOMBDODECAHEDRON.

### Plate V.

The problem of the trapezohedron depends on plane intersection for solution.

In the method of plane intersection, the axial planes play a most important part; the student should therefore endeavour to master and clearly realise their positions and interrelation.

In the isometric system there are three such planes. They are all at right angles—two are vertical, one is horizontal. They intersect in lines coincident with the axes.

The first vertical plane passes through the $a_2$ and $a_3$ axes.

The second vertical plane is at right angles to the first, it passes through the $a_1$ and $a_3$ axes; its intersection with the first plane is a line coincident with the $a_3$ axis.

The third plane is horizontal, it is at right angles to both the others, it passes through the $a_1$ and $a_2$ axes; its intersections with the other two planes are lines coincident with the $a_1$ and $a_2$ axes.

The intersections of these planes with each other and with all other planes will be represented by lines.

The student will recognise that these three planes divide the crystal form into octants, but they are not limited by the edges of that form, they extend beyond it on all sides, indefinitely.

By considering the relations of the several crystal faces to these planes we are able to understand and project the form.

The trapezohedron we will take for illustration has the symbol {211} (see Plate V. Fig. 1).

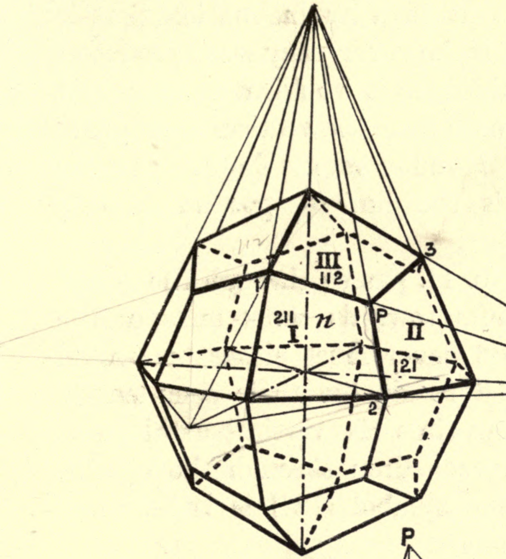
We must first imagine the trapezohedron divided into eight portions by the axial planes. These portions correspond in position to the eight faces of the octohedron. We will consider the construction of the upper right-hand octant only; having solved this problem the construction of the others will be matter of "parallelism" and "repetition".

It will be seen that each octant is formed by three faces. These faces, bounded by edges, are limited portions of three planes which must be imagined to extend beyond the faces.

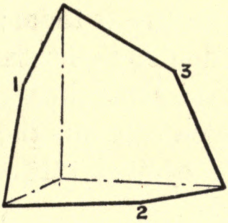
These planes are neither vertical nor horizontal, they are oblique.

If the student has a model of a trapezohedron and will hold it so that one axis is vertical, another horizontal and slightly turned towards the left (reference to a figure of a trapezohedron in any text-book will enable him to get the position), he will see the three oblique faces which compose the upper right-hand octant, and will by a stretch of the imagination be able to realise them as portions of oblique planes which extend beyond their limits.

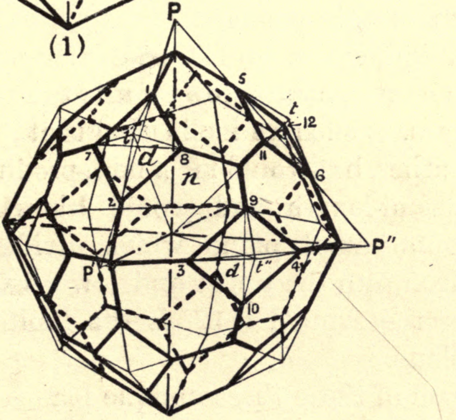
Oblique plane No. I. passes through the + end




(1)




(2)




(3)

## PLATE V.

Isometric System.

Fig. 1. Trapezohedron {211}.
Fig. 2. Auxiliary Construction.
Fig. 3. Trapezohedron and Rhombdodecahedron.
$n$ {211} $d$ {101}.

of the $a_1$ axis, slants towards the + ends of the $a_2$ and $a_3$ axes, but at such an inclination that it never meets them; if, however, they were produced to double their semi-length it would meet them and at the same time would meet the vertical axial plane which contains the $a_3$ and $a_2$ axes. The face having the symbol {211} is the limited portion of this plane.

Oblique plane No. II. passes through the + end of the $a_2$ axis, slants towards the + ends of the $a_1$ and $a_3$ axes, would never meet either but would meet both produced to double their semi-length, meeting at the same time the vertical axial plane that bisects the trapezohedron through the $a_1$ axis. The face having the symbol {121} is the limited portion of this plane.

Oblique plane No III. passes through the + end of the $a_3$ axis, slants downwards towards the other two axes, at such an inclination that it would never meet either, but would meet both produced to double their semi-length and would likewise meet the horizontal axial plane which divides the trapezohedron into the upper and lower half. The face having the symbol {112} is the limited portion of this plane.

Each of these three oblique planes has an intersection, or as it is called, a *trace*, on the three axial planes which pass through the centre of the form.

If we can find two such traces for each plane we can define the octant.

How shall we find these traces?

From the + end of the $a_1$ axis draw a line to cut the + end of the $a_2$ axis produced to twice its semi-length. This line is the horizontal trace of the first of the oblique planes.

Find next the $H$ trace of the second oblique plane. It starts from the + end of the $a_2$ axis to cut the + end of the $a_1$ axis produced to twice its semi-length. Now these two $H$ traces meet in a point 2, the portions between the point of meeting 2 and the + ends of the $a_1$ and $a_2$ axes are the two horizontal edges of the octant.

To find the two front edges we need the traces of the first of the oblique planes and of the third of these planes on the $V$ plane which passes through the $a_1$ axis. These traces are found by a quite similar method.

Join the + end of the $a_1$ axis to a point on the $a_3$ axis produced to twice its semi-length. Join the + end of the $a_3$ axis to the + end of the $a_1$ produced to twice its semi-length. These lines give the desired traces, they meet in a point 1. The portions between the point of meeting and the ends of the axes are the front edges of the octant.

The side edges are found in the same manner. By joining the + ends of the $a_2$ and $a_3$ axes to their semi-lengths produced, the edges are the portions between the point of intersection 3 of the traces and the + ends of the axes.

The octant at present appears as Fig. 2. From point 1, draw a line to the + end of the $a_2$ axis produced to double its semi-length. From point 2 draw a line to the + end of the $a_3$ axis produced to double its semi-length. These lines will intersect in a point $P$ which will be the solid angle of the octant. The point must be joined to point 3 for the intersection of the upper and right-hand lower faces {112} and {121}.

The right-hand upper octant is now complete. The others may be constructed by "parallelism" and "repetition".

The lines in the figure passing upwards, are shown merely to indicate how all the intersections of the planes which meet the $a_1$ and $a_2$ axes at unity would meet the $a_3$ at twice its semi-length. They give a good test for accuracy.

The figure below, Fig. 3, shows a combination of trapezohedron with rhombdodecahedron.

Assume point $P$ to be the apex of the rhomb-dodecahedron.

Having constructed the trapezohedron by rules given, find the points where the rhombdodecahedral faces $d$ cut the edges of the trapezohedron. This is done by drawing the lines $PP'$, $PP''$, and $P'P''$ parallel to octahedral edges; they cut trapezohedral edges in points 1, 2, 3, 4, 5, 6. Next draw lines from the angles of the trapezohedron $t$, $t'$, $t''$ to the centre; the intersections of these with the lines $PP'$, $PP''$, $P'P''$ give points through which lines parallel to the different axes may be drawn, which will cut the edges of the trapezohedron, giving the points 7, 8, 9, 10, 11, 12; lines connecting these with 1, 2, 3, 4, 5, will give the intersections of the two forms and complete the construction of one octant, the others may be found by the usual methods.

## CHAPTER V.

ISOMETRIC SYSTEM (*CONT.*)—TRISOCTAHEDRON—COMBINATION WITH CUBE—TETRAHEXAHEDRON—COMBINATION WITH CUBE—WITH OCTAHEDRON.

### PLATES VI., VII.

THE trisoctahedron offers little difficulty of construction (Plate VI. Fig. 1).

Begin by constructing an octahedron every edge of which will appear in the trisoctahedron.

The completion of the latter form presents one problem for solution; how to find the point where the three oblique planes of each octant meet; the solid angle X.

Plane intersection will be the method to employ.

We have to find the intersections of the three faces with the symbols {212}, {122} and {221}.

As in the trapezohedron we have to find the *H* traces of the planes {212} {122}; but since it is sometimes inconvenient to extend the axes to double their length we may use a proportional method and instead of producing them divide their semi-length into half.

Having carefully so divided the semi $a_1$ and $a_2$ axes join the half points of each to the end of the other. Now these joining lines will show the *direction* of the *H* traces of the planes; we need the *H* traces themselves; they are lines drawn through the end of the $a_1$ axis parallel to that which joins its half to the end of the $a_2$ axis, and through the end of the $a_2$ axis parallel to that which joins its half to the end of the $a_1$. These last drawn lines intersect in point $P$. A line from the intersections of the traces, point $P$, to the + end of the $a_3$ axis gives the intersection of the planes. It remains to find point $X$, the point where the plane {221} meets and obliterates the other two planes.

Form a parallelogram inside the octahedron edges by drawing through the $\frac{1}{2}$ points of the axes lines parallel to those edges. Bisect the lines of this parallelogram and join the bisections to the + end of the $a_3$ axis. A line drawn from the centre of the octahedral edge $g$ parallel to that to the end of the $a_3$ axis will cut the intersection of the planes {212} {122} in point $X$. Join $X$ to the + ends of the three axes and the octant will be complete.

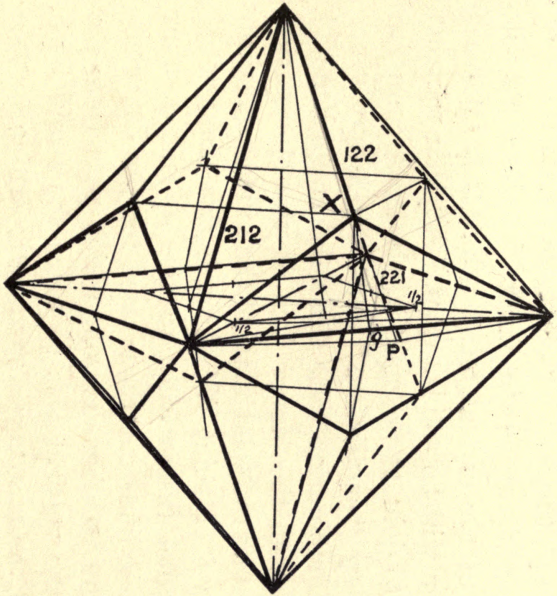
The solid angles of the other seven octants may be found by "parallelism"; by lines parallel to the respective axes cutting lines drawn from the centres of the octahedral edges parallel to those from the inner parallelogram to the + end of the $a_3$ axis.

Plate VI. Fig. 2. The trisoctahedron cut by cube.

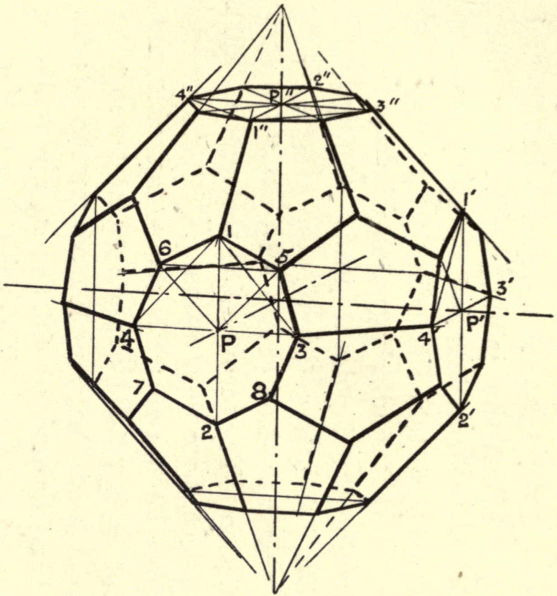
First find the trisoctahedron. It will of course not be necessary to complete all the lines.

Assume the cube faces to cut the axes of the trisoctahedron in points $P$, $P'$, $P''$.

Through these points draw lines parallel to the $a_1$, $a_2$, and $a_3$ axes to cut the octahedral edges at 1, 2, 3, 4; 1′, 2′, 3′, 4′; 1″, 2″, 3″, 4″.




(1)




(2)

PLATE VI.
Isometric System.

Fig. 1. Trisoctahedron {212}.

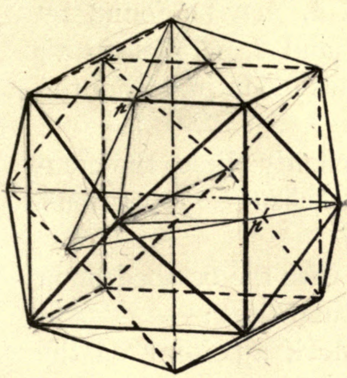
Fig. 2. Trisoctahedron and Cube.

To find points where the cube faces cut the other intersections of the trisoctahedral faces join point 1 to 3 and 4, bisect the joining lines, draw lines through the bisection points and $P$, taking the lines on to cut the trisoctahedral edges in points 5 and 6. Find 7 and 8 similarly. Complete the cubic face by joining the 8 points. Finish the figure by "parallelism" and "repetition".

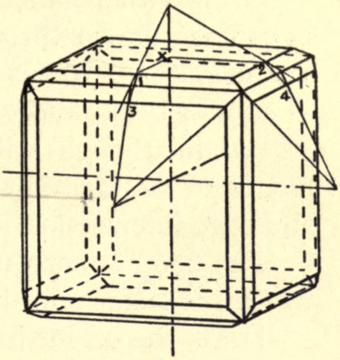
The tetrahexahedron (Plate VII. Fig. 1) is easy of construction. The form having the symbol {201} has been selected for illustration.

Let lines be drawn from the + end of the $a_1$ axis to cut each of the other axes at twice their semi-lengths produced, and lines from the + ends of the $a_2$ and $a_3$ axes to cut the $a_1$ axis produced to twice its semi-length. These lines will intersect in points $pp'$, through which draw lines parallel to the axes as shown in the figure. These lines will be edges of the crystal faces, where they intersect each other will be solid angles. All that remains to do is to join these solid angles and complete the solid by "parallelism" and "repetition".

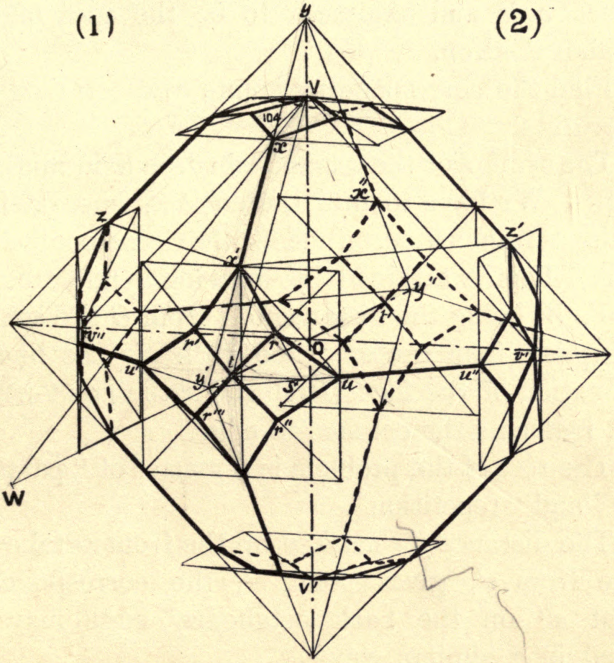
The combination with the cube is constructed thus: Assume the upper face of the obliterating cube to cut off the solid angle of the tetrahexahedron at the level of point $X$; through this point draw lines parallel respectively to the $a_1$ and $a_2$ axes where these lines cut those used for the construction of the tetrahexahedron, which pass from the apex of the vertical axis to twice the semi-length of the $a_1$ and $a_2$ axes produced; will give two points 1 and 2 through which the intersection edges of cube and tetrahexahedron pass.




(1)




(2)




(3)

## PLATE VII.

### Isometric System.

Fig. 1. Tetrahexahedron {201}.
Fig. 2. Tetrahexahedron and Cube.
Fig. 3. Tetrahexahedron {401} and Octahedron.

Other two points, 3 and 4, may be found by short lines drawn through 1 and 2, parallel to an octahedral edge to cut the lines mounting from the + ends of the $a_1$ and $a_2$ axes.

Careful thought will sufficiently direct the completion of the figure by "parallelism," "repetition" and edge intersection.

Fig. 3. The combination with the octahedron in which the latter prevails is found thus :—

Draw the octahedron. Mark a point $V$ on the vertical axis and assume it to be the apex of the tetrahexahedron.

Find the corresponding points $v'$, $v''$ ; $t$, $t'$ on the other axes.

The symbol of the tetrahexahedron is in this case {401}. We have to find the point $x$, where a face of the tetrahexahedron meets the front octahedral edge. This is attained by drawing a line from $V$ to $W$, $W$ being the distance of $t$ from $O$ the centre, multiplied four times. Line $V$ $W$ is the line of inclination of the face {104} and $x$ the point where that face cuts the octahedral edge.

The rest of the problem is a matter of "parallelism" and "repetition".

The distance $y'x$ marked up the front octahedral edge from $y'$, gives point $x'$, the correspondent point $x''$ on the back octahedral edge may be found in a similar way.

$x'$ can be transferred to $z$ and $z'$ by "parallelism".

$u$, $u'$, $u''$, may all be found by "parallelism" and measuring off corresponding equal distances.

Having thus determined all points on octahedral edges where the two forms meet, we have next to find intermediate points such as *r*.

Through points *x*, *u*, *z*, etc., draw lines parallel to the several axes so as to form the lightly drawn squares. Join the points *t*, *t*,′ *v*, *v*′, etc., to the corners of the squares. These diagonals of the squares will be the intersections of the different tetrahexahedral faces; we have to cut off these intersections where they meet the octahedral faces.

For this purpose draw through *t* line *t*, *s* parallel to the $a_2$ axis. *t*, *s* is the line of intersection of the face {401} of the tetrahexahedron with the horizontal axial plane. *s* is the point where this intersection meets the octahedral edge. *s* then is a point which lies in both the face {401} and the octahedral face; it is common to both, therefore line *x*′ *s* is the intersection of these faces, but at *r* this intersection meets the face {410}; we have therefore to join *r* to *u*.

The similar points *r*′, *r*″, *r*‴, may now be easily obtained by "parallelism" and "repetition".

# CHAPTER VI.

## ISOMETRIC SYSTEM (*CONT.*)—HEXOCTAHEDRON.

### PLATE VIII.

THE hexoctahedron is a somewhat complicated form. The student must before undertaking its projection carefully master its proportions and realise the relation of the several faces to those of the octahedron.

The form having the symbol {312} has been chosen for illustration. (Plate VIII.)

It will be well to make this figure an example of how when a crystal is very intricate, much of the work of construction may be done outside the actual drawing by working from the *O* plan.

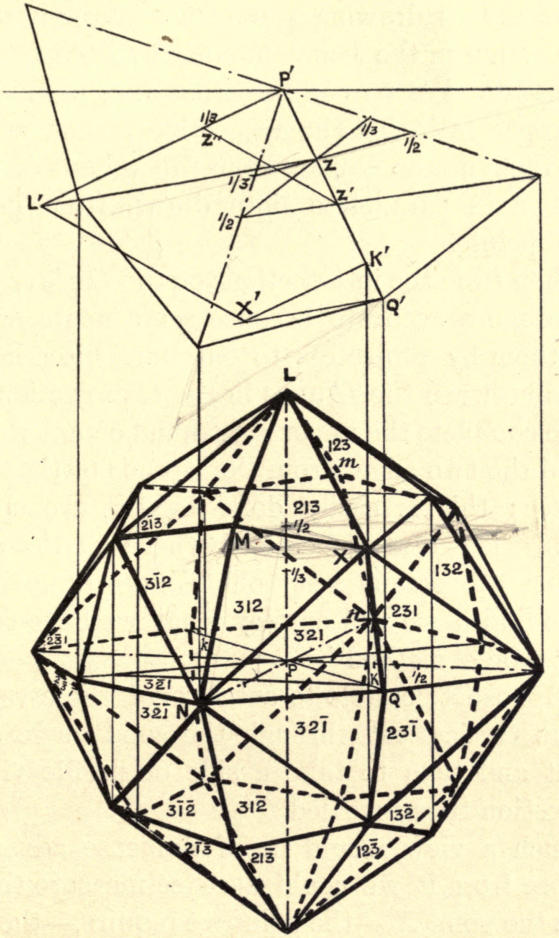
Having found the axes or transferred them, we can at once construct the near and distant central edges *LMN*, *Lmn*, using proportional points to avoid extension of the axes. They will evidently be lines from the + and = ends of the $a_1$ and $a_3$ axes parallel to lines drawn from the $\frac{1}{3}$ to the $\frac{1}{2}$ points of these axes as the several face symbols direct.

We have next to find the horizontal edges. These as we know will be the horizontal traces of the planes to which they belong.

Since these traces cut very obliquely in the *C* projection, their exact intersection being on that

$1 \; \frac{1}{2} \; \frac{1}{3}$

$\frac{1}{2} \; 1 \; \frac{1}{3}$




$\frac{1}{3} \; 1 \; \frac{1}{2}$

$\frac{1}{2} \; \frac{1}{3} \; 1$

$\frac{1}{3} \; \frac{1}{2} \; 1$

**PLATE VIII.**

ISOMETRIC SYSTEM.

Hexoctahedron {312}.

$\frac{1}{3} \; 1 \; \frac{1}{2}$

2 6 3

account, difficult to determine, we may avail ourselves of the $O$ plan.

Instead of drawing these traces directly on the $C$ projection in the usual manner by lines from the ends of the axes to cut the other axes produced at the proper ratio for the respective planes, we may draw them on the $O$ plan, from the + ends of the $a_1$ and $a_2$ axes parallel to lines drawn from proportional points.

Then from the intersections on the $O$ plan, which will be more accurate because more acute, we can drop them by projectors to cut lines bisecting the angles between the $H$ axes in the $C$ projection.

To complete the upper right-hand octant we have to find the two edges from the + ends of the $a_2$ and $a_3$ axes; this is easily done by the proportional points. The central point $X$ where all the planes forming the octant meet offers more difficulty.

To find it we will employ what we have termed the "profile method".

Suppose it possible to cut through the crystal by a clean vertical cut, through the solid angle of the octant and then to take a side or profile view of the section thus revealed.

Such a view would be of immense advantage, because from it we could at once measure the distance the point $X$,—the point we require,—the point of the solid angle—is from the vertical axis and also the height it is above the horizontal plane containing the $a_1$ and $a_2$ axes.

Such a view we can construct reasoning as follows: Our section to obtain the profile passes through line $PQ$ in the $C$ projection; through $P'Q'$

of the $O$ plan. We need only concern ourselves with that portion lying above $PQ$ in the $C$ projection, the lower portion can always be found by "repetition".

We take the line $P'Q'$ of the $O$ plan; it is one true line for the profile. We have another line $PL$ the semi $a_3$ axis; that too is a true line for the profile. One other fact we have towards its construction: these two lines are at right angles to one another. The next step therefore is to draw $P'L'$ equal $PL$ at right angles to $P'Q'$ on the $O$ plan.

Two more lines we require, $L'X'$ and $X'Q'$; they also must be of the true length in the profile. How shall we ascertain their length? If we have a model which is dependable enough we can measure them. The safer means of arriving at their length is by construction.

We know that the planes which have $XQ$ for their intersection both meet the $a_3$ axis at 3 times its own semi-length from the centre, therefore we can find the inclination of the line of intersection. If on the $O$ plan we draw a line from $Q'$ to $P'L'$ produced to 3 times its length we shall have the desired inclination.

To prevent making the drawing extend beyond the limits of our paper we, instead of producing line $P'L'$, draw the line of inclination from $Z$ to $L'$, $Z$ being the proportional point for $Q'$ found by drawing lines through the $\frac{1}{2}$ and $\frac{1}{3}$ points of the $a$ and $a_2$ axes.

A line parallel to $ZL'$ through $Q'$ is the line we want for our profile.

But we need its true length. How shall we arrive at it? By finding the true inclination of the intersection of the planes {213} and {123} and drawing a line from $L'$ at this inclination, to cut off the line through $Q'$.

To find the true inclination of this intersection, first find point $Z'$ on the $O$ plan. $Z'$ is the proportional point on line $P'Q'$ through which the $H$ traces of planes {213} and {123} would pass. $Z''$ is the proportional point of the $a_3$ axis of the profile through which they would both pass. Line $Z''Z'$, gives the inclination of their intersection. A line through $L'$, the apex of the $a_3$ axis of the profile parallel to $Z''Z'$, will cut off the line from $Q'$ in the point $X'$.

