NEWCOMB — PROBABILITY OF CAUSES OF THE PRODUCTION OF SEX IN HUMAN OFFSPRING

A STATISTICAL INQUIRY

INTO

The Probability of Causes of the Production of Sex in Human Offspring

BY

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WASHINGTON, U. S. A.
PUBLISHED BY THE CARNEGIE INSTITUTION OF WASHINGTON
JUNE, 1904

CARNEGIE INSTITUTION OF WASHINGTON

PUBLICATION No. 11

The Lord Baltimore Press
THE FRIEDENWALD COMPANY
BALTIMORE, MD., U. S. A.

PREFATORY NOTE.

The present paper is an attempt to apply a rigorous theory of probable inference to a question of genetic biology, taking statistical data as the basis of the inquiry. If, in making such an investigation, the author may seem to stray outside his professional field, he would reply that the discussion of a biological question was by no means the sole object with which the work has been undertaken. It has appeared to him that the treatment of statistical data generally on a large scale, by the rigorous methods of probable induction, leads one into a field the cultivation of which promises important results to the science of the future; and he hopes the work will show how it is possible by such methods to reach conclusions on questions which elude all direct investigation.

The author has to acknowledge his indebtedness to the Trustees of the Bache fund, who made him a grant to pay the expenses of the necessary examination of genealogical data, and to the Census Bureau, through Mr. W. C. Hunt, chief statistician for population, who supplied statistical data relating to several thousand families culled from the Census records.

WASHINGTON, *November, 1903.*

A STATISTICAL INQUIRY INTO THE PROBABILITY OF CAUSES OF THE PRODUCTION OF SEX IN HUMAN OFFSPRING.

By Simon Newcomb.

1. INTRODUCTORY.

The object of the present investigation is to make an application of certain hitherto undeveloped statistical methods to a class of cases in which the causes elude all direct inquiry. The subject of such methods, although its general principles are well understood, is one which is susceptible of being worked out in detail to a far greater extent than has yet been seriously attempted. The present paper may therefore be regarded as having a double object: one to illustrate and apply certain methods; the other to draw conclusions on a physiological question of the widest scientific and human interest.

The idea that the sex of offspring may depend, to a greater or less extent, on discoverable causes, perhaps even on causes within the control of the parents, is a very natural one. Examples of what might be possible causes are the respective ages of the parents, their vigor or condition of health, their conjugal habits, which may be infinitely varied, or the relation of the time of conception to the periods of menstruation. Besides this, it is possible that some parents, male or female, may be endued with some special faculty for producing children of one sex rather than another, owing to constitutional or other causes which, not being apparent on the surface, might entirely elude direct investigation.

The method of investigating this tendency in the case of recognizable possible causes is well known. The question whether parents having a certain characteristic, A, are more likely than parents of the class not-A to have children of either sex is determined by counting a sufficient number of the offspring of parents of each class and comparing the ratio of male and female children in each class with the average ratio. If we find this ratio to be markedly different in the two classes, we conclude that the characteristic A is associated with some cause having a definite effect upon the production of sex.

Although the author proposes to apply this simple and obvious method to certain cases in the following investigations, his main object is to go farther, and inquire whether there are any causes or conditions whatever, known or unknown, which, in a decided degree, affect the production of sex. When we see one family consisting mainly or wholly of male children, and another consisting mainly or wholly of female children, it is very natural to suspect that the excess, in each case, may be due to some characteristic or faculty of the parents, or some peculiarity of their constitution, which may or may not admit of discovery and investigation. The mere fact of this inequality, taken in itself, does not, however, prove anything, because it may be the natural result of those accidents which determine sex, but of which we know nothing. Granting the existence of constitutional or other tendencies of the kind supposed, we may apply the term *unisexual* to them. We may then make the hypothesis that there is something in the constitution or habits of parents which results in some having a unisexual tendency toward the production of male children, and others toward the production of females, which tendencies nevertheless elude investigation otherwise than by statistical investigation of their effects. The desideratum is to discover a criterion by which we may distinguish between inequalities in the division of a family between the two sexes which are simply the result of chance and those which are the result of a unisexual tendency on the part of the parents. Such a criterion is pointed out in the first four sections of the present paper, and its mathematical theory developed in the Appendix.

2. THE PREPONDERANCE OF MALE BIRTHS.

Certain facts preliminary to this inquiry may be set forth which will serve as points of comparison in coordinating our conclusions. The first of these is the well-known general fact that, in the entire Semitic race, there is a small but well-marked preponderance of male over female births. This preponderance is remarkably uniform in all European and American countries where complete statistics of births are available. Mulhall finds the ratio to be 1052 male to 1000 female births in Europe generally. For our present purpose it will be convenient to express the excess in a different form from this. The result just cited may be expressed by saying that, out of 2052 births, 1052 are males and 1000 females. We may also say that 51.3 per cent are males and 48.7 per cent are females. Altogether, it seems to the writer that the best expression for the sexes is the excess of male over female in 100 births. Since Mulhall's conclusion implies that in 100 births 51.3 per cent are males and 48.7 per cent are females, the number expressing the excess would be 2.6 per cent. The excess thus expressed is represented by the symbol E_m. That is, we put $E_m =$ the excess of male over female births in a total of 100 births.

The slight variations in this excess found in different countries are no greater than may be the result of accident. Although its value is so nearly the same for different countries, there are still some circumstances to be considered in connection with this definition. The most important of these is that it is decidedly larger when still-births are included. In France the male excess of such births is between 4 and 5 per cent of the whole number, a proportion which is probably substantially the same in most European countries. If we desire E_m to express the physiological probability of the production of a male child, still-births should be included. We should then have, in France,

$$E_m = 2.93.$$

Although this expresses the proper physiological probability of the production of male offspring, it will better conduce to our present operations to consider only living births. For these we may take, as a normal ratio, Mulhall's value, which will give

$$E_m = 2.6$$

From the statistics of Massachusetts the ratios are found to be:

Living children, $E_m = 2.8$ All births, $E_m = 3.3$

It appears, therefore, that there is no material difference on the two sides of the Atlantic.

3. IS THE RATIO OF MALE TO FEMALE BIRTHS THE SAME IN ALL RACES?

So far as the author is aware the only races besides the Semitic for which we have statistical data on which to base a conclusion are the Mongolian race of Japan and the negro race in America. In the latter we have only a limited registration of births, but sufficient to at least point to a conclusion. The census of 1900 gives a registration record, and a total record of births of children of the colored race as follows:

Births.	Registration.	Total.
Male	13,526	136,350
Female	13,244	136,625

The registration record shows a preponderance of males less than one-half that of the Semitic race: the census enumeration an actual preponderance of female births.

The census tables also give the number of negro children of each of certain ages at a number of very early ages.

From the census of 1900 we find negro children:

Age.	Male.	Female.
Under one month	10,200	10,322
Under three months	32,840	33,353
Under one year	121,329	123,181

It is difficult to conceive of any cause why the data given to a census taker should be systematically in error as to sex. The number under the age of one month can be nothing else than the number born during that month, less the deaths during the month. If, then, the preponderance of male deaths is no greater in the negro than in the white race, it would seem that an excess of male births does not characterize the negro race, but rather the contrary. Moreover, the uniformity of the excess throughout the first year seems to confirm this conclusion. I find it also to be confirmed by all the statistics of previous censuses since 1870. There is, in general, in all these cases, a slight excess of female negro or colored children under one month of age, which is greater than would be due to the excess of male over female deaths during the early period of life.

