XII. *A Letter from Mr.* John Robertſon, *Lib. R. S. to* James Weſt, *Eſq; Preſident of the Royal Society; containing the Demonſtration of a Law of Motion, in the Caſe of a Body deflected by two Forces tending conſtantly to two fixed Points.*

SIR,

Read April 6, 1769.

THE late Mr. Machin (who was, for many years, ſecretary to the Royal Society, and Greſham profeſſor of aſtronomy) gave to the editor of the Engliſh edition of Sir Iſaac Newton's Principia, publiſhed in the year 1729, a tract entitled, " The Laws of the Moon's Motion " according to Gravity," which was annexed to that impreſſion: Mr. Machin, in the Poſtſcript to that tract, after apologizing for not mentioning the fundamental principles of the demonſtration of the propoſitions relating to the Moon's motions, ſays, " Some " of which, I am apt to think, cannot eaſily be " proved to be either true or falſe, by any methods " which are now in common uſe."

One of theſe principles he gives in the following words:

" There is a law of motion which holds in the " caſe where a body is deflected by two forces, " tending conſtantly to two fixed points.

" Which

" Which is, *That the body, in ſuch a caſe, will* " *deſcribe, by lines drawn from the two fixed points,* " *equal ſolids in equal times, about the line joining the* " *ſaid fixed points.*

And (after obſerving that Sir Iſaac Newton has proved, that Kepler's law of bodies deſcribing equal areas in equal times about the centers of their revolution, cannot hold, whenever the body has a gravity or force to any other than one and the ſame point) further ſays, " there ſeems to be wanting ſome ſuch " law as I have here laid down, that may ſerve to " explain the motions of the Moon and Satellites, " which have a gravity towards two different centers."

About the year 1742, diſcourſing with that eminent mathematician, the late William Jones, Eſq; F. R. S. on the above-mentioned law, he ſhewed me its demonſtration, and permitted me to take a copy thereof; and as I conceive it to be highly worth preſerving, I now offer it to your conſideration, about giving it a place in the Philoſophical Tranſactions. I am,

SIR,

Your moſt humble ſervant,

February 27,
1769.

J. Robertſon.

TAB. IV.

FIG. I.

PROPOSITION.

IF a body (P), projected in a given direction, be constantly drawn towards two fixed points (S and T), which are not both in the same plane with the direction, the triangle (SPT), formed by right lines drawn from the body (P) to those fixed points (S and T), shall describe equal solids (STPP', STP'P''), in equal times, about the right line (ST) joining the said points.

FIG. 2. For, suppose a body projected in the direction PP', and acted upon by two centripetal forces towards the fixed points S and T; the angles P'PS, P'PT lying in different planes. Let the time be divided into equal moments.

In the first moment, suppose the body, by its given force, should move along the line PP'; and in the second moment, if no new force was added, it should continue to move in the same right line along P'*p*=PP'; but when the body has come to P', suppose it acted upon by the two centripetal forces, in the directions P'T, P'S; and let those forces be in proportion to that in the direction PP', as the lines P'*t*, P'*s* to the line P'*p* (=P'P).

With these three right lines P'*p*, P'*t*, P'*s*, complete the parallelopipid P'P''; and the body in P', being acted upon by these three forces, in the directions P'*p*,

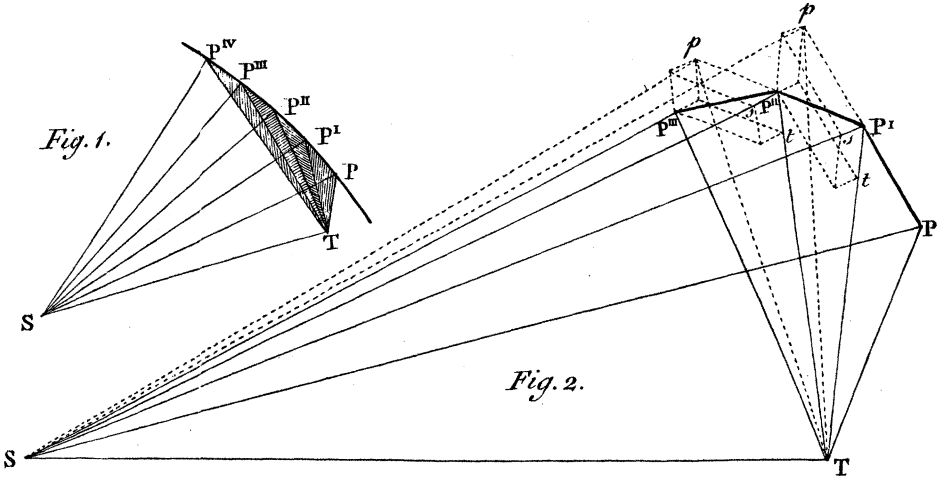
Fig. 1.
P^IV
P^III
P^II
P^I
P
T
S
Fig. 2.
p
p
P^III
P^II
P^I
t
t
P
S
T

$P'p$, $P't$, $P's$, which forces being as theſe three lines, ſhall move along the diagonal of the parallelopipid made by theſe three lines; ſo that, in the ſecond moment of time, the body, inſtead of moving from P' to p, ſhall move from P' to P''.

Draw the lines SP, SP'' and TP', TP'', as alſo Sp, Tp.

Now, the ſolid $STPP'$ = ſolid $STP'p$; for they ſtand upon equal baſes $TP'P$, $TP'p$, and have one common vertex S, or their common altitude is the perpendicular drawn from S to the plane PTp.

And the ſolid $STP'P''$ = ſolid $STP'p$; for they ſtand upon the ſame baſe STP', and lie between the ſame parallel planes pP'', st.

Therefore the ſolid $STPP'$ = ſolid $STP'P''$.

In like manner, in the third moment of time, the body at P'' being acted upon by three forces, in the directions $P'P''$, $P''S$, $P''T$, ſhall move along the line $P''P'''$, ſo as to make the ſolid $STP''P'''$ = ſolid $STP'P''$; and ſo in all ſucceeding equal moments of time, the triangle formed by right lines drawn from the body to the two fixed points S, T, ſhall conſtantly deſcribe little ſolids, each equal to the ſolid $STPP'$.

Therefore, the moments of the ſolids being proportional to the moments of the time in which they are deſcribed; the ſolid itſelf is proportional to the time in which it is deſcribed. Q. E. D.

Some difficulties may, perhaps, ſeem to ariſe upon a ſlight view of only particular caſes of this propoſition; but, it is conceived, all ſuch muſt vaniſh, when the ſame is thoroughly conſidered.

For, as in two bodies T and S; if T is acted upon by S, ſo as to deſcribe a right line, that is, if T falls directly upon S, no area can then be described by the right line connecting T and S; but yet, this is certainly one of the cases whereby S and T may possibly act upon each other.

So in three bodies, S, T, and P; if P moves in the same plane with S and T, no solid can then be described by the plane whose right lined sides are the lines connecting P to T and S; but yet, this must be one of the cases whereby S, T, and P, may possibly act upon each other.