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FREDERICK V. WAUGH, Director of Economic and Statistical Analysis Division, Economic Research Service, a frequent contributor to this journal, is working on a general study of the demand for farm products.

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Agricultural Economics Research

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Ronald L. Mighell has joined the editorial staff of *Agricultural Economics Research* as an Assistant Editor, succeeding M. L. Upchurch, who transferred to the Staff Economists Group in the Office of the Director of Agricultural Economics. Dr. Mighell is Leader of the Pioneering Research Group in the Farm Production Economics Division, Economic Research Service. His group recently completed *Vertical Coordination in Agriculture*, published as Agricultural Economic Report No. 19, U.S. Department of Agriculture. He is the author of the book *American Agriculture—Its Structure and Place in the Economy* published in 1955 and coauthor with the late John D. Black of the book *Interregional Competition in Agriculture* published in 1951. A research economist in the U.S. Department of Agriculture since 1936, he has been a frequent contributor of articles and reviews to *Agricultural Economics Research*.

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Feed-Livestock Relationships: A Model for Analyzing Management Decisions

By R. J. McConnen, C. M. McCorkle, Jr., and D. D. Caton

Economists studying the operation of individual firms have relied in large part on the general theory of production economics as presented in standard references (*3*) (*6*).[1] Usually some changes are required to increase the usefulness of the theory in formulating realistic and testable hypotheses. Generally it is assumed that the goal of the firm is one of maximizing profits. The objective of this paper is to develop a general model especially helpful for analyzing the operation of firms that produce livestock products. This is based on certain modifications and elaborations of the conventional theory of production. For the purpose of illustrating the use of the general model, a California feeder-steer operation is used. The general model should be helpful in evaluating many other types of livestock operations. The major modification is the separation of the livestock operation into production *stages*. "The technical definition of a stage is a matter of both convenience and logic, depending on the importance of the elemental operations and the way in which they fit in with the flow of products and materials. . . . Thus, a stage consists of all productive services—durable or nondurable—that cooperate in performing a single operation or a group of minor but closely related operations" (*4*, p. 545).

MANAGERS OF FIRMS base their decisions on simultaneous consideration of several stages. The three major production stages for livestock firms are (1) feed procurement, (2) feed consumption, and (3) feed conversion. These stages taken together form a sequence in production. More detailed stages can be defined for certain problems. For instance, in the analysis of a feedlot operation, feed procurement might be broken down into feed acquisition, feed storage, feed processing, and feed transportation within the feedlot. These operations may be sufficiently important to justify the explicit definitions of stages for these operations.

Feed procurement, stage 1, consists of those operations necessary to acquire the feed for the livestock firm and to have the feed ready for consumption. The feed may be procured by buying or raising, or both. The decision of how much feed to raise, if any, should be based on a comparison of the marginal cost of production and the purchase price per unit. In order to determine the "best" of alternative methods of feed procurement, it may be necessary to complete the analysis of all three stages before the choice among alternatives can be made.[2] For example, the way in which the forage is grazed (method of consumption) may affect the rate of forage growth (a method of feed procurement). When the alternative methods of feed procurement would result in feeds of like quality and timing, the method which will procure the feed at the lowest cost would be the "best."

Feed consumption, stage 2, comprises all those operations necessary in order for the livestock to consume the feed offered them. The type of grazing management, the type of feed containers, and the timing of the feed operations are major factors affecting the quantity of feed offered to the animal and actually consumed. In a feedlot operation the type of feeding facilities will influence the portion of the feed offered that is consumed. Any investment that increases the ratio of feed consumed to feed offered represents the substitution of capital for feed.

Feed conversion, stage 3, is the process of conversion of the feed consumed by the livestock into a livestock product. The efficiency with which livestock convert the consumed feed into a livestock product will depend on the inherent productivity of each of the individual animals and the past and present environment of the animals. For any class of livestock the most efficient animals

[1] Italic numbers in parentheses refer to Literature Cited, p. 47.

[2] The importance of considering the impact on returns as well as consideration of costs of rations is pointed out in a recent article by W. G. Brown, and G. H. Arscott (*2*).

may not always be the most profitable. If the price differential is great enough, a manager may find it more profitable to buy less efficient animals. The livestock product may be a final product which is sold, such as slaughter cattle or market milk. The livestock product could be an intermediate product for the firm such as replacement heifers or feeder cattle. The resulting product could also be a joint product, such as a group of calves, part of which are sold and part held over as replacement stock.

Application of the General Model

The general model presented in figure 1 was used as the basis for evaluating range fertilization for a ranch located in the Coastal Range of California. The ranch of 1,008 acres is all rangeland except a small area used in the farmstead. It has adequate stock water and no unusual factors that restrict its use as a livestock ranch. Five land classes based on soil capability ratings, are defined for this ranch (table 1). The land is fenced into pastures which correspond roughly to the land classes. Therefore, different stocking rates for different land classes can be maintained and fertilizer responses by pastures can be determined.

Without range fertilization, the owner usually purchases 350 head of 300 to 400 pound weaner calves in early spring whenever the feed is ready to be grazed (between 15 February and 15 March). He grazes the calves as long as they continue to gain, usually 100 to 120 days. The average gain per head for this period is two pounds per day.

Stage 1 considers alternative quantities of feed offered the animals. The level of forage production per acre with no fertilization for each land class is based on estimates by range technicians and on the amount of livestock output produced. The forage response functions with fertilization were approximated on the basis of data from Soil Conservation Service range fertilization experiments conducted 15 miles from the ranch.[3]

Table 1.—*Distribution of land area in selected California ranch by land classes*

Land class	Area
	Acres
1	46
2	86
3	505
4 [1]	348
5 [2]	23
Total	1,008

[1] Recommendations for use of land class 4 indicate that this land class should not be fertilized. This is primarily because of the steep slopes associated with this class.
[2] Area in this land class was assumed to be of no value for grazing purposes.

Since no alternative sources of feed are considered, the total feed offered to the animals in terms of dry matter can be varied only by varying the level of fertilization. In this case, the total product of Stage 1 can be expressed as:

$$\sum_{i=1}^{4} (Ac_i)Y_{ki} = \sum_{i=1}^{4} (Ac_i)F_{ki} + \sum_{i=1}^{4} (A\acute{c}_i)f_{ki}$$

where Ac_i is the number of acres in the *i-th* land class and $A\acute{c}_i$ is the number of acres fertilized in the *i-th* land class. In this case, the feed is offered to the livestock as standing forage.

Stage 2 comprises those operations in which the livestock consume the forage. It is generally accepted that not all the standing forage offered grazing livestock is consumed (*5*) (*7*) (*10*).

"The weight of clipped herbage gives an estimate of the total herbage offered to animals while the animal production gives an estimate of the value of the herbage eaten by the animals . . ."

[3] The Soil Conservation Service experiments were conducted on a site similar to the land class 1 used here. Attempts to fit statistically reliable fertilizer response functions were not successful. The experimental data, as is typical with rough fertilization experiments (1) showed a large degree of variation in response for any level of fertilization, (2) were based on a limited number of replications, and (3) were lacking in a critical range in the levels of application of fertilizer. The approximated fertilizer response functions did not violate the experimental data and they seemed to give "reasonable" results except at very low levels of fertilization. The general net response function in terms of dry matter per acre for each land class is:

$$f_{ki} = a_i X^{b_i}$$

where X is the rate of fertilization (in increments of 100 pounds of 16–20–0 per acre). The individual parameters are: land class 1, $f_{k1}=40\ X^{.8}$; land class 2, $f_{k2}=30\ X^{.7}$; land class 3, $f_{k3}=25\ X^{.6}$. The total dry matter produced per acre for land class 1 is $Y_{ki}=F_{ki}+f_{ki}$ where F_{ki} is the dry matter produced per acre with no fertilization, where F_{k1} is=1,800; F_{k2}=1,400, F_{k3}=1,000 and F_{k4}=703.

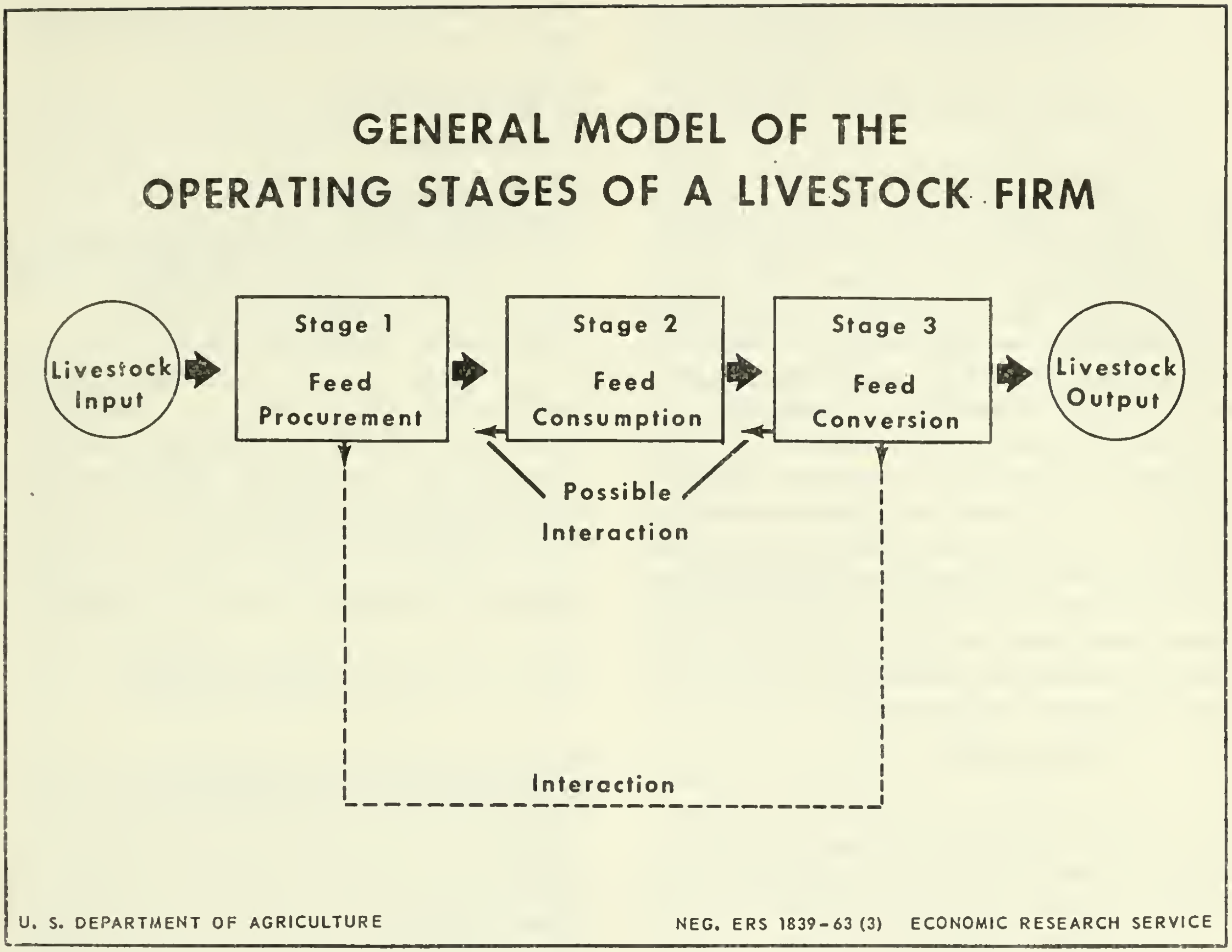

FIGURE 1

(*1*, p. 146). The ratio of these two values is called the clipping-grazing ratio. German and Swiss agronomists often use this ratio as a criterion to judge the efficiency of pasture management systems. This is one measure of what Brown calls the difference between gross forage production and net forage production (*1*, p. 147).

The model used to represent the operation of stage 2 in this study is: $\hat{Y}_k = Y_k\ (1-r)$ where $\hat{Y}_k$ is that amount of the standing forage per acre (Y_k) in terms of pounds of dry matter that is consumed by the livestock and r is the portion of the standing forage not consumed.[4] It

[4] Five different consumption models were considered. One model $\hat{Y}_k = Y_k$ assumes that all the feed offered livestock is consumed. This model is occasionally (and wrongly) used by workers evaluating pasture improvements who assume all clipped forage will be consumed by livestock. The model $\hat{Y}_k = Y_k - R$, where R is some fixed amount of forage left ungrazed, is often used where R is related to either conservation needs or cover required to insure adequate production in subsequent periods. The model $\hat{Y}_k = Y_k(1-r)$ was used for this paper. The model $\hat{Y}_k = Y_k\ [1-f\ (n)]$ recognizes that the portion of the standing forage not used, f(n), is a function of the stocking rate (n). The justification for this model is borne out by Gaalaas and Rogler (*5*). However, data in the present study were not adequate to make even a crude approximation of this function. The model $\hat{Y}_k = Y_k\ [1-f\ (n, t, w, \ldots)]$ considers the impact of stocking rate, topography, stock water availability, and other factors on the portion of standing forage not used. This model is perhaps the most realistic, but difficult to deal with in functional form. The impact of these factors on feed consumption is discussed in Stoddart and Smith (*11*, ch. IX, X, XI).

was assumed, based on the findings of Reuss, that the portion of standing forage consumed was a constant of 0.6=(1−0.4) regardless of either the amount of forage growth or the level of grazing (*10*, p. 11).

In developing the model it was further assumed that (1) no interaction occurs between the different stages, (2) there is no grazing before the time the rangeland reaches grazing readiness, and (3) there is no residual response to fertilization. The simplifying assumptions were incorporated because of data limitations rather than because of any constraints imposed by the general model.

Stage 3 is concerned with the conversion into a livestock product of feed consumed. For the purposes of this paper, the livestock product is assumed to be a final product, all of which is sold.

Much research has been done to determine how feed is converted into a livestock product. Winchester and Hendricks (*13*) have developed the functional relationship of feed conversion to livestock product for beef calves:

$$Z=0.0553\ W^{2/3}\ (1+0.805\ g),$$

where Z is the daily energy intake in terms of pounds of total digestible nutrients, g is the average daily gain or loss in body weight, W is the average weight for the period being equal to $W_i+\frac{Dg}{2}$ where W_i is the initial weight, and D is the number of days in the feeding period. The use of a single aggregate measure of feed value such as total digestible nutrients assumes away many of the problems of feed conversion. However, since only similar feeds are considered in this paper, and since these feeds provide a fairly well balanced ration during the time they are used, the dangers of using a single aggregate as an indicator of feed value are lessened.

The portion of feed offered that is consumed by the livestock is expressed in terms of pounds of dry matter, whereas the feed unit in the conversion function is expressed in terms of pounds of total digestible nutrients. On the basis of secondary information (*8*) and actual livestock production, it is estimated that 55 percent of the dry matter is total digestible nutrients. The percentage of total digestible nutrients is identified as K in the subsequent discussion. It is assumed that this percentage remains constant regardless of the level of fertilization.[5]

It was assumed that all calves purchased will, on the average, perform in accordance with the specification $Z=0.0553\ W^{2/3}\ (1+0.805\ g)$.

Introducing stages into objective features of the firm.—Once the nature of the three operating stages has been defined, they can be brought together. It is assumed that the goal of maximizing profits can be satisfied by maximizing gross ranch profit (GRP) defined as total revenue (TR) minus total variable costs (TVC).

$$GRP=TR-TVC \text{ where } TR=(P_Q)\ (N)\ (W_e) \text{ and}$$

$$TVC=P_iNW_i+\sum_{i=1}^{4}\ [(A\acute{c}_i)X_iP_x+(A\acute{c}_i)A]$$

where P_Q is the price per pound received from the calves when sold, N is the number of calves (it is assumed there is no death loss and that N is a variable), W_e is the average ending weight, P_i is the initial price per pound paid for calves, W_i is the average initial weight. $A\acute{c}_i$ the number of acres fertilized in the *i-th* land class (a variable), X_i is the rate of fertilization in the *i-th* land class (a variable), P_x is the cost of fertilizer per hundred pounds and A is the application cost per acre which is assumed to be constant.[6] Substituting relationships previously defined, gross ranch profit is defined as:

$$GRP=P_QN\left\{\left[\frac{[(1-r)K]\left[\sum_{i=1}^{4}F_{ki}(Ac_i)+\sum_{i=1}^{4}a_ix^{b_i}(A\acute{c}_i)\right]}{ND(a+akg)}\right]+\frac{Dg}{2}\right\}^{3/2}$$

[5] There is evidence that subjects this assumption to some question. The percentage of total digestible nutrients will also be affected by the method of utilization and the level of fertilization. However, adequate data were not available to indicate the nature of these relationships.

[6] For the purposes of this paper, certain of the variables are assumed to have constant values as follows: P_Q=25 cents per pound, P_i=25 cents per pound, W_i=350 pounds, P_x=\$4.50 per hundredweight, and A=\$1 per acre.

$$-\left\{ P_1NW_1+\sum_{i=1}^{4}[(A\acute{c}_i)X_iP_x+(A\acute{c}_i)A] + \sum_{i=1}^{4}[\lambda_i(Ac_i-A\acute{c}_i)] \right\}$$

In this equation, D (days in the grazing period) is equal to 110 for land that is not fertilized and to 130 for land that is fertilized. Since

$$W=\left[\frac{Z}{a+akg}\right]^{3/2}$$

Therefore

$$W_e=W+\frac{Dg}{2}=\left[\frac{Z}{a+akg}\right]^{3/2}+\frac{Dg}{2}$$

Z equals the total pounds of consumed total digestible nutrients divided by the total number of calf days and can be expressed as:

$$Z=\frac{\left[(1-r)K\right]\left[\sum_{i=1}^{4}Y_{ki}Ac_i\right]}{N\ D}$$

$$=\frac{\left[(1-r)K\right]\left[\sum_{i=1}^{4}F_{ki}(Ac_i)+\sum_{i=1}^{4}(a_iX^{b_i})(A\acute{c}_i)\right]}{N\ D}$$

Where Ac_i is the number of acres in the *i-th* land class and $A\acute{c}_i$ is the number of acres in the *i-th* land class that are fertilized. $\sum_{i=1}^{4}\lambda_i(Ac_i-A\acute{c}_i)$ are constraints which say the acres in the *i-th* land class that are fertilized must be equal to the number of acres in that class.

