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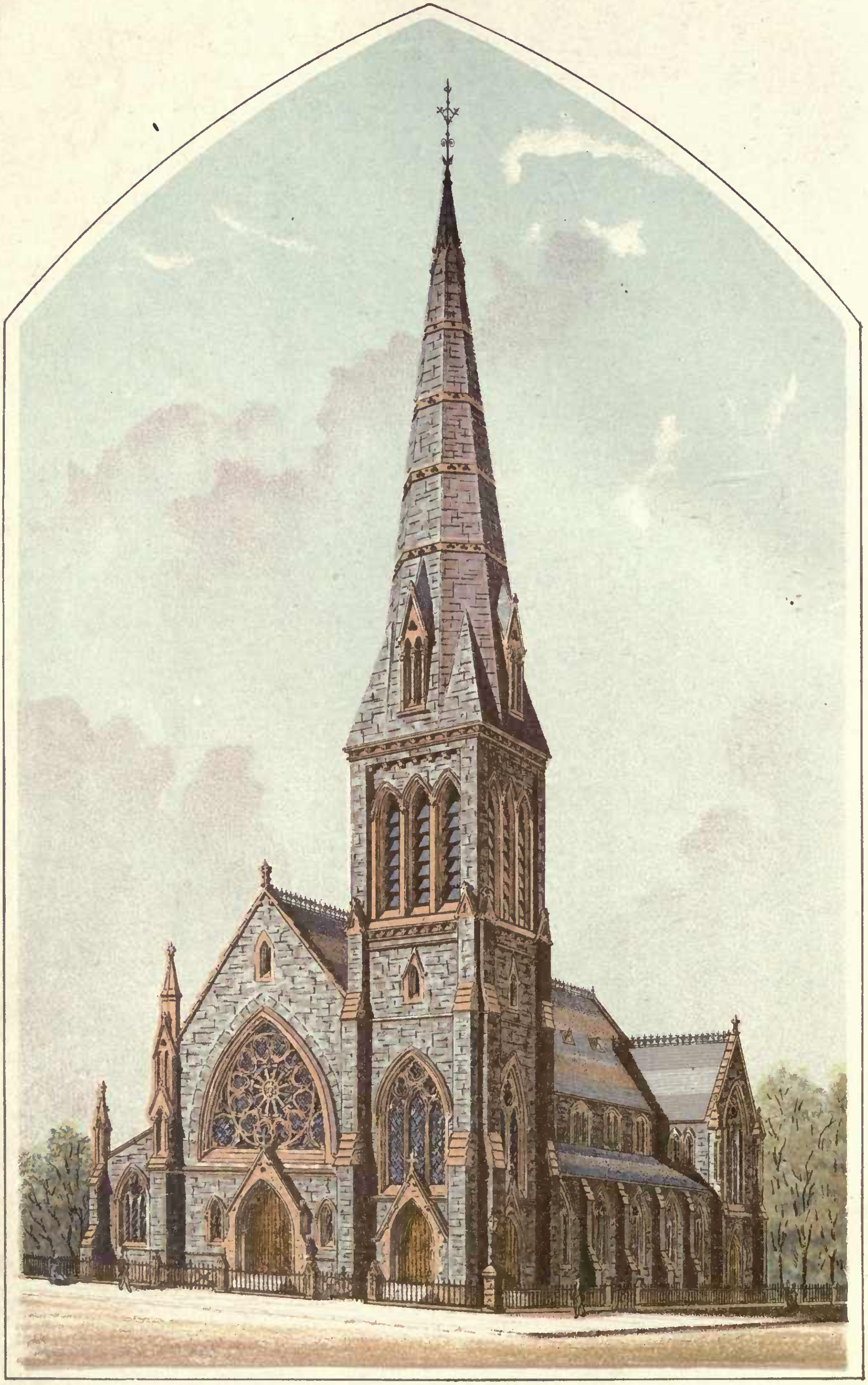

APPLETONS' CYCLOPÆDIA OF TECHNICAL DRAWING

EMBRACING THE PRINCIPLES OF CONSTRUCTION AS APPLIED TO PRACTICAL DESIGN

WITH NUMEROUS ILLUSTRATIONS OF TOPOGRAPHICAL, MECHANICAL, ENGINEERING, ARCHITECTURAL, PERSPECTIVE, AND FREE-HAND DRAWING

EDITED BY

W. E. WORTHEN, C. E.

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PREFACE.

"At the suggestion of the publishers, this work was undertaken to form one of their series of dictionaries and cyclopædias. In this view, it has been the intention to make it a complete course of instruction and book of reference to the mechanic, architect, and engineer. It has not, therefore, been confined to the explanation and illustration of the methods of projection, and the delineation of objects which might serve as copies to the draughtsman, matters of essential importance for the correct and intelligible representation of every form; but it contains the means of determining the amount and direction of strains to which different parts of a machine or structure may be subjected, and the rules for disposing and proportioning of the material employed, to the safe and permanent resistance of those strains, with practical applications of the same. Thus, while it supplies numerous illustrations in every department for the mere copyist, it also affords suggestions and aids to the mechanic in the execution of new designs. And, although the arranging and properly proportioning alone of material in a suitable direction, and adequately to the resistance of the strains to which it might be exposed, would produce a structure sufficient in point of strength for the purposes for which it is intended, yet, as in many cases the disposition of the material may be applied not only practically, but also artistically. . . . 1857."

"There are no changes in the principles of projection as applied to drawing, and no marked improvement in drawing-instruments; but in the present practice finished drawings in shade and colour are exceptional. It is sufficient, for almost every purpose, for the draughtsman to make accurate projections with pencil on paper, and trace them afterward on cloth. The pencil-drawings can be readily altered or amended, and, where there are many repetitions of the same parts, but a single one may be drawn. On the tracing they are made complete, and these are preserved as originals in the office, while sun-prints of them are used for details of construction in the shop, or distributed as circulars among customers.

"Of late years the science of 'graphics' has become of great importance, and is here fully illustrated in its varied applications, showing not only this method of recording the facts of the statistician, and affording comparisons of circumstances and times, the growth of population, the quantities and cost of agricultural and mechanical production, and of their transport, movements of trade, fluctuations of value, the atmospheric conditions, death-rates, etc., but also in its application to the plotting of formulæ for their ready solution, by the draughtsman and designer. For many of the rules in this work the results of the formulæ of various authors have been plotted graphically, and the rule given one deemed of the greatest weight, not always by average, but most consistent with our own experience.

"In astronomical calculations every decimal may have its importance. It is not so in those of the mechanical or architectural designer; solutions by graphics are sufficient for their purpose, and, simpler than mathematical calculations, they are thus less liable to error; it is a very good practice to use one as a check on the other. It is to be remarked that inaccuracy in facts, and carelessness in observation, are not eliminated from an equation by closeness of calculation, and when factors are not established within the limits of units it is useless to extend the results to many places of decimals. It is of the utmost importance to know at first well the object and purposes of the design, the stresses to which its parts are to be subjected, and the strength and endurance of the materials of which it is to be composed. In establishing rules for ourselves, be sure of the facts, and that there are enough of them for a general application. Rules are necessary, but their application and usefulness depend largely on the experience of the user, and life must be a record of applications and effects. It is comparatively easy to make a work strong enough; but to unite economy with proportion is difficult. . . . 1886."

The first edition of this work was suggested by "The Engineer and Machinist's Drawing Book" of Messrs. Blackie & Son, 1855, from which, with the consent of the publishers, much of the text and illustrations were taken. Since then, in the many editions, it has been the aim to keep up with mechanical progress, and matter has been drawn from all sources. Credit, as far as possible, has been given to mechanics for their designs and to experimenters for their results.

Geometrical problems and examples of orthographic projection of the first work are largely retained, but examples of mechanical and architectural construction are brought up to the present age of steel, with the latest illustrations of the applications of steam, and some of electricity. Isometry is retained, perspective has been more fully illustrated, and free-hand drawing now includes the recent processes by which, through photography, the mechanical labour of sketching is diminished, adding to the correctness of detail and improving the effectiveness on paper. This may be called an age of illustration, and the processes have enabled a work like the present drawing book to give better and more illustrations, less text, more comprehensiveness, and greater certainty of detail.

Mr. Robert E. Hawley, brought up in my office, has had charge of the new drawings and has acted as co-editor. W.

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DESCRIPTION OF PLATES.

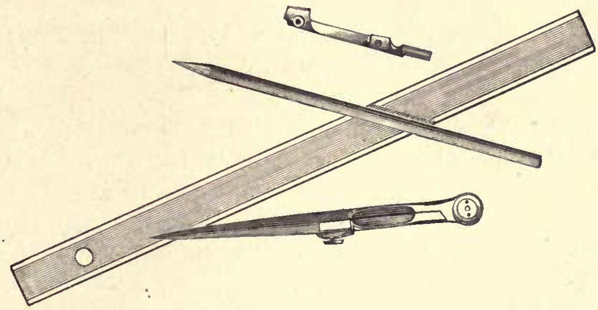
ERRATA.

Page 160, 19th line, omit Plate XI, and for Plates XII and XV, substitute Plates XVI and XVII.

APPLETONS'

CYCLOPÆDIA OF DRAWING.

CONSTRUCTION OF GEOMETRICAL PROBLEMS.



Most persons, at some time, have made use of the simple drawing instruments, as pencils, straight edges or rulers, dividers and compasses with changeable points, and many suppose that there can be no skill in their use; but to one critical in these matters there are great differences, even in common drawings, in the straightness and uniformity of the lines and in the care of the surface of the paper.

Pencils are marked according to their hardness: H (hard), H H, H H H, to 8 H; or H, V (very) H, V V H, M (medium), H M, M B (black), S (soft), M S, V S, V V S; or by numerals, 1, 2, 3, to 8.

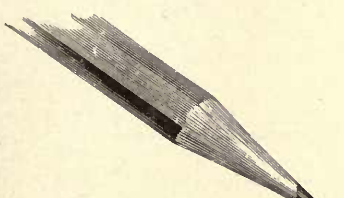


Fig. 1.

Select for the geometrical problems and for usual drawings a No. 3 or H H H pencil. It should be sharpened to a cone-point (Fig. 1). Where a pencil is used for drawing lines only, some draughtsmen sharpen the pencil to a wide edge, like a chisel.

In drawing a straight line, hold the ruler firmly with the left hand; with the right hand hold the pencil lightly but without slackness, and a little inclined in the direction of the line to be drawn, keeping the pencil against the edge of the ruler, and in the same relative position to the edge during the whole operation of drawing the line.

To draw a clean line and preserve the point of the pencil, the part of the cone a little above the point of the pencil should bear against the edge of the ruler, and the pencil should be carried steadily while drawing. Any oscillation will throw the point farther from or nearer the ruler, and the line will not be straight (Fig. 2).

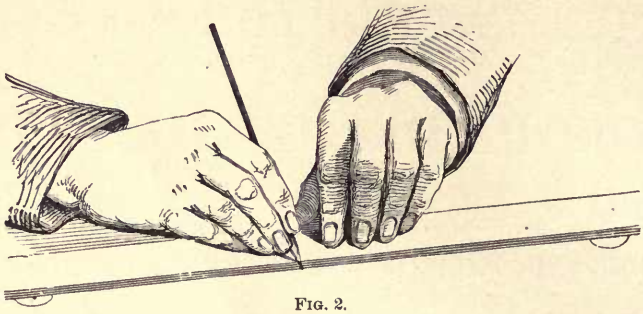


FIG. 2.

In the use of the compasses do not make a hole through the paper with the needle or sharp point, but only into the paper sufficient to maintain the position.

Keep the paper clean, and use rubber as little as possible.

A geometrical point, which is position only, is indicated in drawing by the prick-mark of a needle or sharp point, or the dot of a pencil; sometimes it is inclosed ⊙, sometimes designated by the intersection of two short lines × >. A line, which is extension in length only, is indicated by a visible mark of pencil or pen traced upon the paper.

Geometrically *lines* have but one dimension, *length*, and the direction of a line is the direction from point to point of the points of which the line is composed: in drawing, lines are visible marks of pencil or pen upon paper.

A *straight line* is such as can be drawn along the edge of the ruler, and is one in which the direction is the same throughout. In drawing a straight line through two given points, place the edge of the ruler very near to and at equal distances from the points, as the point of the pencil or pen should not be in contact with the edge of the ruler (Fig. 3).

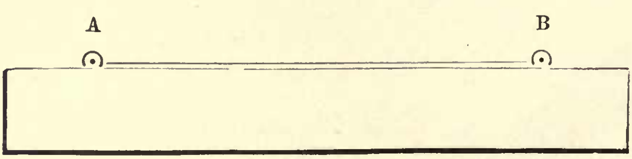


FIG. 3.

Lines in geometry and drawing are generally of limited extent. A *given* or known line is one established on paper or fixed by dimensions. Lines of the same length are *equal*.

Curved Lines.—For the pencil-points of compasses, whittle down the stumps of pencils to suit. Insert the pencil-point in the compasses. With the needle or sharp point resting on the paper describe a line with the pencil around this point; the line thus described is usually called a *circle*—more strictly it is the *circumference* of a circle—the circle being the space inclosed. A portion of a circumference is an *arc.* The point around which the circumference is described is the *centre* of the circle (Fig. 4).

The line embraced between the two points of the compasses is called the *radius* of the circle, and by mechanics a *sweep;* a line passing through the centre and terminating at each end in the circumference is a *diameter*, and is equal in length to two radii; any line not passing through the centre and limited by the circumference is less than a diameter and is a *chord.* The space embraced between a chord and its lesser arc is a *segment.* The space embraced between two radii and its arc is a *sector;* if the arc is the quarter of the circumference, the sector is distinguished as a *quadrant.*

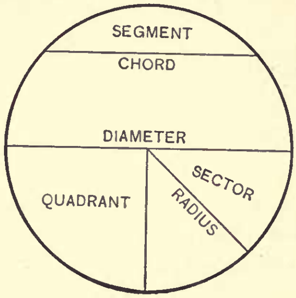
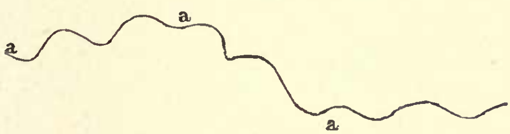


FIG. 4.

It will be observed that *arcs* are lines which are continually changing the directions, and are called curved lines, but there are other curved lines than those described by compasses, of which the construction will be explained hereafter.

Lines which can neither be drawn by rulers or compasses, representing the directions of brooks and rivers, the margins of lakes and seas, points in which are established by surveys, defined on paper, and connected by hand-drawing, are *irregular* or *crooked* lines.

Where it is necessary to distinguish lines by names, we place at their extremities letters or figures, as A———B, 1———2; the line A B, or 1 2. But in lines other than straight, or of considerable extent, it is often necessary to introduce intermediate letters and figures, as a a a.



In the following problems, unless otherwise implied or designated, where lines are mentioned, straight lines are intended.

If a straight line moves sideways in a single direction, it will sweep over a *surface* which is called a *plane.* All drawings are projections on planes of paper or board.

Two lines drawn on paper, and having the same direction, can never come any nearer each other, and must always be at the same distance apart, however far extended. Such lines are called *parallel* lines.

To draw a line parallel to a given line, and at a given distance from it (Fig. 5).

Draw the line A B for the given .line, and take in the compasses the distance A C—the distance at which the other line is to be drawn. On A, as a centre, describe an arc, and on B, as a centre, a similar arc; draw the line C D just touching these two arcs, which will be the parallel line required.

To draw a line parallel to a given line through a given point outside this line (Fig. 6).

Draw the given line A B, and mark the given point C. With C as a centre, find an arc that shall only just touch A B; and with B as a centre, and the

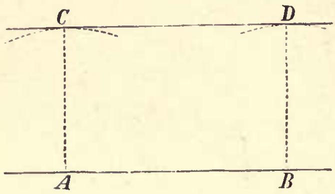


Fig. 5.

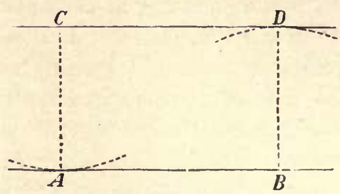


Fig. 6.

same radius, describe an arc D. Draw through the point C a line just touching this last arc, and the line C D will be the parallel line required.

Two lines in the same plane, not parallel to each other, will come together if extended sufficiently far. The inclination or intersection of two lines is called an *angle* (Fig. 7).

If but two lines come together, the angle may be designated by a single letter at the *vertex*, as the angle E.

But, if three or more lines have a common vertex, the angles are designated by the lines of which they are composed, as the angle D B C of the lines D B and B C; the angle A B C of A B and B C; the angle A B D of A B and B D. The letter at the vertex must always be the central letter.

Describe a circle (Fig. 8). Draw the diameter A B. From A and B as centres, with any opening of the compasses greater than the radius, describe two arcs cutting each other as at D. Through the intersection of these arcs and the centre C, draw the line D E. D E makes, with the diameter A B, four angles, viz., A C D, D C B, B C E, and E C A. Angles are equal whose lines

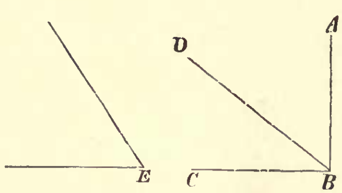


Fig. 7.

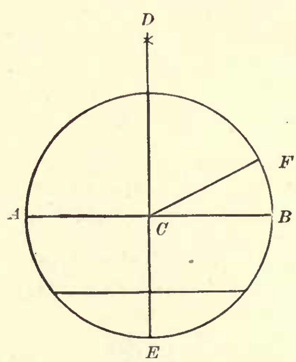


Fig. 8.

have equal inclination to each other, and whose lines, if placed one on the other, would coincide. By construction, the points C and D have, respectively, equal distances from A and B; the line D C can not, therefore, be inclined more to one side than to the other, and the angle A C D must be equal to the angle B C D. Such angles are called *right angles*. The four angles, formed by the intersection of D E with A B, are equal, and are right angles.

The angles A C D and D C B, on the same side of A B, are called *adjacent* angles; as also D C B and B C E, on the same side of D E.

If the *base* line be parallel with the surface of still water, it is called an *horizontal* line, and the line perpendicular to it is called a *vertical* line.

Draw the line C F. It will be observed that the angle F C B is less than a right angle, and it is called an *acute* angle; the angle F C A is greater than a right angle, and it is called an *obtuse* angle. It will be observed that, no matter how many lines be drawn to the centre, the sum of all the angles on the one side of A B can only be equal to two right angles, and, on both sides of A B, can only be equal to four right angles. It will be observed that the angles at the centre include greater or less arcs between their sides, according to the greater or less inclination of their sides to each other; that the right angles intercept equal arcs, and that, no matter how large the circle, the proportion of the circle intercepted by the sides of an angle is always the same, and that the arcs can therefore be taken as the measures of angles. For this purpose the whole circumference is supposed to be divided into three hundred and sixty degrees (360°), each degree subdivided into sixty minutes (60′), and each minute into sixty seconds (60″). Each right angle has for its measure one quarter of the whole circumference ($\frac{360}{4}$), or 90°, and is called a *quadrant.*

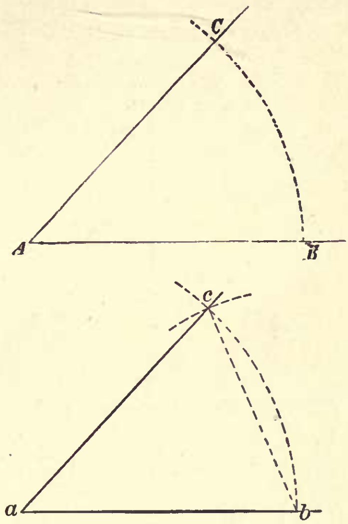


FIG. 9.

To construct an angle equal to a given angle (Fig. 9).

Draw any angle, as C A B, for the given angle, and the line *a b*

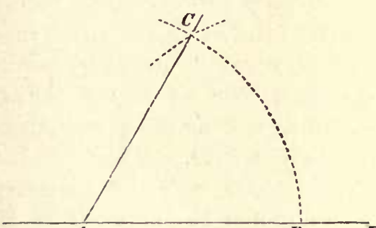


FIG. 10.

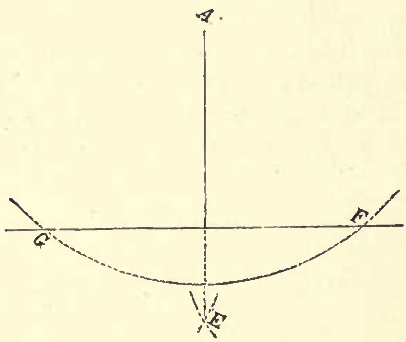


FIG. 11.

as the base of the required angle. From A, with any suitable radius, describe the arc B C, and from *a*, with the same radius, describe the arc *b c*. With the compasses take the length of the chord B C, and, from *b* as centre, describe an arc cutting the arc *b c* at *c*, and draw the line *a c; c a b* is the required angle.

To construct an angle of sixty degrees (Fig. 10).

Lay off any base line, and from A, with any radius, describe an arc, and from B, with the same radius, describe another arc cutting the first, as at C. Draw the line C A, and the angle C A B will be an angle of sixty degrees.

For if, on the circumference of any circle, chords equal to the radius are stepped off successively, six will exactly complete the circle, making 360°.

To draw a perpendicular to a line from a point without the line (Fig. 11).

Draw a line, and mark the given point A. From A as a centre, with a

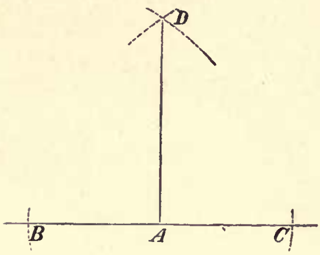


Fig. 12.

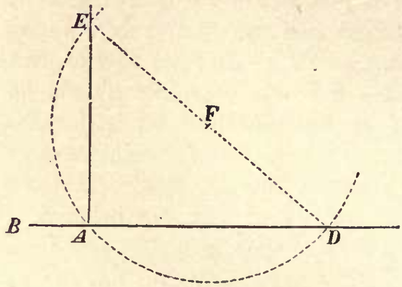


Fig. 13.

suitable radius, describe an arc cutting the line at G and F. From G and F, as centres, describe arcs cutting each other. The line drawn through the point A, and the point of intersection E, will be perpendicular to the line G F.

The radial line A E divides the chord G F and the arc G E F into two equal parts; and, conversely, the line perpendicular to the middle point of a chord of a circle is radial—passes through the centre of that circle.

To draw a perpendicular to a line from a point within that line (Fig. 12).

1st Method.—Draw a line, and take the point A in the line. From A, as a centre, describe arcs cutting the line on each side at B and C. From B and C, as centres, describe intersecting arcs at D. Draw a line through D and A, and it will be perpendicular to the line B C at A.

2d Method (Fig. 13).—Draw the line, and mark the point A as before. From any centre F, without the line, and not directly over A, with a radius equal to F A, describe more than a half-circle cutting the line, as at D. From D, through the centre F, draw a line cutting the arc at E. Draw A E, and it will be the perpendicular to the line A D.

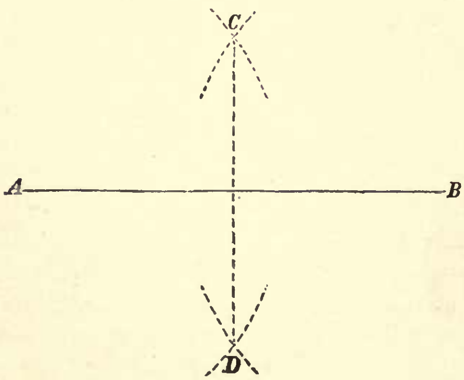


Fig. 14.

The line D E is the diameter of a circle, and the angle D A E, with its vertex at A in the circumference, embraces with its sides half a circle. It has been shown that angles at the centre of a circle have for their measure the arc embraced by their sides. Angles with their vertices in the circumference have for their measure half the arc embraced by their sides; and, consequently, angles embracing half a circumference are right angles.

To draw a perpendicular to the middle point of a line (Fig. 14).

From the extremities A and B of the line, as centres, describe similar intersecting arcs above and below the line. Through these intersections draw the line C D. It will be perpendicular to the line A B, and bisect or divide it into two equal parts.

If the line A B be considered the chord of a circle, its centre would lie in the line C D.

This construction is sometimes used merely to divide a line into two equal parts, or bisect it; it can be more readily done with dividers (Fig. 15).

Place one point of the dividers on one end of the line, and open the dividers to a space as near as may be half the line. Turn the dividers on the

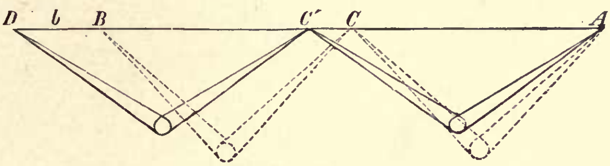


FIG. 15.

central point; if the other point then falls exactly on the opposite extremity of the line, it is properly divided; but, if the point falls either within or without the extremity of the line, divide the deficit or excess by the eye, in halves, and contract or extend the dividers by this measure. Then apply the dividers as before, and divide deficit or excess till a revolution exactly covers the length of the line. By accustoming one's self to this process, the eye is made accurate, and one estimate is sufficient for a correct division of any deficit or excess. By a similar process it is evident that a line can be divided into any number of equal parts, by assuming an opening of the dividers as nearly as possible to that required by the division, and, after spacing the line, dividing the deficit or excess by the required number of parts, contracting or expanding the dividers by one of these parts, and spacing the line again, and so on till it is accurately divided.

To bisect a given angle (Fig. 16).

Construct an angle, and from its vertex A, as a centre, describe an arc cutting the two sides of the angle at B and C. From B and C, as centres, de-

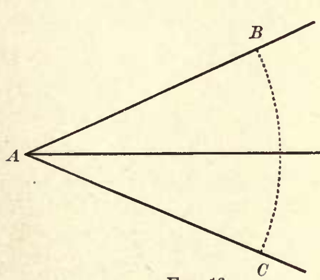


FIG. 16.

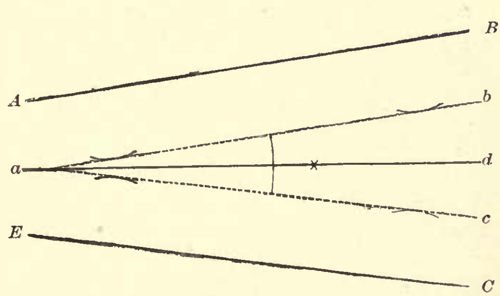


FIG. 17.

scribe intersecting arcs. Draw a line through A and the point of intersection D, and this line will bisect the angle.

To bisect an angle when the vertex is not on the paper (Fig. 17).

Let A B and E C be two lines inclined to each other; at equal distances and parallel to the above lines draw *a b* and *a c*, intersecting lines; bisect the angle *b a c*. A line *a d* drawn through the vertex and the point of bisection is the required line.

Through two given points to describe an arc of a circle with a given radius (Fig. 18).

From B and C, the two given points, with an opening of the dividers equal to the given radius, describe two arcs intersecting at A. From A, as a centre, with the same radius, describe an arc, and it will be the one required.

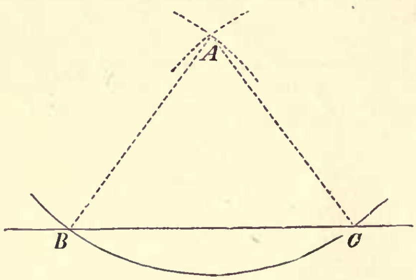


Fig. 18.

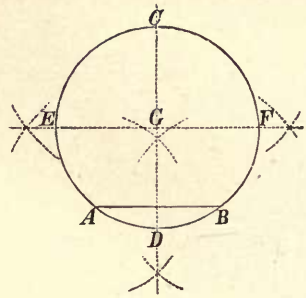


Fig. 19.

To find the centre of a given circle, or of an arc of a circle.

Of a circle (Fig. 19).—Draw the chord A B. Bisect it by the perpendicular C D, whose extremities lie in the circumference, and bisect C D. G, the point of bisection, will be the centre of the circle.

To find the centre of an arc (Fig. 20).—Select the points A, B, and C in the arc, well apart. From A and B as centres, and then from B and C as centres, describe arcs of equal radii cutting each other; draw the two lines D E and F G through their intersections. The point O, where these lines meet, is the centre required.

To describe a circle passing through three given points (Fig. 20).

Proceed, as in the last problem, to find the point O. From O, as a centre, with a radius O A, describe a circle, and it will be the one required.

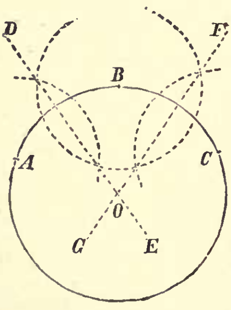


Fig. 20.

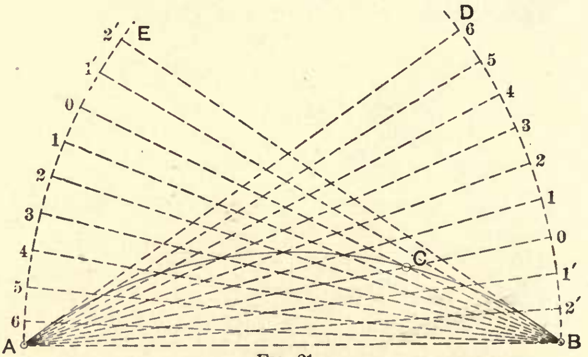


Fig. 21.

To describe an arc of a circle passing through three given points, where the centre is not available (Fig. 21).

From the extreme given points A and B describe arcs A E and B D; through the third given point C draw lines from A and B, intersecting the arcs at O and O; from O and O cut the arcs in either direction by equal divisions, O 1, O 2, O 3, and O 1′, O 2′; draw lines A 1, A 2; A 1′, A 2′; B 1, B 2; B 1′, B 2′. The successive intersections of A 1 by B 1, A 2 by B 2, A 1′ by B 1′ are points in the required arc by the connection of which the problem will be complete.

To describe this arc mechanically (Fig. 22).

Lay off on a piece of cardboard the three points A, C, B, and connect them by lines extended beyond the points A and B; and then cut out the cardboard

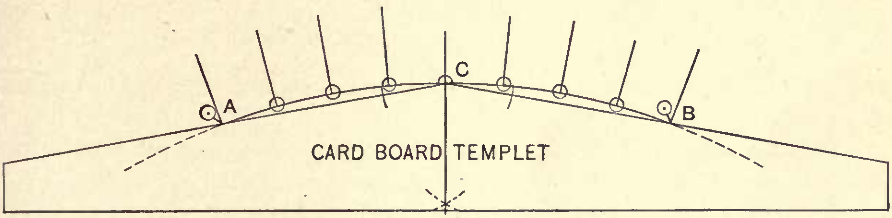


FIG. 22.

along these lines. Insert pins at the points A and B on the drawing, and placing the cardboard templet against these pins, and the angle against the point C, slide the templet each way, dotting in the drawing the angle C in its different positions. These dots will be points in the curve, which are to be connected. By extending the bisecting line in different positions of the templet to the drawing, radial lines are given which will be useful in laying off voussoir joints on segmental arches of large radius. Radial lines are also necessary in perspective drawing, for which an instrument called the centrolinead (Fig. 23) is used. The principle is similar to that of the cardboard templet.

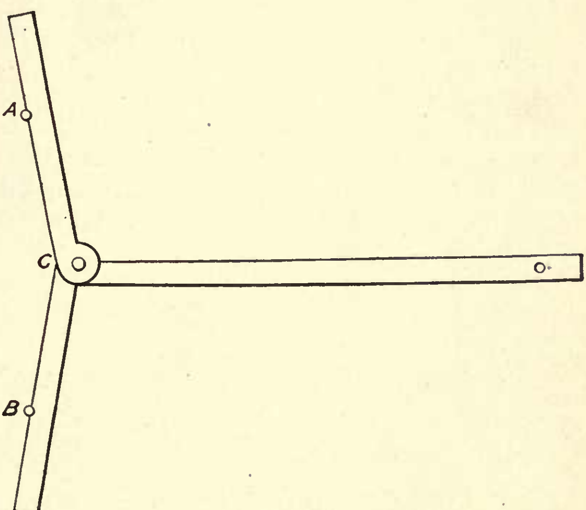


FIG. 23.

To draw a tangent to a circle from a given point in the circumference.

1st Method (Fig. 24).—Through the given point A draw the radial line A C. The perpendicular F G to that line will be the tangent required.

2d Method (Fig. 25).—From the given point A set off equal arcs, A B and A D. Join B and D. Through A draw A E parallel to B D, and it will be the tangent required. This method is useful when the centre is inaccessible.

To draw tangents to a circle from a point without it (Fig. 26).

From the given point A draw A C to the centre of the circle. Bisect A C to find the point D. From D, as a centre, describe an arc with a radius D C,

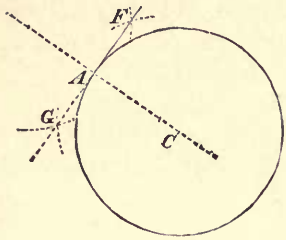


Fig. 24.

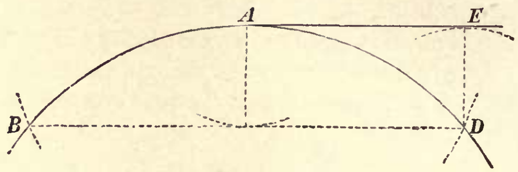


Fig. 25.

cutting the circle at E and F. Draw A E and A F, and they will be the tangents required.

To construct within the sides of an angle a circle tangent to these sides at a given distance from the vertex (Fig. 27).

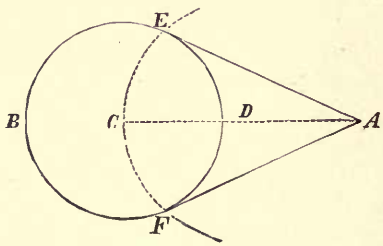


Fig. 26.

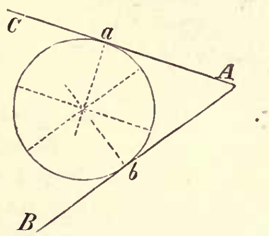


Fig. 27.

Let *a* and *b* be the given points equally distant from the vertex A. Draw a perpendicular to A C at *a*, and to A B at *b*. The intersection of these perpendiculars will be the centre of the required circle.

In the same figure, *to find the centre, the radius being given*, and the points *a* and *b* not known. Draw lines parallel to A C and A B, at a distance equal to the given radius, and their intersection will be the centre required.

To describe a circle from a given point to touch a given circle (Figs. 28 and 29).

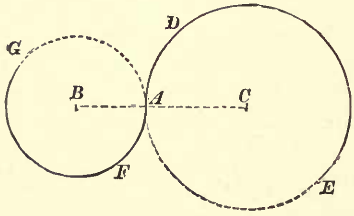


Fig. 28.

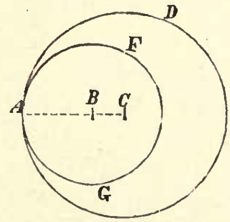


Fig. 29.

D E being the given circle, and B the given point, draw a line from B to the centre C, and produce it, if the point B is within the circle, until it cuts the circle at A. From B, as a centre, with a radius equal to B A, describe the circle F G, touching the given circle, and it will be the circle required.

In all cases of circles tangent to each other, their centres and their points of contact must lie in the same straight line.

To draw tangents to two given circles (Fig. 30).

Draw a straight line through the centres of the two given circles. From the centres A and B draw parallel radii, A D and B E, in the same direction.

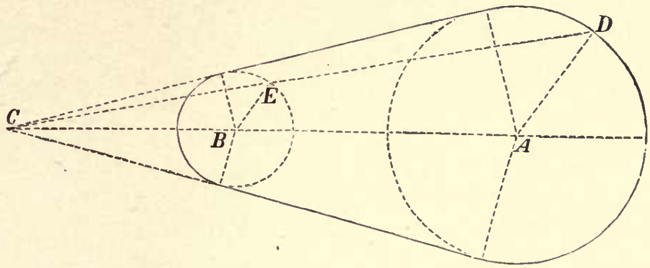


FIG. 30.

Draw a line from D to E, and produce it to meet the centre line at C; and from C draw tangents to one of the circles by Fig. 26. Those tangents will touch *both* circles as required.

To construct a circle through a given point tangent to a second circle at a given point (Fig. 31).

Let A be the given point of a circle A D C, B the point through which the required circle is to be drawn. Connect A and B, extend A O, bisect A B by a perpendicular. The intersection of this perpendicular with A O extended will be the centre of the required circle. The same method of construction would apply if the point B were inside the circle A D C.

Between two inclined lines to draw a series of circles touching these lines and touching each other (Fig. 32).

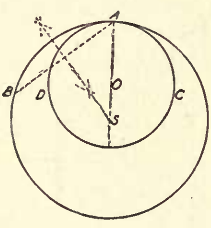


FIG. 31.

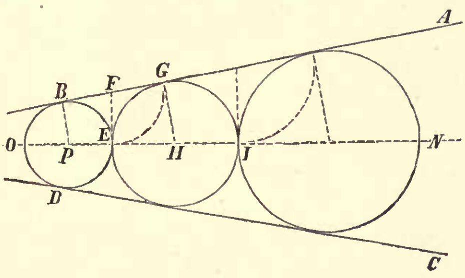


FIG. 32.

Bisect the inclination of the given lines A B and C D by the line N O; this is the centre line of the circles to be inscribed. From a point, P, in this line, draw P B perpendicular to the line A B; and from P describe the circle B D, touching the given lines, and cutting the centre line at E. From E draw E F perpendicular to the centre line, cutting A B at F; from F describe an arc, with a radius F E, cutting A B at G; draw G H parallel to B P, giving H the centre of the second touching circle, described with the radius H E or H G. By a similar process the third circle, I N, is described. And so on.

Inversely, the largest circle may be described first, and the smaller ones in succession.

Note.—This problem is of frequent use in scroll-work.

Between two inclined lines to draw a circular arc to fill up the angle (Fig. 33).

Let A B and D E be the inclined lines. Bisect the inclination by the line F C, and draw the perpendicular A F D to define the limit within which the circle is to be drawn. Bisect the angles A and D by lines cutting at C,

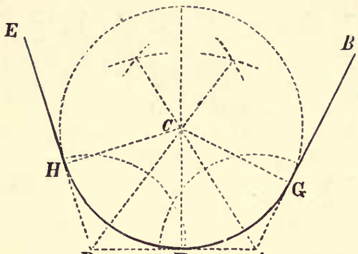


FIG. 33.

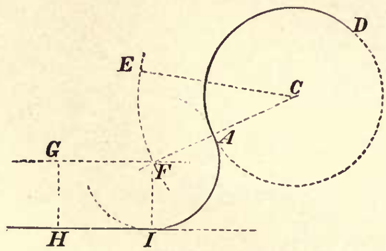


FIG. 34.

and from C, with radius C F, draw the arc H F G, which will be the arc required.

To fill up the angle of a straight line and a circle, with a circular arc of a given radius (Fig. 34).

On the centre C, of the given circle A D, with a radius C E equal to that of the given circle plus that of the required arc, describe the arc E F. Draw G F parallel to the given line H I, at the distance G H, equal to the radius of the required arc, cutting the arc E F at F. Then F is the required centre. Draw the perpendicular F I, and the line F C, cutting the circle at A; and, with the radius F A or F I, describe the arc A I, which will be the arc required.

To fill up the angle of a straight line and a circle, with a circular arc to join the circle at a given point (Fig. 35).

In the given circle B A draw the radius to A, and extend it. At A

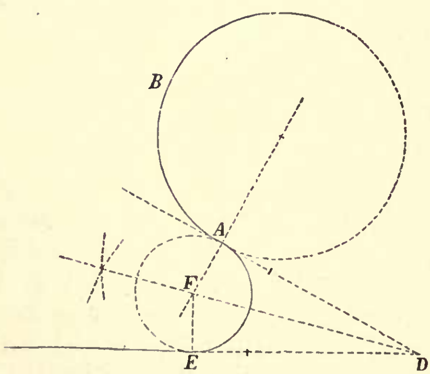


FIG. 35.

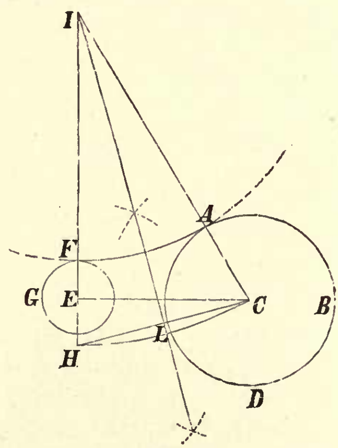


FIG. 36.

draw a tangent, meeting the given line at D. Bisect the angle A D E, so formed, with a line cutting the radius, as extended at F; and, on the centre F, with radius F A, describe the arc A E, which will be the arc required.

To describe a circular arc joining two circles, and to touch one of them at a given point (Fig. 36).

Let A B and F G be the given circles to be joined by an arc touching one of them at F.

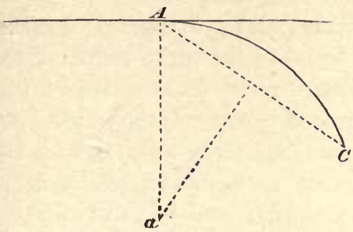


FIG. 37.

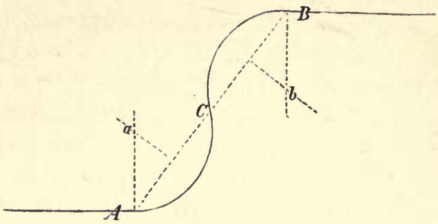


FIG. 38.

Draw the radius E F, and produce it both ways; set off F H equal to the radius, A C, of the other circle; join C to H, and bisect it with the perpendicular L I, cutting E F at I. On the centre I, with radius I F, describe the arc F A, which will be the arc required.

To find the arc which shall be tangent to a given point on a straight line, and pass through a given point outside the line (Fig. 37).

Erect at A, the given point on the given line, a perpendicular to the line. From C, the given point outside the line, draw C A, and bisect it with a perpendicular. The intersection of the two perpendiculars at *a* will be the centre of the required arc.

To connect two parallel lines by a reversed curve composed of two arcs of equal radii, and tangent to the lines at given points (Fig. 38).

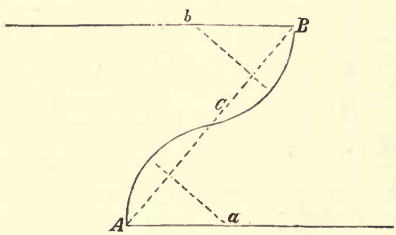


FIG. 39.

Join the two given points A and B, and divide the line A B into two equal parts at C; bisect C A and C B by perpendiculars; at A and B erect perpendiculars to the given lines, and the intersections *a* and *b* will be the centres of the arcs composing the required curve.

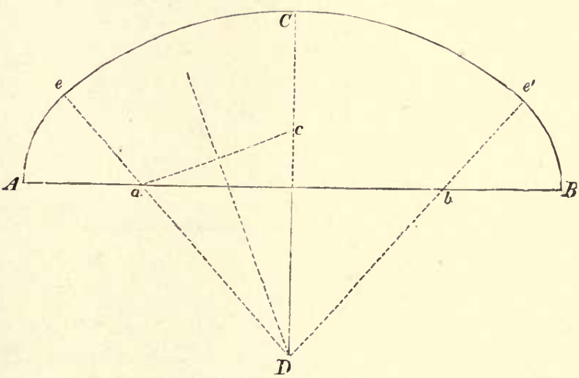


FIG. 40.

To join two given points in two given parallel lines by a reversed curve of two equal arcs, whose centres lie in the parallels (Fig. 39).

Join the two given points A and B, and divide the line A B in equal parts at C. Bisect A C and B C by perpendiculars; the intersections *a* and *b* of the parallel lines, by these perpendiculars, will be the centres of the required arcs.

On a given line, to construct a compound curve of three arcs of circles, the radii of the two side ones being equal and of a given length, and their centres in the given line; the central arc to pass through a given point on the perpendicular, bisecting the given line, and to be tangent to the other two arcs (Fig. 40).

Let A B be the given line, and C the given point. Draw C D perpendicular to A B; lay off A *a*, B *b*, and C *c*, each equal to the given radius of the side arcs; draw *a c*, and bisect it by a perpendicular; the intersection of this line with the perpendicular C D will be the required centre of the central arc *e C e'*. Through *a* and *b* draw the lines D *e* and D *e'*; from *a* and *b*, with the given radius equal to *a* A or *b* B, describe the arcs A *e* and B *e'*. From D, as a centre, with a radius equal to C D, and, consequently, by construction, equal to D *e* and D *e'*, describe the arc *e* C *e'*. The entire curve A *e* C *e'* B is the compound curve required.

PROBLEMS ON POLYGONS AND CIRCLES.

Three lines inclosing a space form a *triangle* (Fig. 41). If two of the sides are of equal length, it is an *isosceles* triangle (Fig. 42); if all three are of equal

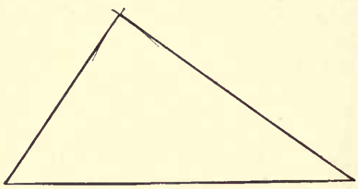


FIG. 41.

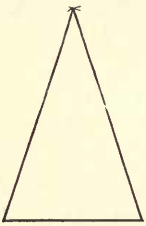


FIG. 42.

length, it is an *equilateral* triangle (Fig. 43). If one of the angles is a right angle, it is a *right-angled* triangle (Fig. 44), and if no two of the sides are of equal length, and not one of the angles a right angle, it is a *scalene* triangle.

To construct an isosceles triangle (Fig. 42).

Draw any line as a base, and, from each extremity as a centre, with equal radius, describe intersecting arcs. Draw

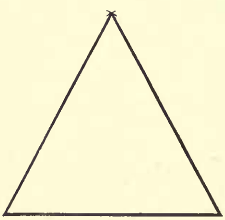


FIG. 43.

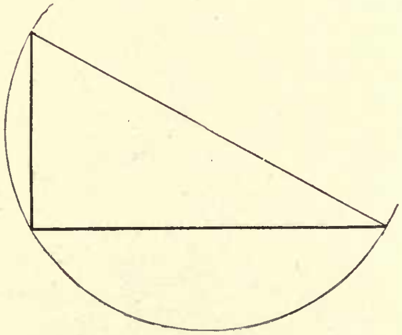


FIG. 44.

a line from each extremity of the base to this point of intersection, and the figure is an isosceles triangle.

To construct an equilateral triangle (Fig. 43).

Draw a base line, and from each extremity as a centre, with a radius equal to the base line, describe intersecting arcs. Draw lines from the extremities of the base to this point of intersection, and the figure is an equilateral triangle.

To construct a right-angled triangle (Fig. 44).

Construct a right angle by any one of the methods before described. Draw a line from the extremity of the one side to the extremity of the other side, and the figure is a right-angled triangle.

It will be evident, in looking at any right-angled triangle, that the side opposite the right angle is longer than either of the other or *adjacent* sides; this side is called the *hypothenuse.*

To construct a triangle equal to a given triangle A B C (Fig. 45).

1st Method (Fig. 46).—Draw a base line, and lay off its length equal to A B; from one of its extremities, as a centre, with a radius equal to A C, describe an arc; and, from its other extremity, with a radius equal to B C, describe an arc intersecting the first. Draw lines from the extremities to the point of intersection, and the triangle equal to A B C is complete.

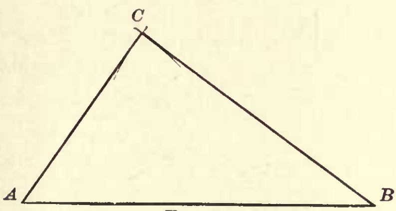


FIG. 45.

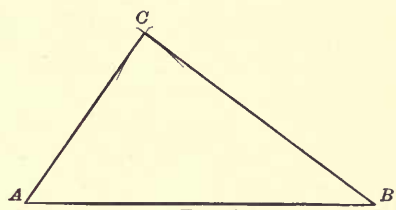


FIG. 46.

2d Method (Fig. 47).—Draw a base line, as before, equal to A B. At one extremity construct an angle equal to C A B, and at the other an angle equal to C B A. The sides of these angles will intersect, and form the required triangle.

3d Method (Fig. 48).—Construct an angle of the triangle equal to any angle of A B C, say the angle A C B. On one of its sides measure a line equal to C A, and on the other side one equal to C B; connect the two extremities by a line, and the triangle equal to A B C is complete.

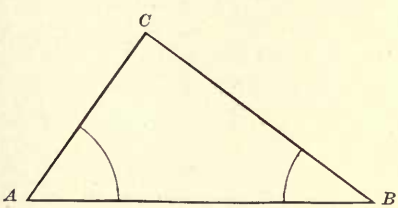


FIG. 47.

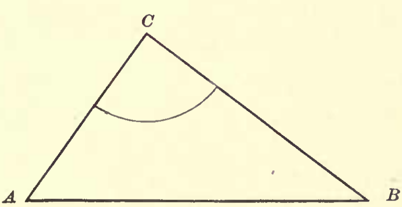


FIG. 48.

From the above constructions it will be seen that, if the three sides of a triangle, or two sides and the included angle, or one side and the two adjacent angles are known, the triangle can be constructed.

Construct a triangle, A B C (Fig. 49). Extend the base to E, and draw B D parallel to A C. As A C has the same inclination to C B that B D has, the angle C B D is equal to the angle A C B. As A C has the same inclination to A E that B D has, the angle D B E is equal to C A B. That is, the two angles formed outside the triangle are equal to the two inside at A and C; and the three angles at B are equal to the three angles of the triangle, and their sum is equal to two right angles. Therefore, the sum of the three angles of a triangle is equal to two right angles.

On one side of a triangle (Fig. 50) construct a triangle equal to the first. The exterior lines of the two triangles form a four-sided or *quadrilateral* figure, of which the opposite sides are equal and parallel, and the opposite an-

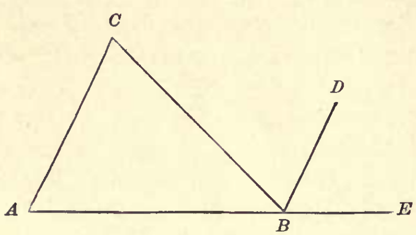


Fig. 49.

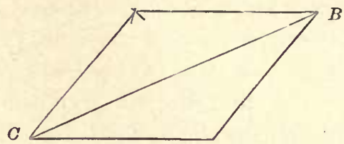


Fig. 50.

gles equal. This figure is called a *parallelogram*, and the line C B, extending between opposite angles, is a *diagonal.*

On the hypothenuse of a right-angled triangle (Fig. 51) construct another equal to it, and the exterior lines form a parallelogram, which, as all the angles are right angles, is called a *rectangle.* If the four sides are all equal, it is called a *square.*

A parallelogram in which all the sides are equal, but the angles not right angles, is called a *rhombus* (Fig. 52); if only the opposite sides are equal, it is

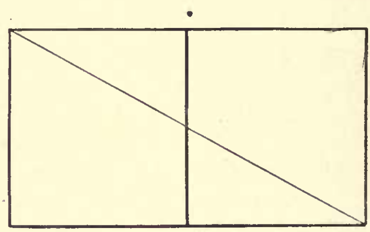


Fig. 51.

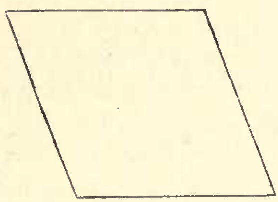


Fig. 52.

called a *rhomboid* (Fig. 50); if only two sides are parallel, the figure is a *trapezoid* (Fig. 53).

Take any figure (Fig. 54) bounded by straight lines and from any interior point draw lines to all the angles. There will be as many triangles as sides, and the sum of the angles of the figure will be equal to as many times two right

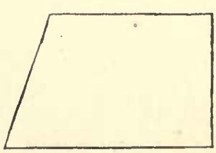


Fig. 53.

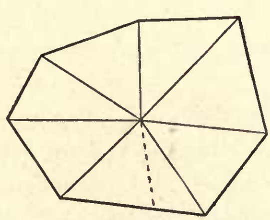


Fig. 54.

angles as sides less the four right angles at the centre, the sum of the angles of any triangle being equal to two right angles. If a line be drawn from the interior point to one side, another triangle is added to the collection and two right angles to the sum of the angles.

Polygons are figures of many angles, which if equal and of equal sides are

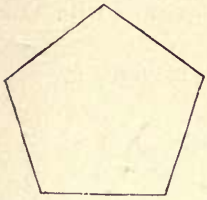


FIG. 55.

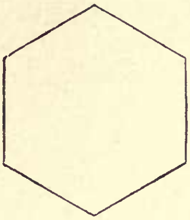


FIG. 56.

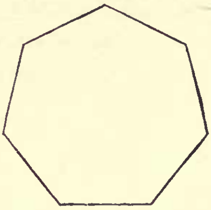


FIG. 57.

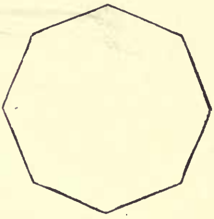


FIG. 58.

called regular polygons, and are designated by the number of their sides, as pentagons, hexagons, octagons, nonagons, decagons, etc.

To describe a circle about a triangle (Fig. 59).

Bisect two of the sides A B, A C, of the triangle at E, F; from these points draw perpendiculars cutting at K. From the centre K, with K A as radius, describe the circle A B C, as required.

To inscribe a circle in a triangle (Fig. 60).

Bisect two of the angles A, C, of the triangle A B C, by lines cutting at D; from D draw a perpendicular D E to any side, as A C; and with D E as radius, from the centre D, describe the circle required.

When the triangle is equilateral, the centre of the circle is more easily found by bisecting two of the sides, and drawing perpendiculars. Or, draw a perpendicular from one of the sides to the opposite angle, and from the side set off one third of the length of the perpendicular.

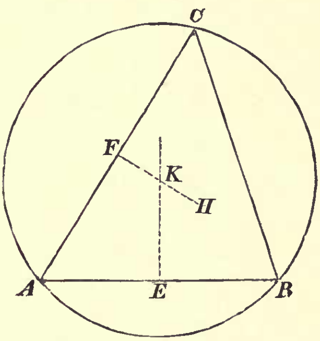


FIG. 59.

To inscribe a square in a circle; and to describe a circle about a square (Fig. 61).

To inscribe the square. Draw two diameters, A B, C D, at right angles, and join the points A, C, B, D, to form the square as required.

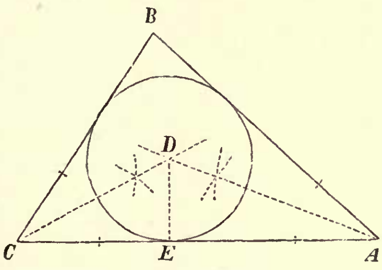


FIG. 60.

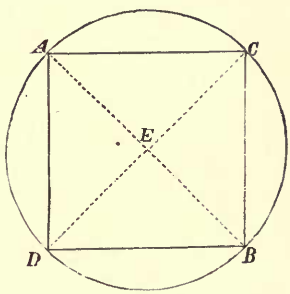


FIG. 61.

To describe the circle. Draw the diagonals A B, C D, of the given square, cutting at E; on E as a centre, with E A as radius, describe the circle as required.

In the same way, a circle may be described about a rectangle.

To inscribe a circle in a square; and to describe a square about a circle (Fig. 62).

To inscribe the circle. Draw the diagonals A B, C D, of the given square, cutting at E; draw the perpendicular E F to one of the sides, and with the radius E F, on the centre E, describe the circle.

To describe the square. Draw two diameters A B, C D, at right angles,

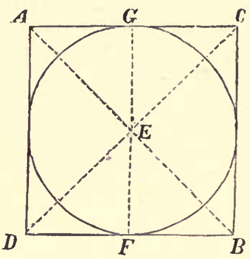


Fig. 62.

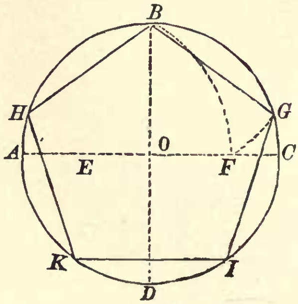


Fig. 63.

and produce them; bisect the angle D E B at the centre by the diameter F G, and through F and G draw perpendiculars A C, B D, and join the points A, D, B, C, where they cut the diagonals, to complete the square.

To inscribe a pentagon in a circle (Fig. 63).

Draw two diameters, A C, B D, at right angles; bisect A O at E, and from E, with radius E B, cut A C at F; from B, with radius B F, cut the circumference at G and H, and with the same radius step round the circle to I and K; join the points so found to form the pentagon.

To construct a regular hexagon upon a given straight line (Fig. 64).

From A and B, with a radius equal to the given line, describe arcs cutting at *g*; from *g*, with the radius *g* A, describe a circle; with the same radius set

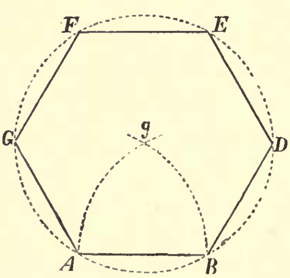


Fig. 64.

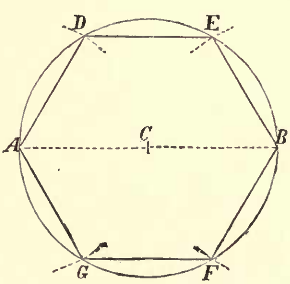


Fig. 65.

off from A the arcs A G, G F, and from B the arcs B D, D E. Join the points so found to form the hexagon.

To inscribe a regular hexagon in a circle (Fig. 65).

Draw a diameter, A B; from A and B as centres, with the radius of the circle A C, cut the circumference at D, E, F, G; draw straight lines A D, D E, etc., to form the hexagon.

To describe a regular hexagon about a circle (Fig. 66).

Draw a diameter, A B, of the given circle. With a radius A D from A as a centre, cut the circumference at C; join A C, and bisect it with the radius D E; through E draw F G parallel to A C, and with the radius D F describe

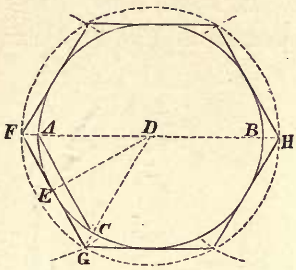


FIG. 66.

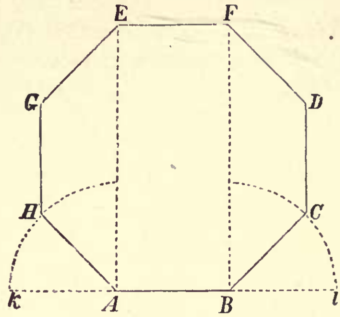


FIG. 67.

the circle F H. Within this circle describe a regular hexagon by the preceding problem; the figure will touch the given circle as required.

To construct a regular octagon upon a given straight line (Fig. 67).

Produce the given line A B both ways, and draw perpendiculars A E, B F;

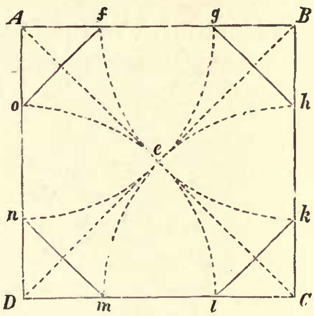


FIG. 68.

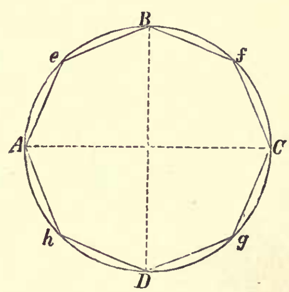


FIG. 69.

bisect the external angles at A and B by the lines A H, B C, each equal to A B; draw C D and H G parallel to A E and equal to A B; and from the centers G, D, with a radius equal to A B, cut the perpendiculars at E, F, and draw E F to complete the octagon.

To make a regular octagon from a square (Fig. 68).

Draw the diagonals of the square intersecting at *e;* from the corners A, B, C, D, with A *e* as radius, describe arcs cutting the sides at *g h*, etc.; join the points so found to complete the octagon.

To inscribe a regular octagon in a circle (Fig. 69).

Draw two diameters, A C, B D, at right angles; bisect the arcs A B, B C, etc., at *e*, *f*, etc.; and join A *e*, *e* B, etc., for the inscribed figure.

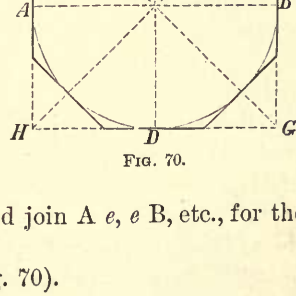


FIG. 70.

To describe a regular octagon about a circle (Fig. 70).

Describe a square about the given circle A B; draw perpendiculars *h k*, etc., to the diagonals, touching the circle.

Or, to find the points *h*, *k*, etc., cut the sides from the corners of the square, as in Fig. 68.

To inscribe a circle within a regular polygon.

When the polygon has an even number of sides, as in Fig. 71, bisect two

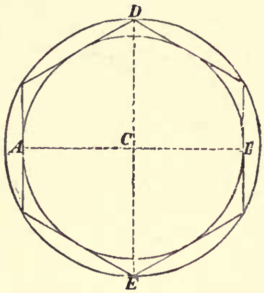


FIG. 71.

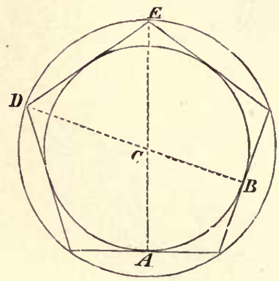


FIG. 72.

opposite sides at A and B, draw A B, and bisect it at C by D E drawn between opposite angles; with the radius C A describe the circle as required.

When the number of sides is odd, as in Fig. 72, bisect two of the sides at A and B, and draw lines A E, B D, to the opposite angles, intersecting at C; from C, with C A as radius, describe the circle as required.

To describe a circle about a regular polygon.

When the number of sides is even, draw two diagonals from opposite angles, like D E (Fig. 71), to intersect at C; and from C, with C D as radius, describe the circle required.

When the number of sides is odd, find the centre C (Fig. 72) as in last problem, and, with C D as radius, describe the circle.

For the construction of the regular polygons Fig. 73 will be found useful.

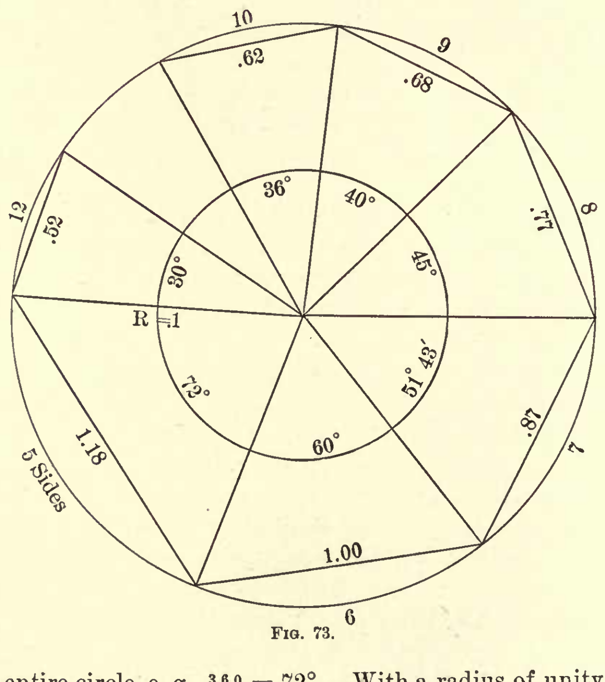


FIG. 73.

Divide the interior circle into the number of degrees corresponding to the proportion of the sides of the polygon to the entire circle, e. g., $\frac{360}{5} = 72°$. With a radius of unity describe an exterior circle and extend radii through the divisions of the interior circle. The chords of the arcs intersected correspond to the sides of the different polygons.

The figure gives the polygons such as are usually found in practice, but a similar figure can be constructed increasing the number of sides as far as may be required.

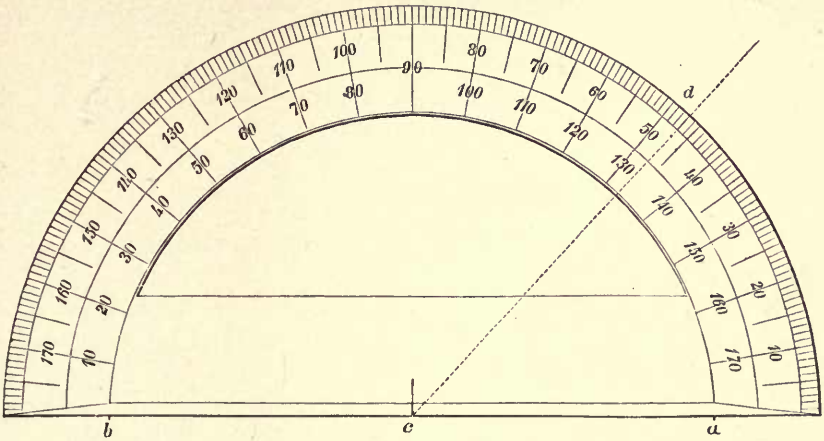


FIG. 74.

For the laying out of angles the protractor is used. In its simplest form it consists of a semicircle of metal or horn of which the edge is divided into 180 degrees.

To lay off a given angle—say 47° (Fig. 74)—place the edge of the protractor, *a b*, along the given line and make the centre of the protractor coincide with the vertex *c* of the angle to be laid off; mark off on the edge the division 47°, remove the protractor, and through this mark and the vertex *c* draw a line; the angle *a c d* will be equal to 47°, and *b c d* to 133°. These two are *supplements* of each other, or what each requires to make up the sum of 180°.

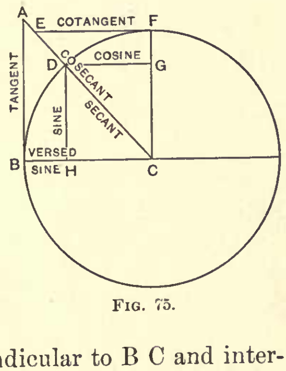
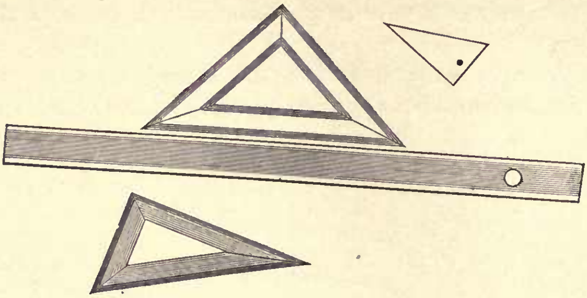


FIG. 75.

Fig. 75 represents the terms used in defining angles, and of which tables are given in the Appendix by which angles may be constructed without the use of the protractor.

Considering B C D the angle, the perpendicular D H dropped from the radius at D and intersecting the diameter at H is the *sine*, the line B A perpendicular to B C and intersecting the extended radius at A is the *tangent*, the extension of the radius C D to the intersection of the tangent at A the *secant*; the *versed sine* is the line B H extending from the sine to the tangent. The *cosine*, *cotangent*, *cosecant*, and *coversed sine* are respectively the sine, tangent, secant, and versed sine of the angle D C F, the *complement* of B C D, having the number of degrees necessary to complete the quadrant of 90 degrees.



Use of Triangle and Square.

Right-angled triangles constructed of wood, hard rubber, celluloid, or metal are very useful in connection with a straight-edge, or ruler, in drawing lines parallel or perpendicular to other lines.

To draw lines parallel to each other, place any edge of the triangle in close contact with the edge of the ruler. Hold the ruler (Fig. 76) firmly with the

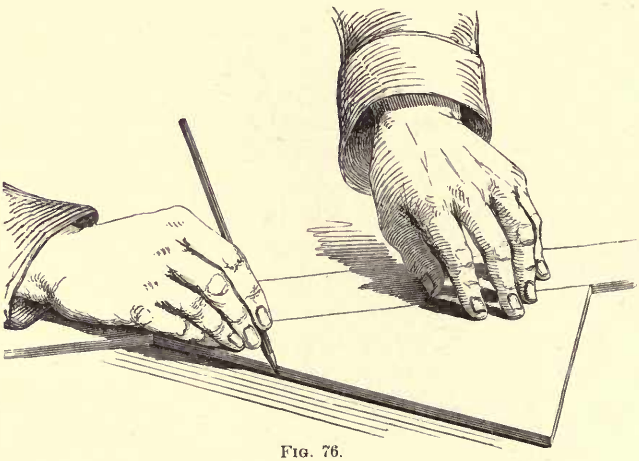


Fig. 76.

thumb and little finger of the left hand, and the triangle with the other three fingers; with a pencil or pen in the right hand, draw a line along one of the free edges of the triangle; withdraw the pressure of the three fingers upon the triangle, and slide it along the edge of the ruler, keeping the edges in close contact; a line drawn along the same edge of the triangle, as before, will be parallel to the first line. If the edge of the hypothenuse of the triangle be placed in contact with the ruler, lines drawn along one edge of the triangle will be at right angles to those drawn along the other.

Through a given point to draw a line parallel to a given line (Fig. 77).

Place one of the shorter edges of the triangle along the given line A B, and bring the ruler against the hypothenuse; slide the triangle up along the edge of the ruler until the upper edge of the ruler is sufficiently near to the given point C to allow a line to be drawn through it. Draw the line, and it will be parallel to A B.

If the triangle be slid farther up along the edge of the ruler, and a line be drawn through C along the other edge of the triangle (Fig. 78), this line will be perpendicular to A B. If the triangle be slid still farther up along the

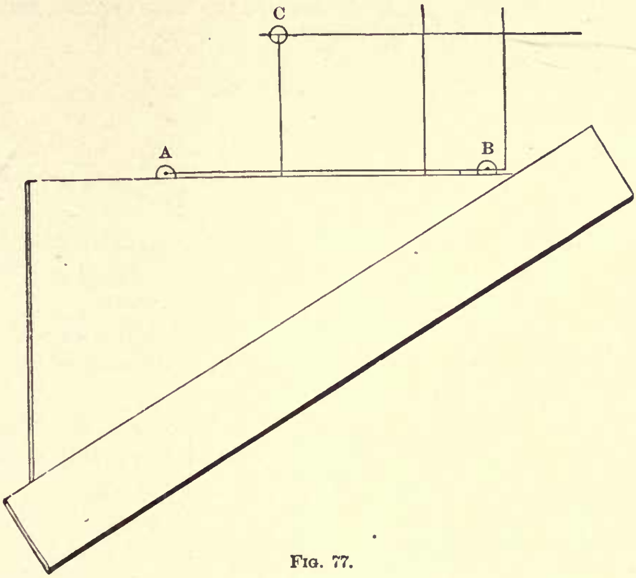


FIG. 77.

edge of the ruler, and a third line be drawn touching A B, the figure constructed will be a rectangle; and if E D be laid off on A B, equal to C E, the figure inclosed is a square (Fig. 79).

It will be seen that the triangle and ruler afford a much readier way of

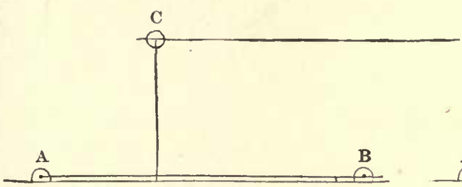


FIG. 78.

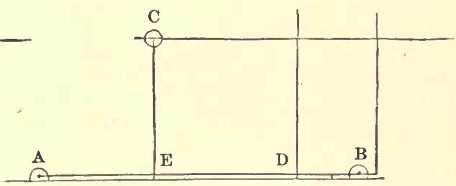


FIG. 79.

drawing parallel lines, and lines at right angles, than the compasses and ruler, and may be used in solving the following problems:

The *area* of a figure is the quantity of space inclosed by its lines.

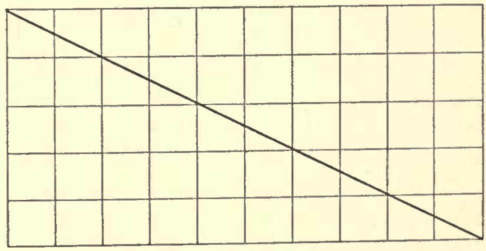


FIG. 80.

Construct a right angle (Fig. 80). Divide the base and the perpendicular by dividers into any number of equal spaces; for instance, ten on the one and five on the other. Construct a rectangle with this base and perpendicular, and through the points of division lay off lines parallel to the base and perpendicular. The rectangle will be divided into fifty equal squares, and its measure in squares will be the divisions ten in the base, multiplied by the five in the perpendicular. If the division were inches, then the area of this rectangle would be fifty square inches; if feet, then fifty square feet. If there

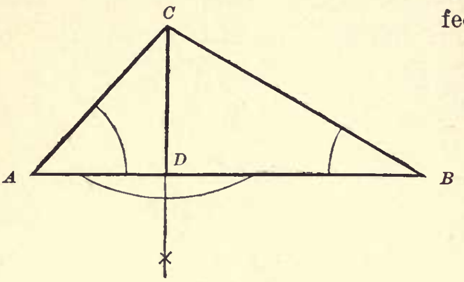


FIG. 81.

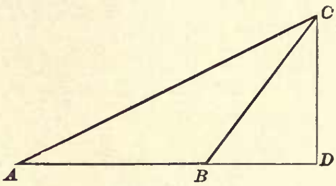


FIG. 82.

were but five divisions in the base and five in the perpendicular, the surface would be twenty-five squares. Therefore, a rectangle has for its measure the base multiplied by its adjacent side or height.

Draw a diagonal, and the rectangle is divided into two equal triangles. Each triangle must therefore have for its measure the base multiplied by half the perpendicular, or, as is usually said, by half the altitude.

Take any triangle (Fig. 81), and from its apex draw a line perpendicular to the base. The triangle is divided into two right-angled triangles, which must have for their measure A D × ½ C D, and D B × ½ C D, and the sum of the two must be A B × ½ C D.

If the perpendicular from the apex falls outside the triangle (Fig. 82), then the triangle B D C and A D C will have for their measure B D × ½ C D and A D × ½ C D, consequently their difference, A B C, must have for its measure A B × ½ C D. Any polygon can be divided into triangles (see Fig. 54), and its area is made up of the sum of the areas of the triangles. By graphic construction the sum of the areas of the different triangles composing a polygon may be resolved readily into a single triangle and its area taken. For instance, take a six-sided polygon (Fig. 83), draw a line from A to C, and a line parallel to A C, at B intersecting the extended base at B′, a diagonal drawn from A to B′ will give one side of the triangle; draw a diagonal from A to E, extend the side E D of the polygon indefinitely, draw a line at F parallel to A E, and intersect the extended side at *e*, draw the line A D and a parallel to this at *e*, intersecting the extended base at E′. A diagonal drawn from A to E′ will, with the side previously obtained and the base, give a triangle equal in area to the polygon.

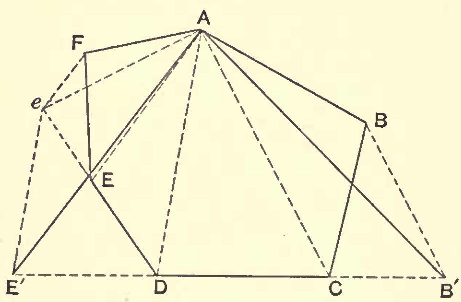


FIG. 83.

SCALES.

The distances given in Fig. 80 may represent feet, yards, miles, or any other unit of measure. Thus, if they represent miles, the figure represents an area of fifty square miles. With a scale of equal parts, each part may represent any unit of measure, and a drawing on paper to that scale represents the object from which it is drawn in a reduced form, from which measures in detail by the scale may be more readily and as accurately taken as from the natural object in the shop or on the estate, and if designs are made to a scale they can be executed conformably and accurately in all their parts in either enlarged or reduced size.

Practically a two-foot rule, with its divisions into inches, halves, quarters, eighths, and sixteenths, may be made use of as a scale of equal parts, any division being taken as the unit to represent a foot, a yard, or a mile; but among drawing instruments scales especially adapted to the purpose are found in great variety of forms, divisions, and material. Fig. 84 represents a convenient form of scale, as it contains, in addition to the simply divided scales, a protractor along its edges.

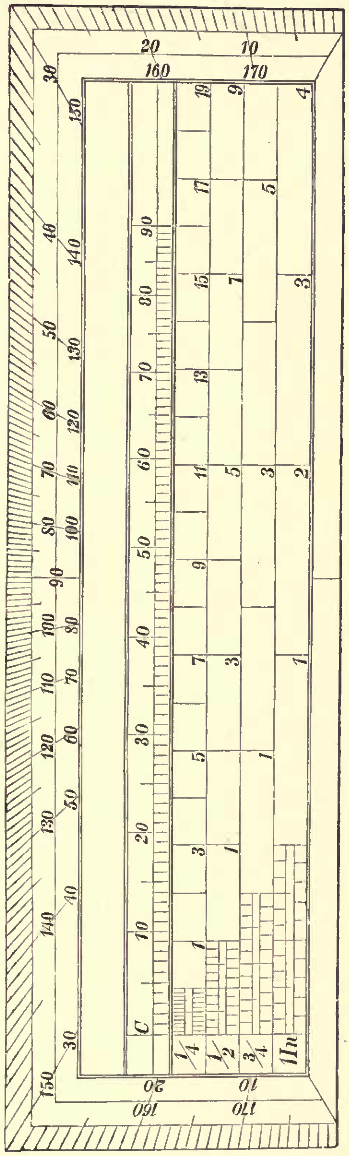


Fig. 84.

The simply divided scales consist of a series of equal divisions of an inch, which are numbered 1, 2, 3, etc., beginning from the second division on the left hand; the upper part of the left division in each is subdivided into twelve equal parts, and the lower part into ten equal parts. The scales are marked at the left 1 inch, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{4}$, and when used in drawing the scale is written as 1 inch, $\frac{3}{4}$, $\frac{1}{2}$, or $\frac{1}{4}$ inch to a foot, rod, or mile, or whatever may be the unit of actual measure. When the unit is the inch the first scale will be full size, the second $\frac{3}{4}$, the third $\frac{1}{2}$, and the fourth $\frac{1}{4}$ full size. If the scale adopted is such a part of an inch to the foot, then the upper subdivisions will represent inches.

Above the simply divided scales there is a scale marked C, which is a scale of chords; taking a radius equal to C–60, the chords of the different angles are represented by the division; thus an angle of 20° the chord will be C–20.

SIMILAR TRIANGLES.

Triangles which are equiangular are similar, and have their homologous sides—that is, their sides adjacent to the equal angles—proportional; conversely, two triangles which have their homologous sides proportional are equiangular.

Two triangles which have their sides parallel (Fig. 85) or perpendicular to

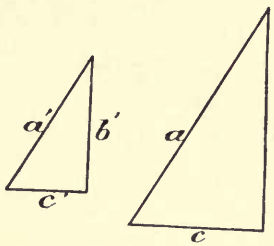


FIG 85.

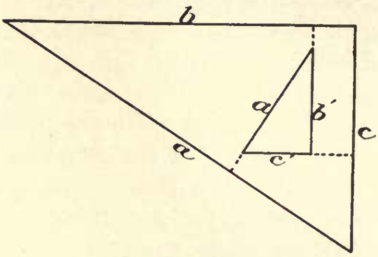


FIG. 86.

each other (Fig. 86) are similar. A line *c* drawn parallel to one side *c'* of a triangle (Fig. 87) forms a triangle *a c b* whose sides are proportional to the original triangle.

In Fig. 88 a polygon is divided into triangles by lines from an interior point to its angles and these lines intersected by lines parallel to the sides of the polygon. The figure thus constructed is a polygon similar to the original polygon composed of triangles similar to the triangles into which it was divided.

In the figure the parallel lines are drawn across the sides of the triangles at one half their length, and the areas of the small triangles are therefore equal to the square of one half or to one quarter that of the original triangles; consequently the area of the interior polygon is one quarter that of the exterior one. As this construction obtains at any intersecting length, it affords a means of reducing the scale of the original polygon.

To a scale of ½ inch (Fig. 89)

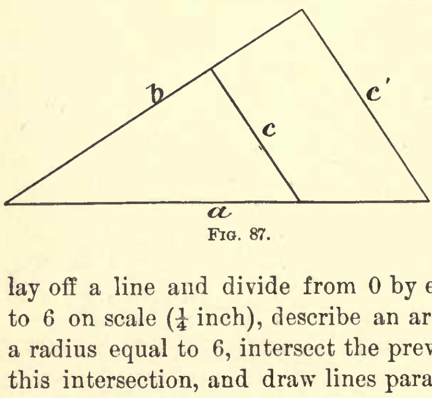


FIG. 87.

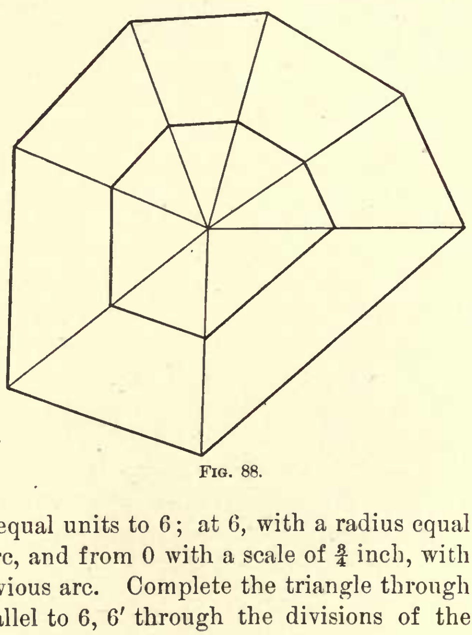


FIG. 88.

lay off a line and divide from 0 by equal units to 6; at 6, with a radius equal to 6 on scale (¼ inch), describe an arc, and from 0 with a scale of ⅜ inch, with a radius equal to 6, intersect the previous arc. Complete the triangle through this intersection, and draw lines parallel to 6, 6′ through the divisions of the first line; the triangle will be divided into six similar triangles of which the homologous sides are proportional and represented on their different scales by the same number of units.

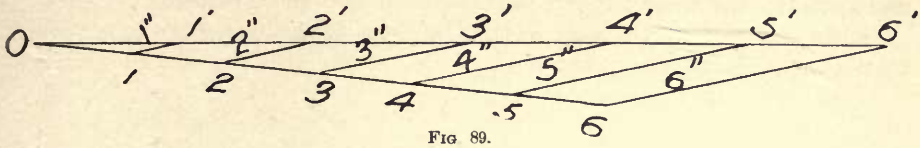


FIG 89.

Fig. 90 illustrates the application of scales to the measurement of lines which are inaccessible. Thus the lines *a b* and *a c*, with their inclosed angle, can be measured, and, if plotted to any scale, the line *c b* can be measured on the same scale.

The height of an object may be obtained by the application of similar triangles, or by the length of the shadow cast, which is merely another application of the same method. The observer measures off, say, 60 feet from a flag pole (Fig. 91), and a rod is held at, say, 12 feet from the observer; a sight is then taken to the top and bottom of the flag pole at the 60-feet distance, and the points at which the sights intersect the rod are found to be 10 feet apart. Then by construction the height of the flag pole is found by scale to be 50 feet. By means of shadows, if the length of the shadow is found to be 40 feet and the shadow cast by a 10-foot rod is 8 feet, then by plotting the height is found, as before, 50 feet.

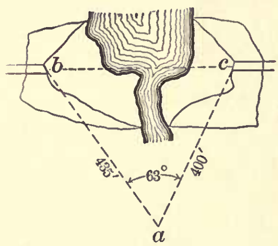


FIG. 90.

The value of the above solution of geometrical problems depends on the accuracy of the drawing.

To construct a square equal to one half of a given square (Fig. 92).

Let *a b c d* be the given square, and draw diagonals in it. The square, *e b f d*, constructed on one half of one of these diagonals, will be equal to one half the given square.

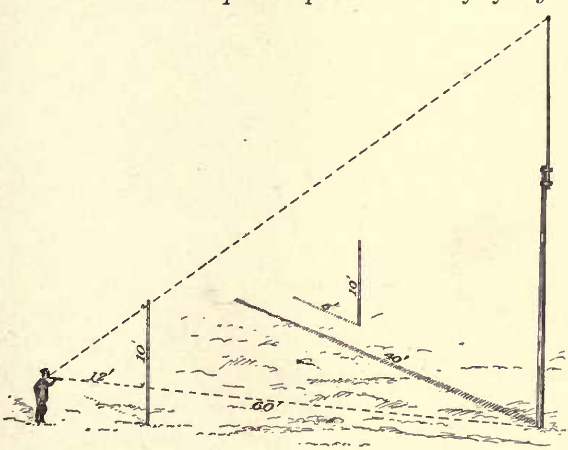


FIG. 91.

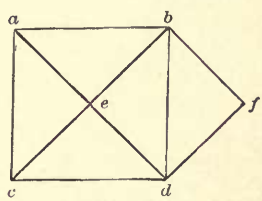


FIG. 92.

To construct a square equal to double a given square (Fig. 93).

Construct a square on one of the diagonals in the given square, or inclose the square with parallels to the diagonals of the square.

To construct a square equal to three times a given square (Fig. 94).

Extend the base of the given square to the length of its diagonal. Draw a line from the point at which this line ends to the extreme angle of the square, and upon this line erect a square, which will be the square required.

For a square four times the size of a given square, make the base double that of the given square.

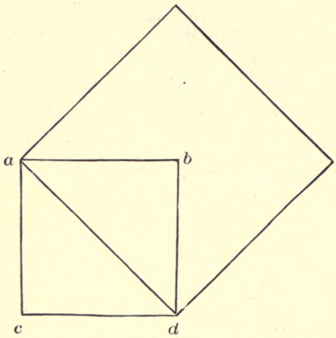


FIG. 93.

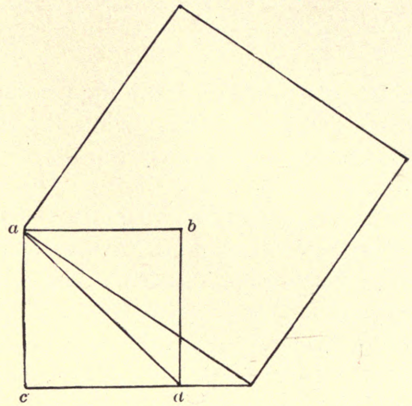


FIG. 94.

To construct a square equal to five times a given square (Fig. 95).

Extend the base of the given square, making the extension to *d e* equal to *c d*. From *e* draw a line to *a*, and on this line construct a square, which will be the square required.

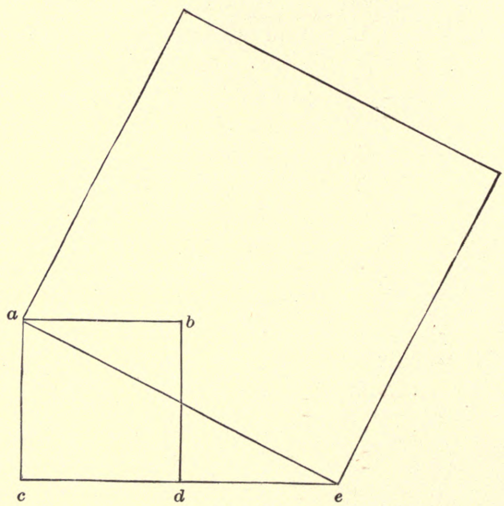


FIG. 95.

Assuming the side of the given square in Figs. 92, 93, 94, and 95 to be the radius (or diameter) (Fig. 96) of a given circle, then the side of the square to be constructed half, twice, three, four, or five times the size of the given square will be the radii (or diameters) of the circles half, twice, three, four, or five times the size of the given circle.

It will be seen by Fig. 93 that the square constructed on the diagonal of a square is equal to double that of the original square.

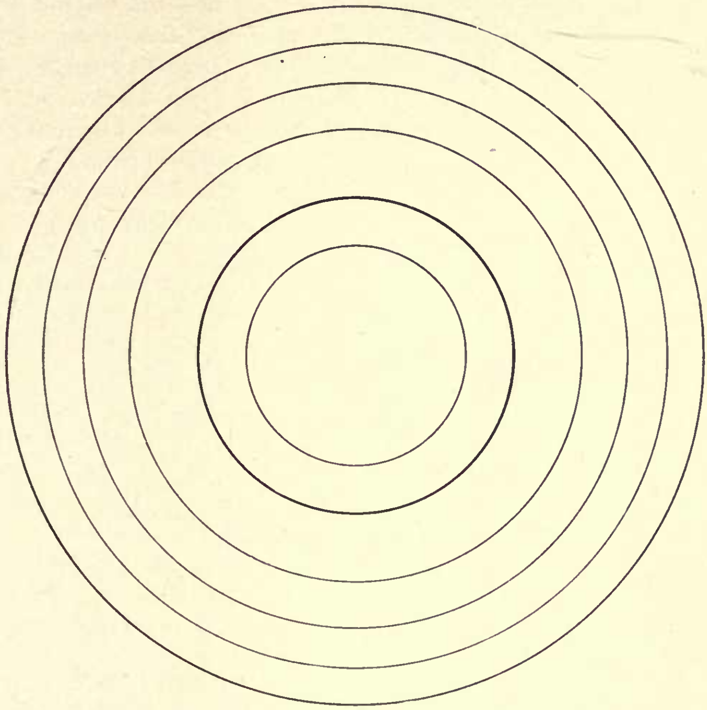


FIG. 96.

On any right-angled triangle A C B (Fig. 97) let fall a perpendicular from the vertex of the right angle to the hypothenuse A B; the triangle will be divided into two similar triangles, similar to each other and to the original triangle, and

$$AD : AC :: AC : AB; \text{ that is, } AC^2 = AD \times AB;$$
$$BD : BC :: BC : AB; \text{ that is, } BC^2 = BD \times AB.$$

$AD + BD = AB$, and the sum of the two equations is $AB^2 = AC^2 + BC^2$.

Therefore the square constructed on the hypothenuse of a right-angled triangle is equal to the sum of the squares of the other two sides (Fig. 98).

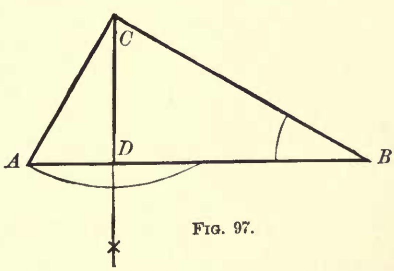


FIG. 97.

To determine how much is added to a given square by extending its base and constructing a square thereon (Fig. 99).

Let a represent the side C D of the given square. The area of the square is $a \times a$ or a^2.

Extend the side C D by a length, D G, represented by b. Then the new square $(a + b) \times (a + b)$ will be made up of the old square, or a^2, and two rectangles, D G E H and P E K L, or $2\ (a \times b)$ or $2\ a\ b$, and one square, E H K J, $b \times b$, or b^2. The area $(a + b)^2 = a^2 + 2\ a\ b + b^2$.

To determine how much is taken from the area of a given square, by reducing its base and constructing a square (Fig. 99).

Let a represent C G, the side of the given square.

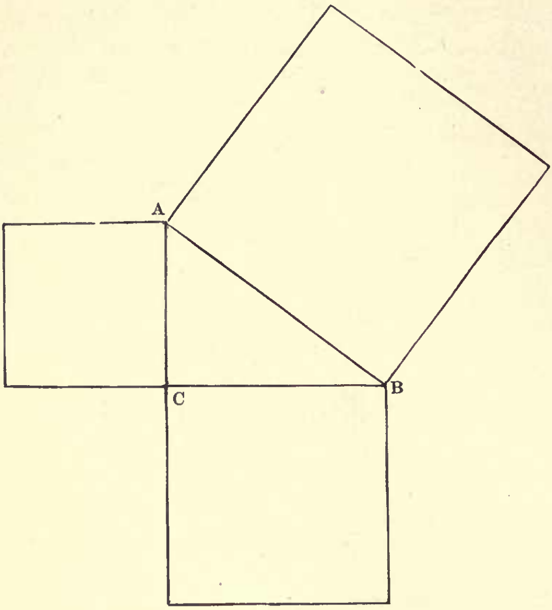


FIG. 98.

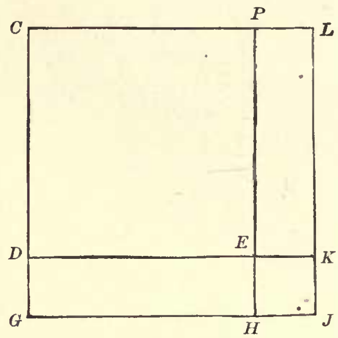


FIG. 99.

Reduce C G, the length G D, represented by b. The new square $(a - b)^2$ is the given square, or a^2, diminished by two rectangles, D G J K and P L J H, or $-\ 2\ a\ b$ excepting one square, E H J K, $b \times b$ or $+\ b^2$. The area $(a - b)^2 = a^2 - 2\ a\ b + b^2$.

The last two constructions, in default of a table of squares, may often be found of use.

CONSTRUCTION OF THE ELLIPSE, PARABOLA, HYPERBOLA, AND SPIRAL.

An *ellipse* is an oval-shaped curve (Fig. 100), in which, if from any point, P, lines be drawn to two fixed points, F and F′, called *foci*, their sum will always be the same. The line A B passing through the foci is the *major* axis, and the perpendicular C D at the centre of it is the *minor* axis.

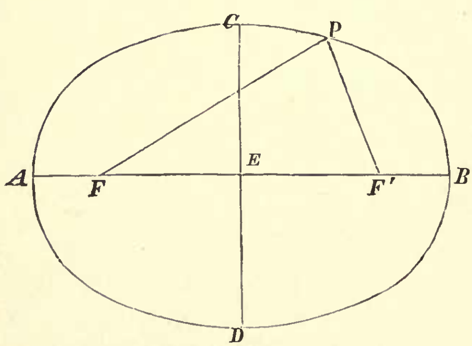


FIG. 100.

To construct an ellipse, the axes being known (Fig. 100).

1st Method.—Let the two axes be the lines A B and C D. From C as a centre, with a radius equal to E B (half the major axis), describe an arc cutting this axis at two points, F and F′, which are the foci. Insert a pin in each of the foci, and loop a thread upon them, so that, when stretched by a pencil inside the loop, the point of the pencil will coincide with C. Move the pencil round, keeping the loop evenly stretched, and it will describe an ellipse. This construction follows the definition above given of an ellipse, that the sum of the distances of every point of the curve from the foci is equal. It is seldom used by the draughtsman, as it is difficult to keep a thread evenly stretched; but for gardeners, laying out beds or plots, it is very convenient and sufficiently accurate.

There are many forms of ellipsographs for drawing ellipses, and various sizes of ellipses in hard wood and rubber on sale. Pattern-makers usually lay out ellipses by means of a trammel (Fig. 101), which consists of a rectangular cross, with guiding grooves in which movable rods attached to sliders on a bar are fitted, so as to move easily and uniformly. In describing an ellipse place the trammel with its grooves on the lines of the axes with the bar on the line of the major axis; set the pencil or marker on the extremity of this axis, and slip the outer rod to the crossing of the grooves and clamp it to the bar. Now slide the rod down the minor axis, and, with the pencil at the extremity of this axis, clamp the intermediate rod to the bar at the crossing of the guides. Revolve the bar, the intermediate rod following the major-axis groove, and the extreme rod that of the minor axis, and the pencil will describe the ellipse. Light trammels are made for the use of draughtsmen, but, as the necessity of drawing ellipses is not frequent, it can be readily done by the use of a strip of cardboard (Fig. 102). Lay off the major and minor axes on the paper; these represent the grooves of the trammel. Now take a strip of cardboard with a straight edge, lay it along the line of the major axis, and mark the positions a at the extremity of this axis, and c at the crossing of the axes; place the mark a on the extremity of the minor axis, and mark on the edge of the card at b the crossing of the axes. Revolve the card as described for the trammel, mark the positions of a by points, and connect them for the curve.

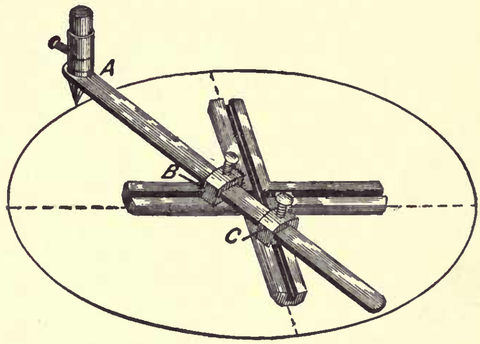


FIG. 101.

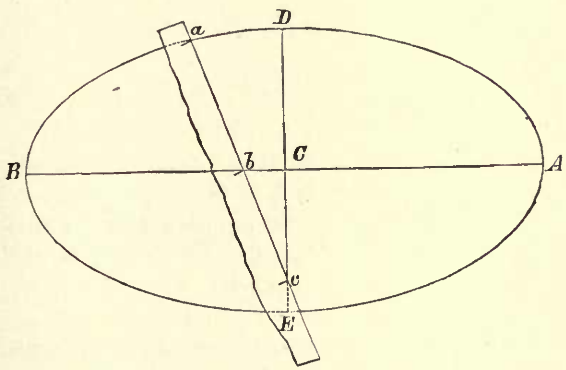


FIG. 102.

To construct an approximate semi-ellipse by means of five arcs of circles.

Let A B (Fig. 103) be the major axis, and O D the semi-minor axis. Draw the semicircles A C B and d D d'. Divide these semicircles into equal parts by the radial lines O *e*, O *f*, O *e'*, O *f'*. From the points of intersection of these radial lines with the semicircumference draw *g b*, *h a*, *h' a'*, *g' b'*, parallel to the major axis. From *e*, *f*, *e'*, *f'*, intersections of the radial lines with the

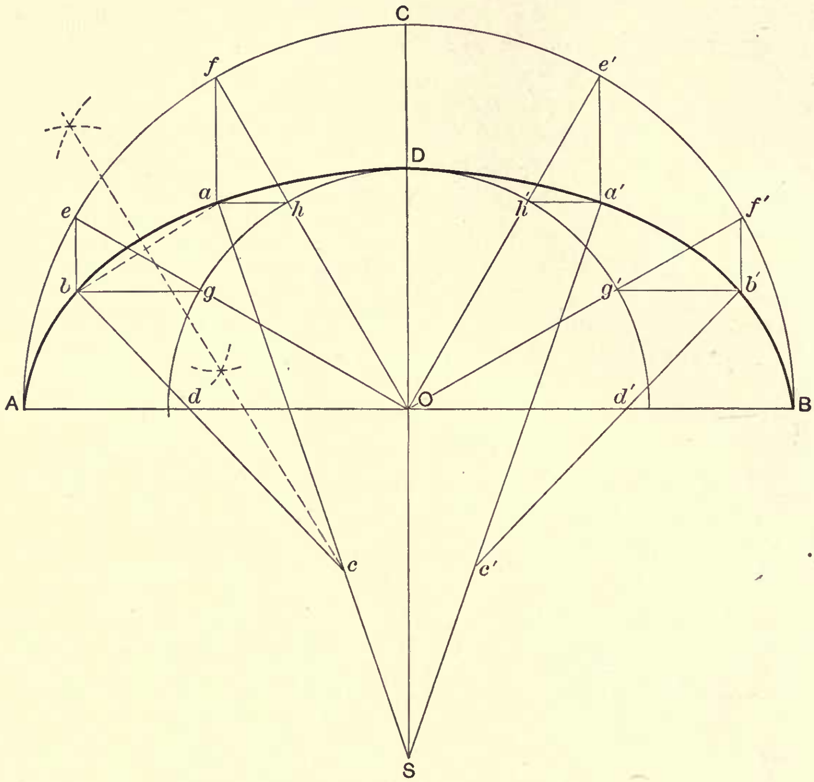


Fig. 103.

semicircumference A C B, draw *e b*, *f a*, *e' a'*, and *f' b'* parallel to the minor axis. The intersections of these lines with *b g*, *a h*, etc., will be points on the ellipse. Now through the three points *a*, D and *a'* construct an arc of a circle. Connect *a* and *b* with a chord, bisect it with a perpendicular; where this perpendicular intersects *a* S at *c* is the centre of the arc *a b*.

Connect *b* and *c*; *d*, the intersection of *b c* with A B, will be the centre of the arc *b* A. Arcs through *a' b'* and B can be constructed in the same way, or the centres can be transferred.

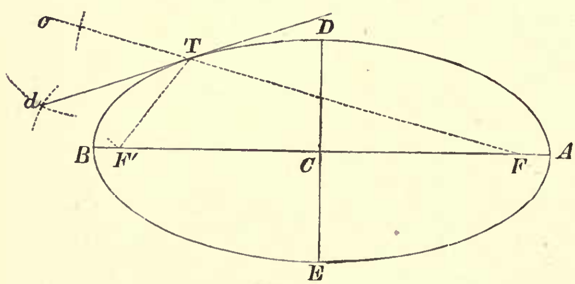


Fig. 104.

The ellipse can in the same way be made up of any number of arcs of circles.

To draw a tangent to an ellipse through a given point in the curve (Fig. 104).

From the given point T draw straight lines to the foci, F, F′; produce F T beyond the curve to *c*, and bisect the exterior angle *c* T F′ by the line T *d*. This line T *d* is the tangent required.

To draw a tangent to an ellipse from a given point without the curve (Fig. 105).

From the given point T as a centre, with a radius equal to its distance from the nearest focus F, describe an arc; from the other focus F′, with the major axis as radius, cut the arc at K, L, and draw K F′, L F′, touching the curve at M, N; then the lines T M, T N, are tangents to the curve.

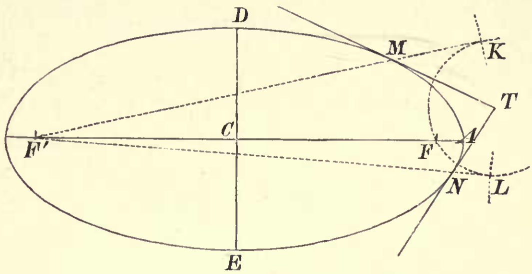


Fig. 105.

The Parabola.

The *parabola* may be defined as an ellipse whose major axis is infinite; its characteristic is that every point in the curve is equally distant from the *directrix* E N and the *focus* F (Fig. 106).

To construct a parabola when the focus and directrix are given.

1st Method (Fig. 106).—Through the focus F draw the axis A B perpendicular to the directrix E N, and bisect A F at *e*, the *vertex* of the curve. Through a series of points, C, D, E, on the directrix, draw parallels to A B; connect these points, C, D, E, with the focus F, and bisect by perpendiculars the lines F C, F D, F E. The intersections of these perpendiculars with the parallels will give points, C′, D′, E′, in the curve, through which trace the parabola.

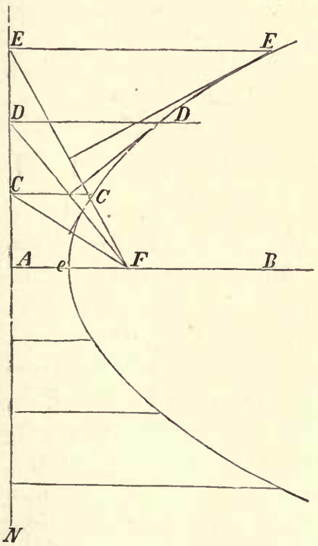


Fig. 106.

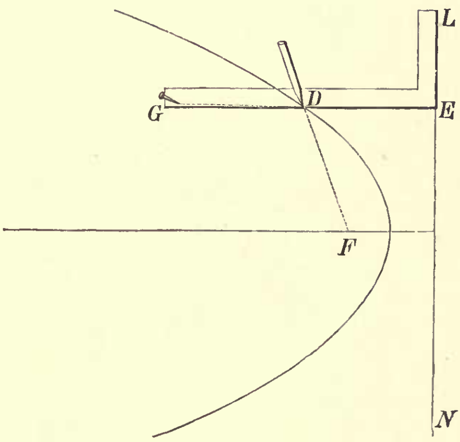


Fig. 107.

2d Method (Fig. 107).—Place a straight-edge to the directrix E N, and apply to it a square L E G; fasten at G one end of a cord, equal in length to

E G; fix the other end to the focus F; slide the square steadily along the straight-edge, holding the cord taut against the edge of the square by a pencil, D, and it will describe the curve.

To construct a parabola when the vertex, the axis, and a point of the curve are given (Fig. 108).

Let A be the vertex, A B the axis, and D the point in the curve. Construct the rectangle A B D C; divide D C into, say, four equal parts at 1 2 3, and A C into the same number at 1′ 2′ 3′; draw diagonals, A 3, A 2, A 1; and

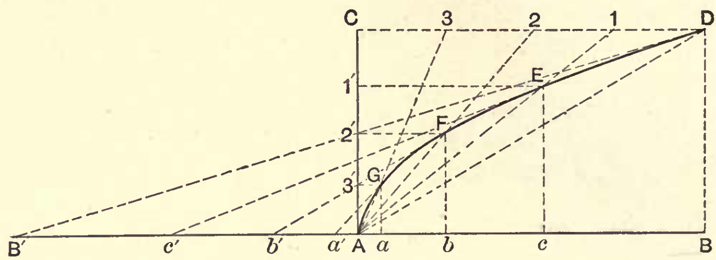


FIG. 108.

parallels to the axis through 1′ 2′ 3′. The intersection of the diagonals A 3, A 2, A 1 with the parallels 3′, 2′, 1′ at G, F, E will be points in the required curve.

Extend the axis to B′, making A B′=A B; draw perpendiculars to the axis from G, F, E, D; lay off toward B′, *a*′=A *a*, A *b*′=A *b*, A *c*′=A *c*; and draw B′ D, *c*′ E, *b*′ F and *a*′ G. These lines will be tangents to the curve at D, E, F, G, and lines perpendicular to the tangents at these points will be perpendicular to the curve.

The Hyperbola.

An *hyperbola* is a curve from any point, P, in which, if two straight lines be drawn to two fixed points, F, F′, the *foci*, their difference will always be the same.

To describe an hyperbola (Fig. 109).

From one of the foci, F, with an assumed radius, describe an arc, and from the other, focus F′, with another radius exceeding the former by the given difference, describe two small arcs, cutting the first as at P and *p*. Let this operation be repeated with two new radii, taking care that the second shall exceed the first by the same difference as before, and two new points will be determined; and this determination of points in the curve may thus be continued till its track is obvious. By making use of the same radii, but transposing, that is, describing with the greater about F, and the less about F′, we have another series of points equal-

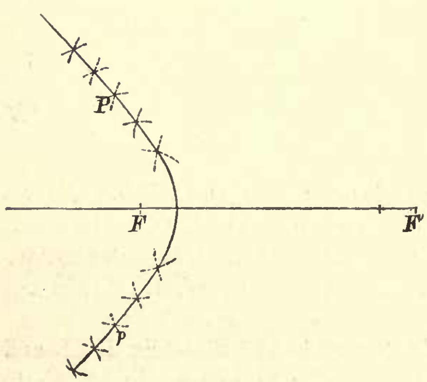


FIG. 109.

ly belonging to the hyperbola, and answering the definition; so that the hyperbola consists of two separate branches.

The curve may be described mechanically (Fig. 110) by fixing a ruler to one focus, F′, so that it may be turned round on this point, and connecting the other extremity of the ruler, R, to the other focus, F, by a cord shorter than the whole length of the ruler by the given difference; then a pencil, P,

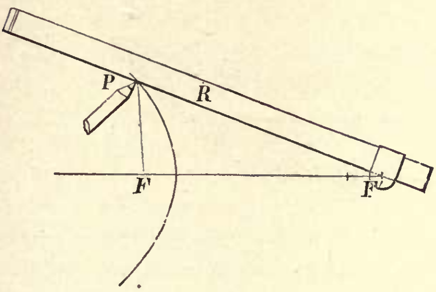


FIG. 110.

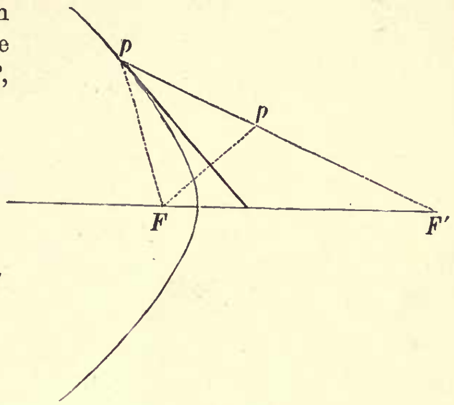


FIG. 111.

keeping this cord always stretched, and at the same time pressing against the edge of the ruler, will, as the ruler revolves around F′, describe an hyperbola.

To draw a tangent to any point of an hyperbola (Fig. 111).

Let P be the point. On F′ P lay off P *p*, equal to F P; draw the line F *p;* from P let fall a perpendicular on this line, P *p*, for the tangent.

To describe a spiral (Fig. 112 and Fig. 113, the primary on a larger scale).

Divide the circumference of the primary into any number of equal parts, say not less than eight. To these points of division *o*, *e*, *f*, *i*, etc., draw tangents,

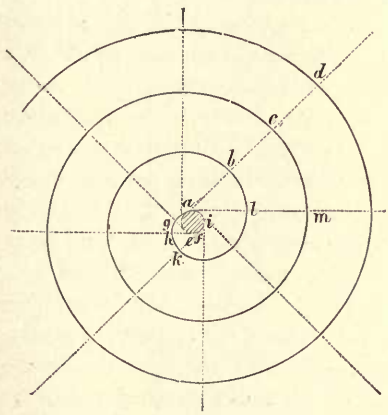


FIG. 112.

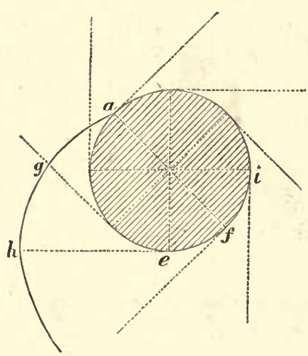
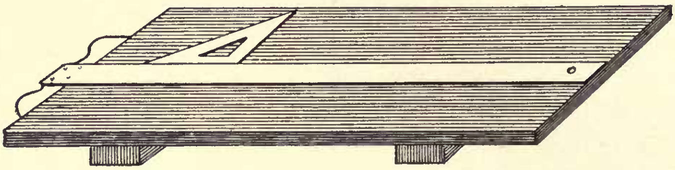


FIG. 113.

and from these points draw a succession of circular arcs; thus, from *o* lay off *o g*, equal to the arc *a o* reduced to a straight line, and connect *a* and *g* by a curve; from *e*, with the radius *e g*, describe the arc *g h;* from *f* the next arc, and so on. Continue the use of the centres successively and repeatedly to the extent of the revolutions required.

DRAWING INSTRUMENTS.



THE simple drawing instruments illustrated and applied in the construction of the preceding problems, together with scales of equal parts, a protractor, and a drawing-pen, are all the instruments essential for topographical or mechanical drawing. It is often convenient, for facility in working, to have compasses of various sizes and modifications, and these, together with an assortment of rulers, triangles, squares, scales, and protractors adapted to varied work, are included in boxes of drawing instruments furnished by dealers. The smaller rulers and triangles are generally of hard rubber, and the larger of wood. As it is often inconvenient to carry long rulers, or straight-edges, and difficult to procure them ready-made, the draughtsman may have to depend on a carpenter or joiner for them.

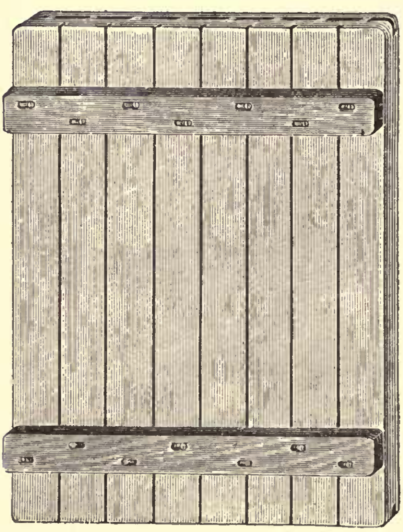


FIG. 114.

The drawing-board in its simplest form consists merely of narrow strips of thoroughly seasoned white-pine wood, free from knots, closely joined and glued, and held together either with a ledge at each end or with battens screwed to the back. For small boards, the former kind is in some ways the best, as it admits of being planed on all four edges.

Fig. 114 is more elaborate and one of the best drawing boards, possessing all the qualities of a first-class board. It is made of pine wood, glued up to the required width, with the heart side of each piece of wood at the surface. A pair of hard-wood battens are screwed to the back, the screws passing through the ledges in oblong slots that are bushed with brass, which fit closely under the heads, and yet allows the screws to move freely when drawn by the contraction of the board. To give the battens power to resist the tendency of the surface to warp, a series of grooves are sunk, half the thickness of the board, over the entire back. These grooves take the transverse strength out of the wood and allow it to be controlled by the battens, leaving at the same time the longitudinal strength of the wood nearly unimpaired.

To make the two working edges perfectly smooth, allowing an easy movement of the square, a slip of hard wood is let into the end of the board. The slip is afterward sawed apart at about every inch to admit of contraction. The drawing-board should be truly rectangular and have perfectly straight sides, for the use of the T square. Two sizes are sufficient for ordinary use—41×30 inches for double elephant paper, and 31×24 inches for imperial and smaller sizes. Boards smaller than these are too light, and unsteady in handling.

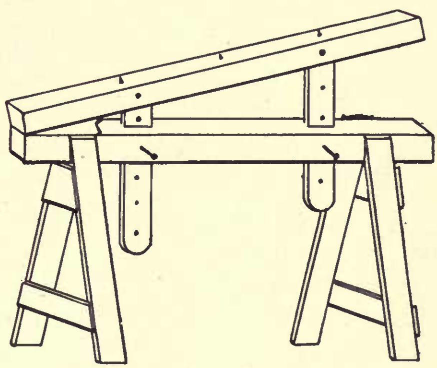


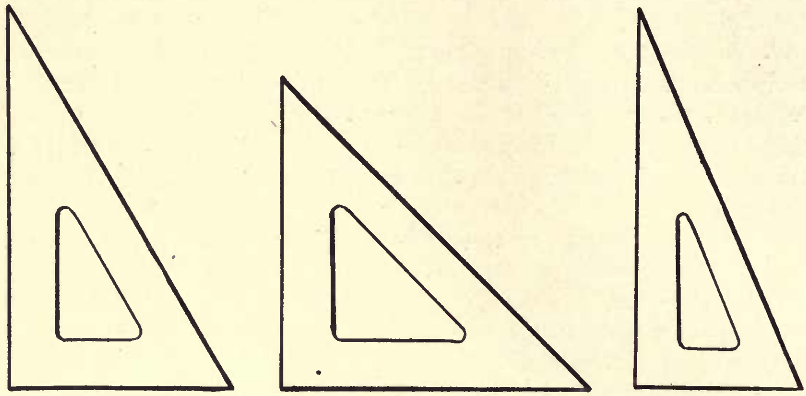
FIG. 115.

The drawing-table should be about 6 feet long and 4 feet wide, of 1½ inch stuff, constructed similarly to the drawing-board, and it is usually supported by a pedestal the height and inclination of which is adjustable, or on trestles, or a strong frame at such height that the draughtsman may not have to stoop to his work.

Fig. 115 shows an excellent form of trestle; the upper part of the horses is attached to hard-wood supports, which slide through the body of the trestle and are provided with numerous holes; by means of strong pins passing through the body of the horses and the holes the board may be set at various angles, the steel points in the top preventing the drawing-board from sliding or slipping off.

Straight-edges are made of close-grained, thoroughly seasoned wood, such as mahogany, maple, pear, etc.; also of celluloid, hard rubber, steel, or German silver. Those made of maple or pear wood answer every purpose and have the advantage of soiling the paper less than rubber or metal. No varnish of any description should be applied to any of the instruments used in drawing, as varnish will retain dust and soil the paper. Use the wood in its natural state, keeping it carefully wiped. Straight-edges should be about ⅛ inch thick in the square or slightly rounded edges and 1 to 2½ inches wide, according to their length. As the accuracy of a drawing depends greatly on the straightness of the lines, the edge of the ruler should be perfectly straight. To test this, place a sheet of paper on a perfectly smooth board; insert two very fine needles in an upright position through the paper into the board, distant from each other nearly the length of the ruler to be tested; bring the edge of the ruler against these needles, and draw a line from one needle to the other; reverse the ruler, bringing the same edge on the opposite side and against the needles, and again draw a line. If the two lines coincide, the edge is straight; but if they disagree, the ruler is inaccurate, and must be rejointed. When one ruler has been tested, others can be examined by placing their edges against the correct one, and holding them between the eye and the light.

Triangles may be made of the same kinds of wood as the ruler, somewhat thinner, and of various sizes. They should be right-angled, with acute angles of 45°, or of 60° and 30°. The most convenient size for general use measures



from 3 to 6 inches on the side. A larger size, from 8 to 10 inches long on the side, is convenient for making drawings to a large scale. In the smaller triangles circular openings are made in the body for the insertion of the end of the finger, to give facility in sliding the triangle on the paper. Triangles are sometimes made as large as 15 to 18 inches on the side; but in this case they are framed in three pieces, about $1\frac{1}{4}$ inch wide, leaving the centre of the triangle open. The value of the triangle in drawing perpendicular lines depends on the accuracy of the right angle. To test this (Fig. 116), draw a line with an accurate ruler on paper. Place the right angle of the triangle near the centre of this line, and make one of the adjacent sides to coincide with the line; now draw a line along the other adjacent side, which, if the angle is strictly a right angle, will be perpendicular to the first line. Turn the triangle on this perpendicular side, bringing it into the position A B C′; if now the sides of the triangle agree with the line B C′ and A B, the angle is a right angle, and the sides are straight. If they do not agree, they must be made to do so with a plane, if right angles are to be drawn by the triangle. The straightness of the hypothenuse or longest side can be tested like a common ruler.

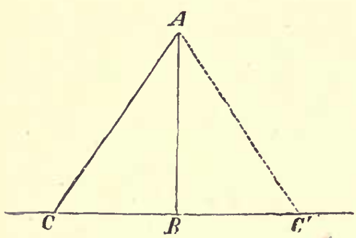


Fig. 116.

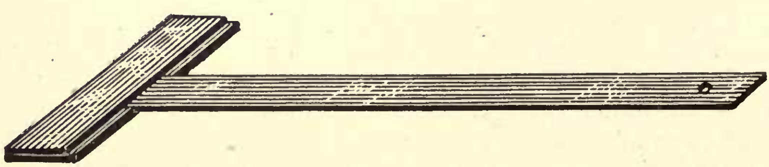


Fig. 117.

The T *square* is a thin straight-edge or ruler (Fig. 117), fitted at one end with a stock, applied transversely at right angles. The stock being so formed as to fit and slide against one edge of the drawing-board, the blade reaches over the surface, and presents an edge of its own at right angles to that of the board, by which parallel straight lines may be drawn upon the paper. The stock should be long enough to give sufficient bearing on the edge of the board, and heavy enough to act as a balance to the blade, and to relieve the operation of handling the square. The blade should be sunk flush into the upper half of the stock on the inside, and very exactly fitted. It should be inserted full breadth, as shown in the figure; notching and dovetailing is a mistake, as it weakens the blade, and adds nothing to the security. The upper half of the stock should be about $\frac{1}{4}$ inch broader than the lower half, to rest firmly on the board and secure the blade lying flatly on the paper.

One half of the stock *c* (Fig. 118) is in some cases made loose, to turn upon a brass swivel to any angle with the blade *a*, and to be clenched by a screwed

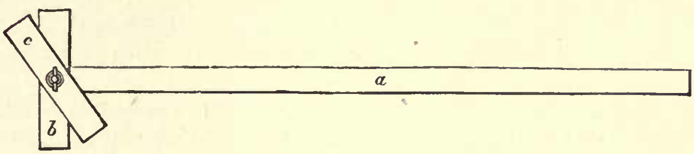
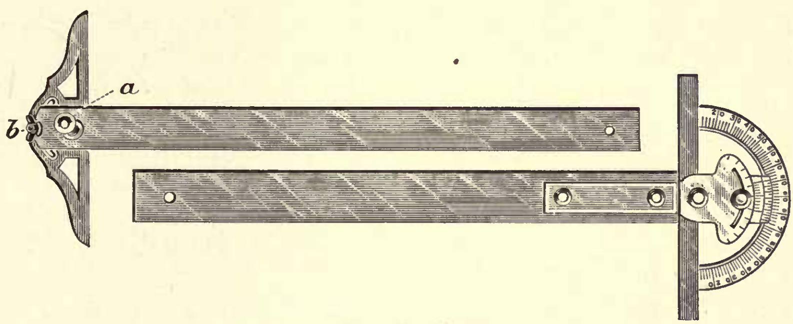


FIG. 118.

nut and washer. The loose stock is useful for drawing parallel lines obliquely to the edges of the board, such as the threads of screws, oblique columns, or connecting-rods of steam-engines.

T squares are also made with a single movable head, shown in Fig. 119;



FIGS. 119, 120.

the blade, turning on *a*, is clamped in position by the thumb-nut *b*. Fig. 120 illustrates a T square with a protractor at the head, convenient for laying off lines of designated angles.

In many drawing-cases will be found the parallel ruler (Fig. 121), consist-

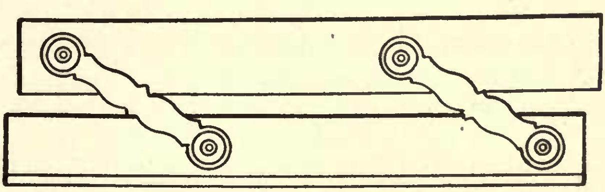


FIG. 121.

ing of two rulers connected by two bars moving on pivots, so adjusted that the rulers, as they open, form the sides of a parallelogram. The edge of one of the rulers being retained in a position coinciding with, or parallel to, a given line, when the other ruler is moved, lines drawn along its edge are also parallel to the given line. This instrument is only useful in drawing small parallels, and in accuracy and convenience does not compare with either the triangle and ruler or T square.

Another form of parallel ruler (Fig. 122) consists of a strip of wood with bevelled edges, having two holes to receive two broad wheels, *a*, *a*, which are

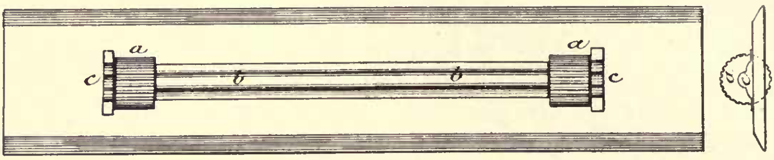


FIG. 122.

connected by an axle passing under the metal cover, *b*, *b*, and revolving in the supports, *c*, *c*; the wheels come slightly below the surface of the wood, as shown in the end elevation. In drawing parallel lines the fingers are placed with a firm pressure about the centre of the metal cover, and the ruler is moved in the proper direction. This ruler is more easily applied than the former, but is more liable to error.

VARIABLE CURVES.

For drawing arcs of a large radius, beyond the range of ordinary compasses, and lines varying in curvature, thin slips of wood, termed curves, are usually employed. These forms are of very general application, but others of almost every form, and made of hard rubber, pear wood, or celluloid, can be purchased. Whatever be the nature of the curve, some portion of the instrument will be found to coincide with its commencement, and it can be continued throughout its extent by applying, successively, such parts as are suitable, care being taken that the parts are tangent to each other, and that the continuity is not injured by unskilful junction.

Fig. 123 shows an adjustable curve ruler, the main features of which are a hard-rubber face, *a*, which holds the form of the required curve by a bar of

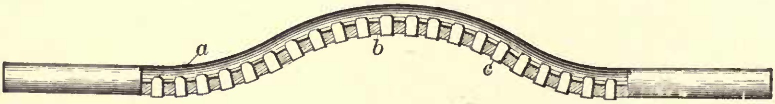
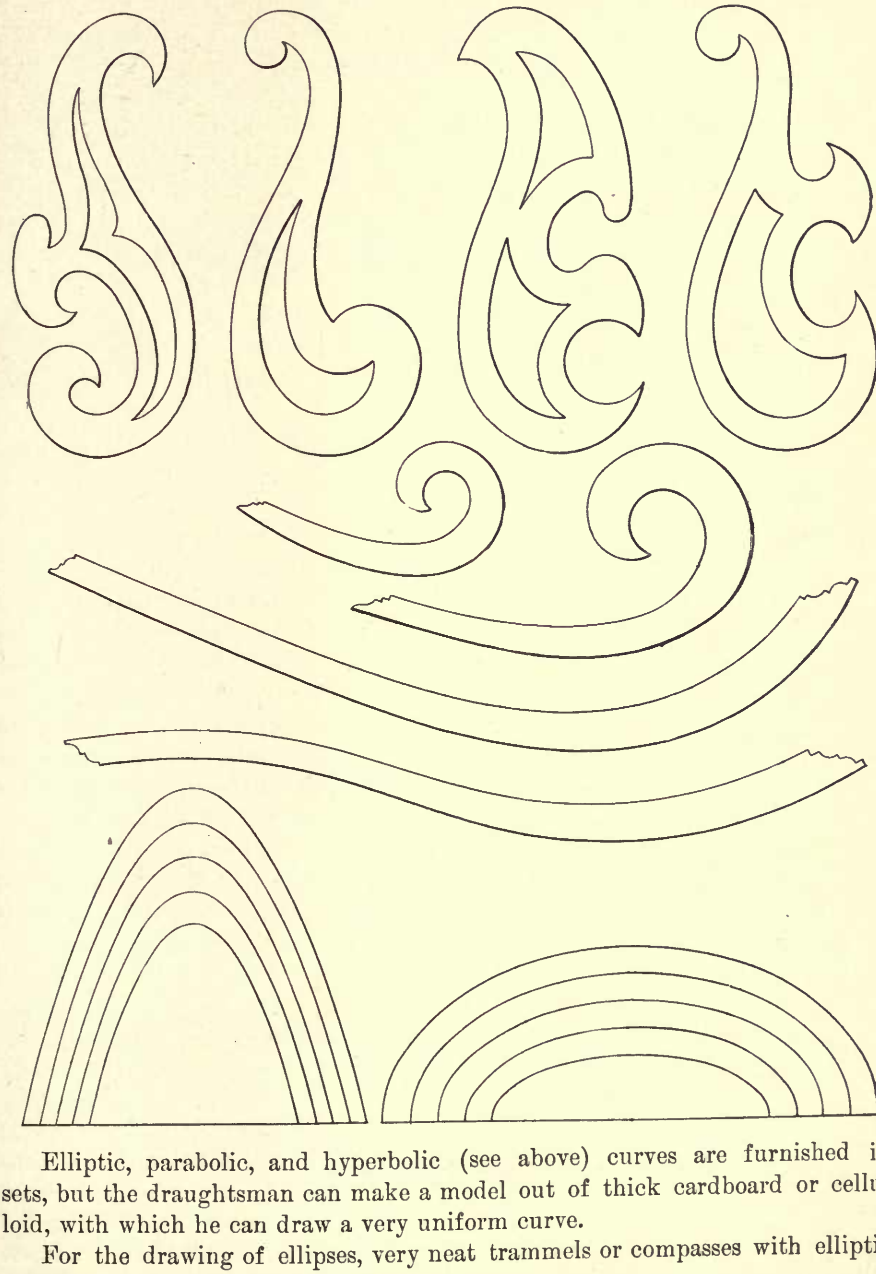


FIG. 123.

soft lead, *b*, kept in contact with the rubber face by the fasteners, *c*, and a flat spring inside these fasteners. This curve, while useful in the coarser kinds of draughting, does not do as neat or accurate work as the separate curves above given.

Thin splines are also to be had, which, held in position by leaden weights, serve admirably for a guide to the pen in describing curves (Fig. 124). For the same purpose a thin, hard-rubber ruler, with soft-rubber backing, answers well, and, as it can be readily rolled up, is extremely portable.

The weights above shown are very convenient in holding the drawing-paper on the board, but *thumb-tacks* (Fig. 125), steel points with large, flat heads, are in general use. They can be readily forced into the wood, and as readily raised, but thumb-tack lifters can be purchased.



Elliptic, parabolic, and hyperbolic (see above) curves are furnished in sets, but the draughtsman can make a model out of thick cardboard or celluloid, with which he can draw a very uniform curve.

For the drawing of ellipses, very neat trammels or compasses with elliptic guides or patterns may be purchased.

The *drawing* or *right-line pen* (Fig. 126) consists of two blades with steel points, fixed to a handle; and they are so bent that a sufficient cavity is left between them for the ink. The blades are set with the points more or less open by means of a mill-headed screw, so as to draw lines of any required fineness or thickness. For red inks, the blades of the pen should be nickle-plated or German silver. One of the blades is framed with a joint, so that by taking out the screw the blades may be completely opened, and the points effectively cleaned after use. The ink is put between the blades by a common pen. In using the pen, it should be slightly inclined in the direction of the line to be drawn, and care should be taken that both points touch the paper. These observations equally apply to the pen-points of the compasses. The drawing-pen should be kept close to the ruler or straight-edge, and in the same direction during the whole operation of drawing the line. Care must be taken to hold the straight-edge firmly with the left hand, that it does not change its position.

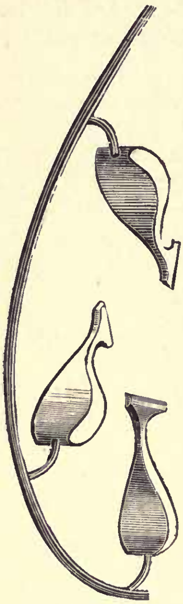


FIG. 124.

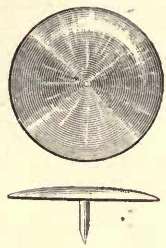


FIG. 125.

For drawing close parallel lines in mechanical and architectural drawings, or to represent railroads, canals, or roads, a *railroad pen* (Fig. 127) is frequently used, a double pen with an adjusting screw to set the pens to any required small distance. This instrument is also made with pencil points (Fig. 128).

FIG. 126.

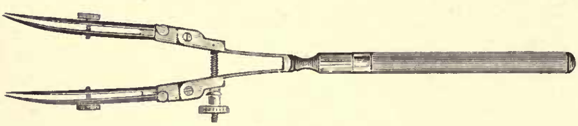


FIG. 127.

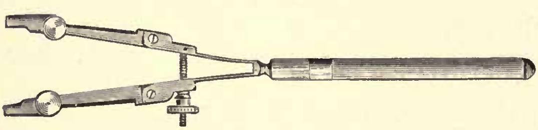


FIG. 128.

FIG. 129.

FIG. 130.

Border-pens (Fig. 129), for drawing broad lines, are double pens with an intermediate blade, and are applicable to the drawing of map-borders. The same work may be done by drawing heavy outer lines with the common drawing-pen, and filling in with a brush or writing-pen.

The *curve-pen* (Fig. 130) is especially designed for the ready drawing of curved lines. The axis of this pen is carried through the handle and fastened by a nut on top, allowing the pen to revolve, and thus more easily follow the curve. This instrument, made with two pens (Fig. 131), is called a *railroad curve-pen*.

The *dotting-pen* (Fig. 132) has on the back blade a pivot, on which may be placed a dotting-wheel, resembling the rowel of a spur; the screw is for open-

FIG. 131.

ing the blades to remove the wheel for cleaning after use or replace it with one of another character of dot. A variety of dotting-wheels accompanies the instrument, each producing a different-shaped dot. These are used as distinguishing marks for different classes of boundaries on maps; for instance, one kind of dot distinguishes county boundaries, another kind town boundaries, a third kind distinguishes that which is both a county and a town boundary, etc.

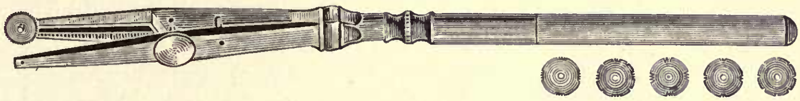


FIG. 132.

In using this instrument, the ink must be inserted between the blades above the dotting-wheel, so that, as the wheel revolves, the points pass through the ink, each carrying with it a drop, and marking the paper as it passes. It sometimes happens that the wheel will revolve many times before it begins to deposit its ink on the drawing, thereby leaving the first part of the line blank, and, when it is gone over again, the first-made dots are liable to get blotted. This evil may be avoided by placing a piece of blank paper over the drawing to the very point the dotted line is to commence at, and drawing the wheel over the blank paper first, so that by the time it reaches the proper point the ink begins to flow.

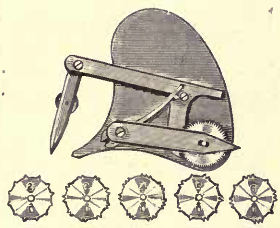


FIG. 133.

The *dotting-instrument* (Fig. 133) works on the principle of the drawing-pen. The outer wheel is rolled on the edge of a T square or straight-edge, and turns a ratchet wheel which causes the pen to move up and down. The flat point close to the pen must slide on the paper. To change the pattern of the dotted lines, the spring which holds the wheels on the axle is thrown back, and the proper ratchet wheel inserted.

The best *pricking-point* is a fine needle held as in Fig. 134, and is used to transfer drawings by pricking through at the points of a drawing into the paper

FIG. 134.

placed beneath. The handle of the ordinary drawing-pen often contains a pricking-point, which may be used by unscrewing the pen where it is joined to the handle.

When drawings are transferred by tracing—a prepared black sheet being placed between the drawing and the paper to receive the tracing—the eye end of the needle forms a good tracing-point.

The *stylus* (Fig. 135) is a piece of polished agate placed in a handle, and is used as a tracing-point.

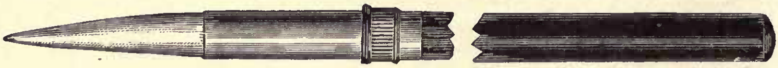


Fig. 135.

Compasses are fitted with ink-points and with lengthening bars for drawing larger circles. Compasses should have joints in the legs, so that the points, pencil, and pen may be set perpendicular to the planes in which the circles are described (Fig. 136). Compasses of this general form may be had in sizes of 3½ to 7 inches.

For the measurement and laying off of small spaces, and the describing of small circles, there are small *bow compasses* (Fig. 137). These are sometimes made with an adjusting screw between the legs.

For the measurement or laying off

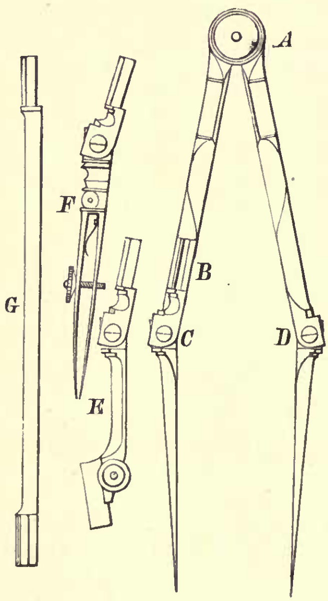


Fig. 136.

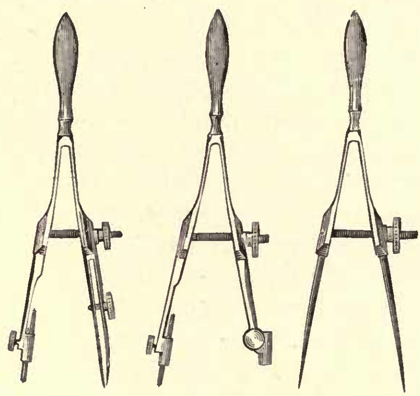


Fig. 137.

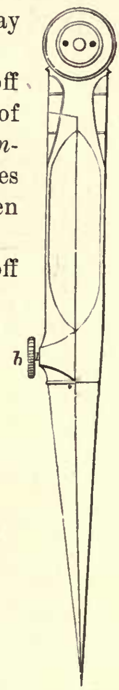


Fig. 138.

of distances the plain dividers are convenient, but for ready and close adjustment the *hair dividers* (Fig. 138) are most suitable. The only difference is that in the hair dividers one of the points is attached to the body by a

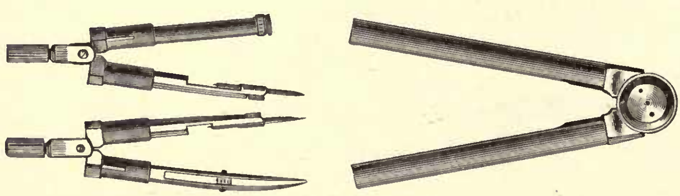


Fig. 139.

spring, and by means of the screw *b* it can be moved a very little toward or from the fixed point more accurately than by closing or opening the dividers. In dividing a line into equal parts especially, it enables one to divide the excess or deficit readily.

For convenience of carrying in the pocket, there are *portable* or *turn-in compasses* (Fig. 139). There is a small attachment for a common pencil which enables it to be used like compasses.

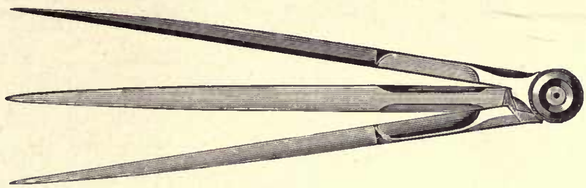


FIG. 140.

Three-legged dividers (Fig. 140) are convenient, while transferring measures from a drawing to a copy on an equal scale, for locating a third point when two are established.

For setting off very long lines, or describing circles of large radius, *beam compasses* are used (Fig. 141). These consist of a mere strip of wood, A, and two brass or German silver boxes, B and C, which can easily be attached to the beam; connected with the brass boxes are the two points of the instrument, G and H. The object of this instrument is the nice adjustment of the points G and H at any definite distance apart; at F is a slow-motion screw, by which the point G may be moved any very minute distance after the distance from H to G has been adjusted as nicely as possible by the hand alone. The wheel attachment, I, is to carry the weight of the beam. The metal parts of this instrument occupy but little space.

There are beam compasses in which the beam is graduated, and in which the boxes corresponding to B and C are fitted with vernier or reading plates, to afford the means of minutely subdividing the divisions on the beam.

Beam compasses are also made of small round German-silver bars, one screwing into the other, on which are slides adapted for carrying pen or pencil and points.

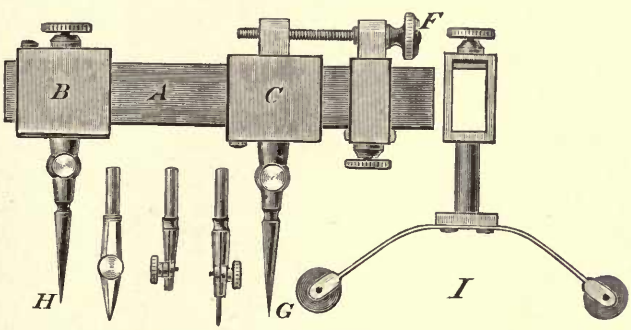


FIG. 141.

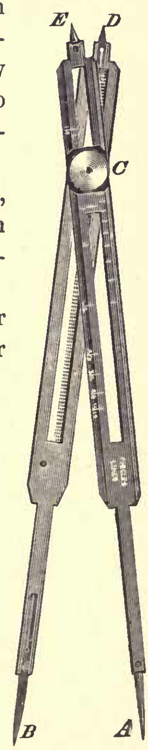


FIG. 142.

Proportional dividers (Fig. 142), for copying and reducing drawings, are found in most cases of instruments.

Closing the dividers and loosening the screw C, the slide may be moved up in the groove until the mark on the index corresponds with the required number; then clamping the screw, the space inclosed between the long points, A B, will be as many times that between the short points, E D, as is shown by the number opposite the index. If the lines are to be reduced, the distances are measured with the long points, and set off by the short ones; if the lines are to be enlarged, then *vice versa.*

Proportional dividers are also used for dividing the circumference of a circle into a number of parts. A special scale along the graduated edge, marked circles, is used, it being only necessary to move the slider to the proper number on this scale to obtain a chord of the proper length. It often happens that the length of the points becomes reduced by use or accident. In this case it is only necessary to loosen the screw holding the shortened point, take it out, grind to a point, and set to its former length.

Scales.—The application of simple scales to the construction of diagrams has been explained; but among drawing instruments scales especially adapted to plotting are to be found in great varieties of form, divisions, and material. It is usual, especially in topographical drawings, for the draughtsman to construct a scale upon the finished sheet on account of its ready application to the determination of measures, and when the drawing is to be reduced or enlarged by photographing it is indispensable. Moreover, paper expands and contracts under hygrometric changes; the scale should be subject to those same changes. To remedy this inconvenience Mr. Charles Holzapfel has introduced paper scales, which are portable and cheap; but as all kinds of paper are not equally susceptible to changes of condition on the atmosphere, the detached paper scale affords only a partial correction.

The scale should be written or drawn in all drawings; also the date of completion and name or initials of the draughtsman, as these data may be of value in the identification of the drawing.

In all working architectural and mechanical drawings, use as large a scale as possible; and even then do not depend upon the mechanics employed in the construction measuring correctly, but write in the dimensions as far as practicable. For architectural plans, the scale of $\frac{1}{4}$ of an inch to the foot is in very general use and is convenient for the mechanic, as the common two-foot rule carried by all mechanics is subdivided into $\frac{1}{4}$ths, $\frac{1}{8}$ths, and sometimes sixteenths of an inch, and the distances on a drawing to this scale can therefore be easily measured by them. This fact should not be lost sight of in working drawings. When the dimensions are not written, make use of such scales that the distances may be measured by the subdivisions of the common two-foot rule; thus, in a scale of $\frac{1}{2}$ or $\frac{1}{4}$ full size, 6 inches or 3 inches represent one foot; in a scale of an inch to the foot or twelfth full size, each $\frac{1}{2}$ an inch represents 6 inches, $\frac{1}{4}$ of an inch, 3 inches; but when $\frac{1}{3}$ or $\frac{1}{10}$ an inch to the foot, or any similar scale, is adopted, it is evident that these divisions can not be taken by the two-foot rule.

Plotting scales (Fig. 143) are scales of equal parts, with the divisions usually on a bevelled edge, by which any length may be marked off on the paper without using dividers. There are also small *offset* scales, for use of which see "Topographical Drawing."

Sometimes these scales are made with edges bevelled on both sides, and graduated to four different scales. Sometimes the section of the scale is triangular (Fig. 144), with six scales on the different edges. To avoid confusion

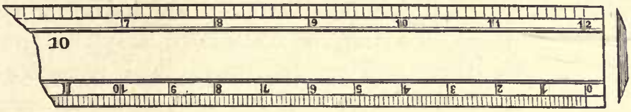


Fig. 143.

from having many scales on one ruler, the triangular scale has a small slip of metal, A, readily put on, which covers partially the scales not in use.

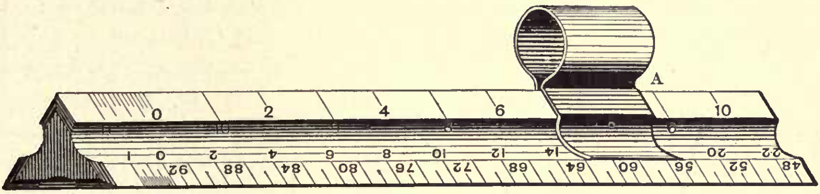


Fig. 144.

To divide a given line into any number of equal parts (Fig. 145).

Let A B be the line, and the number of parts be ten. Draw a perpendicular at one extremity, A, of the line; with a plotting scale place the zero at the other extremity, B, of the line; make the mark 10 on the scale coincide with the perpendicular; draw a line along the edge of the scale, and mark the line at each division of the scale 1 to 9; draw perpendiculars through these marks to the line A B, and they will divide A B into ten equal parts.

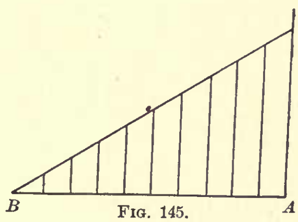


Fig. 145.

The above figure illustrates the construction of *diagonal scales.* The simply divided scales give only two denominations, primaries and tenths, or twelfths; but more minute subdivision is attained by the diagonal scale, which consists of a number of primary divisions, one of which is divided into tenths, and subdivided into hundredths by diagonal lines (Fig. 146). This scale is constructed in the following manner: Eleven parallel horizontal lines are ruled, inclosing ten equal spaces; from one end set off the primary unit divisions, 0, 1, 2, 3, and draw vertical lines through these points; subdivide the extreme unit to the left on the upper and lower lines into ten equal parts, 1, 2, 3, etc.; connect 0 on the upper line with 1 on the lower line

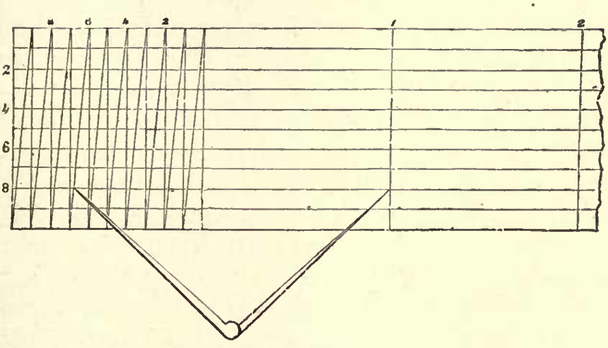


Fig. 146.

by a diagonal, and draw lines parallel to it through the other subdivisions. To take a measurement of, say, 168, we place one foot of the dividers on the primary 1, and carry it down to parallel 8, and then extend the other foot to the intersection of the diagonal which falls from the subdivision 6 with this parallel. The primaries may, of course, be considered as yards, feet, or inches; and the subdivisions as tenths and hundredths of these respective denominations. If the number of parallel spaces be eight and the subdivision be twelve, we can measure feet, inches, and eighths. In the diagonal scale the vertical subdivisions are often omitted.

The diagonals may be applied to a scale where only one subdivision is required. Thus, if seven lines be ruled (Fig. 147), inclosing six equal spaces, and the length be divided into primaries, as A B, B C, etc., the first primary, A B, may be subdivided into twelfths by two diagonals running from 6, the middle of A B, to 12 and 0. We have here a very convenient scale of feet and inches. From C to 6 is 1 foot 6 inches; and from C on the several parallels to the various intersections of the diagonals we obtain 1 foot and any number of inches from 1 to 12.

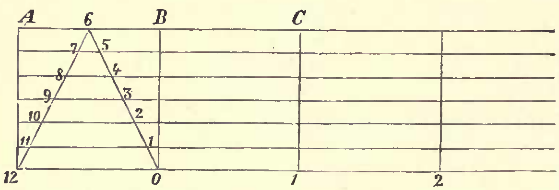


FIG. 147.

For the designing of machinery, it is very convenient to have some scale of reference by which to proportion the parts; for this purpose a vertical and horizontal scale may be drawn on the walls of the room.

Vernier scales are preferred by some to the diagonal scale already described. To construct a vernier scale (Fig. 148) by which a number to three places may be taken, divide all the primary divisions into tenths, and number these sub-

FIG. 148.

divisions 1, 2, 3, from left to right. Take off now with the compasses eleven of these subdivisions, set the extent off backward from the end of the first primary division, and it will reach beyond the beginning of this division, or zero point, a distance equal to one of the subdivisions. Now divide the extent thus set off into ten equal parts, marking the divisions on the opposite side of the divided line to the lines marking the primary divisions and the subdivisions, and number them 1, 2, 3, etc., backward from right to left. Then, since the extent of eleven subdivisions has been divided into ten equal parts, so that these ten parts exceed by one subdivision the extent of ten subdivisions, each one of these equal parts, or, as it may be called, one division of the vernier scale, exceeds one of the subdivisions by a tenth part of a subdivision, or a hundredth part of a primary division; thus, if the subdivision be considered 10, then from 0 to the first division of the vernier will be 11; to the second, 22; to the third, 33; to the fourth, 44; to the fifth, 55, and so on, 66, 77, 88, 99.

To take off the number 253 from this scale, place one point of the dividers at the third division of the vernier; if the other point be brought to the primary division 2, the distance embraced by the dividers will be 233, and the dividers must be extended to the second subdivision of tenths to the right of 2. If the number were 213, then the dividers would have to be closed to the second subdivision of tenths to the left of 2. The number, thus taken, may be 253, 25·3, 2·53, according as the primary divisions are taken as hundreds, tens, or units.

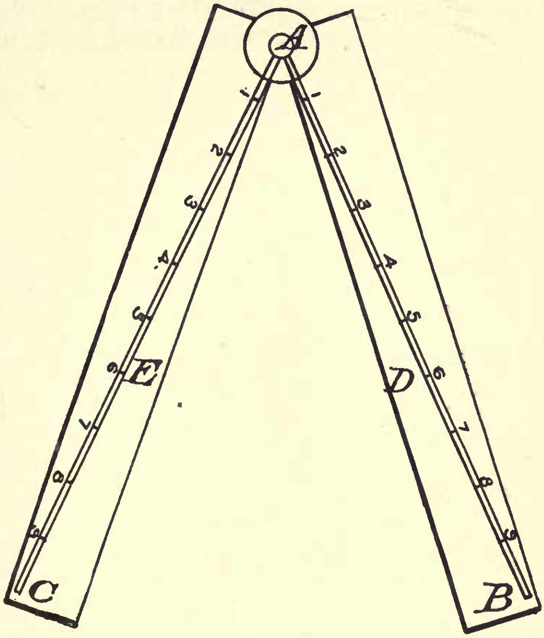


FIG. 149.

The construction of this scale is similar to that of the verniers of theodolites and surveying instruments.

The sector in its old form carried several scales on its faces. As given in Fig. 149, there are only double scales starting from the centre joint, which, without drawing, may be applied to the solution of problems on similar triangles.

Let the lines A B, A C, represent the legs of the sector, and A D, A E, two equal sections from the centre; then, if the points B C and D E be connected, the lines B C and D E will be parallel; therefore, the triangles A B C, A D E,

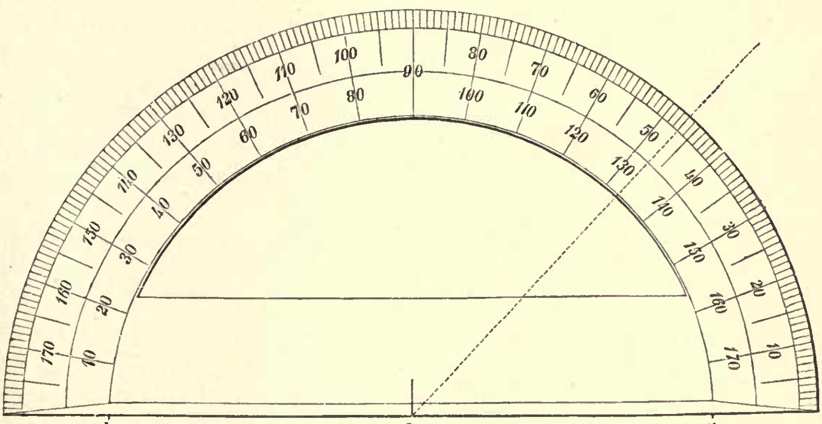


FIG. 150.

will be similar, and, consequently, the sides A B, B C, A D, D E, proportional —that is, as A B : B C : : A D : D E; so that if A D be the half, third, or fourth part of A B, then D E will be a half, third, or fourth part of B C; and the same holds of all the rest. Hence, if D E be the chord, sine, or tangent of any arc, or of any number of degrees to the radius A D, then B C will be the same to the radius A B. Thus, at every opening of the sector, the *transverse* distances D E and C B from one ruler to another are proportional to the *lateral* distances, measured on the lines A B, A C. It is to

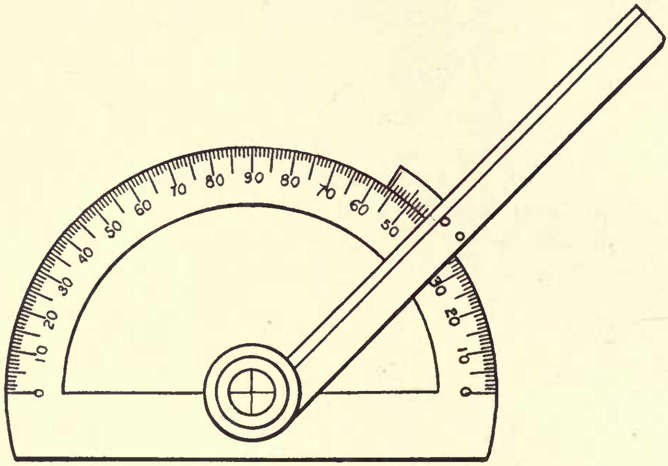


FIG. 151.

be observed that all measures are to be taken from the inner lines, since these only run accurately to the centre.

On the scale in common boxes of drawing instruments, the edges of the sides are divided as a *protractor* (Fig. 84) for the laying out of angles. The ordinary protractor consists of a semicircle of thin metal or horn (Fig. 150), whose circumference is divided into 180 degrees (180°). In the larger protractors each of these divisions is subdivided.

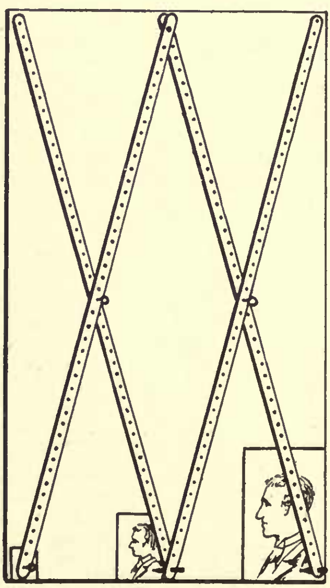


FIG. 152.

Application of the protractor.—To lay off a given angle from a given point on a straight line, let the straight line *a b* of the protractor coincide with the given line, and the point *c* with the given point; now mark on the paper against the division on the periphery coinciding with the angle required; remove the protractor, and draw a line through the given point and the mark.

Fig. 151 is a protractor with a straight-edge revolving on a horn centre. Where the straight-edge is intersected by the edge of the protractor a vernier is attached, and will be found useful in close work for dividing the degrees into tenths. This protractor is often extended to full circles.

For plotting field-notes expeditiously, drawing-paper can be obtained with large, full circular protractors printed thereon, on which the courses can be readily marked, and thus transferred to the part of the paper required by a parallel ruler, or by triangle and ruler. These sheets are of especial use in plotting at night the day's work, as, on account of the large size of the protractor, angles can be laid off with greater accuracy than by the usual protractor of a drawing-instrument case, with less confusion of courses, and more expeditiously.

The *pantagraph* is used for the copying of drawings, either on the same scale, on a reduced scale, or on an enlarged scale.

Fig. 152 shows its simplest form and its application. The lower left-hand point, around which the frame is turned, is fixed, and the proportion of the drawing is determined by the position of the screw eyes in the holes of the arms.

Fig. 153 is another form, of more finished construction. It consists of a set of jointed rulers, A, B, and another, C, D, about one half the length of the

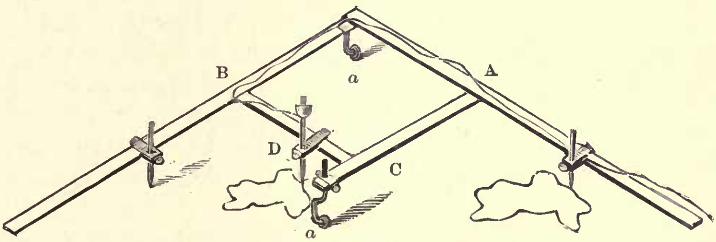


FIG. 153.

former. The free ends of the smaller set are jointed to the larger at about the centre. Casters are placed at *a a*, etc., to support the instrument and to allow an easy movement.

DRAWING-PAPERS.

Papers adapted to drawing may be obtained of various qualities, thicknesses, and dimensions, either in sheets, pads, or rolls. Machine-made papers are generally used, and are to be had from stock in rolls up to 62 inches in width, but to order much wider. They are generally made from cotton for the more finished drawings; but stronger papers for working drawings and details are of manilla or of coarse heavy stock.

Roll and sheet papers can be had mounted or backed with cotton cloth, which prevents them from being torn, and permits of their being hung to the walls as maps.

Hand-made drawing-papers are usually made in certain standard sizes, about as follows:

Cap	13 inches by	17 inches.		Imperial	22 inches by	30 inches.
Demy	15 "	20 "		Atlas	26 "	34 "
Medium	17 "	22 "		Double Elephant	27 "	40 "
Royal	19 "	24 "		Antiquarian	31 "	53 "
Super Royal	19 "	27 "				

Tracing-paper is a prepared tissue paper, transparent and qualified to receive ink lines and tinting without spreading. When placed over a drawing already executed, the drawing is distinctly visible through the paper, and it may be copied directly or traced in pencil or ink.

Tracing-paper often becomes tender with age, is apt to break in the folds, is not easily rolled. It is not suitable, therefore, for permanent drawings; but the tracing can be readily transferred to drawing-paper by means of *transfer paper.* Place the fair sheet on the drawing-board, above it the transfer sheet with the prepared face down, then the tracing, and steady the whole by weights or by thumb-tacks fixed into the drawing-board. A fine, smooth point is then passed over each boundary and line on the tracing with a pressure of the hand sufficient to cause a clear line to be left by the transfer paper on the fair sheet. Finish these lines in ink. The copyist should be careful in his manipulation that no unnecessary lines or smutches be left on the fair sheet. Transfer paper can be readily obtained in sheets, either in black, blue, vermilion, or graphite, or it can be made by smearing with a piece of flannel one surface of thin paper with a coating of lard and graphite and, after a day's drying, wiping off the superfluous portion with a soft rag.

Parchment papers are much stronger than tracing-papers, and are usually transparent enough to serve the same purpose; the thicker kinds are well adapted for drawing and engrossing. De La Rue's process for the manufacture of parchment paper is to plunge unsized paper for a few seconds into sulphuric acid, diluted with half to a quarter its bulk of water, the solution being of the same temperature as the air, and afterward wash with weak ammonia.

A drawing may be made to accompany a letter by saturating the letter paper with benzine till it becomes transparent, then using it as tracing-paper, copying the design in pencil, and finishing in copying-ink after the benzine has evaporated, so that it can be transferred with the descriptive writing to the letter book.

Transparent tracing-cloth can be had in wide and long rolls. It is much stronger than tracing-paper, and serves a permanent purpose. Should the tracing-cloth refuse to take ink lines well, almost any fine white powder will remedy this, such as chalk, fuller's earth, pipe clay, or plaster of Paris sprinkled on and rubbed in well.

It is usual to draw on the dull side of the cloth, except where colour is to be put in, when the ink lines are drawn on the glossy side and the colour on the dull back. Designs and finished drawings made in pencil on paper are traced on cloth in ink, and in this form are preserved as originals and can be copied by the heliographic process, either wholes or details as needed. When a white sheet of paper is placed behind the tracings, the drawing may be readily photographed on a reduced or enlarged scale, and much more cheaply than by any other process; and such negatives may be used in process engraving for book illustrations.

Heliographic paper can readily be had in sheets or rolls, and the mixture for the preparation of the paper can also be purchased, or can be made by dissolving $1\frac{7}{8}$ ounce of common citrate of iron in 8 ounces of water, and $1\frac{1}{4}$ ounce of red prussiate of potash in 8 ounces of water, and then mixing them just previous to using. Papers and mixtures must be kept from the light or they will lose their sensitiveness. The above is a mixture for the most common form of sun prints, called the ferro-prussiate or blue process, in which white lines are developed on a blue ground. By the cyanotype process blue lines are developed on a white ground; by the nigrosine process, black lines on a white ground; by the chromide dry process, dark lines on a tinted ground. Papers for all of the above processes are on sale, with directions for use. If none can be had, and it is desired to prepare some, use the ferro-prussiate process as the simplest, of which a recipe has been given above, the paper should be chemically neutral, of even material, and capable of being washed. Inks are to be had especially adapted for the tracings in bottles and cakes. It is necessary for a good print that the lines should be of a deep black. If not sufficiently opaque, burnt Sienna, burnt umber, or gamboge added to the ink improves the prints.

For the manipulation there is needed plate glass and a blanket a little larger than the drawing, also a shallow tray, that the drawing can be placed in flat for washing.

Lay down the blanket on the drawing-board, above that the ferro-prussiate paper, next the drawing, and then the glass. Expose to the sunlight until the background is a metallic gray. The length of exposure may be from five minutes up, depending on the intensity of the sunlight, the age of the prepared paper, and the transparency of the tracing. Now lay the ferro-prussiate paper in the tray, cover with water, and leave it for five to ten minutes; wash thoroughly and dry.

The usual form of printing-frame, as purchased of dealers, shown in Fig. 154, consists of a frame into which fits a sheet of glass, preferably of plate glass,

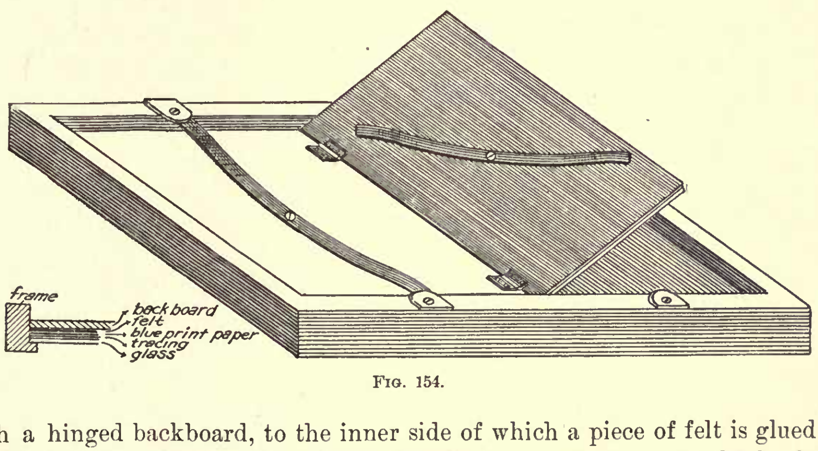


FIG. 154.

with a hinged backboard, to the inner side of which a piece of felt is glued in the smaller sizes, while in the larger the felt is separate; on the back of this board are two brass springs fitting under the metal catch, making a close contact with the glass. As shown in the small sectional drawing, the frame is turned upside down. The glass is placed in first, then, successively, the tracing with its face to the glass, the prepared paper with the prepared side to the tracing, and the felt; then the backboard is placed and held in position by the springs, the frame is turned up, and the directions given above as to exposure and washing are properly carried out.

When it is necessary to make additions and alterations on blue prints, a special ink can be procured; lines made with this preparation on the blue ground turn white. (See FREE-HAND DRAWING.)

The helios process is useful for copying not only drawings, but contracts, estimates, tables, etc., when they are written on transparent paper or cloth; and so is the nigrosine process.

Bristol board is a cardboard with a very fine surface. It can be obtained of various thicknesses and of the same dimensions as sheet drawing-papers. It is adapted to water-colours, pen-and-ink sketches, and fine line-drawings; the Patent Office requires sheets 10 by 15 inches, and these can be obtained with border and wording of witness, inventor, and attorney properly printed in.

Mouth glue, for the sticking of the edges of drawing-paper to the board, is made of glue and sugar or molasses; it melts at the temperature of the mouth, and is convenient for the draughtsman.

Drawing-paper may be fixed down on the drawing-board by thumb-tacks at the corners, by weights, or by gluing or pasting the edges. The first is sufficient when no shading or colouring is to be applied, and if the sheet is not to be a very long time on the board; and it has the advantage of preserving the paper in its natural state. For shaded or tinted drawings, the paper must be damped.

Damp-stretching is done as follows: The edges of the paper should first be cut straight, and, as near as possible, at right angles to each other. The sheet should be enough larger than the intended drawing and its margin to admit of being afterward cut from the board, leaving the pasted or glued border.

The paper must first be placed on the drawing-board and thoroughly and equally damped with a sponge and clean water on the side on which the drawing is to be made. This done, lay a straight flat ruler on the paper, with its edge parallel to, and about half an inch from, one of its edges. The ruler must now be held firm, while the projecting half inch of paper is being turned up along its edge; then a piece of mouth glue, having its edge partially dissolved by holding it in warm water for a few seconds, must be passed once or twice along the turned-up edge of the paper, after which, by sliding the ruler over the glued border, it will be again laid flat, and, the ruler being pressed down upon it, that edge of the paper will adhere to the board. If sufficient glue has been applied, the ruler may be removed directly, and the edge finally rubbed down by an ivory book-knife or by the bow of a common key, rubbing it on a slip of paper placed on the drawing-paper, so that the surface of the latter may not be soiled; this will firmly cement the paper to the board. Another edge of the paper is then treated in like manner, and the remaining edges in succession. Sometimes strong paste or mucilage is used instead of glue.

The wetting of the paper is done for the purpose of expanding it; and the edges, being fixed to the board in its enlarged state, act as stretchers upon the paper, while it contracts in drying, which it should be allowed to do gradually. All creases or undulations by this means disappear from the surface, and it forms a smooth plane to receive the drawing.

After the drawing is finished, cut off the paper inside the pasted edge, and remove the edge by warm water and the knife.

MOUNTING PAPER AND DRAWINGS, VARNISHING, ETC.

When paper of the requisite quality or dimension can not be purchased already backed, it may be mounted on muslin. The cloth should be well stretched upon a smooth flat surface, being damped for that purpose, and its edges glued down, as was recommended in stretching drawing-paper. Then with a brush spread strong paste, beating it in till the grain of the cloth be all filled up; for this, when dry, will prevent it from shrinking when subsequently removed; then, having cut the edges of the paper straight, paste one side of every sheet, and lay them upon the muslin sheet by sheet, overlapping each other slightly. If the drawing-paper is strong, it is best to let every sheet lie five or six minutes after the paste is put on it, for, as the paste soaks in, the paper will stretch, and may be better spread smooth upon the cloth; whereas, if it be laid on before the paste has moistened the paper, it will stretch afterward and rise in blisters when laid upon the cloth. The paper should not be cut off from its extended position till thoroughly dry, which should not be hastened. Leave it in a dry room to do so gradually, if time permit; if not, it may be exposed to the sun; in the winter season the help of a fire may be necessary; but it should not be placed too near a scorching heat.

In joining two sheets of paper together by overlapping, it is necessary, in order to make a neat joint, to feather-edge each sheet; this is done by carefully cutting with a knife half way through the paper near the edges on the sides which are to overlap each other and then stripping off a feather-edged slip from each, which, if done dexterously, will produce a very neat and efficient joint.

For mounting and varnishing drawings or prints, stretch a piece of linen on a frame, to which give a coat of isinglass or common size; paste the back of drawing, which leave to soak; and then lay it on the linen. When dry, give it at least four coats of well-made isinglass size, allowing it to dry between each coat. Take Canada balsam diluted with the best oil of turpentine, and with a clean brush give it a full flowing coat.

When drawings are not mounted on muslin, the edges may be protected from tearing by binding with gummed tape, or strips of paper which may be cut or purchased.

Drawings, as far as possible, should be preserved flat in drawers, and this is especially desirable for tracings which are to be often sun-printed.

The classification of drawings is varied. The common method is to devote a separate drawer to the drawings of each machine, or of each group or class of machine; another is to have drawers of various sizes and arrange the drawings according to sizes.

MANAGEMENT OF THE INSTRUMENTS.

In constructing preparatory pencil-drawings, it is advisable, as a rule of general application, to make no more lines upon the paper than are necessary to the completion of the drawing in ink; and also to make these lines just dark enough to be sufficiently distinct.

It is often beneficial to ink in one part of a drawing before touching other parts at all; it prevents confusion, makes the first part easy of reference, and allows of its being better done, as the surface of the paper inevitably contracts dust and becomes soiled in the course of time, and therefore the sooner it is done with the better.

Circles and circular arcs should, in general, be inked in before straight lines, as the latter may be more readily drawn to join the others than can the former. When a number of circles are to be described from one centre, the smaller ones should be inked first, while the centre is in better condition. When a centre is required to bear some fatigue, it should be protected with a thickness of stout card glued or pasted over it, to receive the compass-leg.

India-rubber is the ordinary medium for cleaning a drawing and for correcting errors of the pencil. For slight work it is quite suitable; that substance, however, operates to destroy the surface of the paper; and, by repeated application, it so ruffles the surface as to spoil it for fine drawing, especially if ink shading or colouring is to be applied. It is much better to leave trivial errors alone, if corrections by the pencil may be made alongside without confusion, and not clear away superfluous lines till the inking is finished.

When ink lines have to be erased to any considerable extent, the best way is to use an ink-erasing rubber. Single lines may be erased by cutting a long narrow slit in a piece of thin cardboard or celluloid and erasing through it.

For cleaning a drawing, a piece of bread two days old is preferable to India-rubber, as it cleans the surface well and does not injure it. A sponge rubber may also be used for this purpose. For ordinary small erasures of ink lines, a sharp rounded pen-blade, applied lightly and rapidly, does well, and the surface may be smoothed down by the thumb-nail. In drawings intended to be highly finished, particular pains should be taken to avoid the necessity for corrections, as everything of this kind detracts from the appearance.

The best work can only be accomplished by keeping the instruments in good order; their working parts should be carefully preserved from injury. The scales must be kept scrupulously clean; the inking tools should have especial care, and the blades kept well set, for which a small oil-stone is convenient.

To dress up the tips of the blades of the pen or of the bows, as they usually become worn unequally, they may be screwed up into contact in the first place, and passed along the stone, turning upon the point in a directly perpendicular plane, till they acquire an identical profile. Being next unscrewed and examined to ascertain the parts of unequal thickness round the nib, the blades are laid separately upon their backs on the stone, and rubbed down at the points, till they be brought up to an edge of uniform fineness. It is well to screw them together again, and to pass them over the stone once or twice more, to bring up any fault; to retouch them also on the outer and inner side of each blade, to remove barbs or fraying; and, finally, to draw them across the palm of the hand.

India ink, which is commonly used for line-drawing, should be rubbed down in water to the degree that avoids the sloppy aspect of light lining without making the ink too thick to run freely from the pen. This medium degree may be judged of after a little practice by the appearance of the ink on the palette. The best quality of ink has a soft feel when wetted and smoothed, being then free from grit or sediment, and has a musky smell.

Slabs of many forms and different materials are used in grinding down the ink. The one shown in Fig. 155 is a square slab of slate, with a countersunk circular recess and a well in the centre to hold the ink; the cover is a piece of heavy glass.

A quantity of ink may be prepared at one time, but it must be kept well covered to exclude dust and prevent evaporation. The pen should be filled by

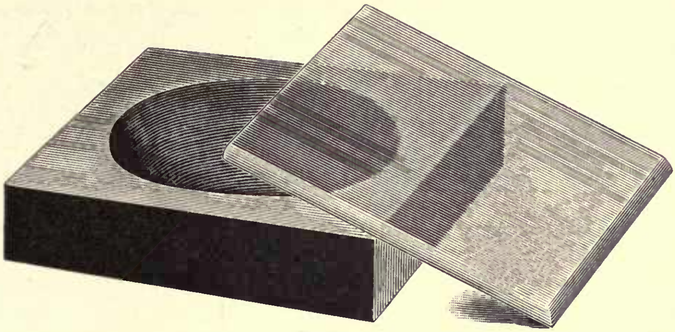


Fig. 155.

a narrow strip of paper, dipped in the ink and inserted between the blades.

India ink and ink of various colours can be purchased in bottles, and this answers very satisfactorily for most work. Waterproof ink, which admits of being washed over, can be bought in sticks or in bottles.

It is of primary importance to keep the blades of the inking tools free from obstruction; this may be readily accomplished without unscrewing the pen by passing a slip of paper between the blades, or by drawing the point firmly over a piece of paper or on the fleshy part of the hand.

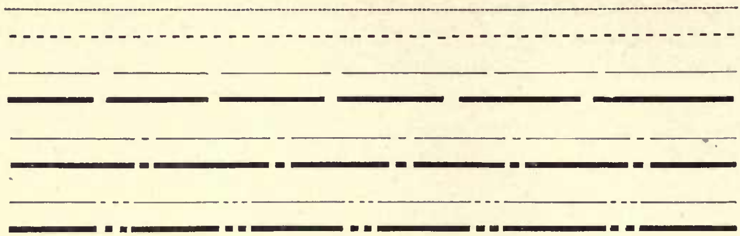
Exercises with the Drawing-Pen.

Before proceeding to the construction of finished drawings, skill should be acquired in the use of the drawing-pen, supplemented often by the writing-pen. Beginning with lines, outlines of figures, alphabets, and the like, the draughtsman should strive to acquire the habit of readily drawing clean, uniform lines, without abruptness or breaks where straight lines connect with curved ones. Draw straight lines of different grades:

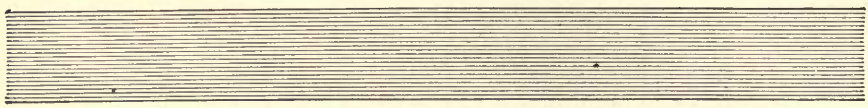
as, fine
medium
coarse

at first, lines of indefinite length, taking care that they are drawn perfectly straight and of uniform width or grade. Then draw lines of definite length between assumed points, taking care to terminate the lines exactly at these points. Lines as above are *full* lines. The grades depend on the effect which the draughtsman wishes to give.

Draw *dotted lines*, *broken lines*, and *broken and dotted* lines, of different grades:



Draw fine lines at uniform distances from each other:



To give uniform appearance, the lines must be of uniform grade and equally spaced. Practice in lines of this sort is important, as they are much used in drawing to represent sections, shades, and conditions, as soundings on charts, density or characteristics of population, areas of rain, temperature, and the like. Draw lines as in Fig. 156. These lines are diagonal with the border-lines, and are used to represent sections of materials. In the figure, lines differently inclined represent different pieces of the same material.

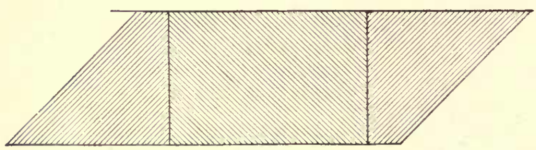


FIG. 156.

Instruments called *section-liners* are to be had for drawing these lines, but for the usual needs of a drawing office the triangle and straight-edge, with the drawing-pen, will be sufficient.

Sections of different materials may be represented in different kinds of lines (see page 177).

To represent cylindrical surfaces (Fig. 157).

Draw a semi-circumference, and mark on it a number of points, at equal distances apart, and through these points draw lines perpendicular to the diameter across the surface to be represented. It is not absolutely necessary that the central space should be equal to the others; it will be more effective to leave out two of the lines, and make it to this extent wider.

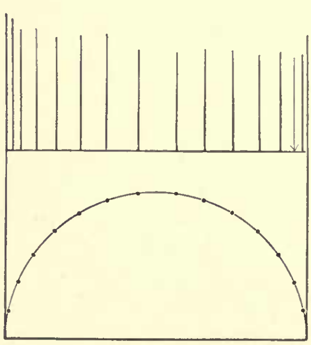


FIG. 157.

To construct a mass of equal squares (Fig. 158).

Lay off a right angle, and on its sides mark as many points at equal distances apart as may be necessary; through these points draw lines parallel to the sides.

Or, construct a rectangle; mark on its sides as many points, at equal distances apart, as may be necessary; through these points draw the lines.

To construct the squares diagonally to the base (Fig. 159).

Mark on the sides of the right angle as many points, at distances apart equal to the diagonal of the required squares, as may be necessary. Connect these points by lines as shown, and through the same points draw lines at right angles to the others.

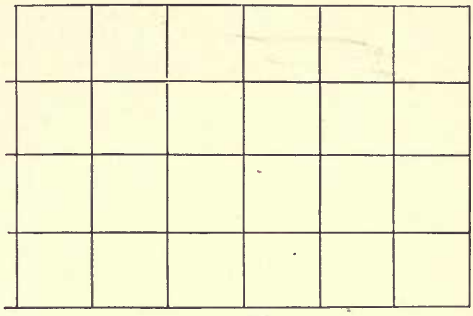


FIG. 158.

To cover a surface with equilateral triangles (Fig. 160).

Construct an angle of 60°, and mark on its sides points at distances apart equal to the side of the triangle. Connect these points; and through these points draw lines parallel to the sides of the angle.

Two such triangles joined at the base form a *lozenge.* Six triangles may be arranged as a hexagon. The whole surface may be arranged in lozenges or hexagons.

To cover a surface with octagons and squares (Fig. 161).

Lay off the surface in squares having sides equal to the width of the octagons. Corner the outer squares to form octagons (Fig. 68). Extend the sides of these octagons across the other squares, and similar corners will be cut off, and the octagons and squares required will be complete.

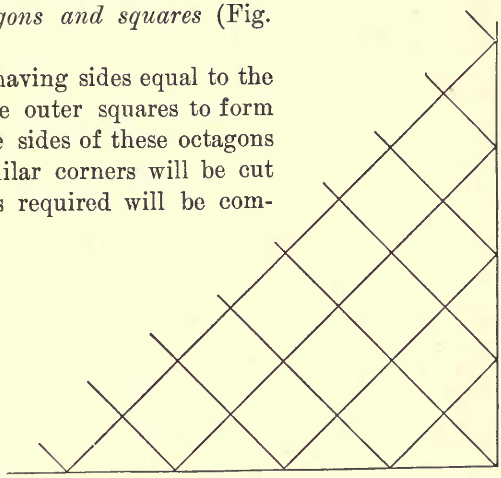


FIG. 159.

With the aid of paper thus covered with squares, triangles, or lozenges, various geometrical designs may be readily constructed, pleasing in their effect, and affording good practice to young draughtsmen.

Any design can be copied by covering it and the clean sheet with squares. Mark the positions of points in the design or the sides of cor-

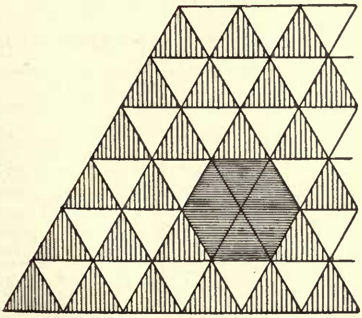


FIG. 160.

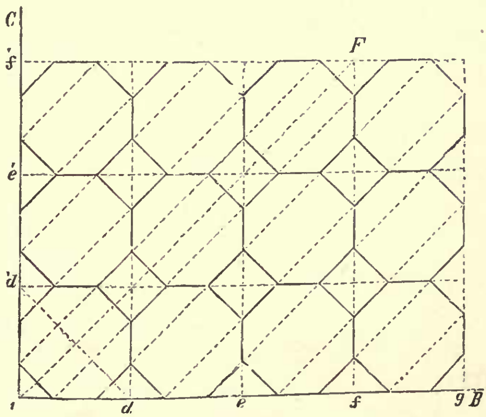


FIG. 161.

responding squares, and draw the connecting lines. To enlarge or reduce the design, make the squares or triangles proportionately larger or smaller.

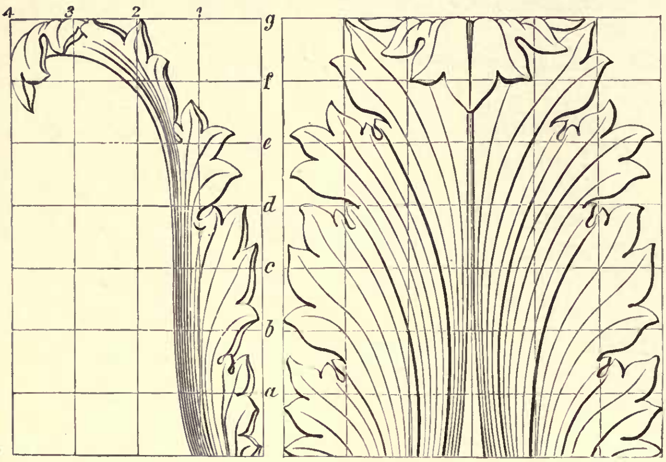


Fig 162.

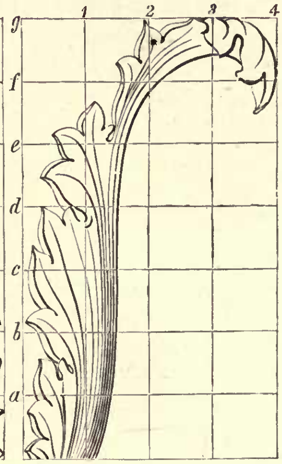


Fig. 163.

In transferring designs and drawings from books or plates, on which squares can not be drawn, it is very convenient to have a square of glass, with squares upon it, which may be laid on the drawing, and thus serve the same purpose as if squares had been drawn. The glass may be readily prepared by painting one of its surfaces with a thin coat of gum, and drawing squares upon it with the drawing-pen; if every fifth or tenth line be made fuller or in a different colour, it will be still more convenient for reference.

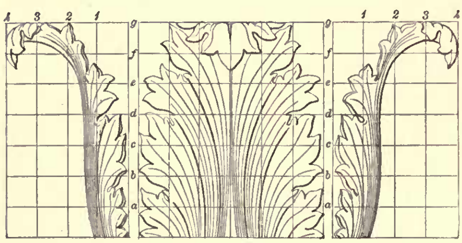
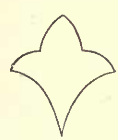


Fig. 164.

Fig. 162 gives the front and side views of an acanthus leaf, the surface being covered with squares, and on a ground of like squares in Fig. 163 the side view is transferred, but in a reversed position. This is done by making the position of the outline and then of the interior lines with reference to the squares, as designated by letters and numerals. Fig. 164 is a transfer of both figures on a reduced scale.

Designs for woven goods, oil cloths, ceiling and wall ornamentation, and the like are usually based on geometrical figures, and in certain proportions symmetry and subordination of one part to another fall within the term of artistic.



The following are designs in which the ruling figure is a trefoil:

"In the equilateral triangle (Fig. 165), each side is divided by a dot, and from the centre of the triangle lines are drawn to each angle, and from the dot in the middle of each side to the opposite sides of the figure. The geometrical plan of the design is thus laid out, and the figure is easily filled in by drawing simple curves from the centre of the form to the dot on each side of it, and, lastly, filling in the form of the trefoil a little below the point of each corner of the triangle.

"The square (Fig. 166), which is the next form, is developed in much the same manner. The sides are bisected, and from a point in the centre lines are

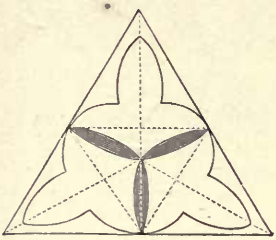


Fig. 165.

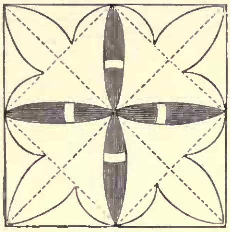


Fig. 166.

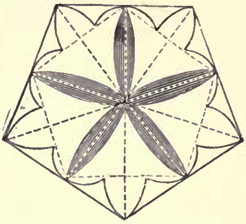


Fig. 167.

carried to each angle, and to all the dots on the sides. As in the preceding figure, slight curves are made on either of the side-lines, and the trefoil is added to each angle, with the base of the middle leaf touching the transverse working-lines between the sides. It will be seen that the pentagon (Fig. 167) and the hexagon (Fig. 168) also are formed in the same general manner, but the proportion of the top of the trefoil varies from its sides.

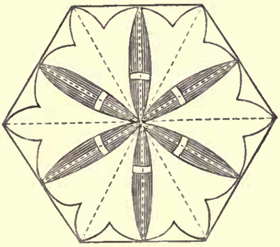


Fig. 168.

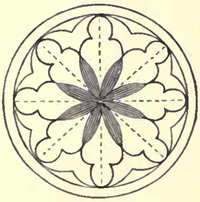


Fig. 169.

"In drawing the circular rosette (Fig. 169), the circumference should be constructed on a vertical and a horizontal diameter, with two other diameters bisecting it at equal angles, which divide it into eight sections, the half diameters, upon

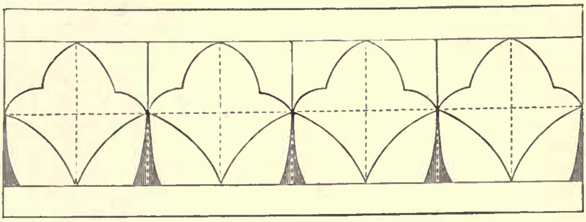


Fig. 170.

all of which curved lines and the top of the trefoil are made. A series of arcs may be added at the pleasure of the designer. In the two pieces of moulding (Figs. 170 and 171) the trefoil is inserted vertically to the sides in one and horizontally in the other. In the latter, a half of the trefoil is added upon the sides to enrich the elementary figure; and the double line and the transverse lines which form the squares are repeated for the sake of symmetry, and as affording an impression of agreeable repose.

"It is from such a basis as this that all these various patterns are derived, and they produce a result which an inexperienced eye, unaccustomed to analyze designs, could scarcely resolve into its elements."

Figs. 172–175 are other illustrations of the same principle, of varieties of rosettes constructed on a similar plan.

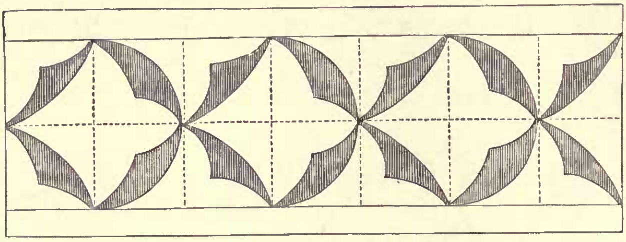


Fig. 171.

All of these designs can be constructed mechanically, but more grace is given to the design by the filling in with free hand, and it is an excellent prac-

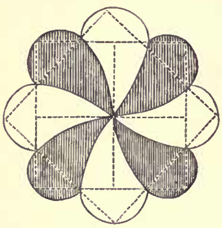


Fig. 172.

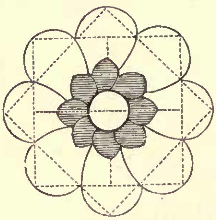


Fig. 173.

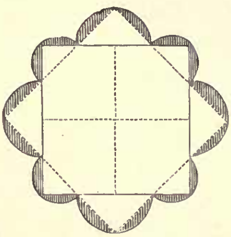


Fig. 174.

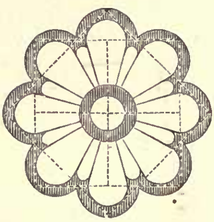


Fig. 175.

tice in the execution of the more elaborate Saracenic and Moorish diaper. In all of these where there are repetitions of the same figures it is usual to draw but one, and then transfer this, with the finish in crayon or pencil.

Lettering.

In Fig. 176 are examples of block letters constructed on squares, a rudimentary form of mechanical letters, which can be made with the aid of cross-section paper.

Although lettering admits of an endless variety of forms, the draughtsman should comprehend that there are rules on which letters should be constructed before he undertakes the free-hand method.

Fig. 177 gives the designation of various parts of a letter to which reference is made in the description. In the Roman letters the square is taken as the scale of construction. Fig. 178 gives the scale of proportionate width. W takes the whole square, its height and width being equal; I is one quarter as wide; A, five sixths, etc. To obtain the width of any letter according to this scale, the height may be marked off on the vertical O 12. Where the horizontal line from this point intersects the diagonals of the desired letter the width is measured.

The thickness of the body stroke of the letters is about one fifth the height; the thickness of the body curve is slightly in excess of this, and the excess is added outside the letter; otherwise in comparison with the straight body strokes the curved stroke would appear too thin.

All letters are of the same height, except those curved or pointed at the top or bottom, such as C, G, J, O, S, U, A, V, W. When the curved or pointed parts are at the top they must extend a little above the line and when at the bottom

FIG. 176.

below the line, otherwise they will look smaller than those of square outline. The lower feet of letters and the feet of T extend about one third the height, but the upper feet are a trifle smaller.

The intermediate horizontal hair stroke in B, E, F, H, and R is a little

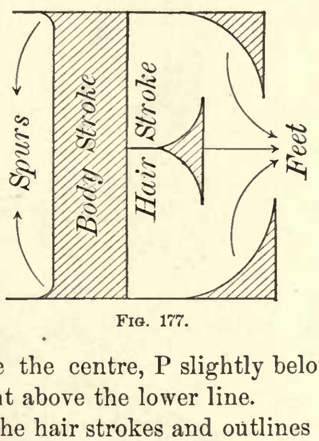


FIG. 177.

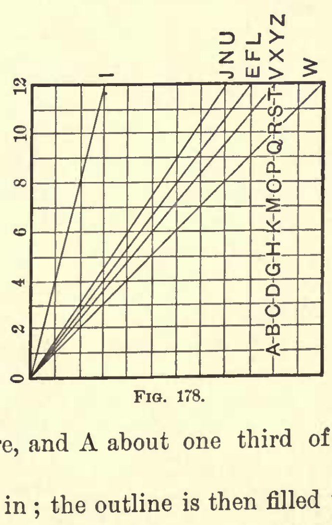


FIG. 178.

above the centre, P slightly below the centre, and A about one third of the height above the lower line.

The hair strokes and outlines are first put in ; the outline is then filled with a writing-pen, toothpick, or brush.

However well proportioned the letters may be, an even effect is not produced unless a proper space is made between them. In letters of square form the spacing is equal, but where such combinations as LT, AT, AA, and numerous others occur the spacing must be less. No general rule can be given for this, and it must be left to the practised eye of the draughtsman.

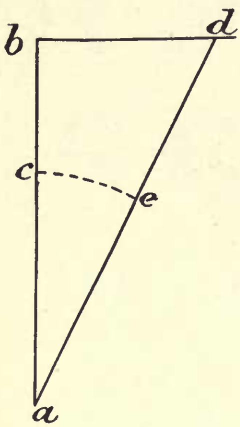


FIG. 179.

The only rule necessary for the construction of small Roman letters is that for ascertaining their height as compared with the capitals: Let a vertical line *a b* (Fig. 179), equal to the height of the capital, be drawn, and a line at right angles at the top of this line, equal to one half its length; connect *d* to *a*, and lay off the length of the line *d e*, equal to *b d*; then *a e* will be the height required. When the learner has acquired some dexterity in lettering, the upper and lower line alone will be necessary for his guidance; he may then attempt the execution of the curves, without the compass, by free hand.

In Italic letters the proper angle for their slant is 23° from the vertical; the proportions are the same as in the Roman letters.

In stump letters the capitals are the same as Italics. The small letters differ somewhat, and are made with one bold stroke of the pen, the hair line gliding imperceptibly into the body stroke.

Block letters, of which an example has been given, constructed on squares, are one of the most valuable types, being very distinct and readily drawn by the drawing-pen. The letters are of the same proportional width as the Roman, except that the M is a square. The height and width of the letters are varied to suit their application. There are *lettering triangles* (Fig. 180) made to give the angles with the verticals of inclined letters.

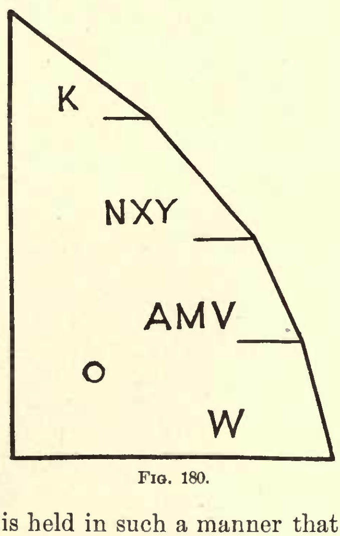


FIG. 180.

Old English and German text and other letters of a similar character may be quickly and neatly written by the use of a wooden toothpick for the body strokes, the hair lines and terminations being afterward put in with a fine pen.

An old style of writing that has lately gained considerable popularity in this country is round writing. This resembles ordinary writing in that one letter is joined to the next, and each word written as a whole. There are special pens made for this writing, which are very useful; but in place of these the ordinary stub pen can be used. This writing consists of a very few elements, merely shaded semicircles, straight shaded lines, and diagonal hair strokes; the pen is held in such a manner that you are able to draw a fine hair line at an angle of 45° with both nibs of the pen, and the pen is held in this position for all letters.

ROMAN

ABCDEFGHIJK

LMNOPQRSTUV

abcde WXYZ fghij

klmnopqrstuvwxyz

ITALIC

ABCDEFGHIJK

LMNOPQRSTUV

abcdeWXYZ fghij

klmnopqrstuvwxyz

acefghikmsvwxyz

I II III IV V VI VII VIII IX XLCDM

0 1 2 3 4 5 6 7 8 9

ABCDEGHJKLMNOPRSTUWY

ABCDEGHJKLMNOPRSTUVWY

ABCDEFGHJKLMNOPRSTUVWY

ABCDEFGHIJKLMNOPQRSTUVWY

ABCDEFGHIJKL

MNOPQRSTUVW

abcdefgXYZhijklmn

opqrstuvwxyz

Round Writing

A B C D E F G H

I K L M N O P

Q R S T U V W

abcde X Y Z fghi

jklmnopqrstuvwx

yz. 1 2 3 4 5 6 7 8 9 0

ENGLISH GOTHIC.

ABC DE FGH IJ KLMN OP
QRST UV WX YZ
1234567890

ITALIC.

ABC DE FGH IJ KLMN OP QRST
UV WX YZ
abc de fgh ij klmn op qrst uv wx yz

TUSCAN.

ABC DE FGH IJ KLMN OP QRST
UV WX YZ
1 2 3 4 5 6 7 8 9 0

ABC DE FGH IJ
KLMN OPQRST
UV WX YZ
abc de fgh ij klmn op
qrst uv wx yz

ENGLISH CHURCH TEXT.

ABC DE FGH IJ KLMN
OP QRST UV WX YZ
abc de fgh ij klmn op qrst uv wx yz

MEDIÆVAL.

ABC DE FGH IJ KLMN OP QRST
UV WX YZ
abc de fgh ij klmn op qrst uv wx yz

ABC DE FGH IJ KLMN OP QRST
UV WX YZ
abc de fgh ij klmn op qrst uv wx yz

COAST CHART No. 20

NEW YORK BAY AND HARBOR

NEW YORK

HYDROGRAPHIC MAP

OF THE

BYRAM, BRONX, CROTON AND HOUSATONIC RIVERS,

AVAILABLE FOR THE WATER SUPPLY OF

NEW YORK CITY.

Scale 1 in. = 6 miles.

BOSTON WATER WORKS

SUDBURY RIVER CONDUIT

CHARLES RIVER BRIDGE

DEC. 1875.

SHEET 2

TRENT RIVER, N.C.

FROM TRENTON TO UPPER QUAKER BRIDGE

SCALE OF MILES

Specification Contract

PLAN FRONT ELEVATION

REAR SIDE TRANSVERSE

LONGITUDINAL SECTION

The character and size of the letters should be in accordance with the drawing on which they are to appear. Thus in engineering or mechanical drawing nothing is so appropriate as the block and Roman letter.

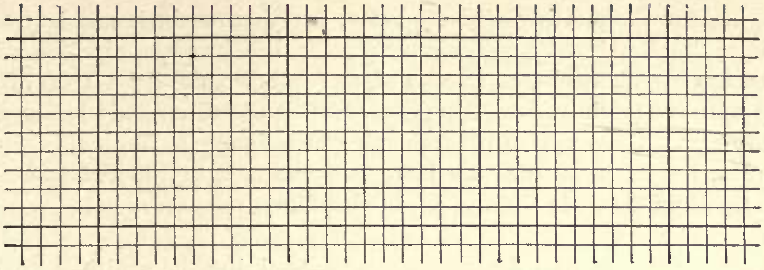
In topographical or map drawing several styles of various sizes are used; Roman capitals of different sizes are used in designating States, countries, and cities, while State boundaries and towns, large villages, summits, and boundaries of countries are written with an initial capital and small Roman letters. Large bodies of water are denoted in Italic capitals, smaller bodies, such as streams, creeks, small lakes, and ponds, with an initial Italic capital and stump letters, the general direction of the letters following the course of the stream. Capital block letters differing in size represent ranges of mountains, railroads, stations, streets, and prominent objects on a map. Oblique capital block letters represent railroads and canals.

Old English and German text are the styles most employed for the engrossing of certificates and similar uses.

Letters on topographical drawings are written horizontally, so as to be read from the lower right-hand corner of the map, except such as follow the course of a stream or a railroad; and these can usually be arranged to be read from the same direction.

A number of specimen titles are given, illustrating the use of the different styles of letters, but it will be found that the plainer the letters the better the effect produced, and for this reason it is generally well to omit fancy letters. No better example of neat and dignified titles can be found than those on the maps issued by the United States Coast Survey, the chief beauty of which consists in the admirable adjustment of the sizes of letters, the few styles, and the almost perfect execution.

The illustrations of letters and titles given are taken from those in general use, but the draughtsman should make a large collection of titles of maps and books, drawings of machinery, advertisements, and business cards, copying carefully such as may suit him, by which he will gain ease of manipulation and taste in selection, to give character and finish to his own drawings.



Profile and Cross-section Paper.

Paper printed in squares is used by designers of figures for calicoes, silks, and woollens. For the engineer, there is a class of papers called *profile* and *cross-section papers*, sold in sheets or rolls, and of various scales. In the first, which is almost entirely applicable to lines of surveys of railroads and highways, the vertical scale is to the horizontal as 20 to 1. This is the usual distortion to make grades, with the cuts and fills apparent. The latter originally

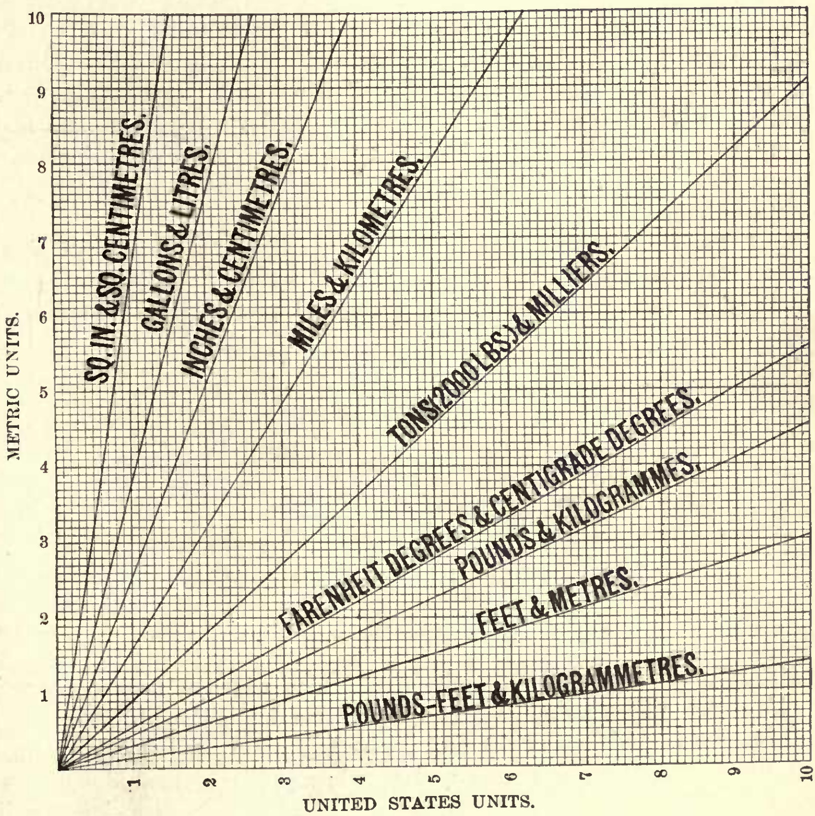


Fig. 181.

intended, as the name implies, for cross-sections of railway or canal cuts, but now extensively employed by the architectural and mechanical designer for the rough sketches of works either executed or to be executed; by the sanitarian, for the plotting of death-rates; for thermometric and hygrometric readings; by the broker and merchant, for the graphic representation of the prices of gold, stocks, or articles of merchandise, during a term of years; by the railway superintendent, for the movement of trains; and for a multitude of other uses. Cross-section papers most generally applicable are in divisions of tenths; but as mechanics are more conversant with the two-foot rule, of which the divisions are in eighths, paper with like divisions are more convenient, and designs on it more intelligible to them.

Fig. 181 shows a graphical method of determining the equivalent values of the metric system of measurements in United States units, or *vice versa.* The vertical scale represents the metric units, and the horizontal the common or United States units.

Example.—What is the equivalent value of seven kilometres in miles? Read upward on the metric scale to 7, then read on that horizontal line to the point of intersection with the line designated "Miles and Kilometres," that is, at the point on the United States scale of units representing 4·35, and you find that seven kilometres are equal to 4·35 miles.

What is the value of five pounds in kilogrammes? The process is the same as the foregoing, except that, to change United States units into the metric units, you first read horizontally, then upward. In this case five pounds is found equal to 2·25 kilogrammes. The divisions may represent single units, ten units, one hundred units, etc. Of late it has been common here and in England, to write ft. lbs., instead of lbs. ft., but where French weight and measures obtain, the rule is to say kilogrammetres, that is the weight before the distance moved.

Fig. 182 shows the method of finding the average of a number of observations, to determine the velocity of a current of water. The figure represents the path of a float in a wooden flume or channel, of rectangular section, from

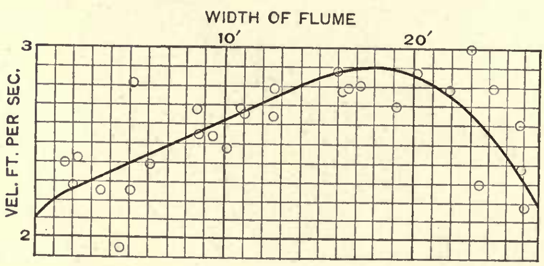


FIG. 182.

Francis's "Lowell Hydraulic Experiments." The width of the cut represents the width of the flume, each abscissa being one foot; the ordinates are the speeds of float in divisions of 0·1 foot per second; the small circles represent the floats in their observed path and speed; and the curved line shows the average velocity in the different threads of the stream, from which the lines of average velocities of the entire width of flume are deduced.

The velocities were taken by tin tubes loaded so as to float within about an inch of the bottom of the flume, with the top plugged and projecting a few inches above the surface of the water. The results were checked by flows measured over a weir; but for all practical purposes the velocities as taken by the floats may be considered averages on each thread of the stream. A full set of tubes were prepared adapted to the depths of the water, and taken to the flumes while experimenting. For general use a cylinder may be adapted as a float with an open pipe sliding down it adjustable to the depth.

Fig. 183 is a diagram illustrating graphically the difference between the charge on a ton of merchandise per mile on the New York Central and Hudson River Railroad and the Erie Canal for every year between 1857 and 1880. The higher values in every case represent the railroad rates and the lower the canal rates. The black band shows the difference between them.

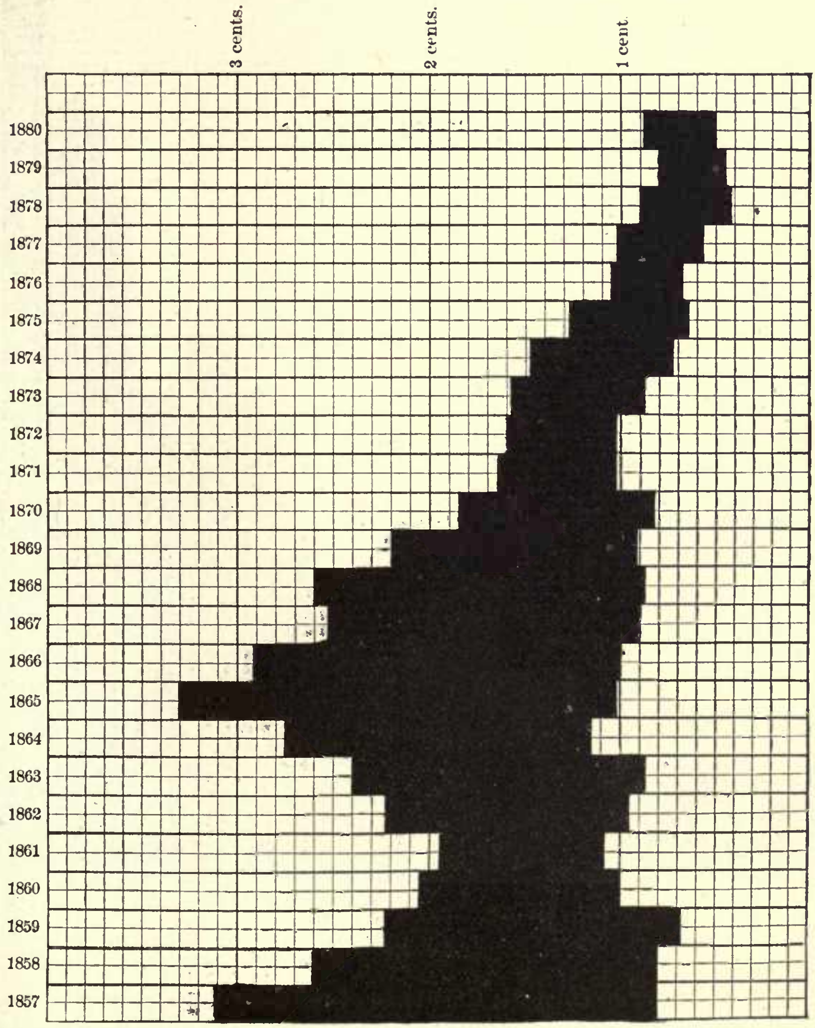


Fig. 183.

Fig. 184 is a graphic representation made up from the records of "The Engineering and Mining Journal," exhibiting the amount of pig iron made in the United States per year for thirty-one years. In constructing the diagram cross-section paper was used, but in tracing the vertical cross lines were omitted.

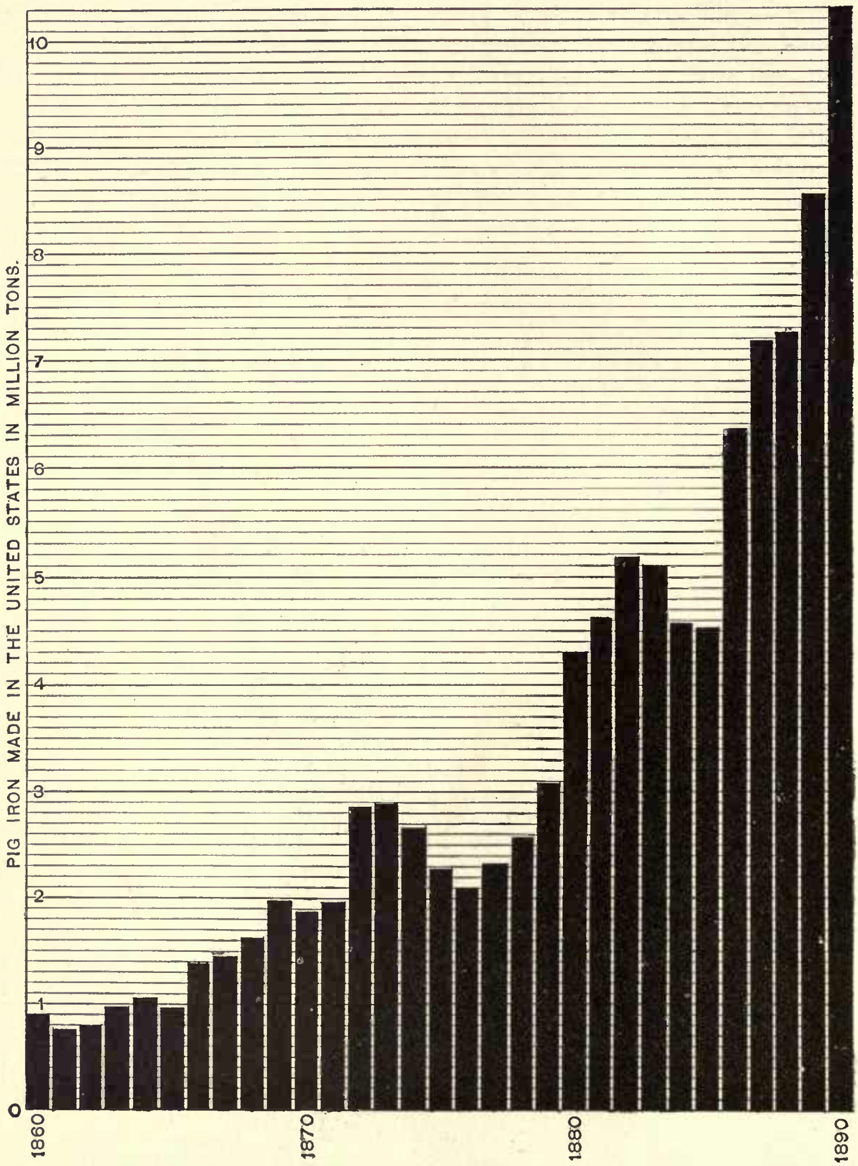


FIG. 184.

Fig. 185 is made up from the time-table of the New York, New Haven and Hartford Railroad, showing the movement of trains, two from New York and two from New London, the horizontal lines being cut off on a scale of miles for each station, and the vertical lines being a scale of hours. If the speed had been uniform, the line showing the movement of trains would have been straight, but the line represents the practical running time.

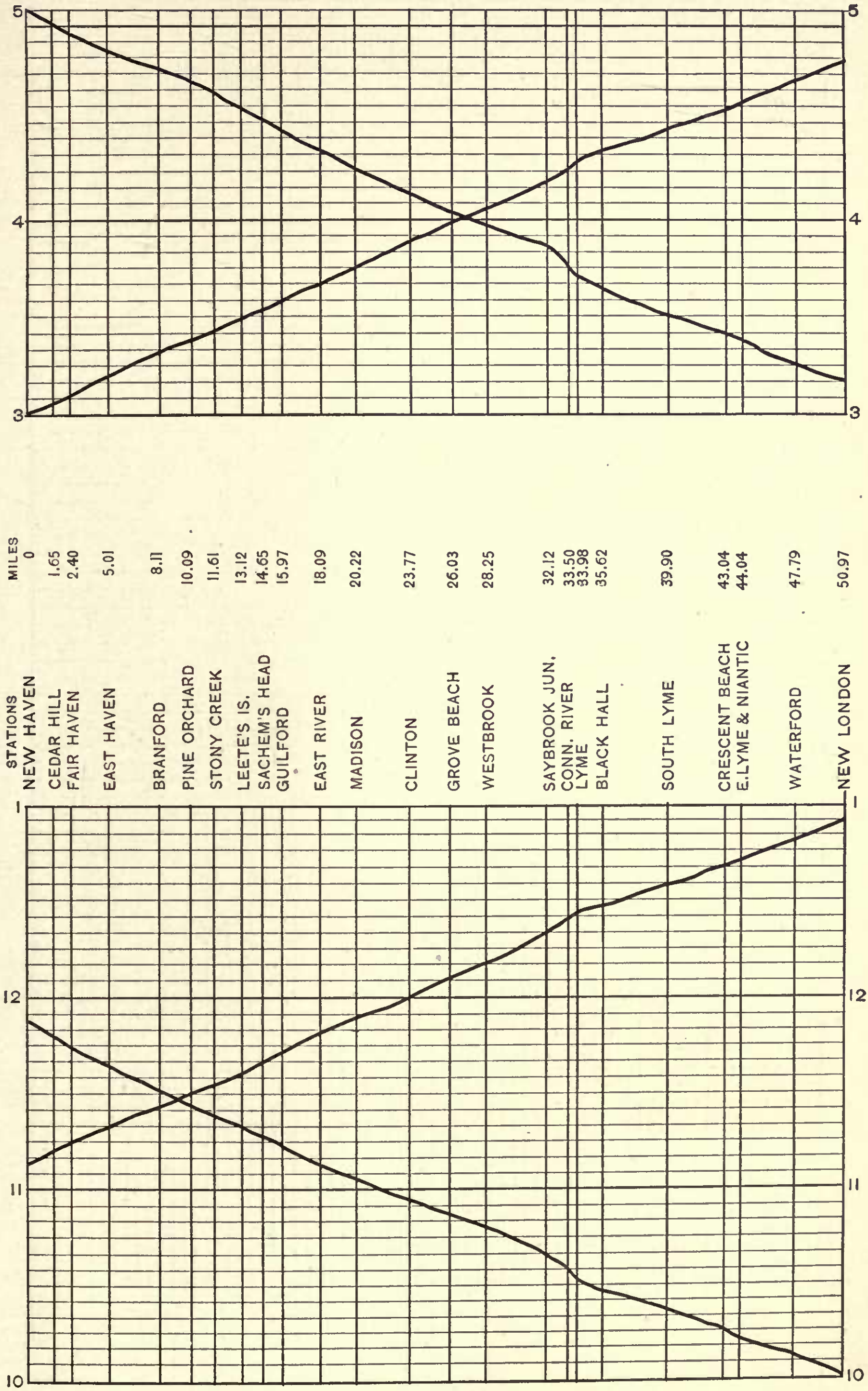

5
4
3
STATIONS
MILES
NEW HAVEN 0
CEDAR HILL 1.65
FAIR HAVEN 2.40
EAST HAVEN 5.01
BRANFORD 8.11
PINE ORCHARD 10.09
STONY CREEK 11.61
LEETE'S IS. 13.12
SACHEM'S HEAD 14.65
GUILFORD 15.97
EAST RIVER 18.09
MADISON 20.22
CLINTON 23.77
GROVE BEACH 26.03
WESTBROOK 28.25
SAYBROOK JUN. 32.12
CONN. RIVER 33.50
LYME 33.98
BLACK HALL 35.62
SOUTH LYME 39.90
CRESCENT BEACH 43.04
E.LYME & NIANTIC 44.04
WATERFORD 47.79
NEW LONDON 50.97
1
12
11
10

FIG. 185.

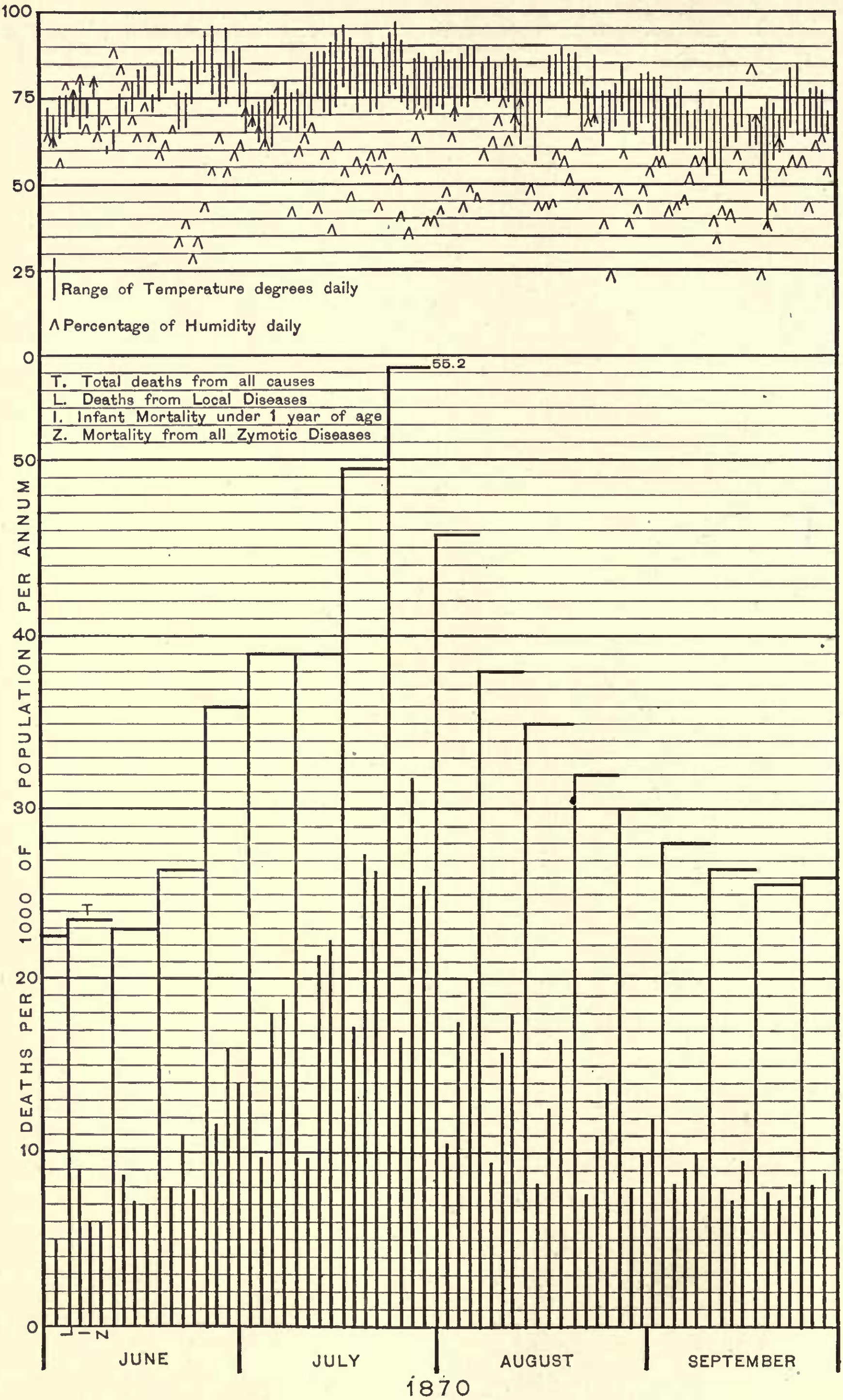
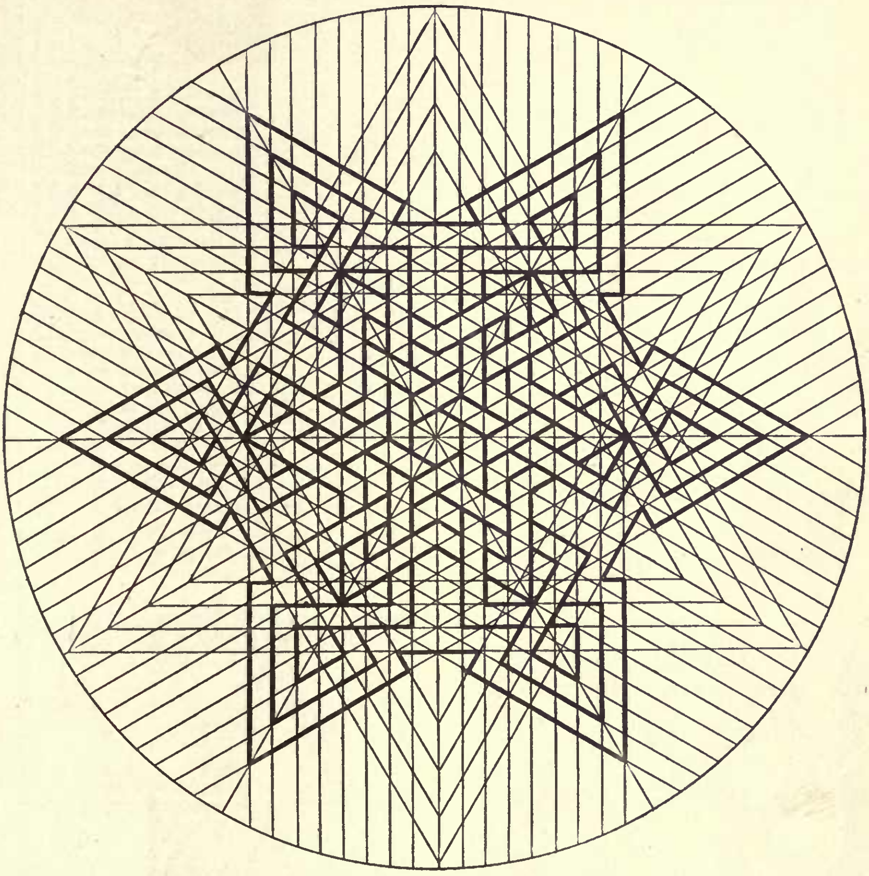

100
75
50
25
0
Range of Temperature degrees daily
Λ Percentage of Humidity daily
T. Total deaths from all causes
L. Deaths from Local Diseases
I. Infant Mortality under 1 year of age
Z. Mortality from all Zymotic Diseases
55.2
50
40
30
20
10
0
DEATHS PER 1000 OF POPULATION PER ANNUM
T
N
JUNE
JULY
AUGUST
SEPTEMBER
1870

Fig. 186.

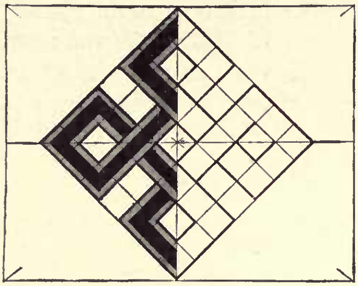
Fig. 186 is a graphic representation of the mortality and general classes of diseases as registered by the New York City Board of Health for the months of June, July, August, and September, 1870; with the daily records of temperature and humidity; to complete these diagrams there should be one of the daily rainfalls. For meteorological purposes it is usual to take at observatories the commencement, termination, and amount of rainfall.

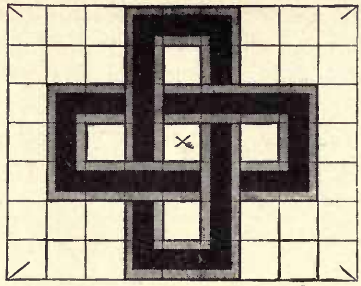


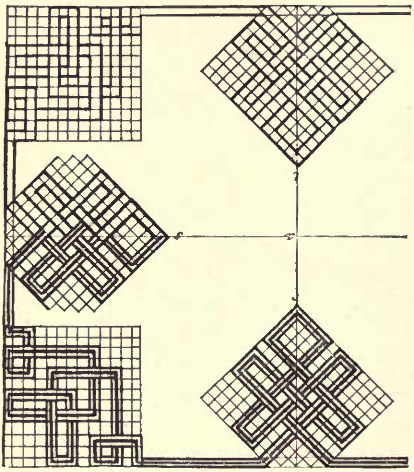
Above is an ornamental design in straight lines on the bases of lines parallel to the sides of an equilateral triangle.

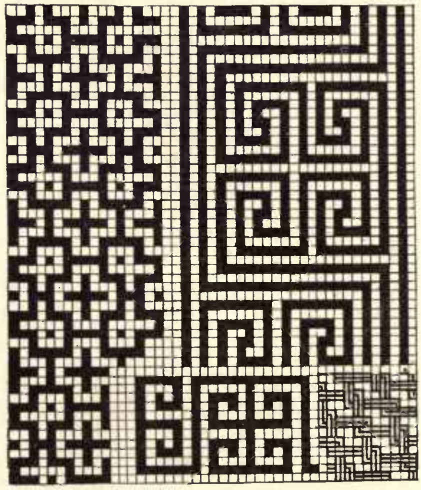
In pages 78, 79 are given designs on the bases of lines ruled in rectangles and lozenges. The figure, page 80, illustrates how colors may be represented in a design.

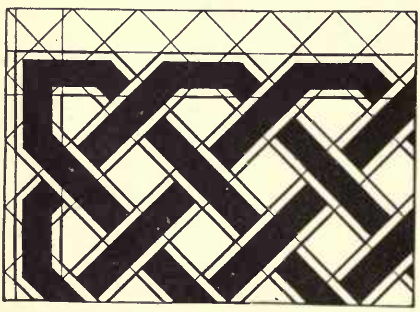
In pages 81, 82 are illustrations in single lines of the tracery of Gothic windows of which the lines of construction are left to assist the draftsman in completion of the design, which will afford excellent practice in the intricate work of making lines tangent to each other.

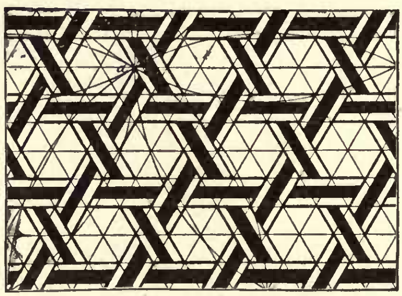


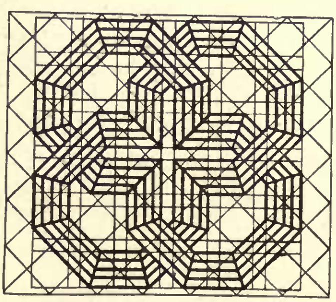


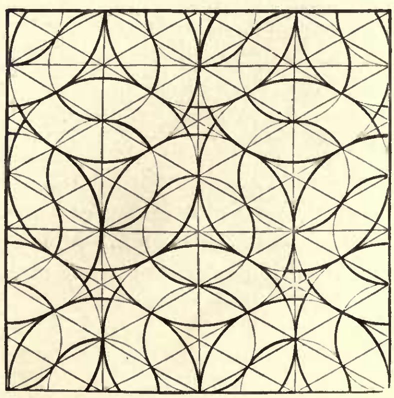


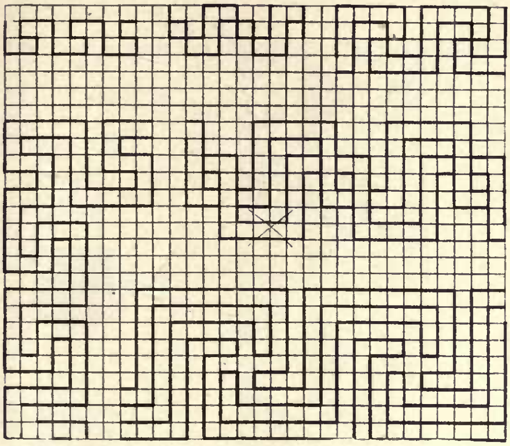


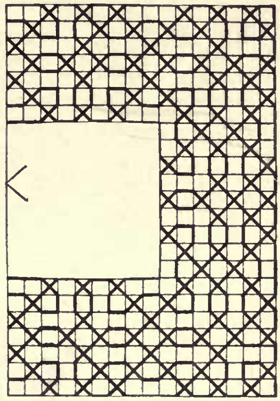


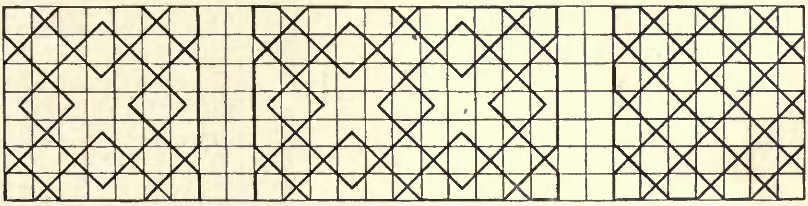


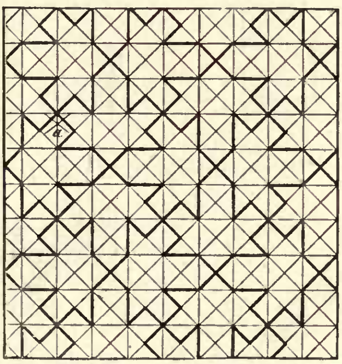


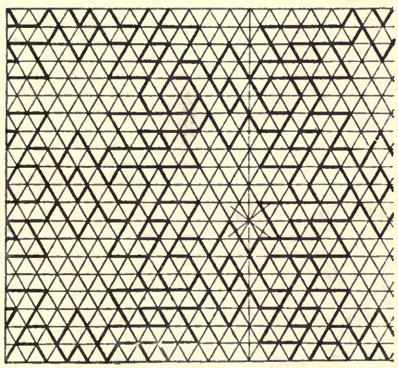


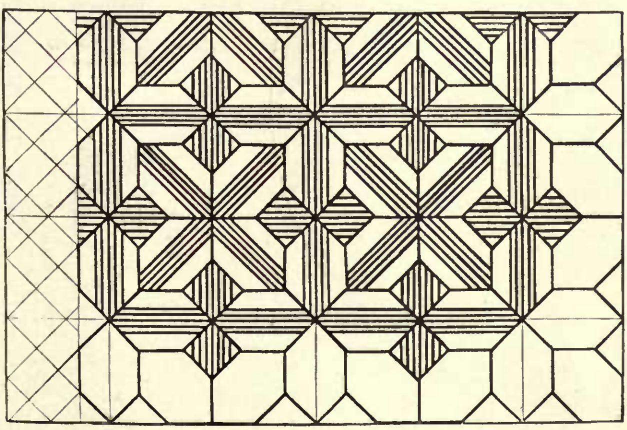


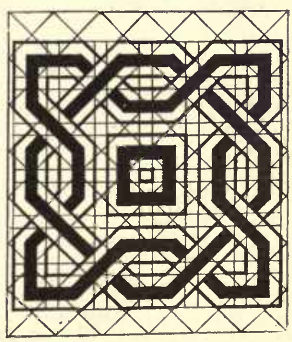


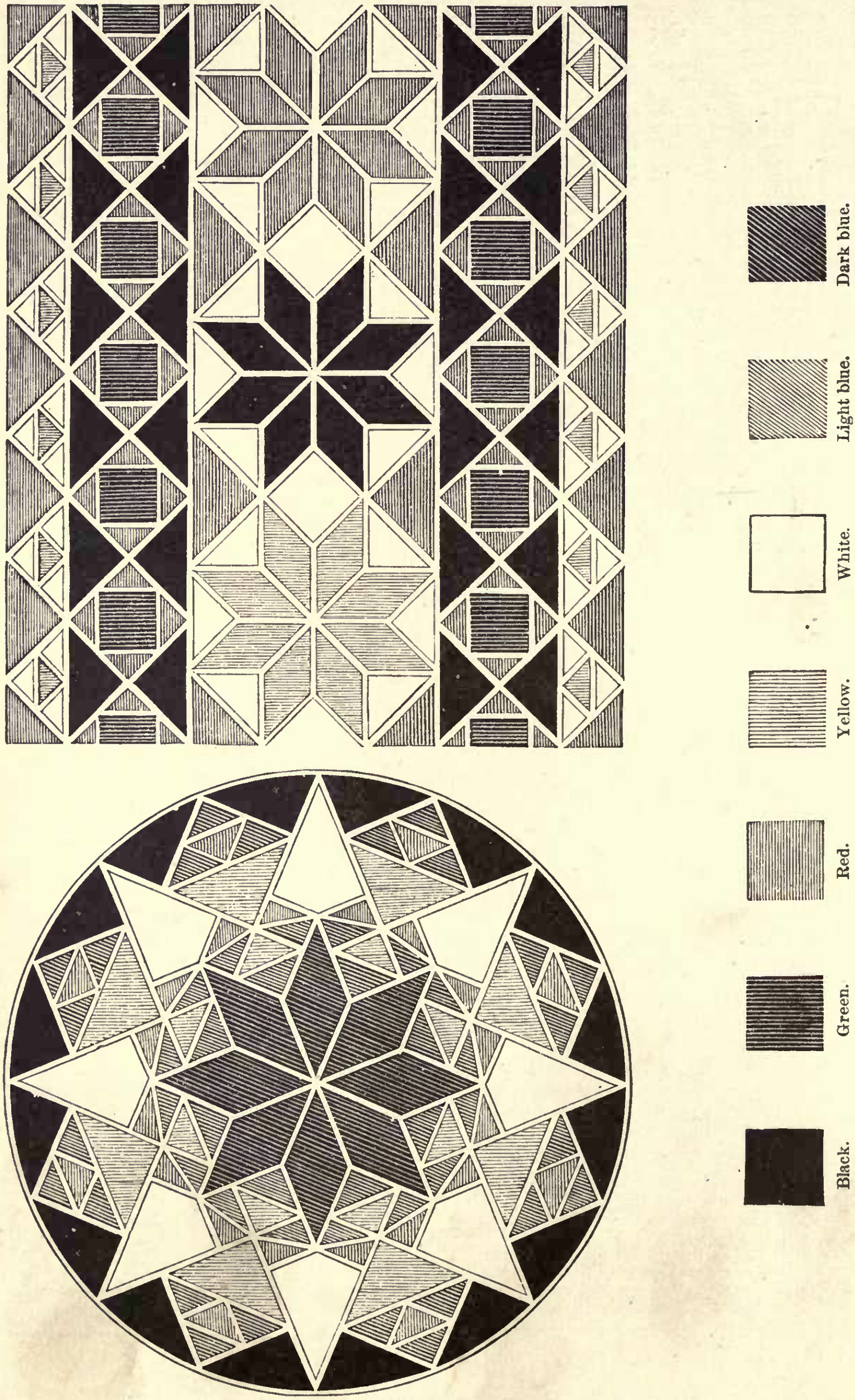


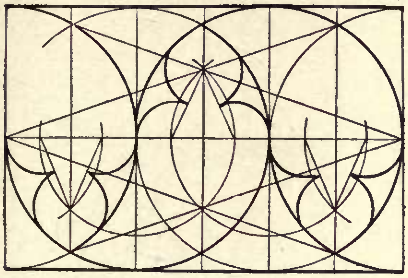

a

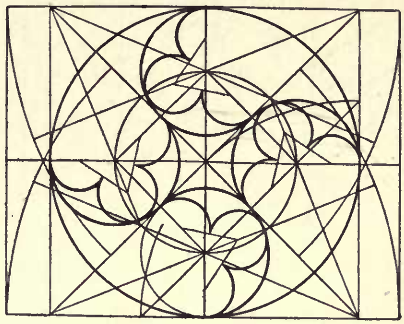


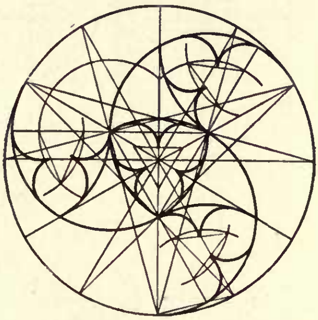


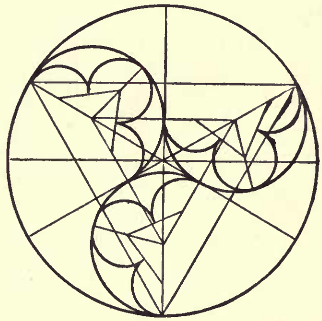


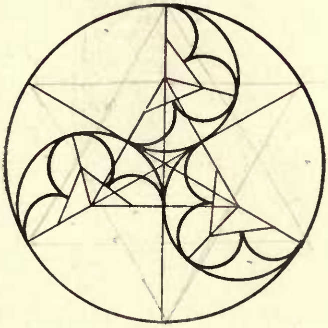

Dark blue.
Light blue.
White.
Yellow.
Red.
Green.
Black.

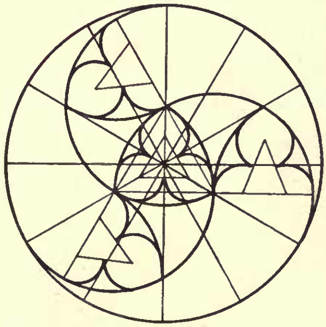


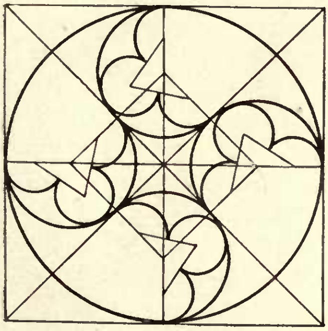


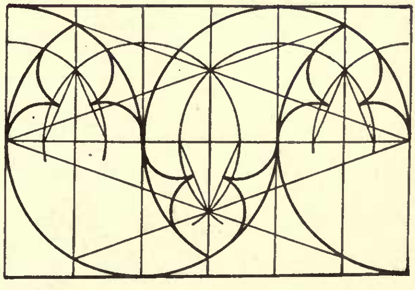


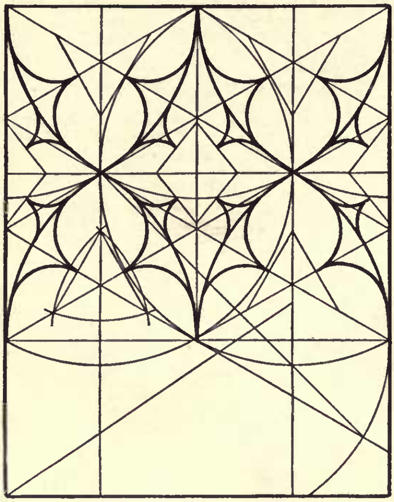


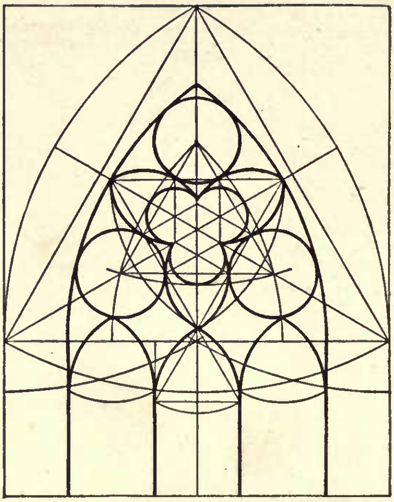


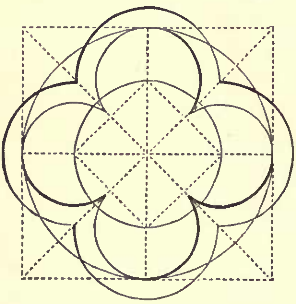


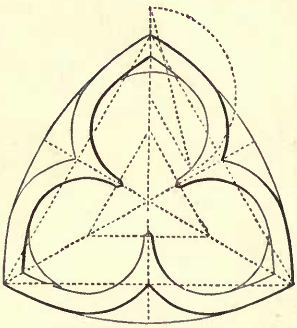












PLOTTING.

PLOTTING is the laying out on paper in plan, or horizontal projection, the boundaries of portions of the earth's surface of greater or less extent, from the notes of surveys or other records. When the extents are large, embracing degrees of latitude and longitude, the plots are designated as maps; but if of small extent, as lots, estates, and farms, they are usually designated as plans or plots. After completing the outlines, it is usual to fill up the plot with the characteristic features, geographical, geological, agricultural, industrial, and domestic, which are expressed conventionally, as will be shown under the head of "Topographical Drawing."

Scales.—The choice of the scale for the plot depends on the purpose for which the drawing is intended. It should be large enough to express all desirable details which it is intended to illustrate, and the place it is to occupy.

Plans of house-lots and small plots of farm surveys are usually so many feet to the inch; maps of surveys of States, so many miles to the inch; and maps of railway surveys, so many feet to the inch or so many inches to the mile. Formerly the lines of farms were measured by the four-rod chain. Two to three chains to the inch was then a very common scale.

On the United States Coast Survey all the scales are expressed fractionally and decimally. The original surveys are generally on a scale of one to ten or twenty thousand, but in some the scale is larger or smaller. The public surveys embrace three general classes: 1. Small harbour charts. 2. Charts of bays, sounds, etc. 3. General coast charts.

The scales of the first class vary from $\frac{1}{5000}$ to $\frac{1}{60000}$, according to the nature of the harbour and the objects to be represented.

The scale of the second class is usually fixed at $\frac{1}{80000}$. Preliminary charts, are, however, issued of various scales, from $\frac{1}{80000}$ to $\frac{1}{200000}$.

Of the third class the scale is fixed at $\frac{1}{400000}$ for the general chart of the coast from Gay Head to Cape Henlopen, although considerations of the proximity and importance of points on the coast may change the scales of charts of other portions of our extended coast.

On all plots of large surveys, it is very desirable that the scales adopted should bear a definite numerical proportion to the linear measurement of the ground to be mapped, and that this proportion should be expressed fractionally on the plan, even if the scale be drawn or expressed some other way, as miles to the inch. The decimal system has the most to recommend it, and is generally adopted in government surveys.

For railroad surveys, the New York general railroad law directs the scale of map which is to be filed in the State Engineer's office to be 500 feet to $\frac{1}{10}$ foot, or $\frac{1}{5000}$.

For canal maps, a scale of two chains to the inch, $\frac{1}{1584}$, is employed. In England plans and sections for projected lines of inland communication, or generally for public works requiring the sanction of the legislature, are required by a standing order to be drawn to scales not less than four inches to the mile, 1 : 15,840, for the plan, and 100 feet to the inch, 1 : 1,200, for the profiles.

In the United States engineer service the following scales are prescribed:

General plans of buildings	10 feet to the inch,	1 : 120
Maps of ground with horizontal curves 1 foot apart .	50 " "	1 : 600
Topographical maps, 1½ mile square	1 mile to 2 feet,	1 : 2,640
Topographical maps, 3 miles square	1 " 1 foot,	1 : 5,280
Topographical maps, between 4 and 8 miles . .	1 " 6 in.,	1 : 10,560
Topographical maps, 9 miles square	1 " 4 "	1 : 15,840
Maps not exceeding 24 miles square	1 " 2 "	1 : 31.680
Maps comprising 50 miles square	1 " 1 inch,	1 : 63,360
Maps comprising 100 miles square	1 " ½ "	1 : 126,720
Surveys of roads and canals.	50 feet to 1 "	1 : 600

Many government maps are made on a scale of $\frac{1}{5000}$ or $\frac{1}{10000}$.

It is always desirable that the scale should be drawn on maps and plans, as they are often reduced by photography.

In cities and towns, plots of lots and squares are generally rectangular, and they can be readily plotted on any convenient scale.

Fig. 187 is a plan of the usual New York city lot, 25×100, on a scale of 20 feet to the inch, or $\frac{1}{240}$.

Fig. 188 is a city square containing thirty-two of these lots, on a scale of 100 feet to the inch, or $\frac{1}{1200}$. The most accurate way is to plot the large rectangle 400×200 feet, and then subdivide it.

Fig. 189 is a plan of the city squares, with the inclosing streets, on a scale 200 feet to the inch, or $\frac{1}{2400}$.

But many lots and most estates are not rectangular, and these, the angles being recorded, must be plotted by the aid of a protractor.

If the survey has been made by triangles, the principal triangles are first laid down in pencil by the intersection of their sides, the length being taken from the scale and described with compasses. In general, when the surveys have been conducted without instruments, the positions of the points on paper are determined by the intersection and construction of the same lines as has been done in the field.

Surveys are mostly conducted by measuring the inclination of lines to a meridian or to each other by the compass or by the transit. In the survey of farms, where great accuracy is not required, the compass is mostly used. The compass gives the direction of a line with reference to the magnetic meridian. The true meridian can be obtained from the observations of the polar star or from the magnetic meridian corrected from the records of the variations by the geodetic surveys of the United States at the place and time.

The plane table is very convenient for filling in the details of a survey when the principal points have been determined by triangulation, and its records are readily transferred to the drawing.

At the left of Fig. 190 are the notes of a compass survey, from which the figure is plotted by drawing a meridian through each station and laying off the

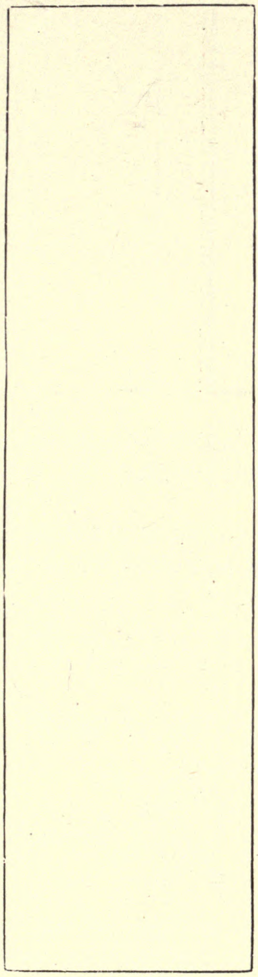


FIG. 187.

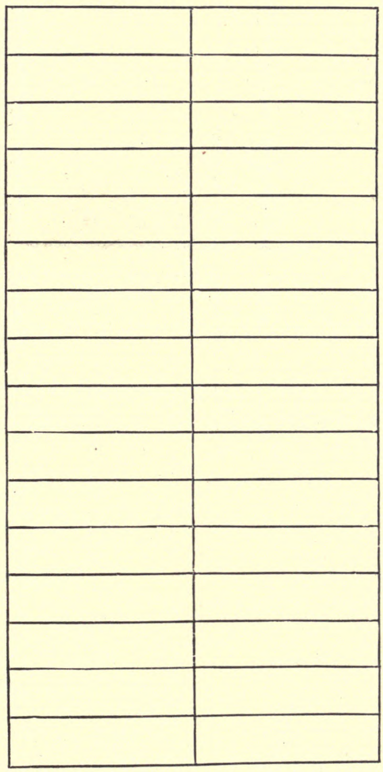


FIG. 188.

angle of deflection. In small drawings it is more convenient, as shown in Fig. 191, to plot all the bearings on a single meridian and then transfer them to the places where they are wanted by any instrument for drawing parallel lines, or to lay off on a single meridian as many bearings as convenient and then transfer the meridian for another plot.

If, as in Fig. 192, the plot fails to close—that is, if the termination a of the last line does not join the commencement of the first line at 1, either the survey or the plotting is incorrect. If the latter be correct, the error of the survey must be *balanced*, or distributed through the lines and angles of the plot (Fig. 192). Connect 1 with a, and draw lines parallel to 1 a through 2, 3, 4, 5, of the plot. Draw an indefinite line, 1 *a* (Fig. 193), and on this, with any convenient scale, lay off consecutively the lines of the survey, 1–2, 2–3, 3–4,

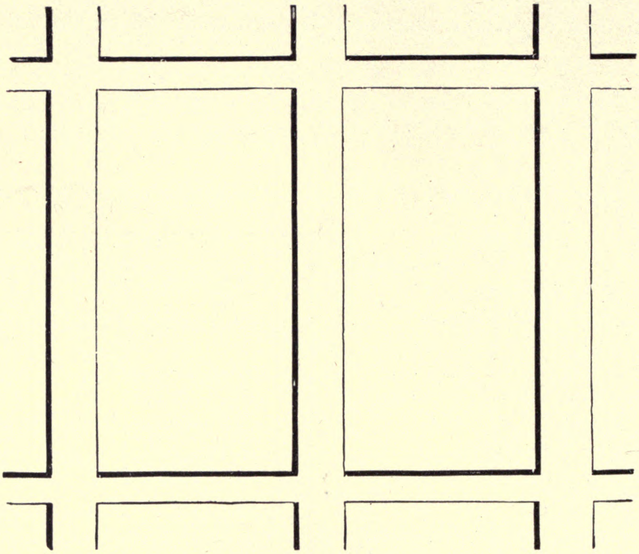


FIG. 189.

4–5, 5–*a*. Erect perpendiculars at the extremities of the lines, 2, 3, 4, 5, and *a*. On the perpendicular *a b*, lay off 1 *a* from the plot and connect *b* 1. The

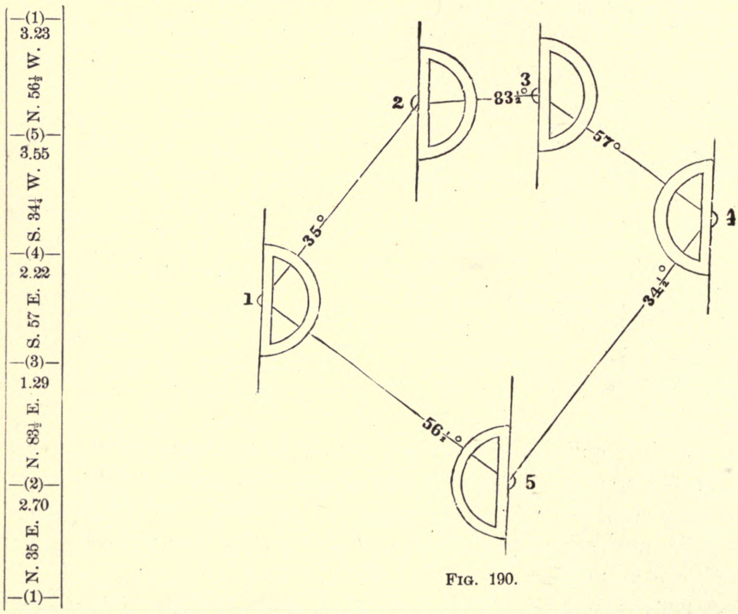


FIG. 190.

intersections of the perpendiculars by this line will determine how much each of the points of the plot is to be moved on the parallels to 1 *a* to distribute the error. The dotted lines on the figure show the corrected outline. If the amount by which the plot fails to close is large, the plot should be resurveyed.

By the aid of the *Traverse Table* a plot of a survey may be balanced. The Traverse Table (see appendix) is a table of differences of latitudes and

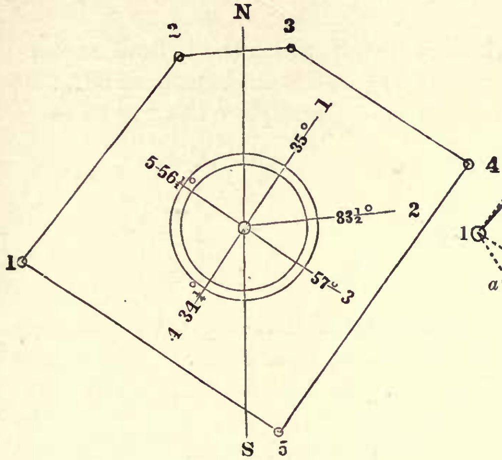


FIG. 191.

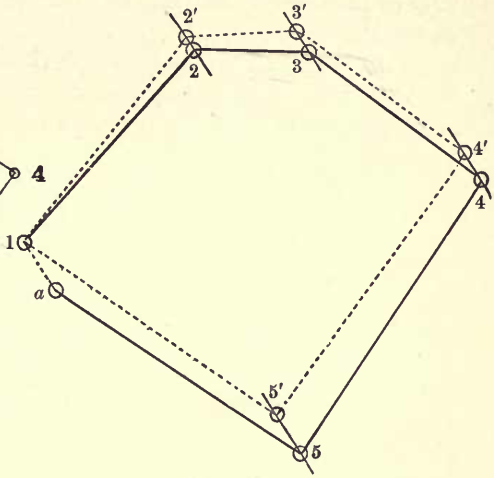


FIG. 192.

departures, the difference of latitude between two stations being the difference north and south between them; the difference of departure, the difference east and west.

Thus, N S (Fig. 194) being the meridian, and A B the course, A C is the difference of latitude, and A D the departure.

The differences vary according to the length of A B, and the angle it makes with the meridian.

Taking the field notes of the following sur-

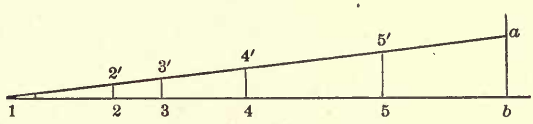


FIG. 193.

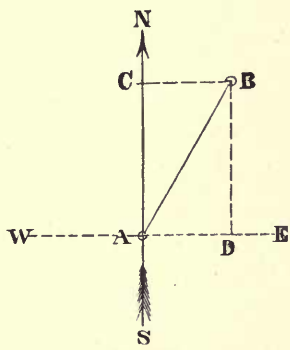


FIG. 194.

vey, we make a table as follows of the stations, bearings, and distances, leaving columns for latitudes and departures:

STATION.	Bearing.	Distance.	LATITUDES.		DEPARTURES.	
			N.	S.	E.	W.
1................	N. 52° E.	1,063	655	...	838	
2................	S. 29¾° E.	410	...	356	203	
3................	S. 31¾° W.	769	...	654	...	405
4................	N. 61° W.	713	346	...	...	624

Find by the Traverse Table the number of degrees of the angle or bearing on the left-hand side of the page if less than 45°, and on the right-hand side if more. The numbers on the same line running across the page are the latitudes and departures for that angle and for the distances which may represent any unit, as feet, chains, links, metres, etc. The traverse table gives the latitudes or departures for a single unit; 10, 100, 1,000, or any other decimal quantity may be obtained by moving the decimal point one, two, three, or more places to the right.

Thus let us take station 1 in previous survey, in which the latitude and departures of a course having a distance of 1,063 feet and a bearing of 52°, and then take out the latitude and departure for 1, 6, 3, and place them as below:

Distance.	Latitudes.	Departures.
1,000	616·0°	788·0
60	36·94	47·28
3	1·847	2·364
1,063	654·787	837·644

If the survey has been accurately performed, the northings and southings of the latitude and the eastings and westings of the departures will balance and the survey will close. In the preceding survey they do not balance; it therefore becomes necessary to balance it. This operation consists in correcting the latitudes and departures of the courses so that the sums of the northings and southings of the latitudes and the eastings and westings of the departures shall be equal. This is done by distributing the difference of their sums among the courses in proportion to their length.

The difference between the northings and southings of latitude, which is 9, is divided by the total length, 2,955, which gives the amount per foot to be added to the lesser column and subtracted from the greater column in proportion to the length of the courses to cause the northings and southings to balance. The departures are balanced in a similar manner.

The following table gives the original latitudes and departures and the corrected ones:

Stations.	Bearings.	Distances.	Latitudes.		Departures.		Corrected Latitudes.		Corrected Departures.	
			N.	S.	E.	W.	N.	S.	E.	W.
1	N. 52° E.	1,063	655	.....	838	.....	658	.....	834	
2	S. 29¾° E.	410	.....	356	203	.....	.....	355	201	
3	S. 31¾° W.	769	.....	654	.....	405	.....	651	.....	408
4	N. 61° W.	713	346	.....	.....	624	348	.....	.....	627
		2,955	1,001	1,010	1,041	1,029	1,006	1,006	1,035	1,035

After the latitudes and departures have been corrected, it is necessary to make a drawing of the plot. How this is done will be readily seen by the accompanying illustration of this survey (Fig. 195). This figure also illustrates a very convenient and accurate method of determining the area of the survey mathematically. By means of the latitudes and departures the area of the full parallelogram is taken, and the triangles on the four sides of the plot are deducted from it, leaving 489,245 square feet as the area of the figure.

The use of the compass is now confined to the surveying of land areas of large extent and little value, or as a means of checking the bearings as taken by the theodolite or transit. Forcing should not be attempted with the latter instruments. If the survey does not balance almost exactly, it should be resurveyed; otherwise it could not be used in a court of law except as approximately correct. The steel tape divided decimally is almost exclusively used in accurate work.

The system of plotting by traverse is the same for the survey by transit as

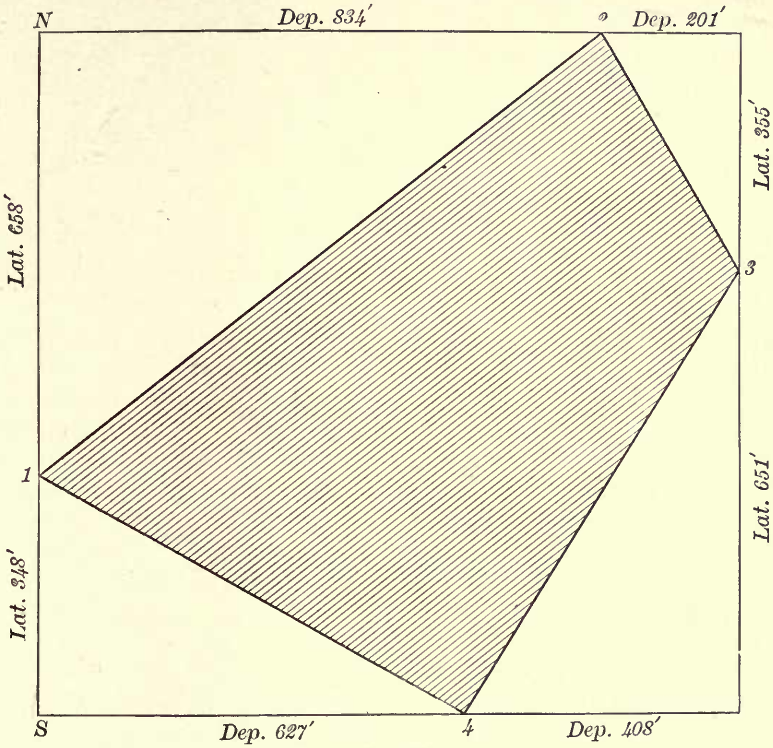


Fig. 195.

by compass, except as the angles are taken to minutes and seconds. The latitudes and departures are to be taken by logarithmic sines and cosines; if these are not obtainable, the table for natural sines and cosines in the appendix will give minutes, and seconds can be obtained by interpolation of differences.

Fig. 196 is the plot of a survey by transit. Any side of the plot may be assumed as a meridian. The bearings are always taken to the right of it. After it has been plotted it can be checked by the traverse. Meridians described through each of the angles will show the meridional angle. The latitudes and departures may be obtained as follows:

Angle.	Nat. sine.		Distance.		Departure.
41° 40′	0·66480	×	350	=	233
Angle.	**Nat. cosine.**		**Distance.**		**Latitude.**
41° 40′	0·74703	×	350	=	261

In the third line of the third column the angle, instead of 120°, is set down at 30°, because, when an angle is in excess of 90°, 180°, or 270°, the excess is the angle of which sine and cosine are to be found.

There can be no division of distance into latitude and departure when the course is either at right angles to or parallel with the meridian. It is then

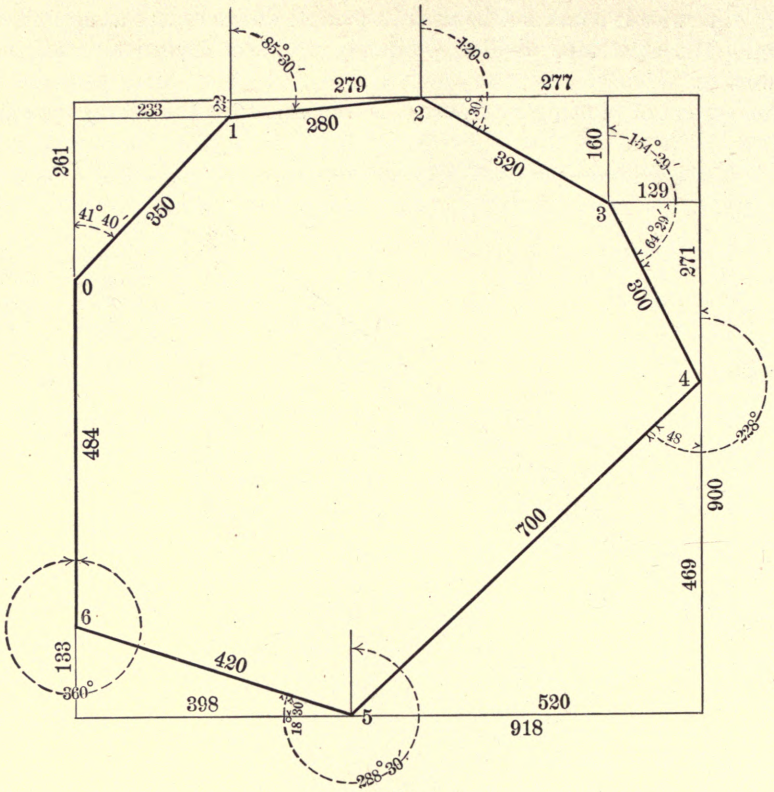


FIG. 196.

either all departure or all latitude. In the tables the latitudes are identical for angles and for their supplements, and so are the departures. Reference to Fig. 75 will illustrate this, calling the sine latitude and the cosine departure.

Course.	Azimuth.	Angle.	Distance.	Latitudes.	Departures.	Triangles deducted.	Rectangles deducted.
0–1	41° 40′	41° 40′	350	261	233	$\frac{261 \times 233}{2} = 30{,}406$	$233 \times 22 = 5{,}126$
1–2	85 30	85 30	280	22	279	$\frac{22 \times 279}{2} = 3{,}069$	
2–3	120 00	30 00	320	−160	277	$\frac{277 \times 160}{2} = 22{,}160$	$129 \times 160 = 20{,}640$
3–4	154 29	64 29	300	−271	129	$\frac{129 \times 271}{2} = 17{,}479$	
4–5	228 00	48 00	700	−469	−520	$\frac{520 \times 469}{2} = 121{,}940$	
5–6	288 30	18 30	420	133	−398	$\frac{133 \times 398}{2} = 26{,}467$	
6–0	360 00	00 00	484	484	000		
						221,521	25,766

Circumscribed rectangle, 918 × 900 = 826,200
Deduct 221,521 + 25,766 = 247,287

578,913 square feet.

When it is difficult to measure along the boundaries of an estate, the surveys should be made along more convenient and accessible lines, which may be by a closed plot, as in Fig. 196, or by base lines from which the intersections of boundaries are established by offsets carefully determined by measures and angles from points of the survey.

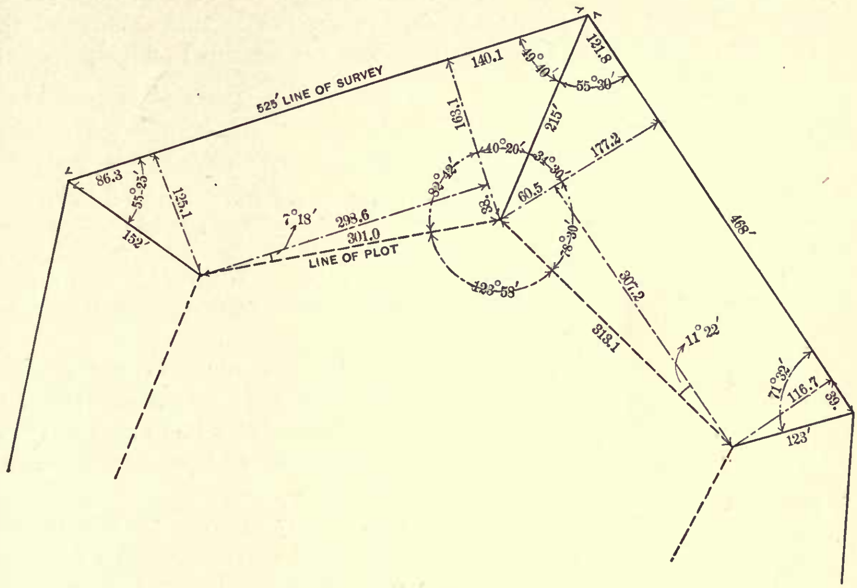


Fig. 197.

The first work of the draughtsman is to complete the plot along the lines of the survey; work up the estimate of contents by traverse, and check by measures on the plot.

Fig. 197 is an illustration.

Find the cosine 49° 40′ distance 215 = 140·1 sine 163·1

" " 55° 25′ " 152 = 86·3 " 125·1

226·4 38

Line 525—226·4 = 298·6

$\sqrt{(298{\cdot}6^2 + 38^2)} = 301{\cdot}0$ the line of plot.

$\frac{38}{298{\cdot}6} = {\cdot}12706 =$ sine of 7° 18′

$90° - 7° \ 18' = 82° \ 42' \quad 90° - 49° \ 40' = 40° \ 20'$.

By similar construction 34° 30′ and 78° 30′ are calculated.

$360° - (82° \ 42' + 40° \ 20' + 34° \ 30' + 78° \ 30') = 123° \ 58'$.

The line of the plot, 301, and the adjacent angle, 123° 58′, are thus obtained, and all lines and angles of the plot are established in the same way. The plot is then transferred to a clean sheet, with lines and angles as calculated, but without any lines of survey.

When the lines of a plot are irregular, as in Fig. 198, divide the plot into equal spaces, and draw parallel lines across the figure through the points of division, add together the two extreme lines, divide the sum by two, and to this dividend add the lengths of the other lines and multiply their sum by the

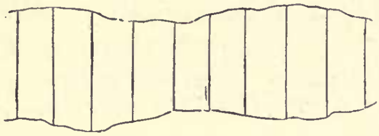


Fig. 198.

equal vertical distance between the parallel lines, which will give very closely the entire area.

Having completed the plot, that is, the main lines of the survey, the filling of other points may in general be done on paper in the same way as they have been established in the field. Intersections of the main lines by roads, streams, fences, and the like are measured off; other points

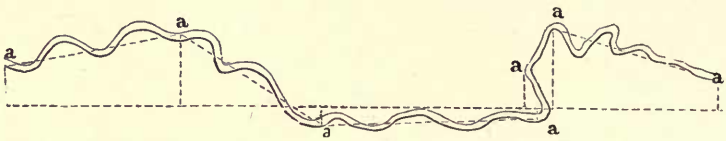


Fig. 199.

not intersecting, are usually fixed by triangles or by *offsets*, or lines run on purpose by angles from the main lines.

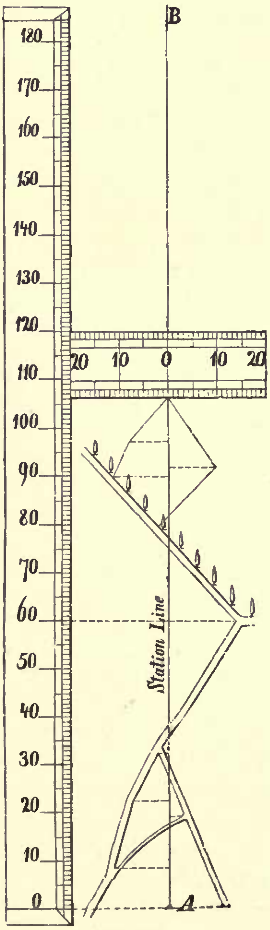


Fig. 200.

In case of unimportant lines, as the crooked brook, for instance (Fig. 199), offsets are taken to the most prominent angles, as a a a, and the intermediate bends are sketched by eye into the field-book, and similarly on the plan.

The most rapid way of plotting the offsets is by the use of a plotting and offset scale (Fig. 200), the one being fixed parallel to the line A B from which the offsets are to be laid off, at such a distance from it, that the zero-line on the movable or offset scale coincides with it, while the zero of its own scale is on a line perpendicular to the position of the station A from which the distances were measured. In the field-book all the offsets are referred to the point of beginning on any one straight line. Move the offset scale to the first distance by the scale at which an offset has been taken, and mark off the length of the offset on its corresponding side of the line; establish thus repeated points, and join the points by lines as they are on the ground. It may not always be possible to obtain the same scales as those of the plan; but they may be made of thick drawing-paper or pasteboard. For extensive plotting, as in government surveys, the offset scale may be made to slide in a groove upon the plotting scale.

In protracting the triangles of an extended trigonometrical survey in which the sides have been calculated or measured, it is better to lay down the triangles from the length of their sides; for ordinary surveys, the triangulation is most expeditiously plotted by the means of a protractor.

The outlines of the survey having been balanced and plotted in, with the subsidiary points, as established by offsets and by triangles, the filling in of the interior detail, with the natural features of the ground, from the skeleton or suggestions in the field-book or other records, is done according to conventional signs, to be shown under "Topographical Drawing."

The public lands of the United States are surveyed, mapped, and divided into nearly square tracts, according to the following system:

Ranges.—Standard lines must first be determined from which to measure. Accordingly, in each land-district some meridian line is run due north and south; this is called the *principal meridian.* From some point of the principal meridian is also run a line due east and west, called the *base line.*

Other lines are then run in the same direction as the principal meridian, at distances of six miles, measured on the base line, on each side of it. The strip between the principal meridian and the first line, thus run east of it, is known as Range 1 East, the second strip as Range 2 East, etc. And so on the west. This division is shown in Fig. 201.

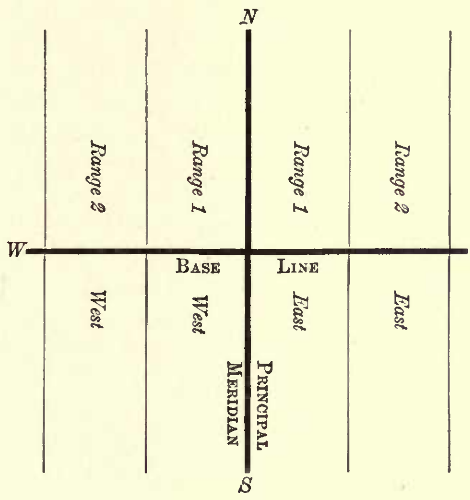


Fig. 201.

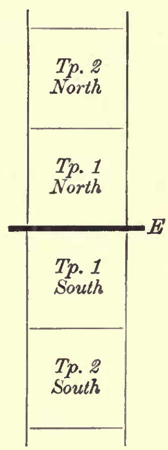


Fig. 202.

Townships.—In like manner, lines are run north and south of the base line at intervals of six miles. These lines cut at right angles those which separate the ranges, and with them form squares six miles on each side, called *townships.* Each township contains thirty-six square miles.

The township nearest the base line on the north is known as *Township 1 North*, of the particular range it is in; the next farther north is *Township 2 North*, of that range, and so on. In like manner, going south from the base line, we have in succession *Township 1 South*, *Township 2 South*, etc. (Fig. 202).

Sections.—Each township is divided into thirty-six squares, called *sections*, each one mile long and one mile wide, and therefore having an area of one square mile. The sections of a township are numbered 1, 2, 3, etc., up to 36, beginning at the northeast, and running alternately from right to left and from left to right, as shown in Fig. 203.

A section may be subdivided into half-sections, quarter-sections, eighths, and sixteenths, designated as in the example that follows:

Let *F G* (Fig. 204) be Section 3 of Township 2 North, in Range 1 West; then—

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

FIG. 203.

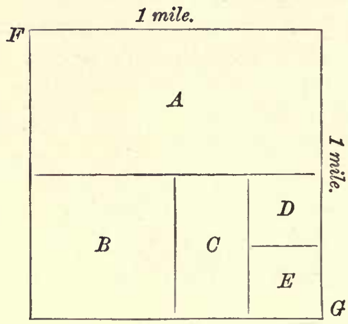


FIG. 204.

A is N. (north) ½ of Section 3, Township 2 North, Range 1 West.

B is S. W. (southwest) ¼ of Section 3, Township 2 North, Range 1 West.

C is W. (west) ½ of S. E. (southeast) ¼ of Section 3, Township 2 North, Range 1 West.

D is N. E. ¼ of S. E. ¼ of Section 3, Township 2 North, Range 1 West.

E is S. E. ¼ of S. E. ¼ of Section 3, Township 2 North, Range 1 West.

Correction Lines.—If the meridian lines were parallel to each other, the townships and sections would be exact squares. But as these lines gradually converge toward the north, meeting at the pole, the townships deviate somewhat from squares, being narrower on the north than on the south; and the northern sections of a township are a little smaller than the southern ones. In order that the townships of a range may not thus keep getting smaller and smaller as we go toward the north, a new base line, called a *correction line*, is taken at intervals, differing in length in different land-districts, and new north-and-south lines are run at distances of six miles measured on the correction lines.

The system of survey described above is not used in Texas, the public lands there being State property.

TOPOGRAPHICAL DRAWING.

TOPOGRAPHICAL DRAWING is the delineation of the surface of a locality, with the natural and artificial objects, as houses, roads, rivers, hills, etc., upon it in their relative dimensions and positions, giving, as it were, a miniature copy of the farm, field, district, etc., as it would be seen by the eye moving over it. Many of the objects thus to be represented can be defined by regular and mathematical lines, but many other objects, from their irregularity of outline, it would be very difficult thus to distinguish; nor are the particular irregularities necessary for the expression. Certain conventional signs have therefore been adopted in general use among draughtsmen, some of which resemble, in some degree, the objects for which they stand, while others are purely conventional. These signs may be expressed by lines, by tints, or by both.

Figs. 205 and 206 represent meadow or grass land, the short lines being

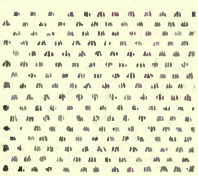


FIG. 205.

FIG. 206.

FIG. 207.

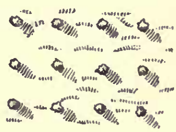


FIG. 208.

supposed to represent tufts of grass; the bases of the tufts should always be parallel to the base of the drawing, whatever may be the shape of the inclosure.

Figs. 207, 208, 209, and 210 give various methods of representing trees. Figs. 207 and 208 represent in plan a forest and an orchard respectively. The

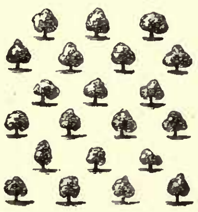


FIG 209.

FIG. 210.

FIG. 211.

method of Figs. 209 and 210, showing the same in elevation, while it is not consonant with the projection of the plan, to many is more expressive and intelligible.

Fig. 211 represents cultivated land. The lines are supposed to represent plough-furrows, and adjacent fields should be distinguished from each other by different inclinations of lines.

Figs. 212 and 213 represent marsh or bog land. Fig. 212 is the more ordinary mode of representing fresh-water bog; Fig. 213, salt marsh.

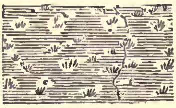


Fig. 212.

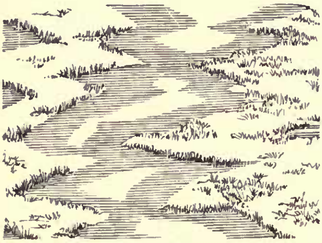


Fig. 213.

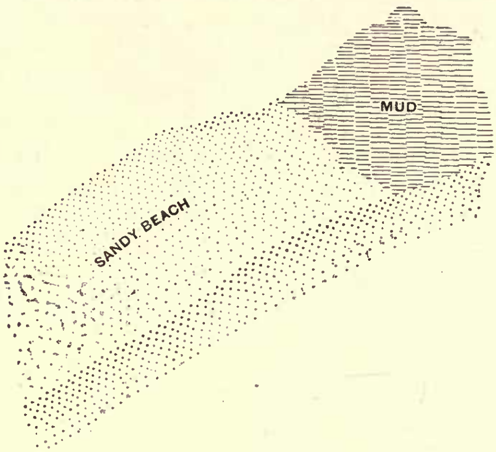


Fig. 214.

Fig. 214 represents a river, with mud and sand banks. Sand is designated by fine dots, made with the point of the pen; mud by a series of short parallel lines. Gravel is represented by coarser dots, and stones by irregular angular forms.

Water is almost invariably represented, except in connection with bogs, by drawing a line parallel to the shore, following its windings and indentations closely, then another parallel a little lighter and a little more distant, a third still more so, and so on. Brooks, and even rivers when the scale is small, are represented by one or two lines. Fig. 215 gives a plan and sectional view of water, in which the white curves represent the character and direction of the

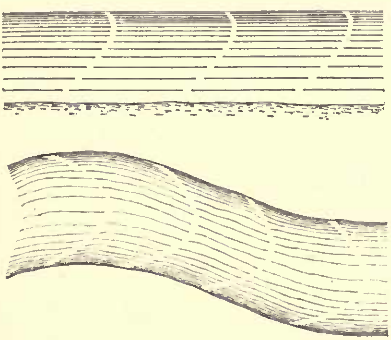


Fig. 215.

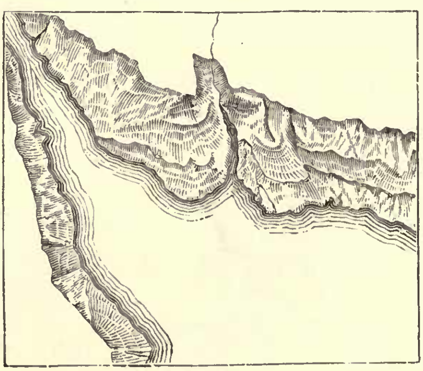


Fig. 216.

flow of streams, retarded at bottom and sides, and more rapid near the surface and at centre. The direction of the current may also be shown by arrows.

Fig. 216 represents a bold shore bounded by cliffs.

Fig. 217 represents a turnpike. If the toll-bar and marks for a gate be omitted, it is a common highway. Fig. 218 represents a road as sunk or cut through a hill; Fig. 219, one raised upon an embankment. Fig. 220 is a rail-

Fig. 217.

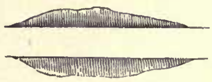


Fig. 218.

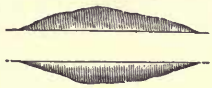


Fig. 219.

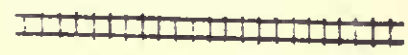


Fig. 220.

road, often represented without the cross-ties by two heavy parallel lines, sometimes by but one.

Fig. 221 represents a bridge with a single pier; Fig. 222, a swing or draw bridge; Fig. 223, a suspension bridge; and Fig. 224, a ford. Fig. 225 is a lock

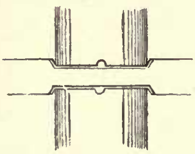


Fig. 221.

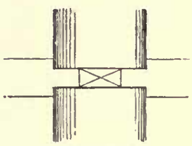


Fig. 222.

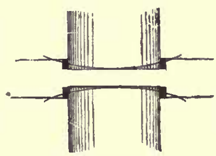


Fig. 223.

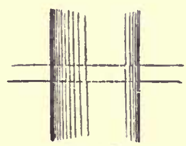
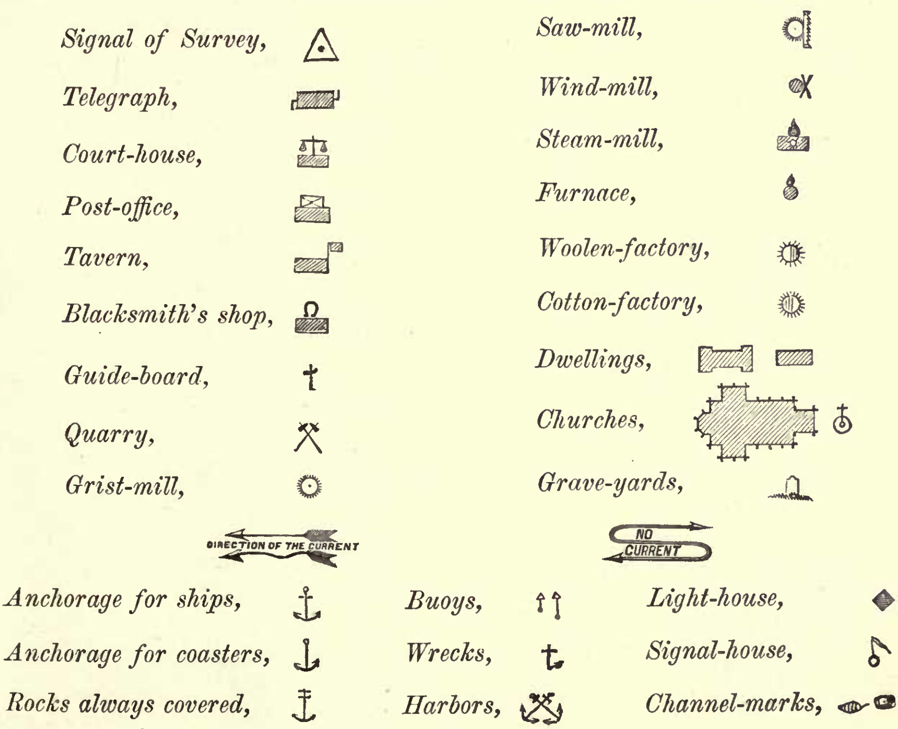


Fig. 224.

of a canal. Canals may be represented like roads, except that in the latter the side from the light is the shaded line; in the former, the side to the light. Or by 

Fig. 225.

Conventional signs for the more important objects that are likely to need representation on a map are:



The localities of mines may be represented by the signs of the planets, which were anciently associated with the various metals, and a black circle is used for coal, thus, ☿ Mercury, ♀ Copper, ♄ Lead, ☽ Silver, ☉ Gold, ♂ Iron, ♃ Tin, ● Coal.

The Representation of Hills.—The two methods in general use for representing with pen or pencil the slopes of ground are known as the vertical and the horizontal. In the former (Fig. 226), the strokes of the pen follow the course that water would take in running down these slopes. In the second (Fig. 227), they represent horizontal lines traced round them, such as would be shown on the ground by water rising progressively by stages, 1, 2, 3, 4, 5, 6, up the hill. The last is the more correct representation of the general character and features of the ground, and, when vertical levels or contours have been traced by level at equal vertical distances over the surface of the ground, they should be so represented; or when, by any lines of levels, these contours can be traced on the plans with accuracy, the horizontal system should be adopted: but where, as in most plans, the hills are but sketched in by the eye, the vertical system should be adopted; it affords but proximate data to judge of the slope, whereas, by the contour system, the slope may be measured exactly. It is a good maxim in topographical drawing not to represent as accurate anything which has not been rigorously established by surveys. On this account, for general plans, when the surface of the ground has not been levelled, nor is required to be determined with mathematical precision, use the vertical system of representing slopes.

On drawing hills on the vertical system, it is very common to draw contour-lines in pencil as guides for the vertical strokes. If the horizontal lines be traced at fixed vertical intervals, and vertical strokes be drawn between them in the line of quickest descent, they supply a sufficiently accurate representation of the face of the country for ordinary purposes. It is usual to make the vertical strokes heavier the steeper the inclination, and systems have been proposed and used by which the inclination is defined by the comparative thickness of the line and of the intervening spaces.

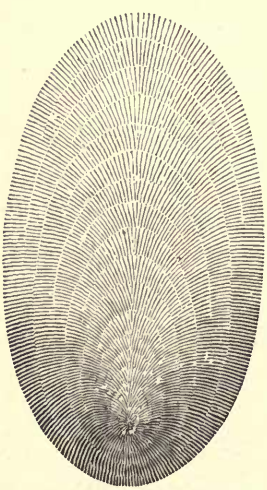


Fig. 226.

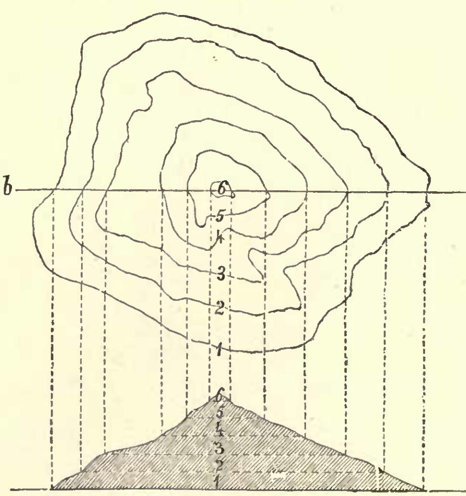


Fig. 227.

In describing ground with the pen, the light is generally supposed to descend in vertical rays, and the illumination received by each slope is diminished in proportion to its divergence from the plane of the horizon. Thus, in Fig. 228, it will be seen that a horizontal surface receives an equal portion of light with the inclined surface resting upon it, and, as the inclined surface is of greater extent, it will be darker than the horizontal in proportion to the inclination and consequent increase of the surface, and on this principle varied forms of ground are represented by proportioning the thickness of stroke to the steepness of the slope.

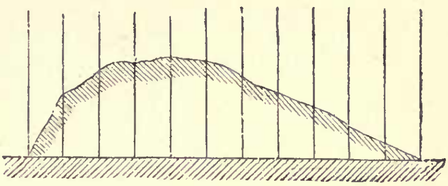


Fig. 228.

In the German system proposed by Major Lehmann for representing the slopes of ground by a scale of shade, the slope, at an angle of 45°, is indicated by black, the horizontal plane by white.

A modification of Lehmann's method, proposed by the United States Coast

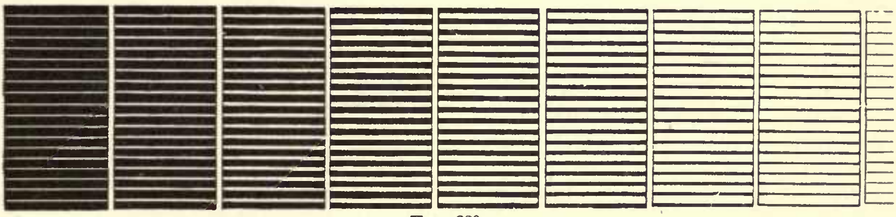
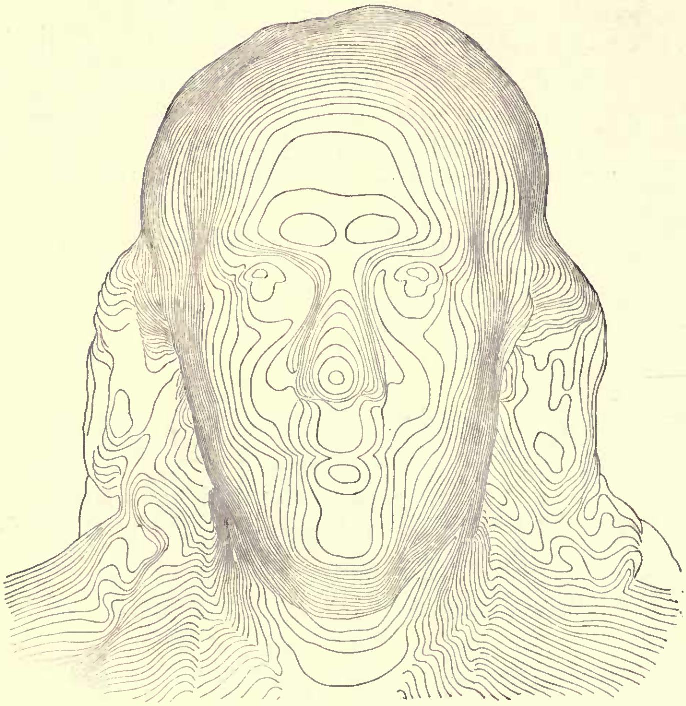


Fig. 229.

Survey, has the advantage of discriminating between slopes of greater inclination than 45°. The table gives the proportions of black and white for different inclinations, and the construction may easily be understood from Fig. 229.

Slope.	Proportion of Black.	Proportion of White.
2½° or 2¾°	1	10
5 or 6	2	9
10 or 11	3	8
15 or 18	4	7
25 or 26	5	6
35	6	5
45	7	4
60	8	3
75	9	2

Contour-Lines. — Conceive a hill to be completely covered with water. Then suppose the water to be drawn down, say five feet at a time. Each line of contact of the hill and the water will be a *contour-line*, or a line every point of which is at the same height or level above a fixed horizontal plane, called the *datum-plane*. For a small hill, stake out the ground in squares of say fifty feet to the side, and take levels at each point of these squares, and as many intermediates as the change of slope makes necessary. To draw the map, lay off these squares to a scale, and mark the elevation of each point and the intermediates in pencil. Then by the eye draw in the contours at such vertical distances apart as the requirements of the map call for. For a large survey, say of a mountain, such a method is impracticable. In this case, the surveyor fixes a number of points at the same level, the points being absolutely established by the transit or compass, so that they can be plotted accurately. Connect all points on each level, and fill in the distances between by the eye, on the supposition that the slope is uniform between these lines. The lines absolutely established and those merely sketched in must not be confounded, and should be distinguished apart either by colour, by size of lines, or by dotting. The contour-lines denoting every five, ten, etc., feet above the datum or plane of reference may be numbered with such height. This is an effective way of rep-



resenting hills, but is only to be recommended when lines have been traced and it becomes a record of facts. Fig. 230 represents, on double the scale, the

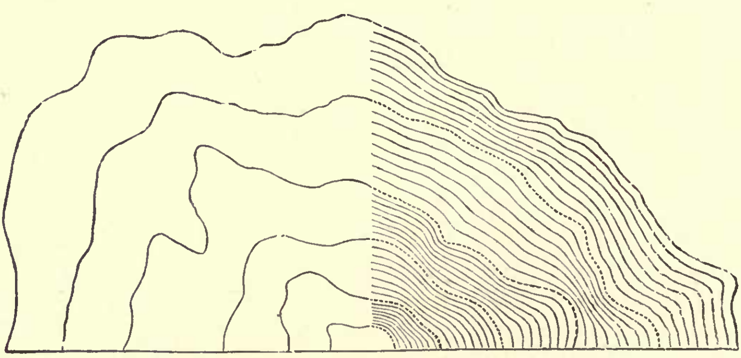


FIG. 230.

half of the hill (Fig. 227), with one half completed by drawing the intermediate contour-lines.

The objection to the drawing of hills by any system is that the depths of shade representing different slopes conflict with the lights and shades of the drawing, and are therefore confusing. The plan adopted by Von Eggloffstein in his maps was to form a model by cutting out of sheet-wax under the needle of a sewing machine, on the lines of contours, and then properly superimposing them on one another. A mould was then taken from them in plaster. A

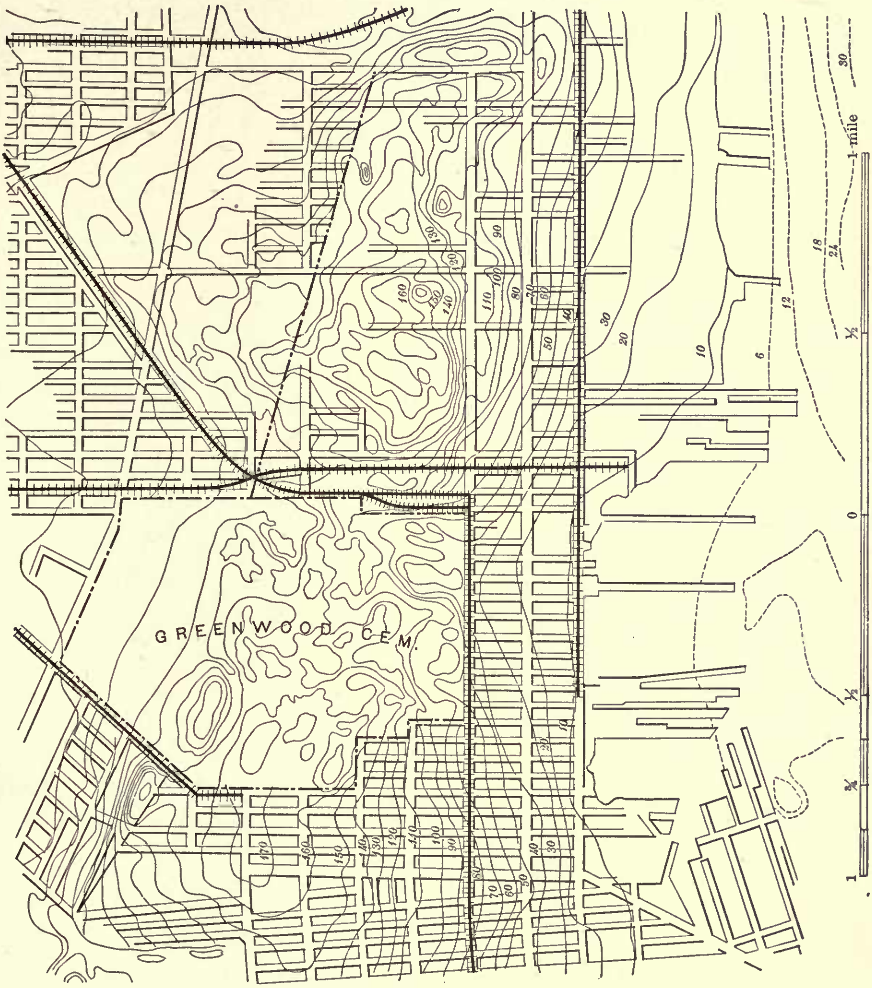


Fig. 231.

model from the mould, also in plaster, was then taken. This was watered while fresh by a vertical rain from a water-pot, which broke down the vertical edge of the contours, and gave natural lines of watershed. This model was then photographed under an inclined light, and gave an admirable projection.

Fig. 231 is a contoured map of Greenwood Cemetery and vicinity, Brooklyn, N. Y.

Fig. 232 is a map of the harbour and city of New Haven, reduced from the charts of the United States Coast Survey.

Plate VI is a map of a farming country. These two maps illustrate the practical applications of topographical conventionalities.

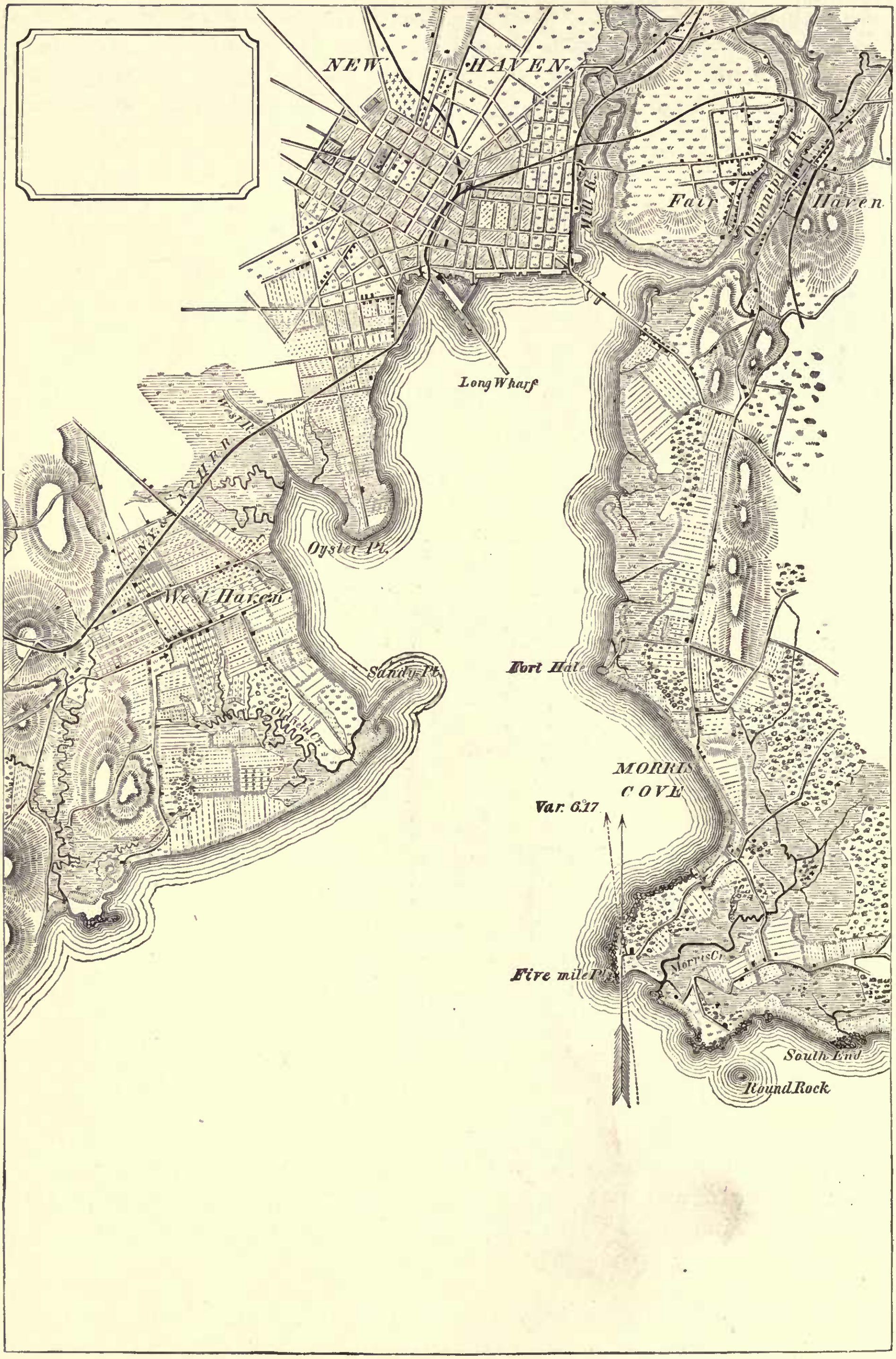

NEW HAVEN.
Fair Haven
Mill R.
Quinnipiac R.
Long Wharf
Oyster Pt.
West Haven
N.Y. & N.H.R.R.
Sandy Pt.
Fort Hale
MORRIS COVE
Var. 6.17
Five mile Pt.
Morris Cr.
South End
Round Rock

FIG. 232.

Railway Surveys are usually plotted by tangents. The curves are then put in, and the topographical features for the width necessary. The curves are designated by degrees, as a curve of 1°, 2°, 3°, etc., according as the angle subtended at the centre by a 100-feet chord is 1°, 2°, 3°, etc.

Degree.	Radii, ft.	Central ordinate.
1°	5729·65	0·218
2	2864·93	0·436
3	1910·08	0·655
4	1432·69	0·873
5	1146·28	1·091
6	955·37	1·309
7	819·02	1·528
8	716·78	1·746
9	637·27	1·965
10	573·69	2·183

Knowing the tangent points, it is easy to plot in the curve, as the centre of the curve must be the intersection of the perpendiculars to the tangents at these points; or, with one point of tangency, erect a perpendicular at this point, and lay off the radius on it to get the centre of the curve.

When the curves are larger than can be described by the dividers or beam compasses, they can be plotted as shown in geometrical problems, or points of a curve may be obtained by calculation of their ordinates, and the curves drawn from point to point by variable curves. Knowing the central ordinate of the curve between two points, the central ordinate of one half that curve will be approximately one quarter of the first; but the greater the number of degrees in the arc, the less near to the truth is the rule.

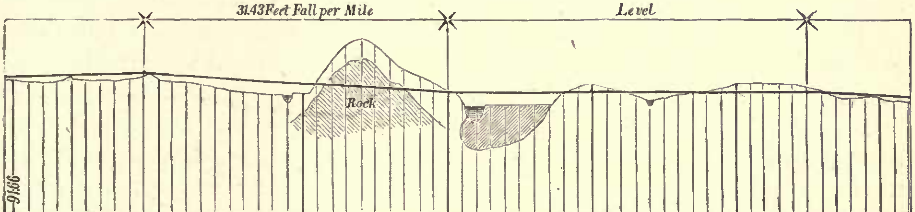


FIG. 233.

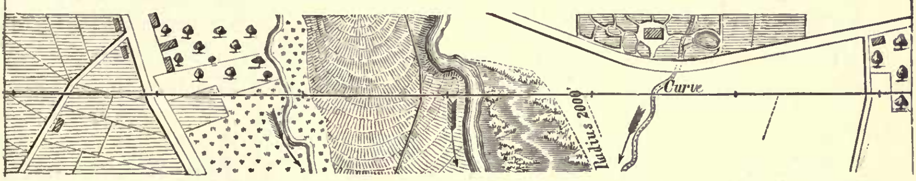


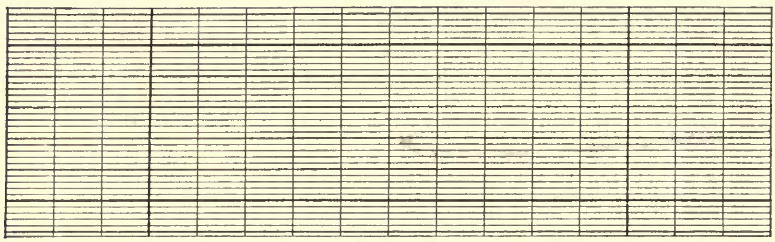
FIG. 234.

Fig. 234 represents a plot of a railway line; in this plot the curve is represented as a straight line, the radius of curvature being written in. This method is sometimes adopted when it is desirable to confine the plot within a limited space upon the sheet, and it is convenient plotted thus directly beneath the profile or longitudinal section (Fig. 233).

In plotting the section, a horizontal or base line is drawn, on which are laid off the stations or distances at which levels have been taken; at these points perpendiculars, or ordinates, are erected; upon them are marked the heights of the ground above the base; and the marks are joined by straight lines.

Where borings or soundings have been made, and it is necessary to indicate the character and define the limit of the material, the rock may be shown by diagonal hatchuring, streams as in Figs. 222 and 223, and other substances by a combination of lines and dots, resembling as nearly as possible the material which it is to represent, and the name inserted. If there is a bog or mud in which soundings have been made, the position and depth of soundings should be given; but when work is to be done by contract, characteristics, unless well established, should not be definitely marked.

Since it would be in general impossible to express the variations of the surface of the ground in the same scale as that adopted for the plan, it is customary to make the vertical scale larger than the horizontal, usually in the proportion of 10 or 20 to 1. Thus, if the horizontal scale of the plan be 400



feet to the inch, the vertical scale would be 40 or 20 feet to the inch. For the purpose of facilitating the plotting of profiles, profile-paper can be obtained from stationers, on which are printed horizontal and vertical lines.

In the plotting of sections across the line which are extended but little beyond the line of the cut or embankment, equal vertical and horizontal scales are adopted; these plots are mostly to determine the position of the slope, or to assist in calculating the excavation. When cross sections are extended to show the grade of cross-road, or changes of level at considerable distance from the line of rail, the same scales, vertical and horizontal, are adopted as in the longitudinal section or profile.

In Fig. 233 the upper or heavy line represents the line of the rail, the grades being written above; this is the more usual way, but sometimes, as in Fig. 235, the profile and plan are combined; that is, the heights and depths above and below the grade line of the road are transferred to the plan, and re-

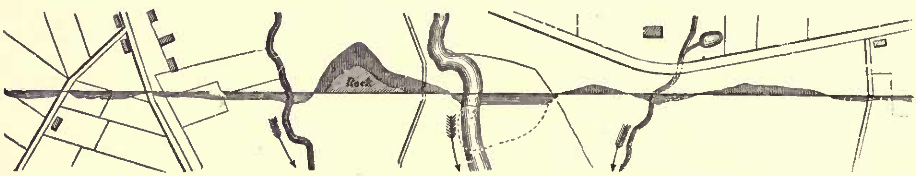


Fig. 235.

ferred to the line in plan, which becomes thus a representation both in plan and elevation.

Cross sections, for grades of cross-roads, etc., are usually plotted beneath or above the profile or across the line when plan and profile are combined.

Besides the complete plans, as above, giving the details of the location, land plans are required, showing the position and direction of all lines of fences and boundaries of estates, with but very few of the topographical features. The centre line of the road is represented in bold line, and at each side, often in red, are represented the boundaries required for the purposes of way. In general, a width of 100 feet is the amount of land set off, lines parallel to the central line being at a distance of 50 feet on each side; but when, owing to the depth of the cut or embankment, the slopes run out beyond this limit, the extent is determined by plotting a cross section and transferring the distances thus found to the plan, and inclosing all such points somewhat within the limits as set off for railway purposes. These plans are generally filed in the register's office for the county through which the line passes.

Hydrometrical or Marine Surveys.—In plotting hydrometrical or marine surveys, the depths of soundings are seldom expressed by sections, but by figures written on the plan, expressing the sounding or depth below a datum

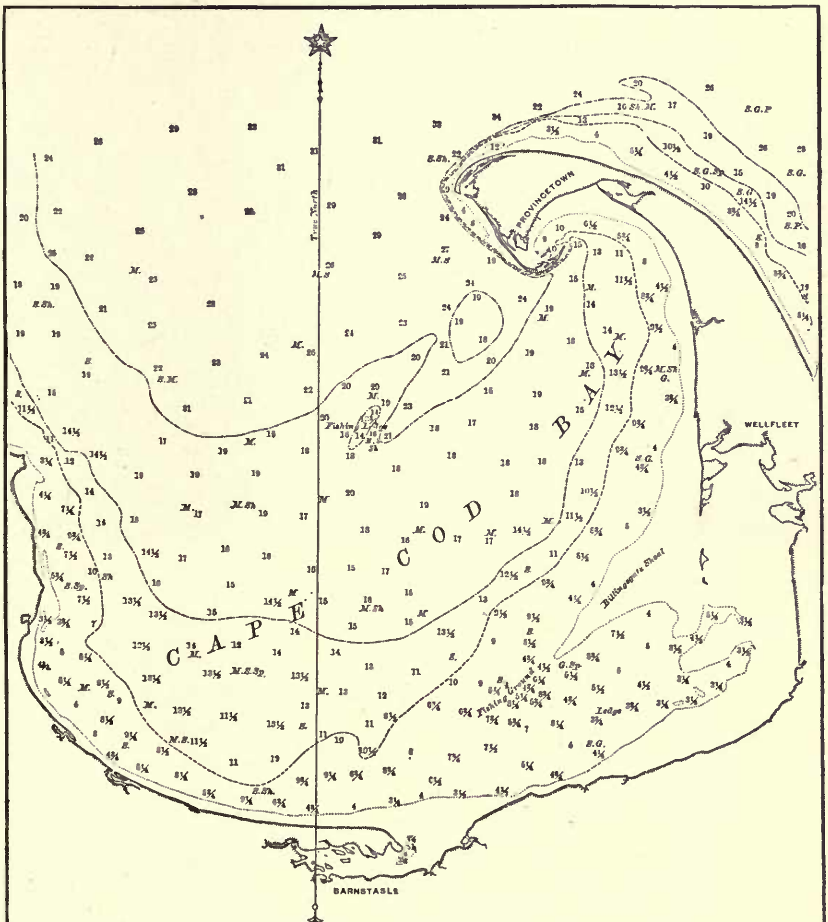


FIG. 236.

line, generally that of high water, the low-water line being usually represented by a single continued line. The soundings are expressed in fathoms or in feet. Fig. 236 is a map of Cape Cod Bay plotted by this method. The depths are expressed in feet, and the dotted lines are contour-lines or lines of equal depths.

An effective way of making a marine chart is to express the different depths by lines varying in direction, distance apart, width, etc. Fig. 237 is a chart of the Isle of Wight and the surrounding water, with the depths expressed as shown at the bottom of the cut. Sections are often used for rivers, especially rivers like those of the West, that have a very changeable bottom. By plotting sections, taken at different times, over one another, distinguishing

Fig. 237.

them apart by a difference in colour and variety of line, the changes that take place in the bottom of the river, and the erosion of the banks, are boldly shown.

In a geological profile the different rocks or formations are sometimes distinguished by colours, explained by marginal notes and squares, but more often by marks, dots, or cross-hatchings.

The geological and statistical features of a country may be expressed similarly and graphically in lines, as in Fig. 238, a preliminary survey of Kentucky, illustrating the principal geological features; and in Fig. 239, in a broader form, giving a larger extent of country, including the portion of the United States east of the Rocky Mountains and the southeasterly portion of Canada.

These maps give the larger geological divisions, and are suited for books, but are not as effective nor comprehensive of the smaller subdivisions, of which Plate X is an example of a portion of a map of the State of New Jersey.

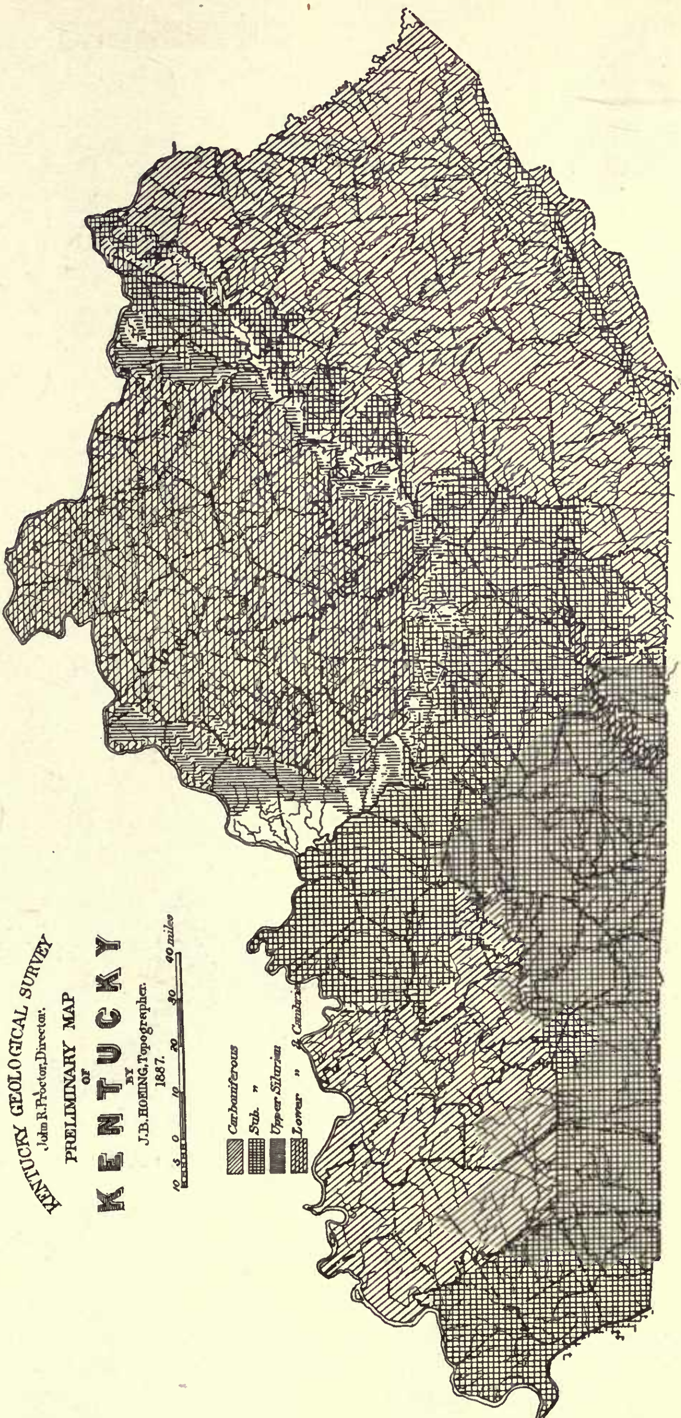

KENTUCKY GEOLOGICAL SURVEY
John R. Procter, Director.
PRELIMINARY MAP
OF
KENTUCKY
BY
J.B. HOEING, Topographer.
1887.
10 5 0 10 20 30 40 miles
Carboniferous
Sub. "
Upper Silurian
Lower " & Cambrian

FIG. 238.

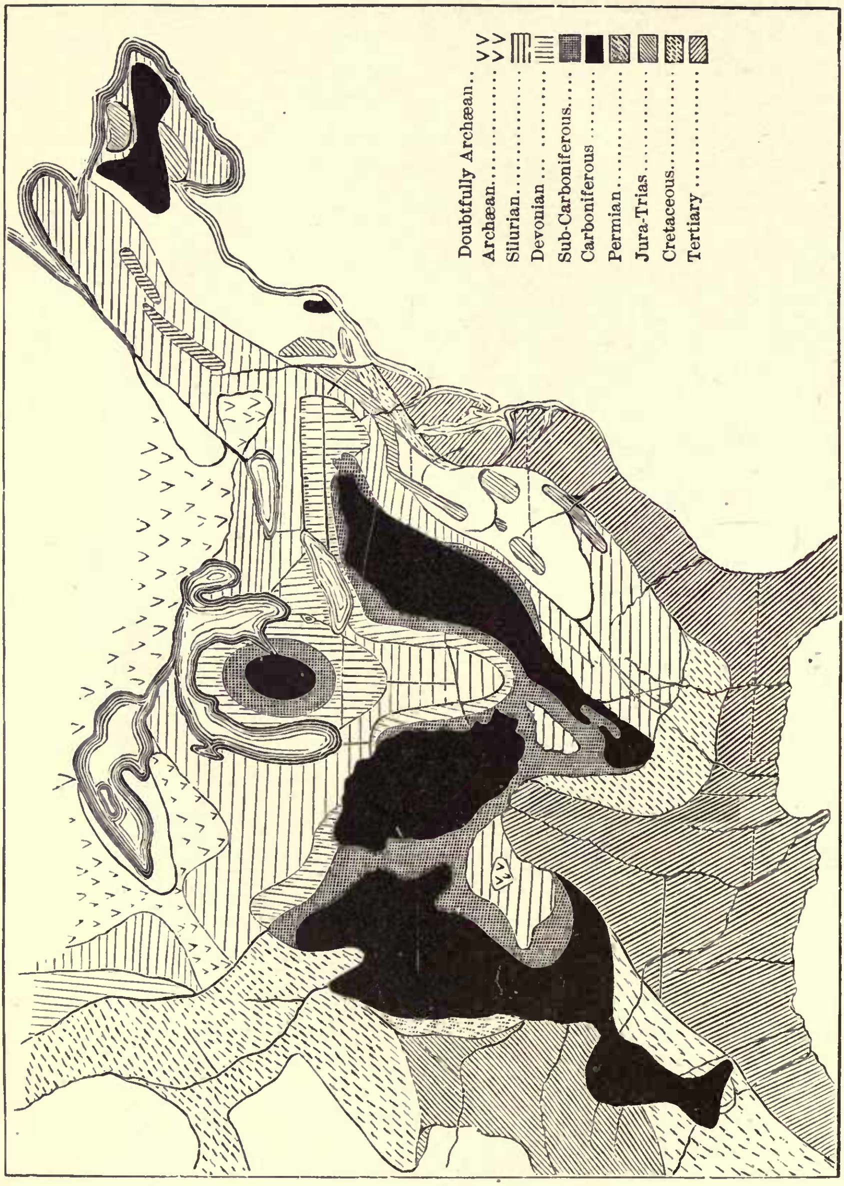
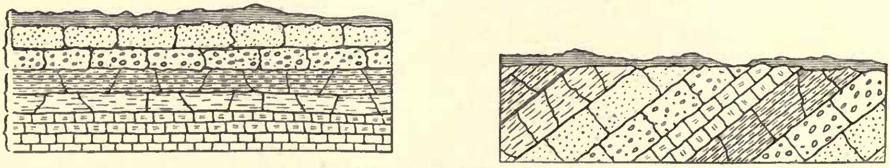


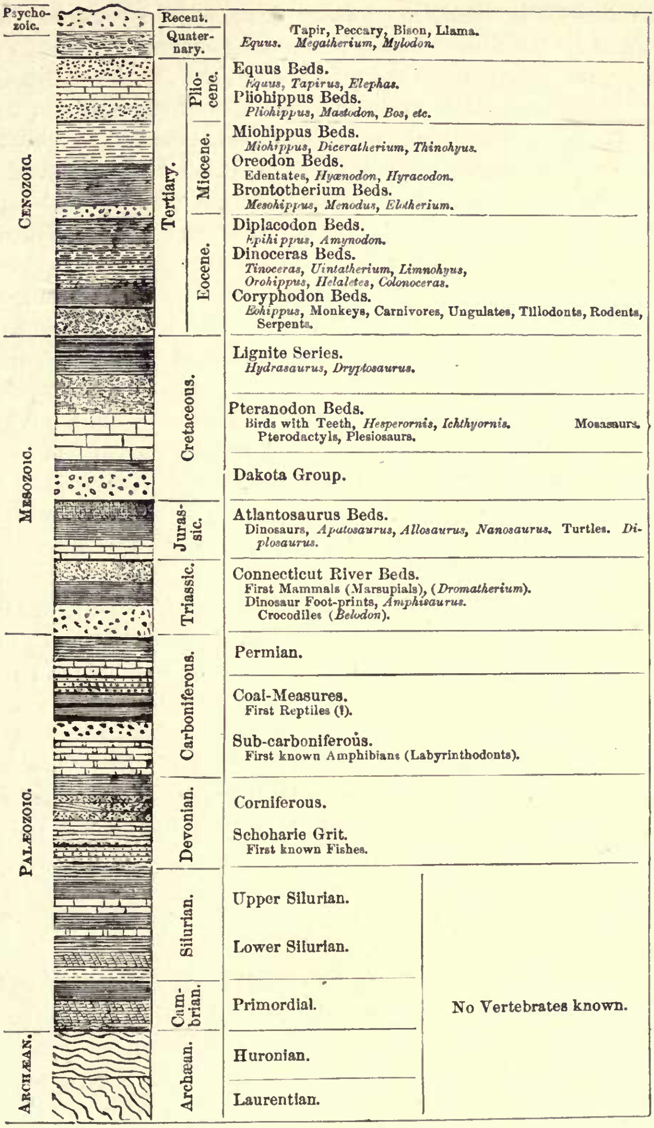
FIG. 239.

STRATIFIED ROCKS



Sections of horizontal and inclined strata.

FIG. 240.



Section of the earth's crust to illustrate vertebrate life in America.

FIG. 241.

Fig. 241 is an ideal section from Le Conte, in which all the most important American strata occurring in different places are brought together and arranged in the order of time.

In the diagram the different rock-systems are placed one on top of the other, and the vertical black spaces represent by their breadth the relative dominance of different classes at different times.

The subdivisions of these again into periods and epochs are founded on more local unconformities, and especially on less important changes in the species.

Transferring.—It is usual, in plotting from a field-book, to make first a rough draft, and then a finished copy on another sheet.

In the copy, only the established points and lines for the outlines are to be transferred. Many lines of construction, balances of surveys, and trial lines of the rough draft are to be omitted, and it is well to sketch in roughly the natural features on the rough draft for aid in the completion of the finished copy.

The most common way of transferring, for a fair copy, is by superposition of the plan above the sheet intended for the copy, and pricking through every intersection of lines on the plan and all points necessary to preserve. The clean paper should be laid and fastened smoothly on the drawing-board; the rough draft should be laid on smoothly, and retained in its position by weights, glue, or tacks. The needle must be held perpendicular to the surface of the plan, and pressed through both sheets; begin at one side and work with system, so as to prick through each point but once, nor omit any; make the important points a trifle the larger. For the irregular curves, as of rivers, make frequent points, but very small ones. On removing the plan, select the important points, those defining leading lines; draw in these, and the other points will be easily recognised from their relative position to these lines. When any point has not been pricked through, its place may be determined by taking any two established points adjacent to the one required, and with radii equal to their distance on the plan from the point required, describing arcs on the copy on the same side of both points; the arcs will intersect at the point desired. In this way, as in a trigonometrical survey, having established the two extremes of a base, a whole plan may be copied. In extensive drawings it is very common to prick off but a few of the salient points, and fill in by intersections, as above, or by copying detached portions on tracing-paper and transferring them to the copy; the position of each sketch being determined by the points pricked off, the transfer is made by pricking through, as above, or by transfer-paper placed between the tracing and the copy.

Plans may be copied, on a reduced or enlarged scale, by means of the pantagraph or by the method of squares.

Map Projections.—For a farm or other small survey, the surface of the earth is conceived to be flat, and the map as a horizontal projection of the plane surface on a reduced scale. But for large maps, as of countries, States, rivers, or the like, where the meridians and parallels of latitude are represented, such a system would be erroneous. The surface of the earth being a sphere, it is incapable of development on a plane, so that it becomes necessary to make the best approximation possible in form, relation, and proportional area of the portions to be represented on a map or chart. There are many different kinds of projection, all more or less imperfect.

In general, map projections may be divided into two classes: perspective and developments. The most useful of the perspective projections are the *orthographic*, the *stereographic*, and the *globular* or *equidistant*.

Orthographic Projection (Figs. 242 and 243).—In this projection the eye is supposed to be at an infinite distance, and the plane of projection perpendicular to the line of sight. Let the circle A L A′ M (Fig. 242) represent a hemisphere; draw the diameters A A′ and L M at right angles; divide the arc A L and A M into nine parts, making the parallels thus 10° apart; and through these divisions draw lines across the sphere parallel to A A′; mark off consecutively on a strip of paper the points where the parallels intersect the central meridian L M, and apply and mark off these points along the equatorial diameter A A′. Draw meridians with centres on the extension of the line of the equator intersecting the points marked off on this line, and meeting at the poles.

Stereographic Projection (Figs. 244 and 245).—As in the last example, draw a circle with a central and an equatorial diameter (Fig. 244); on the central meridian describe a circle O E D F, with a diameter equal to the radius of the

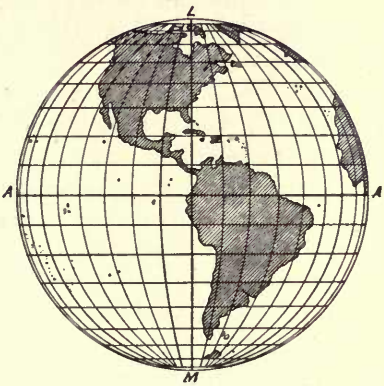


FIG. 242.

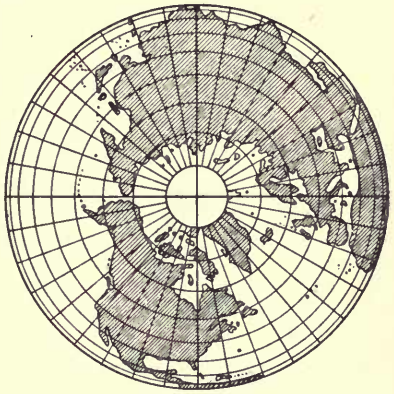


FIG. 243.

larger circle; through the centre of this smaller circle draw a diameter E F, and divide the arcs E O and O F into nine equal parts; and from the pole D, through each of these points, draw lines intersecting the equator. These lines divide the equator into eighteen parts, each containing 10° of longitude, and through these points and the poles meridians are described. Mark off the equatorial divisions on a slip of paper and transfer them to the central meridian;

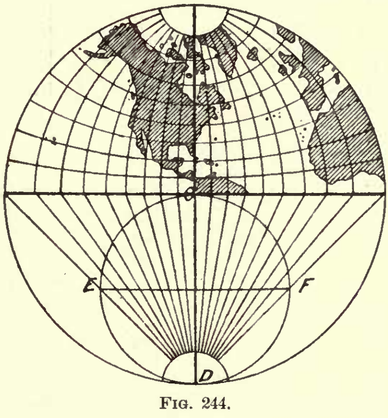


FIG. 244.

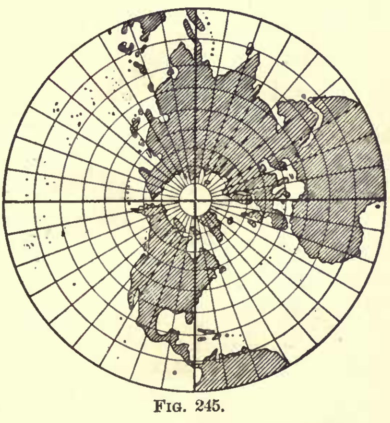


FIG. 245.

divide each quarter of the circumference of the sphere into nine equal parts, though these points and the divisions on the central meridian describe arcs.

Globular or Equidistant Projection (Fig. 246).—A good practical projection can be made by drawing a circle with a diameter for the equator and another at right angles for the central meridian, dividing each quadrant and each radius into nine equal parts. Meridians, with centres on the line of the equatorial diameter, can now be drawn passing through the equatorial divisions, and the poles and parallels of latitude on the line of the central meridian through the divisions of this line and those of the quadrant.

The foregoing perspective projections in their application to astronomy and geography are usually confined to the representation of the hemisphere, and but rarely to smaller surfaces; they are therefore of but little practical use to the engineer or surveyor, and for land and sea charts on a large scale and of limited extent they are not well adapted. To render developments possible a cylindric or conic surface is substituted in place of the ordinary plane of projection, which surface is afterward developed in a plane. The eye is supposed either at the centre of the sphere or else its position is arbitrary. This conception gives rise to two kinds of developed projections, one kind employing a cylinder, tangent generally at the equator, the other employing a cone, tangent generally at the middle parallel.

FIG. 246.

The more useful of these projections are Mercator's, the conic, Bonne's, and the polyconic.

Mercator's Projection (Fig. 247).—This is especially valuable to the navi-

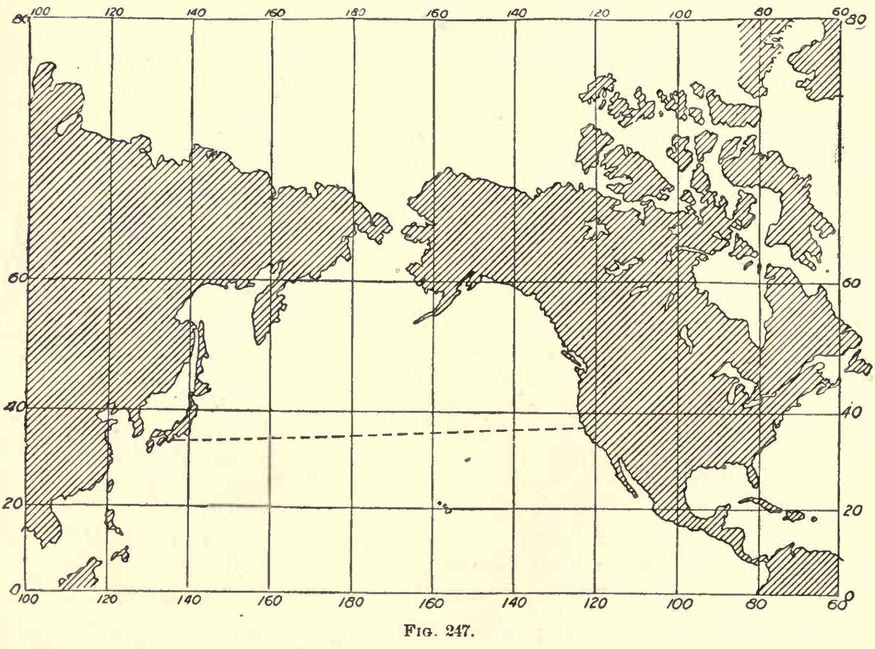


FIG. 247.

gator, as by it he can lay off his course on a straight line. In this projection the surface of the sphere is developed on a tangent cylinder. Conceive the outlines of the continents to be drawn on this, and afterward the surface to be unrolled and laid flat; the result is a chart on Mercator's projection.

Let P Q P′ R (Fig. 248) be the projection of a sphere, P P′ and R Q its diameters, and U Y X Z a portion of the tangent cylinder in the axis of which

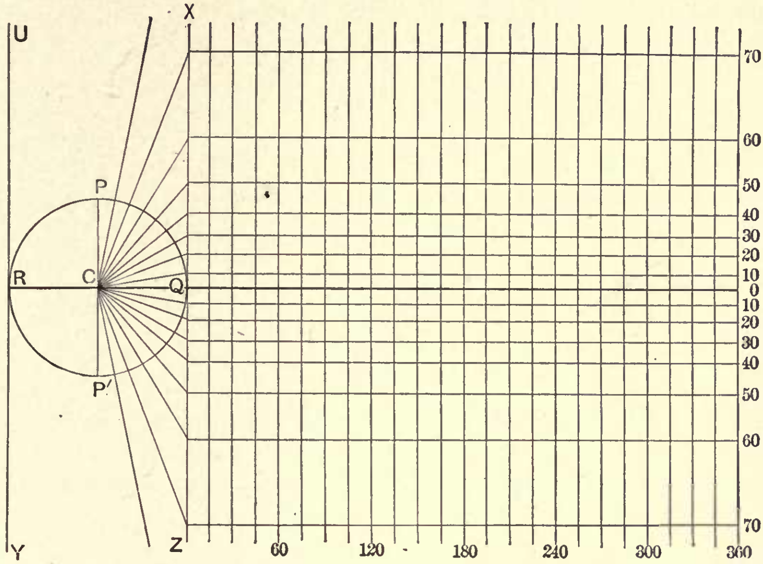


FIG. 248.

is contained the axis of the sphere. Divide the quadrant P Q into 10° parts, a line drawn from C through these points, intersecting the tangent X Z, will be the point for the parallel of that degree. Make the equator of the projection 3·14 times the diameter of the cylinder, and divide it into twenty-four equal parts; these points will be 15° apart, or hour distances. Verticals are drawn through these points, giving us parallels of latitude.

Conic Projection.—Instead of on the surface of a sphere, the map is projected on the surface of a cone. The projection may be developed either on a tangent cone or an intersecting cone. The developed arc of the middle latitude is employed for the graduation of longitudes. Fig. 249 gives an illustration of the development on a tangent cone in which P O B is the sphere and A M is the distance from the apex of the cone to the middle parallel and point of contact; A M will be the radius for the central parallel and A the point from which all parallels are described. Fig. 250 shows the cone developed on a plane surface. The length of a degree on a meridian of the earth is 69·41 miles at the poles and 68·70 at the equator.

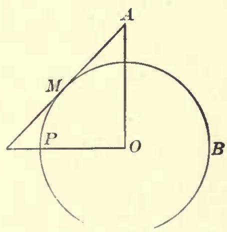


FIG. 249.

Bonne's Projection.—In this projection a central meridian and a central parallel are employed, the latter being the development of the circle of contact of the tangent cone. The other parallels are concentric arcs, as in the simple conic projection drawn through the graduation of the central meridian. Each parallel, however, is divided in accordance with the varying lengths of a degree of longitude in the different latitudes (see table) and an arc passed through

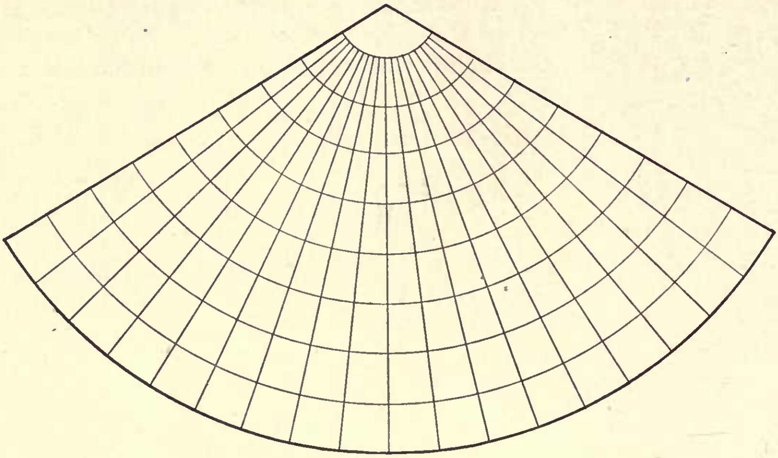


FIG. 250.

these points. The map is slightly distorted at the corners on account of the parallels, as projected, being concentric arcs. The great advantage of Bonne's projection is that true proportions of areas are preserved. This method is al-

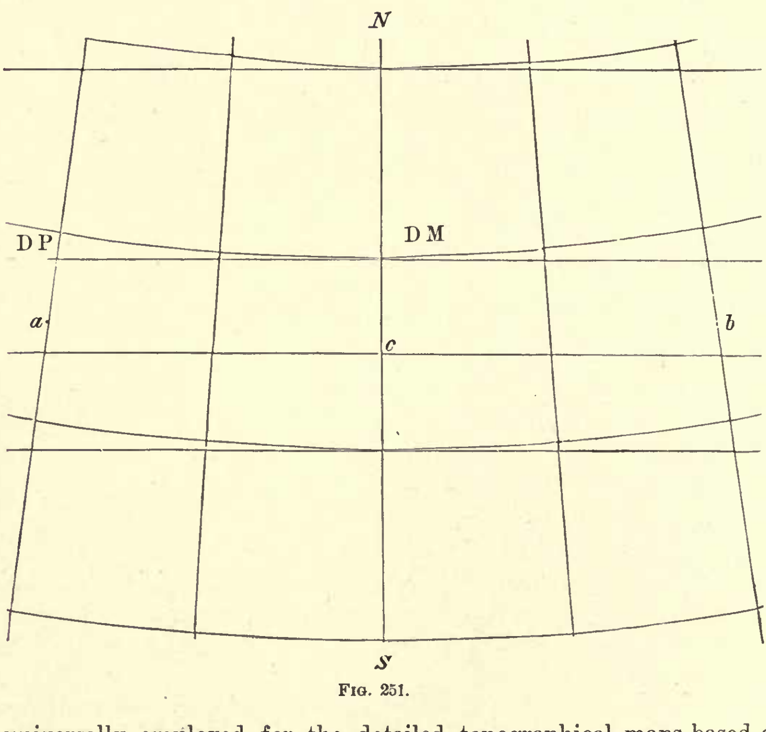


FIG. 251.

most universally employed for the detailed topographical maps based on the trigonometrical surveys of the different countries of Europe.

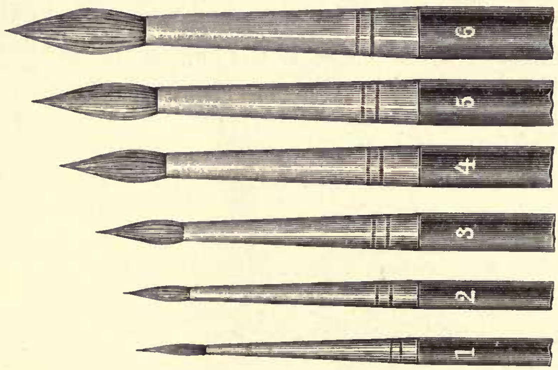
Polyconic Projection.—This employs a tangent cone for every parallel. Each parallel of latitude therefore is independently developed. This has the effect of increasing the length of the degrees of latitude in proportion as we recede from the central meridian. To draw a map according to the tables, lay off (Fig. 251) on the straight line N S, representing the middle meridian, the lengths representing the 10° of latitude between 20°, 30°, 40°, etc. Through these points draw circular arcs with the radii designated by R in the table. On these arcs lay off the lengths of 10° of longitude for each corresponding 10° of latitude on each side of the central meridian. Through the points thus formed draw the meridians, which will be found slightly concave toward the middle one. If the scale is so large that it is impossible to draw the circular arcs with beam compasses, erect perpendiculars at the points 20°, 30°, 40°, and 50°, and on them lay off the values D M from the tables. At each of the points so found erect perpendiculars, and set off on them the corresponding values of D P. Through the points thus found draw the parallels and meridians. By this projection there is little distortion at any portion of the map; a scale of degrees and minutes of the parallels and meridians, by means of which positions, determined by their latitudes and longitudes, may be inserted in the maps; the use of a linear scale in any portion or direction; and the intersection of parallels and meridians at nearly right angles.

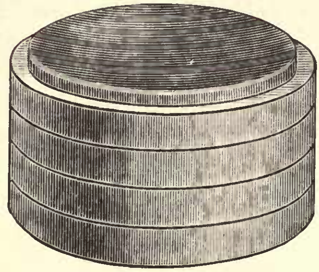
Co-ordinates of Curvature in Miles for Maps of Large Extent.

Longitude.	Latitude 20°.		Latitude 24°.		Latitude 28°.		Latitude 32°.	
	D. M.	D. P.	D. M.	D. P.	D. M.	D. P.	D. M.	D. P.
2°	130·0	0·8	126·4	0·9	122·2	1·0	117·4	1·1
4	260·0	3·1	252·8	3·6	244·4	4·0	234·8	4·3
6	390·0	6·9	379·2	8·1	366·5	9·0	352·0	9·8
8	520·0	12·4	505·5	14·4	488·6	16·0	469·3	17·3
10	649·8	19·4	631·7	22·4	610·4	25·0	586·3	27·1
12	779·7	27·8	757·9	32·2	732·4	36·0	703·5	39·1
14	909·2	38·0	883·6	43·9	853·7	49·0	819·6	53·1
16	1039·2	49·6	1009·9	57·4	975·7	64·1	936·8	69·5
18	1168·1	62·8	1134·8	72·6	1096·0	80·9	1051·9	87·8
20	1298·0	77·6	1261·2	89·7	1218·8	100·1	1169·2	108·6
R.	10892		8905		7458		6348	
Longitude.	**Latitude 36°.**		**Latitude 40°.**		**Latitude 44°.**		**Latitude 48°.**	
	D. M.	D. P.	D. M.	D. P.	D. M.	D. P.	D. M.	D. P.
2°	112·0	1·2	106·1	1·2	99·7	1·2	92·7	1·2
4	224·0	4·6	212·2	4·8	198·9	4·8	185·4	4·8
6	335·9	10·3	318·1	10·7	298·7	10·9	277·9	10·8
8	447·7	18·4	423·9	18·9	398·0	19·3	370·3	19·2
10	559·2	28·7	529·4	29·7	497·1	30·2	462·3	30·0
12	670·5	41·3	634·7	42·8	595·9	43·4	554·1	43·2
14	781·6	56·2	739·7	58·2	694·3	59·1	645·6	58·8
16	892·3	73·4	844·3	76·0	792·3	77·1	736·5	76·7
18	1002·6	92·8	948·5	96·1	889·9	97·5	827·0	97·0
20	1112·5	114·5	1052·3	118·5	986·9	120·2	916·9	119·6
R.	5461		4729		4110		3575	

Length of a Degree of Longitude at Different Latitudes, and at Sea-Level.

Deg. of Lat.	Miles.	Deg. of Lat.	Miles.	Deg. of Lat.	Miles.	Deg. of Lat.	Miles.	Deg. of Lat.	Miles.	Deg. of Lat.	Miles.
0	69·16	14	67·12	28	61·11	42	51·47	56	38·76	70	23·72
2	69·12	16	66·50	30	59·94	44	49·83	58	36·74	72	21·43
4	68·99	18	65·80	32	58·70	46	48·12	60	34·67	74	19·12
6	68·78	20	65·02	34	57·39	48	46·36	62	32·55	76	16·78
8	68·49	22	64·15	36	56·01	50	44·54	64	30·40	78	14·42
10	68·12	24	63·21	38	54·56	52	42·67	66	28·21	80	12·05
12	67·66	26	62·20	40	53·05	54	40·74	68	25·98	82	9·66





COLOURED TOPOGRAPHY.

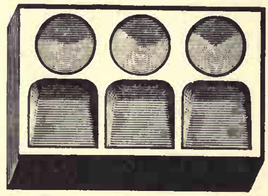
Topographical features may be represented effectively and expeditiously by means of the brush and water-colours, either by India ink alone, or by various tints, or by the union of both. (For preparing colours for tints and their application, see page .)

The most important colours for conventional tints are, besides India ink, indigo, carmine or crimson lake, and gamboge, used separately or compounded, and burnt sienna, yellow ochre, and vermilion, generally used alone.

The following conventional colours are used by the French military engineers in their coloured topography: Woods, *yellow*, using gamboge and a very little indigo; grass land, *green*, made of gamboge and indigo; cultivated land, *brown*, made of lake, gamboge, and a little India ink or burnt sienna will answer. Adjoining fields should be slightly varied in tint. Sometimes furrows are indicated by strips of various colours. Gardens are represented by small rectangular patches of brighter *green* and *brown;* uncultivated land, *marbled green* and *light brown;* brush, brambles, etc., *marbled green* and *yellow;* heath, furze, etc., *marbled green* and *pink;* vineyards, *purple*, composed of lake and indigo; sands, *light brown*, made of gamboge and lake, or yellow ochre will do; lakes and rivers, *light blue*, with a darker tint on their upper and left-hand sides; seas, *dark blue*, with a little yellow added; marshes, the *blue* of water, with spots of grass *green*, the touches all lying horizontally; roads, *brown*, between the tints for sand and cultivated ground, with more India ink; hills, *greenish brown*, made of gamboge, indigo, lake, and India ink. Woods may be finished up by drawing the trees and colouring them green, with touches of gamboge toward the light (the upper and left-hand side), and of indigo on the opposite side.

In addition to the conventional colours, an imitation of the conventional signs is introduced in colour with the brush, and shadows are almost invariably introduced. The light is usually supposed to come from the upper left-hand corner, and to fall sufficiently oblique to allow of a decided light and shade to the slopes of hills, trees, etc. After the shadow has been painted, the outline of the object is strengthened by a heavy black line on the side opposite the light. The flat tints are first laid on as above, and then the conventional signs are drawn in with a pencil and coloured in with appropriate and more intense tints; the shadows are generally represented in India ink.

For the shading of hills, wash the surface first with the proper flat tint, trace in with a pencil the outlines, then lay on in India ink tints proportioned in intensity to the height of the hills and steepness of the slopes. To soften the tints, a water brush is used; the tints are laid on with the colour brush, and softened by passing the water brush rapidly along the edges. The water brush must not have too much water, as it would in that case lighten the tint more than is intended, and leave a ragged, harsh edge. Tints may be applied in very light shades, one over another, with the boundary of the upper tint not reaching the extreme limit of the tint below it. When depth of shade is required, it is best produced by application of several light tints in succession; no tint is to be laid over the other until the first is dry.



In shading by contours it is usual to increase the intensity of the shade beyond that of the mere superposition of one shade on another; where there are high altitudes and numerous contours, the lower ones should be put in with a different tint, as burnt sienna or sepia, with increasing shades, and above these graduated shades of India ink, beginning at the same intensity as that of the colour last put on (Plate IX).

When woods have to be represented, the shading used for the trees, instead of interfering with the shadows due to the slopes, may be made to harmonize with them, and contribute to the general effect by presenting greater or less depth, according to the position of the woods on the sides or summits of the hills.

An expeditious and effective way of representing hills with a brush, imitating hills drawn with a pen on the vertical system, is effected by pressing out the brush flat to a comb-like edge, drawing this over a nearly dry surface of India ink, and then brushing lightly or more heavily between the contours, according to the steepness of the slope, each of the comb-like teeth making its mark (Plate VII).

Rivers and masses of water may be shaded in with a colour and water brush; or, by superposition of light tints, a shadow may be thrown from the bank toward the light, and the outline of this bank strengthened with a heavy black line; the tints are to be in indigo.

Topographical drawings may be made in water-colour with but one tint, as India ink, or ink mixed with a little sepia. The conventional signs are made in imitation of pen drawings, the hills in softened tint, or drawn with the comb-edged brush.

Artistic and effective drawings are made of hills as they would appear in nature under an oblique light, the sides of the hills next the light receiving it more brilliantly according as they are inclined nearer to right angles with its rays, and the shades on the sides removed from the light increasing in intensity as the slopes increase in steepness.

Having damp-stretched the paper upon the drawing-board, first draw in the lines in pencil, and afterward repeat them with a very light ink-line; a soft sponge, well saturated, should then be passed quickly over the surface of the drawing in order to remove any ink that would be liable to mix with the tint and mar its uniformity.

The moistened surface will prevent the tint from drying too rapidly at the edges. In tinting, never allow the edge to dry until the whole surface is covered; leave a little superfluous colour along the edge while filling the brush. In applying a flax tint to large surfaces, let the drawing-board be inclined at an angle of five or six degrees, to allow the colour to flow downward over the surface. With a moderately full brush, commence at the upper outline, and carry the colour along uniformly from left to right and from right to left in horizontal bands, taking care not to overrun the outlines, in approaching which the point of the brush should be used, and at the lower outline let there be only sufficient colour in the brush to complete the tinting.

The colour should not be allowed to accumulate in inequalities of the paper, but should be evenly distributed over the whole surface.

Too much care can not be given to the first application of colour; attempting to remedy a defect by washing or applying fresh tints generally makes bad worse.

Erasers should never be used on a tinted drawing, as the paper, when scratched, receives the tint more readily, and retains a larger portion of colour than other parts, thereby causing a darker tint.

Marbling is done by using two separate tints, and blending them at their edges. A separate brush is required for each tint; before the edge of the first is dry, pass the second tint along the edge, blending one tint into the other, and continue with each tint alternately.

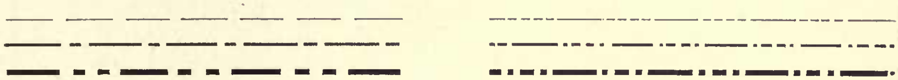
In reference to general effect in tinted topographical drawings, intensity and everything else should be subordinate to clearness. No tint should be prominent or obtrusive. Tints that are of small extent must be a little more intense than large surfaces, or they will appear lighter in shade. Keep a general tone throughout the whole drawing. Beginners will find it best to keep rather low in tone, strengthening their tints as they acquire boldness of touch.

Plate VIII gives an example of coloured topography.

The plan is usually so drawn that the top may represent the north; the upper left-hand corner is then the northwest.

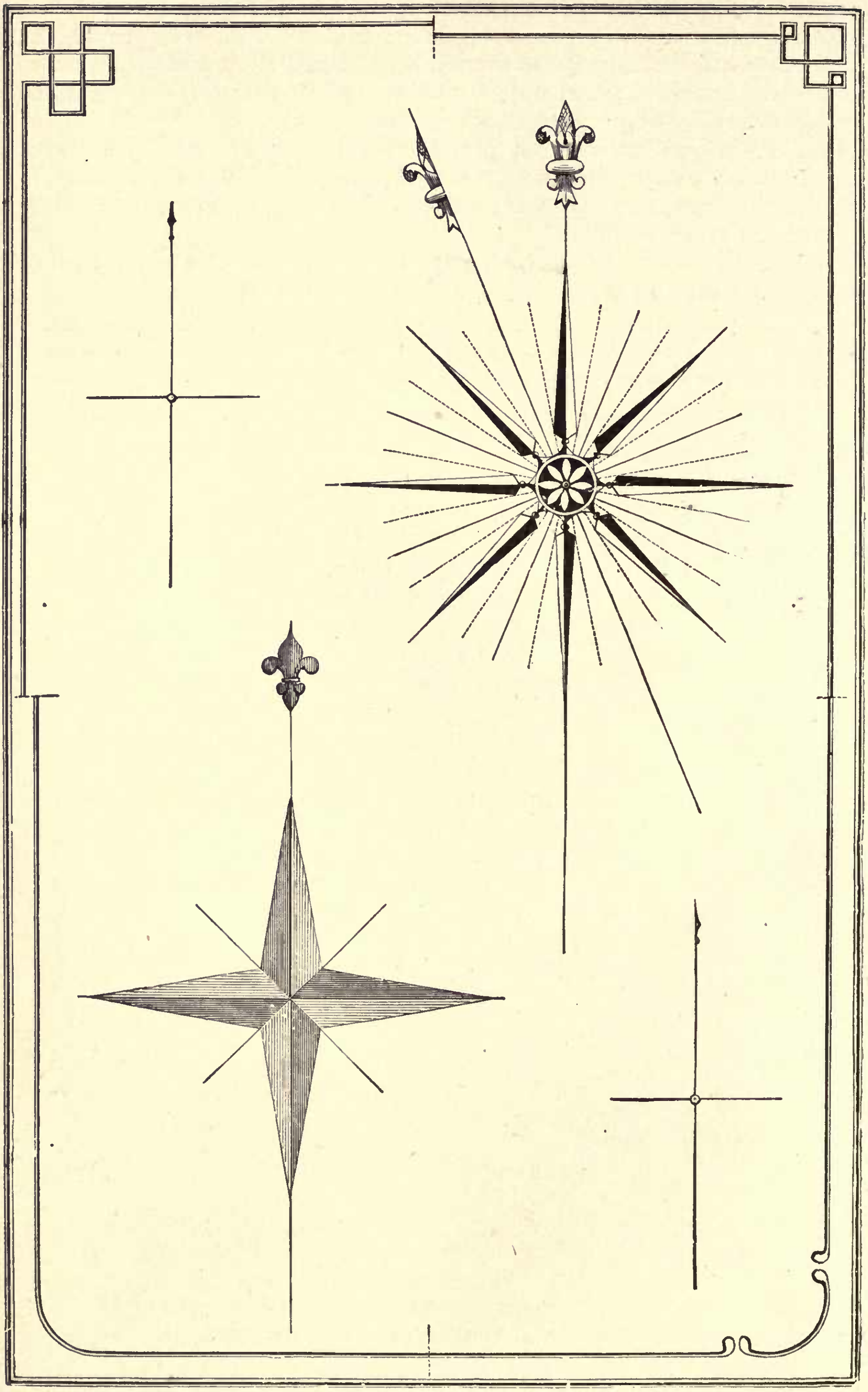
In inking in, commence first with the light lines, since a mistake in these lines may be covered by the shade-lines. Describe all curves before drawing the straight lines, for it is easier to join neatly a straight line to a curve than the opposite. Ink in with system, commencing, say, at the top; ink in all light lines running easterly and westerly, then all light lines running northerly and southerly, then commence in the same way and draw in the shade lines. Elevated objects have their southern and eastern outline shaded, while depressions have the northern and western; thus, in conventional signs, roads and canals are shaded on opposite sides. Having inked in all lines that are drawn with a ruler or described with compasses, commence again at one corner to fill in the detail, keeping all parts of the plan except what you are actually at work upon covered with paper to preserve it from being soiled. The curved lines of brooks, fences, etc., are sometimes drawn with a drawing-pen, sometimes with a steel pen or goose-quill.

Boundary lines of private properties, of townships, of counties, of States, etc., are indicated by various combinations of short lines and dots, thus:

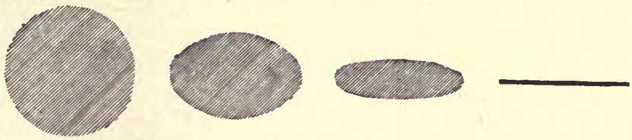


All plans should have meridian lines drawn on them; also scales, and the dates on which the plans were finished. Page 120 gives several designs for meridians and borders. In these diagrams it will be observed that both true and magnetic meridians are drawn; this is desirable when the variation is known, but in many surveys merely the magnetic meridian is taken; in these cases this line is simply represented with half of the barb of the arrow at the north point, and on the opposite side of the line from the true meridian.

Scales on drawings which are to be reproduced by photography should always be drawn in; on others, the proportion of the plan to the ground should be expressed decimally, as $\frac{1}{5000}$, $\frac{1}{10000}$, or by stating the number of feet, chains, etc., to the inch.



ORTHOGRAPHIC PROJECTION.



Architectural and mechanical drawings are usually the delineation of bodies by *orthographic projection*, which is the representation on a sheet of paper, having only two dimensions, length and breadth, of solids, having three, length, breadth, and thickness, on such scales that dimensions can be taken from the parts, and structures and machines constructed therefrom.

Place any surface, for instance, a sheet of paper or a drawing-board, at right angles to the sun's rays. This may be readily done by inserting a pin into the surface, and making it vertical to the surface in every direction by a right-angled triangle; then place the surface in the direct rays of the sun, and in such a position that there will be no shadow on the surface from the pin; the sun's rays are then perpendicular to the surface. Take a wafer or a circular bit of paper and hold it over the paper by means of a long pin or wire, and we obtain shadows, as above, varying with the inclination of the wafer to the plane of the paper. When parallel with the plane, the shadow is a complete circle; when at right angles, a line; and it varies between them as the wafer is inclined. These shadows are the *orthographic* projections of the wafer; no line can be longer than it is naturally, but, if inclined or vertical, it is reduced in length till it becomes a point only. The orthographic projection of the pin which has determined the position of the surface is merely the shadow of the head. If the pin be inclined at all, the body of the pin is projected as a shadow by a line; if the pin be laid on the surface, its projection is the whole length of the pin. The sun's rays act as perpendiculars, which will be hereafter spoken of as projecting the points of an object upon a surface which will represent the object itself in drawing; and, should any confusion occur to the draughtsman of how an object is to be projected or drawn, if he can make the outline of the object on any convenient scale in wire and get its shadows by the sun's vertical rays on a plane, he can readily see how the object should be drawn.

Since the surfaces of all bodies may be considered as composed of points, the first step is to represent the position in space of a point, by referring it to planes whose position is established. The projection of a point upon a plane is the foot of the perpendicular let fall from the point to the plane.

If on two planes not parallel to each other, whose positions are known, we have the projection of a point, the position of this point is determined by erecting perpendiculars from each plane at the projected points: their intersection will be the point.

If from every point of an indefinite straight line, A B (Fig. 252), placed in any manner in space, perpendiculars be let fall on a plane, L M N O, whose position is given, then all the points in which these perpendiculars meet the plane will form another indefinite straight line, *a b*: this line is called the projection of the line A B on this plane. It is only necessary to project two points of the line, and the straight line drawn through the two projected points will be the projection of the given line. The projection of a straight line, itself perpendicular to the plane, is the point in which it meets the plane.

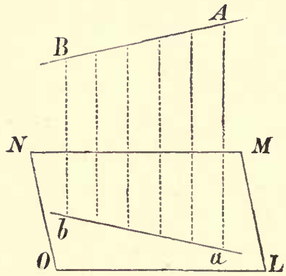


Fig. 252.

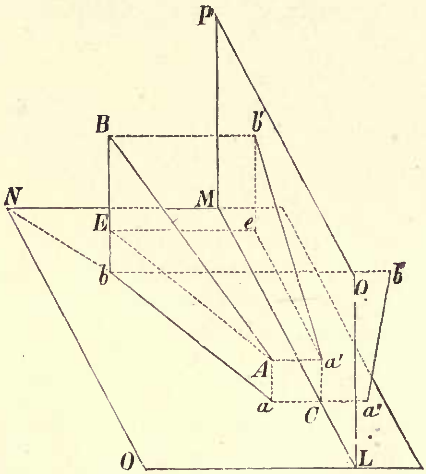


Fig. 253.

If the projections *a b* and *a' b'* of a straight line on the two planes L M N O and L M P Q (Fig. 253) are known, this line A B is determined; for if, through one of its projections, *a b*, a plane be drawn perpendicularly to L M N O, and if through *a' b'* another plane be drawn perpendicular to L M P Q, the intersection of the two planes will be the line A B.

To delineate a solid, it must be referred to three planes, at right angles to each other.

Thus, let *a b c* (Fig. 254) be a parallelopiped in an upright position, of which the plane *a b* is horizontal, and the planes *a c* and *c b* vertical. Let *d e*, *d f*, and *d g* be the planes of projection. The sides of the body being parallel to these planes, each to each, let the figure of the parallelopiped be projected on them. Draw parallel lines from the angles of the body perpendicular to the planes, as indicated by the dotted lines; then upon the plane *d e*, *a' b'* is the projection of the surface *a b*, the *plan* of the object; upon the plane *d f*, *a' c'*, the projection of the surface *a c*, the *front elevation*; and upon the plane *d g*, the projection *b' c'* of the surface *b c*, the *side elevation*. Thus, three distinct views of the regular solid *a b c* are delineated on plane surfaces, any two of which are a sufficient description of the object. From the two figures *a' c'*, *b' c'*, for example, the third figure *a' b'* may be compounded, by drawing the vertical lines *c' h b' i* and *a' k*, *c' l* to meet the plane *d e*, and by producing them horizontally till they meet and form the figure *a' b'*. Similarly, the figure *b' c'* may be deduced from the other two by the aid of the lines *h*, *i*, from *a′ b′* and the lines *m*, *n*, from *a′ c′*.

It is in this way that a third view of any piece of machinery is to be found from two given views; and in many cases two elevations, or one elevation and

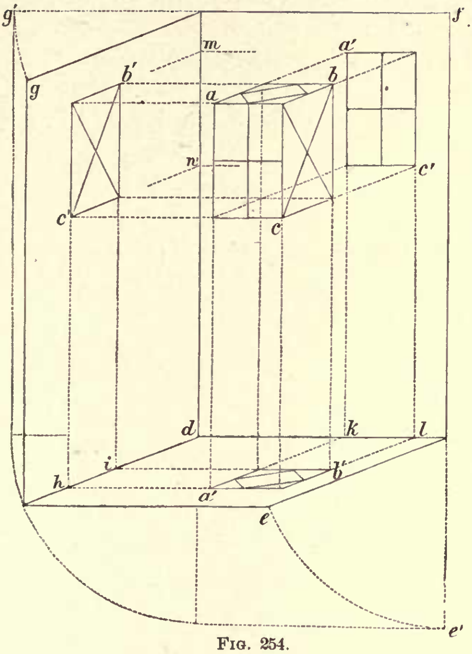


Fig. 254.

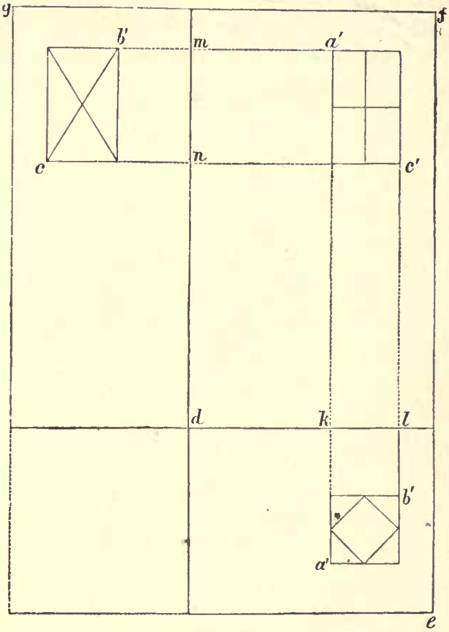


Fig. 255.

a plan, may afford a sufficiently complete idea of the construction of a machine. When parts are inclosed by others, views of the interior are required, in which case the machine is supposed to be cut across by planes, vertical, horizontal, or inclined, to reveal its structure. Such views are termed *sections*, and distinguished, with reference to the planes of section, as *vertical* or *horizontal* or *inclined* sections.

In practice, the drawings are done upon one common surface, the plane of paper. Suppose the plane *d g* (Fig. 254) revolved back into the position *d g′*, and *d e* also moved to *d e′*, both of these positions being in the plane of *d f*. This done, the three views are depicted on one plane surface (Fig. 255); *d l* and *d m* are the ground and vertical lines; the positions of the same points in *a′ c′* and *a′ b′* are in the same perpendicular from the ground-line; and the position of a point in the plane may be found by applying the edge of the square to the same point as represented in the elevation. The same is true as between the two elevations, and establishes a method of drawing several views of one machine upon the same surface of paper in strict agreement with each other.

PROJECTIONS OF SIMPLE BODIES.

Right projections of a regular hexagonal pyramid (Fig. 256).—Two distinct geometrical views are necessary to convey a complete idea of the form of the object: an elevation to represent the sides of the body, and to express its height; and a plan to express the form horizontally.

Draw a horizontal line L T through the centre of the sheet to represent the ground line. Then draw a perpendicular to the ground line, S S′, to represent the axis of the pyramid.

To construct the plan, from any point, S′, on the line S S′, as a centre, construct the hexagonal base; the lines A′ S′, B′ S′, etc., represent the projections of its edges in the plan.

Since the base of the pyramid rests upon the horizontal plane, it must

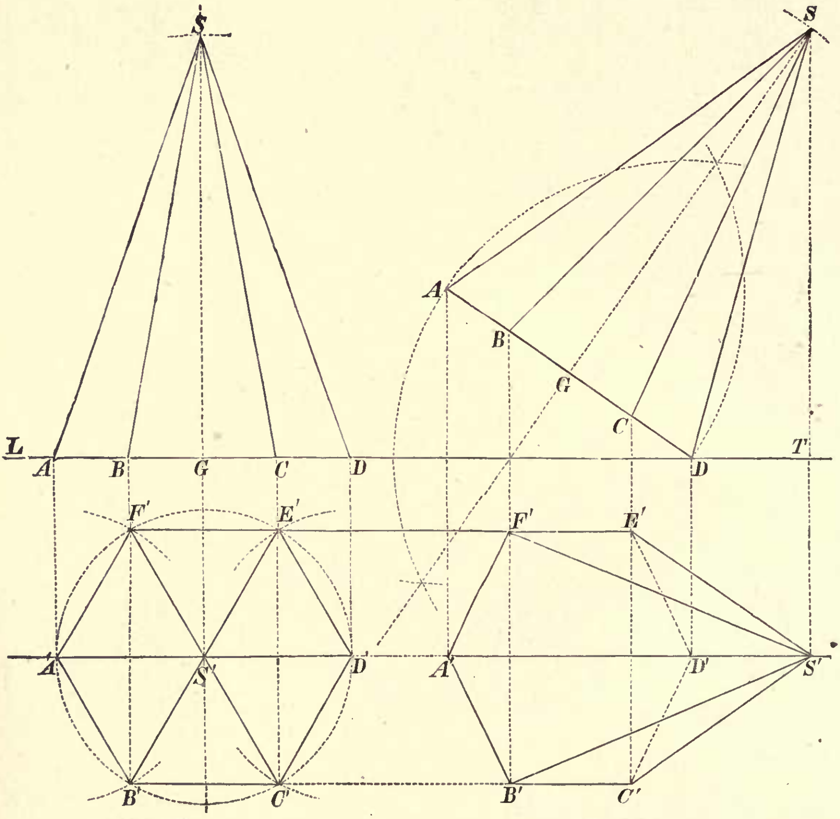


FIG. 256. FIG. 257.

be projected vertically upon the ground line. From each of the angles at A′, B′, C′, and D′ erect perpendiculars to that line. The points of intersection, A, B, C, and D, are the true positions of all the angles of the base; and it only remains to lay off the height of the pyramid, from the point G to S, and to draw S A, S B, S C, and S D, which are the only edges of the pyramid visible in the elevation; S A and S D, being in the vertical plane, are seen in their true length; the points F′ and E′ being situated in the lines B B′ and C C′, the lines S B and S C are each the projections of two edges of the pyramid.

To construct the projections of the same pyramid, having its base set in an inclined position, but with its edges S A *and* S D *still in the vertical plane* (Fig. 257).

With the exception of the inclination, the vertical projection of this solid is precisely the same as in the preceding example, and it is only necessary to copy that elevation. To do this, fix the position of the point D upon the ground line, through which draw D A, making with L T the desired inclination of the base of the pyramid. Make D A equal to the A D of the preceding figure, and on this erect the vertical projection S A D of that figure.

Since the edges S A and S D are still in the vertical plane, and the point D remains unaltered, the projection A′ of the point A will still be in the line A′ S′. The remaining points, B′, C′, etc., in the projection of the base, are found by the intersections of perpendiculars let fall from the corresponding points in the elevation, with lines drawn parallel to A′ S′, at a distance equal to the width of the base. Joining all the contiguous points, we obtain A′ B′ C′ D′ E′ F′, the horizontal projection of the base, two of its sides being concealed by the body of the pyramid. The vertex S having been similarly projected to S′, and joined by straight lines to the several angles of the base, the projection of the solid is completed.

To find the horizontal projection of a transverse section of the same pyramid, made by a plane perpendicular to the vertical, but inclined at an angle to the horizontal plane of projection, letting all the sides of the base be inclined to the ground line (Fig. 258).

Since none of the sides of the base are to be parallel with the ground line, draw a line A′ D′ making the required angle with that line, and from the points A′ and D′ set out the angular points of the hexagon. To obtain the projections of the edges of the pyramid, join the angular points which are diametrically opposite and project the figure thus obtained upon the vertical plane, as shown in the elevation.

If the cutting plane be represented by the line *a d* in the elevation, it will expose, as the section of the pyramid, a polygon whose angular points, being the intersections of the various edges with the cutting plane, will be projected in perpendiculars drawn from the points where it meets these edges respectively; from the points *a*, *f*, *b*, etc., let fall the perpendiculars *a a*′, *f f*′, *b b*′, etc.; join their contiguous points of intersection with the lines A′ D′, F′ C′, B′ E′, etc.; and the resulting six-sided figure represents the section required. The edges F S and E S, being concealed in the elevation, but necessary for the construction of the plan, are expressed by dotted lines, as also is the portion of the pyramid situated above the cutting plane, supposed to be removed, but necessary in order to draw the lines representing the edges. The ordinary method of expressing sections in

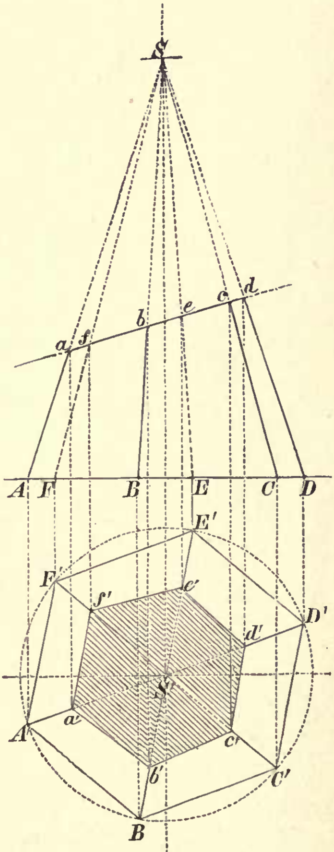


FIG. 258.

purely line drawings is by filling up the spaces comprised within their outlines with a number of parallel straight lines drawn at equal distances, called *section lines.*

To represent in plan and elevation a regular six-sided prism in an upright position (Fig. 259).

Lay down the ground line G K and draw the axis of the prism S S′. Describe the hexagonal plan A′ B′ C′ D′ E′ F′, as in the previous example. From each of the angular points, A′, B′, etc., erect perpendiculars, and on one of these perpendiculars set off A G, the height of the prism, and draw a parallel to the ground line, A D, which completes the vertical projection. The face, B C H I, being parallel to the vertical plane, is seen in its true size, while G H and I K

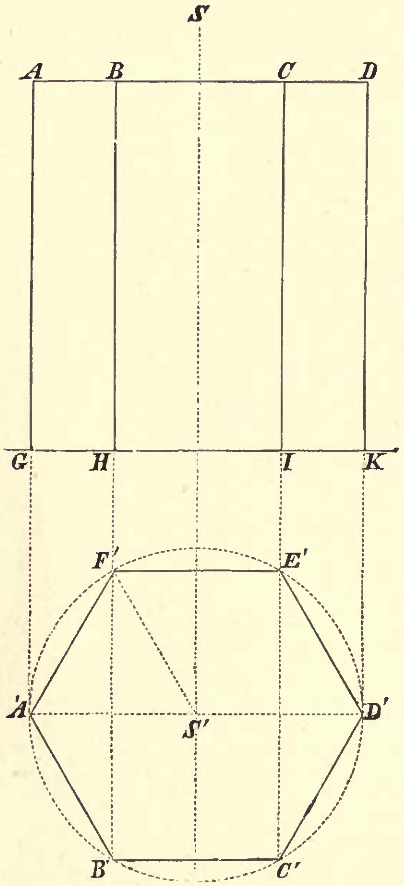


Fig. 259.