On the other hand, the recent Japanese census shows, in a number of births exceeding one million, an excess of males practically the same as in European countries. It would seem therefore that there is no difference in this respect between the Semitic and the Mongolian races.

As the numbers relating to the negro race in America are not beyond the possibility of doubt, and especially as those of actual births registered show a male excess, the suspicion, sometimes expressed, that this excess may run through the whole order of mammals is at least worthy of examination. But the statistics of horses kept with much detail in England and Germany show, on the whole, an almost equal division between the sexes, the general tendency being toward a preponderancy on the female side.

It is also a curious fact that in European countries where complete statistics are available, the excess of male births is smaller for illegitimate than for legitimate children. The problem of explaining this difference, which we can scarcely believe to be real, is one which the writer must leave to others.

4. INQUIRY WHETHER ANY UNISEXUAL TENDENCY, PERMANENT IN THE INDIVIDUAL IN EITHER DIRECTION, EXISTS AMONG PARENTS.

The well-known fact being that inequalities in the proportion of the two sexes are almost the universal rule, some families consisting mainly or entirely of male children, others mainly or entirely of female children, the question before us is whether these inequalities are simply the result of chance, or show unisexual tendencies on the part of the respective parents. What we want is a criterion for distinguishing between these two cases. To make clear the principle of the proposed criterion, we begin with an illustration of its application. Let us suppose that we select at random 100 families of two children each. Granting that this selection corresponds to the general average, and leaving out of consideration the small preponderance of male births, we shall have, in these families, 50 cases in which the first-born was a male, and 50 in which it was a female. Of these two sets of 50 each, if there is no unisexual tendency, the second child will be a male in one-half, or 25 cases, and a female in the other cases. If there is no tendency of the kind sought, the final result will be:

25 families of 2 females each;
25 families of 2 males each;
50 of 1 male and 1 female.

That is to say, in the great mass the number of families comprising two children of the same sex will be the same as that of families comprising two children of opposite sexes.

Now let us suppose that, in consequence of some cause not known in advance, some parents have a tendency toward the production of male and others toward the production of female children. To fix the ideas, let us suppose that in the case of one-half of the parents, which we call Class A, there is a probability of three-fifths in favor of a male child, and in the remaining half, called Class B, a similar preponderance in favor of a female child. The method presupposes that we have no clue to a decision as to which of these classes any given parents belong. The first-born will still be a male in one-half and a female in the other half of the cases. But, in case of the second child, the 50 parents of Class A will have 30 male and only 20 female children. The 50 of Class B will have 20 male and 30 female children. The probable distribution of children between the two classes will be as follows: Of the 50 parents of Class A, 30 will have male first-born children, and 20 will have female first-born. The preponderance being the same in the cases of the second child, the 30 males will be followed by 18 males and 12 females. The 20 females will be followed by 12 males and 8 females.

The numbers will be the same for Class B, only substituting female for male preponderance. Thus the total outcome will be:

	Class A	Class B	Total
2 males	18	8	26
Male-female	12	12	24
Female-male	12	12	24
2 females	8	18	26

That is, in 100 families of 2 children each, we shall have 52 with children of one sex, and 48 with children of different sexes, instead of 50 of each class.

The result will be yet more decisive when we consider families of a greater number of children. Whatever the number of the family, the theory of probabilities and combinations gives a certain distribution of male and female children, which would be the most likely result of pure chance. For example, in families of 3, the most likely chance distribution would be three cases in which the children were of different sexes for one in which all three were of the same sex. But, were there any tendency among special parents to a production of children of one sex, the proportion of families in which all three were of the same sex would be greater than that given by the law of chance distribution. To show the preponderance, let us suppose the tendency to be the same as in the preceding example, and the number of families of 3 children each to be 500, or 250 of each class. The result will be:

	Class A	Class B	Total
3 males	54	16	70
2 males, 1 female	108	72	180
1 female, 2 males	72	108	180
3 females	16	54	70

Assuming no unisexual tendency on the part of parents, the probable result would be a proportion of three families not all of the same sex to one of the same sex. The results would then compare thus:

Families	Actual	Probable
3 children of 1 sex	140	125
3 children of 2 sexes	360	375

The result is an excess of 15 unisexual families.

Let us now pass on to the general problem of which the preceding examples are special cases. We have cited the well-known fact of a general or average unisexual tendency in the male direction among parents of the Semitic race. The question before us is whether this tendency of this race is the same among all parents. What we know to start with is that, if some parents have a tendency greater than the normal to produce male children, then there must be a corresponding tendency among other parents to produce female children. It is this combined tendency toward the production of children of one sex in some cases and the other sex in other cases that, for the present purpose, I term unisexual.

The data for the investigation in question have been derived from two sources. Mr. Hunt, chief of the Division of Population Statistics in the Census Office, very courteously made for me a count of 2000 families in which the parents were of various nationalities, enumerated in the census of 1900. I have also had counts made from a genealogy including all the known families descended from one Andrew Newcomb, who died about the year 1650, and which served a valuable purpose as probably including a wider range of conditions than those affecting the families enumerated in the Census. This was further extended to include a great number of other family genealogies. The entire list may therefore be taken to include the widest possible range of ordinary conditions which might affect the sex of offspring.

In the following summary of families, the first column of figures gives numbers for the white families as supplied by the Census Office. The second gives the corresponding numbers for families taken from the genealogies. The third and fourth columns give the data for the negro and Indian families, as supplied by the Census. The fifth gives the sum for all the families.

This is followed by the probable numbers given by the theory of chances, in case that there is no unisexual tendency among parents.

In each column the first line is the total number of families, the second the number of those families of which the children are all of the same sex, whether male or female. The following lines give the number of bisexual families of each class, each division of the number between the two classes being combined. For example, in families of 4 children, the line marked 3 and 1 gives the combined number of families comprising 3 males and 1 female, together with those comprising 1 male and 3 females. The totality of the families enumerated is too small to give any value to the separate enumeration of males and females. A combination is therefore made in order to reduce the results to the smallest number of distinct data. Thus the reader can see at a glance to what extent, if any, a bisexual tendency can be found in the families enumerated.

FAMILIES OF 2 CHILDREN.

Class	White Census	White Genealogy	Negro	Indian	Total	Probable
Number of families	670	1051	56	6	1783	
Same sex	322	547	30	2	901	892
Opposite sexes	348	504	26	4	882	892

In the case of the Census families, there are fewer pairs of children of the same than of opposite sexes, which result is the opposite of that of a unisexual tendency. In the case of the genealogical families, the excess is in the unisexual direction, but is in part due to an excess of male offspring recorded in the genealogies.

FAMILIES OF 3 CHILDREN.

	White Census	White Genealogy	Negro	Indian	Total	Probable
Number	436	914	38	4	1392	
Same sex	112	219	13	2	346	348
Different sexes	324	695	25	2	1046	1044

In a chance distribution the unisexual families should be one-fourth the entire number. The actual number is slightly below this, so that no unisexual tendency is shown.

FAMILIES OF 4 CHILDREN.

	White Census	White Genealogy	Negro	Indian	Total	Probable
Number	322	729	24	5	1080	
4 of same sex	40	110	4	0	154	135
3 and 1	174	336	14	3	527	540
2 and 2	108	283	6	2	399	405

In a chance distribution the numbers of the three classes should be in the proportion 1 : 4 : 3. The actual number of unisexual families in the entire list is 19 in excess of the probable number. This excess is, however, no greater than might well be the result of chance. The number of families having 3 out of 4 children of the same sex shows the opposite of a unisexual tendency.