From $X'$ draw $X'K'$ at right angles to line $P'Q'$. $K'$ will be the point in the $O$ plan vertically below $X$, the solid angle we require. Drop $K'$ by a vertical projector to the $C$ projection to cut the line $PQ$ in $K$; then from $K$ mark on the vertical projector a height $KX = K'X'$, which is the true height of the solid angle $X$ above the horizontal plane through the $H$ axes. Join $X$ to the other points of the octant already found. The octant is then complete.

Complete the other octants by "parallelism" and "repetition".

The angle of the left-hand back octant, for example, is found by making $Pk = PK$ and from $k$ raising a vertical $= KX$.

The student will perhaps realise the profile construction better, if he imagines a clean cut made through the points $LXQ$ and then imagines it possible to turn the cut open on line $PQ$ acting as a hinge. This has been done on the $O$ plan. $P'Q'$ is the hinge line, the face of the clean cut, the profile, is bounded by lines $P'L'X'Q'$.

# CHAPTER VII.

ISOMETRIC SYSTEM (*cont.*)—PYRITOHEDRONS + AND – —COMBINATIONS— + PYRITOHEDRON OBLITERATED BY OCTAHEDRON— – PYRITOHEDRON OBLITERATED BY OCTAHEDRON—DIPLOID, COMPLEMENTARY FORM—COMBINATION.

## PLATES IX. TO XI.

THE construction of the + pyritohedron is quite easy (Plate IX. Fig. 1). It is worked by plane intersection. The form with the symbol {210} has been chosen for demonstration. In this case the actual lengths, not proportional ones, have been used for the plane intersection.

A word of direction may be needed as to the manner of finding line *XY* the intersection of planes {102} and {210}.

Raise a vertical *OO′* at *O*, the point of intersection of the line drawn from the + end of the $a_1$ axis to cut the $a_2$ produced. This vertical, shown at *OO′*, is the intersection of plane {210} with the vertical axial plane through the $a_3$ and $a_2$ axes. Produce the edge of the pyritohedron *LP* to cut the vertical at *O′*; then a line drawn from *O′* to *X* will be the intersection of the planes {210} and {102}, because *X* and *O′* lie in both those planes.

To find the point *Y* and the intersection *PY* produce the edge through the + end of the $a_2$ axis to meet line *VZ* drawn parallel to the $a_2$ axis through the $a_1$ axis produced to twice its semi-length.

Join $Z$, which is a point common to both the planes {021} and {102}, to $P$. The line joining will cut $XO'$ in $Y$; this is the required point.

Complete the figure by "parallelism" and "repetition".

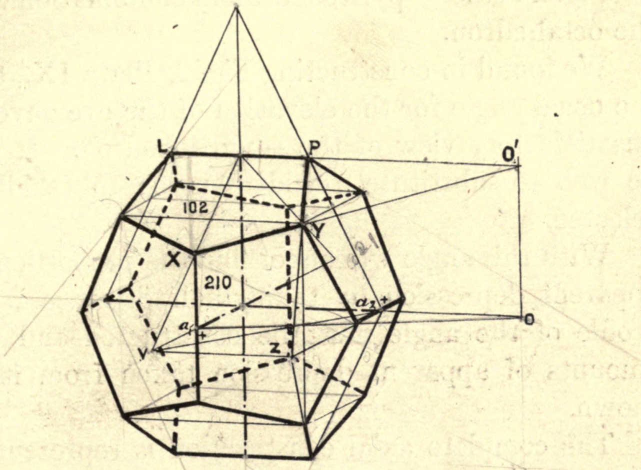
The minus pyritohedron is constructed similarly by plane intersection (Plate IX. Fig. 2).

Point $O$ is found by producing the edge through the + end of the $a_1$ axis to cut a line drawn parallel to the $a_1$ through the $a_2$ produced to twice its semi-length. Join the point of intersection $I$ thus found to $J$. Then through the + end of the $a_3$ axis produced to twice its semi-length draw a line parallel to the $a_2$ to meet the produced vertical edge through the + end of the $a_2$ axis in $L$. Join the point of intersection to $K$; the line to $K$ will cut the line from $J$ to $I$ in $O$, the point required.

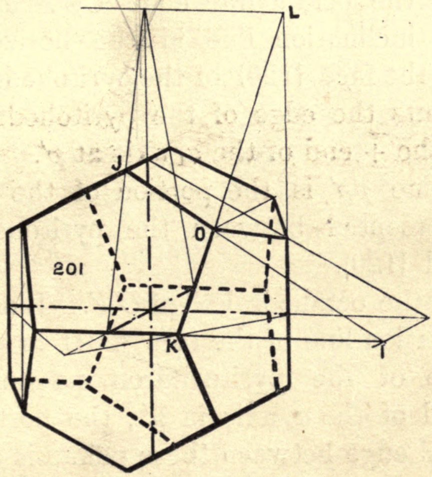
The student should carefully realise each step of the plane intersection employed for these two figures.

The last figure is not quite satisfactory. Two faces, the left-hand top face $\{0\bar{1}2\}$ and the right-hand lower face $\{01\bar{2}\}$ appear as single lines. It is a case when an angle other than the conventional one, might be preferred for the elevation of the point of vision.

Plate X. Fig. 1, shows the + pyritohedron {210} combined with an octahedron. The two forms are in what is termed equilibrium, that is, are equally present. It will be seen at once how the edges of intersection are found, by joining those angles of the pyritohedron which meet in any of the axial planes. Angles not meeting in these planes are obliterated by the octahedral faces.




(1)




(2)

PLATE IX.

ISOMETRIC SYSTEM.

FIG. 1. + Pyritohedron {210}.
FIG. 2. − Pyritohedron {201}.

Fig. 2 is the – pyritohedron in combination with the octahedron.

We found in constructing Fig. 2, Plate IX., that the usual angle for the elevation of the eye gave an unsatisfactory view of the – pyritohedron. It will be well to substitute a wider angle; 15° shall be selected.

With this angle we cannot find the proportionate apparent depression by taking half distances; the profile of the angle must be constructed and the amounts of apparent depression taken from it as shown.

The complete axial construction is represented.

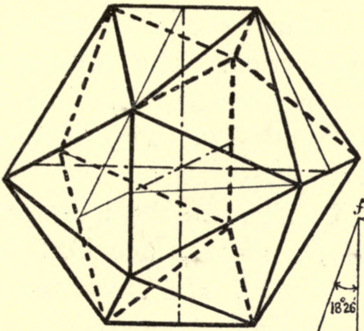
We will assume the left-hand angle of the octahedral face $\{1\bar{1}1\}$ to be at *C*.

Draw the octahedral edge *CC′*. It cuts line *k*, *l*, the inclination line in the horizontal axial plane, of the face $\{1\bar{2}0\}$ of the pyritohedron in point *p*, and cuts the edge of the pyritohedron passing through the + end of the $a_1$ axis at *p′*.

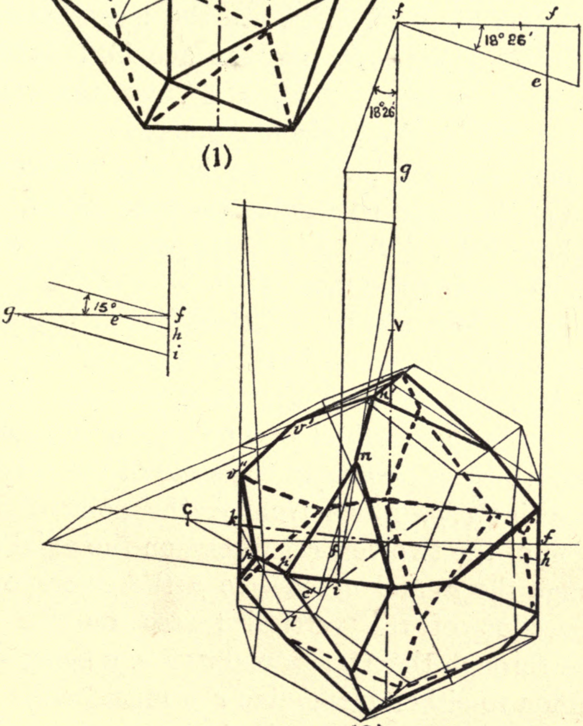
The line *pp′* is the portion of the octahedral edge intercepted between the pyritohedral faces {201} and $\{1\bar{2}0\}$.

Draw the octahedral edge *C″V*. Line *C″V* cuts the inclination line of the face {201} in point *n* and the edge of the pyritohedron passing through the + end of the $a_3$ axis in *n′*; the portion of the octahedral edge between these points is the portion required.

By joining *C* to *V* we get point *v′*. *v′v″* is a portion of another octahedral edge present in the combination. Joining these points *pp′*, *nn′*, *v′v″*, we have an octahedral face. The others can be obtained by



(1)




(2)

# PLATE X.

ISOMETRIC SYSTEM.

FIG. 1. + Pyritohedron and Octahedron in equilibrium.
FIG. 2. – Pyritohedron and Octahedron.

similar reasoning and by "parallelism" and "repetition".

Plate XI. Fig. 1, shows the construction for the diploid by plane intersection. The form having the symbol {321} has been selected. The intersections of planes {213} {213} and {132} with the three axial planes are shown complete.

Lines $XX'$, $XX''$, $X'X''$, are the intersections or traces of the plane containing the face {321} with the three axial planes. $XX'$, $X'X''$, are the vertical traces. $XX''$ is the horizontal trace.

Lines $ZZ'$, $ZZ''$, $Z'Z''$ are in like manner the traces of the plane containing the face {213}.

Lines $YY'$, $YY''$, $Y'Y''$, are the traces of the plane containing the face {132}.

The student will have little difficulty in comprehending the relative positions of these planes from their intersections with the axial planes. He will see at once the method of finding the intersecting edges, which is shown in the case of the edge $e, f$ by the line $e, g$, drawn from $e$, the point of intersection of the two vertical traces, on the axial plane through the $a_1$ and $a_3$ axes, to $g$ the corresponding intersection of the vertical traces on the axial plane through the axes $a_2$ and $a_3$. $e, g$ being points common to either plane, line $e, g$ must be the intersection of the planes and of the faces which are portions of the planes.

Fig. 2. The complementary form is constructed in exactly the same manner by finding the traces of the several planes involved, on the axial planes, by reference to the respective face symbols.

Fig. 3. The combination of the two forms is a

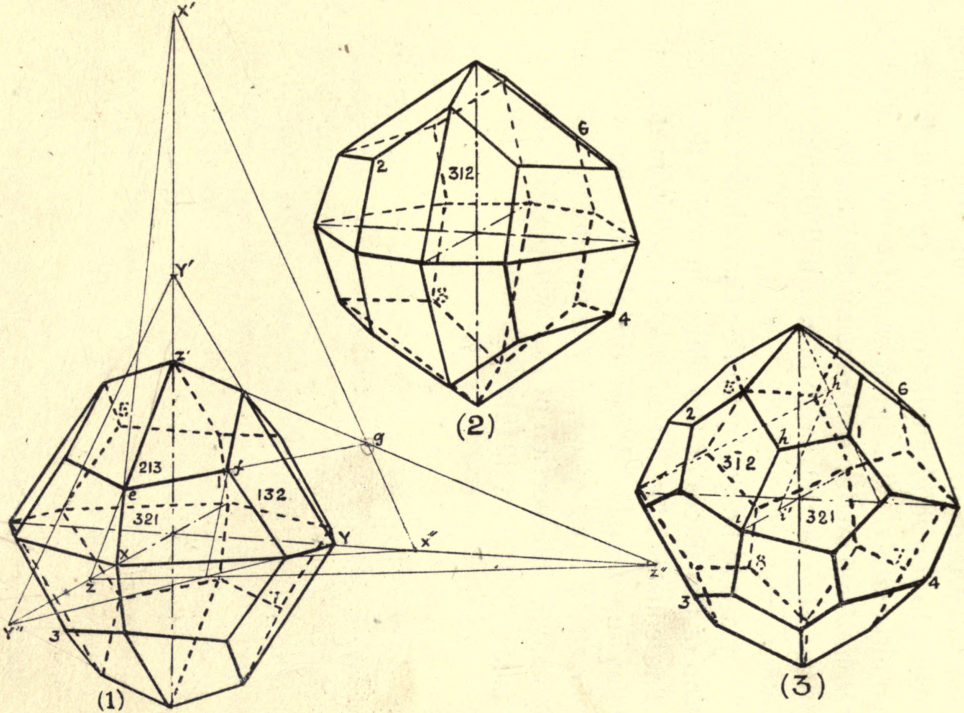



PLATE XI.

ISOMETRIC SYSTEM.

FIG. 1. Diploid {321}. FIG. 2. Complementary form {312}. FIG. 3. Combination of the two.

4*

little more intricate. It is worked in the same way.

First the solid angles which are numbered may be found; these correspond, as indicated by the numbers, to the solid angles of the original forms, taken alternately. The positions of the edges starting from these angles will remain unchanged. The edges meeting in the axial planes will change their direction.

They are to be got by finding the points of intersection of the traces of the planes in which they lie.

The construction for edge *h*, *i* is given in dotted lines as an example. Proportional points have been employed to save space; *i*, *h* is parallel to *i'h'*, the proportional intersection of the two planes containing faces $\{321\}$ and $\{3\bar{1}2\}$.

## CHAPTER VIII.

### ISOMETRIC SYSTEM (*CONT.*)—TETRAHEDRON—TRISTETRAHEDRON—TETRAGONAL TRISTETRAHEDRON—HEXAKISTETRAHEDRON.

### Plates XII., XIII.

The tetrahedrons, positive and negative, are very easy of construction (see Plate XII. Figs. 1 and 2).

The edges are drawn through the ends of the axes parallel to the edges of the octahedron.

For the positive tetrahedron {111} the edge through the + end of the $a_3$ axis is parallel to the octahedral edge from the + end of the $a_1$ to the + end of the $a_2$ axis.

For the negative tetrahedron $\{11\bar{1}\}$ the edge through the – end of the $a_3$ axis is parallel to that octahedral edge.

The positive tristetrahedron {211} may be constructed by plane intersection from the positive tetrahedron.

From each angle draw lines to intersect the axes produced to twice their semi-length; where these lines cut each other will be the solid angles of the figure.

It is not necessary to draw lines to the ends of all three axes for each point; two such lines will suffice but the third ensures accuracy.

Plate XIII. Fig. 1 gives the tetragonal tristetrahedron {221} (deltoid dodecahedron).

Draw lightly the upper half of the octahedron.

The next step is to find the points $X$, $X'$, $X''$, $X'''$.

To find point $X$. It will be situated somewhere on the normal to the octahedral face {111}. The normal will be a line passing from the centre of the axes $O$, through the central point of the face {111}. The centre of the face is the intersection of two lines joining the middle points of any two edges of the face to the opposite angles.

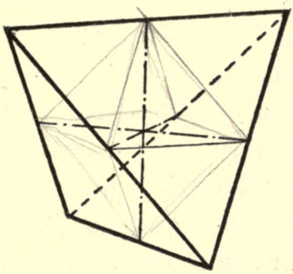
Next draw the $H$ trace of the plane {122} from the + end of the $a_2$ axis to the + end of the $a_1$ produced to twice its semi-length $h$, $t$.

To cut this trace draw line $Oc$. Join $c$ to the + end of the $a_3$ axis; the joining line will be the intersection of planes {212} {122}; the point where it cuts the normal will be the solid angle $X$.

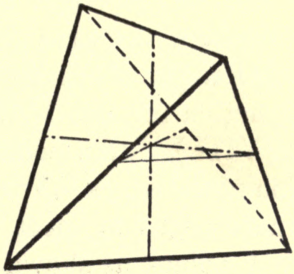
Draw line $xx'$ parallel to $Oc$; mark point $x'$ as far to the left of the $V$ axis as $x$ is to the right.

Mark point $e'$ on the $V$ axis as far below the centre as $e$ is above. Through $e'$ draw two lines $x''$, $x'''$ and $f$, $g$, parallel respectively to $xx'$ and to the octahedral edge $st$. $g$ is a point vertically under $x$ and by drawing $gx''$ parallel to the $a_2$ axis we get point $x''$, and marking the distance $ex''$ to the right of the $V$ axis we get $x'''$.

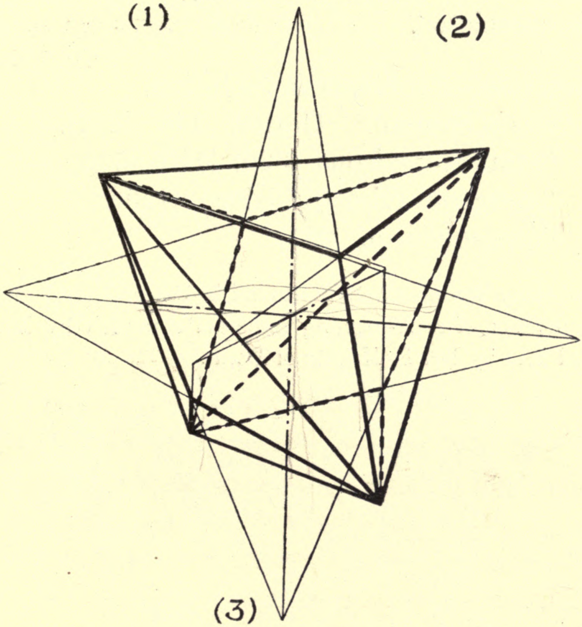
We now have the four solid angles $x$, $x'$, $x''$, $x'''$. Draw from these several points, lines through the central points of octahedral edges; where these lines cut will be the remaining angles required $k$, $i$, $l$, etc. $i$ has however been found preferably, by drawing a line from $k$, parallel to $st$, to cut the line from $x$; the line from $x'$ cuts too obliquely.



(1)



(2)



(3)

PLATE XII.

ISOMETRIC SYSTEM.

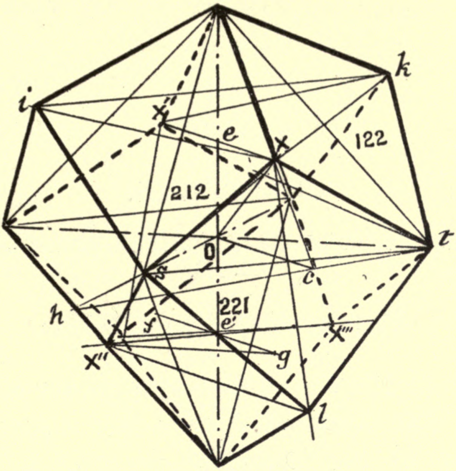
FIG. 1. Tetrahedron, positive $\{111\}$.
FIG. 2. Tetrahedron, negative $\{11\bar{1}\}$.
FIG. 3. Tristetrahedron $\{211\}$.

The construction of the hexakistetrahedron, Fig. 2, is soon described. The form selected is {312}. Find first all the edges of the hexoctahedron, that pass through the axes and the points *X*, *X*, *X*, *X*, the solid angles of alternate octants as directed in the construction for the hexoctahedron (see Chap. VI. Plate VIII.). Join points *X* to the points of intersection of the edges. From all the *X* points, produce alternate edges to meet alternate edges from the other *X* points, as shown in the figure and thus the form is completed.

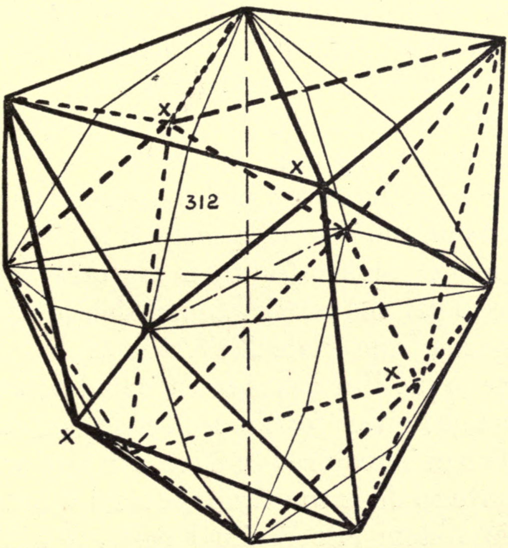
Exercises—Problems for solution.

Isometric System.[1] Garnet {110} {211}.
Tetrahexahedrons {320} {410} {530}.
Tetragonal tristetrahedron {443}.
Trapezohedron {311}.

[1] All these combinations are found in the collection of 100 crystal models of rock-forming minerals arranged by Dr. K. Busz and sold by Dr. F. Krantz of Bonn.




(1)




(2)

PLATE XIII.

ISOMETRIC SYSTEM.

FIG. 1. Tetragonal Tristetrahedron {221}.
FIG. 2. Hexakistetrahedron {312}.

# CHAPTER IX.

## TETRAGONAL SYSTEM—AXES—SIMPLE FORMS—COMBINATIONS.

### PLATES XIV. TO XVIII.

THE axes of the tetragonal system are all at right angles. The horizontal axes are equal and are lettered as in the isometric system, $a_1$, $a_2$. The vertical axis varies in length and is lettered $c$.

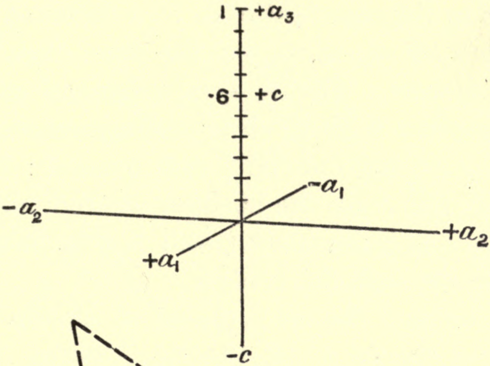
Since the $a_1$ and $a_2$ axes in the tetragonal system are at the same angle and in the same ratio as in the isometric, that is, are at right angles and equal to one another, it is evident that no different construction for finding them is needed.

The $c$ axis, however, which takes the place of the $a_3$ axis of the isometric system, is of a different length, longer or shorter than the other two.

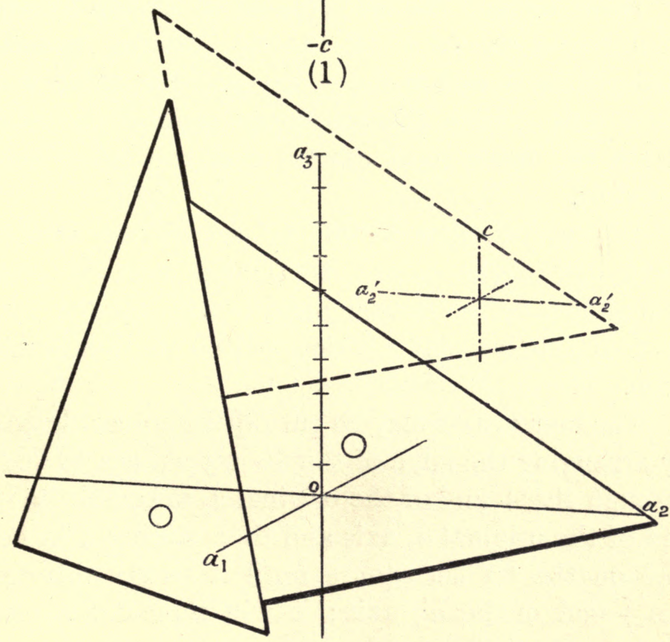
How to get the relative proportion of the $c$ axis is easily seen from Plate XIV. Fig. 1.

Suppose the three axes constructed according to the directions given for the isometric system. Assume the $c$ axis required to have the proportion ·6; all we have to do is to divide the semi $a_3$ axis of the isometric system into decimal parts, ·6 will be the + end of the $c$ axis which is marked off below the centre for the $-\ c$.

Fig. 2, Plate XIV. shows a convenient method by which having once constructed the axes we may




(1)




(2)

PLATE XIV.

TETRAGONAL SYSTEM.

FIG. 1. Method of obtaining proportion of $c$ axis.

FIG. 2. Method of finding axes proportional to given axes by "parallelism".

draw sets of axes either longer or shorter, having the $c$ axis in the same ratio, or in a different ratio.

Let lines $O$ $a_1$, $a_2$, $a_3$ be the semi-axes of the isometric system. Divide $a_3$ into decimal parts.

Suppose we wish to draw tetragonal axes of $\frac{1}{3}$ the length, the ratio of the $c$ axes being 1 : ·6.

Draw a line $a'_2$ in the position desired parallel to the $a_2$ axis but $\frac{1}{3}$ the length, and a line parallel to the $a_1$ axis through the centre.

Now arrange the set squares as shown by the solid lines; the edge of the 45° passing exactly through the + end of the $a_2$ axis and the point ·6 of the $a_3$ axis. Slide the set square against the edge of the 60°, being most careful that the latter does not shift.

When the edge of the 45° set square reaches the + end of the $a'_2$ axis, the new $a_2$ axis, the position shown by the dotted lines, it will cut the new vertical axis in point $c$ which will be in the correct ratio.