Choosing the "best" combination of inputs.—The optimum combination of inputs is defined as that combination which maximizes gross ranch profits consistent with the constraints imposed on the acreage in each land class. The inputs which were permitted to vary in this case are the number of calves, the average gain per day per head, and the level of fertilization for each land class. The usual next step would be to find the derivatives of the GRP with respect to those inputs which are allowed to vary and solve for the values of these inputs which would maximize the objective function. Certain secondary conditions would also have to be met. In this particular case, the calculus was not used to find the optimum values for the variable inputs. This occurred because some of the first order derivatives exist as fifth degree equations. It would have been possible to use a digital computer to approximate an optimum solution by numerical exploration of the model. However, this was not done. The GRP function was used to generate the gross ranch profit for a finite number of combinations of the inputs which were allowed to vary. These combinations of the inputs and the resulting gross ranch profits were used as a basis for defining the activities in a linear programming model.[7]

The programming model, with gross ranch profit defined as the objective function, was used as a device for choosing the "best" of input combinations which were considered. Because the inputs were assumed to be variable with infinite density in the GRP function, it would be possible to find the optimum combination of inputs if the calculus could be used. In the programming model, only a finite number of combinations of inputs were considered. Therefore, the programming solution can only be regarded as an approximation of the optimum solution and is referred to as the "best" rather than optimum combination of inputs.

As the programming model used (table 2) is of standard design, it will not be explained in great detail. Activities P_1 through P_5 are disposal activities. Activity P_6 indicates that one acre of land class 1 is fertilized at the rate of 200 pounds of 16–20–0, stocked at the rate of 1.7 head per acre, requires $159 working capital to purchase the needed cattle and to purchase and apply the fertilizer, results in an average gain per head per day of 1.4 pounds, and gives $72 in gross ranch profits. Activities P_7 through P_{21} are constructed in the same way as P_6. The details of the activities are given in table 3. A working capital constraint of $50,000 was used.

A portion of the final programming tableau is presented in table 4. The items in the P_j column indicate which activities occur in the final program. The values in the P_0 column indicate the levels at which the activities in the final program are operated. The Z_i element in the P_0 column

[7] Linear programming studies have been used previously to analyze the problem of integrating crop and livestock enterprise. For example, see Peterson (*9*, pp. 546–554) and Swanson (*12*, pp. 1249–1259).

TABLE 2.—*Linear program for a California range feeder-steer operation* [1]

Item	Quantity available	Disposal activities					Real activities															
		P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{17}	P_{18}	P_{19}	P_{20}	P_{21}
Land class 1	46 acres	1	0	0	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0
Land class 2	86 acres	0	1	0	0	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
Land class 3	505 acres	0	0	1	0	0	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0
Land class 4	348 acres	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
Working capital [2]	$50,000	0	0	0	0	1	159	96	53	125	76	43	235	129	72	185	104	60	54	42	33	21
Gross ranch profit	Z_i	0	0	0	0	0	72	38	13	79	42	15	100	42	10	111	47	12	34	27	24	14

[1] P_1, P_2, P_3, P_4, and P_5 are disposal activities. The rate of stocking, (n) which is the number of head grazed per acre on each land class, the average gain per head per day, (g) and the rate of fertilization for each of the activities (X) are shown in table 3. The grazing period is 130 days long for the fertilized range (starting in February) and 110 days long for the unfertilized range.

[2] Included in working capital requirements are (1) the cost of the fertilizer, (2) the cost of the application, and (3) the cost of the livestock.

($24,625). The Z_i-C_i values listed under the disposal activities P_1 through P_5 indicate the marginal value of product of the four land classes and working capital. There were $8,573 of the $50,000 available working capital which were not used. Therefore, the marginal value of product of added working capital is zero. Only land classes 1 and 2 would be fertilized and at the heavier rate, in the final program. The final program called for the heavier rate of gain and the lower stocking rate. The gross ranch profit with no fertilization would be $19,363 and require $30,069 in working capital.

Conclusion

The schematic diagram of three stages outlined in figure 1 was used as a basis for developing a model of a particular livestock firm. Only three inputs, the number of livestock, the average daily gain, and the level of fertilization per acre for three land classes were allowed to vary. The "best" of alternative combinations of these three inputs were chosen with the use of a programming model.

Maximization of the GRP function was not a straight-forward exercise in the calculus. The breakdown of the operations of the livestock firm into the three operating stages was the basis for conceptualizing the problem. While this type of conceptualization is of much greater value to the researcher than to the actual manager, it does provide a clearer understanding of the important relationships between variables affecting the profits of a livestock operation. Managers of livestock firms may benefit directly as the result of improved research into the economic structure of their operations. Alternative combinations of the variables considered significant can be synthesized. Combinations can be used that are both different and more numerous than those alternative combinations found in either operating livestock firms or experimental work. The development of the three stages, and their subsequent integration suggests areas in which experimentation may have considerable economic relevance.

In the development of the three stages, it was necessary to make numerous assumptions about the nature of the stages and the nature of the relationship between them because of lack of adequate data. When the assumptions become uncomfortable for the researcher to live with (in this study, many of them were of this nature), a plea must be made for more knowledge. It was found, however, that much more useful information was available than is often utilized in economic studies involving physical production problems of this kind. For instance, in the illustration used, the form in which experimental data on range fertilization was made available to the rancher would not permit a systematic consideration of the impact of different levels of stocking and fertilization on profits. In order to accomplish this, considerable secondary information must be combined with the experimental data in an appropriate economic model.

A simple livestock firm and a limited number of alternative input combinations were used to demonstrate the use of a schematic model. Addi-

TABLE 3.—*Rate of stocking (n), average gain per head per day (g), and rate of fertilization (X) for activities P_6–P_{21}*

Activity	Rate of stocking per acre (n)	Gain per head per day (g)	Rate of fertilization (X)
	Head	*Pounds*	*Pounds*
P_6	1.70	1.4	200
P_7	.99	1.4	200
P_8	.49	1.4	200
P_9	1.31	2.0	200
P_{10}	.76	2.0	200
P_{11}	.37	2.0	200
P_{12}	2.47	1.4	400
P_{13}	1.26	1.4	400
P_{14}	.60	1.4	400
P_{15}	1.90	2.0	400
P_{16}	.97	2.0	400
P_{17}	.46	2.0	400
P_{18}	.79	2.0	0
P_{19}	.62	2.0	0
P_{20}	.49	2.0	0
P_{21}	.31	2.0	0

tional variables could have been considered, such as alternative methods of feed procurement, various price levels, expected price spreads, various initial weights, and different classes of livestock. But the purpose was only to illustrate the usefulness of one approach to problems of a livestock firm.

The same technique could be useful in analyzing dairy operations, feed lot operations and "hog factory" operations. In some of these cases it may be helpful to specify more detailed stages. Certain research projects may deal with either specific stages or with the entire sequence. Since any livestock firm consists of a series of operating stages, ignoring either their existence or their interaction results too often in the solutions of many managerial problems by implicitly assuming them away.

TABLE 4.—*A portion of the final programming tableau for a California range feeder-steer operation*

Final P_j activities	Operating levels			Disposal activities				
----------	P_0	----	----	P_1	P_2	P_3	P_4	P_5
P_{15}	46	----	----	----	----	----	----	----
P_{16}	86	----	----	----	----	----	----	----
P_{20}	505	----	----	----	----	----	----	----
P_{21}	348	----	----	----	----	----	----	----
P_5	8,573	----	----	----	----	----	----	----
Z_1	24,625	----	----	----	----	----	----	----
Z_i–C_i	--------	----	----	111	47	24	14	0

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Using Discrete Programming

By Clark Edwards

In many important economic problems, a variable is maximized subject to constraints. In a subset of such problems, a linear combination of decision variables is maximized subject to linear constraints. The latter subset is amenable to linear programming analysis. Efforts to expand the usefulness of linear programming methods usually involve incorporating nonlinear elements either in the criterion function or in the constraints. Such efforts frequently result in discovering ways to incorporate the nonlinear element in some acceptable linear form, thus retaining the usual linear programming procedure but broadening the researcher's capacity to apply the method to important economic problems (*9*).[1] Discrete programming is a case in point. Discrete programming problems and ordinary linear programming problems are about the same, except that a side condition is imposed that some of the decision variables must take on discrete values, usually nonnegative integers. The resultant, noncontinuous nature of the criterion function or of the constraints places discrete programming in the class of nonlinear programming (*10*). Sufficient conditions for a solution to discrete programming problems have been known for several years (*15*). Recently, systematic procedures for solving discrete programming problems have been put forward (*14, 16*). This paper discusses one of them. Decks and tapes for solving such problems on high-speed computers are not yet abundant, but it would be easy to supply them should the demand arise.

A LINEAR PROGRAMMING problem is one in which elements of a decision vector x are to be chosen in such a way as to maximize Q

(1) $Q = c'x$

subject to

(2) $Ax \leqq b$

and to

(3) $x \geqq 0$

Integer programming is done under the additional constraint that

(4) some elements of x are integers.

If Q is net revenue in a farm management problem and x_j is the swine activity, to insist by (4) that x_j be an integer is simply to require that if some sows are to be farrowed in the optimal farm plan, an integral number must be farrowed. If, without the discrete restriction, the optimal number of sows were estimated at 17.682, the question arises (in this case, perhaps, a somewhat trivial question) as to whether some limited resources should be withdrawn from other activities to increase sow numbers to 18 (or 19) or whether it would be more profitable to farrow 17 (or 16) sows and release some limited resources for other uses.

Some integer programming restrictions are shown in figure 1. Line ABCD represents the feasible, noninteger boundary for pairs of x_1 and x_2 given levels for activities x_3 to x_n. Inside ABCD, the boundary for integer pairs of values is marked with a dotted line. With RR depicting the price ratio, or choice indicator, B marks the high profit point of the noninteger solution. E marks the high profit point for x_1 and x_2 integers but x_3 to x_n held constant. In a simultaneous integer solution of all n activities, the optimal level of, say, x_7 might change sufficiently that the best combination of x_1 and x_2 would shift away from E, say, to F or G.

The marginal utility of adding a discrete restriction to a programming problem may be negligible when the magnitude of the decision variable is large relative to the size of the discrete jump. For example, it may be less interesting to know if a number like 103.467 should round up to 104 or down to 103 than to know if a number like 0.674 should round up to 1.0 or down to zero. It is the systematic way in which discrete programming handles relatively large jumps that makes it useful and interesting.

[1] Italic numbers in parentheses refer to Literature Cited and Selected References, p. 59.

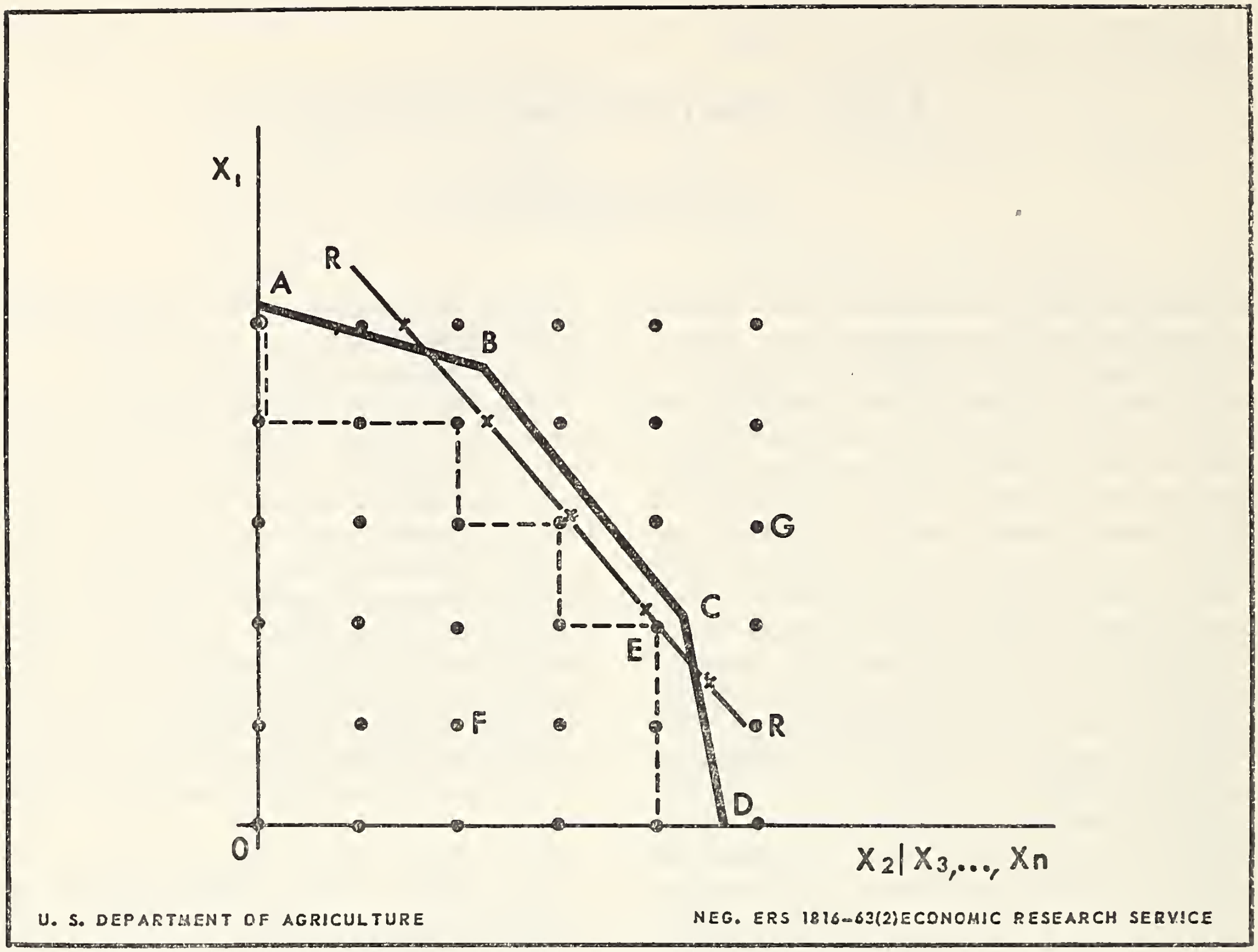

FIGURE 1.—Some integer programming restrictions.

The integer programming method can handle several kinds of problems (*8*):

1. It may ensure that variables in the solution are positive integers.

2. It may incorporate all-or-nothing-at-all restrictions such as owning a combine or not; and as using the full wheat acreage allotment or producing no wheat. Lump-sum costs or resource requirements associated with the all-or-nothing decision may be allowed for. This sort of restriction usually employs a variable confined to two values, 0 and 1.

3. Admissible ranges for levels of inputs or outputs may be established by integer programming, as when a poultry enterprise must have at least 1,000 layers if poultry is included in the optimal farm plan. Variations in net revenues or resource requirements with respect to ranges of values for specific variables may be incorporated.

4. Either-or choices may be analyzed in the integer programming framework as when one would either milk dairy cows or feed beef cows, but not both.

5. Multiple choice problems in which one element is chosen from a set of elements are amenable to integer programming. Choosing among equidistant numbers on the real line is a trivial example and includes choosing among real integers, even numbers, numbers which are multiples of 10, and so on. Choosing among a finite set of numbers not equidistant from one another is associated with some decisions concerning size of enterprise or level of resource use. In an economies-of-scale study, one could analyze a feeder cattle enterprise with the following alternative sizes: 0, 25, 100, 500, and 1,000. Choices among finite sets not usually measured numerically may also be incorporated in integer programs such as choosing

among five crop rotations considered for a farm; or, choosing among three feeding methods considered for a hog enterprise.

Writing the Discrete Restrictions

Writing a discrete restriction in a programming problem frequently involves devising a scheme for restating a nonlinear, discontinuous problem in a linear, continuous form, equations (1), (2), and (3), and then imposing the added restriction that one or more variables are integers, equation (4). Some examples follow:

Example 1.—Let x_j be acres of corn and x_k a (0,1) variable representing a fixed complement of machinery and equipment necessary for handling a corn crop. Net revenue, equation (1), includes the terms

$$c_j x_j + c_k x_k \tag{5}$$

where c_k is the annual cost of making the machinery and equipment available and c_j is the change in net revenue associated with growing an acre of corn given that the machinery has been made available.

A complete statement of the corn-machinery problem could include the statement that

$$\begin{gathered} x_j > 0 \rightarrow x_k = 1 \\ x_k = 0 \rightarrow x_j = 0 \end{gathered} \tag{6}$$

which says that if corn is grown, machinery must be made available, and if machinery is not made available, corn can not be grown. That machinery might be made available without growing corn is feasible according to (6) but not economic according to (5). Growing corn without equipment is not feasible according to (6).

While (6) states the discrete restriction, it is not in the form of equation (2) and is not amenable to ordinary programming procedures. The restriction may be translated to a usable form by introducing a dummy parameter based on the knowledge that the variable x_j has an upper bound imposed by other constraints to the problem. For instance, one of the restrictions in (2) may state that only 100 acres of corn land are available in which case any real number not less than 100 may be used as an upper bound for x_j. Let us call the dummy parameter α_j. Now we write the restriction

$$\begin{gathered} x_j \leq \alpha_j x_k \\ x_k \text{ an integer} \end{gathered} \tag{7}$$

which conforms to (2) and (4) and can be shown to satisfy (6). The terms (5) would appear in equation (1) and the inequality in (7) would appear as one of the equations in (2). According to (7), it is not feasible to grow corn unless machinery is made available. If corn is not grown on the farm, the profit maximizing criterion requires that no machinery is made available. If corn is grown, then (7) requires that at least one set be made available and the profit maximizing criterion requires that not more than one set of machinery is acquired. Thus x_k will be either 0 or 1 in the optimal solution. Many enterprises require an initial, overhead cost in addition to a variable, unit cost. Discrete programming is an efficient way to distribute fixed charges. The term $c_k x_k$ in (5) represents a fixed charge in terms of a cash outlay, or a lump sum reduction in net revenue.