FAMILIES OF 5 CHILDREN.

	White Census	White Genealogy	Negro	Indian	Total	Probable
Number of families	191	620	18	3	832	
5 of same sex	10	49	1	0	60	52
4 and 1	56	177	4	1	238	260
3 and 2	125	394	13	2	534	520

Here again there is an excess of 8 unisexual families over the normal number of 52. The effect of a unisexual tendency would be to produce a number smaller than the probable one of families consisting of 3 children of one sex and 2 of another, but there is an excess in the case of these families. We can, therefore, only attribute the deviation to chance.

FAMILIES OF 6 CHILDREN.

	White Census	White Genealogy	Negro	Indian	Total	Probable
Number of families	94	626	8	2	730	
6 of same sex	6	16	0	0	22	23
5 and 1	18	141	1	2	162	137
4 and 2	43	287	6	0	336	342
3 and 3	27	182	1	0	210	228

The deviations from the normal are in all cases rather less than we might expect as a result of chance deviation.

FAMILIES OF 7 CHILDREN.

	White Census	White Genealogy	Negro	Indian	Total	Probable
Number of families	62	517	10	0	589	
7 of one sex	1	11	0	0	12	9
6 and 1	4	59	0	0	63	65
5 and 2	23	165	3	0	191	193
4 and 3	34	282	7	0	323	322

FAMILIES OF 8 CHILDREN.

Number of families	24	417	6	0	447	
8 of one sex	0	3	1	0	4	3
7 and 1	1	27	0	0	28	28
6 and 2	7	85	3	0	95	98
5 and 3	12	185	1	0	198	196
4 and 4	4	117	1	0	122	122

FAMILIES OF 9 CHILDREN.

Number of families	8	306	3	0	317	
9 of one sex	0	1	0	0	1	1
8 and 1	1	9	0	0	10	11
7 and 2	0	52	1	0	53	45
6 and 3	3	99	0	0	102	104
5 and 4	4	145	2	0	151	156

FAMILIES OF 10 CHILDREN.

Number of families	5	295	0	0	300	
10 of 1 sex	0	0	0	0	0	0.6
9 and 1	0	7	0	0	7	5.9
8 and 2	1	37	0	0	38	26.3
7 and 3	3	67	0	0	70	70.3
6 and 4	0	117	0	0	117	123.0
5 and 5	1	67	0	0	68	73.9

Families of 11 to 16 Children.

	Geneal.	Prob.
209 families of 11 children:		
11 of one sex	0	0.2
10 and 1	6	2.2
9 and 2	16	11.2
8 and 3	32	33.7
7 and 4	68	67.3
6 and 5	87	94.2
124 families of 12 children:		
12 of one sex	0	0.1
11 and 1	2	0.7
10 and 2	10	4.1
9 and 3	18	13.3
8 and 4	39	30.0
7 and 5	35	48.0
6 and 6	20	28.0
50 families of 13 children:		
13 of one sex	0	0.0
13 and 1	0	0.2
11 and 2	1	0.9
10 and 3	4	3.4
9 and 4	9	8.7
8 and 5	15	15.7
7 and 6	21	21.0
25 families of 14 children:		
14 of one sex	0	0.0
13 and 1	0	0.0
12 and 2	0	0.3
11 and 3	3	1.1
10 and 4	5	3.0
9 and 5	9	6.1
8 and 6	6	9.1
7 and 7	2	5.1
11 families of 15 children:		
15 of one sex	0	0.0
14 and 1	0	0.0
13 and 2	0	0.1
12 and 3	0	0.3
11 and 4	1	0.9
10 and 5	2	2.0
9 and 6	5	3.4
8 and 7	3	4.4
7 families of 16 children:		
16 of one sex	0	0.0
15 and 1	0	0.0
14 and 2	0	0.0
13 and 3	0	0.1
12 and 4	0	0.4
11 and 5	1	1.0
10 and 6	1	1.7
9 and 7	3	2.4
8 and 8	2	1.4

In the case of families of 7 or more, a unisexual tendency would be shown by a deficiency in the numbers of families with a nearly equal number of males and females. These are given in the last line of each series of families. We find such a deficiency to be actually shown. The numbers are:

Actual:	101, 35, 46, 19, 25, 4, 7, 2
Probable:	107, 39, 51, 23, 23, 7, 5, 1

Actual sum = 239
Probable sum = 256

Deficit........... 17

This deviation is not large enough to base a conclusion upon. It is partly due to a deficit in the reports of female children in the genealogies, the respective recorded numbers of male and female being:

Number of male children	3,339
Number of female children	3,020
Excess of male children	319

It thus appears that about 110 males are reported against 100 females; in other words, 52.5 per cent of the whole number are males. This excess over the normal is to be attributed to the greater difficulty of tracing female than male children, owing to the greater liability of the former to be omitted from a record, especially when they die young.

We shall therefore expect to see a slight unisexual indication in the numbers from this cause alone, so that we may regard the deviation as explained without supposing any actual tendency of the kind sought.

The general result of the count of 2838 families embracing 13,257 children is that the distribution of male and female offspring follows the statistical laws of chance within the limits of probable deviation, the actual deviations being as great in one direction as in the opposite one. Consequently, there are no unisexual tendencies on the part of parents sufficiently great to be of practical importance. We are not, however, justified in concluding from these numbers alone that there can be absolutely no unisexual differences in the human race, nor that no possible conditions are productive of such a difference. Our conclusions only preclude conditions affecting the sex of a child which may occur with a certain frequency. For example, if one-tenth of the parents in the whole list practiced any habit, or possessed any characteristic, which would lead to two-thirds of their children being of one sex, the effect would show in the statistics. But if only a single pair of parents in the whole list had a unisexual tendency, or if, in the great majority of cases, this tendency was exceedingly small, the effect would not be shown.

As to this latter case, the fact of any unisexual tendency, however minute, would be of great scientific interest, but of no practical importance. It would hardly be worth while for any parent or any community to lay great stress on any cause which would result only in increasing the chances of male or female offspring by 3 or 4 per cent. It seems highly improbable that any very rare or highly artificial cause would produce a unisexual tendency, if no ordinary cause produced it. The absence of any strongly marked unisexual tendency in the families we have examined, therefore, justifies us in concluding, at least with a high degree of probability, that the causes of sex are beyond artificial control.

5. THE UNISEXUAL TENDENCY IN MULTIPLE BIRTHS.

We have next to consider a sort of family in which the conditions are peculiarly favorable for drawing conclusions on the general question of the cause of sex. These are families consisting of children of a single birth in twins or triplets. Considering first the case of twins, we begin with the effects which would result on two extreme hypotheses as to the cause of sex.

HYPOTHESIS 1. *The distinction of male and female exists in original germs, antecedent to conception, presumably supplied by the father.*

In this case we should find the same random distribution of twin children between the two sexes that we find to exist in families of two. In four births of twins we should have one of two males, one of two females, and two bisexual. Only in two ways could this conclusion be avoided. One is by supposing that a germ of either sex is more likely than not to have one of the same sex in physical juxtaposition with it. This view seems inadmissible because, even if such juxtaposition did exist in any case at any moment, it would not be permanent. The other supposition is that at certain periods there is an abnormal excess of male germs and at other periods a similar excess of female germs in the same father. In the absence of permanent unisexualism on the part of any one father, which was shown in the preceding section, such an inequality can not be permanent. It therefore seems to me that this supposition is also too artificial and unlikely to be considered. We may therefore consider the statistical distribution of twin children between the sexes to afford a test of the above hypothesis.