The new $a_1$ axis may be cut off the correct length by arranging the edge of the 45° set square to pass through the + end of the original $a_1$ axis and the + end of the original $a_2$ axis and then sliding it as before on the 60° set square until it passes through the + end of the $a'_2$ axis; it will then cut off the new $a_1$ axis the proper length.

This is an example of "parallelism" used to obtain a proportion. The student will soon discover many further useful adaptations of the method.

Plate XV. shows the simple forms of the tetragonal system. They offer no difficulties and from the figures the constructions will be amply plain.

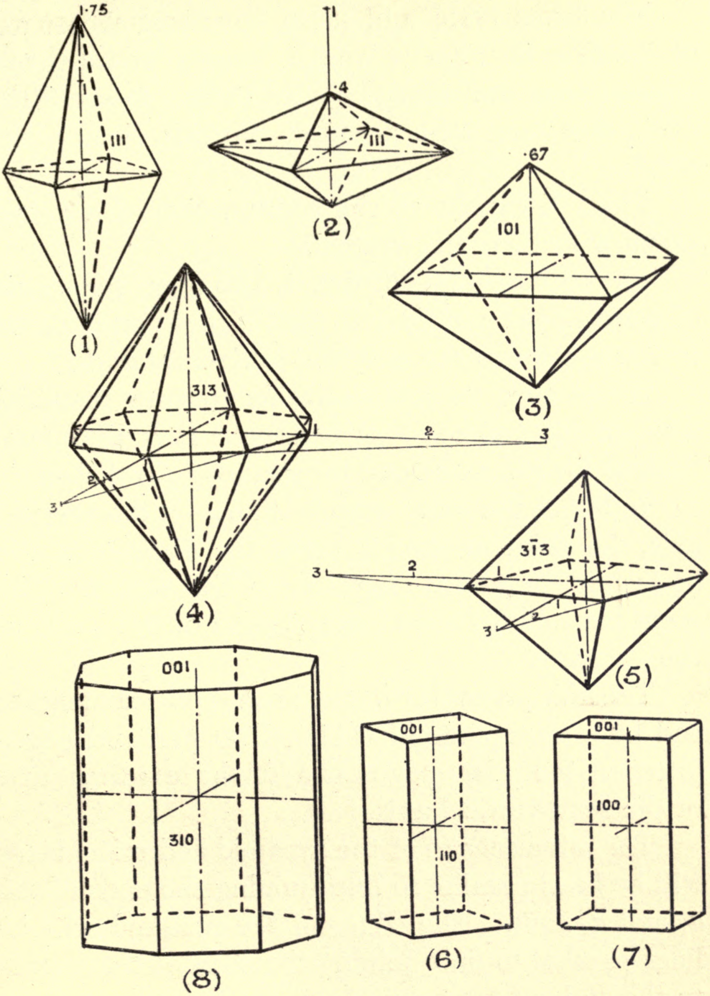



PLATE XV.

TETRAGONAL SYSTEM.

FIG. 1. Unit Pyramid (1st order) {111}; $c$ axis = 1·75.
FIG. 2. Unit Pyramid (1st order) {111}; $c$ axis = ·4.
FIG. 3. Diametral Pyramid (2nd order) {101}; $c$ axis = ·67.
FIG. 4. Ditetragonal Pyramid (Zirconoid) {313}.
FIG. 5. Pyramid of the 3rd order {$3\bar{1}3$}.
FIG. 6. Unit Prism {110} with base {001}.
FIG. 7. Diametral Prism {100} with base {001}.
FIG. 8. Ditetragonal Prism {310} with base {001}.

The forms {313} and {3$\bar{1}$3} have been chosen for illustration in Figs. 4 and 5, merely because the intersections are sharper than in the forms {212} {2$\bar{1}$2} they, on that account, furnish clearer examples.

Plate XVI. shows the combination of the unit prism with the unit pyramid.

The example taken, Fig. 1, is zircon, the *c* axis of which, has the ratio ·64; that is to say, it is as ·64 to 1 when compared with the horizontal axes.

It has been thought desirable to show the axial ratio in the centre of each figure; it is thus convenient for reference and for use by "parallelism".

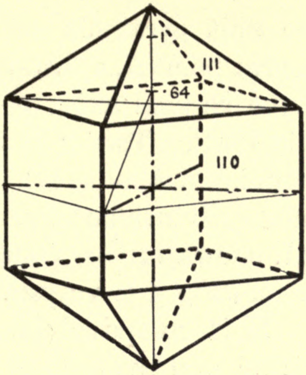
In Fig. 1 the axial ratio is shown on the vertical axis, the point marked 1 being the distance of the horizontal semi-axis length from the centre. The point marked ·64 shows the position of the + end of the *c* axis.

The unit prism {110} has been replaced at either end by a unit pyramid {111}. The pyramid edges intersect the prism edges at a height less than that of the semi-vertical axis.

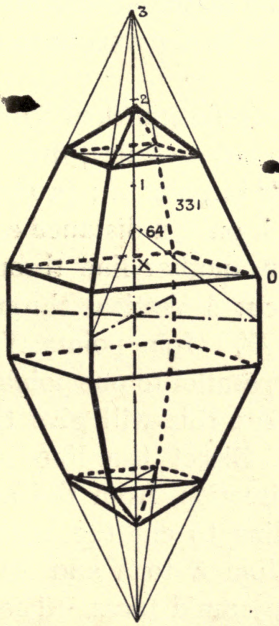
The intersections of the pyramid and prism faces will be lines parallel to lines joining the ends of the $a_1$ and $a_2$ axes. The edges of the pyramid will be lines parallel to lines joining the ends of the *a* axes to the ends of the *c* axis.

One advantage of having the axial ratios shown in the centre of each figure is that edges of unit pyramids can at once be obtained by "parallelism".

Fig. 2 shows a combination also of zircon; axis $c$ = ·64; composed of unit prism {110} and unit pyramids, but it is somewhat more complicated than




(1)




(2)

PLATE XVI.

TETRAGONAL SYSTEM.

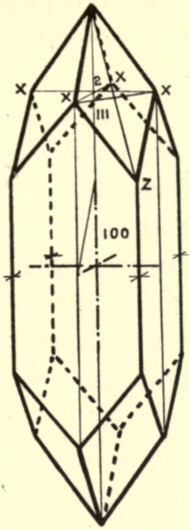
FIG. 1. Zircon $c$ axis = ·64. Prism {110}; Pyramid {111}.
FIG. 2. Zircon $c$ axis = ·64. Prism {110}; Pyramids {331}, {111}.

Fig. 1, as two unit pyramids are present. The prism gives place at point $X$, a distance less than the semi-height of the vertical axis, to a unit pyramid whose symbol is {331}. This in its turn is replaced by another pyramid, symbol {111}. The construction will be sufficiently evident from the figure.

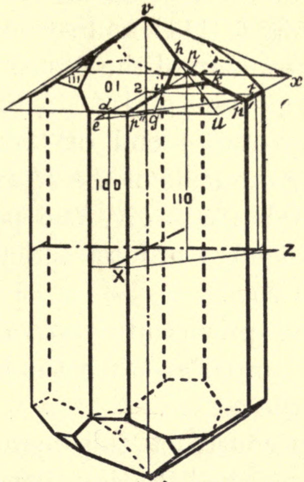
Plate XVII. shows further combinations in the tetragonal system. Fig. 1 is a crystal of apophyllite; $c$ axis = 1·25. It is composed of a prism of the second order {100} and pyramid of the first {111}. The intersection of lines, drawn through the ends of the $a$ axes, give the positions for the vertical edges of the prism. The prism is replaced at a point twice the height of the semi $c$ axis from the centre by the unit pyramid.

Through the point marked 2 draw a line parallel to the $a_1$ axis to cut a vertical drawn through the + end of that axis in point $X$; produce the line backwards and mark on it a distance equal line 2 $X$ beyond 2. Draw another line through 2 parallel to the $a_2$ axis to cut a vertical through the + end of the $a_2$ axis in $X'$. Join points $X$ and $X'$. From $X$ draw a line parallel to one joining the + ends of the $a_1$ and $c$ axes, this will give the front edge of the pyramid. Bisect the line joining $XX'$ and through the bisection, from the apex of the pyramid, draw a line to cut the vertical edge of the prism in $Z$. Join $Z$ to $X$ and $X'$ for intersections of prism and pyramid faces. The figure can then be easily finished by "parallelism" and "repetition".

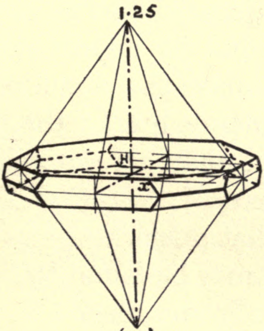
Fig. 2 is a crystal of rutile. The ratio of the $c$ axis is ·64. It is composed of prisms of the first



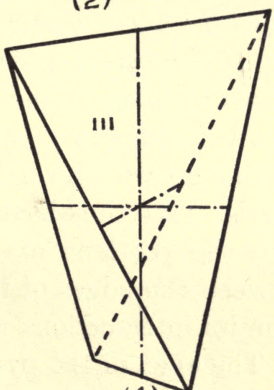

(1)




(2)




(3)




(4)

PLATE XVII.

TETRAGONAL SYSTEM.

FIG. 1. Apophyllite $c$ axis = 1·25. Prism 2nd order {100}; Unit Pyramid {111}.

FIG. 2. Rutile $c$ axis = ·64. Prisms 1st and 2nd order {110}, {100}; Pyramids 1st and 2nd order {111}, {101}.

FIG. 3. Apophyllite $c$ axis = 1·25. Prism 2nd order {100}; Pyramid 1st order {111}; Basal Pinacoid {001}.

FIG. 4. Sphenoid assumed $c$ axis = 1·5.

{110} and second orders {100}, replaced by pyramids of the first {111} and second {101} orders.

The prism of the first order meets the + end of the $a_2$ axis in point $Z$. A line parallel to the $a_2$ axis through the + end of the $a_1$ axis, cutting a line joining the ends of the $H$ axes of the prism of the first order, will give the point $X$ through which a vertical edge of intersection of the two prisms may be drawn.

The prisms are replaced by a pyramid of the second order at twice the height of the semi $c$ axis from the centre.

To construct this pyramid, draw through the point marked 2 a line parallel to the $a_1$ axis, and where it cuts a vertical line through the + end of that axis, in point $d$, draw a line $d$, $u$ parallel to the $a_2$ axis to meet a line $u$, $x$ parallel to the $a_1$ axis drawn through a point $t$, found by intersection of the line 2 $t$ with a vertical through the + end of the $a_2$ axis. We thus have the base edges of the pyramid, the portions of which, that are intercepted between the edges of the faces of the prism {100}, form its intersections with that prism.

The apex of the pyramid may be found by "parallelism" in the manner already described.

Having the apex we can draw the edges of the pyramid, and where the edge $u$, $v$ meets a vertical drawn through the centre of the face of the prism of the first order, will give a point $p$, to which we can draw the intersections, as $p$, $p'$, $p$, $p''$ of the pyramid with that prism.

The crystal shows also small faces of a unit pyramid; this meets the line $d$ 2 in a point $e$.

The intersecting edge of the two pyramids may be found by drawing the line *e, g*, which is parallel to that joining the + ends of the $a_1$ and $a_2$ axes. This line will cut the base edge of the pyramid of the second order in *g*, and from the point where it cuts, a line may be drawn parallel to the front edge of the pyramid {111} marked *g, h*. This line will cut the edge of the second order pyramid and give the intersection of the faces of the two pyramids.

The intersection of the unit pyramid with the unit prism *i, k*, will be parallel to the line joining the + ends of the $a_1$ and $a_2$ axes.

These points found, the crystal can easily be completed by "parallelism" and "repetition".

Fig. 3 is a crystal of apophyllite; *c* axis = 1·25, consisting of a prism of the second order {100}, a pyramid of the first order {111} and the basal pinacoid {001}.

The basal pinacoid cuts the prism at a point *H*.

A line drawn through point *H* parallel to the $a_2$ axis to cut a vertical through the + end of that axis will give a point through which to draw the intersection of base and prism.

A face of the pyramid cuts the intersection of prism and base in point *x*.

The intersection of pyramid and base will be a line parallel to one joining the + ends of the *H* axes.

The intersection of pyramid with prism will be parallel to the edges of the pyramid. The intersections with the front face of the prism, being parallel to the side edges of the pyramid, those with the side faces of prism, to the front and back edges of the pyramid. The reason is evident. The front face of the prism being parallel to the vertical axial plane through the $a_2$ and $c$ axes will show parallel intersections.

Fig. 4 shows the form called a sphenoid {111}. The axial ratio for the $c$ axis has been taken as 1·5. The edges, as in the similar form of the isometric system, the tetrahedron, are parallel to lines, edges of the pyramid {111}.

Plate XVIII. shows a somewhat complicated crystal of zircon, the ratio of the $c$ axis being ·64.

The forms present are a prism of the first order, the edges of which are truncated by one of the second order, whose faces intersect the $a$ axes of the first prism at points $p$ and $p'$.

The ends of the crystal are formed by pyramids, the pyramid {111}, prevailing. Small faces of the form {311} cut off the angles at the ends of the horizontal axes of the first pyramid.

The unit pyramid {111} replaces the prisms at a point $g$.

The construction of the prisms and unit pyramid {111}, should now offer no difficulty, but the intersections of these with the ditetragonal pyramid {311} need explanation.

The ditetragonal pyramid cuts the prism of the second order in the point $S$, the point where the prism face meets the $a_1$ axis of the unit pyramid.

An $O$ plan of the axes of the unit pyramid is constructed above the figure, a vertical taken upwards from point $S$ of the $C$ projection will cut the axis $DC$ of the plan in the point $S'$ where the $H$ trace of the ditetragonal pyramid meets it.

Lines $SXX$ on the $O$ plan are the $H$ traces of

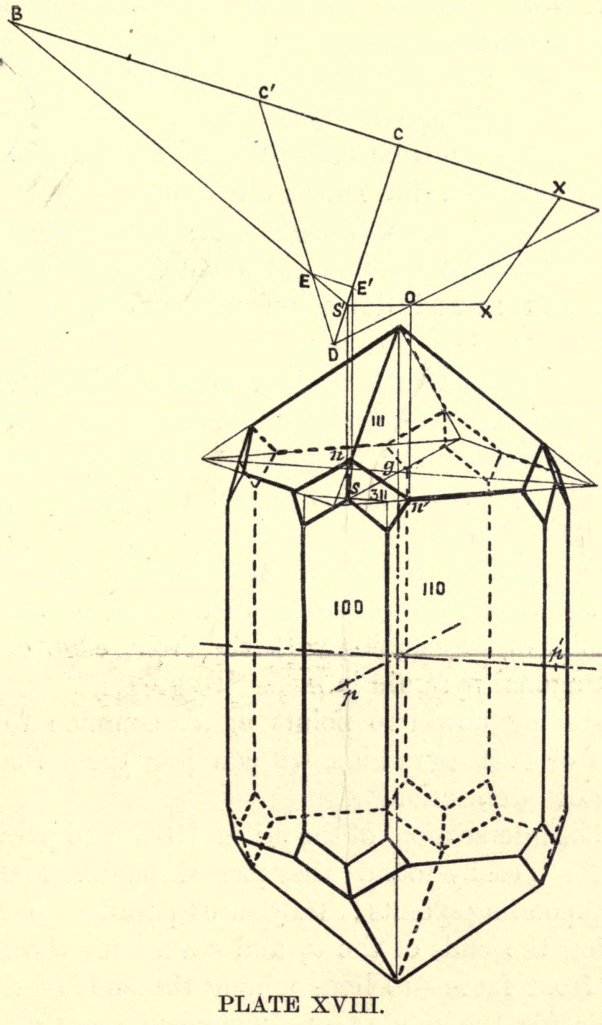



PLATE XVIII.

Tetragonal System.

Zircon $c$ axis = ·64. Prisms of 1st {110} and 2nd {100} orders; Unit Pyramid {111} and Ditetragonal Pyramid {311}.

two faces of the ditetragonal pyramid. They cross the *H* trace of the unit pyramid in point *O*. Another point where the edges of the two pyramids intersect is found by the "profile" method thus:—

The right-angled triangle *CC′D* is the profile cut through the front edge of the unit pyramid and turned down on line *DC* as hinge line.

The right-angled triangle *CSB* is the profile of the ditetragonal pyramid {311} cut through its front edge and turned down on line *CS* as a hinge line.

*E* is the point where the two front edges of the two pyramids cut each other; a line drawn from *E* at right angles to *CD* gives a point immediately below *E*, point *E′*. *E′* is in fact the plan of point *E* on the *O* plan.

Dropped by a vertical projector to the *C* projection it cuts the front edge of the unit pyramid at its point of intersection with the front edge of the ditetragonal pyramid in *n*.

Having now two points *n*, *n′*, common to the faces of both pyramids, we can join them for the edge of intersection.

The intersections of the prism {100} and pyramid {311} are easily found; they pass through *n′* and the corresponding points; they are parallel to lines joining the ends of the $a_2$ and *c* axes for the back and front faces—to lines joining the ends of the $a_1$ and *c* for the side faces. This will seem a little strange; careful consideration of the indices of the symbols involved will make it clear. The intersections are parallel to those of the particular pyramid face to which they belong, with that axial plane, with which the prism face they intersect is parallel.

Exercises—Problems for solution.

Tetragonal System—

Rutile $c$ axis = 0·64. {110}; {100}; {111}.

## CHAPTER X.

### HEXAGONAL SYSTEM—CONSTRUCTION FOR AXES—SIMPLE FORMS—COMBINATIONS.

### PLATES XIX. TO XXIII.

IN the hexagonal system there are four axes. Three of these are horizontal, equal, and make with each other angles of 60°. They are lettered respectively $a_1$, $a_2$, $a_3$.

The fourth axis is vertical, therefore at right angles to the other three, its length is variable. It is lettered $c$.

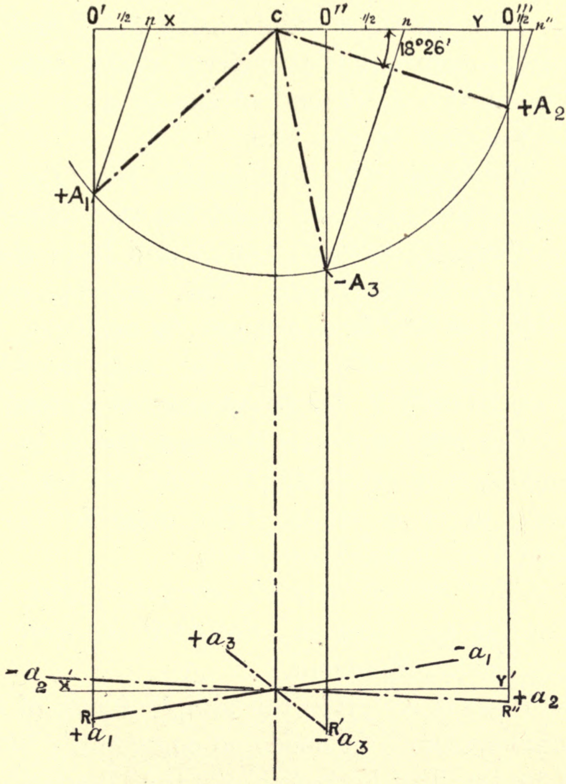
Plate XIX. shows the method of finding the hexagonal axes in $C$ projection.

As for the isometric and tetragonal systems, we begin by making an $O$ plan of the axes above.

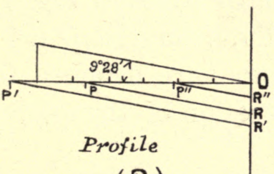
Only the half plan is required. We construct it thus ;—

Draw the horizontal line $XY$. Select a point $c$ for centre and draw line $c, + A_2$, making the angle 18° 26′ with $XY$; cut it off the length desired for the horizontal axes. It is the plan of the + half of the $a_2$ axis.

To get the + half of the $a_1$ and − half of the $a_3$ axes, we describe a circle with $c, + A_2$ as radius, from $c$ as centre, then with the same radius and the + end of the $A_2$ axis as centre, we cut the circle in a point which will be the − end of the




(1)




*Profile*

(2)

PLATE XIX.

HEXAGONAL SYSTEM.

FIG. 1. Construction of axes.
FIG. 2. Profile showing apparent depression.

$A_3$ axis, join the point to $c$ for the $-$ half of the $A_2$ axis.

With the same radius, using the $-$ end of the $A_3$ axis as centre, cut the circle again; this will give a point for the $+$ end of the $a_1$ axis, join it to $c$ for the half of the axis.

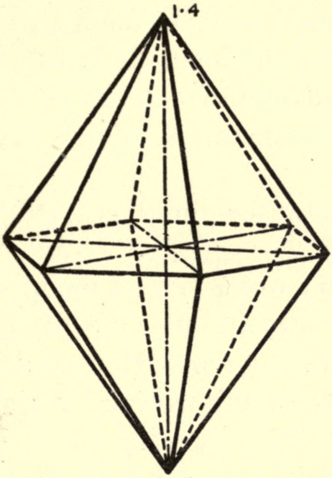
We obtain the $C$ projection from the $O$ plan in exactly the same way as for the two former systems by dropping verticals from $c$ and from the ends of the axes of the $O$ plan. The apparent depressions for the several axes are got by taking $\frac{1}{2}$ distances between $On$, $O''n'$ and $O'''n''$ and marking them down from $X'Y'$ on the respective verticals.

The vertical $c$ axis is found just as in the tetragonal system by finding its ratio, the horizontal axes being taken as unity. The ratio for beryl ·8 has been used in the illustration.

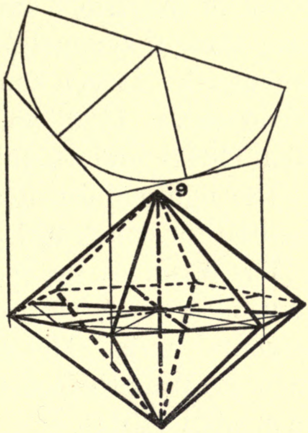
The student should carefully note the axial lettering and bear in mind that the *near* end of $a_3$ axis is the minus end.

Plate XX. gives the simple forms of the hexagonal system. It needs no comment. The method of finding the horizontal pyramidal edges of Fig. 2, the pyramid of the second order, from the $O$ plan may be noted. Vertical projectors are dropped to cut lines drawn through the bisection of the angles made by the axes.

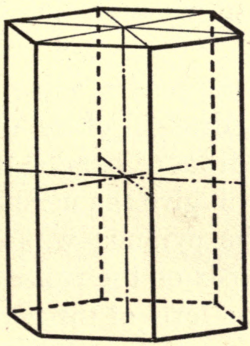
Plate XXI. Fig. 1 shows the construction for the dihexagonal pyramid $\{21\bar{3}3\}$ and the pyramid of the third order Fig. 2. Both are worked from the $O$ plan because it gives sharper intersections than plane intersection of the $C$ projection. Proportional points have been used.



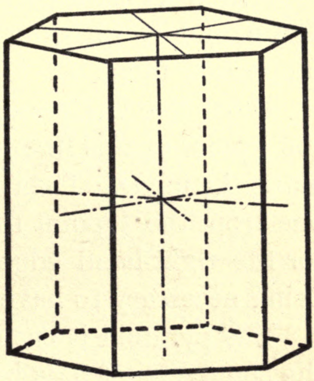

(1)




(2)



(3)



(4)

PLATE XX.

HEXAGONAL SYSTEM.

FIG. 1. Pyramid of the 1st order $\{10\bar{1}1\}$.
FIG. 2. Pyramid of the 2nd order $\{11\bar{2}2\}$.
FIG. 3. Prism of the 1st order $\{10\bar{1}0\}$.
FIG. 4. Prism of the 2nd order $\{11\bar{2}0\}$.

Plate XXII. shows a somewhat complicated crystal of iodyrite, *c* axis ratio = ·8, belonging to a lower class of symmetry in which half the faces are suppressed. It is a combination of a prism of the second order $\{11\bar{2}0\}$ and two pyramids of the first order $\{40\bar{4}1\}$ and $\{40\bar{4}5\}$.

The figure illustrates how few lines are really essential for construction.

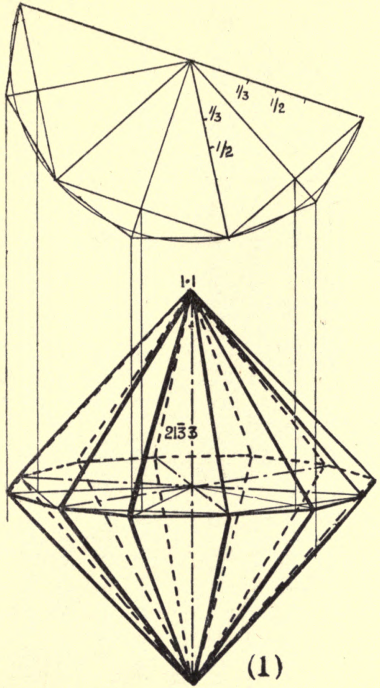
The prism of the second order is first found in the usual way by verticals through the ends of the horizontal axes. The prism is replaced at a distance *TS* from the centre by a unit pyramid having the symbol $\{40\bar{4}1\}$.