If the fixed cost were measured in terms of using up a lump sum of limited resources such as building space or labor, rather than in terms of directly reducing net revenue, a similar restriction results. Interpreting (7) as above, set $c_k = 0$ to show that net revenue is not directly affected by a change in the (0,1) variable, x_k. Then, letting the i^{th} equation in (2) represent the labor restriction, let a_{ik} reflect the fixed labor requirement. See appendix table 1, line 5, for an illustration. Now if $x_k = 1$ the fixed labor supply is reduced by a_{ik}, but if $x_k = 0$ the labor supply is not affected.

Example 2.—In example 1, the discrete variable takes one of two values: 0 and 1. In other problems, the discrete variable may assume any nonnegative integer. For instance, perhaps the number of storage bins is to be an integer and there must be at least one bin for each 10,000 bushels of grain; or the number of machines employed in an activity must be an integer and there must be at least 1 machine for each 10 units of labor.

Let x_j be an activity providing labor and x_k one providing machines and suppose the number of machines supplied must be an integer. The supplies of x_j and x_k may be perfectly elastic with unit costs represented by c_j and c_k. In this event, neither variable would have an upper bound as did the variable in the previous example. Appropriate coefficients in the labor restriction equation would ensure that an adequate quantity of labor,

x_j, is made available. In addition, an equation saying that $x_j \leqq \alpha x_k$ as in equation (7), will ensure that there will be at least one machine for each α units of labor. Hence, while other restrictions in (2) would lead to the optimal labor utilization, (7) would ensure an adequate supply of machinery.

Example 3.—Sometimes it is desirable to impose the restriction that if an activity is used, it is used at least at a minimal level. For instance, if wheat is to be grown then at least 15 acres must be grown; and, if cows are to be milked then at least 25 head must be in the herd. Let us say that for a dairy activity, x_j, it is known from other constraints in the problem that 100 is an upper bound for x_j. Furthermore, we know from prior economic analysis that if there is to be a dairy activity in the final solution, the herd must have at least 25 cows. That is, we wish to impose

$$\text{(8)} \qquad \begin{array}{ll} \text{either} & 25 \leqq x_j \leqq 100 \\ \text{or} & x_j = 0 \end{array}$$

This nonlinear, discontinuous condition can be written in a form suitable for ordinary programming methods by writing

$$\text{(9)} \qquad \begin{array}{l} x_j \leqq \alpha_j x_k \\ x_j \geqq \beta_j x_k \\ x_k \text{ an integer} \end{array}$$

where α_j is the upper bound, 100, and β_j is the lower bound, 25. If introducing the first 25 cows involved an overhead cost, this could be reflected by a value for c_k in the profit equation. If the first 25 cows affected the i^{th} restriction differently from the additional cows, as by requiring a lump sum labor requirement in addition to the variable labor requirement per cow, this could be reflected by a value for a_{ik} in the i^{th} equation of (2). In (9), it follows that if $x_k=0$, $x_j=0$. If $x_k=1$, $25 \leqq x_j \leqq 100$ as required in (8). An uneconomic-size herd of, say, 10 head is not feasible because $x_j=10$ violates the first inequality of (9) if $x_k=0$ and violates the second inequality if $x_k=1$.

Example 4.—Discrete variables may assume one of several values which are not equidistant from one another. One might want to consider wheat acreages of 0, 15, and 40 acres or consider dairy enterprises of 0-, 25-, 60-, and 100-cow herds without examining other activity levels. In the dairy example, each herd size may be considered efficient for alternative sets of buildings and equipment, qualities of cows, rates of feeding, and requirements of labor.

A restriction of the form

$$\text{(10)} \qquad \begin{array}{l} x_j = 25x_k + 60x_m + 100x_n \\ x_k,\ x_m,\ x_n \text{ are integers} \end{array}$$

would permit examination of the four dairy alternatives. If each of the integer variables were zero, x_j would also be zero.

If, in the profit maximizing process, more than one herd appeared in the final solution and at most one herd was considered admissible, the further restriction would be imposed that

$$\text{(11)} \qquad 1 \geqq x_k + x_m + x_n$$

Forcing the equality in (11) would result in exactly one herd and allowing the inequality would allow none. Frequently, an equation of type (11) will not be violated even when not explicitly imposed due to the nature of the criterion function (1) and to other restrictions in (2).

As in prior examples, values for c_k, c_m, c_n, a_{ik}, a_{im}, and a_{in} may be incorporated to account for lump sum increments in costs, returns, or resource use associated with alternative levels of x_j.

Example 5.—If a farm organization could include either Herefords or Holsteins but not both, farrow either one litter of pigs per year or two litters but not both, or plant either corn or soybeans in a field but not both, then a restriction of the form

$$\text{(12)} \qquad x_i \cdot x_j = 0$$

would require that at least one of the activities not enter into the solution. This nonlinear condition may be imposed by two linear inequalities in an integer program. Let α_i and α_j be upper bounds for x_i and x_j, respectively. Such bounds are always implied by other constraints in (2). Then incorporate the restriction that

$$\text{(13)} \qquad \begin{array}{l} x_i \leqq \alpha_i x_k \\ x_j \leqq \alpha_j (1 - x_k) \\ x_k \text{ an integer} \end{array}$$

with the understanding that x_k is confined to the integers 0 and 1. Should difficulties in computation arise from $x_k \geqq 2$, simply impose the additional restriction that $x_k \leqq 1$. See appendix table 1, lines 6 and 7, for an illustration.

Displaying the Discrete Solution

Let the integer programming problem be stated in the form of equations (1) to (4) with (2) including equations such as (7), (9), (10) and (13). The first step in displaying the integer solution is to solve equations (1), (2), and (3), ignoring (4) to get a first approximation (*14*).

The first approximation allows variables which are supposed to be integers to take on noninteger values and therefore need not be a feasible solution. For example, one of the (0,1) variables might have the value of .90 in the first approximation. This might mean that only 90 percent of a combine or 90 percent of a dairy barn is allowed for in the solution. The next step is to add an additional equation to the noninteger solution matrix, incorporating information as to which of the variables are to be integers, and to proceed with the computations.

The rules for discrete programming are simpler if equation (4) is changed to read "all x_j are integers." Let us examine first the complete integer solution and then take up the partial integer solution.

The Complete Integer Solution

Let equation (14) represent the i^{th} of m restrictions in the first approximation, or the noninteger solution of a linear program for which an integer solution is desired.

$$(14) \qquad \sum_{j=1}^{n} a_{ij}x_j = b_i \qquad (1 \leq i \leq m)$$

where it is understood that

$$(15) \qquad \begin{array}{ll} a_{ij}=1 & \text{for the coefficient of the } x_j \text{ having the value } b_i \text{ in the current basis, say } x_k \\ a_{ij}=0 & \text{for coefficients of all other } x_j \text{ included in the current basis} \\ -\infty < a_{ij} < \infty & \text{for coefficients of all } x_j \text{ not included in the current basis} \end{array}$$

Some of the x_j in (14) are real activities, some are slack variables and others were introduced through equations such as (7). Any of the m equations may be chosen for (14) provided b_i is not, in fact, an integer in the first approximation. See appendix table 1 line 12 for an example.

The assertion that all x_j are integers requires that all b_i in (14) are integers. But (14) was derived without imposing the integer restriction. If, on inspection, the b_i are integers, there is no further integer problem. If some b_i are not integers, an additional constraint is required. The additional constraint will expand the solution matrix from an m by n to an m+1 by n+1 matrix. The additional constraint is devised by operating on equation (14).

Let $[b_i]$ equal the largest integer less than or equal to b_i and define δ_i such that

$$(16) \qquad \delta_i = b_i - [b_i]$$

Substituting (14) into (16),

$$(17) \qquad -\sum_{j \neq k}^{n} a_{ij}x_j + [b_i] - x_k = -\delta_i$$

where x_k is the activity in the current basis assigned the value b_i.

Imposing the condition that all x_i are integers on (17) implies that a change in any a_{ij} by an integer amount changes the product a_{ij} x_j by an integer amount. Let a^*_{ij} equal a_{ij} plus a (positive or negative) integer such that

$$(18) \qquad 0 \leq a^*_{ij} < 1$$

then, from (17),

$$(19) \qquad -\sum_{j \neq k}^{n} a^*_{ij}x_j + x_{n+1} = -\delta_j$$

where, if all x_j are integers, x_{n+1} is an integer. The term x_{n+1} is a collection of the integers $[b_i]$ and x_k and the integer changes in the $a_{ij}x_j$.

Furthermore, x_{n+1} is a nonnegative integer. Inasmuch as (20)

$$(20) \qquad \sum_{j \neq k}^{n} a^*_{ij}x_j \geq 0$$

it follows from (19) that

$$(21) \qquad \delta_i + x_{n+1} \geq 0$$

But since δ_i is nonnegative and less than 1 and since x_{n+1} is an integer, it follows from (21) that x_{n+1} is a nonnegative integer.

Equation (19) contains the information that the x_j should be integers and (19) is the additional constraint needed to display the integer solution. x_{n+1} is the additional, slack variable. The new equation and the new variable are added to the matrix containing (14) and at least one more iteration is calculated. There is not a direct illustration of (19) in the appendix table because the problem there allows some variables to remain nonintegers. However, line 13 in the table illustrates the analogous restriction for the mixed integer problem as per equation (36).

Equation (19) is not satisfied at the time it is added to the system. In the first place, not all the x_j are integers as required. In the second place, the slack variable x_{n+1} equals $-\delta_i$ in the current basis which violates the nonnegativity restriction (3). The first computation should be one which re-imposes (3). Then proceed as usual until the second approximation is reached.

With $x_{n+1}<0$ in the current basis, the usual simplex procedure needs a broader than usual interpretation in order to assure that the negative slack is removed from the basis and in order to select an x_j to enter the basis which will not drive some other x_i negative. This is easy to do if the computation is being done with pencil and paper or a desk calculator. Most linear program routines for high speed computers are not designed to handle a negative variable properly. An iteration to remove the negative variable may precede reloading the problem on the machine. Modifications are needed in existing routines to generate the $(m+1)^{st}$ equation and the $(n+1)^{st}$ variable and to handle the negative x_{n+1}. Such modifications are not particularly complicated.

Alternatively, the expanded matrix may be reloaded immediately by multiplying equation (19) by -1 and adding a dummy slack with an indefinitely large negative c value in the criterion function. This procedure adds one equation and two slacks to the original matrix and is illustrated by line 13, table 1 in the appendix.

Sometimes it will happen that the second approximation will satisfy conditions (1) through (4) and display the optimal, integer solution. However, it also sometimes happens that the second approximation contains some unwanted nonintegers as in the example in the appendix. This could result when a_{ij} for a nonbasis integer variable in the first approximation is between zero and one such that $a^*_{ij}=a_{ij}$ as well as when an integer variable is already in the basis. In either event, the new equation would not contain the required information concerning the integer variable. If the second approximation fails to display the required integer solution, generate an additional equation $(m+2)$ and an additional slack variable (x_{n+2}) and proceed as before.

The method of complete integer programming may be outlined in three overtly simple steps:

A. Solve the problem without imposing the integer restriction (4).

B. Add a new slack variable, x_{n+1}, and a new equation of the form:

$$-\sum_{j=1}^{n} f_{ij}x_j+x_{n+1}=-\delta_i \tag{22}$$

where

δ_i = the fractional part of an integer variable which assumed a noninteger value in the current basis

and where

$$f_{ij}=\begin{cases}0 \text{ for all } j \text{ such that } x_j \text{ is in the current basis}\\ a^*_{ij} \text{ for all } j \text{ such that } x_j \text{ is not in the current basis}\end{cases}$$

and where

$$a^*_{ij}=\begin{cases}\text{the fractional part of } a_{ij} \text{ for } a_{ij}\geq 0\\ \text{one minus the fractional part of the absolute value of } a_{ij} \text{ for } a_{ij}<0.\end{cases}$$

C. Continue computing until either the optimal, integer solution appears or until another noninteger optimum appears. In the latter event, repeat steps B and C.

The Partial Integer Solution

The partial, or mixed integer solution of a programming problem requires that some x_j are integers whereas the complete integer solution requires that all x_j are integers (*12*). Consider again equation (14) as the i^{th} of m restrictions in the first approximation, or the noninteger solution of an integer problem. The i^{th} equation may be chosen from the m restrictions in any manner provided that b_i should be an integer in the final solution but is not an integer in the current basis. See line 12 of the appendix table for an example. Defining δ_i as in (16), the additional restriction required for the partial integer solution may be developed by operating on (17)

$$-\sum_{j\neq k}^{n} a_{ij}x_j+[b_i]-x_k=-\delta_i \tag{17}$$

where x_k is the activity in the current basis assigned the value b_i.

Breaking the summation on the left hand size of (17) into two parts according to whether a_{ij} is negative or nonnegative, we have

$$(23) \qquad \sum_{j\neq k}^{n} a_{ij}x_j = \sum_{a_{ij}\geq 0} a_{ij}x_j + \sum_{a_{ij}<0} a_{ij}x_j$$

For alternative values of the x_j, the summation on the left hand side of (23) will be either negative or nonnegative. If it is nonnegative, considering that x_k is an integer and that the summation must therefore differ from δ_i by an integer amount, it follows that

$$(24) \qquad \sum_{j\neq k}^{n} a_{ij}x_j \geq \delta_i$$

or

$$(25) \qquad \sum_{a_{ij}\geq 0} a_{ij}x_j + \sum_{a_{ij}<0} a_{ij}x_j \geq \delta_i$$

which implies

$$(26) \qquad \sum_{a_{ij}\geq 0} a_{ij}x_j \geq \delta_i$$

If, on the other hand, the summation in (23) is negative, it follows that

$$(27) \qquad \sum_{j\neq k}^{n} a_{ij}x_j \leq \delta_i - 1$$

or

$$(28) \qquad \sum_{a_{ij}\geq 0} a_{ij}x_j + \sum_{a_{ij}<0} a_{ij}x_j \leq \delta_j - 1$$

which implies

$$(29) \qquad \sum_{a_{ij}<0} a_{ij}x_j \leq \delta_i - 1$$

multiply both sides by -1

$$(30) \qquad -\sum_{a_{ij}<0} a_{ij}x_j \geq 1 - \delta_i$$

and multiply both sides by a constant containing δ_i

$$(31) \qquad -\left(\frac{\delta_i}{1-\delta_i}\right) \sum_{a_{ij}<0} a_{ij}x_j \geq \delta_i$$

and rearrange

$$(32) \qquad \sum_{a_{ij}<0} \left(\frac{\delta_i}{\delta_i - 1}\, a_{ij}\right) x_j \geq \delta_i$$

Combining (26) and (32) leads to the assertion that

$$(33) \qquad \sum_{a_{ij}\geq 0} a_{ij}x_j + \sum_{a_{ij}<0} \left(\frac{\delta_i}{\delta_i - 1}\, a_{ij}\right) x_j \geq \delta_i$$

and it is on the inequality (33) that the added restriction to the partial integer programming problem is based. Introducing a nonnegative slack variable into (33) produces the equation

$$(34) \qquad -\sum_{a_{ij}\geq 0} a_{ij}x_j - \sum_{a_{ij}<0} \left(\frac{\delta_i}{\delta_i - 1}\, a_{ij}\right) x_j + x_{n+1} = -\delta.$$

In obtaining (34), the variable x_k was required to be an integer. If x_k is the only integer variable in the problem, equation (34) is the additional restriction required. If some variables in the current basis in addition to x_k are required to be integers, additional restrictions similar to (34) need to be generated. Such information can not be incorporated in (34) because the a_{ij}'s for x_j's included in the current basis are each zero.

If some variables not in the current basis are required to be integers, this information can be introduced into equation (34) in a manner entirely analogous to the adjustments used to transform equation (17) into (19) in the complete integer problem. The procedure is to adjust the left hand side of (33) resulting in a stronger inequality. We shall use the property that the smaller the coefficient of an x_j in (33) the stronger the inequality.

For an integral x_j not in the current basis, its coefficient a_{ij} in (23) may be changed by an integer amount to a_{ij}^*. Such a change in a coefficient would not destroy the fact that when x_k is an integer the summation in (23) differs from δ_i by an integer amount. We wish to change the coefficient of x_j in (23) by an integer amount in a way which results in the inequality in (33) becoming as strong as possible. Among the nonnegative numbers differing from a_{ij} by an integer amount, a coefficient less than 1 leads to the strongest possible inequality. Among the negative numbers differing from a_{ij} by an integer amount, a coefficient greater than -1 leads to the strongest possible inequality.

Therefore, defining a_{ij}^* as a_{ij} plus an integer amount such that

$$(18) \qquad 0 \leq a_{ij}^* < 1$$

as before, the choice for a coefficient for x_j in (23) narrows to

$$(35) \qquad \text{either} \quad a_{ij}^* \qquad \text{or} \quad a_{ij}^* - 1$$

whichever leads to the strongest inequality in (33).

If we choose a_{ij}^* for the coefficient of x_j in (23), x_j has the coefficient a_{ij}^* in (33). On the other hand, if we choose $a_{ij}^* - 1$ as the coefficient in (23),

x_j has the coefficient $\delta_i(a_{ij}^*-1)/(\delta_i-1)$ in (33).