HYPOTHESIS II. *Sex is entirely determined by the conditions to which the germ is subject during the early stages of its development.*

All these conditions are the same *ab initio* for the two members of the pair. The result of the hypothesis would therefore be that twin children would always be of the same sex.

The statistics show that neither of these hypotheses is correct taken singly; the actual result being an intermediate one between those of the two hypotheses. A child of one sex is more likely than not to have a twin of the same sex; but there is only a certain preponderance of probability for this.

The negation of the first hypothesis leads to the conclusion that the original germs supplied by the father, if not completely asexual, can at most have no other sexual quality than a slightly greater tendency to develop into one sex than into the other. While the existence of such a tendency is not out of the question, the more likely conclusion would seem to be that the part of the father is completely asexual, and that the determination of sexes is entirely the function of the mother. It is true that this contravenes certain supposed conclusions from statistics which will be considered in the next section. But these seem to me open to misconstruction.

We pass next to the actual numbers shown by the statistics of France and Germany. In the following table the first line shows the number of births giving rise to two males; the second to the number of bisexual births; the third of births of two females. In the next two lines are the total number of male and female children resulting from all the births. The preponderance of males is, on the whole, the normal one, showing that, in the production of twins, there are no causes affecting sex which act in any way differently from those in the ordinary cases. Then is given the normal number of bisexual births as it would have been were the determination of sex, in the case of the two children, completely independent, as required by hypothesis I. The proportional deficiency of the actual number of bisexual births over this probable number may be used to define the unisexual tendency in the case of twins. The observed fact may be set forth thus: The probable proportions of unisexual and bisexual twins, if the sex of each child were determined independently of the other, and the percentage of pairs of the several classes would be:

2 m., 0.260; m. and f., 0.500; 2 f., 0.240,

while the actual proportions are:

2 m., 0.332; m. and f., 0.354; 2 f., 0.314.

Sex of Twins—France and Berlin.

	France 1898-1900	Berlin 1853-1880	Total
2 males	9,537	2,968	12,505
Bisexual	9,826	3,489	13,315
2 females	8,949	2,852	11,801
Total males	28,900	9,425	38,325
Total females	27,724	9,193	36,917
M. to 100 F.	104.2	102.5	103.8
E_m	2.1	1.2	1.9
Normal bisexual	14,149	4,603	18,752
Deficit	4,323	1,114	5,437

The formal discussion, by algebraic methods, of the unisexual tendency implied in these numbers will be found in the Appendix. These methods are not, however, necessary to convey an idea of the principles by which the results are to be explained. What makes a discussion of these principles of interest is that the numbers derived from the statistics of twins may be applied to the case of triplets, and a comparison of the actual statistics of triplets with those derived from the statistics of twins will be of interest.

The processes which we presuppose are these: During an unknown period of time, commencing with the moment of conception, the two germs are exposed to a series of common influences, either in the male or female direction, tending to make them of the same sex. As we can, without appreciable error, make abstraction of the small normal preponderance toward the male sex, we may say that, in the general average, this unisexual tendency will be as often in one direction as in the other. Thus, in one-half the cases, which we term group A, the common influences preponderate in the male direction, and in other cases, which we call group B, in the female direction.

But these preponderating influences do not completely determine the sexes. There are accidental causes operating differently on the two growing organisms, which may result in their becoming of opposite sexes. Numerically stated, the conclusion to which we are led is that the statistics of twins may be explained by supposing that, in Group A there is a probability of 0.77 in favor of either of the organisms taken at random becoming male, and therefore 0.23 in favor of its becoming female; while in group B there are similar probabilities in the opposite direction. This, I say, is a conclusion from the statistics of twins, assuming that there is no interaction between the two organisms tending to make them of the same sex. I may remark, however, that this assumption in the case of twins would have no special significance. The combination of probabilities would lead to the same result whether we supposed it or not. The main point is that there is some preponderating tendency of the pair of organisms towards one sex in some cases and the opposite sex in the remaining cases. Stating probable results in percentages, they would be:

In group A, probability of 2 males..........$0.77 = 59.3$ per cent.
In group A, probability of 2 females........$0.23 = 5.3$ per cent.

Total unisexual percentage....................64.6
Bisexual 35.4

In group B the results would be the same, interchanging male and female.

These numbers, it will be noticed, show the percentages actually given by the statistics. The mathematical method developed in the Appendix shows that the results may be accounted for by assuming a certain unisexual tendency represented by a fraction ι having the value

$$a = 0.27$$

This coefficient may be considered to express the efficiency of all the causes tending to produce sex which are common to the two twin members of the family. In other words, assuming this unisexual coefficient, the result will be 77 per cent of twins of the same sex, and 23 per cent of twins of different sexes, these being the actual results of observation.

A most interesting fact is that, by the methods developed in the Appendix, we may apply this coefficient a to determine how the sexes in families of triplets should be divided. There are two ways of proceeding. We may assume the unisexual tendency to be the same in triplets as we have found it to be in twins, as naturally ought to be the case. From this we can determine what proportion of triplets should be unisexual, and compare the result with statistics. The other method consists in determining the value of the unisexual tendency from the statistics of the triplets in order to see how much it differs from that determined from the statistics of twins. Adopting the first method the problem is: We have three organisms subject to such conditions that, in the case of each, there is a probability of 0.77 that it will prove of one sex and of 0.23 that it will prove of the other. What are the respective probabilities that the three organisms will be unisexual; that is, all three of the same sex; and that one shall be of one sex and two of the opposite? These probabilities are found in the Appendix to be:

Unisexual: percentage, 46.9
Bisexual: " 53.1

To compare the statistics I have collected the sexes of triplets found in the French and German tables of births with the following results:

	France 1858-1900	Berlin 1853-1880	Total
3 males	342	32	374
3 females	304	28	332
Total unisexual	646	60	706
Bisexual	667	43	710

Observed proportion of unisexual triplets..........49.9 per cent.

It appears from these numbers that when we compare the probabilities derived from the case of twins with the actual facts in the case of triplets, there is a discrepancy. From the facts in the case of twins, we should conclude that 46.9 per cent of all triplets should be unisexual; we actually find that 49.9 per cent is unisexual, an excess of 3 per cent.

The discrepancy may take the other form by determining the amount of unisexual preponderance in the case of triplets as we have done in the case of twins. This preponderance is 0.79 instead of 0.77. That is to say, grouping the triplets as we have the twins, there is a probability of 0.79 that any one organism of a triplet of group A will develop into a male, and that one of group B will develop into a female. The coefficient of unisexual tendency is, therefore, for triplets,

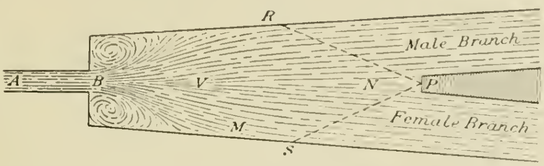
$$a = 0.29$$

Now, we should suppose, *a priori*, that the ratio of the unisexual preponderance to the effects of the accidental causes which finally determine the sex would be the same with twins and triplets. It is true that the discrepancy between 0.27 and 0.29, or between 46.9 and 49.9 per cent is not greater than might easily have been the result of fortuitous deviation. Still it is larger than we should expect. If we may regard it as expressing a real law, we may suppose that, besides the independent causes at action tending toward one sex or the other, there is an interaction between the two organisms, by which the sex of one influences that of the other in its own direction.