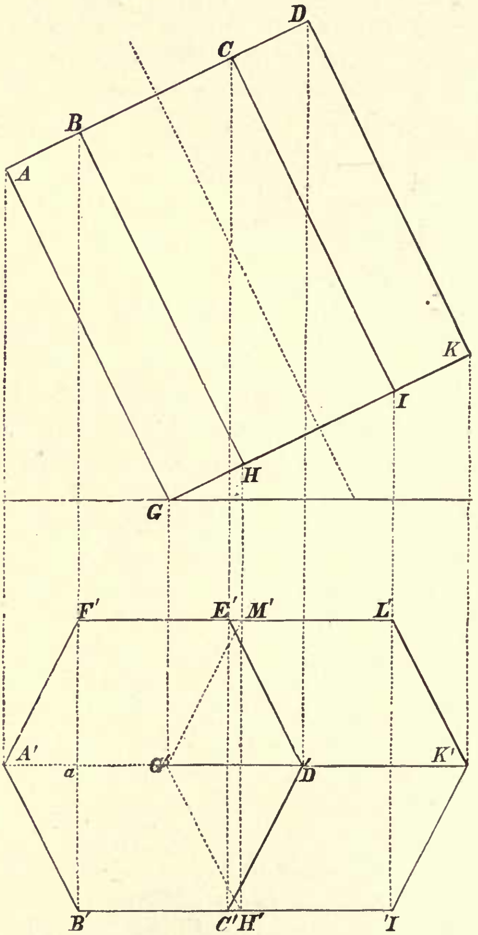


Fig. 260.

are each equal to one half of H I, which enables us to draw the elevation without constructing the plan—a fact to be remembered in the drawing of nuts, bolt-heads, etc., in machine drawing.

To form the projections of the same prism, supposing it to have been moved round the point G *in a plane parallel to the vertical plane* (Fig. 260).

Copy the elevation (Fig. 259) on the inclined base G K; let fall perpendiculars from all the angles in the elevation; and join the contiguous points of intersection with the horizontal lines appropriate to these points respectively. The plan remaining the same width as before, the polygon A′ B′ C′ D′ E′ F′ is the projection of the upper surface, and G′ H′ I′ K′ L′ M′ that of the base of the prism. All the edges are represented in the horizontal projection by equal straight lines, as D′ K′, A′ G′, etc., and the sides A′ B′, G′ H′, etc., remain still parallel to each other, which affords the means of verifying the accuracy of the drawings. The upper surface and the base, seen obliquely in this projection, do not appear as true hexagons in the plan.

Required the projections of the same prism set into a position inclined to both planes of projection (Fig. 261).

Assuming the inclination of the prism upon the horizontal plane to be as in the preceding figures, copy the plan of Fig. 260 on an axis A′ K′ inclined to the vertical plane of projection. Since the prism preserves its former inclination to the horizontal plane, every point in it, as A, in assuming its new position, simply moves in a horizontal plane, and will therefore be at the same distance above the ground line that it was in the elevation (Fig. 260), and it will also be in the perpendicular A′ A; the point of intersection A is, therefore, its projection in the elevation. Determine the remaining angular points in this view and join the contiguous points and the corresponding angles of the upper and lower surface and the figure is complete.

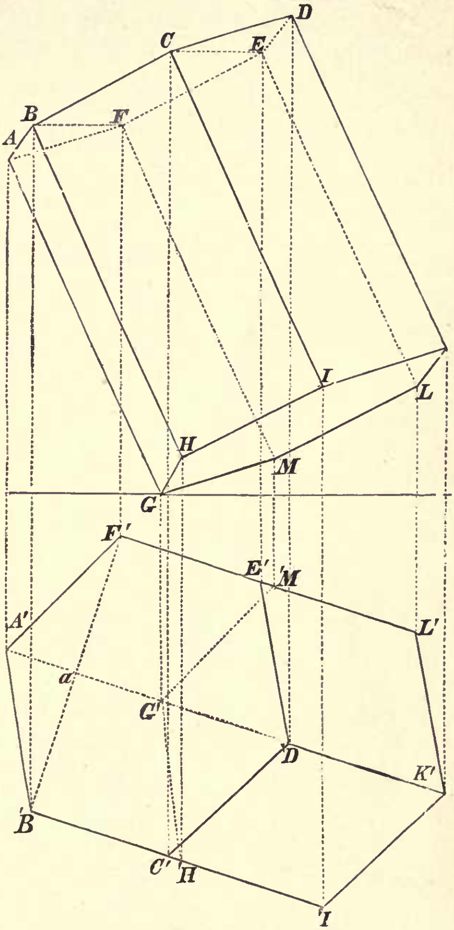


Fig. 261.

CONIC SECTIONS.

The *plan* of the cone (Fig. 262) is simply a circle, described from the centre S′, with a diameter equal to that of the base. Its *elevation* is an isosceles triangle, obtained by drawing tangents A′ A, B′ B, perpendicular to and intersecting the ground line; then set off upon the centre line the height C S, and join S A, S B. These lines are called the exterior *elements* of the cone.

Given the projections of a cone, and the direction of a plane X X, *cutting it perpendicularly to the vertical, and obliquely to the horizontal plane; required* *to find, first, the horizontal projection of this section; and, secondly, the outline of the ellipse thus formed* (Figs. 262, 263).

Through the vertex of the cone draw a line S E to any point within the base A B; let fall a perpendicular from E, cutting the circumference of the base

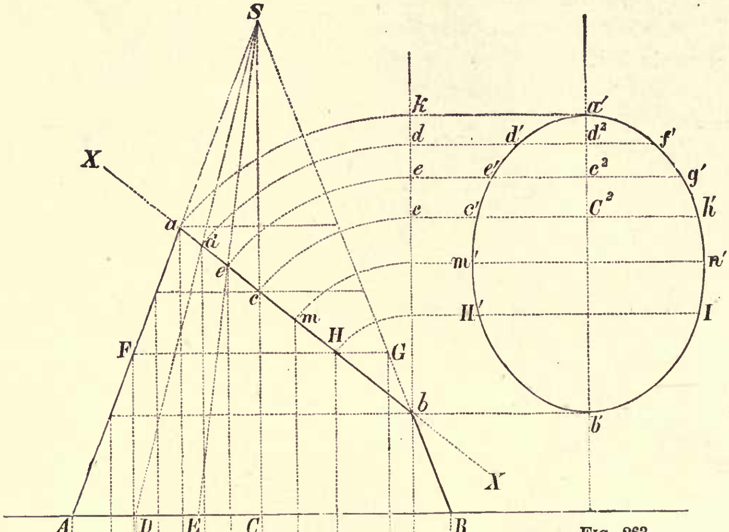


FIG. 263.

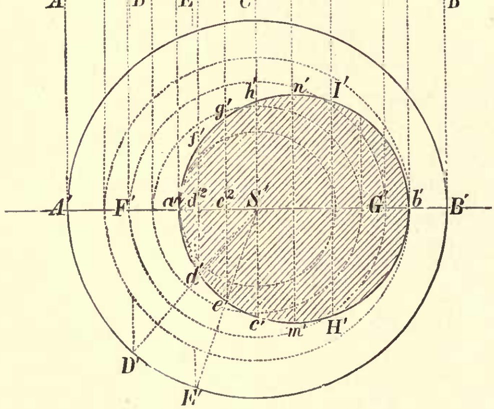


FIG. 262.

in E′, and join E′ S′; then another perpendicular let fall from *e* will intersect E′ S′ in a point *e*′, which will be the horizontal projection of a point in the curve required; and so on for any number of points.

The exterior generatrices A S and B S being both projected upon the line A′ B′, the extreme limits of the curve sought will be at the points *a*′ and *b*′ on that line, which are the projections of the points of intersection *a* and *b* of the cutting plane with the outlines of the cone. And since the line *a*′ *b*′ divides the curve symmetrically into two equal parts, the points *f*′, *g*′, *h*′, etc., will be obtained by setting off above that line, and on their respective perpendiculars, the distances d' d^2, e' e^2, etc. A sufficient number of points having thus been determined, the curve drawn through them (which will be found to be an ellipse) will be the outline of the section required.

This curve may be obtained by another method, depending on the principle that all sections of a cone by planes parallel to the base are circles. Thus, let the line F G represent such a cutting plane; the section which it makes with the cone will be denoted on the horizontal projection by a circle drawn from the centre S′, with a radius equal to half the line F G; and by projecting the point of intersection H of the horizontal and oblique planes by a perpendicular H H′, and noting where this line cuts the circle above referred to, two points H′ and I′ are determined in the curve. Additional points are obtained similarly.

The preceding methods exhibit the section as fore-shortened. To solve the second question proposed, let the cutting plane X X be conceived to turn upon the point *b*, so as to coincide with the vertical line *b k*, and (to avoid con-

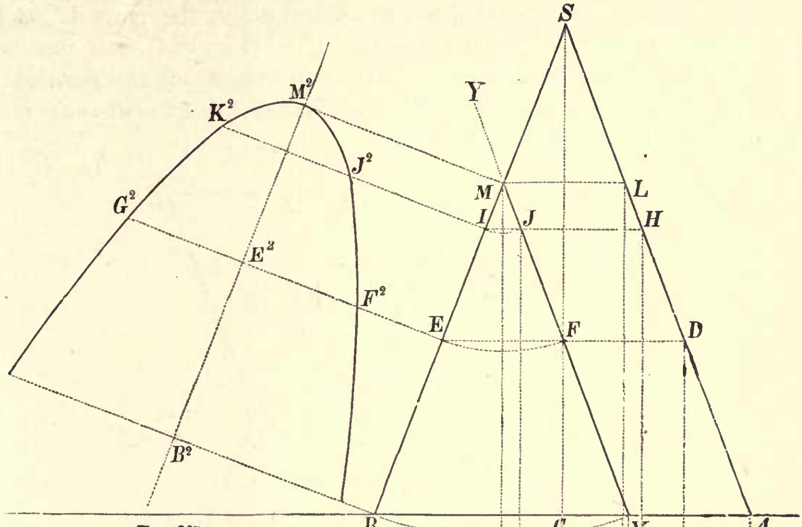


Fig. 265.

fusion of lines) let *b k* be transferred to *a′ b′*, which will represent, as before, the extreme limits of the curve required; take any point, as *d*, in this new position of the cutting plane, it will be represented by d^2, and, if the cutting plane were turned upon *a′ b′* (Fig. 263) as an axis till it is parallel to the vertical plane, the point which had been projected at d^2 would then have described round *a′ b′* an arc of a circle, whose radius is the distance d' d^2 (Fig. 262). This distance, therefore, set off at *d′* and *f′* on each side of *a′ b′*, gives two points in the curve sought. The curve drawn through any number of points thus obtained will be an ellipse of the true form and dimensions of the section.

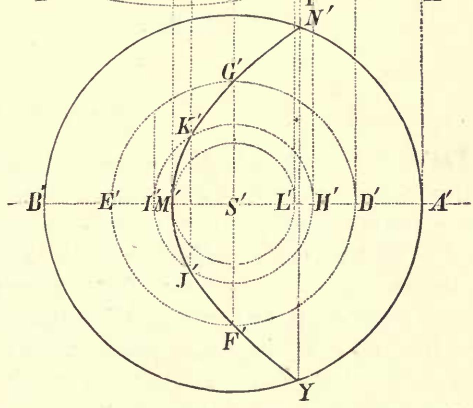


Fig. 264.

To find the horizontal projection and actual outline of the section of a cone, made by a plane Y Y *parallel to one side or element, and perpendicular to the vertical plane* (Figs. 264, 265).

Determine by the second method laid down in the preceding problem any number of points, as F′, G′, J′, K′, etc., in the curve representing the horizontal projection of the section specified. The horizontal plane passing through M gives only one point M′, which is the vertex of the curve sought.

To determine the actual outline of this curve, suppose the plane Y Y to turn as upon a pivot at M, until it has assumed the position M B, and transfer M B to the parallel M^2 B^2 (Fig. 265). The point F will thus have first described the arc F E till it reaches the point E, which is then projected to E^2; suppose the given plane, now represented by M^2 B^2, to turn upon that line as an axis, until it assumes a position parallel to the vertical plane, the point E^2, which is distant from the axis M′ B′ by the distance F′ S′ (Fig. 264), will now be projected to F^2 and G^2, two points in the curve required, which is a *parabola.*

To draw the vertical projection of the sections of two opposite cones made by a plane parallel to their axis (Fig. 266).

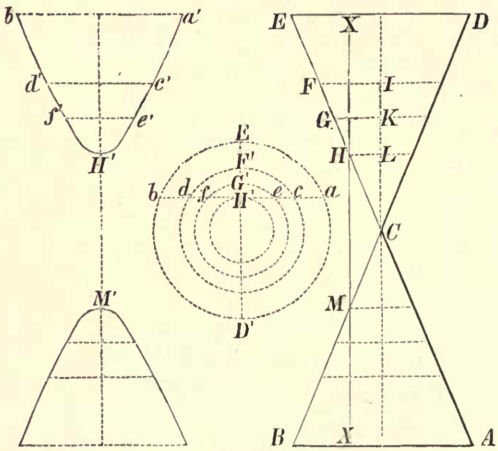


FIG. 266.

Let C E D and C B A be the two cones, and X X the position of the cutting plane. Project in plan either of the cones, as *b* E′ D′; from its centre, with a radius equal to L H, describe a circle, and draw the tangent *b a*; *b a* will be the horizontal projection of the cutting plane. Draw the line H′ M′ parallel to the cutting plane; H′, M′ corresponding in position to the intersections H, M, of the plane with the cones. From H′ and M′ lay off distances equal to L K, K I, and the length of the cone, and through these points draw perpendiculars, as *f′ e′*, *d′ c′*, *b′ a′*, etc., which must be made equal to the chords *f e*, *d c*, *b a*, made by the cutting plane *a b*, with circles whose radii are G K, I F, and the radius of the base of the cone. Through the points *a′*, *c′*, *e′*, H′, *f′*, *d′*, *b′*, draw the curve for the projection required. A similar construction will give the sectional projection of the opposite cone at M′. The curve thus found is the *hyperbola.*

PENETRATIONS OR INTERSECTIONS OF SOLIDS.

Represent the projections of two cylinders of unequal diameters meeting each other at right angles (Fig. 267). The one, the rectangle A B E D for its vertical, and the circle A′ H′ B′ T′ for its horizontal projection; the other, being horizontal, is indicated in the former by the circle L P I N, and in the latter by the rectangle L′ I′ K′ M′. From the position of these two solids the curves formed by their junction will be projected horizontally in the curves O′ H′ P′, R′ T′ S′, and vertically in L P I N.

But, if the position of these bodies be changed into that represented by Fig. 268, the lines of their intersection will assume in the vertical projection a totally different aspect, and may be determined as follows:

Through any point taken upon the plan of Fig. 268 draw a horizontal line *a′ b′*, indicating a plane cutting both cylinders parallel to their axes; this

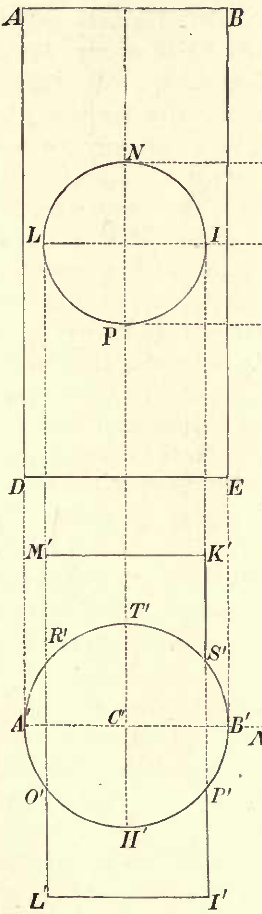


FIG. 267.

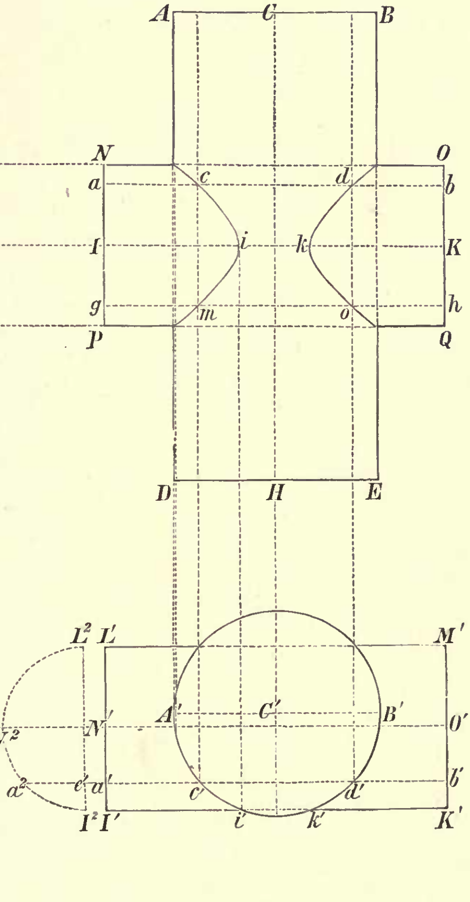


FIG. 268.

plane would cut the vertical cylinder in lines drawn perpendicularly through the points *c′ d′*. To find the vertical projection of its intersection with the other cylinder, conceive its base I′ L′, after being transferred to I^2 L^2, to be revolved about I^2 L^2 as an axis parallel to the horizontal plane; this is expressed in part by simply drawing a semicircle of the diameter I^2 L^2. Produce the line *a′ b′* to a^2; then set off the distance a^2 *e′* on each side of the axis I K, and draw straight lines through these points parallel to it. These lines *a b*, *g h*, denote the intersection of the plane *a′ b′* with the horizontal cylinder, and therefore the points *c*, *d*, *m*, *o*, where they cut the perpendiculars *c c′*, *d d′*, are points in the curve required. By passing other horizontal planes similar to *a′ b′* through both cylinders, and operating as before, any number of points may be obtained. The vertices *i* and *k* of the curves are projected directly from *i′* and *k′*, the intersections of the outlines of both cylinders. When the cylinders are of unequal diameters, as in the present case, the curves of penetration are hyperbolas.

When the diameters of the cylinders are equal (Fig. 269), and when they

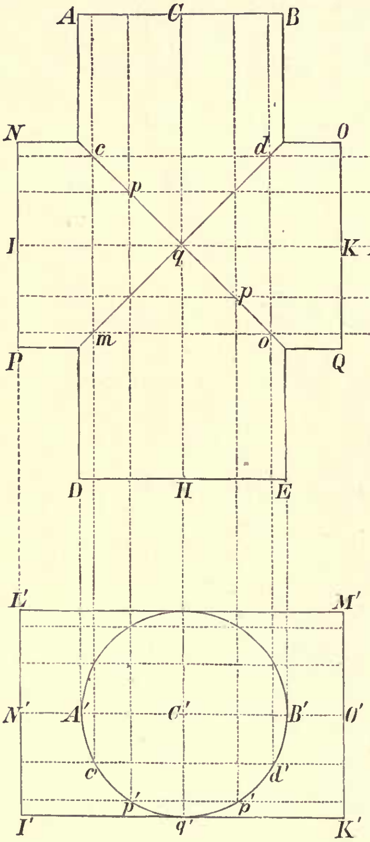


FIG. 269.

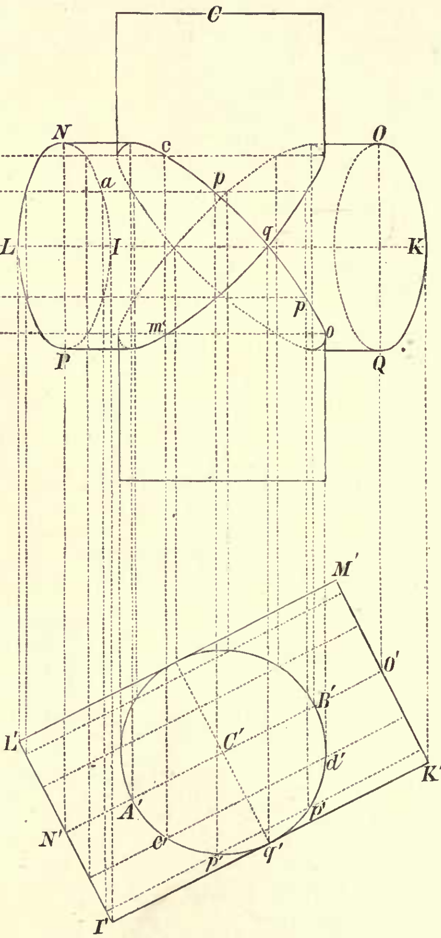


FIG. 270.

cut each other at right angles, the curves of penetration are projected vertically in straight lines perpendicular to each other.

To delineate the intersections of two cylinders of equal diameters at right angles, when one of the cylinders is inclined to the vertical plane (Fig. 270).

Suppose the two preceding figures to be drawn, the projection *c* of any point, as *c′*, must be situated in the perpendicular *c′ c*. Since the distance of this point (projected at *c* in Fig. 269) from the horizontal plane remains unaltered, it must also be in the horizontal line *c c*. Upon these principles all the points indicated by literal references in Fig. 270 are determined; the curves of penetration resulting therefrom intersect each other at two points projected upon the axial line L K, of which that marked *q* alone is seen. The ends of the horizontal cylinder are ellipses.

To find the curves resulting from the intersection of two cylinders of unequal diameters, meeting at any angle (Fig. 271).

Suppose the axes of both cylinders to be parallel to the vertical plane, and let A B E D and N O Q P be their projections upon that plane. In constructing, in the first place, their horizontal projection, observe that the upper end A B of the larger cylinder is represented by an ellipse A′ K′ B′ M′, which may easily be drawn by the help of the major axis K′ M′ equal to the diameter of the cylinder, and of the minor A′ B′, the projection of the diameter. The visible portion of the base of the cylinder being similarly represented by the semi-ellipse L′ D′ H′, its entire outline will be completed by drawing tangents L′ M′ and H′ K′. The upper extremity P N of the smaller cylinder will also be projected in the ellipse *p′* N′.

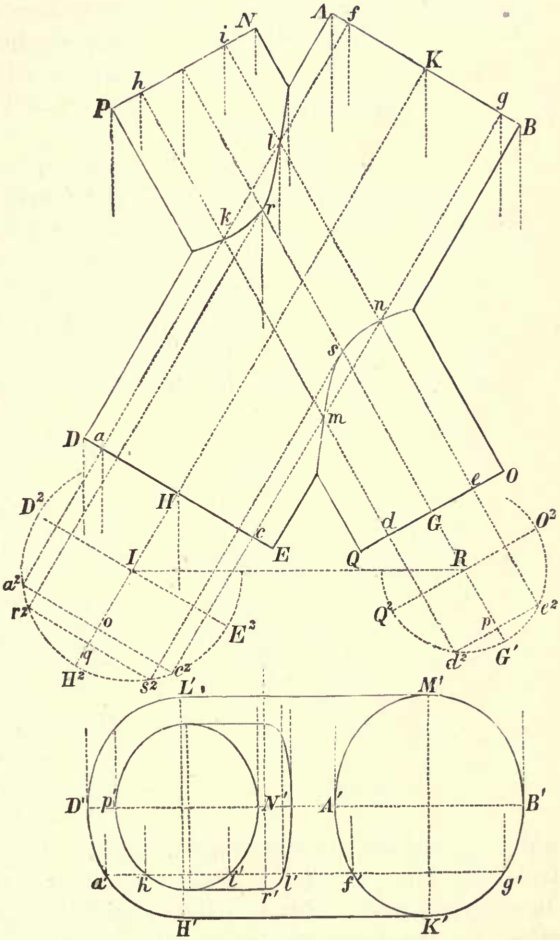


Fig. 271.

Conceive a plane, as *a′ g′*, to pass through both cylinders parallel to their axes; it will cut the surface of the larger cylinder in two straight lines, passing through the points *f′* and *g′* on the upper end of the cylinder; these lines will be represented in the elevation by projecting the points *f′* and *g′* to *f*, *g*, and drawing *a f* and *c g* parallel to the axis. The plane *a′ g′* will in like manner cut the smaller cylinder in two straight lines, which will be represented in the vertical projection by *d h* and *e i*, and the intersections of these lines with *a f* and *c g* will give four points *l*, *k*, *m*, and *n*, in the curves of penetration. Of these points, one only, *l*, is visible, *l′*, in the plan.

To find the curves of penetration in the elevation without the aid of the plan (Fig. 271).

Let the bases D E and Q O of both cylinders be revolved parallel to the vertical plane after being transferred to any convenient distance, as D^2 E^2 and Q^2 O^2, from the principal figure; they will then be vertically projected in the circles D^2 H^2 E^2 and Q^2 G' O^2. Draw a^2 c^2 parallel to D E, and at any suitable distance from the centre I; this line will represent the intersection of the base of the cylinder with a plane parallel to the axes of both, as before. The intersection of this plane with the base of the smaller cylinder will be found by setting off from R a distance R *p*, equal to I *o*, and drawing through the point *p* a straight line parallel to Q O. The intersection of the supposed plane with the convex surfaces of the cylinders will be represented by the lines *a f*, *c g*, and *d h*, *e i*; and, consequently, the intersections of these lines indicate points in the curves sought. These points may be multiplied by conceiving other planes to pass through the cylinders.

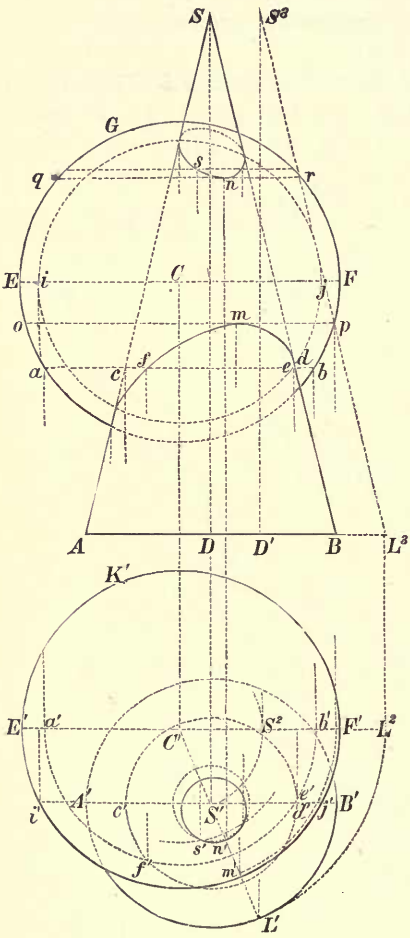


FIG. 272.

To find the curves of penetration of a cone and sphere (Fig. 272).

Let D S be the axis of the cone, A' L' B' the circle of its base, and the triangle A B S its projection on the vertical plane; and let C, C', be the projections of the centre, and the equal circles E' K' F' and E G F those of the circumferences of the sphere.

This problem can be solved only by the aid of imaginary intersecting planes. Let *a b* represent the projection of a horizontal plane; it will cut the sphere in a circle whose diameter is *a b*, and which is partially drawn from the centre C' in the plan, as *a' f' b'*. Its intersection with the cone is also a circle described from the centre S' with the diameter *c d* as *c' f' d'*; the points *e'* and *f'*, where these two circles cut each other, are the horizontal projections of two points in the lower curve, which is entirely hidden by the sphere. The points referred to are projected vertically upon the line *a b* at *e* and *f*. The upper curve, which is seen in both projections, is obtained by a similar process; but it is to be observed that the horizontal cutting planes must be taken in such positions as to pass through both solids in circles which shall intersect each other. In this respect it will be necessary, first, to determine the vertices *m* and *n* of the curves of penetration. For this purpose, conceive a vertical plane passing through the axis of the cone and the centre of the sphere; its horizontal projection will be the straight line C′ L′, joining the centres of the two bodies. Suppose this plane to be turned upon the line C C′ as on an axis, until it becomes parallel to the vertical plane; the points S′ and L′ will now have assumed the positions S^2 and L^2, and consequently the axis of the cone will be projected vertically in the line D′ S^3, and its side in S^3 L^3, cutting the sphere at the points *p* and *r*. Conceive the solids to have resumed their original relative positions, the vertices or adjacent limiting points of the curves of penetration must be in the horizontal lines *p o* and *r q*, drawn through the points determined as above; their exact positions on these lines may be ascertained by projecting vertically the points *m′* and *n′*, where the arcs described by the points *p* and *r*, in restoring the cone to its first position, intersect the line S′ L′.

It is of importance, further, to ascertain the points at which the curves of penetration meet the outlines A S and S B of the cone. The plane which passes through these lines, being projected horizontally in A′ B′, will cut the sphere in a circle whose diameter is *i′ j′*; this circle, described in the elevation from the centre C, will cut the sides A S and S B in four points, at which the curves of penetration are tangent to the outlines of the cone.

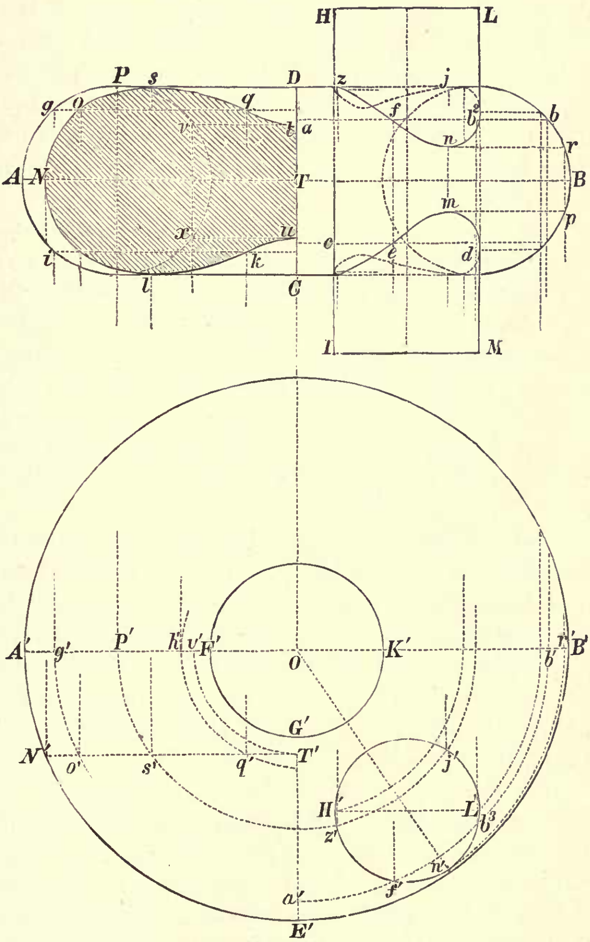


FIG. 273.

To find the lines of penetration of a cylinder and a cylindrical ring, or annular torus (Fig. 273).

Let the circles A′ E′ B′, F′ G′ K′, represent the horizontal, and the figure A C B D the vertical projection of the torus, and let the circle H′ *f′* L′, and the rectangle H I M L be the analogous projections of the cylinder, which passes perpendicularly through it. Conceive, as before, a plane, *a b*, to pass horizontally through both solids; it will cut the cylinder in a circle which will be projected in the base H′ f' L′ itself, and the ring in two other circles, of which one only, part of which is represented by the arc f' b^3 b', will intersect the cylinder at the points f' and b^3, which, being projected vertically, will give two points f and b^3 in the upper curve of penetration.

Another horizontal plane, taken at the same distance below the centre line A B as that marked a b is above it, will cut the ring in circles coinciding with those already obtained; consequently the points f' and b^3 indicate points in the lower as well as in the upper curves of penetration, and are projected vertically at d and e. Thus, by laying down two planes at equal distances on each side of A B, four points in the curves required are determined.

To determine the vertices m and n, following the method explained in the preceding problem, draw a plane O n, passing through the axis of the cylinder and the centre of the ring, and conceive this plane to be revolved about the point O until it has assumed the position O B′, parallel to the vertical plane; the point n', representing the extreme outline of the cylinder in plan, will now be at r', and, being projected vertically, that outline will cut the ring in two points p and r, which would be the limits of the curves of penetration in the supposed relative position of the two solids; and by drawing the two horizontal lines r n and p m, and projecting the point n' vertically, the intersections of these lines, m and n, are the vertices of the curves in the actual position of the penetrating bodies.

The points at which the curves are tangents to the outlines H I and L M of the cylinder may readily be found by describing arcs of circles from the centre O through the points H′ and L′, which represent these lines in the plan, and then proceeding, as above, to project the points thus obtained upon the elevation. Lastly, to determine the points, as j, z, etc., where the curves are tangents to the horizontal outlines of the ring, draw a circle P′ s' j' with a radius equal to that of the centre line of the ring, namely, P D; the points of intersection z' and j' are the horizontal projections of the points sought.

Required to represent the section which would be made in this ring by a plane, N′ T′, *parallel to the vertical plane.*

Such a section will be represented in its actual form and dimensions in the elevation. To determine its outlines, let two horizontal planes, g q and i k, equidistant from the centre line A B, be supposed to cut the ring; their lines of intersection with it will have their horizontal projections in the two circles g' o' and h' q', which cut the given plane N′ T′ in o' and q'. These points being projected vertically to o, q, k, etc., give four points in the curve required. The line N′ T′ cutting the circle A′ E′ B′ at N′, the projection N of this point is the extreme limit of the curve.

The circle P′ s' j', the centre line of the rim of the torus, is cut by the planes N′ T′ at the point s', which, being projected vertically upon the lines D P and C l, determines s and l, the points of contact of the curve with the horizontal outlines of the ring. Finally, the points t and u are obtained by drawing from the centre O a circle, T′ v', tangent to the given plane, and projecting the point of intersection v' to the points v and x, which are then to be replaced upon C D by drawing the horizontals v t and x u.

Required to delineate the lines of penetration of a sphere and a regular hexagonal prism whose axis passes through the centre of the sphere (Fig. 274).

The centres of the circles forming the two projections of the sphere being upon the axis C C′ of the upright prism, which is projected horizontally in the regular hexagon D′ E′ F′ G′ H′ I′, and all the lateral faces of the prism being equidistant from the centre of the sphere, their lines of intersection with it will be circles of equal diameters. The perpendicular face, represented by the line E′ F′ in the plan, will meet the surface of the sphere in two circular arcs, E F and L M, described from the centre C, with a radius equal to *c′ b′* or *a′ c′*. And the intersections of the two oblique faces D′ E′ and F′ G′ will obviously be each projected in two arcs of an ellipse, whose major axis *d g* is equal to *e′ f′*, and minor axis the vertical projection of *e′ f′*. As it is necessary to draw small portions only of these curves, the following method may be employed: Draw D G through the points E, F; divide the portions E F and F G respectively into the same number of equal parts, and, drawing perpendiculars through the points of division, set off from F G the distances from

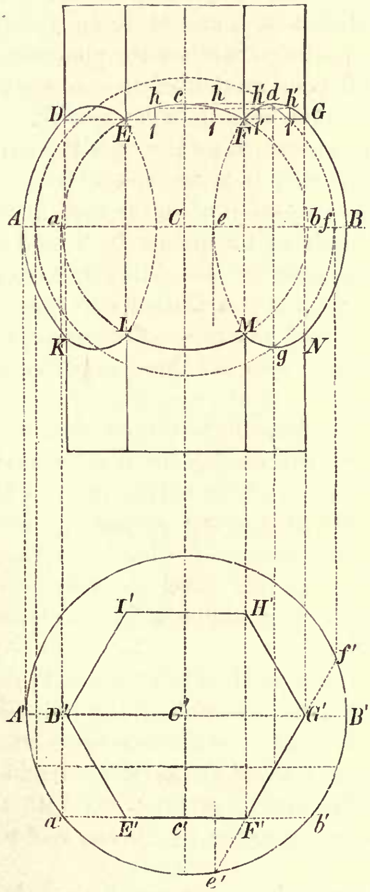


Fig. 274.

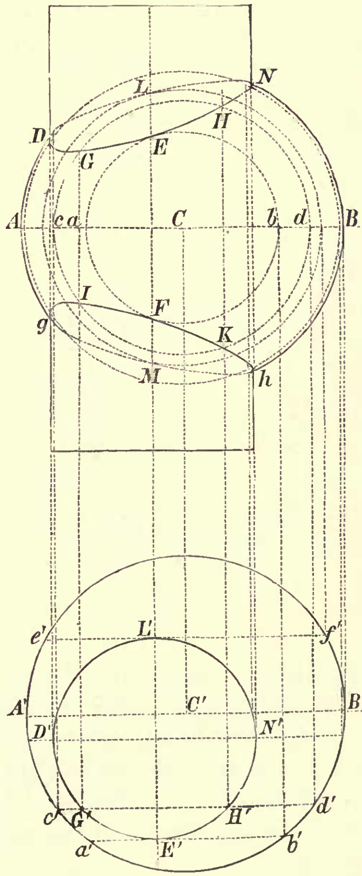


Fig. 275.

the corresponding points in E F to the circular arc E C F, as points in the elliptical arc required. The remaining elliptical arcs can be traced by the same method.

Required to draw the lines of penetration of a cylinder and a sphere, the centre of the sphere being without the axis of the cylinder (Fig. 275).

Let the circle D′ E′ L′ be the projection of the base of the given cylinder, and let A B be the diameter of the given sphere. If a plane, as *c′ d′*, be drawn parallel to the vertical plane, it will cut the cylinder in two straight lines, G G′, H H′. This plane will also cut the sphere in a circle described from the centre C with a radius of half the line *c′ d′*; its intersection with the lines G G′ and H H′ will give so many points in the curves sought, viz., G, H, I, K.

The planes *a′ b′* and *e′ f′*, which are tangents to the cylinder, furnish respectively only two points in the curves; of these points, E and F alone are visible, the other two, L and M, being concealed by the solid; therefore the planes drawn for the construction of the curves must be all taken between *a′ b′* and *e′ f′*. The plane which passes through the axis of the cylinder cuts the sphere in a circle whose projection upon the vertical plane will meet at the points D, N, and *g*, *h*, the outlines of the cylinder, to which the curves of penetration are tangents.

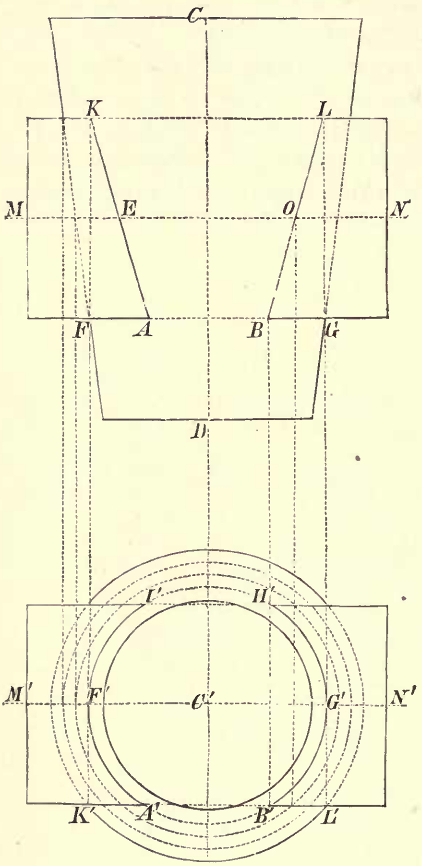


Fig. 276.

To find the lines of penetration of a frustum of a cone and a prism (Fig. 276).

The frustum is represented in the plan by two circles described from the centre C′; and the horizontal lines M N and M′ N′ are the projections of the axis of a prism of which the base is square, and the faces respectively parallel and perpendicular to the planes of projection.

In laying down the plan of this solid, it is supposed to be inverted, in order that the smaller end of the cone and the lines of intersection of the lower surface, F G, of the prism may be exhibited. According to this arrangement, the letters A′ and B′ ought, strictly speaking, to be marked at the points I′ and H′, and conversely; but, as the part above M′ N′ is exactly symmetrical with that below it, the distribution of the letters of reference in the figures can lead to no confusion.

The intersection of the plane F G with the cone is projected horizontally in a circle described from the centre C′, with the diameter F′ G′. The arcs I′ F′ A′ and H′ G′ B′ are the only parts of this circle which require to be drawn. In the vertical projection the extreme points K, L, A, B need only be found, for the lines of intersection are here projected straight.

To describe the curves formed by the intersection of a cylinder with the frustum of a cone, the axes of the two solids cutting each other at right angles (Fig. 277).

The projections of the solids are laid down in the figure precisely as in the preceding example. The intersections of the outlines in elevation furnish four points in the curves of penetration. These points are all projected horizontally upon the line A′ B′. Now pass a plane, as *a b*, horizontally through both solids; its intersection with the cone will be a circle of the diameter *c d*, while the cylinder will be cut in two parallel straight lines, represented in the elevation by *a b*, and whose horizontal projection may be determined in the following manner: Conceive a vertical plane, *f g*, cutting the cylinder at right angles to its axis, and let the circle *g e f* thereby formed be described from the intersection of the axes of the two solids; the line *j h* will now represent, in this position of the section, the distance of one of the lines sought from the axis of the cylinder. Set off this distance on both sides of the point A′, and through the points *k* and *a′* thus obtained, draw straight lines parallel to A′ B′; the intersections of these lines with the circle drawn from the centre C′ of the diameter *c′ d′* will give four points *m′*, *p′*, *n*, and *o*, which, being projected vertically upon *a b*, determine two points, *m* and *p*, in the curves required.

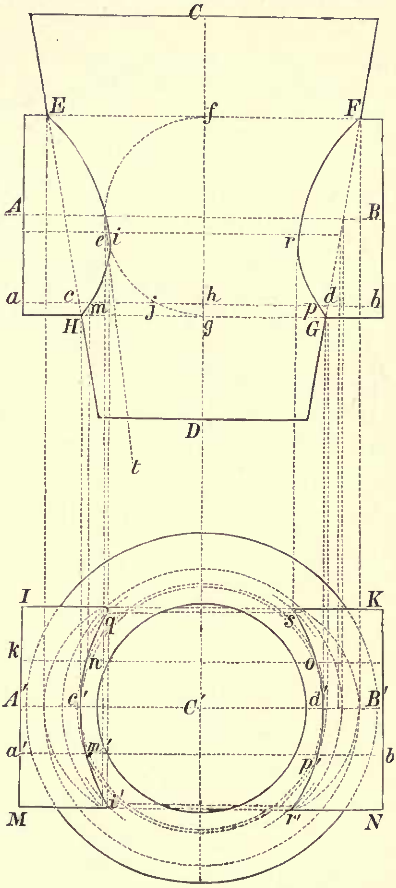


Fig 277.

In order to obtain the vertices or adjacent limiting points of the curves, draw from the vertex of the cone a straight line, *t e*, touching the circle *g e f*, and let a horizontal plane be supposed to pass through the point of contact *e*. Proceed according to the method given above to determine the intersections of this plane with each of the solids in question; the four points *i′*, *r′*, *q*, and *s*, projected vertically upon the line *e r*, determine the vertices *i* and *r* required.

THE HELIX.

A *helix* is the curve described upon the surface of a cylinder by a point revolving round it, and at the same time moving parallel to its axis by a certain invariable distance during each revolution. This distance is called the *pitch* of the helix or screw.

Required to construct the helical curve described by the point A′ *upon a cyl-*

inder projected horizontally in the circle A′ C′ F′, *the pitch being represented by the line* A^1 A^3 (Fig. 278).

Divide the pitch A^1 A^3 into any number of equal parts, say eight; and through each point of division, 1, 2, 3, etc., draw straight lines parallel to the ground line. Then divide the circumference A′ C′ F′ into the same number of equal parts; the points of division, A′, B′, C′, E′, F′, etc., will be the horizontal projections of the different positions of the given point during its motion round the cylinder. Thus, when the point is at B′ in the plan, its vertical projection will be the point of intersection B of the perpendicular drawn through B′ and the horizontal drawn through the first point of division. Also, when the point arrives at C′ in the plan, its vertical projection is the point C, where the perpendicular drawn from C′ cuts the horizontal passing through the second point of division, and so on for all the remaining points. The curve A^1 B C E F A^3, drawn through all the points thus obtained, is the helix required.

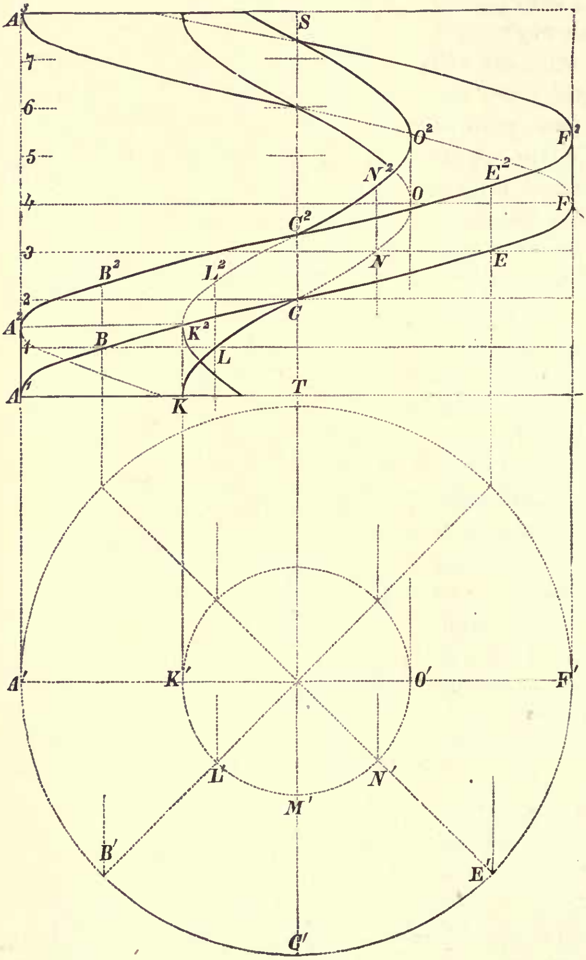


Fig. 278.

To draw the vertical elevation of the solid contained between two helical surfaces and two concentric cylinders (Fig. 278).

A *helical surface* is generated by the revolution of a straight line round the axis of a cylinder, its outer end moving in a helix, and the line itself forming with the axis a constant and invariable angle.

Let A′ C′ F′ and K′ M′ O′ represent the concentric bases of the cylinders, whose common axis S T is vertical; the curve of the exterior helix A^1 C E F A^3 is the first to be drawn according to the method above shown. Then, having set off from A^1 to A^2 the thickness of the required solid, draw through A^2 another helix equal and similar to the former. Now construct, as above, similar helices, K C O and K^2 C^2 O^2, of the same pitch as the last, but on the interior cylinder.

The lines A′ K′, B′ L′, C′ M′, etc., represent the horizontal projections of the various positions of the generating straight line, which, in the present example, has been supposed to be horizontal; and these lines are projected vertically at A¹ K, B L, etc.

In the position A¹ K the generating line is projected in its actual length, and at the position C′ M′ its vertical projection is the point C. The same remark applies to the generatrix of the second helix.

To determine the vertical projection of the solid formed by a sphere moving in a helical curve (Fig. 279).

Let A′ C′ E′ be the base of a cylinder, upon which the centre point C′ of a sphere whose radius is *a*′ C′ describes a helix, which is projected on the vertical plane in the curve A C E J, determined as before. From the various points A, B, C, D . . . . . , in this curve, as centres, describe circles with the radius *a*′ C′; these denote the various positions of the sphere during its helical motion; and, if lines be drawn touching them, the curves thereby formed will constitute the figure required. One of these curves disappears at O, but reappears again at I. The exterior and interior circles of the plan represent the horizontal projection of the solid in question.

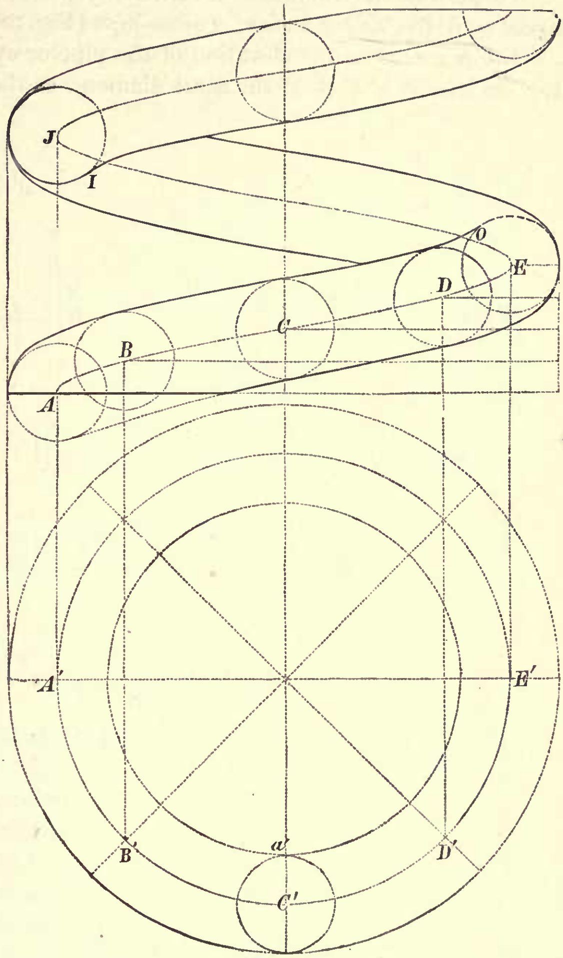


FIG. 279.

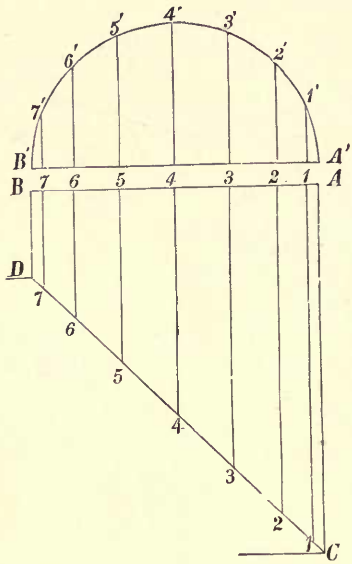
The *conical helix* differs from the cylindrical one in that it is described on the surface of a cone instead of on that of a cylinder; but the construction differs but slightly from the one described. By following out the same principles, helices may be represented as lying upon spheres or upon any other surfaces of revolution.

DEVELOPMENT OF SURFACES.

The *development* of the surface of a solid is the drawing or unrolling on a plane the form of its covering, the form that cut out of paper would exactly fit and cover the surface of the solid. Frequently, in practice, the form of the surface of a solid is found by applying paper or thin sheet-brass directly to the solid and cutting it to fit.

To develop the surface of a cylinder formed by the intersection of another equal cylinder, as the knee of a stove-pipe (Fig. 280).

Let A B C D be the elevation of the pipe or cylinder. Above A B describe the semicircle A′ 4′ B′ of the same diameter as the pipe; divide this semicircle



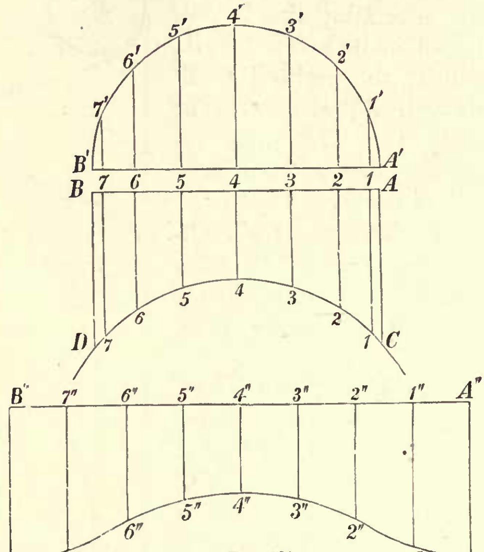


Fig. 281.

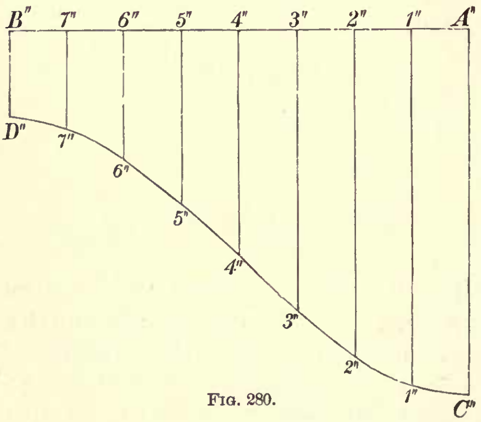


Fig. 280.

into any number of equal parts, eight for instance; through these points, 1′, 2′, 3′, etc., draw lines parallel to the side A C of the pipe, and cutting the line C D of the intersection of the two cylinders. Lay off A″ B″ equal to the semicircle A′ 4′ B′, and divided into the same number of equal parts; through these points of division erect perpendiculars to A″ B″, and on these perpendiculars lay off the distances A″ C″, 1″ 1″, 2″ 2″, 3″ 3″, and so on, corresponding to A C, 1 1, 2 2, 3 3, etc., in preceding figure. Through the points C″, 1″, 2″ . . . . . , D″, draw connecting lines, which gives but one half of the surface of the pipe, the other being exactly similar to it.

To develop the surface of a cylinder intersected by another cylinder, as in the formation of a T-*pipe* (Fig. 281).

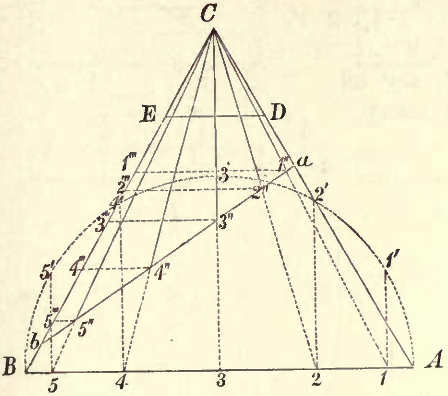
The construction is similar to the preceding.

To develop the surface of a right cone (Fig. 282).

From C′ as a centre, with a radius, C′ A′, equal to the inclined side A C of the cone, describe an arc of a circle, and on this arc lay off the distance A′ B′ A″, equal to the circumference of the base of the cone; connect A′ C′ and C′ A″, and A′ B′ A″ C′ is the developed surface required.

To develop the surface of the frustum of a cone, D A B E (Fig. 282).

D′ E′ D″ is the development of the cut-off cone C D E as shown by the preceding construction, and A′ B′ A″ D″ E′ D′ the developed surface of the frustum.



To develop the surface of a frustum of a cone, when the cutting plane a b is inclined to the base (Fig. 282).

On A B, the base, describe the semicircle A 3′ B; divide the semicircle into any number of equal parts, six for instance; from each point of division, 1′, 2′, 3′, 4′, 5′, let fall perpendiculars to the base at 1, 2, 3, 4, 5; connect each of these last points with

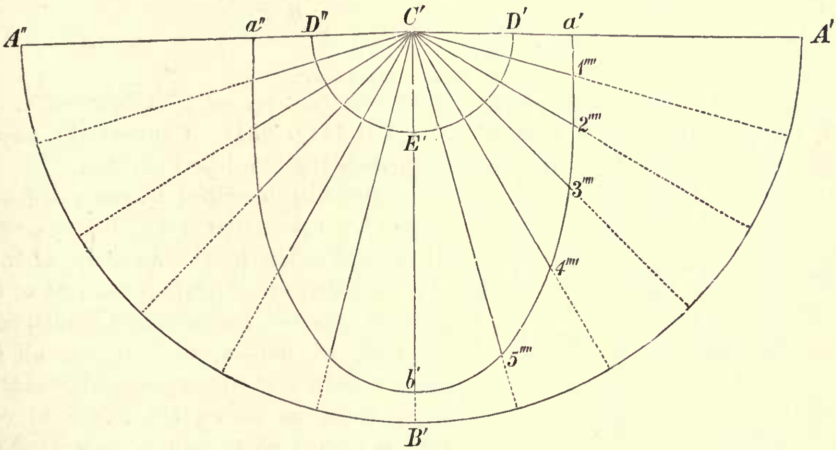


FIG. 282.

the apex C. Divide now the arc A′ B′ A″, equal to the circumference of the base A 3 B, into twelve equal parts; each of these parts by the construction is equal to the arc A 1′, 1′ 2′; connect these points of division with the point C′; on C′ A′ take C′ *a*′ equal to C *a*, *a* being the point at which the plane cuts the inclined side of the cone; in the same way on C′ B′, lay off C′ *b*′ equal to C *b*.

All the lines connecting the apex C with the base, included within the two inclined sides, are represented as less than their actual length, and must be projected on the inclined sides to determine their absolute dimensions; project, therefore, the points 1″, 2″, 3″, 4″, 5″, at which the cutting plane intersects the lines C 1, C 2, C 3, C 4, C 5, by drawing parallels to the base through these points to the inclined side C B. Now lay off C′ 1‴′, C′ 2‴′, etc., equal to C 1′ ″, C 2′ ″, etc.; connect the points *a*′, 1″, 2″, ... *b*′, ... *a*″, and *a*′ A′ B′ A″ *a*″ *b*′ is the developed surface.

To develop the surface of a sphere or ball (Figs. 283, 284).

The surface can not be accurately represented on a plane, but only approximately by gores. Let C A B (Fig. 284) be the eighth of a hemisphere; on C D describe the quarter circle D A *c*; divide this arc into any number of equal parts, six for instance; from the points of division 1, 2, 3, . . . let fall perpendiculars on C D, and from the intersections with this line describe arcs 1′ 1″, 2′ 2″, 3′ 3″, . . . cutting the line C B at 1″, 2″, 3″, ... ; on the straight line C′ D′ (Fig. 283), lay off C′ D′ equal to the arc D A *c*, with as many

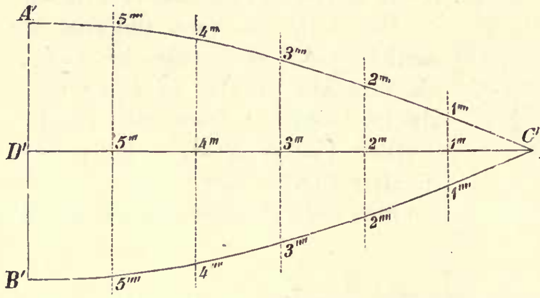


FIG. 283.

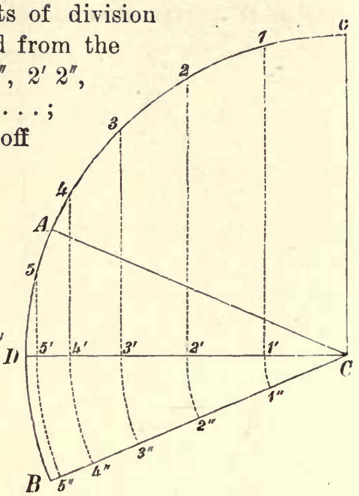


FIG. 284.

equal divisions; then from either side of this line lay off 1‴ 1⁗, 2‴ 2⁗ . . . . D′ B′ equal to the arcs 1′ 1″, 2′ 2″ . . . . D B (Fig. 284). Connect the points C′, 1⁗, 2⁗, ... and C′ A′ B′ is approximately the developed surface.

In the preceding demonstrations the forms are described to cover the surface only; in construction, allowance is to be made for lap by the addition of margins on each side. And on account of the difficulty, in the formation of hemispherical ends of boilers, of bringing all the gores together at the apex, it is usual to make them, as shown (Fig. 285), by cutting short the gores, and surmounting the centre with a cap-piece.

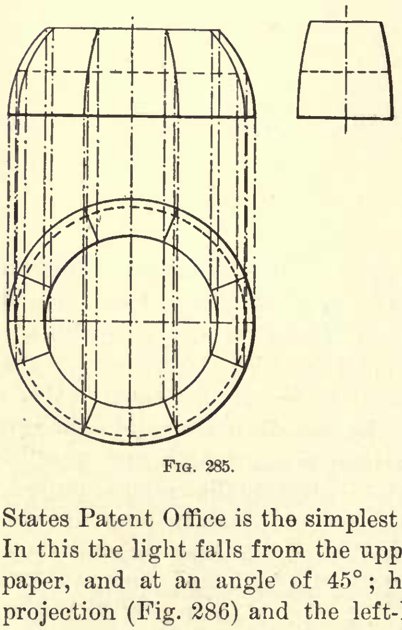


FIG. 285.

For small boilers and air chambers the spherical ends are dished.

SHADE LINES.

The effectiveness of outline drawings is increased by the use of shade lines; the method is wholly conventional. The rule used in preparing drawings for the United States Patent Office is the simplest and the one generally used in this country. In this the light falls from the upper left-hand corner of the sheet of drawing-paper, and at an angle of 45°; hence, the right-hand and lower edge of a projection (Fig. 286) and the left-hand and upper edge of a recess appear in heavy lines (Fig. 287). In the case of a circular object the shade is ter-

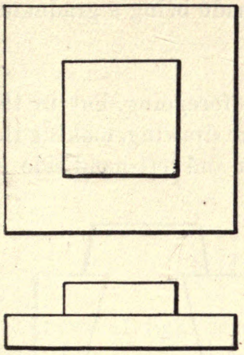


FIG. 286.

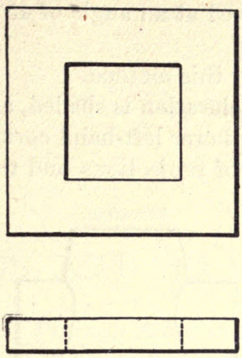


FIG. 287.

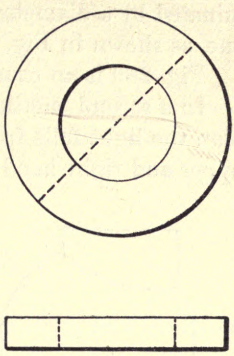


FIG. 288.

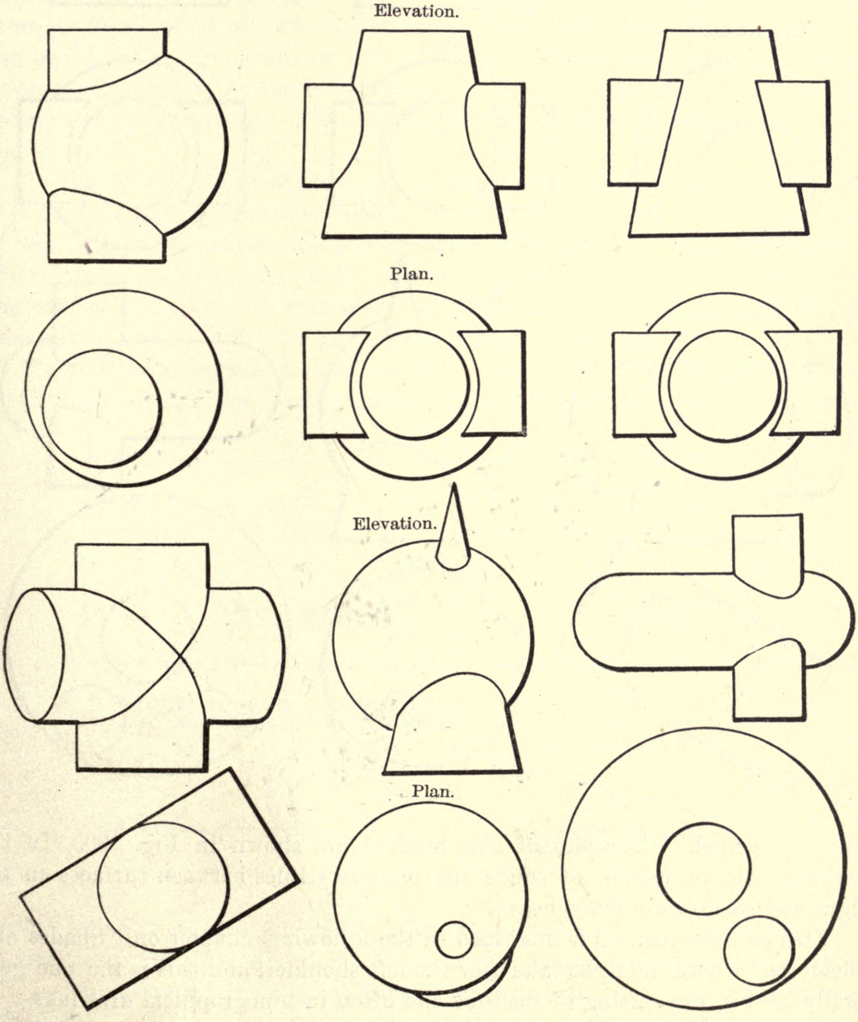


FIG. 289.

minated by a diameter inclined at an angle of 45°, the shade being a graduated one, as shown in Fig. 288.

Fig. 289 is an example of this method.

In a second method the elevation is shaded, as in the foregoing, but in the plan the light falls from the lower left-hand corner of the drawing, making the upper and right-hand edges of projections and the lower and left-hand side of

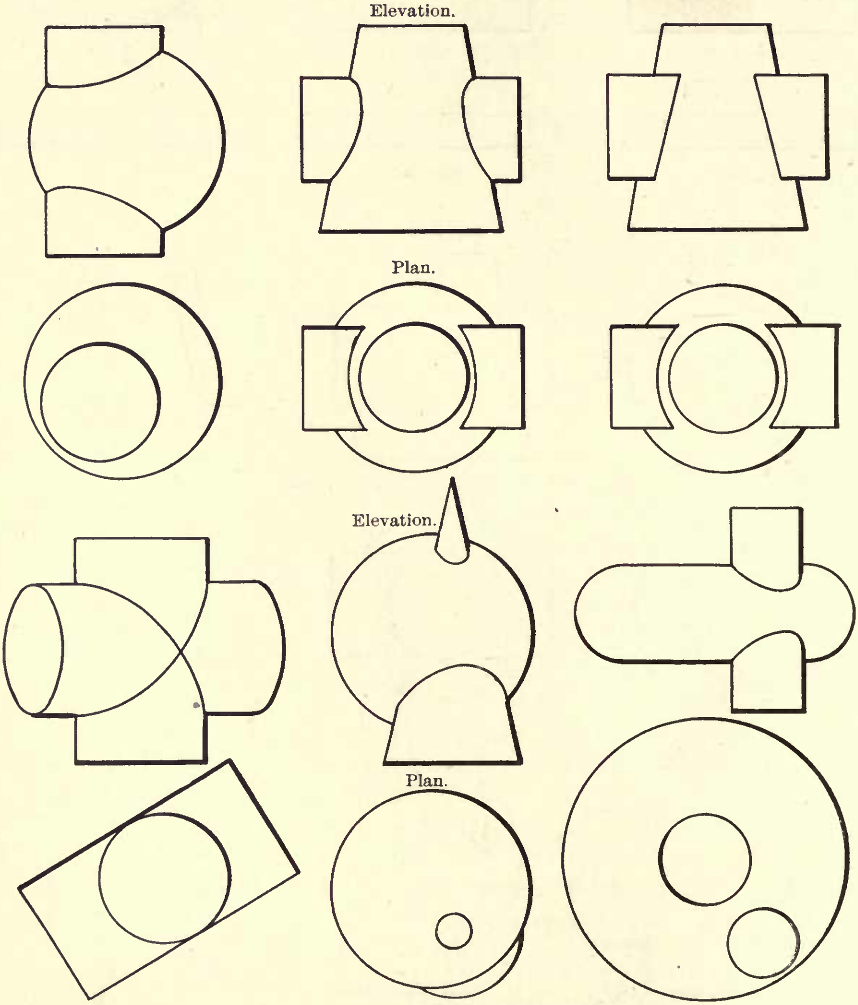


Fig. 290.

recesses shaded. Examples of this method are shown in Fig. 290. In the orthographic projection of solids the boundary lines between surfaces in the light and shadow are made heavy.

Still another method is described in the following chapter on "Shades and Shadows," where the light falls over the left shoulder, and this is the one generally used in the casting of shadows and often in topographical drawings.

SHADES AND SHADOWS.

Rays of light are diffused through space in straight lines, and the direct rays of the sun may be regarded as parallel. The light on an object may come directly from the source of light, or by reflection from other objects or surfaces exposed to light. The surfaces of an object under direct light are the most strongly illuminated, while other surfaces, from their form or position receiving less light, are in shade or direct shadow. Shadows are cast by an object upon another object by the interception of light, or upon any surface by projections or undercutting. The limit of the line of direct shadow is called the *line of shade.*

In the delineation of shadows, the most convenient mode of regarding the rays of light is, in all cases, as falling in the direction of the diagonal of a cube, of which the sides are parallel to the planes of projection. The projections of the ray form each an angle of 45° with the ground line. This is not true of the ray itself in space, for that forms an angle of 54° 44′ with the ground line, and an angle of 35° 16′ with each of the planes of projection.

To find the shadow of a point, as A, A′ (Fig. 291), on either plane of projection, the vertical, for instance, draw a line through the horizontal projection

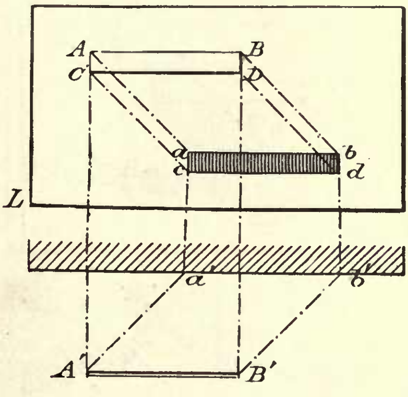


Fig. 291.

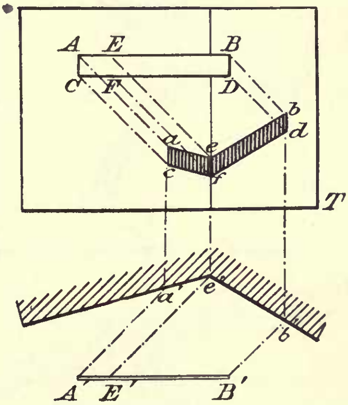


Fig. 292.

of the point A′ at an angle of 45° with the ground line, L T, and at the point of intersection of those lines, a', erect a perpendicular to intersect the vertical projection of the ray through A, which will be at the point a, the shadow in question.

The converse of this method determines the shadow of the point on the horizontal plane. The shadow thrown by B of the given line falls at *b*, and the straight line *a b*, which joins these two points, is the shadow required.

The line *a b* is equal and parallel to the given line A B; this results from the circumstance that the latter is parallel to the vertical plane.

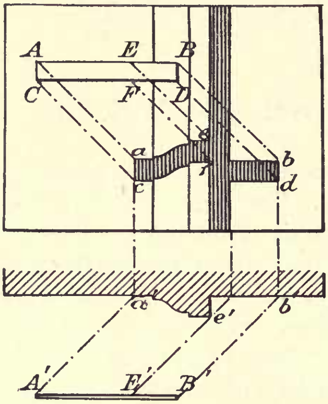


FIG. 293.

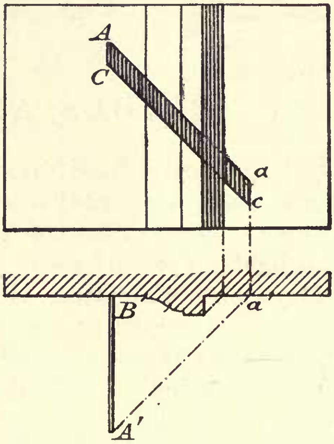


FIG. 294.

The shadows of a rectangular slip of paper or wood, A B C D, cast upon the same vertical plane and parallel to it, is the rectangle A B C D (Fig. 291).

When the object is not parallel to the given plane, the shadow cast is no longer a figure equal and similarly placed, but the method of determining it is similar (Fig. 292).

By a combination of the foregoing principles, the shadow of a slip of moulding placed on the vertical plane (Fig. 293) is determined.

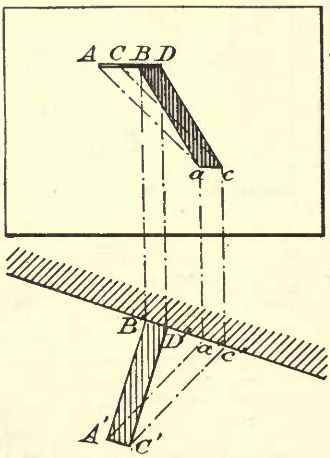


FIG. 295.

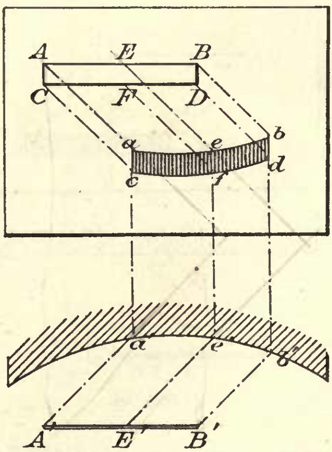


FIG. 296.

Fig. 294 shows the shadow cast by a rectangular slip of cardboard at right angles to the vertical plane; Fig. 295, the shadow cast by a slip parallel to the ground plane and at right angles to the vertical plane, which is itself set at a horizontal angle; Fig. 296, the shadow of the slip cast on a concave surface; Fig. 297, the shadow cast on a vertical plane by a circle parallel to it; Figs. 298, 299, 301, the shadows on a vertical plane by circles in various positions

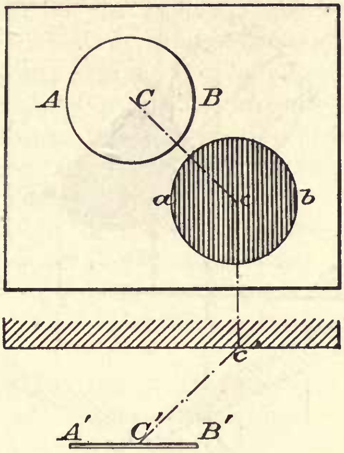


Fig. 297.

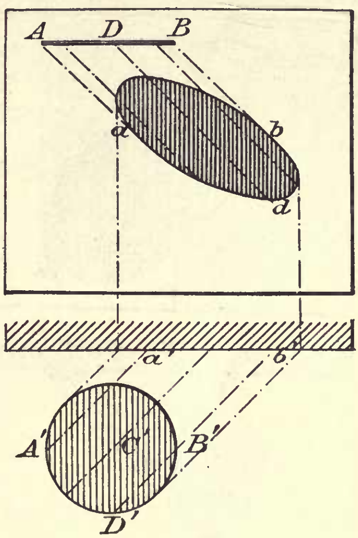


Fig. 298.

relative to this plane, in all which the shadow assumes the form of an ellipse; Fig. 300, the shadow cast by a circle horizontal to the ground plane on a vertical convex surface; Fig. 302, the shadow cast by a circle parallel to the vertical

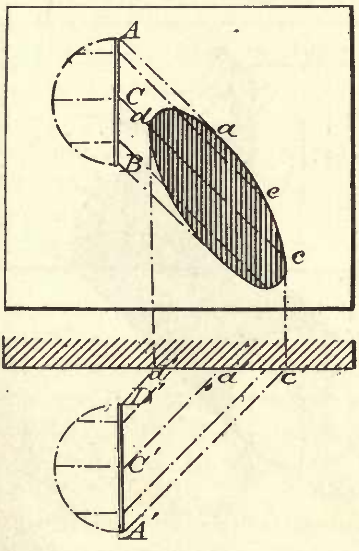


Fig. 299.

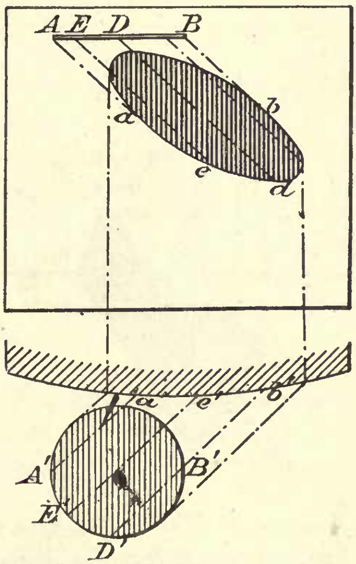


Fig. 300.

plane of the projection, the shadow being thrown upon two plane surfaces at an angle to each other.

In every drawing where the shadows are to be inserted, it is of the utmost importance that the projections which represent the object whose shadow is required, and the surface upon which this shadow is cast, should be exactly de-

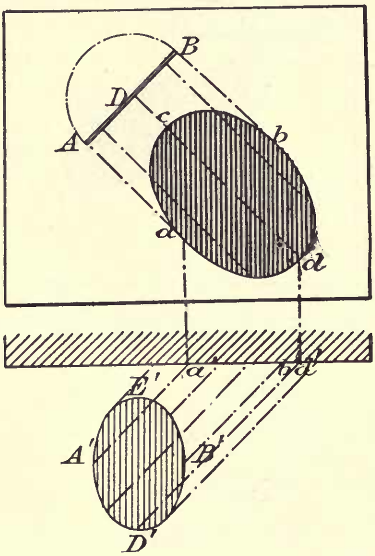


Fig. 301.

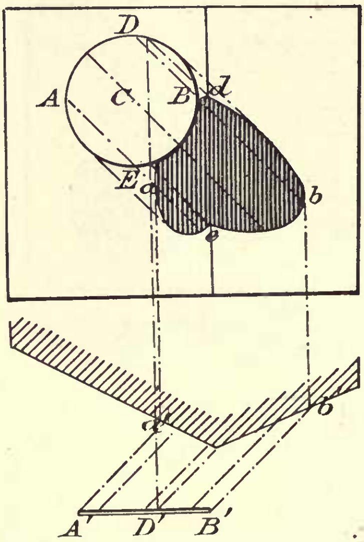


Fig. 302.

fined by lines drawn lightly in India ink, and, to prevent mistakes, erase all pencil marks before proceeding to the operations determining the lines of shadow.

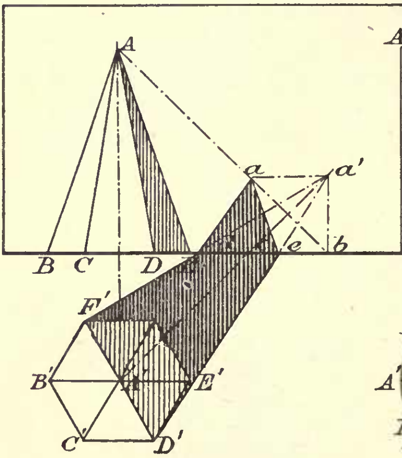


Fig. 303.

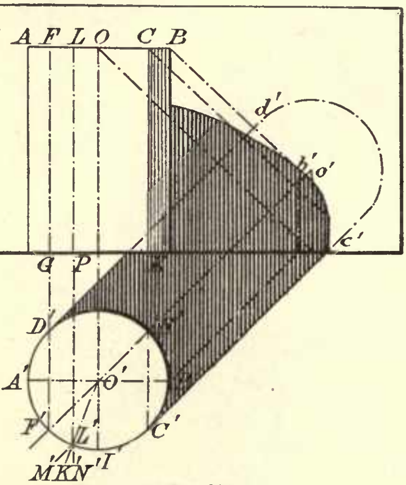


Fig. 304.

In Fig. 303 the three sides of the hexagonal pyramid, A′ B′ F′, A′ B′ C′, and A′ C′ D′, alone receive the light; consequently the edges A′ F′ and A′ D′ are the lines of shade. To determine the shadow cast by these two lines, draw from the projections of the vertex of the pyramid the lines A *b* and A′ *a*′ parallel to the ray of light; then erect at the point *b* a perpendicular to the ground line, which gives at *a′* the shadow of the vertex on the horizontal plane on the other side of the ground line; and finally join this last point, *a′*, with the points D′ and F′; the lines D′ *a′* and F′ *a′* are the outlines of the required shadow on the horizontal plane. But, as the pyramid is sufficiently near the vertical plane to throw a portion of its shadow upon it, this portion may be found by erecting at the point *c*, where the line A′ *a′* cuts the ground line, a perpendicular *c a*, intersecting the line A *b* in *a*; the lines *a d* and *a e* joining this point with those where the horizontal part of the shadow meets the ground line, will be its outline upon the vertical plane.

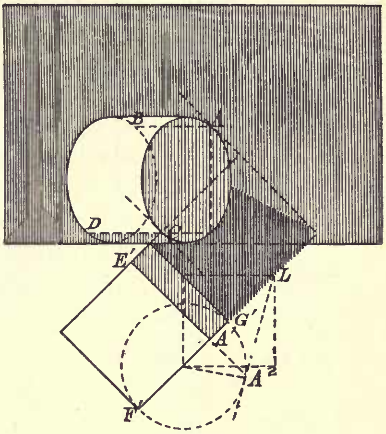


Fig. 305.

Fig. 304 represents the shade on a cylinder placed vertically, and its shadow cast on two planes of projection.

Draw the tangents D′ *d′* and C′ *c′* parallel to the rays of light; these are the outlines of the shadow cast upon the horizontal plane. Through the point of contact C′ draw the vertical line C C′; this line denotes the line of shade upon the surface of the cylinder.

If the shadow of this cylinder were entirely cast upon the horizontal plane, it would terminate in a semicircle drawn from the centre *o′*, with a radius equal to that of the base; but a part of the shadow of the upper part is thrown upon the vertical plane, and its outline is defined by an ellipse drawn in the manner indicated in Fig. 298.

Fig. 305 represents the line of shade and the shadow of a horizontal cylinder inclined to the vertical plane. The construction in this case is the same as that explained by Fig. 304. Of the horizontal lines of shade, C D alone is visible in the elevation, while A B alone is seen in the plan, where it may be found by drawing a perpendicular from A meeting the base F′ G′ in A′. The line A′ E′, drawn parallel to the axis of the cylinder, is the line of shade required. Project the shadow of the line A B on the vertical plane, as in previous examples, to define the outline of the shadow of the cylinder.

The example here given presents the particular case in which the bases of the cylinder are parallel to the direction of the rays of light. In this case, to determine the line A′ E′ lay off the angle A′ L A^2, equal to 35° 16′, so that the side A^2 L shall be tangent to the circle F′ A^2 G′, representing the base of the cylinder laid down on the horizontal plane; through the point of tangency, A^2, draw a line, A′ E′, parallel to the axis of the cylinder, for the line of shade.

Fig. 306 represents a cylinder upon which a shadow is thrown by a rectangular cap, of which the sides are parallel to the planes of projection. The shadow in this case is derived from the edges A′ D′ and A′ E′, the first of which, being perpendicular to the plane of projection, gives a straight line at an angle of 45° for the outline of its shadow, whereas the side A′ E′ being parallel to that plane, its shadow is determined by a portion of a circle, *a b c*, described from the centre, O.

If the cap be hexagonal (Fig. 307), or circular (Fig. 308), the mode of construction is similar. Commence by finding the points which indicate the main

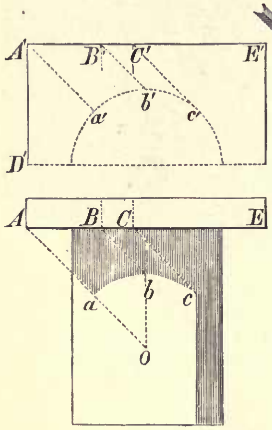


Fig. 306.

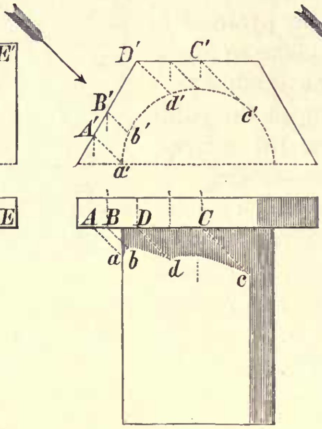


Fig. 307.

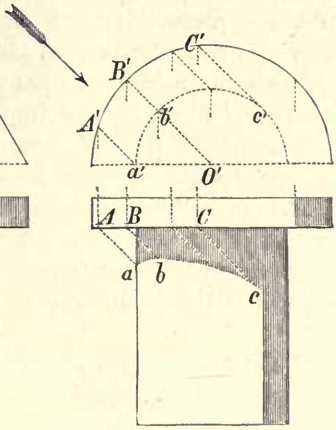


Fig. 308.

direction of the outline. To ascertain the point *a* at which the shadow commences, draw from *a*′ the line *a*′ A′ at an angle of 45°, and projected vertically to *a* A. Then the highest point *b* (Fig. 308) is determined by the intersection of the radius O′ B′, drawn parallel to the ray, with the circumference of the base of the cylinder on which the shadow is cast; the point *c*, where the outline of the cast shadow intersects the line of shade, is determined by a like process.

In Figs. 309, 310, and 311 a hexagonal prism is substituted for the cylinder.

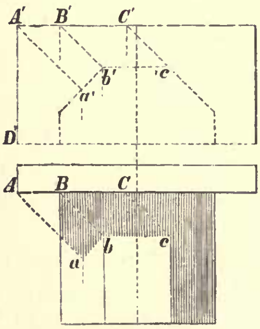


Fig. 309.

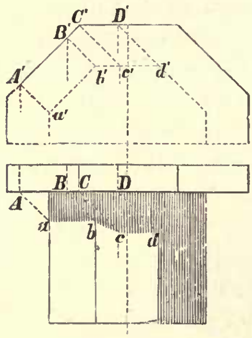


Fig. 310.

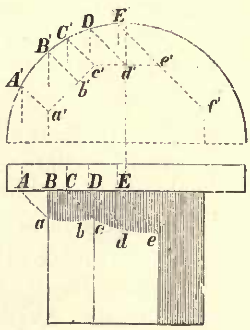


Fig. 311.

Fig. 312 shows the section of a steam-cylinder by a plane passing through its axis, with its piston and rod in full. To define its shadows, conceive the piston P to be removed; the shadow cast into the interior of the cylinder will then consist of that projected by the vertical edge B C, and by a portion of the horizontal edge B A. To find the first, draw through B′ a line, B′ *b*′, at an angle of 45° with B′ A′; the point *b*′, where this line meets the interior surface of the cylinder, being projected vertically, gives the line *b f* as the outline of the shadow sought. Then, parallel to the direction of the light, draw a tangent at F′ to the inner circle of the base; its point of contact, being projected to F in the elevation, marks the commencement of the outline of the shadow cast by the upper edge of the cylinder. The point *b*, where it terminates, will be the intersection of the straight line *f b*, already determined, with a ray, B *b*, from the upper extremity of the edge B C; and any intermediate point in the curve, as *e*, may be found in the same way. The outline of the shadow required will then be the curve F *e b* and the straight line *b f*. Insert the piston P and its rod T into the cylinder, as shown. The lower surface of the piston will then cast a shadow upon the interior surface of the cylinder, of which the outline D *d h o* may be formed as above. The piston-rod T, being cylindrical and vertical, casts a shadow into the interior of the cylinder, consisting of the rectangle *i j l k* drawn parallel to the axis.

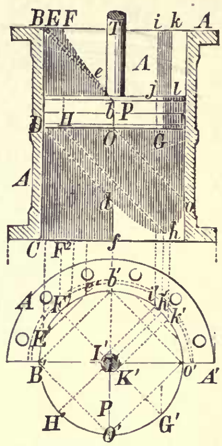


Fig. 312.

In Fig. 313 is shown the interior of a cylinder closed at the top with the exception of a central aperture through which the light is admitted. Follow previous construction, but to determine what parts of the upper and lower edges of the central aperture cast their shadows into the interior of the cylinder, establish the position of the point of intersection, *c*, of the two curves *b c f* and *a c e*, shadows of these edges, which is the cast shadow of the lowest point, C, in the curve D C, previously laid down in the circular opening of the cover.

For the shadow cast in the interior of a cylinder, in section, inclined to the horizontal plane (Fig. 314), on a convenient part of the paper draw the diag-

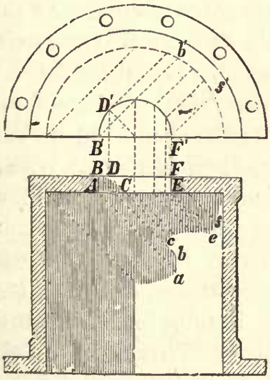


Fig. 313.

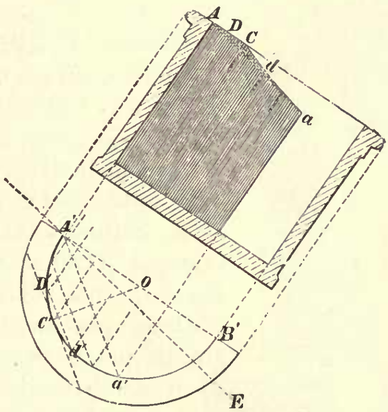


Fig. 314.

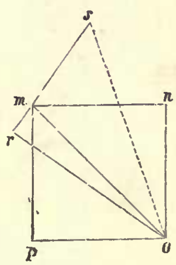


Fig. 315.

onal *m o*, parallel to the line of light A′ E, and construct a square, *m n o p* (Fig. 315); from one of the extremities, *o*, draw the line *o r*, parallel to A′ B′, and through the opposite extremity, *m*, draw a perpendicular, *r s*, to this line, and set off on the perpendicular the distance *r s* equal to the side of the square, and join *s o*. Now, draw through the point A′, in the original figure, a line, A′ *a*′,

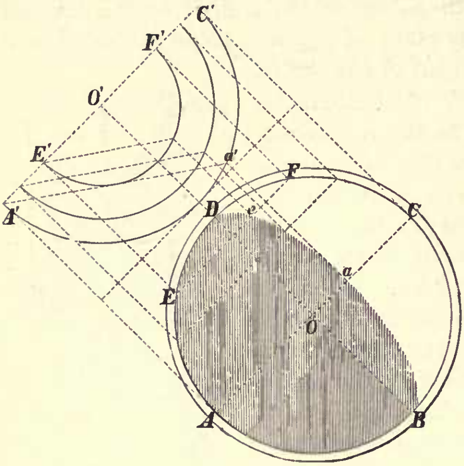


Fig. 316.

parallel to *s o*, intersecting the circle A′ *a′* B′ in the point *a′*, which, being projected by a line parallel to the axis of the cylinder, and meeting the line drawn from A at an angle of 45°, gives the first point *a* in the curve C *d a*. The other points are obtained in like manner, by drawing at pleasure other lines, such as D′ *d′*, parallel to A′ *a′*.

For the shadow cast into the interior of a hollow hemisphere (Fig. 316), let A B C D represent the horizontal projection of a concave hemisphere. Draw through the centre a line, A C, perpendicular to the ray of light; the points B and D will at once give the extremities of the curves sought. On any point of B D produced, as O′, construct the semicircle A′ *a′* C′ with a radius, A′ O′, equal to A O. At A′ draw the line A′ *a′*, making an angle of 35° 16′ with A′ C′. The angle made by the ray of light in space with the planes of projection, *a′*, the point of intersection of the line with the semicircle, projected to *a*, gives a point of the outline of the shadow. Similar sections, as E F parallel to A C, will give other points. As this outline is an ellipse whose axes are B D and twice O *a*, it may be constructed, when the point *a* is determined, by the ordinary methods for ellipses.

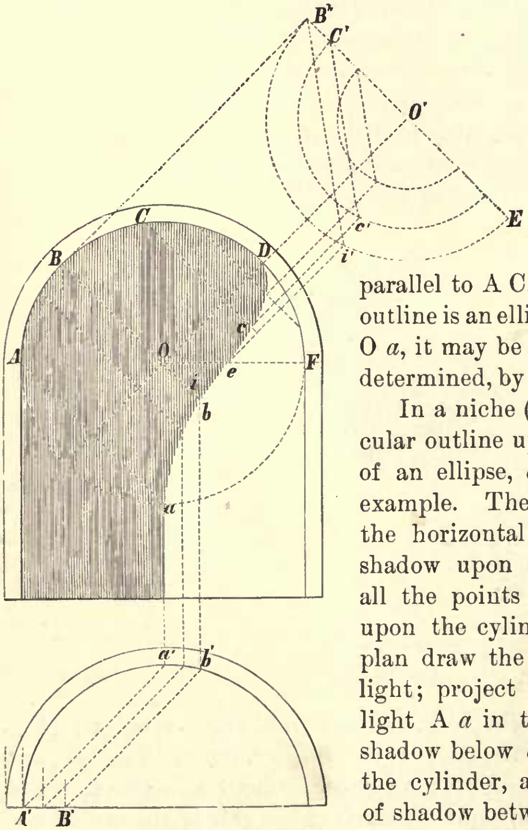


Fig. 317.

In a niche (Fig. 317) the shadow of the circular outline upon the spherical portion is part of an ellipse, *e c* D, found as in the previous example. The point *e*, where this ellipse cuts the horizontal diameter A F, limits the cast shadow upon the spherical surface; therefore all the points beneath it must be determined upon the cylindrical part. Through A′ in the plan draw the line A′ *a′* parallel to the ray of light; project *a′* till it intersects the line of light A *a* in the elevation at *a*. The line of shadow below *a* is the shadow of the edge of the cylinder, and is a straight line. The line of shadow between *a* and *e* is the outline of the circular part falling on a cylindrical surface.

exterior of the ring. Draw rays of light through $E^4 F^4$, draw a diagonal ray tangent at e, transfer $e^1 C^1$ to $C^2 e^2$; $C^3 e^2$ will be half the minor axis, and $E^2 F^2$ the major axis of the ellipse which defines the limit of shadow for the interior of the ring.

The shadow on the surface of a grooved pulley (Fig. 320) is cast by the circumference of the edge A′ B′. Draw central lines through C of the plan and describe a circle C b with the radius of the least diameter of the pulley; from b draw a line b a at an angle of 45°; project a to a', and from a' draw a' b' as a limiting point in the line of shade, for another point draw a horizontal line from b' intersecting the centre line at b^2; project c and d on the plan to the elevation, and intersect the projection of d at d' by a ray of light from c'; d' is the highest point in the curve. Take any horizontal line E F in the elevation and describe from C on the plan an arc with a radius equal to one half this line; draw from C′ a ray intersecting E F at e, which project to e', and from e' as a centre describe an arc with a radius equal to C B; the point of intersection of this arc with the circumference of the plan E F is projected to f'. The limit of shade is then drawn through the points b', d', b^2, f', and g.

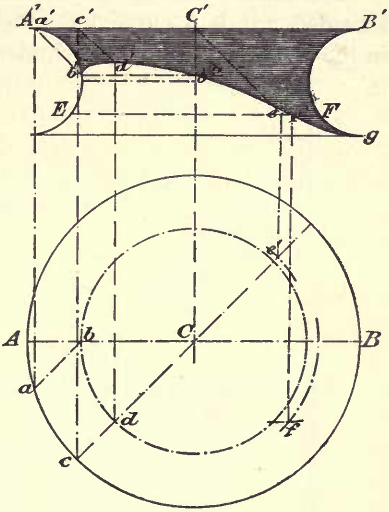


Fig. 320.

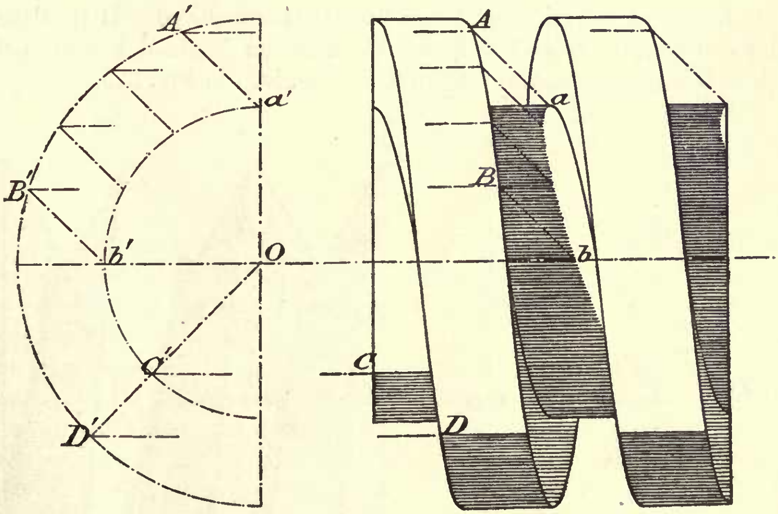


Fig. 321.

Fig. 321 represents the projections of a screw with a single square thread, and placed in a horizontal position, A′ a' being the direction of the ray of light. The shadow is simply that cast by the outer edge, A B, of the thread upon the surface of the inner cylinder; its outline is delineated in the same manner as in treating of a cylinder surmounting another of smaller diameter (Fig. 308).

The shadow cast by the helix A B C upon the concave surface of the square-threaded nut is a curve, *a* C (Fig. 322), determined in the same way as that in the interior of a hollow cylinder. The same rule applies to the edges A A²

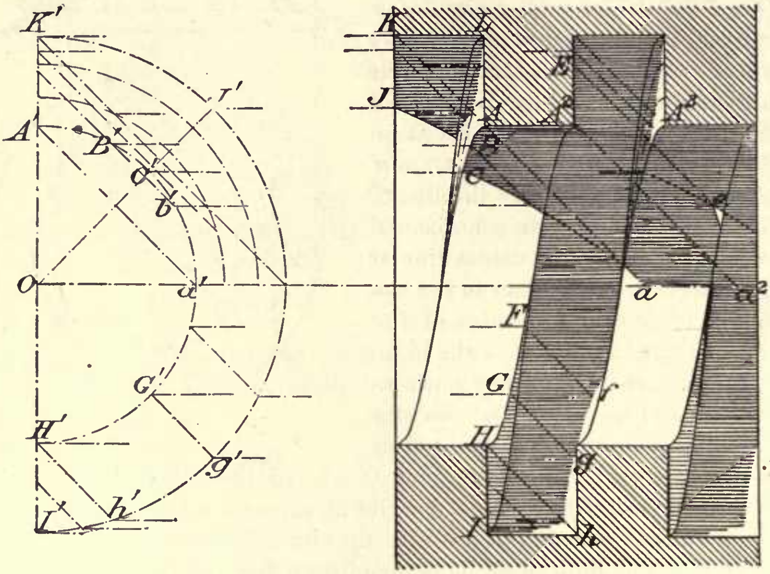


FIG. 322.

and A² E, as well as to those of the helix F G H and the edge H I. The shadow of the two edges J K and K L, thrown upon an inclined helical surface, of which A L is the generatrix, follows the rules given in Fig. 323.

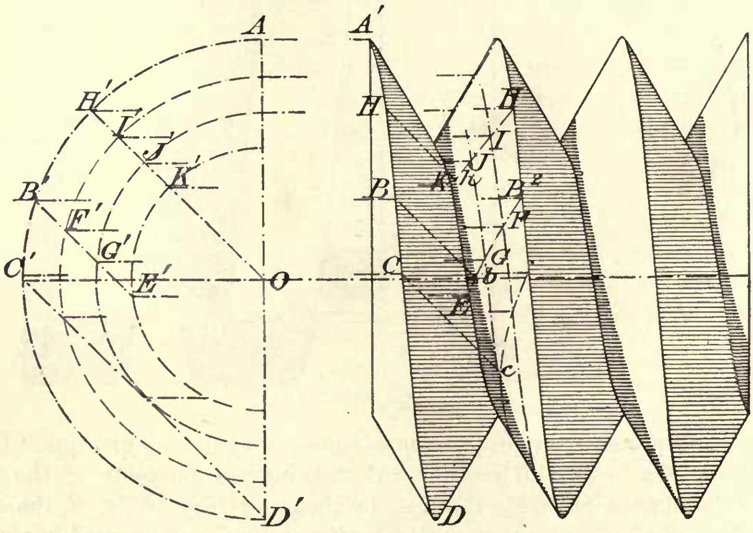


FIG. 323.

In the case of a triangular-threaded screw (Fig. 323) the outer edge, A′ C D, of the thread projects its shadow upon a helical surface inclining to the left, of which the generatrix is known.

Describe from the centre O a number of circles representing the bases of so many cylinders, on the surfaces of which suppose helical lines to be traced of the same pitch as those which form the exterior edges of the screw. Draw any line parallel to the ray of light, as B′ E′, cutting the circles described in the plan in the points B′, F′, G′, E′, which are projected to their corresponding helical lines in the elevation at B^2, F, G, and E. Transferring the point B′ to its appropriate position B on the edge A′ C D, and drawing through the latter a line, B *b*, at an angle of 45°, its intersection with the curve B^2 G E will give one point in the curve of the shadow required. By constructing other curves, as H J K, the remaining points in the curve, as *h*, may be found.

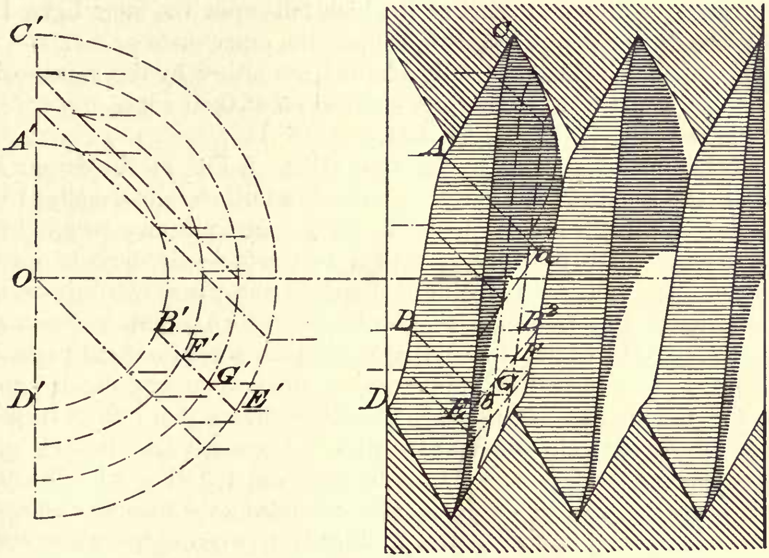


FIG. 324.

The same processes are requisite to determine the outlines of the shadows cast into the interior surfaces of the corresponding nut (Fig. 324). These shadows are derived not only from the helical edge, A B D, but also from that of the generatrix, A C.

The principles here laid down and illustrated are a sufficient guide for the delineation of the shades and shadows of nearly all ordinary forms and combinations of machinery and architecture. Students should not copy the figures as here represented, but should adopt some convenient scale somewhat larger, and construct drawings according to the description.

MANIPULATION OF SHADING AND SHADOWS, AND METHODS OF TINTING.

The intensity of a shade or shadow is regulated by the forms of bodies, the position that they occupy in relation to the light, and the distance from the eye.

Surfaces in the Light.—Flat surfaces wholly exposed to the light, and at all points equidistant from the eye, receive a uniform tint.

Of two parallel surfaces, the one nearer the eye receives a lighter tint. Every surface exposed to the light, but not parallel to the plane of projection, having no two points equally distant from the eye, receives unequal tints, gradually increasing in depth as the parts recede from the eye. Of two surfaces unequally exposed to the light, the one more nearly perpendicular to the rays receives the fainter tint.

Surfaces in Shade.—When a surface entirely in the shade is parallel to the plane of projection, it receives a uniform dark tint. Of two objects in the shade parallel to each other, the one nearer the eye receives the darker tint. On a surface in the shade, inclined to the plane of projection, the parts nearest the eye receive the deepest tint.

If two surfaces exposed to the light, but unequally inclined to its rays, have a shadow cast upon them, the part of it which falls upon the more lighted surface should be darker than where it falls upon the other surface.

The methods of shading generally adopted are either by the superposition of any number of flat tints, or by tints softened off at their edges.

(See Plates I, II, III, IV, V, XI, XII, and XV.)

In the shading of a prism by flat tints (Plate I, Fig. 4), the face, *a b c d*, being parallel to the plane of projection, receives a uniform tint usually of India ink or sepia. When the surface to be tinted is large, put on a very light tint first, and then go over the surface with a tint sufficiently dark to give the desired tone. The face, *b g h c*, being inclined to the plane of projection, receives a graduated tint from the line *b c* to the line *g h*. This is obtained by laying on a succession of flat tints. Divide the plan *b′ g′* into equal parts, as at the points 1′, 2′, and from these points project lines upon, and parallel to the sides of, the face *b g h c*. These lines should be drawn very lightly in pencil, as they merely serve to circumscribe the tints. A grayish tint is then spread over the portion of the face between the lines *b c* and 1, 1 (Fig. 2). When this is dry, the same tint is laid on again, and extended over the space comprised within the lines *b c* and 2, 2 (Fig. 3). A third tint, covering the whole surface *b c h g* (Fig. 4), imparts the desired graduated shade to that side of the prism. The number of tints in the graduated shade depends upon the size of the surface, and the depth of tint must vary according to the number used. The greater the number, the softer the appearance and the less harsh the lines which border the different tints. This method is preferable to that sometimes employed of first covering the whole surface *b g h c* with a faint tint, then putting on a second tint *b* 2 2 *c*, followed by a narrow wash *b* 1 1 *c*, because the outline of each wash remains untouched and presents a prominence and harshness.

The face *a d f e*, also inclined to the plane of projection, should, as it is entirely in the light, be covered by a series of much fainter tints than the surface *b g h c*, which is in the shade, darkening, however, toward the line *e f*. The gradation of tint is effected as on the face *b g h c*.

In shading a cylinder by means of flat tints (Figs. 5–12), the line of separation between the light and shade, *a b* (Fig. 6), is determined by the radius O *a′* (Fig. 5), drawn perpendicular to the rays of light R O. The part of the elevation of the cylinder which is in the shade is comprised between the lines *a b* and *c d*. This portion, then, should be shaded as a surface in the shade inclined to the plane of projection. All the remaining part that is visible of the cylinder presents itself to the light; but, in consequence of its curvature, the rays of light form angles varying at every part of its surface, which should receive a graduated tint. Determine the part of the surface that is most directly affected by the light, situated about the line *e i* (Fig. 12). The visual rays are perpendicular to the vertical plane and parallel to V O; the part which appears clearest to the eye may be limited by the line T O, which bisects the angle V O R. Project the points *e'* and *m'*, and draw the lines *e i* and *m n* (Fig. 12); the surface comprised between these lines will represent the lightest part of the cylinder.

This part should have no tint upon it whatever if the cylinder is polished —as a turned iron shaft or a marble column; but if the surface of the cylinder be rough, as a cast-iron pipe, then a very light tint, considerably lighter than on any other part, may be given it.

Divide the half-plan of the cylinder *f' m' a' c'* into any number of equal parts by faint pencil-lines, and begin the shading by laying a tint over *a c d b* (Fig. 6). When this is dry, put on a second tint covering the line *a b* of separation of light and shade, and extending over one division (Fig. 7). Proceed in this way, as shown in Figs. 6–12, until the whole of that part of the cylinder which is in the shade is covered. Treat in a similar manner the part *f e i g* (Fig. 12), and complete the operation by covering the whole surface of the cylinder, excepting the surface *e m n i*, with a very light tint.

In shading the frustum of a hexagonal pyramid (Plate II), the face *a b c d* should receive a uniform flat tint, as in Plate I, or the tint may be slightly deepened toward the top of the pyramid, as that surface is not quite parallel to the vertical plane.

The face *b g h c*, being inclined and in the shade, should receive a dark tint; darkest where it meets the line *b c*, and gradually becoming lighter as it approaches the line *g h*. To produce this effect, apply a narrow strip of tint at *b c* (Fig. 6), and then, qualifying the tint in the brush with a little water, join another strip to this, and finally, by means of another brush moistened with water, soften off this second strip toward the line 1, 1, which may be taken as the limit of the first tint.

When the first tint is dry, cover it with a second, which must be similarly treated, and should extend up to the line 2, 2 (Fig. 7). Proceed in this manner with other tints, until the whole face *b g h c* is shaded (Fig. 8). In the same way the face *e a d f* is to be covered, though with a considerably lighter tint, for the rays of light fall upon it almost perpendicularly.

The tint on these two faces should be slightly graduated from *e a* to *f d*, and from *c h* to *b g*; but this graduation may be disregarded until some proficiency in shading has been acquired.

In shading a cylinder by means of softened tints (Plate II), the boundary of each tint being indicated, as in Plate I, the first strip of tint must cover the line of extreme shade *a b*, and then be softened off on each side. Other successively wider strips of tint follow, and receive the same treatment as the one first put on.

If, after shading a figure by the foregoing method, any very apparent inequalities present themselves in the shades, such defects may be remedied, in some measure, by washing off excesses of tint with a brush or a damp sponge, and by supplying a little colour to those parts which are too light.

Dexterity in shading figures by softened tints is facilitated by practising upon large surfaces.

Whatman's best rough-grained drawing-paper is better adapted for receiving colour than any other. Of this paper, the double-elephant size is preferable, as it possesses a peculiar consistence and grain. The face of the paper to be used is the one on which the water-mark is read correctly.

The paper for a coloured drawing ought always to be damp-stretched (page 54).

The size of the brushes used depends upon the scale to which the drawing is made. Long, thin brushes, however, should be avoided. Those possessing corpulent bodies and fine points are to be preferred, as they retain a greater quantity of colour, and are more manageable.

Sable brushes are more durable and better than camel's hair, but more expensive. Economy may be practised by using camel's hair for the larger sizes and red or black sable for the smaller.

During the process of laying on a flat tint, if the surface be large, the drawing may be slightly inclined, and the brush well charged with colour, so that the lower edge of the tint may be kept moist until the whole surface is covered. In tinting a small surface, the brush should never have much colour in it, otherwise the surface will unavoidably present coarse, ragged edges, and an uneven appearance throughout.

All objects with curved outlines have a certain amount of reflected light imparted to them. Bodies, whatever may be their form, are affected by reflected light; but, with a few exceptions, this light is only appreciable on curved surfaces.

In proportion to its degree of polish or brightness is the amount of reflected light which a body spreads over adjacent objects, and also its own susceptibility of illumination under the reflection from other bodies. A polished steam-cylinder or a white porcelain vase both receives and imparts more reflected light than a rough casting or a stone pitcher.

Shade, even the most inconsiderable, ought never to extend to the outline of any smooth circular body. On a polished sphere the shade should be delicately softened off just before it meets the circumference, and, when the shading is completed, the body-colour intended for the sphere may be carried on to its outline. This will give a clearness to the part of the sphere influenced by reflected light which it could not have possessed if the shade-tint had been extended to its circumference. Very little shade should be suffered to reach the outlines even of rough circular bodies, lest the colouring present a harsh, unreal appearance.

Shadows become lighter as they recede from the bodies which cast them, and appear to increase in depth as their distance from the spectator diminishes. In Nature this increase is only appreciable at considerable distances. But it is important for the effective representation of machinery that the variation in the distance of each part from the spectator should strike the eye, and an exaggeration in expressing the varying depths of the shadows is one means of effect-ing that object. The shadows on the nearest and most prominent parts of a machine should be made as dark as colour can render them, the colourist being thus enabled to exhibit a marked difference in the shadows on the other parts of the machine as they recede from the eye. The same direction is applicable to shades. The shade on a cylinder, situated near the spectator, ought to be darker than on one more remote. As a general rule, the colour on a machine should become lighter as the parts on which it is placed recede from the eye.

Plates III and IV present some examples of finished shading and shadows of the solids given under the head of "Orthographic Projection."

The direction of the shades and shadows in the elevation is from the upper left-hand corner, and in the plan from the lower left-hand corner.

The shadow on a concave surface is darkest toward its outline, and becomes lighter as it nears the edge of the object. Reflection from the part of the surface on which the light falls causes this gradual diminution in the depth of the shadow, the greatest amount of reflection being opposite the greatest amount of light. No brilliant or extreme light should be left on concave surfaces, as such lights tend to render it doubtful whether the objects presented are concave or convex. After the body-colour has been put on, a faint wash should be passed very lightly over the whole concavity. This will not only modify and subdue the light, but soften asperities in the tinting, which are particularly unsightly on a concave surface.

The lightest part of a sphere is confined to a mere point, around which the shade gradually increases as it recedes. This point is not indicated on the figure because the shade-tint on a sphere ought not to be spread over a greater portion of its surface than is shown there. The very delicate and hardly perceptible progression of the shade in the immediate vicinity of the light-point should be effected by means of the body-colour, either by lightening it toward the light point for polished or light-coloured curved surfaces, or deepening the body-colour for unpolished surfaces from the light point until it meets the shade tint over which it is spread uniformly.

To shade a sphere effectively, put on two or three softened-off tints in the form of crescents converging toward the light-point, the first one being carried over the point of deepest shade.

A ring (Plate IV, Fig. 7) is a difficult object to shade. To change with accurate and effective gradation the shade from the inside to the outside of the ring, to leave with regularity a line of light upon its surface, and to project its shadow with precision, requires attention and care.

It should be noted that the depth of a shadow on any object is in proportion to the degree of light which it encounters on the surface of that object. In the plan (Fig. 6) the shadow of the apex of the cone falls upon the lightest point of the sphere and is the darkest part of the shadow. The deepest portion of the shadow of the cone on the cylinder in the plan (Fig. 4) is where it coincides with the line of extreme light. Flat surfaces are similarly affected, the shadows thrown on them being less darkly expressed according as their inclination to the plane of projection increases. The body-colour on a flat surface should, on the contrary, increase in depth as the surface becomes more inclined to this plane.

Reflected light is incident to shadows as well as to shades. This is observable where the shadow of the cone falls upon the cylinder, though to a less extent, on other parts of these figures. The reflected light on the cone from the sphere or cylinder adds greatly to the effect of the shadows and to the appearance of the objects themselves.

The peculiarities and effects of light, shade, and shadow may be seen in the examples of screws (Plate V).

In topographical and architectural drawing artistic effect may often be introduced, but in mechanical drawing distinctness of outline and accuracy of expression are essential; though, to maintain harmony in the colouring and to equalize the appearance of the drawing, large shades should be coloured less dark than small ones, at equal distances from the eye, and no very dark shading is permissible.

In preparing colours for tints, great care should be used in grinding. The end of the cake should be slightly wetted and rubbed on a porcelain palette, then transferred by a wet brush to another saucer, and water added to bring to the required tint. Mixed colours should be intimately blended by the brush. Grind more than enough of the tint required, and let it stand in the saucer till the grosser particles have settled and the liquid is of clear, uniform tint. It is common to make little boxes or bags of waste drawing-paper to hold the colours instead of saucers; the gross matters, settling on the bottom, are not then so readily disturbed.

Instead of hard cakes of colour, moist colours are used, either in pans or tubes, which saves the trouble of grinding. For flat tints or washes, aniline colours, dissolved in water and kept bottled, are the readiest means of colouring, but are not applicable to finished work.

If the surface of the paper is greasy and resists colours, dissolve a piece of ox-gall, the size of a pea, in a tumbler of water, and use this solution with the colours instead of plain water.

When the brush is too full, as it comes toward the limit of the tint, take up the surplus moisture on a wet sponge or piece of cloth or blotting-paper.

An expeditious way of shading a cylinder, or of delineating the shores of a stream or lake, is by drawing with a brush full of the darkest tint along the sides, and then, with a wet brush, modifying this tint toward the light from the sides, so as to give a shaded appearance. For this purpose, two brushes will be necessary—one with colour, the other with water; also, a tumbler of water, and a piece of blotting-paper, to take up the excess of moisture from paper or brush. Often a single line of dark colour blended this way will express all that is necessary, but the effect may be improved by a sort of stippling with the colour-brush and by extending the line of shade.

The same effect is obtained better by drawing two faint pencil-lines on the elevation of the cylinder, for instance, to indicate the extremes of light and shade on its surface and passing the brush, moderately full of the darkest tint, down the line of deepest shade, spreading the colour more or less on either side, according to the diameter of the cylinder; then, before this layer of tint is dry, if possible, toward the line of extreme light, beginning at the top, and encroaching slightly over the edge of the first tint, lay on another not quite so dark, but about double its width. Put on the second tint before the first is thoroughly dry, that its edges may be softened by the application of the second. While this second tint is still damp, with a much lighter colour in the brush, proceed in the same manner with a third tint, and so on to nearly the line of extreme light. Repeat this process on the other side of the first tint, approaching the outline of the cylinder with a very faint wash, so as to represent the reflected light which progressively modifies the shade as it nears that line. Then let a darkish narrow strip of tint meet and pass along the outline of the cylinder on the other side of the extreme line of light, after which gradually fainter tints should follow, treated in the manner already described, and becoming almost imperceptible just before arriving at the line of light.

But it is not possible, by the above-described means alone, to impart a sufficient degree of well-regulated rotundity to the appearance of such an object. It may be necessary to equalize the superfluities and deficiencies of colour to some extent by a species of gross stippling. This is done by spreading a little colour over the parts where it is deficient, and then passing the brush, supplied with a very light wash, very lightly over nearly the whole width of the shade. This process may be repeated to suit the degree of *finish* which it is desired to give the drawing. The shading of all curved surfaces is treated in the same manner.

The shades having been put in, the shadows follow. Draw the outline of the shadow in pencil, and along its inner line, the line which forms a portion of the figure of the object whose shadow is to be represented, lay on a strip of the darkest tint, wide or narrow, according to the width of the shadow, and then, before it is dry, soften off its outer edge.

The finish is made by a light wash or two of the body-colour, and in passing over the shades and shadows care must be taken to manœuvre the brush at such parts quickly and lightly.

The shades and shadows of a machine being modified in intensity as their distance from the eye increases, its body-colour should be treated in a similar manner, becoming less bright as the parts of the machine which it covers recede from the spectator.

When the large circular members of a machine have been shaded, the shadows, and even the body-colour on those parts farthest removed from the eye, are to follow, and the proportion of India ink in the tint used should increase as the part to be coloured becomes more remote. A little washing, moreover, of the most distant parts is allowable, as it gives a pleasing appearance of atmospheric remoteness, or depth, to the colour thus treated.

The amount of light and reflection on the members of a machine should diminish in intensity as the distance of such objects from the spectator increases. As it is necessary, for effect, to render on the parts of a machine nearest the eye the contrast of light and shade as intense as possible, so, for the same object, the light and shade on the remotest parts should be subdued and blended according to the extent or size of the machine.

To add to the definiteness of a coloured mechanical drawing, it is well to make the lines of light and shade distinct.

After having marked in pencil the position of the extreme light, take the drawing-pen, filled with a just perceptible tint, and draw a line of colour on one side of the line of light, almost touching it; then with the brush, filled with similar light tint, join this line of colour while still wet, and fill up the space unoccupied by the shade-tint, within which the very light colour in the brush will disappear. Treat the part of the object on the other side of the line of light in the same way. The extreme depth of shade may, with great effect, be indicated by filling the pen with dark shade-tint, and drawing it exactly over the line representing the deepest part of the shade. On either side, joining this strip of dark colour, another, of lighter tint, is to be drawn. Others successively lighter follow, until, on one side, the line of the body is joined, and on the other the lightest part of the body is nearly reached. The line of light is then to be shown, and the faint tint used for this to be spread with the brush lightly over the whole of the part of the body that is situated on either side of this line, thus blending into smooth rotundity the graduated strips of tint drawn by the pen.

In all tinted drawings the important parts should be more conspicuously expressed than the mere adjuncts. Thus, if the drawing be to explain the construction of the machine, the tint of the edifice may be more subdued than those of the machine; and if the machine be unimportant, it may be represented in mere outline, while the edifice is brought out conspicuously.

With regard to washings, the soft sponge is an excellent means of correcting great errors in drawing or colouring, but care must be taken not to rub the surface. In removing or softening colour, for large surfaces, use the sponge; for small spots, the brush. While colouring, keep a clean, moist brush by you to remove or modify a colour.

The immediate effect of washing is to soften a drawing, an effect often very desirable in architectural and mechanical drawings, and the process is simple and easily acquired; keep the sponge or brush and the water clean; after the washing is complete, take up the excess of moisture with the sponge or brush or with a piece of clean blotting-paper. Where vigour is required, let the borders of the different tints be distinct.

There are no conventional tints that draughtsmen have agreed upon to be uniformly used to represent different materials. India ink is not a black, but a brown, making with a blue a greenish cast, and with gamboge a smear. A coloured drawing is better without the use of any India ink at all; any depth of colour may be as well obtained with blue as with black. There is the objection to gamboge that it is gummy, and does not wash well, and a better effect is obtained with yellow ochre. For the reds, the madder colours are the best, as they stand washing. For the shade-tint of almost any substance a neutral tint is required, such as Payne's gray, or madder brown subdued with indigo.

MATERIALS.

VARIED materials enter into the composition of structures and machines, or form their supports, which are to be represented by the draughtsman. That of the earths and rocks, in their natural position, are shown under the head of "Topographical Drawing," or by a closer imitation of Nature, with or without colour.

Fig. 325 represents a plan and section of an earth-bank of a canal, with a paved rock-slope.

A base of rock may be represented by stratifications (Fig. 326).

Rocks, gravels, sands, muds, etc., either in their natural or structural positions, are shown in "Engineering Drawing."

Earth, when first dug, occupies more space than when in its natural condition, but, after a time, it shrinks and becomes more compact. The earth dug out of a hole, when settled, will not fill the hole. Sand, gravel, loam, and clay, will occupy from 8 to 12 per cent less space than when in the natural cut.

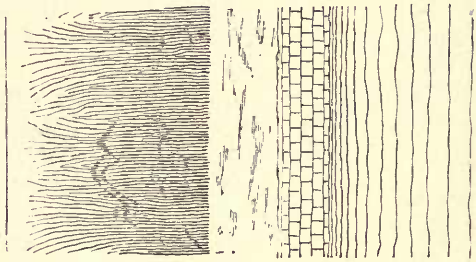


FIG. 325.

Loose, dry sand weighs from 90 to 100 pounds per cubic foot; compacted, 110; gravel, about the same; clay, in the bank, 120 pounds. Sands and gravels are excellent material for embankments and fills. The slopes in cuts and fills are usually $1\frac{1}{2}$ horizontal to 1 perpendicular. Sands and gravels are readily drained, and, when dry, are but little affected by frost. The clays are hard to drain, heave with the frost when wet, and, under the influence of a thaw or excess of water, become fluid, but well rammed they are used as puddle walls in the centre of reservoir embankments for stanchness. In these positions it is recommended that the clay should be compacted dry. Very fine sand, with gravel, and perhaps some admixture of clay, a *glacier till*, is known as *hard-pan* by engineers, very difficult to be moved with the pick, and often requiring blasting. The same material wet, but without gravel, forms a quicksand—a jelly-like material—from which, if a spadeful be taken out, the hole closes up at once, and excavation shows but little visible sign of a depression, the space being made good from the entire mass. There is another material, called quicksand, which is rather a running sand—even when not wet, it rests with a very flat slope; the particles are very fine, and flow like the sands in an hour-glass.

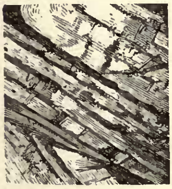


FIG. 326.

Sands and gravels are large components of mortars, *bétons*, and concrete; and burnt with clay, of brick, tile, and pottery.

BUILDING MATERIALS.

The natural building materials of civilized communities are wood and stone, which are to be worked or fashioned to the purposes to which they are to be applied.

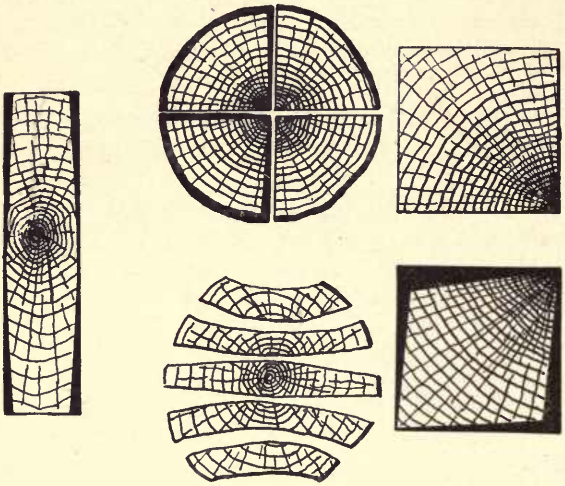


Fig. 327.

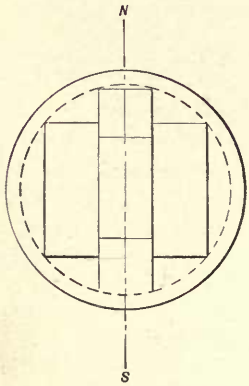


Fig. 328.

Fig. 327 shows various sections of timber in which the rings or yearly growth are very strongly shown, and the effect of shrinkage by black margins and distortion, according to the form of cut.

The strongest timber lies about one third the radius from the pith in the butt log; in the top log the heart position seems strongest; but in important bridge and floor timbers the heart should be excluded (Fig. 328); exterior rings or sap are soft and liable to decay.

For beams or girders, the timber should be cut so that the stress will be parallel with, and not across, grain. For posts in compression, and lengthwise of the fibre, the section may be from any part.

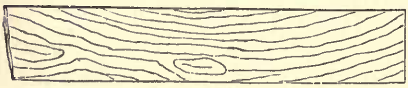


Fig. 329.

Fig. 330.

Figs. 329, 330, and 331 are drawings of wood, longitudinal and sectional, in which the grain of the wood is imitated, but wood is more often represented in plain outline, and the cross-section of a timber thus (Fig. 332), or by mere hatching. When distinguished by colour, burnt sienna is used commonly for wood, but sometimes the colour and grain of the wood is imitated.

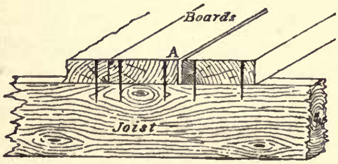


Fig. 331.

Fig. 332.

The draughtsman, for his designs, will probably have to confine himself to the timber within his reach. But he should know what is best for his purpose, reference being had to economy in cost and maintenance. For most purposes, wood should be seasoned, so that joints may not open under this operation after the material is in the structure. But, for work under water, wood should be but slightly seasoned, as a swelling of the wood may be disastrous. Seasoning of timber may be done by exposure for a time to outer air-currents; if in a kiln, it can be done speedily with heated air, or by steam. For beams, girders, and the like, there should be few knots, especially on the outer edges—for posts, small ones are not objectionable; while for sidings and under-floors, firm, large knots do not impair the work; but no smooth work can be made with knotty lumber. In most specifications, lumber is "to be square-edged, without sap, and large or loose knots."

In selecting lumber for a permanent structure, the life and endurance of the material are to be considered. Most of the woods, sheltered from the wet and exposed to air-currents, will last for a very long time; but many will check and warp and become distorted. All lumber in earth beneath the level of water will last indefinitely. In salt water, above the earth, all are subject to the attacks of the worm—the *Teredo* and *Limnoria*—and, where the water is pure, the destruction is very rapid. Sewer-water and fresh water are both destructive to the worm. Green or wet timber, in positions from which the air is excluded, soon fail through dry rot, and even seasoned timber under unfavourable conditions.

The life of timber exposed to wet or dry rot can be prolonged by filling the pores with creosote, pyrolignite of iron, solutions of chlorides of mercury or zinc and various other antiseptics. There are works in which the timber is first steamed in close cylinders to remove the sap and rarefy the air in the pores, and then injecting the preserving fluids. Open tanks of plank can be readily constructed, in which the soaking of lumber in cold solution, especially that of corrosive sublimate, is effective.

CHARACTERISTICS AND USE.

White Pine.—A wood of the most general application in the market; is light, stiff, easily worked, nails are easily driven into it, and takes paint well, warps and checks but little in seasoning, endures well in exposed situations; clear stuff, of best quality, useful for patterns and models, for interior finish of houses, doors, window sashes, furniture. It forms the base or inner core of the best veneered work, holds glue well, and the composite structure is better than single solid wood. The cheaper kinds of pine are used for frames of buildings, posts, girders, and beams. Even with large knots is well adapted for boardings, and is extensively used for goods-boxes.

Southern Pine.—A heavy, strong, resinous, lasting wood, clear and mostly without knots, hard to be worked by hand-tools, and when seasoned difficult to nail. The surfaces, from their resinous character, do not hold paint well. It is used very largely for girders, beams, and posts of mills and warehouses, and for floors of the same, when exposed to heavy work or travel. For the first, it can be obtained of almost any dimension to suit; for floors, it is sold in long strips, from two to six inches wide, of varied lengths, tongued and grooved, and when laid is *blind-nailed*, *toeing* the nail through the tongue, so that the nail-head does not show.

Experiments of the Forestry Division of the United States Department of Agriculture prove that the extracting of the turpentine from the long-leaf yellow-pine trees does not in any material sense injure them for use as lumber. The bled timber is heavier in the bottom cuts by about two pounds per cubic foot.

Canadian Red, *Norway*, *and Silver Pines* are resinous woods, like the Southern pine, and are used for similar purposes, but are not as valuable—woods less straight in the grain, and with more knots.

Spruce.—A light, straight-grained wood, with but few knots, which are small and often decayed. It does not last well exposed to the weather, and checks and warps badly in seasoning. It is the most common wood here for floor-beams and common floors, but it must be well braced and nailed, and is not fitted for joiner-work.

Hemlock is similar to the spruce, and, when selected, is less liable to check and twist in seasoning. It is often of a very poor quality, *brash* and shaky. Exposed, it is but little better, if any, than the spruce. For stables, it is well adapted for grain-boxes, as the fibre prevents the gnawing of rats.

Ash.—Some of the ashes are of exceeding toughness. A straight, close-grained wood. It is used for carriage and machine frames, and for interiors, doors, wainscot, floors, when no paint is used.

Chestnut.—Somewhat like the ash in appearance, but coarser-grained, and very enduring in exposed positions. It is most largely used for cross-ties of railways. As a roof-frame exposed in the inside, and in general interior finish without paint, the effect is very good. The closer-grained woods are very often thus used.

Black Walnut is, in the trunk, a straight-grained, gummy wood, clogging the plane a little in its working; the knots are useful for veneer. Were the wood cheap enough, it would undoubtedly make a good frame. It is used here for desks and counters, for furniture and interior finish, as an ornamental wood.

Butternut.—Similar to the black walnut, less commonly used, but fully equal as an ornamental wood.

Hickory.—A strong, tough wood; is used for cogs of mortise-wheels, handspikes, axe-helves, and wheelwrights' work.

Beech.—A close-grained wood, but of little application in this market. Sometimes used for cogs of wheels, for small tool-handles, and in marquetry.

Oak, Live.—A very strong, tough, enduring wood, used industrially almost entirely for ship-building. Ornamentally, in marquetry and panels.

Oak, White.—A very valuable, strong, tough wood, with great endurance. It is heavy, and hard to work, and was formerly used largely for the frames of houses, but has been superseded by the white pine. It is used in ship-yards and in water-works—for the frames of flumes, penstocks, and dams, and for the planking of the latter, for dock-buffers and piles, and for railway and warehouse platforms. The red and black oaks may in general be considered a cheaper and poorer quality of the white oak. All have a handsome grain, that adapts them to ornamental work.

Bass, Poplar, White-wood, are light woods, mostly used in the manufacture of furniture, for drawer-bottoms, cabinet-backs, panels; they are very clear stock, easily worked, and can be readily obtained in thin, wide boards.

Cedar.—A straight-grained, light wood, of great endurance, valuable for posts, sills, shingles; used for pails and domestic utensils. The red variety, from its odour, is admirable for drawers and chests, preserving their contents from moths.

Locust is in the market only in small sticks; is of extreme endurance. It is used almost invariably here for the sills of the lowest floors of buildings, where there can be no ventilation, and for treenails of ship-planks.

Elm.—Although a tree of wide diffusion, is but little used as lumber. It is kept for an ornamental tree, beyond its usefulness for any other purpose but fuel. Well selected, it is said to be an enduring timber, useful for piles and places exposed to wet.

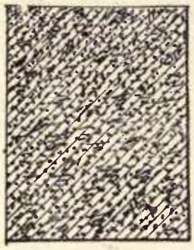
Maples are tough, close-grained woods, rather to be considered among the ornamental woods, for furniture and interior finish. The same may be said of the cherry, plum, and apple tree, of which the denser woods are admirably adapted for the handles of small tools, for bushings of spools and bobbins.

The list of imported woods is extremely large, mostly for ornamental purposes; but the mahogany is one of the very best of woods for patterns and small models, as it changes but little in seasoning; and the lignum-vitæ, a very hard and heavy wood, is used for pulley-sheaves, packing-rings of pumps, water-wheel steps, and shaft-bushings.

Timbers of the same kind vary much in their weight, strength, and endurance, according to the localities in which they are grown, the season at which they are cut, and how seasoned. Tables are given in the Appendix in detail, their varied resistance under stresses and their specific gravities.

Of late years paper in sheets and pulp has been used instead of woods, and serves well many purposes on account of its little shrinkage, incombustibility, strength, and endurance.

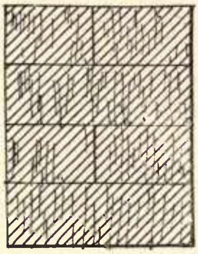
Earth.



Concrete.

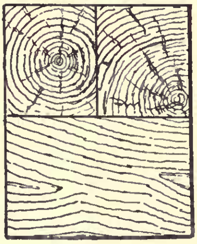
Brick.

Cut stone.



Rubble.

Timber.



STONES.

In selecting the form of construction, and the stones of which it is to be composed, the draughtsman must be governed by the fitness for the purpose and the cost. He must select from what he can readily get, and arrange the form to suit the material. He must know what is to be the exposure, and what the effect will be on the stones. Almost any stone will stand in a protected wall, but many of the sandstones and slates disintegrate and exfoliate under the influence of the weather, heat, cold, frost, and moisture. Even the granites are liable to serious decomposition when the feldspars are alkaline; and the limestones (dolomites), of which the English Houses of Parliament are composed, have failed in the sulphurous air of London smoke, while at Southwell Minster they have stood for over 800 years. Chemical tests of stone to determine endurance are deceptive. The safe way is to see how the material has stood in like situations to the one in which it is to be employed, or go to the quarry, and see how the stones have weathered.

The strength of stones to resist crushing, as determined by experimental cubes, is even in the weaker stones much in excess of what would be required in structures, but most stones are weak under cross-strains, and failures in construction are more likely to occur by faulty workmanship or design, by which the stones are subjected to unequal strains, and for which they are not adapted. The weight should not be brought on the outer edges or *arrises*, as the faces will chip readily; nor should most stones be used for wide-span lintels, unless relieved by the masonry above the opening.

TECHNICAL TERMS OF MASONRY.

Agreeably to the nomenclature recommended in "Transactions of the American Society of Civil Engineers," November, 1877:

Rubble masonry includes all stones which are used as they come from the quarry, prepared at the work by roughly knocking off their corners. It is called *uncoursed rubble* (Fig. 333) when it is laid without any attempt at regular courses; *coursed rubble*, when levelled off at specified heights to a horizontal surface (Fig. 334).

Square-stoned Masonry.—Square stones cover all stones that are roughly squared and roughly dressed on bed and joints.

Quarry-faced stones are those which are left untouched as they come from the quarry.

Pitch-faced stones are those on which the *arris* is clearly defined beyond which the rock is cutting away by pitching-tool.

Drafted stones are those in which the face is surrounded by a chisel-draft.

If laid in regular courses of about the same rise throughout, it is *range-work* (Fig. 335). If laid in courses that are not continuous, it is *broken range* (Fig. 336).

FIG. 333.

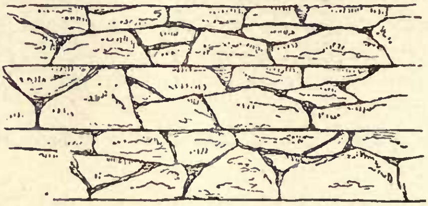


FIG. 334.

Cut stones or ashlar covers all squared stones with smoothly-dressed bed and joints. Generally, all the edges of cut stone are drafted, if the face is not entirely fine cut, but

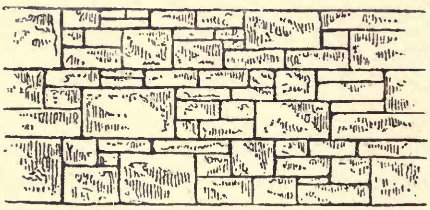


FIG. 335.

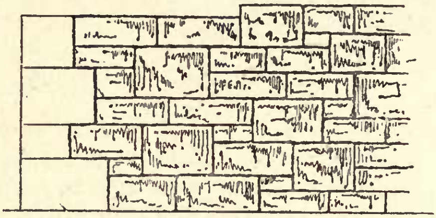


FIG. 336.

they may be quarry-faced or pitch-faced; as a rule, the courses are continuous (Figs. 337, 338), but, if broken by the introduction of smaller stones of the same kind, it is called *broken ashlar* (Fig. 336). If the courses are less than one foot in height, it is *small ashlar* (Fig. 337).

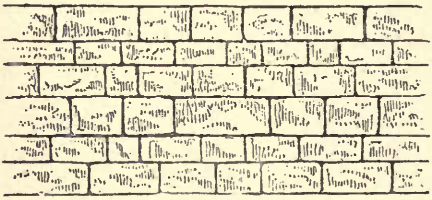


FIG. 337.

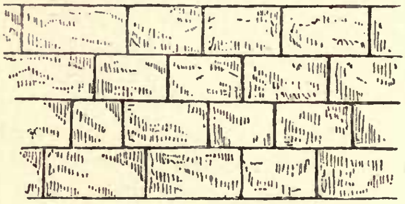


FIG. 338.

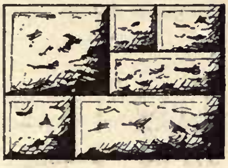
Squared-stoned Masonry.—The joints in one course should not come directly over those of another; there should be a lap or *bond*, and, in connecting the front or face with the backing, headers must be introduced for bond. Headers are stones extending into the wall, stretchers running with the face.

The backing is of rubble, sometimes laid dry, but as from its many and large joints it settles more than the face, it should be set in mortar to provide uniformity of support.

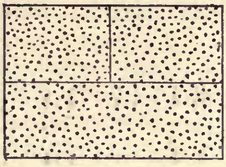
In addition there is a class of ornamental stone work, specified as "close jointed, hammer-dressed rubble," in which there are no courses and no pinners. The joint of each stone is carefully fitted to those beneath it.

For rubble-work, all varieties of sound stone are used, and of almost any size. In dry work, for foundations and for heavy revetment-walls, the stones are laid with derricks, but they must have fair beds and builds. If boulders, they must be split, and cobbles in the filling are worse than useless, as they are unstable, and in settlement act as wedge to increase the movement.

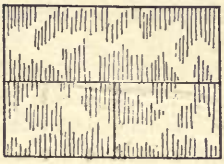
Drafted Quarry Face.



Bush Hammered.



Axed.



GRANITIC STONES.

Granite and syenite are by builders classed as granites. The granite in general rifts in any direction, and works well under the hammer and points. From these circumstances it is more desirable than the syenites, which are much harder to be worked. Both are admirable stones for heavy dock-walls, bridge-abutments, river-walls, either as rubble-squared stones or cut work, and are very enduring. They are also used for the faces of important buildings, either as fine-cut, quarry, or pitched-face. Ornamental work of the simpler kind is readily produced; more elaborate is expensive, but it is about the only stone in this climate in which foliage and sharp undercut work will stand the weather without exfoliating. These stones, especially the syenites, admit of a high polish, and are used considerably for columns and panels in buildings, and in monumental work. Gneiss is of the granitic order, but a cheaper, poorer stone. It splits with difficulty, except parallel with line of bed. It has a foliated structure, and is not adapted for ashlar, but is very good for squared-stone masonry and rubble-work, and often used for sidewalk-covers of vaults.

ARGILLACEOUS STONES.

The slates or stones thus designated by builders were formerly in very common use as roofing material, and were almost entirely from Wales, but latterly they are taken from Vermont and Pennsylvania, and other parts of the United States. They are also used, in thicknesses of one inch and above, for floors, platforms, facing of walls, mantels, and for wash-tubs by plumbers. Soap-stone may be classed under the clay stones; also, used for tubs, for stoves, and for the lining of grates and furnaces.

The Ulster, or North River blue stone of this market, is a coarser slate, a very strong and enduring stone; it can be quarried of varying thickness up to twelve inches, and of any dimension that can be transported. It can be readily cut, hammer-dressed, axed, planed, and rubbed. Is generally used for sidewalks under these various forms. It is used as bond-stones in brick piers, for caps, sills, and string-courses.

THE SANDSTONES.

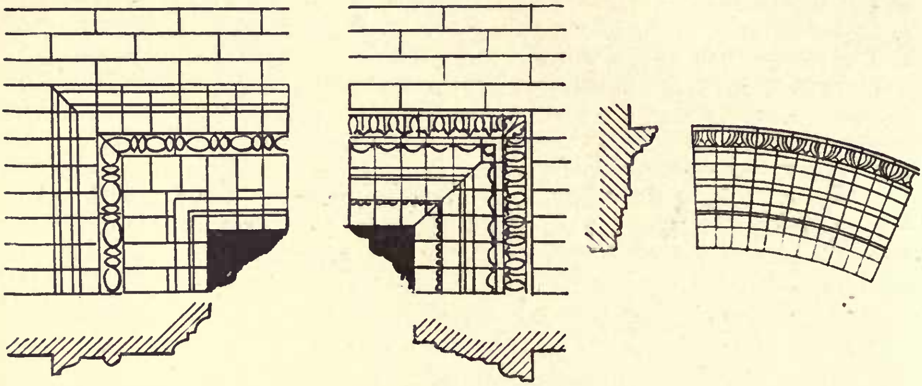
Sandstones, called also freestones, from the ease with which they are worked; and from their colours, are very popular for the fronts of edifices. In general, they are not very enduring stones, and when laid must be set parallel to their natural beds, as otherwise they flake off under the influence of the weather. The sandstones are not all of the same quality; those in which the cementing material is nearly pure silex, are strong, enduring stones, but not those in which the cementing material is alumina, or lime. By examining a fresh fracture, the character of the stone can generally be detected. A clay, shining surface with sharp grains indicate a good stone; while rounded grains, a dull, mealy surface, indicate a soft, perishable stone. None of the sandstones in this locality are used for heavy pier or abutment work and the like, but there are sandstones in other localities adapted to it.

LIMESTONE.

The coarser calcareous stones are of great variety; some are well adapted for building stones, being hard and compact, while others are soft and friable. They are more easily worked than granite, but are not considered as enduring. They are well adapted to the same class of heavy work, and the locks of the Erie and Northern Canals and the dam across the Mohawk, at Cohoes, are built from limestone on the line of the canals.

The finer kinds of limestones are classed under the head of marbles. They are easily worked, sawed, turned, rubbed, and polished. Marble is not popular as a building material, although more enduring than most sandstones, but is susceptible to the action of sulphurous gases in the smoky air of cities; and it is said that the Capitol at Washington, D. C., built of marble, is suffering from disintegration. But, for interior finish, as tiles, wainscots, architraves, mantels, linings of walls, it is admirably adapted, and from its richness, cleanliness, and variety of colour, it is very ornamental and effective.

ARTIFICIAL BUILDING MATERIAL.



The most common and useful are bricks. They are generally made of clay, with an admixture of sand, well incorporated together, and mixed with water to the consistence of a smooth, strong, viscous mud, pressed into moulds, dried, and burned, the best quality being those in the interior of the kiln. The exteriors are light, friable bricks adapted to walls supporting but little weight and not exposed to wet. The brick forming the arches are very hard-burned, dark in colour, often swelled and cracked; but, by proper selection, they can be used for foot-walks. A good brick is well burned throughout; when struck, it gives a ringing sound, and is of uniform shape.

Bricks vary somewhat in size and weight in different localities—from 8 to 8½ inches long × 3½ to 4 inches broad × 2 to 2½ inches thick; in general, the thickness of a wall with the joints is called some multiple of 4″, as 8″, 12″, 16″ . . . walls, and the number of bricks in such wall is estimated by multiplying the number of brick in a square foot of face by 2, 3, 4, . . . In some cases bricks are laid by the thousand and there are custom allowances for corners, openings, and indents. The best face or front brick are pressed and are of great variety of colours, but uniform in tint. Of late there has been a class of face brick of which the edges are taken off by a set, leaving the face like rock-faced ashlar without draft.

For variety, Pompeian face brick 1½″ × 4″ × 12″, Roman 2″ × 4″ × 10″, faces plain glazed and enamelled. Moulded brick for base, cornices, caps, sills, corners, and arches are in common use.

Bricks are laid in mortar, of lime, lime and cement, or cement only—all with an admixture of sand; in common walls, in lime; in walls of heavy buildings, above-ground, in lime and cement; beneath, and in wet, exposed positions, in cement only. The common bond of the different courses of brick is by header-courses every fifth or seventh course. When bricks are laid in arches they are set on edge, and turned in 4-inch rings, without any bond between the different rings; or with a bond of brick lengthways, when two courses come on the same line, or with radial joints, or alternate 4″ and 8″ courses of brick work in masses laid header and stretcher; however laid, the strength depends on full joints in strong cement mortar.

Bricks set on edge, as in arches or in a level course, are here termed *rowlocks.*

Arch brick, between iron beams, to reduce the weight, are often made hollow, and laid in flat arches; that is, the joints are radial, but the upper and lower surfaces are level. Hollow brick are also used for walls and partitions.

Fire-brick can be made of any size and pattern, but are usually 9 × 4½ × 2¾. They are used for the lining of furnaces, flues, and chimneys, exposed to the action of flame or great heat. Fire-clay, with an admixture of sawdust, which is burned out in the firing, leaves a light, porous, spongy mass, which can be sawed in sheets or strips, and is well adapted for covering the exposed parts of iron beams and girders, and, as it admits of nailing, is convenient for partitions.

Enamelled Brick.—The English size is that of fire-brick—the American is that of common brick. The brick, on the faces to be exposed, are covered with glaze of varied colors and designs, and fired. They make a handsome ornamental face for walls, do not absorb moisture, and can be washed.

Tile are a species of brick, with or without enamel. The latter were originally used for roof-covering, but now are used in flooring walks and the like. The enameled or encaustic tile are generally in squares, 4″ × 4″, 6″ × 6″, 8″ × 8″, but there are smaller ones for tessellation, and rectangular strips for borders. They can be obtained of any color or design, forming beautifully ornamented floors and wall-panels.

Terra-cotta, a kind of brick, is now largely used for exterior decoration. It is molded in every variety of capitals, cornices, caps, friezes, and panels. It is a good, strong brick, with all the good qualities of such a material.

Mortars.—Brick are never laid dry, except in the under part of drains, to admit of the removal of ground-water. Stone-work, except in rough, heavy, rubble-work, is also generally laid in mortar. Where cut-work is backed with rubble, the joints in the latter should be as close as possible, and full of mortar, that the settling of the wall in itself may not be more in the backing than in the face. Some lay the rubble dry, and fill in with cement grout, or cement mortar made liquid to flow into the interstices, but the sand is apt to separate and get to the bottom of the course.

By mortar, is usually understood a mixture of quicklime and sand, but mortar may have an addition of cement to the lime, or it may be cement only with sand.

Lime, or properly quicklime, is made by the calcination of limestone, shells, and substances composed largely of carbonate of lime, carbonic-acid gas, water of crystallization, and organic coloring-matter. Quicklime, brought in contact with water, rapidly absorbs it, with a great elevation of temperature, and bursting of the lime into pieces, reducing it to a fine powder, of from two to three and a half times the volume of the original lime. This is *slaked* lime. It may be slaked slowly by exposure to the air, from which it will take the moisture. This is *air-slaked* lime. Barrels of lime exposed to rain often take fire from the heat caused by slaking. The paste of slaked lime may be kept uninjured for a considerable time, if protected from the air, and this may readily be done by a covering of sand, and it is customary, in some places, to hold it over one season, as an improvement to the uniformity of quality in the paste. But, in general, the lime is used soon after slaking, and is thoroughly mixed with sand, in various proportions, generally about two of sand to one of lime. The theory of the mixture is, that the lime should fill the void spaces in the sand, and the space occupied by the mortar is a little in excess of that occupied by the sand alone.

The sand should be sharp, clean, silicious grains, from one twelfth to one sixtieth of an inch in diameter. Close brick-joints do not admit of as coarse sand as those of cut stone work, and, in rubble-work, sand coarser than the above can be used, and there will be considerable saving of lime in using a mixture of coarse and fine sand.

The hydraulic limes contain a small proportion of silica, alumina, and magnesia; slake with but little heat, and small increase of volume; are more or less valuable, according to the property which they have for hardening under water; but, in this particular, are not equal to the hydraulic cements.

There are two great distinctions of cements—natural, as quarried from the rock and burnt, and artificial, like Portland, definite proportions from known materials, generally preferred from their uniformity of composition; but in this country the natural cements, from old quarries and responsible makers, are satisfactory.

Cement is used in all masonry in exposed and wet situations. With a small admixture of lime, it works better under the trowel, and for brick-work it does not sensibly impair its value. Cement adds to the strength of lime-mortar.

The value of a cement depends very largely on its fineness; the residue thrown out from a 2,500-per-inch mess should not exceed 10 per cent, the diameter of the wire is about one half the width of the mesh. This residue is reckoned as sand. The voids in the sand should be filled by the cement. To determine the amount of voids in the sand, fill a tight box of known capacity with the sand and then pour in all the water that it will hold. The whole may be considered as unity, and the quantity of water the percentage of voids. In the same way the voids of gravel or broken stone may be determined for concrete. In mixing cement mortar, it is important that it should be thorough, that the sand should be clean and damp, and that each particle should be covered with cement, and if to be used for concrete or *béton*, that the stone or gravel should be clean and damp, and their surfaces should be completely covered with the mortar. Suppose the proportions be as below:

1 part cement, without voids	1.0
3 parts sand, 30 per cent voids	2.1
6 parts broken stone, 50 per cent voids	3.0
Parts in mixture	6.1

The parts of sand and broken stone to cement are larger than in common use, or, say:

Portland	1, 3, 5
Natural	1, 2, 5

But the increase is rather as an offset to coarseness of cement and defects in mixing. As a rule, from the time the water is added to the concrete, it should be kept damp for a month after it is laid.

The Danish patent sand cement is sand ground with Portland cement to a uniform fineness, and then used like pure cement with great economy of construction and without impairing the strength of the mortar.

Concrete is used for the base course or foundations of walls, and is formed *in situ*, that is, depositing and ramming it in the trench where it is to be left; or by forming in moulds, in immense blocks, for docks or break-water, or in the forms of brick.

The bituminous cements are formed of natural bitumens, or artificial from coal-tar mixed with various proportions of gravel and inert material. The mixture is usually heated, put down in layers, and rolled or rammed. It is used for roads and sidewalks, and for water-proof covering of vaults. For the covering of roofs, coarse paper, saturated with bitumen, is put on in layers, one over the other, breaking joints, cemented with the bitumen, the last coat being of bitumen, in which gravel is imbedded. For an anti-damp course in a wall, or for the joints in the bricks of a wet cellar-floor, or on top of a roof, hot bitumen is used as a cementing material with dry bricks.

Plastering.—Coarse-stuff is nothing more than common brick-mortar, with an admixture of bullock's hair. When time can not be given for the setting it is *gauged*, that is, mixed with some plaster of Paris. *Fine-stuff* is made of pure lump-lime with an admixture of fine sand, and perhaps plaster of Paris. *Hard-finish* is composed of fine-stuff and plaster of Paris. *One-coat work* is of coarse-stuff, which may be *rendered*, that is, put on masonry, or *laid* on laths. *Two-coat work* is a coat of coarse-stuff, or scratch-coat; that is, after the coat is partially dry it is scratched for a back for the fine coat. In three coats, the first coat is a scratch-coat, the second the brown-coat, and the third hard-finish. Keene's cement, for the last finish, gives a hard surface, which admits of washing.

A single brick weighs between 4 and 5 pounds; but a cubic foot, well laid in cement, with full joints, will weigh about 112 pounds. They have resisted, in an experimental test, as high as 13,000 pounds to the square inch, but 12 tons should be the limit to the load per square foot; and the brick should be uniform, well burned, and closely laid in cement. In lime mortar, the load should not exceed 3 tons per square foot.

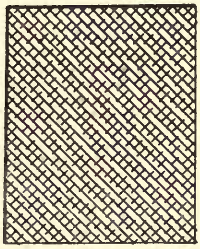
The granites weigh from 160 to 180 pounds per cubic foot; the limestones from 150 to 175; the sandstones from 130 to 170; the slates from 160 to 180; mortar, set, about 100 pounds; masonry, laid full in mortar, according to the quality of the stone and the percentage of mortar, from 150 to 170 pounds. But, for practical purposes, common mortar-rubble is not equal in strength to a brick wall, as it is seldom laid with equal care, with joints not as well filled or the load as evenly distributed; but cut stones will sustain more, and ashlar, up to 50 tons per square foot for sound, strong stones.

METALS.

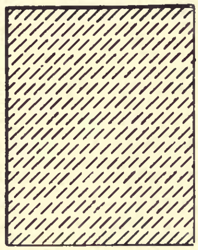
Metals are often to be shown distinctively by the draughtsman. If he can use colour, he will in a measure imitate that of the material. For cast-iron, India-ink, with indigo, and a slight admixture of lake; for wrought-iron, the same colours, with stronger predominance of the blue; steel, in Prussian-blue; brass, in a mixture of gamboge and burnt sienna; copper, gamboge and crimson lake. In drawings where no colour is admissible as for photographing, or to be reproduced in printing, some conventional hatchings are used to represent sections of metals, but none have been so established as to have a universal application. The following are submitted to represent the most common industrial metals:

Brass or Bronze.

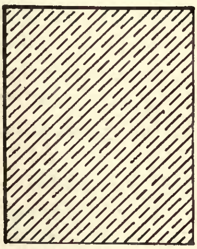
Lead.



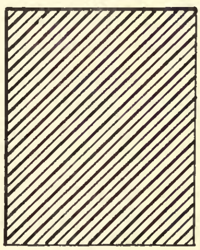
Copper.



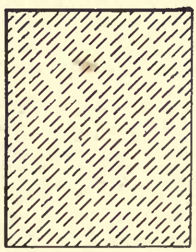
Steel.



Wrought Iron.



Cast Iron.



Under the term iron may be included cast-iron, wrought-iron, and steel, differing from each other in the percentage of carbon contained, and in the uses to which they are applied. Cast-iron contains more carbon than the others, say from two to five per cent. It can be cast in varied forms in moulds, but can not be welded or tempered. The usual moulds are in sand or loam, in which the pattern is imbedded, and when drawn out the space is filled with molten metal. The drawing of patterns for molding involves a knowledge of the art of founding. The shrinkage of the metal, usually about one per cent, for which provision must be made in increased size of pattern, is provided for by the pattern-maker, the draughtsman giving finished sizes, but the draughtsman must know whether the pattern can be drawn from the sand, and by what system of cores voids can be left; or it may often happen that castings, designed as a whole, will have to be made in a number of pieces, involving flanges and bolts. In cooling, the shrinkage takes place the soonest in the thinnest parts, and, if great care be not taken by the molder in exposing the thicker parts to the air first, the parts will shrink unequally, and there will be a strain induced which will materially weaken the casting, and it may even break in the mold. The draughtsman, in his design, should make the parts of as uniform thickness as possible.

Castings cool from the outside inward, in annular crystals perpendicular to the face, as in Figs. 339 and 340. Now, if the casting consist of a right angle (Fig. 341), there will evidently be a weak place along the line A B, but, if the angle be eased by a curve, the

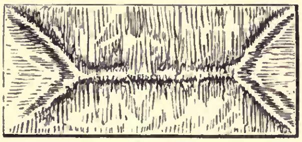


FIG. 339.

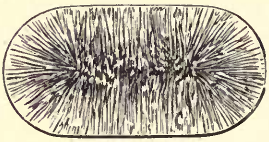


FIG. 340.

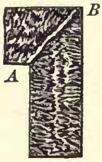


FIG. 341.

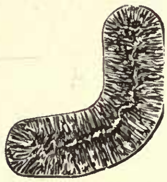


FIG. 342.

crystallization takes place as in Fig. 342, and the line of weakness is avoided. This is effected by a very small easement of the angle, and a cove is almost invariably introduced. In castings, in almost all metals, the same effects result from cooling, and therefore the changes of direction should not be abrupt.

When castings are ordered for important structures, iron of certain tensile strength is called for, and specimens of the metal, in small rectangular bars, are required, cast at the same time and under as nearly the same conditions as the casting which may be subjected to test.

If the casting be made in dry sand, it cools slowly, and the surface is comparatively soft; if in greensand—sand somewhat moist—the surface becomes harder; but if cast on an iron plate, or *chill*, some irons become as hard as the hardest steel, useful in surfaces exposed to heavy wear, as the treads of railway-wheels. Cast-iron, in general, is brittle under the blows of a hammer, but some mixtures, under a process of annealing, become *malleable iron*, used largely for steam-fittings, parts of agricultural machines, forms requiring the toughness of wrought-iron, but difficult to forge.

Wrought-iron is produced from cast-iron by removing the carbon and impurities by puddling, squeezing, heating, and rolling. As a material, it is sold in all sizes of wire, rods, shafts, bars, plates, shapes—girders and beams, chains and anchors. Its application industrially is well known. When hot, it can be welded, forged, drawn, and swaged into almost any required shape. Under the steam-hammer, the largest shafts, anchors, and cranks can be built, or by hand or by machinery it can be wrought into tacks, nuts, bolts, nails, or drawn into the finest wire.

For shafts of mills it is generally turned in a lathe and polished, but of late it can be bought, up to six inches diameter, *cold-rolled*, which adds very considerably to the strength, and is ready for use.

Bessemer and Siemens-Martin metals are made by burning out the carbon from a melted iron, and then reintroducing a known quantity, say from 0·03 to 0·6 per cent of carbon. There are other patents covering somewhat different irons, but the above are the best known. All are commonly classed as steel, but by many are called homogeneous metal;

first-class iron, of very uniform texture and great strength, but not equal to that of the best steel.

Steel is produced from pure wrought-iron by what is called cementation—heating the bars in contact with charcoal, by which a certain amount of carbon is taken up. The bars, when taken out, are covered with blisters, apparently from the expansion of minute bubbles within; hence called *blistered steel.* From this *shear-steel* can be produced by piling, heating, and hammering, or *cast-steel* from melting in a crucible.

Steel, when broken, does not show the fibrous character of wrought-iron. The fracture of shear-steel is fine, with a crystalline appearance. The fracture of cast-steel is very fine, requiring very close inspection to show the crystals or granulations; its appearance is that of a fine, light, slaty-gray tint, almost without luster. Steel is stronger than any of the other iron products, and especially applicable for the piston-rods of steam-engines, and positions requiring great strength and stiffness, with the minimum of space. But it is the way in which steel can be hardened and tempered which adapts it to its peculiar applications.

When the malleable metals are hammered or rolled, they generally increase in hardness, elasticity, and denseness, and some kinds of steel springs are made by the process of hammer-hardening; but the usual process of hardening and tempering is by heating the steel to a degree required by the use to which it is to be applied, and cooling it more or less suddenly by immersing in water or oil. The greater the difference between the heated steel and the cooling medium, the greater the hardness, but too much heat may burn the steel, and too sudden cooling make it too brittle. Steel, in tempering, is heated from 430° Fahr. to 630°. The temperature is shown by the color—from a pale yellow to deeper yellow, light purple to a dark purple, dark blue to a light blue, with a greenish tinge.

Steel is used for the edges of all cutting-tools, faces of hammers and anvils, and is generally welded to bodies of wrought-iron, but often composing the entire tool; for saws, springs, railway tires, pins, and can be bought in the form of wire, rods, bars, sheets, and plates, in varied forgings and castings.

All irons are very liable to rust, and must be protected where exposed to moisture. Polished surfaces are kept wiped and oiled, others painted, others galvanized or plated with some less oxidizable metal, generally tin, zinc, or nickel. Of late, a process has been introduced of coating them with black oxide, but is yet of no general application.

Antimony, *bismuth*, *copper*, *lead*, *tin*, *and zinc*, are used more or less industrially, and alloys of them are extremely useful. They may be hardened somewhat by the process of rolling and hammering, but can not be welded. Joinings are made by soldering or brazing or burning—that is, melting together.

Antimony expands by cooling. With tin, in equal proportions, it makes speculum-metal, and is used, with lead, to make type. Type metal makes a very good bearing for shafts and axles.

Bismuth is chiefly used as a constituent of fusible metal: 3 bismuth, 5 lead, and 3 tin, is an alloy which melts at 212°. Other mixtures are made, increasing the melting-point to adapt the metal for fusible plugs in boilers, or lowering the melting-point, so that, in case of fire in a building, a heat of say 140° melts the joint made by the metal, and lets water through sprinklers, to automatically put out the fire.

Copper is very malleable and ductile. In sheets, it is used for the cover of roofs, gutters, leaders, lining of bath-tubs, kettles, stills, and kitchen utensils. It is worked more easily than iron, and is stronger than lead or zinc, but it is much more costly than either of these metals, and its oxide is so poisonous that, without great care and cleaning, it can not be used to transmit or contain anything that may be used as food, without a cover of tin. It oxidizes slowly, and is used extensively for ships' fastenings and for bottom-sheathing. It is the most important element in all the brass and bronze alloys.

Brass, in common use, covers most of the copper alloys, no matter what the other components are, whether zinc, tin, or lead, or all three.

Copper and zinc will mix in almost any proportions. The ordinary range of good yellow brass is from 4½ to 9 ounces of zinc to the pound of copper. With more zinc it becomes more crystalline in its structure, but, as zinc is very much cheaper than copper, the founder is apt to increase the percentage of zinc, with the addition of a small percentage of lead. Muntz metal, in its best proportion, contains 10⅔ ounces of zinc to the pound of copper.

Copper and tin mix in almost any proportion. The composition of ancient bronzes is from 1 to 3 ounces of tin to the pound of copper. Ten parts of tin to 90 of copper is the usual mixture for field-pieces, and this is used in steam-engine work, often under the name of composition. Bell-metal is from 4 to 5 ounces of tin to the pound of copper; Babbit-metal, for journal-boxes, 90 of tin to 10 of copper.

Copper and lead mix in any proportion up to nearly one half lead, when they separate in cooling.

An addition of from one quarter to one half ounce of tin to the pound of yellow brass renders it sensibly harder. A quarter to one half ounce of lead makes it more malleable.

German-silver is 50 copper, 25 zinc, and 25 nickel.

Holzapfel gives the following alloys:

1½ ounce tin, ½ ounce zinc, to 16 ounces copper, for works requiring great tenacity.

1½ to 1¾ ounces tin, 2 ounces brass, to 16 ounces copper, for cut wheels.

2 ounces tin, 1½ ounce brass, to 16 ounces copper, for turning-work.

2¼ ounces tin, 1½ ounce brass, to 16 ounces copper, for coarse-threaded nuts and bearings.

2½ ounces tin, 2½ ounces zinc, to 16 ounces copper, Sir F. Chantry's mixture, from which a razor was made, nearly as hard as tempered steel.

Professor R. H. Thurston, at the Stevens Technological Institute, tested various alloys of copper, tin, and zinc, and, by a graphic method (Fig. 343), exhibits the results, enabling others to judge the probable strength of other mixtures. The apices of the triangle marked copper, tin, and zinc, represent the points of pure metal, 100 per cent. The lines opposite the apex of any metal represent the 0 of such metal—thus the base opposite copper represents an alloy of tin and zinc only, without any copper, and every line drawn above this base, and parallel to it, will contain a percentage of copper increasing by regular scale, from the base to the apex, and so with lines opposite tin and zinc; the first contains only copper and zinc, the latter tin and copper, and the percentages of tin and zinc increase with the distance from their opposite lines to their vertices. The intersections of these percentage parallels define the percentages of each metal, their sum always making 100 per cent. If the strength of such alloy, as obtained by test, represent an ordinate or elevation, on any convenient scale, at its opposite intersection of percentage, a contour map, as in the figure, may be drawn and a model made from it. The summit, 65,000 on the figure, represents the position of the strongest alloy found: if through the scales marked copper on each side, we find the parallel to the base, which passes through this summit, it will be found to be about 55—that is, 55 per cent copper. In like manner, the parallel to the *o* zinc base, intersecting this summit, will be about 43 per cent zinc; and, in the same way, tin is 2 per cent.

To find the probable strength of any mixture, it is only necessary to find the contour intersected by the triple parallels representing the percentages. It is said *probable* strength, because the care and manipulation of the founder are such important factors in the result.

Aluminum is about two and a half times the specific gravity of water. With the reduction in the cost of manufacture, its uses have become more extended, and its alloys now occupy a high position in practical industry. With a few per cent of silver, titanium or copper aluminum wire can be made of a tensile strength of 80,000 pounds to the square inch, and an electrical conductivity, weight for weight with copper wire, of

170 to 100. A few per cent of aluminum added to other metals gives valuable properties; with 8 to 12 per cent of copper, it makes one of the finest and strongest metals

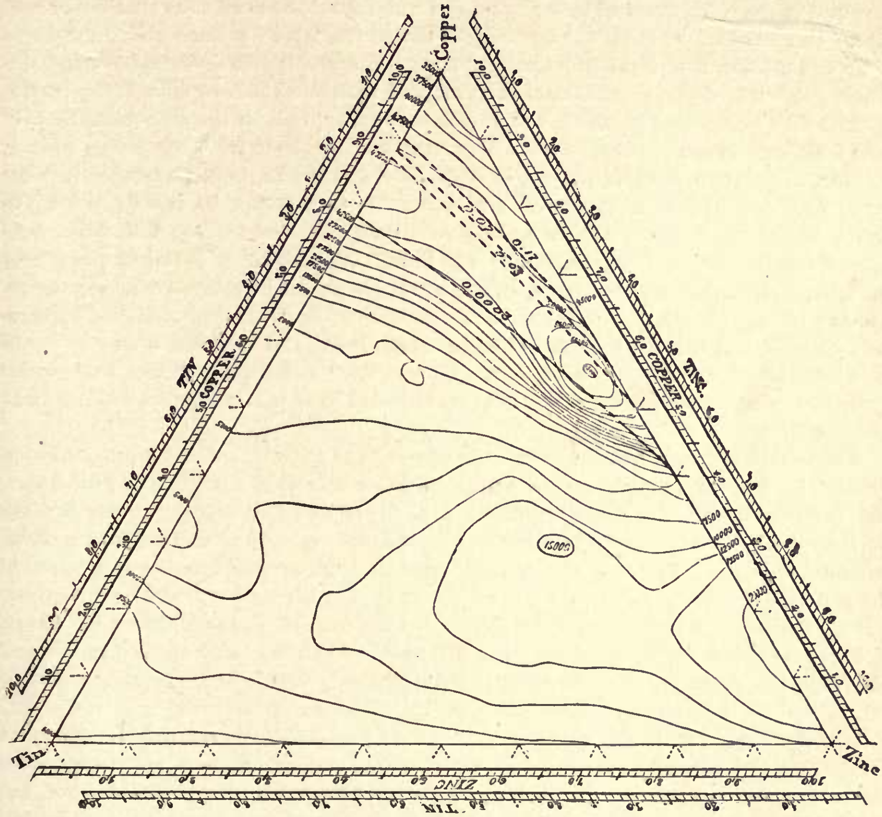


Fig. 343.

known; a 10-per-cent aluminum bronze can be made to fill a specification of 130,000 pounds tensile strength and 5 per cent elongation in 8 inches; and in the proportion of one third to three fourths of a pound to a ton of steel prevents blow holes and unsound tops of ingots.

There are various other alloys, as phosphate bronze, aluminum bronze, Sterro-metal, of which the strength are given hereafter in the appendix.

Lead is a very soft metal, that can be readily rolled into sheets and drawn into pipes, and is so flexible that it can be readily fitted in almost any position. It is, therefore, especially adapted to the use of plumbers, for the lining of cisterns and tanks, and for pipes for the conveyance of water and waste. For pipes for conveying pure water for drinking purposes, or for cisterns containing it, it is objectionable, as it oxidizes, and the oxide is a dangerous and a cumulative poison, but, in common waters which are more or less hard, the insides of the pipes become covered with a deposit which protects them. It is well, before drinking from a lead pipe in which the water has stood for a time, to draw off all the water, and, in lead-lined cisterns exposed more or less to the air, to protect them by a coating of asphalt varnish. Lead expands readily, and has so little tenacity that, in many positions, if heated, it has not strength in cooling to bring it back to its original position. It remains in wrinkles on roofs, and, for pipes conveying hot water, unless continuously supported, it will hang down in loops, continuously increasing under variations of temperature, to rupture. But it makes a very good plating for sheet-iron for roofs, and its oxides are the most valuable of all pigments.

Tin, in a pure state, is used for domestic utensils, as block-tin, and has also been used for pipes in the conveyance of water by parties who feared the poisonous qualities of lead pipe. But its chief use is for the covering of sheet-iron, which is sold under the name of tin or tin-plate, and is of universal application for architectural, industrial, and domestic purposes. It is so little affected by air and moisture that roofs, in many places, covered with it, need no painting, and oxidization takes place in the iron beneath only from deficiency in the plating, or from the abrasion or breaks in it.

Zinc, in the pure form of spelter, is crystalline and brittle, but at a temperature between 210° and 300° it is so ductile and malleable that it can be readily rolled into sheets, and of late has been used as a cheap substitute for sheet-copper; but, under considerable variations of temperature, as for lining of bath-tubs, it takes permanent wrinkles, and, for coverings of roofs, suitable provision must be made for its expansion. But as a plating of iron, under the name of galvanizing, it affords an admirable protection, cheaply, and extends the use of iron in sheets, bolts, and castings, where it would not otherwise be applicable. Zinc, as a pigment, does not discolour, like lead, under the action of sulphuretted hydrogen, but it is objected to by painters for its want of body or cover.

Sulphur, when used in sufficiently large masses as to show on a drawing, may be represented by a reddish-yellow tint, or some distinctive hatching. It melts at 248° Fahr., and, from its fluidity, answers admirably for the filling of joints between stones, beneath the base plates of iron columns, between wood and stone, and around anchor-bolts in stone, forming, when cold, a strong, uniform bearing, and adapting itself to the roughness of the material, and is detached with difficulty. It is used largely for the bases of engines, and for the joints of the cap-stones of dams. On the dam across the Mohawk, at Cohoes, N. Y., many tons were used in these joints, the depth of sulphur being about 6 inches, and now, after about twenty years' use, it has been renewed, and there has been no injurious effect from the sulphur on the limestone, of which the apron or capping is composed. It is better for most of the above purposes than lead, being cheaper, more fluid when molten, shrinks less in cooling, is less affected by temperature, does not creep under pressure, and its crushing strength is adequate to any of the positions of use above, but it is brittle under blows. It sometimes rusts the bolts or iron with which it is brought in contact, but this is prevented by an addition of about 20 per cent of coal tar. This mixture is used as a cement to fasten lights in illuminating tile and vault covers.

When heated to about 300°, sulphur begins to grow viscid, and at 428° it has the consistence of thick molasses. Above this, it begins to grow thin again. Heated to 518°, and thrown into cold water, it becomes for a time plastic, and is used for taking moulds or casts.

Sulphur, in powder, mixed in proportions of one sal-ammoniac, two sulphur, and fifty of iron-filings, makes a mastic which is used for calking the joints of iron pipes, especially gas-pipes. The joint is called a rust-joint.

Pure silver is too soft for general purposes; it alloys readily with lead, zinc, bismuth, gold, and copper. The last is the most important in an industrial point of view in its use for plate, coin, and ornaments, which invariably contain a certain amount of copper.

Silver is used for plating by the electro-plate process, or by fire plating, in which the sheet is soldered or sweated to some other metal, as iron, German silver, composition, copper, etc.

Gold is used industrially for chemical laboratories; sometimes as plate or for ornament, but mostly for gilding the baser metals by electro-plating or by fire gilt. As gold leaf, it is very largely employed for ornamental purposes; leaf can be beaten beyond any requirements in the arts; a single grain of gold was spread to the extent of 75 square inches, and the same weight of silver to 98 square inches. Platinum is used largely for various apparatus in chemical laboratories, as it withstands high temperatures and is proof against a large number of chemicals. It is to be had in wire sheets, crucible and dishes, and in stiles of large capacity for the concentration of sulphuric acid.

Glass, in drawing, is represented by a bluish tint or by different shades or hatchings, expressive of the effect of light upon it, whether the light is reflected or transmitted.

Fig. 344 represents a portion of a mirror when the light is reflected. The exterior of windows is often represented in the same way, but with deeper shades, and often with a piece of curtain behind in white with dim outline. A window viewed from inside is represented in shades less than in the figure, or as transparent, which is conveyed by the dimness of outline of figures or skies seen beyond.

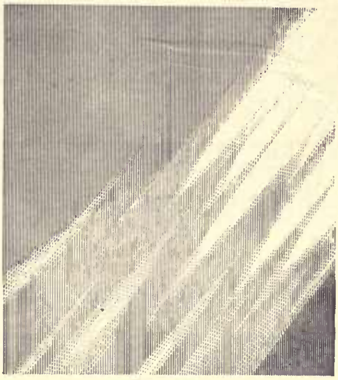


FIG. 344.

Fig. 345 represents a glass flask.

Fig. 346 represents a glass box with glass sides.

Fig. 347 represents a glass jar containing fluids of different densities.

Figs. 348 and 349 represent spars, which may be taken for any transparent substances, as glass, ice, and the like.

Common window-glass is blown in the form of cylinders (hence called cylinder-glass), flatted out, and cut in lights of varying dimensions, from 6×8 up to 30×30

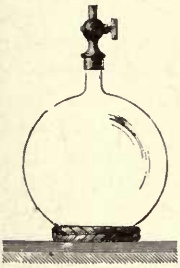


FIG. 345.

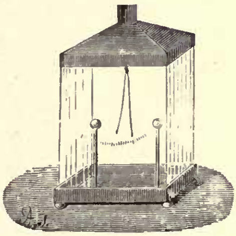


FIG. 346.

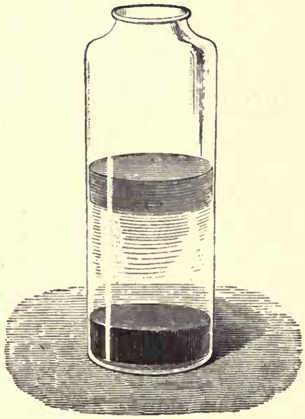


FIG. 347.

inches, and put up in boxes containing about fifty square feet. It is classed as single-thick (bout $\frac{1}{16}$ inch) and double-thick ($\frac{1}{8}$ inch). When the squares are large, or used for sky-lights, they should be the latter. Plate-glass—polished plate is used for windows of stores and first-class buildings. It can be got of almost any dimensions, and of a thickness from $\frac{3}{16}$ to $\frac{3}{4}$ of an inch. Rough plate is largely used for floor-lights and sky-lights. It is cut to required sizes, and of a thickness from $\frac{3}{8}$ to one inch.

Single thick cylinder-glass cuts off from about 8 to 15 per cent of the light.

Double-cylinder, from 12 to 20 per cent of the light.

Polished plate, three sixteenths inch thick, from 5 to 7 per cent of the light.

Rough plate, one half inch thick, from 20 to 30 per cent of the light.

Rough plate, one inch thick, from 30 to 40 per cent of the light.

Ribbed glass, one eighth inch thick, known as "factory glass," gives by diffusion a more effective illumination than clear, plain glass or the crystal-ribbed glass when either is screened with shades. On any exposure but the southern, the crystal-ribbed glass may be used without window-shades.

This is when the glass is clean; but there is always a film of moisture on its surface, which attracts dust, and impairs very much the transmitted light. Rough plate more readily retains the dirt, and, when it is used as floor-lights, becomes scratched. It is therefore usual, in the better class of buildings, to use a cast white glass, set in iron frames. In outer, or platform lights, these lights are in the form of lenses, set in cast-iron frames with an asphalt putty, or resting on iron frames and imbedded in Portland cement.

FIG. 348.

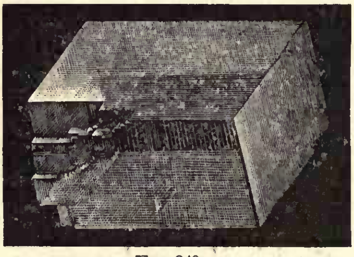


FIG. 349.

Rubber, mixed and ground with sulphur, subjected to heat, becomes *vulcanized*, and is not affected by moderate variations in temperature. Soft rubber, most extensively used for industrial purposes, is subjected to a heat of from 265° to 300°, and for a time can withstand a temperature a little below this without losing its elasticity; after a time it will harden. Soft rubber is classed as pure rubber, and fibrous rubber, or rubber with cloth. Pure rubber contains about fifty per cent of rubber and fifty per cent of compound, white lead and sulphur. It is used for the buffers and springs of railway-carriages, and for the faces of valves and seats of water-pumps, but it is not well suited for the pumping of hot water, especially above 212°, as it is liable to lose its elasticity; and, although some valves may stand a considerable time, it is almost impossible to secure uniformity in the rubber. Fibrous rubber—rubber ground with cotton or other fibre, or spread on cloth, on more or less thicknesses—is used for the packing of faced joints of pipes and gaskets for water or steam. It makes a stanch joint, and, even when hardened under heat, it still preserves it. Rubber cloth is also used for belting and hose-pipes. When used for the conveyance of steam, the inner coat is the first affected, and it may be some time before the whole pipe suffers. In buying rubber, explain the purpose to which it is to be applied, and depend on the guarantee of the vender. Rubber is often to be designated by the draughtsman, which it may be by a bluish-black tint, or by lines across it parallel to its length.

Paints are used for a twofold purpose—for covering and preserving the material to which they are applied, and for ornamentation. The best and the most general is white-lead ground with linseed-oil, either used by itself or mixed with various other pigments, as ochre, chrome, lamp-black, etc. It is often adulterated with barytes. For the covering of iron, or for the packing of close joints in it, nothing is better than pure red-lead, but many of the oxides of iron, red or yellow, form good covers of iron, and, as cheap and good paints, are used on tin roofs. All the leads and pigments are ground in oil: if the oil is raw, it dries slowly; driers, as litharge, are added to hurry the process, but, with boiled oil, no drier is necessary. Almost any inert substance, as cement, chalk, or sand, if fine enough, can be ground with oil for a paint, and make a good cover, and for these fish-oil will answer. The general specification for painting is "paint with—good coats of white-lead, of such colour as may be directed." The priming-coat of new wood-work requires more oil than paint. For the next coats, one-half pound of paint to the square yard would be considered a good coat. If the paint is on old work, or that which has been already painted, there will be a little less lead required. Wood should be fairly dry before the application of paint, so that it may properly adhere and not inclose moisture that may rot the wood. The knots should be *killed*, that is, covered with shellac varnish or similar preparation, to prevent the exuding of the resin. The heads of nails should be sunk, and the holes and cracks filled with putty, and the surface of the wood smoothed.

Coals and other minerals are represented like rocks or stones, in varied shades of tones and colours. Fig. 350 represents the fire-box of a locomotive, with coal in the state of ignition in its usual type. In colour, flame is represented in streaks of red-yellow, with dark tints for smoke. Water occupies the lower half of the boiler; but, as

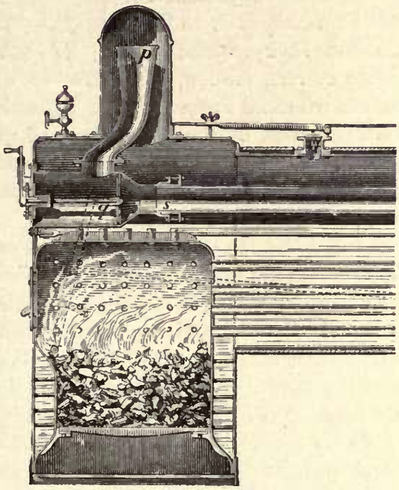


Fig. 350.

Fig. 351.

steam under pressure is invisible like gas, the space occupied by it is shown as empty. If the direction of its movement is desired, it is indicated by arrows. Steam issuing into atmosphere, or boiling in an open kettle, has the appearance of a very light smoke or cloud (Fig. 351).

There are many substances used in such masses in construction, or to be shown in the processes of manufacture, that must be graphically represented by the draughtsman by a general imitation of their natural appearance, or conventionally with explanatory marginal blocks and legends.

MECHANICS.

THE draughtsman in designing a structure should be conversant not only with the nature of the material, but also with the forces to which it is to be subjected—their magnitude, direction, and points of application, and their effects; that is, he should know the first principles of mechanics, the science of rest, motion, and force—to wit, Statics, Dynamics, and Kinematics. Statics treats of balanced forces, or rest; dynamics, of unbalanced forces, where motion ensues; and kinematics, of the comparison of motions with each other. Considering statical forces simply in the abstract, the bodies to which they are applied are assumed to be perfectly rigid, without breaking, binding, twisting, or in any wise changing upon the application of such forces.

Force is a cause tending to change the condition of a body as to rest or motion. Force is measured by weight. In England and the United States the unit of force is the pound; on the Continent, the gramme. All bodies fall, or tend to fall, to the earth. This force is called the attraction of gravitation. Its direction is that of a string from which a weight is suspended (Fig. 352). It is called a vertical line, and its direction is toward the centre of the earth. Practically, all such lines are considered parallels. Let a mass, P (Fig. 353), be suspended by a cord. Each particle is acted on by *gravity*, and the *resultant* of all these parallel forces is the force resisted by the cord, or the entire weight of the body. If a mass (Fig. 354) be suspended from two different points, P and Q, the directions of the string will meet at a point C, which is called the *centre of gravity*, where all the weight may be considered to be concentrated. When a body of uniform density has

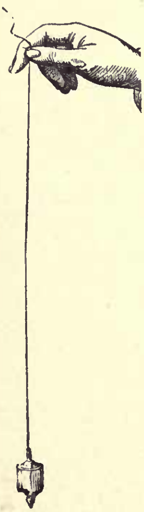


FIG. 352.

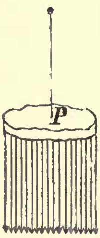


FIG. 353.

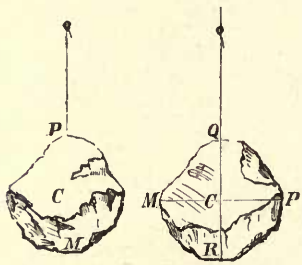


FIG. 354.

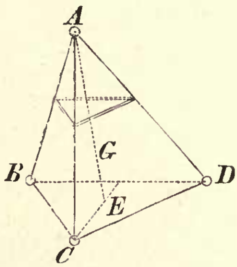


FIG. 355.

a centre of symmetry (a point which bisects all straight lines drawn through it), this point coincides with the centre of gravity. The middle of a straight line, the centre of a circle, the intersection of the diagonals of a parallelogram, the intersection of lines drawn from any two angles of a triangle to the centres of the opposite sides, are the centres of symmetry; in solids, the centre of a sphere, the middle point of the axis of a cylinder, and the intersection of the diagonals of a parallelopiped.

The centre of gravity of the triangular pyramid (Fig. 355) is in the straight line A E, connecting the apex A with the centre of symmetry of the base triangle B C D, and distant $\frac{1}{4}$ of the length of the line A E from E.

The centre of gravity of solids which may be divided into symmetrical figures and pyramids, as for all practical purposes most may be, can be found by determining the centre of gravity of each of the solids of which it is composed, and then compounding them. The centre of gravity of bodies inclosed by more or less regular contours, as a ship, for instance, is determined by dividing it into parallel and equidistant sections, finding the centre of gravity of each, and compounding them.

The centre of gravity of a body may be determined practically, as shown above, by its suspension from different points. It can be done generally more readily by balancing the body in horizontal positions on different lines of support; the centre of gravity will lie in the intersection of planes perpendicular to these lines. A body, unless the vertical line from the centre of gravity falls within the base of support (Fig. 356), will fall over (Fig. 357). A person carrying a weight insensibly throws a portion of the body forward, backward, or laterally, to balance the load. Thus, in Fig. 358, the body is thrown

FIG. 356.

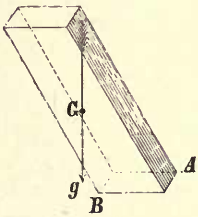


FIG. 357.

FIG. 358.

back, so that the vertical from the centre of gravity *g*, compounded of the centre of gravity G of the woman and H of the load, falls within the base of the feet.

When a figure rests in such a position that its centre of gravity is in its lowest position, it is said to be in *stable equilibrium*. A ball may rest in any position, as the centre of gravity is neither depressed nor raised by movement; but in the toy (Fig. 359) any movement tends to raise the centre of gravity, and, on the cessation of the force, the body returns to its original position. The ellipsoidal form (Fig. 360), placed on its pointed end, is balanced, but the slightest movement lowers the centre of gravity, and, without the application of an outside force, it can not be raised, and therefore falls.

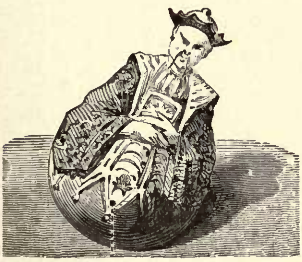


FIG. 359.

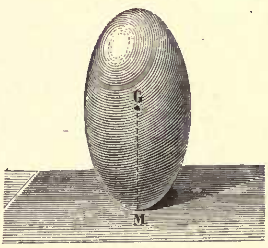


FIG. 360.

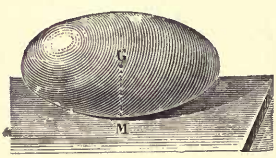


FIG. 361.

This is called *unstable equilibrium*, while in the position shown in Fig. 361 it is in *stable equilibrium*. In the toy (Fig. 362) the body of the figure is light, and the weight of the balls brings the centre below the point of support. This will admit of great oscillation, and return to its original position.

When two parallel forces, F F′, are applied at the extremities of a straight line (Fig. 363), they have a resultant, R, equal to their sum, and acting at a point, C, which divides the line inversely in proportion to the forces. If the forces are equal, the point C will be at the centre of the line; if the force F is double that of F′, C A will be equal to one half C B. This is called the principle of the *lever*.

Levers, in practice, are called of the first (Fig. 364), the second (Fig. 365), and the third class (Fig. 366), according to the positions of the three forces, the weight, W, the power applied, P, and the *fulcrum*, or support or turning-point, F, of the lever. The two extreme forces must always act in the same direction; the middle one must act in the opposite direction, and be equal to the sum of the other two; and the magnitude of the extreme forces is inversely proportional to their distances from the middle one. Let the middle force, *c*, be measured by a spring-balance (Fig. 367), it will mark the sum of the weights *a* and *b*.

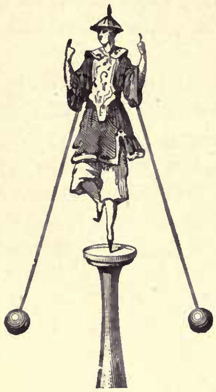


FIG. 362.

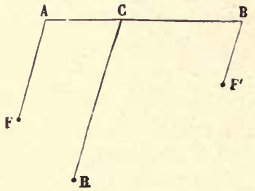


FIG. 363.

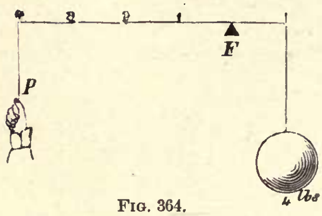


FIG. 364.

Call the distance from *a* to *c*, *x*, and from *b* to *c*, *y*, then the weight *a* will be to the weight *b* as *y* is to *x*, or $a\,x = b\,y$. Suppose the weight *a* to be 6 pounds and at *b* 3 pounds, at *c* it will be 9 pounds, and *x* will be to *y* as 6 to 3, or, if the lever is 48 inches, *b c* will be 16 inches and *a c* 32 inches.

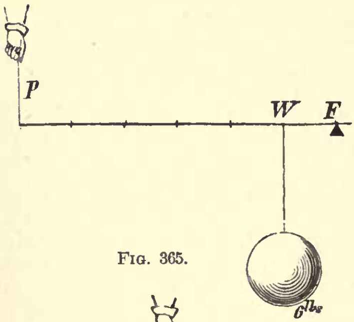


FIG. 365.

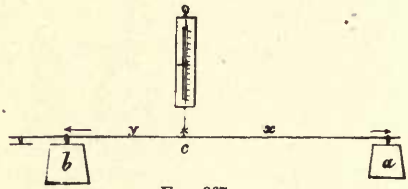


FIG. 367.

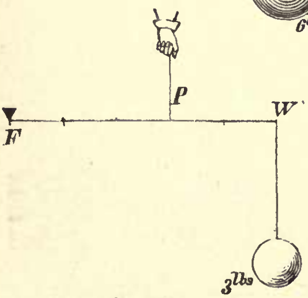


FIG. 366.

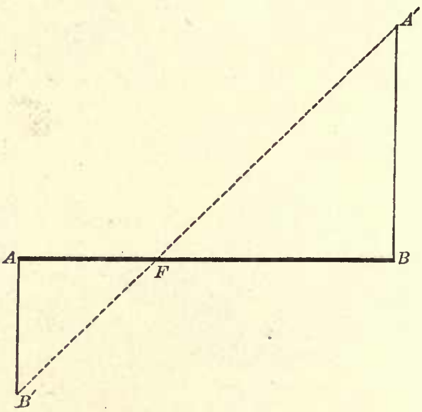


FIG. 368.

To find graphically the fulcrum, or point, at which a lever should be supported to sustain in equilibrium weights, or equivalent forces, acting at the extremities of the lever. Let A B (Fig. 368) be the lever. At A and B let fall and erect perpendiculars to the lever. Lay off from A, on any convenient scale, A B′, corresponding to the weight applied at B; and at B, on the same scale, B A′, the weight applied at A; draw the line A′ B′; its intersection, F, with the lever will be the position of the fulcrum. This is on the hypothesis that there is no weight to the lever, or that, after determining the position of the fulcrum, the lever itself is balanced on the point by the addition of weight on the short arm F A, or the reduction of weight on the long one F B. If the lever is of uniform weight, on perpendiculars to C, the centre of the lever (Fig. 369), and to F, the fulcrum, as before determined, lay off F C′, the weight of the lever, and C F′, the sum of the weights applied at A and B; draw C′ F′. Its intersection, F″, will be the actual fulcrum, taking into consideration the weight of the lever in addition to the weights suspended at the extremities.

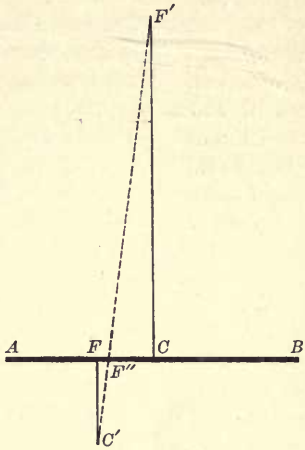


Fig. 369.

The Wheel and Axle.—If a weight, P, be suspended from the periphery of a wheel (Fig. 370), while another weight, W, is suspended on the opposite side of a barrel or axle attached to the wheel, the principle of action is the same as that of the lever. P multiplied by its length of lever, the radius *c a* of the wheel, is equal to W multiplied by its length of lever, the radius of the axle *c b*; the axle *c* is the fulcrum. If a movement downward be communicated to P, as shown by the dotted line, a rotary motion is given to the wheel and axle; the cord of P is unwound while that of W is wound up, while P is still suspended from *a* and W from *b*; the leverage, or distance from the fulcrum, of each is the same as at first. The wheel and axle is a lever of continuous action. Since the wheel has a larger circumference than the axle, by their revolution more cord will be unwound from the former than is wound up on the latter; P will descend faster than W is raised in the proportion of the circumference of the wheel to that of the axle, or of their radii—that is, as *c a* to *c b*. When P has reached the position P′, W will have reached W′. If *c a* be four times *c b*, then P will have moved four times the distance that W has. The movement is directly as the length of the levers, or the radii of the points of suspension. Therefore, to move a large weight by the means of a smaller one, the smaller must move through the most space, and that the spaces described are as the opposite ends of the lever, or inversely as the weights.

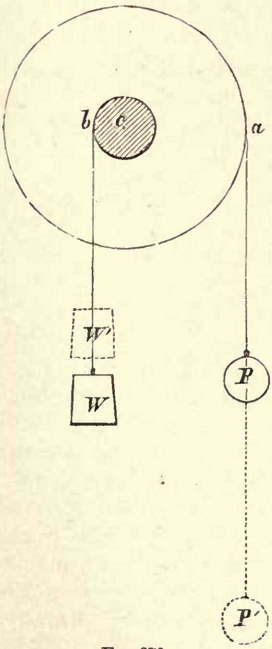


Fig. 370.

It is the fundamental principle of the action of all mechanical powers, that whatever is gained in power is lost in space travelled; that, if a weight is to be raised a certain number of feet, the force exerted to do this must always be equal to the product of the weight by the height to which it is to be raised; thus, if 200 pounds are to be raised 50 feet, the force exerted to do this must be equal to a weight which, if multiplied by its fall, will be equal to the product 200×50, or 10,000; this force may be a weight of 10,000 pounds falling 1 foot, or of 1 pound falling 10,000 feet.

It is now common to refer all forces exerted to a unit of *pounds-feet* (see p. 72, Fig. 181), that is, 1 pound falling 1 foot; and the effect to the same unit of pounds-feet, 1 pound raised 1 foot. Thus, in the example above, the force exerted or power is 10,000 pounds-feet falling; the effect 10,000 pounds-feet raised. In practice, the pounds-feet of force exerted must always be more than the pounds-feet of effect produced; there must be some excess of the former to produce movement and to overcome resistance and friction of parts.

The measure of any force, as represented by falling weight, is termed the *absolute power* of that force; the resulting force, or useful effect for the purposes for which it is applied, is called the *effective power*.

The Pulley.—The single fixed pulley (Fig. 371) consists of a single grooved wheel movable on a pin or axis, the strap through which the pin passes being attached to some fixed object. A rope passes over the wheel in the groove; on one side the force is exerted, and on the other the weight is attached and raised. It may be considered a wheel and axle of equal diameters, or as a lever in which the two sides are equal, the pin being the fulcrum. P, the force exerted, must therefore be equal to the weight W, raised; and, if movement takes place, W will rise as much as P descends.

The fixed pulley is used for its convenience in the application of the force; it may be easier to pull down than up, for instance; but the pounds-feet of force must be equal to the pounds-feet of effect. The tension on the rope is equal to either the force or weight.

Fig. 372 is a combination of a fixed pulley, A, and a movable pulley, B. The simplest way to arrive at the principle of this combination is to consider its action. Let P be pulled down, say, two feet; the length of rope drawn to this side of the pulley must be furnished from the opposite side. On that side there is a loop, in which the movable pulley, with the weight W attached, is suspended. Each side of this loop, 2 and 3, must go to make up the two feet for the side or end 1. Cords 2 and 3 will therefore furnish each one foot. As these cords are shortened one foot, the weight W is raised one foot, and, as the movement of W is but one foot for the two feet of P, W must be twice P.

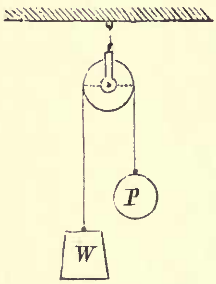


Fig. 371.

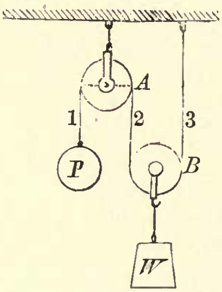


Fig. 372.

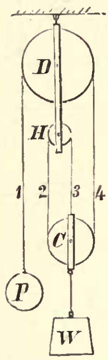



Fig. 373.

Fig. 374.

In the combination of pulleys (Fig. 373), let P be pulled, say, three feet; then this length of rope, drawn from the opposite side of the pulley, is distributed over the three cords, 2, 3, 4, and the weight W is raised one foot; consequently, W is three times P. The cord 1 supports P, the cords 2, 3, 4, the weight W, or three times P; consequently, the tension on every cord is alike. The same rope passing freely around pulleys must have the same tension throughout; so that, to determine the relation of W to P, count the number of cords which sustain the weight. Thus, in Fig. 374, the weight is sustained by four cords; consequently, it is four times the tension of the cord, or four times the force P. In order not to confuse the cords, the pulleys are represented as in the figures; but, in construction, the pulleys, or sheaves, are usually of the same diameter, and when connected, as A and B, and C and D, they run on the same pin.

The Inclined Plane.—To support a weight by means of a single fixed pulley, the force must be equal to the weight. Suppose the weight, instead of hanging freely, to rest upon an inclined plane $b\,d$ (Fig. 375); if motion ensue, to raise the weight W the height $a\,b$, the rope transferred from the weight side of the pulley will be equal to $b\,d$, and P will have, consequently, fallen this amount; thus, if $b\,d$ be six feet, and $a\,b$ one foot, while W is raised one foot, P has descended six feet; and, as pounds-feet of power must equal pounds-feet of effect, P will be one sixth of W; thus, P is to W as *a b* is to *b d*, or as the height of the incline is to its length. If the end of the plane *d* be raised, till it becomes horizontal, the whole weight would rest on the plane, and no force would be necessary at P to keep it in position; if the plane be revolved on *b*, till it becomes per-

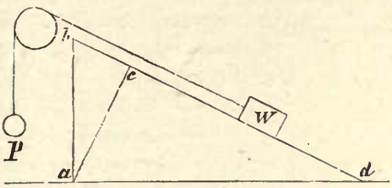


Fig. 375.

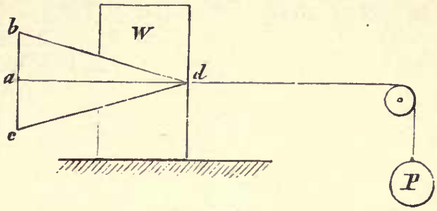


Fig. 376.

pendicular, then the weight is not supported by the plane at all, but it is wholly dependent on the force P, and is equal to it. Between the limits, therefore, of a level and a perpendicular plane, to support a given weight W, the force P varies from nothing to an equality with the weight.

The construction (Fig. 376) illustrates the principle of the *wedge*, which is but a movable inclined plane; if the wedge be drawn forward by the weight P, and the weight W be kept from sliding laterally, the fall of P a distance equal to *a d* will raise the weight W a height *c b*. P will therefore be to W as *c b* is to *a d*. For example, if the length of the wedge *a d* be ten feet, and the back *c b* two feet, then P will be to W as two to ten, or one fifth of it.

Let the inclined plane *a b d* (Fig. 376) be bent round, and attached to the drum A (Fig. 377), to which motion of revolution on its axis is given, by the unwinding of the turns of a cord from around its periphery, through the action of a weight P suspended from a cord passing over a pulley. If the weight W be retained in its vertical position, by the revolution of the drum it will be forced up the incline, and when the cord has unwound one half turn from the drum, and consequently the weight P descended a distance, *c e*, equal to one half the circumference of the drum, the weight W has been raised to the height *a b* by the half revolution of the plane; P must therefore be to W as *a b* is to one half the circumference. Extend the inclined plane so as to encircle the drum (Fig. 378). The figure illustrates the mechanism of the screw, which may be considered as formed by wrapping a fillet-band or thread around a cylinder at a uniform inclination to the axis. In practice, the screw or nut, as the case may be, is moved by means of a force applied at the extremity of a lever; a complete revolution raises the weight the distance from the top of one thread to the top of the one above, or *the pitch*. If the force be always exerted at right angles to the lever (Fig. 379), the lever may be considered the radius of a wheel, at the circumference of which the force is applied. Thus, if the lever be three feet long, the diameter of the circle would be six feet, and the circumference 6 × 3·1416, or $18\frac{85}{100}$ feet; if the pitch be one inch, or one

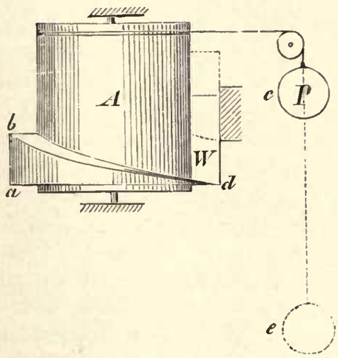


Fig. 377.

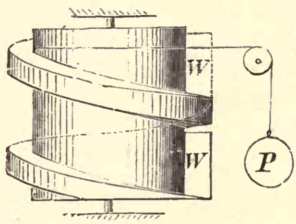


Fig. 378.

twelfth of a foot, then the force would be to the weight as one twelfth is to 18·85; and if the force be one pound, the weight would be 226·20 pounds.

The resultant of two forces of exertion, as has been shown, is their sum, and counterbalances the force of resistance, which must be applied at a point intermediate between, and distant from each of them inversely as the forces exerted.

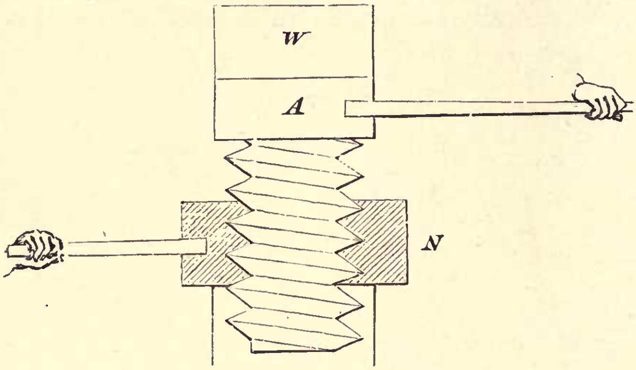


FIG. 379.

The resultant of any number of parallel forces acting in one direction is equal to their sum acting in the same direction at some intermediate point; that is, the effect of all the forces is just the same as if there were but one force, equal to their sum, acting at this point, and is balanced by an equal force acting in the opposite direction. This central point may be determined by finding the resultant, i. e., the sum, and the point of application for any two of the forces, as shown graphically in Figs. 368, 369, and then of the other two, the resultants thus determined being again added together like simple forces.

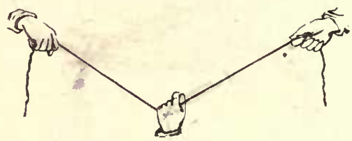


FIG. 380.

Inclined Forces are those whose directions are inclined to each other. When two men of equal strength pull directly opposite to each other, the resultant is nothing. Let a third take hold of the centre of the rope (Fig. 380), and pull at right angles to the rope; he will make an angle in the rope, and the other two now pull in directions inclined to each other. The less the force exerted at the centre, the less the flexure in the rope; but

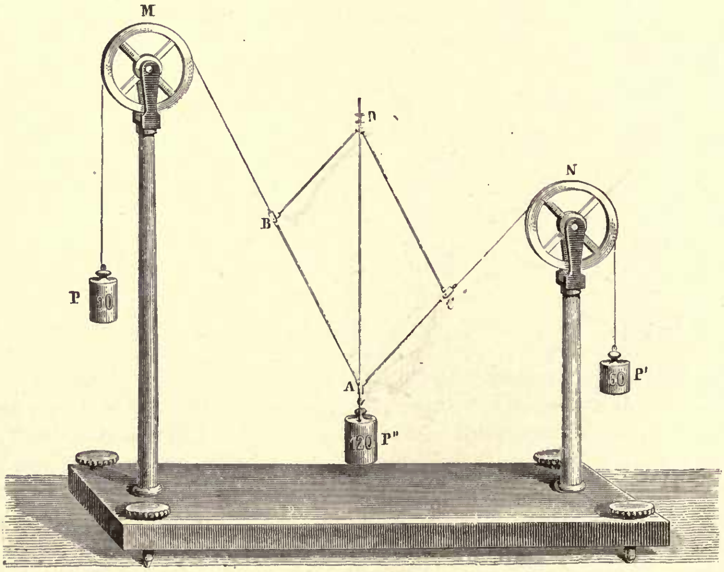


FIG. 381.

when it becomes equal to the sum of the forces at the ends, the two, to balance it, must pull directly against it, bringing the ends of the rope together, and acting as parallel forces. Between the smallest force and the largest that can be exerted at the centre and maintain a balance or equilibrium, the ends of the rope assume all varieties of angles, which angles bear definite relations to the forces.

Represent these forces by weights (Fig. 381). Let P and P′ be the extreme forces acting over the pulleys M and N, and tending to draw the rope straight, which the weight P″ prevents. Lay off the weight of P (90 pounds) along A B, and the weight of P′ (60 pounds) along A C. Draw B D parallel to A C, and C D parallel to A B. Connect D with A. If this is measured with the same scale that A B and A C were laid off with, it will be found that it equals 120 pounds, which will be found to be the same as the weight P″. A D, therefore, gives the amount and direction of the resultant of the two forces P and P′, which resultant is balanced by P″. In the same way the resultant of any number of inclined forces (Fig. 382) may be found by compounding the resultant of any two forces with a third, and so on.

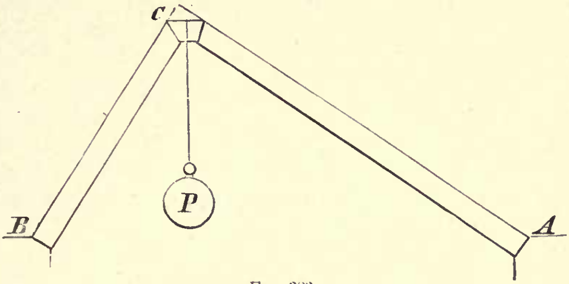


Fig. 383.

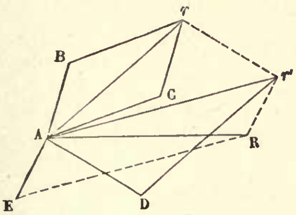


Fig. 382.

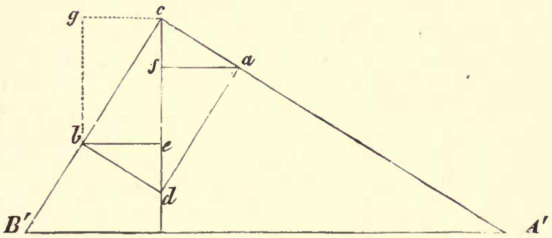


Fig. 384.

As two forces may be compounded into a single resultant, so conversely one force may be resolved into two components; thus, let the weight P (Fig. 383) be supported by two inclined rafters, C A and C B. Each resists a part of the force exerted by the weight P. To find the force exerted against the abutments A and B, in the direction of C A and C B, draw *c* A′ (Fig. 384) parallel to C A, *c* B′ to C B, and *c d*, a parallel to the line C P, the direction in which the weight P acts; lay off *c d* from a scale of equal parts, a length which will represent the number of pounds, or whatever unit of weight there may be in the weight P; draw *d a* parallel to *c* B′, and *d b* parallel to *c* A′; *c a*, measured on the scale of equal parts adopted, will represent the pounds or units of weight exerted against A in the direction of C A, and *c b* the pounds or units of weight exerted against B in the direction of C B.

This method of finding the *resultant* of two forces, or the *components* of one force, is called the *parallelogram of forces.* If two sides of a parallelogram represent two forces in magnitude and direction, the resultant of these two forces will be represented in magnitude and direction by the diagonal of the parallelogram and conversely.

The sum of *a c* and *c b* is greater than *c d*; that is, the weight P exerts a greater force in the direction of the lines C A and C B, against A and B, than its own weight; but the down pressure upon A and B is only equal to the weight of P and of the rafters which support it, which last, in the present consideration, is neglected. Resolve *c b*, the force acting on B in the direction of *c* B′, into *g b* or *c e* the downward pressure, and *c g* or *e b* the horizontal thrust on the abutment B, and *c a* into *c f* and *f a*. To decompose a force, form a triangle, with the direction of the other forces, upon the line representing the magnitude and direction of the given force; *c e* represents the weight on B, *c f* the weight on A; *c d*, or *c e* + *d e*, the whole weight P; therefore, the weight upon the two abutments A and B is equal to the whole weight of P.

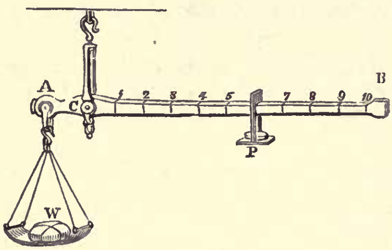


Fig. 385.

The steelyard (Fig. 385) is a lever, from the short arm of which a dependent hook or scale supports the article to be weighed; while, on the long arm, a fixed weight, P, is slid in or out from the fulcrum till it balances the article; the distance as marked on a scale on the long arm determines the weight. In platform-scales, when very heavy weights are balanced by small weights on a graduated arm,

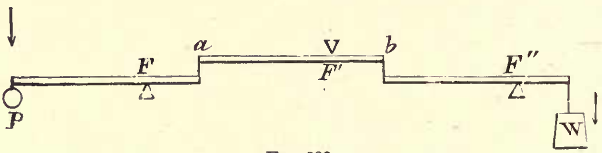


Fig. 386.

combinations of levers are used, the principle of which can be understood from Fig. 386. Thus, suppose P F to be 7″, *a* F 2″, *a* F′ 9″, *b* F′ 2″, *b* F″ 11″, F″ W 3″.

P is to force *a* as *a* F to P F, or as 2 to 7
Force *a* is to *b* as *b* F′ to *a* F, or as 2 to 9
b is to W as F″ W to *b* F″, or as 3 to 11

P is to W as 12 to 693

The differential axle, or Chinese capstan, consists of an axle with two different diameters (Fig. 387), the weight W being suspended in the loop of a cord wound around

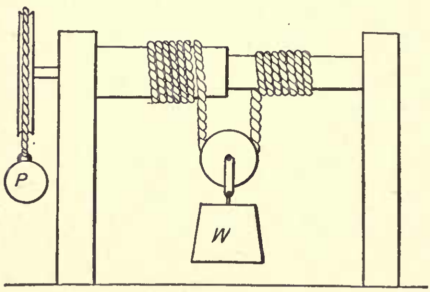


Fig. 387.

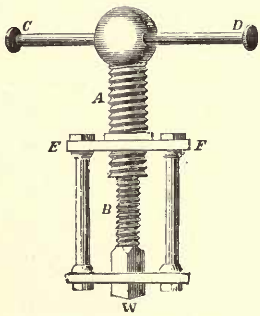


Fig. 388.

these axles in opposite directions by a single turn of the axle. The weight is only raised or lowered by the difference between these two circumferences; one takes up while the other lets out, and the P, to balance W, must be as these differences of circumference of axles is to the circumference of the wheel from which P is suspended.

The differential screw (Fig. 388) consists of an exterior screw, A, and an interior screw, B. By the revolution of the arm, the screw A is moved through the plate D in proportion to its pitch, but the interior screw B moves inward its pitch, and the movement of W is only the pitch of A less that of B, and the power applied is to the weight moved as the difference of these pitches is to the circumference described by the power.

As the lever (Fig. 389) moves under the action of power or weight, the lever becomes inclined to the direction of the forces, but the forces are still parallel. The relations of the forces to each other are not changed, but the absolute action of each is only that due to the length *a b* and *b c*, to which the directions of the forces are perpendicular. In the bent lever (Fig. 390) the action of the forces is estimated on lengths of arms,

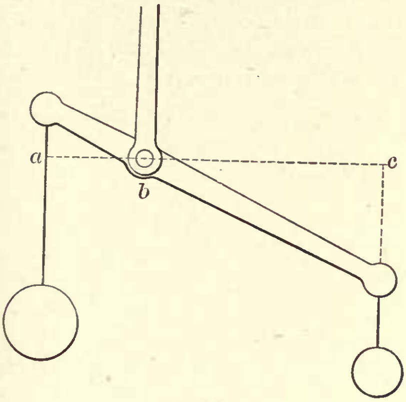


FIG. 389.

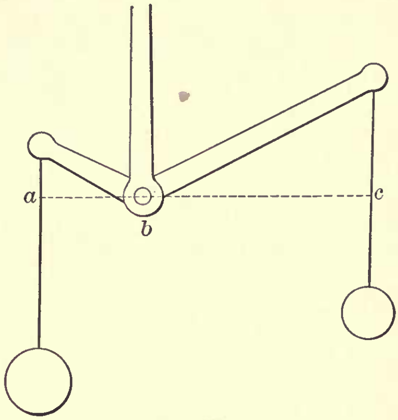


FIG. 390.

determined by the perpendiculars *a b* and *b c* let fall from the fulcrum on the directions of the forces.

The toggle-joint (Fig. 391) is much used for presses. The force is exerted in the direction of the arrow at C, and the effective force is to separate the plates A and B. The action is as shown in Fig. 391*a*. Equal movements, as C–1, 1–2, 2–3, correspond to unequal movements at A and B, as A a', $a'\,a^2$, $a^2\,a^3$. The nearer the force C is to the line A B, the less the movement $a^2\,a^3$; and, consequently, the force C exerts greater effects in intensity, but the latter is less in movement.

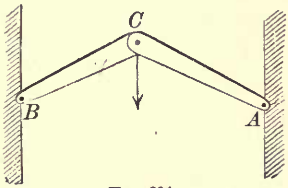


FIG. 391.

Fig. 392 exhibits the principle of the hydraulic press. The small plunger or piston may be considered the application of the force, and the large one the weight to be raised to balance each other; the pressure per square inch of surface must be the same, and the force must be to weight as the surface of its piston is to that of the weight-piston. If motion takes place, the force will move through space corresponding to the area of weight-piston, while the weight will move that of the area of the force-piston. And this is the great principle of all mechanism in the transmission of force: there can be no total gain. What is gained in force is lost in movement, and in many complicated machines the theoretical comparison of force applied and resultant force may be ascertained by the measures of their movements.

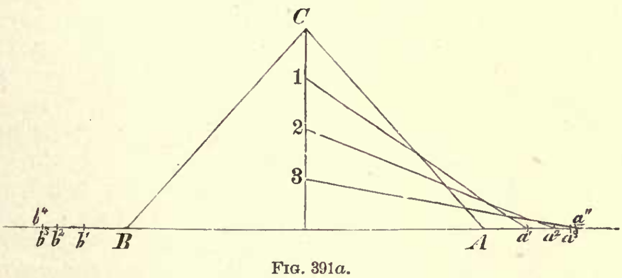


FIG. 391*a*.

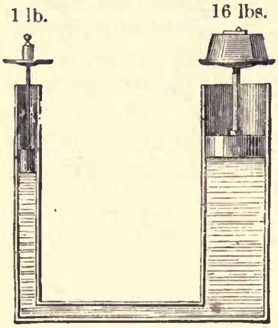


Fig. 392.

The resultant effects of forces, as heretofore treated, have been without motion, or static. But when motion is produced, the forces are called *dynamic.* A weight suspended or supported exerts a force, which is balanced by the resistance of the suspending or supporting medium; but a falling weight acquires an increasing velocity with every unit of time or space passed. All bodies would fall with the same velocities were it not for the different resistances from the air due to their different bulk in proportion to their weight. Dense articles, as stones and metals, acquire a velocity in this latitude of about 32·2 feet in each second, called the intensity of gravity, or g. The value of g at the equator is 32·088; at the poles, 32·253. A body

		Ft.	Tot. Fall.
Starting with a velocity	0		
Falls during the 1*st second*	16 =	16	16
Acquiring a velocity of	32 feet per second.		
Falls during the 2*d second*	32 + 16 =	48	64
Acquiring a velocity of twice 32, or	64 feet per second.		
Falls during the 3*d second*	32 + 32 + 16 =	80	144
Acquiring a velocity of 3 × 32 =	96 feet per second.		
Falls during the 4*th second*	32 + 32 + 32 + 16 =	112	256
Acquiring a velocity of 4 × 32 =	128 feet per second.		
Falls during the 5*th second*	32 + 32 + 32 + 32 + 16 =	144	400
Acquiring a velocity of 5 × 32 =	160 feet per second.		

Calling s the space passed over, v the terminal velocity in feet, t the time in seconds of falling, $s = \frac{1}{2}gt^2$, $v = gt$ or $= \sqrt{64.4s}$. In determining the velocity of issuing water under a head h, corresponding to s in the equation, it is generally near enough to reckon v as eight times the square root of the head ($\sqrt{h}$).

The motion of falling bodies is a uniformly accelerated one, but there are also uniformly retarded motions in which the velocity is decreased by equal losses in equal times. There are also uniform motions when bodies are impelled by a constant force and opposed by constant resistances.

In Fig. 393, *o s* represents the trace of a body impelled horizontally by a uniform, but falling through the action of gravity with an accelerated, force. This curve, a parabola, represents approximately the curve of the thread of stream issuing from an orifice, or flowing.

It will be seen that to produce twice the velocity the body must fall through four times the space; that there is four times the force stored in the body. But to maintain this velocity uniformly, only twice the force is necessary. The *momentum* of a body is its mass multiplied by its velocity, but its *inertia* is as the square of the velocity. It is an established principle of mechanics that the results must be proportional to the causes: if a body has to be raised four feet to obtain a double velocity in falling, the destructive result of that fall must also be four times.

Under statics, it has been shown that forces may be resolved and compounded. The same may be done dynamically—that which has been treated as weight must now be considered as momentum.

In treating of dynamic forces the resultants have been considered as equal to the exertion, without any losses by resistances. This never happens in practice; the resistances are a very large element. Resistances from the medium in which the bodies are moved are from the surfaces on which the bodies are supported; resistances due to the displacement of the fluid in which the bodies move, and frictional resistances, or what is termed skin-resistances, of bodies moving through air or water; and the surface-resistance of bodies sliding or rolling on each other. Suppose a weight to rest on a horizontal surface—it will take a certain force to move the insistent weight depending on the amount of this weight and the kind of surfaces in contact, and the force that will just cause motion overcomes the friction, or frictional force, and is equal to it. The frictional force is only a percentage of the insistent force of the body, and this percentage is called the *coefficient of friction*. If the horizontal surface of support be raised at one end, so as to make the surface inclined, it will after a time become so steep that the insistent body will slide down the surface. Thus, in Fig. 394, if the body Q is ready to slip on the surface A B, the angle B A C, which represents the angle of the surface with the horizontal, is called the *angle of repose, or limiting angle of frictional resistance;* or thus (Fig. 395), if the force acting in the direction P″ M is

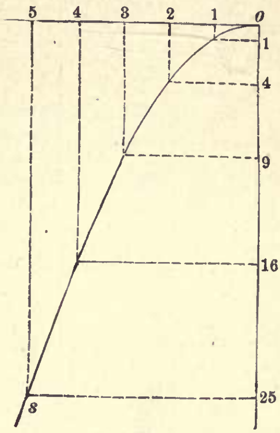


Fig. 393.

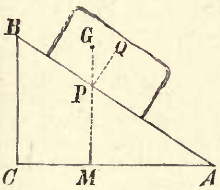


Fig. 394.

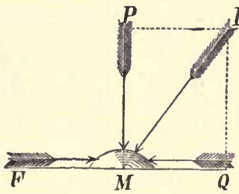


Fig. 395.

just sufficient to produce motion of the mass M along the plane F Q, the angle P M P″ is the limiting angle of resistance.

General Morin has made an elaborate course of experiments on friction, some of the results of which are given in the table on the following page.

In Morin's experiments the surfaces of the woods were first planed and those of the metals filed and polished with the utmost care. When the friction without lubrication was to be determined, any unctuosity was especially provided against. For unctuous surfaces, the unguent was carefully wiped off, so that no layer of it should prevent their intimate contact.

When lubricated, the resistance from the viscidity of the lubricant may be overlooked, compared with the friction. As respects the nature of the substance used in lubrication, it was observed, by comparison of the coefficient of the friction of motion, that with hog's lard and olive oil surfaces of wood on metal, wood on wood, metal on wood, and metal on metal, had all very nearly the same friction—between 0·07 and 0·08. With tallow the coefficient was the same except in the case of metals upon metals.

Practically the sliding of wooden surfaces on each other or on metal surfaces is confined within small limits. As far as possible, sliding surfaces in most machines or mechanical appliances are of metals, and when surfaces are liable to be affected by rust and adhere, they are made of brass or bronze. But in the moving of houses or the launching of ships the ways are of wood, and in this country of Southern pine, as being without knots and extremely stiff.

Sliding Friction of M. Morin's Experiments on Plane Surfaces.

Sliding surface. s.	Surface at rest. s′.	State of the Surfaces.		Friction of Motion. Coefficient of friction.	Friction of Motion. Limiting angle of resistance.	Friction of Quiescence. Coefficient of friction.	Friction of Quiescence. Limiting angle of resistance.
Oak......	Oak......	Fibres s s′ parallel to motion.	Without lubrication	0·478	25°33′	0·625	32° 1′
Oak......	Oak......	Fibres s perpendicular to motion...	Without lubrication	0·324	17 58	0·540	28 23
			Unctuous..........	0·143	8 9	0·314	17 26
Oak......	Oak......	Fibres s s′ perpendicular to motion.	Without lubrication	0·336	18 35		
Oak......	Elm......	Fibres s s′ parallel to motion.	Without lubrication	0·246	13 50	0·376	20 37
			Unctuous..........	0·136	7 45		
Elm......	Oak....	Fibres s s′ parallel to motion.	Without lubrication	0·432	23 22	0·694	34 46
			Unctuous..........	0·119	6 48	0·420	22 47
		Fibres s perpendicular to motion.	Without lubrication	0·450	24 16	0·570	29 41
Beech.....	Oak......	Fibres s s′ parallel to motion.......	Without lubrication	0·360	19 48	0·530	27 56
			Unctuous..........	0·330	18 16		
Cast-iron..	Oak......	Fibres s s′ parallel to motion.......	Without lubrication	0·490	26 7		
			Unctuous..........	0·107	6 7	0·100	5 43
Oak......	Cast-iron .	Fibres of the wood perpendicular to motion.	Without lubrication	0·372	20 25		
Cast-iron..	Elm......	Fibres of the wood parallel to motion.	Without lubrication	0·195	11 3		
			Surface unctuous...	0·125	7 8		
Wrought-iron.	Wrought-iron.	Fibres s s′ parallel to motion.......	Without lubrication	0·138	7 52	0·137	7 49
			Surfaces unctuous..	0·177	10 3		
Wrought-iron.	Cast-iron .	Fibres s s′ parallel to motion.......	Without lubrication	0·194	10 59	0·194	10 59
			Surfaces unctuous..	.....	......	0·118	6 44
			Lubricated with olive oil.........	0·066	......	0·100	5 43
Cast-iron..	Wrought-iron.	Fibres s s′ parallel to motion.......	Surfaces unctuous..	0·143	8 9		
			Without lubrication	0·152	8 39	0·162	9 13
			Surfaces unctuous..	0·144	8 12		
Cast-iron..	Cast-iron .	Fibres s s′ parallel to motion.......	Lubricated with water.....	0·314			
			Lubricated with soap......	0·197			
			Lubricated with tallow.....	0·100	5 43	0·100	5 43
			Lubricated with lard......	0·070	......	0·100	
			Lubricated with olive oil.	0·064			
			Lubricated with lard and plumbago	0·055			
Wrought-iron.	Bronze...	Fibres s s′ parallel to motion.......	Without lubrication	0·172	9 46		
			Surfaces unctuous..	0·160	9 6		
Bronze....	Wrought-iron.	Fibres s s′ parallel to motion.......	Without lubrication	0·161	9 9		
			Surfaces unctuous..	0·166	9 26		
Cast-iron..	Bronze ...	Fibres s s′ parallel to motion.......	Without lubrication	0·147	8 22		
			Surfaces unctuous..	0·132	7 22		
Bronze....	Cast-iron .	Fibres s s′ parallel to motion.......	Without lubrication	0·217	12 15		
			Surfaces unctuous..	0·107	6 7		
Bronze....	Bronze ...	Fibres s s′ parallel to motion.......	Without lubrication	0·201	11 22		
			Surfaces unctuous..	0·134	7 38	0·164	9 19
Steel......	Cast-iron .	Fibres s s′ parallel to motion.	Without lubrication	0·202	11 26		

In the launching of a vessel, the width of the ways must be proportioned to its weight, not to exceed three tons per square foot. A slope of $\frac{5}{8}''$ to the foot permits the control of the vessel if necessary. On a slope of $\frac{3}{4}''$ to the foot, a vessel will move steadily and gently down without attaining a high speed, and is the one adopted where other circumstances do not affect the choice. A slope of $\frac{7}{8}''$ to the foot is often used for large vessels; the speed attained, however, is very high, and in narrow water both dangerous and inconvenient. A slope of 1 in 12 may be used where the distance to be traversed is short, and the ship launched broadside in.

Before striking the dog-shore, which prevents the ship from sliding down the ways, they are thoroughly slushed with some greasy mixture, like tallow and soap.

It was formerly held that friction was directly as the weight, without regard to the amount of surface or velocity of movement, and Morin's experiments, referred mostly to the friction of quiescence and slow movements, come within this rule; the results give coefficients that may be considered maxima, but in practice it has been found that the coefficient of friction with unguents is reduced by increase of velocity and of temperature, that extent of surface may be prejudicial, and that careful selection of unguents, according to the work to be done, will reduce friction.

Axle and Rolling Friction.—Axle friction has been generally supposed to follow the laws of sliding friction, with the exception that its coefficient is a smaller fraction of the total pressure applied. There are but few experimental data relating to this branch of the subject. The results of some experiments by Morin are given in the following table:

Ratios of Friction to Pressure for Axles in Motion in their Bearings.

I.—ACCORDING TO MORIN'S EXPERIMENT.

Designation of Surfaces in Contact.	State of surfaces and nature of lubrication.							
	Dry or slightly greasy.	Greasy, and wet with water.	Lubricated, and wet with water.	Oil, tallow, or hogs' lard.		Purified very soft grease.	Hogs' lard with plumbago.	Greasy, very soft to the touch.
				Supplied in the ordinary manner.	The grease continually renewed.			
Bronze on bronze	......	......	......	0·079	.....	......	......	......
Bronze on cast-iron	......	......	......	.....	0·049	......	......	......
Iron on bronze	0·251	0·189	......	0 075	0·054	0·090	0·111	......
Iron on cast-iron	......	......	......	0·075	0·054	......	......	......
Cast-iron on cast-iron	......	0·137	0·079	0·075	0·054	......	......	......
Cast-iron on bronze	0·194	0·161	......	0·075	0·054	0·065	......	0·137
Iron on lignum-vitæ	0·188	......	......	0·125	.....	......	......	0·166
Cast-iron on lignum-vitæ	0·185	......	......	0·100	0·92	......	0·109	0·140
Lignum-vitæ on cast-iron	......	......	......	0·116	.....	......	......	0·153
Lignum-vitæ on lignum-vitæ	......	......	......	.....	0·170	......	......	......

It is well known that the obstruction which a cylinder meets in rolling along a smooth plane is quite distinct in its character, and far less in amount, to that which is produced by the friction of the same cylinder drawn lengthwise along a plane.

In the composition of machines attrition should be avoided as much as possible, and rolling motions substituted.

On this principle depend the advantages of the application of *friction-wheels* and *friction-rollers.* The extremity of an axle *c*, Fig. 395*a*, instead of resting in a cylindrical socket, is made to rest on the circumferences of two wheels, A and B, to the axles of which, *a* and *b*, the friction is transferred, and consequently diminished in the ratio of the radius of the wheel A to the radius of the axle *a*.

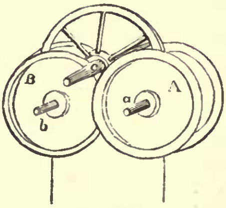


Fig 395*a*.

At speeds and pressures usual in machinery, the resistance from friction decreases as the journals and bearings heat up to limits which the engineer and mechanic would consider safe.

With very heavy pressures and slow speeds, the lubricant may be forced out and journal and lubricant be brought into close bearing, while with light pressure and low speeds the journals float on the film of fluid, and the frictional resistance is independent of the materials of which journal and bearing are composed.

Mr. C. J. H. Woodbury, in his experiments on the driving of cotton spindles, found the coefficient of friction to be from 7 to 20 per cent, the load being from 1 to 5 pounds per square inch, while Prof. Thurston, with heavy loads of 1,000 pounds per square inch, as on the crank-pins of the North River steamboat engines, found the coefficient of friction was one half of one per cent, the unguent being sperm oil. Practically, it may be said that the coefficient of friction for light-running spindles should not exceed 10 per cent, and for the usual work in shops, of say 100 to 200 pounds, should not exceed from 2 to 3 per cent.

For lubricants under heavy pressure and slow speeds, use graphite, soapstone, tallow, lard, and greases. For the journals of calender rolls in paper mills, it is common to lay on the top of the journals a large piece of salt pork, skin up.

Cast iron holds oil better than any other metal or alloy, and is the best metal to use for light bearing, perhaps for heavy.

It has been proved by Mr. Waite's experiments that a highly polished bearing is more liable to friction than any surface finely lined by filing. The lines left by the file serve as reservoirs for the oil, while the high polish leaves no room for the particles between the metal surfaces.

Mr. Waite's experiments on very heavy bearings at Manchester, N. H., go far "to prove that a considerable quantity of thin, fine oil keeps the bearing much cooler, and requires less power than a smaller quantity of thick, viscous oil. No vegetable oil is fit to use as a lubricant, and castor oil is the worst of all, because the most viscous, and all vegetable oils in connection with fine vegetable fibre, like cotton, are liable to spontaneous combustion.

"The rule of best lubrication is to use an oil that has the greatest adhesiveness to metal surfaces, and the least adherence as to its own particles. Fine mineral oils stand first in this respect, sperm second, neat's-foot third, lard fourth."

Experiments on rolling friction usually include that of the axles. The resistance of a wheel rolling on a smooth and resistant plane is very different, and much less than that of the same wheel fixed and sliding on the same plane; a moving railroad train is brought to a stop by the brake reducing or stopping the rotation of the wheels.

Between the usual diameters of wheels the resistance has not been found to increase inversely as the diameter, but rather as their square roots of the diameter, and less on hard and well-graded roads.

Of the resistance dependent on the material and character of the roads there have been many experiments. The following formula and table is from Prof. Thurston on "Rolling Friction."

$R = f\frac{W}{r}$ in which R = resistance, W the total weight, and r the radius of the wheel:

Kind of road.	Value of friction.
Well paved	0·02
Hard, smooth ground	0·02
Well macadamized and rolled	0·015
Smooth wooden pavement	0·01
Ordinary railroads	0·003
Best possible railroad	0·001

"On railway trains a minimum resistance is reached usually at a speed between 10 and 15 miles per hour, but the increase is not great at higher speeds where the common system of lubrication is practised; in ordinary work, the resistance varies as low as from 4 pounds per ton of train up to 25, and sometimes above 30 pounds."

At a speed of 25 miles per hour, Chanute makes the increase of resistance due to curvature about 0·4 pound per degree per ton.

The amount of resistance is measured by the weight resting on the axles, multiplied by c, which is dependent on the condition of the surface of the rails:

Perfectly dry	$c = \frac{1}{5}$
Average condition	$\frac{1}{6}$ to $\frac{1}{7}$
Damp in rain, fog, or tunnel	$\frac{1}{12}$
Greasy or iced	$\frac{1}{20}$

With head winds the resistance is increased with the square foot of front exposed and as the square of the velocity in miles per hour.

Side winds, by forcing the flanges of the wheels against the rails, adds very seriously to the resistance of draught.

On street railways the resistance to the movement of cars is greater than upon railroads, and may be considered from 3 to 5 times as much.

From his experiments in 1840, Morin states the resistance applied at the circumference of the wheel on pavements and macadamized roads to be inversely proportional to the diameter of the wheel independent of the breadth of the tire and increasing with the velocity.

By the Studebacker experiments in South Bend, Ind., it was found that there was no reduction in resistance on hard roads in increasing the width of the tire, but rather the contrary, nor in soft mud or slush; but on sandy roads or across fields the resistance is very much less with a broad tire.

In many States there are laws regulating the width of tire in proportion to the load, and even on the harder roads it is supposed that the broad tire deteriorates the track less than the narrow one, and in fact acts as a roller and improves the roadway, and for the same purpose the front wheels are of less gauge than the hind ones, that they may not track in the same rut.

It is of importance that the line of draught should be horizontal, as with an oblique pull the effort of the animal may either be exerted in carrying the load or by increasing the resistance of it on the roadway.

MECHANICAL WORK OR EFFECT.

Mechanical work is the effect of the simple action of a force upon a resistance which is directly opposed to it, and which it continuously destroys, giving motion in that direction to the point of application of the resistance; it is therefore the product of two indispensable qualities or terms:

First.—The effort, or pressure exerted.

Second.—The space passed through in a given time, or the velocity.

The unit of force in England and here is represented by the pound, and the unit of space by the foot.

The amount of mechanical work increases directly as the increase of either of these terms, and in the proportion compounded of the two when both increase. If, for example, the pressure exerted be equal to 4 pounds, and the velocity one foot per second, the amount of work will be expressed by $4 \times 1 = 4$. If the velocity be double, the work becomes $4 \times 2 = 8$, or double also; and if, with the velocity double, or 2 feet per second, the pressure be doubled as well—that is, raised to 8 pounds—the work will be $8 \times 2 = 16$ pounds feet. It is more usual to write *foot-pounds;* but as explained before the Continental idiom of *kilogrammetre* is followed, in which the unit of force, kilogramme, precedes that of space, the metre, it should be pounds-feet.

In comparison of motors with each other, it is usual to speak of them as so many horse-power equivalent to 550 pounds-feet per second, or 33,000 pounds-feet per minute. The Continental horse-power is equal to 75· kilogrammetres or 542·48 pounds-feet per second.

It is very common to use other units of force and space, as tons-miles; and train-miles, in railway practice.

The *time* must also be expressed or understood. It is impossible to express or state intelligibly an amount of mechanical effect without indicating all the three terms—force, space, and time.

The motors generally employed in manufactures and industrial arts are of two kinds —living, as men and animals; and inanimate, as water and steam.

What may be termed the amount of a day's work, producible by men and animals, is the product of the force exerted, multiplied into the distance or space passed over, and the time during which the action is sustained. There will, however, in all cases be a certain proportion of effort, in relation to the velocity and duration, which will yield the largest possible product or day's work for any one individual, and this product may be termed the maximum effect. In other words, a man will produce a greater mechanical effect by exerting a certain effort at a certain velocity than he will by exerting a greater effort at a less velocity, or a less effort at a greater velocity, and the proportion of effort and velocity which will yield the maximum effect is different in different individuals.

In the manner and means in which the strength of men and animals is applied there are three circumstances which demand attention:

1. The power, when the strength of the animal is exerted against a resistance that is at rest.

2. The power, when the stationary resistance is overcome, and the animal is in motion.

3. The power, when the animal has attained the highest amount of its speed.

In the first case the animal exerts not only its muscular force or strength, but at the same time a very considerable portion of its weight or gravity. The power, therefore, from these causes must be the greatest possible. In the second case some portion of the power of the animal is withdrawn to maintain its own progressive motion; consequently the amount of useful labour varies with the variations of speed. In the third case the power of the animal is wholly expended in maintaining its locomotion; it therefore can carry no weight.

Weisbach calls the mean effort of an animal one fifth its weight, which may serve as a general rule, but, in practice, will be considerably modified, when applied to the individual, depending upon the exertions, and the conditions and circumstances under which it is made. A man-power is usually estimated at one sixth of a horse-power (H. P.); yet, if the muscular force of a man be required for an effort of short duration, it will exceed one horse-power. Thus, a horse-power is equal to 33,000 pounds-feet per minute, or 550 pounds-feet per second; and, if a man weighing 150 pounds move upstairs at the rate of four feet per second, he exerts a force of 600 pounds-feet, which he can readily double for a few seconds.

The force of a man is utilized mechanically through levers, as in pumping or rowing, or at a vertical capstan, or at a crank, carrying or dragging loads, shovelling, etc. In continuous work at the lever he will exert from 25 to 30 pounds; at the crank, from 15 to 20 pounds.

The muscular force of horses is utilized in the draught of carriages, in hoisting, and in horse-powers, either moving in a circle round a central shaft or on a revolving platform, or on an endless belt. The draught of a horse varies with the speed of movement and its duration. Trautwine gives the draught of a horse at two and a half miles per hour for 10 hours per day, 100 pounds; 8 hours, 125 pounds; 6 hours, 166⅔ pounds; 5 hours, 200 pounds. The omnibus-horses here average nearly six miles per hour, and make 16 to 24 miles per day; the average will not exceed 16 miles. At the Manhattan Gas Works a span of horses hoist from the lighter 200 tons gross in 10 hours to the height of say 25 feet, with charges of 6 to the ton, in a bucket weighing 150 pounds, the rope passing over a single block and through a snatch-block. On a horse-power, the force exerted by a single horse is from 125 to 175 pounds, at an average speed of about three miles per hour, and for eight hours per day. Beyond a speed of four miles per hour the pounds-foot of work of a horse will decrease in an increasing ratio up to the limits of his speed, when the whole work done will be used up in locomotion. In proportioning levers, cranks, traces, chains, through which animal force is transmitted to machines, or for mechanical purposes, it is not safe to estimate the stress as the average force; there are impulses and stresses in action which will exceed the weight of the animal.

Water-Power.—Water, used for the purposes of power, moves machinery either by its weight, by pressure, by impact, or by reaction, and is applied through various forms of wheels.

Fig. 396 is what is termed a tub wheel with a vertical shaft, the wheel running in a bottomless, wooden case or tub to which the water is conveyed by a wooden trunk or spout and acting on the floats or buckets by impact. The wheel was commonly used for grist mills, and the power was applied directly to the stones through the vertical shaft. The same wheel on a horizontal shaft, without the tub, was called a flutter wheel, and the power was transferred by a crank on the shaft through a wooden connecting rod or pitman to the saw frame of a saw mill. A similar wheel, but of larger diameter, was suspended over the surface of a stream, the floats being dipped into the water sufficiently to give motion and power.

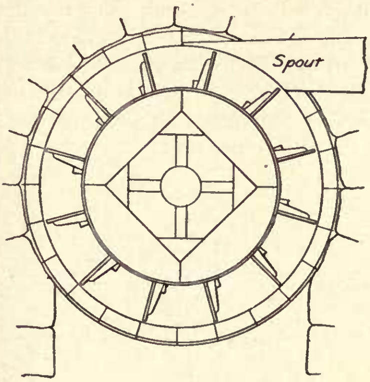


Fig. 396.

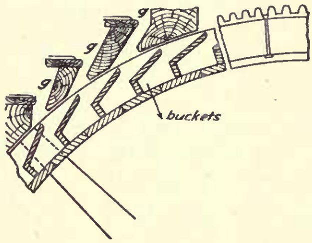


Fig. 397.

Fig. 397 is the section of the upper part of a breast wheel, in which the water is admitted through the gates *g g g*, acting after its delivery into the buckets through gravity; the power is transferred through the circumferential gear to a jack shaft, usually placed below the gates; a plank case called the breast is set outside the wheel with but little clearance from the gates to within a few feet of the bottom of the wheel. Wheels of this class, where water is brought over the top of the wheel and discharged into the buckets without a breast, is called an overshot wheel, and when the water is discharged beneath the wheel, an undershot. The overshot and breast, when properly

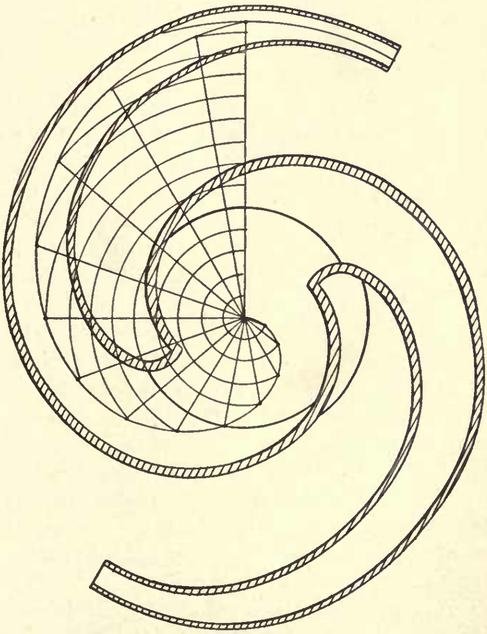


Fig. 398.

proportioned, give a good percentage of effect, but occupy considerable space, and the speed of the gear is slow.

Fig. 398 is one of the earliest improvements of the reactor, the Scotch wheel; the principle is that of the sky-rocket. It is very simple in construction; the central curve of the hollow arm or adjutage is constructed like a cam, in which the horizontal movement is a percentage of the radial velocity due to the head, and the cross sections of the arm a contracted vein.

The above wheels belong rather to history, but there are positions and circumstances which make them the most available.

The wheels now in use are mostly of cast iron or bronze, occupying small space, and the shafts giving a velocity nearly proportioned to the speed of the machinery which it is to drive.

Fig. 399 is a Fourneyron, called in this country a Boyden wheel, from the improvements which he made upon it. The water flowing downward through a pipe is diverted by guides at the bottom, which give it an outward direction with a tangential whirl as it strikes the buckets of the wheel, which are formed to give a reactive impulse.

In Fig. 400, the Jonval, there are fixed guides in the tube that give motion to the water against guides in the wheel beneath and a resulting reaction.

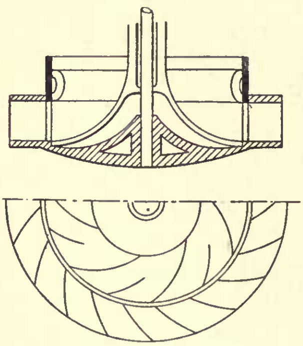


Fig. 399.

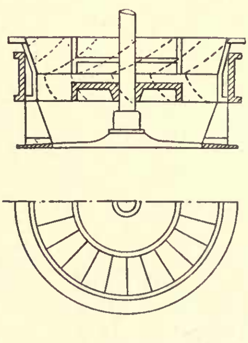


Fig. 400.

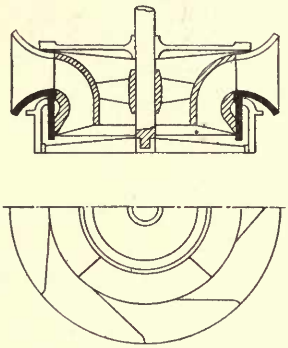


Fig. 401.

Fig. 401 is the type of wheel commonly used; the fixed guides are at the outside of the wheel and the motion of the water is inward and downward against the buckets. The shaft of this wheel is generally vertical, and the power transmitted generally through bevel gears, but it is often preferable to set the shaft horizontally and transmit the power through belting. These wheels, whether horizontal or vertical, can be set above the tail race and the entire fall utilized through draught tubes below the wheels, and admit of the use of belts. The greatest length of draught tube from the centre of horizontal shaft to tail water at Manchester, N. H., is 26 feet.

Wheels can be furnished by many makers from stock from 6 to 84 inches in diameter, the smaller ones for very extreme heads of water, and the larger to the limit of 100 feet. The power for the different wheels depends on the head, the speed of the shaft on the size of the wheels. For the best percentage of effect, the wheel must be of good design, material, and workmanship, and the makers will give their opinion as to which of their wheels they consider best adapted for any given position.

Fig. 402 is the elevation of a Pelton wheel, in which the action of the water on the bucket is by impact, and Fig. 403 a section of the bucket showing the action of the water upon it; they work under heads as high as 1,000 feet and to 2,000 H. P., giving a large percentage of efficiency.

However used, the mechanical effect inherent in water is the product of its weight into the height from which it falls; but there are many losses incurred in its application, so that only a portion of the mechanical effect becomes available; and the comparative efficiency of any water-wheel or motor is represented by this percentage of the absolute effect of the water applicable to power.

The watershed of the Merrimac River at Lowell is 4,085 square miles; and the permanent water equal to one cubic foot per second per square mile of shed during working

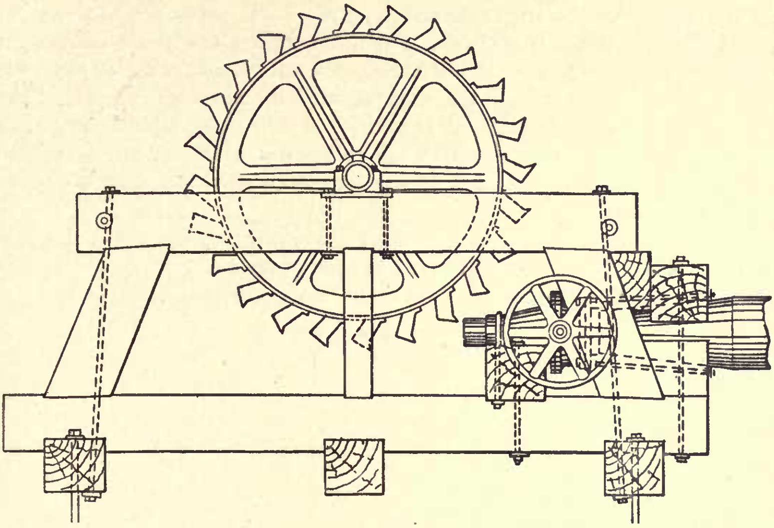


FIG. 402.

hours; gross fall, $34\frac{1}{2}$ feet; loss in canal, say $1\frac{1}{2}$ foot; and in flumes and races, 1 foot; net fall, 32 feet; $\frac{4{,}085 \times 62{\cdot}4 \text{ pounds} \times 32 \text{ feet}}{550 \text{ pounds}} = 14{,}830$ H. P.

Taking the effective power on the wheel at 80 per cent, which is obtained from the wheels at full gate, $14{,}830 \times {\cdot}80 = 11{,}864$ effective H. P. A very close rule to determine the E. H. P. is, multiply the number of cubic feet per second by the fall in feet and divide by 11, thus: $\frac{4{,}085 \times 32}{11} = 11{,}884$.

Wind is applied for the purposes of power; but, as there is no constancy in its action, its use is mostly confined to the purpose of raising water by means of pumps into cisterns or reservoirs.

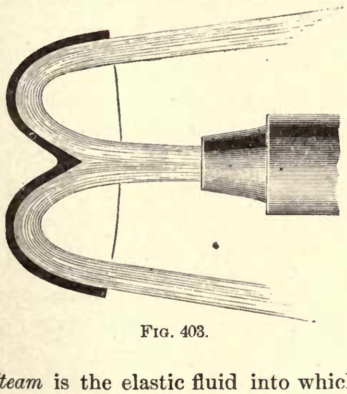


FIG. 403.

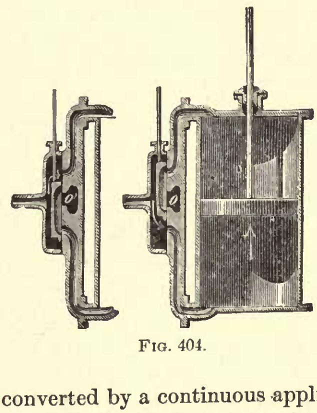


FIG. 404.

Steam is the elastic fluid into which water is converted by a continuous application of heat. It is used to produce mechanical action almost invariably by means of a piston movable in a cylinder. Thus, in Fig. 404, the steam entering through the lower channel-way, or *port*, presses against the under side of the piston in the direction of the arrow, the piston is forced upward, the steam above the piston escaping through the exhaust-channel *o*. When the piston reaches the top of the cylinder, the valve is changed by mechanism, the steam enters above the piston, and the steam below it escapes through the exhaust; in this way a reciprocating motion is established.

It is not practicable at a single instant to let the whole pressure of the steam in the boiler into the cylinder, nor maintain it uniform during the whole stroke; there are losses of pressure due to friction through the pipes, valves, and channel-ways, and back pressure from the same causes in the exhaust, condensation from exposed surfaces, and consequent reduction of pressure and volume of steam, and there are gains of effect by cutting off the introduction of steam to the cylinder at some point of a partial stroke, which is completed by the expansion of the inclosed steam, but with a constantly diminishing pressure.

Under expansion, the volume of a confined gas is inversely proportional to the pressure to which it is exposed. This is called the law of Mariotte, or the law of expansion, under equal temperatures for all gases, and is shown graphically in Fig. 405.

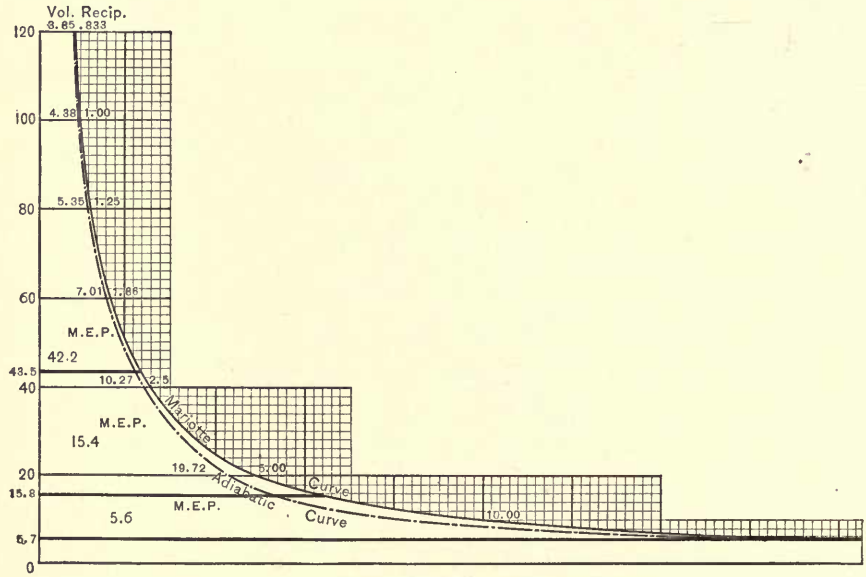


FIG. 405.

To determine the action of the steam within a steam-engine cylinder, an indicator is used. It consists of a small cylinder connected by a steam pipe with the main cylinder, with its piston controlled by a graduated spring, and recording by a pencil the varying pressures in the main cylinder on a paper card wrapped around a reciprocating cylindrical tube; to this, movement is given by a cord attached to some mechanism connected with the steam-engine piston, but with a reduced motion.

Indicator cards show the efficiency of a steam engine by a comparison with theoretical ones, the mean effective power exerted (M. E. P.), the volume of steam expended, losses of pressure by imperfect passages or channels, leaks in valves and around pistons and by radiation.

Fig. 406 is an indicator card from a non-condensing engine. The steam exhausts directly into atmosphere, and the back pressure, 2 to 3 pounds, is shown by the line just above that of the atmosphere line.

In Fig. 407 the exhaust is into the condenser, where a vacuum is maintained by the air pump of from 2 to 3 pounds. The M. E. P. of the condensing engine is greater than

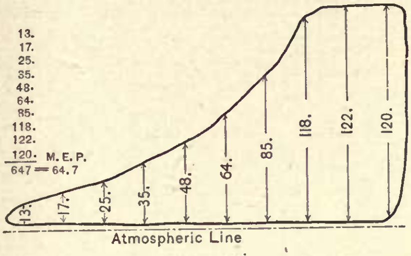


FIG. 406.

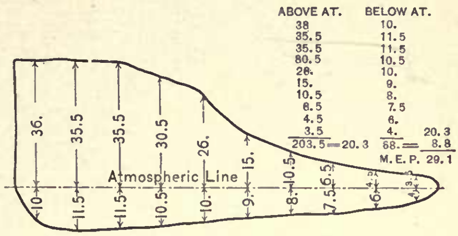


FIG. 407.

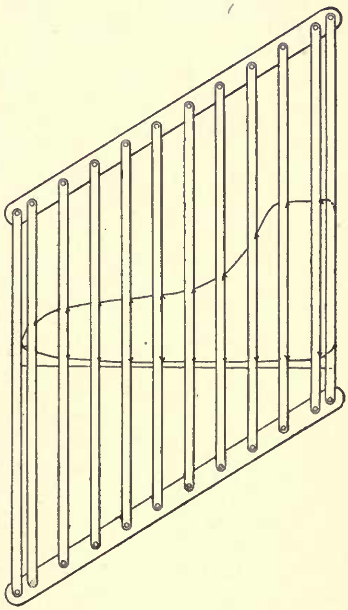
that of the non-condensing, but this is not obtained without expense of power in running the air pump, and more condensation in the cylinder due to the lower temperature at which the steam leaves it. See Table of Temperatures of Steam, Appendix.

The ends of both cards are rounded by the early opening of the steam valve in the out stroke, and by the earlier closing of the exhaust valve in the return stroke. This last shuts up the balance of steam in the cylinder, as shown by the curve of compression, and retains a volume of some importance in the next stroke, especially in quick-running engines regulated by movable eccentrics, and cushions the piston at the end of the stroke.

Both cards are divided into ten equal spaces, and subdivided for the average pressures, which are given in figures on the scale of pounds of the indicator. The sum of the pressures are then divided by 10, which gives the M. E. P., which, multiplied by the area in square inches of the piston by the travel in feet per minute and divided by 33,000, gives the H. P.

Assume the diameter of the cylinder in Fig. 406 to be 15″ (area, 176·7); stroke, 3 feet; revolutions, 90 per minute. $90 \times 3 \times 2 = 540$ feet travel per minute.

Then, $\frac{64{\cdot}7 \times 176{\cdot}7 \times 540}{33{,}000} = 187$ H. P.



The indicator card may be divided into twenty equal spaces, and the first, third, fifth, . . . nineteenth taken as subdivision ordinates, or it may be divided by a flexible grid; but where ordinates are not required the M. E. P. may be taken by planimeter.

The M. E. P. of a steam cylinder can be estimated approximately from the initial pressure and the cut-off. The reduction of pressure by the cut-off is shown in the diagram (Fig. 408) constructed from the formula of Rankine for dry saturated steam. Multiply the initial pressure by the decimal on the vertical line of the diagram at its intersection by the curve of cut-off, and the result is the M. E. P. of the entire card to 0; from which to find the effective pressure as given by the indicator card there must be subtracted the loss of pressure between the lower line of card and the 0 line. Assume the initial pressure in Fig. 406 to be 124 pounds above atmosphere, and the vacuum 14·7, or, together, 138·7 cut-off

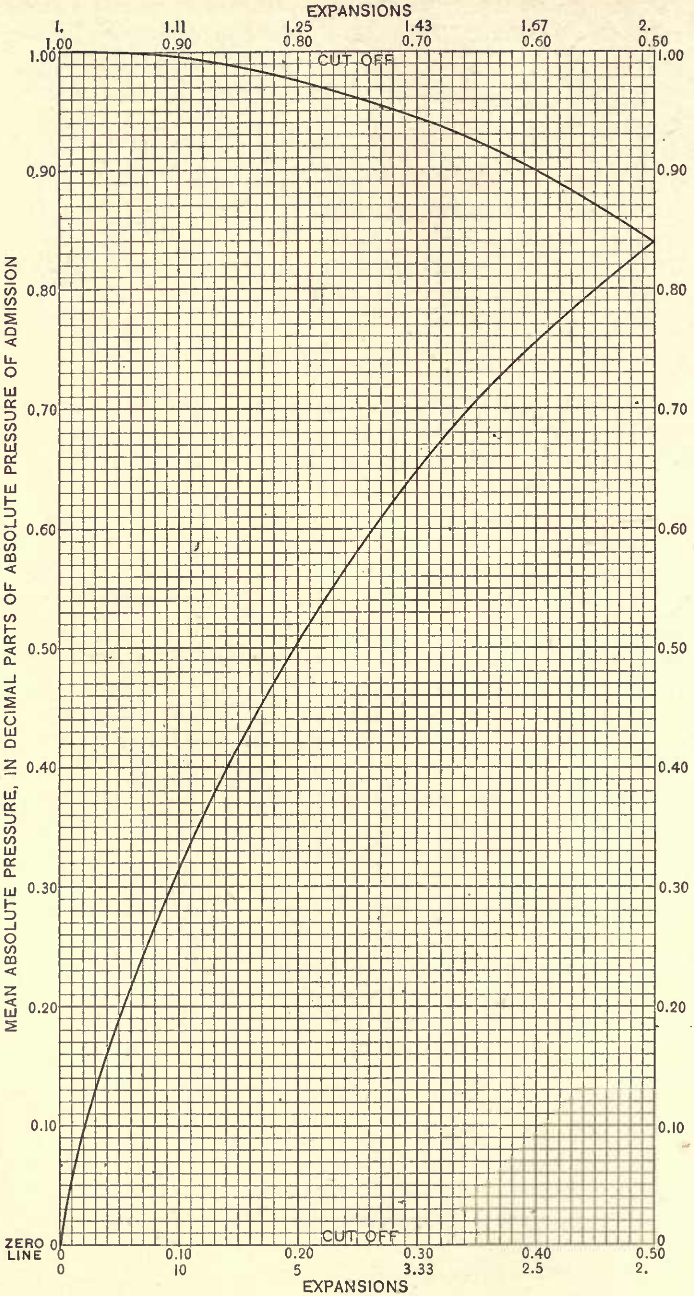


FIG. 408.

at $\frac{1}{4}$, or ·25 of the stroke, 4 expansions by diagram ·582 × 138·7 = 80·7 pounds, less 3 pounds above atmosphere and 14·7 below,

$$\begin{array}{r} 80{\cdot}7 \\ 17{\cdot}7 \\ \hline 63{\cdot}0 \end{array}$$

$$\frac{63 \times 176{\cdot}7 \times 540}{33{,}000} = 182{\cdot}2 \text{ H. P.}$$ A result a little less than that of the card.

In the appendix will be found a table of both dry and moderately moist steam from the same authority.

At the commencement of each stroke there is a space between the piston and the cylinder head which, together with the space between it and the valve, are called clearances, and, as it causes an expenditure of steam, is reckoned in percentages of the stroke.

To enable one to judge of the economy of the steam engine, it is necessary to know how much weight of steam is used. In the example the pressure at cut-off is taken at 139 pounds, and the weight of a cubic foot of steam at this tension, by tables of saturated steam in appendix, is ·3092 pound; the area of cylinder = 1·227 square feet.

Stroke 3 × ·25 cut-off = ·75
Clearance, 3 per cent of stroke = ·09

·84 foot.

1·227 × ·84 × ·3092 = ·319. ·319 × 180 strokes × 60 minutes = 3,445 pounds per hour. If the evaporation be 8 pounds of water per pound of coal, then the consumption of coal would be 431 pounds, and $\frac{431}{187} = 2{\cdot}30$ pounds of coal per H. P. per hour.

Of late years, the principle of expansion has been very much extended by the construction of compound engines. Fig. 409 shows the general arrangement, without valves, of a simple compound engine of two cylinders—a high-pressure (h. p. c.) and a low-pressure (l. p. c.) one. The h. p. c. (A B C D) draws its steam from the boiler and exhausts into the l. p. c. (A′, B′, C′, D′); the top of the h. p. c. into the bottom of the l. p. c., and *vice versa*, so that the pressure on the pistons of the two cylinders is in the same direction.

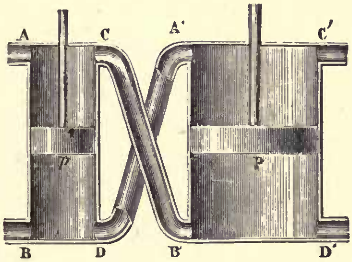


Fig. 409.

To determine whether a steam engine is working properly, it is necessary to compare the absolute card with the theoretical one.

Figs. 410 and 411 represent indicator cards, taken from a condensing and non-condensing engine. On these are shown the construction of the isothermal curve. It will be observed that there is a line, A B, to the top of the card. The space between this and the card represents the *clearance*, which, estimated in percentages of the capacity of the cylinder, is plotted on the indicator card. On the indicator card, as taken by the instrument, the absolute 0 can not be taken, but only that of the atmosphere, the 0 will be at a distance below this, corresponding to the

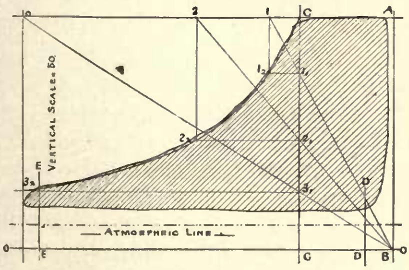


Fig. 410.

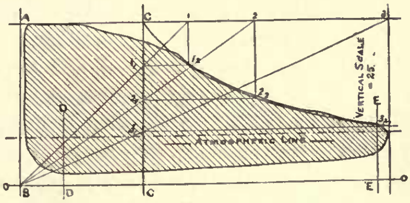


Fig. 411.

barometric pressure, usually 14·7 pounds. Draw the 0 line parallel to the atmospheric line, the clearance line perpendicular to it, a line parallel to the 0 line, at the height of the initial pressure, and a line parallel to the clearance line at the point of cut-off on the initial pressure line. Any point on the expansion line, as 1_2, 2_2, 3_2, may be determined by drawing lines B 1, B 2, B 3, and then horizontal lines $1_2\,1_1$, $2_2\,2_1$, $3_2\,3_1$ from their intersections 1_1, 2_1, 3_1. With the cut-off line, parallel to the 0 line, and perpendiculars from 1, 2, 3, the intersections of these two lines, 1_2, 2_2, 3_2, will be the points in the curve. Inversely, the indicator card may be tested by the construction of parallelograms on the curve, and if their diagonals intersect at a point on the 0 line it may be considered that this represents the amount of clearance. The curves in the outline of the cards, at the times of admission, cut-off, and exhaust, show the action of the valves and time occupied in change of condition. The stroke commences at A, cuts off at C, commences to exhaust at E; about D the exhaust valve closes.

To enable a draughtsman to comprehend the action of the steam in multiple cylinders and calculate approximately the diameter and capacity of the different cylinders, a curve of expansion is constructed, Fig. 405. The ratio of expansion is on the Mariotte curve and is as the reciprocal of the pressures. Taking as the unit the abscissa of the pressure of 100 pounds, at 120 it is ·833, at 40 pounds 2·50, and on this principle the curve is drawn, taking cross-section paper, for the ease with which it may be laid down and as more intelligible in its explanation.

In Fig. 405 is illustrated a triple compound with an initial pressure (I. P.) of 120 pounds and a final expansion of twenty-one times. To divide the expansions between the three cylinders, $\sqrt[3]{21} = 2{\cdot}76$ expansions in the first cylinder, $2{\cdot}76 \times {\cdot}833 = 2{\cdot}30$ abscissa at end of stroke of first cylinder.

(1) $\dfrac{120\ (\text{I. P.})}{2{\cdot}76.} = 43{\cdot}48$ final pressure, ordinate corresponding to an abscissa of 2·30.

(2) $2{\cdot}76^2 = 7{\cdot}617$. $7{\cdot}617 \times {\cdot}833 = 6{\cdot}345\ \dfrac{120}{7{\cdot}62} = 15{\cdot}76$ pounds.

(3) $2{\cdot}76^3 = 21$. $21 \times {\cdot}833 = 17{\cdot}5\ \dfrac{120}{21} = 5{\cdot}71$ pounds.

With a cut-off at ·833, and an expansion of 2·76, the stroke will be 2·3, which is the uniform stroke in the three cylinders. Calling the diameter of the H. P. cylinder 15″ A 176·7 □″, the area of the I. P. cylinder will be 176·7 × 2·76 = 487·69 square inches = 25″ diameter, and that of the low pressure 487·69 × 2·76 = 1,346 square inches = 41½″ diameter. Taking the M. E. P. of the different cards by averages or by planimeter, in which the area of the card in square inches is divided by the length of card in inches and multiplied by the vertical scale, we find the M. E. P. for first cylinder 42·2, second cylinder 15·4, third cylinder 5·6. The pounds-feet per stroke will be:

First cylinder,	176·7	× 2·3	× 42·2	=	17,150
Second "	487·69	× 2·3	× 15·4	=	17,274
Third "	1,346·	× 2·3	× 5·6	=	17,336

The M. E. P. of the whole card was 16·6, stroke 17·5, area $176{\cdot}7 = \dfrac{51332}{3} = 17{,}110$ pounds-feet, a result very nearly that of the single cylinder, but as the areas measured by the planimeter includes only that between the lines of final pressure, while the absolute card in the low-pressure cylinder should be carried to the line of exhaust, or say 3 pounds above 0, the pounds-feet of effect should be more than in either of the others, which it would be impossible to equalize with the same stroke and the same rates of expansion. All these results are obtained from the plotted isothermal curve, but in expanding steam does not maintain the same temperature, and occupies less space than shown by the above curve, and making allowance for this a curve is developed which is called the adiabatic curve.

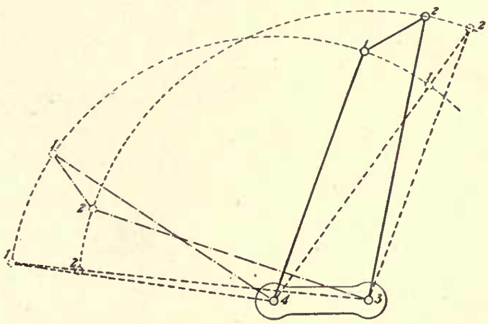
To plot this on the diagram take the abscissa at 120 as equal to 3·65 (or the volume of steam at this pressure), construct a scale taking the volumes at the different pressures from the table in the Appendix; but if the stroke and the areas of cylinders is considered the same as in the previous calculations, although the area of the cards is less, yet as the length of card is less, the M. E. P. remains nearly the same, and consequently the pounds-feet of results, estimating pounds-feet of work done in the several cylinders by the mean pressures as taken from Rankine's tables in the Appendix. In these cases the average pressure is taken from the 0, from which has been subtracted the initial pressure in the next cylinder.

Thus:

Average pressure in first cylinder	88·04
Initial pressure in second cylinder	43·5
M. E. P. in first cylinder	44·54
Average pressure in second cylinder	31·8
Initial pressure in third cylinder	15·8
M. E. P. in second cylinder	16·00
Average pressure in third cylinder	11·56
Pressure at end of expansion	5·7
M. E. P. in third cylinder	5·86

For dry steam the results are very nearly the same as by the Mariotte curve. In none of the calculations is the difference as great as will be found in practice, where it is found desirable to jacket the steam cylinders and heat the steam in the intermediate chambers to prevent condensation from the walls of the cylinders and passages, and to restore the heat which has been converted into work.

The above card refers to an engine in which there is an intermediate chamber between the cylinders.



MOTION.

In the designing of machines it is often important to show the changes in the moving parts, that they may not conflict with each other in the practical working, and to obviate the effect of dead-points through which motion may have to be transferred.

To describe the path of the crosshead of the piston of the steam engine, its connecting-rod, and crank, Fig. 412, draw the path of the crosshead which, controlled by guides, is a straight line; divide into equal parts 0 to 6; on the same line lay off the length of the connecting-rod from 0 to 0′, and the length of the crank to the centre C; from C as a centre with a radius equal to the length of the crank describe a circle which will be the path of the crank-pin. From the points 1, 2, 3, 4, 5, and a radius equal to that of the connecting-rod, describe arcs cutting the crank path at 1′, 2′, 3′, 4′, 5′, 0′ and 6′ on the line of the crosshead path, the commencement and end of the strokes. The path of the crosshead is divided into equal spaces, but that of the crank-path is not. The point 3′ corresponding to the mid stroke 3 of the crosshead is not that of the half circumference of the crank-path. These irregularities are due to the angularity of the connecting-rod, which is here made less than its usual proportion to that of the crank.

Moreover, when the crank-pin and the axle-centre are in the line with the connecting-rod, as at 0 0′ or 6 6′, the driving force passes through the fixed axis and no motion is

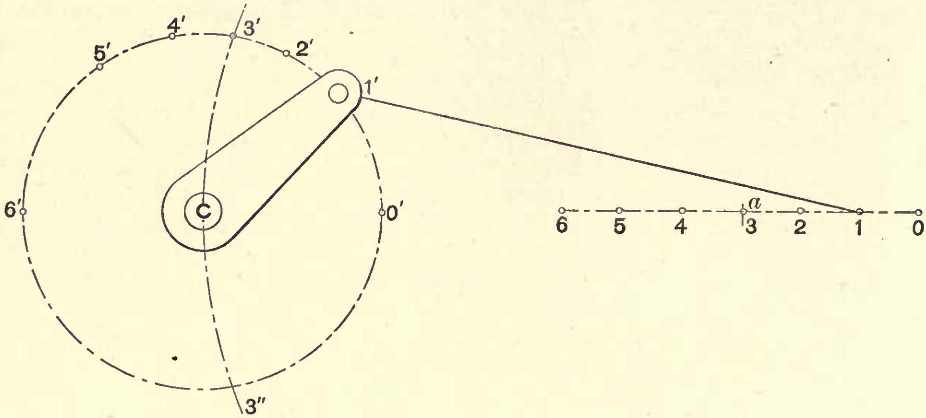


FIG. 412.

possible. Some other mechanism is necessary to pass over the dead-points; usually this is by means of the fly-wheel, in which the force stored in the mass by rotation continues the motion, and rightly proportioned nearly with uniformity, and the irregularities are transferred to the stroke, Fig. 413.

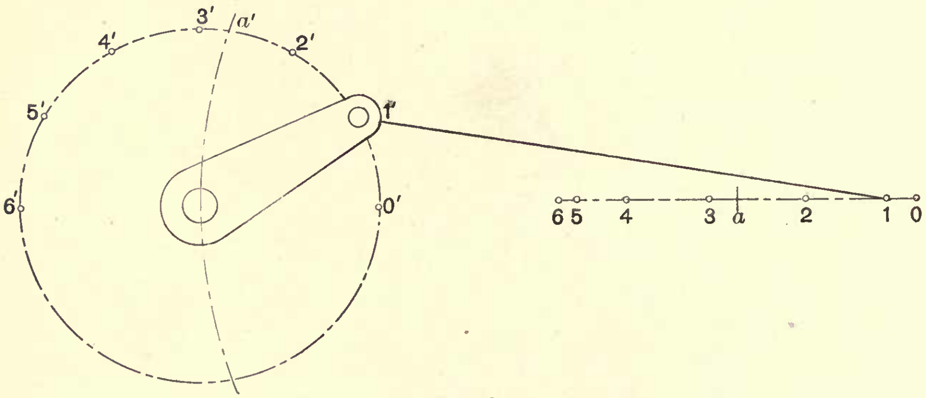


FIG. 413.

If the crank is the driver there will be no dead-point; the circular movement of the crank-pin will be converted into the rectilinear and reciprocating one of the crosshead.

In locomotives the cranks from the cylinders are put at right angles, that there may be no dead-point.

STANHOPE LEVERS.

In the Stanhope hand press (Fig. 414) the extreme power is obtained by a combination of levers in which the greatest pressure is exerted when the platen has reached the type. A is a fixed point on which the bell-crank lever B revolves, and to the extremity of which the power is applied, and which describes the circle designated by equal arcs, 0–10; at the angle *b* of B there is a connection C with the lever D revolving on a fixed centre, *d*; under this action the point of the lever D, at which the pressure is exerted, passes through the small arc 0–10, corresponding to the arcs of the extremity of the lever B, but always with decreasing lengths, as shown on the figure, and consequently increasing pressure.

To obtain the positions of the lever from the path of the lever B, from A as a centre, and with a radius equal to A 0, describe an arc the path of the extremity of B, and

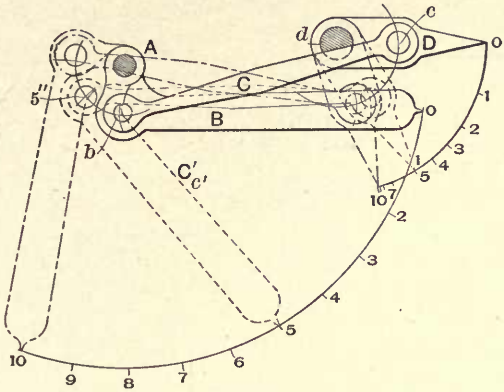


FIG. 414.

divide into equal arcs 0–10; on A as a centre describe another arc, A *b*, and from any point, say 5, with a radius equal to *b* 0, describe an arc 5″, intersecting the arc *b*; from *d* as a centre, with a radius equal to *d c*, describe an arc, and from 5″, with a radius equal to *b c*, intersect the previous arc, and through this point draw a radial line from *d*; with a radius equal to *d* 0, describe an arc for the path of the extremity of the lever D; the intersection of the radial with this arc will give the point 5 on the smaller arc, corresponding to 5 of the larger. In the same manner other positions can be determined.

WHITWORTH'S QUICK RETURN MOTION.

Fig. 415 is known as Whitworth's Quick Return Motion, and consists of a driven crank, *a*; moving at a uniform speed, to the end of the crank is a block, *d*, sliding in a slotted link; this link is fastened to a fixed centre, *e*, and has a vibrating motion. A

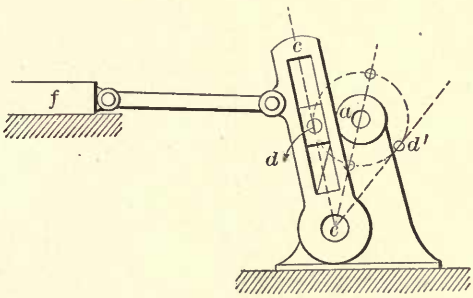


FIG. 415.

tangent drawn from each side of the crank-path to the point *e* will give the extreme movements of the link *c*, and a line at right angles to this line and from the centre of the crank-shaft on each side will divide the crank-path in two parts. The tool-holder, *f*, or planer platen, travels over an equal space while the crank moves from *d′* to *d* as from *d* to *d′*; it follows, therefore, as the crank is moving uniformly, the motion of the tool-holder while cutting is slow and the return quick.

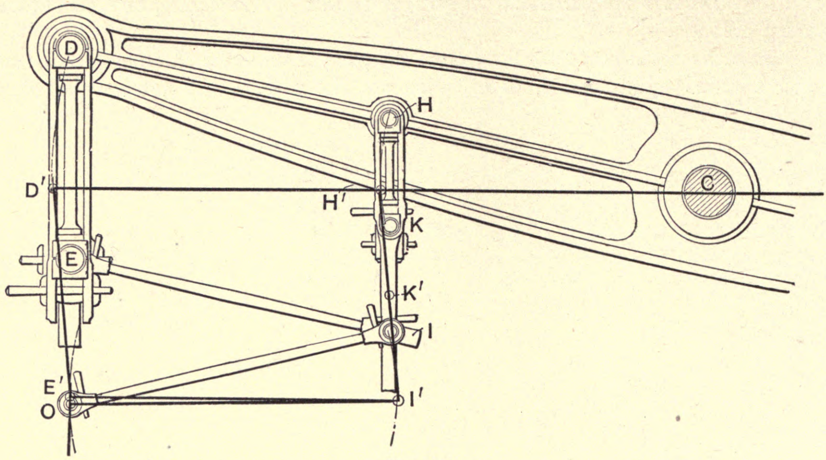


FIG. 416.

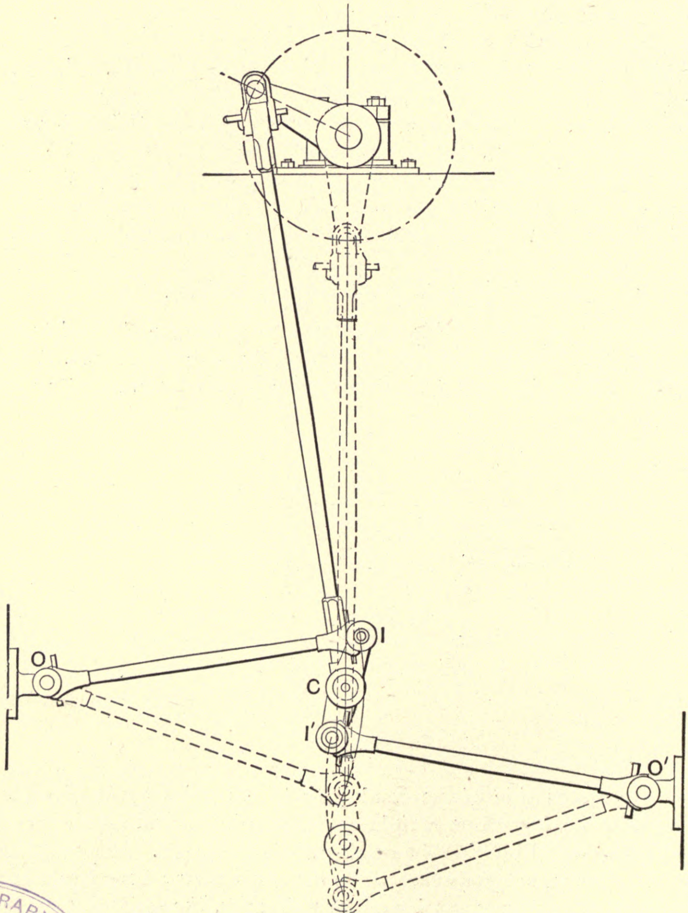


FIG. 417.

WATT'S PARALLEL MOTION.

Watt's parallel motion is shown in Fig. 416, the object of this motion being to obtain a rectilinear motion of the piston-rod and air-pump rod under the varying angularity of the beam.

In the figure the line C D indicates the centre of the beam; from the point H, situated midway between C and D, and from D, are suspended two links, H I and D E, of equal length. Connect E and I by a rod equal in length to D H; the point I is then attached to the fixed centre at O, and is equal in length to E I, with an angle E I O equal to the angular motion of the beam. By the movement of the beam the point H is thrown outward from C and I inward an equal amount; the centre point, K, connected with the air pump, takes a nearly vertical movement, and the same may be said of the point E; the heavy lines illustrate the position of the various levers when the beam is in the middle of its travel.

It is the practice in this country to use crossheads and guides for the piston rather than parallel motions.

Fig. 417 represents another form of parallel motion to preserve the perpendicularity of the piston-rod against the varying angle of the connecting-rod.

This motion consists of two pairs of equal radial bars, O I and O′ I′, moving in planes parallel to that of the circle of revolution of the crank, and on opposite sides of the piston-rod and connected by a link I I′, while the centres O and O′ are fixed at points at equal distances from the centre of motion and on opposite sides; the communication with the piston-rod is at C, which point moves nearly perpendicularly.

The dotted lines show the positions of rods and levers when the crank is at the extreme point of its revolution.

JANNEY CAR COUPLER.

Fig. 418 is a drawing of the Janney car-coupling device, the lower portion being in section to explain the internal mechanism, and the upper portion a top view, showing the exterior appearance.

The section and top view of the coupler, in dotted lines, represent the couplers approaching each other for coupling, and the section and portion in solid lines show the coupling completed.

The knuckle B represents a pinion with two teeth B′ B², and the drawhead A and tooth or nose, B³, of the other coupler representing the rack. When the couplers are brought together the teeth engage each other, the hub revolves, and the long tooth B′ is carried around to a locking position, the catch C being forced back by the circular end of the tooth B′ in passing, after which the catch is returned to its locking position by the catch spring F.

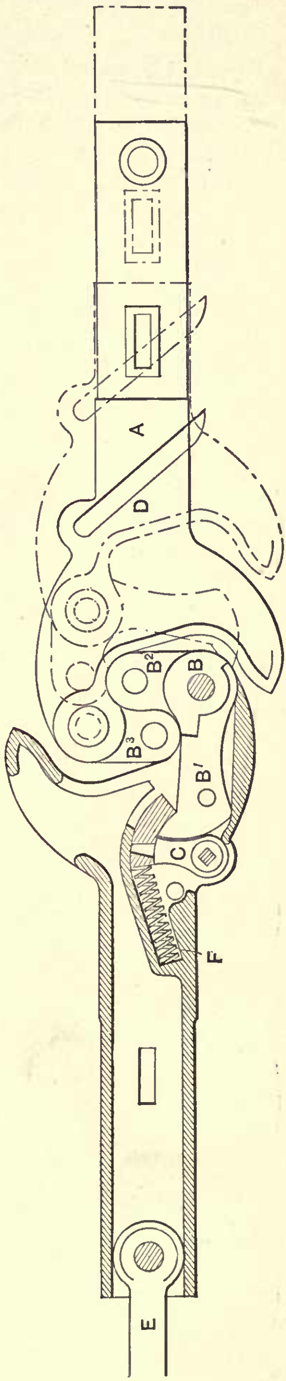


FIG. 418.

The uncoupling is effected by throwing over a platform lever, which in turn forces around the coupler lever D and catch C, allowing the tooth B′ to revolve outward.

VALVE MOTION.

To establish the reciprocating motions of the piston, valves must be moved so as to alternately let the steam into one end of the cylinder and permit it to escape at the other. The mechanism by which the valves are moved are usually by means of an eccentric on the crank-shaft, and a strap and rod connecting it with the valve rod.

The motion of the rod is the same as if the eccentric were a small crank.

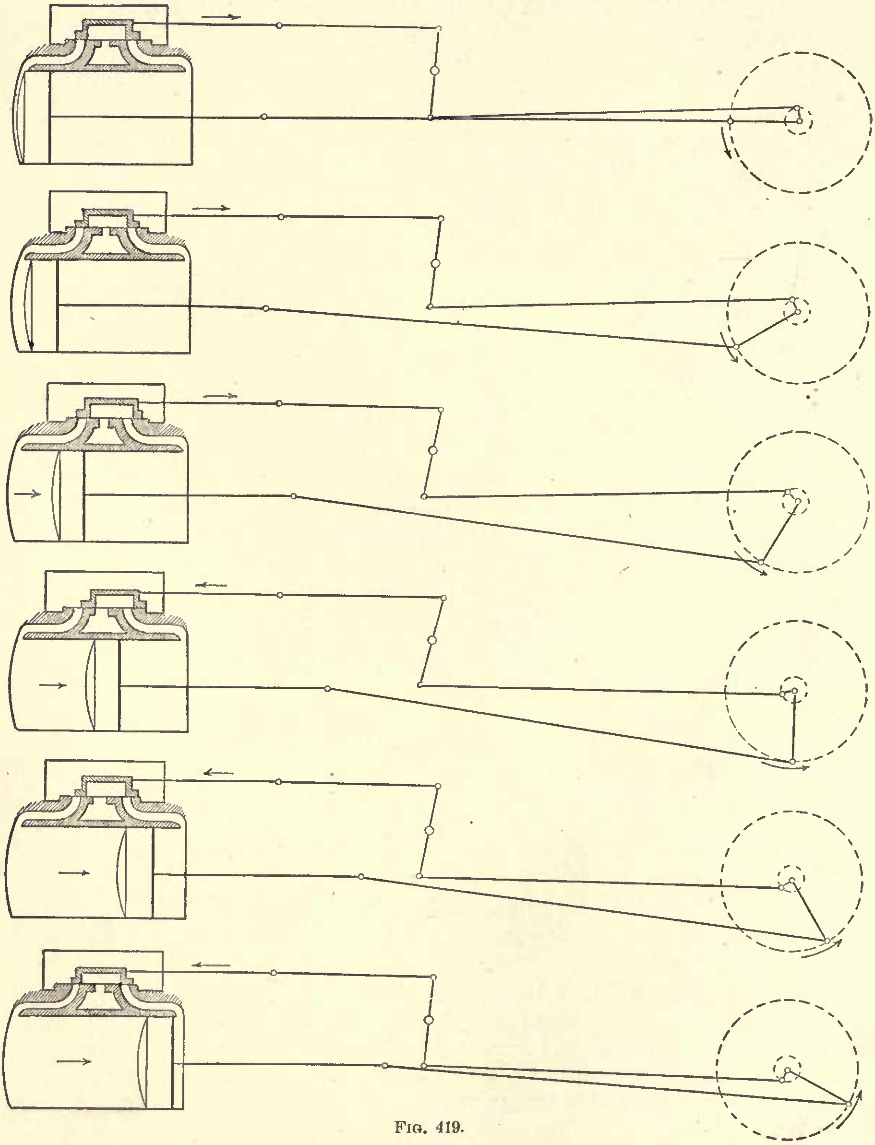


FIG. 419.

Fig. 419 shows six positions of one of the old slide valves and piston, together with the position of the crank and eccentric for each movement of the valve.

In Fig. 419 the valve is merely sufficient to cover the ports, and at the slightest movement in either direction passages will be opened for steam to the cylinder, and the escape of the exhaust from it; under these conditions there can be no cut-off, and consequently no economy from expansion of steam.

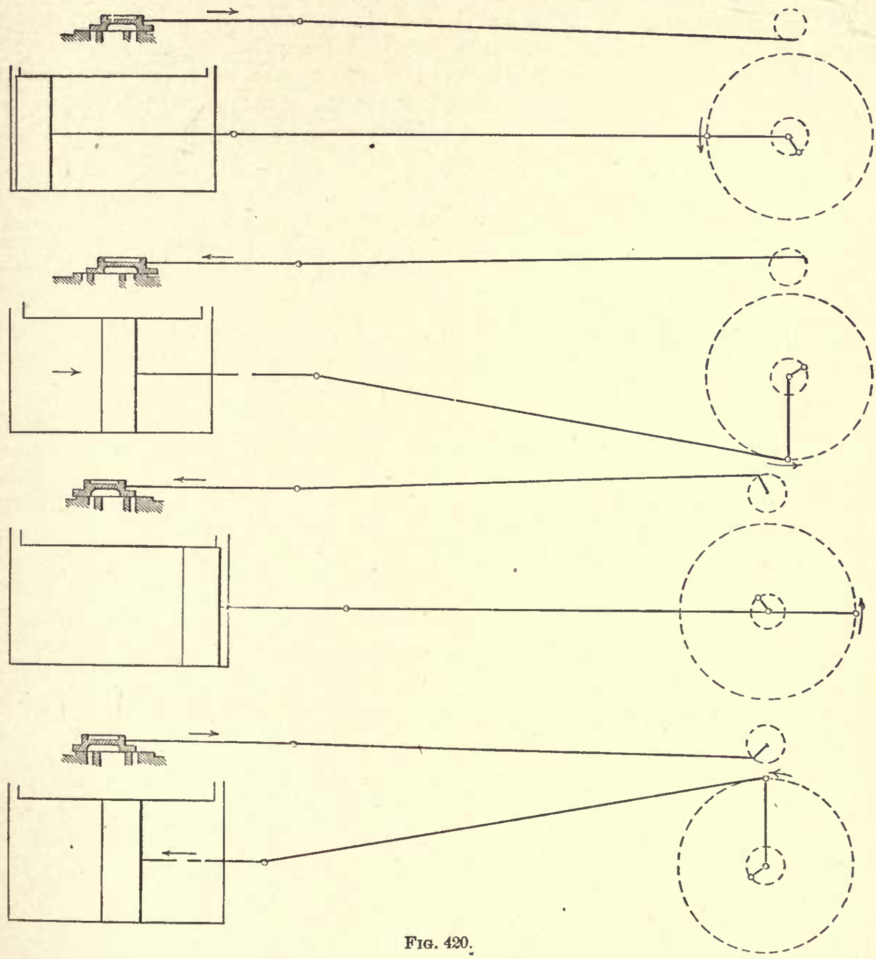


FIG. 420.

For common engines the cut-off is effected by the lap and lead of the valve (Fig. 420). When the valve is placed centrally over the ports, the portion of the valve overlapping the steam ports is known as lap: when on the steam side, it is designated as outside lap, and as inside lap when on the exhaust side. In quick-running engines, both steam and exhaust ports are opened before the completion of the stroke; on the steam side there is a cut-off or closing of the valve before the completion of the stroke to admit of expansion, and before the completion of exhaust for compression, thus saving heat and relieving the pressure and, consequently, the friction of the slide valve.

The channels, usually three in number, alternately exposed and covered during the movement of the valves, are called *ports.* The ones admitting steam to the ends of the cylinder are the *steam ports;* the central one giving exit of the steam from the cylinder is the *exhaust port;* the spaces between the ports are called bridges. The working surfaces on valve and cylinder are *valve* and *valve seat faces.*

The width of opening of the steam ports at the beginning of the stroke for the admission or release of steam is termed *lead;* on the steam side *outside lead;* on the exhaust side *inside lead.* When the valve is placed at half stroke over the ports, the amount which it overlaps each port, either internally or externally, is the lap; on the steam side *outside lap;* on the exhaust side *inside lap.* Laps and leads by themselves refer to outside laps and leads. The amount by which the valve has travelled beyond its middle position when the piston is at the end of the stroke is the *linear advance.*

The proper understanding of the positions of valve under the different positions during the stroke are of great importance to the draughtsman. It is very common in a shop to have a model of an eccentric in which the throw can be readily changed and applied with a small rod for connection to a simple section of valve and ports. But the valve motion may be illustrated by diagram.

VALVE DIAGRAMS.

The separate drawings of the movements of a valve may be shown in one general diagram containing the continuous movement of the valve; for illustration, the valve of the last engraving is taken.

Draw a perpendicular line (Fig. 421), E D, called the datum line; from the centre, C, of this line describe a circle equal in radius to that of the eccentric; for the first position

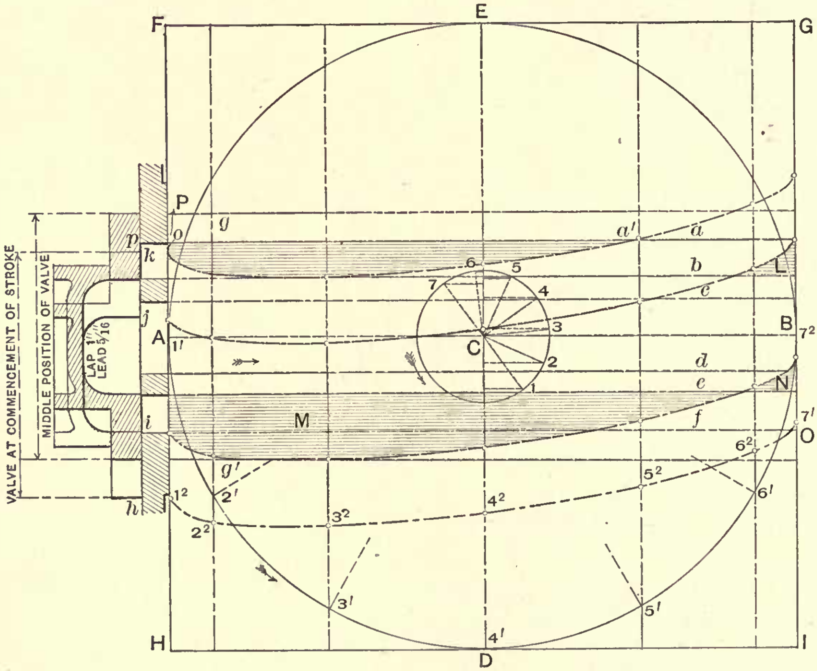


FIG. 421.

of the eccentric draw a line perpendicular to D E of one inch and five sixteenths (one inch lap and five sixteenths lead) till it intersects the path of the eccentric at 1. Draw a line from C to 1; this is the first position of the eccentric; from this point divide half the circle into six equal parts, representing six positions of the eccentric; from each of these points draw a horizontal line till it intersects the datum line; these lines give the linear movement of the eccentric, and therefore the travel of the valve. From the centre C draw a circle equal in radius to that of the crank; commencing at 1′, this being the first position of the crank, divide half the circle into six equal parts, the positions numbered 1′ 2′ 3′ on the crank-path corresponding to 1, 2, 3 of the eccentric path; on the crank circle construct a square, F, G, H, I, and through each of the points of the crank-path draw lines parallel to the side of the square. Draw a horizontal dotted line A B through C, and from A as the centre of the exhaust port lay off the ports and bridges. Above and below this draw parallel lines *a b c d e f* across the square, indicating the limits of the exhaust opening and ports on both sides of the drawing, also horizontal lines *g* and *g′* to define the position of the valve when placed centrally over the ports, and draw a section of the valve according to the dimensions given. Lay off the distance from D E to the first position of the eccentric below *g′* on the line F H, and draw an outline of the valve as moved into this position. Lay off on each successive perpendicular line 2′, 3′, etc., from *g′*, the corresponding positions of the eccentric as measured from D E, and draw a curve through these points. Lay off the position of the edges of the valve *h i j k* on the lines 2′, 3′, etc., above the determined points 2^2, 3^2, etc., and draw curves through these points.

In this position of the valve the steam port has been opened by a distance equal to *p k*; through *p* draw a horizontal line intersecting the elliptical curve at *a′*, draw shade lines from *a* to the curve below; this shade represents the continuance of the steam in the cylinder; the intersection *a′* is the point of cut-off and commencement of expansion. On the other side the cylinder is wide open to the exhaust, as shown by M, and is closed where the elliptical curve intersects the line *e*. The waste steam becomes compressed and is shown at N, and is utilized on the return stroke, as shown at L. The period of admission against the piston is shown where the line of the curve intersects *a* at *o*, and is shown by the small space O at the commencement of the back stroke and at P for the forward stroke.

This diagram entirely disregards the angularity of the connecting-rod.

With a single valve it is difficult to secure economy in the cut-off; it is usual in the larger sizes of stationary engines to have steam and exhaust valves moved by separate mechanisms, and independent of each other, and to regulate the engine by a direct connection of the steam valve with the governor.

Among this class of engines, the widest known is the Corliss, of which Fig. 422 is a side elevation of one of the simplest and earliest forms.

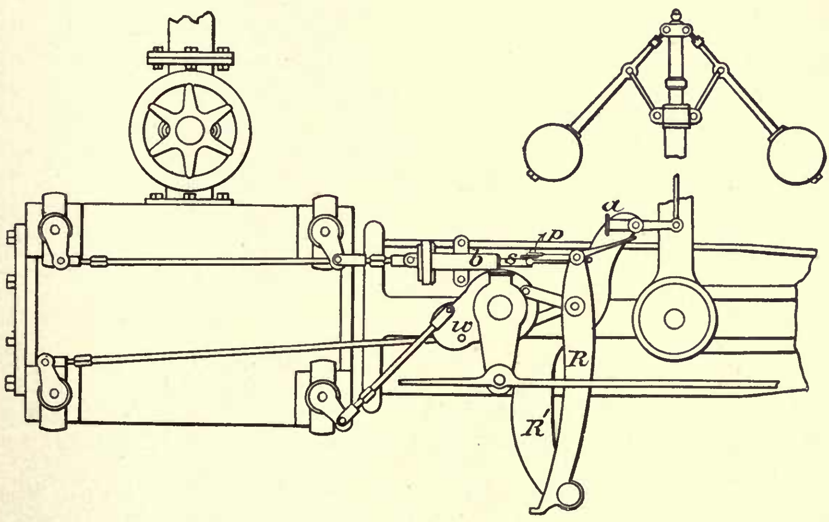


Fig. 422.

The exhaust valves at the bottom of the cylinder are connected positively with the wrist-plate *w*, vibrated by a hooked connection with an eccentric on the engine shaft. Connected with the wrist plate are two vibrating levers, R R′, to the upper ends of which lever pawls, *p p*, are attached, which rest on the stems, *s*, of the steam valves. On the stems are notches against which the pawls strike, push back the stems, compress the inclosed spring, and open the valves, and this continues till the outer end of the pawl lever, coming in contact with the head of the lever *a*, controlled by the governor, releases the spring which closes the valve. The cases, *b*, are cylindrical, with air cushions at the ends.

Fig. 813 shows sections of the valves of which the ports are very long and narrow. In their construction, the valves may be considered cylindrical plugs, of which portions near the ports are cut away for the prompt admittance and exhaust of steam; the valves are fitted on the lathe and the seat by boring. The motion given to the valves is rocking, but the valves are not firmly connected to the rocking shaft; in the figure the valves are shown shade lined, and the shaft or stem plain; the valves are not affected by the packing of the valve stem, but always rest upon the face of the ports.

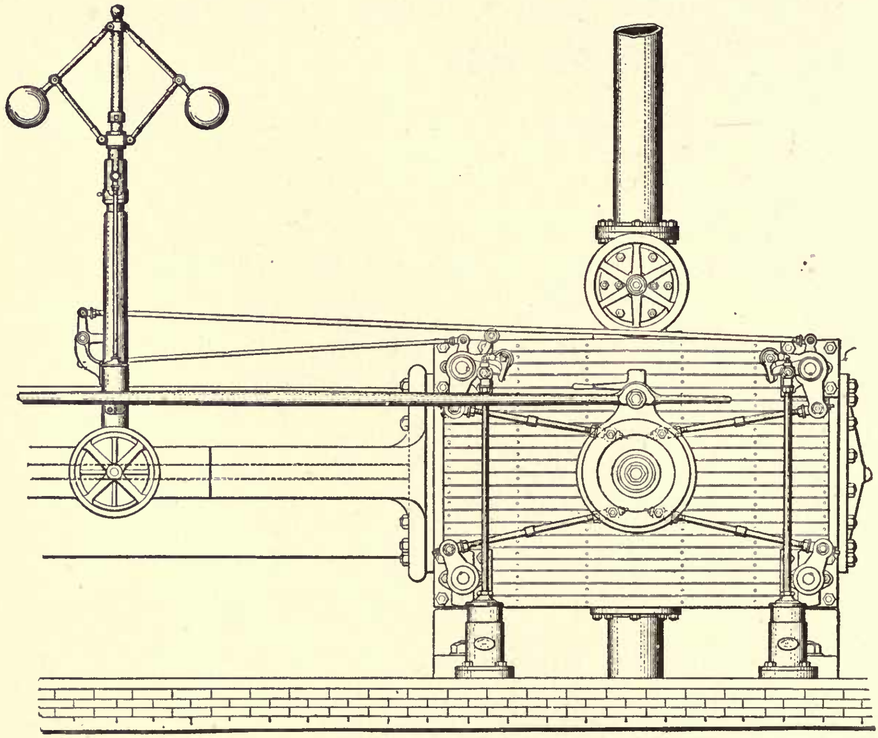


FIG. 423.

Since the lapse of the Corliss patent, it may be considered the most popular form of design, and its manufacture has been taken up by many shops, which, while in the matter of valves and cut-off coming within the claims of the old patent, have been much improved in the details of mechanism. The dash-pot is now set vertically; the plunger acts without springs by gravity.

Fig. 423 is the side elevation of a Corliss engine of the Fishkill Landing Machine Company, of which the details of the dash-pot are given in Fig. 424. It consists of a cylinder, A, of two different diameters, to which is fitted the double-diameter plunger, with grooved air packing, the one in the cylinder, the other on the lower plunger. At the commencement of the lift the plunger is at the bottom; as it is raised, a vacuum is formed beneath the plunger, partial, as air is supplied from the annular space *x x* through the channelways *e e′*, but the vacuum increases with the increase of the space *y* at the cut-off.

The plunger's descent is rapidly retarded by a partial vacuum in *e e′*, and at the end of its stroke by the compression of the air in *y*, and seats without pounding. By the

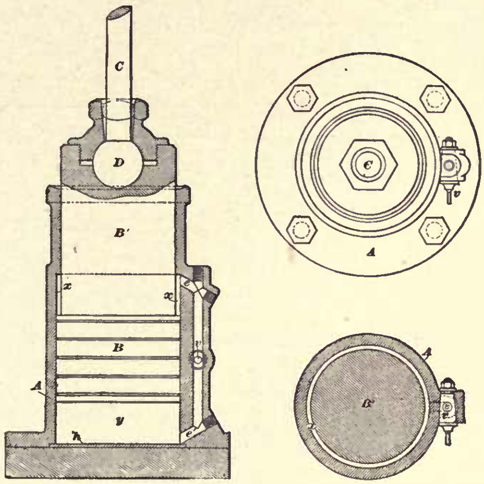


FIG. 424.

valve *v* in the passage *e e′* the flow of air between the upper and lower end of the cylinder is controlled to produce this effect.

Figs. 425, 426, and 427 are details of the releasing mechanism of the same engine.

To the valve stem A is attached a single-armed lever, to which is suspended the dash-pot connection Y, and at the extremity the steel catch-plate *c*. The double crank

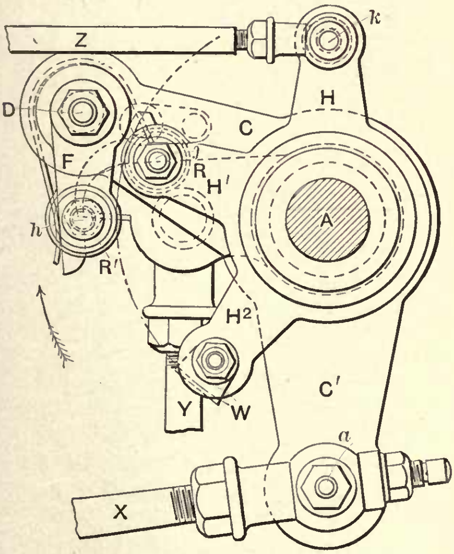


FIG. 425.

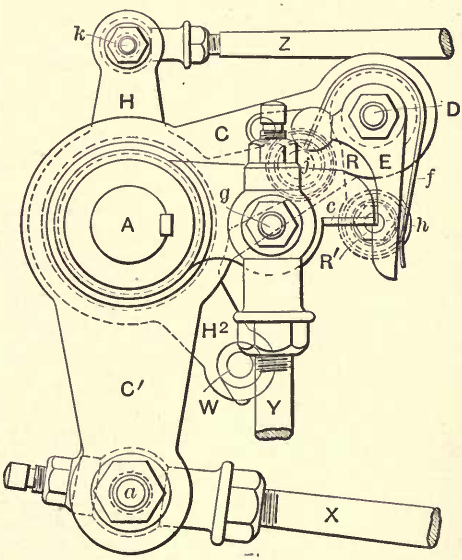


FIG. 426.

C C′ rocks on the valve stem; to its lower end there is a connection X with the wrist-plate, from which it receives motion; at its upper extremity C there is a small rocking lever and hook E which is pressed inward by the spring *f*, and the hook E, engaged with the catch-plate *c*, gives the motion of the wrist-plate to C C′, opens the valve, and

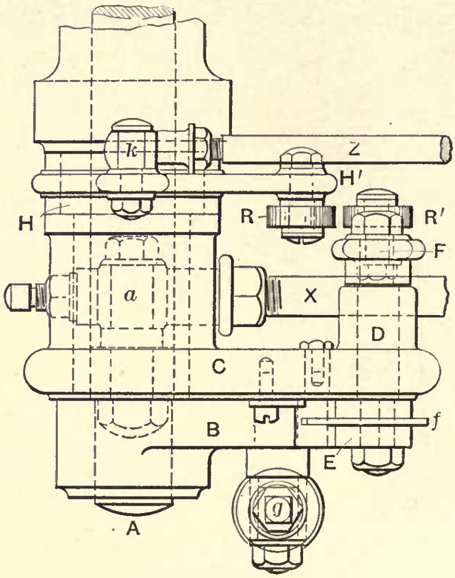


FIG. 427.

raises the piston of the dash-pot. The governor-rod Z moves the triple crank H H′ H², rocking on the valve stem A; at the extremity of H′ there is a roller R which is thrown by the governor into such a position that it comes in contact with the roller R′ on the hook E in its motion upward, and is thrown outward, disengaging the hook E from the catch *c*; this arm, which is connected with the dash-pot, instantly falls, the valve closes,

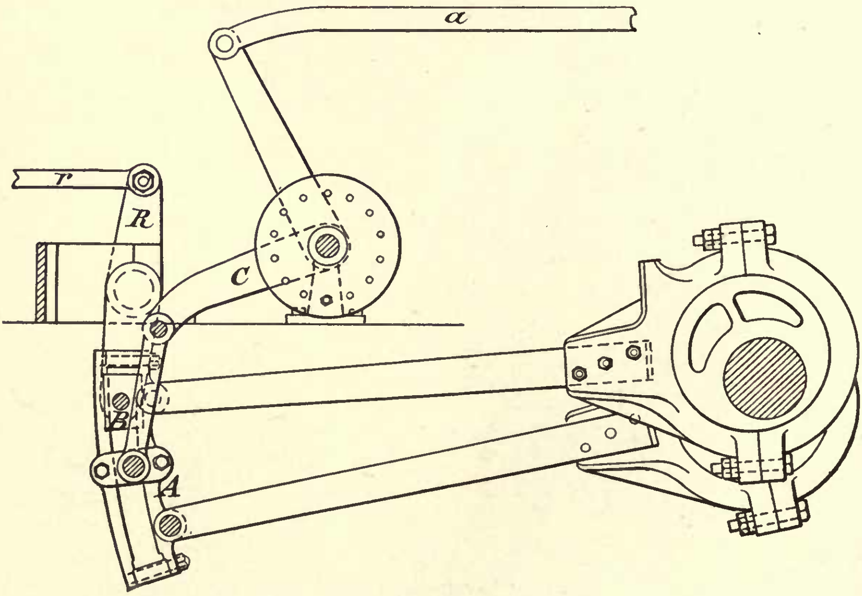


FIG. 428.

and steam is cut off sharply; the return stroke again engages the catch and hook, and the release is again effected by the position of the governor.

On the arm H^2 there is an adjustable cam W, which serves as an automatic safety stop-motion. When the engine is at its lowest normal speed, and the hook E is at the point of engagement with the valve lever B, the roller R′ comes nearly in contact with the cam W. Should the balls fall below the point corresponding to the lowest normal speed, the bell-crank H will move in the direction of the arrow; the cam W will come in the way of the roller R′, which will ride on the top of it, thus preventing the hook E from engaging with the end of the valve lever B, and the valve will remain closed. No steam being admitted, the engine will stop.

Link motion (Fig. 428) is a mechanism by which the travel of the valve is changed and the motion of the piston reversed at the option of the engineer. It consists of two eccentrics at nearly opposite points on the crank-shaft, each of which is connected by rods to the opposite ends of a shifting link A; within the link is a sliding block B; the block B is attached to a rocker R. The link is suspended from a bell-crank C, controlled by the engineer through the rod *a*. As the link A is raised or lowered it slides on the block B, of which the movement becomes more or less, or is changed in direction according to its position in the link, and this motion is transferred to the valve through the rod *r*.

This is a common form of link motion, but the same effect is obtained by a fixed link and a movable block without a rocker shaft connected by a link to the valve rod.

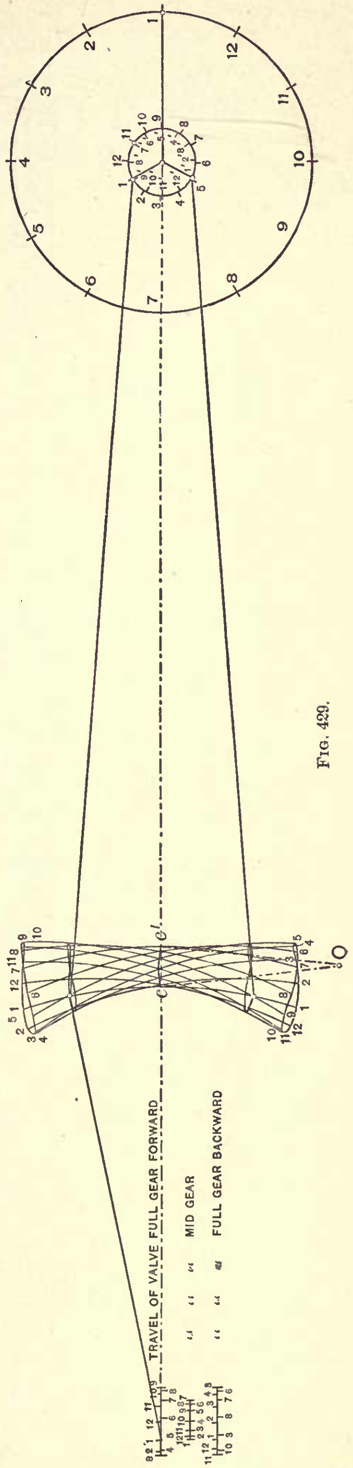


Fig. 429.

DIAGRAM OF LINK MOVEMENT.

The horizontal and vertical movement of the different positions of the sliding block of the stationary link in connection with the eccentrics is shown in Fig. 429.

The link is attached to a radius rod attached to a centre O, which compels the centre of the link to vibrate in an arc *c c′*. The crank-path is divided into a number of equal parts, say 12, commencing at its first position, and the forward and backward eccentric paths are similarly divided, also commencing at the first position. The length of the eccentric rod is then taken in the compasses and an arc described from the point 1 of the fore eccentric, and another from 1′ of the back eccentric. The arcs for the fore eccentric are struck above the centre horizontal line, and for the back eccentric are struck below this line. An arc is now struck from the centre of the link,

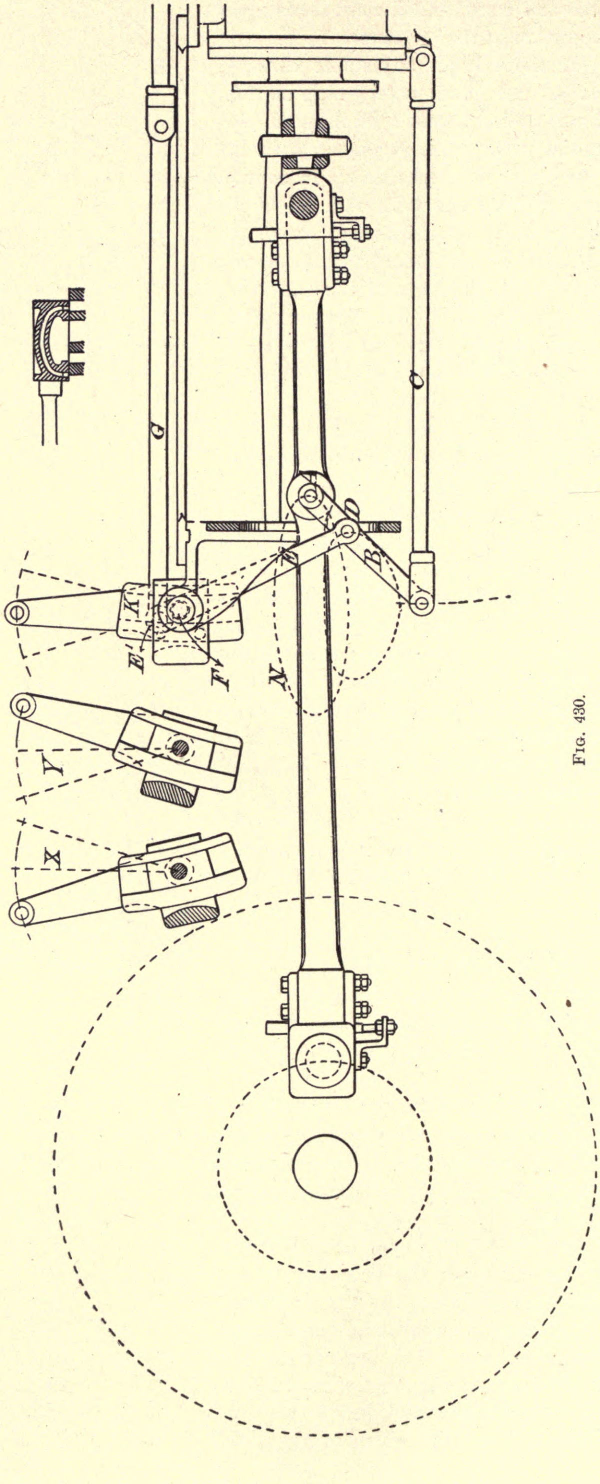


Fig. 430.

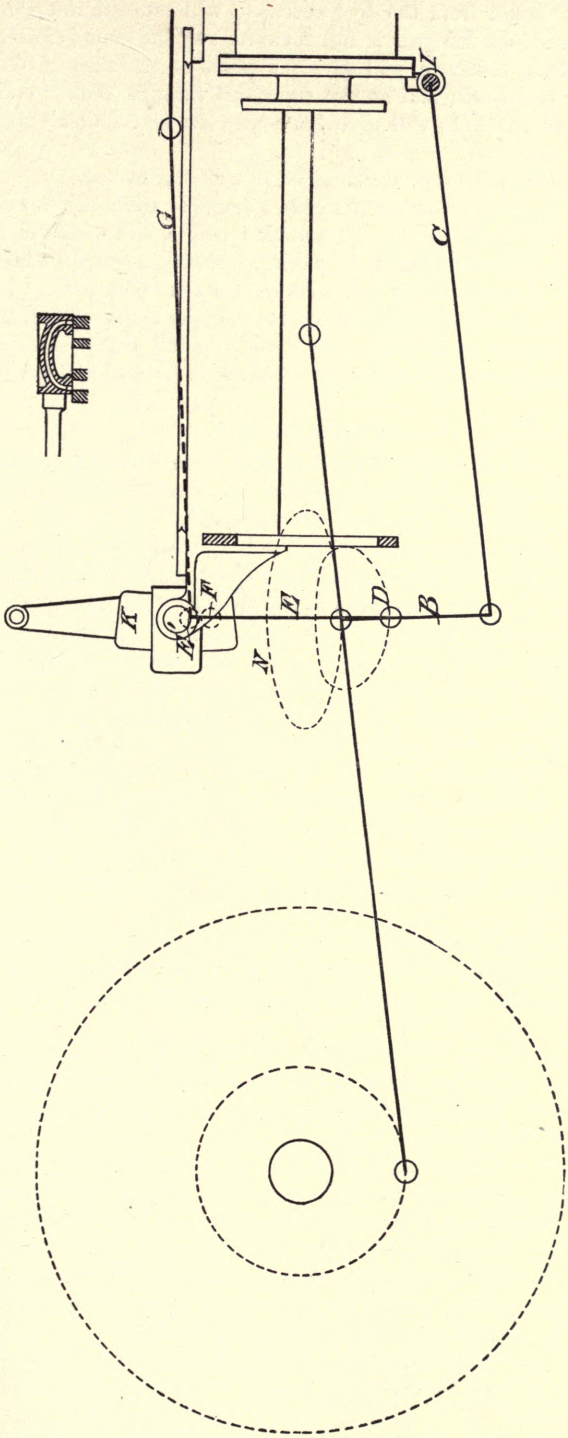


FIG. 431.

with a radius equal to the height at which the sliding block is in the link; where this arc intersects that struck from the fore eccentric will indicate the point at which the sliding block will be when the gear is full forward, and the same arc described, cutting the arc from the back eccentric, will give the point of the sliding block in the link when in full gear backward; and an arc described through these two points, with a radius equal that of the link, will give the centre line of the link when the eccentrics are in the position indicated.

The other positions of the link are obtained in a similar manner.

In describing the centre line of the link, numerous trials are necessary to get the proper centre of the arc passing through the two points, and where many positions of the link are necessary it will expedite the work by making a templet of the link in card-board, which can be readily applied to the several series of points.

The upper line on the link movement connecting the series of points indicates full forward gear. The arc *c c′*, mid-gear. The lower line, full gear backward.

This diagram also shows the position of the valve in forward and backward stroke.

JOY'S VALVE GEAR.

Figs. 430 and 431 are drawings of Joy's valve gear in different positions. This gear differs from the link motion in that the valve motion, instead of being given by means of eccentrics, is imparted directly from the connecting-rod.

At a point A of the connecting-rod a link B is attached, the movement of this link being controlled by the radius rod C attached to a stationary point I of the engine; from a point D of the link, movement is given to the lever E; from the upper end E′ of this lever, motion is given to the valve spindle G. The centre F has also a vertical movement due to the vibration of the connecting-rod at A; the centre or fulcrum F of the lever E is therefore carried in blocks sliding in slots of the link K, which has a radius equal to the length of the valve spindle G. In Fig. 431 the link is shown at mid-gear; this link can be partially rotated by a point on the outside of the link corresponding to the point F of the lever E, shown in Fig. 430. When the link is thus inclined, as shown at X and Y, the vertical movement of E causes the blocks of the link and the centre F to traverse a path inclined to a vertical line. The centre F has therefore a horizontal movement, the amount of which depends on the obliquity of the link. It is by this means that the cut-off of steam and the forward and backward movement of the engine is controlled.

The dotted ellipse A N indicates the path of the point A of the connecting-rod, and the dotted curve beneath shows the path of the pin D, which partakes of the motion of the point A of the connecting-rod and of the extremity of the radius rod E.

WALSCHAERT VALVE GEAR.

Figs. 432, 433, 434, 435 are drawings of the Walschaert valve gear. In this gear the valve derives its motion partly from the piston-rod crosshead and partly from a single eccentric moving a vibrating link.

To the crosshead A is attached a fixed arm B; to this arm is attached the link C, which communicates its motion to the combination arm D. To the upper end of this arm is connected the valve stem *d*; just below this connection the combination arm is pivoted to a centre *e*; the valve derives the other element of its motion from the radius rod E attached to the centre *e* and to the slider in the vibrating slotted link F, which is hung externally at its centre *f*; the link receives its motion from the eccentric rod G attached to its lower end and connecting with the eccentric *g*.

The sliding of the rod F in the slotted link by means of the levers, shown in dotted lines, is the means adopted for giving an earlier or later cut-off and forward or backward gear.

Skeleton diagrams (Figs. 433 to 435) illustrate the movement of this gear :

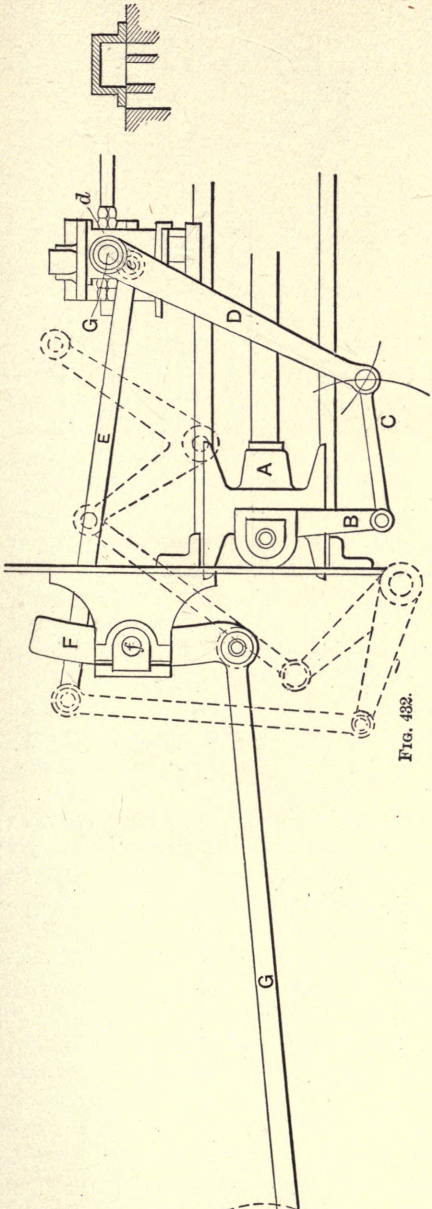


Fig. 432.

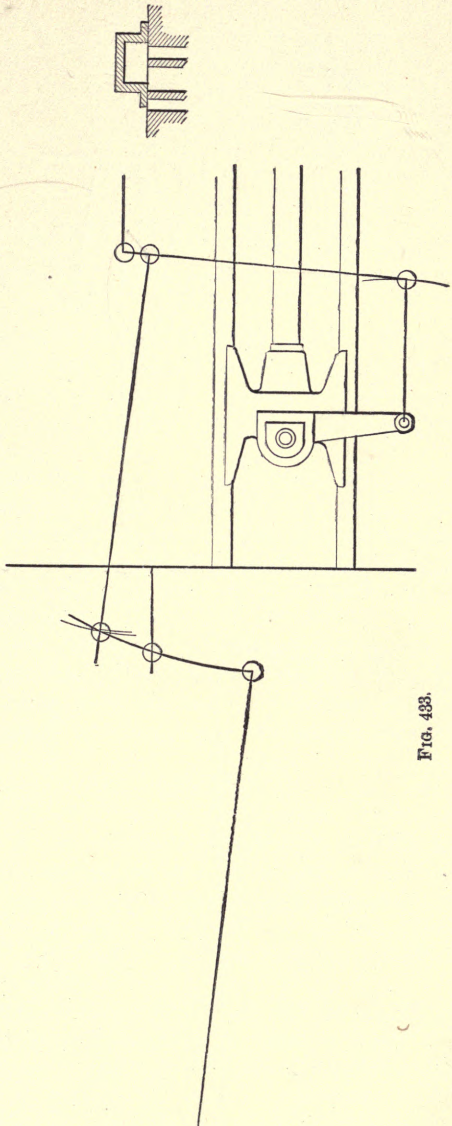


Fig. 433.

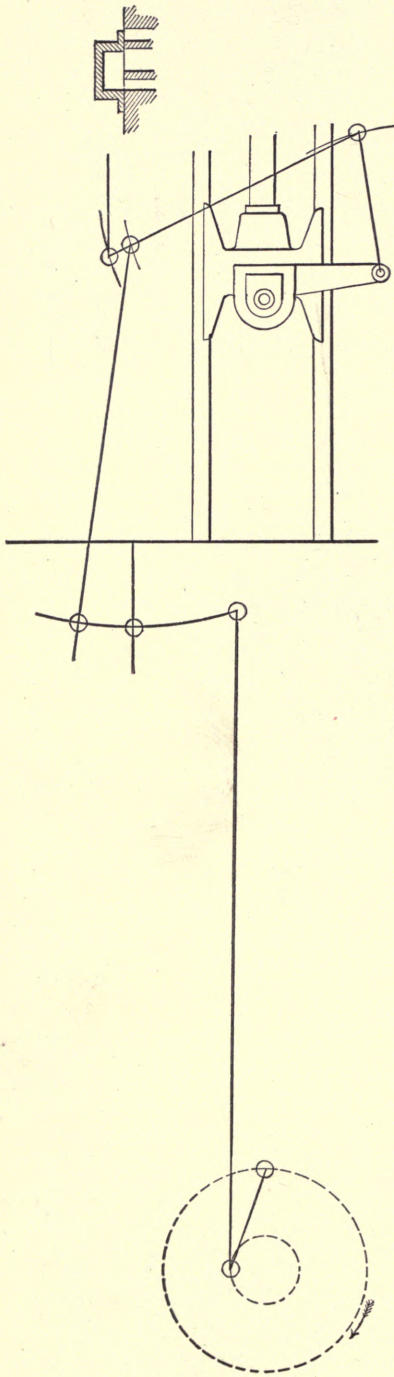


FIG. 434.

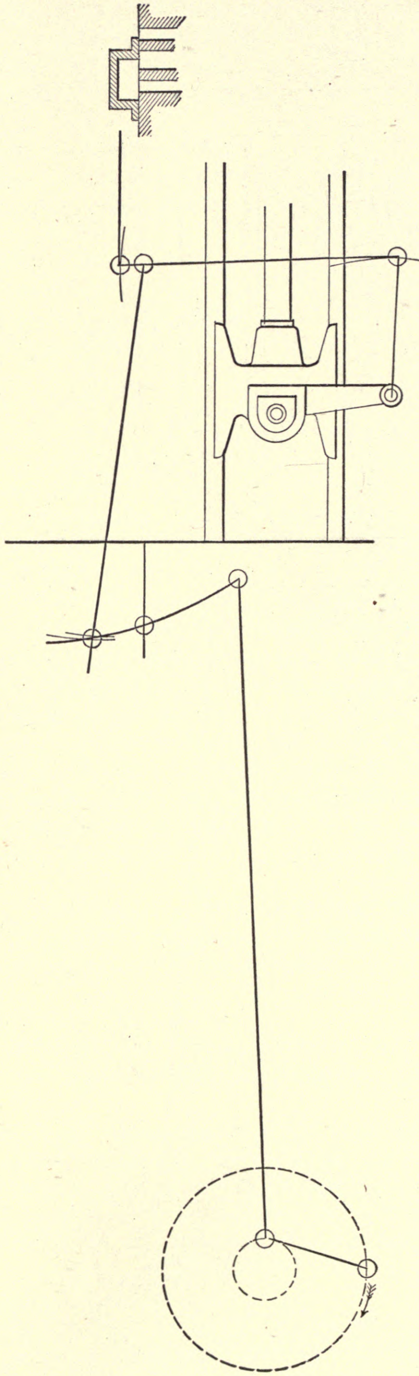


FIG. 435.

MACHINE DESIGN AND MECHANICAL CONSTRUCTIONS.

In the designing of new machines and mechanical constructions, the draughtsman must draw from his knowledge of well-known forms and parts, and combine them; but, to proportion them properly, and adapt them to the purposes required, he must understand the stresses to which they are to be subjected, and the action and endurance of the material to be used, to withstand these stresses.

In the present technical application of the term, *stress* is confined to a force exerted between two bodies or parts of a body, such as a pull, push, or twist. Strain is the alteration produced by a stress. Stress is the cause, strain the effect; the first is measured by the load, the latter by the deformation of the body produced by the first. A stress, not greater than the elastic limit of the material acted upon, produces a strain which disappears as soon as the load is removed: up to this limit the strain is proportional to the stress; beyond, the strain increases faster than the stress, up to the point of rupture. The elastic limit is a percentage of the breaking strain, varying with the kind of material and application of stress. Stress is usually designated as load, meaning thereby the sum of all the external forces acting on the member or structure, together with its weight.

Dead load, or weight, is a steady, unchangeable load. *Live loads* are variable, alternately imposed and removed, or varying in intensity or direction. It is usual, in designing constructions, to proportion the parts to resist a much greater load than will be brought on them in the structure; the load is multiplied by a factor termed *factor of safety*, as a security against imperfections in material and workmanship, contingencies of settlement, and other incidental stresses. But it must be observed that these imperfections are such as can not be seen and met; there can be no factor of safety to provide for poor and unknown material and defective workmanship.

The factor of safety adopted for dead loads varies but little with the same kind of material; but for live loads the factor varies not only with the material, but with the character of the stresses, whether they are applied and relieved gradually or suddenly; whether they only vary in intensity, or also in direction, alternately compressive or tensile. In this latter case the load should never be considered less than the sum of the stresses, with a large factor of safety. Vibrations, shocks, and changes in the direction of stresses concentrate the strains at the weakest point of the construction, and rupture takes place at these points, which would be adequate to the strain if the form throughout were uniform with that at these points. Thus, boiler-plates show wear just at the edge of the lap of the sheets, and car-axles (Fig. 436), with sharp angles at the journals, are known to break after a time, while under the same stresses an axle of uniform size with the journal would not break; nor if a slight cove ¼ inch radius (Fig. 437) be made in the angle to distribute stress.

Besides provisions for strength, the draughtsman should understand the necessities of the construction, and the character of the material to be used. He should know what parts of the design are to be forged, cast, framed, and how it is to be done. He should know what wear is to be met, and what waste, as rust or rot, to be provided for.

This knowledge can only be arrived at by reference to examples of practice and by observation of results under similar conditions of use and time.

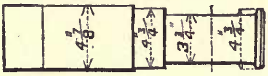


FIG. 436.

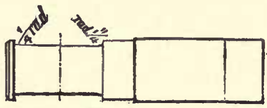


FIG. 437.

The stresses to which constructions and parts of constructions are subjected are the *tensile* or stretching stress, tending to lengthen a body in the direction of the stress; the *compressive* or crushing stress, tending to shorten a body in the direction of the stress; the *shearing* or cutting stress, tending to elongate, compress, and deflect; the *torsional* or twisting stress, the effect being an angular deflection of the parts of the body; and the *transverse* or lateral stress, tending to bend the body or break it across.

In Appendix is given a table of the strength of various metals to resist compression and tensional stresses, and examples will hereafter be given of varied constructions, with their usual or required factors of safety; but, for a practical rule for the common necessities of the above stresses, under dead loads, 10,000 pounds per square inch for wrought-iron and 12,000 for steel may be considered perfectly safe.

Posts in structures are subjected to compressive stresses; but, as the action is modified somewhat by a tendency to bend, depending on the proportion of the length to the diameter, and the material of which they are composed, the usual tables of crushing strength are not generally applicable, and the formulæ to be depended on are those deduced from practical tests. The best tests of wooden posts are those made by Professor Lanza, for the Boston Manufacturers' Mutual Fire-Insurance Company, and the following are the results:

"That the strength of a column of hard pine or oak, with flat ends, the load being uniformly distributed over the ends, is practically independent of the length, such columns giving way by direct crushing, the deflection, if any, being very small. Tests were on columns 6″ to 10″ diameter × 12 feet. The average crushing strength of very highly seasoned, hard pine was 7,386 pounds per square inch. Some very slow-growth and highly seasoned, 9,339 pounds; very wet and green, 3,015 pounds; seasoned about three months, 3,400 pounds; not very well seasoned and not very green, 4,400 to 4,700 pounds. The average of two specimens of thoroughly seasoned white-oak, 7,150 pounds; for green and knotty, average, 3,200 pounds. Spruce, nearly 5,000 pounds. Whitewood, 3,000 pounds.

"That it is a mistake to turn columns, taper, or even turn them at all, square columns being much stronger, cheaper, and better, and that accuracy of fitting is of great consequence, that the stress may be directly vertical." The professor recommends that longitudinal holes be bored through the centre of columns to allow of the circulation of air (in the experiments the holes were 1·7″ diameter), and that iron caps be used instead of wooden bolsters, as the wooden bolster will fail at a pressure far below that which the column is capable of resisting, and the unevenness of pressure brought about by the bolster is sometimes so great as to crack the column. He also recommends horizontal holes in the iron caps to connect the longitudinal ones in the column with the outer air.

From the whole of the experiments, we estimate the safe load, for fair-grained, well-seasoned oak or yellow-pine columns, to be from 1,000 to 1,500 pounds per square inch; for the more imperfect and green specimens, from 300 to 500 pounds; for good specimens of whitewood, about 300 pounds; and of spruce, about 500 pounds.

Cast-Iron.—For the columns of buildings where the load is dead, cast-iron is very generally used. They are, in interiors, mostly of circular section, but for outer columns forms are used suited to the necessities of their position or style of architecture. They admit of considerable ornamentation and finish direct from the mould; but, as they are liable to defects not readily detected in the process of casting, the factor of safety is usually taken as high as 5. To protect them against the effects of fire and water in conflagrations, they should be covered with a shell of light refractory material, as porous brick or tile.

The experiments of Hodgkinson are the usual basis of all formulæ on the strength of circular cast-iron columns, and the ends of all columns are now required to be faced by architects and by the rules of building departments, since Mr. Hodgkinson states this rule, that "in all long columns, of the same dimensions, the resistance to fracture by flexion is three times greater when they are flat and firmly bedded than when they are rounded and capable of moving."

Table of the safe load of solid cylindrical columns, with flat ends calculated with a factor of safety of 5.

TABLE OF SAFE LOADS FOR SOLID CAST-IRON COLUMNS, WITH FLAT ENDS.

Diam.	8′ 1,000 lbs.	9′ 1,000 lbs.	10′ 1,000 lbs.	11′ 1,000 lbs.	12′ 1,000 lbs.	13′ 1,000 lbs.	14′ 1,000 lbs.	15′ 1,000 lbs.	16′ 1,000 lbs.	17′ 1,000 lbs.	18′ 1,000 lbs.	19′ 1,000 lbs.	20′ 1,000 lbs.	21′ 1,000 lbs.	22′ 1,000 lbs.	23′ 1,000 lbs.	24′ 1,000 lbs.
3″	29·	23·	20·	17·	14·	13·	11·	10·	9·	8·	7·	7·	6·	6·	5·	5·	4·
3¼″	40·	31·	26·	22·	19·	17·	15·	13·	12·	11·	10·	9·	8·	7·	7·	6·	6·
3½″	50·	41·	34·	29·	25·	22·	19·	17·	15·	14·	12·	11·	10·	10·	9·	8·	8·
3¾″	63·	54·	43·	37·	32·	28·	24·	22·	19·	18·	16·	15·	13·	12·	11·	11·	10·
4″	77·	66·	54·	46·	40·	35·	31·	27·	24·	22·	20·	18·	17·	15·	14·	13·	12·
4¼″	92·	80·	70·	57·	49·	43·	38·	34·	30·	27·	25·	23·	21	19·	18·	16·	15·
4½″	110·	96·	84·	74·	61·	53·	47·	41·	37·	33·	30·	28·	25·	23·	22·	20·	19·
4¾″	130·	113·	99·	88·	73·	64·	56·	50·	45·	41·	37·	34·	31·	28·	26·	24·	23·
5″	152·	133·	117·	103·	92·	77·	68·	60·	54·	49·	44·	40·	37·	34·	31·	29·	27·
5¼″	176·	154·	136·	121·	108·	97·	81·	72·	64·	58·	53·	48·	44·	40·	37·	35·	32·
5½″	201·	177·	157·	140·	125·	113·	95·	85·	76·	68·	62·	57·	52·	48·	44·	41·	38·
5¾″	230·	203·	180·	161·	144·	130·	118·	99·	89·	80·	73·	66·	61·	56·	52·	48·	45·
6″	260·	230·	205·	183·	165·	149·	135·	115·	103·	93·	84·	77·	71·	65·	60·	56·	52·
6¼″	292·	260·	232·	203·	187·	169·	154·	140·	119·	108·	98·	89·	82·	75·	69·	64·	60·
6½″	327·	292·	261·	234·	212·	192·	174·	159·	146·	124·	112·	102·	94·	86·	80·	74·	69·
6¾″	364·	326·	292·	263·	238·	216·	197·	180·	165·	141·	128·	117·	107·	99·	91·	85·	79·
7″	404·	362·	325·	293·	266·	242·	221·	202·	186·	171·	146·	133·	122·	112·	104·	96·	90·
7¼″	445·	400·	361·	326·	296·	269·	246·	226·	208·	192·	177·	151·	138·	127·	118·	109·	101·
7½″	489·	441·	3 8·	361·	328·	299·	274·	251·	231·	214·	198·	170·	156·	143·	133·	123·	114·
7¾″	536·	484·	433·	298·	362·	331·	303·	278·	257·	237·	220·	204·	175·	161·	149·	138·	128·
8″	584·	529·	480·	436·	398·	364·	334·	308·	284·	263·	244·	227·	196·	180·	167·	155·	144·
8½″	689·	626·	571·	521·	477·	437·	402·	371·	343·	318·	296·	275·	257·	241·	207·	192·	178·
9″	802·	733·	670·	614·	564·	519·	479	442·	410·	381·	354·	331·	309·	290·	272·	235·	218·
9½″	926·	819·	780·	717·	660·	609·	563·	522·	484·	451·	420·	393·	367·	345·	324·	305·	265·
10″	1058·	975·	898·	829·	765·	708·	656·	609·	566·	528·	493·	461·	432·	406·	382·	360·	340·
10½″	1195·	1108·	1026·	957·	892·	823·	779·	740·	693·	658·	610·	580·	546·	511·	485·	459·	433·
11″	1359·	1264·	1159·	1083·	1017·	950·	889·	846·	798·	751·	703·	665·	627·	589·	561·	542·	513·
11½″	1517·	1413·	1319·	1226·	1147·	1080·	1018·	956·	904·	852·	810·	758·	727·	691·	655·	613·	587·
12″	1674·	1583·	1470·	1380·	1289·	1221·	1142·	1074·	1018·	973·	916·	871·	746·	701·	667·	645·	611·

Solid columns are very seldom used in constructions; they are almost invariably made hollow, the shell being ½″ to 2″ thick. To determine the safe load of a hollow column, it will be sufficiently accurate to take from the table the safe load of a column equal to that of the exterior diameter, and subtract from this the safe load of a column of a diameter equal to the core.

Example.—To find the safe load of a column 12 feet long, 8″ exterior diameter, shell ¾″.

Safe load of 8″ column....................................	398,000 lbs.
" " " 6½″ "	212,000 "
" " required column	186,000 "

For square box-columns, it will be safe to estimate that a square column will support as much as a round one, the side of the one being equal to the diameter of the other, and the thickness of shell the same.

For a star-column (Fig. 438), the load should be about ⅕ less than on a cylindrical column of same diameter and same area of section.

There is a convenience in the use of cast-iron that the brackets for the support of beams and girders can be readily cast on, but it must be done with care in the design and in the moulding. It will be seen (Fig. 341) that weak places occur in the cooling when the junctures are at right angles, which must be avoided by easements, or coves, in the patterns, and by intelligence in the moulding, pouring, and cooling. For many years cast-iron was the only metal used for posts, and with but few accidents from imperfect construction or from conflagrations.

Wrought-Iron Columns.—With the decrease in the cost of the manufacture of shapes in wrought-iron, columns of this material have largely superseded those of cast-iron in

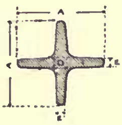


FIG. 438.

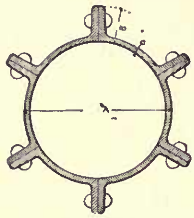


FIG. 439.

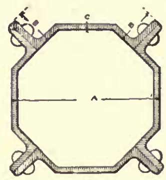


FIG. 440.

constructions liable to varying loads and shocks. Fig. 439 shows the section of a Phœnix column, Fig. 440 of the Keystone.

TABLE OF PHŒNIX COLUMNS.

MARK OF COLUMN.	Thickness in inches.	Area in square inches.	Weight in pounds per foot.	Internal diameter.
A	1/8	2·8	9·3	3 5/8
4 segments	5/16	5·8	19·4	
B	3/16	5·0	16·7	4 13/16
4 segments	5/8	14·8	51.	
B²	3/16	5·8	19·4	5 15/16
4 segments	5/8	17·	58·6	
C	3/16	8·8	30·3	7 3/16
4 segments	1 3/16	40·	138.	
D	1/4	14·0	48·2	9 1/8
5 segments	5/8	26·	89·7	
E	1/4	16·	55·2	11
6 segments	1 3/16	60·	207·	
F	3/8	24·5	84·5	13
7 segments	5/8	36·4	125·6	
G	5/16	24·	82·8	14 5/8
8 segments	1 5/16	80·	276·	

BUILT COLUMNS.

Open columns should be used in positions exposed to dampness and rust on account of their accessibility to painting and inspection. Built columns present advantages in the facility with which connections can be made to floor beams or bracing rods.

In determining upon the proper sections for these columns, the material should be so disposed that the tendency to yield will not be greater in one direction than in the other to secure the full strength of the column.

The rivets at the ends should be pitched not more than three inches apart, and the maximum distance between rivets in the line of stress should not exceed sixteen times the thickness of the plate.

Figs. 459 and 460 are plan and elevation of a latticed column.

Spacing of the Lattice or Lacing Bars.—The object of these bars is to join the two channels composing the post or chord, and thus cause them to act together; they should be attached at intervals so close that there shall be no danger of failure of the channels between the points of attachment, which never should be more than 0·6 of the interval, and lacing bars are not allowed to make an angle of more than 60° with each other, or less than 60° with the flanges.

FIG. 441.

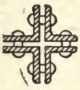


FIG. 442.

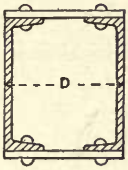


FIG. 443.

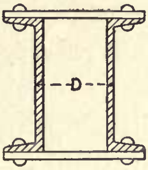


FIG. 444.

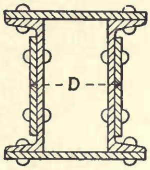


FIG. 445.

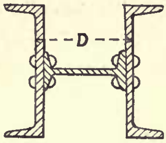


FIG. 446.

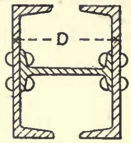


FIG. 447.

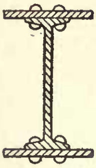


FIG. 448.

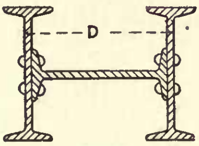


FIG. 449.

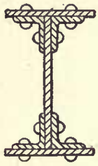


FIG. 450.

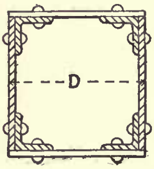


FIG. 451.

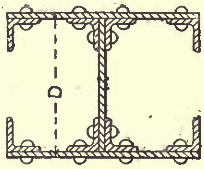


FIG. 452.

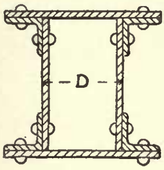


FIG. 453.

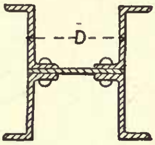


FIG. 454.

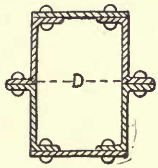


FIG. 455.

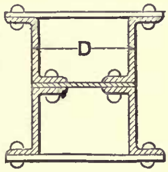


FIG. 456.

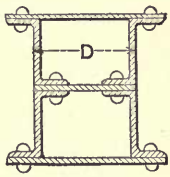


FIG. 457.

On the Strength of Wrought-Iron Columns.—In the former editions of this work the strength of wrought columns was shown by curves of the average breaking loads (Fig. 461), as determined by experiments on the Phœnix, Keystone, Piper, and open columns with flat ends. Vertical distances represent the pounds of load per square inch, the horizontal the proportion of the length of the columns to the diameter $\frac{L}{D}$; D is taken as the least outside diameter as marked on the varied sections above. The lower curves represent the safe loads, under factors of safety of 3, 4, and 5. In looking at these curves, it will be observed that, within the common limits of practice, of 15 to 35 $\frac{\text{length}}{\text{diameter}}$, these lines may be considered straight; that with iron of a breaking strength of 52,000 pounds per square inch, and within the above limits, and a factor of safety of 3, the safe load may be taken at 11,000 per square inch; with a factor of safety of 4, at 8,000 pounds; with a factor of safety of 5, at 6,500 pounds; and that for common and usual purposes 10,000 pounds per square inch is a safe load.

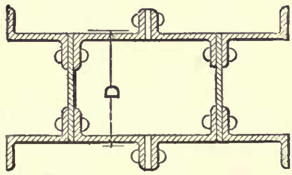


FIG. 458.

At present there are few wrought-iron columns manufactured; they are almost invariably steel, but the diagram still represents in their working proportions the action of columns under stress. The steel column is about 20 per cent stronger. When, as ob-

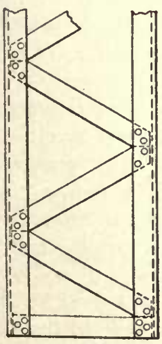


FIG. 459.

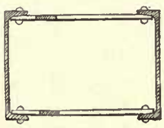


FIG. 460.

served above, the common and usual safe load on iron is 10,000 pounds per square inch, that on steel is 12,000 pounds.

For struts, angle irons, or a combination of them forming an angle or cross (Figs. 441 and 442), with or without separators, are generally used, and in these cases for D in $\frac{L}{D}$ take D equal to 0·8 of the shortest leg or legs.

It has generally been considered that columns with pin or cylindrical ends had about

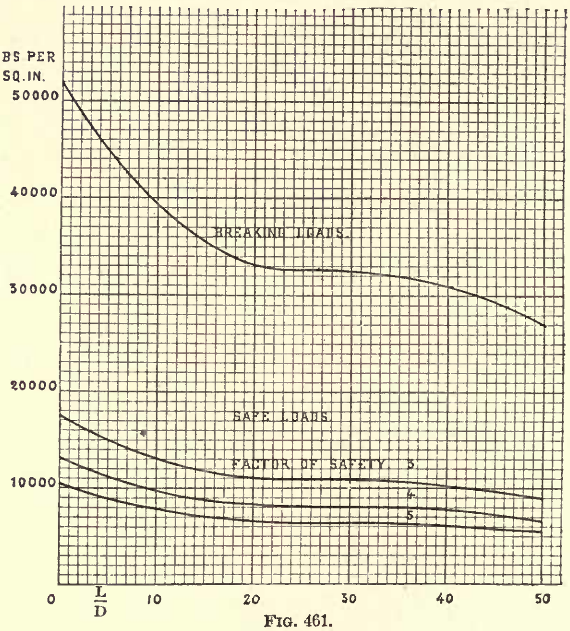


FIG. 461.

three fourths of the resisting strength of flat ends, but if the pin ends are closely fitted, so that the strains are uniformly in the direction of the length of the column, the difference is but little between the two kinds of ends.

Shearing Stresses.—Parts of machines and of constructions subjected to these stresses have often the resistances modified by friction, combined with other stresses. The sizes of parts necessary to resist such stresses practically, as in the cases of bolts, rivets, and the like, will be hereafter illustrated by examples and determined by particular rules. In general, the strength to resist shearing stress is, in wrought-iron and steel, from 70 to 80 per cent of its tensile strength; in cast-iron, about 40 per cent of its crushing strength. The softer woods, as spruce, white pine, hemlock, resting on walls or girders, will safely sustain a load of 200 to 300 pounds per square inch of bearing surface, and the harder woods, as oak and Southern pine, 300 to 500 pounds. By experiment, oak treenails, 1″ to 1¾″ diameter, were found to have an ultimate shearing strength of about two tons per square inch of section; but, according to Rankine, the planks thus connected together should have a thickness of at least three times the diameter of the treenails. In 3″ planks, 1¾″ treenails bore only 1·43 tons per square inch of section; in 6″ plank, 1·73 tons.

Torsional Stress.—Every shaft through which power is transmitted, whether through gears, cranks, or pulleys, is subjected to a torsional stress, of which the power acting tangentially to the shaft in one direction is resisted by the load in an opposite direction. When this stress exceeds a certain limit depending on the material, the fibres are twisted asunder, but much below this limit the elasticity of the shaft may be too great to transmit power uniformly.

The *length* of the axle subjected to torsion does not affect the actual amount of pressure required to produce rupture, but only the angle of torsion which precedes rupture, and therefore the space through which the pressure must be made to act.

A practical limit of torsional deflection is 1° in a length equal to twenty diameters

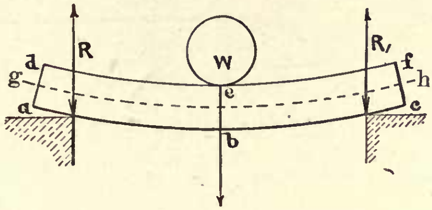


FIG. 462.

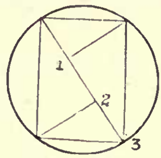


FIG. 463.

of the shaft—or $\frac{1}{360}$ part of a full turn. D. K. Clark gives the following rule: "To find the diameter of a shaft capable of transmitting a given torsional stress within good working limits. Divide the torsional stress in foot-pounds by 18·5 for cast-iron; 27·7 for wrought-iron; and 57·2 for steel. The cube root of the quotient is the diameter of the shaft in inches.

Example.—On the teeth of a 4½-foot gear, the force exerted is 2,800 pounds. What should be the diameter of a wrought-iron shaft to transmit this force safely ?

The torsional stress will be 2,800 pounds multiplied by the radius of leverage, 2¼ feet, or 6,300 foot-pounds $= \frac{6{,}300}{27 \cdot 7} = 228, \ \sqrt[3]{228} = 6 \cdot 1$.

BEAMS.

Transverse Stress.—The strength of a beam is influenced by the manner in which its ends are supported. Where the ends are simply supported—that is, resting upon the abutments—and the beam loaded with a weight W (Fig. 462), the beam is subjected to a bending movement, or transverse stress, composed of a tensile stress on the lower part of the beam and compressive on the upper part. In addition, the weight of the beam and its supported load act on the abutments as shearing stresses.

If the ends of a beam are fixed in the wall, however, the transverse stresses are considerably relieved throughout the beam, which is thereby capable of sustaining a heavier load. Figs. 464 to 471 are examples of beams with both ends simply supported and both ends fixed, or one end supported and one end fixed, and a comparison of their strength for a centre load and a load uniformly distributed.

The strength of a square or rectangular beam to resist transverse stress is as the breadth and the square of the depth; and inversely as the length, or the distance from or between the points of support. Thus a beam twice the breadth of another, other proportions being alike, has twice the strength; or twice the depth, four times the strength; but twice the length, only half the strength.

It is evident, therefore, that, with the same area of section, the deeper a beam the stronger it will be, if the breadth is sufficient to prevent lateral buckling.

To cut the best beam from a log, Fig. 463, the section of which is a circle: draw a diameter, divide it into three equal parts, erect perpendiculars at the points of division 1, 2, and they will intersect the circumference at the corners of the beam, of which the extremities of the diameter are the other two.

For the transverse strength of rectangular beams the general formula is $W = \frac{S\,b\,d^2}{l}$, in which W is the breaking weight; S, a number determined by experiment on different materials; b, the breadth, and d, the depth in inches; and l, the length in feet.

Figs. 464 to 471 represent the usual methods of loading beams, and the loads as drawn represent the comparative strength of beams under these different conditions. Thus, in Fig. 464, the beam supports but one unit of load, while Fig. 465 supports twice as

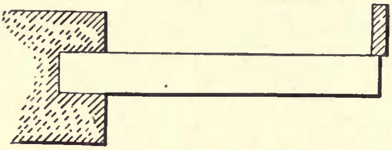


FIG. 464.

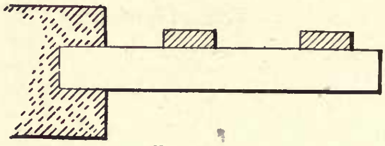


FIG. 465.

much. The formulæ given represent the safe dead loads with a factor of safety of 6, deduced from experiments of Mr. C. J. H. Woodbury on Southern pine. For spruce the

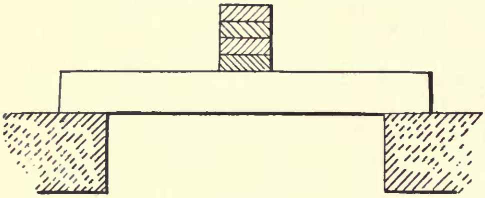


FIG. 466.

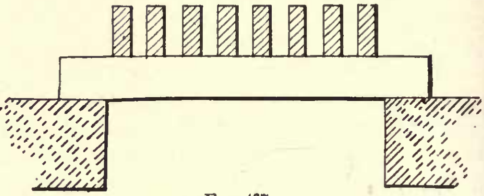


FIG. 467.

coefficient would be about $\frac{1}{5}$ less, and for live loads the factor of safety should be 12.

Beams fixed at one end and loaded at the other (Fig. 464).

Safe load $= 30 \frac{b\,d^2}{l}$.

Beams fixed at one end and load distributed uniformly, not as represented in the figure, as the two units of weight would be spread over the whole length of the beam (Fig. 465).

Safe load $= 60 \frac{b\,d^2}{l}$.

Beams supported at the extremities and loaded at the middle (Fig. 466).

Safe load $= 120 \frac{b\,d^2}{l}$.

Beams supported at the extremities and the load uniformly distributed (Fig. 467).

Safe load $= 240 \frac{b\,d^2}{l}$.

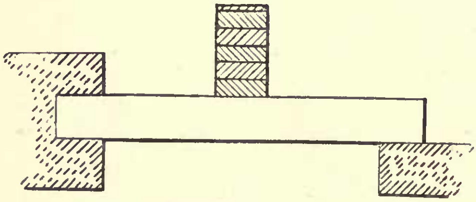


FIG. 468.

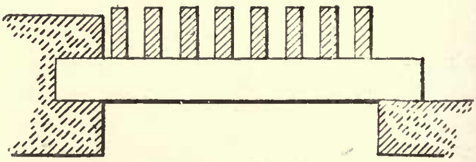


FIG. 469.

Beams, one end firmly fixed, the other supported, and loaded at the middle (Fig. 468).

Safe load $= 160 \frac{b\,d^2}{l}$.

Beams with one end fixed, the other supported, and load uniformly distributed (Fig. 469).

Safe load $= 240 \frac{b\,d^2}{l}$.

Beams with both ends fixed, and loaded at centre (Fig. 470).

Safe load $= 240 \frac{b d^2}{l}$.

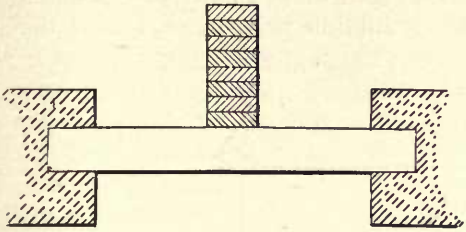


FIG. 470.

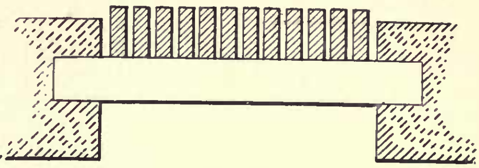


FIG. 471.

Beams with both ends fixed, and load uniformly distributed (Fig. 471).

Safe load $= 360 \frac{b d^2}{l}$.

If the loads on a beam be neither at the centre nor uniformly distributed, but at intermediate points, it is important to determine the load which placed at the centre will produce an equivalent stress.

Thus on a beam of 30 feet span, if a weight of 800 pounds be placed at 10 feet from an end. Lay off on any convenient scale (Fig. 472) a horizontal line

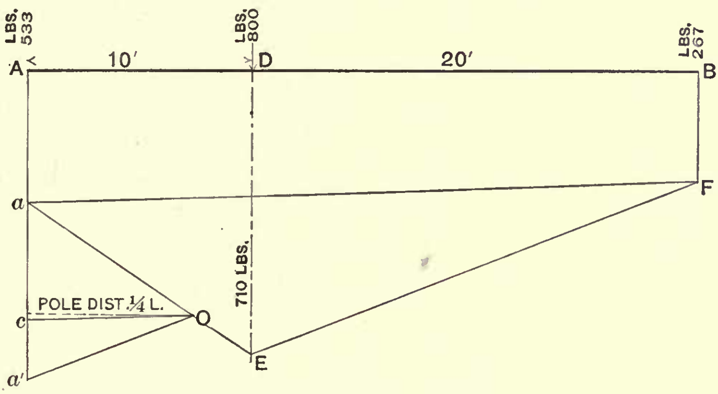


FIG. 472.

of 30 feet, and at a point 10 feet from one end, draw a vertical or ordinate from this point, and at the ends A and B.

From any convenient point on the ordinate beneath A draw a line parallel to A B and set off to O, a distance equal to ¼ of the span for the pole distance of the force polygon about to be constructed, which is established at ¼ of the span, in order to determine readily the equivalent central load. From the point *a* in the ordinate lay off on any scale $a\,a' = 800$ pounds, draw lines O *a* and O *a′*, and the force polygon is complete.

From *a* extend the line *a* O till it intersects the weight ordinate from D at E. Draw E F parallel to O *a′* to intersect the ordinate B; connect F with *a*; F *a* is called the closing line of the equilibrium polygon.

The weight 800 pounds is supported by the abutments A and B. Draw a line O *c* in the force polygon parallel to F *a*; the distance above this line in pounds will be the amount of the load on A, 533 pounds, $c\,a'$, below the line, 267 pounds will be that on B.

If the weight be placed centrally (Fig. 473) and its material sufficiently elastic to be considered uniformly distributed for its whole length; to determine its effect under these conditions, draw the equilibrium polygon a E F as if the

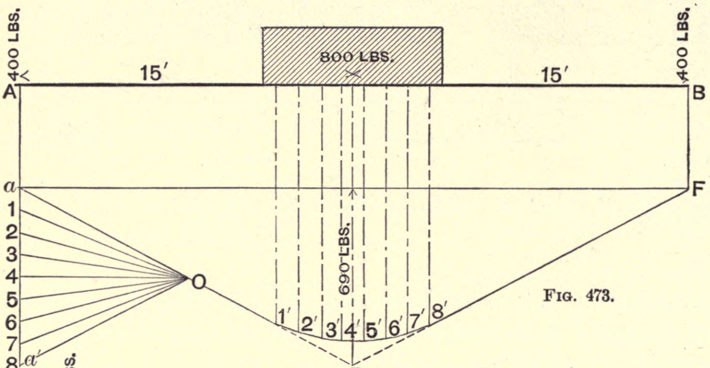


FIG. 473.

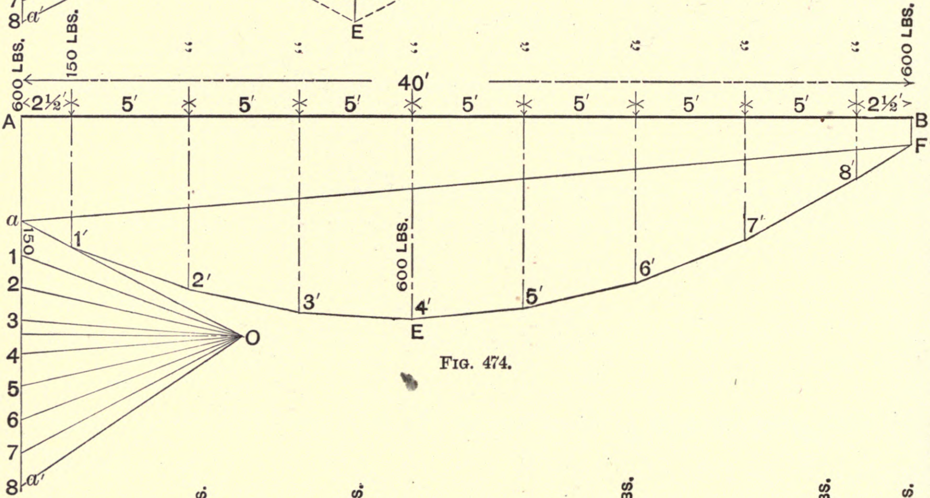


FIG. 474.

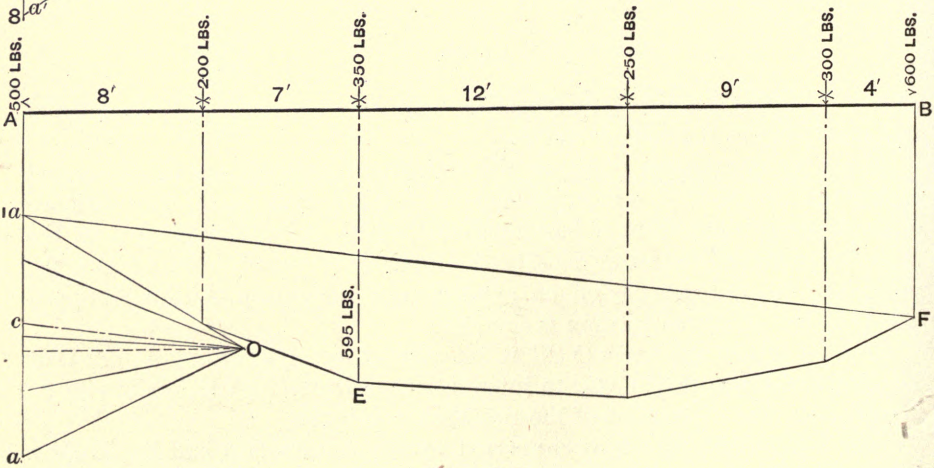


FIG. 475.

weight of 800 pounds were suspended from a single central point; divide the weight into any number of equal parts (say 8, of 100 pounds each), and draw ordinates through the centre of these parts. Draw through 1′ a line parallel to O 1 and consecutively through 2′ 3′ 4′ 5′ 6′ 7′ 8′ lines parallel to O 2, O 3, etc. The central weight as measured on the longest ordinate is 690 pounds.

If the weight 1,200 pounds (Fig. 474) be distributed on a beam of 40 feet—40 ÷ say 8 = 5. Lay off at each extremity one half part or 2½ feet, and for the intermediate lengths, 7 parts of 5 feet each, and draw ordinates through these divisions; follow the diagram construction as in the preceding, each weight being 150 pounds, the longest ordinate is 600 pounds, or one half the total weight of 1,200 pounds, and each abutment load is 600 pounds.

In Fig. 475 the weights are many and unequally distributed; construct diagram as before, always making the pole distance equal to a quarter of the span; measure the longest ordinate, which is 595 pounds, the equivalent central load. Draw O *c* parallel to the closing line *a* F; the total of the weights 1,100 pounds will be supported by abutment A, 500 pounds, and by abutment B, 600 pounds. In all the figures the weight of the beam itself is not taken into account, but, if considered, it is almost invariably of one section and therefore distributed, and its central load will be one half the total weight.

The following table for the strength of yellow-pine beams is calculated for a safe central load, as determined by the graphic constructions or by taking one half the uniformly distributed load if this is given.

TABLE OF THE SAFE CENTRAL LOAD OF YELLOW-PINE BEAMS, CALCULATED FROM THE FORMULA $120 \frac{b d^2}{l}$.

Span in feet.	DEPTH IN INCHES OF YELLOW-PINE BEAMS, ONE INCH WIDE.													
	3 ins.	4 ins.	5 ins.	6 ins.	7 ins.	8 ins.	9 ins.	10 ins.	11 ins.	12 ins.	13 ins.	14 ins.	15 ins.	16 ins.
4	270·	480·	750·	1080·	1470·	1920·	2430·	3000·	3630·	4320·	5070·	5880·	6750·	7680·
5	216·	384·	600·	864·	1176·	1536·	1944·	2400·	2904·	3456·	4056·	4704·	5400·	6144·
6	180·	320·	500·	720·	980·	1280·	1620·	2000·	2420·	2880	3380·	3920·	4500·	5120·
7	154·	274·	430·	616·	840·	1097·	1389·	1714·	2074·	2469·	2897·	3360·	3857·	4388·
8	135·	240·	375·	540·	735·	960·	1215·	1500·	1815·	2160·	2535·	2940·	3375·	3840·
9	120·	213·	333·	480·	653·	853·	1080·	1333·	1613.	1920·	2253·	2613·	3000·	3413·
10	108·	192·	300·	432·	588·	768·	972·	1200·	1452.	1728·	2028·	2352·	2700·	3072·
11		175·	273·	392·	535·	700·	882·	1092·	1320·	1571·	1844.	2140·	2457·	2793·
12		160·	250·	360·	490	640·	810·	1000·	1210·	1440·	1690·	1960·	2250·	2560·
13			230·	332·	452·	592·	747·	923·	1117·	1328·	1560·	1808·	2070·	2363·
14			215·	308·	420·	548·	693·	860·	1037·	1234·	1448·	1680·	1928·	2192·
15				288·	392·	512·	648·	800·	968·	1155·	1352·	1568·	1800·	2048·
16				270·	368·	480·	607·	748·	907·	1080·	1267·	1470·	1688·	1920·
17				254·	346·	452·	566·	704·	854·	1016·	1193·	1384·	1588·	1808·
18					327·	427·	540·	668·	806·	960·	1126·	1307·	1500·	1707·
19						404·	512·	632·	764·	909·	1067·	1238·	1422·	1616·
20						384·	486·	600·	726·	864·	1014·	1176·	1350·	1536·
21							463·	572·	691·	823·	966·	1120·	1287·	1463·
22							442·	546·	660·	785·	922·	1070·	1228·	1395·
23								522·	631·	752·	882·	1023·	1178·	1329·
24								500·	605·	720·	845·	980·	1125·	1280·
25									581·	691·	811·	940·	1080·	1230·
26									558·	665·	780·	904·	1035·	1182·

The table gives the load which the beam can sustain, allowing a certain factor of safety, but the strength given is in excess of stiffness, and in permanent construction it is necessary to proportion the beams to bear its load with a certain limited deflection. Cross-lines on the tables represent those limits, but, as usually the weight of construction is much less than that of the changeable loads, and as the change of deflection is that to be guarded against, the weight of construction may be neglected, and it will only be necessary to consider the amount of movable loads above these lines.

The table is deduced from Mr. Woodbury's experiments on yellow pine, of good quality and practical sizes. For spruce he takes loads of about one fifth less.

The table is intended to be used as a unit of width by which the strength of timber of usual depths and spans can be estimated, by multiplying by such widths as are found in practice; widths of less than two inches are not used. Mr. Woodbury established the limit of deflection in wooden beams at three quarters of an inch for 25-feet span, and his formula is $E = \frac{432\ W\ l^3}{b\ h^3 d}$, in which E, the modulus of elasticity per square inch, is for Southern pine 2,000,000, and for spruce 1,200,000: W central load in pounds, l the span in feet, b the breadth, h the depth, and d the deflection of beam, all in inches. Using this formula, marks are drawn in each column of depth, above which the loads will be supported stiffly, and below less than recommended; the formula is only applicable to seasoned wood. Mr. Woodbury's results are confirmed by late experiments on the long-leaved pine by Prof. Johnson for the Forestry Division of the United States Department of Agriculture.

He gives the shearing strength measured by the cross-section of the beam at about 600 pounds to the square inch, and the crushing strength across the grain at about 1,150 pounds. In the resting of the beam on abutments of masonry it will be difficult to exert a clean shearing stress, and the strength would be in excess of any likely to occur, but the stress would be a crushing one, and be met by the area in square inches resting upon the wall.

In the above tables there is a factor of safety of six—that is, the rupture should not take place except at a load six times that given. This is to provide for some unknown weakness of the timber, or some sudden excess of load, depreciation by age, etc., but this factor does not make up for want of inspection or knowledge of the material. It is well to load some of the timbers, as a test to the elastic limit, which can be done by placing two timbers at convenient distances apart and loading with pig iron or barrels of sand.

The early cast-iron beams used in framing have been almost entirely superseded by wrought ones, as cheaper and more reliable. In the application of the former, the sections were adapted to the stresses, as shown in Figs. 476–478. Practical examples of cast-iron beams are given, as there may be conditions under which it may be necessary to make use of them.

A beam subjected to a transverse stress, as shown in Fig. 462, one side is compressed, while the other side is extended; and therefore, where extension terminates and compression begins, there is a lamina or surface, $g\ h$, which is neither extended nor compressed, called the *neutral surface.* As the strains are proportional to the distance from this surface, the material of which the beam is composed should be concentrated as much as possible at the outer surfaces, as can readily be done in beams of cast and wrought iron. Mr. Hodgkin-

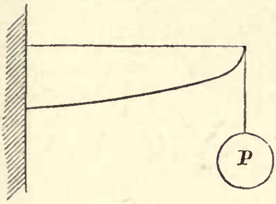


Fig. 476.

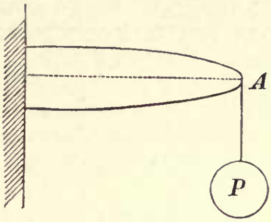


Fig. 477.

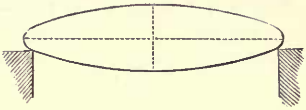


Fig. 478.

son found that the strength of cast-iron to resist compression is about six times that to resist extension; the top web is therefore made only one sixth the area of the lower one. The depth of the beam is generally about one sixteenth of its length, the deeper of course the strouger; the thickness of the stem or the upright part should be from $\frac{1}{2}$ an inch to $1\frac{1}{2}$ inch, according to the size of the beam. The rule for finding the ultimate strength of beams of the above section is: Multiply the sectional area of the bottom flange in square inches by the depth of the beam in inches, and divide the product by the distance between the supports in feet, and 2·42 times the quotient will be the breaking weight in tons (2,000 pounds). The section thus determined is that of the greatest strain, and can be reduced toward the points of support, either by reducing the width of the flanges to a parabolic form (Fig. 479), or by reducing the *thickness* of the bottom flange; the reduction of the girder in *depth* is not in general as economical or convenient.