It happens that if $a_{ij}^*=\delta_i$, either choice in (35) would lead to identical results. Consequently, the inequality (33) will be as strong as possible if we choose the coefficient of x_j in (23) for x_j an integer variable according to the rule:

If $a_{ij}^*\leq\delta_i$, let the coefficient in (23) of $x_j=a_{ij}^*$

and

If $a_{ij}^*>\delta_i$, let the coefficient in (23) of $x_j=a_{ij}^*-1$

The method for partial integer programming may be outlined in three overtly simple steps:

(A) Solve the problem without imposing the integer restriction (4)

(B) Add a new slack variable, x_{n+1}, and a new equation of the form (36):

$$(36) \qquad -\sum_{j=1}^{n} f_{ij}x_j+x_{n+1}=-\delta_i$$

where

δ_i=the fractional part of an integer variable which assumed a noninteger value in the current basis

and where

$$f_{ij}=\begin{cases} 0 & \text{for all j such that } x_j \text{ is in the current basis} \\ a_{ij} & \text{for all j such that } x_j \text{ is a noninteger variable not in the current basis and } a_{ij}\geq 0 \\ \frac{\delta_i}{\delta_i-1}a_{ij} & \text{for all j such that } x_j \text{ is a noninteger variable not in the current basis and } a_{ij}<0 \\ a_{ij}^* & \text{for all j such that } x_j \text{ is an integer variable not in the current basis and } a_{ij}^*\leq\delta_i \\ \frac{\delta_i}{\delta_i-1}(a_{ij}^*-1) & \text{for all j such that } x_j \text{ is an integer variable not in the current basis and } a_{ij}^*>\delta_i \end{cases}$$

and where

$$a_{ij}^*=\begin{cases} \text{the fractional part of } a_{ij} \text{ for } a_{ij}\geq 0 \\ \text{one minus the fractional part of the absolute value of } a_{ij} \text{ for } a_{ij}<0 \end{cases}$$

(C) Continue computing until either the optimal, integer solution appears or until another noninteger optimum appears. In the latter event, repeat steps B and C.

Appendix

By way of illustrating the integer programming procedure, let us seek to maximize profits for a farm producing oats, hay, milk, and beef where

$$(37) \qquad \pi=15x_1+6x_2+175x_3+76x_4-1000x_5$$

subject to

$$(38) \qquad \begin{aligned} 1.50x_1 + 2.00x_3 + .50x_4 + x_6 &= 120 \\ .50x_2 + 3.00x_3 + 3.50x_4 + x_7 &= 140 \\ 6.00x_1+3.50x_2+70.00x_3+40.00x_4+400.00x_5 + x_8 &= 3500 \\ 1.00x_3 - 44.00x_5 + x_9 &= 0 \\ + 1.00x_4 + 40.00x_5 + x_{10} &= 40 \end{aligned}$$

and to

$$(39) \qquad x_i\geq 0 \qquad (i=1, 2, \ldots, 10)$$

and to

(40) x_3, x_4 x_5, x_9, and x_{10} are integers

where the x's are interpreted as follows:

x_1 is tons of oats produced
x_2 is tons of hay produced
x_3 is number of milk cows
x_4 is number of beef cows
x_5 is a (0,1) variable
x_6 is slack acres of crop land
x_7 is slack acres of hay and pasture land
x_8 is slack hours of labor
x_9 is slack milk cow capacity
x_{10} is slack beef cow capacity

Equations (37) to (40) are counterparts to equations (1) to (4), respectively. In this example, we require that livestock units in the solution are integers and we require that the solution may include either milk cows or beef cows but may not include both. The first 2 equations in (38) ensure that the land used in producing crops and livestock does not exceed total land available on the farm. The third equation in (38) is the labor restriction. This equation indicates that 400 hours of labor are required for each unit of x_5; x_5 is a (0,1) variable which is to have the value of 1 if cows are milked. Thus, if cows are milked, a 400-hour fixed labor requirement is charged against the available supply of labor. At the same time, if cows are milked a fixed cost of $1,000 per year is charged against net revenue for providing services of specialized dairy equipment, according to equation (37). Distributing fixed charges in integer programming is discussed in connection with example (1), page 51.

The two final equations in (38) are counterparts of equation (13) in example (5) page 52. They ensure that the final solution does not contain both beef and dairy cattle. These equations recognize upper bounds for the dairy and beef enterprises of 44 cows and 40 cows, respectively. For the dairy cattle, as many as 60 head could be handled with the 120 acres of cropland available; as many as 46.666 head with the 140 acres of pasture and hay land available; but not more than 44.2857 head with the available supply of labor. Thus, 44 is the largest integer feasible for dairy herd size. Similarly, the hay and pasture land available limits to 40 head the maximum feasible beef herd.

The fourth equation in (38) says if $x_5=1$, the number of cows milked (x_3) plus the slack milk cow capacity (x_9) must total the maximum feasible dairy herd size. On the other hand, if $x_5=0$, x_3 and x_9 must each be zero. The fifth equation in (38) says that if $x_5=0$, the number of beef cows (x_4) plus the slack beef cow capacity (x_{10}) must total the maximum feasible beef herd size. If $x_5=1$, x_4 and x_{10} must each be zero.

Table 1 shows the steps required to reach the mixed integer solution by the ordinary simplex method. The upper section of the table shows the initial basis. The second section shows an approximate, noninteger solution which is obtained by solving equations (37), (38), and (39) without regard to (40). In the first approximation, oats, milk, and beef activities are used to produce a net revenue of $6,820. Three of the basis variables in the approximation, x_3, x_4, and x_5 are supposed to be integers; x_5, the (0,1) variable, has the value 0.9328. This means net revenue fails to reflect about $67 per year of the fixed cost of specialized dairy equipment and it means about 27 hours of the fixed dairy labor requirement are not charged against the fixed labor supply.

Using line (12) in table 1 as the counterpart of equation (14), line (13) is generated to incorporate the information that some variables are to be integers. Line (13) is the counterpart to equation (36). The fractional part of the basis variable in line (12) is $\delta_t=0.9328$. Following the rules for mixed integer programming listed on page 56:

$$f_{t2}=a_{t2} \qquad =.0022$$

$$f_{t6}=\frac{\delta_t}{\delta_t-1}\,a_{t6} \qquad =(-13.8889)(-.0025) \qquad =.0345$$

$$f_{t8}=a_{t8} \qquad =.0006$$

$$f_{t9}=\frac{\delta_t}{\delta_t-1}\,(a^*_{t9}-1)=(-13.8889)(.9614-1.000)=.5355$$

$$f_{t10}=\frac{\delta_t}{\delta_t-1}\,(a^*_{t10}-1)=(-13.8889)(.9764-1.000)=.3282$$

To avoid a negative basis variable in line (13), and thereby to facilitate reloading the problem on a highspeed computer, line (13) reflects -1 times equation (36) as per the discussion on page 54 of the text. x_{11} is the counterpart of the slack variable x_{n+1} in equation (36) and x_{12} is a dummy slack. The $1,000 negative coefficient of x_{12} in the criterion function, line (2), is sufficiently large to preclude x_{12} from the final solution.

Line (14) of table 1 shows the derivatives of the profit equation. Elements of line (14) are frequently referred to as C_j-Z_j values in programming literature. Before adding line (13), each derivative was negative indicating that the solu-

TABLE 1.—*Steps in the solution of a mixed integer programming problem* [1]

		Oats	Hay	Milk	Beef	Dairy equipment	Idle cropland	Idle hay and pasture land	Idle labor	Idle milk capacity	Idle beef capacity				
(1)	X_0	Non-integer X_1	Non-integer X_2	Integer X_3	Integer X_4	Integer X_5	Non-integer X_6	Non-integer X_7	Non-integer X_8	Integer X_9	Integer X_{10}	Non-integer X_{11}	Non-integer X_{12}	Non-integer X_{13}	Non-integer X_{14}
(2)		$15	$6	$175	$76	-$1000	$0	$0	$0	$0	$0	$0	-$1000	$0	-$1000
	INITIAL BASIS														
(3)	120.0000	1.5000		2.0000	.5000		1.0000								
(4)	140.0000		.5000	3.0000	3.5000			1.0000							
(5)	3500.0000	6.0000	3.5000	70.0000	40.0000	400.0000			1.0000						
(6)	0			1.0000		−44.0000				1.0000					
(7)	40.0000				1.0000	40.0000					1.0000				
	FIRST NONINTEGER APPROXIMATION														
(8)	24.3781	1.0000	−.0987				.7794		−.0282	.4146	.7380				
(9)	7.4027		.5174				−.0199	1.0000	.0050	−3.3085	−3.6891				
(10)	41.0448		.0958	1.0000			−.1095		.0274	−.6965	−1.0398				
(11)	2.6866		−.0871		1.0000		.0995		−.0249	1.5423	1.9453				
(12)	.9328		.0022			1.0000	−.0025		.0006	−.0386	−.0236				
(13)	.9328		.0022				.0345		.0006	.5355	.3282	−1.0000	1.000		
(14)			$1.69				$31.97		−$1.23	$495.48	$327.69	$1000.00			
	SECOND APPROXIMATION														
(15)	23.6559	1.0000			−1.0753		.7527				−.5014	−2.3226	2.3226		
(16)	13.2258				5.6878		.1935	1.0000	−.1429		4.0265	10.2028	−10.2028		
(17)	42.2581			1.0000	1.0565		−.0645				.4435	1.7419	−1.7419		
(18)	0		1.0000		−10.7143				.2857		−10.7143	−30.8571	30.8571		
(19)	1.0000				.0250	1.0000					−.0250				
(20)	1.7419				.0435		.0645			1.0000	.6565	−1.7419	1.7419		
(21)	.2581				.0565		.0224				.1427	1.7419	.6057	−1.0000	1.0000
(22)					$53.03		$22.24		−$1.71		$113.24	$1657.05	−$309.45	$1000.00	
	MIXED INTEGER SOLUTION														
(23)	15.0000	1.0000			−2.9687						−7.0822	−60.7483	−18.0000	33.5407	−33.5407
(24)	11.0000				5.2000			1.0000	−.1429		2.3574	−4.8210	−15.4286	8.6248	−8.6248
(25)	43.0000			1.0000	1.2187						1.0000	6.7500		−2.8749	2.8749
(26)	0		1.0000		−10.7143				.2857		−10.7143	−30.8571	30.8571		
(27)	1.0000				.0250	1.0000					.0250				
(28)	1.0000				−.1187					1.0000	1.0000	−6.7500		2.8749	−2.8749
(29)	11.5000				2.5156		1.0000				8.6235	77.6228	27.0000	−44.5612	44.5612
(30)					−$3.46				−$1.71		−$96.34	−$84.88	−$915.14	$0.	−$1000.00

[1] Numbers in this table are rounded to 4 decimal places, but computations were made carrying more decimals, so some computations may appear not to add up.

tion was optimal given the restrictions imposed. With the addition of line (13), several of the derivatives are positive indicating that, under the added restriction, some other feasible farm organization than the current one would be more profitable.

The third section of table 1 shows the second approximation obtained by seeking to maximize profits subject to the restrictions represented by lines (8) to (13) of the table. In the second approximation, oats and milk activities are used to produce a net revenue of $6,750, x_5 is an integer as required. However, x_3 and x_9, also in the basis, are not integers. Evidently the counterpart of inequality (33) was not strong enough to produce the desired integer solution—the process must be repeated.

Using line (17) as the counterpart to equation (14), the rules for mixed integer programming were used to generate line (21). Line (22) shows the $C_j - Z_j$ values. Solving the expanded, 7 by 14 matrix led to the solution in the lower section of the table in which oats and milk activities are used to produce a net revenue of $6,750, the same revenue as obtained in the previous approximation. The derivatives in line (30) are each negative, indicating that this is the mixed integer solution which maximizes (37) subject to (38, 39, and 40).

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Utilization of Wheat for Food

By H. Wayne Bitting and Robert O. Rogers

Raw materials used in the food processing industry must have specific properties for specified end uses. As specialization of food processing increases and as new food products are developed, the raw materials used in producing these food products are less easily substituted. The following article illustrates how this trend affects wheat utilization. This has implications in agricultural policy, marketing, and research. A lack of recognition of product properties can lead to loss of export markets, reduced farm income, and a continued decline in per capita consumption of wheat products. Economists often are unaware of product properties. As a result, some food consumption data include food items within a group which require different agricultural raw materials. In this paper an attempt is made to separate wheat foods according to the types of wheat required to produce them. While this research relates to wheat utilization in the domestic market, it has significance to markets for other agricultural commodities as well as wheat exports.

WHEAT is one of the oldest and most important food crops. The ability and capacity of the American wheat farmer to produce exceed our domestic market needs. Wheat production efficiency and know-how have outpaced the development of new markets. How can the markets for wheat be expanded? What can research do to help expand these markets? Utilization research has obtained information which should be useful in answering some of these questions.

All types of wheat cannot be used to make all kinds of wheat foods. For example, the following wheats possess specific properties: *Hard red spring and hard red winter* (high in protein and strong in gluten)—essential for quality yeast breads and hard rolls; *white and soft red winter*—necessary for good cakes and crackers; and *durum*—a special type used for good macaroni and spaghetti. For certain food uses, one type of wheat may be partially or completely substituted and still a quality food product can be made. For other food uses, different types of wheat cannot be substituted. As a result, there are certain years when there is a surplus of one and a shortage of another kind of wheat.

Getting the desired wheat properties for specific food uses is even more an exacting process than the selection of wheat on the basis of class. During the growing season the amount of moisture affects the properties which a given variety of wheat will have in any particular year. Even with the same variety, a wet season, accompanied by high yields, lowers the protein level as compared with a dry year and low yields. Likewise, the same variety produced under irrigation, versus nonirrigation, produces different properties. Location also affects wheat properties. A hard red winter wheat grown in a soft red winter area yields dissimilar properties from the same variety grown in a hard red winter area. In addition to the difference in properties, associated with the conditions under which wheat is grown, there are also variations in the properties demanded in wheat flour due to the baking methods used and the management skills of the baker. Add to these variations the fact that a desirable bread in one country may not be considered preferable by consumers in another country.

Despite all of these problems, it is essential that wheat growers endeavor to produce wheat which has the properties needed to make acceptable food products in the markets where the wheat is to be sold. If this is not done, wheat will suffer severe price discounts in the market place. Growers will plan their production more intelligently if they know which wheat varieties in their particular location produce suitable properties for specific food uses, and how many bushels are needed to satisfy these markets at home and abroad.

Domestic Food Market

In examining the domestic food market for wheat, it is essential to consider the end uses and the wheat properties desired for each use. Since commercial bakers are the primary users of wheat flour, the demands of the baking industry largely determine the flour characteristics for each end-product use.

As a first approximation, the amount of flour required for products purchased by consumers in retail stores has been divided among hard, soft, and durum wheats. Hard wheat (hard red win-

TABLE 1.—*Domestic food use of wheat, by type, United States, 1959–1960* [1]

Product	Wheat (million bushels)			
	Hard	Soft	Durum	Total
Bread	200. 15	-------	-------	200. 15
Rolls	10. 27	-------	-------	10. 27
Biscuits and muffins	-------	2. 87	-------	2. 87
Crackers	3. 40	18. 53	-------	21. 93
Cakes	-------	6. 11	-------	6. 11
Pies	-------	2. 52	-------	2. 52
Other sweet goods	11. 38	8. 10	-------	19. 48
Alimentary paste products	8. 20	-------	22. 00	30. 20
Flour:				
All purpose	85. 57	72. 33	-------	157. 90
Whole wheat	2. 38	-------	-------	2. 38
Cake	-------	12. 21	-------	12. 21
Prepared mixes	-------	22. 34	-------	22. 34
Wheat cereals	9. 65	1. 99	-------	11. 64
Total	331. 00	147. 00	22. 00	500. 00

[1] Robert J. Lavell, formerly with Economic Research Service, developed the estimates of flour consumption for individual States, based upon the consumption data from the 1955 Household Food Consumption Survey and related demographic data. Robert E. Post, formerly with ERS, provided corresponding data for total wheat disappearance by hard, soft, and durum types. The breakdown by type of flour for each of the major food product categories was developed by the following committee: Robert J. Lavell; Robert E. Post; Lawrence Zeleny, Chief, Standardization and Testing Branch, Grain Division, Agricultural Marketing Service; Philip Talbott, Executive Secretary, Grain Defense Planning Committee, Grain Division, Agricultural Stabilization and Conservation Service; Edward F. Seeborg, Cereal Technologist, Grain and Feed Division, Foreign Agricultural Service; Robert O. Rogers, Assistant Director, and H. Wayne Bitting, Staff Specialist (Food), Product and Process Evaluation Staff, Office of Administrator, Agricultural Research Service. These estimates were based upon the percentage composition of the several types of flour normally used in each food item of the product food group. The flour consumption data, by States, and end-product uses were adjusted to fit the total wheat disappearance, by class of wheat.

ter and hard red spring) accounts for 66.2 percent of the total domestic food usage; soft wheat, 29.4 percent; and durum, 4.4 percent. How are these classes of wheat used?

Hard wheats are used for the yeast-leavened products—primarily bread, rolls, and sweet goods. For the chemically leavened products—cakes, pies, cookies, doughnuts, biscuits, some pastries, and crackers—flour from soft wheat is used. The relative importance of each end-product use of wheat flour and the amounts of hard, soft, and durum wheats used by each food product are shown in table 1. Of the wheat used for food in 1961, wheat flour constituted 97.8 percent of the total, and wheat cereals the remaining 2.2 percent.

Can Research Expand the Market for Wheat?

Research can help the wheat farmer by lowering marketing and production costs and developing new or improved wheat products. A reduction in marketing or production costs, or both, could lead to increased returns to growers without changing the demand for wheat products. On the other hand, the development of new or improved wheat products could expand the demand for wheat. With expansion in demand, consumers would be willing to buy more wheat at the same price or pay more for a given quantity of wheat. Both the possibilities of reducing marketing costs and expanding the demand for wheat need to be explored for domestic and foreign markets.