Apart from this, the general conclusions from triplets confirm that from twins [1] —there are not male and female germs. It would seem that we have in this a practically conclusive negation of the theory of completely determined sex in the original germs and may provisionally accept that of complete asexuality on the part of such germs, subject, however, to farther statistical tests.

6. PROCESSES IN THE DETERMINATION OF SEX SUGGESTED BY THE STATISTICS OF MULTIPLE BIRTHS.

The view that, if the sex is not completely determined in the original formation of a germ, it must be determined at some definite moment of development—that there can be no intermediate state between complete asexuality and complete sexuality—is one which, at first sight, seems almost axiomatic. And yet, the preceding statistics of multiple births seem to show that such is not the case, and that there may be a series of causes acting first in one direction and then in the other, each of which tends to make one sex or the other more probable until, gradually, the sex is definitely determined. An analogue to this determination by a succession of accidental causes may be constructed in the following way: Let A be a large pipe or aqueduct, from the mouth B of which a stream flows into a gradually widening river V. At a certain distance below



the exit B the river is divided into two branches by a promontory P. On one side of this promontory, which we may call the male side, the river is slightly broader than on the other. Between the exit and the promontory, the river flows over a rough bottom with many eddies, but the ultimate result must be that every drop of water which comes from the conduit ultimately passes on one side of the promontory or the other. But the side on which it shall pass is not determined at any one moment. As a drop, or, to give the analogy a more complete form, a small particle suspended in the water, leaves the conduit, it is equally likely to pass into one branch of the river or the other. If it chance to incline to the right after leaving the conduit, there will be a greater probability of its passing into the right branch, but this will be only a probability until a certain point of the course is reached. A particle reaching the point M, for example, will be likely to go into the female branch, but yet may be carried by an eddy across to the opposite side before it reaches it. One at N, although farther down, will still be uncertain; possibly its course may not be decided until it almost reaches P. A particle on one bank or the other will be more and more likely to pass into the corresponding branch the farther down it is found. When the particle once crosses one of the dotted lines PR and PS the branch it will take will be completely determined.

A case of twins or of triplets has its analogue in the case of two or three particles emerging from the conduit in contiguity. They are more likely to keep together and enter the same eddies than if they were widely separated in the beginning. To speak with numerical exactness there is a probability of 0.77 that they will pass on the same side of the promontory and of 0.23 that they will separate. In the case of triplets the corresponding probability would be 0.79; but these are only probabilities. At any moment any two particles may widen their distance and be drawn into different parts of the stream, never to reunite.

We may thus say that the question, which branch of the river a particle, emerging from the conduit, is to flow into, will be determined by a series of accidents tending in one direction or the other; and the most plausible conclusion from the statistics of twins is that sex is determined in an analogous way.

7. INFLUENCE OF THE AGE OF THE PARENT ON SEX.

The changes produced by age in the human system are such that we may most plausibly look to them as causes affecting the sex of offspring. The question of the influence of the age of the parent has been studied by several investigators, especially by Rosenfeld, Sadler and Bertillon. I have not been able to refer to the original work of Bertillon and shall therefore confine myself to citing, in its proper place, one of his conclusions bearing on the case. Dr. Rosenfeld gives the following table of the sexes of more than thirty thousand births in Vienna, arranged according to the age of the father. I add the percentage E_m for each age:

VIENNA STATISTICS OF BIRTHS.

Age of father	Male children	Female children	100 M : F	E_m
Under 25	873	767	113.7	6.5
25 to 30	6,090	5,717	106.5	3.3
30 35	11,987	11,291	106.2	3.1
35 40	3,605	3,559	101.3	0.6
40 50	622	502	128.9	10.7
Over 50				

The table shows a decided preponderance of male children in the case of young and old fathers as compared with those in middle life. The conclusion thence drawn is that male unisexuality is at its maximum in young and old people.

Francke, from the statistics of Norway, reached the same conclusions as regards young fathers, but the opposite as regards old ones. His numbers for the ratio of male to female births, arranged according to the age of the father, are as follows:

NORWEGIAN STATISTICS OF BIRTHS.

Age of father	100 M : F	E_m
Below 20	117.0	7.9
20 to 25	101.5	0.7
25 30	109.0	4.4
30 35	105.9	2.9
35 40	102.6	1.3
40 45	104.6	2.2
45 50	103.8	1.8
50 55	98.4	—0.8
55 60	97.9	—1.0
Over 60	99.8	—0.1

Rosenfeld gives a similar classification, arranged according to the age of the mother, as follows:

VIENNA STATISTICS OF BIRTHS.

Age of mother	Male children	Female children	100 M : F	E_m
Below 17	19	7	271.4	46.1
17 to 20	366	341	107.3	3.6
20 25	4,444	4,161	106.8	3.3
25 30	7,287	6,759	107.8	3.7
30 40	9,907	9,356	105.9	2.8
Over 40	1,412	1,396	101.1	0.5

The enormous preponderance of male births in the case of mothers under 17 years of age is probably the result of accident and not expressive of a general law, the births, 26 in number, being too few to base a determinate conclusion upon. If we combine all the mothers under 20 years of age, the result will be:

$$100 \text{ M} : \text{F} = 110.6 ; E_m = 5.0$$

The numbers now show a marked preponderance of male children borne by very young mothers, which drops to the normal at the age of 20 and falls below it at the age of 40.

It may be noted in this connection that the ratio of male and female children is, in the general average, somewhat above the normal, possibly indicating an imperfection of the record by not including all female children. This, however, will not alter the conclusion.

All these conclusions as regards the age of the parent seem to me to lack a solid foundation, from the fact that the ages of the two parents are not completely distinguished. I shall discuss this difficulty after setting forth the results of the genealogical statistics collected by myself.

The effect of difference of age between father and mother was investigated by Sadler, who laid down the general law that the older parent has a preponderating influence in the direction of determining children of his or her own sex. But Ahlfeldt reached the opposite conclusion, finding that when the father was more than 10 years older than the mother, there was a preponderance of female children instead of the normal excess of male children. His numbers are, however, too few to base any conclusions upon; and the same is probably true of the statistics used by Sadler.

I have not attempted to investigate this subject by age because the data are not at hand for the purpose. Instead of doing this, I have taken the order of progression of children in families as found in the genealogies already cited. In each family the sex of the several children was tabulated in the order, first born, second born, third born, etc. Then the total number of first-born children of each sex, the second born, and so on, was taken. The results are shown in the following table in which the first column gives the order of birth. This is followed by the respective number of male and female first-born children. In the same line the numbers are given for the second born, and so on. In families of more than 14 children, the fourteenth and those following are all tabulated together, as their separate numbers are too small to base a conclusion upon.

The fourth column gives the total number of children; the fifth the excess of males, and the sixth the percentage of this excess.

Comparison of Male and Female Children in the Order of Birth in American Families.