The apex of this pyramid might be found by marking off the *c* axial ratio four times on the vertical from point *O*, but this entails making the figure unduly high, so instead we may find it by proportion thus :—

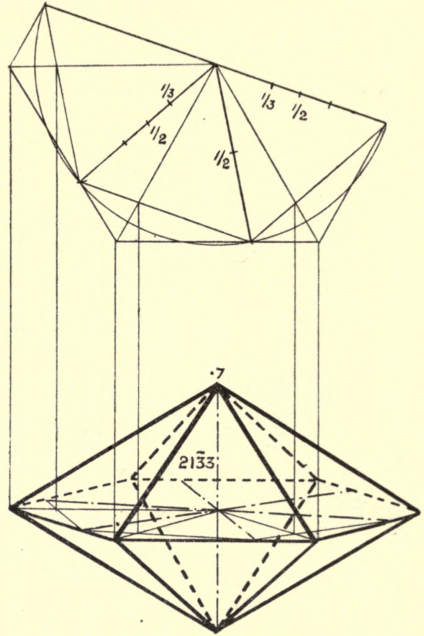
Draw line *OO′* parallel to the semi $a_2$ axis. Join the $\frac{1}{4}$ point of this line to *H* the height of the *C* axis marked from *O*. Then a line from *O′* parallel to the one from the $\frac{1}{4}$ point to *H*, will give the inclination for the right-hand edge of the pyramid without it being necessary to have the apex on the paper.

The pyramid is cut off at a level of three times the semi-vertical axis by the basal plane $\{00\bar{0}1\}$.

A line drawn through the centre parallel to the $a_2$ axis at this level will cut the edge of the pyramid already drawn, in *L*. The remaining pyramidal edges may be found by lines in the basal plane parallel to the three horizontal axes, intersected by lines parallel to those joining the ends of the horizontal axes. The intersections give us the points




(1)




(2)

PLATE XXI.

HEXAGONAL SYSTEM.

FIG. 1. Dihexagonal Pyramid $\{21\bar{3}3\}$.

FIG. 2. Pyramid of the 3rd order $\{21\bar{3}3\}$.

$l'$ $l''$ $l'''$ $l''''$ $l'''''$ on the basal plane to which the several pyramidal edges can be drawn.

Even when the basal plane is not present in a crystal, it is clear that a horizontal plane may be assumed and thus the pyramidal edges found without using the apex.

We have next to obtain the intersections of prismatic and pyramidal faces; proceed as follows:—

Draw on the *O* plan, used for constructing the axes, the *H* traces of the prism, they will of course be lines at right angles to the axes. Drop a vertical from point *X*, the intersection of two of the *H* traces; cut the vertical in a point *P* by a line passed through the centre of line $SS'$ and the centre of the basal edge parallel to it, $l'l''$. Point *P* is one of the points of intersection which may be joined to *S* and $S'$, two other such points; thus we get two edges of intersection.

The other intersection edges of prism and pyramid may be found by drawing from *P*, line $PP'$ parallel to $NN'$. Make the distance $P'Y = PY$. Point $P''$ and corresponding points, may be found similarly.

We now have only to construct the unit pyramid at the lower end to complete the crystal.

The symbol for this form is $\{40\bar{4}5\}$. It replaces the prism at the same distance from the centre below, as the other pyramid above. The height then of its semi *c* axis, will be not ·8 of line *CD* as for the prism, but ·8 of $\frac{4}{5}$ of *CD*, since the axial ratio of the pyramid is $\{40\bar{4}5\}$; that is, 1·25 : 0 : 1·25 : 1.

From point *R*, found by making *TR* equal *ST*,

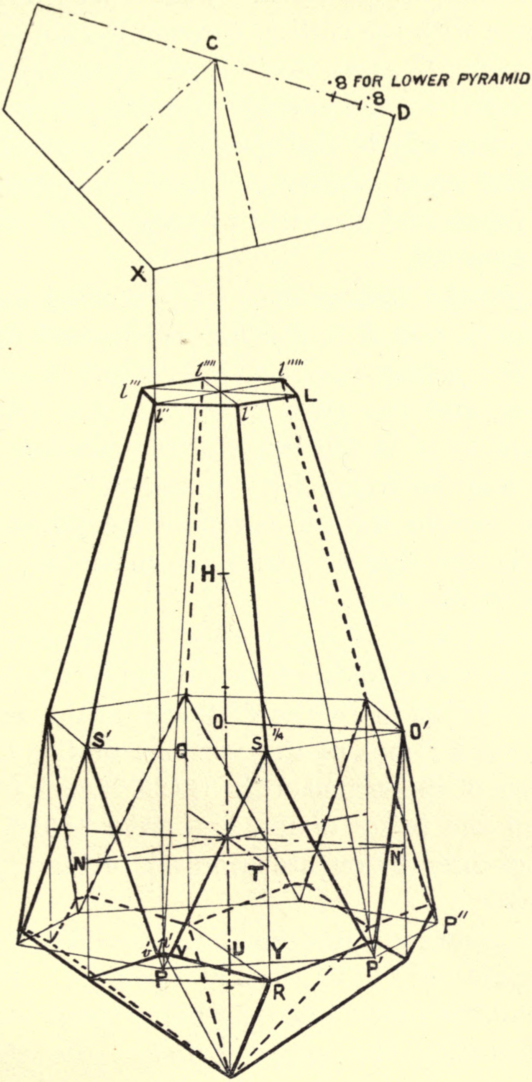



PLATE XXII.

Hexagonal System.

Iodyrite $c$ axis = ·8.
Prism of the 2nd order $\{11\bar{2}0\}$ ; Pyramid 1st order $\{40\bar{4}1\}$.
Pyramid 1st order $\{40\bar{4}\bar{5}\}$ ; Base $\{000\bar{1}\}$.

draw a line parallel to the $a_3$ axis and from its intersection with the vertical through the centre of the prism, point $U$, mark downwards a length equal the semi $c$ axis; for the lower pyramid ·8, shown on the $O$ plan, this will be the apex of the pyramid. The pyramidal edges join the points of the several faces of the prism correspondent to point $R$, to the apex of the pyramid.

To get the intersections of pyramidal and prismatic faces, join $R$ to the next correspondent point, bisect the joining line and pass a line through the bisection and the apex to cut the line $PQ$ in $p'$. $R$ joined to $p'$ is one edge of intersection. The others may be found by "repetition".

The reasons for the several steps taken in constructing the figure should be apparent without further explanation.

Plate XXIII. shows a crystal of connelite. All the construction lines are not shown in the figure.

The $O$ plan gives the $H$ traces of the prism of the first order and of the portions of the faces of the prism of the second order $\{11\bar{2}0\}$ which truncate those of the prism of the first order $\{11\bar{1}0\}$. The points of intersection are dropped to the $C$ projection below.

The prisms are replaced by a berylloid or dihexagonal pyramid $11 : 2 : \overline{13} : 3$ at a point $X$. Through this point the centre lines parallel to the $H$ axes and lines bisecting the angles between the axes are drawn.

The proportional points $11 : 2$ are found on the $a_1$ and $a_2$ axes of the $O$ plan, and the horizontal traces $g\ i\ h$ drawn by "parallelism," $i\ h$ being parallel to

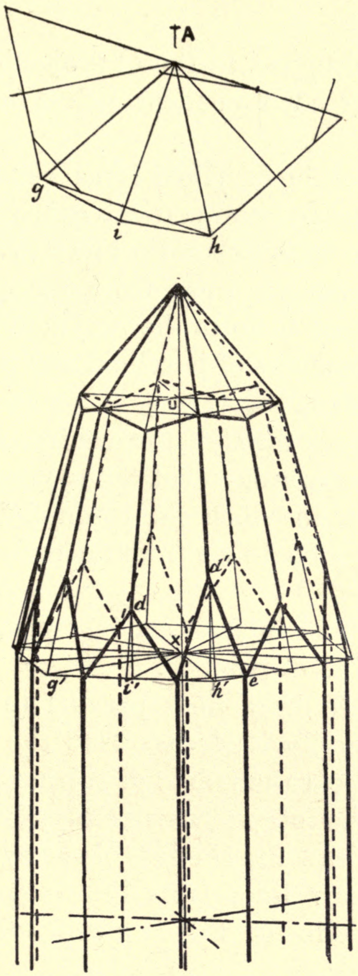



PLATE XXIII.

HEXAGONAL SYSTEM.

Connelite $c$ axis = 1·16.
Prisms 1st order {11$\bar{1}$0}, 2nd {11$\bar{2}$0}.
Pyramid 1st order {11$\bar{1}$1} ; Dihexagonal Pyramid (Berylloid), Axial Ratio 11 : 2 : $\overline{13}$ : 3.

the line from points found in proper proportion on the axes.

*g i h* are dropped to the *C* projection in *g′ i′ h′*, and the other correspondent points are found by " repetition ".

The apex of the berylloid is marked at *A*, a point 4 times and $\frac{1}{3}$ the semi-vertical axis from point *X*, in accordance with the ratio $11 : 2 : \overline{13} : 3$, and towards this point *A*, lines from the points *g′ h′ i′*, etc., are directed to form the berylloid edges.

The intersections of berylloid and prism faces is the next problem.

Vertical lines through the centres of the prism faces, drawn upwards to cut the pyramidal edges at *dd′* give points to which the prism and pyramid intersections may be drawn from the ends of the prism edges at *e*, etc.

The berylloid in its turn is replaced by a unit pyramid $\{11\bar{1}1\}$ at point *U*.

Through *U* draw lines parallel to the *H* axes to cut the berylloid edges. Find the apex of the pyramid and join the points of cutting to it.

To find the intersections of berylloid and pyramidal faces, draw the *H* edges of the pyramid, bisect them and draw lines from the apex through the bisections, producing them downwards to cut the berylloid edges.

The points so found must be joined to the meeting points of pyramidal and berylloid edges; the joining lines will be the desired intersections.

The figure can now be achieved by "parallelism".

## CHAPTER XI.

### HEXAGONAL SYSTEM (*CONT.*)—RHOMBOHEDRON—SCALENOHEDRON—TRAPEZOHEDRON.

### PLATES XXIV. TO XXVII.

PLATE XXIV. Fig. 1 is the + rhombohedron $\{10\bar{1}1\}$ of hematite $c$ axis = 1·36. Its faces correspond to alternate faces of the unit pyramid.

Having drawn the axes by the construction given on Plate XIX., join the + end of the $a_1$ axis to the − end of the $a_3$, producing the line on either side. Also join the − end of the $a_2$ to the + end of the $a_3$ axis, producing the line to the left to cut the first drawn line.

These two lines are the $H$ traces of planes $\{10\bar{1}1\}$ and $\{0\bar{1}11\}$. Where they cut is a point common to both planes; the apex of the vertical axis is another point common to both. Join these two common points, the joining lines are the intersections of the planes forming two faces of the rhombohedron. Join the + end of the $a_1$ axis to the − end of the $a_2$. Bisect the line joining them; through the point of bisection and the − end of the $c$ axis draw a line to cut the intersection of the planes in point $i$.

Now find in a similar way the intersection of planes $\{10\bar{1}1\}$ and $\{\bar{1}101\}$ by the intersection of the $H$ traces, giving a point common to both planes, another point common to both being the apex of the

$c$ axis. Cut off the intersection of the planes in point $s$ by a line from the $-$ end of the $c$ axis drawn through the bisection of the line joining the $- a_3$ and $+ a_2$ axes.

From $s$ draw a line $s$, $t$ parallel to $i$, $v$ and make it an equal length on each side of the $- a_3$ axis.

Line $ts$ is another edge of the rhombohedron which may be easily completed by "parallelism" and "repetition".

Fig. 2 shows a $-$ rhombohedron $\{01\bar{1}1\}$ of calcite $c$ axis $= \cdot 85$. It is constructed similarly to the + rhombohedron, the alternate planes omitted in the + form being now developed.

Fig. 3 shows a combination of the + rhombohedron $\{10\bar{1}1\}$ with the base $\{000\bar{1}\}$.

The basal planes cut the vertical axis in points $XX'$. The intersections with the rhombohedral planes are found by joining the axis through which the edge of the rhombohedron passes to the end of the $c$ axis, then passing through $X$ or $X'$ a line parallel to the $H$ axis to cut the line going to the $c$ axis. The place where they cut, is a point through which may be drawn the intersection of the rhombohedral face with the basal plane. The several intersections will be parallel to the several axes.

Fig. 4 shows a $-$ rhombohedron $\{02\bar{2}1\}$ of hematite $c$ axis $= 1 \cdot 36$, whose edges are truncated by the + rhombohedron $\{10\bar{1}1\}$.

To understand the relation of the truncated rhombohedron to the other, is of great assistance in the construction of this combination. It is this, the + rhombohedron to truncate the minus form, has twice the latter's length of vertical axis.

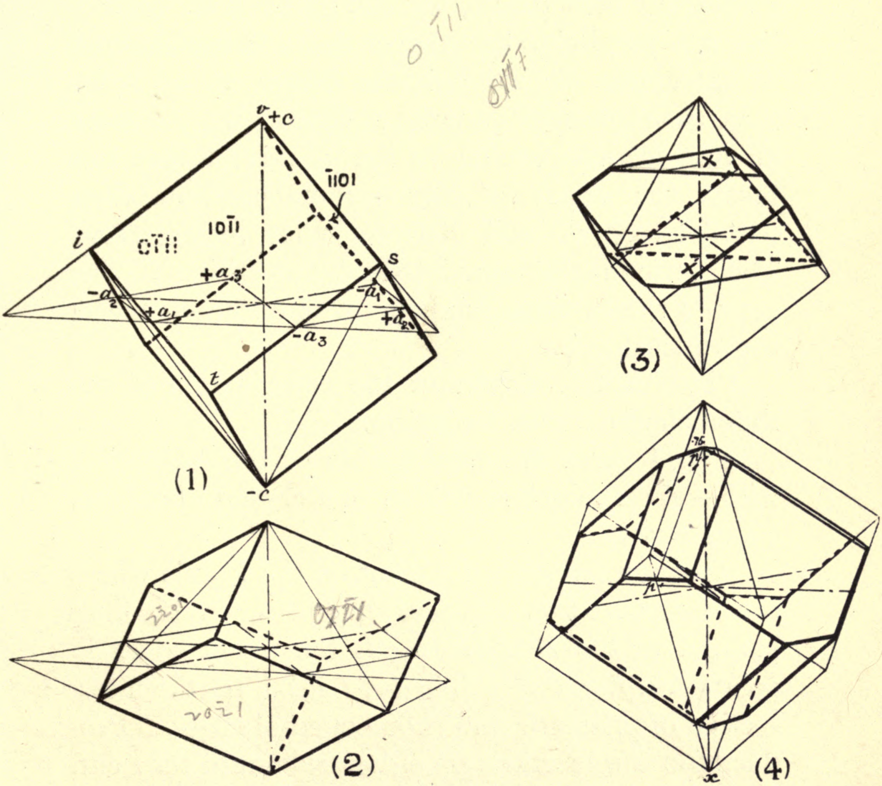



PLATE XXIV.

HEXAGONAL SYSTEM.

FIG. 1. Hematite $c$ axis = 1·36 ; + Rhombohedron {10$\bar{1}$1}.
FIG. 2. Calcite $c$ axis = ·85 ; − Rhombohedron {01$\bar{1}$1}.
FIG. 3. Calcite + Rhombohedron cut by Basal Plane.
FIG. 4. + Rhombohedron {10$\bar{1}$1} truncating edges of − Rhombohedron {02$\bar{2}$1}.

Having found the point on the vertical axis, in this case ·75 from the centre, through which the truncating face passes, we merely have to draw a line $p, p'$ parallel to the edge of the – rhombohedron to cut a line $p'x$ through the centre of the lower face, then through the point of cutting we can draw an intersection parallel to the $a_2$ axis; where this meets the edges of the – rhombohedron, lines parallel to $p$, $p'$ will give us other intersections. The figure may be completed by "repetition". The apex of the combination being where line $pp'$ meets the vertical axis.

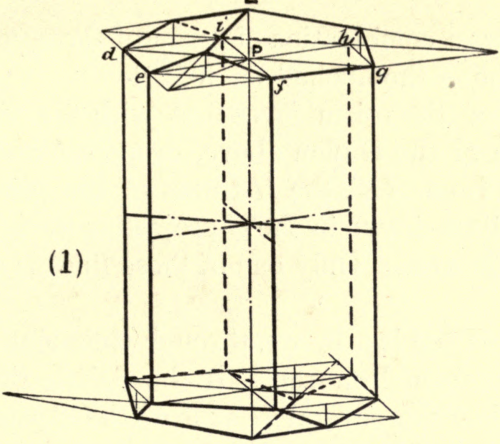
Plate XXV. illustrates the combination of the rhombohedron with the prism.

Fig. 1 gives the combination of the unit-prism with – rhombohedron $\{01\bar{1}2\}$, in a crystal of calcite, $c$ axis = ·85.

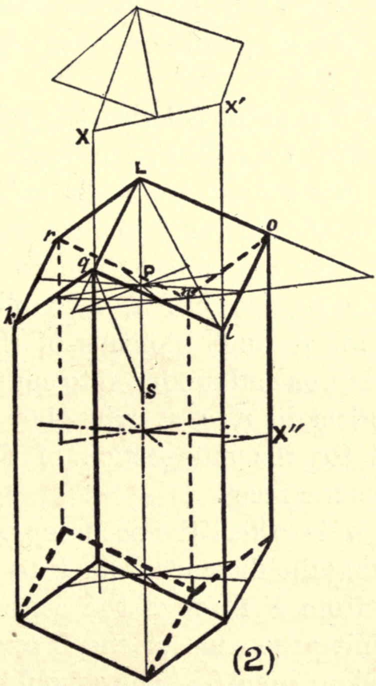
Through point $P$, the point on the $V$ axis where the rhombohedron replaces the prism, draw lines parallel to the $H$ axes to cut the prism edges in points $d$, $e$, $f$, $g$, $h$, $i$. Join $d$-$e$, $f$-$g$, $h$-$i$, for the intersection of prismatic and rhombohedral edges. Produce the intersections on either side until they cut. Join the point where they cut to one half the height of the semi-vertical axis in accordance with the symbol $\{01\bar{1}2\}$. Cut off the lines so drawn by verticals through the centres of the prism faces erected through the bisection points of lines $e$-$f$, $g$-$h$, etc.

We then have all the needed intersection edges and can complete the figure as shown, treating the lower end of the crystal in a similar manner.

Fig. 2 is a crystal of dioptase, vertical axis = ·53.




(1)




(2)

## PLATE XXV.

### Hexagonal System.

Fig. 1. Calcite $c$ axis = ·85. Unit Prism $\{11\bar{1}0\}$ and – Rhombohedron $\{01\bar{1}2\}$.

Fig. 2. Dioptase $c$ axis = ·53. Prism 2nd order $\{11\bar{2}0\}$ and – Rhombohedron $0\bar{2}21\}$.

It shows a combination of prism of the second order with the − rhombohedron $\{02\bar{2}1\}$.

Draw the prism faces first with the aid of the portion of the *O* plan above, dropping vertical projectors from *XX′*, the *H* traces of the prism faces, to cut lines drawn in the *C* projection at right angles to the *H* axes. Only one of these lines is shown at *X″*.

As in the last figure, through the point *P* where the rhombohedron replaces the prism, draw lines parallel to the several *H* axes of the crystal; these will give the *H* axes of the rhombohedron. Join the ends of the $+a_1$ and $-a_3$ axes; bisect the joining line.

Next find point *L* twice the height of the semi-vertical axes from *P* in accordance with the symbol $\{02\bar{2}1\}$. Draw lines from this point *L* through the bisections of the lines joining the *H* axes of the rhombohedron; produce them to cut the vertical edges of the prism at *k, l, n*. Oblique lines drawn from *k, l, n*, through the ends of the *H* axes of the rhombohedron and produced to cut the vertical edges of the prism in *v, q, r*, will give the other points required for the intersection of the rhombohedral and prismatic faces.

It is well to test the accuracy of this latter step by marking the distance from *P* to *L*, downwards in *S*, then from *S* through the central points of the lines joining the ends of the *H* axes draw lines to cut the edges from *L*. They should cut in the points *q, r, o*. One such line is shown for example, line *q, s*.

The lower portion of the crystal can be got by "parallelism" and "repetition".

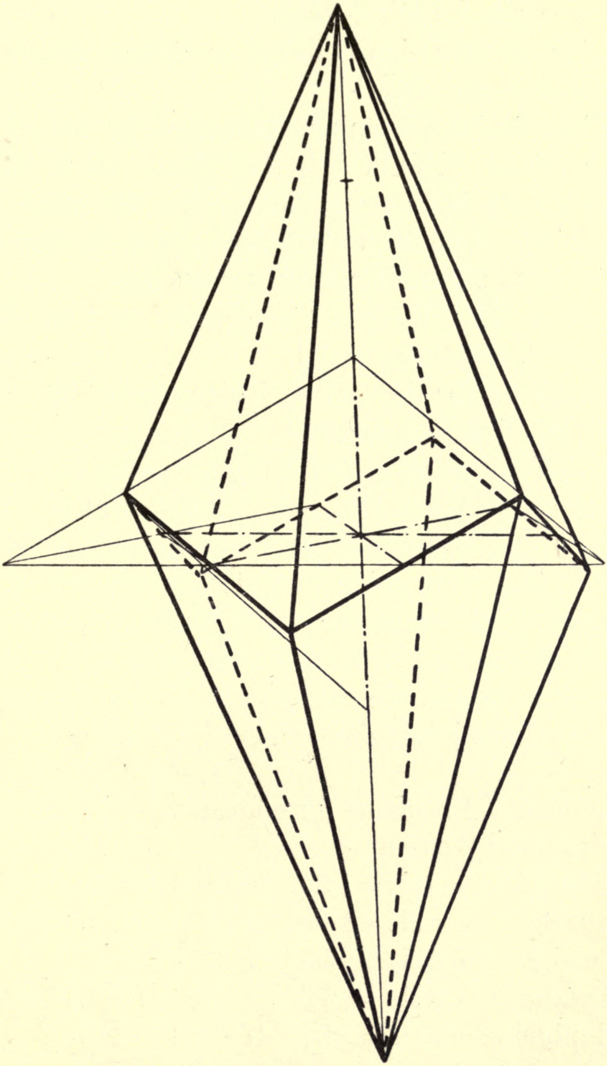


PLATE XXVI.

HEXAGONAL SYSTEM.

Calcite $c$ axis = ·85 ; + Scalenohedron $\{21\bar{3}1\}$.

Plate XXVI. shows the scalenohedron, calcite $c$ axis = ·85 $\{21\bar{3}1\}$, + form.

The construction is based on that of the unit + rhombohedron $\{10\bar{1}1\}$.

First find the rhombohedral edges by the method given (Plate XXIV. Fig. 1). Then mark off the vertical axis, in this case three times the axial ratio of calcite. Join the ends of each rhombohedral edge to the apex; the form is then complete.

Plate XXVII. gives a + trapezohedron $\{21\bar{3}1\}$ of quartz, the $c$ axis = 1·1.

Start as for the scalenohedron with the rhombohedral edges but retain and produce only alternate ones. Find points $e, f, g$, the intersections of the $H$ traces of planes $\{21\bar{3}1\}$ with $\{\bar{3}211\}$ and of $\{21\bar{3}1\}$ with $\{1\bar{3}21\}$. Join the points of intersection to the apex. The lines going to the apex will cut off the rhombohedral edges the right length on one side. Mark off the length on the other side of the point where the axis meets the edge, thus completing the form by lines joining the ends of the edges.

Exercises—Problems for solution.