Research may contribute to lowering marketing costs, primarily by reducing transportation costs. If wheat were produced and consumed within the same area, transportation costs would be minimized. Is this what we find? Five distinct classes of wheat are grown—hard red spring, hard red winter, soft red winter, white, and durum.

We know from our domestic utilization pattern that local wheat under present milling and baking practices do not always produce all the wheat foods consumed locally (table 1). For example, in the soft wheat regions, hard wheats or hard wheat flours are imported to produce bread, rolls, general-purpose, and whole-wheat flour. In the hard wheat regions, soft wheats or soft wheat flours are imported for cakes, cookies, crackers, and soft-wheat flours. To the extent that air classification of wheat flour enables local wheats to be used for a wider variety of baked products, a savings in transportation costs could result. With air classification, high protein-low starch fractions of flour can be separated from low protein-high starch fractions by use of air streams. Conceivably, this would enable flour millers consistently to tailormake flours for specific end uses without blending wheat varieties to obtain the desired protein-starch combinations as practiced under conventional methods. However, the potential savings in transportation are limited because 80 percent of the population resides in areas where only 30 percent of the wheat is produced. To examine this situation more specifically, see figure 1—a map showing the production and utilization of hard wheat, by individual States.

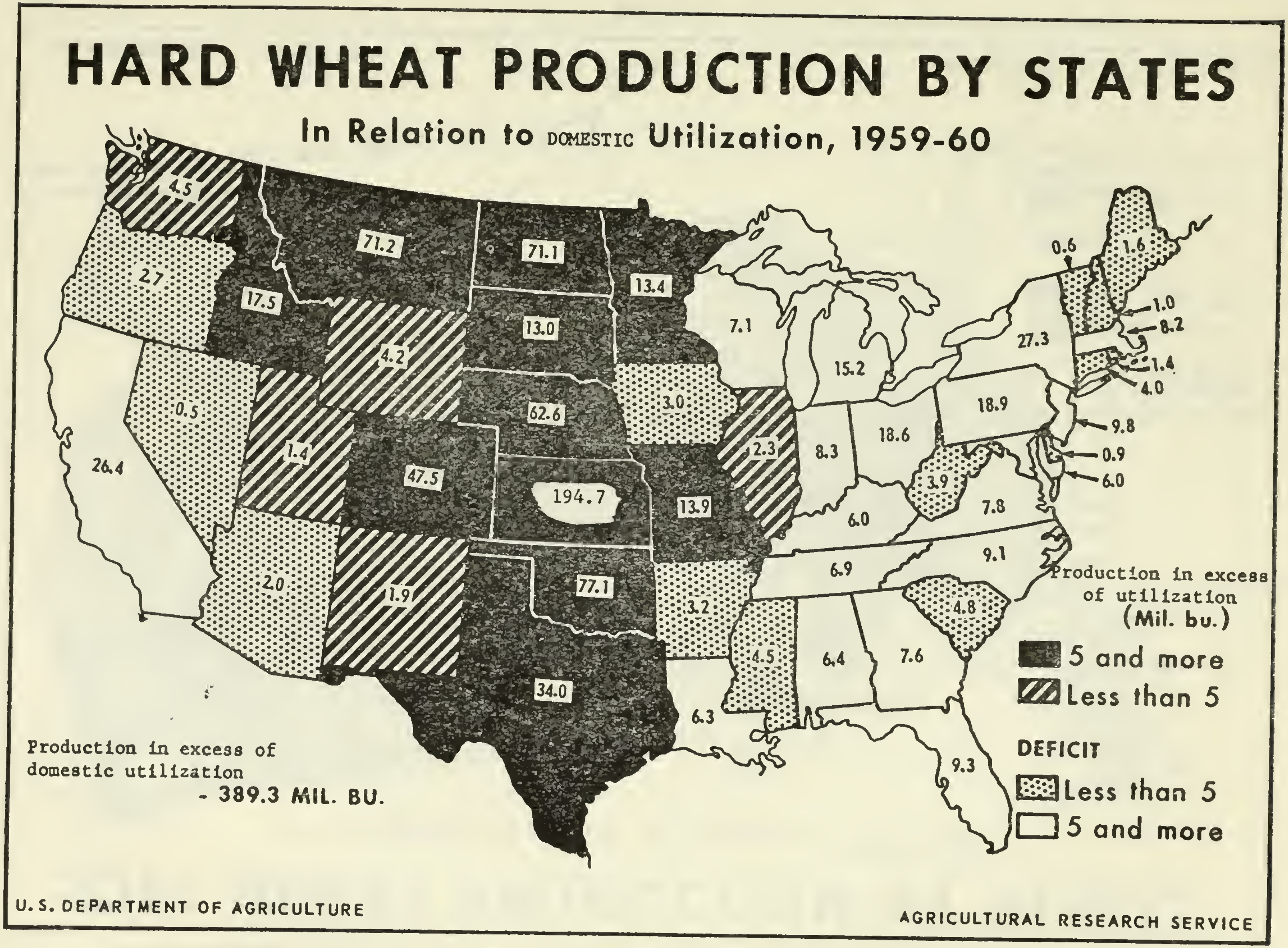
HARD WHEAT PRODUCTION BY STATES
In Relation to DOMESTIC Utilization, 1959-60
4.5
2.7
71.2
71.1
13.4
17.5
13.0
4.2
7.1
15.2
27.3
0.6
1.6
1.0
8.2
1.4
4.0
0.5
62.6
3.0
18.9
1.4
2.3
8.3
18.6
9.8
0.9
6.0
26.4
47.5
194.7
13.9
3.9
7.8
6.0
6.9
9.1
2.0
1.9
77.1
3.2
4.8
4.5
6.4
7.6
34.0
6.3
9.3
Production in excess of utilization (Mil. bu.)
5 and more
Less than 5
DEFICIT
Less than 5
5 and more
Production in excess of domestic utilization
- 389.3 MIL. BU.
U.S. DEPARTMENT OF AGRICULTURE
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FIGURE 1

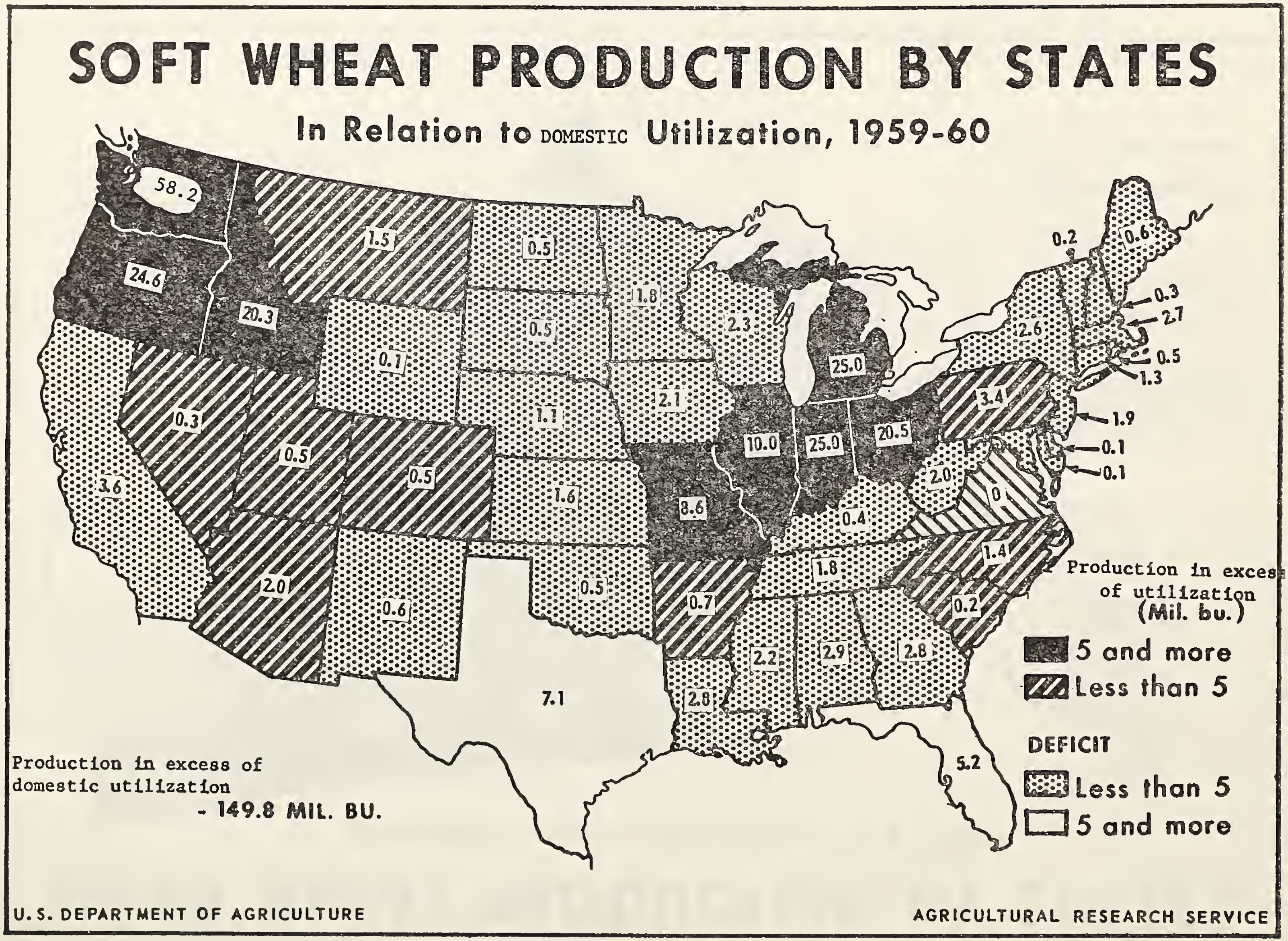
SOFT WHEAT PRODUCTION BY STATES
In Relation to DOMESTIC Utilization, 1959-60
58.2
24.6
20.3
1.5
0.5
1.8
2.3
0.5
0.1
0.3
3.6
0.5
0.5
1.1
2.1
25.0
10.0
25.0
20.5
8.6
1.6
0.4
2.0
0
2.0
0.6
7.1
0.5
0.7
1.8
1.4
0.2
2.2
2.9
2.8
2.8
5.2
0.2
0.6
0.3
2.7
0.5
1.3
2.6
3.4
1.9
0.1
0.1
Production in excess of domestic utilization
- 149.8 MIL. BU.
Production in excess of utilization (Mil. bu.)
5 and more
Less than 5
DEFICIT
Less than 5
5 and more
U.S. DEPARTMENT OF AGRICULTURE
AGRICULTURAL RESEARCH SERVICE

FIGURE 2

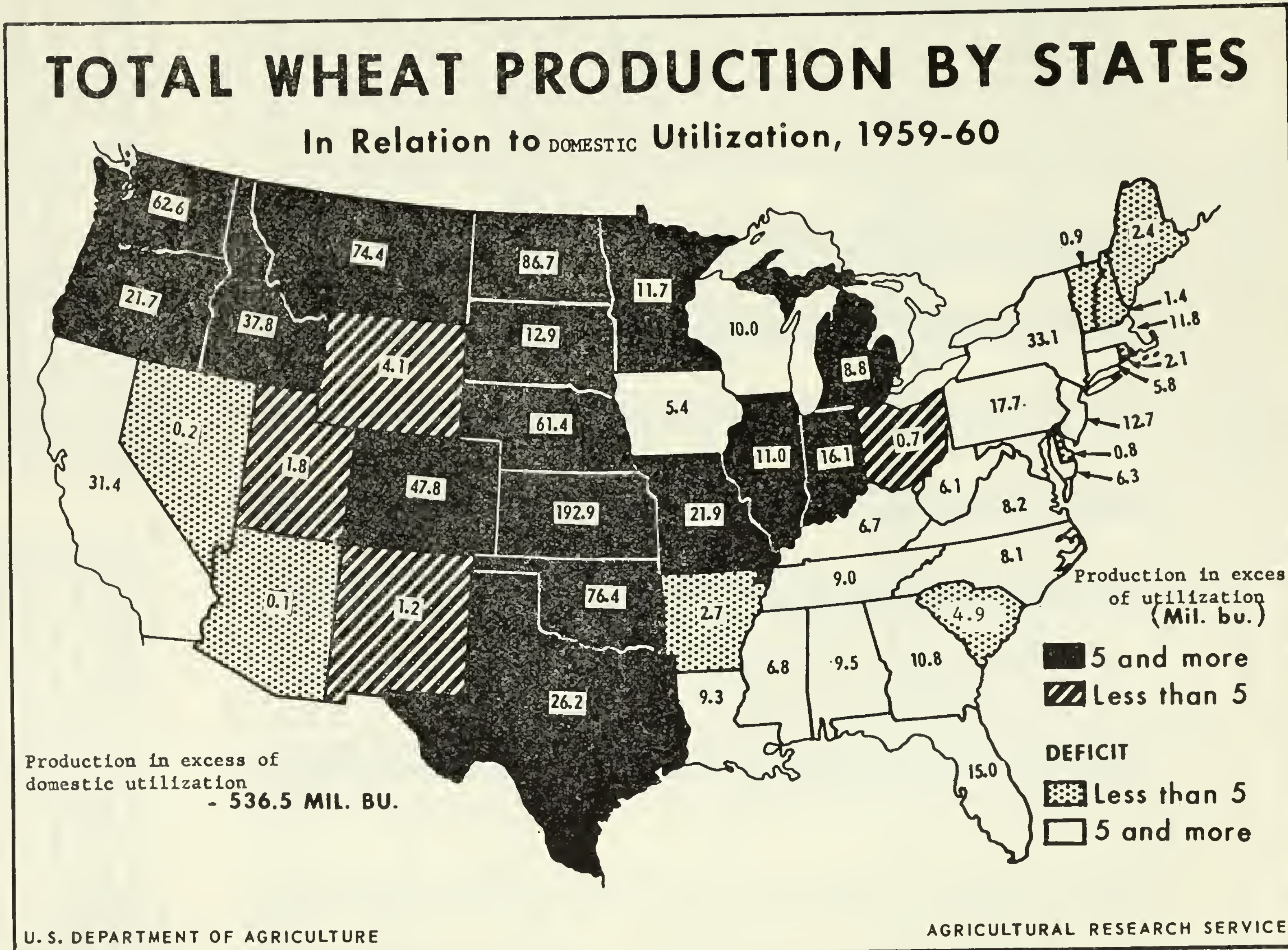
TOTAL WHEAT PRODUCTION BY STATES
In Relation to DOMESTIC Utilization, 1959-60
62.6
21.7
74.4
37.8
86.7
11.7
12.9
4.1
10.0
5.4
61.4
0.2
1.8
47.8
192.9
21.9
11.0
16.1
8.8
0.7
31.4
0.1
1.2
76.4
2.7
26.2
9.3
6.8
9.5
10.8
15.0
9.0
6.7
6.1
8.2
8.1
4.9
33.1
17.7
0.9
2.4
1.4
11.8
2.1
5.8
12.7
0.8
6.3
Production in excess of utilization (Mil. bu.)
5 and more
Less than 5
DEFICIT
Less than 5
5 and more
Production in excess of domestic utilization - 536.5 MIL. BU.
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Figure 3