Order of Birth	Males	Females	Sum	Excess of Males	E_m	Corrected E_m
1	3,906	3,265	7,171	641	8.9	6.7
2	3,261	2,987	6,248	274	4.4	2.2
3	2,605	2,532	5,137	73	1.4	—0.8
4	2,145	2,024	4,169	121	3.0	0.8
5	1,766	1,651	3,417	115	3.4	1.2
6	1,406	1,338	2,744	68	2.5	0.3
7	1,102	1,020	2,122	82	3.9	1.7
8	782	754	1,536	28	1.8	—0.4
9	592	513	1,105	79	7.3	5.1
10	407	356	763	51	6.7	4.5
11	246	221	467	25	5.3	3.1
12	142	108	250	34	13.6	11.4
13	71	52	123	19	15.4	13.2
14 to 17 ..	51	42	93	9	9.8	7.6
Total...	18,482	16,863	35,345	1,619	4.6	

It will be seen that the excess of male births in the general average markedly exceeds its normal value. We must regard this divergence as unreal and attribute it to the greater liability of a female child to be omitted from the record. As this omission would be probably about equal in the case of all the successive children, we may assume that the values of E_m are all equally in error from this cause. The normal value from the statistics of birth being about 2.4, while the count gives 4.6, we subtract the excess, 2.2, from each separate value of E_m and thus obtain corrected values of the percentage of excess, which are found in the last column.

It will be seen from the numbers of this column that the excess of males among first-born children exceeds 6 per cent. This shows that there are about eight males to seven females of this class. But, in the case of the second child, the percentage of excess drops to 2.2, which is slightly blow the normal and, in the case of the third child, it becomes negative, showing that, after we correct the supposed defect of the record, there is actually a slight excess of female births.

The rapidity of the drop from 6.7 in the case of the first birth to 2.2 in the case of the second and then to a negative quantity in the case of the third, seems to show quite conclusively that the excess of males in the number of the first-born children is not attributable to the age of the mother, but to the fact that it is a first child, irrespective of age. That the fall is too rapid to be the effect of age is shown in the following way: The difference of age at the birth of the first and third child is not likely to have been more, in the general average, than three years. Now a drop of 4 in the percentage in three years would imply a drop of twice this amount between the ages of 17 and 24, which we may take as the probable range in the case of a first child. The approach to uniformity in the percentage in these cases where the marriage must have been at such different ages, precludes the supposition that age is the main factor in the case.

Continuing our study of the table, we find a remarkable uniformity in the number of male and female births up to the eighth child. In the case of the second child the excess is still fairly well marked. Thus we may conclude that the tendency toward male excess, though greatly diminished, is probably not wholly obliterated in the case of the second child. But from the fourth to the eighth inclusive, the deviations are so small that we may regard them as the effects of accident. In the case of the six children from the third to the eighth, it would seem that the birth of the two sexes is equally probable. Then, from the ninth child onward we find an excess of males which generally exceeds the normal all through. But it is not at all certain that this arises from a unisexual tendency in the case of older parents. It is quite possible that it may be attributed to first-born children after remarriage, the table having been constructed without any reference to the mother and giving only children in families by the same father. It must also be noted that the total numbers beyond the tenth child become too small to predicate a very certain conclusion upon. A more complete investigation of the subject will therefore be necessary before it can be said with certainty whether the results derived by Rosenthal in the Vienna statistics in the case of old parents are correct, or whether we here have to do with the first-born of second or third wives.

It might appear, at first sight, that these statistics do not decide whether the variation in the proportion of male and female, as the family advances in number, are due to the male or female parent. But a consideration of the ratio between the number of acts on the part of the two parents who are concerned in the case, decides the probabilities in favor of the mother.

A more conclusive investigation than has as yet been made is necessary to absolutely decide whether, as has been suggested in this paper, the part of the father is completely asexual. To make this investigation, it is necessary to compare the statistics of births by mothers of one and the same class with fathers of different ages. Since the ratio of male to female is the same, at least from the third to the eighth birth, the preferable method is to confine the investigations to those births which may be grouped all together, so far as the mother is concerned. We then compare the sex of each child of this class with the age of its father and, by a sufficient accumulation of cases, ascertain whether the ratio varies with that age.

8. EXAMINATION OF CERTAIN OTHER CONDITIONS WHICH HAVE BEEN SUPPOSED TO INFLUENCE THE PRODUCTION OF SEX.

It has sometimes been supposed that the destruction of an important fraction of the male population of a country by war, such as has occasionally been known in history, has resulted in a greater preponderance of male offspring in the country so affected. A very slight analysis of the supposed cause will show that this proposition belongs to a class which require very strong proof. Granting the truth of the proposition: since those who were killed in war could not subsequently have taken part in the propagation of the race, it would follow that those who returned in safety showed a unisexual tendency in the male direction.

That a tendency of this sort could be produced in one man by the mere death of another is a notion that hardly needs to be refuted. If such an effect is real, it would therefore have to be the result of privations and other evils suffered in war, and not of the mere destruction of life, a process which Nature is carrying on all the time. The question would then be whether privations and sufferings generally produce a male unisexual tendency. This idea seems to be conclusively negatived by the fact that the male preponderance is not shown to be a function of the wealth of the country, or the condition of the great mass of the population.

Nevertheless, in order that none of my conclusions might be based on *a priori* reasoning, and in order to answer the objection that there may be something peculiar in the effect of privations suffered in war, which differentiates them from other privations, I have examined the population statistics found in the New York census for 1865, and the United States census for 1870, enumerating the sexes of children who, from their ages, must have been born about the close of the civil war. In the case of the United States census I confined the examination to the Southern States, because there it was that the suffering and privations were the greater. The result, comprising enumeration of the sex of more than 100,000 children, showed that the male preponderance was as nearly as possible the normal one, and that not the slightest influence of the war could be detected.

It has also been maintained that the practice of polygamy has been found productive of the unisexual tendency in the female direction. The data for deciding this question are insufficient; but I find that, in the only region of the United States where such an effect would be likely to be observable, there is the usual preponderance of male births. Analysis will show that this proposition also belongs to the most improbable class. The only polygamous practices which could reasonably be supposed to affect sex are so far from rare that any unisexual tendency arising from it would be brought to light by a very slight examination. The author believes that the preceding paper contains sufficient matter to disprove the supposition, without the necessity of further inquiry.

9. SUMMARY OF CONCLUSIONS.

I do not present the following summary of conclusions as being, in all cases, so well established as not to be worthy of farther investigation. Whether well or ill established, they are those indicated by the statistics, and I earnestly hope that other investigators, more especially concerned with the subject, will take it up with more extended data and test each conclusion separately. With this proviso we may say that the following propositions are indicated by the statistics with a greater or less degree of probability.

I. The preponderance of male over female births probably varies with the race. Although remarkably uniform in all branches of the Semitic race, it seems to be either non-existent or quite small in the Negro race.

II. There are no important differences as regards capacity for producing children of one sex rather than the other which are permanent in the individual. All fathers and all mothers are equally likely to have children of either sex, except for the slight variations that may be due to age. In view of the great variety of conditions on which this conclusion is based, it seems in the highest degree unlikely that there is any way by which a parent can affect the sex of his or her offspring.

III. The most natural inference from all the statistical data is that the functions of the father in generation are entirely asexual, the sex being determined wholly by the mother. If so, it cannot be said that one father is more likely than another to have children of either sex. This conclusion requires to be tested by making a classification of the sex of third born and following children according to the age of the father.

IV. The sex is not absolutely determined at any one moment or by any one act, but is the product of a series of accidental causes, some acting in one direction and some in another, until a preponderance in one direction finally determines it. The statistics of twins and triplets seem to show very strongly that these accidents occur after conception, but throw no light upon the question of the time which they occupy.