Hexagonal System—

| | | | |
|---|---|---|---|
| Hematite | $c$ axis = 1·36. | $\{000\bar{1}\}$; $\{10\bar{1}0\}$. |
| Quartz | $c$ axis = 1·09. | $\{10\bar{1}1\}$; $\{01\bar{1}1\}$; $\{10\bar{1}0\}$. |
| Calcite | $c$ axis = ·85. | $\{10\bar{1}1\}$. |
| Apatite | $c$ axis = ·73. | $\{10\bar{1}0\}$; $\{10\bar{1}1\}$; $\{000\bar{1}\}$. |
| Nepheline | $c$ axis = ·84. | $\{10\bar{1}0\}$; $\{000\bar{1}\}$. |

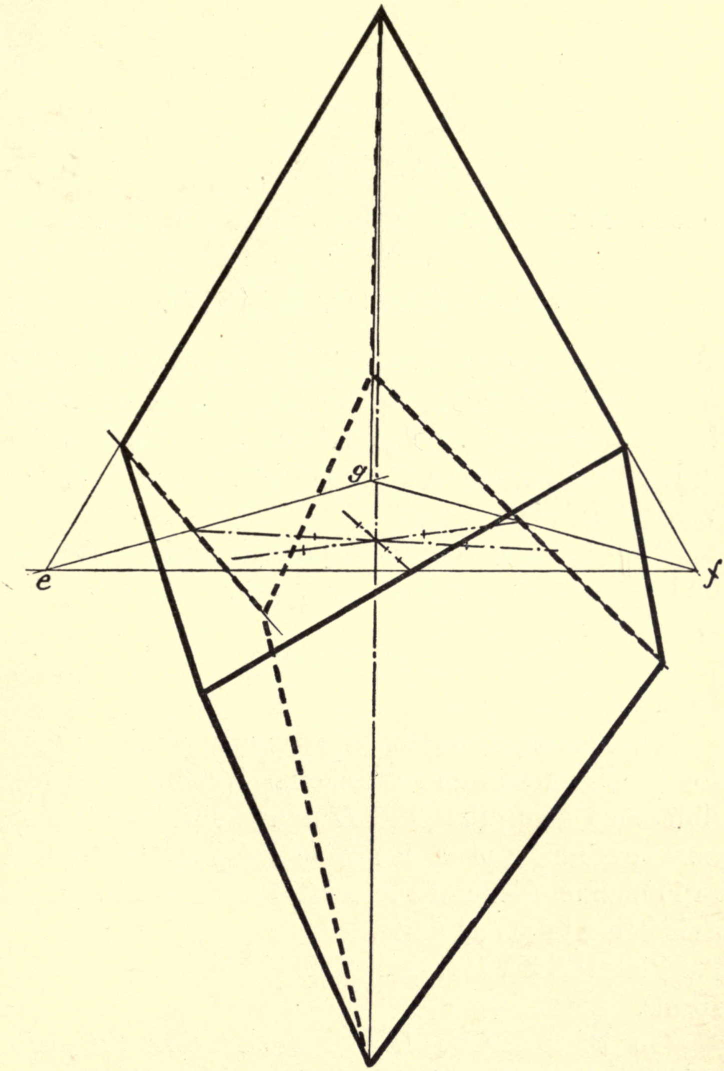



PLATE XXVII.

HEXAGONAL SYSTEM.

Quartz $c$ axis = 1·1 ; Right-handed Trapezohedron {$21\bar{3}1$}.

# CHAPTER XII.

## ORTHORHOMBIC SYSTEM—AXES—SIMPLE FORMS—COMBINATIONS.

### PLATES XXVIII. TO XXXI.

THE axes of the orthorhombic system are all at right angles but of different lengths, they therefore are all lettered differently. The axis correspondent to the $a_1$ is lettered $a$, that correspondent to the $a_2$ is $b$, the vertical axis is $c$.

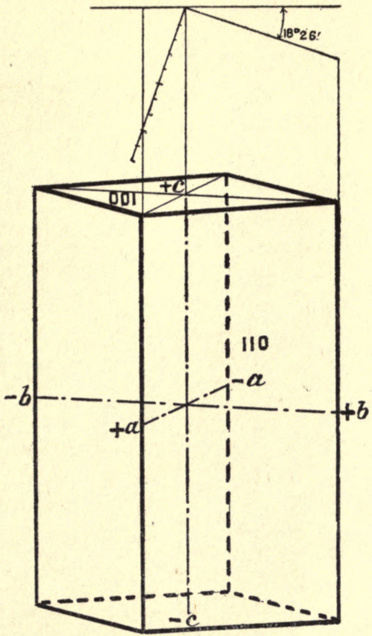
The $a$ axis is the shorter of the horizontal axes and is sometimes called the brachy axis, from the Greek *βρᾰχύς*, short.

The $b$ axis is sometimes called the macro axis, from the Greek *μακρός*, long.

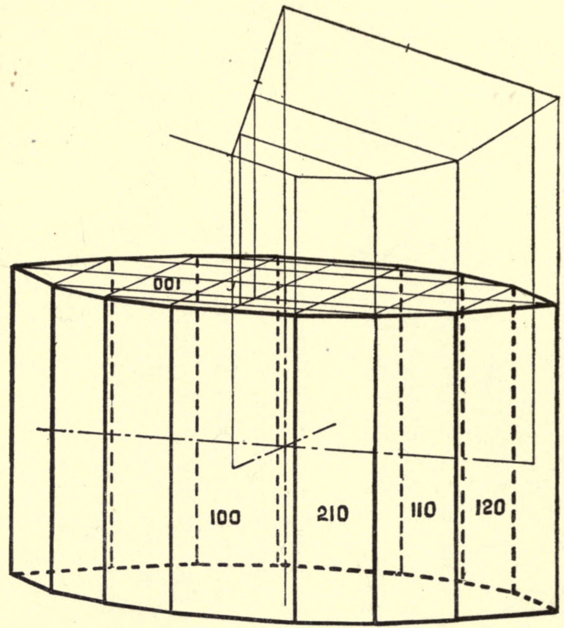
The construction for the axes is precisely that for the isometric and tetragonal systems, the only difference being that the $H$ axes as well as the $V$ axis are made of a length correspondent to the ratio characteristic of the crystal to be drawn, the $b$ axis being taken as unity.

Plates XXVIII. and XXIX. give the simple forms.

On Plate XXVIII. Fig. 1 is shown the unit prism {110} with the basal pinacoid {001}. Fig. 2 illustrates the relations of the macropinacoid {100}, macroprism {210}, unit prism {110}, brachyprism {120} and basal pinacoid {001}.



(1)



(2)

PLATE XXVIII.

ORTHORHOMBIC SYSTEM.

FIG. 1. Unit Prism {110}; Basal Pinacoid {001}; assumed Axial Ratio ·8 : 1 : 1·3.
FIG. 2. Macropinacoid {100}; Macroprism {210}; Unit Prism {110}; Brachyprism {120}.

The figure has preferably been constructed from the $O$ plan, a portion of which is shown above, rather than by plane intersection directly on the $C$ projection, because on the $O$ plan the lines cut each other more acutely.

Plate XXIX. Fig. 1 shows the combination of the three pinacoids: the macropinacoid parallel to the macro axis {100}, the brachypinacoid parallel to the brachy axis {010} and the basal pinacoid parallel to both these axes {001}.

Fig. 2 shows the unit pyramid {111}; the construction needs no comment.

Fig. 3 shows the macrodome {101} in combination with the brachypinacoid {010}.

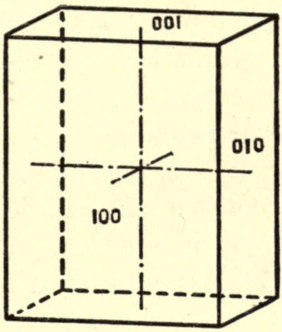
Fig. 4 shows the brachydome {011} combined with the macropinacoid {100}.

Plate XXX. shows several combinations in the orthorhombic system. Fig. 1, a crystal of barite axial ratio ·8 : 1 : 1·3, is composed of a unit prism {110} and macrodome {102}.

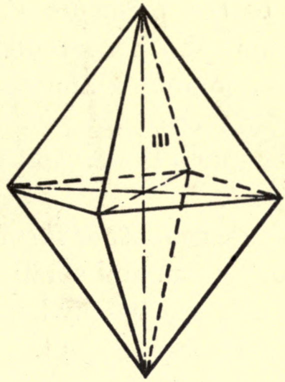
The face of the macrodome meets the $a$ axis of the prism at point $X$.

Join the + ends of the $a$ and $b$ axes, erect the vertical edges of the prism through the ends of the $b$ axis. Draw the intersection edge of the macrodome faces through point $X$ parallel to the $b$ axis. Where the edge of intersection meets the line joining the ends of the $a$ and $b$ axes in $N$ will be one point where the macrodomal and prismatic faces meet.

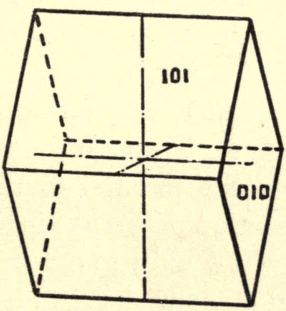
Draw a line from the + end of the $a$ axis to $\frac{1}{2}$ the height of the semi $c$ axis, then a line parallel to it from point $X$ to cut that axis in point $P$. Line $XP$ is the line of inclination of the macrodome face.



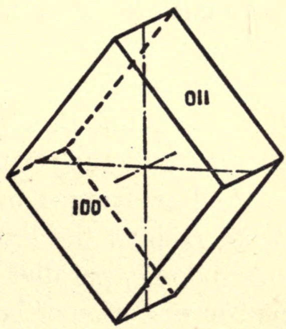

(1)




(2)




(3)




(4)

PLATE XXIX.

ORTHORHOMBIC SYSTEM.

FIG. 1. Macropinacoid {100}; Brachypinacoid {010}; Basal Pinacoid {001}.

FIG. 2. Unit Pyramid {111}.

FIG. 3. Macrodome {101}; Brachypinacoid {010}.

FIG. 4. Brachydome {011}; Macropinacoid {100}.

Through $P$ the edge of macrodome may now be drawn parallel to the $b$ axis; it will cut the vertical edge of the prism at $V$. $V$ then will be a point common to both macrodomal and prismatic faces, so that by joining $V$ and $N$ we obtain the intersection of those faces.

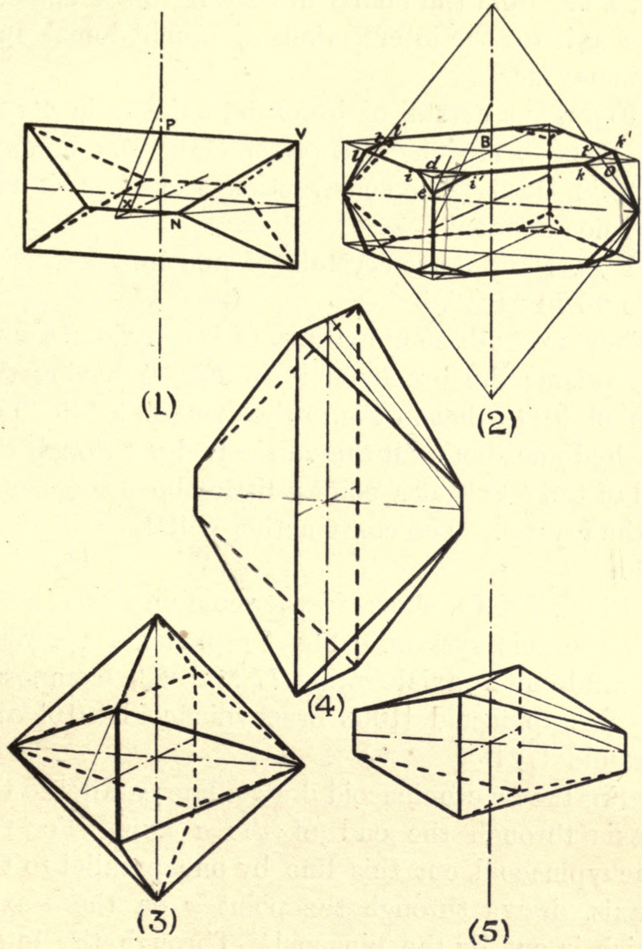
Complete the crystal by "parallelism" and "repetition".

Fig. 2 is another form of barite. In addition to the unit prism and small faces of macrodome shown at $d$, etc., the brachydome $O$ {011} and basal pinacoid {001} are present.

First draw the unit prism, then through point $B$ the point on the vertical axis through which the basal plane passes, draw lines parallel to the $a$ and $b$ axes to cut off the vertical prism edges.

For the macrodome face join the + end of the $a$ axis to $\frac{1}{2}$ the semi height of the $c$ axis, draw a line parallel to this line through point $e$, where the macrodome face cuts the prism edge. Through the point where the line from $e$ meets the parallel to the $a$ axis from $B$ draw the intersection of basal pinacoid and macrodome, parallel to the $b$ axis, to cut the $H$ edges of intersection of basal plane with prism faces, in points $i$, $i'$, then join these points of cutting to $e$ for the intersections of macrodomal and prism faces.

To obtain the brachydome faces join the ends of the $b$ axis to the ends of the $c$ axis, this gives the lines of inclination of the brachydomal faces; where they cut the parallel to the $b$ axis through $B$ are points $t$, $t'$, through which the intersections of brachydome and base may be drawn, parallel to the $a$ axis to cut the $H$ intersections of base and prism in points




## PLATE XXX.

ORTHORHOMBIC SYSTEM.

FIG. 1. Barite. Axial Ratio ·8 : 1 : 1·3 ; Unit Prism {110} ; Macrodome {102}.

FIG. 2. Barite. Unit Prism {110} ; Macrodome {102} ; Brachydome {011} ; Basal Plane {001}.

FIG. 3. Brookite. Axial Ratio ·8 : 1 : ·9 ; Unit Prism {110} ; Brachypyramid {122}.

FIG. 4. Arsenopyrite. Axial Ratio ·67 : 1 : 1·18 ; Unit Prism {110} ; Brachydome {011}.

FIG. 5. Arsenopyrite. Unit Prism {110} ; Brachydome {014}.

*k*, *k'*, *l*, *l'*. Join the points of cutting to the ends of the *b* axis for the intersections of brachydomal and prismatic faces.

Fig. 3 is a crystal of brookite, axial ratio ·8 : 1 : ·9. It is formed by unit prism {110} and brachypyramid {122}. The construction is so simple it will need no comment.

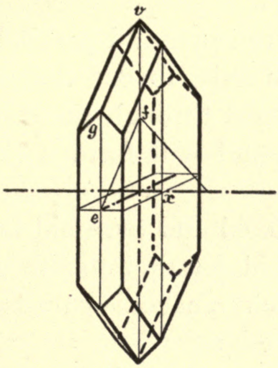
Figs. 4 and 5 are crystals of arsenopyrite, axial ratio ·67 : 1 : 1·18.

They are both combinations of brachydomes with unit prism; the brachydome in Fig. 4 having the symbol {011}, that in Fig. 5 the symbol {014}. The brachydomes both cut the vertical edge through the end of the *b* axis at a point a little above the centre of the crystal. The construction will be evident at sight.

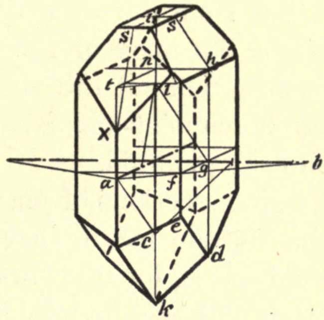
Plate XXXI. shows further combinations in the orthorhombic system. Fig. 1 represents a crystal of childrenite, axial ratio ·77 : 1 : ·53, composed of macropinacoid {100}, brachypinacoid {010} and pyramid {121}.

For the macropinacoid draw a line parallel to the *b* axis through the end of the *a* axis. For the brachypinacoid cut this line by one parallel to the *a* axis, drawn through the point *x* on the *b* axis, which is cut by the pinacoid. Through the intersection of these two lines erect the vertical intersection of the pinacoidal faces.

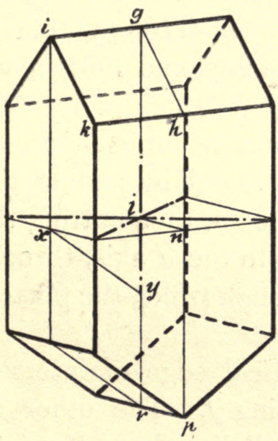
For the pyramid draw vertical lines through the centres of the pinacoid faces, then a line parallel to the front pyramid edge, *e*, *f*, drawn through the point *V* where the pyramid cuts the vertical axis, will cut the vertical through the centre of the




(1)




(2)




(3)

PLATE XXXI.

ORTHORHOMBIC SYSTEM.

FIG. 1. Childrenite. Axial Ratio ·77 : 1 : ·53; Macropinacoid {100}; Brachypinacoid {010}; Pyramid {121}.

FIG. 2. Calamine. Axial Ratio ·78 : 1 : ·48; Prism {110}; Pyramid {12$\bar{1}$}; Brachypinacoid {010}; Brachydome {031}; Macrodome {301}.

FIG. 3. Epsomite. Axial Ratio ·99 : 1 : ·57; Prism {110}; Sphenoid {111}.

macropinacoid face in $g$ and will be the required edge of the pyramid.

A line from the apex of the pyramid parallel to one from the apex of the $c$ axis, to cut the semi $b$ axis at $\frac{1}{2}$, will cut the vertical through the centre of the brachypinacoid face and will form a short edge of the pyramid.

The intersections of pinacoid and pyramid faces will be, for the macropinacoid parallel to the side pyramid edges, for the brachypinacoid parallel to the front and back pyramid edges.

Fig. 2 is a somewhat complicated crystal of calamine, axial ratio ·78 : 1 : ·48, showing combinations of unit prism {110}, pyramid {121}, brachypinacoid {010}, brachydome {031}, macrodome {301} and basal pinacoid {001}.

The intersection of unit prism {110} with the brachypinacoid is found by joining the ends of the $a$ and $b$ axes as shown, then cutting the joining line by a line parallel to the $a$ axis, through the point $g$ on the $b$ axis, through which the pinacoid face {010} passes.

Raise the vertical edge of intersection through the point of cutting $f$. The other edges may be obtained by "parallelism" and "repetition".

The brachydome replaces the prism and pinacoid at a point $p$ on the $c$ axis.

Draw a vertical through the centre of the face of the brachypinacoid, then a line through $p$, parallel to the $b$ axis, to cut the vertical gives the point $h$ through which the intersection edge of dome with pinacoid may be drawn parallel to the $a$ axis.

The inclination of the brachydome may now be obtained by a line from *h* parallel to one from the apex of the *c* axis to the $\frac{1}{3}$ point *g*, of the *b* axis.

The macrodome passes through a point at *t*, we can get its inclination and the point where it meets that vertical prism edge, which passes through the + end of the *a* axis by aid of a line drawn through *t*, parallel to one from the + end of the *c* axis to the $\frac{1}{3}$ point of the *a*, according to the symbol {301}. This line will cut the prism edge in point *X*.

The domes give way to the basal plane at a point on the *c* axis marked *i*. Through *i* lines parallel to the *a* and *b* axes will meet the lines of inclination of the two domes in points *SS′*, through which their intersections with the base may be drawn; that from the brachydome parallel to the brachy axis; that for the macrodome to the macro axis.

We need the intersection edge between the two domes; we can obtain it by drawing lines through *h* and *t* parallel respectively to the *a* and *b* axes; where they meet will be one point *l* in the intersection, the other will of course be where the intersections with the base meet.

One other form is present, the unit pyramid which forms the lower end of the crystal. The symbol is {121} and furnishes the clue for the construction. The pyramid cuts the − *c* axis in point *k*, a line from *k* parallel to one from + end of *a* axis to − end of *c* will give the front edge of pyramid. Lines from *k* on either side parallel to one joining the − end of the *c* axis to the $\frac{1}{2}$ points of the + and − semi *b* will give the side edges. The right side edge meets the pinacoid face at *d*, where it cuts a vertical through the centre of that face. A line from *d* parallel to the front edge gives the intersection of the two forms. This intersection edge meets the prism edge at *e*, from which point the intersection of pyramid with prism can be drawn, to where the two front edges meet.

Fig. 3 represents a crystal of epsomite, axial ratio ·99 : 1 : ·57. It is a combination of unit prism {110} and sphenoid {111}.

First draw prism vertical edges.

To obtain the intersections of prism and sphenoid draw verticals through the centres of the prism faces {110} and $\{1\bar{1}0\}$. Then through apex of crystal *g*, which is twice the semi height of the *c* axis from the centre, draw a line *g*, *i*, parallel to one joining the + ends of the *H* axes. This line will be an edge of the sphenoid; it will cut the vertical through the centre of the prism face $\{1\bar{1}0\}$. Next draw line *g*, *h*, downwards parallel to one joining the apex of the *c* axis and point *n*, *n* being the centre of the line joining the + ends of the *a* and *b* axes, *g*, *h*, will cut the vertical through *n* in a point *h* through which may be drawn an intersection edge of prism and sphenoid faces, parallel to the edge through *g*; where this line meets the vertical edge of prism, will give a point *k*. Join *k* to *i* for intersection of prism face $\{1\bar{1}0\}$ with sphenoid.

Then the other upper face of sphenoid can be got by "parallelism".

The lower faces of the sphenoid will be found by drawing a line through the lowest point of crystal parallel to a line joining + end of *a* and − end of *b* axes or to *l*, *n*, the line bisecting the angle between the + ends of the *a* and *b* axes. Let this line parallel to *l*, *n*, cut the vertical through the centre of the prism face {110}. It will be the lowest edge of the sphenoid. Draw through the point *p* where it cuts the vertical, a line parallel to *x*, *y*; *x*, *y* being drawn from − end of *c* axis to centre of line joining + end of *a* axis to − end of *b*. The line drawn through *p* will be the intersection of sphenoid face {111} and prism face {110}. Draw a line parallel to *x*, *y* through the lowest point of crystal, *r*, it will cut the vertical through the centre of the prism face $\{1\bar{1}0\}$, through the point of cutting will lie an intersecting edge parallel to *l*, *n*. The crystal is easily finished by the usual methods.

Plate XXXII. shows a crystal of chrysolite, axial ratio ·46 : 1 : ·58. It is a combination of the unit pyramid {111}, unit prism {110}, macropinacoid {100}, brachypinacoid {010}, macrodome {101}, brachydome {021} and basal pinacoid {001}.

The construction is shown somewhat fully.

After the axes have been found, the lines joining the *H* axes are put in, then from the *O* plan the points *p*, *q*, *r*, the points of intersection of the prism and pinacoids are dropped by verticals to the *C* projection in *p′*, *q′*, *r′*, and through *p′*, *q′*, the vertical intersection edges are raised.

The prism and pinacoids are replaced at a point *O* on the vertical axis by the domes and unit pyramid.

Through *O* draw lines parallel to the *a* and *b* axes, also lines parallel to those joining the axes. On the latter will lie the intersections of unit prism and pyramid, they will meet the vertical intersections of the pinacoids and prism. Through the points of meeting *e, f*, the intersections of pinacoids and domes may be drawn, parallel respectively, to the macro and brachy axes.

We may next find the inclination lines of the domes. Make *OA* equal the vertical axial height, join *A* to *g*, then draw a line parallel to *Ag*, through point *h*; this line will be the inclination line of the macrodome {101}.

For the inclination of the brachydome {021} join *A* to *k*, *Ok* being equal to $\frac{1}{2}$ the semi *b* axis; from the centre of the intersection of brachypinacoid and dome *i* draw a line parallel to *Ak*. This is the line of inclination of the brachydome.

Between the faces of the macrodome {101} and brachydome {021} is a face of the unit pyramid {111}. Its intersection with the unit prism {110} will evidently be *e, f*.

To find its intersection with the macrodome draw the pyramid edge *AB*, then through *h'*, the point where the inclination line of the macrodome cuts the *c* axis, a line parallel to the *b* axis, to cut the edge *AB*; join the point of cutting *e'* to *e* for the intersection of the forms.

Next find the intersection edge between the pyramid {111} and the brachydome. This is done in a similar way, by joining *f* to *f'*, the point where the pyramid edge and line of inclination of the brachydome meet.

The basal pinacoid {001} cuts the vertical axis produced in *X*. Through *X* draw lines parallel to the *a* and *b* axes to cut the line of inclination of the macrodome and the unit pyramid edge *AB*, in points

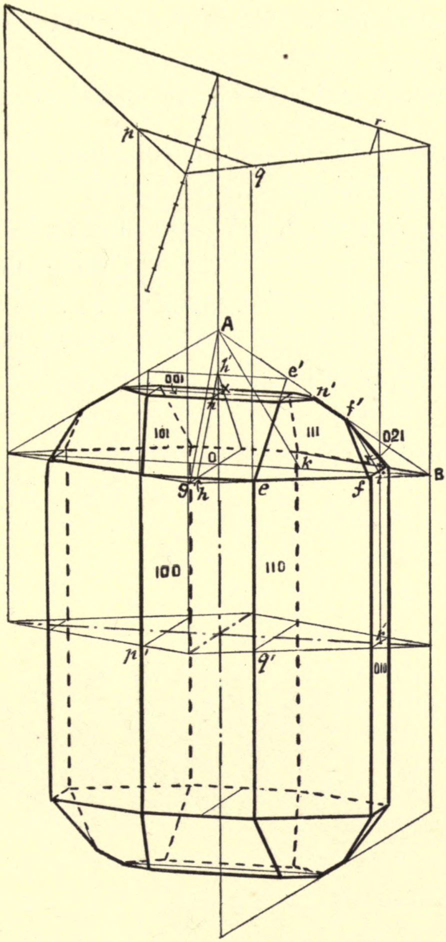



PLATE XXXII.

ORTHORHOMBIC SYSTEM.

Chrysolite. Axial Ratio ·46 : 1 : ·58 ; Macropinacoid {100} ; Macrodome {101} ; Brachypinacoid {010} ; Brachydome {021} ; Unit Prism {110} ; Unit Pyramid {111} ; Basal Pinacoid {001}.