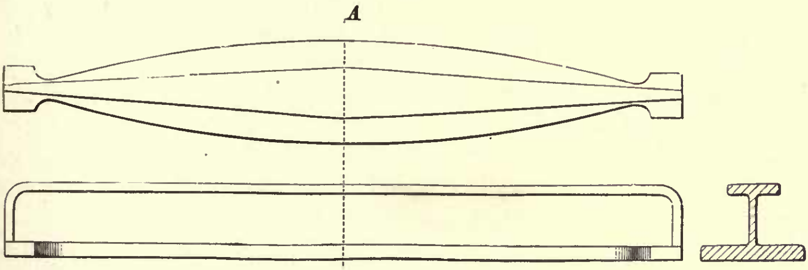


Fig. 479.

For railway structures subject to an impulsive force, Mr. Joseph Cubitt, C. E., recommends that the section of the upper flange should be one third that of the lower.

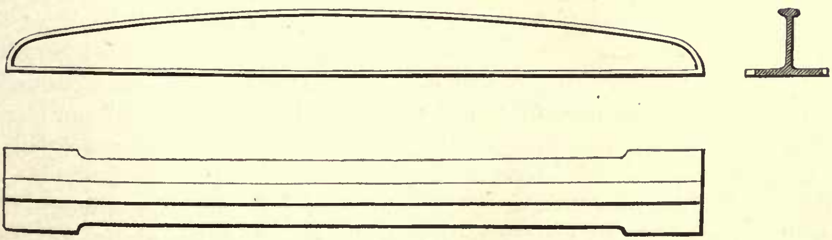


Fig. 480.

Fig. 480 is side elevation, plan, and section of cast-iron girder, adopted by him for railway purposes, a pair of girders for each track, the rails being supported on wooden cross-beams.

DIMENSIONS FOR DIFFERENT SPANS.

Opening.	Bearing on abutment.	Height of girder at center.	Top flange.	Bottom flange at center.	At end.	Thickness of middle web.
12 ft.	1'·6"	1'·3"	3" × 1½"	1'·4" × 1½"	1'·8" × 1½"	1½"
30 ft.	2'·6"	3'·	5" × 2"	1'·6" × 2"	1'·10" × 2"	2"
45 ft.	2'·9"	3'·9"	7" × 2½"	2'· × 2½"	2'· × 2½"	2"

Rolled I-beams (Fig. 481) may be taken as the type in wrought-iron and steel sections. The depths of the beams, A, and the widths, B, of bottom and top flanges do not vary much with the different makers for the same class of beams, but the thickness of the stems varies more.

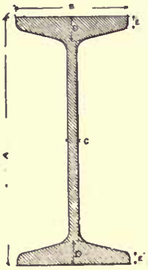


FIG. 481.

The tables of the Strength of Wrought-iron and Steel Beams (pages 243, 244) have been made up by comparison of the tables of different makers, the safeload is taken in units of 100 pounds, and 00 are to be added to the tabulated figures to give the safe distributed load in pounds.

It is assumed in these tables that proper provision is made for preventing the beam from deflecting sideways. They should be held in position at distances not exceeding twenty times the width of the flange; this is usually effected by the brick arches between the beams, or the wooden joists resting on them. The beams will support the loads as given in the tables, but the deflection may be too much for the purposes to be served. A line is drawn in each column in the tables, at which the deflection is $\frac{1}{360}$ beyond that due to the weight of the beams, or one inch for every thirty feet of span, beyond which the deflection is apt to crack the plastering of ceilings.

To find the sectional area of a beam, plate or rod from its weight, multiply weight per foot by 3 and divide by 10; and, conversely, to determine the weight multiply the sectional area by 10 and divide the product by 3.

Thus, if a steel bar 12 feet long weigh 480 pounds, or 40 pounds per foot, its sectional area will be $\frac{3 \times 40}{10}$, or 12 square inches; and a bar of 9 square inches section will weigh $\frac{9 \times 10}{3} = 30$ pounds per foot.

For naval constructions, *deck-beams* (Fig. 482) are from 3" to 12" deep, with varied widths of flanges and thicknesses of stem; lighter than the grades of heavy and light I-beams, but heavier can be rolled to order; bulb angles with terminal bulbs similar to those of deck-beams, on long legs of from 5" to 10", and short legs of 2½" to 3½" can be had of varied thicknesses. Properly proportioned, they are equal in strength to the I-beams.

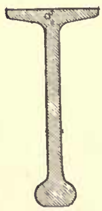


FIG. 482.

Coupled I-Beams.—When the load is beyond the strength of a single I-beam, two or more may be united, as shown in Fig. 483.

TABLE OF THE COMPARATIVE STRENGTH AND STIFFNESS OF DIFFERENT SECTIONS OF WROUGHT-IRON I-BEAMS.

Depth of Beam.		3		4			5		6		7		8		9		10		10½		12		15		
Width of Flange.	INCHES.	2·32	2·52	2·48	2·75	3·	2·75	3·	3·	3·46	3·5	3·91	4·	4·29	4·	5·375	4·32	4·77	4·54	5·	4·64	5·09	5·	5.4	5·81
Thickness of Stem.		0·19	0·39	0·23	0·25	0·313	0·25	0·313	0·25	0·46	0·35	0·53	0·3	0·79	0·26	0·6	0·32	0·77	0·41	0·47	0·51	0·96	0·5	0·7	0·93
Weight per foot.	LBS.	7·	9·	8·	10·	12⅛	10·	13½	13½	18·	18	25·	22·	35·	23¼	50·	30·	45·	31¼	45·	42·	60·	50·	67·	80·
Distance between supports, in feet.	5	33·	38·	50·	60·	73·	78·	97·	131·	152·	210·	248·	280·	362·											
	6	27·	31·	41·	50·	60·	65·	82·	109·	126·	175·	207·	233·	301·											
	7	24·	27·	35·	42·	52·	56·	69·	93·	108·	150·	177·	200·	258·											
	8	21·	24·	31·	37·	45·	49·	60·	82·	95·	131·	155·	175·	226·											
	9	18·	21·	28·	33·	40·	43·	53·	73·	84·	116·	138·	156·	201·											
	10	16·	19·	25·	29·	36·	39·	48·	65·	76·	105·	124·	140·	181·	174·	394·	240·	300·	251·	—	367·	454·	565·	772·	800·
	11	15·	17·	23·	26·	32·	36·	43·	59·	69·	95·	113·	127·	164·	158·	356·	218·	278·	228·	322·	334·	412·	513·	700·	727·
	12	14·	16·	21·	24·	29·	33·	40·	54·	63·	87·	103·	117·	151·	145·	328·	200·	250·	209·	295·	306·	378·	471·	643·	667·
	13			19·	22·	27·	30·	36·	50·	58·	81·	95·	108·	139·	134·	304·	185·	231·	193·	271·	282·	349·	434·	594·	615·
	14			18·	21·	25·	28·	33·	47·	54·	75·	89·	100·	129·	124·	282·	171·	214·	179·	251·	262·	324·	404·	551·	571·
	15			17·	20·	23·	26·	31·	44·	51·	70·	83·	93·	121·	116·	264·	160·	200·	167·	233·	245·	302·	377·	518·	533·
	16						25·	29·	41·	47·	66·	78·	88·	118·	109·	246·	150·	188·	157·	218·	230·	284·	353·	482	500·
	17						23·	27·	38·	45·	62·	73·	82·	106·	102·	232·	141·	176·	148·	204·	216·	267·	332·	453·	471·
	18						22·	25·	36·	42·	58·	69·	78·	100·	96·	218·	133·	167·	140·	192·	204·	252·	314·	429·	444·
	19								34·	40·	55·	65·	74·	95·	91·	206	126·	158·	132·	181·	193·	239·	297·	406·	421·
	20								33·	38·	52·	62·	70·	90·	87·	196·	120·	150·	126·	171·	184·	227·	282·	386·	400·
	21								31·	36·	50·	59·	67·	86·	83·	186·	114·	143·	120·	162·	175·	216·	269·	367·	381·
	22										48·	56·	64·	82·	79·	178·	109·	136·	114·	154·	167·	206·	257·	350·	364·
	23										46·	54·	61·	79·	75·	170·	104·	130·	109·	146·	160·	197·	245·	335·	348·
	24										44·	52·	58·	75·	72·	164·	100·	125·	105·	139·	153·	189·	235·	322·	333·
	25												56·	72·	69·	158·	96·	120·	100·	133·	147·	181·	226·	309·	320·
	26												54·	70·	67·	152·	92·	115·	97·	127·	141·	174·	217·	297·	308·
	27												52·	67·	64·	146·	89·	111·	93·	121·	136·	168·	209·	286·	296·
	28														62·	140·	86·	107·	90·	116·	131·	162·	202·	275·	286·
	29														60·	136·	83·	103·	87·	111·	126·	156·	195·	266·	276·
	30														58·	132·	80·	100·	84·	106·	122·	151·	188·	257·	267·
	31																77·	97·	81·	102·	118·	146·	182·	249·	258·
	32																75·	94·	79·	98·	114·	142·	177·	241·	250·
	33																73·	91·	76·	94·	111·	137·	171·	233·	242·
	34																		74·	91·	108·	133·	166·	225·	235·
	35																		72·	87·	105·	130·	161·	218·	229·
	36																				102·	126·	157·	211·	222·
	37																				99·	123·	153·	204·	216·
	38																						149·	197·	211·
	39																						145·		205·

COMPARATIVE STRENGTH AND STIFFNESS OF STEEL BEAMS.

Depth of Beam.		4		5		6		7		8		9		10		12		15		20		24	
Width of Flanges.	Inches.	2·63	2·75	3·00	3·13	3·50	3·63	4·00	4·25	4·25	4·50	4·50	4·75	4·75	5·00	5·25	5·50	5·50	5·75	6·25	7·00	6·95	7·20
Thickness of Stem.		·20	·24	·22	·26	·23	·26	·23	·27	·25	·27	·27	·31	·32	·37	·35	·39	·40	·45	·50	·60	·50	·75
Weight per foot.	Lbs.	8	10	10	13	13	16	16	20	18	22	21	27	26	33	32	40	41	50	64	80	80	100
Distance between supports, in feet.	5	63·	82·	106·	134·	167·	203·	235·	303·	308·	383·												
	6	52·	68·	88·	112·	139·	169·	196·	252·	257·	320·												
	7	45·	59·	76·	96·	119·	145·	168·	216·	220·	274·												
	8	39·	51·	66·	84·	104·	127·	147·	189·	193·	240·												
	9	35·	46·	59·	74·	93·	113·	131·	168·	171·	213·												
	10	31·	41·	53·	67·	84·	102·	118·	151·	154·	192·	200·	262·	264·	344·	395·	500·	604·	753·	1222·	1546·	1830·	1919·
	11	29·	37·	48·	61·	76·	92·	107·	138·	140·	174·	182·	238·	240·	313·	359·	455·	549·	685·	1111·	1405·	1664·	1799·
	12	26·	34·	44·	56·	70·	85·	98·	126·	128·	160·	167·	218·	220·	287·	329·	417·	503·	628·	1019·	1288·	1525·	1649·
	13	24·	32·	41·	52·	64·	78·	90·	117·	118·	147·	154·	202·	203·	265·	304·	385·	464·	580·	940·	1189·	1408·	1522·
	14	22·	29·	38·	48·	60·	73·	84·	108·	110·	137·	143·	187·	188·	246·	282·	357·	431·	538·	873·	1104·	1307·	1413·
	15	21·	27·	35·	45·	56·	68·	78·	101·	103·	128·	133·	175·	176·	229·	263·	333·	402·	502·	815·	1031·	1220·	1319·
	16	20·	26·	33·	42·	52·	64·	74·	95·	96·	120·	125·	164·	165·	215·	247·	313·	377·	471·	764·	966·	1144·	1237·
	17	19·	24·	31·	39·	49·	60·	69·	89·	91·	113·	118·	154·	155·	202·	232·	294·	355·	443·	719·	909·	1077·	1164·
	18	17·	23·	29·	37·	46·	56·	65·	84·	86·	107·	111·	146·	147·	191·	220·	278·	335·	419·	679·	859·	1017·	1100·
	19	17·	22·	28·	35·	44·	54·	62·	80·	81·	101·	105·	138·	139·	181·	208·	263·	318·	397·	643·	814·	963·	1041·
	20	16·	21·	26·	33·	42·	51·	59·	76·	77·	96·	100·	131·	132·	172·	198·	250·	302·	377·	611·	773·	915·	989·
	21					40·	48·	56·	72·	73·	91·	95·	125·	126·	164·	188·	238·	287·	359·	582·	736·	872·	942·
	22					38·	46·	53·	69·	70·	87·	91·	119·	120·	156·	180·	227·	274·	342·	556·	703·	832·	899·
	23					36·	44·	51·	66·	67·	83·	87·	114·	115·	150·	172·	217·	262·	328·	531·	672·	796·	860·
	24					35·	42·	49·	63·	64·	80·	83·	109·	110·	143·	165·	208·	251·	314·	509·	644·	763·	824·
	25					33·	41·	47·	61·	62·	77·	80·	105·	106·	138·	158·	200·	241·	301·	489·	618·	732·	791·
	26					32·	39·	45·	58·	59·	74·	77·	101·	101·	132·	152·	192·	232·	290·	470·	595·	704·	761·
	27					31·	38·	44·	56·	57·	71·	74·	97·	98·	127·	146·	185·	224·	279·	453·	573·	678·	733·
	28					30·	36·	42·	54·	55·	68·	71·	94·	94·	123·	141·	179·	216·	269·	437·	552·	654·	707·
	29					29·	35·	41·	52·	53·	66·	69·	90·	91·	119·	136·	172·	208·	260·	422·	533·	631·	682·
	30					28·	34·	39·	50·	51·	64·	67·	87·	88·	115·	132·	167·	201·	251·	407·	515·	610·	660·
	31											64·	85·	85·	111·	127·	161·	195·	243·	394·	499·	590·	638·
	32											62·	82·	82·	108·	124·	156·	189·	235·	382·	483·	572·	618·
	33											61·	79·	80·	104·	120·	152·	183·	228·	370·	468·	555·	600·
	34											59·	77·	78·	101·	116·	147·	178·	222·	360·	455·	538·	582·
	35											57·	75·	75·	98·	113·	143·	172·	215·	349·	442·	523·	565·
	36											56·	73·	73·	96·	110·	139·	168·	209·	340·	429·	508·	550·
	37											54·	71·	71·	93·	107·	135·	163·	204·	330·	418·	495·	535·
	38											53·	69·	69·	91·	104·	132·	159·	198·	322·	407·	482·	521·
	39											51·	67·	68·	88·	101·	128·	155·	193·	313·	396·	469·	507·
	40											50·	66·	66·	86·	99·	125·	151·	188·	306·	386·	458·	495·

A cast-iron block, or separator, is inserted between the beams, and two bolts, through them and the block, add lateral strength. The bolt-holes, placed at some distance from the centre of the span, do not reduce the transverse strength.

To strengthen an I-beam or box-girder composed of I-beams, rivet plates on the top and bottom flanges (Figs. 484 and 485), thus adding to the section by the area of the plate, less the rivet-holes. Box-girders, except of the larger sizes, are preferably composed of channel-beams (Fig. 486).

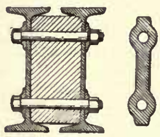


Fig. 483.

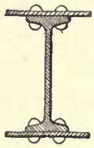


Fig. 484.

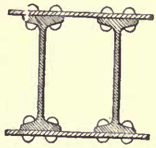


Fig. 485.

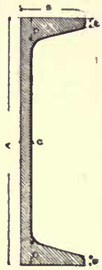


Fig. 486.

The rivet spacing at the ends of a box-girder not over 3″ at the middle 6″.

Channel-beams can be furnished of depths the same as I-beams, from three to fifteen inches, of varied grades of light and heavy.

COMPARATIVE STRENGTH AND STIFFNESS OF STEEL BOX GIRDERS.

Fig. 492.

Depth of Girder.	Inches.	10″		12″		15″		20″	
Weight of [s per foot.	Lbs.	15	24	20	30	32	51	64	80
Size of Plates.	Inches.	12 × ½		14 × ½		16 × ⅝		18 × ¾	
Distance between supports, in feet.	10	694·	758·	1004·	1096·	1836·	2052·	4030·	4500·
	11	631·	689·	912·	997·	1669·	1866·	3664·	4091·
	12	578·	631·	837·	913·	1530·	1710·	3359·	3750·
	13	534·	583·	772·	843·	1412·	1579·	3100·	3462·
	14	496·	541·	717·	783·	1311·	1466·	2879·	3214·
	15	463·	505·	669·	731·	1224·	1368·	2687·	3000·
	16	434·	473·	627·	685·	1147·	1283·	2519·	2813·
	17	408·	446·	591·	645·	1080·	1207·	2371·	2647·
	18	386·	421·	558·	609·	1020·	1140·	2239·	2500·
	19	365·	399·	528·	577·	966·	1080·	2121·	2368·
	20	347·	379·	502·	548·	918·	1026·	2015·	2250·
	21	330·	361·	478·	522·	874·	977·	1919·	2143·
	22	315·	344·	456·	498·	835·	933·	1832·	2045·
	23	302·	329·	436·	477·	798·	892·	1752·	1957·
	24	289·	316·	418·	457·	765·	855·	1679·	1875·
	25	278·	303·	402·	438·	734·	821·	1612·	1800·
	26	267·	291·	386·	422·	706·	789·	1550·	1731·
	27	257·	281·	372·	406·	680·	760·	1493·	1667·
	28	248·	271·	359·	391·	656·	733·	1439·	1607·
	29	239·	261·	346·	378·	633·	708·	1390·	1552·
	30	231·	253·	335·	365·	612·	684·	1343·	1500·
	31	224·	244·	324·	354·	592·	662·	1300·	1452·
	32	217·	237·	314·	343·	574·	641·	1259·	1406·
	33	210·	230·	304·	332·	556·	622·	1221·	1364·
	34	204·	223·	295·	322·	540·	604·	1185·	1324·
	35	198·	216·	287·	313·	525·	586·	1152·	1286·
	36	193·	210·	279·	304·	510·	570·	1120·	1250·
	37	188·	205·	271·	296·	496·	555·	1089·	1216·
	38	183·	199·	264·	288·	483·	540·	1061·	1184·
	39	178·	194·	257·	281·	471·	526·	1033·	1154·
	40	173·	189·	251·	274·	459·	513·	1008·	1125·

TABLE GIVING INCREASE OR DECREASE OF STRENGTH, IN PER CENT, ABOVE OR BELOW THAT GIVEN BY PRECEDING TABLE, FOR DIFFERENT THICKNESSES OF PLATES IN STEEL BOX GIRDERS.

Thickness of Plate.		⅜″	½″	⅝″	¾″	⅞″	1″
Width of Plates.	Size of Channels.						
12″	10″ [15#	−19	0	+20	+41		
12″	10″ [24#	−18	0	+18	+38		
14″	12″ [20#		0	+19	+40		
14″	12″ [30#		0	+18	+36		
16″	15″ [32#		−15	0	+15	+30	
16″	15″ [51#		−13	0	+13	+27	
18″	20″ [64#			−10	0	+10	+20
18″	20″ [80#			−9	0	+9	+20

It may be desirable, on account of position, to finish a box-girder as in Fig. 487; in this case the dimensions must be such as to admit of a helper inside to hold the rivets. Fig. 488 shows a closed box-beam made of channel-bars and plates. The lower channel is first riveted, and the upper one afterward. This

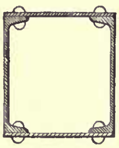


FIG. 487.

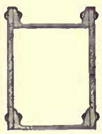


FIG. 488.

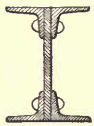


FIG. 489.


FIG. 490.

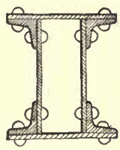


FIG. 491.

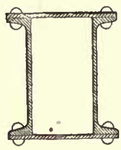


FIG. 492.

form gives a *clean* surface below, but the lower channel-bar can be reversed and riveted the same as the upper.

Where the purpose can be served by I-beams, either single, or coupled, as in Fig. 483, or in numbers, they afford the best and cheapest construction. But, where the spans are large and loads heavy, it is often economical to obtain greater depth by means of plate-girders, as in Figs. 489, 490, 491, 492, or perhaps from requirements of position, as in Fig. 493, subject as above to the necessities of large inside dimensions. These girders are made up of plates of uniform thickness, and angle-irons riveted together.

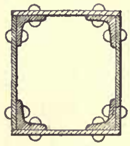


FIG. 493.

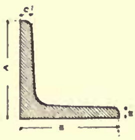


FIG. 494.

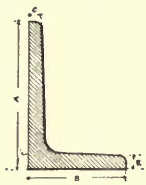
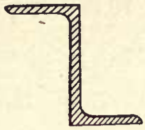


FIG. 495.

Angle-irons are made of varied dimensions, and are classed as *equal-legs* (Fig. 494), *unequal-legs* (Fig. 495), and *square-root angles* when the thickness of the iron is uniform throughout, and consequently the interior angle a complete right angle without rounding.

Angle irons are manufactured of equal legs, of ⅝″, ¾″, ⅞″, 1″, 1⅛″, 1¼″, 1½″, 1¾″, 2″, 2¼″, 2½″, 2¾″, 3″, 3½″, 4″, 5″, 6″, of varied thicknesses, according to the size and necessities of construction, from ⅛″ on the smaller sizes to ⅞″ on the larger ones.

Angles of unequal legs—7″ × 3½″, 6½″ × 4″, 6″ × 4″, 6″ × 3½″, 5″ × 4″, 5″ × 3½″, 5″ × 3″, 4½″ × 3″, 4″ × 3½″, 4″ × 3″, 3½″ × 3″, 3½″ × 2½″, 3½″ × 2″, 3″ × 2½″, 3″ × 2″, 2½″ × 2″, 2¼″ × 1½″, 2″ × 1⅜″, 1⅜″ × 1″, with the same variety of thickness as the equal legs, up to 1 inch for the largest size.



DIMENSIONS AND WEIGHTS OF Z BARS.

Thickness of metal in inches.	SIZE IN INCHES.			Weight per foot.	Thickness of metal in inches.	SIZE IN INCHES.			Weight per foot.
	Flange.	Web.	Flange.			Flange.	Web.	Flange.	
3/8	3 1/2	6	3 1/2	15·6	13/16	3 3/8	5 1/8	3 3/8	28·3
7/16	3 9/16	6 1/16	3 9/16	18·3	1/4	3 1/16	4	3 1/16	8·2
1/2	3 5/8	6 1/8	3 5/8	21·0	5/16	3 1/8	4 1/16	3 1/8	10·3
9/16	3 1/2	6	3 1/2	22·7	3/8	3 3/16	4 1/8	3 3/16	12·4
5/8	3 9/16	6 1/16	3 9/16	25·4	7/16	3 1/16	4	3 1/16	13·8
11/16	3 5/8	6 1/8	3 5/8	28·0	1/2	3 1/8	4 1/16	3 1/8	15·8
3/4	3 1/2	6	3 1/2	29·3	9/16	3 3/16	4 1/8	3 3/16	17·9
13/16	3 9/16	6 1/16	3 9/16	32·0	5/8	3 1/16	4	3 1/16	18·9
7/8	3 5/8	6 1/8	3 5/8	34·6	11/16	3 1/8	4 1/16	3 1/8	20·9
5/16	3 1/4	5	3 1/4	11·6	3/4	3 3/16	4 1/8	3 3/16	22·9
3/8	3 5/16	5 1/16	3 5/16	13·9	1/4	2 11/16	3	2 11/16	6·7
7/16	3 3/8	5 1/8	3 3/8	16·4	5/16	2 3/4	3 1/16	2 3/4	8·4
1/2	3 1/4	5	3 1/4	17·8	3/8	2 11/16	3	2 11/16	9·7
9/16	3 5/16	5 1/16	3 5/16	20·2	7/16	2 3/4	3 1/16	2 3/4	11·4
5/8	3 3/8	5 1/8	3 3/8	22·6	1/2	2 11/16	3	2 11/16	12·5
11/16	3 1/4	5	3 1/4	23·7	9/16	2 3/4	3 1/16	2 3/4	14·2
3/4	3 5/16	5 1/16	3 5/16	26·0					

T-irons (Fig. 496) may be used for top and bottom flanges in the manufacture of plate-girders by riveting a web on one side of the T, or on both sides, with a separator between of the thickness of the stem E; but, as the areas of section of T-irons to be had are small, the flanges will be too slight in proportion to the webs at depths above that of rolled beams. Angle-irons are then to be preferred for flanges. The T-irons are well adapted in many positions as struts or braces, and can be bought of varied dimensions and weights, from widths, B, of from 2 to 5 inches, and equal or less depths, A, and thicknesses from $\frac{3}{16}''$ to $\frac{5}{8}''$.

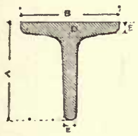


FIG. 496.

Rivets for plate-girders are usually from $\frac{3}{4}''$ to $\frac{7}{8}''$ diameter, and *pitched* or spaced not more than 6″ nor less than 3″ between centres. The number of rivets through flange and stem are the same, but alternating. Usually angle irons and plates can be had of the full length of girder, but, where joints are necessary, they should be butt, with a splicing-piece to make the strength as nearly as possible uniform. Stiffeners are often necessary for the webs, which may be of band, angle, or T-iron, and one should always be placed at each end, where the shearing stress is the greatest.

A common formula for determining the strength of a wrought-iron beam or girder is $W = \frac{8\,D\,(a + \frac{a'}{6})\,S}{L}$, in which W is the load in pounds, equally distributed on the beam, D the effective depth between the centres of gravity of the flanges, and L the clear span, both in the same unit, feet or inches; a the area of the top or bottom flange in square inches; a' the area of the stem.

To construct a diagram from the formula, in which the relation of the factors may be shown. Let S be 10,000 for riveted girders of wrought iron (12,000 for steel), then $W = \frac{D}{L} \times (a + \frac{a'}{6})$ 80,000. On a sheet of cross-section paper, from a corner, O, lay off on the line of ordinates, 5, 10, 15, 20, 25, representing the factor $a + \frac{a'}{6}$. From the same O, on the line of abscissas, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{15}$, $\frac{1}{20}$, $\frac{1}{25}$, $\frac{1}{30}$, $\frac{1}{35}$, $\frac{1}{40}$, representing $\frac{D}{L}$. Suppose $\frac{D}{L} = \frac{1}{40}$, then $W = (a + \frac{a'}{6})$ 2,000. If $a + \frac{a'}{6}$ be = 10, then W = 20,000. From the intersection of ordinate on line of $\frac{1}{40}$, and abscissa line of 10, draw a line to the point O. This line will represent the safe distributed load W, and its intersections of the

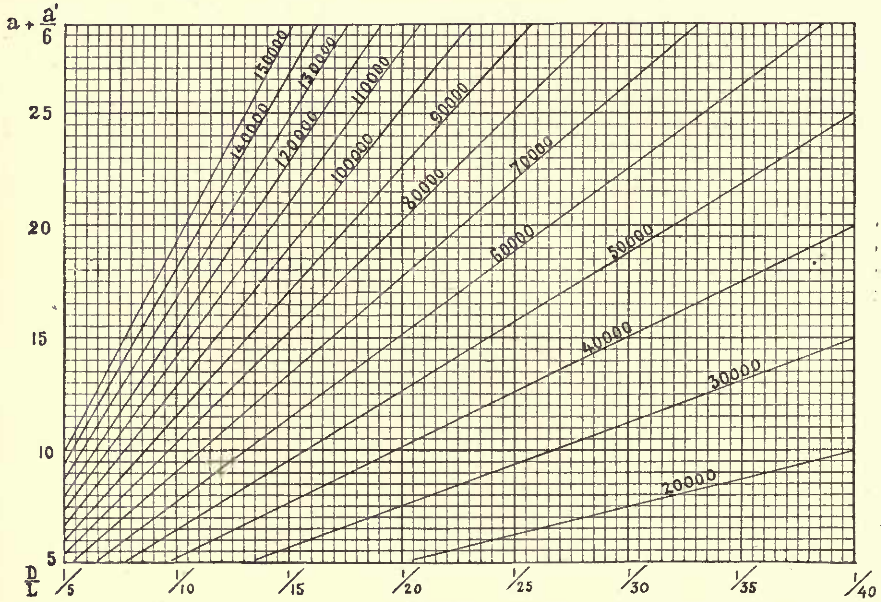


FIG. 497.

ordinates and abscissas will represent the relative proportions of the two factors $\frac{D}{L}$ and $a + \frac{a'}{6}$ under this load. On the abscissa line 15, and ordinate $\frac{1}{40}$, W = 30,000, on line 20, 40,000, and so on, and lines drawn from these intersections to O will represent W.

In the diagram lines below 5 and above 30 on line of ordinates are erased, as within these limits may be found most of the proportions required in practice.

Application of the Diagram.—What will be the area of section $a + \frac{a'}{6}$ of a girder, 40-foot span, depth 32″, distributed load 90,000 pounds?

D in the formula represents the distance between the centres of gravity of the flanges, which will be somewhat less than the depth of beam. Approximately assume it at 30″, $\frac{D}{L} = \frac{480''}{30''} \doteq \frac{1}{16}$, and the intersection of the line of load, 90,000, with the ordinate $\frac{1}{16}$, will be 18, on the line of $a + \frac{a'}{6}$. A fair proportion of a to a' is 5 to 6, therefore $\frac{5a'}{6} + \frac{a'}{6} = 18$ or $a' = 18$. $\frac{18''}{30} = 0{\cdot}6'' =$ thickness of web, and $a = \frac{5}{6}$ of $18 = 15$, or weight per foot of one flange $= \frac{15 \times 10}{3} = 50$ pounds, which is slightly in excess of the weight of two angle-irons $6 \times 4 \times \frac{3}{4}$, compensated by thickness of web outside centres of gravity.

This calculation is sufficiently near for all practical purposes, but D can be found more accurately by plotting the angle-irons, on thick card-board, cutting out, and then balancing for the centre of gravity.

Composite Beams.—Often, in constructions where the beams or girders are of wood, and on account of extent of spans and loads, the stress is beyond the strength and stiffness of beams of this material, of readily available dimensions, it is usual to supplement by some application of iron. A simple form, in which the iron is not exposed to view, is by bolting a plate or *flitch* of wrought-iron between two beams, of the full length and depth of the beams, and of such thickness as may be necessary. In bolting them together, let the bolt-holes be so bored that the weight of the beam may primarily be on the wood; the stress will then be better adjusted between the two materials when in service. It is usual to make the holes zigzag, in two lines about one quarter the depth of beam from each edge, the holes closer together nearer the ends. The safe-distributed load for the iron may be estimated from the formula: $W. = \frac{15000\, b\, h^2}{l}$, b breadth, h depth, l length—all in inches.

Fig. 498 represents a bracing truss of wrought-iron between two beams, which should be let into the wood. As it is held firmly laterally, the factor of

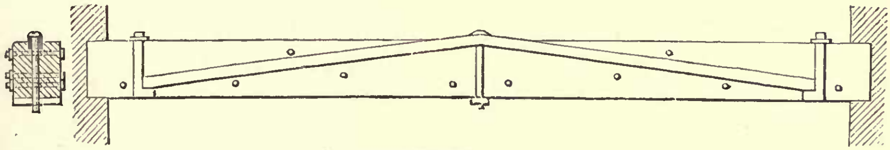


FIG. 498.

safety may be considered about one third of the crushing resistance of the material. The load on each inclined bar will be one half the load on the centre, multiplied by the length of the bar and divided by the rise. Instead of wrought-iron, cast-iron or wood is used.

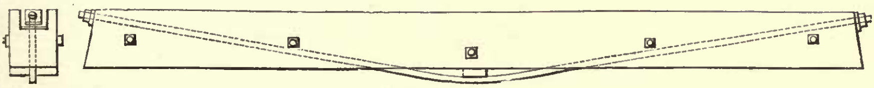


FIG. 499.

In Fig. 499 the beams are strengthened by a tension-rod, of which the strength may be determined by that of the material; allowing the usual factor of safety, the load is obtained as in the example above. The deeper the block beneath the centre of the beam, the less the stress on the rods for the same load. In construction, the beam should not be cambered by the screwing up of the rod; but, if the beams are crowning, the convex side should be placed upward, the nut turned by hand just to a bearing, and the tension put on by the settlement of the beams under the load.

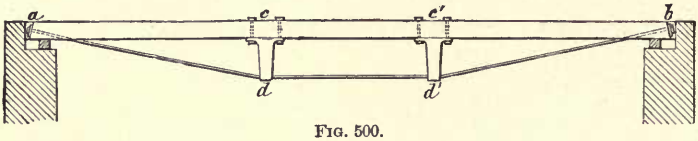


FIG. 500.

Fig. 500 represents the trussing of a beam by two struts and a tension-rod. The stress on the tension-rod is the load on *c*, multiplied by the length *a d*, divided by *c d*.

The theory of trusses will be treated and illustrated under "Bridges" and "Roofs," and the proportions of rivets and forms of plate-iron joints under "Boiler Construction."

Bolts and *nuts* are of such universal application that their manufacture forms the centre of large industries. Much thought has been given to their

FIG. 501.

proportions and the forms of thread, but without producing complete uniformity in the practice of different countries and makers. The old form of thread was the triangular pitch (Fig. 502), still used by some, especially when the threads are cut in a lathe. In this country the standard U. S. thread is

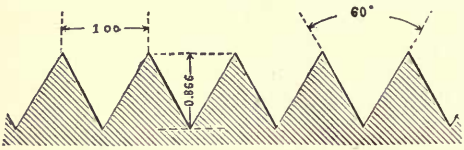


FIG. 502.

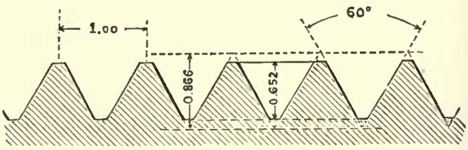


FIG. 503.

that recommended by the Franklin Institute in 1864 (Fig. 503). The angle is 60°, with straight sides and flat surface at top and bottom, equal to one eighth the *pitch*, or distance from centre to centre of threads.

In England, the standard thread for bolts and nuts is the Whitworth (Fig. 504); the angle is 55°, with top and bottom rounded.

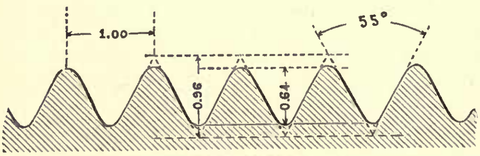


FIG. 504.

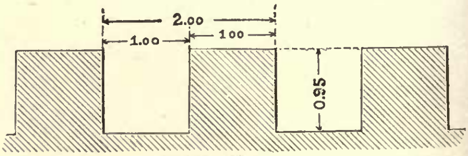


FIG. 505.

The square and rounded threads (Figs. 505 and 506) are only made to order and used in presses and the like as parts of machines.

Figs. 507, 508, and 509 represent the proportions of the various parts of English nuts to the diameters of bolts, as 1, or unity. Fig. 508 is a flange-nut, in which a washer-like flange is forged with the nut.

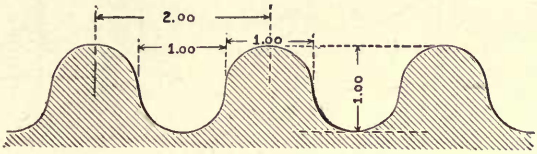


FIG. 506.

Fig. 510 is a cap-nut, in which the thread does not go through the nut, to prevent leaking along the thread, and a soft copper washer is introduced to prevent leakage below the nut.

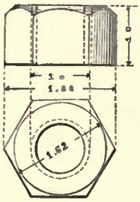


FIG. 507.

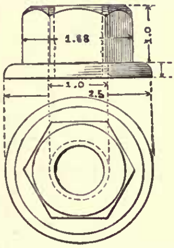


FIG. 508.

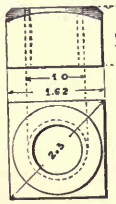


FIG. 509.

Figs. 511 and 512 are circular nuts, in one of which holes are drilled to insert a rod for turning, and in the other grooves for a *spanner*.

Lock-nuts (Fig. 513) are intended to prevent the gradual unscrewing of

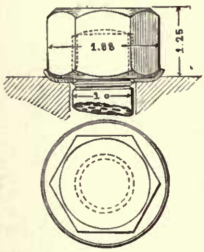


FIG. 510.

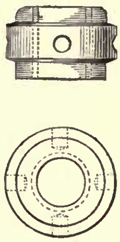


FIG. 511.

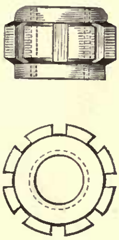


FIG. 512.

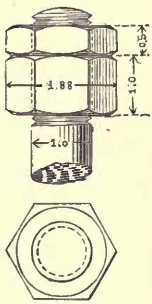


FIG. 513.

nuts subjected to vibration, which is to a great extent prevented by the use of double nuts, the lock-nut being one half the thickness of the common nut. The usual practice is as shown, the lock-nut being outside; the better way is inside.

The following figures are from trade circulars; the limits of sizes given are such as can usually be found in stock.

Figs. 514, 515, and 516 are machine-bolts, from $\frac{1}{4}''$ to $\frac{3}{4}''$ diameter, and 1″ to 4″ long, but not flanged, as in Fig. 514, unless expressly ordered; the dotted line shows the radius of curvature of a finished head. The diagonal lines beneath the head (Figs. 515 and 516) represent square bolts tapering into round bolts, as shown by the curved lines.

Figs. 517 and 518 are tap-bolts and set screws, from $\frac{1}{4}''$ to $\frac{3}{4}''$ diameter, and from $1''$ to $3''$ long.

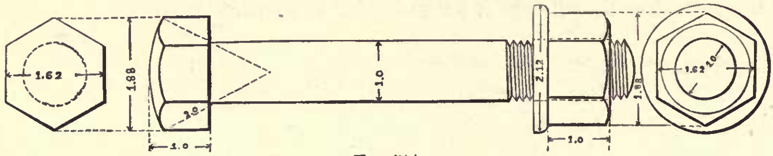


FIG. 514.

Fig. 519 is a carriage-bolt, from $\frac{1}{4}''$ to $\frac{3}{4}''$ diameter, and from $1''$ to $16''$ long.
Fig. 520 is a plough-bolt, from $\frac{3}{8}''$ to $\frac{1}{2}''$ diameter, and from $1''$ to $4''$ long.

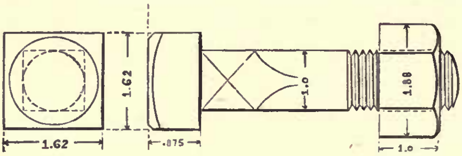


FIG. 515.

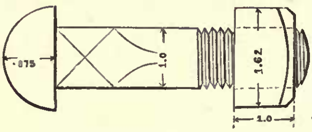


FIG. 516.

Fig. 521 is a stove-bolt, from $\frac{1}{4}''$ diameter and from $\frac{3}{4}''$ to $3''$ long.
Figs. 522 and 523 are machine-screws without nuts; the holes in the metals

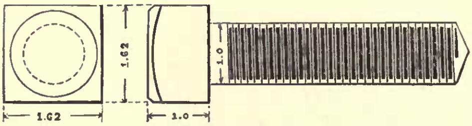


FIG. 517.

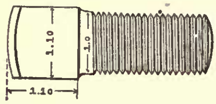


FIG. 518.

are tapped to receive them; Fig. 522 is button-headed; Fig. 523 a counter-sunk head—both slotted to admit of driving by a screw-driver. They are made of various-sized wire and lengths, and sold by the gross like the common wood-

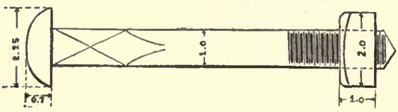


FIG. 519.

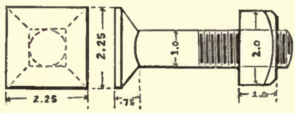


FIG. 520.

screw (Fig. 524). The wood-screw is for connecting pieces of wood together, or metal to wood. They are of very great variety, usually with a gimlet-point,

FIG. 521.

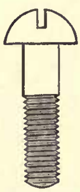


FIG. 522.

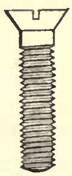


FIG. 523.

FIG. 524.

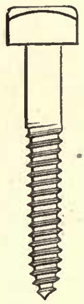


FIG. 525.

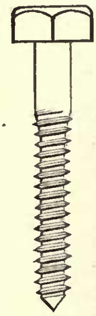


FIG. 526.

so that they can be screwed into the wood, without any holes being previously made. When made of rods, with a square or hexagonal head (Figs. 525 and 526) to admit of the use of a wrench, they are called *lag-screws*. It will be seen that wood-screws differ in their thread from bolts and machine-screws. The thread is a very sharp V, flatter on the upper surface, and the flat space

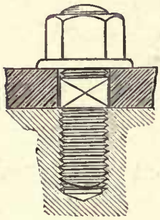


Fig. 527.

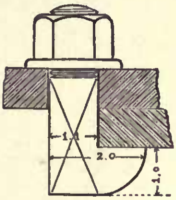


Fig. 528.

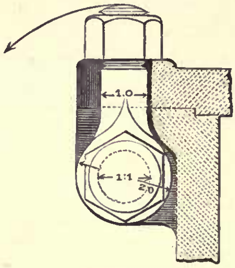


Fig. 529.

between the threads wide as the thread, making it of easier introduction into the wood, and retaining as much strength in the iron as in the wood.

Fig. 527 is a *stud*-bolt, which is screwed firmly into one of the pieces of connected metal; the other is bored so as to slip over the bolt, and the nut then brought down upon it. It is in common use for holding on the bonnets of steam-chests and water-chambers, the bolt remaining permanent.

Fig. 528 is a *hook*-bolt; it relieves the necessity of a bolt through the bottom-piece, and may be turned like a button, to loose or hold the bottom-plate.

Fig. 529 is another kind of button-bolt; the lower end can revolve on a stud or pin if the nut be raised enough to clear the cap or upper plate. By this arrangement there is no necessity of taking off the nut entirely; the bolt lies in a slot in the cap, and the nut bears on three sides.

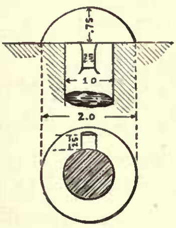


Fig. 530.

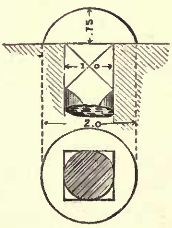


Fig. 531.

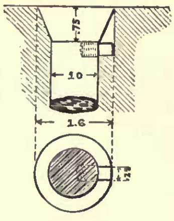


Fig. 532.

Figs. 530, 531, and 532 show expedients to prevent the bolt from turning when the nut is being screwed on or off.

Fig. 533 is an *anchor*-bolt flattened and jagged, introduced into a hole in masonry, and firmly leaded or sulphured in; but later experiments with deep holes have established that it was not necessary to flatten and jag the bolts, and with holes $1\frac{3}{8}''$ and $1\frac{5}{8}''$ diameter and $3'\ 6''$ deep, and bolts $1''$ diameter when leaded or sulphured in, did not develop as uniform adhesion as when the spaces were filled in with Portland cement and allowed a set of two weeks, when the bolts broke under the test. The resistance was 400 to 500 pounds per square inch of surface of bolt exposed.

It is very common to split the bolt at the bottom (Fig. 534) and insert a wedge in the cleft and drive it against the bottom of the bolts, forcing open the cleft and holding the bolt in the hole.

Fig. 535 is a bolt keyed into the stone, corresponding to a form of *lewis* for raising stones in which the key (Fig. 536) is wedged in, and when the stone is

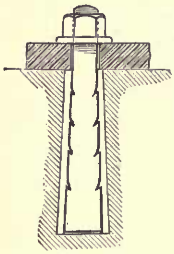


FIG. 533.

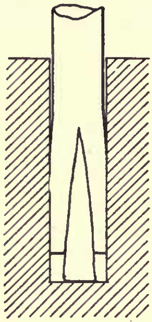


FIG. 534.

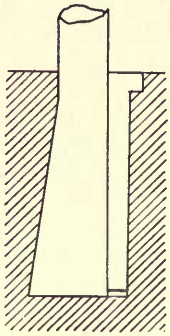


FIG. 535.

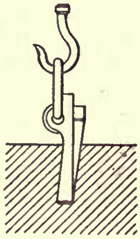


FIG. 536.

set the wedge is struck and the lewis withdrawn. Fig. 537 is a double or chain lewis.

For small holes in brick or stone masonry drill a hole and insert a short piece of lead pipe and force in a wood-screw of a little larger diameter than the bore of the pipe, or drive a wooden pin into the hole, and screw into the wood.

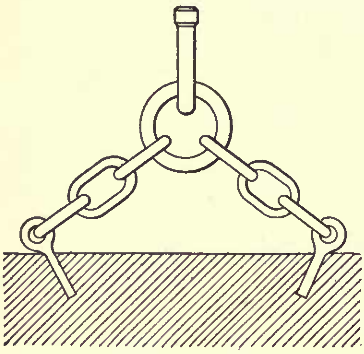


FIG. 537.

Take a clout-nail or picture-nail, turn up the point, cast a lead petticoat on it, and drive it into a loosely fitting hole; by driving, the point and the wedge form of the nail expands the lead and gives a firm hold. Similar forms of points to anchors of copper wire are convenient in holding stones together or face stones of a building to the backing.

Expansion-bolts (Fig. 538) from $\frac{1}{4}''$ to $1''$ diameter are on sale and very convenient in connecting architectural iron-work to masonry, and have this convenience: that they can be removed when necessary; all that is required is a hole of uniform diameter and of suitable size and depth to insert the bolt with nut and jaw; then by turning the head, the iron or composition nut is drawn outward, opening the wedge-shape jaws and causing them to bind strongly against the side. There is a small split-steel band around the jaws to keep them together before insertion in the hole and to free the jaws when they are to be removed. The bolts may have a head at the top, or a screw with a nut, or the lower nut may be formed as a head.

FIG. 538.

Fig. 539 is a bolt with a *fang*-nut or corner turned down and driven into the wood to prevent turning; the screwing to be done at the head.

It is often convenient to use bolts with two nuts, as in Fig. 540, or *collar*-bolts, which are readily made to order, and of any dimensions.

Fig. 541 is a *hanger*-bolt; the lag-screw part is screwed into the wooden beam, the hanger then put over the bolt, and the nut put on.

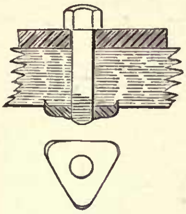


FIG. 539.

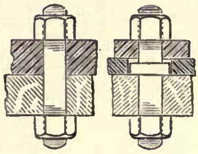


FIG. 540.

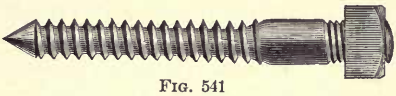


FIG. 541

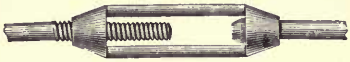


FIG. 542.

Figs. 542 and 543 represent forms of *turn-buckles*, and the *swivel* and *pipe*, sometimes designated as *swivels*. Turn-buckles are very useful in straining tierods, where neither end of the bolt can be got at. By turning the buckle, the rod can readily be made longer or shorter. In the pipe-swivel, *right* and *left* threads are cut on the bolts, so that each turn of the pipe shortens or lengthens the tie by double the pitch of the screw. The turn-buckle is also made in the same way, with two screws instead of a head at one end.

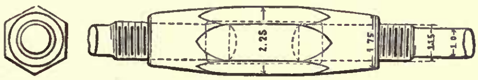


FIG. 543.

Screws, unless otherwise ordered, are made right-handed; that is, turning the nut to screw up, the hand moves from left to right, the apparent motion of the sun.

On the Strength of Bolts.—The strength of a bolt depends on its smallest section—that is, between the bottom of the threads. It is very common, there-

FIG. 544.

fore, to *upset* the screw-end, so that the screw may be cut entirely from this extra boss, or re-enforce. Bolt-ends (Fig. 544) are sold either with or without re-enforce, to be welded to bolts. It will be observed that the ends of the pipe-swivel bolts (Fig. 543) are thus upset.

In the following table the sizes and dimensions of bolts and nuts are from the United States standard, and the strength, or safe-load of the bolts, is computed from the report of the committee on the test of wrought-iron and chain-cables to the United States Government in 1879. Nuts and heads as furnished are either hexagonal or square. Columns 4, 5, and 6 apply equally to either.

There is often an uncertainty in the determination of the load. The effective load due to the forces acting on the machine may be estimated with tolerable accuracy. But that due to the forces used in tightening the nut is uncertain. If the bolt is screwed up so as to develop a reaction between the connected pieces, the additional load may be greater than the effective one.

Washers (Fig. 545)—in common use to provide seatings for nuts which would otherwise rest on rough metallic surfaces, and also to adapt bolts to

Diameter of screw in inches.	Diameter at root of thread.	Thread per inch of length.	Short diameter of nut and head.	Thickness of nut.	Thickness of head.	Safe-load of upset bolts.	Safe-load of plain bolts.
$\frac{1}{8}$			$\frac{5}{16}$	$\frac{1}{8}$	$\frac{5}{32}$		
$\frac{3}{16}$			$\frac{13}{32}$	$\frac{3}{16}$	$\frac{13}{64}$		
$\frac{1}{4}$	·185	20	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$		
$\frac{5}{16}$	·240	18	$\frac{19}{32}$	$\frac{5}{16}$	$\frac{19}{64}$		
$\frac{3}{8}$	·294	16	$\frac{11}{16}$	$\frac{3}{8}$	$\frac{11}{32}$		1,700
$\frac{7}{16}$	·344	14	$\frac{25}{32}$	$\frac{7}{16}$	$\frac{25}{64}$		1,900
$\frac{1}{2}$	·400	13	$\frac{7}{8}$	$\frac{1}{2}$	$\frac{7}{16}$		2,200
$\frac{9}{16}$	·494	12	$\frac{31}{32}$	$\frac{9}{16}$	$\frac{31}{64}$		2,500
$\frac{5}{8}$	·507	11	$1\frac{1}{16}$	$\frac{5}{8}$	$\frac{17}{32}$		2,800
$\frac{11}{16}$			$\frac{37}{32}$	$\frac{11}{16}$	$\frac{37}{64}$		3,200
$\frac{3}{4}$	·620	10	$1\frac{1}{4}$	$\frac{3}{4}$	$\frac{5}{8}$	6,000	3,600
$\frac{13}{16}$			$1\frac{11}{32}$	$\frac{13}{16}$	$\frac{43}{64}$	7,000	4,300
$\frac{7}{8}$	·731	9	$1\frac{7}{16}$	$\frac{7}{8}$	$\frac{23}{32}$	8,000	5,100
1	·837	8	$1\frac{5}{8}$	1	$\frac{13}{16}$	10,000	7,000
$1\frac{1}{8}$	·940	7	$1\frac{13}{16}$	$1\frac{1}{8}$	$\frac{29}{32}$	12,000	9,000
$1\frac{1}{4}$	1·065	7	2	$1\frac{1}{4}$	1	15,000	11,000
$1\frac{3}{8}$	1·160	6	$2\frac{3}{16}$	$1\frac{3}{8}$	$1\frac{3}{32}$	18,000	13,500
$1\frac{1}{2}$	1·284	6	$2\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{3}{16}$	21,000	16,000
$1\frac{5}{8}$	1·389	$5\frac{1}{2}$	$2\frac{9}{16}$	$1\frac{5}{8}$	$1\frac{9}{32}$	24,000	19,000
$1\frac{3}{4}$	1·490	5	$2\frac{3}{4}$	$1\frac{3}{4}$	$1\frac{3}{8}$	28,000	22,300
$1\frac{7}{8}$	1·615	5	$2\frac{15}{16}$	$1\frac{7}{8}$	$1\frac{15}{32}$	32,000	25,500
2	1·712	$4\frac{1}{2}$	$3\frac{1}{8}$	2	$1\frac{9}{16}$	36,000	29,300
$2\frac{1}{8}$			$3\frac{5}{16}$	$2\frac{1}{8}$	$1\frac{21}{32}$	40,000	33,000
$2\frac{1}{4}$	1·962	$4\frac{1}{2}$	$3\frac{1}{2}$	$2\frac{1}{4}$	$1\frac{3}{4}$	45,000	37,000
$2\frac{3}{8}$			$3\frac{11}{16}$	$2\frac{3}{8}$	$1\frac{27}{32}$	50,000	41,500
$2\frac{1}{2}$	2·175	4	$3\frac{7}{8}$	$2\frac{1}{2}$	$1\frac{15}{16}$	55,000	46,000
$2\frac{5}{8}$			$4\frac{1}{16}$	$2\frac{5}{8}$	$2\frac{1}{32}$		
$2\frac{3}{4}$	2·425	4	$4\frac{1}{4}$	$2\frac{3}{4}$	$2\frac{1}{8}$		
$2\frac{7}{8}$			$4\frac{7}{16}$	$2\frac{7}{8}$	$2\frac{7}{32}$		
3	2·629	$3\frac{1}{2}$	$4\frac{3}{8}$	3	$2\frac{3}{16}$		
$3\frac{1}{4}$	2·879	$3\frac{1}{2}$	5	$3\frac{1}{4}$	$2\frac{1}{2}$		
$3\frac{1}{2}$	3·100	$3\frac{1}{4}$	$5\frac{3}{8}$	$3\frac{1}{2}$	$2\frac{11}{16}$		
3	3·317	3	$5\frac{3}{4}$	$3\frac{3}{4}$	$2\frac{7}{8}$		
4	3·567	3	$6\frac{1}{8}$	4	$3\frac{1}{16}$		
$4\frac{1}{4}$	3·798	$2\frac{7}{8}$	$6\frac{1}{2}$	$4\frac{1}{4}$	$3\frac{1}{4}$		
$4\frac{1}{2}$	4·028	$2\frac{3}{4}$	$6\frac{7}{8}$	$4\frac{1}{2}$	$3\frac{7}{16}$		
$4\frac{3}{4}$	4·255	$2\frac{5}{8}$	$7\frac{1}{4}$	$4\frac{3}{4}$	$3\frac{5}{8}$		
5	4·480	$2\frac{1}{2}$	$7\frac{5}{8}$	5	$3\frac{13}{16}$		
$5\frac{1}{4}$	4·730	$2\frac{1}{2}$	8	$5\frac{1}{4}$	4		
$5\frac{1}{2}$	5·053	$2\frac{3}{8}$	$8\frac{3}{8}$	$5\frac{1}{2}$	$4\frac{3}{16}$		
$5\frac{3}{4}$	5·203	$2\frac{3}{8}$	$8\frac{3}{4}$	$5\frac{3}{4}$	$4\frac{3}{8}$		
6	5·423	$2\frac{1}{4}$	$9\frac{1}{8}$	6	$4\frac{9}{16}$		

shorter spaces than their lengths—are sold for bolts up to 2″ diameter. Circular in form, their diameter is slightly in excess of that of the largest diameter of the nut, and the hole that of the bolt, and thickness from $\frac{1}{20}$″ to $\frac{1}{8}$″, according to the diameter of the bolt.

Lock-nut Washers.—Nuts subject to jars are apt to unscrew of themselves; to prevent which many expedients are adopted, as cupping the washer or turning up one corner of a square washer against a face of the nut. The simplest is Shaw's Lock-nut, in which one side of a circular washer is cut through, one edge pressed up and the other down, to form a spring by which a pressure is brought on the thread when the nut is screwed home. In the Billing's lock-nut by cupping a circular washer a similar effect is obtained through the elasticity of the cup.

The square washer is used under both head and nut on surfaces of wood,

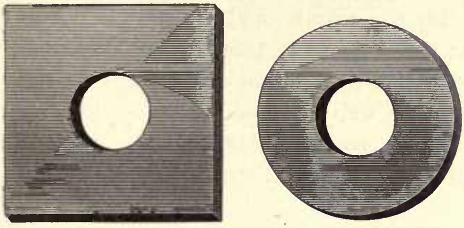


Fig. 545.

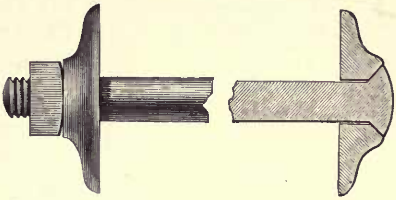


Fig. 546.

and of dimensions suited to the stress. That they may neither sink into the wood nor bend or break, cast-iron is frequently used, and often, as shown in Fig. 546, for roof-frames.

Shafts and Axles.—Short shafts, revolving in bearings or boxes, or fastened with pulleys, drum, or wheels revolving on them, are called axles; but long or heavy revolving bars are usually termed shafts. They may be independent; that is, a single shaft, revolving in its bearings, or coupled, forming what is termed a *line of shafting.* The small shafts, as in clock-work and spinning-machinery, are termed *pins* and *spindles.*

Shafts and axles are made of wood and metal, and of varied sections and form.

Wooden shafts are polygonal, circular, or square section (Fig. 547).

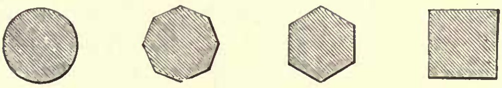


Fig. 547.

Wrought metal, iron, or steel shafts, are almost invariably circular in section, but sometimes square.

Cast-iron is used in great variety of section and form for shafts (Fig. 548); without uniformity longitudinally, but adapted to their position and load.

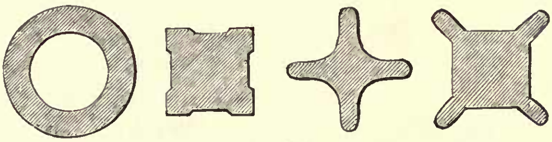


Fig. 548.

Formerly, either wood or cast-iron was invariably used for water-wheel shafts; but a change of motors, from the breast, over-shot and under-shot wheels to reactors or turbines, has involved an entire change of construction, and now only wrought-iron is used. Wooden shafts are often used in machines subject to wet and shocks, or from the greater convenience in obtaining the material, and from this last necessity the journals and boxes are sometimes of wood; but for wooden shafts it is the usual practice to insert cast-iron journals with boxes or bearings of the same material.

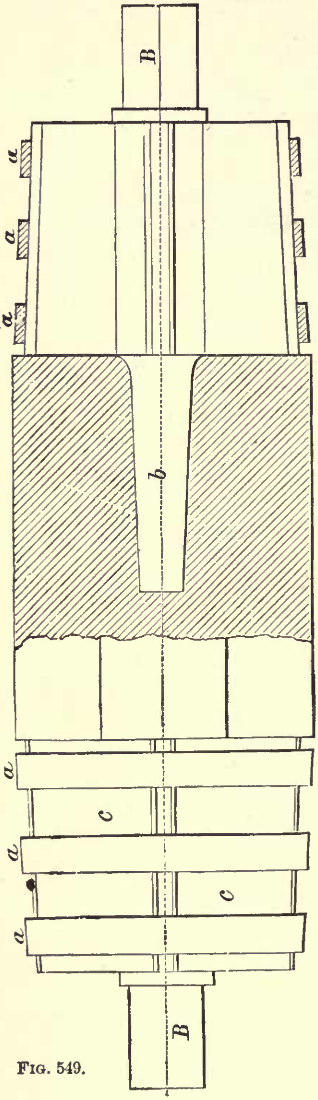


FIG. 549.

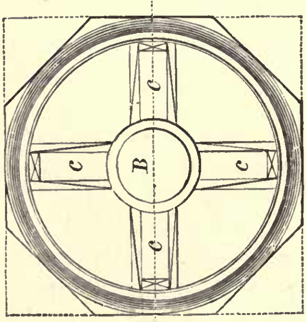


FIG. 550.

Fig. 549 is a side view of a wooden shaft with one end in section, and Fig. 550 an end view of the shaft. On the journal B is cast four wings, *c c*, and a small spindle, *b*. The ends of the shaft are bored for the spindle and grooved to receive the wings; the casting is then drawn into place, hooped with hot ferules, *a a*, and after this hard wood wedges are driven on each side of the wings and iron spikes are sometimes driven into the end of the wood; most millwrights omit the spindle *b*.

Figs. 551, 552, and 553 represent different views of a cast-iron shaft of a water-wheel. Fig. 551 is an elevation of the shaft, with one half in section to show the form of the core; Fig. 552, an end elevation; Fig. 553, a section on the line *c c* across the centre. The body is cylindrical and hollow, and cast with four feathers, *c c*, disposed at right angles to each other, and near the extremities of these feathers four projections, for the attachment of the bosses of the water-wheel or pulley. These projections are made with facets, so as to form the corners of a circumscribing square (Fig. 552), and are planed to receive the keys by which they are fixed to the naves which are grooved to receive them. The shaft is cast in one entire piece, the journals turned, and the feathers of an external parabolic outline to stiffen the shaft.

Shafts like the examples given are for purposes where their loads are nearly constant and for moderate speeds, and cast-iron gives satisfactory results.

The usual length of such journals is from one to one half times the diameters, and the safe load 500 pounds to the square inch, taking the area as d^2, the square of the diameter, the diameter and length of journal being considered equal.

To determine the size of a shaft, considered as a beam merely, but with a shifting load—as by the revolution of the shaft—each longitudinal line of surface has to undergo successively tension and compression. The safe load of wrought-iron is estimated at 6,000 pounds per square inch, and the formula on which the graphic diagram (Fig. 554) is constructed is $d = \cdot 06 \sqrt[3]{w\,l}$, d being diameter, l = length between bearings, both in inches, w the load in pounds; the load is not only the weight of shaft and pulleys or gears, but also the stress in transmitting the power.

Use of Diagram.—Suppose $w = 50{,}000$ pounds, and $l = 6$ feet $= 72''$, then $wl = 3{,}600{,}000$, the ordinate of 3·6 cuts the curve on the abscissa 9·2, which is the required diameter of the shaft in inches.

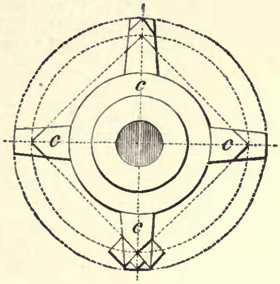


FIG. 552.

Fly-wheel and crank shafts are of forged iron or steel, often forged in steps (Fig. 555) with the largest boss beneath the wheel hub, and sufficiently raised above the next to admit of the planing of the key seats.

The transverse stress upon the shaft, due to the transmission of power, is equal to the H. P. divided by the velocity of surface, whether of belt or of gear, by which it is transmitted, and the same acts by torsion through the leverage of the radius of the pulley or gear. This stress is seldom calculated, as it is sufficiently met by the tables and diagrams for the determination of the diameters of shafts.

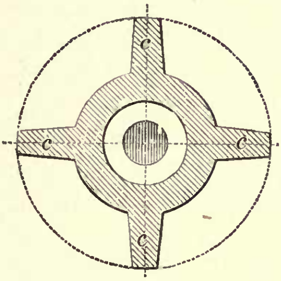


FIG. 553.

Keys are pieces of metal, usually steel, employed to secure the hubs of pulleys, gears, and couplings to shafts. They may be *sunk* keys (Fig. 556), *flat* keys (Fig. 557), and *hollow* keys (Fig. 558). The shaded circle represents the shaft. The breadth of the key (Fig. 559) is uniform, but the thickness is tapered about one eighth of an inch per foot. The shoulder h is for the purpose of drawing out the key. Sunk keys are not necessarily taper. Some prefer them of uniform section, and to force the hub on over the key.

It is good practise in fitting keys that they shall always bind tight sideways, but not necessarily touch either at the bottom of the key-seat or the top of the slot cut in the hub. Such keys depend upon a forcing fit of the wheel upon the shaft so tight as to require screw-pressure to put the wheel in place upon the shaft.

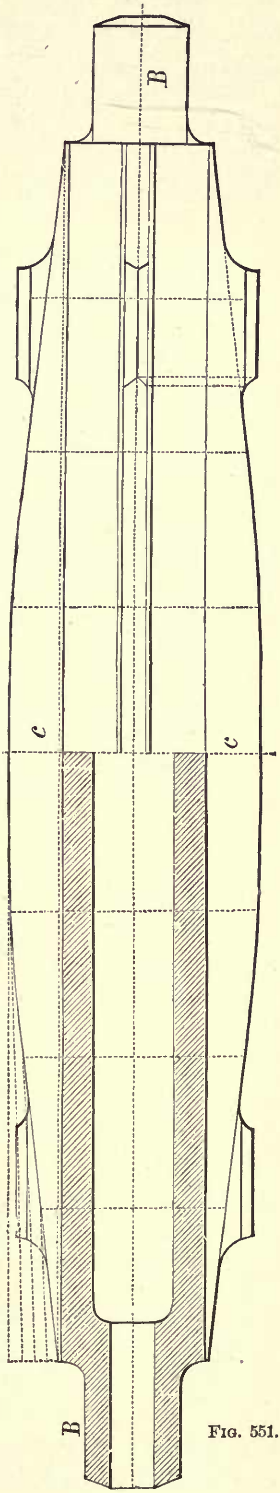


FIG. 551.

PROPORTIONS OF SUNK KEYS.

DIAMETER OF SHAFT, IN INCHES.	1	2	3	4	5	6	7	8	9	10	11	12
Breadth of key.......	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{7}{8}$	$1\frac{1}{8}$	$1\frac{3}{8}$	$1\frac{5}{8}$	$1\frac{7}{8}$	$2\frac{1}{8}$	$2\frac{3}{8}$	$2\frac{5}{8}$	$2\frac{7}{8}$	$3\frac{1}{8}$
Thickness of key.....	·25	·34	·43	·52	·61	·71	·80	·89	·98	1·07	1·16	1·25
Depth sunk in shaft..	·10	·125	·15	·175	·20	·225	·25	·275	·30	·325	·35	·375
Depth sunk in wheel.	·15	·215	·28	·345	·41	·485	·55	·615	·68	·745	·81	·875

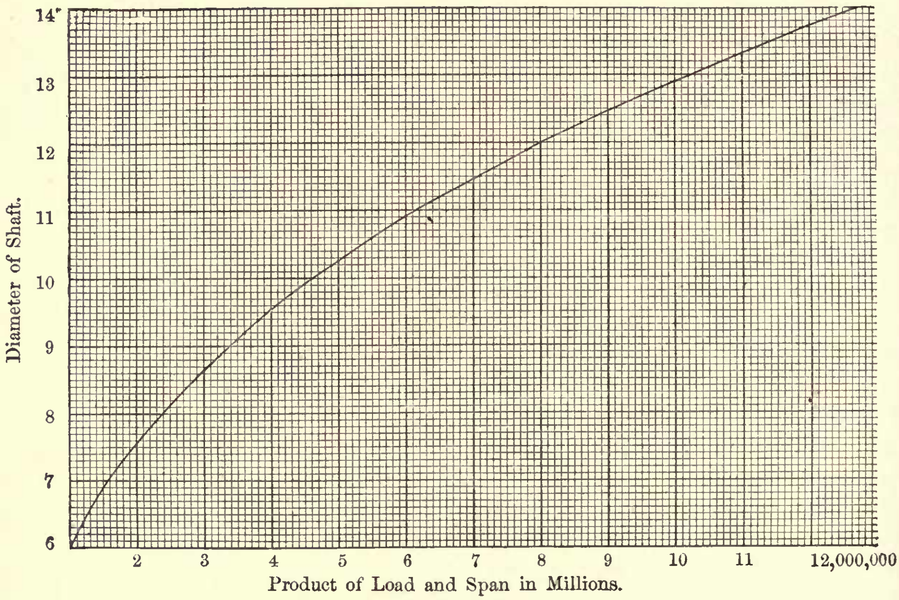


FIG. 554.

Car-Axles.—Fig. 560 is the form and dimensions of axle adopted as standard by the American Master Car-Builders' Association for wrought-iron and steel.

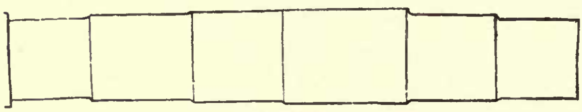


FIG. 555.

Shafting.—Thus far, independent shafts or axles have been treated of, and the dimensions have been established mostly by the load acting transversely;

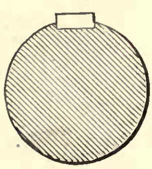


FIG. 556.

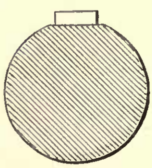


FIG. 557.

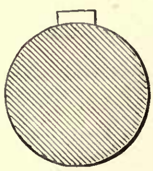


FIG. 558.

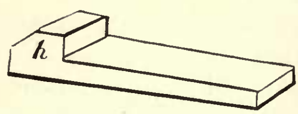
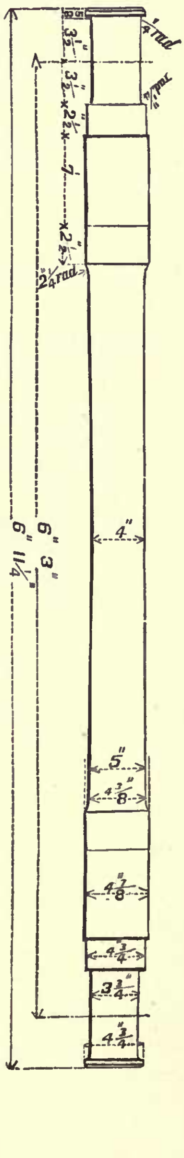


FIG. 559.

but, in transferring power to machines, lines of shafting are necessary, almost invariably of wrought-iron or steel bars, which are subject not only to transverse but also torsional stress. When there are no pulleys or gears on the shafts between the bearings, and the couplings are close to the bearings, there is still an amount of deflection due to the weight of the shaft. James B. Francis, C. E., puts the maximum distances between bearings for shafts of wrought-iron or steel, under these conditions, as follows:

Diameter of shaft.	Distance between bearings.	Diameter of shaft.	Distance between bearings.	Diameter of shaft.	Distance between bearings.
1″	12 ft.	5″	21 ft.	9″	26 ft.
2	15	6	22	10	27
3	18	7	24	11	28
4	20	8	25	12	28

Fig. 560.



The diagram (Fig. 561) is one established by J. T. Henthorn, M. E., to determine the size of wrought-iron shafting, to transmit a fixed amount of horse-power.

Use of Table.—To find the size of a shaft making 150 revolutions, and transmitting 350 horse-power.

The intersection of the ordinate of 350 with the abscissa of 150 is between the diagonals 5 and 5¼, and the diameter of the shaft may be taken safely at 5¼″.

Mr. Francis has constructed a table from his own experiments, of which the following is a synopsis:

"The following table gives the power which can be safely carried by shafts making 100 revolutions per minute. The power which can be carried by the same shafts at any other velocity may be found by the following simple rule:

"*Multiply the power given in the table by the number of revolutions made by the shaft per minute; divide the product by 100; the quotient will be the power which can be safely carried.*"

The diagram and table given are applicable to shafts which are called second movers, subject to no sudden shock. For first movers, Mr. Francis takes but one half the horse-power given in the table for any diameter of shafts. Of late, cold-rolled shafts can be procured in the market, which are much stiffer than turned shafts, but not equal to that given for steel in the table.

It is usual to make the shafts of second and third movers throughout manufactories and shops of uniform diameter, without reduction at the journals, the end-slip being prevented by collars keyed or fastened by set-screws. The usual length between bearings is from 7 to 10 feet; but that they may run smooth, and not spring intermediately, it is desirable that they should never be less than 2 inches diameter, and that the pulleys or gears through which the power is transmitted to the next mover or to the machine should be as near as possible to the bearing.

Horse-power which can be safely transmitted by shafts making 100 revolutions per minute, in which the transverse strain, if any, need not be considered ; if of

Diameter in inches.	Wrought-iron.	Steel.	Diameter in inches.	Wrought-iron.	Steel.	Diameter in inches.	Wrought-iron.	Steel.
1·	2·0	3·2	4·5	182·	291·	7·5	843·	1350·
1·5	6·7	10·7	5·	250·	400·	8·	1024·	1638·
2·	16·0	25·6	5·5	332·	532·	8·5	1228·	1965·
2·5	31·2	50·	6·	432·	691·	9·	1458·	2332·
3·	54·0	86·4	6·5	549·	878·	9·5	1714·	2743·
3·5	85·7	137·	7·	686·	1097·	10·	2000·	3200·
4·	128·	204·						

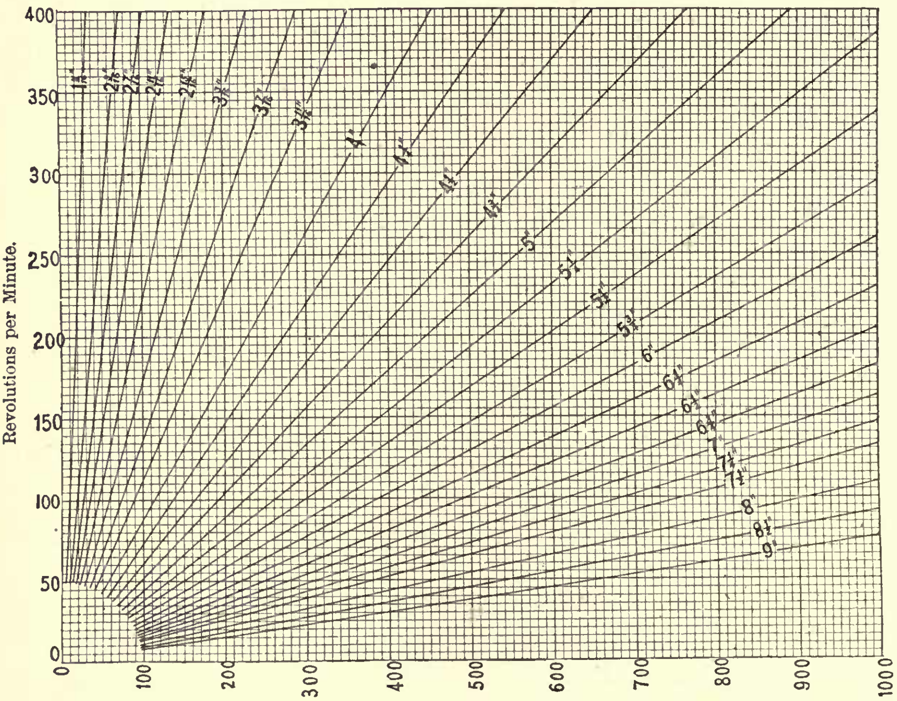


Fig. 561.

Fig. 562 represents a line of shafting. A is an upright shaft; *a a*, bevel-gears; *b b*, bearings for the shafts; *c*, coupling or connection of the several pieces of shafting. These shafts are of wrought-iron or steel, of uniform section. As the power is distributed from this line of shafting, the torsional strain diminishes with the distance from the bevel-gears or first movers, and the diameter of each piece of shafting may be reduced consecutively, if necessary; but uniformity will generally be found to be of more importance than a small saving of material. The drawing given is of a scale large enough to order shafting by, but the dimensions should be written in.

In laying out lines of shafting, the position of the bearings is usually fixed, and the lengths of shafts must be determined thereby, with as few couplings as possible. When there is no necking or reduction of the shafts, which is usually the case, the orders given for shafting will be so many lengths and of such diameters, and so many couplings and hangers. When there is to be a necking, the sketch for the order may be very simple, showing length and diameter of shaft, and position, length, and diameter of bearing.

The couplings and pulleys are to be placed as near the bearings as possible. It frequently happens, therefore, that the coupling and pulley are needed at

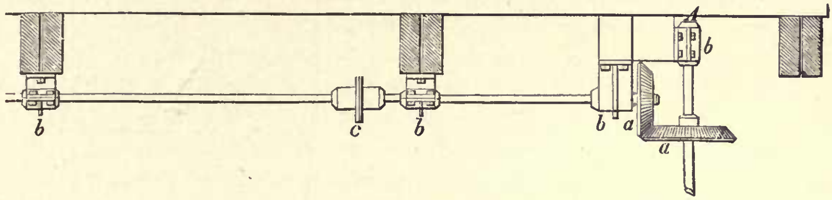


FIG. 562.

the same point; to remedy this, as the position of the pulley depends on the machine which it is required to drive, it frequently can not be moved without considerable inconvenience or loss of room; the shaft will have, therefore, to be lengthened or shortened, to change position of coupling; or, if the couplings are plate couplings, they may be made with faces for belts.

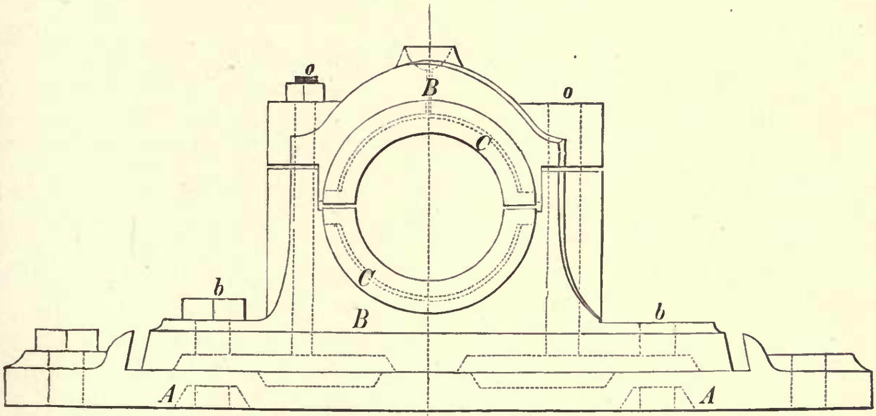


FIG. 563.

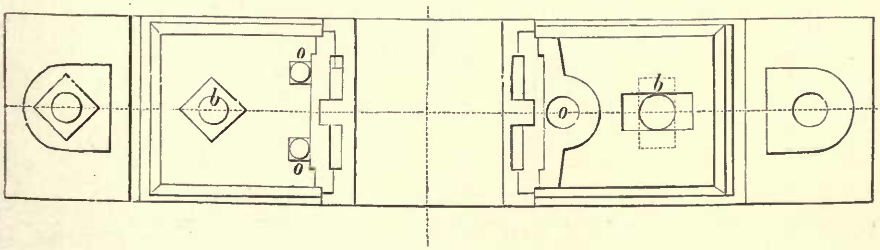


FIG. 564.

When a horizontal shaft is supported from beneath, its bearing is usually called a *pillow-* or *plumber-block*, or *standard;* if suspended, the supports are called *hangers*.

Figs. 563 and 564 are the elevation and plan of a pillow-block. It consists of a base plate, A, the body of the block B, and the box C. The plate is bolted

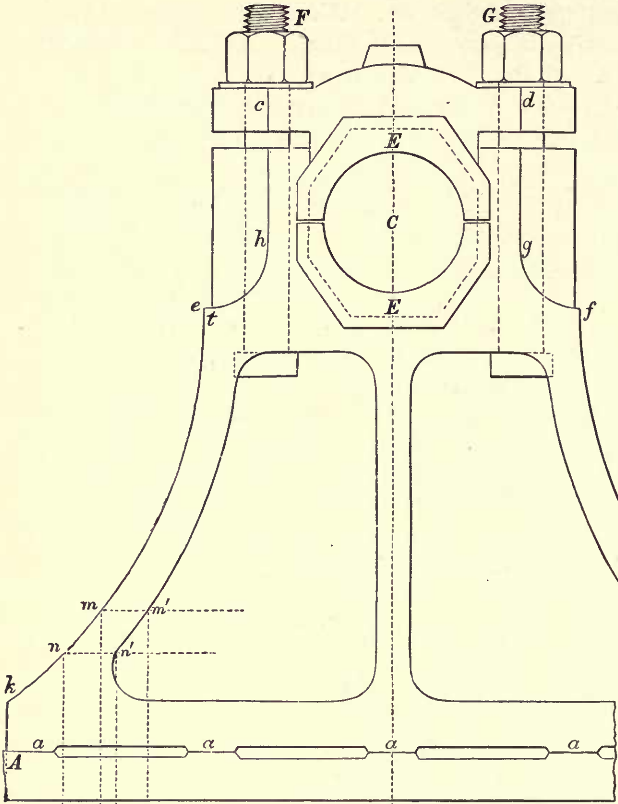


Fig. 565.

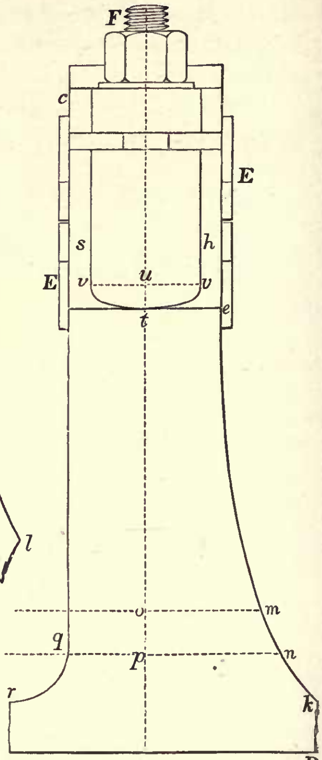


Fig. 567.

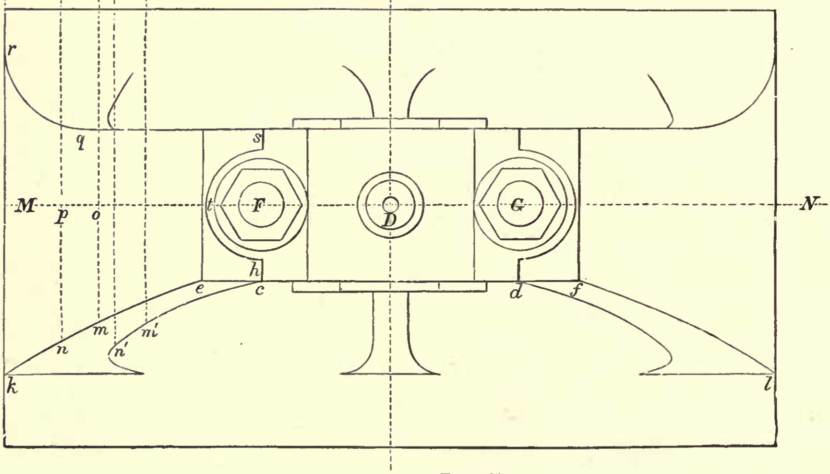


Fig. 566.

securely to its base, the surface on which the block B rests being horizontal. A and B are connected by bolts passing through oblong holes to adjust the position in either direction laterally. The box or bush C is of composition, in two parts or halves, extending through the block, and forming a collar by which it is retained in its place. The cap of the block is retained by the screws *o o o;* in the figure there are two screws on one side and one on the other; often four are used, two on each side, but most frequently but one on each side.

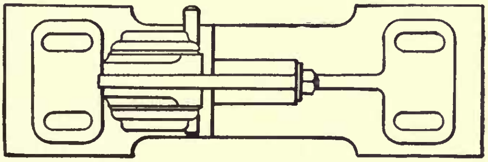
The *standard* is for the support of horizontal shafts at a considerable distance above the foundation-plate. Fig. 565 is a front elevation; Fig. 566, a plan; and Fig. 567, an end elevation of a standard. Like the pillow-block, the plate A is fastened to the foundation itself, and the upper surface is placed perfectly level in both directions. On these bearing surfaces, *a a a*, the body of the standard rests, and can be adjusted in position horizontally, and then clamped by screws to the foundation-plate, or keyed at the ends.

Elevations and plan are usually drawn in such positions to each other that lines of construction can be continued from one to the other, which not only simplifies the drawings, but makes them more readily intelligible. Letters and dotted lines in these figures illustrate this sufficiently.

The sides of the elevations are represented as broken; this is often done in drawing, when the sides are uniform, and economy of space on the paper is required.

Hangers.—Figs. 568, 569, and 570 are the plan side and front elevation of a side hanger especially adapted to a position in which the strain is in one direction and against the upright part.

Fig. 568.



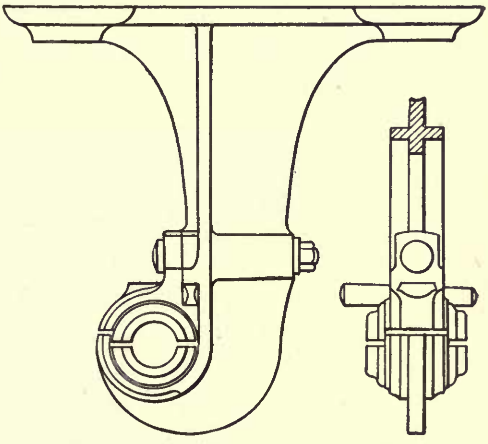


Fig. 569. Fig. 570.

Figs. 571, 572, and 573 are side elevation, plan, and section on line A B of a centre hanger of an old pattern, but simple, adapted to any strain, and if adjusted to a position where the shaft is not likely to be moved, the form is strong and economical.

Fig. 574 is of a later pattern, in which the shaft can be readily adjusted or removed.

Hangers are bolted to the floor-timbers, or to strips placed to sustain them, the centres of the boxes being placed accurately in line, both horizontally and laterally.

Figs. 575–578 represent different views of what may be called a *yoke-hanger*. A is the plate which is fastened to the beam, E is the yoke, and B the stem of the yoke, cut with a thread so as to admit of a vertical adjustment; the box D of the shaft C is supported by two pointed set-screws passing through the jaws of the yoke; this affords a very flexible bearing, and a chance for lateral adjustment.

The last hangers are of the design of William Sellers & Co., who have made improvements in the designs for bearings, pillow-blocks, hangers, shafting, and couplings, which are in use in their own shops and have been extensively copied by others. Some of the distinctive forms are further illustrated.

Fig. 571.

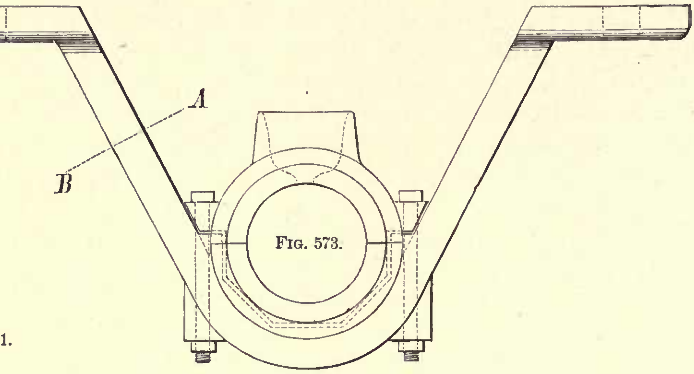


Fig. 573.

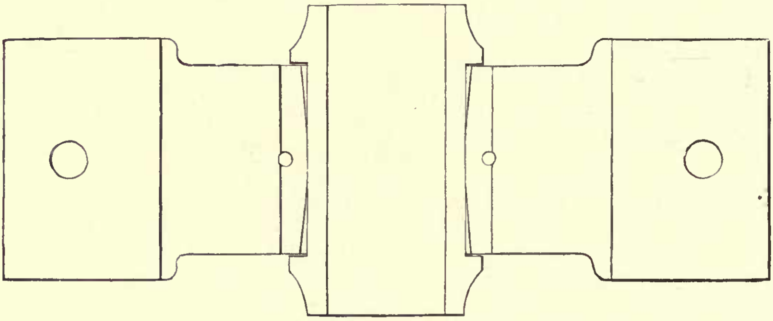


Fig. 572.

Figs. 579 and 580 are the front and side elevations of a pillow-block, one half of each being in section. The length of the box is about four diameters

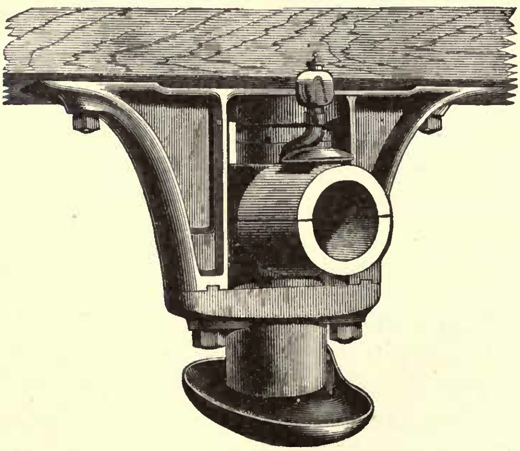


Fig. 574.

of the shaft. The centre bearings are spherical and fit in corresponding recesses in the block and cap, permitting the bearing to adjust itself to the journal of the shaft. Lubrication is ordinarily through the centre of the cap; but the upper box has two cups containing a mixture of oil and tallow which is usually

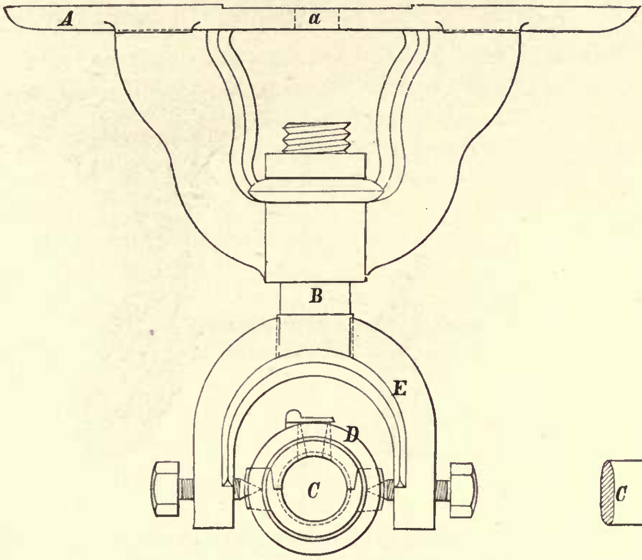


FIG. 575.

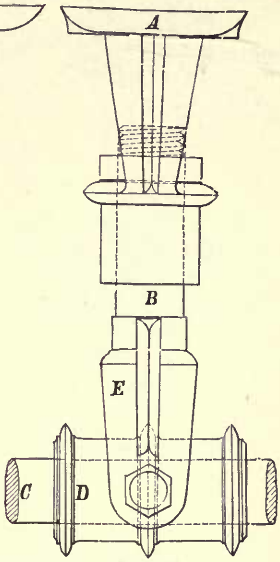


FIG. 576.

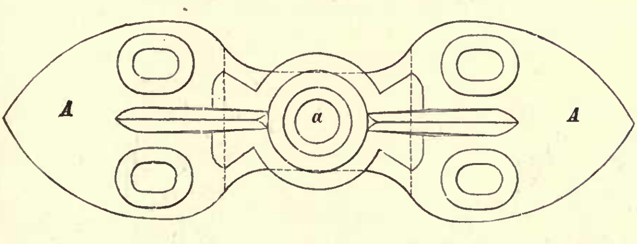


FIG. 577.

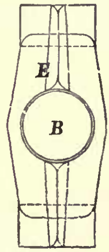


FIG. 578.

solid but melts when the bearing heats. The maximum pressure allowed is 50 pounds per square inch, the diameter multiplied by the length of bearing or 4 D.

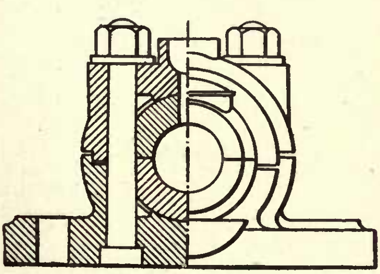


FIG. 579.

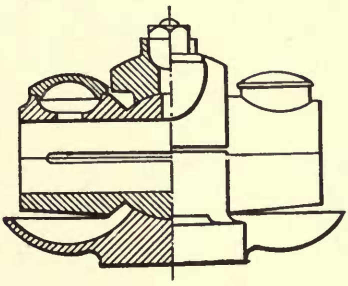


FIG. 580.

Another feature of the Sellers bearings is the spherical-shaped drip-pan to catch the waste oil. By the distribution of the oil the metal of the shaft will not touch that of the box. Any metal can be used for the box; cast-iron is the cheapest and best if the surface is kept oiled, but the poorest if allowed to run dry. The oil cup at the centre of the cap for a shaft $2\frac{1}{2}''$ in diameter making 120 revolutions per

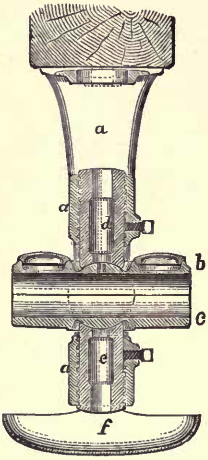


Fig. 581.

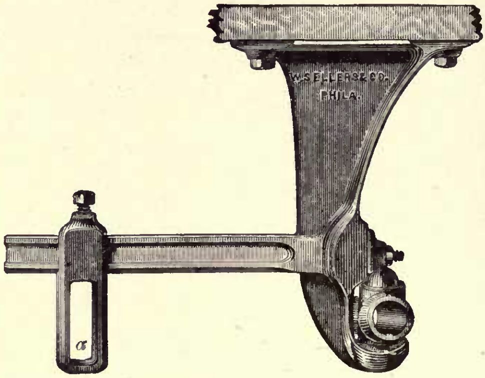


Fig. 582.

minute has a capacity of 2·2 fluid ounces, which is sufficient for six months' run. The above revolutions per minute are Messrs. Sellers & Co.'s practice for machine shops, for wood-working machinery 250, for that of cotton and woollen mills 300 to 400 per minute.

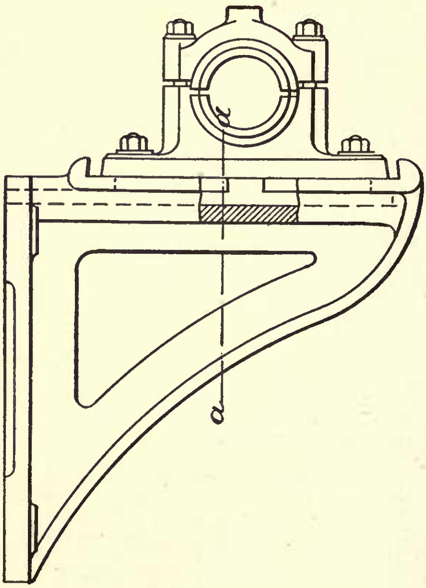


Fig. 583.

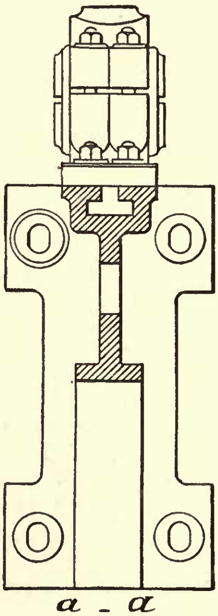


Fig. 584.

Fig. 581 is a ball-and-socket hanger the construction of which is similar to the pillow-block. The centre spherical bearings are adjusted vertically by the screws *d* and *e*, the interior of which are made in hexagonal form and into which a key is fitted for the operation of the screw.

Fig. 582 is a view of a side hanger adapted to a counter-shaft; the square slot *a* is for the shipping bar.

Fig. 583 represents the elevation of a bracket, or the support of a shaft bolted to an upright; the box is movable, and is adjusted laterally by the set-screws. Fig. 584 is a front elevation of the back-plate cast on the post; it will be seen that the holes are oblong, to admit of the vertical adjustment of the bracket. The Sellers pillow-block (Figs. 579 and 580) may be used for the same purpose.

For Upright Shafts.—Footstep, or Step, for an Upright Shaft.—Fig. 585 represents a half elevation and section of the step. It consists of a foundation or bed-plate, A, a box, B, and a cup or socket, C. The plate A is firmly fast-

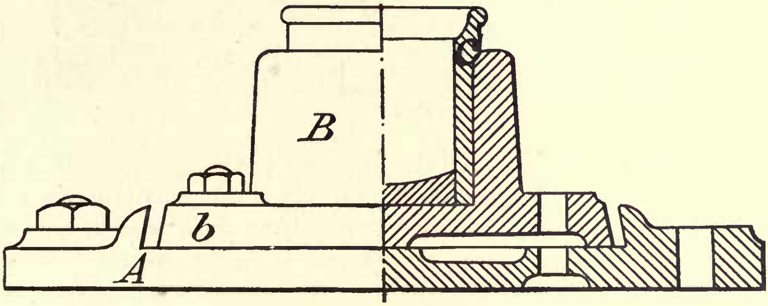


Fig. 585.

ened to the base on which it rests; in the case of heavy shafts, often to a base of granite. The box B is placed on A, the bearing surface being accurately levelled, and fitted either by planing or chipping and filing; the bearing surfaces *b* are commonly called chipping-pieces, which are the bearing surfaces of the bottom of B. A and B are held together by two screws; the holes for these are cut oblong in the one plate at right angles to those of the other; this admits of the movement of the box in two directions to adjust nicely the lateral position of the shaft, after which, by means of the screws, the two plates are clamped firmly to each other. C, the cup or bushing, which should be made of brass, slips into a socket in B. Frequently circular plates of steel (Figs. 586 and 587) are dropped into the bottom of this cup for the step of the shaft. The cup C, in case of its sticking to the shaft, will revolve with the shaft in the box B; if plates are used, these also admit of movement in the cup.

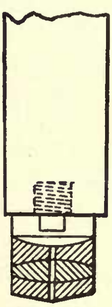


Fig. 586.

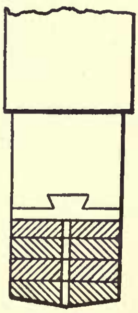


Fig. 587.

Fig. 588 represents the elevation of a bearing for an upright shaft, in which the shaft is held laterally by a box and bracket above the step. The step B is made larger than the shaft, so as to reduce the amount of wear incident to a heavy shaft. The end of the shaft and the cup containing oil are shown in dotted line. The bed-plate A rests on pillars, between which is placed a pillow-block or bearing for horizontal shaft.

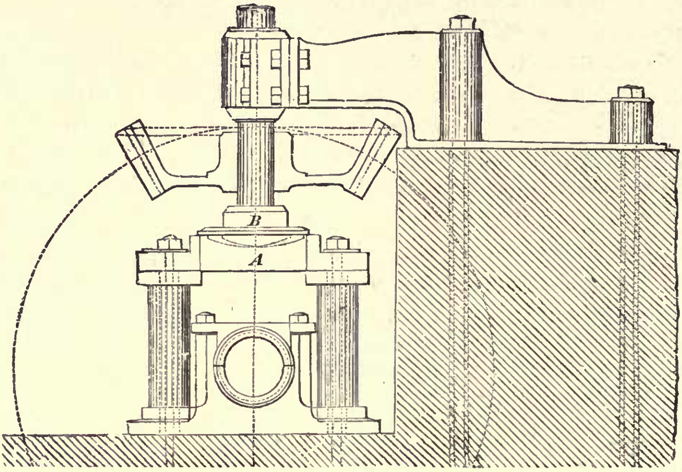


FIG. 588.

Figs. 589 and 590 represent the elevation and vertical section of the suspension bearing used by Mr. Boyden for the support of the shaft of his turbine-wheels. It having been found difficult to supply oil to the step of such wheels, it was thought preferable by him to suspend the entire weight of wheel and shaft, where it could be easily attended to. The shaft (see section) is cut into necks, which rest on corresponding projections cast in the box *b*; the spaces in the box are made somewhat larger than the necks of the shaft, to admit of Babbitting, as it is termed, the box; that is, the shaft being placed in its position

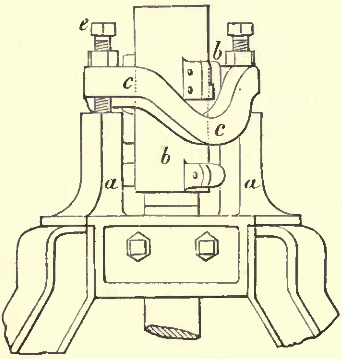


FIG. 589.

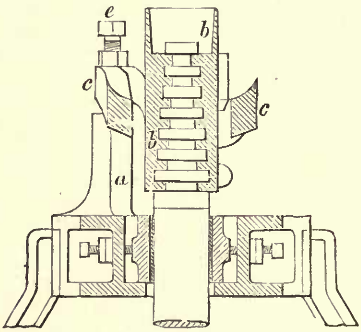


FIG. 590.

in the box, Babbitt, or some other soft metal melted, is poured in round the shaft, and in this way accurate bearing surfaces are obtained; projections or holes are made in the box to hold the metal in its position. The box is suspended by lugs *b*, on gimbals *c*, similar to those used for mariners' compasses, which give a flexible bearing, so that the necks may not be strained by a slight sway of the shaft. The screws *e e* support the gimbals, consequently the shaft and wheel; by these screws the wheel can be raised or lowered, so as to adjust its position accurately; beneath the box will be seen a movable collar, to adjust the lateral position of shafts.

No weight rests on the foot of the shaft, but a cast-iron plate is firmly bolted to the floor of the wheel pit, with side flanges, and set screws by which iron or wooden cushions can be adjusted to preserve the shaft in its central position.

The great care in design and mechanical construction of his wheels and their details enabled Mr. Boyden to obtain large percentages of effect, and led to the general introduction of turbines. In form and construction they have been much simplified, and with economy. Some makers still retain a form of upper hangers and bottom guides, but wooden steps (Fig. 591) are now almost universally adopted. They are made either conical or a portion of a sphere, of various woods, usually lignum-vitæ, but oak and poplar are preferred by some.

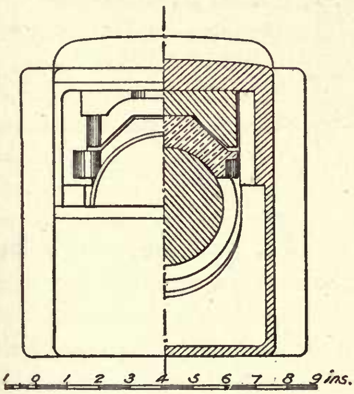


FIG. 592.

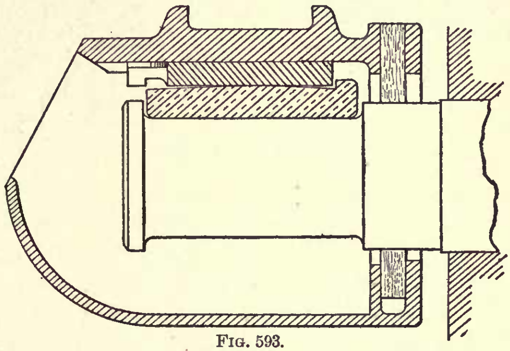


FIG. 593.

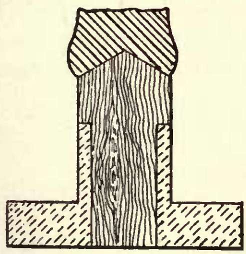


FIG. 591.

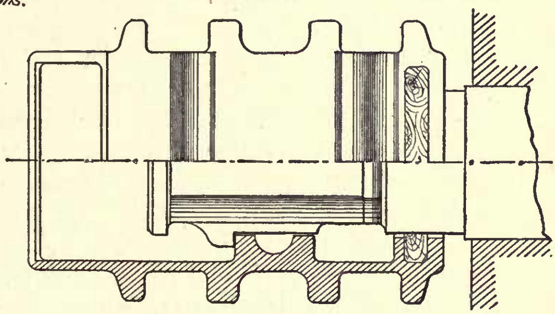


FIG. 594.

The load is from 50 to 75 pounds per square inch. The fibres of the wood are placed vertically, and afford an excellent bearing surface. Water is sometimes introduced into the centre of the wood, or into a box around it, from the upper level of water. When cast-iron or steel is used for the step, it is usual to incase the box and supply oil by leading a pipe, sufficiently high above the surface of the water, to force the oil down.

For long, upright shafts, it is very usual to suspend the upper portion by a suspension-box, and to run the lower on a step, connecting the two portions by a loose sleeve or expansion coupling, to prevent the unequal meshing of the bevel-wheels, incident to an alteration of the length of shaft by variations of temperature. The suspension is frequently made by a single collar at the top of the shaft.

Fig. 592 is a one half outside end view, and one half transverse. Fig. 593 is a section on the centre line of axle, and Fig. 594 sectional plan of box on centre line of axle with a plan of journal and journal bearing of the standard journal box adopted by the Master Car-Builders' Association, 1874.

Thrust Bearings for Screw Propeller Shafts.—The thrust along the shaft of a steamship propelled by a screw is taken up by collar bearings, and through them transmitted to the ship. Small shafts up to 8″ in diameter have generally one thrust collar, but sometimes comparatively small shafts have several thrust collars. In the latter case the bearing may have a brass bush in halves containing grooves to receive the collars on the shaft. The general practice is to fit between the collars cast-iron or cast-steel horseshoe-shaped pieces, clamped between two nuts, which are threaded on a screwed steel bar, supported at its ends by solid bearings cast on the block, as shown in Figs. 595 and 596. In this design each horseshoe piece may be adjusted separately by the nuts on each side, or they may be all moved together by means of the nuts at the ends of the bars.

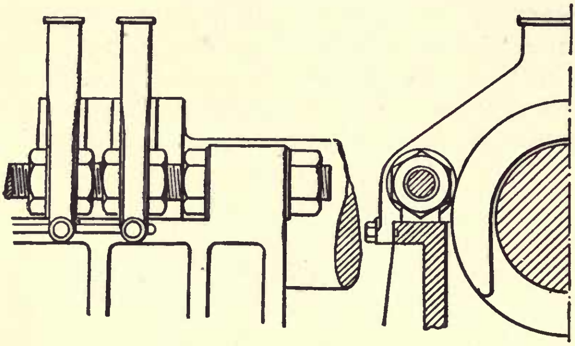


FIG. 595. FIG. 596.

A useful diagram (Fig. 597), with accompanying explanation, by George R. Bate, Assoc. M. Inst. C. E., is presented in the Practical Engineer for Nov. 2, 1894. "In its construction the effective horse power, or the power actually employed in propelling the ship, has been assumed to be equal to two thirds the indicated H. P. of the engines, so that

"I. H. P. = the indicated H. P. of the engines.

"E. H. P. = effective H. P. = I. H. P. $\times \frac{2}{3}$.

"K = speed of the vessel in knots.

"T = total thrust, or load on thrust block in pounds.

"P = pressure on thrust collars, pounds per square inch.

"S = surface of thrust collar, square inches per I. H. P.; then

$\frac{K \times 6{,}080}{60} = K \times 101{\cdot}3$ = speed in feet per minute and work done per minute in foot-pounds.

"$= T \times K \times 101{\cdot}3$, so that $T \times K \times 101{\cdot}3 = \text{E. H. P.} \times 33{,}000$; therefore $T = \frac{\text{E. H. P.} \times 33{,}000}{K \times 101{\cdot}3}$.

"Again, if E. H. P. $= \frac{2}{3}$ I. H. P., we may write

$$T = \frac{21\ \text{H. P.} \times 33{,}000}{3\,K \times 101{\cdot}3} = \frac{\text{I. H. P.} \times 22{,}000}{K \times 101{\cdot}3} = \frac{\text{I. H. P.} \times 217}{K};$$

therefore S, the surface of thrust collars in square inches per I. H. P. $= \frac{217}{K \times P}$.

"The diagram gives the value of S with P, varying from 30 to 80 pounds per square inch. In ordinary practice this pressure is 50 to 60 pounds per square inch in naval, and 40 to 50 pounds per square inch in mercantile steamers, although, in cases where white metal is fitted, it is found that these loads may be safely increased by 25 per cent.

"As an example, suppose the case of a thrust block for a vessel having engines of 3,000 I. H. P., driving her at a speed of 18 knots, to determine the necessary surface of thrust collars that the pressure on them may not exceed 60 pounds per square inch. At the point marked 18 knots on the scale for speed of vessel follow the ordinate up till it cuts the line marked 60 on the scale of pressures on the thrust collars to the left of the diagram, at which point of intersection follow to the right, and read off the corresponding surface of collars per I. H. P.—i. e., 0·201 square inches; then $0·201 \times 3,000 = 603$ square inches, the total thrust surface required for E. H. P. $= 3,000 \times \frac{2}{3} =$ 2,000, and $\frac{2,000 \times 33,000}{18 \times 101·3} = T$, the load on the block equal to 36,180 pounds; then, if pressure per square inch between surfaces is not to exceed 60 pounds, we have total surface of block $= \frac{36,180}{60} = 603$, as per diagram."

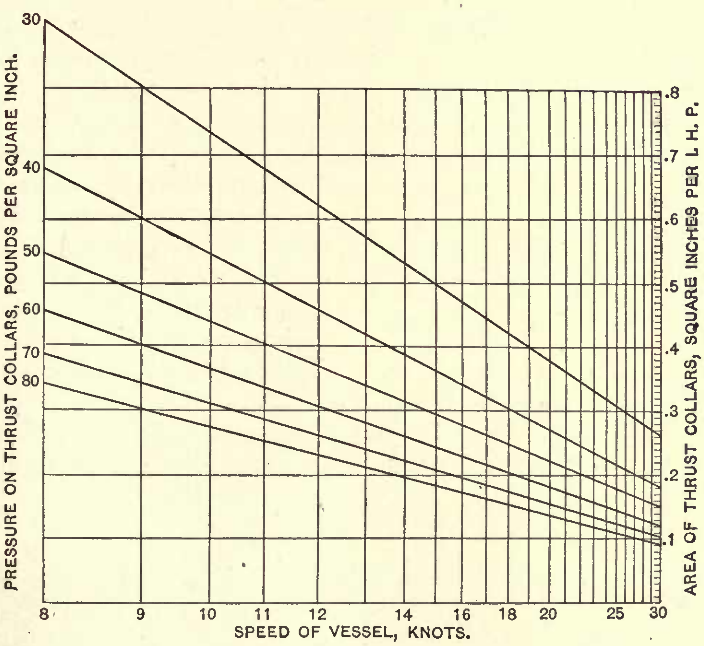


FIG. 597.

Couplings are the connections of shafts, and are varied in their construction and proportions, often distinctive of the mechanic making them.

The *Face Coupling* (Fig. 598), the one in general use for the connecting of wrought-iron shafts, consists of two plates or disks with long, strong hubs, through the centre of which holes are accurately bored to fit the shaft; one half is drawn on to the shaft and tightly keyed; the plates are faced square with the shaft, and the two faces are brought together by bolts. The number and size of the bolts depend upon the size of the shaft; never less than 4 for shafts less than 3 inches diameter, and more as the diameter increases; the size of the bolts varies from $\frac{5}{8}$ to $1\frac{1}{4}$ inch in diameter. The figure shows a usual proportion of parts for shafts of from 2 to 5 inches diameter; for larger than these, the proportion of the diameter of the disk to that of the shaft is too large.

Fig. 599 is a rigid sleeve coupling for a cast-iron shaft; it consists of a solid hub or ring of cast-iron hooped with wrought-iron; the shafts are made with bosses, the coupling is slipped on to one of the shafts, the ends of the two are then brought together; and the coupling slipped back over the joint, and firmly keyed. This is an extremely rigid connection. Some makers use keys without taper, and force the couplings on the shafts.

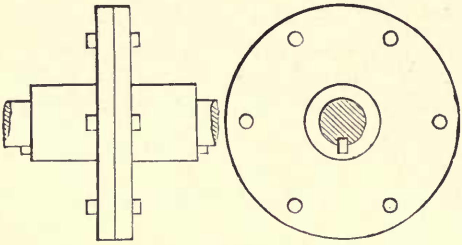


FIG. 598.

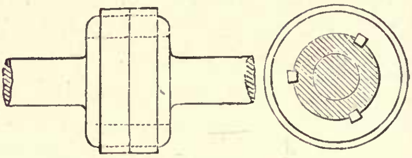


FIG. 599.

Fig. 600 is a screw coupling for the connecting of the lighter kinds of shafts. It will be observed that this coupling admits of rotation but in one direction—the one tending to bring the ends of the shafts toward each other; the reverse motion tends to unscrew, throw them apart, and uncouple them.

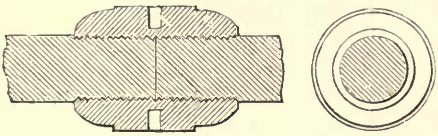


FIG. 600.

Figs. 601 and 602 is a double-cone vice coupling; B is the outer shell or sleeve, C C the two cones, and D the bolts. The sleeve is cylindrical outside, but bored with a double taper inside, smallest at the centre. The cones are bored to fit the shaft, and turned outside to fit the interior cones of the sleeve.

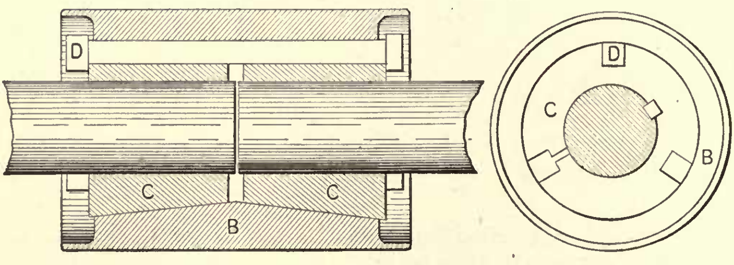


FIG. 601. FIG. 602.

There are three bolt grooves in the cones and sleeves, and one is cut through to give elasticity to the cones. The sleeves and cones are adjusted over the joint of the shafts, leaving it an easy fit, some $\frac{3}{8}$ inch between the ends of the cones.

If the bolts be introduced and screwed up, the cones are brought nearer to each other and the shafts are securely clamped together.

Fig. 603 is a clamp coupling for a square shaft.

In many cases it occurs that rigid couplings, such as have been given, are objectionable; they necessarily imply that, to run with the least strain possible,

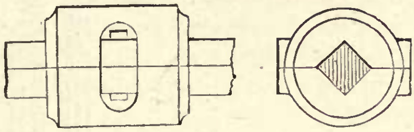


FIG. 603.

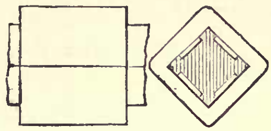


FIG. 604.

the bearings should be in accurate line; *any* displacement involves the springing of the shaft, heating of the journals, and loss by friction. Wherever, from any cause, the alignment can not be very nearly accurate, some coupling that admits of lateral movement should be adopted. The simplest of these is the *box* or *sleeve* coupling (Fig. 604), sliding over the end of two square shafts, keyed to neither, sometimes held in place by a pin passing through the coupling into one of the shafts. For round shafts, the loose sleeve Coupling is a pipe or hub, generally 4 to 6 times the diameter of the shaft in length, sliding on keys fixed on either shaft.

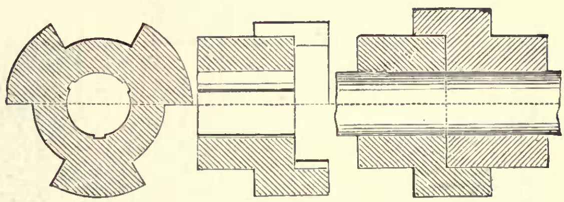


FIG. 605.

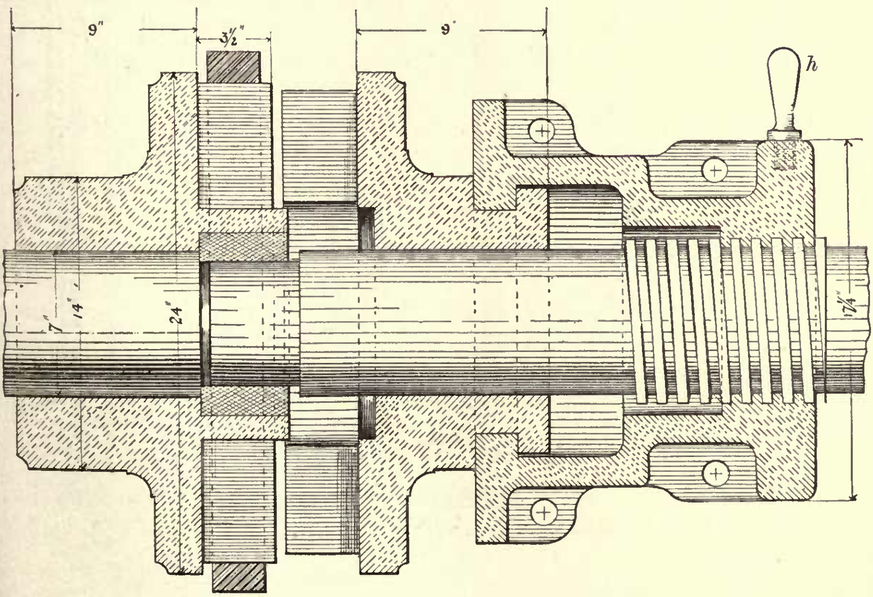


FIG. 606.

Fig. 605 is a *horned* coupling. The two parts of the coupling are counterparts, each firmly keyed to its respective shaft, but not fastened to each other;

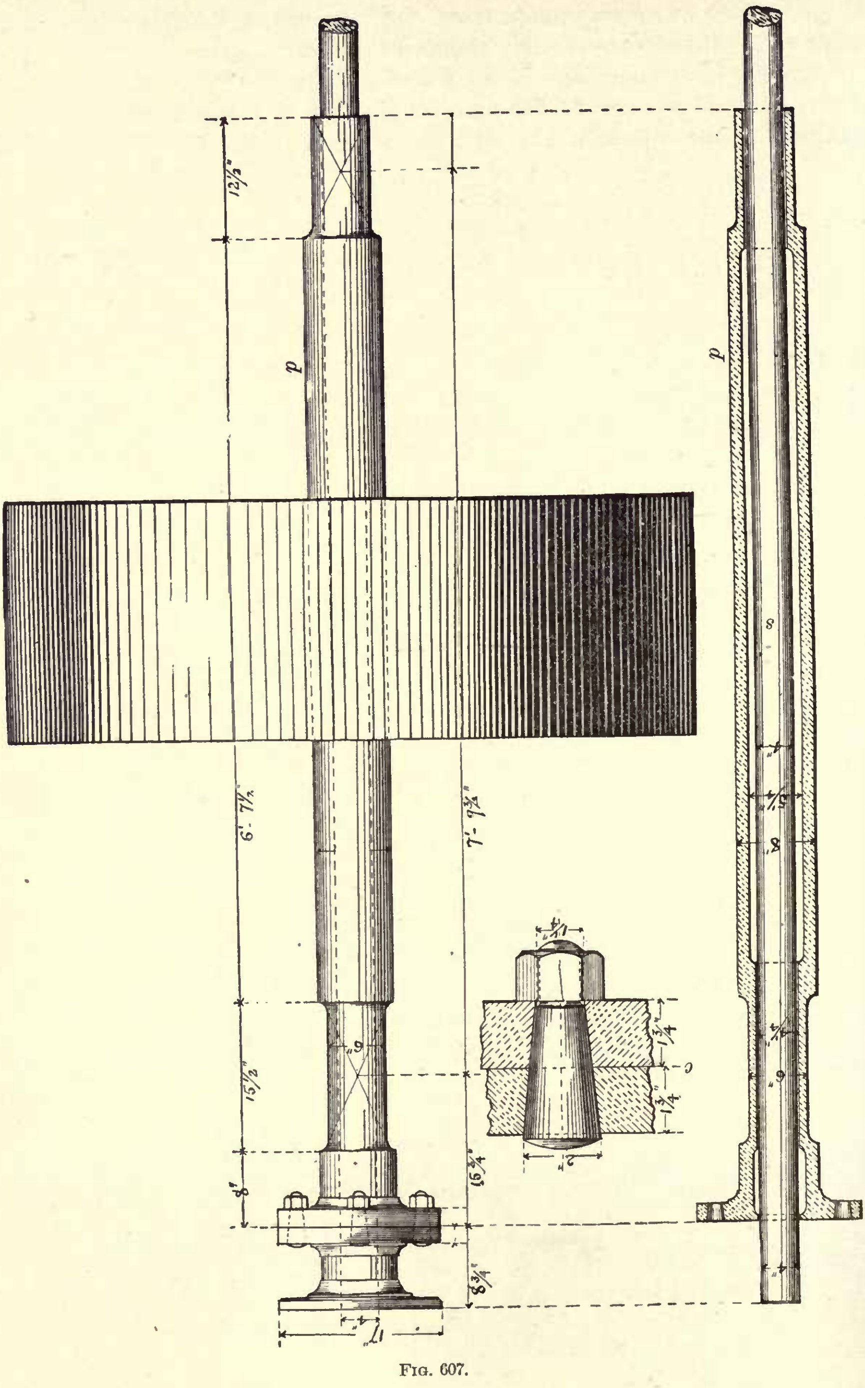


Fig. 607.

the horns of the one slip into the spaces of the other, and, if accurately fitted, it affords an excellent coupling, and is not perfectly rigid.

It often happens that some portion of a shaft or machine is required to be stopped while the rest of the machinery continues in motion. It is evident that, if one half of a horned coupling be permitted to slide lengthwise on the key—the key being fixed in the shaft, forming in this case what is more usually called a feather—when the horns of one half are out of the spaces of the other, communication of motion will cease between the shafts.

Fig. 606 represents a coupling of this sort for a large shaft, from the Corliss Steam-Engine Company. The horns are 8 in number on each part, and are thrown readily in or out of action by the handle *h* turning nut in the loose part of the clutch on the screw cut on the shafts.

Fig. 607 is another form of disengaging a large pulley from a main shaft, from the Corliss Steam-Engine Company. The pulley is fastened to a cast-iron pipe or sleeve *p* through which the main shaft *s* passes. The two are attached by means of the coupling *c*, one half of which is attached to the shaft and the other to the sleeve. When bolted together, the pulley and main shaft move together; but if the bolts be removed, then the pulley becomes stationary even if the shaft is running. Shaft and sleeve have independent bearings. A section of the coupling *c* on a larger scale shows the strong taper of the bolts without head.

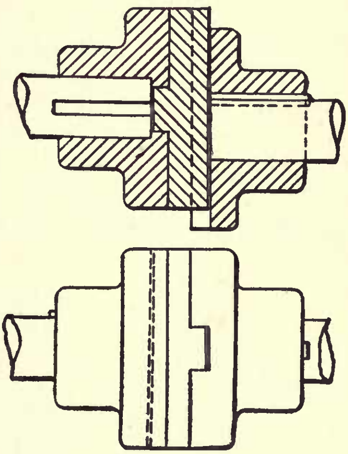


Fig. 608.

It is difficult to maintain shafts in exact line, and slight disarrangements are met by the elasticity of the shafts but, as a further precaution, flexible couplings are used, of which Fig. 608, called Oldham's Coupling, makes a strong connection and admits of considerable variation in the lines of the coupled shafts. It consists of two heads fastened to their several shafts, across the face of these heads two grooves are cut, and between these faces an intermediate plate is inserted with two tongues at right angles to each other, slide-fitted to the grooves in the heads, thus coupling the two shafts.

In Fig. 609 the coupling admits of more motion. The grooves are in the intermediate plate, and one of the tongues is fitted in its head with a T or dovetail groove held in position by a set screw or pin, by the removal of which the tongue can be withdrawn and the shaft uncoupled.

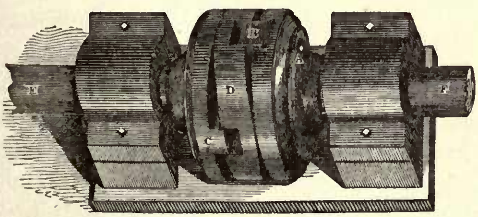


Fig. 609.

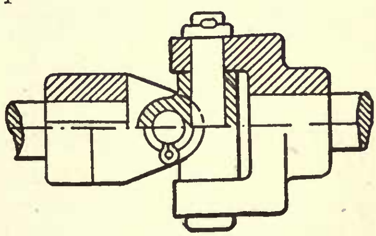


Fig. 610.

Hooke's Joint or Universal Coupling is used to connect two shafts whose axes intersect, and it has the advantage that the angle between the shafts may be varied while they are in motion. Fig. 610 shows the entire coupling partly in section; the shafts to be coupled are forked at their ends; these forked ends carry between them a cross the arms of which are at right angles to each other. The arms of the cross are jointed to the forks so that they may turn freely about their axes. The angular velocities of the shafts will be unequal except at every quarter revolution, but by using a double Hooke's joint, as shown in Fig. 611, the two shafts will have the same angular velocities, if they make equal angles with the intermediate shaft and are in the same plane with it.

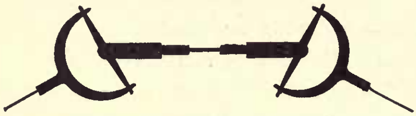


FIG. 611.

It is often necessary to engage shafts, when one is in motion, or disengage when both are in motion. One of the oldest forms of clutch for this purpose is the *slide* or *clutch* couplings, when the motion is required but in one direction (Fig. 612). A represents the half of the coupling that is keyed to the shaft, B the sliding half, *c* the handle or lever which communicates the sliding movement; the upper end of the lever terminates in a fork, inclosing the hub of the coupling, and fastened by two bolts or pins to a collar round the neck of the hub; to support B the end of its shaft extends a slight distance into the coupling A. Shafts can not be engaged with this form of coupling while the driving shaft is in rapid motion without shock. To obviate this, other forms of coupling are requisite; one of these is represented (Fig. 613). On the shaft B is fixed a drum or pulley,

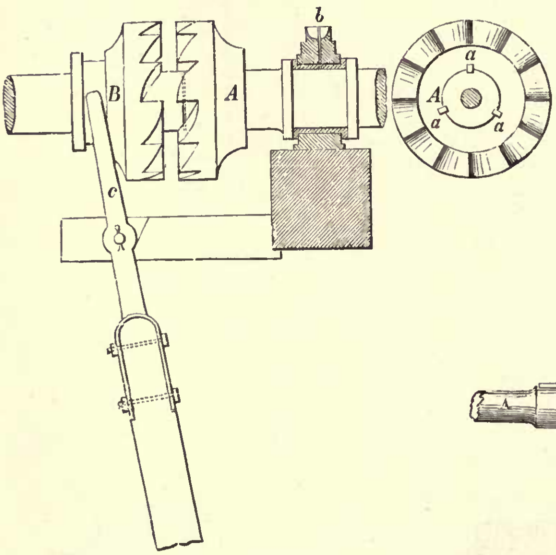


FIG. 612.

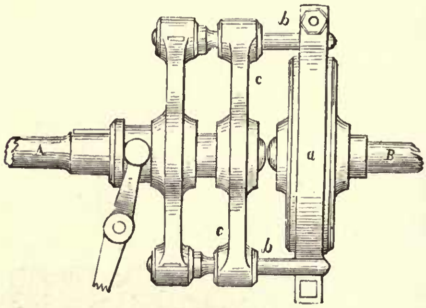


FIG. 613.

which is embraced by a friction band as tightly as may be found necessary; this band consists of two straps of iron, clamped together by bolts, leaving ends projecting on either side; the portion of the coupling on the shaft A is the common form of *bayonet* clutch; the part *c c* is fixed to the shaft, and affords a guide to the prongs or bayonets *b b*, as they slide in and out. Slipping these prongs forward, they are thrown into gear with the ears of the friction band; the shaft A being in motion, the band slips round on its pulley till the friction becomes equal to the resistance, and the pulley gradually attains the motion of the clutch.

But of all slide couplings, to engage and disengage with the least shock and at any speed, the *friction-cone* coupling (Fig. 614) is by far the best. It consists of an exterior and interior cone, *a*, *b*; *a* is fastened to the shaft A, while *b* slides in the usual way on the feather of the shaft B; pressing *b* forward, its exterior surface is brought in contact

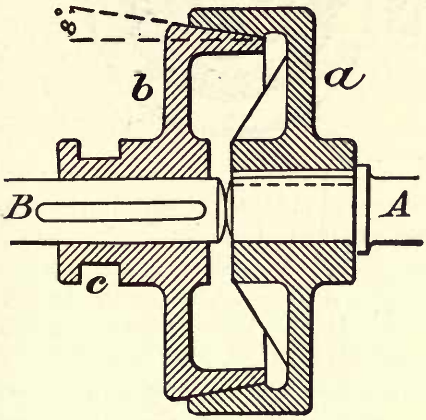


Fig. 614.

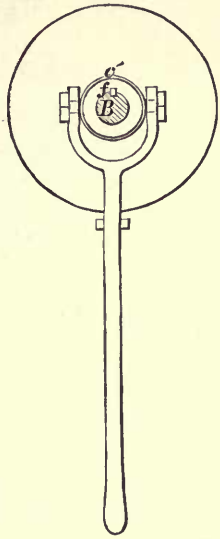


Fig. 615.

with the interior conical surface of *a*; this should be done gradually; the surfaces of the two cones slip on each other till the friction overcomes the resistance, and motion is transmitted comparatively gradually and without danger to the machinery. The longer the taper of the cones, the more difficult the disengagement; but the more blunt the cones, the more difficult to keep the surfaces in contact. An angle of 8° with the line of shaft is a very good one for surfaces of cones of cast-iron on cast-iron. When thrown into gear, the handle of the lever or *shipper* (Fig. 615) is slipped into a notch, that it may not be thrown out by accident.

The objection to this coupling is that it will work out of gear unless the shipper-handle is held firmly in its position, and producing considerable friction against the collar. To obviate this the shipper is made to act on a toggle-joint fastened to the shaft, and, once thrown, the pressure is self-continued and preserved without any action of the shipper, and without friction.

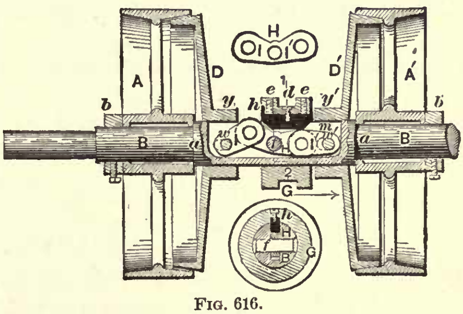


Fig. 616.

Fig. 616 represents a double-friction clutch, of the Weston-Capen patent. The clutch G is slid over the toggle, and the friction cone is forced into the pulley and engaged therewith. In the figure, D′ is thus engaged with A′, while D and A are not in contact.

When from any cause, as in rolling mills, the gearing is subject to sudden shocks, which might be injurious unless some means were adopted to modify the blow, friction couplings may be introduced of which Fig. 617 is an illus-

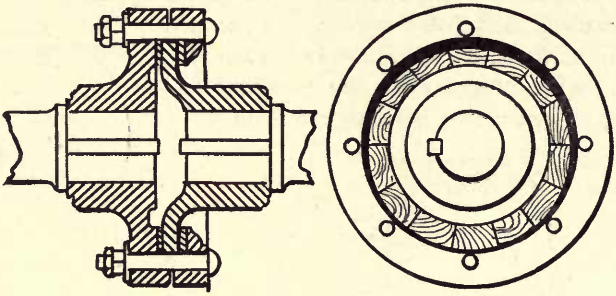


FIG. 617.

tration, in which the frictional resistance is sufficient to transmit the required power, but under sudden shocks yields and slips. It consists of two hubs, both keyed to their shafts; one of the hubs has a wood-lined groove into which the plate of the other is inserted and friction is produced between the two, by the bolts, which bring together the parts of the groove; loosening the bolts removes the frictional contact and disengages the clutch.

Fig. 618 is a longitudinal section and Fig. 619 a transverse section of the Weston clutch. The five iron disks A engage with solid keys on the long boss of the spur wheel E, within which the driving shaft C turns freely when no coupling pressure is applied to the disks. The drum D containing the six intermediate wood disks B slides on feathers on the shaft C, and the groove G on the outer end of the drum receives the forked end of a lever by which the coupling pressure is applied, compressing the disks against the fixed collar F on the shaft, and thereby coupling the spur wheel E to the shaft C.

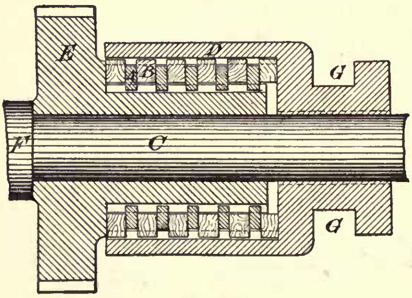


FIG. 618.

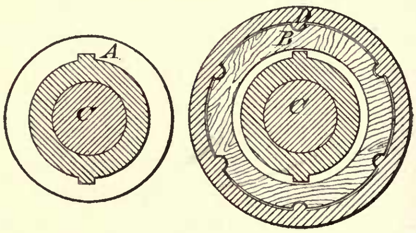


FIG. 619.

In Fig. 620 is shown the cylinder-friction clutch of Koechlin. In this case the clutch movement takes place readily. The part A is a hollow cylinder in which three internal clamp pieces are fitted, each being provided with a bronze shoe. These are thrown in and out of action by means of a sliding collar B′ which operates right- and left hand-screws by means of the lever *b*. The clamps slide in radial grooves. The nuts for the right- and left-hand screws can be closely adjusted and clamped by set screws, so that a radial movement of less than $\frac{1}{16}''$ is sufficient to throw the clutch in or out of action.

Figs. 621 and 622 are sections and elevation of a friction clutch in which the piece A is in the form of a ring of unequal thickness and divided at its thin-

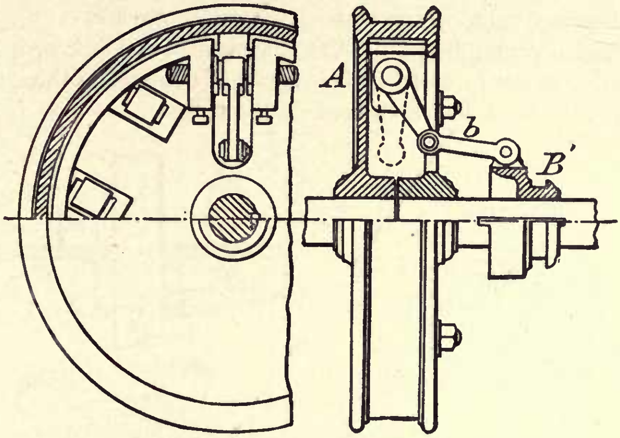


FIG. 620.

nest part. From the thickest part of the ring A a strong arm proceeds to a central boss which is keyed to the shaft. This ring, arm, and boss are all cast in one piece. An outer ring or shell B is bored or turned to fit easily over the

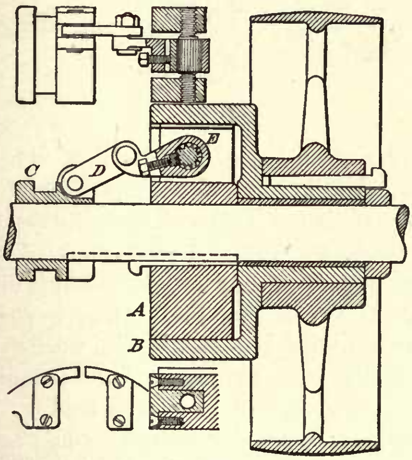


FIG. 621.

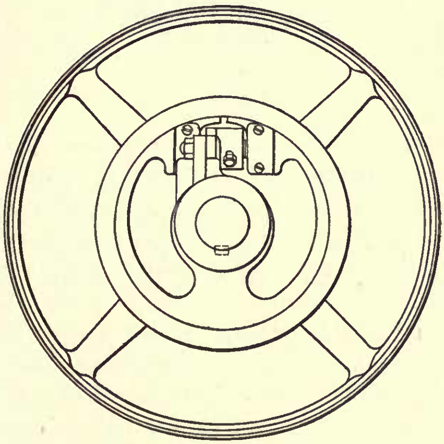


FIG. 622.

ring A. On the back of the piece B is a boss which serves to carry a wheel or pulley. To use the arrangement as a shaft coupling, the boss on the back of B is keyed to one shaft, the boss of A being keyed to the other.

To take the place of fast and loose pulleys, as shown in the figures, the piece B rides loose on the shaft, except when it is bound to the ring A. The pieces A and B are bound together by the expansion of the former caused by the rotation of right- and left-handed screws working in two nuts which fit into sockets in the ring A, one on each side of the line of division. By pushing the sliding boss C along the shaft, movement of rotation is communicated to the screw through the link D and lever E. The lever E is secured to the middle portion of the screw by a grooved key, which is held by a set screw. At the back of this key there is a clearance space sufficient to allow of the key being withdrawn clear of the grooves, so that the lever may be turned on the screw into another position and take up the wear of the screw threads.

Bovet's magnetic coupling (Fig. 623) consists of a block or head keyed to the power shaft, on the face of which is a groove containing the iron-wire core, connected with the brushes which bear on the rings.

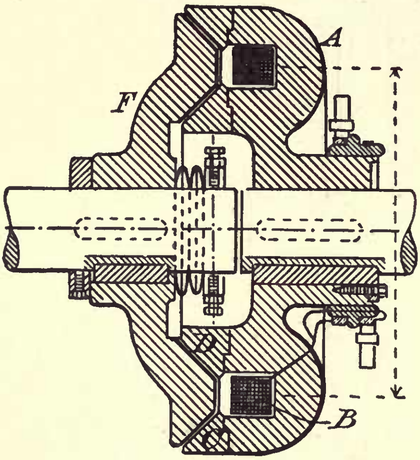


Fig. 623.

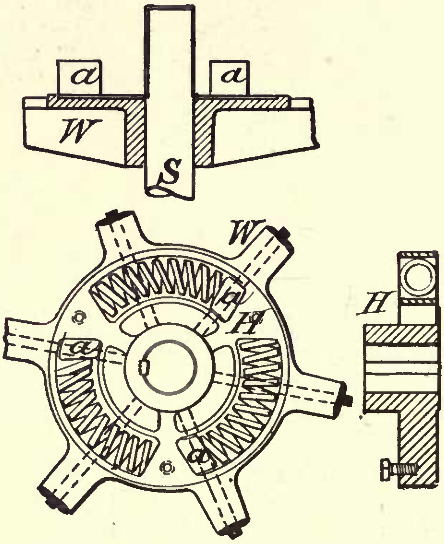


Fig. 624.

The shaft to be clutched is provided with a block F, capable of sliding along and approaching C D until it is in contact. It follows, therefore, that when the wire B is traversed by a current, F is attracted against C D and participates in the motion of A. Adhesion is obtained without any external reaction.

Fig. 624 is a spring hub used on a large rope-driving wheel to take up the shock of starting. The wheel W runs loose on the driving shaft S and is provided with lugs *a*, *a*, *a*, which project into the spring hub H keyed to the driving shaft S. Three heavy springs interposed between the lugs and the hub allow a circumferential movement of four feet on a six-foot-diameter wheel.

Pulleys are used for the transmission of motion from one shaft to another by the means of belts; by them every change of velocity may be effected. The speeds of two shafts will be to each other in the inverse ratio of the diameter of their pulleys. Thus, if the driving shaft make 100 revolutions per minute, and the driving pulley be 18 inches in diameter, while the driven pulley is 12 inches, then,

$$12 : 18 :: 100 : 150;$$

that is, the driven shaft will make 150 revolutions per minute without allowance for slip. Where there is a succession of shafts and pulleys, *to find the velocity of the last driven shaft:* Multiply together all the diameters of the driving pulleys by the speed of the first shaft, and divide the product by the product of the diameters of all the driven pulleys.

Pulleys are made of cast-iron and of every diameter, from 2 inches up to 20 feet. The number of arms vary according to the diameter; for less than 8 inches diameter the *plate* pulley is preferable (Fig. 625); that is, the rim is attached to the hub by a plate; for pulleys of larger diameters, those with arms are used, never less than four in number. The arms are made usually straight (Fig. 626), sometimes curved (Fig. 627).

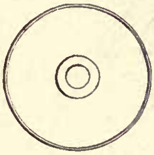


Fig. 625.

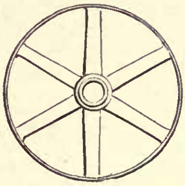


Fig. 626.

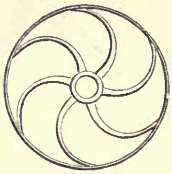


Fig. 627.

Fig. 628 represents a portion of the elevation of a pulley sufficient to show the proportion of the several parts, and Fig. 629 a section of the same. The parts may be compared proportionately with the diameter of shaft; thus the

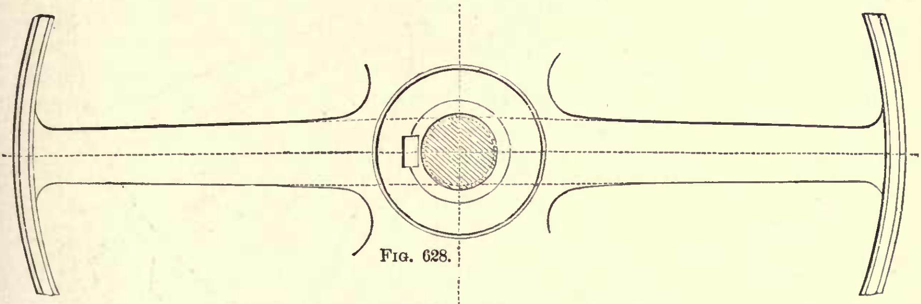


Fig. 628.

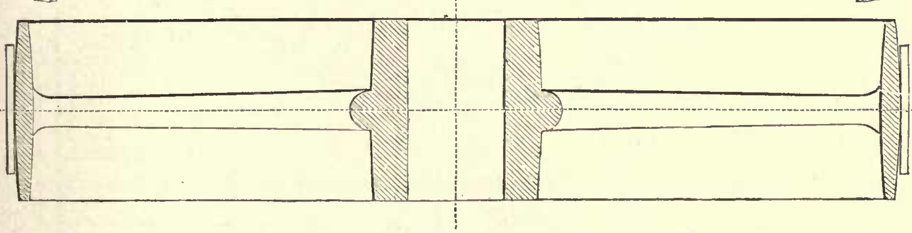


Fig. 629.

thickness of the hub is about $\frac{1}{2}$ the diameter of the shaft; this proportion is also used for the hubs of couplings; the width of the arms from $\frac{3}{4}$ to full diameter; the thickness half the width; the thickness of the rim from $\frac{1}{8}$ to $\frac{1}{6}$ the diameter; the length of hub the same as the width of face.

Fig. 630 is a large pulley of the Southwark Foundry pattern. The hub is cast with four divisions, to admit of contraction in cooling, and the rim is in halves, to admit of the pulley being put on the shaft without removing it from its bearings, a very common practice with large pulleys. Wrought-iron rim-pulleys consist of a spider—that is, the hub and arms—of cast-iron, and a wrought-iron plate-rim is bolted to flanges on the extremities of the arms.

Fig. 631 represents a faced coupling pulley, an expedient sometimes adopted when a joint occurs where a pulley is also required; the two are then combined;

the pulley is cast in halves—two plate pulleys, with plates at the side instead of central, faced and bolted together.

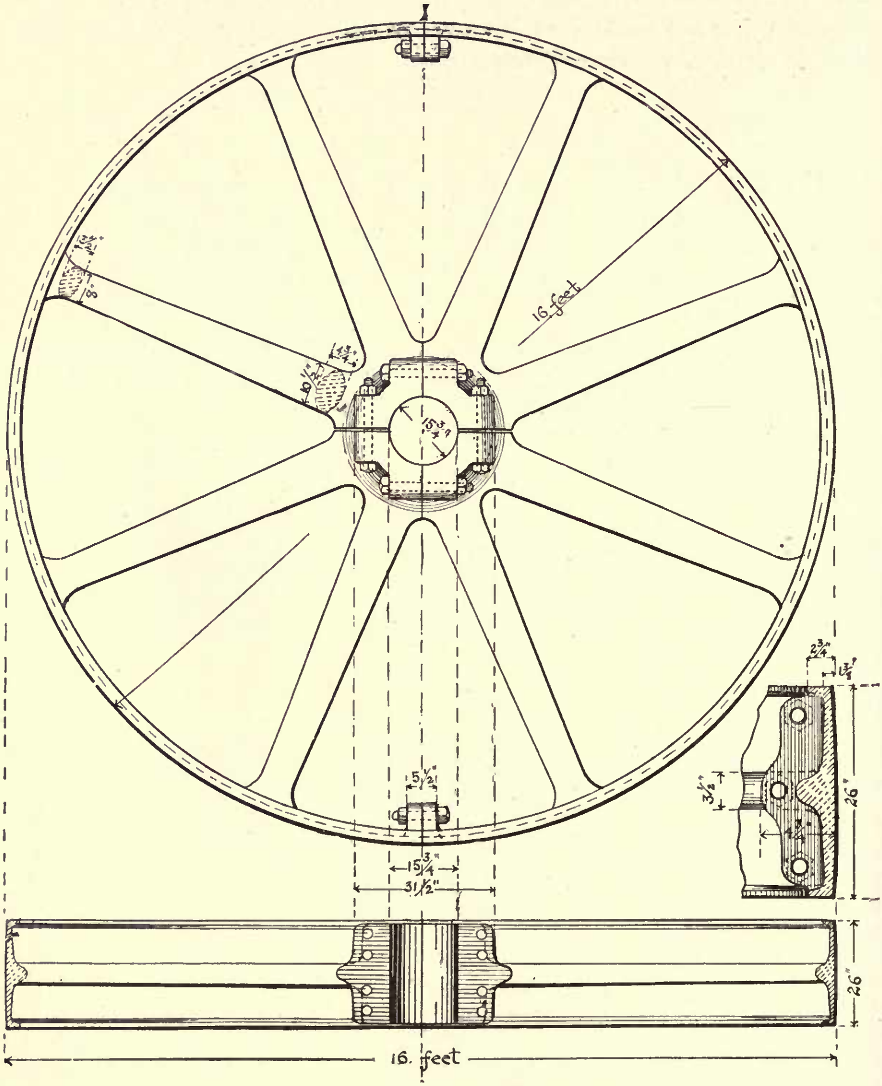


FIG. 630.

Wooden pulleys called *drums* are used for pulleys of very wide face. Fig. 632 represents one form of construction in elevation and longitudinal section. It consists of two cast-iron pulleys A A, with narrow rims; they are keyed on to the shaft at the required distance from each other, and plank or *lagging* is bolted on the rims to form the face of the drum; the heads of the bolts are sunk beneath the surface of the lagging, and the face is turned.

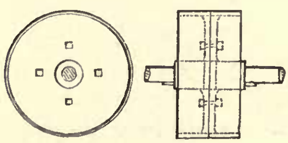


FIG. 631.

Fig. 633 represents a wooden plate pulley, consisting of sectors of inch boards firmly glued and nailed together, the joints of the boards being always broken. The face is formed in a similar way by nailing and gluing arcs of board one to another to the required width of face; these last should be of

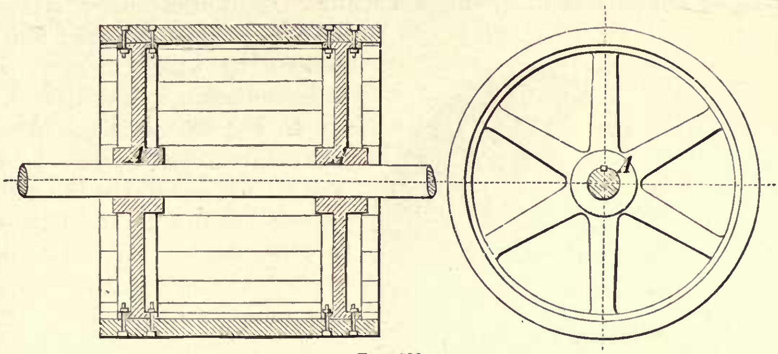


Fig. 632.

clear stuff. The whole is retained on the shaft by an iron hub, cast with a plate on one side, and another separate plate sliding on to the hub; the hub is placed in the centre of the pulley, the two plates are brought in contact with the sides of the pulley, and bolted through; and the pulley turned. A similar arrangement of hub is used for the hanging of grindstones.

Fig. 634 is an elevation of Chase's pulley similar in its rim to that of Fig. 633, but an iron spider supplies the place of the wooden plate. They are built

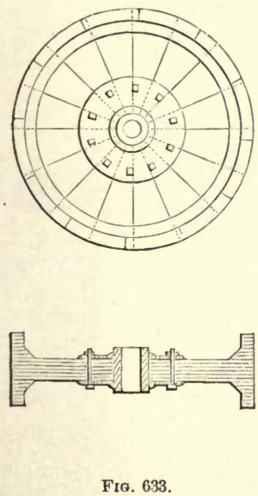


Fig. 633.

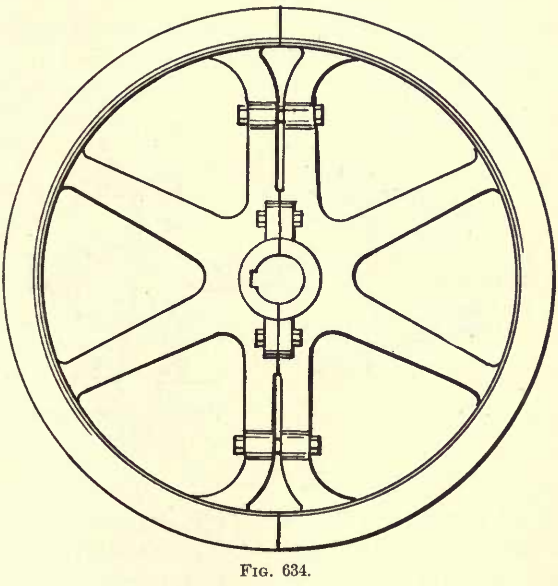


Fig. 634.

up with solid rims or split, as in the figure. They are made of the usual pulley dimensions up to 15 feet in diameter, and any desirable width of face, and able to transmit any amount of H. P. at any speed safe for a belt.

Fig. 635 is a perspective of a pulley with a wrought-iron rim. It is shown with two spiders, but it can be made with a single one or with any number, according to the width of belt required. They are also made in halves, adapted to any position, and safe under all practical requirements.

A counter-shaft is one distinct from the main shaft, but connected with it by a belt for the purpose of driving a machine. On the counter-shaft there is an arrangement of fixed and loose pulleys by which motion can be communicated to it or cut off.

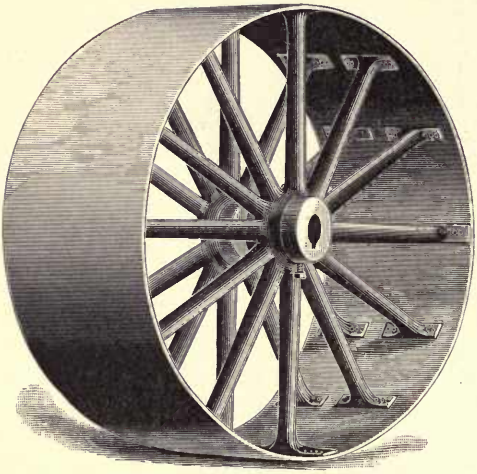


Fig. 635.

In Fig. 636, B is the belt from the main shaft which is being shifted on or off the fast pulley F on the counter by the fingers *f f* on the shipper bar. The belt is shifted to a full position on either the pulleys F or L. When the belt is on the fixed pulley F, the motion of the main shaft is communicated to the counter; when on the loose pulley L, the counter-shaft remains still and the pulley revolves upon it. It sometimes happens that the friction of the loose pulley upon the counter induces its revolution; to prevent which an arrangement is made, as shown in section (Fig. 637), by which the loose pulley moves on a fixed sleeve.

The shipper handle, not shown, is held positively by notches, or by some arrangement attached to the shipper bar. The faces of the pulleys should be flat or but slightly rounded. The fixed sleeve should be kept oiled.

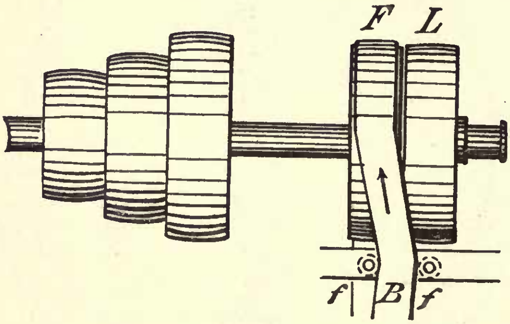


Fig. 636.

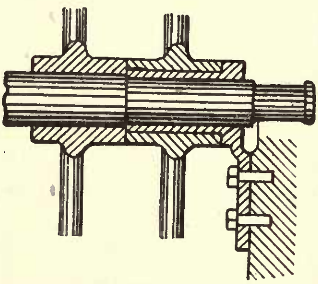


Fig. 637.

At the other end of the shaft cone pulleys are shown which correspond to similar cones on the machine but reversed so that the speed of the machine can be changed by shifting the belt from one set of cones to the other when the machine is stopped.

Cone pulleys may be made continuous (Fig. 638), thus becoming conoids upon which the belt can be shifted to any line by an adjusting guide.

It is often necessary to reverse the motion of a machine. This is readily done by a system of fast and loose pulleys, as shown in the plan and elevation (Figs. 639 and 640), in which A is a drum or wide-faced pulley on the driving-shaft, B a fast pulley on the driven shaft, and C and D loose pulleys on the same. The movement is indicated by the direction of the arrows. The driving-shaft revolves always in the same direction, but on the driven shaft the loose pulley of the straight belt is drawn from the bottom, and partakes of the same motion as the driving-pulley; while by the cross-belt the draft is at the top of its pulley, and the motion reversed. If the straight or *open* belt be shipped on to the fast pulley B, the motion given to the shaft is like that of the driving-shaft; if the cross-belt be shipped on to the fast pulley, the motion of the shaft is reversed. In the elevation, the lower side of the open belt is straight, while there is a sag in the upper; the first is the *tight* or *leading* belt, through which the power is transmitted, while the upper side is the *loose* or *slack* belt.

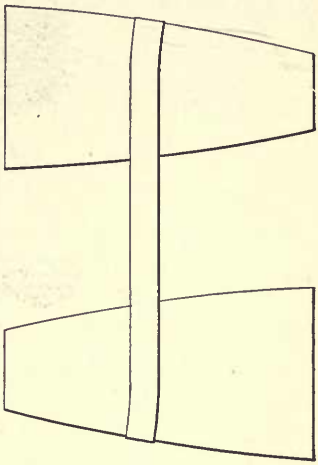


FIG. 638.

When the belt is shifted, while in motion, to a new position on a drum or pulley, or from fast to loose pulley, or *vice versa*, the lateral pressure must be applied on the *advancing* side of the belt, on the side on which the belt is ap-

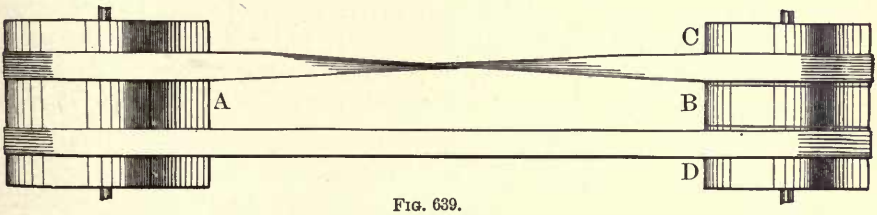


FIG. 639.

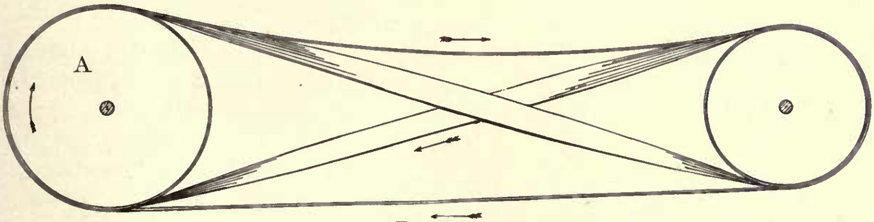


FIG. 640.

proaching the pulley, and not on the side on which it is running off. It is only necessary that a belt, to maintain its position, should have its advancing side in the plane of rotation of that section of the pulley on which it is required to remain, without regard to the retiring side. On this account, the shipper yoke or pins must be on opposite sides of the shipper bar.

When the main shaft is connected directly with a machine, and it has to be thrown out, the belt is often slipped from the pulley (Fig. 641), and hangs loosely from the shaft, by which the belt is worn and often becomes entangled with the shaft or with couplings. It is better to have a hook suspended from the ceiling to catch the belt when thrown off; or, still better, iron suspended bows (Fig. 641), from which it is easier to slip the belt on the pulley again.

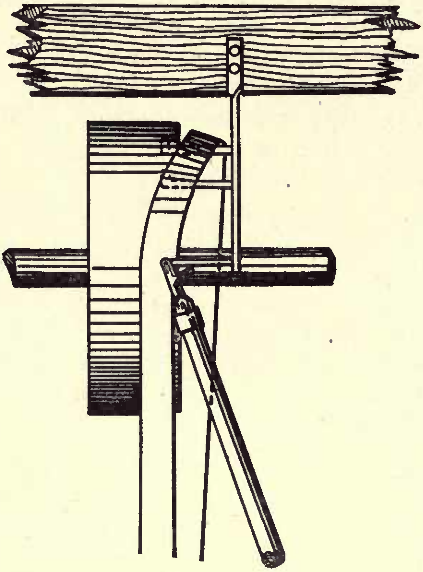


FIG. 641.

Motion may be transmitted by belts to shafts at right angles to each other.

Figs. 642 and 643 is a plan and elevation in which A is the driving-shaft and pulley and B the driven one, at right angles to each other. The arrows show the advancing sides of the belts and their position with regard to the face of the pulleys.

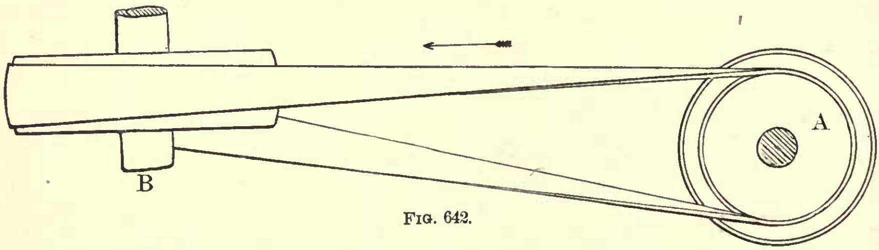


FIG. 642.

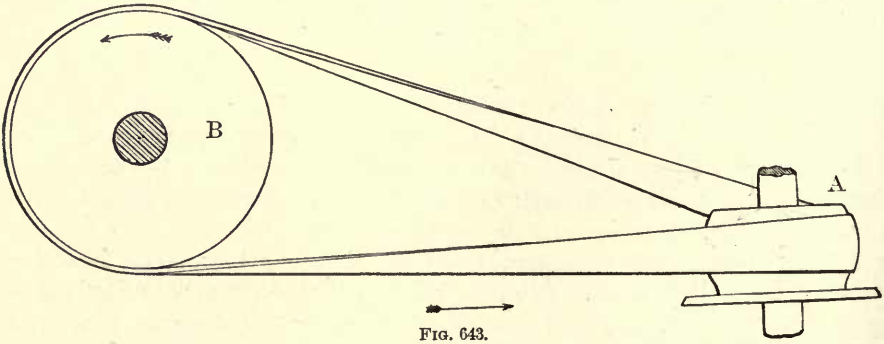


FIG. 643.

In the case of inclined axis (Fig. 644) the leading line falls in the middle plane of each pulley, but the following side of the belt does not, hence such systems can only be run in one direction. The leaving points in the figures

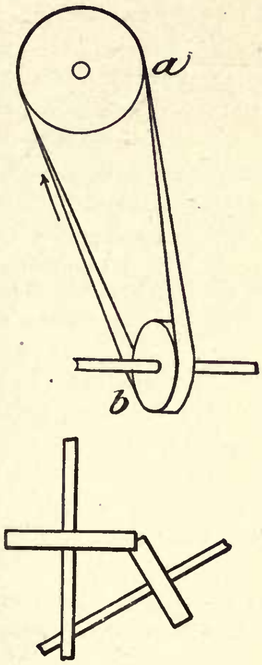


Fig. 644.

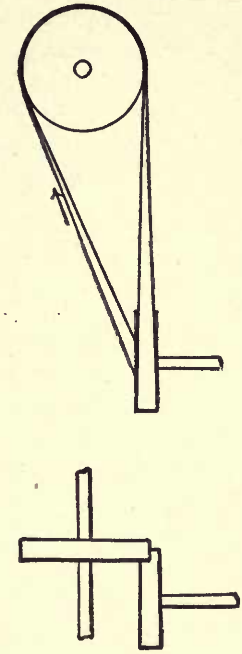


Fig. 645.

are at *a* and *b*. The arrangement gives an open belt when the angles between the planes of the pulleys = 0°, and at cross belt = 180°. In the intermediate positions a partial crossing at the belt is produced, the angle = 90°; the belt is quarter twist (Fig. 645); if = 45°, it is quarter crossed. The maximum leading-off angle is 25°, which occurs when the distance between the axis is equal to twice the diameter of the largest pulley.

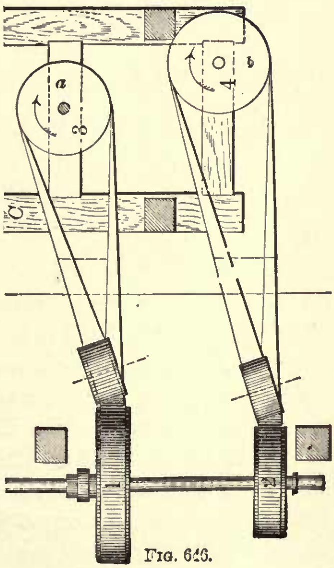


Fig. 646.

Guide pulleys are very useful in belt transmission for shafts at varied angles, and the proper direction is obtained when each guide pulley is placed at the point of departure of its plane with that of the next following pulley.

Fig. 646 is an arrangement adopted in portable grist-mills for driving the vertical shafts, *a*, *b*, of mill-stones, from pulleys on a horizontal shaft. Here it is thought necessary to use guide-pulleys.

Figs. 647 and 648 are the elevation and

plan of another arrangement of pulleys and guide-pulleys; *a b* is the intersection of the middle plane of the principal pulleys. Select any two points *a* and *b* on this line, and draw tangents *a c*, *b d*, to the principal pulleys. Then *c a c*

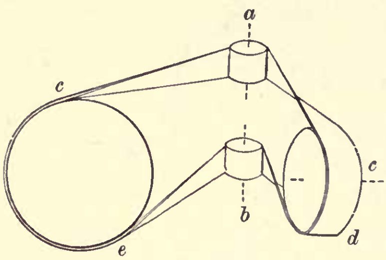


FIG. 647.

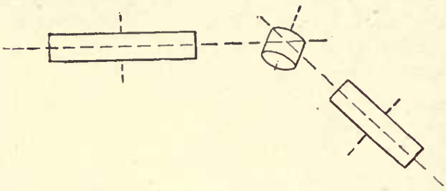


FIG. 648.

and *e b d* are suitable directions for the belt. The guide-pulleys must be placed with their middle planes coinciding with the planes *c a c* and *e b d*. The belt will run in either direction.

In Figs. 649 and 650 are parallel axes with two guide-pulleys. In the first the guide-pulleys are placed in planes tangent to both operating pulleys, and hence driving may occur in either direction. Usually, however, it is required to provide for motion in but one direction, in which case the second form is used as being simpler. The pulley B may be used as one of the guide-pulleys, in which case it may be placed loose upon the same shaft as A, and C or D be made drivers or driven.

It is necessary to stretch the belt over the pulleys to prevent its slip while conveying power. But if the belt is very heavy, and runs nearly horizontally,

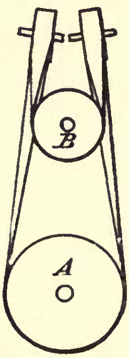


FIG. 649.

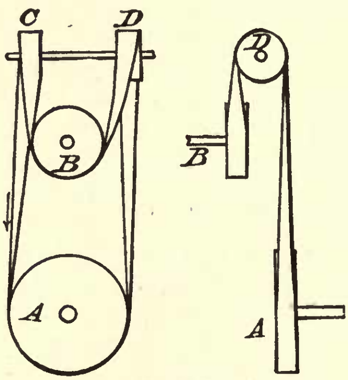


FIG. 650.

its weight will supply a portion of the adhesion which diminishes with the inclination of the belt till it becomes vertical, when the friction of the stretch is the only factor of the adhesion of the belt to the lower pulley; and, as the belt lengthens by use, the value of this friction becomes nothing. This position of pulleys should not obtain if it can be avoided; but if not, the friction-stress should be by means of an idler or binder (Fig. 651) on the loose belt; distinctively the idler rests in a loose frame against the belt, acting by gravity, while the binder is forced against the belt by mechanical appliances. By the relief of the binder the belt becomes slack, the friction of the belt on the pulleys becomes nothing, and motion stops on the driven pulley; but that of the belt may continue by its friction on the driver, from which it can be raised by a rope attachment or by a more positive contrivance. This is not found necessary when the arrangement is used for the engaging or disengaging of machines, but the driver pulley is provided with flanges so that the belt can not slip off. Idlers or binders are of necessity when the two pulleys are near to each other, as in steam hoist engines, either to increase the bearing surface on the pulleys or make up for the slight weight of a short belt.

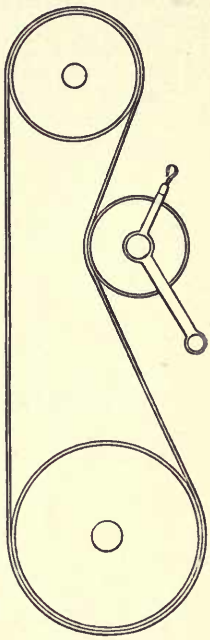


Fig. 651.

Belts run the best when their length and position are such as to give the frictional stress without much stretching on the pulleys, and without binders, and for this purpose the tight side of the belt—that is, the one approaching the driver pulley—should be at its under side.

In determining the necessary length of a belt for any position, the simplest way is to measure it, if the construction is complete; if not, to make a drawing of the pulleys in position to a scale, and measure on the drawing.

The width of the belt should always be a little less than the face of the pulley; both are to be determined by the power to be transmitted and the velocity of movement.

Allowance should be made in calculation of speed of a driven pulley for the slip of the belt, which is always something, but should not fall behind more than one per cent that of the driver; the friction, with too tight a belt is too much, and the slip, with a too slack one. Too small dimension of pulley or too little of arc of contact increases slip. It does no particular harm to have a belt unnecessarily wide, but it does to have it too narrow. If the diameter of the pulleys be increased, the speed of the belt is also increased, and for transmitting the same power, its width decreased.

By experiments of H. B. Gale at the Washington University, St. Louis, the practical limits of speed of belt may be taken at from 3,000 to 7,000 feet per minute. The flesh side of leather possesses much greater tension than the grain or hair side, and on this account, and in the practice of most millwrights, the grain side is put in contact with the pulley, and in double belts the flesh side is placed centrally.

John T. Henthorn's formula for double belts is $\frac{D \times \pi \times R}{450} = \text{H. P. or } \frac{V}{450}$ per inch in width, in which D is the diameter of pulley in feet, R the revolutions per minute. This is expressed graphically in Fig. 652.

Use of Diagram.—To find the horse power that can be transmitted by a 24″ belt on a 20-foot pulley making 100 revolutions per minute: The abscissa line 100 intersects the diagonal 20 on the ordinate line 14; $14 \times 24 = 336 =$ horse power transmissible.

To find the belt necessary to transmit 100 horse power through a 10-foot pulley and 120 revolutions per minute of shaft: The abscissa 120 cuts the diagonal 10 on the ordinate line $8\frac{1}{3}$; $\frac{100}{8\frac{1}{3}} = 12''$ width of belt. If the pulley were 12-foot instead of 10, it will be seen by the diagram that the intersection of diagonal would be at 10, and the width of belt $\frac{100}{10} = 10''$.

This rule referred to single belts capable of transmitting one half the power of the double belt, would require a velocity of 900 feet per minute for a belt of

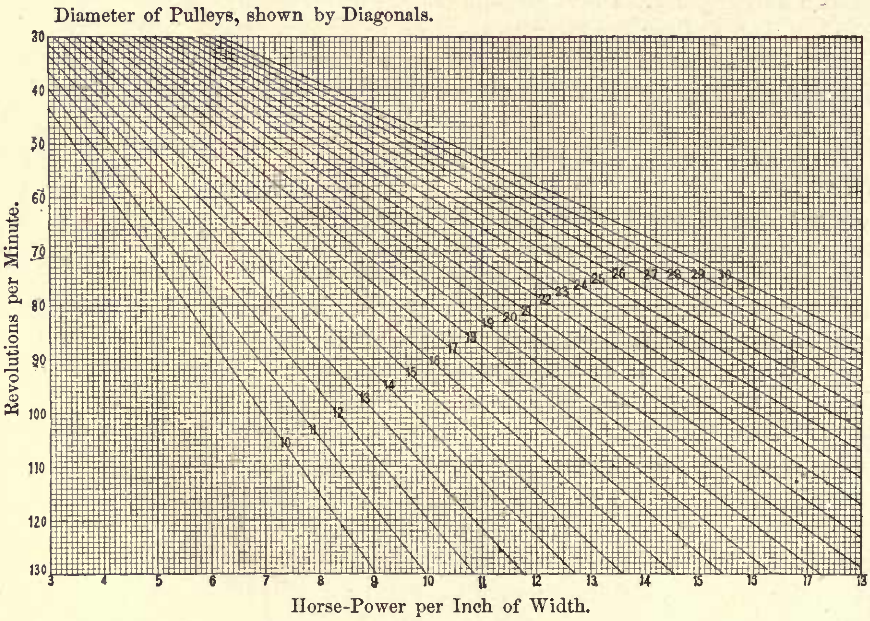


Fig. 652.

1″ in width to transmit 1 horse power, and if the belt were triple, and running with the same velocity, it would transmit 3 horse power, which is a rule in common use.

The above rules are applicable to India rubber and canvas belts, which are largely used. They are made of plies of closely woven duck, stitched and cross-stitched together, with or without rubber between the plies and on the outer surfaces; the rubber belts are the only ones that can be run in wet places. The plain duck belts depend largely on the close stitching of the plies, and can be used as cross belts, or in any place where leather belts can be used.

There is a great difference among mechanics as to the amount of power that may be transmitted by a belt with economy, but the rules as given by Henthorne above are within limits of practice. The Amoskeag Manufacturing Co., Manchester, N. H., run two double belts each 40″ wide, and one 24″ wide, on a 30′ fly-wheel pulley, of 110″ face, making 61 revolutions per minute with 1,950 indicated H. P. on the steam engine and transmitted through the belts, say 1,800 H. P., gives 17·3 H. P. for each inch in width of belt, and Henthorne 12·8. The belts were considered heavily loaded but not overtaxed.

Samuel Webber, in the "American Machinist," February, 1894, reports the case of a belt 30″ wide, $\frac{5}{8}''$ thick, running for six years at a velocity of 3,900 feet per minute on a pulley of 5 feet diameter and transmitting 556 H. P., which gives a velocity of 210 feet per minute per inch width per H. P. By Mr. Fred.

W. Taylor's rule it would be used to transmit only 123 H. P., who as Mem. A. S. C. E., in Vol. XV. of its "Transactions," has given conclusions from his practical use of belts in the running of a machine shop day and night for nine years, a very long life if estimated in day's work of ten hours each, with ample opportunity for repairs, and for narrow belts transmitting power to machinery. He finds by testing the tension of belts by a spring balance between two clamps attached to the ends of the belt while stretching, the most economical average total load for double belting to be 200 to 225 pounds per square inch of section, that a total load of 111 pounds per inch width corresponds to a pulling power of 65 pounds, of 54 pounds to 26 pounds, and that the maximum speed for economy should be from 4,000 to 4,500 feet per minute.

"Belts are more durable and work more satisfactorily when made narrow and thick than wide and thin. Minimum diameter of pulley for a double belt 12″, for a triple 20″, for a quadruple 30″. The ends of a belt should be fastened together by splicing and cementing instead of lacing, wiring, or using hooks or clamps of any kind."

Leather belts may be purchased from stock from 1″ to 48″ in width, round belts from $\frac{1}{4}$″ to $\frac{5}{8}$″ in diameter.

Rubber 3-ply is equal to single leather. The following are stock sizes: 2-ply from 1″ to 28″ width; 6-ply up to 60″; 7- and 8-ply to order.

Full rolls contain 400 to 450 feet, and endless belts are made to order. Solid cotton belts, 2-ply 1″ to 6″ wide; 4-ply 1″ to 22″.

The use of endless ropes instead of belts is of very old application, by single lines of rope with outdoor exposure, and large pulleys at considerable distances apart. In Fig. 653 an arrangement is shown for the transfer of a reciprocat-

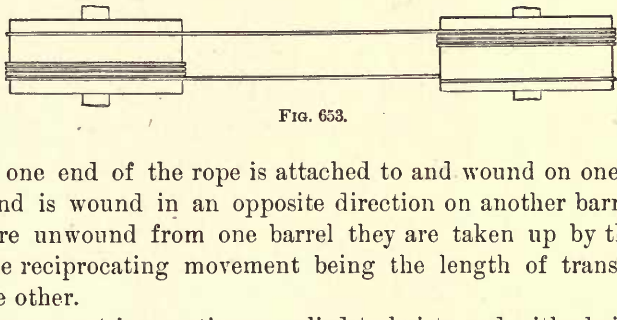


FIG. 653.

ing power; one end of the rope is attached to and wound on one barrel while the other end is wound in an opposite direction on another barrel, so that as the ropes are unwound from one barrel they are taken up by the other, the length of the reciprocating movement being the length of transfer from one barrel to the other.

This arrangement is sometimes applied to hoists, and with chains instead of ropes to the old type of planers.

Of late the use of ropes for the transmission of power has increased very rapidly both in this country and in England, on account of their economy in first cost and maintenance, in transmitting large amounts of power to considerable distances with simplicity in changes of direction and distribution of power, smooth running, and absence of slip.

There are two forms of arrangement, in one of which a single spliced endless rope by its tension gives the necessary adhesion (Fig. 654), and as the rope grows slack by use, taking it up by a fresh splice; in the other the slack is taken up by a tension carriage. In both forms increase of power is met by multiple grooves or pulleys and in the number of loops of rope.

Fig. 655 shows the tension carriage as applied to the driving cable on the Brooklyn Bridge. A is the pulley connected with the steam engine, on which

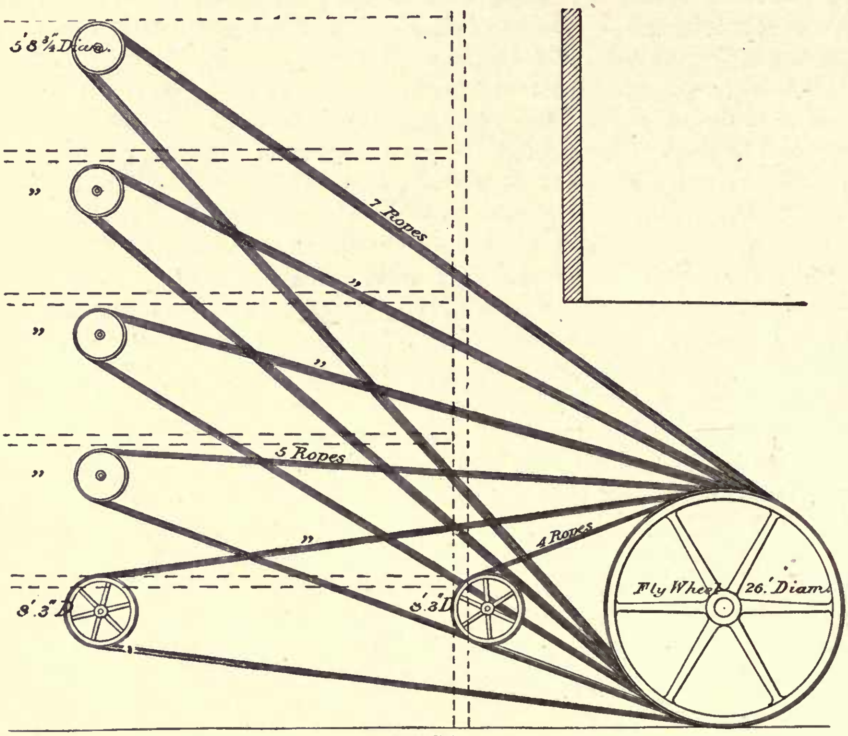


FIG. 654.

there are four grooves and lines of rope for adhesion; one line passes over the large standing 10-foot sheave B, thence round the tension pulley C on a weighted car moving in inclined rails, thence over the sheave D to the line of bridge, over

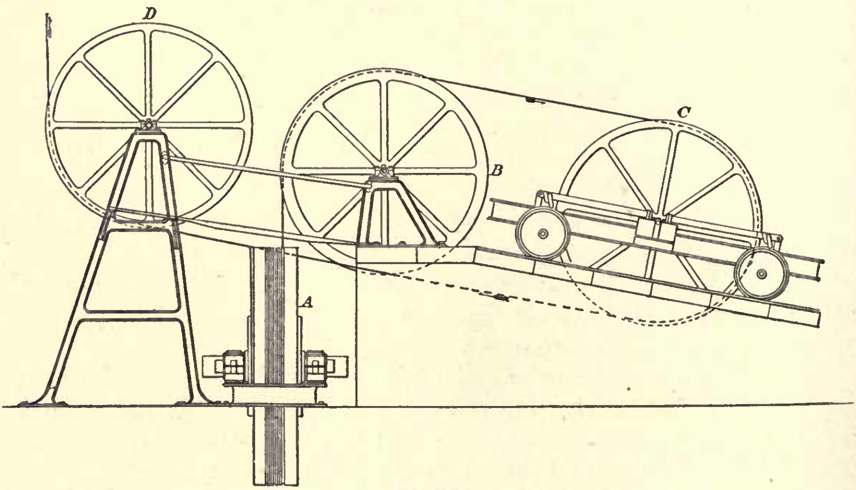


FIG. 655.

the bridge beyond track, and returning the other to the pulley A. The cable in diameter and general arrangement is similar to that for cable roads, and is only given as an illustration of an extreme size of tension car. For transmission of power to machinery the diameter of rope does not exceed 2 inches, and the tension car is usually a light grooved pulley, sliding on vertical or horizon-

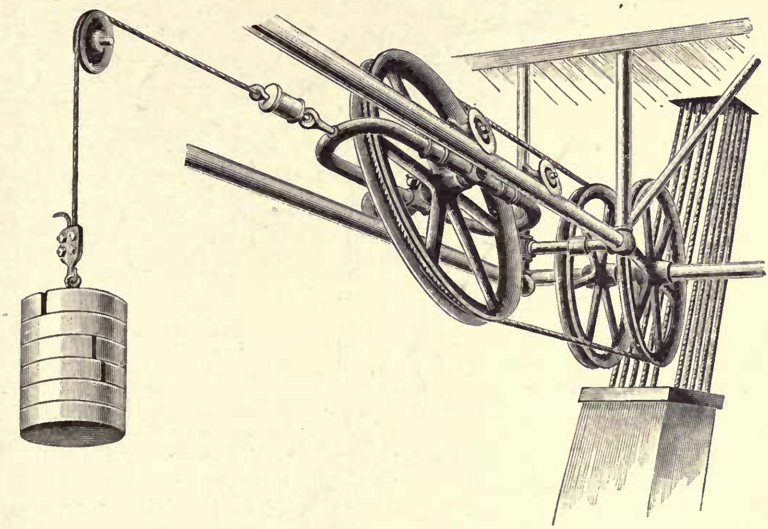


Fig. 656.

tal tracks, with weights attached and moving vertically to give the requisite adhesion.

With most makers the pulleys are single multiple grooves, but the Link Belt Engineering Co. make light pulleys, with a single groove to each, which can be bolted together to make a multiple grooved sheave of the requisite number of ring sections.

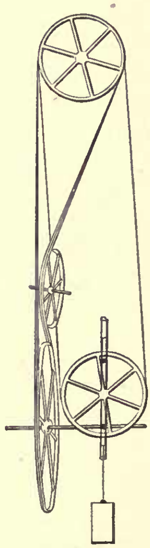


Fig. 657.

"The diameter of the pulleys has an important effect on the wear of the rope. The larger the sheaves, the less the fibres of the rope slide on each other, and consequently there is less internal wear of the rope. The pulleys should not be less than forty times the diameter of the rope for economical wear, and as much larger as it is possible to make them. This rule applies also to the idle and tension pulleys as well as to the main driving-pulley."

Fig. 656 is a view of a horizontal tension carriage; Fig. 657, a half turn with vertical tension; Fig. 658, a portion of the line of shaft of the factory of the same company.

The usual material for rope gearing of mills is either hemp, manilla, or cotton. The ropes are untarred hawser laid—that is, formed with three strands twisted together right-handed; a strand is made by twisting yarns together left-handed.

The "Stevedore" rope of the Link Belt Engineering Co. is a 4-strand rope, manufactured from long-fibre manilla laid in tallow mixed with plumbago (to reduce the friction in the bending of the strands passing around the sheaves), which renders it nearly waterproof, therefore suitable for out-of-door work.

Fig. 659 represents a section of the grooves of a pulley as designed by E. D. Leavitt, M. E.

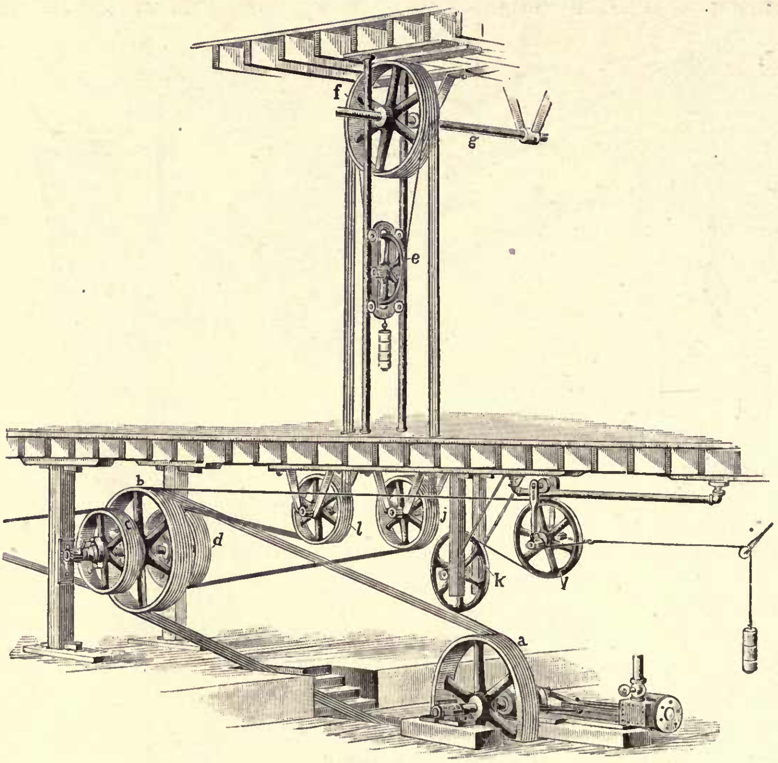


FIG. 658.

For the strength of a rope, the assumption of Mr. C. W. Hunt is that "a rope one inch in diameter should have a working strain of 200 pounds at all speeds. This is about one twentieth of the strength of the splice. This large

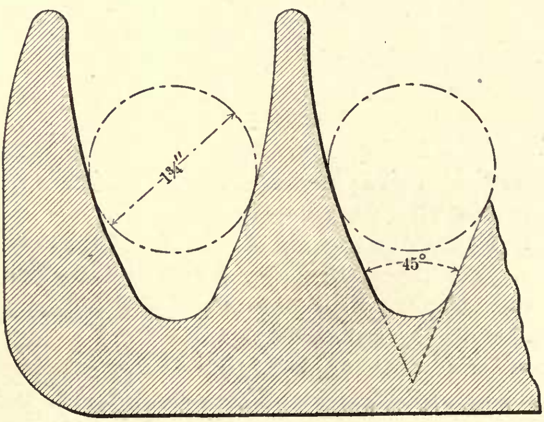


FIG. 659.

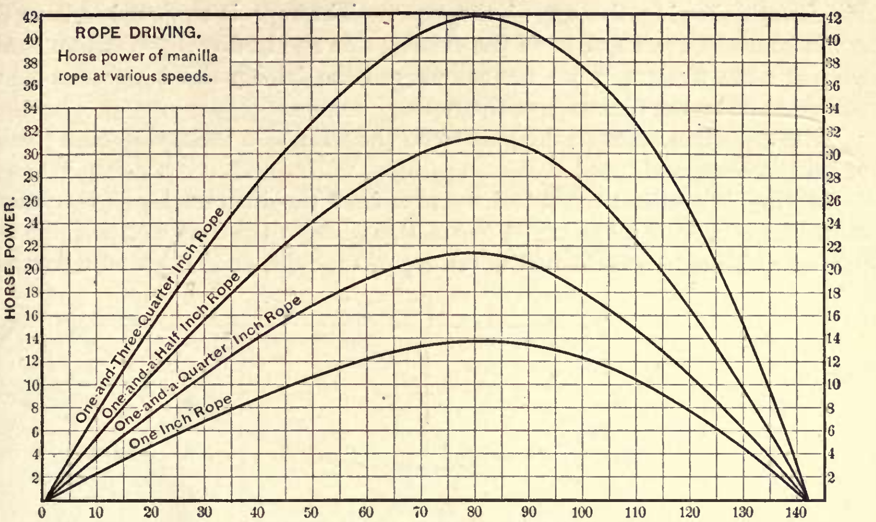


FIG. 660.

margin is to enable the rope to perform a great amount of useful work before it is so weakened by wear that it is necessary to be renewed. There are many strains which can not be computed owing to the irregularities of the power and the work. The diagram (Fig. 660) takes into consideration the effects of the centrifugal force so that the strain on the rope is constant on the driving side in transmitting the tabular horse power, no matter what the speed may be. It shows also the power a rope transmits at various speeds, illustrating the rapidity with which the horse power decreases when the speed gets beyond about eighty feet per second."

It is desirable in all cases of rope transmission to so arrange the drive that the slack side of the rope shall be on the upper part of the pulley, thus increasing the arc of contact, as the two sides will then approach each other when in motion.

In order that the desired tensions shall be attained in the two parts of a rope, the deflection or sag must be of predetermined values.

Fig. 661 is another diagram of

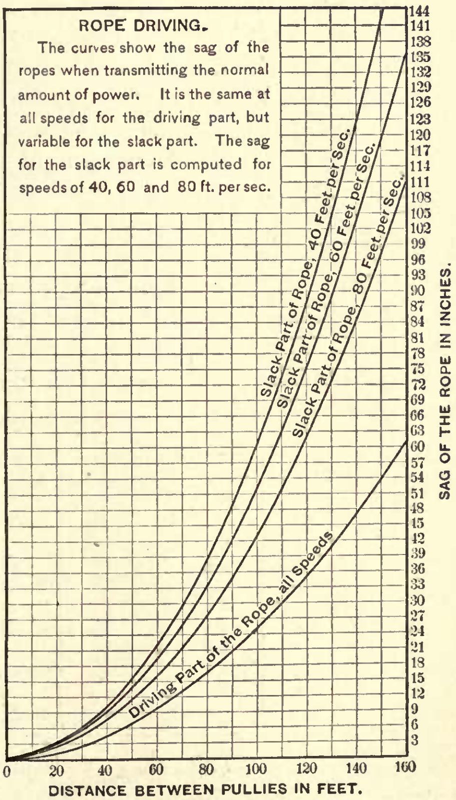


FIG. 661.

Mr. Hunt's showing the sag of the rope. The rope is supposed to have the strain constant at all speeds on the driving side and in direct proportion to the area of cross section, hence the catenary of the driving side is not affected by the speed or by the diameter of the rope.

The deflection between the pulleys on the slack side varies with each change of load or change of speed.

Having determined the sag of the rope from the diagram, lay off the pulleys as in Fig. 662, draw a horizontal line A B, and from the centre of this line and normal to it the line E C equal to the sag of the rope, and construct a parabola

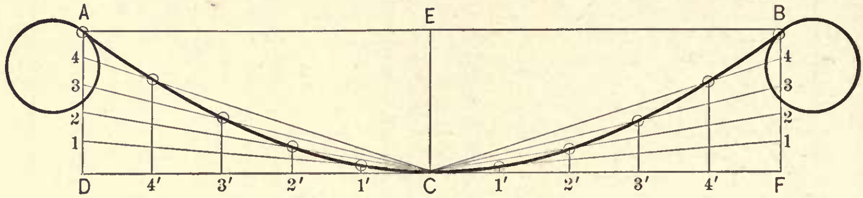


FIG. 662.

by dividing the line C D into, say, five equal parts and the line A D into the same number of equal parts; the intersections of the lines C1, C2, C3, etc., by the perpendiculars at 1′, 2′, 3′, etc., give the points of the curve.

The determination of the sag of ropes whose points of contact with their pulleys are not level may be determined by calculation; but as it can only be for one condition of speed and load, it will be sufficient to determine it practically by taking a cord equal to the whole distance between the points of contact, and that of the sag as given by the Hunt diagram, when the sag is central. Fix one end of the cord and raise the other to the level of the position it is to occupy in running, and the amount of sag and its position will be defined by the cord. If it is necessary to represent it by drawing, construct two parabolas.

Wire-rope power transmission is applicable for distances of from 50 to 400 feet, and withstands weather exposure. It is of much less diameter than hemp rope to transmit the same power and has more endurance. For endurance the diameters of wheel and rope and sag must be proportioned to each other. The table below is from the circular of the Trenton Iron Works Co., gives the proportionate diameters of wheels and ropes, and the H. P. transmission at 100 revolutions:

Diameter of wheel, in feet.	Diameter of rope, in inches.	H. P.	Diameter of wheel, in feet.	Diameter of rope, in inches.	H. P.	Diameter of wheel, in feet.	Diameter of rope, in inches.	H. P.
3	$\frac{3}{8}$	7	7	$\frac{9}{16}$	36	9	$\frac{3}{4}$	82
4	$\frac{3}{8}$	9	6	$\frac{5}{8}$	38	10	$\frac{3}{4}$	91
5	$\frac{3}{8}$	11	7	$\frac{5}{8}$	44	8	$\frac{7}{8}$	99
5	$\frac{7}{16}$	16	8	$\frac{5}{8}$	51	9	$\frac{7}{8}$	112
5	$\frac{1}{2}$	20	7	$\frac{11}{16}$	54	10	$\frac{7}{8}$	124
6	$\frac{1}{2}$	24	8	$\frac{11}{16}$	61	8	1	130
5	$\frac{9}{16}$	26	9	$\frac{11}{16}$	69	9	1	146
6	$\frac{9}{16}$	31	8	$\frac{3}{4}$	73	10	1	162

In applying this table for a given amount of horse power, preference should be given to the larger wheels as most serviceable.

If more H. P. is to be transmitted it may be obtained by increasing the revolution in a direct proportion up to the limit of 80 feet per second, but not by the increase of tension of rope as shown by sag, which may be taken for a span of 100 feet at ·7 feet, and for other spans directly as their squares—that is, for 200 feet it would be $·7 \times 4 = 2·8$ feet. The sag as given is that measured from a horizontal line through the point at which the rope leaves the wheel. If the two wheels are not on the same level the sag must be measured from the level of the point of contact with the lower wheel, and the span to be used in deter-

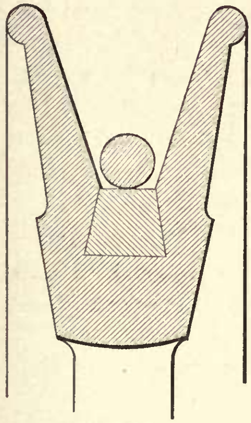


Fig. 663.

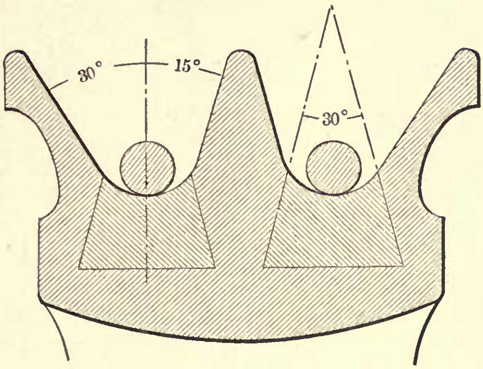


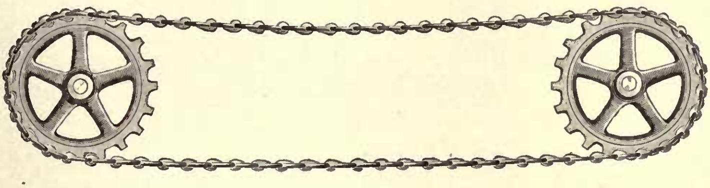
Fig. 664.

mining the sag below this level is the distance along the horizontal line from the wheel to the point at which it again intersects the rope. This point may be ascertained by hanging a wire in place on the wheels before splicing the rope.

If the difference of level is very great, run the rope over intermediate carrying sheaves so placed as to give a level stretch of rope of which the sag can be taken from the rule of sag.

Intermediate supporting pulleys should be avoided as far as practicable, as each one increases the wear on the rope. When they can not be dispensed with they should not be less than one half to two thirds the diameter of the main wheels.

The sag of the rope when doing full work will be one half that when at rest. The driving-sheaves should always be lined with some elastic material, as rubber, leather, or wood. Figs. 663 and 664 give the section of grooves as made at different works.



Power-transmission by Chains.—Fig. 665 is a sectional perspective of a pulley in which there is a groove for the vertical links of the chains and a face for the horizontal ones, which serve merely as guides for hoisting in cranes or saw-mills and the like. By the introduction of ribs on the face (Fig. 666) adapted to the length of the link or pitch of the chain the motion is determinate and is used to transmit power.

Fig. 667 is a sprocket wheel for

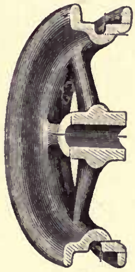


Fig. 665.

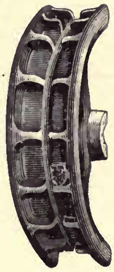


Fig. 666.

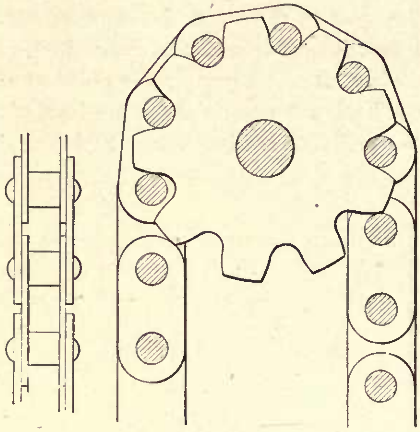


Fig. 667.

punched links with teeth between each link, especially adapted to position where the stress is great and the movement slow, for which they can readily be proportioned.

The same class of wheel and chains are made light and used as in bicycles, and driven at considerable speeds with little friction.

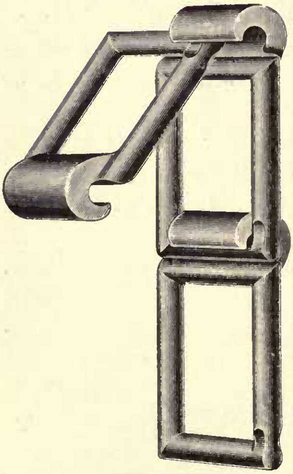


Fig. 668.

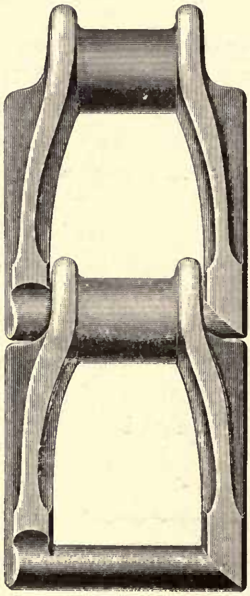


Fig. 669.

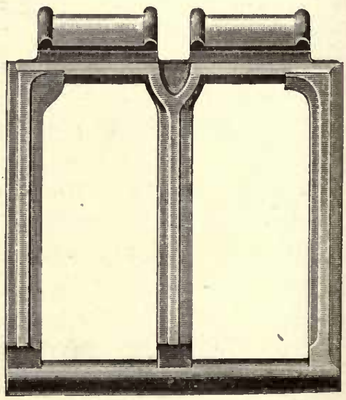


Fig. 670.

The link belting is made with malleable iron links and detachable, so that the belt can be readily lengthened or shortened. The sprocket wheels are machine finished to pitch.

Figs. 668, 669, and 670 are drawings of links of different forms. They are made of all dimensions to suit the purposes and stresses required. At the usual speeds of leather belts, link belting is noisy and wears rapidly, but at moderate speeds—for conveyors of grain, clay, and the like—they are admirably adapted, and in such positions and conditions of speed can be used to transmit power.

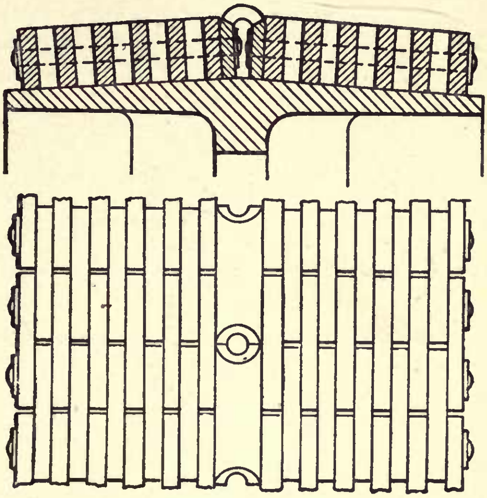


FIG. 671.

Leather link belting consists of links made of leather connected by iron or steel pins. A belt of this design can be made of any width, works freely on a pulley of small diameter, and can be driven at very high speed, which, combined with its great strength, make this form of belt very suitable for driving dynamos. When the link belt runs between guides, or on flanged pulleys, the rivet heads are faced on the outside links with leather after the belt is riveted up.

When a link belt of considerable width works on a curved pulley, either there is contact at the centre only, or the pins are bent where the band is in contact with the pulley. The belt in this case is in two or more longitudinal strips, hinged together, as shown in Fig. 671.

GEARING.

The term gearing, in a general sense, is applied to all arrangements for the transmission of power; but, in a particular sense, to toothed gearing, which may in general be divided into three classes—spur, bevel, and screw. In the former the axis of the driving and driven wheels are parallel to each other; in the bevel they may be situated at any angle; if of equal size and at right angles, they are called mitre gears. In screw gearing a toothed wheel is driven by a screw with their axes usually at right angles to each other. Spur wheels are termed external or internal, according to the disposition of their teeth with regard to the rim of the wheel.

Rack gear and pinion are employed to convert a rotary into a rectilinear motion, or *vice versa*. In this arrangement the pinion is a spur wheel, acting on teeth placed along a straight bar (Fig. 680).

Bevel gearing consists of toothed wheels formed to work together in different planes, their teeth being disposed at an angle to the plane of their faces.

Trundle pins or wheels (Fig. 672) are constructed with cylindrical pieces called staves or pins, instead of teeth. A pinion with double plates is called a lantern; the wheel, a face or crown wheel; this construction is very useful when iron gears can not be easily obtained or repaired.

Fundamental principle.—In order that two circles A and B (Fig. 673) may be made to revolve by the contact of the surfaces of the curves *m m* and *n n* of their teeth precisely as they would by the friction of their circumferences, it is necessary and sufficient that a line drawn from the point of contact t of the teeth to the point of contact c of the circumferences (pitch-circles) should, in

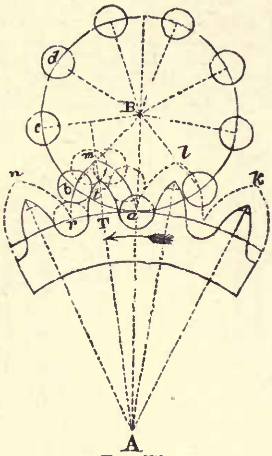


FIG. 672.

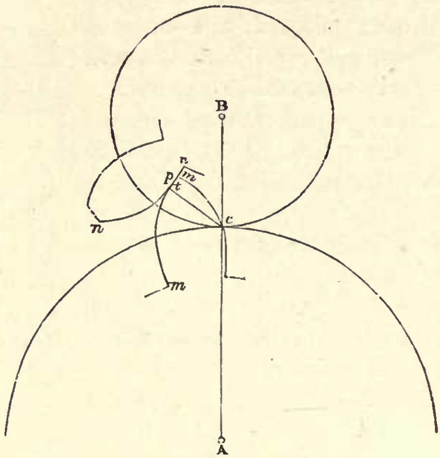


FIG. 673.

every position of the point t, be perpendicular or normal to the surfaces of contact at that point to both the curves $m\,m$ and $n\,n$, a particular property of that curve known as the *cycloid.*

When one wheel conducts the other, it is called the driver or leader, and the other the driven or follower. The angular velocities of the pitch circles is the same, but the number of revolutions of the wheels is inversely as their diameters taken at the pitch circles. If the driver is 24″ diameter, making 120 revolutions per minute, and the driver is required to make 200 revolutions per minute, then the diameter of the driven gear would be

$$\frac{120 \times 24}{200} = 14\frac{4''}{10}.$$

Fig. 674 gives the designation of the various parts of a spur wheel by which names they will hereafter be called.

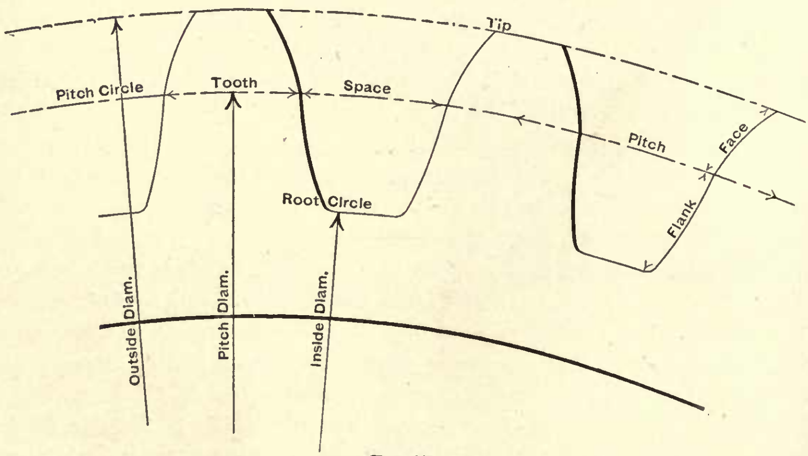


FIG. 674.

There is considerable variation in the proportion of teeth, as—
Thickness of teeth, from ·45 to ·48 pitch.
Space between teeth, from ·55 to ·52 pitch.

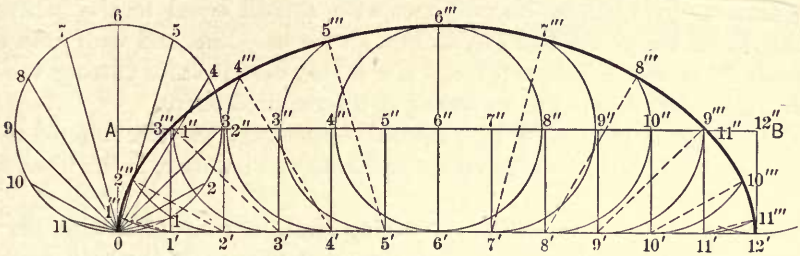


FIG. 675.

Height of teeth outside pitch circle, from ·2 to ·3 pitch.
Depth of teeth inside pitch circle, from ·3 to ·4 pitch.

The above is for cast teeth, used without finishing; the teeth are made narrower than the space, and the height less than the depth on account of the irregularities of a rough casting. The teeth and space, when machine cut, may be made the same or very nearly so.

The cycloid (Fig. 675) is a curve described by any point on the circumference of a circle on a straight line as a base which is the pitch line of the rack in its application to the formation of teeth of a rack and pinion.

Divide the circumference of the generating or rolling circle A into a num-

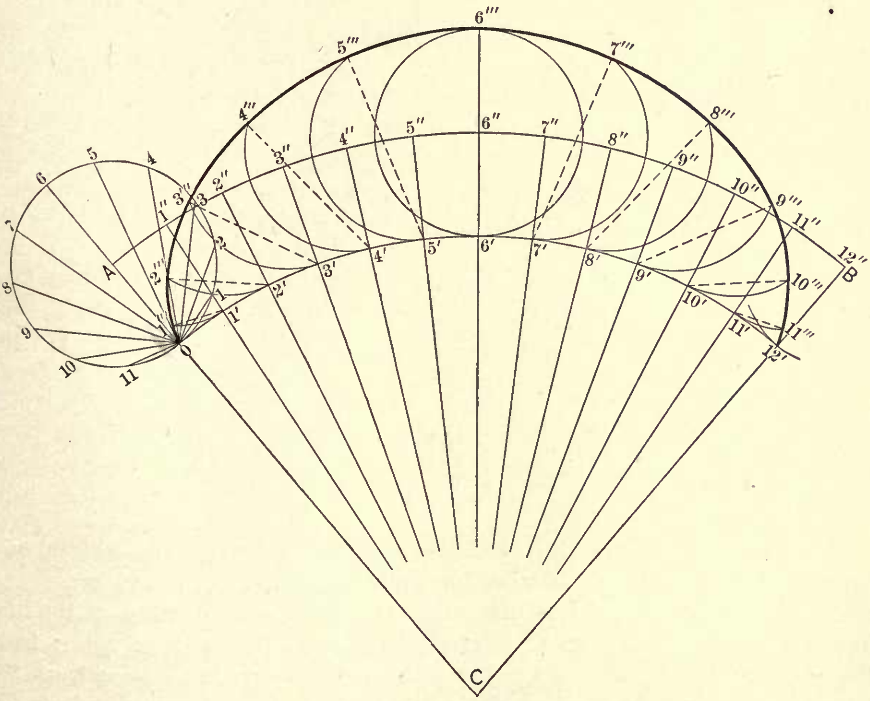


FIG. 676.

ber of equal parts, say 12; draw chords from each of these points to 0; divide the base line into an equal number of parts of the same length as the arcs of the generating circle numbered 0, 1′, 2′, etc., and from each of these points erect a perpendicular intersecting the centre of the circle A B at 1″, 2″, etc.; from each of these points describe arcs with a radii equal to the generating circle. From the points 1′, 2′, 3′, 4′, etc., on the base line, and with radii equal successively to the chords 0 1, 0 2, 0 3, 0 4, etc., describe arcs cutting the preceding, the intersections will be points of the required curve.

The epicycloid (Fig. 676) is a cycloid formed on the circumference of a circle as base, which, in its application to the teeth of wheels, is the pitch circle of external gearing.

The path of the centre of the generating circle is concentric with the base circle. Divide the generating circle from 0 and the arc of the base circle into the same number of equal parts. Radial lines from C, passing through 1′, 2′, etc., and intersecting A B, give the points from which the arcs of the generating circle are described; from the points 1′, 2′, 3′, etc., on the base circle, and with radii equal successively to the chords 0 1, 0 2, 0 3, etc., describe arcs cutting the preceding; the intersections are points in the required curve.

The hypocycloid (Fig. 677) is a cycloid with a base of the interior circumference of a circle corresponding to the pitch circle of internal gearing.

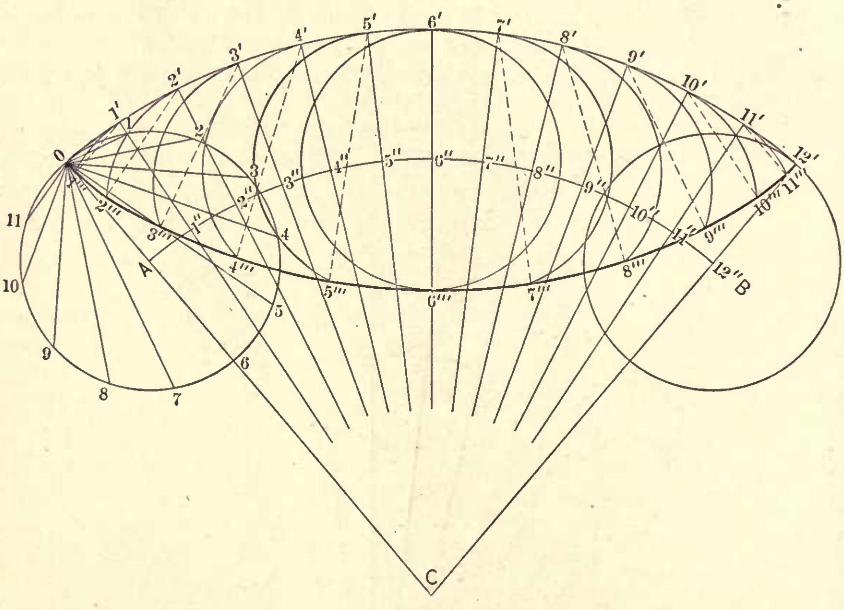


Fig. 677.

Proceed as in the previous example and divide the generating circle and base line into equal parts, describe the path of the centre of the generating circle A B concentric with the base circle, and from each division of the base line draw radial lines from C, intersecting the line A B at 1″, 2″, etc.; from these centres describe arcs of a radius equal to the generating circle; from the points 1′, 2′, 3′, etc., on the base circle, and with radii equal successively to the chords

chords 0 1, 0 2, 0 3, etc., describe arcs cutting the preceding; the intersections are points of the required curve.

INVOLUTE.

The involute (Fig. 678) is the curve described by the end of a string being unwound from the circumference of a circle.

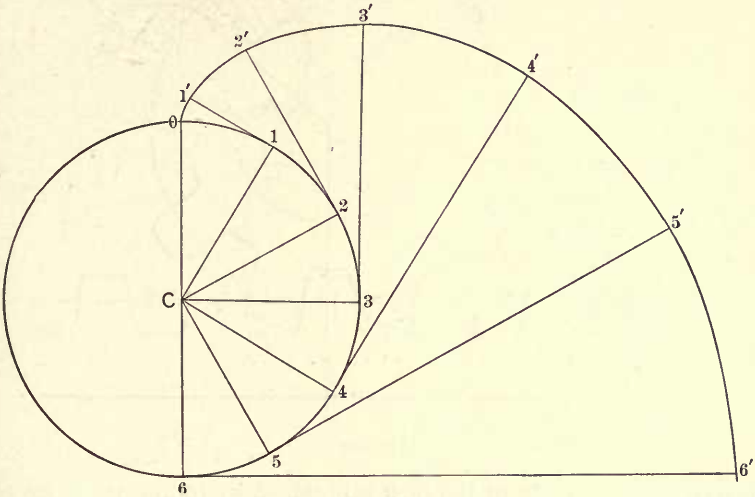


FIG. 678.

Divide the circumference of the given circle into any number of equal parts, as 0, 1, 2, etc. At each of these points draw tangents to the given circle; on the first of these lay off the distance 1–1′, equal to the arc 0–1; on the second lay off 2–2′, equal to twice the arc 0–1 or the arc 0–2; establish in a similar way the points 3′, 4′, 5′, as far as may be necessary, which are points in the required curve.

The involute curve may be described mechanically in several ways. Thus, let A (Fig. 679) be the centre of a wheel for which the form of involute teeth is to be found. Let *m n a* be a thread lapped round its circumference, having a loop-hole at its extremity, *a*; in this fix a pin, with which describe the curve or *involute a b . . . . h*, by unwinding the thread gradually from the circumference, and this curve will be the proper form for the teeth of a wheel of the given diameter.

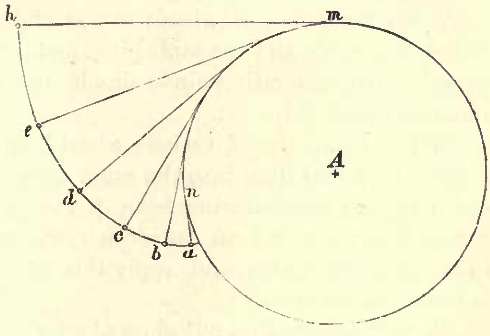


FIG. 679.

In all the problems in which curves have been determined by the position of points, the more numerous the points the more accurately can the curve be drawn.

Spur Wheel and Rack (Fig. 680).—The pitch-circle of the spur wheel is drawn and the proper curve for the flank and face of the teeth obtained by rolling a generating circle on the inside and outside of the pitch-circle; thus the point A′ on the generating circle gives for the flank of the teeth the hypocycloid A′ B′, and for the face the epicycloid A′ C′.

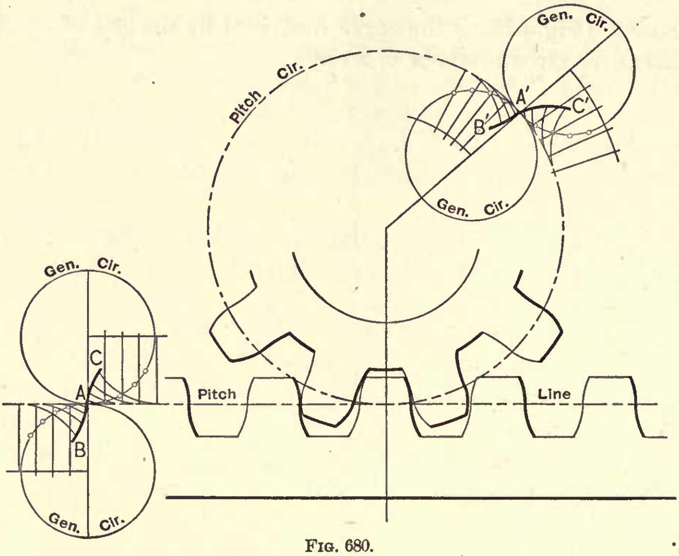


Fig. 680.

The curve for the teeth of the rack is obtained by rolling the same generating circle on the upper and lower side of the rack pitch-line, giving two cycloids, A C and A B, for the face and flank of the teeth. The diagram showing the construction of above curves is, to avoid confusion, separate from the drawing of the spur wheel and rack.

The diameter of the generating circle is to a certain extent arbitrary. In this and the following examples it is taken at a little less than the radius of the smallest spur wheel; if taken at exactly the radius, the flanks of the teeth of this wheel will be radial lines, which is not usually as satisfactory as where the flanks are of hypocycloidal form, as the teeth, being narrower at the root, have a tendency to break at this point.

When a number of wheels are intended to gear together, the same size of generating circle and the same pitch and dimensions of the teeth must be maintained. The generating circle should never be larger than the radius of the smallest wheel of the set.

Where a drawing of a whole wheel is to be made, the circumference can be divided by radial lines into the same number of parts as there are teeth. The curve of one tooth having been found, a templet can be made and applied successively; or find an arc of a circle corresponding as closely as possible to the cycloidal curve, and apply this at the proper divisions; the latter way is the more common.

Spur Wheels.—Fig. 681 shows two wheels gearing together, one of ten and the other of thirty teeth. The diameter of the generating circle is less than the radius of the pitch-circle of the smaller wheel; when the generating circle rolls on the exterior and interior of both pitch-circles, it will in the former case generate the faces of the teeth as shown at A C and A′ C′, and in the latter case their flanks A B and A′ B′. In other respects the construction is similar to the former example.

The simplest illustration of the action of epicycloidal teeth is when they are employed to drive a trundle, as represented in Fig. 672. Let it be assumed that

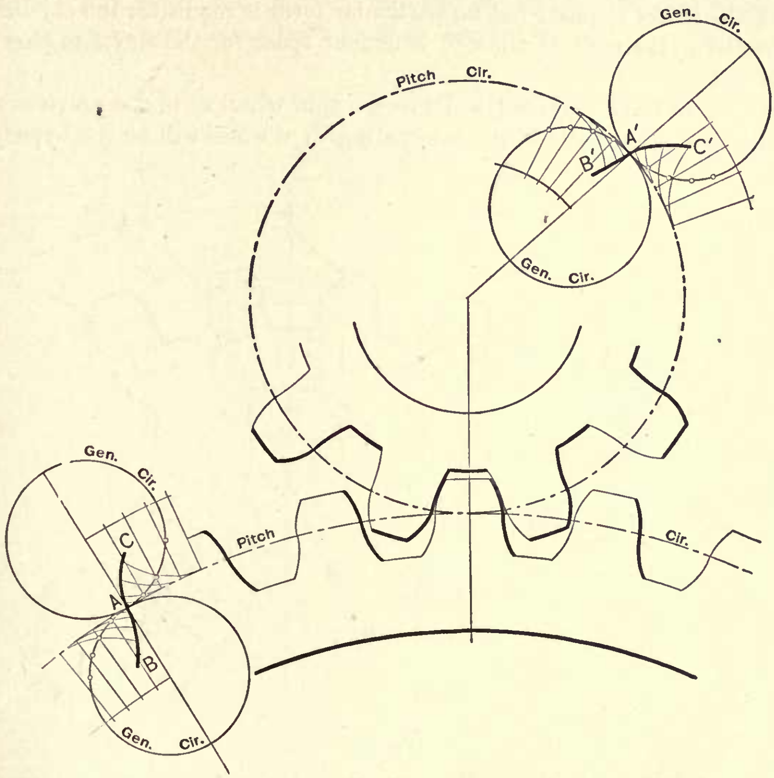


Fig. 681.

the staves of the trundle have no sensible thickness; that the distance of their centres apart, that is their *pitch*, and also their distance from the centre of the trundle, that is their pitch-circle, are known. The pitch-circles of the trundle and wheel being then drawn from their respective centres B and A, set off the pitches upon these circumferences, corresponding to the number of teeth in the wheel and number of staves in the trundle; let five pins, *a b c*, etc., be fixed into the pitch-circle of the trundle to represent the staves, and let a series of epicycloidal arcs be traced with a describing circle, equal in diameter to the radius of the pitch-circle of the trundle, and meeting in the points *k l m n*, etc., alternately from right and left. If motion be given to the wheel in the direction of the arrow, then the curved face *m r* will press against the pin *b*, and move it in the same direction; but as the motion continues, the pin will slide upward till it reaches *m*, when the tooth and pin will quit contact. Before this happens, the next pin *a* will have come into contact with the face *a l* of the next tooth, which, repeating the same action, will bring the succeeding pair into contact; and so on continually.

To allow of the required thickness of staves, it is sufficient to diminish the size of the teeth of the wheel by a quantity equal to the radius of the staves (sometimes increased by a certain fraction of the pitch for clearance) by drawing within the primary epicycloids, at the required distance, another series of curves parallel to these. In practice, a portion must be cut from the points of the teeth, and also a space must be cut out within the pitch-circle of the driver, to allow the staves to pass; but no particular form is requisite; the condition to be attended to is simply to allow of sufficient space for the staves to pass without contact.

Internal Gearing (Fig. 682).—Draw the spur wheel as in the previous examples; the face of the teeth of the internally geared wheel will be the hypocycloid

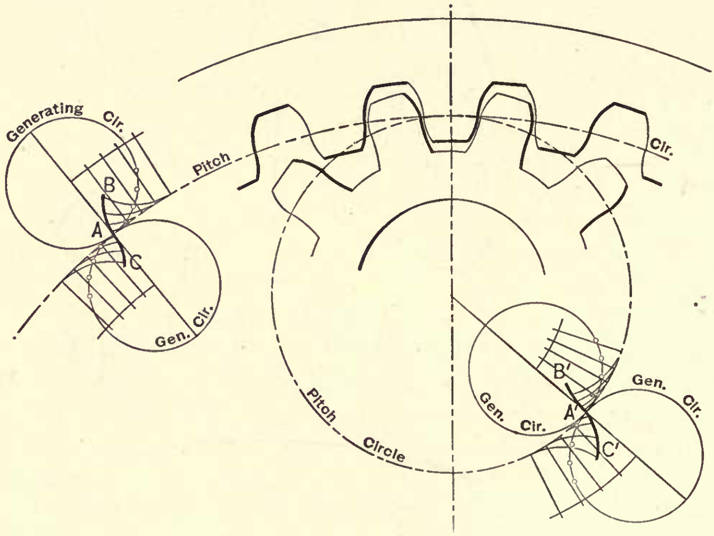


FIG. 682.

A C, formed by the generating circle rolling on the interior of the pitch-circle, and the flanks the epicycloid A B, formed by the generating circle rolling on the exterior of the pitch-circle.

INVOLUTE TEETH.

Fig. 683 is a pair of spur wheels showing the mode of drawing involute teeth. The addendum, pitch, and root-circles of both wheels are first drawn, then the base-circles of the two wheels, which must bear the same proportion to each other as the pitch-circles. To arrive at this proportion a semicircle is constructed on the line A C B with a diameter equal to the radius A C of the pinion; a perpendicular erected where the larger addendum-circle intersects the line A C B intersects the semicircle at E; a line is then drawn from A to E and a normal to it at E of indefinite length; then from B a parallel to A E intersecting the normal at E′; circles concentric to the pitch-circles drawn through E and E′ give the base-circles.

All teeth on the line E E′ will be in contact at the same time, called, therefore, the line of contact. The involute curves of the teeth of both wheels are drawn from their respective base-circles, which correspond to the given circle of

Fig. 678. Both involutes in Fig. 683 start from the point of contact E, hence one curve is described forward, the other backward.

Thus on the normal E′ E there are eight parts, and from the commencement of the next normal seven parts are marked off, the next six, and so on until finally the involute reaches the base-circle. The part of the tooth within the base-circle may be a radius to it and tangent to the involute, and small fillets should be drawn connecting their roots to the root-circle.

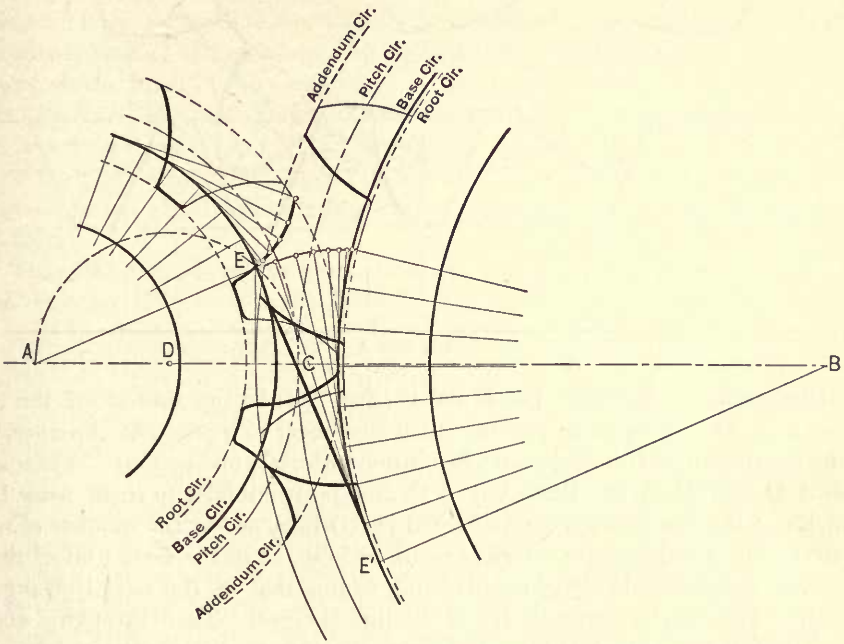


Fig. 683.

A pinion gearing into a rack is shown in Fig. 684. The faces of the teeth of the rack can be taken at an angle of 23° with the perpendicular; this angle gives the longest line of contact, which may be considered the pitch-line of the rack, and, the pitch-circle of the pinion being indicated, the base-circle for the construction of the involute curves is obtained by drawing a line through the point C′ where the two pitch-lines come together at an angle of 23° with the horizontal and where a normal from this line intersects the centre C of the pinion a line is drawn, and through its junction E of the line of contact a base-circle is described; at this point E the involute is drawn as in the previous example. As in cycloidal teeth, the exact curve may be laid down for one tooth, and an arc corresponding as closely as possible to the involute used to describe the remainder.

All involute wheels whose teeth have the same pitch and obliquity to the line of contact work well together, but no wheel should have less than twelve teeth to work well.

Involute wheels can not be made with very long teeth, because the obliquity of the line of contact will be too great.

The diameters of spur wheels are in proportion to the number of their revolutions per minute, but the relative sizes of a pair of bevel wheels is determined by a division of the angle included between the two axes inversely as the ratio

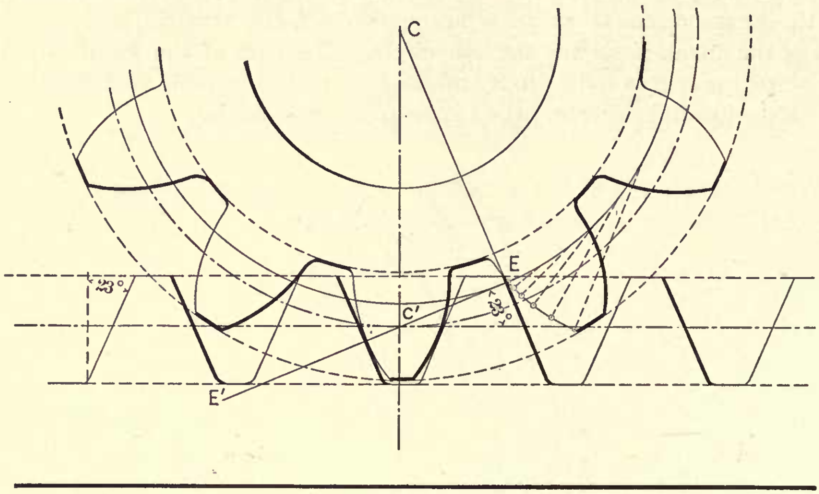


FIG. 684.

of their angular velocities. Let B and C (Fig. 685) be the position of the two given axes, and let them be prolonged till they meet in a point A. Further, let it be required that C makes *seven* revolutions while B makes *four*. From any points D and E in the lines A B, A C, and perpendicular to them, draw D *d* and E *e* of lengths (from a scale of equal parts) inversely as the number of revolutions which the axes are severally required to make in the same unit of time. Thus, the angular velocity of axis B being 4, and that of the axis C being 7, the line D *d* must be drawn = 7, and the line E *e* = 4. Then through *d* and *e* parallel with the axes A B and A C draw *d c* and *e c* till they meet in *c*. A straight line drawn from A through *c* will then make the required division of the angle B A C, and define the line of contact of the two cones, by means of which the two rolling frusta may be projected at any convenient distance from A.

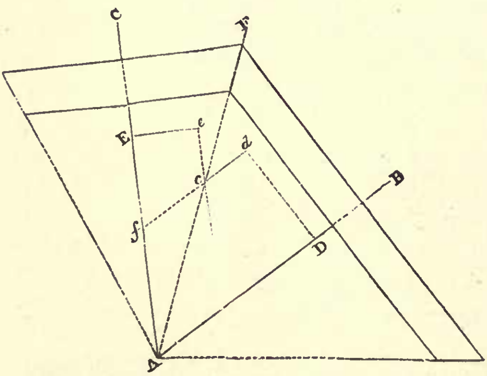


FIG. 685.

If the relative perimeters, diameters, or radii, of the pair have been determined, then the lines D *d* and E *e* are to each other directly as these quantities. B F and C F are radii of the pitch-circle.

The case in which the axes are neither parallel nor intersecting admits of solution by means of a pair of bevels upon an intermediate axis, so situated as to meet the others in any convenient points.

When the contiguity of the shafts is such as to permit of their being connected by a single pair, skewed bevels (Fig. 702) are sometimes employed.

When the axes are at right angles to each other, and do not intersect, the wheel and screw may be employed to connect them. The velocity of motion is in this arrangement immediately deduced from that of the screw, its number of threads, and the number of teeth in its gearing-wheel. Thus, if it be required to transmit the motion of one shaft to another, contiguous and at right angles to it—the angular motions being as 20 to 1—then, if the screw be a single-threaded one, the wheel must have 20 teeth ; but if double-threaded, the number of teeth will be increased to 40, for 2 teeth will be passed at every revolution. If the screw have few threads compared with the number of teeth of the wheel, it must always assume the position of driver on account of the obliquity of the thread to the axis ; and in this respect its action is analogous to that of a travelling rack, moving endwise one tooth, while the screw makes one revolution on its axis.

If the pitch-circle be divided into as many equal parts as there are teeth to be given to the wheel, the length of one of these parts is termed the pitch of the teeth.

The pitch depends on the power to be transmitted or the stress on each tooth. The diagram (Fig. 686) is by John T. Henthorn, M. E., in which pitch and

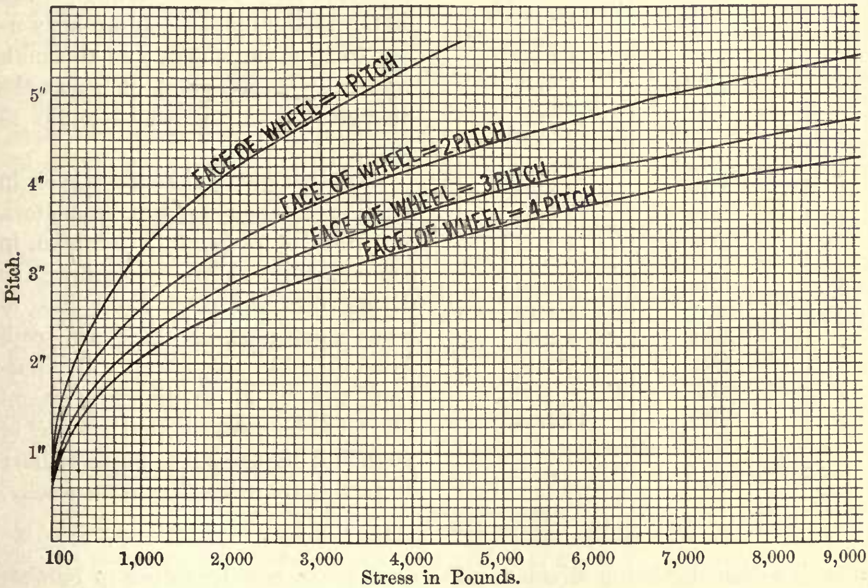


FIG. 686.

face, represented by multiples of the pitch, are proportioned to the stress in pounds.

If the pitch be known, the number of teeth in a wheel can be determined approximately by dividing the circumference of the wheel by the pitch, but there must be no remainder in the quotient—there can be no fraction of a pitch —either the pitch or diameter of wheel must be changed to produce this result ; generally the latter, as gears are usually made of determinate inches and fractions, as given in the table, by which also calculation for diameters and number of teeth is much simplified.

	$D = \frac{P}{\pi} \times N.$	$N = \frac{\pi}{P} \times D.$
PITCH IN INCHES AND PARTS OF AN INCH.	RULE.—To find the diameter in inches, multiply the number of teeth by the tabular number answering to the given pitch.	RULE.—To find the number of teeth, multiply the given diameter in inches by the tabular number answering to the given pitch.
Values of P.	Values of $\frac{P}{\pi}$	Values of $\frac{\pi}{P}$
6	1·9095	·5236
5	1·5915	·6283
$4\frac{1}{2}$	1·4270	·6981
4	1·2732	·7854
$3\frac{1}{2}$	1·1141	·8976
3	·9547	1·0472
$2\frac{3}{4}$	·8754	1·1333
$2\frac{1}{2}$	·7958	1·2566
$2\frac{1}{4}$	·7135	1·3963
2	·6366	1·5708
$1\frac{7}{8}$	·5937	1·6755
$1\frac{3}{4}$	·5570	1·7952
$1\frac{5}{8}$	·5141	1·9264
$1\frac{1}{2}$	·4774	2·0944
$1\frac{3}{8}$	·4377	2·2848
$1\frac{1}{4}$	·3979	2·5132
$1\frac{1}{8}$	·3568	2·7926
1	·3183	3·1416
$\frac{7}{8}$	·2785	3·5904
$\frac{3}{4}$	·2387	4·1888
$\frac{5}{8}$	·1989	5·0266
$\frac{1}{2}$	·1592	6·2832
$\frac{3}{8}$	·1194	8·3776
$\frac{1}{4}$	·0796	12·5664

Example 1.—Given a wheel of 88 teeth, $2\frac{1}{2}$-inch pitch, to find the diameter of the pitch-circle. Here the tabular number in the second column answering to the given pitch is ·7958, which multiplied by 88 gives 70·03 for the diameter required.

2. Given a wheel 33 inches diameter, $1\frac{3}{4}$-inch pitch, to find the number of teeth. The corresponding factor is 1·7952, which, multiplied by 33, gives 59·242 for the number of teeth—that is, $59\frac{1}{4}$ teeth nearly. Now 59 would here be the nearest whole number, but as a wheel of 60 teeth may be preferred for convenience of calculation of speeds, we may adopt that number, and find the diameter corresponding. The factor in the second column answering to $1\frac{3}{4}$ pitch is ·557, and this multiplied by 60 gives 33·4 inches as the diameter which the wheel ought to have.

Another mode of sizing wheels in relation to their pitches, diameters, and number of teeth, is adopted, in some machine shops, by dividing the diameter of the pitch-circle into as many equal parts as there are teeth to be given to the wheel. To illustrate this by an arithmetical example, let it be assumed that a wheel of 20 inches diameter is required to have 40 teeth; then the *diametral* pitch,

$$\frac{20}{40} = \frac{1}{m} = \frac{1}{2} \text{ inch};$$

that is, the diameter being divided into equal parts corresponding in number to the number of teeth in the circumference of the wheel, the length of each of these parts is $\frac{1}{2}$ an inch, consequently $m = 2$; and according to the phraseology of the workshop, the wheel is said to be one of *two pitch*.

A decided advantage is obtained by the use of the diametral-pitch system, since circular pitch $= \frac{\text{diam.} \times 3{\cdot}1416}{\text{No. of teeth}}$, diam. $= \frac{\text{circ. pitch} \times \text{No. of teeth}}{3{\cdot}1416}$, which always brings out the diameter as a number with an inconvenient fraction if the pitch is in even inches or simple fractions of an inch. By the diametral-pitch system the diameter may be in even inches or convenient fractions, and the number of teeth is usually an even multiple of the number of inches in the diameter.

RELATION OF DIAMETRAL TO CIRCULAR PITCH.

Diametral pitch.	Circular pitch.	Diametral pitch.	Circular pitch.	Circular pitch.	Diametral pitch.	Circular pitch.	Diametral pitch.
	In.		In.		In.		In.
1	3·142	11	0·286	3	1·047	15/16	3·351
1½	2·094	12	0·262	2½	1·257	7/8	3·590
2	1·571	14	0·224	2	1·571	13/16	3·867
2¼	1·396	16	0·196	1⅞	1·676	¾	4·189
2½	1·257	18	0·175	1¾	1·795	11/16	4·570
2¾	1·142	20	0·157	1⅝	1·933	⅝	5·027
3	1·047	22	0·143	1½	2·094	9/16	5·585
3½	0·898	24	0·131	1 7/16	2·185	½	6·283
4	0·785	26	0·121	1⅜	2·285	7/16	7·181
5	0·628	28	0·112	1 5/16	2·394	⅜	8·378
6	0·524	30	0·105	1¼	2·513	5/16	10·053
7	0·449	32	0·098	1 3/16	2·646	¼	12·566
8	0·393	36	0·087	1⅛	2·793	3/16	16·755
9	0·349	40	0·079	1 1/16	2·957	⅛	25·133
10	0·314	48	0·065	1	3·142	1/16	50·266

To find the outside diameter of spur-gear blanks, add two parts of the pitch to the pitch diameter. Thus for an 8-pitch gear of 40 teeth the outside diameter of blank is 42/8, equal to 5¼ inches; for a 12-pitch gear of 36 teeth the outside diameter of blank is 38/12, equal to 3⅙ inches; for a 16-pitch gear of 46 teeth the outside diameter of blank is 48/16, equal to 3 inches.

To obtain the distance between the centres of two gears, add the number of teeth together and divide half the sum by the diametral pitch. Thus if two gears have 40 and 30 teeth respectively and are 5 pitch, add 40 and 30, making 70, divide by 2, and then divide this quotient 35 by the diametral pitch 5, and the result, 7 inches, is the distance between centres.

It is a common practice of shops to take as the diameter of the rolling circle the radius of the smallest pinion which will ever be used for gears of this pitch, and constructing the epicycloids for different diameters of this pitch, and allowing arcs of circles corresponding very closely to these epicycloids. On this principle Robert Adcock, C. E., constructed a table of radii for these arcs, for rolling circles of pinions of 8, 10, and 12 teeth. We give the last only as best suited to the usual conditions of practice :

TABLE OF RADII OF ARCS OF CIRCLES FOR GEAR TEETH.

No. of teeth.	Radius of pitch-circle.	Smallest pinion, twelve teeth.				No. of teeth.	Radius of pitch-circle.	Smallest pinion, twelve teeth.				No. of teeth.	Radius of pitch-circle.	Smallest pinion, twelve teeth.			
		Radii of the faces of teeth.		Radii of the flanks of teeth.				Radii of the faces of teeth.		Radii of the flanks of teeth.				Radii of the faces of teeth.		Radii of the flanks of teeth.	
12	1·93	1·88	0·75			27	4·31	4·23	0·84	4·63	1·41	42	6·69	6·60	0·89	6·91	1·20
13	2·09	2·04	0·76	7·45	7·14	28	·46	·39	·85	·37	·38	43	·85	·76	·89	7·06	·20
14	2·25	·19	·77	4·86	4·27	29	·62	·55	·85	·92	·36	44	7·01	92	·89	·22	·19
15	2·40	·35	·78	3·92	3·04	30	·78	·70	·86	5·07	·34	45	·17	7·07	·89	·38	·18
16	2·56	·50	·78	·62	3·53	31	·94	·86	·86	·21	·32	46	·33	·23	·90	·53	·18
17	2·72	·66	·79	·58	2·22	32	5·10	5·02	·86	·37	·30	47	·49	·39	·90	·09	·17
18	2·88	·82	·80	·59	2·02	33	·26	·18	·86	·52	·29	48	·64	·55	·90	·84	·16
19	3·04	·97	·81	·63	1·87	34	·42	·34	·87	·67	·28	49	·80	·71	·90	8·00	·16
20	3·20	3·13	·81	·73	1·76	35	·58	·49	·87	·82	·26	50	·96	·86	·90	·96	·16
21	3·35	·29	·82	·83	1·68	36	·74	·65	·87	·97	·25	51	8·12	8·02	·91	·31	·16
22	3·51	·44	·82	·95	1·61	37	·90	·81	·88	6·13	·24	52	·28	·18	·91	·47	·1[illegible]
23	3·67	·60	·83	4·07	1·56	38	6·05	·97	·88	·29	·23	53	·44	·34	·91	·63	·1[illegible]
24	3·83	·76	·83	·21	1·51	39	·21	6·13	·88	·44	·23	54	·60	·50	·91	·79	·14
25	3·99	·91	·84	·34	1.47	40	·37	·28	·88	·60	·22	55	·76	·66	·91	·95	·14
26	4·15	4·07	·84	·48	1·44	41	·53	·44	·89	·75	·21	56	·92	·81	·91	9·10	·14

No. of teeth.	Radius of pitch-circle.	SMALLEST PINION, TWELVE TEETH. Radii of the faces of teeth.		Radii of the flanks of teeth.	
57	9·08	8·97	0·91	9·26	1·13
58	·23	9·13	·91	·42	·13
59	·37	·29	·92	·57	·13
60	·55	·45	·92	·73	·13
61	·71	·61	·92	·89	·12
62	·87	·77	·92	10·05	·12
63	10·03	·92	·92	·20	·12
64	·19	10·08	·92	·36	·12
65	·35	·24	·92	·52	·12
66	·51	·40	·92	·68	·11
67	·67	·56	·92	·84	·11
68	·83	·72	·92	·99	·11
69	·98	·88	·93	11·15	·11
70	11·14	11·04	·93	·31	·11
71	·30	11·20	·93	·47	1·10
72	·46	·35	·93	·63	·10
73	·62	·51	·93	·79	·10
74	·78	·67	·93	·95	·10
75	·94	·83	·93	12·10	·10
76	12·10	11·99	·93	·26	·09
77	·26	12·15		·42	·09
78	·42	·30	·93	·58	·09
79	·58	·47		·74	·09
80	·73	·63	·93	·90	·09
81	·89	·79		13·06	·09
82	13·05	·94	·93	·22	·09
83	·21	13·10		·38	·09
84	·37	·26	·94	·53	·08
85	·53	·42	·94	·69	·08
86	·69	·58		·85	·08
87	·85	·74	·94	14·01	·08
88	14·01	·90	·94	·17	·08
89	·17	14·06		·33	·08
90	·33	·22	·94	·49	·08
91	·49	·38	·94	·65	·08
92	·64	·53		·81	·08
93	·80	·69	·94	·97	·08
94	·96	·85	·94	15·12	·07
95	15·14	15·01		·30	·07
96	·28	·17	·94	·44	·07
97	·44	·33		·60	·07
98	·60	·49	·94	·76	·07
99	·76	·65		·92	·07
100	·92	·81	·95	16·08	·07
101	16·08	·97		·24	·07
102	·24	16·13		·40	
103	·39	·28	·95	·56	·07
104	·55	·44		·72	
105	·71	·60		·87	·07
106	·87	·76	·95	17·03	
107	17·03	·92		·19	·06
108	·19	17·08		·35	
109	·35	·24	·95	·51	·06
110	·51	·40		·67	
111	·67	·56		·83	·06
112	·83	·71	·95	·99	
113	·99	·87		18·15	·06
114	18·15	18·03	·95	·30	
115	·30	·19		·46	·06
116	·46	·35		·26	
117	·62	·51	·95	·62	·06
118	·78	·67		·78	
119	·94	·83	·95	·94	1·06

No. of teeth.	Radius of pitch-circle.	SMALLEST PINION, TWELVE TEETH. Radii of the faces of teeth.		Radii of the flanks of teeth.	
120	19·10	18·99		19·10	
121	·26	19·15		·42	1·06
122	·42	·30	·95	·57	
123	·58	·46		·73	·06
124	·74	·62		·89	
125	·90	·78	·95	20·05	·06
126	20·05	·94		·21	
127	·21	20·10		·37	·05
128	·37	·26	·96	·53	
129	·53	·42		·69	·05
130	·68	·58		·84	
131	·84	·74	·96	21·00	·05
132	21·00	·89		·16	
133	·17	21·05		·32	·05
134	·33	·21	·96	·48	
135	·49	·37		·64	·05
136	·65	·53	·96	·80	
137	·81	·69		·96	·05
138	·96	·85		22·11	
139	22·12	22·01	·96	·27	·05
140	·28	·17		·43	·05
141	·44	·33		·59	
142	·60	·48	·96	·75	·05
143	·76	·64		·91	
144	·92	·80		23·07	·05
145	23·08	·96	·96	·23	
146	·24	23·12		·38	1·04
147	·40	·28		·54	
148	·56	·44		·70	·04
149	·72	·60	·96	·86	
150	·87	·76		24·02	·04
151	24·03	·92		·18	
152	·19	24·07	·96	·34	
153	·35	·23		·50	·04
154	·51	·39		·65	
155	·67	·55	·96	·81	·04
156	·83	·71		·98	
157	·99	·87		25·13	·04
158	25·15	25·03	·97	·29	
159	·31	·19		·45	·04
160	·47	·35		·61	
161	·62	·51	·97	·77	
162	·78	·66		·93	·04
163	·94	·82		26·09	
164	26·10	·98	·97	·25	·04
165	·26	26·14		·42	
166	·42	·30		·56	·04
167	·58	·46		·72	
168	·74	·62	·97	·88	·03
169	·90	·78		27·04	
170	27·06	·94		·22	·03
171	·22	27·10	·97	·36	
172	·38	·25		·52	·03
173	·53	·41		·68	
174	·69	·57	·97	·84	
175	·85	·73		1·00	·03
176	28·01	·89		28·16	
177	·17	28·05	·97	·31	·03
178	·33	·21		·47	
179	·48	·37		·63	
180	·64	·53	·97	·79	·03
181	·80	·69		·95	
182	·97	·84		29·11	1·03

No. of teeth.	Radius of pitch-circle.	SMALLEST PINION, TWELVE TEETH. Radii of the faces of teeth.		Radii of the flanks of teeth.	
183	29·13	29·00	·97	29·27	
184	·28	·16		·43	1·03
185	·44	·32		·59	
186	·60	·48	·97	·74	1·03
187	·76	·64		·90	
188	·92	·80		30·06	
189	30·08	·96	·98	·22	1·03
190	·24	30·12		·38	
191	·40	·28	·98	·54	1·02
192	·55	·43		·70	
193	·71	·59		·86	
194	·87	·75	·98	31·02	1·02
195	31·03	·91		·18	
196	·19	31·07		·33	1·02
197	·35	·23	·98	·49	
198	·51	·39		·65	1·02
199	·67	·55		·81	
200	·83	·71	·98	·97	
201	·99	·87		32·13	
202	32·15	32·02		·29	
203	·30	·18		·45	
204	·46	·34		·61	
205	·62	·50		·77	
206	·78	·66		·92	
207	·94	·82		33·08	
208	33·10	·98		·24	
209	·26	33·14		·40	
210	·42	·30		·56	
211	·58	·46		·72	
212	·74	·61		·88	
213	·90	·77		34·04	
214	34·06	·93		·20	
215	·21	34·09		·36	
216	·37	·25		·51	
217	·53	·41		·67	
218	·69	·57		·83	
219	·85	·73		·99	
220	35·01	·89		35·15	
221	·17	35·05		·31	
222	·33	·20		·47	
223	·49	·36		·63	
224	·65	·52		·79	
225	·80	·68		·95	
226	·96	·84		36·10	
227	36·12	36·00		·26	
228	·28	·16		·42	
229	·44	·32		·58	
230	·59	·48		·74	
231	·75	·64		·90	
232	·91	·79		37·06	
233	37·08	·95		·22	
234	·24	37·11		·58	
235	·40	·27		·54	
236	·56	·43		·69	
237	·72	·59		·85	
238	·87	·75		38·01	
239	38·03	·91		·17	
240	·19	38·07		·33	
241	·35	·23		·69	
242	·51	·38		·85	
243	·67	·54		39·01	
244	·83	·70		·17	
245	·99	·86		·33	

No. of teeth.	Radius of pitch-circle.	SMALLEST PINION, TWELVE TEETH. Radii of the faces of teeth.	Radii of the flanks of teeth.	No. of teeth.	Radius of pitch-circle.	SMALLEST PINION, TWELVE TEETH. Radii of the faces of teeth.	Radii of the flanks of teeth.	No. of teeth.	Radius of pitch-circle.	SMALLEST PINION, TWELVE TEETH. Radii of the faces of teeth.	Radii of the flanks of teeth.
246	39·15	39 02	39·28	265	42·17	42·04	42·31	284	45·19	45·06	45·33
247	·31	·18	·44	266	·33	·20	·46	285	·35	.22	·49
248	·47	·34	·60	267	·49	·36	·62	286	·51	·38	·64
249	·64	·50	·76	268	·64	·52	·78	287	·67	·54	·80
250	·78	·86	·92	269	·80	·68	·94	288	·83	·70	·96
251	·94	·82	40·08	270	·97	·84	43·10	289	·99	·86	46·12
252	40·10	·97	·24	271	43·13	1·00	·26	290	46·15	46·02	·28
253	·26	40·13	·40	272	·28	43·15	·42	291	·30	·17	·44
254	·42	·30	·56	273	·44	·31	·58	292	·46	·33	·60
255	·59	·45	·72	274	·60	·47	·74	293	·62	·49	·76
256	·74	·61	·87	275	·76	·63	·90	294	·78	·65	·82
257	·90	·77	41·03	276	·92	·79	44·05	295	·94	·81	·98
258	41·06	·93	·20	277	44·08	·96	·21	296	47·10	·97	47·13
259	·22	41·09	·36	278	·24	44·11	·37	297	·25	47·13	·29
260	·38	·25	·51	279	·40	·27	·53	298	·42	·29	·45
261	·53	·41	·67	280	·55	·43	·69	299	·58	·45	·61
262	·69	·56	·83	281	·71	·59	·85	300	·74	·61	·77
263	·85	·72	·99	282	·87	·74	45·01	Rack		·129 1·00	0·129 1·00
264	42·01	·89	42·15	283	45·03	·90	·17				

Rule.—Seek in the first column of the table for the number of teeth it is proposed that the wheel shall contain. In a line with such number of teeth take from columns 2, 3, 4, 5, and 6 the numbers that are in them; and in every case multiply such numbers by the pitch. The products will be the number of inches and parts of inches to which the compasses must be opened to describe the circles and parts of circles that are required.

Example.—Suppose that a wheel (Fig. 687) is to be made to contain thirty teeth, and that the pitch of the teeth is to be $2\frac{1}{2}$ inches, proceed as follows:

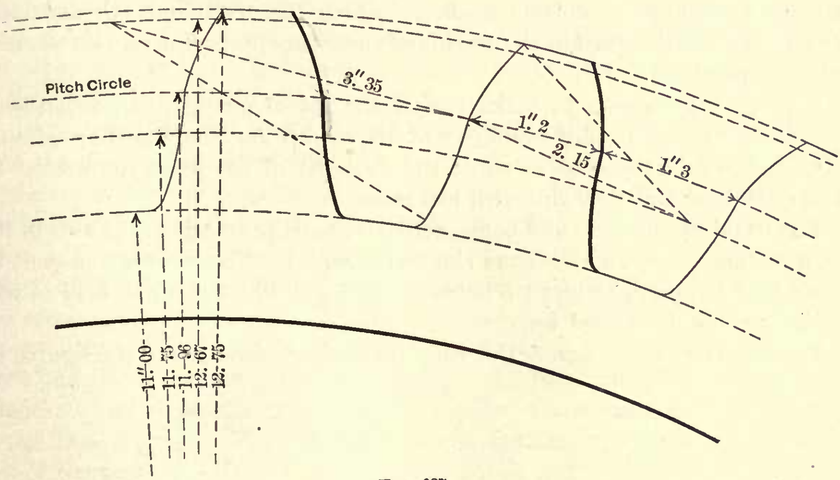


FIG. 687.

Seek in column 1 for 30, the number of proposed teeth, and take from column 2 the numbers 4·78, which multiplied by $2\frac{1}{2}$ inches, the product will be 11·957″. Open the compasses, therefore, to this radius and describe a circle, which will be the "pitch-circle." On an arc of this circle lay off $2{\cdot}5 \times {\cdot}48 = 1{\cdot}2''$ for the thickness of a tooth, and $2{\cdot}5 \times {\cdot}52 = 1{\cdot}3''$ for the space. Having determined the number of teeth and pitch, next, in column 3, and in the same line with 30 teeth, will be found the numbers 4·70, which multiply by 2½ inches—the product will be 11·75. With the compasses opened to this distance, and from the same centre as the last, describe another circle, which will be the paths of centres for the curves of the faces of the teeth. From column 4 similarly take the numbers 0·86 and multiply by 2½ inches. The product is 2·15, to which distance the compasses must be opened to describe the faces of the teeth.

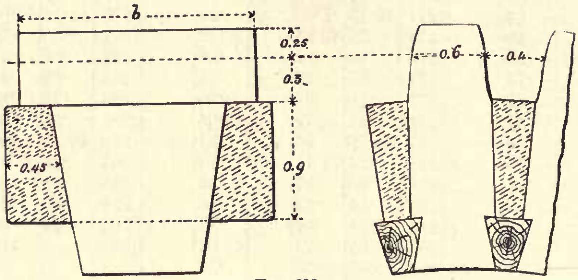


FIG. 688.

Again, in column 5, multiply $5·07 \times 2·5 = 12·675''$, and from the centre, with this radius, describe another circle for the paths of centres of flanks of the teeth, from column 6, $1·34 \times 2·5 = 3·35$, the radius of the flanks of the teeth.

For the height of a tooth a common proportion is $\frac{3}{10}$ of pitch outside of pitch-circle, and $\frac{4}{10}$ of pitch within, which leaves $\frac{1}{10}$ pitch for clearance at the bottom, where usually small arcs are described to connect the teeth with the wheel.

Having described a few teeth of any gear to its full size, the rest may be laid off from a templet, or cutters made by which the teeth may be accurately formed. In the illustration the teeth and spaces are proportioned to a common form (see page 303).

It is not uncommon to make one of the set of gears with wooden teeth, mortices being cast in the periphery of the wheel for the insertion of these teeth—hence called mortise or core; the elasticity of the wood diminishes the effect of shocks, and they run with less noise.

The usual proportions and construction of mortise wheels are shown in Fig. 688, a section across and with the rim of the wheel. The figures represent the proportions to pitch as unity; b is from 2 to 3 p. The teeth are held in position by wooden dovetailed keys.

Fig. 689 is a section across the rim of mortised bevel-gear; the figures are

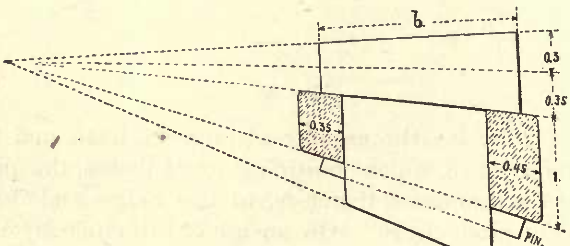


FIG. 689.

as before in ratios to p. In this illustration the teeth are held in by pins, common also to spur-mortise-gears.

It is unusual in drawings to complete gears with teeth according to the examples given; it is sufficient for the purposes of pattern-making that the pitch-circle, pitch and form of one tooth be given. For lines of shafting, spur-gears may be represented, like plain pulleys, tangent to each other, of the diameters of the pitch-circle, with the pitch and number of teeth written in: bevel-gears, as in Fig. 690. In finished drawings, detail is necessary. The following simple forms of describing spur-and bevel-gears will in general answer the purpose, but if more accuracy is required use Adcock's tables.

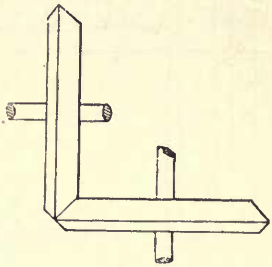


FIG. 690.

Projections of a Spur Wheel.—To draw side elevation (Fig. 691), an edge view (Fig. 692), and a vertical section (Fig. 693) of a spur wheel with 54 teeth and a pitch of two inches:

Determine the radius of the pitch-circle from the table, page 312 ($\cdot 6366 \times 54 = 34 \cdot 376 =$ D. $R = 17 \cdot 19$); draw the central line A C B and the perpendicular D E; on C as a centre, with a radius 17·19, describe the pitch-circle, and divide it into 54 equal parts. To effect this division, without defacing by repeated trials that part of the paper on which the teeth are to be represented, describe from the same centre c, with any convenient radius, a circle $a\,b\,c\,d$; with the same radius divide its circumference into six equal parts, and subdivide each sixth into nine equal parts, and draw radii to the centre c; these radii will cut the pitch-circle at the required number of points. Divide the pitch (2 inches) into 10 equal parts; mark off beyond the pitch-circle a distance equal to 3 of these parts, and within it a distance equal to 4 parts, and from the centre C describe circles passing through these points; these circles are projections of the cylinders bounding the points of the teeth and the roots of the spaces respectively.

In forming the outlines of the teeth, the radii, which, by their intersections with the pitch-circle, divide into the required number of parts, may be taken as the centre lines of each tooth. The thickness of the tooth, measured on the pitch-circle, is $\cdot 46$ pitch $\times 2'' = \cdot 92$, and the width of the space is equal to $\cdot 54\,p \times 2'' = 1 \cdot 08$. These distances being set off, take in the compasses the length of the pitch, and from the centre g describe a circular arc $h\,i$; and from the centre j, with the same radius, describe another arc $h\,k$ touching the former; these arcs, being terminated at the circles bounding the points of the teeth and the bottoms of the spaces respectively, form the curve of one side of a tooth. The other side is formed in a similar manner, by drawing from the centre l the arc $m\,n$, and from the centre p the arc $m\,o$, and so on for all the rest of the teeth.

The teeth having been completed, proceed to the delineation of the rim, arms, and eye of the wheel. The thickness of the rim is usually made equal to that of the teeth, say one half of the pitch, which distance is accordingly set off on a radius within the circle of the bottoms of the spaces, and a circle is decribed from the centre C through the point q thus obtained. Within the rim, a strengthening feather $q\,r$, in depth about three fourths of the thickness of the

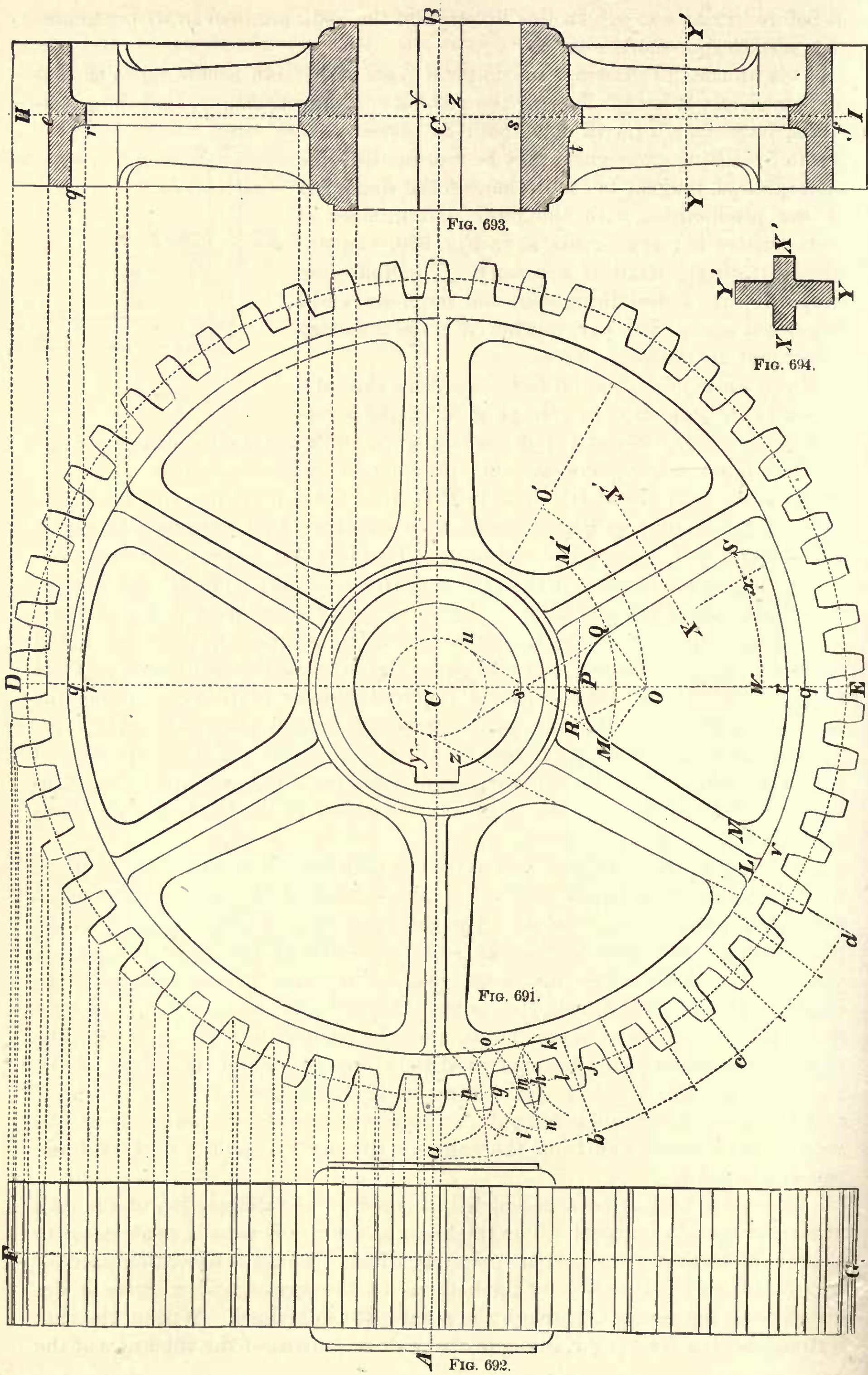


Fig. 693.

Fig. 694.

Fig. 691.

Fig. 692.

rim, is generally formed, as shown in the plate. Describe the *eye*, or central aperture for the reception of the shaft, to the specified diameter, as also the circle representing the thickness of metal round the eye, usually equal to the pitch of the teeth.

To draw the arms, from the centre C, with the radius C *u* equal to the pitch, describe a circle; draw all the radii, as C L, which are to form the centre lines of the arms, and set off the distance L *v*, equal to one third pitch, on each side of these radii at the inner circumference of the rim; and through all the points thus obtained draw tangents to the circle passing through *u*. The contiguous arms are rounded off into each other by arcs of circles (Fig. 691). Draw the central web of the arm by lines parallel to their radii, making the thickness about $\frac{3}{4}$ inch for wheel of about this size.

Having completed the elevation for the edge view and vertical section, draw the perpendiculars F G and H I (Figs. 692 and 693) as central lines in the representations; set off on each side of these lines half the breadth of the teeth, and draw parallels; project the teeth of Fig. 691 upon Fig. 692, by drawing through all the visible angular points straight lines parallel to A B, and terminated at either extremity by the verticals representing the outlines of the breadth of the wheel; project in like manner the circles of the hub; lay off half length on each side of F G, and draw parallels to it. The section (Fig. 693) is made on the line D E of the elevation; project, as in Fig. 692, those portions which will be visible in this section, and shade those parts which are in section. The arms are made tapering in width, and somewhat less than the face of the wheel (Fig. 694); a cross-section of one of them is made by a plane passing through X X′ and Y Y′. The points *y*, *z*, in Fig. 691, and corresponding lines in Fig. 693, represent the edges of *key-seat.*

Oblique Projection of a Spur Wheel.—In drawing an object in an oblique position with respect to the vertical plane of projection, lay down in the first place the elevation and plan as if it were parallel to that plane (Figs. 695 and 696). Then transfer the plan to Fig. 698, giving it the same inclination with the ground line which the wheel ought to have in relation to the vertical plane; and assuming that the horizontal line A B represents the axis of the wheel, both in the parallel and oblique positions, the centre of its front face in the latter position will be determined by the intersection of a perpendicular raised from the point C′ (Fig. 697) with that axis. Take any point, as *a* in Fig. 695, and the projection of that point on Fig. 696 must be in the line *a a*, parallel to A B; this point being projected at *a′* (Fig. 698), must be in the perpendicular *a′ a*; therefore the intersection of these two lines is the point required. Thus all the remaining points *b*, *c*, *d*, etc., may be obtained by the intersections of the perpendiculars raised from the points *b′*, *c′*, *d′*, etc. (Fig. 698), respectively, with the horizontals drawn through the corresponding points in Fig. 695. Since the points *e* and *f*, in the further face of the wheel, have their projections in *a* and *b* (Fig. 695), their oblique projections will be situated in the lines *a a* and *b b*, but they are also at *e* and *f*; consequently the lines *e a* and *f b* are the oblique projections of the edges *a′ e′* and *b′ f′*.

All the circles which, in the rectangular elevation (Fig. 695), have been employed in the construction of this wheel are projected in the oblique view into ellipses; thus, since the plane F′ G′, in which these circles are situated, is verti-

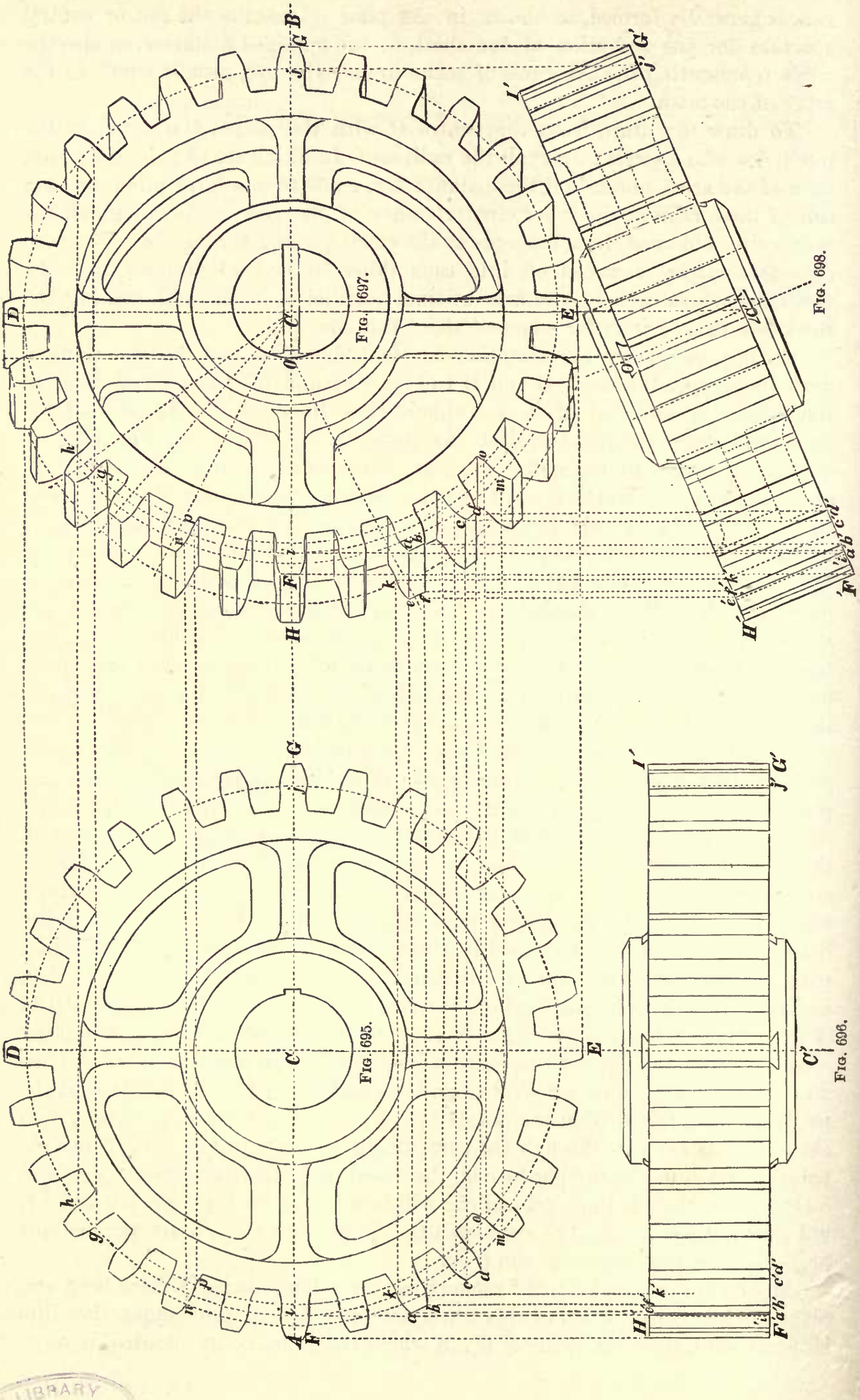


FIG. 697.

FIG. 698.

FIG. 695.

FIG. 696.

cal, the major axes of all the ellipses in question are perpendicular to the line A B, and equal to the diameters of the circles of which they are respectively the projections; and the minor axes, representing the horizontal diameters, will all coincide with the line A B.

To obtain the ellipse into which the pitch-circle is projected, set off upon the vertical D E (Fig. 697) above and below C the radii of the pitch-circle for the major axis, and project $i' j'$ (Fig. 698) to the horizontal axis at i and j (Fig. 697) for the minor axis.

The intersection of the horizontal lines $g\,g$, $h\,h$, etc., with the projection of the pitch-circle gives the thickness of the teeth at the pitch-line; in the same manner the circles bounding the extremities and roots of the teeth may be determined. If strict accuracy is required, a greater number of points is necessary for the construction of the curvature of the teeth, and two additional circles, $m\,n$ and $o\,p$, may be drawn on Fig. 695 and projected to Fig. 697, and the points of their intersection with the curves of the teeth projected to corresponding points as indicated by the same letters.

Projections of a Bevel Wheel.—Fig. 699 is an edge view, Fig. 700 a face view, and Fig. 701 a vertical transverse section. For the determination of the division of the angle of inclination of the axes of a pair of bevel wheels, see Fig. 685; for their size and proportion, the rules given for spur wheels; thus, consider the base of the cone A B (Figs. 700 and 701) as the diameter of the pitch-circle of a spur wheel, and proportion the pitch, form, and breadth of teeth, according to the stress to which they are to be subjected.

Having determined and laid down, according to the required conditions, the axis O S of the primitive cone, the diameter A B of its base, the angle A S O which the side of the cone makes with the axis, and the straight lines A o, D o', perpendicular to A S, and representing the sides of two cones, between which the breadth of the wheel (or length of the teeth) is comprised, the first operation is to divide the primitive circle, described with the radius A C, into a number of equal parts corresponding to the number of teeth or *pitch* of the wheel. Then upon the section (Fig. 701) draw with the radius o A or o B, moving parallel to itself, outside the figure, a small portion of a circle, upon which construct the outlines of a tooth M, and of the rim of the wheel, with the same proportions and after the same manner in reference to spur wheels; set off from A and B the points a, d, and f, denoting respectively the distances from the pitch-line to the points and roots of the teeth, and to the inside of the rim, and join these points to the vertex S of the primitive cone, terminating the lines of junction at the lines D o', E o'; the figure $a\,b\,c\,d$ will represent the lateral form of a tooth, and the figure $c\,d\,f\,e$ a section of the rim of the wheel, by the aid of which the face view (Fig. 700) is constructed.

The points a, b, c, d, and e, having been projected upon the vertical diameter A′ B′, describe from the centre C′ a series of circles passing through the points thus obtained, and draw any radius, as C′ L, passing through the centre of a tooth. On either side of the point L set off the distances L k, L l, making the thickness of the tooth M at the point, and indicate, in like manner, upon the circles passing through the points B′ and d', its thickness at the pitch-line and root; then draw radii through the points i, l, k, g, m, etc., terminating them respectively at the circles forming the projections of the corresponding

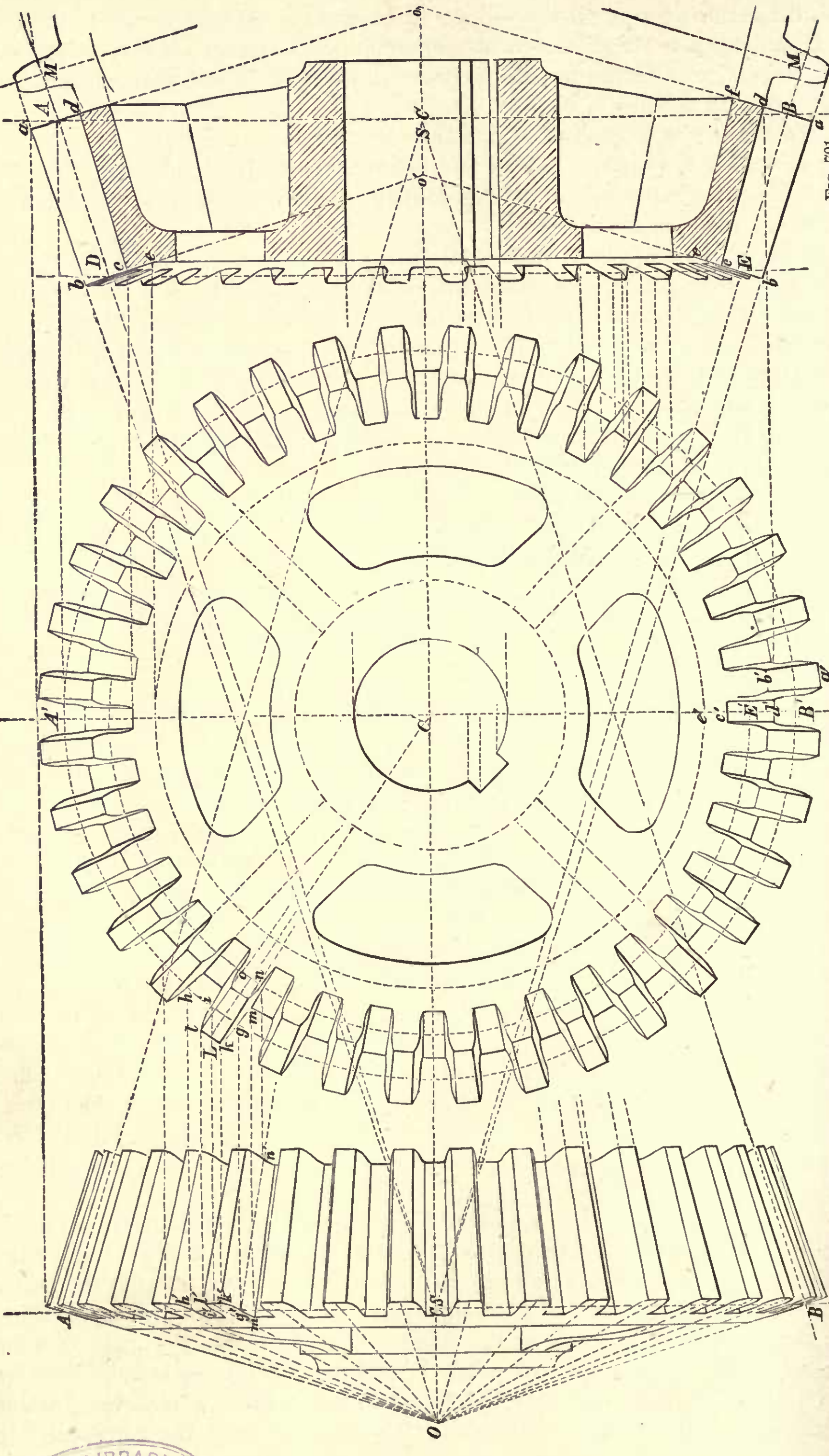


Fig. 701.

Fig. 700.

Fig. 699.

parts at the inner extremity of the teeth; these radial lines will represent the rectilinear edges of all the teeth. The curvilinear outlines may be delineated by arcs of circles, tangents to the radii *g* C′ and *i* C′, and passing through the points obtained by the intersections of the radii and the various concentric circles. The radii of these circular arcs may in general, as in the case of spur wheels, be taken equal to the pitch, and their centres upon the interior and exterior pitch-circles; thus the points *g* and *i*, *n* and *o*, for example, are the centres for the arcs passing through the corresponding points in the next adjacent teeth, and *vice versa.*

The drawing of the teeth in the edge view (Fig. 699), and of such portions of them as are visible in the section (Fig. 701), are explained by inspection of the lines of projection. In the construction observe that every point in the principal figure from which they are derived is situated upon the projection of the circle drawn from the centre C′, and passing through that point. Thus the points *g* and *i*, for example, situated upon the exterior pitch-circle, will be determined in Fig. 699 by the intersection of their lines of projection with the base A B of the primitive cone; and the points *k* and *l* will be upon the straight line passing through *a a* (Fig. 701), and so on. Further, as the lateral edges of all the teeth in Fig. 699 are radii of circles drawn from the centre O′, so in Fig. 700 they are represented by lines drawn through the various points found as above for the outer extremities of the teeth, and converging toward the common apex S; while the centre lines of the exterior and interior extremities themselves all tend to the points *o* and *o′* respectively.

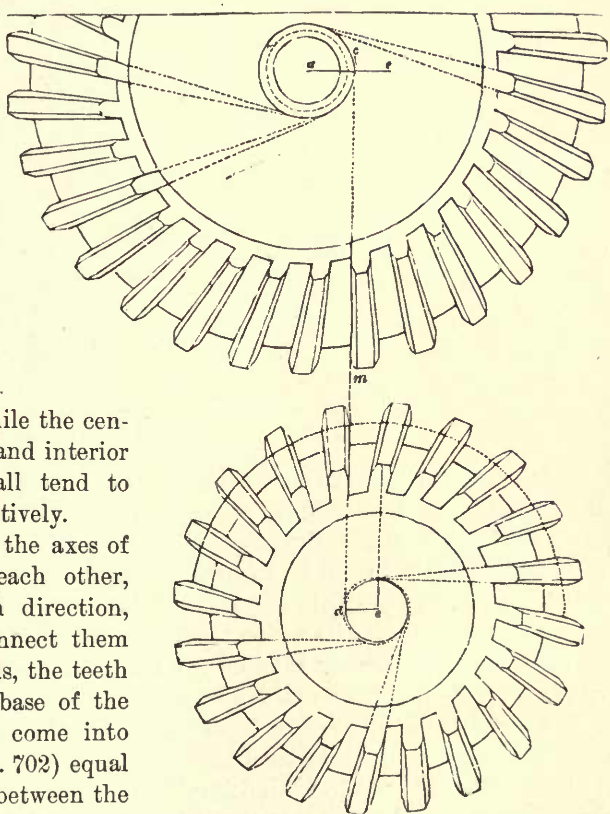


FIG. 702.

Skew-Bevels.—When the axes of wheels are inclined to each other, and yet do not meet in direction, and it is proposed to connect them by a single pair of bevels, the teeth must be inclined to the base of the frusta to allow them to come into contact. Set off *a e* (Fig. 702) equal to the shortest distance between the axes (called the *eccentricity*), and divide it in *c*, so that *a c* is to *e c* as the mean radius of the frustum to the mean radius of that with which it is to work; draw *c m d* perpendicular to *a e*. The line *c m d* gives the direction of the teeth; and, if from the centre *a*, with radius *a c*, a circle be described, the direction of any tooth of the wheel will be a tangent to it, as at *c*. Draw the line *d e* perpendicular to *c m d*, and with a radius *d e* equal to *c e* describe a circle; the direction of the teeth of the second wheel will be tangents to this last, as at *d*.

System composed of a Pinion driving a Rack (Fig. 703).—The pitch-line M N of the rack and the pitch-circle A B D of the pinion being laid down touching one another, divide the latter into twice the number of equal parts that it is to have teeth, and set off the common distance of these parts upon the line M N, as many times as may be required; this marks the thickness of

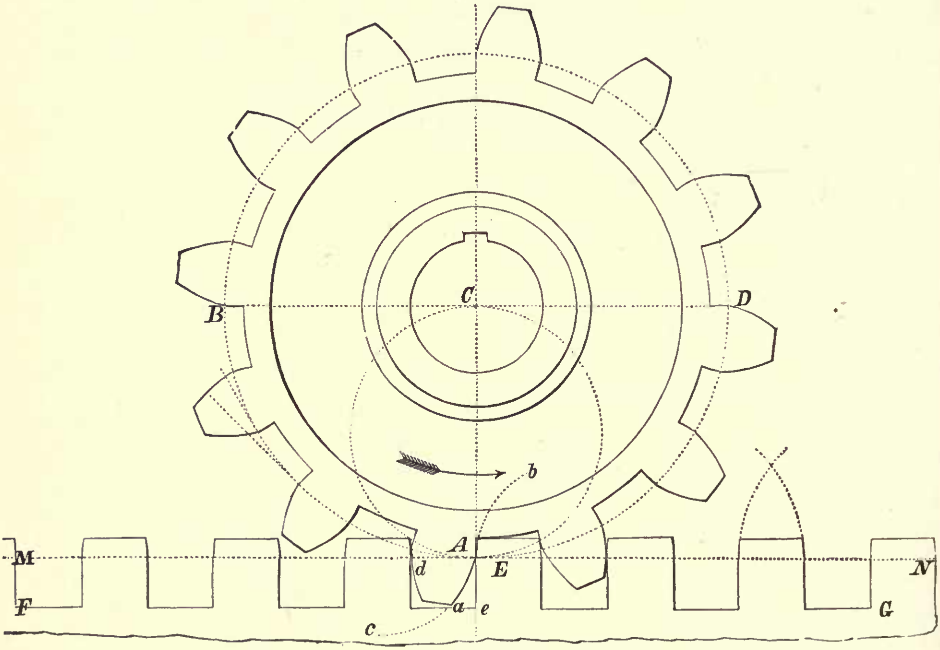


Fig. 703.

the teeth and width of the spaces in the rack. Perpendiculars drawn through all these points to the solid part of the rack will represent the flanks of the teeth upon which those of the pinion are to be developed in succession. The curvature of these latter should be an involute A *c* of the circle A B D. The teeth might be cut off at the point of contact *d* upon the line M N, for at this position the tooth A begins its action upon that of the rack E; but it is better to allow a little more length; in other words, to describe the circle bounding the points of the teeth with a radius somewhat greater than C *d*.

For the form of the spaces in the rack, set off from M N, as at the point *e*, a distance slightly greater than the difference A *a* of the radius of the pitch-circle, and that of the circle limiting the points of the teeth, and through this point draw a straight line F G parallel to M N. From this line the flanks of all the teeth of the rack spring, and their points are terminated by a portion of a cycloid A *b*, which, however, may in most instances be replaced by an arc of a circle. As the depth of the spaces in the pinion depends upon the height of this curved portion of the teeth; their outline is formed by a circle drawn from the centre C, with a radius a little less than the distance from this point to the straight line bounding the upper surface of the teeth of the rack.

System composed of a Rack driving a Pinion.—In this case the construction is identical with that of the preceding example, except that the form proper to be given to the teeth of the rack is a cycloid generated by a point A in the circumference of the circle A E C rolling on the line M N. The curvature of the teeth is an involute as before.

System composed of an Internal Spur Wheel driving a Pinion (Fig. 704).—The form of the teeth of the driving-wheel is in this instance determined by the epicycloid described by a point in the circle A E C, rolling on the concave

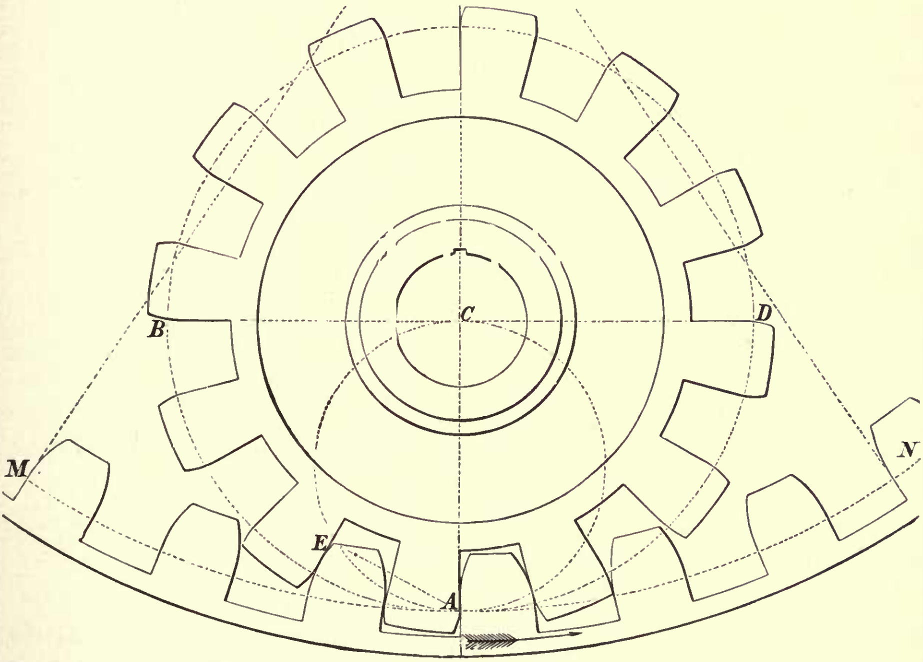


FIG. 704.

circumference of the primitive circle M A N. The points of the teeth are to be cut off by a circle drawn from the centre of the internal wheel, and passing through the point E, indicated by the contact of the curve with the flank of the driven tooth.

The wheel being supposed to be the driver, the curved portion of the teeth of the pinion may be very small. This curvature is a part of an epicycloid generated by a point in the circle M A N rolling upon B A D.

System composed of an Internal Wheel driven by a Pinion.—Fig. 705 involves a different mode of treatment from that employed in the preceding cases. The epicycloidal curve A *a*, generated by a point in the circle having the diameter A O, the radius of the circle M A N, and which rolls upon the circle B A D, can not be developed upon the flank A *b*, the line described by the same point in the same circle in rolling upon the concave circumference

M A N; and for the reason that that curve is situated *without* the circle B A D, while the flank, on the contrary, is *within* it. In order that the pinion

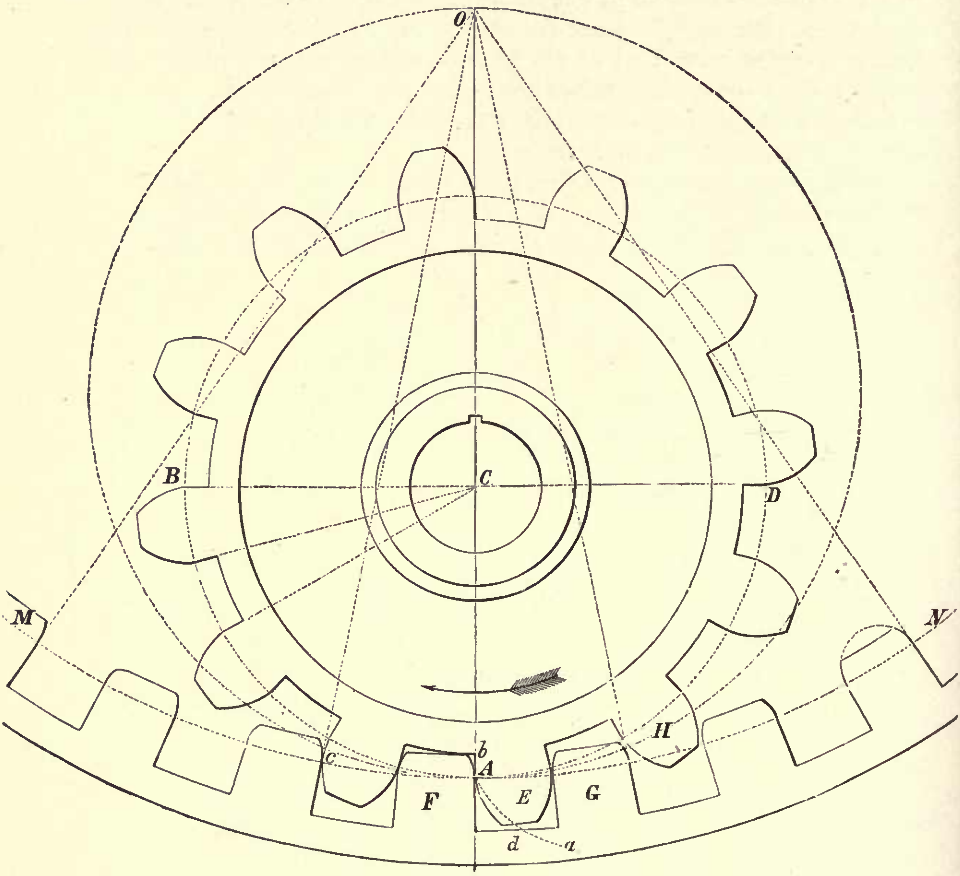
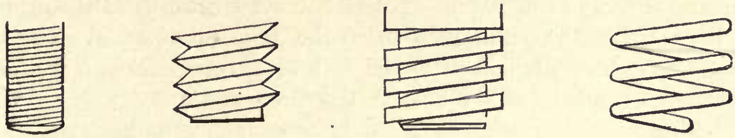


FIG. 705.

may drive the wheel uniformly according to the required conditions, form the teeth so that they shall act always upon one single point in those of the wheel. By taking for the curvature of the teeth of the pinion the epicycloid A *d*, described by the point A in the circle M A N rolling over the circle B A D, as in the preceding examples, the tooth E of the pinion begins its action upon the tooth F of the wheel at the point of contact of their respective primitive circles, and it is unnecessary to be continued beyond the point *c*, because the succeeding tooth H will then have been brought into action upon G; consequently the teeth of the wheel might be bounded by a circle passing through the point *c*. One of the practical advantages which this species of gearing has over wheels working externally is that the surfaces of contact of the wheel and pinion admit of being more easily increased; and, by making the teeth somewhat longer than simple necessity demands, the strain may be distributed over two or more teeth at the same time. The flanks of the teeth of the wheel are formed by radii drawn to the centre O, and their points are rounded off to enable them to enter freely into the spaces of the pinion.

DRAWING OF SCREWS.



Projections of a Triangular-threaded Screw and Nut (Fig. 706).—Draw the ground line A B, and the centre lines C c' of the figures, from O as a centre, with a radius equal to that of the exterior cylinders, describe the semicircle

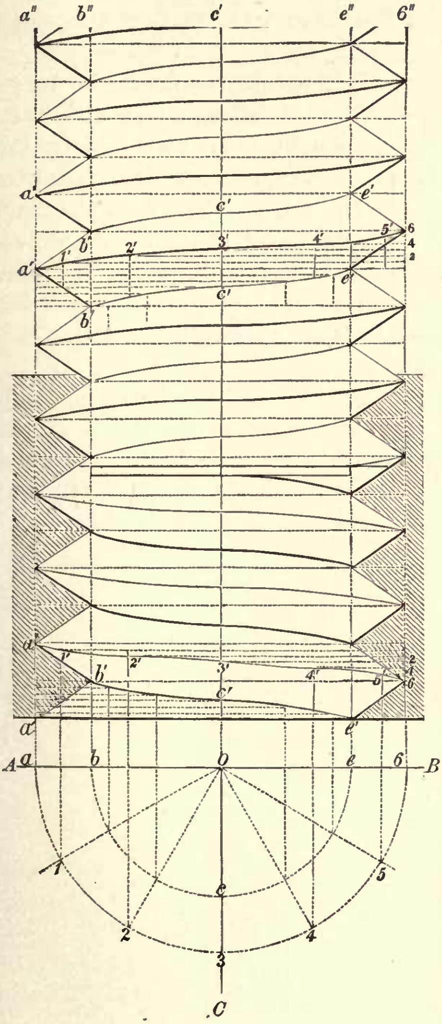


Fig. 706.

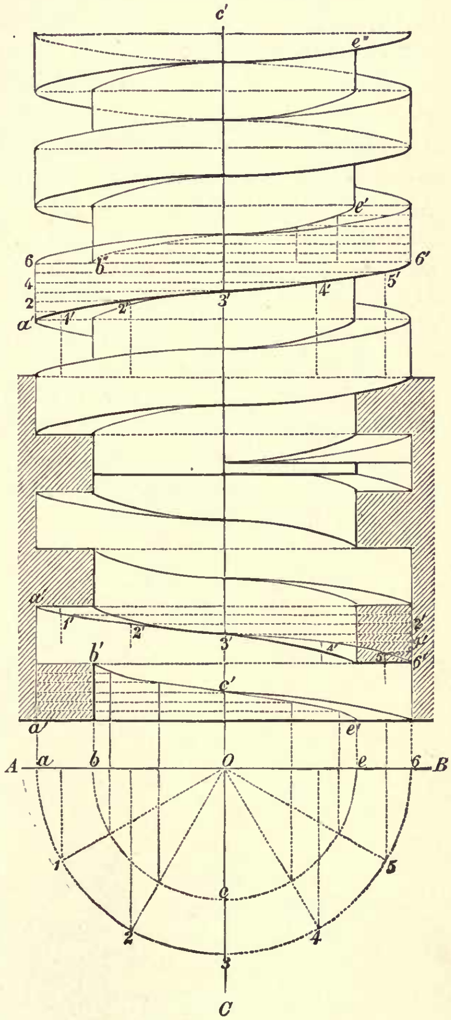


Fig. 707.

a 3 6; and with the radius of the interior cylinder the semicircle *b c e*. Draw the perpendiculars $a\,a''$ and 6 6″, $b\,b''$ and $e\,e''$, to represent the vertical projections of the exterior and interior cylinders; divide the semicircle *a* 3 6 into any number of equal parts, say 6, and through each part draw radii to divide the interior. On the line a' a'' set off the length of the pitch as many times as required; and through the points of division draw lines parallel to the ground line A B. Divide the pitch into twice the number of equal parts that the semicircles have been divided into, and following instructions laid down (page 140), construct the helix a' 3′ 6 both in the screw and nut.

Having obtained the point b', by the intersection of the horizontal line passing through the middle division of a' a with the perpendicular b b'', describe the helix b' c' e', which will represent the bottom of the groove. The apparent outlines of the screw and its nut will then be completed by drawing the lines b' a', a' b', etc., to the curves of the helices; these are not, strictly speaking, straight lines, but their deviation from the straight line is, in most instances, so small as to be imperceptible.

When a long series of threads have to be delineated, they should be drawn mechanically, by means of a *templet* constructed in the following manner: Take a small slip of thin wood or pasteboard, and draw upon it the helix a' 3′ 6 to the same scale as the drawing, and cut the slip carefully and accurately to this line. Applying this templet upon Fig. 706, so that the points a' and 6 on the plate shall coincide with a' and 6 on the drawing, the curve a' 3′ 6 can be drawn mechanically, and so on for the remaining curves of the outer helix. The same templet may be employed to draw the corresponding curves in the screw-nut by simply inverting it; but for the interior helix a separate one must be cut, its outlines being laid off in the same manner.

Projections of a Square-threaded Screw and Nut (Fig. 707).—The depth of the thread is equal to its thickness, and this latter to the depth of the groove. The construction is similar to the preceding; the same letters and figures mark relative parts. The parts of the curve concealed from view are shown in dotted lines.

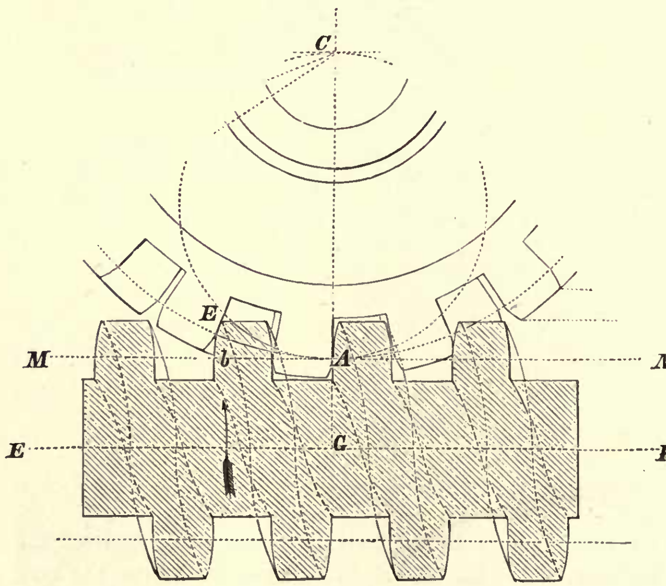


Fig. 708.

System composed of a Wheel and Tangent, or Endless Screw.—In laying out the work, the pitch of the teeth is to be determined by the required stress, as for spur wheels, and the number of the teeth in the wheel by the number of turns of the screw for each revolution of the wheel. With these determined, take C (Fig. 708) to be the centre of the wheel, E F the axis of the screw, C A the radius of the pitch-circle of the wheel, and G A that of the pitch-cylinder of the screw; the line M N drawn through A, parallel to E F, will be the generatrix of that cylinder, which will serve the purpose of determining the form of the teeth. The section is made through the axis, and is the case of a rack driving a pinion; consequently the curve of the teeth, or rather *thread*, of the screw should be simply a cycloid generated by a point in the circle A E C, described upon A C as a diameter, and rolling upon the straight line M N. The outlines of the teeth are helical surfaces described about the cylinder forming the screw, with the pitch A *b* equal to the distance, measured upon the primitive scale, between the corresponding points of two contiguous teeth. These curves are expressed by dotted lines. The teeth of the wheel are set at angle to the plane of its face, and with surfaces corresponding to the inclination and

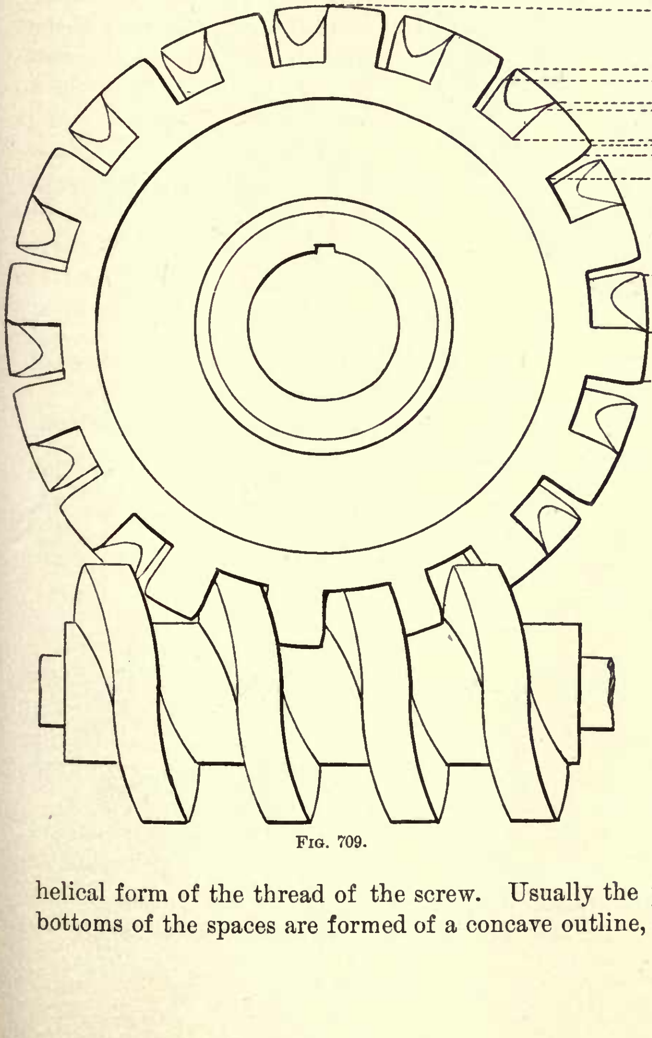


Fig. 709.

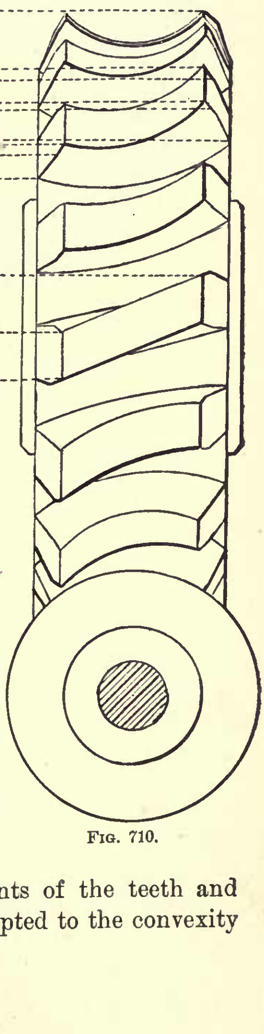


Fig. 710.

helical form of the thread of the screw. Usually the points of the teeth and bottoms of the spaces are formed of a concave outline, adapted to the convexity

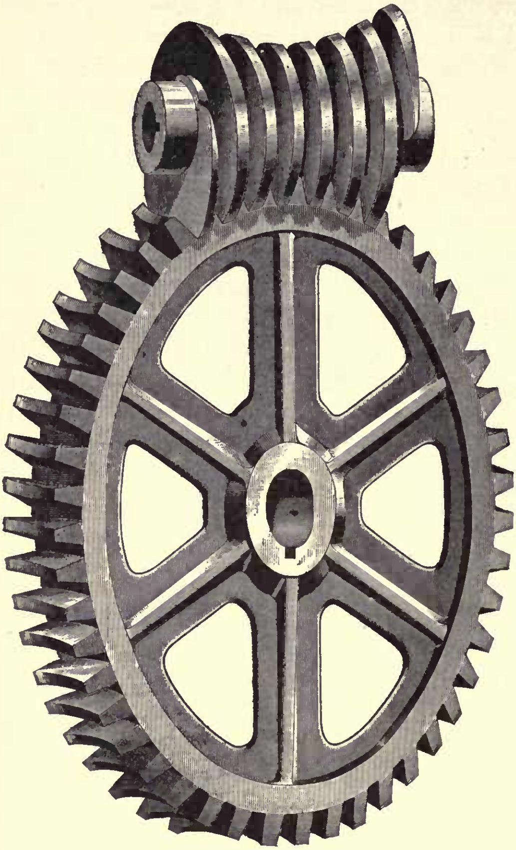


Fig. 711.

of the screw, in order to present as much bearing surface as possible to its action. In this kind of gearing it is almost invariably the screw that imparts the motion.

A screw is designated as right or left hand, according as the forward motion is imparted by turning the screw to the right or left.

In the proportions adopted by the Yale & Towne Manufacturing Co. for worm gearing, the wheel under the weight will revolve the screw slowly. This angle (slightly less than the angle of quiescence, see Morin's table, page 199) of the teeth is found to be the best

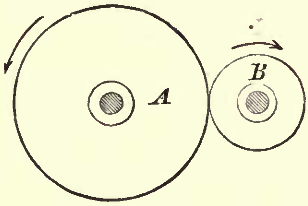


Fig. 712.

adapted for economy of power. In the wheel the teeth in section are those of a spur wheel, cut with a chasing cutter, and in the screw turned in a lathe.

Figs. 709 and 710 are two views, worm and wheel, with such lines of construction dotted as will explain the manner of drawing.

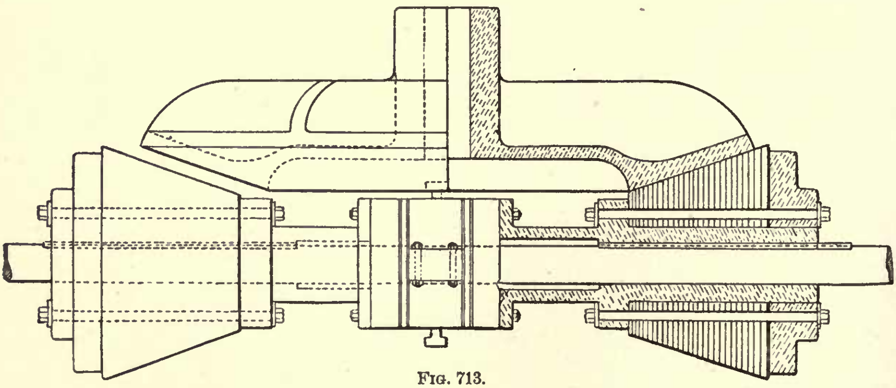


Fig. 713.

Fig. 711 is the Albro worm and worm wheel as used on the cruisers of the United States Government for ship steering and for heavy windlass work.

Frictional Gearing.—When motion is not continuous for a long time, but frequently stopped and started or reversed, frictional gearing is very often used. The starting is with as little shock as with belting, as by the usual appliances this pressure may be applied gradually and is fully as positive. The simplest form of frictional gearing is that in which the surfaces in contact correspond to that of the pitch-circles (Fig. 712).

Fig. 713 is a plan of a bevel frictional gear. One half is shown in section. The surface of the large or driven gear is of cast-iron, that of the pinion of paper, in washers compressed by a hydraulic press and firmly held together by bolts. The bevel in section is in contact with the large wheel-surface, the other is disengaged. A slight motion to the right will throw the one in contact out, and not throw in the other, and motion ceases in the large driven wheel; a still further motion throws in the left pinion, and the motion of the driven wheel is reversed.

The mode in which this is done is shown in the elevation (Fig. 714). The shipper consists of a bell-crank, controlled by a screw. The screw works in a stand, on the top of which is a hand wheel; the hand wheel can be moved in either direction, and any desirable pressure can be brought upon the frictional surfaces by means of the screw. It is not unusual, instead of two pinions to have one pinion, with a little clearance on each side, revolving between two wheels, a slight lateral motion, in either direction, bringing it in contact with one or the other of the wheels. Some provision, by a loose coupling or otherwise, must be made to admit of this lateral movement in the pinion shaft. Straight pulleys, or what would correspond to spur-gears without teeth, are constructed, as in the example given, and are thrown in or out of gear by a lateral motion of the pinion.

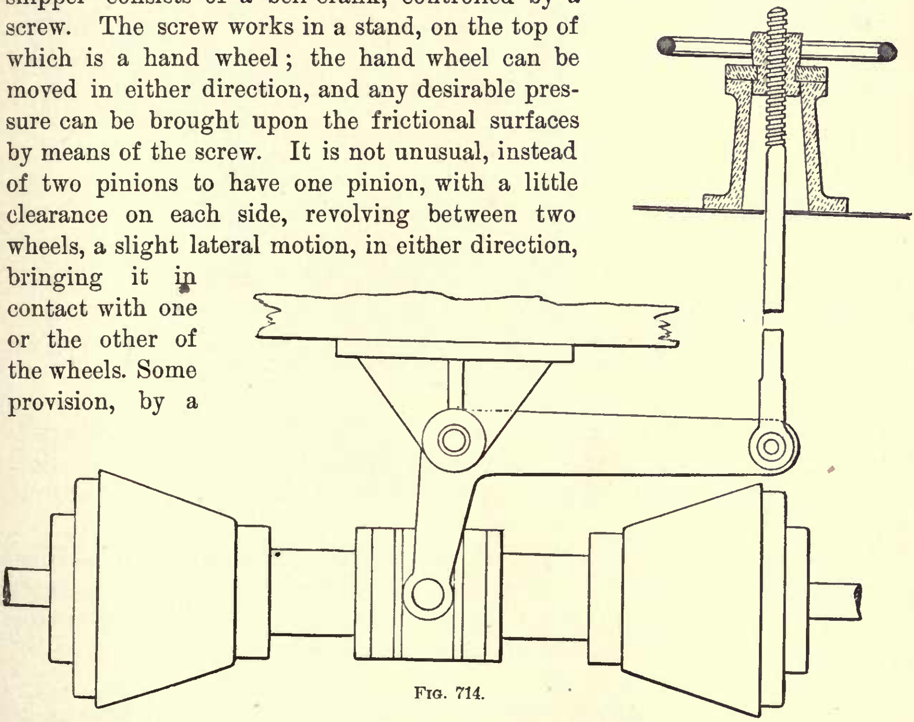


Fig. 714.

In proportioning the face of the pulleys it has been found safe to consider it the same as belts, given in the table (page 292). The pressure can be applied according to the requirements of driving, and there is no falling off in the friction. The frictional surfaces are not always paper; wood, leather, and prepared rubber are frequently used.

Wedge Gearing, or Robertson Grooved-Surface Frictional Gearing.—Fig. 715 is the cross-section of the rims of two wheels of this gearing. The angle recommended by Robertson is 50° (usually not over 30° in our practice), and the pitch to vary somewhat with the velocity and power to be transmitted. For the adhesion, it is safe to make the horizontal face equal to that of a belt under the same circumstances of transfer of power.

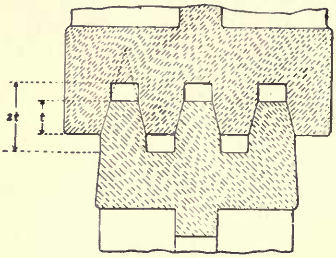


FIG. 715.

Fig. 716 shows the application of wedge gearing to a hoist drum *d*. The power is applied through the shaft *s*; one end of the drum shaft rests in a swivel box, the other in an eccentric box. Motion is given through the eccentric handle by which the drum gear can be engaged with that

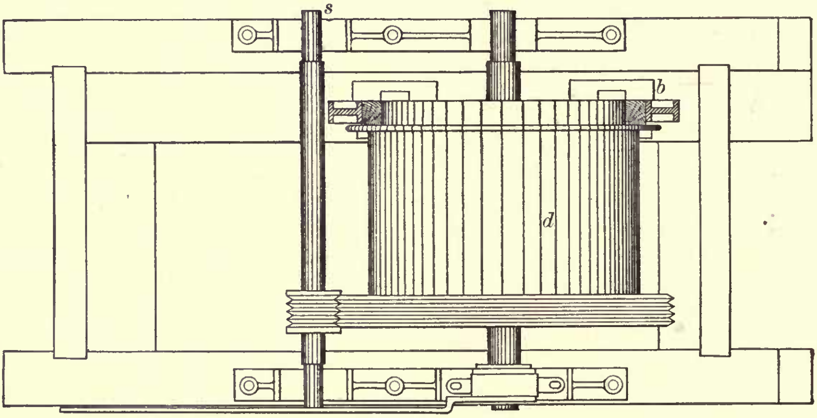


FIG. 716.

of the pinion of the driving shaft; the drum is revolved and the load raised. In lowering, the drum is thrown out by the eccentric, and the brake *b* applied, which is controlled by a system of levers brought within reach of the man at the eccentric lever.

When applied to the driving of a rotary fire pump, the friction pinion on the pump shaft is thrown into gear with the friction wheel of the main shaft by a sliding frame of iron with a screw and hand wheel. With this apparatus the pump can be started without shock or jar and without reducing the speed of the main shaft.

For large, straight pulleys, wooden rims from 6″ to 8″ deep, and built up in segments from 1″ to 2″ thick, so placed that the direction of the fibres shall follow the circumference of the wheel as nearly as possible. The segments are firmly clamped together and secured by glue joints, nails, and bolts, and the rim is bolted to the arms of the pulley with additional outside and deeper rings secured to the rim. The whole is then turned and finished. This pulley may be used with a belt as a friction gear.

Figs. 717 and 718 are sections of wooden bevel friction gears, showing the way in which they are formed by wooden disks planed and fitted, glued, nailed, and bolted, and then turned to the proper angle and face.

For the transmission of small powers, the combination of one conical wheel and one narrow disk wheel with rounded edges, both of iron, is sometimes used

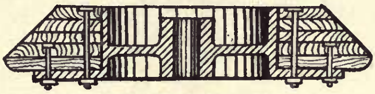


Fig. 717.

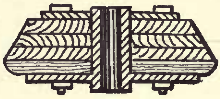


Fig. 718.

(Fig. 719). The pressure is applied to the disk wheel, and is so arranged that it can be shifted along its axis for a variable speed motion. The surfaces in contact are limited, the diameters, therefore, should be as large as possible, with high velocity.

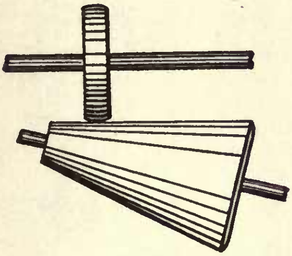


Fig. 719.

Another variable speed gear is made by using, instead of a cone, a crown plate on which the disk rests which can be moved inward or outward from the centre of the driving plate.

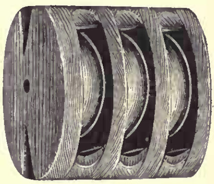


Fig. 720.

Rope for running rigging for derricks and general hoisting work is made of hemp or manilla, but rope of iron or steel wire, mostly with hemp centres, is preferred for many positions.

The shells of the common and lighter *blocks* are made of solid wood, mortised for the reception of the sheaves, and rope-strapped (Fig. 720). Larger and better blocks are made of separate pieces of wood, riveted or bolted together top and bottom, and iron-strapped (Fig. 721); the sheaves (Fig. 722) are usually of lignum-vitæ, metal-bushed; the pin is fastened to the shell, and the sheaves revolve on the pin. To diminish friction under heavy weights, friction rolls are introduced (Fig. 723).

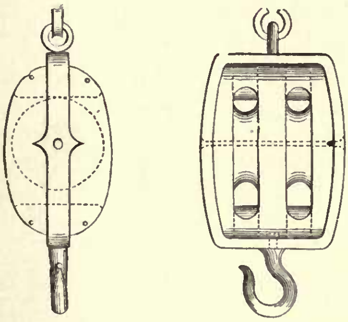


Fig. 721.

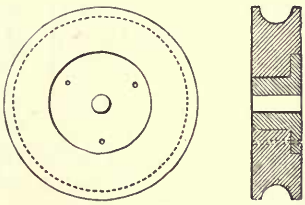


Fig. 722.

To strengthen the pin, blocks are inside-strapped (Fig. 724), or with wrought-iron straps or malleable-iron shells (Fig. 725) and cast-iron sheaves, entirely without woodwork. Most blocks (Figs. 724 and 725) have beckets attached.

In the Appendix will be found a table of the strength and weight of hemp, steel-, and iron-wire rope and chain; but as a simple rule of their working strength, multiply the square of the girth or circumference of a hemp rope by

100, of an iron-wire rope by 600, of steel-wire rope by 1,000, and the square of the diameter of the rods of which a chain is made by 12,000.

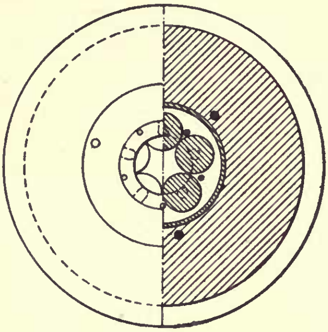


FIG. 723.

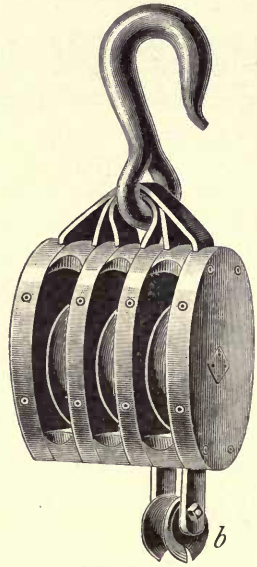


FIG. 724.

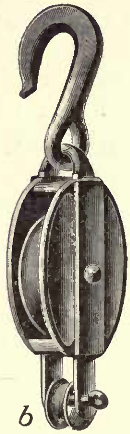


FIG. 725.

The following table of sizes are from the Boston and Lockport Block Company:

DIMENSIONS.			DIMENSIONS.		
Size sheave.	For dia. rope.	Size shell.	Size sheave.	For dia. rope.	Size shell.
$2\frac{1}{4} \times \frac{5}{8} \times \frac{3}{8}$	$\frac{1}{2}$	4 inches.	$8 \times 1\frac{3}{8} \times \frac{3}{4}$	$1\frac{1}{4}$	12 inches.
$3 \times \frac{3}{4} \times \frac{5}{8}$	$\frac{5}{8}$	5 "	$9 \times 1\frac{1}{2} \times \frac{3}{4}$	$1\frac{1}{4}$	13 "
$3\frac{1}{2} \times 1 \times \frac{1}{2}$	$\frac{3}{4}$	6 "	$9\frac{1}{2} \times 1\frac{5}{8} \times \frac{3}{4}$	$1\frac{3}{8}$	14 "
$4\frac{1}{4} \times 1 \times \frac{1}{2}$	$\frac{7}{8}$	7 "	$10 \times 1\frac{5}{8} \times \frac{7}{8}$	$1\frac{1}{2}$	15 "
$4\frac{3}{4} \times 1\frac{1}{8} \times \frac{5}{8}$	1	8 "	$11 \times 2\frac{1}{8} \times 1$	2	16 "
$5\frac{1}{2} \times 1\frac{1}{8} \times \frac{5}{8}$	1	9 "	$12 \times 2\frac{3}{8} \times 1$	$2\frac{3}{8}$	18 "
$6\frac{1}{4} \times 1\frac{1}{4} \times \frac{5}{8}$	$1\frac{1}{8}$	10 "	$14 \times 2\frac{3}{8} \times 1\frac{1}{4}$	$2\frac{1}{2}$	20 "
$7\frac{1}{4} \times 1\frac{1}{4} \times \frac{3}{4}$	$1\frac{1}{8}$	11 "	$15 \times 3\frac{3}{8} \times 1\frac{1}{4}$	3	22 "
			$16 \times 3\frac{3}{8} \times 1\frac{1}{2}$	$3\frac{1}{2}$	24 "

Gin-blocks (Fig. 726) are made with wrought- and malleable-iron frames and wrought swivel-hook.

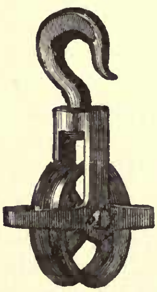


FIG. 726.

Winding-drums or barrels must have their diameters proportioned to the diameters of the rope or chain to be used (see table of sheaves above), and their length to the length of rope or chain to be taken in, and when the coils or turns of the rope are numerous, provision must often be made for keeping the rope or chain so that one coil may not ride on another. This is done by spiral grooves in the barrel, or shifting the barrel or the rope-guide automatically.

Fig. 1806 shows the way in which a chain cable is taken in with but few coils on the barrel. The coils are sufficient for the friction of taking up the cable; the tight cable is wound on the larger part of the barrel, and as the coils are unwound on the slack side the tight coil slips down to a smaller diameter; the weight of the chain on the slack side, as it drops into the locker, is sufficient to preserve the friction; but with a rope and a few turns on the barrels the force of a man is sufficient, and he can readily slack and hold the load in position or lower without changing the direction of motion or the speed of the barrel.

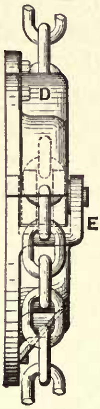


FIG. 729.

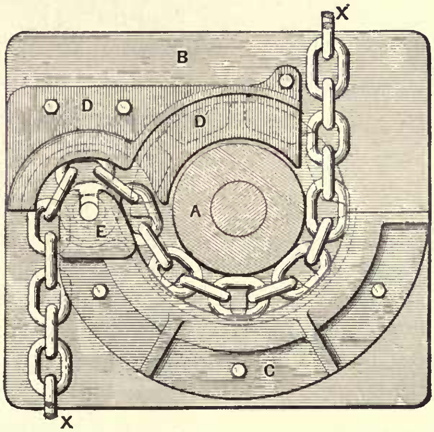


FIG. 728.

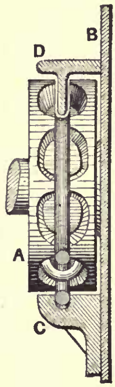


FIG. 727.

Chain-wheels with pockets are especially applicable to the purpose of hoisting, requiring a width only slightly greater than that of the chain, and a diameter sufficient to give the proper engagement with it.

Yale & Towne Manufacturing Company have made a pitch chain, of common form but of uniform links, especially adapted to hoists, and Figs. 727, 728, and 729 illustrate the construction of their chain-wheel. A is a pocketed chain-wheel, made of soft cast-iron, mounted on a frame B. C is the chain-guide enveloping the lower half of the chain-wheel. The inner curved surface of the chain-guide is grooved, and is of such a shape as to leave a space between it and the periphery of the chain-wheel merely sufficient to admit the

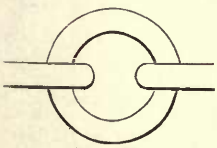


FIG. 730.

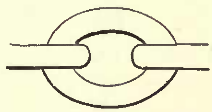


FIG. 731.

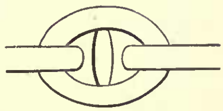


FIG. 732.

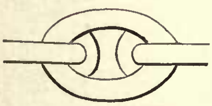


FIG. 733.

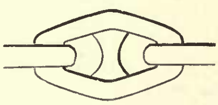


FIG. 734.

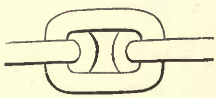


FIG. 735.

chain; it must then enter properly and continue engaged with the chain-wheel. E is a chain-guide roller, that delivers the slack chain into the box or locker.

D is the chain-stripper, bolted also to the plate B, with a tongue or rib projecting into the centre groove of the wheel which disengages the chain.

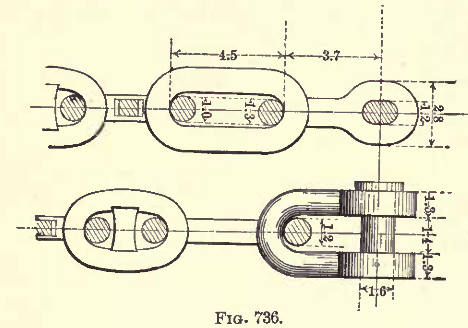


Fig. 736.

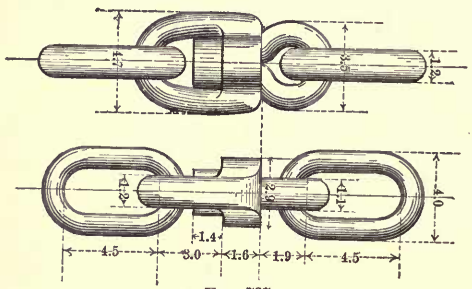


Fig. 737.

Forms of chain-cables are represented by the open circular link (Fig. 730), the open oval (Fig. 731), oval with pointed stud (Fig. 732), oval with broad-headed stud (Fig. 733), an obtuse-angled stud-link (Fig. 734), and the parallel-sided stud-link (Fig. 735). The usual proportions of chain-links are 6 diameters of the iron in length by $3\frac{1}{2}$ in width. The end links, which terminate each 15 fathoms of chain, are 6·5 in length to 4·1 in breadth, and the iron about 1·2 the diameter of the rest of the chain.

Chain Couplings.—Fig. 736 is an ordinary coupling link of an anchor chain. The link is of wrought-iron, the bolt and pin of steel, both galvanized. The next link is made somewhat longer than other links of the chain, so that the coupling link may be more readily introduced. Fig. 737 is a coupling link in which one part is a swivel, so that the chain may have a rotation about its axis of length without twist-

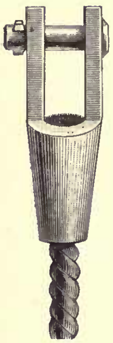
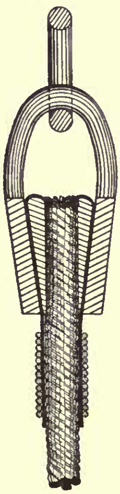


Fig. 738.



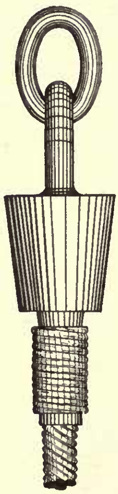


Fig. 739.

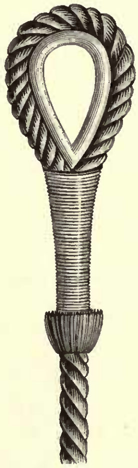


Fig. 740.

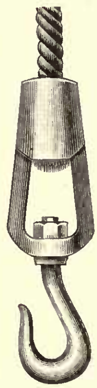


Fig. 741.

ing. Figs. 738, 739, and 741 are sockets for wire-rope connections, shown in section, Fig 739, with a wrought-iron conical key; often the wires of the rope are spread with wrought-iron wedges or nails, and in addition lead is poured in.

In Fig. 739 the conical cone is formed over an iron core by unravelling the rope, passing the wires over it, and serving the ends to the rope below; an iron socket fits over the core. Fig. 740 is a thimble in which the end of the rope is either spliced in or loose and served to the rope, with the end wires turned down. Fig. 741 is an open socket with a swivel hook, to prevent the twisting and untwisting of the rope and thereby weakening it. Fig. 742 is a loop-clamp.

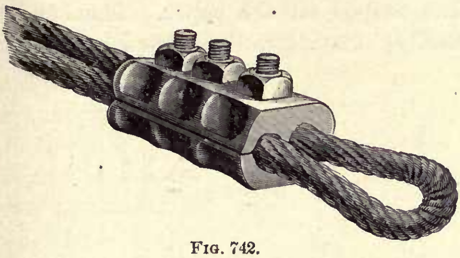


Fig. 742.

Hooks.—Figs 743 and 744 (from Redtenbacher) represent two wrought-iron hooks. The proportions of Fig. 743 are: assuming the neck of the hook as 1, the diameter of journals of the

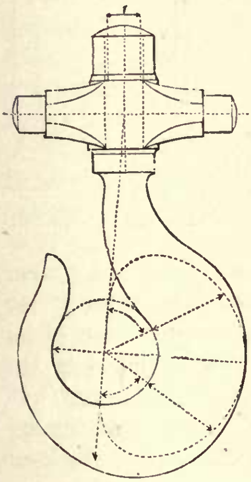


Fig. 743.

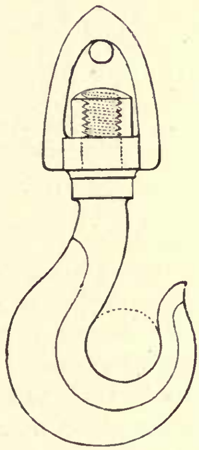


Fig. 744.

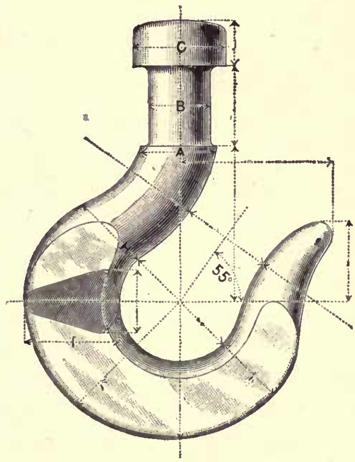


Fig. 745.

traverse are 1·1; width of traverse at centre, 2; distance from the centre of the hook to the centre of the traverse, 7·5; interior circle of the hook, 3·4; greatest thickness of the hook, 2·8. Assuming (Fig. 744) the diameter of the wire of the chain as 1: interior circle of hook is 3·2, and greatest thickness of hook 3·5.

Fig. 745 represents a hook of the Yale & Towne Manuf'g Company as fitted for a cross-head; the diameter at A is that of iron from which the hook is forged, and the section shown hatched is equal to that of the round iron. Hooks, of the proportions but with a much greater load than given in the table below, yield by the gradual opening of the jaw, giving ample notice before rupture.

	Ton.	Ton.	Ton.	Ton.	Ton.	Ton.	Ton.	Ton.	Ton.	Ton.	Ton.	Ton.
Capacity of hook	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	3	4	5	6	8	10
	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.
Dimensions of A...........	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$1\frac{1}{16}$	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{3}{4}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{7}{8}$	$3\frac{1}{4}$
Dimensions of D...........	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$2\frac{1}{4}$	$2\frac{3}{4}$	$3\frac{1}{4}$	$3\frac{1}{2}$	$4\frac{1}{4}$	$5\frac{1}{4}$	$6\frac{1}{4}$

All parts of the hook are drawn in parts of A.

Levers.—Figs. 746 and 747 are side and front elevations of an ordinary straight lever on a shaft; both are shown broken, either because the length is indefinite, or because it is inconvenient to put on the paper. The handle should be from 5 to 6 inches long, and $1\frac{1}{4}''$ diameter. The bar beneath the

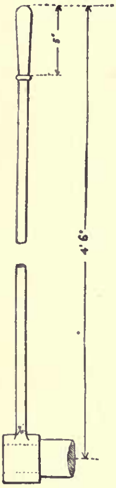


FIG. 746.


FIG. 747.

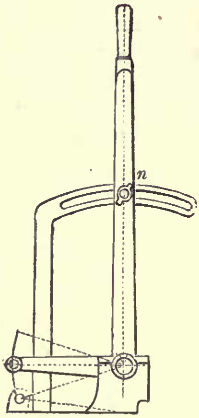


FIG. 748.

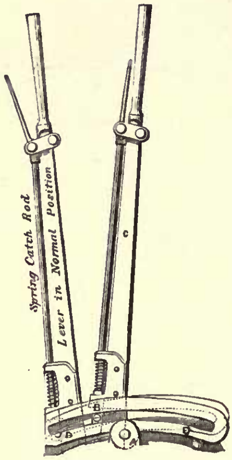


FIG. 749.

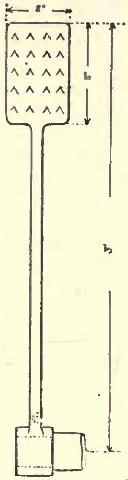


FIG. 750.


FIG. 751.

handle to be square, and of uniform width on one side of the lever and a taper on the other, as shown, of about $\frac{1}{2}''$ in 4 feet on each side. The sides of the square at the handle to be $\frac{1}{4}\sqrt[3]{\text{length}}$ in inches, or say $\frac{3}{4}''$ for $30''$ lever, $\frac{7}{8}''$ for 4 feet, and $1''$ for 5 feet. The neck of the shaft to be, as proportioned in the drawing, about $\frac{8}{10}$ of the greatest width of the lever, and the diameter of hub $1\frac{3}{10}$. The stress exerted by a man may be from 75 to 100 pounds, and the size of the shaft will depend on the torsional stress between the hub of the lever and the point of resistance.

Fig. 748 is a hand-lever forming one arm of a bell-crank—a bolt passing through a slot in the frame and the arm of the lever, and the two are clamped together by a *thumb-nut*, *n*, by which the lever can be held in any position. The same purpose is often effected by notches in the frame, into which the arm of the lever is caught, or by spring latches, as in Fig. 749.

Figs. 750 and 751 are side view and plan of a foot-lever. The foot-plate is $8'' \times 5'' \times \frac{5}{8}''$, and as the lever is subject to double the stress of the hand-lever above, the dimension should be somewhat increased. The side of square next the foot-plate should be, say for a lever of $30''$, $1''$; of 4 feet, $1\frac{1}{8}''$; of 5 feet, $1\frac{1}{4}''$; the form and taper as in the hand-lever.

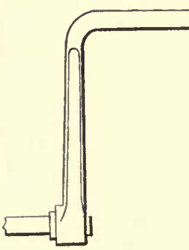


FIG. 752.


FIG. 753.

Figs. 752 and 753 are views of a hand-crank. The diameter of the handles, for convenience in grasping, should not be less than $1\frac{1}{4}''$; if for the force of two men, $1\frac{1}{2}''$, and from the diameter of the handle the rest may be proportioned as in the figure. The length of handle for a single man should be from 10″ to 12″; for two men, from 20″ to 24″; the crank from 15″ to 18″, and the height of shaft above the foot support for the men from 2′ 10″ to 3′ 2″.

In the construction of steam engines the makers adopt simple rules of proportion between the diameter of cylinder and its connections. Thus, if that of the diameter of cylinder be 1, that of the crank-shaft at the journal is ·50, of crank-pin ·25, of crank-pin eye ·6, that of the cylinder may be altered moderately without change in that of connections which may be kept in stock. In large quick-running engines (Fig. 791) the crank-pin is of larger proportions than above.

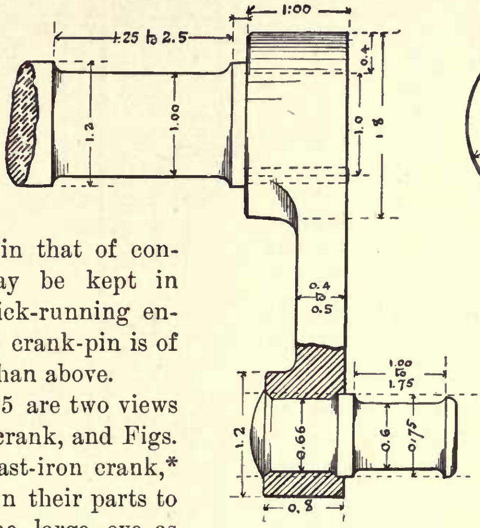


FIG. 754.

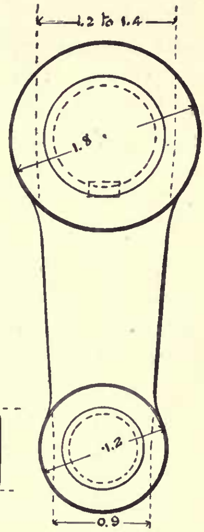


FIG. 755.

Figs. 754 and 755 are two views of a wrought-iron crank, and Figs. 756 and 757 of a cast-iron crank,* both proportioned in their parts to the diameter of the large eye as unity, but, as shown by the diagram and rule following, these figures can only apply to a single throw of crank, as the diameters of the two eyes vary as their distances apart.

Taking the diameter of the large eye of the crank as the unit, Redtenbacher gives in the table the relative sizes of central and end eyes of cranks, depending on the proportion between the length of crank and the diameter of central eye. The first column exhibits the number of times the diameter of eye is contained in the length of crank; the second and third columns give the suitable diameters of crank-pins.

DIAMETER OF EYE, BEING UNITS.

$\frac{l}{d}$	For wrought-iron shafts.	Cast-iron shafts.
2	0·85	0·62
3	0·69	0·51
4	0·60	0·44
5	0·54	0·39
6	0·49	0·36
7	0·45	0·33
8	0·42	0·31
9	0·40	0·29
10	0·38	0·28
11	0·36	0·26
12	0·34	0·25
13	0·33	0·24

The diameters of crank-pins as above given are on the basis of a length of from 1 to $1\frac{1}{8}$ of the diameter; if the length be increased beyond this, the diameter should be increased in the ratio of 1 to the square root of the diameter.

Disk-cranks are circular disks of cast-iron, with crank-pins of iron or steel, and as much strength of metal around the pin as in the crank. They are better than the crank, in that there is no unbalanced crank, and part of the weight

* "Elements of Machine Design," Unwin.

of the connection can be balanced by a proper disposition of metal within the area of the disk.

Fig. 758 is a plan of a double crank-axle, although by the projection the lower axle A appears as a straight shaft. The dimensions given are from an axle in use. In construction the cranks are rectangular in section, of which the width is $\frac{7}{10}$ the depth, and the depth 1·5 the diameter of crank-journal. Double cranks are usually forged solid, and the slot for the crank cut out; that shown in the figure was cast in steel for a double compound engine, 7 × 15 × 15, and has long worked satisfactorily.

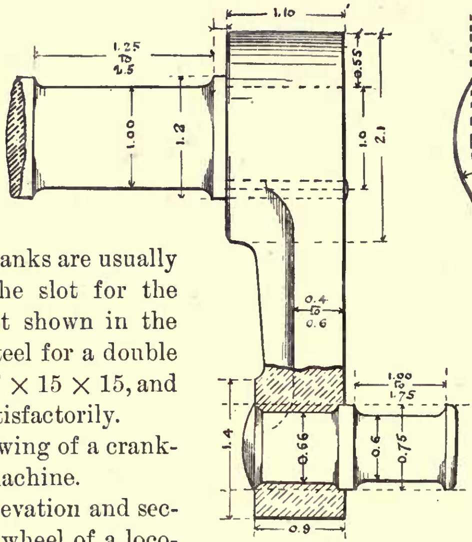


FIG. 756.

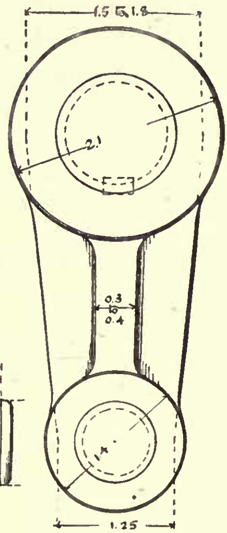


FIG. 757.

Fig. 759 is a drawing of a crank-axle adapted to a machine.

Fig. 760 is an elevation and section of the driving wheel of a locomotive, with a counterbalance opposite the crank-pin, and Fig. 761 is a section of the trailing wheel. The crank-pin of the driver has two journals—one for the main connecting rod, the other for the coupling rod, for which there is a journal on the trailing wheel.

Fig. 762 is a front and side elevation of a return crank, returning back and

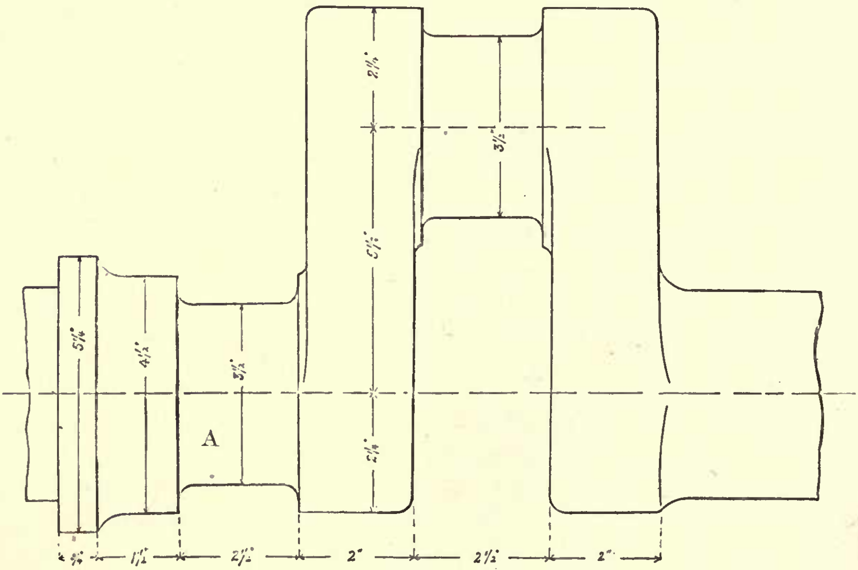


FIG. 758.

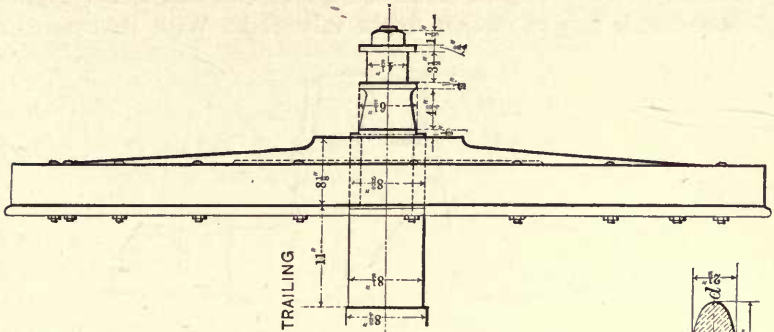


FIG. 761.

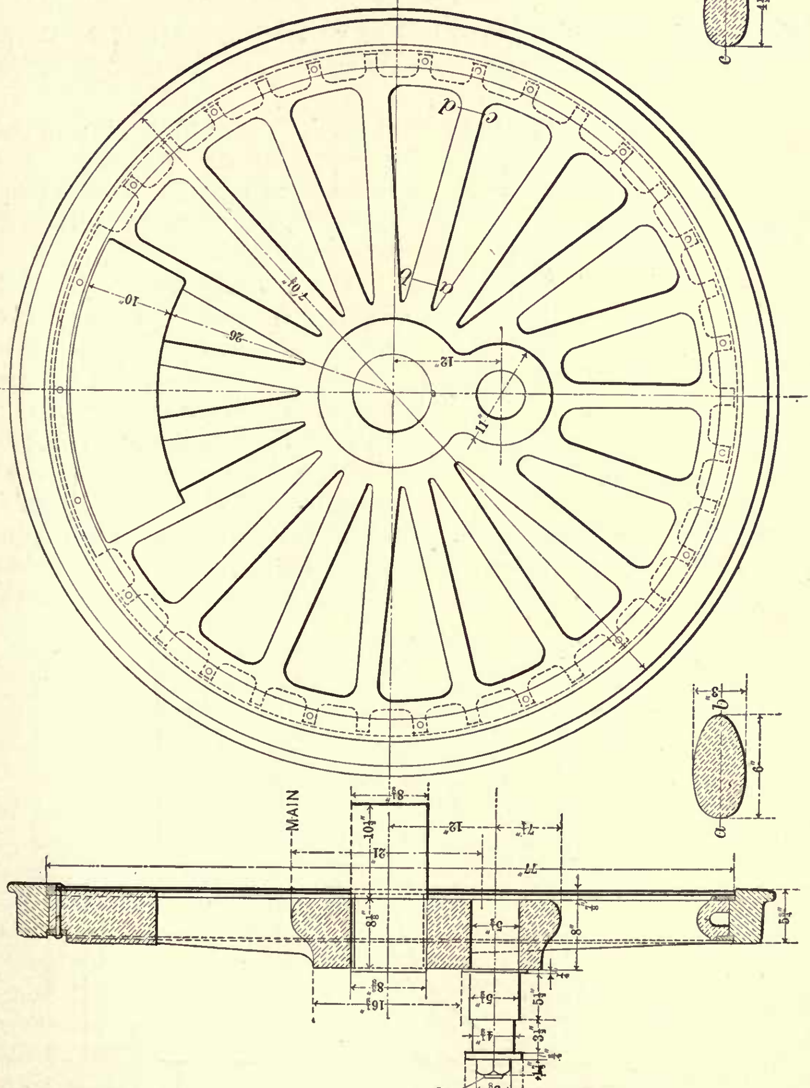


FIG. 760.

having rotation about the main crank-shaft, used on small steam engines instead of an eccentric to give motion to the valve-rod. With its centre on the

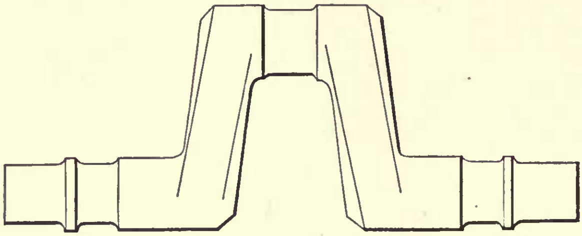


Fig. 759.

central line of the main shaft there will be no motion, and used for the position of an oil cup, the oil flowing down the moving arm for the lubrication of the crank-journal.

Eccentrics.—An eccentric is a modified crank; the crank-pin is enlarged so as to include the crank-shaft; motion is conveyed through the crank to the eccentric, and not through the eccentric to the shaft.

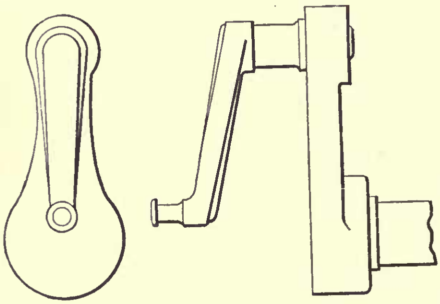


Fig. 762.

Fig. 763 represents a front view, Fig. 764 the side view, and Fig. 765 a section, of a form of eccentric usually adopted in steam engines for giving motion to the valves regulating the action of the steam upon the piston. A ring or hoop, eccentric strap, is accurately fitted within projecting ledges on the outer circumference of the eccentric, so that the latter may revolve freely within it; this ring is connected by an inflexible rod with a system of levers, by which the valve is moved. By the revolution of the shaft to which

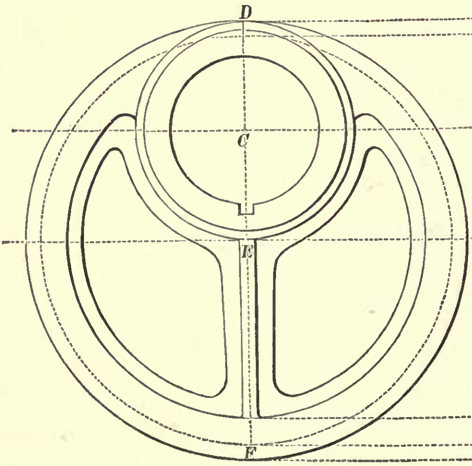


Fig. 763.

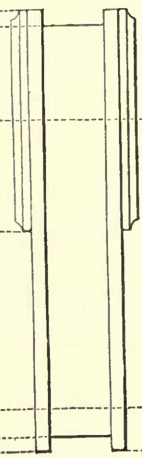


Fig. 764.

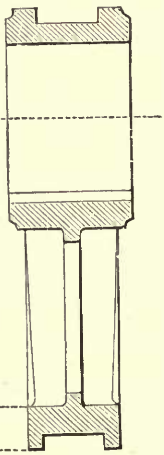


Fig. 765.

the eccentric is fixed an alternating rectilinear motion is impressed upon the rod, its amount being determined by the eccentricity, or distance between the centre of the shaft and that of the exterior circle. The *throw* of the eccentric is twice the eccentricity C E; or the diameter of the circle described by the point E. The alternating motion is identical with that of the crank.

The term *eccentric* is generally confined to the *circular* eccentric, all others, with exception of that last described, being called *cams* or *wipers*, but *eccentric* is often applied to curves composed of points situated at unequal distances from a central point or axis.

Fig. 766.—*To draw the eccentrical symmetrical curve called the heart, which, when revolving with a uniform motion on its axis, communicates to a movable point* A, *a uniform rectilinear motion of ascent and descent.*

Let C be the axis or centre of rotation upon which the eccentric is fixed, and which is supposed to revolve uniformly; and let A A′ be the distance which the point A is required to traverse during a half revolution of the eccentric. From the centre C, with radii respectively equal to C A and C A′, describe two circles; divide the outer one into, say, 16 equal parts, and through these points of division draw the radii C 1, C 2, C 3, etc.; divide the line A A′ into the same number of equal parts as in the semicircle, and through all the points 1′, 2′, 3′, etc., draw circles concentric with the former; the points of their intersection B, D, E, etc., with the respective radii C 1, C 2, C 3, etc., are points in the curve required, its vertex being at the point 8.

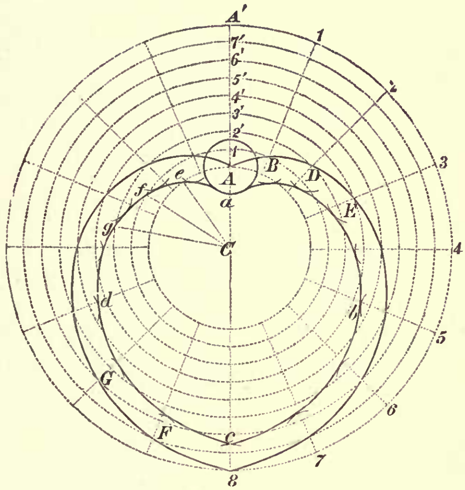


FIG. 766.

When the axis, in its angular motion, shall have passed through one division, the radius C 1 coincides with C A′, the point A, urged upward by the curvature of the revolving body on which it rests, takes the position indicated by 1′; and further, when the succeeding radius C 2 shall have assumed the same position, the point A will have been raised to 2′, and so on till it arrives at A′, after a half revolution of the eccentric. The remaining half, A G F 8, of the eccentric, symmetrical with the other, enables the point A to descend in the same manner as it was elevated.

If the eccentric is turned vertically and the point of a weighted lever rests upon the curve, the lever will take a uniform motion from the curve; but if a groove be cut in the face of a wheel with its outer edge like that of the curve and its inner one parallel to the outer one and a friction roll be inserted in the groove and attached to a lever or rod, the motion of roller and rod will be similar to that of the weighted lever resting on the eccentric. This construction is of frequent use, applicable to a great variety of motions, and is designated as a grooved cam. The grooves may be cut either in the face of a plate or of a cylinder.

In the "Transactions of the American Society of Mechanical Engineers" is an illustration of a grooved cam cut on a machine of W. A. Gabriel, M. E., the motions being first laid out on paper and transferred to wooden or metallic forms, which act as guides to the cutting out of the grooves. The difficulty for the draughtsman is to comprehend the motions required and lay them out on the paper, which in lack of a machine may be transferred to the metal surface and cut by hand, with the aid of drills, chisels, and files.

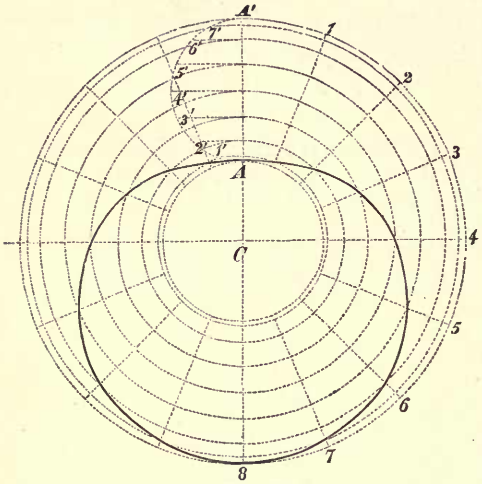


FIG. 767.

Fig. 767.—*To draw a double and symmetrical eccentric curve, such as to cause the point* A *to move in a straight line, and with an unequal motion; the velocity of ascent being accelerated in a given ratio from the starting-point to the vertex of the curve, and the velocity of descent being retarded in the same ratio.*

As in Fig. 766, take C as the centre of motion and A A′ the distance to which the point A is to be moved by a half revolution; with C as centre and a radius C A′ describe a circle and divide the semicircle into eight equal arcs, and draw the radii C 1, C 2, C 3, etc. On A A′ as a diameter describe a semicircle and divide it also into eight equal arcs and project the points of division 1′, 2′, 3′, and 4′; on the diameter A A′, with C as a centre through the several points projected, describe circles; the

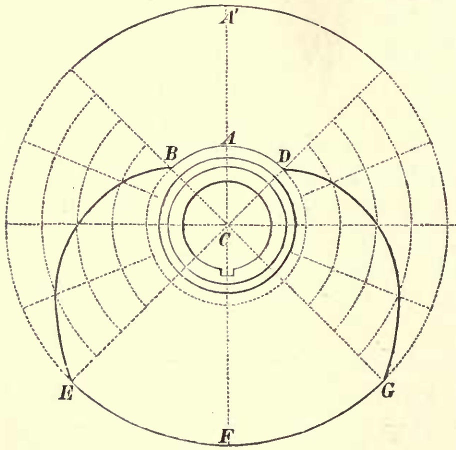


FIG. 768.

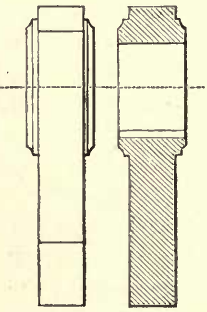


FIG. 769. FIG. 770.

intersection severally of these circles with the radii C 1, C 2, C 3, will give points of the eccentric curve required.

Fig. 768.—*To construct a double and symmetrical eccentric, which shall pro-*

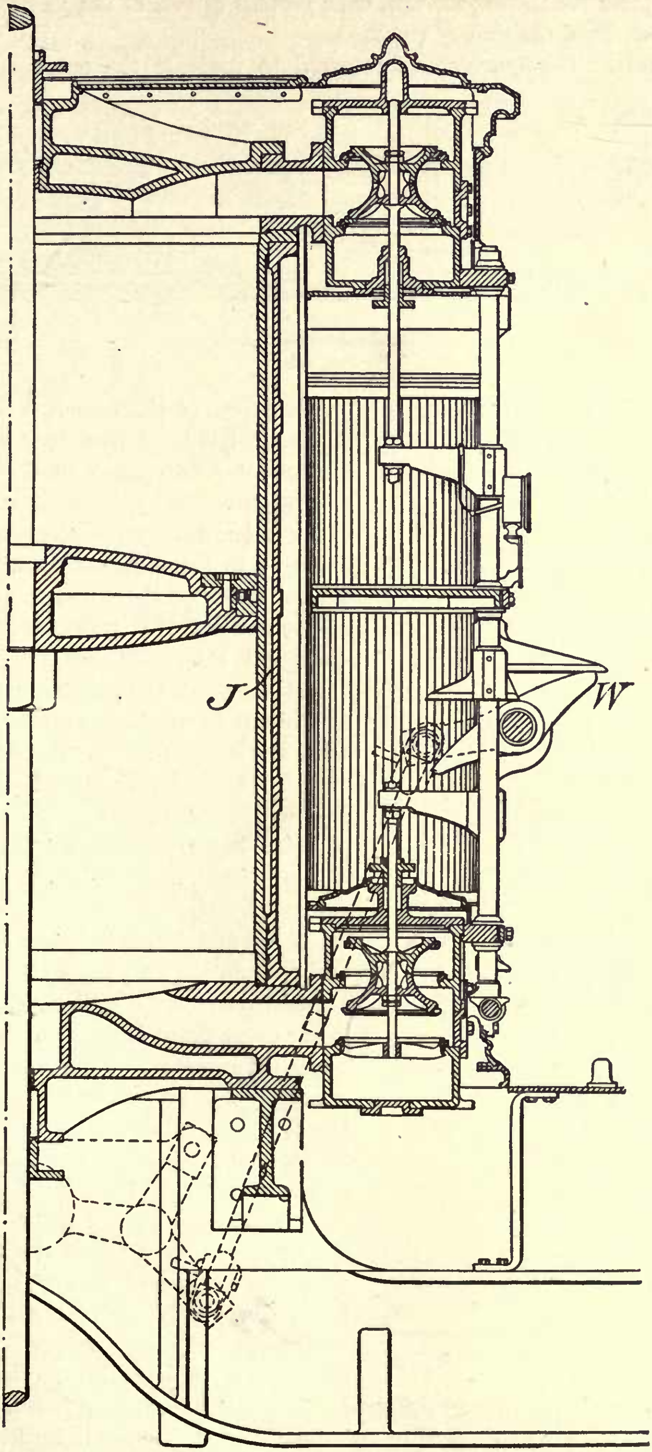


Fig. 771.

duce a uniform rectilinear motion, with periods of rest at the points nearest to, and farthest from, the axis of rotation.

The lines in the figure above referred to indicate the construction of the

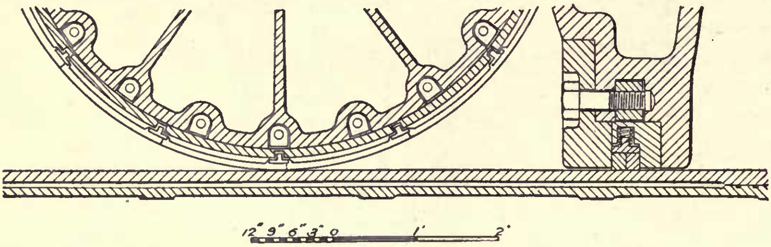


FIG. 772.

curve in question, which is simply a modification of the eccentric represented at Fig. 766. In the present example the eccentric is adapted to allow the movable point A to remain in a state of rest during the first quarter of a revolution B D; then, during the second quarter, to cause it to traverse, with a uniform motion, a given straight line A A′, by means of the curve D G; again, during the next quarter E F G, to render it stationary at the elevation of the point A′; and finally, to allow it to subside along the curve B E, with the same uniform motion as it was elevated, to its original position, after having performed the entire revolution.

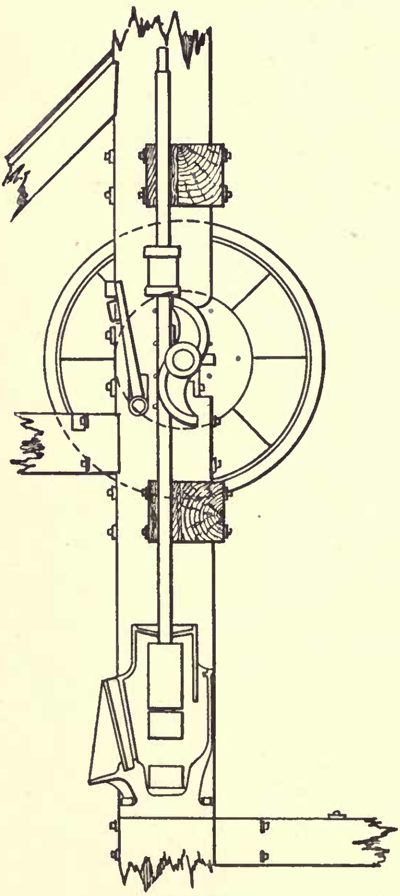


FIG. 773.

Fig. 769 represents an edge view of this eccentric, and Fig. 770 a vertical section of it.

If but one side of this were constructed, and the motion only equal to that of the arc and reciprocating, it would raise and lower every point resting on it, and would be called a wiper. The wiped surface is generally flat, an arm extending out from the rod to be raised, and a curve D G may be formed adapted to the height of lift, and action during the lift.

Fig. 771 is a partial vertical section of the valve motion of one of the high-service pumping engines at St. Louis, Mo., showing the wiper W, or lifting toe, with its connection with the valves. The wiper shaft has a reciprocating motion by which the valve stem is raised and dropped. The valves are double-beat balanced; the cut shows the arrangement of the steam jacket J, a space for live steam inclosing the cylinder, to aid in preserving the temperature and pressure within it.

Fig. 772 exhibits on a larger scale a portion of a section in plan and elevation of the steam piston and mode of packing of the above engine.

Fig. 773 is the elevation of a stamp mill in which a double wiper on a rotary shaft raises the stamp by its contact with a hub or collar on its shaft, and lets it fall suddenly by its weight.

Machines like the above are often used in bleacheries to give a watered finish to goods. The battery of stamps is equal to the width of the cloth which turns in a roll beneath it, and the blows follow in quick succession.

Connections.—Figs. 774 and 775 are sections of cottered joints of wrought-iron bars, the first made with a socket and the end of one of the bars; the

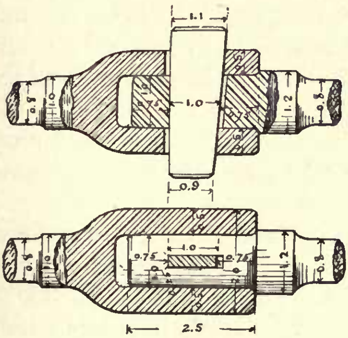


FIG. 774.

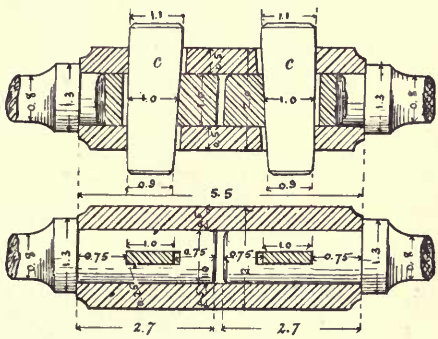


FIG. 775.

latter by a sleeve connecting the two bars. The bars in the socket and sleeve are upset, to give more section than the bars themselves, so that the slots cut for the cotters *c c* will not reduce the strength below that of the bars. The cotters must have sufficient shearing strength and bearing surface, and at the same time diminish as little as possible the section of the parts connected. The proportions given in the figures are drawn to a scale of the diameter of the enlarged part as the unit, and the proportions given in figures are such as obtain in practice for wrought-iron. If the cotter be of steel, its breadth may be three fourths of that given, preserving the other dimensions the same; the thickness is ·25 of the unit.

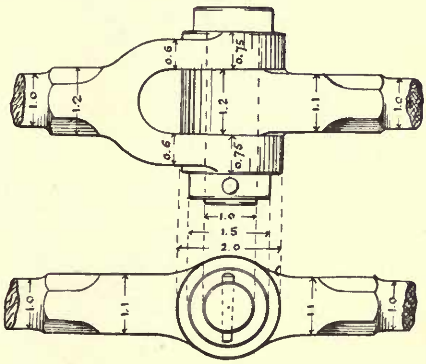


FIG. 776.

The knuckle-joint (Fig. 776) is given in dimensions of the bar as a unit, and adapted to usual work. If there is much motion at the joint, the wearing-surface should be larger, by increasing the width of the eyes and the length of the pin. The pin in the drawing is through the collar; usually the pin is extended, and the pin passes through the bolt outside the collar.

Connecting-rods, in their application to steam engines, are the rods connecting the piston through the cross-head to the crank. When two cranks are connected it is called a *coupling-rod.*

Fig. 777 is the side elevation of the eccentric and strap of a Reynold's 20″ × 48″ Corliss engine. The eccentric rod is made with a shoulder, the stem is fitted to the 1⅞″ socket, and held firmly by a nut in the 4″ groove.

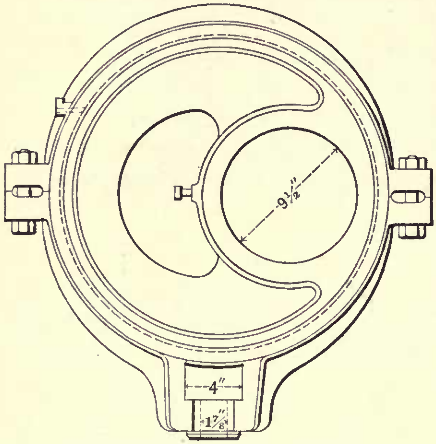


Fig. 777.

Fig. 778 is a double eccentric, used in giving motion to the link in the cut-off. The eccentrics are made in halves for convenience in putting them on the shaft.

Governors are used on quick-running engines (page 407) to control the cut-off through movable eccentrics, but eccentrics adjustable by hand are convenient as applied to pumps in modifying the stroke. With a uniform action of the motor the stroke of the pump is reduced, giving greater pressure on the plunger, say from a domestic to a fire pressure.

Figs. 779 and 780 are plan and section of such an eccentric. The crank-pin is fastened on a slide-rest moving in guides attached to the power shaft by a long hub. To hold the slide firmly to the shaft-hub there are three key-bolts, *n n n*. To move the crank-pin slide these bolts must be slacked, and by means of a hand lever (Fig. 781), using as a fulcrum the pins on one of the guides, and bringing the short end of the lever against a projection, *p*, attached to the crank-pin, it may be moved to any desirable position or stroke and clamped to the hub firmly by the key-nuts.

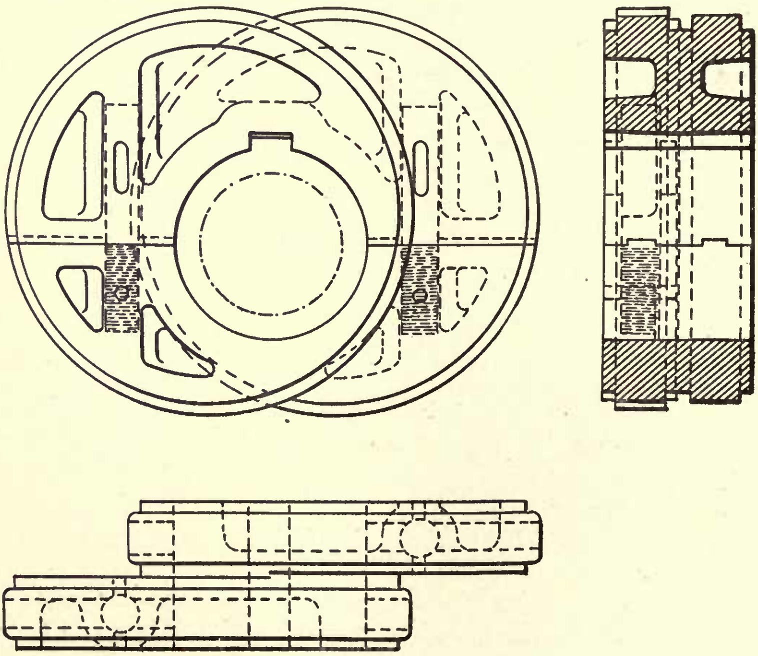


Fig. 778.

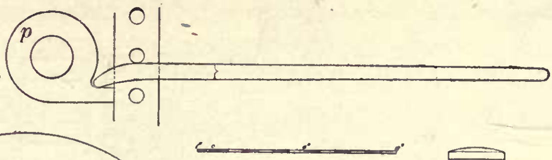


FIG. 781.

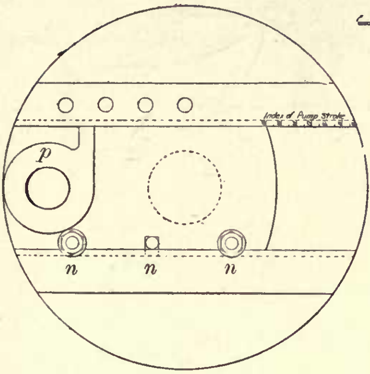


FIG. 779.

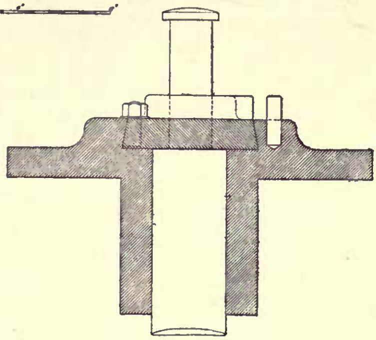


FIG. 780.

Fig. 782 is the elevation, plan, and section of an eccentric strap of a locomotive engine, of cast-iron, with very long bolts, and a very rigid construction. The strap forms a cup-section over the projecting ring on the eccentric, and retains the oil at the bottom for more efficient lubrication and prevents drip.

Figs. 783, 784, and 785 is another cast-iron eccentric strap, in which a box is inserted, fitted with metallic disks, to prevent frequent oiling. The bolts at the large end are bored up to a sectional area of that of the screwed portion to secure equal elasticity.

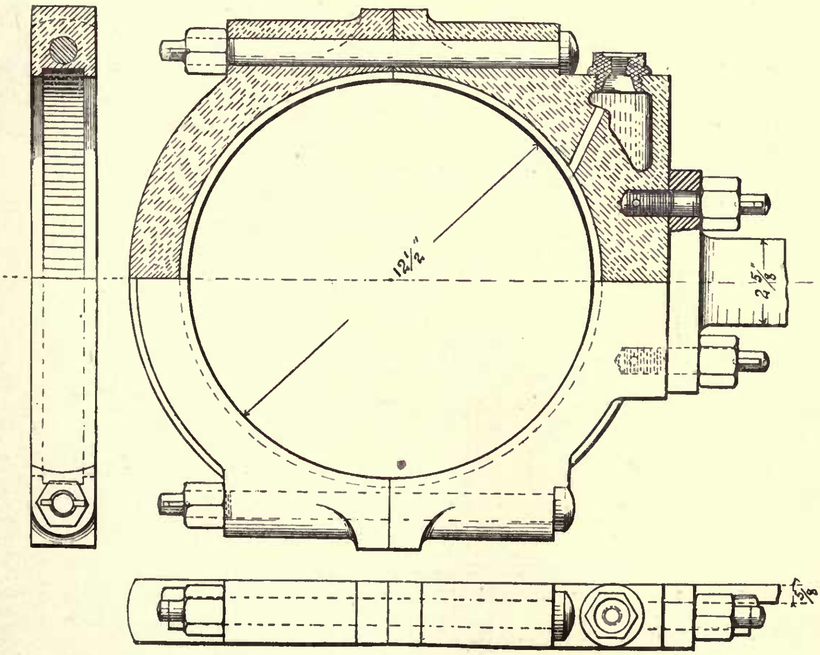


FIG. 782.