V. The first born child of any mother is more likely to be a male in the proportion of about 8 to 7. There is probably a smaller preponderance in the case of the second child. But there is no conclusive evidence that, after a mother has had two children, there is any change in her tendencies.

VI. The observed preponderance of male births in the Semitic race is due mainly to the unisexual tendency of the mother in the case of a first child.

APPENDIX.

MATHEMATICAL THEORY OF THE EFFECT OF A UNISEXUAL TENDENCY.

The statistical theory on which the preceding research is based, being presumably susceptible of other applications than that here made, will now be developed. So far as generality is concerned, nothing will be lost by taking the special problem, considered in section IV preceding, as a basis of investigation. The data of the problem will be as follows:

1. An indefinite number of pairs of parents, each pair of which may have an indefinite number of children of either sex. The treatment of this subject will include the general case of an indefinite number of causes, each of which may, on each trial, be productive of one or the other of two different effects.

2. Taking the general average of the whole mass of couples, there is a certain normal probability, p, that a child, taken at random, will be male, and the probability $1 - p$ that it will be female.

3. It may be that this probability is the same for every individual couple of the whole mass. But it may also be that, for some of the couples, the probability is greater than p. In this case it will necessarily follow that for certain other couples the probability is less than p, the latter quantity being the average for the whole mass.

4. In order not to complicate the problem too greatly, we shall suppose that each of the individual couples belongs to one of three classes; a class for which the probability of having a male child has the normal value p, another for which it is greater than p by an unknown quantity α, and a third for which it is less than p by the same quantity. We designate these classes by A, B and C; A representing couples with probability $p + \alpha$; B, those with probability p; and C, those with the probability $p - \alpha$. The numbers of classes A and C are necessarily equal.

Let us put:

h, the fraction of the total belonging to the two equal classes A and C;

h', the fraction of the whole mass belonging to class B.

We shall then have

$$h + h' = 1.$$

Proceeding according to the method of probabilities, we suppose a parent couple taken at random from the mass. The respective probabilities that this couple will belong to the classes A, B and C are

$$\tfrac{1}{2}h,\ h' \text{ and } \tfrac{1}{2}h.$$

The probabilities of a male child are, in these several classes:

$$\begin{aligned} &\text{For class A, } p + \alpha. \\ &\qquad\quad\ \ \text{B, } p. \\ &\qquad\quad\ \ \text{C, } p - \alpha. \end{aligned} \tag{1}$$

Then by the principles of the theories of probabilities, if a couple be taken at random from the whole mass, the respective combined probabilities that the couple will be of one of the classes, and the child a male, will be:

$$\begin{aligned} &\text{In class A, } \tfrac{1}{2}h(p+a).\\ &\qquad\quad \text{B. } h'\,p. \\ &\qquad\quad \text{C. } \tfrac{1}{2}h(p-a). \end{aligned} \tag{2}$$

of which the sum is p, as it should be.

The problem before us is to find a criterion for deciding whether the quantity a, which we may consider as the unisexual factor, and which we shall call the *coefficient of unisexuality*, is or is not of appreciable magnitude. Such a criterion is afforded by a count of males and females in families of two or more children. The theory requires that, in a family of a given number of children, we express the probable respective numbers of males and females in terms of the factor a.

The problem now assumes the following form: A parent couple, taken at random from the whole mass, has n children; what is the probability that s of these children will be males and $n-s$ females?

Using the notation

$$\begin{bmatrix} n \\ s \end{bmatrix} = \frac{n(n-1)\,(n-2)\,\ldots\,(n-s+1)}{1\,.\,2\,.\,3\,\ldots\,s}$$

we have the well-known theorem that, if the probability of an event on a single trial is μ, the probability of its occurring s times on n trials is

$$P_s^n = \begin{bmatrix} n \\ s \end{bmatrix} \mu^s (1-\mu)^{n-s}$$

Putting for μ the three values of the probabilities given in (1) we find that the probabilities in question are:

$$\begin{aligned} &\text{For class A, } \begin{bmatrix} n \\ s \end{bmatrix}(p+a)^s\,(1-p-a)^{n-s} \\ &\text{For class B, } \begin{bmatrix} n \\ s \end{bmatrix} p^s\,(1-p)^{n-s} \\ &\text{For class C, } \begin{bmatrix} n \\ s \end{bmatrix}(p-a)^s\,(1-p+a)^{n-s} \end{aligned} \tag{3}$$

Multiplying these expressions, as in (2), by the respective factors $\tfrac{1}{2}h$, h' *and* $\tfrac{1}{2}h$, putting for brevity

$$n-s=r$$
$$1-p=q$$

and taking the sum of the products, we find the probability that a family of n children taken at random from the whole mass, will comprise s males and r females to be

$$P^{(n)}_{r,s} = \left\{ \tfrac{1}{2}h((p+a)^s(q-a)^r + (p-a)^s(q+a)^r) + h'p^sq^r \right\} \begin{bmatrix} n \\ s \end{bmatrix} \qquad (4)$$

This expression may now be developed in even powers of a, the coefficients of the odd powers all vanishing. In the form

$$P^{(n)}_{r,s} = (A_0 + A_2a^2 + A_4a^4 + \ldots) \begin{bmatrix} n \\ s \end{bmatrix}$$

the values of the first two coefficients are

$$A_0 = (h + h')\,p^sq^r = p^sq^r$$

$$A_2 = hp^{s-2}q^{r-2} \left(p^2 \begin{bmatrix} r \\ 2 \end{bmatrix} + q^2 \begin{bmatrix} s \\ 2 \end{bmatrix} - rspq \right)$$

$$= \tfrac{1}{2}h\,\{n(n-1)p^2 - 2(n-1)sp + s(s-1)\}\,p^{s-2}q^{r-2}$$

For our present purpose these terms suffice. To investigate unisexual deviations it will also lead to no appreciable error to suppose

$$p = q = \tfrac{1}{2}$$

The value of $P^{(n)}_{r,s}$, that is, the probability that a family of n children will consist of s males and r females now becomes

$$P^{(n)}_{r,s} = \begin{bmatrix} n \\ s \end{bmatrix} \left\{ \frac{1}{2^n} + \frac{(r-s)^2 - n}{2^{n-1}} ha^2 \right\} \qquad (5)$$

We may use this formula to express the probability in question for the case of a family of any number of children, distributed in any way among the two sexes. We shall now form these expressions for families of various numbers of children. In doing this families in which the sexual distributions are the reverse of each other will be combined. For example, the equal probabilities that a family of five will be wholly male and wholly female will be added into one sum; as will the probabilities of 4 of one sex and 1 of the other, whichever sex it may be.

The pair of probabilities thus combined would be rigorously equal when, and only when, there is an equal probability of male and female children. But not only is the error involved in the assumption of inequality unimportant for the present purpose but, resulting as it does in giving too small a probability for a preponderance of male and too large for the preponderance of females, it is nearly self-compensatory when we combine families of inversely distributed sexes.

The computation of the formula (5) is shown in the following table. To enable the essential numbers of this table to be understood without the necessity of going through all the mathematical formulæ, I shall state their significance and application. On the left are found certain possible values of n, the number of children in a family from 2 to 12 inclusive. Each block of numbers connected with a single value of n relates solely to families of that number of children.

In the next column are given all possible distributions between the two sexes which the family can have. Complementary families as regards sex are combined. For example, a family of three children must consist either of three children of one sex, whether male or female, and none of the other; or it comprises two of one sex, whichever it may be, and one of the other. The two lines correspond to these cases.