Table 2.—*Domestic disappearance of wheat flour,*

[*Thousand*

State and class	Bread	Rolls	Flour, other than mixes: All purpose		Flour, other than mixes: Whole wheat	Flour, other than mixes: Cake	Prepared flour mixes	Biscuits and muffins	Crackers	
	Hard		Hard	Soft	Hard	Soft	Soft		Hard	Soft
Idaho	33,192	1,218	12,463	10,540	292	1,776	5,056	241	555	3,020
Washington	142,221	5,550	47,146	39,871	1,104	6,718	20,800	1,204	2,308	12,516
Oregon	88,817	3,452	30,182	25,524	707	4,301	13,096	722	1,448	7,882
California	739,854	29,237	231,231	195,548	5,413	32,949	106,697	6,741	11,932	64,938
Nevada	14,120	541	4,397	3,719	103	627	2,022	144	223	1,216
Arizona	62,286	2,436	19,878	16,811	465	2,833	9,004	530	1,008	5,488
White A	1,080,490	42,434	345,297	292,013	8,084	49,204	156,675	9,582	17,474	95,060
Maine	48,621	2,436	9,445	7,987	221	1,346	4,526	385	839	4,567
New Hampshire	30,232	1,489	5,438	4,599	127	775	2,841	241	528	2,873
Vermont	19,130	947	4,189	3,543	98	597	1,781	144	332	1,805
Massachusetts	251,192	13,265	37,806	31,972	885	5,387	23,785	2,359	4,358	23,721
Rhode Island	44,408	2,369	6,583	5,567	154	938	4,189	433	772	4,199
Connecticut	122,635	6,429	18,942	16,019	443	2,699	11,604	1,107	2,132	11,603
New York	838,237	43,788	132,150	111,757	3,094	18,831	79,108	7,655	14,552	79,215
Michigan	443,970	21,048	114,588	96,905	2,683	16,328	52,578	3,467	6,470	35,236
White B	1,798,425	91,771	329,141	278,349	7,705	46,901	180,412	15,791	29,983	163,219
Texas	382,196	24,297	341,343	288,668	7,991	48,668	38,567	12,712	7,580	41,254
Oklahoma	89,158	5,617	85,628	72,415	2,005	12,202	8,907	2,890	1,800	9,798
Nebraska	78,796	3,722	24,874	21,036	582	3,544	9,341	530	1,184	6,446
Kansas	117,227	5,550	34,267	28,979	802	4,833	13,867	867	1,739	9,466
Missouri	232,517	11,032	67,857	57,386	1,589	9,669	27,589	1,686	3,445	18,748
Iowa	151,159	7,106	49,514	41,873	1,159	7,056	17,911	1,011	2,281	12,413
Wyoming	16,055	609	5,620	4,753	132	801	2,359	144	264	1,437
Utah	44,351	1,760	14,545	12,300	341	2,073	6,500	385	724	3,941
Colorado	84,774	3,316	27,840	23,544	652	3,967	12,374	722	1,381	7,514
New Mexico	44,238	1,692	14,805	12,520	347	2,110	6,500	385	717	3,904
Montana	34,444	1,286	12,280	10,386	288	1,750	5,104	288	562	3,056
Hard red winter	1,274,915	65,987	678,573	573,860	15,888	96,673	149,019	21,620	21,677	117,977
North Dakota	33,933	1,557	12,333	10,430	289	1,757	3,993	193	521	2,836
South Dakota	36,495	1,692	12,905	10,914	302	1,839	4,333	241	555	3,020
Wisconsin	219,822	10,355	64,007	54,129	1,498	9,121	26,048	1,589	3,255	17,717
Minnesota	185,320	8,798	55,863	47,242	1,308	7,960	21,956	1,300	2,761	15,028
Hard red spring	475,570	22,402	145,108	122,715	3,397	20,677	56,330	3,323	7,092	38,601
Illinois	571,615	27,072	142,845	120,811	3,342	20,355	67,699	4,574	8,291	45,121
Indiana	255,633	12,114	71,786	60,708	1,680	10,229	30,286	1,926	3,770	20,516
Ohio	540,871	25,651	139,800	118,226	3,272	19,921	64,086	4,237	7,878	42,874
Pennsylvania	576,796	29,644	97,311	82,294	2,275	13,866	54,120	5,104	10,010	54,477
New Jersey	300,952	15,904	45,300	38,309	1,060	6,455	28,505	2,793	5,225	28,436
Soft red winter, North	2,245,867	110,385	497,042	420,348	11,629	70,826	244,696	18,634	35,174	191,424
Florida	196,535	12,588	161,890	136,908	3,790	23,069	20,078	6,644	3,837	20,885
Mississippi	75,893	4,670	98,924	83,658	2,316	14,096	7,174	2,215	1,651	8,987
Arkansas	64,164	3,993	72,775	61,545	1,704	10,370	6,259	1,974	1,347	7,330
Louisiana	123,888	7,783	119,297	100,887	2,793	16,999	12,374	4,044	2,497	13,592
Delaware	18,447	1,218	15,976	13,510	374	2,276	1,878	578	365	1,989
Maryland	124,173	7,918	104,674	88,521	2,451	14,916	12,615	4,189	2,436	13,260
Virginia	155,999	9,813	150,806	127,534	3,531	21,489	15,600	5,055	3,140	17,091
West Virginia	75,893	4,805	75,689	64,009	1,772	10,785	7,559	2,455	1,543	8,398
North Carolina	168,581	10,490	185,723	157,063	4,348	26,465	16,467	5,296	3,506	19,080
South Carolina	91,493	5,685	96,478	81,590	2,259	13,748	9,004	2,889	1,881	10,240
Georgia	148,540	9,340	147,502	124,740	3,453	21,018	14,782	4,815	3,012	16,391
Kentucky	117,568	7,377	125,932	106,498	2,948	17,945	11,556	3,707	2,430	13,223
Tennessee	132,599	8,257	139,488	117,962	3,266	19,876	13,096	4,237	2,727	14,844
Alabama	120,301	7,512	128,352	108,545	3,005	18,290	11,796	3,804	2,477	13,481
District of Columbia	35,413	2,301	27,320	23,104	640	3,893	3,659	1,204	684	3,720
Soft red winter, South	1,649,487	103,750	1,650,826	1,396,074	38,650	235,235	163,897	53,106	33,533	182,511
Continental U.S.	8,524,754	436,729	3,645,987	3,083,359	85,353	519,516	951,029	122,056	144,933	788,792

Note: Conversion factors used: (1) 42.95 pounds of flour/bushel of wheat, or 71.58 pounds of flour/cwt. of wheat; or (2) 139.7 pounds of wheat/cwt. of flour;

Note that hard wheat must be transported to the East, Northeast, Southeast, and Southwest to meet utilization requirements. The States shown in solid black indicate a production in excess of utilization of more than 5 million bushels. The striped areas show the States producing more hard wheat than they utilize, but under 5-million-bushel excess. Similarly, the deficit States are shown in white and in dots. The figure for each State indicates the amount of excess or deficit in terms of million bushels.

Figure 2 shows soft wheat, by States, in relation to domestic utilization. Note that in this case soft wheat must be moved into the States producing *pounds]*

Cakes	Pies	Other sweet goods		Alimentary paste products (macaroni, spaghetti, noodles)		Wheat cereals		Total		Total
Soft		Hard	Soft	Hard	Durum	Hard	Soft	Hard	Soft	Hard, Soft
819	578	1,692	1,204	1,262	3,470	2,287	456	52,961	23,690	76,651
3,804	2,600	7,445	5,296	5,195	1,429	9,582	1,907	220,551	94,716	315,267
2,311	1,589	4,636	3,298	3,270	8,997	6,012	1,197	138,524	59,920	198,444
20,511	13,771	39,152	27,854	26,558	73,098	49,359	9,825	1,132,736	478,834	1,611,570
385	241	778	554	506	1,395	943	188	21,611	9,096	30,707
1,685	1,156	3,282	2,335	2,250	6,191	4,167	830	95,772	40,672	136,444
29,515	19,935	56,985	40,541	39,041	94,580	72,350	14,403	1,662,155	706,928	2,369,083
1,733	770	2,775	1,974	2,767	7,581	2,572	512	69,676	23,800	93,476
1,059	530	1,726	1,228	1,739	4,770	1,583	315	42,862	14,461	57,323
674	289	1,083	770	1,069	2,929	1,031	205	27,879	9,808	37,687
9,438	4,670	14,991	10,665	14,795	40,546	12,901	2,568	350,193	114,565	464,758
1,685	819	2,673	1,902	2,619	7,175	2,275	453	61,853	20,185	82,038
4,574	2,263	7,275	5,176	7,202	19,734	6,316	1,257	171,374	56,302	227,676
31,153	15,311	49,608	35,292	49,098	134,546	43,264	8,612	1,173,791	386,934	1,560,725
10,786	3,467	28,628	20,366	16,080	44,239	22,439	4,467	655,906	243,600	899,506
61,102	28,119	108,759	77,373	95,369	261,520	92,381	18,389	2,553,534	869,655	3,423,189
13,241	4,045	18,239	12,976	12,125	33,142	12,654	2,519	806,425	462,650	1,269,075
3,081	915	4,230	3,009	2,844	7,774	2,974	592	194,256	113,809	308,065
1,781	530	4,974	3,539	2,832	7,787	4,282	852	121,246	47,599	168,845
2,744	867	7,478	5,320	4,226	11,626	6,188	1,232	177,477	68,175	245,652
5,441	1,685	14,822	10,545	8,381	23,054	12,267	2,442	351,910	135,191	487,101
3,370	1,011	9,509	6,765	5,421	14,907	8,328	1,658	234,477	93,068	327,545
433	289	812	578	596	1,638	1,089	217	25,177	11,011	36,188
1,204	819	2,335	1,661	1,614	4,445	2,983	594	68,653	29,477	98,130
2,311	1,541	4,467	3,178	3,088	8,499	5,703	1,135	131,221	56,286	187,507
1,156	819	2,301	1,637	1,620	4,461	2,986	594	68,706	29,625	98,331
866	578	1,792	1,276	1,288	3,543	2,352	468	54,292	23,772	78,064
35,628	13,099	70,959	50,484	44,035	120,876	61,806	12,303	2,233,840	1,070,663	3,304,503
722	193	2,132	1,517	1,210	3,329	1,949	388	53,924	22,029	75,953
770	241	2,267	1,613	1,303	3,583	2,073	413	57,592	23,384	80,976
5,152	1,589	14,009	9,967	7,927	21,798	11,587	2,306	332,460	127,618	460,078
4,285	1,300	11,776	8,378	6,673	18,352	9,895	1,970	282,394	109,419	391,813
10,929	3,323	30,184	21,475	17,113	47,062	25,504	5,077	726,370	282,450	1,008,820
14,011	4,573	36,954	26,289	20,728	57,029	28,560	5,690	839,407	309,123	1,148,530
6,067	1,925	16,344	11,628	9,228	25,387	13,300	2,647	383,855	145,932	529,787
13,145	4,236	34,854	24,797	19,590	53,896	27,326	5,444	799,242	296,966	1,096,208
21,089	10,157	33,738	24,002	33,497	91,789	29,972	5,971	813,243	271,080	1,084,323
11,315	5,584	17,935	12,759	17,721	48,562	15,543	3,076	419,640	137,232	556,872
65,627	26,475	139,825	99,475	100,764	276,663	114,701	22,828	3,255,387	1,160,333	4,415,720
6,789	2,167	9,407	6,693	6,198	16,936	6,455	1,285	400,700	224,518	625,218
2,696	578	3,452	2,456	2,488	6,810	2,627	523	192,021	122,383	314,404
2,263	578	2,978	2,119	2,075	5,676	2,181	434	151,217	92,872	244,089
4,285	1,252	5,888	4,189	3,954	10,806	4,134	823	270,234	158,445	428,679
626	193	880	626	584	1,594	609	121	38,453	21,797	60,250
4,285	1,348	5,922	4,213	3,923	10,719	4,088	814	255,585	144,161	399,746
5,393	1,589	7,377	5,248	4,979	13,611	5,207	1,037	340,852	200,036	540,888
2,648	770	3,587	2,552	2,431	6,642	2,543	506	168,263	99,682	267,945
5,922	1,541	7,851	5,585	5,439	14,875	5,710	1,137	391,648	238,556	630,204
3,178	867	4,298	3,057	2,939	8,038	3,082	613	208,115	125,186	333,301
5,152	1,493	7,005	4,983	4,749	12,986	4,971	990	328,572	194,364	522,936
4,141	1,107	5,516	3,924	3,782	10,346	3,969	790	269,522	162,891	432,413
4,622	1,252	6,226	4,430	4,261	11,653	4,467	889	301,291	181,208	482,499
4,237	1,156	5,651	4,020	3,870	10,585	4,059	808	275,227	166,137	441,364
1,204	385	1,692	1,204	1,112	3,036	1,156	230	70,318	38,603	108,921
57,441	16,276	77,730	55,299	52,784	144,313	55,258	11,000	3,662,018	2,170,839	5,832,857
260,242	107,227	484,442	344,647	349,106	945,014	422,000	84,000	14,093,304	6,260,868	20,354,172

and (3) 2.328 bushels of wheat/cwt. of flour.

hard red spring and hard red winter wheat, as well as the Northeast, South, Southeast, and California. While deficits are not large, transportation costs are involved.

Figure 3 shows total wheat production in relation to domestic utilization. This is the picture if any kind of wheat could be used for any type of end-use product. The significant item to be noted from this map is that wheat still would have to be moved to the East, Northeast, Southeast, and Southwest. Only four States east of the Mississippi River produce more total wheat than they consume. These are Illinois, Indiana, Michigan, and Ohio.

With the exception of Iowa and Wisconsin, all States in the hard wheat region produce more wheat than they consume. Despite the large surplus production in the hard wheat region, it is a deficit region so far as soft wheat flour requirements are concerned. It would seem that air classification offers a possibility for reducing transportation costs in the hard wheat region insofar as it enables bakers to use hard wheats for more of their end-product requirements.

Under conventional milling procedures some of the hard wheats produced in the Southwest are not used alone in making bread flour. Some wheat with higher protein content and stronger gluten quality has to be brought in for blending purposes. With air classification, the higher protein fraction removed during production of cake flours from local wheats can be used to raise the protein level of local wheat flours for bread when the wheat has sufficient gluten strength. Thus the need for importing high protein wheat can be eliminated and local wheats can be used to satisfy all food uses without producing a surplus of either a high protein fraction or a low protein high starch fraction (cake flour). From this standpoint, Oklahoma and Texas wheats having high quality protein appear to be dual-purpose wheats for air classification purposes. Savings in transportation costs would arise from eliminating the purchasing of high protein wheats as well as importing soft wheats for soft wheat food uses.

It may be of interest to know how air classification of soft wheat flour is working in Australia. Soft wheat flour of 9–10 percent protein is being reground and classified; the high-protein fraction (17–18 percent) is blended with the coarse residue to give an excellent bread flour (12.5–14.0 percent). The low-protein fraction (5–6 percent) is sold to bakers as cake flour, to wet processors for prime starch, to manufacturers of soups and other canned foods as a thickener, and to adhesive producers. Excellent markets are reported for the fortified bread flour, but some difficulty is encountered in disposing of all the low-protein fraction.

We do not yet know what advantages, if any, air classification offers in the export markets. Where countries prefer to do their own processing, air classification of flour for exports may be limited even if there were a potential savings in transportation costs from the United States.

Table 3.—*Estimates of flour composition of food groups, 1959–60*

Product	Wheat flour		
	Hard	Soft	Durum
	Percent	*Percent*	*Percent*
Bread	100		
Rolls	100		
Biscuits and muffins		100	
Crackers	15	85	
Cakes		100	
Pies		100	
Other sweet goods	58	42	
Alimentary paste products	26		74
Flour:			
All purpose	54	46	
Whole wheat	100		
Cake		100	
Prepared mixes		100	
Wheat cereals	83	17	

Source: See footnote 1, table 1.

Expanding the Demand for Wheat

Marketing economists do not provide much encouragement for expanding the demand for wheat foods in the domestic market. Wheat as food has a relatively inelastic demand. This is another way of stating that lowering the retail price of bread will not result in increased bread consumption. We must do something more than just lower the price. In a few cases companies have carved a niche for themselves by providing higher priced products which are sold on the basis of better quality; for example, certain frozen cake and pastry products. A quality bread also is produced by a Nevada bakery which sells for 39 cents per pound loaf along the Pacific coast. This illustrates the fact that consumers will pay for quality products from wheat.

If research leading to new or improved wheat food products could merely stem the decline in per capita flour consumption, it would be the equivalent of finding a new market for over 8 million bushels of wheat each year. An additional 8-million-bushel increase is normally picked up yearly from the increase in population of approximately 3 million people. Up to the present time this increase from population growth has been offset by the average annual decline in per capita flour consumption. If wheat were $2 per bushel, stemming the decline in per capita consumption would add $32 million a year to the wheat farmers' market. Can the air classification of wheat flour contribute to new or improved wheat food products?

New wheat products are a major field of inquiry at the Western Utilization Research and Development Division of the U.S. Department of Agriculture. For example, bulgur, or parboiled wheat, either cracked or in whole-kernel form, is well suited for use in many recipes—soups, main courses, and desserts. New, inexpensive, and convenient ways have been developed to make this product more attractive and useful to domestic and foreign consumers. New or improved wheat foods, many of which are still in the development stage, may help to maintain per capita consumption in the domestic market and expand exports.

On the basis of the calorie and protein needs in many of the countries throughout the world, it would appear that a great potential export market for wheat does exist. We need to know more about these markets—not so much what the needs are, but rather what these countries will accept and pay for. Questions for which we need answers are what it takes to make wheat products desirable in terms of the tastes, customs, and traditions of consumers in specific countries, and what quality specifications and processing requirements are essential for our wheat to meet the demands of specific importing countries. USDA research groups are working with private industry to give the wheat industry the products needed to develop markets abroad as well as at home.

Table 2 summarizes the domestic disappearance of wheat flour, by type of use, State, and class, 1959–60. While these data do not adequately describe the wheat or flour properties required by bakers to produce specific end-use food products, they represent a step toward the recognition of differences between end food use requirements. The data do not illustrate why a shortage of bread type wheats could exist with a large carryover of hard wheats; however, they do reflect differences between regional consumption patterns of wheat foods as well as differences between urban and rural population consumption patterns within regions.

Estimates of flour consumption for individual States are based on consumption data from the 1955 Household Food Consumption Survey and related demographic data. (Conversion factors from product weight to flour equivalent are those used in the 1955 survey.) Total consumption of flour in a State was estimated by combining separate estimates for farm households and nonfarm households, using different consumption rates for each group and matching population estimates. Total flour was then distributed to the types of flour on the basis of the considered judgment of Department specialists and trade information. The same distribution was used for all States.

The Farm Population Branch, Economic Research Service, supplied unofficial estimates of farm population, by States, as of April 1959. Nonfarm population was estimated by subtracting farm population from total population, by States, July 1, 1959, as reported in Series P–25, No. 210, of the Current Population Report, U.S. Bureau of the Census.

State consumption rates for flour and flour products, for lack of other data, are estimated to be the same as rates for the same population group (i.e., farm and nonfarm) for the region as a whole in which the State is located.

Further breakdown by type of flour was done by estimating the overall flour composition of each product group based upon estimates of the several types of flour normally used in each food item of the product mix of these groups (table 3). Finally, minor adjustments of consumption by type of flour were made using a constant factor for each flour type to make the total of consumption in individual States agree with U.S. consumption.

Book Reviews

Agricultural Price Analysis
By Geoffrey S. Shepherd. Iowa State University Press, Ames. 328 pages. 1963. 5th ed. $6.00.

GEOFFREY SHEPHERD'S books on price analysis are well known. They are simple, down to earth, understandable, and practical. They show how to explain the changes that have occurred in agricultural prices in the past, how to predict changes in the future, and how to estimate the effects of new programs to support prices. The student who reads this book need not get lost in a maze of generalized, introspective abstractions. Rather, he should get a feel of how to analyze concrete statistics and to find out how the prices of hogs or potatoes actually do respond to changes in such things as supply and consumer income.

This is the fifth edition of Shepherd's book. He says in the preface that it "includes a number of substantial changes and additions, to keep abreast of new concepts and techniques that are rapidly being developed in the field." I find that much of the book is new, but it maintains its directness and simplicity, for which the previous editions are noted.