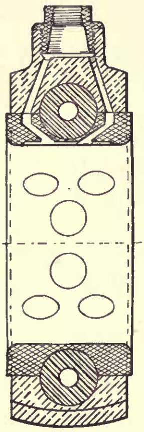


FIG. 785.

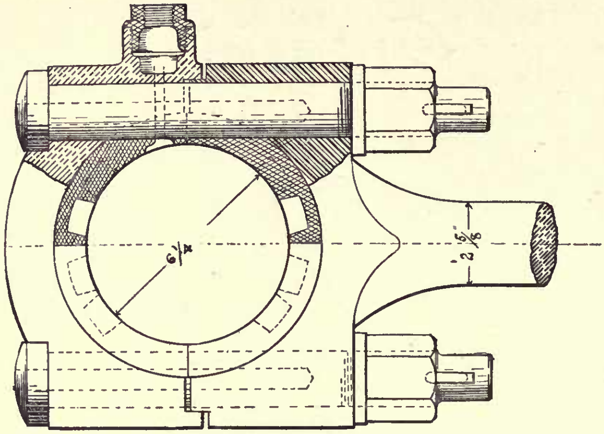


FIG. 783.

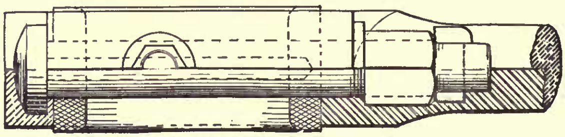


FIG. 784.

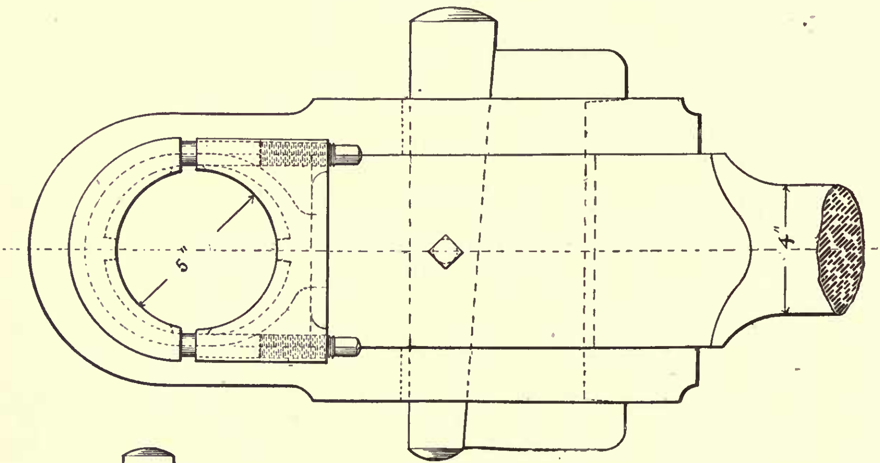


FIG. 786.

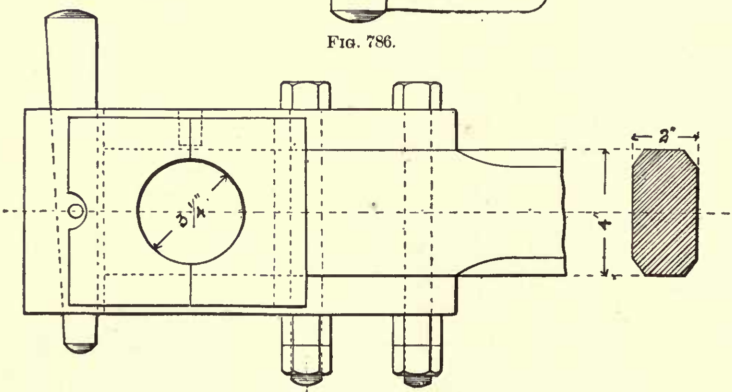


FIG. 787.

In many marine engines the boxes of both crank and cross-head pins are connected with strong and heavy bolts without any other rod.

Fig. 786 is a strap-end of a connecting-rod, from the Corliss Steam-Engine Company. The peculiarity is the adjusting-screws connected with the boxes.

Fig. 787 is the strap-end of a locomotive connecting-rod in which the wear of the boxes is taken up by a cotter at the end of the strap.

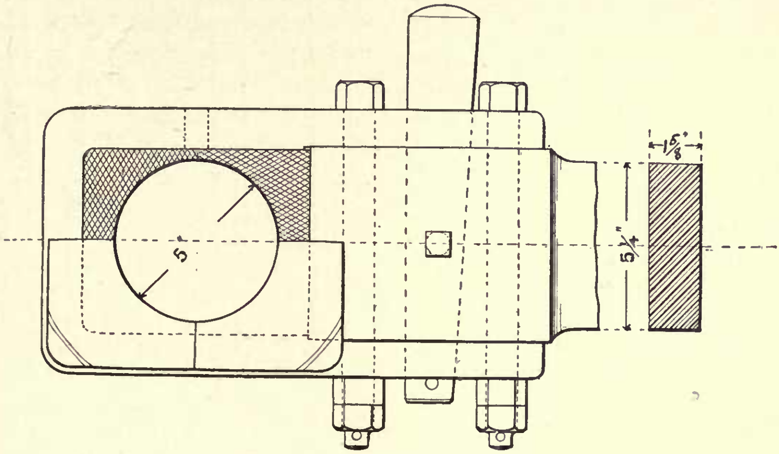


FIG. 788.

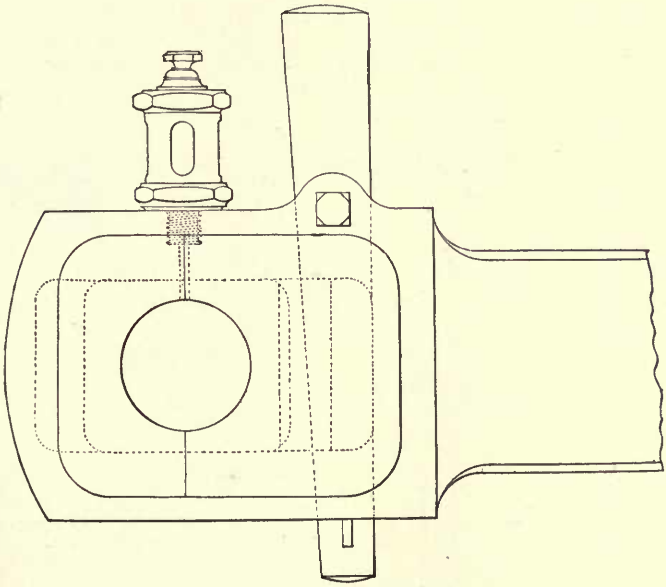


FIG. 789.

In Fig. 788 the key is between the bolts; the weakness from bolt-holes or cotter-slot is compensated by the width of the strap.

Fig. 789 is the box-end of a locomotive-rod, in which the strap gives place to a box forged on the end of the connecting-rod.

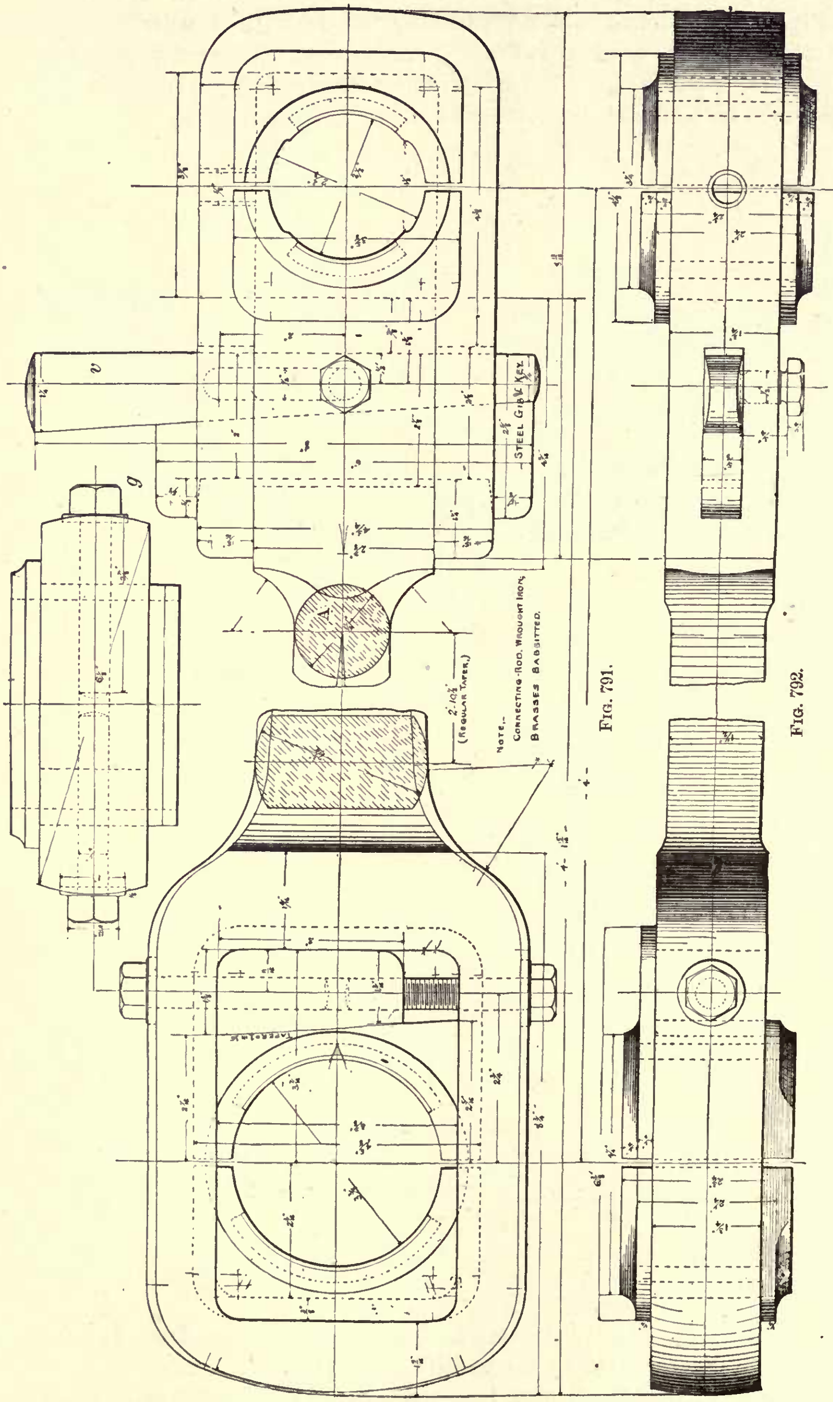


FIG. 790.

FIG. 791.

FIG. 792.

Figs. 790, 791, and 792 are side, plan, and end views of a connecting-rod of the Southwark Foundry, of Philadelphia, used on their fast-running Porter-Allen engines.

The cross-head end is a *strap-end*, while that of the crank is a *box-end*, and of larger diameter than the former on account of the extra wear and size of the crank-pin. The length of the page does not admit of the representation of the full length of the connecting-rod on the scale; it is therefore shown broken, with the dimensions figured in. The sections of the two ends are drawn in on the rods; the circular section A is the same as that of the piston-rod, and both are represented in the conventional hatching of cast-iron, but it is of wrought-iron. The gib *g* and key or cotter *v* at the strap-end are of steel, and the key is fastened when in position by a set-screw through the head. At the box-end, a wedge and screw forces the box into position.

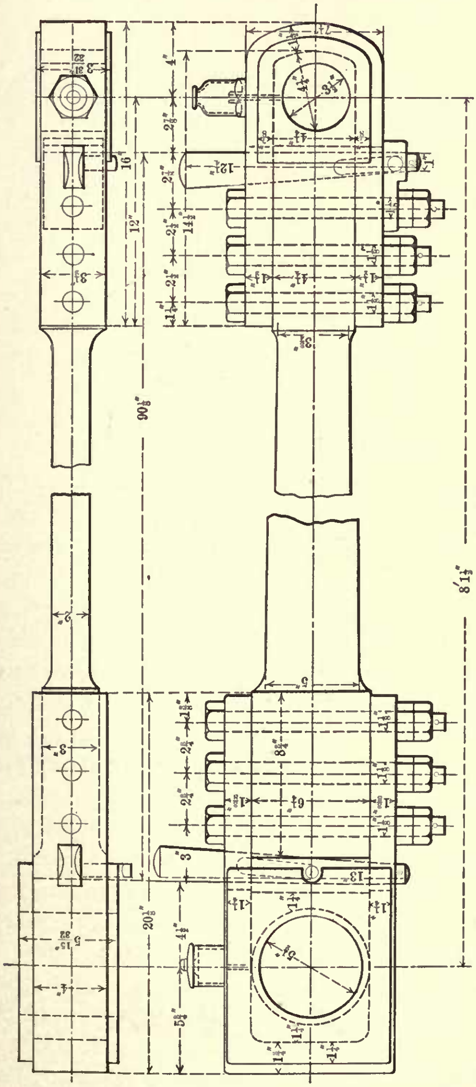


Fig. 793.

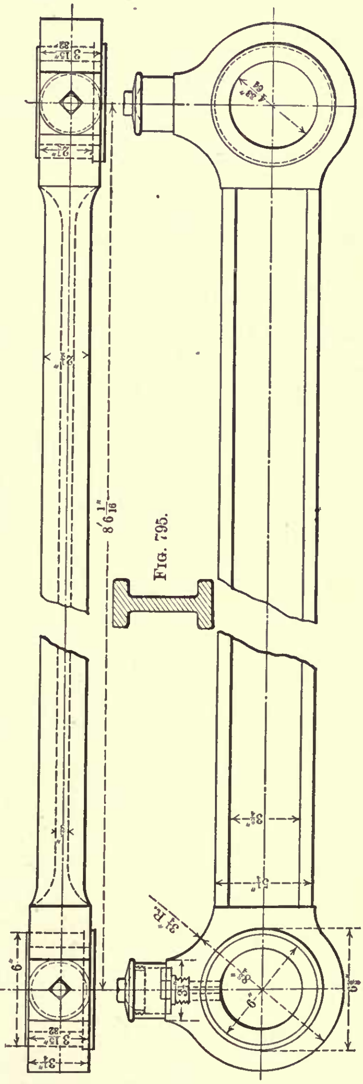


Fig. 795.

Fig. 794.

Figs. 793 and 794 are the sides and top elevations of the connecting- and coupling-rods of the express locomotives of the New York Central and Hudson River Railroad, designed by Mr. Buchanan, and built at the Schenectady Locomotive Works, of which the drawing of driving wheels will be found at Fig. 760, of the frame Fig. 1301, and the boiler Fig. 926, with the general weights and dimensions of the locomotive. Fig. 795 is a section of the coupling-rod.

Fig. 796 is a connecting-rod of the Rider hot-air engine, in which the ends, made of gun metal, are connected together by a tube in which is fitted a rod, extending from the upper to the lower brass, and so arranged that one key, E, capable of nice adjustment by nuts, at once takes up the lost motion on both upper and lower brasses.

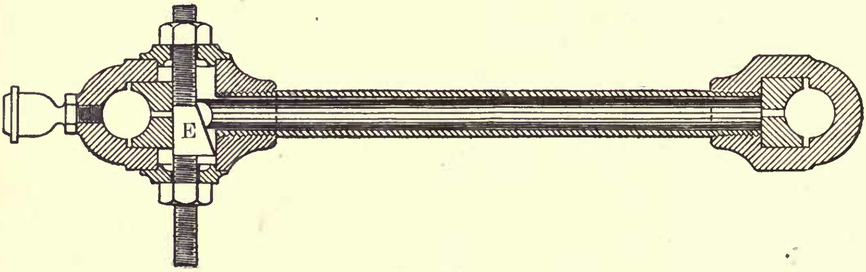


FIG. 796.

Fig. 797 is the stub end of a coupling-rod. The bushes are solid, of brass, and kept from turning round by taper-pins, which are secured by set-screws pressing on the larger end; taper, $\frac{1}{16}$ in 3 inches.

Cast-iron connecting-rods are now very seldom used. In some cases of vertical-beam pumping engines it is necessary that the weight of the steam pistons should be counterbalanced by some mass of material, and it may be convenient to make use of a heavy pump-connection.

The wrought-iron crank connections of American river-boat engines are peculiar in their construction. They are made as light as possible, with very great stiffness. Fig. 798 represents the side elevation of such a connecting-rod. The means adopted to give the required stiffness consist of a double-truss brace, *a a*, of round iron, which is fastened by bolts to the rod near each end; struts *b b*, cut with a screw, and furnished with nuts, pass through the centre of the brace, by which means the braces are tightened. The connecting-rod at its smallest part near the extremities is of the same diameter as the piston-rod; the boss in the centre is from one to two inches more.

Fig. 799 is the front view of the forked end of the rod, which is fitted with the usual straps, gibs, and cotters. Fig. 800 is the side view of the brace-rod.

The working-beam (Figs. 801 and 802), of similar framed construction, was at one time largely used for river boats, but may still be applicable in many places for its lightness and stiffness. It is composed of a skeleton frame of cast-iron round which a wrought-iron strap is fitted and fastened. The strap is forged in one piece, and its extreme ends are formed into large eyes, which are bored to receive the end-pins or journals. The skeleton frame is a single casting, and contains the eyes for the main centre and air-pump journals; the centre hub is strengthened by wrought-iron hoops shrunk upon it. At the points of contact of the strap and skeleton, key-beds are prepared. Small straps connect the frame and main-strap at these points, keyed to the frame—keys riveted over. The frame is further braced by wrought-iron straps, C C, which tie the middle of the long arms to the extremities of the shorter ones. The following are the general dimensions: From centre to centre of end-journals, 26 feet; this is somewhat less than the usual proportion to length of stroke, being slightly less than double the stroke; length of centre hub, 26″, *a a*; diameter of main centre eye *c*, 15⅝″; of air-pump journal-eye *d*, 6¾″; of end-journals *e e*, 8⅛″.

Double plates or flitches of wrought-iron are often used in the construction of working-beams and side-levers. Fig. 803 is the section between the two plates of a beam

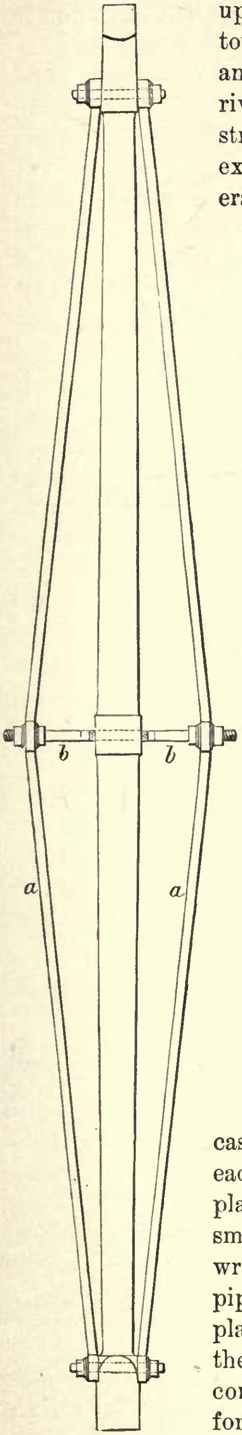


FIG. 798.

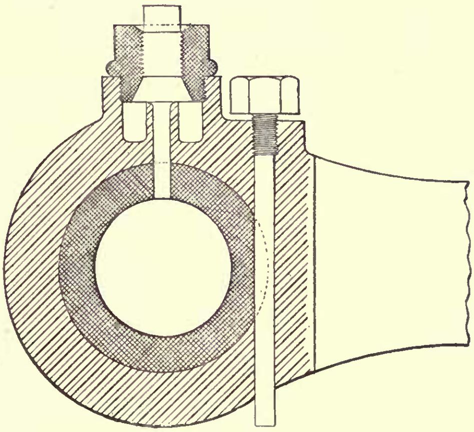


FIG. 797.

of this kind, attached to the compound pumping engines at Milwaukee, Wis. The plates are each 30 feet long, by 6′ 4″ deep at centre, by 1¾″ thick. The connections between the two, shown in section in the figure, are cast-iron pipes with wide flanges at each end riveted or bolted to the plates. The main centre and other small journal-pins are rods of wrought-iron, passing through the pipes, and extending outside the plates to form the journals; *c* is the section of the pin for crank connection, *p* for that of pump, *h* for that of high-pressure cylinder, *l* for that of low-pressure cylinder,

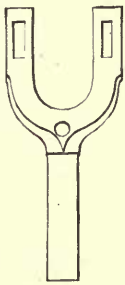


FIG. 799.

FIG. 800.

m for main centre-pin, and *g* for the parallel-motion links. This last is usually the position of the air-pump centre, but in this engine the air-pump is

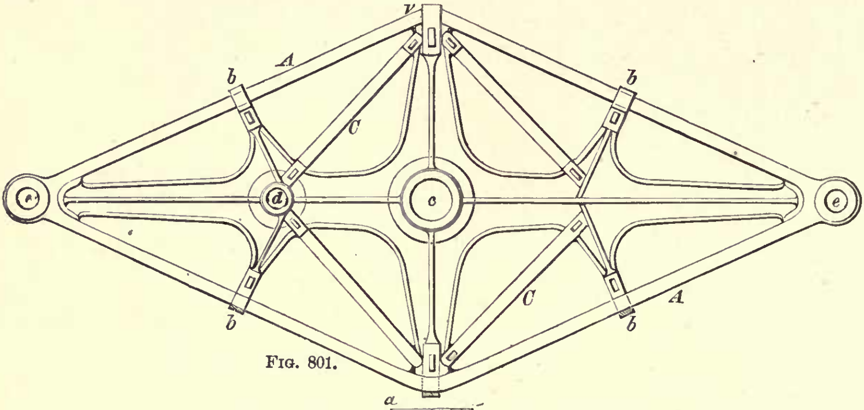


FIG. 801.

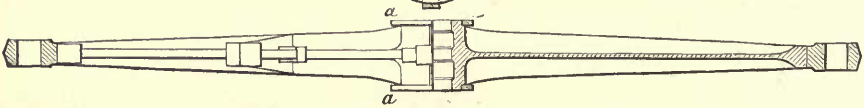


FIG. 802.

below the high-pressure cylinder, and its piston-rod is extended to the air-pump piston. The dimensions are—H. P., cylinder 36″ × 62″; L. P., cylin-

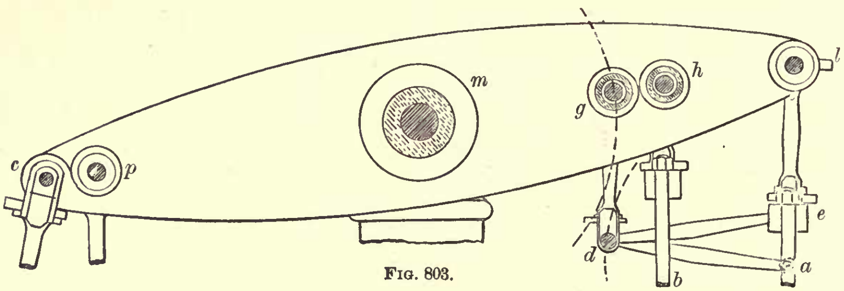


FIG. 803.

der 58″ × 8 feet. The heads of the two cylinders are kept at the same levels by increasing the length of the H. P. piston.

Working-beams with vertical steam engines are now seldom used except in

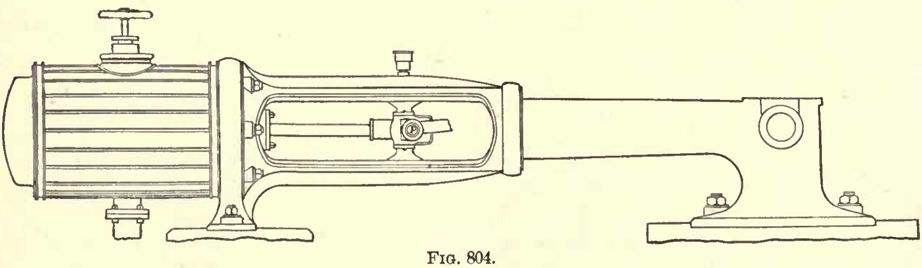


FIG. 804.

connection with large pumps. For power the horizontal engine is now substituted and connected closely with the fly-wheel. Fig. 804 represents such an engine and frame.

The pin *p* is the connection in the *cross-head* (Fig. 804) between the piston-rod of the steam engine and the connecting-rod to the crank.

Figs. 805, 806, and 807 represent the plan, end view, and section of the cross-head adopted by the Southwark Foundry for their high-speed engines. It is of cast-iron, with large, flat faces, the pin *p* for the connecting-rod being in the middle of the length. This pin is of wrought-iron, large and flattened on top and bottom, so that the boxes of the rod can never bind on the pin at the extreme of the vibrations of the rod; usually these pins are round. The pin is formed with large squares at the ends, by which it is fitted into the jaws of the cross-head, where it is secured by a steel pin passing through the cross-head. The bearing surfaces of the head and those of the guide-bars are finished by scraping to true planes; there are no means of adjustment, as there is no wear if kept clean.

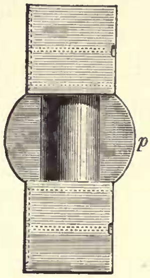


FIG. 806.

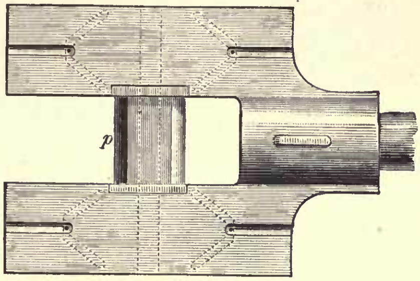


FIG. 805.

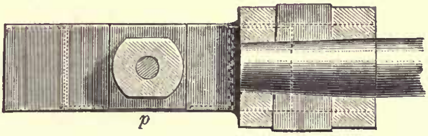


FIG. 807.

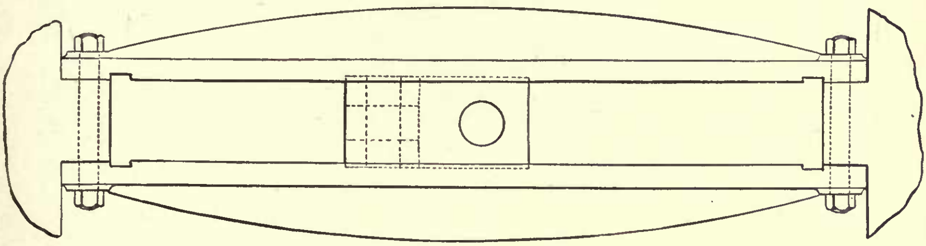


FIG. 808.

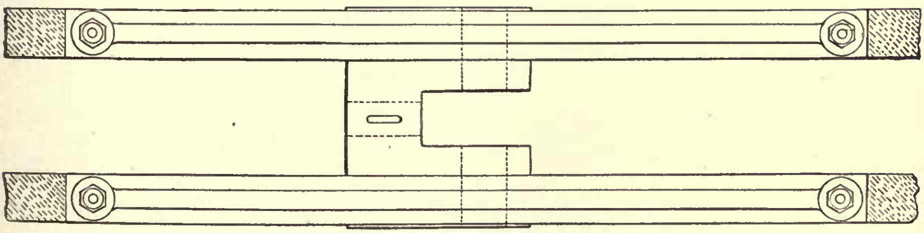


FIG. 809.

It is to be understood that the piston-rod moves in a straight line, and that the stress on the connecting-rod pin is mostly oblique. Guides are to be provided, between which the cross-head slides, to take the oblique stress off the piston-rod.

Figs. 808 and 809 are elevation and plan of *guide-bars* suited to the cross-head above, which are in common use for both vertical and horizontal engines. Lugs or ears are cast on the steam-cylinders, and on the frames to which the bars are bolted, and between which the cross-head slides. The grooves or notches across the guide-bars, at the ends of the stroke, are to throw off any grease or dirt that may be carried along by the head and prevent their accumulation. The stress on the guide-bars is due to the pressure of the steam on the piston acting obliquely on the crank through the connecting-rod, and is the greatest when the crank is at right angles to the piston. It can be determined by multiplying the pressure on the piston by the length of the crank, and dividing the product by the length of the connecting-rod, which will be the stress tending to separate the guides. If the connecting-rod be 3 times the stroke, or 6 times that of the crank, which is the usual proportion, then the stress is $\frac{1}{6}$ the pressure on the piston. Sometimes the proportion of connecting-rod to

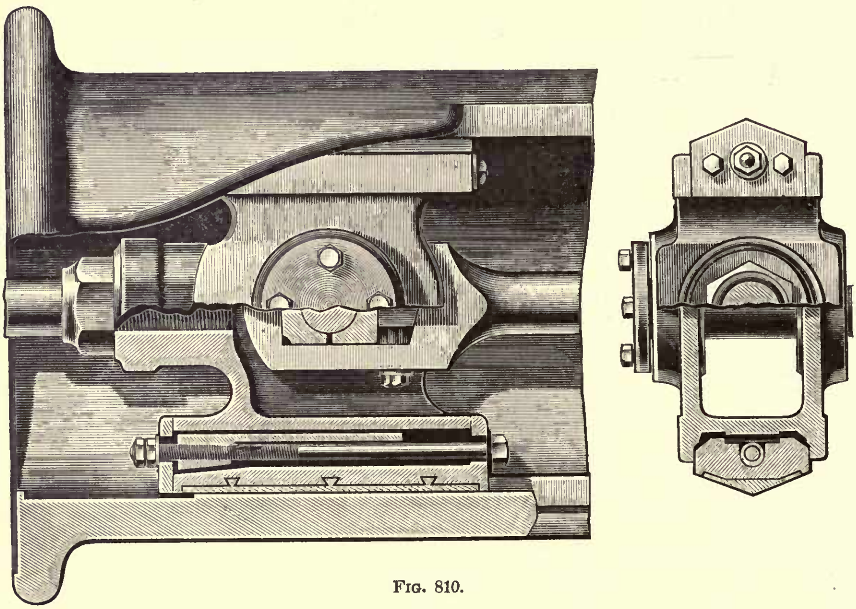


Fig. 810.

stroke is $2\frac{1}{2}$ to 1. When a portion of the force of the steam is opposed directly to the resistance, as in direct-acting pumps, and only the irregularities in the steam-pressure are transmitted through the connecting-rods, the proportion of rod to stroke may be still smaller.

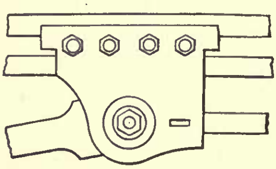


Fig. 811.

Fig. 810 is the cross-head of the Harris-Corliss engine in which the bearing surfaces of guides are adjusted by bolts passing through wedges.

On locomotives it is not unusual to have the guide on one side, as in Fig. 811, where the side-bars are of wrought-iron and the slide-block is fastened between the two plates of the cross-head by bolts. It is the most common practice in this country to use guides with vertical engines, even when the connection is with working-beams.

Steam-Cylinders.—Fig. 812 is a sectional plan of a common form of small steam-cylinder. A is the cylinder, B the piston, *b* the piston-rod, D the slide-valve, *d* the valve-rod, C the valve-chest, *c* the chest-cover, *s s* the steam-ports, *e* the exhaust-port, S the stuffing-box of the piston-rod, *s'* that of the valve-rod. H is the front head and H' the back head of the cylinder. The bolts attaching the heads to the body of the cylinder are not shown.

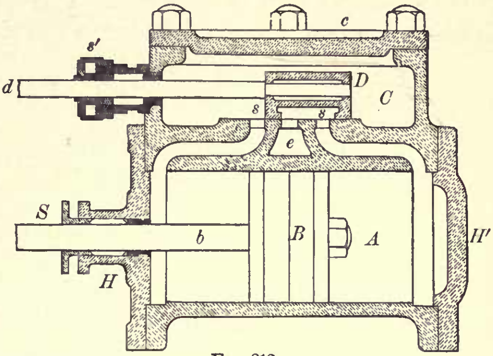


FIG. 812.

Length of Cylinder.—It is the present practice, in the construction of stationary engines for driving machinery, to make the stroke not over twice the diameter of the cylinder, and for diameters above 24″ about 1½ time the diameter of the cylinder, and invariably to place the cylinders horizontally with a direct connection with the crank, without the intervention of a working-beam.

Fig. 813 is a front view partly in section, and Fig. 814 is a transverse section through the centre of a Fishkill Corliss steam-engine cylinder, giving the steam-

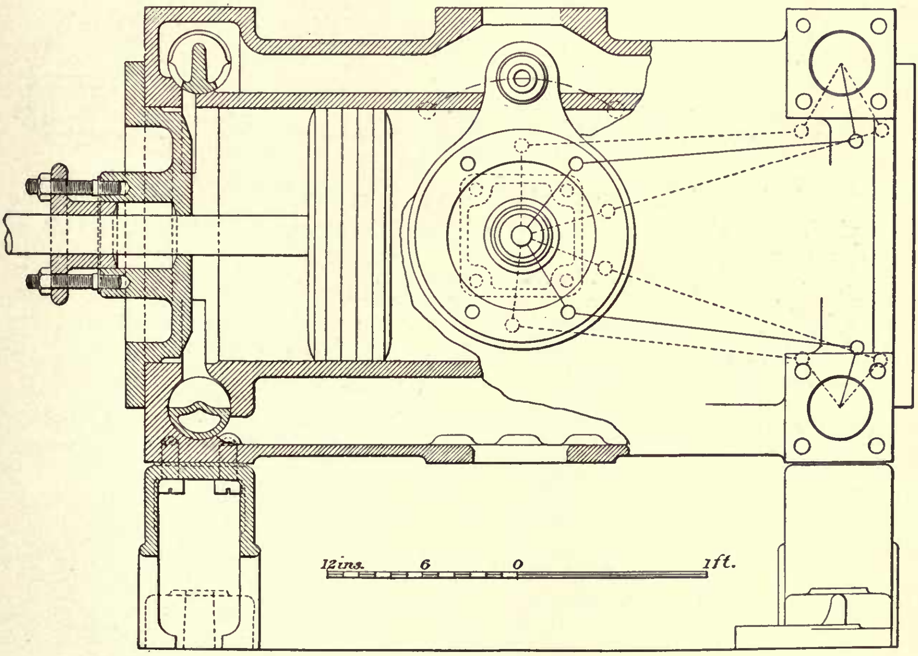


FIG. 813.

valves in section and outside view showing their connection with the wrist-plate and its motion, also the piston-rod and stuffing-box with a scale of reference. The thickness of shell Mr. Henthorne finds, by many examples in Corliss's large practice, to conform to the formula $t = \cdot268\sqrt{d}$, t and d being in inches.

Thus the thickness of the shell of a 16″ cylinder will be $\sqrt{16} \times \cdot 268 = 4 \times \cdot 268 = 1\cdot 072$, a little more than 1″. The thickness of flanges should exceed that of the shell by $\frac{1}{8}$ to $\frac{1}{4}$ its thickness. The bolts should not be less than $\frac{3}{4}$″ and seldom more than 1″. It is better to increase the number of bolts than their diameter, the breadth of flange to be about 3 times the diameter of the bolts, and the pitch of the bolts, or the distance between centres, about 6 times the diameter of the bolts.

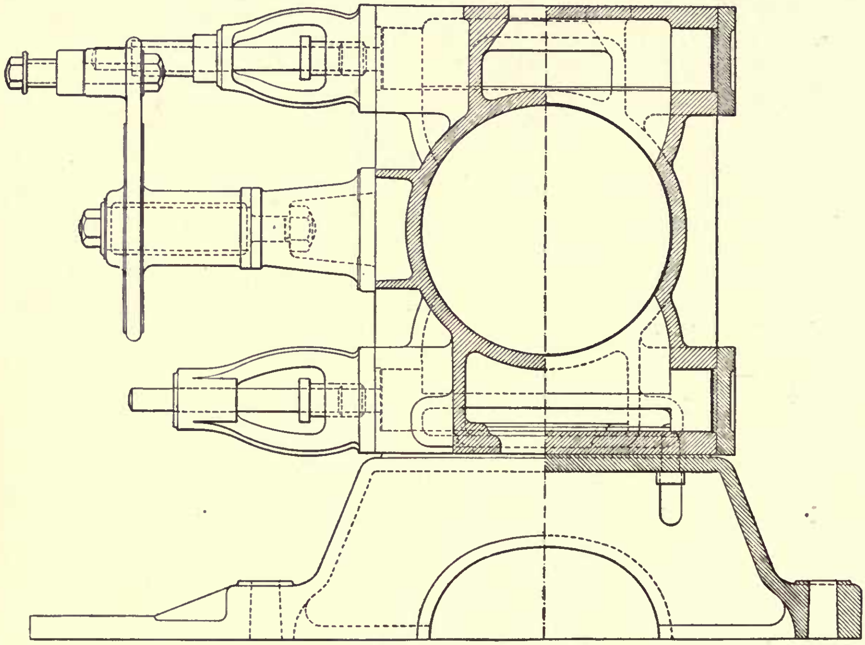


FIG. 814.

Cast-iron is the material chiefly used for pistons, but those of wrought-iron, brass, bronze, and cast-steel are common. The wrought-iron pistons recently introduced in American locomotive practice follow the designs of the older cast-iron pistons.

Fig. 815 is the cast-iron piston of a locomotive. The spring or snap-rings forming the packing are of cast-iron, $1\frac{1}{2}$″ wide by $\frac{1}{2}$″ thick, of uniform section. The split is made with a half lap, and the splits of the two rings are on opposite sides of the piston. The outsides of the rings are turned to a diameter slightly in excess of that of the cylinder, and are sprung into recesses of the piston fitted to receive them.

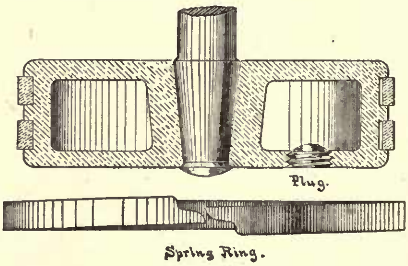


FIG. 815.

Fig. 816 is a half plan and half section and Fig. 817 a full cross-section of piston of a steam-cylinder of Leavitt's design; the packing is Wheelock's, in which the rings are in sections joined to adjust themselves to cylinders that have become worn.

HALF SECTION C D

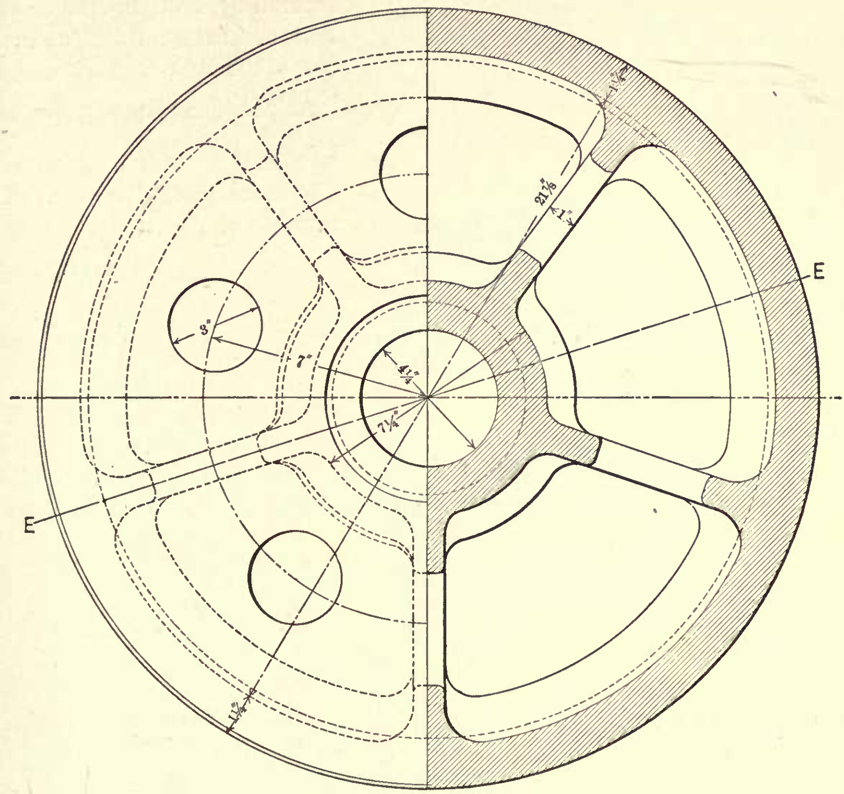


FIG. 816.

SECTION E F

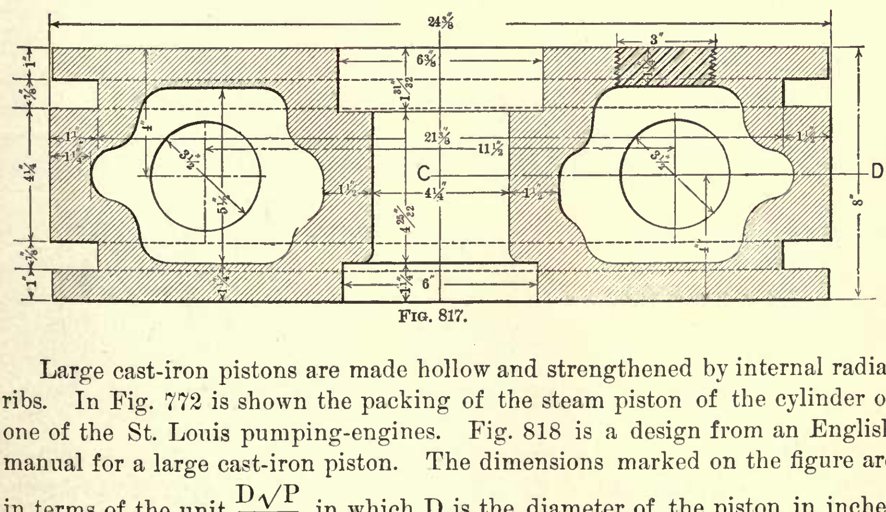


FIG. 817.

Large cast-iron pistons are made hollow and strengthened by internal radial ribs. In Fig. 772 is shown the packing of the steam piston of the cylinder of one of the St. Louis pumping-engines. Fig. 818 is a design from an English manual for a large cast-iron piston. The dimensions marked on the figure are in terms of the unit $\frac{D\sqrt{P}}{100}$, in which D is the diameter of the piston in inches

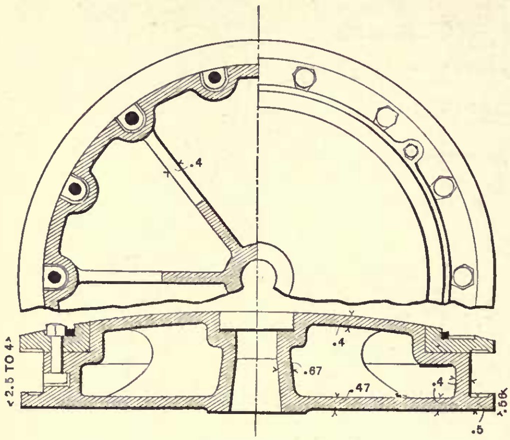


FIG. 818.

and P the initial pressure per square inch. The junk-ring is secured by wrought-iron or steel bolts and brass nuts. The diameter of the junk-ring bolts may be $\frac{\cdot 28\,D\sqrt{P}}{100} + \frac{1}{4}$ inch, and they may be placed at a pitch of seven to ten times their diameter. The number of ribs or webs may be about $\frac{10}{D} + 2$ and their thickness $\frac{\cdot 4\,D\sqrt{P}}{100}$. The size of the space for the packing will depend on the design of packing adopted.

Fig. 819 is a sectional plan and Fig. 820 is a sectional elevation of the interior of a piston-ring, showing another common form of ring packing, which consists of a single interior ring *r* and two exterior rings *r″ r″*, and each cut in

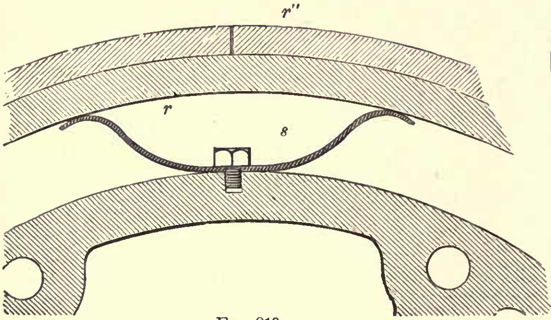


FIG. 819.

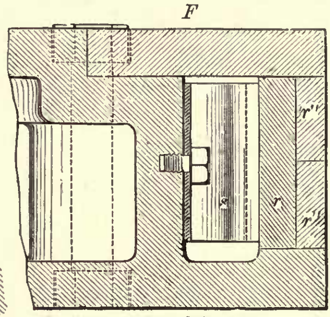


FIG. 820.

two and so fastened that the joints are always broken. The packing is set out by springs *s s*, one of which is shown. F is the follower, which can be taken off for the admission of the rings and springs, and then replaced and bolted to the piston, making a close joint with the end of the rings. The depth of the piston at the exterior is from 3″ to 9″, varying with the diameter of the piston.

Figs. 821, 822, and 823 are sections of the exterior rings of pistons adapted

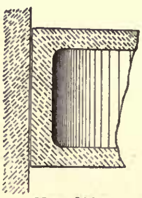


FIG. 821.

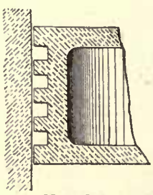


FIG. 822.

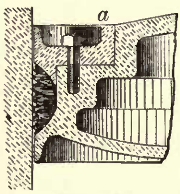


FIG. 823.

more particularly to water-pumps. Fig. 821 depends on the closeness of fit of the exterior of the piston with the inner surface of the cylinder, and when accurately turned and fitted the leak is very inconsiderable, and by the use of grooves

(Fig. 822) it is still less. In Fig. 823 the joint between the piston and the cylinder is made tight by a gasket, usually of hemp, compressed by a joint ring or follower, *a*, in the pocket between piston and cylinder.

Wood-packing put in short staves, as shown in Fig. 824, is often used for pump-pistons and buckets. Make the diameter of the wood-packing a little less than the diameter of the barrel of the pump, to allow for the swelling which takes place when the wood becomes saturated with water.

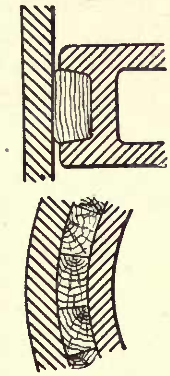


FIG. 824.

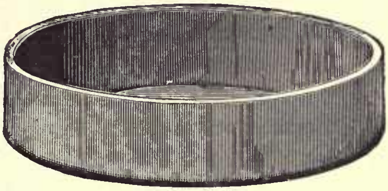


FIG. 825.

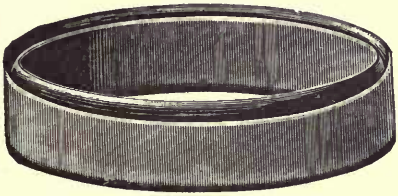


FIG. 826.

Fig. 825 is a cup leather packing, and Fig. 826 is a U-packing for small cylinders of hydraulic presses. The application of the first will be understood from Fig. 827, in which the piston is packed with two cup leathers, in this case to withstand pressure in both directions. Were the piston single-acting, but one cup would be necessary—and if from beneath the piston, this would be the lower cup. The flexible flange is pressed against the inside of the cylinder, and the joint is perfectly tight.

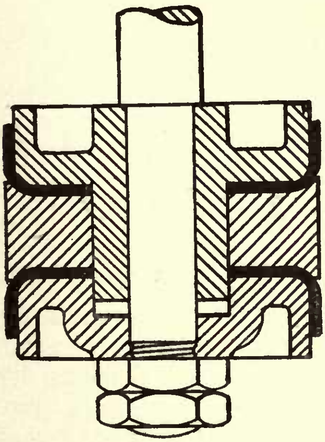


FIG. 827.

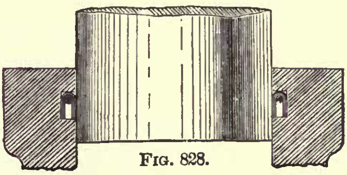


FIG. 828.

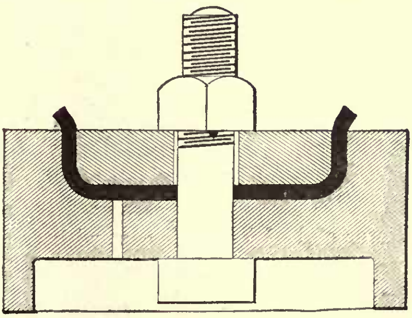


FIG. 829.

Fig. 828 shows the application of the U-packing; it is put into a recess in the cylinder by bending the packing into a saddle-bag form, and then allowing it to spring back into the recess. In English practice hemp packings serve the same purpose, and are necessary when the temperature of the water exceeds 90°.

Packings can be obtained from hydraulic-pump and press manufacturers, and are kept in stock of all the usual sizes. Their depths are from 1″ to 1½″ for diameters varying from 4″ to 14″, and the space occupied by the thickness in the U from ½″ to ⅝″. A filling of flat braided hemp is placed inside the U to keep it tight when not under pressure. The packings are made by steeping the leather in warm water, forcing them into a mould, and leaving them to dry and harden. The moulds (Fig. 829) are made of either metal or wood; frequently the rings are of metal, and the piston over which the cup is formed, of wood.

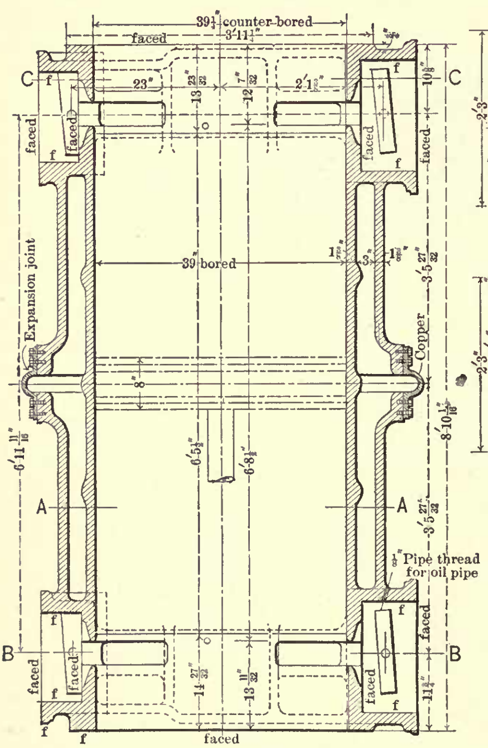


Fig. 830.

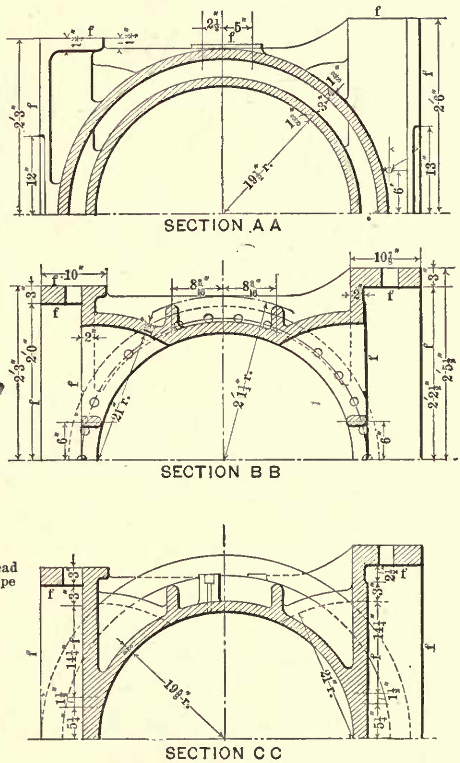


Fig. 831.

The outer cylinder forming the exterior shell of the steam-jacket is fastened to the steam-cylinder, but is allowed to move under changes of temperature. In Figs. 830 and 831 are given longitudinal and transverse sections of a steam-cylinder with jacket, as constructed by E. D. Leavitt, M. E., in which the upper and lower end of jacket is cast with the cylinder, and the connection between the two is by a copper U-ring, which admits the necessary expansion and contraction. All steam-cylinders, whether with or without jackets, should be clothed—that is, covered with some preparation to prevent the escape of heat from contact with air. The usual clothing is hair felt, with a lagging—that is, an exterior shell of some wood, usually black walnut.

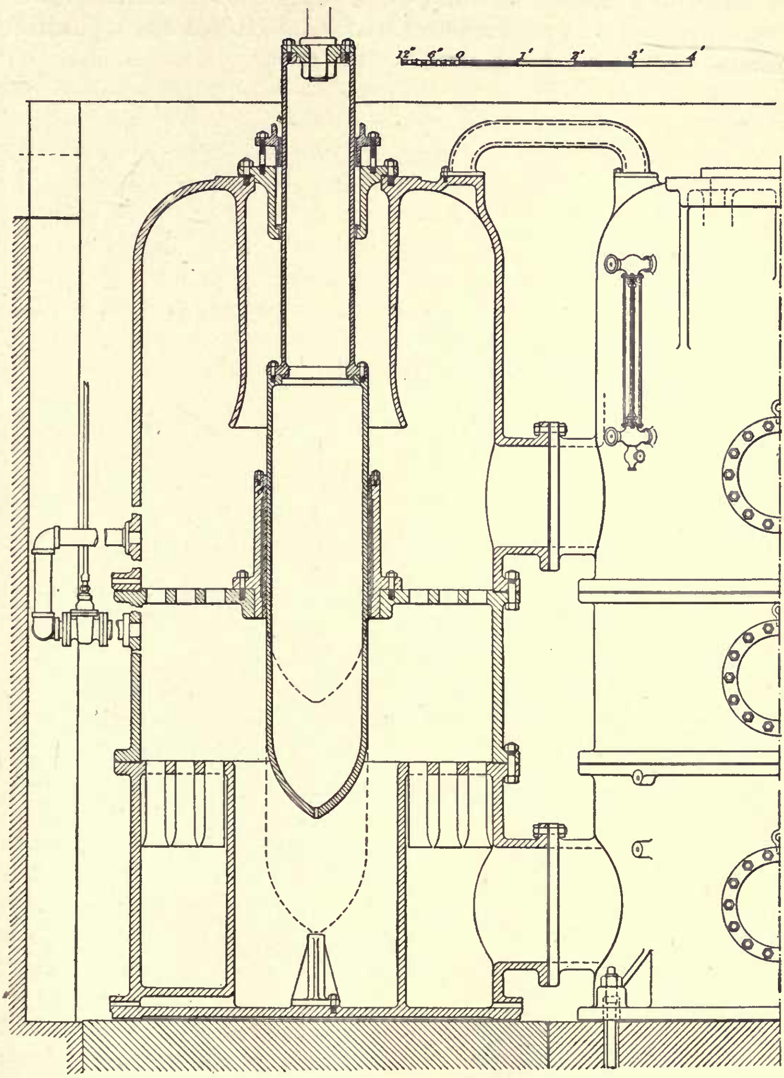


FIG. 832.

Fig. 832 is a section and partial elevation of the air-chamber of one of Leavitt's pumps. It is of the Thames Ditton variety, in which the whole charge of water is drawn into the lower chamber, and a portion corresponding to the difference between the two plungers raised to the full head in the upper chamber; in the down stroke a quantity of water equal to the displacement of the upper plunger is raised to the full head. The packing of the lower plunger is by grooves in the exterior ring; this form of packing by proper fitting makes a tight joint without friction. With this packing the lower chamber of the pump can be drawn off and examined, leaving the water-load in the upper piston. The dash-pot of a Fishkill Landing Corliss engine, with a similar groove packing, was tested to a water pressure of 140 pounds and leaked very little. Like packings are used in air-pump pistons with satisfaction.

The pump-valves are not shown in the drawing, but are Riedler's controlled valves, which are opened by the movement of the water and closed positively by a mechanical connection.

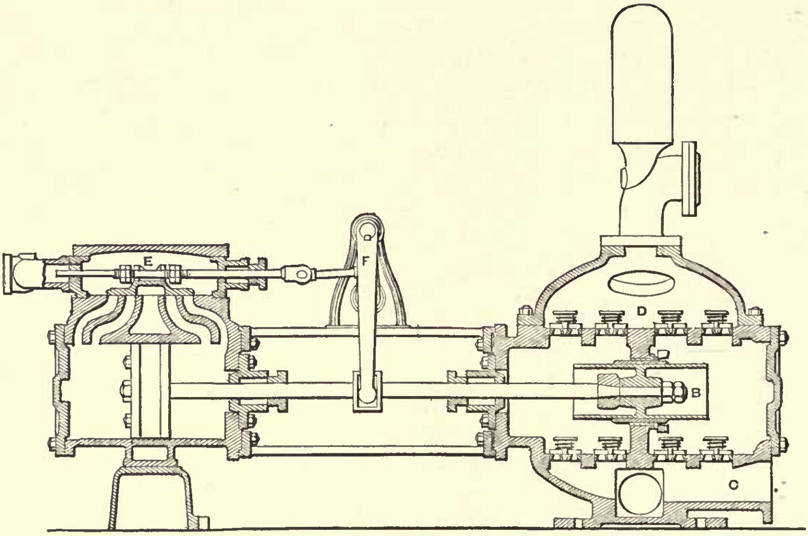


FIG. 833.

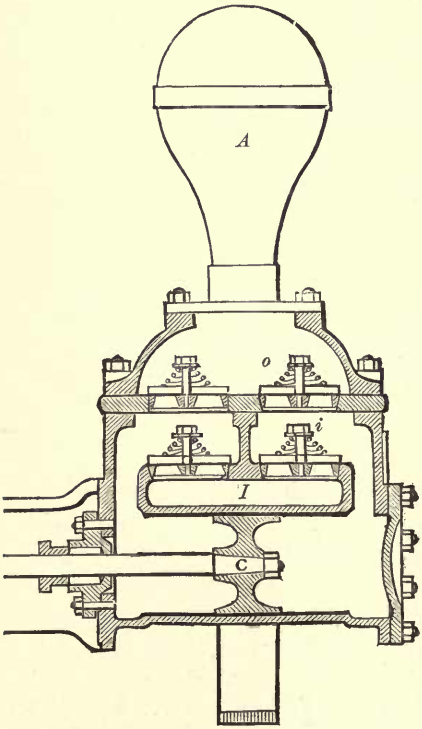


FIG. 834.

Fig. 833 is a section of a single Worthington steam-pump, one of a pair always placed side by side (hence called duplex), combined to act reciprocally on the steam-valves of each other.

In the form of pump shown, the length of barrel is about equal to the diameter of the piston, but the length of the piston is equal to that of the stroke of the pump plus that of the length of the barrel.

In Fig. 834 the pump barrel is long and the piston short.

The duplex was H. R. Worthington's invention and is now the general type of most makers for boiler feed pumps and small water supplies. For the boiler feed especially of locomotives the injector is largely used, and as supplementary to the pump.

The *injector* is an apparatus in which the momentum of a jet of steam issuing from the boiler is transferred to a body of water, producing a resulting velocity sufficient to force the water into the same boiler. Many shops manufacture injectors. The section shown (Fig. 835) is from William Sellers & Co., one of the earliest makers, and the explanation of its working illustrates the principles of a good and successful machine.

To start the injector the valve G is raised to permit the admittance of water into the chamber J; the lever F is then opened sufficiently to allow the steam through the opening *d d*, while the plug *h* still keeps the forcing nozzle *a* closed, thus admitting steam into the annular steam nozzle *i*, which, passing into the water-chamber J and the combining tube through the overflow chamber I and into the overflow E, producing a strong vacuum in the water-chamber J, into which the water from the source of supply is forced by the atmospheric pressure, the characteristic of a lifting injector. The combined jet of water and steam passes through the combining tube; the valve F is then fully raised, admitting steam into the forcing nozzle *a*; this steam, uniting with the jet in the combining tube, accelerates its velocity to such an extent that the valve *g* is forced down, thus allowing the passage of water to the boiler.

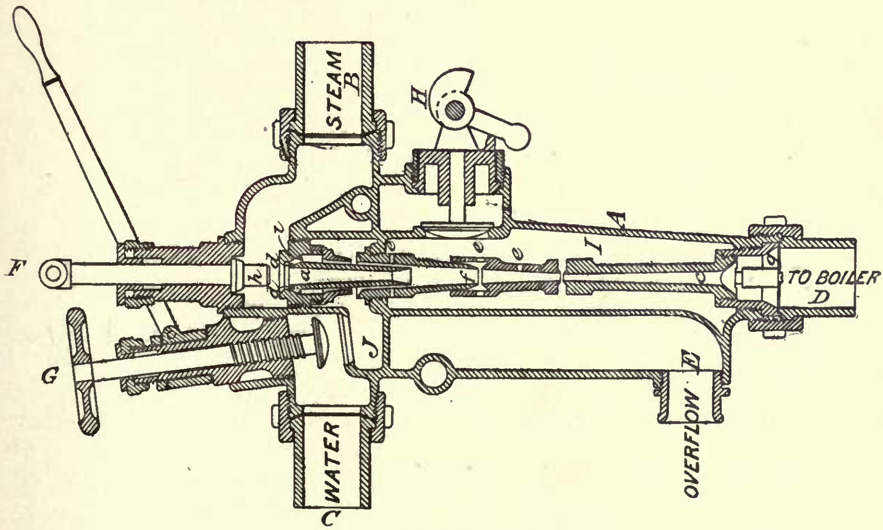


Fig. 835.

The issuing steam and water maintains its velocity by reduction of the areas of channel discharge till the last channel in connection with the boiler, which increases till it meets the valve *g*.

The lower the temperature of the feed-water the greater the capacity of the injector. As the steam strikes the water its momentum is checked and transferred to heat and at once absorbed by the feed, all the energy apparently wasted reappearing in the latent and sensible heat transferred to the feed. It is this absorption of heat that gives the great economy over the feed-pump, but for raising water it is not to be used except where simplicity of construction and convenience of application may offset the waste of power.

To use the injector as a heater to prevent the freezing of the water in the water-tank, the valve G is opened and the eccentric lever H closed; the steam will then have free admittance into the chamber I and the water-tank.

Each size of injector is named from the diameter of the delivery tube in millimetres. In the table given by Strickland L. Kneass the diameter runs from 3·2 to 11·5 millimetres; the steam pressure from 30 to 150 pounds, and the discharge 23·5 to 550 cubic feet per hour; height of lift, 1 foot; temperature of water, 60°.

Ejectors are also used as pumps for the raising of water by suction and pressure, of which Figs. 836 and 837 are of the simplest form. The steam is

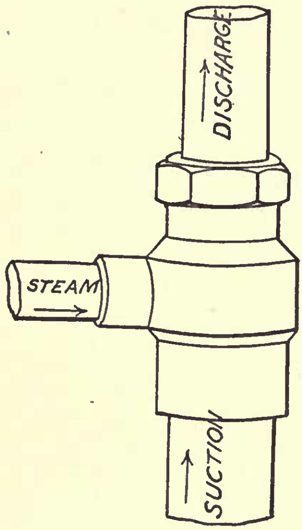


FIG. 836.

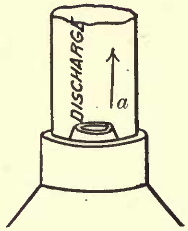


FIG. 837.

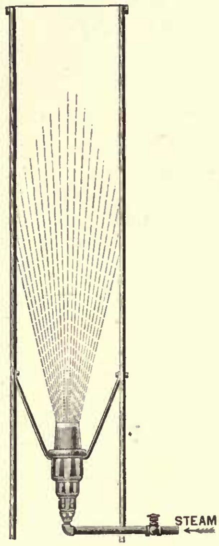


FIG. 838.

ejected through a central nozzle *a*, while the water is raised and discharged through the pipe inclosing it. The same principle of induced current is applied through a current of water, steam, or air in discharging earth from a caisson, the ashes from the boiler hold of a marine engine, water from foundations or from driven wells.

Fig. 838 is a section of a Korting blower to improve the draft in the ash-pit, fire-box, or chimney of a boiler, and Fig. 839 is that of a larger size, showing the construction of the jet in which the force of the steam drawing the air through compound nozzles reduces the intensity of its flow, but increases its quantity to suit the purposes of draft.

M. Mondesir, in the French Exhibition of 1867, effected the ventilation by means of reservoirs of compressed air. Around the exterior of the building there was a large underground gallery into which the exterior air was introduced by sixteen vertical shafts symmetrical in position, and from the main gallery the air was distributed by radial galleries into the interior of the building, into which the air current was introduced by numerous jets of condensed air from the reservoirs. The air was supplied to the jets at a pressure of $29\frac{1}{2}$ to $31\frac{1}{2}$ inches of water. The vitiated air escaped through a ventilator in the roof.

Clearances in cylinders include, in general signification, not only the spaces between the piston and cylinder-heads at the ends of the stroke, but also the spaces between the cylinder and the valves; and as those spaces are voided in a steam-cylinder at each stroke for which adequate work from the steam is not obtained, they are usually made as small as possible. If the steam is fairly dry, from ½″ to 1″ will be sufficient for end-clearances—that is, minimum distance between piston and cylinder-head.

Piston-rods are proportioned to the stress on them, usually one square inch of section to each 5,000 pounds of stress. In Fig. 815 the tapered end fits a taper hole in the piston, and is riveted over. It is more usually held by a nut, some use a shoulder on the inner end of the piston-rod instead of a taper, and the nut brings the piston strongly up against this shoulder.

Piston-rods are made either of steel or hammered iron, some makers of engines preferring one and some the other material.

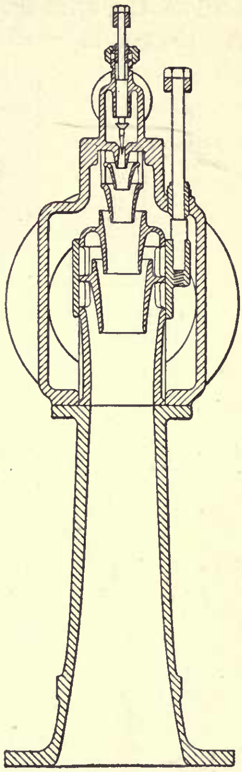


Fig. 839.

Stuffing-boxes are the mechanisms to prevent the leakage of steam, air, or water, in the movement of the piston or other rod out of the cylinder or chest. They consist of an annular chamber around the rod, generally filled with gaskets of hemp, which is forced down by a ring or *gland* into close contact with the rod and the sides of the box. In Fig. 812 there are two stuffing-boxes shown, one for the main piston-rod, the other for the valve-rod. In the latter the cap of the gland is fitted with a screw to connect it with the side of the stuffing-box, by which the gasket may be more or less compressed. This is the general form of stuffing-box for small stems or rods, sometimes with a ring or follower on the top of the gasket, which is forced down by the gland without turning the ring or gasket. In the figure the stuffing-box is made of brass, and screwed into the end of the steam- or valve-chest.

The stuffing-box to the piston is cast with the

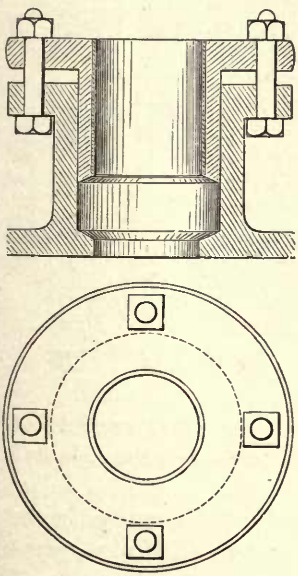


Fig. 840.

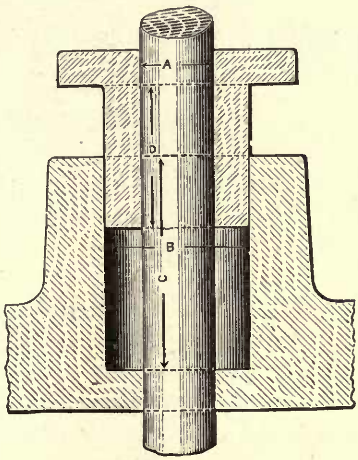


Fig. 841.

head of the cylinder, and is bored out, and a brass bushing fitted and driven into the end of the box. The hole through the bushing in most boxes fits the piston-rod accurately. The gland is of cast-iron, turned to fit the stuffing-box, and bored to fit the piston-rod; after packing the box the gland is forced in and retained by screws.

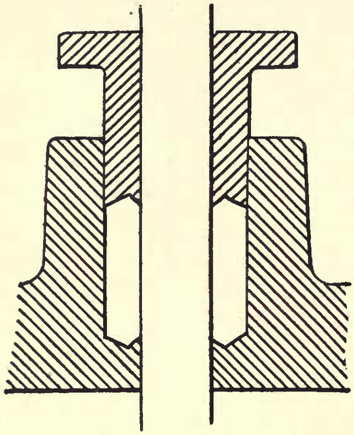


Fig. 842.

Fig. 840 is the plan and section of a common stuffing-box, in which the thickness of packing is from $\frac{1}{2}''$ to $1\frac{1}{2}''$, and the depth from $1\frac{1}{2}$ to 2 times the diameter of the piston-rod. The number of bolts vary with the diameter of the piston—seldom more than four, and usually but two.

Fig. 841 is the section of a stuffing-box of the proportions adopted by the Southwark Foundry. Taking the diameter of piston-rod A as the unit, B is 2, C 3, D 2, all scant up to a 3″ rod, or 22″ cylinder. For a 28″ × 42″ A = 4, with an allowance of $\frac{1}{32}''$ for clearance, B $6\frac{3}{4}$, C 9, D $6\frac{1}{4}''$.

Fig. 842 is a modification with double-cone grooves in the glands and stuffing-box by which the loose fibrous packing is prevented from getting into the joint spaces.

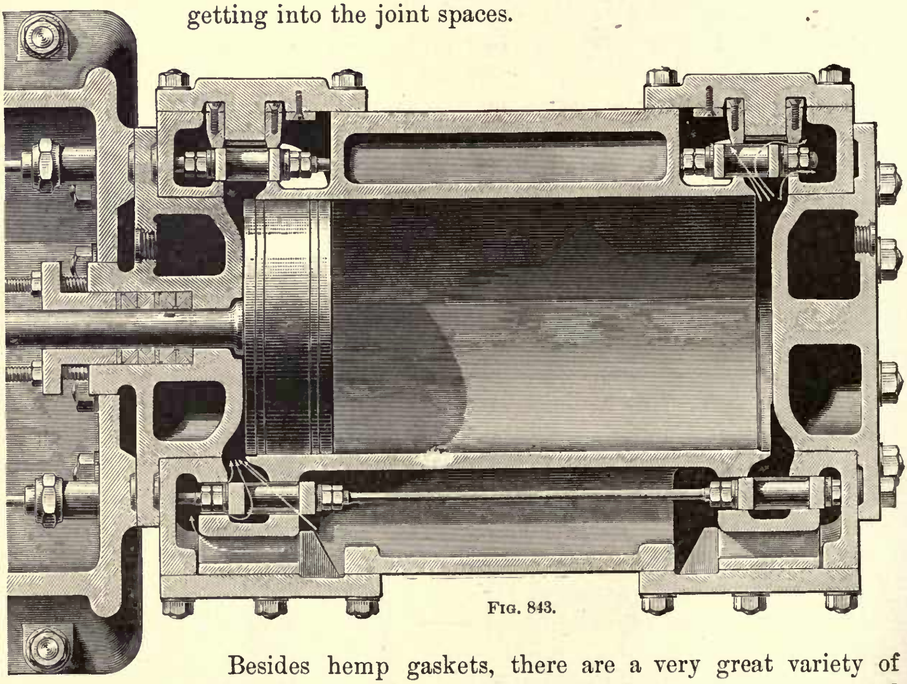


Fig. 843.

Besides hemp gaskets, there are a very great variety of packings, patented or otherwise, which are very good, adapted to common stuffing-boxes, and easily procured.

Valves—Steam-cylinder Valves.—The simplest and most common is the slide D, of which the action is described under the chapter on Motion, pages 216–218.

Of the Size of Ports or Openings.—Under "Steam-pipes" will be given the formula for the flow of steam, but the general rule of proportioning the ports of a cylinder is to consider the velocity of steam 100 feet per second, and of the exhaust 80 feet per second. With the slide-valve the opening and closing are made gradually, thereby throttling the flow of the steam. To avoid this, Mr. Corliss in his engine has made the ports long and narrow; the steam-valves open quickly and close at once by a drop; the exhaust-valves move rapidly from the wrist connection. From the great size and form of the common slide-valve there ensues a great pressure on the surface; various expedients have been adopted to relieve this pressure, which is especially desirable in quick-running engines.

Fig. 843 is a horizontal section of cylinder, through steam- and exhaust-valves, of a Porter-Allen engine, and Fig. 844 a vertical cross-section through

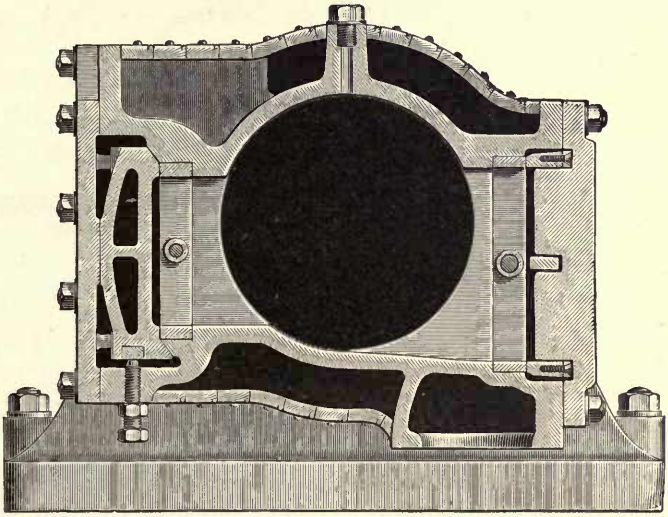


Fig. 844.

cylinder and valves. The valves are four in number, one for admission and one for exhaust, at each end of the cylinder, and on opposite sides. They stand vertically to drain the cylinder. The valves work between opposite parallel seats; the exhaust-valves nearly and the admission-valves wholly in equilibrium. The action of the back plate, and how the wear is taken up, will be understood from the section (Fig. 844), which passes through the middle of one pressure-plate. It is made hollow, and most of the steam supplied to two of the openings passes through it. It is arched to resist the pressure of the steam without deflection. It rests on two inclined supports, one above and the other below the valve. These inclines are so steep that the plate will move down under steam pressure; and that it may be closed up to the valve with only a small vertical movement, the pressure-plate is held in its correct position by projections in the chest on one side and tongues from the cover in the other, which bear against it at the near end, as shown. Between these guides it is capable of motion up and down and back and forth from $\frac{1}{16}''$ to $\frac{1}{8}''$. The pressure of the steam on this plate tends to force it down the inclines to rest on the valve. By the means of the screw the plate is forced up and away from the valve, and can be so nicely adjusted that the valve works freely and perfectly steam-tight. When the pressure is greater in the cylinder than in the chest, the pressure-plate is forced back, to the instant relief of the cylinder.

Cylindrical Valves.—Fig. 845 represents the section of the steam-cylinder of an Armington & Sims steam engine with a cylindrical valve. The steam-

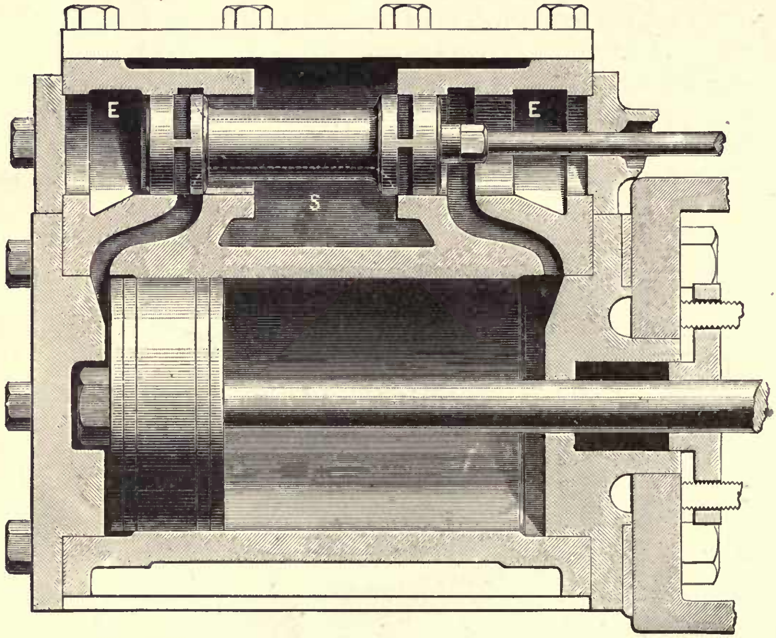


Fig. 845.

chest S is central and incloses the valve; the exhaust chambers E E are at the ends of the valve, and are connected through the hollow stem or body of the valve. The valve depends on its accuracy of fit for its tightness. The valve-chamber is bored out and ground, the valve is turned, ground, and carefully worked by hand, to so close a fit that there is no loss of steam in action, and the valve is completely balanced.

There is a form of balanced valves, called the *double-beat*, much used both for steam and water valves. Fig. 846 is a sectional elevation of a steam valve

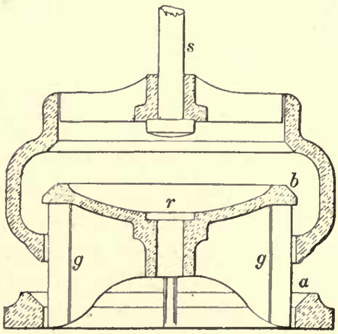


Fig. 846.

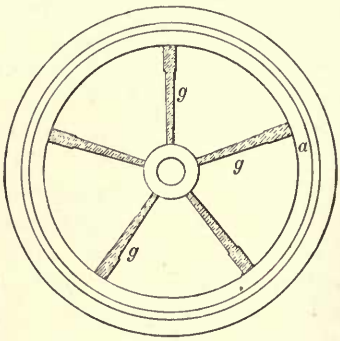


Fig. 847.

of this kind, and Fig. 847 a plan of the lower seat *a*, with the valve-guides *g g* in section. There are two seats, *a* and *b*, and two faces on the valve corresponding to them. The balance depends upon the relative diameters of the bearing-lines of the two faces. In the figure, if the exterior of the bearing at

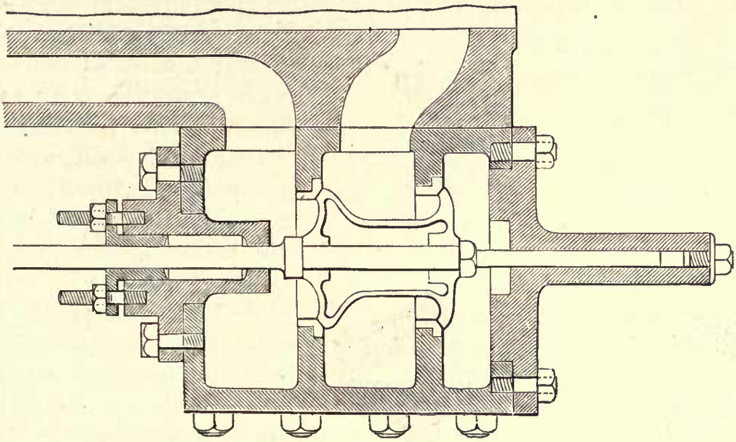


FIG. 848.

b and the interior at *a* are both tight, the valve is balanced under any pressure, except as to its own weight; *s* is the valve-stem, and the hole *r* is for a bolt to fasten the valve-seat to the casting of the steam-chest. The scale is $\frac{1}{8}$ full size.

Fig. 848 is another form of balance, consisting of two equal poppet-valves connected together—the steam passage to the cylinder being central, and the steam-chest at each end, connected.

Automatic valves, that are moved by the action of the fluid in which they are placed.

The double-beat valve (Fig. 846) is sometimes used in large pumping-engines. From its two beats, the lift is about one half that of a plain valve. There must be difference enough in the faces to admit of the lift of the valve by the pressure of water acting on this difference. The seats of the valves are often made of wood, set endways.

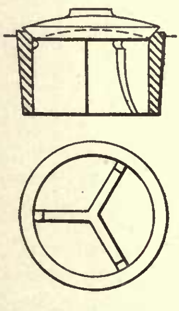


FIG. 849.

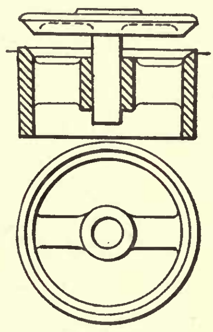


FIG. 850.

Large valves, from their great weight and flow of water through them, are noisy in both seating and lifting. This is met in the balanced valves by slower movements of the piston; but present practice is to obtain outlets by increasing the number of valves, the total area of the apertures, and the speed.

Fig. 849 is a single-beat direct lift-valve guided by three feathers on its under side, which slide in the cylindrical part of the pipe. The feathers are of a screw form, by which a rotary motion is given to the valve through the flow of the water, which prevents its beating on the same parts of the seat; usually the feathers are straight, and often four instead of three. Fig. 850 is a poppet-valve guided by a central stem ; both these valves have conical faces and seats, with generally an inclination of 45° to the axis of the valve. In many valves the faces and seats are flat, one or the other of which is of soft metal or rubber.

Figs. 851 and 852 are elevation and section of a rubber disk-valve in very common use in direct-acting pumps and small pumping-engines ; sometimes with a thimble in the rubber as a guide ; usually, as in the figure, with a metallic plate on top of the rubber for the bearing of the spring ; valve-seat generally of composition, with spindle riveted or screwed into it. Sometimes the rubber is held in a metallic plate or cup. The springs at their backs cushion the blow on the lift, and start the valve downward promptly on the check of the waterflow at the end of the stroke. The great desideratum of water-valves is that there should be little lift, but ample water-way.

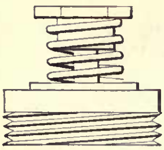


Fig. 851.

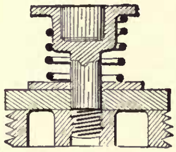


Fig. 852.

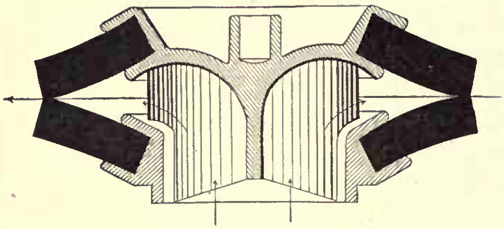


Fig. 853.

Fig. 853 is a section of Field's pump-valves, an English design for high-water service, as fire-pumps, of which the flaps are rubber disks. For lower pressures, as for the pumping of half-stuff in paper-mills, valves are made in the shape of a bishop's mitre slit lengthwise at the top and partly down the sides. The bottom flanges should be held loosely without bolts through them.

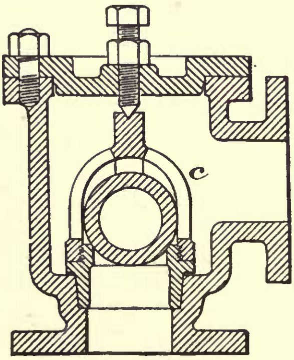


Fig. 854.

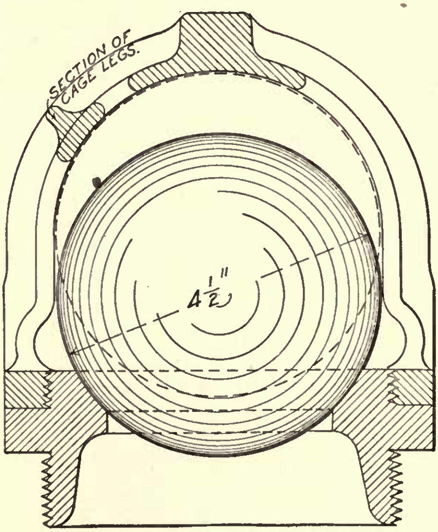


Fig. 855.

Fig. 854 is a ball-valve, guided in its movement by an open guide-cage, *c*, which is held down by a set screw in the cover. Globe-valves with this form are on sale. Ball-valves are usually small metallic balls on metallic or wooden seats, or rubber balls on metallic seats; and cylindrical valves have been made of the same section as in the figure; the body of the valves of brass pipes with rubber jackets.

In Fig. 855 the ball is of rubber and the seat is of composition, screwed into the pipe with a cage of the same metal, screwed to the seat.

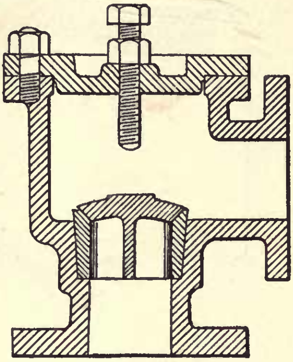


FIG. 856.

Fig. 856 is a section of a poppet-valve; the body is of cast-iron, but the valve and seat are of brass. The valve is guided by three feathers. The lift of the valve may be controlled by a set screw in the cover which admits of adjustment to varied lifts.

Figs. 857 and 858 are the plan and section of a disk-valve for the air pump of a condensing steam engine. The valve consists of a disk of rubber lying on a flat grating or perforated plate of brass, held in position between the grating and a spherical guard by a central bolt. The shape of the guard gives a uniform flexure to the rubber in lifting, and an easy flow to the current of air and water. The rubber is not closely clamped between the guard and plate, as will be seen in the figure. The lower nut, after being screwed home, is riveted, and the upper nut usually pinned to prevent turning. The size of the apertures in the grating are adapted to the thickness of the rubber. With an external diameter of opening of 6″, and

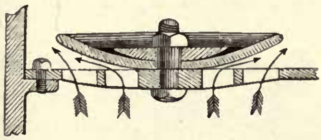


FIG 857.

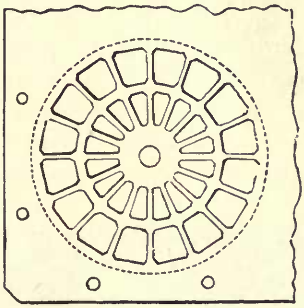


FIG. 858.

rubber $\frac{1}{2}''$ thick, the exterior ring of openings may be $\frac{3}{4}''$ by $\frac{3}{4}''$, the *lands* or spaces between openings $\frac{1}{4}''$ wide, and exterior lap of the rubber $\frac{1}{2}$ inch. With larger diameters and larger openings thicker rubber must be used. This valve is often made of a long strip or flap of rubber, on a suitable grating, with a curved guard attached on one side. For the common air-pump pressure, $\frac{3}{4}''$ rubber is sufficient for apertures $1'' \times 4''$. With the use of backing and face plates on the rubber or leather flaps, gratings may be dispensed with (Fig. 859). Valves of this description—duplicate (Fig. 860) beneath the central pin and half circular in plan—are often used in pumps, and are called butterfly valves. It is the best practice to insert thimbles in the rubber (Fig. 859),

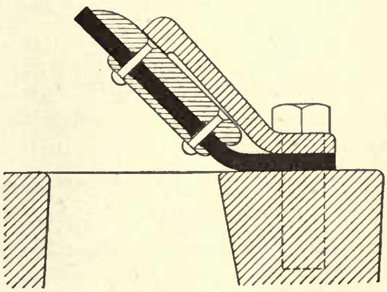


FIG. 859.

and the rivets connecting the plates pass through these thimbles, so that the rubber may be held but not tightly fastened—a rule applicable to all such valves.

Check-valves (Figs. 861 and 862) are placed outside of large pumps to prevent the return of water in cases of accident to the pumps, and for facility of

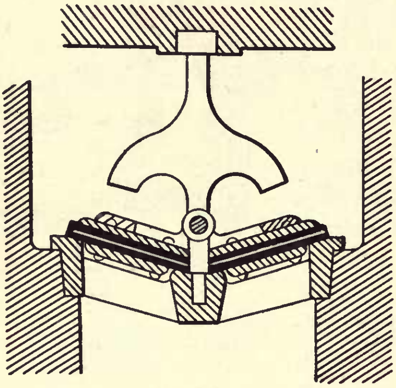


FIG. 860.

SPACE OCCUPIED BY THE VALVES.

SIZE.	Measure-ment from face to face of flange.	Measure-ment from end to end of hub.
Inches.		
4	$11\frac{3}{4}$	$13\frac{3}{8}$
5	$14\frac{1}{2}$	16
6	$14\frac{1}{2}$	16
8	$17\frac{3}{4}$	19
10	$21\frac{1}{2}$	24
12	$24\frac{1}{2}$	$26\frac{1}{2}$
16	29	31
18	33	35
20	$35\frac{1}{4}$	38
24	$39\frac{3}{4}$	39

their examination. Valves of this kind open from the pressure of water beneath, and, from a state of rest, with some suddenness and shock. To prevent this in large valves, there is a valve and small by-pass pipe, from one side to the other of the valves, by opening which the pressure on the two sides of the valve may be equalized, and the excess due to the starting of the pump distributed. At many pumping works the by-pass is kept open except when necessary to get at the pumps. In case of accident to the pumps the flow through the by-pass would be comparatively small, and readily shut off.

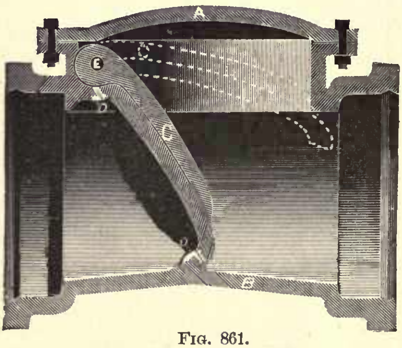


FIG. 861.

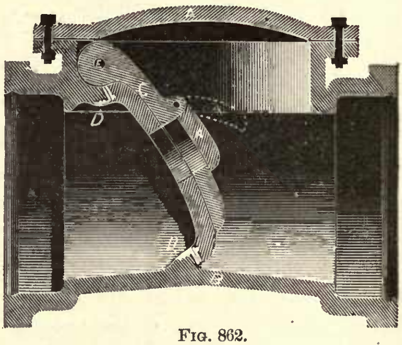


FIG. 862.

Valves controlled by Hand.—Fig. 863 represents a side view of a water *bib-cock*, called a *hose*-bib, because the outlet end is fitted with a screw to adapt it to a hose. Without this screw it is a *plain bib.* If both ends of the cock are in the same line, it is called a *stop-cock.* The ends may not be fitted with screws, as in the figure; the screws are sometimes female screws, and often with taper ends, to solder lead pipe to, or to drive into a cask. These cocks come under the common designation of plug-cocks, from their interior construction, which will be readily understood from the section given in Fig. 864.

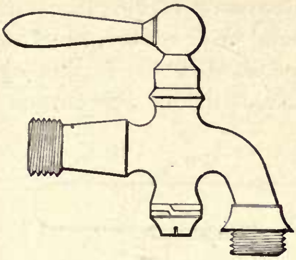


FIG. 863.

FIG. 864.

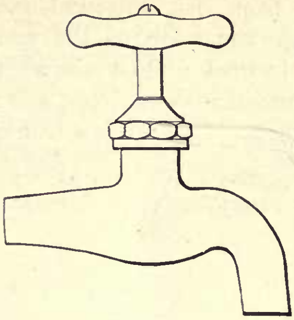


FIG. 865.

They are used in both steam and water pipes, but not in the former when the use is frequent and daily, and then usually not over 2″ in diameter of passage.

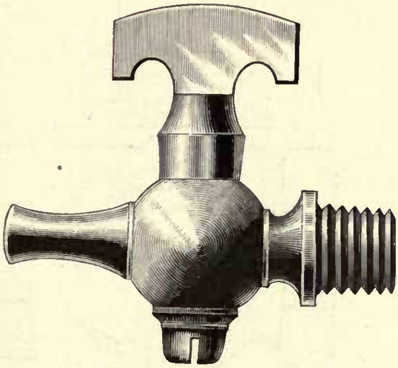


FIG. 866.

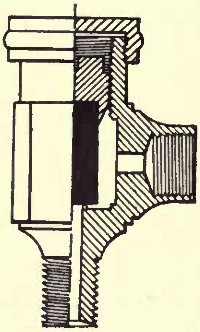


FIG. 867.

Fig. 865 is the side view of a compression water-bib, used when the pressure of the water is great. The section is somewhat similar to that of Fig. 870, in which a rubber disk is forced against a metallic seat to shut off the flow.

Fig. 866 is a side view of a common air-cock for boilers and steam work; they are plugs in their construction, as are the cocks used in gas-fitting; size of vent of air-cocks, $\frac{1}{8}$ to $\frac{1}{2}$ inch diameter.

Fig. 867 is an air-cock in which the valve is a plug of Jenkins's patent composition, mostly rubber and graphite, on a flat seat of small surface.

Figs. 868 and 869 are front views of globe-valves, so called from the shape of the body inclosing the valve. Fig. 868 is an angle globe-valve; Fig. 869, a cross globe-valve used for cutting off the steam supply through the vertical

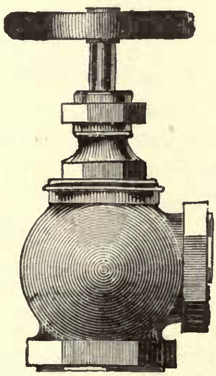


FIG. 868.

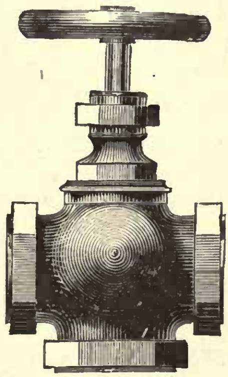


FIG. 869.

pipe from the horizontal one; the same purpose is attained by putting a straightway valve on the vertical pipe. The valve seats of all globe-valves, kept in stock and on sale, are now made of soft metal or of rubber, or of some mixture of it vulcanized; soft rubber for cold water, hard rubber for hot water or steam. Fig. 870 is a front view of a globe-valve, with one side turned off

DIMENSIONS OF GLOBE-VALVES IN COMMON USE, WITH SOFT SEATS.

Diameter of opening in seat.	Length over all.	Diameter of globe.	Body metal.
$\frac{1}{8}$	$2\frac{1}{8}$	$1\frac{3}{8}$	Brass.
$\frac{1}{4}$	$2\frac{1}{8}$	$1\frac{3}{8}$	"
$\frac{3}{8}$	$2\frac{1}{2}$	$1\frac{1}{2}$	"
$\frac{1}{2}$	$2\frac{3}{4}$	$1\frac{3}{4}$	"
$\frac{3}{4}$	$3\frac{1}{4}$	$2\frac{1}{8}$	"
1	$3\frac{3}{4}$	$2\frac{1}{2}$	"
$1\frac{1}{4}$	$4\frac{1}{4}$	3	"
$1\frac{1}{2}$	$4\frac{3}{4}$	$3\frac{1}{2}$	"
2	$5\frac{3}{4}$	$4\frac{3}{8}$	"
$2\frac{1}{2}$	8	$6\frac{1}{4}$	Iron.
3	$9\frac{3}{4}$	$7\frac{1}{4}$	"
$3\frac{1}{2}$	10	$8\frac{1}{4}$	"
4	$11\frac{1}{2}$	$9\frac{1}{4}$	"
5	$13\frac{3}{8}$	$10\frac{7}{8}$	"
6	$14\frac{3}{4}$	$12\frac{1}{2}$	"
7	$16\frac{1}{4}$	14	"
8	$17\frac{1}{4}$	14	"

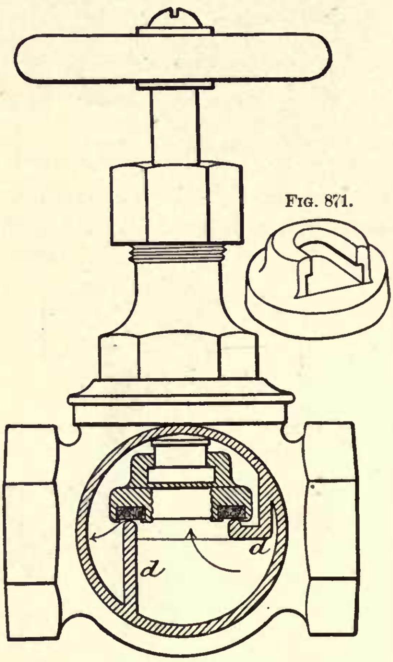


FIG. 871.

FIG. 870.

to show its interior construction. The diaphragm *d d* is in the form of a ⌐—⌐, which divides the interior of the globe; through the flat part is the aperture for the passage of the fluid covered by the valve, controlled by the handle and screw on its stem. The arrows show the direction of flow.

Fig. 871 is a perspective of the valve, showing the grooves through which it is slipped on to the head on the stem and held. The composition or rubber is in the form of a ring slipped into a circular groove in the bottom of the valve. In some valves the rubber ring is slid into a straight groove and retained by a nut on the stem, and the head of the stem is held to the valve by a nipple.

When the seat is of soft metal it is run into a groove in the case, faced, and the valve formed with a circular chisel edge is forced down by the hand wheel into the soft metal. On account of the loss of head by the change of direction in flow—through the valve aperture—it is better to make it of a little larger diameter than that of the pipe.

Figs. 872 and 873 are the plan and section of a steam valve of the Southwark Foundry pattern; the seats and faces are of metal ground to a fit. The valve is guided by three wings, *w w*. The flow through globe-valves, as will be seen by their sections, have three changes of direction; to avoid this, straightway gates are almost invariably used on water mains.

If the double-beat valve (Fig. 846 or 848) be mounted with a screw (like the

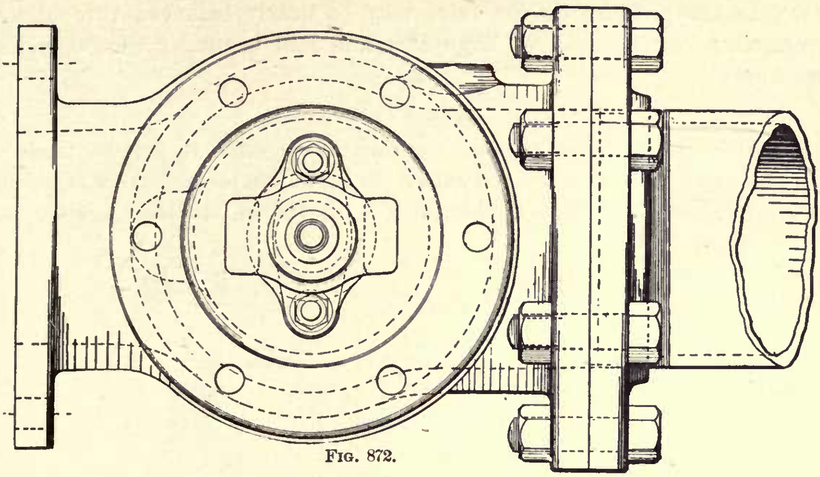


FIG. 872.

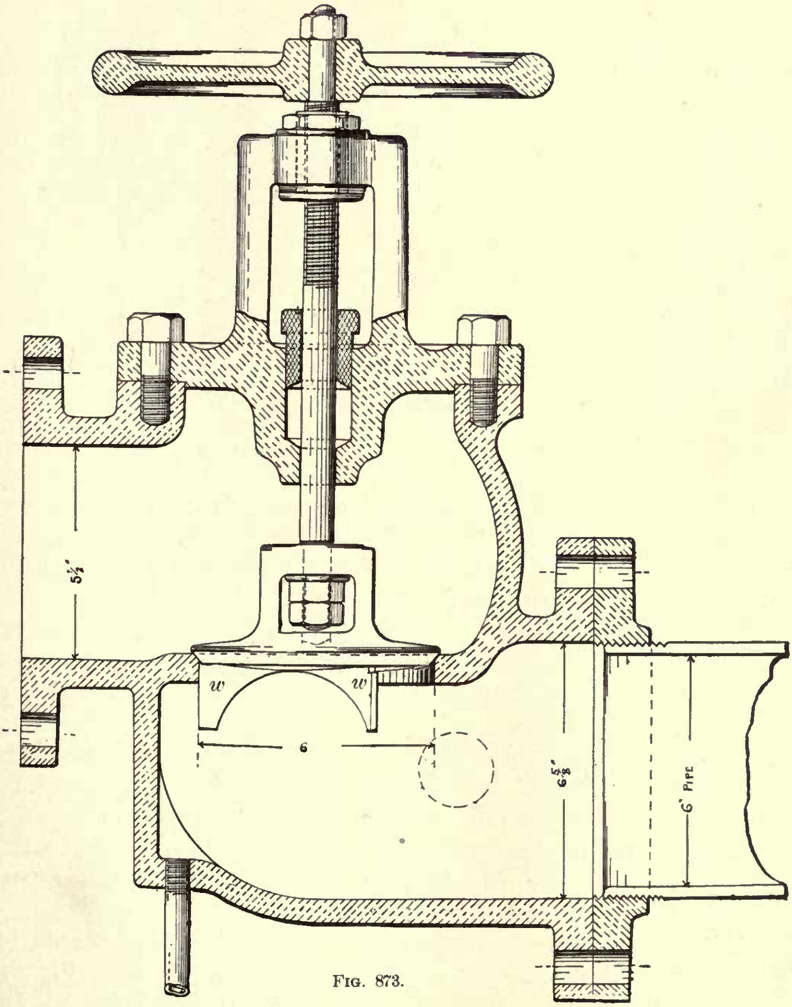


FIG. 873.

above), as the pressure on the valve may be nearly balanced throughout its movement, it can be made of large area, and still be under the control of a hand wheel.

GATE–VALVES.

When the pressure is always on one side of the valve, it may be made as a plain gate, sliding in grooves, and raised up into a chamber; but it is the usual practice to make these gates double, each being forced positively against faces.

The earliest of these forms of gates are the Peet (Fig. 874) and the Coffin valve (Fig. 875).

The first were usually made of small sizes, and for steam-pipes, it is shown in the figure shut; the two side valves or disks are forced against their respective faces by the cone suspended between the disks, forced upward by its stem coming in contact with the bottom of the case; as the gate is raised, the cone drops, the pressure is released, and the valve easily drawn up into the chamber above, giving an unobstructed passage through the body of the valve.

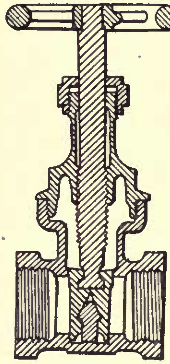


FIG. 874.

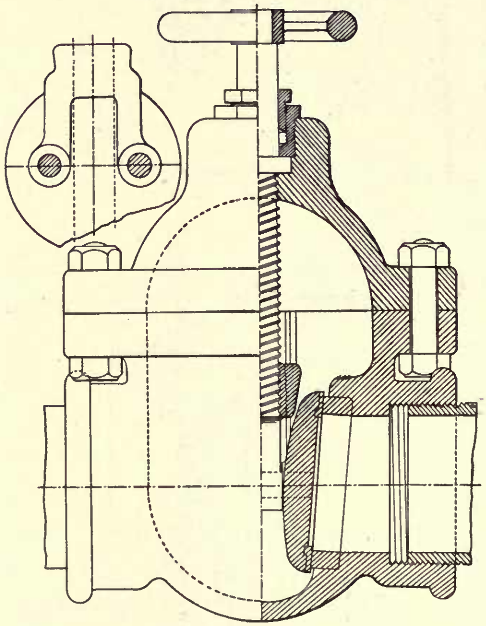


FIG. 875.

In the Coffin valve the disks are suspended by two pivots in their backs to a wedge connected with the same. The wedge by its downward movement forces the disks outward against the seats, while by the upward motion this pressure is relieved.

In the Pratt and Cady valve (Fig. 876) the seats are of soft metal which are cast in a mould and forced into position, making a tight fit with the body of the valve and a seat for the valve with smooth faces. In case of a cutting of the soft metal, it can be readily withdrawn and replaced by a new one.

The above forms of gates, especially those of large sizes, are used as water gates. Steam-valves are mostly of the globe pattern.

Between the boilers and the steam-chest of an engine there should be a valve that can be shut promptly. The simplest is the damper-valve (Fig. 877), which is also used for the control of the draft in the smoke-pipe, but for a

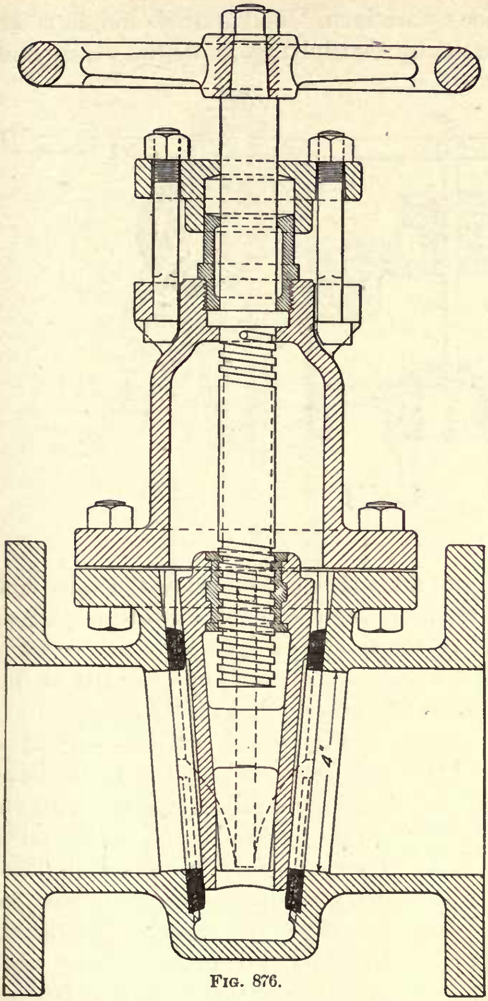


FIG. 876.

steam-pipe it is made with a closer fit; it never can be close, but sufficiently so to control or slow down the engine. A better valve is the usual Corliss steam-cylinder valve, moved by a handle, or the register-valve (Fig. 878), which is held down by a spring, and is opened and shut like a register. These valves may be connected to a governor.

Fig. 879 is a valve used in Nasmyth's works for steam hammers. Opposite the ports there are false ports or slight recesses in the shell. The steam enters at the end of the valve into the spaces *a a*; the endwise pressure is received by a thrust bearing. This valve is so nearly balanced that it is readily moved by hand.

To prevent excessive pressures, either of steam or water, a safety-valve is used (Fig. 880), which consists of a poppet-valve held down by a lever and weight. To determine the weight counterbalancing the pressure, put the valve and lever in position, attach the valve to the stem, and with a spring balance attached to the lever at its connection with the stem, find how much weight it takes to lift the valve-stem and lever; this is a constant, which if divided by the area of valve at its lowest bearing diame-

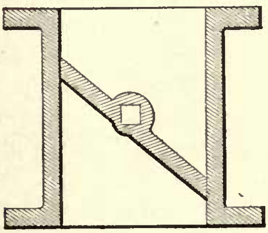


FIG. 877.

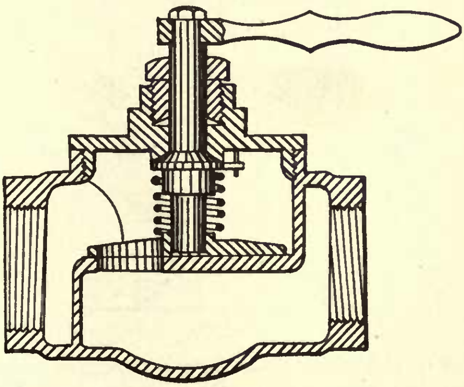


FIG. 878.

ter will give the constant pressure per square inch. The movable weight is the P of a steelyard, and is to be estimated by multiplying fp by the weight and dividing it by fs and the area of the valve, to which is to be added the constant weight per square inch found above.

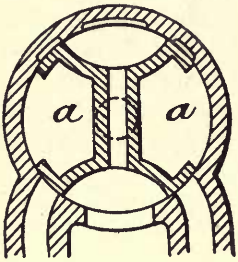


FIG. 879.

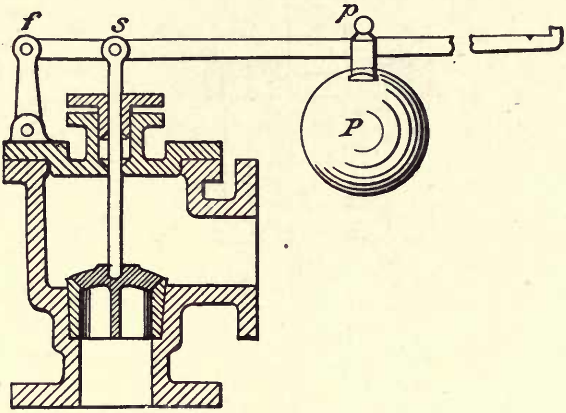


FIG. 880.

The safest and simplest way is to put a blank flange on below the valve and with a force pump inject water to certain pressure, as shown by a steam gauge; balance this pressure by a weight P on the lever and mark its distance from the fulcrum, which will give the weight per square inch on the valve at the position of the P; in the same way determine other points on the lever.

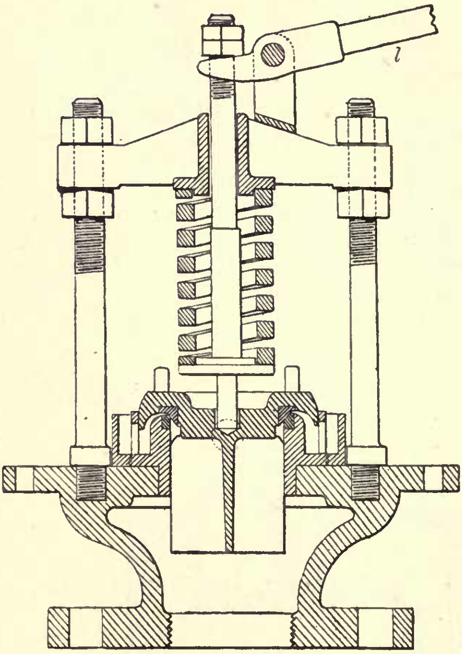


FIG. 881.

Fig. 881 is what is termed a pop safety-valve; the steam issuing as the valve rises, impinges on a cup surface to force the valve farther open. The valve is held down by a spring, but can be raised by the lever l. Valves of this kind are often inclosed in a locked box, that they may not be tampered with.

To determine the weight on the spring, test it by raising the pressure in the boiler, as shown by the gauge, to the height which is deemed safe; adjust the valve to the lifting point by the nuts on the side bolts. If not in position on a boiler, test by a pump.

Hydrants.—For water-service in connection with high-pressure mains.

Fig. 882 is a section of a post-hydrant. The valve v consists of a series of leather disks bolted together and turned conical, which is brought in contact with a corresponding seat by the valve-rod and its screw at the top of the hydrant. The valve is opened by being forced down into the cavity of a branch of the pipe-main; n is the nozzle for the coupling of the hose; outside the main pipe of the hydrant there is a case, extending from near the line of valve to the ground line, called the hydrant or frost case, which prevents the hydrant from being lifted by the frost. Were the water left in the hydrant, it would freeze in most exposures during winter; the hydrant, when not in use, is therefore kept empty. This is effected by a small hole at a, which, when the valve is closed, is opened, and the water in the hydrant, if any, is discharged. This vent is closed by a slide attached to the valve-rod, when this last is moved down to open the main valve. Instead of leather for the valve-face, many valves are fitted with rubber; there is also a great variety of valves for hydrant purposes—slides, poppets, disks—but in the arrangement of hydrants the illustration is the common one, although often without the frost case.

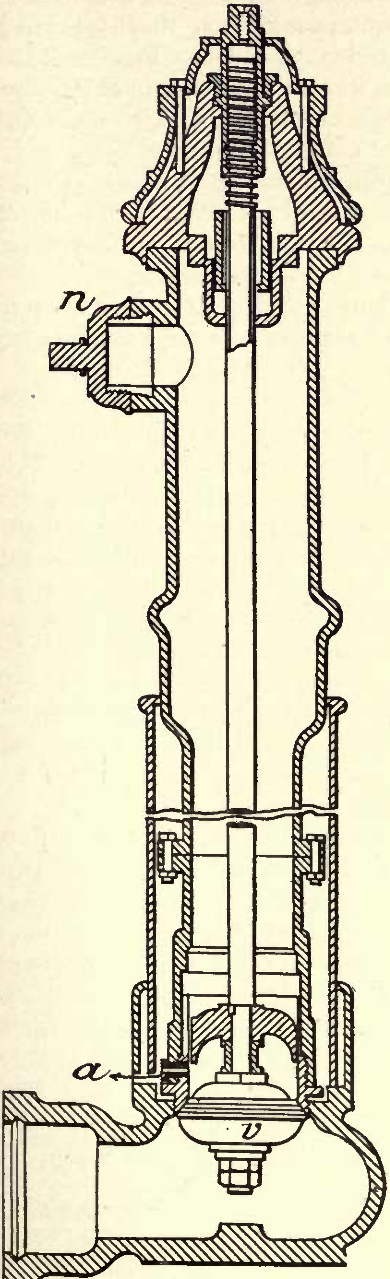


FIG. 882.

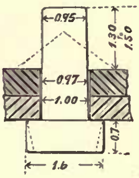


FIG. 883.

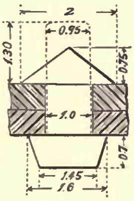


FIG. 884.

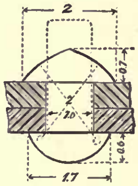


FIG. 885.

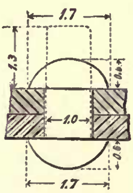


FIG. 886.

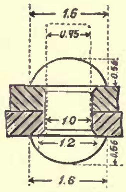


FIG. 887.

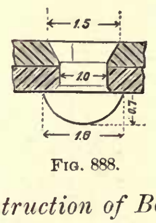


FIG. 888.

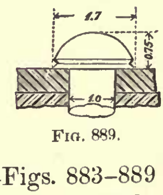


FIG. 889.

Riveted Joints, as used in the Construction of Boilers.—Figs. 883–889 are forms of rivets with their proportions referred to the diameters next the heads. The thickness of the plate connected by rivets will be given in tables hereafter. Figs. 884 and 885 are the usual finish of rivets in hand-riveting; Figs. 886 and 887, when made by machines, in which, as the rivet-hole is slightly counter-sunk, the strength of the head is increased. Fig. 888 is a counter-sunk rivet, the head being flush with the outside of the plate. Fig. 889 is the head of a rivet, in which a narrow strip at the edge is burred down by a chisel, or *calked*, to make the joint between rivet and plate tight.

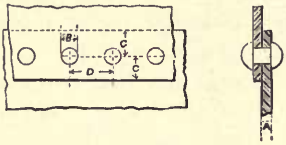


Fig. 890.

Fig. 890 is a plan and section of a single-riveted lap-joint. Joints of this kind fail from the tear of the plate on the line of rivets if the rivets are too close, or the distance of the rivet to the outside of the plate too small, or by the shear of the rivets if they are too few.

Rivet-holes.—Punching has an injurious effect upon plates; but this injury (if the plates are not cracked by the process) is removed by afterward annealing them, or by rymering or drilling to the extent of $\frac{1}{16}$ inch on the diameter.

It is difficult to insure the correct spacing of the holes when they are made by punching. In the best boiler work the rivet-holes are drilled after the plates have been bent or flanged and put together in their proper places. This insures that the corresponding holes in the different plates shall be exactly opposite to one another. After drilling, the plates are taken asunder and any burr that has been formed at the edges of the holes is removed. Riveting may be performed either by hand hammering or by a machine. Hydraulic riveting machines are the best.

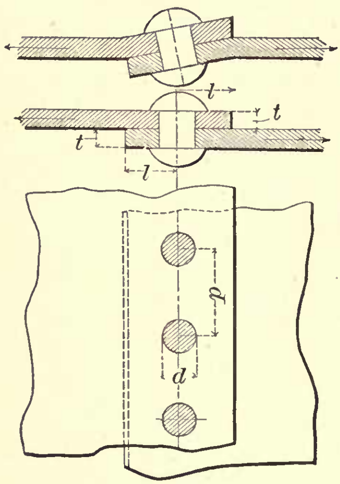


Fig. 891.

For wrought-iron plates a tenacity of 47,000 pounds is estimated per square inch in the direction of the fibre, and 40,000 pounds per square inch across the fibre; steel plates, a tenacity of 65,000 pounds per square inch.

Shearing resistance of wrought-iron rivets is about equal to the tenacity of wrought-iron plates; the shearing resistance of steel rivets is about eight tenths the tenacity of steel plates.

Fig. 891 shows the connection of two plates by means of single-riveted lap-joint. When the plates are arranged as shown in the lower section, the tension in the plates causes a bending action on them at the lap. To avoid this, the plates have sometimes a set at the lap, as shown in the upper section.

DIMENSIONS OF SINGLE-RIVETED LAP-JOINTS FOR BOILER WORK.

Thickness of Plate.		Iron Plates and Iron Rivets.				Steel Plates and Steel Rivets.			
		Diam. of rivet.		Pitch.		Diam. of rivet.		Pitch.	
$\frac{5}{16}$		$\frac{5}{8}$		$1\frac{1}{2}$		$\frac{5}{8}$		$1\frac{7}{16}$	
$\frac{7}{16}$	$\frac{3}{8}$	$\frac{13}{16}$	$\frac{3}{4}$	$1\frac{7}{8}$	$1\frac{3}{4}$	$\frac{7}{8}$	$\frac{3}{4}$	2	$1\frac{11}{16}$
$\frac{9}{16}$	$\frac{1}{2}$	$\frac{15}{16}$	$\frac{7}{8}$	$2\frac{1}{8}$	2	1	$\frac{15}{16}$	$2\frac{1}{8}$	$2\frac{1}{16}$
$\frac{11}{16}$	$\frac{5}{8}$	$1\frac{1}{16}$	1	$2\frac{3}{8}$	$2\frac{1}{4}$	$1\frac{1}{8}$	$1\frac{1}{16}$	$2\frac{3}{8}$	$2\frac{1}{4}$

The efficiency of the joint in the percentage of the strength of the plate is to be taken at the lowest figure, whether in tear or shear. In the above table it is 55 per cent. The distance between the edge of the plate and the centre of the outer rivet, as shown in the figures by l, is invariably one and a half time the diameter of the rivet.

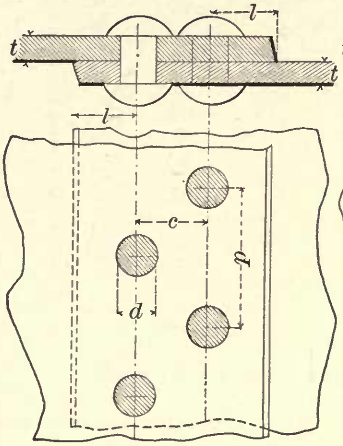


Fig. 892.

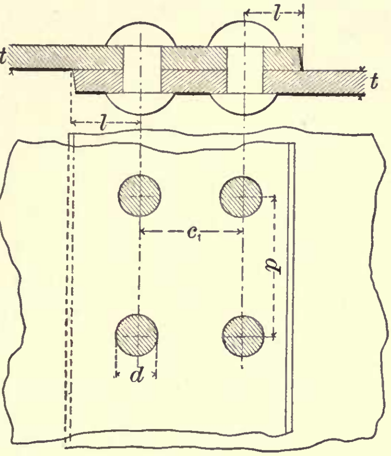


Fig. 893.

The arrangement of the rivets in Fig. 892 is known as zigzag riveting, while that of Fig. 893 is chain riveting.

DIMENSIONS OF DOUBLE-RIVETED LAP-JOINTS.

t.	IRON PLATES AND IRON RIVETS.				STEEL PLATES AND STEEL RIVETS.			
	d.	p.	c.	c_1.	d.	p.	c.	c_1.
$\frac{3}{8}$	$\frac{11}{16}$	$2\frac{5}{8}$	$1\frac{5}{16}$	$1\frac{7}{8}$	$\frac{3}{4}$	$2\frac{5}{8}$	$1\frac{3}{8}$	2
$\frac{7}{16}$	$\frac{3}{4}$	$2\frac{3}{4}$	$1\frac{3}{8}$	2	$\frac{13}{16}$	$2\frac{3}{4}$	$1\frac{7}{16}$	$2\frac{1}{8}$
$\frac{1}{2}$	$\frac{13}{16}$	$2\frac{7}{8}$	$1\frac{1}{2}$	$2\frac{1}{8}$	$\frac{7}{8}$	$2\frac{13}{16}$	$1\frac{1}{2}$	$2\frac{1}{4}$
$\frac{9}{16}$	$\frac{7}{8}$	3	$1\frac{9}{16}$	$2\frac{1}{4}$	$\frac{15}{16}$	$2\frac{7}{8}$	$1\frac{9}{16}$	$2\frac{3}{8}$
$\frac{5}{8}$	$\frac{15}{16}$	$3\frac{1}{8}$	$1\frac{5}{8}$	$2\frac{3}{8}$	1	3	$1\frac{5}{8}$	$2\frac{1}{2}$
$\frac{11}{16}$	1	$3\frac{1}{4}$	$1\frac{3}{4}$	$2\frac{1}{2}$	$1\frac{1}{16}$	$3\frac{1}{8}$	$1\frac{11}{16}$	$2\frac{5}{8}$
$\frac{3}{4}$	$1\frac{1}{16}$	$3\frac{7}{16}$	$1\frac{13}{16}$	$2\frac{5}{8}$	$1\frac{1}{8}$	$3\frac{1}{4}$	$1\frac{3}{4}$	$2\frac{3}{4}$
$\frac{13}{16}$	$1\frac{1}{8}$	$3\frac{9}{16}$	$1\frac{7}{8}$	$2\frac{3}{4}$	$1\frac{3}{16}$	$3\frac{3}{8}$	$1\frac{13}{16}$	$2\frac{7}{8}$
$\frac{7}{8}$	$1\frac{3}{16}$	$3\frac{3}{4}$	2	$2\frac{7}{8}$	$1\frac{1}{4}$	$3\frac{1}{2}$	$1\frac{15}{16}$	3
$\frac{15}{16}$	$1\frac{1}{4}$	$3\frac{7}{8}$	$2\frac{1}{16}$	3	$1\frac{5}{16}$	$3\frac{5}{8}$	2	$3\frac{1}{8}$
1	$1\frac{5}{16}$	4	$2\frac{3}{16}$	$3\frac{1}{8}$	$1\frac{3}{8}$	$3\frac{3}{4}$	$2\frac{1}{8}$	$3\frac{1}{4}$

The efficiency of joints in the above table is for iron plates 68 per cent, and steel, 64 per cent.

In all riveted joints the distance between adjacent rivets, measured from centre to centre, whether in the same or different rows, should not be less than $2d$.

On the results of experiments on riveted joints, Professor Kennedy has stated that the net section of metal in the plate, measured zigzag, should be from 30 to 35 per cent. in excess of that measured straight across. This gives a diagonal pitch of $\frac{2p+d}{3}$.

Treble-riveted Lap-joints.—In Fig. 894 the riveting is zigzag, and in Fig. 895 chain.

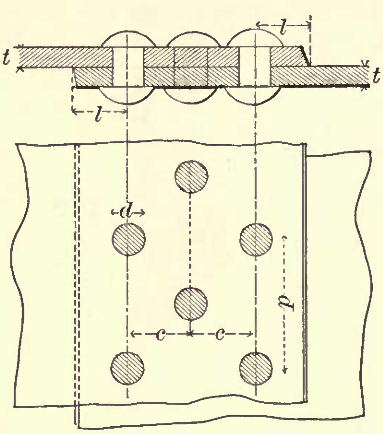


FIG. 894.

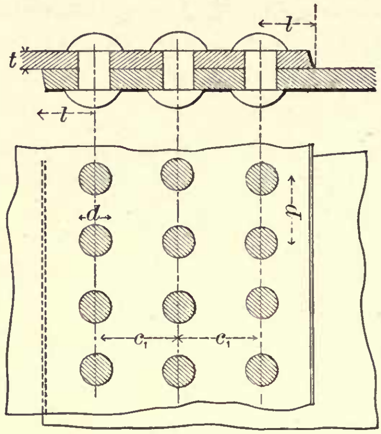


FIG. 895.

DIMENSIONS OF TREBLE-RIVETED LAP-JOINTS.

t.	IRON PLATES AND IRON RIVETS.				STEEL PLATES AND STEEL RIVETS.			
	d.	p.	c.	c1.	d.	p.	c.	c1.
$\frac{5}{8}$	$\frac{13}{16}$	$3\frac{1}{4}$	$1\frac{5}{8}$	$2\frac{1}{8}$	$\frac{7}{8}$	$3\frac{1}{8}$	$1\frac{5}{8}$	$2\frac{1}{4}$
$\frac{11}{16}$	$\frac{7}{8}$	$3\frac{1}{2}$	$1\frac{3}{4}$	$2\frac{1}{4}$	$\frac{15}{16}$	$3\frac{3}{8}$	$1\frac{3}{4}$	$2\frac{3}{8}$
$\frac{3}{4}$	$\frac{15}{16}$	$3\frac{11}{16}$	$1\frac{7}{8}$	$2\frac{3}{8}$	1	$3\frac{1}{2}$	$1\frac{13}{16}$	$2\frac{1}{2}$
$\frac{13}{16}$	1	$3\frac{7}{8}$	$1\frac{15}{16}$	$2\frac{1}{2}$	$1\frac{1}{16}$	$3\frac{11}{16}$	$1\frac{15}{16}$	$2\frac{5}{8}$
$\frac{7}{8}$	$1\frac{1}{16}$	$4\frac{1}{8}$	$2\frac{1}{16}$	$2\frac{5}{8}$	$1\frac{1}{8}$	$3\frac{7}{8}$	2	$2\frac{3}{4}$
$\frac{15}{16}$	$1\frac{1}{8}$	$4\frac{5}{16}$	$2\frac{3}{16}$	$2\frac{3}{4}$	$1\frac{3}{16}$	4	$2\frac{1}{16}$	$2\frac{7}{8}$
1	$1\frac{3}{16}$	$4\frac{1}{2}$	$2\frac{1}{4}$	$2\frac{7}{8}$	$1\frac{1}{4}$	$4\frac{3}{16}$	$2\frac{3}{16}$	3
$1\frac{1}{16}$	$1\frac{1}{4}$	$4\frac{11}{16}$	$2\frac{3}{8}$	3	$1\frac{5}{16}$	$4\frac{3}{8}$	$2\frac{1}{4}$	$3\frac{1}{8}$
$1\frac{1}{8}$	$1\frac{5}{16}$	$4\frac{7}{8}$	$2\frac{1}{2}$	$3\frac{1}{8}$	$1\frac{3}{8}$	$4\frac{1}{2}$	$2\frac{3}{8}$	$3\frac{1}{4}$

The efficiency of joints for iron plates is 74 per cent, and that of steel 70 per cent. The strength of a treble-riveted joint (Figs. 896 and 897) may be increased by making the pitch of the inner row of rivets one half that of the outer.

$d = 1{\cdot}27t$ for iron plates and iron rivets.

$d = 1{\cdot}59t$ for steel plates and steel rivets.

The pitch of a quadruple lap-joint will be the same as in the last example.

A butt joint with a single cover-strap (Fig. 898) is composed of two lap-joints, and is proportioned by the rules previously given for lap-joints. With this form of joint, the tension on the plates will tend to bend the cover-strap. For that reason the cover-strap is made thicker than the plates. If t_1 = thickness of cover-strap and t = thickness of plates, then $t_1 = 1\frac{1}{8}t$. For single-riveted butt-joints (Fig. 899), with double cover-straps, the usual rule for the thickness of each butt-strap is $t_1 = \frac{5}{8}t$.

The diameter of the rivets for different thicknesses of plates may be as follows:

$d = t + \frac{1}{4}$ for iron plates and iron rivets.

$d = t + \frac{5}{16}$ for steel plates and steel rivets.

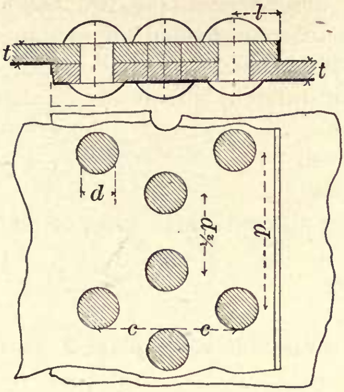


FIG. 896.

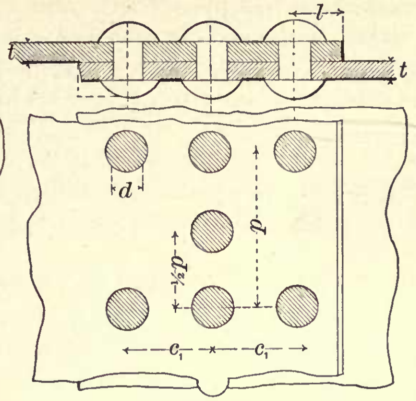


FIG. 897.

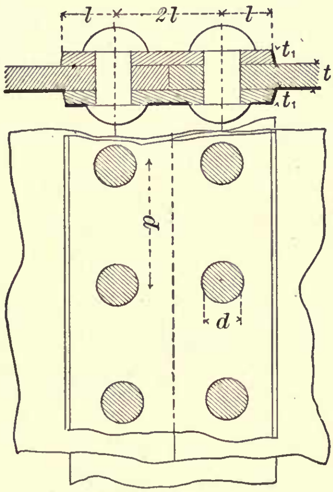


FIG 899.

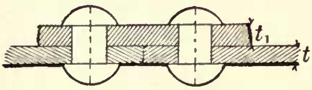


FIG. 898.

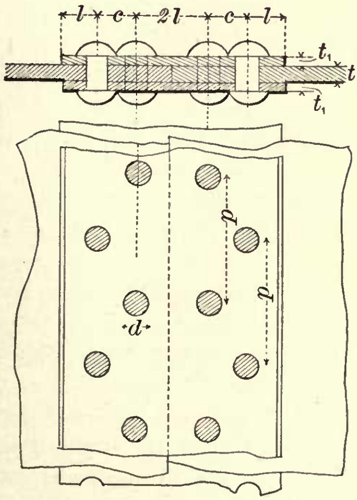


FIG. 900.

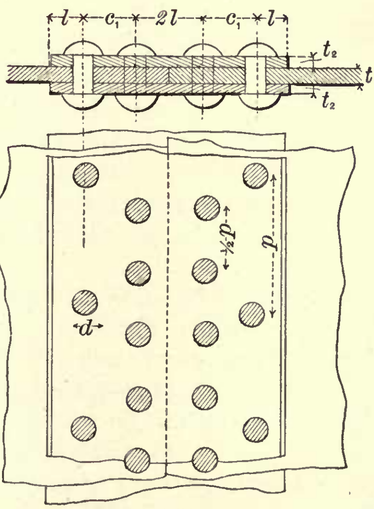


FIG. 901.

Double-riveted Butt-joints with Double Cover-straps (Fig. 900).—The pitch of the rivets are the same in each row. If alternate rivets in the outer rows be removed, as in Fig. 901, the arrangement is stronger.

The diameter of the rivets (Fig. 900) for different thicknesses of plates may be as follows:

$d = t + \frac{3}{16}$ for iron plates and iron rivets.

$d = t + \frac{1}{4}$ for steel plates and steel rivets.

The diameter of the rivets (Fig. 901) for different thicknesses of plates may be as follows:

$d = t + \frac{1}{8}$ for iron plates and iron rivets.

$d = t + \frac{3}{16}$ for steel plates and steel rivets.

Fig. 902 is a triple-riveted butt-joint with double cover-plate, as butt-joints with double covers, one on each side of the plates, increase the shearing resist-

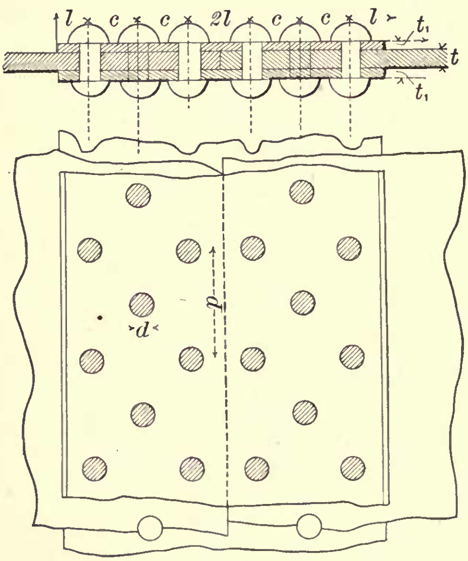


FIG. 902.

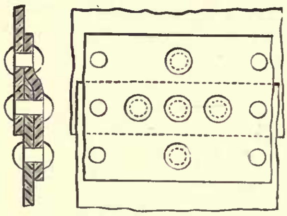


FIG. 903.


FIG. 904.

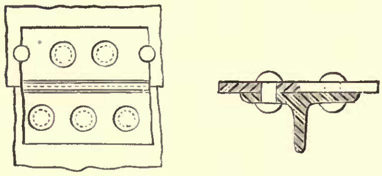


FIG. 905.

ance of the rivets, so that rupture always takes place in the plates; and as these can not bend, and there is considerable frictional resistance between the plates, the strength of the joint has been found to be more than that due to the net section of the plates between the rivets.

Fig. 903 is a plan and section of a combined lap- and butt-joint. The pitch of the exterior rows is double that of the central one; for a $\frac{3}{8}''$ plate, $4''$ for the former and $2''$ for the latter.

Fig. 904 is a section of joint, showing a better arrangement than Fig. 903, requiring less work, more easily calked, and of as much strength.

Fig. 905 is the plan and section of a butt-joint when the cover is of T-iron—a not uncommon form of strengthening flues to resist collapse.

Junction of more than Two Plates, shown in Plans and Sections (Figs. 906, 907, and 908).—These become necessary when cross-joints intersect longitudinal ones. At these joints one or more of the plates are thinned or drawn out by forging.

Fig. 909 is the plan and section of an angular connection of plates by the means of angle-iron; this should be a little thicker than the plates, and its width four times the diameter of the rivets.

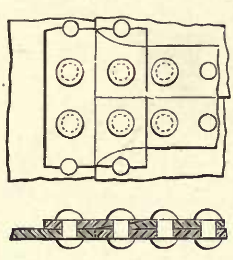


FIG. 906.

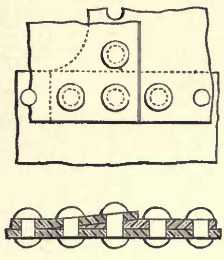


FIG. 907.

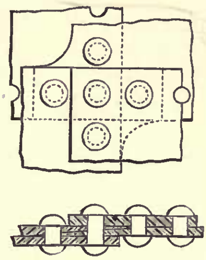


FIG. 908.

Figs. 910, 911, and 912 are sections of angular connections by flanging the plates. The iron should be good and the curvature easy; inside radius at least four times the thickness of the plates.

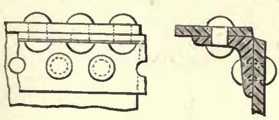


FIG. 909.

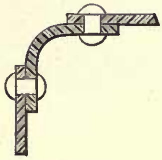


FIG. 910.

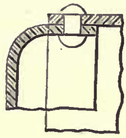


FIG. 911.

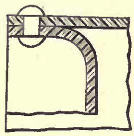


FIG. 912.

Figs. 913 and 914 are sections of joints of cylinders of unequal diameters, or surfaces not in line with each other.

Figs. 915, 916, and 917 are sections of fire-box legs.

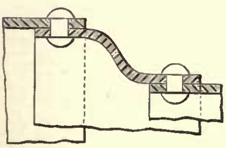


FIG. 913.

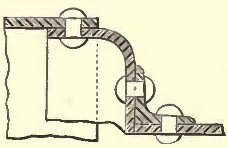


FIG. 914.

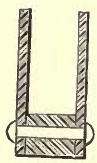


FIG. 915.

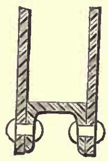


FIG. 916.

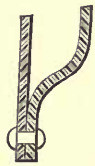


FIG. 917.

In all connections provisions are to be made for the means of holding the head of the rivet, and for riveting and for calking the joints.

Fig. 918 is the perspective view of a horizontal tubular boiler, very largely used with anthracite as a fuel, but with bituminous coal the tubes should be of the larger diameters.

The proportions of the boiler vary with the requirements of their position, and with the views of the mechanical engineer or maker constructing them. Many use a dome, but it is the better practice to increase the diameter of the boiler an inch or two for more steam space, if necessary, and insert a dry pipe in the space. Those in most extensive use are with shells of 4 to 5 feet inside diameter and 3″ to $3\frac{1}{2}$″ tubes, 14 to 16 feet long. The line of the top of the upper tubes is usually about $\frac{1}{10}$ of the diameter of the boiler above its centre; tubes arranged in vertical rows, with distance between tubes $\frac{1}{3}$ of their diameter. By keeping the average distance the same, but making them farther apart at the top row, say $\frac{1}{2}$ diameter, and the lowest $\frac{1}{4}$ diameter, so that the line of tubes is radial instead of vertical, the outside of the tube will meet the flame better, and at the same time be more readily cleaned with a brush.

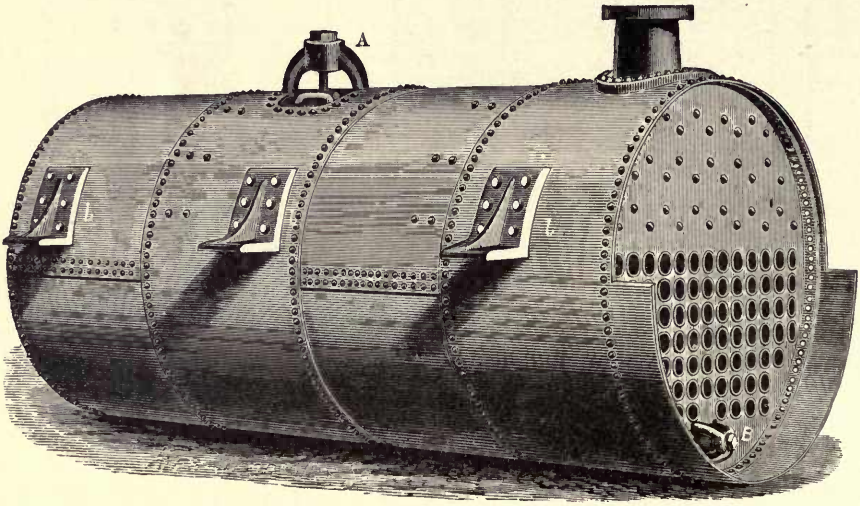


Fig. 918.

The following table is from Barr, showing the greatest number of tubes which should be put in a given head, no tube to come nearer to the shell than 2″ for boilers of small diameter, $2\frac{1}{2}$″ for medium, and 3″ for the larger series:

Diameters of bodies inside, in inches.	Number of Tubes (outside diameter).						
	3 in.	$3\frac{1}{4}$ in.	$3\frac{1}{2}$ in.	$3\frac{3}{4}$ in.	4 in.	$4\frac{1}{2}$ in.	5 in.
36	26	23	20	19	16	12	10
40	34	34	25	23	20	14	14
44	48	36	32	25	25	20	16
48	50	38	36	30	26	21	18
52	57	50	48	38	32	26	21
56	72	57	55	48	41	32	23
60	80	68	62	55	46	36	30

A is the *man-hole*, to enable the mechanic to get into the boiler to examine it. It consists of a cast-iron frame, bolted to the shell of the boiler, with an elliptical opening usually 9″ × 15″ in the clear; the valve laps about 1″ on each side. In closing the opening the valve is passed down into the boiler, and is brought up against the valve-seat, where it is held by its stem passing up through a movable yoke, and brought up tight by a nut and screw. The joint is made with a gasket or with sheet-rubber. The man-hole is often placed in one of the boiler heads, or at one side, above the tubes, for convenience of access. B is the hand-hole, of the same general construction as the man-hole, but smaller, to enable the fireman to clean the boiler. Formerly this hand-hole was quite small, but of late the practice is to place a hand-hole at the rear end of the boiler and a man-hole at the front in the position B. This is for the readier cleaning and repairs of the boiler; it reduces the number of tubes by four, but without detriment to the evaporation of the boiler; and by taking off both covers one can look directly through the boiler. As the hand-hole is exposed to the flame and products of combustion, it is well to make it small, say 3″ × 5″; *l l l* are lugs by which the boilers are supported on the brickwork, but they are in the way in getting the boiler through a confined space, and rest so solidly on the brickwork that it often becomes cracked by the expansion of the boiler. It is preferable that the boiler should be hung as in Fig. 1252. In the head above the tubes there are rivet-heads, and also in the sides back of the first seams at each end. These are for the attachment of diagonal stays. The tubes themselves serve as stays in the lower part of the boiler, but above, the flat surface needs something to prevent the head from moving out under pressure. The stays are made of round or flat iron (see Fig. 1249), bolted directly to the shell, the round part being flattened, and connected by a yoke and pin to a crow-foot or piece of angle-iron attached to the head. The stays are from $\frac{3}{4}$″ to $1\frac{1}{4}$″ diameter or equivalent sections.

To determine the diameter of stays in square inches multiply the area supported by the stays and divide the product by 7,000 for wrought-iron stays not welded; and for steel stays, under same condition, by 9,000; but if welded or otherwise worked after heating, take three fourths of above.

Forms of Boiler Stays.—Fig. 919 is a direct stay in which a hole is drilled through the head of a boiler; the stay has an outside nut of a thickness equal

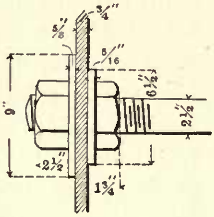


FIG. 919.

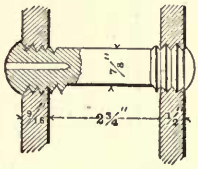


FIG. 920.

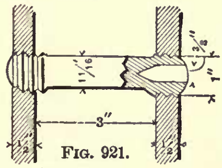


FIG. 921.

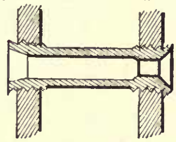


FIG. 922.

to the diameter of the screwed part, and the inside or locked nut three fourths of this thickness. The plates are stiffened by inside and outside washers. If the stay is diagonal it is usual to increase its area in the proportion of the length of the diagonal to that of the horizontal.

The flat parallel surfaces surrounding the fire-boxes of locomotive and marine boilers are secured by means of screwed stays, so called because they are screwed into the plates (Figs. 920, 921, and 922). After being screwed into the plates their ends are riveted. The fracture of the stays is detected by the escape of steam through the small holes which are sometimes drilled through the screwed parts. The screwed stays for locomotive boilers are usually placed about 4 inches apart, centre to centre, and vary in diameter from $\frac{3}{4}$ inch to 1 inch.

In marine boilers the screwed stays are made of steel, and they vary in diameter from $1\frac{1}{4}$ inch to $1\frac{5}{8}$ inch. They are provided with washers and nuts at each end, as shown at Fig. 919. The nuts have a thickness of from five eighths to three fourths the diameter of the stay.

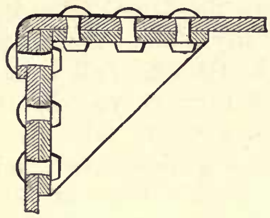


FIG. 923.

In the steel-screwed stays the ends of these stays are drilled as shown, and after they are screwed into place a steel drift is driven into the holes by slight blows to expand the ends tightly into the plates to make steam-tight joints.

Fig. 923 is a *gusset* stay, used in angles, consisting of a triangular plate with the edges flanged and riveted to the shell.

BARR'S PROPORTIONS FOR STAY-BOLTS FOR FLAT SURFACES.

Pressure per square inch.	CENTRE TO CENTRE OF STAY-BOLTS IN SQUARE INCHES.				
	$\frac{1}{4}''$ plate.	$\frac{5}{16}''$ plate.	$\frac{3}{8}''$ plate.	$\frac{7}{16}''$ plate.	$\frac{1}{2}''$ plate.
60	$5\frac{3}{8}$	$6\frac{3}{8}$	$7\frac{1}{4}$	$8\frac{1}{8}$	9
80	$4\frac{5}{8}$	$5\frac{1}{2}$	$6\frac{1}{4}$	$7\frac{1}{8}$	$7\frac{7}{8}$
100	$4\frac{1}{4}$	$4\frac{3}{4}$	$5\frac{1}{2}$	$6\frac{1}{4}$	7
120	$3\frac{7}{8}$	$4\frac{1}{4}$	5	$5\frac{3}{4}$	$6\frac{3}{8}$
140	$3\frac{5}{8}$	$4\frac{1}{8}$	$4\frac{5}{8}$	$5\frac{1}{4}$	6

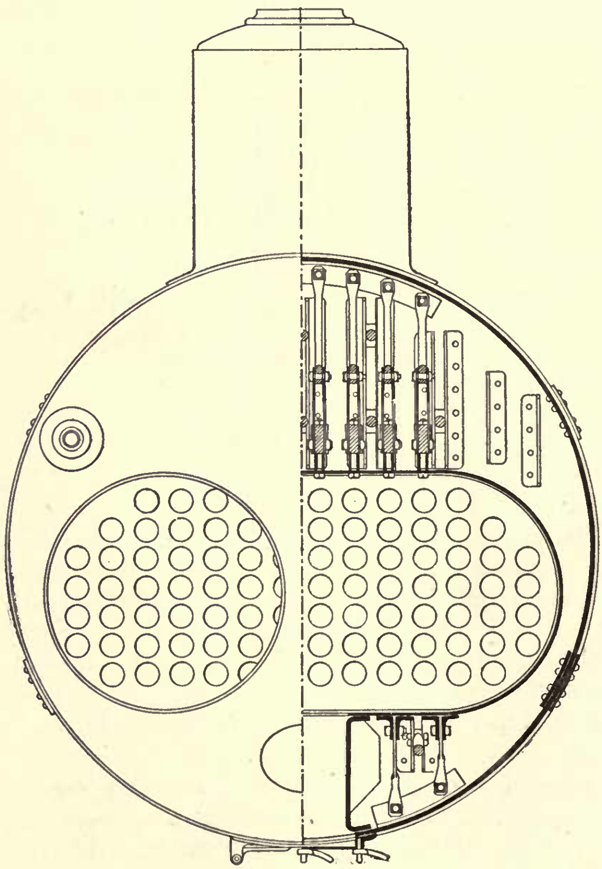


FIG. 924.

Stationary boilers as designed and built under the direction of John E. Codman, M. E., for the Philadelphia Water-Works, of which Fig. 924 is a transverse section and one half cross-section through fire-box, and one half front view without the doors. Fig. 925 is a longitudinal section. These boilers have inside fire-boxes, and the outside is protected by a covering of brick or some clothing of a non-conducting material to prevent radiation. The corrugated furnaces are made in this country by the Continental Iron Works, Brooklyn, of an inside diameter of from 30″ to 60″ and up to 32 feet long.

Rules for calculating the pressure allowa-

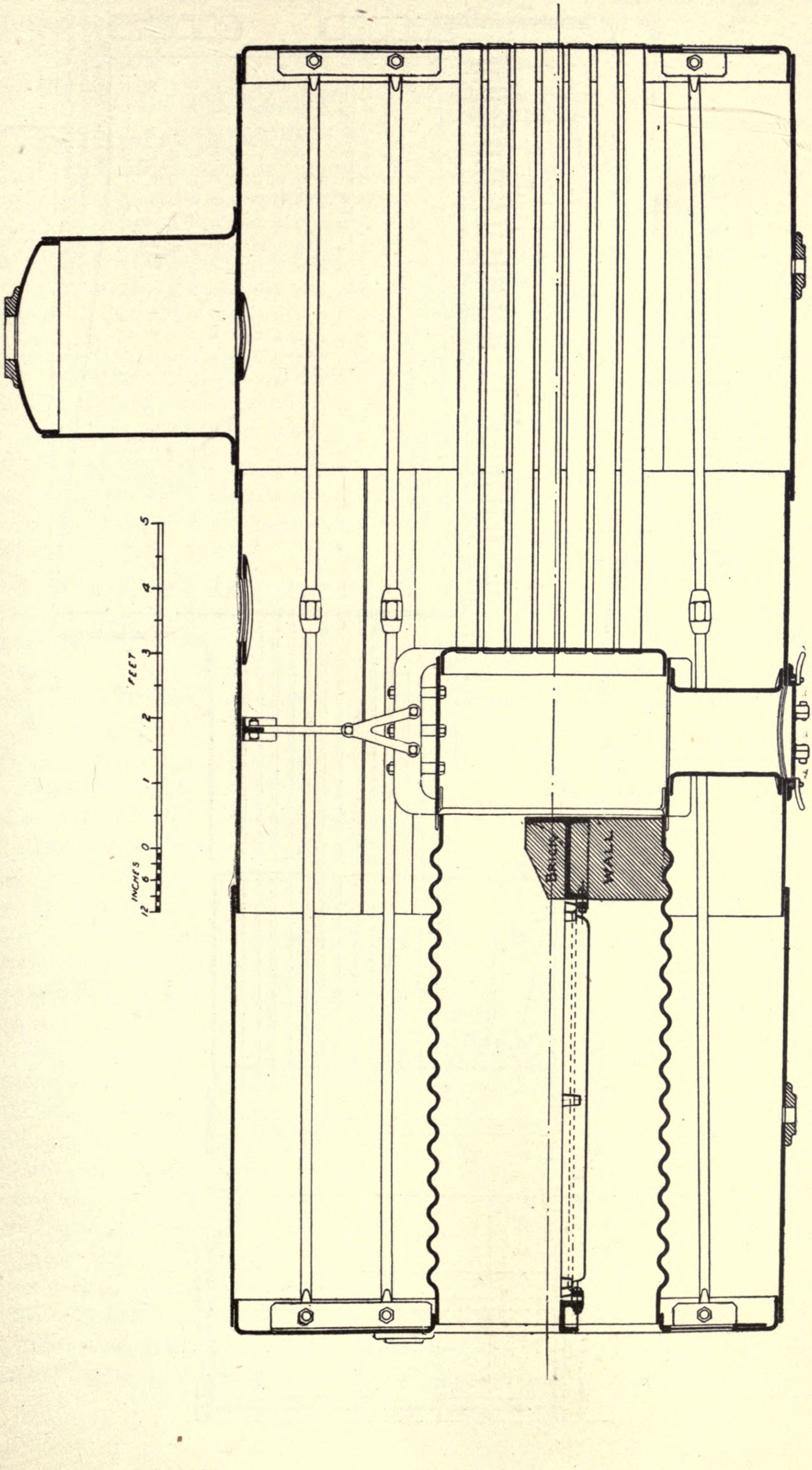


Fig. 925.

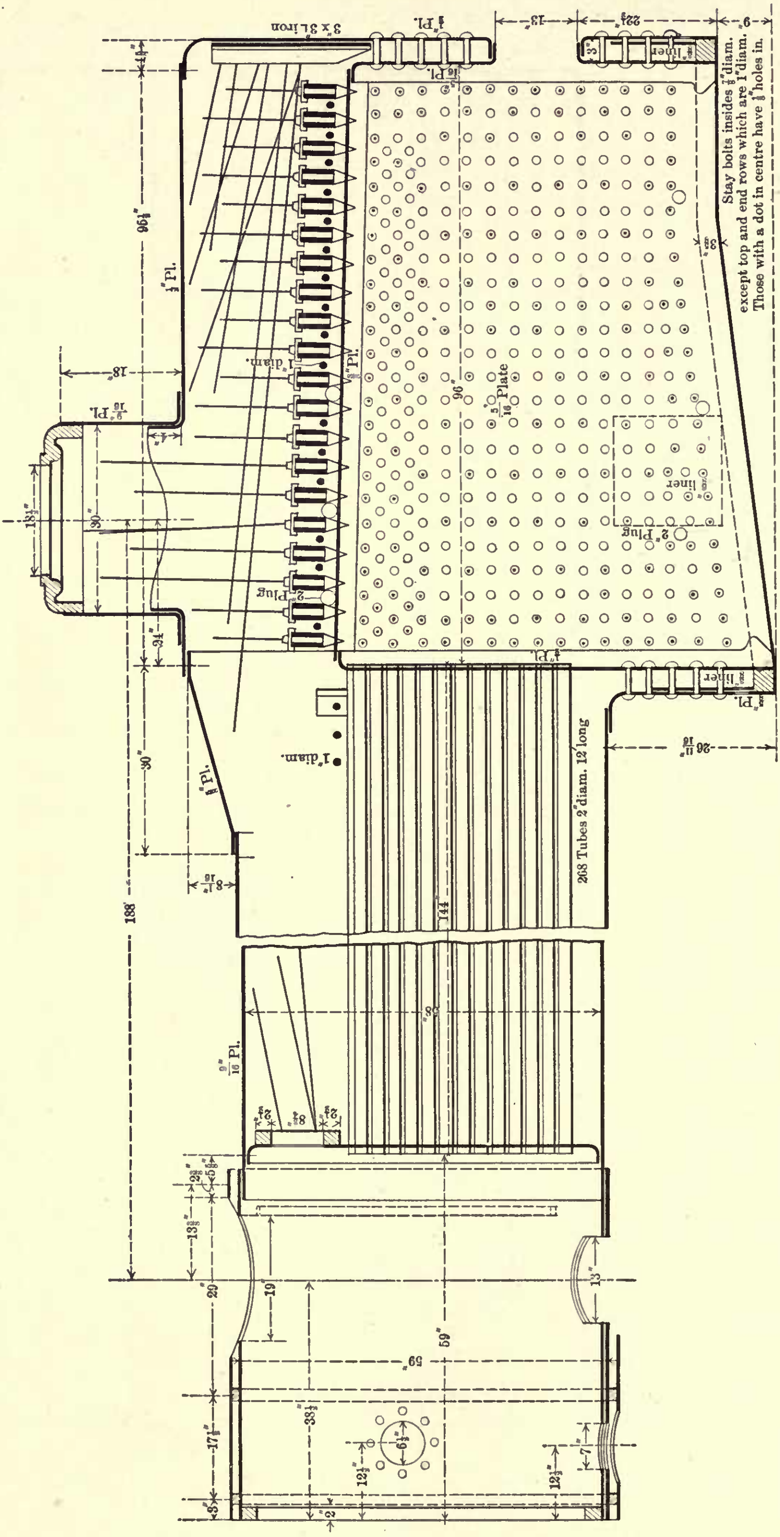

Stay bolts insides ⅞" diam.
except top and end rows which are 1" diam.
Those with a dot in centre have ⅜" holes in.
268 Tubes 2" diam. 12' long
2" Plug
1" diam.
liner

FIG. 926.

ble on corrugated furnaces adopted by the Board of United States Supervising Inspectors:

Corrugations to be 6 inches pitch and $1\frac{1}{2}$ inch deep.

$$\frac{14000}{D} \times T = \text{working pressure in pounds per square inch.}$$

T = thickness in inches.

D = mean diameter in inches.

Example.—Given a corrugated furnace 40 inches mean diameter to carry 175 pounds working pressure, required the thickness of metal.

$$\frac{P. \times D.}{14000} = \text{thickness.} \quad \frac{175 \times 40}{14000}$$

$= \frac{1}{2}$ inch thickness of metal.

Figs. 926 to 929 are drawings of a locomotive boiler as designed and constructed by Mr. Buchanan for express passenger locomotives for the N. Y. C. and H. R. R. R, Fig. 926 is a longitudinal section Fig. 927 a transverse section of one half the fire-box and an elevation of one half of that end. Fig. 928 of the fire-box with cover off, and showing one half of the tubes. Fig. 929 are details of the riveting.

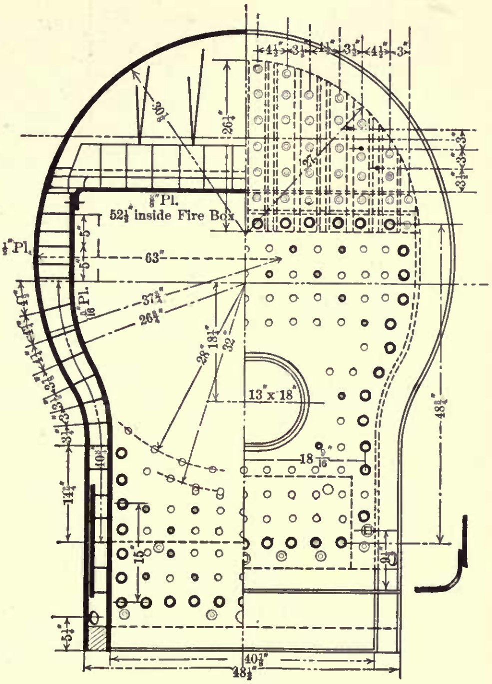


Fig. 927.

Figs. 930 and 931 are drawings of a marine boiler of the United States steamer Minneapolis, showing longitudinal and cross-section of fire-box end. When locomotive or marine boilers are used as stationary their outsides should be protected as the Codman boiler (page 392).

Water-tube boilers are now in extensive use, economical in evaporation, and popular from the comparative safety from explosion. Some of the numerous and varied forms will be found illustrated in the Appendix.

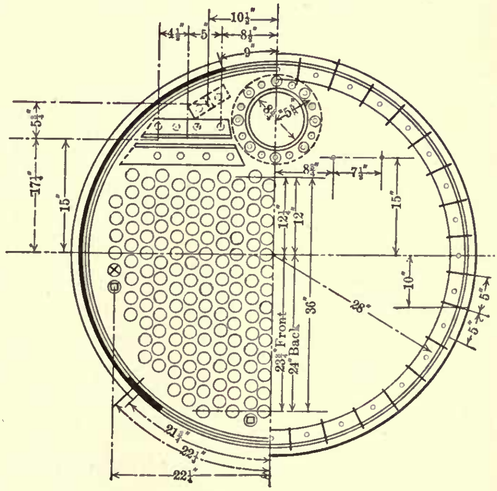


Fig. 928.

Flue Boilers.—Where bituminous coal is used, small tubes become clogged with soot; it was therefore customary to construct boilers with large tubes or flues of boiler-iron riveted together, which sometimes failed from collapse. It may be considered ample to make the tubes subject to outside stress 50 per cent thicker than for bursting, especially for the large drawn tubes now made. Mr. Fairbairn, from his experiments, considered it necessary to make the joints of tubes subject to collapse as in Figs. 932 and 933.

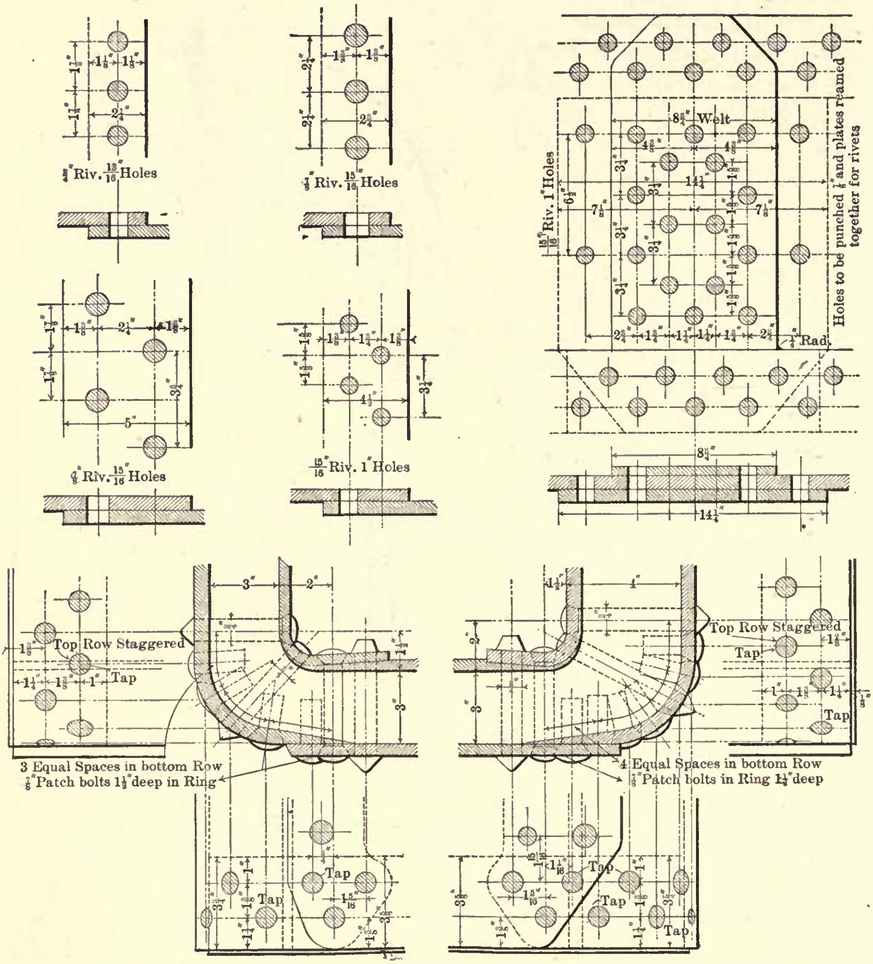


FIG 929.

Fig. 934 is a section of the Shapley boiler, as made by the Knowles Steam-Pump Works—a good form of upright boiler, with the head of the boiler stayed by rods directly to the crown-sheet, beneath which short tubes or nipples connect the fire-box with a cast iron smoke-box around the boiler and the draft is downward through vertical tubes to a smoke-box in the base. The crown-sheet and downward draft tubes are well covered by water. It is an admirable illustration for the draughtsman of how a boiler in action may be represented.

The usual form of upright boiler consists of a fire-box, extending a little above the door, and tubes extending from the crown-sheet to the top-head, over which there is a bonnet to receive the smoke, which is led off by a smoke-

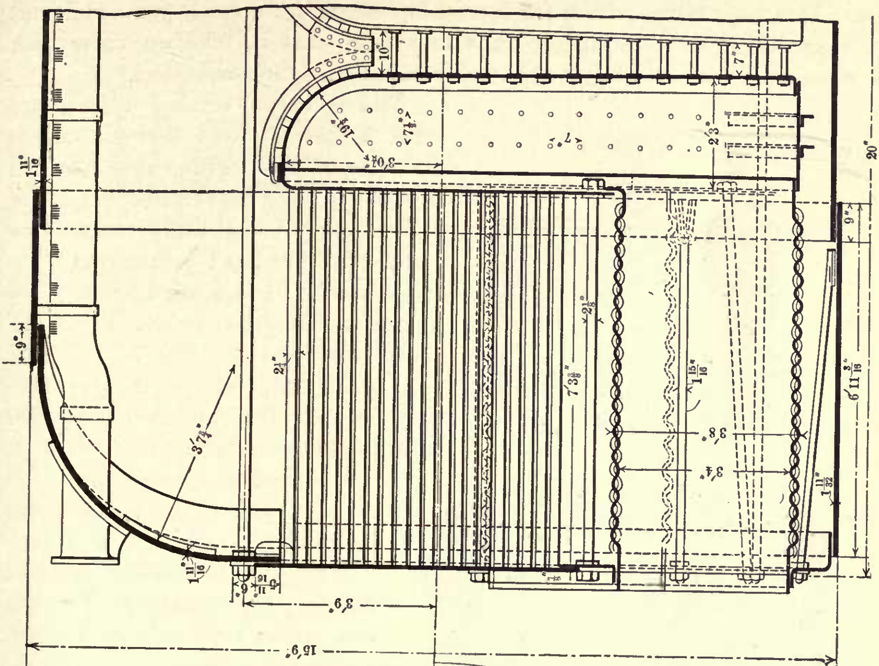


Fig. 931.

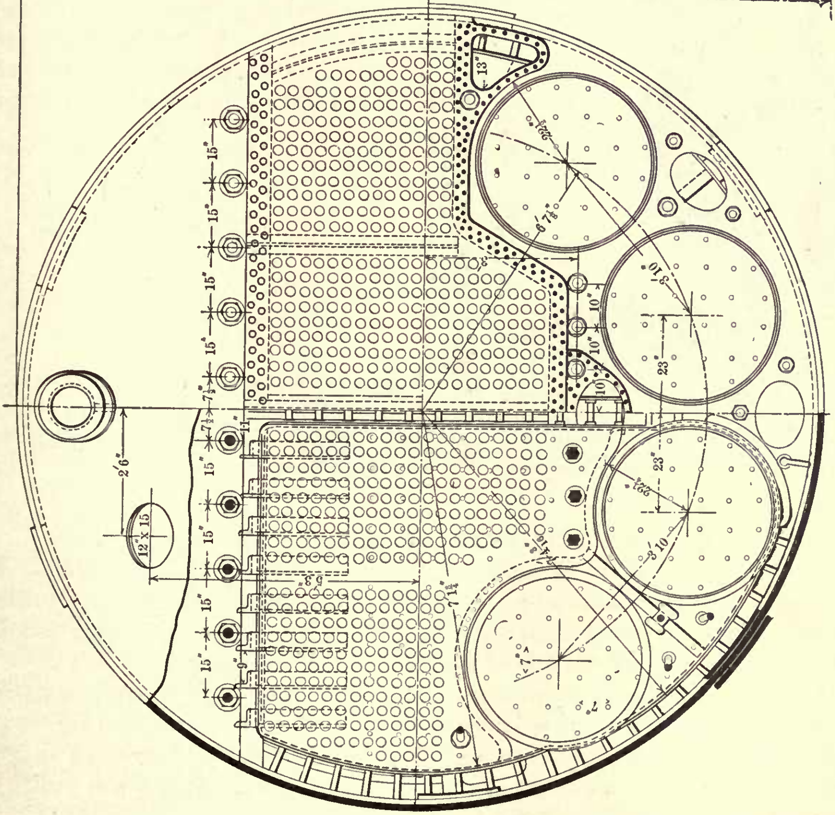


Fig. 930.

pipe. It is a convenient form for furnishing steam for a small power, but not as economical in combustion, and apt to *prime*—that is, take up water with the steam, and leak at the top of the tube exposed in the smoke-box.

The common vertical boilers are from 2 feet 6″ to 4 feet 6″ outside diameter of shell, with water space in legs of $2\frac{1}{2}''$ to 3″; extreme height of boiler from 2 to $2\frac{1}{2}$ times the outside diameter of fire-box; tubes from 2″ to $2\frac{1}{2}''$ diameter, and spaced from 1″ to $1\frac{1}{2}''$ apart. Water-line from 10″ to 15″ above crown-sheet.

There is supposed to be a proportion between the tube sectional area and the grate-surface, say from $\frac{1}{7}$ in the horizontal to $\frac{1}{8}$ in the vertical; but this rule is entirely empirical. There is also a proportion of grate to heating surface; but only the same class of boilers can be compared with each other, as fire-box surface—and that exposed directly to the flame—is much more effective than that of the tubes, and the products of combustion escape at much different temperatures in different boilers.

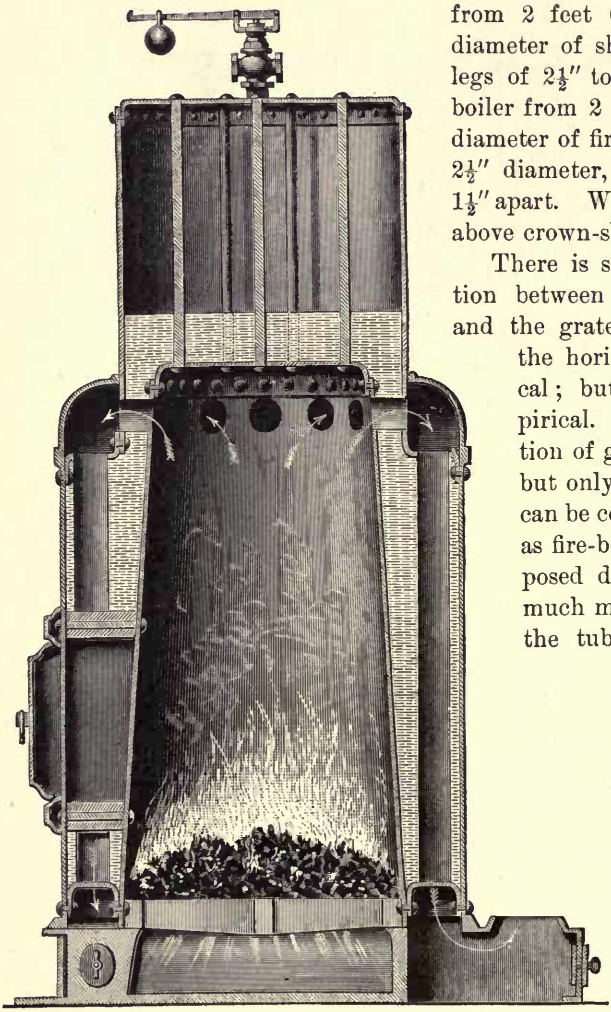


Fig. 934.

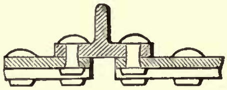


Fig. 932.

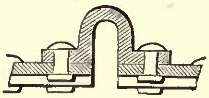


Fig. 933.

Flange Connections for Steam and Water Pipe.—Fig. 935 is a section of a flanged connection of a cast-iron pipe of the most usual form, but some thicken or re-enforce the pipe a little for 1″ to 2″ in length next the flange; but if there is a good fillet in the angle of the flange it is unnecessary.

The flanges are almost invariably faced, and joints made with red and white lead, or a sheet-rubber washer, or with corrugated copper gaskets (Fig. 936) of very thin sheet copper, which are used of full diameter of flanges on rough boiler joints and red-lead putty; but for faced surfaces thin paint will insure a perfect joint inside the bolts.

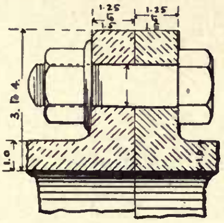


FIG. 935.

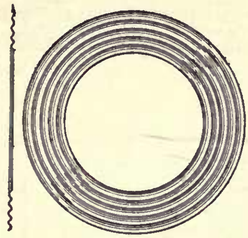


FIG. 936.

DIMENSIONS OF PIPE FLANGES AND CAST-IRON PIPES.

J. E. Codman, M. E., Pro. Engrs. Club, Philadelphia.

Diam. of pipe.	DIAMETER OF FLANGE.		Diameter of bolt circle.	Diam. of bolt.	No. of bolts.	Thickness of flange.	THICKNESS OF PIPE.		Weight per foot without flange.	Weight of flange and bolts.
	Diagr.	Form.					Inches.	Dec.		
2	6 1/4	6 1/2	4 3/4	3/4	4	5/8	3/8	0·373	6·96	4·41
3	7 1/2	....	5 7/8	3/4	4	5/8	13/32	0·396	11·16	5 93
4	9	8 3/4	7	3/4	6	1/16	7/16	0·420	15·84	7·66
5	9 3/4	....	8	3/4	6	3/4	7/16	0·443	21·00	9·63
6	10 3/4	11	9 1/8	3/4	8	3/4	15/32	0·466	26·64	11·82
8	13 1/4	....	11 3/8	3/4	8	13/16	1/2	0·511	39·36	16·91
10	15 1/4	15 1/2	13 1/4	3/4	10	7/8	9/16	0·557	54·00	23·00
12	17 3/4	....	15 3/4	7/8	12	15/16	19/32	0·603	70·56	30·13
14	20	...	18	7/8	14	1	21/32	0·649	89·04	38·34
16	22	22 1/4	20	7/8	16	1 1/16	11/16	0·695	109·44	47·70
18	24	24 1/2	22 1/4	7/8	16	1 1/8	3/4	0·741	131·76	58·23
20	27	26 3/4	24 1/2	1	18	1 3/16	25/32	0·787	156·00	70·00
22	28 3/4	29	26 1/2	1	20	1 1/4	27/32	0·833	182·16	83·05
24	31 1/4	....	28 3/4	1	22	1 5/16	7/8	0·879	210·24	97·42
26	33 1/4	33 1/2	31	1	24	1 3/8	15/16	0·925	240·24	113·18
28	35 1/2	35 3/4	33 1/4	1	24	1 7/16	31/32	0·971	272·16	130·35
30	38	....	35 1/2	1	26	1 9/16	1	1·017	306·00	149·00
32	40	40 1/4	37 1/2	1 1/8	28	1 5/8	1 1/16	1·063	341·76	169·17
34	42 1/4	....	40	1 1/8	30	1 11/16	1 1/8	1·109	379·44	190·90
36	45	44 3/4	42	1 1/8	32	1 3/4	1 5/32	1·155	419·04	214·26
38	47	....	44	1 1/8	32	1 16	1 3/16	1·201	460·56	239·27
40	49	49 1/4	46	1 1/8	34	1 7/8	1 1/4	1·247	504·00	266·00
42	51 1/4	51 1/2	48 1/4	1 1/8	34	1 15/16	1 5/16	1·293	549·36	294·49
44	53 1/2	....	50 1/4	1 1/4	36	2	1 11/32	1·339	596·64	324·78
46	55 3/4	56	52 3/4	1 1/4	38	2 1/16	1 3/8	1.385	645·84	356·94
48	58	58 1/4	55	1 1/4	40	2 1/8	1 7/16	1·431	696·96	391·00

Fig. 937 is a section of the joint used by Sir William Armstrong for the pipes of his accumulator. For a working pressure of 800 pounds per square inch, pipes of 5″ diameter are made 1″ thick and tested to 3,000 pounds per square inch. The flange is elliptical, and there are but two bolts; one pipe slightly enters the other, forming a dovetailed recess in which is placed a gutta-percha ring 1/4″ in diameter.

Figs. 938 and 939 are sections of two other forms of cast-iron flanged pipes, both with projections fitting into grooves. The packing in Fig. 939 is a ring of lead. In Siemens's air reservoirs, where the pressure sustained by steel rings is 1,000 pounds per square inch, the joint is made by turning a V-groove in the face of the rings, and placing in it a ring of annealed copper $\frac{5}{16}''$ diameter. This form is adopted by many mechanics for forming flanged joints even for steam purposes.

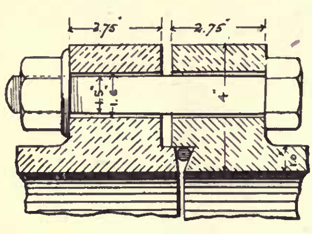


FIG. 937.

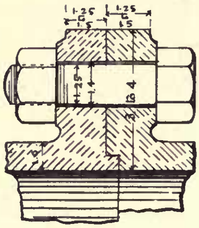


FIG. 938.

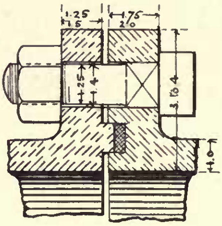


FIG. 939.

Figs. 940, 941, and 942 are steam-pipe joints, as used at the works of the Narragansett Electric Lighting Company, where it is essential to maintain the

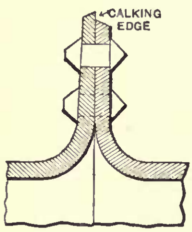


FIG. 940.

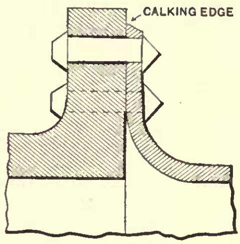


FIG. 941.

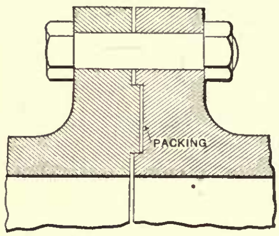


FIG. 942.

full boiler pressure permanently. Fig. 940 is a joint between two wrought-iron pipes; Fig. 941 that between a wrought-iron and cast-iron pipe. In both these the joints are calked. In Fig. 942, between two cast-iron pipes, the joint is made by a gasket of vulcanized asbestos placed in a recess of the female joint.

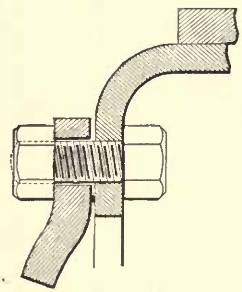
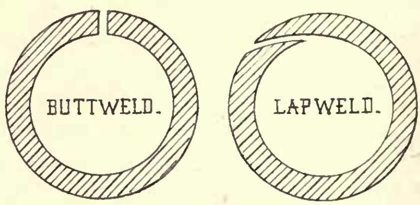


FIG. 943.

Fig. 943 is a connection between wrought-iron plates, in which the joint is made by a copper ring brazed together.

Wrought-Iron Pipe Connections.—With the present cost of wrought-iron pipes, they are almost invariably used for the conveyance of steam, but are more liable to rust for water purposes than cast-iron. Wrought-iron pipes are either butt-welded or lap-welded. It is a mere question of manufacture. It is difficult to make a lap-welded tube less than $1\frac{1}{2}''$ diameter, and therefore below this size they are usually butt-welded; but this size and above, lap-welded.



Wrought-iron pipes of the smaller diameters are connected by socket-sleeve couplings (Fig. 944), of wrought-iron, of large diameters, by cast-iron flanges screwed to the ends of the pipes to be coupled. The screw in the coupling is tapped parallel usually, but the ends of the tubes are cut with a taper thread, uniform with all makers, of 1 in 32 to the axis. The length of the screwed portion varies with the diameter.

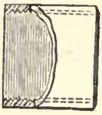


FIG. 944.

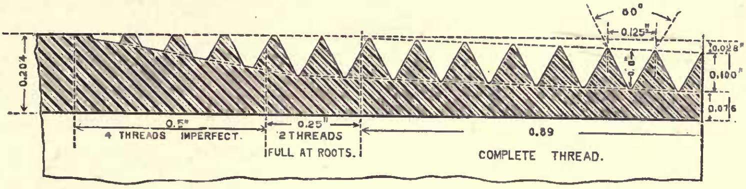


FIG. 945.

Fig. 945 is the longitudinal section of tapering tube-end with the screw thread as actually formed, and considered standard by the late Robert Briggs, C. E., in his "Treatise on Warming Buildings by Steam." It is shown in the figure double full size for a nominal $2\frac{1}{2}''$ tube.

DIMENSIONS OF WROUGHT TUBES AND COUPLINGS.

DIAMETER OF TUBE.			CIRCUMFERENCE.		SCREWED ENDS.		Weight per foot in length.	COUPLINGS.	
Nominal inside.	Actual inside.	Actual outside.	Inside.	Outside.	No. of threads per end.	Length of screw.		Outside diameter.	Length.
In.	In.	In.	In.	In.	In.	In.	Lbs.	In.	In.
$\frac{1}{8}$	0·27	0·41	0·85	1·27	27	0·19	0·24	0·55	$\frac{7}{8}$
$\frac{1}{4}$	0·36	0·54	1·14	1·70	18	0·29	0·42	0·70	1
$\frac{3}{8}$	0·49	0·67	1·55	2·12	18	0·30	0·56	0·83	1
$\frac{1}{2}$	0·62	0·84	1·96	2·65	14	0·39	0·84	1·01	$1\frac{5}{16}$
$\frac{3}{4}$	0·82	1·05	2·59	3·30	14	0·40	1·13	1·24	$1\frac{3}{8}$
1	1·05	1·31	3·29	4·13	$11\frac{1}{2}$	0·51	1·67	1·53	$1\frac{3}{4}$
$1\frac{1}{4}$	1·38	1·66	4·33	5·21	$11\frac{1}{2}$	0·54	2·26	1·89	$1\frac{3}{4}$
$1\frac{1}{2}$	1·61	1·90	5·06	5·97	$11\frac{1}{2}$	0·55	2·69	2·17	2
2	2·07	2·37	6 49	7·46	$11\frac{1}{2}$	0·58	3·67	2·68	$2\frac{1}{4}$
$2\frac{1}{2}$	2·47	2·87	7·75	9·03	8	0·89	5·77	3·19	$2\frac{3}{4}$
3	3·07	3·50	9·64	11·00	8	0·95	7·55	3·87	3
$3\frac{1}{2}$	3·55	4·00	11·15	12·57	8	1·00	9·06	4·40	$3\frac{1}{8}$
4	4·03	4·50	12·65	14·14	8	1·05	10·73	4·99	$3\frac{1}{4}$
$4\frac{1}{2}$	4·51	5·00	14·15	15·71	8	1·10	12·49	5·49	$3\frac{3}{8}$
5	5·04	5·56	15·85	17·47	8	1·16	14·56	6·19	$3\frac{1}{2}$
6	6·06	6·62	19·05	20·81	8	1·26	18·77	7·24	$3\frac{3}{4}$
7	7·02	7·62	22·06	23·95	8	1·36	23·41	8·36	4
8	7·98	8·62	25·08	27·10	8	1·46	28·35	9·49	$4\frac{1}{4}$
9	9·00	9·69	28·28	30·43	8	1·57	34·08	10·54	$4\frac{1}{2}$
10	10·02	10·75	31·47	33·77	8	1·68	40·64	11·72	5

Figs. 946, 947, 948, also from Briggs's treatise, give the dimensions of the parts of elbows, tees, crosses, and branches. Fig. 947 shows the parts of an elbow designated by letters in Fig. 946, and Fig. 948 shows the applicability of the same to tees and crosses. The scale is one quarter full size; if much used, it would be better for the draughtsman to construct one of full size. The dimensions are obtained by measuring from the base or zero to the inclined lines, on ordinates corresponding to the inside diameter of pipe required.

When pipes are thus put together in lengths, with couplings, it is frequently impossible to take out a length of pipe for repairs or alterations without breaking a coupling or fitting; provision is made for disconnections by the insertion of a *union* or unions in the line.

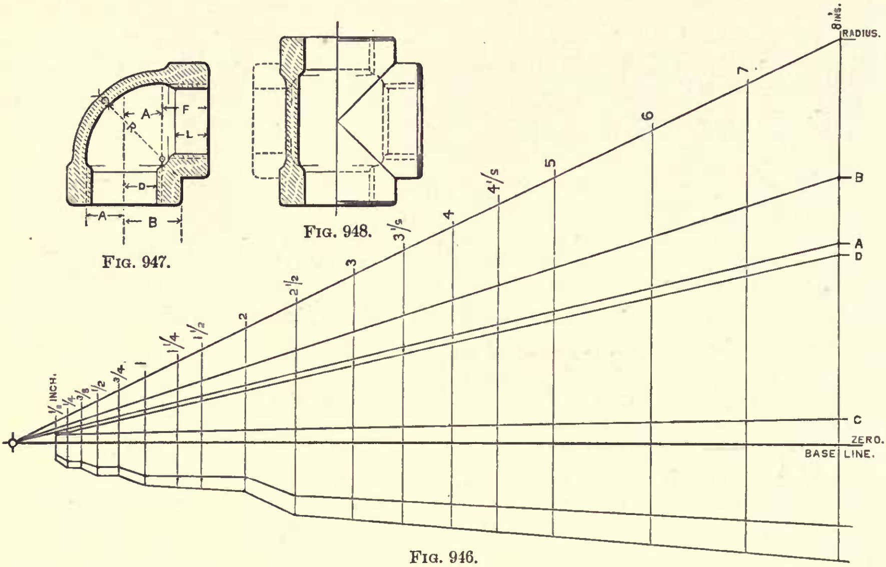


FIG. 947.

FIG. 948.

FIG. 946.

Fig. 949 is an exterior view, and Fig. 950 a section, of the common malleable-iron union; p and p' are the halves into which the tube is screwed, and the joint is made by a male and female coupling. The male, b, turning on a flange on the tube p, is screwed to the other half of the coupling, and the joint is made tight by a rubber washer, shown in black. These unions are used only

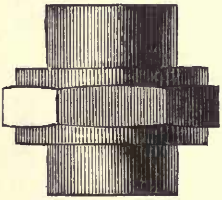


FIG. 949.

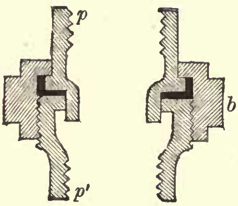


FIG. 950.

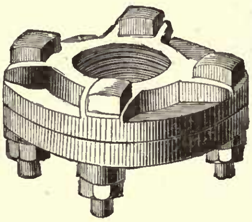


FIG. 951.

in the smaller sizes of pipes. The flange coupling (Fig. 951) is preferred by most fitters, and they are made of diameters up to 14″; the thickness is about one half that of the length of a coupling of the same diameter. The bolts are from $\frac{3}{8}$″ to $\frac{3}{4}$″, and spaced somewhat larger than that given for cast-iron flanges. The width of flange is such as to admit of the head and nut of the bolt without projection beyond the edge of the flange.

Fig. 952 is a common cast-iron flange, and with about the same proportions as in Fig. 951. When the lines are long, and provision can not be made by bends for the expansion and contraction of pipes under changes of temperature, a fitting like a stuffing-box is often used, the end of one of the tubes being attached to the box, and the other sliding in and out like a piston-rod; sometimes expansion is permitted by two flexible flanges, admitting of a sort of bellows-like movement; sometimes by a U-connection between pipes, as in Fig. 933, or a succession of corrugations.

Fig. 953 is a *soldering union;* the ring, *b*, is like that of the male coupling (Fig. 950), which is screwed directly to the wrought-iron pipe, while *a* is a brass tube, with a shoulder on the bottom on which the coupling, *a*, turns, and a lead pipe is soldered to the tube. If it is not necessary to break the joints, a *soldering nipple* (Fig. 954) only is necessary, one end of which is screwed into the wrought-iron pipe, and the other soldered to the lead pipe.

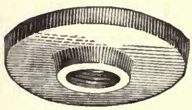


FIG. 952.

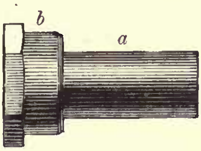


FIG. 953.

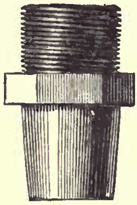


FIG. 954.


FIG. 955.

Fig. 955 is a *close nipple;* Fig. 956 is a *shoulder nipple.*

If the uncut part of the tube is longer than in the figure, it is called a *long* nipple; they serve the purpose of short pipes.

Fig. 957 is a *bushing.* There is a thread cut inside. It is screwed into a coupling, and the pipe that is screwed into the bushing must be smaller in diameter than that connected with the coupling. The service of the bushing

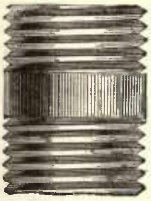


FIG. 956.

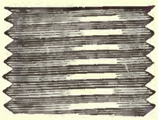


FIG. 957.

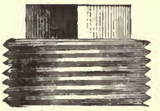


FIG. 958.

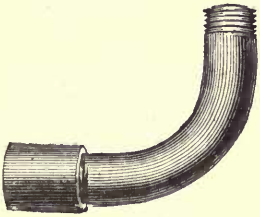


FIG. 959.

is to connect pipes of different diameters, but the reduction of one side or arm of a coupling tee, or cross is better.

Fig. 958 is a *plug* to close up the end of a pipe by screwing it into the coupling; *caps* are used for the same purpose; half-couplings with one end closed, or *blank* flanges—that is, flanges covering the aperture in the pipe—bolted to a flange on the end of a pipe.

It will be seen by Fig. 947 that the cast-iron elbow makes a very short turn, with considerable obstruction to the flow of the fluid through it. Fig. 959 is an elbow in which the obstruction is very much reduced. It consists of a piece of wrought-iron pipe curved to an easy radius; and, as a general rule, it may be said that for the connection of pipes not in a line with each other, it is better to bend the pipe, if possible, than make angles by cast-iron elbows.

Figs. 960, 961, and 962 are oblong, spiral, and flat coils, showing the extent to which pipe can be bent by machinery, and are used largely for heaters and in refrigerating plants.

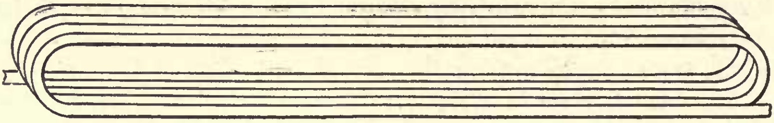



FIG. 960.

Figs. 963 and 964 are a tee and a cross, as used in connections of hydraulic presses, made of composition. The tubes are of wrought-iron, extra thick. The usual dimensions for such are as follows:

Outside diameter..................	$1\frac{3}{16}''$	$\frac{7}{8}''$	$1''$
Inside diameter....................	$\frac{3}{8}''$	$\frac{3}{8}''$	$\frac{1}{2}''$

The joints are made by leather washers, square ends on square seats.

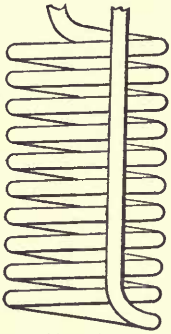


FIG. 961.

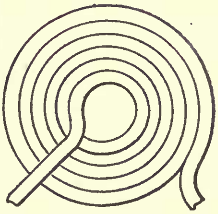


FIG. 962.

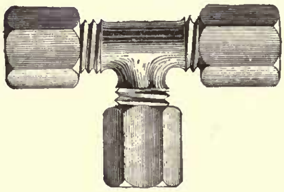


FIG. 963.

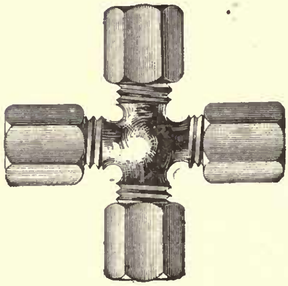


FIG. 964.

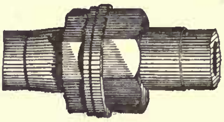


FIG. 965.

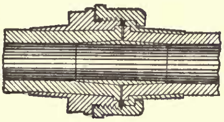


FIG. 966.

In Leland's lead-pipe coupling (Figs. 965 and 966) a double-cone ring is inserted between the ends of the pipes to be coupled, and they are brought together by the common wrought-iron pipe-union, the inner surfaces of which are adapted to the surfaces of the lead pipe, compressing it between the pipe and the cone.

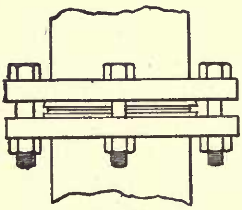


FIG. 967.

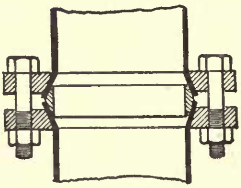


FIG. 968.

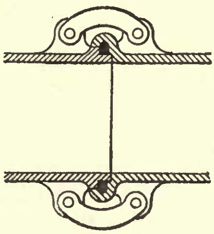


FIG. 969.

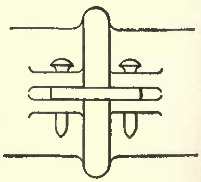


FIG. 970.

Figs. 967 and 968 are a section and plan for a similar joint of steel water pipe.

Figs. 969 and 970 are Petit's pipe joint used in the water system of the camp at Chalons. A rubber ring is inserted in the short bell, one clamp being connected; the other is brought over and secured, compressing the rubber, making a tight and a slightly flexible joint, easily taken apart and put together.

Fig. 971 is a flexible steam joint consisting of a ball and socket carefully turned and then ground to a close fit, to be connected with wrought-iron pipe.

The earlier flexible joints for water mains were turned and fitted, but about

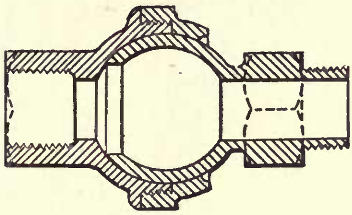


Fig. 971.

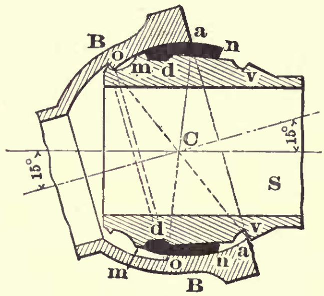


Fig. 972.

1870 John T. Ward, C. E., introduced a ball-and-socket pipe in which the socket was turned but the ball was fitted by a lead packing run in. There was some danger to the socket by the stress of the ball if the movement was in excess of that contemplated. Mr. Ward in his late designs made stops *o o* (Fig. 972) to prevent this. Other engineers have re-enforced the ball by a hoop, of which Fig. 973 is an example, from the "Transactions" A. S. C. E., designed by James C. Duane, C. E., and laid beneath the East River from New York city to Ward's Island.

Fig. 974 is a section and elevation of a flexible joint for a submerged steel water main used at Toronto, Ontario. This joint is 5 feet in diameter, con-

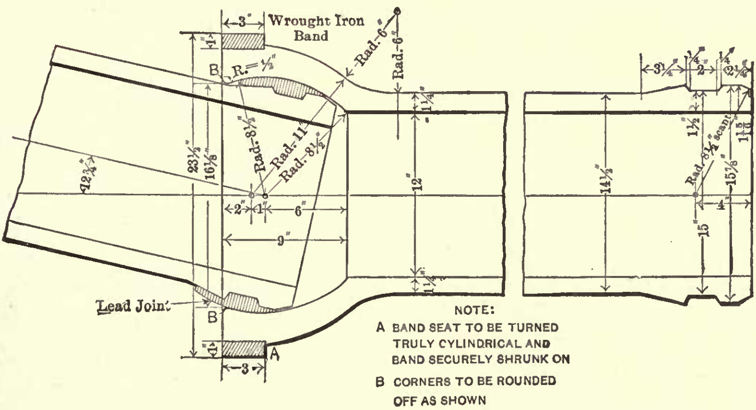


Fig. 973.

sisting of two parts, one part having a turned spherical section riveted to the straight pipe; the other part is a socket, on the inside of which are two U-shaped sections, one riveted to the sheet-iron of the socket and the other to a flange fastened to another flange on the end of the socket; these U-shaped rims are filled with soft pig lead projecting $\frac{1}{8}$ of an inch beyond the rim; the lead joint bears against the spherical section and makes a close yet flexible

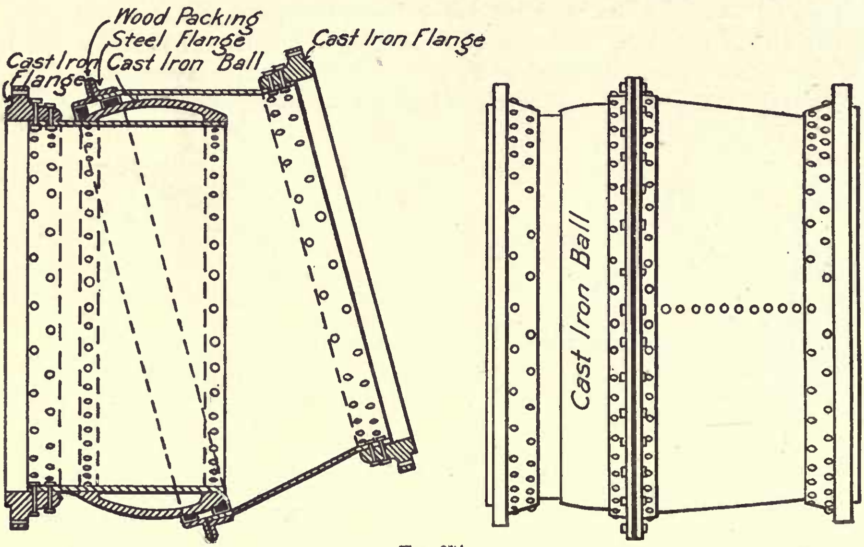


FIG. 974.

joint. The space between the two flanges on the socket end is made tight by a wood packing.

Fig. 975 is the section of a ball-joint used as a connection for the 12″ steel pipe for a temporary supply of water to Liverpool and sunk beneath the Mersey, which was connected for a length of 800 feet on the shore and drawn across the river by the united forces of horses and a steam winch in twenty-eight minutes. The ball is of cast-iron, turned, and has a socket of the same material, with a joint of cast lead, for which two holes are shown—one for running the metal and the other for the vent.

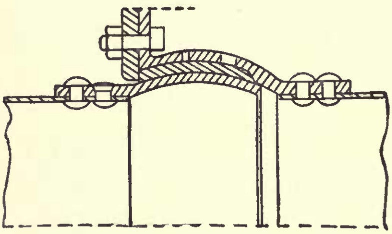


FIG. 975.

Governors.—In the running of all machinery there are variations of speed, due to varying powers and resistances caused by increase or decrease in the pressure producing the power, as of steam or water, or in the resistances of the machinery, from more or less being brought into action, or through inequalities of work done. To maintain the speeds at as much uniformity as possible, governors are used, which, applied to steam engines or water-wheels, open or close valves or gates, and increase or reduce the supply of steam or water to the cylinders or wheels, according to the varying necessities. The ordinary governor (Fig. 976) consists of two heavy balls, suspended by links from a spindle, and caused to revolve by some connection with the shaft of the motor. In the figure the governor is driven by a belt-connection to the pulley, p, bevel-geared to the governor. When at rest, the balls hang close to the spindle, but when in motion the balls rise by the centrifugal force. When the motor is running at its established speed, for which the pulley is to be adjusted, the links assume a position nearly at 45° with the spindle. If the speed falls off, the balls fall, and, acting on the lever, as shown in side view, open the valve or gate controlling the passage of steam to the cylinder or water to the wheel; if the speed rises, the balls rise and close the valve or gate. The lever does not always connect directly with the gate, nor is there always a lever, but the rise or fall of the balls acts on some mechanism which performs the function of reducing or increasing the supply of steam or water.

The size of the balls depends somewhat on the work to be done, the resistance to be overcome in the movement of the gate and connections, and may be much reduced if this work is thrown on some other mechanism, which is usually the case in the regulation of water-wheels; while for steam engines the

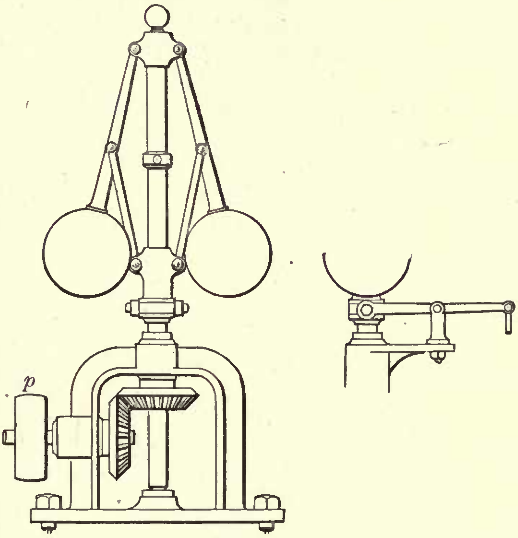


FIG. 976.

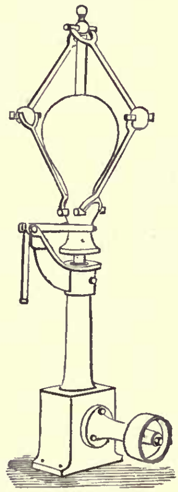


FIG. 977.

work to be done by the governor is reduced by balancing the steam-valve, or to the merely releasing of a trip, cuts off the movement of the valve at any point of stroke.

The common governor (Fig. 976) is sufficient for the regulation of drop cut-off engines like the Corliss (Figs. 422, 423), but for slide-valve engines with throttle regulation the Porter governor (Fig. 977) is better adapted; the balls of this governor are comparatively light, but they are connected to a heavy central weight by levers, the same as those connecting the balls with the spindle.

Of late it has become very common to run engines at very high speeds, beyond that possible to be obtained by drop cut-offs; and light slide-valves are used in which the governors act by shifting the cams (actuating the valves) placed within the pulley or fly-wheel. Fig. 978 is an elevation of one of this class of governors—the Westinghouse. The disk A is cast solid and keyed to one of the cranks. The loose eccentric C is suspended by the arm *c* from the pin *d*, around which it has a motion of adjustment; BB are the governor weights, pivoted on the pins *b b*; one of the weights is connected to the eccentric by the link *f*, and both weights are connected to operate in unison by the link *e*. Coil springs, D D, furnish the centripetal or returning force. The eccentric encircles the shaft S, the opening being elongated to admit of the

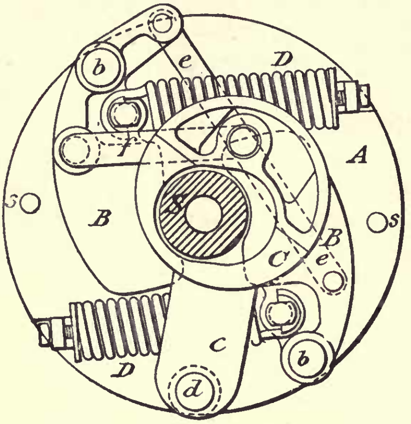


FIG. 978.

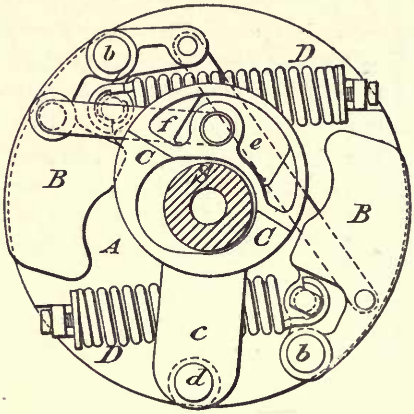


FIG. 979.

proper motion. The stops *s s* limit the motion of the weights. Fig. 979 shows the governor in the position of latest cut-off. The governor weights are shown in the position of rest, whereby the eccentric is thrown over to its position of greatest eccentricity, giving a maximum travel to the valve corresponding to a cut-off of about $\frac{3}{4}$ stroke. The parts of the governor remain in this position till the engine is within a few revolutions of its full speed. The centrifugal force of the weights then overbalances the tension of the springs, and the weights move outward, reducing the travel of the eccentric and valve, consequently shortening the cut-off and closing the exhaust earlier, thus increasing the compression curve and preserving greater economy in running the engine.

Fly-Wheels.—In most machinery there are great inequalities of movements, from the great difference in power exerted or resistances overcome, and in the application of the force, as through cranks. To obviate this, fly-wheels are used, which absorb energy in one part of their revolution and give it out at another, or by their mass in movement overcome resistances, as in the punching, shearing, and rolling of metal, which comes only periodically, and is much in excess of that usually required. In addition, fly-wheels give governors time to act, and consequently the motion is more uniform and constant.

All shafting, pulleys, and machines in movement act as regulators, and where the resistances vary largely on machines they require independent fly-wheels. In addition, friction, hygrometric, and other conditions vary so much at different times, even with the same engines, that it is impossible to get data for an estimate by any mathematical formula embracing the conditions. From the experience of the best mechanical engineers, and from published examples of constructions, are deduced the following rules, applicable to common practice for the fly-wheels of steam engines: The diameter of fly-wheel to be 4 times that of the stroke of the engine, and the entire weight of the wheel 40 times the square root of the diameter, its exterior velocity being about 5,000 feet per minute; if less or more, increase or reduce the weight inversely as the velocity. The rim is generally a little less than $\frac{3}{4}$ of the whole weight, but the arms should be made strong in view of the fact that a great strain may be produced in them by any suddenly interposed obstacle. For rolling-mill engines, Prof. C. B. Richards takes the weight of the fly-wheel at 60 times the square of the diameter of the cylinder, and the diameter of the wheel 5 times that of the stroke, and rim velocity not to exceed 125 feet per second.

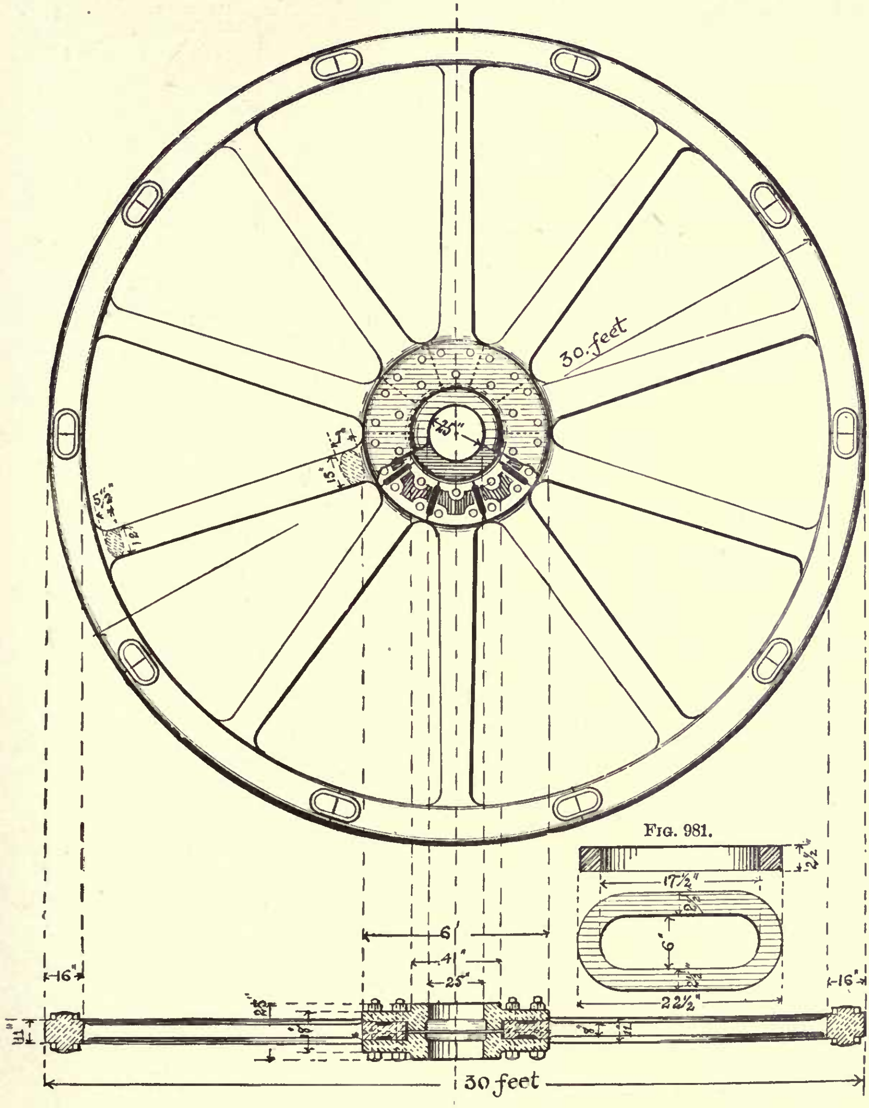


Fig. 980.

In most stationary engines the fly-wheel is a pulley or band-wheel or gear driving the machinery, but often the fly-wheel is independent. Fig. 980 is the elevation and section of a fly-wheel built by the Southwark Foundry. The construction will be understood from the drawings, but the wrought-iron links

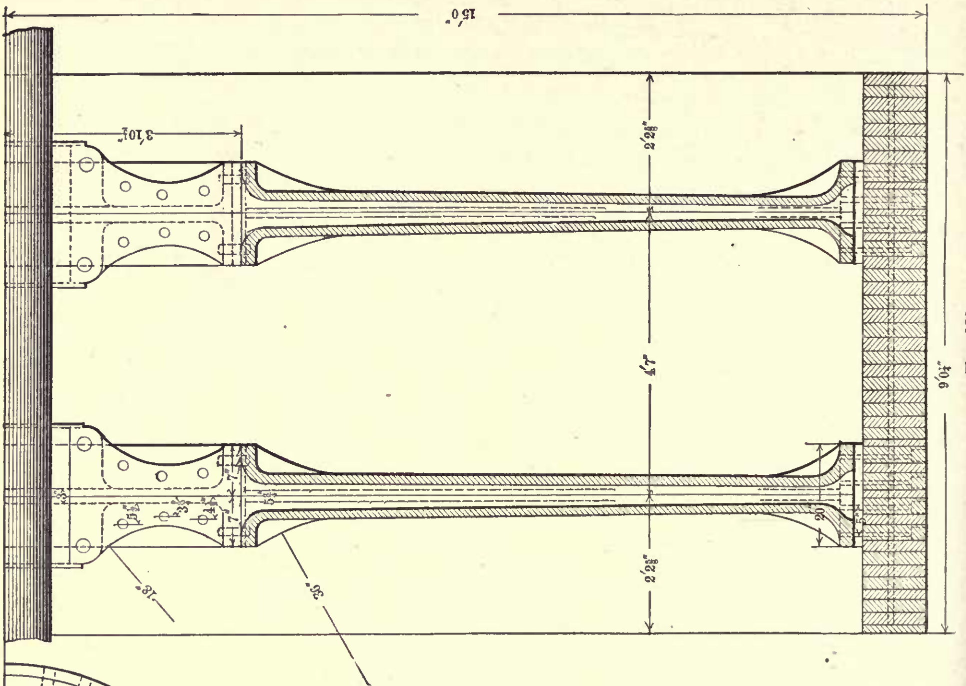


FIG. 983.

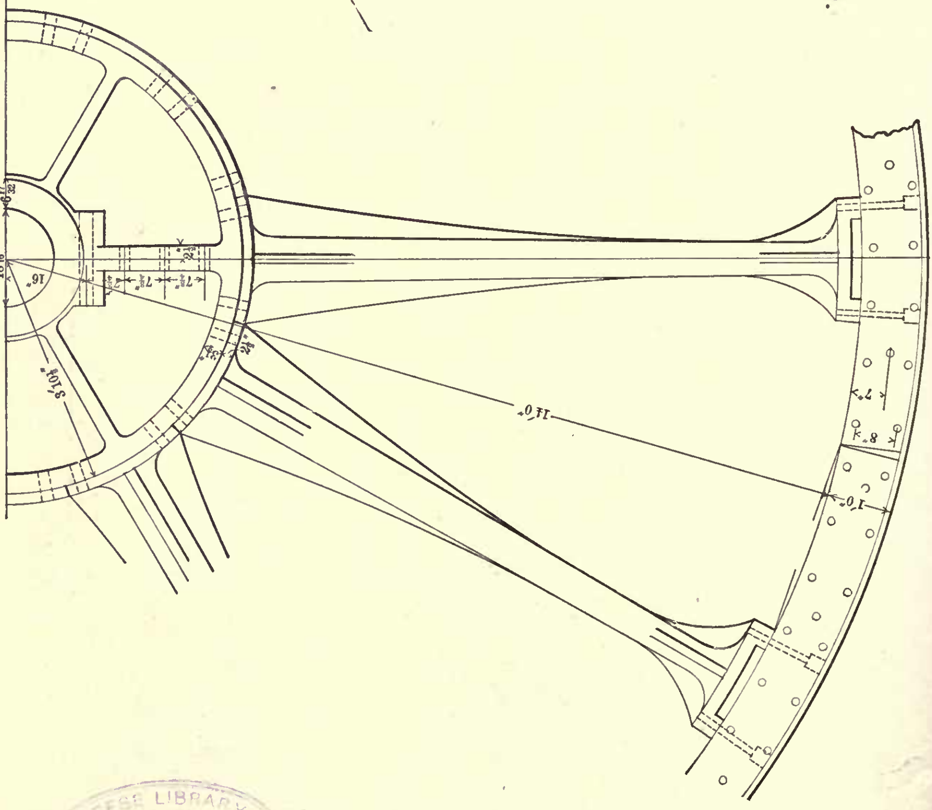


FIG. 982.

connecting the segments, shown on a larger scale (Fig. 981), do not project, but are countersunk in the sides of the rim.

The cast-iron fly-wheel of a steam engine 36″ × 72″, built for the Amoskeag Manufacturing Company in 1883, 30 feet diameter, 110″ face, with one set of 12 arms, and total weight 116,000 pounds, making 61 revolutions per minute; exploded in 1891, and was replaced by a wooden-rimmed pulley with two sets of arms (Figs. 982 and 983). The counterbalance of the cranks and connecting-rods was obtained by placing heavy cast-iron plugs in the outer ends of the three arms directly opposite each crank. Though the total weight of the wheel is not much less than that of the old one, the weight of the rim (31,855 pounds) is only about one half, but has shown itself ample for a very steady speed at much less cost and greater security.

Fly-wheels, and in fact all wheels that have a rapid motion, should be balanced, and if driven by cranks their connections should be balanced dynamically to make the motion as uniform as possible.

Air-Chambers.—The action of the air-chamber is very similar to that of a fly-wheel; it tends to make the outflowing or inflowing pressure of the fluid uniform, and cushions or prevents the reaction that takes place from the fluid in reciprocating pumps, especially crank-pumps; but pumps in which the pistons or plungers start very slowly and stop equally so require but little air-chamber. Cornish engines are usually provided with a stand-pipe instead of an air-chamber—that is, a vertical pipe of considerably larger diameter than that of the pump, and high enough to contain the water-column.

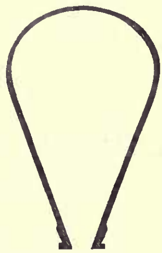


Fig. 984.

Fig. 984 is the section of a copper air-chamber for the smaller size of steam or hand-pumps. It is screwed into the top of the pump-chamber. Fig. 985 is the elevation of an air-chamber for power pumps of larger size of cast-iron or a cast-iron base with a copper chamber. A flange is cast on the top of the pump-chest, and the chamber is bolted to it.

Fig. 986 is the elevation of an air-chamber of one of the older Brooklyn pumping-engines.

The lower end of the small air-chamber (Fig. 984) is necked, or of smaller diameter than the main part of the chamber. This prevents a too sensitive reaction of the air and retards its escape; for the same purpose a diaphragm perforated with holes is put across the inside of the chamber. When the inlet column is long, whether suction or under pressure, put an air-chamber on it.

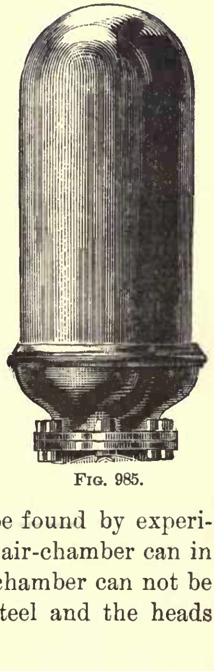


Fig. 985.

Air-chambers should be from ten to fifteen times the capacity of the pump-cylinder, with glass gauges to show the quantity of air in them for large pumps, and some provision to supply and maintain the air at such levels as will be found by experiment suited to the easiest working of the pump. A large air-chamber can in this way be reduced in capacity, while that of a too small chamber can not be increased. Air-chambers are made of wrought-iron or steel and the heads are bumped up or dished. The thickness of the cylindrical part is to be determined by the rules on riveting (page 383). The dished head, struck up on a spherical radius (Fig. 987) equal to the diameter of the cylinder and of the same thickness of plate, is of equal strength.

In the earlier application of water to the distribution of power the air-chamber held the reserve of force, but since the use of power in this form has become of general application, the accumulator (Fig. 988) is the form adopted, in which the pressure of the water in the pipe A raises the piston B from which is suspended the dead weight C sufficient to maintain the pressure required. For the distribution of power throughout the city of London the pressure is about 700 pounds to the inch. At the Forth Bridge Works there are two forms of accumulators in use through which the high-pressure water is pumped; in one form a 16″ cylinder is loaded with dead weights, in the other an 8″ cylinder is loaded with steam.

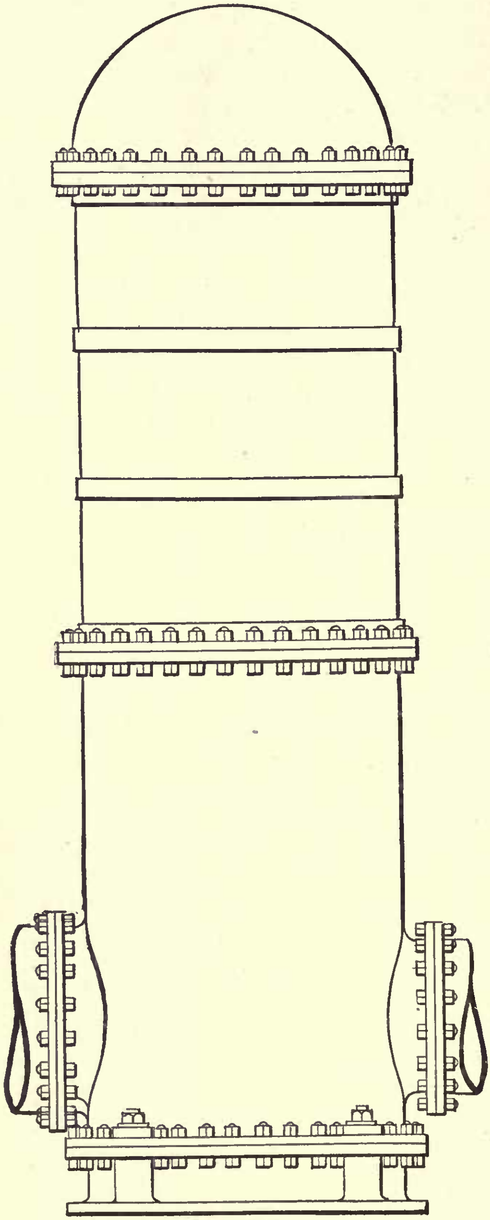


Fig. 986.

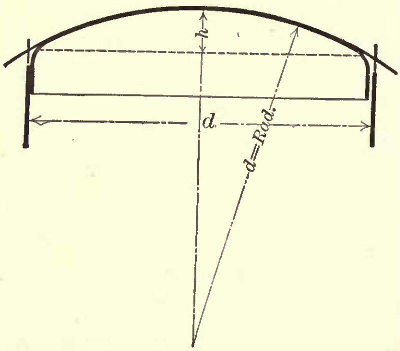


Fig. 987.

To make a hydraulic riveting machine that could be introduced into some of the more complex parts of the structure, it was necessary to increase the pressure of the riveter and reduce its dimensions. This was effected by the multiplier (Fig. 989), in which the usual high pressure is introduced into a large

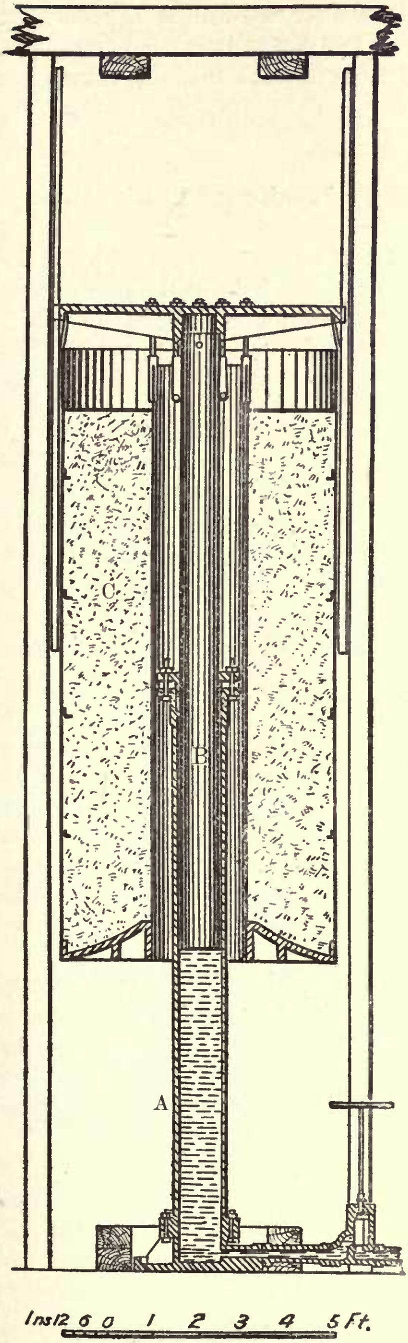


FIG. 988.

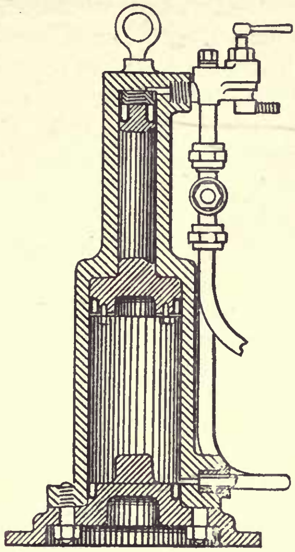


FIG. 989.

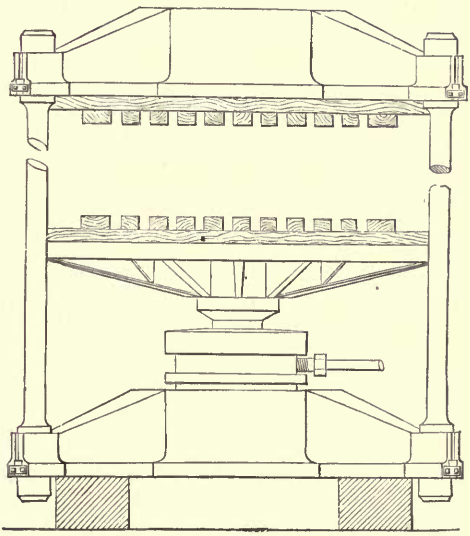


FIG. 990.

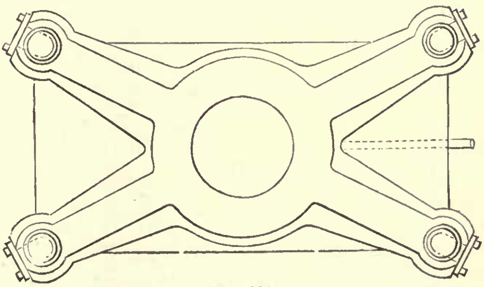


FIG. 991.

cylinder with its piston connected with a plunger of smaller diameter, and the pressure from the smaller cylinder is connected with the riveter.

For hydraulic cylinders the common rules are for cast-iron to a pressure up to 2,000 pounds per square inch and cast-steel to 6,000 pounds per square inch, rarely exceeding 8,000 pounds. For the resistance of thick shells Rankine gives the bursting strain—

$$p = f\,\frac{R^2 - r^2}{R^2 + r^2},$$

in which f is the tenacity of the metal, R and r exterior and interior diameter respectively.

Figs. 990 and 991 are the elevation and plan of a common form of hydraulic press for the baling of goods. The dimensions of the bolts at the four corners should be estimated from the hydraulic pressure, with a factor of safety of 5.

As tools the hydraulic punch, riveter, and jack are in general use.

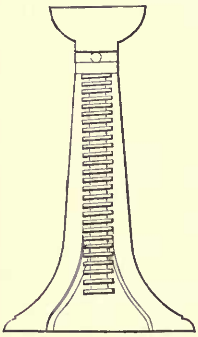


Fig. 992.

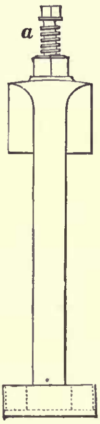


Fig. 993.

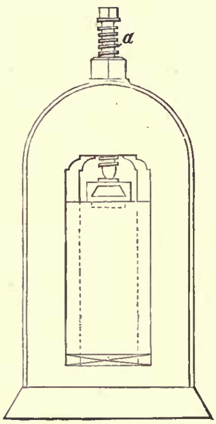


Fig. 994.

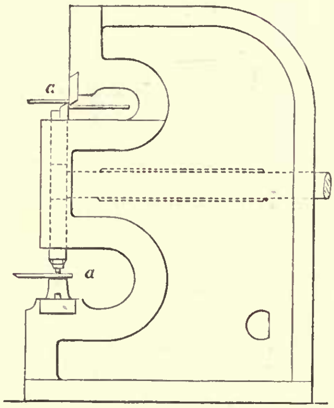


Fig. 995.

Fig. 992 is the section of a common jack-screw in which the screw is turned by a lever of which the hole beneath shows where its end is inserted. The whole weight is taken by the extended base.

Figs. 993 and 994 are side and end views of a cast-iron housing; the screw exerts a pressure on the roll box journals to be resisted by the frame.

Fig. 995 is the side elevation of a cam punch and shear, the action being to spring the jaws, which are reinforced.

The above drawings of machines, not shown elsewhere in the work, are given as illustrations of forms adapted to the stresses for which they are designed.

ENGINEERING DRAWING.

THERE is no part of engineering more important than that of securing a good foundation for the structure. Where likely to be disturbed by frost, the structure should start below it, unless, as in the extreme northern regions where frost is permanent at certain depths, the support should be in it. In preparing the foundation for any structure, there are two sources of failure which must be carefully guarded against: viz., inequality of settlement, and lateral escape of the supporting material; and if these radical defects can be guarded against, there is scarcely any situation in which a good foundation may not be obtained. It is therefore important that, previous to the commencement of the work, soundings should be taken to ascertain the nature of the soil and the lay of the strata, to determine the kind of foundation; and the more important and weighty the superstructure, the more careful and deeper the examination. But it must be understood that in general it is not an *unyielding* but a *uniformly yielding* stratum that is required, and that a moderate settlement is not objectionable, but an inequality of settlement.

In good sand or gravel, the load on foundations per square foot is usually from three to five tons. Many soils are very compressible, not supporting one ton per square foot; if the structure is important, the bearing resistance of the strata should be tested by experiment. The base of the wall is extended to secure the requisite area of bearing-surface, either by a base-stone (Fig. 996), by a bed of concrete (Fig. 997), or by extending the wall by steps (Fig. 998),

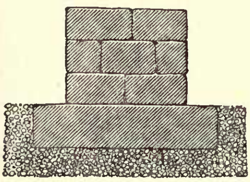


FIG. 996.

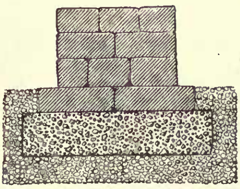


FIG. 997.

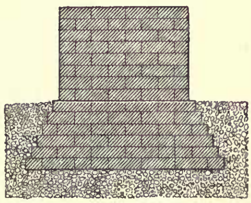


FIG. 998.

with or without concrete base, or the weight may be distributed by inverted arches between walls and piers.

Wooden platforms are often used for foundations, but must be laid beneath the water line where they are kept wet, otherwise rot will take place. These platforms may be of a single course of plank or plank and timber, as in Fig. 999, or may be very much extended, forming rafts or grillages of many courses. A similar foundation has been introduced in Chicago, where the great depth of clay requires extended areas of foundation, but instead of timber it consists of a combination of masonry with common rails or I-beams, affording greater length of offsets and less depth than by timber or masonry. The length of offsets may be calculated from the insistent weight, by the formula of beams fixed at one end and uniformly loaded or one quarter that given in the tables (pages 243 and 244).

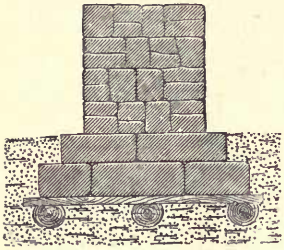


FIG. 999.

Fig. 1000 is a section of one of these foundations beneath a pier, but equally applicable to the support of walls. The space between the beams is filled with cement, mortar, or concrete, which adds to the stiffness of the structure and is a preservative of the metal. The iron grillages for piers are distinct, and each proportioned in bearing-surface to its proposed load. In astronomical observatories it is especially necessary that the foundations for the large and

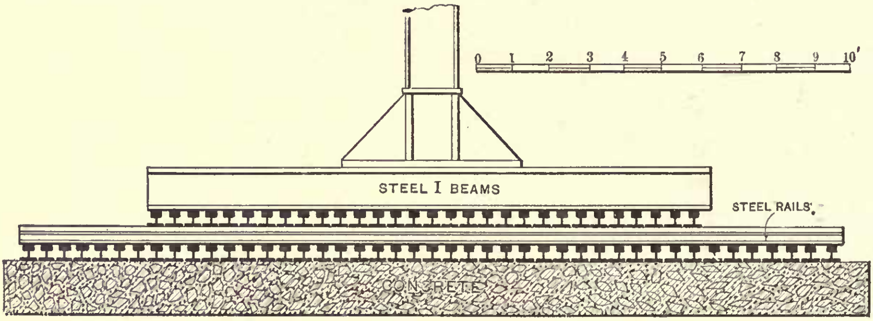


FIG. 1000.

delicate instruments should be detached from those of the building, and also where the noise and jar of the machinery might interfere with the occupancy of the building for other purposes.

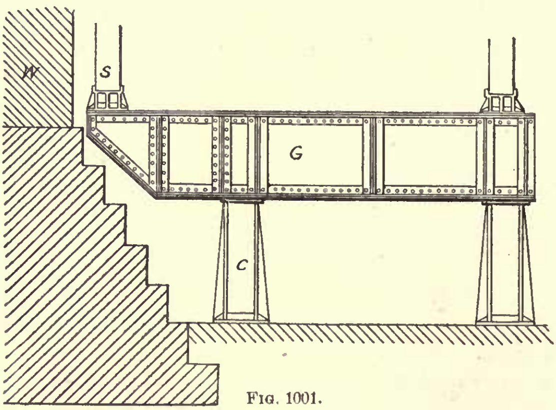


FIG. 1001.

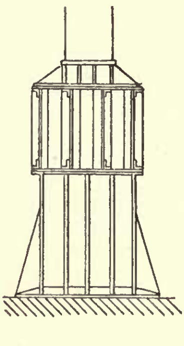


FIG. 1002.

Walls on party lines and confined to these lines are often partially supported by cantilever beams reaching over interior posts. Such a construction is shown in Figs. 1001 and 1002.

It is the safer construction to lay walls in air and open to inspection, and therefore important that their foundations should be freed from water, which can be done by inclosing them with a bank of earth or by a curb of sheet-piling (Figs. 1003 and 1004). Sheet-piling is usually of plank two to three inches thick, set or driven. For driving, the bottom of the plank should be sharpened to a chisel-edge, a little out of centre toward the ranging timber side, and cornered slightly at the outer edge, that it may hug the timber and the plank while being driven·

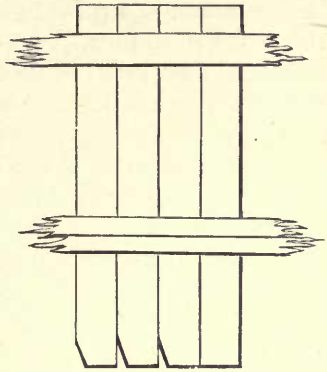


Fig. 1003.

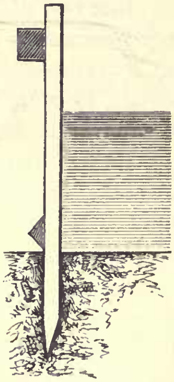


Fig. 1004.

Fig. 1005 is the section of a timber sheet-piling, in which a tongue and groove forms the guide, the grooves being either made in the timber, as shown at *a a*, or planted on, *b b*. The pile should be of uniform thickness, but the widths may be random; six inches thick is a good practical thickness, driving well under short and frequent blows of a ram; the tongue should be of hard, straight-grained wood, 2 inches by 2 inches, and well spiked to the pile.

Frequently, to secure the foundation from water, a wall is constructed of two rows of sheet-piling, driven one within the other, and the space between the two filled with clay or some compact earth. This is called a *coffer-dam;* the two rows of piling are stayed to each other by bolts, and if the wall is wide enough no other stays or braces will be necessary.

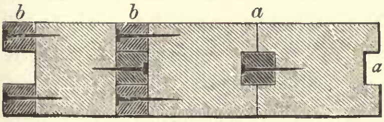


Fig. 1005.

Pile-drivers are now constructed to drive sheet-piling in panels of 6 to 8 feet wide, which serves to preserve the line and make tight joints.

In quicksands it is very common to secure a foundation by consolidation with small stones or rubble worked down by iron bars or driven by rams of a pile-driver, till sufficient resistance is secured for the structure. Some successful experiments have been made in compacting such sands by forcing down cement mortar through pipes. In soft earths piles are generally used for this purpose, and if a firm bottom can be secured at a reasonable depth they are the most economical expedient.

Piles are used either as posts or columns driven through soft earth to a hard bottom, or depending on their skin resistance to give the necessary support, either in earth naturally compact or made so by the driving of the piles. In the first case care must be taken that the piles be driven sufficiently deep into the lower strata to secure their ends from slipping laterally, and soundings should be made carefully to ascertain the dip and character of these strata. In many places, from the hardness and the inclined position of the lower strata, this kind of foundation is inapplicable and unsafe.

For a foundation where no firm bottom can be found within an available depth, piles are driven, to consolidate the mass, a few feet apart over the whole area of the foundation, which is surrounded by a row of sheet-piling to prevent the escape of the soil; the space between the pile-heads is then filled to the depth of several feet with stones or concrete, and the whole is covered with a timber platform.

It is very difficult to establish a rule of general application for the load which a pile will sustain. It is well in untried soil to drive a few piles, noting the settlement under blows, and then load the piles in excess of what they will be required to bear, noting the results from time to time; and if settlement continues, drive deeper, or more piles with less spaces.

Major Saunders, in the "Journal of the Franklin Institute," gave a rule for the safe load of piles depending on the skin resistance, which has been of general application, but of which the factor of safety is unnecessarily large, and is given below modified to 4 instead of 8.

Multiply the weight of the ram in pounds by the distance in which it falls in inches at the last blow, and divide the product by four times the depth driven in inches at this blow.

Weight of ram 2,000 pounds, fall 15′ or 180″, set $1\frac{1}{2}''$.

$$\text{Safe load} = \frac{2000 \times 180}{1\frac{1}{2} \times 4} = 60{,}000 \text{ pounds.}$$

In driving piles the effect of the ram should be carefully noted, the rate of fall under successive blows, the brooming or splitting of the head of the pile, and the rebound of the ram after the blow. Especial note should be taken toward the close of the driving to determine the last set when the formula above given is used.

The usual weight of the ram or hammer employed on our public works varies from 1,400 to 2,400 pounds, and the height of leaders or fall from 20 to 35 feet; but there is a great advantage in reducing the fall, increasing the weight of the hammer, and the frequency of the blows. As generally driven, and of average size, when the whole weight is to be supported by the pile, ten tons may be considered a usual load, but when additional support is received from compacted earth, broken stone, or concrete between piles and caps, this bearing surface should also be taken into consideration.

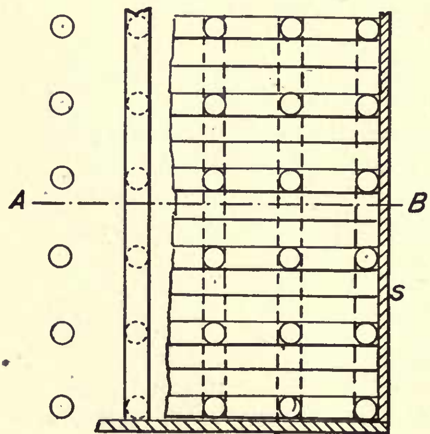


Fig. 1006.

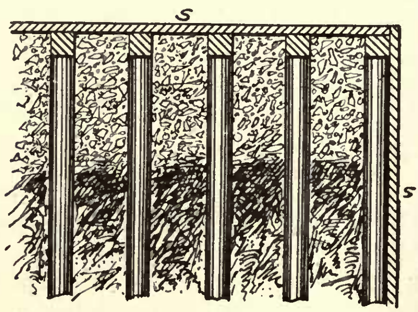


Fig. 1007.

Figs. 1006 and 1007 represent plan and elevation of a pile foundation; the piles are usually from 10″ to 14″ top diameter, and driven at about 3 feet between centres. The tops are cut off square and capped with timber the caps treenailed or ragbolted to the piles, and plank spiked to the timber. In the figure a sheet-piling, *s s*, is shown, inclosing the piles; the spaces between piles and timbers are often filled with concrete, small stone, or closely packed earth.

In compacting some soils it has been found that good results may be obtained by drawing the pile and filling the hole with sand. It would seem that the best result would be obtained by forming these piles of a uniform taper downward, as the consolidation would be more uniform, the withdrawal easier, and less disturbance of the sides of the hole. The consolidation might be still further increased by ramming the sand in thin layers, owing to the ability of the latter to transmit pressure laterally. The sand should be fine, sharp, clean, and the grains of uniform size.

It is often difficult to obtain or drive piles of sufficient length, and they must be spliced. After the first pile is driven, its head is cut level; a wooden dowel or iron pin, penetrating each pile about a foot, is inserted in the centre; the upper pile is then fitted to the lower one, or a cast-iron collar or ring of plate iron 6″ to 12″ wide may be used to strengthen the joint and protect the pins from splitting either head of the piles.

Spliced in this way, the pile is deficient in lateral stiffness. In most positions it is safer to re-enforce the splice by flatting the sides of the piles and nailing on with, say, 8-inch spikes, four pieces 2 or 3 inches thick, 4 or 5 inches wide, and 4 to 6 feet long. In the erection of the bridge over the Hudson River at Poughkeepsie, N. Y., two piles were thus spliced together to form a single one 130 feet long.

Piles may be made of any required length or cross-section by bolting and fishing together, sidewise and lengthwise, a number of squared timbers. Hollow-built piles 40 inches in diameter and 80 feet long were used as guide-piles in constructing the St. Louis bridge. To protect the head of the pile against brooming or splitting it is usual to drive on a tight-fitting wrought-iron ring or cap (Fig. 1008) by a blow of the hammer. In driving into compact gravel or shingle the point of the pile should be protected with iron straps, as shown in Fig. 1009.

Fig. 1008.

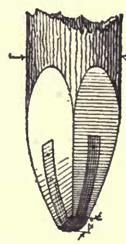


Fig. 1009.

Fig. 1010 is a section of a bulkhead wall on the North River front, in positions where the mud is deep, as designed and constructed by George S. Greene, Jr., Chief Engineer for the Department of Docks, New York city.

The site of the wall is first dredged to hard mud compacted with sand. The vertical piles are then driven, and small cobble-stones mixed with coarse gravel put around and among the piles to the height of the under side of the binding frames, and rip-rap stone placed outside the piles, in front and rear.

The binding frames, then slid down to their places, were made of two pieces of spruce plank 5 × 10 inches, placed edgewise one over the other, and running from front to rear of the piles between the rows. An oak beam 8 × 8 inches is let through these planks in front of the front row and in rear of the rear row of piles, and an oak wedge block fitted and placed by the divers between the oak beam and each pile nearest it.

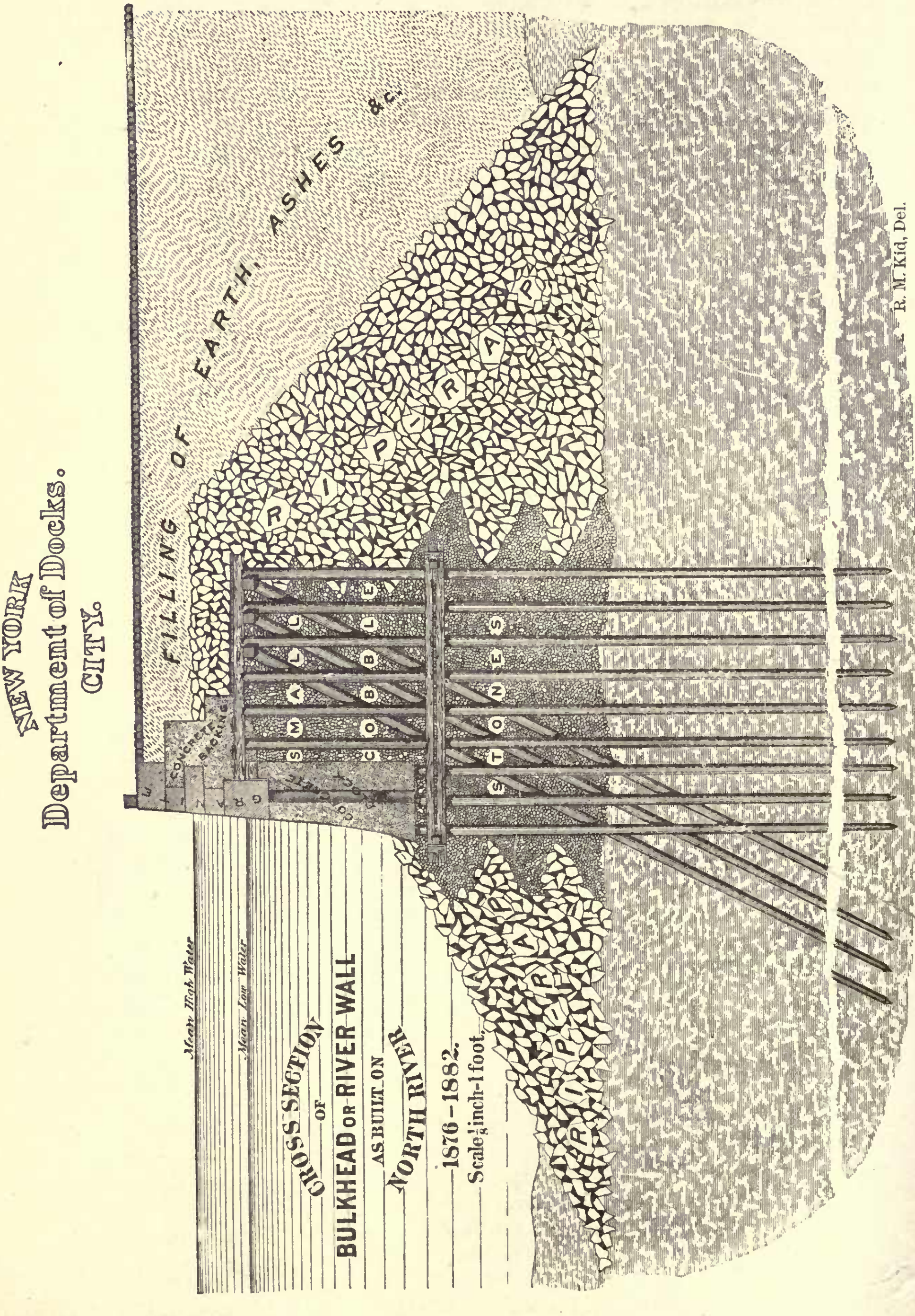

NEW YORK
Department of Docks.
CITY.
FILLING OF EARTH, ASHES &c.
RIPRAP
SMALL COBBLE STONES
CONCRETE BACKING
GRANITE
CONCRETE BLOCK
Mean High Water
Mean Low Water
CROSS SECTION OF BULKHEAD OR RIVER WALL AS BUILT ON NORTH RIVER
1876–1882.
Scale 1/8 inch=1 foot.
R. M. Kid, Del.

FIG. 1010.

These frames hold the front rows of piles firmly, in case there should be any tendency in them to tilt outward. More cobble-stone is then put in to the height of the bottom of the base blocks of the wall, weighting the binding frames and preventing any tendency to floating.

The bracing piles are then driven on a slope of 6 inches horizontal to 12 inches vertical, between the rows of vertical piles, and spaced 3 feet from centre to centre longitudinally and transversely. All the piles are staylathed and adjusted in position as soon as they are driven.

The bracing piles are cut off at right angles to their axis, about 1 foot below mean low water, and capped with 12-inch square timber, running longitudinally. The sides of the caps are kept horizontal and vertical, and a sloping recess or notch made to receive the head of each bracing pile, and give it a good bearing.

The six rear rows of vertical piles are cut off at 2 inches above mean low water, and notched front and rear to give an 8-inch-wide bearing across their tops for the transverse caps.

The three front rows of vertical piles are cut off by a circular saw, suspended in the ways of a pile-driver, at 15·3 feet below mean low-water mark, to receive the concrete base blocks of the wall. It being impossible to cut off piles at this distance below the surface of the water to exactly the same height, and as the bottom of the concrete base-blocks would rest only upon the highest piles of those under them, a mattress of burlap, containing freshly mixed soft mortar, in a layer about 2 inches thick, placed on a network of marline stuff, supported by a plank frame about its edges, is lowered upon the tops of these piles immediately before setting the base-blocks upon them. The diver then cuts the netting between the edge of the mattress and the plank frame, and the frame floats to the surface of the water.

The base-block is then immediately placed in position upon the mattress of mortar resting on the piles, and the excess of mortar is pressed out from between the head of the pile and the bottom of the base-block, until each pile has a well and evenly distributed portion of the load to carry.

The concrete base-blocks for this section are 7 feet wide at the bottom and 5 feet wide at the top; on the front the vertical height is 13 feet, and on the rear 14 feet. The top has a step on the rear of 1 foot height and 1½ foot wide, extending the entire length of the block, for the purpose of giving the mass concrete backing of the granite superstructure a good hold upon the block. For handling, grooves for chains are moulded in the end, and a longitudinal hole, 2 feet in the clear above the bottom, connects them, with the corners rounded, to enable the chain to render easily. The face is curved inward, to save material while giving a broad base; their length is 12 feet.

After the blocks are set, the vertical chain-grooves in each block, coming opposite to each other, are filled in with concrete in bags, well rammed into place. This closes the joints between the blocks, and also acts as a tongue set into the grooves in the blocks.

Fig. 1011 shows a block in section with the central- and side-groove spaces, which are deep enough to admit of the easy slipping in and withdrawal of the chain.

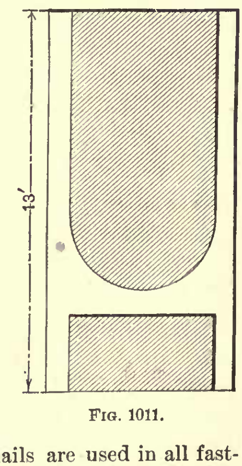


FIG. 1011.

As soon as the base-blocks are set, and the groove filled in, the cross-caps resting on the tops of the vertical piles, and on the longitudinal caps of the bracing piles, reaching about half way across the base-blocks, are placed and fastened. Oak treenails are used in all fastenings. The small cobbles are then filled in around and among the piles to the top of the caps, and the rip-rap placed in the rear of them.

Figs. 1012 and 1013 are the elevation and plan of a crib with dock or pier. Below the level of the water, as here shown, the logs are round and locked

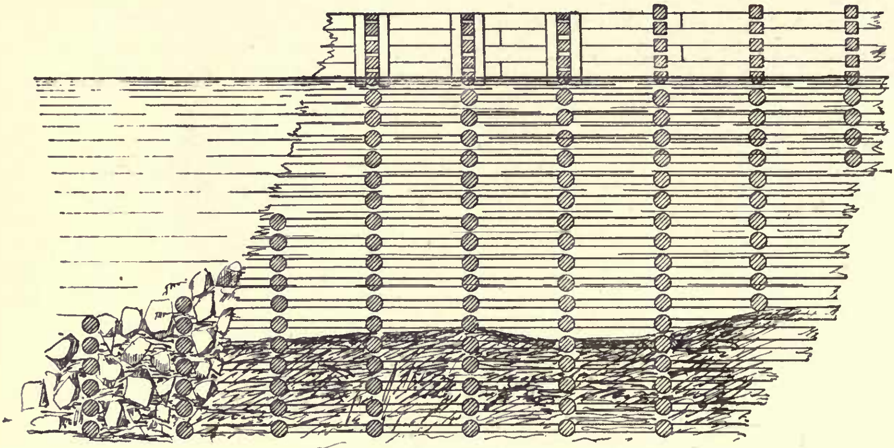


FIG. 1012.

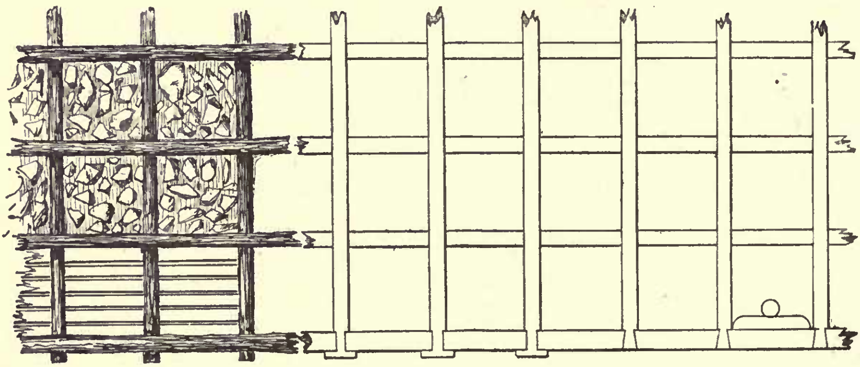


FIG. 1013.

to the cross-timbers; above the water the timber is squared, the exterior walls presenting a tight, smooth surface into which the cross-timbers are dovetailed.

A quarantine station, built for the port of New York, on the west bank consists of an exterior wall of cribwork, of which Fig. 1014 is a section; the inclosed space is filled with sand, comprising an extent of about 228′×448′. The base to low water consists of cribs about 80′ in length, sunk, and loaded with stone; above it the construction is continuous. The base timbers 14″ × 14″, upper 12″ × 12″, interties 12″ × 12″, at intervals of 7 feet centres; the ranging timbers to be secured at every joint to these below, and at every crossing by 1″ square bolts 21″ long. The exterior is close-fendered with oak plank, iron bolted, with three iron bands at the corners.

Fig. 1015 is a transverse section of the river-wall Thames embankment, Middlesex side; a wall of concrete, etc., faced with granite, with a sewer and subway within the same, lined with brickwork. The different material is represented by different shadings and letters: *g*, granite; *b b*, brickwork; *c c*, concrete.

Extracts from specifications:

"The embankment-wall is to be formed within iron caissons or coffer-dams. As soon as the excavations shall have been made to the requisite depths, and the works cleared of water, the trenches shall be filled up with concrete to a level of 12½ feet below datum, and a bed dressed to the proper slope and level for the footings of the brick wall. This wall to be formed thereon, generally in courses at right angles to the face of the wall. The subway shall be formed 7 feet 6 inches high by 9 feet wide in the clear, generally; the side-walls to be 18 inches, the arch 1 foot 1½ inch thick. The subway sewer and river-wall shall be tied into each other, at intervals of 6 feet, by cross or

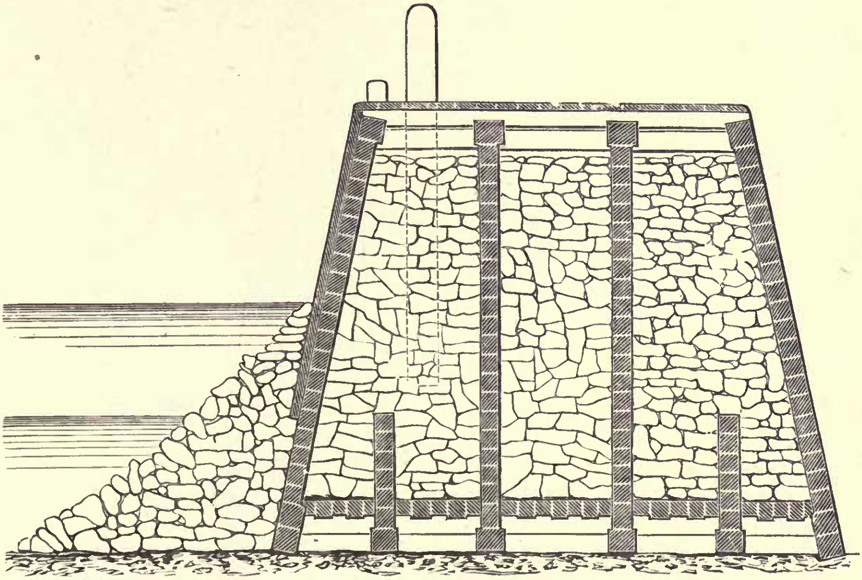


FIG. 1014.

counterfort walls 18 inches thick, extending from the brickwork of the wall to a vertical line 9 inches beyond the side of the sewer farthest from the said wall, and from footings 9 feet below datum, which are to be bedded on a concrete foundation 12 inches thick, up to the under side of the subway. The upper arch of the subway, and all other similar arches, shall be coated on their outside circumference with a 1″ layer of Claridge's patent Seyssel asphalt. The whole of the stones above 11½′ datum to be dowelled together in bed and joints with slate-dowels, not less than 5 for every foot run of wall; each 2½ inches square at least, let fully 2½ inches into each stone, very accurately fitted, and run in with neat cement; the stones to be bedded and jointed in cement, and the joints struck with neat cement."

Fig. 1016 is an isometrical view of the overflow and outlet of the Victoria and Regent Street sewers in the Thames embankment. S is the main sewer, and W the subway shown in Fig. 1015; *s s s* the street-sewers, discharging into the overflow basin O; *w w* the weirs over which the water is discharged into the weir-chamber *c c*; *p* is the penstock-chamber, which is but a continuation of the weir-chamber. Whenever, from storms, the discharge from the street-sewers (*s s s*) is greater than can be carried off by the main sewer (S), the water rises in the overflow-chamber (O), passes over the weirs (*w w*) down into the weir-chamber (*c*), then into the penstock-chamber, and through the flap-gates (*g*) into the river.

Extracts from the specifications:

"The foundation to be of concrete, not less than 2 feet in thickness; upon this brick-work shall be built for the flooring of

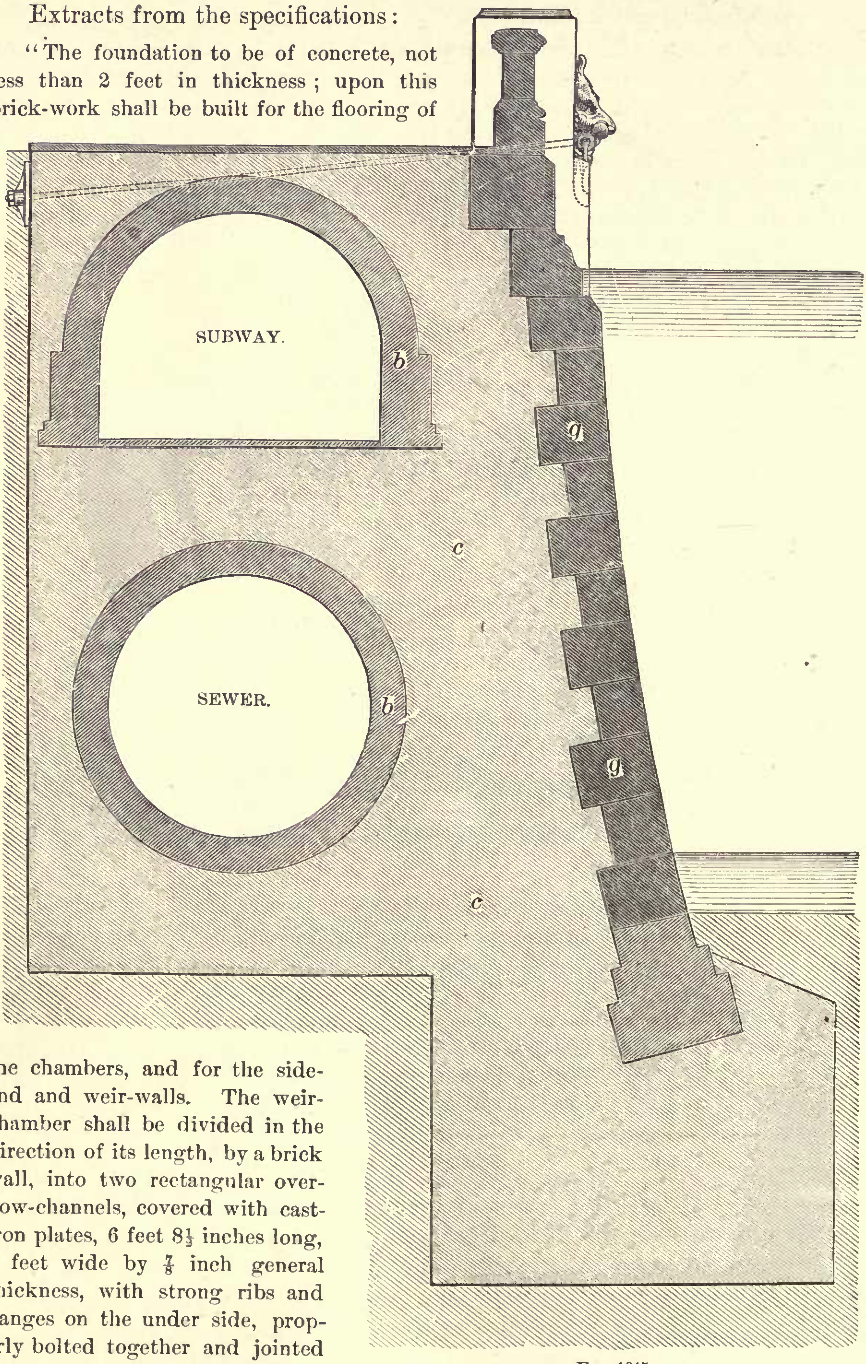


FIG. 1015.

the chambers, and for the side-end and weir-walls. The weir-chamber shall be divided in the direction of its length, by a brick wall, into two rectangular overflow-channels, covered with cast-iron plates, 6 feet 8½ inches long, 3 feet wide by ⅞ inch general thickness, with strong ribs and flanges on the under side, properly bolted together and jointed with iron cement, and bolted down to stones which are to be built into the under side of the brickwork of the basement-chamber. Arches on either side, running parallel thereto, and communicating with this chamber and with the weirs which are to be formed, upon which weir-walls, divided so as to correspond with these arches, are to be built in brickwork, capped with granite

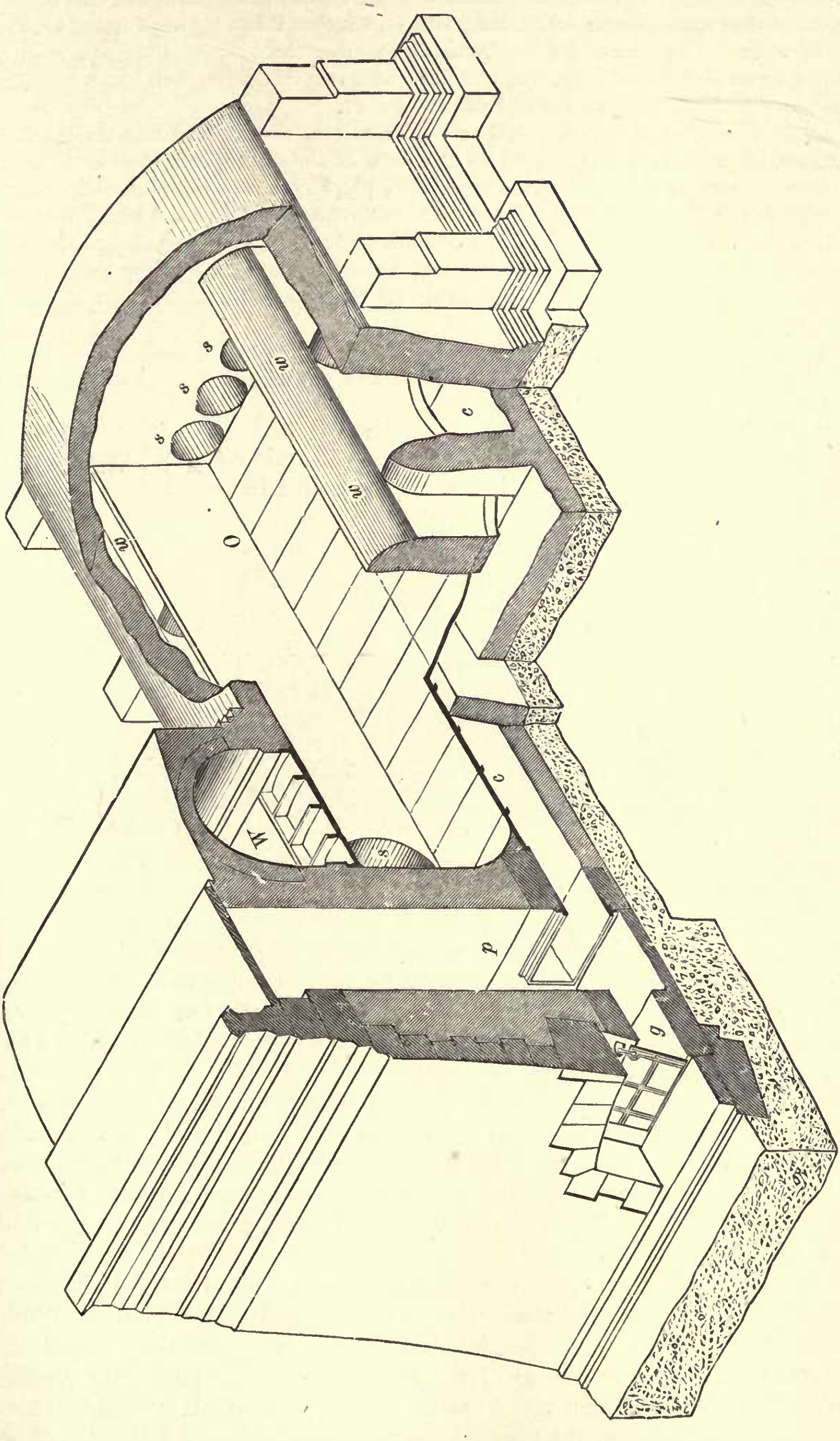


FIG. 1016.

blocks, 4 feet long, 2 feet deep, and 2 feet 3 inches in the bed. The floor of the penstock-chamber to be formed with York landings, 6 inches thick, having a fall of 3 inches to the river. The outlets for the penstock-chamber through the river-wall shall be formed by an arch-recess in granite, and fixed with two tidal flaps, well hung, and firmly secured to the masonry by strong bolts and screws.

"The subway is to be continued over the low-level sewer, and across the overflow-chamber, by cast-iron plates, curved to the form of the arch, ⅞ inch general thickness, with strong ribs and flanges on the upper side, properly bolted together, and strongly bolted down to the brickwork; jointed with iron cement, and covered with brickwork, to form the floor of the subway. From a point of 10 feet 3 inches on either side of the central longitudinal line of the chamber, where the sewer and subway are farthest from the river-wall, these are again to be brought into their general position by two curves, each not less than 80 feet in length.

"The whole of the cast-iron shall receive one coat priming of red lead and linseed oil, and three coats best coal-tar, before fixing; and the accessible surfaces one further coat best coal-tar, when fixed."

Fig. 1017 is the section of the dike or jetty forming a breakwater for the harbour of Boulogne, France. It may be considered in two distinct parts, cor-

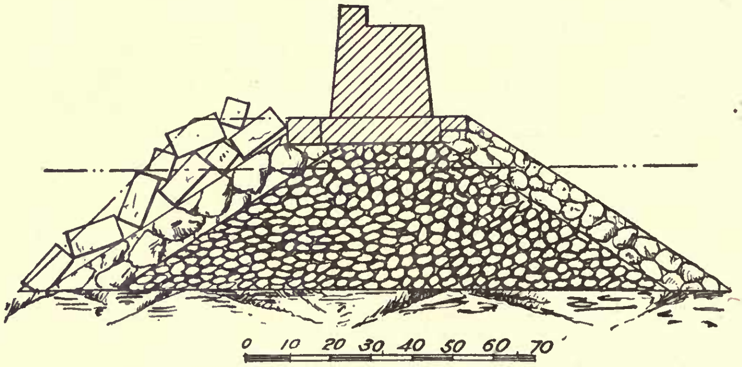


FIG. 1017.

responding to the substructure and the superstructure. The substructure is formed by a mass of natural and artificial rip-rap, with a central core of stones weighing about two hundred and fifty pounds each, resting on the bottom, and rising to a level of one metre above low tide. The shore side is protected by a pitching of stone, each about twelve hundred pounds weight; the sea-side slope, by one of heavy rubble of about seven tons each, and covered by beton-blocks weighing uniformly thirty-three tons each; on the above is built the masonry superstructure. On each side of the wall, on a level with the lower platform, the slopes are consolidated by masonry bermes formed of isolated blocks, which protect the foot of the wall and afford a path for the workmen and materials at low tide.

The *water-jet* is extensively employed on sandy shores for the sinking of piles for foundations of lighthouses, wharves, etc., and in the Southern States it renders the palmetto available, which resists the ravages of the teredo, but is too soft to withstand the blows of the pile-driver. In a simple and effective application it consists of a pile to which a small iron pipe is attached, extending below the bottom of the pile. A flexible hose is attached to the top of the pipe and, water being forced through, the earth is washed away from the bottom and side of the pile, which falls by its own or superadded weight or light blows of a maul or hammer.

The jet has also been employed in a great variety of ways to facilitate the passage of screw and disk piles, cylinders, etc., through earthy material, and as an ejector to remove earth from the inside of caissons, and relieving stranded vessels by removing the sand from their bottoms and sides.

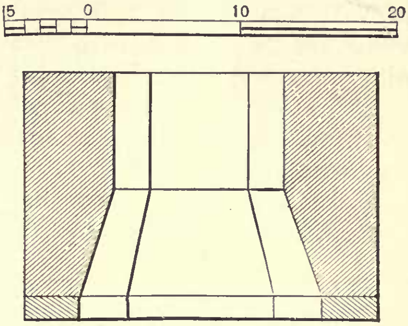


Fig. 1018.

Cast-iron or wrought-iron is used for piles, and, at the present prices of iron, with economy. They can be driven by the pile-driver, the interior earth removed by an augur, by sand-pump, or water-jet. The blows of the ram should be cushioned by a wooden block. For the construction of the iron pier at Coney Island, N. Y., to the bottom of the wrought-iron pipes $8\frac{3}{4}''$ in diameter, in lengths of from 12 to 20 feet, cast-iron disks were fastened by set screws and sunk by an inside water-jet to a depth of 17 feet in the sand.

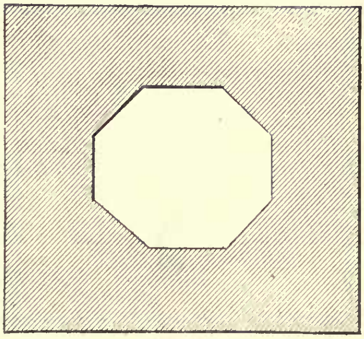


Fig. 1019.

In Chili iron piles were sunk $14\frac{3}{4}''$ diameter with bottom flange of 42″; the pump discharged 12,000 gallons per hour through two 2″ pipes extending 8″ below the base, and would sink two piles 28 feet on an average in 18 hours. The bottom was of coarse compact sand and the pile was worked down by an endless cable passing around a pulley on the pile and giving it a motion of rotation.

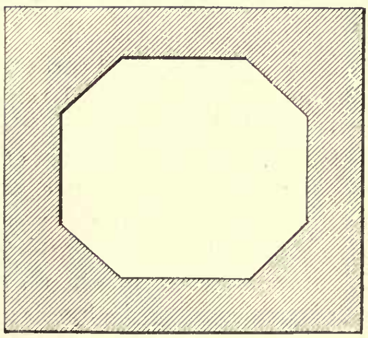


Fig. 1020.

The screw pile usually consists of a wrought shaft from 3″ to 8″ diameter to which is keyed a cast-iron screw from 2 to 5 feet in diameter sunk by turning the shaft by hand or power, mostly used for marine purposes, for the foundations of lighthouses, or anchors for buoys where they resist the upward motion of the waves. As they are sunk without jar or disturbance of the soil, they are adapted to positions where the neighbouring structure might be injured by other methods of sinking foundations.

Figs. 1018, 1019, and 1020 are sections of masonry curbs sunk by water-jets for a quay at Calais, France.

The base is of concrete made in a mould, the rest of the masonry laid in cement mortar.

The sand surface beneath the blocks was exposed to the action of strong jets, and the mixture of sand and water was pumped up by centrifugal pumps; the suction-pipe nozzle was just below the level of the bottom of the block; care was taken that the quantity of water forced in should be the same as that pumped out, and the level of the water in the curb should be just below that in the surrounding sand. When the curb had reached the bottom after a descent of about 15 feet, the sand was allowed to settle and the opening was filled with concrete up to a level where it could be filled with masonry.

The blocks 1, 3, 5 are sunk alternately, and then 2, 4, 6, the blocks being cemented together as shown in Fig. 1021.

It has long been common in India to sink brick wells in clusters by hand labour,

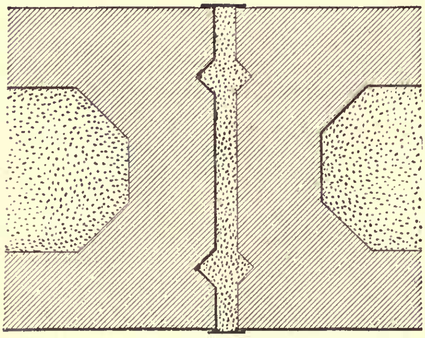


FIG. 1021.

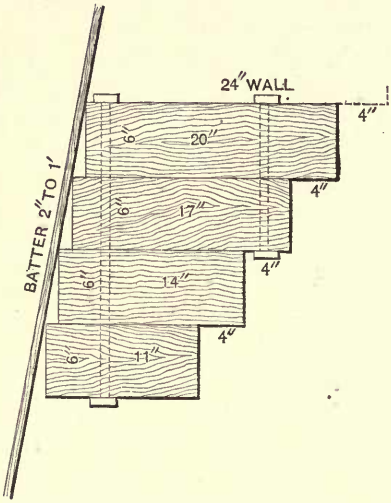


FIG. 1022.

excavating and pumping for the foundations of bridges, but by substitution of bucket-dredges to remove the inner cores of earth greater facility in work and depth of sinking has been secured. This system has been successfully applied to the sinking of iron caissons.

In the sinking of small curbs for wells it is common to make circular plank curbs supported by segmental ribs inside, and load them so that they will sink as the earth is removed from the inside and then lining with masonry.

Fig. 1022 is a partial section of a shoe of the 50-foot diameter well sunk at Long Island City, N. Y. It consists of timber segments bolted together on the top of which a brick wall was laid; on the outside is a board sheathing. As the earth is removed from the inside the curb settles from the weight of the brick masonry, which is built up and settles again and again till the required depth is reached. As the boards are set at a batter of 2″ to 1 foot, the structure settles readily as the earth is removed. Were it not for the batter the earth would press against the sheathing and masonry, and vertical bolts would be necessary in the latter to anchor the courses together and prevent rupture. The sheathing of boards is planed on the outside and firmly attached to the shoe with bound wooden segments above to preserve the form, which are removed as the brickwork reaches them.

Fig. 1023 is a partial section of a steel caisson, exhibiting the cutting edge re-enforced by steel plates and supported by beams to the bottom girder.

Foundations for the abutments of piers and bridges may be constructed on any of the systems illustrated, but as it is generally necessary that they should not extend in width so as to obstruct the current of the stream and increase its velocity, it is usual to inclose the site of the foundations within a coffer-dam, freeing it from water and preparing the foundation ; but this is expensive in deep water, and other means are adopted.

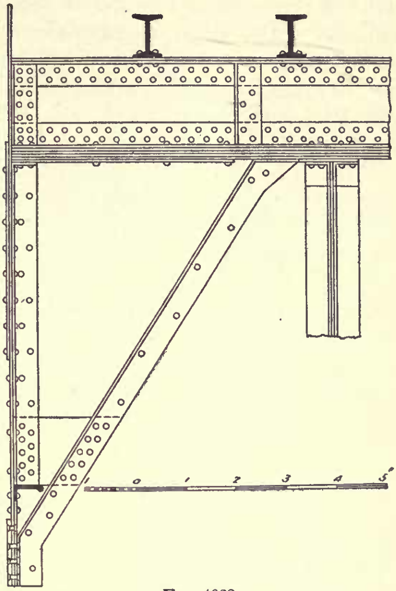


FIG. 1023.

Figs. 1024, 1025, and 1026 are plans and sections of the foundations of one of the Poughkeepsie bridge piers, which may be designated as a crib, although its walls are tighter than usual in such constructions.

It is built of 12″ × 12″ white hemlock timber, except the bottom or cutting course of the shoe, which is white oak. The shoe is carried up in a triangular section of solid lumber around the sides and a central longitudinal division to the height of 20 feet. Top surface at sides 10 feet wide, at ends 9 feet, central 16 feet; above the shoe there are hollow spaces. Cross-timber walls 2 feet thick divide the spaces into pockets, of which those above the shoe are weighting pockets and those open at the bottom are dredging pockets D. The earth is dredged from the pockets D and discharged into the pockets W, and acts as a weight to sink the crib, care being taken to distribute the load to equalize the sinking. As the earth is dredged from the pockets D the crib sinks, and when it reaches the bottom they are cleared out and the space filled with concrete lowered into place in boxes of one cubic yard capacity and unloaded by a trip.

The top of the crib was sunk to a level of 7 feet below water mark and the concrete brought to within 2 feet of this level; this space was filled with broken stone and levelled by divers; a floating caisson, or tight box with timber bottom and sides, was floated over the crib and sunk and the masonry begun. When complete above water the sides of the caisson was removed.

The Chinese anchors used in mooring the cribs of the Poughkeepsie bridge as shown in Fig. 1027 is composed of 3″ × 8″ hemlock planks 10 feet long piled to make the interior dimensions 6′ × 6′ × 6′, which is filled with broken stone, each anchor holding 8 cubic yards; the eye-bolts shown at the corners serve the double purpose of holding the framework together and carrying the slings to which the cable is fastened.

Open caissons, as shown in the description of the Poughkeepsie bridge, are useful when a suitable foundation can be secured either in a uniformly yielding material or by preparing one by means of piles or divers.

Figs. 1028 and 1029 are illustrations of the means adopted by G. A. Parker, C. E., in lowering the caissons for the erection of some of the piers of the Susquehanna bridge.

He commenced by dredging away as much as possible of the material in the bed of the river at the pier site. A $\frac{3}{8}$-inch-thick boiler-iron curb was then sunk and secured in place. The curb was about 30 feet wide and 50 to 60 feet long, and of sufficient height to reach above the bed of the river. The material was then pumped by sand-pumps out of the curb, which gradually undermined, and settled down to the established depth, or to the bed-rock. When stumps, logs, or boulders were met with they were removed by divers working in a bell. After the rock had been thoroughly cleaned off, it was brought to a uniform level by a solid bed of concrete extending over a greater space than the size of the bottom of the pier by the use of the diving bell.

Three guide-piles on each side, and one at each end, were fixed firmly in position. A strong platform of solid timber, the size of the bottom of the pier, was then placed in position over the curb and at the surface of the water. On this was placed a caisson of iron large enough to contain the pier, and with sides and ends high enough to reach to

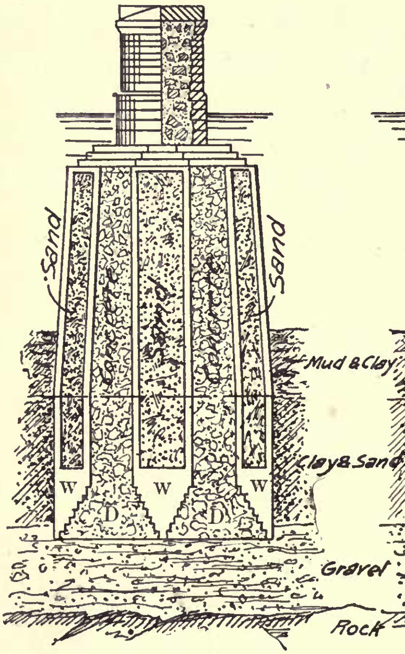


FIG. 1024.

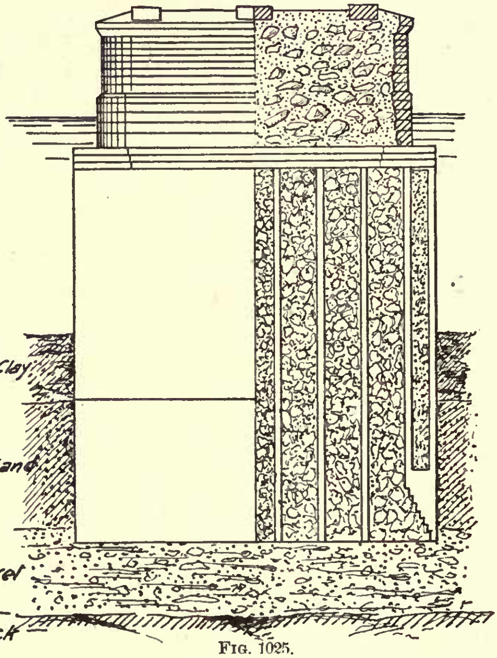


FIG. 1025.

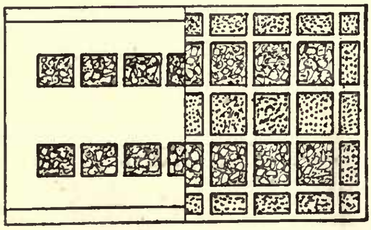


FIG. 1026.

the level of high water after the caisson was landed on the bottom. The caisson was then made water-tight. The bottom was then floored over with masonry and stone, and laid in mortar up the sides of the caisson to the top, thus constituting a stone caisson inside of an iron one. This was secured to the guide piles, and the masonry of the pier proper was laid up, the caisson sinking as the weight of masonry inside increased, until it finally settled upon the bottom which had been prepared for it. At some of the piers (Figs. 1028 and 1029) screw-rods were used to suspend the pier and gearing attached, governed by one man, who could raise or lower without assistance the whole pier. Wooden piles were driven for some piers and cut off by machinery just above the ground, and the platform, with its masonry, lowered upon them.

Piers are sometimes made by sinking a wrought-iron curb, extending from the bottom to above the level of the water, driving within it the usual proportion of piles, and then filling the spaces entirely with concrete. This process was adopted in forming some of the piers of the bridge of the Shore Line Railroad across the Connecticut River at Saybrook, under Cushing's patent.

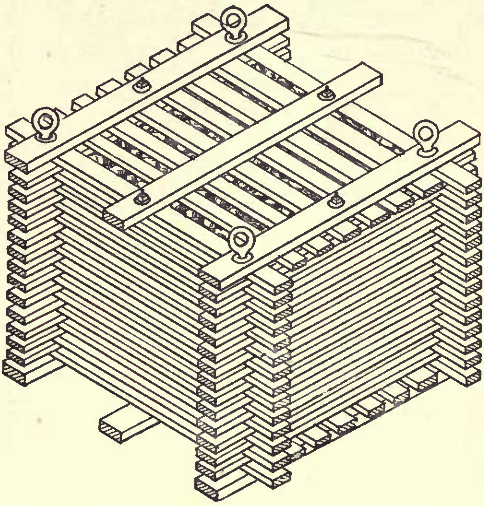


FIG. 1027.

Pneumatic Piles.—The vacuum process in which the hollow pile of cast- or wrought-iron was sunk, by capping it and exhausting the air within, and thus loading it with the pressure of the exterior atmosphere, gave place to the plenum process, which was adopted for the old piers of the Third Avenue bridge across the Harlem River, of which Fig. 1030 is a section. It conists of a pipe in two sections; the upper one is called the air-lock, which can be connected with either the lower chamber or the atmosphere. The lower chamber being under pressure and shut off from the air-lock, the workman passes from the outer air into the lock, closes the door, opens the pipe connection between the two compartments, and, when the pressure becomes equal, opens the lower door and passes down.

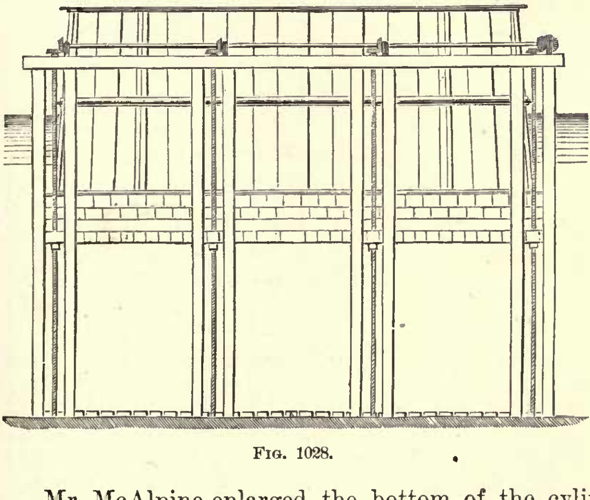


FIG. 1028.

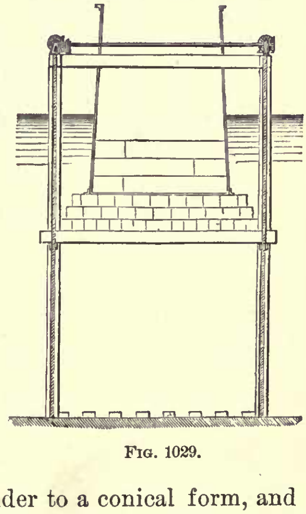


FIG. 1029.

Mr. McAlpine enlarged the bottom of the cylinder to a conical form, and also added largely to the bearing surface by poling-boards driven obliquely outward. After the excavation was completed, a strong course of concrete was laid, and, when set, the air-lock was taken off and the balance of the concrete filling was done in open air.

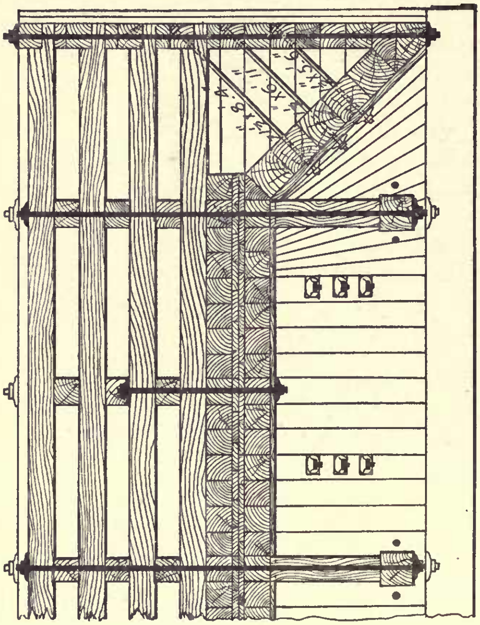


FIG. 1033.

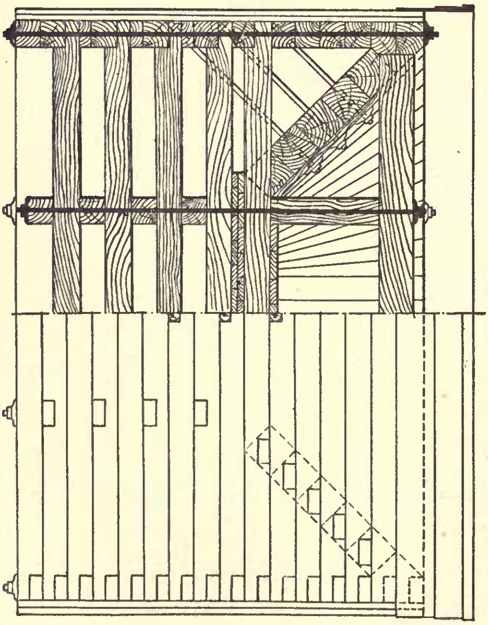


FIG. 1032.

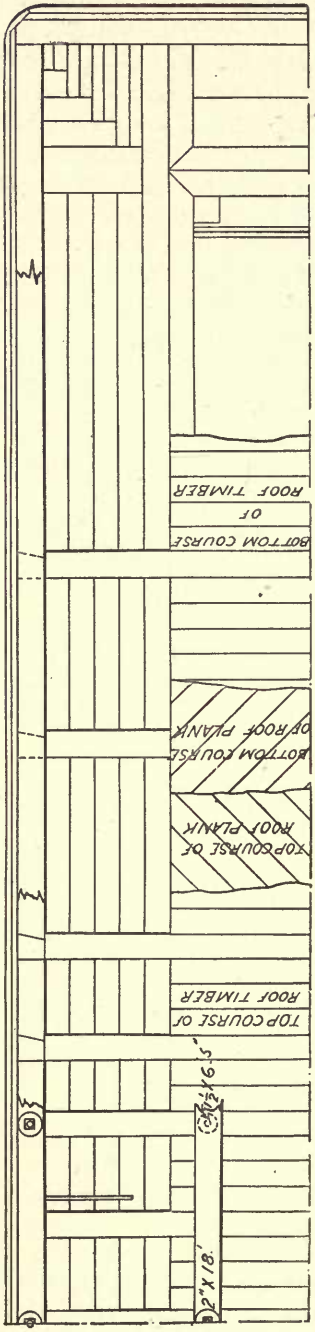


FIG. 1031.

The pneumatic caisson has superseded the pneumatic pile. The system of air-lock is the same, but, instead of sinking several piles and then combining them with a structure, the caisson embraces the whole pier and starts from the bottom. The pneumatic caisson is an inverted boat or diving bell of timber, which forms the working or air chamber, connected with the upper air by pipes and air-locked shafts for the ascent and descent of the workmen and for the removal of waste from, and the delivery of material into, the working chamber.

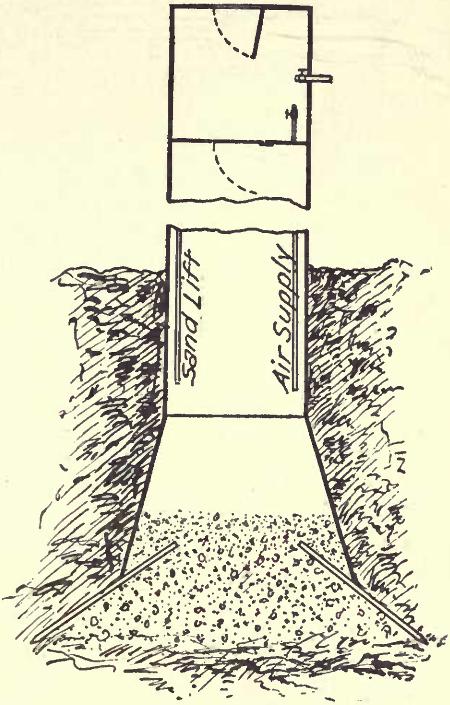


FIG. 1030.

Figs. 1031, 1032, and 1033 are the plan and partial sections of a late form of the framing of a caisson. The ceiling of the working chamber is of plank, and between the first and second courses of beams there is a double-plank floor, which are calked and then coated with pitch, and the spaces between the timbers filled with concrete.

Fig. 1240 is a section of the pier of the Bismarck bridge. The sand was removed from the air chambers by water ejectors. As it is removed from the chamber, the masonry sinks the caisson, and when it reaches the bottom the space is filled with concrete or with sand. If the top of the caisson, when first sunk, does not reach the surface of the water, a curb is formed on the top to a height sufficient to per-

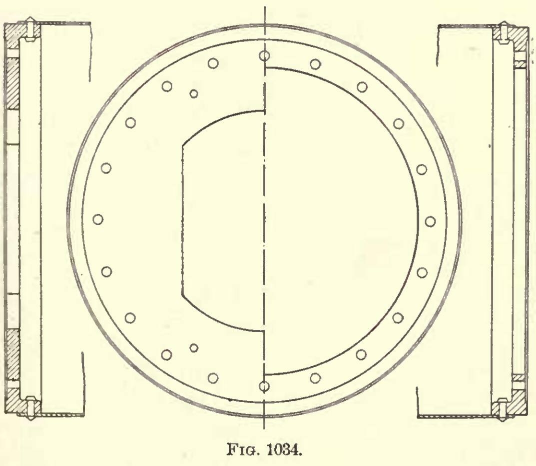


FIG. 1034.

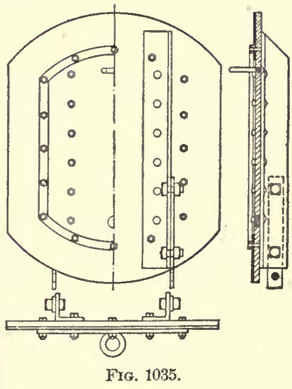


FIG. 1035.

mit the construction of the masonry in the open air. To preserve the position of the air-lock during the whole construction there is an offset in the pipe at

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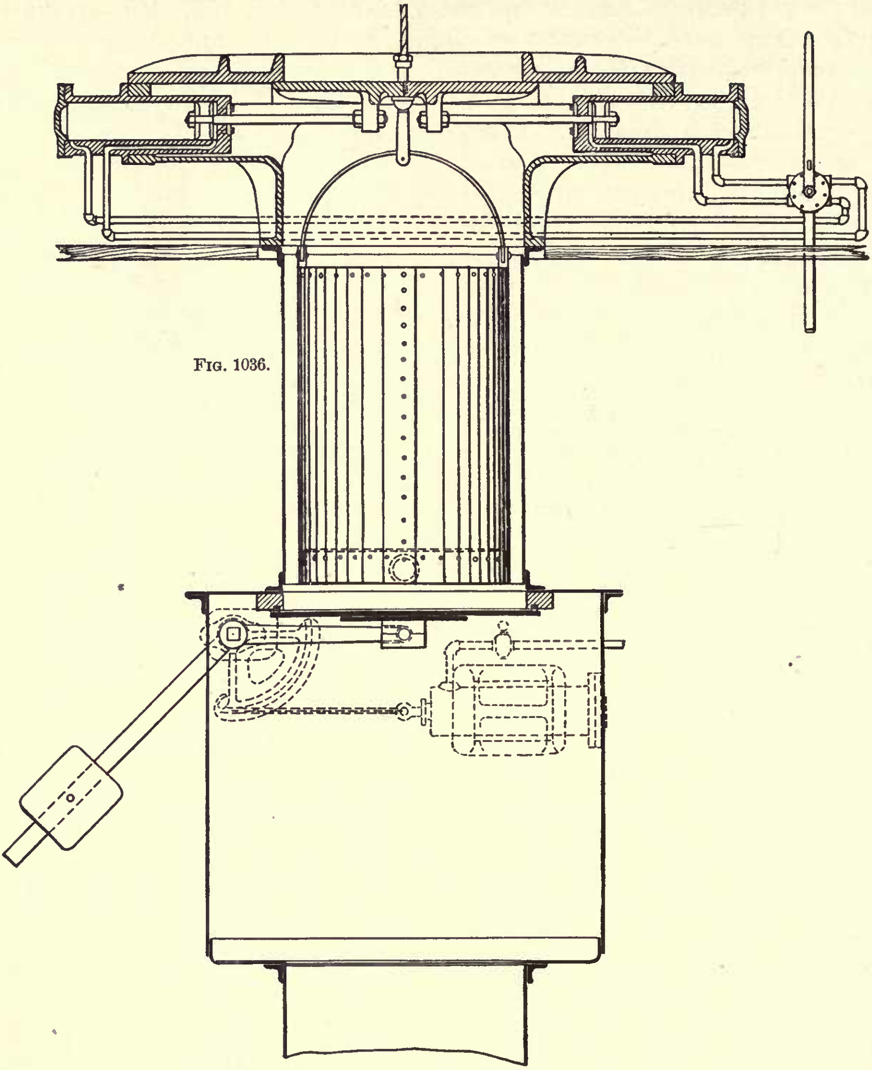


Fig. 1036.

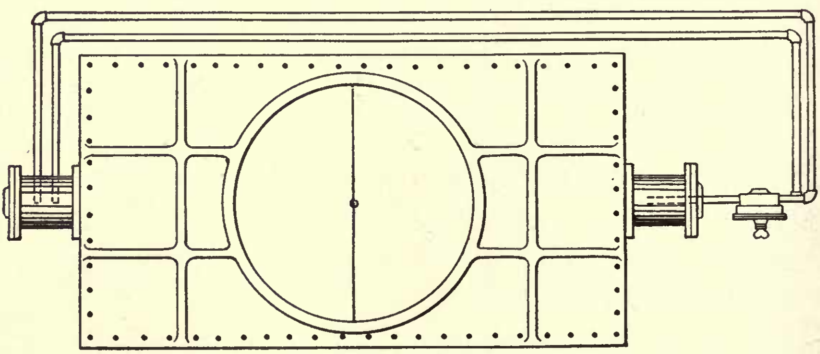


Fig. 1037.

this point, and necessarily an inconvenience in the change of movement of material and workmen, but in later constructions this is avoided.

Fig. 1034 is a recent plan and section of a main shaft, through which access to the caisson is obtained by the workmen; each length consists of a cylindrical shell 4 feet in diameter and 8 feet long, flanged at the bottom, with a head having an opening in it; these lengths are added as fast as the caisson sinks 8 feet. The head of the top and second section are provided with doors (Fig. 1035), thus making the air-lock. When a length has been added the top door is removed and placed on the top of the new length; the lower door may then be taken off and placed where the other has been removed.

Figs. 1036 and 1037 are plan and section of the Barr-Moran air-lock on the excavating shaft. The top doors are double slides, tight-fitting and working on cast-iron guides, operated by pistons driven by steam or by compressed air from the caisson. The lower door is a flap-valve, balanced by a counterweight and operated through a rocker-shaft extending through the air-lock, and a quadrant and chain attached to the piston-rod of a steam or of a pneumatic cylinder in connection with the caisson. In the figures both doors are shut. Under atmospheric pressure the upper doors can be opened for the clear movement of the bucket by a rope attached to the bail, the rope passes through a stuffing-box, around which the doors close when shut. With the upper doors shut, and the air in the lock brought to caisson pressure, the flap-door can be opened and the bucket lowered to the bottom. At the bottom of the shaft there is a flap-door which can be raised when it is necessary to repair the air-lock or to raise it by adding another length of pipe.

The air-lock at the top of the shaft adds to the security of the workmen and to the easier supervision of the machinery, while the single straight pipe gives greater facility to the movement of men and materials.

In the *freezing process*, invented by F. H. Poetsch, of Austria, the site of the foundation is inclosed with large vertical pipes sunk by a water jet; within these pipes, which are closed at the lower end and sunk to the proper depth, a small pipe open at the bottom is inserted, through which is forced a freezing mixture (as chloride of calcium), returning through the outer pipe. In this way a curb of ice is formed, inside which the earth is removed for the foundation. In Finsterwalde, Austria, 12 tubes of 8½″ diameter were sunk through 115 feet of quicksand, in a circle of about 14 feet diameter, for a shaft 8½ feet diameter. After the brick lining was laid the tubes were withdrawn, a hot circulation freeing them from the ice.

This process has been used successfully in this country.

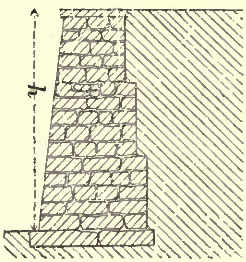


Fig. 1038.

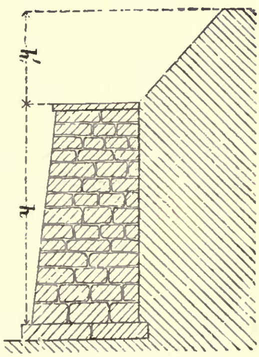


Fig. 1039.

Retaining-walls are such as sustain a lateral pressure from an embankment or head of water (Figs. 1038 and 1039). The width of a retaining-wall depends upon the height of the embankment which it may have to sustain, the kind of earth of which it is composed (the steeper the natural slope at which the earth would stand, the less the thrust against the wall), and the comparative weight of the earth and of the masonry. The formula given by Morin for ordinary earths and masonry is $b = 0{\cdot}285\, h + h'$; that is, to find the breadth of a wall laid in mortar, multiply the whole height of the embankment above the footing by $\frac{285}{1000}$; for dry walls make the thickness one fourth more.

Most retaining-walls have an inclination or *batter* to the face, sometimes also the same in the back, but *offsets* (Fig. 1038) are more common. The usual batter is from one to three inches horizontal for each foot vertical. To determine the thickness of a wall having a batter, "determine the width by the rule above, and make this width at one ninth of the height above the base."

The formulæ for the thickness of retaining-walls are very complicated. Engineers make use of some general rules as above, and depend on their experience for any modification. The top of the wall should not be less than 2 feet, and in climates subject to frost it will be impossible to secure permanently the upper part. Where the soil is saturated with water, it is usual to put weep-holes at or near the bottom to relieve the pressure against the back of walls laid in mortar.

Buttresses and counterforts in the rear of a wall of which the construction requires a uniformity of thickness are only considered equivalent to increasing the strength by the mean amount added to the horizontal section of the wall; but when the buttresses are on the line of the bridge trusses, they add to the strength by the better distribution of the weight of the truss and by the security which the weight of the truss gives at this point to the wall. The buttresses should be well bonded to the wall.

Dams are constructed to pond water for the supply of cities and towns; for inland navigation, by deepening the water over shoals, and the feeding of canals; for power in its application to mills and workshops; and for irrigation. To whatever purpose the water is to be applied, there are two questions to be settled: Whether the level will be raised high enough by the construction, and whether the flow of the stream is sufficient for the purpose required; and, further, it may often be important to know how large a pond will be thus formed, how ample a reservoir to balance unequal flows or intermittent use. If the pond be small, so that the water can not be retained, and the supply is only the natural run of the stream, then the minimum flow of the stream is the measure of its capacity.

The rule that obtains on the Merrimac River, at Lowell and Lawrence, where the pondage is more than the average, is that 1 cubic foot per second per day of 12 hours per square mile of water-shed can be depended on for permanent mill-power. On small streams it happens that comparatively large pondage may be secured, and the supply be equal to one half the rainfall.

Blodgett, in his "Climatology of the United States," says that "in this sense of permanence as a physical fact we may consider the quantity of rain for a year as a surface-stratum, on the Alantic slope and in the Central States of 3½ feet, which may be diminished to half this quantity, or increased to twice as great a depth in the extreme years. The evaporation from a water surface is now usually considered equal to that of a rainfall; therefore in the estimate of the water-shed available for pondage the area of the reservoir is not taken into account; the quantity of rain falling upon it is offset by the evaporation.

The usual form of dams for small streams and but little fall is to build a rubble wall across the stream and secure the up-stream side with an earth of loamy gravel puddled with water to the consistence of mortar or rammed in, the top where the water is to flow over or through being protected by tight planking, around or beneath which the water can not leak.

Lake McMillan dam, built across Pecos River, Colorado, intercepting its entire flow for the purpose of irrigation, is 1,686 feet long, 54 feet high, with an estimated water capacity of 1,000,000,000 cubic feet. Fig. 1040 is a section

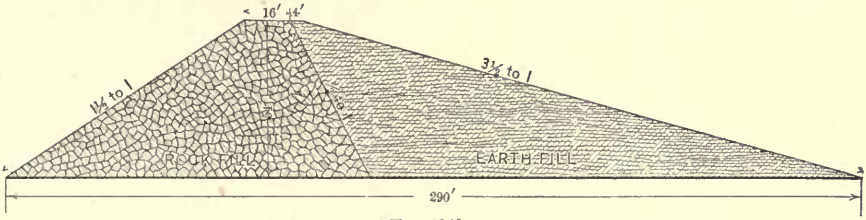
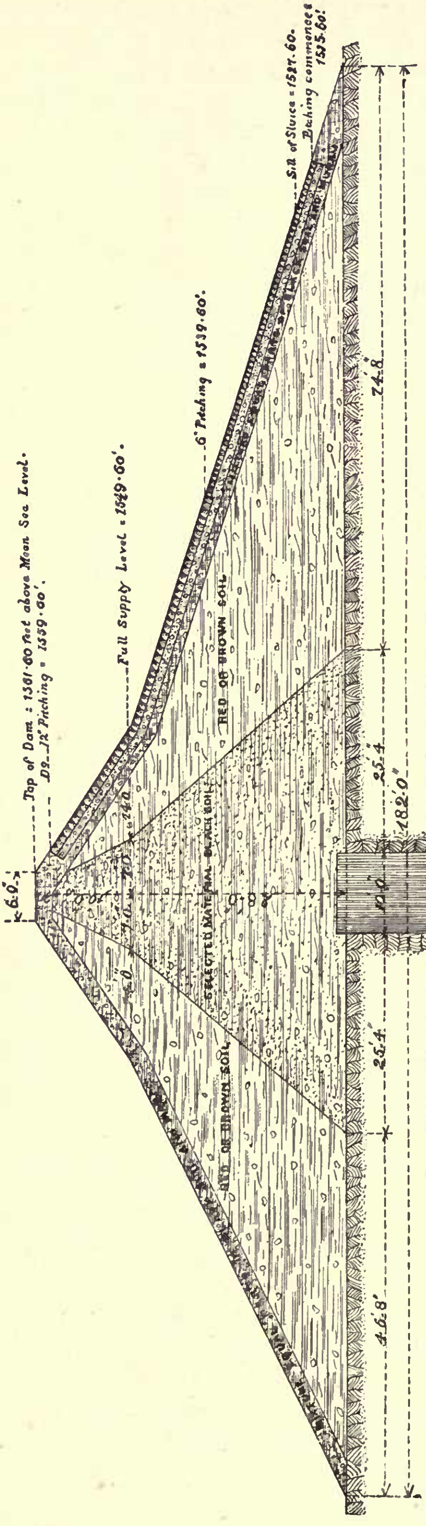


FIG. 1040.

showing its construction, a heavy rock fill with a puddle slope in reservoir front. There is an ample spillway or waste channel by which all surplus water will be discharged, with no flow over the dike. The South Fork, Pa., dam, similar in construction to the above, gave way from a flow over the top of the dam, due to an insufficient waste and an extreme freshet.

In the "Transactions" of the A. S. C. E., December, 1892, James D. Schuyler, a member, gives a description of the asphalt lining of reservoirs for the city of Denver. The excavation consisted of 2′ to 3′ sandy loam, 12′ to 15′ hard clay impregnated with alkali, and 8′ to 12′ of shale. On the completion of the embankment the slopes were sprinkled and rolled with a roller of 5 tons, drawn up and lowered by an engine. Beginning at the bottom, the slopes were laid in horizontal strips of asphalt 10′ wide and about $1\frac{3}{4}''$ thick spread with hot rakes, tamped with hot tampers, and smoothed with hot smoothing-irons. While the sheet was still warm anchor spikes of strap iron $1'' \times \frac{1}{8}''$, 7″ to 8″ long, were driven through it into the bank, in rows about 12″ centres, alternately flush with the surface and projecting $1\frac{1}{2}''$; lumber strips of $2'' \times 4''$ were placed loosely above them on the slope for the workmen. After the finishing coat was applied, the projecting spikes were driven flush and painted over. The bottom thickness was 1″, spread, tamped, and rolled with a cold roller of 5 tons. The finishing coat of refined Trinidad asphalt, fluxed with residuum oil, was poured on from hot buckets, and ironed over with smoothing-irons heated to a cherry-red.

Dikes or dams over which there is no flow of water can be made entirely of earth. It is sufficient that the material be made more compact than the natural earth in which the dam is built, that it be of sufficient section to resist the pressure, and width to obstruct the flow of water through it, and reduce the percolation to a safe and economical limit, the passage of water between the particles of earth being like that through very small broken and crooked pipes.



SECTION OF DAM WHERE MURAM IS SCARCE

FIG. 1041.

Dikes across salt marshes are made of material taken from the marsh at some distance from the site of the dike, well packed in thin layers on a base prepared on the soil without excavation. Sand and gravel, being heavier than the moist material, break through it and settle to the bottom, involving often the construction of a large embankment, while by the use of a homogeneous material the foundation is not displaced but compressed.

Fig. 1041 is a section of the dike or embankment for the Ashti Tank or Reservoir, constructed for retaining water for irrigation purposes in India. The following is an abstract of the description of the work given in the "Minutes of the Proceedings of the Institute of Civil Engineers," vol. lxxvi:

"The net supply available for irrigation may be calculated thus:

Available capacity of tank........	1,348,192,450 cub.ft.
Deduct loss by evaporation, etc.	233,220,240 "
Net supply available for irrigation	1,114,972,210 "

"Area of catchment basin nearly 92 square miles."

The total length of the dam is 12,709 feet; the breadth at the top, which is uniform throughout, 6 feet; breadth at full supply-level, 42 feet; height of the top of the dam above full supply-level, 12 feet; greatest height of dam, 58 feet. The seat of the dike throughout was cleared of vegetable mould, stones, and loose material, all trees and shrubs with their roots being completely grubbed or dug out. The puddle-trench laid in the natural ground is rectangular in cross-section, 10 feet in width, excavated through various materials to a compact water-tight bed, and then filled in with puddle material, consisting of two parts of sand and three parts of black soil, carefully mixed and worked by treading with the feet, and then kneaded into balls and thrown or dashed into the trench in layers up to 12 inches in thickness. The puddle was brought to a level of 1 foot above the ground. Across the river the trench was cut down to the rock and filled with concrete.

The general distribution of the material of the dam is shown in the figure. The central core is formed of the best black soil attainable; on each side, extending to the surface of the mixed material, brown, reddish, or white earth is used. The outer part of the dike is formed of a mixture of equal parts of black soil and sand. The black soil may be described as a clayey earth, tenacious and adhesive when wet—a product of the decomposition of volcanic rock. The brown and reddish soils are of a clayey nature, but contain admixtures of fine sand, nodules, and thin layers of fine grains of lime. The white soil consists of finely powdered particles of a grayish color, similar to wood ashes, which when dry possesses little adhesion, but when wet is adhesive.

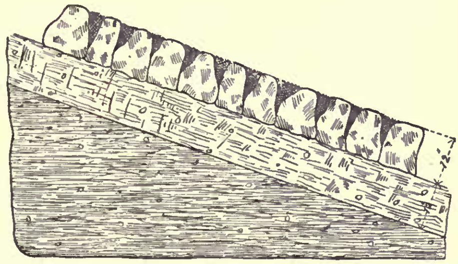


FIG. 1042.

The various soils were laid in the work in layers 8 inches in thickness, every layer being thoroughly watered and rolled with iron rollers. The outer slope was protected by a mixture of equal parts of soil and sand, and with sods of grass, laid about 3 feet apart, which in time extended over the whole slope.

The inner slope is protected from the action of the waves by being pitched or faced with dry stone, set by hand, and laid on a layer of coarse sand. The stones of the pitching were bedded on the slope, and were laid with their broadest end downward (Fig. 1042), each stone being roughly squared with the hammer, and touching for at least 3 or 4 inches. The interstices were then packed with small stone-chippings, and finished off with sand.

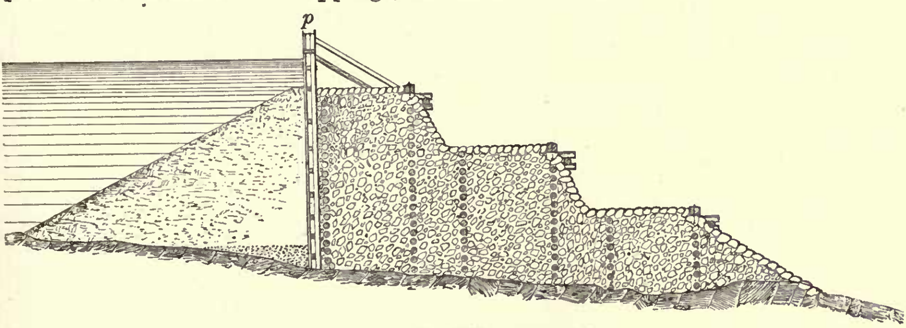


FIG. 1043.

Fig. 1043 is the section of a crib-dam in northeastern Colorado for the pondage of water for the purposes of irrigation. The crib-work is of round logs, 10″ at least in diameter, joined at the end as in ordinary log huts, with dovetail or tongue. Each crib is 18 feet long on the face, and the fastenings are 2″ × 18″ treenails. The cribs are set radially, forming a curve up-stream of 200 to 238 feet radius. The crib gives the stability, but the water-tightness depends on a shutter, *p*, or vertical panel of timber, and the filling of earth on the up-stream side.

For dikes where water does not flow over the top a construction similar to Fig. 1043 is very strong and in many places the most economical, but without any wood, which is likely to decay or rot when exposed to the air. In construction it consists of a mass of masonry laid dry, with a nearly vertical up-stream face pinned and pointed with cement mortar and again faced with a concrete or cement wall in close connection with the wall face and mortar bonded with it. The face of the concrete or cement wall is plastered with a light coat of cement and protected against the wash of the water or thrust of the ice by an earth embankment. This embankment adds to the security of

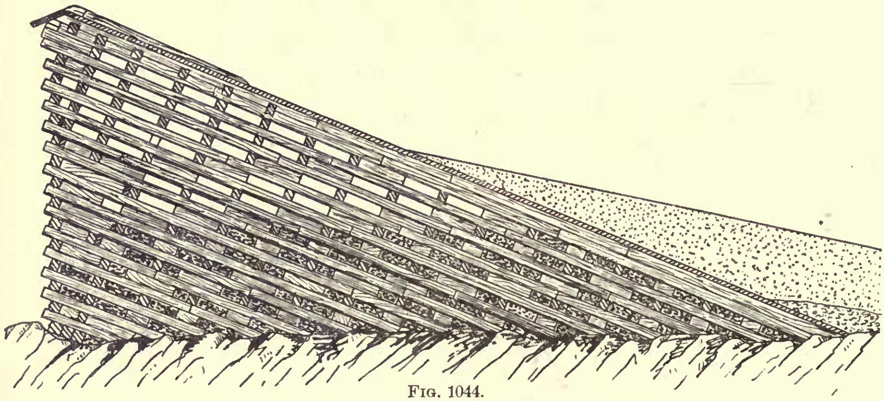


FIG. 1044.

the dam by cutting off seams in the rock foundations and to the stanchness of the cement wall.

Fig. 1044 is a section of the dam across the Connecticut River, at Holyoke, Mass. This dam is 1,017 feet long between abutments, averages 30 feet high by a base of 80 feet and is constructed of timber crib-work, loaded with stone for about one third its height. The foot of each rafter is bolted to the ledge, and all their intersections are treenailed together with 2″ white-oak treenails. The inclined plank face is loaded with gravel, and the joint at the ledge covered with concrete. The lower or base tier of ranging timbers are 15″ × 15″, the other timbers 12″ × 12″. The rafters are placed vertically over each other, in bents of 6 feet between centres. The planking is of hemlock 6″ thick, with oak cross planking at crest of dam 4″ thick at bottom and 8″ at top. The crest is plated with iron ¼″ thick, 5 feet wide. During the construction the dam was planked first about 30 feet on the incline; a space was then left of about 16 feet width by sufficient length, through which the water flowed; and the balance of the dam was then completed. A plank flap was then made for the opening, and when everything was ready it was shut down and the pond filled. The objection to the flap construction is that the space left for the waterway through the dam, after its completion, serves as a duct for air from below which softens and rots the timber-work.

When the dam was first contemplated the longer time and extra cost required for a masonry construction turned the scale in favour of crib-work. Some twenty-five years after its completion it was found that the water overfall from the crest was cutting out the ledge beneath the dam, and a crib-apron was constructed entirely across the river, sheathed with plank on a slope of about $2\frac{1}{2}$ to 1 from the crest downward. From the decay and weakening of the timbers of the old dam, continual repairs have been required, and it is now decided to put in a masonry dam, the wooden dam having served its purpose for some fifty years.

Fig. 1045 is a section of the dam across the Croton River, constructed under the direction of Mr. John B. Jervis, for the supply of the aqueduct for the city of New York. This dam was built on an earth foundation, with curved roll in

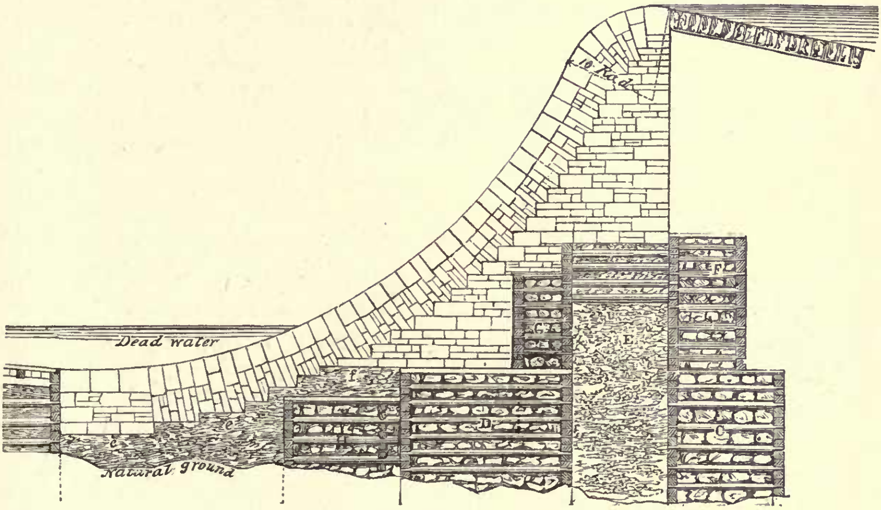


FIG. 1045.

cut stone, extending by a timber-apron some 50 feet, supported by strong crib-work. Originally there was a small supplementary dam farther down to set the water back on to the crib-apron, but this was washed out, and the crib-work is replaced by heavy rock pave. In the erection of this dam all loose material was removed, and then the cribs C and D were built up and the tops were planked; on this planking were carried up the cribs F and G. Between these piers the space E, as well as *e* below and on the cribs, was filled in with concrete; on this the body of the dam was erected in stone-masonry, laid in cement. The face-work of granite is cut to admit of a joint, not exceeding $\frac{3}{16}$ of an inch. The radius of the granite face is 55 feet, and the dam 38 feet high from level of apron to crest of dam. This dam has been in use fifty years and is in very good condition and tight.

Fig. 1046 is a section of a part of the dam across the Merrimac River, at Lowell, built under the direction of Mr. James B. Francis. It was laid dry, with the exception of the upper face and coping, which was laid full in cement. The horizontal joints at the crest were run in with sulphur. The coping-stones were dowelled to the face and together, and clamped to an inclined stone on the lower slope; the end-joint between these stones was broken by making every alternate lower stone longer, and the upper shorter, than shown in the drawings.

The Cohoes dam (Fig. 1047) was built by W. E. Worthen, C. E., directly below an old dam of somewhat similar construction to that of Holyoke. The

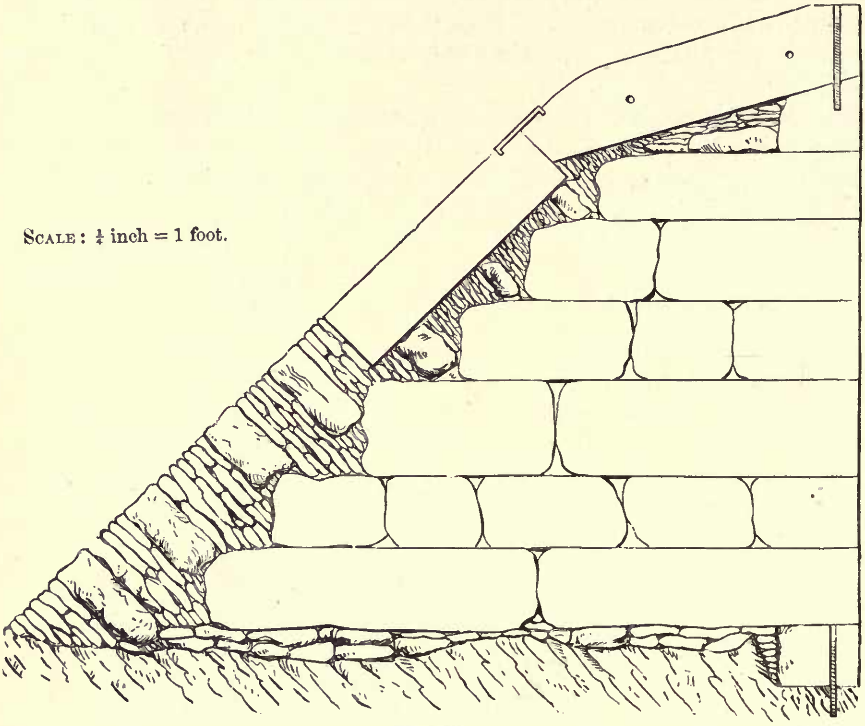


FIG. 1046.

old dam had become very leaky and worn, and the overfall had in many places cut deep into the rock, and in some places within the line of the dam. It was therefore proposed to make the new dam, as a roll to the old one, to discharge the water as far from the foot of the dam as possible, and to keep the old dam for the protection of the new. The exterior of the dam was of rock-faced ashlar; the caps were in single lengths of 10 feet, and none less than 15″ thick and 2 feet wide; they were dowelled together with two galvanized wrought-iron dowels each. The whole work was laid full in cement; the 20″ face was laid first to divert the leak of the old dam from the new work. The whole was brought up to the outline, to receive the cap-stones, which were bedded in cement; the top-joints were then run or grouted in neat cement, to within about 6″ of the top of the stone, which was afterward run in with sulphur. Entire length of overfall, 1,443 feet; average depth below crest of dam, 12 feet.

Where the body of water which may at any time discharge over the dam is large and the fall high, it is especially desirable to secure a location where the overfall can be upon solid rock. If there be a ledge at the side of the river, and none can be found in the channel, it is often better to make a solid dike across the river and above the level of freshets, and cut the overfall out of the bank.

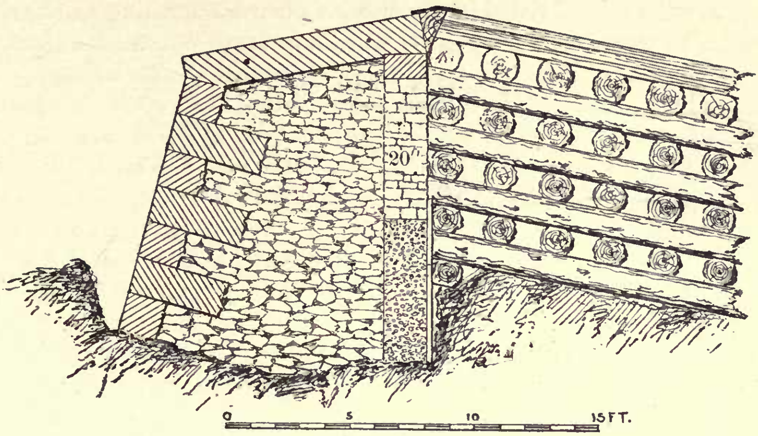


Fig. 1047.

When the dam can have only an earth foundation, an artificial apron, or platform of timber or rock, is to be made, on which the water may fall, or the high fall may be broken up by a succession of steps. In some cases a roll or incline, like that given in the Croton dam, is extended to the bed of the stream, and continued by an apron. The water thus rolls or slides down, and takes a direction, as it leaves the apron, parallel with that of the bed of the stream. But care must be taken to protect the outer extremity of the apron by sheet-piling and heavy paving, as the current, by its velocity, takes with it gravel and all small rocks, and undermines the apron.

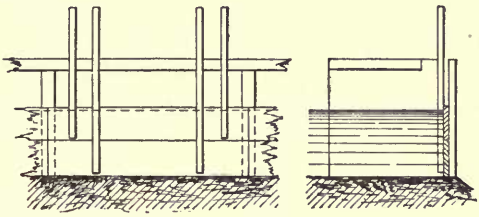


Fig. 1048.

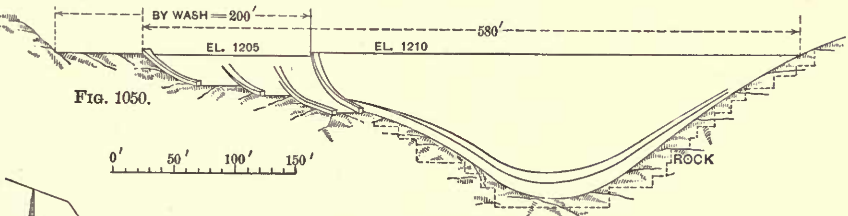


Fig. 1050.

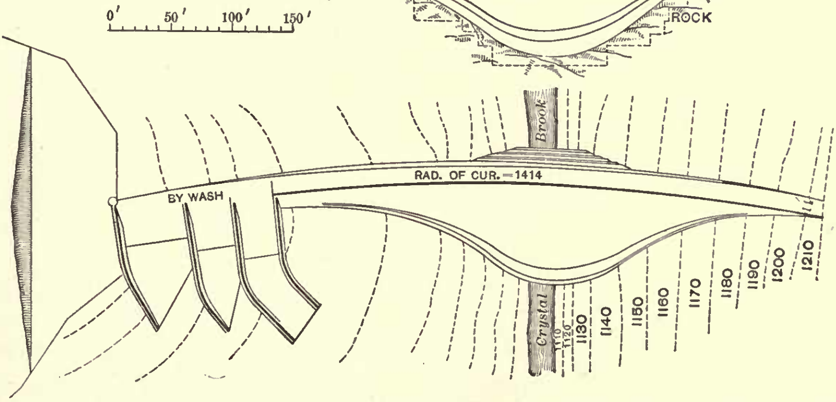


Fig. 1049.

To retain the flow of rivers in dry seasons when the ponding will have little or no effect on works farther up the stream, flash-boards are used, which usually consist of an iron bolt driven into the crest of the dam, against which common boards are raised to be swept off when the river rises unexpectedly. To control the levels of the canal, hand flash-boards are used, as in Fig. 1048, sliding in permanent grooves.

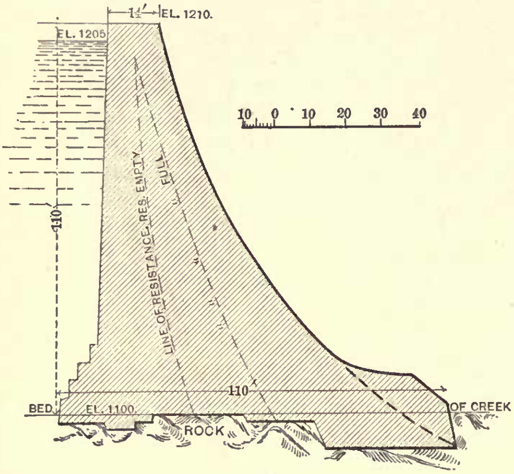


Fig. 1051.

Figs. 1049, 1050, and 1051 are plan, elevation, and section of the Beetaloo dam, South Australia. The dam is built of concrete, 580 feet long, 110 feet high, 110 feet thick on a level with the bed of the creek, and 14 feet thick at the top. The cross-section is in accordance with Prof. Rankine's formula, the horizontal curvature having a radius of 1,414 feet. The structure is founded on rock and has a spillway 200 feet long by 5 feet deep, its channel below being divided by walls into three sections, as shown by the drawings.

Head-gates are constructions necessary to control the flow from the river-pond or reservoir into the canal or conduit by which the water is to be conveyed and distributed for the purposes to which it is to be applied. The top of the works should therefore be entirely above the level of the highest freshets, that no water may pass, except through the gates; and it is better that the opening of the gates should be entirely below the level of the top of the dam, to prevent as much as possible the passage of drift and ice, which are often excluded by booms and racks placed outside the gates.

Figs. 1052, 1053, and 1054 are drawings, in plan and detail, of the head-gates, and the machinery for hoisting them, at the Cohoes Company's dam. There are ten gates, four 8′ × 6′ 6″ and six 8′ × 9′ in the clear; all can be hoisted by machinery connected with a turbine-wheel at *a*, or separately by hand. At *b* there is an overfall, at the same height as the dam, over which any drift that is brought against the gate-house is carried. At *c* there is a similar overfall within the gates, and another at *d*, by which any sudden rise of the level of the canal is prevented. At *e* there is a gate for drawing down the pond, and another at *f*, for drawing off by the canal, both raised and lowered like the head-gates. The head-gates are of solid timber bolted together, moving in cast-iron guides set in grooves in the stone; in front of these grooves there is another set of grooves (*g g*), which are intended for slip-planks or gates, to be put in whenever it is necessary to shut off the water from the gates themselves in case of repairs.

Hoisting Apparatus.—To each gate there are strongly bolted two cast-iron racks, geared into two pinions on a shaft extending across the gate-space, and supported on cast-iron standards on the piers. At one extremity of this shaft

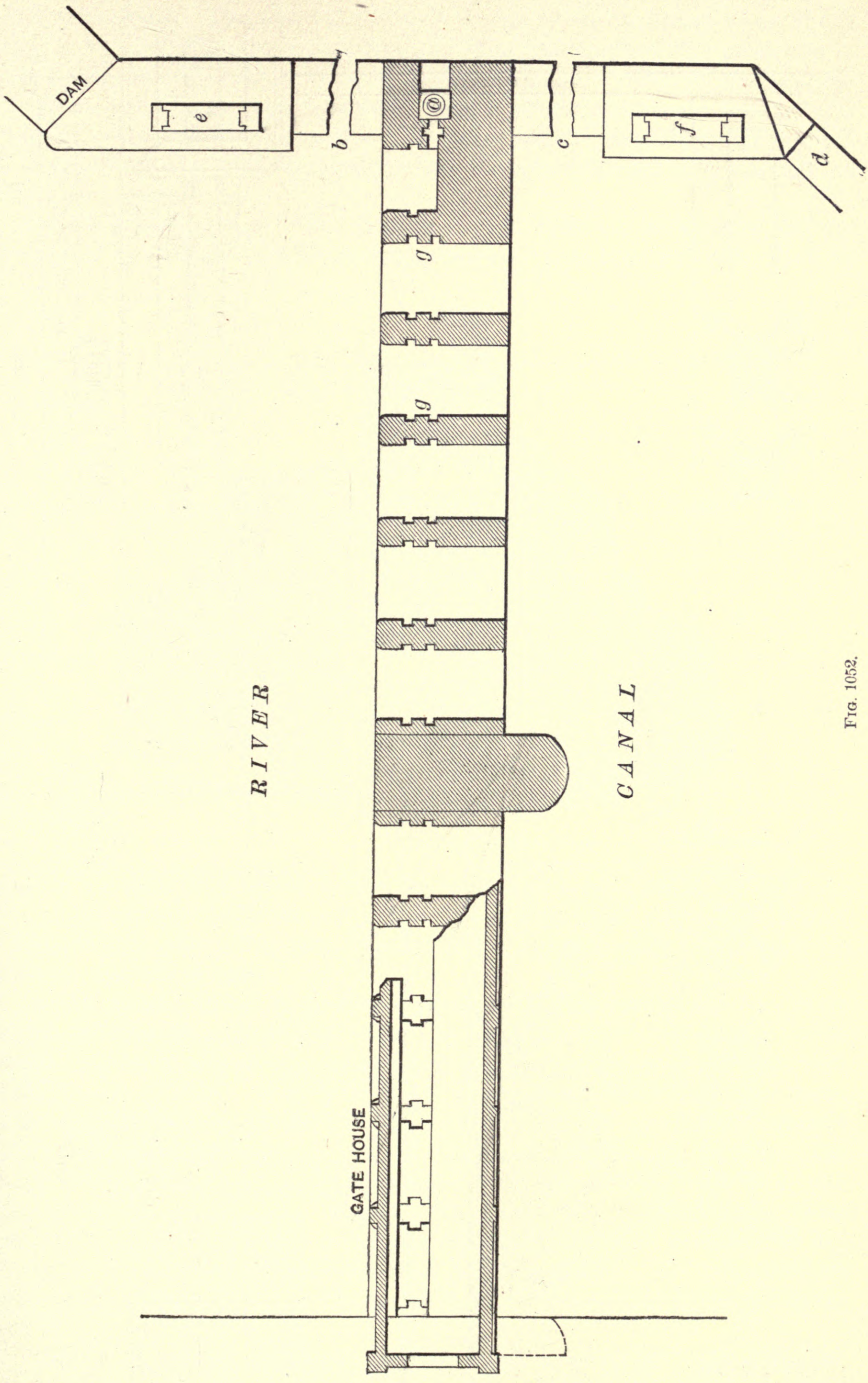


Fig. 1052.

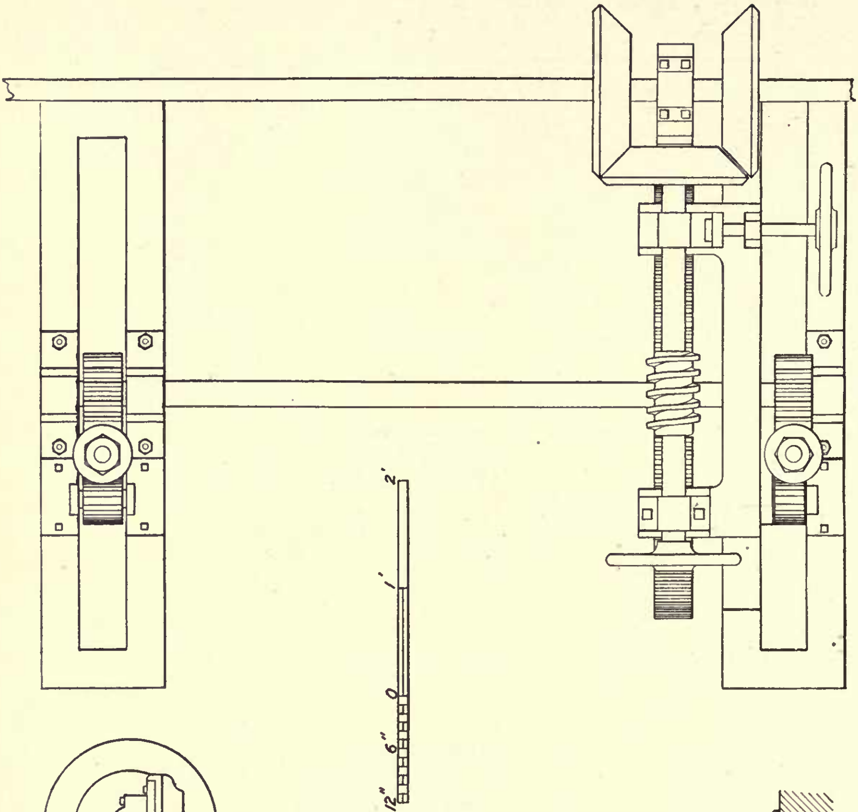


Fig. 1054.

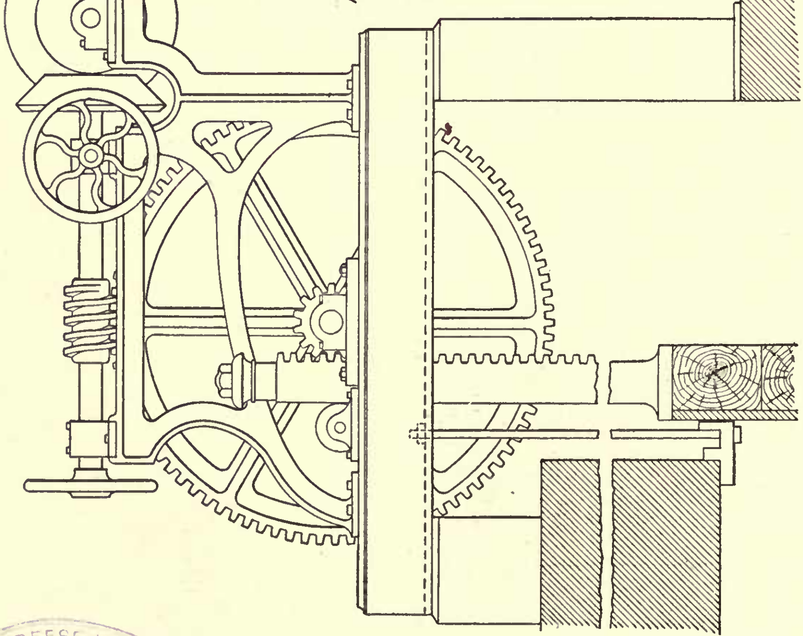


Fig. 1053.

there is a worm-wheel, driven by a worm or screw on a shaft perpendicular to the pinion-shaft. The worm-shaft can be driven either by a hand-wheel at one end, or by the friction-bevel at the other. The friction-bevel can be driven in either direction by being brought in contact with one or other of the friction-bevels on a shaft extending the whole length of the gate-house, and in gear directly with the small turbine at *a*. The small turbine draws its supply through a pipe, built in the walls, and opening into the space between the gates and the slip-plank groove.

In the guard gates at Lowell, instead of racks attached to the gates they are supported by strong rods with screws cut at the upper ends and are raised by

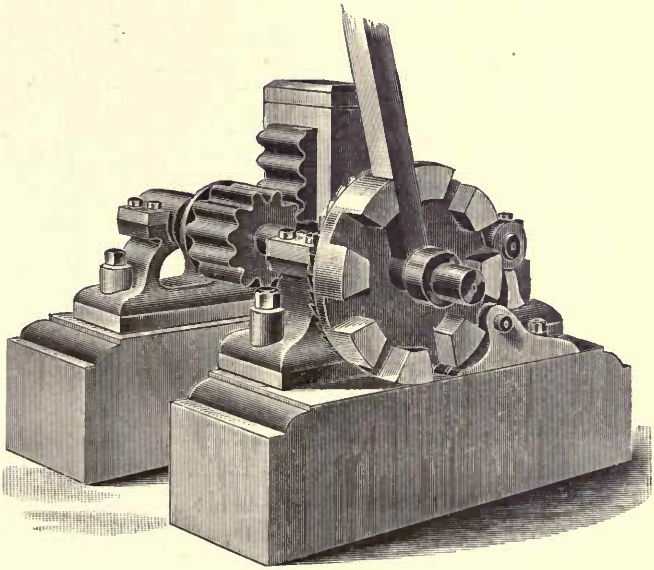
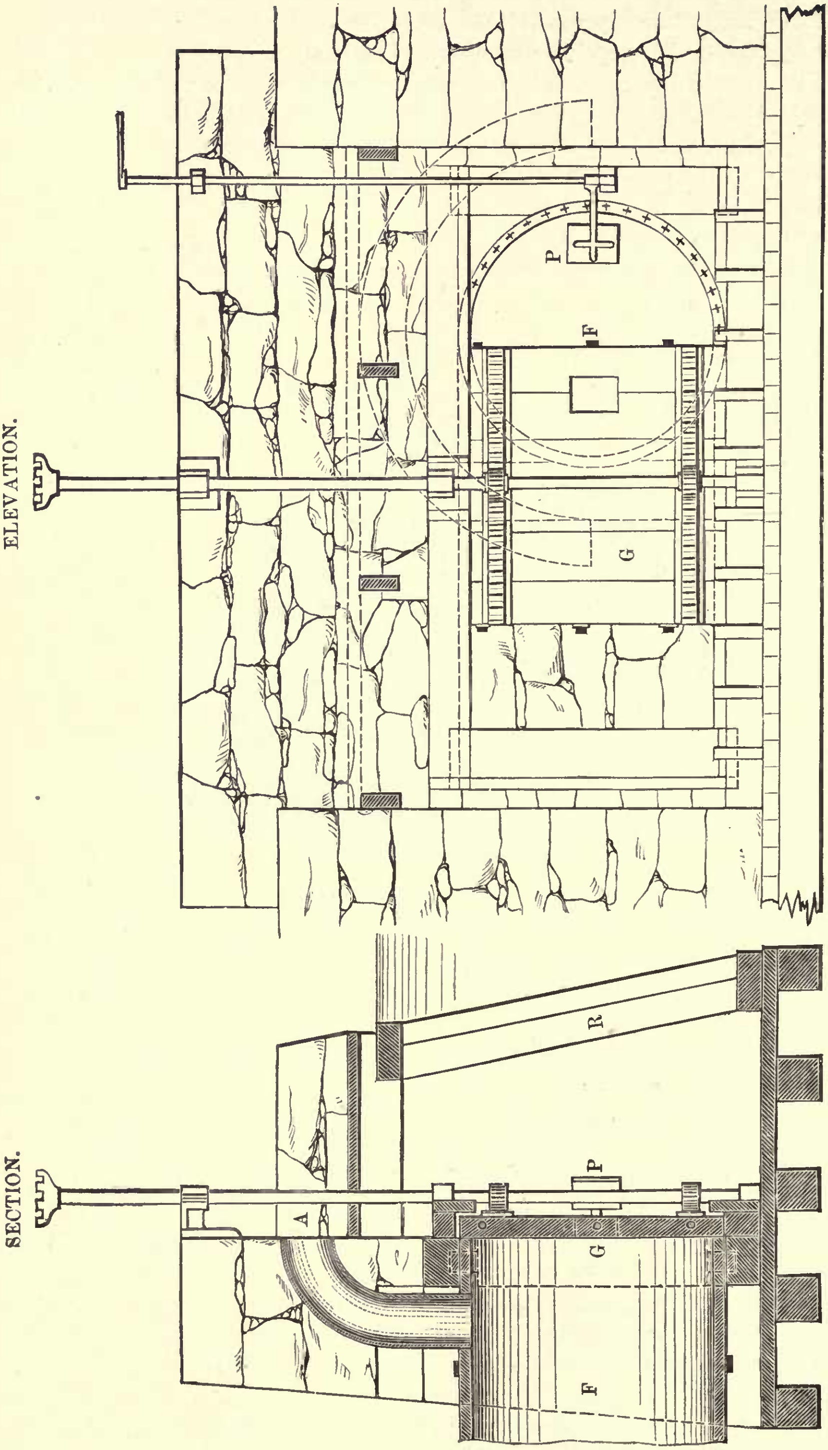


FIG. 1055.

nuts in the hubs of a pair of horizontal gears driven by a pinion between the two on an upright shaft, connected with the gearing of a turbine water-wheel. In late constructions the gates are raised by hydraulic jacks in connection with the city water mains.

Small gates in canals are usually made of wood, with wooden starts on which a rack or racks are planted and hoisted by hand through a crank and pinion shaft. Fig. 1055 is a perspective of the hoisting apparatus of such a gate with a single stem or start. The pinion is driven through a horn wheel and lever. The lever is pressed down and the gate raised; when this movement is stopped a dog catches a ratchet on the back of the wheel and the gate is held. The lever is slipped outward, and is brought in contact with another horn, with another depression and raise of gate. The fulcrum of the lever is the centre pin of the wheel. In dropping the gate the lever holds it in position, the dog is thrown out, the lever thrown out from the horn, and the gate drops from its own weight; if it sticks, it can be forced down by the reverse movement of the lever.

Gates seldom used are raised by chains over a barrel by handspikes, and held by ratchet and dog and dropped as above; but for gates at the head of



Scale, ¼ inch = 1 foot.

Fig. 1057.

flumes leading to wheels the provision for their movement must be positive in both directions, as they are of frequent use.

Figs. 1056 and 1057 are the elevation and section of flume head-gates as manufactured and used at Holyoke, Mass., for such positions. G G are plank gates sliding laterally, moved by two pinions working into racks at the top and bottom of the gates, turned by a capstan bar on a horn wheel head. F is the flume, circular, of wooden staves or wrought-iron plates. P is a paddle gate by which the flume must be filled slowly, and A the pipe for escape of air from the flume during filling.

The Cheney head-gates (Figs. 1058 and 1059), as applied to several of the water gates of the canals at Lowell, are gates supported by wheels, which run

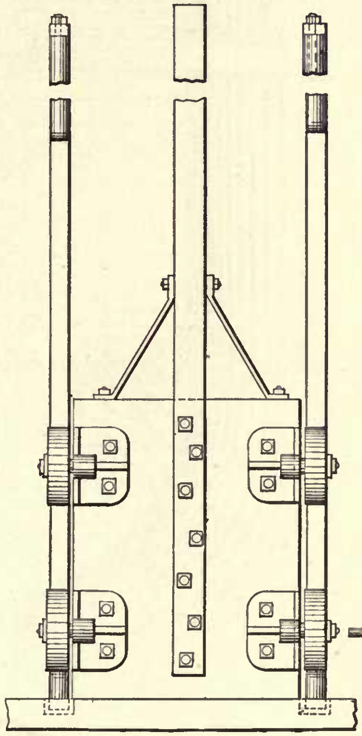


FIG. 1058.

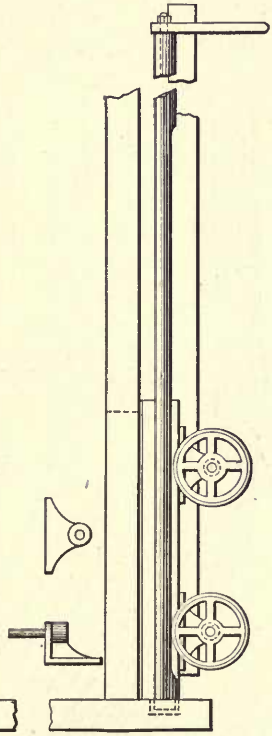


FIG. 1059.

on upright cam shafts on each side of the gate; when the gate is moved either up or down, the cam shafts are turned to raise the gate from the flat-closing surface, and take the whole water pressure, which is, in a measure, relieved by the opening of this joint, and the whole friction is transferred to the wheels and the gate readily raised.

Fig. 1060 is the elevation of a circular tube of plate-iron, as used for a waste gate in the canal of the Connecticut River Company at Windsor Locks. It is of the form of a hollow plug, largely used for bath tubs, and now called a standard waste. The tube is 8 feet diameter and 9 feet high. The joint between the plug and the pipe extending to the river, is made by angle irons A A. The movement of the plug vertically is controlled by radius bars working in

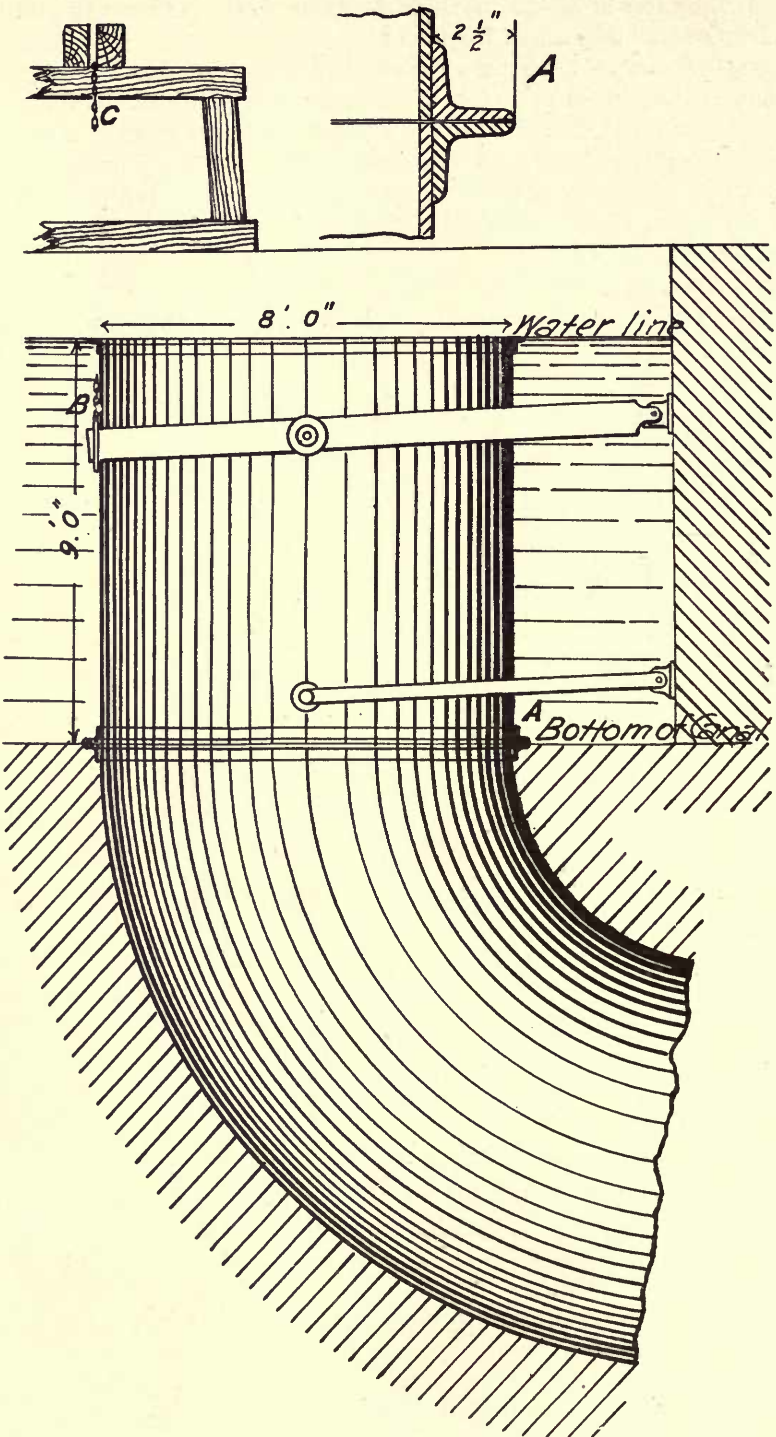


Fig. 1060.

centres on the back wall; they are made of channel bars, each set braced horizontally. The plug is raised by a differential pulley-block suspended at C from a wooden beam across the water. The hoist chain is carried by a yoke to the two radii bars at B; when eased, the plug drops readily to its seat. The level of the canal is about 35 feet above that of the river, and the discharge is so large that it produces a scour in the canal. When the level of the canal rises above the crest of the tube it forms an overfall; a depth of a little over 2 feet will take the whole capacity of the tube.

Figs. 1061 and 1062 are the front elevation and section of the gates of Farm Pond, Sudbury River Conduit, Boston Water-Works. The main web or plate

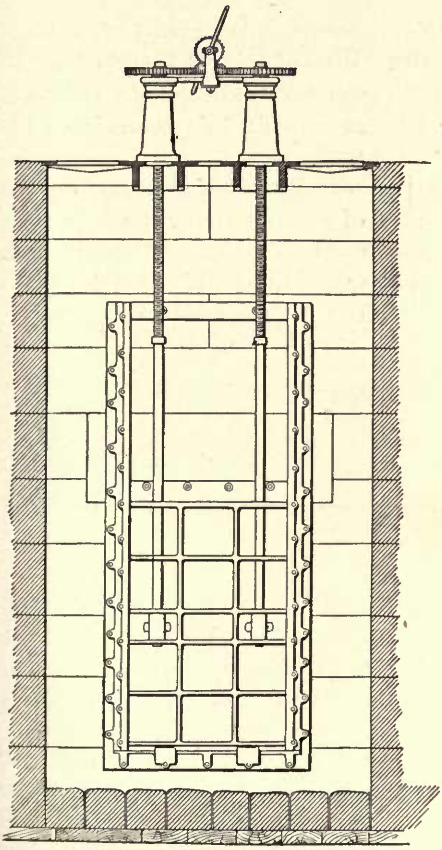


Fig. 1061.

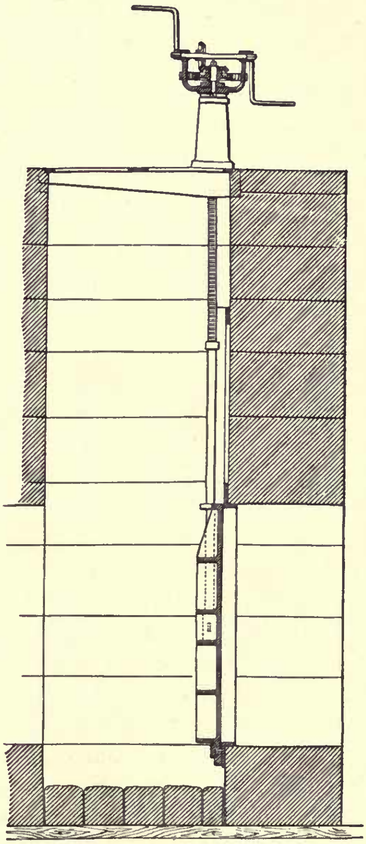


Fig. 1062.

of the gate is $1\frac{1}{4}''$ thick, the ribs 6″ deep, the gate-stems $2\frac{1}{2}''$ diameter. The nuts by which the gates are raised are geared together, and actuated by a double crank. The gates and guides are faced with brass, about $\frac{3}{16}''$ thick.

Similar gates are very common, plates of cast-iron strengthened by ribs; the guides are also of cast-iron, bolted to the masonry, with faces of the gates and guides usually of brass plates, as iron faces become rusty and stick.

Canals.—The sections of canals depend upon the purposes to which they are to be applied, whether for navigation or for power; if for navigation, reference must be had to the class of boats for which they are intended; if for power, to the quantity of water to be supplied, and sundry precautions of construction.

Fig. 1063 is a section of the Erie Canal: width at water-line, 70 feet; at bottom, 28 feet; depth of water, 7 feet; width of tow-path, 14 feet. The

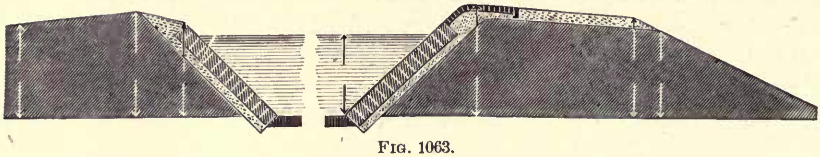


FIG. 1063.

slopes are gravelled and paved, the edge of the tow-path is paved with cobble-paving, and the path gravelled. The smaller canals of this State and of Pennsylvania are generally 40 feet wide at water-line, and 4 feet deep; the Delaware and Raritan, 75′ × 7′; the Chesapeake and Delaware, 66′ × 10′; the ship-canals of Canada, 10 feet deep and from 70 to 190 feet wide.

The dimensions for canals for the supply of mills depend—first, on the quantity of water to be delivered. Their area of cross-section should be such that the average velocity of flow should not exceed two to three feet per second, and in northern climates less velocity than this would be still better; it should always be such that during the winter the canals may be frozen over, and re-

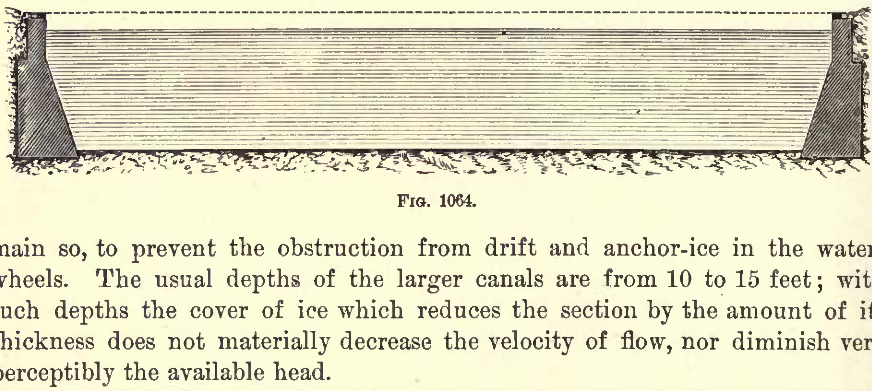


FIG. 1064.

main so, to prevent the obstruction from drift and anchor-ice in the water-wheels. The usual depths of the larger canals are from 10 to 15 feet; with such depths the cover of ice which reduces the section by the amount of its thickness does not materially decrease the velocity of flow, nor diminish very perceptibly the available head.

Fig. 1064 is a section of the Northern Canal, at Lowell, Mass., which may be considered a model for large works. The width at water-line is 103 feet and the depth 16′, and is intended for an average flow of 2,700 cubic feet per second. The fall in the whole length of 4,300 feet is between 2″ and 3″; when covered by ice, about 4″. The sides are walled in dry rubble, and coped by split granite. The portion above, and about three feet below, the water-line, or between the limits of extreme fluctuations of level, is laid plumb, that the ice may have as free a movement as possible vertically.

Fig. 1065 is a section, on a scale of $\frac{1}{8}'' = 1$ foot, of the river-wall of this same canal, where the canal passes out into and occupies a portion of the river channel. The main wall is in dry masonry, faced on river-side with rough-faced ashlar, pointed beds and end-joints. The inside lining is of two courses of cement-wall, to the dry rubble wall pointed with cement, against which is laid the first cement lining, plastered on the inside, and the interior wall is then laid; the granite inside wall, above lining, is laid in cement.

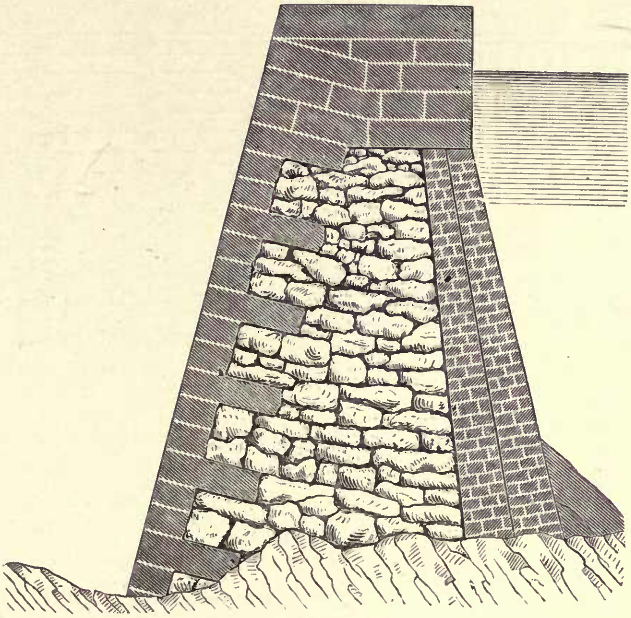


FIG. 1065.

Locks of Canals.—Figs. 1066 and 1067 are portions of plan and vertical section of locks, taken from the general plans for timber locks on the Chemung Canal. They represent the half of upper gates. Fig. 1068 is a section of one side of the lock of the same. Fig. 1069 is the plan of a portion of one of the

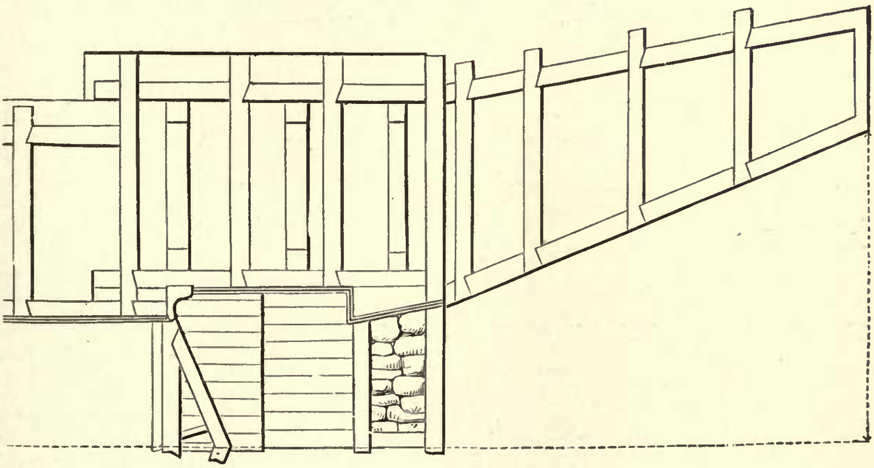


FIG. 1066.

enlarged locks of the Erie Canal, showing one of the upper gates and the side-walls. Fig. 1070 is a cross-section of one of the same locks, showing the culvert in the centre between the locks, used for the supply of the waste of the lower level. The proper height of water in this level is controlled by gates in the upper level.

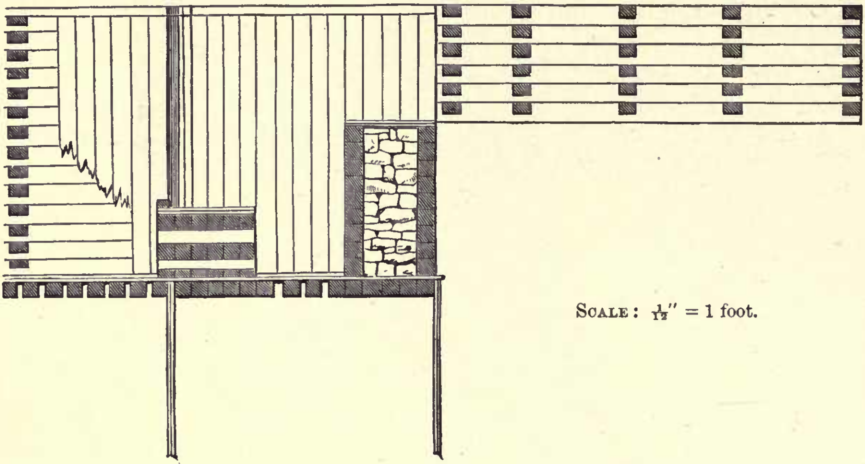


Fig. 1067.

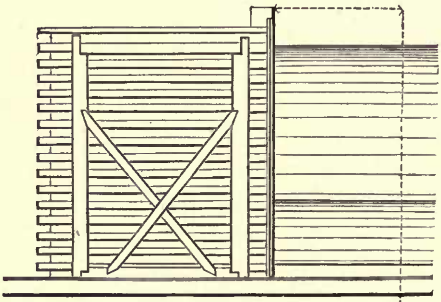


Fig. 1068.

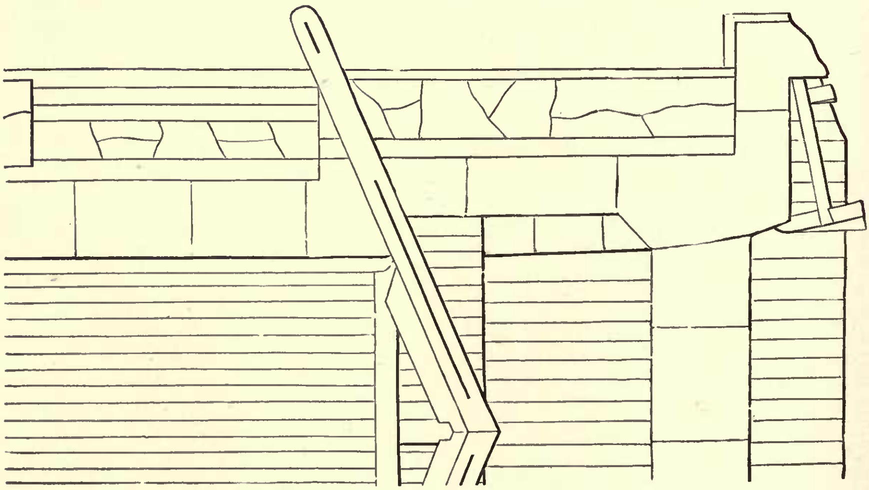


Fig. 1069.

Fig. 1071 is a drawing, in outline, of the hollow quoin of the lock-gate, on a scale of $\frac{1}{24}$ full size (Chemung Canal).

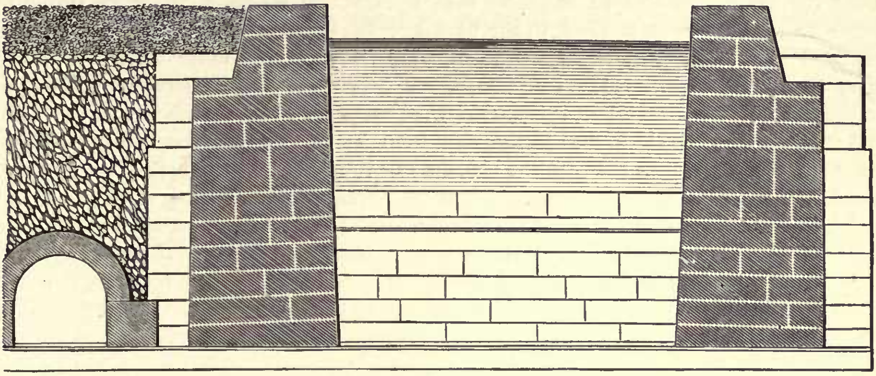


FIG. 1070. SCALE: $\frac{1}{10}''$ = 1 foot.

Fig. 1072 is a plan and elevation of pintal for heel-post of lock, with a section of the bottom of the post. The pintal is imbedded in bottom timber or stone, as the case may be.

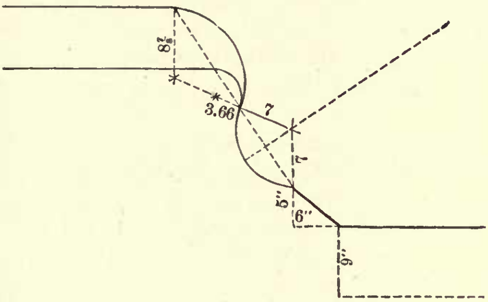
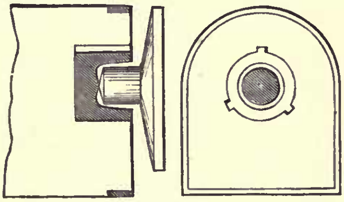


FIG. 1071.



$\frac{1}{16}$ full size.

FIG. 1072.

Fig. 1073 is a plan and elevation of the strap for the upper part of heel-post. Extracts from lock specifications ("New York State Canals," 1854):

"Locks to be composed of hydraulic stone masonry, placed on a foundation of timber and plank. The chamber to be 18′ wide at the surface of the water in the lower level, and 110′ long between the upper and lower gate-quoins. The side-walls to extend 21′ above the upper gate-quoins, and 14′ below lower gate-quoins. If the bottom is of earth, and not sufficient to support the foundation, then bearing-piles of hard wood, not less than 10″ diameter at small end, shall be driven. The foundation timbers to be 12″ × 12″, and of such lengths as will extend from and cover the outside piles, and to be tree-nailed with a 2″ white-oak or white-elm treenail, 24″ long, to each pile.

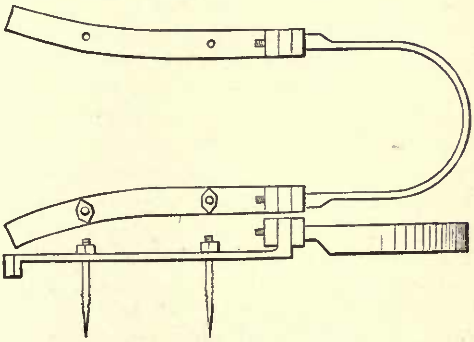


FIG. 1073.

"If without bearing-piles, the foundation to be composed of timber, 12″ thick and not less than 10″ wide, counterhewed on upper side, placed at uniform distance, according to their width, to occupy or cover at least ⅘ of the area of the foundation, and under the lower mitre-sill to be placed side by side: in all cases to be of sufficient length to extend across the lock to the back line of the centre buttresses, and at the head and foot to the rear or back line of wing-walls. The timber under the lower mitre-sill to be of white oak, white elm, or red beech, the other foundation and apron timber to be of hemlock. The foundation to be extended 3′ above the face of the main wall at the head of the lock, and at the foot from 25′ to 30′ below the exterior wing—that portion of the spaces between the timbers in all cases to be filled with clean coarse gravel, well rammed in, or concrete. Where rock composes the bottom of the lock, and is of such a character that timber is not required for the foundation, the same shall be excavated smooth and level, and the first course of stone well fitted to the rock.

"*Sheet-Piling.*—In all cases where rock does not occur, there shall be a course at the head of the foundation, under each mitre-sill, and at the lower end of the wings, and at the lower end of the apron, to be from 4′ to 6′ deep as may be required—in each to extend across the whole foundation.

"*Flooring.*—A course of 2½″ pine or hemlock plank to be laid over the whole of the foundation timbers, except a space, 3′ wide, under the face-line of each wall to be 2½″ white oak; the whole to be well jointed, and every plank to be treenailed with two white-oak treenails at each end, and at every 3′ in length, to enter the timber at least 5″, or with wrought-iron spikes, treenails to fill 1¼″ bore. *Platform for the upper mitre-sill* to be 5′ 10″ wide, and 6′ high above foundation, and to extend across from side-wall to side-wall, to be composed of masonry, coped with white-oak timbers, extending 6″ into each side-wall. *Mitre-sills* to be of best white-oak timber, 9″ thick, to be well jointed, and bolted to the foundation or platform timbers with bolts 20″ long, 1″ × 1″, well ragged and headed.

"*Masonry.*—The main walls, for 21′ 6″ in length, from wing-buttresses at the head, and 32′ at lower end, to be 9′ 8¾″ thick, including recesses, and for the intermediate space, 7′ 8¾″ thick, with three buttresses projecting back 2½′, and 9′ long at equal distances apart. The *quoin-stones*, in which the heel-post is to turn, shall not be less than 5′ 6″ in length in line of the chamber, to be alternately header and stretcher. The *recesses* for the gates to be 20″ wide at top of wall, 12′ long, with sub-recesses, 9″ wide, 6′ high, 10′ long, for the valve-gates.

"*Culvert between Locks.*—The sluice-way shall be made in the head-wall with cut-stone jambs, grooves to be cut in the jambs for the sluice-gates, the bottom of the aperture to be of cut stone, with lower corner beveled off, over which the water will fall into the well, the bottom of which shall be covered with a sheeting of cut stone, 6″ thick.

"*Second flooring* of seasoned 2″ first-quality white-pine plank, to be well jointed, and laid on the foundation between the walls, from the breast-wall to lower end of main wall, and also on the floor of the wall.

"*Gates.*—The framing to be made of best quality white-oak timber; the cross-bar to be framed into heel and toe posts with double tenons, each tenon to be 7″ long, and secured with wrought-iron Ts, well bolted. The heel and toe posts to be framed to the balance-beam by double tenons, and secured by a wrought-iron strap and balance-rod, from the top of the beam to the under side of the upper bar. The lower end of the heel-posts to be banded with wrought-iron bars; the collar and other hangings to be of wrought-iron, secured together with a double nut and screw, and to the coping by bedding the depth of the iron in, and by screw-bolts fastened with sulphur and sand-cement. The pivots and sockets which support the heel-posts to be of best cast-iron; a chilled cast-iron elliptical ball, 2½″ horizontal, and 1″ vertical diameter, to be placed on the pivot and in the socket of each heel-post, gates planked with seasoned 2″ white-pine plank, jointed, grooved, and tongued—tongues of white oak."