The three following columns contain numbers employed in computing the probabilities as found in the expression on the right. The denominators of the fractions which enter into these probabilities are written after the sign $\div$ of division, and, in each set relating to one value of n, the fractions are reduced to the least common denominator, but not to their lowest terms. This form of expression is used for convenience in tracing the law of the numbers and continuing the table. The probability is expressed as the sum of two terms: one a pure number; the other a coefficient of the factor ha^2. The purely numerical term shows what the respective probabilities of the division of sexes found in the second column will be in case of no unisexual tendency. For example, in a family of four children there will then be one chance of all four being of one sex, three chances of one being of one sex and one of the other, and three chances of an equal division, making eight chances in all. Hence, in a great mass of such families, we shall have one-eighth all of the same sex, four-eighths, or one-half, with a preponderance 3 to 1, and three-eighths with an equal division.

The next term shows how this probability is modified in case of a unisexual tendency. The symbol h expresses the fraction of the whole number of parents which have such a tendency. The tendency in one-half of this fraction of cases will be in the male, in the other in the female direction. The symbol a is the unknown amount of this tendency.

These expressions for the probability are rigorous when n is 2 or 3. But, when n has a greater value, terms in the higher powers of a really exist, the highest power being n, or $n - 1$, according as n is even or odd, but, as a must always be a rather small factor, these high powers may be neglected.

Construction of the Numerical Formulæ.

n	r, s	$\begin{bmatrix} n \\ s \end{bmatrix}$	2^{n-1}	$(s-r)^2-n$	*Probability*	
2	2, 0	1	2	2	$1 \div 2$	$+\ 2ha^2$
	1, 1	2		-2	$1 \div 2$	$-\ 2ha^2$
3	3, 0	1	4	6	$1 \div 4$	$+\ 3ha^2$
	2, 1	3		-2	$3 \div 4$	$-\ 3ha^2$
4	4, 0	1	8	12	$1 \div 8$	$+\ 3ha^2$
	3, 1	4		0	$4 \div 8$	
	2, 2	6		-4	$3 \div 8$	$-\ 3ha^2$
5	5, 0	1	16	20	$1 \div 16$	$+\ 5ha^2 \div 2$
	4, 1	5		4	$5 \div 16$	$+\ 5ha^2 \div 2$
	3, 2	10		-4	$10 \div 16$	$-\ 10ha^2 \div 2$
6	6, 0	1	32	30	$1 \div 32$	$+\ 15ha^2 \div 8$
	5, 1	6		10	$6 \div 32$	$+\ 30ha^2 \div 8$
	4, 2	15		-2	$15 \div 32$	$-\ 15ha^2 \div 8$
	3, 3	20		-6	$10 \div 32$	$-\ 30ha^2 \div 8$
7	7, 0	1	64	42	$1 \div 64$	$+\ 21ha^2 \div 16$
	6, 1	7		18	$7 \div 64$	$+\ 63ha^2 \div 16$
	5, 2	21		2	$21 \div 64$	$+\ 21ha^2 \div 16$
	4, 3	35		-6	$35 \div 64$	$-\ 105ha^2 \div 16$
8	8, 0	1	128	56	$1 \div 128$	$+\ 7ha^2 \div 8$
	7, 1	8		28	$8 \div 128$	$+\ 28ha^2 \div 8$
	6, 2	28		8	$28 \div 128$	$+\ 28ha^2 \div 8$
	5, 3	56		-4	$56 \div 128$	$-\ 28ha^2 \div 8$
	4, 4	70		-8	$35 \div 128$	$-\ 35ha^2 \div 8$
9	9, 0	1	256	72	$1 \div 256$	$+\ 9ha^2 \div 16$
	8, 1	9		40	$9 \div 256$	$+\ 45ha^2 \div 16$
	7, 2	36		16	$36 \div 256$	$+\ 72ha^2 \div 16$
	6, 3	84		0	$84 \div 256$	
	5, 4	126		-8	$126 \div 256$	$-\ 126ha^2 \div 16$
10	10, 0	1	512	90	$1 \div 512$	$+\ 45ha^2 \div 128$
	9, 1	10		54	$10 \div 512$	$+\ 270ha^2 \div 128$
	8, 2	45		26	$45 \div 512$	$+\ 585ha^2 \div 128$
	7, 3	120		6	$120 \div 512$	$+\ 360ha^2 \div 128$
	6, 4	210		-6	$210 \div 512$	$-\ 630ha^2 \div 128$
	5, 5	252		-10	$126 \div 512$	$-\ 630ha^2 \div 128$
11	11, 0	1	1024	110	$1 \div 1024$	$+\ 55ha^2 \div 256$
	10, 1	11		70	$11 \div 1024$	$+\ 385ha^2 \div 256$
	9, 2	55		38	$55 \div 1024$	$+\ 1045ha^2 \div 256$
	8, 3	165		14	$165 \div 1024$	$+\ 1155ha^2 \div 256$
	7, 4	330		-2	$330 \div 1024$	$-\ 330ha^2 \div 256$
	6, 5	462		-10	$462 \div 1024$	$-\ 2310ha^2 \div 256$
12	12, 0	1	2048	132	$1 \div 2048$	$+\ 33ha^2 \div 256$
	11, 1	12		88	$12 \div 2048$	$+\ 264ha^2 \div 256$
	10, 2	66		52	$66 \div 2048$	$+\ 858ha^2 \div 256$
	9, 3	220		24	$220 \div 2048$	$+\ 1320ha^2 \div 256$
	8, 4	495		4	$495 \div 2048$	$+\ 495ha^2 \div 256$
	7, 5	792		-8	$792 \div 2048$	$-\ 1584ha^2 \div 256$
	6, 6	924		-12	$462 \div 2048$	$-\ 1386ha^2 \div 256$

The method of using the numbers is, from the statistics for each value of n, to form conditional equations having h and a as unknown quantities. These unknowns are to be determined by a solution of the equations. It will be seen that h and a cannot be determined separately, but only the combination ha^2. We may therefore suppose $h = 1$ without any loss of generality so far as these equations are concerned.

We now make a practical application of this theory by determining the numerical value of the unisexual tendency, a, in the respective cases of twins and triplets, as enumerated in section 5 preceding. The statistics of twins there cited show that, of such pairs, 0.646 are unisexual and 0.354 bisexual. Equating these percentages to the expressions for the probability we find

$$\tfrac{1}{2} + 2ha^2 = 0.646$$
$$\tfrac{1}{2} - 2ha^2 = 0.354$$

Subtracting these from each other we find $4ha^2 = 0.292$, and hence, supposing $h = 1$,

$$a^2 = 0.073$$
$$a = 0.27$$

We may now consider the case of triplets in two ways. Proceeding, as in the case of twins, by equating each probability to the fraction indicating the proportional number of the families to which it relates, we have the equations:

$$\tfrac{1}{4} + 3ha^2 = 0.499 \qquad \tfrac{3}{4} - 3ha^2 = 0.501$$

Solving these we derive, after putting $h = 1$,

$$a^2 = 0.0831 \qquad a = 0.29$$

We may also proceed in another way by substituting in the expressions for the respective probabilities of unisexual and bisexual triplets the value of ha^2 derived from the case of twins. This will give, as has already been stated, the percentage 46.9 for unisexual triplets instead of 49.9, as has been found from observation. It may be added that this relation is not changed by changing the value of h; it is therefore indifferent what value we assign to h.