*n*, *n′*. Through these points the intersections of base with dome and pyramid, may be drawn parallel to the intersections of those forms with prism and pinacoids.

Complete the figure by the usual methods.

Exercises—Problems for solution.

Orthorhombic system—

Andalusite, axial ratio ·98 : 1 : 7.
{110}; {001}; {101}.

Staurolite, axial ratio ·47 : 1 : ·68.
{110}; {010}; {001}; {101}.

Olivine, axial ratio ·47 : 1 : 59.
{110}; {120}; {010}; {021}; {101}; {111}.

Cordierite, axial ratio ·59 : 1 : 56.
{110}; {130}; {010}; {001}.

## CHAPTER XIII.

### MONOCLINIC SYSTEM—CONSTRUCTION OF AXES—SIMPLE FORMS—COMBINATIONS.

### Plates XXXIII. to XXXVI.

The axes of the monoclinic system are all unequal; they are lettered as in the orthorhombic system, $a$, $b$, $c$.

The $a$ axis is at right angles to the $b$. The $b$ is at right angles to the $c$. The $a$ is inclined to the $c$.

The angle the $a$ axis makes with the $c$ is measured from $+c$ to $-a$, it is represented by the Greek letter $\beta$.

The $a$ axis is sometimes called the clinodiagonal axis, the $b$ axis the orthodiagonal.

Plate XXXIII. Fig. 1 shows the construction of the monoclinic axes. The assumed axial ratio is $\cdot 65 : 1 : \cdot 55$. The $\beta$ angle $64^\circ$.

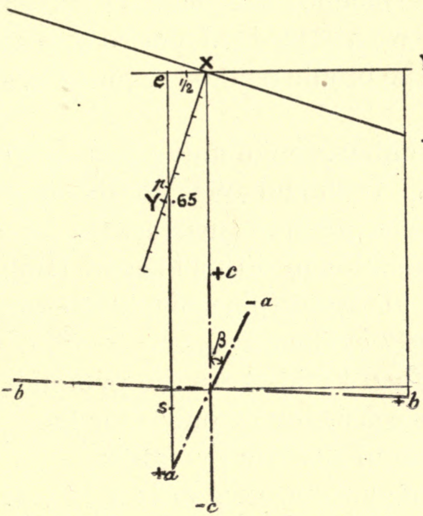
Begin with the $O$ plan in exactly the same way as for the isometric and tetragonal systems. Make the $b$ axis equal 1, also make the line of direction of the $a$ axis equal 1; divide it into tenths and mark the point $\cdot 65$ at $Y$.

Thus far the construction differs only from the isometric and tetragonal in axial ratio, and if axis $a$ were at *right angles* to both $b$ and $c$, $XY$ would represent it in $O$ plan. But axis $a$ is *inclined* to $c$ at an angle $\beta$ upwards and backwards. Looked at sideways, in profile, its position with regard to $c$ is as shown in Fig. 2. If the $a$ axis were at right angles to the $c$ its position seen in profile would be horizontal, it would be coincident with $Y'X'$; but it is inclined, it has taken the position of the line from $+a'$ to $-a'$, it makes the angle $\beta$ with the vertical axis. Now point $+a'$ lies on an arc drawn with the radius $X'Y'$, and if we draw a vertical through it to cut the line $YX$, we find it lies under a point $p'$ on that line; the distance $p'X'$ equals the distance point $+a'$ is from the vertical axis, is in fact equal to the plan of the semi $a$ axis.

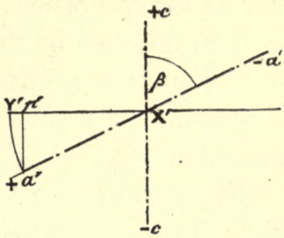
It is this distance from $p'$ to $X'$ that we have to mark off on $XY$ the line of direction of the $a$ axis on the $O$ plan. It gives us point $p$, it is from point $p$ the length of the plan, not from point $Y$ the actual length, that we are to drop the vertical projector to the $C$ projection; the $\frac{1}{2}$ distance we have to mark on it for the apparent depression is half $eX$; thus we get point $S$.

But even in $S$ we have not obtained the position for the + end of the $a$ axis in $C$ projection, for point $S$ would be the position of point $p$ supposing $X, p$ were merely a horizontal line, $X, p$ being the true length; as we have seen $X, p$ is the plan of an inclined line, the end of which is vertically below $p$; to find how far below we again refer to the profile, Fig. 2, $p', +a'$, gives us the distance below.

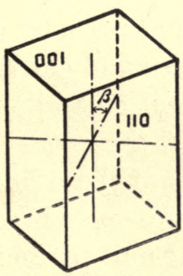
In $C$ projection we know that all vertical distances retain their true length, therefore we take the distance $p', +a'$ and mark it directly on the projector through $S$ downwards; it gives us the position for the + end of the $a$ axis; producing the axis beyond the centre we mark off an equal distance for $-a$.



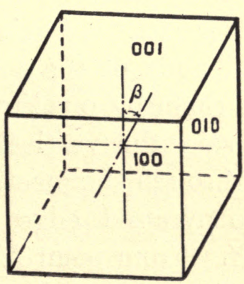

(1)




(2)




(3)




(4)

## PLATE XXXIII.

MONOCLINIC SYSTEM.

FIG. 1. Construction of Axes, assumed Axial Ratio ·65 : 1 : ·55.
FIG. 2. Profile of *a* and *c* Axes.
FIG. 3. Unit Prism {110} ; Base {001}.
FIG. 4. Orthopinacoid {100} ; Clinopinacoid {010} ; Basal Pinacoid {001}.

The $b$ axis being obtained in exactly the same way as for the other systems needs no explanation.

The problem of the inclined axis might be solved by trigonometry, but we are pledged not to appeal to trigonometry and the profile method requires less mental effort.

Fig. 3 shows the combination of unit prism {110} with the base {001}. It should offer no difficulty when once the axis construction is mastered.

Fig. 4 is the combination of orthopinacoid {100} clinopinacoid {010} and basal pinacoid {001}; the construction is perfectly evident.

Plate XXXIV. Figs. 1 and 2 are the pyramids {111} and {$\bar{1}$11} each consisting of only four faces. Fig. 3 is a combination of the two pyramids.

Fig. 4 is a crystal of gypsum, axial ratio ·7 : 1 : ·4, angle $\beta$ = 80° 42′. It is composed of unit pyramid {111}, unit prism {110} and clinopinacoid {010}. The latter cuts the $b$ axis of the unit prism at point $e$. Join the ends of the $a$ and $b$ axes and cut the line joining them by one through point $e$ parallel to the $a$ axis, raise the vertical edge of the pinacoid and prism through the meeting point of these lines.

The pyramid edge meets the prism edge at point $X$. From point $X$ draw the unit pyramid edge to cut the $c$ axis at the proper inclination.

Because the unit pyramid of this system is composed of only four faces, the front edge must be continued beyond and behind the $V$ axis until it meets the edge of the unit prism through the $-a$ axis.

From the point of meeting of pyramid and prism edges, $X$, the intersection edges of the two

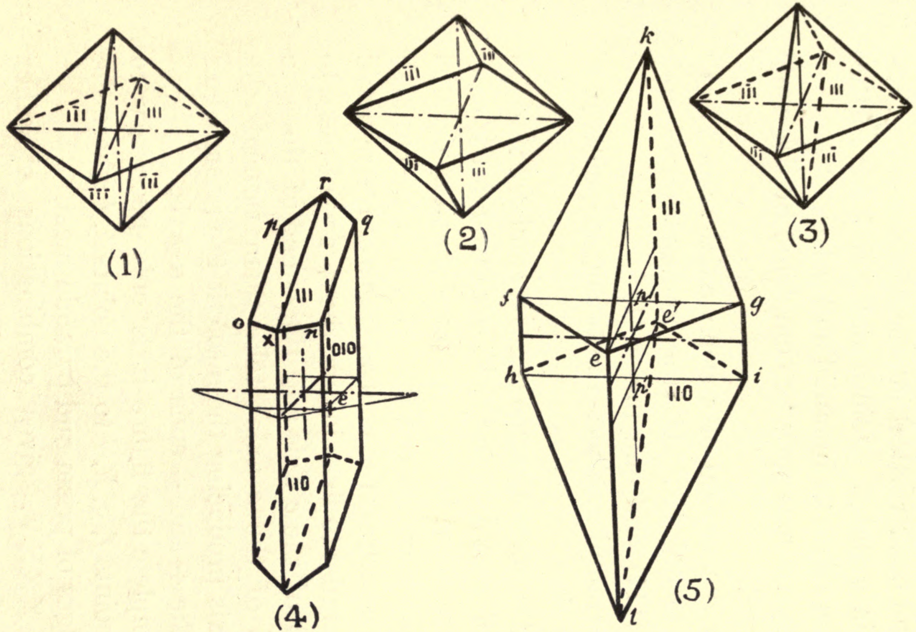



PLATE XXXIV.

MONOCLINIC SYSTEM.

FIG. 1. Pyramid {111}. FIG. 2. Pyramid {$\bar{1}$11}. FIG. 3. Combination of Pyramids.

FIG. 4. Gypsum. Axial Ratio ·7 : 1 : ·4 ; $\beta = 80° 42'$. Unit Prism {110} ; Unit Pyramid {111}; Clinopinacoid {010}.

FIG. 5. Sphene. Axial Ratio ·75 : 1 : ·85 ; $\beta = 60° 17'$. Unit Pyramid {111} ; Unit Prism {110}.

forms may be drawn; they will be lines parallel to the front edges of the unit pyramid *X*, *o*, and *X*, *n*. The intersection edges of pyramid and pinacoid will be parallel to the centre edge of the pyramid; they will cut the back vertical intersections of prism and pinacoid in *p* and *q*. Join *p* and *q* to *r* for further intersection edges of pyramid and prism.

The crystal may now be completed by the usual methods.

Fig. 5 is a crystal of sphene, axial ratio ·75 : 1 : ·85, $\beta$ angle 60° 17′. It is a combination of unit pyramid {111} and unit prism {110}.

The pyramid replaces the prism at points *p*, *p′* on the *c* axis.

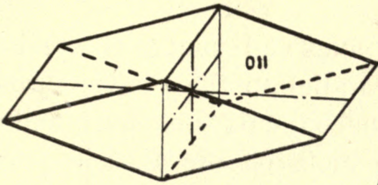
Draw the vertical prism edges *e*, *l* and *e′*, *k* through the ends of the *a* axis and *f*, *h′*, *g*, *i* through the ends of the *b*, cut them by lines through *p* and *p′* parallel to the *a* and *b* axes in points *e*, *e′*, *f*, *g*, *h*, *i*. Through the points *e*, *e′* draw the central edges of the pyramid *e*, *k*, *e′*, *l* parallel to lines joining the + end of the *a* to the + end of the *c* axis and the − end of the *a* to the − end of the *c* axis; take them through the vertical axis producing them to cut the prism edges through the + and − ends of the *a* axis.

To complete the figure it is only necessary to join the points *f*, *g*, *h*, *i* to the points *k*, *l* where the central edges of prism and pyramid meet.

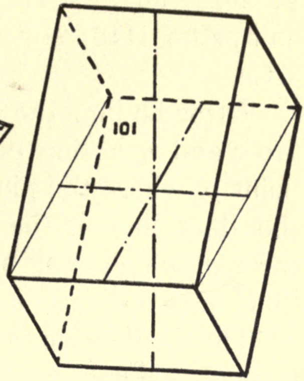
Plate XXXV. gives combinations of other monoclinic forms.

The clinodomes shown in Fig. 1, the orthodomes in Fig. 2, need no direction; the construction is apparent.

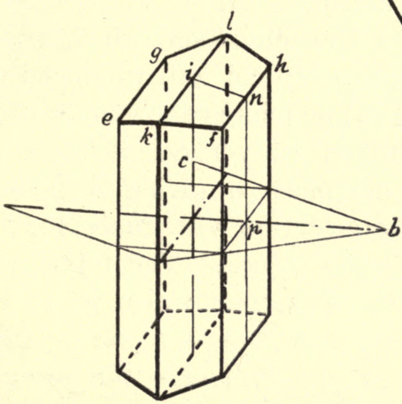
Fig. 3 is a crystal of amphibole, axial ratio ·55 :



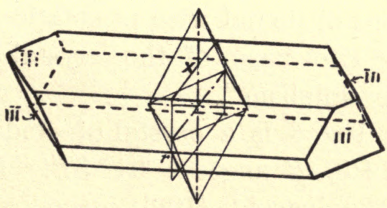

(1)




(2)




(3)




(4)

PLATE XXXV.

MONOCLINIC SYSTEM.

FIG. 1. Clinodomes {011}.
FIG. 2. Orthodomes {101}.
FIG. 3. Amphibole. Axial Ratio ·55 : 1 : ·29 ; $\beta$ 73° 58′. Unit Prism {110} ; Clinopinacoid {010} ; Clinodome {011}.
FIG. 4. Epidote. Axial Ratio 1·58 : 1 : 1·8 ; $\beta$ 64° 37′. Orthopinacoid {100} ; Orthodome {110} ; Basal Pinacoid {001} ; Unit Pyramids $\{\bar{1}11\}$ $\{1\bar{1}1\}$ $\{11\bar{1}\}$ $\{1\bar{1}\bar{1}\}$.

1 : ·29, $\beta$ angle = 73° 58′. It is a combination of unit prism {110}, clinodome {011} and clinopinacoid {010}.

After finding the axes and joining the ends of the *a* and *b*, a line parallel to the *a* axis through point *p*, where the pinacoid cuts the *b* axis, will cut the lines joining the ends of the axes in points, through which may be raised the vertical intersections of prism and pinacoid.

The inclination of the clinodome which passes through a point *i* on the vertical axis produced will be a line *i*, *n* parallel to one joining the + ends of the *b* and *c* axes, it will cut a vertical through the centre of the pinacoid face, so giving a point *n* through which the intersection with that face may be drawn parallel to the *a* axis. The top edge of the dome is drawn through point *i* also parallel to the *a* axis, it cuts the prism edges in *k* and *l*. *k* and *l* joined to points *e*, *f*, *g*, *h*, will give the intersections of domal with prismatic faces.

The base of the crystal can be finished by "parallelism".

Fig. 4 is a crystal of epidote, axial ratio 1·58 : 1 : 1·8, $\beta$ angle = 64° 37′. It is acombination of orthopinacoid {100}, orthodome {110}, basal pinacoid {001}, unit pyramids $\{\bar{1}11\}$ $\{\bar{1}\bar{1}1\}$ $\{11\bar{1}\}$ and $\{1\bar{1}\bar{1}\}$.

It is prismatic in the direction of the ortho axis. The intersection of orthopinacoid and basal pinacoid may be obtained by verticals through the ends of the *a* axis, cut by lines parallel to the *a* axis drawn through point *x* the point where the base intersects the *c* axis.

The dome cuts the orthopinacoid face in point $r$, through which may be drawn their basal intersection. The inclination of the dome will be a line parallel to the – front edge of the unit pyramid.

To find the intersections of the various forms with the unit pyramid, it will be useful to have the edges of the pyramid shown, because all the intersections will be parallel to one or other of these thus :—

The intersection with the base will be parallel to the edges joining the $a$ and $b$ axes.

That with the orthopinacoid, will be parallel to those joining the $b$ and $c$ axes.

That with the dome is parallel to the front and back edges. Thus the crystal may be completed.

Plate XXXVI. gives a crystal of orthoclase, axial ratio ·66 : 1 : ·56 ; angle $\beta = 63° 57'$. It is a combination of unit prism {110}, unit pyramid {$\bar{1}$11}, clinopinacoid {010}, clinoprism {130}, basal pinacoid {001}, orthodomes {$\bar{1}$01} and {$\bar{2}$01}.

The front edge of the unit prism is drawn vertically through the + end of the $a$ axis. The + ends of the $a$ and $b$ axes are joined. The line joining them will be the trace of the unit prism on the axial plane which includes the $a$ and $b$ axes. This plane is parallel to the basal plane {001} ; we may therefore call the intersections of prisms and pinacoids with it, their basal traces.

We have the front and back edges of the unit prism through the + and – ends of the $a$ axis. At a point $U$ on its basal trace the prism {110} meets the prism {130}. We can draw the basal trace of this latter prism from $U$. It will be a line parallel to one drawn from the + end of the *a* axis to the $\frac{1}{3}$ point on the *b* axis. It will meet the basal trace of the clinopinacoid {010}, a line *VW* drawn through point *z*, the point where the pinacoid cuts the *b* axis, parallel to the *a* axis; the point of meeting is lettered *V*. Through *U* and *V* we can raise the vertical intersections of prisms and pinacoid.

The front edge of the unit prism is cut by the base in point *T*. From *T* draw a line parallel to the *a* axis to cut the vertical axis in *S*; this will be the inclination line of the basal plane.

The intersection of basal plane and unit prism will be a line from *T* parallel to the basal trace of the prism. It meets the intersection edge of unit and clinoprisms in point *R*.

The intersection of basal plane and clinoprism will be a line from *R* parallel to *XY*, the basal trace of the prism. It will meet the intersection of clinoprism and pinacoid in point *P*.

The intersection of basal plane with clinopinacoid will be a line from *P* parallel to the *a* axis.

We have next to find the position of the face of the orthodome {$\bar{2}01$} and its intersection edges. Its lower front face {$20\bar{1}$} cuts the front edge of the unit prism in point *O*. Its inclination line is drawn from *O* parallel to a line joining the $\frac{1}{2}$ point of the *a* axis to the – end of the *c* axis.

The intersection of the orthodome face {$20\bar{1}$} with the unit prism is got by drawing through the point *e* where its line of inclination cuts the *a* axis a line *e*, *f*, parallel to the *b* axis, to cut the basal trace of the prism in *f*. Through the point of cutting the required intersection *ON* can be drawn.

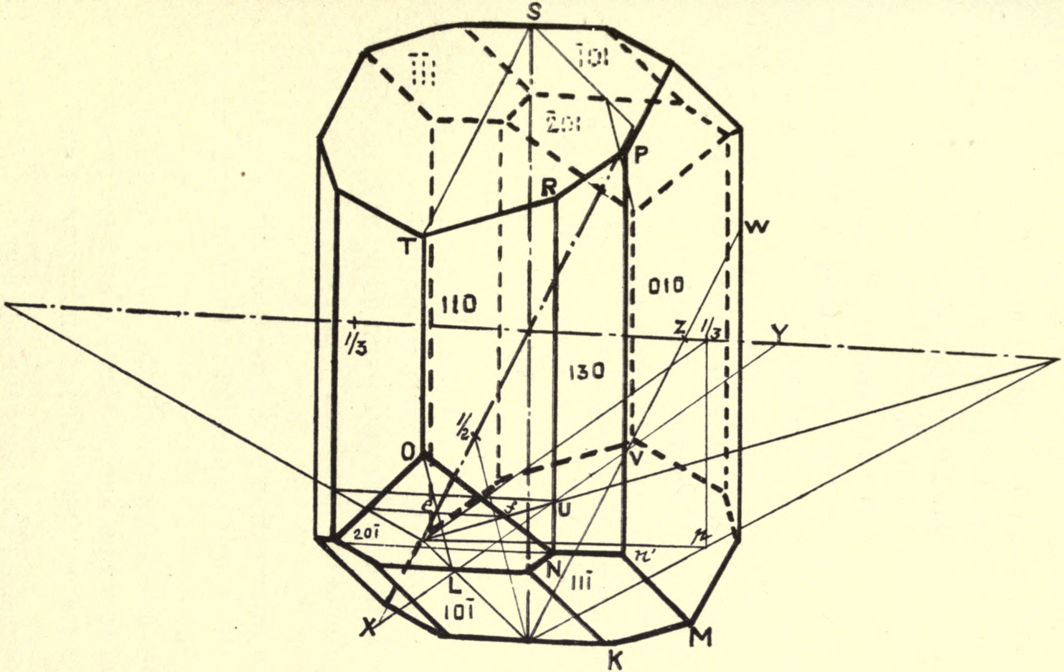



PLATE XXXVI.

MONOCLINIC SYSTEM.

Orthoclase. Axial Ratio ·66 : 1 : ·56; Angle $\beta = 63° 57'$. Unit Prism {110}; Clinoprism {130}; Unit Pyramid {$\bar{1}$11}; Clinopinacoid {010}; Basal Pinacoid {001}; Orthodomes {$\bar{1}$01} {$\bar{2}$01}.

The orthodome face $\{20\bar{1}\}$ gives place to the orthodome face $\{10\bar{1}\}$ at point $L$. The intersection of the two orthodomes will be parallel to the $b$ axis.

Draw the line of inclination of the orthodome $\{10\bar{1}\}$. It joins the + end of the $a$ and – end of the $c$ axis, then through the – end of the $c$ axis its intersection with the basal plane.

One other form is present, the unit pyramid $\{\bar{1}11\}$. Its lower right-hand face $\{11\bar{1}\}$ cuts the clinopinacoid at a point $M$. Its intersection with the pinacoid is of course parallel to its own front edge. Its intersection with the face of the prism $\{130\}$ is found by drawing a vertical through the $\frac{1}{3}$ point of the semi $b$ axis to cut the line joining the + end of the $b$ to the – end of the $c$ axis in $p$; join $p$, the point of cutting to the + end of the $a$ axis, and make the edge of intersection parallel to it $Nn'$.

The intersection of the unit pyramid and the orthodome $\{20\bar{1}\}$ will be parallel to a line joining the point where the inclination line of the orthodome meets the front edge of the pyramid point $L$, to where line $e, f$ cuts the line from + end of $a$ to + end of $b$ axis.

The intersection of the pyramid with the orthodome $\{10\bar{1}\}$ will be parallel to the front edge of the pyramid.

The intersection with the basal plane is $MK$ parallel to a line joining the + $a$ and + $b$ axes.

The rest is "parallelism" and "repetition".

Exercises—Problems for solution.

Monoclinic system—

Gypsum, axial ratio ·69 : 1 : ·4; $\beta$ 80° 42′ $\{010\}$; $\{110\}$; $\{111\}$.

Hornblende, axial ratio ·55 : 1 : ·29 ; $\beta$ 73° 58′.
{110} ; {010} ; {011} ; {$\bar{1}$01}.

Epidote, axial ratio 1·58 : 1 : 1·80 ; $\beta$ 64° 37′.
{100} ; {001} ; {$\bar{1}$01} ; {110}.

Orthoclase, axial ratio ·66 : 1 : ·56 ; $\beta$ 63° 57′.
{010} ; {110} ; {001} ; {$\bar{1}$01} ; {201} ; {011} ; {$\bar{1}$11}.

Orthoclase, {001} ; {010} ; {110} ; {$\bar{2}$01}.

## CHAPTER XIV.

### TRICLINIC SYSTEM—CONSTRUCTION OF AXES—SIMPLE FORMS—CONBINATIONS.

### Plates XXXVII. to XXXIX.

In the triclinic system there are three axes, all unequal and intersecting at oblique angles. They are lettered *a*, *b*, *c*. The angle between the *c* or vertical axis and the *a* axis is measured from the + end of one to the + end of the other, it is lettered $\beta$. The angle between the *c* and *b* axes is measured from the + end of one to the + end of the other; it is lettered $\alpha$. The angle between the *a* and *b* axes is measured from the + end of the one to the + end of the other and is lettered $\gamma$.

Plate XXXVII. shows the construction of the triclinic axes. It is dependent on the *O* plan and profiles of the axes.

The *a* axis is in the same direction as in the isometric system but has this difference, the + end in the triclinic system is either depressed or elevated a certain number of degrees, so that the axis is no longer at right angles to the *c* or vertical axis, its position is similar to the *a* axis of the monoclinic system.

The *b* axis is neither at right angles to the *c* nor to the $a$ axis, therefore its position is quite altered from that of the isometric $a_2$ axis.

We will assume the lengths given on the plate for the three axes, making as usual the $b$ axis = unity. The angles are also assumed $\beta = 140^\circ$, $\alpha = 75^\circ$, $\gamma = 135^\circ$.

We start by finding the direction of the $a$ axis in the usual manner, then find its plan by a profile (see Fig. 2) exactly as for the monoclinic $a$ axis. Having thus obtained the plan of its semi-length, we mark it on the $O$ plan as shown at $a$, $x$.

We next find the plan of the semi $b$ axis, in the same way by a profile, shown at Fig. 3.

Having then the plans of the semi $a$ and $b$ axes, it would at first appear quite easy to construct the $O$ plan by making the plan of the $b$ axis at an angle of 105° with the plan of the $a$ axis.

This, however, would not be correct, the *plan* of the $\gamma$ angle is not the true angle 105°, and it is the plan we require.

To get it we must make use of yet another profile, giving another plan. This plan is of the line that would join the + ends of the $a$ and $b$ axes.

With a little thought we can attain this plan. Make a triangle (see Fig. 4) having the semi $a$ and $b$ axes for sides at an angle to each other equal the true $\gamma$ angle 105°, the other side of the triangle, the hypotenuse, will be the true length of the line joining the axes.