Water, ponded by dams, and conveyed by canals for use as mill-power, is carried within the workshops or manufactories, to be applied on water-wheels, by some form of covered channels usually designated as *flumes*. The common form of a flume for the conveyance of water to breast, overshot, or undershot wheels, is of a rectangular section, framed with sills, side-posts, and cap, and, if a large section is required, with intermediate posts. The sills are set, and earth well rammed in the spaces between them; the bottom plank is then laid, posts and cap framed with tenon and mortice, set and pinned, and the plank is then firmly spiked on the outside of posts and caps. The planks are usually partially seasoned and brought to close joints; the size of timbers will depend on the depth beneath the soil, or the insistent load. Within the mill, and just above the wheel, the flume is framed without a cover, and the posts and side-planks are brought above the level of the water. This open flume is termed the *penstock*, especially necessary, in the class of wheel above referred to, to secure the full head of water.

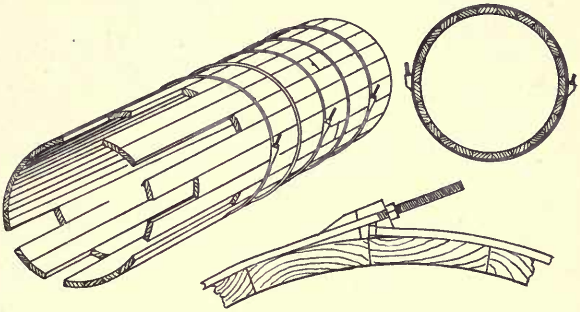


Fig. 1074.

For the conveyance of water to turbine-wheels, wrought-iron pipes are almost invariably used. Cast-iron is also sometimes used, with flange, or hub and spigot-joints. Plank-pipes (Fig. 1074) are also used, made with continuous staves, and hooped with wrought-iron; these constructions are much cheaper, and serve a very good purpose. The head-gates of flumes are placed at the head of the flumes, in a recess back from the face of the canal, with racks in front to prevent the passage of any drift that might obstruct or injure the wheel. The total area of passages through the racks should liberally exceed the area of cross-section of the flume, not only on account of the extra lateral friction of the rack-bars, but also on account of their liability to become obstructed. Sometimes two sets of racks are placed in front of the flumes, especially for turbines and reacting wheels: a coarse rack outside, with wide spaces, say 2″ and a finer one inside, say of $\frac{5}{8}''$ to $\frac{3}{4}''$ spaces.

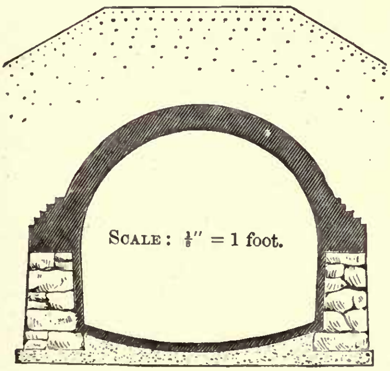


Fig. 1075.

Conduits for the supply of water to cities and towns are of masonry, or cast- or wrought-iron pipes. Their capacity to deliver the required quantity depends upon the area and form of cross-section, and the velocity of flow due to the loss of head or of pressure permissible; this velocity being due primarily to gravity, but largely modified by conditions of structure, as the kind and amount of wetted surface, and length and directness of line.

Fig. 1075 is a cross-section of the main conduit of the Nassau Water-Works for the supply of the city of Brooklyn, Long Island. The width is 10′ at the springing of the arch; the side-walls 3 feet in height; versed sine of invert, 8″; height of conduit in centre, 8′ 8″; fall or inclination of bottom, 1 in 10,000.

The foundations to be of concrete, 15′ wide, on earth, or, if the water was troublesome, on a platform of plank. The side-walls of stone, with an interior lining of 4″ brick; arch brick, 12″, and the invert 4″ thick. The outside of arch, and each wall, were plastered over on the outside with a thick coat of cement mortar. In both cuttings and embankments the arch was covered with 4′ of earth, with a width of 8′ at top, and slopes on each side of 1½ to 1, covered with soil and seeded with grass.

Fig. 1076 is a section of the conduit of the Boston Water-Works. The inside section is equal to a circle 8½ feet diameter, and is uniform throughout except in tunnels. The section given is the general one, resting on concrete, brick lining at sides and invert at bottom, with an 8″ arch at top for a 4′ cover, and 12″ for exceptional depths or under railway-tracks. The lower corners were of special brick.

The inclination of the conduit is 1 foot per mile, and the flow 80,000,000 gallons per 24 hours when full, or 5 feet above centre of invert. The maximum flow takes place when the depth of water is 7′ 2″, the delivery then being 109,000,000 gallons.

Fig. 1077 represents a section of

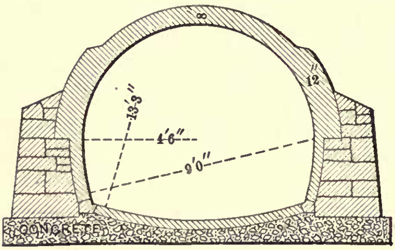


FIG. 1076.

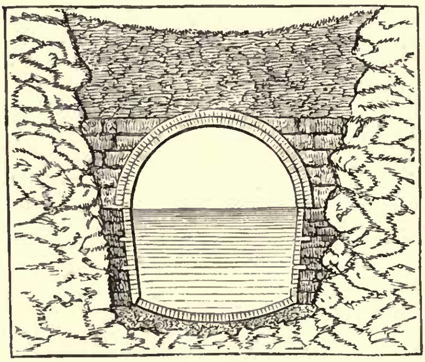


FIG. 1077.

the old Croton Aqueduct, in an open rock-cut. The width at spring of arch, 7′; versed sine of invert, 6″; height of conduit, 8′ 6″; fall or inclination of bottom, about 1 in 5,000.

Fig. 1078 is a two half section of the new Croton Aqueduct, of which one is in earth and rock, and the other entirely in rock. The foundation is in concrete, the lining and arches in brick. At the junction of the invert and side linings there is a special angle block in brick; exterior of brickwork is plastered.

Before the construction of High Bridge water was conveyed for the supply of the city through siphon pipes beneath the Harlem River, afterward through two 3′ cast-iron pipes in the masonry of the bridge. As the demand for water increased in the city, the obstruction caused by lack of capacity in these pipes made the introduction of a larger pipe necessary (Fig. 1079), which has been made of wrought-iron, ½" thick and 7' 6½" in diameter, supported by cast-iron columns which admit of a rocking movement, and slip-joints in the pipe to compensate for any expansion or contraction. The pipes are inclosed in a long chamber, extending the whole length of the bridge, covered by a brick arch, laid in cement with a cover of asphalt, and a brick pavement over all. A A are the arch-stones of the bridge.

In large works, where there is considerable length of conduit, receiving reservoirs, within or near the limits of the city, are necessary as a precaution to guard against accidents which might happen to conduit or dam, and cut off the supply, and also as a sort of balance against unequal or intermittent draught among the consumers. The size of these reservoirs must depend on the necessities of the case, and on the facilities

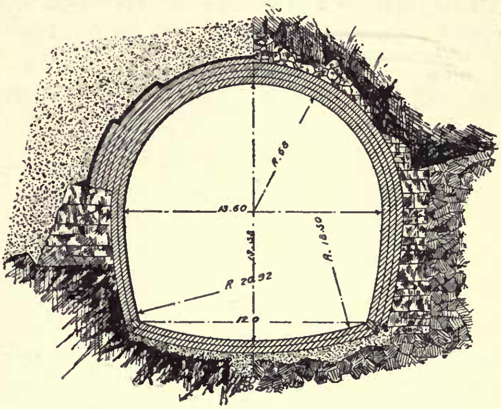


Fig. 1078.

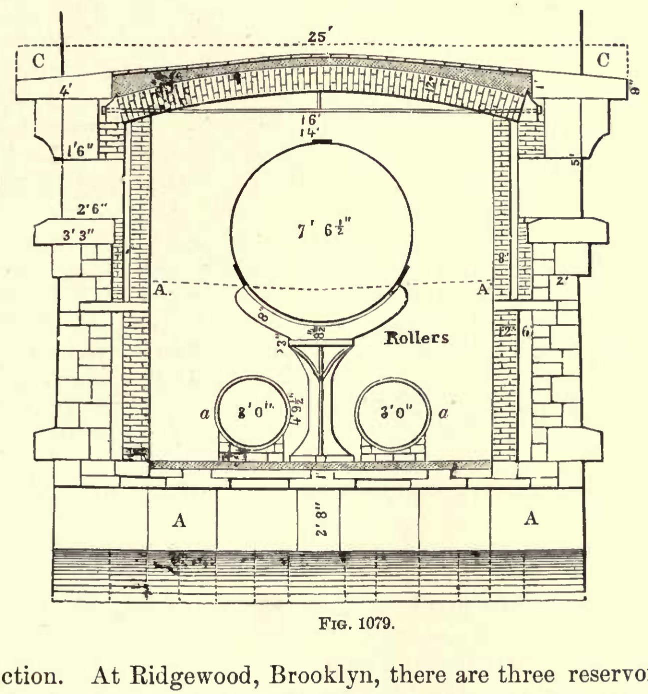


Fig. 1079.

for construction. At Ridgewood, Brooklyn, there are three reservoirs in connection, of total capacity of about 325,000,000 gallons. The capacity of the upper Croton reservoir in Central Park, New York, in two compartments, is about 1,000,000,000 gallons.

Fig. 1080 is a section of the division-bank of the Croton reservoir, made of earth, with a puddled ditch in the centre.

Extracts from the specification:

"All the banks will have the inner and outer slopes of 1½ base to 1 perpendicular. All the inner or water-slopes will be covered with 8″ of broken stone, on which will be placed the stone pavement, 1½ foot thick. The outer slopes will be covered with soil 1

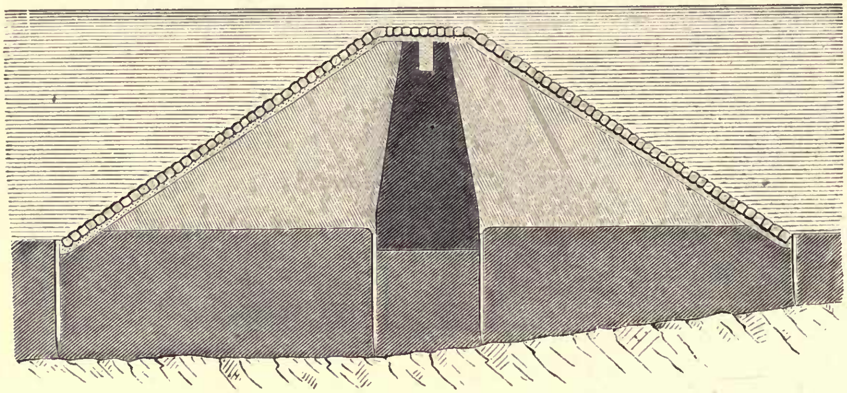


FIG. 1080.

foot thick. The banks, when finished, to be 15 feet on top, exclusive of the soil on the outer slope. The top of the outer bank to be 4 feet above water-line; the top of the division-bank to be 3 feet below water-line. In the centre of all the banks a puddle-bank will be built, extending from the rock to within 2 feet of the top of the outer bank. It will be 6′ 2″ wide at top in division-bank, and 14′ wide at top in exterior bank, and 16′ wide at a plane 38′ feet below top of exterior bank. In the middle of the division-bank there will be built a concrete wall 4′ high, 20″ wide; 8″ thickness of concrete to be laid on the top of the bank, on each side of, and connected with, this wall. On the pavement 18″ thick will be laid in concrete. The slope-wall on each side of the division-bank, 10′ in width, to be laid in cement.

"Puddle-ditches are to be excavated to the rock under the centre of all embankments. The earth within the working-lines of interior slopes to be excavated to the depth of 40′ below top of exterior bank, rock 36′. The puddle-ditch to be formed of clay, gravel, sand, or earth, or such admixture of these materials, or any of them, as the engineer may direct, to be laid in layers of not more than 6″, well mixed with water, and worked with spades by cutting through vertically, in two courses at right angles with each other; the courses to be 1″ apart, and each spading to extend 2″ into the lower course or bed. The puddle to extend to all the masonry and pipes, as the engineer may direct.

"The embankments to be formed in layers of not more than 6″, well packed by carting and rolling, and, where rollers can not be effectually used, by ramming. The embankments to be worked to their full width as they rise in height, and not more than 2′ in advance of the puddle. The interior slopes of all the banks to be covered with 8″ thickness of stone, broken to pass through a 2″ ring. On this to be laid the paving, 18″ in thickness, of a single course of stones set on edge at right angles with the slope, laid dry, and well wedged with pinners."

Fig. 1081 is a drawing of a sheet-iron water pipe as used in the States of the Pacific slope. The bottom joint is a slip-joint of the stove-pipe iron order, which Mr. Hamilton Smith considers good for pressures not exceeding 380 feet. For pressures greater than this, the lead joint, as shown in Fig. 1082 in section, should be used; it consists of an inner sleeve riveted to the inside of one pipe and an outer sleeve covering the joint with a $\frac{3}{8}''$ space, into which lead is run and calked as for cast-iron pipes (see page 463).

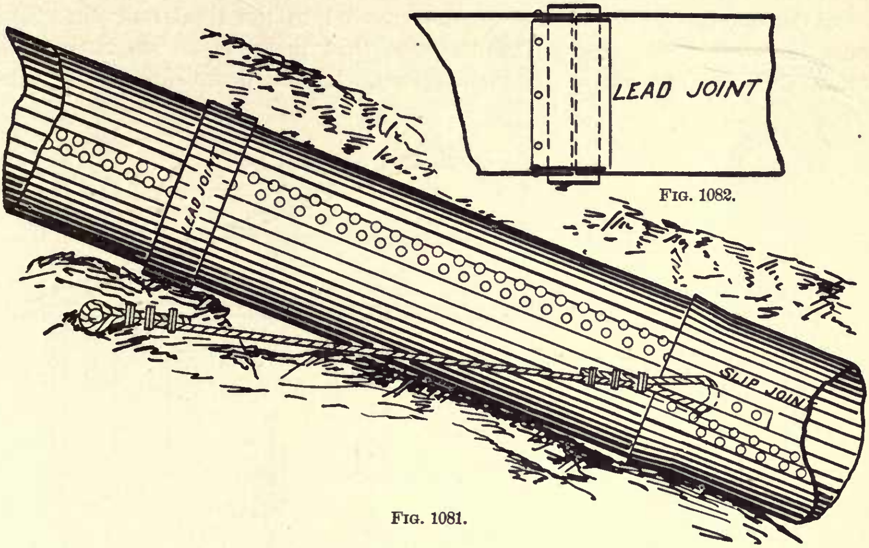


FIG. 1082.

FIG. 1081.

The pipe as drawn is on an incline and is anchored by a wire cable. Under heavy heads and on inclines the pipes should be wired together, and at angles lead joints should be used.

The pipes are invariably coated with asphalt, inside and out, by immersing them for some minutes in the hot liquid. The pipes are double riveted, and the thickness of the iron very much less than used here; the Cherokee pipe, 30 inches diameter up to 203 feet head, is No. 14, or ·083″ thick; head 900 feet, $\frac{3}{8}''$ thick.

At first slip-joints were made for expansion, but were found unnecessary. Time has shown no practical depreciation in the pipes.

For conduits of large size, and under extreme pressure, wrought-iron or steel may be considered the rule in the United States; they are more economical in cost and safer than masonry, less disturbance from settlement, practically without leak, and admitting of prompt and easy connections and repairs.

In construction pipes are made up at shops and riveted like boilers, and are put together in such lengths as can be readily transported and laid. Welded longitudinal joints on the separate lengths can be obtained without much increase in cost, and flanges may be turned on them for bolting or riveting.

For water supplies to locomotives on our railroads, the usual tanks are wooden cisterns strongly hooped and resting on timber frames. Fig. 1083 is a tank of wrought-iron on the Orleans Railway, containing 22,000 gallons, and applicable to small services of towns and villages. The inverted-dome bottom has no supports except at the circumference, where there is a strong iron plate with extra angle irons, as shown in detail. The pipes pass through the bottom, but give it no support. This form has been applied at the Liverpool waterworks in the Norton Tower, where the tank is 82 feet in diameter and contains 651,000 gallons.

Water-works for small towns often draw their supply from driven wells, and for this service stand-pipes or tanks of iron are preferable to reservoirs in earth; but waters from these sources are usually deficient in oxygen, and confervæ develop rapidly in open reservoirs exposed to the light and air. They must therefore be of small dimensions, so that there be no stagnation; the water must circulate and be changed often. These inconveniences, together

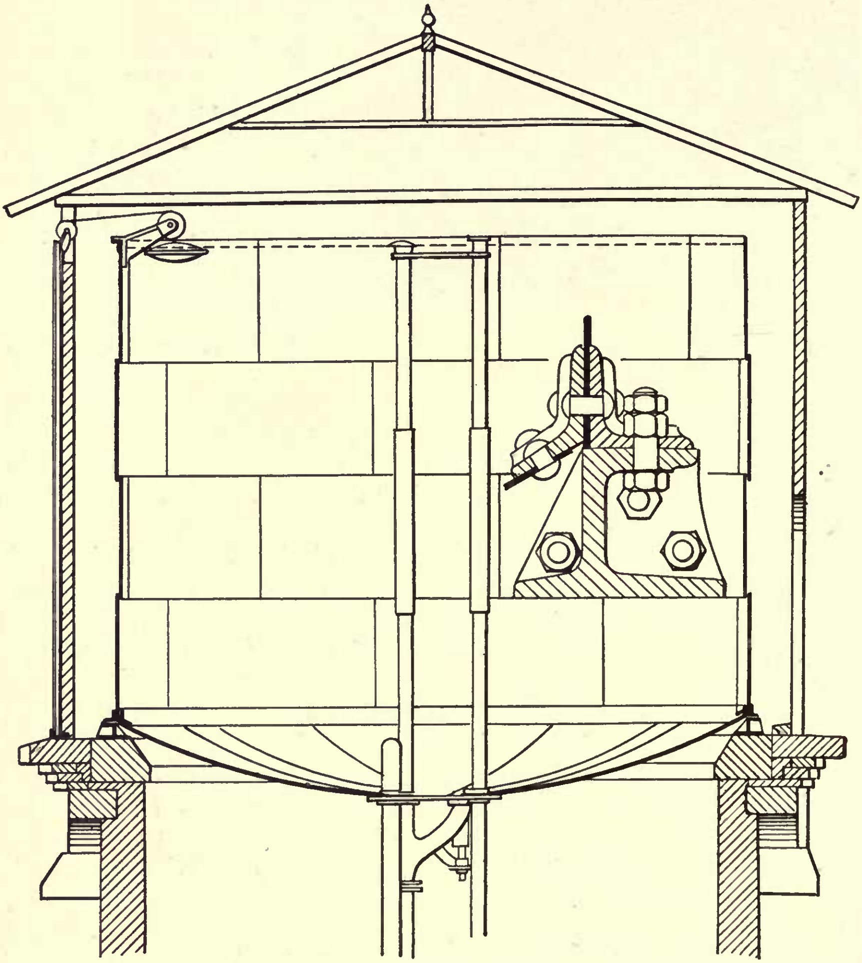


FIG. 1083.

with warmth in such metallic tanks in summer weather, has led to the making of tight tanks as in the elevator service, under a pressure of about 100 pounds to the square inch and earth covered.

Distribution.—Figs. 1084 to 1088 are sections of the spigot and faucet ends of cast-iron water-mains as used in Brooklyn and Philadelphia.

Below is given a table of dimensions, thickness, etc., of pipes, which may be considered fair averages for water-mains; 4″ pipe is the smallest diameter to be used in distribution, and if a hydrant is placed at the extremity of 500 feet of such pipe, the head will be very much reduced in case of fire service.

In cities the minimum diameter is usually 6″, except in the 4″ connection of dead ends for circulation. The thicknesses given in the table are as low as ad-

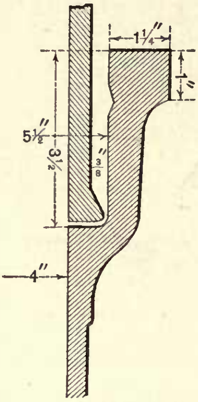


Fig. 1084.

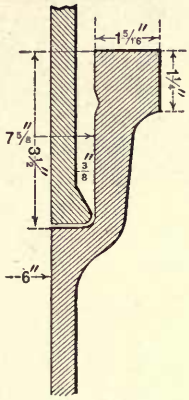


Fig. 1085.

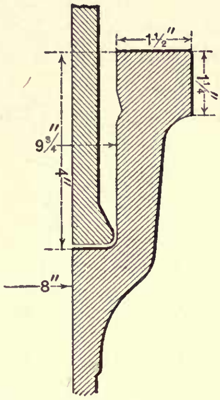


Fig. 1086.

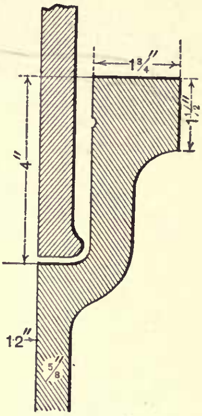


Fig. 1087.

missible for tapping, and if the pipe is not of uniform thickness, is to be put the thickest side up for this purpose.

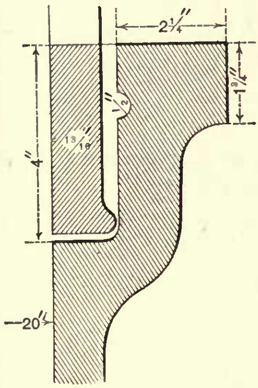


Fig. 1088.

The diameters given in the table are such as can be found in stock, but other sizes will be made to

Diameter.	Thickness.	Weight per foot laid.	Lead-joint weight.	Safe, feet head.
4	0·42	19·7	4	1,609
6	0·47	32·6	7	1,200
8	0·47	42·4	10	900
12	0·61	82·2	16	800
16	0·67	119·7	23	680
20	0·75	164·7	31	600
24	0·80	210·3	40	520
30	0·98	320·0	57	520
36	1·15	450·0	75	500
48	1·44	757·0	120	480

order; if ordered of extra thickness the bore is preserved uniformly, and the extra metal is put on the outside.

After laying, a hemp gasket is forced down to the lower end of the bell to prevent the molten lead from escaping into the pipe; the end of the pipe is then stopped by a clay roll, or a rope covered with clay, or clay alone, or with a metal clamp containing clay, and the molten lead is then poured in through an aperture or gate at the top; after cooling, the joint is made secure and tight by compacting the lead with calking tools made for this purpose.

Wrought-iron pipes have also been used for distribution, generally with cast-iron bells, riveted at one end and a small ring on the other (Fig. 1089) or by flanges secured on both ends and bolted together when laid.

Specials.—All parts of a main except the straight pipes are called special castings.

Fig. 1090 is a 12″ × 8″ 4-way branch, shown full and in section, diagonally.

The horns on the branch are for straps to hold in a plug, cap, or a connected short or curved pipe. The 4-way branches are often called crosses, and the

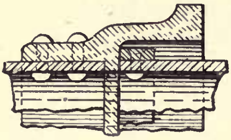


FIG. 1089.

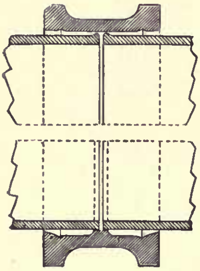


FIG. 1091.

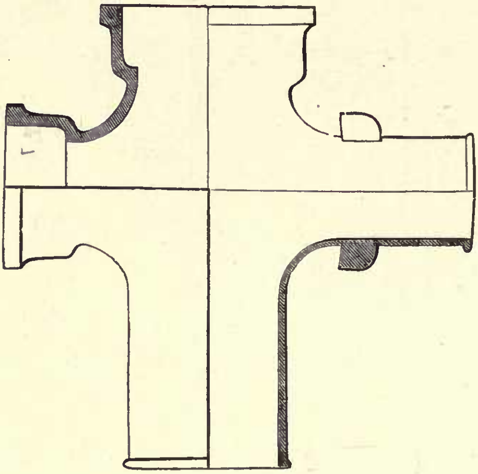


FIG. 1090.

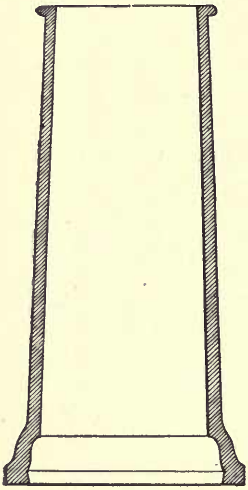


FIG. 1092.

3-way T's, or single branches. The branches may be of any appropriate size. In ordering, designate diameter of main pipe first, and then that of the branches. It is very common in these pipes to make all of the ends bell ends—it saves sleeves when pipes are cut, as they usually are at street intersections.

Fig. 1091 is a section of a sleeve for uniting cut pipes or uncut spigot-ends; a kind of double hub is often used for the former. Sometimes sleeves are made in halves, and bolted together.

Fig. 1092 is the section of a *reducer* for the connection of pipes of unequal diameters.

Fig. 1093 is the section of a *bend;* the horns on the outer circle are for straps between the pipes, as the pressure is unbalanced.

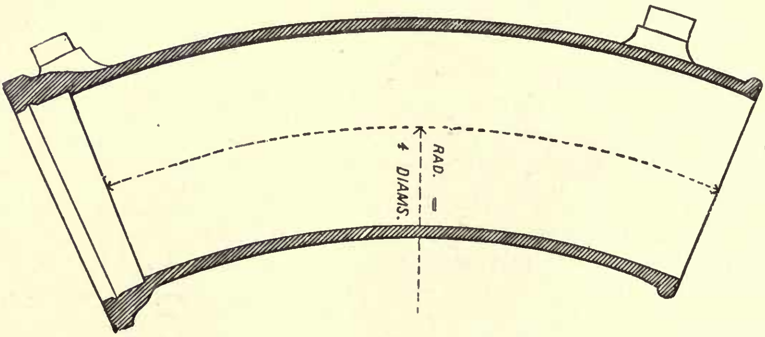


FIG. 1093.

House-services are mostly lead pipes; the taps on the mains for house-connections in New York city are usually 1″.

Fig. 1094 is a clamp sleeve.

Figs. 1095 and 1096 are globe specials of the Builders' Iron Foundry, Providence, a T and a cross branch, which serve the purpose of Fig. 1090 and are much lighter, and Figs. 1097 to 1102 are specials of the same makers.

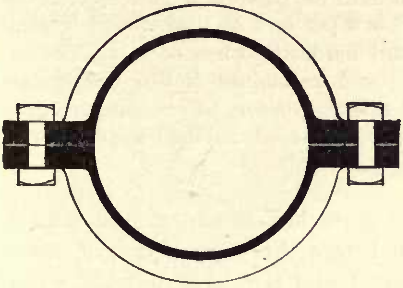


FIG. 1094.

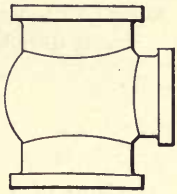


FIG. 1095.

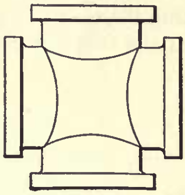


FIG. 1096.

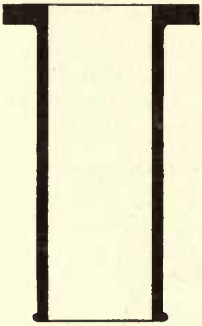


FIG. 1097.

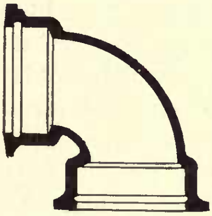


FIG. 1098.

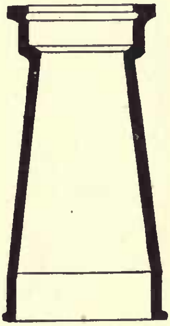


FIG. 1099.

FIG. 1100.

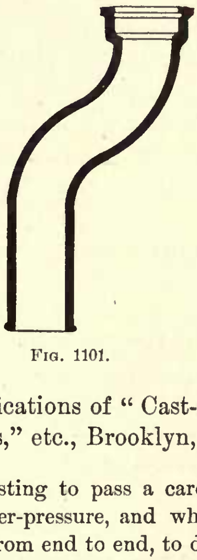


FIG. 1101.

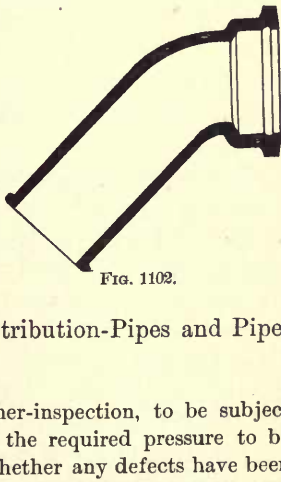


FIG. 1102.

According to the specifications of "Cast-Iron Distribution-Pipes and Pipe-Mains, with their Branches," etc., Brooklyn, L. I.:

Every pipe-branch and casting to pass a careful hammer-inspection, to be subject thereafter to a proof by water-pressure, and while under the required pressure to be rapped with a hand-hammer from end to end, to discover whether any defects have been overlooked. The pipes were to be carefully coated inside and outside with coal-pitch and oil, according to Dr. R. A. Smith's process, as follows:

"Every pipe must be thoroughly dressed and made clean from sand and free from rust. If the pipe can not be dipped promptly, the surface must be oiled with linseed-oil to preserve it until it is ready to be dipped; no pipe to be dipped after rust has set in. The coal-tar pitch is made from coal-tar, distilled until the naphtha is entirely removed and the material deodorized. The mixture of 5 or 6 per cent of linseed-oil is recommended by Dr. Smith. Pitch, which becomes hard and brittle when cold, will not answer. The pitch must be heated to 300° Fahr., and maintained at this temperature during the time of dipping. Every pipe to attain this temperature before being removed from the vessel of hot pitch. It may then be slowly removed and laid upon skids to drip."

Sewers.—For the removal of waste water from houses and rainfall, sewers are very convenient in towns and cities, even before the construction of water-works; but, after the introduction of a liberal and regular supply of water, sewers are indispensable. The ruling principle in the establishment of sewerage-works is, that each day's sewage of each street and of each dwelling should be removed from the limits of city and town promptly before decomposition begins, and that it should not be allowed to stagnate in the sewers, producing noxious gases prejudicial to health. To attain this end, the refuse fluids must be sufficient in quantity to float and carry off the heavier matters of sewage.

There has been considerable discussion of late whether sewage and rainfall should be carried off by a single system of pipes. This must depend largely on the location, economy of construction, and the financial ability to carry out the design. If the rainfall can be provided for by street gutters, the sewers may be for the conveyance of house-waste only and very small. If the sewage is to be discharged into a river of so large a flow that practically it will not pollute it, or into large bodies of water not used for domestic purposes, it is cheaper and better to discharge rainfall and sewage by one channel. The dimensions of the sewer will depend on the quantity to be discharged and the grade for which graphic sheets will be found in the Appendix. The rainfall is here estimated at 1″ in depth per hour for the whole area drained, but this is in many places exceeded for excessive rainfalls, and it is well at such times to have a partial relief by the street gutters. For the sewage flow it is usual to calculate it from the water that is or may be furnished on any length, and that the maximum that may be delivered at any one time is 50 per cent greater than the average flow, and further that at that time the sewer should be one half full.

The value of sewers depends on the correctness of their lines, uniformity of descent, and smoothness of interior surface.

For Washington, Brooklyn, and New York, and many other large cities discharging into large rivers, there is but one system of sewers for rainfall and sewage, and for the great areas drained very large sewers are necessarily constructed. Fig. 1103 is a section of a Washington sewer, the largest in the United States. The bottom course of the sewer, which is exposed to a strong current, is of stone; the ring courses are of brick—three for the 13-foot sewers and two for the 7-foot.

Fig. 1104 is a section of the largest Brooklyn sewer. In general there is no base or side supports of concrete or wall, except where the bottom is of quicksand. The sewer at the upper end is of circular section 10′ diameter, brickwork 12″; next size 12′, then 14′, and 15′—all brickwork 16″. At the discharge into Gowanus Creek the section is nearly rectangular with top of iron beams and brick arches, brick side-walls, and inverts.

The smaller sewers in all cities and towns are of vitrified ware or cement of circular section, or of cement of egg-shaped form, but beyond the diameter of 24 inches or its equivalent section brick is generally used. The vitrified pipe is laid either with (Fig. 1105) or without concrete base and side supports, depending somewhat on the kind of earth through which it is

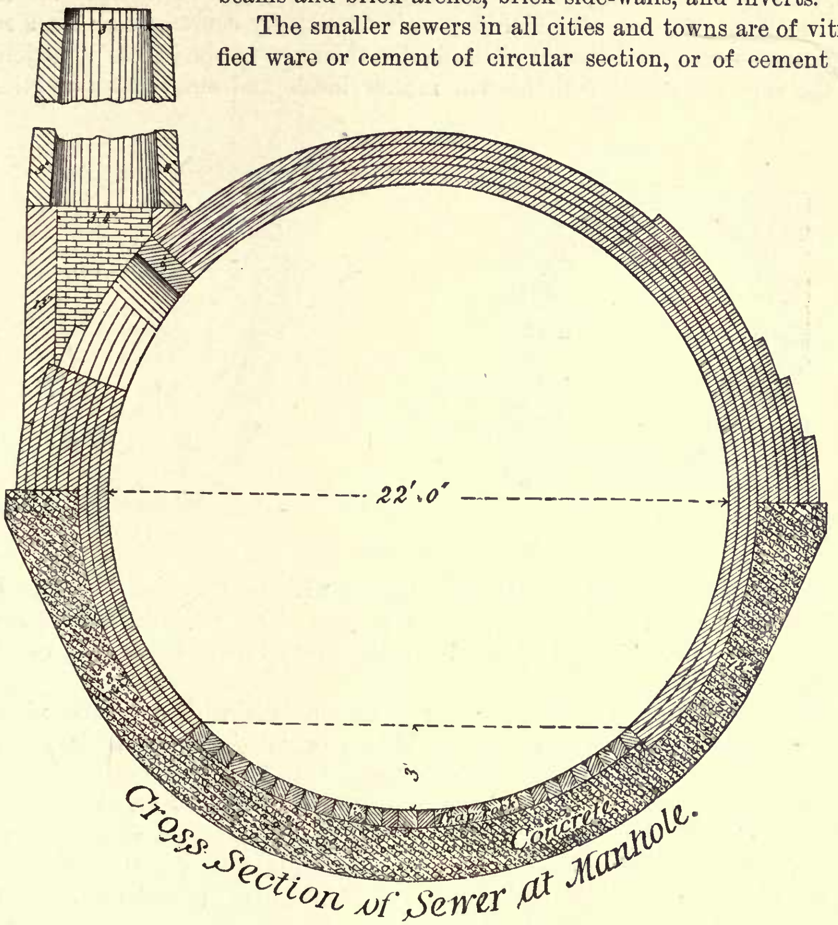


Fig. 1103.

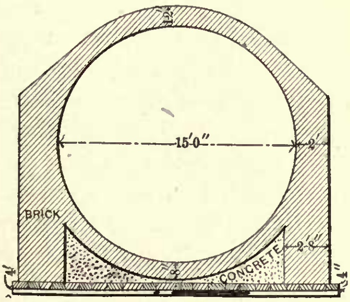


Fig. 1104.

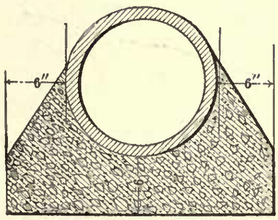


Fig. 1105.

laid and the desirability of uniformity or security of construction, but the joints are laid full in cement with a re-enforce outside and wiped clean on the inside. Up to 15″ the pipes are usually bell and spigot, larger they are laid with collars. The egg-shaped cement pipe is usually constructed with a sole plate, which rests on a base of plank double thickness at the joints. The joints of the pipe are struck with cement mortar inside and out, made very strong

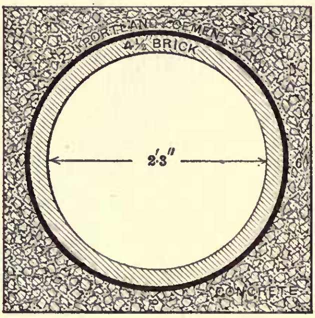


FIG. 1106.

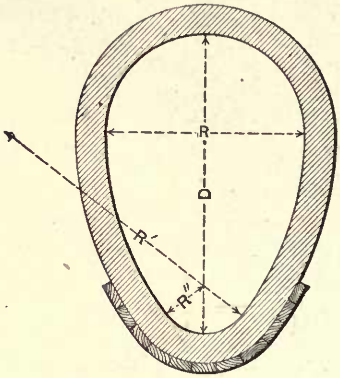


FIG. 1107.

outside and very smooth inside, but usually without concrete backing, but packed solidly in earth. Fig. 1106 is a section of an English form of sewer cheap and strong. The bricks are laid with close joints inside and an exterior coating of cement plaster.

This form of construction is not uncommon here with a central pipe of vitrified stoneware. The concrete backing is very desirable to withstand the shock of heavy moving loads, especially for pipes of large diameter.

Fig. 1107 is the usual form of egg-shaped pipe, of which the proportions are given in the table.

EGG-SHAPED SEWERS.

Equal to a circular sewer of	D inches.	D and R′ inches.	R″ inches.
Diameter.			
24 in.	19·8	29·8	5·0
30 "	24·8	37·2	6·2
36 "	29·8	44·7	7·4
42 "	34·8	52·1	8·7
48 "	39·7	59·5	9·9
54 "	44·6	67·0	11·2
60 "	49·6	74·4	12·4

Thickness of brickwork, 8″; boards shown at bottom only used in cases of soft earth for convenience of construction. For area of egg-shaped sewer of above section, multiply R^2 by 4·6.

In general, brick sewers are only well packed in earth, but when the soil is loose and it is desirable to secure an extended base and side supports, the form Fig. 1108 is adopted, which may be in rubble masonry or concrete, and if the earth cover is light and the structure exposed to heavy moving loads the masonry should be carried above the arch. At the bottom, where the arch is of small radius and it is difficult to get good joints in the brickwork, it is very common to place inverts of vitrified ware which are sold for this purpose.

Fig. 1109 is a section of a sewer constructed *in situ* at Mount Vernon, N. Y., through a long rock-cut. Concrete was rammed in around an inverted centre, and after it was withdrawn the face was plastered with a very thin coat of

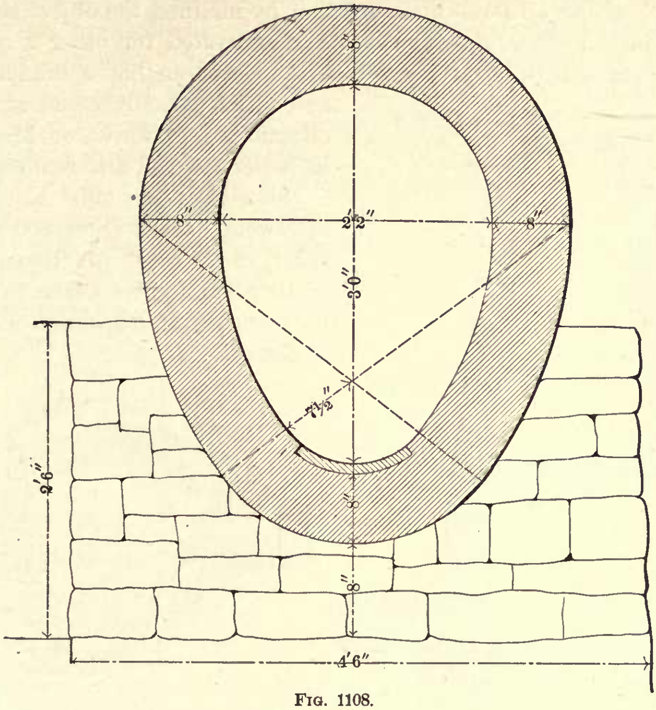


Fig. 1108.

Portland cement; the invert is of vitrified ware. A plumb ⊥ is inserted across the sewer (Fig. 1110) with points for centres at different heights, the radius is uniform, and the arch turned as high as convenient for the rock sides and

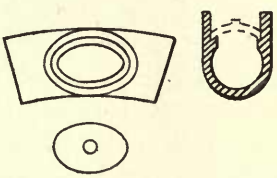


Fig. 1111.

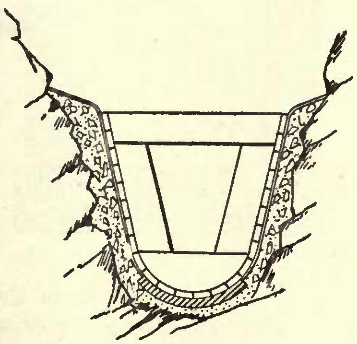


Fig. 1109.

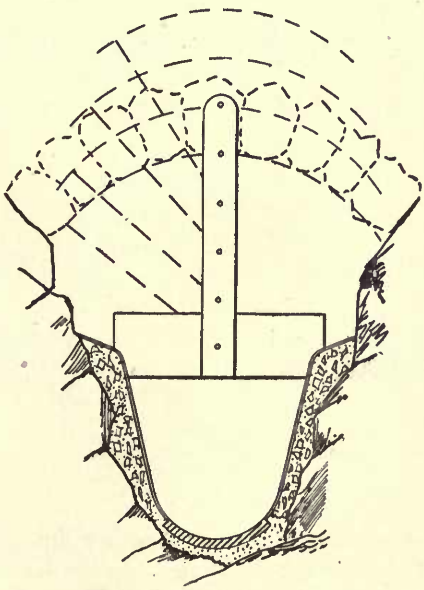


Fig. 1110.

street grades, either on levels and offsets or by inclines, the object being to secure space in the sewer, and save the rock excavated for other city purposes. Fig. 1111 represents the first joint from the sewer on house branches curved and with a cover by means of which any obstruction in sewer or branch could be better reached and removed.

Man-holes are built along the line of sewers, usually from 200 to 400 feet apart, and at every junction and change of direction, to give access to the sewers for purposes of inspection and removal of deposit.

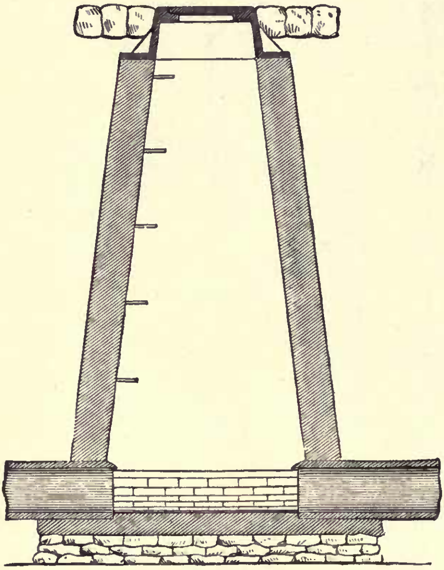
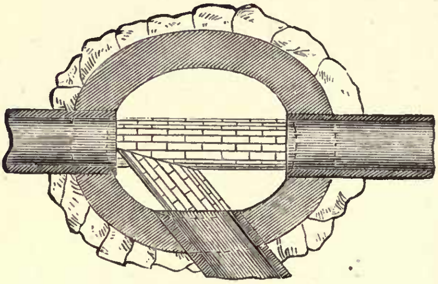


FIG. 1112.



SCALE: $\frac{1}{4}''$ = 1 foot.
FIG. 1113.

In the large Washington sewer the man-hole is constructed on the side of the sewer; in the Brooklyn one the aperture of the man-hole comes within the exterior wall. In most sewers it is directly over them.

Figs. 1112 and 1113 are section and plan of the man-hole at present used by the Croton Sewer Department. It consists of a funnel-shaped brick wall, oval at the bottom 4′, circular at top 2′ diameter.

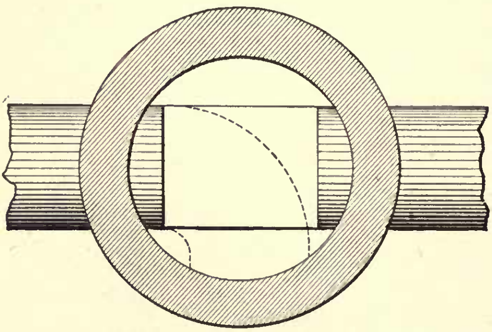


FIG. 1114.

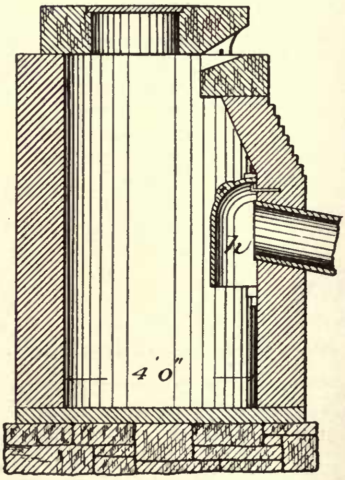


FIG. 1115.

Side-walls, 8″ thick, through which the pipe-sewers pass at the bottom of the well. Across the open space the sewer is formed in brick, whose bottom section corresponds to that of pipe, side-walls carried up perpendicular to top of sewer;

the flat spaces at the sides of sewer are flagged. The top of the sewer is a heavy cast-iron frame, fitted with a strong cover, which may or may not be perforated, for ventilation. In the figure the main sewer is 12″ pipe, with a 12″ branch entering at an acute angle, as all branches and connections with a sewer should. The short lines on the left vertical wall represent sections of **U** staples, built in to serve for a ladder.

When the T is at right angles to the through pipe it is very convenient to form the base, as in Fig. 1114, in concrete to a wooden mould, with the inside plastered. This form enables the sewer to be seen in all directions by a light from man-hole to man-hole. Curved pipes are now seldom laid; angles are made at the man-holes, with straight pipe between. When necessary, catch-basins (of which Fig. 1115 is a vertical section, Fig. 1116 a half plan and a half sectional plan) are placed at the corner of the street or at depressions of the street gutters, to catch any heavy matter that might clog the sewer or fill the dock slips, and is removed by hand. The cast-iron hood makes a seal, preventing the escape of sewer gas through the sewer branch. Fig. 1117 is a perspective drawing of the basin head at the corner of the street.

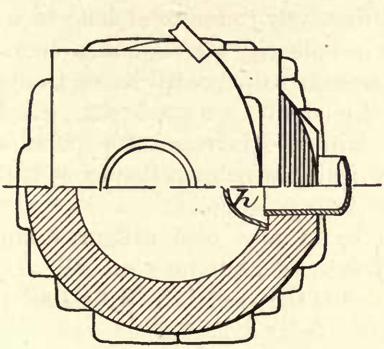


FIG. 1116.

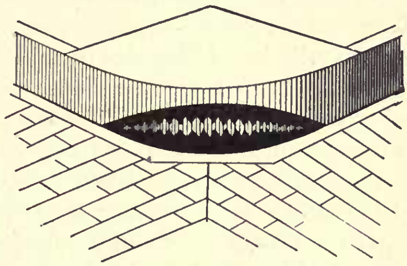


FIG. 1117.

Where, by the difference of grades between the main line of sewer and the branches, the latter are at a considerable height above the base of the man-hole, it is best to lead the branch down by a vertical cast-iron pipe having a cross at the top for access to the branch, and an elbow at the bottom to protect the masonry from the effects of the discharge.

Gas-Supply.—Next in importance to the necessities of a city or town for water-supply and sewerage is the luxury of gas-supply. The gas-works should always be placed remote from the thickly populated part of the city, for under the best regulations some gas (offensive and deleterious) will escape in the manufacture. They should be placed at the lowest level, for gas, being light, readily rises, and the portions of the city below the works are supplied at less pressure than those above. Gas-mains, like those for water, are of cast-iron, and put together in the same way; but, as they have to resist no pressure beyond that of the earth in which they are buried, they are never made of as great thickness as those of water-pipes, but drips must be provided, and the pipe laid with such inclination to them that the condensed tar may be received in them and pumped out.

Owing to the reduction in price complete systems of wrought-iron gas-pipe are now laid, with the usual screw-coupling and tested to a pressure much in excess of that of the gas.

WEIGHT OF GAS-PIPES PER RUNNING FOOT.

3″	12 lbs.	10″	50 lbs.
4″	16 "	12″	62 "
6″	27 "	16″	103 "
8″	40 "	20″	150 "

The holes for access to the valves on gas-pipes usually consist of two pipes, the lower one with a saddle base resting on the pipe, the upper with a cap at the top which is adjustable to the height of the pavement by screws cast on both pipes, the upper one being the extension.

Roads and highways are terms applied distinctively to routes of land travel in the country, though in general they may include city streets and avenues. Lines of canal, river, lake, and ocean transit are often designated as highways.

The first requirements in the opening up of a country are good roads, which must naturally be of the cheapest kind, but with the increase of population they must be made more and more permanent, and the reflex action extends to farming and mechanical facilities and increase in population.

For constant and continued use there can be no good road without drainage; there may be seasons when hauling is practicable, but no road will stay good unless the surplus water is got rid of, and if the road is well drained it will pay the farmer to extend the drainage to the fields adjacent.

Fig. 1118 is a section of a dirt road in which there is a central drain, and above it a 4″ layer of coarse straw, hay, or stubble, the bed for which has been excavated on slopes toward the drain of 1″ to 2″ to the foot. The earth should then be filled above in layers and rolled, with slopes to the sides sufficient to

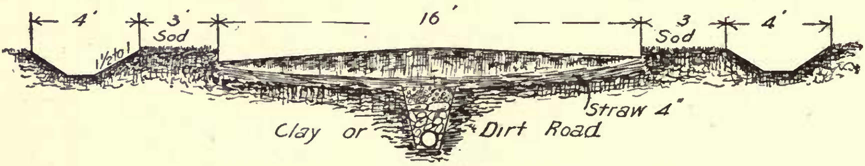


FIG. 1118.

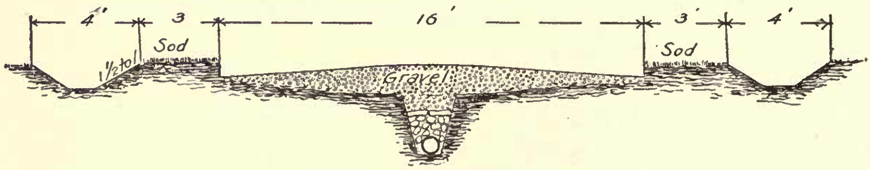


FIG. 1119.

carry off all the water from the centre. The width of a common road should be 16 feet, slopes of about 8 inches to each side, grade not to exceed 9′ to the hundred, or about 5°, and this not continuous, but broken by changes of grade.

Fig. 1119 is a section of a gravel road similar to the dirt road, but, as illustrative of a road with a steep grade, the cut across the turf borders are diagonal to turn the water more readily into the ditches, while in the level road the cuts are at right angles. If not convenient to construct the drains in the centre of the road, they may be placed at the sides with the sub-slopes discharging into them. Throughout our country there is at the present time a strong movement in favour of good roads, and machinery has been designed and constructed by which the cost of construction has been very much reduced, with great improvement in the character of the roads.

Across marsh lands it is important that the surface of sods and roots should not be broken, but that the weight of the roadway should be further distributed by a layer of brush, fagots, or poles, on which the layers of earth should be sods from the marsh taken distant from the roadway, compacted in layers and finished like the dirt road, but without gutters, or at such distance and depth as not to weaken the road float.

For country roads, wagons subject to heavy loads should have wide tires, say 4″ tires for the front wheels and 6″ for the rear ones, and the paths of the wheels should lap but not track, the axles of the rear wheels being longer than those of the fore.

Near the sea, where the soil is sand or gravel and stone is not to be had readily, oyster shells make an admirable road; they are arranged in layers like Macadam, and grinding together beneath rolling and travel, they make a compact and solid cover.

Macadam was the first to reduce the construction of broken-stone roads to a science, and has given the name, in his own and this country, to all this class of roads. He says that "the whole science of artificial road-making consists in making a dry, solid path on the natural soil, and then keeping it dry by a durable water-proof covering." The road-bed, having been thoroughly drained, must be properly shaped, and sloped each way from the centre, to discharge any water that may penetrate it. Upon this bed a coating of 3″ of clean broken stone, free from earth, is to be spread on a dry day. This is then to be rolled, or worked by travel till it becomes almost consolidated; a second 3″-layer is then added, wet down so as to unite more readily with the first; this is then rolled, or worked, and a third and fourth layer, if necessary, added. Macadam's standard for stone was 6 ounces for the maximum weight, corresponding to a cube of 1½″, or such as would pass in any direction through a 2½″ ring. The Telford road is of broken stone, supported on a bottom course or layer of stone set by hand in the form of a close, firm pavement.

Early in the construction of the New York Central Park, after trials of the Macadam and Telford roads, gravel was adopted and still maintains its position for these roads. The gravel road, of which a cross-section of one half is shown (Fig. 1120), consists of a layer of rubble-stones, about 7″ thick, on a well-rolled or packed bed, with a covering of 5′ of clean gravel. C is the catch-basin for

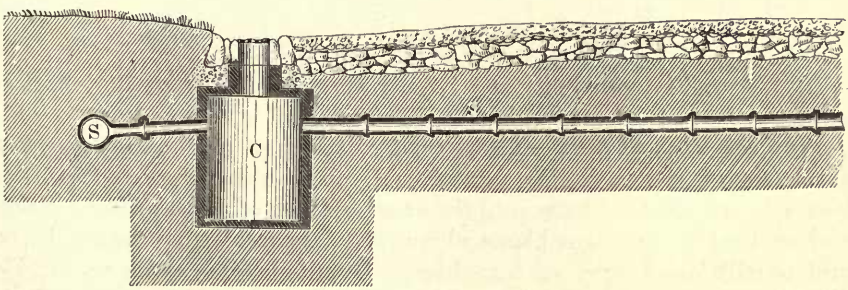


FIG. 1120.

the reception of water and deposit of silt from the gutters; S is the main sewer or drain, and *s* a sewer-pipe leading to a catch-basin on opposite side of the road. In wider roads each side has its own main drain, and there is no cross-pipe *s*. The road-bed was drained by drain-tiles of from 1½″ to 4″ bore, at a depth of 3′ to 3½′ below the surface. The maximum grade of the Park roads is 1 in 20.

In New York the streets above Fourteenth Street running north and south are called avenues, and those at right angles, streets, and *boulevard* is applied to very wide avenues in which there are rows of trees. The terms street and avenue, as laid out, are the established bounds within which no buildings may be erected. The street, therefore, technically includes the street or travelled way for carriages, and the sidewalks and front areas. New York streets above Fourteenth Street are 60 and 100 feet wide, avenues 100 feet, of which the carriage-way occupies one half (Fig. 1121), and the sidewalks and area one quarter on each side. The space occupied by areas is from 5 to 8 feet, which may be inclosed by iron fence; the space beneath it and the sidewalk to the curb and used for vaults can be leased from the city. The stoop-line extends into the sidewalk beyond the area-line some 1′ to 18″, fixing the limit for the first step and newel to a high stoop or platform. The boulevard in the old line of upper Broadway and the Bloomingdale Road is 150 feet wide, of which 100 feet are to be carriage-way, and 25 feet on each side for sidewalk and area, the latter not to exceed 7 feet; one row of trees to be set within the sidewalk, about 2 feet from the curb.

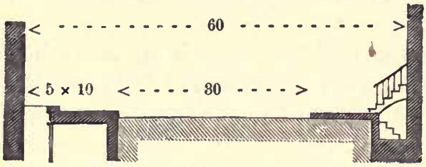


Fig. 1121.

The best grade is from 1 in 50 to 1 in 100; this gives ample descent for the flow of water in the gutters. Many of our street-gutters have a pitch not exceeding 1 foot in the width of a block, or 200 feet.

The grade of a road is described as 1 in so many; so many feet to the mile, or such an angle with the horizon:

Inclination.	Feet per mile.	Angle.	Inclination.	Feet per mile.	Angle.
1 in 10	528	5° 43′	1 in 30	176	1° 55′
1 " 11	462	5°	1 " 40	132	1° 26′
1 " 14	369	4°	1 " 50	106	1° 9′
1 " 20	264	2° 52′	1 " 57	92	1°
1 " 29	184	2°	1 " 100	53	35″

The foot-walks in this city and vicinity are generally formed of flags, or what is here termed blue-stone, laid on a bed of sand or cement-mortar. The flags are from 2″ to 4″ thick. Stone thicker, select in quality, upper surface bush hammered or planed, close jointed, and covering the whole width of the sidewalk in one stone, add much to the exterior finish of large houses. Bricks are often used in towns, or places where good flagging can not be readily obtained, usually laid flatways on a sand-bed. Granite is often employed for sidewalks in lengths equal to the width of the sidewalk, and making a cover for the vault beneath; the objection to it is that by wear and under certain atmospheric conditions it becomes slippery.

Cement face, with a base of concrete and laid in squares to admit of expansion, makes a good and permanent walk. For the country a composition of coal tar, or asphalt and gravel, is economical and satisfactory.

In Paris there is no area (Fig. 1122); the sidewalk comes up to the house or street-line, and there is a space for trees between sidewalk and street-curb.

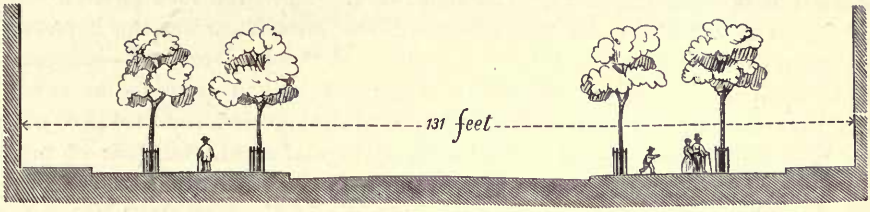


FIG. 1122.

This space is available for pedestrians, a part being a gravel, asphalt, or flagged walk. The following are the dimensions according to the law of June 5, 1856:

Entire width of boulevard and avenues.	Width of carriage-way.	Width of sidewalk.	Width for trees.	Rows of trees.	DISTANCE OF ROW FROM	
					Street-line.	Street-curb.
Metres.	Metres.	Metres.	Metres.		Metres.	Metres.
26 to 28	12	...	....	1	5·5 to 6·5	1·5
30 " 34	14	...	....	1	6·5 " 8·5	1·5
36 " 38	12 to 13	3·5	8·0 to 8·5	2	5·0 " 5·5	1·5
40	14	3·5	9·5	2	6·5	1·5

1 metre = 3·281 feet.

GRANITE BLOCK PAVEMENT WITH FOUNDATION AGREEABLY TO NEW YORK CITY SPECIFICATIONS (Fig. 1123).

Stone blocks of granite, measuring on the upper surface not less than 8″ nor more than 12″ in length, not less than $3\frac{1}{2}''$ nor more than $4\frac{1}{2}''$ in width, not

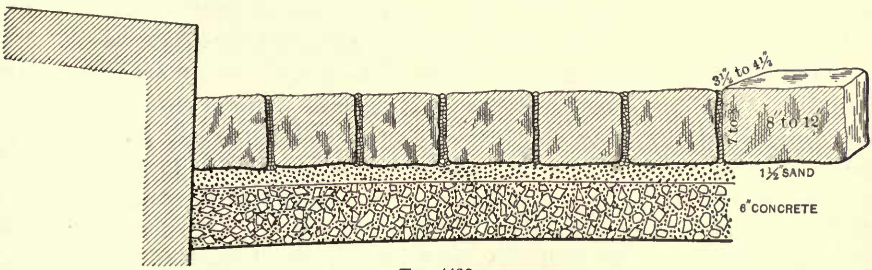


FIG. 1123.

less than 7″ nor more than 8″ in depth, blocks to be dressed to form when laid, close-end joints, and side joints not exceeding 1″ in width.

Bridge or Crossing Stones.—Each stone to be not less than 4′ nor more than 8′ long and 2′ wide; thickness, from 6″ to 8″; dressed to an even face on top, bottom bedded, sides square and full, ends cut to a bevel of 6″ in 2′ not parallel with the line of vehicle travel.

Curbstones shall not be less than 3′ in length, 5″ thick, 20″ deep, matched width, the top cut to a bevel of 1″, front cut to a fair line to a depth of 14″, ends from top to bottom squared.

The subsoil to be excavated and removed to a depth of 16″ below the top line of the proposed pavement; roadbed shall be truly shaped and trimmed to the required grade, and rolled to ultimate resistance with a roller weighing not less than ten tons. When the roller can not reach any portion of the roadbed it shall be tamped or rolled with a small roller. Upon the foundation a 6″ bed of concrete, except in the space between the rails known as the horse-ways, where no concrete shall be laid. Concrete of Portland cement—one part of cement, three parts of sharp sand, seven parts of broken stone by measure; or one part of cement, three parts of sand, four of broken stone, and three of gravel. Of Rosendale cement—one part of cement, two of sand, and four of broken stone; or one of cement, two of sand, two of broken stone, and two of gravel.

On this concrete foundation, and on the foundations of the horse-ways of street railways (where no concrete shall be laid) shall be laid a bed of clean, sharp sand, perfectly free from moisture, not less than 1½″ thick, to the depth necessary to bring the pavement to the proper grade when properly rammed. Upon this bed of sand cross-walks to be laid, stone blocks in courses at right angles with the line of street, except in intersections of streets, when the courses shall be laid diagonally. Each course of blocks shall be of uniform depth and width, so laid that all end joints shall be broken by a lap of at least 3″; joints between courses not more than 1″ in width. As the blocks are laid they shall be covered immediately with clean, hard, hot, dry gravel, of proper size, which shall be brushed into the joints until all the joints become filled; gravel shall be free from sand. Blocks shall be thoroughly rammed to an unyielding bearing, with uniform surface, true to the roadway grade. Before pouring the paving cement, the joints and gravel filling must be made dry and free from dirt. Paving cement shall be poured into the joints while the gravel is still hot (paving cement to be heated to 300° Fahr.) until the joints are filled flush with the top of the blocks.

Paving cement to be composed of twenty parts refined Trinidad asphalt and three parts of residuum oil, mixed with a hundred parts of coal tar.

Gutter stones are not now used, the paving extending to the curbstone.

Granite Pavement without Concrete Bed—Similar to the above, excepting that the subsoil shall be excavated and removed to the depth of 10″ below the top line of the proposed pavement when rolled or rammed. Upon this foundation shall be laid a bed of clean sharp sand, or clean fine gravel, to the depth necessary to bring the pavement and cross-walks to the proper grade. No ramming shall be done within 25 feet of the face of the work being laid. Whenever the pavement shall have been constructed it shall be covered with a good and sufficient second coat of clean sharp sand, and immediately thoroughly rammed until the work is made solid and secure; no paving cement in the joints of the blocks.

Asphalt Pavement with Concrete Foundation (Fig. 1124).—The subsoil or other matter shall be excavated and removed to the depth of 9 inches below the top line of the proposed Trinidad asphalt pavement, which shall have a crown not to exceed the rate of 5 inches on a roadway of 30 feet, 9½ inches below the top of the proposed asphalt pavement, and 13½ inches below the top of the stone block pavement adjoining the rails, man-hole heads, and stop-cock boxes,

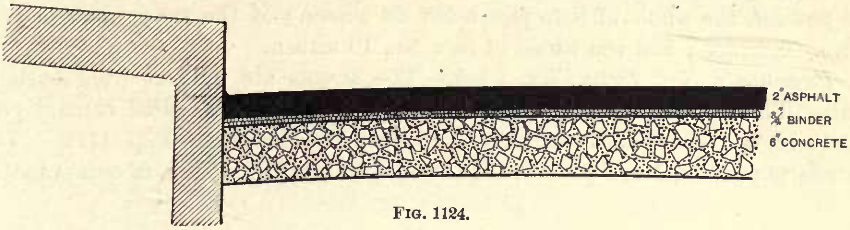


Fig. 1124.

the entire road-bed to be thoroughly rolled with a heavy roller. Upon the foundation thus prepared shall be laid a bed of hydraulic cement concrete, 6 inches in thickness, which, if necessary, must be protected from the action of the sun and wind until set. Upon this foundation must be laid a fine bituminous concrete or binder of clean, broken stone not exceeding 1¼ inch in their largest dimensions, thoroughly screened, and coal-tar residuum, commonly known as No. 4 paving composition. The stone to be heated by passing through revolving heaters, and thoroughly mixed by machinery with the paving composition in the proportion of 1 gallon of paving composition to 1 cubic foot of stone. The binder to be spread with hot iron rakes to true grade of the pavement and to such thickness that, after being thoroughly compacted by tamping and hand rolling, the surface shall have a uniform grade and cross-section, and the thickness of the binder at any point shall be not less than three fourths of an inch, the upper surface to be parallel with the surface of the pavement to be laid. Upon this foundation must be laid the wearing surface, the basis of which, and of paving cement, must be pure asphaltum, unmixed with any of the products of coal tar. The wearing surface to be composed of refined asphaltum, heavy petroleum oil, fine sand containing not more than one per cent of hydrosilicate of alumina and fine powder of carbonate of lime; asphalt from the Pitch Lake, on the Island of Trinidad; petroleum oil to be freed from all impurities and brought to a specific gravity of from 18 to 22° Beaume, and a fire test of 250° Fahr. The cement from these two hydrocarbons shall have a fire test of 250° Fahr., to be composed of 100 parts of pure asphalt and from 15 to 20 parts of petroleum oil.

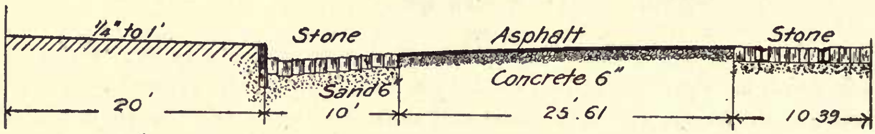
The pavement mixture to be composed of:

Asphaltic cement	12 to 15 parts.
Sand	83 to 70 "
Pulverized carbonate of lime	5 to 15 "

Sand and asphaltic cement to be heated separately to about 300° Fahr., the carbonate of lime while cold to be mixed with the hot sand and then with the asphaltic cement. The pavement mixture, at a temperature of about 250°, shall then be carefully spread by means of hot iron rakes in such manner as to give a uniform and regular grade. The surface shall then be compressed by hand rollers, after which a small amount of hydraulic cement shall be swept over it, and it shall then be thoroughly compressed by a steam roller weighing not less than 250 pounds to the inch run, the rolling to be not less than five hours for every 1,000 yards of surface. After its ultimate compression the pavement must have a thickness of not less than 2 inches.

Of the powdered carbonate of lime, 5 to 15 per cent shall be of an impalpable powder, the whole of it to pass a No. 26 screen; of the sand, none to pass a No. 80 screen; and the whole of it, a No. 10 screen.

Pavements, Salt Lake City, Utah.—The streets are 92 feet from curb to curb. The pavement is practically in two portions—an asphalted central portion and two stone-block surfaces—arranged as shown in Fig. 1125. The foundation of the block pavement is 10 feet wide on each side, is composed of



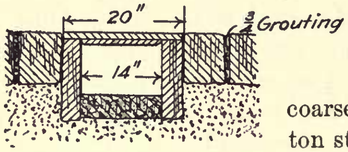


FIG. 1125.

coarse sand laid on a foundation compacted by a 10-ton steam roller; the sand stratum is 6″ thick. The blocks are of very hard sandstone, and vary from 7″ × 3½″ × 7″ to 10″ × 4½″ × 7″; they form a pavement 7″ thick, with joints not over ⅜″ wide, filled with fine-screened gravel and grouted pitch. Before the joints are poured the blocks are rammed with hammers weighing from 75 to 80 pounds until brought to a firm bed at the proper level. The asphalted portion of the street is laid on a concrete foundation 6″ thick.

Methods of laying Brick Pavements (Fig. 1126).—After excavation the ground is compacted with rollers weighing about 2 tons. A foundation course

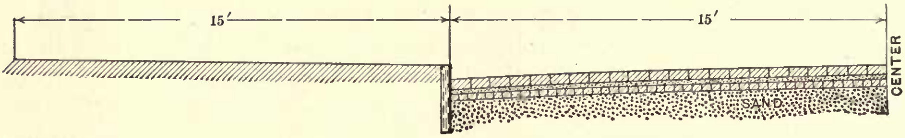


FIG. 1126.

of sand or broken stone is thoroughly compressed with the roller to the exact form and crown of the finished pavement. On this foundation a layer of brick is laid flatwise, with the longest dimension longitudinal with the roadway; on this layer sufficient sand is spread to fill all joints and then rolled or rammed; then a cushion of 1″ or 2″ of sand, and on this a layer of brick is set on edge, with the longest dimension across the roadway, which layer is either rolled or rammed. After rolling is completed, any broken bricks are replaced with whole ones; this is also done in the first layer. By some a foundation is preferred composed of 8″ to 15″ of broken stone, made compact with rollers weighing from 12 to 15 tons, and upon this is spread a 3″ cushion of sand. A crown of 4″ is allowed in a roadway 50 feet wide, and 3″ in a roadway 40 feet wide.

Modulus of rupture in compression: First-quality brick, 1,700 pounds per square inch; absorption, 1·6 per cent; abrasion under test to be equal to granite.

Modulus of rupture in compression: Second-quality, 1,500 pounds per square inch; absorption, 5 per cent; abrasion not to exceed twice that of granite.

Of wooden pavements, cedar block is the most general (Fig. 1127). The

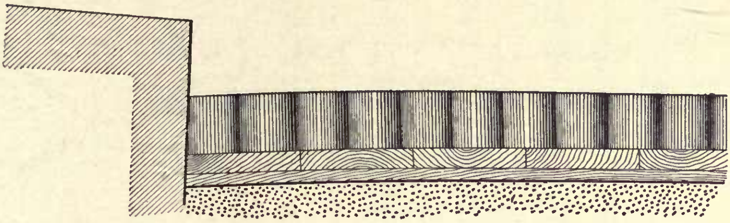


FIG. 1127.

curbstone is set to the true grade, and the ground between is graded to the true cross-section of the street. After this the surface is rolled by a heavy steam roller; next, 3″ of coarse sand is spread over the entire surface. Stringers 1″ × 12″ are then laid across the street from curb to curb, usually 8″ apart and bedded in the sand to the true section of the roadway; after the stringers are in place and well tamped, sand is levelled between flush to the top of the stringers. On the stringers and sand the foundation planks, 1″ to 2″ thick, well seasoned and dry, are laid close together lengthwise of the street, the ends abutting on the stringers. Sometimes the foundation boards are dipped in hot coal tar before laying. The paving blocks are sawed from peeled cedar fence posts, and run from 4″ to 8″ diameter. The standard length is 6″, but some cities use blocks 7″ or 8″ long.

The cedar blocks are packed on end close together upon the plank foundation, and with the spaces between adjoining blocks three-sided rather than more sides. Against the curb, or any other straight vertical surface, each alternate block should be split in halves and the straight side placed against the curb. After the blocks are in place the spaces between them are filled with gravel rammed down by iron rods. Some require that a paving cement, composed of coal tar and asphalt, be poured over the whole surface of the pavement, and run into the spaces between the blocks. The entire surface of the pavement is then covered with about 1″ of fine roofing gravel.

RAILROADS.

The necessity of a well-drained road-bed is as important beneath rails as on a highway. The cuts should be excavated to a depth of at least 2 feet below grade, with ditches at the sides still deeper, for the discharge of water. The embankments should not be brought within 2 feet of grade; this depth to be left in cut and on embankment for the reception of ballast. The best ballast is Macadam stone, in which the cross-ties are to be bedded, and finer broken stone packed between them. Good coarse gravel makes very good ballast; but sand, although affording filtration for the water, is easily disturbed by the passage of the trains, raising a dust, an annoyance to travellers, and an injury to the rolling-stock by getting into boxes and bearings. The average length of sleepers on the 4′ 8½″ gauge railways is about 8 feet; bearing surface, 7″; distance between centres, 2′ to 2′ 6″, except at joints, where they are as close to each other as the necessity of tamping beneath them will admit. Average width of New York railways of same gauge as above, for single lines, in cuts 18′, banks 13′; for double lines, cuts 31′, banks 26½′.

Figs. 1128 and 1129 are two standard sections of the permanent way of the

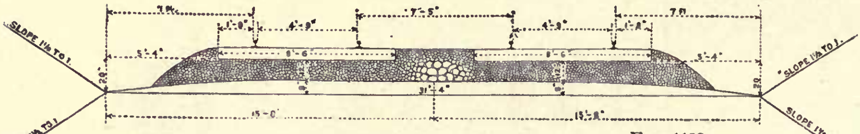


Fig. 1128.

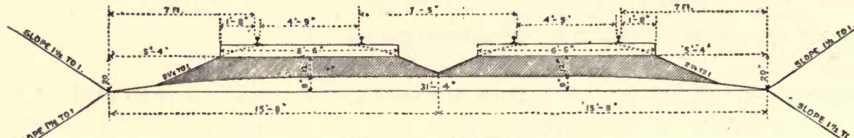


Fig. 1129.

Pennsylvania Railroad, in which the width of cuts and top of embankments are the same, 31′ 4″, and other dimensions equally ample.

Rail sections are of infinite variety and weights, adapted to the class of railroads on which they are to be used, and the loads and speed of trains to which they are to be subjected. For roads of the common gauge, the weight of rails is from 56 to 100 pounds per yard. The joints are made with a fish-plate.

Figs. 1130, 1131, and 1132 are the elevation, section, and plan of the standard rail-joint of the West Shore Railroad.

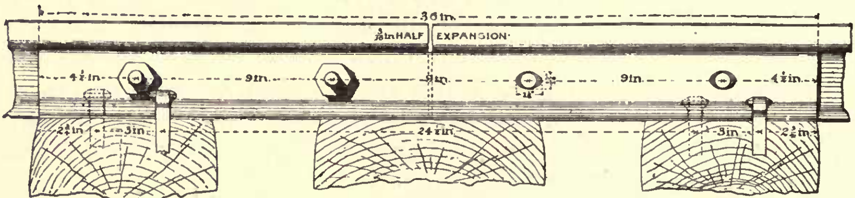
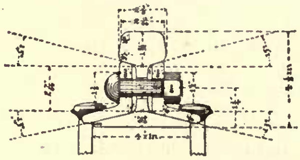


Fig. 1130.



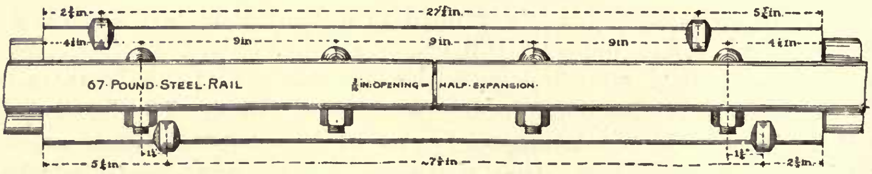


Fig. 1131.

With the increase of speed on railways, the tracks on the larger roads are being laid with rails of 100 pounds to the yard, and wrought-iron ties are being tested, but experiments on them with very heavy trains and high speeds have not yet been conclusive as to their adoption.

DIMENSIONS OF STANDARD RAILS.*

Pounds per Yard.	A.	B.	C.	D.	E.	F.	G.
40	$3\frac{1}{2}$	$3\frac{1}{2}$	$1\frac{7}{8}$	$\frac{5}{8}$	$1\frac{55}{64}$	$1\frac{1}{64}$	$\frac{25}{64}$
45	$3\frac{11}{16}$	$3\frac{11}{16}$	2	$\frac{21}{32}$	$1\frac{31}{32}$	$1\frac{1}{16}$	$\frac{27}{64}$
50	$3\frac{7}{8}$	$3\frac{7}{8}$	$2\frac{1}{8}$	$\frac{11}{16}$	$2\frac{1}{16}$	$1\frac{1}{8}$	$\frac{7}{16}$
55	$4\frac{1}{16}$	$4\frac{1}{16}$	$2\frac{1}{4}$	$\frac{23}{32}$	$2\frac{11}{64}$	$1\frac{11}{64}$	$\frac{15}{32}$
60	$4\frac{1}{4}$	$4\frac{1}{4}$	$2\frac{3}{8}$	$\frac{49}{64}$	$2\frac{17}{64}$	$1\frac{7}{32}$	$\frac{31}{64}$
65	$4\frac{7}{16}$	$4\frac{7}{16}$	$2\frac{13}{32}$	$\frac{25}{32}$	$2\frac{3}{8}$	$1\frac{9}{32}$	$\frac{1}{2}$
70	$4\frac{5}{8}$	$4\frac{5}{8}$	$2\frac{7}{16}$	$\frac{13}{16}$	$2\frac{15}{32}$	$1\frac{11}{32}$	$\frac{33}{64}$
75	$4\frac{13}{16}$	$4\frac{13}{16}$	$2\frac{15}{32}$	$\frac{27}{32}$	$2\frac{35}{64}$	$1\frac{27}{64}$	$\frac{17}{32}$
80	5	5	$2\frac{1}{2}$	$\frac{7}{8}$	$2\frac{5}{8}$	$1\frac{1}{2}$	$\frac{35}{64}$
85	$5\frac{3}{16}$	$5\frac{3}{16}$	$2\frac{9}{16}$	$\frac{57}{64}$	$2\frac{3}{4}$	$1\frac{35}{64}$	$\frac{9}{16}$
90	$5\frac{3}{8}$	$5\frac{3}{8}$	$2\frac{5}{8}$	$\frac{59}{64}$	$2\frac{55}{64}$	$1\frac{19}{32}$	$\frac{9}{16}$
95	$5\frac{9}{16}$	$5\frac{9}{16}$	$2\frac{11}{16}$	$\frac{15}{16}$	$2\frac{63}{64}$	$1\frac{41}{64}$	$\frac{9}{16}$
100	$5\frac{3}{4}$	$5\frac{3}{4}$	$2\frac{3}{4}$	$\frac{31}{32}$	$3\frac{5}{64}$	$1\frac{45}{64}$	$\frac{9}{16}$

* Dimensions not given in table are constant, and are to be taken from the standard section of a 70-pound rail.

Fig. 1132 is a drawing of a standard 70-pound rail.

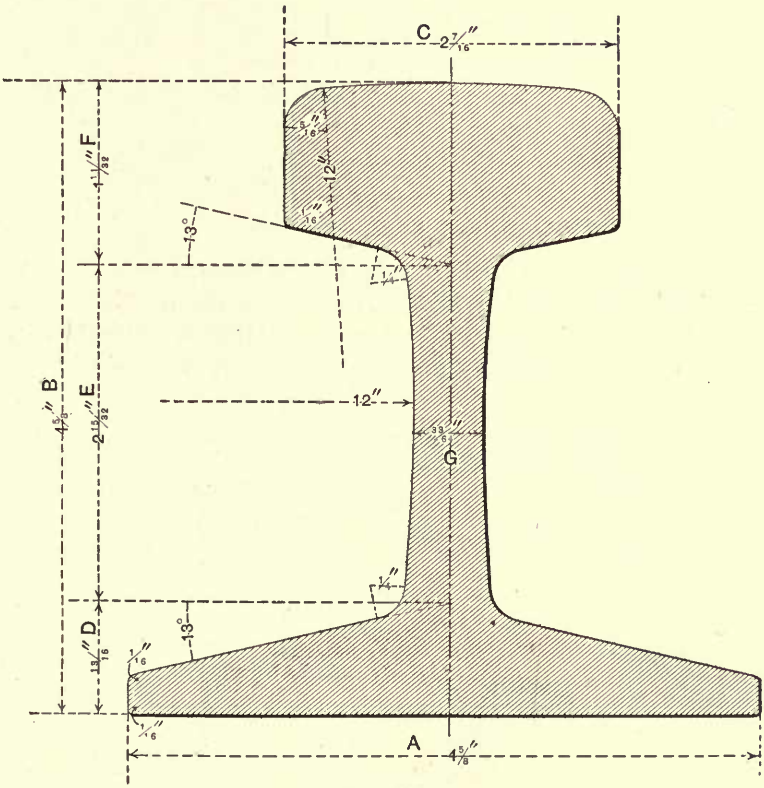


Fig. 1132.

Figs. 1133, 1134, and 1135 are the front elevation, plan, and side elevation of a New York city cable conduit adapted to Love's electric traction, one half of each, showing the construction of the hand holes *h* on each side at every third yoke about 15 feet centres, the other half showing the intermediate construction. Details of hand hole are shown in Figs. 1136, 1137, and 1138.

There are two copper-wire conductors supported and well insulated beneath each hand hole, which affords access to the insulator and wire. The support for the conductor is formed by two iron rods made fast inside the yoke, carrying the mica block into which the hook-shaped conductor is inserted.

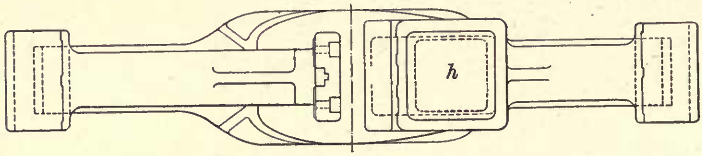
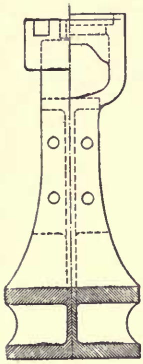


FIG. 1135.



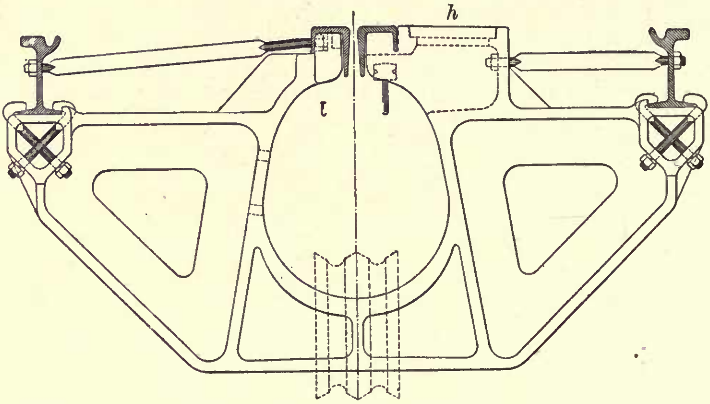


FIG. 1134. FIG. 1133.

Under the main body of the car is a thin, broad plough (Figs. 1139 and 1140), side and front elevation, that passes through the slot between the rails with wires (as shown in figures) leading to the trolley, which is brought in contact with the conductor by making connections with both wires as it is

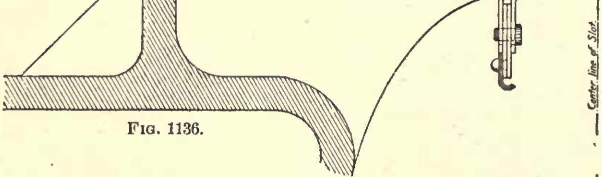


FIG. 1136.

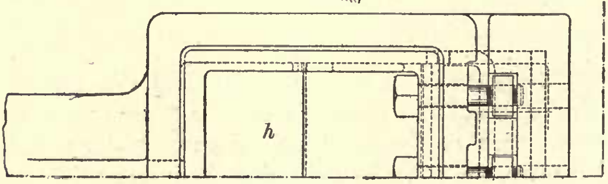


FIG. 1138.

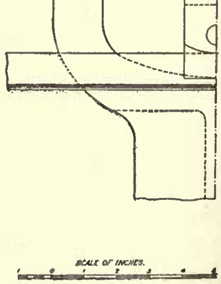


FIG. 1137.

pressed against them. The current forms a complete circuit, passing up through one wire and one blade of the plough to the motor; thence it returns through the other blade and wire to the power house. None of the current returns through the rails or escapes through the ground.

The pulleys shown (Fig. 1133) are those belonging to the cable traction, see Appendix. Manholes into the conduit are placed at intervals for inserting the lower bar of the plow, the tongue of which is raised through the slot, and fastened by the pin p to the head attached to the car.

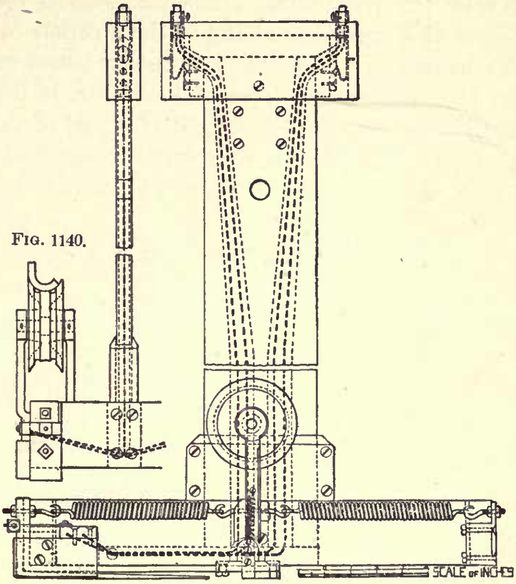


FIG. 1140.

FIG. 1139.

ROOFS AND BRIDGES.

At pages 490 and 491 will be found illustrations of the trussing of wooden beams. These are simple forms, which may be used in roofs or bridges, and rules are given for the proportion of parts. Rolled I-beams or plate-girders will serve also for floor-beams and moderate spans, but with modern necessities much more complicated structures are required.

On the General Principles of Bracing.—Let Fig. 1141 be the elevation of a common roof-truss, and let a weight, W, be placed at the foot of one of the suspension-rods. Now, if the construction consisted merely of the rafter C′ B, and the collar-beam C′ C, resting against some fixed point, then the point B would support the whole downward pressure of the weight; but in consequence of the connection of the parts of the frame, the pressure must be resolved into components in the direction C′ A and C′ B; C′ b will represent the pressure in the direction C′ B, C′ w the portion of the weight supported at B, C′ a the pressure in the direction C′ A, and w W the portion of the weight supported on A. The same resolution obtains to determine the direction and amount of force exerted on a bridge-truss of any number of panels, by a weight placed at any point p of its length (Fig. 1142). In either case, the effect of the oblique form C′ A upon the angle C is evidently to force it upward; that is, a weight placed at one side of the frame has, as in case of the arch, a tendency

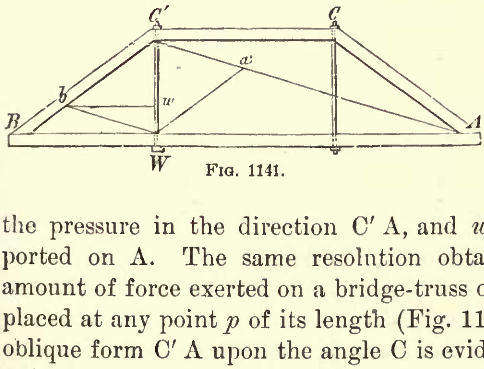


FIG. 1141.

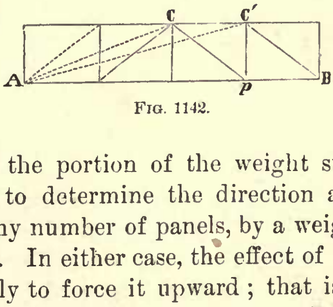


FIG. 1142.

to raise the other side. The effect of this upward force is a tension on a portion of the braces, according to the position of the weight; but as braces, from the manner in which they are usually connected with the frame, are not capable of opposing any force of extension, it follows that the only resistance is that which is due to the weight of a part of the structure.

Figs. 1143 and 1144 illustrate the effects of overloading at single points such forms of construction. Such an unequal loading on trusses requires that a

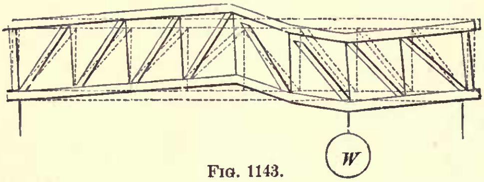


FIG. 1143.

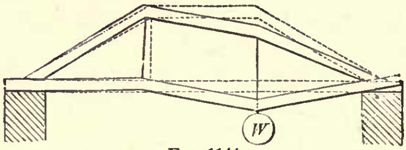


FIG. 1144.

portion of the load W be transferred to *each point of support* inversely proportionate to the distances of the weight from each support. The above trusses are not prepared to transfer this weight to but one support. To remedy the difficulty, it will be necessary to add braces running in the opposite direction as shown by dotted lines (Fig. 1145), at every point subject to the above distortion. These are called *counter-braces.*

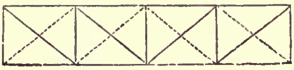


FIG. 1145.

To prevent the *braces* from becoming loose when the counter-braces are in action, it is always customary to give the braces and counter-braces an initial compression, by putting a moderate tension on the suspension-rods. In this case, therefore, the passage of a load produces no additional strain upon any of the timbers, but tends to relieve the counters. The counter-braces do not assist in sustaining the weight of the structure; on the contrary, the greater the weight of the structure itself, the more will the counter-braces be relieved.

If, instead of the counter-braces, the braces themselves are made to act both as ties and struts, as in some iron bridges and trusses, then the upward force will be counteracted by the tension of the brace.

If a system be composed of a series of suspension-trusses (Fig. 1146) in which

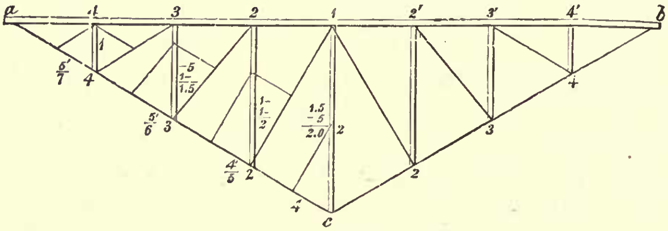


FIG. 1146.

the load is uniformly distributed, represent the load at each of the points, 4, 3, 2, 1, 2′, etc., by 1, then the load at 4 will be supported $\frac{1}{2}$ upon a and $\frac{1}{2}$ upon 3; hence the strut 3 will have to support a load of $1 + \cdot 5 = 1\cdot 5$; of this $\frac{2}{3}$ will be supported by 2 and $\frac{1}{3}$ by a; $\frac{2}{3}$ of $1\cdot 5 = 1$, $1 + 1 = 2$, load on strut 2; $\frac{3}{4}$ of this load, or 1·5, will be supported at 1, and since from the opposite side there is an equal force exerted at 1, therefore the strut 1 supports $1 + 1{\cdot}5 + 1{\cdot}5 = 4$.

$$\text{The tension on the rod } c\text{–}2 = 2\ \frac{c\,a}{c\,1}.$$

"	"	$2\text{–}3 = 2\tfrac{1}{2}$	"
"	"	$3\text{–}4 = 3$	"
"	"	$4\text{–}a = 3\tfrac{1}{2}$	"

If this construction be reversed as in a roof-truss, the parts which now act as ties become braces, and the braces ties. The force exerted on the several parts may be estimated in a similar way as for the suspension-truss. Neither of these constructions would serve for a bridge-truss, subject to the passage of heavy loads, but are only fit to support uniform and equally distributed loads.

To frame a construction so that it may be completely braced—that is, under the action of any arrangement of forces—the angles must not admit of alteration, and consequently the shape can not. The form

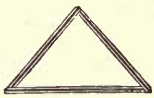


Fig. 1147.

Fig. 1148.

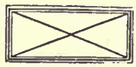


Fig. 1149.

should be resolvable into either of the following elements (Figs. 1147, 1148, and 1149):

In these figures, lines ═══ represent parts required to resist compression; lines ——— parts to resist tension only; lines ≡≡≡ parts to resist both tension and compression.

In a triangle (Fig. 1147), an angle can not increase or diminish without the opposite angles also increasing or diminishing. In the form Fig. 1148 a diagonal must diminish; in Fig. 1149 a diagonal must extend, in order that any change of form may take place. Consequently, all these forms are completely braced, as each does not permit of an effect taking place, which would necessarily result from a change of figure. Hence, also, any system composed of these forms, properly connected, breaking joint as it were into each other, must be braced to resist the action of forces in any direction; but as in general all bridge-trusses are formed merely to resist a downward pressure, the action on the top chord being always compression, it is not necessary that these chords should act in both capacities.

Most city roofs are flat; the timbers are placed like those of the floor beneath, but at greater distances between centres, as they have less load to sustain. The roof covering is inclined to discharge the rain water; and the timbers beneath are arranged to admit of this inclination, which may be quite small, $\frac{1}{4}$ of an inch to the foot; but it should be positive and uniform, to prevent the retention of the water in puddles. It is an advantage to hold the snowfall, as it relieves the street of a great encumbrance.

Figs. 1150, 1151, and 1152 represent elevations or portions of elevations of the usual form of framed roofs. The same letters refer to the same parts in all the figures. T T are the *tie-beams*, R R the *main rafters*, *r r* the *jack-rafters*, P P the *plates*, *p p* the *purlins*, K K the *king-posts*, *k k king-bolts*, *q q queen-bolts*—both are called *suspension-bolts*—C the *collar* or *straining beam*, B B *braces* or *struts*, *b b ridge-boards*, *e corbels*.

The *pitch* of the roof is in the inclination of the rafters, and is usually designated in reference to the span, as $\frac{1}{3}$, $\frac{1}{4}$, $\frac{3}{8}$, etc., pitch; that is, the height of

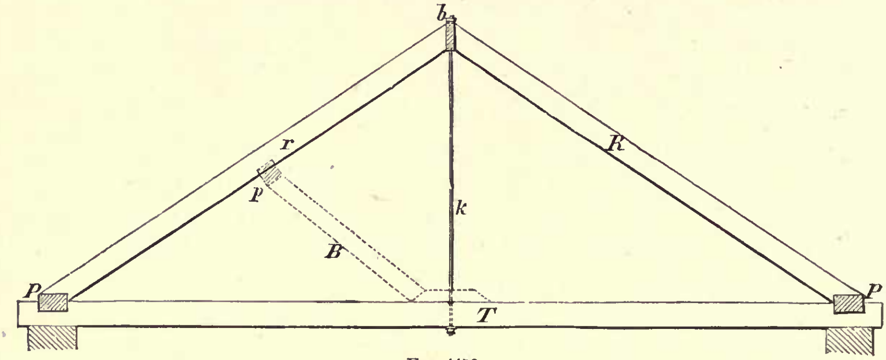


FIG. 1150.

the ridge above the plate is $\frac{1}{3}$, $\frac{1}{4}$, $\frac{3}{8}$, etc., of the span of the roof at the level of the plate. The steeper the pitch of the roof, the less the thrust against the

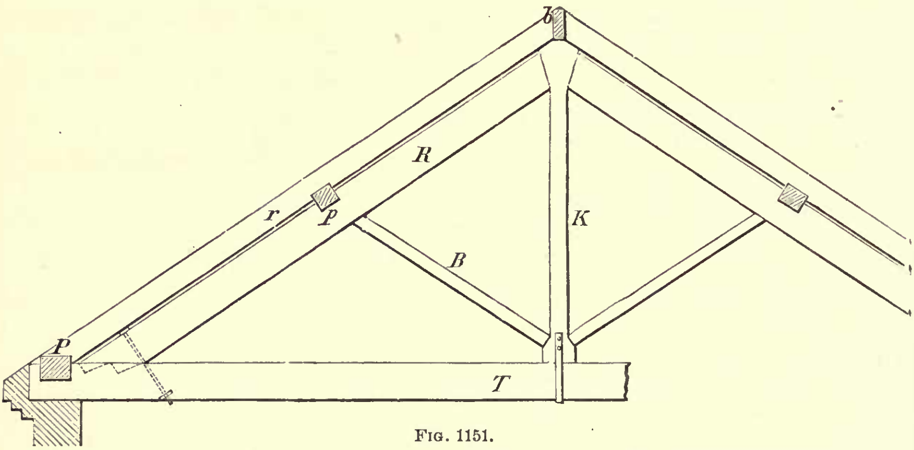


FIG. 1151.

side-walls, the less likely the snow or water to lodge, and consequently the tighter the roof. For roofs covered with shingles or slate, in this portion of the country, it is not advisable to use less than $\frac{1}{4}$ pitch; above that, the pitch should be adapted to the style of architecture adopted. The pitch in most common use is $\frac{1}{3}$ the span.

Fig. 1150 represents a simple form of framed roof; it consists of rafters, resting upon a plate framed into the tie- or ceiling-beam; this beam is supported by a suspension-rod, *k*, from the ridge, if supported from below, the rod is omitted. If neither ceiling nor attic floor are necessary, an iron tie-rod connecting the plates is sufficient. The rafters are to be spaced from 1 to 2 feet centres, and the tie-beams at intervals of from 6 to 8 feet; the roof cover to be of boards nailed to the rafters. This form of construction is sufficient for any roof of less than 25 feet span, and of the usual pitch, and may be used for a 40-foot span by increasing the depth of the rafters; deep rafters should always be bridged. By the introduction of a purlin extending beneath the centre of the rafter, supported by a brace to the foot of the suspension-rod, as shown in dotted line, the depth of the rafters may be reduced. If the tie-beam,

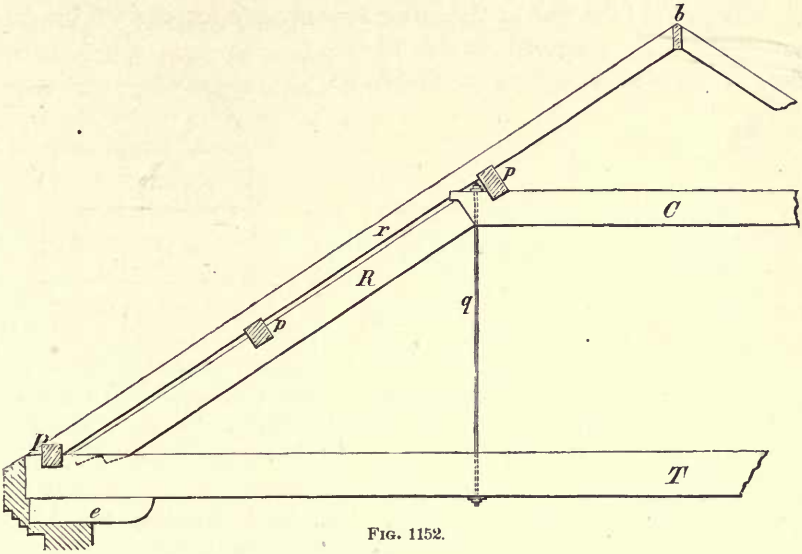


FIG. 1152.

which is also a ceiling and floor beam, be below the plate some 2 to 4 feet the thrust of the roof is resisted (Fig. 1153) by bolts, *b b*, passing through the plate and the beam, and by a collar-plank, C, spiked on the sides of the rafters, high enough above the beam for head-room. For roofs ⅝ pitch and under 20 feet span the bolts are unnecessary, the collar alone being sufficient.

Fig. 1151 represents a roof, a larger span than Fig. 1150; the frame may be made very strong and safe for roofs of 60 feet span. King-bolts or suspension-rods are now oftener used than posts, with a small triangular block of hard wood or iron, at the foot of the bolts, for the support of the braces. The objection to this form of roof is that the framing occupies all the space in the attic; on this account the form, Fig. 1152, is preferred for roofs of the same span, and is also applicable to roofs of at least 75 feet span, by the addition of a brace to the rafter from the foot of the queen-bolt. The collar-beam (Fig. 1155) is also trussed by the framing similar to Fig. 1151.

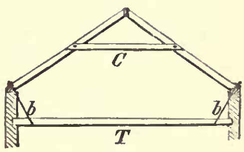


FIG. 1153.

In many church and barn roofs the tie-beam is cut off (Fig. 1154),

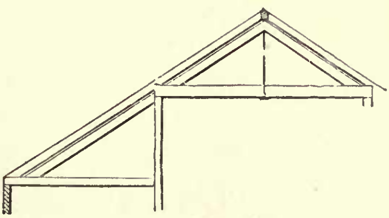


FIG. 1154.

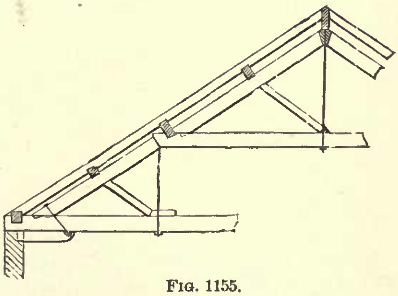


FIG. 1155.

the queen-post being supported on a post, or itself extending to the base, with a short tie-rod framed into it from the plate.

Figs. 1156 and 1157 are representations of the feet of rafters on an enlarged scale. In Fig. 1156 the end of the rafter does not project beyond the face of the plate; the cove is formed by a small triangular, or any desirable form of plank, se-

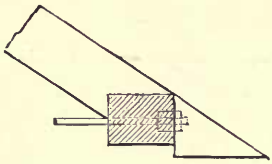


Fig. 1156.

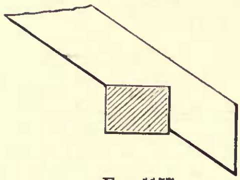


Fig. 1157.

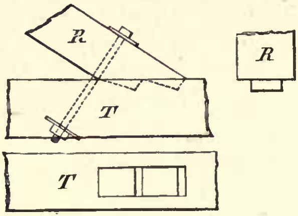


Fig. 1158.

cured to the plate. The form given to the foot of the rafter is called a *crow-foot.* In Fig. 1157 the rafter itself projects beyond the plate to form the coving. Fig. 1158 represents a front and side elevation and plan of the foot of a main rafter, showing the form of tenon, in this case double; a bolt, passing through the rafter and beam, retains the foot of the former in its place. Fig. 1159 represents the foot of a main rafter, with a wooden shoe too short at *a*, outside of the rafter; it should be framed as in Fig. 1158.

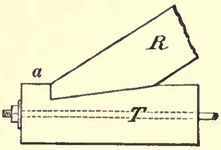


Fig. 1159.

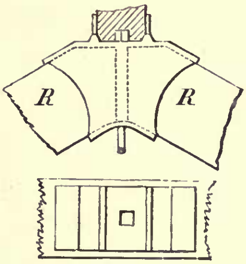


Fig. 1160.

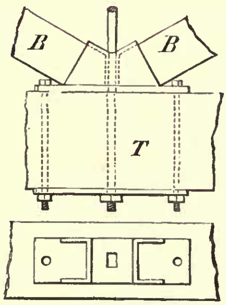


Fig. 1161.

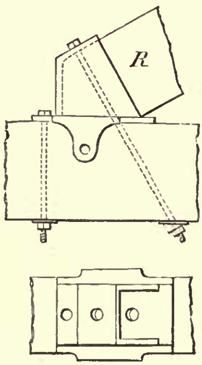


Fig. 1162.

Roofs may be very neatly and strongly framed by the introduction of cast-iron shoes and abutting plates for the ends of the braces and rafters. Fig. 1160

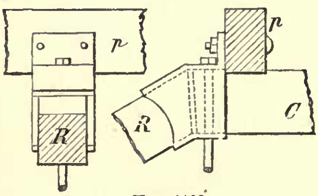


Fig. 1163.

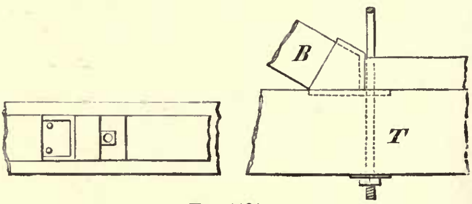


Fig. 1164.

represents the elevation and plan of a cast-iron king-head for a roof similar to Fig. 1151; Fig. 1161, that of the brace-shoe; Fig. 1162, that of the rafter-shoe for the same roof; Fig. 1163, the front and side elevation of the queen-head of roof similar to Fig. 1152; and Fig. 1164, elevation and plan of queen brace-shoe. Fig. 1165 represents the section of a rafter-shoe for a tie-rod; the side flanges are shown in dotted line.

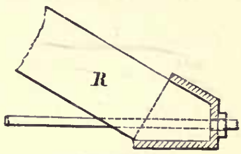


FIG. 1165.

On *the size and the proportions* of the different members of a roof: Tie-beams, usually serving a double purpose, are affected by two strains: one in the direction of their length, from the thrust of the rafters; the other a cross-strain, from the weight of the floor and ceiling. In estimating the size necessary for the beam the thrust need not be considered, because it is always abundantly strong to resist this strain, and the dimensions are to be determined as for a floor-beam merely, each point of suspension being a support. When tie-rods are used, the strain is in the direction of their length only, and their dimensions can be calculated, knowing the pitch, span, and weight of the roof per square foot, and the distance apart of the ties, or the amount of surface retained by each tie.

The weight of the wood-work of the roof may be estimated at 40 pounds per cubic foot; slate at 7 to 9 pounds, shingles at $1\frac{1}{2}$ to 2 pounds per square foot. The force of the wind may be assumed at 15 pounds per square foot. The excess of strength in the timbers of the roof, as allowed in all calculations, will be sufficient for any accidental and transient force beyond this. Knowing the weights, pressures, and their directions on parts of a roof, their stresses may be determined by the parallelogram of forces and dimensions proportioned to the strength of the materials of which the roof is composed. It will generally be sufficient for the draughtsman to have practical examples of construction to draw from. Dimensions are therefore given of the parts of wooden roofs already illustrated. Beams are usually proportioned to the weight that they are to sustain in floors and load, but where tie-rods are used, the stress upon them may be determined by the following rule:

Rule.—Multiply one half the weight of the roof and load by one half the span, and divide the product by the rise or height of ridge above eaves.

Gwilt, in his "Architecture," recommends the following dimensions for portions of a roof:

Span.	Form of Roof.	Rafters.	Braces.	Posts.	Collar-beams.
Feet.		Inches.	Inches.	Inches.	Inches.
25	Fig. 1151,	5 × 4	5 × 3	5 × 5	
30	"	6 × 4	6 × 3	6 × 6	
35	Fig. 1152,	5 × 4	4 × 2	4 × 4	7 × 4
45	"	6 × 5	5 × 3	6 × 6	7 × 6
50	2 sets of queen-posts,	8 × 6	5 × 3	{ 8 × 8 / 8 × 4 }	9 × 6
60	" "	8 × 8	6 × 3	{ 10 × 8 / 10 × 4 }	11 × 6

These dimensions, for rafters, are somewhat less than the usual practice in this country; no calculations seem to have been made for using the attic. An average of framed roofs here would give the following dimensions nearly: 30 feet span, 8 × 5 inches; 40 feet, 9 × 6; 50 feet, 10 × 7; 60 feet, 11 × 8; collar-beams the same size as main rafters. Roof-frames from 8 to 12 feet from centre to centre.

Dimensions for jack-rafters, 15 to 18 inches apart:

For a bearing of 12 feet.... 6×3 inches.	For a bearing of 8 feet... 4×3 inches.
" " 10 " .... 9×3 "	" " 20 " ... 10×3 "

Purlins:

Length of bearing.	Distances apart in Feet.			
Feet.	6	8	10	12
8	7×5	8×5	9×5	9×6
10	9×5	10×5	10×6	11×6
12	10×6	11×6	12×7	13×8

The pressure on the plates is transverse from the thrust of the rafters, but in all forms except Fig. 1150, owing to the notching of the rafters on the purlins, this pressure is inconsiderable. The usual size of plates for Figs. 1150 and 1151 is 6 × 6 inches. The dimensions of rafters depend on the distances between their supports and between centres. The depth in all such cases to be greater than the width; 2 to 6 inches may be taken as the width, 8 to 12 for the depth.

In the framing of roofs it is now customary, for roofs of mills, to omit purlins, jack-rafters, and plates, and make the roof-boards of plank stiff enough to supply their places, from 2″ to 3″ thick (according to the space between the frames), tongued and grooved, and strongly spiked to the main rafters. There is no need of plates; the plank forms a deep beam, and, if the ends of the frame are secured, there is no need of intermediate ties.

Joinings.—As timber can not always be obtained of sufficient lengths for the different portions of a frame, or to tie the walls of a building, it is often necessary to unite two or more pieces together by the ends, called scarfing or lapping. Fig. 1166 is a most common means of lapping or halving employed when there is not much longitudinal stress, and when a post is to be placed beneath the lower joint.

For beams with a butt joint under the last condition joint bolts (Fig. 1167) are often used in which the ends of the timber are squared and held together by bolts inserted in holes central of the beams with the nuts in side-pockets, in which one is screwed up by turning the bolt and the other by a cold chisel.

Fig. 1168 is a long scarf, in which the parts are bolted through and strapped, suitable for tie-beams. Joints (Figs. 1169, 1170, and 1171) are also often made

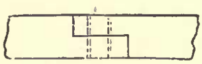


FIG. 1166.

FIG. 1167.

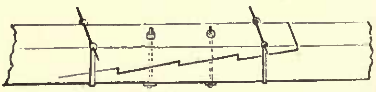


FIG. 1168.

by abutting the pieces together and bolting splicing-pieces on each side; still further security is given by cutting grooves in both timbers and pieces and driving in keys, *k k*.

Iron Roofs.—Roofs of less than 30 feet span are often made of corrugated iron alone, curved into a suitable arc, and tied by bolts passing through the iron about 2 to 4 feet above the eaves.

Fig. 1172 represents the half elevation of an iron roof of a forge at Paris; Figs. 1173, 1174, and 1175, details on a larger scale. This is a common type of

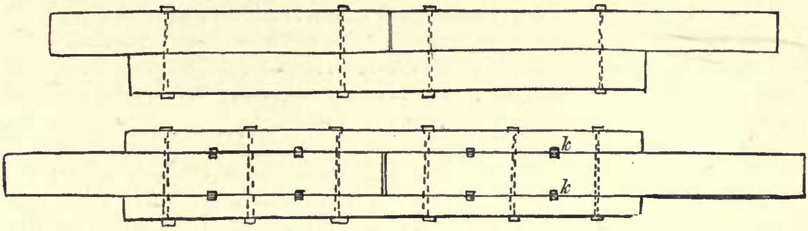


Fig. 1169.

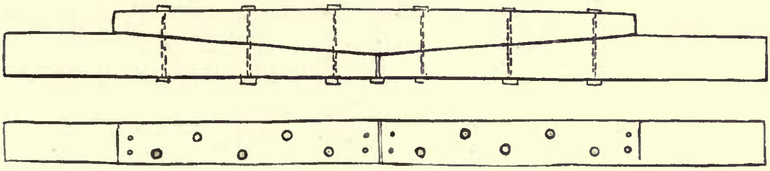


Fig. 1170.

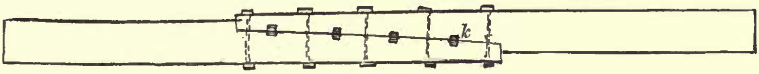


Fig. 1171.

iron roof, consisting of main rafters, R, of the I-section (Fig. 1175), trussed by a suspension-rod, and tied by another rod. The purlins are also of I-iron, secured to the rafters by pieces of angle-iron on each side; and the roof is covered with either sheet-iron resting on jack-rafters, or corrugated iron extending

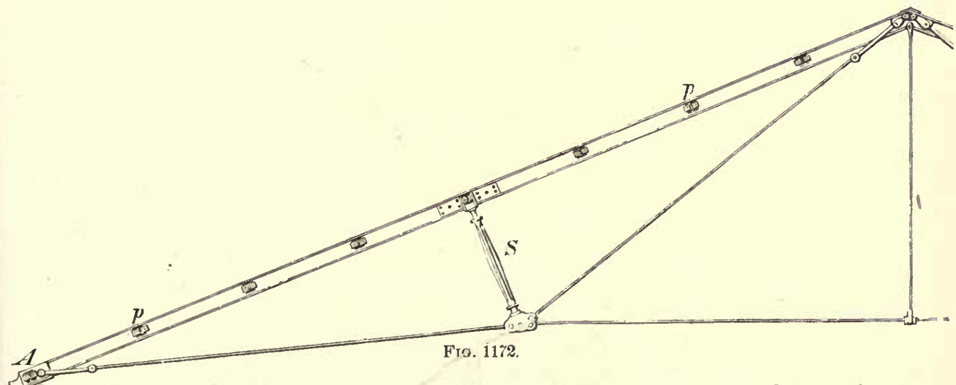


Fig. 1172.

from purlin to purlin. The rafter-shoe, A, and the strut, S, are of cast-iron; all the other portions of the roof are of wrought-iron. In American practice it is usual to make the strut of wrought-iron, with a single pin connection at its foot, instead of as in the figure.

The surface covered by this particular roof is 53 metres (164 feet) long and 30 metres (98½ feet) wide. There are eleven frames, including the two at the ends, which form the gables.

The following are the details of the dimensions and weights of the different parts :

	Pounds.
2 rafters, 0·72 feet deep; length together, 99·1 feet	1,751
5 rods, 0·13 feet diameter; length together, 131·4 feet	882
16 bolts, 0·13 feet diameter	79
8 bridle-straps, 0·24 × 0·05	123
2 pieces, 0·46 thick, connecting the rafters at the ridge, } 4 pieces, 0·46 thick, at the foot of the strut }	88
4 pieces, 0·36 thick, uniting the rafters at the junction in the strut—together with their bolts and nuts	176
2 cast-iron struts	308
2 rafter-shoes	287
Total of one frame	3,694
16 purlins, 1 ridge-iron, each 0·46 deep, 17·2 long	2,985
Bolts for the same	64
16 jack-rafters, I-iron, 0·16 deep	2,489
Weight of iron covering, including laps, per square foot	2·88

Roofs are sometimes made with deep corrugated main rafters with flat iron between, or purlins and corrugated iron for the covering. The great objection to iron roofs lies in the condensation of the interior air by the outer cold, or, as it is termed, sweating; on this account, they are seldom used for other buildings

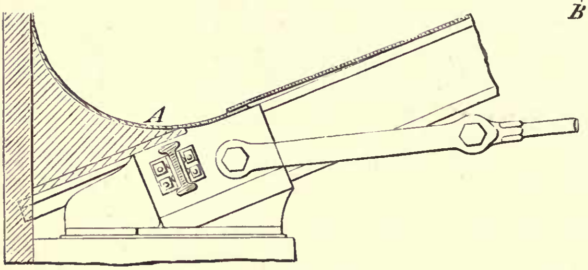


Fig. 1173.

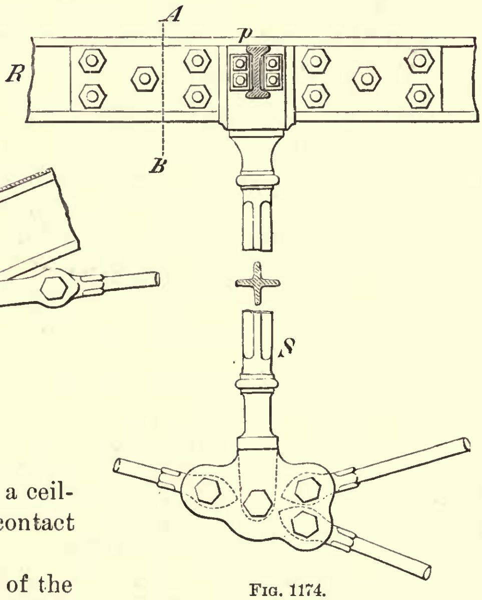


Fig. 1174.

than boiler-houses or depots, except a ceiling be made below to prevent the contact of the air inside with the iron.

Fig. 1176 is an elevation of one of the three panels of one of the cast-iron girders for connecting the columns, and carrying the transverse main gutters, which supported the roof of the Crystal Palace of the English Exposition of 1851. Figs. 1177 to 1181 are sections of various parts on an enlarged scale. The depth of the girder was 3 feet, and its length was 23 feet $3\frac{3}{4}$ inches. The sectional area of the bottom rail and flange in the centre (Fig. 1178) was $6\frac{1}{4}$ square inches; the width of both bottom and top rail (Fig.

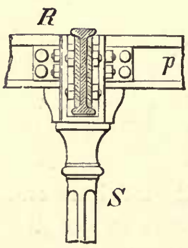


Fig. 1175.

1177) was reduced to 3 inches at their extremities. The weight of these girders was about 1,000 pounds, and they were proved by a pressure of 9 tons, distributed on the centre panel.

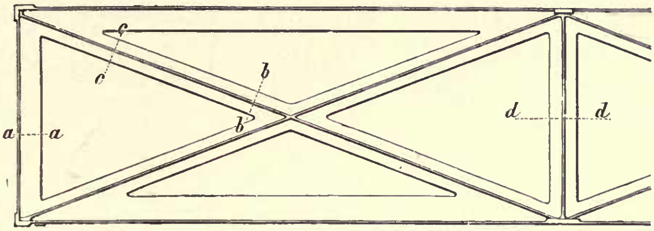


FIG. 1176.

A second series of girders were made of similar form, but of increased dimensions in the section of their parts. Their weight averaged about 1,350

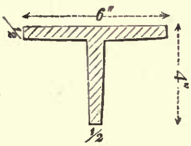


FIG. 1177.

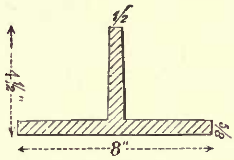


FIG. 1178.

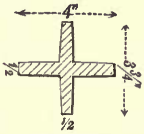


FIG. 1179.

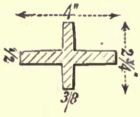


FIG. 1180.

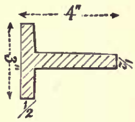


FIG. 1181.

pounds, and were proved to 15 tons. A third series weighed about 2,000 pounds, and were proved to 22½ tons.

Figs. 1182 and 1183 are sections and details of the trusses for sustaining the roof and floor of the English High and Latin School Gymnasium, Boston, Massachusetts. The object of sustaining the gymnasium-floor by rods was to secure a drill-hall for the military exercises of the school, and trusses were designed to have sufficient strength to resist the vibration of the floor. As the trusses were to be in sight, a central cloumn of cast-iron was introduced to sustain the centre of the top chord, with lattice between the main diagonals to enable them to act as counters, and a 3½-inch gas-pipe for horizontal bracing-struts. The floor-sustaining rods all have upset ends, and their tops pass through ornamental foliated castings, but their connection with the trusses is wholly of wrought-iron.

The top chords consist of two 9-inch channel-irons weighing 50 pounds per

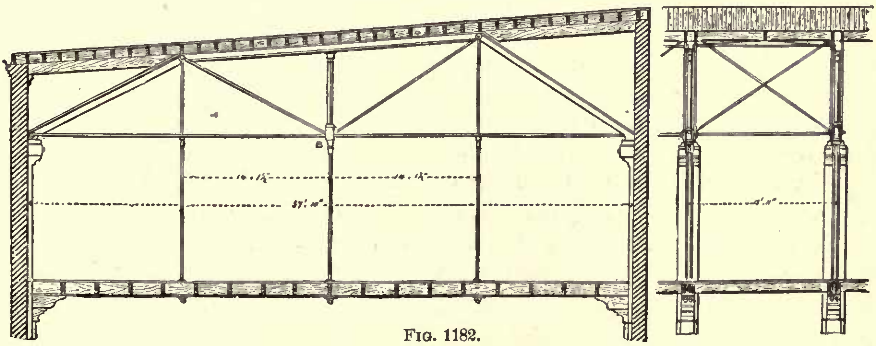


FIG. 1182.

yard, and one plate 12 × $\frac{3}{8}$ inches. The end-posts have the same section. The bottom chord consists of four bars $2\frac{1}{2}$ × 1 inch at the shallow end of the

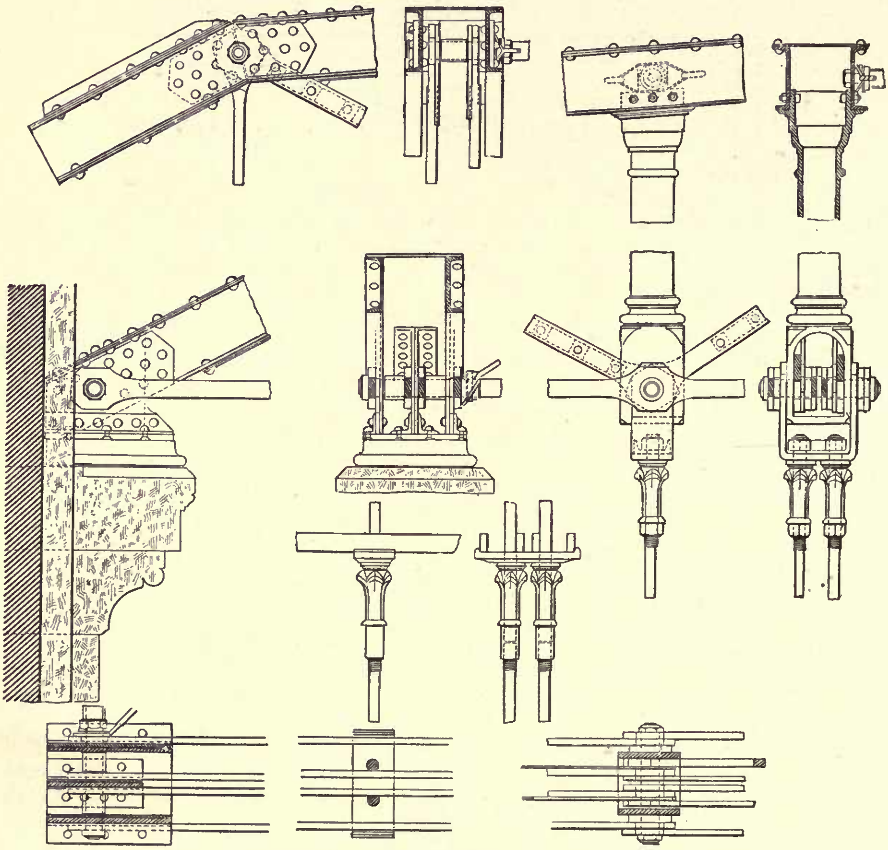


FIG. 1183.

truss, and four bars $2\frac{1}{2}$ × $\frac{3}{4}$ of an inch at the deep end of the truss. The diagonals are two bars 3 × 1 inch at deep end of truss, and two bars 3 × $\frac{1}{2}$ inch at shallow end of truss. The pins are all $2\frac{1}{2}$ inches diameter. The trusses were designed and constructed by D. H. Andrews, C. E., of the Boston Bridge Works.

In order to secure free space in the room beneath the roof, a roof or bridge truss may be constructed above, and the roof framed as a floor suspended from it, with such pitch as is requisite to shed rainfall.

Figs. 1184 to 1189 are the elevation and details for an iron roof-truss, for wood, slate, or corrugated iron covers, built by the Missouri Valley Bridge and Iron Works, A. S. Tulloch, engineer.

Fig. 1190 is a half cross-section of a two-story freight-shed for the New York, Lake Erie and Western Railroad, a simple and cheap construction of wood, readily framed and put together. The shed rests upon a pile-dock. The platform for the reception of freight is 4 feet above the dock-planking and about 26 feet wide, with occasional inclined runs for the transfer of freight to or from vessels.

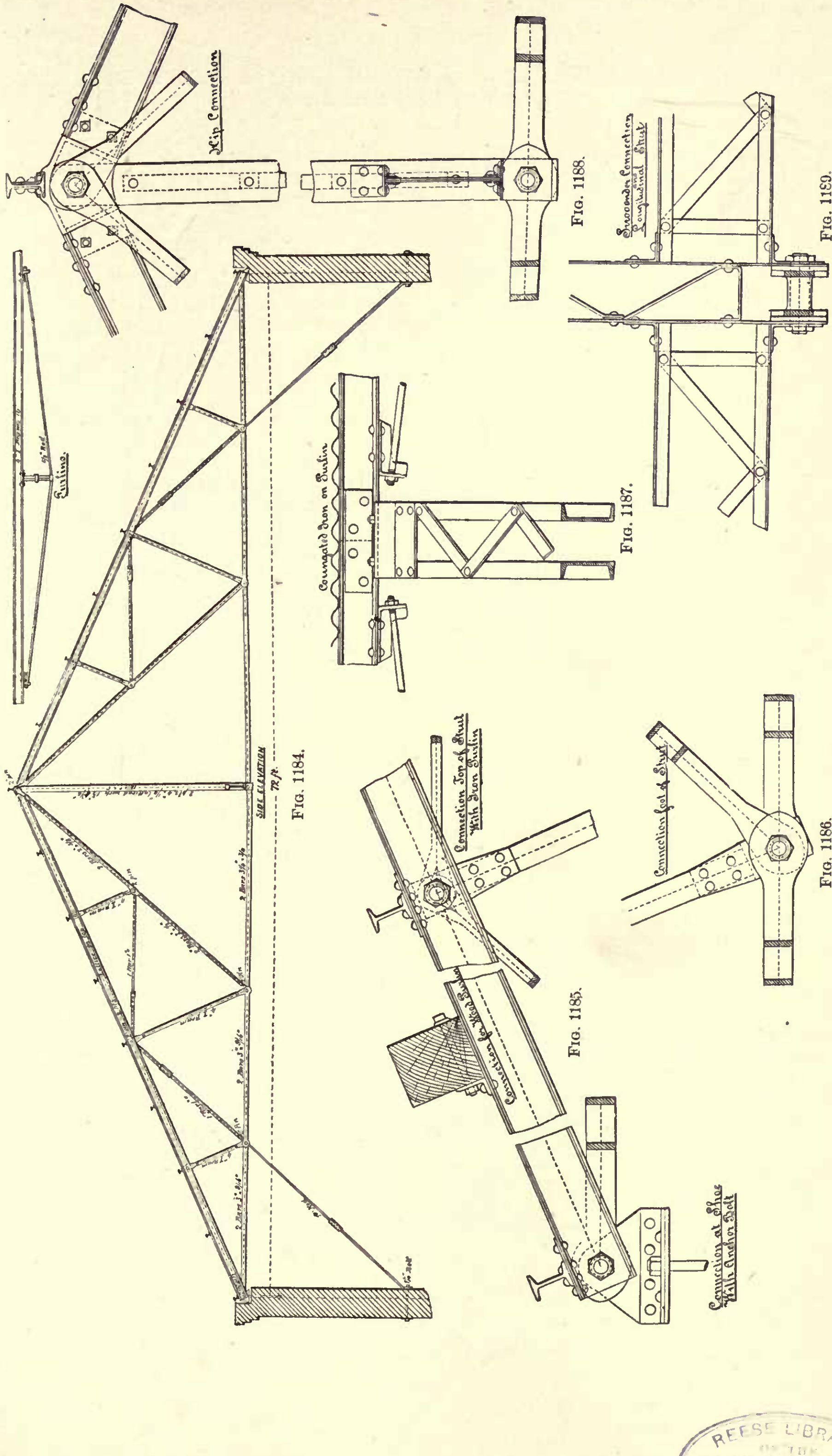


FIG. 1184.

FIG. 1185.

FIG. 1186.

FIG. 1187.

FIG. 1188.

FIG. 1189.

CROSS-SECTION OF ONE HALF OF A FREIGHT-SHED, NEW YORK, LAKE ERIE AND WESTERN RAILROAD.

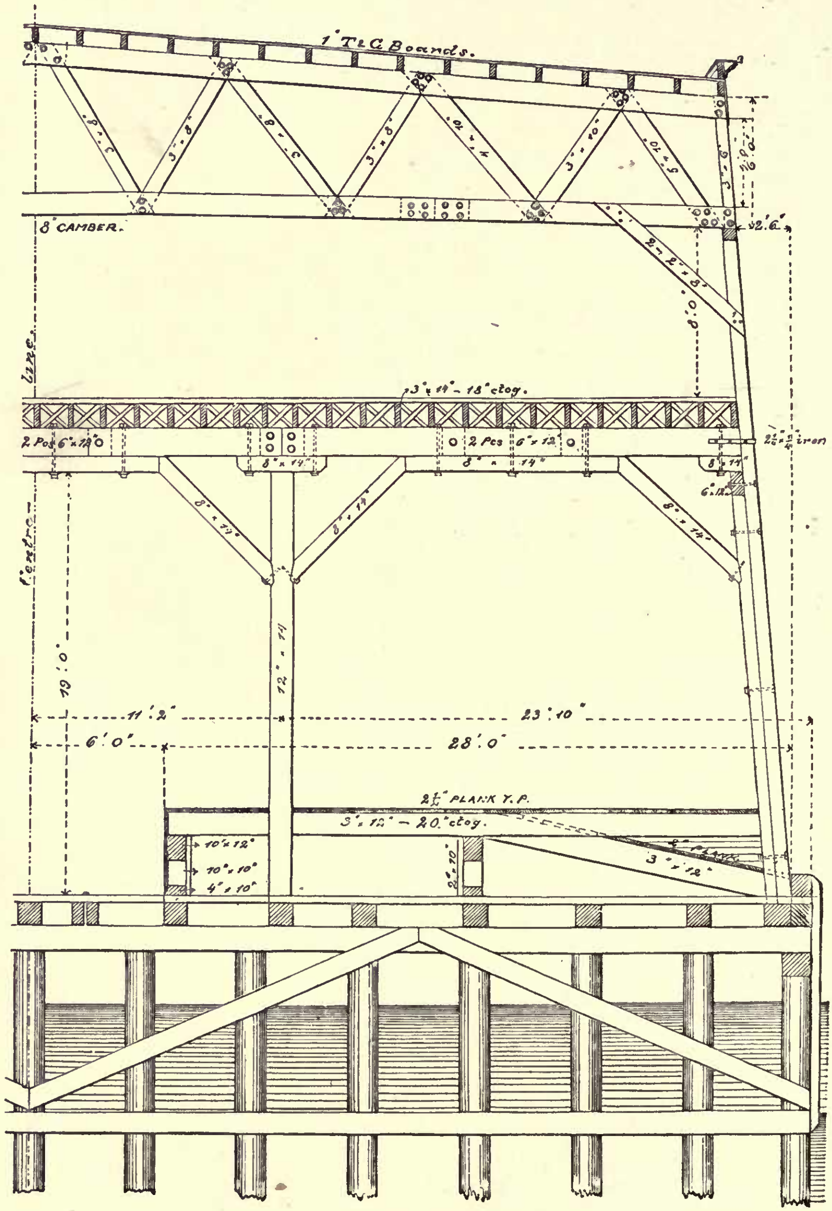


FIG. 1190.

Fig. 1191 is a section of the Baltimore and Ohio coaling bins for locomotives.

Piers.—Fig. 1192 is an elevation of a pile-pier for a bridge. Tenons are cut on the top of the piles, and a cap (*a*) mortised on. The two outer piles

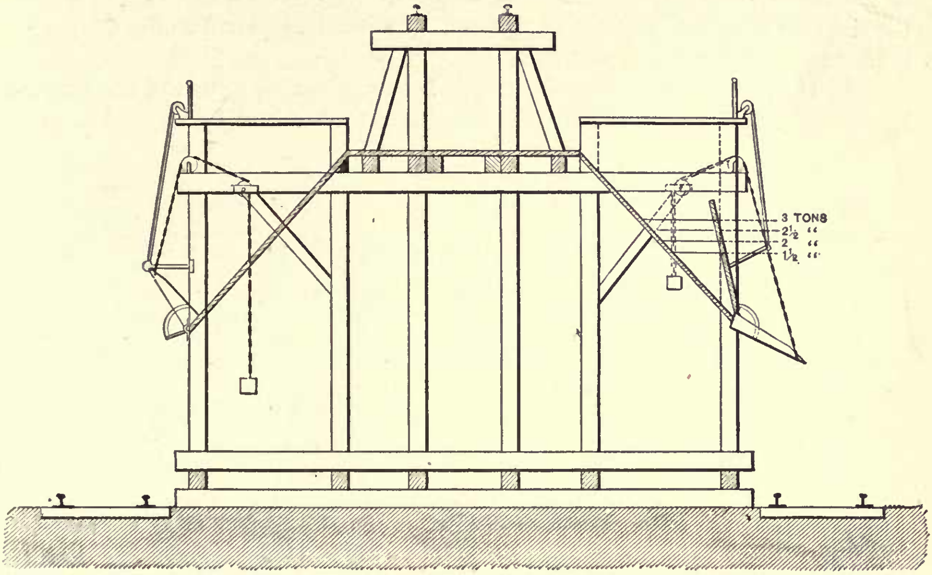


FIG. 1191.

driven in an inclined position, and the heads bolted to the piles adjacent. The piles are made into a strong frame laterally by the planks *b* and *c*, and plank-braces *d d* on each side of the piles, bolted through. The string-pieces of the

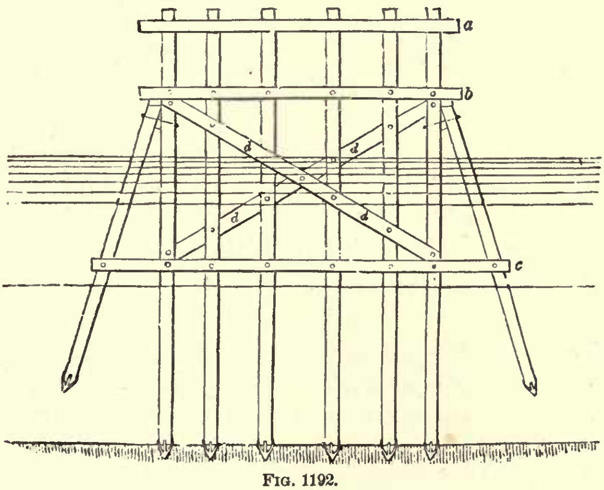


FIG. 1192.

bridge rest on the cap. Longitudinal braces are often used, their lower ends resting on the plank *b*—which should be, then, notched on to the piles—and their upper ends coming together, or with a straining-piece between, beneath the string-pieces, acting not only as supports to the load, but also as braces to prevent a movement forward of the frames. As the tendency of a moving train is to push forward the structure on which it is supported, in railway-bridges especially, great care is taken to brace the structure in every way—vertically and horizontally, laterally and longitudinally. If the plank *c* be a timber-sill, and the piles beneath be replaced by a masonry-pier, the structure will represent a common form of trestle.

Fig. 1193 is an illustration of a trestle-bent supporting a span of the Brooklyn Elevated Railroad over its coal-yard. The timber is of yellow pine of the

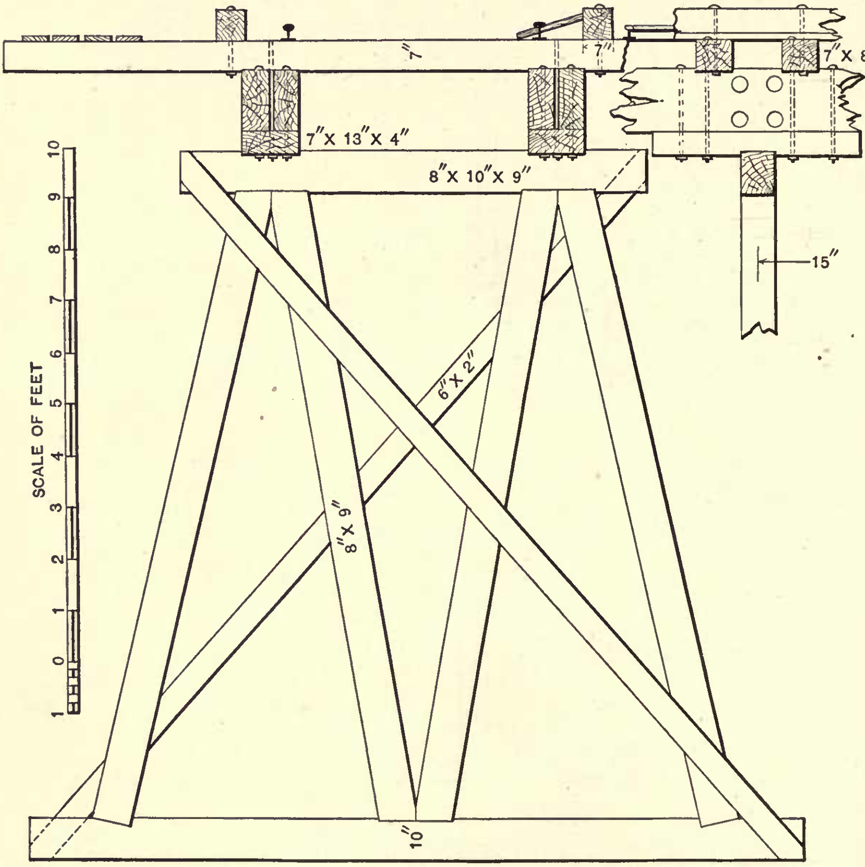


Fig. 1193.

dimensions shown ; the ties are extended beyond the guard-rail on one side to support a plank walk. The bent is about 15 feet high.

Bridge-Trusses.—Whatever may be the form of truss or arrangement of the framing, provided its weight is supported only at the ends, the tension of the lower chord, or the compression of the upper chord at centre, may be determined by this common rule :

Rule.—The sum of the total weight of the truss, and the maximum distributed load which it will be called on to bear, multiplied by the length of the span, and divided by 8 times the depth of the truss in the middle, the quotient will be the tension of lower chord and compression of upper at the middle. In nearly all the forms of diagonal bracing, if the uniform load be considered as acting from the centre toward each abutment, each tie or brace sustains the whole weight between it and the centre, and the strain is this weight multiplied by the length of tie or brace, divided by its height. Any diagonals, equally distant from the centre, sustain all the intermediate load : if rods, as in Fig. 1195, by tension ; if braces, as in Fig. 1194, by compression.

It follows, therefore, that in all these trusses the upper and lower chords

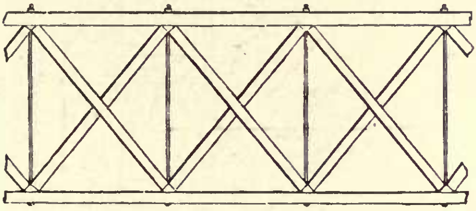


Fig. 1194.

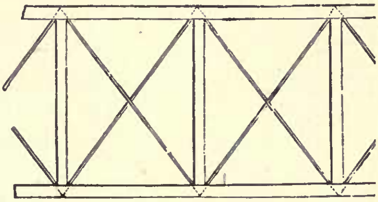


Fig. 1195.

should be stronger at the centre than at the ends, while diagonals should be largest at the abutments. Unless the weight of the bridge is great compared with the moving loads, counter-braces become necessary.

The general rule adopted in the construction of the Howe truss is, to make the height of the truss one eighth of the length up to 60 feet span; above this span the trusses are 21 feet high, to admit of a system of lateral bracing, with head clearance for a person standing on the top of a freight-car. From 175 feet to 250 feet span, height of truss gradually increased up to 25 feet. Moving load for single-track railroad-bridge calculated at 2 tons per running foot.

Wooden Truss-Bridges.—Fig. 1194 is the elevation of a few panels of a Howe truss, and Fig. 1195 of a Pratt truss. The Howe truss is by far the most popular of all wooden trusses, being readily framed and put together, uniting great strength with simplicity of construction.

Fig. 1196 is the side elevation of three of the five panels of a Howe truss highway-bridge of the New York, Lake Erie and Western Railroad. Fig. 1197 is a cross-section. There is a section of 3″ plank laid close, and another beneath, laid with spaces; these planks are laid diagonally across the floor-beams, and at right angles to each other, to act as lateral bracing. Fig. 1198 is the details of the abutment end of bridge; the foot of the brace rests on a cast-iron shoe. The length over all—that is, including the portions on the abutment—is 81′ 2″, or 75 feet between abutments, usually designated as the span.

Figs. 1199, 1200, and 1201 are the side elevation, floor cross-section, and plan of floor and bottom chord of three of the twelve panels of a single-track railway Howe truss. Their length is each 10′ $10\frac{5}{16}$″. The centre braces are two, 7″ × 10″; the centre rods three, $1\frac{1}{2}$″ diameter. The counters, each one 6″ × 8″; lateral brace, top and bottom, 6″ × 6″; rods $1\frac{1}{4}$″; top chord, four pieces, 7″ × 12″; bottom chord, four pieces, 7″ × 15″; floor-beams, 7″ × 16″. The shoes, splices, and blocks between chord-timbers are of cast-iron. In the earlier practice the angle-blocks were of oak, and the splices made as in Fig. 1202. Both were satisfactory.

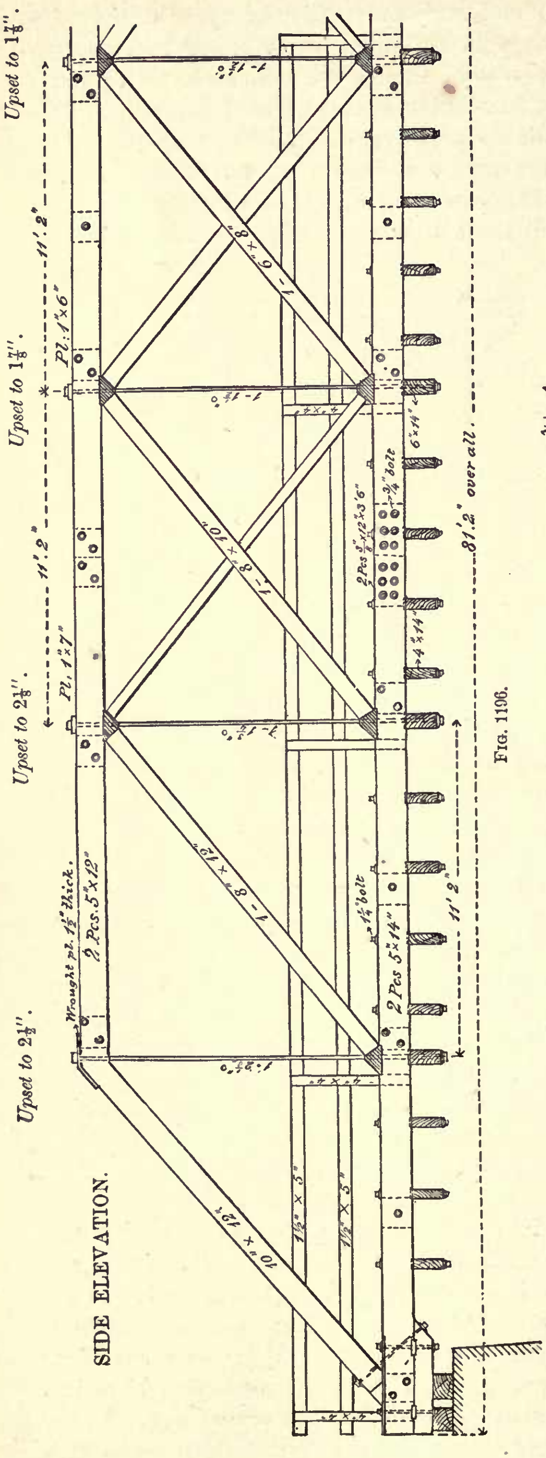


FIG. 1196.

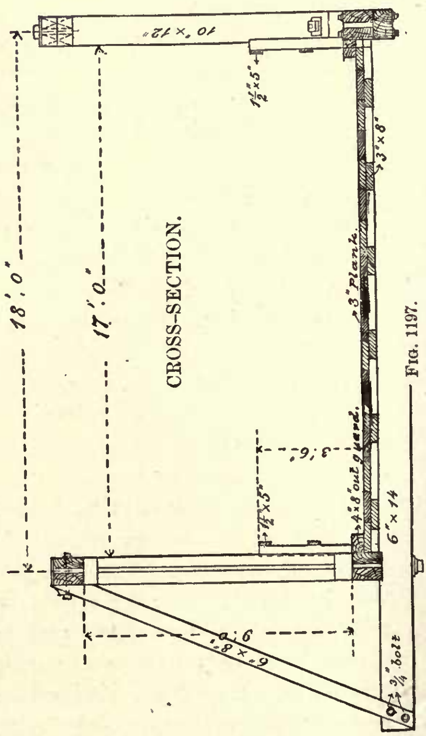


FIG. 1197.

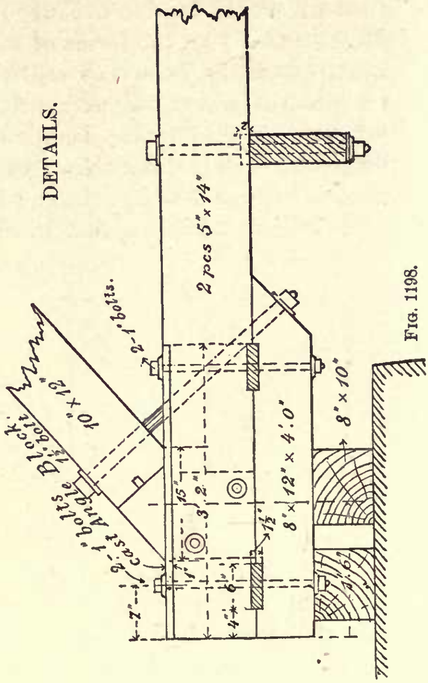


FIG. 1198.

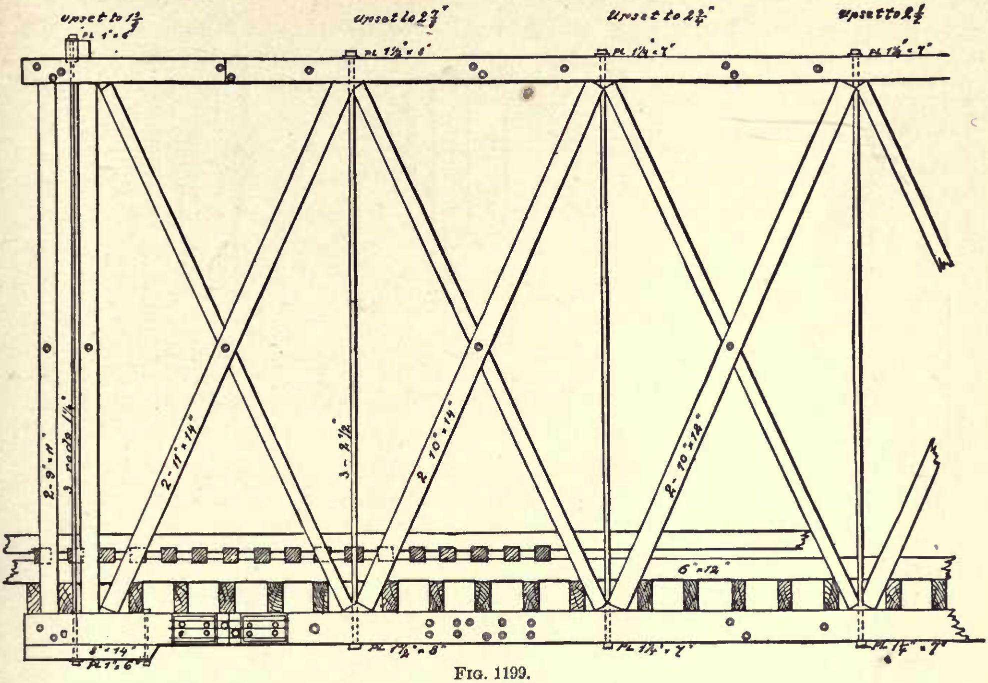


FIG. 1199.

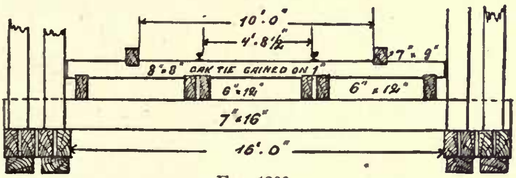


FIG. 1200.

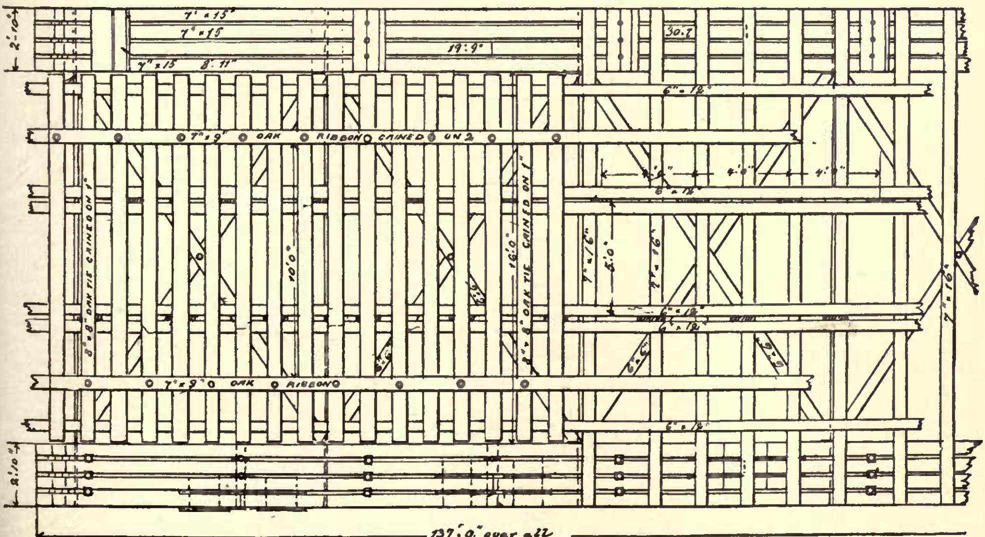


FIG. 1201.

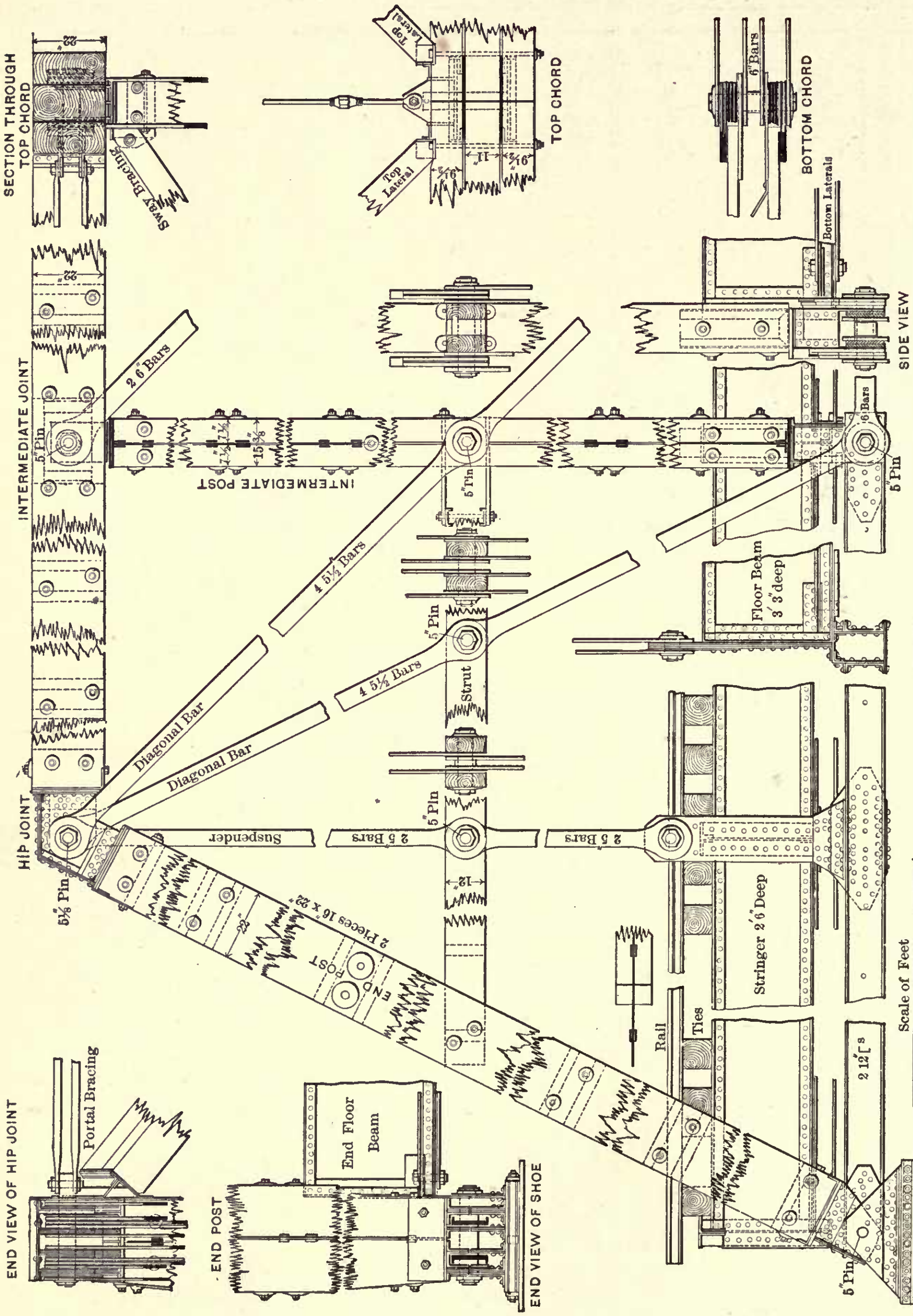

SECTION THROUGH TOP CHORD
22"
Sway Bracing
Top Lateral
Top Lateral
11"
TOP CHORD
6" Bars
BOTTOM CHORD
Bottom Laterals
SIDE VIEW
INTERMEDIATE JOINT
5" Pin
2 6" Bars
INTERMEDIATE POST
5" Pin
4 5½ Bars
5" Pin
4 5½ Bars
Strut
Floor Beam
3' 3" deep
Diagonal Bar
Diagonal Bar
HIP JOINT
5⅝ Pin
Suspender
2 5 Bars
5" Pin
2 5 Bars
12"
2 Pieces 16 x 22"
22"
END POST
Stringer 2' 6" Deep
Rail
Ties
2 12" [s
Scale of Feet
1 0 1 2 3 4 5
END VIEW OF HIP JOINT
Portal Bracing
END POST
End Floor Beam
END VIEW OF SHOE
5" Pin
SHOE AT ROLLER END

FIG. 1203.

Combination Truss.—Fig. 1203 is the elevation and details of a combination or composite truss, which owes its name to the use of the two materials, wood and wrought-iron, the tension members being of iron and the compression of wood. This class is entirely American in practice, and embodies an essentially American feature, *pin* connections.

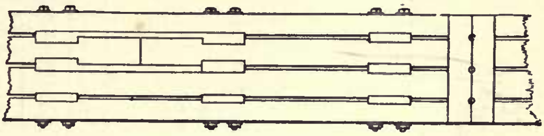


FIG. 1202.

The bridge illustrated is a double intersection combination through bridge, designed by L. L. Buck, C. E., for the Northern Pacific Railroad crossing the Yakima River in the State of Washington. The span is 300 feet between centres of end pins, and is divided into 15 panels of 20 feet each. The height from centre to centre of chords is 40 feet, and width 20 feet from centre to centre.

The cheapness of timber and the distance from which iron had to be secured made it advisable to construct this form of bridge. The timbers are heavy and framed, so that no two pieces are in direct contact, and provision has been made by which iron can at any time be substituted for the wood.

Iron Bridges.—When the span is of moderate extent, the load can be safely carried by beams put together at the works and transferred to the road in complete form. Plate or lattice girders are used, put together with rivets.

Fig. 1204 is a plan side and end elevation of a plate-girder bridge on the New York, New Haven and Hartford Railroad. The bridge is for a single line of rails, crossing a highway on a skew. The following is the bill of material:

LIST OF MATERIAL FOR PLATE-GIRDER BRIDGE, WOODMONT, CONN.

NUMBER.	Section.	Length.	Weight, pounds.
32	6″×6″ ×$\frac{11}{16}$″ Ls.	47′ 1″	40,530
8	5″×3½″×$\frac{5}{8}$″ Ls.	15′ 10″	2,140
4	5″×3″ ×$\frac{7}{16}$″ Ls.	19′ 9″	884
8	4″×3″ ×$\frac{3}{8}$″ Ls.	15′ 10″	1,063
68	3″×3″ ×$\frac{3}{8}$″ Ls.	15′ 10″	7,538
14	" " " "	28′ 10″	2,826
4	" " " "	23′ 0″	644
4	" " " "	27′ 0″	756
4	3½″×$\frac{11}{16}$″ bars.	24′ 6″	786
38	3″ ×$\frac{11}{16}$″ "	24′ 6″	6,405
	Total Ls and bars..........	....	63,572
16	48″ ×$\frac{7}{16}$″ plate.	15′ 8½″	17,590
8	48″ ×$\frac{3}{8}$″ "	15′ 5½″	7,419
16	20¼″×$\frac{7}{8}$″ "	2′ 0″	1,880
1	15″ ×$\frac{3}{8}$″ "	25′ 6″	478
1	" " "	10′ 3″	192
6	14″ ×$\frac{3}{8}$″ "	18′ 3″	1,916
1	" " "	23′ 0″	402
16	" " "	21′ 7″	6,043
16	" " "	4′ 1″	1,143
16	" " "	1′ 5″	396
16	14″ ×$\frac{7}{16}$″ "	26′ 10″	8,766
32	" " "	23′ 5¾″	15,342
2	" " "	29′ 0″	1,184
4	12½″×$\frac{7}{16}$″ "	24′ 6″	1,786
2	12″ ×$\frac{3}{8}$″ "	22′ 6″	675
Total weight of bridge, exclusive of rivet heads................			128,784

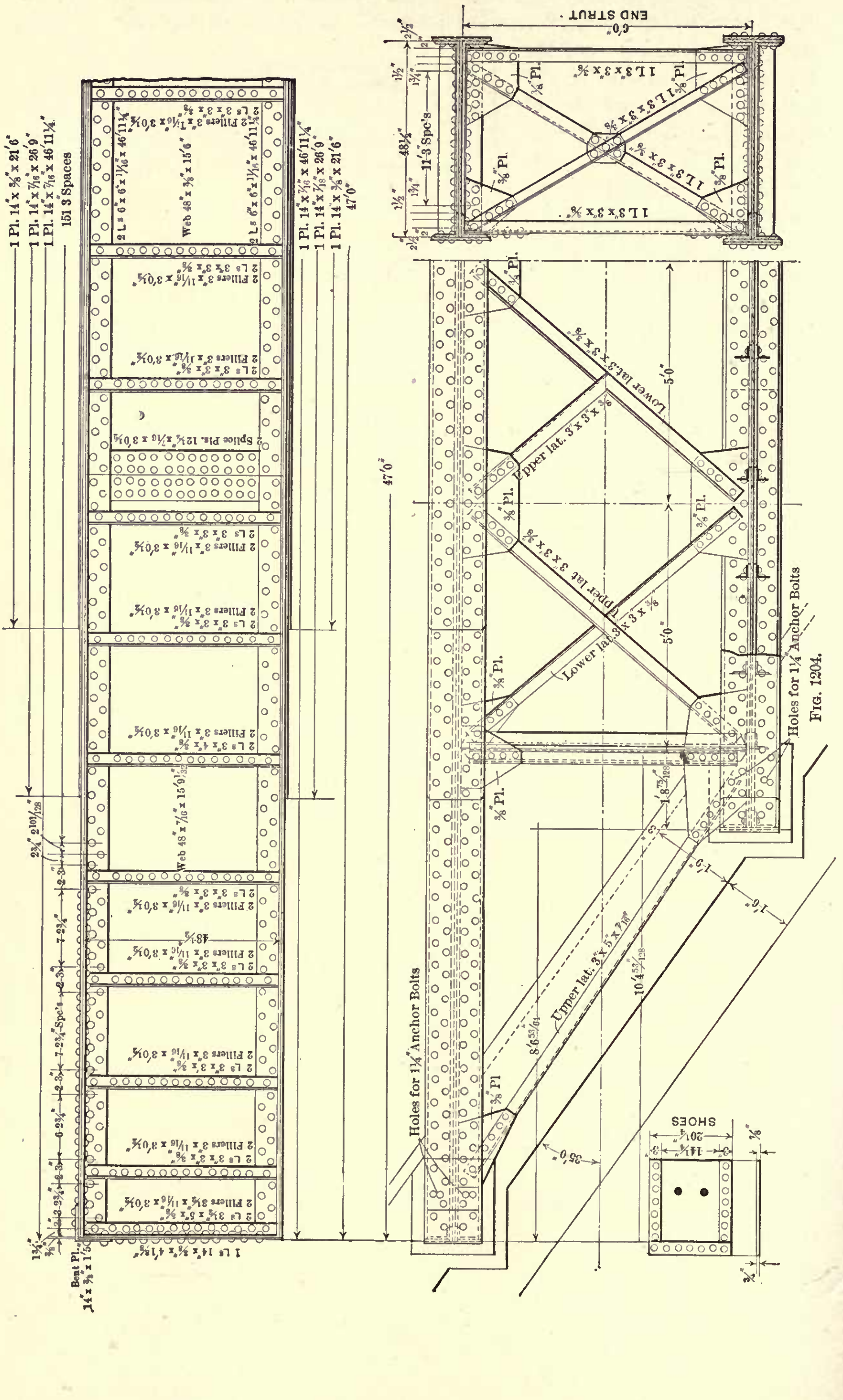

END STRUT
Holes for 1¼" Anchor Bolts
Holes for 1¼" Anchor Bolts
SHOES

FIG. 1204.

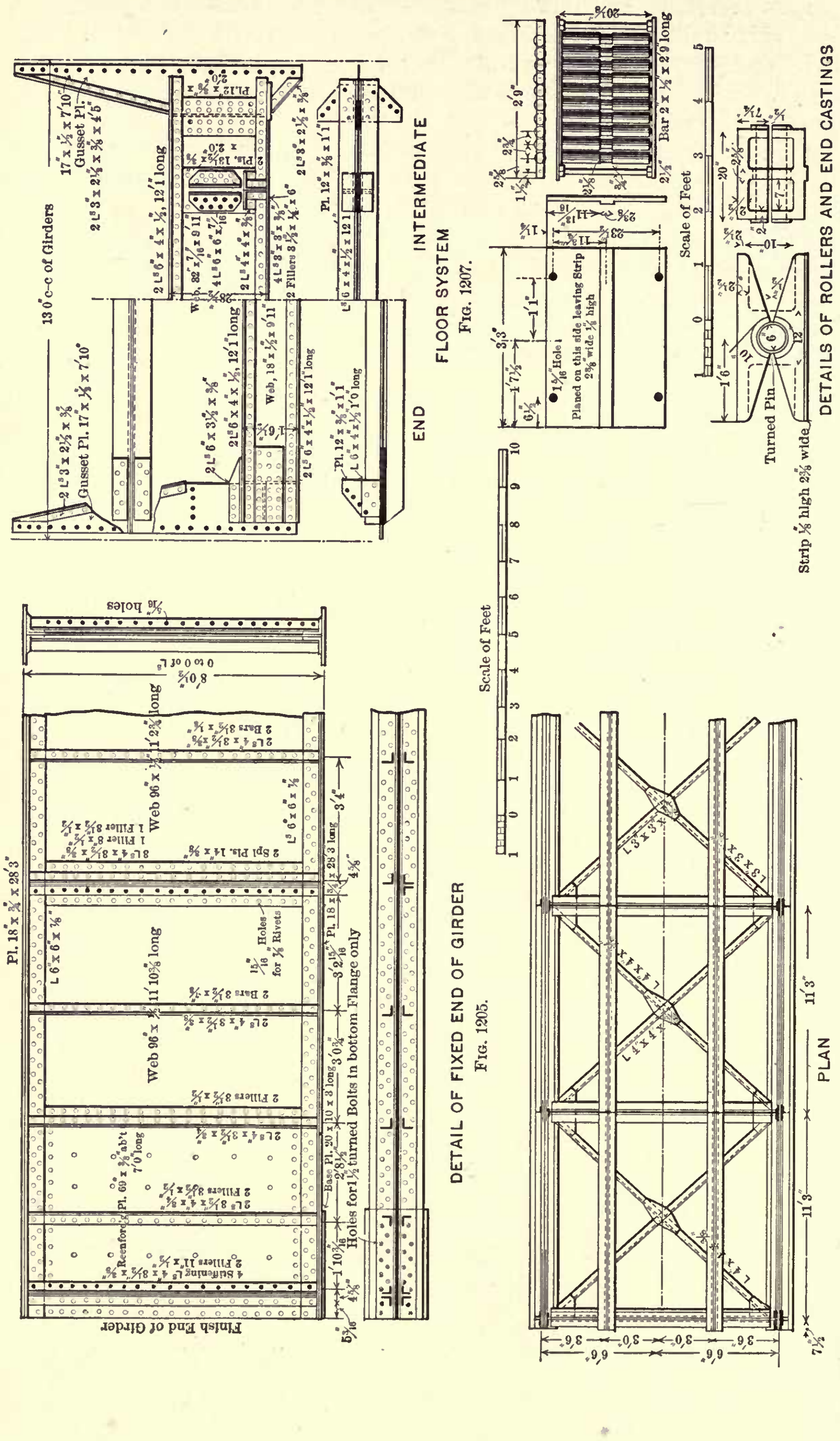
Figs. 1205, 1206, and 1207 are plans, elevations, and details of a plate-girder river bridge built for the New York, New Haven and Hartford Railroad by the Berlin Iron Bridge Company within the limits of the builders' yards. It is a single-track bridge 102′ 9″ from end to end of girders fixed at one end and supported on rollers at the other.

Figs. 1208, 1209, and 1210 are respectively the outside elevation, top lateral bracing, and cross-frame of a single-track lattice-deck span designed by the Phœnix Bridge Company. The bridge is 70 feet long over all, and divided into four panels, two of which are shown in the figure; there are five cross-frames attached to the vertical posts. Each truss is built in two sections, spliced as shown, the blackened circles indicating holes through which the connecting rivets will be driven in the field. The girders are 7′ 11″ deep, and placed 6′ 6″ apart. The ties of the railroad rest immediately on the top chord.

BILL OF MATERIAL.

Two lattice girders and bracing (Figs. 1208, 1209, 1210).

Description.	Sections.	Sizes.	Length.	Weight per foot.	No. of pieces.	Remarks.	Weight, pounds.
Top chords...........	Angles.	6×4	32′ 4″	16·3	4	....	2,106
" "	"	6×4	37′ 7″	16·3	4	....	2,452
Bottom chords........	"	6×4	32′ 4″	15·0	4	....	1,938
" "	"	6×4	37′ 7″	15·0	4	....	2,256
Diagonals............	"	6×4	11′ 7″	16·7	8	Cut.	1,547
"	"	$5 \times 3\frac{1}{2}$	11′ 3″	9·0	8	"	792
"	"	$5 \times 3\frac{1}{2}$	11′ 3″	9·0	8	"	792
Verticals.............	"	4×3	7′ 10″	6·7	20	....	1,049
Top laterals..........	"	$5 \times 3\frac{1}{2}$	9′ 4″	14·0	2	Cut.	261
" "	"	$5 \times 3\frac{1}{2}$	9′ 4″	11·3	6	"	633
Connecting angles for end bottom laterals.	"	$5 \times 3\frac{1}{2}$	0′ 9″	10·0	2	"	14
Struts................	"	3×3	5′ 5″	6·0	4	Top lateral.	129
Bottom laterals	"	3×3	9′ 7″	6·0	2	....	115
" "	"	3×3	9′ 3″	6·0	6	....	333
Cross bracing.........	"	3×3	8′ 9″	6·0	10	....	525
Top and bottom struts.	"	3×3	6′ 2″	6·0	10	....	372
Web for top chord....	Plates.	$12 \times \frac{3}{4}$	32′ 5″	30·0	2	....	1,945
" " " "	"	$12 \times \frac{3}{4}$	37′ 8″	30·0	2	....	2,262
Web for bottom chord.	"	$12 \times \frac{3}{4}$	15′ 9″	30·0	2	....	945
" " " "	"	$12 \times \frac{3}{4}$	21′ 0″	30·0	2	....	1,260
" " " "	"	$12 \times \frac{3}{4}$	2′ 0″	30·0	4	....	240
Splice plates..........	"	$12 \times \frac{7}{16}$	2′ 2″	17·5	4	....	152
" "	"	$11 \times \frac{3}{8}$	2′ 2″	13·8	8	....	242
" "	"	$8 \times \frac{1}{2}$	2′ 2″	13·3	8	....	234
Diagonals............	"	$6 \times \frac{11}{16}$	11′ 6″	13·8	8	....	1,270
Wall plates........	"	$12 \times \frac{3}{4}$	1′ 8″	30·0	4	....	204
Gussets..........	"	$10 \times \frac{3}{8}$	1′ 4″	12·5	2	Cut.	25
"	"	$10 \times \frac{3}{8}$	1′ 2″	12·5	2	"	23
"	"	$10 \times \frac{5}{16}$	1′ 0″	10·4	20	Sway frame.	208
"	"	$9 \times \frac{3}{8}$	1′ 4″	11·3	4	Cut.	38
"	"	$9 \times \frac{3}{8}$	0′ 6″	11·3	4	Top lateral cut.	16
"	"	$9 \times \frac{5}{16}$	1′ 0″	9·4	6	Bottom lateral cut.	40
"	"	$7 \times \frac{5}{16}$	0′ 8″	7·3	5	Sway frame.	24
"	"	$10 \times \frac{3}{8}$	2′ 1″	12·5	2	Top lateral.	52
"	"	$9 \times \frac{3}{8}$	2′ 1″	11·3	2	Top lateral cut.	39
"	"	$9 \times \frac{5}{16}$	1′ 9″	9·4	4	Bottom lateral.	66
Fillers...............	"	$6 \times \frac{11}{16}$	0′ 9″	13·8	8	....	83
"	"	$6 \times \frac{1}{2}$	1′ 0″	10·0	8	....	80
"	"	$3 \times \frac{3}{16}$	0′ 9″	1·9	8	....	13
1,482 pairs rivet heads at 4..........							593
Total..........							25,368



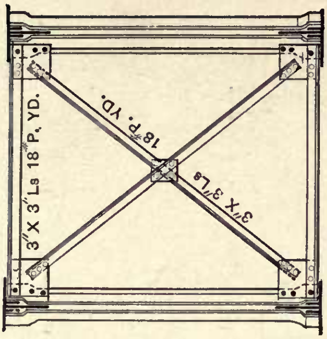
INTERMEDIATE — END

FLOOR SYSTEM
FIG. 1207.

DETAILS OF ROLLERS AND END CASTINGS
FIG. 1206.

DETAIL OF FIXED END OF GIRDER
FIG. 1205.

PLAN



CROSS FRAME.
FIG. 1210.

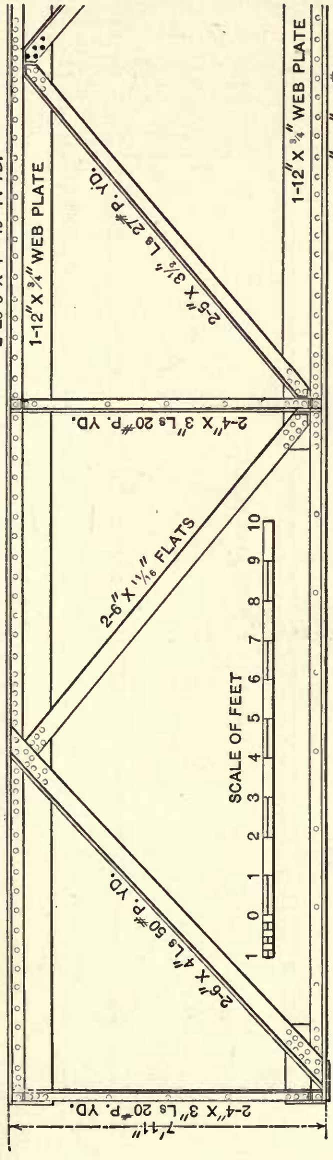


FIG. 1208.

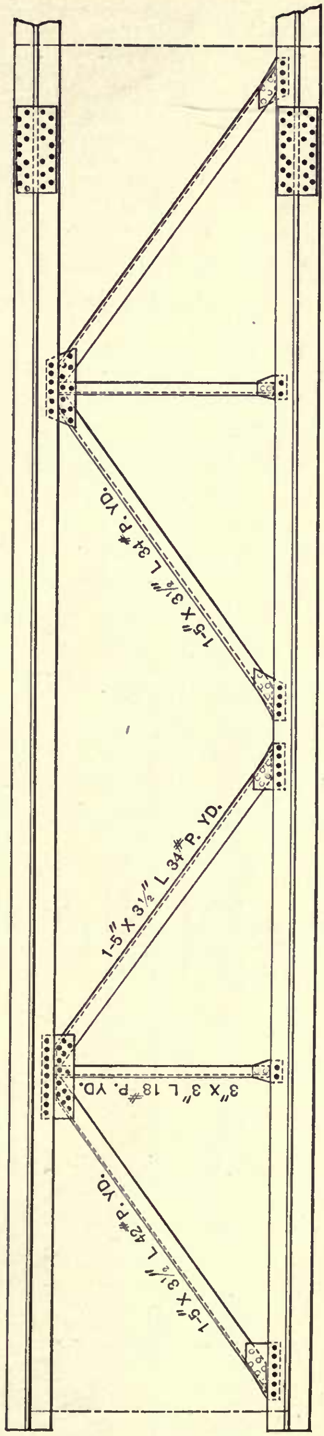


FIG. 1209.

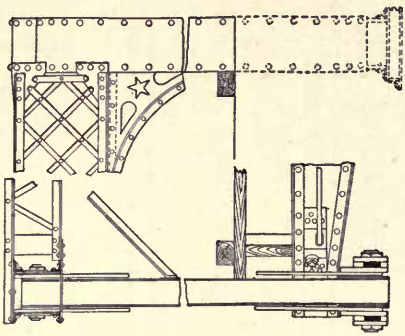


Fig. 1212.

Fig. 1213.

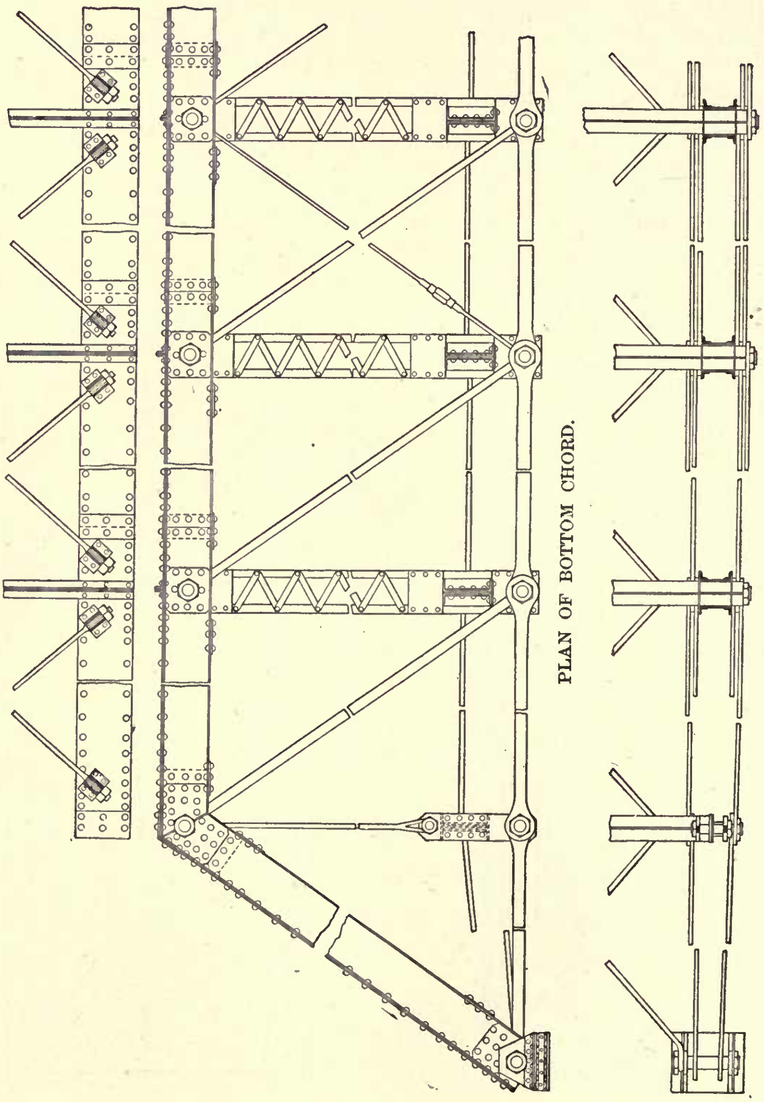


Fig. 1211.

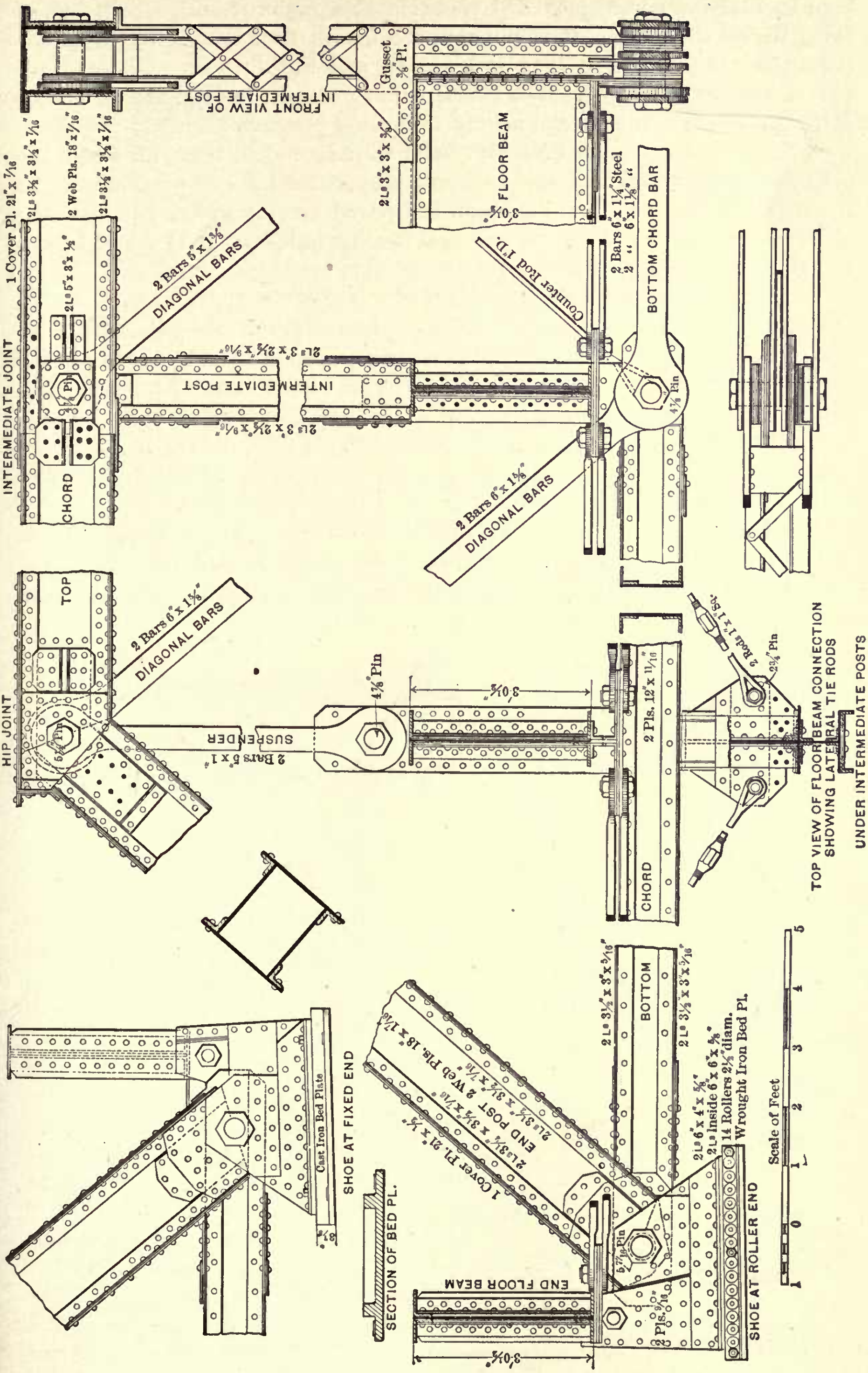

FRONT VIEW OF INTERMEDIATE POST
Gusset ⅜" Pl.
FLOOR BEAM
BOTTOM CHORD BAR
Counter Rod 1" D.
INTERMEDIATE JOINT
1 Cover Pl. 21" x 7/16"
2 Web Pls. 18" x 7/16"
DIAGONAL BARS
INTERMEDIATE POST
CHORD
HIP JOINT
TOP
SUSPENDER
2 Bars 5" x 1"
2 Bars 6" x 1⅛"
4⅞" Pin
2 Pls. 12" x 11/16"
TOP VIEW OF FLOOR BEAM CONNECTION SHOWING LATERAL TIE RODS UNDER INTERMEDIATE POSTS
SHOE AT FIXED END
Cast Iron Bed Plate
SECTION OF BED PL.
END POST
BOTTOM
END FLOOR BEAM
SHOE AT ROLLER END
14 Rollers 2½" diam.
Wrought Iron Bed Pl.
Scale of Feet

FIG. 1214.

Fig. 1211 is a partial plan and elevation of a highway bridge built by the King Bridge Company. It is pin connected, with floor-beams riveted to the posts above the pin. Fig. 1212 is a section, and Fig. 1213 the end post, with part of the portal bracing. The lateral bracing of both top and bottom chords is designed to take the stresses arising from wind pressure. Each system forms with the respective chords of both trusses an independent truss, for which the stresses are calculated and the members proportioned. The wind stresses of the upper system are carried through the portal bracing and end posts to the abutments, and those of the lower lateral bracing immediately to the end shoes and abutments; the floor-beams are compression members.

Fig. 1214 contains drawings in detail of a railroad bridge through span on the Lima and Oroya Railroad, and is a good example of the usual American practice. It is a single-intersection Pratt truss through span, built of iron and steel. The length from centre to centre of end pins is 180 feet, with eight panels, each 22 feet 6 inches; depth of truss 26 feet and distance between trusses 16 feet. The bridge is of the pin-connected type, the tension members being steel eye-bars, and the compression members built sections of plates and angles. The floor-beams are riveted to the posts above the pins.

Fig. 1215 is a side elevation of an angle connection of end brace and top chord.

Conventional signs of riveting will be found in the Appendix.

Fig. 1216 is a standard pin-nut, on sale in sizes from $1\frac{15}{16}''$ to $7\frac{9}{16}''$ diameter

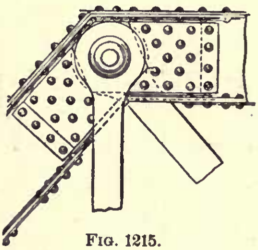


FIG. 1215.

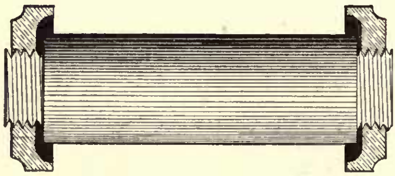


FIG. 1216.

of pin; Fig. 1215 is an example of its use. Fig. 1217 is a side and top elevation of a standard clevis, in sizes, changing by eighths from $\frac{7}{8}''$ to $3''$ diameter of bar.

Figs. 1218, 1219, and 1220 are elevation, plan, and detail of a turn-table as made by the Passaic Rolling Mill Company. The table is entirely centre-bearing, and rests on a hard steel base. The girders are strongly built of plates and angles and cover-plates, and connected to each other by frames and lateral angle bracing. The sizes vary from 40 to 60 feet in length.

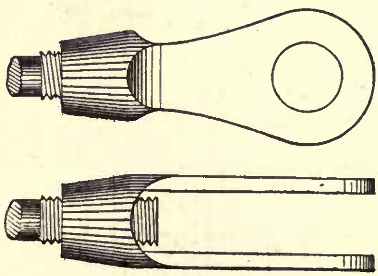


FIG. 1217.

Figs. 1221, 1222, and 1223 are section and details of a turn-table as made by the Greenleaf Manufacturing Company. The conspicuous feature of this centre is the nest of rollers, shown on a larger scale in Fig. 1222. The rolls and bearings are made of good quality tool steel and are $2\frac{7}{16}$ inches in length and of the

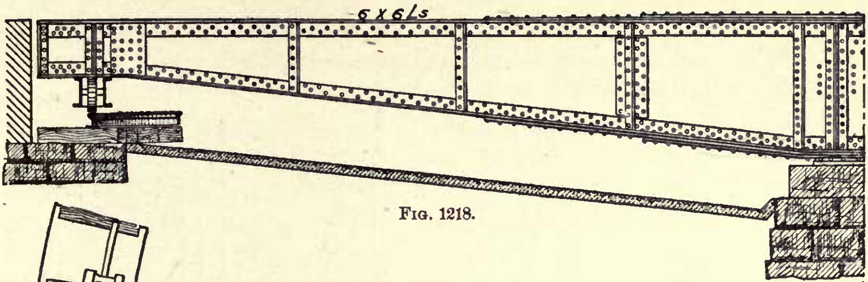


FIG. 1218.

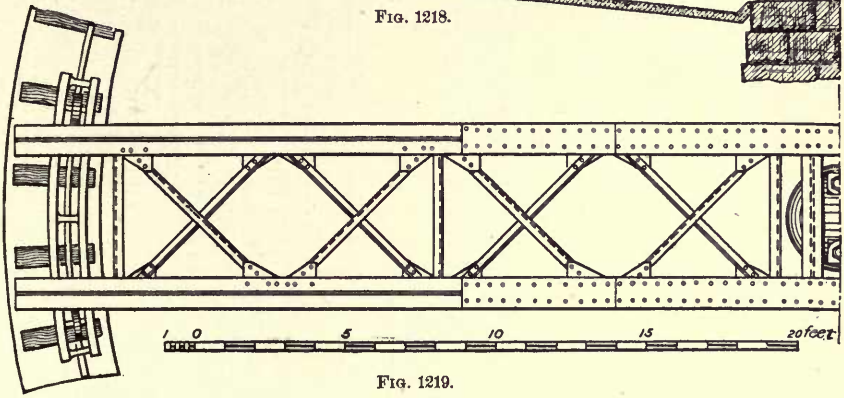


FIG. 1219.

same diameter on the large end and $1\frac{7}{16}$ inch in diameter on the small end, and run free in the annular groove. Fig. 1223 is the end carriage of the same table.

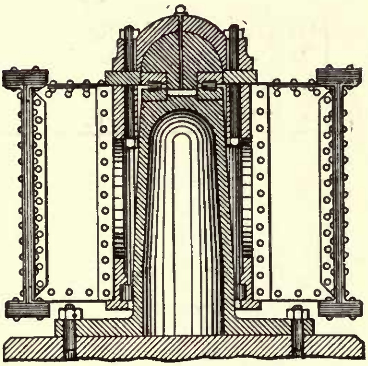


FIG. 1221.

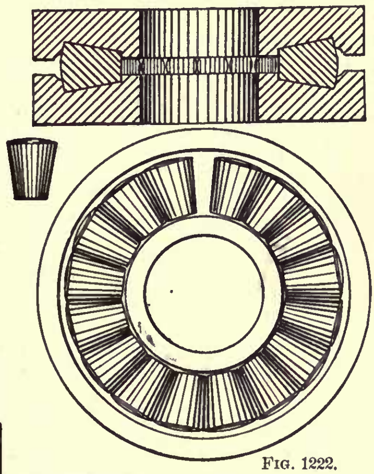


FIG. 1222.

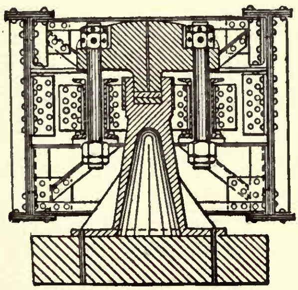


FIG. 1220.

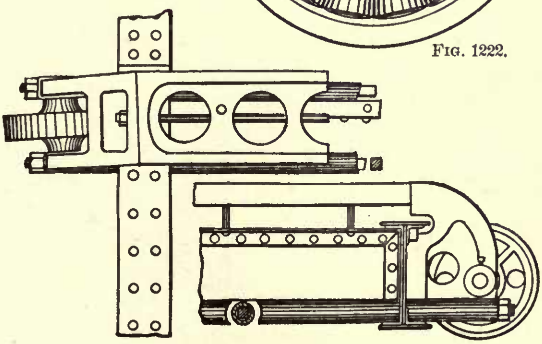


FIG. 1223.

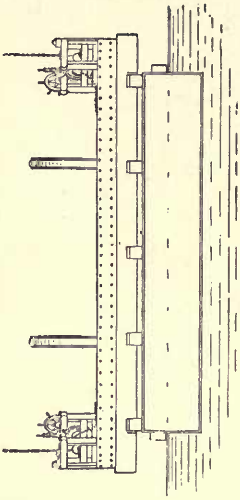


FIG. 1225.

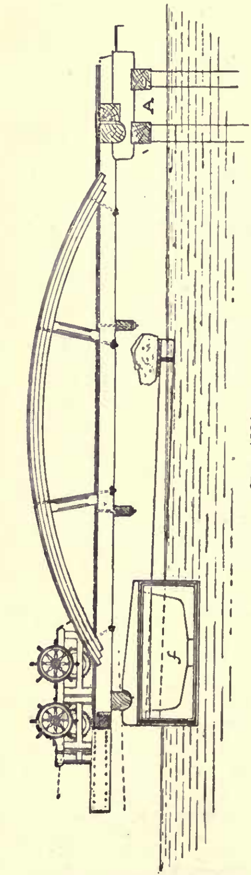
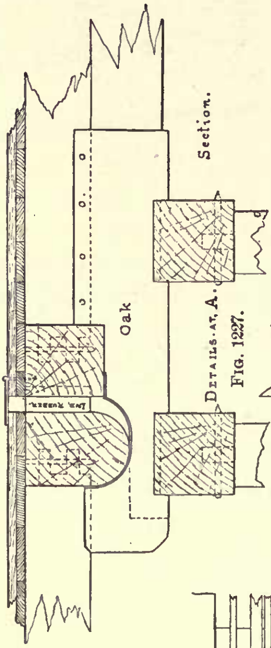


FIG. 1224.



DETAILS AT A.
FIG. 1227.

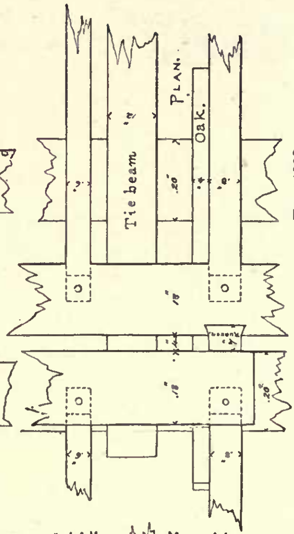


FIG. 1228.

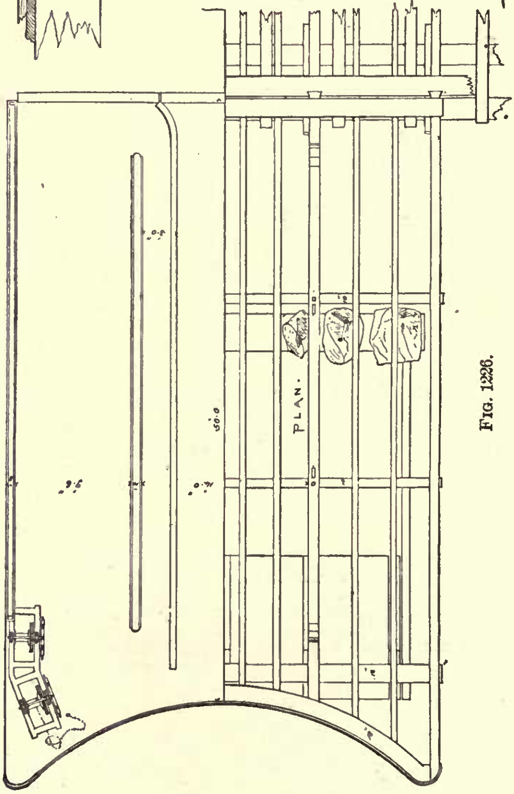


FIG. 1226.

Figs. 1224 to 1228 are illustrations of a landing-bridge common at New York city ferries. Fig. 1224 is a longitudinal section, showing a section of the float, *f*, with its lever shown at one side of the float and stone counterpoise to balance the weight of the bridge. Fig. 1225 is the front elevation, and Fig. 1226 the plan, one half in planking and one half in framing. There are two chain-barrels, on each side of the bridge,

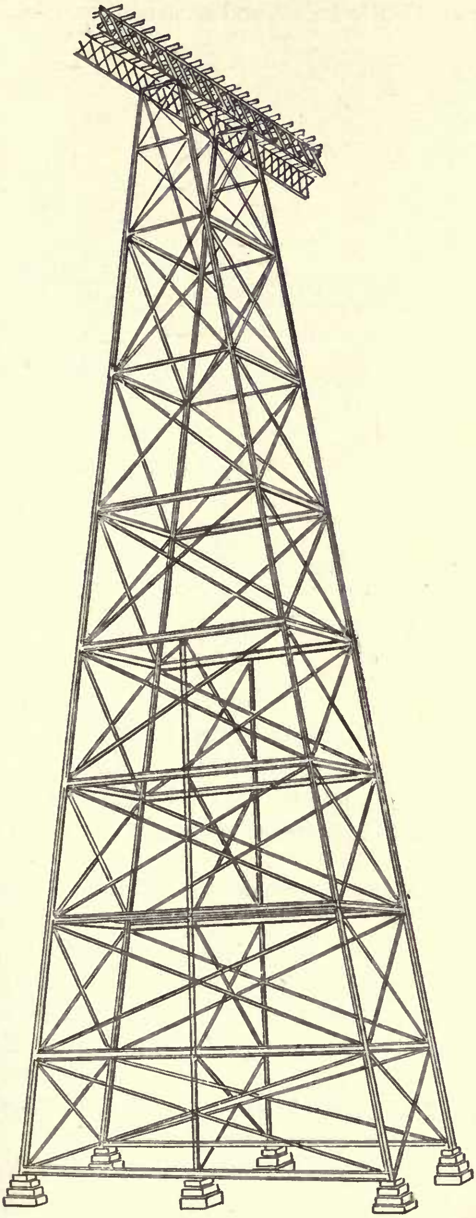


Fig. 1229.

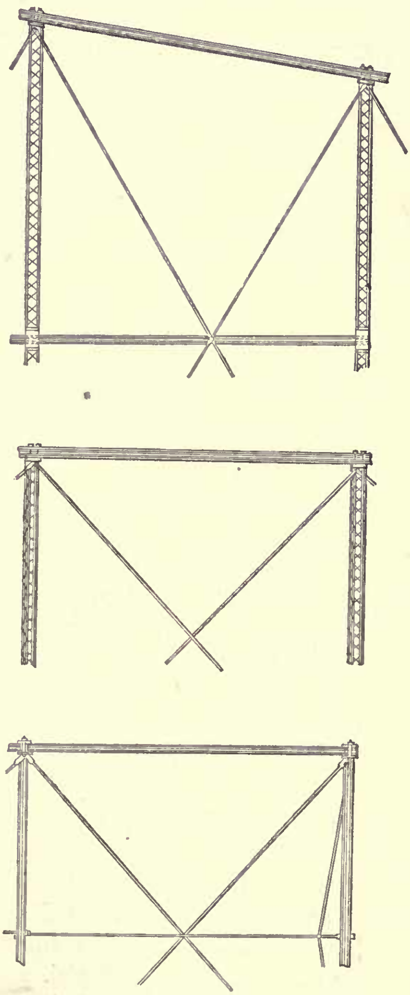


Fig. 1230.

worked by hand-wheels; on the outer ones are the chains by which the boat is drawn up to the bridge; on the inner ones the chains by which the bridge is adjusted to the load on the boat, and by which a part of the weight of the bridge is held, the upper ends of the chains being attached to the frame over-head. The details (Figs. 1227 and 1228) in section and plan explain the construction of the land-hinge; a cushion of rubber is introduced into the joint to modify the shock caused by the boat striking the bridge, and a flap of wrought-iron to cover the joint, for protection to travel, and security from dirt.

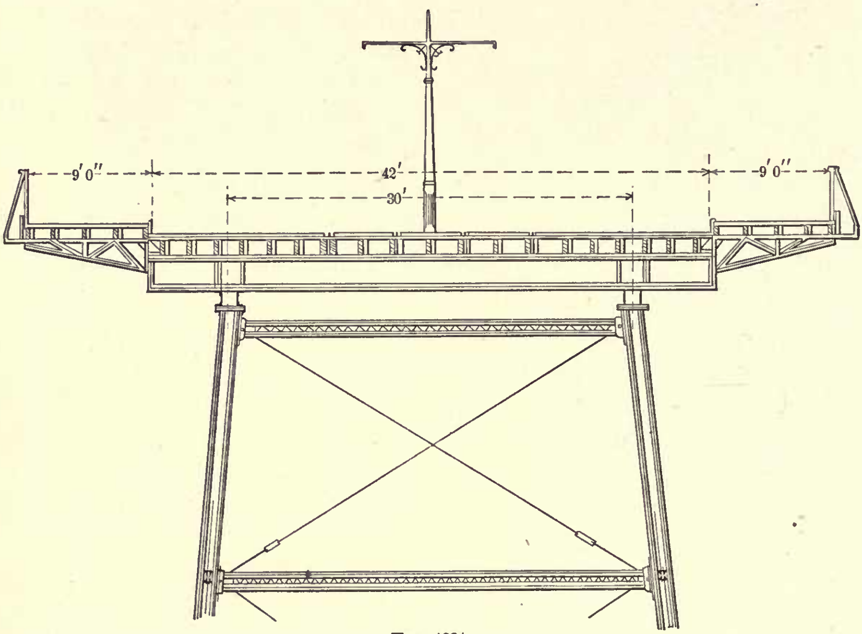


FIG. 1231.

For motives of economy in cost or space, it has become very common among engineers to construct bridges with frequent wrought-iron piers or trestles of great height rather than extended truss spans; one of the most remarkable of these is the Kinzua Viaduct, of which Fig. 1229 represents a single pier in perspective, with details, Fig. 1230.

Fig. 1231 is a section of the roadway of the Rivermont Bridge, Lynchburg, Va., supported by similar piers.

Fig. 1232 is the elevation of a bridge over the Rio Galisteo, Apache Cañon, on the line of the N. M. & S. P. R. R. The ends of the bridge rest on abutments on opposite sides of the cañon. The centre line of the track is on a ten-degree curve, and is 14½ inches off the centre line of the plate girders at the centre and at both ends of the span.

Fig. 1233 is an elevation of one bent of the Third Avenue Elevated Road, and Fig. 1234 of post and foundation on a line of cross-section.

The foundations for the columns supporting the iron structure are simply blocks of concrete, capped with a block of granite, to which the bases of the wrought-iron columns are bolted. In soft material nine piles were driven, capped with 12-inch timbers, and concrete filled in around them. On sandy bottom concrete was placed about 6 feet below the surface. On a portion of the line a right of way 50 feet wide was secured; the tracks were supported on plate girders resting upon piers of brick masonry, with foundations of concrete below the surface of the ground. The brick piers were capped with granite.

Fig. 1235 is an elevation and half plan of the foundations of a post on the Brooklyn Elevated Railroad, with a base of cast-iron, and Fig. 1236 a similar

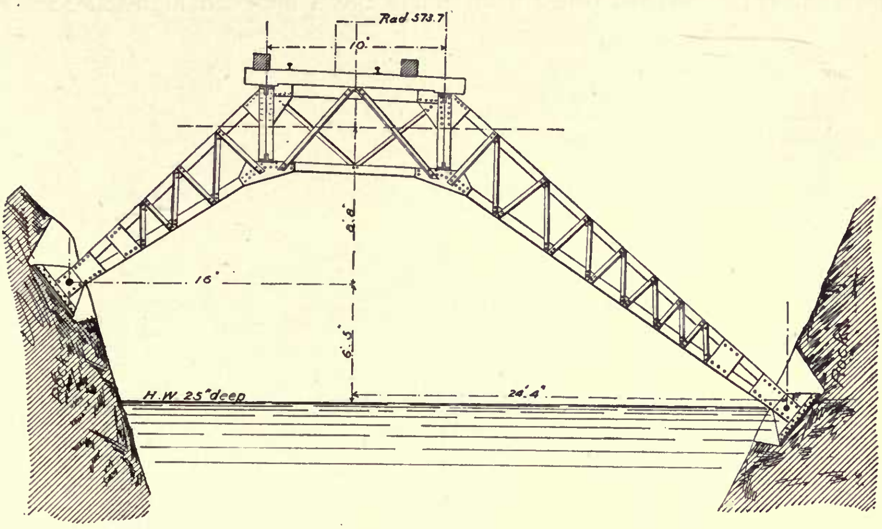


FIG. 1232.

one, with a wrought-iron base. In neither is there any cap except that connected with the post base, and the anchorage is to cast-iron plates wholly within the masonry, which is of concrete.

Fig. 1237 is a plan of one of the stone piers of the railway bridge across the Susquehanna at Havre de Grace. To lessen as much as possible the ob-

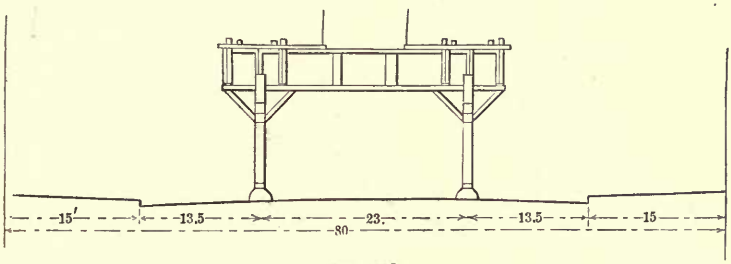


FIG. 1233.

struction to the flow of the stream, it is usual to make both extremities of the piers pointed or rounded. Sometimes the points are right angles; sometimes angles of 60°; often a semicircle, the width of the pier being the diameter; occasionally pointed arches, of which the radii are the width of the pier, the centres being alternately in one side, and their arcs tangent to the opposite side. None of the stones break joint at the angle; this is important in opposing resistance to drift-wood and ice. It is not unusual, in very exposed places, to make distinct ice-breakers above each pier usually of strong crib-work, with a plank-slope upstream of 45°, and with a width somewhat greater than that of the pier.

Fig. 1238 is the plan and Fig. 1239 the side elevation of a pier of a bridge across the Missouri, on the Northern Pacific Railroad at Bismarck, designed and constructed by George S. Morison, C. E. In this design both ends of the pier are rounded, but the upper extremity is extended beyond the main body of the pier, and is inclined and plated with iron between low- and high-water mark.

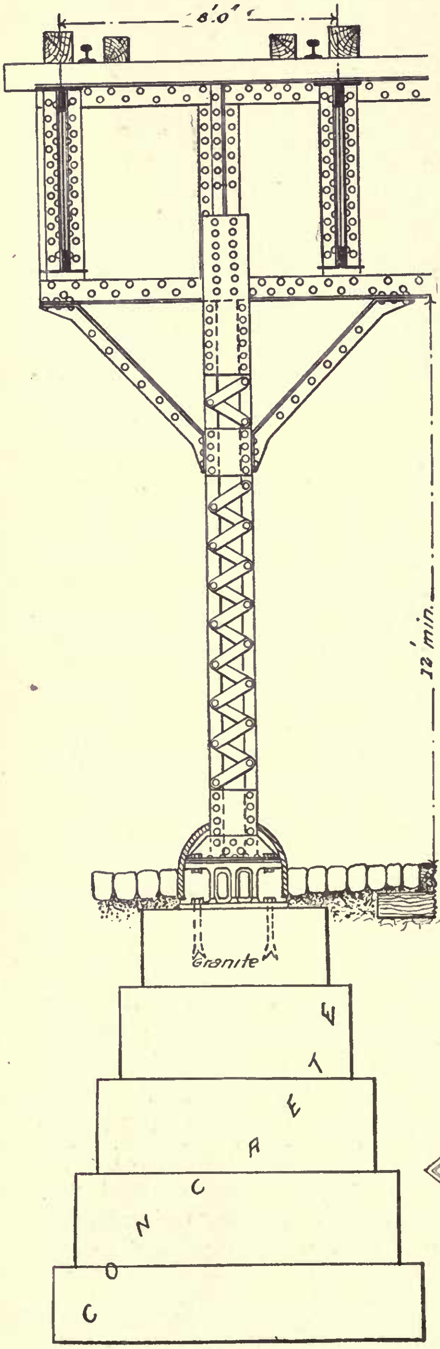


FIG. 1234.

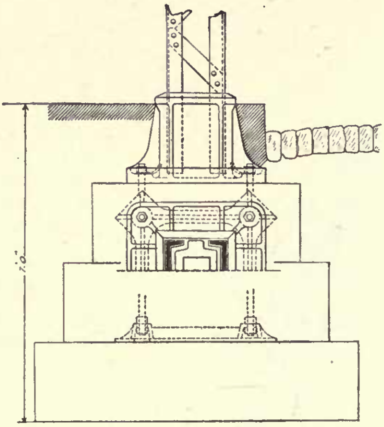


FIG. 1235.

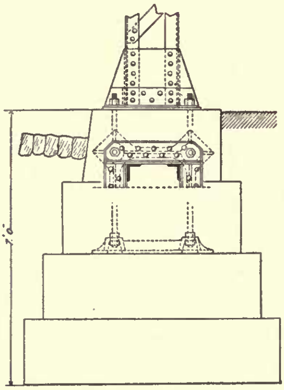


FIG. 1236.

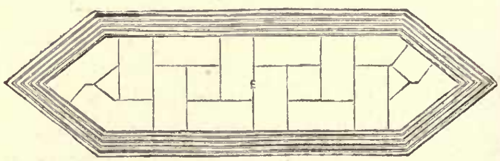


FIG. 1237.

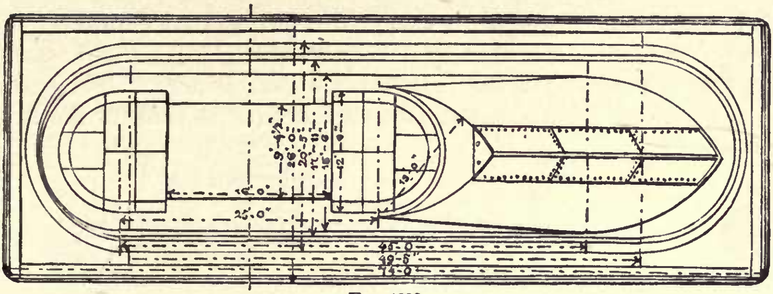


FIG. 1238.

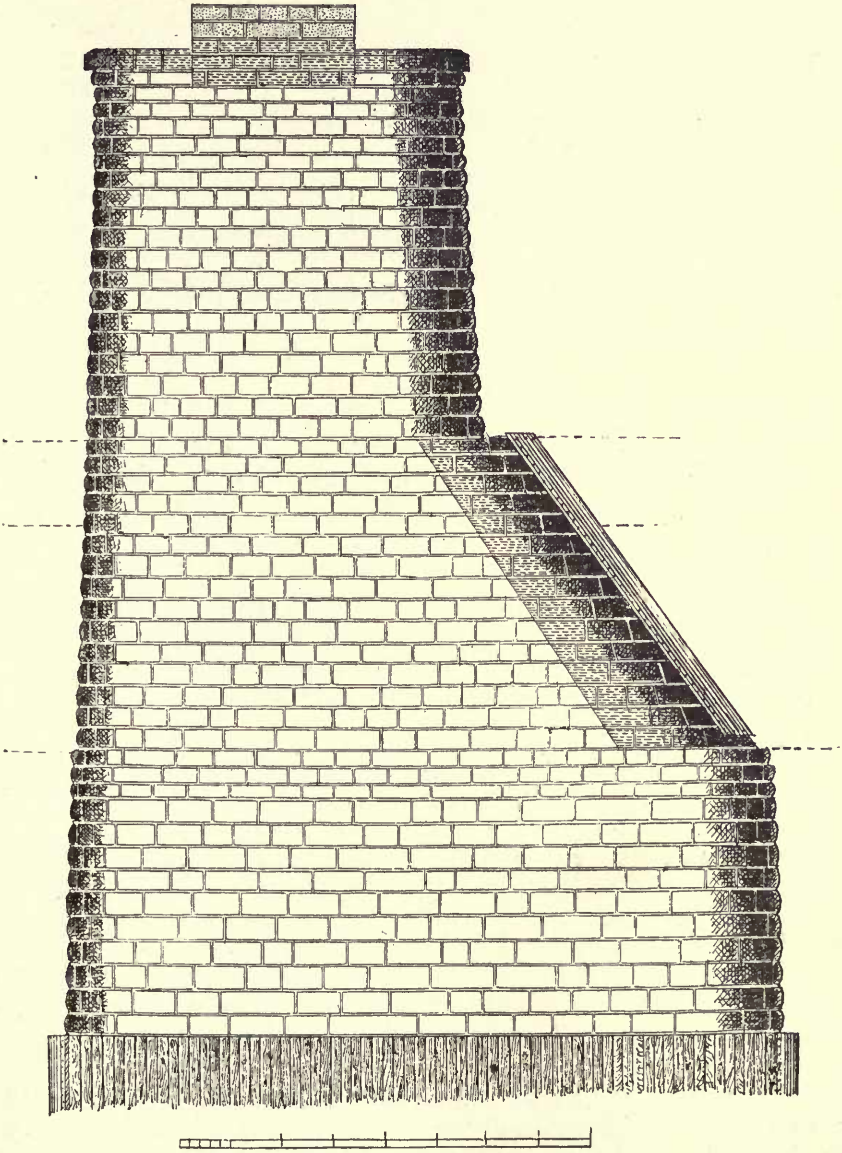


FIG. 1239.

This is intended not only to turn aside drift, but as an ice-breaker; the ice, moving up the incline, is broken by its own weight.

Fig. 1240 is a section of the foundation of the Bismarck bridge, showing the construction of the inverted caisson, similar to that used for the Brooklyn

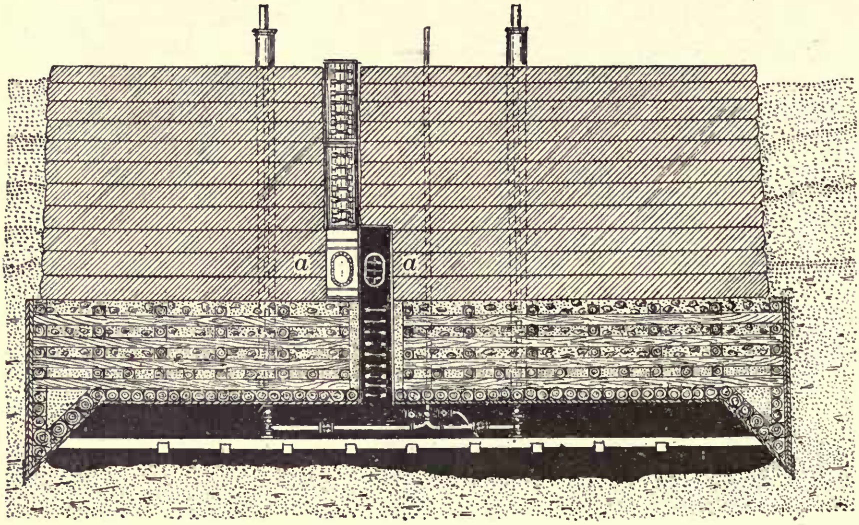


Fig. 1240.

Bridge piers. The caissons are 74′ long, 26′ wide, and 17′ high outside; the working-chamber 7 feet high. The caissons are built of pine, sheathed with two thicknesses of 3″ oak-plank. Above is crib-work filled in with Portland cement concrete; *a a* are the air-locks. The sand was removed from the caissons by water-ejectors.

Arch bridges are of stone, brick, concrete, or metal; the parts of the arch exert a direct thrust upon the abutments, resisted by the inherent weight of the latter, or its absolute fixed mass, as in the case of natural-rock abutments.

Arch bridges, in masonry, are arcs of circle, semicircular (Fig. 1241), segmental (Fig. 1242), elliptical, or described from three or five centres (page 32).

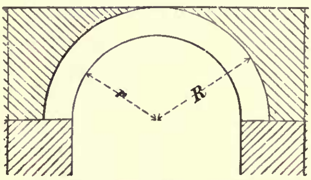


Fig. 1241.

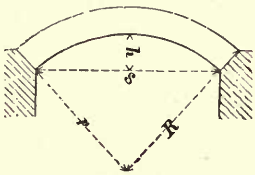


Fig. 1242.

The stones forming the arch are called *voussoirs*, or arch-stones; those forming the exterior face are called ring-stones, the inner line of arch the *intrados*, exterior line the *extrados*. The stones at the top, which are those set last and complete the arch, are *key-stones*. The courses from which the arches spring are called *skew-backs*, and the first course the *springing-course*. The masonry on the shoulders of the arch is called the *spandrel-courses*, or *spandrel-backing*. The weight at the crown of a semicircular arch tends to raise the haunches.

This is counteracted by the spandrel-backing, and by the earth-load, which should be carefully distributed on each side of the arch.

To determine the depth of the key-stone, Rankine gives the following empirical rule, which applies very well to most of the above examples:

Depth at key, for an arch of a series, in feet, $= \sqrt{\cdot 17 \times \text{radius at crown}}$.

For a single arch, $= \sqrt{\cdot 12 \times \text{radius at crown}}$.

To find the radius at crown of a segmental arch, add together the square of half the span and the square of the rise, and divide their sum by twice the rise—

$$R = \frac{\frac{1}{2} S^2 + r^2}{2\,r}$$

Thus, the Blackwall Railway bridge has a span of 87 feet, and a rise of 16—

$$\frac{43{\cdot}5^2 + 16^2}{2 \times 16} = \frac{1892{\cdot}25 + 256}{32} = 67{\cdot}1.$$

To find the radius of an elliptical arch, on the hypothesis that it is an arch of five centres (Fig. 103, page 32), the half-span is a mean proportional between the rise and the radius. Thus, for example, the Great Western Railway bridge is 128′ span, and 24·25′ rise—

$$64^2 = 24{\cdot}25 \times R$$

$$R = \frac{4096}{24{\cdot}25} = 169 \text{ feet.}$$

To find the depth of key-stone, by rule above, as in one of a series—

$$d = \sqrt{\cdot 17 \times 169} = \sqrt{28{\cdot}73} = 5{\cdot}33.$$

The depth of the voussoir is increased in most bridges from the key-stone to the springing-course, but not always; it is safer to increase the depth.

If an arch be loaded too heavily at the crown, the lines of pressure pass above the extrados of the crown and below the line of intrados at the haunches, depressing the crown and raising the haunches, separating the arch into four pieces, and *vice versa* if the arches are overloaded at the haunches. To prevent such effects, especially from moving loads, in construction the arches are loaded with masonry and earth, and this constant load in such excess that there will be no dangerous loss of equilibrium by accidental changes of load.

The horizontal thrust may be determined, according to Rankine, by the following approximate rule, which seldom errs more than 5 per cent:

The horizontal thrust is nearly equal to the weight supported between the crown and that part of the soffit whose inclination is 45°. This thrust is to be resisted by the masonry of the abutment and the earth-load behind it.

Thus, if Fig. 1243 be a section of an abutment of an arch, the horizontal thrust exerted at T is resisted by the mass of masonry of the abutment; the tendency is to slide back the abutment on its base A C, or turn it over on the point A. The sliding motion is resisted by friction, being a percentage, say from $\frac{1}{2}$ to $\frac{3}{4}$ of the weight of the abutment and of half the arch which is supported by this base; but, in turning over the abutment on the point A, the action may be considered that of a lever, the force T acting with a lever T C to raise the weight of the abutment on a lever A B (G being the centre of gravity, and G B the perpendicular let fall on the base), and the weight of half the arch on the lever A C. That is, to be in equilibrium, the horizontal thrust

T × T C must be less than the sum of the weights of the abutment multiplied by A B, and the weight of the arch multiplied by A C.

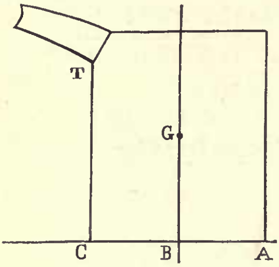


FIG. 1243.

Skew bridges are those in which the abutments are parallel, but not at right angles to the centre line, and the arches oblique. To construct these in cut stone requires intelligence and education both in the designer and stone-cutter; but, when the work is laid full in cement, so that the joints are as strong as the material itself, this refinement of stone-cutting is not necessary. The arch may safely be constructed as a regular cylinder of a diameter equal to the rectangular distance between the abutments, with its extremity cut off parallel to the upper line of road. For such an arch hard-burned brick is the best material, the outer voussoirs being cut stone.

In the rules above given no consideration is paid to the strength of the cement in which the stones are bedded. When the cement is thoroughly set, the structure is in a measure monolithic, and the thrust is inconsiderable.

Fig. 1244 is the elevation of one of the stone arches of the Minneapolis Union Railway Viaduct, with the timber centres on which the arch was turned. The arch is nearly semicircular, 97·82 feet span, 50 feet rise; width, 28 feet;

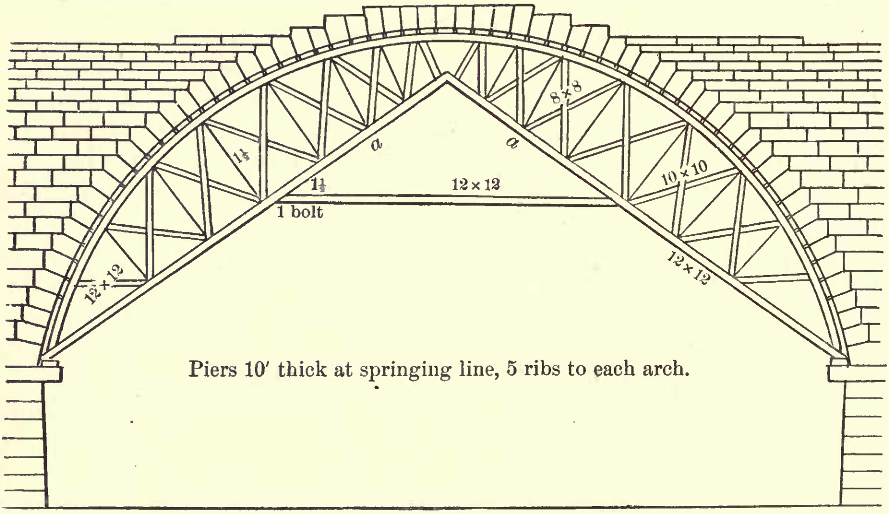


FIG. 1244.

depth of arch at spring, 40″; at key, 36″. The piers are 10 feet thick at springing line; their up-stream ends are at angles to the main body of the piers, and parallel to the thread of the stream. The whole length is 2,100 feet, of which there are 3 arches of 40 feet span, 16 of 80 feet, and 4 of 100 feet. Height above water, 65 feet; total height, 82 feet.

The centres were very light frames, five to each arch; the chords, timber arches, and ties were each 12″ × 12″, the central braces 10″ × 10″, and the shorter side-braces 8″ × 8″; the bolts, single, 1½″ diameter.

The bridge was constructed after the designs and under the direction of Charles C. Smith, C. E., Chief Engineer of the St. Paul, Minneapolis and Manitoba Railway, and is an example of a very economical and stable construction. The piers are of Minnesota granite, but above springing line the masonry is of magnesian limestone. It was commenced in February, 1882, and completed in November, 1883.

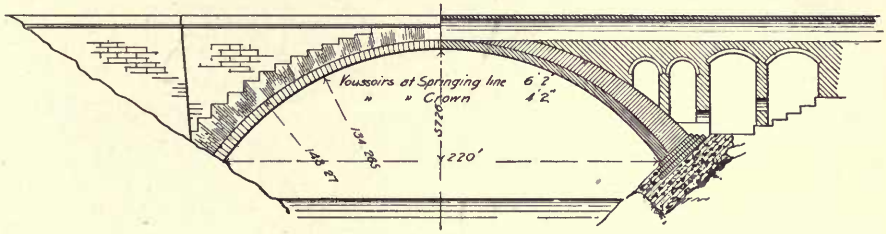


FIG. 1245.

Fig. 1245 is a half-elevation and half-section of the Cabin John Bridge used for supporting the Potomac conduit on the line of supply to the city of Washington.

LOCATION.	Material.	Form of arch.	Span.	Rise.	Depth at crown.	Depth at spring.
Manchester and Birmingham Railroad.	Brick.	Semicircular.	18	9	1·6	Uniform.
" " " "	"	"	63	31·6	3	"
London and Brighton Railroad.......	"	"	30	15	1·6	2·3
" " Blackwall "	"	Segmental.	87	16	4·1½	Uniform.
Great Western Railroad	"	Elliptical.	128	24·3	5	7·1½
Chestnut Street (Philadelphia) Railroad	"	Segmental.	60	18	2·6	
High Bridge, Harlem River, New York	Stone.	Semicircular.	80	40	2·8	
St. Paul, Minneapolis & Manitoba Railroad (largest arch), at Minneapolis..	"	Segmental.	97·8	50	3	3·4
Cabin John Washington Aqueduct....	"	"	220	57·3	4·2	
Licking Aqueduct and Ohio Canal....	"	Elliptical.	90	15	2·10	
Monocacy "	"	"	54	9	2·6	
Hutcheson "	"	Segmental.	79	13·6	3·6	4·6
Chemin du Fer du Nord, sur l'Oise....	"	"	82·5	13·5	4·6	
D'Enghien Railroad du Nord.........	"	Semicircular.	24·4	12·2	1·4	
Du Crochet Railroad	"	"	13·2	6·6	1·7½	
Experimental arch, designed and built by M. Vaudray, Paris..............	"	Segmental.	124	6·11	2·8	3·7

The arch last in the list was a very bold specimen of engineering, built as an experiment, preliminary to the construction of a bridge over the Seine. It was made of cut stone, laid in Portland cement, with joints of $\frac{3}{8}''$, and left to set four months; the arch was 12′ wide; the centres rested on posts in wrought-iron boxes filled with sand, and, as the centring was eased by the running out of the sand, the crown came down $\frac{6}{10}''$; the joints of one of the skew-backs opening $\frac{7}{1000}''$ during the first day, it came down $\frac{3}{100}''$. It was then loaded with a distributed weight of 360 tons; under this load the crown settled $\frac{3}{10}''$ more. Since then nothing has stirred, although it was afterward tested by allowing five tons to fall vertically 1′ 6″ on the roadway over the key-stone. This bridge will not come within any of the rules laid down for other constructions. The rise is about $\frac{1}{18}$ the span, although the usual practice for segmental and elliptical arches is more than $\frac{1}{6}$, or within the limits of $\frac{1}{4}$ and $\frac{1}{8}$.

Melan concrete arch, Stockbridge, Mass. (Figs. 1246 and 1247), is a segmental arch of 100 feet span and $\frac{1}{10}$ rise.

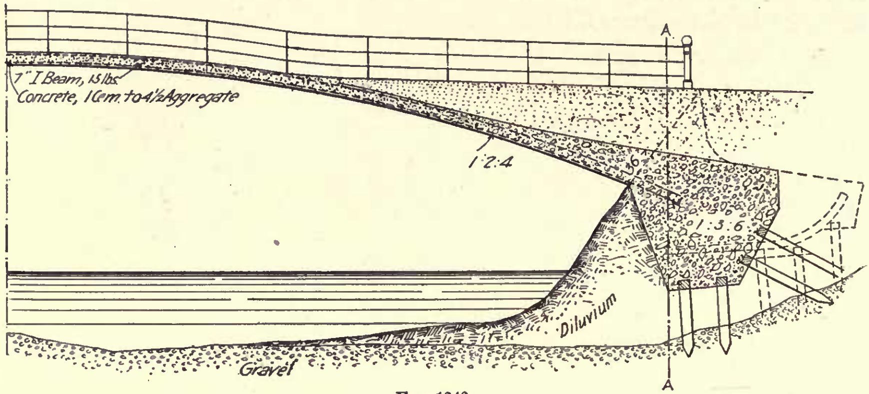


FIG. 1246.

The iron in the arch consisted of bent ribs of 7-inch, 15-lb. I-beams, spaced 28 inches apart and raised about $1\frac{1}{2}$ to 2 inches from the soffit. The concrete in the abutments (1 cement, 3 sand, and 6 stone) was put in first with the adjoining wing walls to such a height as to inclose the ends of the iron ribs, which abutted against an iron cross-angle previously bolted to them.

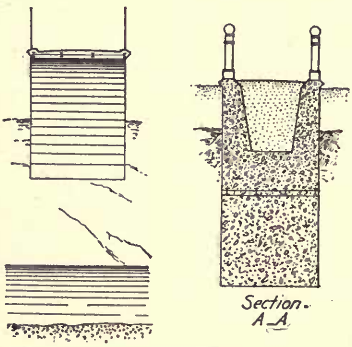


FIG. 1247.

The concrete in the arch is 9 inches thick at the crown and 30 inches at the haunches. The concreting of the arch was done in one day. Then the concrete filling, the face and wing walls, and finally the coping were built.

The bridge is a concrete monolith, the coping forming one piece with the arch. The iron railing is embedded in the concrete.

After striking the centres the arch settled about $\frac{1}{2}$ inch at the abutments and $\frac{3}{4}$ inch at the centre.

The bridge was built for $1,475.

In suspension-bridges the platform of the bridge is suspended from cables, or chains, the ends of which are securely anchored within the natural or artificial abutments.

The curve of a suspended chain is that known as the catenary, and, if the whole weight of the structure were in the chain itself, this would be the curve of the chains of a suspension-bridge; but, as a large part of the weight and the whole of the loading lies in the platform, the curve approaches that of the parabola, and in all calculations it is so regarded.

Let Fig. 1248 represent a suspension-bridge, in which A, B, C, are points in a parabolic curve.

Rule.—Add together four times the square of the deflection $(E\,B)^2$ and the square of half-span $(A\,E)^2$, and take the square root of this sum; multiply this

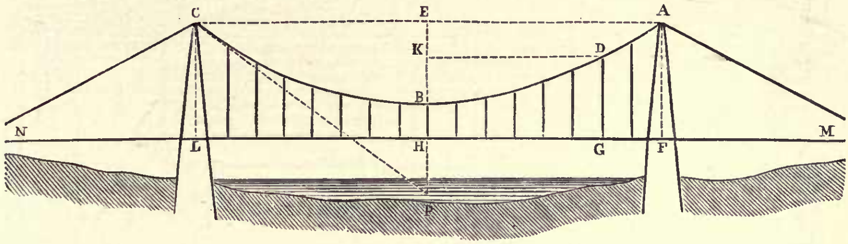


FIG. 1248.

result by the total weight of *one* chain and all that is suspended from it, including the distributed load, and divide this product by four times the deflection (E B) of the cable at the centre, and the result will be the tension on one chain, at each point of support, A and C. The angles made by the chain at the point of support—viz., angle P C L and the angle of the back-stays, or continuation of the chain (angle L C N)—should be equal to each other, in order that there be no tendency to overturn the tower C L and A F.

BRIDGES.	Main spans.	Deflection of chain or cable.	No. of chains and cables.	Total effective section of cable in square inches.	Mean weight of cable per foot of span (pounds).	Fixed load per ft. of span (lbs.).	Breadth of platform in feet.
Menai	570	43	16	260	880		28
Chelsea......	348	29	4	230	767		47
Pesth........	666	47·6	4	507	1,690	9,892	46
Bamberg.....	211	14·1	4	40·2	137	1,581	30·5
Freyburg	870	63	4	49	167	760	21·25
Niagara Falls.	821	54 and 64	4	241·6	820	2,032	24
Cincinnati....	1,057	89	2	172·6	516	2,580	36
Brooklyn.....	1,595·5	128·5	4	188	501·3	5,365	85

BOILER-SETTING.

Fig. 1249 is a longitudinal section, Fig. 1250 a plan with section of wall, and Fig. 1251 an elevation half-front and half-sectional of a boiler and setting as recommended by the Hartford branch of the Hartford Steam-Boiler and Inspection Company, showing the interior bracing, steam and water connections, and brickwork. There are ten braces in each head, secured to pieces of T-iron, placed radially, as shown in dotted lines (Fig. 1251). The feed-pipe is through the front-head, just above the line of tubes, extending to the back of the boiler, with a perforated branch across it, that the water may be warmed in its passage and distributed. The front is a *projecting cut-away* front, the boiler-head being nearly on a line with the front below, different from that given in Fig. 918, where the lower part of the shell projects beyond the head of the boiler, and the cast-iron front covers the end. The doors giving access to the tubes are usually semicircular, and hung on the top diameter, but it will be found more convenient to form them in two quadrants, and hung so as to move horizontally. The boilers are to be protected against radiation by a covering of ashes, or a brick arch, resting on the side-walls.

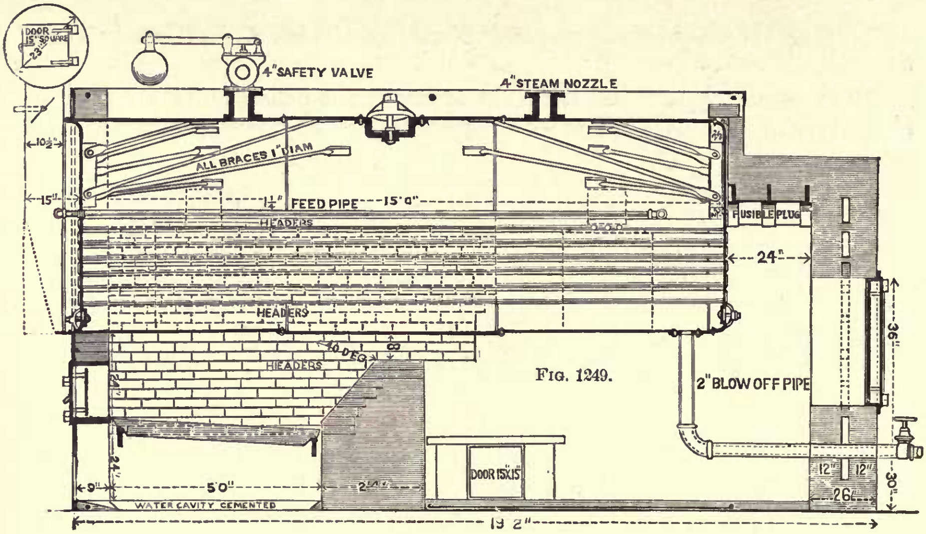


Fig. 1249.

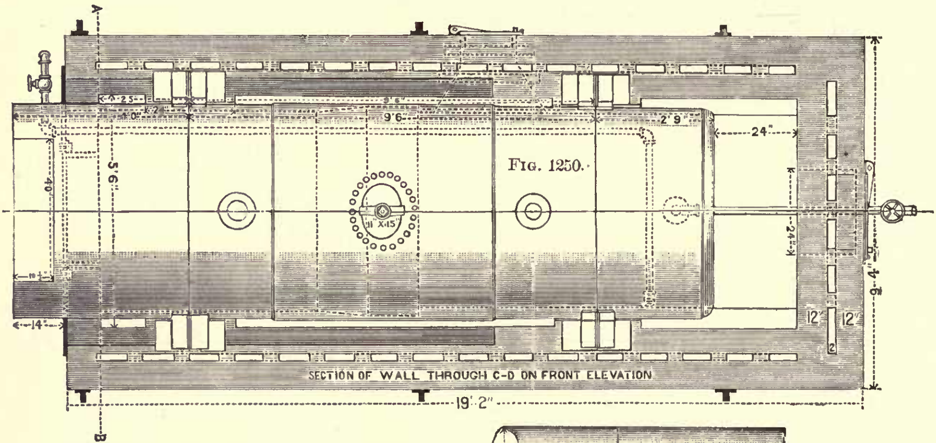


Fig. 1250.

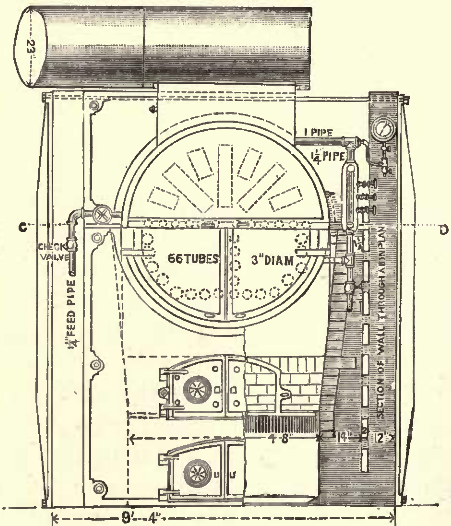


Fig. 1251.

In the figure the man-hole frame is riveted to the inside of the boiler; frequently it is on the outside. In many positions the man-hole should be placed in the back-head, as easier of access. With the blow-off in the flue (Fig. 1249) it is well to make it a circulating pipe by connecting an inch-pipe inside the valve with the upper water-space of the boiler.

Figs. 1252, 1253, and 1254 are drawings of boiler as made by the Fishkill Landing Machine Company. The boilers are suspended between vertical side-walls. The side-walls here are composed of bearing walls, air spaces, and an inside 4″ wall, which is exposed

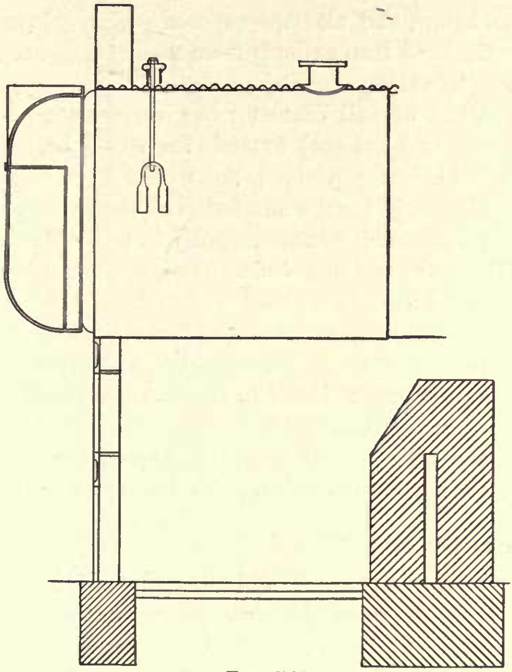


Fig. 1253.

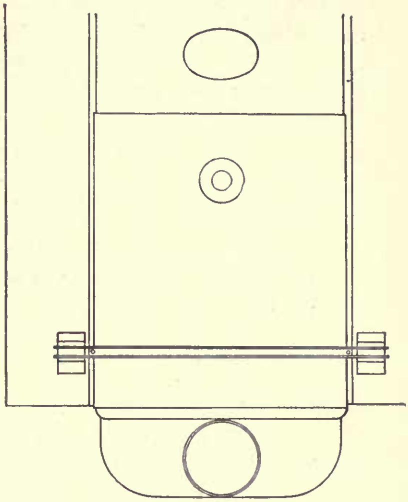


Fig. 1254.

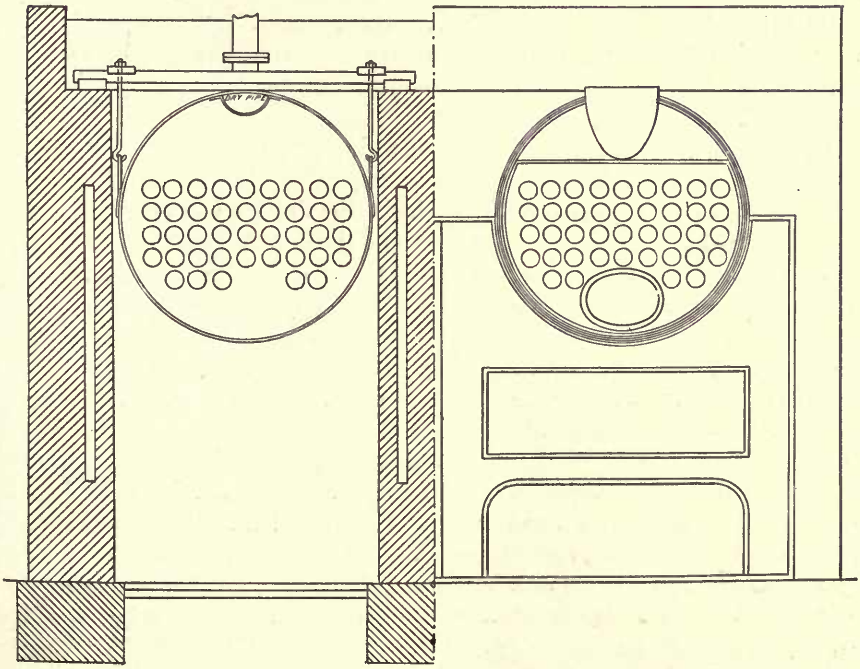


Fig. 1252.

to the heat of the gases from combustion. The fire-box is lined with fire-brick to the height of the centre of the boiler, and also the exposed surface of the bridge wall. A covering of thin corrugated iron extends from wall to wall over the boiler, and supports a thick cover of ashes.

The boiler is built without dome, but of full diameter to give steam capacity; the dry pipe of light iron (gutter or U shape) extends the whole length of the upper part of the boiler, to which it is riveted, but with a washer between it so as to leave open joints of about $\frac{1}{8}''$ for the admission of steam. The pipes are laid out in fan lines to offer as much surface as possible to the flame and to admit of ready cleaning. There are two man-holes, one above the tubes where easiest of access, and one beneath in the front-head with a hand-hole in the back-head. The breeching is of cast-iron, attached to the boiler, with doors opening laterally; the smoke-pipe leads out from the top with a damper at the bottom; the size of the smoke-pipe is proportioned to the number and size of tubes, according to the tables given in the Appendix.

The furnace doors are hung to the sides of the frames to expose the full width of opening for cleaning, and smaller doors are hung on the main doors for firing.

CHIMNEYS.

Figs. 1255 and 1256 are sections of a small circular chimney, about 100 feet high, in which the buttresses are within the outer shell, supporting but not attached to a central flue; these flues may be made of brick glazed ware or concrete. This chimney is without outside buttresses, panels, or ornaments, and offers the least possible resistance to wind.

For chimneys of small diameter it is difficult to obtain circular brick, but the chimney may be made octagonal of any face, with a strong bond, by corner brick, from brickyards and terra-cotta works. Chimneys of the above section, 100 feet high, with bottom corners rounded, will give ample draft with an area of chimney-flue of two square inches for every pound of anthracite per hour burned upon the grate.

If the chimney is of less height it is well to increase the section in proportion to the reduction, but the top should be always above buildings, trees, etc., and remote from obstructions that would check the draft. Chimneys should have flues at least 16″ diameter, and there is no objection to flues of larger area than given by rule above; but it is indispensable for a sure draft that all flues should have corners rounded without abrupt changes in area or direction.

Fig. 1257 is a sectional elevation of a chimney 160 feet high, from John T. Henthorne, M. E., with a cross-section (Fig. 1258) midway of the height.

Figs. 1259, 1260, and 1261 are sectional elevation, front elevation, and cross-sections of a chimney of large dimensions, built by Mr. Henthorne for the Narragansett Electric Light Works, Providence, R. I.

Fig. 1262 is a vertical section of a chimney at the Ridgewood Pumping-engine House, Brooklyn, N. Y., and Fig. 1263 an elevation at the point where the square base is changed into an octagonal.

Fig. 1264 is the cross-section of a buttressed chimney at 100 feet above base, built for the Calumet & Hecla Mining Company, and designed by E. D. Leavitt, Jr., M. E. The whole height of the chimney is 150 feet. The buttress walls are 16″ and 12″ thick, that of the body 12″ and 8″, and of the cen-

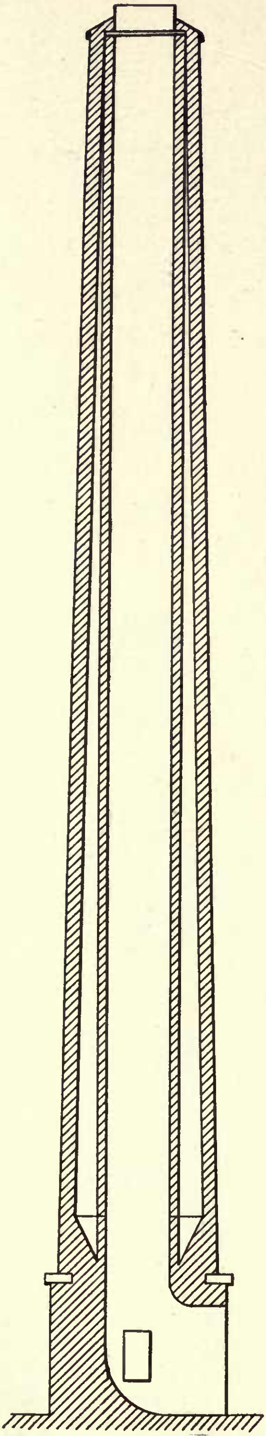


FIG. 1255.

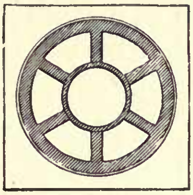


FIG. 1256.

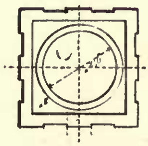


FIG. 1258.

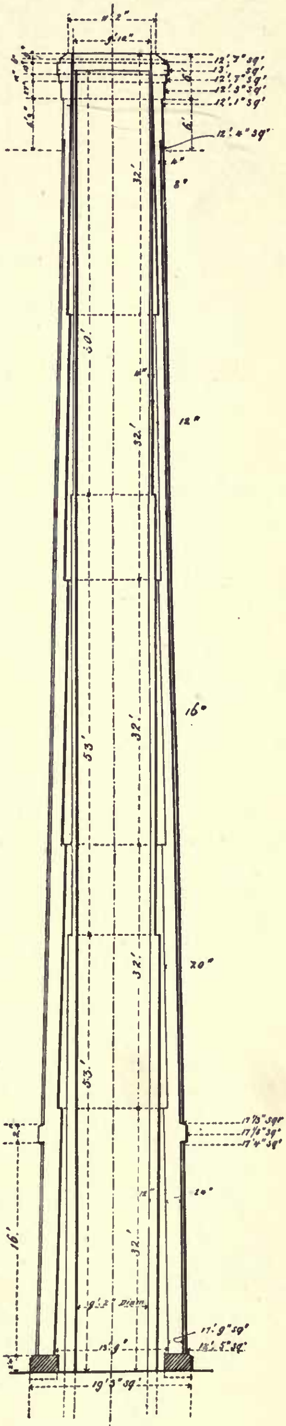


FIG. 1257.

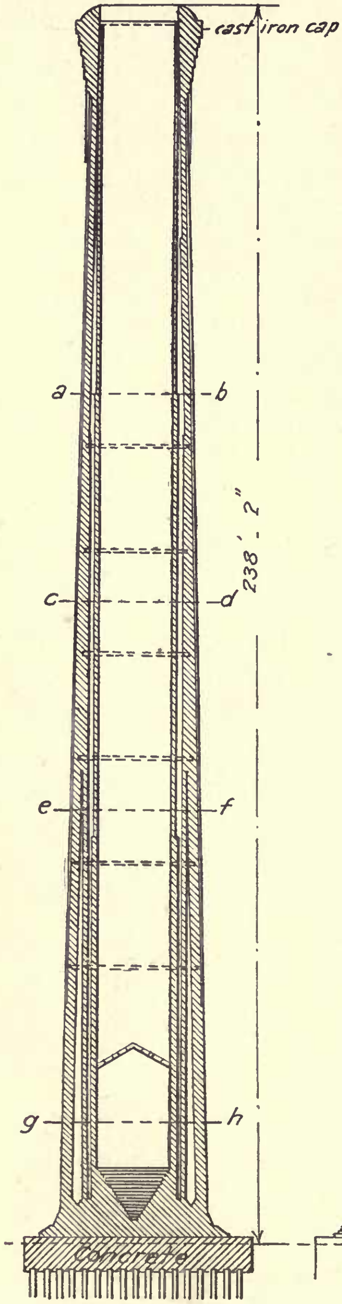


Fig. 1259.

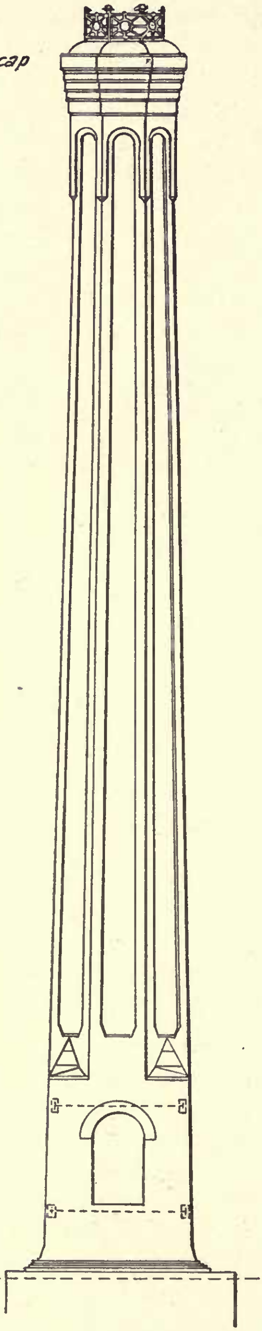


Fig. 1260.

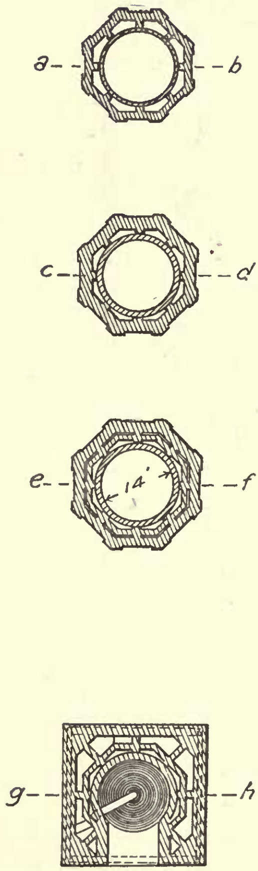


Fig. 1261.

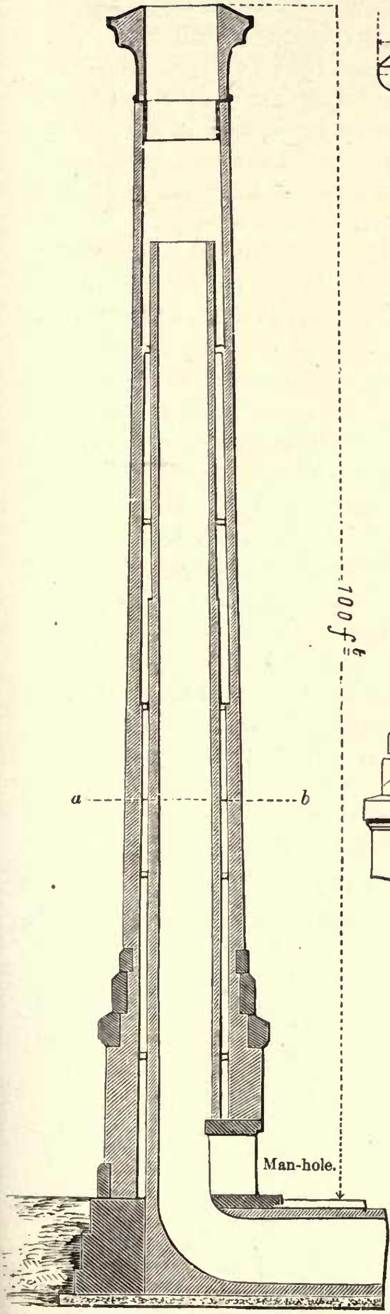


FIG. 1262.

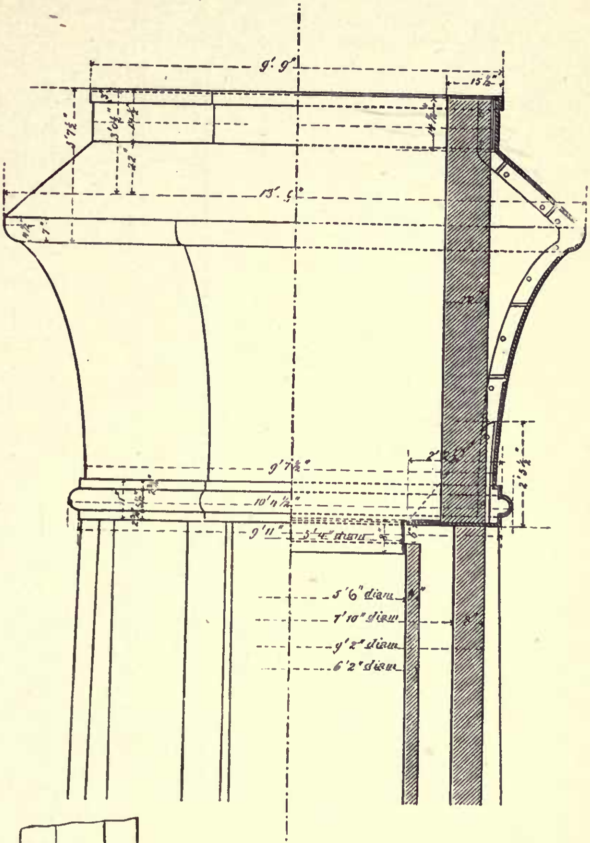


FIG. 1265.

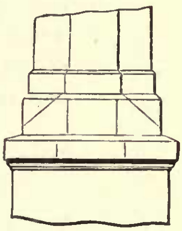


FIG. 1263.

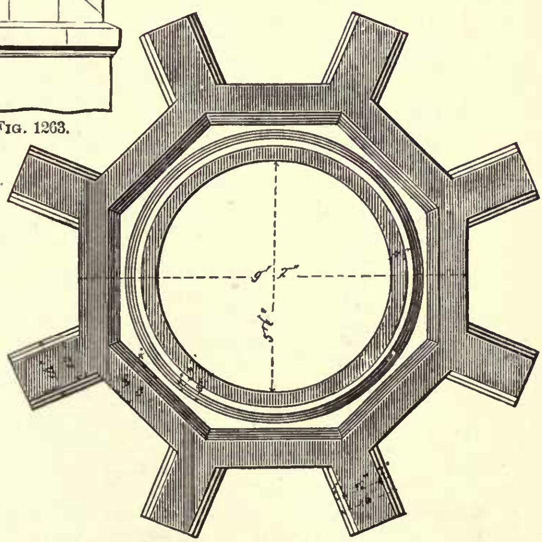


FIG. 1264.

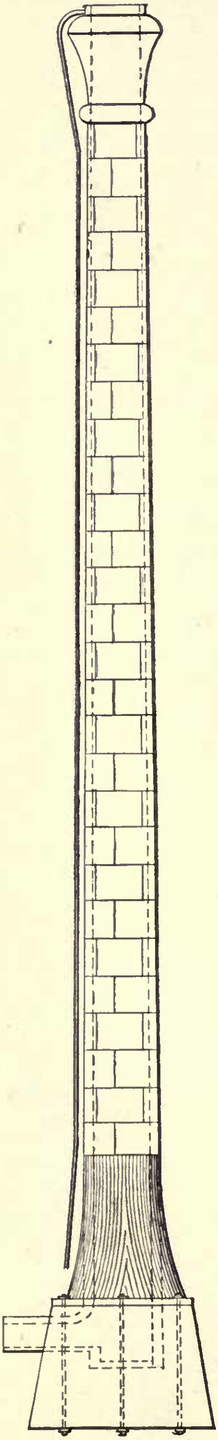


Fig. 1266.

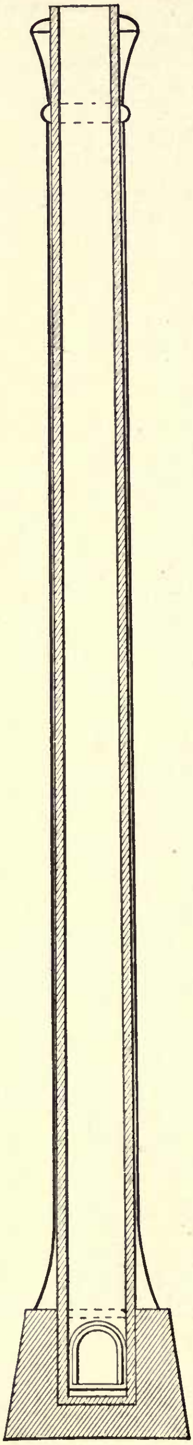


Fig. 1267.

tral flue 8″ and 4″, offsetting into each other by 1″ offsets; the taper is 4 inches to 10 feet on each side. Fig. 1265 is a half elevation and half section of the cap and the cover of the interior flue by which its expansion is permitted.

Figs. 1266 and 1267 are elevation and section of a wrought-iron chimney stack, such as are now in common use; they are brick-lined, with a spread base well anchored and without other stays.

In most chimneys an interior and independent flue is used, which may expand without disturbing and cracking the exterior shell. Chimneys are built of various sections, uniform throughout, with flues tapering to the top, or increasing in section, bell-mouthed, some, like a lamp chimney, contracted at the entrance; all draw well if without abrupt changes in section and direction, and adapted to the position. Chimneys are less likely to be overturned by the wind, the nearer the section to the circular and the smoother in outline, and the fewer the projections.

ON THE LOCATION OF MACHINES.

The construction of buildings for mills and manufactories (if any æsthetic effect is intended) is usually left to the architect, but the necessities of the construction, the weights to be supported, and the space to be occupied, must be supplied by the mechanical engineer or millwright.

In the arrangement of a manufactory or workshop it is of the utmost importance to know how to place the machinery, both as to economy of space and also of working. Where a new building is to be constructed for a specific purpose of manufacture, it will be found best to arrange the necessary machines as they should be, and then build the edifice to suit them. For defining the position of a machine, the space it occupies in plan and elevation, the position of the driven pulley or gear, of the operative, and the spaces for the working and access to parts, are required. To illustrate this subject, take a two-story weaving-room, of which Fig. 1268 is an elevation and Fig. 1269 a plan.

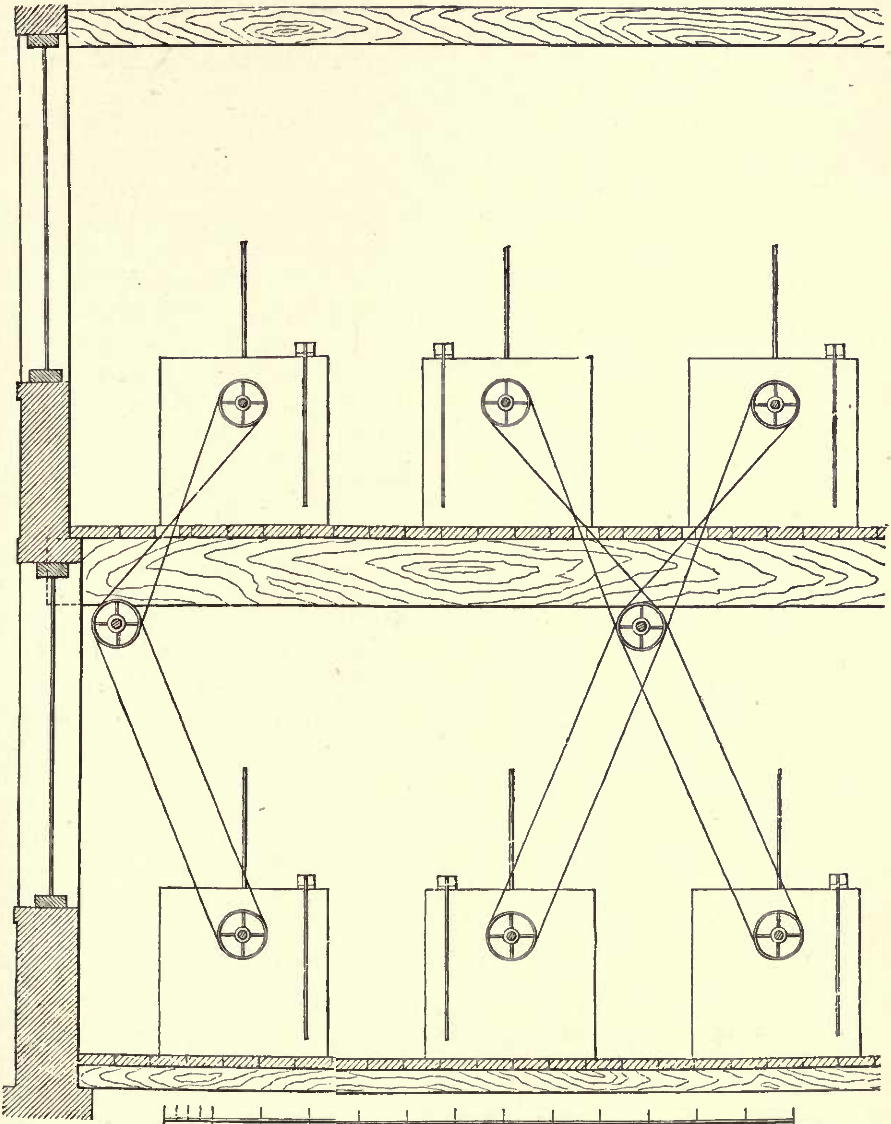


Fig. 1268.

Lay down the outlines of an interior angle of the building, and dot in, or draw in red or blue, the position and width of beams. This last is of importance, as no driving-pulley can come beneath the beam, and this is also the posi-

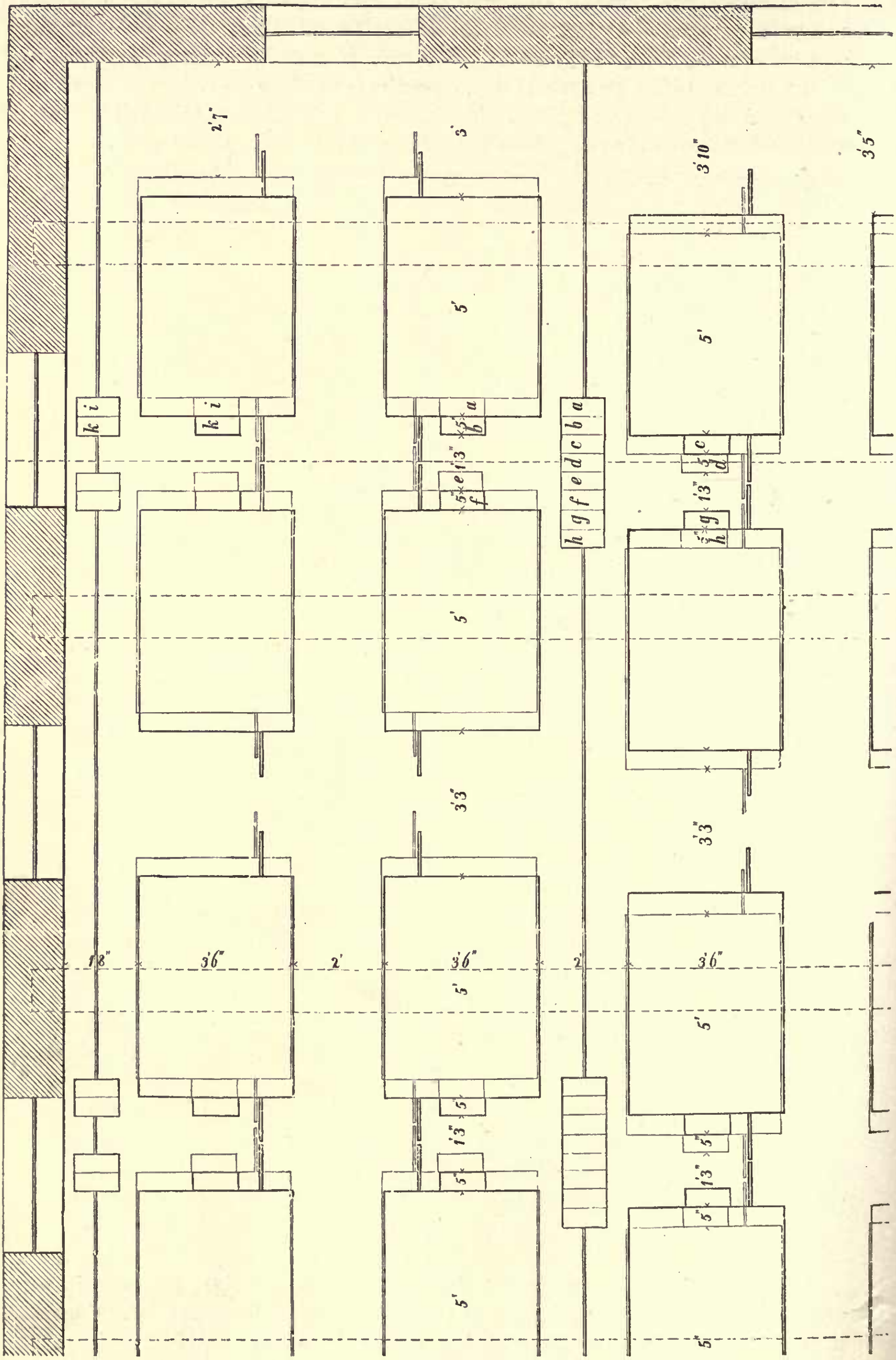


Fig. 1269.

tion for the hanger. Lay off the width of the alleys and of the machines. The first alley, or nearest the side-wall, is a back alley; that is, where the operative does not stand, and so on alternate alleys. Draw the lines of shafting central to the alleys, as in this position the belts are least in the way. One operative usually tends four looms; they are therefore generally arranged in sets of four, two on each side of the main alley, where the operative stands; the twos are placed as close to each other as possible, say one inch between the lays, a small cross-alley being left between them and the next set. Lay off next the alley necessary at the end of the room, and space off the length of two rows of looms with alleys at the end of alternate looms, and mark the position of the pulleys. Looms are generally rights and lefts, so that the pulleys of both looms come in the space where there is no alley. Should the pulley come beneath a beam, the loom must be either moved to avoid it, or the pulley may be shifted to the opposite end of the loom. Parallel with the pulleys on the looms draw the driving-pulleys on the shafts, that is, *k* parallel with *k*, *b* with *b*, *f* with *f*, and so on. Draw the third and fourth rows of looms; since the second and third rows are driven from the same shaft; if they are placed on the same line, it will be impossible to drive both from the same end, and, as this is important, move the third row the width of the pulley *b*, and, for the sake of uniformity, the fourth row also. Lay off the length of looms and position of pulleys as before, and parallel with the pulleys the driving-pulleys on the shaft, that is, *c* against *c*, *g* against *g*, and so on. Having in this way plotted in all the looms, every alternate set being on a line with the third and fourth row, if there are to be looms on the story above, proceed to lay down the position of the looms on this floor; and since for economy of shafting it is usual to drive from the lines in the lower rooms, to avoid errors, interference of belts and pulleys, plot the upper room on the same paper or board as the lower room, in two different colored inks, or drawing the machines in one room in deep and in the other in light line, as shown in Fig. 1269. If the width of the rooms is the same, the lateral lines of looms and alleys are the same, and it is only necessary, therefore, to fix the end lines. As the first loom in the outer row of looms, in the lower room, occupies for its belt the position *k* on the shaft, the loom in the upper room must be moved either one way or the other to avoid this; thus the position *i* of the pulley on the loom must be made parallel to the pulley *i* on the shaft, so in the other looms *a* to *a*, *e* to *e*, *d* to *d*, and *h* to *h*.

Besides the plan, it is often necessary, and always convenient, to draw a sectional elevation (Fig. 1268) of the rooms, with the relative positions of the driving-pulleys and those on the machines, to determine the length of the belts, and also to see that their position is in every way the most convenient possible. In the figure, one of the lower belts should have been a cross-belt, and one of the upper ones straight: had the belts to the second row of looms in the upper story been drawn (as they should have been) straight, the belt would have interfered a little with the alley, and it would have been better to have moved the driving-shaft a trifle toward the wall.

From this illustration of the location of machines, knowing all the requirements, in a similar way any machinery may be arranged with economy of space, materials, power, and attendance. These last two items are of the more im-

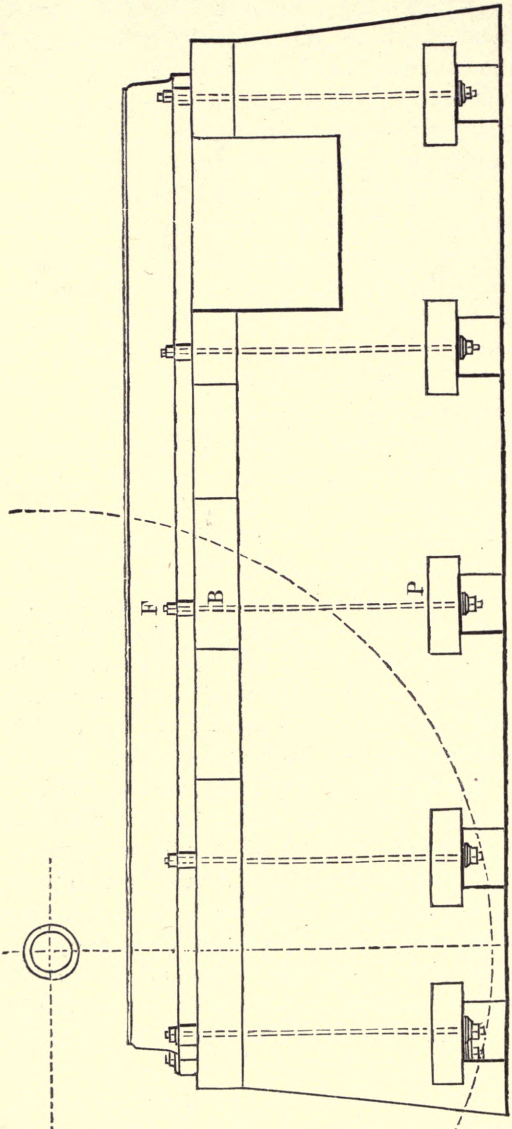


Fig. 1270.

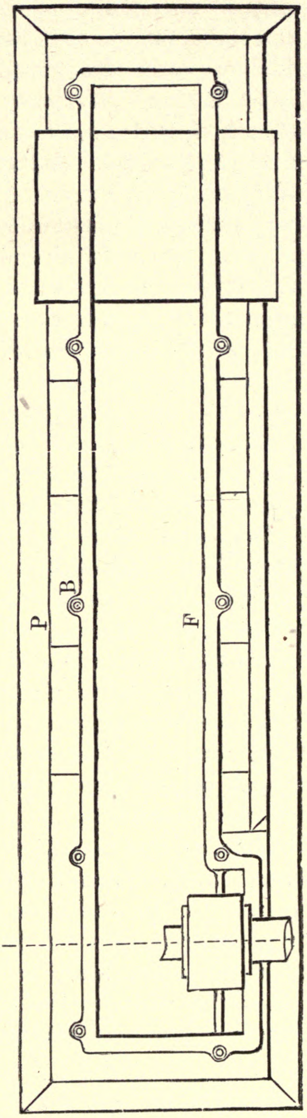


Fig. 1272.

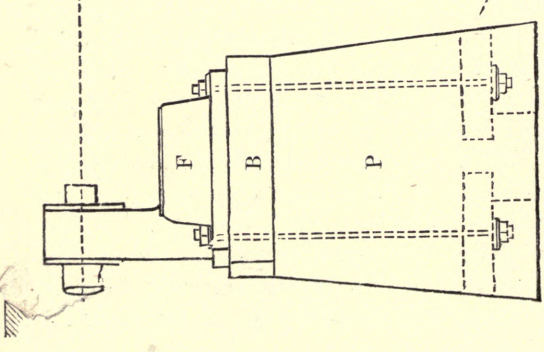


Fig. 1271.

portance as they involve a daily expense, where the others are almost entirely in the first outlay.

Machine Foundations.—Figs. 1270, 1271, and 1272 are side, end elevations, and plan of the foundation of the stationary steam-engine. F is the cast-iron frame or bed-plate of the engine; B the granite bed of the engine, or coping of foundation; P the stone or brick pier, laid full in cement. The sides and surfaces of granite exposed are usually fine-hammered, the upper bed or build to receive the engine-frame, hammer-dressed and set level. Strong wrought iron bolts pass through frame, bed, and pier, with nuts at each end, and the whole is strongly bolted together. Pockets are left in the pier near the bottom for access to nuts, and these pockets are covered by granite caps or iron plates.

Few stationary steam engines are now built with bed-plates extending the whole length of the engines, but the illustration is applicable to the partial plates supporting the cylinder and pillow-block, and to engines and machines for which heavy foundations are necessary. It is not an uncommon practice, instead of granite caps, to use timber, to cushion the shocks and blows incident to most machinery.

Tunnels.—Figs. 1273 to 1282 are illustrations, with description, taken from "Tunneling," a standard work on this subject by H. S. Drinker, C. E.

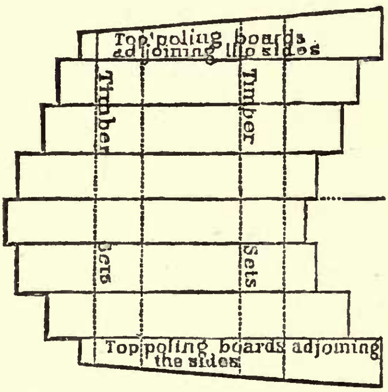


FIG. 1273.

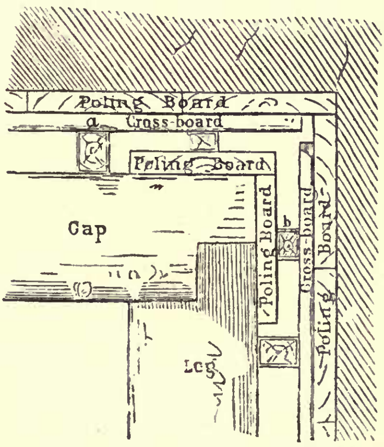


FIG. 1274.

Figs. 1273 to 1278 illustrate the principles of timbering applied to driving a gallery through running material. Figs. 1273 and 1274 are parts of the construction on a large scale, with the technical names of the parts. Each frame is called a *timber-set.* Suppose a leading set (Figs. 1275 and 1276) is in place, close to the face, and that the leading ends of the poling-boards resting above this leading set are held up from the collar by wedges sufficiently high to allow the insertion of the new poling-boards. In Fig. 1276 the sets *e e*, standing midway between the front and the hind ends of the poling-boards, serve as middle sets between the main sets *d d*. By referring to the plan (Fig. 1278) of a gallery thus timbered it will be seen that the side-*poling* has to be wedged *out* at its leading end, just as the roof-poling is wedged *up*, and the space to be filled across the top by the roof-poling is wider over a front main-set than over a back one. The two outer top poling-boards (Fig. 1273) are therefore made wider at their leading ends than at their back ends. The miners begin by inserting the roof-poling at either corner of the face, removing the extreme end-wedges between the collars and the poling, and driving into this space the new poling-boards (i. e., the ones shown in Fig. 1273). Though the wedges between the collar and the poling-boards serve well enough to keep back the material, it would be dangerous thus to take any of them out were there no other guard for the poling, as the board just above the wedge removed would be pressed down; a run might also be started, and all the other wedges forced out, when the poling-boards would snap down on the leading collar, and perhaps break

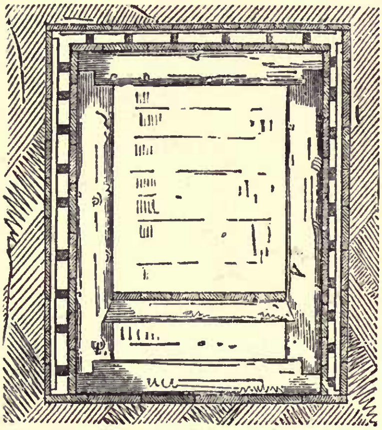


Fig. 1275.

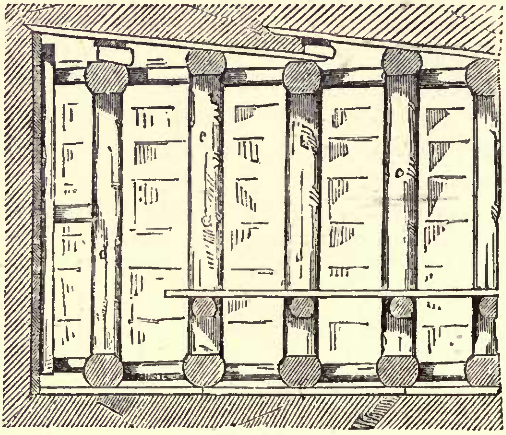


Fig. 1276.

off; in any event, it would be a matter of great trouble to get them wedged up again. In order to guard against this trouble, a cross-board or plank *a* (Fig. 1274) is placed just under the poling-boards, and over the wedges. Then, when one wedge is removed, this cross-connection holds in place the poling-board that is immediately above the wedge removed, until the new board can be

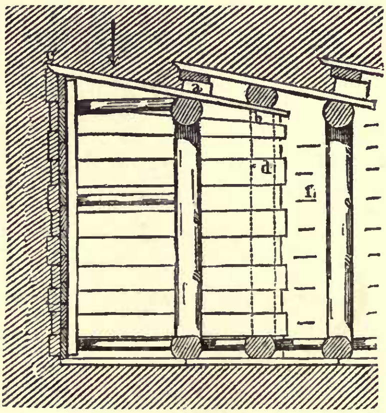


Fig. 1277.

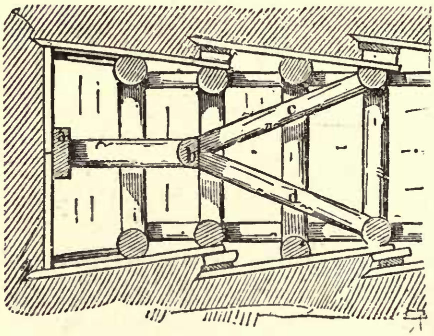


Fig. 1278.

put in; it also stays the tendency to any general movement. The new poling-board being inserted, it is driven ahead six or twelve inches, and then tem-

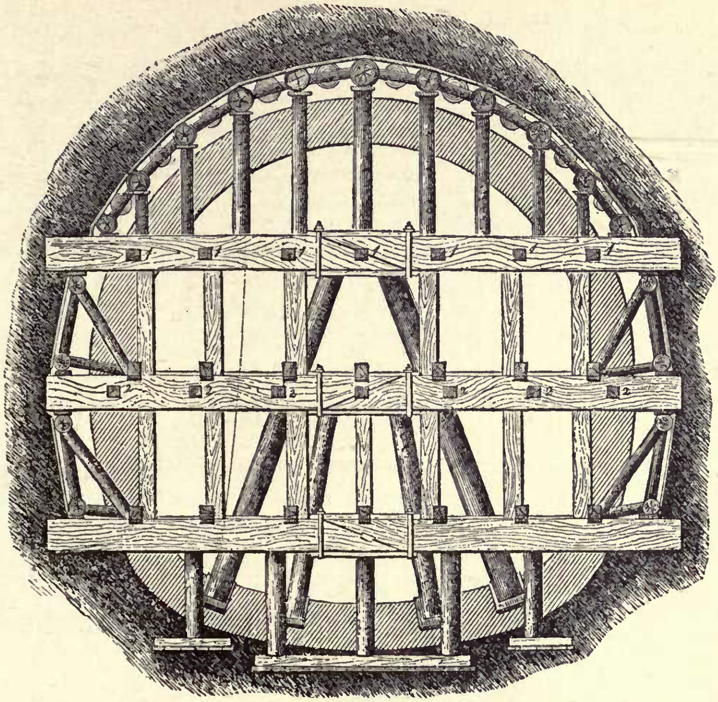


Fig. 1279.
(Section of Fig. 1281, through A B, looking west.)

HOOSAC TUNNEL.

Timbering and arching through soft ground at the West End. Scale, 10′ = 1″.

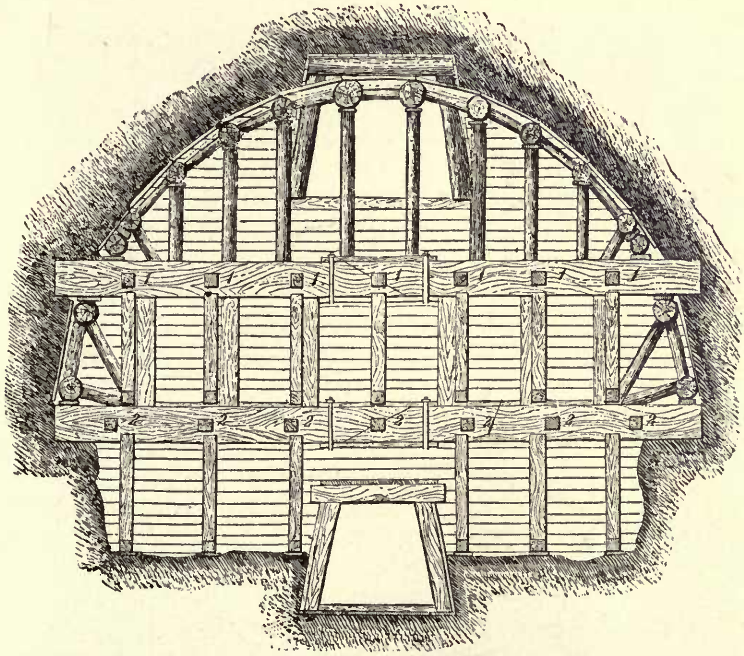


Fig. 1280.

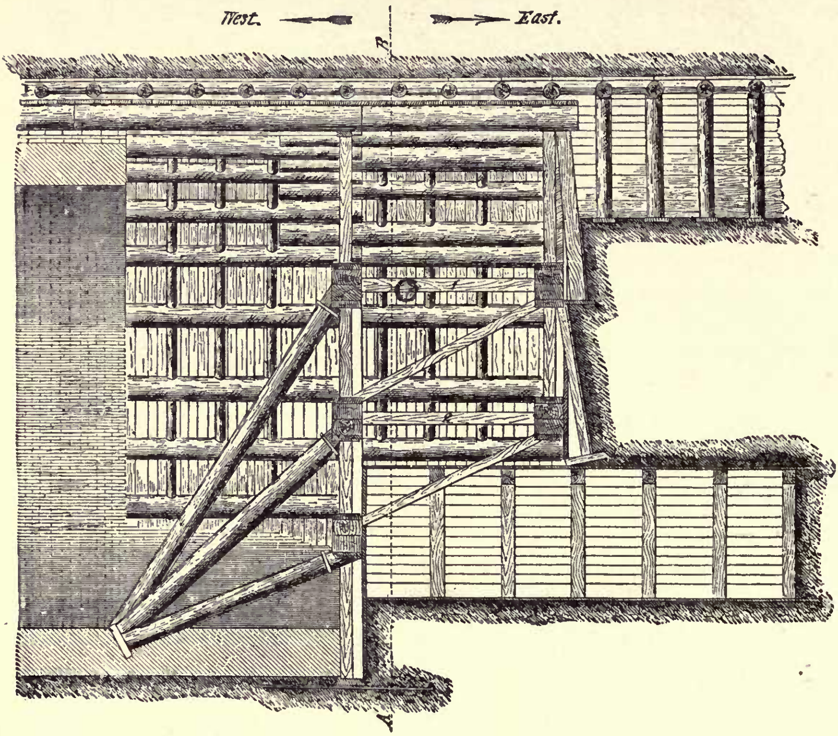


FIG. 1281.
HOOSAC TUNNEL.
West End. Scale, 10′ = 1″.

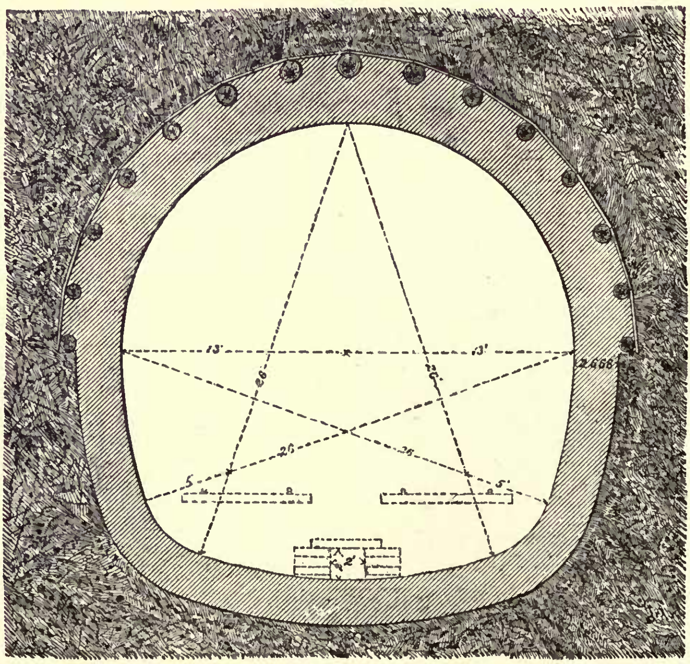


FIG. 1282.

porarily stayed by wedges, *b* (Fig. 1277). The corner roof-polings being thus in place, the middle ones (Fig. 1273) are similarly inserted. Then the top retaining-board in the face is cut out, and the material allowed to flow into the heading through the space. As room is thus given ahead, the poling-boards are gradually driven forward, say 24 or 30 inches, or about half the length of a board. Whenever they are thus tapped, the wedges *b* (Fig. 1274) must be loosened, and then tightened again after the driving. The side-poling is similarly advanced; as space is gained ahead, it must be protected by new face-boarding, stayed by stretchers. Thus the work can be gradually carried down to the floor of the heading by successively taking out the face-boards. Often the floor of the gallery also has to be planked, and, in very extreme cases, to be poled similarly to the roof and sides.

While inserting the new poling-board for half its length the boards have been held in place by the double support offered by *a* and *b*. The face retaining-boards are kept back by a vertical plank laid across them, and stayed by stretchers. On this newly excavated chamber the outside pressure will be great, acting, as it does, on the front half length of the poling-board *c a*, and, if the remaining work is not rapidly executed, the front ends of the boards may be snapped beyond *a*; then, if it were attempted to drive the remaining portion of the board on, as soon as its back end left *b* it would snap between *a* and *b*. A middle set is therefore required at once. The middle set being in position, the work of excavating the face can be proceeded with as before. The face-boards are removed, one by one, from top to bottom, and the polings are driven in to their full length; then in the new length ahead the next main set is erected. Such are the general principles of heading-driving through running ground, or *sheet-piling* in tunnelling.

Figs. 1279 to 1282 show the English system of bar-timbering, as used at the Hoosac Tunnel for the soft ground at the west end. The material was of the worst character, and was exceedingly difficult to drive through. Figs. 1279 and 1280 are cross-sections, the one looking west from A B, the other east. Fig. 1281 is a longitudinal section. Fig. 1282 is a cross-section of the tunnel as completed with an invert, and the bars not drawn but bricked in.

Railway Stock.—Figs. 1283 and 1284 are the elevation and plan of a standard box-car of the New York Central and Hudson River Railroad.

Figs. 1285 to 1288 are the plan and elevations of the truck for the same car.

Figs. 1289, 1290, and 1291 are end-elevations and cross-sections, Figs. 1292 and 1293 longitudinal sections, and Fig. 1294 plan of a standard passenger-car of the Pennsylvania Railroad.

Fig. 1295 is a plan and section of a pair of wrought-iron plates which support a car body in the centre of the truck. There are two—the body centre-plate and the truck centre-plate. The centre-pin or king-bolt, not shown, carries none of the strain except in emergencies.

Figs. 1296 to 1299 are elevations, in full and parts, and Fig. 1300 a plan of the trucks of the above car.

In the figures, both of standard box and passenger cars, the elevations and plans are usually broken, to show the construction. When the two sides or two ends of a car or truck are similar, it has not been considered necessary to

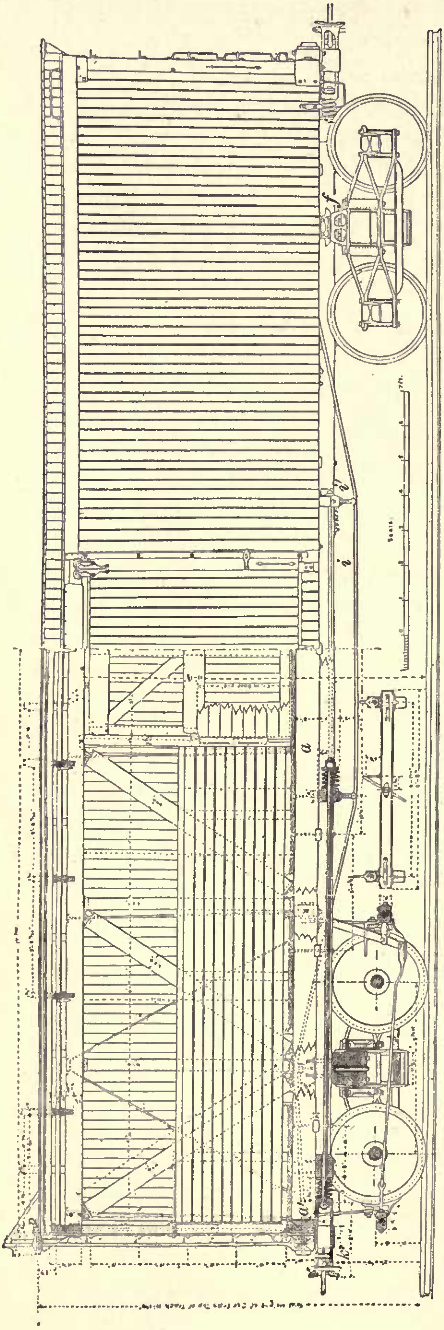


Fig. 1283.

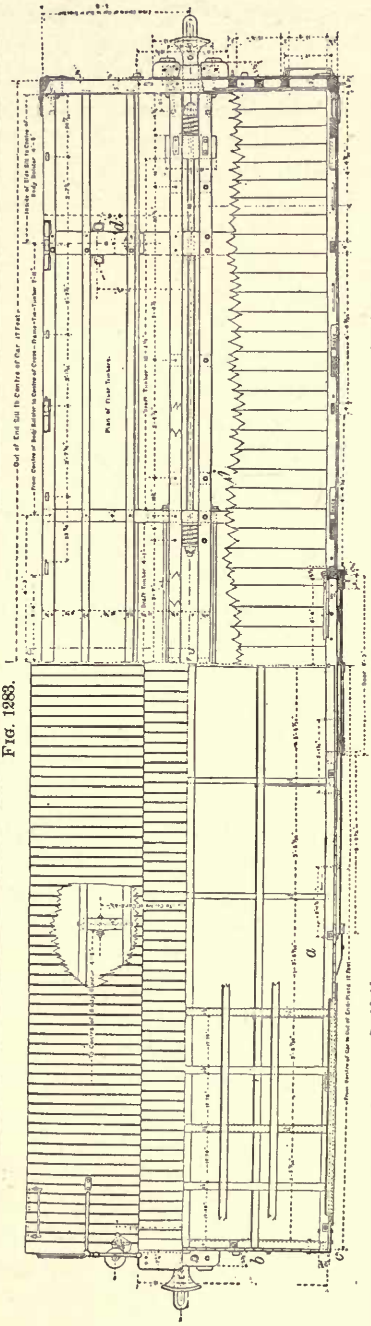


Fig. 1284.

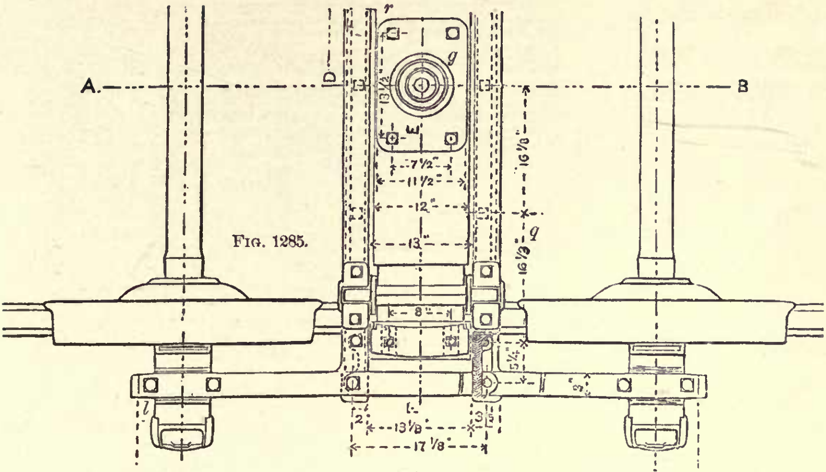


FIG. 1285.

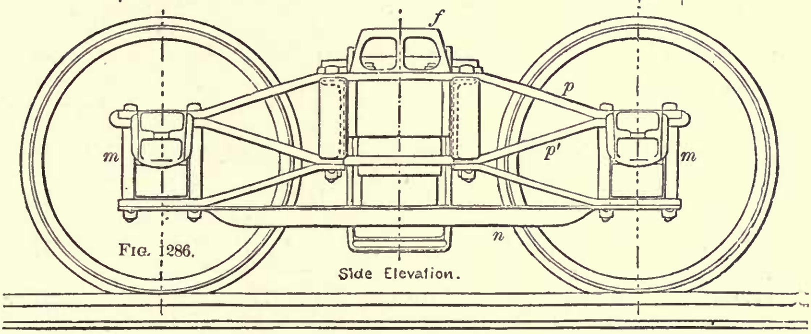


FIG. 1286.

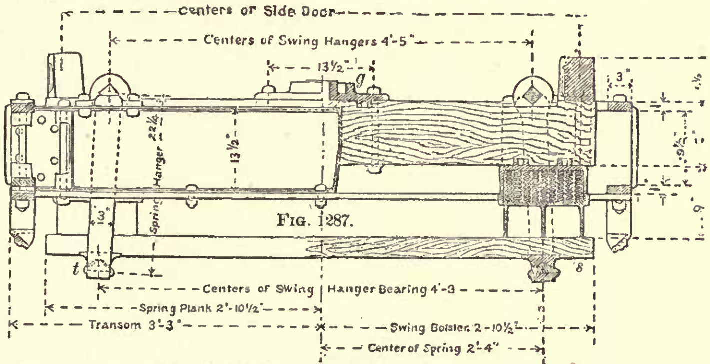


FIG. 1287.

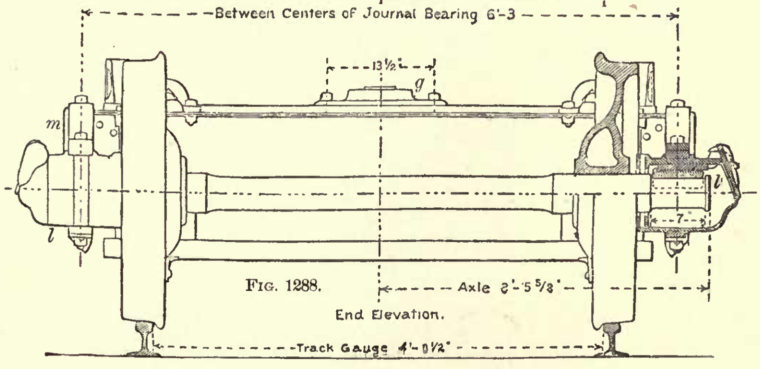


FIG. 1288.

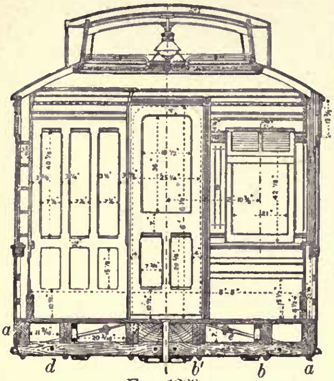


FIG. 1289.

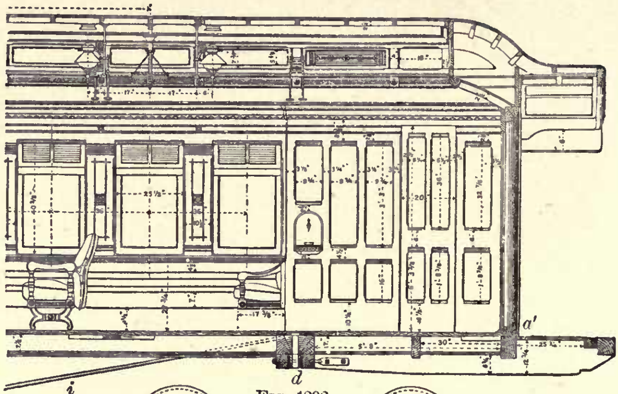


FIG. 1292.

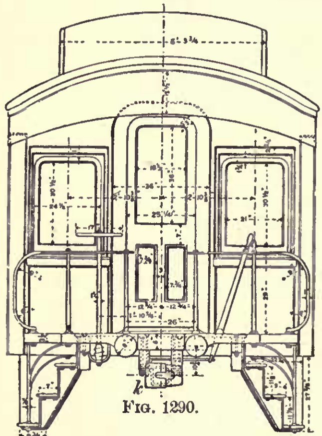


FIG. 1290.

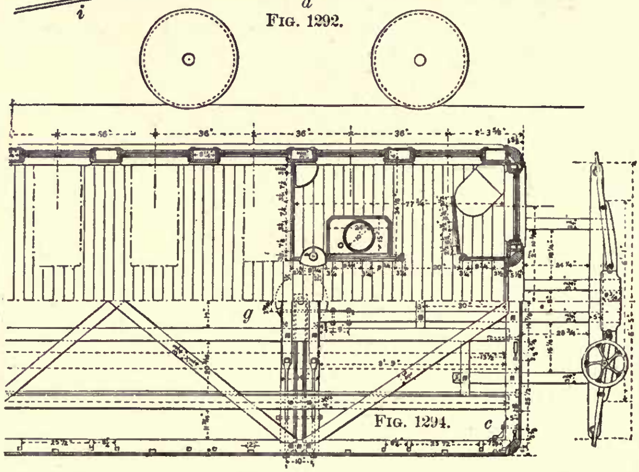


FIG. 1294.

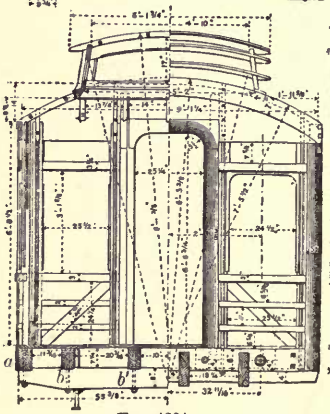


FIG. 1291.

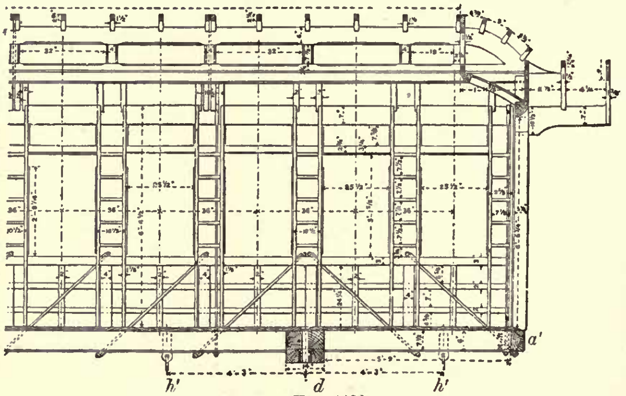


FIG. 1293.

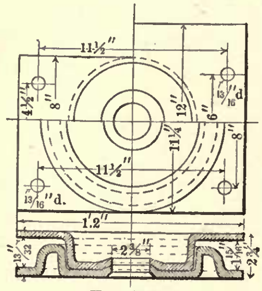


FIG. 1295.

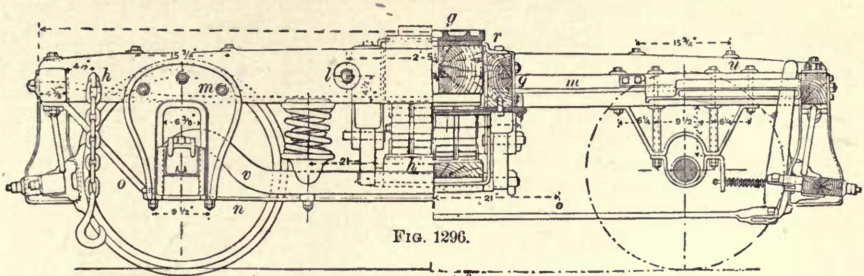


FIG. 1296.

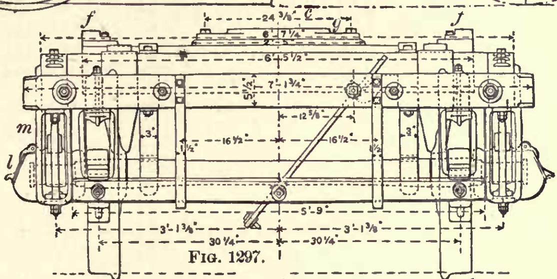


FIG. 1297.

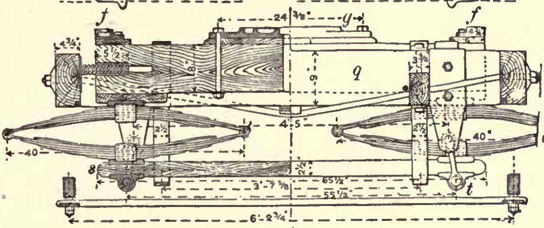


FIG. 1298.

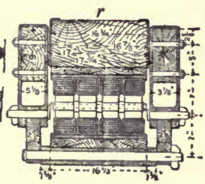


FIG. 1299.

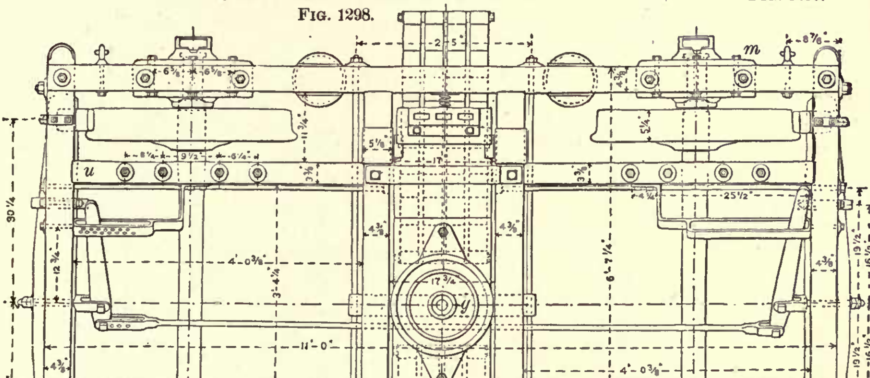


FIG. 1300.

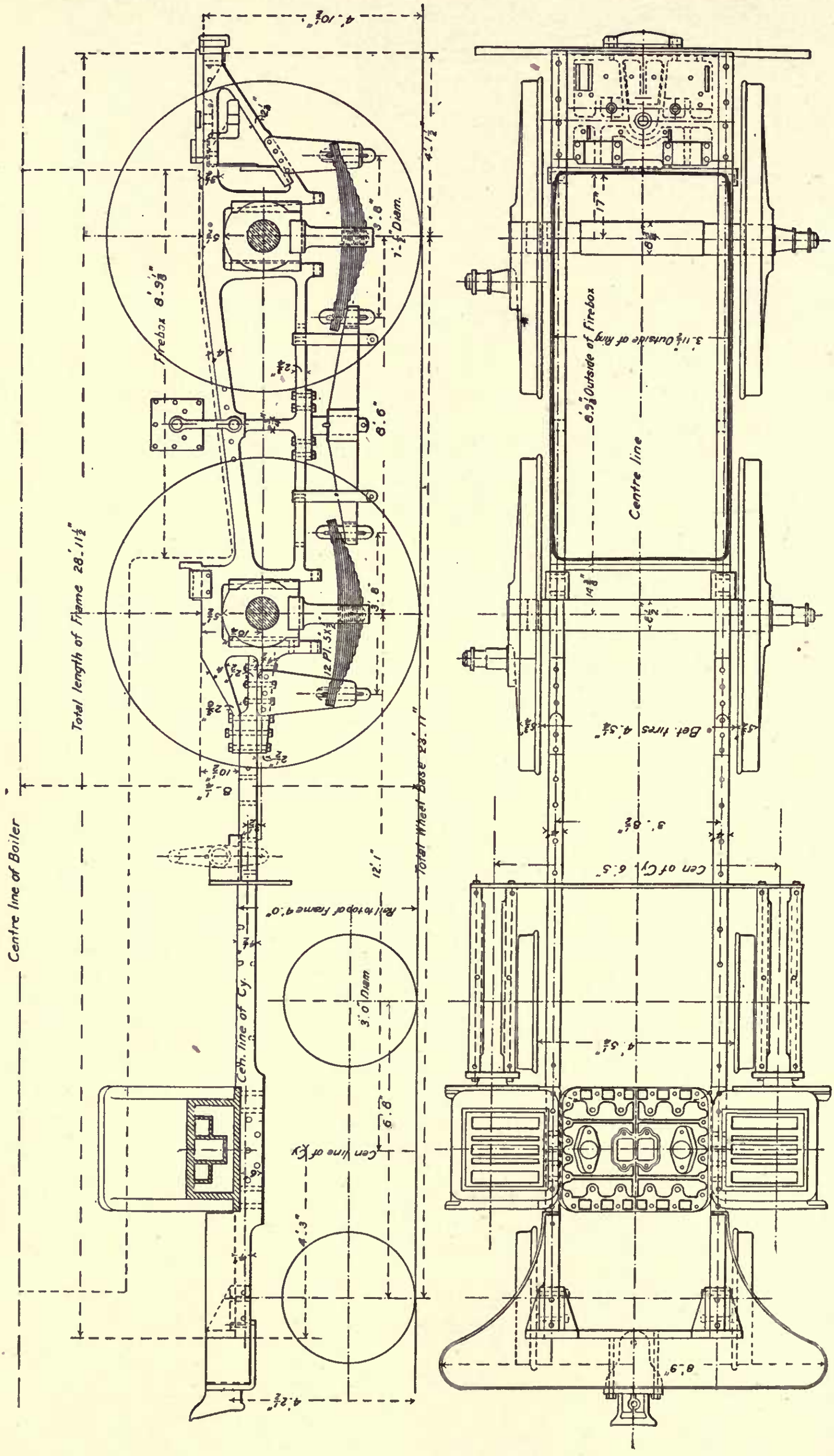

Centre line of Boiler
Total length of Frame 28'.11½"
Total Wheel Base 23'.11"
Firebox 8'.9⅞"
Rail to top of Frame 4'.0"
Cen. line of Cy.
3'.0" Diam
Cen line of Cy
Centre line
6'.9⅞" Outside of Firebox
3'.11½" Outside of Ring
Bet. tires 4'.5½"
Cen of Cy. 6'.5"

FIG. 1301.

show both, but the figure is completed with a section of the other part, through a different plane.

The following letters of reference and technical names of similar parts apply equally to all the figures :

a, Sill.
a', End-sill.
b, Intermediate floor-timbers.
b', Centre floor-timbers.
c, Sill knee-iron or strap.
d, Body bolster.
e, Body bolster truss-rod.
f, Truck side-bearing.
g, Centre plate, body or truck.
h, Check-chain on the truck, hooking into
h', Check-chain eye on the car.
i, Body truss-rod.
i', Body truss-rod queen-post.
j, Cross-frame tie-timber.
k, Draw-bar.
l, Journal-box.
m, Pedestal.
n, Pedestal tie-bar.
o, Pedestal stay-rod.
p, Pedestal arch-bar.
p', Pedestal inverted arch-bar.
q, Transom.
r, Truck bolster.
s, Spring-plank.
t, Swing-hanger.
u, Safety-beam.
v, Equalizing-bar.

Fig. 1301 is a plan and elevation of the frame of a locomotive as designed by Mr. Robert Buchanan of the New York Central and Hudson River Railroad. The distinguishing feature is the American " bar " frame, while the plate frame may be called the English practice. The latter admits of a wider fire-box between the frames, but in the American example the fire-box is entirely above the frames.

The Wave-line Principle of Ship-Construction, from Russell's "Naval Archi-

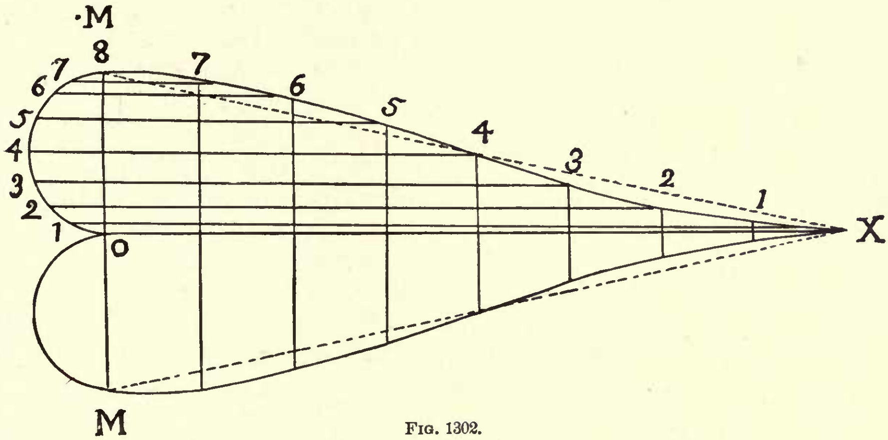
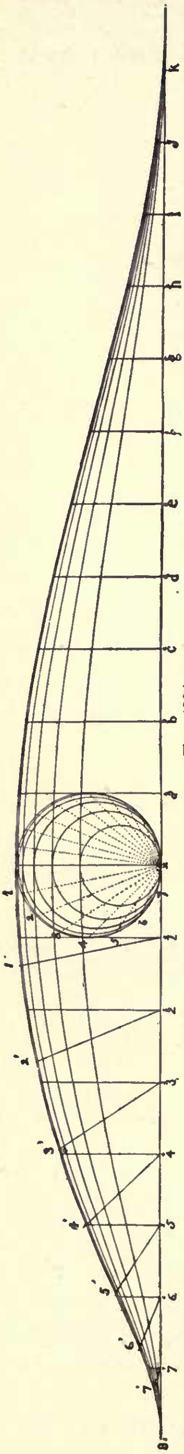


FIG. 1302.

tecture."—The general doctrines arrived at by J. Scott Russell, F. R. S., from numerous and long-continued experiments and practical tests, is " that the form of least resistance for the water-line of the bow is horizontally the curve of versed sines, and that the form of least resistance for the stern of the vessel is the cycloid ; and you can either adopt the said cycloid vertically or horizontally, or you can adopt it partly vertically and partly horizontally, according to the use of the vessel or the depth of water."

" That the length of entrance, or fore body, should be $\frac{3}{5}$, and that of the run, or after body, $\frac{2}{5}$."



BOW.

FIG. 1304.

STERN.

"When it is required *to construct the water-lines of the bow of a ship of which the breadth and the length of the bow are given*, so as to give the vessel the form of least resistance to passage through the water, or to obtain the highest velocity with a given power: Take the greatest breadth, M M (Fig. 1302), on the main section of construction at midship-breadth, and halve this breadth, M O; at right angles to M M at O draw the centre line of the length of the bow, O X; on each half-breadth describe a half-circle, dividing its circumference into, say, eight equal parts. Divide the length O X into the same number of equal parts. The divisions of the circle, reckoned successively from the extreme breadth, indicate the breadths of the water-line at the successive corresponding points of the line of length. Through the divisions of the

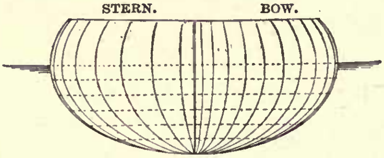


FIG. 1303.

circles draw lines parallel to O X, and through the divisions of O X lines parallel to M M. These, intersecting one another, show the successive points in the required water-line. The line traced through all these points is the *wave water-line of least resistance* for a given length of bow and breadth of body."

To construct the water-lines of the after body or run of a ship (Fig. 1304), the mid-section (Fig. 1303) being given: The bow is constructed as in Fig. 1302, but with 12 divisions on the centre line; for the run lay off 8 divisions, each equal to those of the bow; divide the half circle into 8 equal parts, and draw chords to these divisions from 0 to 1, 2, 3, 4. From the point 1 on the centre line lay off an inclined line equal and parallel to the chord 0 1; the point 1′ will be in the water-line. In the same way from the point 2 draw an inclined line parallel and equal to the chord 0 2, for 2′, and determine in the same way the points 3′, 4′, 5′, 6′, 7′. The other circles drawn in the figure are described on semi-diameters of the mid-section at different levels, and the points of their wave-lines are determined on the same inclined lines 1 1′, 2 2′, but the lengths are those of the chords of the different circles. In Fig. 1303, the elevations of the mid body, the curved lines of sections are projected from the plan.

Fig. 1305 is a body plan of a vessel adapted to speed; Fig. 1306 of one adapted to freight.

"To determine the after body it is expedient to construct a vertical wave-line on the run as well as a horizontal one, and in designing shallow vessels to give more weight to the vertical wave-line."

"The wave system destroys all idea of any proportion of breadth to length being required for speed. An absolute length is required for entrance and run, but, these being formed in accordance with the wave principle for any given speed, the breadth may have any proportion to that which the uses of the ship and the intentions of the constructor require."

"The wave system allows us to give the vessel as much length as we please. It is by this means that we can give to a vessel of the wave form the capacity we may require, but which the ends may not admit. Thus, the Great Eastern,

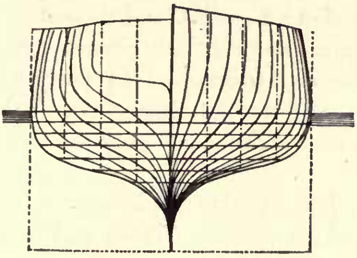


Fig. 1305.

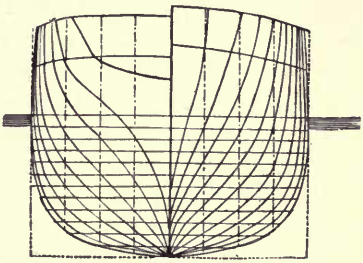


Fig. 1306.

which is a pure example of the wave form, has an entrance or fore body of 330′, a run or after body of 220′, and a middle body of 120′, which was made of this length merely to obtain the capacity required. The lengths of the fore and after body are indicated by the required speed, and if the beam is fixed, it is only by means of a due length of middle body that the required capacity, stability, and such other qualities are to be given as will make a ship, as a whole, suit its use."

Length of entrance of a vessel for a 10-mile speed should be 42 feet, of run 30 feet; for a 20-mile speed, 168 and 120 feet; that is, the lengths increase as the squares of the speed.

Under Isometrical Drawing is given an illustration of a vessel constructed on wave-lines.

ARCHITECTURAL CONSTRUCTION.

It is the duty of an architect to design a building to be suitable and convenient for the purposes for which it is intended ; to select and dispose of the materials of which it is composed to withstand securely and permanently the stresses and wear to which they may be subjected ; to arrange the parts to produce the artistic effects consistent with the use of the building and its location, and to apply such appropriate ornament as may express the purpose and harmonize with the construction.

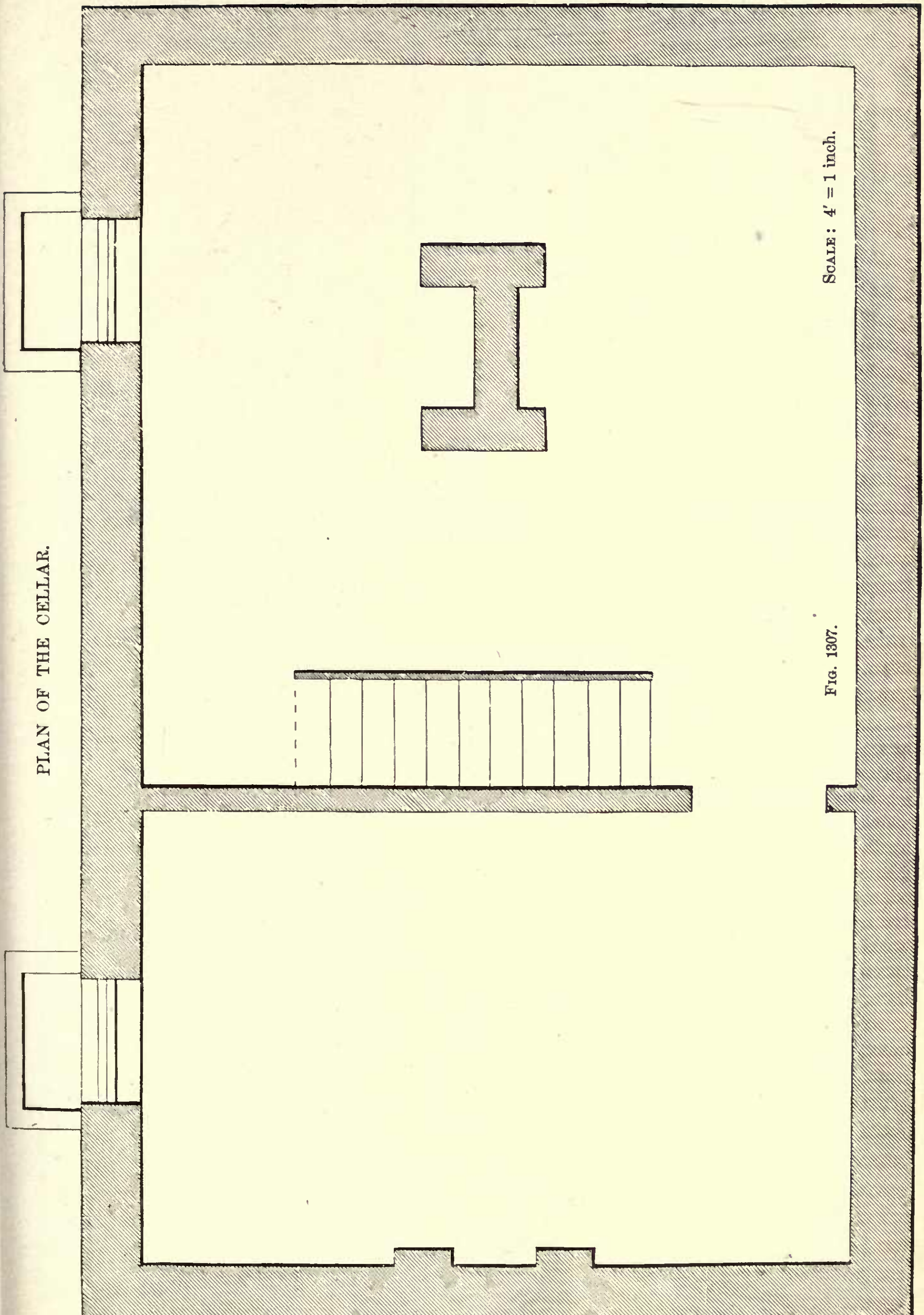
In domestic architecture, by far the most extensive branch of the profession, most persons can give some idea of the kind of building which they wish to have constructed, and perhaps express by line the general arrangement of rooms ; but it is left to the architect to settle the style of building appropriate to the position, to adapt the dimensions and positions of rooms and passages to the requirements, to determine the thickness of walls and partitions, and arrange for drainage, heating, and ventilating. The graphical representation is left to the draughtsman, and his assistance is the more valuable if he is not only conversant with practical details, but understands the best proportions of parts, the necessities of construction, and the requirements of building laws.

The draughtsman usually commences his education with the copying of drawings. For this purpose, in Figs. 1307 to 1310, inclusive, are given plans and elevations of a simple house, showing the drawings necessary to get a clear comprehension of plans and elevation ; but for an estimate and for constructive purposes it is necessary in addition to draw elevations of the remaining sides and one or more longitudinal and transverse sections showing the framing and general construction ; details drawn to a large scale are also required, from which and the specifications the building may be erected.

The size of the page has compelled the titles to be put within the body of the drawings ; after copying, place them outside, and give ample margin. In all scale drawings the scale should form a part of the title. On Fig. 1311 the section and end-elevation are given together. This is also for economy of space, but should be copied by the draughtsman in two distinct drawings, each of the full width of the building.

Instead of hatching the walls and partitions, as in the examples given, these are often indicated in solid black, or in colour, brick walls by carmine, wooden walls and partitions by burnt sienna, other materials by colours as nearly representing them as possible, which may be purchased in pans under the name of technical water colours.

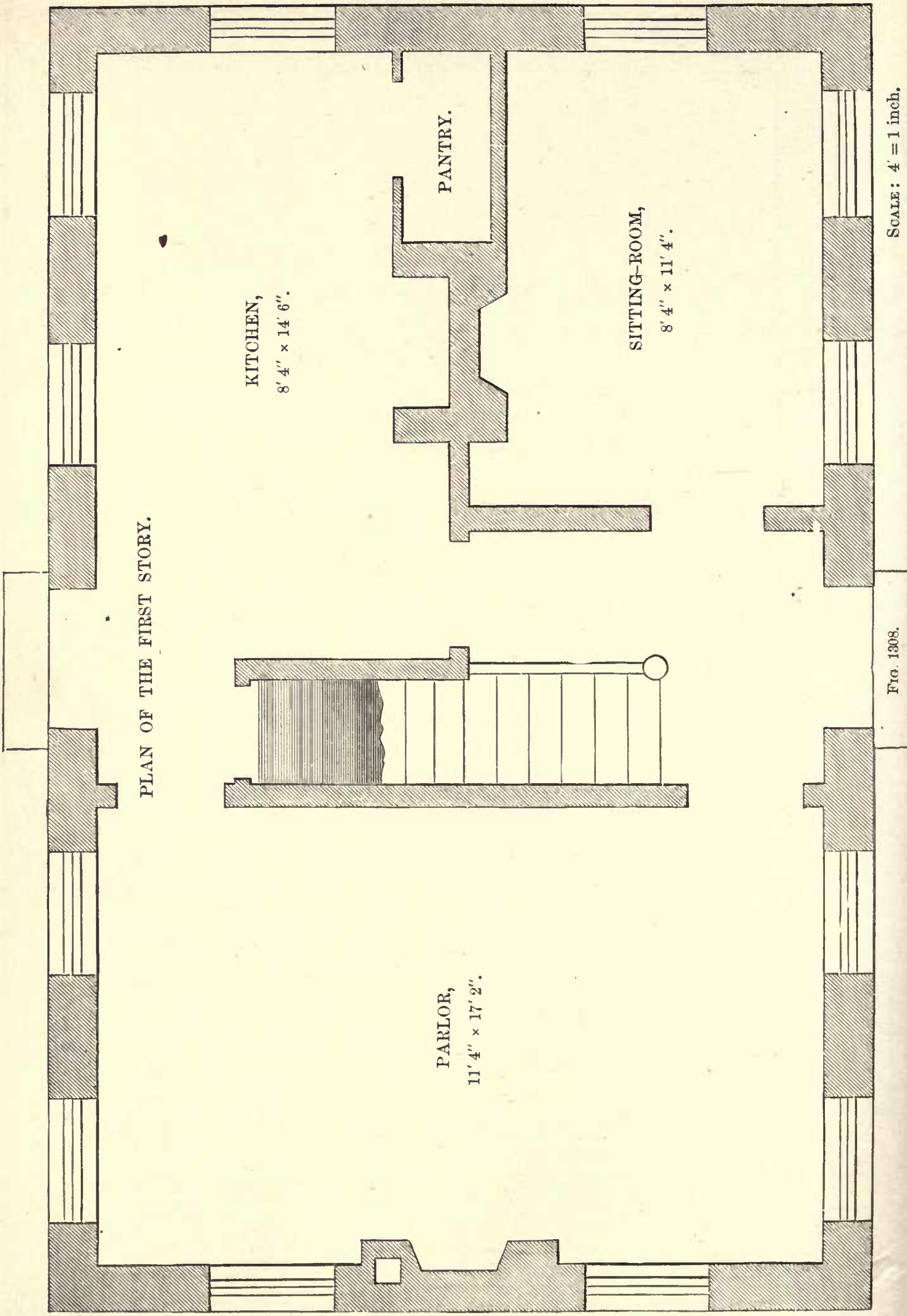
Plate XIII is an illustration in colour of a plan and ceiling from a design of Arthur Gilman, architect.



PLAN OF THE CELLAR.

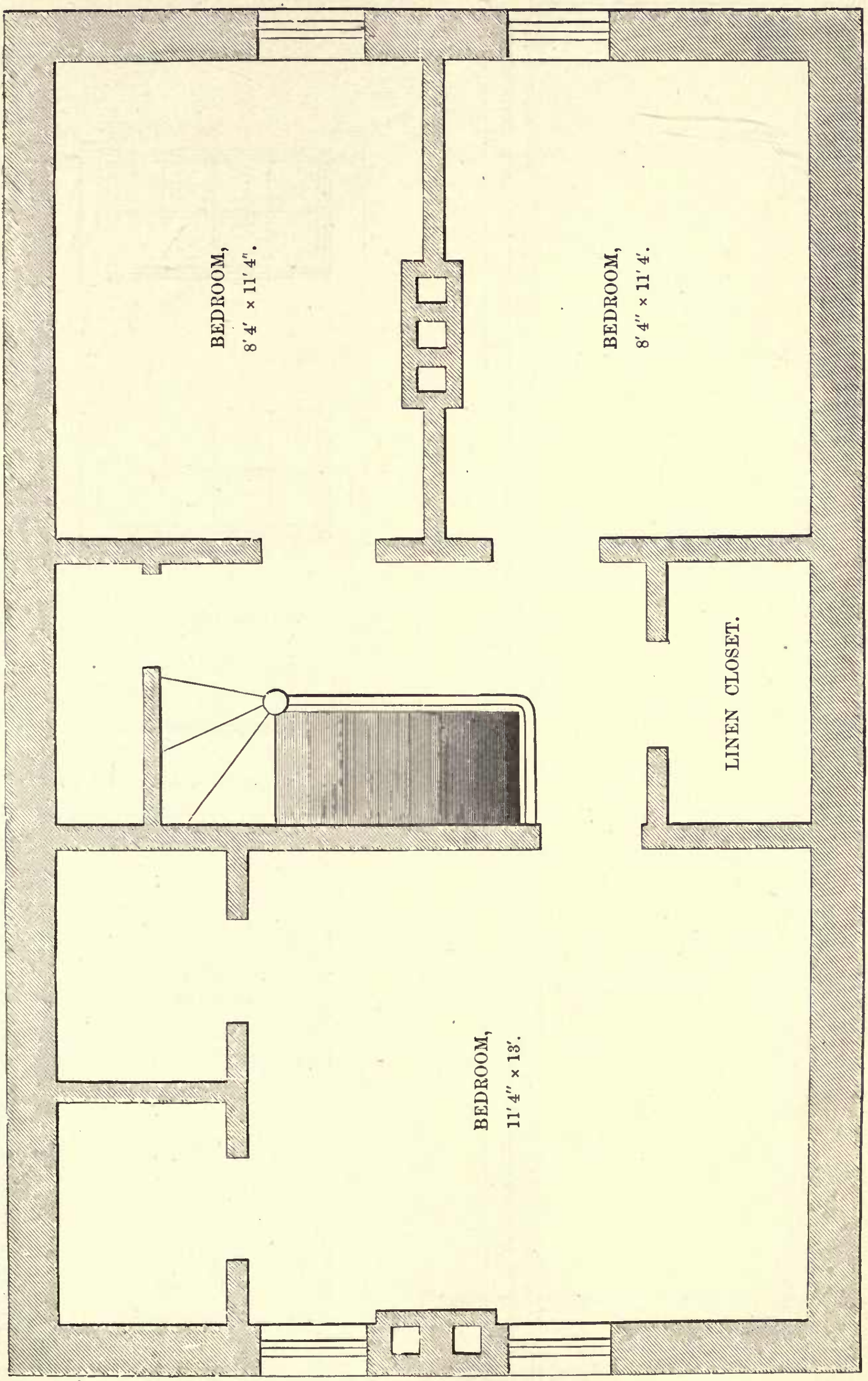
Scale: 4′ = 1 inch.

Fig. 1307.



SCALE: 4' = 1 inch.

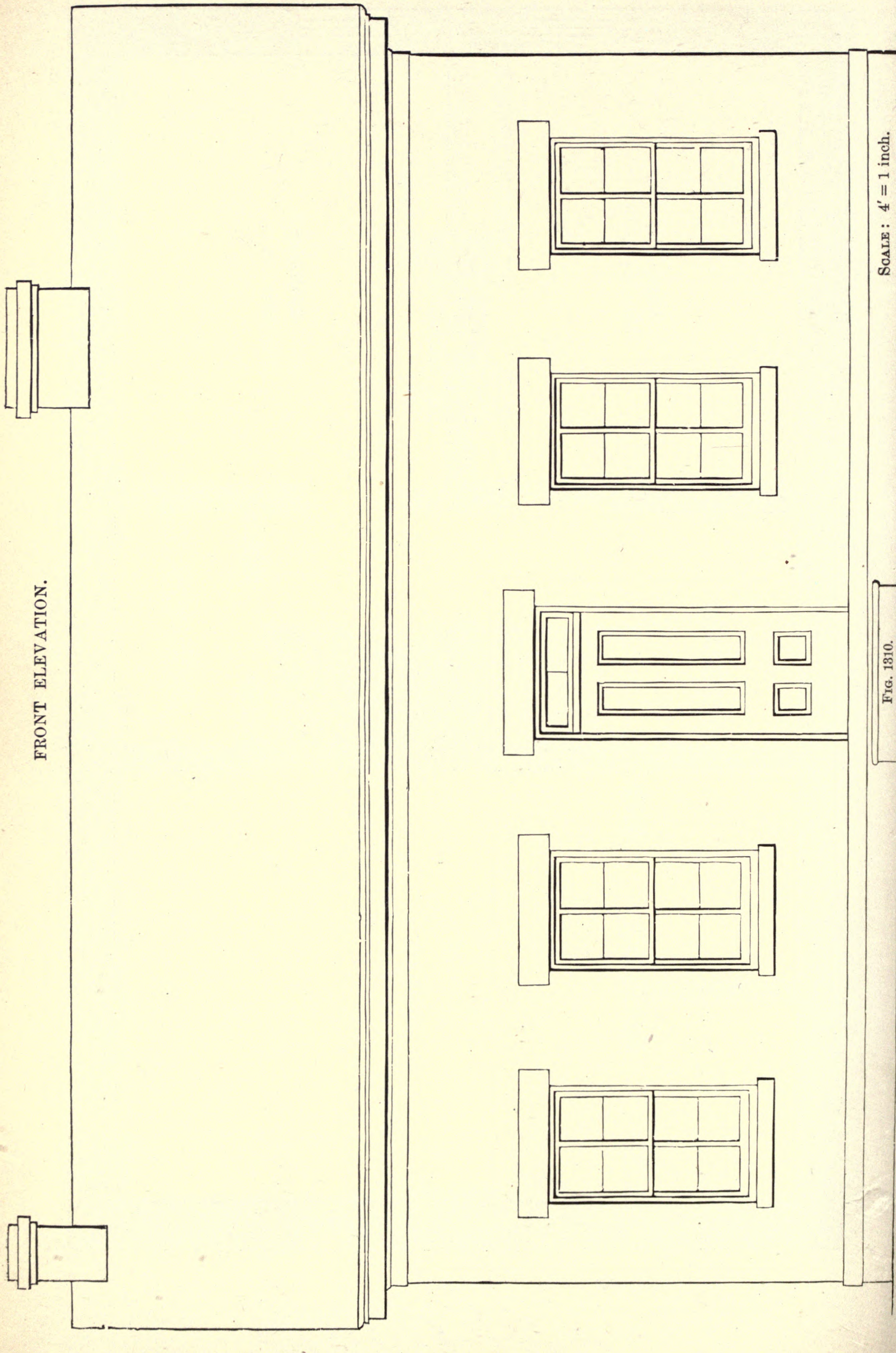
FIG. 1308.



PLAN OF THE ATTIC.

SCALE: 4' = 1 inch.

FIG. 1309.



FRONT ELEVATION.

SCALE: 4′ = 1 inch.

FIG. 1310.

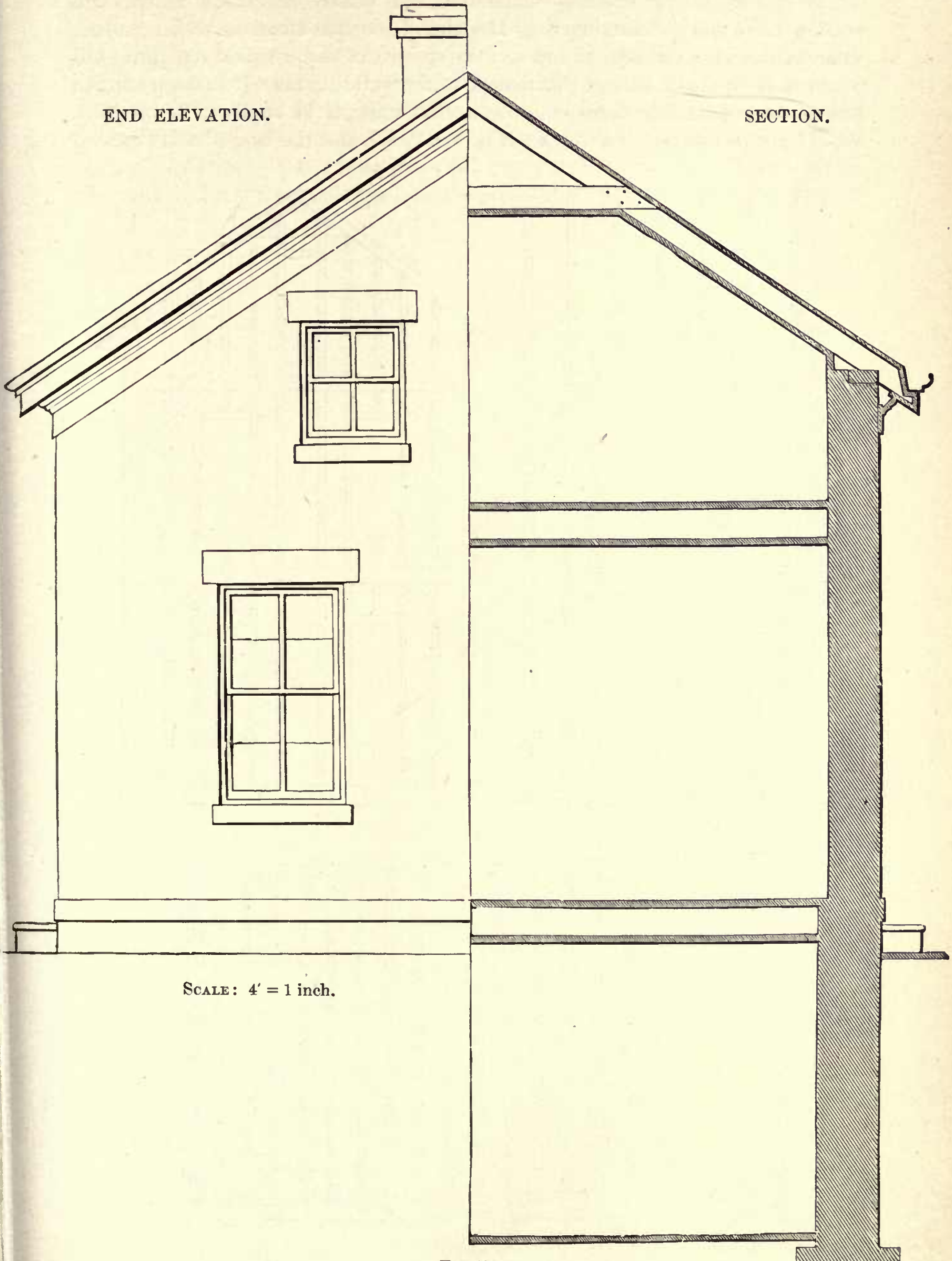


FIG 1311.

Details of Construction.—Foundations in variety and requirements are treated fully under "Engineering Drawing," but for common structures the draughtsman, if there are examples in the vicinity of the proposed structure, will conform to the teachings of practice, or to the building laws, if there are any in force. In general, for small buildings, cellar-walls, if of stone laid in mortar, should not be less than 18″ thick; if of brick, 16″, and the base 6″ to 12″ wider.

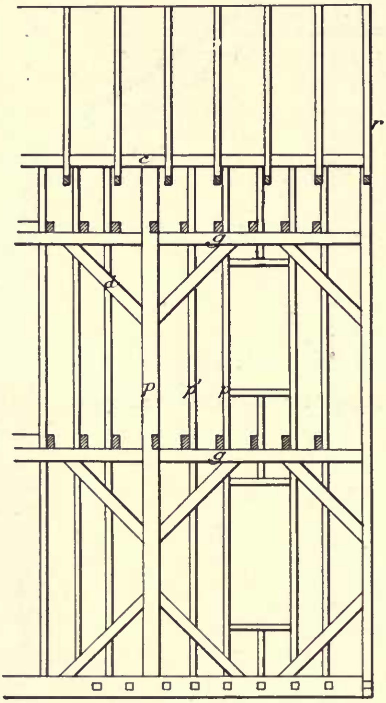


FIG. 1312.

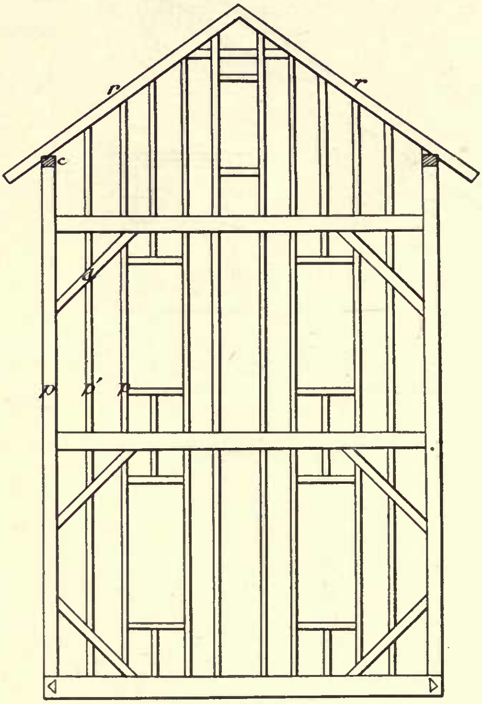


FIG. 1313

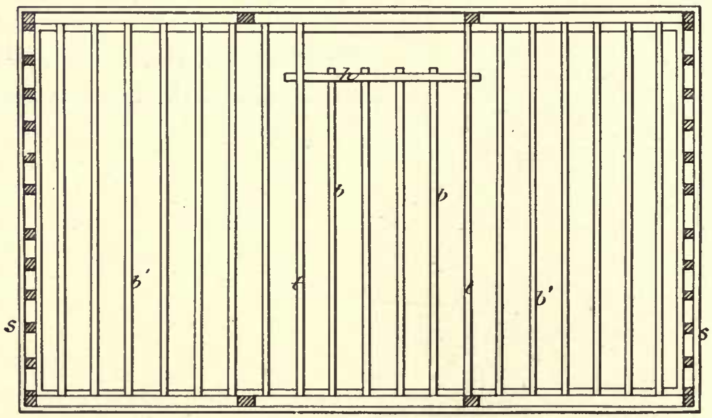


FIG. 1314.

Figs. 1312 to 1314 represent the side and end elevations and plan of a timber frame building, of common construction, supported on brick or stone walls, in which *s s* are the sides, *b' b'* the floor-beams, *t t* trimmer-beams, and *h* the header. The beams, *b b*, framed into the header are called tail-beams; *p p* are posts, which are distinguished by their position, as corners, intermediate and window posts; *p' p'* studs, *g g* girts, *c c* plates, *d d* braces, and *r r* rafters.

Usual dimensions of timber for frame of common dwelling-houses: sills 6″ × 8″, posts 4″ × 8″, studs 2″ × 4″ or 3″ × 4″, girts 6″ × the depth of floor-joists, plates 4″ × 6″; the floor-joists (J, Fig. 1315) are notched into the girts. The posts and studs are *tenoned* into the sills and girts. Fig. 1316 represents a

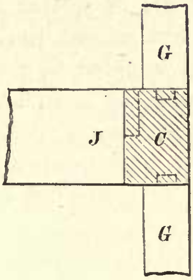


Fig. 1315.

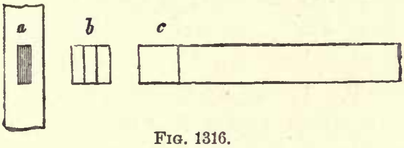


Fig. 1316.

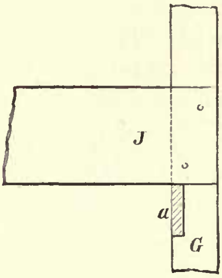


Fig. 1318.

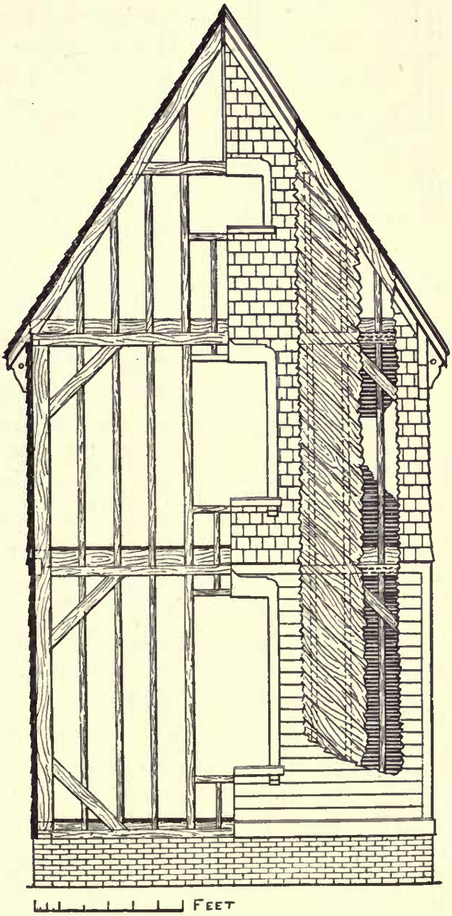


Fig. 1317.

tenon, *b c*, in side and end elevation, and *mortise*, *a*; the portions of the end of the stud resting on the beam are called the *shoulders* of the tenon.

The frame is covered with boards usually 1″ thick, laid either horizontally or diagonally, and nailed strongly to the posts or studs. Fig. 1317 is the elevation of the end frame of a house, showing by breaks the diagonal cover of boards and the inner lathing. The lower story is sheathed or ceiled with narrow boards, the upper shingled.

In the balloon or spike frame the stiffness depends on the nailing, and mor-

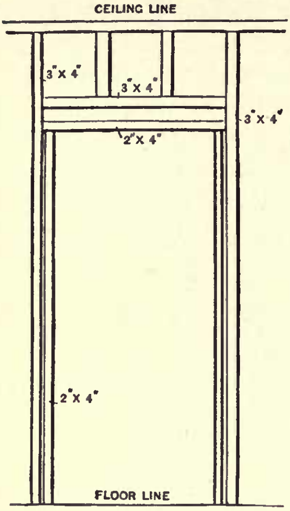


FIG. 1319.

tise and tenon are omitted, and the girts supplied by a board *a* 3″ or 4″ × 1″ let into the studs (Fig. 1318) and firmly nailed to them. The studs are nailed to the plates and sills.

The studs at all door-openings should be set at least 2″ wider and 3″ higher than the size of the finished opening. It is not unusual to have double studs (2″ × 4″) to inclose these openings (Fig. 1319). This leaves the doorway more or less independent of the partition. Partition studs are of smaller dimensions than those of the frame, but are set like them and usually 12″ and 16″ centres, adapted to the length of the lath (48″). The sizes of the studs are generally 2 × 4, 3 × 5, or 3 × 6 inches, according to the height of the partition; for very high partitions, greater depth may be required for the studs, but three inches will be sufficient width.

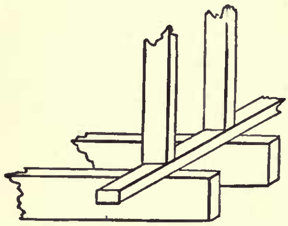


FIG. 1320.

Partitions are usually cut in between sills placed on the floor-beams (Fig. 1320), and similar caps above, beneath the beams. Where partitions of the second story are directly above those on the first story it is better to foot the studs on the caps of the latter, and not on the beams (Fig. 1321). Where there are double floors, the sills are placed on the bottom floor, or on the floor without a sill. It may be important that the partitions should be self-sustaining. This is effected by simple bridging, well nailed to the studs, as shown in Fig. 1322, or by herring-bone bridge, as shown in plan of floor (Fig. 1330), or by a system of trussing, as in Fig. 1323. This method of trussing must vary with the position of opening. The foot of the braces should rest on a positive support. The bridging should be accurately cut and firmly nailed. Bridging distributes the weight of the partition, but trussing concentrates it at the ends of the braces.

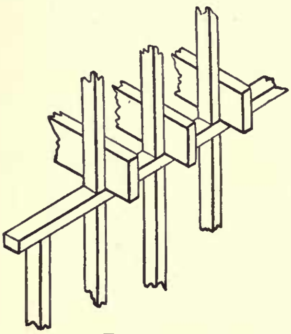


FIG. 1321.

Walls in Masonry.—For walls above the cellar, it will be found difficult to lay stone walls in mortar,

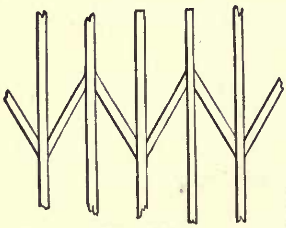


FIG. 1322.

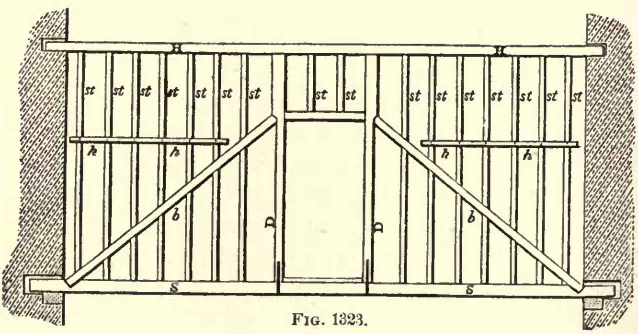


FIG. 1323.

with fair bond and face, less than 16″ thick. Brick walls may be as thin as 8″ for exteriors, and for partitions 4″. Brick walls are usually bonded by heading-courses every fifth to seventh course. Where the outside course is pressed or face brick, these are laid on stretchers, and the bond with the backing may be thin strap-iron, laid in the joints, or, by cutting off the interior corners of the face-course, say every fifth course, and laying common brick diagonally of the wall resting in this clipped corner (Fig. 1324). The face of buildings is often built of thin ashlar, which is secured with iron anchors to the brick backing.

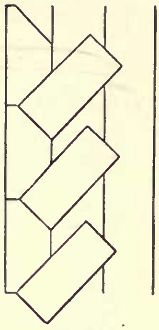


FIG. 1324.

In most cities there are building acts in force defining the kinds of material and thickness of walls and foundations, to which all constructions within their limits must conform.

Openings in masonry-walls are covered by lintels or arches, or both. It is usual to place a stone or cast-iron lintel in the exterior face over openings for doors and windows, with a wooden lintel inside (Fig. 1325), and a relieving arch above. For larger openings, brick arches are turned on cast-iron skew-backs, of which the thrust is resisted

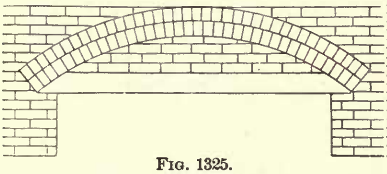


FIG. 1325.

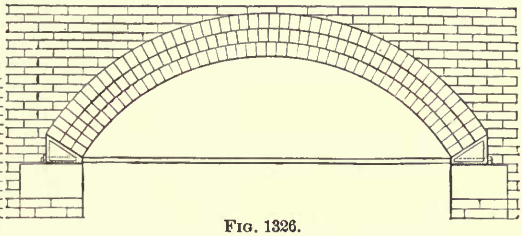


FIG. 1326.

by a tie-bolt (Fig. 1326), or cast-iron lintels, box girders, ⊥- or rolled I-beams. But it is to be observed that, when the cement is set, there is little or no thrust from the arch. The whole *dead work*, or masonry without an opening, forms a monolithic beam, and, if there is depth enough of this, the arch is of no account. It is the custom in the north of Italy to construct flat lintels of brick, of considerable span, depending entirely on the mortar for strength.

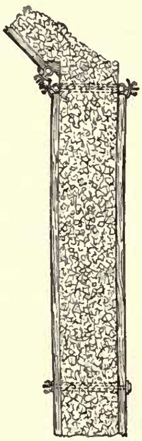


FIG. 1327.

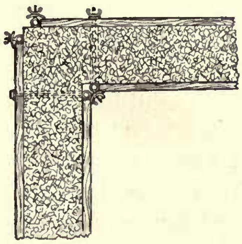


FIG. 1328.

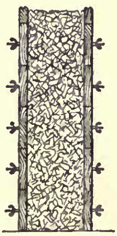


FIG. 1329.

To distribute the weight of piers over the foundation of walls with openings, it is very common to construct inverted arches beneath spans or openings. In old houses it was not unusual to make the exterior arches of an opening flat or rectangular in outline, with the joints radial. This is now relegated to ornamental construction.

Concrete Walls.—It is common in many places where brick and stone are expensive and gravel is abundant to make walls of concrete, in proportions of

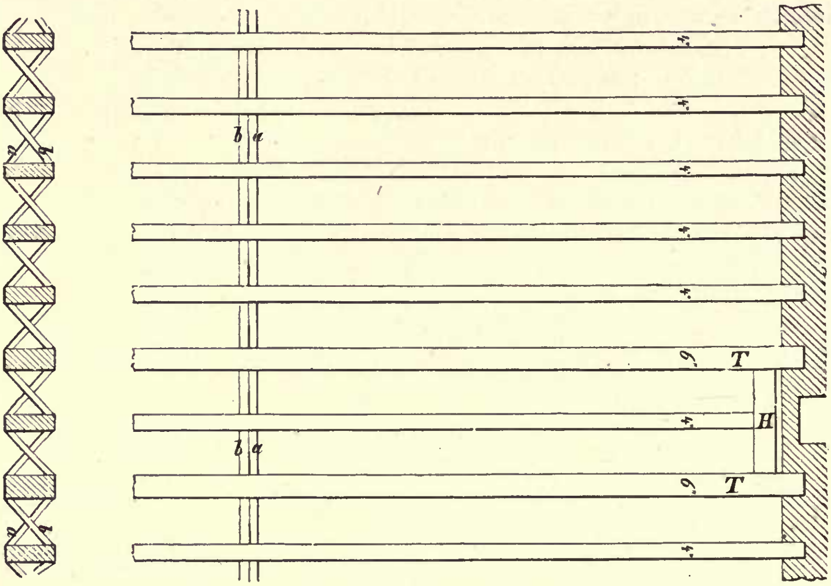


FIG. 1330.

one of cement to five to seven of gravel. The space requisite for the wall is inclosed with plank, and is filled in with concrete, well rammed. Figs. 1327 and 1328 are plans of concrete walls with inclosing plank, and Fig. 1329 an elevation. The planks are held by bolts passing through wall and plank, all of which are removed after the wall is set, and the bolt-holes are then filled with cement. The thickness of walls should be a little in excess of those of brick.

Flooring.—The timbers which support the flooring-boards and ceiling of a room are called the naked flooring.

The simplest form of flooring, and the one usually adopted in the construction of city houses and stores, is represented in plan and section (Fig. 1330). It consists of a single series of beams or deep joists, reaching from wall to wall. As a lateral brace between each set of beams a system of *bridging* is adopted, of which the best is the *herring-bone* bridging, formed of short pieces of joists about 2 × 3, crossing each other, and nailed securely to the tops and bottoms of the several beams, represented by *a* and *b*; and wherever a flue occurs, or a stairway or well-hole prevents one or more joists from resting on the wall, a *header*, H, is framed across the space into the outer beams or *trimmer-beams* T T, and the beams cut off or *tail-beams* are framed into the header.

Whenever the distances between the walls exceed the length that can safely be given to floor-joists in one piece, an intermediate beam or girder, running longitudinally, is introduced, on which the joist may be set (Fig. 1331), notched on (Fig. 1332), or boxed in (Fig. 1333), or both boxed and notched. They may also be framed in with tenon and mortise; the best form is the *tusk*-tenon

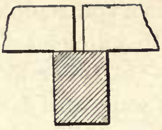


Fig. 1331.

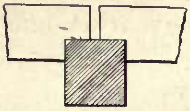


Fig. 1332.

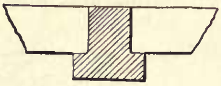


Fig. 1333.

(Fig. 1334). Flooring is still further varied, by framing with girders longitudinally; beams crosswise, and framed into or resting on the girders; and joists framed into the beams, running the same direction as the girders. When the joists are not flush or level with the bottom of the beams or girders, either in the finish the beams will show, or ceiling-joists or furrings will have to be introduced.

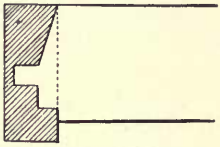


Fig. 1334.

The width of beams as sold in the market is from 2″ to 6″; those for common houses and spans are 2″, but for more important buildings and structures 3″ to 4″. The depth of beams and distances between centres may be determined from the weight to be supported (that is, load per square foot by the number of feet, including that of the floor), and from the table, page 239; but this table gives the central load, which is one half the distributed load.

The following table gives the load per square foot for different characters of buildings; for floors of—

Dwellings.	40	pounds per square foot.
Churches and public halls	80	" " " "
Warehouses and general merchandise	250	" " " "
Factories	200 to 400	" " " "
Snow, 30 inches deep	16	" " " "
Maximum wind pressure	50	" " " "
Roofs for wind and snow	30	" " " "
For slate	45	" " " "
Plastering	8	" " " "

Timber on sale is seldom found sawed to fractional dimensions of an inch or in lengths to fractions of feet.

Trimmer-beams and headers should be of greater width than the other beams, depending on the distance of the headers from the wall, and the number of tail-beams framed into the trimmers; by the New York Building Act all headers must be hung in stirrup-irons (Fig. 1335), and not framed.

Beams must be anchored to the walls and to each other when there are two lengths meeting on girders. The straps to be not less than $1\frac{1}{2}'' \times \frac{3}{8}''$, spiked to sides, on top, or bottom of beam. The anchors and beams to make a

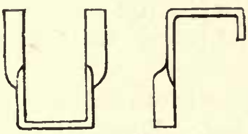


Fig. 1335.

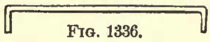


Fig. 1336.

Fig. 1337.

tie across the building about every 6 feet. Figs. 1336 and 1337 are common forms of ties between beams or between beams and walls. Fig. 1338 is an anchor between beams and embracing a column. Fig. 1339 is a spear-anchor between beams and wall; the angle is driven into the beam. In warehouses it is usual to carry anchors through the wall with large washers, often ornamental, and nut on the outside.

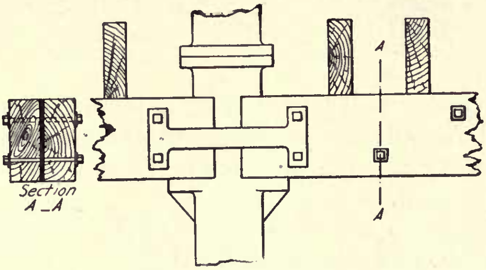


Fig. 1338.

Fig. 1339.

Wooden beams built into walls must have an air space round them with ends cut diagonally and anchored to the walls (Fig. 1340). The air spaces at the sides are for ventilation, and with the diagonal cuts at the ends permit the beams weakened by fire to fall without disturbing the walls. Cast-iron boxes and plates for ends of beams can be purchased, with flanges for anchors to walls and beams.

Floors.—In New York it is usual to lay single floors of tongued and grooved boards directly on the beams, but in the Eastern States double floors are more common. The first floor consists of an inferior kind of boards, as hemlock, unmatched, laid during the progress of the work as a sort of staging for the carpenter and mason, and, in finishing, a second course is laid on them of better material, generally tongued and grooved, but sometimes only jointed. Ceilings should always be *furred*, and the laths be nailed to the strips. Furring-strips usually are of inch board, 2″ wide, and 12″ from centre to centre, nailed cross-wise from joist to joist.

In dwellings it is desirable to isolate the floors of each story, so that noise, vermin, smells, and fire may be cut off. The first is usually done by deafening, which consists (Fig. 1341) of a sub-floor, resting on cleats, nailed to the beams and about 4″ below the floor. This space is filled with lime-mortar, weakened by a mixture of sand or gravel, but strong

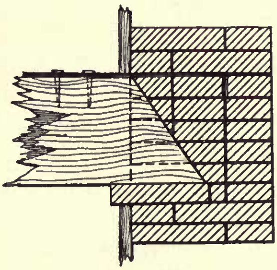


Fig. 1340.

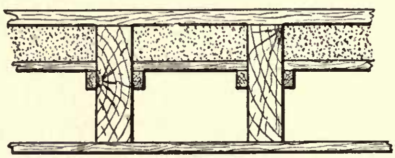


Fig. 1341.

enough to set. If the timber is green, the contact of the mortar is apt to produce dry rot.

Deafening may be secured to great extent by double floors, with two or three thicknesses of carpet paper or sheathing quilt between. When small vermin, like cockroaches, can pass, stenches and fire can find their way. To prevent which, begin at the cellar and cut off all access to space between plastering and floor; if the ceiling is plastered, fill in between studs tightly with strips of board, and above them a course of brick in cement. Fibrous hemlock boards prevent the gnawing of rats and mice.

The fireproof of old builders consisted simply of plain cylindrical or groined arches (Fig. 1342) in stone or brick masonry or concrete.

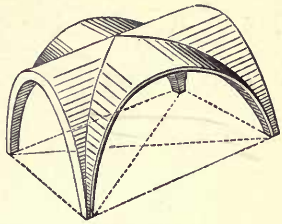


Fig. 1342.

Figs. 1343 to 1347 are illustrations of Roman constructions in masonry, from "Dictionnaire Raisonné de l'Architecture," par M. Viollet Le Duc.

Fig. 1343 is a perspective view of a cylindrical arch in process of construction. The centres A and lagging B are quite light, as the full load of the arch

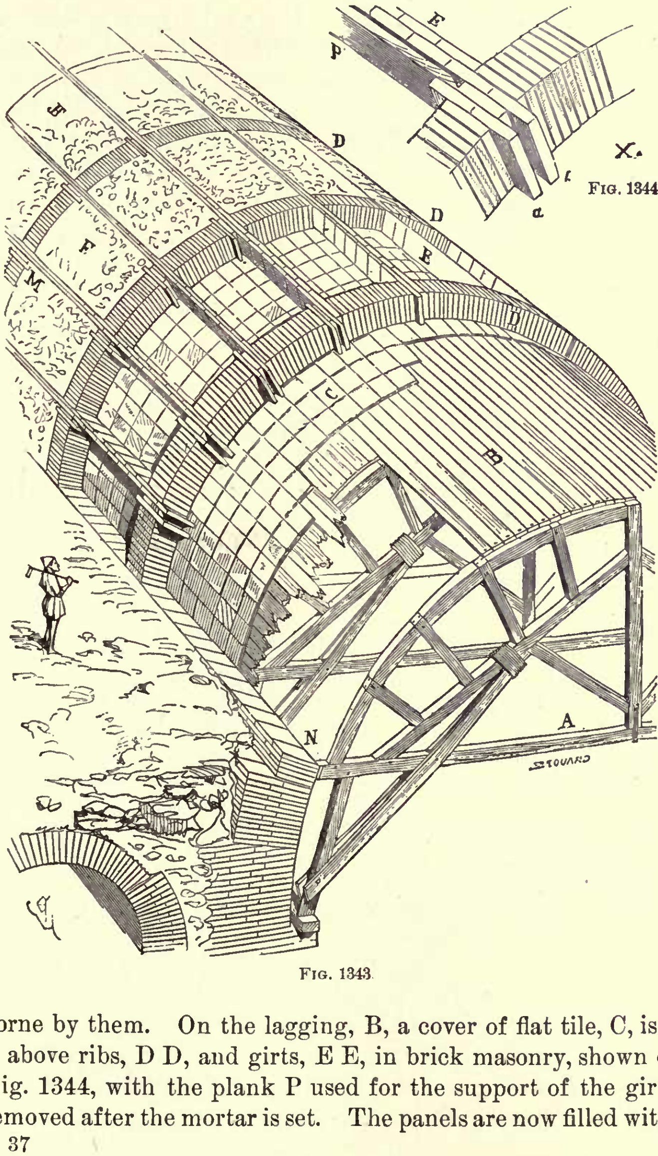


Fig. 1344.

Fig. 1343

is never borne by them. On the lagging, B, a cover of flat tile, C, is laid in cement, and above ribs, D D, and girts, E E, in brick masonry, shown on a larger scale in Fig. 1344, with the plank P used for the support of the girt bricks E, which is removed after the mortar is set. The panels are now filled with concrete.

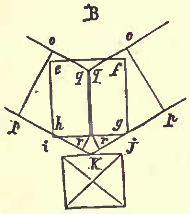


Fig. 1346.

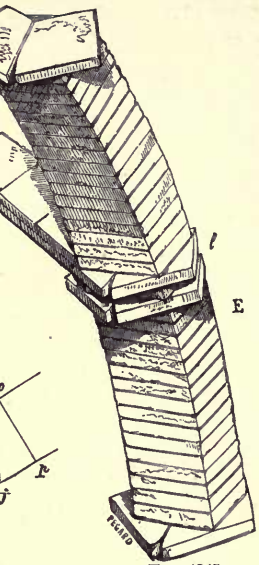


Fig. 1345.

Fig. 1345 represents rib and portions of girts of a groin shown in plan, Fig. 1346, *e f g h* being that of the rib; *K*, a timber of the centre.

A similar construction also obtained for domes, the girts being of the same width as the ribs, and sunk panels formed by furring up on the wooden lagging of the centres.

Fig. 1347 is a perspective of a dome, in which the brick skeleton, ribs, and girts are curved, with panels, B B, of concrete.

In Italy ceilings are made in single courses of brick, groined, and laid without centres, the arcs being described on the side-walls, and the bricks laid in plaster to a line. The spandrels may be levelled up with concrete, when rooms above are to be occupied, but often there is only the brick arch forming the ceiling of the principal rooms, with a light wooden roof above.

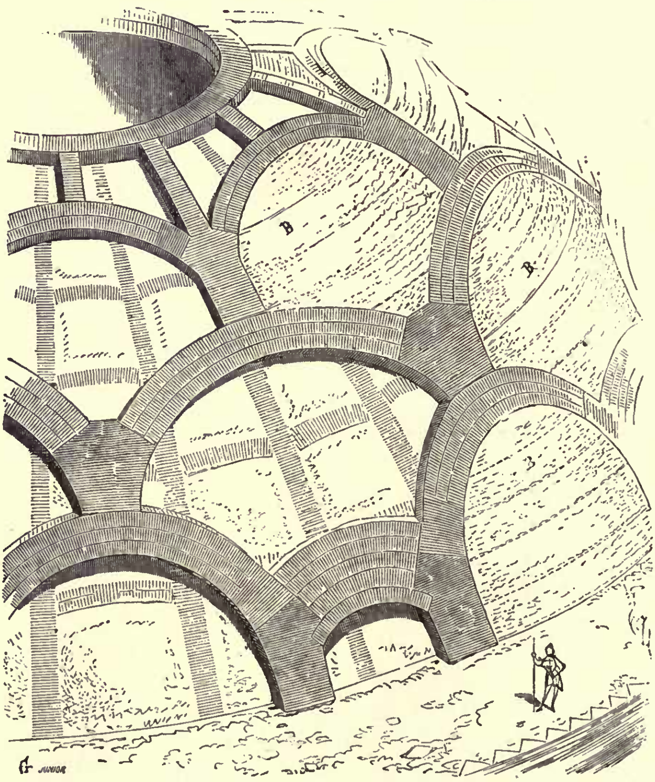


Fig. 1347.

The sizes of Italian brick are 2″ × 4″ × 10″ and 1½″ × 4″ × 12″; when there is no load above, these are laid flatwise in the arch.

In one of the warehouses of the Appleton Manufacturing Company at Brooklyn the floor was constructed of groined arches in concrete, supported on brick side-walls and piers. Piers were 2′ square and 13′ centres; the arches finished for a floor above were 21″ deep at the spring and 9″ at the key. Ceiling was plastered and the room beneath occupied as a warehouse. In the room above a wooden floor was laid and used as a printery, and has been occupied for many years. There has been found one objection to the concrete—that water in quantity spilt on the upper floor sipes through, to prevent which there should have been on the concrete floor a thin coat of Portland cement or of asphalt.

The first fireproof buildings in use here were like the example above of the Appleton warehouse, with supports and arches entirely in masonry. The latter were usually cylindrical or segmental, plain or groined. The depth taken for the construction and the floor space occupied by the supports were objectionable, but as fireproof constructions they were the best.

The erection of high buildings, the large and valuable stocks often carried in stores and warehouses, have involved the necessity of forms of fireproof constructions which will prevent the spreading of conflagrations and secure the contents of a building, but it must be understood that no construction has yet been invented that will prevent the destruction of a building by fire whose contents are large masses of combustibles; the buildings should be called fire-resisting rather than fireproof. The first buildings designed for this purpose consisted of iron I-beams, brick arches, and concrete spandrels, with wooden floors above. This, aside from its weight, was satisfactory, but a skew-back tile and plaster were necessary to protect the bottom flange of the iron beam. The tie-rod, when necessary, was concealed in the arch.

In another form of construction, instead of using a movable centre, a crimped sheet-iron arch was used, which was left in and made a part of the structure, above which there was a concrete filling to the level of the top of the beam.

To reduce the weight of brick and increase their fire resistance, brick are now made hollow, with flat surfaces below for the reception of plastering, and above for the wooden floors (Fig. 1348). Such floorings weigh only about one half that of solid brick arches, and therefore admit of beams of less weight per

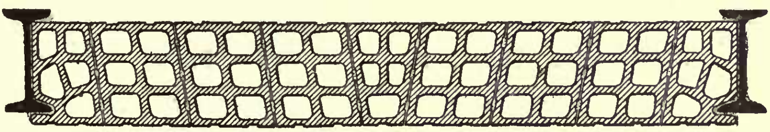


Fig. 1348.

square foot of floor, either by the reduction of the weight of the beam or an increase of span. A thin layer of concrete is put on the top of the brick, and wooden strips embedded to nail the floor to.

By mixing sawdust with the clay, which is burned out in the firing, the product becomes a very light, porous, and firm substance, which can be cut with the saw and will hold a nail. Porous brick, hollow, with thicker ribs than the tiles, are sawed to voussoir joints and laid between I-beams as flat arches.

FIG. 1349.

FIG. 1350.

For ceilings, porous tiles about 2″ thick rest on iron straps suspended from wooden beams (Fig. 1349), or on the flanges of ⊥-irons in a bed of mortar (Fig. 1350), the thickness of the plaster finish beneath affording some protection to the iron straps and flanges from fire.

Iron posts not protected are not as safe as wooden ones. Cast-iron posts may be cast with a surface of nails or projections for plaster of common mortar, or of cement, with a finish of Keene's cement, which admits of washing. Figs. 1351 and 1352 are elevation and section of a Phœnix column with a porous terra-cotta covering; Figs. 1353 and 1354, a similar covering applied to a square and round post.

Fig. 1355 shows in perspective the applications of hollow brick to the lining of exterior walls bonding with the common-sized brick and equal in crushing strength; they supply the place of a course, and the moisture will not strike through.

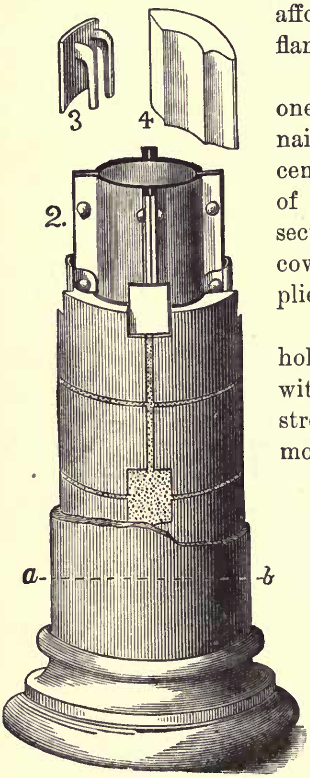


FIG. 1351.

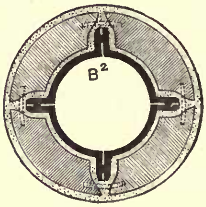


FIG. 1352.

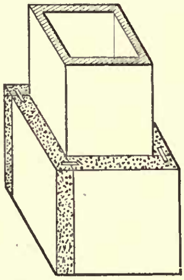


FIG. 1353.

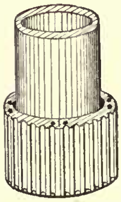


FIG. 1354.

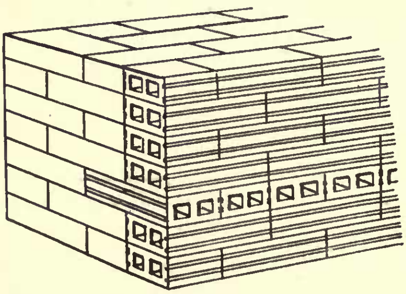


FIG. 1355.