To my mind, some of the most interesting ideas and suggestions in this book are those dealing with deflation and with geographical price differentials. Among the fraternity of economists and statisticians, it has become customary to "deflate" prices and incomes by dividing them by some sort of index, commonly the CPI. Shepherd is right, I think, in criticizing such routine practices of deflation. In fact, I think there is a great deal too much routine in modern price analysis generally. Shepherd wisely keeps away from much of it by knowing the commodity markets he deals with, and by using graphic analysis, especially in the preliminary phases of the study. And he properly discusses in some detail how to make appropriate allowances for the effects of "the general price level."

Economists and statisticians in this country have not paid much attention to geographical price differentials. They have typically worked with national aggregates and national averages. This covers up a multitude of interesting and important economic problems. Examples are the determination of milksheds, interregional competition, and the setting up of geographical differentials in loan rates by the Commodity Credit Corporation. Shepherd has done good work in this area. His chapter on geographical price surfaces is most interesting and suggests problems for further detailed analysis.

One can always find details to criticize in a book of this kind. I have never cared for the concept of "arc elasticity" to which Professor Shepherd devotes most of a chapter. If such a concept is used at all, I would think the statistician should take the smallest possible arc. Professor Shepherd takes the extreme opposite position. He indicates, for example, on page 54 that the arc elasticity should be measured between "two points at the ends of the line." Further, he says, "It is the elasticity of the line as a whole that is to be measured." If one has the whole line (that is, the whole demand curve) from the point where quantity is zero to the point where price is zero, the arc elasticity measured between the ends of the line will always be -1, regardless of the slope or shape of the line. Similarly, if the elasticity of a curve is $-\frac{1}{2}$ at every point, the arc elasticity will be between $-\frac{1}{2}$ and -1, and will approach -1 as the arc increases.

Chapter 10 is a good, well-balanced discussion of the relative merits of simultaneous equations vs. least squares. But I think that Shepherd's defense of least-squares regressions in agricultural price analysis is unnecessarily weak. He, like many others, says that the so-called "least-squares bias" is small, and least-squares analysis is cheaper than simultaneous equations. I think too many statisticians have been taken in by the propaganda about "least-squares bias." It is a well-established fact that a least-squares regression is an unbiased estimate of the dependent variable. For example, it gives unbiased estimates of expected prices associated with given, or assumed, values of marketings, consumer income, etc. Usually this is what the practical economist and statistician wants.

But these are minor differences of opinion. Shepherd's fifth edition—like the previous four editions—is a good, well-written, interesting book on a difficult, but important, subject.

Frederick V. Waugh

Human Resources of Central America, Panama and Mexico, 1950–1980, in Relation to Some Aspects of Economic Development

By Louis J. Ducoff. United Nations, Economic Commission for Latin America, New York. 155 pages. 1960. $2.00.

THE AUTHOR STATES: "The major purpose of this study is to make a broad comparative survey of current and future trends in population, labor force and related socioeconomic conditions, as an aid to the programming of economic development in the countries of the region, and the implementation of Central American economic integration policy."

The report consists of 6 chapters including text, 87 tables, 19 maps and figures plus a statistical appendix of 51 tables. The recent demographic situation is inventoried in terms of population density and composition and such items as social and cultural characteristics, education, and marital status. The distribution and growth rates of both rural and urban population are reviewed. The major determinants of future population trends, birth and death rates, are reviewed, and deficienies in the basic data concerning these two factors are noted.

Low, medium, and high rates of population increase are assumed for projection purposes. For each rate, projections are made for each country by 5-year periods up to 1980, and are broken down by sex by age groups. Rural and urban projected populations are broken down by age groups but only for the medium rate.

Projections are made to give an indication of the probable labor force under certain conditions. Country projections (medium rate) by 5-year intervals to 1980 are made of the economically active labor force by agricultural and nonagricultural activities by sex. The projected labor force (medium and high rates) is also classified by sex by age groups. The 1950 labor force is broken down by sex within industry groups. Labor force participation rates are given by age and sex for 1950 and 1980. Replacement rates and ratios are presented by provinces and departments to show the excess male labor supply in the 1950–60 decade and to point out the urgency for creating job opportunities in the present and following decades.

Using the "medium" assumption of population increase as a base, rates of growth in real national product required to maintain levels of living by 1980 are projected.

Future school age population is projected and the attending needs in the way of nutrition, recreation, housing, and health and educational facilities are mentioned.

The land requirements for a rapidly increasing farm labor supply are pointed out and attention is called to the fact that a large percentage of the present farmland is in relatively few hands. A brief reference to the land situation barely touches on one of the major problems of economic development, land reform. In addition to this problem many others related to economic development will come to the reader's mind. The author has adhered closely to his stated objective and left related problems to the specialists and planners.

The author has accomplished a formidable task in bringing together demographic data for the area and presenting it in forms useful in economic planning. In common with many other types of data for underdeveloped countries, the basic demographic data have shortcomings, and these are acknowledged by the author. They detract from some of the analyses made, but the procedures demonstrated represent useful tools.

Granted that many of the projections are based on arbitrary assumptions, they do provide some valuable guidelines on future population trends. These may be used for points of departure by planners who may wish to refine and revise specific data for their own particular area of interest. They provide a valuable base which may be updated with 1960 census data and other material. Students and planners interested in Central America will find in this thought-provoking report both profitable reading and a valuable reference. It should be useful for orientation purposes in an area that continues to experience a population explosion in a period when great efforts are being made to improve health, education, and economic opportunities.

James L. Paschal

The Alliance for Progress: Problems and Perspectives

Edited by John C. Drier. The Johns Hopkins Press, Baltimore. 146 pages. 1962. $3.95.

THE JOHNS HOPKINS School of Advanced International Studies sponsored a series of lectures on the Alliance for Progress in the spring of 1962. Five lectures were selected as essays for this publication to contribute to the success of the Alliance though enlightened public opinion and support.

Authors are Milton S. Eisenhower, Raúl Prebisch, Jośe Figueres, Teodoro Moscoso, and Dean Rusk, who deal with historic roots, economic aspects, political goals, social change, and the context in world affairs of the Alliance, respectively. The text of the Charter of Punta del Esta which established the Alliance is also included.

"Economic Aspects of the Alliance," constituting about half of the essay material and of primary concern to economists, makes a case for planning. Industrialization is the key to economic development and must play two dynamic roles: Supply-needed goods the region cannot afford to import; and the absorption of excess manpower from agriculture and other sectors.

Past industrialization patterns were characterized by static monopolies and the failure to fulfill both roles; new and unguided industrialization efforts are likely to be characterized by the adoption of labor-saving technology of the developed countries and the failure to fulfill the manpower absorption role.

Associated problems are agricultural development, education and technical training, income redistribution and capital accumulation, and combining internal and external resources.

The solution is planning with the quantification of targets and resources, along with the necessary government intervention, to reduce existing monopolies and to guide new efforts to a balance between labor-saving and labor-absorbing investment.

The other four essays are concerned with facets of the Alliance related to its central purpose of increased economic growth to raise living standards. The Alliance is viewed as the positive result of a long evolutionary process. Development financing problems are held small, political problems almost unsurmountable. Needed is a sense of urgency for the most important common venture of the Hemisphere.

The publication measures up to objectives and should prove stimulating and useful to developed economists.

Charles R. Davenport

International Economic Instability—The Experience After World War II.

By Joseph D. Coppock. McGraw-Hill, New York. 184 pages. 1962. $7.95.

CONCERN over international economic instability is not new; it has been evident at least since the end of World War I. But the emergence of new nations with the attendent worldwide revolution of rising expectations has now forced countries to take steps, and rapidly, to remove as much of the instability as possible through stabilizing economic arrangements. Post-World War II institutions such as the International Monetary Fund and the International Bank for Reconstruction and Development are but two of many mechanisms which over the years have been developed to reduce instability. Today concern is directed primarily to those popularly called underdeveloped countries where extreme shifts of export earnings, often arising out of the proceeds of one or two commodities, cause volatile swings in the national income with serious unstabilizing effects. In *International Economic Instability* the author attempts to analyze the problems created by international economic instability. His approach is statistical.

The author has chosen export proceeds as the "primary dependent variable" in measuring international economic instability and, adopting the log variance method of measurement, develops for the years 1946–1958, indexes of instability for 83 countries based on the movement in export proceeds. This measure of instability is also used for several other variables which seem to bear on instability of export proceeds. The index measures typical year-to-year relative changes corrected for trend influences. Removal of trend influences is necessary since they were so strong in the postwar period that without correction to remove, it would not have been possible to isolate the fluctuations important to the author's analysis. Annual figures are used, thus eliminating purely seasonal influences, the author properly recognizing, however, that annual data clearly understate the actual fluctuations.

Instability indexes are developed for all the major countries, both for exports and for imports. The author carries his analysis beyond the individual country to regional and political bloc groupings. Some mildly surprising results emerge. The instability resulting from swings in exports is not nearly so high for Latin America as is commonly assumed. Argentina is an exception with an instability index of 36.8 compared to Iran with the highest, 73.8, and to Switzerland with the lowest, 6.2. The United States has a relatively low index of 16.8, but because of the magnitude of its international trade relative to total trade of supplying countries makes a high contribution to instability.

Export instability tends to be greater in Asia and Africa than in other continents, although the range of dispersion within each continent is wide. As one might expect, imports are much more unstable in Asia, South America, and Africa than in Europe and North America. The median instability index for Asia (29.6) is twice as large as that for North America (14.5). Even to note the entire range of the author's analysis is beyond the scope of this review. One or two important generalizations can be made:

(1) There is a distinction between the incidence among countries of high instability of trade (exports, imports, or both) and the incidence of a high percentage share of world instability of trade, and (2) the United States is not as important a factor in international economic instability as is widely assumed.

Thus it seems obvious that it is one thing to take measures to reduce the instability of exports or imports, or both, of countries with high instability indexes, but it is something very much different to take measures to reduce the instability of world trade generally. The two characteristics do not show a high positive correlation among countries.

Almost the entire book is given over to the statistical analysis. This analysis is both rigorous and elegant, but not entirely satisfying, as too few of the important conclusions arise out of the analysis. The author recognizes this in his concluding chapter and admits that his empirical study of international economic instability provides few positive prescriptions for governmental or intergovernmental policy for reducing such instability. Statistical analysis is not needed to know that the stabilization of prices *and* quantities will stabilize proceeds. But the author's work does show that manipulation of some single, indirectly connected variable, i.e., other than price and quantity, is very unlikely to have much influence on export instability.

The author, in his service with the United States Department of State from 1945 to 1953 as an economic advisor, and in 1961–62 as Director, Foreign Economic Advisory Staff, Office of the Under Secretary, has dealt directly with the problem of international economic instability. He indicates his awareness of plans, now under consideration in international economic circles, for compensatory financing when the export proceeds of a country fall by more than a reasonable amount. His policies for dealing with international economic instability show a practical concern with and knowledge of the problem. Despite the interest of his statistical work, the tenuously related practical conclusions are probably his more satisfying contribution to an ultimate solution.

Warrick E. Elrod, Jr.

Factors Affecting the United States Balance of Payments

Compilation of Studies prepared for Subcommittee on International Exchange and Payments, Joint Economic Committee, 87th Congress, 2d Session. U.S. Government Printing Office, Washington, D.C. 561 pp. 1962. $1.75.

THIS VOLUME of 24 papers by 20 authors is a timely exploratory analysis of the U.S. trade and balance of payments problem. The wide attention our balance of payments has received since 1958 is the result of record payments deficits. A loss of monetary gold reserves totaling $6 billion and a buildup of dollar assets abroad now totaling over $25 billion resulted. Even with the improved payments position in 1962, the

United States is still faced with the problem of considering domestic policies in terms of how they affect the external value of the dollar.

Balance of payments trends are analyzed in depth by Seymour E. Harris and Edward M. Bernstein. Capital movements and the Euro-money market's significance to the U.S. payments position are discussed by Philip W. Bell and Oscar L. Altman, respectively. Walter Lederer's paper explores some of the more difficult conceptual problems of the balance of payments. One problem is the definition of a surplus or deficit. The most useful definition, Mr. Lederer believes, is measuring the changes in our capability to defend the exchange value of the dollar. Because this defense is the responsibility of U.S. monetary authorities, measuring the change in our financial resources and the liquid claims against our assets is the most meaningful institutional measure of the Nation's external liquidity position.

In his paper, Charles P. Kindleberger points out that U.S. trade policy is developing along lines previously adopted by France and Britain. This policy is one of exporting to protected markets through tied loans, shifting military procurement from low-cost sources abroad to high-cost firms at home, and using high-cost surplus farm commodities as part of our foreign aid. In terms of long-run U.S. economic growth, he suggests financing be furnished directly to less developed countries to buy goods in Europe and Japan; these countries would then buy other goods from the United States.

Robert E. Baldwin finds the United States in the position wherein no adjustment process for righting the payments problem appears politically acceptable. But like several other authors, he further suggests that in maintaining our "international economic viability" we should not only move toward freer trade but prepare to change our exchange rate. James E. Meade suggests that Western countries should make freer use of alterations in the rates of exchange between their national currencies under a reformed system for international payments. George N. Halm advocates the introduction of a system of flexible exchange rates because such a move would greatly reduce the need for high international reserves.

James C. Ingram believes the payments position could be strengthened through a closer degree of financial integration within the Atlantic Community. He sets out several prerequisites for attaining a closer integration of capital markets. These are rigidly fixed exchange rates, removal of all legal restrictions on international transactions, and taking steps to remove market imperfections which inhibit international capital transactions.

Fritz Machlup reviews past and present plans for the centralization of the world's monetary reserves. George N. Halm's review of the supranational bank issue includes an informative analysis of how the International Monetary Fund has gained strength in the international financial world through concluding special borrowing arrangements with the industrial countries.

Under Secretary Roosa has conveniently stated prior remedial proposals and in turn presents the objection to each. Dollar devaluation through increasing the price of gold would be a temporary expedient because this action would certainly be followed by similar actions on the part of other countries. A dollar guarantee on the other hand merely assures the rest of the world that devaluation will not occur, but such a guarantee would mean greater control of domestic policies by foreign interests. Lastly, the supranational bank, advocated to increase the world's liquidity, would generate the "most high-powered" money ever created by a man-made institution; however, it would have no super government to insure the validation of claims among nations.

Part 2 of this volume should be of particular interest to those following developments in the Common Market. Lawrence B. Krause in analyzing the effects on U.S. agriculture concludes that prospects for agricultural exports are not encouraging, due to the Common Market's move toward self-sufficiency for a wide range of crops and higher internal prices coupled with greater trade barriers to outside countries. In the aggregate, he estimates that the loss of agricultural export values might well reach 30 percent of current levels of U.S. agricultural trade with these countries.

This volume provides a penetrating analysis on the Nation's payments difficulties. While many of the proposals made are not new, their inclusion and evaluation in one volume makes an extremely valuable reference book for those interested in the

U.S. trade and payments problem. The reviewer feels that this volume provides the general public and students of international trade and finance with the most provocative information of its type available.

McGehee H. Spears

Rural Land Tenure in the United States

Edited by Alvin L. Bertrand and Floyd L. Corty. Louisiana State University Press, Baton Rouge. 313 pages. 1962. $7.50.

LAND TENURE conditions in the United States and problems associated with them hardly stand still long enough in these days of rapid change to permit a portrait to be drawn. But, in *Rural Land Tenure in the United States*, Bertrand, Corty, and a group of 10 associates have made the needed effort. The authors were affiliated with the former Southwest Land Tenure Research Committee which sponsored the volume.

The work appears to be intended primarily as a text for students. A broad spectrum of topics in the field of tenure is presented. These range from the legal principles of leasing, through a review of present tenure patterns and trends, to a discussion of public policies affecting land tenure and the needs and techniques of tenure research.

As a means of "presenting tenure problems in their total social aspect," each chapter has been written or reviewed by both economists and sociologists. An attempt has been made to introduce an integrative conceptual frame derived largely from sociology, but the success of this varies with so many authors being involved. As a result, the reviewer deems the book more valuable for the merit and information of its individual chapters than for any degree of uniqueness as an integrated whole.

In a summary chapter on tenure research needs the authors give a perceptive list of 21 areas and trends that currently are "being looked into or need looking into." To the reviewer, a disappointment of the book is that many of the topics on this list are barely mentioned elsewhere in the work. For example, the list includes "tenure problems of minority groups." However, there is only passing mention elsewhere of the exceptional tenancy status of Negro farmers, despite the body of census data available and the far reaching effect on the future of the Negro farmer that current tenure changes are having. The subjects "minority group" and "Negro" do not appear in the index.

Similarly, although the "impact of Federal action programs on tenure" is listed as a research need, the text gives only an unpointed history of production control and soil bank programs without presenting existing knowledge or thoughts about the effects of these programs on tenure.

Withal, *Rural Land Tenure in the United States* fills a need for many teachers and is a useful general reference. It should also be an encouraging reminder to other regional committees working on socioeconomic projects that it is possible to produce a monographic study from the oftimes tedious process of the regional committee system.

Calvin L. Beale

Selected Recent Research Publications in Agricultural Economics Issued by the United States Department of Agriculture and Cooperatively by the State Universities and Colleges[1]

BADGER, HENRY T. THE IMPACT OF TECHNOLOGICAL CHANGE ON MARKETING COSTS AND GROWER'S RETURNS—CASE STUDIES FOR POTATOES, SNAP BEANS, ORANGES, LEMONS. U.S. Dept. Agr., Mktg. Res. Rpt. 573, 31 pp., illus. Dec. 1962.

Case studies were conducted on potatoes, snap beans, oranges, and lemons retailed in Washington, D.C., in 1959–60 to determine the impact of added processing on prices and marketing costs. Data were collected on the fresh product and at least two processed forms of the product. These data consisted of retail prices; retail, wholesale, packer, and processor gross margins; transportation charges; and returns to growers.