We have the true length, but it is the plan of this line we require to complete our $O$ plan.

To obtain the plan we need to know the height the end of the $b$ axis is above the level of the end of the *a* axis. The + end of the *a* axis is the distance *r*, *t* below the centre of the vertical axis; the + end of the *b* axis is the height *u*, *s* above the centre. The height then of the end of the *b* axis above the end of the *a* is $us + rt$.

Now we have all the factors necessary for finding the plan of the line joining the ends of the *a* and *b* axes. We find it thus:—

Draw a horizontal line *hh'* (see Fig. 5) at *h*, raise a perpendicular $hg = us + rt$, from *g* with radius the true length of the line taken from Fig. 4 cut *hh'* in *i*. Join *i* to *g*, then *h*, *i* will be the plan of *g*, *i*.

We can now finish our *O* plan of the axes. We have already found the plan of the semi *a* axis, from the end *x* with radius = plan of semi *b* axis make an arc, from the other end *a* with radius = plan of line joining ends of *a* and *b* axes, cut the arc in *e*, the *O* plan is now complete.

We drop the ends of the axes by vertical projectors to the *C* projection as usual, find points *r* and *u* which would be the ends of the plans of the axes if they were horizontal; to get the actual positions mark from *r* downwards the distance *t*, *r* taken from the profile Fig. 2 of the *a* axis and for the *b* axis mark upwards from *u* the height *u*, *s* taken from Fig. 3 the profile of the *b* axis.

Plate XXXVIII. shows the simple forms of the triclinic system, each composed of only two faces.

Figs. 1, 2, 3, 4 are the four pyramids $\{111\}$ $\{1\bar{1}\bar{1}\}$ $\{11\bar{1}\}$ $\{1\bar{1}1\}$.

Figs. 5 and 6 are the prisms $\{1\bar{1}0\}$ $\{110\}$.

Figs. 7 and 8 are the pinacoids $\{100\}$ and $\{010\}$.

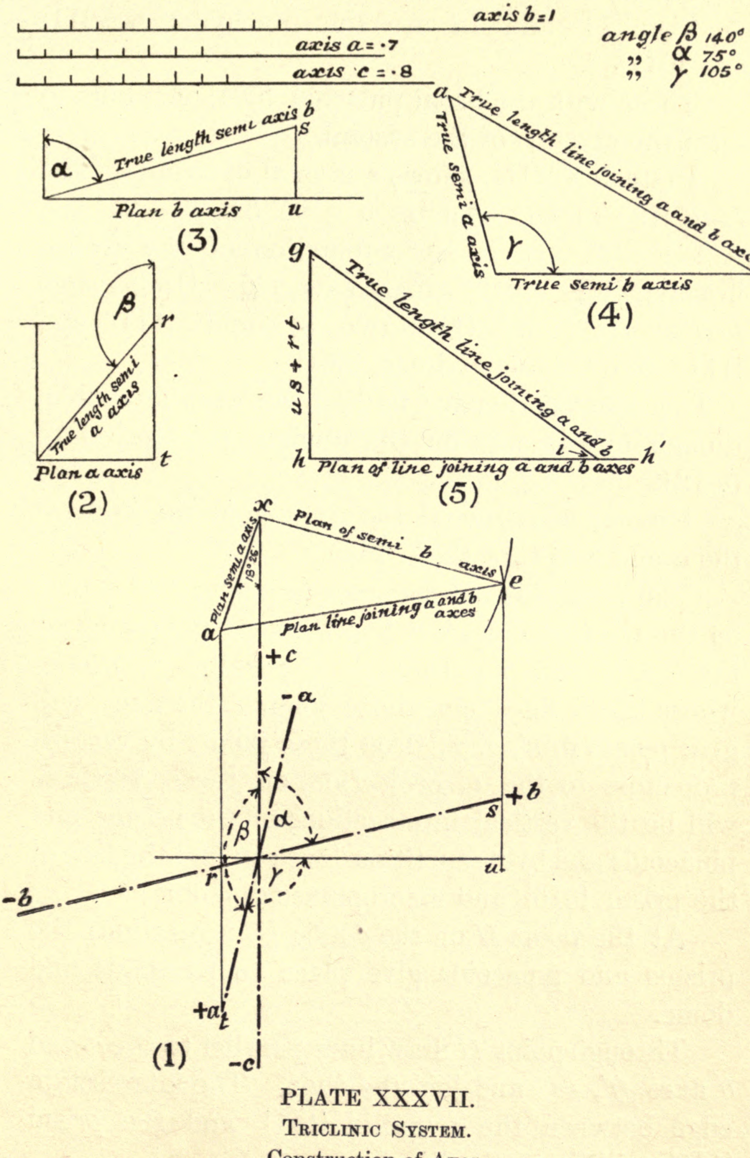



PLATE XXXVII.

TRICLINIC SYSTEM.

Construction of Axes.

Figs. 9 and 10 are macrodomes {101} and {10$\bar{1}$}.

Figs. 11 and 12 are the brachydomes {011} and {0$\bar{1}$1}.

These with the basal pinacoid {001} combine to form the crystals of the system.

Plate XXXIX. shows a crystal of axinite, axial ratio ·49 : 1 : ·48. Angles $\alpha = 82^\circ\ 54'$, $\beta = 91^\circ\ 52'$, $\gamma$ 131° 32′. It is a combination of the forms, brachyprism {010}, macropinacoid {100}, two prisms {110} and {1$\bar{1}$0}, two pyramids {111} and {1$\bar{1}$1} and the macrodome {201}.

The construction for finding the axes is shown, though for description the student is referred to p. 121.

Having constructed the axes, join the ends of the *a* and *b* axes of the *O* plan.

The pinacoids cut the axes in points *PP′* shown on the *O* plan. Draw a line through *P* parallel to the *b* axis and one through *P′* parallel to the *a*, where these lines cut those joining the axes will give points *n*, *n′*, *g*, *g′*, drop these points by vertical projectors to the *C* projection, on these verticals will be the vertical intersections of the prisms and pinacoids and by "repetition" we can find the face of the prism {0$\bar{1}$0} and macropinacoid {$\bar{1}$00}.

At the point *H* on the *c* axis (*C* projection) the prisms and pinacoids give place to pyramids and dome.

Through point *H* draw lines parallel to the *a* and *b* axes *ff′*, *ee′*, and join the ends. The intersection edge between the pyramid {1$\bar{1}$1} and the prism {1$\bar{1}$0} will lie on the line from *e* to *f*.

We will next follow carefully the construction

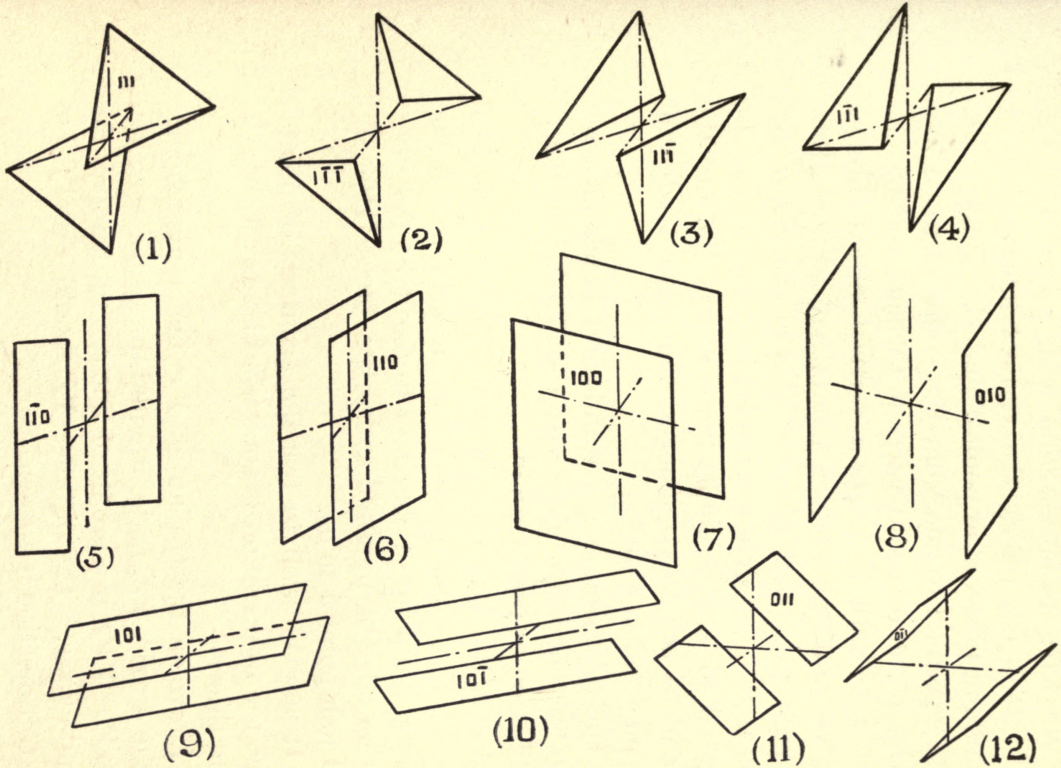



PLATE XXXVIII.

TRICLINIC SYSTEM.

FIG. 1. Pyramid $\{111\}$.
FIG. 2. Pyramid $\{1\bar{1}\bar{1}\}$.
FIG. 3. Pyramid $\{11\bar{1}\}$.
FIG. 4. Pyramid $\{1\bar{1}1\}$.
FIG. 5. Prism $\{1\bar{1}0\}$.
FIG. 6. Prism $\{110\}$.
FIG. 7. Pinacoid $\{100\}$.
FIG. 8. Pinacoid $\{010\}$.
FIG. 9. Macrodome $\{101\}$.
FIG. 10. Macrodome $\{10\bar{1}\}$.
FIG. 11. Brachydome $\{011\}$.
FIG. 12. Brachydome $\{0\bar{1}1\}$.

for the several intersection edges of the pyramid face $\{1\bar{1}1\}$ with the other forms.

We have its intersection with prism $\{1\bar{1}0\}$, line *e*, *f*. It is cut off by the vertical intersections of prism and pinacoid in point *g′* and *n′*. Draw a line from *e* parallel to one from the + end of the *a* axis to the + end of the *c*, carry it through and beyond the *c* axis, until it meets a vertical raised through the centre of the further macropinacoid face $\{\bar{1}00\}$, through the point of meeting we can draw the intersection of pyramid and pinacoid, it will be parallel to a line joining the – end of the *b* to the + end of the *c* axis, it meets the vertical edges of the pinacoid in *h* and *o*.

The intersection of pyramid and brachypinacoid $\{0\bar{1}0\}$ will be line *g*, *g′* drawn parallel to *e*, *i* the line of inclination of the pyramid.

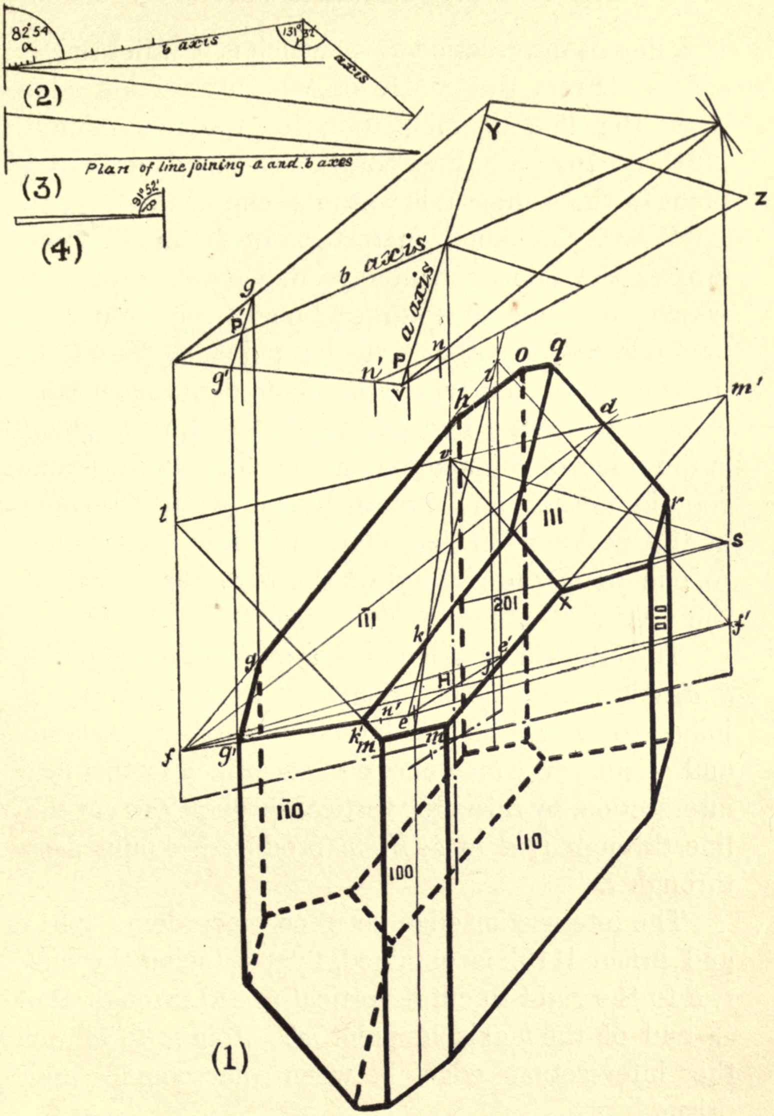
Carry the line of inclination to cut a vertical through the – end of the *a* axis in point *i*. A line joining the point of cutting *i* to *f* will be the intersection edge of pyramid and prism face $\{1\bar{1}0\}$ and will join point *h* to *g*.

It may be well to test the accuracy of point *i*, since the intersection is somewhat acute; it may be done thus:—

Draw on the *O* plan at *Y* a line perpendicular to the *a* axis, *YZ*, make it equal the semi *c* axis, join *Z* to *V*. *YVZ* is a profile of line *e*, *i* cut through the *a* and *c* axes. The distance *YZ* should equal *j*, *i* of the *C* projection if correct.

The pyramid face $\{1\bar{1}1\}$ is cut by the macrodome $\{201\}$.

The macrodome meets the macropinacoid $\{100\}$




## PLATE XXXIX.

### TRICLINIC SYSTEM.

FIG. 1. Axinite. Axial Ratio ·49 : 1 : ·48 ; $\alpha$ 82° 54′ ; $\beta$ 91° 52′ ; $\gamma$ 131° 32′. Macropinacoid {100} ; Two Unit Prisms {110} $\{1\bar{1}0\}$ ; Macrodome {201} ; Two Unit Pyramids {111} $\{1\bar{1}1\}$ ; Brachyprism {010}.

FIGS. 2, 3, 4. Construction for Axes.

in a line of intersection *m*, *m′* which is parallel to the *b* axis. From the centre of this intersection edge draw the line of inclination for the macrodome, {201}, it will be a line parallel to one from the $\frac{1}{2}$ point of the semi *a* axis to the + end of the *c*.

Where this line of inclination of the macrodome crosses the line *e*, *i* will give one point *k* through which to draw the intersection of pyramid and macrodome. Another will be point *d*, found by drawing through *v*, where the macrodome meets the *c* axis produced, a line parallel to the *b* axis, which is the trace of the macrodome on the vertical plane containing the *c* and *b* axes, at *d* it meets the trace of the pyramid on that plane, the line *f*, *d* parallel to one joining the – end of the *b* to the + end of the *c* axis.

The intersection edge of pyramid and macrodome *k*, *d*, will meet that of prism and pyramid in *k*. *k* joined to *m* will be the intersection of macrodome and prism. We may check the accuracy of this last intersection, by raising a vertical through *f* to cut the line through *v*, *d* in *l*; *k′*, *m* produced should pass through *l*.

The intersection edge between macrodome {201} and prism {110} is obtained, by producing the line *v*, *d* to the right to cut a vertical raised through the + end of the *b* axis in point *m′*. Join *m* to *m′* for the intersection edge between macrodome and prism.

One other form is present. The pyramid {111} cuts the prism in point *X*. From *X* a line parallel to that joining the + ends of the *a* and *b* axes will give the intersection of pyramid and prism.

The intersection of this pyramid with the brachypinacoid will be parallel to line *e*, *i*.

The intersection of the pyramid with the dome {201} will join *X* to *v* the point where both cut the *c* axis produced.

The intersection of pyramids {111} and {1$\bar{1}$1} will be parallel to line *e*, *i*, it will meet the short intersection edge between the pyramid {111} and the prism {1$\bar{1}$0}, *o*, *q*, which is parallel to *f*, *k*.

A line from *q* to *r* will complete the crystal. This latter line is the intersection edge between the pyramid {111} and the prism {1$\bar{1}$0}. It should be parallel to a line joining *f′* to point *i′* if worked correctly.

Exercises—Problems for solution.

Triclinic system—

Plagioclase, axial ratio ·63 : 1 : ·56 ; $\alpha$ 94° 3′ ; $\beta$ 116° 29′; $\gamma$ 88° 9′. {010}; {110}; {1$\bar{1}$0}; {001}; {10$\bar{1}$}.

Kyanite, axial ratio ·9 : 1 : ·7 ; $\alpha$ 90° 5′ ; $\beta$ 101° 2′; $\gamma$ 105° 44′. {100}; {010}; {001}; {1$\bar{1}$0}; {110}; {210}.

## A CHAPTER ON TWINS.

TWINNED CRYSTALS IN THE ISOMETRIC—TETRAGONAL—HEXAGONAL—ORTHORHOMBIC—MONOCLINIC—AND TRICLINIC SYSTEMS.

### PLATES XL. TO XLIV.

FOR the construction of twinned forms the profile method is most useful.

Plate XL. shows a twinned octahedron or spinel twin. It is a twin of the isometric system. The twinning plane is parallel to an octahedral face.

The small figure 3 shows the twinning plane *in situ.* The twinning axis is indicated by the finely dotted line.

To draw the twinned crystal, begin by drawing an octahedron, making an *O* plan of the *H* axes above. Bisect the edges joining the + end of the $a_1$ axis to the – end of the $a_2$, the + end of the $a_2$ to the – end of the $a_1$ the edges joining the + ends of the $a_1$ and $a_2$ axes to the + end of the $a_3$, the edges joining the – ends of the $a_1$ and $a_2$ axes to the – end of the $a_3$. The bisection of the edges will give us six points through which to draw the edges of the twinning plane, shown shaded in the small figure 3.

We know that the twinned portion of the crystal is revolved on an axis at right angles to this plane, through an angle of 180°. That is to say, suppose we made a clean cut through the octahedron in the

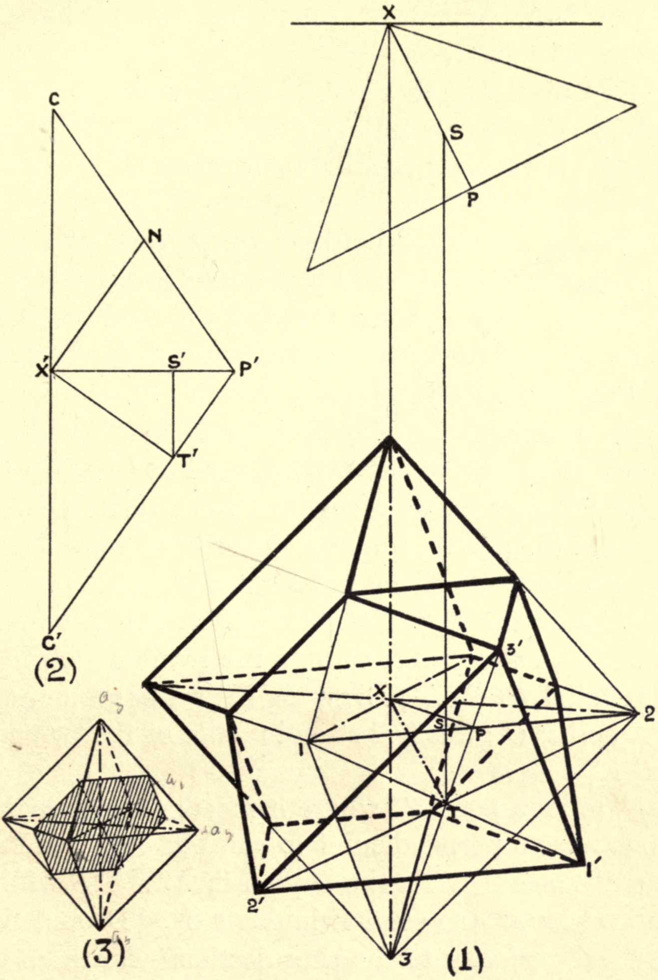



PLATE XL.

ISOMETRIC SYSTEM.

FIG. 1. Spinel, Twinned Octahedron. FIG. 2. Profile.
FIG. 3. Shows Positions of Twinning Axis and Plane.

direction of the plane and twisted the separated lower half, half way round, we should have a twinned octahedron.

We must find this axis of rotation, this twinning axis; to do this we make a profile. The profile shall be taken in a plane parallel to the face {110} of the rhombdodecahedron.

Bisect the angle between the + ends of the $a_1$ and $a_2$ axes both of the *O* plan and *C* projection; the profile or section passes through the bisecting line. It lies in a plane of symmetry both of the twinned and untwinned crystal.

Construct the profile thus (see Fig. 2): draw a line $X'P' = XP$ of the *O* plan, the line bisecting the angle between the axes; at $X'$ raise a vertical, make it equal the $a_3$ axis, join the ends $CC'$ to $P'$. This is the profile through the octahedron at line $XP$.

Divide the line $CP'$ into half at $N$. Join $N$ to $X'$, $NX'$ is the profile of half the twinning plane, it is parallel to $P'C'$. Draw $X'T'$ at right angles to $NX$ and $P'C'$; this is the semi-profile of the twinning axis.

Through point $T'$ where it meets the octahedral face, raise a perpendicular to cut $X'P'$ in $S'$. Mark the distance $X'S'$ on the *O* plan in $XS$. $XS$ will be the *O* plan of the semi-twinning axis. Drop $S$ by a vertical projector to the *C* projection. Mark on the projector downwards from $S$ of the *C* projection the distance $S'T'$ taken from the profile in $T$. $T$ will be the point where the twinning axis meets the octahedral face in *C* projection.

Join the angles 1, 2, 3 of the octahedron to point $T$ producing each line beyond $T$, making $T1'$

equal $T1$, $T2'$ equal $T2$, $T3'$ equal $T3$. Join points 1′, 2′, 3′ to each other and to the points where the twinning plane cuts the edges of the octahedron, the figure will then be complete.

We can certify the construction by this consideration. The face 1′, 2′, 3′ of the twinned portion lies in the same plane as the face 1, 2, 3 of the original octahedron. Point $T$ therefore is common to both faces, is the centre of both faces, therefore the distances $T1$, $T2$, $T3$ must equal the distances $T1'$, $T2'$, $T3'$.

This is one of the characteristics of a form of which the experienced crystal draughtsman takes quick advantage; the student should always be on the alert for such welcome short cuts through intricate problems.

The plane of twinning is in all cases a plane of symmetry in the twinned but not in the untwinned crystal.

Plate XLI. shows twins from the tetragonal and orthorhombic systems.

Fig. 1, a crystal of cassiterite, $c$ axis = ·67, gives a combination of unit prism {110}, a prism of the second order {100}, a unit pyramid {111} and a pyramid of the second order {101}.

The twinning plane parallel to a face of the pyramid of the second order, is shown shaded Fig. 1.

Fig. 2 shows the construction for the twinned crystal.

Having first drawn the crystal in the usual position the twinned portion may be found thus: Through point $O$, the point on the vertical axis cut by the twinning plane, draw a line $OU$ parallel to a line from the – end of the $a_2$ axis to the + end of the $c$ axis, let it cut a vertical through the centre of the prism face {010} at $v$. Through the point of cutting draw a short line parallel to the $a_1$ axis to cut the vertical edges of the face at $Z'$ and $W'$.

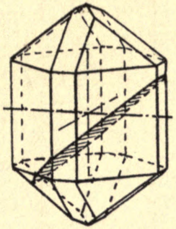
Through the point $O$ draw another line parallel to the $a_1$ axis to cut a vertical through the centre of the prism face {100}, through the point of cutting pass a short line parallel to $OU$ to cut the vertical edges of the face {100} at $Y$ and $Z$.

Repeat for corresponding back face of prism {$\bar{1}$00}. Join the points thus found $Z$ to $Z'$, $W$ to $W'$, the lines joining will be the intersections of the twinning plane with the faces of the prism of the first order {110}.