Fig. 1356 is another form applied directly to the face of a wall secured by flat-headed nails driven into the joints of the brickwork.

Fig. 1357 shows the mode of setting hollow brick for a partition; where it is necessary that nails should be driven, porous brick should be inserted.

Improvements in material, especially the use of rolled steel instead of wrought-iron, led to a skeleton construction which consists of rolled steel, wrought- or cast-iron columns supporting a frame of rolled steel, beams and girders, set and framed before any of the masonry except the foundation walls are laid. The next in order of construction are the side and end walls, of which the masonry may be wholly or partially self-sustaining or merely a screen or curtain extending from the top of any wall girder to the under side of the next girders above. The outside posts in

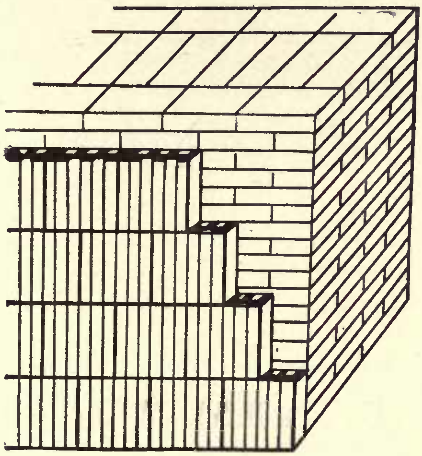


Fig. 1356.

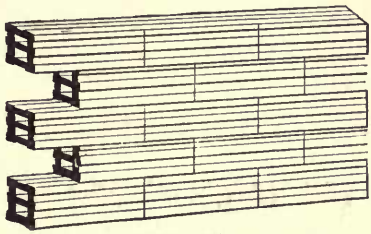


Fig. 1357.

the last case as well as the inside, therefore, sustain all the dead and live load. The dead loads include material used in actual construction, or fixtures and machinery which form a permanent part of the necessities of occupation. Live loads are the weight of occupants, furniture, goods, stores, and movables.

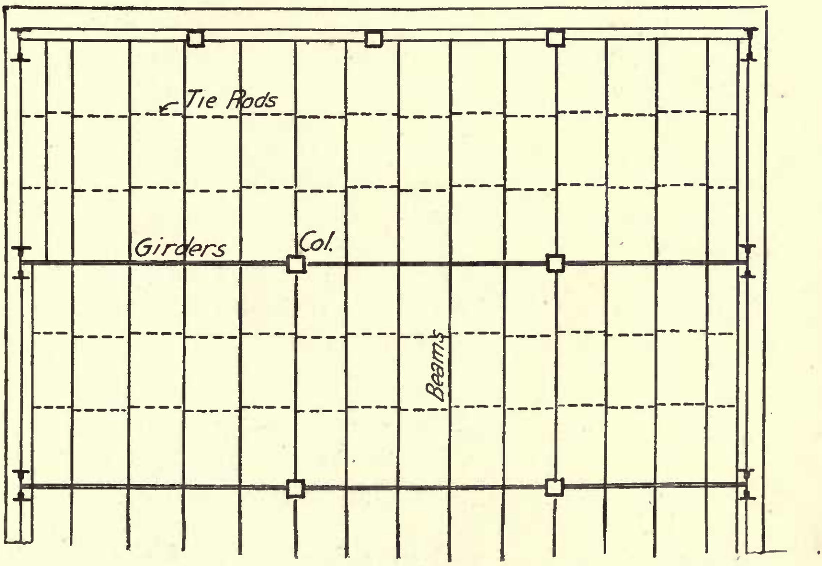


Fig. 1358.

Fig. 1358 is a floor plan of girders and beams of the standard form; columns are preferably of rolled steel, but often of cast-iron. In all the walls, centrally between walls and columns, and between columns, girders are framed on which the beams rest and to which they are usually fastened.

Under the head of "Engineering Drawing" foundations are shown with the bases of these columns and the iron grillage on which they are supported. All the parts are proportioned to the loads which they are to sustain.

The iron surface of all columns, girders, and beams must be protected from

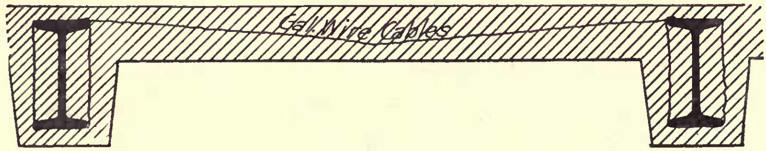


FIG. 1359.

fire, commonly by brick—common, hollow, or porous, and of thickness not less than 4″. The floors are usually flat arches of hollow brick (Fig. 1348) in which the skew-back extends below the flanges of beams and girders to sustain the plaster protection.

Besides the flat arches of hollow brick and of porous brick, concrete has been used for beams and flooring, but, as the material has comparatively little tensile strength, the bottom tension of a concrete beam has been met by steel rods or wire anchored and embedded in the material. Twisted bars of wrought-iron from $\frac{1}{4}$″ to 2″ square have been introduced by E. R. Ransome, of San Francisco, to strengthen flat floors of considerable span, and, as they can not slip, the anchorage is well and uniformly distributed. Experiments by Kirkaldy for Mr. Hyatt demonstrated that the expansion and contraction of concrete and of iron is equal under changes of temperature.

Metropolitan Fire Proofing Company of this city make use of the I-beam frame and tension rods or wires of galvanized iron (Fig. 1359) which are embedded in a concrete composed very largely of plaster-of-Paris cement and a little sand, with crushed coke, cork, or sawdust.

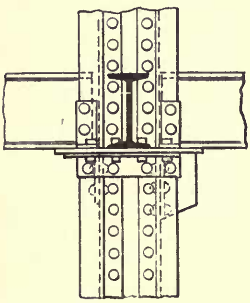


FIG. 1361.

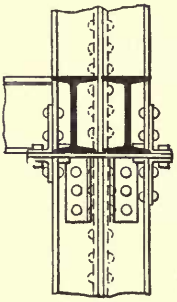


FIG. 1362.

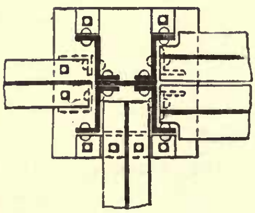


FIG. 1360.

Figs. 1360, 1361, and 1362 are plan and elevations of one of the standard connections of I-beams and Z-bars from the hand-book of the Carnegie Steel Company.

Figs. 1363 and 1364 is a cross-pintle connection of girders and struts of the Phœnix Iron Company with their usual steel column.

Figs. 1365 and 1366 is a cast-iron pintle connection of the same company.

Figs. 1367, 1368, and 1369 are drawings of a cast-iron joint detail.

In the present form of framing steel skeletons, rivets are preferred to bolts, and much depends on their strength in shear.

Fig. 1370 is a plate of the standard connection of angles for I-beams from the Carnegie Company.

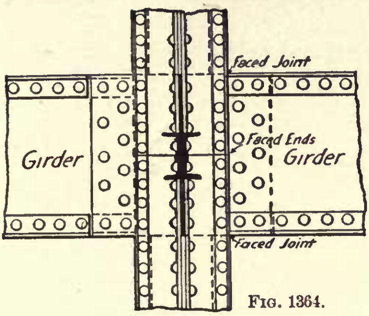


FIG. 1364.

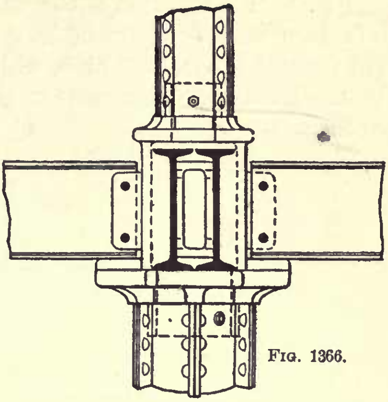


FIG. 1366.

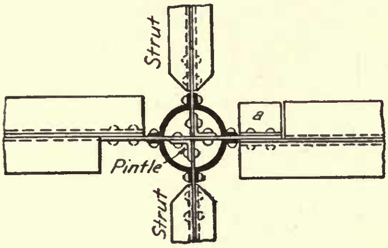


FIG. 1363.

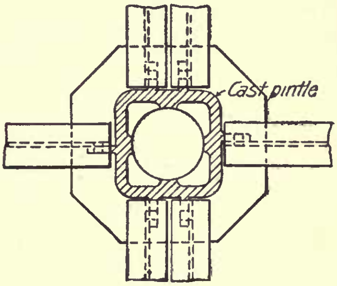
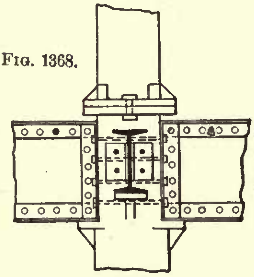


FIG. 1365.

FIG. 1368.



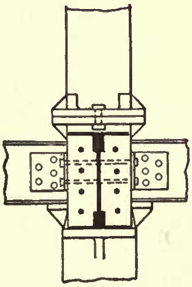


FIG. 1369.

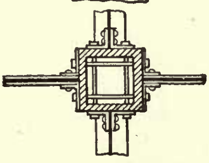


FIG. 1367.

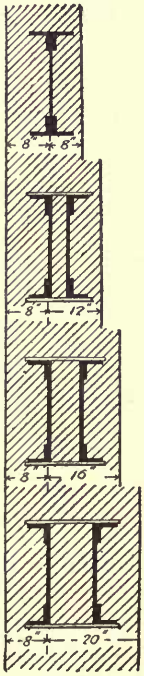


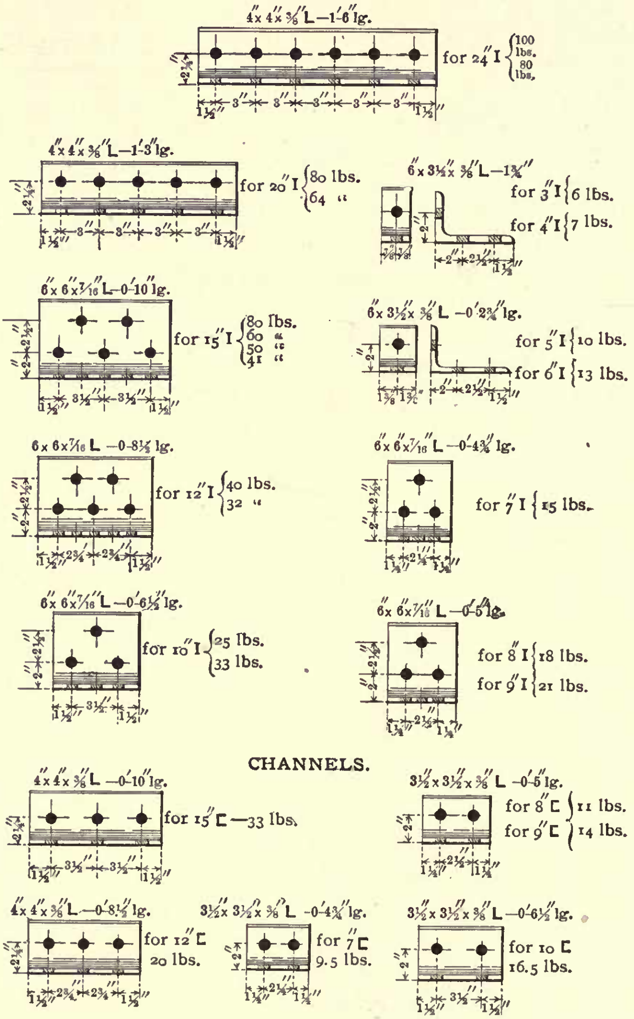
FIG. 1371.

Fig. 1371 is a section of the wall of the New Street front of the Manhattan Life Insurance Company of New York, showing the position of wall girders. The front is 176′ × 254′ high, with five posts, including the corner ones.

The great objections made to the skeleton-frame construction are that the

STANDARD CONNECTION ANGLES.

FOR I BEAMS.



Connections for 3″, 4″, 5″, and 6″ I-beams apply also to Channels.

All holes for ¾″ Bolts or Rivets.

Fig. 1370.

iron is liable to rust in position where it can not be seen, but it is met by care in cleansing the iron, in boiling in oil and painting, or by the Smith process used in protecting cast-iron pipes, and, second, the want of protection against wind strains and a proper system of bracing without interfering with the occupancy of the building. The section of walls (Figs. 1372 and 1373) shows the dimensions required by the Building Department of New York city for the skeleton construction and for plain walls without it.

If the wall construction follows promptly that of the steel frame, and it is temporarily held by tie rods, a mortar set will be secured which will resist the usual wind stresses. For buildings entirely of brick masonry, it is usual to introduce wooden braces—as the work progresses—which are removed when fully closed in.

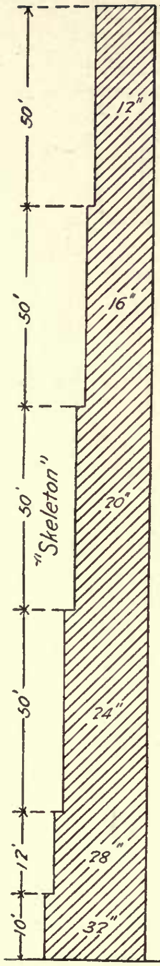


FIG. 1372.

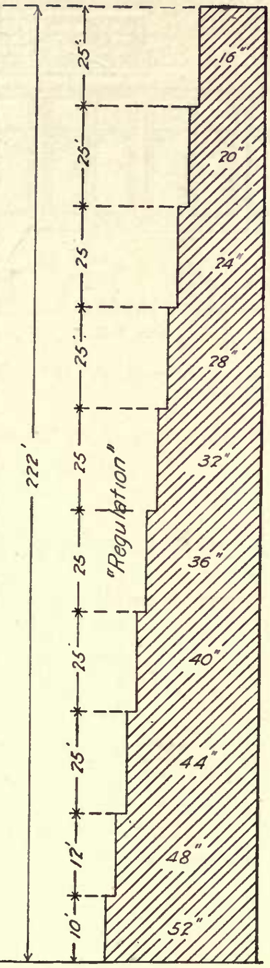


FIG. 1373.

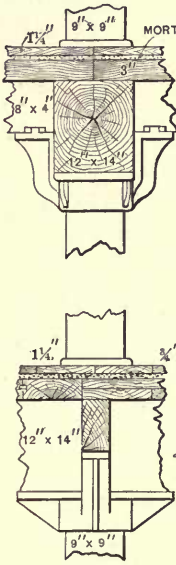


FIG. 1374.

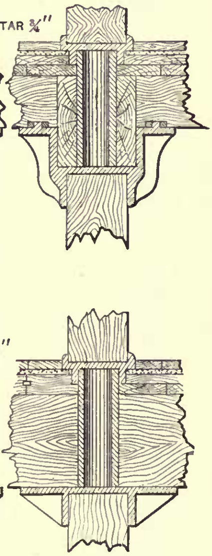


FIG. 1375.

In New England there has been introduced what are called fire-retarding constructions for mills, of which the beams, posts, and floors are of wood. The

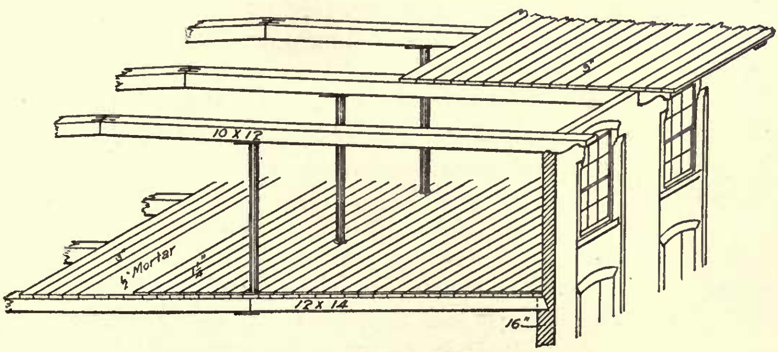


FIG. 1376.

principle of the construction is to consolidate the material in such a way that a fire can be held long enough in any room in which it may originate till the water and appliances under the management of an established fire depart-

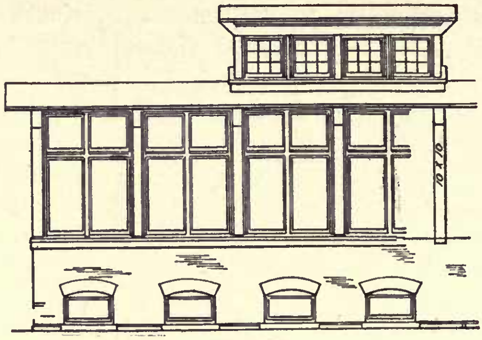
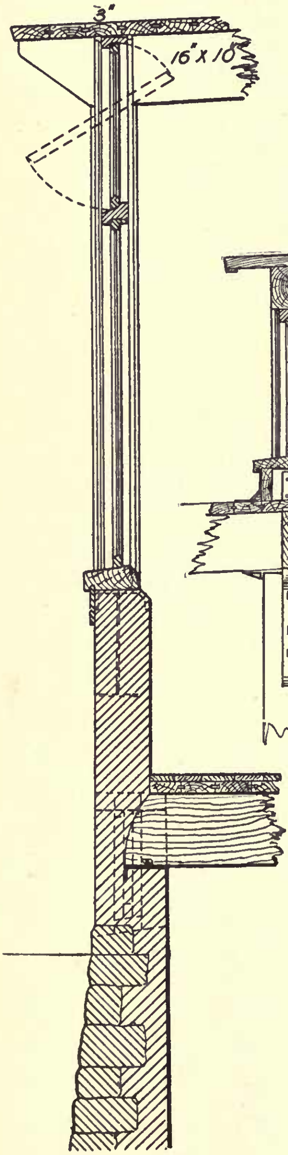


Fig. 1377.

ment connected with the mills or public can get it under control.

Figs. 1374 and 1375 are elevation and section of post, beams, and floors which show the details of con-



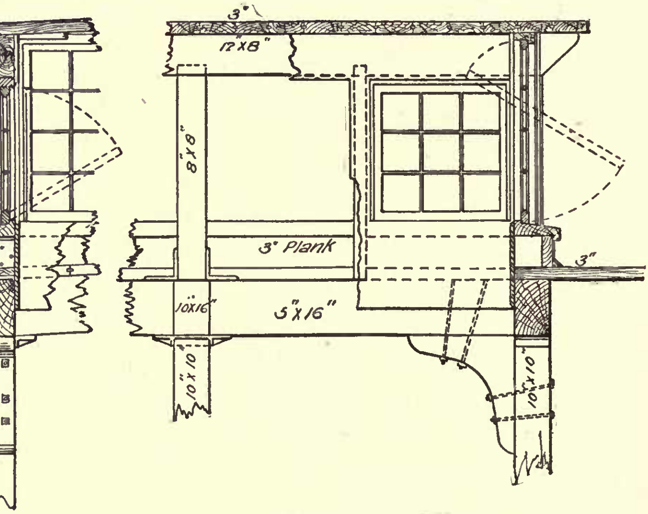


Fig. 1379.

struction, and Fig. 1376 of a mill showing a floor and a roof. The ceiling must be high posted and the windows wide, and set well up to the ceiling to admit of light into a wide mill. When there is plenty of ground space, it is in most industries safer from fire and more economical in management to have a one-story mill (Fig. 1377), of which details are shown in Figs. 1378 and 1379. A one-story mill may be of any dimension required, as the central spaces are lighted by monitor decks. The principle of all this construction is that there should be no spaces where dust may collect; in the floors the material is close, but if the

Fig. 1378.

timber be not seasoned or kiln-dried, as there is no circulation of air, it may dry rot. The posts have a bore through the centre and holes connecting with it at

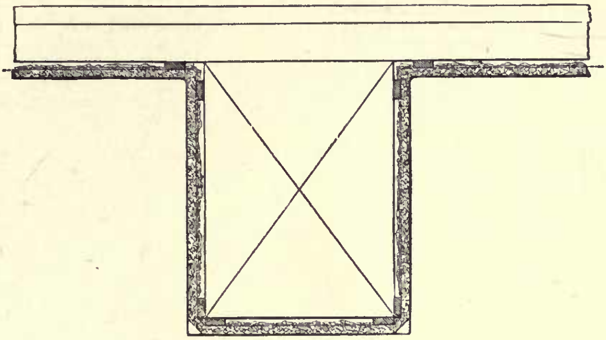


FIG. 1380.

top and bottom to provide air circulation, and it is better not to paint any of the exposed timber until all the moisture is dried out.

In mills sheathing is preferable to plastering, as this last may break and pieces fall into the machinery; but Fig. 1380 shows a form of construction that has served well for warehouses, and is safer and more ornamental than the usual timber constructions. The lathing consists of wire cloth fastened with staples, stapled through furring strips $\frac{3}{8}''$ to $\frac{1}{2}''$, and then the usual three-coat plaster.

Doors.—In stud-partitions, the openings for doors are framed as in Fig. 1319, the door-frame being independent of the studs.

Fig. 1381 represents the elevation and Fig. 1382 the horizontal section of a common inside-door. A A are the *stiles*, B, C, H, D, the *bottom*, *lock*, *parting*, and *top rail*, E the *panels*, and F the *muntin;* the combination of mouldings and offsets around the door, G, is called the *architrave;* in the section, *a a* are the partition-studs, *b b* the door-*jambs*.

Fig. 1383 represents the forms of the parts of a door, and the way in which they are put together. When the tenons are to be slipped into the mortises, they are covered with hot glue, and, after being closed up, keys are driven in.

With regard to the proportions of internal doors, they should depend in some degree on the size of the apartments; in a small room a large door always gives it a diminutive appearance, but doors leading from the same room or passage, which are brought into the same view, should be of uniform height. The smaller doors which are found on sale are 2 feet 4 inches × 6 feet; for water-closets, or very small pantries, they are sometimes made as narrow as 15 inches, but any less height than 6 feet will not afford requisite head-room; 2 feet 9 inches × 7 feet, 3 feet × 7 feet 6 inches, or 3 feet 6 inches × 8 feet, are well-proportioned, six-panelled doors. But the apparent proportions of a door may be varied by the omission of the parting-rail, making the door four-panelled, or narrowed still more by the omission of the lock-rail, making a two-panelled door. Sometimes the muntin is omitted, making but one panel; but this, of course, will not add to the appearance of width, but the reverse. Wide panels are objectionable, as they are apt to shrink from the mouldings and crack. The mouldings are generally planted on, and nailed to the stiles and rails, but sometimes formed on the latter.

When the width of the door exceeds four feet, it is generally made in two parts, each part being hung to its side of the frame, or one part hung to the other, so as to fold back like a shutter; or the parts may be made to slide back into pockets

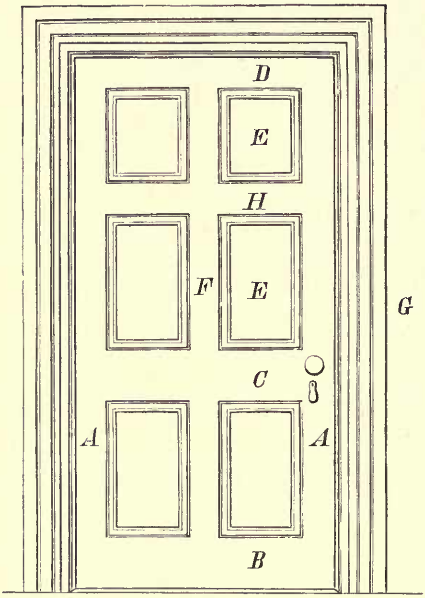


Fig. 1381.

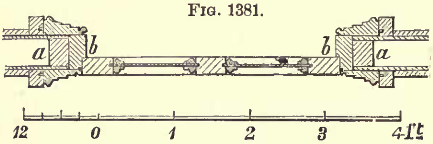


Fig. 1382.

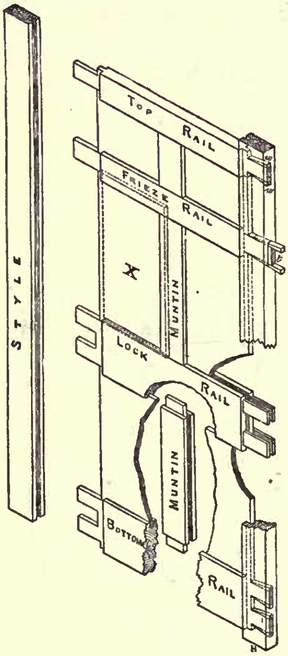


Fig. 1383.

or grooves in the partition. The doors may be supported on wheels, and run on tracks at the floor-level; or the tracks may be above the doors, and the doors suspended; or they may be supported by levers, and be moved parallel without rollers; all these appliances can be purchased.

Figs. 1384, 1385, and 1386 are the elevation, vertical and horizontal sections of a pair of sliding-doors. There are no knobs, but countersunk pulls to the doors, that they may be slid entirely within the pockets, with a special handle in the locks at the edges of the doors for withdrawing them.

Figs. 1387 and 1388 are vertical and horizontal sections of the same doors hung on butts or hinges.

Figs. 1389 and 1390 are the elevation and horizontal section of an antæ-finished outside-door, with the *side-lights* C C, and a *top*, *fan*, or *transom light* B. The bar A is called a transom, and this term is applied generally to horizontal bars extending across openings, or even across rooms.

Fig. 1391 is the elevation of an outside folding-door. The plan (Fig. 1392) shows a vestibule, V, and an interior door. The outer doors open, as shown by the arcs, and fold back into the pockets or recesses, *p p*, in the wall. This is a very common form of doors for first-class houses in this city. The fan-lights are made semicircular, and also the head of the upper panels of the door; these panels in the interior or vestibule door are of glass. Of late the outer doors are extensively used as storm doors, glazed with plate glass exposing the vestibule, and hung with spring butts and ornamental in finish.

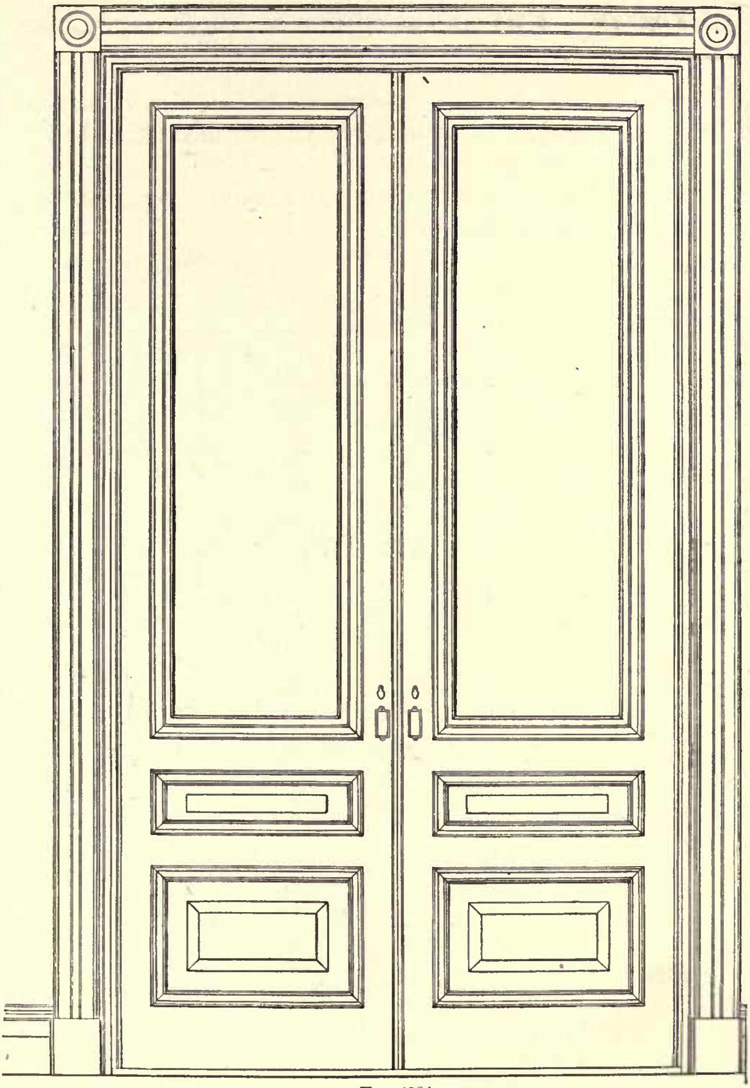


Fig. 1384.

Fig. 1385.

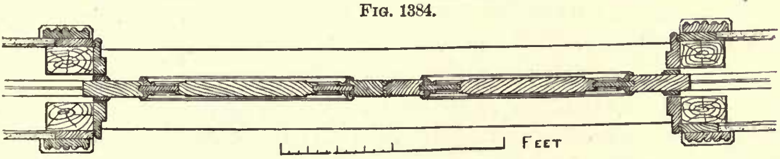


Fig. 1386.

Windows are usually understood to be glazed apertures. The sashes may be stationary, but for most positions they are made to open either by sliding vertically, or laterally, or like doors. The first is the common form of window, and the sashes are generally balanced by weights; the second, except in a cheap

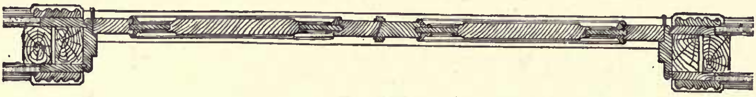


FIG. 1388.

form in mechanics' shops, are seldom used; the third, often used for access to balconies or between rooms, are called casements, or French windows.

Figs. 1393 and 1394 are the outside elevation and horizontal section of one half of a common

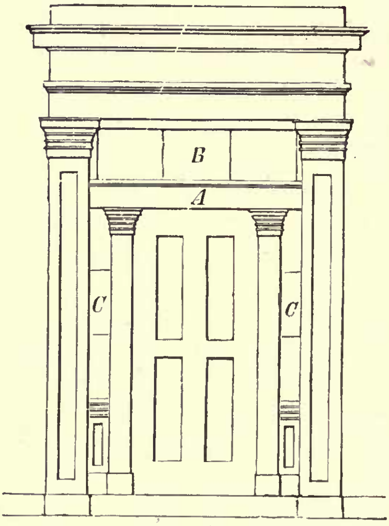


FIG. 1389.

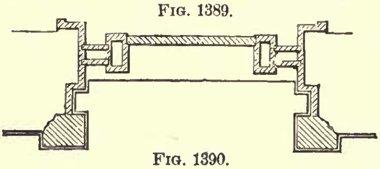


FIG. 1390.

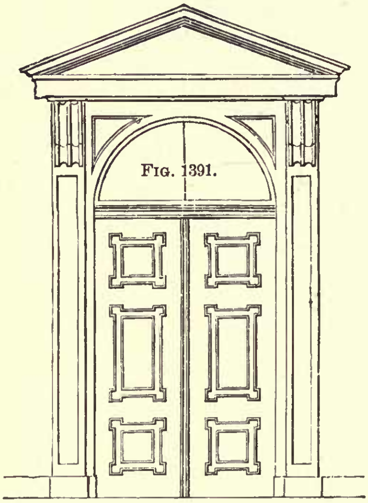


FIG. 1391.

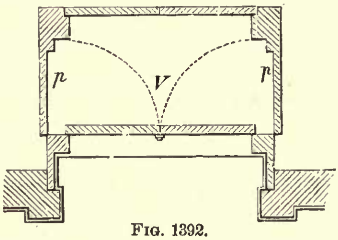


FIG. 1392.

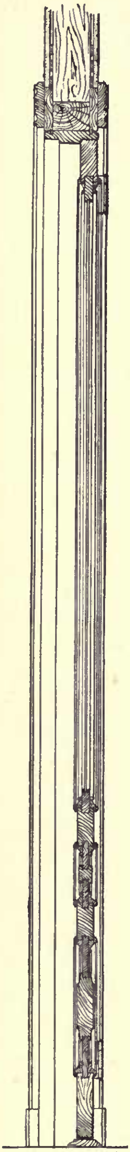


FIG. 1387.

box-frame, and Fig. 1395 a vertical section of the same in a wooden frame house. S is the sill of the sash-frame, W the frame-sill, with a wash to discharge the water, B the bottom rail of the sash, M the meeting rails, T the top rail, H the head of the sash-frame, and A the architrave similar to that around doors. Instead of two sills, S and W, one is often used, and inclined to form the wash. D is the common outside blind. In the sectional plan (Fig. 1400), C C′ are the *window-stiles*, F the *pulley-stile*, *w w* the sash-weights, *p* the parting strip, and D D double-fold shutters.

Figs. 1396 and 1397 are the interior elevation and vertical section of a box-frame window in a masonry wall; Fig. 1398 is an exterior view of the same

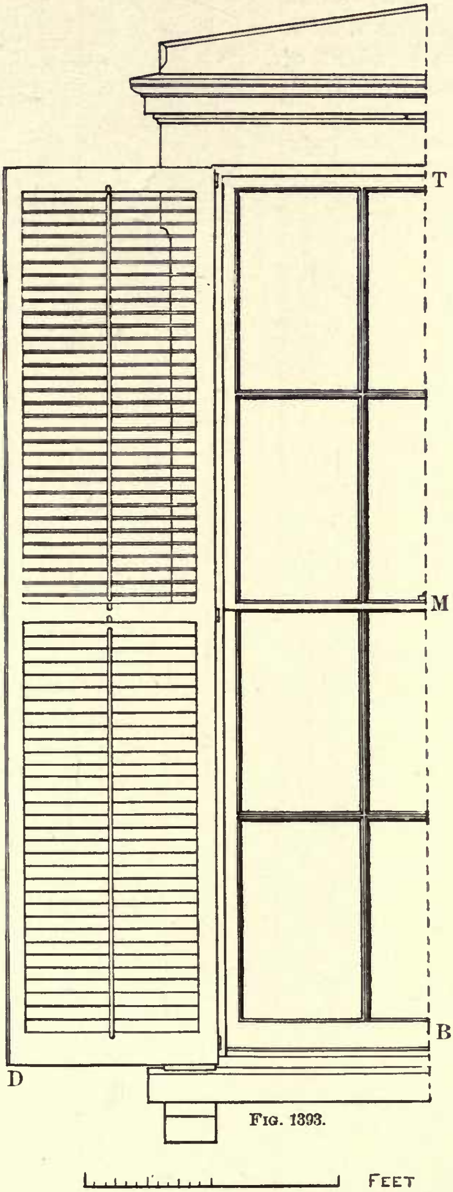


FIG. 1393.

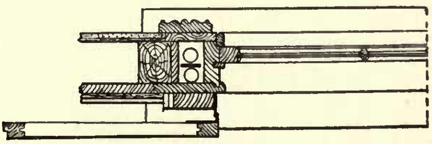


FIG. 1394.

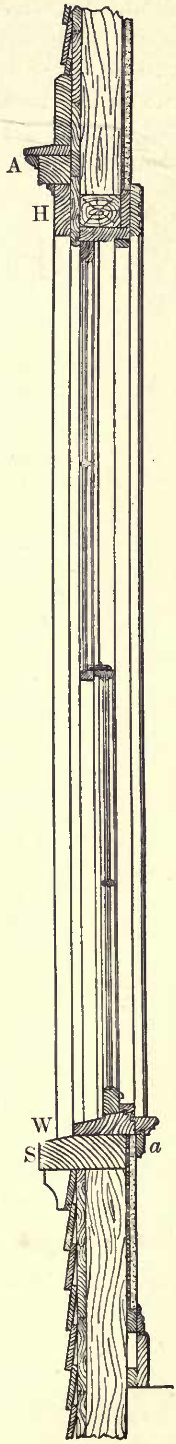


FIG. 1395.

window, and Fig. 1399 a horizontal section. In masonry walls the sills are usually of stone, as shown in Fig. 1397, with a lap of the window frame on it. In setting the sill, one course of brick is left out beneath the central portion to admit of settlement in the walls without stress to the sill.

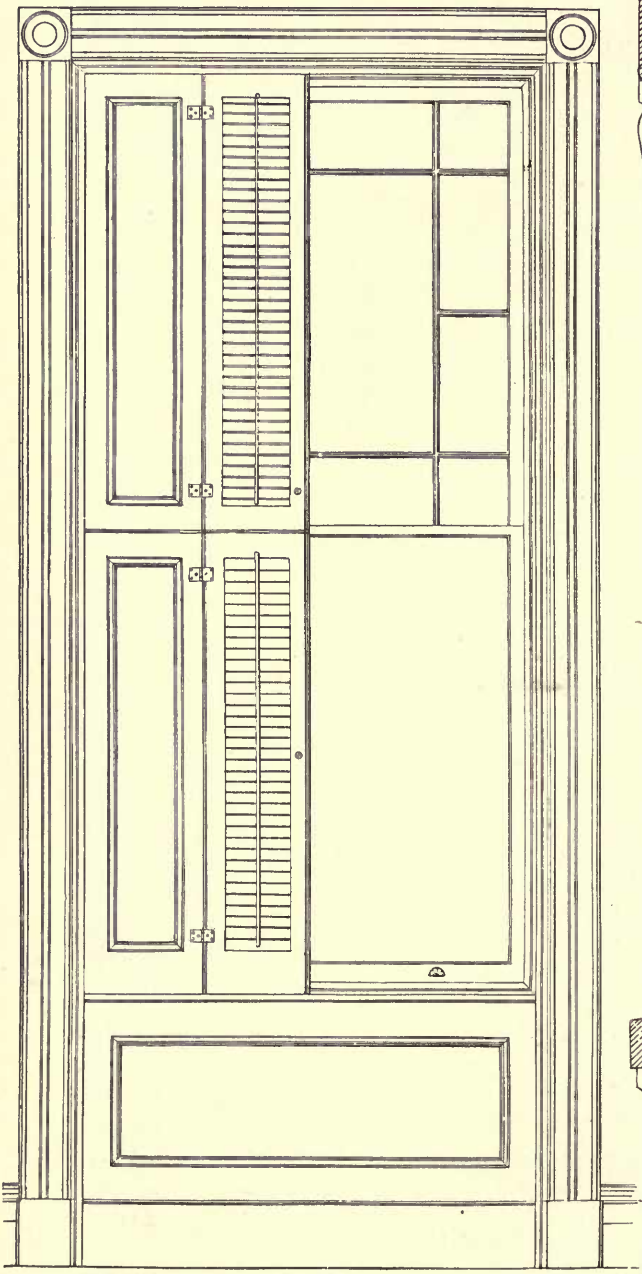


Fig. 1396.

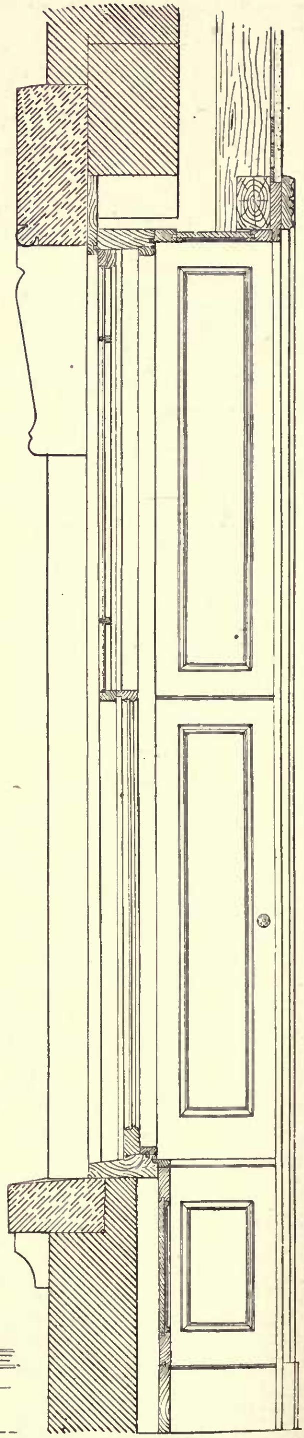


Fig. 1397.

In brickwork, the height of sill and cap corresponds to a determinate number of courses, so that it may not be necessary to split brick in setting them.

Unless the windows begin from, or nearly from, the floor, the point *a* (Fig. 1395) may be fixed at a height of about 30 inches above the floor, and the top of the window sufficiently below the ceiling to allow space for the architrave or other finish above the window, and for the cornice of the room, if there be any; a little space between these adds to the effect. For common windows, the width of the sash is 4 inches more than that of the glass, and the height 6 inches more; thus the sash of a window 3 lights wide and 4 lights high, of 12″ × 16″ glass, is 3 feet 4 inches wide and 5 feet 10 inches high. In plate-glass windows more width is taken for the stiles and rails.

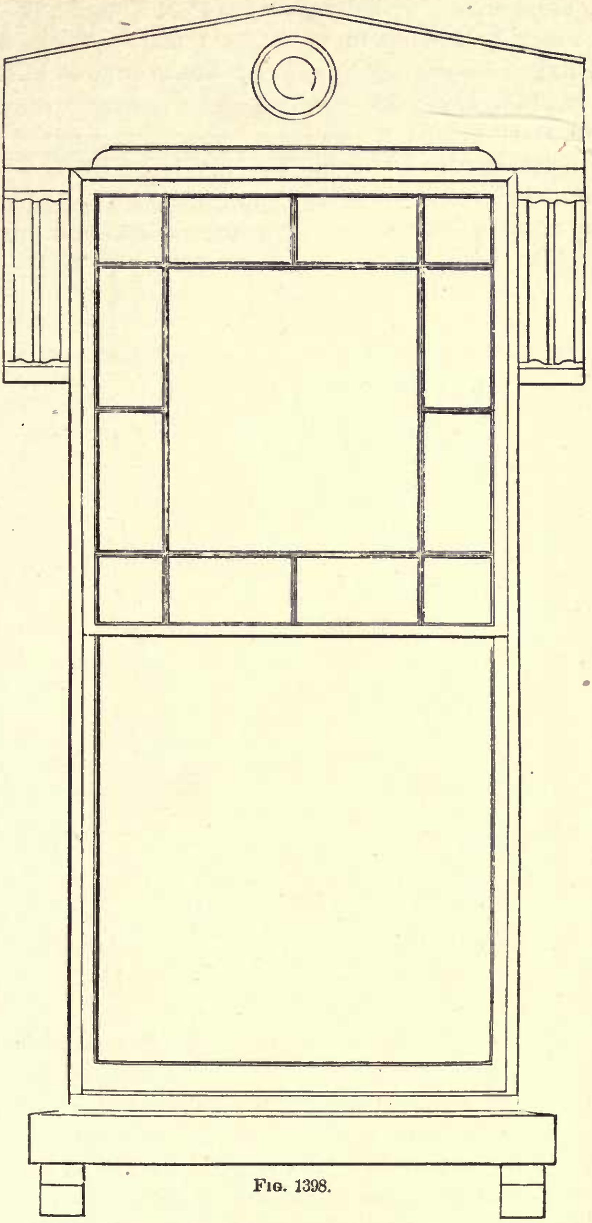


Fig. 1398.

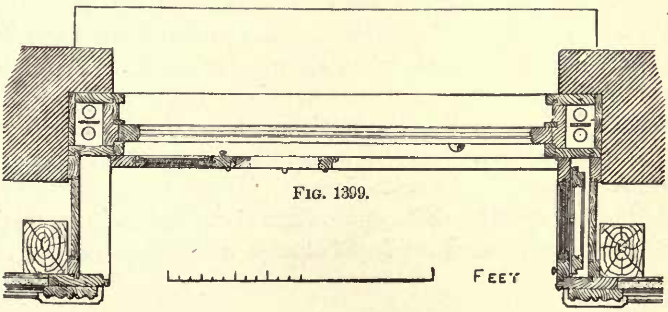


Fig. 1399.

The usual sizes of cylinder glass are 7″ × 9″ up to 24″ × 36″, but single thick glass may be had up to 40″ × 60″; double thick, 48″ × 62″. Plate glass, polished or rough, may be had of a size as large as 14 × 8 feet.

In Fig. 1393 the blind D is hinged to the hanging stile, and is closed within the opening in the masonry. The slats are movable on pin tenons, and those of each half, upper and lower, are connected by a central bar, so that they are moved together, and adjusted at any angle to the light. In Fig. 1399 the blinds are inside, 4-fold, and folding back into pockets. It is more usual to make the pockets for the blinds inclined to the window, as in Fig. 1400, giving to the interior more light, or ampler space for curtains.

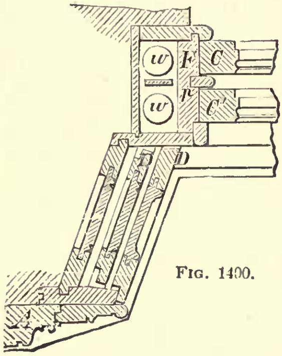


FIG. 1400.

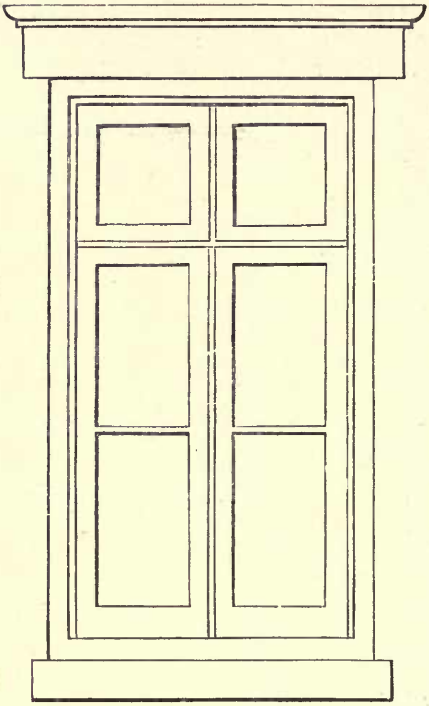


FIG. 1401.

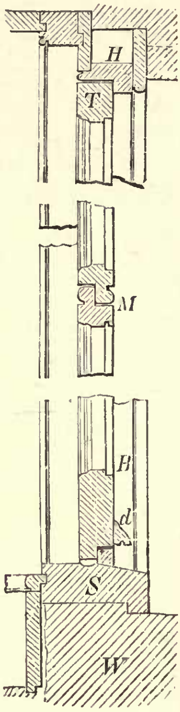


FIG. 1402.

Fig. 1401 is the outside elevation of a French window or casement.

Fig. 1402 represents the sectional elevation of the same window, in broken lines, and on a larger scale; the same letters designate similar parts as in Fig. 1395. A transom-bar is often framed between the meeting-rails, and in this case the upper sash may be movable; in Fig. 1402 it is fixed. An upright, called a *mullion*, is often introduced in the centre, against which the sash shuts.

For use as doors, the lower sashes should not be less than 5 feet 6 inches high. In these forms of sash the rails and stiles are wide, and for equal apertures. French windows when closed admit light. The chief objection to this window lies in the difficulty of keeping out the rain at the bottom in a driving storm. To obviate this, the small moulding *d*, with a drip or undercut, is nailed to the bottom rail; but the more effectual means is the patent weather-strip, the same as used on outside doors.

Dormer or attic windows are framed and set as in an upright stud-partition.

Stairs consist of the *tread* or *step* on which to set the feet, and *risers*, upright pieces supporting the treads—each tread and riser forms a stair. If the treads are parallel they are called *fliers;* if less at one end than the other, they are called *winders*, *f* and *w* (Fig. 1409). The top step, or any intermediate wide step, for the purpose of resting, is called a *landing;* the height from the top of the nearest step to the ceiling above the *headway;* the rounded edge of the step a *nosing* (*a*, Fig. 1403); and if a small hollow or cavetto (*b*) be glued in the angle of the nosing and riser, it is called a *moulded* nosing. The pieces which support the ends of the stairs are the strings (Fig. 1404); that against the wall the *wall-string*, the other the *outer string*. Besides these strings, pieces of timber are framed and placed beneath the fliers, when the stairs are

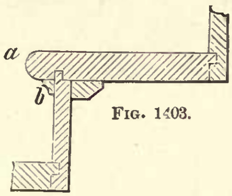


FIG. 1403.

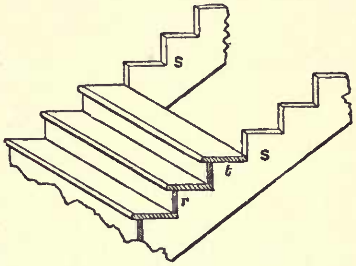


FIG. 1404.

wide (Fig. 1405), called *carriages.* Sometimes the strings, instead of being notched out to receive the steps, have the upper and lower edges parallel, with grooves cut in the inner faces to receive the ends of their steps and

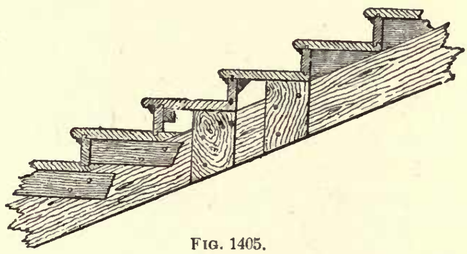


FIG. 1405.

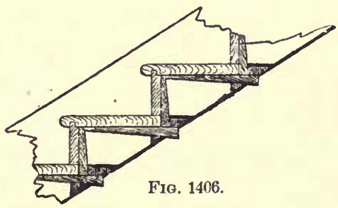


FIG. 1406.

risers (Fig. 1406). These are called *housed strings.* Steps and risers are secured in the grooves by wedges covered with glue, and driven in. For the rough, strong strings of warehouses the carriages are made of plank, with grooves to receive plank-treads, and without risers.

Figs. 1407 and 1408 are elevation and plan of a straight run of stairs, both partly in section. N is the *newel-post*, *n* a *baluster*, *h* the hand-rail, *w* the well. In the section of the floors, cleats are shown nailed to the beams; on these short boards are nailed to form a box for the reception of mortar for the deafening. The opening represented in the plan (which must occur between the outer strings, if they are not perpendicular over each other) is called the *well* (W, Fig. 1409).

The breadth of tread in general use is from 9 to 12 inches; in the best staircases, it should never be less than 11 inches, nor more than 15. The height of the riser should be the more, the less the width of the tread; for a 15-inch tread the riser should be 5 inches high; for 12 inches, $6\frac{1}{2}$; for 9 inches, 8. In laying out the plan of stairs, having determined the starting-point, either at bottom or top, as the case may be, find exactly the height of the story; divide this by the height you suppose the riser should be. Thus

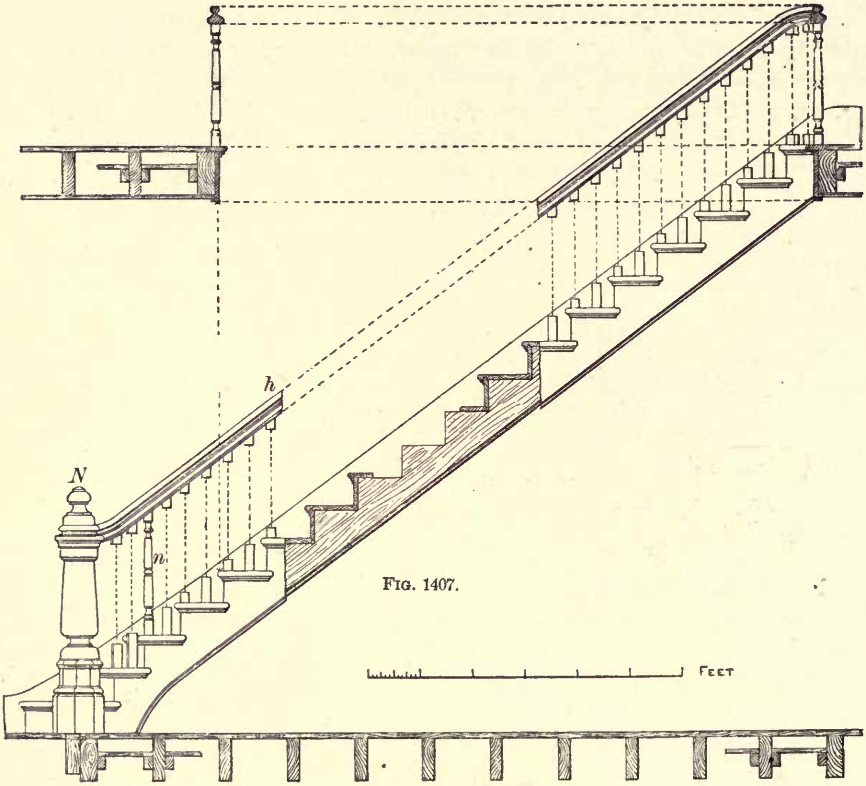


FIG. 1407.

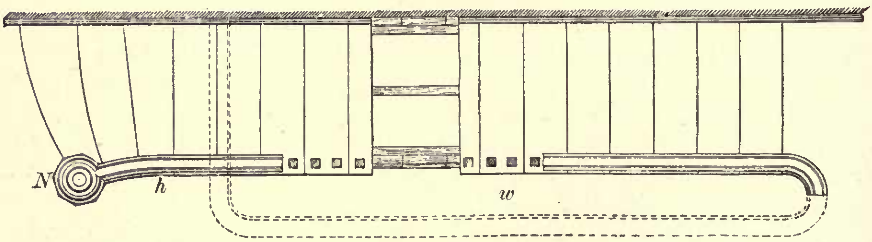


FIG. 1408.

(Fig. 1410), if the height of the story and thickness of floor be 9 feet, and we suppose the riser should be 7 inches high, then 108 inches, divided by $7 = 15\frac{3}{7}$.

It is clear that there must be a full number of steps, either 16 or 15; to be near the supposed height of the riser, adopt 15, then—

$$\frac{108}{15} = 7\tfrac{3}{15} \text{ inches, height of riser.}$$

For this particular case, assume the breadth of the step as 10 inches, and the length at 3 feet, a very usual length, seldom exceeding 4 feet in the staircases of private houses. For the plan—lay off the outside of the stairs, two parallel lines 3 feet apart, and space off from the point of beginning 14 treads of 10 inches each, and draw the cross-parallel lines. To construct the elevation, project the lines of the steps in plan, and divide the height, either on a perpendicular or by an inclined line, into the number of risers (15), and draw

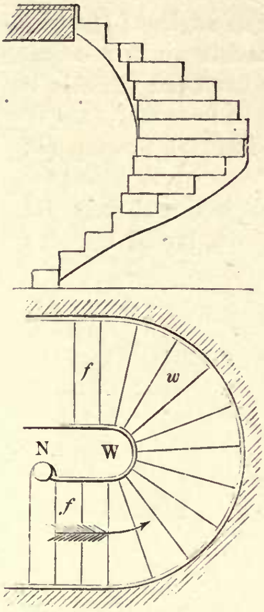


FIG. 1409.

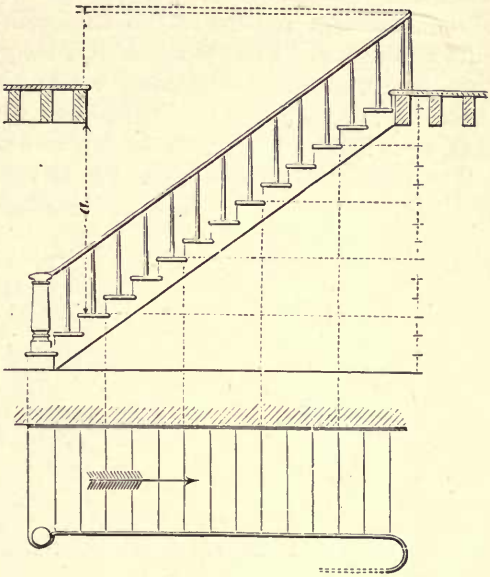


FIG. 1410.

horizontal lines through these points; or the same points may be determined by intersection of the projections of the plan with a single inclined line drawn along the nosing of top and bottom steps. The number of treads is always one less than the number of risers, the reason of which appears in the elevation.

For the framing plan the drawing of the elevation of stairs is in general necessary, to determine the opening to be framed in the upper floor, to secure proper headway. Thus (Fig. 1410), the distance, *a*, between the nearest stair and the ceiling should not be less than 6 feet 6 inches; a more ample space improves the look of the stairway; but if confined in our limits, this determines the position of one header; the other will be of course at the top of the stairs. When one flight is placed over another, the space required for timber and plastering, under the steps, is about 6 inches for ordinary stairs.

When the stairs are circular, or consist in part of winders and fliers, as in Fig. 1409, the width of the tread of the winders should be measured on the central line. The construction of the elevation is similar to that of the straight run (Fig. 1410), dividing the space between the stories by a number of parallel lines equal to the number of risers, and intersecting the parallels by projections from the plan. The objection to all circular stairs of this form, or with a small well-hole or a central shaft, is that there is too much difference between the width of the tread, but a small portion being of a suitable size. The handsomest and easiest stairs are straight *runs*, divided into landings, intermediate of the stories, and either continuing then in the same line, or making a full return at right angles. It is at times fashionable to make the *newel* a prominent feature in the hall, often occupying valuable space. It is sufficient that it be large and stiff enough for a support to the hand-rail and may be equally ornamental.

The top of the hand-rail should, in general, be about 2′ 8″ to 3′ above the nosing, and should follow the general line of the steps. The angles of the hand-rail should always be eased off. A hand-rail, affording assistance in ascending or descending, should not be wider than the grasp of the hand (Fig. 1411); but where, for architectural effect, a more massive form may be necessary, it is very convenient to have a sort of double form, with the hand-rail at the top (Fig. 1412), or as in Fig. 1413, with the groove outside.

To a draughtsman conversant with the principles of projection already given, it will not be difficult to draw in the hand-rail of stairs, or to lay off the mould

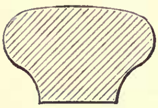


FIG. 1411.

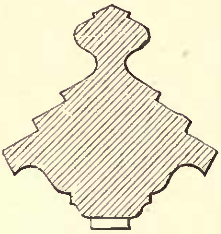


FIG. 1412.

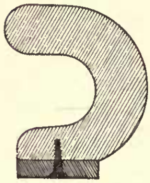


FIG. 1413.

for its construction. It will follow the line of stair-nosing, and where there are changes of pitch they are made to connect by curves tangent to these pitches, except where the landings are square, and newels set at the head of the land-

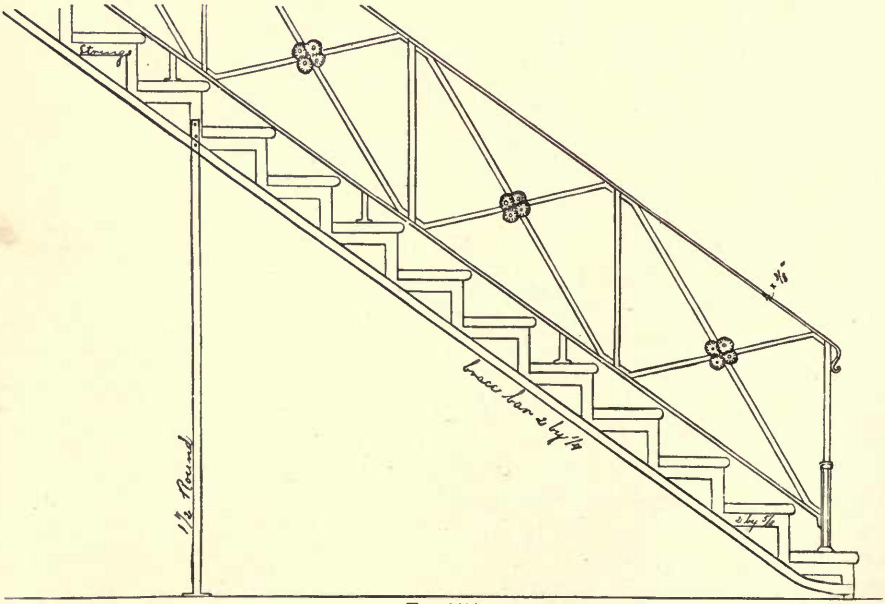


FIG. 1414.

ings, the rail is framed into the newel. At the bottom the rail is curved to the horizontal, when it comes into or upon top of the newel.

Balusters are of great variety—usually turned forms—attached to the treads by dovetails, covered with the returned nosing, or with pin-ends and holes in treads and under side of caps. Sometimes (especially in iron-work) the baluster

ELEVATION.

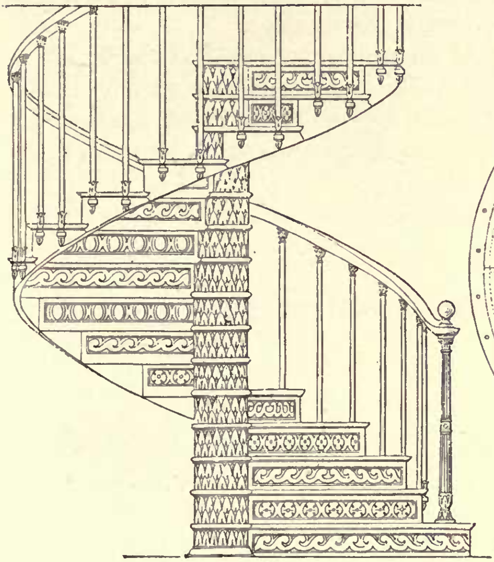


Fig. 1415.

PLAN.

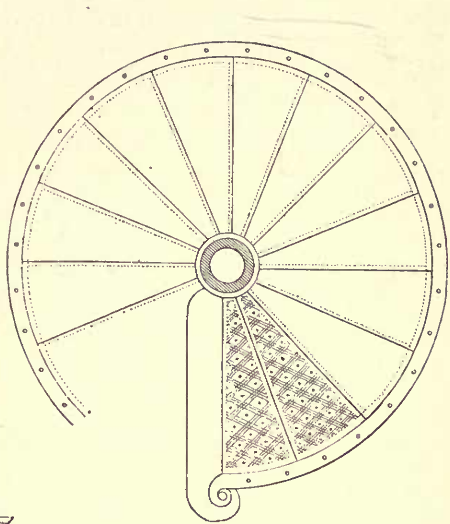


Fig. 1416.

is set in a bracket from the face of the string, as in Fig. 1415; or the balusters may be cast with the bracket.

Fig. 1414 is the side elevation of a stairs with wrought-iron string and rail. The string is made of wrought-iron knees, welded together continuously, with

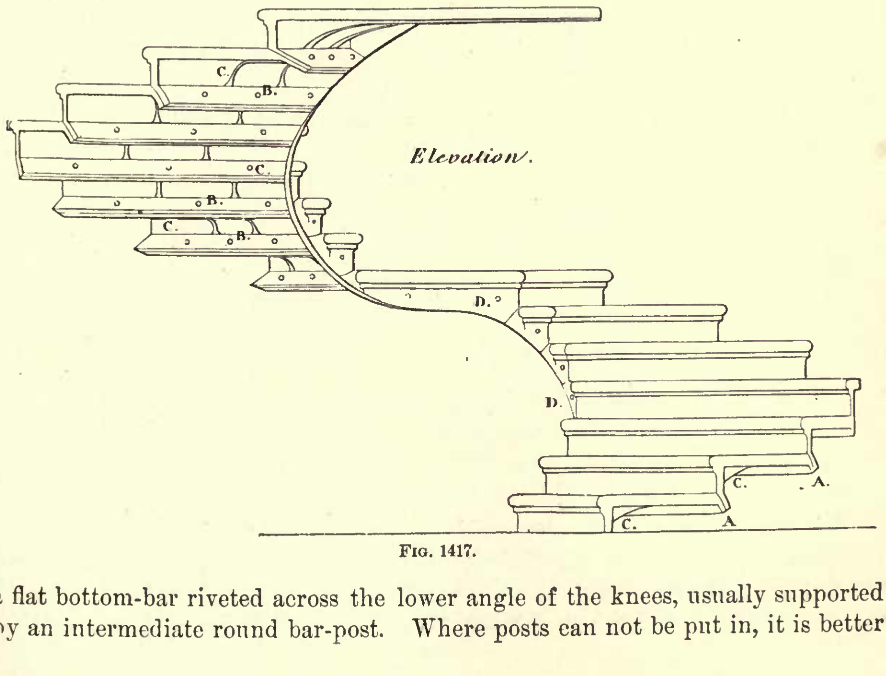


Fig. 1417.

a flat bottom-bar riveted across the lower angle of the knees, usually supported by an intermediate round bar-post. Where posts can not be put in, it is better that the bottom bar should be a carriage or beam of I or channel-iron, with knees or cast-iron angle-blocks riveted on the top of the beam. It is not unusual to make housed strings of plate-iron, with angle-irons riveted on to receive the treads and risers. If the plate-iron be wide enough to serve instead of balusters, it makes a very strong and stiff carriage.

Figs. 1415 and 1416 are the plan and elevation of a cast-iron stairs, with a central post or *newel* (this term is applied also to the first post of any stairs). The newel-ring, tread, and riser of each step are cast in one piece, and they are put together by placing one newel-ring upon that below and bolting the outer extremity of the riser to the tread below.

Fig. 1417 is a form of cast-iron stairs with a well instead of a newel; the step and riser are bolted together by the flanges. It will be seen that one tread is wider than the others; this is a landing.

Fireplaces.—Fireplaces for wood are made with flaring jambs of the form shown in plan (Fig. 1418); the depth from 1 foot to 15 inches, the width of opening in front from 2 feet 6 inches to 4 feet, according to the size of the room to be warmed; height 2 feet 3 inches to 2 feet 9 inches, the width of back about 8 inches less than in front; but at present fireplaces for wood are seldom used, stoves and grates having superseded the fireplace. The space requisite for the largest grate need not exceed 2 feet in width by 8 inches in depth. The requisite depth is given by the projection of the grate, and the mantel-piece. Ranges require from 4 feet 4 inches to 6 feet 4 inches wide × 12 inches to 20 inches deep; jambs 8 inches to 12 inches.

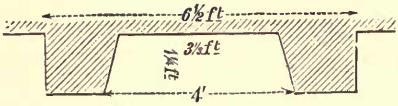


FIG. 1418.

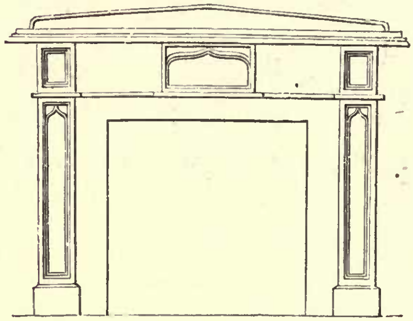


FIG. 1419.

Fig. 1419 represents the elevation of a mantel-piece of very usual proportions. The length of the mantel is 5 feet 5 inches, the width at base 4 feet 6 inches, the height of opening 2 feet 7 inches, and width 2 feet 9 inches. A portion of this opening is covered by the iron sides or architrave of the grate, and the actual open space would not probably exceed 18 inches in width by 2 feet in height. In brick or stone houses the flues are formed in the thickness of the wall, but when distinct they have an outside shell of a half-brick or 4 inches, and sometimes 8″ (Fig. 1420); the *withs* or division-walls always 4″.

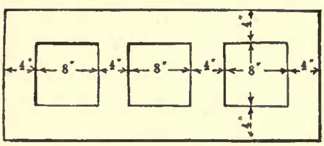


FIG. 1420.

The size of house flues is usually 8″ × 8″, but some are 4″ × 8″, 4″ × 12″, and 8″ × 12″. The flues of different fireplaces should be distinct. Those from the lower stories pass up through the jambs of the upper fireplaces, and, keeping side by side with but 4-inch brickwork between them, are topped out above the roof, sometimes in a double and often in a single line 16 inches wide by a breadth required by the number of flues, as in Fig. 1420 or in Fig. 1421. The latter is an illustration of how far flues may be diverted from a vertical line, but it is to be observed that the construction must be stable, as any settling or cracks not only injures the draught of the chimney, but impairs

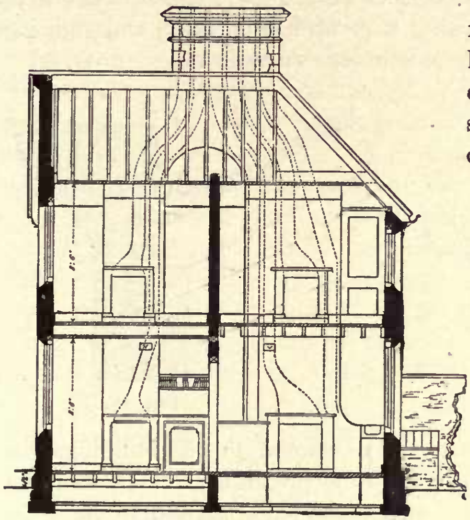


FIG. 1421.

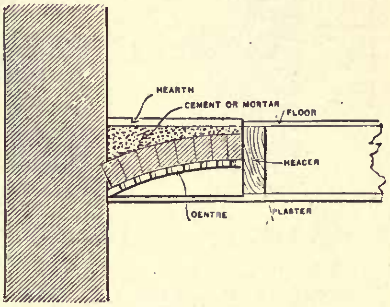


FIG. 1422.

the security of the building against fire. Changes of direction of flues should never be abrupt. The back of the fireplace may be perpendicular through its whole height, but it is usual to incline the upper half inwardly toward the room, making the throat to the flue long and narrow. It is very common to form the upper 3″ to 4″ of the inclined back by an iron plate, which can be turned back or forward to increase or diminish the draught. Fig. 1422 represents the arrangement of frame and brick arch for the support of the hearth. The chimney is generally capped with stone, sometimes with tile or cement pots. As an architectural feature, the chimney is often made to add considerably to the effect of a design.

Roofs.—Framed roofs have been illustrated (page 486). City roofs are usually flat, and timbered similarly to floors, but not so strongly, with a slight pitch to discharge rainfall. Roofs of country dwellings are usually framed like stud-partitions, with inclined studs somewhat deeper than if they were vertical, depending on the inclination from the vertical; if flat, depth like that of a floor. The theory of the construction of the gambrel or Mansard roof (Fig. 1423), a

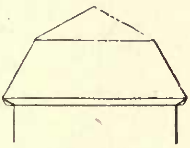


FIG. 1423.

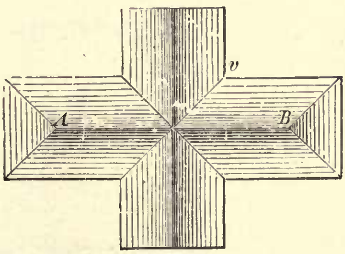


FIG. 1424.

roof with two kinds of pitch, is that of the polygon of rods, and self-supporting; but, in general, they have central support from partitions, and their outlines are much varied by curves in the lower rafters cut from plank.

Fig. 1424 is the plan of a roof as usually drawn, shaded strongly at the ridges. The transept roof is *hipped* at A and B.

Gutters are generally formed in the cornice (Fig. 1425); sometimes on the roof (Fig. 1426), and sometimes by raising a parapet (Fig. 1427) and forming a *valley*. The intersection of two roofs as at *v* forms a valley.

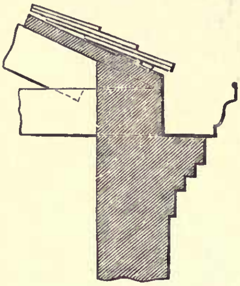


FIG. 1425.

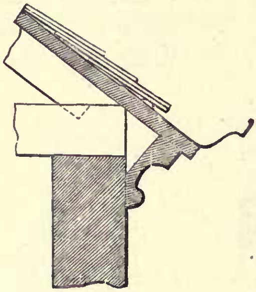


FIG. 1426.

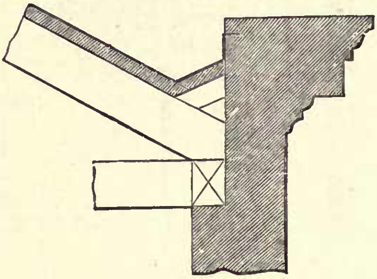


FIG. 1427.

Fig. 1425 represents a form of gutter very common to city buildings, the roof boarding extending over the gutter; but it is preferable to make the roof pitch from both rear and front to the centre of the building, and to carry the leader down in the interior, where it may serve as a soil-pipe for the water-closets, basins, and baths, affording ventilation in fair weather and a scour in rains.

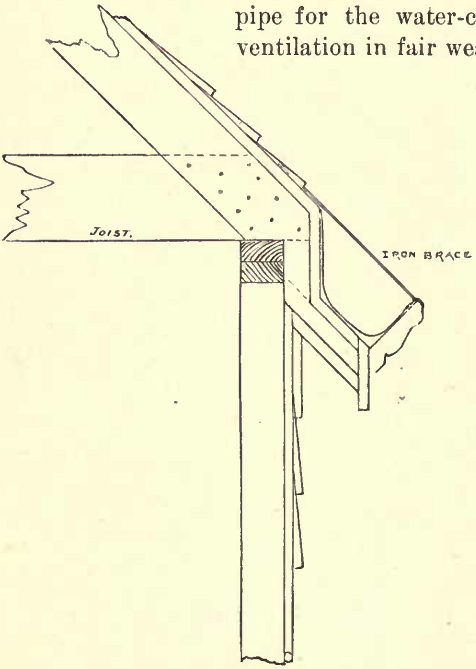


FIG. 1428.

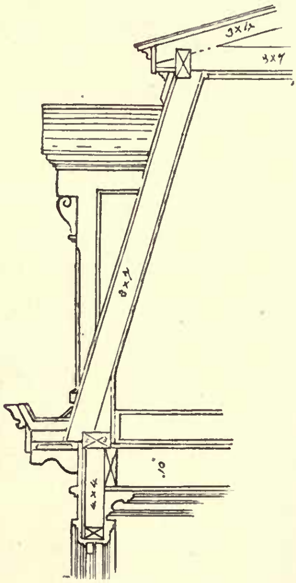


FIG. 1429.

Fig. 1428 is a gutter of a cottage roof.

Fig. 1429 is the section of a Mansard roof, showing the side elevation of a dormer-window, with the gutter below its sill.

The sheet-metal forming the gutter must extend well up or back beneath the shingles or felt, or be soldered to the tin of the roof, to prevent water finding its way into the interior; and at the sides *flashings* of tin must extend on the walls above the roof and into the joints of the brick.

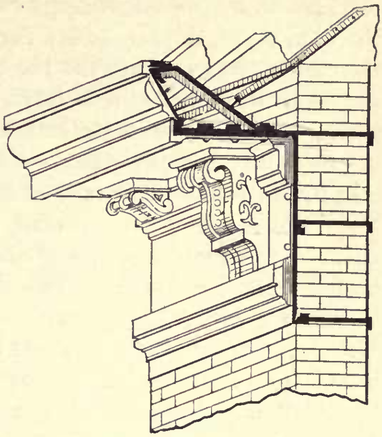


FIG. 1430.

Sheet-metal cornices, at their first introduction, were put up on wood lookouts, cut to the form of the cornice, but it is now the practice to use metal supports and fastenings to the entire exclusion of wood—in many cases cheaper and always safer against fire and rotting, but the iron used must be protected against rust by galvanizing or heavy coats of paint or by both. Fig. 1430 shows a section of a galvanized cornice with bar-iron frames anchored to the wall and roof and riveted to the cornice; the joints of the cornice should be both riveted and soldered. The pitch of the gutter is secured by variations in the bend of the roof braces.

Plastering.—To prevent damp striking through the plastering of outer walls, and cracks in ceilings, it is usual to *fur* walls and beams; that is, to nail

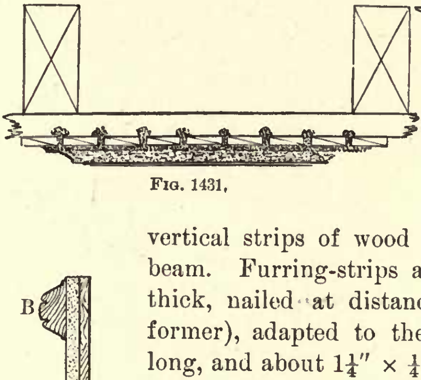


FIG. 1431.

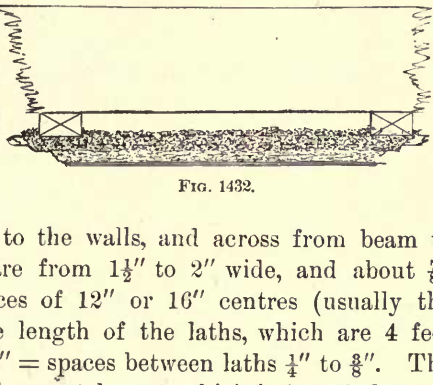


FIG. 1432.

vertical strips of wood to the walls, and across from beam to beam. Furring-strips are from $1\frac{1}{2}''$ to $2''$ wide, and about $\frac{7}{8}''$ thick, nailed at distances of $12''$ or $16''$ centres (usually the former), adapted to the length of the laths, which are 4 feet long, and about $1\frac{1}{4}'' \times \frac{1}{4}''$ = spaces between laths $\frac{1}{4}''$ to $\frac{3}{8}''$. The first coat of mortar is the scratch-coat, which is forced through the interstices between the laths, to make a lock to retain it.

FIG. 1433.

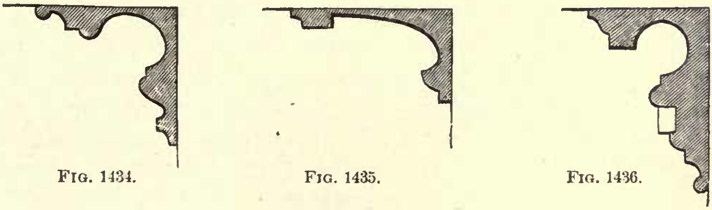


FIG. 1434. FIG. 1435. FIG. 1436.

This coat is about $\frac{1}{4}''$ thick. The next or brown coat is about $\frac{1}{2}''$ thick, and if the last coat is a sand-finish, it will be less than $\frac{1}{8}''$ thick; while, if the last coat is a hard finish, its thickness will be almost imperceptible. Figs. 1431 and 1432 are sections of furring and plastering.

The brown coat is usually carried down to the floor. Over this is nailed the base-board, A (Fig. 1433), for the finish around the bottom of the walls of the room. To guard against the crack formed between the floor by the shrinking of the base, a tenon is formed along the outer edge of the latter with a groove for its reception in the floor. This is termed dadoing. Above the base is a moulding forming a part of the base; above this, there may be a moulded rail, B, called the chair-rail, or surbase, and between a panel, termed a *dado*. The walls of stores are generally ceiled up as high as the surbase. For the finish of the angle of the wall and ceiling, it is usual in the better rooms to form a cornice in plaster. The cornices are mouldings of varied forms, with or without *enrichments*—that is, plaster ornaments. Figs. 1434, 1435, and 1436 are sections of cornices. If the rooms are low, the cornice should extend but little on the wall, but well out on the ceiling.

In all architectural finish *mouldings* are a necessity, the simpler forms of which are taken from Greek or Roman examples.

Greek and Roman Mouldings.—The regular Greek mouldings are eight in number: the Fillet or Band, Torus, Astragal or Bead, Ovolo, Cavetto, Cyma Recta or Ogee, Cyma Reversa or Talon, and Scotia.

The *fillet* (*a*, Fig. 1437) is a small rectangular member, on a flat surface, whose projection is usually made equal to its height.

The *torus* and *astragal* are semicircles in form, projecting from vertical diameters, as in Fig. 1438. The astragal is distinguished from the torus in the same order by being made smaller. The torus is generally employed in the bases of columns; the astragal, in both the base and capital.

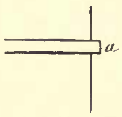


FIG. 1437.

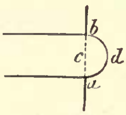


FIG. 1438.

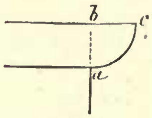


FIG. 1439.

The *ovolo* is a member strong at the extremity, and intended to support. The Roman ovolo consists of a quadrant or a less portion of a circle (Fig. 1439). The Greek ovolo is elliptic.

To describe the Greek ovolo (Fig. 1440): Draw *d f* from the lower end of the proposed curve, at the required inclination; draw the vertical *g e f* to define the projection, the point *e* being the extreme point of the curve. Draw *e h* parallel to *d f*, and draw the vertical *d h k*, such that *d h* is equal to *h k*. Divide *e h* and *e f* into the same number of equal parts; from *d* draw straight lines to the points of division in *e f*, and from *k* draw lines through the divisions in *e h* to meet those others successively. The intersections so found are points in the curve, which may be traced accordingly.

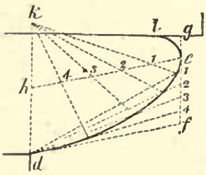


FIG. 1440.

The *cavetto* is described like the Roman ovolo—by circular arcs, as shown in Figs. 1441 and 1442. Sometimes it is composed of two circular arcs united (Fig. 1443); set off *b e*, two thirds of the projection, draw the vertical *b d* equal to *b e*, and on *d* describe the arc *b i*. Join *e d* and produce it to *p*; draw *i n* perpendicular to *e d*, set off *n o* equal to *n i*, and draw the horizontal line *o p* meeting *e p*; on *p* describe the arc *i o* to complete the curve.

The *ogee*, or cyma recta (Fig. 1444), is compounded of a concave and a convex surface. Join *a* and *b*, the extremities of the curve, and bisect *a b* at *c*; on *a* and *c*, as centres, with the radius *a c*, describe arcs cutting at *d*; and on *b* and *c*, describe arcs cutting at *e*. On *d* and *e*, as centres, describe the arcs *a c*, *c b*, composing the moulding.

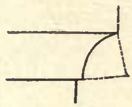


FIG. 1441.

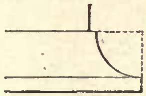


FIG. 1442.

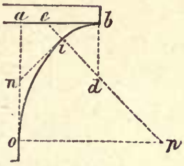


FIG. 1443.

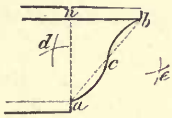


FIG. 1444.

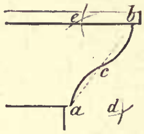


FIG. 1445.

The cyma reversa, or talon (Fig. 1445), is a compound curve, distinguished from the ogee by having the convex part uppermost.

If the curve be required to be made quicker, a shorter radius than *a c* must be employed. The projection of the moulding *n b* (Fig. 1444) is usually equal to the height *a n*.

To describe the Greek talon: Join the extreme points *a*, *b* (Fig. 1446); bisect *a b* at *c*, and on *a c* and *c b*, describe the semicircles *b d c* and *c a*. Draw perpendiculars *d o*, etc., from a number of points in *a c* and *c b*, meeting the circumferences; and from the same points set off horizontal lines equal to the respective perpendiculars: *o n* equal to *o d*, for example. The curve line *b n a*, traced through the ends of the lines, will be the contour of the moulding.

To describe a scotia: Divide the perpendicular *a b* (Fig. 1447) into three equal parts, and with the first, *a e*, for radius, on *e* as a centre, describe the arc *a f h*, in the perpendicular *c o* set off *c l* equal *a e*, join *e l*, and bisect it by the perpendicular *o d*, meeting *c o* at *o*, on the centre *o*, with *o c* for radius, complete the figure by the arc *c h*.

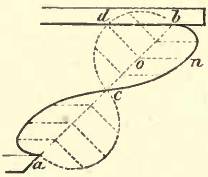


FIG. 1446.

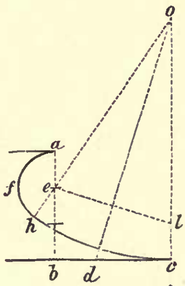


FIG. 1447.

These mouldings, and combinations of them, are *stuck* in wood, and are to be purchased in every variety. Fig. 1448 represents some of the common forms always to be had, and of suitable sizes.

Proportions and Distribution of Rooms and Passages.—Rooms of dwelling-houses are to be proportioned and arranged according to the necessities of position and use, the space that can be occupied, the financial means available, and often to suit the peculiar wishes of owners or occupants. In cities, the limits of the lot restrict the arrangements to a small ground-space, and require an increase in the number of stories. Use has established certain forms often peculiar to different cities, beyond which there is little change; but in the country, where there is plenty of ground-space, and where many stories are usually injurious to the æsthetic effect, and where there are few canons in archi-

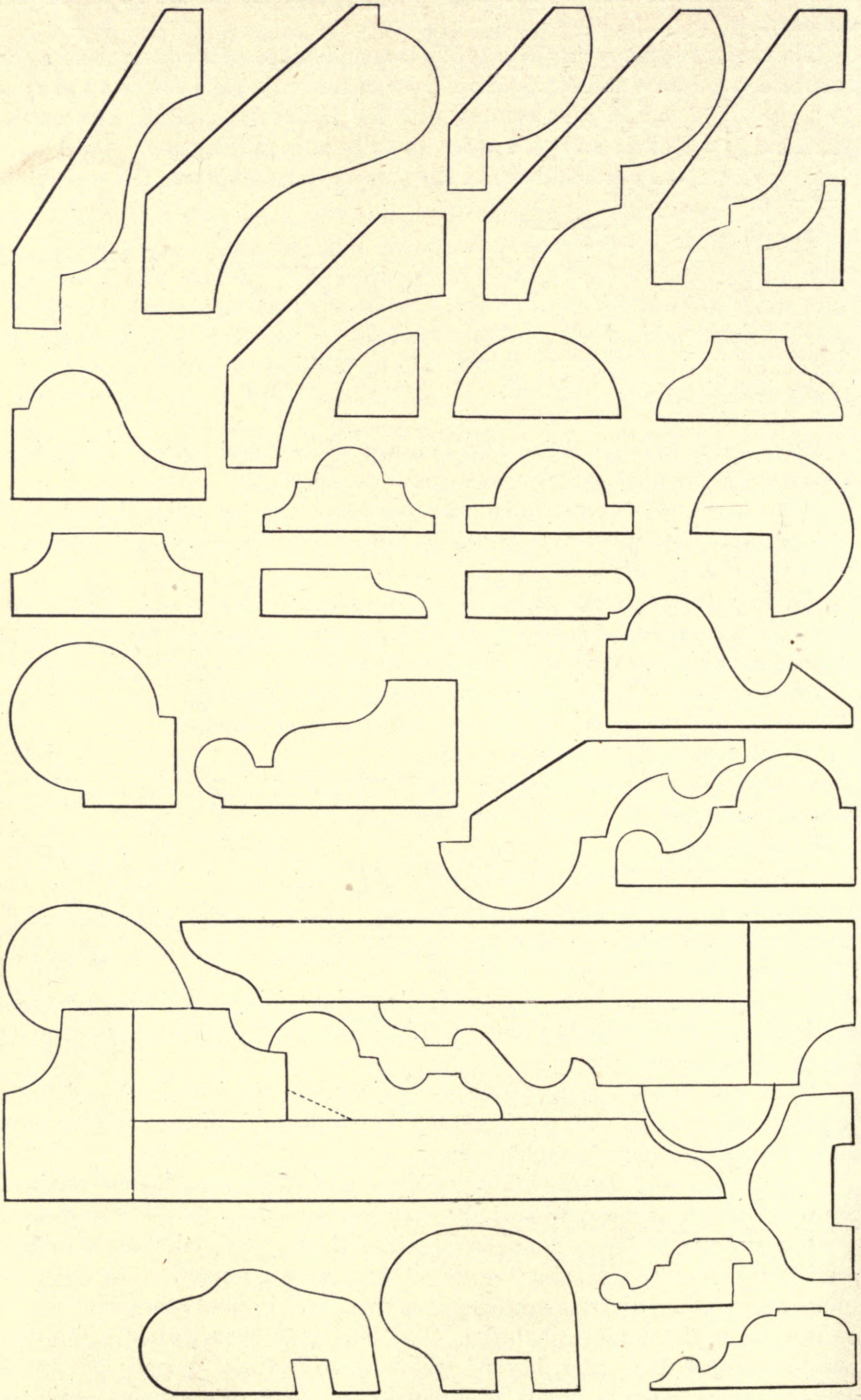


Fig. 1448.

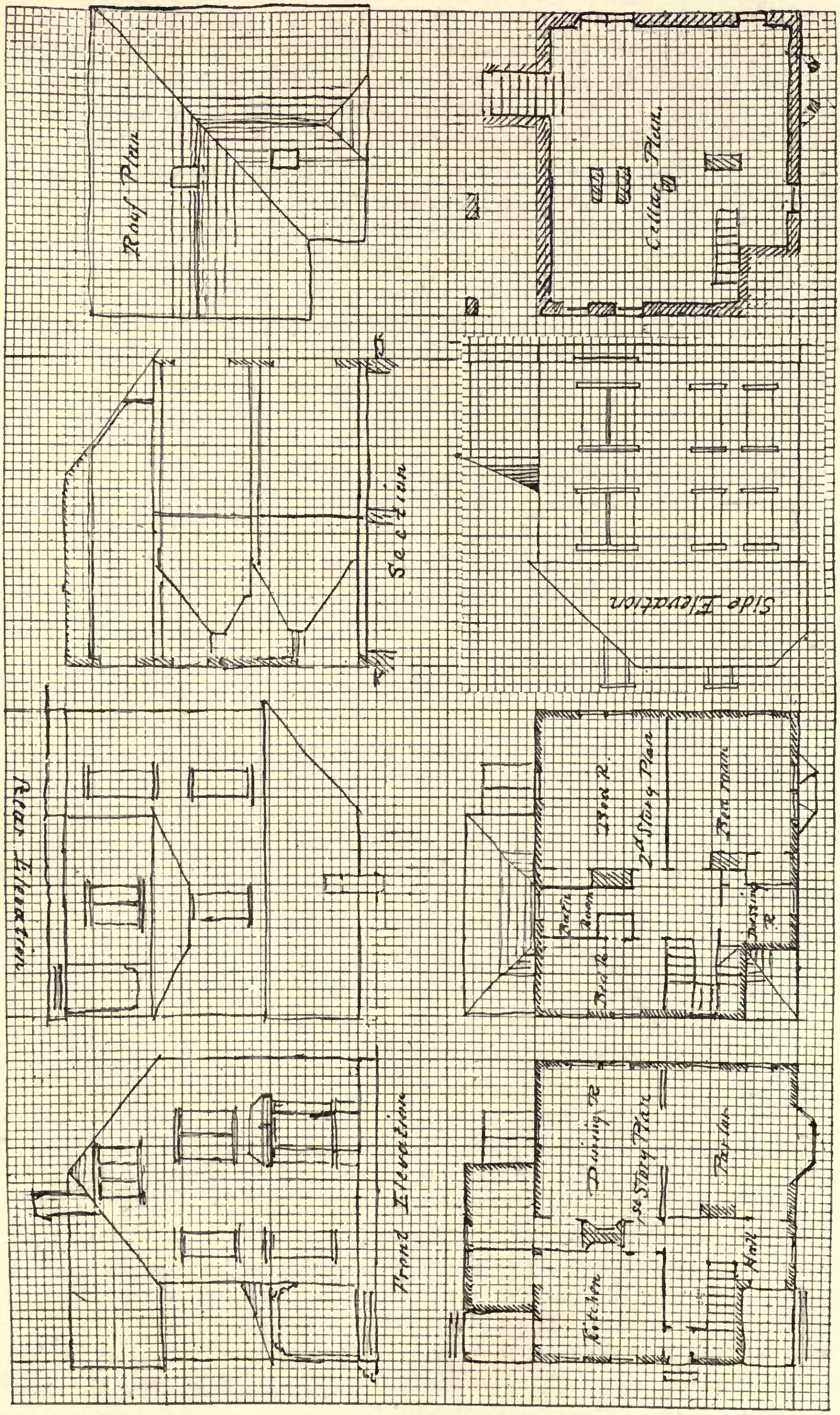
tecture to be observed, there is little limit to the variety of forms and arrangements of country-houses.

In designing a country-house, where one is not restricted in space, it is often convenient to mark out the rooms of the desired size on slips of paper, according to some scale, then cut them out and arrange them in as convenient an order as possible, and modify the arrangement by the necessities of construction and economy. Thus, the greater the outside surface in proportion to the included area, and the greater the number of chimneys and space used for passages, the greater the cost. The kitchen should be of convenient access to the dining-room, both should have large and commodious pantries, and all rooms should have an access from an entry, without being compelled to pass through other rooms; this is particularly applicable to the communication of the kitchen with the front door. Outside doors for common and indiscriminate access should not open into important rooms. All rooms to be occupied as living or bedrooms should have flues opening directly or indirectly to the outer air for ventilation.

As to the size of the different rooms, they must of course depend on the purposes to which they are to be applied, the class of house, and the number of occupants. The *kitchen* for the poorer class of houses is also used as an eating-room, and should therefore be of considerable size to answer both purposes; for the richer houses, size is necessary for the convenience of the work. In the older New York private houses the average was about 16 × 20 feet; for medium houses in the country generally less, say 12 × 16. A back kitchen, scullery, or laundry, should be attached to the kitchen, and may serve as a passage-way out.

The Dining or Eating Rooms.—The common width of dining-tables varies from 4 to 5 feet 6 inches; the space occupied by the chair and person sitting at the table is about 18 inches; the table-space, for comfort, should be not less than 2 feet for each person at the sides of the table, and considerable more at the head and foot; hence the space that will be necessary for the family and its visitors at the table may be calculated. Allow a further space of 2 feet at each side for passages, and some 3 to 5 at the head for the extra tables or chairs, for the minimum of space required; but, if possible, do not confine the dining-room to meagre limits, unless for very small families; let not the parties be lost in the extent of space, nor let them appear crowded.

The parlours should be made according to the rules given below, not square, but the length about once and a half the width; if much longer than this, break up the walls by transoms or projections. As to the particular dimensions, no rules can be given; they must depend on every person's taste and means; 20 × 16 is a very fair size for a regular living-room parlour, not a drawing-room. The same size is ample for a *sleeping-room.* The usual width of single beds is 2′ 8″ to 3′ 6″; of three-quarter, 4′ to 4′ 6″; of whole, 5 to 6 feet; the length, 6′ 6″; and as the other furniture may be made to consist of but very few pieces, if adequate means of ventilation are provided parties may live snugly in small quarters. The bed should not stand too near the fire, nor between two windows; its most convenient position is head against an interior wall, with a space on each side of at least 2 feet. To the important bedrooms of first-class houses, dressing-rooms should be attached, and, if there is water



FIGS. 1449-1456.

and sewer service, fitted with set bowls and baths and water-closets. If possible, there should be windows opening to the outer air, but always with flue-ventilation.

Pantries.—Closets for crockery should not be less than 14 inches in depth in the clear; for the hanging up of clothes, not less than 18 inches, and should be attached to every bedroom. For medium houses, the closets of large sleeping-rooms should be at least 3 feet wide, with hanging-room, and drawers and shelves. There should also be blanket-closets, for the storing of blankets and linen; these should be accessible from the entries, and may be in the attic. Store-closets should also be arranged for groceries and sweetmeats.

Passages.—Front entries are usually 6 feet wide in the clear; common passage-ways, 3 feet; these are what are required, but ample passages give an important effect to the appearance of the house. The width of principal stairs should be not less than 3 feet, and all first-class houses, especially those not provided with water-closets and slop-sinks on the chamber-floor, should have two pairs of stairs, a front and a back pair; the back stairs need not necessarily be over 2 feet 6 inches in width.

The Height of Stories.—It is usual to make the height of all the rooms on each floor equal; it can be avoided by furring down, or by the breaking up of the stories, by the introduction of a mezzonine or intermediate story over the smaller rooms. Both remedies are objectionable.

The average height of the stories for common city dwellings is: Cellar, 6 feet 6 inches; common basement, 8 to 9 feet; English basement, 9 to 10 feet; principal story, 12 to 15 feet; first chamber floor, 10 to 12 feet; other chamber-floors, 8 to 10 feet—all in the clear. For country-houses, the smaller of the dimensions are more commonly used. Attic stories are sometimes but a trifle over 6 feet in height, but are objectionable.

Privies, Water-Closets, and Out-Houses.—The size of privies must depend greatly on the uses of the building to which they are to be attached, its position, and the character of its occupants. Allowing nothing for evaporation and absorption, the entire space necessary for the excrementitious deposits of each individual, on an average, will be about seven cubic feet for six months, of which seven eighths is fluid. In the country, vaults are usually constructed of dry rubble-stone, and the fluid matters are expected to be filtered through the earth, the same as in cesspool-waste; but great care must be taken that they neither vitiate the water-supply nor the air of the house. A brick and cement vault, air and water tight, with a ventilating-pipe into a hot chimney-flue, is the best preventive, and may even be built within the house. In all other cases there should be free air-space between the house and privy. In the city, where there is adequate water-supply and sewerage, the water-closet should be adopted. The water-closet, or privy, with a single seat, should occupy a space not less than $4' \times 2'\,6''$. The rise of seat should be about $17''$ high; and the hole egg-shaped, $11'' \times 8''$. The earth-closet, when properly taken care of, is an extremely useful appendage to a country-house, and the space requisite for it is the same as that of a water-closet.

The forms of modern water appliances, and the means to get rid of house-waste, will be illustrated hereafter, under the heads of Ventilation and Plumbing.

For Wood or Coal Sheds or Bins.—In estimating the size of these accessories, it may only be necessary to state that a cord of wood contains 128 cubic feet, and a ton of anthracite egg coal occupies a space of about 40 cubic feet, of bituminous coal about 45 cubic feet, of coke from 45 to 50 cubic feet.

On the Size and Proportion of Rooms in general.—"Proportion and ornament," according to Ferguson, "are the two most important resources at the command of the architect, the former enabling him to construct ornamentally, the latter to ornament his construction." A proportion to be good must be modified by every varying exigence of a design; it is of course impossible to lay down any general rules which shall hold good in all cases; but a few of its principles are obvious enough. To take first the simplest form of the proposition, let us suppose a room built, which shall be an exact cube—of say 20 feet each way—such a proportion must be bad and inartistic; and, besides, the height is too great for the other dimensions. As a general rule, a square in plan is least pleasing. It is always better that one side should be longer than the other, so as to give a little variety to the design. Once and a half the width has been often recommended, and with every increase of length an increase of height is not only allowable, but indispensable. Some such rule as the following meets most cases: "The height of the room ought to be equal to half its width plus the square root of its length"; but if the height exceed the width the effect is to make the room look narrow. Again, by increasing the length we diminish, apparently, the other two dimensions. This, however, is merely speaking of plain rooms with plain walls; it is evident that it will be impossible, in any house, to construct all the rooms and passages to conform to any one rule of proportion, nor is it necessary, for in many rooms it would not add to their convenience, which is often the most desirable end; and, if required, the unpleasing dimensions may be counteracted by the art of the architect, for it is easy to increase the apparent height by strongly marked vertical lines, or bring it down by horizontal ones. Thus, if the walls of two rooms of the same dimensions be covered with the same strongly marked striped paper, in one case the stripes being vertical and in the other horizontal, the apparent dimensions will be altered very considerably. So also a deep, bold cornice diminishes the apparent height of a room. If the room is too long for its other dimensions, this can be remedied by breaks in the walls, by the introduction of pilasters, etc. So also, as to the external dimensions of a wall, if the length is too great it is to be remedied by projections, or by breaking up the lengths into divisions.

Understanding the general necessities of a dwelling, the proportions of rooms, forms of construction, and space to be occupied, the draughtsman is prepared to undertake designing, and for this purpose cross-section paper will be found of very great use. Taking the side of a small square as a unit—one foot, for instance—he can readily pencil in rooms and passages, and alter and modify at pleasure.

Figs. 1445 to 1456 are illustrations of this form of designing. Partitions are to be as much as possible one over the other, and the posts or walls arranged in the cellar, for the support of these lines of partitions. For the sketch, it is sufficient to make door and window openings 3 feet, unless for some particular purpose double-fold doors or bow or mullioned windows are required. In arranging the stairs, the whole run may be taken as 1½ time the height of the story between floor and floor; as square landings have one riser,

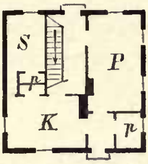


Fig. 1457.

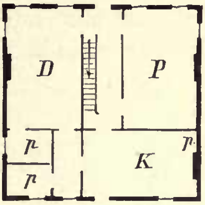


Fig. 1458.

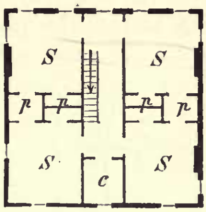


Fig. 1459.

the rise will be equal to the square less one tread, say, for the design 1 foot. The length of opening in the frame, say 12 feet for a straight run.

Figs. 1447 to 1472 are plans of familiar forms of houses, drawn to the scale of 32 feet to the inch, as illustrations to the student and as examples to be copied on a larger scale. The same letters of reference are used on all the plans. Thus, K K designate kitchens, cooking-rooms, or laundries; D D eating-rooms; S S sleeping-rooms; P P drawing-rooms, parlours, or libraries; *p p* pantries, china or store closets, or clothes-presses; *c c* water-closets and bath-rooms. These last are not shown in the plans of country houses, but are recognised as a necessity in the best of this class. The space occupied by them is given on page 593.

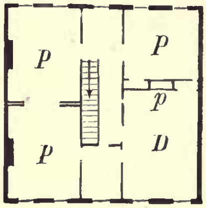


Fig. 1460.

Figs. 1457, 1458, and 1460 are first-story plans of houses of square outline. Fig. 1459 is the second story of Fig. 1458.

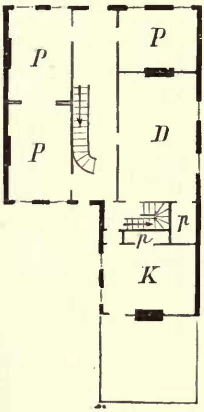


Fig. 1461.

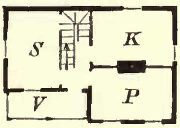


Fig. 1462.

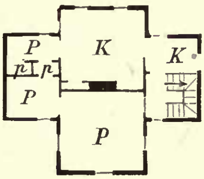


Fig. 1463.

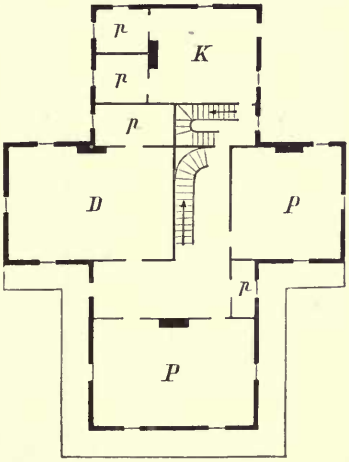


Fig. 1464.

This form of house has the greatest interior accommodations for the outside cover, and, although not picturesque in its elevation, is a very convenient and economical structure. The kitchen (Fig. 1460) is in the basement, and the connection with the dining-room is by a dumb-waiter in the pantry (*p*). In Fig. 1461 the plan is the same as in Fig. 1460, but the kitchen (*K*) is in an L attached to the house; there is a small opening between the pantry (p) and kitchen, through which dishes are passed to and from the dining-room.

Fig. 1462 is the plan of a very small but convenient floor, of prettier outline than the square; V is a portico or veranda. No chimney is shown in the

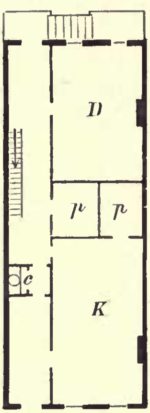


FIG. 1465.

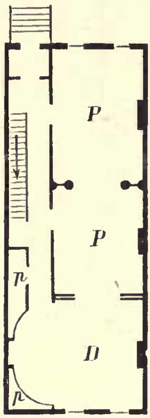


FIG. 1466.

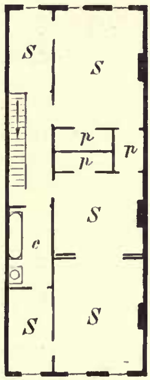


FIG. 1467.

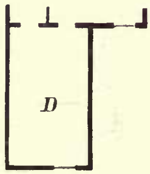


FIG. 1468.

sleeping-room S; there should be one either against the stairs or the back wall.

Figs. 1463 and 1464 are first-story plans of houses still more extensive. All of the above are adapted to the country, dependent on lights on all sides, and ample spaces.

In the cities, houses are mostly confined to one form in their general out-

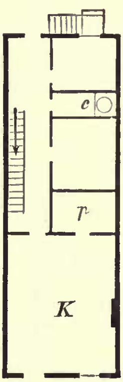


FIG. 1469.

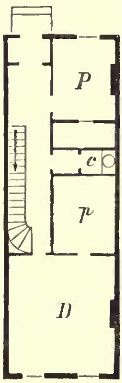


FIG. 1470.

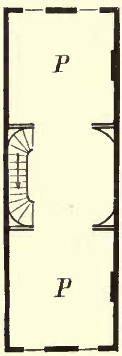


FIG. 1471.

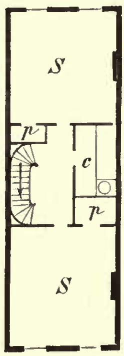


FIG. 1472.

line—a rectangle. Figs. 1465 and 1469 may be taken as the usual type of New York city houses. Figs. 1465, 1466, and 1467 are the basement, first and second floor plans of a three-rooms-deep, high-stoop house as the first floor is reached by an outside flight of steps about 6 feet high. There is usually a cellar beneath the basement, but in some cases there are front vaults, entered beneath the steps to the front door; the entrance to the basement itself is also

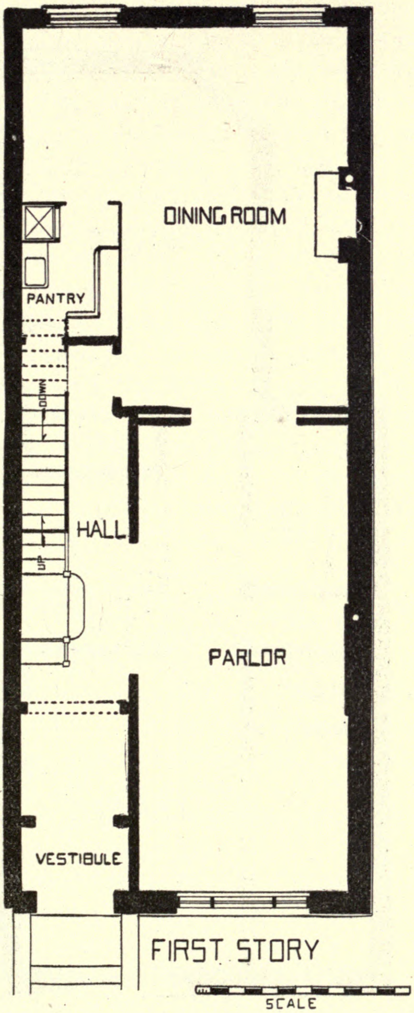


FIG. 1473.

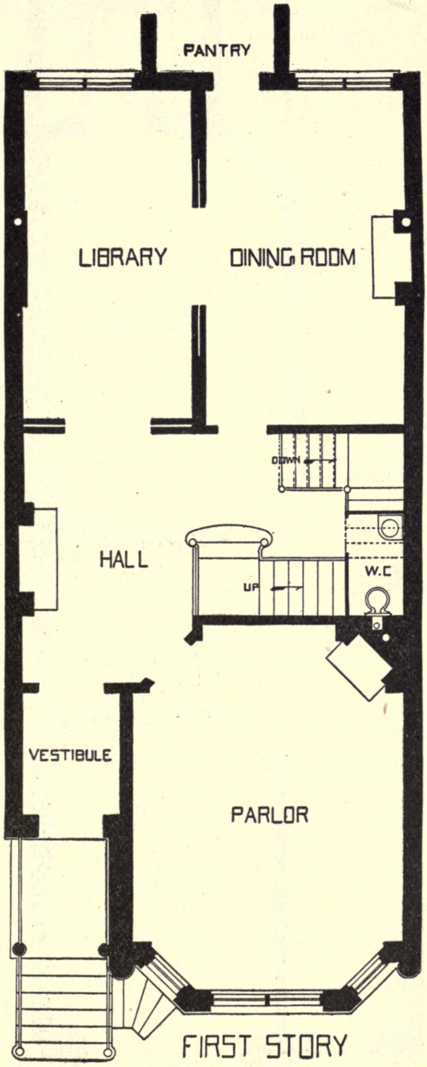
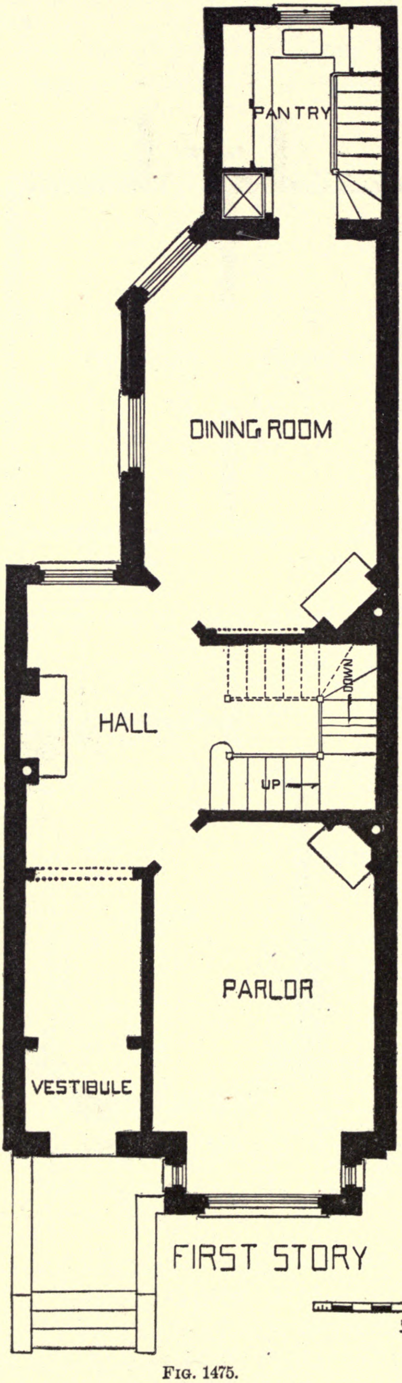


FIG. 1474.



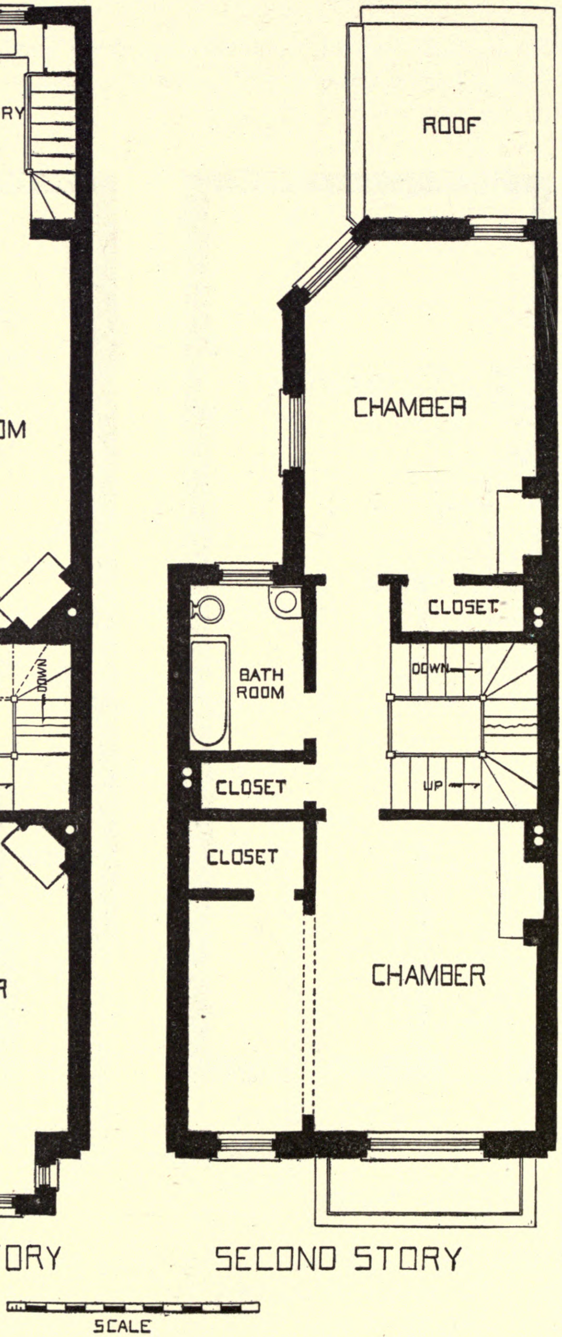


Fig. 1475.

Fig. 1476.

beneath the steps. The front room of the basement may be used as an eating-room, for the servants' sleeping-room, billiards, or library. The usual dining-room is on the first floor, a dumb-waiter being placed in the butler's pantry, *p*, for convenience in transporting dishes to and from the kitchen. The objection to three-rooms-deep houses is that the central room is too dark, being lighted by sash folding-doors between that and the front or rear rooms, or both. Fig. 1468 is a modification to avoid this objection, the dining-room, or tea-room, as it is generally called, being built as an L, so that there is at least one window in the central room opening directly outdoors. This was an old fashion here, and has lately been revived.

Figs. 1469 to 1470 are plans of the several floors of an English-basement house, so called, distinguished from the former in that the principal floor is up one flight of stairs. The first story or basement is but one or two steps above the street, and contains the dining-room, with its butler's pantry and dumb-waiter, a small sitting-room, with, in some cases, a small bedroom in the space in the rear of it. The kitchen is situated beneath the dining-room, in the sub-basement. The grade of the yard is in general some few steps above the floor of the kitchen. Vaults for coal and provisions are excavated either beneath the pavement in front or beneath the yard. The advantages of this form of house are the small reception-room on the first floor, which in small families and in the winter months is the most frequently occupied as a sitting-room of any in the house; the spaciousness of its dining-room and parlours in proportion to the width of the house, which is often but 16 feet 8 inches in width, or three houses to two lots, and not infrequently of even a less width. The objections to the house are the stairs, which it is necessary to traverse in passing from the dining-rooms or kitchen to the sleeping-rooms, but this objection would, of course, lie against any house of narrow dimensions, where floor-space is supplied by height.

In New York, outside access to the kitchen is from the front, as there is no back street or alley. In Philadelphia, where the lots are deeper, and there is a street in the rear, the kitchen is usually in a rear L, on the level of the first floor, with the dining-room above it on a mezzanine or half-story between the first and second floors.

Figs. 1477 to 1483 are plans and elevations of a country-house in the Flemish or Queen Anne style.

Fig. 1486 is the front elevation of a high-stoop house, in New York city, of brown stone, a comparatively old but still popular design.

To accommodate the poor and people of small means in all cities, it was, and to some extent still is, the custom to divide houses which were intended for single families into small apartments for many, or to let rooms singly for this purpose. This was found to be objectionable to both occupants and owners, and houses have been constructed especially for parties of limited means. Virtually, they are now nearly all apartment-houses, each family having distinct rooms or suites to itself. But the term tenement-houses is applied to the cheaper kind of apartments, occupied by the poorer class, and situated in the least expensive localities. The common form of tenement-house consists of two buildings, one in the front and one in the rear of the lot, with an outer or air space between. A hall leads through the first story to the central area; on each side

PLAN OF FIRST FLOOR.

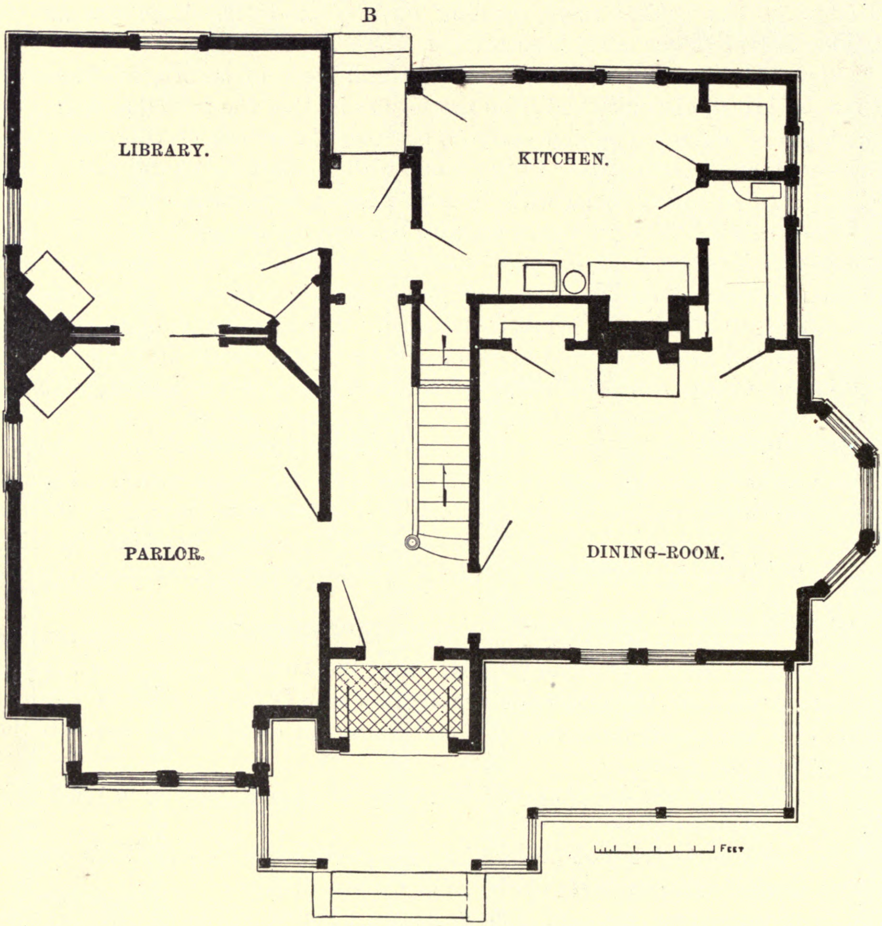


FIG. 1477.

PLAN OF SECOND FLOOR.

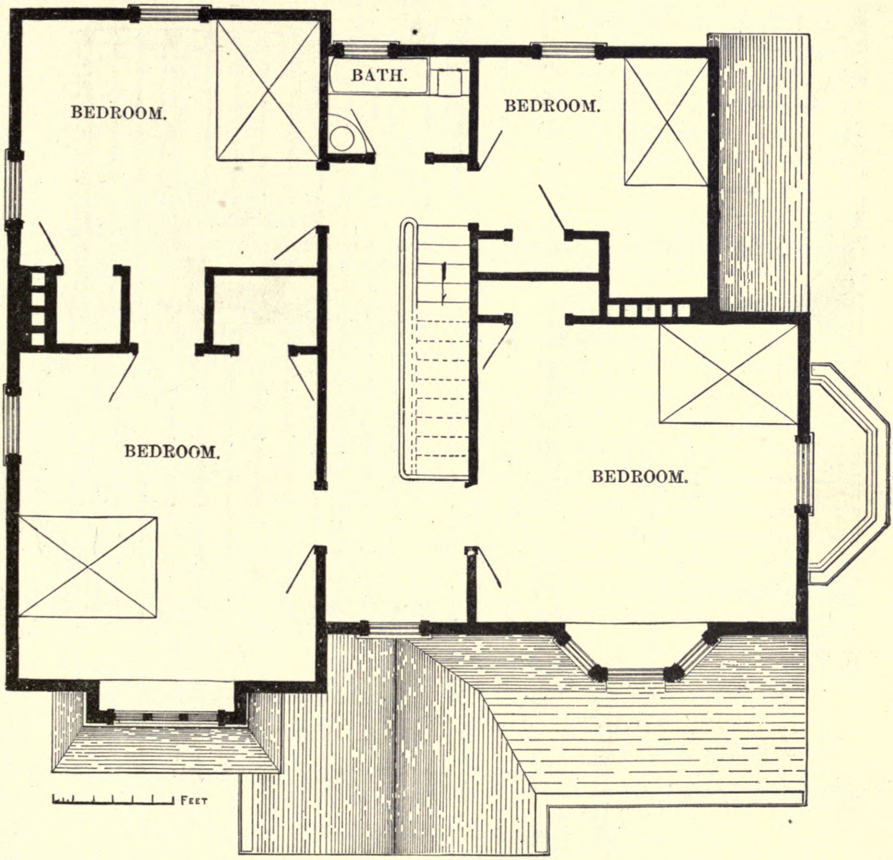


FIG. 1478.

FRAMING-PLAN OF FIRST FLOOR.

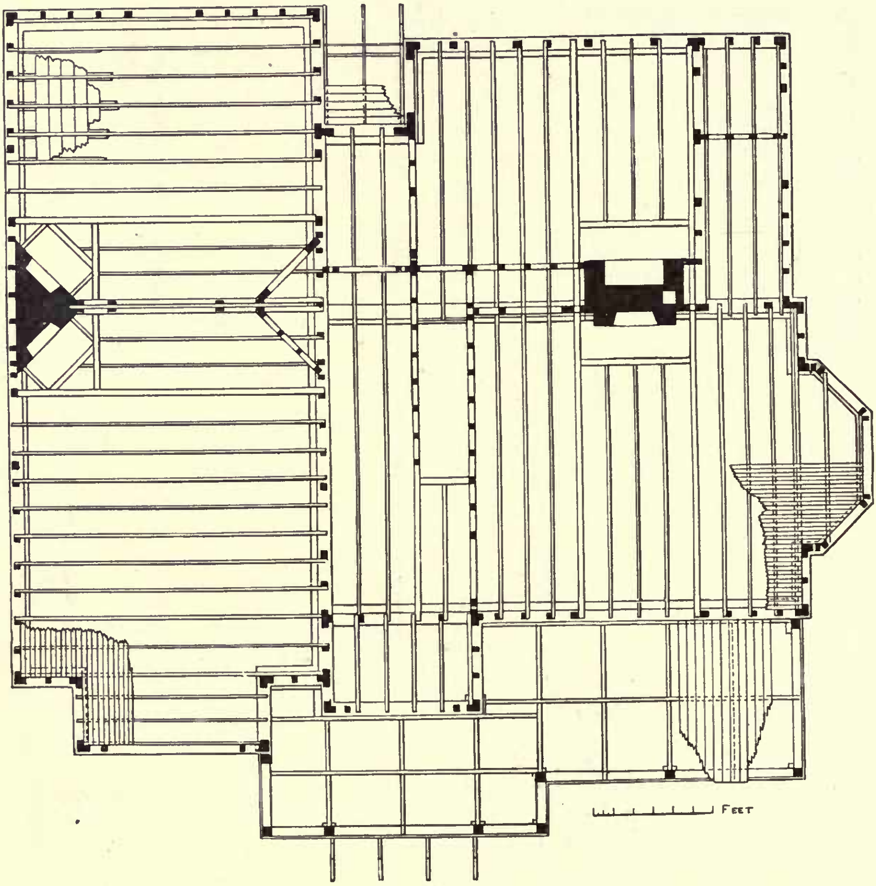


Fig. 1479.

SECTIONAL ELEVATION ON A. B.

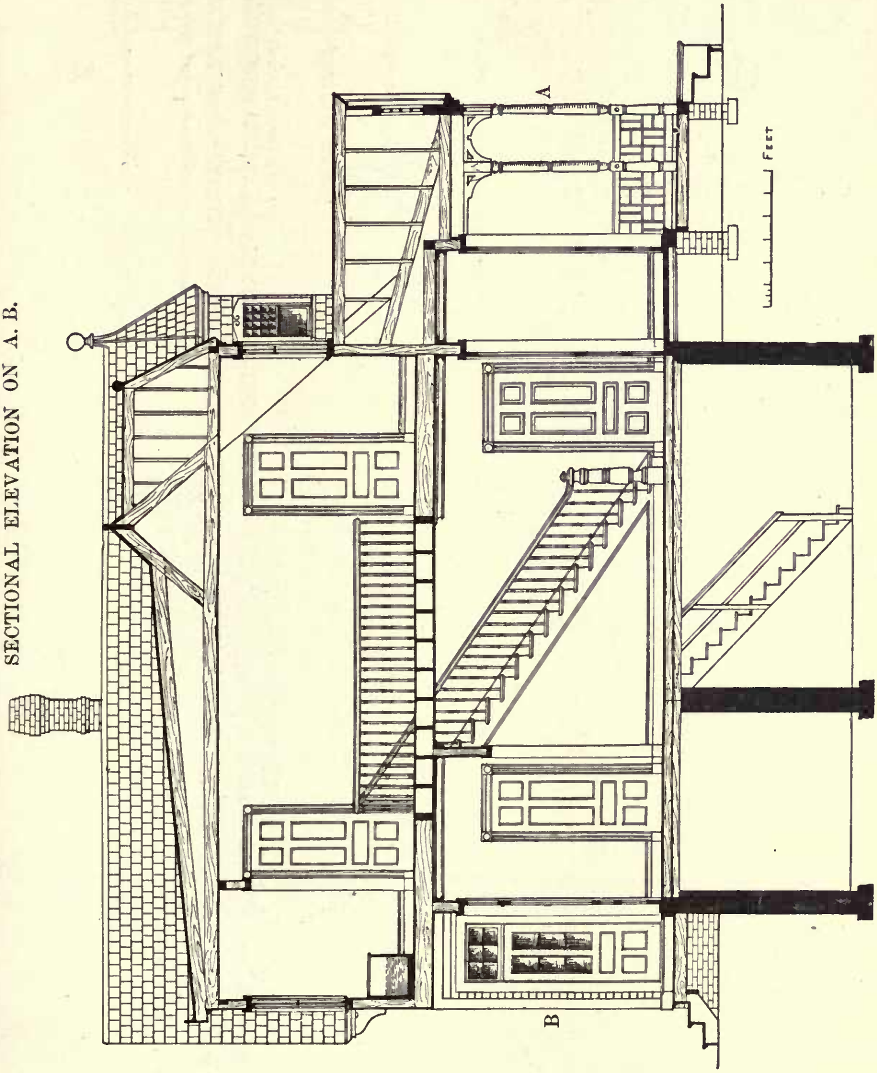


Fig. 1480.

ELEVATION OF CHIMNEY OF DINING-ROOM.

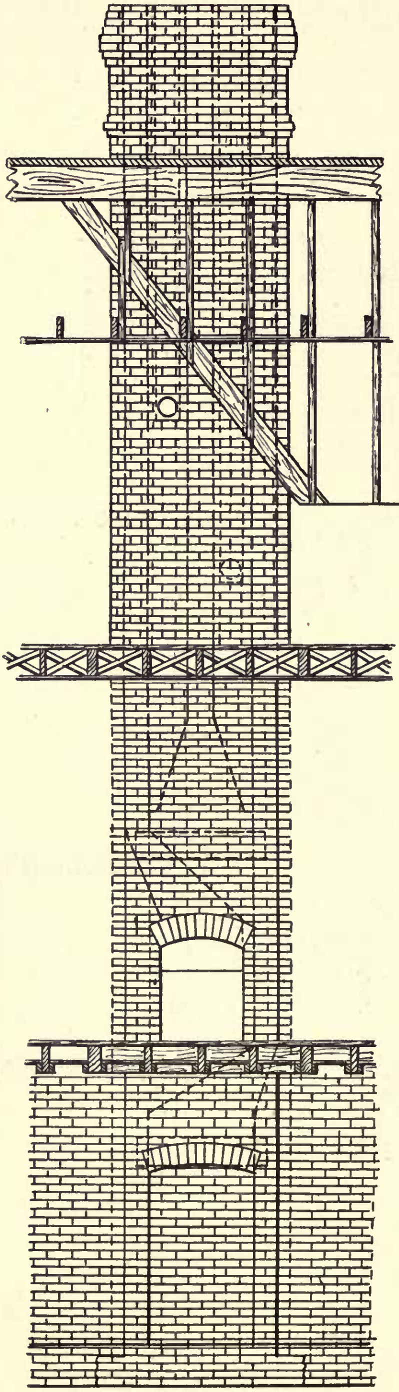


FIG. 1481.

SECTION.

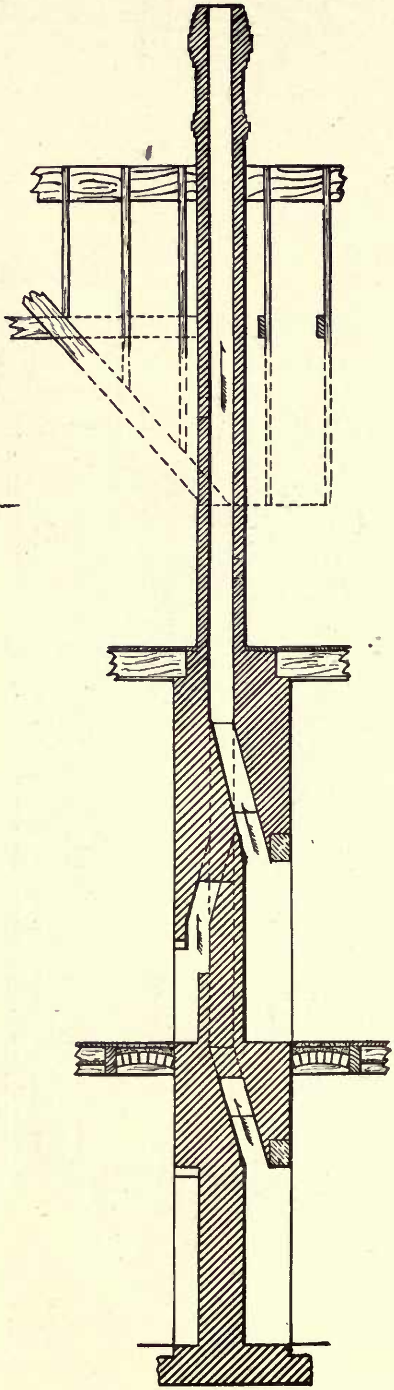


FIG. 1482.

FEET

FRONT ELEVATION.

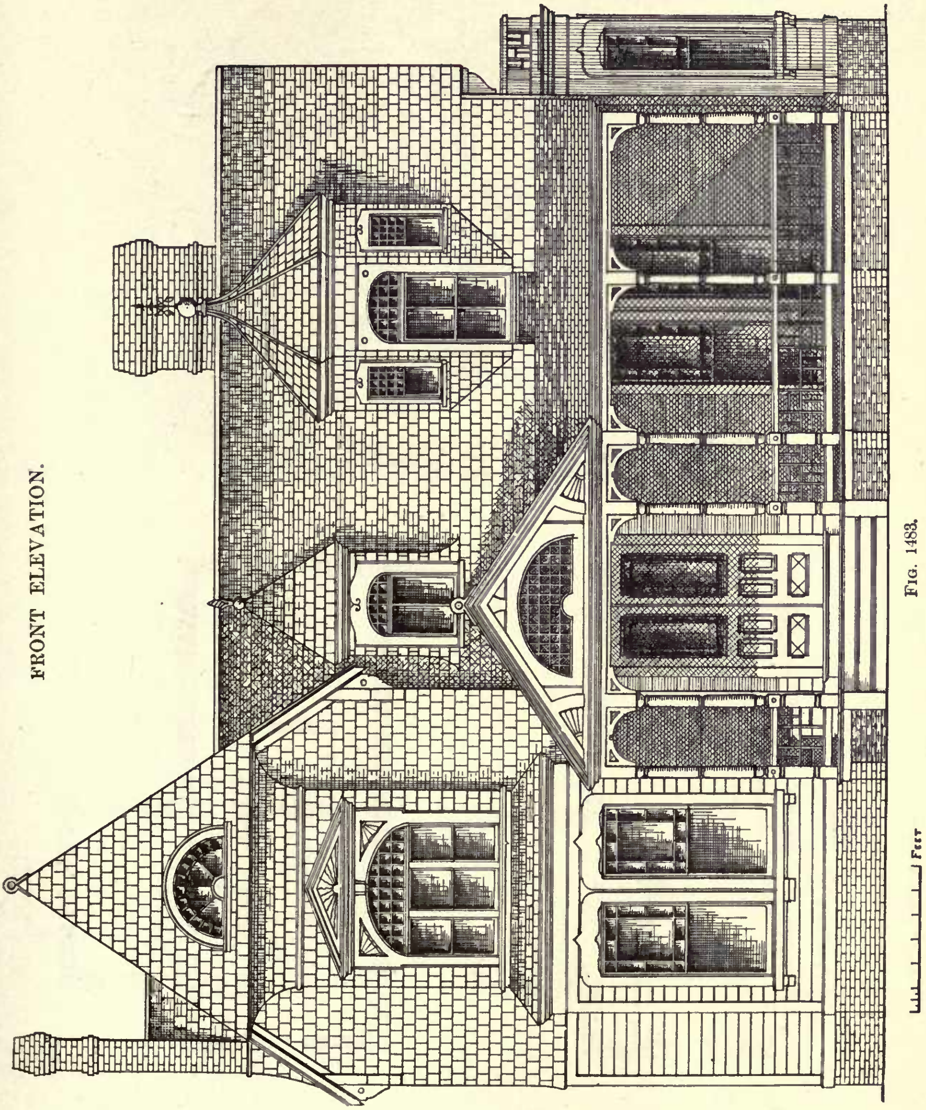


Fig. 1483.

ENGLISH RURAL STYLE.

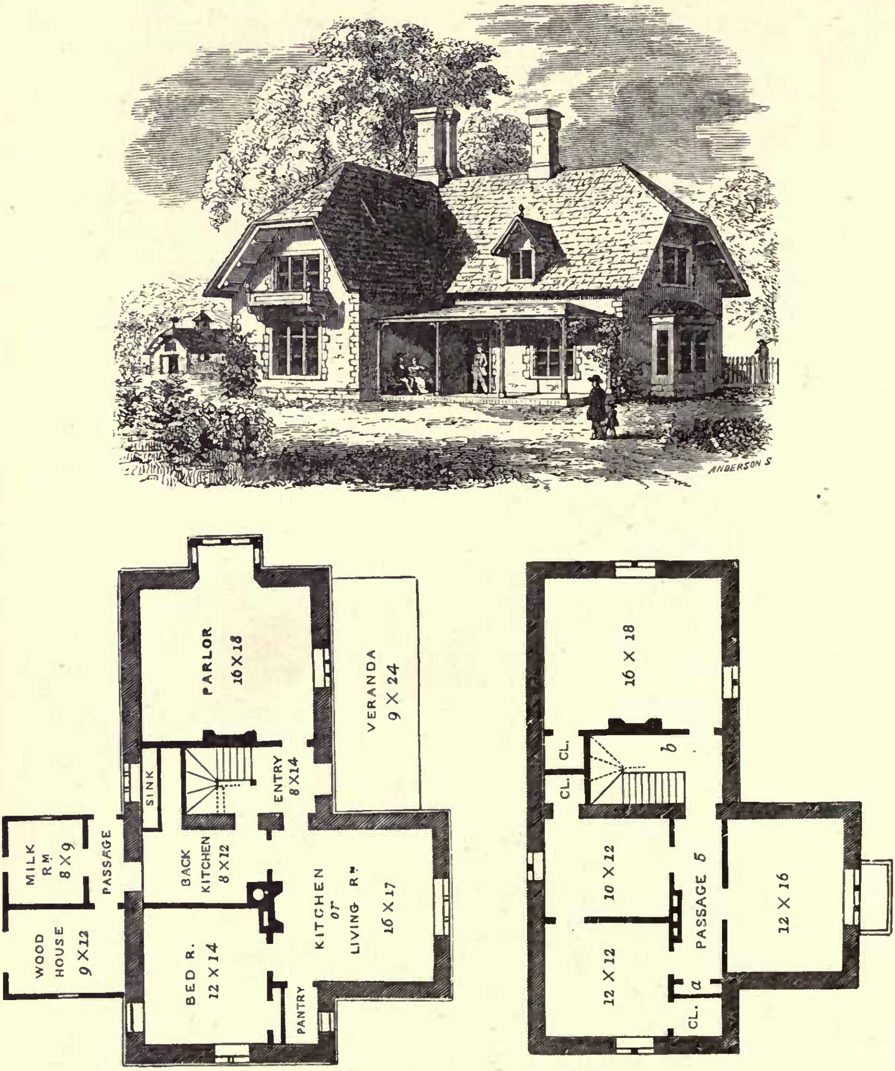


Fig. 1484.

ITALIAN VILLA, BY UPJOHN.

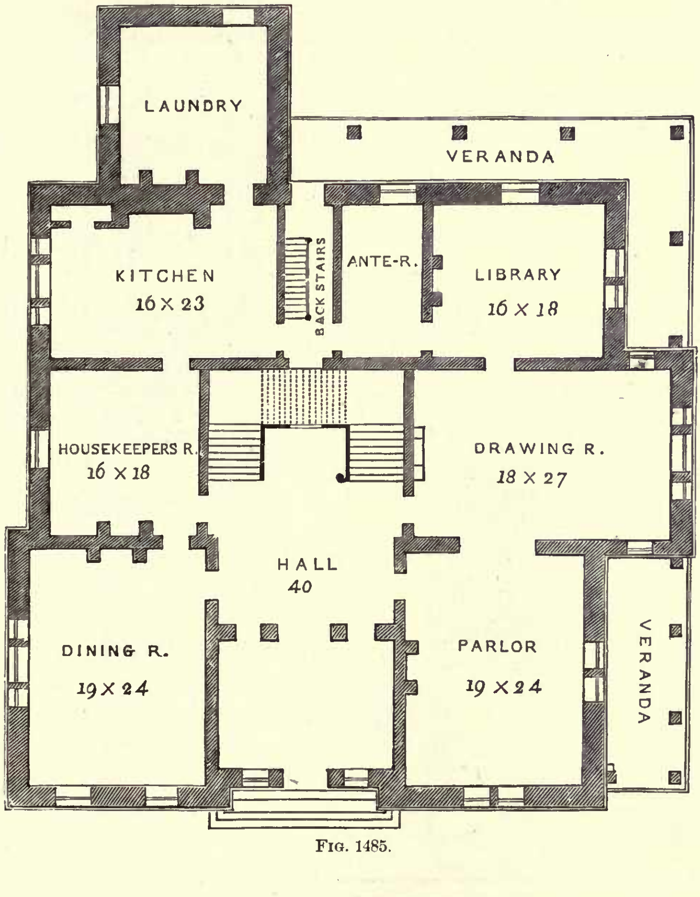


Fig. 1485.

FIG. 1486.

of this hall there may be small stores and apartments. Stairs from the hall lead to the apartments above. The 25 feet is divided in two, making two living-rooms on each front; these are the only rooms opening directly into the outer air. Bedrooms are attached to each of these rooms, but take their light and air from the staircases, or small light-wells. In the rear houses there are two tenements to each story; they take their light and air from the central and back areas. Water-closets are in the central area. These tenements are mostly occupied by work-people, largely of foreign birth, dependent directly on small wages. There is a large class, of limited means, to whom these accommodations are insufficient; parties who can not well afford an entire house, but still wish for the privacy of one. Within the limits of a lot 25′ × 100′ it has been found difficult to secure all the necessaries of light and ventilation, with the number of suites of apartments adapted to the means of the occupants, and satisfactory as an investment to the owners. Fig. 1487 is a plan of one of the best of these designs. It provides for four families

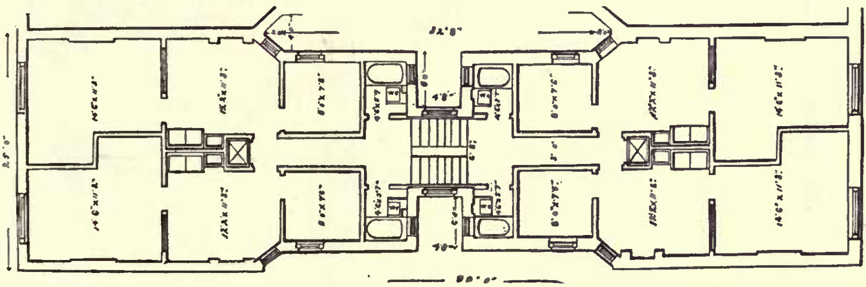


Fig. 1487.

on each story, although it will be observed by the plan of the stairs that the front and rear tenements are not on the same level; they are separated by the half flight of stairs. By means of the cross-shaped court between the adjacent houses, every room, including the bath-room, has a window to the open air. This is the most commendable feature of the plan. It is remarkable, also, however, for providing more conveniences than have been customary in dwellings of this class, as, for instance, a small bath-tub as well as a water-closet for each family, and two wash-tubs as well as a sink; also, a dumb-waiter (common to two families on each level) for bringing up fuel, provisions, etc. The large rooms have recesses for beds, which provide for an extra bedroom, while detracting but little from their value as parlors, as the recess may be curtained off in the daytime, or the bed turned up. The dimensions of the rooms, as marked on the plans, are the average length and breadth. These suites are much too restricted for a very large class, but apartment-houses somewhat on this model are constructed in desirable localities, where the accommodations and conveniences are equal to those of any private house, and not bounded by the limits of a single lot nor single story, many unsurpassed in luxury of finish and appointments.

The larger apartment-houses are often designated as flats. The suites should be supplied with water, gas, and steam heat; should be entirely distinct in their ventilation and protected against fire; some are now lighted by electric light.

Fig. 1488 is an illustration of a "flat" situated on the corner of a street, and one suite takes its light exteriorly from the streets while the other depends

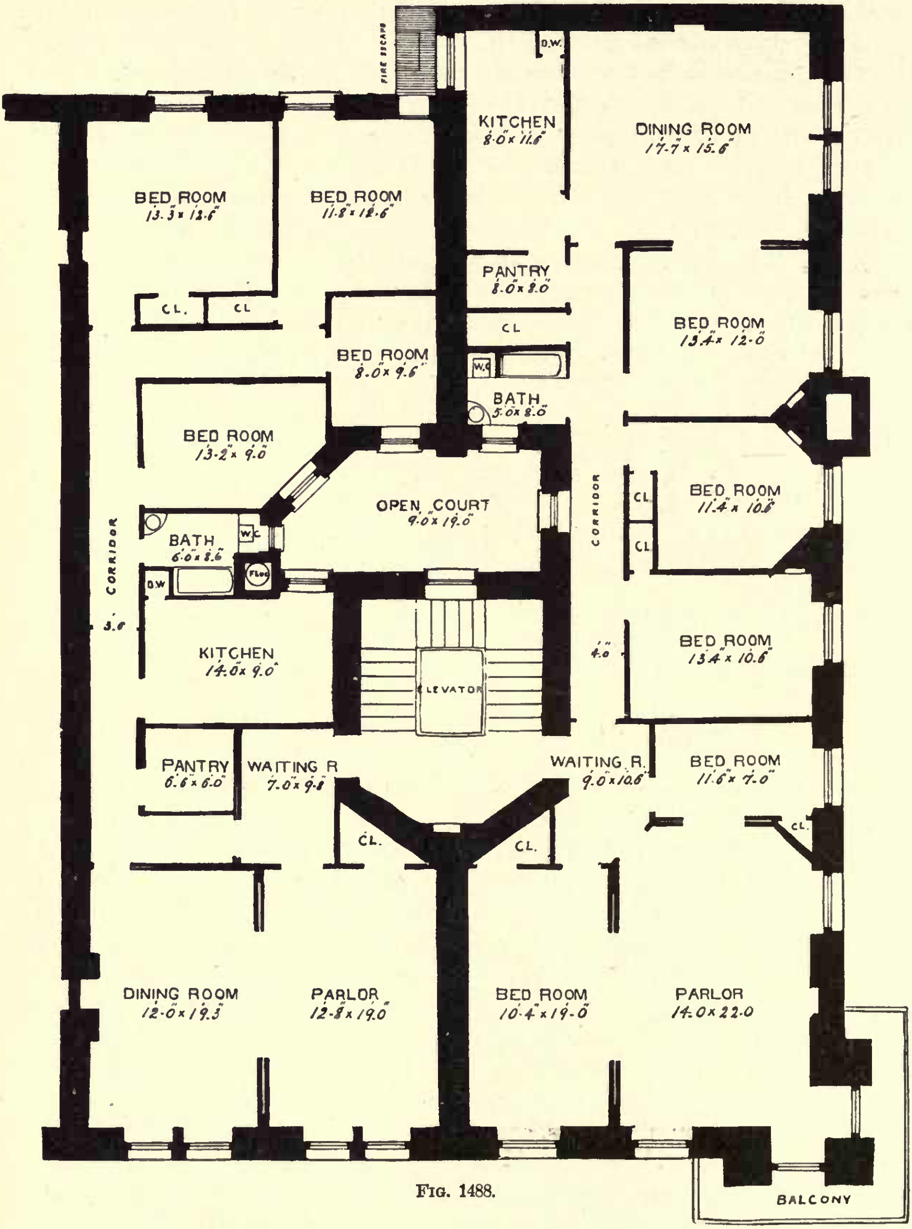


FIG. 1488.

in a measure on the court, with a ventilating passage from the rear beneath the fire-escape grating. Kitchens, in the figure, are attached to the suites; the laundries are in the upper story. Many flats are without kitchens or laundries, and meals are furnished either from without or from restaurants in the building. It then corresponds very nearly to a hotel without transient cus-

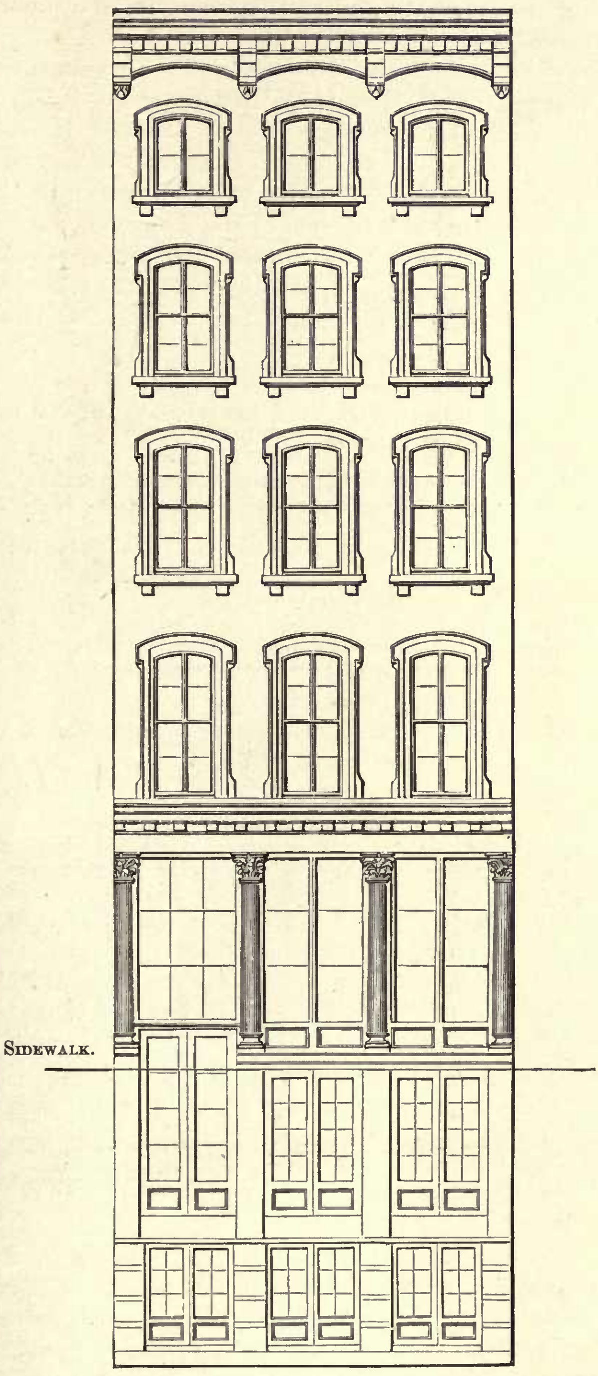


Fig. 1489.

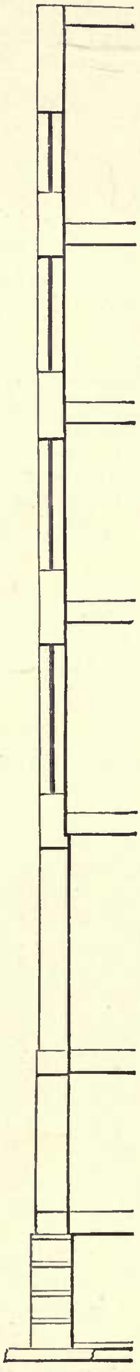


Fig. 1490.

tom, with ample and separate suites. It would seem that boarding-houses might be built on such plans—less extensive in their arrangements and adapted to small families of moderate means; but boarding-houses are almost invariably private houses, but little modified for the more public use.

Stores and Warehouses.—Fig. 1489 is the front elevation of a common type of New York city store, occupying a single lot of 25 feet in width. It will be observed that there are two stories beneath the level of the sidewalk, the basement and sub-cellar, and this construction still obtains largely; but deep basements only are considered preferable by some, with extra stories at the top rather than in the cellar. Fig. 1490 is a section of the front wall, showing heights of stories, which of late years have been increased over former practice, say to 16′ for the first story, 13′ for the second, and 12′ and 11′ for others, the light for the interior being taken almost universally from the front and rear, and sky lights done away with.

Fig. 1491 is a plan of the first-story floor, with basement in front dotted in; five feet of this space, or that usually allotted for areas, is covered with

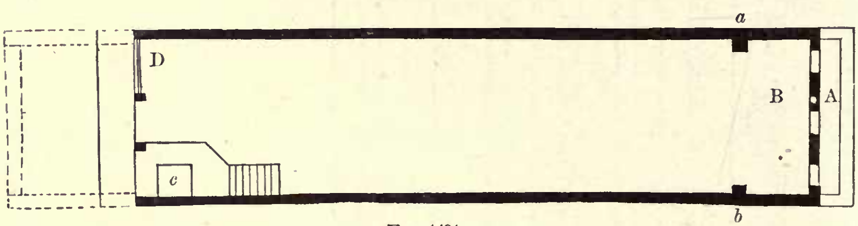


Fig. 1491.

illuminating tile (Fig. 1492), that is, small glass lenses, set in iron frames, the whole water-tight; or lenses on a skeleton frame of cast-iron set in Portland cement. In the extreme rear there is a small area, A, open to the air, of about

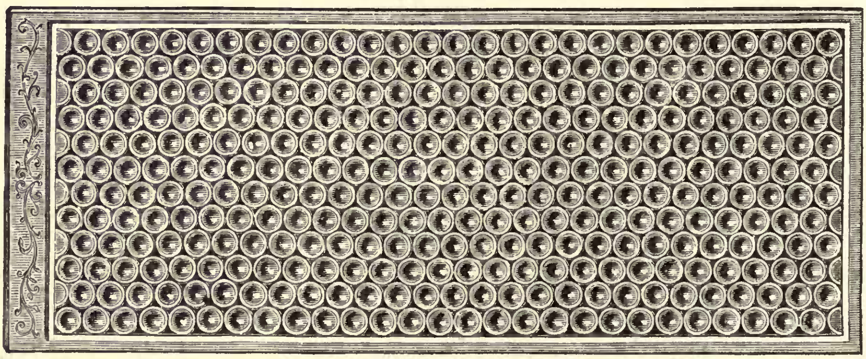


Fig. 1492.

5 feet, for light and air to the basement and cellar. The offices of the first story are situated at B, over which there is usually a curved lean-to of illuminating tile. The main wall above this story is on the line *a b*—plain brick—with iron shutters. When shutters are used to close the first-story front they are mostly rolling shutters of sheet-steel. The hoist-way to the upper stories is at *c*, a position somewhat objectionable as interfering with the use of the

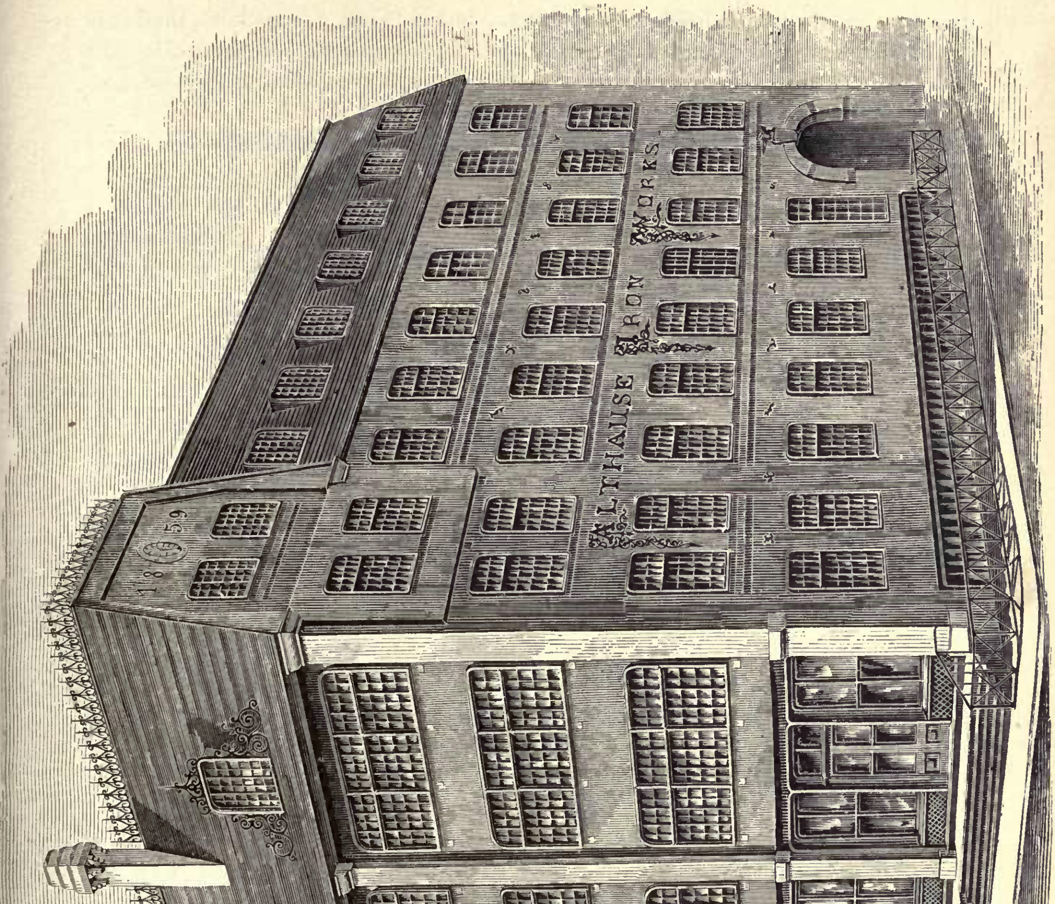


Fig. 1494.

stairs, when a common hoist-wheel is used; but if it is a power-hoist, then it is put close to the wall, guarded by a rail, with a passage round to the stairs. In 50-feet-front stores the hoist is put on the opposite corner from the stairs, as

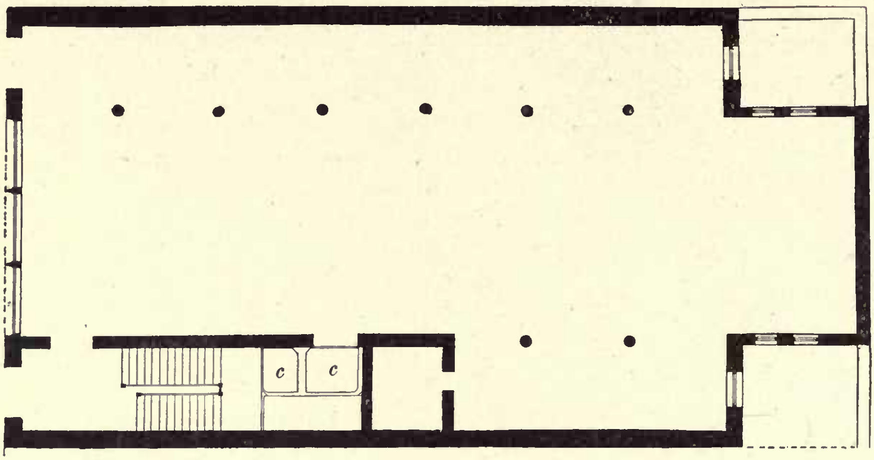


Fig. 1493.

at D, but this cuts off considerable light from the first-story front. In some the arrangement is as in Fig. 1493, in which the hoists *c c* are in the rear of the stairs. The arrangement for offices in the rear of the first story is in a T, with spaces at the sides for the ventilation and light of the lower stories. It will be observed that there is no central door, as in the elevation (Fig. 1489), which last most usually obtains for wholesale stores. Formerly illuminating tile on iron quadrant frames over rear extensions of stores were common, but were objectionable from the inside condensation and drip. It is very common to leave open areas at the sides, inclosed by brick walls (Fig. 1493), with the windows protected by iron shutters. For deep stores the area should be at one side and central, say from 30 to 40 feet long and 6 feet wide, which may be covered in the first story with glass. If this recess is on the side occupied by the staircases, it does not detract from the inside finish of the stores.

Hoists now in large stores are power-hoists—that is, worked by either steam, water, or electricity. The platform of a freight-hoist is usually 5 feet square; for passenger-hoists, in wholesale stores, somewhat less—4′ × 5′. For the raising of goods from the basement or sub-cellar to the sidewalk there is a hatch in the front light platform, opposite some window, and the space is like that of freight-hoists, 5′ × 5′; these may be power or hand hoists. For the delivery of goods into these lower stories there is often a slide or incline, iron-plated, ending at the bottom with an easy curve to the horizontal, down which boxes and bales are slid.

Fig. 1494 is a perspective view of a city machine and blacksmith shop. It was built for a purpose, and to express the purpose constructionally and economically. As regards convenience and strength, it was found to be, on occupation, all that could be wished. Posts, lintels, window-frames, sashes, and ornamental letters were of iron, and painted a very deep green; the structure was of brick, with sills and bands of rubbed Ulster bluestone, roof of Welsh

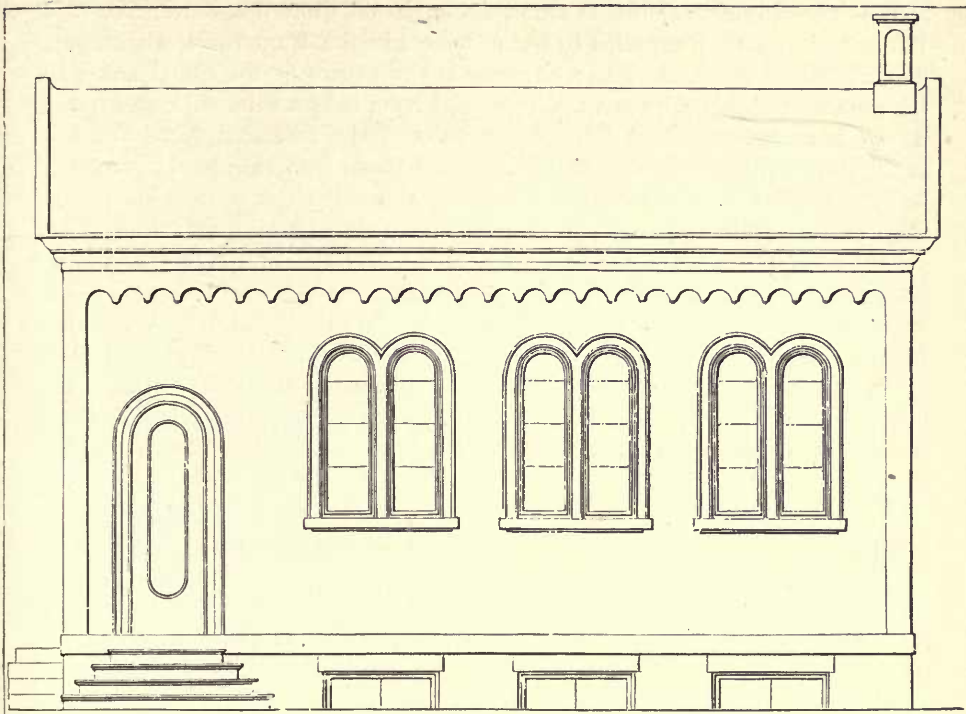


FIG. 1495

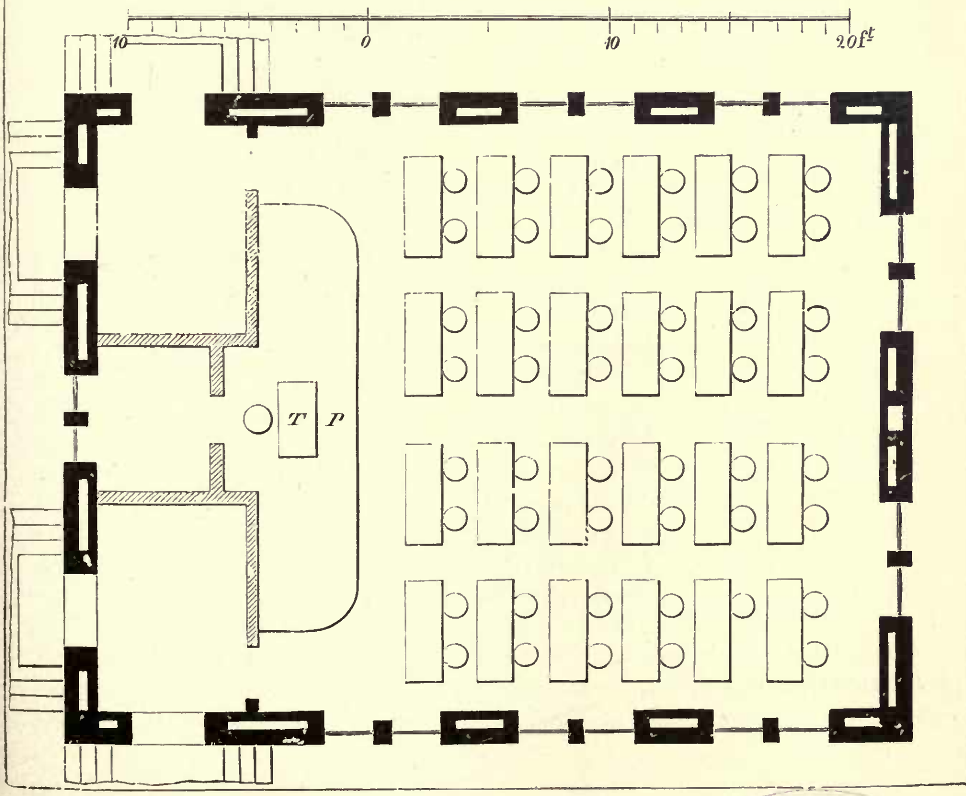


FIG. 1496.

slate. The chimneys shown in front, although not dummies, were never used. Power and heat were supplied by steam-boilers in the front vault, with a long flue, slightly rising, leading to a chimney at the centre of the side blank wall. On each side of this chimney, and separated from it by a thin *with*, there were flues. Forges occupied all the exterior walls of the basement, front and side areas, and the draught was upward and then down into the nearly horizontal flues connected with the central flues, and the draught was invariably good. Care was taken that all angles, horizontal and vertical, were rounded.

School-Houses.—Figs. 1495 and 1496 are an elevation and plan of a country district school-house, with seats for forty-eight scholars. There are two entrances, one for each sex, with ample accommodations of entry or lobby-room for the hanging up of hats, bonnets, and cloaks. A side door leads from each entry into distinct yards, and an inside door opens into the school-room. The desk, T, of the teacher, is central between the doors, on a platform, P, raised some 6″ or 8″ above the floor. In the rear of the teacher's desk is a closet or small room, for the use of the teacher. The seats are arranged two to each desk, with two alleys of 18″ and a central one of 2′. The passages around the room are 3′.

Figs. 1497 and 1498 are the elevation in perspective and plan of an English country school-house, introduced as suggestive—whether a one-story plan might not be better suited, and of more beautiful effect in our own country towns, where there is plenty of ground space, than the imitation of city edifices of many stories.

On the Requirements of a School-House.—Every scholar should have room enough to sit at ease, his seat should be of easy access, so that he may go to and fro, or be approached by the teacher without disturbing any one else. The seat and desk should be properly proportioned to each other and to the size of the scholar for whom it is intended, who should not sit in a cross-light, the light should come from a single direction as near as possible over the left shoulder. The seats, as furnished by the different makers of school furniture, vary from 9″ to 14″ in height; and the benches from 17″ to 28″; measuring on the side next the scholar. The average width of the desk is about 18″, and it is formed with a slope of from 1½″ to 2½″, with a small horizontal piece of from 2″ to 3″ at top. There is a shelf beneath for books, but it should not come within about 3″ of the front. The width of the seat varies from 10″ to 14″, with a sloping back, like that of a chair; it should, in fact, be a comfortable chair. In the figure, two scholars occupy one bench. Fig. 1499 represents another arrangement, in which each scholar has a distinct bench; this is more desirable, but not quite so economical in room. In primary schools desks are not necessary; and in many of the intermediate schools the seat of one bench is formed against the back of the next bench; but seats distinct are preferable.

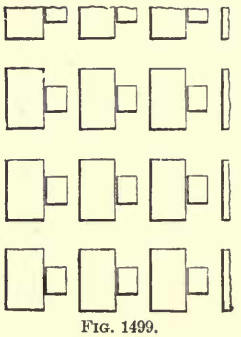


FIG. 1499.

The teacher's seat is invariably on a raised platform, and had better be against a dead wall than where there are windows. Blackboards and maps should be placed along the walls. Care should be taken in the warming and ventilation;

warm air should be introduced in proportion to the number of scholars, and ventiducts should be formed to carry off the impure air.

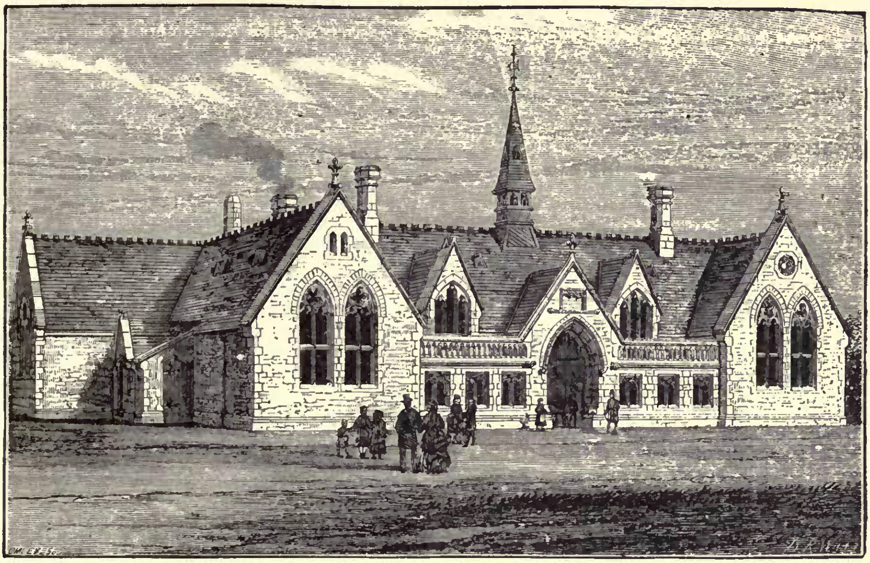


Fig. 1497.

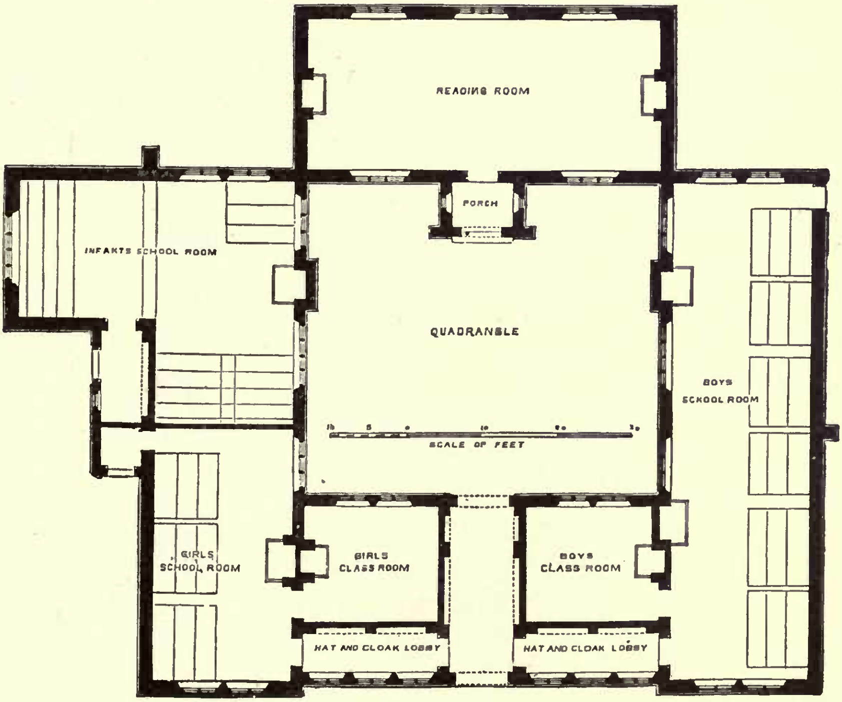


Fig. 1498.

In cities and large towns it is almost indispensable to build school-houses many stories in height, dividing the rooms in each story according to the neces-

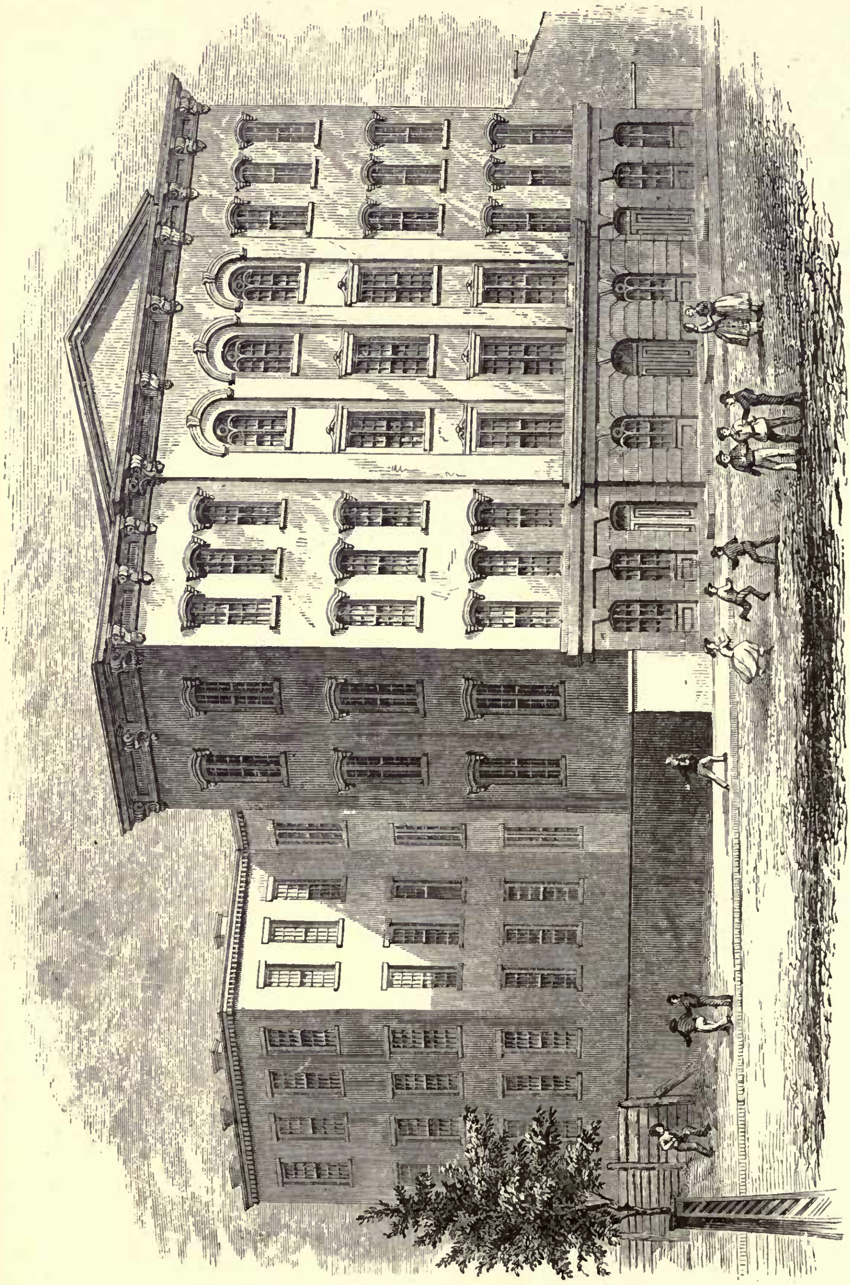


Fig. 1500.

sities of their occupancy. The management of schools differs in different localities. This will be seen in the illustrations given below, showing the arrangements of school-houses in the city of New York and of Cleveland, Ohio.

Fig. 1500 is an elevation in perspective of one of the largest of the New York city schools, showing the yards around it. Fig. 1501 is the plan of the

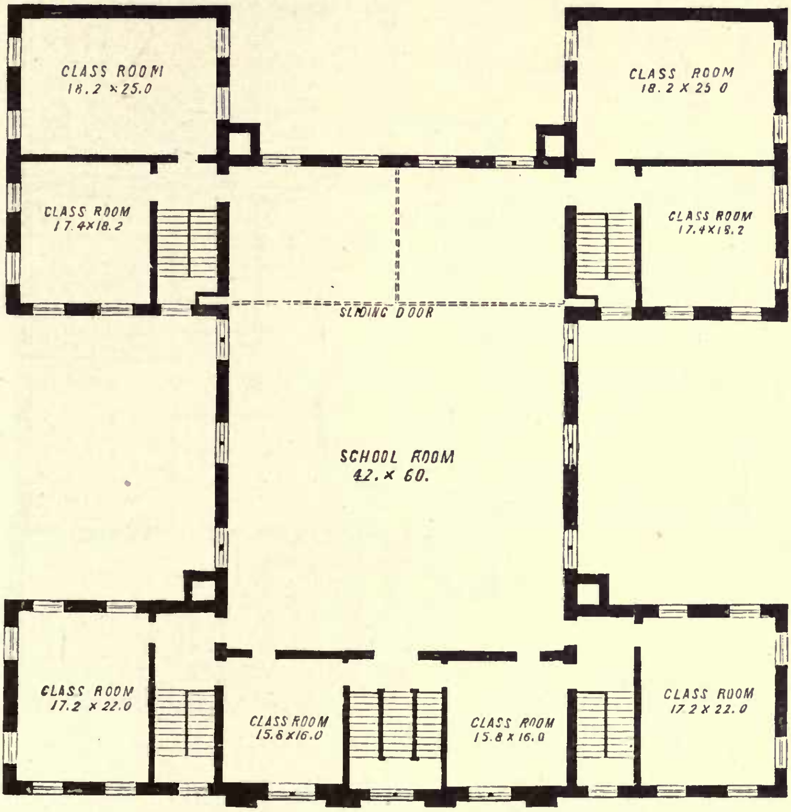


FIG. 1501.

grammar-department floors of this house; and Fig. 1502 the plan of the same floors of another house of a different outline.

Figs. 1503 to 1506 are plans of school-houses, built at Cleveland, Ohio, a type inaugurated under the supervision of the then superintendent, Mr. A. J. Rickoff. Figs. 1503, 1504, and 1505 are plans of the High-School house. Fig. 1503 is the plan of the third story; Figs. 1504 and 1505 of those portions of the second and first stories which differ from that of the third. There is a rear vestibule in the first story to correspond with the one in front, shown in the figure. In the whole building there are 14 session-rooms, each 37′ × 30′ × 16′; each having its connecting cloak-room; one general assembly-room, 94′ × 56′ × 38′ high, with a seating capacity for at least 1,000 persons; one lecture-room, with seats for 100, with an apparatus-room; one room for drawing, 30′ × 55′, with a room for models, drawing-boards, etc.; two rooms for the principal and reception-room; five rooms for library and recitation-rooms.

Fig. 1506, a plan of one half of one story of the Walton Avenue School, on a larger scale, explains more fully the arrangement of seats and the ventilation. Four ventilating educts, of 8 square feet of section each, may be heated to any required temperature for the purposes of circulation by four upright 2″ steam-pipes; six ducts of 1 square foot section lead from different points in the floor

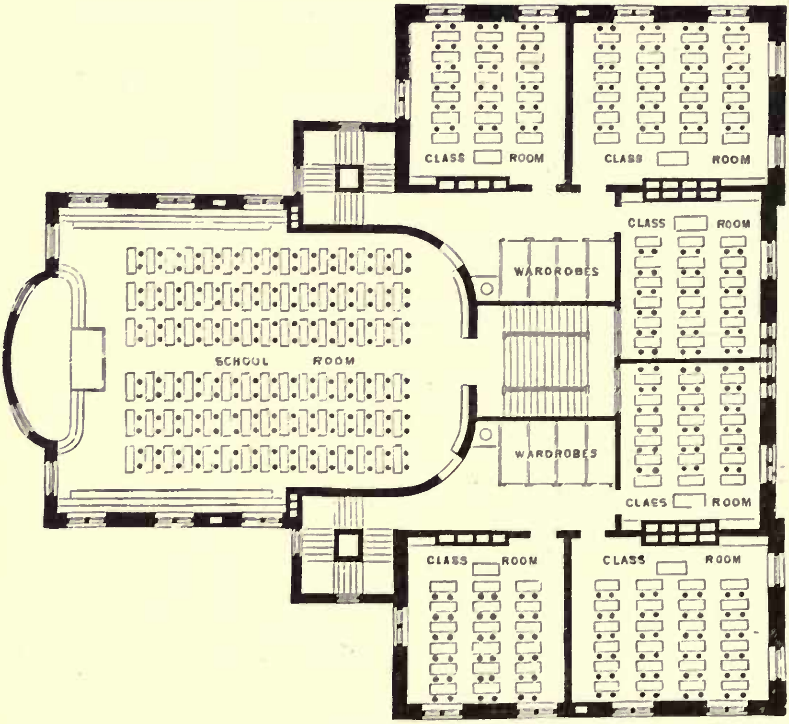


FIG. 1502

of each session-room (as shown in dotted lines in the figure) into the ventilating educts. There are besides other registers opening directly into the educts. The building is heated by steam coils or radiators placed under the windows of the rooms, with provision for the admission of fresh air under the stone sills behind the radiators. The main light of every room is admitted at the left hand of the pupil, so that in writing the shadow of the hand does not fall on the space to be written on. There are none of the cross-lights that so seriously impair the vision. The wall facing the pupil and behind the teacher is unbroken by windows, affording large and convenient spaces for blackboards.

Churches, Theatres, Lecture-Rooms, Music and Legislative Halls.—To the proper construction of rooms or edifices adapted for these purposes some knowledge of the general principles of acoustics, and their practical application, is necessary. In the case of lecture-rooms and churches, the positions of the speaker and the audience are fixed; in theatres, one portion of the inclosed space is devoted to numerous speakers and the other to the audience; in legislative halls, the speakers are scattered over the greater part of the space, and also form the audience.

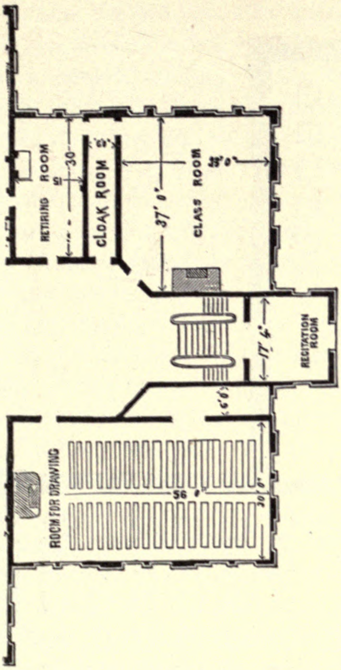


FIG. 1504.

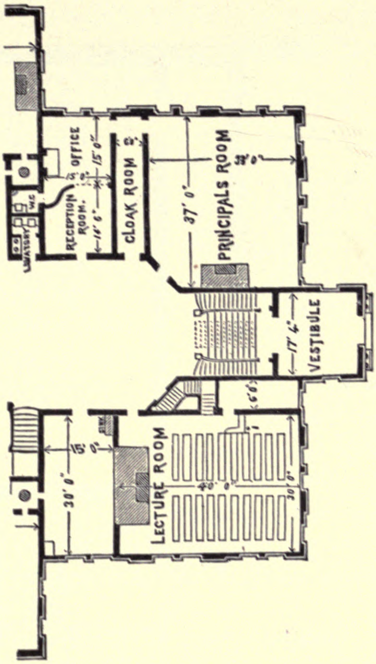


FIG. 1505.

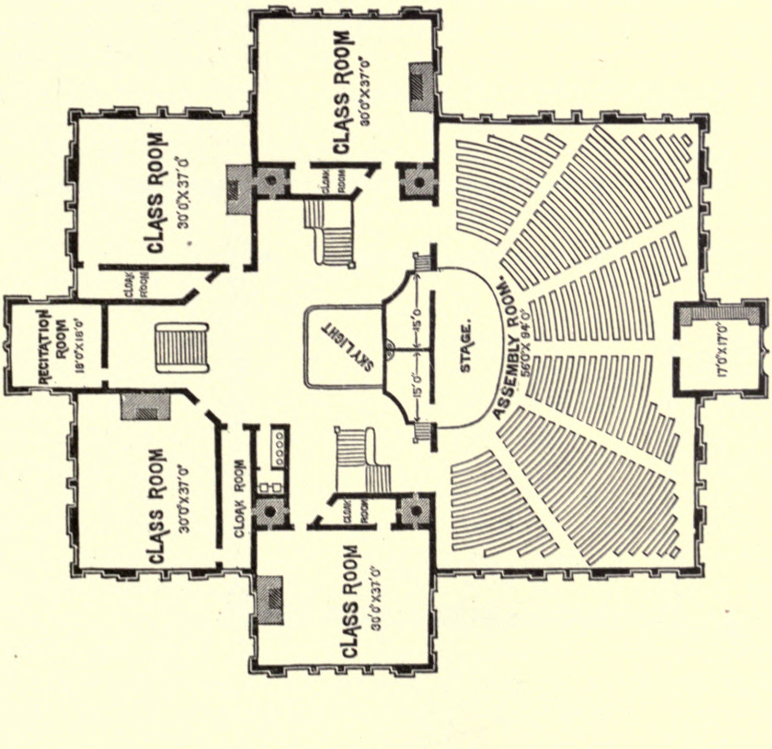


FIG. 1503.

PLAN OF HALF OF ONE STORY OF WALTON AVENUE SCHOOL, CLEVELAND, OHIO.

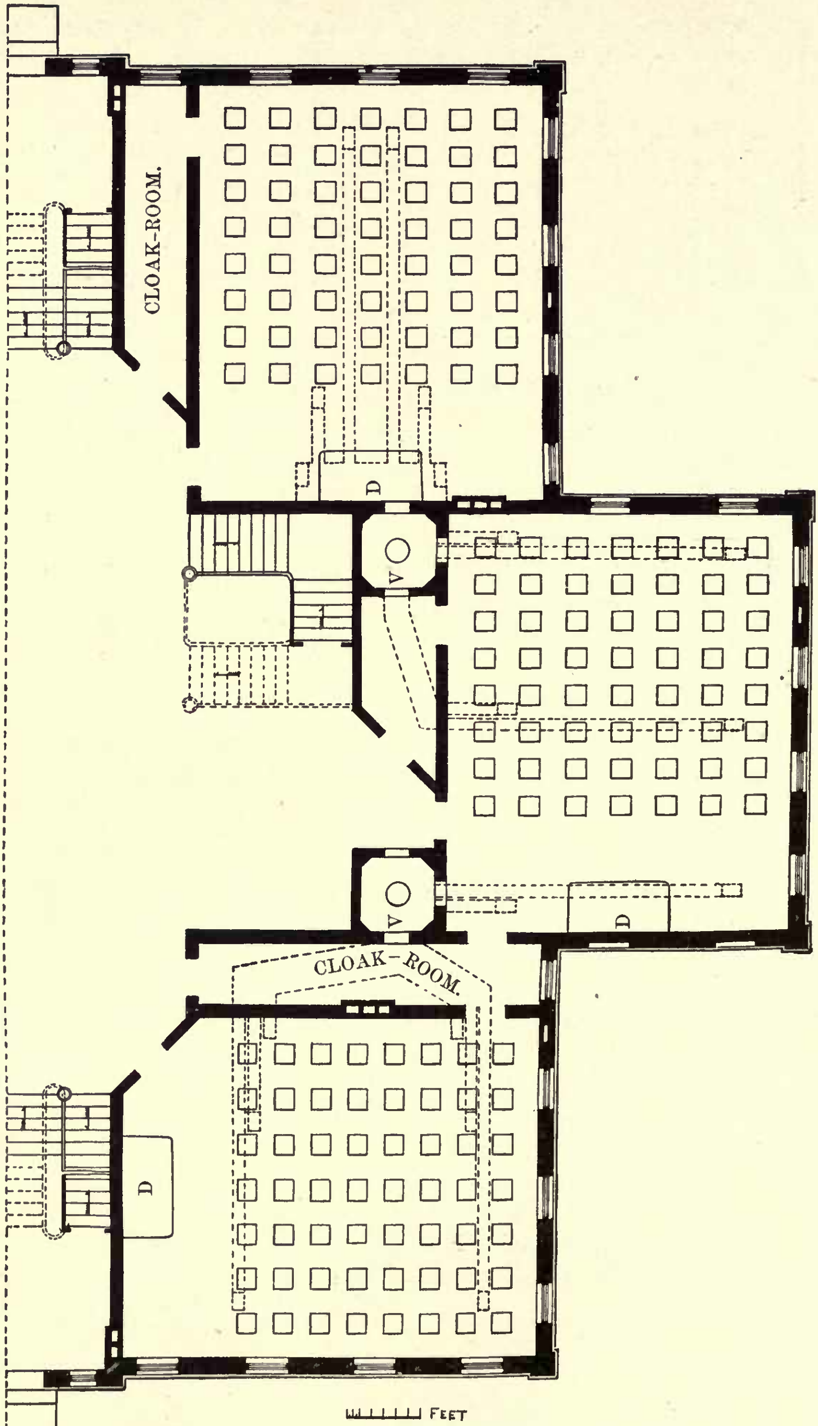


FIG. 1506.

The transmission of sound is by vibrations, illustrated by the waves formed by a stone thrown into still water; but direction may be given to sound, so that the transmission is not equally strong in every direction; thus, Saunders found that a person reading at the centre of a circle of 100 feet in diameter, in an open meadow, was heard most distinctly in front, not as well at the sides, but scarcely at all behind. Fig. 1507 shows the extreme distance every way at which the voice could be distinctly heard: 92 feet in front, 75 feet on each side, and 31 feet in the rear. The waves of sound are subject to the same laws as those of light, the angles of reflection are equal to those of incidence; therefore, in every inclosed space there are reflected sounds, more or less distinct, according to the position of the hearer, and to the form and condition of the surfaces against which the waves of sound impinge. Thus, of all the sounds entering a parabolic sphere, the reflected sounds are collected at the focus. Solid bodies reflect sound, but draperies absorb it. As, in all rooms, the audience can never be concentrated at focal points, nor is it possible in any construction to make calculation for all positions, it is in general best to depend on nothing but the direct force of the voice, and not to construct larger than can be heard directly without aid from reflected sounds.

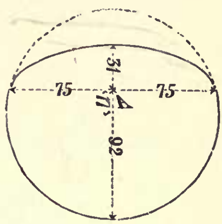


FIG. 1507.

There is great difference in the strength of voice of different speakers; the limits as given in the figure are for ordinary reading in an open space. In inclosed spaces, owing to the reflected sounds or some other cause, there are certain pitches or keys peculiar to every room, and to speak with ease the speaker must adapt his tone to those keys. The larger the room, the slower and more distinct should be the articulation.

It has been observed that the direction of the sound influences the extent to which it may be heard. The direction of the currents of air through which the sound passes affects the transmission of the sound, and this may be made useful when the rooms are heated by hot air, by introducing the air near the speaker and placing the ventilators or educts at the outside of the rooms, and by placing their apertures rather nearer the bottom of the room than at the top. It would seem much better and easier to make a current of air a vehicle of sound rather than depend on reflection.

In the "Baltimore Academy of Music," designed by Mr. J. Crawford Neilson, architect, the ventilation was arranged to obstruct the formation of air-currents of unequal density. The whole supply of fresh air is admitted at the back of the stage, is there warmed, crosses the stage horizontally, and passes through the proscenium and then, somewhat diagonally toward the roof, across the auditorium in one grand volume and with gentle motion so as to almost entirely prevent the formation of minor air-currents. It is exhausted partly by an outlet in the roof and partly by numerous registers in the ceilings of the galleries. From this central outlet and from the large flues of the registers the air passes into the ventilating-tower over the great chandelier, which supplies, in its heat, a part of the motive power of the circulation. It is further expelled from the tower by means of large valves, offering no obstacle to the *egress of air*, but completely *cutting off its entrance.*

The direction of the air-currents within the house was determined by thistle balls, and the quantity, as found by anemometers, was about 15,000 cubic feet per minute. This amount, sufficient to ventilate the house, is that required to impress the proper movement on its atmosphere. It is amply sufficient for ventilation, as is shown by the fact that the thermometers of the upper circle do not vary perceptibly from those of the orchestra circle. The seating capacity of the house is about sixteen hundred persons. The acoustics are satisfactory.

On the Space occupied by Seats in general.—A convenient arm-chair occupies about 20″ × 20″, the seat itself being about 18″ in depth, and the slope of the back 2″; 18″ more affords ample space for passage in front of the sitter. In churches the seats are arranged by pews or stalls, the width of each pew in general being about 2′ 10″. In the arrangement of theatre seats the bottom turns up (Figs. 1508 and 1509), and 29″ only is allowed for both seat and passage-way, and 18″ for the width of seat, which may be taken as the average allowance in width to each sitter in comfortable public rooms. In lecture-rooms, benches and settees are often used, the space there occupied by seat and passage being about 2′ 6″.

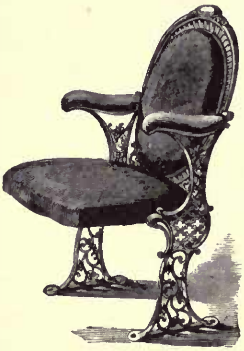


FIG. 1508.

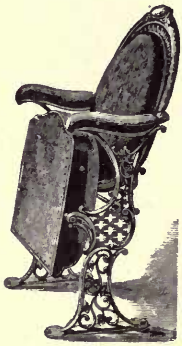


FIG. 1509.

In the earlier churches, ceremonies and rites formed a very large part of the worship, the sight was appealed to rather than the hearing, and for this purpose churches were constructed of immense size, and with all the appliances of ornament and construction, with pillars, vaults, groins, and traceried windows. In the churches of this country, the great controlling principle in the construction of a church is its adaptation to the comfortable hearing and seeing the preacher. In this view alone, the church is but a lecture-room; the ceiling should be low, and pillars and transoms should be little used; but since even the character of the building may tend to devotional feelings in the audience, and since certain styles and forms of architecture have long been used for church edifices, it has been the custom to follow these time-honoured examples, adapting them to modern requirements of church worship, with adequate means of heating and ventilation.

Fig. 1511 is a plan of an ancient basilicon or Romanesque church. Fig. 1510 is a sectional elevation of the same. Fig. 1512 is a plan of a Gothic church, in which C is the chancel, usually at the eastern extremity, T T the transept, and N the nave. In general elevation the Gothic and Romanesque agree: a high central nave and low side aisles. In the later Romanesque the transept is also added.

The basilicas aggregated within themselves all the offices of the Romish church. The circular end or apse, with the raised platform, or dais, in front appropriated to the altar; in the rear the confessional and the sacristy; beneath was the crypt, where were placed the bodies of the saints and martyrs, and pulpits were placed in the nave, from which the services were said or sung by the inferior order of clergy.

The plan (Fig. 1512) is that of the original Latin cross, the eastern limb or chancel being the shortest, and the nave the longest. Sometimes the eastern

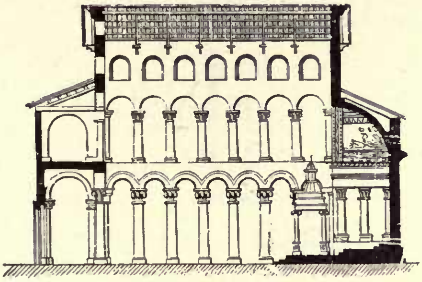


FIG. 1510.

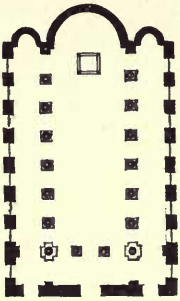


FIG. 1511.

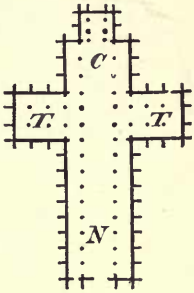


FIG. 1512.

limb was made equal to that of the transept, sometimes even longer, but never to exceed that of the nave. In the Greek cross all the limbs are equal. In most of the French Gothic churches the eastern end is made semicircular, often inclosed by three or more apsidal chapels, that is, semi-cylinders, surmounted by semi-domes.

The Byzantine church consisted internally of a large square or rectangular chamber, surmounted in the centre by a dome, which rested upon massive piers; an apse was formed at the eastern end. Circular churches were built in the earlier ages for baptisteries, and for the tombs of saints and emperors.

The Greek, Roman, and English churches conform in their cathedrals and larger edifices nearly to the Romanesque or Gothic models. But as the general requirements for church services now are those of a lecture-room, modern churches are constructed adapted to these purposes, and, in cities, to the size and form of the lots, with some ecclesiastical accessories of towers and steeples, windows and doors and interior finish.

Fig. 1513 is the plan of the English church at The Hague.

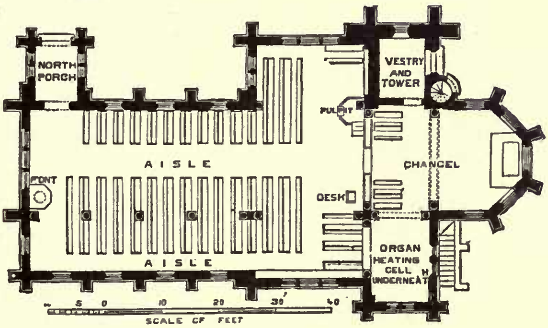


FIG. 1513.

Fig 1514 is the plan of a Wesleyan chapel in London; the requirements of the service have been well adapted to the necessities of the lot.

Fig. 1515 is the cross-section of a common form of small country church, with nave *n*, aisles *a a*, and clear-story *c*. The effect, both inside and out, is

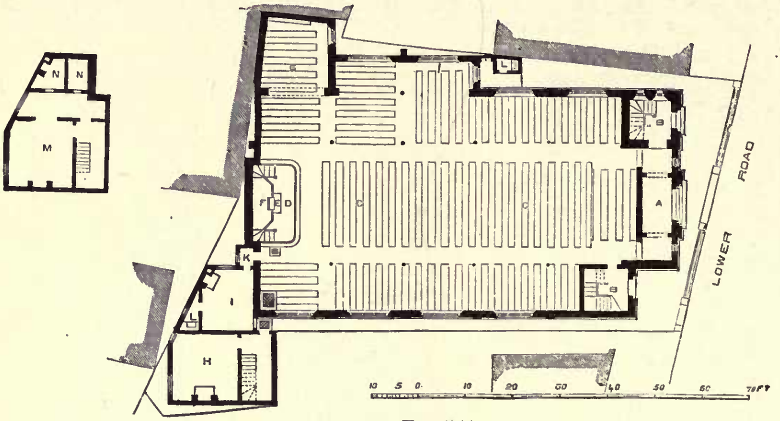


FIG. 1514.

good, but there are objections to large or masonry-columns, which cut off the view of the desk and the altar from many sitters, and to the windows of the

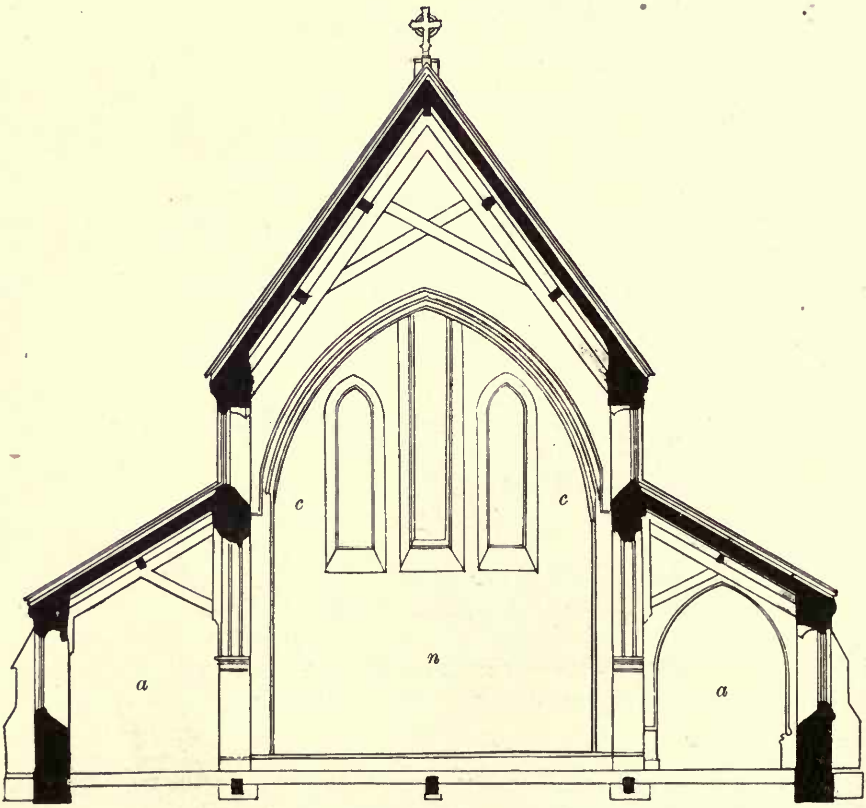


FIG. 1515.

clear-story, which in winter act as coolers to the air descending in draughts upon the heads of the congregation beneath them. Neither columns nor clear-story are constructively necessary; the span can readily be met by a single roof, and sufficient light can be obtained from the sides.

Figs. 1516, 1517, and 1518 are examples of open-timbered Gothic roofs of churches.

The technical names (Fig. 1516) are: 1, Principals; 2, Purlines; 3, Collars; 4, Braces; 5, Wall-pieces; 6, Wall-plates; 7, Struts; 8, Rafters. 4 and 5 are shown in section.

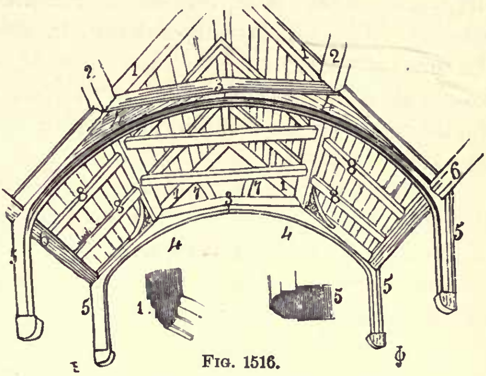


Fig. 1516.

The length of pews is various, being of two sizes, adapted to either small or large families, say from 7′ 6″ to 12′ 6″, 18″ being allowed for each sitter. In arrangement it is always considered desirable that there should be a central aisle, and if but four rows of pews (often of two sizes in the same church), an aisle against each wall; if six rows, one row on each side will be wall-pews. Formerly it was the universal practice to construct pews with doors, but of late it is more customary to omit the doors, making the pews open stalls.

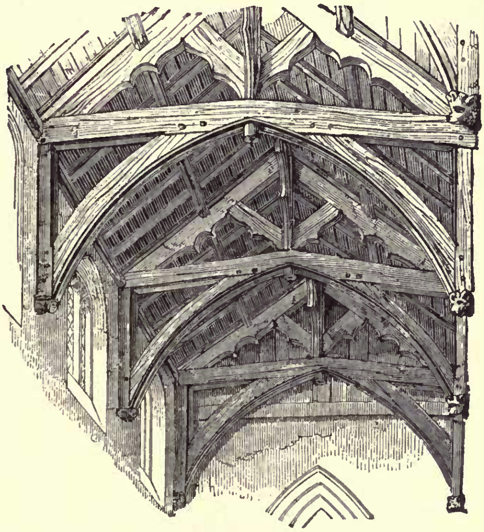


Fig. 1517.

Few churches are now without an organ; its dimensions should of course depend on the size of the church. In form it may be adapted somewhat to the place which may be appropriated to it—either in a gallery over the main entrance or above the pulpit or at the side of the chancel, as in Fig. 1513. Sometimes there are two, one at each extremity of the church, one organist playing on both by electric connection. In general, it is oblong in form, the longer side being with the keys. The dimensions suited to a medium-sized church are about 9′ × 15′, and 12′ in height.

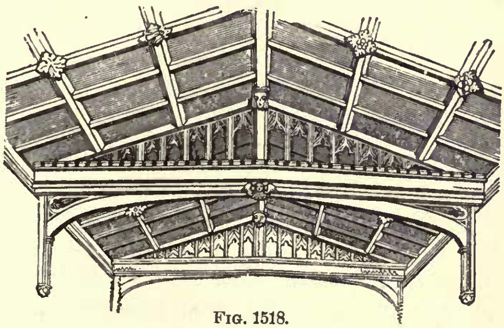


Fig. 1518.

The vestry-room, if used for the purposes of its meetings, should be adapted in size to the purpose; but if only for a withdrawing or robing room for the clergyman, it may be of very small dimensions, and should be accessible from without. The Sunday-school room, in general, requires in plan about half the area of the church. From motives of economy it is usually placed in the basement of the church; but, in the country especially, it is better that it should be a separate building, and form one of the group of church, parsonage, and Sunday-school house.

In elevation, city churches are generally Romanesque and Gothic, occasionally Byzantine. The Greek have no tower, but often a spire above the portico; the Romanesque and Gothic generally one tower, over the central door of entrance, or at one corner; sometimes two, one at each side of the principal door, almost invariably surmounted by spires, high and tapering, usually of wood, but in some instances of stone.

Theatres.—In theatres and opera-houses it is not only necessary that the audience should have a good position for hearing and seeing the performance upon the stage, but also to see each other. The most approved form, now, for the body of a dramatic theatre is a circular plan, the opening for the stage occupying from one fourth to one fifth of the circumference, the sides of the proscenium being short tangents; but for a lyric theatre, where music only is performed, and where, consequently, hearing is easier, the curve is elongated into an ellipse, with its major axis toward the stage.

In the general position of the stage, proscenium, orchestra, orchestra seats, parquette, and boxes, but one plan is followed. The line of the front of the stage, at the footlights, is generally slightly curved, with a sweep, say, equal to the depth of the stage, and the orchestra and parquette seats are arranged in circles concentric with it: of the space occupied by seats we have already spoken. The entrance to the parquette may be through the boxes, near the proscenium, and centrally, but better at the sides, dividing the boxes into three equal benches; the seats in the boxes are usually concentric with the walls, and more roomy than those of the parquette. The orchestra seats are of a height to bring the shoulders of the sitter level with the floor of the stage, and the floor of the parquette rises to the outside, 1 in 15 to 18. The floor of the first row of boxes is some 2 to 3 feet above the floor of the parquette at the front centre, and rises, by steps at each row, some 4 inches; in the next tier of boxes the steps are considerably more in height, and so on in the boxes above. In general, three rows of boxes are all that is necessary; in front, above the second, the view of the stage is almost a bird's-eye view. The floor of the stage descends to the footlights at the rate of about 1 in 50. In large theatres it is of the utmost importance that all the lobbies or entries should be spacious, and the means of exit numerous and ample—the staircases broad, in short flights and square landings, and not circular, as, in case of fright, the pressure of persons behind may precipitate those in front the whole length of the flight. Ladies' drawing-rooms should be placed convenient to the lobbies, of a size adapted to that of the theatre, also rooms for the reception of gentlemen's canes and umbrellas, both with usual water arrangements. The box-office should be near the entrance and arranged to interfere as little as possible with the approach to the doors of the house. At the entrance there should be a very spacious lobby, or hall, so that the audience may wait sheltered from the weather; if possible, there should be a long portico over the sidewalk, to cover the approach to the carriages. Only single entrances are necessary to distinct parts of the house, but the greater the number of, and the more ample places for exit at the conclusion of the piece, or for the contingency of fire, the better.

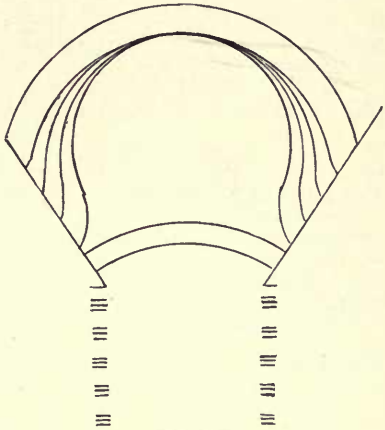


Fig. 1519.

Fig. 1519 is a plan suggested by Ferguson of keeping the centre of the balconies perpendicular over one another, and then, by throwing back the sides of each balcony till the last is a semicircle, the whole audience would sit more directly facing the stage, would look at it at a better angle, and the volume of sound be considerably increased by its freer expansion immediately on leaving the stage.

Figs. 1520 and 1521 are a plan and section of Wagner's theatre.

In cities, the auditoria of dramatic theatres conforming to the shape of the lots are rectangular in their outline, and seldom exceed a seating capacity of 1,500. Lyric theatres are much larger, both in seating and scenic capacity. Lecture-rooms are usually arranged with the audience-floor flat, room rectangular, with reading-desk or platform raised, and with or without galleries. The same form usually obtains for music-halls, only they are much greater in extent, the first being capable of containing from 500 to 1,000 persons; whereas some music-halls will contain 2,500, and Ferguson thinks that a music-hall might be arranged so that even 10,000 might hear as well as in those of present construction. The lecture and music halls are seldom devoted to a single purpose, but are used for political meetings, for fairs, and dances, and the constructon must be such as to serve these other purposes.

PLAN.

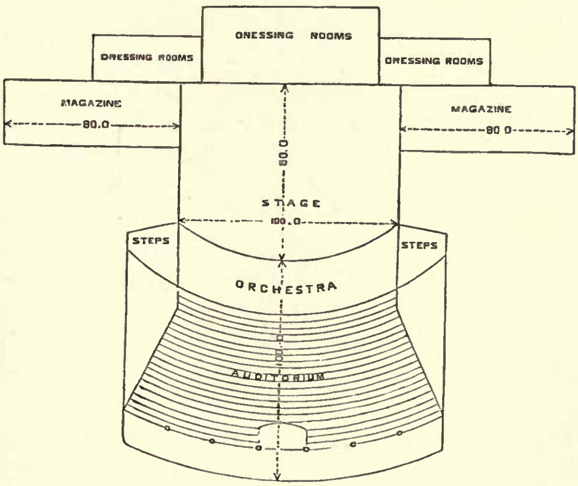


Fig. 1520.

SECTION.

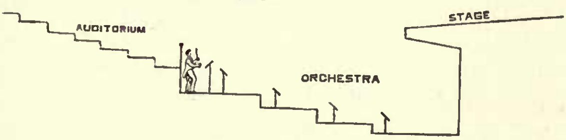


Fig. 1521.

COMPARATIVE TABLE OF THE DIMENSIONS OF A FEW THEATRES.

Name and Location.	Distance, in feet.						Height, in feet.	
	Between boxes and footlights.	Between footlights and curtain.	Between curtain and back of stage.	Greatest breadth of pit.	Breadth of curtain.	Breadth of stage between side-walls.	From floor of pit to cornice.	From floor of pit to centre of ceiling.
Alexandre, St. Petersburg	65	11	84	58	56	75	53	58
———, Berlin	62	16	76	51	41	92	43	47
La Scala, Milan	77	18	78	71	49	86	60	64
San Carlo, Naples	77	18	74	74	52	66	81	83
Grand Théâtre, Bordeaux	46	10	69	47	37	80	50	57
Salle Lepelletier, Paris	67	9	82	66	43	78	52	66
Covent Garden, London	66*	..	55	51	32	86	54	..
Drury Lane, London	64*	..	80	56	32	48	60	..
Boston, Boston	53	18	68	..	46	87	55½	58
Academy of Music, New York	74	13	71	62	48	83	74	..
Grand Opera-House, New York	54	8½	63½	48	44	76	52	67
Opera-House, Philadelphia	61	17	72	66	48	90	64½	74

* These dimensions include the distance between the footlights and curtain.

Legislative Halls.—Although much has been written about their construction in relation to acoustic principles, there is great disagreement in practical examples, and in the deductions of scientific men. The Chamber of French Deputies was constructed, after a report of most celebrated architects, in a semicircular form, surmounted by a flat dome, but as the member invariably addresses the house from the tribune, at the centre, in its requirements it is but a lecture-room. Mr. Mills, architect, of Philadelphia, recommends for legislative or forensic debate a room circular in its plan, with a very slightly concave ceiling. Dr. Reid, on the contrary, in reference to the Houses of Parliament, gave preference to the square form, with a low, arched ceiling. The Hall of Representatives at Washington is 139 feet long by 93 feet wide, and about 36 feet high, with a spacious retiring gallery on three sides, and a reporters' gallery behind the Speaker's chair. The members' desks are arranged in a semicircular form. The ceiling is flat, with deep-sunk panels, openings for ventilation, and glazed apertures for the admission of light. The ventilation is intended, in a measure, to assist the phonetic capacity of the hall, the air being forced in at the ceiling and drawn out at the bottom.

In reviewing the general principles of acoustics, it will be found that those rooms are the best for hearing in which the sound arrives directly to the ear, without reflection; that the sides of the room should neither be reflectors nor sounding-boards, and that surfaces absorbing sound are less injurious than those that reflect. Slight projections, such as ornaments of the cornices and shallow pilasters, tend to destroy sound, but deep alcoves and recessed rooms produce echoes. Let the ceiling be as low as possible, and slightly arched or domed; all large external openings should be closed; as M. Meynedier expresses it, in his description of an opera-house, "Let the hall devour the sound; as it is born there, let it die there."

Hospitals.—In large cities, hospitals, by necessity, are confined to narrow spaces, but they should be placed, if possible, on river fronts or on open parks, to secure as much open-air ventilation as possible. They are usually many stories in height, with large wards one above the other. Sir J. T. Simpson alleges a very high rate of mortality in hospitals after surgical operations as compared with the mortality after the same operations when performed at the homes of the patients, and asserts that the mortality after operations performed in hospitals containing more than 300 beds is in excess of that in hospitals containing less; that great hospitals are great evils in exact proportion to their magnitude, and suggests the construction of smaller hospitals.

Figs. 1522 and 1523 are an elevation and plan of an English country hospital.

Stables.—Under this general name are included the barn, or the receptacle of hay and fodder, the carriage-house, and the stable proper, or lodging-house for horses and cows. The first two may be included under one roof, the carriages on the first floor, and hay in the loft; but the lodging-place should be distinct, in a wing attached to the barn, that the odours from the animals may not impregnate their food, or the cloth-work of the carriages, or the ammonia tarnish their mountings.

Hay in bulk, in the mow, occupies about 340 cubic feet per ton; bales average $2' 4'' \times 2' 6'' \times 4'$, and weigh from 220 to 320 pounds. The door-space for a load of hay in the bulk should be from 12 to 13 feet high and 12 feet wide. The floor beneath the hay should be tight, so that dust and seed may not drop on the carriage. A door for carriages should be 10 feet 6 inches high by 9 feet wide.

The horse is to be treated with greater care than any other domestic animal. His stable is to be carefully ventilated, that he may have fresh air without being subject to cross-draughts. Preferably the floor should be on the ground, that there may be no cold from beneath. He should stand as near as possible level; and for this purpose a grated removable floor, with small interstices, should be laid over a concrete bottom, with a drip toward the rear of the stall, and the urine should be collected in a drain and discharged into a trapped manure-tank outside the stable. In Fig. 1524 the pitch of bottom of stalls is to the centre and outward. The manure should never be deposited beneath the stable, but should be wheeled out and deposited in a manure-yard or tank daily. It is as essential that all excrements should be removed entirely from the stable as that the privy should be placed outside the house.

The breadth of stalls should be from 4 feet 6 inches to 5 feet in the clear; the length, 7 feet 6 inches to 8 feet; the rack and feed-box require two feet in addition, to which access is given in the best stables by a passage in front. Rack and feed-boxes are often made of iron, and the upper part of stalls fitted with wrought-iron guards. Box-stalls, in which horses are shut up but not tied in cases of sickness or foaling, are about 10 feet square.

In large stables in cities the first floors are often occupied by the carriages, while the horse-stalls are in the basement or upper stories, with inclined ways of access. In the basement provision must be made for light and ventilation. In the upper stories these may be secured more readily, but the floors must be made tight and deafened, that the urine may not leak through, nor the cold come through from below to make too cool a bed for the horse.

Fig. 1524 is an elevation in perspective of two first-class stalls, a box shown

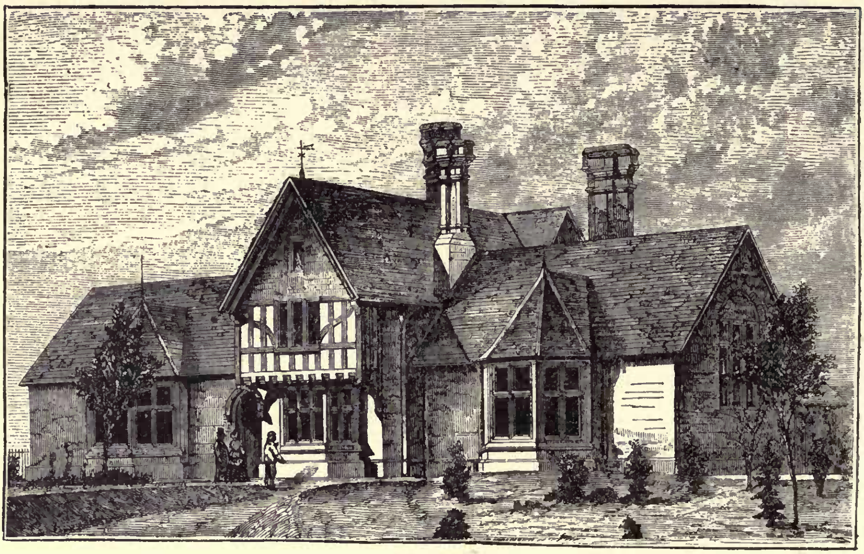


FIG. 1522.

GROUND PLAN.

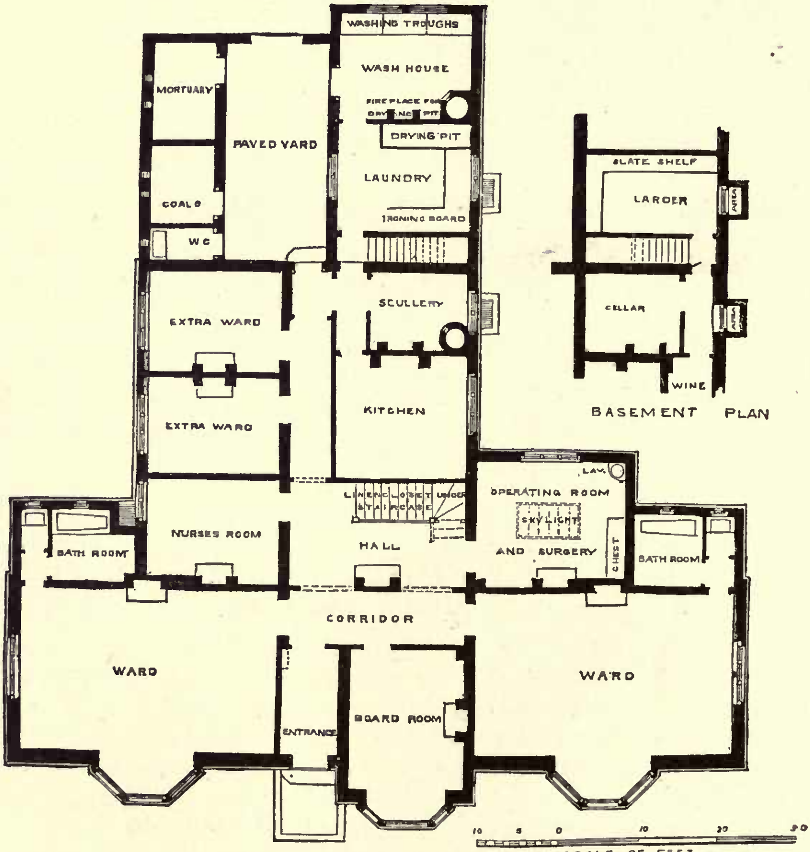


FIG. 1523.

with the door open, and a single stall. The lower part of the inclosures is of plank, with wrought-iron guards and ramp above. The posts are of oak, and

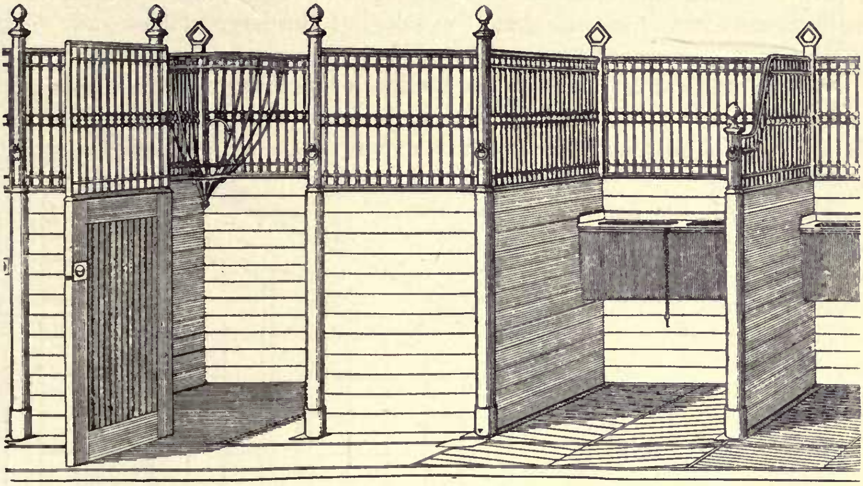


FIG. 1524.

the hay-boxes or mangers of cast-iron; the hay-rack in the box-stall is of wrought-iron. These are of common manufacture, and are of varied patterns; but in the country they are usually made of wood, and connected with the stall.

Fig. 1525 is the plan of a small country stable, showing the desirable pas-

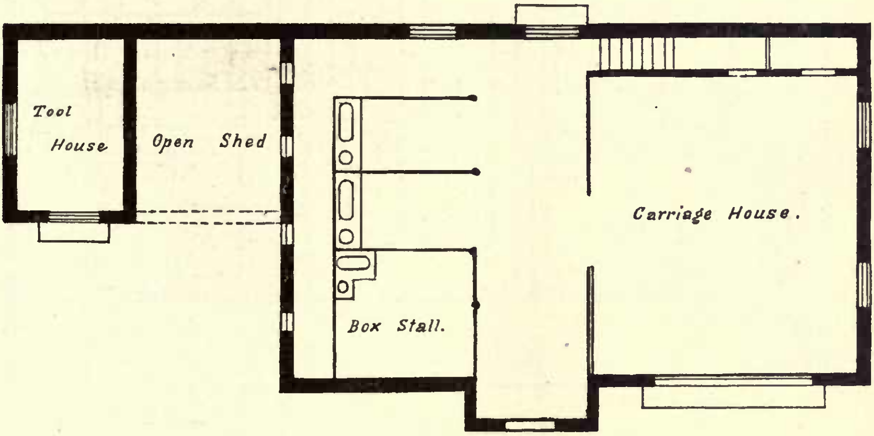


FIG. 1525.

sages around the stalls and exterior windows in front of each stall, that the horses may not only have light and air, but can see out.

Cow-houses, for cows giving milk, should be constructed with care for ventilation, light, and cleanliness. Other cattle are usually left out, with sheds under which they can go for shelter. For those housed, the spaces occupied should be about the same per head as the single horse-stall. The manger should be on the floor, 12″ to 18″ high, and about 18″ wide. It is not usual to have partitions, but there ought to be between every pair, reaching from the manger half-way to the gutter behind. The floor should be level, grated, with a drip beneath, and cleansed by washing out. The partition and mangers are often of cast-iron, and on sale, but for large stables and in the country they are commonly of wood.

Greenhouses.—Fig. 1526 is the section of a greenhouse, with shelves for plants. The floor is of concrete and the walls are of masonry; the northern exposure is a blank wall.

Fig. 1527 is the details of windows. The sides are box-sash, hung with weights (*w*, *w*, Fig. 1528). The lower roof sash is firmly fixed, but the upper

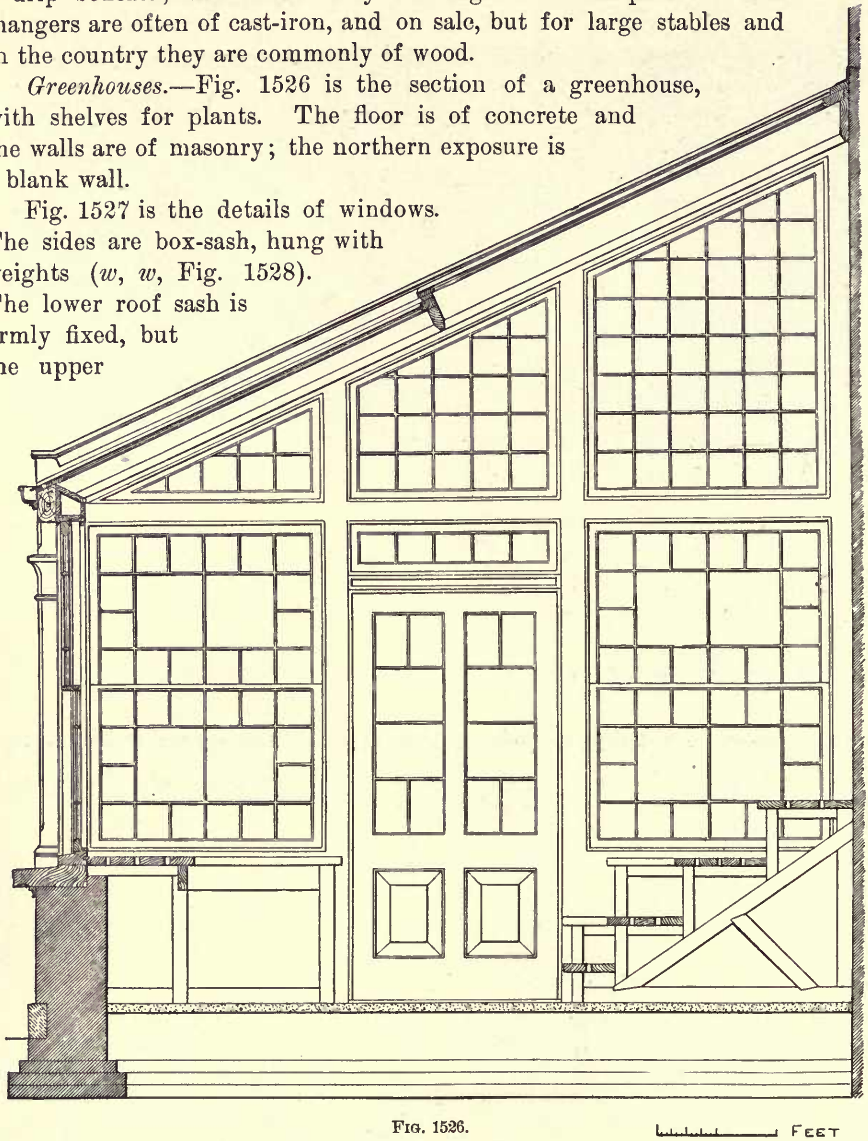


Fig. 1526.

one can be slid down; it is usually retained in place by a cord attached to the lower part of the sash, passing over a pulley on the upper bar of the frame, with the loose end within reach of the gardener, who can fasten it to a cleat.

Ventilation and Warming.—The purposes of ventilation are not changes of air merely, but the removal of foul and vitiated air, and the substitution therefor of pure air; and this air may be warm or cool according to the necessities of the season and personal requirements. Open space is not necessarily well ventilated; there must be circulation, outward and inward—the latter from purer sources than the former. With an equal discharge and supply of pure air, the smaller the rŏom, the more frequent the change of air, the better its distribution and the better the ventilation. But if the means of removal, supply, and distribution of air be proportioned to the size of the room, then the larger the room the better. Apertures do not necessarily mean circulation; a flue may draw or it may not draw, it may be inert, or the air may come

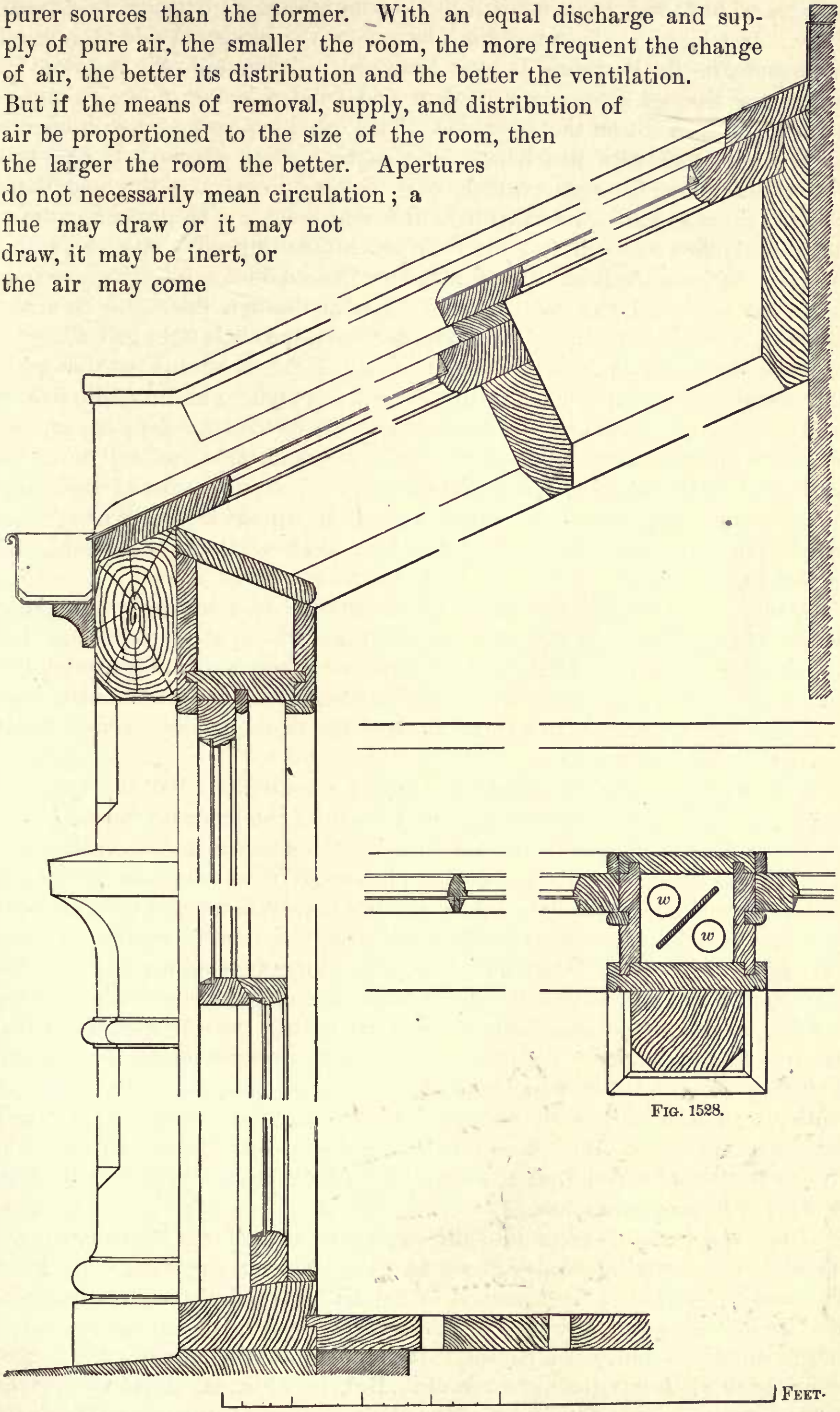


FIG. 1528.

FIG. 1527.

down; a window may be open, with little or no inward or outward movement of air. In a house exposed to a fresh breeze, on the windward side there is an air-pressure, on the leeward side there is an eddy or vacuum. Air is forced in on the first through every crack of door and window—often down chimney-flues—and drawn out on the other side. This often happens even with fires in the chimneys, and with insufficient heat in ventilating educts. If one will make an experiment in cold weather, when the windows are closed and there are fires in some rooms, he will often find that there is cold air coming down the unused flues, and will feel the cold current flowing down the stairs and along the floors to the fires. Architects have placed kitchens in the basement and in the attic, and the smell of cooking rises through the house from the former, and usually descends from the latter when the air is light and muggy.

Every room should have its distinct flue; if the current is not upward it will probably be downward, affording a fresh supply of air for ventilation if there is an escape elsewhere. A chimney-flue may be too large for the purposes of a fire; for most fires a flue 8″ × 8″ is amply sufficient, and will serve for ventilation in the common occupation of a house. If the throat of the chimney be made with rounded corners and a diverging sectional area, or a damper hinged at the bottom, for like effect, it should increase upward, and prevent back draught.

Small circular flues of from 6″ to 8″ diameter, or of equivalent rectangular section, are now made in concrete or stoneware, which, as they are smoother and with less joints than brickwork, give greater velocities of current with less section, and, laid with care, changes in direction afford but little obstruction. In an ordinary chimney with natural draught the velocity of ascending current is about six feet per second.

It is usual to depend largely on windows for ventilation, but the space on which they open may be too circumscribed to afford the requisite change of air, or the outer air itself may be too hot, or too cold, or too malarial or offensive, to make the change of air sanitary or pleasant. In tenement or apartment houses care should especially be taken that the inner windows on different flats open into as large air-shafts as possible, and that these shafts should have free opening to the outer air below and at the top, without skylights; and that the floors should be tight, so that the smells may not pass from one flat to another. Nothing more surely shows faults in ventilation than the diffusion of kitchen smells or tobacco smoke. Distinct flues should be constructed for each room, extending independently well above the roof; and not into an attic with a ventilating louvre, as the air may ascend one flue and descend another, and not out of the louvre. Pipe flues may lead into a single stack if each branch is given the direction of the main current at its connection, without obstructing its flow, as in sewer branches.

The quantity of air taken into and expired from the lungs by a single individual is quite small, probably about 14 cubic feet on an average per hour. The usual gas-burner delivers from 4 to 6 cubic feet per hour, under a pressure of 1″ and 2″ of water. It will be seen, therefore, how small apertures are necessary to supply the lungs of a person, if it could be provided directly to him and taken away without vitiating other air. But, in addition, air is vitiated by personal emanations and consumed by lights. These last can readily be arranged in connection with flues, not only to remove all their products of combustion, but also improve the ventilation of the room.

All systems of ventilation are based on the idea that so many individuals within a room and so many lights burning vitiate so much air, and that consequently a very large quantity of outer air must be introduced to reduce the percentage of vitiation, and generally with very little consideration as to the distribution of this air, although it is in every one's experience that the air in some portions may be fresh, in others stifling; that in hospital wards there are often dead ends where the air does not circulate, and where patients do not as a rule recover. The system is to provide, somewhere in a room, air enough and trust to chance for its distribution.

Some architects make the educts at the ceiling, some at the floor, some at both, with registers to control the openings. For sleeping apartments, if there is a fireplace this is all that will be necessary; if the air goes up or comes down it does not make draughts about the heads of the occupants.

To make flues draw, various forms of chimney-tops or cowls are adopted. The best and simplest are the Emerson (Fig. 1529) and a modification of the same (Fig. 1530); there are also various forms of self-acting flaps, turn-cowls, etc., the principle being to take advantage of the wind to make a draught. With the wind blowing across the top of a chimney, a bit of square-ended iron pipe extending above the chimney will answer as an expirator, but without a wind the draught must depend on circumstances within the dwelling and artificial draught. When sufficient circulation can not be obtained from natural differences of temperature in the atmosphere, or from winds, it is usual to have recourse to fans to force air into or draw it from a building, or by heat applied to the air in flues, ducts, or chambers in the hot-air furnaces. Both the air and the heat are necessary.

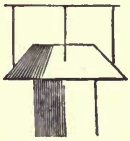


Fig. 1529.

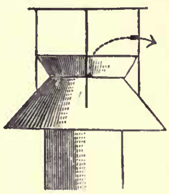


Fig. 1530.

"No systematic ventilation, however well devised and constructed, however extensive its supply of fresh air, however regularly or judiciously operated, can afford to dispense with the repeated displacement of the air of rooms and substitution of entirely fresh air through open windows and doors, at times, during all seasons of the year" (Briggs).

Methods of Heating.—The open fireplace grate heats by radiation, communicating heat to objects, which by contact transfer it to the air. Persons coming in contact with rays are themselves heated, while the air around them is cool and invigorating for breathing; the bright glow has a cheering and animating effect upon the system, somewhat like that of sunlight. As a ventilator, an open fire is one of the most important, drawing in air not only for the support of combustion, but also, by the heat of the fire and flue, making a very considerable current through the throat of the chimney above the fire. From this cause, although there is a constant change of air, yet there arises one great inconvenience of disagreeable draughts, especially along the floor, if the air-supply be drawn directly from the outer cold air; but in connection with properly regulated furnaces or stoves, the open fireplace becomes the most perfect means of heating and ventilation. As a heater merely, the open grate in very cold weather is not satisfactory; its influence is only felt in its immediate vicinity, and but from 10 to 15 per cent. of the heat of the fuel is rendered available.

Fig. 1531 represents an old form of open fire used in a tavern bar-room and office, which answered admirably for heating and ventilation, and admitted of access to many persons. It consisted of a circular grate at the level of the floor in the centre of the room. In the cellar beneath was an ash-pit, *a*, in brickwork, with an opening, *o*, to supply air for the combustion of the fuel. Above the grate was a counter-weighted sheet-iron hood, *h*, connected by a pipe with the chimney, which could be raised or lowered to suit the required draught. Around the grate was a ring-guard to rest the feet on, and the customers ranged themselves in a circle round the fire.

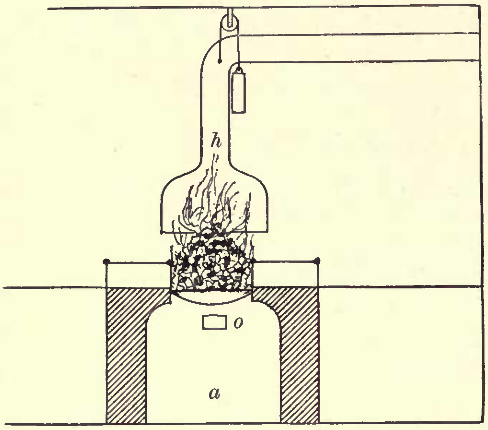


FIG. 1531.

Stoves.—Open stoves heat by direct radiation, and by heating the air in contact with them, and close stoves by the latter way only; as economical means of heating the latter are the best, and, when properly arranged, give both a comfortable and wholesome atmosphere. There should be some dish of water upon them to supply a constant evaporation, sufficient to compensate for increased capacity of the air for moisture due to its increased heat. In the hall there will be no objection to a close stove, letting it draw its supply of air as it best can; but in close rooms the open stove is best, on the plan of the old Franklin stove, or, if a close stove, somewhat on the plan of a furnace, with an outer air-supply for combustion and ventilation.

Stoves are made of sizes adapted to large and small rooms, in every style and with all possible appliances for comfort, convenience, and economy: self-feeders, in which the coal is furnished to the fire in a close chute from the top downward and in proportion to the coal consumption below; base-burners, in which the draught is reversed so that the base becomes a portion of the heating surface; doors with mica panels around the circumference, by which the fire is seen with the advantage of radiant heat and easy access to the fire-pot. Stoves are usually coal-burners, but plain box-stoves in cast or sheet iron are well adapted for wood-burners and for holding fire and retaining heat. They have appliances for controlling draught by dampers or by opening the smoke-pipe or flue to the room, thereby reducing the velocity of draught, but not throwing the products of combustion outward into the room, as is done by dampers.

Hot-air furnaces are close cast-iron stoves, inclosed in air-chambers of brick or metal, into which external air is introduced, heated, and distributed by metal pipes to the different rooms of a house. Furnaces have been of late very much decried, but under proper regulation they are a very cheap, economical, and even healthful means of ventilation and warming. The heating surface should be very large, the pot thick, or even incased with fire-brick, that it may not become too hot; there should be a plentiful supply of water in the chamber for evaporation, perhaps also beneath the opening of each register; the air supply should always be drawn from the outer air and unobjectionable sources, through ample and tight ducts, without any chance of draught from the cellar; the pot and all joints in the radiator should be perfectly gas-tight so that nothing may escape from the combustion into the air-chamber. With these provisions on a sufficient scale, and proper means for distribution of the heated air and escape of foul air, almost any edifice may be very well heated and ventilated. The air should be delivered through the floor or the base-board of the room, and at the opposite side from the flue for the escape of foul air, making as thorough a current as possible across the room, and putting the whole air in motion. In dwelling-houses the fireplace will serve the best means of exit; in public rooms distinct flues will have to be made for this purpose, and they should be of ample dimensions and well distributed, with openings at the floor and ceiling with registers, and means should be provided for heating the flues. An architect, in laying out flues for heating and ventilation, should, both in plan and elevation, fix the position of hot and foul air flues, and trace in the current of air, always keeping in mind that the tendency of hot air is to rise; he will then see that, if the exit-opening be directly above the entrance-flue, the hot air will pass out, warming the room but little; if the exit-opening be across the room and near the ceiling, the current will be diagonal, with a cold corner beneath, where there will be very little circulation or warmth. To heat the exit-flue, a very simple way is to make the furnace-flue of iron, and let it pass up centrally through the exit-flue; but the current may be obstructed by a high wind.

Fig. 1532 is one of the many forms of furnaces which consist of the most approved stoves, with a large heating-chamber above—in the figure of cast-iron, and composed of numerous flues, but very often a drum of wrought-iron. The whole is inclosed in a brick chamber; in those denominated portable furnaces the case is of galvanized iron. The figure contains the usual appliances for feeding and clearing fires, with a check draught opening from outside into the smoke flue, and dust damper and flue so arranged that when the grate is shaken no ashes or dust comes into the hot-air chamber. The air is introduced at the bottom of the case, passes up and around the stove, and out through the ducts to different parts of the building. The water-pan is indispensable to the hot-air furnace, and should be of capacity enough for a day's supply, or have automatic means of keeping up the supply.

Air in winter is very dry, but as its volume is enlarged by heat it draws a supply of moisture from everything with which it comes in contact—from the skin and lungs, creating that parched and feverish condition experienced in many furnace-heated houses; from furniture and woodwork, snapping joints and making unseemly cracks. Thus, taking the air at 10° and heating it to 70°, the ordinary temperature of our rooms requires about nine times the moisture contained in the original external atmosphere, and, if heated to 100°, as most of our hot-air furnaces heat the air, it would require about twenty-three times.

The portable furnace is not so economical as the furnace set in brickwork, as more heat escapes through the metallic case. The former are usually made

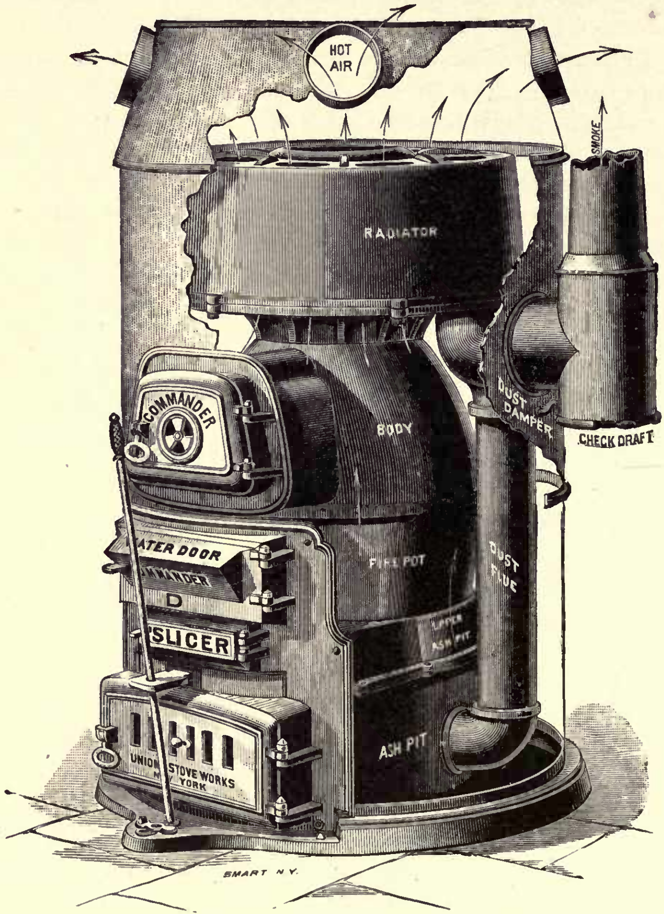


FIG 1532.

from 12″ to 36″ diameter of pot, from 2′ to 6′ outside diameter, and 5′ to 6′ height of case. The brick-set furnaces are from 20″ to 32″ pot, outside brickwork from 5′ to 6′ square, walls 4″ thick, height 6′ to 7′. It is difficult to give any rule for the heating capacity. A 22″ pot should be adequate for the heating of a common 25′ × 60′ city house, and the higher the air-duct the less its diameter.

The total sectional area of the hot-air educts should be equal to that of the fire-pot; that to the first floor should be larger than to the other floors, since the column of hot air is shorter it will have less velocity. Air ducts that have outlets at the same level under the same conditions should have greater area if the horizontal pipes are longer. The cold-air duct should have about the same area as the grate, and the inlet should be above the level of the street or back area to avoid dust. If air ducts lead from both sides of the building to the furnace-chamber, the current can be controlled according to the wind, and the hot air distributed more equably through the building.

The Baltimore heater (Fig. 1533) was the earliest union of the stove with the furnace. A stove is set in the fireplace of a room in a lower story, of which the exterior or ornamental half is exposed for the heating of this room, while the inner half acts as a furnace for the upper rooms. The smoke-pipe passes up into a chimney above, and is inclosed by an air pipe or jacket to which heat is communicated and distributed by drum pipes and registers to upper stories.

Steam and hot-water circulation are applied to the heating of buildings by means of wrought- or cast-iron pipes connected with boilers. In the simplest form, as common in workshops and factories, steam is made to give warmth without ventilation by direct radiation from wrought-iron pipes. The general arrangement is by rows of 1″ to 1½″ pipe hung against the walls of the room, or suspended from the ceilings, 3′ of 1″ pipe being considered adequate to heat 200 cubic feet of space; if there are many windows in the room, or the building is very much exposed, more length should be allowed.

Steam, as a means of heating, is the most convenient and surest in its application to extensive buildings and works. From boilers, located at some central point, steam can be conveyed to points so remote that in many cities it is a matter of sale both for heating and power purposes. The limits of the extension of steam-pipes economically have not yet been determined, but within the range of the buildings occupied by any single textile manufacturing industry steam-heating has proved satisfactory, and is almost universally adopted. For stores, warehouses, large buildings of all sorts, where there are extensive or numerous rooms to be heated, steam has been long used, and the appliances for its use can be as readily obtained in all our cities and large towns as stoves or grates. Steam is used for heating at either high or low pressures; under 5 or 6 pounds would be considered low pressure. A low-pressure apparatus may draw direct from a boiler, or be supplied from the exhaust of a steam-engine; if from

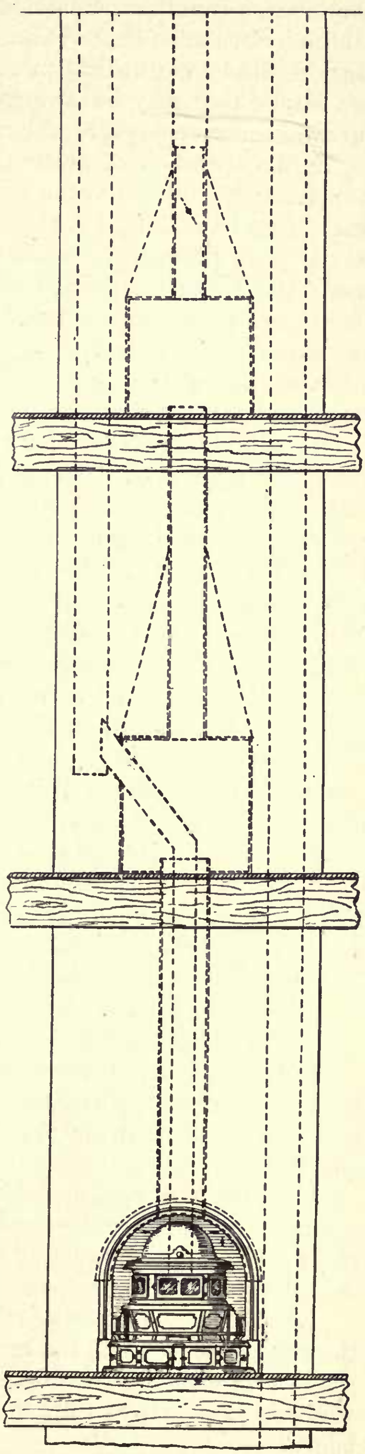


Fig. 1533.

the latter, a certain amount of back pressure must be put on the engine to establish a circulation in the steam-heating pipes. But the loss in power is more than repaid by the utilization of the heat in the steam. If the heating-pipes are ample, they may be arranged to act as condensers, reducing the back pressure below atmosphere and supplying low steam for heating.

In the operation of heating by steam, the steam, in giving off its latent heat through the pipes to the air of the room, returns to water; the apparatus would then be nothing but pipes to convey the steam to radiators to condense it, and pipes to return the water to the boiler, were it not for air invariably in water and steam. This necessitates a more complicated circulation; there should be a regular flow outward of steam from the boiler, and inward of water and steam to it, which must be provided for in the design and by care in construction that there are no corners or angles forming eddies where air can lodge, and with provision for its continuous movement.

When hot water is used for heating, there must be circulation throughout the system; the water flows out from the top of the boiler, gives out its heat, and returns, practically of the same bulk, cold to the bottom of the boiler, and any radiator out of the line of this current is of no use.

Both steam and water are used for heating rooms either *directly* or *indirectly.* Direct heating is like that of common stoves, without any considerations for ventilation; indirect heating, like that of hot-air furnaces. Radiators are inclosed in a box or chamber, into which air is drawn or forced, and then distributed by ducts to the rooms to be warmed and ventilated. With steam or hot-water heating, the metallic surfaces brought in contact with the air usually range from 212° to 250°, while the pot of the air-furnace is often from 900° to 1000°. In a sanitary point of view hot-water or low-steam coils in air-chambers are a more surely healthy means of warming and ventilation; the greatest objection is their expense, the care requisite in attending them, and the danger of freezing and bursting the pipes if worked intermittently in winter. In the arrangement it is usual in dwelling-houses to place the coils at different points in the cellar, as near as possible beneath the rooms to be heated. In public buildings frequently a very large space in the cellar is occupied by the coils, into which the air is forced by a fan, and then distributed by flues or ducts throughout the building.

All inlet or outlet ventilating flues should be provided with dampers or registers to control the supply or discharge of air, cutting it off when sufficient heat is secured, or retaining the warmth when ventilation is not required.

Fig. 1534 is an elevation showing the usual arrangement of mains, *s s*, and returns, *r r*, when the horizontal distance from the boiler is small and the risers few. The inclination of the mains is toward the boiler, and their condensed water returns by them to the boiler.

Fig. 1535 is the better practice, and necessary if the steam is high pressure, the mains extended, and the branches numerous. The inclination of the mains, *s s*, is from the boiler, and the condensed water flows down to the lowest angle, where it is connected with the return, *r*, and is by this brought back to the boiler.

The size of the boiler for a steam-heating apparatus is based on the amount of radiating surface, which must include that of the steam-mains and of the returns.

In Fig. 1535 the steam riser, *s*, descends from the boiler to the last riser, which is connected at this point with the return, and this should obtain in all forms of steam-heating, keeping the flow of condensed water as far as possible in the direction of the flow of the steam, and removing it from the steam-pipes.

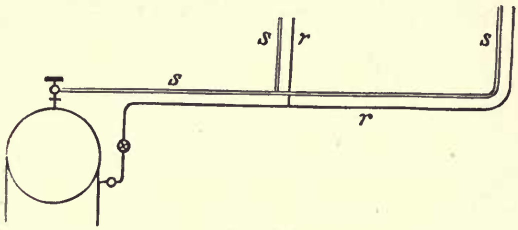


FIG. 1534.

Figs. 1536 to 1538 are common forms of distributing steam and return pipes of different systems of heating.

In Fig. 1536 the riser proceeds from the boiler and leads directly to the highest point of service. Provision is made for the condensation in the pipe by a direct connection.

In Fig. 1537 the risers are as in Fig. 1536, but there are separate returns for each radiator.

In Fig. 1538 the main riser is carried directly to the highest story to be warmed, and the distributing mains are led from it with a pitch from the riser, and the descending pipes conveying the steam to the different radiators and the condensed water to the hot well and boiler. This should be the quietest circulation, as the steam, except in the main riser, does not interfere with the flow of the condensed water.

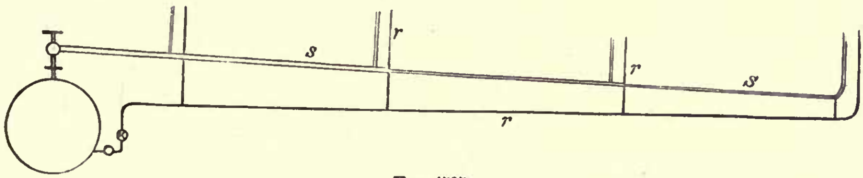


FIG. 1535.

Valves are introduced in the mains or returns where necessity or convenience may require the shutting off of any of the radiators or mains, and air-cocks to relieve stagnation in angles or pockets where circulation must be established before the whole of the heating surface can be utilized.

The amount of radiating surface depends on the cubic feet of air to be heated and the number of degrees to which it is to be heated. These are determined from the outer exposure of the building or room, its plan, material, and construction, whether there is more or less window surface, and its occupancy; whether for living rooms or for business, sedentary or active; in the store the bookkeeper needs much more heat than the salesman.

On an average, 100 to 150 cubic feet of room space can be heated from 0° to 70° by 1 square foot of radiating surface—say 3′ of 1″ pipe. This covers an average glass exposure which may be taken, according to Mr. Briggs, at 100 cubic feet of space to each square foot of glass.

The effect of glass under air exposure is to be noticed in the different con-

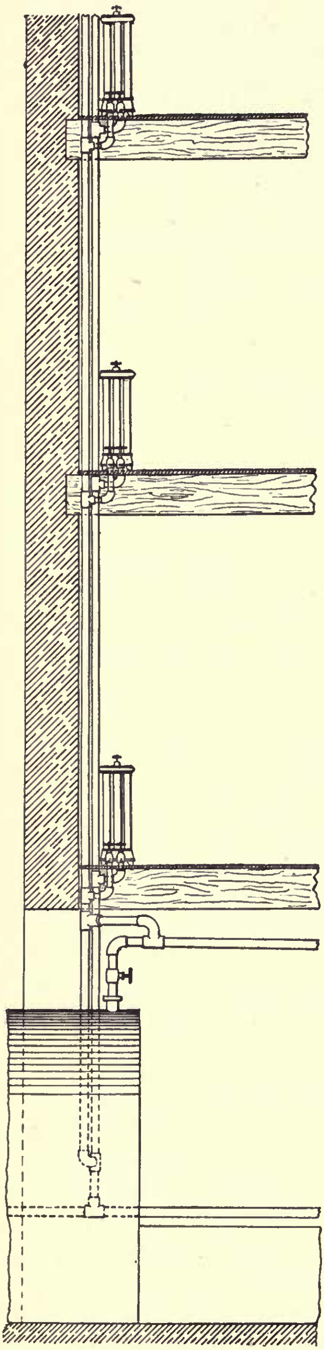


FIG. 1536.

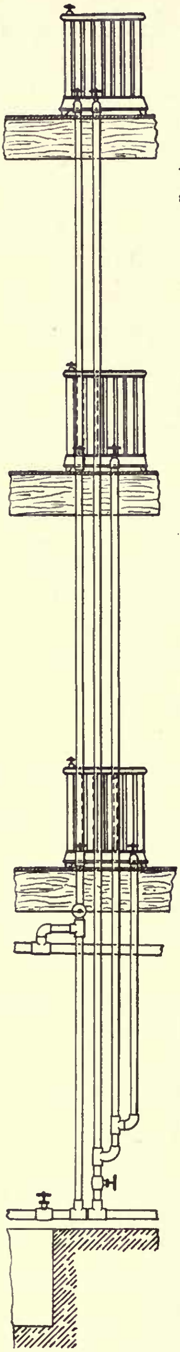


FIG. 1537.

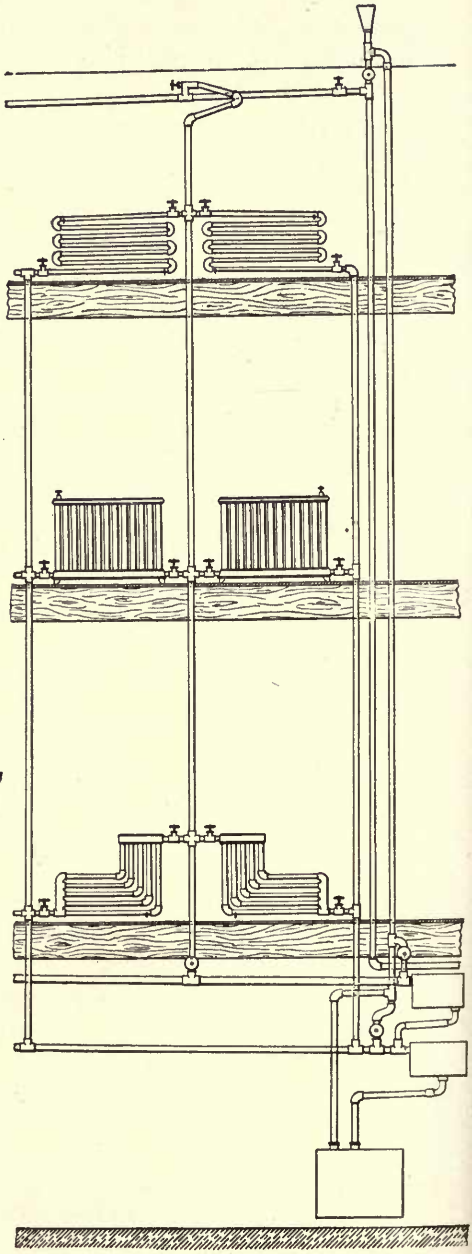


FIG. 1538.

ditions experienced in cars while in motion and when stopped in cold weather, and the advantages of double windows in these conveyances.

Proportions of mains to radiating surface: one of 1″ diameter will serve for 75 feet of radiating surface, including that of the mains. One and a half inch diameter for 250 square feet, 2″ diameter for 500 square feet, 3″ diameter for 1,250 square feet, 4″ diameter for 2,500 square feet.

For the returns, one size less than that of the steam mains is the rule; thus, a ¾″ return for a 1″ pipe, but no pipe of less diameter than ¾″ is used; for a 2½″ steam a 2″ return, and a larger than 2″ is seldom used. It may not be always practicable to return the condensed water, as shown in the figures above, by gravitation, but there are various forms of receivers or traps in which the water is collected and returned by hand or automatically pumping to the boiler.

Fig. 1539 is a float trap, in which *p* is the pot with a tight cover, *f* an open float sliding on the stem, *s*, at the foot of which is a valve, *v*; the condensed water flows in through the inlet, *i*, raises the float, *f*, and closes the valve, *v*; eventually the condensed water overflows into *f* till it sinks and opens the valve, *v*, and the condensed water flows out through the valve, *v*, under the pressure of steam at the inlet; when blown out, *f* rises and *v* closes for another charge. The condensed water is either wasted or returned to the boiler by a pump. The hand valve, *h*, is an independent relief to the trap.

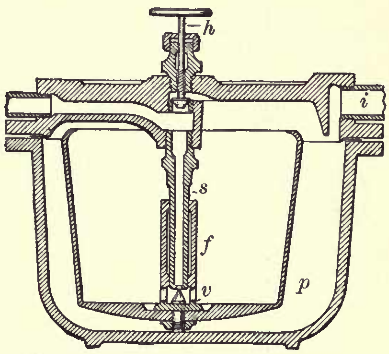


FIG. 1539.

There are traps which return the condensed water directly to the boiler, in which the condensed water is forced into a chamber above the level of the water in the boiler and the steam pressure then brought upon the chamber, and the water flows down from it into the boiler.

Heating by indirect radiation is like that by hot-air heaters; the heaters are inclosed in chambers to which cold air is introduced and the heated air conveyed into different rooms by pipes. It is usual to place the hot-air chambers in the cellar and basement, as directly beneath the room to be heated as possible, and extending the cold-air duct to the hot chamber. The chamber is usually made of galvanized iron in a wooden box, and often suspended from the ceiling of the cellar. Where the heating is indirect, as there are more cubic feet of air to be heated, the radiating surface is to be increased, usually to about three times that of the direct heating.

Heating by Hot Water.—The principle of it is based on the fact that water when heated becomes of greater volume and less density, and rises; coming in contact with cooler surfaces, it loses its heat and descends; the water circulates by the addition and reduction of heat. The possibility of a rapid and thorough circulation gives efficiency to the apparatus. One of its greatest advantages is, that when necessary the circulation will continue with a water temperature at an extremely low point, say 110°, and a proportionate consumption of coal.

Fig. 1540 exhibits the application of hot water to the heating of a building. The boiler shown will serve as an illustration of the water circulation; it is of cast-iron, of which there are numerous forms in this material, as well as wrought-iron.

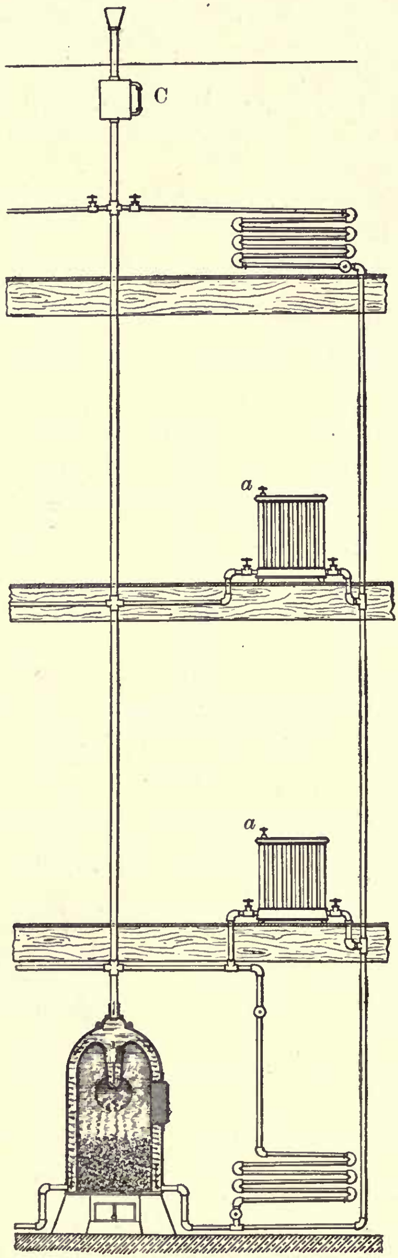


FIG. 1540.

The main rises directly to the top, where there must be an expansion chamber, C, in connection with it, to provide for the increase of volume in the water due to the heat. It is usual to have this chamber open at the top, and water may be poured down through the funnel nozzle. A glass at the side shows the level of the water.

The mains are of a little larger diameter than in steam, and reduced in size as the current is distributed into the radiators on which valves are placed to control the current; these valves should be gates, to provide for full water-way. The returns are to be of the same diameter as the rising mains. The surface of the radiator should be greater than for steam, as the temperature of the water is less. When the radiators are vertical, as shown in the first and second story, there must be air-cocks, *a*, as shown, for air effectually cuts off circulation. Hot-water circulation can be used for direct or indirect heating with like appliances as for steam.

Boilers are of such varied forms and proportions to the area of grate that it is impossible to determine the value of the heating surface except by actual test. As a unit it is better to refer to the area of grate as a measure of the capacity of the boiler, the evaporation of water by the combustion of pounds of coal being the standard of efficiency.

Under the head of chimneys it is stated that two square inches of flue is sufficient for the combustion of each pound of coal per hour. This has been found in excess for a 5-foot diam. chimney, where provision has been made for avoiding eddies by the rounding of corners, and where there is a good natural draught, but there must be a factor of safety when care has not been taken in the location and construction. For the small flues connected with the heating of common buildings this rule will not obtain; flues for this purpose should be at least 8″ × 12″. But as a large area of chimney flue does not interfere with the draught, and as the necessities of chimneys usually increase by the extension of works, it is safer to make the flue larger, but without omitting care in construction.

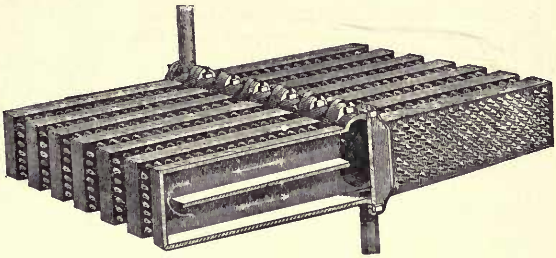


Fig. 1541.

The combustion of coal on small grates may be taken at 4 to 6 pounds of anthracite coal per square foot of surface; on grates 4′ × 4′, 8 pounds; on grates of larger area, 10 pounds, which would be a fair average of this class, and the vents of large chimneys may be calculated on this data, although the draught under equal conditions is in favour of the chimneys of large diameter.

With forced draught by fans, ejectors, or, as in locomotives, by the exhaust, very large quantities of coal can be consumed on grates.

The air-ducts from boiler and heaters for smaller and private dwellings have a natural draught; but for schoolhouses, churches, and other public edifices a forced draught is usual, is under surer control, and with adequate indirect heaters the quantity of air can be readily furnished. It is common to introduce local fans driven by electric motors to supply air at varied points and in quantities suited to the needs of the position. By the use of dampers or valves air of any temperature, from that of the outside to that of the radiator, may be distributed.

In Figs. 1536, 1537, 1538, and 1540 illustrations are given of the usual radiators: the wall coil with return bends, which may be more or less open (with branch Ts and multiple coils they are called box coils), wall coils with branch

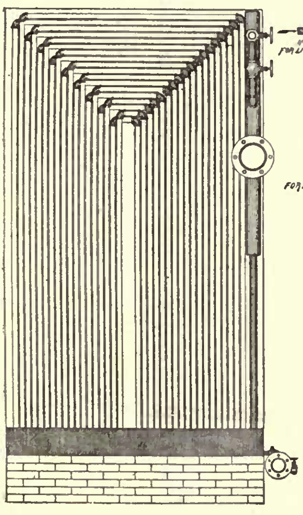


Fig. 1542.

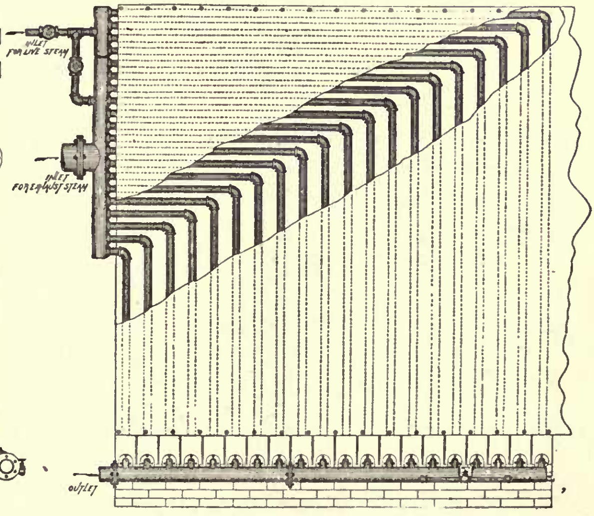


Fig. 1543.

Ts, vertical tube radiators with box bases of effective heating surface, cast-iron radiators of similar design, ornamented and of great variety. Indirect radiators, Fig. 1541, in cast-iron with pin or iron projections, and wrought-iron pipes covered with compound coils of wrought-iron ribbons, increase heating surface. In measuring the surface of circulating coils include the lengths of angles and all fittings; in the vertical radiators include the base.

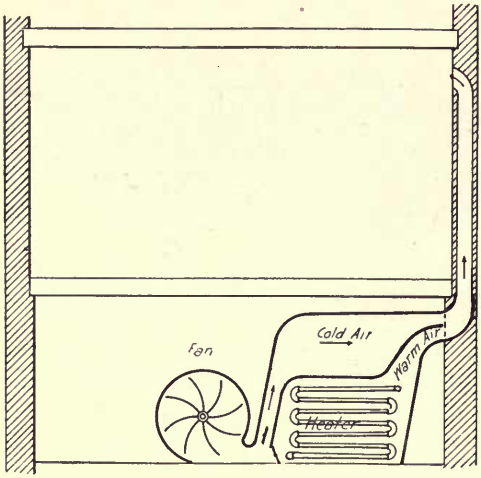


Fig. 1544.

Figs. 1542 and 1543 are the end and side elevation of a radiator for live or exhaust steam, inclosed in a case for indirect heating. Air is forced through the case into the building by a fan.

By the location of the radiators in an independent chamber and a valve, Fig. 1544, the air may be forced through it hot, or mixed with cold air or without connection with the chamber cold.

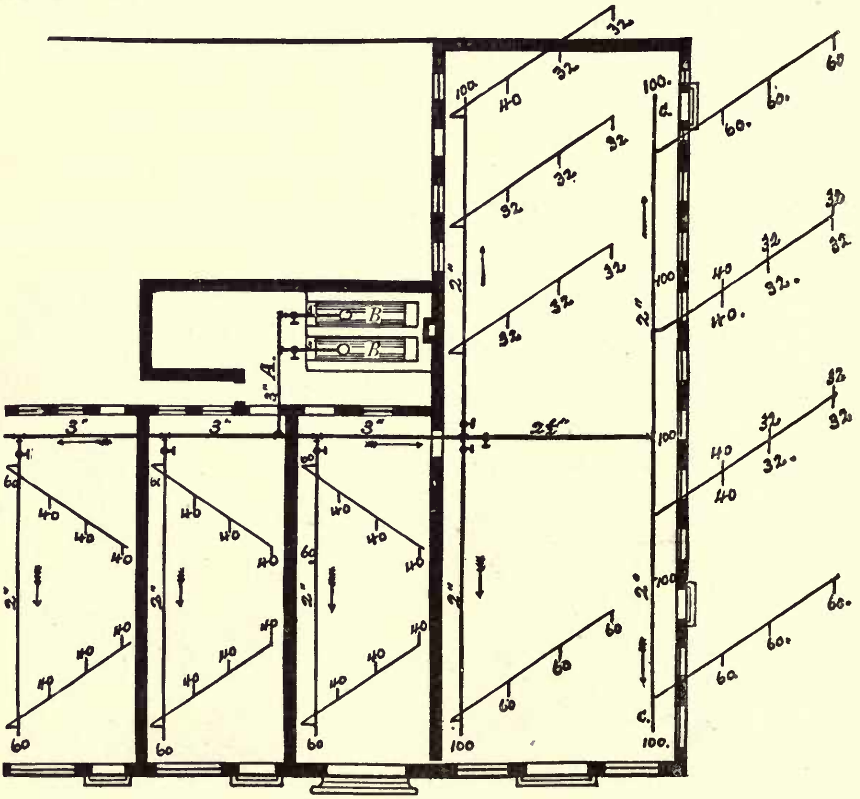


Fig. 1545.

Fig. 1545 is the plan of a portion of a large building heated by steam. B B are two boilers, either of which would be sufficient for the purpose; the steam mains are shown by black lines following those of the building, with the sizes marked upon them; the risers by inclined lines, with the square foot of radiating surface on each story marked. This is a very convenient form of drawing, explanatory of the system. It is usual to draw the steam mains and risers in red and the returns in blue, with the diameters on each.

Plumbing.—The conveniences for comfort in modern buildings require the introduction of water and its removal. Most cities have water-supplies and a system of sewers, and the plumber makes the connections with both. In the country, where there are not these public conveniences, their places are largely supplied by pumps and elevated tanks and by cesspools. The quantity used in each household varies with the wants and habits of the occupants. An average bath will take 25 gallons; each use of a water-closet from 1 to 3 gallons. A wash-tub will hold from 10 to 20 gallons. If the water is to be pumped by hand, from 7 to 10 gallons may be reckoned as the daily use by each person; if from aqueduct, 30 to 50 gallons is ample. With the popular style of water-closets the use of water has been largely increased, and by carelessness in the selection and use of fixtures the waste has become greater.

The regulation size of taps for city mains is from ½″ to 1″, and the pipes

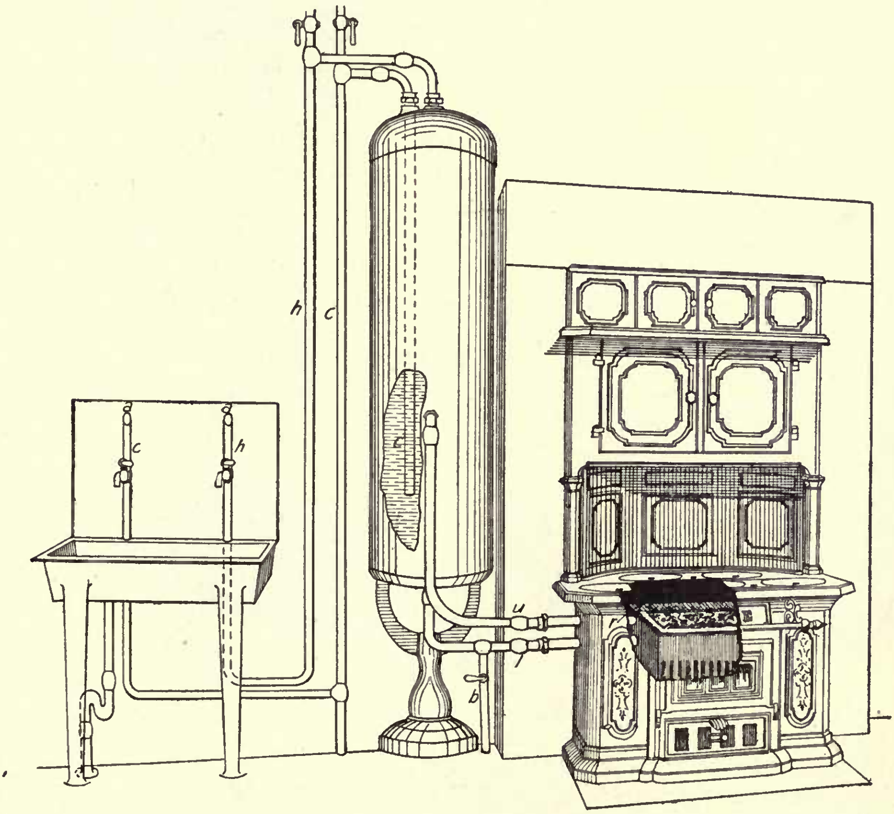


Fig. 1546.

leading into the house from $\frac{3}{4}''$ to 1″ diameter. The pipes are usually of lead, as most waters are not affected sensibly by lead, if the pipes are always kept full, but water which has stood for some time in the pipe should not be used for drinking, and lead-lined tanks should be coated with asphalt varnish. In some cases block-tin pipes are used; or iron, galvanized, or coated with some preparation of asphalt, or glass-lined.

The soil or house-sewer pipe connections with the main sewer or cesspool are usually vitrified stoneware pipe, from 4″ to 6″ diameter, the large size is for the discharge of the sewage, and the rainfall from the roof. Within the house the pipe is either of stoneware or cast-iron; invariably of the latter if the pipe is exposed. The rising pipe to the roof is here, also, usually of cast-iron, and 4″ diameter may be considered ample for a common house; branches as small as 2″ are usually of lead.

Fig. 1546 is the perspective of a kitchen-range boiler and sink: *c* is the cold-water pipe leading to the sink and to the boiler; it enters the top of the boiler, and is led down nearly to the bottom. The hot water is drawn from the top, through the pipe *h*, is led down to the sink and up for distribution through the house. The water is heated in the boiler by the water-back, which consists of a closed-box casting, forming the back of the range, *r*, with two connections with the boiler, the one at the bottom introducing cold water, and the one at the extreme top discharging it heated into the boiler above, the circulation taking place as in hot-water heating; the water flows through the pipe, *l*, is connected with the lower part of the water-back, and returns by the pipe, *u*, from the top of the water-back to a higher point in the boiler; *b* is the blow-off pipe. Stoves are arranged with water-backs.

The pipes, *a a*, are carried above the draw-cocks over the sink, forming air-chambers, to cushion the blow of the water-hammer when the cocks are shut quickly. Beneath the sink there is a trapped connection with the sewer-pipe.

Fig. 1547 is the elevation of a galvanized-iron boiler, but those in general use here are of copper.

Fig. 1548 is the perspective drawing of a cast-iron sink,

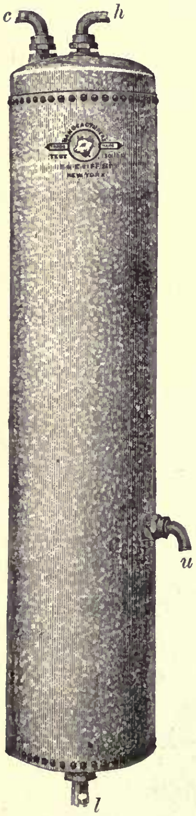


FIG. 1547.

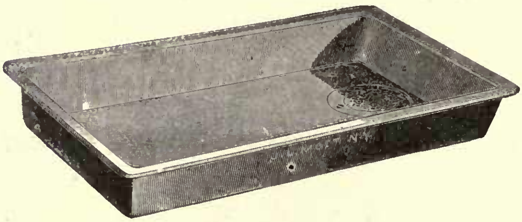


FIG. 1548.

of the usual form and material. They are to be obtained of all suitable dimensions, rectangular, from 16″ × 12″ × 5″ deep, to 96″ × 24″ × 10″ deep; also, half-circle and corner sinks, and deep and slop sinks.

In the kitchen, or a laundry-room adjacent, tubs are set for washing, with hot and cold water service. The water-pipe connections are usually $\frac{3}{4}''$, the waste connections 2″. The tubs themselves are mostly of wood, but there are many of cast-iron (Fig. 1549), galvanized or enamelled, of slate, of earthenware, and of soapstone.

In the butler's pantry there is usually a sink of planished tinned-copper, with hot and cold water connections. In the chambers and dressing-rooms

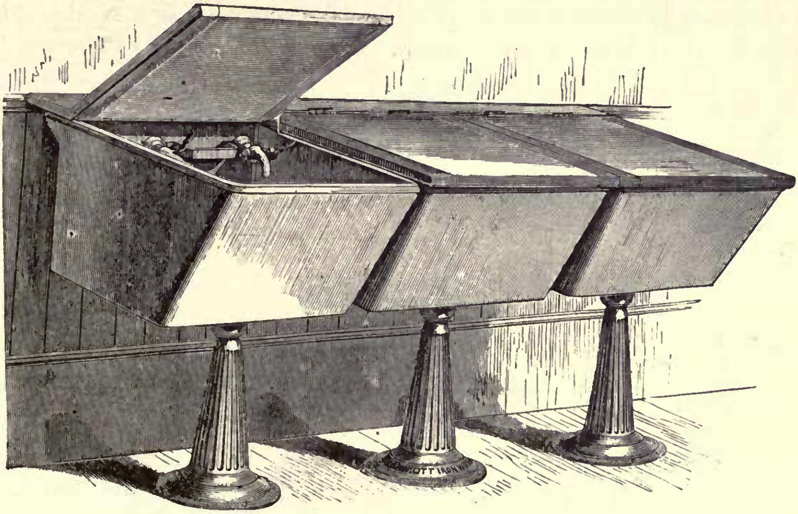


FIG. 1549.

wash-basins, usually of porcelain or porcelain-lined cast-iron, are set with like connections. The sizes of basins vary from 12″ to 18″ outside diameters.

Fig. 1550 shows the usual form of setting of a wash-basin in a countersunk marble slab, with a back of the same material; swing faucets for the supply of hot and cold water; self-closing faucets prevent waste, and compression cocks are best suited for high pressures. The waste is closed by a metal or rubber plug, attached to a chain, with the other end fastened to a pin in the marble slab. The sides are inclosed with wood, forming a closet beneath the basin, with usually small drawers for towels at each side of the closet. It is cleaner and neater to support the slabs and

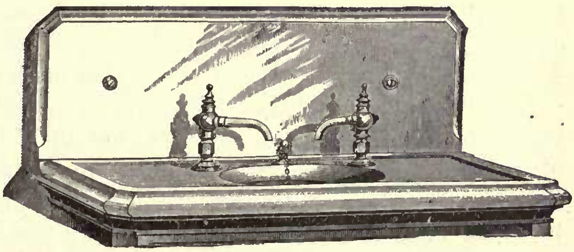


FIG. 1550.

basins by a metal frame and posts or brackets, with the traps and pipes exposed. A later form of wash-basin is an oval (largest size 19″ × 15″, outside measure), which admits of the washing of the head and shoulders, with a standard waste or hollow pipe overflow, preventing offence in the oxidation of the soapy waste which obtains from pipes, formed on the basin, and which can not be readily cleaned. Fig. 1551 is a plan, and Fig. 1552 a section, of a cast-iron bath-tub porcelain-lined; there are cocks for hot and cold water; for the discharge there is a standard waste, which can be taken out entirely, but there is often a common plug and an overflow by an independent pipe.

The dimensions of tubs are varied to suit the rooms in which they are to be placed; the largest are 6′ × 24″, 18″ and 19″ deep. They may be reduced to 3′ 6″ in length, but should then be made deeper. Tubs of porcelain are set up on blocks; porcelain-lined, on cast-iron legs, about 6″ in height. Bath-tubs are more generally made of planished tinned-copper in a wooden box for support, and inclosed by wooden panels. In most bath-rooms there are basin

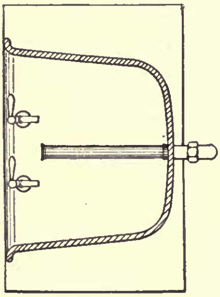


Fig. 1552.

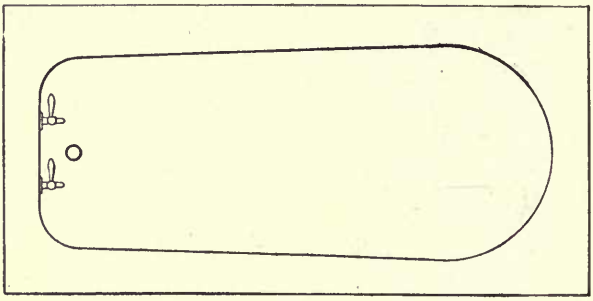
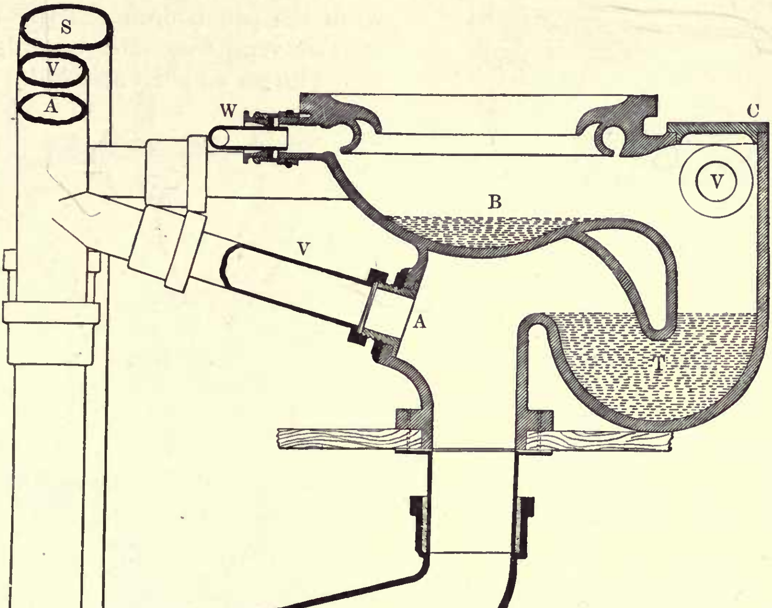


Fig. 1551.

and water-closet—often a foot-bath and bidet-pan—but it is preferable to make the water-closet a separate room, distinct, with its own water and sewer-service and means of ventilation.

The construction of one form of water-closet, with all the modern appliances for the removal of soil and for ventilation, will be understood from the section (Fig. 1553). The seat is not shown, but is just above the basin, B, which contains some water to receive the defecations, to prevent the soil attaching to the side of the basin, and in a measure to check its offensive smell. T is the trap or water-seal which prevents the smell from the soil-pipe S passing up through the basin. The water-discharge from the pipe W is through a rim-flush around the edge of the basin. The sudden discharge washes out the basin B into the trap T, which is also cleaned by the rush of water. The soil-pipe S extends up through the roof, and may or may not also serve as a rain-leader. A sudden flow of water down the soil-pipe often acts as an ejector to draw the water out of the trap T, and break the water-seal; to prevent this there is a back-air connection, A, leading also to the top of the house. But as the offence of a water-closet is largely due to its recent use, and as smell once getting into the room is with difficulty removed, but generally diffused, there is a ventilating-pipe, V, connecting the basin B with a ventilating-flue; this is the most important part of the apparatus; connected with a chamber commode, it would remove all smell, and if there were no trap to the soil-pipe, or were the water-seal broken, it would still prevent any offensive smell from penetrating the house. If the soil-pipe be made also a ventilating-pipe, as is frequently done by its connection with the hot-air flue, then the trap and pipes A and V are unnecessary.

Many sanitary engineers object to the back-air pipe as imperfect in its action (a sudden suction will draw the water out of the trap before the suction can be relieved through a back-air pipe), that it is expensive, and that there are many better ways of protecting the seats by anti-siphon traps or diverging Y-



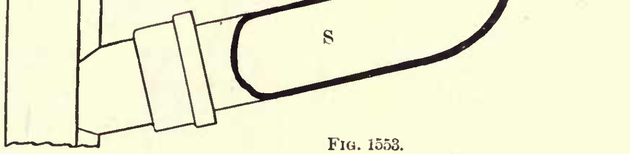


Fig. 1553.

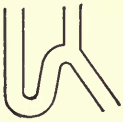


Fig. 1554.

branches. In Fig. 1554 is shown a branch by which the flow from the upper pipe dripping into the lower trap preserves the water seal. As the water-closet must be ventilated, it is better to have a regular flue in the wall with an induced ventilation by heat in some form, or electric fans, and air supplied from outer rooms or windows.

Fig. 1555.

Fig. 1555 is an elevation of the simplest form of closet—the *hopper*-closet—and in many respects the best. A standard waste in the cistern *c* will serve both for the disk-valve and the overflow. A rim-flush is supplied through the pipe W, controlled by a plain cock or by a handle *h*, as in Fig. 1556, lifts the disk valve, closing automatically; or by cock connecting with the seat, which, when down for occupancy, opens the flush, and, as the sitter rises, the counterbalanced lid rises and closes the cock. The service box beneath the cistern continues a temporary flow after the valve is dropped. V is a ventilator branch.

Fig. 1557 is the section of a *pan*-closet, for many years the most popular closet. The copper pan, when shut, cuts off the view of the trap below and any odour from it; with a small flow of water the basin is readily kept clean, but soil is apt to lodge in the iron receiver, and the odour to arise from it when the pan is down. There is an annular ventilating-tube beneath the seat, with an air-shaft attached, but of

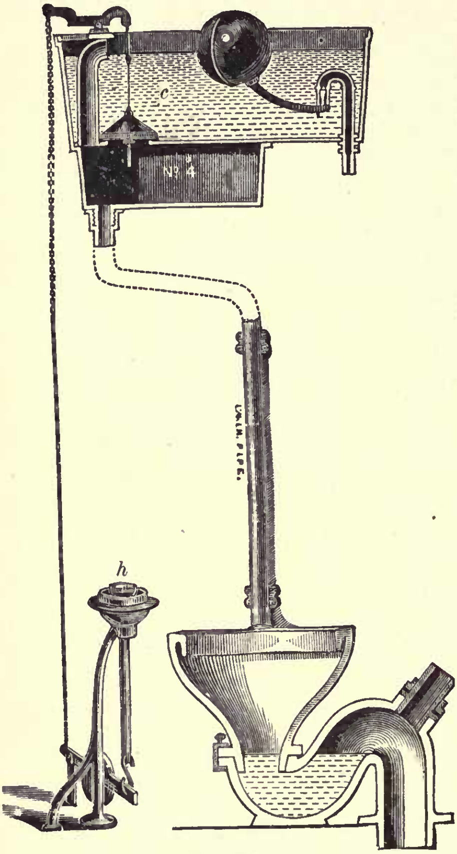


Fig. 1556.

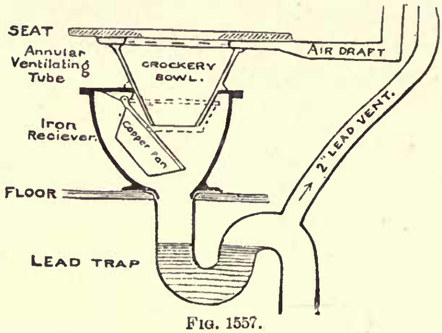


Fig. 1557.

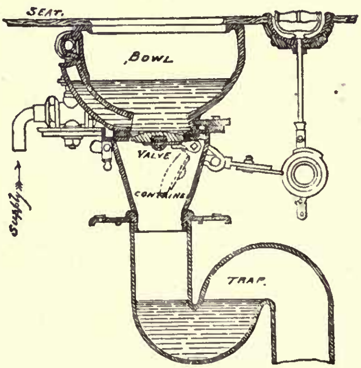


Fig. 1558.

altogether inadequate dimension for the purpose, as may be said of all such vents attached to water-closets. There is also the air-vent to prevent the water being drawn from the trap. No water connections are shown in the figure.

If the 2″ vent be removed, and the air-draught pipe be enlarged and connected with a positive draught-flue, all offence from either recent or former use of the closet will be cut off.

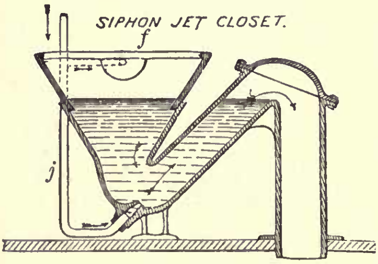


Fig. 1559.

Fig. 1558 is the section of a *flap*-closet, in which a flap-valve supplies the place of a pan.

Fig. 1559 is the section of a siphon-jet closet. In addition to the *fan* flush, *f*, into the basin, it has a jet-pipe, *j*, at its bottom, inducing a current in the direction of the inclined leg of the trap, and by flush and jet the water is siphoned from the basin.

Traps are varied in their form, but all to cut off the air-connection of the soil-pipe with the room in which the appliance is placed. The smaller traps are invariably lead, the larger cast-iron.

Figs. 1560 to 1567 represent the usual forms of lead traps. There are screw-plugs at the bottom of the traps, which can be taken out to remove

S.


FIG. 1560.

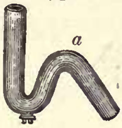
¾S.


FIG. 1561.

½S.

FIG. 1562.

RUNNING.

FIG. 1563.

RUNNING Y.

FIG. 1564.

BAG.

FIG. 1565.

SHORT BEND.

FIG. 1566.

LONG BEND.

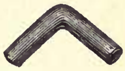



FIG. 1567.

any obstruction. As the water may be drawn out of any trap by the passage of water down the pipe with which it is connected, air-vents, as already described, in the water-closet trap, are put on these small traps. But if upper waste pipes be inserted, as in Figs. 1560 and 1561, at *a a*, and shown in section (Fig. 1554, p. 653), loss of seal is cut off.

Figs. 1568 and 1569 are cast-iron traps, with a cap that may be removed to clean the trap, or the aperture may be used for air-vent connection.

Fig. 1570 is the section of a *bell*-trap, used on sinks, with a strainer, S, above it.

Fig. 1571 is a plate with plug, for the bottom of basins and bath-tubs.

S-TRAP.

FIG. 1568.

TRAP WITH SIDE OUTLET.

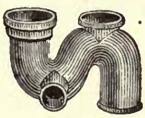



FIG. 1569.

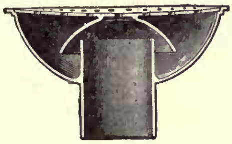


FIG. 1570.

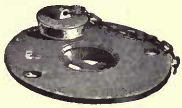


FIG 1571.

Figs. 1572 to 1577 are common cast-iron bends or angles.

Figs. 1578 to 1583 are cast-iron branches. The T-branch and cross are objectionable, as the flows from the branches and mains are at right angles, and mutually obstructive; whereas in the Y, especially in the full Y, the flows are at acute angles with each other, and the currents converge. Similar fittings are used for water, but they are much heavier.

QUARTER BEND.

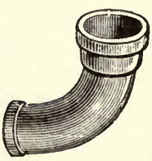



FIG. 1572.

DOUBLE HUB, QUARTER BEND.

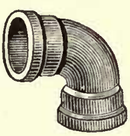



FIG. 1573.

EIGHTH BEND.

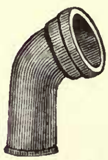



FIG. 1574.

SIXTH BEND.

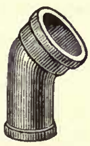



FIG. 1575.

SIXTEENTH BEND.

FIG. 1576.

RETURN BEND.

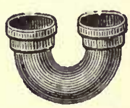



FIG. 1577.

Most water-closet basins are inclosed by a lidded seat and riser, but the less wood-work about a basin the better. The seat is generally hung with hinges of brass or composition, so that it can be raised, and the basin makes the

T-BRANCH.

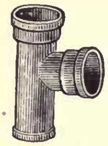


FIG. 1578.

CROSS-HEAD.

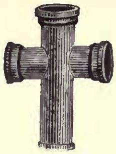


FIG. 1579.

Y-BRANCH.

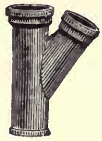


FIG. 1580.

HALF Y-BRANCH.

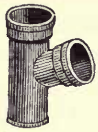


FIG. 1581.

DOUBLE Y-BRANCH.

FIG. 1582.

DOUBLE HALF Y-BRANCH.

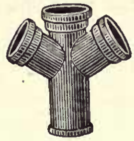


FIG. 1583.

best urinal for men, the upper edge of the basin being covered with an earthenware tray, sloping toward the basin. Instead of the tray, if the rim of the basin be made square, of sufficient width to support the seat, sloping to the basin, and overhanging, it will make the better urinal, and afford space for an adequate ventilating pipe.

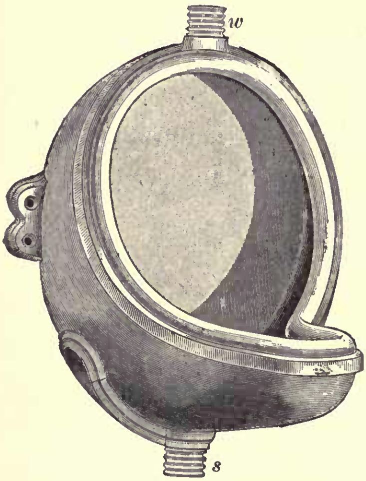


FIG. 1584.

Urinals, of which one form for males is shown (Fig. 1584), are often used in public buildings, and in open stalls. Although they have water connections, *w*, and a rim flush, it is almost impossible to keep them sweet; a cake of carbolic soap is often put in the basin, but the most effectual means adopted on many railway-cars is a piece of ice.

As direct supply from the service is uncertain if there is a draught in another quarter, it is now common to have small cisterns for closets, shown in Fig. 1556. The water from the service pipe is discharged well below the surface overflow, generally through a pipe attached to the goose-neck on the ball-valve pipe, to avoid the noise of running water.

Lighting is one of the present necessities of civilization, and for a great many years gas has been used for lighting in domestic and industrial buildings.

Gas fittings are in all forms—brackets and pendants, wide branches with fixed, swing, and slide joints; and burners in great variety—bat wings, fishtail tips, and Argand burners, and many patents for increased light and economy of consumption.

Service mains are seldom placed in buildings less than $\frac{3}{4}''$ diameter for 10 burners and 100 feet of pipe, to $1\frac{3}{4}''$ and 200 feet length.

Electric lighting, although not superseding gas for common use, is now largely employed in industrial operations, and for all shows and brilliant illuminations.

The three accompanying drawings (Fig. 1584, A, B, C) illustrate three methods of wiring for electric lighting.

Fig. A is a series installation. The lamps are in series and dynamo series wound. The wires may be led in any direction, care being taken to insulate them properly and avoid proximity to inflammable substances. The current leaves the dynamo at +, passes through each lamp, and returns to the dynamo at —. This current is maintained at a constant strength required by the construction of each style of lamp, and determined by the E. M. F. of the dynamo, and the full resistance of the entire circuit, including the dynamo, the lamps, and the conducting wires. The strength of the current required varies greatly, ranging from ten to twenty ampères for series installations in the various leading arc-light systems for which this method is used.

Fig. B.—One method of wiring is shown for incandescent electric lighting, and is known as the multiple-series installation. Between two main conductors extending from the dynamo are placed, in parallel, several short lines connect-

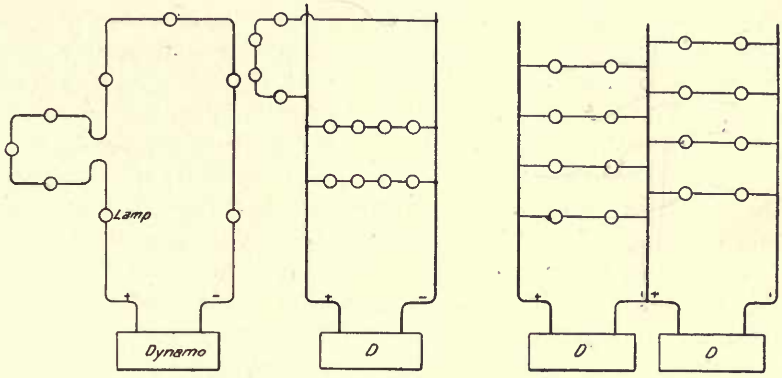


Fig. 1584.

ing with the lamps, the current being thus divided among the lamps in proportion to their number and resistance, while in the series system the entire current passes through every lamp. This method is common to both the direct and alternating current system, and is convenient for lighting a large room or hall where many lamps are required at one time. A simpler and better arrangement for most purposes is the parallel system, in which two mains are led from the dynamo, and each lamp is placed on a separate branch between the mains, so that it can be lighted or extinguished without interfering with the remaining lamps.

Fig. C shows the Edison three-wire system for incandescent lighting. Two dynamos are joined in series as shown. Each lamp has the same advantage of independent connection with the dynamo as in the two-wire system, shown at B. If an equal number of lamps are burning simultaneously in each row the current will flow through the parallel branches from one dynamo to the other, the central wire remaining neutral; but if the number is varied by the extinguishing of lamps or otherwise, the third wire furnishes a path for the surplus current required by the row having the greater number lighted, which will flow to that side in consequence of the reduced resistance resulting from the greater number of branches open through the lighted lamps.

The chief advantage of this system is in the reduced amount of copper used for conductors, three wires instead of four being used, and each only one half the area, a saving of five eighths being thereby effected.

A diagram in the Appendix illustrates a graphic method for obtaining the size of wire to be used in electric lighting.

43

GREEK AND ROMAN ORDERS OF ARCHITECTURE,

as examples of proportions of graceful curves and outlines, are useful as studies and manual practice for the draughtsman.

The Tuscan, Doric, Ionic, Corinthian, and Composite orders are systems or assemblages of parts subject to certain uniform established proportions, regulated by the office each part has to perform, consisting of two essential parts, a column and entablature, subdivided into three parts each: the first into the base, the shaft, and the capital; the second into the architrave, or chief beam, C (Fig. 1585), which stands immediately on the column; the frieze, B, which lies on the architrave; and the cornice, A, which is the crowning or uppermost member of an order. In the subdivisions certain horizontal members or mouldings are used: thus, the ogee (*a*), the corona (*b*), the ovolo (*c*), the cavetto (*d*), with the fillets, compose the cornice; the fasciæ (*f f*), the architrave; the abacus (*g*), the ovolo (*c*), the astragal (*i i*), and the neck (*h*), are the capital of the column; the torus (*k*) and the plinth (*l*) (Fig. 1587) are the base. The character of an order is displayed not only in its column, but in its general forms and details, whereof the column is, as it were, the regulator; the expression being of strength, grace, elegance, lightness, or richness. Though a building be without columns, it is nevertheless said to be of an order, if its details be regulated according to the method prescribed for such order.

In all the orders a similar unit of reference is adopted for the construction of their various parts. Thus, the lower diameter of the column is taken as the proportional measure for all other parts and members, for which purpose it is subdivided into sixty parts, called minutes, or into two modules of thirty minutes each. Being proportional measures, modules and minutes are not fixed ones like feet and inches, but are variable as to the actual dimensions which they express—larger or smaller, according to the actual size of the diameter of the column. For instance, if the diameter be just five feet, a minute, being one sixtieth, will be exactly one inch. To draw an elevation of any one of the orders, determine the diameter of the column, and from that form a scale of equal parts by sixty divisions, and then lay off the widths and heights of the different members according to the proportions of the required order, as marked in the body or on the sides of the figures.

Figs. 1585 to 1589 are illustrations of the Tuscan order: *e*, in the frieze corresponding to the Doric triglyph, may or may not be introduced. Fig. 1585 is an elevation of the capital and entablature; Fig. 1587 of the base; and Fig. 1586 of another capital.

A slightly convex curvature, or *entasis*, is given in execution to the outline of the shaft of a column, by classic architects, to counteract a fancied appearance of concave curvature, which might cause the middle of the shaft to appear thinner than it really is.

Fig. 1588 represents the form of a half-column from the Pantheon at Rome. In Fig. 1589, another example, the lower third of the shaft is uniformly cylindrical. The *entasis* of the two thirds is constructed by dividing the arc, *a b*, into equal parts, and the columns into the same number, and projecting the divisions of the arc on to those of the column. The upper diameter of column or chord at *b* is 52 minutes.

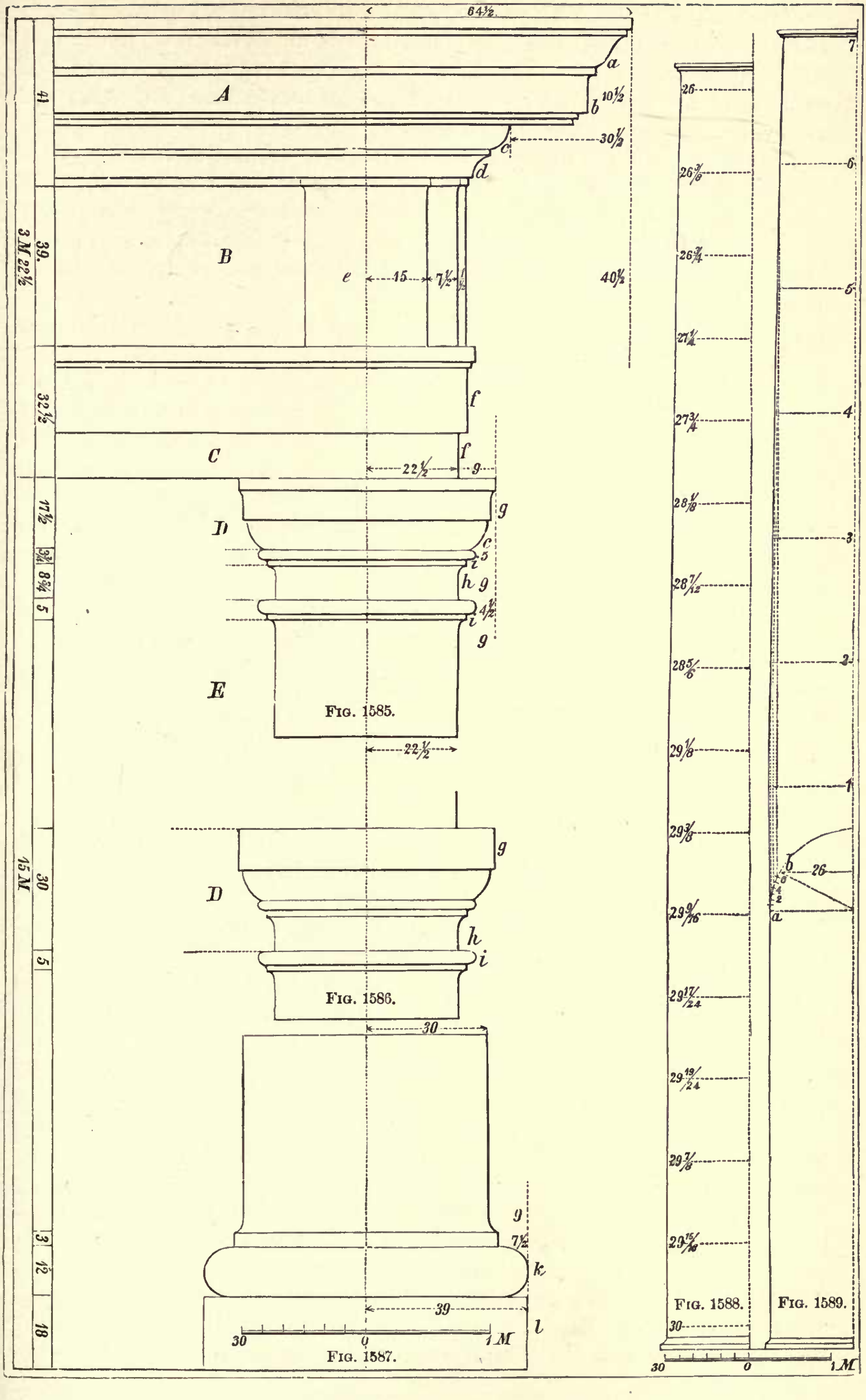


FIG. 1585.

FIG. 1586.

FIG. 1587.

FIG. 1588.

FIG. 1589.

Figs. 1590 to 1594 exhibit an example of the Doric order, from the Temple of Minerva, in the Island of Egina. Fig. 1590 is an elevation of the capital and the entablature; Fig. 1591 of the base; Fig. 1592 shows the forms of the flutes at the top of the shaft, and Fig. 1593 at the base; Fig. 1594 the outline of the capital on an enlarged scale.

The mutules, *a a*, the triglyphs, *b b*, the guttæ or drops, *d d*, of the entablature, the echinus, *f*, and the annulets, *g g*, of the capital, may be considered characteristic of the Doric. The triglyph is placed over every column, and one or more intermediately over every *intercolumn* (or span between two columns), at such a distance from each other that the metopes, *c*, or spaces between the triglyphs, are square.

In the best Greek examples of the order there is only a single triglyph over each intercolumn. The end triglyphs are placed quite up to the edge or outer angle of the frieze. The mutules are thin plates attached to the under side or soffit of the corona, over each triglyph and each metope, with the former of which they correspond in breadth, and their soffits or under surfaces are wrought into three rows of guttæ or drops, conical or otherwise shaped, each row consisting of six guttæ, or the same number as those beneath each triglyph. The shaft of the Doric column was generally fluted; the number of channels is either sixteen or twenty, afterward increased in the other orders to twenty-four, a centre flute on each side of the column.

Figs. 1595 to 1598 exhibit an example of the Ionic order, taken from the Temple of Minerva Polias, at Athens. Fig. 1595 is an elevation of the capital and entablature; Fig. 1596, of the base; Fig. 1597 is a sectional half of the plan of the column at the base and the top; Fig. 1598 an elevation of the baluster side of the capital. It differs from the Doric in the more slender proportions of its shaft, and the addition of a base; but the capital is the indicial mark of the order.

When a colonnade was continued in front and along the flanks of the building, this form of capital in the end column occasioned an offensive irregularity; for while all the other columns on the flanks showed the volutes, the end one showed the baluster side. It was necessary that the end column should, therefore, have two adjoining volute faces, which was effected by placing the volute at the angle diagonally.

Figs. 1599 and 1600 represent an example of the Corinthian order, from the Arch of Hadrian, at Athens. This order is distinguished from the Ionic more by its deep and foliaged capital than by its proportions. The capital is considerably more than a diameter in height, varying in different examples from one to one and a half diameter, upon the average about a diameter and a quarter, and has two rows of leaves, eight in each row, so disposed that of the taller ones, composing the upper row, one comes in the middle, beneath each face of the abacus, and the lower leaves *alternate* with the upper ones, coming between the stems of the latter; so that in the first or lower tier of leaves there is in the middle of each face a space between two leaves occupied by the stem of the central leaf above them. Over these two rows is a third series of eight leaves, turned so as to support the small volutes which, in turn, support the angles of the abacus. Besides these outer volutes, invariably turned diagonally, there are two other smaller ones, termed *caulicoli*, which meet each other be-

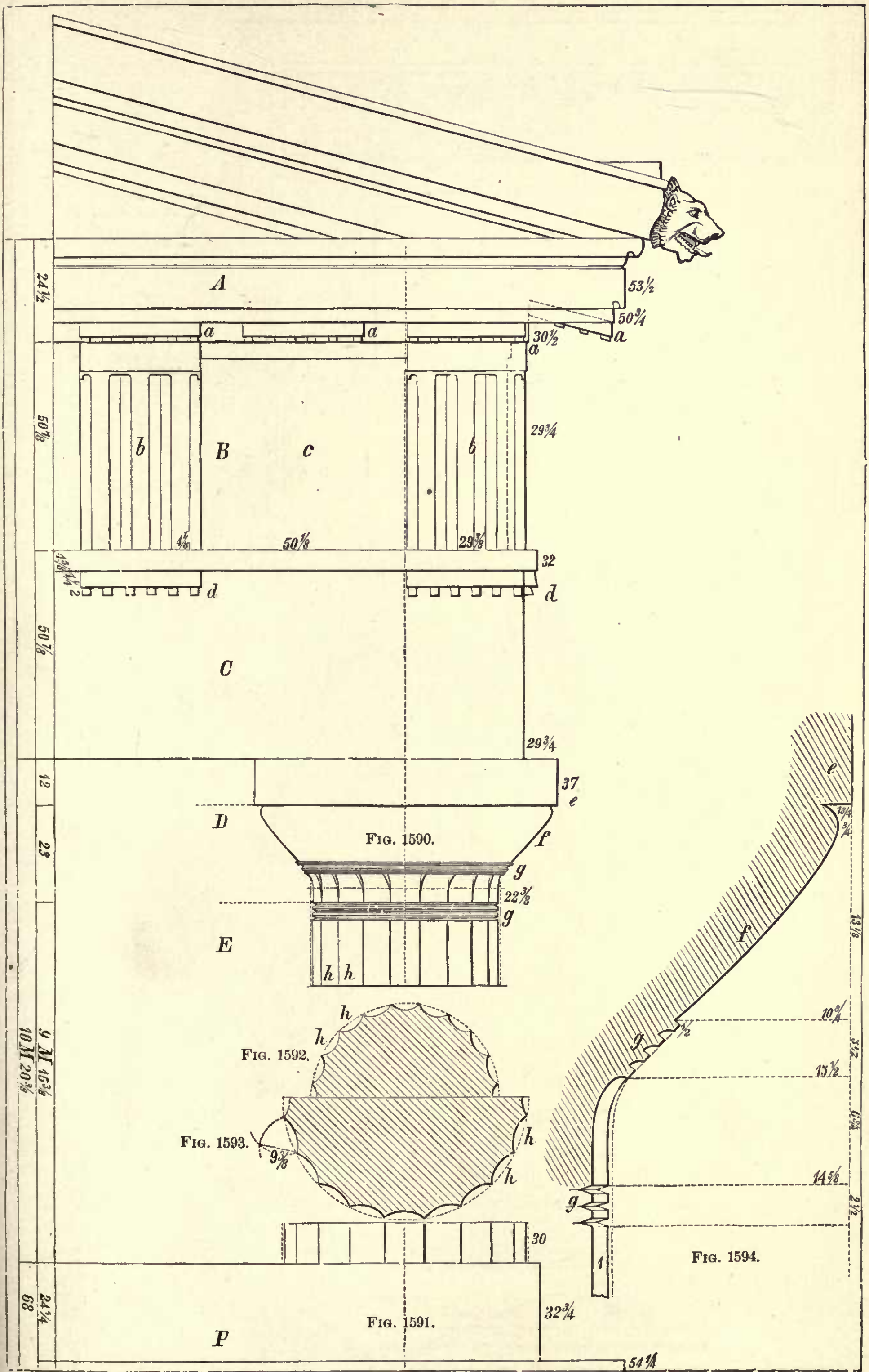


Fig. 1590.

Fig. 1591.

Fig. 1592.

Fig. 1593.

Fig. 1594.

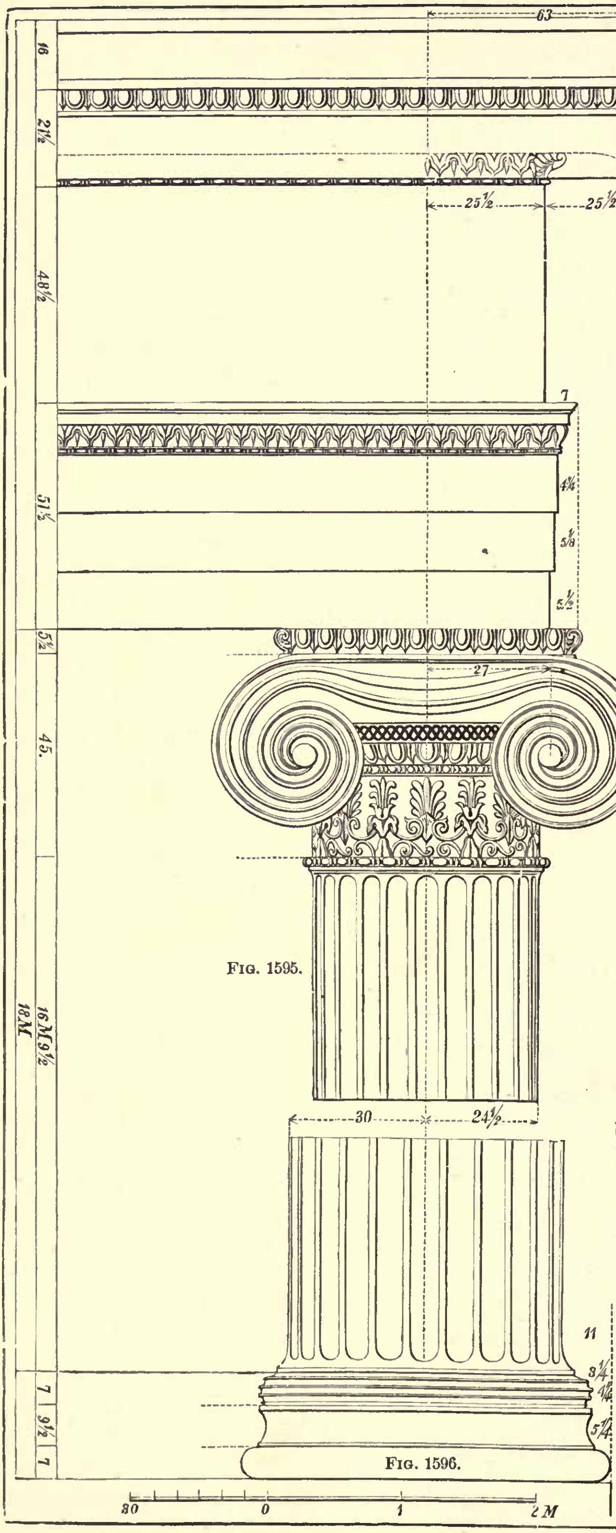


FIG. 1595.

FIG. 1596.

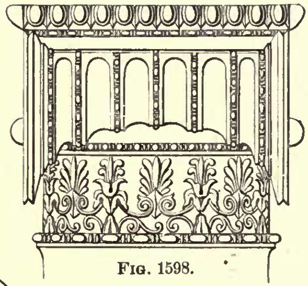


FIG. 1598.

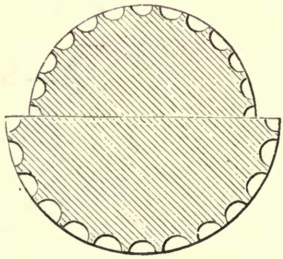


FIG. 1597.

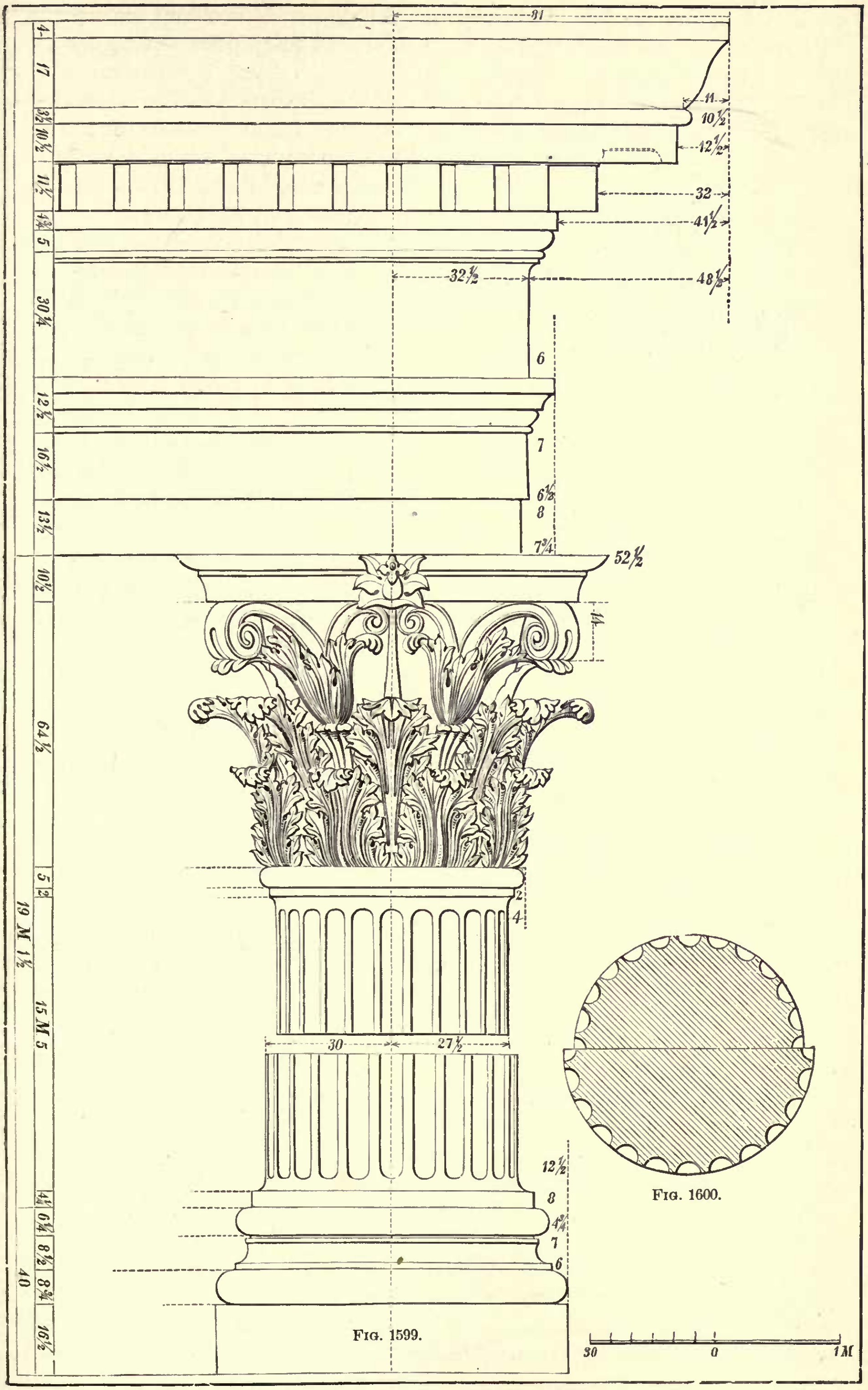


FIG. 1599.

FIG. 1600.

neath a flower on the face of the abacus. The sides of the abacus are concave in plan, being curved outward so as to produce a sharp point at each corner, which is usually cut off.

Fig. 1601 represents one of the capitals of the Tower of the Winds, showing the earliest formation of the Corinthian capital. In this example the abacus is square, and the upper row of leaves, of the kind called *water-leaves*, are broad and flat, and merely carved upon the vase or body of the capital.

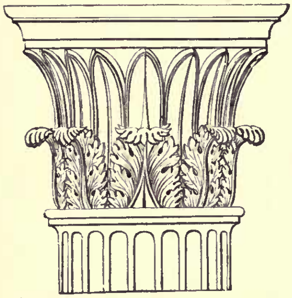


Fig. 1601.

The shaft is, in general, fluted, similarly to that of the Ionic column, but sometimes the flutes are *cabled;* that is, the channels are hollowed out for only about two thirds of the upper part of the shaft, and the remainder cut so that each channel has the appearance of being partly filled up by a round staff or piece of rope.

The cornice is very much larger than in the other orders, in height and in projection, consisting of a greater number of mouldings beneath the corona, for that and the cymatium over it are invariably the crowning members. In Fig. 1599 square blocks or *dentels* are introduced, but often to the dentels is added a row of *modillions* (Fig. 1719), immediately beneath, and supporting the corona; and between them and the dentels, and also below the latter, are other mouldings, sometimes cut, at others left plain.

The Composite Order is a union of the Ionic and Corinthian orders. Its capital consists of a Roman Ionic one, superimposed upon a Corinthian foliaged base, in which the leaves are without stalks, placed directly upon the body of the base.

The spacing between the columns, or intercolumn, is from one to one and one half diameters, but modern architects have coupled the columns, making a wide intercolumn between every pair of columns, so that as regards the average proportion between solids and voids, that disposition does not differ from what it would be were the columns placed singly. Supercolumniation, or the system of piling up orders, or different stages of columns one above another, was employed for such structures merely as were upon too large a scale to admit of the application of columns at all as their decoration, otherwise than by disposing them in tiers.

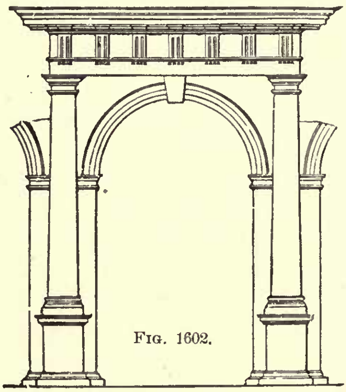


Fig. 1602.

The Greeks seldom employed human figures to support entablatures or beams; the female figures, or Caryatides, are almost uniformly represented in an erect attitude, without any apparent effort to sustain any load; while the male figures, Telamones or Atlantes, display strength and muscular action. Besides entire figures, either Hermes pillars or Termini are occasionally used as substitutes for columns of the usual form, on a moderate scale. The first mentioned consist of a square shaft with a bust or human head for its capital; the latter of a half-length figure rising out of, or terminating in, a square shaft tapering downward. Hermes pillars are frequently employed by modern architects for the decoration of window architraves.

The Romans introduced circular forms and curves, not only in elevation and section, but in plan. The true Roman order consists, not in any of the columnar ordinances, but in an arrangement of two pillars (Fig. 1602) placed at a distance from one another nearly equal to their own height, and having a very long entablature, which, in consequence, required to be supported in the centre by an arch springing from piers.

Figs. 1603, 1604, and 1605, from the Palace of Diocletian at Spalatro, are illustrations of the different modes of treatment of the arch and entablature.

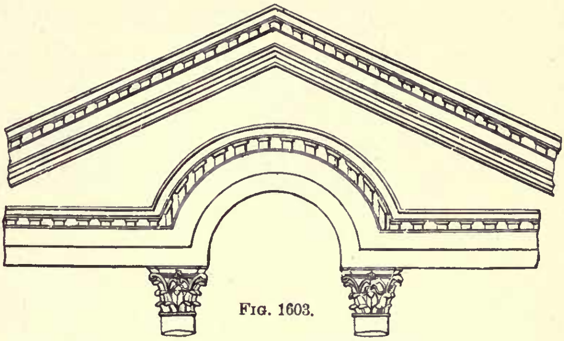


FIG. 1603.

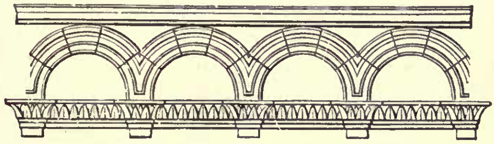


FIG. 1604.

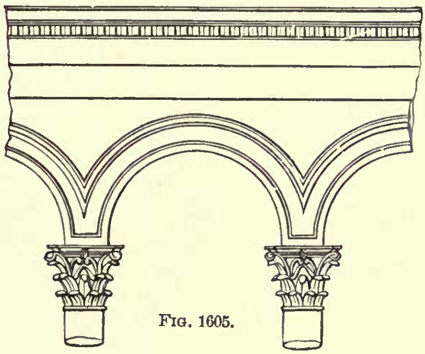


FIG. 1605.

Perhaps the most satisfactory works of the Romans are those which we consider as belonging to civil engineering rather than to architecture—their aqueducts and viaducts, all of which, admirably conceived and executed, have furnished practical examples for modern constructions, of which High Bridge across Harlem River may be taken as an illustration.

The history of Roman architecture is that of a style in course of transition, beginning with purely pagan or Grecian, and passing into a style almost wholly Christian. The first form of Christian art was the Romanesque, which afterward branched off into the Byzantine and the Gothic.

The Romanesque and Byzantine, as far as regards the architectural features, are almost synonymous; in the earlier centuries there is an ornamental distinction. In its widest signification, the Romanesque is applied to all the earlier round-arch developments, in contradistinction to the Gothic or later pointed arch varieties of the North. In this view the Norman is included in the Romanesque.

The general characteristics of the Gothic are its essentially pointed or ver-

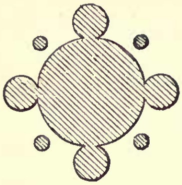


Fig. 1606.

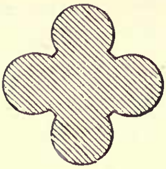


Fig. 1607.

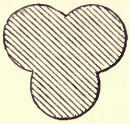


Fig. 1608.

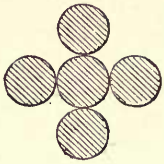


Fig. 1609.

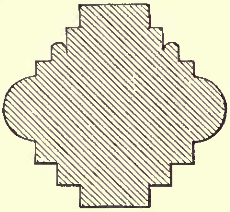


Fig. 1610.

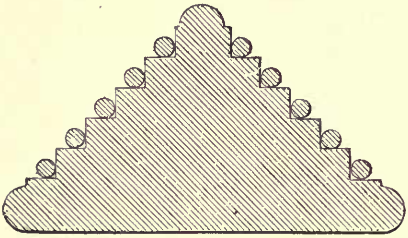


Fig. 1611.

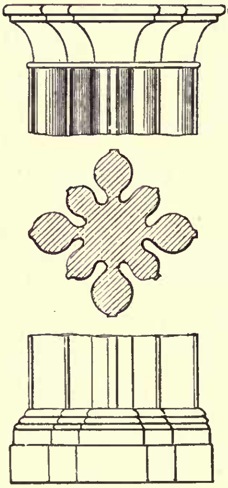


Fig. 1612.

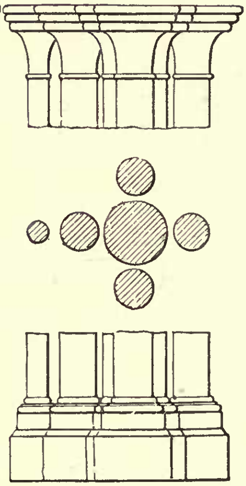


Fig. 1613.

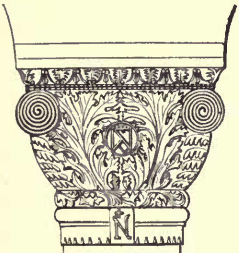


Fig. 1614.

Fig. 1616.

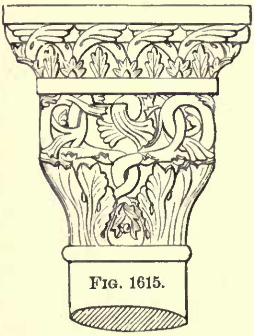


Fig. 1615.

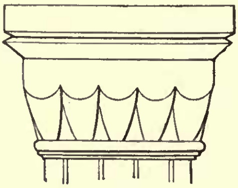


Fig. 1617.

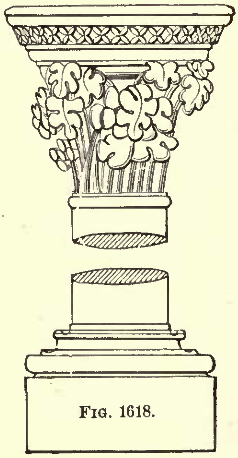


Fig. 1618.

tical tendency, its geometrical details, its window-tracery, its openings, its cluster of shafts and bases, its suits of mouldings, the universal absence of the dome, and the substitution of the pointed for the round arch.

The Romanesque pillars are mostly round or square, and, if square, generally set evenly, while the Gothic square pillar is set diagonally.

Figs. 1606 to 1610 represent sections of Gothic pillars. Fig. 1611 is half of one of the great western piers of the Cathedral of Bourges, measuring 8 feet on each side. Figs. 1612 and 1613 are elevations of capitals and bases, and sections of Gothic pillars; one from Salisbury, the other from Lincoln Cathedral.

Figs. 1614, 1615, and 1616 are examples of Byzantine capitals; Fig. 1617 a Norman one, from Winchester Cathedral; and Fig. 1618 a Gothic capital and base, from Lincoln Cathedral.

Arches are generally divided into the triangular-headed arch, the round-headed arch, and the pointed arch. Of the round-headed arch, there are semicircular, segmental, stilted (Fig. 1619), and horseshoe (Fig. 1620). Of

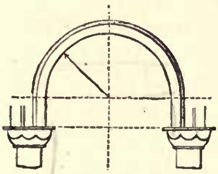


FIG. 1619.

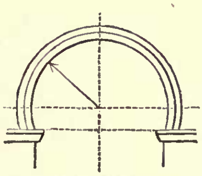


FIG. 1620.

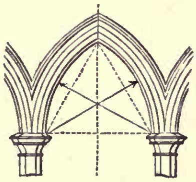


FIG. 1621.

the two-centred pointed, the equilateral (Fig. 1621), the lancet, and the obtuse. Of the first, the radii of the segments forming the arch are equal to the breadth of the arch, of those of the lancet longer, and of the obtuse shorter.

Of the complex arches, there are the ogee (Fig. 1622) and the Tudor (Fig. 1623). The Tudor arch is described from four centres, two on a level with the spring and two below it.

Of foiled arches, there are the round-headed trefoil (Fig. 1624), the pointed

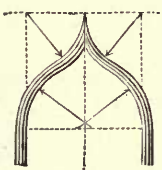


FIG. 1622.

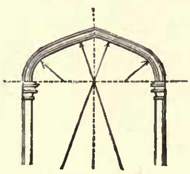


FIG. 1623.

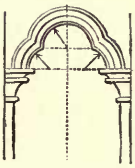


FIG. 1624.

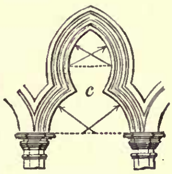


FIG. 1625.

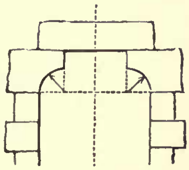


FIG. 1626.

trefoil (Fig. 1625), and the square-headed trefoil (Fig. 1626). The points *c* are termed cusps.

The semicircular arch is the Roman Byzantine and Norman arch; the ogee and horseshoe are the profiles of many Turkish and Moorish domes; the pointed and foliated arches are Gothic.

Domes and Vaults.—The Greek vaulting consisted wholly of spherical surfaces, the Roman of cylindrical ones. Figs. 1627 and 1628 illustrate this distinction, Fig. 1627 being the elevation of a Roman cylindrical cross-vault, and Fig. 1628 the elevation of the roof of the church of St. Sophia at Constantinople; and the sprouting of domes out of domes continues to characterize the

Byzantine style. As a constructive expedient the cross-vault is to be preferred, as the whole pressure and thrust are collected in four definite resultants, ap-

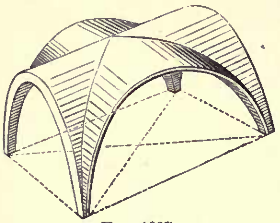


FIG. 1627.

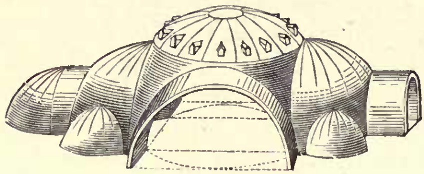


FIG. 1628.

plied at the angles only, so that it might be supported by four flying buttresses, placed in the direction of the resultants, and strong enough to resist the pressure.

Fig. 1629 represents a compartment of the simplest Gothic vaulting—*a*, *a*, groin ribs; *b*, *b*, *b*, side ribs.

The Romans introduced side ribs, appearing on the inside as flat bands, and harmonizing with the similar form of pilasters in the walls, but they never used groin ribs; the Gothic builders introduced these, and deepened the Roman ribs. The impenetration of vaults, either round or pointed, produces elliptical groin lines, or else lines of double curvature; yet the early Gothic architects made their groin ribs usually simple pointed arches of circular curvature, thrown diagonally across the space to be groined, and the four side arches were equally simple, the only care being that all the arches should have their vertices at the same level. The strength depended on the ribs, and the shell was made quite light, often not more than six inches, while Roman vaults of the same span would have been three or four feet. The Romans made their vault *surfaces* geometrically regular, and left the groins to take their chance; while the early Gothic architects made their groins geometrically regular, and let the intermediate surfaces take their chance.

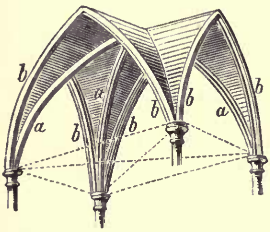


FIG. 1629.

In the next step the groin ribs were elliptical, and when intermediate ribs or *tiercerons* were inserted, these ribs had also elliptical or cylindrical curvatures, different from the groins, and the ribs were placed near each other, in order that the portion of the vault between each pair might practically be almost cylindrical. In the formation of the compound circular ribs three conditions were to be observed: 1. That the two arcs should have a common tangent at the point of meeting. 2. That the feet of all the ribs should have the same radius, up to the level at which they completely separate from each other. 3. That from this point upward their curvature should be so adjusted as to make them all meet their fellows on the same horizontal plane, so that all the ridges of the vaults may be on one level.

The geometrical difficulty of such works led to what is called *fan-tracery vaulting*. If similar arches spring from each side of the pillars (Fig. 1629), the portion of vault springing from each pillar would have the form of an inverted concave-sided pyramid, its horizontal section at every level being square.

The later architects, by converting this section into a circle, the four-sided pyramid became a conoid, and all the ribs forming the conoidal surface became alike in curvature, so that they all might be made simple circular arcs; these ribs are continued with unaltered curvature till they meet and form the ridge; but in this case the ridges are not level, but gradually descend every way from the centre point (Fig. 1630).

In the figure this is not fully carried out, for no rib is continued higher than those over the longer sides of the compartment, so that a small lozenge is still left, with a boss at its centre. When the span of the main arch *b a* was large in proportion to that of *b c*, the arch *b c* became a very acute lancet arch, scarcely admitting windows of an elegant or sufficient size. To obviate this, the compound curve was again introduced.

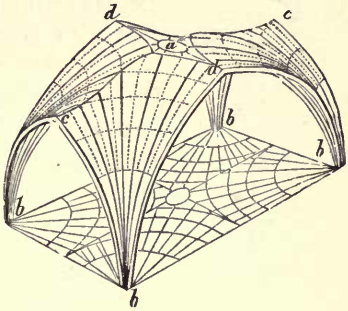


Fig. 1630.

The four-centred arch is not necessarily flat or depressed; it can be made of any proportion, high or low, and always with a decided angle at the vertex. In general, the angular extent of the lower curve is not more than 65°, nor less than 45°. The radius of the upper curve varies from twice to more than six times the radius of the lower. The projecting points of the trefoil arch, or *cusps*, are often introduced for ornament merely, but serve constructively, both in vaults and arches, as a load for the sides, to prevent them rising from the pressure on the crown.

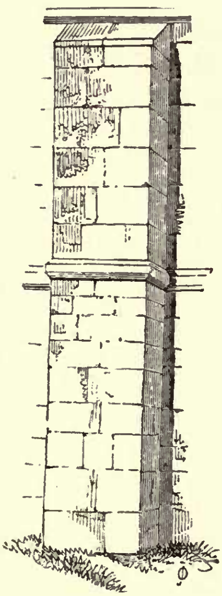


Fig. 1631.

As vaultings, in general, were contrived to collect the whole pressure of each compartment into four single resultants, at the points of springing, leaving the walls so completely unloaded that they are required only as inclosures or screens, they might be entirely omitted or replaced by windows. Indeed, the real supporting walls are broken into narrow strips, placed at right angles to the outline of the building, and called *buttresses*, and the *inclosing* walls may be placed either at the outer or inner edge of the buttresses. The first, that adopted by the French architects, gave deep recesses to the interiors, while the other, or English method, served to produce external play of light and shade.

The Norman buttress (Fig. 1631) resembles a flat pilaster, being a mass of masonry with a broad face, slightly projecting from the wall. They are, generally, of but one stage, rising no higher than the cornice, under which they often, but not always, finished with a slope. Sometimes they are carried up to, and terminate in, the corbel table.

Fig. 1632 represents a buttress in two stages, with slopes as set-offs.

Fig. 1633 is a buttress of the early English style, having a plain triangular or pedimental head. The angles were sometimes chamfered off, and sometimes

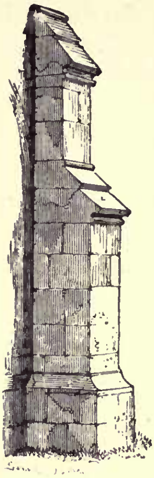


FIG. 1632.

ornamented with slender shafts. In buttresses of different stages, the triangular head or gable is used as a finish for the intermediate stages.

In the Decorated style, the outer surfaces of the buttresses are ornamented with niches, as in Fig. 1634. In the Perpendicular style the outer surface is often partially or wholly covered with panel-work tracery (Fig. 1635).

The buttress was a constructive expedient to resist the thrust of vaulting; but to resist the thrust of the principal vault, or that over the nave or central part of the church, buttresses of the requisite depth would have filled up the side aisles entirely. To obviate this, the system of flying but-

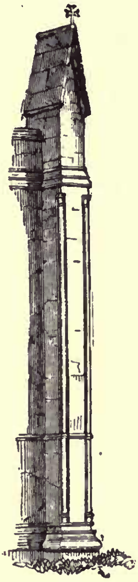


FIG. 1633.

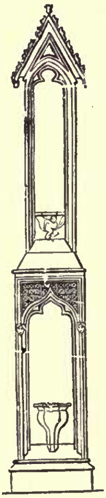


FIG. 1634.

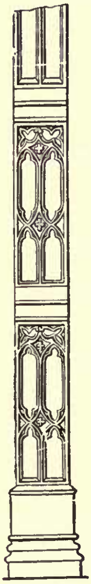


FIG. 1635.

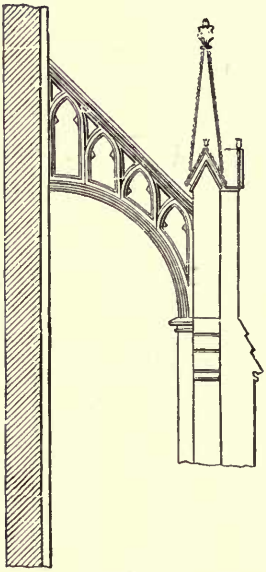


FIG. 1636.

tresses was adopted; that is, the connection of the interior with the outer buttress, by an arch or system of arches, as shown in Fig. 1636. The outer piers were surmounted by pinnacles, to render them a sufficiently steady abutment to the flying arches.

The earlier towers of the Romanesque style were constructed without spires. All are square in plan, and extremely similar in design. Fig. 1637 is an elevation of the tower attached to the church of Sta. Maria, in Cosmedin, and is one of the best and most complete examples of this style. It is 15 feet broad and 110 feet high. These towers are the types of the later Italian campaniles, generally attached to some angle of churches; if detached, so placed that they still form a part of the church design. Sometimes they are but civic constructions, as belfries, or towers of defence. The campanile is square, carried up without break or offset to two thirds, at least, of its

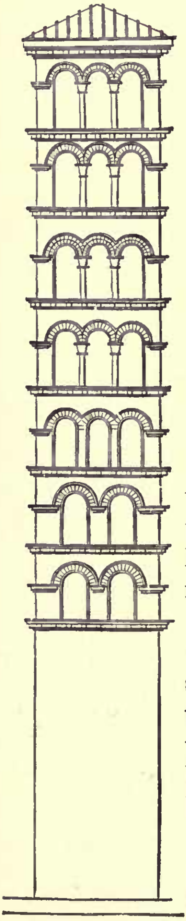


FIG. 1637.

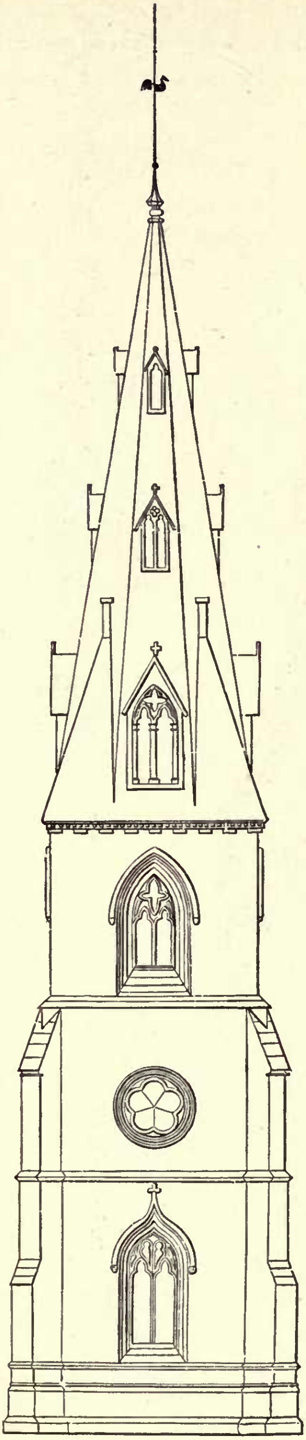


Fig. 1638.

intended height; it is generally solid to a considerable height, or with only such openings as serve to admit light to the staircases. Above this solid part one round window is introduced in each face; in the next story, two; in the one above this, three; then four, and lastly five; the lights being separated by slight piers, so that the upper story is virtually an open loggia.

The Gothic towers have projecting buttresses, frequent offsets, lofty spires, and a general pyramidal form. Fig. 1638 is the front elevation of a simple English Gothic tower; here the plain pyramidal roof, rising at an equal slope on each of the four sides, is intersected by an octagonal spire of steep pitch. The first spires were simple quadrangular pyramids; afterward the angles were cut off, and they became octagonal, and this

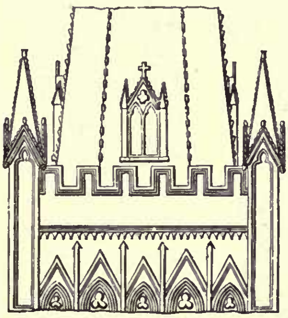


Fig. 1639.

is the general Gothic form of spire. Often, instead of intersecting the square roof, as in the figure, the octagonal spire rests upon a square base, and the angles of the tower are carried up by pinnacles, or the sides by battlements, or by both, as in Fig. 1639, to soften the transition between the perpendicular and sloping part.

In general the spires of English churches are more lofty than those on the Continent, the angle at the apex in the former being about 10°, and in the latter about 15°. The apex angle of the spires of Chichester and Lichfield are from 12° to 13°, or a mean between the two proportions, and, according to Ferguson, more pleasing than either. Although having more lofty spires, yet the English construction is much more massive in appearance than the Continental; the apertures are less numerous, and the surfaces are

Fig. 1642.

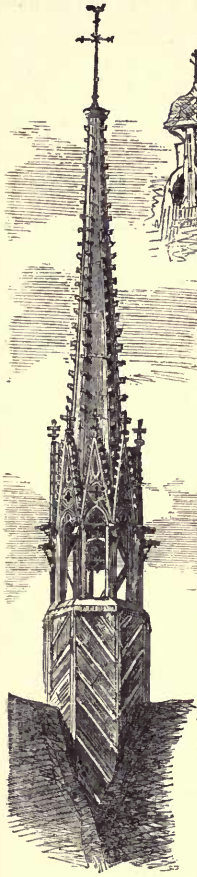


Fig. 1640.

Fig. 1643.

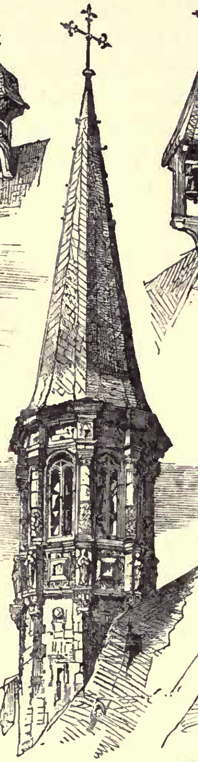


Fig. 1641.

Fig. 1645.

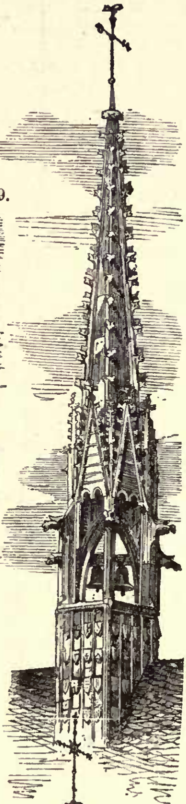


Fig. 1646.

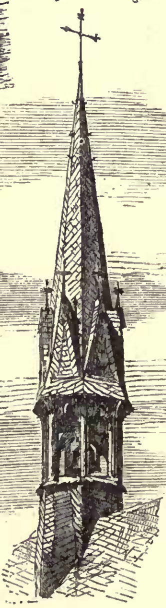


Fig. 1647.

Fig. 1649.

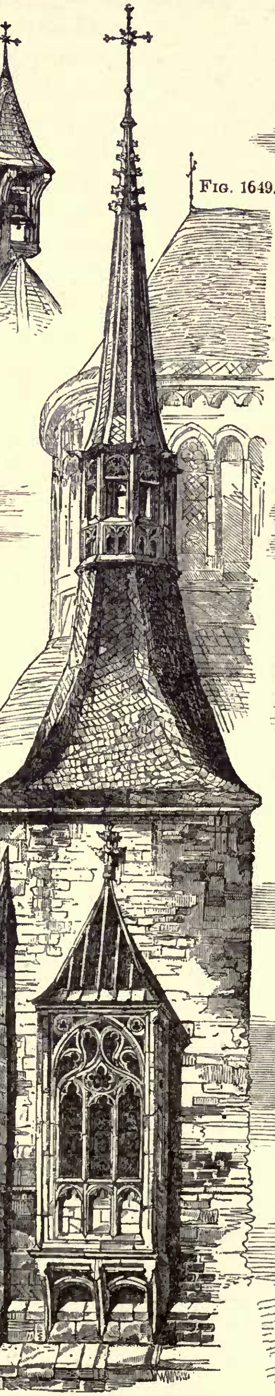


Fig. 1644.

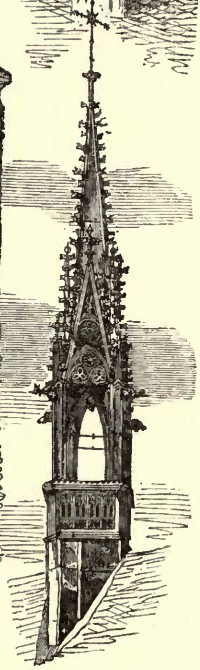


Fig. 1648.

less cut up and covered with ornaments. The spires of Freiberg Church, and many others on the Continent, are open work.

Figs. 1640 and 1641 are bell-cots. Figs. 1642 to 1648 are spires. Fig. 1649 is an *apse*, or circular end of a church, from German Gothic examples.

FIG. 1650.

FIG. 1651.

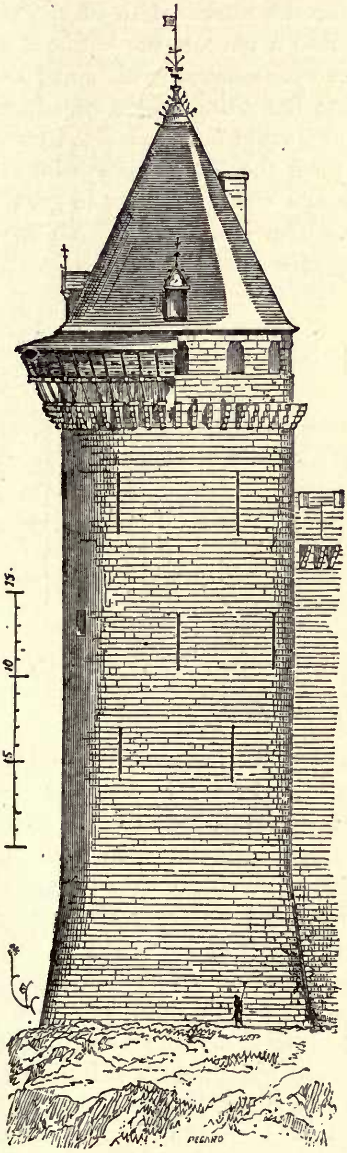


FIG. 1652.

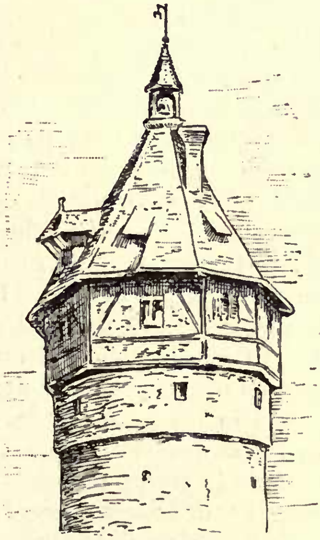


FIG. 1653.

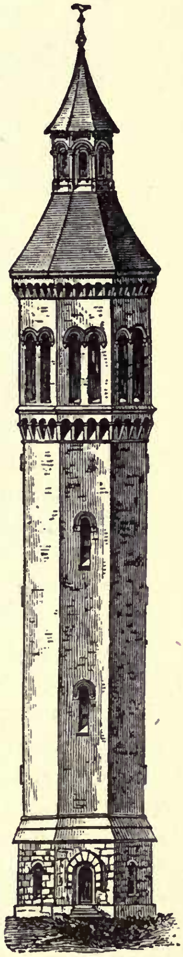


FIG. 1654.

Figs. 1650 and 1651 are examples of spire finials, with weather-cocks.

Figs. 1652 and 1653 are examples of towers not connected with church edifices.

Fig. 1654 is a tower of very recent construction, and is applied to the utilitarian purpose of sustaining a water-tank for the highest service of the Croton in New York city.

Fig. 1655 represents the upper portion of the tower of Ivan Veliki, at Moscow. The Russian towers are generally constructed independent of their churches, and are intended for the reception of their massive bells.

Windows.—Before the use of painted glass, as very small apertures sufficed for the introduction of the required quantity of light into a church, the windows of the Romanesque churches were generally small, and devoid of tracery; and as the Byzantine architects, adorning the walls with paintings, could not use stained glass, they followed in general form the Romanesque window, apertures with circular heads, either single or in groups (Fig. 1656 or Fig. 1657). The Norman windows were also small, each consisting of a single light, semicircular

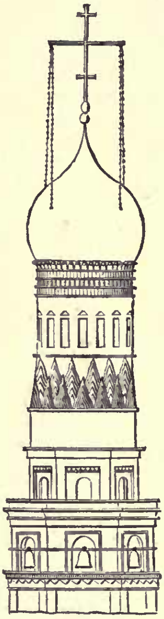


Fig. 1655.

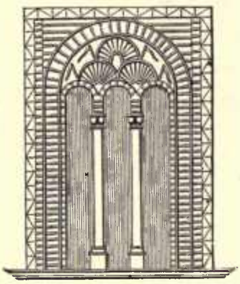


Fig. 1656.

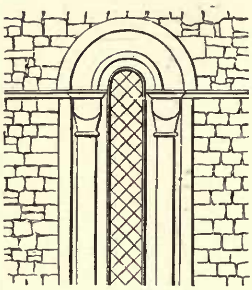


Fig. 1657.

in the head, and placed as high as possible above the ground; at first splayed on the inside only, afterward the windows began to be recessed with mouldings and jamb-shafts in the angles, as in Fig. 1657.

The Lancet, in general use in the early Gothic period, was of the simplest arrangement: in these windows the glass was brought within three or four inches of the outside of the wall, and the openings were widely splayed in the interior. The proportions of these windows vary considerably, in some the height being but five times the width, in others as much as eleven; eight or nine times may be taken as the average. Lancet windows occur singly (Fig. 1658), or in groups of two, three, five, and seven, rarely of four and six. The triplet (Fig. 1659) is the most beautiful arrangement of lancet windows. It was customary to mark with greater importance the central light, by giving it additional height, and in most cases increased width also. In some examples the windows of a lancet triplet are placed within one drip-stone forming a single arch, thus bearing a strong resemblance to a single three-light window. The first approximation to tracery appears to have been the piercing of the space over a double lancet window comprised within a single drip-stone (Fig. 1660).

A traceried window is a distinctive characteristic of Gothic architecture; with the establishment of the principle of window tracery the mullions were recessed from the face of the wall in which the window arch was pierced, and the fine effect thus produced was speedily enhanced by the introduction of distinct orders of mullions, and by recessing certain portions of the tracery from the face of the primary mullions and their corresponding tracery bars.

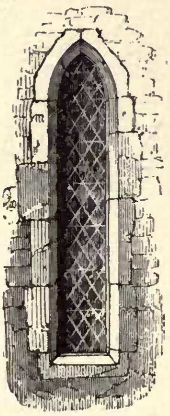


Fig. 1658.

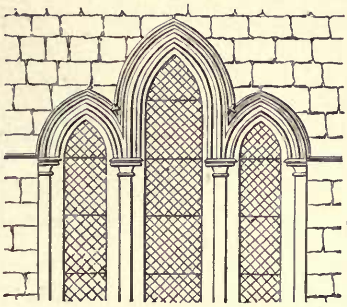


Fig. 1659.

Fig. 1660.

Examples of window tracery, showing its constructive centres and lines, are given on pages 81 and 82, illustrating the chief varieties. The following figures are more complete in position and with architraves, Geomet-

Fig. 1661.

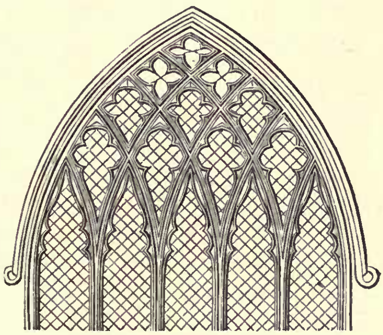


Fig. 1662.

rical and Flowing; the former consisting of geometrical figures, as circles, trefoils, quatrefoils, curvilinear triangles, lozenges, etc.; while in flowing tracery these figures, though still existing, are gracefully blended together in one design.

Fig. 1661 represents an example of the earlier decorated tracery window-head, consisting of two foiled lancets, with a pointed quatrefoil in the spandrel between them. One half of the windows in this, as in some of the following figures, is drawn in skeleton, to explain their construction. Fig. 1662 is another example of Decorated tracery.

Fig. 1663 is an example of the English leaf tracery; Fig. 1664, of the French flamboyant. The difference between the two styles is, that while the upper ends of the English loops or leaves are round, or simply pointed, the upper ends of the latter terminate, like their lower ones, in angles of contact, giving a flamelike form to the tracery bars and form pieces.

In England the Perpendicular style succeeded the Decorated; the mullions, instead of diverging in flowing or curvilinear lines, are carried up straight through the head of the windows; smaller mullions spring from the head of the prin-


Fig. 1663.

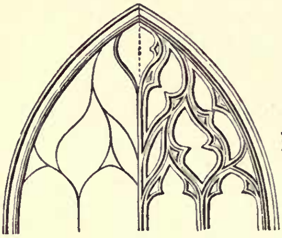


Fig. 1664.

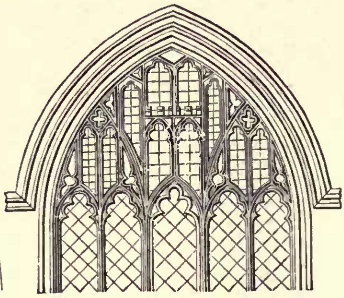


Fig. 1665.

cipal lights, and thus the upper portion of the window is filled with panel-like compartments. The principal as well as the subordinate lights are foliated in their heads, and large windows are often divided horizontally by transoms. The forms of the window arches vary from simple pointed to the complex four-centred, more or less depressed.

Fig. 1665 is an example of a Perpendicular window.

Fig. 1666 is a square-headed window, such as were usual in the clear-stories of Perpendicular architecture.

Figs. 1667 and 1668 are quadrants of circular windows, used more espe-

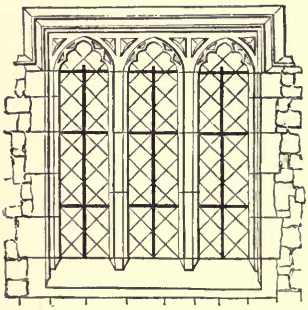


Fig. 1666.

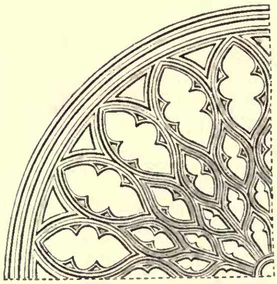


Fig. 1667.


Fig. 1668.

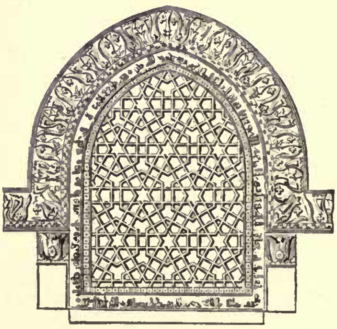


Fig. 1669.

cially in France for the adornment of the west ends and transepts of the cathedrals.

Besides the tracery characteristic of Gothic architecture, there is a tracery peculiar to the Saracenic and Moorish style, of which Fig. 1669 may be taken as an example—it being a window of one of the earliest mosques. The general form of the window and door-heads of this style is that of the horse-shoe, either circular or pointed.

Doorways.—Fig. 1670 is the elevation of a circular-headed doorway, which may be considered the type of many entrances both in Romanesque, Gothic, and later styles. It consists of two or more recessed arches, with shafts or mouldings in the jambs. In the earlier styles the arches were circular, in the later Gothic,

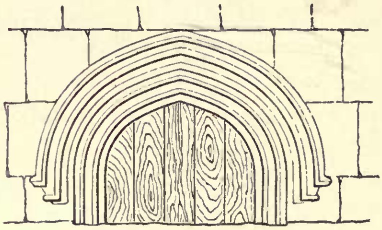


FIG. 1671.

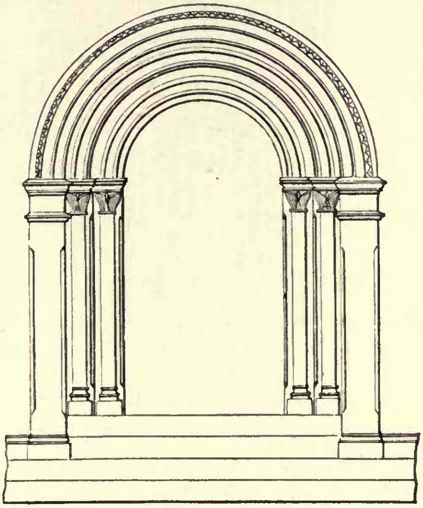


FIG. 1670.

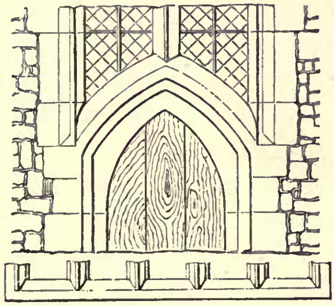


FIG. 1672.

generally pointed, but sometimes circular; in the earlier, the angles in which the shafts are placed are rectangular; in the latter, the shaft is often moulded on a *chamfer* plane—that is, a plane inclined to the face of the wall, generally at an angle of 45°; often the chamfer and rectangular plane are used in connection.

Fig. 1671 is a simple head of a depressed four-centred or Tudor-arched doorway, with a hood moulding.

Fig. 1672 represents the incorporation of a window and doorway. Sometimes the doorway pierces a buttress; in that case, the buttress expands on either side, forming a sort of porch. The Gothic architects placed doors where they were necessary, and made them subservient to the beauty of the design.

Fig. 1673 is an example of a gabled doorway with crockets and finial.

Fig. 1674 is an example of a perpendicular doorway, with a label or hood moulding above, and ornamented spandrels.

Fig. 1675 is an example of a Byzantine, and Fig. 1676 of a Saracenic doorway.

The Renaissance style was, originally, but the revival or a fair rendering of the classical orders of architecture, with ornaments from the Byzantine and Saracenic styles.

Garbett divides this style into three Italian schools, the Florentine, Venetian, and Roman. The Florentine admits of little apparent ornament, but any degree of real richness, preserving in its principal forms severe *contrast;* powerful masses self-poised without corbeling, without arching; *breadth* of everything, of light, of shade, of ornament, of plain wall; *depth* of recess in the openings, of perspective in the whole mass, of projection in the cornice.

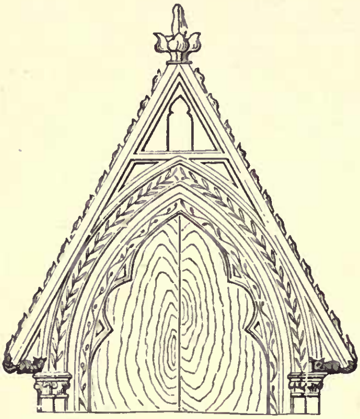


Fig. 1673.

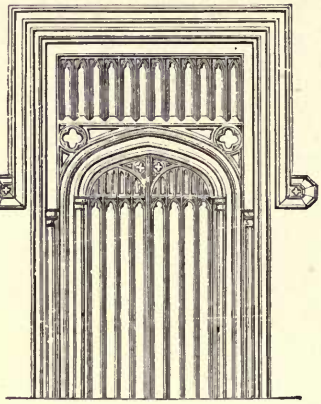


Fig. 1674.

Absence of features useless to convenience or stability, admitting of great plainness, or of very florid enrichment.

The aim of the Venetian school was splendour, variety, show, and ornament; not so much real as effective ornament. Thus, it rarely contains as

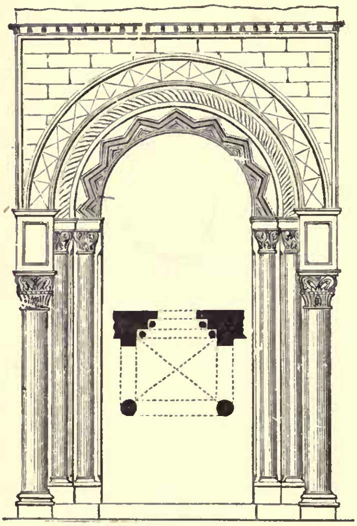


Fig. 1675.

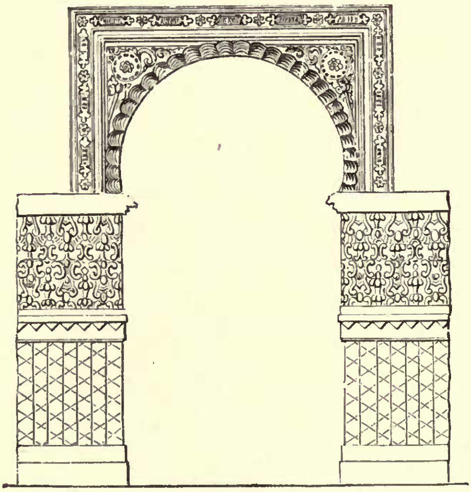


Fig. 1676.

much carving or minute enrichment as the Florentine admits; but it has larger ornaments, constructed (or built) ornaments, great features useless except for ornament, such as inaccessible porticoes, detached columns, and architraves supporting no ceiling, towers built only for breaking an outline.

The Roman school is intermediate in every respect between the two other schools. It is better adapted to churches than to any other class of buildings. This fitness arises from the grand, simple, and unitary effect of one tall order, generally commencing at or near the ground, obliterating the distinction of two or three stories, making a high building appear a single story.

Mouldings.—"All classical and Romanesque architecture is composed of bold independent shafts, plain or fluted, with bold detached capitals forming arcades or colonnades where they are needed, and of walls whose apertures are surrounded by courses of parallel lines called mouldings, and have neither shafts nor capitals. The shaft system and moulding system are entirely separate; the Gothic architects confounded the two; they clustered the shafts till they looked like a group of mouldings, they shod and capitalled the mouldings till they looked like a group of shafts." The mouldings appear in almost every conceivable position; from the bases of piers and piers themselves, to the ribs of the fretted vaults which they sustain.

In the earliest examples of Norman doorways the jambs are mostly simply squared back from the walls; recessed jambs succeeded, and are common in both Norman and Gothic architecture; and when thus recessed, detached shafts were placed in each angle (Fig. 1677). In the later styles the shafts were almost invariably attached to the structure. The angles themselves were often cut or chamfered off, and the mouldings attached to the chamfer-plane. The arrangement of window jambs, during the successive periods, was in close accordance with that of doorways.

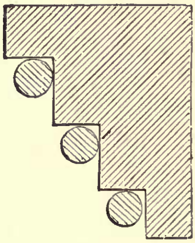


FIG. 1677.

In the richer examples small shafts were introduced, which, rising up to the springing of the window, carried one or several of the arch mouldings. Yet mouldings are nevertheless not essential accessories; many windows of the richest tracery have their mullions and jambs composed of simple chamfers.

Figs. 1678 to 1686 are examples of *arch and architrave mouldings*, which,

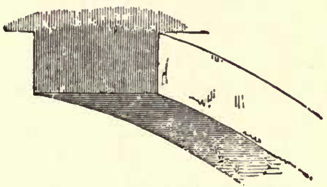


FIG. 1678.

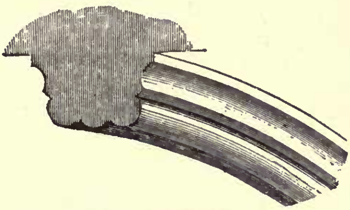


FIG. 1679.

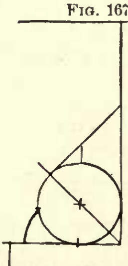


FIG. 1680.

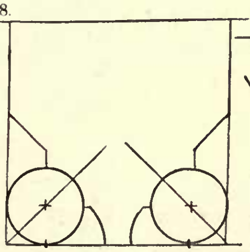


FIG. 1681.

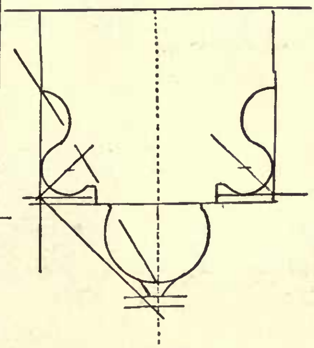


FIG. 1682.

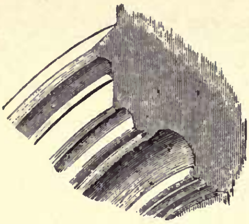


FIG. 1683.

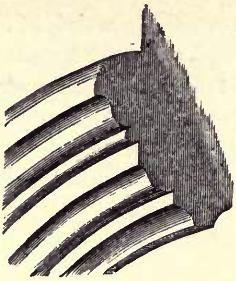


FIG. 1684.

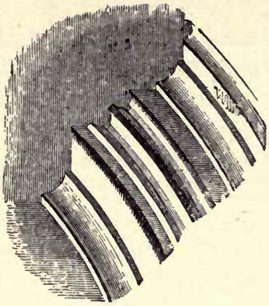


FIG. 1685.