BALLINGER, ROY A., AND LARKIN, L. C. SWEETENERS USED BY FOOD PROCESSING INDUSTRIES IN THE UNITED STATES—THEIR COMPETITIVE POSITION IN THE CANNING INDUSTRY. U.S. Dept. Agr., Agr. Econ. Rpt. 20, 16 pp. illus. Nov. 1962.

One of the largest consumers of sugar and corn sweeteners in the United States, the canning industry, used 9.6 percent of sweeteners consumed in the United States in 1961, compared with 7.7 percent in 1952. This is the first of a group of publications on the use of sweeteners and their competitive position in the various food industries.

BEALE, CALVIN L., AND BOGUE, DONALD J. RECENT POPULATION TRENDS IN THE UNITED STATES WITH EMPHASIS ON RURAL AREAS. U.S. Dept. Agr., Agr. Econ. Rpt. 23, 48 pp. Jan. 1963.

Unless a sharp downturn in birth rate occurs, the population of the United States will exceed 210 million by 1970. The movement of people has been heavy to the Pacific Southwest, the Gulf of Mexico, and Atlantic Coasts, and to metropolitan areas in general. Never before have so many rural areas experienced declining population or presented such disparities in the age distribution of farm and nonfarm populations. Because of heavy outmigration of young adults, births in some rural areas have declined to the point that they no longer exceed deaths.

BROWN, LESTER R. FOOD CONSUMPTION AND EXPENDITURES: INDIA, JAPAN, UNITED STATES. U.S. Dept. Agr., Econ. Res. Serv., ERS-Foreign–42, 15 pp., illus. Nov. 1962.

Diets in India, Japan, and the United States vary greatly in both quantity and composition. Variations are attributable to differences in income and climate. The economy of Japan is expanding at an unparalleled rate, and the rate of population increase is low. Impressive per capita income gains are producing a rapid rise in the intake of animal products and fruit, while consumption of starchy foods appears to be declining.

[1] State publications may be obtained from the issuing agencies of the respective States.

BROWN, W. HERBERT. COSTS AND RETURNS, COMMERCIAL COTTON FARMS, 1961. U.S. Dept. Agr., Econ. Res. Serv., FCR–8, 10 pp., illus. Nov. 1962.

Study gives costs and returns on typical cotton farms in important cotton-producing regions of the United States—Southern Piedmont, Mississippi Delta, Texas, San Joaquin Valley, and the Southern Coastal Plains. Incomes vary with size, resources, technology, and location of the farms. In both 1960 and 1961 they were highest on the large farms in the San Joaquin Valley and lowest on the relatively small farms in the Southeast.

CROP REPORTING BOARD, U.S. STATISTICAL REPORTING SERVICE. DIRECTORY OF REFRIGERATED WAREHOUSES IN THE UNITED STATES. U.S. Dept. Agr., Statis. Rptg. Serv., SRS–1, 51 pp. Dec. 1962.

This is the fifth directory of the kind published since the first, in 1947. Data are arranged alphabetically by States, cities within States, and warehouses within cities.

DEWOLFE, MILDRED R. FOR-HIRE MOTOR CARRIERS HAULING EXEMPT AGRICULTURAL COMMODITIES—NATURE AND EXTENT OF OPERATIONS. U.S. Dept. Agr., Mktg. Res. Rpt. 585, 92 pp. Jan. 1963.

Deals with size of exempt motor carriers, length of time in business, type of commodities hauled, and origins and destinations of hauls. Findings are based on the replies of 1,514 truck operators throughout continental United States.

FRYE, ROBERT E., BOYD, HARPER W., JR., AND WESTFALL, RALPH. ADVERTISING PROCEDURES AND PRACTICES OF AGRICULTURAL COMMODITY GROUPS. U.S. Dept. Agr., Mktg. Res. Rpt. 567, 32 pp. Nov. 1962.

Some 1,100 farm groups or organizations in 1958 were engaged in promotion activities, spending about $75 million annually to maintain the strength of their markets. This study provides these groups with a broad perspective of administrative processes through which advertising and promotion programs can best be conceived and carried out.

GAVETT, EARLE E. TRUCK CROP PRODUCTION PRACTICES, COLQUITT COUNTY, GEORGIA. U.S. Dept. Agr., Econ. Res. Serv., ERS–82, 45 pp., illus. Nov. 1962.

Georgia is a leading State in the production of truck crops grown for fresh-market use, ranking eighth in harvested acreage and 15th in value of production in 1961. Colquitt County is located in the center of the major producing area. In 1959, information on production of truck crops was obtained by interviewing 125 farm operators in the county. This report presents information on the 10 vegetables most widely grown in the county.

Gray, Leo R. retail price specials for frying chickens in selected u.s. cities, 1960–61. U.S. Econ. Res. Serv., ERS–101, 1963, 20 pp., illus. Jan. 1963.

This preliminary report, based on interviews with and records of private firms, newspapers, and Government agencies, covers extent and frequency of specials and analyses some effects of price specials as related to selected elements of the marketing system.

Harlow, Arthur A. factors affecting the price and supply of hogs. U.S. Dept. Agr., Econ. Res. Serv., Tech. Bul. 1274, 85 pp., illus. Dec. 1962.

Hog production and prices have exhibited fairly regular recurrent cycles since 1949. An explanation of these cycles is developed, using the supply and demand functions for the industry and incorporating the time lags inherent in hog production. A system of six equations is fitted statistically to measure the relative effect of various factors upon variables in the hog industry.

Havas, Nick, and Smith, Hugh M. customers' shopping patterns in retail food stores—an exploratory study. U.S. Dept. Agr., Econ. Res. Serv., ERS–99, 14 pp. illus. Dec. 1962.

This study of the shopping patterns of 3,200 customers in 13 retail food supermarkets in a northeastern city reveals that customers were exposed to about 64 percent of the store's display locations, spend less than 23 minutes shopping, made 13 purchases per store visit at an average value of 56 cents per purchase.

Manning, Travis W., and Nelson, Ralph E. procurement policies and practices of dairy manufacturing plants in eastern south dakota. part ii. managerial decision making. S. D. Agr. Expt. Sta. Bul. 498, 51 pp. (Econ. Res. Serv. cooperating.)

Second in a two-part series, this report concludes that decision making could be improved though (1) lessened emphasis on competitive strategy where it conflicts with marketing efficiency, (2) use of more reliable information, (3) improved management training, and (4) better understanding of dairy marketing problems and procedures by directors, members, and patrons of the 11 producer-integrated butter manufacturing plants studied.

MacPherson, D. C. milk distributors' operations—analysis of growth, sales distribution, costs and profits. U.S. Dept. Agr., Econ. Res. Serv., ERS–84, 62 pp., illus. Nov. 1962.

Beginning in the fall of 1956, the USDA has issued quarterly data on the costs of processing and distributing fluid milk. This report is made up of the special analyses that appeared in the quarterly reports. They are arranged by subject matter rather than in chronological order.

McGrath, Edward J., and Kerr, Howard W., Jr. dehydrofrozen apple slices: their potential in selected markets. U.S. Dept. Agr., Mktg. Res. Rpt. 578, 20 pp. Jan. 1963.

Study of bakers' acceptance conducted in Baltimore, Philadelphia, and Washington, D.C. Approximately 9 out of 10 bakers indicated anvantages in using dehydrofrozen apple slices.

Ogdon, Montell. the new british commonwealth economic and commercial policies as related to agricultural production and trade. U.S. Dept. Agr., Econ. Res. Serv., For. Agr. Econ. Rpt. 5, 98 pp., illus. Oct. 1962.

The Commonwealth, composed of the United Kingdom and 15 other member countries, their dependent territories, and several self-governing territories, still carry on a large volume of intra-Commonwealth and sterling area trade. But the United Kingdom and the other component parts of the system are becoming less dependent upon each other, and both the older dominions and the newer members are looking more and more to countries other than the United Kingdom for outlets for their farm commodities.

Rush, John D. farm accidents in the united states. U.S. Dept. Agr., Agr. Econ. Rpt. 17, 62 pp. Oct. 1962.

Farm-accident fatalities are not declining in proportion to the decline in farm population. This is partly due to the increasing average age of people on the farms. Motor vehicles are listed as the agency of injury most frequently associated with accidents to farm people. Perhaps 80 percent of farm accidents result from carelessness or failure to deal with hazards safely.

Schertz, Lyle P., and Learn, Elmer W. administrative controls on quantities marketed in feed-livestock economy. Minn. Agr. Expt. Sta. Bul. 241, 72 pp., illus. (Interregional publications for State Agr. Expt. Stas.)

Study examines several different mechanics that might be included in any direct control program of the feed-livestock economy.

Shaw, Lawrence H., and Durost, Donald D. measuring the effects of weather on agriculture output. procedures for constructing weather indexes. U.S. Dept. Agr., Econ. Res. Serv., ERS–72, 1962, 49 pp., illus. Oct. 1962.

How much of the dynamic increase in agricultural output in recent years is due to weather and how much to technology? This study presents a procedure for separating the effects of these two factors. Weather indexes for corn yields and production in Iowa from 1929 to 1960 were constructed using a plot data approach. The procedures can be used to construct indexes suitable as deflators for single and aggregate measures of crop production now published for farm production regions in the United States.

Spaeth, David H. individual incentive prods red china to change commune system. U.S. Econ. Res. Serv., ERS-Foreign-40, 8 pp. Dec. 1962.

Focusses on the procedures, under the current organization of the rural work force in Communist China, by which the individual peasant establishes his claim against the aggregate output of the economy. This abstract way of describing the nature of personal income is used since, in the absence of the private ownership of capital, the sources of income in Red China are not comparable with those of the Free World. The report discusses the marketing and wage systems, valuation of labor, and management problems.

Spielmann, Heinz. fruit and vegetable marketing problems associated with rural development in western montana. Mont. Agr. Expt. Sta. Bul. 573. Oct. 1962. (Econ. Res. Serv. cooperating.)

Though fruit and vegetable producing areas on West Coast have market advantages, Montana potato and sweet cherry producers can improve their competitive position by developing a homogeneous product of high quality for sale in carlot or larger units.

Starbird, I. R. and Brown, W. H. changes in costs and efficiency of cotton production—a preliminary report. U.S. Dept. Agr., Econ. Res. Serv., 27 pp., illus. Nov. 1962.

All major production regions registered some increase in efficiency from 1947–49 to 1959–61. The increase was nearly 20 percent for the U.S. as measured by direct inputs per bale.

Starbird, Irving R., and Vermeer, James. crop production practices and costs by size of farm, delta area, mississippi, 1957–58. U.S. Dept. Agr., Agr. Econ. Rpt. 21, 71 pp., illus. Nov. 1962.

Gives results of a study of crop production practices, direct costs, and estimated net returns to unpaid labor, land, and management by size of farm in the Delta area of Mississippi. Five crops are studied—cotton, soybeans, corn, oats, and wheat. The study was designed to show which elements of costs differ by size of farm, and the extent of that difference.

Stocker, Frederick D. revenues and expenditures of state and local governments in the great plains. U.S. Dept. Agr., Agr. Econ. Rpt. 22, 30 pp. Jan. 1963.

Expenditures and revenues of the Plains States conform in many respects to a common pattern; this pattern differs in significant ways from that outside the Plains. Generally, State and local governments spend substantially more per capita in the Plains States than the U.S. average. The same is true of expenditure in relation to per capita personal income. To a large extent this is met by revenue from the Federal Government. Sales and income taxes are generally lower in the Plains than elsewhere in the United States.

Turney, Jack R., and Ellis, Harold H. state water-rights laws and related subjects—a bibliography. U.S. Dept. Agr., Econ. Res. Serv., Misc. Pub. 921, 199 pp. Dec. 1962.

This bibliography was prepared to aid persons searching available literature on State water laws. Both authors are members of the bar. The growing interest in laws concerning State water rights and related subjects has caused State and Federal agencies, universities, legislative and executive study committees, and others to prepare a number of publications on the subject. Vitally concerned are public officials charged with administration of water laws and water resource programs. Also interested are courts, lawyers, and farm leaders.

Umstott, Haven D., and Hollon, Dan S. rediwheat—a new canned cooked bulgur. market position and consumer acceptance in wichita, kansas. U.S. Dept. Agr., Mktg. Res. Rpt. 574, 75 pp., illus. Dec. 1962.

A new process for cooking and canning debranned whole kernel wheat provides a versatile, easy-to-prepare, convenient form of cooked wheat. Redi-wheat was developed by U.S. Agricultural Research Service. Market tested at Wichita, Kansas, in March–September 1961, the product attained a favorable sales position relative to the sales for 57 well-established products such as canned Spanish rice, dry rice specialities, quick-cooking rice, dry rice, wild rice, dry spaghetti and macaroni complete dinners, canned speciality bean products, and dry bulgur.

U.S. Agricultural Stabilization and Conservation Service. wheat facts. U.S. Dept. Agr., Agr. Stab. and Conserv. Serv., PA–551, 21 pp. Jan. 1963.

Facts and statistics for wheat growers. Background reports on conditions that affect the crop, its markets, and the Wheat Program for 1964.

U.S. Department of Agriculture. food consumption and dietary levels of households of different sizes—by regions. U.S. Dept. Agr., Agr. Res. Serv., and Econ. Res. Serv. cooperating. Rpt. 17, Household Food Consumption Survey 1955, 168 pp., illus. Jan. 1963.

Report presents survey data for U.S. households at different levels of income and for various urbanization groups and parts of the country.

U.S. Department of Agriculture. the wheat program for 1964: an economic analysis. U.S. Dept. Agr., 45 pp., illus. Jan. 1963.

Describes the 1964 program and provides estimates of wheat prices, farm income, and other factors in the event wheat growers decide, by a two-thirds "yes" vote, to put it into operation. It also provides estimates of what may be expected to happen to wheat prices, to farm income, and to wheat production if more than one-third of the growers vote "no." The study was prepared by a staff of the Department of Agriculture and was reviewed by a group of agricultural economists from land grant universities in the major wheat growing areas.

U.S. Economic Research Service. indices of agricultural production for the 20 latin american countries (plus country tables for jamaica and trinidad). U.S. Dept. Agr., Econ. Res. Serv., ERS-Foreign-44, 28 pp. Nov. 1962.

Reports, in tabular form, indices of volume of agricultural and livestock production and of food production, total and per capita, in Latin America for the years 1957–58 through 1961–62 (preliminary). Includes a table for each country (plus Jamaica and Trinidad) with indices of production of major commodities.

U.S. Economic Research Service. marketing margins for white bread. U.S. Dept. Agr., Econ. Res. Serv., Misc. Pub. 712, 15 pp., illus. Nov. 1962.

Bread prices have risen every year since 1945. Consumers in 1961 paid an average price of 20.9 cents for a 1-pound looaf of bread, 55 percent above the 13.5 cents paid in 1947–49. This report shows the changes in bread prices, margins, and costs for those years.

U.S. Economic Research Service. statistics on the european economic community. vol. 1. agricultural trade and finance. U.S. Dept. Agr., Econ. Res. Serv., ERS-Foreign-43, 236 pp. Dec. 1962.

Report compiles data pertaining to agricultural trade and finance of the six original Common Market members —Belgium, France, Italy, Luxembourg, Netherlands, and West Germany—the associate member, Greece, and four applicants for membership—Denmark, Ireland, Norway, and the United Kingdom.

U.S. Economic Research Service. the 1962 crop outlook in communist countries. U.S. Dept. Agr., Econ. Res. Serv., ERS-Foreign-41, 7 pp. Nov. 1962.

The Soviet Union and other Eastern European countries have suffered another disappointing crop year, but in Communist China weather conditions have been better than those of the past 3 years, and the crop outlook for 1962 improved. Poor growing conditions in Eastern Europe were a major contributing factor in the mediocre crop outlook. Stagnation in Soviet agricultural production has continued for the fourth consecutive year.

U.S. Economic Research Service. the 1963 world agricultural situation. U.S. Dept. Agr., Econ. Res. Serv., Western Hemisphere Supplement 1, 50 pp. Feb. 1963.

Agricultural output rose about 3 percent in 1962–63 above 1961–62, but was only slightly greater than growth in population, and fell below industrial output. As a result, price adjustments in 1962 were mostly favorable to agriculture. The agricultural situation by commodities and by regions is discussed.

VanDress, Michael G. estimated number of days' supply of food and beverages in retail stores, 1962—a civil defense study. U.S. Dept. Agr., Mktg. Res. Rpt. 577, 80 pp. Dec. 1962.

Estimates number of days' supply of food and concentrated beverages in retail food stores in continental United States. Supply ranges from 14.5 days per person in region III to 19.4 days in region VIII. Tables show estimated supplies in eight regions by States and counties.

Vosloh, Carl J., Jr. labor and capital for mixing formula feeds. U.S. Dept. Agr., Mktg. Res. Rpt. 564, 28 pp., illus. Oct. 1962.

Contains information on standards for costs, labor, and equipment in two models of mixing centers with capacities of 80 tons and 200 tons per shift per day. The models were developed from records of feed manufacturers in 34 States. Since World War II, farmers have increased their demands on the industry for both feeds and services.

Statistical Compilations

Crop Reporting Board, U.S. Statis. Rptg. Serv. vegetables for processing—cucumbers for pickles, acreage, yield, production, price, value, and stocks, by states, 1951–60. U.S. Dept. Agr., Statis. Bul. 299, 11 pp. Nov. 1962.

Crop Reporting Board, U.S. Statis. Rptg. Serv. milk production and dairy products. annual statistical summary 1962. U.S. Statis. Rptg. Serv. Da 3(63), 31 pp. Feb. 1963.

(Continued from inside front cover.)

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McGehee H. Spears, International Monetary Branch, Development and Trade Analysis Division, Economic Research Service, is working on the significance of using agricultural commodities as part of total economic aid and the effect on the Nation's balance of payments.

Calvin L. Beale is Head of the Farm Population Analysis Section in the Economic and Statistical Analysis Division, Economic Research Service. He has been interested in the effects of changing tenure patterns on the size and composition of the farm population.

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