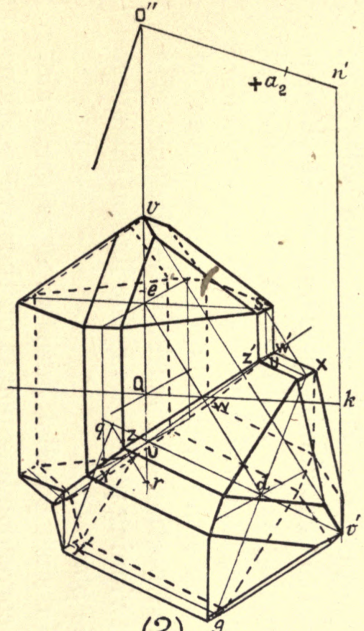
We next proceed to find the $c$ axis of the twinned portion (this must not be confounded with the twinning axis or axis of rotation which is at right angles to the twinning plane but need not be shown). The twinned $c$ axis we find by a profile seen at Fig. 3.

This profile we take through the $a_2$ and $c$ axes because the vertical plane which contains them contains also the twinned $c$ axis.

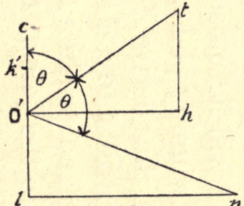
To construct the profile, first draw a vertical line $l$, $c$, this represents a portion of the $c$ axis, through point $O'$, the point where the $c$ axis is cut by the twinning plane, draw line $O't$, making with the vertical axis the same angle $\theta$ as the twinning plane makes; to get this angle draw the horizontal line $O'h$ equal the semi $a_2$ axis, and the vertical $ht$ equal the semi $c$ axis. Join $O'$ to $t$, $O't$ will be the profile of a portion of the twinning plane.



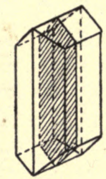
(1)



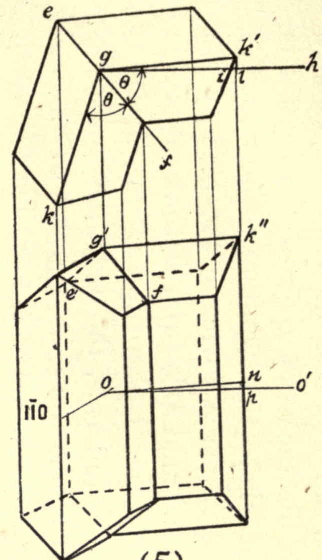

(2)




(3)



(4)




(5)

PLATE XLI.

Tetragonal and Orthorhombic Systems.

Tetragonal:—

Fig. 1. Cassiterite $c$ axis = ·67, showing Twinning Plane parallel to Face {101}.

Fig. 2. Twinned Crystal.

Fig. 3. Profile Construction.

Orthorhombic:—

Fig. 4. Aragonite. Axial Ratio ·66 : 1 : ·72. Twinning Plane parallel to Prism of 60°.

Fig. 5. Twinned Crystal.

Draw line $O'n$ making the angle $Ot'n$ equal $cO't$, line $O'n$ will be the profile of the new $c$ axis, make $O'n$ equal the distance the apex of the crystal is from the point where the twinning plane cuts the $c$ axis. The distance $n, l$ will be the plan of the portion of the twinned axis from $O'$ to $n$, mark this plan distance on the $O$ plan from $O''$ to $n'$. Drop $n'$ to the $C$ projection at $v'$, the exact position of $v'$ being obtained by marking the distance $k'l = k'o + ol$ from the profile downwards from the line $Qk$, which is a production of the $a_2$ axis. The point $v$ thus found is the apex of the twinned portion of the crystal; the $c$ axis may be drawn from $v'$ through $O$.

Join the apex of the crystal to the apex of the twinned portion $v'$. The joining line will be at right angles to the twinning plane, therefore parallel to the twinning axis. A line from $e$ drawn parallel to this joining line will enable us to get point $d$,—the point where the pyramids replace the prisms of the twinned portion—by marking the distance of $e$ from the twinning plane, along the line on the other side of the plane to $d$.

Now in such parallel lines we have great aid, since all corresponding points of the two portions of the twin will lie on such lines at equal distances from the twinning plane.

We can thus derive $X$ from $S$ by drawing through $S$ a short parallel to $e, d$ to cut the central line of the twinning plane produced and marking the distance that $S$ is to the left of the point of cutting, to the right in $X$.

Join $X$ to $d$ and producing the line an equal distance beyond $d$ we have the point $g$. The completion of this end of the twinned portion will be easy.

For the other end, to get point $q$, the point on the new $c$ axis where the pyramids replace the prisms, we have only to draw a parallel to $de$ through $r$ ($r$ being obtained by making $Qr = Qe$) to cut the twinned $c$ axis produced beyond $O$ in $q$. Through $q$ draw a line $qX'$ parallel to $dX$.

This end of the twinned portion can now be easily finished by "parallelism" and "repetition".

Fig. 5 shows a twin of the orthorhombic system. A crystal of aragonite, axes ·66 : 1 : ·72. It is a combination of the unit prism {110}, brachypinacoid {010} and brachydome {011}. It is twinned on a plane parallel to the face $\{1\bar{1}0\}$. The twinning plane is shown shaded at Fig 4.

The construction offers little difficulty. Draw the crystal and $O$ plan. Draw the line $e, f$ on the $O$ plan parallel to the face $\{1\bar{1}0\}$; $e, f$ is the plan of the twinning plane. Make angle $\theta$ equal angle $\theta$ and $gk'$ equal $gk$ and complete the plan of the twinned portion as shown. Cut the upper edge of the dome by a projector dropped from $g$ in $g'$. Cut the intersection edges of pinacoidal and domal faces by projectors from $e$ and $f$ in $e'f'$. Join $e'f'$ to $S'$ for the intersection of the twinned domes. Vertical lines through $e'$ and $f$ will be the intersections of the twinned pinacoid faces.

There is, however, one slight difficulty. At first sight it may not be apparent how the direction of the edge $g'k''$, the uppermost edge of the twinned portion of the dome is determined.

To obtain it we may use the same method of construction as for finding the axes.

We will first find a line *o, n* through the centre of the crystal in *C* projection, parallel to which $g'k''$ may be drawn.

For this purpose draw a horizontal line *g, h* through the centre of the *O* plan. Draw a vertical projector from $k'$ to cut an *H* line $oo'$ drawn through the centre of the *C* projection, in point *p*.

The vertical projector cuts the line *g, h* in point *l*. Draw the short line *k, i* parallel to *g, k*, that is parallel to the *a* axis.

With the dividers convey half the distance *i, l* to the *C* projection and mark it upwards from point *p* on the vertical projector in *n*, it will be the amount of apparent elevation the point $k'$ will assume above the horizontal, for since $k'$ lies behind the plane of projection it will appear *elevated* not *depressed*. Join *o* to *n*, *on* is a horizontal line through the centre of the crystal having the direction the upper edge of the dome will take, to which the edge of the dome may be drawn parallel.

The figure is completed by the usual methods.

Plates XLII. and XLIII. give the construction for a twinned + scalenohedron $\{21\bar{3}1\}$ of calcite, *c* axis = ·84, belonging to the hexagonal system.

It is twinned about the − rhombohedron $\{02\bar{2}1\}$.

Fig. 1 Plate XLII. is the crystal drawn in *C* projection in the customary position. The twinning plane $\{02\bar{2}1\}$ is shown shaded.

When the scalenohedron has been constructed according to the directions given on Plate XXVI.

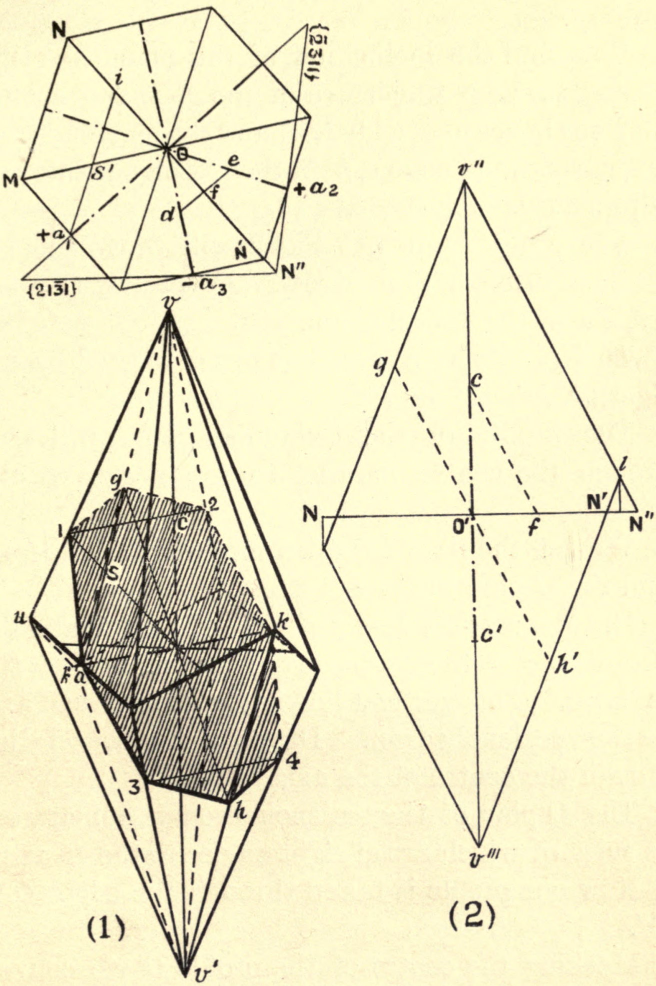



**PLATE XLII.**

Hexagonal System.

Calcite, $c$ axis = ·85.

Fig. 1. + Scalenohedron {21$\bar{3}$1}, showing Twinning Plane parallel to Face of − Rhombohedron {02$\bar{2}$1}.

Fig. 2. Profile.

we may find the intersections of the twinning plane thus :—

First find the inclination of the plane. A line passing through the centre of the axes and cutting the front lower long edge $k$, $v'$, and the corresponding upper back long edge $v$, $k'$ in their centre points $g$, $h$ will give this inclination.

The profile shown at Fig. 2 will prove this ; it is taken through the edges $NON'$ of the $O$ plan ; $k$, $v$, $k'v'$ of the $C$ projection.

This profile is a parallelogram, formed by the edges $vk$, $kv'$, $vk'$ and $k'v'$.

The profile of the twinning plane will pass through the centre parallel to the edges $vk$ and $k'v'$.

We get the data for drawing the parallelogram from the $O$ plan thus :—

Draw a vertical line $v''v'''$ (see Fig. 2), make the portion $cO'c'$ = the $c$ axis, make the line $v''v'''$ = the $c$ axis $\times$ 3. This vertical line is the geometrical axis of the scalenohedron. Draw a horizontal line through the centre of the axis.

The $O$ plan of the scalenohedron is a hexagon, the sides of which are at right angles to the $H$ axes.

Now our profile is taken through the edges $ON$, $ON'$.

Marking the centre of the profile $O'$ we convey the measurements $ON$, $ON'$ to the horizontal line through $O'$.

Next find $N''$. This is obtained by producing the $H$ traces of the planes $\{21\bar{3}1\}$ and $\{\bar{2}3\bar{1}1\}$ on the $O$ plan. They meet in point $N''$. $ON''$ is therefore the intersection edge of those planes on the $O$

plan. Convey point $N''$ to the profile. Join $N''$ to $v''$ for the edge $v, k$.

A vertical raised through point $N'$ to cut $v''N''$ will give point $l$, the point where the upper and lower edges meet; the parallelogram $v''Nv'''N''$ can now be completed.

The next step is to find the line $cf'$, the line of inclination of the twinning plane; to do this we refer again to the $O$ plan.

In accordance with the symbol $\{02\bar{2}1\}$ of the twinning plane we draw the line $de$ on the $O$ plan, joining the $\frac{1}{2}$ points of the $-a_3$ and $+a_2$ axes. $de$ cuts the line $NO$ in $f$. Convey $Of$ to the profile in $O'f$, $f$ joined to $c$ gives us the line of inclination of the twinning plane in profile, which is parallel to the edge $v''l$.

For since $de$ cuts the line $ON''$ at the $\frac{1}{3}$ point, and $O'c$ in accordance with the symbols of the planes is $\frac{1}{3}$ $O'v''$, the triangles $O'cf$ and $O'v''N''$ are similar.

The line $g, h'$ drawn parallel to the line of inclination is the profile of the twinning plane in position and since it passes through the centre of the parallelogram $Nv''N''v'''$, parallel to the sides, it must bisect the sides $Nv''$ and $N''v'''$.

Having ascertained their position on the edges of the scalenohedron, from the profile we can at once mark the two points $g, h$ in $C$ projection.

We have to find six other points where the twinning plane meets the edges. We have two more in the ends of the $a_1$ axis, since the plane passes through that axis. The other four points must lie somewhere on the edges of the scalenohedron between the points already obtained.

They are easily found. Take the one on the edge $u$, $v$, we get it thus: join the + end of the $a_1$ axis to $g$, bisect the joining line and draw through the centre of the scalenohedron and the bisection a line, producing it to cut $u$, $v$ in 1; 1 is another of the required points, the others can be found by "parallelism"; a line from 1 parallel to the $a_1$ axis will cut the edge corresponding to $u$, $v$ in 2, etc.

The explanation of the method used for finding point 1 is as follows:—

Point $i$ on the $O$ plan is the plan of $g$, the point where the twinning plane meets the edge $vk'$. Join $i$ to the + end of the $a_1$ axis. The bisection of $i$, + $a_1$, point $S'$, falls somewhere vertically below the edge $MO$, *i.e.*, $u$, $v$ in $C$ projection. If now we imagine a vertical plane passed through that edge it will contain the centre point $O$, the point of bisection $S$, and consequently a line passed through the points $O$ and $S$, which line will also lie in the twinning plane, since $O$ and $S$ are in that plane; clearly then where such a line meets the edge will be a point where the twinning plane cuts the edge as shown at point 1 in $C$ projection.

To construct the crystal twinned about this plane is the next problem.

But this position of the scalenohedron is not suitable for showing the twinned form well. It will be better to rotate the crystal a little to the right until the $a_1$ axis takes the position of the $a_1$ axis of the isometric system (Plate XLIII. Fig. 1).

In Fig. 1 this has been done by putting the $O$ plan with the $a_1$ axis in the desired position.

Draw the $C$ projection in the new position,

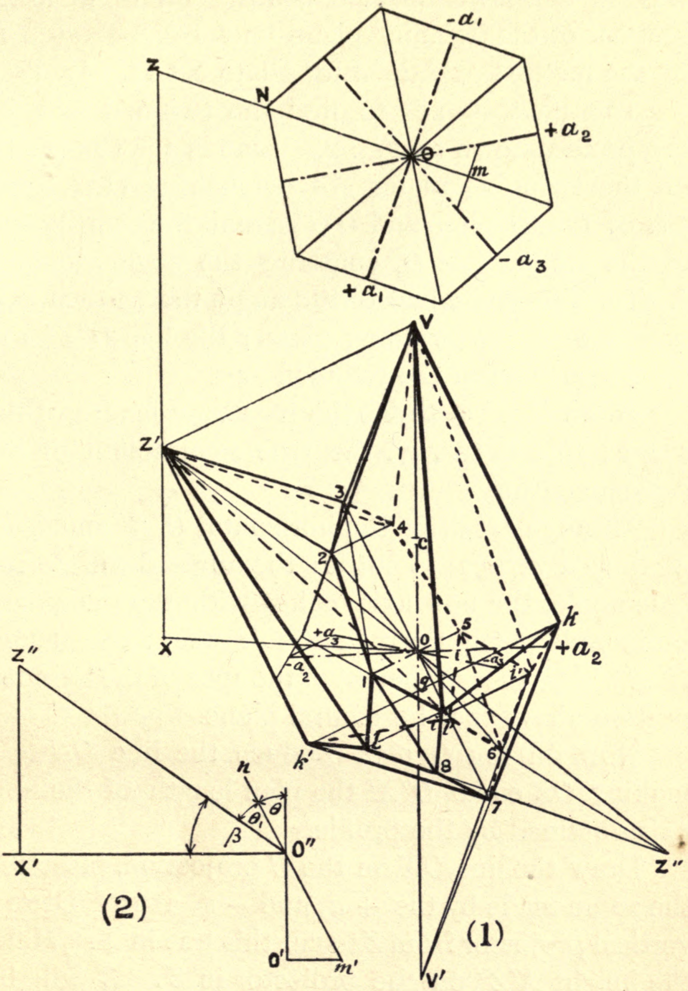



PLATE XLIII.

HEXAGONAL SYSTEM.

Calcite *c* axis = ·85.

FIG. 1. + Scalenohedron $\{21\bar{3}1\}$, Twinning Plane – Rhombohedron $\{02\bar{2}1\}$.

FIG. 2. Profile for finding *c* axis of Twinned Portion.

leaving the lower half in pencil. Find the intersections of the twinning plane, lines 1-2, 3-4, 5-6, 7-8, by the method explained for Plate XLII. Fig. 1.

The next step is to find the twinned axis by the profile shown at Fig. 2. Line $m'n$ is the profile of the twinning plane, got by making $O'm'$ equal $Om$ of the $O$ plan and $O'O''$ equal the semi-height of the $c$ axis. At $O''$ construct the angle $\theta'$, equal $\theta$, that is to say, equal to the angle the vertical axis makes with the twinning plane; the line $O''Z''$ will be the position of the twinned axis.

Make $O''Z''$ = thrice the true semi-length of the $c$ axis, that is to say, the true semi-height of the scalenohedron.

Through $O''$ draw the horizontal $O''X'$ and from $Z''$ drop a vertical to cut it; the angle $\beta$ will be the true angle, the new axis makes with the imaginary horizontal plane, through the $H$ axes; the height $X'Z''$ will be the true height of the apex of the twinned portion above that horizontal plane.

Now on the $O$ plan produce the line $ON$ to $Z$ making $OZ$ equal $O''X'$ the plan length of the new axis obtained by the profile.

Draw the line $OX$ on the $C$ projection, bisecting the angle made by the $+a_3$ and $-a_2$ axes. Drop a vertical projector from $Z$ to cut this line in $X$. Mark the height $X'Z''$ on the projector in $Z'$. $Z'$ will be the apex of the twinned portion of the crystal in $C$ projection.

To the apex thus found we can at once join the points 2, 3, 4.

Carry the new axis beyond $O$ to the right and mark off the distance $OZ'''$ from the other end.

To find point $k'$ of the twinned portion, join the apex $Z'$ of the twinned portion to the apex $V$ of the other part; then from point $k$ draw a line parallel to that joining the apices and make the distance from $k'$ to $q$, the point where this parallel cuts the twinning plane, equal $qk$. Join $k'$ to point 7.

To get points $l$ and $l'$, through points 6 and 8, the points where the twinning plane meets scalenohedron edges, draw lines from the apex $Z''$ to meet lines drawn from $i$ and $i$ parallel to that joining the apices.

The figure is now complete.

Plate XLIV. shows twinning in the monoclinic and triclinic systems. Fig. 1 is a crystal of gypsum, axes ·69 : 1 : ·4, $\beta$ angle 84° 42′, with the twinning plane parallel to the vertical axis. The plane is shown shaded.

Fig. 2 shows the twinned crystal, composed of unit prism {110}, clinopinacoid {010} and unit pyramid {111}.

Find the axes according to the construction given for the monoclinic system (see Plate XXXIII.), showing the position of point $e$, the level of the + end of the $a$ axis if it were not inclined.

Draw the crystal in pencil, finding the points $g$, $h$, $i$, $l$ through which the vertical intersections of the prism {110} and the clinopinacoid {010} pass.

The front portion is the twinned portion. From point $f$, where the front unit pyramid edge meets the back edge of the prism, draw a line parallel *not* to the $a$ axis $Oa$, but to line $Oe$, the position of the axis if it were horizontal. Where this line cuts the front edge of the prism in $f'$ will be the twinned position of $f$.

Lines parallel to $ff'$ from $n$, $p$, $m$ will find the corresponding twinned points $n'$ $p'$ $m'$. The edges may then be drawn by "parallelism".

Fig. 4 is a triclinic crystal, labradorite, axial ratio ·63 : 1 : ·55, angles $\alpha = 93° 23'$, $\beta = 116° 29'$, $\gamma = 89° 59'$.

The twinning plane shown shaded is $\{010\}$, parallel to the brachypinacoid.

The figure is a combination of brachypinacoid $\{010\}$, base $\{001\}$, macrodome $\{20\bar{1}\}$ and unit prism $\{1\bar{1}0\}$.

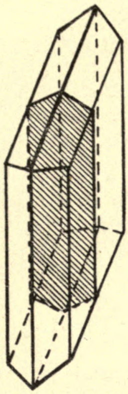
Having found the axes according to the construction for the triclinic system (see Plate XXXVII.), the intersection $d$, $e$ of unit prism $\{1\bar{1}0\}$ and base is drawn parallel to the line joining the + end of the $a$ and − end of the $b$ axes.

The intersection of prism and pinacoid $d$, $f$ is a vertical line. Joining $e$ to $f$, forms the intersection edge between macrodome $\{20\bar{1}\}$ and prism, which if correct will be parallel to line $k$, $k'$, the intersection of the planes $\{1\bar{1}0\}$ and $\{20\bar{1}\}$, found by drawing the line $n$, $k$ parallel to the $b$ axis through the $\frac{1}{2}$ point of the + $a$ axis, to cut line $a$, $b$, from the + end of the $a$ to the − end of the $b$ axes in $k$.

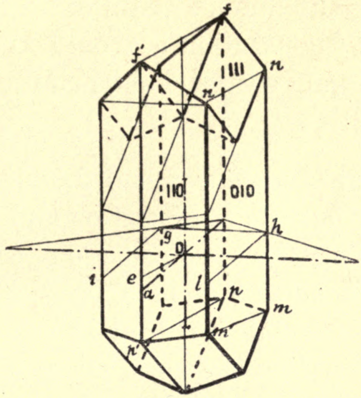
$k$, $n$ and $a$, $b$ are the basal traces respectively of the planes $\{1\bar{1}0\}$ and $\{20\bar{1}\}$. $k'$ is found by drawing $b$, $k'$ a vertical through the − end of the $b$ axis to cut $k'n$, a line parallel to the $b$ axis through the − end of the $c$ axis.

These last lines are respectively the vertical traces of the planes $\{1\bar{1}0\}$ and $\{20\bar{1}\}$ on the axial plane through the $b$ and $c$ axes; $k$, $k'$ is their intersection.

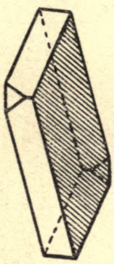
The twinned portion is simple of construction.



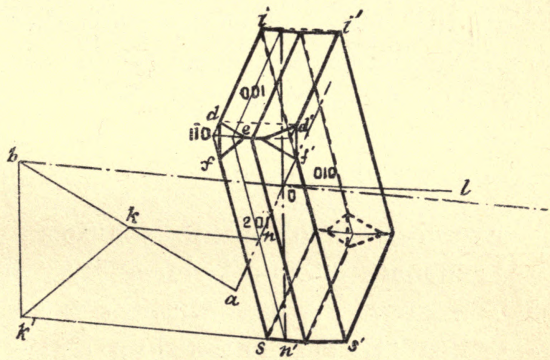
(1)




(2)



(3)




(4)

PLATE XLIV.

MONOCLINIC AND TRICLINIC SYSTEMS.

FIG. 1. Gypsum. Axial Ratio ·69 : 1 : ·4 ; $\beta$ 80° 42′ ; showing Twinning Plane, Orthopinacoid {100}.

FIG. 2. Twinned Crystal.

FIG. 3. Labradorite. Axial Ratio ·63 : 1 : ·55 ; $a$ 93° 23′ ; $\beta$ 116° 29′ ; $\gamma$ 89° 59′ ; showing Twinning Plane, Pinacoid {010.

FIG. 4. Twinned Crystal.

Point $i'$, the corresponding point to $i$, is found by drawing the line $i$, $i'$ parallel *not* to the $b$ axis but to line $Ol$, the position it would take if not inclined. All similarly corresponding points are found in like manner, as $d'f's'$.

If the student has had the patient perseverance to work out the construction and follow the reasoning in all the problems on crystal forms here given, he should be in a position—with the aid of careful thought—to draw in $C$ projection any form or combination of forms, he, as a mineralogist, may wish to represent, supposing he can obtain the necessary data forangles and measurements.

On the other hand, should he only wish to follow the subject of crystal projection far enough to enable him to pass school examinations, he need only concern himself with the more common forms, the constructions for which, he may easily master and commit to memory.

Exercises—Problems for solution. Twins.

Isometric system—Twinned Pyritohedron.

Tetragonal system—Rutile $c$ axis = ·64 ; {110} ; {100} ; {111}, twinned on {101}.

Tetragonal system—Rutile, {110} ; {100} ; {111} ; {101}, twinned on {301}.

Hexagonal system—Calcite $c$ axis = ·85 ; $\{10\bar{1}1\}$, twinned on $\{01\bar{1}2\}$.

Monoclinic system — Hornblende, axial ratio ·55 : 1 : ·29 ; $\beta$ 73° 58′ ; {110} ; {010} ; {011} ; $\{\bar{1}01\}$, twinned on {100}.

THE ABERDEEN UNIVERSITY PRESS LIMITED