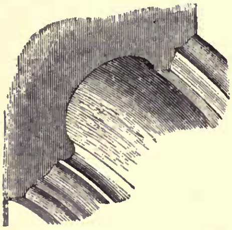


FIG. 1686.

even when not continuous, partook of the same general arrangement as those in the jambs, with greater richness of detail. When shafts were employed, they carried groups of mouldings more elaborate than those of the jambs, though still falling on the same planes.

Capitals were either moulded or carved with foliage, animals, etc.; they always consisted of three distinct parts (Fig. 1687)—the head mould (A), the bell (B), and the neck mould (C). In Norman examples the head mould was almost invariably square; in the later styles it is circular, or corresponding to the form of the pillar.

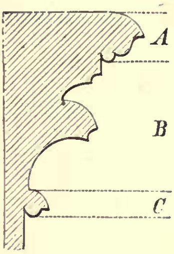


FIG. 1687.

Bases consist of the plinth and the base mouldings. The plinth was square in the Norman style, afterward octagonal; then, assuming the form of the base mouldings, it bent in and out with the outline of the pier. Base mouldings were also extensively used round the buttresses, towers, and walls of churches.

String Courses, of which Figs. 1688 to 1693 are examples, were horizontal courses in the face of a wall, the most usual position being under the windows. In the Norman styles they were usually heavy in the outline; in the later styles they were remarkably light and elegant; free from restraint or horizontality, they now rose close under the sill

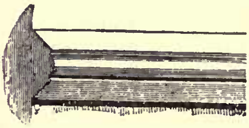


FIG. 1688.

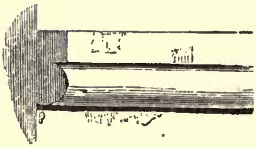


FIG. 1689.

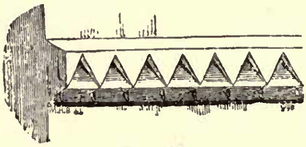


FIG. 1690.

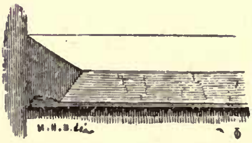


FIG. 1691.

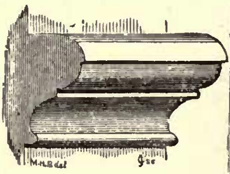


FIG. 1692.

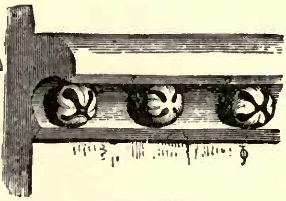


FIG. 1693.

of the window, and then suddenly dropping to accommodate themselves to the arch of a low doorway, and again rising to run immediately under the adjoining window. In this way the string courses frequently served the purpose of a *drip-stone* or hood moulding over doors; occasionally the hood mould was continued from one window to the other.

Cornices are not an essential feature in Gothic architecture. In the Norman and early English styles the cornice was a sort of enlarged, projecting string course, forming a drip-stone beneath the roof, which, if supported on brackets or *corbels*, was termed the corbel table.

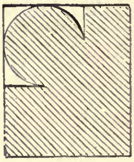


Fig. 1694.

The earliest moulding in Norman work is a circular bead strip, worked out of the edges of a recessed arch, called a circular *bowtel* (Fig. 1694). From a circular form the bowtel soon became pointed, and, by an easy transition, into the bowtel of one, two, or three fillets.

Figs. 1695 to 1700 are sections of Romanesque drip- or cap-stones, adapted to different pitches of roof.

Fig. 1701 is the *scroll* moulding; a simple filleted bowtel, with the fillet

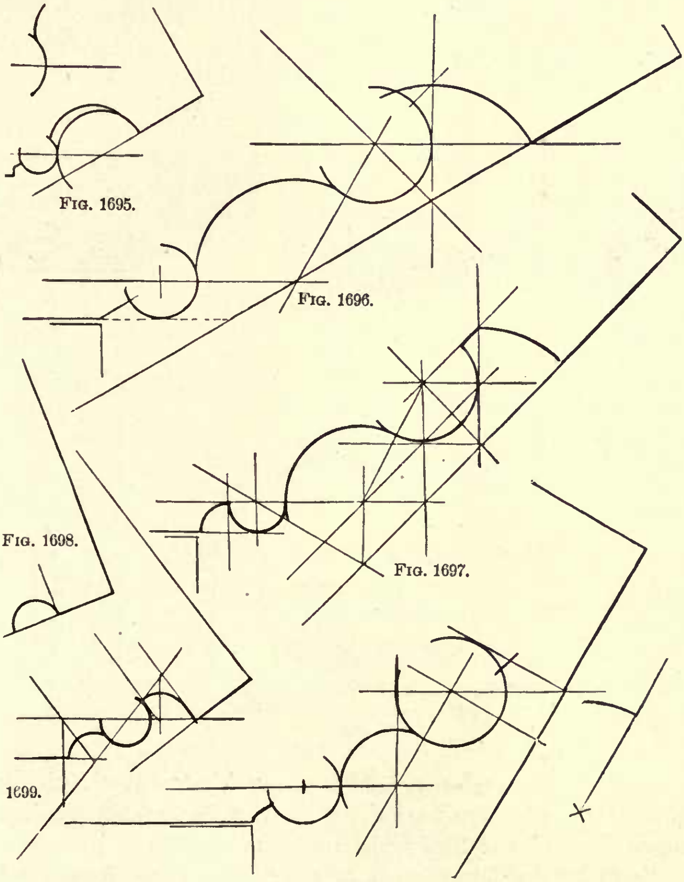


Fig. 1695. Fig. 1696. Fig. 1698. Fig. 1697. Fig. 1699. Fig. 1700.

undeveloped on one side, as shown by the dotted lines. If this moulding be cut in half, through the centre of the fillet, we have on the developed side the moulding now termed by carpenters the rule joint, which, by rounding off the corners by reverse curves, becomes the wave moulding.

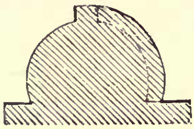


Fig. 1701.

Fig. 1702 is a Gothic example of the filleted bowtel with prominent alternate hollows.

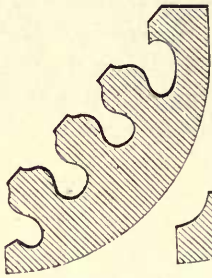


Fig. 1702.

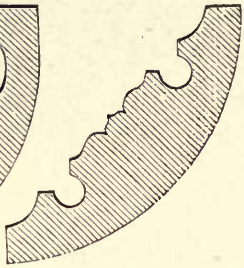


Fig. 1703.

Fig. 1703 is an example of the perpendicular style, an insignificant hollow separating groups of mouldings.

Figs. 1704 to 1709 are examples of moulded timbers, used largely in open-timbered roofs and for exposed beams. It is still the custom, when the framing is not covered in with plastering or ceiling, to corner the edges of the joists and beams, at an angle of 45°, for about 1″ on each face, but not extending close to the joint or wall; this is called stop-chamfering.

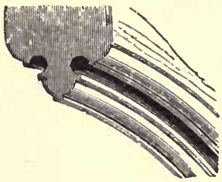


Fig. 1704.

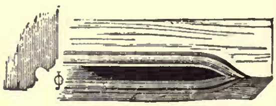


Fig. 1705.

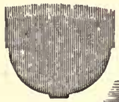
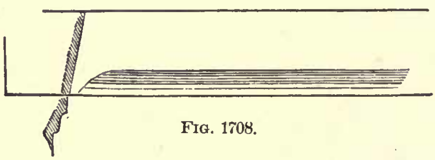


Fig. 1706.

Fig. 1707.



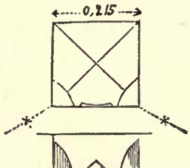
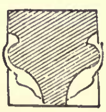


Fig. 1708.



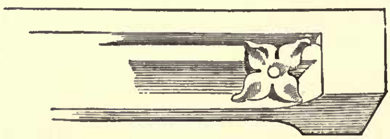


Fig. 1709.

Ornament.—Architectural ornament is of two kinds, constructive and decorative. By the former is meant all those contrivances, such as capitals, brackets, vaulting-shafts, and the like, which serve to explain or give expression to the construction; by the latter, such as mouldings, frets, foliage, etc., which give grace and life, either to the actual constructive form, or to the constructive decoration. Mouldings of the different styles have been already treated of; it is proposed to give now what are even more purely decorations of a style.

In the Grecian orders the Doric (Fig. 1590) has the triglyph mutules and guttæ; the Ionic (Fig. 1596) has various mouldings of the cornice, frieze, abacus, and neck of the column enriched. The principal ornament of the neck of the column is the anthemion, commonly known, in its most simple form, as the honeysuckle or palmetto; in the anthemion, as represented in the figure, the palmetto alternates with the lily or some analogous form. The ornament of the abacus is the egg and dart (Fig. 1710); the ornament of the

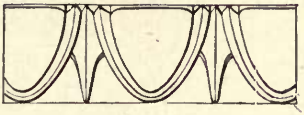


Fig. 1710.

Fig. 1711.

frieze and cornice (Fig. 1711). The fret (Fig. 1712) and the guilloche (Fig. 1713) are also common Greek ornaments, used to adorn the soffits of beams and ceilings. The acanthus is the distinctive ornament of the Corinthian, of which a leaf is represented in front and side view (Figs. 1714 and 1715).

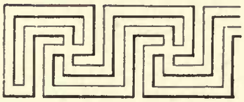


Fig. 1712.

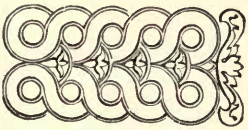


Fig. 1713.

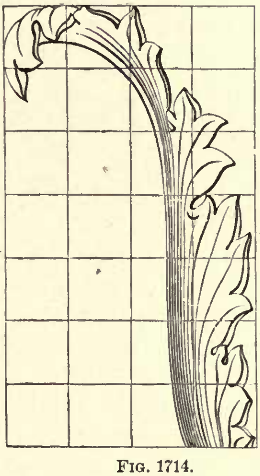


Fig. 1714.

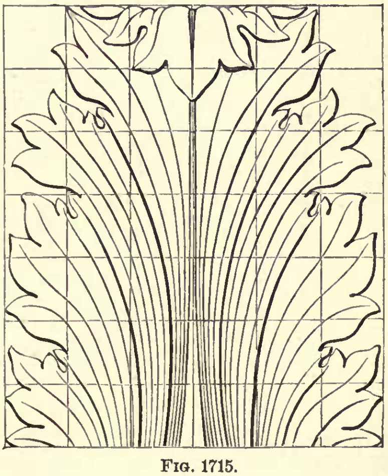


Fig. 1715.

Figs. 1716, 1717, and 1718 are the side elevation, front elevation, and section of a Greek bracket, the principal ornaments of which are taken from the anthemion and acanthus.

Fig. 1719 is an elevation of a portion of an enriched cornice from the temple of Jupiter Stator, at Rome, of the Corinthian order of architecture. Fig. 1720 is the under side of the modillion, on a larger scale.

The chief characteristic of Roman ornament is its uniform magnificence, an enrichment of the Greek. The most used elements of the Roman decorations are the scroll and the acanthus. The acanthus of the Greeks is the narrow prickly acanthus; that of the Roman, the soft acanthus. For capitals the Roman acanthus is commonly composed of conventional clusters of olive-leaves. Fig. 1721 represents a Roman acanthus scroll.

The free introduction of monsters and animals is likewise a characteristic of Greek and Roman ornament, as the sphinx, the triton, the griffin, and others; they occur, however, more abundantly in the Roman.

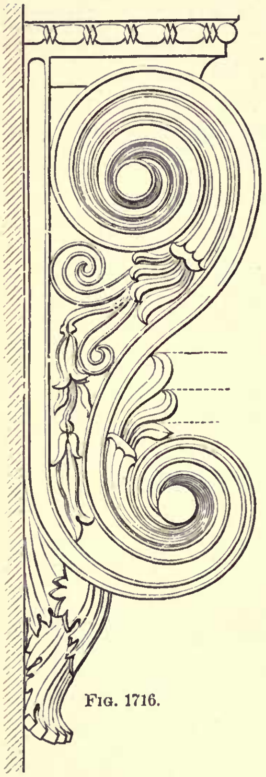


Fig. 1716.

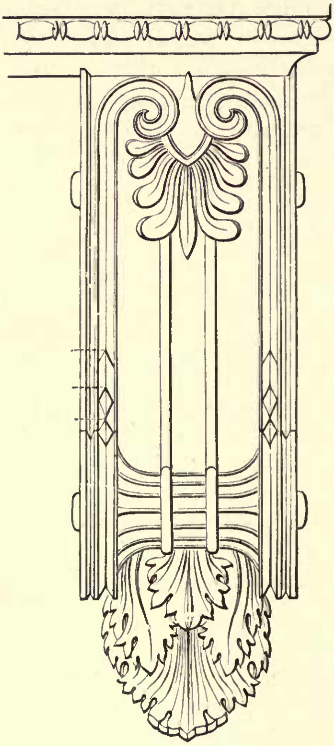


Fig. 1717.

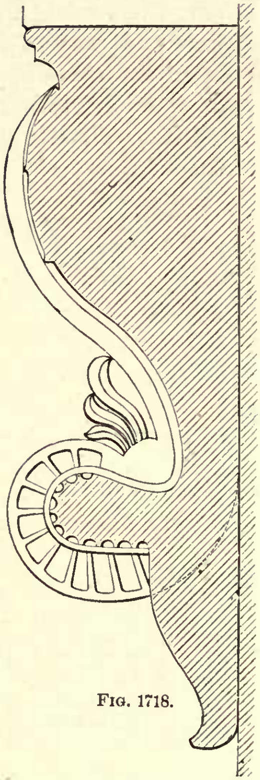


Fig. 1718.

Symbols are the foundation of decorations in the Byzantine and Romanesque. The early symbols were the monogram of Christ, the lily, the cross, the serpent, the fish, the aureole, or vesica piscis, and the circle or nimbus, the trefoil and quatrefoil, the first having reference to the Trinity, the second to the four Evangelists. Occasionally the symbolic images of the Evangelists, the angel, the lion, the ox, and the eagle,

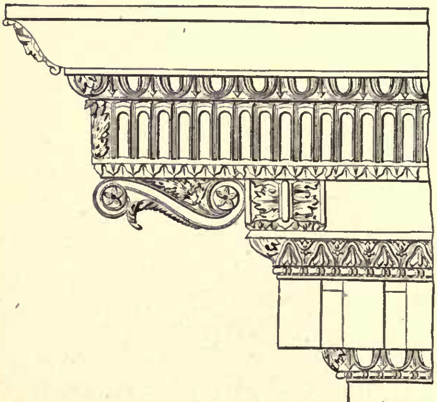


Fig. 1719.

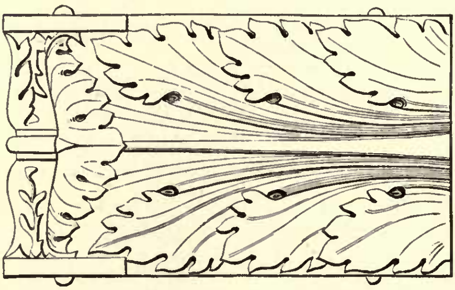


Fig. 1720.

are represented within these circles. The hand in the attitude of benediction, and the lily (the *fleur-de-lis*), the emblem of the virgin and purity, are com-

FIG. 1721.

mon; also a peculiarly formed leaf, somewhat resembling the leaf of the ordinary thistle. The serpent figures largely in Byzantine art as the instrument of the fall, and one type of the redemption.

FIG. 1722.

Pagan ornaments, under certain symbolic modifications, were admitted into Christian decorations. Thus the foliations of the scroll were terminated by lilies, or by leaves of three, four, and five blades, the number of blades being significant; and in a similar way the anthemion and every other ancient ornament. In the Byzantine, all their imitations of natural forms were invariably conventional; it is the same even with animals and the human figure; every saint had his prescribed colours, proportions, and symbols.

The Saracenic was the period of gorgeous diapers (Figs. 1722 and 1723), for their habit of decorating the entire surfaces of their apartments was highly favourable to the development of this class of design. The Alhambra displays almost endless specimens, and all are in relief and enriched with gold and colour, chiefly blue and red. The religious cycles and symbolic figures of the Byzantine are excluded. Mere curves and angles or interlacings were now to bear the chief burden of a design, but distinguished by a variety of colour. The curves, however, very naturally fell into standard forms and floral shapes, and the lines and angles were soon developed into a very characteristic species of tracery, or interlaid strap-work, very agreeably diversified by the ornamental

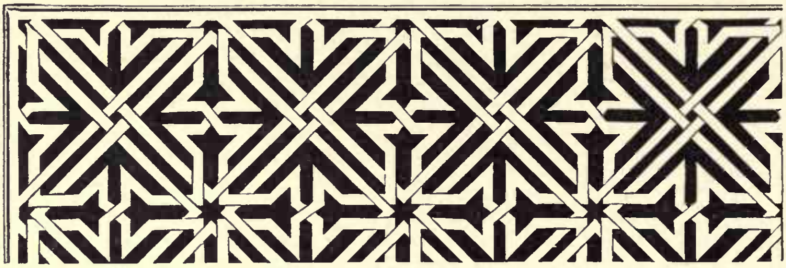


FIG. 1723.

introduction of the inscriptions, which last custom of elaborating inscriptions with their designs was peculiarly Saracenic. Although flowers were not palpably admitted, yet the great mass of the minor details of Saracenic designs are composed of flower forms disguised—the very inscriptions are sometimes thus grouped as flowers; still, no actual flower ever occurs, as the exclusion of all natural images is fundamental to the style in its purity.

All the symbolic elements of the Byzantine are continued in the Gothic. Ornamentally, the Gothic is the geometrical and pointed element elaborated to the utmost; its only peculiarities are its combinations of details; at first the conventional and geometrical prevailing, and afterward these combined with the elaboration of natural objects in its decoration. The most striking feature of all Gothic work is the wonderful elaboration of its geometric tracery; vesicas, trefoils, quatrefoils, cinquefoils, and an infinity of geometric varieties besides. The tracery is so paramount a characteristic that the three English varieties, the early English, the decorated, and the perpendicular, and the French flamboyant, are distinguished almost exclusively by this feature. (See Figs. 1661 to 1665.)

The ornamental mouldings used in the decorative details are numerous, among which the more common is the chevron or zigzag (Fig. 1724), simple as the indented, or duplicated, triplicated, or quadrupled; the billet, the prismatic billet, the square billet, and the alternate billet (Fig. 1725); the star (Fig. 1726), the fir-cone; the cable (Fig. 1727); the embattled (Fig. 1728); the nail-head (Fig. 1729); the dog-tooth (Fig. 1730); the ball-flower (Fig. 1731); and the serpentine vine-scroll.

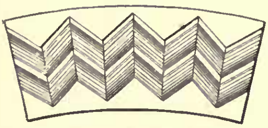


FIG. 1724.

FIG. 1725.

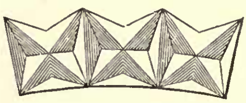


FIG. 1726.

The crocket, in its earliest form, was the simple arrow-head of the episcopal pastoral staff; subsequently finished with a trefoil, and afterward still further enriched. Figs. 1732 and 1733 are early English crockets; Fig. 1734 a decorated one. Fig. 1735 is a finial of the same style. Both finials and crockets in detail display a variety of forms.

The parapets of the early English style are often a simple horizontal course, supported by a corbel table, sometimes relieved by a series of sunk blank trefoil-

Fig. 1728.

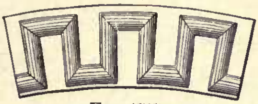


Fig. 1727.

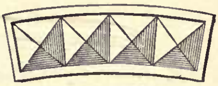


Fig. 1729.

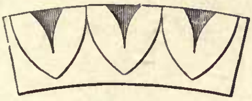


Fig. 1730.

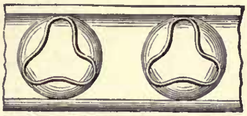


Fig. 1731.

headed panels; sometimes a low embattled parapet crowns the wall. In the decorated style the horizontal parapet is sometimes pierced with trefoils, sometimes with wavy, flowing tracery (Fig. 1736). Grotesque spouts or gargoyles discharge the water from the gutters. The para-

Fig. 1732.

Fig. 1734.

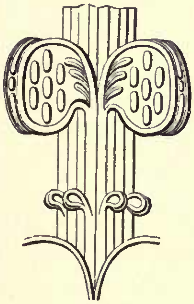


Fig. 1733.

Fig. 1735.

pets of the perpendicular style are frequently embattled (Fig. 1737), covered with sunk or pierced panelling, and ornamented with quatrefoil, or small tre-

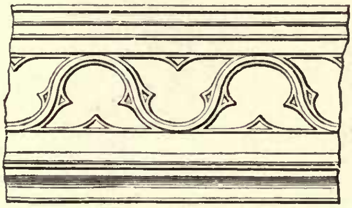


Fig. 1736.

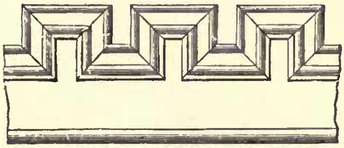


Fig. 1737.

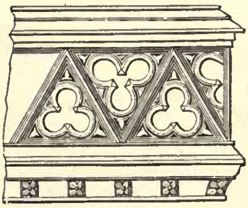


Fig. 1738.

foil-headed arches; sometimes not embattled but covered with sunk or pierced quatrefoils in circles, or with trefoils in triangular spaces, as in Fig. 1738.

Among the varieties of ornamental work, the mode of covering small plain surfaces with diapering (Fig. 1739) was sometimes used, the design being in exact accordance with the architectural features and details of the style. The rose (Fig. 1740), the badge of the houses of York and Lancaster, is often met with in the perpendicular style;

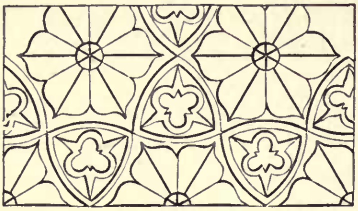


Fig. 1739.

Fig. 1740.

and tendrils, leaves, and fruit of the vine are carved in great profusion in the hollows of rich cornice mouldings, especially on screen-work in the interior of a church. Fig. 1741, in its original type a Byzantine ornament, an alternate lily and cross, is a common finish to the cornice of rich screen-work in the latest Gothic, and is known under the name of the Tudor flower.

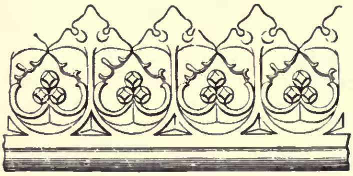


Fig. 1741.

Sculptured foliage (Figs. 1742 to 1747) is much used in capitals, brackets, corbels, bosses, and crockets. Among the forms of foliage the trefoil is most predominant.

The Ornaments of the Renaissance.—The term Renaissance is used in a double sense; in a general sense implying the revival of art, and specially signifying a peculiar style of ornament. It is also sometimes, in a very confined sense, applied in reference to ornament of the style of Benvenuto Cellini; or, as it is sometimes designated, the Henry II (of France) style.

The mixture of various elements is one of the essentials of this style. These

Fig. 1742.

Fig. 1743.

Fig. 1744.

Fig. 1745.

Fig. 1746.

Fig. 1747.

elements are the classical ornaments; unnatural and natural flowers and foliage; men and animals, natural and grotesque; cartouches, or pierced and scrolled shields, in great prominence; tracery independent, and developed from the scrolls of the cartouches; and jewel forms (Figs. 1748 and 1749).

Fig. 1748.

The Elizabethan is a partial elaboration of the same style; the present Elizabethan exhibits a very striking preponderance of strap and shield work; but the earlier is much nearer allied to the Continental styles of the time, classical ornaments but rude in detail, occasional scroll and arabesque work, and strap-work, holding a much more prominent place than the pierced or scrolled shields. Fig. 1750 is an example of the style from the old guard chamber, Westminster.

Of the earliest and transition styles of Renaissance ornament are the Tricento and the Quatrecento. The great features of the first are its intricate tracery and delicate scroll-work of conventional foliage, the style being but a slight remove from the Byzantine and Saracenic; of the second, elaborate natural imitations of fruit, flowers, birds, or animals (Fig. 1751), all disposed simply with a view to the ornamental; also occasional cartouches, or scrolled shield-work.

Fig. 1749.

The Renaissance is something more approximative to a combination of previous styles than a revival of any in particular, developed solely on æsthetic principles, from a love of the forms and harmonies themselves, as varieties of effect and arrangements of beauty, not because they had any particular signification, or from any superstitious attachment to them as heirlooms.

Fig. 1752 is an example of ornament in the Cinquecento style. The arabesque scroll-work is the most prominent feature of the Cinquecento, and with this in its elements, it combines every other feature of classical art, with the unlimited choice of natural and conventional imitations from the entire animal and vegetable kingdom, both arbitrarily disposed and combined. Absolute works of art, such as vases and implements, and instruments of all kinds, are prominent elements of the Cinquecento arabesque, but cartouches and strap-work wholly

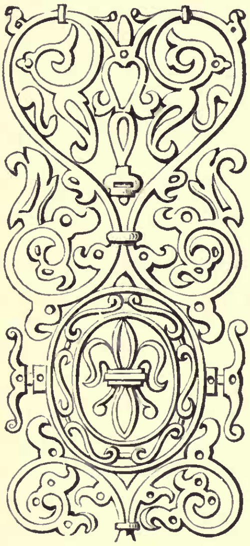


Fig. 1750.

Fig. 1751.

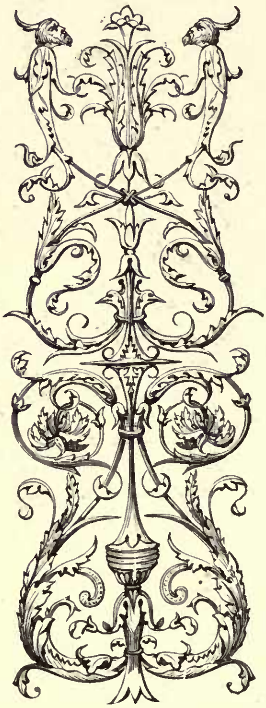


Fig. 1752.

disappear from the best examples. Another chief feature of the Cinquecento is the admirable play of colour in its arabesques and scrolls; and it is worthy of note that the three secondary colours—orange, green, and purple—perform the chief parts in all the coloured decorations.

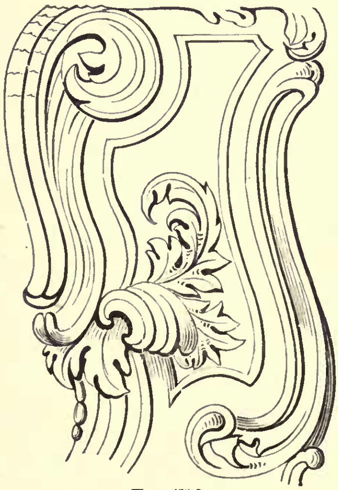


Fig. 1753.

Fig. 1753 is an example of the Louis Quatorze style of ornament. The great medium of this style was gilt stucco-work, and this absence of colour seems to have led to its most striking characteristic, infinite play of light, of shade; colour, or mere beauty of form in detail, having no part in it whatever. Flat surfaces are not admitted; all are concave or convex: this constant varying of the surface gives every point of view its high lights and brilliant contrasts.

The Louis Quinze style differs from that of Louis Quatorze chiefly in its absence of symmetry; in many of its examples it is an almost random dispersion of the scroll and shell, mixed only with that peculiar crimping of shell-work, the coquillage.

The ornaments of which we have thus given examples are, in general, applied to interior decorations, to friezes, pilasters, panels, architraves, the faces and soffits of arches, ceilings, etc., to furniture, and to art-manufactures in general. For exteriors these ornaments are sparingly applied; shield and scroll-work, of the later Elizabethan or Renaissance style, is sometimes used, but very seldom tracery.

Principles of Design.—Professedly treating of architecture only in its most mechanical phase of drawing, the history of it as an art, and the distinctions of styles, have been but briefly treated. To one anxious to acquire knowledge in this department we refer, as the very best compendium within our knowledge, to Ferguson's "Hand-Book of Architecture." The study of this work will give direction to a person's observation, but, without referring to actual examples, mere reading will be of little use. Drawings give general ideas of the character of buildings, but no idea of size or of the surroundings of a building. Many a weak design, especially in cast-iron buildings, acquires a sort of strength by the number of its repetitions, giving an idea of extent; and many a beautiful design on paper has failed in its execution, being dwarfed by its surroundings. With regard to the style of a building, there are none of the ancient styles in their purity adapted to present requirements; our churches and theatres are more for the gratification of the ear than the eye, and the comforts of our domestic architecture, and the requirements of our stores and warehouses, are almost the growth of the present century. For a design, look first to the requirements of the structure, the purposes to which it is to be applied; sketch the plan first, arrange the divisions of rooms, the openings for doors and windows, construct the sections, and then the elevations, first in plain outline; modify each by the exigencies of construction.

"Construction, including in the term the disposition of a building in reference to its uses, is by some supposed to be the common part of the art of architecture, but it is really the bone, muscle, and nerve of architecture, and the arts of construction are those to which the true architect will look, rather than to rules and examples, for the means of producing two at least of the three essential conditions of building well, *commodity*, *firmness*, and *delight*, which conditions have been aptly said to be the end of architecture as of all creative arts.

"The two great principles of the art are: First, that there should be no features about a building which are not necessary for convenience, construction, or propriety; second, that all ornament should consist of enrichment of the essential construction of the building.

"The neglect of these two rules is the cause of all the bad architecture of the present time. Architectural features are continually tacked on buildings with which they have no connection, merely for the sake of what is termed effect, and ornaments are continually constructed instead of forming the decoration of construction to which in good taste they should always be subservient. The taste of the artist ought to be held merely ancillary to truthful disposition for structure and service. The soundest construction is the most apt in the production, or the reproduction, it may be, of real art. The Eddystone Lighthouse is well adapted to its uses; it is commodious, firm and stable almost to a miracle, and its form is as beautiful in outline to the delight of the eye as it is well adapted to break and mitigate the force of the sea in defence of its own structure. The English Exhibition Building of 1851 was most commodious for the purposes of an exhibition, firm enough for the temporary purpose required of it, and there was delight in the simplicity and truth of its combinations; and all this may be said to have grown out of propriety of construction, as applied to the material, cast-iron. The use of unfitting material, or fitting material inappropriately, leads almost entirely to incommodiousness, infirmity, and offence, or some of them.

"Out of truth in structure, and that structure of a very inartificial sort, grow the beautiful forms of the admirable proportions found in the works of the Greeks; and out of truth in structure, with the strictest regard to the necessities of the composition and of the material employed, and that structure as full of artifice as the artifice employed is of truth and simplicity, grew the classical works vulgarly called Gothic, but now characteristically designated as Pointed, from the arch which is the basis of the style. Structural untruth is not to be justified by authority; neither Sir Christopher Wren, nor the Athenian exemplars of Doric or Ionic in the Propylæum and in the Minerva Polias, with their irregular and inordinately wide intercolumniation, can persuade even the untutored eye to accept weakness for strength, or what is false for truth.

"The Greek examples offer the most beautiful forms for mouldings, and the Grecian mode of enriching them is unsurpassed. It should be borne in mind that the object in architectural enrichment is not to show ornament, but to enrich the surface by producing an effective and pleasing variety of light and shade; but still, although ornament should be a secondary consideration, it will develop itself, and therefore should be of elegant form and composition."

We have quoted thus at some length from the article "Architecture," "Encyclopædia Britannica," because with many authority is necessary, and they distrust their own powers of observation and analysis; all must feel the truth of the above, but in practice it is very little appreciated or carried out. The present taste in architecture, as in the theatre, is for the spectacular; breadth or dignity of effect is not popular; edifices are not only covered with, but built up in ornament; and construction is but secondary. The French, having a building-stone that is very easily worked, cut merely the joints, leaving the rough outer surface to be worked after it is laid; chopping out mouldings and ornaments almost as readily as though it were in plaster, and the surface when finished is covered with enrichments in low relief. The fashion thus set is imitated in this country at immense cost, in the most unfitting materials, marble and granite. Our architectural buildings express fitly our condition—a rich country, recent and easily acquired wealth, and a desire and rivalry to exhibit it, or a display as a means of advertising, and in this truth of expression will have an archæological interest; although it does not contribute much to present excellence in construction, it still has this value: that the architect or constructor need be governed by no rules or principles—he can make experiments on a pretty extensive scale, and out of much bad construction even forms and ornament may spring up which will stand the test of time, and form a nucleus of a new style adapted to the present wants.

Cast-iron as a building material, with the exception of exhibition buildings, has seldom been treated distinctively; buildings erected with it have been copies of those in stone, and have been even imitated in colour. For the first story of stores, where space is necessary for light and the exhibition of wares, cast-iron columns are almost invariably used, but are objected to architecturally, that they look too weak for the support of the piles of brick and stone above them. The objection should not be to the use, but that the truth of the adequate strength of the cast-iron is not conveyed by the form or colour. No one objects that the ankles of Atlas look too light to support the massive figure and globe, or wishes him seated to give the idea of stability; so if the columns and lintels were some other form than Greek or Roman with immense intercolumniations, and coloured fitly, the appearance of weakness would be entirely lost sight of.

Improvements in the manufacture of iron and steel have led up to the skeleton construction (page 565)—frames of cast or rolled iron and steel framed and set before any of the masonry except that of the foundations is laid. All the columns, girders, and beams are bolted together, built into, and covered with masonry to add to the rigidity of the structure and for protection against fire. The framing is square, but, for variety and design, arches, soffits, and ornamental clothing is made in masonry. Little in exterior form can be considered a necessity of construction, and there is as yet no standard of finish. The function of the metal, like the bone in the animal structure, is to give strength to sustain it; the masonry is the muscle to stiffen, protect, and ornament it. Constructive expedients, like roofs, reduce the appearance of height, and are objectionable from the necessity of gutters and leaders.

In conclusion, the draughtsman should be conversant with classic and later styles; still, as he must design to suit the necessities of the times, and the requirements of present tastes and fashions of buildings, he should keep himself posted on what is being done, and he will find it very convenient to have a scrap-book of cuts from which to draw parts of a design, and afford him ready means of combinations. He will find much in illustrated magazines and newspapers, many cuts unpromising as a whole, yet fruitful in suggestions of parts; many an agreeable outline unsatisfactorily filled up; many that are only valuable as showing dimensions requisite for certain uses. But the larger the collection the better for the draughtsman; it will save time to know, as far as possible, what has been done, that he may judge what forms and proportions it will be best for him to use, and what to avoid.

It has been our practice to select, from papers and magazines, cuts which we considered of value, and arrange them in scrap-books with appropriate headings. In the Appendix a few pages of "scraps" are given as illustrations.

ISOMETRICAL DRAWING.

PROFESSOR FARISH, of Cambridge, has given the term Isometrical Perspective to a particular projection which represents a cube, as in Fig. 1754. The words imply that the measure of the representations of the lines forming the sides of each face are equal.

The principle of isometric representation consists in selecting, for the plane of the projection, one equally inclined to three principal axes, at right angles to each other, so that all straight lines coincident with or parallel to these axes are drawn in projection to the same scale. The axes are called isometric axes, and all lines parallel to them are called isometric lines. The planes containing the isometric axes are isometric planes; the point in the object projected, assumed as the origin of the axes, is called the *regulating-point.*

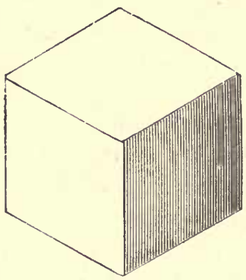


FIG. 1754.

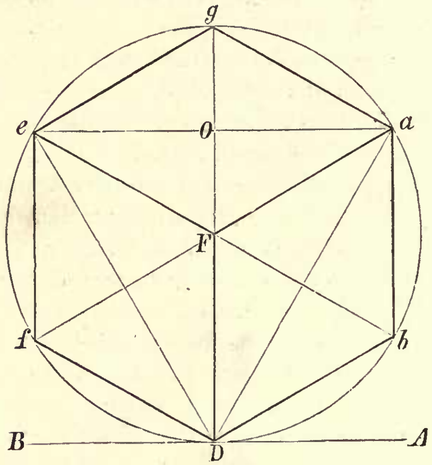


FIG. 1755.

To draw the isometrical projection of a cube (Fig. 1755), draw the horizontal line A B indefinitely; at the point D erect the perpendicular D F, equal to one side of the cube required; through D draw the lines D *b* and D *f* to the right and left, making *f* D B and *b* D A each equal an angle of 30°. Consequently, the angles F D *f* and F D *b* are each equal to 60°. Make D *b* and D *f* each equal to the side of the cube, and at *b* and *f* erect perpendiculars, making *b a* and *f e* each equal to the side of the cube; connect F *a* and F *e*, and draw *e g* parallel to *a* F, and *a g* parallel to F *e*, and we obtain the projection of the cube.

If from the point F, with a radius F D, a circle be described, and commencing at the point D, radii be laid off around the circumference, forming a regular inscribed hexagon, and the points D *a e* be connected with the centre of the circle F, we have an isometrical representation of a cube. The point D is called the *regulating-point.*

On page 123, Fig. 255, is shown the orthographic projection of a parallelopipedon on the several planes, and Fig. 254 the revolution of these planes and their cubical representation. Fig. 1756 is the representation of the same solid in isometric perspective, producing nearly the same effect. Measures are transferred directly from plans and elevations in orthographic projections to those in isometry. The isometric scale adopted applies only to isometric lines, as F D, F *a*, and F *e* (Fig. 1755), or lines parallel thereto; the diagonals which are equal to each other, and longer than the sides of the cube, are the one less, the other greater.

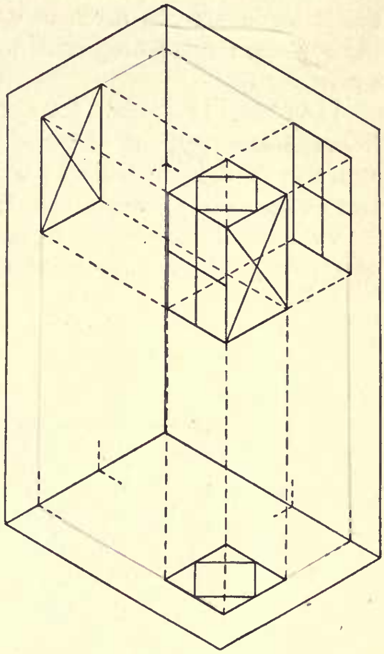


Fig. 1756.

Understanding the isometrical projection of a cube, any surface or solid may be similarly constructed, since it is easy to suppose a cube sufficiently large to contain within it the whole of the model intended to be represented, and, as hereafter will be further illustrated, the position of any point on or within the cube, the direction of any line, or the inclination of any plane to which it may be cut, can be easily ascertained and represented.

In Figs. 1754 and 1755 one face of the cube appears horizontal, and the other two faces appear vertical. If now the figures be inverted, that which

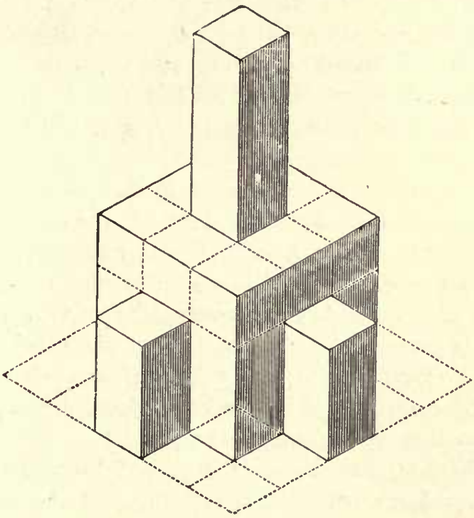


Fig. 1757.

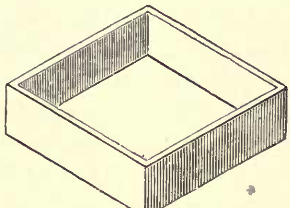


Fig. 1758.

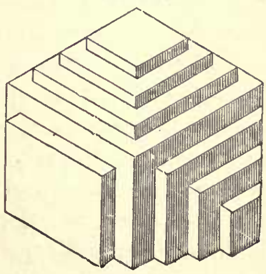


Fig. 1759.

before appeared to be the top of the object will now appear to be its under side.

The angle of the cube formed by the three radii meeting in the centre of the hexagon may be made to appear either an internal or external angle, in the one case the faces representing the interior, and in the other the exterior of a cube.

Figs. 1757, 1758, and 1759 illustrate the application of isometrical drawing to simple combinations of the cube and parallelopipedon. The mode of construction of these figures will be easily understood by inspection, as they contain no lines except isometrical ones.

To draw Angles to the Boundary Lines of an Isometrical Cube.—Draw a square (Fig. 1760) whose sides are equal to those of the isometrical cube A

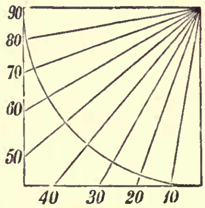


FIG. 1760.

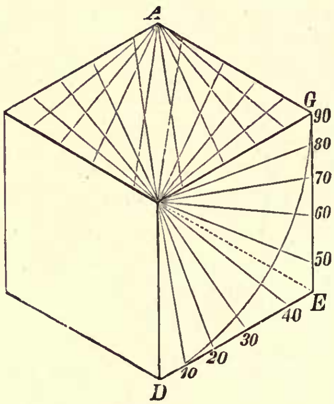


FIG. 1761.

(Fig. 1761), and from any of its angles describe a quadrant, which divide into 90°, and draw radii through the divisions meeting the sides of the square. These will then form a scale to be applied to the faces of the cube; thus, on D E, or any other, by making the same divisions along their respective edges.

As the figure is bounded by twelve isometrical lines, and the scale of tangents may be applied two ways to each, it can be applied therefore twenty-four ways in all, affording a simple means of drawing, on the isometrical faces of the cube, lines at any angles with their boundaries.

Figs. 1762 to 1767 show the section of the cube by single planes, at various inclinations to the faces of the cubes. Figs. 1768 and 1769 are the same cube, but turned round, with pieces cut out of it. Fig. 1770 is a cube cut by two planes forming the projection of a roof. Fig. 1771 is a cube with all of the angles cut off by planes, so as to leave each face an octagon. Fig. 1772 represents the angles cut off by planes perpendicular to the base of the cube, forming thereby a regular octagonal prism. By drawing lines from each of the angles of an octagonal base to the centre point of the upper face of the cube, we have the isometrical representation of an octagonal pyramid.

As the lines of construction have all been retained in these figures, they will be easily understood and copied, and are sufficient illustrations of the method of representing any solid by inclosing it in a cube.

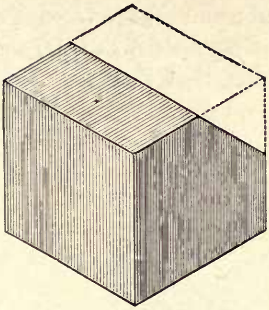


Fig. 1762.

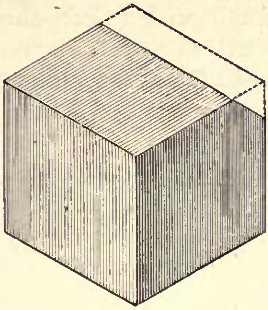


Fig. 1763.

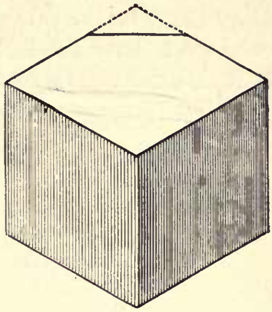


Fig. 1764.

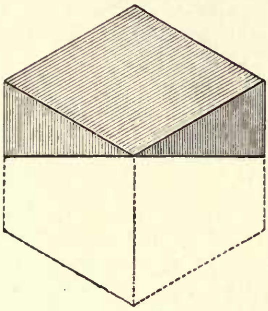


Fig. 1765.

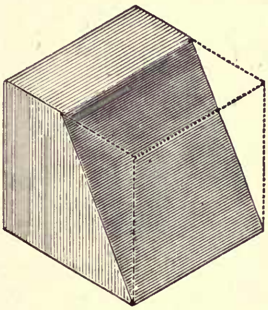


Fig. 1766.

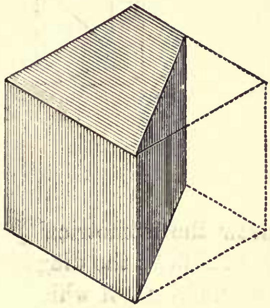


Fig. 1767.

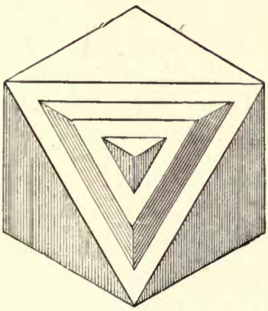


Fig. 1768.

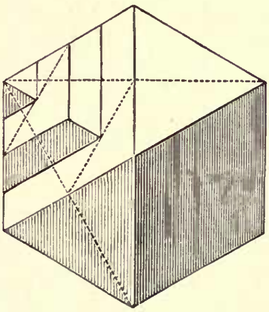


Fig. 1769.

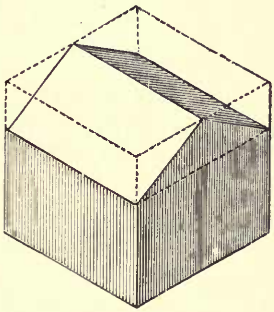


Fig. 1770.

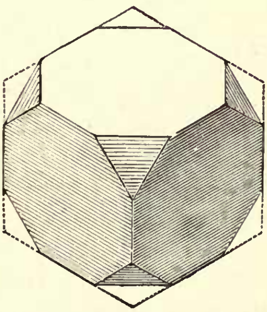


Fig. 1771.

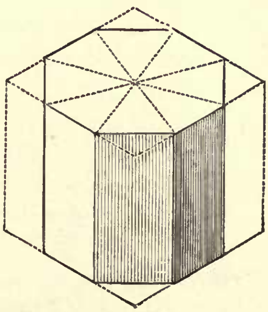


Fig. 1772.

In the application of this species of projection to curved lines, let A B (Fig. 1773) be the side of a cube with a circle inscribed; and all the faces of a cube are to have similarly inscribed circles. Draw the diagonals A B, C D, and at their intersection with the circumference, lines parallel to A C, B D. Now

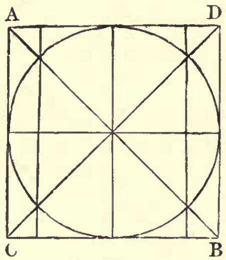


Fig. 1773.

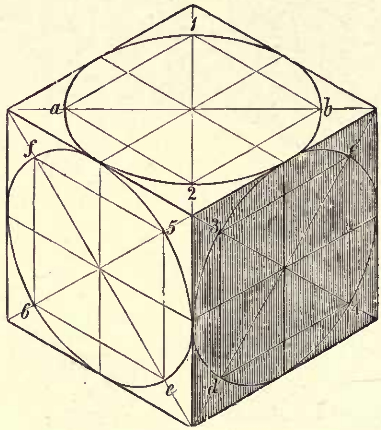


Fig. 1774.

draw the isometrical projection of the cube (Fig. 1774), and lay out on the several faces the diagonals and the parallels; the projection of the circle will be an ellipse, of which the diagonals being the axes, their extremities are defined by their intersections *f* 6, *e* 5, *a* 2, *b* 1, *d* 3, *c* 4, with the parallels; having thus the major and minor axes, construct the ellipse by the trammel, or, since the curve is tangent at the centre of the sides, we have eight points in the curve; it may be put in by curves or by the hand.

To divide the Circumference of a Circle.—First method: On the centre of the line A B (Fig. 1775) erect a perpendicular, C D, making it equal to C A or C B; then from D, with any radius, describe an arc and divide it in the ratio

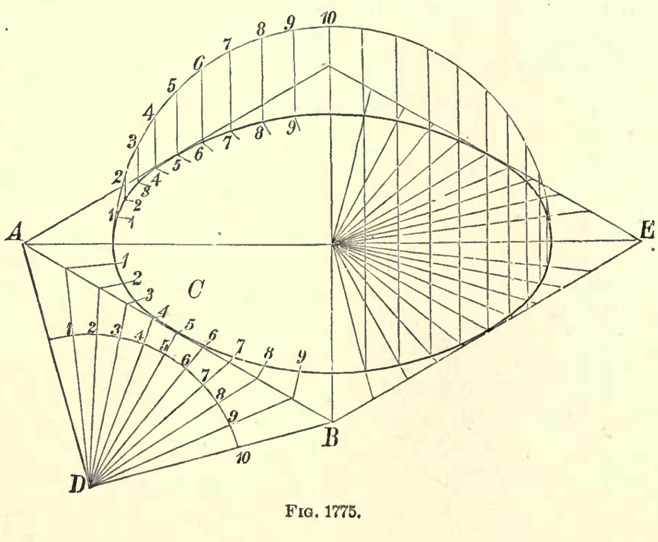


Fig. 1775.

required, and draw through the divisions radii from D meeting A B; then from the isometric centre of the circle draw radii from the divisions on A B, cutting the circumference in the points required.

Second method: On the major axis of the ellipse describe a semicircle, and divide it in the manner required. Through the points of division draw lines

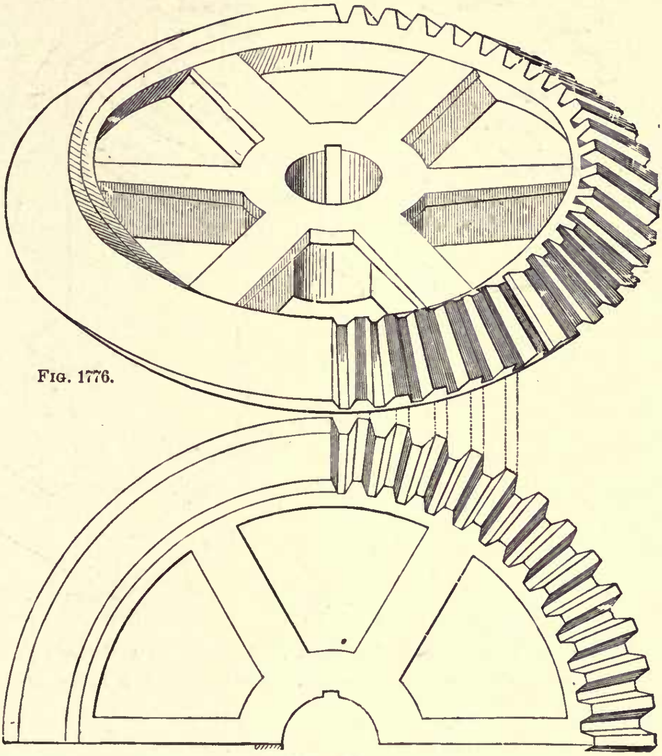


FIG. 1776.

FIG. 1777.

perpendicular to A E, which will divide the circumference of the ellipse in the same ratio. On the right hand of the figure both methods are shown in combination, and the intersections of the lines give the points in the ellipse.

Fig. 1776 is an isometrical projection of a bevel-wheel, with a half-plan (Fig. 1777) beneath, and projected lines explanatory of the method to be adopted in drawing the teeth, and of which only half are shown as cut. It will be seen, by reference to the second method given above for the division of the circumference of a circle, that the semicircle is described directly on the major axis of the ellipse. In practice it will be found more convenient, when a full drawing is to be made, to draw the semicircle on a line parallel to the major axis, and entirely without the lines of the main drawing; and also, as in the example of the bevel-gear, complete on the semicircle, or half-plan, the drawings of all lines, the intersections of which with circles it will be necessary to project on the isometrical drawing.

Fig. 1778 is an isometrical projection of a complete pillow-block, with its hold-down bolts. By reference to Fig. 700 and Figs. 563 and 564, it will be

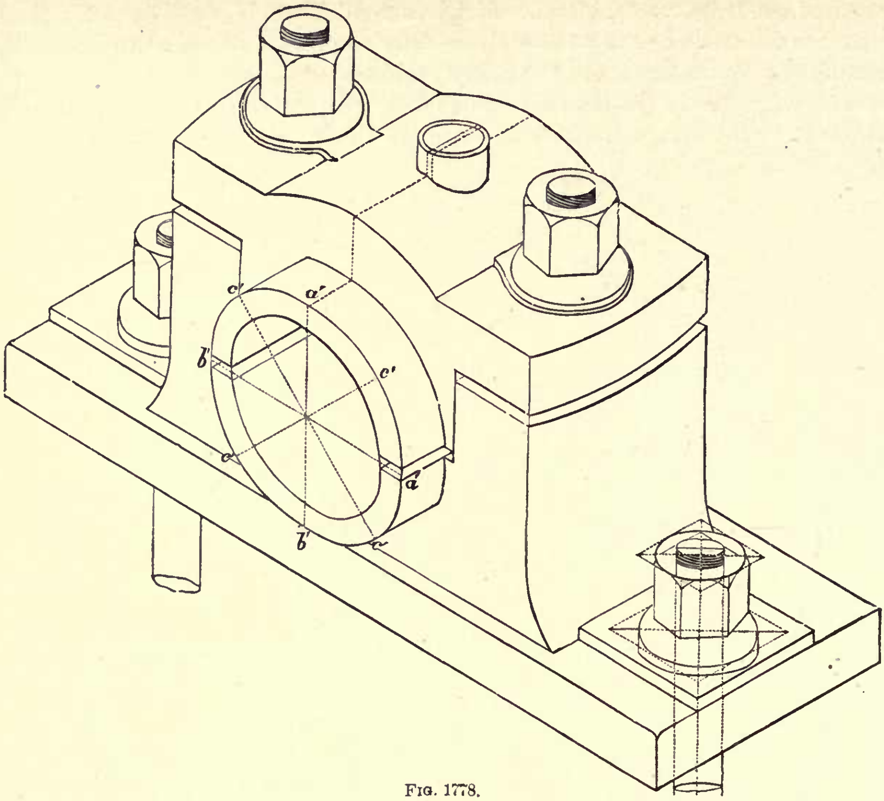


FIG. 1778.

seen how graphically these forms of gearing are given by isometry. Fig. 1779 is an isometrical projection of a water-closet cistern with a standard waste.

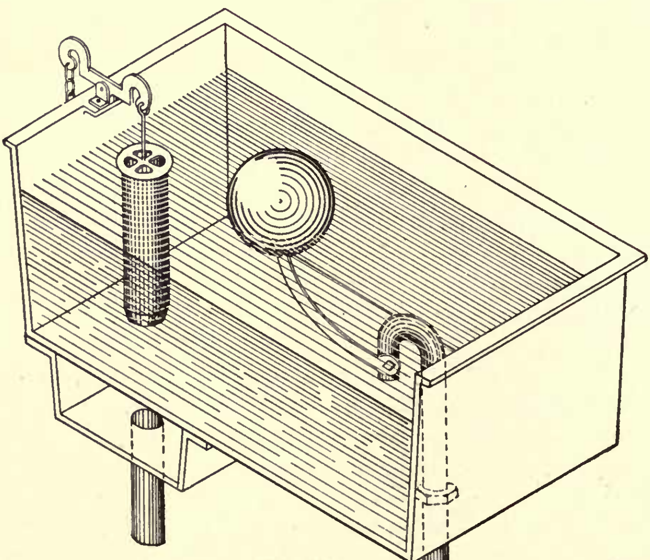


FIG. 1779.

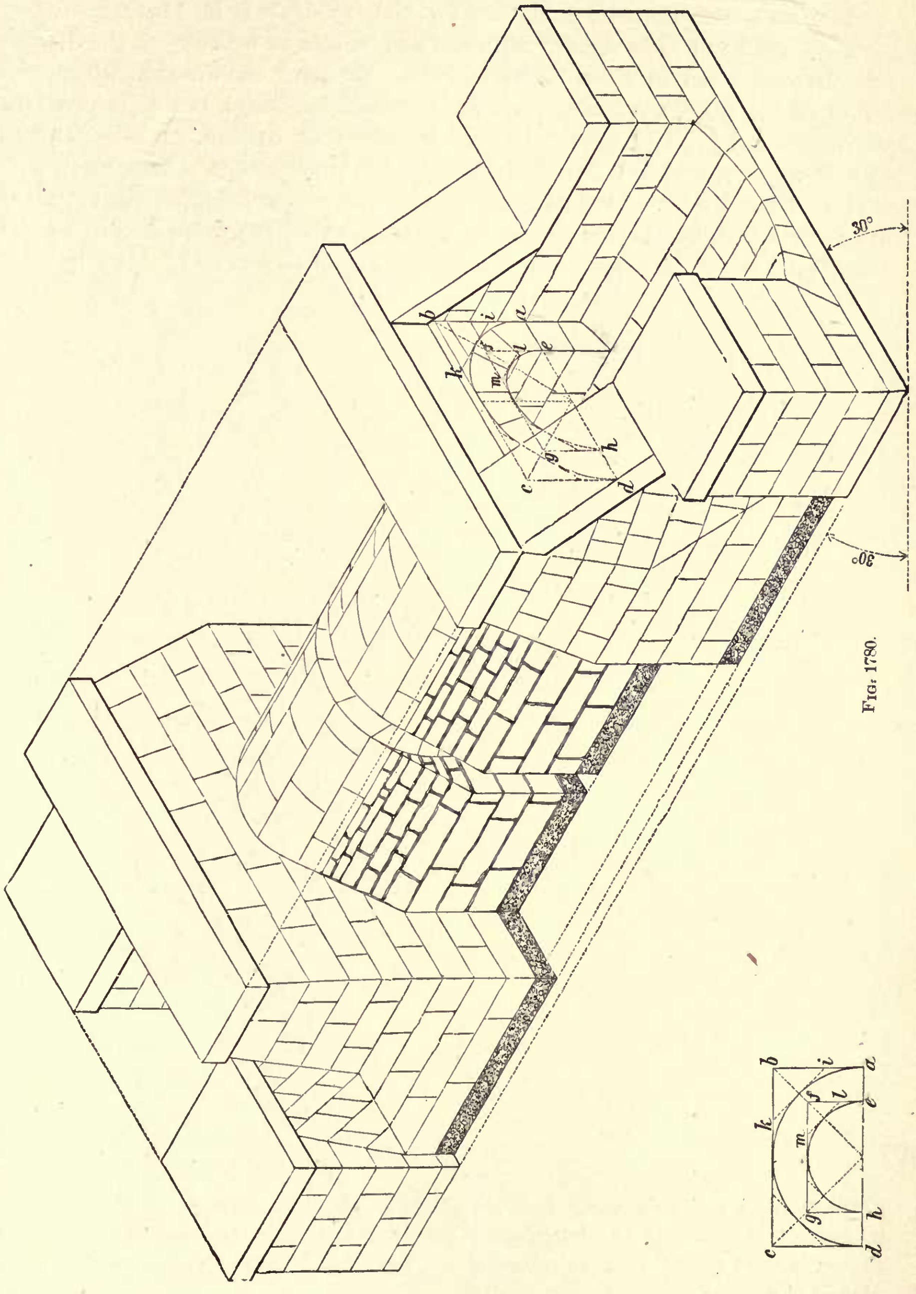


Fig. 1780.

Fig. 1780 is an isometrical projection of a culvert, such as were built beneath the Croton Aqueduct.

Fig. 1016 is an isometrical view of the overflow and outlet of the Victoria and Regent Street sewers in the Thames embankment.

Fig. 1781 is an isometrical elevation of the roof-truss (Fig. 1151).

Figs. 1782 and 1783 are the elevation and section in isometry of the district school-house given in Figs. 1495 and 1496. To bring the drawing within the limits of the page, the scale has been necessarily reduced, but it is given in the figure as it should always be, either drawn or written, on all drawings to a scale, not intended for mere pictures or illustrations. The section is drawn at the height of 8 feet above the base course, and higher than is usual in such sections, but it was necessary on account of the extra height of the window-sill above the floor, desirable in all school-rooms. Fig. 1783 is more

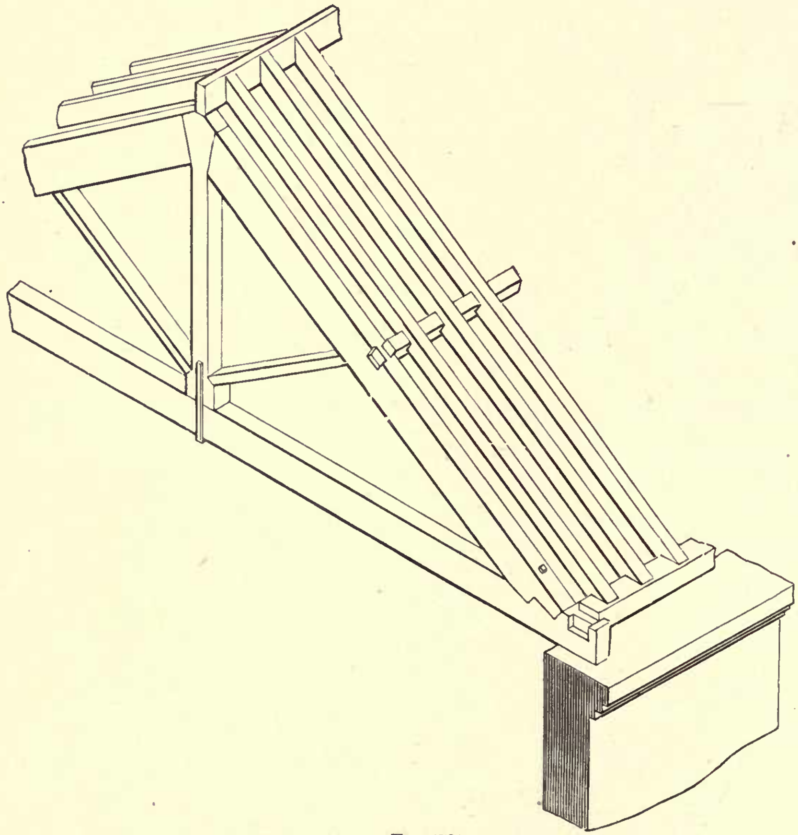


Fig. 1781.

graphic than the plan (Fig. 1496), especially when more detail is to be shown, but there is nothing in the present drawing that can not be nearly as well shown by the plan; and to a mechanic, for the purposes of construction, the plan is the simpler.

By comparing the elevation (Fig. 1782) with the perspective (Fig. 1799), the former appears distorted and out of drawing, but it is much more readily

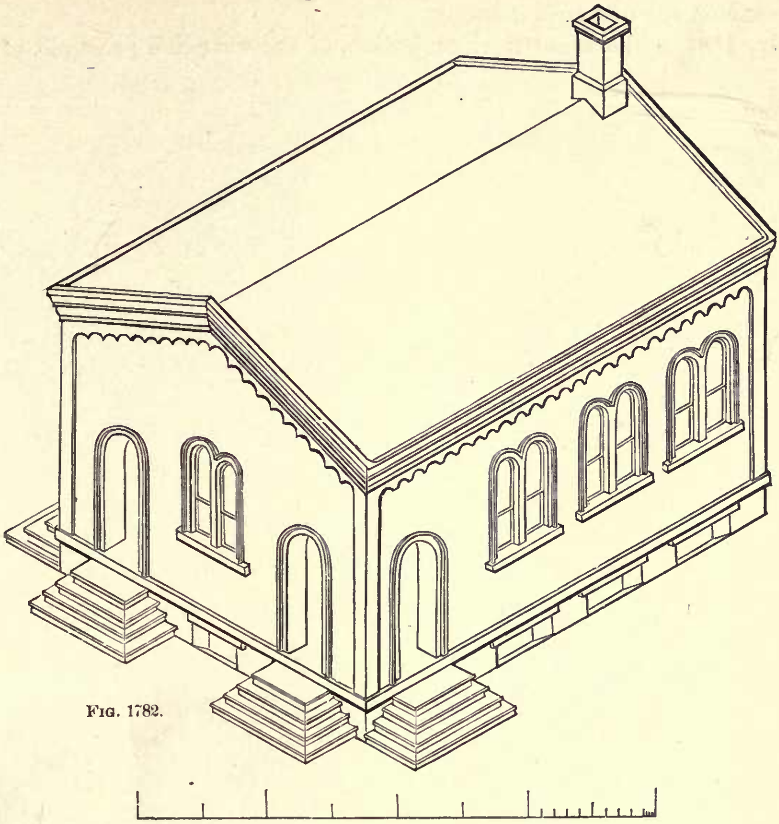


Fig. 1782.

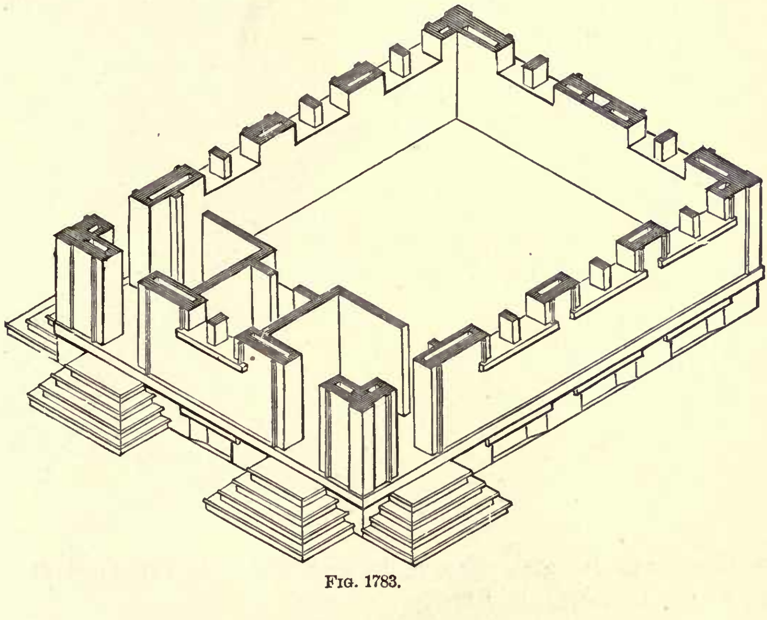


Fig. 1783.

drawn, and has this great convenience, that it is drawn to and can be measured by a scale on the isometric lines.

Fig. 1784 is the isometrical projection, of the wave-line principle, of ship

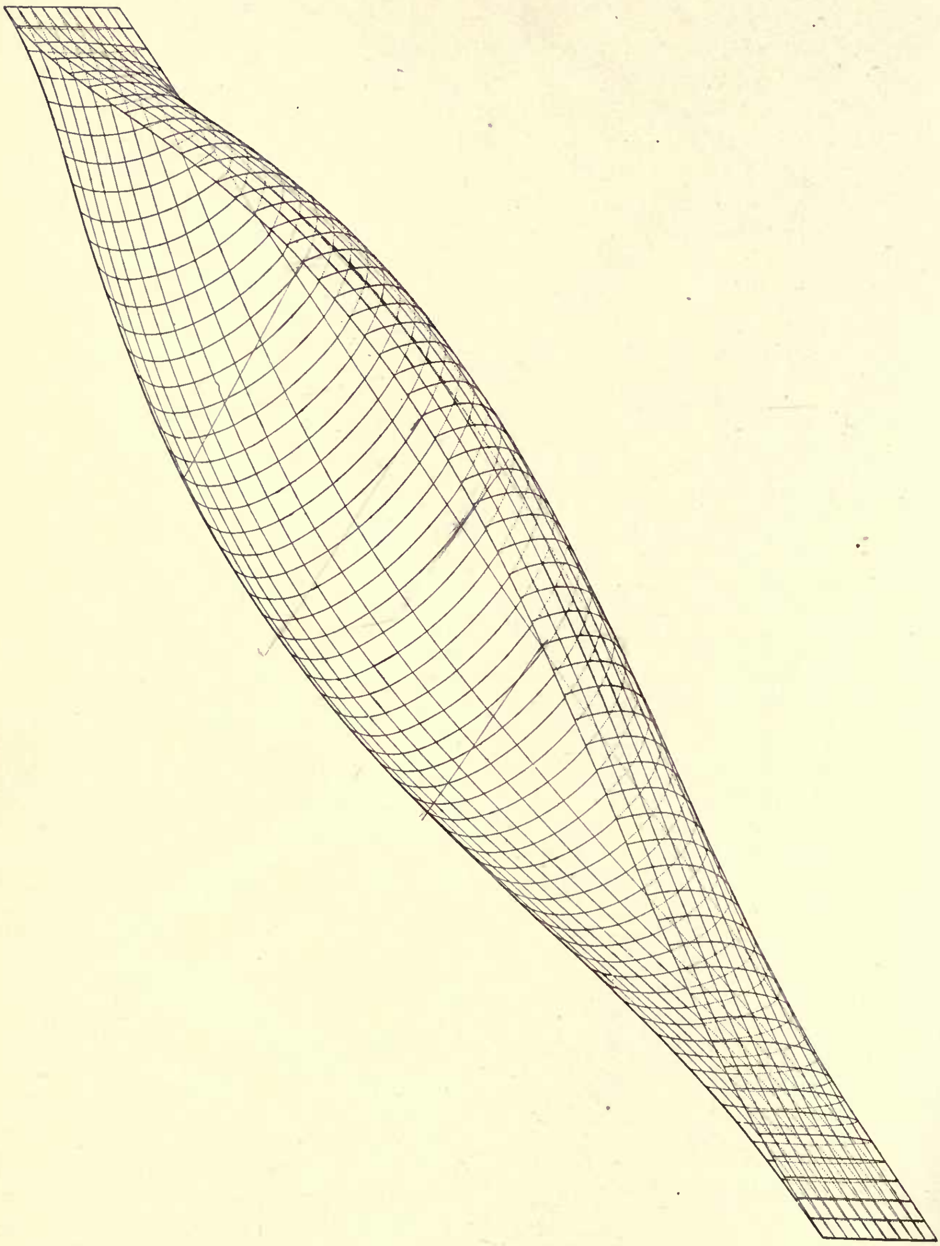
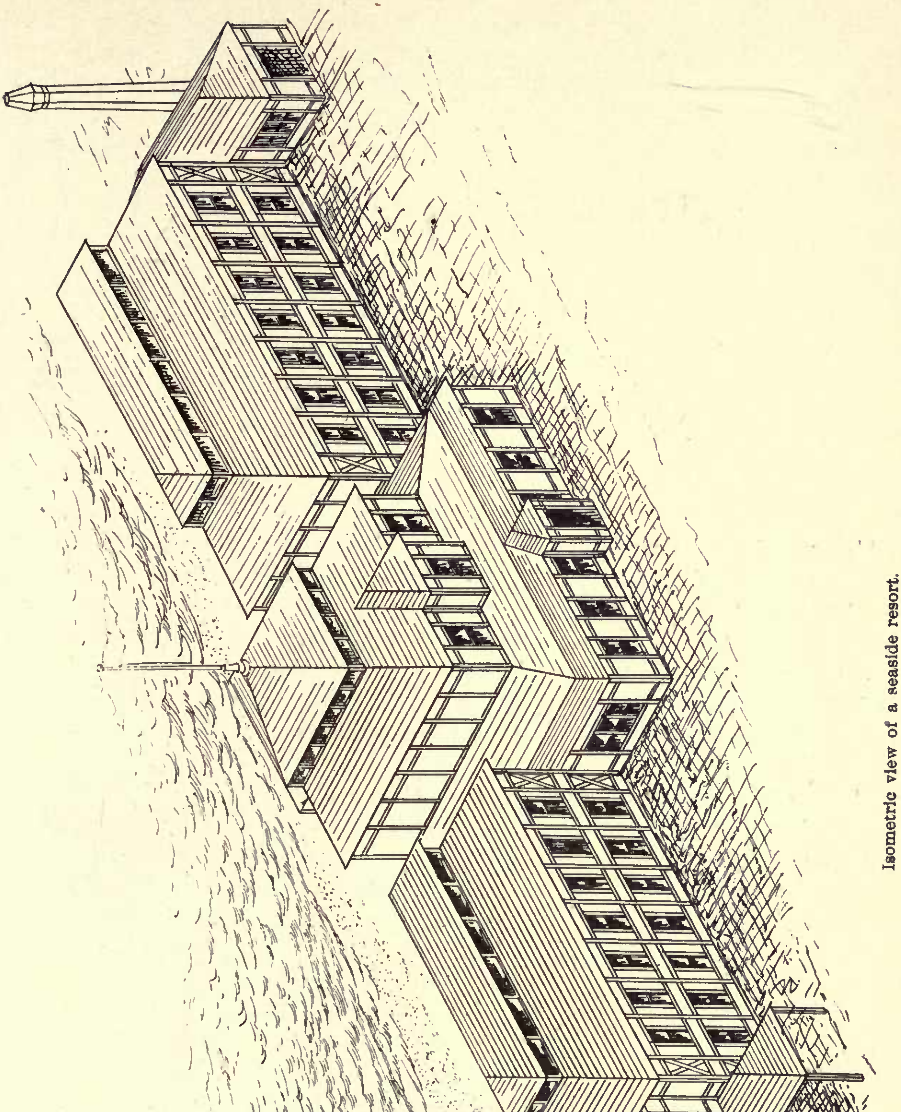


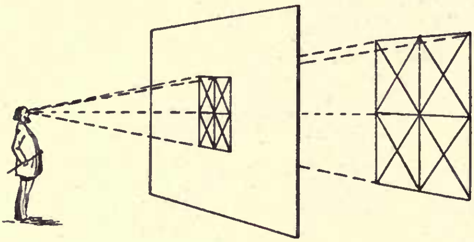
Fig. 1784.

construction, from Russell's "Naval Architecture"—as explained and illustrated on pages 545, 546, and 547.



Isometric view of a seaside resort.

PERSPECTIVE DRAWING.



THE science of Perspective is the representation by geometrical rules, upon a plane surface, of objects as they appear to the eye, from any point of view.

All the points of the surface of a body are visible by means of luminous rays proceeding from these points to the eye. Thus, let the line A B (Fig. 1785) be placed before the eye, C, the lines drawn from the different points 1, 2, 3, 4, etc., represent the visual rays emanating from each of these points. If in the place of a line a surface is substituted, the result will be a pyramid of rays.

The greatest angle under which one or more objects can be distinctly seen is one of 90°. If between the object and the eye there be interposed a transparent plane, the intersections of this plane with the visual rays are termed perspectives of the points from which the rays emanate. In the operations of projection, several important planes are employed in perspective, as:

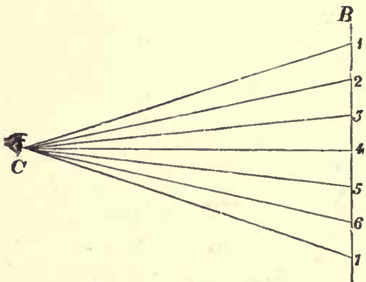


FIG. 1785.

1. The horizontal plane A B (Fig. 1786), on which the spectator and the object viewed are supposed to stand, for convenience supposed perfectly level, is termed the *ground plane.*

2. The plane G N, which has been considered as a transparent plane placed in front of the spectator, on which the objects are delineated, is called the *plane of perspective* or the *plane of the picture.* The intersection G L of the first and second planes is called the line of projection, the *ground*, or *base line of the picture.*

3. The plane E F passing horizontally through the eye of the spectator, and cutting the plane of the picture at right angles, is called the horizontal plane, and its intersection at D D with the plane of the picture is called the *horizon line*, the *horizon* of the picture, or simply the *horizon.*

4. The plane S T passing vertically through the eye of the spectator, and cutting each of the other planes at a right angle, is called the *central plane.*

Point of view, or *point of sight*, is the point where the eye is supposed to be placed to view the object, as at C, and is the vertex of the optical pyramid. Its projection on the ground plane at S is termed the *station point.*

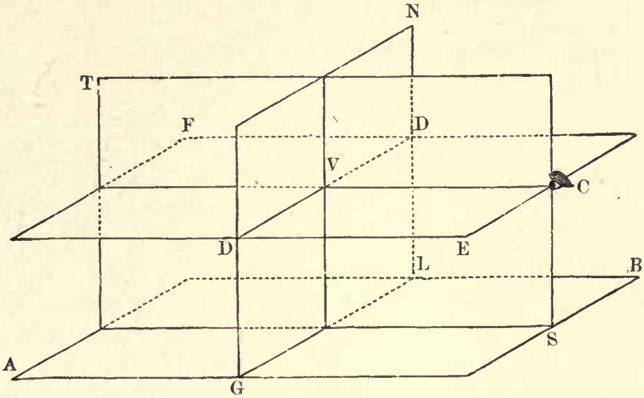


FIG. 1786.

The projection of any point on the ground plane is called the seat of that point.

Centre of view, or centre of picture (commonly, though erroneously, called the point of sight), is the point V where the central vertical line intersects the horizon line; a line drawn from this point to the eye would be in every way perpendicular to the plane of the picture.

Points of distance are points on the horizon as remote from the centre of view as the eye.

Vanishing points are points in a picture to which the perspective of all lines converge that in the original object are parallel to each other.

The vanishing point of any line or system of parallel lines is found by passing a line through the point of sight parallel to the given line, or system of lines. Its intersection with the plane of the picture will be the vanishing point desired. Therefore the vanishing points of all horizontal lines lie on the horizon, and the vanishing points of horizontal lines making an angle of 45° with the ground line are at the points of distance.

Parallel Perspective.—An object is said to be seen in parallel perspective when one of its sides is parallel to the plane of the picture.

Angular Perspective.—An object is said to be seen in angular perspective when none of its sides are parallel to the picture.

The vanishing points of all lines parallel to the plane of perspective are at infinity, or, in other words, such lines have no vanishing points.

The perspectives of lines parallel to the perspective plane are parallel to their projections on that plane.

The process of finding the perspective of lines, plane figures, and solids consists merely in finding the perspectives of established points and connecting them.

To find the Perspective of Two Squares in the Ground Plane whose Sides *are Parallel and Perpendicular to the Perspective Plane* (Fig. 1787).—Let G L be the ground line, *a b d c* and *e f h g* the horizontal projections of the two squares; S′ is the horizontal, and S the vertical projection of the point of sight; *a b* in the ground line is the vertical projection of the two squares.

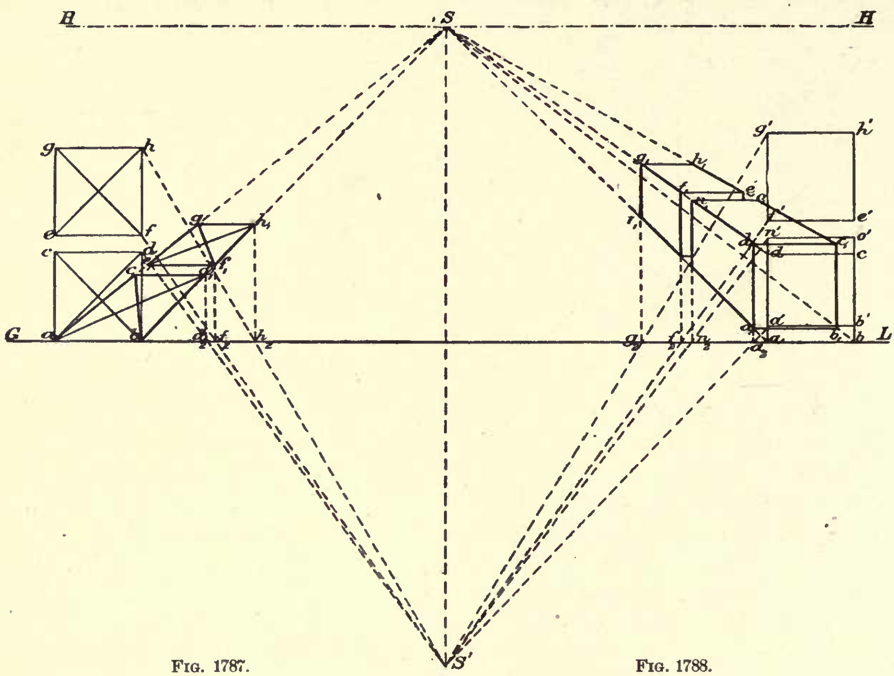


FIG. 1787. FIG. 1788.

Draw *a* S and *b* S; these will be the indefinite perspectives of the sides of the squares perpendicular to the perspective plane. Draw *h* S′; where this line intersects the ground line, project it vertically to h_1, on *b* S, which is the perspective of *h*; the line *g h*, being parallel to the perspective plane, is parallel to the ground line, and is shown at g_1 h_1. In the same way e_1 f_1 and c_1 d_1 are found. The perspectives of the diagonals are the lines connecting the corners.

The line *a b* is in the perspective plane, and is its own perspective. All lines or plane figures lying in the perspective plane appear in perspective in their true form and size.

To find the Perspectives of Two Cubes whose Sides are Parallel and Perpendicular to the Perspective Plane (Fig. 1788).—*a b c d* is the vertical, and *a′ b′ o′ n′* and *f′ e′ h′ g′* the horizontal, projections of the cubes. Draw *a* S, *b* S, *c* S, and *d* S. These will be the indefinite perspectives of the edges of the cubes perpendicular to the perspective plane. Draw S′ *g′*; where this strikes the ground line, project to r_1 and g_1 on S *a* and S *d*, which will be the perspectives of the two corners horizontally projected at *g′* and vertically projected at *a* and *d*; g_1 h_1, drawn parallel to the ground line and limited by S *c* and S *a*, is the perspective of *g′ h′*. In a similar manner the perspectives of all other points and edges are found.

From the drawing of a square in parallel perspective, we deduce rules for the construction of a scale in perspective. Let D G L D (Fig. 1789) be the plane of the picture. From S′ lay off the distance o S′ equal to some unit of measure, as may be most convenient; from o draw the diagonal to D the point

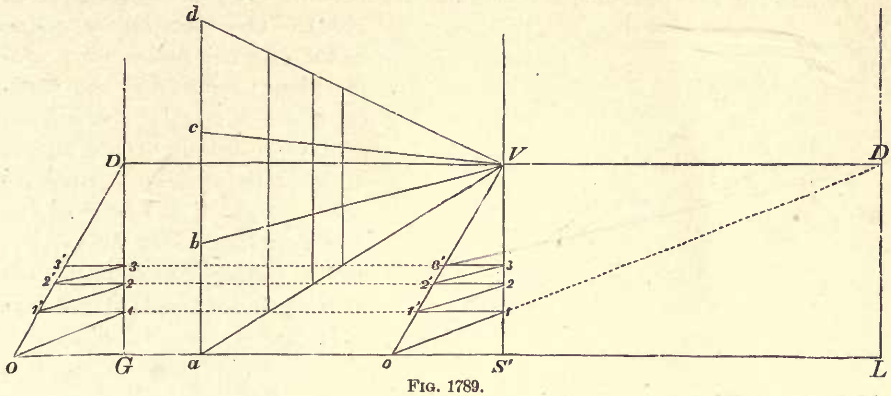


FIG. 1789.

of distance; now draw 1 1′ parallel to the ground line G L, again draw from 1′ the diagonal 1′ D, and lay off the parallel 2 2′, proceed in the same way with the diagonal 2′ D and the parallel 3 3′, and extend the construction as far as may be necessary. It is evident o S′ 1 1′, 1′ 1 2 2′, 2′ 2 3 3′ are the perspective projections of equal squares, and therefore o S′, 1 1′, 2 2′ 3 3′, etc., and S′ 1, 1 2, 2 3, etc., are equal to each other, and that if o S′ is set off to represent any unit of measure, as one foot, one yard, or ten feet, etc., each of these lines represents the same distance, the one being measured parallel to the base line, the others perpendicular to it. In making a perspective drawing a scale thus placed will be found convenient; which in the centre of the picture might interfere with the construction lines of the object to be put in perspective, is better transferred to the side of the picture a G o, the diagonals to be laid off to a point to the right of D equal to the point of distance.

The scales thus projected are for lines in the base or ground plane; for lines perpendicular to this plane the following construction is to be adopted: Upon any point of the base line removed from S′, as a, for instance, erect a perpendicular, $a\ d$; on this line, lay off as many of the units o S′ as may be necessary; in this example three have been laid off, that is, $a\ d = 3\ o$ S′. From a and d draw lines to the centre of view, and extend the parallels 1 1′, 2 2′, 3 3′; at the intersection of these lines with a V erect perpendiculars. The portions comprehended between the lines a V and d V will be the perspective representations of the line $a\ d$, in planes at distances of 1, 2, 3, o S′ from the base line, and as b, c, d are laid off at intervals equal to o S′, by drawing the lines c V and b V nine equal squares are constructed, of which the sides correspond to the unit of measure o S′.

To find the Scale for the Perspective of any Line Oblique to the Perspective Plane (Fig. 1790).—Let A B be the perspective of any line oblique to the perspective plane and V its vanishing point; through the extremity A draw any line A 7; divide it into as many parts as is desired to divide the perspective line A B, here seven. Draw 7 B. Connect the remaining points 6, 5, 4, etc., with V; where these lines intersect the line 7 B they are projected on A B, parallel to A 7. The divisions A 6″, 6″ 5″, etc., are the perspectives of the seven equal divisions of A B. If the line A B is fourteen feet long, the divisions A 6″, 6″ 5″, 5″ 4″ are each equal to two feet.

To find the Perspective of a Hexagonal Prism with a Pyramidal Top (Fig. 1791).—The perspective plane in the original position is a profile plane; *a b f i e* is the vertical and *c′ b′* c_0' d_0' *e′ d′* the horizontal projection of the figure. M N is the profile perspective plane in which the edge of the prism *e i* lies. S is the vertical and S′ the horizontal projection of the point of sight. Draw rays of light at S *a* and S′ *a′*, S *c*, S′ *c′*, S *d*, S′ *d′*, etc. These rays pierce the profile perspective plane at points a_1 c_1 d_1, etc.

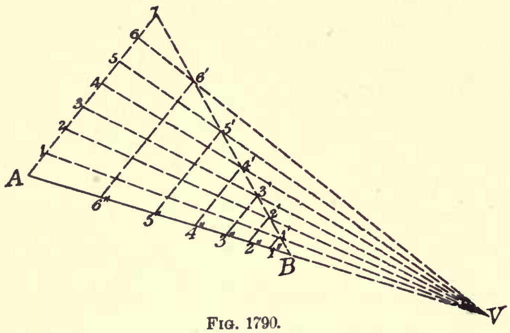


Fig. 1790.

All the points in sight being thus fixed, the profile perspective plane is then transferred to the position M_1 N_1 and revolved about its vertical trace into the plane of the paper. Thus, the point projected at *a″* and *a′*, the perspective of the point originally projected at *a* and *a′* is transferred to *a‴* and revolved to a^{iv}, which

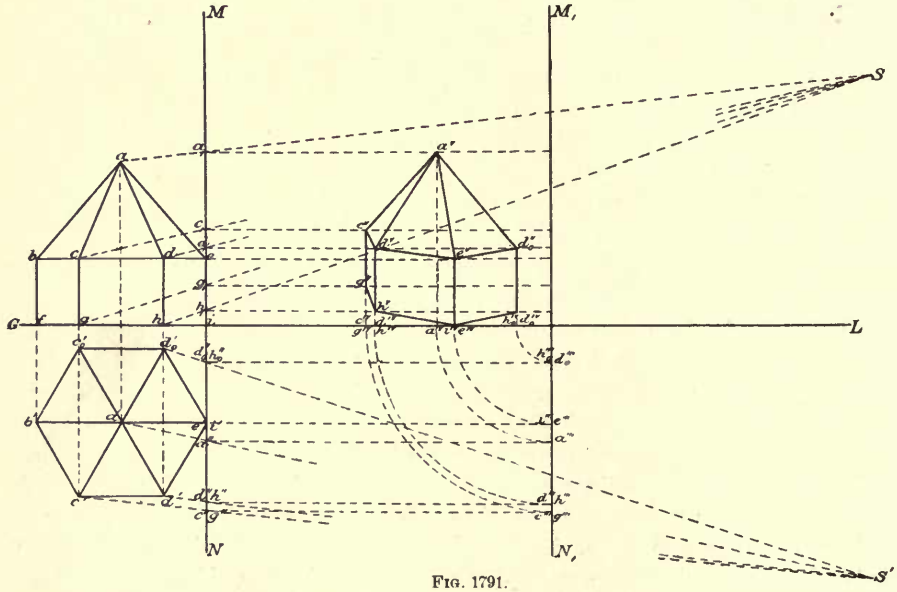


Fig. 1791.

is its final position. All other points are treated in the same way. The edge *e i* lying in the perspective plane is the same length in perspective as in projection.

The transposition of the profile plane from M N to M_1 N_1 is only necessary to avoid complicating the perspective with the projections.

Method of Perpendiculars and Diagonals, to find the Perspective of a Pavement in Parallel Perspective (Fig. 1792).—*a b d c* is the pavement, the square blocks running diagonally across it.

The projection of the pavement being revolved so as to appear below the ground line, diagonals drawn through points of it to the right will vanish in the point of distance to the left and *vice versa.*

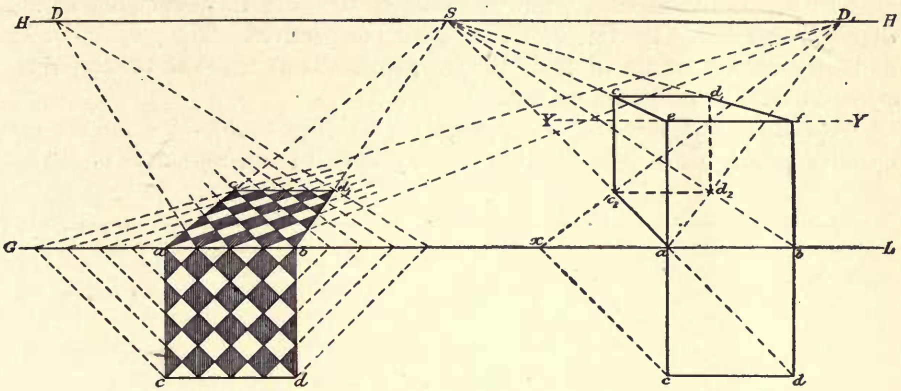


Fig. 1792.

Fig. 1793.

The sides of the pavement, being perpendicular to the perspective plane, are their own perpendiculars. Thus S *a* and S *b* are their perspectives. The lines of division of the blocks, being at 45°, are their own diagonals and vanish at D and D_1.

To find the Perspective of a Cube in Parallel Perspective (Fig. 1793), *a b d c* is the horizontal projection of the cube. The face *a b f e* being in the plane of perspective, appears in full size. From the point *c* the diagonal *c x* disappears at D_1, and the perpendicular at S. Their intersection c_2 is the perspective of the lower corner; d_2 is found in the same way; *c* and *d* are found by projecting up vertically from c_2 and d_2 and intersecting S *e* and S *f*, or they could be found by constructing diagonals in the plane Y Y of the top.

To find the Perspective of the Frustrum of a Right Square Pyramid by Per-

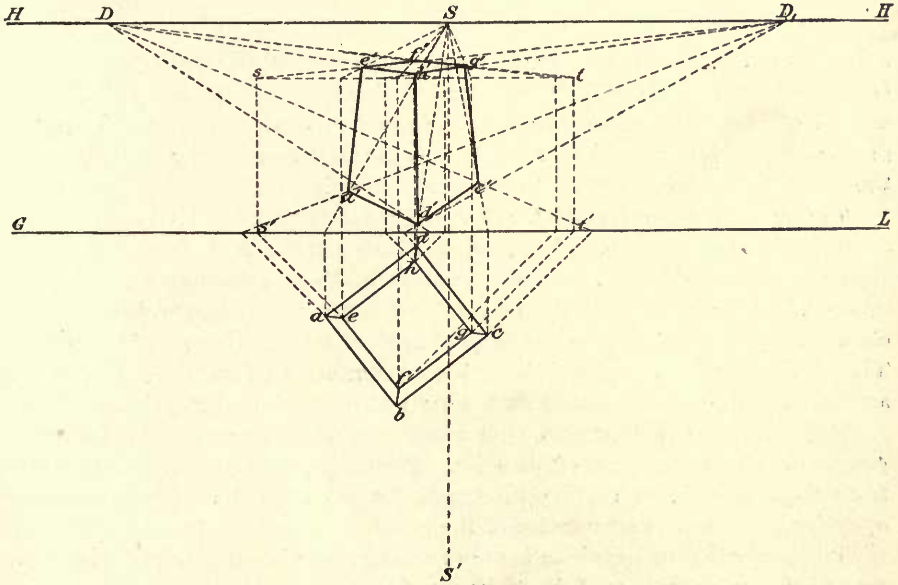


Fig. 1794.

pendiculars and Diagonals (Fig. 1794).—*a b c d* is the horizontal projection of the base and *e f g h* of the top of the pyramid. The point of sight is projected at S′ and S; D and D_1 are the points of distance.

The perspective of each point is found by drawing its perpendicular and diagonal and fixing the intersection of their perspectives. The perpendiculars and diagonals must lie in their proper plane. Thus those at the top of the pyramid must be projected up to the plane *s t*.

To find the Perspective of a Horizontal Circle (Fig. 1795).—To find the perspective of any curve it is merely necessary to find the perspectives of enough

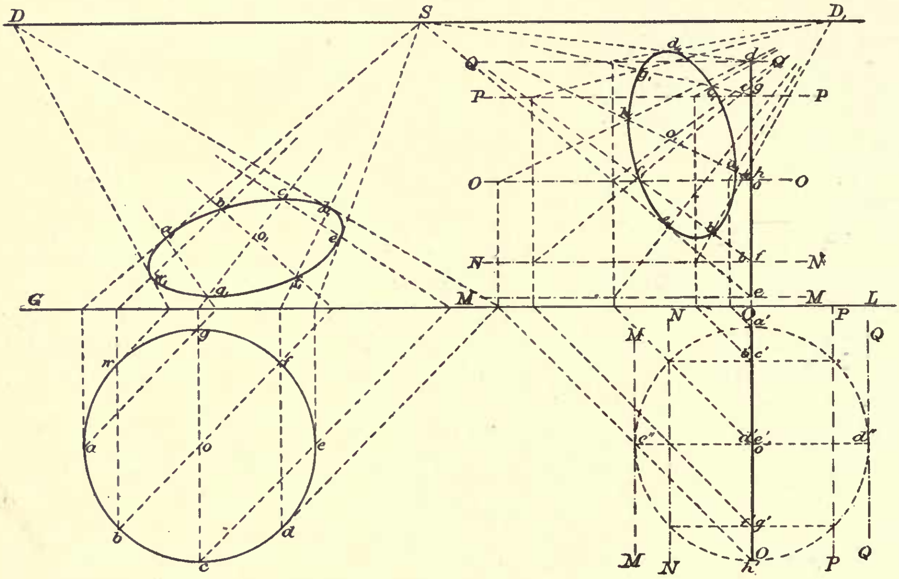


FIG. 1795. FIG. 1796.

points on it to enable the perspective curve to be traced through them; *a g e c* is the horizontal circle. Take any number of equidistant points on it, as *a b c*, etc. Their perspectives are found by the usual method of perpendiculars and diagonals at a_1 b_1 c_1, etc. The curve traced through these perspective points is the perspective of the circle. This curve is an ellipse.

To find the Perspective of a Circle in a Profile Plane (Fig. 1796).—*a′ h′* is the horizontal and *e d* the vertical projection of the circle. As before, a number of points are taken on the circle, the location of these points on the horizontal and vertical projections of the circumference are found by revolving the circle about its horizontal diameter parallel to the horizontal plane of projection. The vertical distance of all points on the circumference from the diameter *a′ h′* are thus ascertained and set off on the vertical projection of the circle.

M M, N N, O O, P P, and Q Q are the vertical traces of the horizontal planes in which the points taken lie. Perpendiculars and diagonals drawn from these points and carried up to get the points in the correct planes give a_1 c_1 d_1 g_1, etc., as the perspective of the circle.

The projection of the circle is only necessary to give the height above the ground line of the planes M M, N N, etc., in which lie the points taken on the circle. Had the circle been in any vertical plane other than a profile plane, the mode of proceeding would have been the same.

To find the Perspective of a Cylinder whose Axis is Horizontal and at an Angle of 45° with the Perspective Plane (Fig. 1797).—*a e j f* is the horizontal

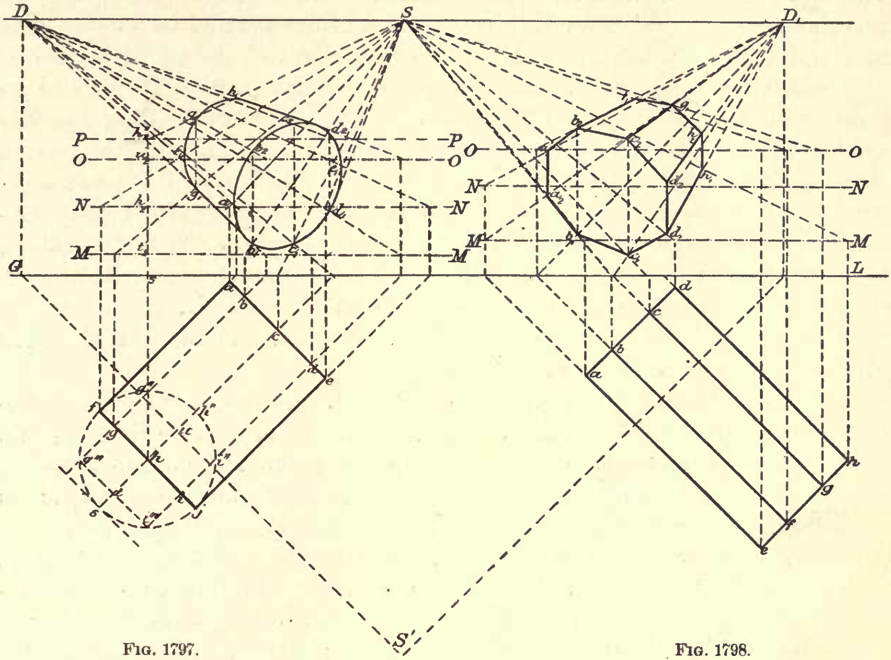


FIG. 1797. FIG. 1798.

projection of the cylinder. The base *f j* is revolved parallel to the horizontal plane and shown at *s f h″ j* and equidistant points taken on it. Set off above the ground line the heights *s t*, *s h*, *s u*, and *s h″* and draw horizontal lines through them. These lines will be traces on the perspective plane of horizontal planes passing through the assumed points and corresponding points on the other base. Then finding the perspectives of diagonal and perpendicular lines passing through these points, the perspectives of corresponding points on the bases through which the perspective curves can be drawn.

To find the Perspective of an Octagonal Prism with its Axis Horizontal and making an Angle of 45° with the Perspective Plane (Fig. 1798).—*a d h e* is the horizontal projection of the prism. M M, N N, and O O are the traces on the perspective plane of the horizontal planes containing the edges of the prism. The perspectives of the points *a b c d*, etc., from the perspectives of the edges vanishing at D_1 and of the perpendiculars vanishing at S. The intersections of these perspectives give the points desired.

To draw the Elevation of a Building in Angular Perspective (Fig. 1799).—The plan of the two sides which are to appear in perspective are drawn, all openings, projections, and roof plan being indicated. This plan, *c*, *a*, *b*, is placed at the top and about the centre of the drawing board in any desired position; a line P P′, known as the picture plane or plane of measures, is drawn of indefinite length. For convenience in measuring heights it is usual, though not necessary, to draw P P′ through the point *a* of the building.

The station point S is selected, largely a matter of judgment, and lines drawn through S parallel to the sides of the building and intersecting the picture plane at P P′; from P and P′ perpendiculars are dropped of indefinite length; the ground line G L, parallel to P P′, is drawn at any convenient place, and the horizon drawn parallel and about six feet above, and intersecting the perpendiculars at V V′, the vanishing points, all lines parallel to *a c* vanishing at V and those parallel to *a b* at V′; lines are drawn from all points in the plan which are to appear in the perspective to S intersecting P P′ and then transferred by vertical lines to the portion of the paper reserved for the perspective drawing; the horizontal measures are thus obtained. For the vertical ones the sides of the plan *a c* and *a b* are extended till they cut P P′; at *d* and *e* perpendiculars are dropped from these points which are lines of heights. Elevations drawn to the same scales as the plan are placed to the right and left of space reserved for the perspective on G L; heights are transferred from the elevation on the left to line of heights *e e′* and vanish at V, heights on the right to *d d′* and vanish at V′. The remaining lines and filling in of details will be understood from the drawing.

Fig. 1800, the building sketched on page 592, is shown in angular perspective. The general construction will be understood from the description of Fig. 1799, the same letters being used to describe similar lines. Additional lines of measures must be taken where there are recesses, projections, chimneys, etc., to locate. Thus, take the chimney *f*; the line of heights for this is obtained by continuing the line of chimney till it intersects P P′ at *f′*, a perpendicular is dropped from this point, and the height transferred to this line from the elevation; a vanishing point drawn from this intersection to V intersects the horizontal limits of the chimney and gives the height; the bay window similarly.

At page 906 is a general diagram showing the principal constructive lines necessary in rendering a building in perspective.

Fig. 1800 A illustrates a method whereby the orthographic plan is dispensed with and a perspective plan substituted, from which the vertical lines and horizontal measures may be taken directly.

The horizon, vanishing points, and point of sight are determined as in the previous examples. Describe arcs from the vanishing points through the point of sight. The intersection of these arcs with the horizon are the points of distance. A line of measures is taken parallel to and at any convenient distance above or below the horizon, and a point, indicating the corner of the building, located where desired on it; to the right and left of this point the horizontal measures of the two sides of the building are laid off to scale; from the corner of the building to each vanishing point lines are drawn representing the sides of the plan. The extreme limits of the building laid off on the line of measures vanish at the points of distance; that to the left of the corner to the point of distance to the right, and *vice versa*. Where these lines intersect the sides of the building are the limits of the perspective plan. These lines may then be transferred by perpendiculars to the perspective elevation.

The perspective plan described above is shown on a larger scale in Fig. 1800 B, the dimensions of the doors and windows of the plan being laid off on the line of measures and carried to their respective points of distance as described in obtaining the perspective limits of the building.

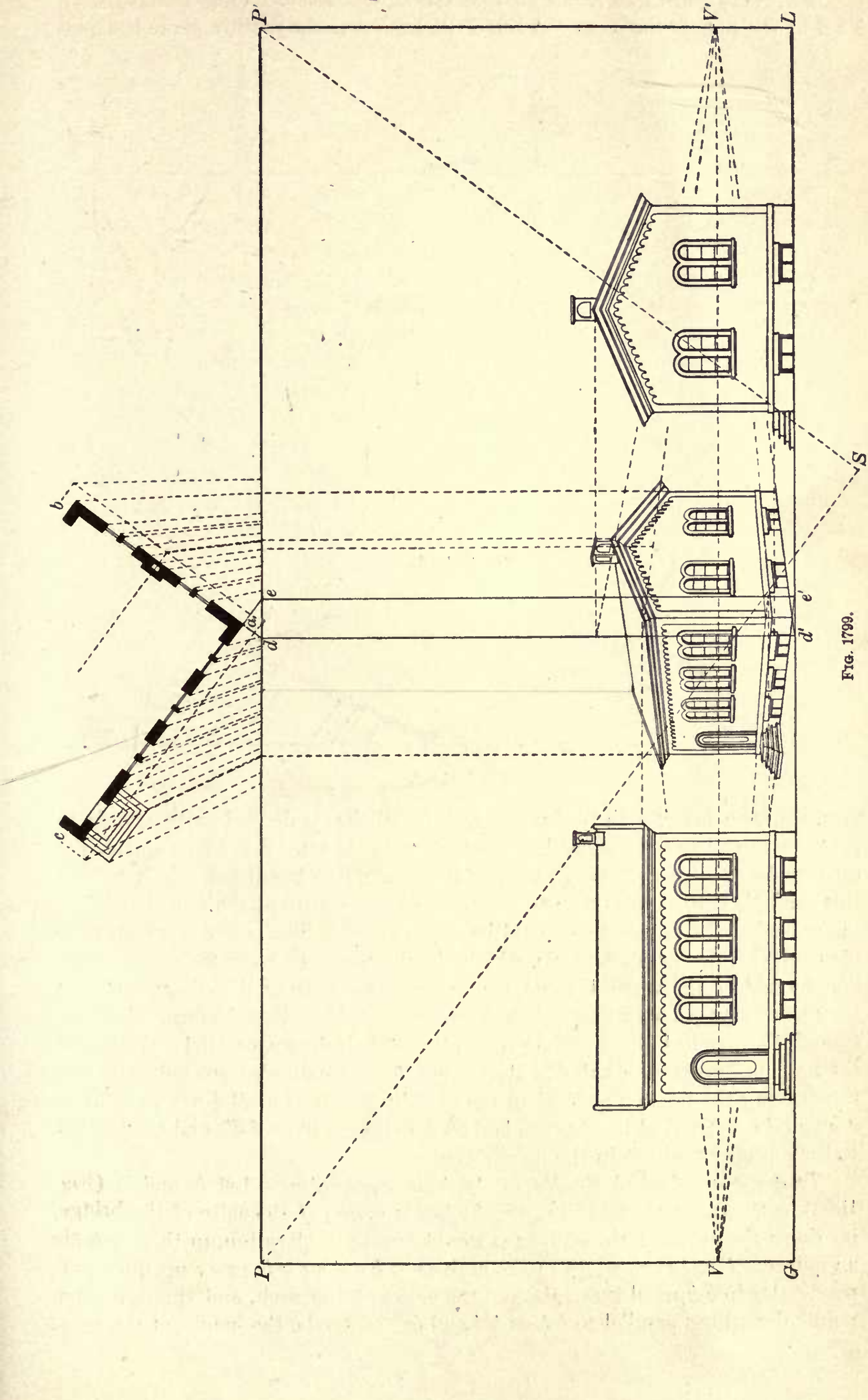


FIG. 1799.

To draw in Parallel Perspective the Interior of a Room. (Fig. 1801).—Let *a b c d* be the plan of the room. A line P P′, known as the picture plane of meas-

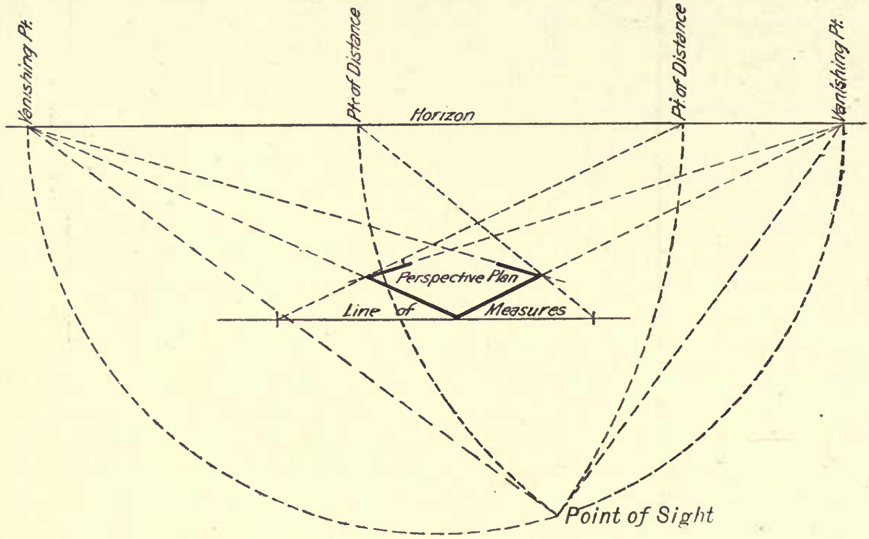


FIG. 1800.—A.

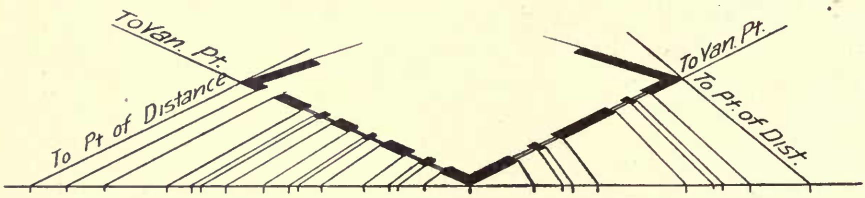


FIG. 1800.—B.

ures, is drawn far enough forward to include all that is desired to be shown in perspective and usually parallel to the rear of the room. S is taken outside the room and as far away from the plan as the size of the board will admit of. In this example S has been taken to the right of the centre of the room; if taken in the centre, the perspective would be symmetrical. The points *a* and *d* of the plan subtend the greatest angle, which should never be over 60°. The same may be said of the vertical angle, a greater angle causing distortion. Dotted lines show rays from various points intersecting P P′ and establishing the position of the vertical lines. The ground line G L is drawn parallel to P P′, and the horizon drawn at a suitable elevation. A perpendicular erected at S and intersecting the horizon at V gives the vanishing point for all lines parallel to *a b* and *d c*. Vertical heights are laid off on either *a a′* or *d d′*, and transferred to their proper position in the perspective.

To draw an Arched Bridge in Angular Perspective.—Let A and B (Fig. 1802) be the plans of the piers; on the line *a p*, one of the sides of the bridge, lay down the curve of the arch as it would appear in elevation, in this example an ellipse. Divide the width of the arch as at *b c d e f g h*, carry up lines perpendicular to *b h* until they intersect the curve of the arch, and through these points draw lines parallel to *b h* as *k l* and *m*; let *o r* be the height of the para-

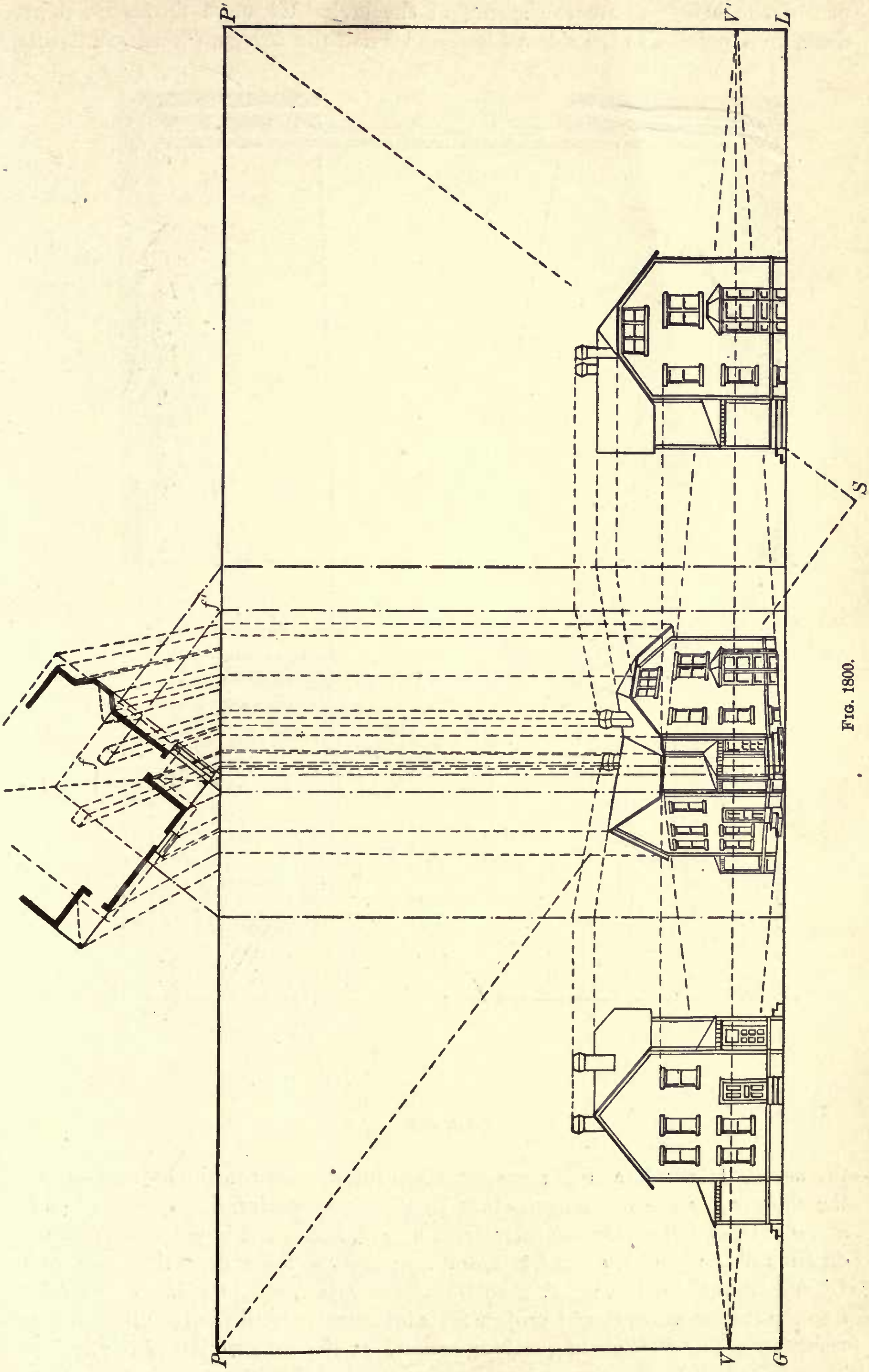


FIG. 1800.

pet of the bridge above the spring of the arch. Through the station point draw lines parallel to the side *a h* and end *a a* of the bridge, till they intersect

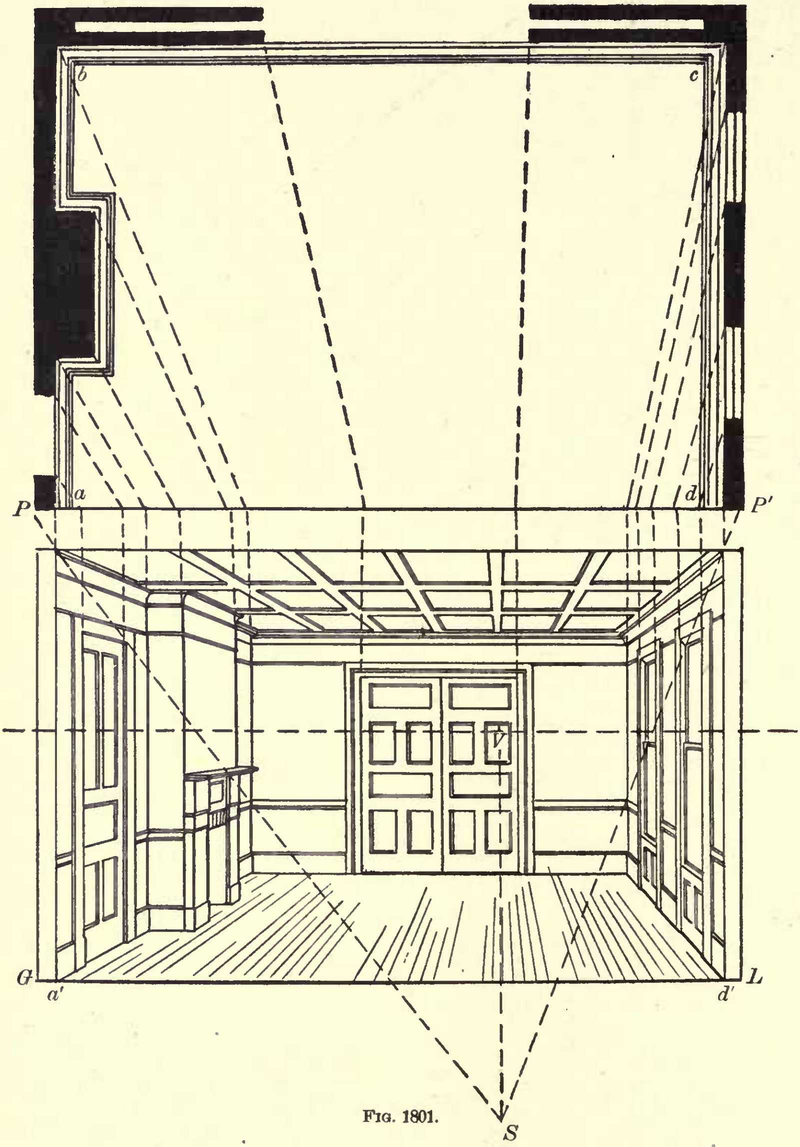


FIG. 1801.

the assumed base line M M; project these intersections to the horizon line of the picture for the vanishing points D, D′ of perspective lines parallel to *a h* and *a a*. Let fall a perpendicular from *a* to *a′*, and on this perpendicular set off from *a′* the heights *s k*, *s l*, *s m*, and *s r*; from *a′* and *r′* draw lines to D and D′, and from the points *m′*, *l′*, *k′* to D′. Draw rays from the points *a b c d e f g h* to the station point S, and project their intersection with the base lines to the perspective line *a′* D′ as in previous examples; the intersection of the lines *k′* D′, *l′* D′, *m′* D′ by the perpendiculars thus projected will establish the points of the curve of the arch on the side nearest the spectator. To determine the position of the opposite side of the arch, from a'', the perspective of the corner of the pier a'' draw a'' D′, and from h' draw lines to D; the line h' p' will be

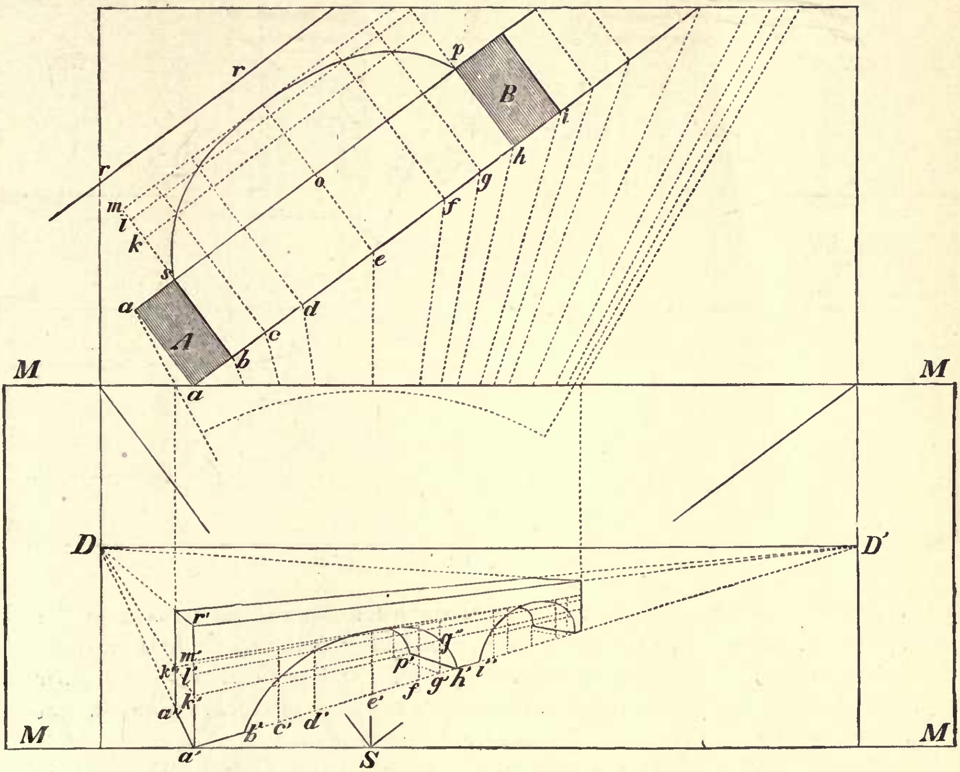


FIG. 1802.

the perspective width of the pier; draw k' D; and from k'', k'' D′; from g'' the intersection of the curve of the arch by the perpendicular to g', draw g'' D, the intersection with k'' D′ will be one point in the curve of the arch on the opposite side of the bridge; in the same way, from any point in the nearer arc draw lines to D, and the intersection with lines in the same planes on the opposite side of the bridge will furnish points for the further arch; all below the first only will be visible to the spectator.

To draw in Perspective a Flight of Stairs (Fig. 1803).—Lay off the base line, horizon, centre of view, and point of distance of the picture; construct the solid *a b c d*, *e f g h*, containing the stairs, and in the required position in the plane of the picture; divide the rise *a c* into equal parts according to the number of stairs, nine, for instance; divide perspectively the line *a b* into one less (eight) number of parts; at the points of division of this latter erect perpendiculars, and through the former draw lines to the centre of view; one will form the rise and the other the tread of the steps. From the top of the first step to the top of the upper continue a line *a d*, till it meets the perpendicular S′ V prolonged in *v*; this line will be the inclination or pitch of the stairs; if through the top of the step at the other extremity a similar line be drawn, it will meet the central perpendicular at the same point *v*, and will define the length of the lines of nosing of the steps, and the other lines may be completed.

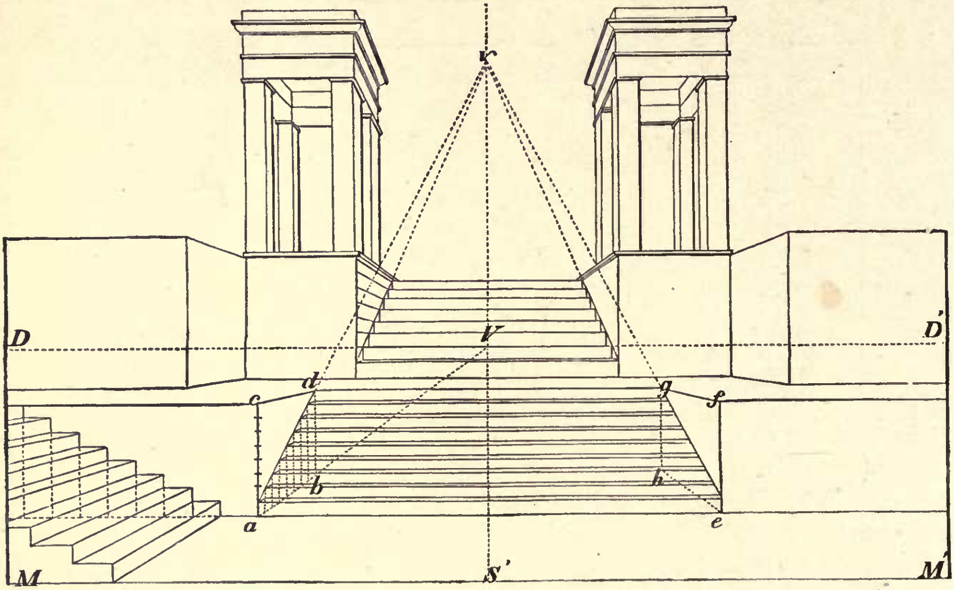


Fig. 1803.

As the pitch lines of both sides of the stairs meet the central vertical in the same point, in like manner *v* will be the vanishing point of all lines having a similar inclination to the plane of the picture. The projection of the other flight of stairs will be easily understood from the lines of construction perpendicular to the base line or parallel thereto, lying in planes.

To find the Reflection of Objects in the Water.—Let B (Fig. 1804) be a cube suspended above the water; find the reflection of the point *a* by letting fall a perpendicular from it, and setting off the distance *a′ w* below the plane of the water equal to the line *a w* above this line; the line *w f′* will also be equal

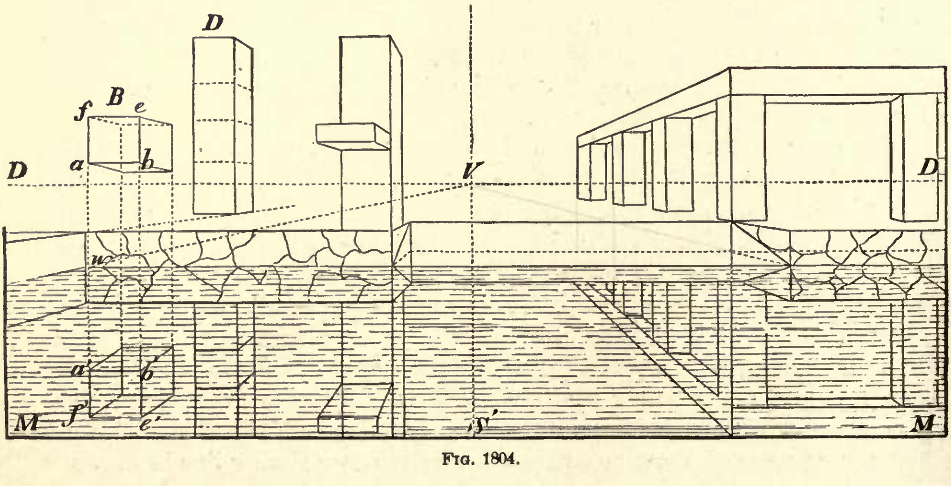


Fig. 1804.

to the line $w\,f$; find in the same way the points b' and e', through these points construct perspectively a cube in this lower plane, for the reflection of the cube above.

To find the reflection of the square pillar D removed from the shore: suppose the plane of the water extended beneath the pillar, and proceed as in the previous example.

The lines of an object which meet in the centre of view V, in the original, have their corresponding reflected lines converging to the same point. If the originals converge to the points of distance, the reflected ones will do the same. To find the reflection of any inclined line, find the reflection of the rectangle of which it is the diagonal, if the plane of the rectangle is perpendicular to the plane of the picture. If the line is inclined in both directions inclose it in a parallelopiped and project the reflection of the solid.

To find the Perspective Projection of Shadows (Fig. 1805).—Let the construction points and lines of the picture be plotted. Let A be the perspective projection of a cube placed against another block, of which the face is parallel to the plane of the picture; to find the shadow upon the block and upon the ground plane, supposing the light to come at such an angle as to cause the projections of it (both vertical and horizontal) to make an angle of 45° with the ground line. Since the angle of light is the diagonal of a cube, construct another cube similar to A, and adjacent to the face $b\,c\,g$; draw the diagonal $b\,k$, it will be the direction of the ray of light, and k will be the shadow of b; connect $f\,k$ and $c\,k$, $f\,k$ must be the shadow of the line $b\,f$, and $c\,k$ of $b\,c$; the one upon the horizontal plane and the other in a vertical one: the former will have its direction, being a diagonal, toward the point of distance D′, the other being a diagonal in a plane parallel to that of the picture, will be always projected upon this plane in a parallel direction.

Let B be a cube similar to A; to find its projection upon a horizontal plane, the shadow of the point b' may be determined as in the preceding example, but the shadow of the point c', instead of falling upon a plane parallel to the picture, falls upon a horizontal one; its position must be determined as before by b. Construct the cube and draw the diagonal $c'\,l$; in the same way determine the point m the shadow of d'; connect $c\,k'\,l\,m\,n$, and for the shadow of the cube in perspective on a horizontal plane.

On examination of these projected shadows, it will be found that as the rays of light fall in a parallel direction to the diagonal of the cube, the vanishing point of these rays will be in one point V′ on the line D′ L, prolonged, at a distance below D′ equal V D′; and since the shadows of vertical lines upon a horizontal plane are always directed toward the point of distance, the extent of the shadow of a vertical line may be determined by the intersection of the shadow of the ground point of the line by the line of light, from the other extremity. Thus, the point k, cube A, is the intersection of f D′ by b V′; the points k', l, m are the intersections of c D′, o D′, n D′ by b' V′, c' V′ d' V′. Similarly on planes parallel to that of the picture, k, cube A is intersection of the diagonal $c\,k$, by the ray of light b V′.

Applying this rule to the frame C, from r, s, p, draw lines to D′; from r', s', p', draw rays to V′; their intersections define the outline of the shadow of the post. To draw the shadow of the projection, the shadow upon the post

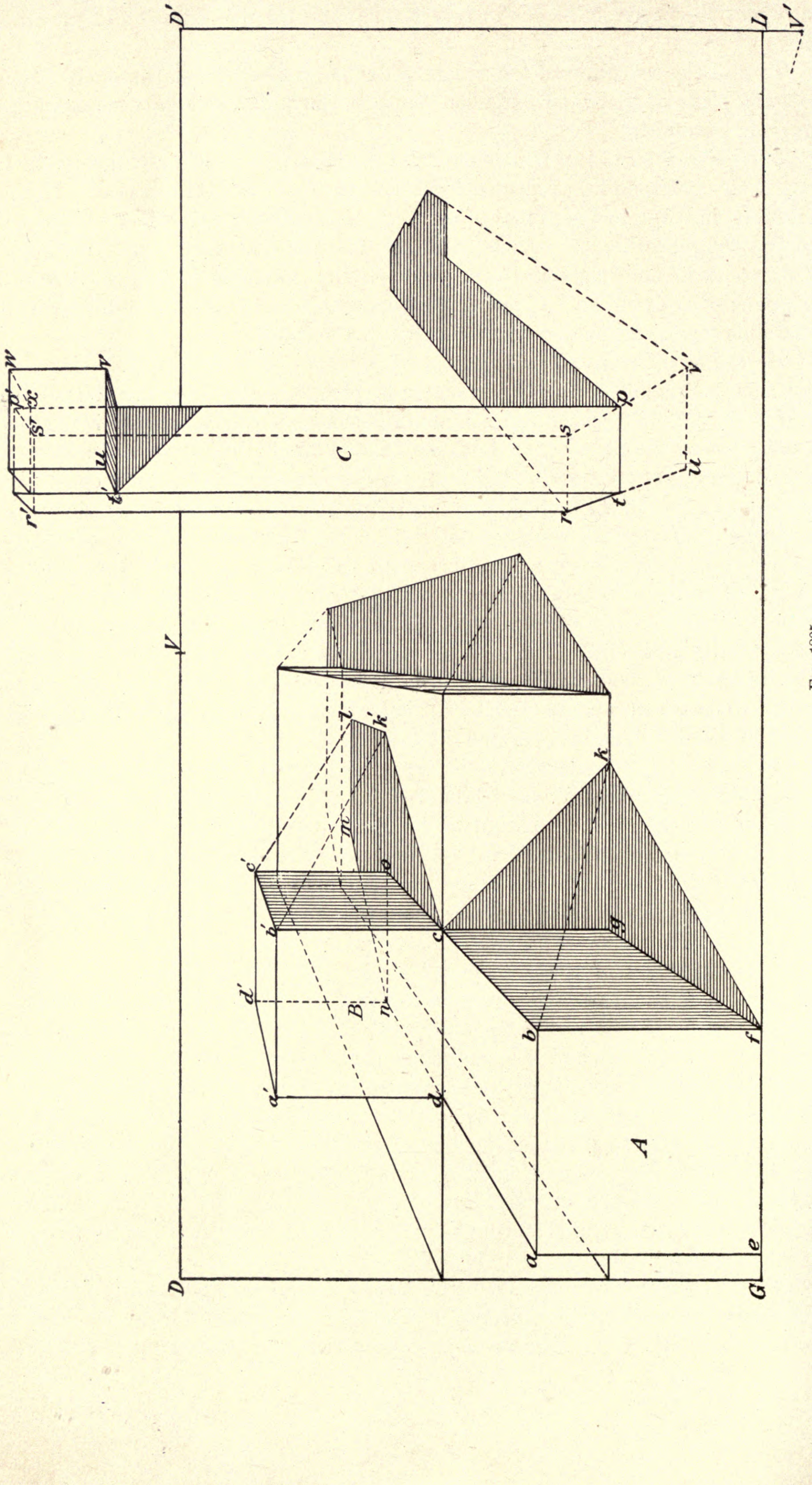


FIG. 1805.

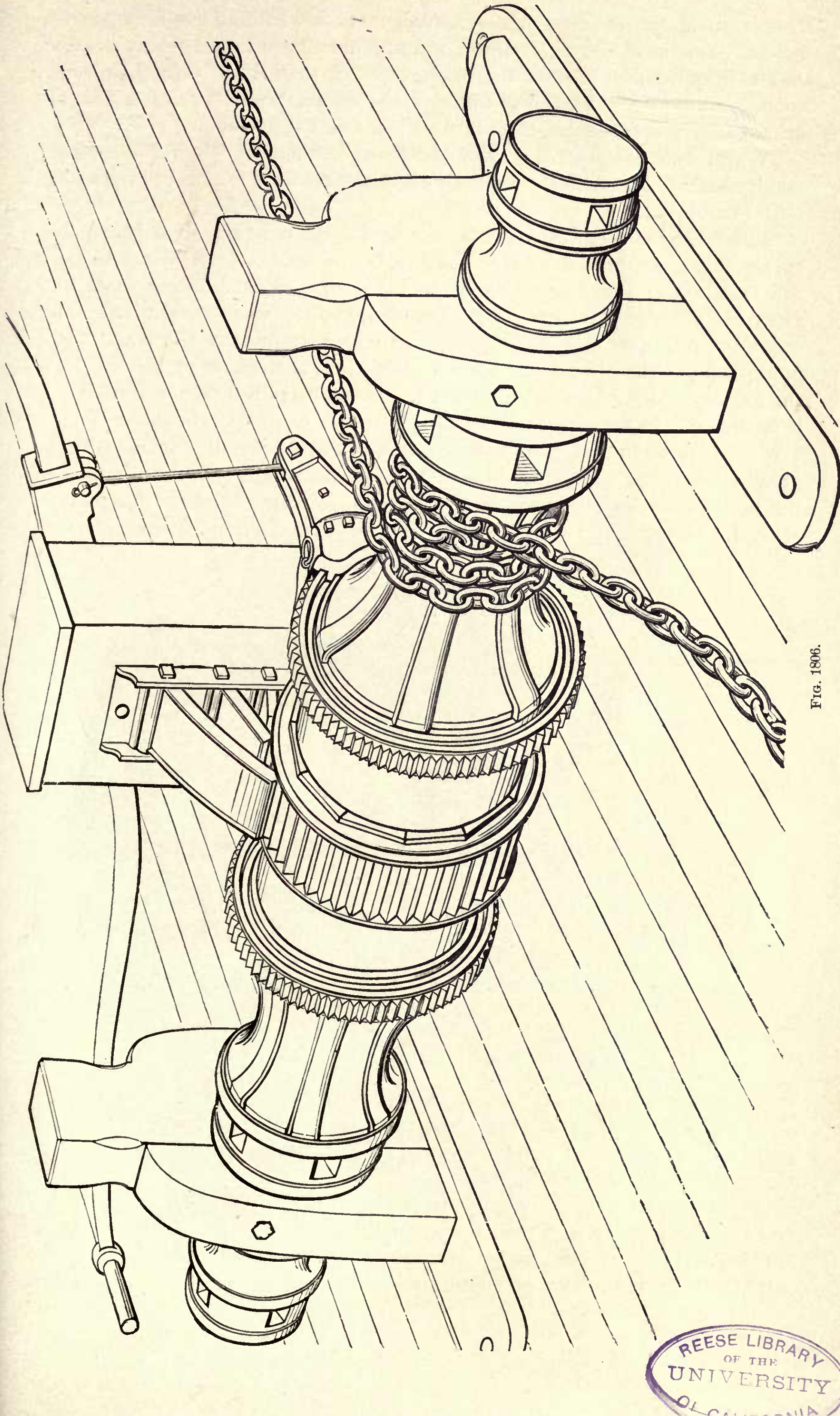


Fig. 1806.

from t will follow the direction of the diagonal $c\,k$. Project u and v upon the ground plane at u' and v'; from $t\,u'\,v'$ and p draw lines to D′; from t', u, v, w, and x draw rays to V′, and the intersection of these lines with their corresponding lines from their bases will give the outline required; as v and w are on the same perpendicular, their rays will intersect the same line v' V′.

With reference to the intensity of "shade and shadow," and the necessary manipulation to produce the required effect, the reader is referred to the article on this subject.

In treating of Perspective it has been considered not from an artistic point, as enabling a person to draw from Nature, but rather as a useful art to assist the architect or engineer to complete his designs, by exhibiting them in a view such as they would have to the eye of a spectator when constructed. Our examples, owing to size of the page, have been limited in the scale of the figures, and in the distance of the point of view, or distance of the eye from the plane of the picture, unimportant to the mathematical demonstration. It is unnecessary in these particular points that the examples should be copied. The most agreeable perspective representations are generally considered to be produced by fixing the angle of vision at from 45° to 50°, and the distance of the horizon above the ground-line at about one third the height of the picture.

In the early edition of this work there were illustrations of machinery on

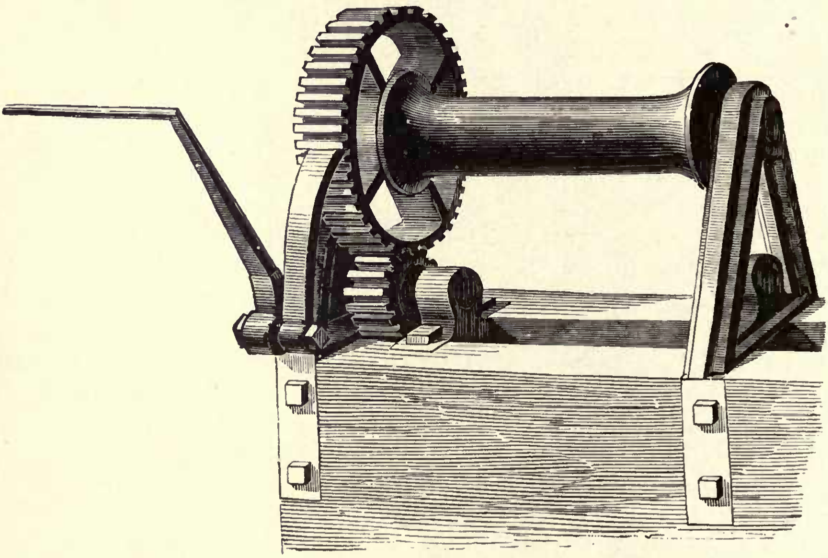


Fig. 1807.

sale in a kind of perspective, of which two, Figs. 1806 and 1807, specimens of ship work, a windlass and centreboard winch, are reproduced. They are graphic and natural in appearance, and similar illustrations will be found in the collection of "Scraps."

With the introduction of photography and the ready transfer of the negatives to the positive and permanent condition of prints and plates, one is enabled to judge of sizes and dimensions from their surroundings, or by the introduction into the view of appropriate marks or bounds of known distance affording lines for measures. In fact, photography has been applied to surveys with records of angles and topographical views of lines. Views of buildings, circulars of machinery, and objects on sale or of interest are illustrated by the aid of photography. The figure below represents the office of the publishers of this work taken by photography, printed on plain salted paper. The pen-work is done in water-proof ink on the photograph; the print is then washed in a chemical solution, removing the photographed lines and leaving those in ink. (See Free-Hand Drawing.)

FREE-HAND DRAWING.

A draughtsman, who has made himself conversant with the rules of projection as laid down in the preceding pages, and has applied these rules to practice, will be capable of representing correctly such objects as have been illustrated, or make up similar combinations of his own invention and design for the comprehension of others. But natural objects, as animals, trees, rocks, clouds, etc., can not be imitated on paper with the aid of drawing instruments; outlines so varied can not be copied in this mechanical way; it can only be done by hand drawing, an educated eye that can recognise proportion and position, and an educated hand that can execute and portray naturally things recognised by the eye, with the aid of pencil, pen, crayon, brush, or the various tools that now obtain with draughtsmen. But it is by education that one acquires the facilities of such an eye and hand. As the writer acquires facilities by the copying of pothooks, letters, and flourishes which may develop into a valuable distinctive hand, so the draughtsman by the study of examples, first of drawings and copying, the learning of proportions, comparison of the works of different artists, observations of effect in drawing and nature, will commence an education which will produce pleasant and successful pictures distinctive of the educated artist, appreciated by the public and of mercantile value.

An educated free hand adds largely to the effect on most drawings, where close measures are not requisite, giving grace and beauty to mechanical designs, and is especially applicable to architectural ornaments and accessories. It will be found impossible to draw many of these in any other way, and there are few drawings that do not require some patching by hand—short curves, which can be thus done much more readily, and connections of lines, which can not be done by drawing instruments.

The pencil or pen should be held by the thumb and first finger, and supported and guided by the second. The two fingers touching the pencil should be placed firmly on it, and be perfectly straight, the end of the middle finger at least one inch above the point of the pencil. In drawing, it is well to commence, as in writing, with straight lines. Lines vertical, horizontal, and inclined, parallel to each other and at angles, light and strong—short and long lines, straight and curved, with pen, pencil, or crayon on paper, or chalk on a board. Dot points, and draw lines between them, at a single movement, without going over them a second time, and without patching. Besides direction, lines have a definite length, and the draughtsman must practise himself in drawing lines of equal lengths, or in certain proportions to each other.

Lines equal to each other :

Lines twice another line :

Divide a line into any number of equal parts :

The accuracy of these divisions may be tested by a strip of paper applied along the line, marking off the divisions upon it, and then slipping it along one division, and noting if the divisions on the paper and line still agree. By practice, the eye will be able to make these divisions almost accurately. Having acquired this skill, apply it to the construction of the Geometrical Problems, in the earlier part of the book, in their proper proportions, both in straight and curved lines. The construction should be dependent entirely on eye and hand; but it will be found, whether the draughtsman draws from copy or nature, that it is almost impossible to get along well without defining positions by some points in the pictures, and sketching in some defined lines which may serve as guides.

Following this practice of guide lines, it will be well to copy the outlines of architectural mouldings, of which most of the ornaments are conventional representations of natural objects.

At page 60 will be found an application to the drawing of acanthus leaves within the guiding lines of squares and the designing for calicoes and woven goods, oilcloths, ceiling, and wall ornamentation based on geometrical figures.

In such designs " a true artistic end has been accomplished when well-observed features of natural objects have been chronicled within the conventionalized limits of a few geometric rules that include proportion, symmetry, and a proper subordination of one part to another."

To acquire still further readiness in free hand, extend the practice to " lettering."

Material.

Paper.—The different papers manufactured by Whatman are excellent for sketching purposes, and can be purchased either in sheets or in pads of various sizes. Any toothed paper answers the purpose very well, such as common newspaper; a sketching paper with either a rough grain or a canvas grain may be made by pinning a sheet of thin typewriting paper over a piece of sandpaper or a canvas book in the same manner.

For pen-and-ink work a hard, unyielding surface is needed; nothing answers the purpose as well as the best quality of Bristol board, although excellent results are obtained on Whatman's H. P. (Hot Pressed) paper.

Pencils.—Faber's or Dixon's pencils of medium hardness are best for sketching, but for drawing on Bristol board the harder grades are better.

Lithographic chalks are now coming into use; they are much superior to the ordinary chalks, crayons, and charcoal in their not being readily smeared. They find their best medium in Whatman's paper, either H. P. or " not," and in grained scratch-out cardboards they give greater intensity than lead pencil, and reproduce with more certainty. Of course they can not be used where much detail is required, but for generalities they are excellent.

Pens.—The use and selection of pens must be left largely to the draughtsman's own judgment. The ordinary school pen is excellent; Gillott's 303, or the same maker's mapping pens, are also good. The amateur is cautioned against the diminutive crow quill and lithographic pens; the pen is merely a secondary matter, some illustrators doing excellent work with a brush used as a pen, a reed pen, a toothpick, or, indeed, with anything that happens to be at hand.

Ink.—The best ink for reproductive purposes is India ink; that which has a dull appearance when dry is the best for this purpose; India ink can be purchased in the stick and ground down with water or ready prepared in bottles, either waterproof or otherwise.

The first step in free-hand drawing is its application to simple objects, of which one must learn to determine the relative proportion of their parts and to lay them down on paper in their proper position and this entirely by eye. Having thus drawn one object, one proceeds to increase the group of objects. As has been shown in "Perspective," objects appear smaller as they are more remote from the spectator, who must know how much the relative scale is changed, not only in objects remote from each other, but also as to what parts of objects can be seen and how they are seen. The rules of perspective give an idea of what can be seen and the proportions, and serve to make the eye intelligent in its observations to be confirmed and strengthened by practice.

In drawings of the human frame there are numerous charts and rules which may be said to be established which may assist the learner in fixing the form and proportions of parts within certain classical or normal limits. Outlines from these charts may be traced for a brief time by the learner to acquire ideas of the forms, proportions, and positions when at rest; but when in action limbs are moved, muscles increase under action, and the parts present different lines of sight which must be studied by themselves. Although the length of limbs is not increased, it may be more or less foreshortened.

"*Proportions of the Human Frame.*" *By Joseph Bonomi.*

The following, with the illustrations, are taken from the above work:

"The human frame is (Figs. 1808 and 1809) divided into four equal measures, by very distinctly marked divisions on its structure and outward form:

"1. From the crown of the head to a line drawn across the nipples.

"2. From the nipples to the pubes.

"3. From the pubes to the bottom of the patella (knee-pan).

"4. From the bottom of the patella to the sole of the foot.

"Again, four measures, equal in themselves, and equal to those just described, and as well marked in the structure of the human body, are seen when the arms are extended horizontally. They are the following:

"From the tip of the middle or longest finger to the bend of the arm is one fourth of the height of the person.

"From the bend of the arm to the pit of the neck is another fourth.

"These two measures, taken together, make the half of the man's height, and with those of the opposite side equal the entire height.

"In the figures, the differences in width between the male and female figures are given from the tables of the Count de Clarac of the Apollino and the Venus de Medici. The male figure is in thicker line than the female, and the measurements referring to it are on your right hand, and those referring to the female on your left.

"The measurements of length, according to Vitruvius and Leonardo da Vinci, are the same in both sexes, and expressed in long horizontal lines running through both the front and profile figures.

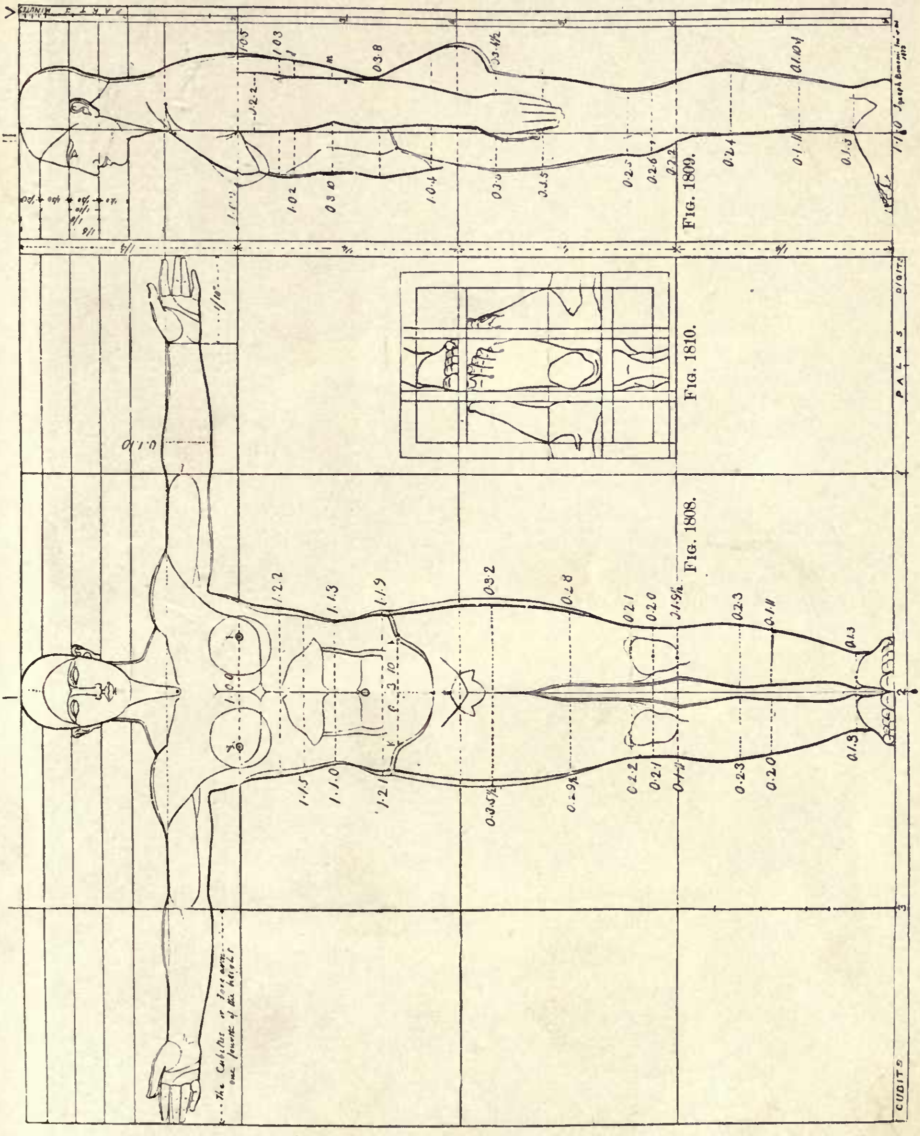


FIG. 1809. FIG. 1810. FIG. 1808.

"Almost innumerable are the varieties of character to be obtained by the alterations of widths, without making any change in the measurements of length; nevertheless, some ancient statues differ slightly in these measurements of length.

"No measurement is given in the figure of the width of the foot; its normal proportion should be one sixteenth of the height. The views of the foot are those of the female.

"The scale, V, used is 8 heads to the height; parts, $\frac{1}{4}$ of a head; and minutes, $\frac{1}{12}$ of a part.

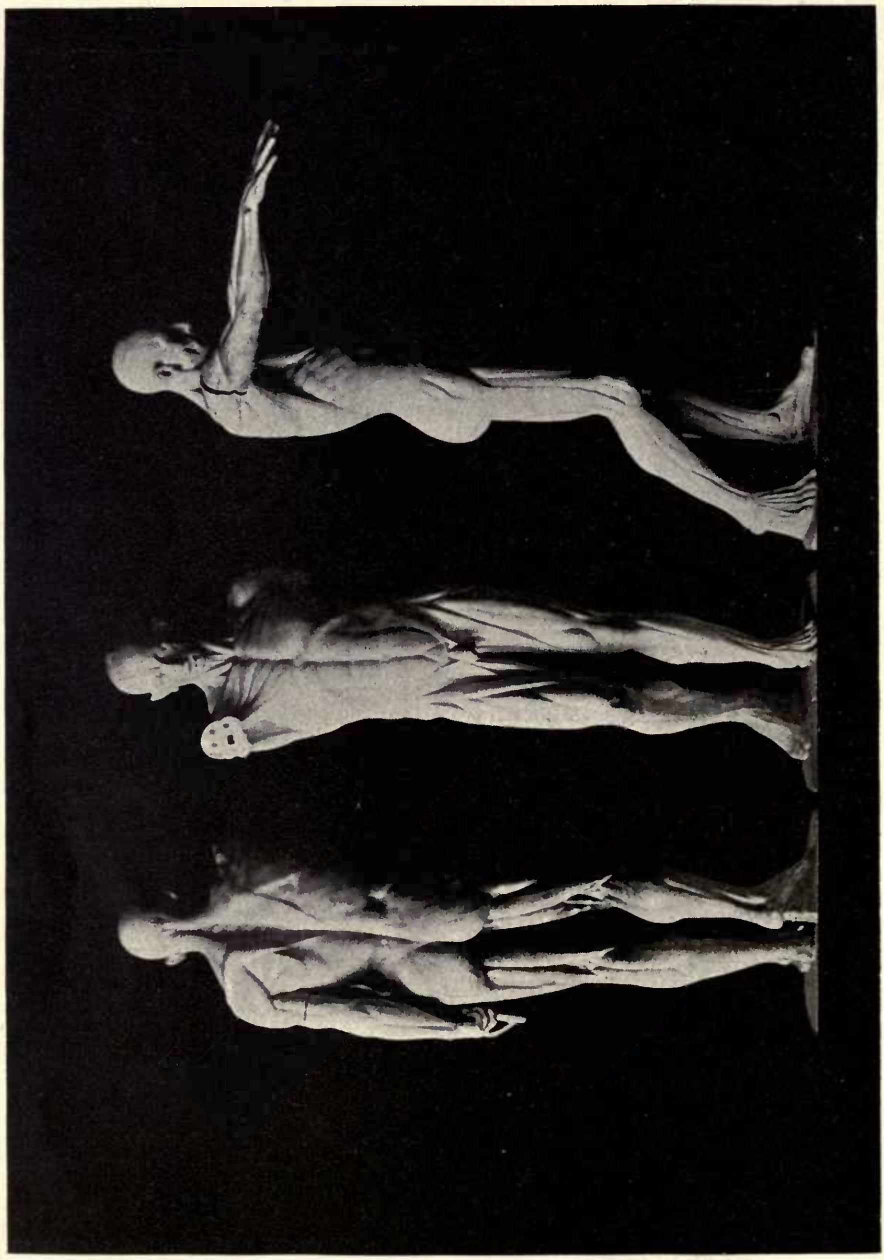


Fig. 1811.

"The whole height is usually taken at 8 heads, but there are slight differences in the classic statues; the height of the Venus de Medici is equal to 7 heads, 3 parts, 10 minutes, that of the Apollino of Florence, 7 heads, 3 parts, 6 minutes.

"When the student is acquainted with the forms of the body and limbs in two aspects—viz., the front and side views—and the normal proportions they bear to each other, then will follow the study of the characteristic features of childhood, youth, and mature age, and those niceties of character that the ancients invariably observed in the statues of their divinities, so that in most cases a mere fragment of a statue could be identified as belonging to this or that divinity—as, for instance, the almost feminine roundness of the limbs of the youthful Bacchus, the less round and distinctly marked muscles of the Mercury, and of the statues of the Athletæ."

Fig. 1811 is a half-tone reproduction of a photograph showing three views of a plaster model, *écorché*—i. e., the body with the skin removed, showing the muscles. In the half-tone process a ruled screen of glass is interposed between the drawing or object to be photographed and the negative. The screen of glass is closely ruled with lines crossing at right angles and etched with hydrofluoric acid; into the grooves thus produced printing ink is rubbed. It is these lines which produce the crisscross appearance seen in the resulting picture. This process is commonly used in reproducing wash drawings and photographs.

Figs. 1812, 1813, and 1814 are three views of the above figure drawn in line. A large negative was taken and the print made on "plain salted paper"—that is, paper prepared without albumen, which gives to the ordinary print its glossy appearance. This paper is made by being soaked in a solution of

Chlorate of ammonia....................	100 grains;
Gelatin..................................	10 "
Water....................................	10 ounces.

The figures thus made may now be drawn in with pen and water-proof India ink. The pen work should not attempt the fulness of detail given in the photograph. When the drawing has been finished it may be immersed in a solution composed of 1 ounce of bichloride of mercury dissolved in 8 ounces of water, which removes all trace of the photograph, leaving the drawing uninjured on white paper. Omissions may now be supplied, but if there are any conspicuous, the photograph may again be brought out by immersing in a solution of hyposulphite of soda in water.

A readier way is to draw with water-proof ink upon photographs printed on ferro-prussiate paper or blue print paper, the directions for making which will be found on page 52, or it can be purchased. It can then be sent for reproduction as it is, as the blue will not photograph, or the blue may be bleached by immersing the print in a dish of water in which a small piece of washing soda has been dissolved; then wash the print in clean water.

Both pen drawing and opaque water colour can be used on the ordinary photograph by mixing a small piece of ox gall with the liquid.

Figs. 1815 and 1816 are two views of Sandow taken from his photographs, the pen work being executed as above and the photograph washed out.

In the "Dictionnaire Raisonné de l'Architecture" of M. Viollet-le-Duc are given drawings from an album of the middle of the thirteenth century. Certain mechanical processes are given to facilitate the composition and design of figures. According to these sketches, geometry is the generator of movements

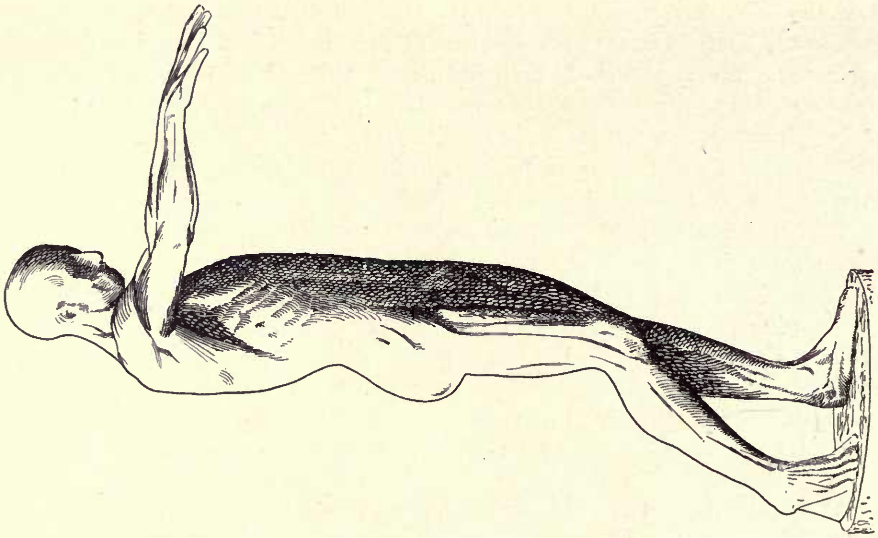


Fig. 1814.

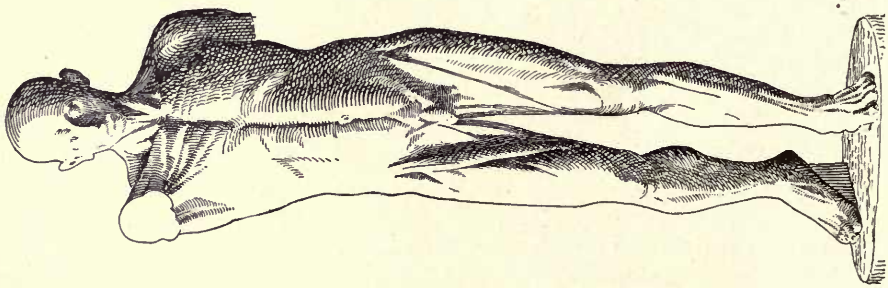


Fig. 1813.

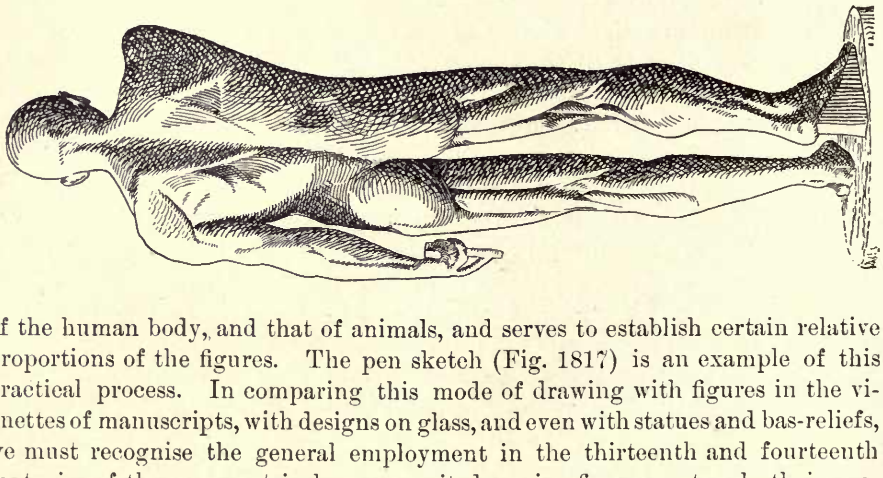


Fig. 1812.

of the human body, and that of animals, and serves to establish certain relative proportions of the figures. The pen sketch (Fig. 1817) is an example of this practical process. In comparing this mode of drawing with figures in the vignettes of manuscripts, with designs on glass, and even with statues and bas-reliefs, we must recognise the general employment in the thirteenth and fourteenth centuries of these geometrical means, suited to give figures not only their proportions but also the justness of their movement and bearing. Rectifying the canon of Villard in its proportions by comparison with the best statues, notably those in the interior of the western façade of the Cathedral of Reims, we obtain the Fig. 1818. The line A B, the height of the human figure, is di-

FIG. 1815.

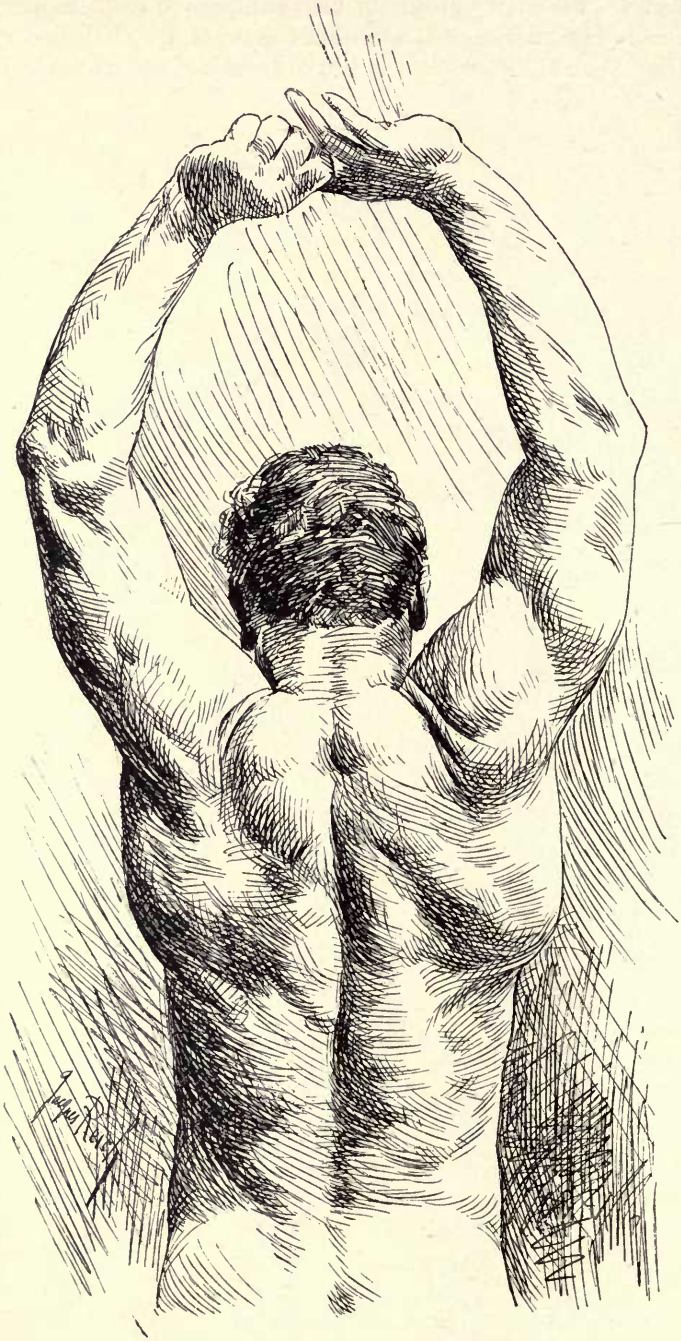


FIG. 1816.

vided into seven equal parts. The upper division is from the top of the head to the shoulders. Let C D be the axis of the figure, the line at the breadth of the shoulders is $\frac{2}{9}$ of the whole height A B. The point E is the centre of the line C D; draw through this point two lines, *a f* and *b e*, and from the point *g* two other lines, *g e* and *g f*. The line *b h* is the length of the humerus, and

Fig. 1817.

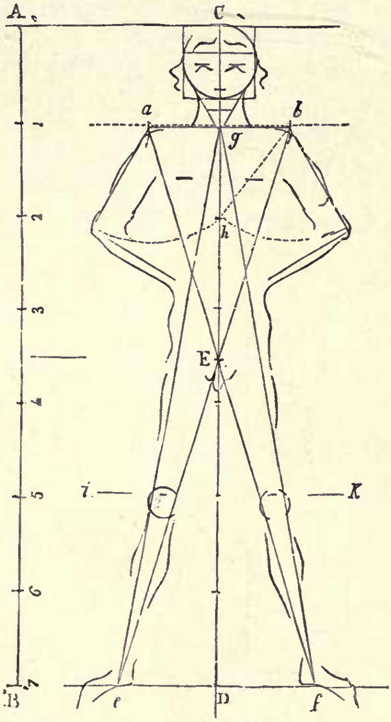


Fig. 1818.

the line of the knee-pan is on *i k*. The length of the foot is $\frac{5}{9}$ of a division, A 1. Having established these proportions, it will be seen by the following cuts how the artisan gave movements to these figures when the movements were not in absolute profile.

Suppose the weight of the figure to be borne upon one leg (Fig. 1819), the line *g e* becomes perpendicular, and the axis *o p* of the figure is inclined. The movement of the shoulders and trunk follow this inflection; the axis of the head and the right heel are in the same vertical line.

In stepping up (Fig. 1820), the axis of the figure is vertical, and the right heel raised is on the inclined line *s t*, while the line of the neck is on the line *l m*, and the trunk is vertical.

In Fig. 1821 it will be seen how a figure can be submitted to a violent movement and yet preserve the same geometrical trace. The figure is fallen, supported on one knee and one arm, while the other wards off a blow; the head is vertical.

In Fig. 1822, the left thigh being in the line *a f*, to determine the position of the heel *c* on the ground, supposed to be level, an arc is to be described from the knee-pan; the line *e f* is horizontal.

It is clear that, in adopting these practical methods, all the limbs can be developed geometrically without shortening. The above will supply to many a ready means of sketching the human figure in various attitudes, naked, or in the close-fitting dresses of the present fashion; but in the arrangement of

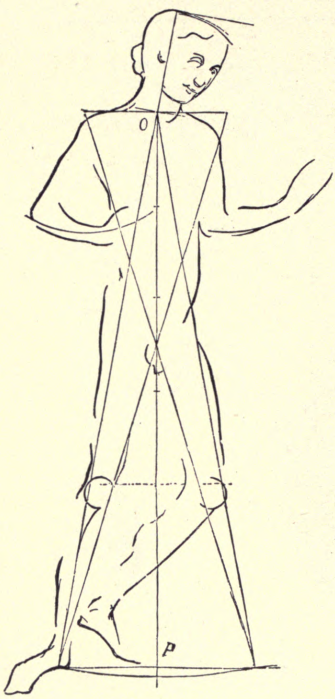


Fig. 1819.

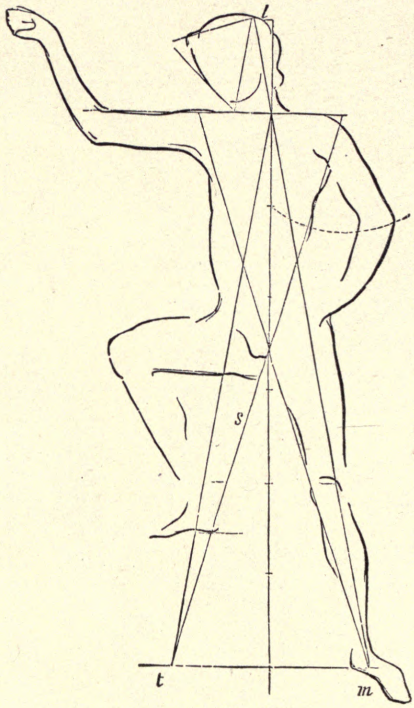


Fig. 1820.

drapery upon a figure, care must be taken that the drapery should fall in graceful folds. " It is necessary to give the body certain inflections which would be ridiculous in a person walking naked. The walk should be from the hips, with

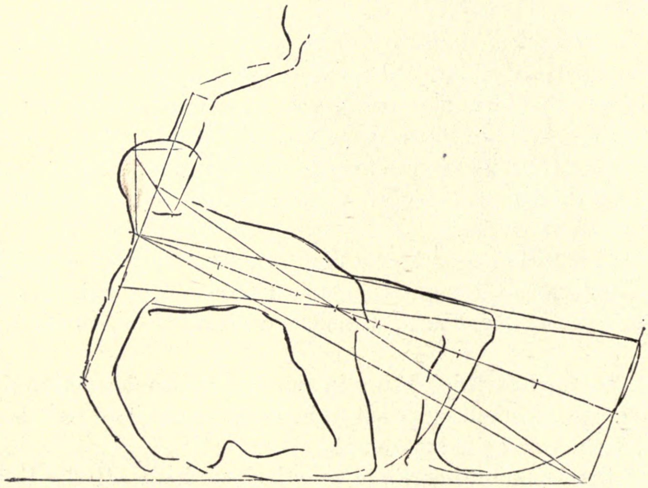


Fig. 1821.

wide-spread legs, and, by the movements of the trunk, make the drapery cling on certain parts and float on others."

In figures in repose, their centres of gravity must fall within the points of support, but the body can be sustained by muscular exertion, and this should be expressed in such cases by the tension of the muscles on which the position depends. In the act of running, the body inclines forward, its weight assists the movement, and the motions prevent its falling.

In drawing figures it will be understood that the part that lies behind another can not be seen, and that one side of a limb or of the body can not be turned toward you without turning the other from you, and that the length of a body or limb can only be shown in its full length and proportion when it is

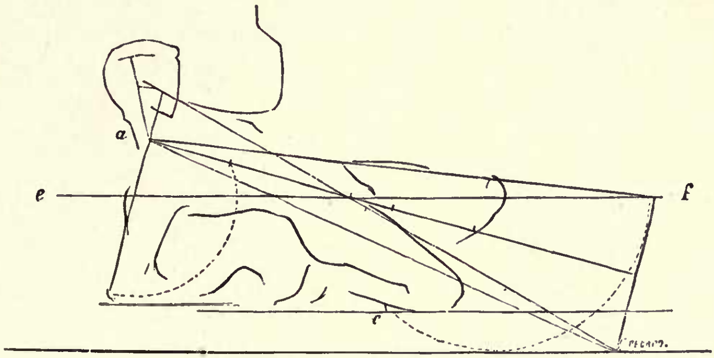


FIG. 1822.

perpendicular to the line of sight—that is, if the arm, for instance, is directed toward the eye, the hand will be the prominent object in view, that the arm will only be shown by the portions prominent beyond the outlines of the hand, that limbs or portions more or less inclined to the line of sight will be more or less foreshortened or show less than their natural length.

It is very common in the drawing of figures to indicate merely the centre lines of the various portions and then clothe them as shown in several of the figures.

It is very common with modern draughtsmen to adopt a somewhat similar form of sketching as given in the "Dictionnaire Raisonné de l'Architecture"—a framework of bones in positions of action—and then clothing them with flesh or drapery; or manikins and lay figures in which the limbs can be set or fixed as desired, and then drawn from as models. Figs. 1823 and 1824 are illustrations from Dr. Rimmer's "Elements of Design" of skeleton lines and of manikins.

Fig. 1825 is taken from photographs of a manikin in our office, which, although maimed as to its hands, is one of the best forms of these jointed figures, of which the limbs and body can be set in any required position; but the suggestion is obvious that by the salted-paper or blue-print process, nude figures can be used instead of manikins, and photographs promptly secured without fatigue to the model and in positions of motion impossible in sketching. As a further illustration of this process, a pen drawing is given of the Venus de Milo (Figs. 1826–1827) from two points of view, taken from a plaster model, and a portrait of Alexandre Dumas (Fig. 1858) from a photograph.

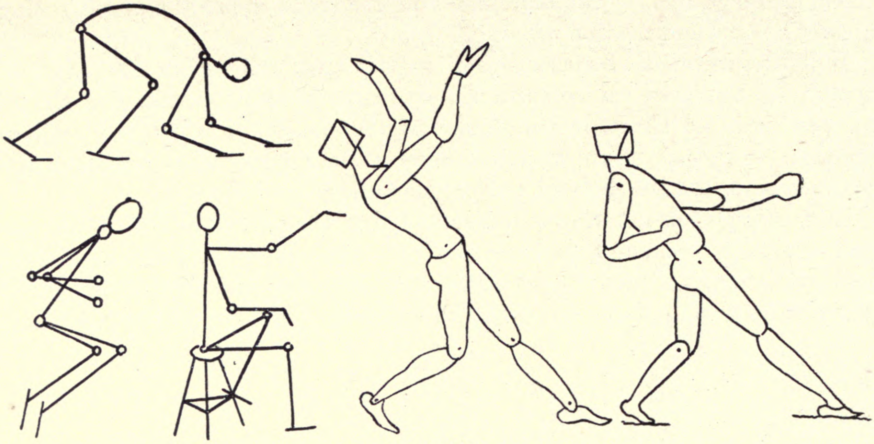


FIG. 1823.

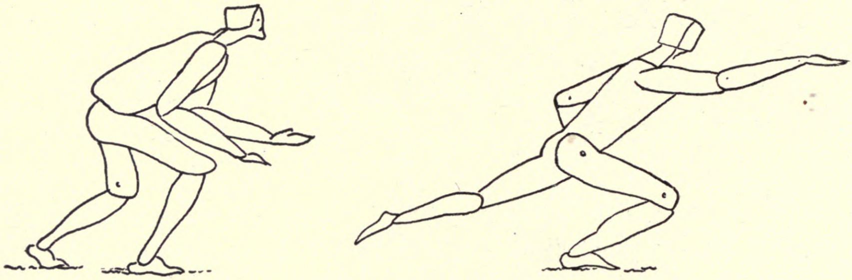


FIG. 1824.

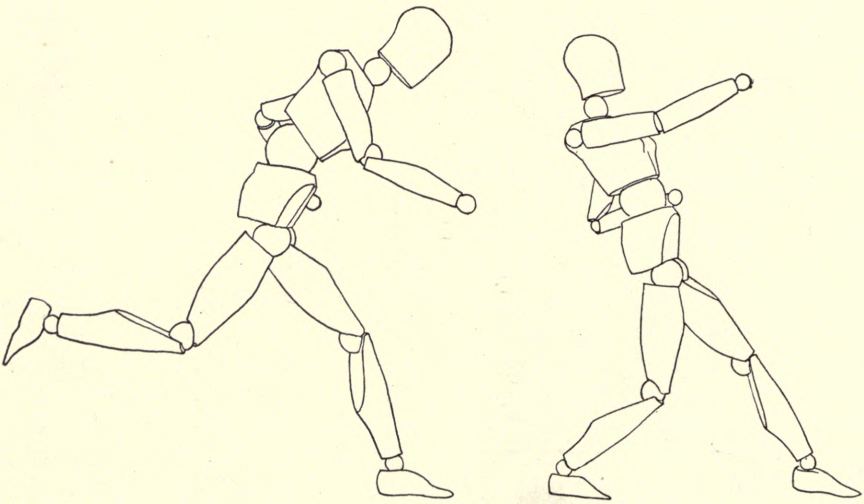


FIG. 1825.

Artists object that photography is too exact a reproduction, but it is well that they should understand what is an exact reproduction. Tint and colour may produce pleasant impressions without conformity to laws of perspective, but if the picture is to be taken as whole it should be natural, and with the present processes of photography it is well to throw the mechanical drudgery on it.

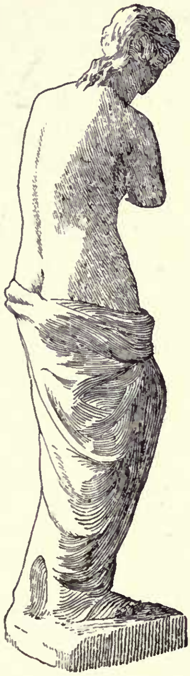


FIG. 1826.

FIG. 1827.

Figs. 1828–1831 are drawings of male hands, Figs. 1832–1838 of legs and feet, with guide-lines to assist the draughtsman.

Figs. 1839–1841 are drawings of female hands and arms.

Figs. 1842–1845 are hands and feet of children.

Figs. 1846 and 1847 are illustrations of the human head and face.

FIG. 1828.

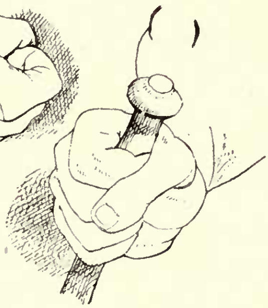


FIG. 1829.

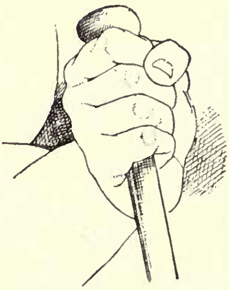


FIG. 1830.

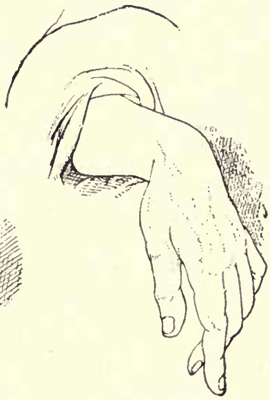


FIG. 1831.

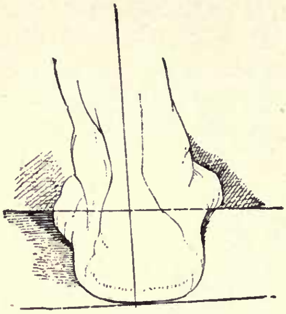


Fig. 1832.

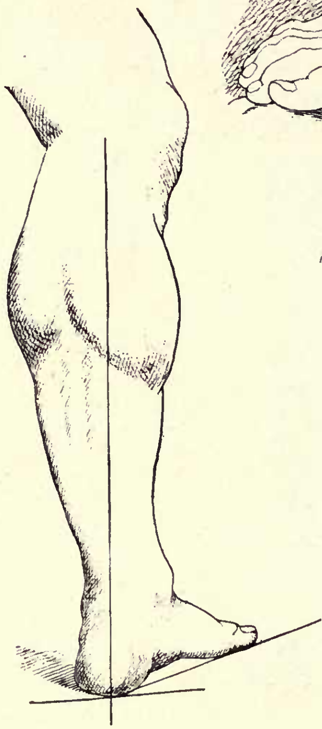


Fig. 1833.

Fig. 1834.

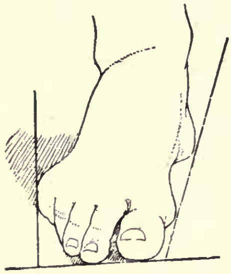


Fig. 1835.

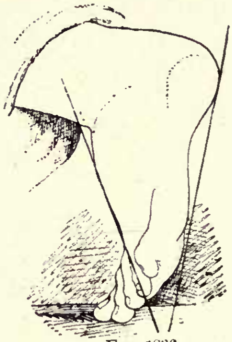


Fig. 1836.

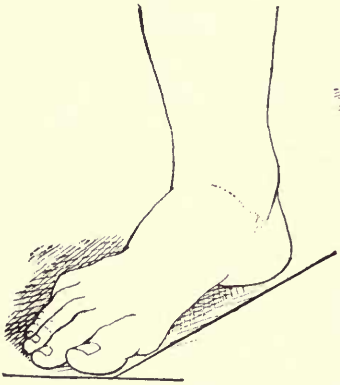


Fig. 1837.

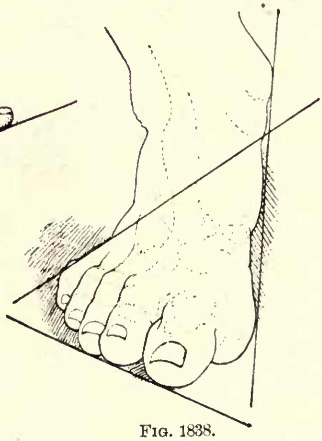


Fig. 1838.

Fig. 1839.

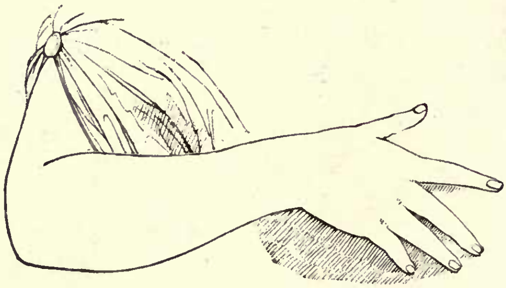


Fig. 1840.

Fig. 1841.

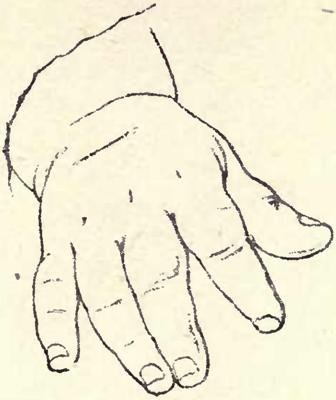


Fig. 1842.

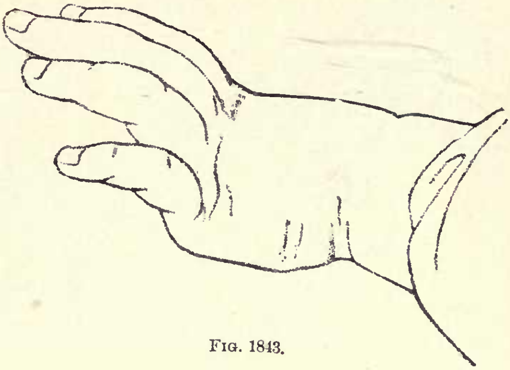


Fig. 1843.

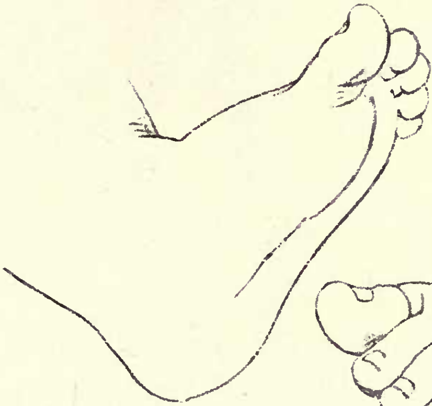


Fig. 1844.

Fig. 1845.

Fig. 1846.

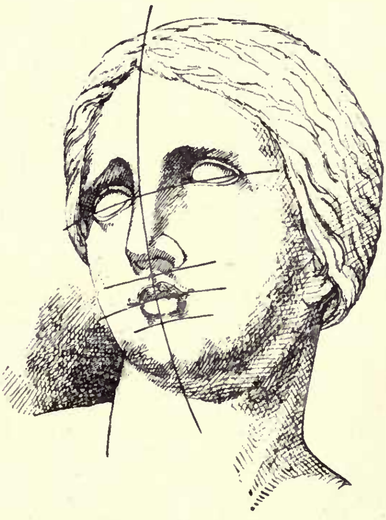
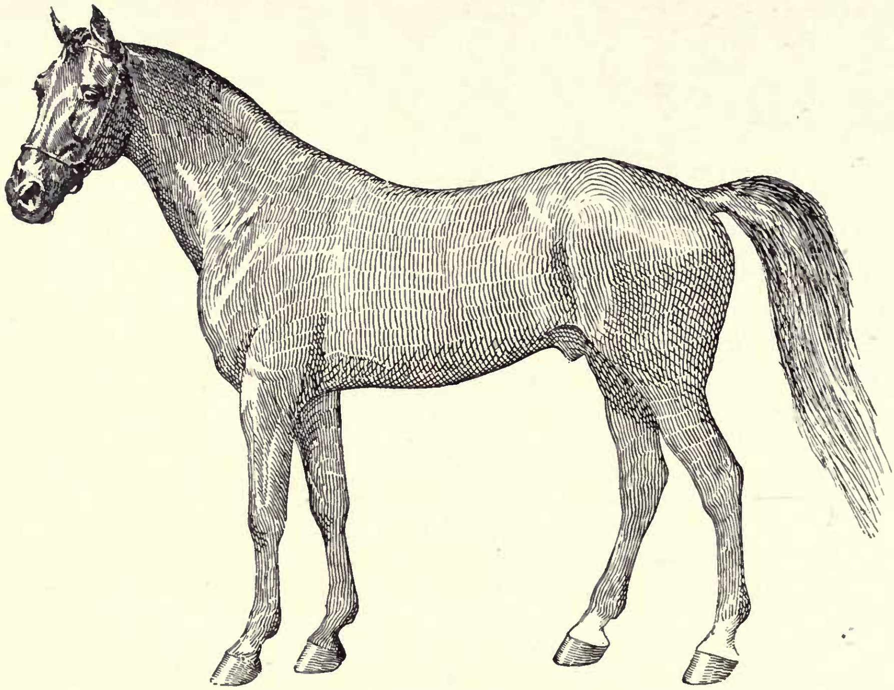


Fig. 1847.



ELECTIONEER.

The Forms of Animals.—The bodies of most quadrupeds can be included in rectangles as guide-lines, which may be drawn around the illustration of the cow and horse (Figs. 1848 and 1849). The action of the limbs of quadrupeds is chiefly directly forward or directly backward, the power of lateral motion being limited. The hinder limbs always commence progressive motion, as in the first position of the walk, the fore foot of the same side advances next, then the hind foot of the opposite side, and lastly the fore foot on that side, and so on. In the trot, the hinder leg of one side and the fore leg of the other are raised together. In the canter or gallop, both fore legs and one hind leg are raised together; when rapidly moving, the two fore legs and two hind legs appear to advance together. In fact, all the movements are rather resultants, as they appear to us, but when instantaneously photographed the legs are wonderfully mixed.

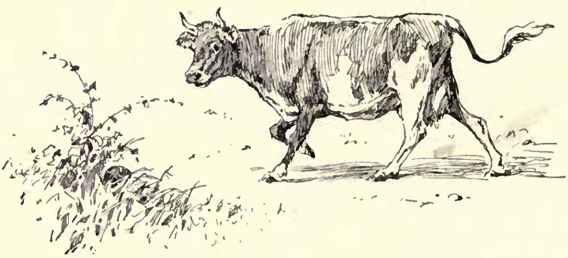


FIG. 1848.

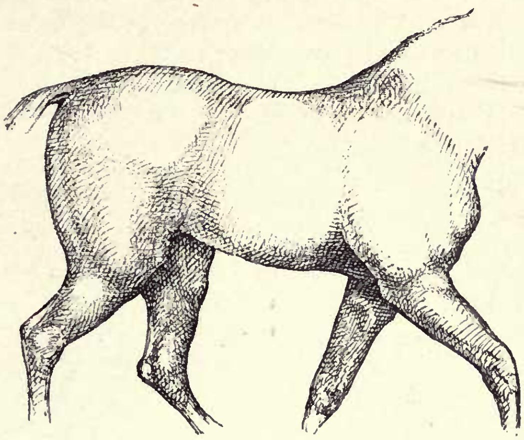


Fig. 1849.

Fig. 1850.

Fig. 1850 is one of Landseer's sketches.

The forms of feet range under two great divisions—hoofs (Fig. 1851) and paws (Fig. 1852). All hoofs, whether whole or cloven, approximate to a right-angled triangle, and all paws to a rhomboid.

The Noses of Animals.—Fig. 1853 represents that of the horse; Fig. 1854, that of the ox and deer tribe; Fig. 1855, those of the carnivori; Fig. 1856, those of the camel, sheep, and goat tribes; and Fig. 1857, those of the hog tribes. The muzzles of nearly all quadrupeds will be found to range under one or other of these classes, with minute variations to characterize the different species and individuals.

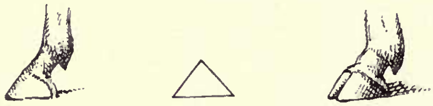


FIG. 1851.

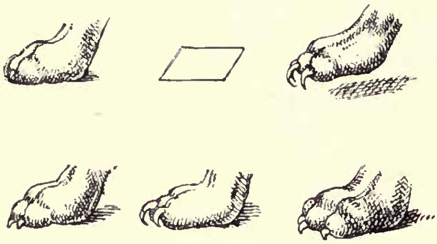


FIG. 1852.

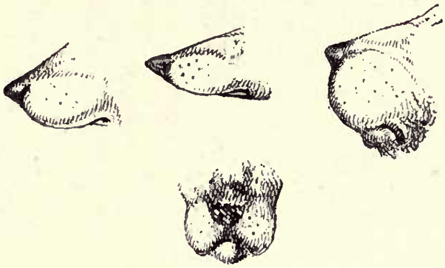


FIG. 1855.

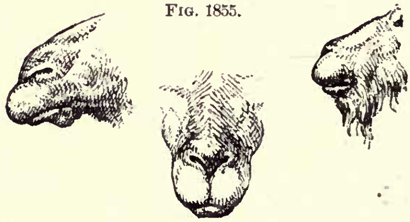


FIG. 1856.

FIG. 1853.

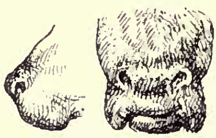


FIG. 1854.

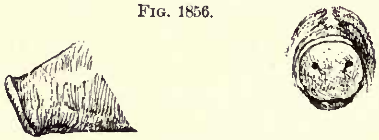


FIG. 1857.

In looking over the many sketches and engravings of Landseer which have been published, it will be noticed in how varied a manner they were executed. Sometimes in mere outline with lead-pencil, sometimes with a camel's-hair pencil charged with India ink or sepia for the outlines, giving effect to the subject by slight tints or washes of the same colour; in others, pen and ink have been alone employed; some are in oils, others in water-colours; frequently chalks, both black and coloured. "As we look at some of these, we are tempted to believe that, of all the instruments that can be used by the artist, there is none quite so wonderful as the pen. A simple sketch with a pen or lead-pencil is naked, unadorned truth, bearing witness to the skill or its opposite of the hand which produced it."

Strength and boldness in outline are acquired by large scale in drawing; and in copying, if suitable originals can be obtained, copy them, but if you are confined to the illustration of books and periodicals, recollect that they have usually been reproduced on a reduced scale, and make your drawings two or three times their lineal dimensions.

The directions and illustrations already given may be considered copying, which is absolutely necessary as an introduction to free-hand drawing, and examples have been selected from figure drawing to give the draughtsman a strong bold hand and an education in proportions.

Sketching from Nature.—It is useless to give detailed rules for sketching, as each has his own way and perhaps equally good though dissimilar. If one is inside the house, what one sees through a window is a picture, and it may be transferred to paper if the eye is kept in a single position by a sight fastened to the sash at a convenient distance; and if the pane of glass be prepared in squares, as described on page 60, the location of different points can be readily established. A simple plan to begin with is to set out carefully the most conspicuous outline and draw the others with reference to it. If doubtful as to the distances and length of lines, stretch out your arm, holding your pencil vertically, horizontally, or aslant, as the case may be, and shutting one eye, mark off the measure from the end of the pencil by placing your thumb on the spot; then compare that line or space with any other needed. By noting the relative position of one object with reference to another, all will fall into place almost without thought. Thus you have sketched a house, notice the next object and observe where it is projected against the building, as at such a window or door, and draw it in. Every artist has his own method of handling the sketch, of using his pen or pencil, charcoal or brush; the aim of each should be to express what he sees by cross-lines and hatching if the pen is used, or by scribbling if the work is in pencil or charcoal. A general rule is to adapt your strokes as far as possible to the modelling of the subjects you are drawing. Thus if you are representing water, it is natural to do it with horizontal lines. If the water is in motion, the lines, though still horizontal, will be broken and irregular. The reflections which in still water will be represented by vertical lines, in running water are indicated by horizontal lines with closer shading to give depth of tone.

In drawing the trunk of a tree the characteristics of the bark must be observed. The trunk of an oak is rough and broken, while that of the birch requires curved lines across the thickness of the tree. Nature sufficiently indicates the treatment. In many cases one must not be content with an exact reproduction. The scene must be interpreted.

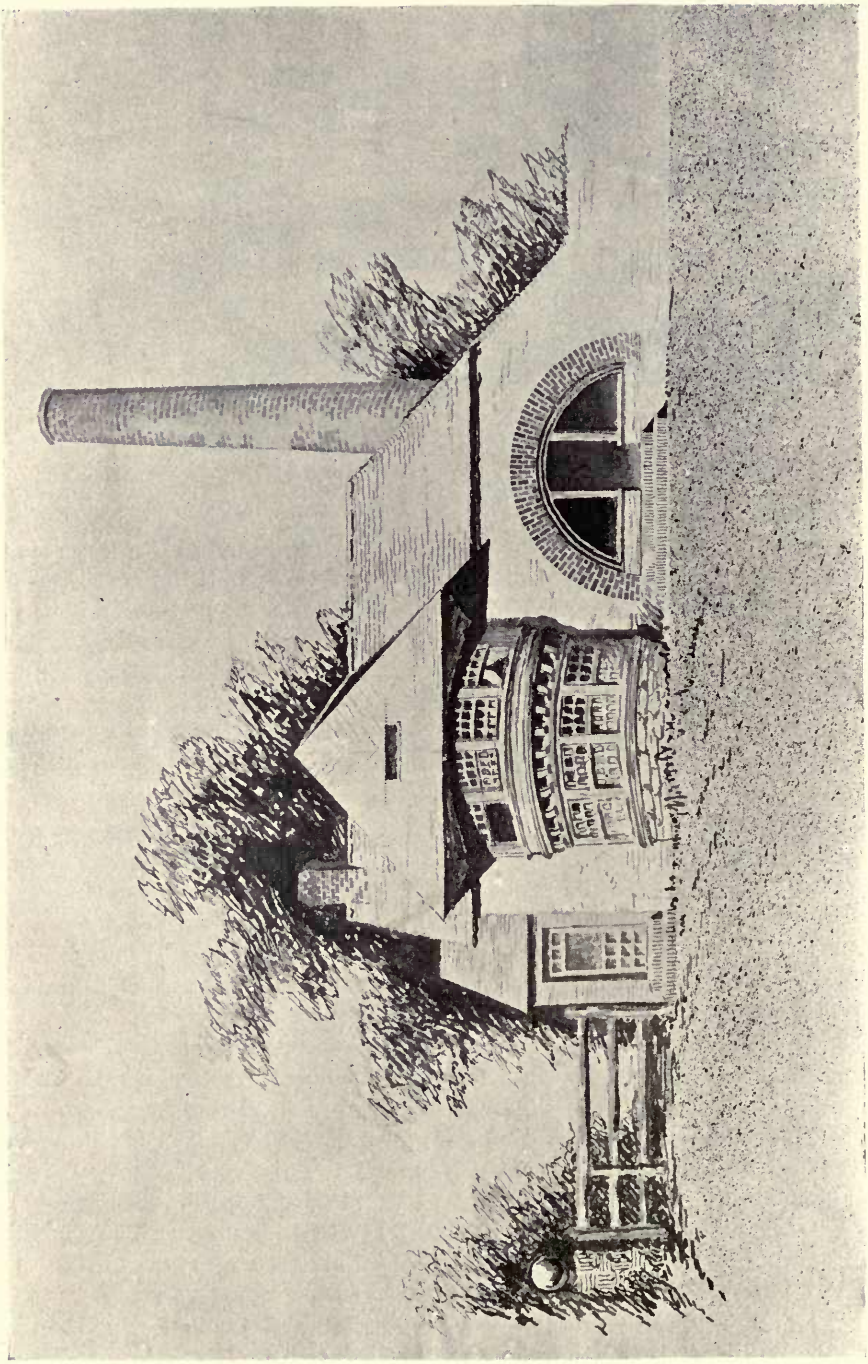


FIG. 1860.

In all sketching from Nature the lines must be crisply and unhesitatingly drawn, the forms clearly defined, and the masses decisively indicated. In rapid work a mere outline must suffice, but it must be clearly and cleanly pencilled in. When an elaborate drawing is required, begin by indicating the general arrangement, and then fill in as much detail as may be necessary; but it is to be remembered that the simplest expression is the best and at the same time the most difficult, the effort being to concentrate the effect on the object to be emphasized, all others being subordinate to it.

When a sketch has been faithfully made from Nature, as a general rule it is better not to try to improve it afterward, as one is apt to lose the crispness and vigour of the original. It is better to modify or amplify the first notes on another sheet of paper. Neither should there be many erasures, as they impart to the paper a dingy appearance and the sketch loses its sharpness.

The depth of tone of shaded portions, as clouds, in some hasty sketches is indicated by numerals or letters, 1 or A representing the lightest; but it is better if time is afforded to put the shading in the sketch. In sketches of landscapes not intended for reproduction the addition of colour often adds very much to the effect and value of the sketch, but the colour must be light and transparent.

The foregoing applies chiefly to landscape sketching. If you desire to

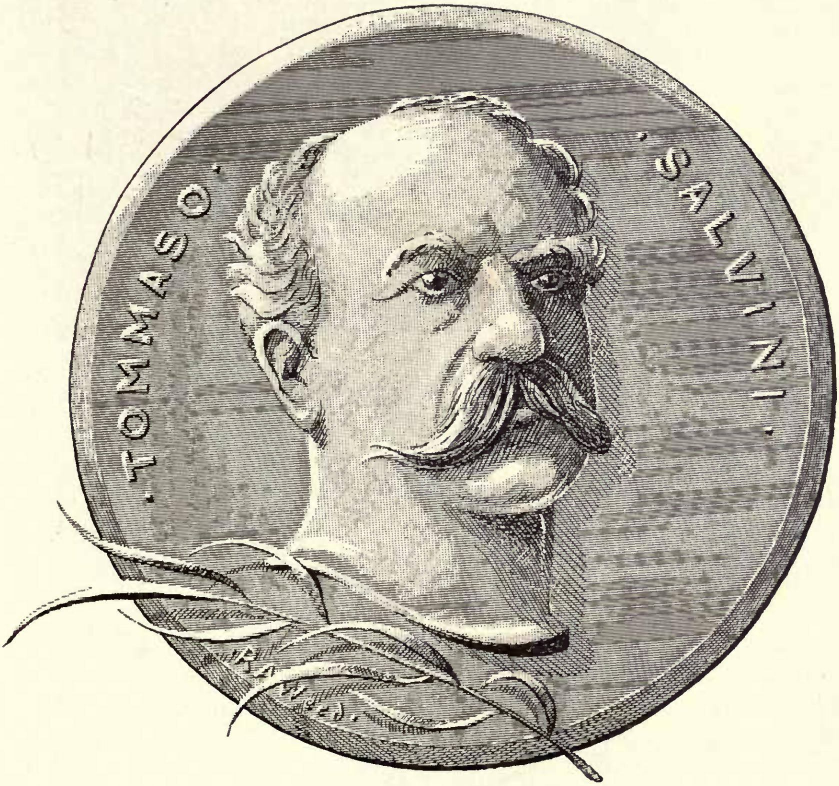


FIG. 1861.

sketch figures or animals, the character of the individual is what you must try to seize; if an animal in motion, one must work quickly and await the repetition of the action or particular movement you are sketching. Meanwhile em-

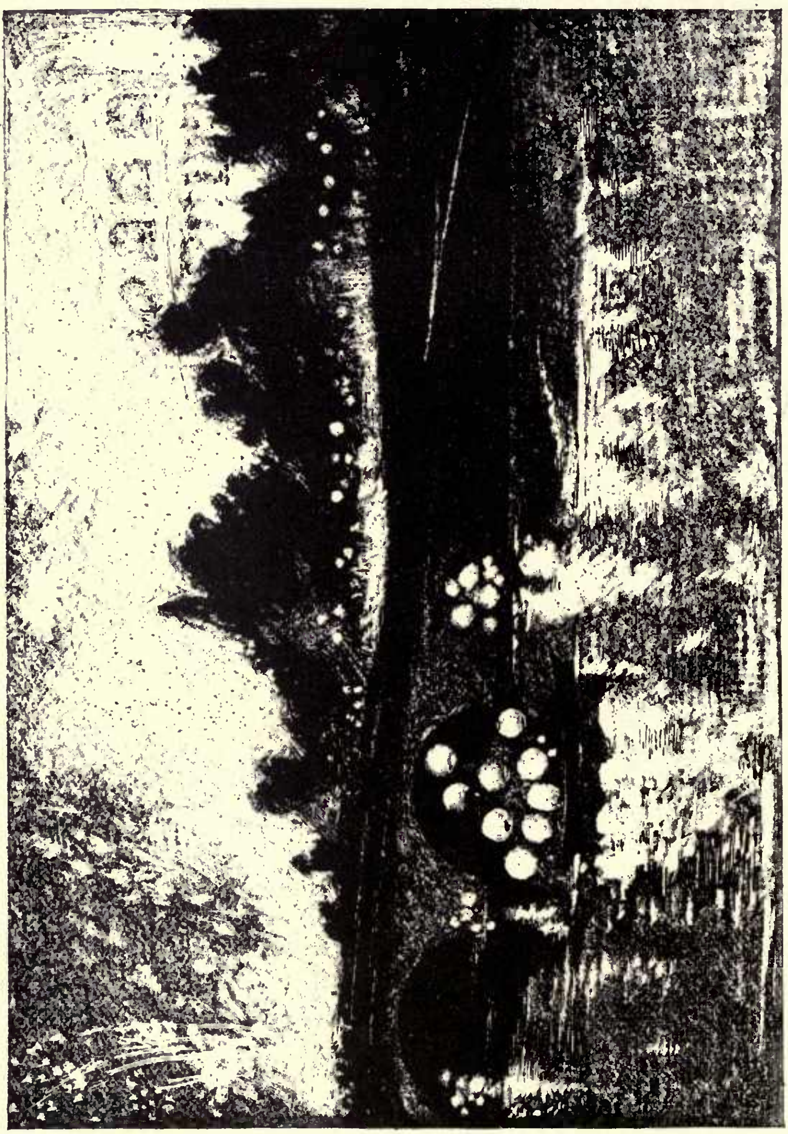


Fig. 1862.

ploy your pencil on those portions that remain longer in one position, excepting to work on the moving limb the instant it has regained the required position. The sketching of animals in rapid motion is still more difficult and is to a great extent a matter of careful observation and memory, for the limbs are not the only parts that change their position; the whole attitude of the body is changed. In addition to memorizing all possible of the movement, considerable aid may be obtained in observing the animal at rest, which will enable you to understand the details of the structure.

All sketches made should be preserved for future use, as they furnish the best material for original work that one can have. Sketches made in pencil or other materials which smear may be fixed with a solution composed of one part of gum mastic and seven parts of methylated spirits of wine, and is best applied with an atomizer.

In transferring your sketches to the paper on which the pen drawing is to be made, commence by making a careful drawing with a hard pencil in outlines, confining yourself to those of shadows as much as possible, to save the surface of the paper, as rubbing spoils it and grays the ink; or make the drawing on another sheet of paper and transfer it by means of graphite paper to the fair sheet, then ink in.

If you want a clean, sharp line, the ink must be perfectly black and must stand out alone on the paper. If you want a gray line, it will not be obtained by using light ink, but by making thin lines. A single thin line will come out in the reproduction much blacker than is the intention, for though a number of lines will stand together, a single one will have to be thickened in the type metal by the photo-engraver.

FIG. 1858.

FIG. 1859.

Fig. 1858, portrait of Alexander Dumas. For description of process see page 738.

Fig. 1859 is a portrait of Erik Werenskiold, the artist, drawn by himself on Whatman's paper.

Wash drawings are made with a brush on either Bristol board or water-colour paper, the wash consisting of India ink and water, of various degrees of intensity. A number of illustrations are given of this method on page 750, by Paul de Longpre, and reproduced in half tone.

Fig. 1860 is a design for a small pumping station, drawn with India ink and a toothpick, after W. R. Emerson, in the "Technology Architectural Review."

There are many devices which may be used with effect in pen-and-ink drawing, such as splatter work, which is done by using a small stiff bristle brush (a toothbrush answers the purpose very well), inking it, and holding the bristles downward and inclining toward the drawing and stroking the bristles toward one with a match stick. All parts not intended to be splattered should be covered with paper masks, and even then a waterproof ink must be used in case it is necessary to paint out some portions with Chinese white. The lower portion of Fig. 1860 is in splatter. An inked thumb is also a novel and effective means of representing a background or an imitation of velvet, the lines on the skin being marked on the paper and reproduced excellently.

An invention of recent years is known among artists as stipple paper or clay board. The surfaces of these cardboards are of various kinds, but are all prepared with a surface of china clay. The simplest variety is that prepared with a plain white surface, upon which the drawing is executed with pen and ink or brush; the lights are taken out with a sharp ink eraser. It is usual to work upon these boards with a pigmental ink, such as lampblack, ivory black, or India ink. More liquid inks have a tendency to soak through the prepared surface rather than rest upon it, rendering the board useless for scratch-out purposes; other kinds of boards are impressed with a grain or with plain indented lines, which are used similar to the above. Scratch-out boards are difficult of manipulation and are not to be recommended to the amateur.

Fig. 1861, a portrait of Salvini, is an example of the use of stipple paper. The background or middle tone shows the board in its natural state. The high lights and shadows are obtained respectively by erasing and adding India ink.

Fig. 1862, "A Venetian Fête on the Seine," is another specimen of work on clay board. In this case the board was entirely black, the various tones being obtained by various degrees of erasing.

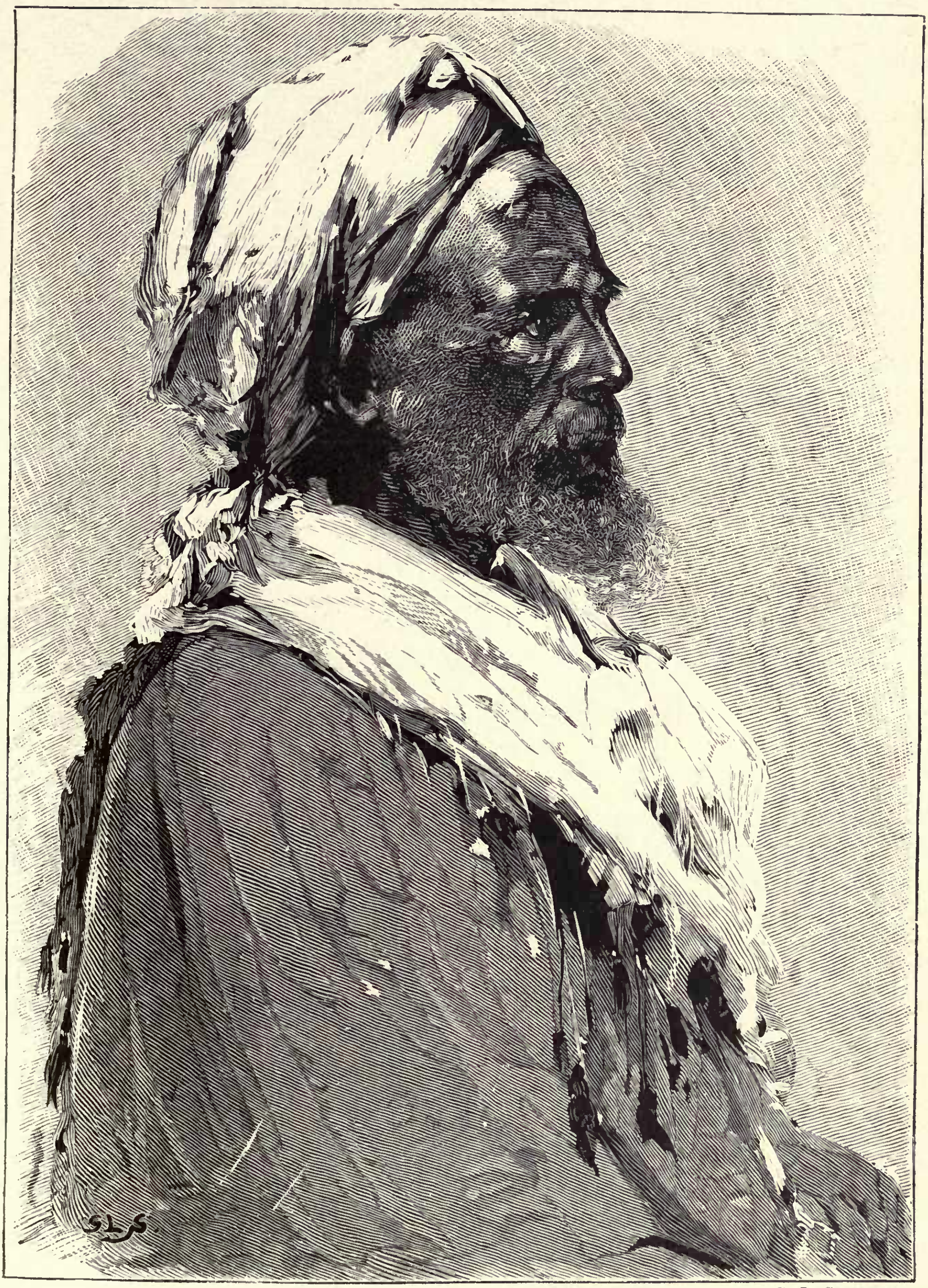
The appearance a drawing will present when reduced may be approximately judged by the use of a "diminishing glass"—that is, a concave glass.

To remove blots or make erasures, use an ink eraser or simply paste a piece of paper over and join the lines at the edges; or a neater way, cut out the blotted part and paste a piece of paper underneath.

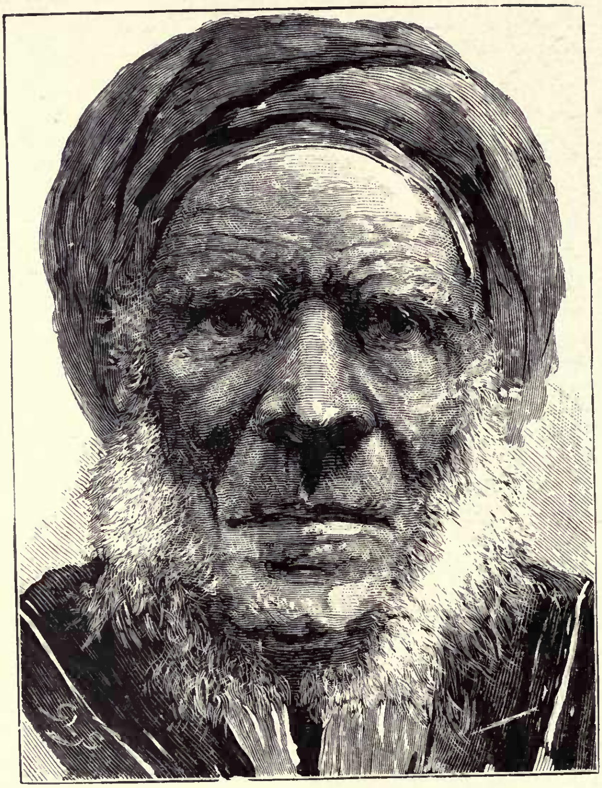
To clean pen-and-ink drawings use bread one or two days old and not rubber.

Aërial perspective, or the tones of lights and shadows according to their distance from the observer and the sources of the light, will be acquired by studies of pictures and observations of Nature. The rule in drawing from Nature is to draw only what you see and express it in the most truthful form.

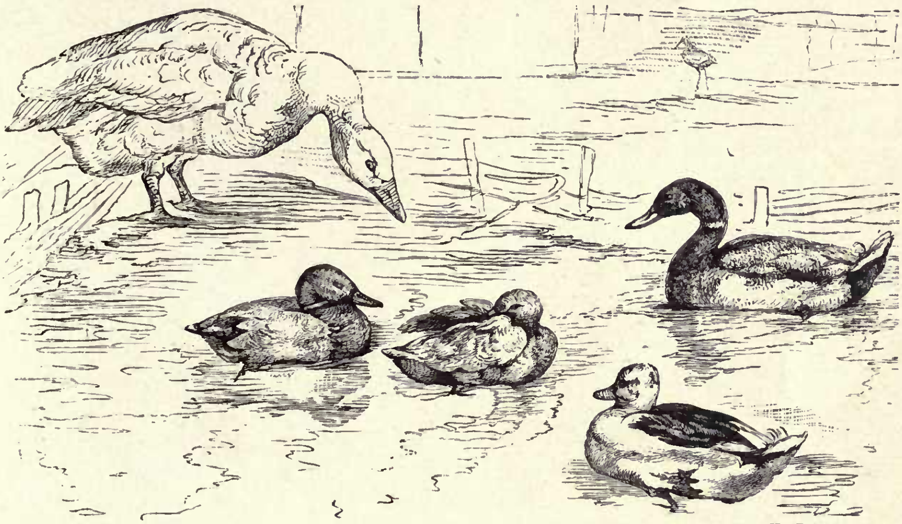
After a Pen-and-ink Design, by FORTUNY.



G. L. SEYMOUR.

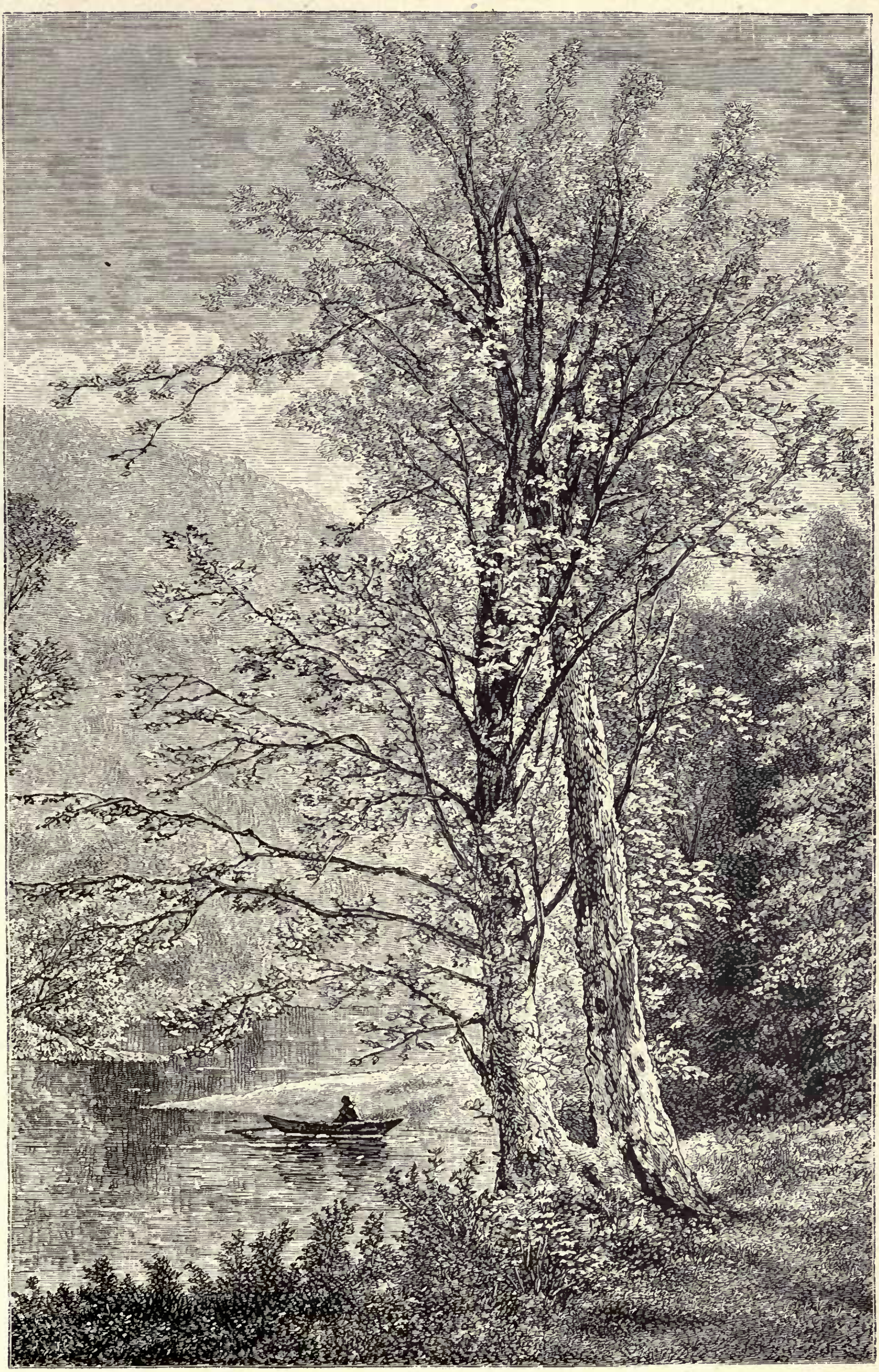


E. LANDSEER.



E. LANDSEER.

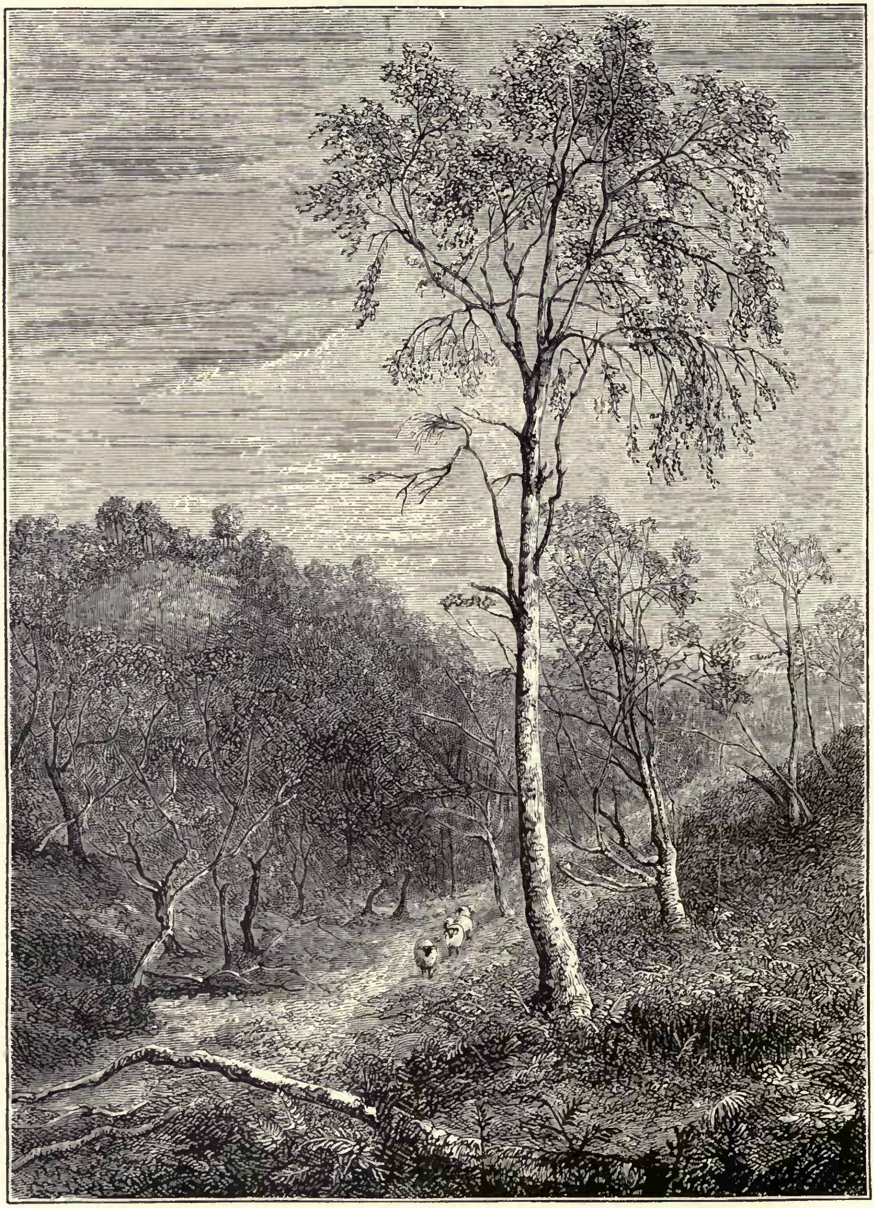
Study of Oak-Trees. E. LANDSEER.



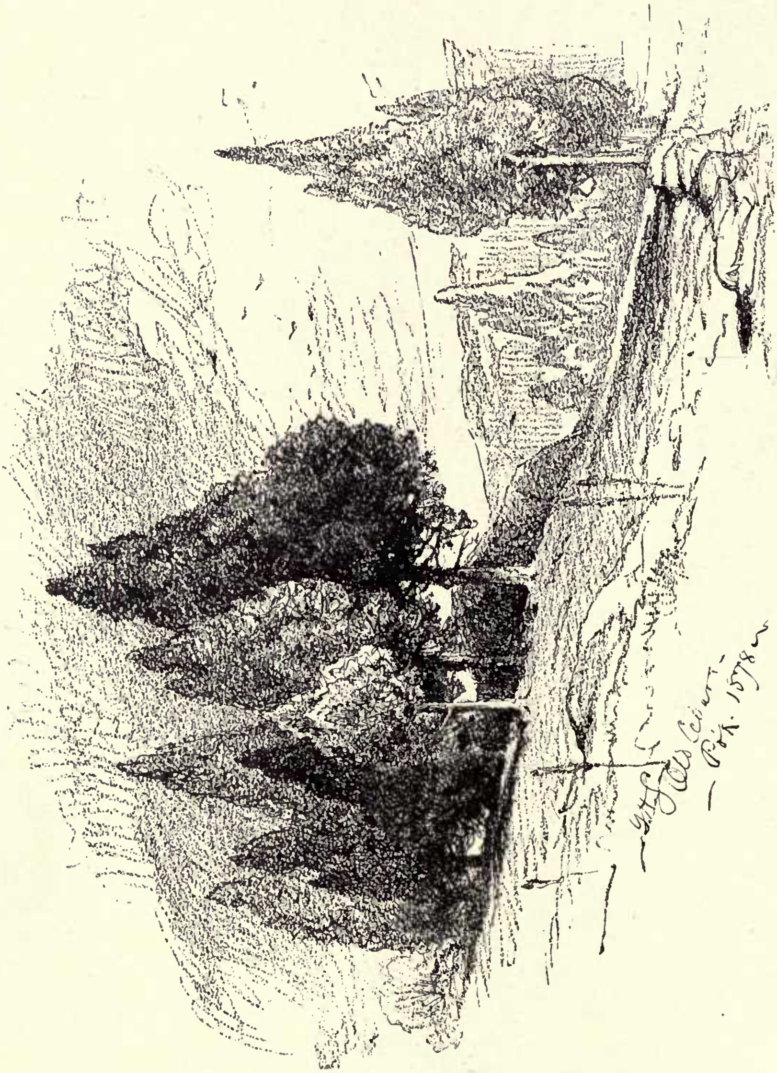
Morning. H. W. Robbins.

Cattle going Home. JAMES M. HART.

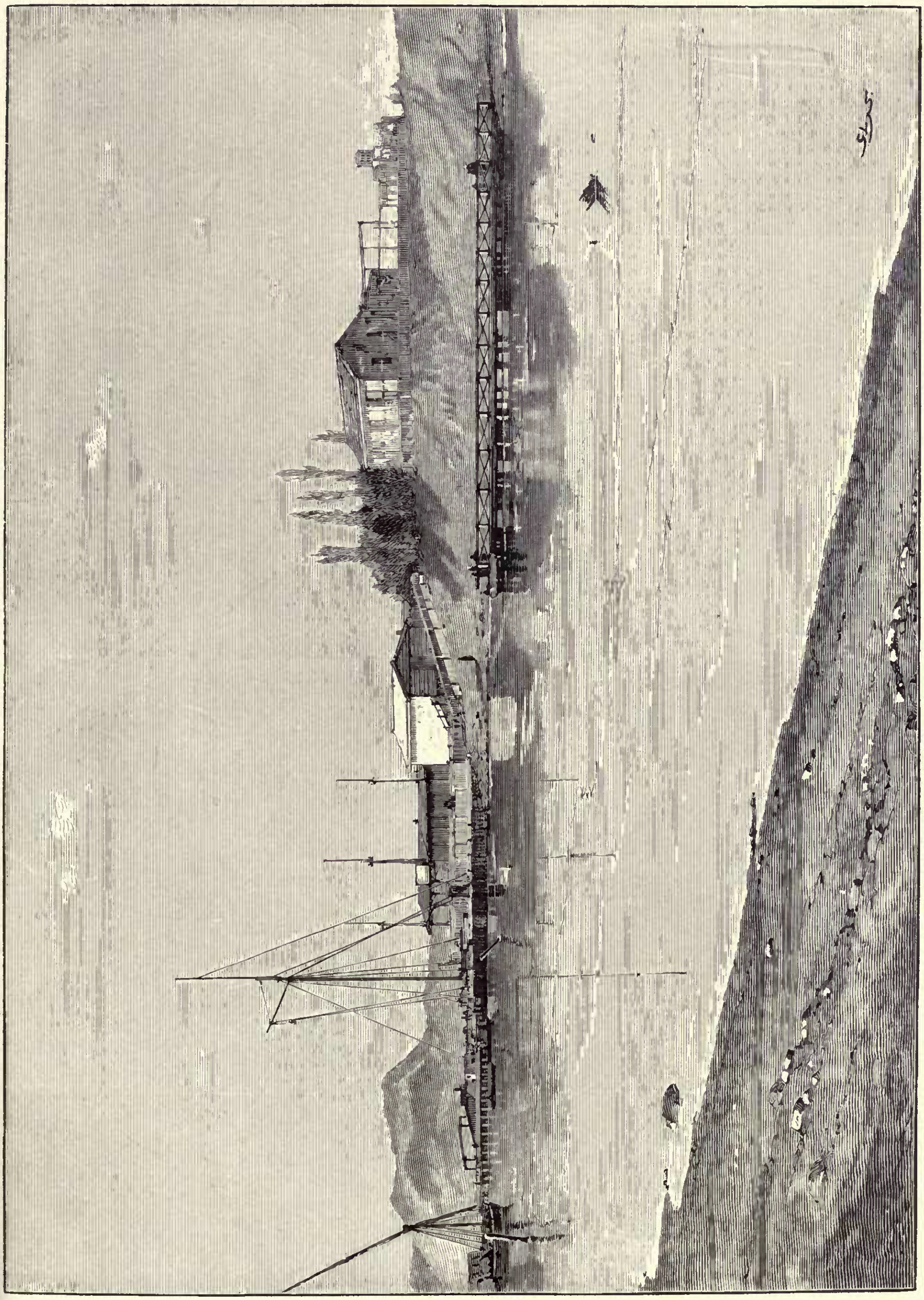
Apple-Blossoms. A. T. Bricher.



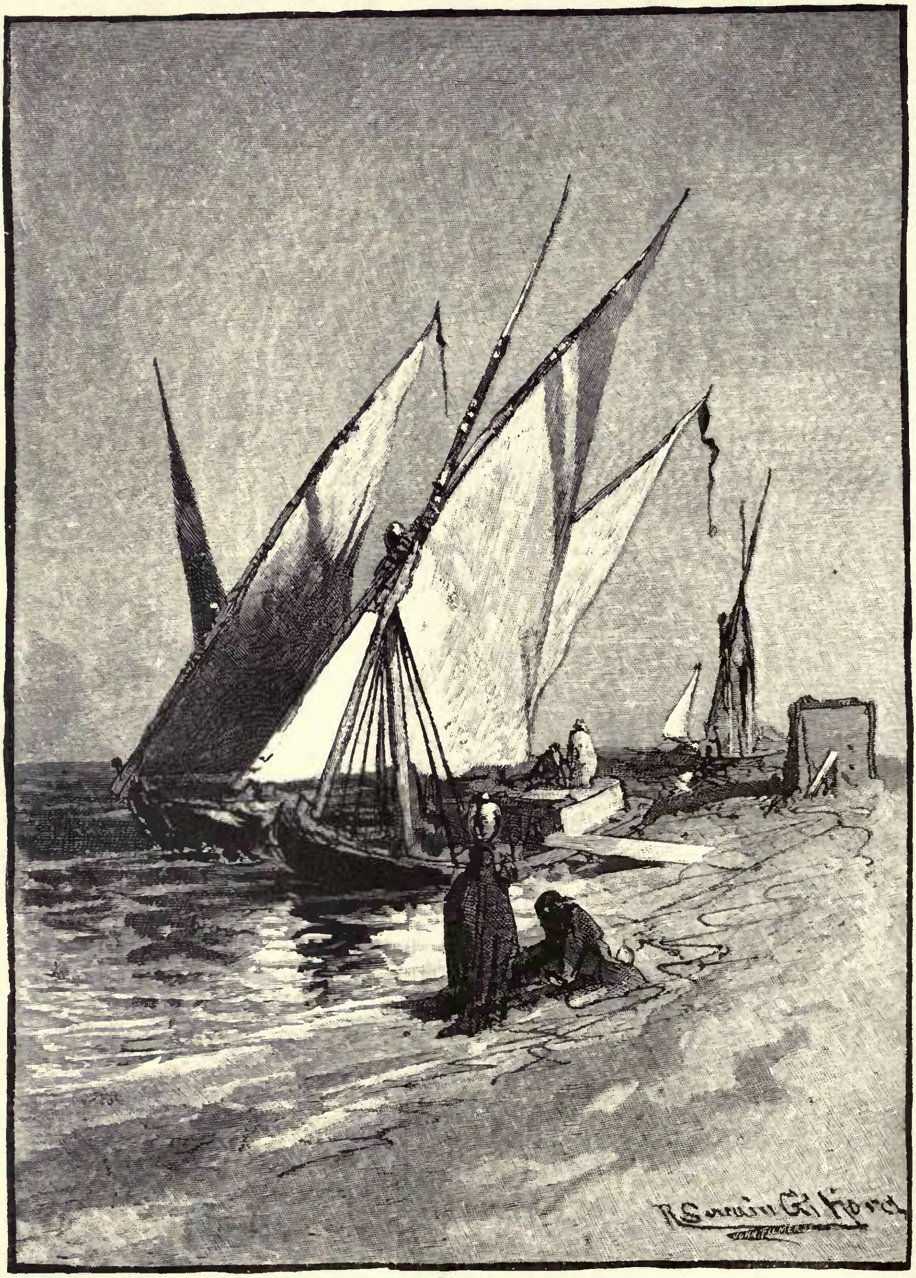
The Lady of the Woods.



Sketch in Chalk.



The Station at Kantarah, on the Suez Canal.



PL. I.

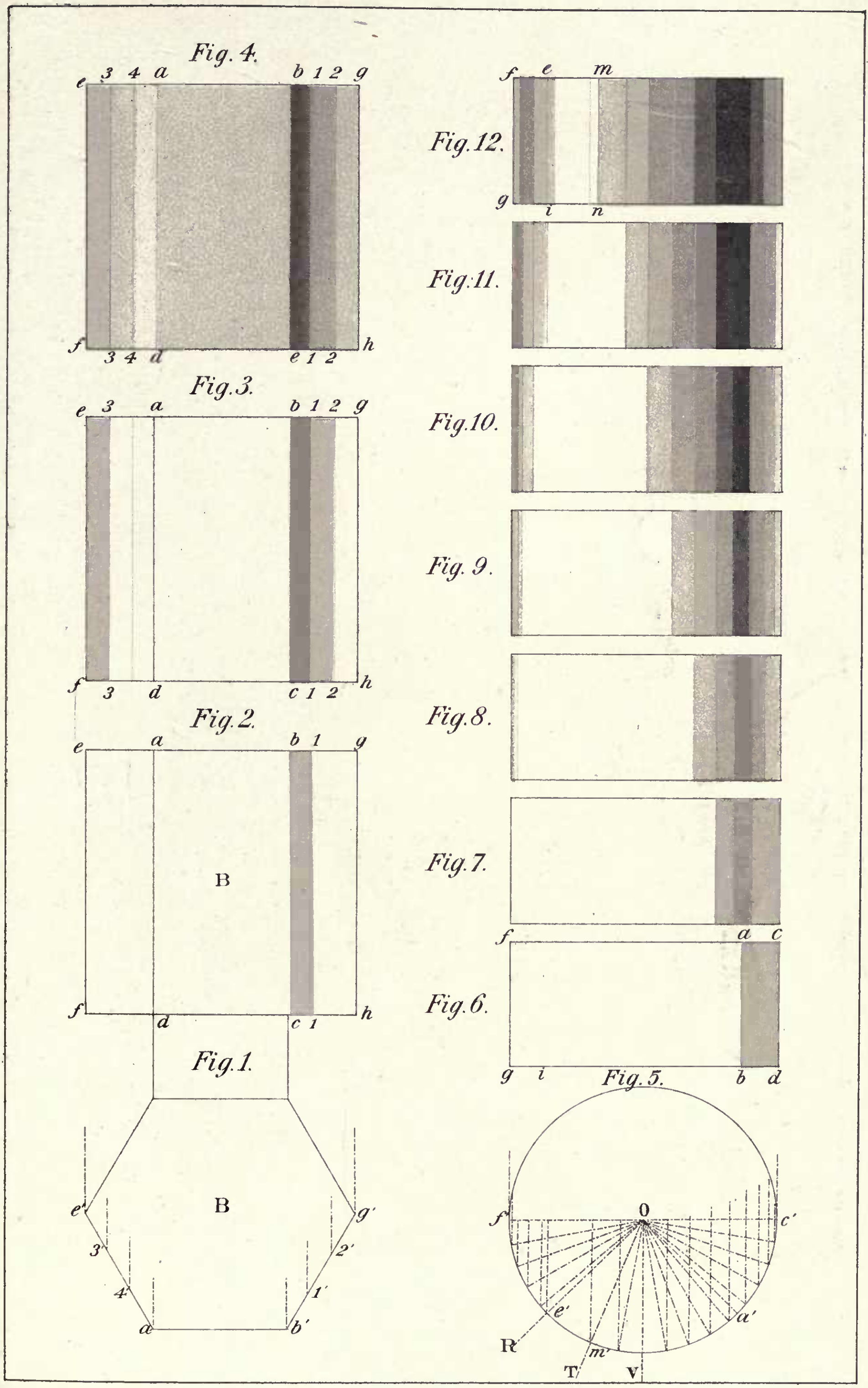


Fig. 4.
e 3 4 a b 1 2 g
f 3 4 d e 1 2 h
Fig. 3.
e 3 a b 1 2 g
f 3 d c 1 2 h
Fig. 2.
e a b 1 g
B
f d c 1 h
Fig. 1.
B
e' 3' 4' a b' 1' 2' g'
Fig. 12.
f e m
g i n
Fig. 11.
Fig. 10.
Fig. 9.
Fig. 8.
Fig. 7.
f a c
Fig. 6.
g i b d
Fig. 5.
0
f' c'
e' a'
R m'
T V

Fig. 4.

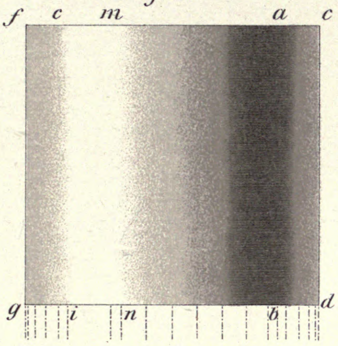


Fig. 3.

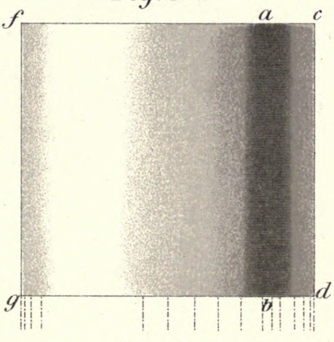


Fig. 2.

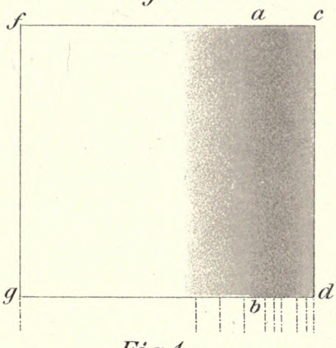


Fig. 1.

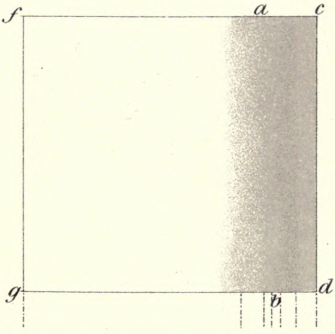


Fig. 8.

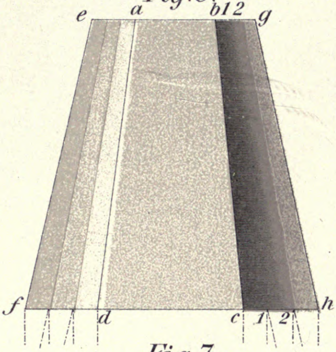


Fig. 7.

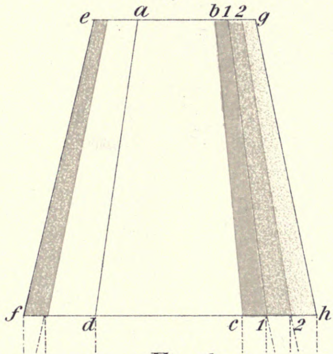


Fig. 6.

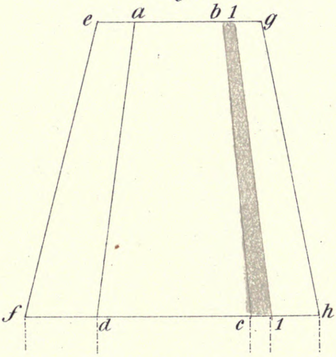
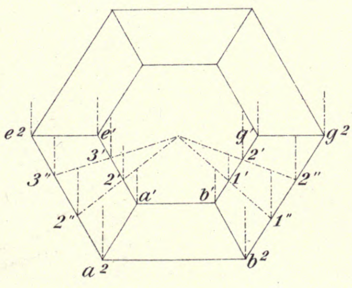
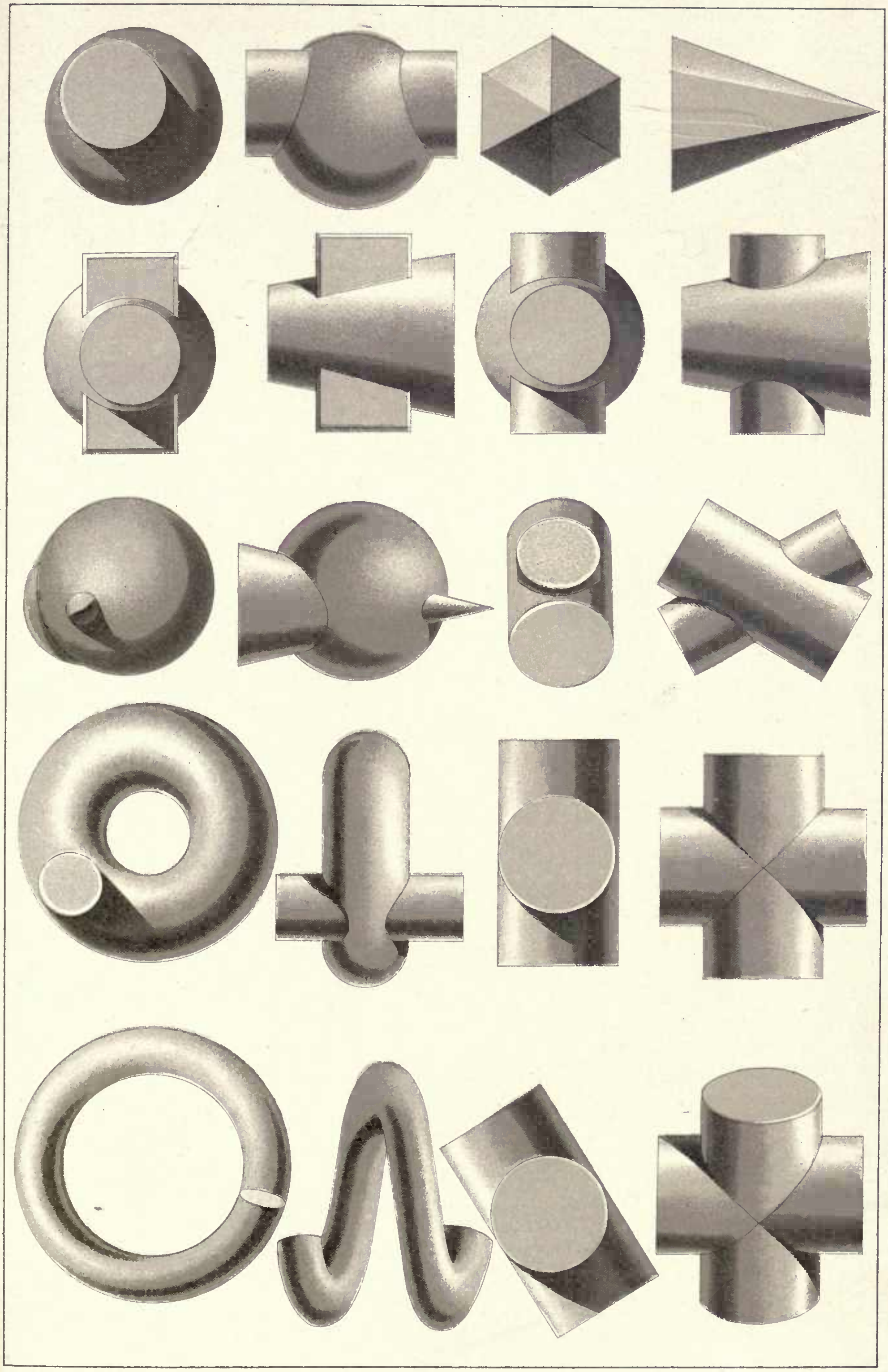
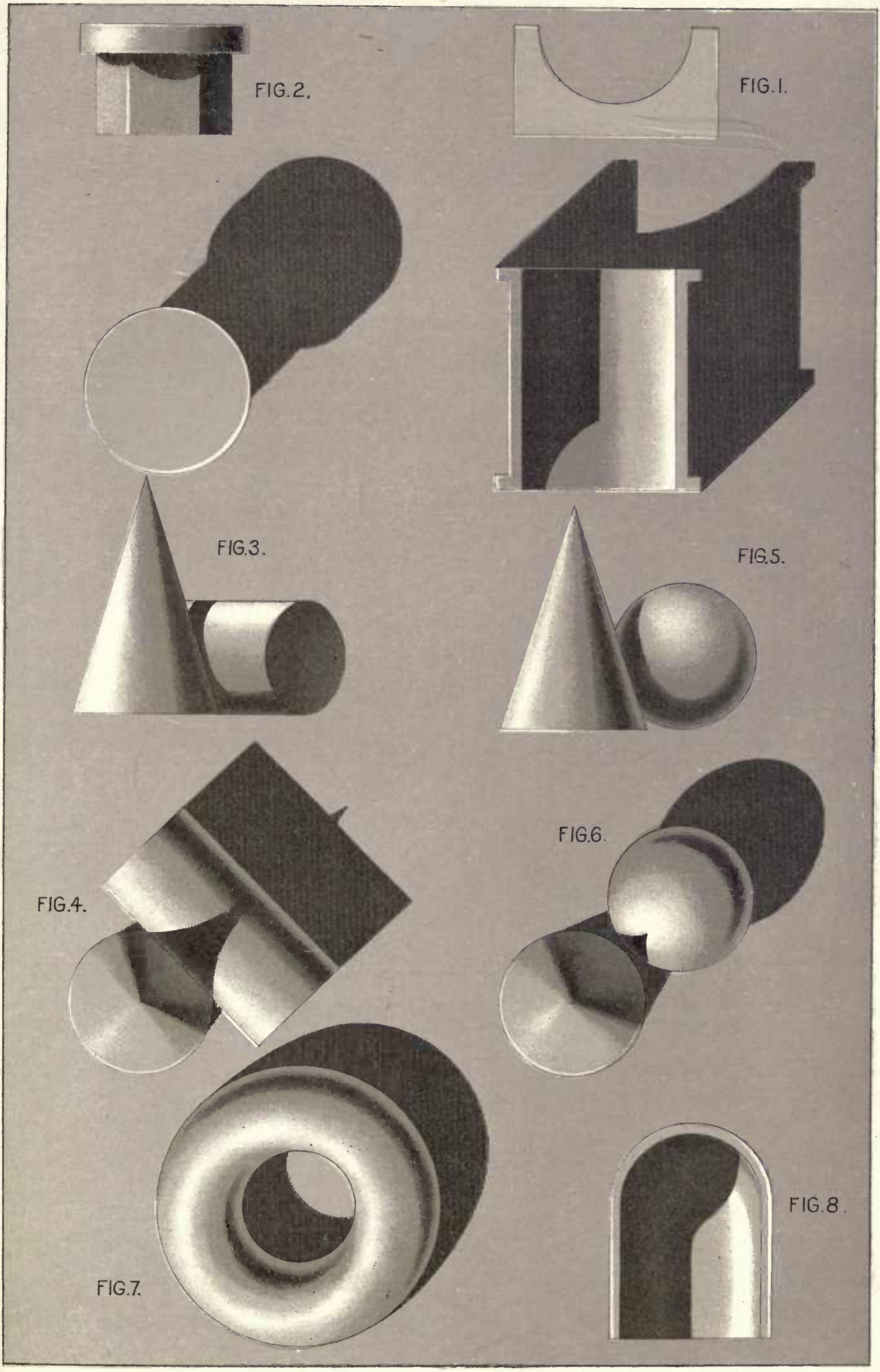


Fig. 5.

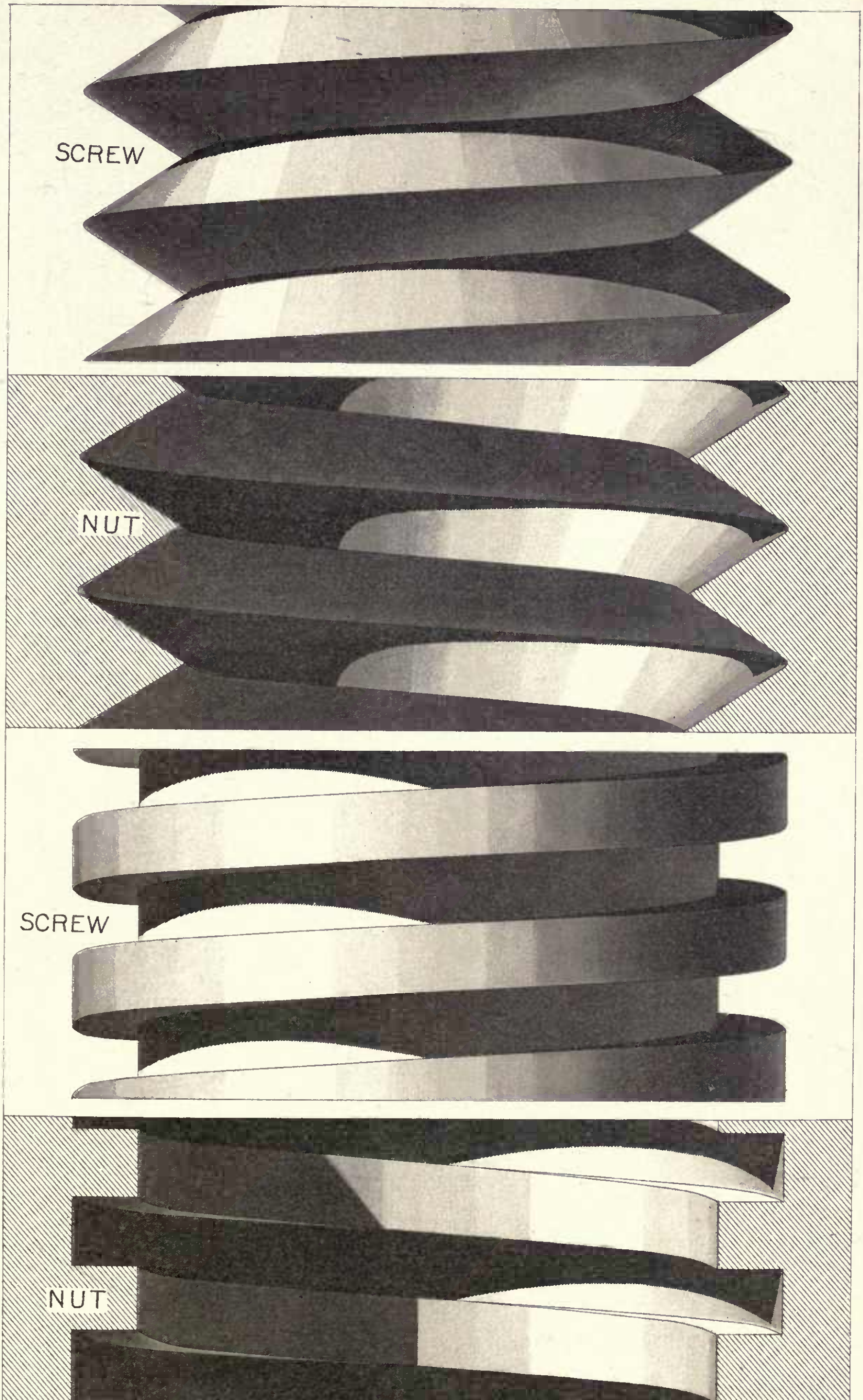




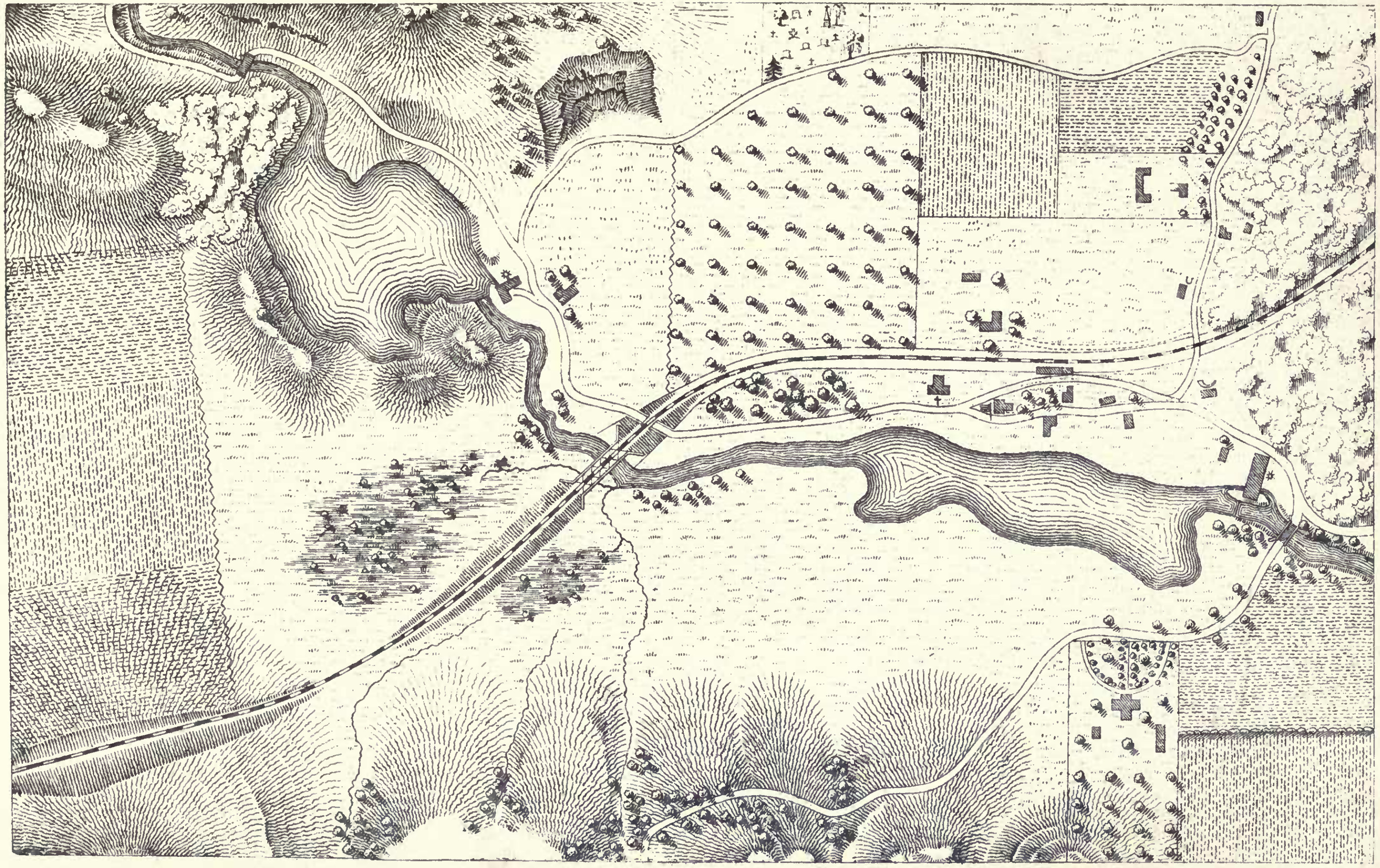
PL. IV.



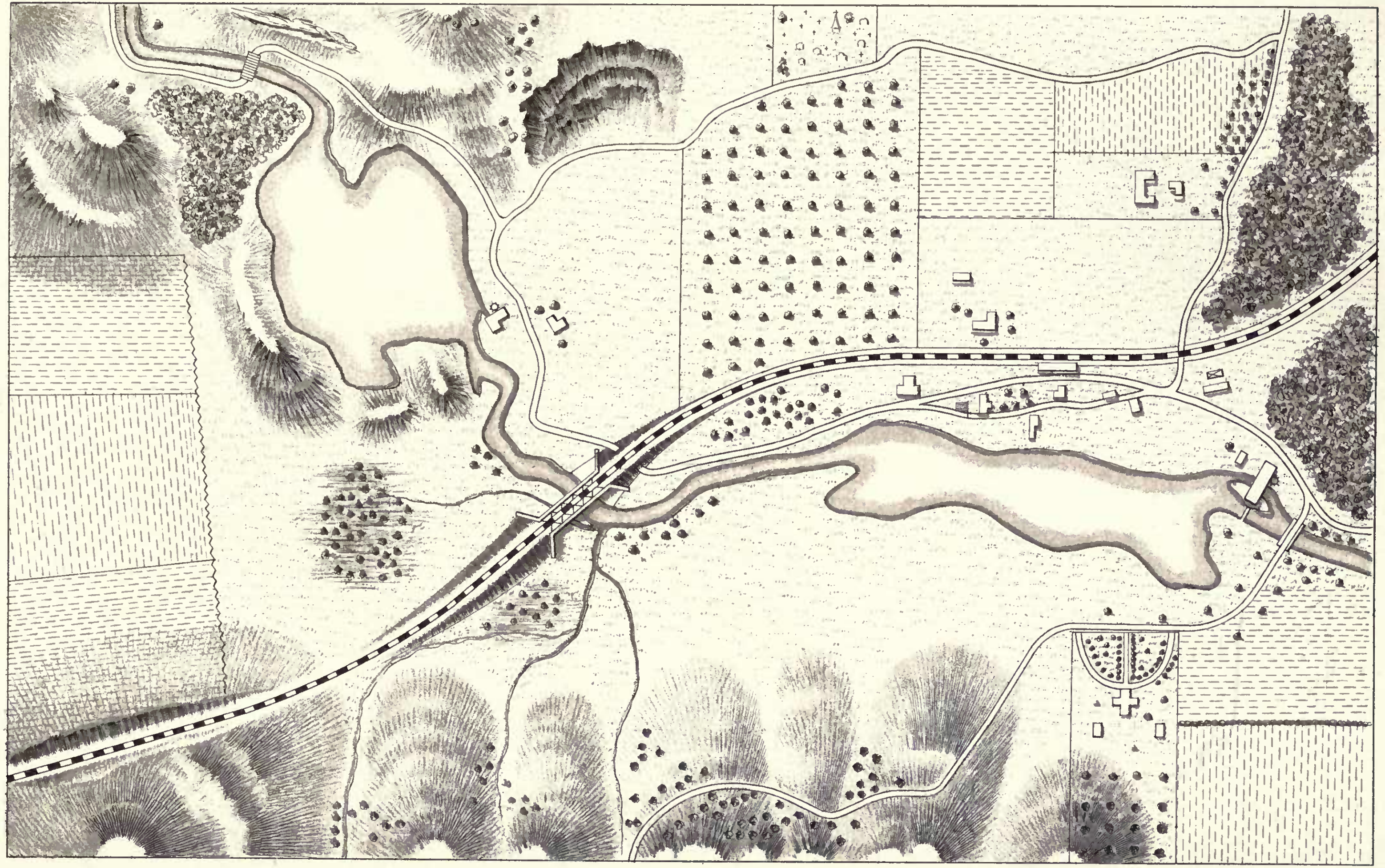
PL.V.

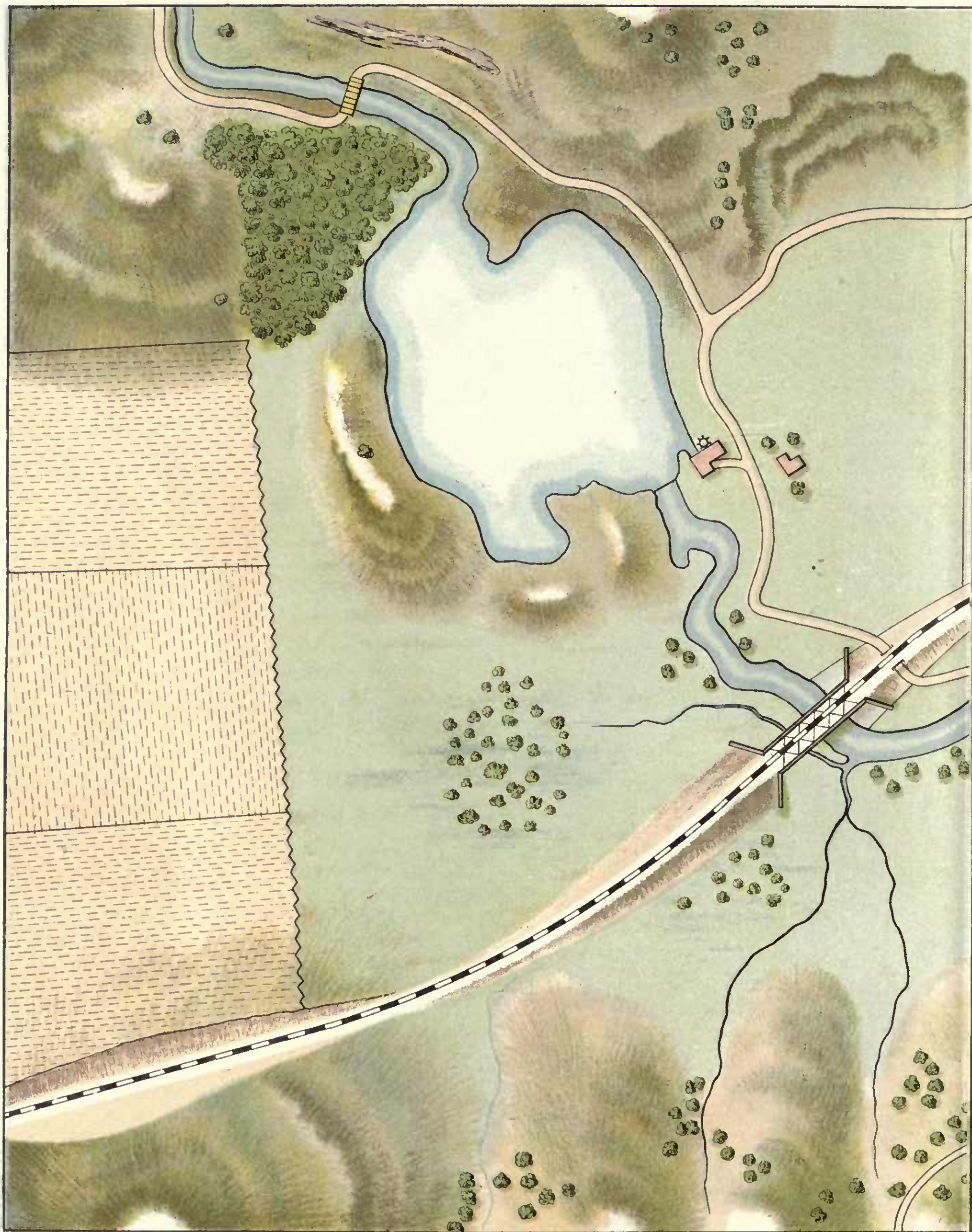


PL.VI.

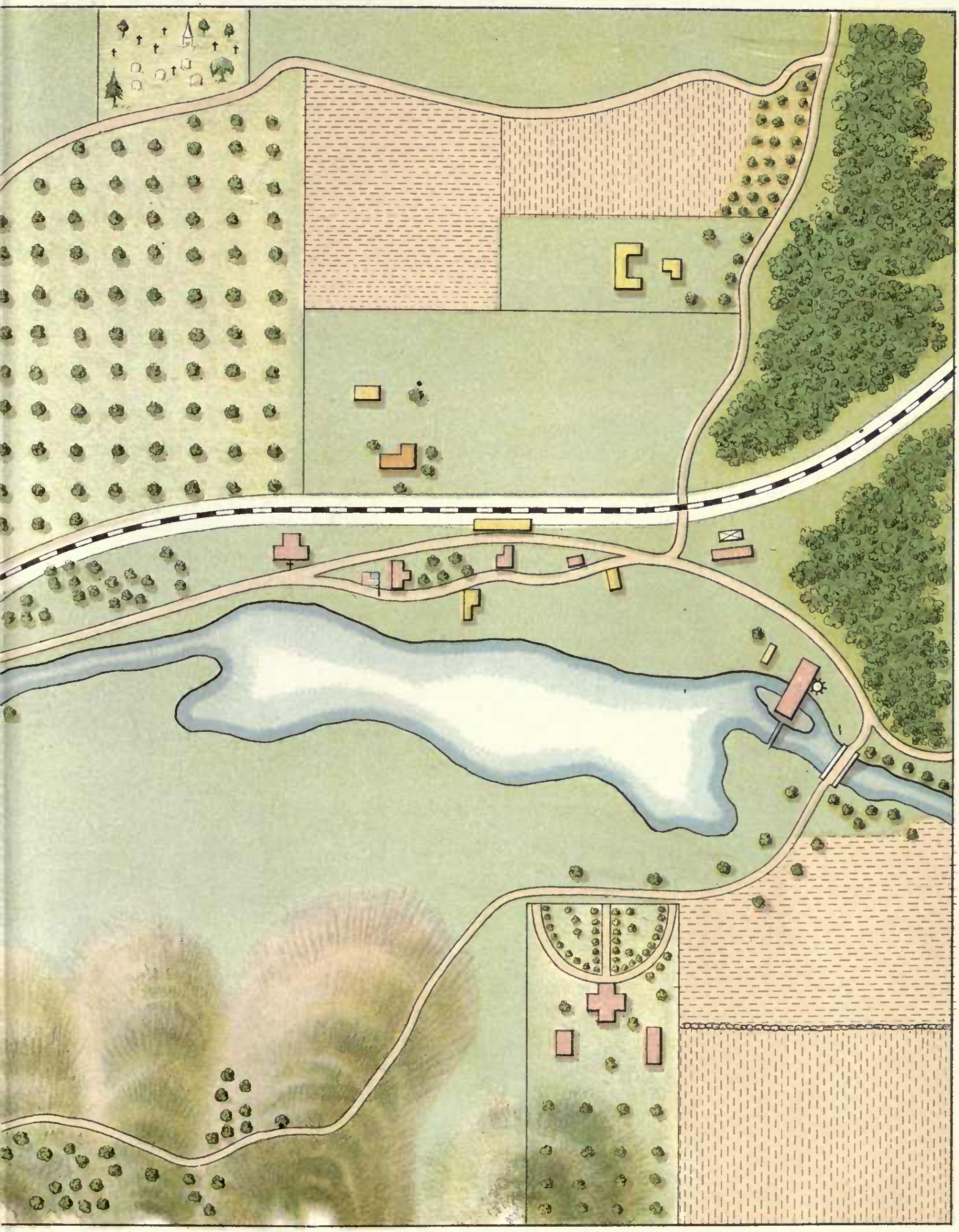


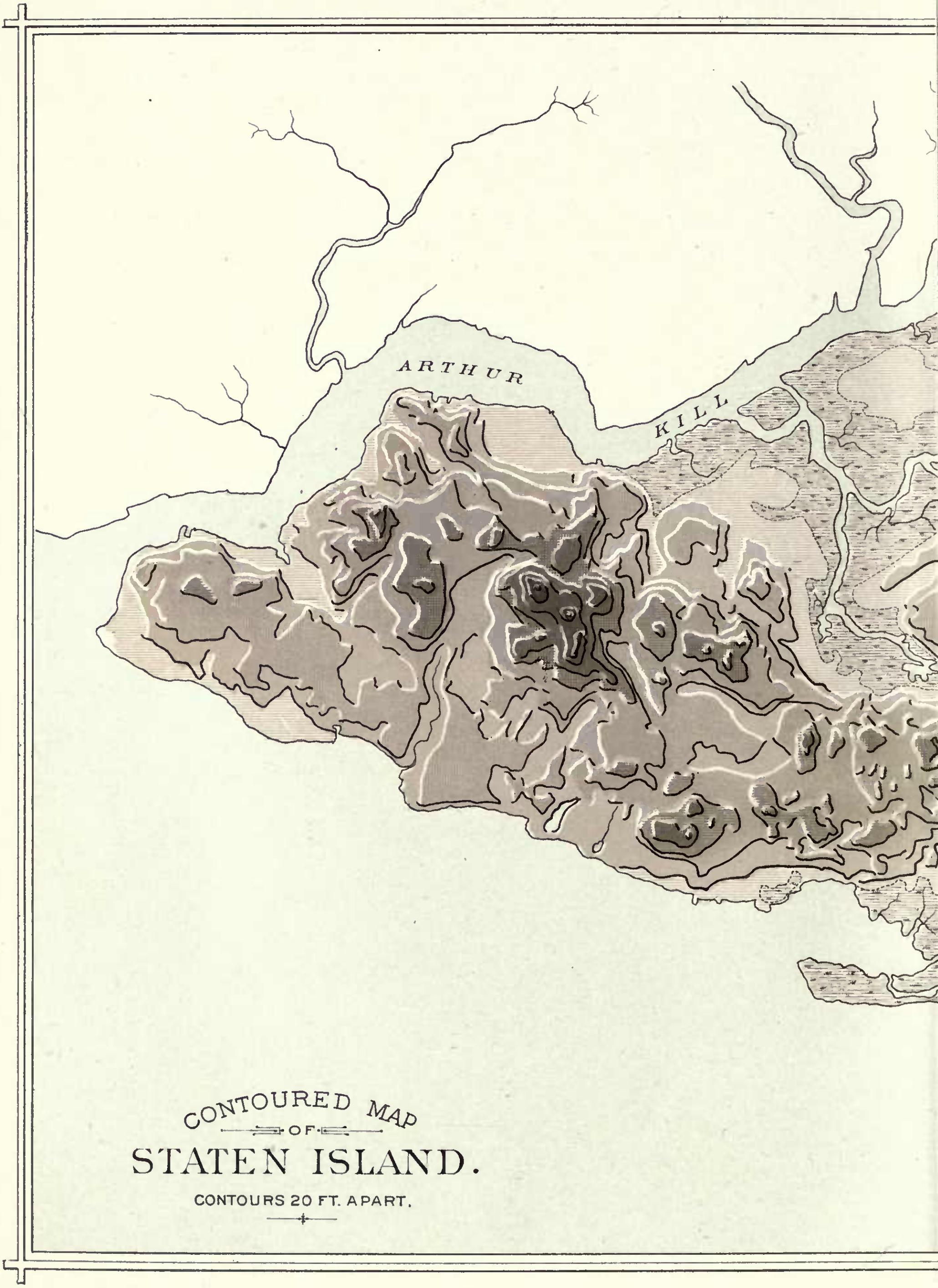
PL.VII.





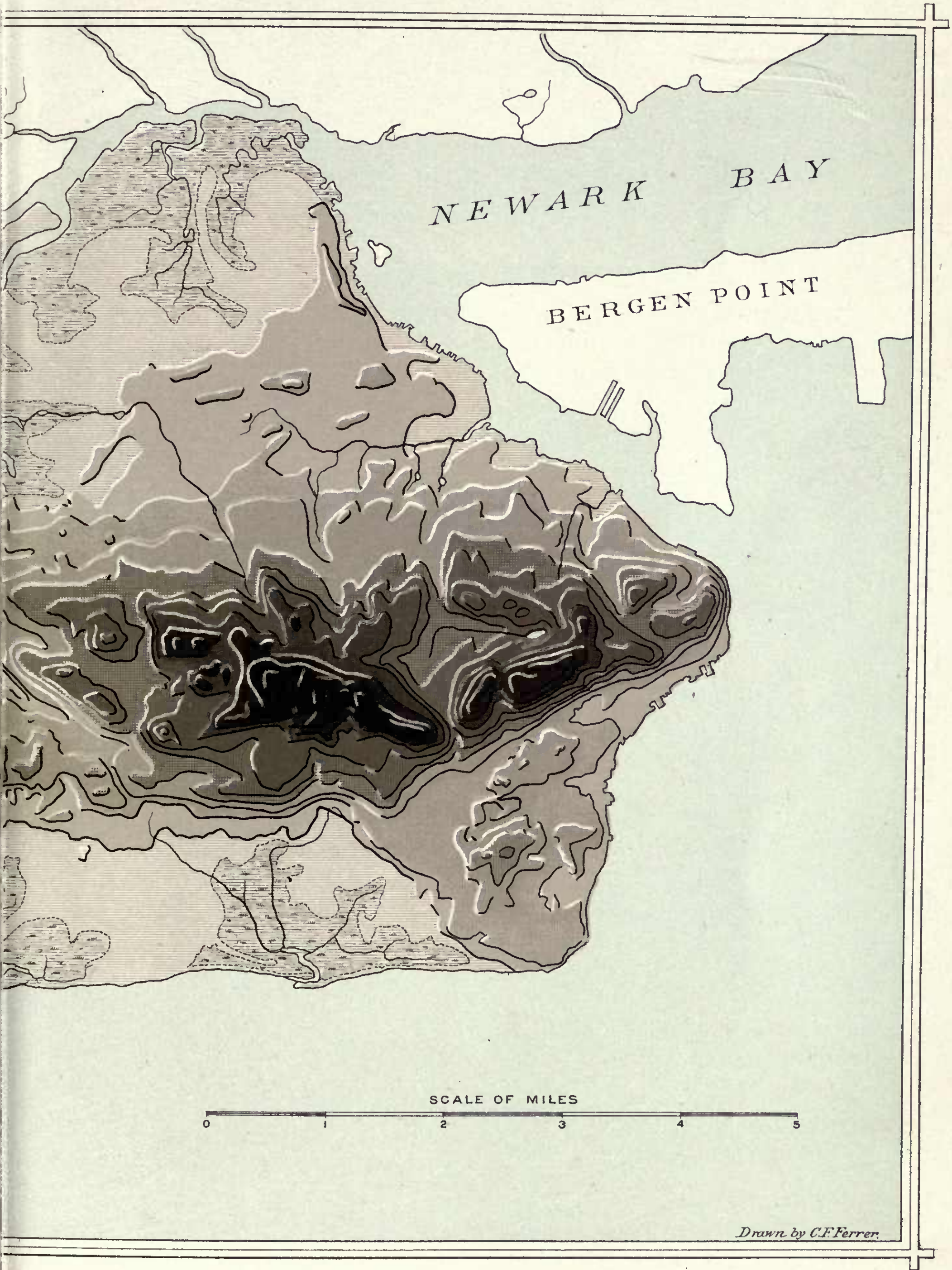
III.





CONTOURED MAP
OF
STATEN ISLAND.
CONTOURS 20 FT. APART.

IX.

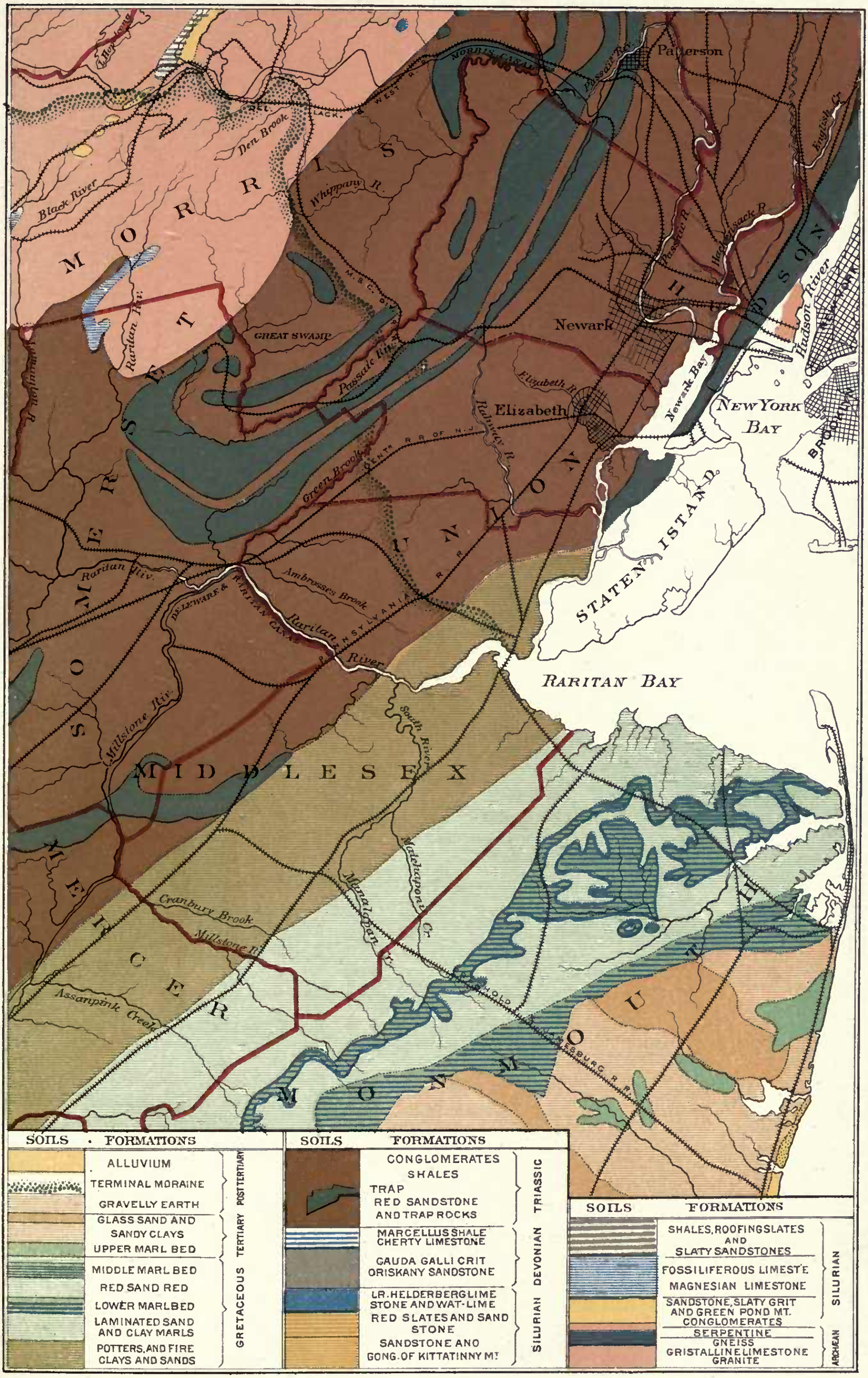


NEWARK BAY
BERGEN POINT
SCALE OF MILES
0
1
2
3
4
5
Drawn by C.F.Ferrer.

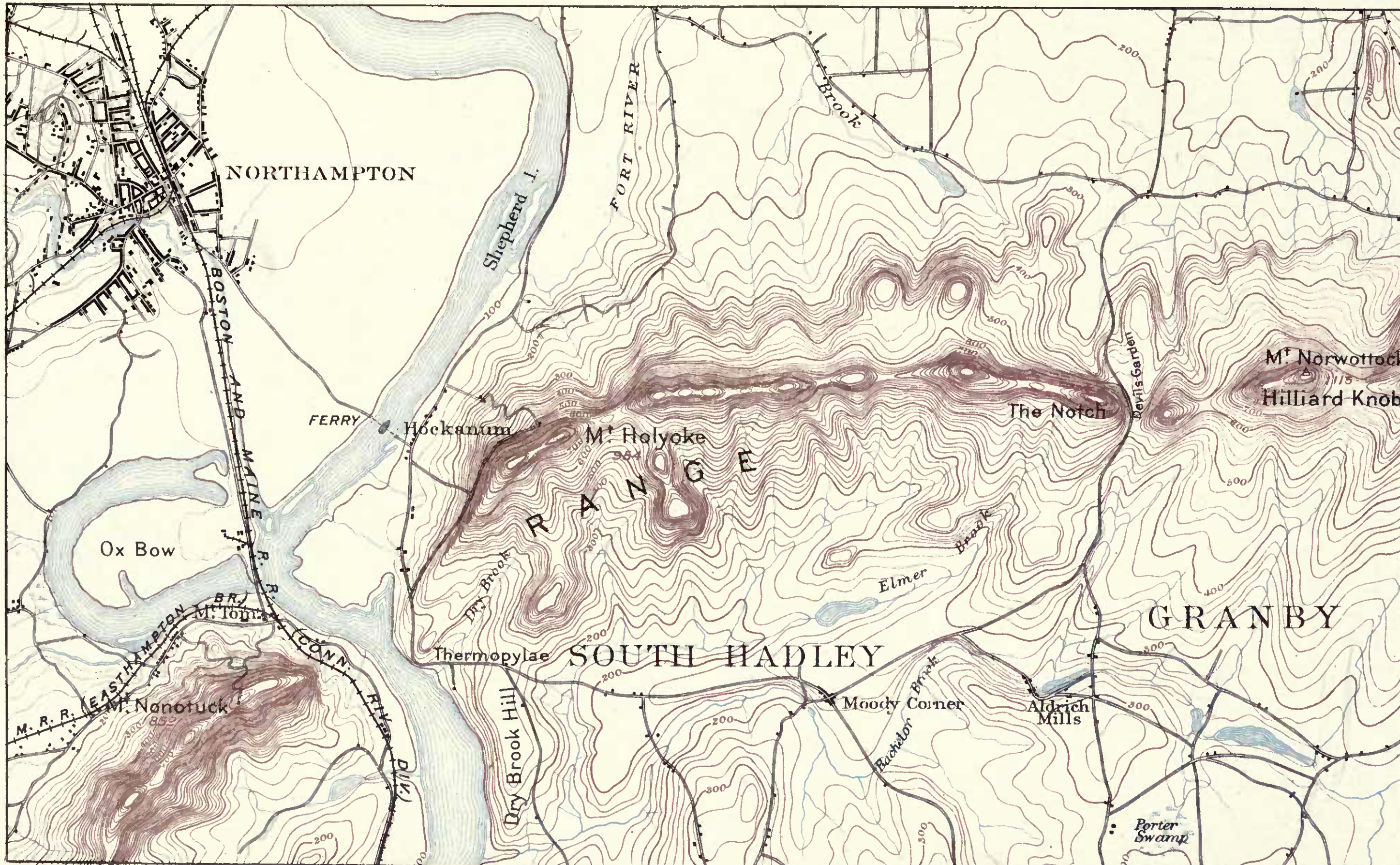
PL.X.

GEOLOGICAL MAP OF
NEW JERSEY

GEORGE.H.COOK,STATE GEOLOGIST.



PL. XI.

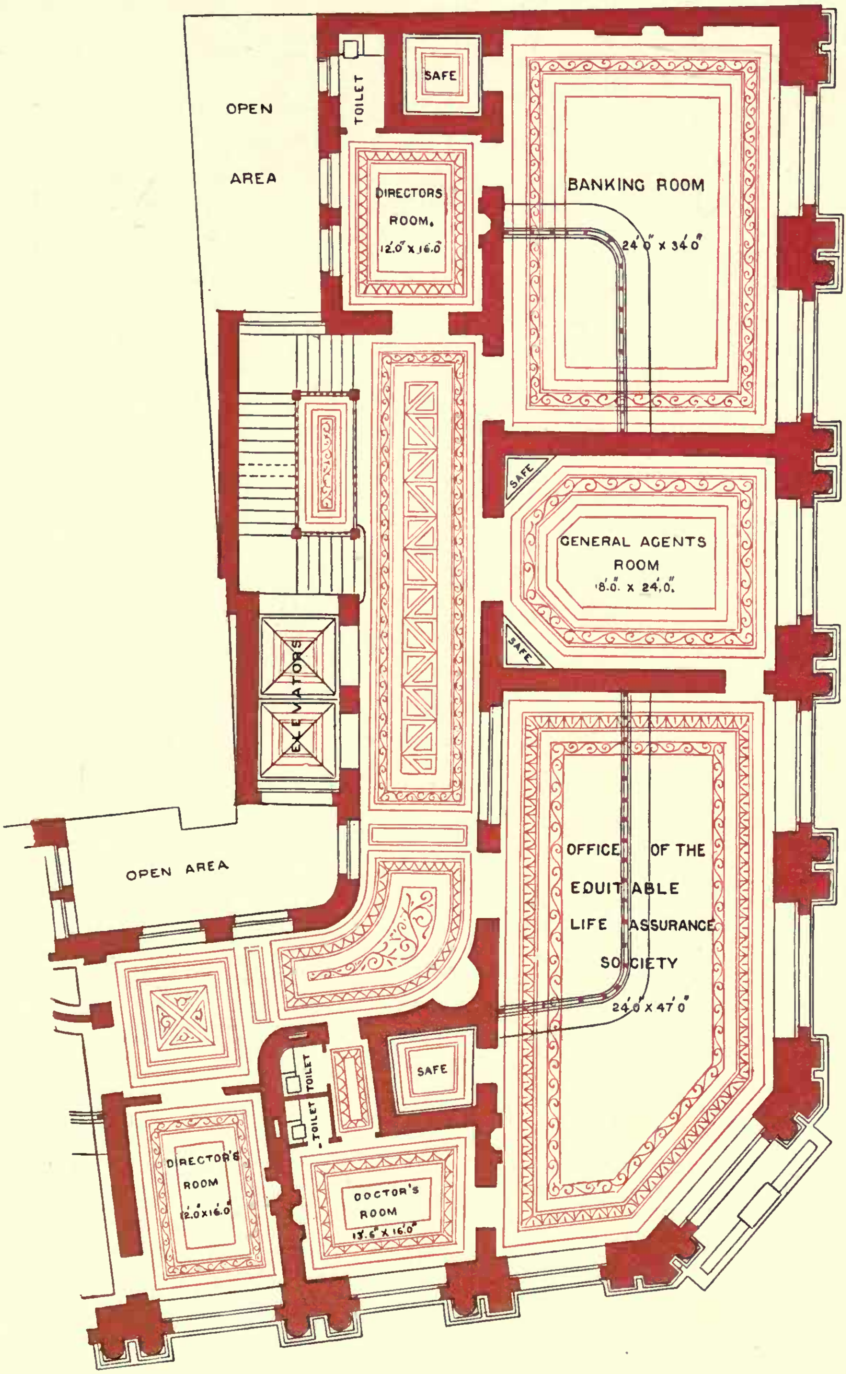


"Topographical Map of Massachusetts."

PL. XII.

"Topographical Map of Massachusetts."

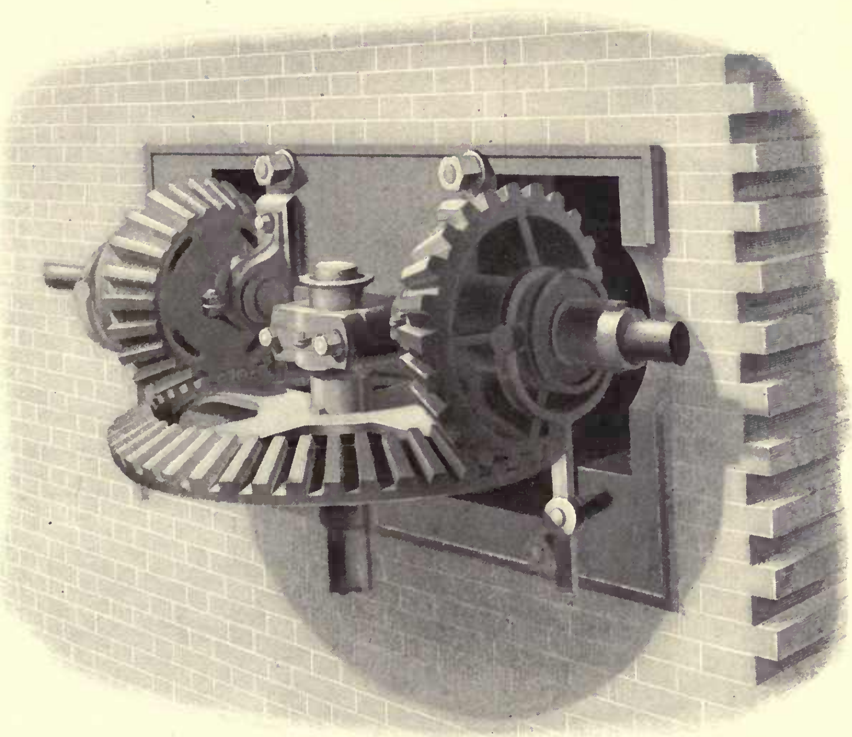
PL. XIII.



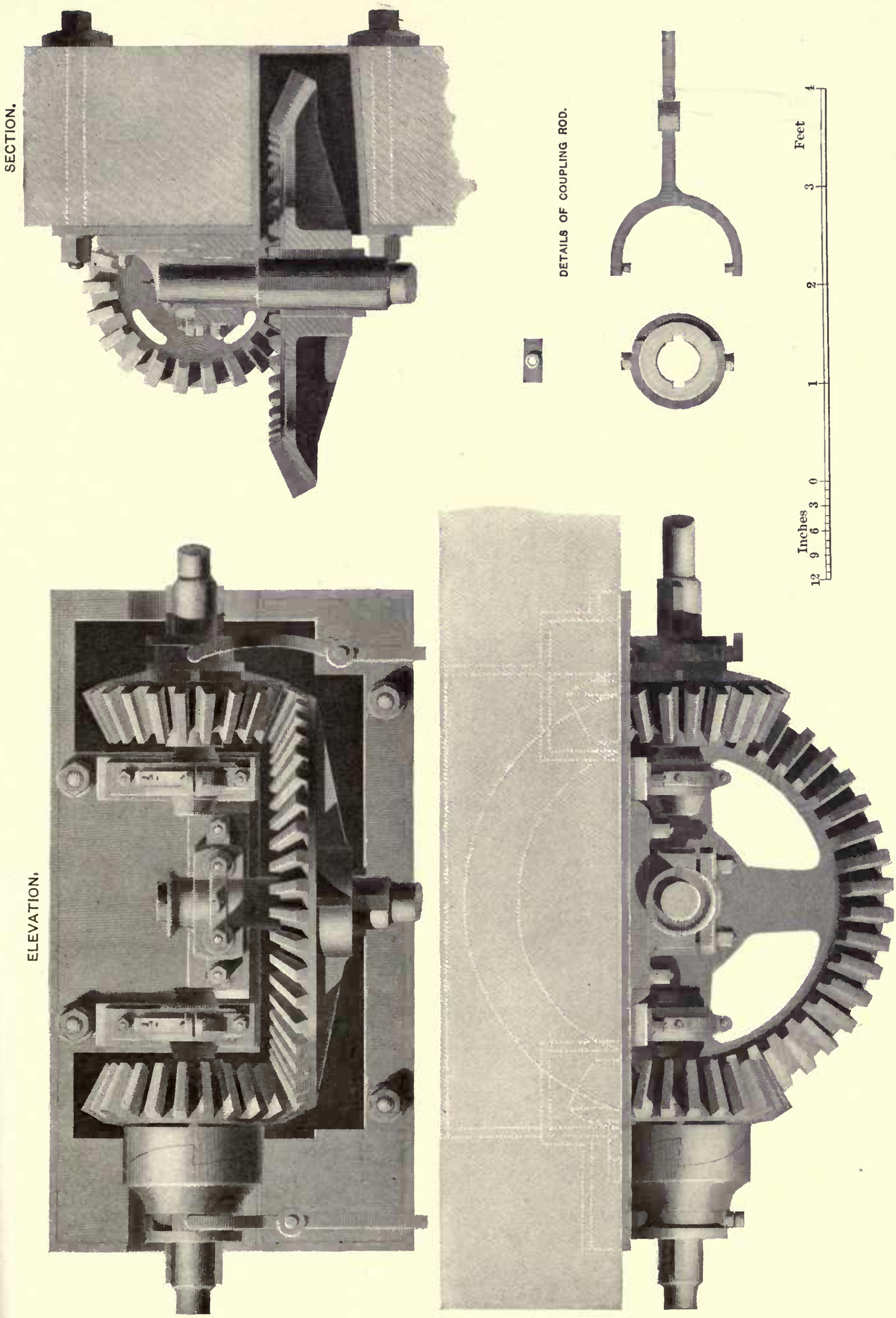
OPEN
AREA
TOILET
SAFE
BANKING ROOM
24' 0" x 34' 0"
DIRECTORS
ROOM.
12.0" x 16.0"
SAFE
GENERAL AGENTS
ROOM
18.0" x 24.0"
SAFE
ELEVATORS
OPEN AREA
OFFICE OF THE
EQUITABLE
LIFE ASSURANCE
SOCIETY
24' 0" x 47' 0"
TOILET
TOILET
SAFE
DIRECTOR'S
ROOM
12.0 x 16.0
DOCTOR'S
ROOM
13.6" x 16.0"

PL.XV.

PL.XVI.



PL.XVII.


SECTION.
DETAILS OF COUPLING ROD.
ELEVATION.
PLAN.
Feet
Inches
12 9 6 3 0 1 2 3 4

APPENDIX.

PATENT–OFFICE DRAWINGS

must be made upon pure white paper, of a thickness corresponding to three-sheet Bristol board, with surface calendered and smooth. India ink alone must be used.

The size of the sheet must be exactly 10 by 15 inches. 1″ from its edges single marginal lines are to be drawn, leaving the "sight" precisely 8″ by 13″. Within this margin all work must be included. Measuring downward from the marginal line of one of the shorter sides, a space of not less than 1¼ inch is to be left blank for the heading of title, name, number, and date.

All drawings must be made with the pen only. All lines and letters must be absolutely black, clean, sharp, and solid, and not too fine or crowded. Surface shading should be open, and used only on convex and concave surfaces sparingly. Sectional shading should be made by oblique parallel lines, which may be about $\frac{1}{20}$″ apart.

Drawings should be made with the fewest lines possible consistent with clearness. The plane upon sectional views should be indicated on the general view by broken or dotted lines. Heavy lines on the shade sides of objects should be used, except where they tend to thicken the work and obscure letters of reference: light to come from the upper left-hand corner, at an angle of 45°.

The scale of the drawing to be large enough to show the mechanism without crowding; but the number of sheets must never be increased unless it is absolutely necessary.

Letters and figures of reference must be carefully formed, and, if possible, measure at least ⅛″ in height, and so placed as not to interfere with a thorough comprehension of the drawing, and therefore should rarely cross the lines. Upon shaded surfaces a blank space must be left in the shading for the letter. The same part of an invention must always be represented by the same character, and the same character must never be used to designate different parts.

The signature of the inventor, by himself or by his attorney, is to be placed at the lower right-hand corner of the sheet, and the signature of two witnesses at the lower left-hand corner, all within the marginal line. The title is to be written with pencil on the back of the sheet. The permanent names and title will be supplied subsequently by the office in uniform style.

Drawings should not be folded for transmission to the office.

REGISTRATION OF PRINTS AND LABELS.

A label is a device or representation borne by an article of manufacture or vendible commodity. A print is a device or representation not borne by an article of manufacture or vendible commodity, but in some fashion pertaining thereto. A label can not be registered if it bear a device capable of sequestration as a trade-mark until after such device is registered as a trade-mark. Both labels and prints, in order to be entitled to registry, must be intellectual productions in the degree required by the copyright law.

MENSURATION.

The principles of measurement have been quite fully explained under the heads of the Construction of Geometrical Problems and Plotting, but for ready reference there are many rules which are of general application and very necessary in designing and calculation, and are given briefly as follows:

To find

The area of a parallelogram. Multiply the length by the height or perpendicular by the breadth. Multiply the product of two contiguous sides by the natural sine of the included angle. (See Appendix, Table of Natural Sines.)

The area of a triangle. Multiply the base by the perpendicular height and take half the product. Multiply half the product of two contiguous sides by the natural sine of the included angle.

The area of a trapezoid. Multiply half the sum of the parallel sides by the perpendicular distance between them.

The area of any quadrilateral figure. Divide the quadrilateral into two triangles; the sum of the areas of the triangles is the area.

The area of any polygon. Divide the polygon into triangles and take the sum of their areas.

The circumference of a circle. Multiply the diameter by $3 \cdot 1416 = \pi$.

The diameter of a circle. Multiply the circumference by the reciprocal of π.

The area of a circle. Multiply the square of the diameter by ·7854, or the circumference by one fourth of the diameter.

The length of an arc of a circle. Multiply the number of degrees in the arc by the radius, and by ·01745.

NOTE.—The length of an arc of one degree = radius × ·017453.
" " " " " " " minute = " × ·000291.
" " " " " " " second = " × ·000005.

The area of a sector of a circle. Multiply the length of the arc of the sector by half the radius.

The area of a segment of a circle. From the area of a sector subtract the area of the triangle formed by the radial sides of the sector and its chord. The area of this triangle is the product of the natural sine and cosine by the square of the radial side.

The area of regular polygons. Find the area of one triangle, and multiply by the number of triangles composing the polygon:

Or, multiply the total of cosines for the periphery by one half the sine, by the square of the radius for the area.

	½ Chord.		No. of Sides.		Vertical.		Product.
Pentagon	·5878	×	5	×	·8090	=	2·3776
Hexagon	·5000	×	6	×	·8660	=	2·5980
Heptagon	·4357	×	7	×	·9032	=	2·7478
Octagon	·3827	×	8	×	·9238	=	2·8284
Nonagon	·3420	×	9	×	·9396	=	2·8925
Decagon	·3090	×	10	×	·9510	=	2·9389
Undecagon	·2817	×	11	×	·9594	=	2·9739
Dodecagon	·2588	×	12	×	·9660	=	3·0000
Circle						= π =	3·1416

To find

The area of the cycloid. Multiply the area of the generating circle by 3.

To find

The area of the parabola. Multiply the base by the height; two thirds of the product is the area.

The circumference of an ellipse. Multiply the square root of half the sum of the squares of the two axes by 3·1416.

The area of an ellipse. Multiply the product of the two axes by $\cdot 7854 = \frac{1}{4}\pi$.

NOTE.—The area of an ellipse is equal to the area of a circle of which the diameter is a mean proportional between the two axes.

To find the area of any curvilineal figure, bounded at the ends by parallel straight lines (Fig. 198). Divide the length of the figure into any number of equal parts and draw ordinates through the points of division, to touch the boundary lines. Or, add together the first and last ordinates, making the sum A; and add together all the intermediate ordinates, making the sum B. Let L = the length of the figure, and n = the number of divisions, then

$$\frac{A + 2B}{2n} \times L = \text{area of figure.}$$

That is to say, twice the sum of the intermediate ordinates, plus the first and last ordinates, divided by twice the number of divisions, and multiplied by the length, is equal to the area of the figure. This method is sufficiently exact for most purposes.

To find

The surface of a prism or cylinder. To the product of the perimeter of the end by the height, add twice the area of an end.

The cubic contents of a prism or a cylinder. Multiply the area of the base by the height.

The surface of a pyramid or cone. Multiply the perimeter of the base by half the slant height, and add the area of the base.

The cubic contents of a pyramid or a cone. Multiply the area of the base by one third of the perpendicular height.

The surface of a frustum of a pyramid or a cone. Multiply the sum of the perimeters of the ends by half the slant height, and add the areas of the ends.

The cubic contents of the frustum of a pyramid or a cone. Add together the areas of the two ends and the mean proportional between them (that is, the square root of their product) and multiply the sum by one third of the perpendicular height.

The cubic contents of a wedge. To twice the length of the base add the length of the edge; multiply the sum by the breadth of the base, and by one sixth of the height.

The cubic contents of a prismoid (a solid of which the two ends are unequal but parallel plane figures of the same number of sides). To the sum of the area of the two ends add four times the area of a section parallel to and equally distant from both ends; and multiply the sum by one sixth of the length.

The surface of a sphere. Multiply the square of the diameter by 3·1416.

The surface of a sphere is equal to four times the area of one of its great circles.

The surface of a sphere is equal to the convex surface of its circumscribing cylinder.

The surfaces of spheres are to one another as the squares of their diameters.

The curve surface of any segment or zone of a sphere. Multiply the diameter of the sphere by the height of the zone or segment, and by 3·1416.

The cubic contents of a sphere. Multiply the cube of the diameter by $\cdot 5236 = \frac{1}{6}\pi$.

The cubic contents of the segment of a sphere. From three times the diameter of the sphere subtract twice the height of the segment; multiply the difference by the square of the height, and by ·5236.

The cubic contents of a frustum or zone of a sphere. To the sum of the squares of the radii of the ends add $\frac{1}{3}$ of the square of the height; multiply the sum by the height and by $1 \cdot 5708 = \frac{1}{2}\pi$.

The cubic contents of a spheroid. (A solid body generated by the revolution of an ellipse around one of its axes.) Multiply the square of the revolving axis by the fixed axis and by ·5236.

LINEAL MEASURE.

Inches.	Feet.	Yards.	Fathoms.	Links.	Rods.	Chains.	Furlongs	Statute miles.	Nautical miles.	Metres.
1 =	·08333	·02778	·0139	·126	·005	·00126	·000126	·000016	....	·0254
12 =	1	·333	·1667	1·515	·0606	·0151	·00151	·00019	....	0·3048
36 =	3	1	·5	4·545	·182	·0454	·00454	·00057	....	0·9144
72 =	6	2	1	9·1	·364	·091	·0091	·00114	....	1·8289
7·92 =	0·66	·22	·11	1	·04	·01	·001	·000125	....	·2012
198 =	16½	5½	2¾	25	1	·25	·025	·003125	....	5·0294
792 =	66	22	11	100	4	1	·10	·0125	....	20·118
7920 =	660	220	110	1000	40	10	1	·125	....	201·18
63360 =	5280	1760	880	8000	320	80	8	1	0·86755	1609·41
	6086·07	2028·69		....	....	....	..	1·1527	1	1855·11
39·3685 =	3·2807	1·0936	·5468	....	....	....	..	0·000621	...	1

Latin prefixes, as milli-, centi-, deci-, to the French units of length (metre), surface (are), weight (gramme), or volume (litre), signify $^1/_{1000}$, $^1/_{100}$, or $^1/_{10}$ of the unit; as, millimetre, $^1/_{1000}$ of a metre, decigramme, $^1/_{10}$ of a gramme. Greek prefixes, as kilo, hekto, deka, multiples of the unit by 1,000, 100, or 10, as kilometre = 1000 metres.

MEASURES OF SURFACE.

Sq. inches.	Sq. feet.	Sq. yards.	Sq. rods.	Roods.	Acres.	Sq. miles.	Sq. metres.	Ares.
1 =	·00694	....	....	....	....	....	....	....
144 =	1	·111	·0037	....	....	....	·0929	·0009
1296 =	9	1	·033	....	....	....	·8361	·0084
....	272¼	30¼	1	·025	·00625	....	25·293	0·253
....	10890	1210	40	1	·25	....	....	....
....	43560	4840	160	4	1	·00156	4046·86	40·47
....	27878400	3097600	....	....	640	1	....	25899
1549·8 =	10·763	1·196	·0395	·0009	·000247	....	1	·01
....	1076·31	119·60	....	....	·02471	....	100	1

BOARD AND TIMBER MEASURE.

In board measure boards are assumed at one inch in thickness. To obtain the number of feet, board measure (B. M.), of a board or stick of square timber, multiply together the length in feet, the breadth in feet, and the thickness in inches.

To Compute the Volume of Round Timber.—When all dimensions are in feet, multiply the length by one quarter of the product of the mean girth and diameter, and the product will give the measurement in cubic feet.

For Square Timber.—When all dimensions are in feet, multiply together the length, breadth, and depth: the product will be the volume in cubic feet. When one dimension is given in inches, divide by 12; when two dimensions are given in inches, divide by 144; when all three dimensions are given in inches, divide by 1728.

MEASURES OF CAPACITY.

LIQUID MEASURE.

Gills.	Pints.	Quarts.	Gallons.	Imp. gallons.	Litres.	Cubic feet.	Cubic in.	Lbs. water at 62°.
1 =	0·25	0·125	·03125	·026	·1183	·0042	7·219	·26
4 =	1	0·5	0·125	·1041	·4731	·01671	28·875	1·0412
8 =	2	1	0·25	·2083	0·9463	·03342	57·75	2·0825
32 =	8	4	1	0·8331	3·7852	0·1337	231	8·33
38·4096 =	9·6024	4·8012	1·2003	1	4·5435	0·1605	277·27	10·00
8·4534 =	2·1133	1·0567	0·26417	0·2201	1	0·0353	61·0279	2·2007
239·36 =	59·84	29·92	7·48	6·232	28·320	1	1728	62·321
·138528 =	·034632	·017316	·004329	·0036	0·01639	0·000579	1	·03606
....	....	......	......	....	......	·01604	27·727	1

1 barrel = 31½ gallons. Reciprocal = ·03175.

DRY MEASURE.

Pints.	Quarts.	Gallons.	Pecks.	Bushels.
1 =	0·50	0·125	·0625	0·01562
2 =	1	0·25	0·125	0·0312
8 =	4	1	0·50	0·125
16 =	8	2	1	0·25
64 =	32	8	4	1

The standard bushel contains 2150·42 cubic inches.

WEIGHTS.

APOTHECARIES'.

Grains.	Scruples.	Drachms.	Ounces.	Pounds.
1 =	·05	·0167	·0021	·00018
20 =	1	·333	·042	·0035
60 =	3	1	·125	·0104
480 =	24	8	1	·083
5760 =	288	96	12	1

TROY.

Grains.	Pennyweights.	Ounces.	Pounds.
1 =	·042	·0021	·00018
24 =	1	·05	·0042
480 =	20	1	·083
5760 =	240	12	1

AVOIRDUPOIS.

Drachms.	Ounces.	Pounds.	Hundred-weights.	Tons.	French grammes.
1 =	·0625	·0039	·000035	·00000174	1·771836
16 =	1	·0625	·000558	·000028	28·34938
256 =	16	1	·00893	·000446	453·59
28672 =	1792	112	1	·05	50802·
573440 =	35840	2240	20	1	1016041·6

It is common usage here to omit hundred-weights (cwt.) and rate tons at 2,000 pounds as net, and 2240 lbs. as gross; 7,000 troy or apothecaries' weight equal 1 pound avoirdupois.

COMPARISON OF WEIGHT.

Pounds apothecaries'.	Pounds Troy.	Pounds avoirdupois.	Kilo-gramme.
1 =	1	0·8229	0·37324
1 =	1	0·8229	0·37324
1·2153 =	1·2153	1	0·4536
2·6792 =	2·6792	2·2046	1

DYNAMIC TABLE.

Pounds, feet.	Kilogramme-metre.	Horse-power.	French horse-power.
1 =	0·13825	·00003	·000031
7·2331 =	1	·000219	·000222
Per min. 33·000 =	4562·3	1	1·01386
32548·9 =	4500	0·98633	1

CUBIC OR SOLID MEASURE.

Cubic inches.	Cubic feet.	Cubic yards.	Cubic metres.	United States gallon.
1 =	·00058	·000021	·000016	·004329
1728 =	1	0·037	0·0283	7·48
46656 =	27	1	0·7646	201·97
61016 =	35·31	1·3078	1	264·141
231 =	0·1337	·00495	·00379	1

SHIPPING MEASURE.

Register Ton.—For register tonnage or for measurement of the entire internal capacity of a vessel:

100 cubic feet = 1 register ton.

Shipping Ton.—For the measurement of cargo:

40 cubic feet = { 1 U. S. shipping ton. / 31·16 Imp. bushels. / 32·143 U. S. "

42 cubic feet = { 1 British shipping ton. / 32·719 Imp. bushels. / 33·75 U. S. "

Carpenter's Rule.—Weight a vessel will carry = length of keel × breadth at main beam × depth of hold in feet ÷ 95 (the cu. ft. per ton). The result will be the tonnage. For a double-decker, instead of the depth of the hold take half the breadth of the beam.

TABLE OF INCHES AND SIXTEENTHS IN DECIMALS OF A FOOT.

Inches.		1/16	2/16	3/16	4/16	5/16	6/16	7/16	8/16	9/16	10/16	11/16	12/16	13/16	14/16	15/16
0....	·000	·005	·010	·016	·021	·026	·031	·036	·042	·047	·052	·057	·062	·068	·073	·078
1....	·083	·089	·094	·099	·104	·109	·115	·120	·125	·130	·135	·141	·146	·151	·156	·161
2....	·167	·172	·177	·182	·187	·193	·198	·203	·208	·214	·219	·224	·229	·234	·240	·245
3....	·250	·255	·260	·266	·271	·276	·281	·286	·292	·297	·302	·307	·312	·318	·323	·328
4....	·333	·339	·344	·349	·354	·359	·365	·370	·375	·380	·385	·391	·396	·401	·406	·411
5....	·417	·422	·427	·432	·437	·443	·448	·453	·458	·464	·469	·474	·479	·484	·490	·495
6....	·500	·505	·510	·516	·521	·526	·531	·536	·542	·547	·552	·557	·562	·568	·573	·578
7....	·583	·589	·594	·599	·604	·609	·615	·620	·625	·630	·635	·641	·646	·651	·656	·661
8....	·667	·672	·677	·682	·687	·693	·698	·703	·708	·714	·719	·724	·729	·734	·740	·745
9....	·750	·755	·760	·766	·771	·776	·781	·786	·792	·797	·802	·807	·812	·818	·823	·828
10....	·833	·839	·844	·849	·854	·859	·865	·870	·875	·880	·885	·891	·896	·901	·906	·911
11....	·917	·922	·927	·932	·937	·943	·948	·953	·958	·964	·969	·974	·979	·984	·990	·995

ELECTRICAL UNITS.

C. G. S.

Unit of space, 1 centimetre, C. ; of mass, 1 gramme, G. ; of time, 1 second, S.

The definitions of units as adopted at the International Electrical Congress at Chicago in 1893, established by Act of Congress of the United States, July 12, 1894, are as follows:

The *ohm* (the unit of resistance, represented by R) is equal to 10^9 (or 1,000,000,000) units of resistance of the C. G. S. system, and is represented by the resistance offered to an unvarying electrical current by a column of mercury at 32° Fahr. (14·4521 grammes in mass) of a constant cross-sectional area, and of the length of 106·3 centimetres.

The *ampère* (the unit of current strength, or rate of flow, represented by C) is one tenth of the unit of current of the C. G. S. system, and is the equivalent of the unvarying current which, when passed through a solution of nitrate of silver in water in accordance with standard specifications, deposits silver at the rate of ·001118 gramme per second.

The *volt* (the unit of electro-motive force, or difference of potential, represented by E) is the electro-motive force that, steadily applied to a conductor whose resistance is 1 ohm, will produce a current of one ampère, and is equivalent to $\frac{1000}{1434}$ (or ·6974) of the electro-motive force between the poles or electrodes of a Clark's cell at a temperature of 15° C., and prepared in the manner described in the standard specifications.

The *coulomb* (or ampère-second, the unit of quantity, Q) is the quantity of electricity transferred by a current of one ampère in one second.

The *farad* (the unit of capacity represented by K) is the capacity of a condenser charged to a potential of one volt by one coulomb of electricity.

The *joule* (volt-coulomb, the unit of energy or work, W) is equal to 10,000,000 units of work in the C. G. S. system, and is practically equivalent to the energy expended in one second by an ampère in one ohm.

The *watt*, or ampère-volt (the unit of power, P), is equal to 10,000,000 units of power in the C. G. S. system, and is practically equivalent to the work done at the rate of one joule per second; 746 watts = 1 H. P.

The *henry* (the unit of induction) is the induction in a circuit when the electro-motive force induced in this circuit is one volt, while the inducing current varies at the rate of one ampère per second.

The *ohm*, volt, etc., as above defined, are called the "international" ohm, volt, etc., to distinguish them from the "legal" ohm, B. A. unit, etc.

The value of the ohm, determined by a committee of the British Association in 1863, called the B. A. unit, was the resistance of a certain piece of copper wire preserved in London. The so-called "legal" ohm as adopted by the International Congress of Electricians in Paris in 1884, was a correction of the B. A. unit, and was defined as the resistance of a column of mercury 1 square millimetre in section and 106 centimetres long, at a temperature of 32° F.

1 legal ohm	= 1·0112 B. A. units,	1 B. A. unit	= 0·9889 legal ohm.
1 international ohm	= 1·0136 " "	1 " "	= 0·9866 int. "
1 " "	= 1·0023 legal ohm,	1 legal ohm	= 0·9977 " "

UNIT OF HEAT.

The British Thermal Unit (B. T. U.) is that quantity of heat required to raise the temp. of 1 pound of pure water 1° F. at or near 39·1° F., the maximum density of water.

The French thermal unit, or calorie, is that quantity of heat which is required to raise the temperature of 1 kilogramme of pure water 1° C., at or about 4° C., which is equivalent to 39·1° F.

1 French calorie = 3·968 British thermal units, 1 B. T. U. = ·252 calorie.

FIFTH POWERS, TABLE OF.

	Fifth Power.		Fifth Power.		Fifth Power.		Fifth Power.
11	1 61051	33	391 35393	55	5032 84375	77	27067 84157
12	2 48832	34	454 35424	56	5507 31776	78	28871 74368
13	3 71293	35	525 21875	57	6016 92057	79	30770 56399
14	5 37824	36	604 66176	58	6563 56768	80	32768 00000
15	7 59375	37	693 43957	59	7149 24299	81	34867 84401
16	10 48576	38	792 35168	60	7776 00000	82	37073 98439
17	14 19857	39	902 24199	61	8445 96301	83	39390 40643
18	18 89568	40	1024 00000	62	9161 32832	84	41821 19424
19	24 76099	41	1158 56201	63	9924 36543	85	44370 53125
20	32 00000	42	1306 91232	64	10737 41824	86	47042 70176
21	40 84101	43	1470 08443	65	11602 90625	87	49842 09207
22	51 53632	44	1649 16224	66	12523 32576	88	52773 19168
23	64 36343	45	1845 28125	67	13501 25107	89	55840 59449
24	79 62624	46	2059 62976	68	14539 33568	90	59049 00000
25	97 65625	47	2293 45007	69	15640 31349	91	62403 21451
26	118 81376	48	2548 03968	70	16807 00000	92	65908 15232
27	143 48907	49	2824 75249	71	18042 29351	93	69568 83693
28	172 10368	50	3125 00000	72	19349 17632	94	73390 40224
29	205 11149	51	3450 25251	73	20730 71593	95	77378 09375
30	243 00000	52	3802 04032	74	22190 06624	96	81537 26976
31	286 29151	53	4181 95493	75	23730 46875	97	85873 40257
32	335 54432	54	4591 65024	76	25355 25376	99	95099 00499

CAST-IRON BALLS, VOLUME AND WEIGHT OF.

Diameter.	Volume.	Weight.	Diameter.	Volume.	Weight.	Diameter.	Volume.	Weight.
Inches.	Cu. inches.	Pounds.	Inches.	Cu. inches.	Pounds.	Inches.	Cu. inches.	Pounds.
2	4·19	1·09	5½	87·1	22·7	9	381·7	99·4
2½	8·18	2·13	6	113·1	29·5	9½	448 9	116·9
3	14·1	3·68	6½	143·8	37·5	10	523·6	136·4
3½	22·5	5·85	7	179·6	46·8	11	696·9	181·8
4	33·5	8·73	7½	220·9	57·5	12	904·8	235·9
4½	47·7	12·4	8	268·1	69·8	13	1150·3	299·6
5	65·5	17·0	8½	321·5	83·7	14	1436·8	374·6

Weight for other metals vary as their specific gravities and for diameters as their cubes.

CAST-IRON PIPES, STANDARD WEIGHTS OF.

Size of Pipe.	Thickness of Metal.	Weight of Cylinder 1 foot in length.	Weight of 1 foot of Pipe, laid, including bells.	Weight of Pipe to lay 12 feet, including bells.	Ultimate strength of Pipe, made of 18 000 lb. iron.	One-fifth ultimate strength of pipe.	Capacity.
Inches.	Inches.	Pounds.	Pounds.	Pounds.	Pounds.	Pounds.	U. S. gal.
4	0·40	17·27	18·75	225	3600	720	·6528
6	0·42	26·46	28·92	347	2515	503	1·469
8	0·45	37·33	40·50	486	2025	405	2·611
10	0·50	51·54	56·17	673	1800	360	4·081
12	0·55	67·76	73·75	885	1650	330	5·876
14	0·58	83·02	90·67	1088	1490	298	7·997
16	0·60	97·78	106·75	1281	1350	270	10·440
18	0·64	117·11	126·67	1520	1280	256	13·22
20	0·70	142·25	153·43	1841	1260	252	16·32
24	0·80	194·77	210·33	2524	1200	240	23·50
30	0·90	273·00	285·33	3524	1080	216	36·72
36	1·00	363·22	390·50	4686	1000	200	52·88

From "Water Works," by Howland and Ellis.

TABLE OF WEIGHT OF ONE FOOT IN LENGTH OF ROLLED IRON.

Breadth or diameter in inches.	Round Iron.	Thickness in Inches. ⅛	¼	⅜	½	⅝	¾	⅞	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼	3½	3¾	4	4½	5	5½	6	7	8	9	10	11	12
⅛	0·04	·05																													
¼	0·17	·10	·21																												
⅜	0·37	·15	·31	·47																											
½	0·66	·21	·42	·63	·84																										
⅝	1·03	·26	·53	·79	1·05	1·31																									
¾	1·49	·31	·63	·95	1·26	1·58	1·90																								
⅞	2·02	·36	·73	1·10	1·47	1·84	2·21	2·58																							
1	2·65	·42	·84	1·26	1·68	2·11	2·53	2·95	3·37																						
1¼	4·13	·53	1·05	1·58	2·11	2·63	3·16	3·68	4·21	5·26																					
1½	5·95	·63	1·26	1·90	2·53	3·16	3·79	4·42	5·05	6·32	7·58																				
1¾	8·10	·74	1·47	2·21	2·95	3·68	4·42	5·16	5·89	7·37	8·84	10·31																			
2	10·58	·84	1·68	2·53	3·37	4·21	5·05	5·89	6·74	8·42	10·10	11·79	13·47																		
2¼	13·39	·95	1·90	2·84	3·79	4·74	5·68	6·63	7·58	9·47	11·37	13·26	15·16	17·05																	
2½	16·53	1·05	2·11	3·16	4·21	5·26	6·32	7·37	8·42	10·53	12·63	14·74	16·84	18·95	21·05																
2¾	20·01	1·16	2·32	3·47	4·63	5·79	6·95	8·10	9·26	11·58	13·89	16·21	18·52	20·84	23·16	25·47															
3	23·81	1·26	2·53	3·79	5·05	6·32	7·58	8·84	10·10	12·63	15·16	17·68	20·21	22·74	25·26	27·79	30·31														
3¼	27·94	1·37	2·74	4·11	5·47	6·84	8·21	9·58	10·95	13·69	16·43	19·17	21·90	24·63	27·37	30·11	32·84	35·57													
3½	32·41	1·47	2·95	4·42	5·89	7·37	8·84	10·32	11·79	14·74	17·68	20·64	23·58	26·53	29·48	32·42	35·37	38·32	41·26												
3¾	37·20	1·58	3·16	4·74	6·32	7·89	9·47	11·05	12·63	15·79	18·95	22·11	25·26	28·42	31·58	34·74	37·89	41·05	44·21	47·37											
4	42·33	1·68	3·37	5·05	6·74	8·42	10·10	11·79	13·47	16·84	20·20	23·58	26·95	30·32	33·69	37·06	40·42	43·79	47·16	50·53	53·89										
4½	53·57	1·90	3·79	5·68	7·58	9·47	11·37	13·26	15·16	18·95	22·75	26·52	30·31	34·10	37·89	41·68	45·47	49·26	53·05	56·84	60·63	68·20									
5	66·13	2·11	4·21	6·32	8·42	10·53	12·63	14·74	16·84	21·05	25·26	29·48	33·68	37·89	42·10	46·31	50·52	54·73	58·94	63·15	67·36	75·78	84·20								
5½	80·02	2·32	4·63	6·95	9·26	11·58	13·89	16·21	18·52	23·16	27·78	32·42	37·04	41·67	46·31	50·95	55·58	60·21	64·84	69·47	74·10	83·36	92·63	101·90							
6	95·23	2·53	5·05	7·58	10·10	12·63	15·16	17·68	20·21	25·26	30·31	35·37	40·42	45·47	50·52	55·57	60·63	65·68	70·73	75·78	80·84	90·95	101·05	111·15	121·25						
7	129·60	2·94	5·90	8·84	11·79	14·74	17·68	20·64	23·58	29·47	35·36	41·25	47·14	53·03	58·92	64·81	70·70	76·60	82·50	88·39	94·28	106·07	117·86	129·65	141·43	165·00					
8	169·30	3·36	6·74	10·10	13·48	16·84	20·21	23·58	26·95	33·69	40·43	47·17	53·90	60·64	67·38	74·12	80·86	87·60	94·33	101·07	107·80	121·28	134·76	148·24	161·71	188·66	215·60				
9	214·30	3·79	7·58	11·36	15·16	18·95	22·75	26·52	30·32	37·89	45·47	53·05	60·63	68·20	75·78	83·36	90·94	98·52	106·10	113·67	121·25	136·40	151·56	166·72	181·88	212·20	242·50	272·80			
10	264·50	4·21	8·42	12·64	16·84	21·05	25·26	29·48	33·68	42·10	50·52	58·94	67·36	75·78	84·20	92·62	101·04	109·46	117·88	126·30	134·72	151·56	168·40	185·24	202·08	235·76	269·44	303·12	336·80		
11	320·10	4·64	9·26	13·90	18·52	23·16	27·78	32·42	37·04	46·30	55·56	64·82	74·08	83·34	92·60	101·87	111·13	120·39	129·65	138·91	148·17	166·70	185·23	203·75	222·27	259·31	296·35	333·40	370·45	407·50	
12	380·90	5·05	10·10	15·16	20·21	25·26	30·31	35·37	40·42	50·52	60·63	70·73	80·83	90·94	101·04	111·14	121·25	131·35	141·45	151·55	161·65	181·86	202·07	222·28	242·50	282·92	323·34	363·76	404·17	444·59	485·00

WEIGHTS OF WROUGHT-IRON AND BRASS PLATES AND WIRE, SOFT ROLLED.

Birmingham Gauge.		No. of gauge.	American Gauge (Brown & Sharpe).				
				Plates per square foot.		Wire per lineal foot.	
Plate iron.	Thickness of each number.		Thickness of each number.	Wrought iron.	Brass.	Wrought iron.	Brass.
Lbs.	Inch.		Inch.	Lbs.	Lbs.	Lbs.	Lbs.
17·025	·454	0000	·46	17·25	19·68	·5607	·6051
15·9375	·425	000	·4096	15·361	17·53	·4447	·4799
14·25	·38	00	·3648	13·68	15·61	·3527	·3806
12·75	·34	0	·3248	12·182	13·90	·2797	·3018
11·25	·3	1	·2893	10·848	12·38	·2218	·2393
10·65	·284	2	·2576	9·661	11·02	·1759	·1898
9·7125	·259	3	·2294	8·603	9·81	·1395	·1505
8·925	·238	4	·2043	7·661	8·74	·1106	·1193
8·25	·22	5	·1819	6·822	7·78	·0877	·0946
7·6125	·203	6	·1620	6·075	6·93	·0695	·0750
6·75	·18	7	·1442	5·410	6·17	·0551	·0595
6·1875	·165	8	·1284	4·818	5·49	·0437	·0472
5·55	·148	9	·1144	4·291	4·89	·0347	·0374
5·025	·134	10	·1018	3·820	4·36	·0275	·0296
4·5	·12	11	·0907	3·402	3·88	·0218	·0235
4·0875	·109	12	·0808	3·030	3·45	·0173	·0186
3·5625	·095	13	·0719	2·698	3·07	·0137	·0148
3·1125	·083	14	·0640	2·403	2·74	·0109	·0117
2·7	·072	15	·0570	2·140	2·44	·00863	·00931
2·4375	·065	16	·0508	1·905	2·17	·00684	·00758
2·175	·058	17	·0452	1·697	1·93	·00542	·00585
1·8375	·049	18	·0403	1·511	1·72	·00430	·00464
1·575	·042	19	·0358	1·345	1·53	·00341	·00368
1·3125	·035	20	·0319	1·198	1·36	·00271	·00292
1·2	·032	21	·0284	1·067	1·21	·00215	·00231
1·05	·028	22	·0253	·9505	1·08	·00170	·00183
·9375	·025	23	·0225	·8464	·9660	·00135	·00145
·825	·022	24	·0201	·7537	·8602	·00107	·00115
·75	·02	25	·0179	·6712	·7661	·00085	·000916
·675	·018	26	·0159	·5977	·6822	·000673	·000726
·6	·016	27	·0141	·5323	·6075	·000534	·000576
·525	·014	28	·0126	·4740	·5410	·000423	·000457
·4875	·013	29	·0112	·4221	·4818	·000336	·000362
·45	·012	30	·0100	·3759	·4290	·000266	·000287
·375	·01	31	·0089	·3348	·3821	·000211	·000228
·3375	·009	32	·0079	·2981	·3402	·000167	·000180
·3	·008	33	·00708	·2655	·3030	·000132	·000143
·2625	·007	34	·00630	·2364	·2698	·000105	·000113
·1875	·005	35	·00561	·2105	·2402	·0000836	·00009015
·15	·004	36	·005	·1875	·214	·0000662	·0000715
		37	·00445	·1669	·1905	·0000525	·00005671
		38	·00396	·1486	·1697	·0000416	·00004496
		39	·00353	·1324	·1511	·0000330	·00003566
		40	·00314	·1179	·1345	·0000262	·00002827

Copper is about 5 per cent heavier than brass. Lead is about 47 per cent heavier than wrought iron. Zinc is about 7 per cent lighter than wrought iron. Sheet copper is rated by weight at so many ounces per square foot, and sheet lead at so many pounds per square foot.

TABLE OF DIMENSIONS AND WEIGHT OF WROUGHT-IRON WELDED TUBES.

Nominal diameter.	External diameter.	Thick-ness.	Internal diameter.	Internal circum-ference	External circum-ference.	Length of pipe per square foot of internal surface.	Length of pipe per square foot of external surface.	Internal area.	Weight per foot.	No. of threads per inch of screw.
Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Feet.	Feet.	Inches.	Lbs.	
1/8	·40	·068	·27	·85	1·27	14·15	9·44	·057	·24	27
1/4	·54	·088	·36	1·14	1·7	10·5	7·075	·104	·42	18
3/8	·67	·091	·49	1·55	2·12	7·67	5·657	·192	·56	18
1/2	·84	.109	·62	1·96	2·65	6·13	4·502	·305	·84	14
3/4	1·05	.113	·82	2·59	3·3	4·64	3·637	·533	1·13	14
1	1·31	·134	1·05	3·29	4·13	3·66	2·903	·863	1·67	11 1/2
1 1/4	1·66	·14	1·38	4·33	5·21	2·77	2·301	1·496	2·26	11 1/2
1 1/2	1·9	·145	1·61	5·06	5·97	2·37	2·01	2·038	2·69	11 1/2
2	2·37	·154	2·07	6·49	7·46	1·85	1·611	3·355	3·67	11 1/2
2 /2	2·87	·204	2·47	7·75	9·03	1·55	1·328	4·783	5·77	8
3	3·5	·217	3·07	9·64	11·	1·24	1·091	7·388	7·55	8
3 1/2	4·	·226	3·55	11·15	12·57	1·08	0·955	9·887	9·05	8
4	4·5	·237	4·07	12·69	14·14	·95	0·849	12·73	10·73	8
4 1/2	5·	·247	4·51	14·15	15·71	·85	0·765	15·939	12·49	8
5	5·56	·259	5·04	15·85	17·47	·78	0·629	19·99	14·56	8
6	6·62	·28	6·06	19·05	20·81	·63	0·577	28·889	18·77	8
7	7·62	·301	7·02	22·06	23·95	·54	0·505	38·737	23·41	8
8	8·62	·322	7·98	25·08	27·1	·48	0·444	50·039	28·35	8
9	9·69	·344	9·	28·28	30·43	·42	0·394	63·633	34·08	8
10	10·75	·366	10·02	31·47	33·77	·38	0·355	78·838	40·64	8

Nominal diameter.	Thickness, extra strong.	Thickness, double extra strong.	Actual inside diameter. Extra strong.	Actual inside diameter. Double extra strong.
Inches.	Inches.	Inches.	Inches.	Inches.
1/8	0·100	.....	0·205	.....
1/4	0·123	.....	0·294	.....
3/8	0·127	.....	0·421	.....
1/2	0·149	0·298	0·542	0·244
3/4	0·157	0·314	0·736	0·422
1	0·182	0·364	0·951	0·587
1 1/4	0·194	0·388	1·272	0·884
1 1/2	0·203	0·406	1·494	1·088
2	0·221	0·442	1·933	1·491
2 1/2	0·280	0·560	2·315	1·755
3	0·304	0·608	2·892	2·284
3 1/2	0·321	0·642	3·358	2·716
4	0·341	0·682	3·818	3·136

BOILER TUBES.

External diameter.	Thickness, wire gauge.	Average weight.	External diameter.	Thickness, wire gauge.	Average Weight.
Inches.	No.	Lbs. per foot.	Inches.	No.	Lbs. per foot.
1 1/4	16	1·	3	11	3·5
1 1/2	15	1·16	3 1/4	11	4·
1 3/4	14	1·63	4	8	6·4
2	13	2·	5	7	9·1
2 1/4	12	2·16	6	6	12·3
2 1/2	12	2·56	7	6	15·2
2 11/16	11	2·2	8	6	16·

HEAVY PIPE FOR DRIVEN WELLS.

Tested at 1200 pounds hydraulic pressure. Furnished in five-foot lengths.

Size (inches)	1¼	1½	2	2½	3	3½	4
Weight per foot, lbs	3·62	2·75	3·75	6·00	7·75	9·25	11·00

HEAVY WROUGHT GALVANIZED IRON SPIRAL RIVETED PIPES,

WITH FLANGED CONNECTIONS.

Tested at 150 pounds hydraulic pressure. Regalvanized after riveting.

Inside diameter (inches)	3	4	5	6	7	8	9	10	11	12
Wire gauge, Nos	20	20	20	18	18	18	18	16	16	16
Nominal weight per foot, lbs	2½	4	5	6	7	8	9	12	13	14

Manufactured lengths, 20 feet or less. Elbows and other fittings, cast iron.

LIGHT PIPE, SUITABLE FOR HOUSE LEADERS, VENTILATING, AIR, AND BLOWER PIPES, ETC.

Inside diameter (inches)	2	2½	3	3½	4	4½	5	5½	6
Nominal weight per foot, lbs	⅝	¾	⅞	1	1¼	1⅜	1½	1⅝	1¾

TABLE OF COPPER AND BRASS RODS ONE FOOT IN LENGTH.

To find the weight of copper or brass pipe, take the weight of the exterior diameter from the table, and subtract from it the weight of a rod equal to that of the interior diameter, or bore.

Diamet'r in inches.	Copper.	Brass.	Diamet'r in inches.	Copper.	Brass.	Diamet'r in inches.	Copper.	Brass.
1/8	·047	·045	1 5/8	7·993	7·593	4 1/4	55·62	52·27
3/16	·106	·101	1 11/16	8·630	8·198	4 3/8	58·94	55 39
1/4	·189	·179	1 3/4	9·270	8·806	4 1/2	62·36	58·60
5/16	·296	·281	1 13/16	9·950	9·452	4 5/8	65·87	61·90
3/8	·426	·405	1 7/8	10·642	10·110	4 3/4	69·48	6
7/16	·579	·550	1 15/16	11·370	10·801	4 7/8	73·19	68·77
1/2	·757	·719	2	12·108	11·503	5	77·43	72·76
9/16	·958	·910	2 1/8	13·668	12·985	5 1/8	80·89	76·00
5/8	1·182	1·123	2 1/4	15·325	14·559	5 1/4	84·88	79·76
11/16	1·431	1·360	2 3/8	17·075	16·221	5 3/8	88·97	83·60
3/4	1·703	1·618	2 1/2	18·916	17·970	5 1/2	93·15	87·53
13/16	1·998	1·898	2 5/8	20·856	19·808	5 5/8	97·44	91·56
7/8	2·318	2·202	2 3/4	22·891	21·746	5 3/4	101·81	95·68
15/16	2·660	2·527	2 7/8	25·019	23·768	5 7/8	106·29	99·88
1	3·027	2·876	3	27·243	25·881	6	110·85	104·15
1 1/16	3·417	3·246	3 1/8	29·559	28·081	6 1/4	120 30	113·04
1 1/8	3·831	3·639	3 1/4	31·972	30·373	6 1/2	130·10	122·26
1 3/16	4·269	4·056	3 3/8	34·481	32·757	6 3/4	140·32	131·85
1 1/4	4·723	4·487	3 1/2	37·081	35·227	7	150·86	141·76
1 5/16	5·214	4·953	3 5/8	39·777	37·788	7 1/4	161·87	152·10
1 3/8	5·723	5·437	3 3/4	42·568	40·440	7 1/2	173·22	162·77
1 7/16	6·255	5·943	3 7/8	45·455	43·182	7 3/4	184·97	173·81
1 1/2	6·811	6·470	4	48·433	46·000	8	197·03	185·14
1 9/16	7·390	7·020	4 1/8	52·40	49·24			

NUMBER OF BURDEN'S RIVETS IN ONE HUNDRED POUNDS.

Lengths.	Diameter. ½	⅝	11/16	¾	Lengths.	⅞ S. B.
¾	1,092	665	....	....	5	90
⅞	1,027	597	....	....	5½	85
1	940	538	450	....	6	80
1⅛	840	512	415	....	6½	75
1¼	797	487	389	356	7	70
1⅜	760	460	370	329	7½	67
1½	730	440	357	280	8	65
1⅝	711	420	340	271	8½	61
1¾	693	390	325	262	9	57
1⅞	648	375	312	257	9½	54
2	608	360	297	243	10	51
2⅛	573	354	....	....	10½	47
2¼	555	347	280	232		
2½	525	335	260	220		
2¾	500	312	242	208		
3	460	290	224	197		
3¼	433	267	212	180		
3½	413	248	201	169		
3¾	395	241	192	160		
4	....	230	184	158		
4¼	....	220	177	150		
4½	....	210	171	146		
4¾	....	200	166	138		
5	....	190	161	135		
5¼	....	180	156	130		
5½	....	172	151	124		
5¾	....	164	145	120		
6	....	157	140	115		
6¼	....	150	138	111		
6½	....	146	134	107		
6¾	....	143	129	104		
7	....	140	125	100		

WROUGHT SPIKES—NUMBER TO A KEG OF ONE HUNDRED AND FIFTY POUNDS.

Length.	¼″	5/16″	⅜″	7/16″	½″
Inches					
3	2,250	....	....	....	....
3½	1,890	1,208	....	....	....
4	1,650	1,135	....	....	....
4½	1,464	1,064	....	....	....
5	1,380	930	742	....	....
6	1,292	868	570	....	....
7	1,161	662	482	445	306
8	....	635	455	384	256
9	....	573	424	300	240
10	....	....	391	270	222
11	....	....	....	249	203
12	....	....	....	236	180

LENGTHS OF CUT NAILS AND SPIKES, AND NUMBER IN A POUND.

Size.	Length.	No.	Size.	Length.	No.	Size.	Length.	No.
	Inches.			Inches.			Inches.	
3*d.*	1¼	420	8*d.*	2½	100	30*d.*	4	24
4	1½	270	10	3	65	40	4½	20
5	1¾	220	12	3¼	52	60		
6	2	175	20	3½	28			

APPROXIMATE NUMBER OF WIRE NAILS PER POUND.

Wire Gauge. B. W. G.	Length, Inches.																	
	¼	⅜	½	⅝	¾	1	1¼	1½	1¾	2	2½	3	3½	4	4½	5	6	7
00	.....	.....	.....	.....	.....	.....	33	27	23	20	16	14	12	10	9	8	7	6
0	.....	.....	.....	.....	.....	.....	34	29	25	21	17	15	13	11	10	9	8	7
1	.....	.....	.....	.....	.....	57	45	38	32	28	23	19	16	14	13	11	10	8
2	.....	.....	.....	.....	.....	65	52	44	37	32	26	22	19	16	14	13	11	9
3	.....	.....	.....	.....	100	76	60	50	43	38	30	25	22	19	17	15	13	11
4	.....	.....	.....	.....	120	90	72	60	51	45	36	30	26	23	20	18	15	13
5	.....	.....	211	169	141	106	85	71	60	53	42	35	30	26	24	21	18	15
6	.....	.....	247	197	164	123	99	82	71	62	50	41	35	31	28	25	21	18
7	.....	.....	299	239	200	149	120	100	85	75	60	50	43	37	33	30	25	
8	.....	.....	345	275	229	172	137	115	98	86	69	57	49	43	39	35	29	
9	.....	.....	414	331	276	207	165	138	118	103	82	69	59	52	46	41		
10	.....	663	496	397	333	248	198	165	142	124	99	83	71	62	55	50		
11	.....	837	628	502	418	314	251	209	179	157	125	105	90	79	70			
12	.....	1096	822	658	548	411	329	274	235	204	164	137	117	103				
13	.....	1429	1072	857	714	536	429	357	306	268	214	178	153					
14	2840	1893	1420	1136	947	710	568	473	406	350	284	236						
15	3504	2336	1752	1402	1168	876	701	584	500	438	350							
16	4571	3048	2280	1828	1523	1143	913	761	653	571								
17	6233	4156	3116	2495	2077	1558	1246	1038	890	779								
18	8276	5517	4138	3310	2758	2069	1655	1379	1182									
19	10668	7112	5334	4267	3556	2667	2133	1778										
20	15000	10000	7500	6000	5000	3750	3000											
21	17777	11850	8888	7111	5926	4444												
22	22856	15237	11428	9143	7618													

B. W. G.	8	9	10	11	12
0	5	4½	4	3¾	3¼
1	5½	5	4⅓	4	3½
2	7	6	5½	5	4½
3	8	7	6½	6	5½
4	10	8	7½		
5	11	10	9		

TESTS OF TELEGRAPH WIRE.

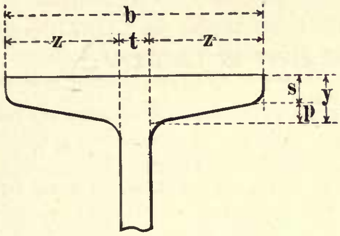
The following data are taken from a table given by Mr. Prescott relating to tests of E. B. B. galvanized wire furnished the Western Union Telegraph Company:

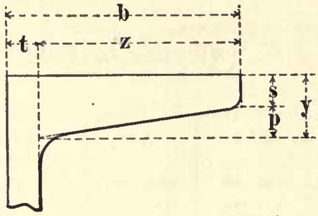
Size of Wire.	Diameter Parts of One Inch.	Weight.		Length, Feet per Pound.	Resistance. Temp., 75·8° Fahr.		Ratio of Breaking Weight to Weight per Mile.
		Grains, per Foot.	Pounds per Mile.		Feet per Ohm.	Ohms per Mile.	
4	·238	1043·2	886·6	6·00	958	5·51	
5	·220	891·3	673·0	7·85	727	7·26	
6	·203	758·9	572·2	9·20	618	8·54	3·05
7	·180	596·7	449·9	11·70	578	10·86	3·40
8	·165	501·4	378·1	14·00	409	12·92	3·07
9	·148	403·4	304·2	17·4	328	16·10	3·38
10	·134	330·7	249·4	21·2	269	19·[illegible]	3·37
11	·120	265·2	200·0	26·4	216	24·42	2·97
12	·109	218·8	165·0	32·0	179	29·60	3·43
14	·083	126·9	95·7	55·2	104	51·00	3·05

SIZES OF GALVANIZED WIRE USED IN TELEGRAPH AND TELEPHONE LINES.

No. 4. Now used on important lines where the multiplex systems are applied. No. 5. Little used. No. 6. Used for important circuits between cities. No. 8. Medium size for circuits of 400 miles or less. No. 9. For similar locations to No. 8, but on somewhat shorter circuits ; until lately was the size most largely used in this country. Nos. 10, 11. For shorter circuits, railway telegraphs, private lines, police and fire-alarm lines, etc. No. 12. For telephone lines, police and fire-alarm lines, etc. Nos. 13, 14. For telephone lines and short private lines : steel wire is used most generally in these sizes.

The grades of line wire are generally known to the trade as "Extra Best Best" (E. B. B.), "Best Best" (B. B.), and "Steel."





STANDARD I BEAMS.

Depth of beam.	Min. wt. per foot.	Min. b.	Min. t.	z.	s.	p.	y.	WEIGHT PER FOOT. Intermediate.	Increase of b and t for each lb. inc. of weight.	Max.
24″	80·00	7·00	·50	3·250	·60	·542	1·142	Vary by 5 lbs.	·0123	100·00
20″	64·00	6·25	·50	2·875	·55	·479	1·029	65 lbs. then by 5 lbs.	·015	75·00
15″	42·00	5·50	·41	2·545	·41	·424	0·834	45 " " 5 "	·020	55·00
12″	31·50	5·00	·35	2·325	·35	·388	0·738	....	·025	35·00
10″	25·00	4·66	·31	2·175	·31	·363	0·673	Vary by 5 lbs.	·029	40·00
9″	21·00	4·33	·29	2·020	·29	·337	0·627	25 lbs. then by 5 lbs.	·033	35·00
8″	18·00	4·00	·27	1·865	·27	·311	0·581	Vary by 2½ lbs.	·037	25·50
7″	15·00	3·66	·25	1·705	·25	·284	0·534	" " 2½ "	·042	20·00
6″	12·25	3·33	·23	1·550	·23	·258	0·488	" " 2½ "	·049	17·25
5″	9·75	3·00	·21	1·395	·21	·233	0·443	" " 2½ "	·059	14·75
4″	7·50	2·66	·19	1·235	·19	·206	0·396	" " 1 lb.	·074	10·50
3″	5·50	2·33	·17	1·080	·17	·180	0·350	" " 1 "	·098	7·50

STANDARD CHANNELS.

Depth of channel.	Min. wt. per foot.	Min. b.	Min. t.	z.	s.	p.	y.	WEIGHT PER FOOT. Intermediate.	Increase of b and t for each lb. inc. of weight.	Max.
15″	33·00	3·40	·40	3·00	·40	·500	·900	35 lbs. then by 5 lbs.	·020	55·00
12″	20·50	2·94	·28	2·66	·28	·443	·723	25 " " 5 "	·025	40·00
10″	15·00	2·60	·24	2·36	·24	·393	·633	Vary by 5 lbs.	·029	35·00
9″	13·25	2·43	·23	2·20	·23	·367	·597	15 lbs. then by 5 lbs.	·033	25·00
8″	11·25	2·26	·22	2·04	·22	·340	·560	Vary by 2½ lbs.	·037	21·25
7″	9·75	2·09	·21	1·88	·21	·313	·523	" " 2½ "	·042	19·75
6″	8·00	1·92	·20	1·72	·20	·287	·487	" " 2½ "	·049	15·50
5″	6·50	1·75	·19	1·56	·19	·260	·450	" " 2½ "	·059	11·50
4″	5·25	1·58	·18	1·40	·18	·233	·413	" " 1 lb.	·074	7·25
3″	4·00	1·41	·17	1·24	·17	·207	·377	" " 1 "	·098	6·00

During the preparation of this work the Association of American Steel Manufacturers has been formed, from whose circular the above table has been extracted.

GRADES OF STEEL.

Rivet Steel.—Ultimate strength, 48,000 to 58,000 pounds per square inch. Elongation, 26 per cent.

Soft Steel.—Ultimate strength, 52,000 to 62,000 pounds per square inch.

Elongation, 25 per cent.

Medium Steel.—Ultimate strength, 60,000 to 70,000 pounds per square inch.

Elongation, 22 per cent.

Elastic limit, not less than one half the ultimate strength.

Bending test, 180 degrees flat on itself, or equal to thickness of piece tested, without fracture on outside of bent portion.

WEIGHTS OF LEAD PIPE PER FOOT IN LENGTH.

Caliber.	Mark.							
	AAA	AA	A	B	C	D	E	
	Lbs. oz.	Lbs. oz.	Lbs. oz.	Lbs oz.	Lbs. oz.	Lbs. oz.	Lbs. oz.	Lbs. oz.
1/8	.. ..	.. ..	.. ..	.. ..	.. ..	.. .	.. ..	0 2
1/4	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..	0 2	.. ..
3/8	1 12	1 5	1 2	1 0	0 14	0 7	.. ..	0 10
7/16	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..	0 9½
1/2	3 ..	2 0	1 10	1 3	1 0	0 10	.. ..	.. ..
	2 8	.. ..	.. ..	.. ..	.. ..	0 12	.. ..	.. ..
5/8	3 8	2 12	2 8	2 0	1 7	1 4	0 12	.. ..
3/4	4 14	3 3	3 0	2 3	1 12	1 3	1 0	.. ..
1	6 ..	4 8	4 0	3 4	2 8	2 4	2 0	1 8
1¼	6 12	5 12	4 11	3 11	3 0	2 8	2 0	.. ..
1½	8 0	7 0	6 4	5 0	4 4	3 8	2 0	.. ..
	.. ..	.. ..	.. ..	.. ..	.. ..	3 0	.. ..	.. ..
1¾	.. ..	8 8	6 7	5 0	4 0	3 10	.. ..	.. ..
2	10 11	8 14	7 0	6 0	5 0	4 0	.. ..	.. ..
	.. ..	.. ..	.. ..	.. ..	.. ..	3 0	.. ..	.. ..
	Thickness.				Waste.			
	3/8	5/16	1/4	3/16				
2½	16 11	13 10	10 10	7 3	6 0	4 0	.. ..	.. ..
3	19 9	16 0	12 9	9 4	5 0	3 8	.. ..	.. ..
3½	22 8	18 7	14 8	10 12	.. ..	.. ..	.. ..	.. ..
4	25 6	20 14	16 7	12 2	8 0	6 0	.. ..	.. ..
4½	.. ..	.. ..	18 6	13 9	.. ..	.. ..	10 8	7 6
5	31 3	.. ..	20 5	15 0	.. ..	.. ..	10 8	.. ..
6	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..	12 0	.. ..

TABLE OF THE WEIGHT OF A CUBIC FOOT OF WATER AT DIFFERENT TEMPERATURES.

Fahrenheit.	Centigrade.	Weight in pounds.	Fahrenheit.	Centigrade.	Weight in pounds.	Fahrenheit.	Centigrade.	Weight in pounds.
Degrees.	Degrees.		Degrees.	Degrees.		Degrees.	Degrees.	
32	0	62·42	95	35	62·06	167	75	60·87
39	4	62·42	104	40	61·95	176	80	60·68
41	5	62·42	113	45	61·83	185	85	60·48
50	10	62·41	122	50	61·69	194	90	60·27
59	15	62·37	131	55	61·55	203	95	60·04
68	20	62·32	140	60	61·39	212	100	59·83
77	25	62·25	149	65	61·23			
86	30	62·16	158	70	61·06			

THE FLOW OF WATER.

With the increased consumption of water by the mills at Lowell, Mass., there was found a necessity for the accurate gauging of the quantities severally used, and, as at times the total was beyond the flow of the stream, that it should be properly distributed and that none should be wasted. At this time the late James B. Francis was the engineer of the Locks and Canals Company, to whom the charge of the canals and water distribution was committed. He decided that the weir was the form in which the water from the several wheels should be measured as most economical in application and accurate in results.

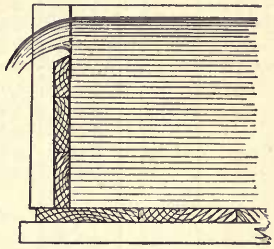


Fig. 1863.

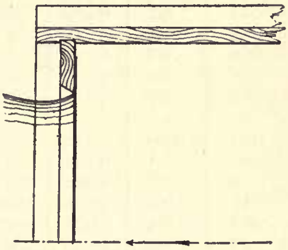


Fig. 1864.

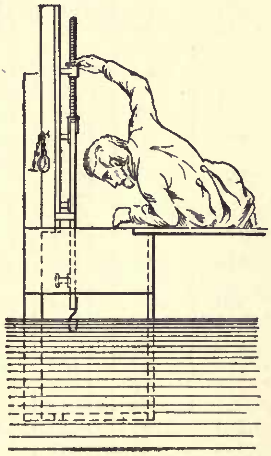


Fig. 1865.

Figs. 1863 and 1864 are the sectional elevation and plan of the common form of weir, in which the lower edges, bottom, and sides are chamfered off, with edges about $\frac{1}{8}''$ wide, making a perfect rectangle. In his experiments Mr. Francis made the bottom plate of cast iron with the upper edge planed and set accurately horizontal and the sides planed for the vertical edges and for the joints with the bottom plate. The iron rectangle could be accurately measured, but it was necessary to determine the wetted rectangle from the height of the water above the bottom edge. This was done by the hook gauge, in which the hook was submerged in the water and gradually raised by a micrometer screw till it showed the sign of a rising at the surface of the water (Fig. 1865). The 0 of the scale of the gauge was accurately referred to the edge of the weir. It was necessary that the surface of the water at the hook be kept perfectly still. It was therefore submerged in a tight box which had communication with the water in the flume by a very small hole or by a small pipe on the bottom of the flume, across it parallel to the weir and at a slight distance from it. In this pipe at equal intervals in the width of the weir small holes were drilled. The effect of this pipe was to give an average water level in the gauge box.

The general formula established by the experiments, on which the following table is calculated, is $Q = 3{\cdot}33\,(l - {\cdot}2h)\,h^{\frac{3}{2}}$, in which Q is the discharge in cubic feet per second, l the length of the weir, and h the height of water above the crest of the weir, both in feet.

For complete end contractions the side of the weir should be at least equal to the depth of the water on it from the side of the canal. The bottom contraction, also complete, is shown by a body of air below the crest of the weir. Where there is no end contraction, provision should be made for introducing and maintaining free communications of the air beneath the water sheet. For large velocities of approach to the weir, divide the area of water section above it by its discharge from the table for v, in the equation $h' = \frac{v^2}{2g}$, and add h' to previous depth for corrected discharge.

In the table, the discharge is given for one foot in length; but as in weirs there are usually two end contractions, virtually reducing the length, and met in the formula above by $- {\cdot}2h$, a column of correction has been added, which is to be subtracted from the product of discharge, as shown in example.

DISCHARGE, IN CUBIC FEET PER SECOND, OF A WEIR ONE FOOT LONG, WITHOUT CONTRACTION AT THE ENDS; FOR DEPTHS FROM 0·200 TO 0·999 FEET.

Correction for contractions.	Depth.	0	1	2	3	4	5	6	7	8	9
·012	0·20	0·298	0·300	0·302	0·305	0·307	0·309	0·311	0·314	0·316	0·318
·013	·21	0·320	0·323	0·325	0·327	0·330	0·332	0·334	0·337	0·339	0·341
·015	·22	0·344	0·346	0·348	0·351	0·353	0·355	0·358	0·360	0·362	0·365
·017	·23	0·367	0·370	0·372	0·374	0·377	0·379	0·382	0·384	0·387	0·389
·019	·24	0·391	0·394	0·396	0·399	0·401	0·404	0·406	0·409	0·411	0·414
·021	·25	0·416	0·419	0·421	0·424	0·426	0·429	0·431	0·434	0·436	0·439
·023	·26	0·441	0·444	0·447	0·449	0·452	0·454	0·457	0·459	0·462	0·465
·025	·27	0·467	0·470	0·472	0·475	0·478	0·480	0·483	0·485	0·488	0·491
·028	·28	0·493	0·496	0·499	0·501	0·504	0·507	0·509	0·512	0·515	0·517
·030	·29	0·520	0·523	0·525	0·528	0·531	0·534	0·536	0·539	0·542	0·544
·033	0·30	0·547	0·550	0·553	0·555	0·558	0·561	0·564	0·566	0·569	0·572
·036	·31	0·575	0·577	0·580	0·583	0·586	0·589	0·591	0·594	0·597	0·600
·039	·32	0·603	0·606	0·608	0·611	0·614	0·617	0·620	0·623	0·625	0·628
·042	·33	0·631	0·634	0·637	0·640	0·643	0·646	0·649	0·651	0·654	0·657
·045	·34	0·660	0·663	0·666	0·669	0·672	0·675	0·678	0·681	0·684	0·687
·048	·35	0·689	0·692	0·695	0·698	0·701	0·704	0·707	0·710	0·713	0·716
·052	·36	0·719	0·722	0·725	0·728	0·731	0·734	0·737	0·740	0·743	0·746
·056	·37	0·749	0·752	0·755	0·759	0·762	0·765	0·768	0·771	0·774	0·777
·059	·38	0·780	0·783	0·786	0·789	0·792	0·795	0·799	0·802	0·805	0·808
·063	·39	0·811	0·814	0·817	0·820	0·823	0·827	0·830	0·833	0·836	0·839
·067	0·40	0·842	0·846	0·849	0·852	0·855	0·858	0·861	0·865	0·868	0·871
·072	·41	0·874	0·877	0·881	0·884	0·887	0·890	0·893	0·897	0·900	0·903
·076	·42	0·906	0·910	0·913	0·916	0·919	0·923	0·926	0·929	0·932	0·936
·081	·43	0·939	0·942	0·945	0·949	0·952	0·955	0·959	0·962	0·965	0·969
·085	·44	0·972	0·975	0·978	0·982	0·985	0·988	0·992	0·995	0·998	1·002
·090	·45	1·005	1·009	1·012	1·015	1·019	1·022	1·025	1·029	1·032	1·035
·095	·46	1·039	1·042	1·046	1·049	1·052	1·056	1·059	1·063	1·066	1·070
·100	·47	1·073	1·076	1·080	1·083	1·087	1·090	1·094	1·097	1·100	1·104
·106	·48	1·107	1·111	1·114	1·118	1·121	1·125	1·128	1·132	1·135	1·139
·111	·49	1·142	1·146	1·149	1·153	1·156	1·160	1·163	1·167	1·170	1·174
·118	0·50	1·177	1·181	1·184	1·188	1·191	1·195	1·199	1·202	1·206	1·209
·124	·51	1·213	1·216	1·220	1·223	1·227	1·231	1·234	1·238	1·241	1·245
·130	·52	1·249	1·252	1·256	1·259	1·263	1·267	1·270	1·274	1·278	1·281
·136	·53	1·285	1·288	1·292	1·296	1·299	1·303	1·307	1·310	1·314	1·318
·143	·54	1·321	1·325	1·329	1·332	1·336	1·340	1·343	1·347	1·351	1·355
·150	·55	1·358	1·362	1·366	1·369	1·373	1·377	1·381	1·384	1·388	1·392
·157	·56	1·395	1·399	1·403	1·407	1·410	1·414	1·418	1·422	1·425	1·429
·164	·57	1·433	1·437	1·441	1·444	1·448	1·452	1·456	1·459	1·463	1·467
·171	·58	1·471	1·475	1·478	1·482	1·486	1·490	1·494	1·498	1·501	1·505
·178	·59	1·509	1·513	1·517	1·521	1·524	1·528	1·532	1·536	1·540	1·544
·186	0·60	1·548	1·551	1·555	1·559	1·563	1·567	1·571	1·575	1·579	1·583
·194	·61	1·586	1·590	1·594	1·598	1·602	1·606	1·610	1·614	1·618	1·622
·202	·62	1·626	1·630	1·633	1·637	1·641	1·645	1·649	1·653	1·657	1·661
·210	·63	1·665	1·669	1·673	1·677	1·681	1·685	1·689	1·693	1·697	1·701
·218	·64	1·705	1·709	1·713	1·717	1·721	1·725	1·729	1·733	1·737	1·741
·227	·65	1·745	1·749	1·753	1·757	1·761	1·765	1·769	1·773	1·777	1·781
·236	·66	1·785	1·790	1·794	1·798	1·802	1·806	1·810	1·814	1·818	1·822
·245	·67	1·826	1·830	1·834	1·838	1·843	1·847	1·851	1·855	1·859	1·863
·254	·68	1·867	1·871	1·875	1·880	1·884	1·888	1·892	1·896	1·900	1·904
·263	·69	1·909	1·913	1·917	1·921	1·925	1·929	1·934	1·938	1·942	1·946
·273	0·70	1·950	1·954	1·959	1·963	1·967	1·971	1·975	1·980	1·984	1·988
·283	·71	1·992	1·996	2·001	2·005	2·009	2·013	2·017	2·022	2·026	2·030
·293	·72	2·034	2·039	2·043	2·047	2·051	2·056	2·060	2·064	2·068	2·073
·303	·73	2·077	2·081	2·085	2·090	2·094	2·098	2·103	2·107	2·111	2·115
·314	·74	2·120	2·124	2·128	2·133	2·137	2·141	2·146	2·150	2·154	2·159
·324	·75	2·163	2·167	2·172	2·176	2·180	2·185	2·189	2·193	2·198	2·202

DISCHARGE, IN CUBIC FEET PER SECOND, OF A WEIR ONE FOOT LONG, WITHOUT CONTRACTION AT THE ENDS; FOR DEPTHS FROM 0·200 TO 0·999 FEET.

(*Continued.*)

Correction for contractions.	Depth.	0	1	2	3	4	5	6	7	8	9
·335	·76	2·206	2·211	2·215	2·219	2·224	2·228	2·232	2·237	2·241	2·246
·346	·77	2·250	2·254	2·259	2·263	3·267	2·272	2·276	2·281	2·285	2·290
·358	·78	2·294	2·298	2·303	2·307	2·312	2·316	2·320	2·325	2·329	2·334
·369	·79	2·238	2·343	2·347	2·351	2·356	2·360	2·365	2·369	2·374	2·378
·381	0·80	2·383	2·387	2·392	2·396	2·401	2·405	2·410	2·414	2·419	2·423
·393	·81	2·428	2·432	2·437	2·441	2·446	2·450	2·455	2·459	2·464	2·468
·406	·82	2·473	2·477	2·482	2·486	2·491	2·495	2·500	2·504	2·509	2·513
·413	·83	2·518	2·523	4·527	2·532	2·536	2·541	2·545	2·550	2·554	2·559
·431	·84	2·564	2·568	2·573	2·577	2·582	2·587	2·591	2·596	2·600	2·605
·444	·85	2·610	2·614	2·619	2·623	2·628	2·633	2·637	2·642	2·646	2·651
·457	·86	2·656	2·660	2·665	2·670	2·674	2·679	2·684	2·688	2·693	2·698
·470	·87	7·702	2·707	2·712	2·716	2·721	2·726	2·730	2·735	2·740	2·744
·484	·88	2·749	2·754	2·758	2·763	2·768	2·772	2·777	2·782	2·786	2·791
·498	·89	2·796	2·801	2·805	2·810	2·815	2·819	2·824	2·829	2·834	2·838
·512	0·90	2·843	2·848	2·853	2·857	2·862	2·867	2·872	2·876	2·881	2·886
·526	·91	2·891	2·895	2·900	2·905	2·910	2·915	2·919	2·924	2·929	2·934
·541	·92	2·938	2·943	2·948	2·953	2·958	2·963	2·967	2·972	2·977	2·982
·555	·93	2·986	2·991	2·996	3·001	3·006	3·011	3·015	3·020	3·025	3·030
·570	·94	3·035	3·040	3·044	3·049	3·054	3·059	3·064	3·069	3·074	3·078
·586	·95	3·083	3·088	3·093	3·098	3·103	3·108	3·113	3·117	3·122	3·127
·601	·96	3·132	3·137	3·142	3·147	3·152	3·157	3·162	3·166	3·171	3·176
·677	·97	3·181	3·186	3·191	3·196	3·201	3·206	3·211	3·216	3·221	3·226
·632	·98	3·231	3·235	3·240	3·245	3·250	3·255	3·260	3·265	3·270	3·275
·648	·99	3·280	3·285	3·290	3·295	3·300	3·305	3·310	3·315	3·320	3·325

Example.—Let the weir, with end contractions, be 5·3 feet long, and depth of water, or $h = 0{\cdot}612$.

By table the discharge for one foot in length is........................	1·594
	5·3
	8·4482
Correction...	·196
Discharge in cubic feet per second..................................	8·2522

The measures of the discharge of the different wheels was accurately determined; it was equally important to determine the quantities used by the different corporations. Mr. Francis's arrangements for this purpose were the construction of rectangular plank flumes in the different canals or feeders, of which the widths were divided into feet marked on the upstream faces of the timbers, extending above and across the ends of the flume, which enabled the average thread of the float in its passage through the flume to be ascertained. The depth of the water in the flume was measured by a gauge placed centrally on one side of the flume. The floats measuring the velocity of the currents were adjusted to the depths of water, having some two inches clearance from the bottom. The tubes were of two to three inches diameter, loaded at the bottom, and marked with their water line. They were brought to the flume and those of length appropriate to its depth were selected. From the notes thus taken the average path of the float was determined, on which was plotted its velocity, as represented by the different circles (Fig. 182). Through the average of these circles on the lines of flow is drawn the full line of averages from which the average of the total flow was determined.

In a similar way gaugings may be made of natural streams where the sections are approximately smaller. Soundings are to be made across the stream and sections drawn. The average velocities may be taken through these sections, which multiplied by the products of the sections and nine tenths of their sum will give approximately the flow of the streams. For this kind of measure I have used two rubber balls connected by an adjustable soft cotton cord, the lower ball being filled with water. A light strong thread is attached to the upper ball with the ends in the hands of the observers on each side of the stream, by which the ball may be guided in the thread of its appropriate section.

It was customary to find the average flow of a stream from the surface velocities in different threads of it according to the sections by means of apples loaded with shingle nails to nearly the specific gravity of the water, casting them into the stream, and taking the time of transit between two cords stretched across it, and taking as an average about eighty per cent that of the whole observations by the area of the entire section. Surface velocities only to be considered as loose approximations, as they are very much influenced by the direction and strength of the wind and the uniformities of flow in the different sections and diversions or eddies.

The miner's inch is designated as the flow through one square inch, but under various heads in different states. P. J. Flynn in "Irrigation Canals" gives one cubic foot per second as the equivalent flow through fifty miner's inches under a mean head of four inches.

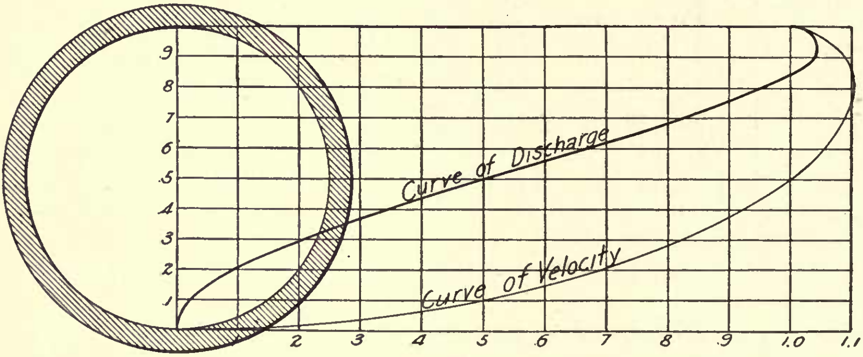


FIG. 1866.

FLOW OF WATER IN PIPES AND CONDUITS.

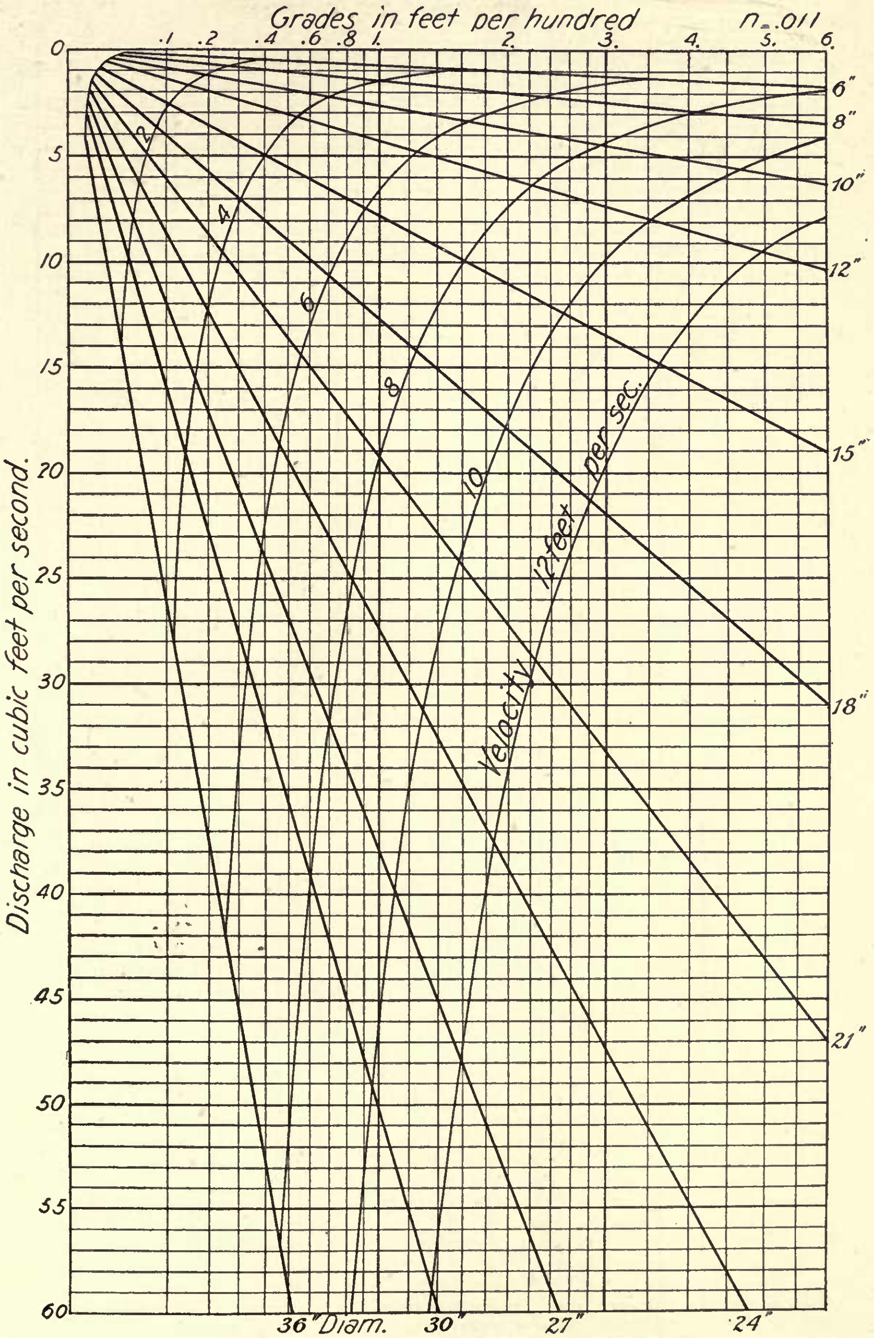
The general formula for the flow of water in all channels is $v = c\sqrt{Rs}$, in which v is the velocity, c a coefficient determined for the particular form of channel, R the hydraulic mean depth—that is, the area of water cross-section—divided by its wetted perimeter of channel; s is the slope of difference of level or pressure in feet between two points divided by the length in feet.

Fig. 1866 is a section of a water pipe in which the capacities are shown of its different sections from the bottom to a full section without any head. The friction of the sides of the full pipe reduces the velocity, and the maximum is at about eighty per cent of full, and the maximum of the curve of discharge or velocity by area of section a little above ninety per cent. It must be observed that the velocity at half section is the same as when the pipe is full.

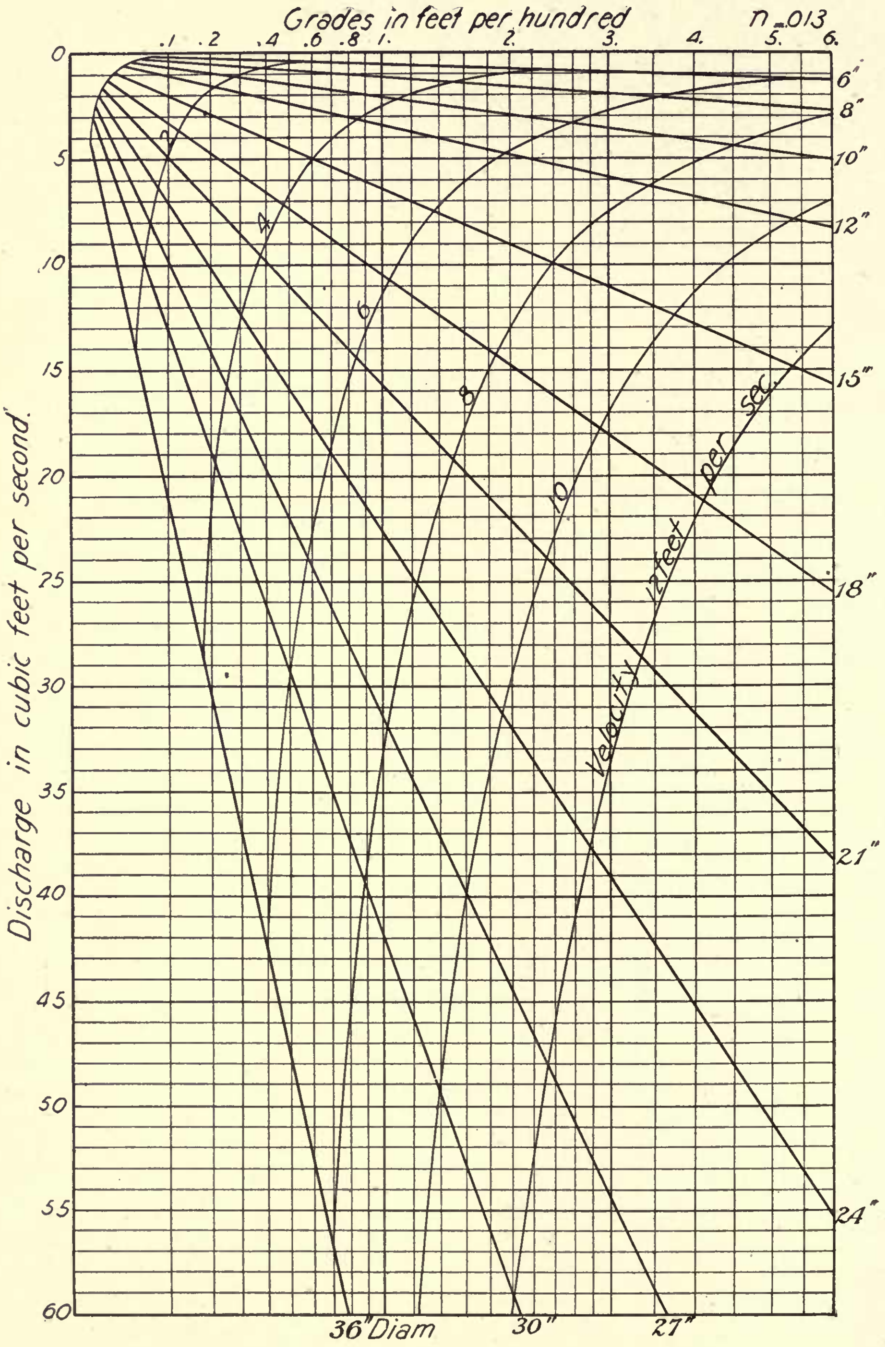
Messrs. Ganguillet and Kutter have established formulæ, of which there is a modification according to the character of the surface of the wetted perimetre.

The Kutter formula, as it is called, is very complicated, but it has been simplified graphically by Mr. Rudolph Hering and published in the "Transactions of the A. S. C. E.," January, 1879, from which the figures have been redrawn.

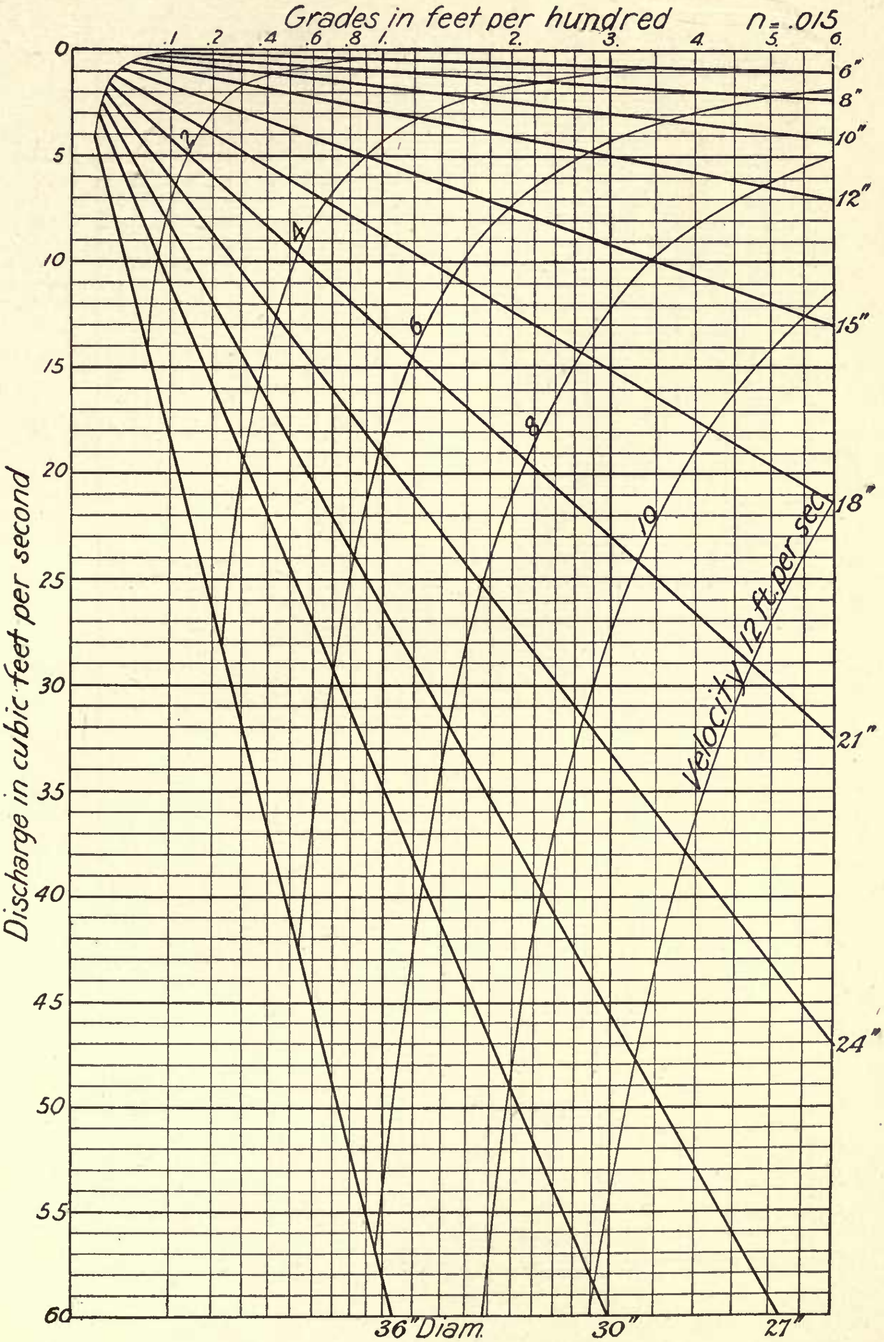
IRON; CEMENT; TERRA COTTA PIPES, WELL-JOINTED AND IN BEST ORDER; CAREFULLY PLASTERED SURFACE.



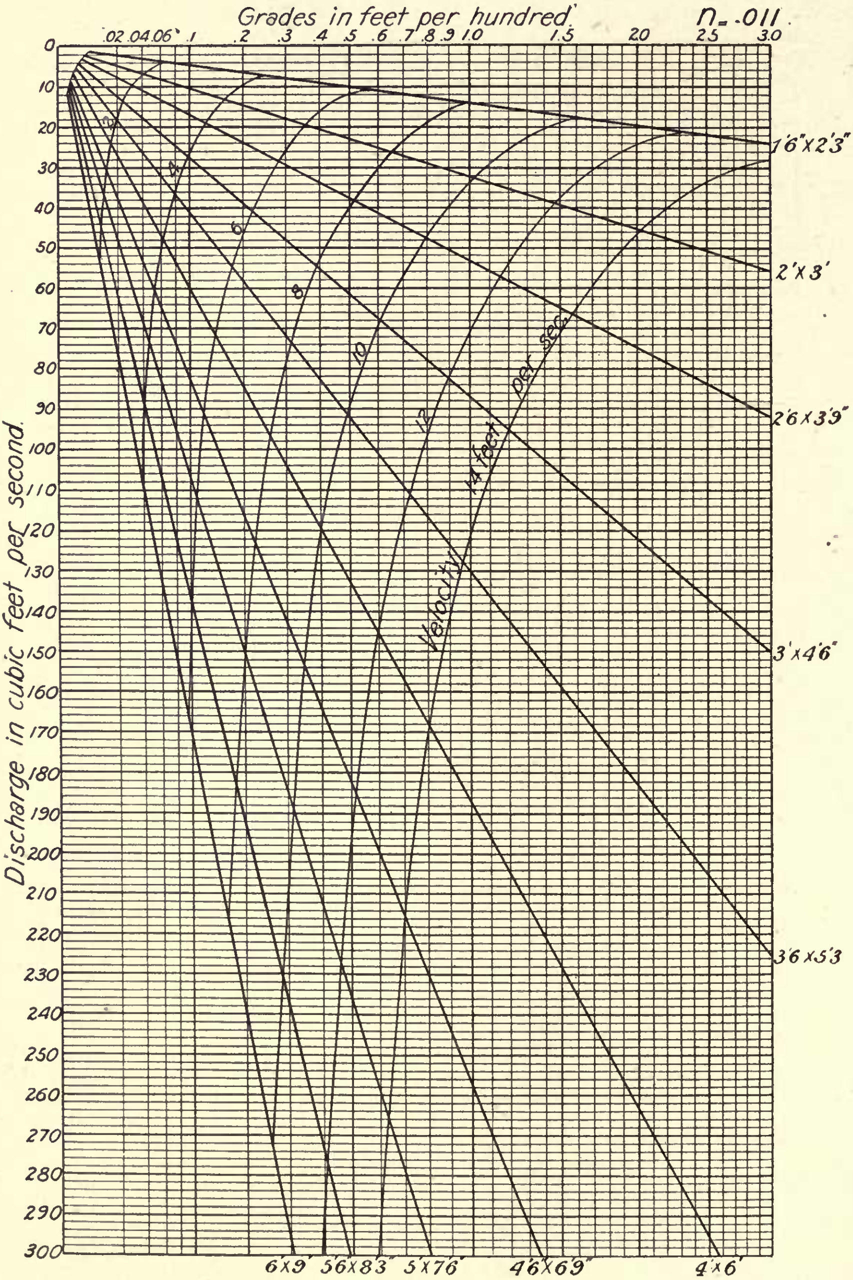
OLD IRON ; CEMENT AND TERRA COTTA PIPES NOT WELL-JOINTED NOR IN PERFECT ORDER ; WELL LAID BRICKWORK.



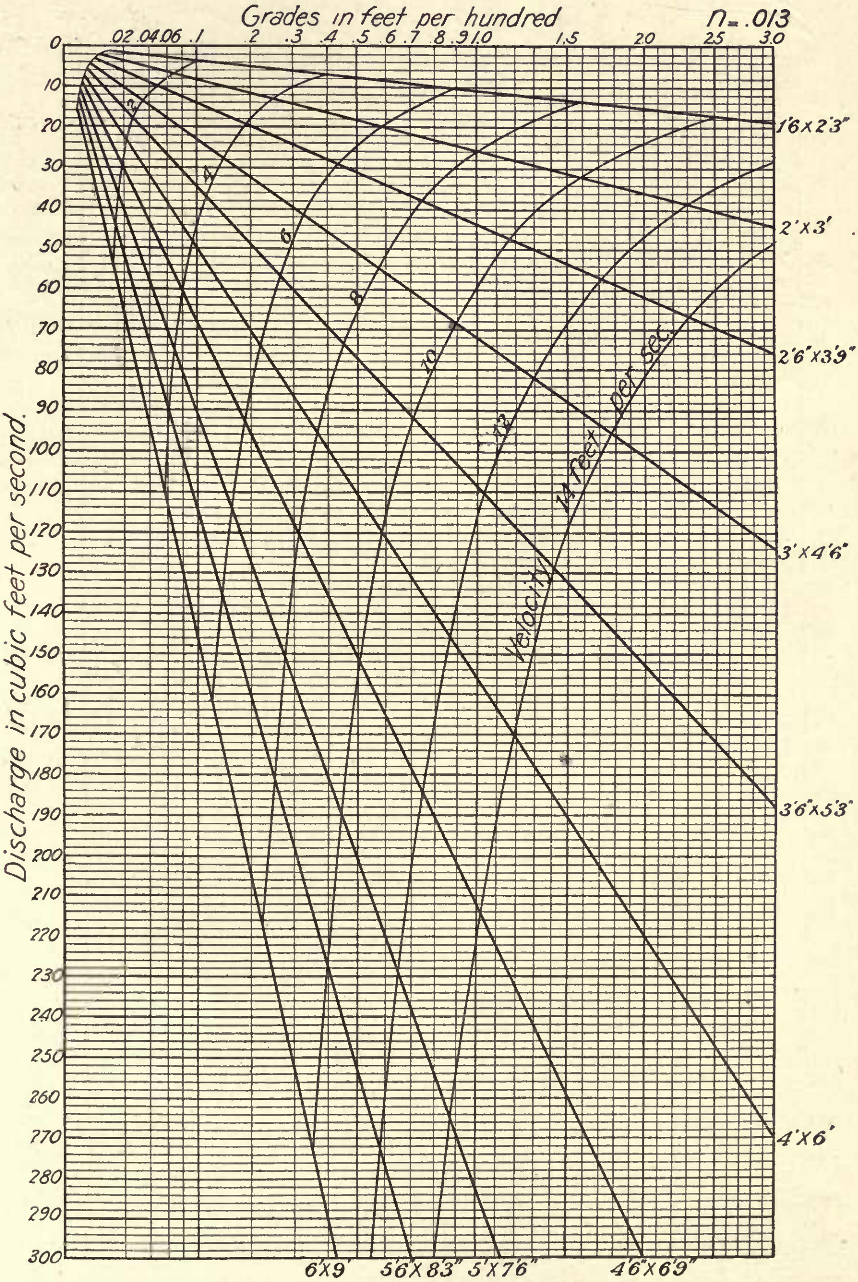
FOUL AND SLIGHTLY TUBERCULATED IRON; CEMENT AND TERRA COTTA PIPES WITH IMPERFECT JOINTS, AND IN BAD ORDER; WELL-DRESSED STONEWORK AND SECOND CLASS BRICKWORK.



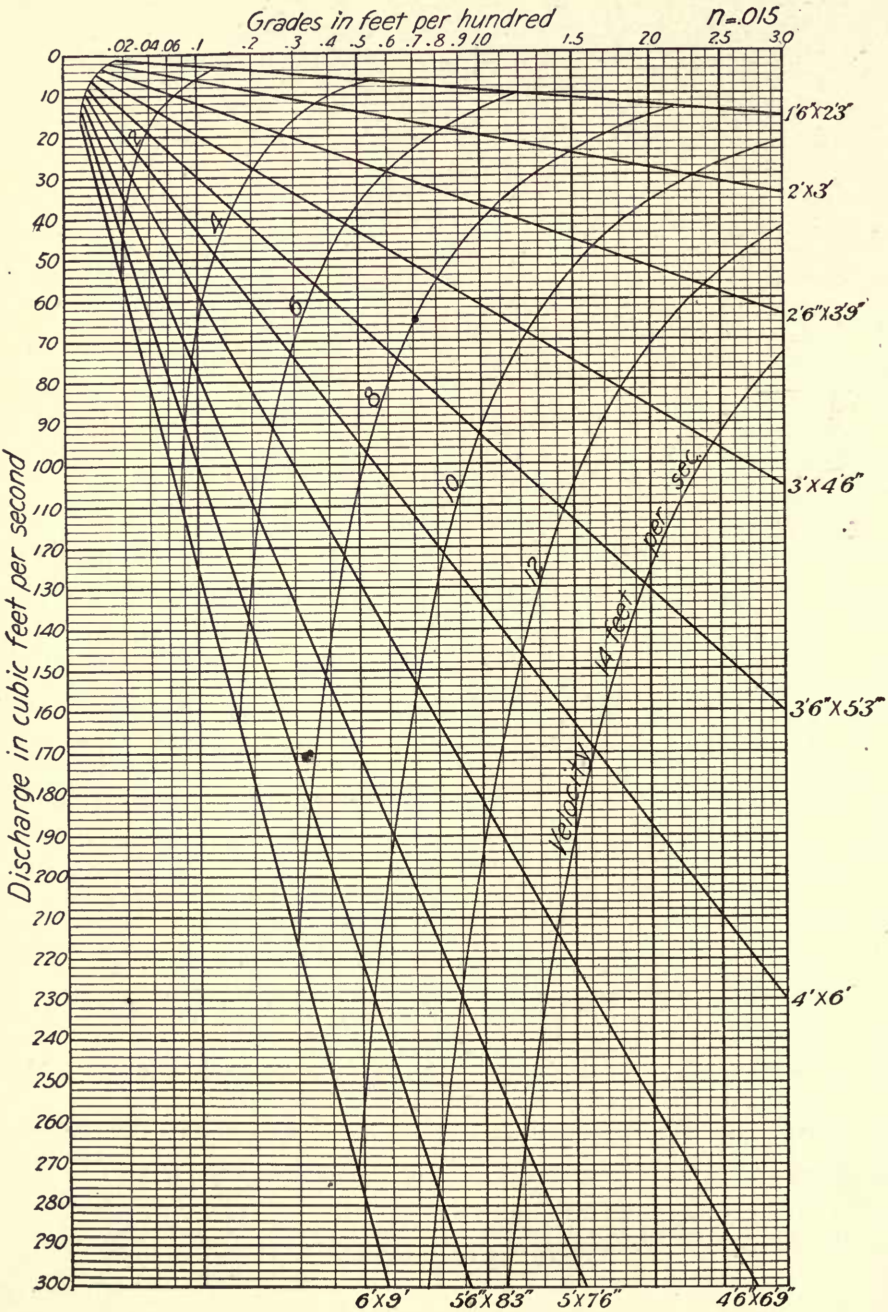
IRON ; CEMENT ; TERRA COTTA PIPES, WELL-JOINTED AND IN BEST ORDER.



OLD IRON ; CEMENT AND TERRA COTTA PIPES NOT WELL-JOINTED NOR IN PERFECT ORDER ; WELL LAID BRICKWORK.



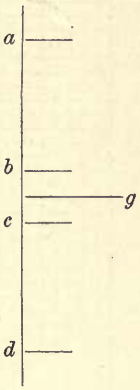
FOUL AND SLIGHTLY TUBERCULATED IRON; CEMENT AND TERRA COTTA PIPES WITH IMPERFECT JOINTS, AND IN BAD ORDER; WELL-DRESSED STONEWORK AND SECOND CLASS BRICKWORK.



For tables of the Kutter formula see "Irrigation Canals," by P. J. Flynn, C. E.

It will be observed that n depends on the condition of the wetted surface, and that with iron surfaces it must vary largely with the time exposure of these surfaces and the characteristics of the water passing through.

Where pipes of iron can not be cleaned, it is impossible for the engineer to form reliable judgment of their condition after years of use. They can be tested by measuring flows through a hydrant which discharges from a single length of pipe and determining the s in that length.



For a main with no available gate the following is suggested:

Let $a\ d$ be the main; establish s between $a\ b$ and $c\ d$ by reliable gauges. At g put in a branch controlled by a gate. After $s\ s$ are established as above, open the gate and measure its discharge accurately by a weir. With this factor see what s is between a and b and check the quantities by the changes between c and d and compare with flows at different values of n.

Should the values of n, as thus established by experiment, be outside of the limits of $n = \cdot 011$, $\cdot 013$, $\cdot 015$, as given by the diagrams, curves can be established from these values and extended to include those of the experiments, which may answer as approximations.

FLOW OF AIR.

The flow of air is subject to the same laws as the flow of water. The preceding diagrams may therefore be used in finding the discharge of air in cubic feet per second, under the same heads of water in feet, by multiplying by 27·6, the square root of 761, the difference in density of a cubic foot of the two fluids.

The theoretical discharge of a pipe is as the square root of the fifth power of the diameter, from which the following table, derived from the circular of the B. F. Sturtevant Company, is based:

TABLE FOR EQUALIZING THE DIAMETER OF PIPES.

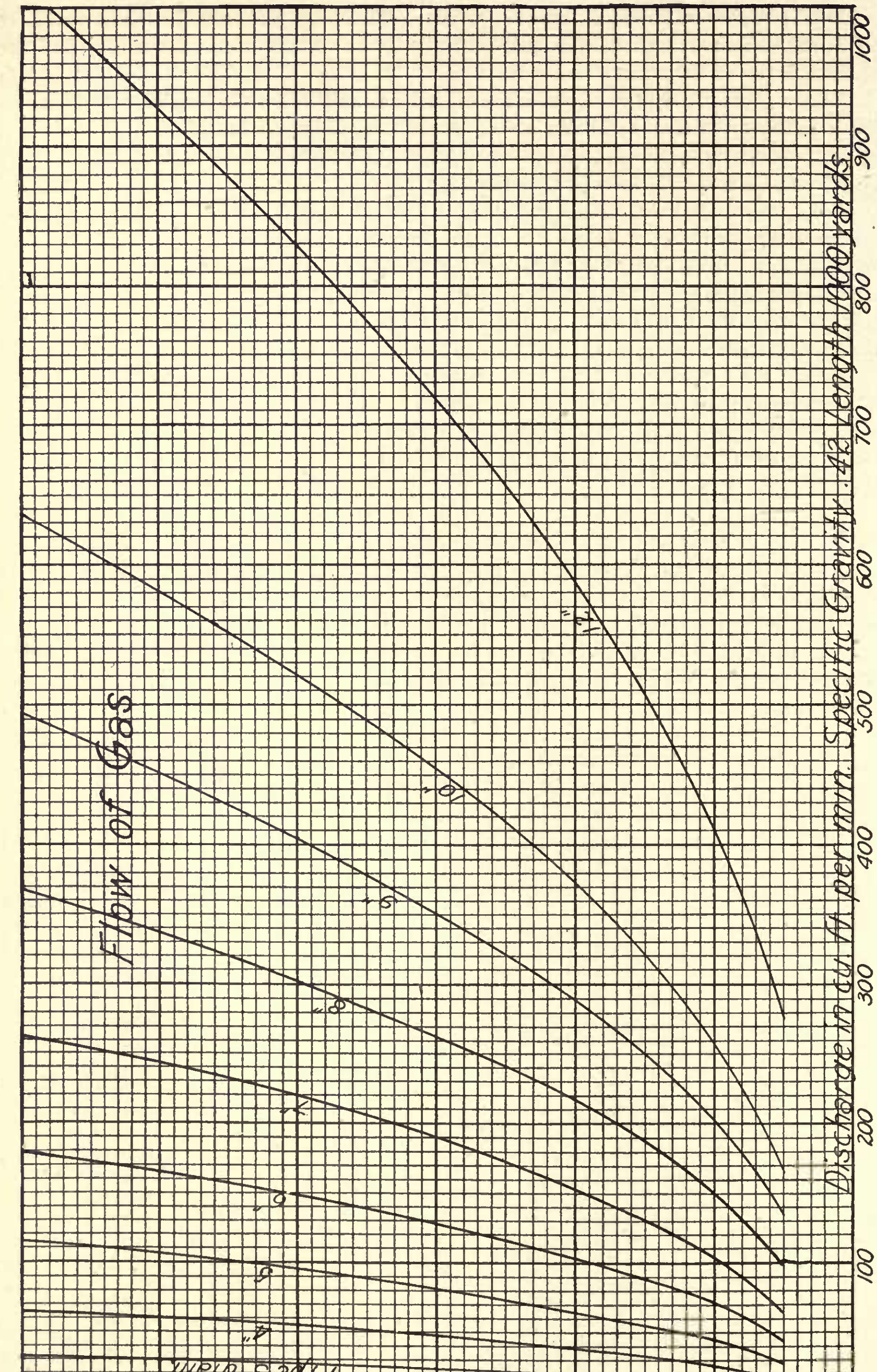
Diameter of main Blast Pipe, in inches.																														
1	1																													
2	5.7	2																												
3	16	2.7	3																											
4	32	5.7	2.	4																										
5	56	9.8	3.6	1.8	5																									
6	88	16	5.7	2.8	1.6	6																								
7	129	23	8.3	4.1	2.3	1.5	7																							
8	180	32	12	5.7	3.2	2.1	1.4	8																						
9	244	42	16	7.6	4.3	2.8	1.9	1.3	9																					
10	317	56	20	9.9	5.7	3.6	2.4	1.7	1.3	10																				
11	402	71	26	12	7.0	4.5	3.1	2.2	1.7	1.3	11																			
12	501	88	32	16	9.0	5.7	3.8	2.8	2.0	1.6	1.2	12																		
13	613	107	39	19	11	6.9	4.7	3.4	2.5	1.9	1.5	1.2	13																	
14	737	129	47	23	13	8.3	5.7	4.1	3.0	2.3	1.8	1.5	1.2	14																
15	876	152	56	27	16	9.9	6.7	4.8	3.6	2.8	2.2	1.8	1.4	1.2	15															
16	1026	180	65	32	18	11	7.9	5.7	4.2	3.2	2.6	2.1	1.7	1.4	1.2	16														
17	1197	208	76	37	21	13	9.2	6.6	4.9	3.8	2.9	2.4	2.0	1.6	1.4	1.2	17													
18	1375	239	88	43	24	16	10	7.7	5.7	4.3	3.4	2.8	2.3	1.9	1.6	1.3	1.2	18												
19	1580	275	100	49	28	18	12	8.8	6.5	5.	3.9	3.2	2.6	2.2	1.8	1.5	1.3	1.2	19											
20	1797	313	114	56	32	20	14	9.9	7.4	5.7	4.5	3.6	2.9	2.5	2.1	1.7	1.5	1.3	1.1	20										
22	2284	398	145	71	41	26	18	13	9.3	7.2	5.7	4.5	3.7	3.1	2.6	2.2	1.9	1.7	1.4	1.3	22									
24	2834	493	180	88	50	32	22	16	12	8.9	7.6	5.7	4.6	3.8	3.2	2.9	2.4	2.1	1.8	1.6	1.2	24								
26	3474	605	219	108	62	39	27	19	14	11	8.6	6.9	5.7	4.7	4.0	3.4	2.9	2.5	2.2	1.9	1.5	1.2	26							
28	4165	725	265	129	74	48	32	23	17	13	10	8.3	6.8	5.7	4.8	4.1	3.5	3.0	2.6	2.3	1.8	1.5	1.2	28						
30	4963	864	315	154	88	56	38	28	20	16	12	9.9	8.0	6.7	5.7	4.7	4.1	3.6	3.0	2.6	2.2	1.7	1.4	1.2	30					
36	7818	1361	497	243	139	88	60	45	32	25	19	16	13	11	8.9	7.6	6.5	5.7	5.0	4.5	3.4	2.7	2.2	1.9	1.6	36				
42	11488	2000	730	358	205	129	88	63	47	36	29	23	19	18	13	11	9.6	8.5	7.3	6.4	5.0	4.1	3.3	2.8	2.3	1.5	42			
48	15989	2792	1081	492	282	180	123	88	66	50	39	32	26	22	18	16	13	12	10	8.9	7.0	5.7	4.7	3.8	3.2	2.1	1.4	48		
54	21560	3753	1368	671	384	244	166	119	88	68	53	43	35	29	24	21	18	16	15	12	9.4	7.6	6.2	5.2	4.3	2.8	1.9	1.3	54	
60	27913	4879	1781	872	499	314	215	154	115	88	69	56	46	38	32	27	23	20	18	16	12	9.9	8.1	6.7	5.7	3.8	2.4	1.8	1.3	

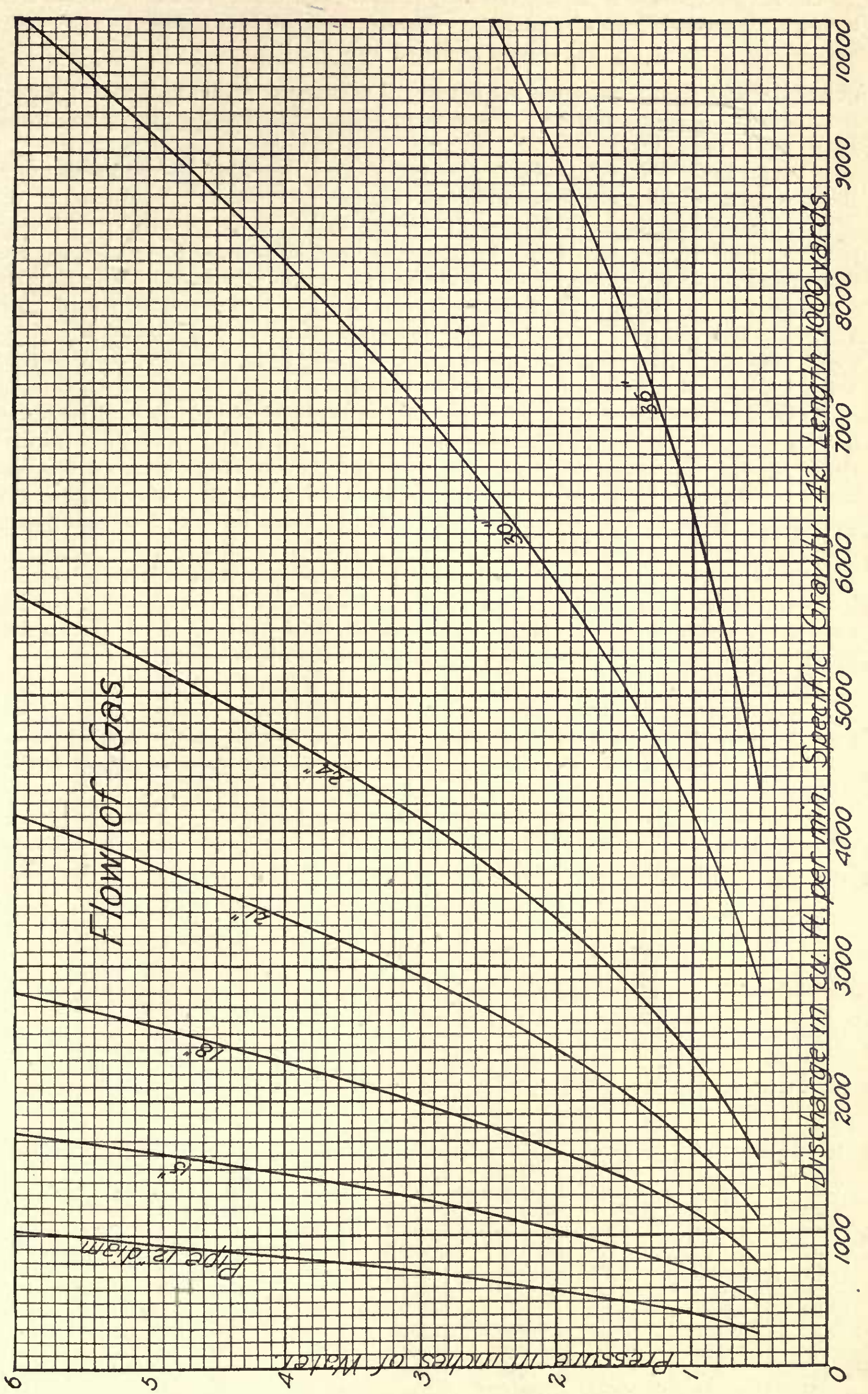
Parties putting up blast pipes are very liable to think, because the combined area of four 6-inch pipes is the same as one 12-inch pipe, that the four pipes will convey the same quantity of air with the same ease and freedom that the 12-inch will, whereas it actually does take 5·7—almost six 6-inch pipes. Again, 16 3-inch pipes have the combined area of one 12-inch pipe, but in actual practice it takes just 32 3-inch pipes to do the work of one 12-inch.

This is due to the excess of friction for every cubic foot of air in the small pipes over that in the large.

The large figures at the top of each column give the diameters in inches of the branch pipes.

The figures at the intersection of the horizontal line with the vertical give the number of pipes, of the diameter given at the top of the column, that will be equal in capacity for conveying air to one given opposite in the first column.


Flow of Gas
Discharge in cu. ft. per min. Specific Gravity .42 Length 1000 yards.
Pressure in inches of Water
Pipe 3" diam.
4"
5"
6"
7"
8"
9"
10"
12"
0
100
200
300
400
500
600
700
800
900
1000
1
2
3
4
5
6


Flow of Gas
Discharge in cu. ft. per min. Specific Gravity .42 Length 1000 yards.
Pressure in inches of Water
Pipe 12" diam.
15"
18"
21"
24"
30"
36"
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
1
2
3
4
5
6

The foregoing diagrams of the flow of gas have been made from the formula contained in "Practical Treatise on Heat," by Thomas Box, and is as follows :

$$Q = (H \times (3{\cdot}7D)^5 \div L)^{\frac{1}{2}},$$

in which Q = discharge cubic feet per minute; H = pressure (grade) in inches of water; D = diameter of pipe in inches; L = length in yards. The specific gravity (G) of the gas in this formula is ·42.

Formula in previous editions of this work was $Q = 1350\ D^2\sqrt{\frac{H\ D}{G\ L}}$, and agrees very closely with the results from the diagrams given below.

The diagrams for the flows of water (pages 785–790) in pipes are equally applicable to the movement of gaseous fluids, the velocities being directly as the square roots of their specific gravities. Thus, the water discharge of a conduit 2 feet in diameter under a grade of .0833′ per 100 feet in the diagram $n =$ ·015 is 5·5 cubic feet per second. By Tables of the Weight of Water (page 780) compared with that of air below it will be seen that at 62° F. the weight of a cubic foot of water is to that of air as 1 to 820. $5{\cdot}5 \times \sqrt{820} = 5{\cdot}5 \times 28{\cdot}636 = 157$ cubic feet per second.

As the diagrams for the flow of water are in feet per hundred and those of air or gas in inches, to convert the grades of the former into those of the latter, multiply the discharge by the square root of ·0833′ (1″) or ·2889′. Taking above example, diagram under 1:100 gives a discharge of 19 cubic feet per second, which × ·2889 × 28·636 = 157 cubic feet per second; but as the flow of air is usually given in cubic feet per minute, multiply by 60 and we obtain 9,420 cubic feet of air per minute.

In the movement of compressed air by the action of the pump the air is heated, and in its passage through the pipe this heat is gradually dissipated with a change of its specific gravity, for which allowance is to be made in velocity of movement. With illuminating gas the specific gravity of ·42 adopted in the diagram is a common one.

The products of combustion escaping into a large chimney may be taken as of the same specific gravity as illuminating gas at ·42, but the velocity in the general rules is much less than that given by the diagrams. There is no objection to large chimneys except in their cost and back draughts, which last may be met by uniformity of section without eddies or by a Venturi converging and diverging tube, at the bottom.

Insufficient draught in short chimneys, induced by positions or necessities of construction, is met by fans discharging directly into a fire-room or ash-pit, or by steam-jet blowers, as illustrated on page 368.

VOLUME AND WEIGHT OF DRY AIR

At Different Temperatures under a Constant Atmospheric Pressure of 29·92 Inches of the Barometer, the Volume at 32° F. being the Unit.

Temp. F.	Volume.	Weight per cub. ft., Pounds.	Temp. F.	Volume.	Weight per cub. ft., Pounds.	Temp. F.	Volume.	Weight per cub. ft., Pounds.
0°	·935	·0864	132°	1·204	·0671	325°	1·597	·0506
12	·960	·0842	142	1·224	·0659	350	1·648	·0490
22	·980	·0824	152	1·245	·0649	375	1·689	·0477
32	1·000	·0807	162	1·265	·0638	400	1·750	·0461
42	1·020	·0791	172	1·285	·0628	450	1·852	·0436
52	1·041	·0776	182	1·306	·0618	500	1·954	·0413
62	1·061	·0761	192	1·326	·0609	550	2·056	·0384
72	1·082	·0747	202	1·347	·0600	600	2·150	·0376
82	1·102	·0733	212	1·367	·0591	650	2·260	·0357
92	1·122	·0720	230	1·404	·0575	700	2·362	·0338
102	1·143	·0707	250	1·444	·0559	800	2·566	·0315
112	1·163	·0694	275	1·495	·0540	900	2·770	·0292
122	1·184	·0682	300	1·546	·0522	1000	2·974	·0268

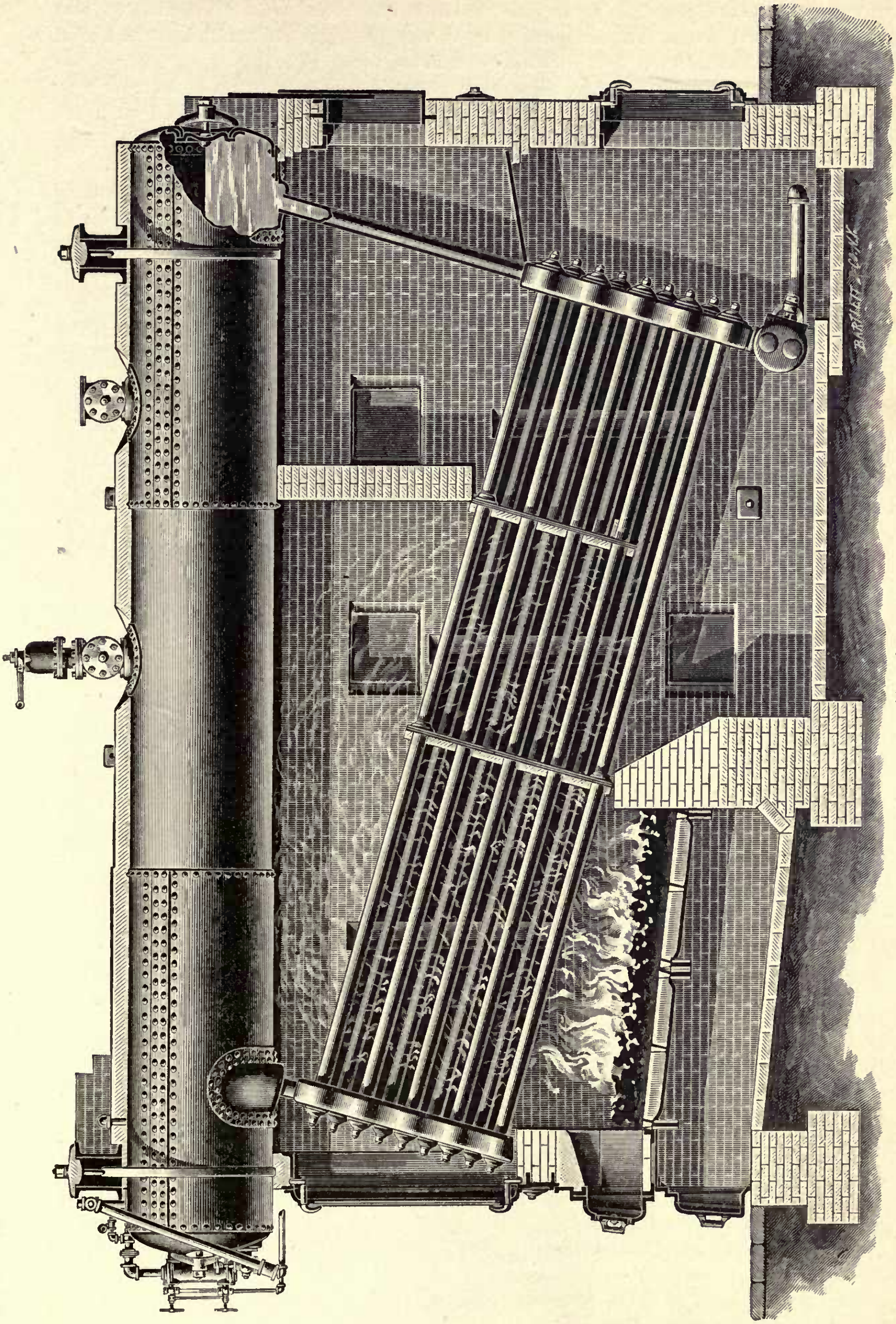


Fig. 1867.

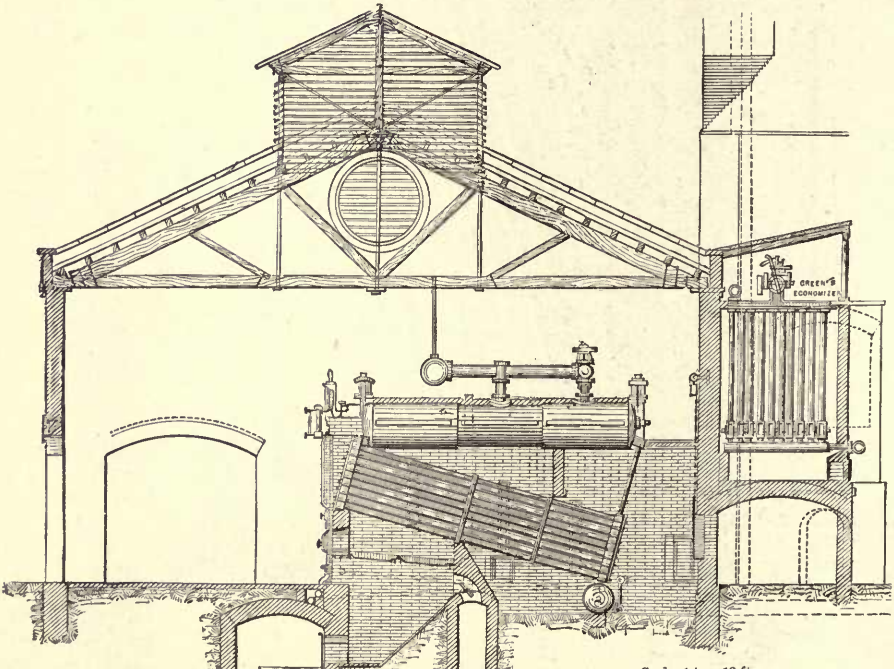
A Babcock and Wilcox water tube boiler (Fig. 1867) is composed of lap-welded wrought-iron tubes placed in an inclined position and connected with each other and with a horizontal steam and water drum, by vertical passages at each end, while a mud drum is connected to the rear and lowest point in the boiler.

The end connections are in one piece for each vertical row of tubes, and are of such form that the tubes are staggered. The sections thus formed are connected with the drum, and with the mud drum also, by short tubes expanded into bored holes. The openings for cleaning, opposite the end of each tube, are closed by hand-hole plates.

To utilize waste heat heaters are set in a chamber in connection with the flues leading to the chimney. Fig. 1868 is an elevation of one of these forms of apparatus, the Green economizer, consisting of ranges of vertical pipes, connected at the top and bottom with horizontal pipes, into which the feed water is introduced at the bottom and leaves at the top. The whole is inclosed in a brick chamber with the products of combustion passing among the pipes. The outsides of the pipes are cleaned by automatic scrapers. Where the heat is necessary to insure draft in the smoke flue there can be no economy in the apparatus, but an obstruction to the draft.

In operation, the fire is made under the front and higher ends of the tubes, and the products of combustion pass up between the tubes into a combustion chamber under the steam and water drum; from thence they pass down between the tubes, then once more up through the spaces between the tubes and off to the chimney. The water inside the tubes, as it is heated, tends to rise toward the higher end, and as it is converted into steam, the mingled column of steam and water rises through the vertical passages into the drum above the tubes, where the steam separates from the water, and the latter flows back to the rear and down again through the tubes in a continuous circulation.

The steam is taken out at the top of the steam drum, near the back end of the boiler, after it has thoroughly separated from the water.



Scale, 1 in.=13 ft.

FIG. 1868.

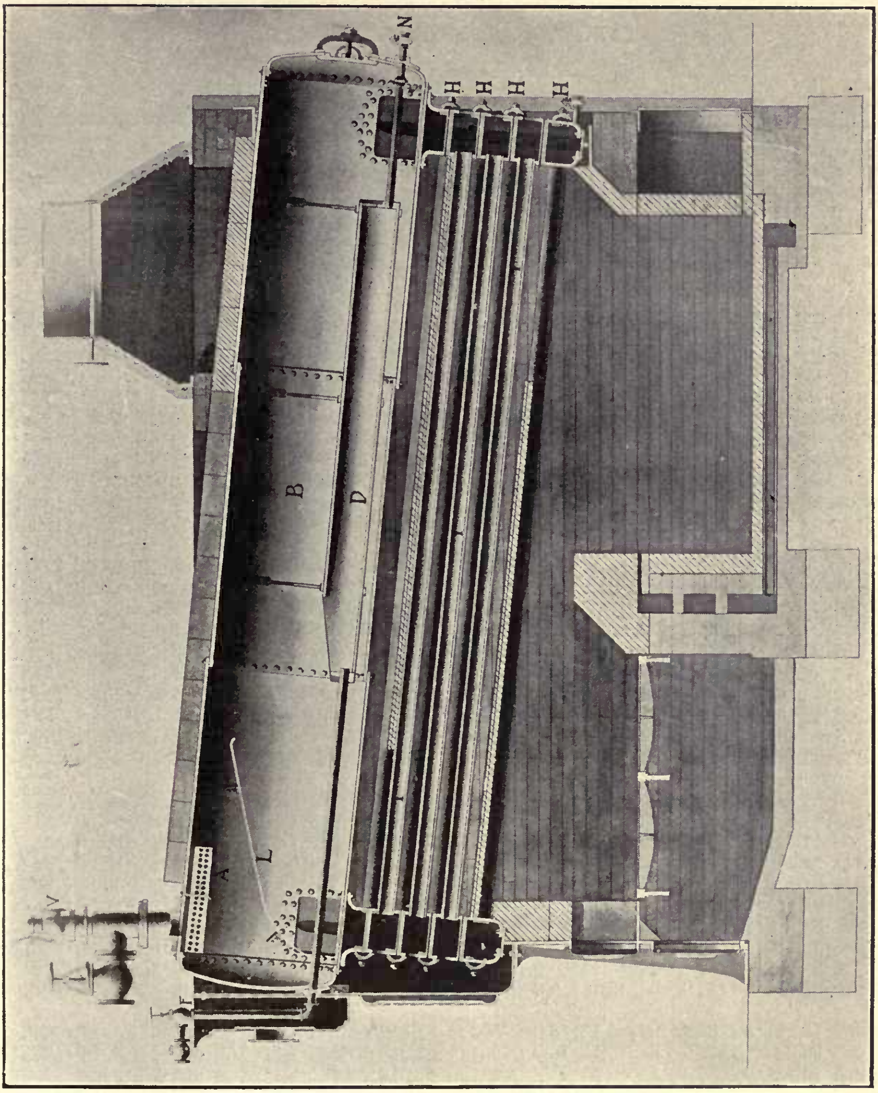


Fig. 1869.

The Heine boiler (Fig. 1869) is composed of wrought-iron tubes extending between the inside of two "water legs," or end connections, between the tubes and a steam and water drum placed above them. These end chambers are of approximately rectangular shape, drawn in at the top to fill the curvature of the shells. Each is composed of a head plate and a tube sheet, flanged all around and joined at bottom and sides by a butt strap of the same material, strongly riveted to both. They are further stayed by hollow stay bolts of hydraulic tubing of large diameter, so placed that two stays support each tube and hand hole. The water legs are joined to the shell by flanged and riveted joints, and the shells are cylinders with heads dished. The steam space in front is about two thirds the diameter of the shell, while at the rear the water occupies two thirds of the shell, the whole contents being equally divided between steam and water. On the top of the shell, near the front end, is riveted a nozzle for a steam and safety valve. A flue, or breeching, connects the furnace to the chimney.

TABLE OF SATURATED STEAM.

ENGLISH UNITS.

CECIL H. PEABODY, B. S.

Pressure, Pounds per Sq. Inch. p	Temperature, Degrees Fahr. t	Heat of the Liquid. q	Total Heat. λ	Heat of Vaporization. r	Specific Volume. s	DENSITY. Weight, in Pounds, of One Cubic Foot. γ	Pressure, Pounds per Sq. Inch. p	Temperature, Degrees Fahr. t	Heat of the Liquid. q	Total Heat. λ	Heat of Vaporization. r	Specific Volume. s	DENSITY. Weight, in Pounds, of One Cubic Foot. γ
1	101·99	70·0	1113·1	1043·0	334·6	0·00299	51	282·10	251·5	1168·0	916·5	8·259	0·1211
2	126·27	94·4	1120·5	1026·1	173·6	0·00576	52	283·32	252·7	1168·4	915·7	8·110	0·1233
3	141·62	109·8	1125·1	1015·3	118·4	0·00844	53	284·53	253·9	1168·7	914·8	7·968	0·1255
4	153·09	121·4	1128·6	1007·2	90·31	0·01107	54	285·72	255·1	1169·1	914·0	7·829	0·1277
5	162·34	130·7	1131·5	1000·8	73·22	0·01366	55	286·89	256·3	1169·4	913·1	7·696	0·1299
6	170·14	138·6	1133·8	995·2	61·67	0·01622	56	288·05	257·5	1169·8	912·3	7·568	0·1321
7	176·90	145·4	1135·9	990·5	53·37	0·01874	57	289·19	258·6	1170·1	911·5	7·443	0·1344
8	182·92	151·5	1137·7	986·2	47·07	0·02125	58	290·31	259·7	1170·5	910·8	7·323	0·1366
9	188·33	156·9	1139·4	982·5	42·13	0·02374	59	291·42	260·8	1170·8	910·0	7·208	0·1387
10	193·25	161·9	1140·9	979·0	38·16	0·02621	60	292·51	261·9	1171·2	909·3	7·096	0·1409
11	197·78	166·5	1142·3	975·8	34·88	0·02866	61	293·59	263·0	1171·5	908·5	6·987	0·1431
12	201·98	170·7	1143·6	972·9	32·14	0·03111	62	294·65	264·1	1171·8	907·7	6·882	0·1453
13	205·89	174·6	1144·7	970·1	29·82	0·03355	63	295·70	265·2	1172 1	906·9	6·779	0·1475
14	209·57	178·3	1145·8	967·5	27·79	0·03600	64	296·74	266·2	1172·4	906·2	6·680	0·1497
15	213·03	181·8	1146·9	965·1	26·15	0·03826	65	297·77	267·2	1172·7	905·5	6·583	0·1519
16	216·32	185·1	1147·9	962·8	24·59	0·04067	66	298·78	268·3	1173·0	904·7	6·490	0·1541
17	219·44	188·3	1148·9	960·6	23·22	0·04307	67	299·77	269·3	1173·3	904·0	6·401	0·1562
18	222·40	191·3	1149·8	958·5	22·00	0·04547	68	300·76	270·3	1173·6	903·3	6·314	0·1584
19	225·24	194·1	1150·7	956·6	20·90	0·04786	69	301·74	271·2	1173·9	902·7	6·228	0·1606
20	227·95	196·9	1151·5	954·6	19·91	0·05023	70	302·71	272·2	1174·3	902·1	6·144	0·1628
21	230·55	199·5	1152·3	952·8	19·01	0·05259	71	303·66	273·2	1174·6	901·4	6·063	0·1649
22	233·06	202·0	1153·0	951·0	18·20	0·05495	72	304·61	274·1	1174·9	900·8	5·984	0·1671
23	235·47	204·5	1153·7	949·2	17·45	0·05731	73	305·54	275·1	1175·2	900·1	5·908	0·1693
24	237·79	206·8	1154·4	947·6	16·76	0·05966	74	306·46	276·0	1175·4	899·4	5·834	0·1714
25	240 04	209·1	1155·1	946·0	16·13	0·06199	75	307·38	276·9	1175·7	898·8	5·762	0·1736
26	242·21	211·2	1155·8	944·6	15·55	0·06432	76	308·28	277·8	1176·0	898·2	5·691	0·1757
27	244·32	213·4	1156·5	943·1	15·00	0·06666	77	309·18	278·7	1176·2	897·5	5·621	0·1779
28	246·36	215·4	1157·1	941·7	14·49	0·06899	78	310·06	279·6	1176·5	896·9	5·554	0·1801
29	248·34	217·4	1157·7	940·3	14·03	0·07130	79	310·94	280·5	1176·8	896·3	5·488	0·1822
30	250·27	219·4	1158·3	938·9	13·59	0·07360	80	311·80	281·4	1177·0	895·6	5·425	0·1843
31	252·15	221·3	1158·8	937·5	13·18	0·07590	81	312·66	282·3	1177·3	895·0	5·362	0·1865
32	253·98	223·1	1159·4	936·3	12·78	0·07821	82	313·51	283·2	1177·6	894·4	5·301	0·1886
33	255·76	224·9	1159·9	935·0	12·41	0·08051	83	314·36	284·1	1177·8	893·7	5·240	0·1908
34	257·50	226·7	1160·4	933·7	12·07	0·08280	84	315·19	285·0	1178·1	893·1	5·182	0·1930
35	259·19	228·4	1161·0	932·6	11·75	0·08508	85	316·02	285·8	1178·3	892·5	5·125	0·1951
36	260·85	230·0	1161·5	931·5	11·45	0·08736	86	316·84	286·7	1178·6	891·9	5·069	0·1973
37	262·47	231·7	1162·0	930·3	11·16	0·08964	87	317·65	287·5	1178·8	891·3	5·014	0·1994
38	264·06	233·3	1162·5	929·2	10·88	0·09191	88	318·45	288·4	1179·1	890·7	4·961	0·2016
39	265·61	234·8	1163·0	928·2	10·62	0·09417	89	319·25	289·2	1179·3	890·1	4·909	0·2037
40	267·13	236·4	1163·4	927·0	10·37	0·09644	90	320·04	290·0	1179·6	889·6	4·858	0·2058
41	268·62	237·9	1163·9	926·0	10·13	0·09869	91	320·83	290·8	1179·8	889·0	4·808	0·2080
42	270·08	239·3	1164·3	925·0	9·906	0·10090	92	321·60	291·6	1180·0	888·4	4·760	0·2101
43	271·51	240·8	1164·8	924·0	9·690	0·10320	93	322·37	292·4	1180·3	887·9	4·712	0·2122
44	272·91	242·2	1165·2	923·0	9·484	0·10540	94	323·14	293·2	1180·5	887·3	4·665	0·2144
45	274·29	243·6	1165·6	922·0	9·287	0·10770	95	323·89	294·0	1180·7	886·7	4·619	0·2165
46	275·65	245·0	1166·0	921·0	9·097	0·10990	96	324·64	294·8	1181·0	886·2	4·574	0·2186
47	276·99	246·3	1166·4	920·1	8·914	0·11220	97	325·38	295·6	1181·2	885·6	4·530	0·2208
48	278·30	247·6	1166·8	919·2	8·740	0·11440	98	326·12	296·4	1181·4	885·0	4·486	0·2229
49	279·58	248·9	1167·2	918·3	8·573	0·11660	99	326·86	297·1	1181·6	884·5	4·444	0·2250
50	280·85	250·2	1167·6	917·4	8·414	0·11880	100	327·58	297·9	1181·9	884·0	4·403	0·2271

TABLE OF SATURATED STEAM—*Continued.*

Pressure, Pounds per Sq. Inch. p	Temperature, Degrees Fahr. t	Heat of the Liquid. q	Total Heat. λ	Heat of Vaporization. r	Specific Volume. s	Density. Weight, in Pounds, of One Cubic Foot. γ	Pressure, Pounds per Sq. Inch. p	Temperature, Degrees Fahr. t	Heat of the Liquid. q	Total Heat. λ	Heat of Vaporization. r	Specific Volume. s	Density. Weight, in Pounds, of One Cubic Foot. γ
101	328·30	298·6	1182·1	883·5	4·362	0·2293	151	358·78	330·5	1191·4	860·9	2·992	0·3342
102	329·02	299·4	1182·3	882·9	4·322	0·2314	152	359·30	331·1	1191·5	860·4	2·973	0·3363
103	329·73	300·1	1182·5	882·4	4·282	0·2335	153	359·82	331·6	1191·7	860·1	2·955	0·3384
104	330·43	300·9	1182·7	881·8	4·244	0·2356	154	360·34	332·2	1191·8	859·6	2·937	0·3405
105	331·13	301·6	1182·9	881·3	4·206	0·2378	155	360·86	332·7	1192·0	859·3	2·919	0·3426
106	331·83	302·3	1183·1	880·8	4·169	0·2399	156	361·37	333·3	1192·2	858·9	2·901	0·3447
107	332·52	302·0	1183·4	880·4	4·132	0·2420	157	361·88	333·8	1192·3	858·5	2·884	0·3467
108	333·20	303·8	1183·6	879·8	4·096	0·2441	158	362·39	334·3	1192·5	858·2	2·867	0·3488
109	333·88	304·5	1183·8	879·3	4·061	0·2462	159	362·90	334·9	1192·7	857·8	2·850	0·3509
110	334·56	305·2	1184·0	878·8	4·026	0·2484	160	363·40	335·4	1192·8	857·4	2·833	0·3530
111	335·23	305·9	1184·2	878·3	3·992	0·2505	161	363·90	335·9	1193·0	857·1	2·816	0·3551
112	335·89	306·6	1184·4	877·8	3·959	0·2526	162	364·40	336·4	1193·1	856·7	2·799	0·3572
113	336·55	307·3	1184·6	877·3	3·926	0 2547	163	364·90	337·0	1193·3	856·3	2·783	0·3593
114	337·20	308·0	1184·8	876·8	3·894	0·2568	164	365·39	337·5	1193·4	855·9	2·767	0·3614
115	337·86	308·7	1185·0	876·3	3·862	0·2589	165	365·88	338·0	1193·6	855·6	2·751	0·3635
116	338·50	309·4	1185·2	875·8	3·831	0·2610	166	366·37	338·5	1193·7	855·2	2·736	0·3655
117	339·14	310·0	1185·4	875·4	3·801	0·2631	167	366·85	339·0	1193·9	854·9	2·721	0·3675
118	339·78	310·7	1185·6	874·9	3·770	0·2653	168	367·33	339·5	1194·0	854·5	2·706	0·3695
119	340·42	311·4	1185·8	874·4	3·740	0·2674	169	367·81	340·0	1194·2	854·2	2·691	0·3716
120	341·05	312·0	1186·0	874·0	3·711	0·2695	170	368·29	340·5	1194·3	853·8	2·676	0·3737
121	341·67	312·7	1186·2	873·5	3·683	0·2715	171	368·77	341·0	1194·4	853·4	2·661	0·3758
122	342·29	313·3	1186·3	873·0	3·655	0·2736	172	369·24	341·5	1194·6	853·1	2·647	0·3778
123	342·91	314·0	1186·5	872·5	3·627	0·2757	173	369·71	342·0	1194·7	852·7	2·632	0·3799
124	343·52	314·6	1186·7	872·1	3·599	0·2779	174	370·18	342·5	1194·8	852·3	2·618	0·3820
125	344·13	315·2	1186·9	871·7	3·572	0·2800	175	370·65	343·0	1195·0	852·0	2·603	0·3841
126	344·73	315·9	1187·1	871·2	3·546	0·2820	176	371·12	343·5	1195·1	851·6	2·589	0·3862
127	345·33	316·5	1187·3	870·8	3·520	0·2841	177	371·59	344·0	1195·3	851·3	2·575	0·3883
128	345·93	317·1	1187·4	870·3	3·494	0·2862	178	372·05	344·4	1195·4	851·0	2·561	0·3904
129	346·53	318·7	1187·6	869·9	3·469	0·2883	179	372·51	344·9	1195·6	850·7	2·548	0·3925
130	347·12	318·4	1187·8	869·4	3·444	0·2904	180	372·97	345·4	1195·7	850·3	2·535	0·3945
131	347·71	319·0	1188·0	869·0	3·419	0·2925	181	373·43	345·9	1195·9	850·0	2·522	0·3966
132	348·29	319·6	1188·2	868·6	3·395	0·2946	182	373·88	346·4	1196·0	849·6	2·508	0·3987
133	348·87	320·2	1188·4	868·2	3·371	0·2967	183	374·33	346·8	1196·1	849·3	2·495	0·4008
134	349·45	320·8	1188·5	867·7	3·347	0·2988	184	374·78	347·3	1196·2	848·9	2·482	0·4029
135	350·03	321·4	1188·7	867·3	3·323	0·3009	185	375·23	347·8	1196·4	848·6	2·470	0·4049
136	350·60	322·0	1188·9	866·9	3·300	0·3030	186	375·68	348·2	1196·5	848·3	2·457	0·4070
137	351·17	322·6	1189·0	866·4	3·277	0·3051	187	376·12	348·7	1196·6	847·9	2·445	0·4090
138	351·73	323·2	1189·2	866·0	3·255	0·3072	188	376·56	349·2	1196·8	847·6	2·432	0·4111
139	352·29	323·8	1189·4	865·6	3·234	0·3092	189	377·00	349·6	1196·9	847·3	2·420	0·4132
140	352·85	324·4	1189·5	865·1	3·212	0·3113	190	377·44	350·1	1197·1	847·0	2·408	0·4153
141	353·40	325·0	1189·7	864·7	3·191	0·3134	191	377·88	350·5	1197·2	846·7	2·396	0·4174
142	353·95	325·6	1189·9	864·3	3·170	0·3155	192	378·32	351·0	1197·3	846·3	2·385	0·4194
143	354·50	326·1	1190·1	864·0	3·149	0·3176	193	378·75	351·4	1197·4	846·0	2·373	0·4215
144	355·05	326·7	1190·2	863·5	3·128	0·3197	194	379·18	351·9	1197·6	845·7	2·361	0·4236
145	355·59	327·2	1190·4	863·2	3·107	0·3218	195	379·61	352·4	1197·7	845·3	2·349	0·4257
146	356·13	327·8	1190·6	862·8	3·087	0·3239	196	380·04	352·8	1197·8	845·0	2·337	0·4278
147	356·67	328·3	1190·7	862·4	3·068	0·3259	197	380·47	353·3	1198·0	844·7	2·325	0·4298
148	357·20	328·9	1190·9	862·0	3·049	0·3280	198	380·89	353·7	1198·1	844·4	2·314	0·4318
149	357·73	329·4	1191·0	861·6	3·030	0·3300	199	381·31	354·1	1198·2	844·1	2·304	0·4338
150	358·26	330·0	1191·2	861·2	3·011	0·3321	200	381·73	354·6	1198·4	843·8	2·294	0·4359

EXPANSIVE WORKING OF STEAM.

Cut-off in percentage of stroke.	Number of expansions.	MEAN ABSOLUTE PRESSURE, IN DECIMAL PARTS, OF ABSOLUTE PRESSURE OF ADMISSION.		Cut-off in percentage of stroke.	Number of expansions.	MEAN ABSOLUTE PRESSURE, IN DECIMAL PARTS, OF ABSOLUTE PRESSURE OF ADMISSION.	
		Dry saturated steam.	Moderately moist steam.			Dry saturated steam.	Moderately moist steam.
·800	1¼	·9760	·9784	·1	10	·3145	·3303
·667	1½	·9403	·9366	·095	10½	·3035	·3189
·571	1¾	·8857	·8910	·091	11	·2933	·3089
·500	2	·8394	·8465	·087	11½	·2839	·2991
·444	2¼	·7960	·8050	·083	12	·2751	·2904
·400	2½	·7560	·7664	·080	12½	·2669	·2818
·364	2¾	·7200	·7316	·077	13	·2592	·2742
·333	3	·6870	·6997	·074	13½	·2520	·2666
·308	3¼	·6570	·6705	·071	14	·2452	·2599
·286	3½	·6300	·6437	·069	14½	·2388	·2532
·267	3¾	·6051	·6192	·067	15	·2327	·2472
·250	4	·5820	·5965	·065	15½	·2270	·2411
·235	4¼	·5608	·5757	·063	16	·2216	·2358
·222	4½	·5410	·5564	·061	16½	·2164	·2303
·211	4¾	·5230	·5386	·059	17	·2115	·2255
·200	5	·5060	·5218	·057	17½	·2069	·2205
·191	5¼	·4904	·5063	·056	18	·2024	·2161
·182	5½	·4760	·4918	·054	18½	·1982	·2116
·174	5¾	·4620	·4781	·053	19	·1942	·2181
·167	6	·4490	·4653	·051	19½	·1903	·2034
·160	6¼	·4370	·4533	·050	20	·1866	·1998
·154	6½	·4250	·4418	·048	21	·1796	·1926
·148	6¾	·4150	·4311	·045	22	·1736	·1860
·143	7	·4046	·4208	·043	23	·1672	·1790
·138	7¼	·3950	·4112	·042	24	·1610	·1741
·133	7½	·3857	·4020	·040	25	·1566	·1688
·129	7¾	·3770	·3933	·038	26	·1518	·1638
·125	8	·3688	·3849	·037	27	·1474	·1591
·121	8¼	·3608	·3769	·036	28	·1431	·1547
·117	8½	·3533	·3694	·034	29	·1392	·1506
·114	8¾	·3461	·3622	·033	30	·1355	·1467
·111	9	·3392	·3552				
·108	9¼	·3326	·3494				
·105	9½	·3274	·3422				
·103	9¾	·3203	·3359				

TABLE OF FACTORS OF EVAPORATION.

Pressure Pounds per Sq. Inch.	Boiling Point T_1. Fahr.	INITIAL TEMPERATURE OF FEED WATER, T_2.										
		32°	50°	68°	86°	104°	122°	140°	158°	176°	194°	212°
14·70	212°	1·19	1·17	1·15	1·13	1·11	1·10	1·08	1·06	1·04	1·02	1·00
20·78	230	1·20	1·18	1·16	1·14	1·12	1·10	1·08	1·06	1·04	1·02	1·01
28·82	248	1·20	1·18	1·16	1·14	1·13	1·11	1·09	1·07	1·05	1·03	1·01
39·26	266	1·21	1·19	1·17	1·15	1·13	1·11	1·09	1·07	1·06	1·04	1·02
52·56	284	1·21	1·20	1·18	1·16	1·14	1·12	1·10	1·08	1·06	1·04	1·02
69·27	302	1·22	1·20	1·18	1·16	1·14	1·12	1·11	1·09	1·07	1·05	1·03
89·95	320	1·22	1·21	1·19	1·17	1·15	1·13	1·11	1·09	1·07	1·05	1·03
115·22	338	1·23	1·21	1·19	1·17	1·15	1·14	1·12	1·10	1·08	1·06	1·04
145·75	356	1·23	1·22	1·20	1·18	1·16	1·14	1·12	1·10	1·08	1·06	1·04
182·27	374	1·24	1·22	1·20	1·18	1·17	1·15	1·13	1·11	1·09	1·07	1·05
225·56	392	1·24	1·23	1·21	1·19	1·17	1·15	1·13	1·11	1·09	1·07	1·06
276·54	410	1·25	1·23	1·22	1·20	1·18	1·16	1·14	1·12	1·10	1·08	1·06
336·26	428	1·25	1·24	1·22	1·20	1·18	1·16	1·14	1·12	1·11	1·09	1·07

The Twenty-eighth Street Central Station of the United Electric Light and Power Company. By H. W. York, "Trans. A. S. C. E.," March 18, 1896.

In this station 20,000 H. P. of engines, together with the boilers, condensing apparatus, dynamos, and switchboard, and storage for 6,000 tons of coal, are all on a plot of ground 160 feet 11 inches by 197 feet 6 inches. All machinery, including the boilers, is on the ground floor, and yet there is plenty of light, air, and ample space for working around all the apparatus. Fig. 1870 is a plan of the foundation walls and piers of the

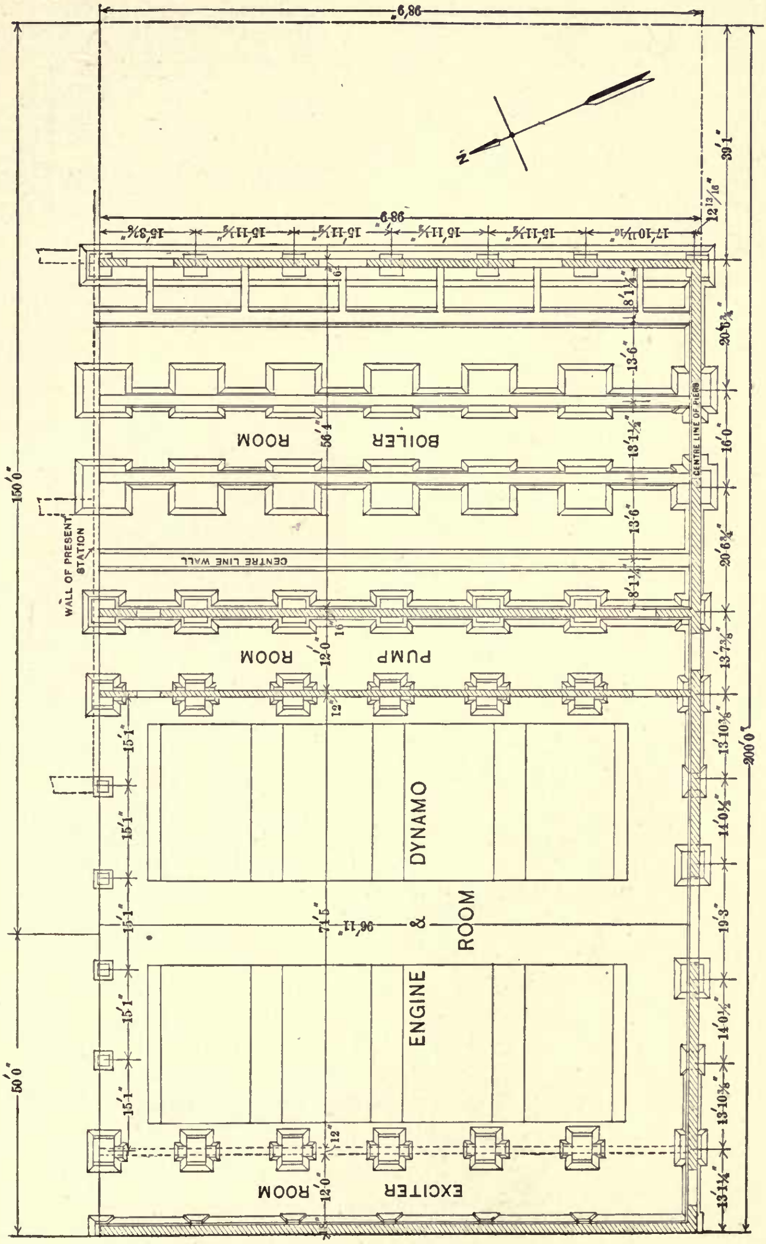


Fig. 1870.

building. The entire front wall is hollow and carried up above the roof to prevent the noise of the machinery annoying the patients in Bellevue Hospital, which is directly across the street. Fig. 1871 is a cross-section of the entire structure.

Loop System.—The main steam and exhaust piping is shown in plan on Fig. 1872. A 16-inch header is run the length of the boiler-room, and a similar header is run the length of the engine room between the two rows of foundations and parallel to the boiler-room header. Each of these headers are divided into five sections by means of four gate-

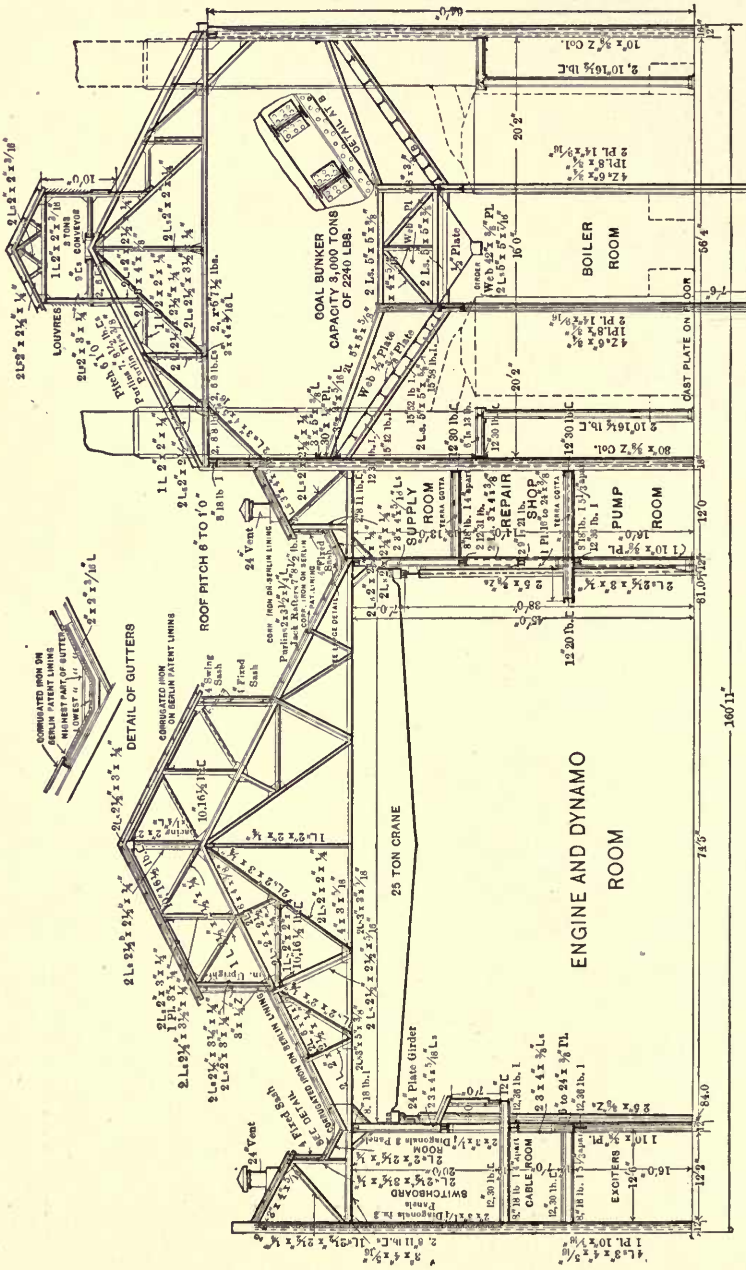


FIG. 1871.

valves, and each section of the boiler-room header is connected to the corresponding section of the engine-room header by a 14-inch branch rising from the top of one and discharging into the top of the other, a valve being placed on each end where a connection is made to the header. Each boiler has an independent connection to the boiler-room header, supplied with two stop-valves, one in the customary position just beyond the safety-valves, and the other at the point where the pipe enters the header. This second valve has its stem extended through the wall into the repair shop, so that in case of trouble any boiler may be cut out of a room having no communication with the boiler house.

Each engine on the east side of the engine room is connected to one of the 14-inch

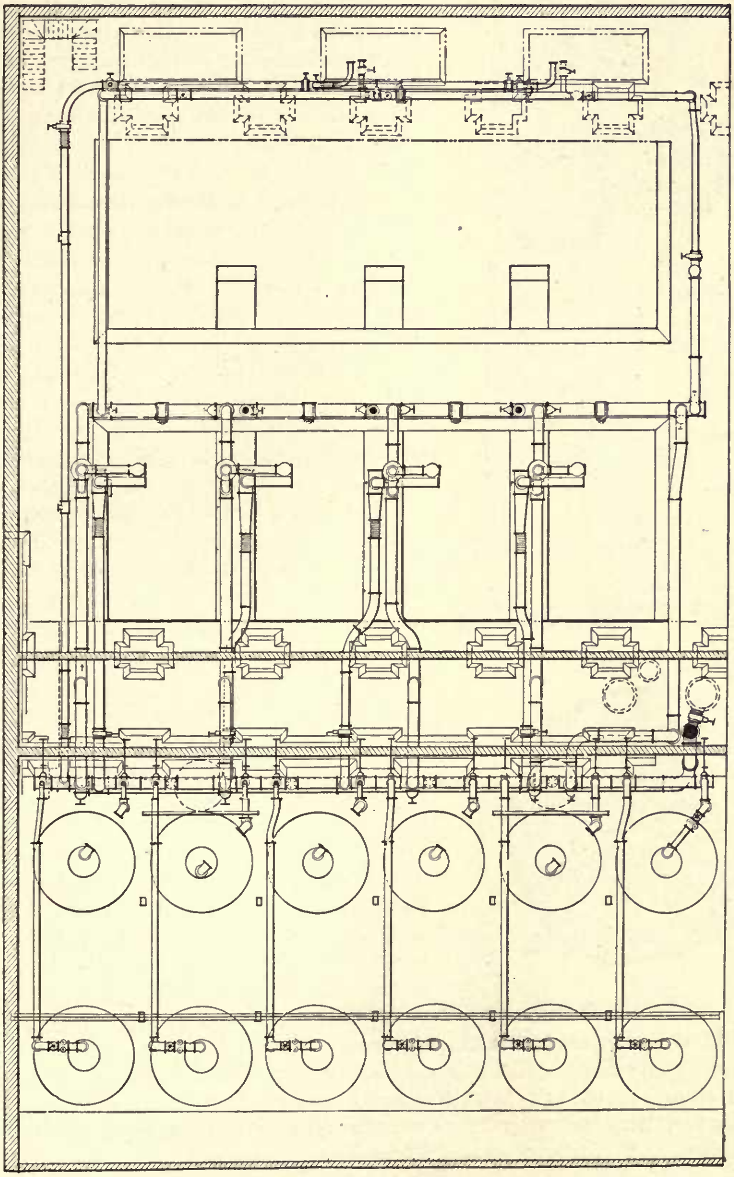
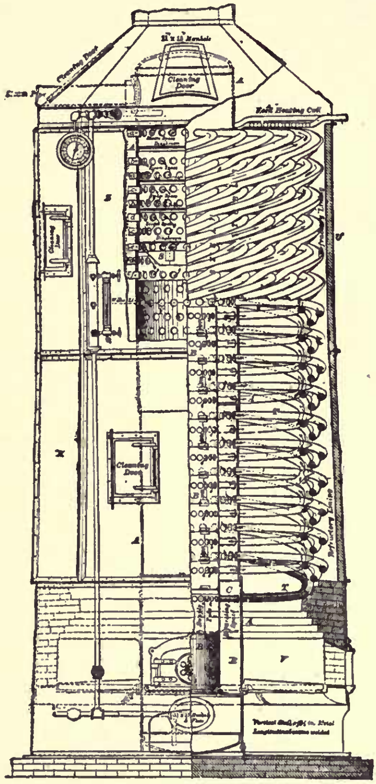
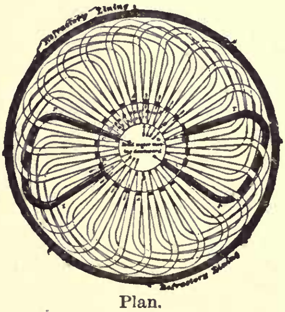


FIG. 1872.

branches previously mentioned, while outlets are left on the engine-room header for connections to the west row of the engines as soon as they are placed in position. In case any section of the engine-room header is cut out, one engine connected to this section can be fed directly from the 14-inch branch, leaving only one which can not be run, and in case any section of the boiler-room header is cut out no engine need be shut down, as the one connected to the 14-inch branch can be fed back from the engine-room header.



Elevation.



Plan.
FIG. 1873.

The boilers are of the upright water-tube type of the Clonbrock pattern, and are in 600 H.-P. units, occupying little ground room per unit of capacity (Fig. 1873).

The conveyer for handling coal and ashes consists of an endless chain of gravity buckets, which are loaded by means of a filler and can be dumped at any desired point. The driver is in the north end of the ventilator over the coal bunker. The coal filler is in a vault under the sidewalk, and the coal is dumped into this apparatus through a grating situated about the street level. After being deposited in the buckets the coal is carried up into the ventilator over the coal bunker and dumped into any portion of the bunker desired. From the hoppers in the bottom of the bunker the coal is spouted to the different boilers. The arrangement is such that the coal trims itself and will continue running down the spouts as required and without assistance so long as any remains in the bunker.

Under each boiler is an ash hopper delivering the ashes to a second movable filler, which deposits them in the buckets of the conveyer when it is not used for coal. The conveyer dumps the ashes at a point from which they are spouted over to a tank in the southeast corner of the coal bunker.

The engines are Westinghouse double-acting "Columbian steepled compounds." Fig. 1874 shows one of these engines in section. The low pressure is placed over the high pressure, and both pistons are connected to the same rod. The crank is inclosed in the same manner as the Westinghouse engines. The low-pressure valve is operated by a fixed eccentric placed inside the crank case, while the high-pressure valve receives its motion from a shifting eccentric outside the crank case, operated automatically by the governor, which is placed on the shaft outside of the eccentric. The low-pressure valve is of the slide-valve type, while that for the high-pressure cylinder is a hollow piston valve, being constructed in this manner to allow the exhaust from the lower end of the high-pressure cylinder to pass up through it. Diameter high-pressure cylinder, $21\frac{1}{2}$ inches; diameter low-pressure cylinder, 37 inches; stroke, 22 inches. The speed is 200 revolutions per minute and the rated horse power 1,200 when operating condensing, with 150 pounds initial steam pressure.

Each main engine is directly connected to a 600-kilowatt Westinghouse alternator by a rigid coupling, both engine and generator being set on a firm cast-iron bedplate. The generator has but one bearing, the armature being swung between the engine and this single support.

For exciting the fields of the alternators, 75-kilowatt direct-current Westinghouse dynamos of the railway generator type are used.

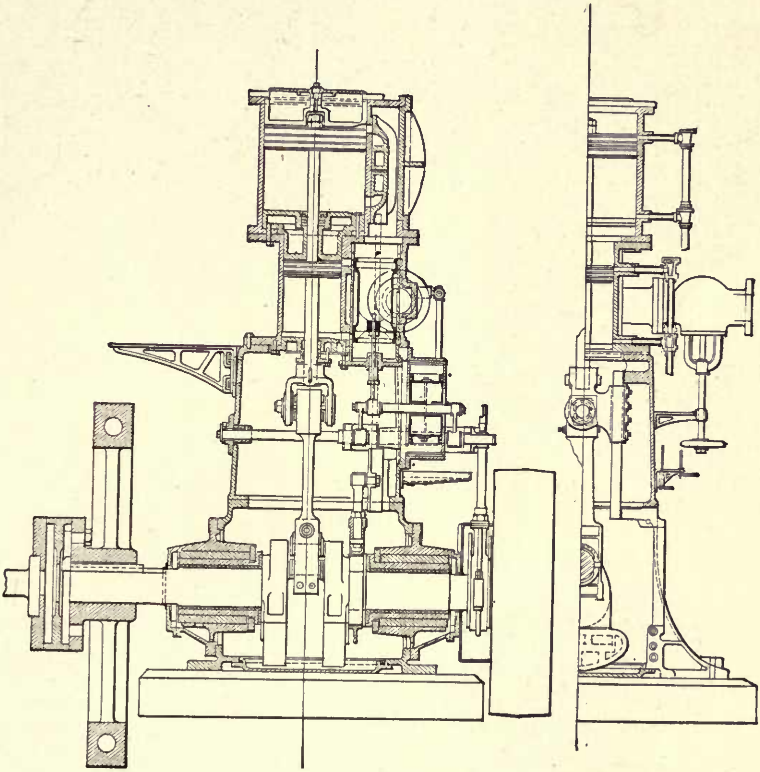
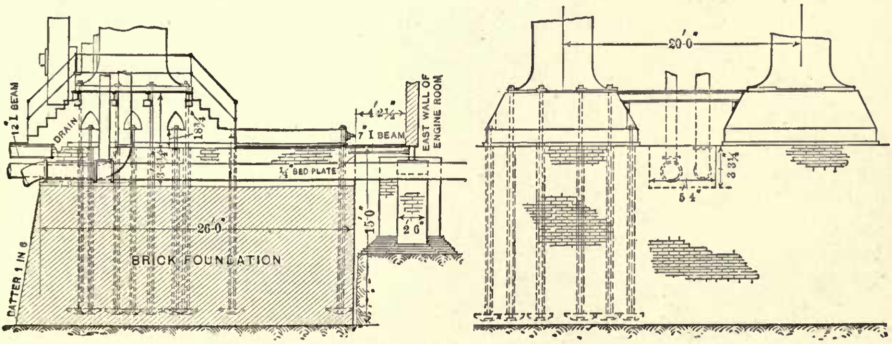
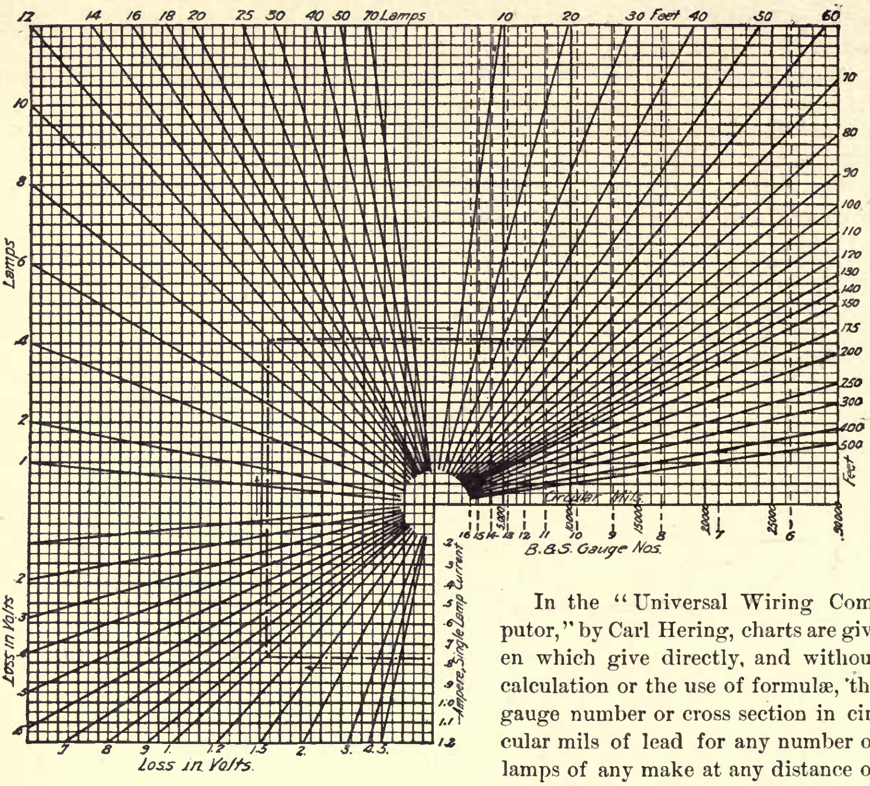


Fig. 1874.



FOUNDATION FOR VERTICAL ENGINES.



In the "Universal Wiring Computor," by Carl Hering, charts are given which give directly, and without calculation or the use of formulæ, the gauge number or cross section in circular mils of lead for any number of lamps of any make at any distance or for any loss. One of these tables is given above as an illustration on "graphics."

Follow the general direction of the broken line and the arrows from one set of diagonals to the next.

Example: What size wire is required for 10 lamps of ·775 ampères each, at 50 feet, for a loss of one volt?

Solution: Starting with the current for one lamp, ·775 ampères (see scale below centre), follow it to the left until it intersects the diagonal representing one volt loss, thence up to the diagonal representing 10 lamps, thence to the right to the diagonal representing 50 feet, and from here down to the scale of the circular mils or gauge numbers, on which the reading is found to be about 8,200 circular mils, or a No. 11 B. S. wire.

Fig. 1875 is an incandescent electric lamp socket of the Bryant Electric Company.

Fig. 1876 is a switch of the Hart and Hegeman Company.

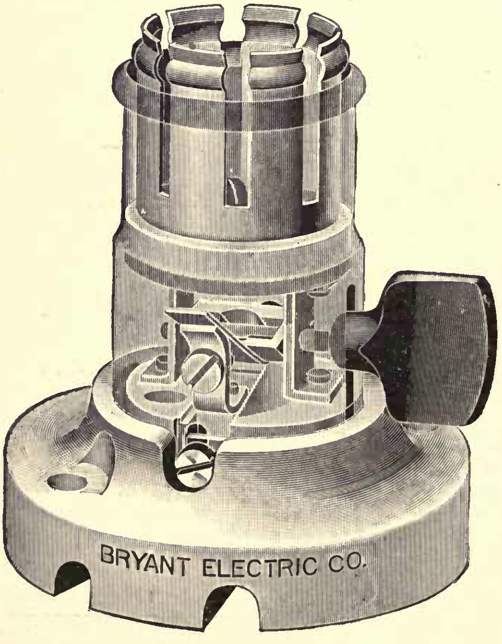


Fig. 1875.

All of the above drawings are made in a style in which the exterior shell is transparent, showing the interior mechanism, and known as "ghost cuts."

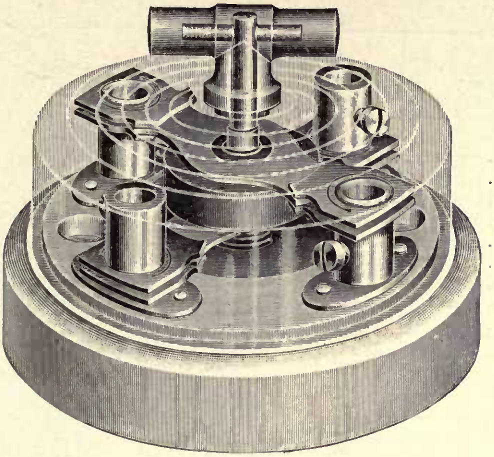
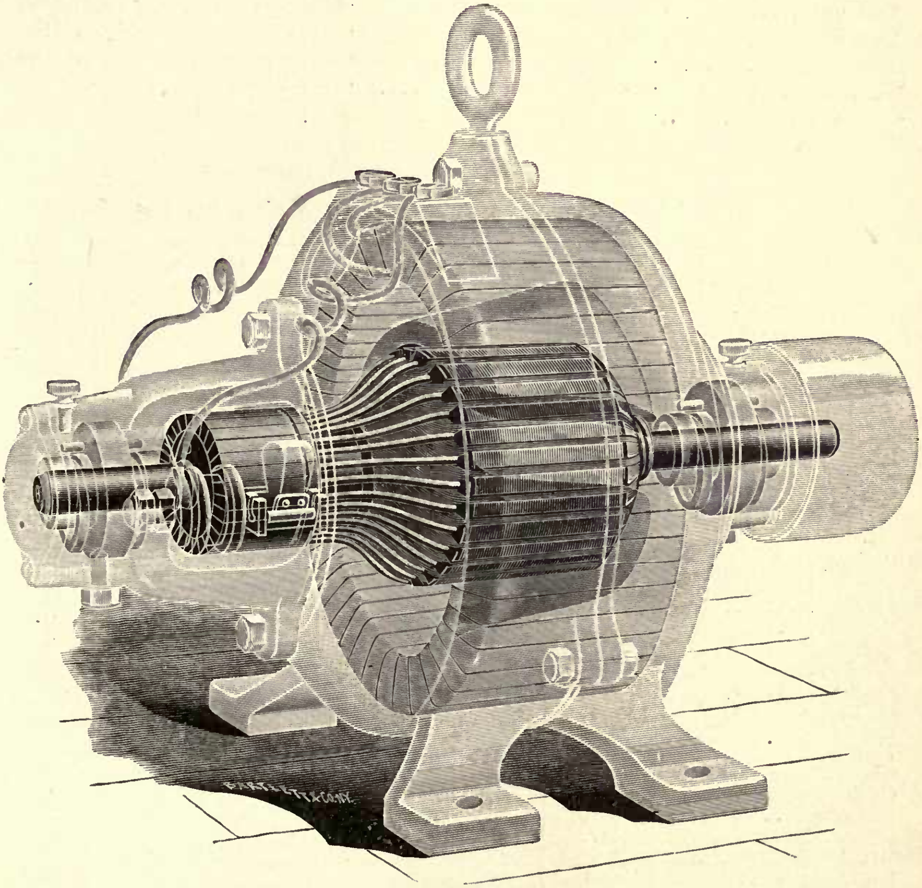


FIG. 1876.

No. B. & S. Guage.	Diameter, Mils.	Area. Circular Mils (d^2) 1 mil = ·001 in.	Ohms pr 1000 ft. at 70° F. Current allowed by Underwriter's Code.
0000	460·000	211600·00	175
000	409·640	167805·00	145
00	364·800	133079·04	120
0	324·860	105534·03	100
1	289·300	83694·20	95
2	257·630	66373·00	70
3	229·420	52633·40	60
4	204·310	41742·57	50
5	181·940	33102·00	45
6	162·020	26250·50	35
7	144·280	20816·70	30
8	128·490	16509·00	25
9	114·430	13094·00	
10	101·890	10381·00	20
11	90·742	8234·10	
12	80·808	6529·90	15
13	71·961	5178·40	
14	64·084	4106·80	10



LUNDELL MOTOR.

TABLE OF DENSITY OF GASES AND VAPOURS, AIR AT THE SAME TEMPERATURE AND PRESSURE BEING 1·0; ALSO THE WEIGHT OF A CUBIC FOOT AT 62° FAHR., UNDER AN ATMOSPHERIC PRESSURE OF 29·92 INCHES OF MERCURY.

	Density of air.	Specific gravity.	Weight of cubic foot in pound.	Cubic feet.
Air (atmospheric)	1·00000	·001221	·07610	13·14
Hydrogen gas	·06926	·000085	·00527	189·70
Oxygen gas	1·10563	·001350	·08414	11·88
Nitrogen gas	·97137	·001185	·07383	13·54
Carbonic-acid gas	1·52901	·001870	·11636	8·59
Carbonic-oxide gas	·9674	·00118	·07364	13·60
Vapour of water	·6235	·000761	·04745	21·07
" " alcohol	1·589	·00194	·12092	8·27
" " sulphuric ether	2·586	·00316	·19680	5·08
" " oil of turpentine	4·760	·00581	·36224	2·76

SPECIFIC GRAVITY OF LIQUIDS AT 60° FAHR.

Acid, muriatic	1·200	Ether, sulphuric	·720	Oil, whale	·920
Acid, nitric	1·217	Oil, linseed	·940	Tar	1·000
Acid, sulphuric	1·849	Oil, olive	·920	Vinegar	1·080
Alcohol, pure	·794	Oil, petroleum	·780–·880	Water	1·000
Ammonia, 27·9 per cent	·891	Oil, turpentine	·870	Water, sea	1·026–1·030
Carbon disulphide	1·260				

SPECIFIC GRAVITIES AND WEIGHTS OF EARTHS, ETC.

Substance.	Common specific gravity.	Average weight per cubic foot.	Substance.	Common specific gravity.	Average weight per cubic foot.
Alabaster	2·73	171	Gypsum	2·24	140
Asbestos	3·57	192	Lime, quick	·80	50
Asphalt, California	1·13	70	Limestone	2·88	180
" Trinidad	1·40	87	Magnesia carbonate	2·4	150
Barytes	4·0–4·86	250–304	Marble	2·72	170
Basalt	2·74	171	Masonry, concrete	2·08	130
Borax	1·71	107	" rubble	2·08	130
Brick, masonry	1·6–1·8	100–115	" granite	2·72	170
Bricks	1·6–2·16	100–125	" limestone	2·72	170
" fire	2·3	140–150	" sandstone	2·24	140
Cement, Portland	1·28	80	Mica	2·80	175
" Rosendale	·96	60	Porphyry	2·80	175
Clay	1·93	121	Pumice stone	·96	60
Coal	1·40	80–95	Quartz	2·64	165
Coal tar	1·24–1·30	77–81	Red lead	8·96	560
Coke	1·0	62	Resin	1·09	68
Concrete	2·08	120–140	Rubber, pure	·95	60
Earth	....	80–110	Salt, common and rock	2·16	135
Emery	4·0	250	Sand	1·60	100
Feldspar	2·60	162	Sandstone	2·32	145
Glass	....	150–200	Slate	2·80	175
Gneiss and granite	....	160–170	Soapstone	2·72	170
Graphite	2·20	138	Sulphur	2·03	127
Gravel	....	110	Terra cotta	1·95	122
Grindstone	2·14	135	Trap	2·88	180
Gutta-percha	·98	61			

SPECIFIC GRAVITY AND WEIGHT OF WOOD.

Wood.	Specific gravity.	Average weight per cubic foot.	Wood.	Specific gravity.	Average weight per cubic foot.
Apple	·73– ·80	47	Lignum-vitæ	·65–1·33	62
Ash	·60– ·84	45	Lime	·80	50
Bamboo	·31– ·40	22	Linden	·60	37
Beech	·62– ·85	46	Locust	·73	46
Birch	·56– ·74	41	Mahogany	·56–1·06	51
Box	·91–1·33	70	Maple	·57– ·79	42
Butternut	·38	24	Oak, live	·96–1·10	64
Cedar	·49– ·75	39	" white	·69– ·86	48
Cherry	·61– ·72	41	" red	·73– ·75	46
Chestnut	·46– ·66	35	Palmetto	....	..
Cork	·24	15	Pine, long leaf	·70	44
Cypress	·41– ·66	33	" white	·35– ·55	25
Ebony	1·13–1·33	76	" yellow	·46– ·76	38
Elm	·55– ·78	38	Poplar	·38– ·58	30
Fir	·48– ·70	37	Spruce	·40– ·50	28
Gum	·84–1·00	57	Sycamore	·59– ·62	37
Hackmatack	·59	37	Tamarack	·40	25
Hemlock	·36– ·41	24	Teak	·66– ·98	51
Hickory	·69– ·94	48	Walnut	·50– ·67	36
Hornbeam	·76	47	" black	·50	31

PROPERTIES OF METALS.

Metal.	Specific gravity.	Weight per cubic foot.	Melting point, degrees Fahr.	Tensile strength, pounds per square inch.	Expansion in 100 ft. from temp. 32° to 212° F.
Aluminum	2·63	166	1,400	26,000	
Antimony	6·76	421	842	....	·1083
Bismuth	9·82	612	510	6,400	·1392
Cadmium	8·65	539			
Calcium	1·58				
Chromium	5·00				
Cobalt	8·60				
Copper	8·7 to 8·9	450	1,930	20,000 to 30,000	·1722
Gold, pure	19·30	1,215	1,915	....	·1552
" 22 carats	17·49				
" 20 "	15·71				
Iron, cast	7·20	450	2,000 to 2,200	....	·1109
" wrought	7·68	480	2,700 to 2,900	46,000 to 48,000	·1220
Lead	11·36	709	625	....	·2848
Magnesium	1·70	106	1,200		
Manganese	8·00	499			
Mercury, 60°	13·58	847	662		
Nickel	8·3 to 8·9	537	3,000		
Platinum	21·50	1,341	....	....	·0857
Potassium	0·86	....	144		
Selinium	4·30	....	220		
Silver	10·50	655	1,750	....	·1909
Sodium	0·97				
Steel	7·84	490	2,500	High grade, 70,000 to 80,000. Medium grade, 60,000 to 70,000. Soft grade, 52,000 to 60,000.	·1079
Strontium	2·54				
Tellium	6·11				
Thallium	11·85				
Tin	7·35	458	442	3,500	·2173
Tungsten	17·00				
Uranium	18·33				
Zinc	7·14	445	780	5,000 to 6,000	·2942

SOLDERS.

	Copper.	Tin.	Lead.	Zinc.	Antim'ny.	Silver.	Nickel.
Lead, melts at 441°	..	33	67	..	..	..	..
Tin, melts at 340°	..	67	33	..	..	..	..
Spelter, soft	50	..	..	50	..	..	..
" hard	65	..	..	35	..	..	..
Steel	13	..	..	5	..	82	..
Brass or copper	50	..	..	50	..	..	..
Fine brass	47	..	..	47	..	6	..
Iron	66	..	..	33	1	..	..
Copper	53	47	..	..	..	..	..
German silver	38	..	..	54	..	..	8

Solders of lead and tin are termed soft solders, in plumbers' work the joints are wiped and the solder is in a mass. In soldering cornices, where there is a strain, the metal should be riveted as well. Solders containing copper are used for brazing.

Sheet lead for sulphuric-acid chambers is joined by burning the sheets by a blowpipe. The welding of various metals is now effected by electricity. In joining one metal to another, one must be fluid; and mercury, being usually fluid, combines with most metals except iron and platinum. An amalgam of tin or cold solder can be applied by friction to polished iron and form a surface which admits of being soldered.

Fusible Alloys.—A fusible alloy composed of 50 bismuth, 25 tin, and 25 lead melts at 212°; the melting point may be still further reduced by a larger percentage of bismuth, and it is used either as a solder or a fusible plug. In automatic fire extinguishers the cap of the sprinkler is soldered with this alloy, and melts at about 170°. For fusible plugs in boilers, the United States supervising inspectors specify Banca tin, which melts at about 445° F. The plug must be at least ½″ smallest diameter.

ALLOYS AND COMPOSITIONS.

	Copper.	Zinc.	Tin.	Nickel.	Lead.	Aluminum.	Steel.		Antimony.	Tensile strength per sq. inch.
Aluminum, bronze	90·0	.....	....	.....	....	10·0	.....	..	....	75.000–90,000
" brass	33⅓	33⅓	Add 33⅓ per cent aluminum bronze..						.....	78,327
" composition	.....	.....	10·0	.....	....	90·0	.....	.....	....	......
Babbitt's metal, light duty	1·8	.....	89·3	.....	....	...	.....	.....	8·9	......
" " for bearings	3·7	.....	88·9	.....	....	...	.....	.....	7·4	......
Britannia metal	2·0	1·0	81·0	.....	....	...	.....	.....	16·0	......
Chrome steel	.....	....	....	.....	....	...	98·0	2·0*	....	......
German silver	51·1	31·9	3·2	13·8	....	...	.....	.....	....	......
Gun metal	.....	.....	....	.....	....	...	.....	.....	....	42,560
Muntz metal	6·25	93·75	....	.....	....	...	.....	.....	....	49,280
Manganese alloy †	67·3	13·0	....	.....	....	1·2	.....	18·0‡	·5‖	57,000
Nickel steel	.....	.....	....	3·25	....	...	96·75	.....	....	......
Sheathing metal	56·0	44·0	....	.....	....	...	.....	.....	....	......
Speculum "	66·0	.....	22·0	.....	....	12·0	.....	.....	....	......
" "	50·0	21·0	29·0	.....	....	...	.....	.....	....	......
Sterrometal	61·0	35·0	2·0	.....	....	...	Iron. 2·0	.....	Antimony.	85,120
Type metal	.....	.....	....	.....	75·0	...	.....	.....	25·0	......
" "	.....	.....	....	.....	87·5	...	.....	.....	12·5	......
White "	7·4	7·4	28·4	.....	....	...	.....	.....	56·8	......
" " hard	69·8	25·8	4·4	.....	....	...	.....	.....	....	......

* Chromium, 2·0. † Twenty-five per cent manganese to copper doubles its tensile strength without diminishing its ductility. ‡ Manganese, 18·0. ‖ Silicon, ·5.

TABLES OF THE CIRCUMFERENCES OF CIRCLES TO THE NEAREST FRACTION OF PRACTICAL MEASUREMENT ; ALSO, THE AREAS OF CIRCLES, IN INCHES AND DECIMAL PARTS, LIKEWISE OF FEET AND DECIMAL PARTS.

Circumference in feet and inches.	Diameter in inches.	Area in square inches.	Area in square feet.	Circumference in feet and inches.	Diameter in inches.	Area in square inches.	Area in square feet.
				1 6⅞	6	28·27	·196
·20	$\frac{1}{16}$	·003		1 7¼	6⅛	29·46	·204
·39	⅛	·012		1 7⅝	6¼	30·68	·212
·59	$\frac{3}{16}$	·028		1 8	6⅜	31·92	·220
·78	¼	·049		1 8⅜	6½	33·18	·228
·98	$\frac{5}{16}$	·077		1 8¾	6⅝	34·47	·237
1·18	⅜	·110		1 9¼	6¾	35·78	·246
1·37	$\frac{7}{16}$	·150		1 9½	6⅞	37·12	·256
1·57	½	·196		1 10	7	38·48	·267
1·77	$\frac{9}{16}$	·248		1 10⅜	7⅛	39·87	·277
1·96	⅝	·307		1 10¾	7¼	41·28	·287
2·16	$\frac{11}{16}$	·371		1 11⅛	7⅜	42·72	·297
2·36	¾	·442		1 11½	7½	44·18	·307
2·55	$\frac{13}{16}$	·518		1 11⅞	7⅝	45·66	·318
2·75	⅞	·601		2 0⅜	7¾	47·17	·328
2·94	$\frac{15}{16}$	·690		2 0¾	7⅞	48·71	·338
3⅛	1	·785	·0054	2 1⅛	8	50·26	·349
3½	1⅛	·994	·0069	2 1½	8⅛	51·85	·360
3⅞	1¼	1·23	·0085	2 1⅞	8¼	53·46	·371
4¼	1⅜	1·48	·0103	2 2¼	8⅜	55·09	·383
4⅝	1½	1·77	·0123	2 2⅝	8½	56·74	·394
5⅛	1⅝	2·07	·0144	2 3	8⅝	58·43	·406
5½	1¾	2·40	·0167	2 3⅜	8¾	60·13	·428
5⅞	1⅞	2·76	·0192	2 3⅞	8⅞	61·86	·430
6¼	2	3·14	·0218	2 4¼	9	63·62	·442
6⅝	2⅛	3·55	·0246	2 4⅝	9⅛	65·40	·455
7	2¼	3·98	·0276	2 5	9¼	67·20	·467
7⅜	2⅜	4·43	·0307	2 5⅜	9⅜	69·03	·480
7⅞	2½	4·91	·0341	2 5¾	9½	70·88	·493
8¼	2⅝	5·41	·0376	2 6¼	9⅝	72·76	·506
8⅝	2¾	5·94	·0412	2 6⅝	9¾	74·66	·519
9	2⅞	6·49	·0450	2 7	9⅞	76·59	·532
9⅜	3	7·07	·0490	2 7⅜	10	78·54	·545
9¾	3⅛	7·67	·0532	2 7¾	10⅛	80·51	·559
10¼	3¼	8·29	·0576	2 8⅛	10¼	82·52	·573
10⅝	3⅜	8·95	·0621	2 8½	10⅜	84·54	·587
11	3½	9·62	·0668	2 9	10½	86·59	·601
11⅜	3⅝	10·32	·0716	2 9⅜	10⅝	88·66	·615
11¾	3¾	11·04	·0766	2 9¾	10¾	90·76	·630
12⅛	3⅞	11·79	·0818	2 10⅛	10⅞	92·88	·645
1 0½	4	12·57	·087	2 10½	11	95·03	·660
1 1	4⅛	13·36	·093	2 10⅞	11⅛	97·21	·675
1 1⅜	4¼	14·19	·099	2 11¼	11¼	99·40	·690
1 1¾	4⅜	15·03	·105	2 11¾	11⅜	101·62	·705
1 2⅛	4½	15·90	·111	3 0⅛	11½	103·87	·720
1 2½	4⅝	16·80	·118	3 0½	11⅝	106·14	·736
1 2⅞	4¾	17·72	·124	3 0⅞	11¾	108·43	·752
1 3¼	4⅞	18·66	·130	3 1¼	11⅞	110·75	·768
1 3⅝	5	19·63	·136	3 1⅝	12	113·10	·785
1 4⅛	5⅛	20·63	·143	3 2	12⅛	115·47	·802
1 4½	5¼	21·65	·150	3 2½	12¼	117·86	·819
1 4⅞	5⅜	22·69	·157	3 2⅞	12⅜	120·28	·836
1 5¼	5½	23·76	·165	3 3¼	12½	122·72	·853
1 5⅝	5⅝	24·85	·173	3 3⅝	12⅝	125·19	·870
1 6	5¾	25·97	181	3 4	12¾	127·68	·887
1 6⅜	5⅞	27·11	·189	3 4⅜	12⅞	130·19	·904

TABLES OF THE CIRCUMFERENCES OF CIRCLES, ETC.—(*Continued.*)

Circumference in feet and inches.	Diameter in inches.	Area in square inches.	Area in square feet.	Circumference in feet and inches.	Diameter in feet and inches.	Area in square inches.	Area in square feet.
3 4¾	13	132·73	·922	5 2⅞	20	314·16	2·182
3 5⅛	13⅛	135·30	·939	5 3¼	20⅛	318·10	2·209
3 5⅝	13¼	137·89	·956	5 3⅝	20¼	322·06	2·237
3 6	13⅜	140·50	·974	5 4	20⅜	326·05	2·265
3 6⅜	13½	143·14	·992	5 4⅜	20½	330·06	2·293
3 6¾	13⅝	145·80	1·011	5 4¾	20⅝	334·10	2·321
3 7⅛	13¾	148·49	1·030	5 5⅛	20¾	338·16	2·349
3 7⅝	13⅞	151·20	1·050	5 5½	20⅞	342·25	2·377
3 8	14	153·94	1·069	5 6	21	346·36	2·405
3 8⅜	14⅛	156·70	1·088	5 6⅜	21⅛	350·50	2·434
3 8¾	14¼	159·49	1·107	5 6¾	21¼	354·66	2·463
3 9⅛	14⅜	162·30	1·126	5 7⅛	21⅜	358·84	2·492
3 9½	14½	165·13	1·146	5 7½	21½	363·05	2·521
3 9⅞	14⅝	167·99	1·166	5 7⅞	21⅝	367·28	2·550
3 10¼	14¾	170·87	1·186	5 8¼	21¾	371·54	2·580
3 10¾	14⅞	173·78	1·206	5 8¾	21⅞	375·83	2·610
3 11⅛	15	176·71	1·227	5 9⅛	22	380·13	2·640
3 11½	15⅛	179·67	1·247	5 9½	22⅛	384·46	2·670
3 11⅞	15¼	182·65	1·267	5 9⅞	22¼	388·82	2·700
4 0¼	15⅜	185·66	1·288	5 10¼	22⅜	393·20	2·730
4 0⅝	15½	188·69	1·309	5 10⅝	22½	397·61	2·761
4 1	15⅝	191·75	1·330	5 11	22⅝	402·04	2·792
4 1½	15¾	194·83	1·352	5 11½	22¾	406·49	2·823
4 1⅞	15⅞	197·93	1·374	5 11⅞	22⅞	410·97	2·854
4 2¼	16	201·06	1·396	6 0¼	23	415·48	2·885
4 2⅝	16⅛	204·22	1·418	6 0⅝	23⅛	420·00	2·917
4 3	16¼	207·39	1·440	6 1	23¼	424·56	2·949
4 3⅜	16⅜	210·60	1·462	6 1⅜	23⅜	429·13	2·981
4 3¾	16½	213·82	1·484	6 1¾	23½	433·74	3·013
4 4¼	16⅝	217·08	1·507	6 2¼	23⅝	438·36	3·045
4 4⅝	16¾	220·35	1·530	6 2⅝	23¾	443·01	3·077
4 5	16⅞	223·65	1·553	6 3	23⅞	447·69	3·109
4 5⅜	17	226·98	1·576	6 3⅝	2 0	452·39	3·142
4 5¾	17⅛	230·33	1·599	6 4⅛	2 0¼	461·86	3·207
4 6⅛	17¼	233·70	1·622	6 4⅞	2 0½	471·44	3·273
4 6½	17⅜	237·10	1·645	6 5¾	2 0¾	481·11	3·341
4 6⅞	17½	240·53	1·669	6 6½	2 1	490·87	3·408
4 7⅜	17⅝	243·98	1·693	6 7¼	2 1¼	500·74	3·477
4 7¾	17¾	247·45	1·718	6 8⅛	2 1½	510·71	3·547
4 8⅛	17⅞	250·95	1·743	6 8⅞	2 1¾	520·77	3·617
4 8½	18	254·47	1·767	6 9⅝	2 2	530·93	3·687
4 8⅞	18⅛	258·02	1·792	6 10½	2 2¼	541·19	3·758
4 9¼	18¼	261·59	1·817	6 11¼	2 2½	551·55	3·830
4 9¾	18⅜	265·18	1·842	7 0	2 2¾	562·00	3·904
4 10⅛	18½	268·80	1·868	7 0⅞	2 3	572·56	3·976
4 10½	18⅝	272·45	1·893	7 1⅝	2 3¼	583·21	4·050
4 10⅞	18¾	276·12	1·918	7 2⅜	2 3½	593·96	4·124
4 11¼	18⅞	279·81	1·943	7 3⅛	2 3¾	604·81	4·200
4 11⅝	19	283·53	1·969	7 3⅞	2 4	615·75	4·276
5 0	19⅛	287·27	1·995	7 4¾	2 4¼	626·80	4·352
5 0⅜	19¼	291·04	2·021	7 5½	2 4½	637·94	4·430
5 0⅞	19⅜	294·83	2·047	7 6¼	2 4¾	649·18	4·508
5 1¼	19½	298·65	2·074	7 7	2 5	660·52	4·586
5 1⅝	19⅝	302·49	2·101	7 7⅞	2 5¼	671·96	4·666
5 2	19¾	306·36	2·128	7 8⅝	2 5½	683·49	4·747
5 2⅜	19⅞	310·25	2·155	7 9½	2 5¾	695·13	4·827

TABLES OF THE CIRCUMFERENCES OF CIRCLES, ETC.—(*Continued.*)

Circumference in feet and inches.	Diameter in feet and inches.	Area in square inches.	Area in square feet.	Circumference in feet and inches.	Diameter in feet and inches.	Area in square inches.	Area in square feet.
7 10¼	2 6	706·86	4·908	11 6¼	3 8	1520·5	10·56
7 11	2 6¼	718·69	4·990	11 7	3 8¼	1537·9	10·68
7 11¾	2 6½	730·62	5·073	11 7¾	3 8½	1555·3	10·80
8 0⅝	2 6¾	742·64	5·157	11 8½	3 8¾	1572·8	10·92
8 1⅜	2 7	754·77	5·241	11 9⅜	3 9	1590·4	11·04
8 2⅛	2 7¼	766·99	5·326	11 10⅛	3 9¼	1608·1	11·17
8 2⅞	2 7½	779·31	5·411	11 10⅞	3 9½	1626·0	11·29
8 3¾	2 7¾	791·73	5·498	11 11¾	3 9¾	1643·9	11·41
8 4½	2 8	804·25	5·585	12 0½	3 10	1661·9	11·54
8 5⅜	2 8¼	816·86	5·673	12 1¼	3 10¼	1680·0	11·67
8 6⅛	2 8½	829·58	5·761	12 2	3 10½	1698·2	11·79
8 6⅞	2 8¾	842·39	5·849	12 2⅞	3 10¾	1716·5	11·92
8 7⅝	2 9	855·30	5·939	12 3⅝	3 11	1734·9	12·05
8 8½	2 9¼	868·31	6·029	12 4⅜	3 11¼	1753·4	12·18
8 9¼	2 9½	881·41	6·120	12 5¼	3 11½	1772·0	12·30
8 10	2 9¾	894·62	6·212	12 6	3 11¾	1790·8	12·43
8 10¾	2 10	907·92	6·305	12 6¾	4 0	1809·6	12·57
8 11½	2 10¼	921·32	6·398	12 7½	4 0¼	1828·5	12·70
9 0⅜	2 10½	934·82	6·491	12 8⅜	4 0½	1847·4	12·83
9 1⅛	2 10¾	948·42	6·586	12 9⅛	4 0¾	1866·5	12·96
9 1⅞	2 11	962·11	6·681	12 9⅞	4 1	1885·7	13·09
9 2¾	2 11¼	975·91	6·777	12 10¾	4 1¼	1905·0	13·23
9 3½	2 11½	989·80	6·874	12 11½	4 1½	1924·4	13·36
9 4¼	2 11¾	1003·8	6·970	13 0¼	4 1¾	1943·9	13·50
9 5	3 0	1017·9	7·069	13 1	4 2	1963·5	13·63
9 5⅞	3 0¼	1032·1	7·167	13 1⅞	4 2¼	1983·2	13·77
9 6⅝	3 0½	1046·3	7·266	13 2⅝	4 2½	2003·0	13·91
9 7½	3 0¾	1060·7	7·366	13 3⅜	4 2¾	2022·8	14·05
9 8¼	3 1	1075·2	7·466	13 4¼	4 3	2042·8	14·19
9 9	3 1¼	1089·8	7·567	13 5	4 3¼	2062·9	14·32
9 9⅞	3 1½	1104·5	7·669	13 5¾	4 3½	2083·1	14·46
9 10⅝	3 1¾	1119·2	7·772	13 6½	4 3¾	2103·3	14·61
9 11⅜	3 2	1134·1	7·876	13 7⅜	4 4	2123·7	14·75
10 0⅛	3 2¼	1149·1	7·979	13 8⅛	4 4¼	2144·2	14·89
10 0⅞	3 2½	1164·2	8·085	13 8⅞	4 4½	2164·7	15·03
10 1¾	3 2¾	1179·3	8·189	13 9¾	4 4¾	2185·4	15·18
10 2½	3 3	1194·6	8·295	13 10½	4 5	2206·2	15·32
10 3¼	3 3¼	1209·9	8·403	13 11¼	4 5¼	2227·0	15·46
10 4	3 3½	1225·4	8·509	14 0	4 5½	2248·0	15·61
10 4⅞	3 3¾	1241·0	8·617	14 0⅞	4 5¾	2269·1	15·76
10 5⅝	3 4	1256·6	8·727	14 1⅝	4 6	2290·2	15·90
10 6⅜	3 4¼	1272·4	8·836	14 2⅜	4 6¼	2311·5	16·05
10 7¼	3 4½	1288·2	8·946	14 3¼	4 6½	2332·8	16·20
10 8	3 4¾	1304·2	9·056	14 4	4 6¾	2354·3	16·35
10 8¾	3 5	1320·2	9·169	14 4¾	4 7	2375·8	16·50
10 9½	3 5¼	1336·4	9·211	14 5½	4 7¼	2397·5	16·65
10 10⅜	3 5½	1352·6	9·394	14 6⅜	4 7½	2419·2	16·80
10 11⅛	3 5¾	1369·0	9·506	14 7⅛	4 7¾	2441·1	16·95
10 11⅞	3 6	1385·4	9·62	14 7⅞	4 8	2463·0	17·10
11 0¾	3 6¼	1402·0	9·73	14 8¾	4 8¼	2485·0	17·26
11 1½	3 6½	1418·6	9·84	14 9½	4 8½	2507·2	17·41
11 2¼	3 6¾	1435·4	9·96	14 10¼	4 8¾	2529·4	17·56
11 3	3 7	1452·2	10·08	14 11	4 9	2551·8	17·72
11 3⅞	3 7¼	1469·1	10·20	14 11⅞	4 9¼	2574·2	17·88
11 4⅝	3 7½	1486·2	10·32	15 0⅝	4 9½	2596·7	18·03
11 5½	3 7¾	1503·3	10·44	15 1⅜	4 9¾	2619·3	18·19

TABLES OF THE CIRCUMFERENCES OF CIRCLES, ETC.—(*Continued.*)

Circumference in feet and inches.	Diameter in feet and inches.	Area in square inches.	Area in square feet.	Circumference in feet and inches.	Diameter in feet and inches.	Area in square inches.	Area in square feet.
15 2¼	4 10	2642·1	18·35	18 10⅛	6 0	4071·5	28·27
15 3	4 10¼	2664·9	18·51	18 10⅞	6 0¼	4099·8	28·47
15 3¾	4 10½	2687·8	18·66	18 11¾	6 0½	4128·2	28·67
15 4½	4 10¾	2710·8	18·82	19 0½	6 0¾	4156·8	28·87
15 5¼	4 11	2734·0	18·98	19 1¼	6 1	4185·4	29·07
15 6⅛	4 11¼	2757·2	19·15	19 2⅛	6 1¼	4214·1	29·27
15 6⅞	4 11½	2780·5	19·31	19 2⅞	6 1½	4242·9	29·47
15 7¾	4 11¾	2803·9	19·47	19 3⅝	6 1¾	4271·8	29·67
15 8½	5 0	2827·4	19·63	19 4½	6 2	4300·8	29·87
15 9¼	5 0¼	2851·0	19·80	19 5¼	6 2¼	4329·9	30·07
15 10	5 0½	2874·8	19·96	19 6	6 2½	4359·2	30·27
15 10¾	5 0¾	2898·6	20·13	19 6¾	6 2¼	4388·5	30·47
15 11⅝	5 1	2922·5	20·29	19 7½	6 3	4417·9	30·68
16 0⅜	5 1¼	2946·5	20·46	19 8⅜	6 3¼	4447·4	30·88
16 1¼	5 1½	2970·6	20·63	19 9⅛	6 3½	4477·0	31·09
16 2	5 1¾	2994·8	20·80	19 9⅞	6 3¾	4506·7	31·30
16 2¾	5 2	3019·1	20·96	19 10¾	6 4	4536·5	31·50
16 3½	5 2¼	3043·5	21·13	19 11½	6 4¼	4566·4	31·71
16 4¼	5 2½	3068·0	21·30	20 0¼	6 4½	4596 3	31·92
16 5⅛	5 2¾	3092·6	21·48	20 1⅛	6 4¾	4626·4	32·13
16 5⅞	5 3	3117·2	21·65	20 1⅞	6 5	4656·6	32·34
16 6⅝	5 3¼	3142·0	21·82	20 2⅝	6 5¼	4686·9	32·55
16 7½	5 3½	3166·9	21·99	20 3½	6 5½	4717·3	32·76
16 8¼	5 3¾	3191·9	22·17	20 4¼	6 5¾	4747·8	32·97
16 9	5 4	3217·0	22·34	20 5	6 6	4778·3	33·18
16 9¾	5 4¼	3242·2	22·51	20 5¾	6 6¼	4809·0	33·40
16 10⅝	5 4½	3267·5	22·69	20 6½	6 6½	4839·8	33·61
16 11⅜	5 4¾	3292·8	22·87	20 7⅜	6 6¾	4870·7	33·82
17 0⅛	5 5	3318·3	23·04	20 8⅛	6 7	4901·6	34·04
17 1	5 5¼	3343·9	23·22	20 8⅞	6 7¼	4932·7	34·25
17 1¾	5 5½	3369·6	23·40	20 9⅝	6 7½	4963·9	34·47
17 2½	5 5¾	3395·3	23·58	20 10½	6 7¾	4995·1	34·69
17 3⅜	5 6	3421·2	23·76	20 11¼	6 8	5026·5	34·91
17 4⅛	5 6¼	3447·2	23·94	21 0⅛	6 8¼	5058·0	35·12
17 4⅞	5 6½	3473·2	24·12	21 0⅞	6 8½	5089·5	35·34
17 5⅝	5 6¾	3499·4	24·30	21 1⅝	6 8¾	5121·2	35·56
17 6½	5 7	3525·1	24·48	21 2⅜	6 9	5153·0	35·78
17 7¼	5 7¼	3552·0	24·67	21 3¼	6 9¼	5184·8	36·01
17 8	5 7½	3578·5	24·85	21 4	6 9½	5216·8	36·23
17 8¾	5 7¾	3605·0	25·03	21 4¾	6 9¾	5248·8	36·45
17 9⅝	5 8	3631·7	25·22	21 5⅝	6 10	5281·0	36·67
17 10⅜	5 8¼	3658·4	25·40	21 6⅜	6 10¼	5313·2	36·89
17 11⅛	5 8½	3685·3	25·59	21 7⅛	6 10½	5345·6	37·12
17 11⅞	5 8¾	3712·2	25·78	21 7⅞	6 10¾	5378·0	37·35
18 0¾	5 9	3739·3	25·96	21 8¾	6 11	5410·6	37·57
18 1½	5 9¼	3766·4	26·15	21 9½	6 11¼	5443·2	37·80
18 2¼	5 9½	3793·7	26·34	21 10¼	6 11½	5476·0	38·03
18 3⅛	5 9¾	3821·0	26·53	21 11⅛	6 11¾	5508·8	38·26
18 3⅞	5 10	3848·5	26·72	21 11⅞	7 0	5541·7	38·48
18 4⅝	5 10¼	3876·0	26·92	22 0⅝	7 0¼	5574·8	38·71
18 5½	5 10½	3903·6	27·11	22 1⅜	7 0½	5607·9	38·94
18 6¼	5 10¾	3931·4	27·30	22 2¼	7 9¾	5641·1	39·17
18 7	5 11	3959·2	27·49	22 3	7 1	5674·5	39·41
18 7¾	5 11¼	3987·1	27·69	22 3¾	7 1¼	5707·9	39·64
18 8⅜	5 11½	4015·2	27·88	22 4½	7 1½	5741·4	39·87
18 9⅜	5 11¾	4043·3	28·08	22 5¼	7 1¾	5775·0	40·10

TABLES OF THE CIRCUMFERENCES OF CIRCLES, ETC.—(*Continued.*)

Circumference in feet and inches.	Diameter in feet and inches.	Area in square inches.	Area in square feet.
22 6⅛	7 2	5808·8	40·34
22 6⅞	7 2¼	5842·6	40·57
22 7¾	7 2½	5876·5	40·80
22 8½	7 2¾	5910·5	41·04
22 9¼	7 3	5944·6	41·28
22 10⅛	7 3¼	5978·9	41·52
22 10⅞	7 3½	6013·2	41·76
22 11⅝	7 3¾	6047·6	42·00
23 0⅜	7 4	6082·1	42·24
23 1¼	7 4¼	6116·7	42·48
23 2	7 4½	6151·4	42·72
23 2¾	7 4¾	6186·2	42·96
23 3⅝	7 5	6221·1	43·20
23 4⅜	7 5¼	6256·1	43·44
23 5⅛	7 5½	6291·2	43·68
23 6	7 5¾	6326·4	43·93
23 6¾	7 6	6361·7	44·18
23 7½	7 6¼	6397·1	44·43
23 8¼	7 6½	6432·6	44·67
23 9⅛	7 6¾	6468·2	44·92
23 9⅞	7 7	6503·8	45·17
23 10⅝	7 7¼	6539·6	45·41
23 11⅝	7 7½	6575·5	45·66
24 0¼	7 7¾	6611·5	45·91
24 1	7 8	6647·6	46·16
24 1¾	7 8¼	6683·8	46·42
24 2½	7 8½	6720·0	46·67
24 3¼	7 8¾	6756·4	46·92
24 4⅛	7 9	6792·9	47·17
24 4⅞	7 9¼	6829·4	47·43
24 5¾	7 9½	6866·1	47·68
24 6½	7 9¾	6902·9	47·94
24 7¼	7 10	6939·7	48·19
24 8	7 10¼	6976·7	48·45
24 8¾	7 10½	7013·8	48·71
24 9⅝	7 10¾	7050·9	48·96
24 10⅜	7 11	7088·2	49·22
24 11¼	7 11¼	7125·5	49·48
25 0	7 11½	7163·0	49·74
25 0¾	7 11¾	7200·5	50·00
25 1½	8 0	7238·2	50·26
25 2⅝	8 0¼	7275·9	50·53
25 3⅛	8 0½	7313·8	50·79
25 3⅞	8 0¾	7351·7	51·05
25 4¾	8 1	7389·8	51·32
25 5½	8 1¼	7427·9	51·58
25 6¼	8 1½	7466·2	51·85
25 7	8 1¾	7504·5	52·11
25 7⅞	8 2	7542·9	52·38
25 8⅝	8 2¼	7581·5	52·65
25 9⅜	8 2½	7620·1	52·92
25 10¼	8 2¾	7658·8	53·19
25 11	8 3	7697·7	53·46
25 11¾	8 3¼	7736·6	53·73
26 0½	8 3½	7775·6	54·00
26 1¼	8 3¾	7814·7	54·27
26 2⅛	8 4	7853·9	54·54
26 5¼	8 5	8011·9	55·64
26 8⅜	8 6	8171·3	56·75
26 11½	8 7	8332·3	57·86
27 2¾	8 8	8494·9	58·99
27 5¾	8 9	8659·0	60·13
27 9	8 10	8824·7	61·28
28 0⅛	8 11	8892·0	62·44
28 3¼	9	9160·9	63·62
28 6⅜	9 1	9331·3	64·80
28 9½	9 2	9503·3	66·00
29 0⅝	9 3	9676·9	67·20
29 3¾	9 4	9852·1	68·42
29 7	9 5	10028·8	69·64
29 10⅛	9 6	10207·1	70·88
30 1¼	9 7	10386·9	72·13
30 4⅜	9 8	10568·3	73·39
30 7½	9 9	10751·3	74·66
30 10¾	9 10	10935·9	75·94
31 1¾	9 11	11122·0	77·24
31 5	10	11309·8	78·54
31 8⅛	10 1	11499·0	79·85
31 11¼	10 2	11689·9	81·18
32 2⅜	10 3	11882·3	82·52
32 5½	10 4	12076·3	83·86
32 8⅝	10 5	12271·9	85·22
32 11¾	10 6	12469·0	86·59
33 2⅞	10 7	12667·7	87·97
33 6⅛	10 8	12868·0	89·36
33 9¼	10 9	13069·8	90·76
34 0⅜	10 10	13273·3	92·17
34 3½	10 11	13478·2	93·60
34 6⅝	11	13684·8	95·03
34 9¾	11 1	13892·9	96·48
35 0⅞	11 2	14142·6	97·93
35 4⅛	11 3	14313·9	99·40
35 7¼	11 4	14526·8	100·88
35 10⅜	11 5	14741·2	102·37
36 1½	11 6	14957·2	103·87
36 4⅝	11 7	15174·7	105·38
36 7¾	11 8	15393·8	106·90
36 10⅞	11 9	15614·5	108·43
37 2	11 10	15836·8	109·98
37 5¼	11 11	16060·6	111·53
37 8⅜	12	16286·0	113·10
37 11½	12 1	16513·0	114·67
38 2⅝	12 2	16741·6	116·26
38 5¾	12 3	16971·7	117·86
38 8⅞	12 4	17203·4	119·47
39 0	12 5	17436·7	121·09
39 3¼	12 6	17671·5	122·72
39 6⅜	12 7	17907·9	124·36
39 9½	12 8	18145·9	126·01
40 0⅝	12 9	18385·4	127·68
40 3¾	12 10	18626·6	129·35
40 6⅞	12 11	18869·2	131·04

TABLE OF DIAMETERS, CIRCUMFERENCES, AND AREAS OF CIRCLES.

Diameter.	Circumference.	Circular area.	Diameter.	Circumference.	Circular area.	Diameter.	Circumference.	Circular area.
1	3·1416	0·7854	51	160·22	2042·82	101	317·30	8011·85
2	6·28	3·14	52	163·36	2123·72	102	320·41	8171·28
3	9·42	7·07	53	166·50	2206·18	103	323·58	8332·29
4	12·57	12·57	54	169·65	2290·22	104	326·73	8494·87
5	15·71	19·63	55	172·79	2375·83	105	329·87	8659·01
6	18·85	28·27	56	175·93	2463·01	106	333·01	8824·73
7	21·99	38·48	57	179·07	2551·76	107	336·15	8992·02
8	25·13	50·27	58	182·21	2642·08	108	339·29	9160·88
9	28·27	63·62	59	185·35	2733·97	109	342·43	9331·32
10	31·42	78·54	60	188·50	2827·43	110	345·57	9503·32
11	34·56	95·03	61	191·64	2922·47	111	348·72	9676·89
12	37·70	113·10	62	194·78	3019·07	112	351·86	9852·03
13	40·84	132·73	63	197·92	3117·25	113	355·00	10028·75
14	43·98	153·94	64	201·06	3216·99	114	358·14	10207·03
15	47·12	176·71	65	204·20	3318·31	115	361·28	10386·89
16	50·26	201·06	66	207·34	3421·19	116	364·42	10568·32
17	53·41	226·98	67	210·49	3525·65	117	367·57	10751·32
18	56·55	254·47	68	213·63	3631·68	118	370·71	10935·88
19	59·69	283·53	69	216·77	3739·28	119	373·85	11122·02
20	62·83	314·16	70	219·91	3848·45	120	376·99	11309·73
21	65·97	346·36	71	223·05	3959·19	121	380·13	11499·01
22	69·11	380·13	72	226·19	4071·50	122	383·27	11689·87
23	72·26	415·48	73	229·34	4185·39	123	386·42	11882·29
24	75·40	452·39	74	232·48	4300·84	124	389·56	12076·28
25	78·54	490·87	75	235·62	4417·86	125	392·70	12271·85
26	81·68	530·93	76	238·76	4536·46	126	395·84	12468·98
27	84·82	572·56	77	241·90	4656·63	127	398·98	12667·69
28	87·96	615·75	78	245·04	4778·36	128	402·12	12867·96
29	91·11	660·52	79	248·19	4901·67	129	405·26	13069·81
30	94·25	706·86	80	251·33	5026·55	130	408·41	13273·23
31	97·39	754·77	81	254·47	5153·00	131	411·55	13478·22
32	100·53	804·25	82	257·61	5281·02	132	414·69	13684·78
33	103·67	855·30	83	260·75	5410·61	133	417·83	13892·91
34	106·81	907·92	84	263·89	5541·77	134	420·97	14102·61
35	109·96	962·11	85	267·03	5674·50	135	424·11	14313·88
36	113·10	1017·88	86	270·18	5808·80	136	427·26	14526·72
37	116·24	1075·21	87	273·32	5944·68	137	430·40	14741·14
38	119·38	1134·11	88	276·46	6082·12	138	433·54	14957·12
39	122·52	1194·59	89	279·60	6221·14	139	436·68	15174·68
40	125·66	1256·64	90	282·74	6361·73	140	439·82	15393·80
41	128·80	1320·25	91	285·88	6503·88	141	442·96	15614·50
42	131·95	1385·44	92	289·03	6647·61	142	446·11	15836·77
43	135·09	1452·20	93	292·17	6792·91	143	449·25	16060·61
44	138·23	1520·53	94	295·31	6939·78	144	452·39	16286·02
45	141·37	1590·43	95	298·45	7088·22	145	455·53	16513·00
46	144·51	1661·90	96	301·59	7238·23	146	458·67	16741·55
47	147·65	1734·94	97	304·73	7389·81	147	461·81	16971·67
48	150·80	1809·56	98	307·88	7542·96	148	464·96	17203·36
49	153·94	1885·74	99	311·02	7697·69	149	468·10	17436·62
50	157·08	1963·50	100	314·16	7853·98	150	471·24	17671·46

TABLE OF DIAMETERS, CIRCUMFERENCES, AND AREAS OF CIRCLES.

Diameter.	Circumference.	Circular area.	Diameter.	Circumference.	Circular area.	Diameter.	Circumference.	Circular area.
151	474·38	17907·86	201	631·46	31730·87	251	788·54	49480·87
152	477·52	18145·84	202	634·60	32047·39	252	791·68	49875·92
153	480·66	18385·39	203	637·74	32365·47	253	794·82	50272·55
154	483·80	18626·50	204	640·88	32685·13	254	797·96	50670·75
155	486·95	18869·19	205	644·03	33006·36	255	801·11	51070·52
156	490·09	19113·45	206	647·17	33329·16	256	804·25	51471·86
157	493·23	19359·28	207	650·31	33653·53	257	807·39	51874·76
158	496·37	19606·68	208	653·45	33979·47	258	810·53	52279·24
159	499·51	19855·65	209	656·59	34306·98	259	813·67	52685·29
160	502·65	20106·19	210	659·73	34636·06	260	816·81	53092·96
161	505·80	20358·34	211	662·88	34966·71	261	819·96	53502·11
162	508·94	20611·99	212	666·02	35298·94	262	823·10	53912·87
163	512·08	20867·24	213	669·16	35632·73	263	826·24	54325·21
164	515·22	21124·07	214	672·30	35968·09	264	829·38	54739·11
165	518·36	21382·46	215	675·44	36305·03	265	832·52	55154·59
166	521·50	21642·43	216	678·58	36643·61	266	835·66	55571·63
167	524·65	21903·97	217	681·73	36983·61	267	838·80	55990·25
168	527·79	22167·08	218	684·87	37325·26	268	841·95	56410·44
169	530·93	22431·76	219	688·01	37668·48	269	845·09	56832·20
170	534·07	22698·01	220	691·15	38013·27	270	848·23	57255·53
171	537·21	22965·83	221	694·29	38359·63	271	851·37	57680·43
172	540·35	23235·22	222	697·43	38707·56	272	854·51	58106·90
173	543·50	23506·18	223	700·57	39057·07	273	857·65	58534·94
174	546·64	23778·71	224	703·72	39408·14	274	860·80	58964·55
175	549·78	24052·82	225	706·86	39760·78	275	863·94	59395·74
176	552·92	24328·49	226	710·00	40115·00	276	867·08	59828·49
177	556·06	24605·79	227	713·14	40470·78	277	870·22	60262·82
178	559·20	24884·56	228	716·28	40828·14	278	873·36	60698·72
179	562·34	25164·94	229	719·42	41187·07	279	876·50	61136·18
180	565·49	25446·90	230	722·57	41547·56	280	879·65	61575·22
181	568·63	25730·43	231	725·71	41909·63	281	882·79	62015·82
182	571·77	26015·53	232	728·85	42273·27	282	885·93	62458·00
183	574·91	26302·20	233	731·99	42638·48	283	889·07	62901·75
184	578·05	26590·44	234	735·13	43005·26	284	892·21	63347·07
185	581·19	26880·25	235	738·27	43373·61	285	895·35	63793·97
186	584·34	27171·63	236	741·42	43743·54	286	898·49	64242·43
187	587·48	27464·59	237	744·56	44115·03	287	901·64	64692·46
188	590·62	27759·11	238	747·70	44488·09	288	904·78	65144·07
189	593·76	28055·21	239	750·84	44862·73	289	907·92	65597·24
190	596·90	28352·87	240	753·98	45238·93	290	911·06	66051·99
191	600·04	28652·11	241	757·12	45616·71	291	914·20	66508·30
192	603·19	28952·92	242	760·26	45996·06	292	917·34	66966·19
193	606·33	29255·30	243	763·41	46376·98	293	920·49	67425·65
194	609·47	29559·26	244	766·55	46759·47	294	923·63	67886·68
195	612·61	29864·77	245	769·69	47143·52	295	926·77	68349·28
196	615·75	30171·86	246	772·83	47529·16	296	929·91	68813·45
197	618·89	30480·52	247	775·97	47916·36	297	933·05	69279·19
198	622·03	30790·75	248	779·11	48305·13	298	936·19	69746·50
199	625·18	31102·55	249	782·26	48695·47	299	939·34	70215·38
200	628·32	31415·93	250	785·40	49087·39	300	942·48	70685·83

TABLE OF DIAMETERS, CIRCUMFERENCES, AND AREAS OF CIRCLES.

Diameter.	Circumference.	Circular area.	Diameter.	Circumference.	Circular area.	Diameter.	Circumference.	Circular area.
301	945·62	71157·86	351	1102·70	96761·84	401	1259·78	126292·81
302	948·76	71631·45	352	1105·84	97314·76	402	1262·92	126923·48
303	951·90	72106·62	353	1108·98	97867·68	403	1266·06	127553·73
304	955·04	72583·36	354	1112·12	98422·96	404	1269·20	128189·55
305	958·19	73061·66	355	1115·26	98979·80	405	1272·34	128824·93
306	961·33	73541·54	356	1118·41	99538·22	406	1275·49	129461·89
307	964·47	74022·99	357	1121·55	100098·21	407	1278·63	130100·42
308	967·61	74506·01	358	1124·69	100659·27	408	1281·77	130740·52
309	970·75	74990·60	359	1127·83	101222·90	409	1284·91	131382·19
310	973·89	75476·76	360	1130·97	101787·60	410	1288·05	132025·43
311	977·03	75964·50	361	1134·11	102353·87	411	1291·19	132670·24
312	980·18	76453·80	362	1137·26	102921·72	412	1294·34	133316·63
313	983·32	76944·67	363	1140·40	103491·13	413	1297·48	133964·58
314	986·46	77437·12	364	1143·54	104062·12	414	1300·62	134614·10
315	989·60	77931·13	365	1146·68	104634·67	415	1303·76	135265·20
316	992·74	78426·72	366	1149·82	105208·80	416	1306·90	135917·86
317	995·88	78923·88	367	1152·96	105784·49	417	1310·04	136572·10
318	999·03	79422·60	368	1156·11	106361·76	418	1313·19	137227·91
319	1002·17	79922·90	369	1159·25	106940·60	419	1316·33	137885·29
320	1005·31	80424·77	370	1162·39	107521·01	420	1319·47	138544·24
321	1008·45	80928·21	371	1165·53	108102·99	421	1322·61	139204·70
322	1011·59	81433·22	372	1168·67	108686·54	422	1325·75	139866·85
323	1014·73	81939·80	373	1171·81	109271·66	423	1328·89	140530·51
324	1017·88	82447·96	374	1174·96	109858·35	424	1332·03	141195·74
325	1021·02	82957·68	375	1178·10	110446·62	425	1335·18	141862·54
326	1024·16	83468·98	376	1181·24	111036·45	426	1338·32	142530·92
327	1027·30	83981·84	377	1184·38	111627·86	427	1341·46	143200·86
328	1030·44	84496·28	378	1187·52	112220·83	428	1344·60	143872·38
329	1033·58	85012·28	379	1190·66	112815·38	429	1347·74	144545·46
330	1036·73	85529·86	380	1193·80	113411·49	430	1350·88	145220·12
331	1039·87	86049·01	381	1196·95	114009·18	431	1354·03	145896·35
332	1043·01	86569·73	382	1200·09	114608·44	432	1357·17	146574·15
333	1046·15	87092·02	383	1203·23	115209·27	433	1360·31	147253·52
334	1049·29	87615·88	384	1206·37	115811·67	434	1363·45	147934·46
335	1052·43	88141·31	385	1209·51	116415·64	435	1366·59	148616·97
336	1055·57	88668·31	386	1212·65	117021·18	436	1369·73	149301·05
337	1058·72	89196·88	387	1215·80	117628·30	437	1372·88	149986·70
338	1061·86	89727·03	388	1218·94	118236·98	438	1376·02	150673·93
339	1065·00	90258·74	389	1222·08	118847·24	439	1379·16	151362·72
340	1068·14	90792·03	390	1225·22	119459·06	440	1382·30	152053·08
341	1071·28	91326·88	391	1228·36	120072·46	441	1385·44	152745·02
342	1074·42	91863·31	392	1231·50	120687·42	442	1388·58	153438·53
343	1077·57	92401·31	393	1234·65	121303·96	443	1391·73	154133·60
344	1080·71	92940·88	394	1237·79	121922·07	444	1394·87	154830·25
345	1083·85	93482·02	395	1240·93	122541·75	445	1398·01	155528·47
346	1086·99	94024·73	396	1244·07	123163·00	446	1401·15	156228·26
347	1090·13	94569·01	397	1247·21	123785·82	447	1404·29	156929·62
348	1093·27	95114·86	398	1250·35	124410·21	448	1407·43	157632·55
349	1096·42	95662·28	399	1253·49	125036·17	449	1410 57	158337·06
350	1099·56	96211·28	400	1256·64	125663·71	450	1413·72	159043·13

TABLE OF DIAMETERS, CIRCUMFERENCES, AND AREAS OF CIRCLES.

Diameter.	Circumference.	Circular area.	Diameter.	Circumference.	Circular area.	Diameter.	Circumference.	Circular area.
451	1416·86	159750·77	501	1573·94	197135·72	551	1731·02	238447·67
452	1420·00	160459·99	502	1577·08	197923·48	552	1734·16	239313·96
453	1423·14	161170·77	503	1580·22	198712·80	553	1737·30	240181·83
454	1426·28	161883 13	504	1583·36	199503·70	554	1740·44	241051·26
455	1429·42	162597·06	505	1586·50	200296·17	555	1743·58	241922·27
456	1432·57	163312·55	506	1589·65	201090·20	556	1746·73	242794·85
457	1435·71	164029·62	507	1592·79	201885·81	557	1749·87	243668·99
458	1438·85	164748·26	508	1595·93	202682·99	558	1753·00	244544·61
459	1441·99	165468·47	509	1599·07	203481·74	559	1756·15	245422·00
460	1445·13	166190·25	510	1602·21	204282·06	560	1759·29	246300·86
461	1448·27	166913·60	511	1605·35	205083·95	561	1762·43	247181·30
462	1451·42	167638·53	512	1608·49	205887·42	562	1765·57	248062·30
463	1454·56	168365·02	513	1611·64	206692·45	563	1768·72	248946·87
464	1457·70	169093·08	514	1614·78	207499·05	564	1771·86	249832·01
465	1460·84	169822·72	515	1617·92	208307·23	565	1775·00	250718·73
466	1463·98	170553·92	516	1621·06	209116·97	566	1778·14	251607·01
467	1467·12	171286·70	517	1624·20	209928·29	567	1781·28	252496·87
468	1470·26	172021·05	518	1627·34	210741·18	568	1784·42	253388·30
469	1473·41	172756·97	519	1630·49	211555·63	569	1787·57	254281·30
470	1476·55	173494·45	520	1633·63	212371·66	570	1790·71	255175·86
471	1479·69	174233·51	521	1636·77	213189·26	571	1793·85	256072·00
472	1482·83	174974·14	522	1639·91	214008·43	572	1796·99	256969·71
473	1485·97	175716·35	523	1643·05	214829·17	573	1800·13	257868·99
474	1489·11	176460·12	524	1646·19	215651·49	574	1803·27	258769·85
475	1492·26	177205·46	525	1649·34	216475·37	575	1806·42	259672·27
476	1495·40	177952·37	526	1652·48	217300·82	576	1809·56	260576·26
477	1498·54	178700·86	527	1655·62	218127·85	577	1812·70	261481·83
478	1501·68	179450·91	528	1658·76	218956·44	578	1815·84	262388·96
479	1504·82	180202·54	529	1661·90	219786·61	579	1818·98	263297·67
480	1507·96	180955·74	530	1665·04	220618·32	580	1822·12	264207·94
481	1511·11	181710·50	531	1668·19	221451·65	581	1825·26	265119·79
482	1514·25	182466·84	532	1671·33	222286·53	582	1828·41	266033·21
483	1517·39	183224·75	533	1674·47	223122·98	583	1831·55	266948·20
484	1520·53	183984·23	534	1677·61	223961·00	584	1834·69	267864·76
485	1523·67	184745·28	535	1680·75	224800·59	585	1837·83	268782·80
486	1526·81	185507·90	536	1683·89	225641·75	586	1840·97	269702·59
487	1529·96	186272·10	537	1687·04	226484·48	587	1844·11	270623·86
488	1533·10	187037·86	538	1690·18	227328·77	588	1847·26	271546·70
489	1536·24	187805·19	539	1693·32	228174·66	589	1850·40	272471·12
490	1539·38	188574·10	540	1696·46	229022·10	590	1853·54	273397·10
491	1542·52	189344·57	541	1699·60	229871·12	591	1856·68	274324·66
492	1545·66	190116·62	542	1702·74	230721·71	592	1859·82	275253·78
493	1548·80	190890·24	543	1705·88	231573·86	593	1862·96	276184·48
494	1551·95	191665·43	544	1709·03	232427·59	594	1866·11	277116·75
495	1555·09	192442·19	545	1712·17	233282·89	595	1869·25	278050·59
496	1558·23	193220·51	546	1715·31	234139·76	596	1872·39	278985·99
497	1561·37	194000·42	547	1718·45	234998·20	597	1875·53	279922·97
498	1564·51	194781·89	548	1721·59	235858·21	598	1878·67	280861·53
499	1567·65	195564·93	549	1724·73	236719·79	599	1881·81	281801·65
500	1570·80	196349·54	550	1727·88	237582·94	600	1884·96	282743·34

TABLE OF SQUARES, CUBES, SQUARE AND CUBE ROOTS OF NUMBERS.

Squares.	Cubes.	No.	Square roots.	Cube roots.	Squares.	Cubes.	No.	Square roots.	Cube roots.
1	1	1	1·000	1·000	4096	262144	64	8·000	4·000
4	8	2	1·414	1·259	4225	274625	65	8·062	4·020
9	27	3	1·732	1·442	4356	287496	66	8·124	4·041
16	64	4	2·000	1·587	4489	300763	67	8·185	4·061
25	125	5	2·236	1·709	4624	314432	68	8·246	4·081
36	216	6	2·449	1·817	4761	328509	69	8·306	4·101
49	343	7	2·645	1·912	4900	343000	70	8·366	4·121
64	512	8	2·828	2·000	5041	357911	71	8·426	4·140
81	729	9	3·000	2·080	5184	373248	72	8·485	4·160
100	1000	10	3·162	2·154	5329	389017	73	8·544	4·179
121	1331	11	3·316	2·223	5476	405224	74	8·602	4·198
144	1728	12	3·464	2·289	5625	421875	75	8·660	4·217
169	2197	13	3·605	2·351	5776	438976	76	8·717	4·235
196	2744	14	3·741	2·410	5929	456533	77	8·774	4·254
225	3375	15	3·872	2·466	6084	474552	78	8·831	4·272
256	4096	16	4·000	2·519	6241	493039	79	8·888	4·290
289	4913	17	4·123	2·571	6400	512000	80	8·944	4·308
324	5832	18	4·242	2·620	6561	531441	81	9·000	4·326
361	6859	19	4·358	2·668	6724	551368	82	9·055	4·344
400	8000	20	4·472	2·714	6889	571787	83	9·110	4·362
441	9261	21	4·582	2·758	7056	592704	84	9·165	4·379
484	10648	22	4·690	2·802	7225	614125	85	9·219	4·396
529	12167	23	4·795	2·843	7396	636056	86	9·273	4·414
576	13824	24	4·898	2·884	7569	658503	87	9·327	4·431
625	15625	25	5·000	2·924	7744	681472	88	9·380	4·447
676	17576	26	5·099	2·962	7921	704969	89	9·433	4·464
729	19683	27	5·196	3·000	8100	729000	90	9·486	4·481
784	21952	28	5·291	3·036	8281	753571	91	9·539	4·497
841	24389	29	5·385	3·072	8464	778688	92	9·591	4·514
900	27000	30	5·477	3·107	8649	804357	93	9·643	4·530
961	29791	31	5·567	3·141	8836	830584	94	9·695	4·546
1024	32768	32	5·656	3·174	9025	857374	95	9·746	4·562
1089	35937	33	5·744	3·207	9216	884736	96	9·797	4·578
1156	39304	34	5·830	3·239	9409	912673	97	9·848	4·594
1225	42875	35	5·916	3·271	9604	941192	98	9·899	4·610
1296	46656	36	6·000	3·301	9801	970299	99	9·949	4·626
1369	50653	37	6·082	3·332	10000	1000000	100	10·000	4·641
1444	54872	38	6·164	3·361	10201	1030301	101	10·049	4·657
1521	59319	39	6·244	3·391	10404	1061208	102	10·099	4·672
1600	64000	40	6·324	3·419	10609	1092727	103	10·148	4·687
1681	68921	41	6·403	3·448	10816	1124864	104	10·198	4·702
1764	74088	42	6·480	3·476	11025	1157625	105	10·246	4·717
1849	79507	43	6·557	3·503	11236	1191016	106	10·295	4·732
1936	85184	44	6·633	3·530	11449	1225043	107	10·344	4·747
2025	91125	45	6·708	3·556	11664	1259712	108	10·392	4·762
2116	97336	46	6·782	3·583	11881	1295029	109	10·440	4·776
2209	103823	47	6·855	3·608	12100	1331000	110	10·488	4·791
2304	110592	48	6·928	3·634	12321	1367631	111	10·535	4·805
2401	117649	49	7·000	3·659	12544	1404928	112	10·583	4·820
2500	125000	50	7·071	3·684	12769	1442897	113	10·630	4·834
2601	132651	51	7·141	3·708	12996	1481544	114	10·677	4·848
2704	140608	52	7·211	3·732	13225	1520875	115	10·723	4·862
2809	148877	53	7·280	3·756	13456	1560896	116	10·770	4·876
2916	157464	54	7·348	3·779	13689	1601613	117	10·816	4·890
3025	166375	55	7·416	3·802	13924	1643032	118	10·862	4·904
3136	175616	56	7·483	3·825	14161	1685159	119	10·908	4·918
3249	185193	57	7·549	3·848	14400	1728000	120	10·954	4·932
3364	195112	58	7·615	3·870	14641	1771561	121	11·000	4·946
3481	205379	59	7·681	3·892	14834	1815848	122	11·045	4·959
3600	216000	60	7·745	3·914	15129	1860867	123	11·090	4·973
3721	226981	61	7·810	3·930	15376	1906624	124	11·135	4·986
3844	238328	62	7·874	3·957	15625	1953125	125	11·180	5·000
3969	250047	63	7·937	3·979	15876	2000376	126	11·224	5·013

TABLE OF SQUARES, CUBES, SQUARE AND CUBE ROOTS OF NUMBERS—(*Continued*).

Squares.	Cubes.	No.	Square roots.	Cube roots.	Squares.	Cubes.	No.	Square roots.	Cube roots.
16129	2048383	127	11·269	5·026	36100	6859000	190	13·784	5·748
16384	2097152	128	11·313	5·039	36481	6967871	191	13·820	5·758
16641	2146689	129	11·357	5·052	36864	7077888	192	13·856	5·768
16900	2197000	130	11·401	5·065	37249	7189517	193	13·892	5·778
17161	2248091	131	11·445	5·078	37636	7301384	194	13·928	5·788
17424	2299968	132	11·489	5·091	38025	7414875	195	13·964	5·798
17689	2352637	133	11·532	5·104	38416	7529536	196	14·000	5·808
17956	2406104	134	11·575	5·117	38809	7645373	197	14·035	5·818
18225	2460375	135	11·618	5·129	39204	7762392	198	14·071	5·828
18496	2515456	136	11·661	5·142	39601	7880599	199	14·106	5·838
18769	2571353	137	11·704	5·155	40000	8000000	200	14·142	5·848
19044	2628072	138	11·747	5·167	40401	8120601	201	14·177	5·857
19321	2685619	139	11·789	5·180	40804	8242408	202	14·212	5·867
19600	2744000	140	11·832	5·192	41209	8365427	203	14·247	5·877
19881	2803221	141	11·874	5·204	41616	8489664	204	14·282	5·886
20164	2863288	142	11·916	5·217	42025	8615125	205	14·317	5·896
20449	2924207	143	11·958	5·229	42436	8741816	206	14·352	5·905
20736	2985984	144	12·000	5·241	42849	8869743	207	14·387	5·915
21025	3048625	145	12·041	5·253	43264	8998912	208	14·422	5·924
21316	3112136	146	12·083	5·265	43681	9129329	209	14·456	5·934
21609	3176523	147	12·124	5·277	44100	9261000	210	14·491	5·943
21904	3241792	148	12·165	5·289	44521	9393931	211	14·525	5·953
22201	3307949	149	12·206	5·301	44944	9528128	212	14·560	5·962
22500	3375000	150	12·247	5·313	45369	9663597	213	14·594	5·972
22801	3442951	151	12·288	5·325	45796	9800344	214	14·628	5·981
23104	3511008	152	12·328	5·336	46225	9938375	215	14·662	5·990
23409	3581577	153	12·369	5·348	46656	10077696	216	14·696	6·000
23716	3652264	154	12·409	5·360	47089	10218312	217	14·730	6·009
24025	3723875	155	12·449	5·371	47524	10360232	218	14·764	6·018
24336	3796416	156	12·489	5·383	47961	10503459	219	14·798	6·027
24649	3869893	157	12·529	5·394	48400	10648000	220	14·832	6·036
24964	3944312	158	12·569	5·406	48841	10793861	221	14·866	6·045
25281	4019679	159	12·609	5·417	49284	10941048	222	14·899	6·055
25600	4096000	160	12·649	5·428	49729	11089567	223	14·933	6·064
25921	4173281	161	12·688	5·440	50176	11239424	224	14·966	6·073
26244	4251528	162	12·727	5·451	50625	11390625	225	15·000	6·082
26569	4330747	163	12·767	5·462	51076	11543176	226	15·033	6·099
26896	4410944	164	12·806	5·473	51529	11697083	227	15·066	6·100
27225	4492125	165	12·845	5·484	51984	11852352	228	15·099	6·109
27556	4574296	166	12·884	5·495	52441	12008989	229	15·132	6·118
27889	4657463	167	12·922	5·506	52900	12167000	230	15·165	6·126
28224	4741632	168	12·961	5·517	53361	12326391	231	15·198	6·135
28561	4826809	169	13·000	5·528	53824	12487168	232	15·231	6·144
28900	4913000	170	13·938	5·539	54289	12649337	233	15·264	6·153
29241	5000211	171	13·076	5·550	54756	12812904	234	15·297	6·162
29584	5088448	172	13·114	5·561	55225	12977875	235	15·329	6·171
29929	5177717	173	13·152	5·572	55696	13144256	236	15·362	6·179
30276	5268024	174	13·190	5·582	56169	13312053	237	15·394	6·188
30625	5359375	175	13·228	5·593	56644	13481272	238	15·427	6·197
30976	5451776	176	13·266	5·604	57121	13651919	239	15·459	6·205
31329	5545233	177	13·304	5·614	57600	13824000	240	15·491	6·214
31684	5639752	178	13·341	5·625	58081	13997521	241	15·524	6·223
32041	5735339	179	13·379	5·635	58564	14172488	242	15·556	6·231
32400	5832000	180	13·416	5·646	59049	14348907	243	15·588	6·240
32761	5929741	181	13·453	5·656	59536	14526784	244	15·620	6·248
33124	6028568	182	13·490	5·667	60025	14706125	245	15·652	6·257
33489	6128487	183	13·527	5·677	60516	14886936	246	15·684	6·265
33856	6229504	184	13·664	5·687	61009	15069223	247	15·716	6·274
34225	6331625	185	13·601	5·698	61504	15252992	248	15·748	6·282
34596	6434856	186	13·638	5·708	62001	15438249	249	15·779	6·291
34969	6539203	187	13·674	5·718	62500	15625000	250	15·811	6·299
35344	6644672	188	13·711	5·728	63001	15813251	251	15·842	6·307
35721	6751269	189	13·747	5·738	63504	16003008	252	15·874	6·316

TABLE OF SQUARES, CUBES, SQUARE AND CUBE ROOTS OF NUMBERS—(*Continued*).

Squares.	Cubes.	No.	Square roots.	Cube roots.	Squares.	Cubes.	No.	Square roots.	Cube roots.
64009	16194277	253	15·905	6·324	99856	31554496	316	17·776	6·811
64516	16387064	254	15·937	6·333	100489	31855013	317	17·804	6·818
65025	16581375	255	15·968	6·341	101124	32157432	318	17·832	6·825
65536	16777216	256	16·000	6·349	101761	32461759	319	17·860	6·832
66049	16974593	257	16·031	6·357	102400	32768000	320	17·888	6·839
66564	17173512	258	16·062	6·366	103041	33076161	321	17·916	6·847
67081	17373979	259	16 093	6·374	103684	33386248	322	17·944	6·854
67600	17576000	260	16·124	6·382	104329	33698267	323	17·972	6·861
68121	17779581	261	16·155	6·390	104976	34012224	324	18·000	6·868
68644	17984728	262	16·186	6·398	105625	34328125	325	18·027	6·875
69169	18191447	263	16·217	6·406	106276	34645976	326	18·055	6·882
69696	18399744	264	16·248	6·415	106929	34965783	327	18·083	6·889
70225	18609625	265	16·278	6·423	107584	35287552	328	18·110	6·896
70756	18821096	266	16·309	6·431	108241	35611289	329	18·138	6·903
71289	19034163	267	16·340	6·439	108900	35937000	330	18·165	6·910
71824	19248832	268	16·370	6·447	109561	36264691	331	18·193	6·917
72361	19465109	269	16·401	6·455	110224	36594368	332	18·220	6·924
72900	19683000	270	16·431	6·463	110889	36926037	333	18·248	6·931
73441	19902511	271	16·462	6·471	111556	37259704	334	18·275	6·938
73984	20123643	272	16·492	6·479	112225	37595375	335	18·303	6·945
74529	20346417	273	16·522	6·487	112896	37933056	336	18·330	6·952
75076	20570824	274	16·552	6·495	113569	38272753	337	18·357	6·958
75625	20796875	275	16·583	6·502	114244	38614472	338	18·384	6·965
76176	21024576	276	16·613	6·510	114921	38958219	339	18·411	6·972
76729	21253933	277	16·643	6·518	115600	39304000	340	18·439	6·979
77284	21484952	278	16·673	6·526	116281	39651821	341	18·466	6·986
77841	21717639	279	16·703	6·534	116964	40001688	342	18·493	6·993
78400	21952000	280	16·733	6·542	117649	40353607	343	18·520	7·000
78961	22188041	281	16·763	6·549	118336	40707584	344	18·547	7·006
79524	22425768	282	16·792	6·557	119025	41063625	345	18·574	7·013
80089	22665187	283	16·822	6·565	119716	41421736	346	18·601	7·020
80656	22906304	284	16·852	6·573	120409	41781923	347	18·627	7·027
81225	23149125	285	16·881	6·580	121104	42144192	348	18·654	7·033
81796	23393656	286	16·911	6·588	121801	42508549	349	18·681	7·040
82369	23639903	287	16·941	6·596	122500	42875000	350	18·708	7·047
82944	23887872	288	16·970	6·603	123201	43243551	351	18·734	7·054
83521	24137569	289	17·000	6·611	123904	43614208	352	18·761	7·060
84100	24389000	290	17·029	6·619	124609	43986977	353	18·788	7·067
84681	24642171	291	17·058	6·626	125316	44361864	354	18·814	7·074
85264	24897088	292	17·088	6·634	126025	44738875	355	18·841	7·080
85849	25153757	293	17·117	6·641	126736	45118016	356	18·867	7·087
86436	25412184	294	17·146	6·649	127449	45499293	357	18·894	7·093
87025	25672375	295	17·175	6·656	128164	45882712	358	18·920	7·100
87616	25934836	296	17·204	6·664	128881	46268279	359	18·947	7·107
88209	26198073	297	17·233	6·671	129600	46656000	360	18·973	7·113
88804	26463592	298	17·262	6·679	130321	47045831	361	19·000	7·120
89401	26730899	299	17·291	6·686	131044	47437928	362	19·026	7·126
90000	27000000	300	17·320	6·694	131769	47832147	363	19·052	7·133
90601	27270901	301	17·349	6·701	132496	48228544	364	19·078	7·140
91204	27543608	302	17·378	6·709	133225	48627125	365	19·104	7·146
91809	27818127	303	17·406	6·716	133956	49027896	366	19·131	7·153
92416	28094464	304	17·435	6·723	134689	49430863	367	19·157	7·159
93025	28372625	305	17·464	6·731	135424	49836032	368	19·183	7·166
93636	28652616	306	17·492	6·738	136161	50243409	369	19·209	7·172
94249	28934443	307	17·521	6·745	136900	50653000	370	19·235	7·179
94864	29218112	308	17·549	6·753	137641	51064811	371	19·261	7·185
95481	29503609	309	17·578	6·760	138384	51478848	372	19·287	7·191
96100	29791000	310	17·606	6·767	139129	51895117	373	19·313	7·198
96721	30080231	311	17·635	6·775	139876	52313624	374	19·339	7·204
97344	30371328	312	17·663	6·782	140625	52734375	375	19·364	7·211
97969	30664297	313	17·691	6·789	141376	53157376	376	19·390	7·217
98596	30959144	314	17·720	6·796	142129	53582633	377	19·416	7·224
99225	31255875	315	17·748	6·804	142884	54010152	378	19·442	7·230

TABLE OF SQUARES, CUBES, SQUARE AND CUBE ROOTS OF NUMBERS—(*Continued*).

Squares.	Cubes.	No.	Square roots.	Cube roots.	Squares.	Cubes.	No.	Square roots.	Cube roots.
143641	54439939	379	19·467	7·236	195364	86350888	442	21·023	7·617
144400	54872000	380	19·493	7·243	196249	86938307	443	21·047	7·623
145161	55306341	381	19·519	7·249	197136	87528384	444	21·071	7·628
145924	55742968	382	19·544	7·255	198025	88121125	445	21·095	7·634
146689	56181887	383	19·570	7·262	198916	88716536	446	21·118	7·640
147456	56623104	384	19·595	7·268	199809	89314623	447	21·142	7·646
148225	57066625	385	19·621	7·274	200704	89915392	448	21·166	7·651
148996	57512456	386	19·646	7·281	201601	90518849	449	21·189	7·657
149769	57960603	387	19·672	7·287	202500	91125000	450	21·213	7·663
150544	58411072	388	19·697	7·293	203401	91733851	451	21·236	7·668
151321	58863869	389	19·723	7·299	204304	92345408	452	21·260	7·674
152100	59319000	390	19·748	7·306	205209	92959677	453	21·283	7·680
152881	59776471	391	19·773	7·312	206116	93576664	454	21·307	7·685
153664	60236288	392	19·798	7·318	207025	94196375	455	21·330	7·691
154449	60698457	393	19·824	7·324	207936	94818816	456	21·354	7·697
155236	61162984	394	19·849	7·331	208849	95443993	457	21·377	7·702
156025	61629875	395	19·874	7·337	209764	96071912	458	21·400	7·708
156816	62099136	396	19·899	7·343	210681	96702579	459	21·424	7·713
157609	62570773	397	19·924	7·349	211600	97336000	460	21·447	7·719
158404	63044792	398	19·949	7·355	212521	97972181	461	21·470	7·725
159201	63521199	399	19·974	7·361	213444	98611128	462	21·494	7·730
160000	64000000	400	20·000	7·368	214369	99252847	463	21·517	7·736
160801	64481201	401	20·024	7·374	215296	99897344	464	21·540	7·741
161604	64964808	402	20·049	7·380	216225	100544625	465	21·563	7·747
162409	65450827	403	20·074	7·386	217156	101194696	466	21·587	7·752
163216	65939264	404	20·099	7·392	218089	101847563	467	21·610	7·758
164025	66430125	405	20·124	7·398	219024	102503232	468	21·633	7·763
164836	66923416	406	20·149	7·404	219961	103161709	469	21·656	7·769
165649	67419143	407	20·174	7·410	220900	103823000	470	21·679	7·774
166464	67917312	408	20·199	7·416	221841	104487111	471	21·702	7·780
167281	68417929	409	20·223	7·422	222784	105154048	472	21·725	7·785
168100	68921000	410	20·248	7·428	223729	105823817	473	21·748	7·791
168921	69426531	411	20·273	7·434	224676	106496424	474	21·771	7·796
169744	69934528	412	20·297	7·441	225625	107171875	475	21·794	7·802
170569	70444997	413	20·322	7·447	226576	107850176	476	21·817	7·807
171396	70957944	414	20·346	7·453	227529	108531333	477	21·840	7·813
172225	71473375	415	20·371	7·459	228484	109215352	478	21·863	7·818
173056	71991296	416	20·396	7·465	229441	109902239	479	21·886	7·824
173889	72511713	417	20·420	7·470	230400	110592000	480	21·908	7·829
174724	73034632	418	20·445	7·476	231361	111284641	481	21·931	7·835
175561	73560059	419	20·469	7·482	232324	111980168	482	21·954	7·840
176400	74088000	420	20·493	7·488	233289	112678587	483	21·977	7·846
177241	74618461	421	20·518	7·494	234256	113379904	484	22·000	7·851
178084	75151448	422	20·542	7·500	235225	114084125	485	22·022	7·856
178929	75686967	423	20·566	7·506	236196	114791256	486	22·045	7·862
179776	76225024	424	20·591	7·512	237169	115501303	487	22·068	7·867
180625	76765625	425	20·615	7·518	238144	116214272	488	22·090	7·872
181476	77308776	426	20·639	7·524	239121	116930169	489	22·113	7·878
182329	77854483	427	20·663	7·530	240100	117649000	490	22·135	7·883
183184	78402752	428	20·688	7·536	241081	118370771	491	22·158	7·889
184041	78953589	429	20·712	7·541	242064	119095488	492	22·181	7·894
184900	79507000	430	20·736	7·547	243049	119823157	493	22·203	7·899
185761	80062991	431	20·760	7·553	244036	120553784	494	22·226	7·905
186624	80621568	432	20·784	7·559	245025	121287375	495	22·248	7·910
187489	81182737	433	20·808	7·565	246016	122023936	496	22·271	7·915
188356	81746504	434	20·832	7·571	247009	122763473	497	22·293	7·921
189225	82312875	435	20·856	7·576	248004	123505992	498	22·315	7·926
190096	82881856	436	20·880	7·582	249001	124251499	499	22·338	7·931
190969	83453453	437	20·904	7·588	250000	125000000	500	22·360	7·937
191844	84027672	438	20·928	7·594	251001	125751501	501	22·383	7·942
192721	84604519	439	20·952	7·600	252004	126506008	502	22·405	7·947
193600	85184000	440	20·976	7·605	253009	127263527	503	22·427	7·952
194481	85766121	441	21·000	7·611	254016	128024064	504	22·449	7·958

TABLE OF SQUARES, CUBES, SQUARE AND CUBE ROOTS OF NUMBERS—(*Continued*).

Squares.	Cubes.	No.	Square roots.	Cube roots.	Squares.	Cubes.	No.	Square roots.	Cube roots.
255025	128787625	505	22·472	7·963	322624	183250432	568	23·832	8·281
256036	129554216	506	22·494	7·968	323761	184220009	569	23·853	8·286
257049	130323843	507	22·516	7·973	324900	185193000	570	23·874	8·291
258064	131096512	508	22·538	7·979	326041	186169411	571	23·895	8·296
259081	131872229	509	22·561	7·984	327184	187149248	572	23·916	8·301
260100	132651000	510	22·583	7·989	328329	188132517	573	23·937	8·305
261121	133432831	511	22·605	7·994	329476	189119224	574	23·958	8·310
262144	134217728	512	22·627	8·000	330625	190109375	575	23·979	8·315
263169	135005697	513	22·649	8·005	331776	191102976	576	24·000	8·320
264196	135796744	514	22·671	8·010	332929	192100033	577	24·020	8·325
265225	136590875	515	22·693	8·015	334084	193100552	578	24·041	8·329
266256	137388096	516	22·715	8·020	335241	194104539	579	24·062	8·334
267289	138188413	517	22·737	8·025	336400	195112000	580	24·083	8·339
268324	138991832	518	22·759	8·031	337561	196122941	581	24·103	8·344
269361	139798359	519	22·781	8·036	338724	197137368	582	24·124	8·349
270400	140608000	520	22·803	8·041	339889	198155287	583	24·145	8·353
271441	141420761	521	22·825	8·046	341056	199176704	584	24·166	8·358
272484	142236648	522	22·847	8·051	342225	200201625	585	24·186	8·363
273529	143055667	523	22·869	8·056	343396	201230056	586	24·207	8·368
274576	143877824	524	22·891	8·062	344569	202262003	587	24·228	8·372
275625	144703125	525	22·912	8·067	345744	203297472	588	24·248	8·377
276676	145531576	526	22·934	8·072	346921	204336469	589	24·269	8·382
277729	146363183	527	22·956	8·077	348100	205379000	590	24·289	8·387
278784	147197952	528	22·978	8·082	349281	206425071	591	24·310	8·391
279841	148035889	529	23·000	8·087	350464	207474688	592	24·331	8·396
280900	148877000	530	23·021	8·092	351649	208527857	593	24·351	8·401
281961	149721291	531	23·043	8·097	352836	209584584	594	24·372	8·406
283024	150568768	532	23·065	8·102	354025	210644875	595	24·392	8·410
284089	151419437	533	23·086	8·107	355216	211708736	596	24·413	8·415
285156	152273304	534	23·108	8·112	356409	212776173	597	24·433	8·420
286225	153130375	535	23·130	8·118	357604	213847192	598	24·454	8·424
287296	153990656	536	23·151	8·123	358801	214921799	599	24·474	8·429
288369	154854153	537	23·173	8·128	360000	216000000	600	24·494	8·434
289444	155720872	538	23·194	8·133	361201	217081801	601	24·515	8 439
290521	156590819	539	23·216	8·138	362404	218167208	602	24·535	8·443
291600	157464000	540	23·237	8·143	363609	219256227	603	24·556	8·448
292681	158340421	541	23·259	8·148	364816	220348864	604	24·576	8·453
293764	159220088	542	23·280	8·153	366025	221445125	605	24·596	8·457
294849	160103007	543	23·302	8·158	367236	222545016	606	24·617	8·462
295936	160989184	544	23·323	8·163	368449	223648543	607	24·637	8·467
297025	161878625	545	23·345	8·168	369664	224755712	608	24·657	8·471
298116	162771336	546	23·366	8·173	370881	225866529	609	24·677	8·476
299209	163667323	547	23·388	8·178	372100	226981000	610	24·698	8·480
300304	164566592	548	23·409	8·183	373321	228099131	611	24·718	8·485
301401	165469149	549	23·430	8·188	374544	229220928	612	24·738	8·490
302500	166375000	550	23·452	8·193	375769	230346397	613	24·758	8·494
303601	167284151	551	23·473	8·198	376996	231475544	614	24·779	8·499
304704	168196608	552	23·494	8·203	378225	232608375	615	24·799	8·504
305809	169112377	553	23·515	8·208	379456	233744896	616	24·819	8·508
306916	170031464	554	23·537	8·213	380689	234885113	617	24·839	8·513
308025	170953875	555	23·558	8·217	381924	236029032	618	24·859	8·517
309136	171879616	556	23·579	8·222	383161	237176659	619	24·879	8·522
310249	172808693	557	23·600	8·227	384400	238328000	620	24·899	8·527
311364	173741112	558	23·622	8·232	385641	239483061	621	24·919	8·531
312481	174676879	559	23·643	8·237	386884	240641848	622	24·939	8·536
313600	175616000	560	23·664	8·242	388129	241804367	623	24·959	8·540
314721	176558481	561	23·685	8·247	389376	242970624	624	24·979	8·545
315844	177504328	562	23·706	8·252	390625	244140625	625	25·000	8·549
316969	178453547	563	23·727	8·257	391876	245314376	626	25·019	8·554
318096	179406144	564	23·748	8·262	393129	246491883	627	25·039	8·558
319225	180362125	565	23·769	8·267	394384	247673152	628	25·059	8·563
320356	181321496	566	23·790	8·271	395641	248858189	629	25·079	8·568
321489	182284263	567	23·811	8·276	396900	250047000	630	25·099	8·572

TABLE OF SQUARES, CUBES, SQUARE AND CUBE ROOTS OF NUMBERS—(*Continued*).

Squares.	Cubes.	No.	Square roots.	Cube roots.	Squares.	Cubes.	No.	Square roots.	Cube roots.
398161	251239591	631	25·119	8·577	481636	334255384	694	26·343	8·853
399424	252435968	632	25·139	8·581	483025	335702375	695	26·362	8·857
400689	253636137	633	25·159	8·586	484416	337153536	696	26·381	8·862
401956	254840104	634	25·179	8·590	485809	338608873	697	26·400	8·866
403225	256047875	635	25·199	8·595	487204	340068392	698	26·419	8·870
404496	257259456	636	25·219	8·599	488601	341532099	699	26·438	8·874
405769	258474853	637	25·238	8·604	490000	343000000	700	26·457	8·879
407044	259694072	638	25·258	8·608	491401	344472101	701	26·476	8·883
408321	260917119	639	25·278	8·613	492804	345948408	702	26·495	6·887
409600	262144000	640	25·298	8·617	494209	347428927	703	26·514	8·891
410881	263374721	641	25·317	8·622	495616	348913664	704	26·532	8·895
412164	264609288	642	25·337	8·626	497025	350402625	705	26·551	8·900
413449	265847707	643	25·357	8·631	498436	351895816	706	26·570	8·904
414736	267089984	644	25·377	3·635	499849	353393243	707	26·589	8·908
416025	268336125	645	25·396	8·640	501264	354894912	708	26·608	8·912
417316	269586136	646	25·416	8·644	502681	356400829	709	26·627	8·916
418609	270840023	647	25·436	8·649	504100	357911000	710	26·645	8·921
419904	272097792	648	25·455	8·653	505521	359425431	711	26·664	8·925
421201	273359449	649	25·475	8·657	506944	360944128	712	26·683	8·929
422500	274625000	650	25·495	8·662	508369	362467097	713	26·702	8·933
423801	275894451	651	25·514	8·666	509796	363994344	714	26·720	8·937
425104	277167808	652	25·534	8·671	511225	365525875	715	26·739	8·942
426409	278445077	653	25·553	8·675	512656	367061696	716	26·758	8·946
427716	279726264	654	25·573	8·680	514089	368601813	717	26·776	8·950
429025	281011375	655	25·592	8·684	515524	370146232	718	26·795	8·954
430336	282300416	656	25·612	8·688	516961	371694959	719	26·814	8·958
431649	283593393	657	25·632	8·693	518400	373248000	720	26·832	8·962
432964	284890312	658	25·651	8·697	519841	374805361	721	26·851	8·966
424281	286191179	659	25·670	8·702	521284	376367048	722	26·870	8·971
435600	287496000	660	25·690	8·706	522729	377933067	723	26·888	8·975
436921	288804781	661	25·709	8·710	524176	379503424	724	26·907	8·979
438244	290117528	662	25·729	8·715	525625	381078125	725	26·925	8·983
439569	291434247	663	25·748	8·719	527076	382657176	726	26·944	8·987
440896	292754944	664	25·768	8·724	528529	384240583	727	26·962	8 991
442225	294079625	665	25·787	8·728	529984	385828352	728	26·981	8·995
443556	295408296	666	25·806	8·732	531441	387420489	729	27·000	9·000
444889	296740963	667	25·826	8·737	532900	389017000	730	27·018	9·004
446224	298077632	668	25·845	8·741	534361	390617891	731	27·037	9·008
447561	299418309	669	25·865	8·745	535824	392223168	732	27·055	9·012
448900	300763000	670	25·884	8·750	537289	393832837	733	27·073	9·016
450241	302111711	671	25·903	8·754	538756	395446904	734	27·092	9·020
451584	303464448	672	25·922	8·759	540225	397065375	735	27·110	9·024
452929	304821217	673	25·942	8·763	541696	398688256	736	27·129	9·028
454276	306182024	674	25·961	8·767	543169	400315553	737	27·147	9·032
455625	307546875	675	25·980	8·772	544644	401947272	738	27·166	9·036
456976	308915776	676	26·000	8·776	546121	403583419	739	27·184	9·040
458329	310288733	677	26·019	8·780	547600	405224000	740	27·202	9·045
459684	311665752	678	26·038	8·785	549081	406869021	741	27·221	9·049
461041	313046839	679	26·057	8·789	550564	408518488	742	27·239	9·053
462400	314432000	680	26·076	8·793	552049	410172407	743	27·258	9·057
463761	315821241	681	26·095	8·797	553536	411830784	744	27·276	9·061
465124	317214568	682	26·115	8·802	555025	413493625	745	27·294	9·065
466489	318611987	683	26·134	8·806	556516	415160936	746	27·313	9·069
467856	320013504	684	26·153	8·810	558009	416832723	747	27·331	9·073
469225	321419125	685	26·172	8·815	559504	418508992	748	27·349	9·077
470596	322828856	686	26·191	8·819	561001	420189749	749	27·367	9·081
471969	324242703	687	26·210	8·823	562500	421875000	750	27·386	9·085
473344	325660672	688	26·229	8·828	564001	423564751	751	27·404	9·089
474721	327082769	689	26·248	8·832	565504	425259008	752	27·422	9·093
476100	328509000	690	26·267	8·836	567009	426957777	753	27·440	9·097
477481	329939371	691	26·286	8·840	568516	428661064	754	27·459	9·101
478864	331373888	692	26·305	8·845	570025	430368875	755	27·477	9·105
480249	332812557	693	26·324	8·849	571536	432081216	756	27·495	9·109

TABLE OF SQUARES, CUBES, SQUARE AND CUBE ROOTS OF NUMBERS—(*Continued*).

Squares.	Cubes.	No.	Square roots.	Cube roots.	Squares.	Cubes.	No.	Square roots.	Cube roots.
573049	433798093	757	27·513	9·113	672400	551368000	820	28·635	9·359
574564	435519512	758	27·531	9·117	674041	553387661	821	28·653	9·363
576081	437245479	759	27·549	9·121	675684	555412248	822	28·670	9·367
577600	438976000	760	27·568	9·125	677329	557441767	823	28·687	9·371
579121	440711081	761	27·586	9·129	678976	559476224	824	28·705	9·375
580644	442450728	762	27·604	9·133	680625	561515625	825	28·722	9·378
582169	444194947	763	27·622	9·137	682276	563559976	826	28·740	9·382
583696	445943744	764	27·640	9·141	683929	565609283	827	28·757	9·386
585225	447697125	765	27·658	9·145	685584	567663552	828	28·774	9·390
586756	449455096	766	27·676	9·149	687241	569722789	829	28·792	9·394
588289	451217663	767	27·694	9·153	688900	571787000	830	28·809	9·397
589824	452984832	768	27·712	9·157	690561	573856191	831	28·827	9·401
591361	454756609	769	27·730	9·161	692224	575930368	832	28·844	9·405
592900	456533000	770	27·748	9·165	693889	578009537	833	28·861	9·409
594441	458314011	771	27·766	9·169	695556	580093704	834	28·879	9·412
595984	460099648	772	27·784	9·173	697225	582182875	835	28·896	9·416
597529	461889917	773	27·802	9·177	698896	584277056	836	28·913	9·420
599076	463684824	774	27·820	9·181	700569	586376253	837	28·930	9·424
600625	465484375	775	27·838	9·185	702244	588480472	838	28·948	9·427
602176	467288576	776	27·856	9·189	703921	590589719	839	28·965	9·431
603729	469097433	777	27·874	9·193	705600	592704000	840	28·982	9·435
605284	470910952	778	27·892	9·197	707281	594823321	841	29·000	9·439
606841	472729139	779	27·910	9·201	708964	596947688	842	29·017	9·442
608400	474552000	780	27·928	9·205	710649	599077107	843	29·034	9·446
609961	476379541	781	27·946	9·209	712336	601211584	844	29·051	9·450
611524	478211768	782	27·964	9·213	714025	603351125	845	29·068	9·454
613089	480048687	783	27·982	9·216	715716	605495736	846	29·086	9·457
614656	481890304	784	28·000	9·220	717409	607645423	847	29·103	9·461
616225	483736625	785	28·017	9·224	719104	609800192	848	29·120	9·465
617796	485587656	786	28·035	9·228	720801	611960049	849	29·137	9·468
619369	487443403	787	28·053	9·232	722500	614125000	850	29·154	9·472
620944	489303872	788	28·071	9·236	724201	616295051	851	29·171	9·476
622521	491169069	789	28·089	9·240	725904	618470208	852	29·189	9·480
624100	493039000	790	28·106	9·244	727609	620650477	853	29·206	9·483
625681	494913671	791	28·124	9·248	729316	622835864	854	29·223	9·487
627264	496793088	792	28·142	9·252	731025	625026375	855	29·240	9·491
628849	498677257	793	28·160	9·256	732736	627222016	856	29·257	9 494
630436	500566184	794	28·178	9·259	734449	629422793	857	29·274	9·498
632025	502459875	795	28·195	9·263	736164	631628712	858	29·291	9·502
633616	504358336	796	28·213	9·267	737881	633839779	859	29·308	9·505
635209	506261573	797	28·231	9·271	739600	636056000	860	29·325	9·509
636804	508169592	798	28·248	9·275	741321	638277381	861	29·342	9·513
638401	510082399	799	28·266	9·279	743044	640503928	862	29·359	9·517
640000	512000000	800	28·284	9·283	744769	642735647	863	29·376	9·520
641601	513922401	801	28·301	9·287	746496	644972544	864	29·393	9·524
643204	515849608	802	28·319	9·290	748225	647214625	865	29·410	9·528
644809	517781627	803	28·337	9·294	749956	649461896	866	29·427	9·531
646416	519718464	804	28·354	9·298	751689	651714363	867	29·444	9·535
648025	521660125	805	28·372	9·302	753424	653972032	868	29·461	9·539
649636	523606616	806	28·390	9·306	755161	656234909	869	29·478	9·542
651249	525557943	807	28·407	9·310	756900	658503000	870	29·495	9·546
652864	527514112	808	28·425	9·314	758641	660776311	871	29·512	9·550
654481	529475129	809	28·442	9·317	760384	663054848	872	29·529	9·553
656100	531441000	810	28·460	9·321	762129	665338617	873	29·546	9·557
657721	533411731	811	28·478	9·325	763876	667627624	874	29·563	9·561
659344	535387328	812	28·495	9·329	765625	669921875	875	29·580	9·564
660969	537367797	813	28·513	9·333	767376	672221376	876	29·597	9·568
662596	539353144	814	28·530	9·337	769129	674526133	877	29·614	9·571
664225	541343375	815	28·548	9·340	770884	676836152	878	29·631	9·575
665856	543338496	816	28·565	9·344	772641	679151439	879	29·647	9·579
667489	545338513	817	28·583	9·348	774400	681472000	880	29·664	9·582
669124	547343432	818	28·600	9·352	776161	683797841	881	29·681	9·586
670761	549353259	819	28·618	9·356	777924	686128968	882	29·698	9·590

TABLE OF SQUARES, CUBES, SQUARE AND CUBE ROOTS OF NUMBERS—(*Continued*).

Squares.	Cubes.	No.	Square roots.	Cube roots.	Squares.	Cubes.	No.	Square roots.	Cube roots.
779689	688465387	883	29·715	9·593	894916	846590536	946	30·757	9·816
781456	690807104	884	29·732	9·597	896808	849278123	947	30·773	9·820
783225	693154125	885	29·748	9·600	898704	851971392	948	30·789	9·823
784996	695506456	886	29·765	9·604	900601	854670349	949	30·805	9·827
786769	697864103	887	29·782	9·608	902500	857375000	950	30·822	9·830
788544	700227072	888	29·799	9·611	904401	860085351	951	30·838	9·833
790321	702595369	889	29·816	9·615	906304	862801408	952	30·854	9·837
792100	704969000	890	29·832	9·619	908209	865523177	953	30·870	9·840
793881	707347971	891	29·849	9·622	910116	868250664	954	30·886	9·844
795664	709732288	892	29·866	9·626	912025	870983875	955	30·903	9·847
797449	712121957	893	29·883	9·629	913936	873722816	956	30·919	9·851
799236	714516984	894	29·899	9·633	915849	876467493	957	30·935	9·854
801025	716917375	895	29·916	9·636	917764	879217912	958	30·951	9·857
802816	719323136	896	29·933	9·640	919681	881974079	959	30·967	9·861
804609	721734273	897	29·949	9·644	921600	884736000	960	30·983	9·864
806404	724150792	898	29·966	9·647	923521	887503681	961	31·000	9·868
808201	726572699	899	29·983	9·651	925444	890277128	962	31·016	9·871
810000	729000000	900	30·000	9·654	927369	893056347	963	31·032	9·875
811801	731432701	901	30·016	9·658	929296	895841344	964	31·048	9·878
813604	733870808	902	30·033	9·662	931225	898632125	965	31·064	9·881
815409	736314327	903	30·049	9·665	933156	901428696	966	31·080	9·885
817216	738763264	904	30·066	9·669	935089	904231063	967	31·096	9·888
819025	741217625	905	30·083	9·672	937024	907039232	968	31·112	9·892
820836	743677416	906	30·099	9·676	938961	909853209	969	31·128	9·895
822649	746142643	907	30·116	9·679	940900	912673000	970	31·144	9·898
824464	748613312	908	30·133	9·683	942841	915498611	971	31·160	9·902
826281	751089429	909	30·149	9·686	944784	918330048	972	31·176	9·905
828100	753571000	910	30·166	9·690	946729	921167317	973	31·192	9·909
829921	756058031	911	30·182	9·694	948676	924010424	974	31·208	9·912
831744	758550528	912	30·199	9·697	950625	926859375	975	31·224	9·915
833569	761048497	913	30·215	9·701	952576	929714176	976	31·240	9·919
835396	763551944	914	30·232	9·704	954529	932574833	977	31·256	9·922
837225	766060875	915	30·248	9·708	956484	935441352	978	31·272	9·926
839056	768575296	916	30·265	9·711	958441	938313739	979	31·288	9·929
840889	771095213	917	30·282	9·715	960400	941192000	980	31·304	9·932
842724	773620632	918	30·298	9·718	962361	944076141	981	31·320	9·936
844561	776151559	919	30·315	9·722	964324	946966168	982	31·336	9 939
846400	778688000	920	30·331	9·725	966289	949862087	983	31·352	9·943
848241	781229961	921	30·347	9 729	968256	952763904	984	31·368	9·946
850084	783777448	922	30·364	9·732	970225	955671625	985	31·384	9·949
851929	786330467	923	30·380	9·736	972196	958585256	986	31·400	9·953
853776	788889024	924	30·397	9·739	974169	961504803	987	31·416	9·956
855625	791453125	925	30·413	9·743	976144	964430272	988	31·432	9·959
857476	794022776	926	30·430	9·746	978121	967361669	989	31·448	9·963
859329	796597983	927	30·446	9·750	980100	970299000	990	31·464	9·966
861184	799178752	928	30·463	9·753	982081	973242271	991	31·480	9·969
863041	801765089	929	30·479	9·757	984064	976191488	992	31·496	9·973
864900	804357000	930	30·495	9·761	986049	979146657	993	31·511	9·976
866761	806954491	931	30·512	9·764	988036	982107784	994	31·527	9·979
868624	809557568	932	30·528	9·767	990025	985074875	995	31·543	9·983
870489	812166237	933	30·545	9·771	992016	988047936	996	31·559	9·986
872356	814780504	934	30·561	9·774	994009	991026973	997	31·575	9 989
874225	817400375	935	30·577	9·778	996004	994011992	998	31·591	9·993
876096	820025856	936	30·594	9·782	998001	997002999	999	31·606	9·996
877969	822656953	937	30·610	9·785	1000000	1000000000	1000	31·622	10·000
879844	825293672	938	30·626	9·788	1000201	1003003001	1001	31·638	10·003
881721	827936019	939	30·643	9·792	1004004	1006012008	1002	31·654	10·006
883600	830584000	940	30·659	9·795	1006009	1009027027	1003	31·670	10·009
885481	833237621	941	30·675	9·799	1008016	1012048064	1004	31·685	10·013
887364	835896888	942	30·692	9·802	1010025	1015075125	1005	31·701	10·016
889249	838561807	943	30·708	9·806	1012036	1018108216	1006	31·717	10·019
891136	841232384	944	30·724	9·809	1014049	1021147343	1007	31·733	10·023
893025	843908625	945	30·740	9·813	1016064	1024192512	1008	31·749	10·026

TABLE OF RECIPROCALS.

0	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9
0	∞	10·000	5·0000	3·3333	2·5000	2·0000	1·6667	1·4286	1·2500	1·1111
1	1·00000	·90909	·83333	·76923	·71428	·66667	·62500	·58823	·55555	·52631
2	·50000	·47619	·45454	·43478	·41667	·40000	·38461	·37037	·35714	·34483
3	·33333	·32258	·31250	·30303	·29412	·28571	·27778	·27027	·26316	·25641
4	·25000	·24390	·23810	·23256	·22727	·22222	·21739	21277	·20833	·20408
5	·20000	·19608	·19231	·18868	·18519	·18182	·17857	·17544	·17241	·16949
6	·16667	·16393	·16129	·15873	·15625	·15385	·15152	·14925	·14706	·14493
7	·14286	·14085	·13889	·13699	·13514	·13333	·13158	·12987	·12821	·12658
8	·12500	·12346	·12195	·12048	·11905	·11765	·11628	·11494	·11364	·11236
9	·11111	·10989	·10870	·10753	·10638	·10526	·10417	·10309	·10204	·10101
10	·10000	·09901	·09804	·09709	·09615	·09524	·09434	·09346	·09259	·09174
11	·09091	·09009	·08929	·08850	·08772	·08696	·08621	·08547	·08475	·08403
12	·08333	·08264	·08197	·08130	·08065	·08000	·07937	·07874	·07813	·07752
13	·07692	·07634	·07576	·07519	·07463	·07407	·07353	·07299	·07246	·07194
14	·07143	·07092	·07042	·06993	·06944	·06897	·06849	·06803	·06757	·06711
15	·06667	·06623	·06579	·06536	·06494	·06452	·06410	·06369	·06329	·06289
16	·06250	·06211	·06173	·06135	·06098	·06061	·06024	·05988	·05952	·05917
17	·05882	·05848	·05814	·05780	·05747	·05714	·05682	·05650	·05618	·05587
18	·05556	·05525	·05495	·05464	·05435	·05405	·05376	·05348	·05319	·05291
19	·05263	·05236	·05208	·05181	·05155	·05128	·05102	·05076	·05051	·05025
20	·05000	·04975	·04950	·04926	·04902	·04878	·04854	·04831	·04808	·04785
21	·04762	·04739	·04717	·04695	·04673	·04651	·04630	·04608	·04587	·04566
22	·04545	·04525	·04505	·04484	·04464	·04444	·04425	·04405	·04386	·04367
23	·04348	·04329	·04310	·04292	·04274	·04255	·04237	·04219	·04202	·04184
24	·04167	·04149	·04132	·04115	·04098	·04082	·04065	·04049	·04032	·04016
25	·04000	·03984	·03968	·03953	·03937	·03922	·03906	·03891	·03876	·03861
26	·03846	·03831	·03817	·03802	·03788	·03774	·03759	·03745	·03731	·03717
27	·03704	·03690	·03676	·03663	·03650	·03636	·03623	·03610	·03597	·03584
28	·03571	·03559	·03546	·03534	·03521	·03509	·03497	·03484	·03472	·03460
29	·03448	·03436	·03425	·03413	·03401	·03390	·03378	·03367	·03356	·03344
30	·03333	·03322	·03311	·03300	·03289	·03279	·03268	·03257	·03247	·03236
31	·03226	·03215	·03205	·03195	·03185	·03175	·03165	·03155	·03145	·03135
32	·03125	·03115	·03106	·03096	·03086	·03077	·03067	·03058	·03049	·03040
33	·03030	·03021	·03012	·03003	·02994	·02985	·02976	·02967	·02959	·02950
34	·02941	·02933	·02924	·02915	·02907	·02899	·02890	·02882	·02874	·02865
35	·02857	·02849	·02841	·02833	·02825	·02817	·02809	·02801	·02793	·02786
36	·02778	·02770	·02762	·02755	·02747	·02740	·02732	·02725	·02717	·02710
37	·02703	·02695	·02688	·02681	·02674	·02667	·02660	·02653	·02646	·02639
38	·02632	·02625	·02618	·02611	·02604	·02597	·02591	·02584	·02577	·02571
39	·02564	·02558	·02551	·02545	·02538	·02532	·02525	·02519	·02513	·02506
40	·02500	·02494	·02488	·02481	·02475	·02469	·02463	·02457	·02451	·02445
41	·02439	·02433	·02427	·02421	·02415	·02410	·02404	·02398	·02392	·02387
42	·02381	·02375	·02370	·02364	·02358	·02353	·02347	·02342	·02336	·02331
43	·02326	·02320	·02315	·02309	·02304	·02299	·02294	·02288	·02283	·02278
44	·02273	·02268	·02262	·02257	·02252	·02247	·02242	·02237	·02232	·02227
45	·02222	·02217	·02212	·02208	·02203	·02198	·02193	·02188	·02183	·02179
46	·02174	·02169	·02165	·02160	·02155	·02151	·02146	·02141	·02137	·02132
47	·02128	·02123	·02119	·02114	·02110	·02105	·02101	·02096	·02092	·02088
48	·02083	·02079	·02075	·02070	·02066	·02062	·02058	·02053	·02049	·02045
49	·02041	·02037	·02033	·02028	·02024	·02020	·02016	·02012	·02008	·02004
	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9

TABLE OF RECIPROCALS—*Continued.*

	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9
50	·02000	·01996	·01992	·01988	·01984	·01980	·01976	·01972	·01969	·01965
51	·01961	·01957	·01953	·01949	·01946	·01942	·01938	·01934	·01931	·01927
52	·01923	·01919	·01916	·01912	·01908	·01905	·01901	·01898	·01894	·01890
53	·01887	·01883	·01880	·01876	·01873	·01869	·01866	·01862	·01859	·01855
54	·01852	·01848	·01845	·01842	·01838	·01835	·01832	·01828	·01825	·01821
55	·01818	·01815	·01812	·01808	·01805	·01802	·01799	·01795	·01792	·01789
56	·01786	·01783	·01779	·01776	·01773	·01770	·01767	·01764	·01761	·01757
57	·01754	·01751	·01748	·01745	·01742	·01739	·01736	·01733	·01730	·01727
58	·01724	·01721	·01718	·01715	·01712	·01709	·01706	·01704	·01701	·01698
59	·01695	·01692	·01689	·01686	·01684	·01681	·01678	·01675	·01672	·01669
60	·01667	·01664	·01661	·01658	·01656	·01653	·01650	·01647	·01645	·01642
61	·01639	·01637	·01634	·01631	·01629	·01626	·01623	·01621	·01618	·01616
62	·01613	·01610	·01608	·01605	·01603	·01600	·01597	·01595	·01592	·01590
63	·01587	·01585	·01582	·01580	·01577	·01575	·01572	·01570	·01567	·01565
64	·01563	·01560	·01558	·01555	·01553	·01550	·01548	·01546	·01543	·01541
65	·01538	·01536	·01534	·01531	·01529	·01527	·01524	·01522	·01520	·01517
66	·01515	·01513	·01511	·01508	·01506	·01504	·01502	·01499	·01497	·01495
67	·01493	·01490	·01488	·01486	·01484	·01481	·01479	·01477	·01475	·01473
68	·01471	·01468	·01466	·01464	·01462	·01460	·01458	·01456	·01453	·01451
69	·01449	·01447	·01445	·01443	·01441	·01439	·01437	·01435	·01433	·01431
70	·01429	·01427	·01425	·01422	·01420	·01418	·01416	·01414	·01412	·01410
71	·01408	·01406	·01404	·01403	·01401	·01399	·01397	·01395	·01393	·01391
72	·01389	·01387	·01385	·01383	·01381	·01379	·01377	·01376	·01374	·01372
73	·01370	·01368	·01366	·01364	·01362	·01361	·01359	·01357	·01355	·01353
74	·01351	·01350	·01348	·01346	·01344	·01342	·01340	·01339	·01337	·01335
75	·01333	·01332	·01330	·01328	·01326	·01325	·01323	·01321	·01319	·01318
76	·01316	·01314	·01312	·01311	·01309	·01307	·01305	·01304	·01302	·01300
77	·01299	·01297	·01295	·01294	·01292	·01290	·01289	·01287	·01285	·01284
78	·01282	·01280	·01279	·01277	·01276	·01274	·01272	·01271	·01269	·01267
79	·01266	·01264	·01263	·01261	·01259	·01258	·01256	·01255	·01253	·01252
80	·01250	·01248	·01247	·01245	·01244	·01242	·01241	·01239	·01238	·01236
81	·01235	·01233	·01232	·01230	·01229	·01227	·01225	·01224	·01222	·01221
82	·01220	·01218	·01217	·01215	·01214	·01212	·01211	·01209	·01208	·01206
83	·01205	·01203	·01202	·01200	·01199	·01198	·01196	·01195	·01193	·01192
84	·01190	·01189	·01188	·01186	·01185	·01183	·01182	·01181	·01179	·01178
85	·01176	·01175	·01174	·01172	·01171	·01170	·01168	·01167	·01166	·01164
86	·01163	·01161	·01160	·01159	·01157	·01156	·01155	·01153	·01152	·01151
87	·01149	·01148	·01147	·01145	·01144	·01143	·01142	·01140	·01139	·01138
88	·01136	·01135	·01134	·01133	·01131	·01130	·01129	·01127	·01126	·01125
89	·01124	·01122	·01121	·01120	·01119	·01117	·01116	·01115	·01114	·01112
90	·01111	·01110	·01109	·01107	·01106	·01105	·01104	·01103	·01101	·01100
91	·01099	·01098	·01096	·01095	·01094	·01093	·01092	·01091	·01089	·01088
92	·01087	·01086	·01085	·01083	·01082	·01081	·01080	·01079	·01078	·01076
93	·01075	·01074	·01073	·01072	·01071	·01070	·01068	·01067	·01066	·01065
94	·01064	·01063	·01062	·01060	·01059	·01058	·01057	·01056	·01055	·01054
95	·01053	·01052	·01050	·01049	·01048	·01047	·01046	·01045	·01044	·01043
96	·01042	·01041	·01040	·01038	·01037	·01036	·01035	·01034	·01033	·01032
97	·01031	·01030	·01029	·01028	·01027	·01026	·01025	·01024	·01022	·01021
98	·01020	·01019	·01018	·01017	·01016	·01015	·01014	·01013	·01012	·01011
99	·01010	·01009	·01008	·01007	·01006	·01005	·01004	·01003	·01002	·01001
	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9

LATITUDES AND DEPARTURES.

Bearing.	1		2		3		4		5	Bearing.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
0°	1·000	0·000	2·000	0·000	3·000	0·000	4·000	0·000	5·000	90°
0¼	1·000	0·004	2·000	0·009	3·000	0·013	4·000	0·017	5·000	89¾
0½	1·000	0·009	2·000	0·017	3·000	0·026	4·000	0·035	5·000	89½
0¾	1·000	0·013	2·000	0·026	3·000	0·039	4·000	0·052	5·000	89¼
1°	1·000	0·017	2·000	0·035	3·000	0·052	3·999	0·070	4·999	89°
1¼	1·000	0·022	2·000	0·044	2·999	0·065	3·999	0·087	4·999	88¾
1½	1·000	0·026	1·999	0·052	2·999	0·079	3·999	0·105	4·998	88½
1¾	1·000	0·031	1·999	0·061	2·999	0·092	3·998	0·122	4·998	88¼
2°	0·999	0·035	1·999	0·070	2·998	0·105	3·998	0·140	4·997	88°
2¼	0·999	0·039	1·998	0·079	2·998	0·118	3·997	0·157	4·996	87¾
2½	0·999	0·044	1·998	0·087	2·997	0·131	3·996	0·174	4·995	87½
2¾	0·999	0·048	1·998	0·096	2·997	0·144	3·995	0·192	4·994	87¼
3°	0·999	0·052	1·997	0·105	2·996	0·157	3·995	0·209	4·993	87°
3¼	0·998	0·057	1·997	0·113	2·995	0·170	3·994	0·227	4·992	86¾
3½	0·998	0·061	1·996	0·122	2·994	0·183	3·993	0·244	4·991	86½
3¾	0·998	0·065	1·996	0·131	2·994	0·196	3·991	0·262	4·989	86¼
4°	0·998	0·070	1·995	0·140	2·993	0·209	3·990	0·279	4·988	86°
4¼	0·997	0·074	1·995	0·148	2·992	0·222	3·989	0·296	4·986	85¾
4½	0·997	0·078	1·994	0·157	2·991	0·235	3·988	0·314	4·985	85½
4¾	0·997	0·083	1·993	0·166	2·990	0·248	3·986	0·331	4·983	85¼
5°	0·996	0·087	1·992	0·174	2·989	0·261	3·985	0·349	4·981	85°
5¼	0·996	0·092	1·992	0·183	2·987	0·275	3·983	0·366	4·979	84¾
5½	0·995	0·096	1·991	0·192	2·986	0·288	3·982	0·383	4·977	84½
5¾	0·995	0·100	1·990	0·200	2·985	0·301	3·980	0·401	4·975	84¼
6°	0·995	0·105	1·989	0·209	2·984	0·314	3·978	0·418	4·973	84°
6¼	0·994	0·109	1·988	0·218	2·982	0·327	3·976	0·435	4·970	83¾
6½	0·994	0·113	1·987	0·226	2·981	0·340	3·974	0·453	4·968	83½
6¾	0·993	0·118	1·986	0·235	2·979	0·353	3·972	0·470	4·965	83¼
7°	0·993	0·122	1·985	0·244	2·978	0·366	3·970	0·487	4·963	83°
7¼	0·992	0·126	1·984	0·252	2·976	0·379	3·968	0·505	4·960	82¾
7½	0·991	0·131	1·983	0·261	2·974	0·392	3·966	0·522	4·957	82½
7¾	0·991	0·135	1·982	0·270	2·973	0·405	3·963	0·539	4·954	82¼
8°	0·990	0·139	1·981	0·278	2·971	0·418	3·961	0·557	4·951	82°
8¼	0·990	0·143	1·979	0·287	2·969	0·430	3·959	0·574	4·948	81¾
8½	0·989	0·148	1·978	0·296	2·967	0·443	3·956	0·591	4·945	81½
8¾	0·988	0·152	1·977	0·304	2·965	0·456	3·953	0·608	4·942	81¼
9°	0·988	0·156	1·975	0·313	2·963	0·469	3·951	0·626	4·938	81°
9¼	0·987	0·161	1·974	0·321	2·961	0·482	3·948	0·643	4·935	80¾
9½	0·986	0·165	1·973	0·330	2·959	0·495	3·945	0·660	4·931	80½
9¾	0·986	0·169	1·971	0·339	2·957	0·508	3·942	0·677	4·928	80¼
10°	0·985	0·174	1·970	0·347	2·954	0·521	3·939	0·695	4·924	80°
10¼	0·984	0·178	1·968	0·356	2·952	0·534	3·936	0·712	4·920	79¾
10½	0·983	0·182	1·967	0·364	2·950	0·547	3·933	0·729	4·916	79½
10¾	0·982	0·187	1·965	0·373	2·947	0·560	3·930	0·746	4·912	79¼
11°	0·982	0·191	1·963	0·382	2·945	0·572	3·927	0·763	4·908	79°
11¼	0·981	0·195	1·962	0·390	2·942	0·585	3·923	0·780	4·904	78¾
11½	0·980	0·199	1·960	0·399	2·940	0·598	3·920	0·797	4·900	78½
11¾	0·979	0·204	1·958	0·407	2·937	0·611	3·916	0·815	4·895	78¼
12°	0·978	0·208	1·956	0·416	2·934	0·624	3·913	0·832	4·891	78°
12¼	0·977	0·212	1·954	0·424	2·932	0·637	3·909	0·849	4·886	77¾
12½	0·976	0·216	1·953	0·433	2·929	0·649	3·905	0·866	4·881	77½
12¾	0·975	0·221	1·951	0·441	2·926	0·662	3·901	0·883	4·877	77¼
13°	0·974	0·225	1·949	0·450	2·923	0·675	3·897	0·900	4·872	77°
13¼	0·973	0·229	1·947	0·458	2·920	0·688	3·894	0·917	4·867	76¾
13½	0·972	0·233	1·945	0·467	2·917	0·700	3·889	0·934	4·862	76½
13¾	0·971	0·238	1·943	0·475	2·914	0·713	3·885	0·951	4·857	76¼
14°	0·970	0·242	1·941	0·484	2·911	0·726	3·881	0·968	4·851	76°
14¼	0·969	0·246	1·938	0·492	2·908	0·738	3·877	0·985	4·846	75¾
14½	0·968	0·250	1·936	0·501	2·904	0·751	3·873	1·002	4·841	75½
14¾	0·967	0·255	1·934	0·509	2·901	0·764	3·868	1·018	4·835	75¼
15°	0·966	0·259	1·932	0·518	2·898	0·776	3·864	1·035	4·830	75°
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
Bearing.	1		2		3		4		5	Bearing.

LATITUDES AND DEPARTURES.

Bearing.	5	6		7		8		9		Bearing.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
0°	0·000	6·000	0·000	7·000	0·000	8·000	0·000	9·000	0·000	90°
0¼	0·022	6·000	0·026	7·000	0·031	8·000	0·035	9·000	0·039	89¾
0½	0·044	6·000	0·052	7·000	0·061	8·000	0·070	9·000	0·079	89½
0¾	0·065	5·999	0·079	6·999	0·092	7·999	0·105	8·999	0·118	89¼
1°	0·087	5·999	0·105	6·999	0·122	7·999	0·140	8·999	0·157	89°
1¼	0·109	5·999	0·131	6·998	0·153	7·998	0·175	8·998	0·196	88¾
1½	0·131	5·998	0·157	6·998	0·183	7·997	0·209	8·997	0·236	88½
1¾	0·153	5·997	0·183	6·997	0·214	7·996	0·244	8·996	0·275	88¼
2°	0·174	5·996	0·209	6·996	0·244	7·995	0·279	8·995	0·314	88°
2¼	0·196	5·995	0·236	6·995	0·275	7·994	0·314	8·993	0·353	87¾
2½	0·218	5·994	0·262	6·993	0·305	7·992	0·349	8·991	0·393	87½
2¾	0·240	5·993	0·288	6·992	0·336	7·991	0·384	8·990	0·432	87¼
3°	0·262	5·992	0·314	6·990	0·366	7·989	0·419	8·988	0·471	87°
3¼	0·283	5·990	0·340	6·989	0·397	7·987	0·454	8·986	0·510	86¾
3½	0·305	5·989	0·366	6·987	0·427	7·985	0·488	8·983	0·549	86½
3¾	0·327	5·987	0·392	6·985	0·458	7·983	0·523	8·981	0·589	86¼
4°	0·349	5·985	0·419	6·983	0·488	7·981	0·558	8·978	0·628	86°
4¼	0·371	5·984	0·445	6·981	0·519	7·978	0·593	8·975	0·667	85¾
4½	0·392	5·982	0·471	6·978	0·549	7·975	0·628	8·972	0·706	85½
4¾	0·414	5·979	0·497	6·976	0·580	7·973	0·662	8·969	0·745	85¼
5°	0·436	5·977	0·523	6·973	0·610	7·970	0·697	8·966	0·784	85°
5¼	0·458	5·975	0·549	6·971	0·641	7·966	0·732	8·962	0·824	84¾
5½	0·479	5·972	0·575	6·968	0·671	7·963	0·767	8·959	0·863	84½
5¾	0·501	5·970	0·601	6·965	0·701	7·960	0·802	8·955	0·902	84¼
6°	0·523	5·967	0·627	6·962	0·732	7·956	0·836	8·951	0·941	84°
6¼	0·544	5·964	0·653	6·958	0·762	7·952	0·871	8·947	0·980	83¾
6½	0·566	5·961	0·679	6·955	0·792	7·949	0·906	8·942	1·019	83½
6¾	0·588	5·958	0·705	6·951	0·823	7·945	0·940	8·938	1·058	83¼
7°	0·609	5·955	0·731	6·948	0·853	7·940	0·975	8·933	1·097	83°
7¼	0·631	5·952	0·757	6·944	0·883	7·936	1·010	8·928	1·136	82¾
7½	0·653	5·949	0·783	6·940	0·914	7·932	1·044	8·923	1·175	82½
7¾	0·674	5·945	0·809	6·936	0·944	7·927	1·079	8·918	1·214	82¼
8°	0·696	5·942	0·835	6·932	0·974	7·922	1·113	8·912	1·253	82°
8¼	0·717	5·938	0·861	6·928	1·004	7·917	1·148	8·907	1·291	81¾
8½	0·739	5·934	0·887	6·923	1·035	7·912	1·182	8·901	1·330	81½
8¾	0·761	5·930	0·913	6·919	1·065	7·907	1·217	8·895	1·369	81¼
9°	0·782	5·926	0·939	6·914	1·095	7·902	1·251	8·889	1·408	81°
9¼	0·804	5·922	0·964	6·909	1·125	7·896	1·286	8·883	1·447	80¾
9½	0·825	5·918	0·990	6·904	1·155	7·890	1·320	8·877	1·485	80½
9¾	0·847	5·913	1·016	6·899	1·185	7·884	1·355	8·870	1·524	80¼
10°	0·868	5·909	1·042	6·894	1·216	7·878	1·389	8·863	1·563	80°
10¼	0·890	5·904	1·068	6·888	1·246	7·872	1·424	8·856	1·601	79¾
10½	0·911	5·900	1·093	6·883	1·276	7·866	1·458	8·849	1·640	79½
10¾	0·933	5·895	1·119	6·877	1·306	7·860	1·492	8·842	1·679	79¼
11°	0·954	5·890	1·145	6·871	1·336	7·853	1·526	8·835	1·717	79°
11¼	0·975	5·885	1·171	6·866	1·366	7·846	1·561	8·827	1·756	78¾
11½	0·997	5·880	1·196	6·859	1·396	7·839	1·595	8·819	1·794	78½
11¾	1·018	5·874	1·222	6·853	1·425	7·832	1·629	8·811	1·833	78¼
12°	1·040	5·869	1·247	6·847	1·455	7·825	1·663	8·803	1·871	78°
12¼	1·061	5·863	1·273	6·841	1·485	7·818	1·697	8·795	1·910	77¾
12½	1·082	5·858	1·299	6·834	1·515	7·810	1·732	8·787	1·948	77½
12¾	1·103	5·852	1·324	6·827	1·545	7·803	1·766	8·778	1·986	77¼
13°	1·125	5·846	1·350	6·821	1·575	7·795	1·800	8·769	2·025	77°
13¼	1·146	5·840	1·375	6·814	1·604	7·787	1·834	8·760	2·063	76¾
13½	1·167	5·834	1·401	6·807	1·634	7·779	1·868	8·751	2·101	76½
13¾	1·188	5·828	1·426	6·799	1·664	7·771	1·902	8·742	2·139	76¼
14°	1·210	5·822	1·452	6·792	1·693	7·762	1·935	8·733	2·177	76°
14¼	1·231	5·815	1·477	6·785	1·723	7·754	1·969	8·723	2·215	75¾
14½	1·252	5·809	1·502	6·777	1·753	7·745	2·003	8·713	2·253	75½
14¾	1·273	5·802	1·528	6·769	1·782	7·736	2·037	8·703	2·291	75¼
15°	1·294	5·796	1·553	6·761	1·812	7·727	2·071	8·693	2·329	75°
Bearing.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Bearing.
	5	6		7		8		9		

LATITUDES AND DEPARTURES.

Bearing.	1		2		3		4		5	Bearing.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
15°	0·966	0·259	1·932	0·518	2·898	0·776	3·864	1·035	4·830	**75°**
15¼	0·965	0·263	1·930	0·526	2·894	0·789	3·859	1·052	4·824	74¾
15½	0·964	0·267	1·927	0·534	2·891	0·802	3·855	1·069	4·818	74½
15¾	0·962	0·271	1·925	0·543	2·887	0·814	3·850	1·086	4·812	74¼
16°	0·961	0·276	1·923	0·551	2·884	0·827	3·845	1·103	4·806	**74°**
16¼	0·960	0·280	1·920	0·560	2·880	0·839	3·840	1·119	4·800	73¾
16½	0·959	0·284	1·918	0·568	2·876	0·852	3·835	1·136	4·794	73½
16¾	0·958	0·288	1·915	0·576	2·873	0·865	3·830	1·153	4·788	73¼
17°	0·956	0·292	1·913	0·585	2·869	0·877	3·825	1·169	4·782	**73°**
17¼	0·955	0·297	1·910	0·593	2·865	0·890	3·820	1·186	4·775	72¾
17½	0·954	0·301	1·907	0·601	2·861	0·902	3·815	1·203	4·769	72½
17¾	0·952	0·305	1·905	0·610	2·857	0·915	3·810	1·220	4·762	72¼
18°	0·951	0·309	1·902	0·618	2·853	0·927	3·804	1·236	4·755	**72°**
18¼	0·950	0·313	1·899	0·626	2·849	0·939	3·799	1·253	4·748	71¾
18½	0·948	0·317	1·897	0·635	2·845	0·952	3·793	1·269	4·742	71½
18¾	0·947	0·321	1·894	0·643	2·841	0·964	3·788	1·286	4·735	71¼
19°	0·946	0·326	1·891	0·651	2·837	0·977	3·782	1·302	4·728	**71°**
19¼	0·944	0·330	1·888	0·659	2·832	0·989	3·776	1·319	4·720	70¾
19½	0·943	0·334	1·885	0·668	2·828	1·001	3·771	1·335	4·713	70½
19¾	0·941	0·338	1·882	0·676	2·824	1·014	3·765	1·352	4·706	70¼
20°	0·940	0·342	1·879	0·684	2·819	1·026	3·759	1·368	4·698	**70°**
20¼	0·938	0·346	1·876	0·692	2·815	1·038	3·753	1·384	4·691	69¾
20½	0·937	0·350	1·873	0·700	2·810	1·051	3·747	1·401	4·683	69½
20¾	0·935	0·354	1·870	0·709	2·805	1·063	3·741	1·417	4·676	69¼
21°	0·934	0·358	1·867	0·717	2·801	1·075	3·734	1·433	4·668	**69°**
21¼	0·932	0·362	1·864	0·725	2·796	1·087	3·728	1·450	4·660	68¾
21½	0·930	0·367	1·861	0·733	2·791	1·100	3·722	1·466	4·652	68½
21¾	0·929	0·371	1·858	0·741	2·786	1·112	3·715	1·482	4·644	68¼
22°	0·927	0·375	1·854	0·749	2·782	1·124	3·709	1·498	4·636	**68°**
22¼	0·926	0·379	1·851	0·757	2·777	1·136	3·702	1·515	4·628	67¾
22½	0·924	0·383	1·848	0·765	2·772	1·148	3·696	1·531	4·619	67½
22¾	0·922	0·387	1·844	0·773	2·767	1·160	3·689	1·547	4·611	67¼
23°	0·921	0·391	1·841	0·781	2·762	1·172	3·682	1·563	4·603	**67°**
23¼	0·919	0·395	1·838	0·789	2·756	1·184	3·675	1·579	4·594	66¾
23½	0·917	0·399	1·834	0·797	2·751	1·196	3·668	1·595	4·585	66½
23¾	0·915	0·403	1·831	0·805	2·746	1·208	3·661	1·611	4·577	66¼
24°	0·914	0·407	1·827	0·813	2·741	1·220	3·654	1·627	4·568	**66°**
24¼	0·912	0·411	1·824	0·821	2·735	1·232	3·647	1·643	4·559	65¾
24½	0·910	0·415	1·820	0·829	2·730	1·244	3·640	1·659	4·550	65½
24¾	0·908	0·419	1·816	0·837	2·724	1·256	3·633	1·675	4·541	65¼
25°	0·906	0·423	1·813	0·845	2·719	1·268	3·625	1·690	4·532	**65°**
25¼	0·904	0·427	1·809	0·853	2·713	1·280	3·618	1·706	4·522	64¾
25½	0·903	0·431	1·805	0·861	2·708	1·292	3·610	1·722	4·513	64½
25¾	0·901	0·434	1·801	0·869	2·702	1·303	3·603	1·738	4·503	64¼
26°	0·899	0·438	1·798	0·877	2·696	1·315	3·595	1·753	4·494	**64°**
26¼	0·897	0·442	1·794	0·885	2·691	1·327	3·587	1·769	4·484	63¾
26½	0·895	0·446	1·790	0·892	2·685	1·339	3·580	1·785	4·475	63½
26¾	0·893	0·450	1·786	0·900	2·679	1·350	3·572	1·800	4·465	63¼
27°	0·891	0·454	1·782	0·908	2·673	1·362	3·564	1·816	4·455	**63°**
27¼	0·889	0·458	1·778	0·916	2·667	1·374	3·556	1·831	4·445	62¾
27½	0·887	0·462	1·774	0·923	2·661	1·385	3·548	1·847	4·435	62½
27¾	0·885	0·466	1·770	0·931	2·655	1·397	3·540	1·862	4·425	62¼
28°	0·883	0·469	1·766	0·939	2·649	1·408	3·532	1·878	4·415	**62°**
28¼	0·881	0·473	1·762	0·947	2·643	1·420	3·524	1·893	4·404	61¾
28½	0·879	0·477	1·758	0·954	2·636	1·431	3·515	1·909	4·394	61½
28¾	0·877	0·481	1·753	0·962	2·630	1·443	3·507	1·924	4·384	61¼
29°	0·875	0·485	1·749	0·970	2·624	1·454	3·498	1·939	4·373	**61°**
29¼	0·872	0·489	1·745	0·977	2·617	1·466	3·490	1·954	4·362	60¾
29½	0·870	0·492	1·741	0·985	2·611	1·477	3·481	1·970	4·352	60½
29¾	0·868	0·496	1·736	0·992	2·605	1·489	3·473	1·985	4·341	60¼
30°	0·866	0·500	1·732	1·000	2·598	1·500	3·464	2·000	4·330	**60°**
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
Bearing.	1		2		3		4		5	Bearing.

LATITUDES AND DEPARTURES.

Bearing.	5 Dep.	6 Lat.	6 Dep.	7 Lat.	7 Dep.	8 Lat.	8 Dep.	9 Lat.	9 Dep.	Bearing.
15°	1·294	5·796	1·553	6·761	1·812	7·727	2·071	8·693	2·329	75°
15¼	1·315	5·789	1·578	6·754	1·841	7·718	2·104	8·683	2·367	74¾
15½	1·336	5·782	1·603	6·745	1·871	7·709	2·138	8·673	2·405	74½
15¾	1·357	5·775	1·629	6·737	1·900	7·700	2·172	8·662	2·443	74¼
16°	1·378	5·768	1·654	6·729	1·929	7·690	2·205	8·651	2·481	74°
16¼	1·399	5·760	1·679	6·720	1·959	7·680	2·239	8·640	2·518	73¾
16½	1·420	5·753	1·704	6·712	1·988	7·671	2·272	8·629	2·556	73½
16¾	1·441	5·745	1·729	6·703	2·017	7·661	2·306	8·618	2·594	73¼
17°	1·462	5·738	1·754	6·694	2·047	7·650	2·339	8·607	2·631	73°
17¼	1·483	5·730	1·779	6·685	2·076	7·640	2·372	8·595	2·669	72¾
17½	1·504	5·722	1·804	6·676	2·105	7·630	2·406	8·583	2·706	72½
17¾	1·524	5·714	1·829	6·667	2·134	7·619	2·439	8·572	2·744	72¼
18°	1·545	5·706	1·854	6·657	2·163	7·608	2·472	8·560	2·781	72°
18¼	1·566	5·698	1·879	6·648	2·192	7·598	2·505	8·547	2·818	71¾
18½	1·587	5·690	1·904	6·638	2·221	7·587	2·538	8·535	2·856	71½
18¾	1·607	5·682	1·929	6·629	2·250	7·575	2·572	8·522	2·893	71¼
19°	1·628	5·673	1·953	6·619	2·279	7·564	2·605	8·510	2·930	71°
19¼	1·648	5·665	1·978	6·609	2·308	7·553	2·638	8·497	2·967	70¾
19½	1·669	5·656	2·003	6·598	2·337	7·541	2·670	8·484	3·004	70½
19¾	1·690	5·647	2·028	6·588	2·365	7·529	2·703	8·471	3·041	70¼
20°	1·710	5·638	2·052	6·578	2·394	7·518	2·736	8·457	3·078	70°
20¼	1·731	5·629	2·077	6·567	2·423	7·506	2·769	8·444	3·115	69¾
20½	1·751	5·620	2·101	6·557	2·451	7·493	2·802	8·430	3·152	69½
20¾	1·771	5·611	2·126	6·546	2·480	7·481	2·834	8·416	3·189	69¼
21°	1·792	5·601	2·150	6·535	2·509	7·469	2·867	8·402	3·225	69°
21¼	1·812	5·592	2·175	6·524	2·537	7·456	2·900	8·388	3·262	68¾
21½	1·833	5·582	2·199	6·513	2·566	7·443	2·932	8·374	3·299	68½
21¾	1·853	5·573	2·223	6·502	2·594	7·430	2·964	8·359	3·335	68¼
22°	1·873	5·563	2·248	6·490	2·622	7·417	2·997	8·345	3·371	68°
22¼	1·893	5·553	2·272	6·479	2·651	7·404	3·029	8·330	3·408	67¾
22½	1·913	5·543	2·296	6·467	2·679	7·391	3·061	8·315	3·444	67½
22¾	1·934	5·533	2·320	6·455	2·707	7·378	3·094	8·300	3·480	67¼
23°	1·954	5·523	2·344	6·444	2·735	7·364	3·126	8·285	3·517	67°
23¼	1·974	5·513	2·368	6·432	2·763	7·350	3·158	8·269	3·553	66¾
23½	1·994	5·502	2·392	6·419	2·791	7·336	3·190	8·254	3·589	66½
23¾	2·014	5·492	2·416	6·407	2·819	7·322	3·222	8·238	3·625	66¼
24°	2·034	5·481	2·440	6·395	2·847	7·308	3·254	8·222	3·661	66°
24¼	2·054	5·471	2·464	6·382	2·875	7·294	3·286	8·206	3·696	65¾
24½	2·073	5·460	2·488	6·370	2·903	7·280	3·318	8·190	3·732	65½
24¾	2·093	5·449	2·512	6·357	2·931	7·265	3·349	8·173	3·768	65¼
25°	2·113	5·438	2·536	6·344	2·958	7·250	3·381	8·157	3·804	65°
25¼	2·133	5·427	2·559	6·331	2·986	7·236	3·413	8·140	3·839	64¾
25½	2·153	5·416	2·583	6·318	3·014	7·221	3·444	8·123	3·875	64½
25¾	2·172	5·404	2·607	6·305	3·041	7·206	3·476	8·106	3·910	64¼
26°	2·192	5·393	2·630	6·292	3·069	7·190	3·507	8·089	3·945	64°
26¼	2·211	5·381	2·654	6·278	3·096	7·175	3·538	8·072	3·981	63¾
26½	2·231	5·370	2·677	6·265	3·123	7·160	3·570	8·054	4·016	63½
26¾	2·250	5·358	2·701	6·251	3·151	7·144	3·601	8·037	4·051	63¼
27°	2·270	5·346	2·724	6·237	3·178	7·128	3·632	8·019	4·086	63°
27¼	2·289	5·334	2·747	6·223	3·205	7·112	3·663	8·001	4·121	62¾
27½	2·309	5·322	2·770	6·209	3·232	7·096	3·694	7·983	4·156	62½
27¾	2·328	5·310	2·794	6·195	3·259	7·080	3·725	7·965	4·190	62¼
28°	2·347	5·298	2·817	6·181	3·286	7·064	3·756	7·947	4·225	62°
28¼	2·367	5·285	2·840	6·166	3·313	7·047	3·787	7·928	4·260	61¾
28½	2·386	5·273	2·863	6·152	3·340	7·031	3·817	7·909	4·294	61½
28¾	2·405	5·260	2·886	6·137	3·367	7·014	3·848	7·891	4·329	61¼
29°	2·424	5·248	2·909	6·122	3·394	6·997	3·878	7·872	4·363	61°
29¼	2·443	5·235	2·932	6·107	3·420	6·980	3·909	7·852	4·398	60¾
29½	2·462	5·222	2·955	6·093	3·447	6·963	3·939	7·833	4·432	60½
29¾	2·481	5·209	2·977	6·077	3·474	6·946	3·970	7·814	4·466	60¼
30°	2·500	5·196	3·000	6·062	3·500	6·928	4·000	7·794	4·500	60°
Bearing.	5 Lat.	6 Dep.	6 Lat.	7 Dep.	7 Lat.	8 Dep.	8 Lat.	9 Dep.	9 Lat.	Bearing.

LATITUDES AND DEPARTURES.

Bearing.	1		2		3		4		5	Bearing.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
30°	0·866	0·500	1·732	1·000	2·598	1·500	3·464	2·000	4·330	**60°**
30¼	0·864	0·504	1·728	1·008	2·592	1·511	3·455	2·015	4·319	59¾
30½	0·862	0·508	1·723	1·015	2·585	1·523	3·447	2·030	4·308	59½
30¾	0·859	0·511	1·719	1·023	2·578	1·534	3·438	2·045	4·297	59¼
31°	0·857	0·515	1·714	1·030	2·572	1·545	3·429	2·060	4·286	**59°**
31¼	0·855	0·519	1·710	1·038	2·565	1·556	3·420	2·075	4·275	58¾
31½	0·853	0·522	1·705	1·045	2·558	1·567	3·411	2·090	4·263	58½
31¾	0·850	0·526	1·701	1·052	2·551	1·579	3·401	2·105	4·252	58¼
32°	0·848	0·530	1·696	1·060	2·544	1·590	3·392	2·120	4·240	**58°**
32¼	0·846	0·534	1·691	1·067	2·537	1·601	3·383	2·134	4·229	57¾
32½	0·843	0·537	1·687	1·075	2·530	1·612	3·374	2·149	4·217	57½
32¾	0·841	0·541	1·682	1·082	2·523	1·623	3·364	2·164	4·205	57¼
33°	0·839	0·545	1·677	1·089	2·516	1·634	3·355	2·179	4·193	**57°**
33¼	0·836	0·548	1·673	1·097	2·509	1·645	3·345	2·193	4·181	56¾
33½	0·834	0·552	1·668	1·104	2·502	1·656	3·336	2·208	4·169	56½
33¾	0·831	0·556	1·663	1·111	2·494	1·667	3·326	2·222	4·157	56¼
34°	0·829	0·559	1·658	1·118	2·487	1·678	3·316	2·237	4·145	**56°**
34¼	0·827	0·563	1·653	1·126	2·480	1·688	3·306	2·251	4·133	55¾
34½	0·824	0·566	1·648	1·133	2·472	1·699	3·297	2·266	4·121	55½
34¾	0·822	0·570	1·643	1·140	2·465	1·710	3·287	2·280	4·108	55¼
35°	0·819	0·574	1·638	1·147	2·457	1 721	3·277	2·294	4·096	**55°**
35¼	0·817	0·577	1·633	1·154	2·450	1·731	3·267	2·309	4·083	54¾
35½	0·814	0·581	1·628	1·161	2·442	1·742	3·257	2·323	4·071	54½
35¾	0·812	0·584	1·623	1·168	2·435	1·753	3·246	2·337	4·058	54¼
36°	0·809	0·588	1·618	1·176	2·427	1·763	3·236	2·351	4·045	**54°**
36¼	0·806	0·591	1·613	1·183	2·419	1·774	3·226	2·365	4·032	53¾
36½	0·804	0·595	1·608	1·190	2·412	1·784	3·215	2·379	4·019	53½
36¾	0·801	0·598	1·603	1·197	2·404	1·795	3·205	2·393	4·006	53¼
37°	0·799	0·602	1·597	1·204	2·396	1·805	3·195	2·407	3·993	**53°**
37¼	0·796	0·605	1·592	1·211	2·388	1·816	3·184	2·421	3·980	52¾
37½	0·793	0·609	1·587	1·218	2·380	1·826	3·173	2·435	3·967	52½
37¾	0·791	0·612	1·581	1·224	2·372	1·837	3·163	2·449	3·953	52¼
38°	0·788	0·616	1·576	1·231	2·364	1·847	3·152	2·463	3·940	**52°**
38¼	0·785	0·619	1·571	1·238	2·356	1·857	3·141	2·476	3·927	51¾
38½	0·783	0·623	1·565	1·245	2·348	1·868	3·130	2·490	3·913	51½
38¾	0·780	0·626	1·560	1·252	2·340	1·878	3·120	2·504	3·899	51¼
39°	0·777	0·629	1·554	1·259	2·331	1·888	3·109	2·517	3·886	**51°**
39¼	0·774	0·633	1·549	1·265	2·323	1·898	3·098	2·[illegible]31	3·872	50¾
39½	0·772	0·636	1·543	1·272	2·315	1·908	3·086	2·544	3·858	50½
39¾	0·769	0·639	1·538	1·279	2·307	1·918	3·075	2·558	3·844	50¼
40°	0·766	0·643	1·532	1·286	2·298	1·928	3·064	2·571	3·830	**50°**
40¼	0·763	0·646	1·526	1·292	2·290	1·938	3·053	2·584	3·816	49¾
40½	0·760	0·649	1·521	1·299	2·281	1·948	3·042	2·598	3·802	49½
40¾	0·758	0·653	1·515	1·306	2·273	1·958	3·030	2·611	3·788	49¼
41°	0·755	0·656	1·509	1·312	2·264	1·968	3 019	2·624	3·774	**49°**
41¼	0·752	0·659	1·504	1·319	2·256	1·978	3·007	2·637	3·759	48¾
41½	0·749	0·663	1·498	1·325	2·247	1·988	2·996	2·650	3·745	48½
41¾	0·746	0·666	1·492	1·332	2·238	1·998	2·984	2·664	3·730	48¼
42°	0·743	0·669	1·486	1·338	2·229	2·007	2·973	2·677	3·716	**48°**
42¼	0·740	0·672	1·480	1·345	2·221	2·017	2·961	2·689	3·701	47¾
42½	0·737	0·676	1·475	1·351	2·212	2·027	2·949	2·702	3·686	47½
42¾	0·734	0·679	1·469	1·358	2·203	2·036	2·937	2·715	3·672	47¼
43°	0·731	0·682	1·463	1·364	2·194	2·046	2·925	2·728	3·657	**47°**
43¼	0·728	0·685	1·457	1·370	2·185	2·056	2·913	2·741	3·642	46¾
43½	0·725	0·688	1·451	1·377	2·176	2·065	2·901	2·753	3·627	46½
43¾	0·722	0·692	1·445	1·383	2·167	2·075	2·889	2·766	3·612	46¼
44°	0·719	0·695	1·439	1·389	2·158	2·084	2·877	2·779	3·597	**46°**
44¼	0·716	0·698	1·433	1·396	2·149	2·093	2·865	2·791	3·582	45¾
44½	0·713	0·701	1·427	1·402	2·140	2·103	2·853	2·804	3·566	45½
44¾	0·710	0·704	1·420	1·408	2·131	2·112	2·841	2·816	3·551	45¼
45°	0·707	1·707	1·414	1·414	2·121	2·121	2 828	2·828	3·536	**45°**
Bearing.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Bearing.
	1		2		3		4		5	

LATITUDES AND DEPARTURES.

Bearing.	5	6		7		8		9		Bearing.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
30°	2·500	5·196	3·000	6·062	3·500	6·928	4·000	7·794	4·500	**60°**
30¼	2·519	5·183	3·023	6·047	3·526	6·911	4·030	7·775	4·534	59¾
30½	2·538	5·170	3·045	6·031	3·553	6·893	4·060	7·755	4·568	59½
30¾	2·556	5·156	3·068	6·016	3·579	6·875	4·090	7·735	4·602	59¼
31°	2·575	5·143	3·090	6·000	3·605	6·857	4·120	7·715	4·635	**59°**
31¼	2·594	5·129	3·113	5·984	3·631	6·839	4·150	7·694	4·669	58¾
31½	2·612	5·116	3·135	5·968	3·657	6·821	4·180	7·674	4·702	58½
31¾	2·631	5·102	3·157	5·952	3·683	6·803	4·210	7·653	4·736	58¼
32°	2·650	5·088	3·180	5·936	3·709	6·784	4·239	7·632	4·769	**58°**
32¼	2·668	5·074	3·202	5·920	3·735	6·766	4·269	7·612	4·802	57¾
32½	2·686	5·060	3·224	5·904	3·761	6·747	4·298	7·591	4·836	57½
32¾	2·705	5·046	3·246	5·887	3·787	6·728	4·328	7·569	4·869	57¼
33°	2·723	5·032	3·268	5·871	3·812	6·709	4·357	7·548	4·902	**57°**
33¼	2·741	5·018	3·290	5·854	3·838	6·690	4·386	7·527	4·935	56¾
33½	2·760	5·003	3·312	5·837	3·864	6·671	4·416	7·505	4·967	56½
33¾	2·778	4·989	3·333	5·820	3·889	6·652	4·445	7·483	5·000	56¼
34°	2·796	4·974	3·355	5·803	3·914	6·632	4·474	7·461	5·033	**56°**
34¼	2·814	4·960	3·377	5·786	3·940	6·613	4·502	7·439	5·065	55¾
34½	2·832	4·945	3·398	5·769	3·965	6·593	4·531	7·417	5·098	55½
34¾	2·850	4·930	3·420	5·752	3·990	6·573	4·560	7·395	5·130	55¼
35°	2·868	4·915	3·441	5·734	4·015	6·553	4·589	7·372	5·162	**55°**
35¼	2·886	4·900	3·463	5·716	4·040	6·533	4·617	7·350	5·194	54¾
35½	2·904	4·885	3·484	5·699	4·065	6·513	4·646	7·327	5·226	54½
35¾	2·921	4·869	3·505	5·681	4·090	6·493	4·674	7·304	5·258	54¼
36°	2·939	4·854	3·527	5·663	4·115	6·472	4·702	7·281	5·290	**54°**
36¼	2·957	4·839	3·548	5·645	4·139	6·452	4·730	7·258	5·322	53¾
36½	2·974	4·823	3·569	5·627	4·164	6·431	4·759	7·235	5·353	53½
36¾	2·992	4·808	3·590	5·609	4·188	6·410	4·787	7·211	5·385	53¼
37°	3·009	4·792	3·611	5·590	4·213	6·389	4·815	7·188	5·416	**53°**
37¼	3·026	4·776	3·632	5·572	4·237	6·368	4·842	7·164	5·448	52¾
37½	3·044	4·760	3·653	5·554	4·261	6·347	4·870	7·140	5·479	52½
37¾	3·061	4·744	3·673	5·535	4·286	6·326	4·898	7·116	5·510	52¼
38°	3·078	4·728	3·694	5·516	4·310	6·304	4·925	7·092	5·541	**52°**
38¼	3·095	4·712	3·715	5·497	4·334	6·283	4·953	7·068	5·572	51¾
38½	3·113	4·696	3·735	5·478	4·358	6·261	4·980	7·043	5·603	51½
38¾	3·130	4·679	3·756	5·459	4·381	6·239	5·007	7·019	5·633	51¼
39°	3·147	4·663	3·776	5·440	4·405	6·217	5·035	6·994	5·664	**51°**
39¼	3·164	4·646	3·796	5·421	4·429	6·195	5·062	6·970	5·694	50¾
39½	3·180	4·630	3·816	5·401	4·453	6·173	5·089	6·945	5·725	50½
39¾	3·197	4·613	3·837	5·382	4·476	6·151	5·116	6·920	5·755	50¼
40°	3·214	4·596	3·857	5·362	4·500	6·128	5·142	6·894	5·785	**50°**
40¼	3·231	4·579	3·877	5·343	4·523	6·106	5·169	6·869	5·815	49¾
40½	3·247	4·562	3·897	5·323	4·546	6·083	5·196	6·844	5·845	49½
40¾	3·264	4·545	3·917	5·303	4·569	6·061	5·222	6·818	5·875	49¼
41°	3·280	4·528	3·936	5·283	4·592	6·038	5·248	6·792	5·905	**49°**
41¼	3·297	4·511	3·956	5·263	4·615	6·015	5·275	6·767	5·934	48¾
41½	3·313	4·494	3·976	5·243	4·638	5·992	5·301	6·741	5·964	48½
41¾	3·329	4·476	3·995	5·222	4·661	5·968	5·327	6·715	5·993	48¼
42°	3·346	4·459	4·015	5·202	4·684	5·945	5·353	6·688	6·022	**48°**
42¼	3·362	4·441	4·034	5·182	4·707	5·922	5·379	6·662	6·051	47¾
42½	3·378	4·424	4·054	5·161	4·729	5·898	5·405	6·635	6·080	47½
42¾	3·394	4·406	4·073	5·140	4·752	5·875	5·430	6·609	6·109	47¼
43°	3·410	4·388	4·092	5·119	4·774	5·851	5·456	6·582	6·138	**47°**
43¼	3·426	4·370	4·111	5·099	4·796	5·827	5·481	6·555	6·167	46¾
43½	3·442	4·352	4·130	5·078	4·818	5·803	5·507	6·528	6·195	46½
43¾	3·458	4·334	4·149	5·057	4·841	5·779	5·532	6·501	6·224	46¼
44°	3·473	4·316	4·168	5·035	4·863	5·755	5·557	6·474	6·252	**46°**
44¼	3·489	4·298	4·187	5·014	4·885	5·730	5·582	6·447	6·280	45¾
44½	3·505	4·280	4·206	4·993	4·906	5·706	5·607	6·419	6·308	45½
44¾	3·520	4·261	4·224	4·971	4·928	5·681	5·632	6·392	6·336	45¼
45°	3·536	4·243	4·243	4.950	4·950	5·657	5·657	6·364	6·364	**45°**
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
Bearing.	5	6		7		8		9		Bearing.

NATURAL SINES AND COSINES.

′	0°		1°		2°		3°		4°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	00000	Unit.	01745	99985	03490	99939	05234	99863	06976	99756	60
1	00029	Unit.	01774	99984	03519	99938	05263	99861	07005	99754	59
2	00058	Unit.	01803	99984	03548	99937	05292	99860	07034	99752	58
3	00087	Unit.	01832	99983	03577	99936	05321	99858	07063	99750	57
4	00116	Unit.	01862	99983	03606	99935	05350	99857	07092	99748	56
5	00145	Unit.	01891	99982	03635	99934	05379	99855	07121	99746	55
6	00175	Unit.	01920	99982	03664	99933	05408	99854	07150	99744	54
7	00204	Unit.	01949	99981	03693	99932	05437	99852	07179	99742	53
8	00233	Unit.	01978	99980	03723	99931	05466	99851	07208	99740	52
9	00262	Unit.	02007	99980	03752	99930	05495	99849	07237	99738	51
10	00291	Unit.	02036	99979	03781	99929	05524	99847	07266	99736	50
11	00320	99999	02065	99979	03810	99927	05553	99846	07295	99734	49
12	00349	99999	02094	99978	03839	99926	05582	99844	07324	99731	48
13	00378	99999	02123	99977	03868	99925	05611	99842	07353	99729	47
14	00407	99999	02152	99977	03897	99924	05640	99841	07382	99727	46
15	00436	99999	02181	99976	03926	99923	05669	99839	07411	99725	45
16	00465	99999	02211	99976	03955	99922	05698	99838	07440	99723	44
17	00495	99999	02240	99975	03984	99921	05727	99836	07469	99721	43
18	00524	99999	02269	99974	04013	99919	05756	99834	07498	99719	42
19	00553	99998	02298	99974	04042	99918	05785	99833	07527	99716	41
20	00582	99998	02327	99973	04071	99917	05814	99831	07556	99714	40
21	00611	99998	02356	99972	04100	99916	05844	99829	07585	99712	39
22	00640	99998	02385	99972	04129	99915	05873	99827	07614	99710	38
23	00669	99998	02414	99971	04159	99913	05902	99826	07643	99708	37
24	00698	99998	02443	99970	04188	99912	05931	99824	07672	99705	36
25	00727	99997	02472	99969	04217	99911	05960	99822	07701	99703	35
26	00756	99997	02501	99969	04246	99910	05989	99821	07730	99701	34
27	00785	99997	02530	99968	04275	99909	06018	99819	07759	99699	33
28	00814	99997	02560	99967	04304	99907	06047	99817	07788	99696	32
29	00844	99996	02589	99966	04333	99906	06076	99815	07817	99694	31
30	00873	99996	02618	99966	04362	99905	06105	99813	07846	99692	30
31	00902	99996	02647	99965	04391	99904	06134	99812	07875	99689	29
32	00931	99996	02676	99964	04420	99902	06163	99810	07904	99687	28
33	00960	99995	02705	99963	04449	99901	06192	99808	07933	99685	27
34	00989	99995	02734	99963	04478	99900	06221	99806	07962	99683	26
35	01018	99995	02763	99962	04507	99898	06250	99804	07991	99680	25
36	01047	99995	02792	99961	04536	99897	06279	99803	08020	99678	24
37	01076	99994	02821	99960	04565	99896	06308	99801	08049	99676	23
38	01105	99994	02850	99959	04594	99894	06337	99799	08078	99673	22
39	01134	99994	02879	99959	04623	99893	06366	99797	08107	99671	21
40	01164	99993	02908	99958	04653	99892	06395	99795	08136	99668	20
41	01193	99993	02938	99957	04682	99890	06424	99793	08165	99666	19
42	01222	99993	02967	99956	04711	99889	06453	99792	08194	99664	18
43	01251	99992	02996	99955	04740	99888	06482	99790	08223	99661	17
44	01280	99992	03025	99954	04769	99886	06511	99788	08252	99659	16
45	01309	99991	03054	99953	04798	99885	06540	99786	08281	99657	15
46	01338	99991	03083	99952	04827	99883	06569	99784	08310	99654	14
47	01367	99991	03112	99952	04856	99882	06598	99782	08339	99652	13
48	01396	99990	03141	99951	04885	99881	06627	99780	08368	99649	12
49	01425	99990	03170	99950	04914	99879	06656	99778	08397	99647	11
50	01454	99989	03199	99949	04943	99878	06685	99776	08426	99644	10
51	01483	99989	03228	99948	04972	99876	06714	99774	08455	99642	9
52	01513	99989	03257	99947	05001	99875	06743	99772	08484	99639	8
53	01542	99988	03286	99946	05030	99873	06773	99770	08513	99637	7
54	01571	99988	03316	99945	05059	99872	06802	99768	08542	99635	6
55	01600	99987	03345	99944	05088	99870	06831	99766	08571	99632	5
56	01629	99987	03374	99943	05117	99869	06860	99764	08600	99630	4
57	01658	99986	03403	99942	05146	99867	06889	99762	08629	99627	3
58	01687	99986	03432	99941	05175	99866	06918	99760	08658	99625	2
59	01716	99985	03461	99940	05205	99864	06947	99758	08687	99622	1
60	01745	99985	03490	99939	05234	99863	06976	99756	08716	99619	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	89°		88°		87°		86°		85°		′

NATURAL SINES AND COSINES.

′	5°		6°		7°		8°		9°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	08716	99619	10453	99452	12187	99255	13917	99027	15643	98769	60
1	08745	99617	10482	99449	12216	99251	13946	99023	15672	98764	59
2	08774	99614	10511	99446	12245	99248	13975	99019	15701	98760	58
3	08803	99612	10540	99443	12274	99244	14004	99015	15730	98755	57
4	08831	99609	10569	99440	12302	99240	14033	99011	15758	98751	56
5	08860	99607	10597	99437	12331	99237	14061	99006	15787	98746	55
6	08889	99604	10626	99434	12360	99233	14090	99002	15816	98741	54
7	08918	99602	10655	99431	12389	99230	14119	98998	15845	98737	53
8	08947	99599	10684	99428	12418	99226	14148	98994	15873	98732	52
9	08976	99596	10713	99424	12447	99222	14177	98990	15902	98728	51
10	09005	99594	10742	99421	12476	99219	14205	98986	15931	98723	50
11	09034	99591	10771	99418	12504	99215	14234	98982	15959	98718	49
12	09063	99588	10800	99415	12533	99211	14263	98978	15988	98714	48
13	09092	99586	10829	99412	12562	99208	14292	98973	16017	98709	47
14	09121	99583	10858	99409	12591	99204	14320	98969	16046	98704	46
15	09150	99580	10887	99406	12620	99200	14349	98965	16074	98700	45
16	09179	99578	10916	99402	12649	99197	14378	98961	16103	98695	44
17	09208	99575	10945	99399	12678	99193	14407	98957	16132	98690	43
18	09237	99572	10973	99396	12706	99189	14436	98953	16160	98686	42
19	09266	99570	11002	99393	12735	99186	14464	98948	16189	98681	41
20	09295	99567	11031	99390	12764	99182	14493	98944	16218	98676	40
21	09324	99564	11060	99386	12793	99178	14522	98940	16246	98671	39
22	09353	99562	11089	99383	12822	99175	14551	98936	16275	98667	38
23	09382	99559	11118	99380	12851	99171	14580	98931	16304	98662	37
24	09411	99556	11147	99377	12880	99167	14608	98927	16333	98657	36
25	09440	99553	11176	99374	12908	99163	14637	98923	16361	98652	35
26	09469	99551	11205	99370	12937	99160	14666	98919	16390	98648	34
27	09498	99548	11234	99367	12966	99156	14695	98914	16419	98643	33
28	09527	99545	11263	99364	12995	99152	14723	98910	16447	98638	32
29	09556	99542	11291	99360	13024	99148	14752	98906	16476	98633	31
30	09585	99540	11320	99357	13053	99144	14781	98902	16505	98629	30
31	09614	99537	11349	99354	13081	99141	14810	98897	16533	98624	29
32	09642	99534	11378	99351	13110	99137	14838	98893	16562	98619	28
33	09671	99531	11407	99347	13139	99133	14867	98889	16591	98614	27
34	09700	99528	11436	99344	13168	99129	14896	98884	16620	98609	26
35	09729	99526	11465	99341	13197	99125	14925	98880	16648	98604	25
36	09758	99523	11494	99337	13226	99122	14954	98876	16677	98600	24
37	09787	99520	11523	99334	13254	99118	14982	98871	16706	98595	23
38	09816	99517	11552	99331	13283	99114	15011	98867	16734	98590	22
39	09845	99514	11580	99327	13312	99110	15040	98863	16763	98585	21
40	09874	99511	11609	99324	13341	99106	15069	98858	16792	98580	20
41	09903	99508	11638	99320	13370	99102	15097	98854	16820	98575	19
42	09932	99506	11667	99317	13399	99098	15126	98849	16849	98570	18
43	09961	99503	11696	99314	13427	99094	15155	98845	16878	98565	17
44	09990	99500	11725	99310	13456	99091	15184	98841	16906	98561	16
45	10019	99497	11754	99307	13485	99087	15212	98836	16935	98556	15
46	10048	99494	11783	99303	13514	99083	15241	98832	16964	98551	14
47	10077	99491	11812	99300	13543	99079	15270	98827	16992	98546	13
48	10106	99488	11840	99297	13572	99075	15299	98823	17021	98541	12
49	10135	99485	11869	99293	13600	99071	15327	98818	17050	98536	11
50	10164	99482	11898	99290	13629	99067	15356	98814	17078	98531	10
51	10192	99479	11927	99286	13658	99063	15385	98809	17107	98526	9
52	10221	99476	11956	99283	13687	99059	15414	98805	17136	98521	8
53	10250	99473	11985	99279	13716	99055	15442	98800	17164	98516	7
54	10279	99470	12014	99276	13744	99051	15471	98796	17193	98511	6
55	10308	99467	12043	99272	13773	99047	15500	98791	17222	98506	5
56	10337	99464	12071	99269	13802	99043	15529	98787	17250	98501	4
57	10366	99461	12100	99265	13831	99039	15557	98782	17279	98496	3
58	10395	99458	12129	99262	13860	99035	15586	98778	17308	98491	2
59	10424	99455	12158	99258	13889	99031	15615	98773	17336	98486	1
60	10453	99452	12187	99255	13917	99027	15643	98769	17365	98481	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	84°		83°		82°		81°		80°		′

NATURAL SINES AND COSINES.

′	10° Sine.	10° Cosine.	11° Sine.	11° Cosine.	12° Sine.	12° Cosine.	13° Sine.	13° Cosine.	14° Sine.	14° Cosine.	′
0	17365	98481	19081	98163	20791	97815	22495	97437	24192	97030	60
1	17393	98476	19109	98157	20820	97809	22523	97430	24220	97023	59
2	17422	98471	19138	98152	20848	97803	22552	97424	24249	97015	58
3	17451	98466	19167	98146	20877	97797	22580	97417	24277	97008	57
4	17479	98461	19195	98140	20905	97791	22608	97411	24305	97001	56
5	17508	98455	19224	98135	20933	97784	22637	97404	24333	96994	55
6	17537	98450	19252	98129	20962	97778	22665	97398	24362	96987	54
7	17565	98445	19281	98124	20990	97772	22693	97391	24390	96980	53
8	17594	98440	19309	98118	21019	97766	22722	97384	24418	96973	52
9	17623	98435	19338	98112	21047	97760	22750	97378	24446	96966	51
10	17651	98430	19366	98107	21076	97754	22778	97371	24474	96959	50
11	17680	98425	19395	98101	21104	97748	22807	97365	24503	96952	49
12	17708	98420	19423	98096	21132	97742	22835	97358	24531	96945	48
13	17737	98414	19452	98090	21161	97735	22863	97351	24559	96937	47
14	17766	98409	19481	98084	21189	97729	22892	97345	24587	96930	46
15	17794	98404	19509	98079	21218	97723	22920	97338	24615	96923	45
16	17823	98399	19538	98073	21246	97717	22948	97331	24644	96916	44
17	17852	98394	19566	98067	21275	97711	22977	97325	24672	96909	43
18	17880	98389	19595	98061	21303	97705	23005	97318	24700	96902	42
19	17909	98383	19623	98056	21331	97698	23033	97311	24728	96894	41
20	17937	98378	19652	98050	21360	97692	23062	97304	24756	96887	40
21	17966	98373	19680	98044	21388	97686	23090	97298	24784	96880	39
22	17995	98368	19709	98039	21417	97680	23118	97291	24813	96873	38
23	18023	98362	19737	98033	21445	97673	23146	97284	24841	96866	37
24	18052	98357	19766	98027	21474	97667	23175	97278	24869	96858	36
25	18081	98352	19794	98021	21502	97661	23203	97271	24897	96851	35
26	18109	98347	19823	98016	21530	97655	23231	97264	24925	96844	34
27	18138	98341	19851	98010	21559	97648	23260	97257	24954	96837	33
28	18166	98336	19880	98004	21587	97642	23288	97251	24982	96829	32
29	18195	98331	19908	97998	21616	97636	23316	97244	25010	96822	31
30	18224	98325	19937	97992	21644	97630	23345	97237	25038	96815	30
31	18252	98320	19965	97987	21672	97623	23373	97230	25066	96807	29
32	18281	98315	19994	97981	21701	97617	23401	97223	25094	96800	28
33	18309	98310	20022	97975	21729	97611	23429	97217	25122	96793	27
34	18338	98304	20051	97969	21758	97604	23458	97210	25151	96786	26
35	18367	98299	20079	97963	21786	97598	23486	97203	25179	96778	25
36	18395	98294	20108	97958	21814	97592	23514	97196	25207	96771	24
37	18424	98288	20136	97952	21843	97585	23542	97189	25235	96764	23
38	18452	98283	20165	97946	21871	97579	23571	97182	25263	96756	22
39	18481	98277	20193	97940	21899	97573	23599	97176	25291	96749	21
40	18509	98272	20222	97934	21928	97566	23627	97169	25320	96742	20
41	18538	98267	20250	97928	21956	97560	23656	97162	25348	96734	19
42	18567	98261	20279	97922	21985	97553	23684	97155	25376	96727	18
43	18595	98256	20307	97916	22013	97547	23712	97148	25404	96719	17
44	18624	98250	20336	97910	22041	97541	23740	97141	25432	96712	16
45	18652	98245	20364	97905	22070	97534	23769	97134	25460	96705	15
46	18681	98240	20393	97899	22098	97528	23797	97127	25488	96697	14
47	18710	98234	20421	97893	22126	97521	23825	97120	25516	96690	13
48	18738	98229	20450	97887	22155	97515	23853	97113	25545	96682	12
49	18767	98223	20478	97881	22183	97508	23882	97106	25573	96675	11
50	18795	98218	20507	97875	22212	97502	23910	97100	25601	96667	10
51	18824	98212	20535	97869	22240	97496	23938	97093	25629	96660	9
52	18852	98207	20563	97863	22268	97489	23966	97086	25657	96653	8
53	18881	98201	20592	97857	22297	97483	23995	97079	25685	96645	7
54	18910	98196	20620	97851	22325	97476	24023	97072	25713	96638	6
55	18938	98190	20649	97845	22353	97470	24051	97065	25741	96630	5
56	18967	98185	20677	97839	22382	97463	24079	97058	25769	96623	4
57	18995	98179	20706	97833	22410	97457	24108	97051	25798	96615	3
58	19024	98174	20734	97827	22438	97450	24136	97044	25826	96608	2
59	19052	98168	20763	97821	22467	97444	24164	97037	25854	96600	1
60	19081	98163	20791	97815	22495	97437	24192	97030	25882	96593	0
′	79° Cosine.	79° Sine.	78° Cosine.	78° Sine.	77° Cosine.	77° Sine.	76° Cosine.	76° Sine.	75° Cosine.	75° Sine.	′

NATURAL SINES AND COSINES.

′	15°		16°		17°		18°		19°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	25882	96593	27564	96126	29237	95630	30902	95106	32557	94552	60
1	25910	96585	27592	96118	29265	95622	30929	95097	32584	94542	59
2	25938	96578	27620	96110	29293	95613	30957	95088	32612	94533	58
3	25966	96570	27648	96102	29321	95605	30985	95079	32639	94523	57
4	25994	96562	27676	96094	29348	95596	31012	95070	32667	94514	56
5	26022	96555	27704	96086	29376	95588	31040	95061	32694	94504	55
6	26050	96547	27731	96078	29404	95579	31068	95052	32722	94495	54
7	26079	96540	27759	96070	29432	95571	31095	95043	32749	94485	53
8	26107	96532	27787	96062	29460	95562	31123	95033	32777	94476	52
9	26135	96524	27815	96054	29487	95554	31151	95024	32804	94466	51
10	26163	96517	27843	96046	29515	95545	31178	95015	32832	94457	50
11	26191	96509	27871	96037	29543	95536	31206	95006	32859	94447	49
12	26219	96502	27899	96029	29571	95528	31233	94997	32887	94438	48
13	26247	96494	27927	96021	29599	95519	31261	94988	32914	94428	47
14	26275	96486	27955	96013	29626	95511	31289	94979	32942	94418	46
15	26303	96479	27983	96005	29654	95502	31316	94970	32969	94409	45
16	26331	96471	28011	95997	29682	95493	31344	94961	32997	94399	44
17	26359	96463	28039	95989	29710	95485	31372	94952	33024	94390	43
18	26387	96456	28067	95981	29737	95476	31399	94943	33051	94380	42
19	26415	96448	28095	95972	29765	95467	31427	94933	33079	94370	41
20	26443	96440	28123	95964	29793	95459	31454	94924	33106	94361	40
21	26471	96433	28150	95956	29821	95450	31482	94915	33134	94351	39
22	26500	96425	28178	95948	29849	95441	31510	94906	33161	94342	38
23	26528	96417	28206	95940	29876	95433	31537	94897	33189	94332	37
24	26556	96410	28234	95931	29904	95424	31565	94888	33216	94322	36
25	26584	96402	28262	95923	29932	95415	31593	94878	33244	94313	35
26	26612	96394	28290	95915	29960	95407	31620	94869	33271	94303	34
27	26640	96386	28318	95907	29987	95398	31648	94860	33298	94293	33
28	26668	96379	28346	95898	30015	95389	31675	94851	33326	94284	32
29	26696	96371	28374	95890	30043	95380	31703	94842	33353	94274	31
30	26724	96363	28402	95882	30071	95372	31730	94832	33381	94264	30
31	26752	96355	28429	95874	30098	95363	31758	94823	33408	94254	29
32	26780	96347	28457	95865	30126	95354	31786	94814	33436	94245	28
33	26808	96340	28485	95857	30154	95345	31813	94805	33463	94235	27
34	26836	96332	28513	95849	30182	95337	31841	94795	33490	94225	26
35	26864	96324	28541	95841	30209	95328	31868	94786	33518	94215	25
36	26892	96316	28569	95832	30237	95319	31896	94777	33545	94206	24
37	26920	96308	28597	95824	30265	95310	31923	94768	33573	94196	23
38	26948	96301	28625	95816	30292	95301	31951	94758	33600	94186	22
39	26976	96293	28652	95807	30320	95293	31979	94749	33627	94176	21
40	27004	96285	28680	95799	30348	95284	32006	94740	33655	94167	20
41	27032	96277	28708	95791	30376	95275	32034	94730	33682	94157	19
42	27060	96269	28736	95782	30403	95266	32061	94721	33710	94147	18
43	27088	96261	28764	95774	30431	95257	32089	94712	33737	94137	17
44	27116	96253	28792	95766	30459	95248	32116	94702	33764	94127	16
45	27144	96246	28820	95757	30486	95240	32144	94693	33792	94118	15
46	27172	96238	28847	95749	30514	95231	32171	94684	33819	94108	14
47	27200	96230	28875	95740	30542	95222	32199	94674	33846	94098	13
48	27228	96222	28903	95732	30570	95213	32227	94665	33874	94088	12
49	27256	96214	28931	95724	30597	95204	32254	94656	33901	94078	11
50	27284	96206	28959	95715	30625	95195	32282	94646	33929	94068	10
51	27312	96198	28987	95707	30653	95186	32309	94637	33956	94058	9
52	27340	96190	29015	95698	30680	95177	32337	94627	33983	94049	8
53	27368	96182	29042	95690	30708	95168	32364	94618	34011	94039	7
54	27396	96174	29070	95681	30736	95159	32392	94609	34038	94029	6
55	27424	96166	29098	95673	30763	95150	32419	94599	34065	94019	5
56	27452	96158	29126	95664	30791	95142	32447	94590	34093	94009	4
57	27480	96150	29154	95656	30819	95133	32474	94580	34120	93999	3
58	27508	96142	29182	95647	30846	95124	32502	94571	34147	93989	2
59	27536	96134	29209	95639	30874	95115	32529	94561	34175	93979	1
60	27564	96126	29237	95630	30902	95106	32557	94552	34202	93969	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	74°		73°		72°		71°		70°		′

NATURAL SINES AND COSINES.

′	20°		21°		22°		23°		24°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	34202	93969	35837	93358	37461	92718	39073	92050	40674	91355	60
1	34229	93959	35864	93348	37488	92707	39100	92039	40700	91343	59
2	34257	93949	35891	93337	37515	92697	39127	92028	40727	91331	58
3	34284	93939	35918	93327	37542	92686	39153	92016	40753	91319	57
4	34311	93929	35945	93316	37569	92675	39180	92005	40780	91307	56
5	34339	93919	35973	93306	37595	92664	39207	91994	40806	91295	55
6	34366	93909	36000	93295	37622	92653	39234	91982	40833	91283	54
7	34393	93899	36027	93285	37649	92642	39260	91971	40860	91272	53
8	34421	93889	36054	93274	37676	92631	39287	91959	40886	91260	52
9	34448	93879	36081	93264	37703	92620	39314	91948	40913	91248	51
10	34475	93869	36108	93253	37730	92609	39341	91936	40939	91236	50
11	34503	93859	36135	93243	37757	92598	39367	91925	40966	91224	49
12	34530	93849	36162	93232	37784	92587	39394	91914	40992	91212	48
13	34557	93839	36190	93222	37811	92576	39421	91902	41019	91200	47
14	34584	93829	36217	93211	37838	92565	39448	91891	41045	91188	46
15	34612	93819	36244	93201	37865	92554	39474	91879	41072	91176	45
16	34639	93809	36271	93190	37892	92543	39501	91868	41098	91164	44
17	34666	93799	36298	93180	37919	92532	39528	91856	41125	91152	43
18	34694	93789	36325	93169	37946	92521	39555	91845	41151	91140	42
19	34721	93779	36352	93159	37973	92510	39581	91833	41178	91128	41
20	34748	93769	36379	93148	37999	92499	39608	91822	41204	91116	40
21	34775	93759	36406	93137	38026	92488	39635	91810	41231	91104	39
22	34803	93748	36434	93127	38053	92477	39661	91799	41257	91092	38
23	34830	93738	36461	93116	38080	92466	39688	91787	41284	91080	37
24	34857	93728	36488	93106	38107	92455	39715	91775	41310	91068	36
25	34884	93718	36515	93095	38134	92444	39741	91764	41337	91056	35
26	34912	93708	36542	93084	38161	92432	39768	91752	41363	91044	34
27	34939	93698	36569	93074	38188	92421	39795	91741	41390	91032	33
28	34966	93688	36596	93063	38215	92410	39822	91729	41416	91020	32
29	34993	93677	36623	93052	38241	92399	39848	91718	41443	91008	31
30	35021	93667	36650	93042	38268	92388	39875	91706	41469	90996	30
31	35048	93657	36677	93031	38295	92377	39902	91694	41496	90984	29
32	35075	93647	36704	93020	38322	92366	39928	91683	41522	90972	28
33	35102	93637	36731	93010	38349	92355	39955	91671	41549	90960	27
34	35130	93626	36758	92999	38376	92343	39982	91660	41575	90948	26
35	35157	93616	36785	92988	38403	92332	40008	91648	41602	90936	25
36	35184	93606	36812	92978	38430	92321	40035	91636	41628	90924	24
37	35211	93596	36839	92967	38456	92310	40062	91625	41655	90911	23
38	35239	93585	36867	92956	38483	92299	40088	91613	41681	90899	22
39	35266	93575	36894	92945	38510	92287	40115	91601	41707	90887	21
40	35293	93565	36921	92935	38537	92276	40141	91590	41734	90875	20
41	35320	93555	36948	92924	38564	92265	40168	91578	41760	90863	19
42	35347	93544	36975	92913	38591	92254	40195	91566	41787	90851	18
43	35375	93534	37002	92902	38617	92243	40221	91555	41813	90839	17
44	35402	93524	37029	92892	38644	92231	40248	91543	41840	90826	16
45	35429	93514	37056	92881	38671	92220	40275	91531	41866	90814	15
46	35456	93503	37083	92870	38698	92209	40301	91519	41892	90802	14
47	35484	93493	37110	92859	38725	92198	40328	91508	41919	90790	13
48	35511	93483	37137	92849	38752	92186	40355	91496	41945	90778	12
49	35538	93472	37164	92838	38778	92175	40381	91484	41972	90766	11
50	35565	93462	37191	92827	38805	92164	40408	91472	41998	90753	10
51	35592	93452	37218	92816	38832	92152	40434	91461	42024	90741	9
52	35619	93441	37245	92805	38859	92141	40461	91449	42051	90729	8
53	35647	93431	37272	92794	38886	92130	40488	91437	42077	90717	7
54	35674	93420	37299	92784	38912	92119	40514	91425	42104	90704	6
55	35701	93410	37326	92773	38939	92107	40541	91414	42130	90692	5
56	35728	93400	37353	92762	38966	92096	40567	91402	42156	90680	4
57	35755	93389	37380	92751	38993	92085	40594	91390	42183	90668	3
58	35782	93379	37407	92740	39020	92073	40621	91378	42209	90655	2
59	35810	93368	37434	92729	39046	92062	40647	91366	42235	90643	1
60	35837	93358	37461	92718	39073	92050	40674	91355	42262	90631	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	69°		68°		67°		66°		65°		′

NATURAL SINES AND COSINES.

′	25°		26°		27°		28°		29°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	42262	90631	43837	89879	45399	89101	46947	88295	48481	87462	60
1	42288	90618	43863	89867	45425	89087	46973	88281	48506	87448	59
2	42315	90606	43889	89854	45451	89074	46999	88267	48532	87434	58
3	42341	90594	43916	89841	45477	89061	47024	88254	48557	87420	57
4	42367	90582	43942	89828	45503	89048	47050	88240	48583	87406	56
5	42394	90569	43968	89816	45529	89035	47076	88226	48608	87391	55
6	42420	90557	43994	89803	45554	89021	47101	88213	48634	87377	54
7	42446	90545	44020	89790	45580	89008	47127	88199	48659	87363	53
8	42473	90532	44046	89777	45606	88995	47153	88185	48684	87349	52
9	42499	90520	44072	89764	45632	88981	47178	88172	48710	87335	51
10	42525	90507	44098	89752	45658	88968	47204	88158	48735	87321	50
11	42552	90495	44124	89739	45684	88955	47229	88144	48761	87306	49
12	42578	90483	44151	89726	45710	88942	47255	88130	48786	87292	48
13	42604	90470	44177	89713	45736	88928	47281	88117	48811	87278	47
14	42631	90458	44203	89700	45762	88915	47306	88103	48837	87264	46
15	42657	90446	44229	89687	45787	88902	47332	88089	48862	87250	45
16	42683	90433	44255	89674	45813	88888	47358	88075	48888	87235	44
17	42709	90421	44281	89662	45839	88875	47383	88062	48913	87221	43
18	42736	90408	44307	89649	45865	88862	47409	88048	48938	87207	42
19	42762	90396	44333	89636	45891	88848	47434	88034	48964	87193	41
20	42788	90383	44359	89623	45917	88835	47460	88020	48989	87178	40
21	42815	90371	44385	89610	45942	88822	47486	88006	49014	87164	39
22	42841	90358	44411	89597	45968	88808	47511	87993	49040	87150	38
23	42867	90346	44437	89584	45994	88795	47537	87979	49065	87136	37
24	42894	90334	44464	89571	46020	88782	47562	87965	49090	87121	36
25	42920	90321	44490	89558	46046	88768	47588	87951	49116	87107	35
26	42946	90309	44516	89545	46072	88755	47614	87937	49141	87093	34
27	42972	90296	44542	89532	46097	88741	47639	87923	49166	87079	33
28	42999	90284	44568	89519	46123	88728	47665	87909	49192	87064	32
29	43025	90271	44594	89506	46149	88715	47690	87896	49217	87050	31
30	43051	90259	44620	89493	46175	88701	47716	87882	49242	87036	30
31	43077	90246	44646	89480	46201	88688	47741	87868	49268	87021	29
32	43104	90233	44672	89467	46226	88674	47767	87854	49293	87007	28
33	43130	90221	44698	89454	46252	88661	47793	87840	49318	86993	27
34	43156	90208	44724	89441	46278	88647	47818	87826	49344	86978	26
35	43182	90196	44750	89428	46304	88634	47844	87812	49369	86964	25
36	43209	90183	44776	89415	46330	88620	47869	87798	49394	86949	24
37	43235	90171	44802	89402	46355	88607	47895	87784	49419	86935	23
38	43261	90158	44828	89389	46381	88593	47920	87770	49445	86921	22
39	43287	90146	44854	89376	46407	88580	47946	87756	49470	86906	21
40	43313	90133	44880	89363	46433	88566	47971	87743	49495	86892	20
41	43340	90120	44906	89350	46458	88553	47997	87729	49521	86878	19
42	43366	90108	44932	89337	46484	88539	48022	87715	49546	86863	18
43	43392	90095	44958	89324	46510	88526	48048	87701	49571	86849	17
44	43418	90082	44984	89311	46536	88512	48073	87687	49596	86834	16
45	43445	90070	45010	89298	46561	88499	48099	87673	49622	86820	15
46	43471	90057	45036	89285	46587	88485	48124	87659	49647	86805	14
47	43497	90045	45062	89272	46613	88472	48150	87645	49672	86791	13
48	43523	90032	45088	89259	46639	88458	48175	87631	49697	86777	12
49	43549	90019	45114	89245	46664	88445	48201	87617	49723	86762	11
50	43575	90007	45140	89232	46690	88431	48226	87603	49748	86748	10
51	43602	89994	45166	89219	46716	88417	48252	87589	49773	86733	9
52	43628	89981	45192	89206	46742	88404	48277	87575	49798	86719	8
53	43654	89968	45218	89193	46767	88390	48303	87561	49824	86704	7
54	43680	89956	45243	89180	46793	88377	48328	87546	49849	86690	6
55	43706	89943	45269	89167	46819	88363	48354	87532	49874	86675	5
56	43733	89930	45295	89153	46844	88349	48379	87518	49899	86661	4
57	43759	89918	45321	89140	46870	88336	48405	87504	49924	86646	3
58	43785	89905	45347	89127	46896	88322	48430	87490	49950	86632	2
59	43811	89892	45373	89114	46921	88308	48456	87476	49975	86617	1
60	43837	89879	45399	89101	46947	88295	48481	87462	50000	86603	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	64°		63°		62°		61°		60°		′

NATURAL SINES AND COSINES.

′	30°		31°		32°		33°		34°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	50000	86603	51504	85717	52992	84805	54464	83867	55919	82904	60
1	50025	86588	51529	85702	53017	84789	54488	83851	55943	82887	59
2	50050	86573	51554	85687	53041	84774	54513	83835	55968	82871	58
3	50076	86559	51579	85672	53066	84759	54537	83819	55992	82855	57
4	50101	86544	51604	85657	53091	84743	54561	83804	56016	82839	56
5	50126	86530	51628	85642	53115	84728	54586	83788	56040	82822	55
6	50151	86515	51653	85627	53140	84712	54610	83772	56064	82806	54
7	50176	86501	51678	85612	53164	84697	54635	83756	56088	82790	53
8	50201	86486	51703	85597	53189	84681	54659	83740	56112	82773	52
9	50227	86471	51728	85582	53214	84666	54683	83724	56136	82757	51
10	50252	86457	51753	85567	53238	84650	54708	83708	56160	82741	50
11	50277	86442	51778	85551	53263	84635	54732	83692	56184	82724	49
12	50302	86427	51803	85536	53288	84619	54756	83676	56208	82708	48
13	50327	86413	51828	85521	53312	84604	54781	83660	56232	82692	47
14	50352	86398	51852	85506	53337	84588	54805	83645	56256	82675	46
15	50377	86384	51877	85491	53361	84573	54829	83629	56280	82659	45
16	50403	86369	51902	85476	53386	84557	54854	83613	56305	82643	44
17	50428	86354	51927	85461	53411	84542	54878	83597	56329	82626	43
18	50453	86340	51952	85446	53435	84526	54902	83581	56353	82610	42
19	50478	86325	51977	85431	53460	84511	54927	83565	56377	82593	41
20	50503	86310	52002	85416	53484	84495	54951	83549	56401	82577	40
21	50528	86295	52026	85401	53509	84480	54975	83533	56425	82561	39
22	50553	86281	52051	85385	53534	84464	54999	83517	56449	82544	38
23	50578	86266	52076	85370	53558	84448	55024	83501	56473	82528	37
24	50603	86251	52101	85355	53583	84433	55048	83485	56497	82511	36
25	50628	86237	52126	85340	53607	84417	55072	83469	56521	82495	35
26	50654	86222	52151	85325	53632	84402	55097	83453	56545	82478	34
27	50679	86207	52175	85310	53656	84386	55121	83437	56569	82462	33
28	50704	86192	52200	85294	53681	84370	55145	83421	56593	82446	32
29	50729	86178	52225	85279	53705	84355	55169	83405	56617	82429	31
30	50754	86163	52250	85264	53730	84339	55194	83389	56641	82413	30
31	50779	86148	52275	85249	53754	84324	55218	83373	56665	82396	29
32	50804	86133	52299	85234	53779	84308	55242	83356	56689	82380	28
33	50829	86119	52324	85218	53804	84292	55266	83340	56713	82363	27
34	50854	86104	52349	85203	53828	84277	55291	83324	56736	82347	26
35	50879	86089	52374	85188	53853	84261	55315	83308	56760	82330	25
36	50904	86074	52399	85173	53877	84245	55339	83292	56784	82314	24
37	50929	86059	52423	85157	53902	84230	55363	83276	56808	82297	23
38	50954	86045	52448	85142	53926	84214	55388	83260	56832	82281	22
39	50979	86030	52473	85127	53951	84198	55412	83244	56856	82264	21
40	51004	86015	52498	85112	53975	84182	55436	83228	56880	82248	20
41	51029	86000	52522	85096	54000	84167	55460	83212	56904	82231	19
42	51054	85985	52547	85081	54024	84151	55484	83195	56928	82214	18
43	51079	85970	52572	85066	54049	84135	55509	83179	56952	82198	17
44	51104	85956	52597	85051	54073	84120	55533	83163	56976	82181	16
45	51129	85941	52621	85035	54097	84104	55557	83147	57000	82165	15
46	51154	85926	52646	85020	54122	84088	55581	83131	57024	82148	14
47	51179	85911	52671	85005	54146	84072	55605	83115	57047	82132	13
48	51204	85896	52696	84989	54171	84057	55630	83098	57071	82115	12
49	51229	85881	52720	84974	54195	84041	55654	83082	57095	82098	11
50	51254	85866	52745	84959	54220	84025	55678	83066	57119	82082	10
51	51279	85851	52770	84943	54244	84009	55702	83050	57143	82065	9
52	51304	85836	52794	84928	54269	83994	55726	83034	57167	82048	8
53	51329	85821	52819	84913	54293	83978	55750	83017	57191	82032	7
54	51354	85806	52844	84897	54317	83962	55775	83001	57215	82015	6
55	51379	85792	52869	84882	54342	83946	55799	82985	57238	81999	5
56	51404	85777	52893	84866	54366	83930	55823	82969	57262	81982	4
57	51429	85762	52918	84851	54391	83915	55847	82953	57286	81965	3
58	51454	85747	52943	84836	54415	83899	55871	82936	57310	81949	2
59	51479	85732	52967	84820	54440	83883	55895	82920	57334	81932	1
60	51504	85717	52992	84805	54464	83867	55919	82904	57358	81915	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	59°		58°		57°		56°		55°		′

NATURAL SINES AND COSINES.

′	35°		36°		37°		38°		39°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	57358	81915	58779	80902	60182	79864	61566	78801	62932	77715	60
1	57381	81899	58802	80885	60205	79846	61589	78783	62955	77696	59
2	57405	81882	58826	80867	60228	79829	61612	78765	62977	77678	58
3	57429	81865	58849	80850	60251	79811	61635	78747	63000	77660	57
4	57453	81848	58873	80833	60274	79793	61658	78729	63022	77641	56
5	57477	81832	58896	80816	60298	79776	61681	78711	63045	77623	55
6	57501	81815	58920	80799	60321	79758	61704	78694	63068	77605	54
7	57524	81798	58943	80782	60344	79741	61726	78676	63090	77586	53
8	57548	81782	58967	80765	60367	79723	61749	78658	63113	77568	52
9	57572	81765	58990	80748	60390	79706	61772	78640	63135	77550	51
10	57596	81748	59014	80730	60414	79688	61795	78622	63158	77531	50
11	57619	81731	59037	80713	60437	79671	61818	78604	63180	77513	49
12	57643	81714	59061	80696	60460	79653	61841	78586	63203	77494	48
13	57667	81698	59084	80679	60483	79635	61864	78568	63225	77476	47
14	57691	81681	59108	80662	60506	79618	61887	78550	63248	77458	46
15	57715	81664	59131	80644	60529	79600	61909	78532	63271	77439	45
16	57738	81647	59154	80627	60553	79583	61932	78514	63293	77421	44
17	57762	81631	59178	80610	60576	79565	61955	78496	63316	77402	43
18	57786	81614	59201	80593	60599	79547	61978	78478	63338	77384	42
19	57810	81597	59225	80576	60622	79530	62001	78460	63361	77366	41
20	57833	81580	59248	80558	60645	79512	62024	78442	63383	77347	40
21	57857	81563	59272	80541	60668	79494	62046	78424	63406	77329	39
22	57881	81546	59295	80524	60691	79477	62069	78405	63428	77310	38
23	57904	81530	59318	80507	60714	79459	62092	78387	63451	77292	37
24	57928	81513	59342	80489	60738	79441	62115	78369	63473	77273	36
25	57952	81496	59365	80472	60761	79424	62138	78351	63496	77255	35
26	57976	81479	59389	80455	60784	79406	62160	78333	63518	77236	34
27	57999	81462	59412	80438	60807	79388	62183	78315	63540	77218	33
28	58023	81445	59436	80420	60830	79371	62206	78297	63563	77199	32
29	58047	81428	59459	80403	60853	79353	62229	78279	63585	77181	31
30	58070	81412	59482	80386	60876	79335	62251	78261	63608	77162	30
31	58094	81395	59506	80368	60899	79318	62274	78243	63630	77144	29
32	58118	81378	59529	80351	60922	79300	62297	78225	63653	77125	28
33	58141	81361	59552	80334	60945	79282	62320	78206	63675	77107	27
34	58165	81344	59576	80316	60968	79264	62342	78188	63698	77088	26
35	58189	81327	59599	80299	60991	79247	62365	78170	63720	77070	25
36	58212	81310	59622	80282	61015	79229	62388	78152	63742	77051	24
37	58236	81293	59646	80264	61038	79211	62411	78134	63765	77033	23
38	58260	81276	59669	80247	61061	79193	62433	78116	63787	77014	22
39	58283	81259	59693	80230	61084	79176	62456	78098	63810	76996	21
40	58307	81242	59716	80212	61107	79158	62479	78079	63832	76977	20
41	58330	81225	59739	80195	61130	79140	62502	78061	63854	76959	19
42	58354	81208	59763	80178	61153	79122	62524	78043	63877	76940	18
43	58378	81191	59786	80160	61176	79105	62547	78025	63899	76921	17
44	58401	81174	59809	80143	61199	79087	62570	78007	63922	76903	16
45	58425	81157	59832	80125	61222	79069	62592	77988	63944	76884	15
46	58449	81140	59856	80108	61245	79051	62615	77970	63966	76866	14
47	58472	81123	59879	80091	61268	79033	62638	77952	63989	76847	13
48	58496	81106	59902	80073	61291	79016	62660	77934	64011	76828	12
49	58519	81089	59926	80056	61314	78998	62683	77916	64033	76810	11
50	58543	81072	59949	80038	61337	78980	62706	77897	64056	76791	10
51	58567	81055	59972	80021	61360	78962	62728	77879	64078	76772	9
52	58590	81038	59995	80003	61383	78944	62751	77861	64100	76754	8
53	58614	81021	60019	79986	61406	78926	62774	77843	64123	76735	7
54	58637	81004	60042	79968	61429	78908	62796	77824	64145	76717	6
55	58661	80987	60065	79951	61451	78891	62819	77806	64167	76698	5
56	58684	80970	60089	79934	61474	78873	62842	77788	64190	76679	4
57	58708	80953	60112	79916	61497	78855	62864	77769	64212	76661	3
58	58731	80936	60135	79899	61520	78837	62887	77751	64234	76642	2
59	58755	80919	60158	79881	61543	78819	62909	77733	64256	76623	1
60	58779	80902	60182	79864	61566	78801	62932	77715	64279	76604	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	54°		53°		52°		51°		50°		′

NATURAL SINES AND COSINES.

′	40°		41°		42°		43°		44°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	64279	76604	65606	75471	66913	74314	68200	73135	69466	71934	60
1	64301	76586	65628	75452	66935	74295	68221	73116	69487	71914	59
2	64323	76567	65650	75433	66956	74276	68242	73096	69508	71894	58
3	64346	76548	65672	75414	66978	74256	68264	73076	69529	71873	57
4	64368	76530	65694	75395	66999	74237	68285	73056	69549	71853	56
5	64390	76511	65716	75375	67021	74217	68306	73036	69570	71833	55
6	64412	76492	65738	75356	67043	74198	68327	73016	69591	71813	54
7	64435	76473	65759	75337	67064	74178	68349	72996	69612	71792	53
8	64457	76455	65781	75318	67086	74159	68370	72976	69633	71772	52
9	64479	76436	65803	75299	67107	74139	68391	72957	69654	71752	51
10	64501	76417	65825	75280	67129	74120	68412	72937	69675	71732	50
11	64524	76398	65847	75261	67151	74100	68434	72917	69696	71711	49
12	64546	76380	65869	75241	67172	74080	68455	72897	69717	71691	48
13	64568	76361	65891	75222	67194	74061	68476	72877	69737	71671	47
14	64590	76342	65913	75203	67215	74041	68497	72857	69758	71650	46
15	64612	76323	65935	75184	67237	74022	68518	72837	69779	71630	45
16	64635	76304	65956	75165	67258	74002	68539	72817	69800	71610	44
17	64657	76286	65978	75146	67280	73983	68561	72797	69821	71590	43
18	64679	76267	66000	75126	67301	73963	68582	72777	69842	71569	42
19	64701	76248	66022	75107	67323	73944	68603	72757	69862	71549	41
20	64723	76229	66044	75088	67344	73924	68624	72737	69883	71529	40
21	64746	76210	66066	75069	67366	73904	68645	72717	69904	71508	39
22	64768	76192	66088	75050	67387	73885	68666	72697	69925	71488	38
23	64790	76173	66109	75030	67409	73865	68688	72677	69946	71468	37
24	64812	76154	66131	75011	67430	73846	68709	72657	69966	71447	36
25	64834	76135	66153	74992	67452	73826	68730	72637	69987	71427	35
26	64856	76116	66175	74973	67473	73806	68751	72617	70008	71407	34
27	64878	76097	66197	74953	67495	73787	68772	72597	70029	71386	33
28	64901	76078	66218	74934	67516	73767	68793	72577	70049	71366	32
29	64923	76059	66240	74915	67538	73747	68814	72557	70070	71345	31
30	64945	76041	66262	74896	67559	73728	68835	72537	70091	71325	30
31	64967	76022	66284	74876	67580	73708	68857	72517	70112	71305	29
32	64989	76003	66306	74857	67602	73688	68878	72497	70132	71284	28
33	65011	75984	66327	74838	67623	73669	68899	72477	70153	71264	27
34	65033	75965	66349	74818	67645	73649	68920	72457	70174	71243	26
35	65055	75946	66371	74799	67666	73629	68941	72437	70195	71223	25
36	65077	75927	66393	74780	67688	73610	68962	72417	70215	71203	24
37	65100	75908	66414	74760	67709	73590	68983	72397	70236	71182	23
38	65122	75889	66436	74741	67730	73570	69004	72377	70257	71162	22
39	65144	75870	66458	74722	67752	73551	69025	72357	70277	71141	21
40	65166	75851	66480	74703	67773	73531	69046	72337	70298	71121	20
41	65188	75832	66501	74683	67795	73511	69067	72317	70319	71100	19
42	65210	75813	66523	74664	67816	73491	69088	72297	70339	71080	18
43	65232	75794	66545	74644	67837	73472	69109	72277	70360	71059	17
44	65254	75775	66566	74625	67859	73452	69130	72257	70381	71039	16
45	65276	75756	66588	74606	67880	73432	69151	72236	70401	71019	15
46	65298	75738	66610	74586	67901	73413	69172	72216	70422	70998	14
47	65320	75719	66632	74567	67923	73393	69193	72196	70443	70978	13
48	65342	75700	66653	74548	67944	73373	69214	72176	70463	70957	12
49	65364	75680	66675	74528	67965	73353	69235	72156	70484	70937	11
50	65386	75661	66697	74509	67987	73333	69256	72136	70505	70916	10
51	65408	75642	66718	74489	68008	73314	69277	72116	70525	70896	9
52	65430	75623	66740	74470	68029	73294	69298	72095	70546	70875	8
53	65452	75604	66762	74451	68051	73274	69319	72075	70567	70855	7
54	65474	75585	66783	74431	68072	73254	69340	72055	70587	70834	6
55	65496	75566	66805	74412	68093	73234	69361	72035	70608	70813	5
56	65518	75547	66827	74392	68115	73215	69382	72015	70628	70793	4
57	65540	75528	66848	74373	68136	73195	69403	71995	70649	70772	3
58	65562	75509	66870	74353	68157	73175	69424	71974	70670	70752	2
59	65584	75490	66891	74334	68179	73155	69445	71954	70690	70731	1
60	65606	75471	66913	74314	68200	73135	69466	71934	70711	70711	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	49°		48°		47°		46°		45°		′

LOGARITHMS OF NUMBERS.

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102	* 8600	9026	9451	9876	•300	0724	1147	1570	1993	2415	424
103	01 2837	3259	3680	4100	4521	4940	5360	5779	6197	6616	419
104	* 7033	7451	7868	8284	8700	9116	9532	9947	•361	0775	416
105	02 1189	1603	2016	2428	2841	3252	3664	4075	4486	4896	412
106	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
107	* 9384	9789	•195	0600	1004	1408	1812	2216	2619	3021	404
108	03 3424	3826	4227	4628	5029	5430	5830	6230	6629	7028	400
109	* 7426	7825	8223	8620	9017	9414	9811	•207	0602	0998	396
110	04 1393	1787	2182	2576	2969	3362	3755	4148	4540	4932	393
111	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	389
112	* 9218	9606	9993	•380	0766	1153	1538	1924	2309	2694	386
113	05 3078	3463	3846	4230	4613	4996	5378	5760	6142	6524	382
114	* 6905	7286	7666	8046	8426	8805	9185	9563	9942	•320	379
115	06 0698	1075	1452	1829	2206	2582	2958	3333	3709	4083	376
116	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	372
117	* 8186	8557	8928	9298	9668	•038	0407	0776	1145	1514	369
118	07 1882	2250	2617	2985	3352	3718	4085	4451	4816	5182	366
119	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363
120	* 9181	9543	9904	•266	0626	0987	1347	1707	2067	2426	360
121	08 2785	3144	3503	3861	4219	4576	4934	5291	5647	6004	357
122	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
123	* 9905	•258	0611	0963	1315	1667	2018	2370	2721	3071	351
124	09 3422	3772	4122	4471	4820	5169	5518	5866	6215	6562	349
125	* 6910	7257	7604	7951	8298	8644	8990	9335	9681	•026	346
126	10 0371	0715	1059	1403	1747	2091	2434	2777	3119	3462	343
127	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	340
128	* 7210	7549	7888	8227	8565	8903	9241	9579	9916	•253	338
129	11 0590	0926	1263	1599	1934	2270	2605	2940	3275	3609	335
130	3943	4277	4611	4944	5278	5611	5943	6276	6608	6940	333
131	* 7271	7603	7934	8265	8595	8926	9256	9586	9915	•245	330
132	12 0574	0903	1231	1560	1888	2216	2544	2871	3198	3525	328
133	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
134	* 7105	7429	7753	8076	8399	8722	9045	9368	9690	•012	323
135	13 0334	0655	0977	1298	1619	1939	2260	2580	2900	3219	321
136	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
137	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	315
138	* 9879	•194	0508	0822	1136	1450	1763	2076	2389	2702	314
139	14 3015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
140	6128	6438	6748	7058	7367	7676	7985	8294	8603	8911	309
141	* 9219	9527	9835	•142	0449	0756	1063	1370	1676	1982	307
142	15 2288	2594	2900	3205	3510	3815	4120	4424	4728	5032	305
143	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
144	* 8362	8664	8965	9266	9567	9868	•168	0469	0769	1068	301
145	16 1368	1667	1967	2266	2564	2863	3161	3460	3758	4055	299
146	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022	297
147	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
148	17 0262	0555	0848	1141	1434	1726	2019	2311	2603	2895	293
149	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291
150	6091	6381	6670	6959	7248	7536	7825	8113	8401	8689	289
151	* 8977	9264	9552	9839	•126	0413	0699	0985	1272	1558	287
152	18 1844	2129	2415	2700	2985	3270	3555	3839	4123	4407	285
153	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
154	* 7521	7803	8084	8366	8647	8928	9209	9490	9771	•051	281
155	19 0332	0612	0892	1171	1451	1730	2010	2289	2567	2846	279
156	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
157	5900	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
158	* 8657	8932	9206	9481	9755	•029	0303	0577	0850	1124	274
159	20 1397	1670	1943	2216	2488	2761	3033	3305	3577	3848	272
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160	20 4120	4391	4663	4934	5204	5475	5746	6016	6286	6556	271
161	6826	7096	7365	7634	7904	8173	8441	8710	8979	9247	269
162	* 9515	9783	•051	0319	0586	0853	1121	1388	1654	1921	267
163	21 2188	2454	2720	2986	3252	3518	3783	4049	4314	4579	266
164	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221	264
165	7484	7747	8010	8273	8536	8798	9060	9323	9585	9846	262
166	22 0108	0370	0631	0892	1153	1414	1675	1936	2196	2456	261
167	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
168	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630	258
169	* 7887	8144	8400	8657	8913	9170	9426	9682	9938	•193	256
170	23 0449	0704	0960	1215	1470	1724	1979	2234	2488	2742	254
171	2996	3250	3504	3757	4011	4264	4517	4770	5023	5276	253
172	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795	252
173	* 8046	8297	8548	8799	9049	9299	9550	9800	•050	0300	250
174	24 0549	0799	1048	1297	1546	1795	2044	2293	2541	2790	249
175	3038	3286	3534	3782	4030	4277	4525	4772	5019	5266	248
176	5513	5759	6006	6252	6499	6745	6991	7237	7482	7728	246
177	* 7973	8219	8464	8709	8954	9198	9443	9687	9932	•176	245
178	25 0420	0664	0908	1151	1395	1638	1881	2125	2368	2610	243
179	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031	242
180	5273	5514	5755	5996	6237	6477	6718	6958	7198	7439	241
181	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
182	26 0071	0310	0548	0787	1025	1263	1501	1739	1976	2214	238
183	2451	2688	2925	3162	3399	3636	3873	4109	4346	4582	237
184	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937	235
185	7172	7406	7641	7875	8110	8344	8578	8812	9046	9279	234
186	* 9513	9746	9980	•213	0446	0679	0912	1144	1377	1609	233
187	27 1842	2074	2306	2538	2770	3001	3233	3464	3696	3927	232
188	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232	230
189	6462	6692	6921	7151	7380	7609	7838	8067	8296	8525	229
190	* 8754	8982	9211	9439	9667	9895	•123	0351	0578	0806	228
191	28 1033	1261	1488	1715	1942	2169	2396	2622	2849	3075	227
192	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
193	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
194	7802	8026	8249	8473	8696	8920	9143	9366	9589	9812	223
195	29 0035	0257	0480	0702	0925	1147	1369	1591	1813	2034	222
196	2256	2478	2699	2920	3141	3363	3584	3804	4025	4246	221
197	4466	4687	4907	5127	5347	5567	5787	6007	6226	6446	220
198	6665	6884	7104	7323	7542	7761	7979	8198	8416	8635	219
199	* 8853	9071	9289	9507	9725	9943	•161	0378	0595	0813	218
200	30 1030	1247	1464	1681	1898	2114	2331	2547	2764	2980	217
201	3196	3412	3628	3844	4059	4275	4491	4706	4921	5136	216
202	5351	5566	5781	5996	6211	6425	6639	6854	7068	7282	215
203	7496	7710	7924	8137	8351	8564	8778	8991	9204	9417	213
204	* 9630	9843	•056	0268	0481	0693	0906	1118	1330	1542	212
205	31 1754	1966	2177	2389	2600	2812	3023	3234	3445	3656	211
206	3867	4078	4289	4499	4710	4920	5130	5340	5551	5760	210
207	5970	6180	6390	6599	6809	7018	7227	7436	7646	7854	209
208	8063	8272	8481	8689	8898	9106	9314	9522	9730	9938	208
209	32 0146	0354	0562	0769	0977	1184	1391	1598	1805	2012	207
210	2219	2426	2633	2839	3046	3252	3458	3665	3871	4077	206
211	4282	4488	4694	4899	5105	5310	5516	5721	5926	6131	205
212	6336	6541	6745	6950	7155	7359	7563	7767	7972	8176	204
213	* 8380	8583	8787	8991	9194	9398	9601	9805	•008	0211	203
214	33 0414	0617	0819	1022	1225	1427	1630	1832	2034	2236	202
215	2438	2640	2842	3044	3246	3447	3649	3850	4051	4253	202
216	4454	4655	4856	5057	5257	5458	5658	5859	6059	6260	201
217	6460	6660	6860	7060	7260	7459	7659	7858	8058	8257	200
218	* 8456	8656	8855	9054	9253	9451	9650	9849	•047	0246	199
219	34 0444	0642	0841	1039	1237	1435	1632	1830	2028	2225	198
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LOGARITHMS OF NUMBERS.

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220	34 2423	2620	2817	3014	3212	3409	3606	3802	3999	4196	197
221	4392	4589	4785	4981	5178	5374	5570	5766	5962	6157	196
222	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110	195
223	* 8305	8500	8694	8889	9083	9278	9472	9666	9860	•054	194
224	35 0248	0442	0636	0829	1023	1216	1410	1603	1796	1989	193
225	2183	2375	2568	2761	2954	3147	3339	3532	3724	3916	193
226	4108	4301	4493	4685	4876	5068	5260	5452	5643	5834	192
227	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744	191
228	7935	8125	8316	8506	8696	8886	9076	9266	9456	9646	190
229	* 9835	•025	0215	0404	0593	0783	0972	1161	1350	1539	189
230	36 1728	1917	2105	2294	2482	2671	2859	3048	3236	3424	188
231	3612	3800	3988	4176	4363	4551	4739	4926	5113	5301	188
232	5488	5675	5862	6049	6236	6423	6610	6796	6983	7169	187
233	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	186
234	* 9216	9401	9587	9772	9958	•143	0328	0513	0698	0883	185
235	37 1068	1253	1437	1622	1806	1991	2175	2360	2544	2728	184
236	2912	3096	3280	3464	3647	3831	4015	4198	4382	4565	184
237	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
238	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	182
239	* 8398	8580	8761	8943	9124	9306	9487	9668	9849	•030	181
240	38 0211	0392	0573	0754	0934	1115	1296	1476	1656	1837	181
241	2017	2197	2377	2557	2737	2917	3097	3277	3456	3636	180
242	3815	3995	4174	4353	4533	4712	4891	5070	5249	5428	179
243	5606	5785	5964	6142	6321	6499	6677	6856	7034	7212	178
244	7390	7568	7746	7923	8101	8279	8456	8634	8811	8989	178
245	* 9166	9343	9520	9698	9875	•051	0228	0405	0582	0759	177
246	39 0935	1112	1288	1464	1641	1817	1993	2169	2345	2521	176
247	2697	2873	3048	3224	3400	3575	3751	3926	4101	4277	176
248	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025	175
249	6199	6374	6548	6722	6896	7071	7245	7419	7592	7766	174
250	7940	8114	8287	8461	8634	8808	8981	9154	9328	9501	173
251	* 9674	9847	•020	0192	0365	0538	0711	0883	1056	1228	173
252	40 1401	1573	1745	1917	2089	2261	2433	2605	2777	2949	172
253	3121	3292	3464	3635	3807	3978	4149	4320	4492	4663	171
254	4834	5005	5176	5346	5517	5688	5858	6029	6199	6370	171
255	6540	6710	6881	7051	7221	7391	7561	7731	7901	8070	170
256	8240	8410	8579	8749	8918	9087	9257	9426	9595	9764	169
257	* 9933	•102	0271	0440	0609	0777	0946	1114	1283	1451	169
258	41 1620	1788	1956	2124	2293	2461	2629	2796	2964	3132	168
259	3300	3467	3635	3803	3970	4137	4305	4472	4639	4806	167
260	4973	5140	5307	5474	5641	5808	5974	6141	6308	6474	167
261	6641	6807	6973	7139	7306	7472	7638	7804	7970	8135	166
262	8301	8467	8633	8798	8964	9129	9295	9460	9625	9791	165
263	* 9956	•121	0286	0451	0616	0781	0945	1110	1275	1439	165
264	42 1604	1768	1933	2097	2261	2426	2590	2754	2918	3082	164
265	3246	3410	3574	3737	3901	4065	4228	4392	4555	4718	164
266	4882	5045	5208	5371	5534	5697	5860	6023	6186	6349	163
267	6511	6674	6836	6999	7161	7324	7486	7648	7811	7973	162
268	8135	8297	8459	8621	8783	8944	9106	9268	9429	9591	162
269	* 9752	9914	•075	0236	0398	0559	0720	0881	1042	1203	161
270	43 1364	1525	1685	1846	2007	2167	2328	2488	2649	2809	161
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272	4569	4729	4888	5048	5207	5367	5526	5685	5844	6004	159
273	6163	6322	6481	6640	6799	6957	7116	7275	7433	7592	159
274	7751	7909	8067	8226	8384	8542	8701	8859	9017	9175	158
275	* 9333	9491	9648	9806	9964	•122	0279	0437	0594	0752	158
276	44 0909	1066	1224	1381	1538	1695	1852	2009	2166	2323	157
277	2480	2637	2793	2950	3106	3263	3419	3576	3732	3889	157
278	4045	4201	4357	4513	4669	4825	4981	5137	5293	5449	156
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282	45 0249	0403	0557	0711	0865	1018	1172	1326	1479	1633	154
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284	3318	3471	3624	3777	3930	4082	4235	4387	4540	4692	153
285	4845	4997	5150	5302	5454	5606	5758	5910	6062	6214	152
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288	* 9392	9543	9694	9845	9995	•146	0296	0447	0597	0748	151
289	46 0898	1048	1198	1348	1499	1649	1799	1948	2098	2248	150
290	2398	2548	2697	2847	2997	3146	3296	3445	3594	3744	150
291	3893	4042	4191	4340	4490	4639	4788	4936	5085	5234	149
292	5383	5532	5680	5829	5977	6126	6274	6423	6571	6719	149
293	6868	7016	7164	7312	7460	7608	7756	7904	8052	8200	148
294	8347	8495	8643	8790	8938	9085	9233	9380	9527	9675	148
295	* 9822	9969	•116	0263	0410	0557	0704	0851	0998	1145	147
296	47 1292	1438	1585	1732	1878	2025	2171	2318	2464	2610	146
297	2756	2903	3049	3195	3341	3487	3633	3779	3925	4071	146
298	4216	4362	4508	4653	4799	4944	5090	5235	5381	5526	146
299	5671	5816	5962	6107	6252	6397	6542	6687	6832	6976	145
300	7121	7266	7411	7555	7700	7844	7989	8133	8278	8422	145
301	8566	8711	8855	8999	9143	9287	9431	9575	9719	9863	144
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303	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731	143
304	2874	3016	3159	3302	3445	3587	3730	3872	4015	4157	143
305	4300	4442	4585	4727	4869	5011	5153	5295	5437	5579	142
306	5721	5863	6005	6147	6289	6430	6572	6714	6855	6997	142
307	7138	7280	7421	7563	7704	7845	7986	8127	8269	8410	141
308	8551	8692	8833	8974	9114	9255	9396	9537	9677	9818	141
309	* 9958	•099	0239	0380	0520	0661	0801	0941	1081	1222	140
310	49 1362	1502	1642	1782	1922	2062	2201	2341	2481	2621	140
311	2760	2900	3040	3179	3319	3458	3597	3737	3876	4015	139
312	4155	4294	4433	4572	4711	4850	4989	5128	5267	5406	139
313	5544	5683	5822	5960	6099	6238	6376	6515	6653	6791	139
314	6930	7068	7206	7344	7483	7621	7759	7897	8035	8173	138
315	8311	8448	8586	8724	8862	8999	9137	9275	9412	9550	138
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319	3791	3927	4063	4199	4335	4471	4607	4743	4878	5014	136
320	5150	5286	5421	5557	5693	5828	5964	6099	6234	6370	136
321	6505	6640	6776	6911	7046	7181	7316	7451	7586	7721	135
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325	1883	2017	2151	2284	2418	2551	2684	2818	2951	3084	133
326	3218	3351	3484	3617	3750	3883	4016	4149	4282	4414	133
327	4548	4681	4813	4946	5079	5211	5344	5476	5609	5741	133
328	5874	6006	6139	6271	6403	6535	6668	6800	6932	7064	132
329	7196	7328	7460	7592	7724	7855	7987	8119	8251	8382	132
330	8514	8646	8777	8909	9040	9171	9303	9434	9566	9697	131
331	* 9828	9959	•090	0221	0353	0484	0615	0745	0876	1007	131
332	52 1138	1269	1400	1530	1661	1792	1922	2053	2183	2314	131
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337	7630	7759	7888	8016	8145	8274	8402	8531	8660	8788	129
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346	* 9076	9202	9327	9452	9578	9703	9829	9954	⋆079	0204	125
347	54 0329	0455	0580	0705	0830	0955	1080	1205	1330	1454	125
348	1579	1704	1829	1953	2078	2203	2327	2452	2576	2701	125
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351	5307	5431	5555	5678	5802	5925	6049	6172	6296	6419	124
352	6543	6666	6789	6913	7036	7159	7282	7405	7529	7652	123
353	7775	7898	8021	8144	8267	8389	8512	8635	8758	8881	123
354	* 9003	9126	9249	9371	9494	9616	9739	9861	9984	⋆106	123
355	55 0228	0351	0473	0595	0717	0840	0962	1084	1206	1328	122
356	1450	1572	1694	1816	1938	2060	2181	2303	2425	2547	122
357	2668	2790	2911	3033	3155	3276	3398	3519	3640	3762	121
358	3883	4004	4126	4247	4368	4489	4610	4731	4852	4973	121
359	5094	5215	5336	5457	5578	5699	5820	5940	6061	6182	121
360	6303	6423	6544	6664	6785	6905	7026	7146	7267	7387	120
361	7507	7627	7748	7868	7988	8108	8228	8349	8469	8589	120
362	8709	8829	8948	9068	9188	9308	9428	9548	9667	9787	120
363	* 9907	⋆026	0146	0265	0385	0504	0624	0743	0863	0982	119
364	56 1101	1221	1340	1459	1578	1698	1817	1936	2055	2174	119
365	2293	2412	2531	2650	2769	2887	3006	3125	3244	3362	119
366	3481	3600	3718	3837	3955	4074	4192	4311	4429	4548	119
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368	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909	118
369	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	118
370	8202	8319	8436	8554	8671	8788	8905	9023	9140	9257	117
371	* 9374	9491	9608	9725	9842	9959	⋆076	0193	0309	0426	117
372	57 0543	0660	0776	0893	1010	1126	1243	1359	1476	1592	117
373	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755	116
374	2872	2988	3104	3220	3336	3452	3568	3684	3800	3915	116
375	4031	4147	4263	4379	4494	4610	4726	4841	4957	5072	116
376	5188	5303	5419	5534	5650	5765	5880	5996	6111	6226	115
377	6341	6457	6572	6687	6802	6917	7032	7147	7262	7377	115
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379	8639	8754	8868	8983	9097	9212	9326	9441	9555	9669	114
380	* 9784	9898	⋆012	0126	0241	0355	0469	0583	0697	0811	114
381	58 0925	1039	1153	1267	1381	1495	1608	1722	1836	1950	114
382	2063	2177	2291	2404	2518	2631	2745	2858	2972	3085	114
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384	4331	4444	4557	4670	4783	4896	5009	5122	5235	5348	113
385	5461	5574	5686	5799	5912	6024	6137	6250	6362	6475	113
386	6587	6700	6812	6925	7037	7149	7262	7374	7486	7599	112
387	7711	7823	7935	8047	8160	8272	8384	8496	8608	8720	112
388	8832	8944	9056	9167	9279	9391	9503	9615	9726	9838	112
389	* 9950	⋆061	0173	0284	0396	0507	0619	0730	0842	0953	112
390	59 1065	1176	1287	1399	1510	1621	1732	1843	1955	2066	111
391	2177	2288	2399	2510	2621	2732	2843	2954	3064	3175	111
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393	4393	4503	4614	4724	4834	4945	5055	5165	5276	5386	110
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397	8791	8900	9009	9119	9228	9337	9446	9556	9665	9774	109
398	* 9883	9992	⋆101	0210	0319	0428	0537	0646	0755	0864	109
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409	1723	1829	1936	2042	2148	2254	2360	2466	2572	2678	106
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412	4897	5003	5108	5213	5319	5424	5529	5634	5740	5845	105
413	5950	6055	6160	6265	6370	6476	6581	6686	6790	6895	105
414	7000	7105	7210	7315	7420	7525	7629	7734	7839	7943	105
415	8048	8153	8257	8362	8466	8571	8676	8780	8884	8989	105
416	* 9093	9198	9302	9406	9511	9615	9719	9824	9928	◆032	104
417	62 0136	0240	0344	0448	0552	0656	0760	0864	0968	1072	104
418	1176	1280	1384	1488	1592	1695	1799	1903	2007	2110	104
419	2214	2318	2421	2525	2628	2732	2835	2939	3042	3146	104
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426	* 9410	9512	9613	9715	9817	9919	◆021	0123	0224	0326	102
427	63 0428	0530	0631	0733	0835	0936	1038	1139	1241	1342	102
428	1444	1545	1647	1748	1849	1951	2052	2153	2255	2356	101
429	2457	2559	2660	2761	2862	2963	3064	3165	3266	3367	101
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436	* 9486	9586	9686	9785	9885	9984	◆084	0183	0283	0382	99
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449	2246	2343	2440	2536	2633	2730	2826	2923	3019	3116	97
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454	7056	7152	7247	7343	7438	7534	7629	7725	7820	7916	96
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457	* 9916	◆011	0106	0201	0296	0391	0486	0581	0676	0771	95
458	66 0865	0960	1055	1150	1245	1339	1434	1529	1623	1718	95
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461	3701	3795	3889	3983	4078	4172	4266	4360	4454	4548	94
462	4642	4736	4830	4924	5018	5112	5206	5299	5393	5487	94
463	5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	94
464	6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	94
465	7453	7546	7640	7733	7826	7920	8013	8106	8199	8293	93
466	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	93
467	* 9317	9410	9503	9596	9689	9782	9875	9967	*060	0153	93
468	67 0246	0339	0431	0524	0617	0710	0802	0895	0988	1080	93
469	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	93
470	2098	2190	2283	2375	2467	2560	2652	2744	2836	2929	92
471	3021	3113	3205	3297	3390	3482	3574	3666	3758	3850	92
472	3942	4034	4126	4218	4310	4402	4494	4586	4677	4769	92
473	4861	4953	5045	5137	5228	5320	5412	5503	5595	5687	92
474	5778	5870	5962	6053	6145	6236	6328	6419	6511	6602	92
475	6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	91
476	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	91
477	8518	8609	8700	8791	8882	8973	9064	9155	9246	9337	91
478	* 9428	9519	9610	9700	9791	9882	9973	*063	0154	0245	91
479	68 0336	0426	0517	0607	0698	0789	0879	0970	1060	1151	91
480	1241	1332	1422	1513	1603	1693	1784	1874	1964	2055	90
481	2145	2235	2326	2416	2506	2596	2686	2777	2867	2957	90
482	3047	3137	3227	3317	3407	3497	3587	3677	3767	3857	90
483	3947	4037	4127	4217	4307	4396	4486	4576	4666	4756	90
484	4845	4935	5025	5114	5204	5294	5383	5473	5563	5652	90
485	5742	5831	5921	6010	6100	6189	6279	6368	6458	6547	89
486	6636	5726	6815	6904	6994	7083	7172	7261	7351	7440	89
487	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
488	8420	8509	8598	8687	8776	8865	8953	9042	9131	9220	89
489	* 9309	9398	9486	9575	9664	9753	9841	9930	*019	0107	89
490	69 0196	0285	0373	0462	0550	0639	0728	0816	0905	0993	89
491	1081	1170	1258	1347	1435	1524	1612	1700	1789	1877	88
492	1965	2053	2142	2230	2318	2406	2494	2583	2671	2759	88
493	2847	2935	3023	3111	3199	3287	3375	3463	3551	3639	88
494	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	88
495	4605	4693	4781	4868	4956	5044	5131	5219	5307	5394	88
496	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	87
497	6356	6444	6531	6618	6706	6793	6880	6968	7055	7142	87
498	7229	7317	7404	7491	7578	7665	7752	7839	7926	8014	87
499	8101	8188	8275	8362	8449	8535	8622	8709	8796	8883	87
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501	* 9838	9924	*011	0098	0184	0271	0358	0444	0531	0617	87
502	70 0704	0790	0877	0963	1050	1136	1222	1309	1395	1482	86
503	1568	1654	1741	1827	1913	1999	2086	2172	2258	2344	86
504	2431	2517	2603	2689	2775	2861	2947	3033	3119	3205	86
505	3291	3377	3463	3549	3635	3721	3807	3895	3979	4065	86
506	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	86
507	5008	5094	5179	5265	5350	5436	5522	5607	5693	5778	86
508	5864	5949	6035	6120	6206	6291	6376	6462	6547	6632	85
509	6718	6803	6888	6974	7059	7144	7229	7315	7400	7485	85
510	7570	7655	7740	7826	7911	7996	8081	8166	8251	8336	85
511	8421	8506	8591	8676	8761	8846	8931	9015	9100	9185	85
512	* 9270	9355	9440	9524	9609	9694	9779	9863	9948	*033	85
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514	0963	1048	1132	1217	1301	1385	1470	1554	1639	1723	84
515	1807	1892	1976	2060	2144	2229	2313	2397	2481	2566	84
516	2650	2734	2818	2902	2986	3070	3154	3238	3323	3407	84
517	3491	3575	3650	3742	3826	3910	3994	4078	4162	4246	84
518	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	84
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522	7671	7754	7837	7920	8003	8086	8169	8253	8336	8419	83
523	8502	8585	8668	8751	8834	8917	9000	9083	9165	9248	83
524	* 9331	9414	9497	9580	9663	9745	9828	9911	9994	•077	83
525	72 0159	0242	0325	0407	0490	0573	0655	0738	0821	0903	83
526	0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	82
527	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	82
528	2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	82
529	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	82
530	4276	4358	4440	4522	4604	4685	4767	4849	4931	5013	82
531	5095	5176	5258	5340	5422	5503	5585	5667	5748	5830	82
532	5912	5993	6075	6156	6238	6320	6401	6483	6564	6646	82
533	6727	6809	6890	6972	7053	7134	7216	7297	7379	7460	81
534	7541	7623	7704	7785	7866	7948	8029	8110	8191	8273	81
535	8354	8435	8516	8597	8678	8759	8841	8922	9003	9084	81
536	9165	9246	9327	9408	9489	9570	9651	9732	9813	9893	81
537	* 9974	•055	0136	0217	0298	0378	0459	0540	0621	0702	81
538	73 0782	0863	0944	1024	1105	1186	1266	1347	1428	1508	81
539	1589	1669	1750	1830	1911	1991	2072	2152	2233	2313	81
540	2394	2474	2555	2635	2715	2796	2876	2956	3037	3117	80
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546	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	79
547	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	79
548	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	79
549	* 9572	9651	9731	9810	9889	9968	•047	0126	0205	0284	79
550	74 0363	0442	0521	0600	0678	0757	0836	0915	0994	1073	79
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552	1939	2018	2096	2175	2254	2332	2411	2489	2568	2646	79
553	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	78
554	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	78
555	4293	4371	4449	4528	4606	4684	4762	4840	4919	4997	78
556	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
557	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	78
558	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	78
559	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	78
560	8188	8266	8343	8421	8498	8576	8653	8731	8808	8885	77
561	8963	9040	9118	9195	9272	9350	9427	9504	9582	9659	77
562	* 9736	9814	9891	9968	•045	0123	0200	0277	0354	0431	77
563	75 0508	0586	0663	0740	0817	0894	0971	1048	1125	1202	77
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567	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	77
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570	5875	5951	6027	6103	6180	6256	6332	6408	6484	6560	76
571	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
572	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	76
573	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836	76
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584	6413	6487	6562	6636	6710	6785	6859	6933	7007	7082	74
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587	8638	8712	8786	8860	8934	9008	9082	9156	9230	9303	74
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591	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	73
592	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	73
593	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	73
594	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
595	4517	4590	4663	4736	4809	4882	4955	5028	5100	5173	73
596	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	73
597	5974	6047	6120	6193	6265	6338	6411	6483	6556	6629	73
598	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354	73
599	7427	7499	7572	7644	7717	7789	7862	7934	8006	8079	72
600	8151	8224	8296	8368	8441	8513	8585	8658	8730	8802	72
601	8874	8947	9019	9091	9163	9236	9308	9380	9452	9524	72
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603	78 0317	0389	0461	0533	0605	0677	0749	0821	0893	0965	72
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605	1755	1827	1899	1971	2042	2114	2186	2258	2329	2401	72
606	2473	2544	2616	2688	2759	2831	2902	2974	3046	3117	72
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611	6041	6112	6183	6254	6325	6396	6467	6538	6609	6680	71
612	6751	6822	6893	6964	7035	7106	7177	7248	7319	7390	71
613	7460	7531	7602	7673	7744	7815	7885	7956	8027	8098	71
614	8168	8239	8310	8381	8451	8522	8593	8663	8734	8804	71
615	8875	8946	9016	9087	9157	9228	9299	9369	9440	9510	71
616	* 9581	9651	9722	9792	9863	9933	*004	0074	0144	0215	70
617	79 0285	0356	0426	0496	0567	0637	0707	0778	0848	0918	70
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620	2392	2462	2532	2602	2672	2742	2812	2882	2952	3022	70
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624	5185	5254	5324	5393	5463	5532	5602	5672	5741	5811	70
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626	6574	6644	6713	6782	6852	6921	6990	7060	7129	7198	69
627	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	69
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630	9341	9409	9478	9547	9616	9685	9754	9823	9892	9961	69
631	80 0029	0098	0167	0236	0305	0373	0442	0511	0580	0648	69
632	0717	0786	0854	0923	0992	1061	1129	1198	1266	1335	69
633	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	69
634	2089	2158	2226	2295	2363	2432	2500	2568	2637	2705	69
635	2774	2842	2910	2979	3047	3116	3184	3252	3321	3389	68
636	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	68
637	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	68
638	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
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642	7535	7603	7670	7738	7806	7873	7941	8008	8076	8143	68
643	8211	8279	8346	8414	8481	8549	8616	8684	8751	8818	67
644	8886	8953	9021	9088	9156	9223	9290	9358	9425	9492	67
645	* 9560	9627	9694	9762	9829	9896	9964	*031	0098	0165	67
646	81 0233	0300	0367	0434	0501	0569	0636	0703	0770	0837	67
647	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
648	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	67
649	2245	2312	2379	2445	2512	2579	2646	2713	2780	2847	67
650	2913	2980	3047	3114	3181	3247	3314	3381	3448	3514	67
651	3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	67
652	4248	4314	4381	4447	4514	4581	4647	4714	4780	4847	67
653	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	66
654	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175	66
655	6241	6308	6374	6440	6506	6573	6639	6705	6771	6838	66
656	6904	6970	7036	7102	7169	7235	7301	7367	7433	7499	66
657	7565	7631	7698	7764	7830	7896	7962	8028	8094	8160	66
658	8226	8292	8358	8424	8490	8556	8622	8688	8754	8820	66
659	8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	66
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666	3474	3539	3605	3670	3735	3800	3865	3930	3996	4061	65
667	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	65
668	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
669	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	65
670	6075	6140	6204	6269	6334	6399	6464	6528	6593	6658	65
671	6723	6787	6852	6917	6981	7046	7111	7175	7240	7305	65
672	7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	65
673	8015	8080	8144	8209	8273	8338	8402	8467	8531	8595	64
674	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239	64
675	9304	9368	9432	9497	9561	9625	9690	9754	9818	9882	64
676	* 9947	*011	0075	0139	0204	0268	0332	0396	0460	0525	64
677	83 0589	0653	0717	0781	0845	0909	0973	1037	1102	1166	64
678	1230	1294	1358	1422	1486	1550	1614	1678	1742	1806	64
679	1870	1934	1998	2062	2126	2189	2253	2317	2381	2445	64
680	2509	2573	2637	2700	2764	2828	2892	2956	3020	3083	64
681	3147	3211	3275	3338	3402	3466	3530	3593	3657	3721	64
682	3784	3848	3912	3975	4039	4103	4166	4230	4294	4357	64
683	4421	4484	4548	4611	4675	4739	4802	4866	4929	4993	64
684	5056	5120	5183	5247	5310	5373	5437	5500	5564	5627	63
685	5691	5754	5817	5881	5944	6007	6071	6134	6197	6261	63
686	6324	6387	6451	6514	6577	6641	6704	6767	6830	6894	63
687	6957	7020	7083	7146	7210	7273	7336	7399	7462	7525	63
688	7588	7652	7715	7778	7841	7904	7967	8030	8093	8156	63
689	8219	8282	8345	8408	8471	8534	8597	8660	8723	8786	63
690	8849	8912	8975	9038	9101	9164	9227	9289	9352	9415	63
691	* 9478	9541	9604	9667	9729	9792	9855	9918	9981	*043	63
692	84 0106	0169	0232	0294	0357	0420	0482	0545	0608	0671	63
693	0733	0796	0859	0921	0984	1046	1109	1172	1234	1297	63
694	1359	1422	1485	1547	1610	1672	1735	1797	1860	1922	63
695	1985	2047	2110	2172	2235	2297	2360	2422	2484	2547	62
696	2609	2672	2734	2796	2859	2921	2983	3046	3108	3170	62
697	3233	3295	3357	3420	3482	3544	3606	3669	3731	3793	62
698	3855	3918	3980	4042	4104	4166	4229	4291	4353	4415	62
699	4477	4539	4601	4664	4726	4788	4850	4912	4974	5036	62
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701	5718	5780	5842	5904	5966	6028	6090	6151	6213	6275	62
702	6337	6399	6461	6523	6585	6646	6708	6770	6832	6894	62
703	6955	7017	7079	7141	7202	7264	7326	7388	7449	7511	62
704	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	62
705	8189	8251	8312	8374	8435	8497	8559	8620	8682	8743	62
706	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	61
707	9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	61
708	85 0033	0095	0156	0217	0279	0340	0401	0462	0524	0585	61
709	0646	0707	0769	0830	0891	0952	1014	1075	1136	1197	61
710	1258	1320	1381	1442	1503	1564	1625	1686	1747	1809	61
711	1870	1931	1992	2053	2114	2175	2236	2297	2358	2419	61
712	2480	2541	2602	2663	2724	2785	2846	2907	2968	3029	61
713	3090	3150	3211	3272	3333	3394	3455	3516	3577	3637	61
714	3698	3759	3820	3881	3941	4002	4063	4124	4185	4245	61
715	4306	4367	4428	4488	4549	4610	4670	4731	4792	4852	61
716	4913	4974	5034	5095	5156	5216	5277	5337	5398	5459	61
717	5519	5580	5640	5701	5761	5822	5882	5943	6003	6064	61
718	6124	6185	6245	6306	6366	6427	6487	6548	6608	6668	60
719	6729	6789	6850	6910	6970	7031	7091	7152	7212	7272	60
720	7332	7393	7453	7513	7574	7634	7694	7755	7815	7875	60
721	7935	7995	8056	8116	8176	8236	8297	8357	8417	8477	60
722	8537	8597	8657	8718	8778	8838	8898	8958	9018	9078	60
723	9138	9198	9258	9318	9379	9439	9499	9559	9619	9679	60
724	* 9739	9799	9859	9918	9978	*038	0098	0158	0218	0278	60
725	86 0338	0398	0458	0518	0578	0637	0697	0757	0817	0877	60
726	0937	0996	1056	1116	1176	1236	1295	1355	1415	1475	60
727	1534	1594	1654	1714	1773	1833	1893	1952	2012	2072	60
728	2131	2191	2251	2310	2370	2430	2489	2549	2608	2668	60
729	2728	2787	2847	2906	2966	3025	3085	3144	3204	3263	60
730	3323	3382	3442	3501	3561	3620	3680	3739	3799	3858	59
731	3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	59
732	4511	4570	4630	4689	4748	4808	4867	4926	4985	5045	59
733	5104	5163	5222	5282	5341	5400	5459	5519	5578	5637	59
734	5696	5755	5814	5874	5933	5992	6051	6110	6169	6228	59
735	6287	6346	6405	6465	6524	6583	6642	6701	6760	6819	59
736	6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	59
737	7467	7526	7585	7644	7703	7762	7821	7880	7939	7998	59
738	8056	8115	8174	8233	8292	8350	8409	8468	8527	8586	59
739	8644	8703	8762	8821	8879	8938	8997	9056	9114	9173	59
740	9232	9290	9349	9408	9466	9525	9584	9642	9701	9760	59
741	* 9818	9877	9935	9994	*053	0111	0170	0228	0287	0345	59
742	87 0404	0462	0521	0579	0638	0696	0755	0813	0872	0930	58
743	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	58
744	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	58
745	2156	2215	2273	2331	2389	2448	2506	2564	2622	2681	58
746	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	58
747	3321	3379	3437	3495	3553	3611	3669	3727	3785	3844	58
748	3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
749	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	58
750	5061	5119	5177	5235	5293	5351	5409	5466	5524	5582	58
751	5640	5698	5756	5813	5871	5929	5987	6045	6102	6160	58
752	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	58
753	6795	6853	6910	6968	7026	7083	7141	7199	7256	7314	58
754	7371	7429	7487	7544	7602	7659	7717	7774	7832	7889	58
755	7947	8004	8062	8119	8177	8234	8292	8349	8407	8464	57
756	8522	8579	8637	8694	8752	8809	8866	8924	8981	9039	57
757	9096	9153	9211	9268	9325	9383	9440	9497	9555	9612	57
758	* 9669	9726	9784	9841	9898	9956	*013	0070	0127	0185	57
759	88 0242	0299	0356	0413	0471	0528	0585	0642	0699	0756	57
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762	1955	2012	2069	2126	2183	2240	2297	2354	2411	2468	57
763	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	57
764	3093	3150	3207	3264	3321	3377	3434	3491	3548	3605	57
765	3661	3718	3775	3832	3888	3945	4002	4059	4115	4172	57
766	4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	57
767	4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	57
768	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	57
769	5926	5983	6039	6096	6152	6209	6265	6321	6378	6434	56
770	6491	6547	6604	6660	6716	6773	6829	6885	6942	6998	56
771	7054	7111	7167	7223	7280	7336	7392	7449	7505	7561	56
772	7617	7674	7730	7786	7842	7898	7955	8011	8067	8123	56
773	8179	8236	8292	8348	8404	8460	8516	8573	8629	8685	56
774	8741	8797	8853	8909	8965	9021	9077	9134	9190	9246	56
775	9302	9358	9414	9470	9526	9582	9638	9694	9750	9806	56
776	* 9862	9918	9974	♦030	0086	0141	0197	0253	0309	0365	56
777	89 0421	0477	0533	0589	0645	0700	0756	0812	0868	0924	56
778	0980	1035	1091	1147	1203	1259	1314	1370	1426	1482	56
779	1537	1593	1649	1705	1760	1816	1872	1928	1983	2039	56
780	2095	2150	2206	2262	2317	2373	2429	2484	2540	2595	56
781	2651	2707	2762	2818	2873	2929	2985	3040	3096	3151	56
782	3207	3262	3318	3373	3429	3484	3540	3595	3651	3706	56
783	3762	3817	3873	3928	3984	4039	4094	4150	4205	4261	55
784	4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	55
785	4870	4925	4980	5036	5091	5146	5201	5257	5312	5367	55
786	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	55
787	5975	6030	6085	6140	6195	6251	6306	6361	6416	6471	55
788	6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	55
789	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	55
790	7627	7682	7737	7792	7847	7902	7957	8012	8067	8122	55
791	8176	8231	8286	8341	8396	8451	8506	8561	8615	8670	55
792	8725	8780	8835	8890	8944	8999	9054	9109	9164	9218	55
793	9273	9328	9383	9437	9492	9547	9602	9656	9711	9766	55
794	* 9821	9875	9930	9985	♦039	0094	0149	0203	0258	0312	55
795	90 0367	0422	0476	0531	0586	0640	0695	0749	0804	0859	55
796	0913	0968	1022	1077	1131	1186	1240	1295	1349	1404	55
797	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	54
798	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	54
799	2547	2601	2655	2710	2764	2818	2873	2927	2981	3036	54
800	3090	3144	3199	3253	3307	3361	3416	3470	3524	3578	54
801	3633	3687	3741	3795	3849	3904	3958	4012	4066	4120	54
802	4174	4229	4283	4337	4391	4445	4499	4553	4607	4661	54
803	4716	4770	4824	4878	4932	4986	5040	5094	5148	5202	54
804	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742	54
805	5796	5850	5904	5958	6012	6066	6119	6173	6227	6281	54
806	6335	6389	6443	6497	6551	6604	6658	6712	6766	6820	54
807	6874	6927	6981	7035	7089	7143	7196	7250	7304	7358	54
808	7411	7465	7519	7573	7626	7680	7734	7787	7841	7895	54
809	7949	8002	8056	8110	8163	8217	8270	8324	8378	8431	54
810	8485	8539	8592	8646	8699	8753	8807	8860	8914	8967	54
811	9021	9074	9128	9181	9235	9289	9342	9396	9449	9503	54
812	* 9556	9610	9663	9716	9770	9823	9877	9930	9984	♦037	53
813	91 0091	0144	0197	0251	0304	0358	0411	0464	0518	0571	53
814	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	53
815	1158	1211	1264	1317	1371	1424	1477	1530	1584	1637	53
816	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169	53
817	2222	2275	2328	2381	2435	2488	2541	2594	2647	2700	53
818	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	53
819	3284	3337	3390	3443	3496	3549	3602	3655	3708	3761	53

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821	4343	4396	4449	4502	4555	4608	4660	4713	4766	4819	53
822	4872	4925	4977	5030	5083	5136	5189	5241	5294	5347	53
823	5400	5453	5505	5558	5611	5664	5716	5769	5822	5875	53
824	5927	5980	6033	6085	6138	6191	6243	6296	6349	6401	53
825	6454	6507	6559	6612	6664	6717	6770	6822	6875	6927	53
826	6980	7033	7085	7138	7190	7243	7295	7348	7400	7453	53
827	7506	7558	7611	7663	7716	7768	7820	7873	7925	7978	52
828	8030	8083	8135	8188	8240	8293	8345	8397	8450	8502	52
829	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	52
830	9078	9130	9183	9235	9287	9340	9392	9444	9496	9549	52
831	* 9601	9653	9706	9758	9810	9862	9914	9967	*019	0071	52
832	92 0123	0176	0228	0280	0332	0384	0436	0489	0541	0593	52
833	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	52
834	1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	52
835	1686	1738	1790	1842	1894	1946	1998	2050	2102	2154	52
836	2206	2258	2310	2362	2414	2466	2518	2570	2622	2674	52
837	2725	2777	2829	2881	2933	2985	3037	3089	3140	3192	52
838	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	52
839	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	52
840	4279	4331	4383	4434	4486	4538	4589	4641	4693	4744	52
841	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	52
842	5312	5364	5415	5467	5518	5570	5621	5673	5725	5776	52
843	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	51
844	6342	6394	6445	6497	6548	6600	6651	6702	6754	6805	51
845	6857	6908	6959	7011	7062	7114	7165	7216	7268	7319	51
846	7370	7422	7473	7524	7576	7627	7678	7730	7781	7832	51
847	7883	7935	7986	8037	8088	8140	8191	8242	8293	8345	51
848	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	51
849	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	51
850	9419	9470	9521	9572	9623	9674	9725	9776	9827	9879	51
851	* 9930	9981	*032	0083	0134	0185	0236	0287	0338	0389	51
852	93 0440	0491	0542	0592	0643	0694	0745	0796	0847	0898	51
853	0949	1000	1051	1102	1153	1204	1254	1305	1356	1407	51
854	1458	1509	1560	1610	1661	1712	1763	1814	1865	1915	51
855	1966	2017	2068	2118	2169	2220	2271	2322	2372	2423	51
856	2474	2524	2575	2626	2677	2727	2778	2829	2879	2930	51
857	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	51
858	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943	51
859	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	51
860	4498	4549	4599	4650	4700	4751	4801	4852	4902	4953	50
861	5003	5054	5104	5154	5205	5255	5306	5356	5406	5457	50
862	5507	5558	5608	5658	5709	5759	5809	5860	5910	5960	50
863	6011	6061	6111	6162	6212	6262	6313	6363	6413	6463	50
864	6514	6564	6614	6665	6715	6765	6815	6865	6916	6966	50
865	7016	7066	7117	7167	7217	7267	7317	7367	7418	7468	50
866	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	50
867	8019	8069	8119	8169	8219	8269	8320	8370	8420	8470	50
868	8520	8570	8620	8670	8720	8770	8820	8870	8920	8970	50
869	9020	9070	9120	9170	9220	9270	9320	9369	9419	9469	50
870	9519	9569	9619	9669	9719	9769	9819	9869	9918	9968	50
871	94 0018	0068	0118	0168	0218	0267	0317	0367	0417	0467	50
872	0516	0566	0616	0666	0716	0765	0815	0865	0915	0964	50
873	1014	1064	1114	1163	1213	1263	1313	1362	1412	1462	50
874	1511	1561	1611	1660	1710	1760	1809	1859	1909	1958	50
875	2008	2058	2107	2157	2207	2256	2306	2355	2405	2455	50
876	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950	50
877	3000	3049	3099	3148	3198	3247	3297	3346	3396	3445	49
878	3495	3544	3593	3643	3692	3742	3791	3841	3890	3939	49
879	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	49
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881	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419	49
882	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	49
883	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	49
884	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	49
885	6943	6992	7041	7090	7140	7189	7238	7287	7336	7385	49
886	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
887	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	49
888	8413	8462	8511	8560	8609	8657	8706	8755	8804	8853	49
889	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	49
890	9390	9439	9488	9536	9585	9634	9683	9731	9780	9829	49
891	* 9878	9926	9975	*024	0073	0121	0170	0219	0267	0316	49
892	95 0365	0414	0462	0511	0560	0608	0657	0706	0754	0803	49
893	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	49
894	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	49
895	1823	1872	1920	1969	2017	2066	2114	2163	2211	2260	48
896	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744	48
897	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	48
898	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711	48
899	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194	48
900	4243	4291	4339	4387	4435	4484	4532	4580	4628	4677	48
901	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158	48
902	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640	48
903	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	48
904	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	48
905	6649	6697	6745	6793	6840	6888	6936	6984	7032	7080	48
906	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	48
907	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	48
908	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	48
909	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994	48
910	9041	9089	9137	9185	9232	9280	9328	9375	9423	9471	48
911	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947	48
912	* 9995	*042	0090	0138	0185	0233	0280	0328	0376	0423	48
913	96 0471	0518	0566	0613	0661	0709	0756	0804	0851	0899	48
914	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374	47
915	1421	1469	1516	1563	1611	1658	1706	1753	1801	1848	47
916	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322	47
917	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	47
918	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268	47
919	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741	47
920	3788	3835	3882	3929	3977	4024	4071	4118	4165	4212	47
921	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684	47
922	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	47
923	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	47
924	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095	47
925	6142	6189	6236	6283	6329	6376	6423	6470	6517	6564	47
926	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	47
927	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	47
928	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	47
929	8016	8062	8109	8156	8203	8249	8296	8343	8390	8436	47
930	8483	8530	8576	8623	8670	8716	8763	8810	8856	8903	47
931	8950	8996	9043	9090	9136	9183	9229	9276	9323	9369	47
932	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835	47
933	* 9882	9928	9975	*021	0068	0114	0161	0207	0254	0300	47
934	97 0347	0393	0440	0486	0533	0579	0626	0672	0719	0765	46
935	0812	0858	0904	0951	0997	1044	1090	1137	1183	1229	46
936	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	46
937	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	46
938	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	46
939	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	46
N.	0	1	2	3	4	5	6	7	8	9	D.

LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	D.
940	97 3128	3174	3220	3266	3313	3359	3405	3451	3497	3543	46
941	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005	46
942	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	46
943	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	46
944	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46
945	5432	5478	5524	5570	5616	5662	5707	5753	5799	5845	46
946	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	46
947	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	46
948	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	46
949	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	46
950	7724	7769	7815	7861	7906	7952	7998	8043	8089	8135	46
951	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	46
952	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	46
953	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	46
954	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	46
955	98 0003	0049	0094	0140	0185	0231	0276	0322	0367	0412	45
956	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	45
957	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	45
958	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	45
959	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226	45
960	2271	2316	2362	2407	2452	2497	2543	2588	2633	2678	45
961	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	45
962	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	45
963	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	45
964	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	45
965	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	45
966	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	45
967	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	45
968	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	45
969	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	45
970	6772	6817	6861	6906	6951	6996	7040	7085	7130	7175	45
971	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	45
972	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	45
973	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	45
974	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	45
975	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405	45
976	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	44
977	* 9895	9939	9983	*028	0072	0117	0161	0206	0250	0294	44
978	99 0339	0383	0428	0472	0516	0561	0605	0650	0694	0738	44
979	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182	44
980	1226	1270	1315	1359	1403	1448	1492	1536	1580	1625	44
981	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	44
982	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	44
983	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	44
984	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	44
985	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833	44
986	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	44
987	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
988	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	44
989	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	44
990	5635	5679	5723	5767	5811	5854	5898	5942	5986	6030	44
991	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
992	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	44
993	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	44
994	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	44
995	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	44
996	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	44
997	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	44
998	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	44
999	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43
N.	0	1	2	3	4	5	6	7	8	9	D.

The Application of Logarithms.—The logarithm of a number is set down as a decimal, and addition of ciphers to numbers does not change the logarithm; it is the same for 11, 110, 1100, but the value of the number is established by figures to the left of the decimal point; thus, if the number is among the units, the characteristic is 0; if in the tens, 1; in the hundreds, 2; thousands, 3; tens of thousands, 4, and so on; if the number is a decimal fraction and the first figure a tenth, the characteristic is $\bar{1}$, if hundredths $\bar{2}$, thousandths $\bar{3}$.

Multiplication of two numbers is performed by the addition of their logarithms and characteristics, and finding the number corresponding to their sum; thus, to multiply 119 by 2760.

Characteristic of 119	2, logarithm.		2·075547
" 2760	3, "		3·440909
			5·516456
	3284		403
	401	D =	132)53(401
	328440·1		528
			200
			132
			68

As the characteristic is 5, the result is 6 figures of whole numbers.

Division is performed by subtracting the logarithm of the divisor from that of the dividend, and finding the logarithm of the remainder for the quotient. But if the divisor is the larger, then the characteristic of the remainder is —.

Thus, to divide 500 by 63008.

Logarithm of 500		2·698970
Logarithm of 63000 = 4·799341		
D = 69	$\frac{8 \times 69}{10} =$ 55·2	
Logarithm of 63008		4.799396
Corresponding number ·007935 =		$\bar{3}$·899574

Numbers are raised to any power by multiplying their logarithm by the exponents, and roots are extracted by dividing the logarithm. Thus, to get the square of any number, its logarithm is multiplied by 2, for the cube by 3, for the 4th power by 4; in like manner, to obtain the square root of the number, divide the logarithm by 2; by 3 for $\sqrt[3]{}$; by 4 for $\sqrt[4]{}$.

The roots of numbers are better expressed by fractional exponents, thus: $\sqrt{a}$ by $a^{\frac{1}{2}}$, $\sqrt[3]{a}$ by $a^{\frac{1}{3}}$.

The raising of numbers to different powers is extremely simple, by logarithms, when the numbers are whole numbers, but becomes somewhat more complicated when the numbers are decimals.

Thus, to find the 4th power of ·07.

Logarithm ·07		$\bar{2}$·845098
		4
	$\bar{8}$	3·380392
Number ·00002401		$\bar{5}$·380392

To extract the 4th root of ·07—

Logarithm ·07	$\bar{2}$·845098
Add $\bar{2}$ to the characteristic to make it divisible by 4, and a positive 2 to the logarithm to balance it.	$\bar{2}$·2·845098
	4)$\bar{4}$·2·845098
Number ·5143	$\bar{1}$· 711274

The exponent of a root is often a decimal; thus the $\sqrt[4]{\cdot 07}$ may be expressed by $\cdot 07^{\cdot 25}$.

Logarithm ·07	$\bar{2}$·845098
	·25
	4225490
	1690196
	·$\bar{5}$·21127450
	·$\bar{5}$·5
Number ·5143	$\bar{1}$·71127450

Note.—In this example, ·$\bar{5}$ is added to the resultant characteristic to bring it to an integer, and an equal positive amount to the logarithm to balance it.

The same logarithm as by dividing by 4· and corresponding to the number ·5143. The rule is to consider the logarithm as a plus quantity, and multiply by the exponent and the characteristic as minus, and, after similar multiplication, subtract it from the first product. When a characteristic has a minus sign ($\bar{3}$), and it is to be subtracted, the sign is changed and added.

Thus, to divide 10· by $\frac{1}{10}$.

Logarithm 10·	1·00000
$\frac{1}{10}$	$\bar{1}$
Logarithm of 100·	2·000
To divide $\frac{1}{10}$ Logarithm	$\bar{1}$·00000
by $\frac{1}{100}$	$\bar{2}$·
Logarithm of 10·	1·0000
To divide $\frac{1}{1000}$ Logarithm	$\bar{3}$·00000
by 100	2
Logarithm of ·00001	$\bar{5}$

Table of Reciprocals, pages 828 and 829.

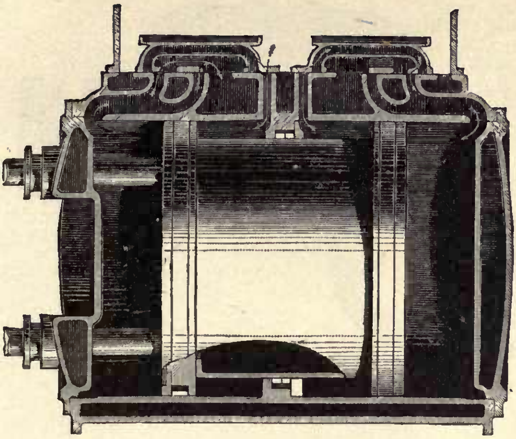
Use of Reciprocals.—Reciprocals may be conveniently used to facilitate computations in long divisions. Instead of dividing as usual, multiply the dividend by the reciprocal of the divisor. The method is especially useful when many different dividends are required to be divided by the same divisor. In this case find the reciprocal of the divisor, make a small table of its multiples up to nine times, and use this as a multiplication table, instead of actually performing the multiplication in each case.

SCRAPS.

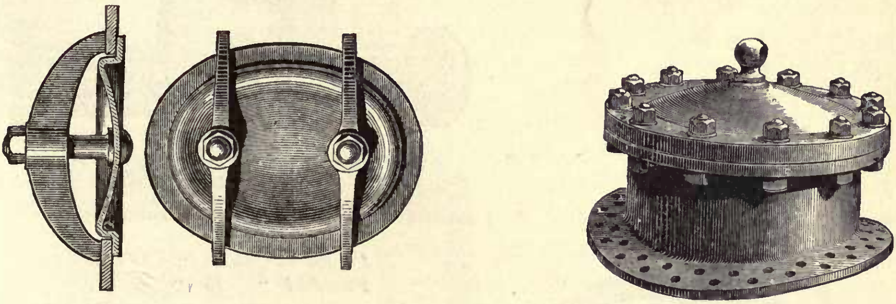
It is good practice to collect, from the circulars of manufacturers, and from illustrated newspapers and magazines, varied illustrations of tools and machines, engineering structures, buildings, etc., and arrange them under their appropriate heads in scrap-books. They will be found very useful in designing, not only enabling one the more readily to make drawings, but to convey to the draughtsman the character and proportions of the design which is to be made. And those parts which are of common use and purchasable in the market can be readily arranged in position and executed more economically than from a new design. There is a saving in the matter of drawing, and also in the cost of construction.

A proper combination and arrangement of parts which have served a purpose will afford material for a more practical and satisfactory design than can usually be made from attempts at originality. Knowledge of what has been done is economy in all labour. If the construction or machine can not be seen, its picture can supply its place, and its details can be studied at leisure; and as the education of the eye is of essential importance to the draughtsman, let him see as much as he can practically, and at the same time acquire a good collection of scraps from which to design. There are few constructions from which something of education can not be drawn, parts if not a whole.

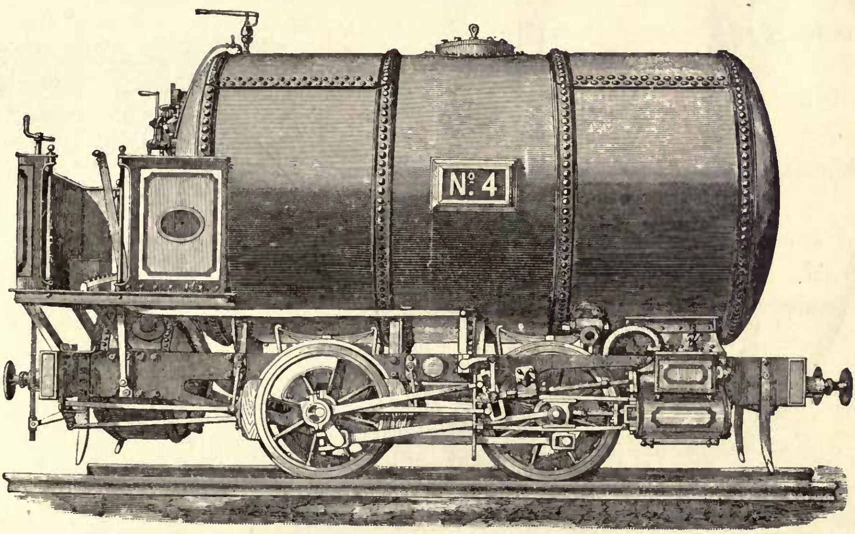
In this view a small collection of scraps has been made pertinent to the book. Its page does not admit of the sizes which will be found in the illustrated papers and magazines—the quarto will be found much more generally useful—and a library of such scrap-books will furnish material for a draughtsman which can not be found in any encyclopædia.



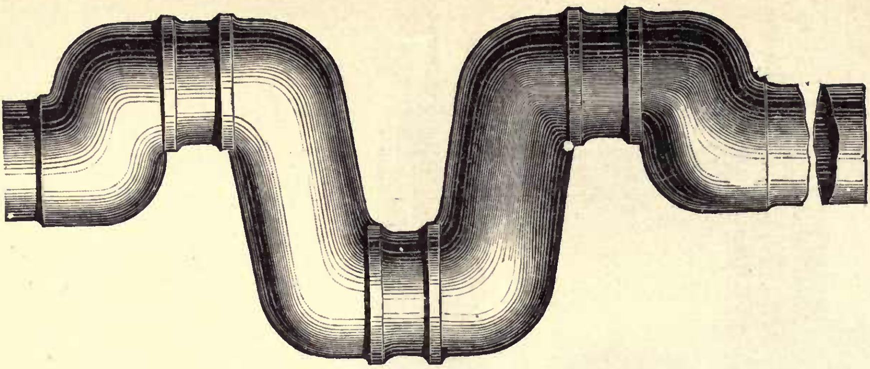
Compound Steam Cylinders. H. M. S. Spartan.



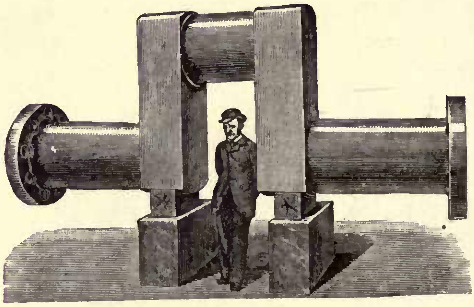
Wrought-Iron Plates and Covers.



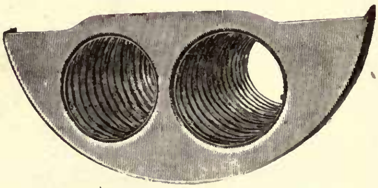
Compressed-Air Locomotive, St. Gothard Tunnel.



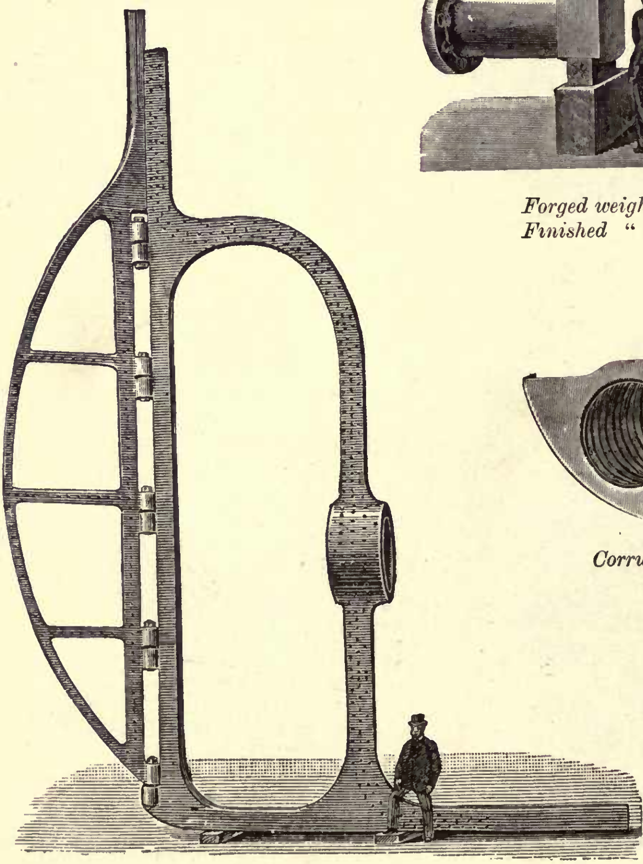
Three-Throw Crank.



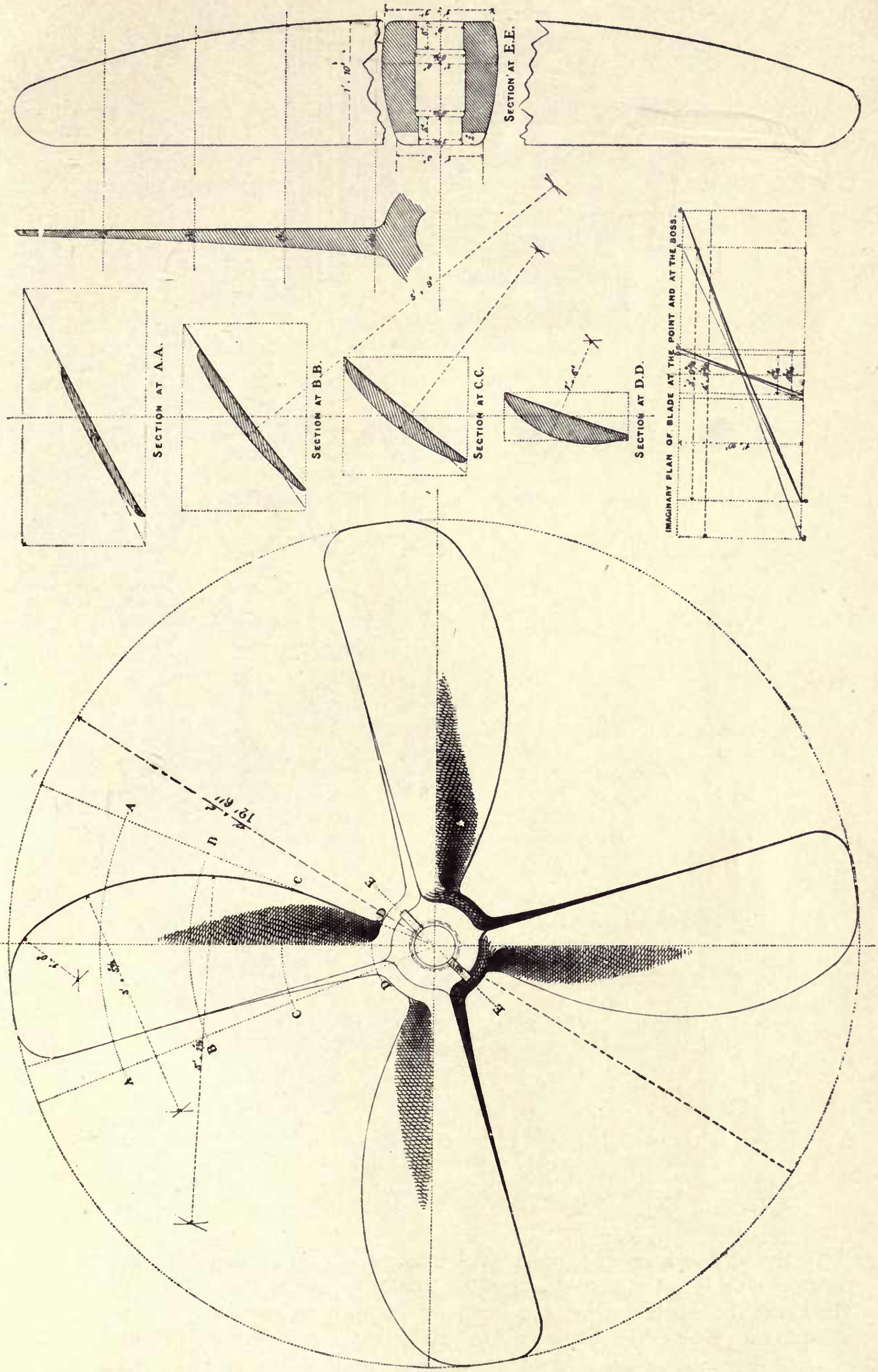
Forged weight, 24 tons 11 cwt.
Finished " 15 " 8 "



Corrugated Boiler-Flues.

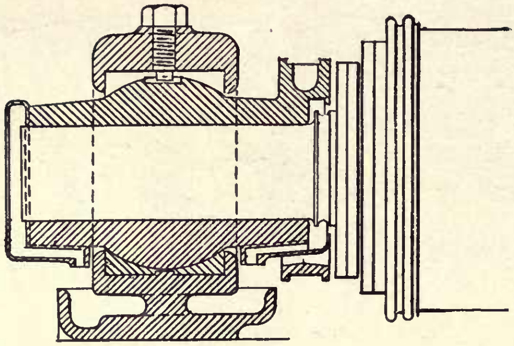


Weight, 25 tons 10 cwt.

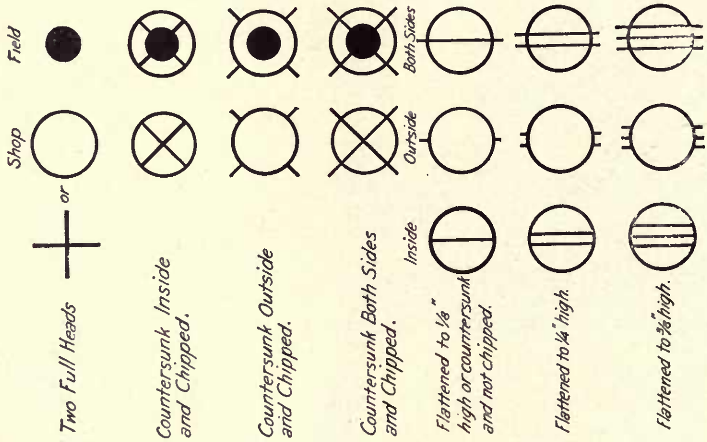


Screw Propeller.

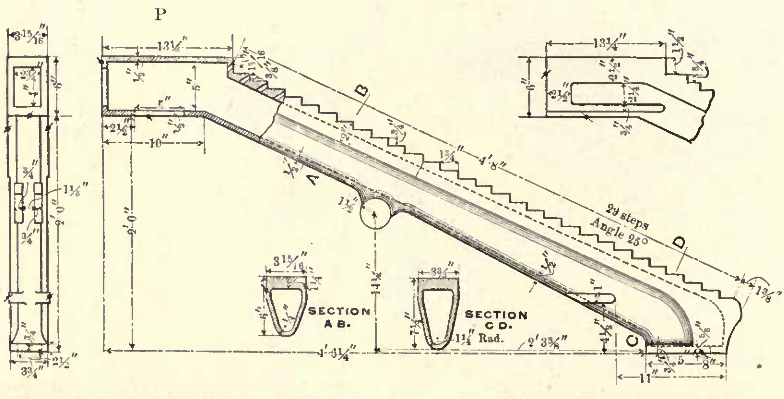
Vessel, 1400 gross tons. Engines, 130 nominal English horse-power



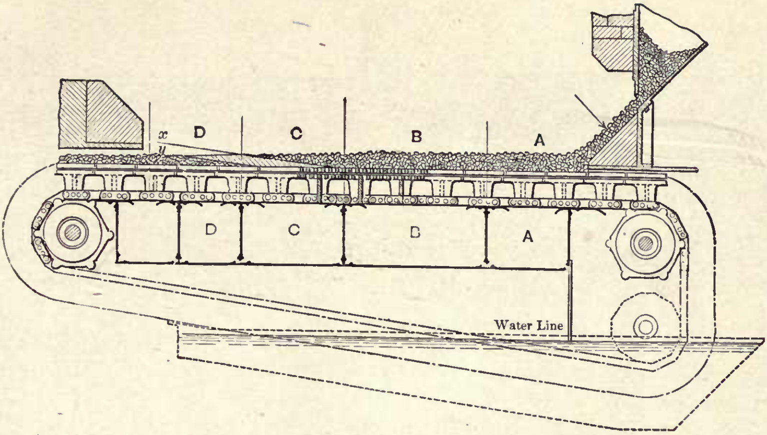
Spherical Bearing.



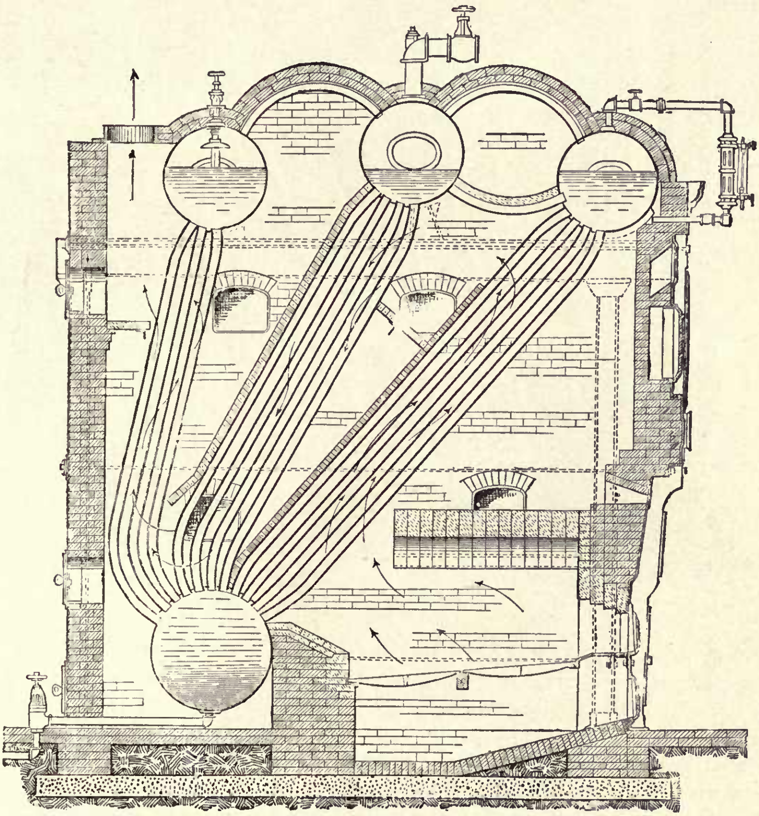
Conventional Signs of Riveting.



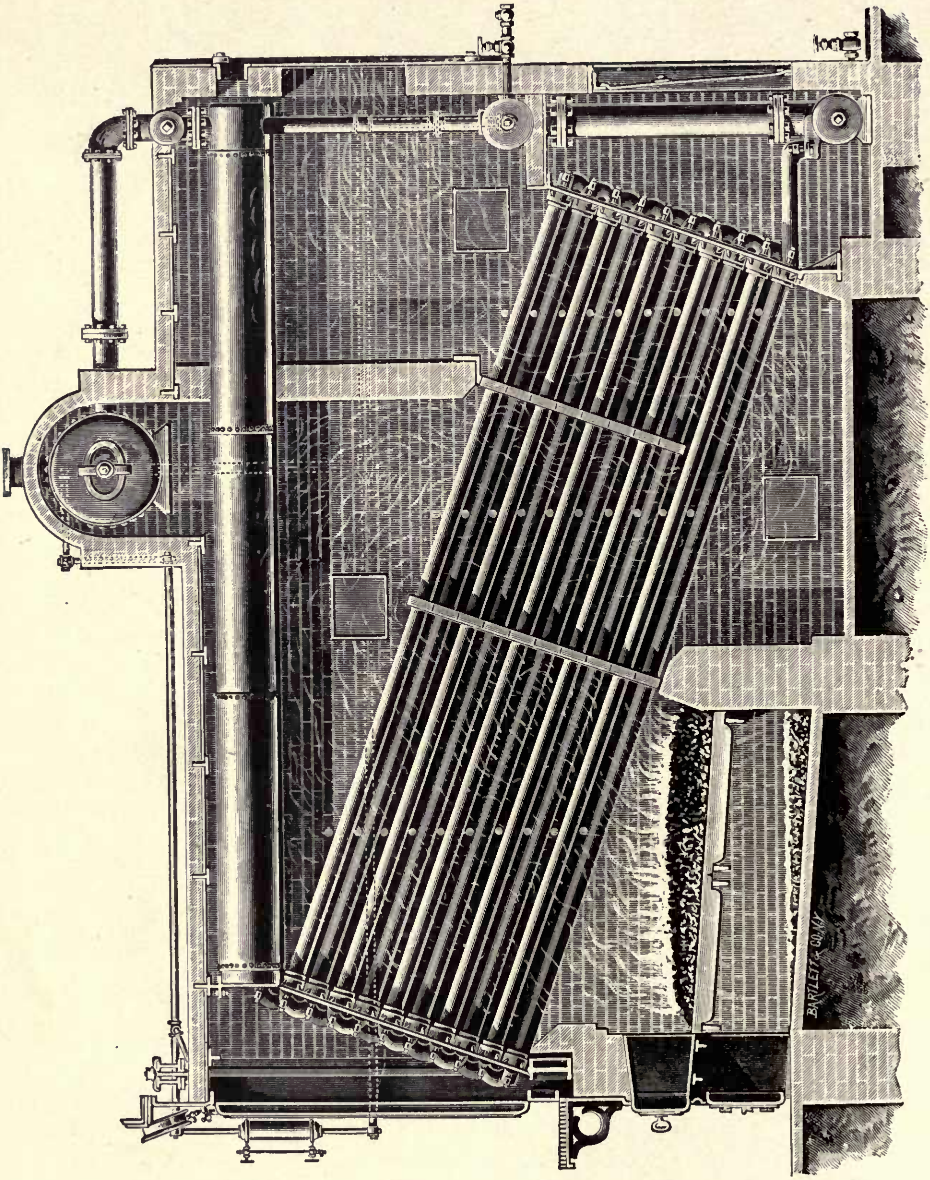
In the *Wilkinson stoker* the coal is fed mechanically through an inclined pipe on to the dead plate P and slides down upon the bars, which are hollow and set at an angle. The top of the bars is stepped, and *tuyère*-shaped openings about $\frac{1}{4} \times 3$ inches are provided in each riser. The bars are carried at their ends on hollow boxes, and are 4-inch centres. For the feeding, adjacent bars move in opposite directions by a system of toggles driven by the stoker engine.



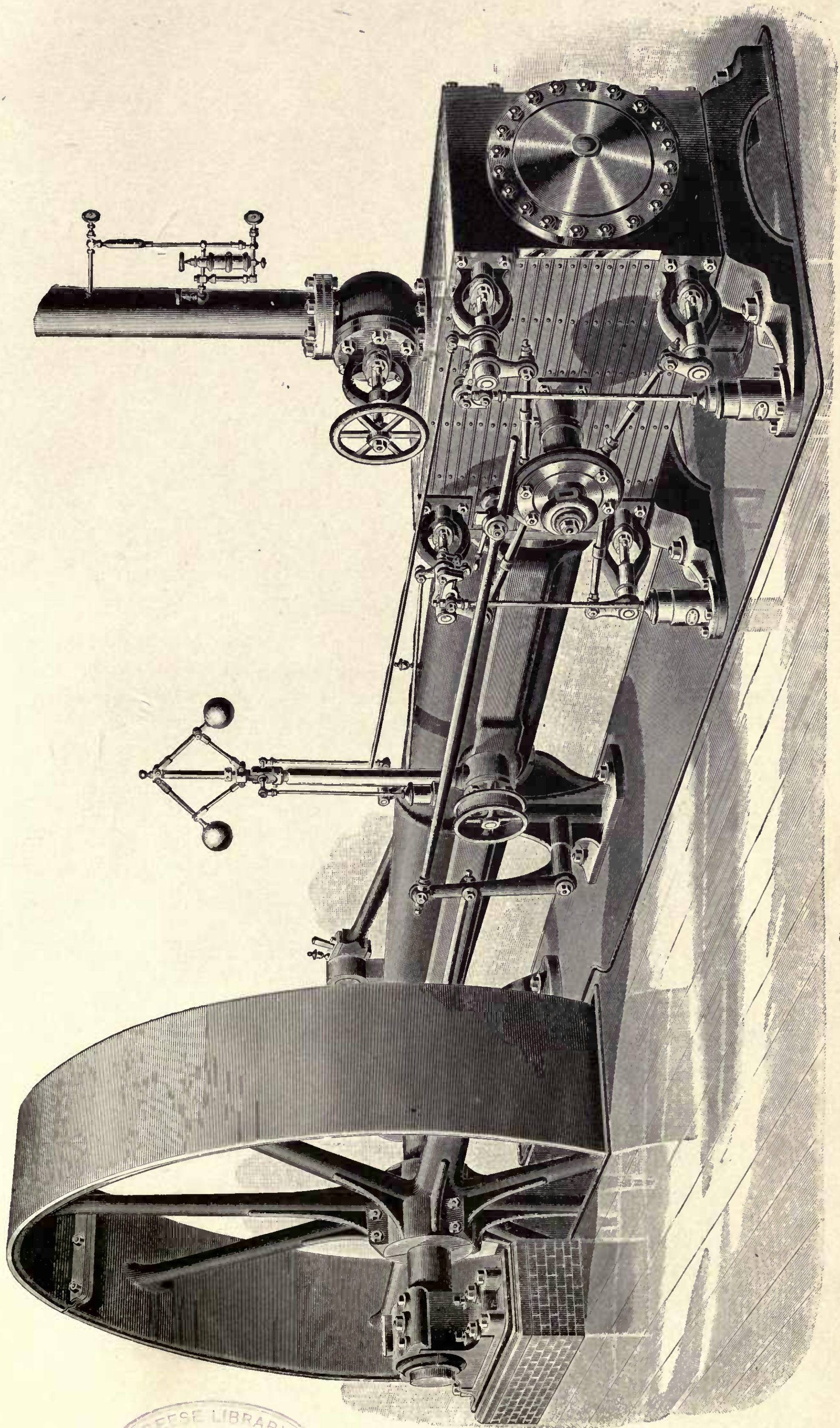
The *Coxe stoker* consists of a travelling grate with fire on its upper surface. The coal is fed by the motion of the grate at the front. There are four blast compartments, A, B, C, D, under the fire connected by dampers. The sides of the furnace are protected by wrought-iron water backs, through which the water circulates under slight pressure.



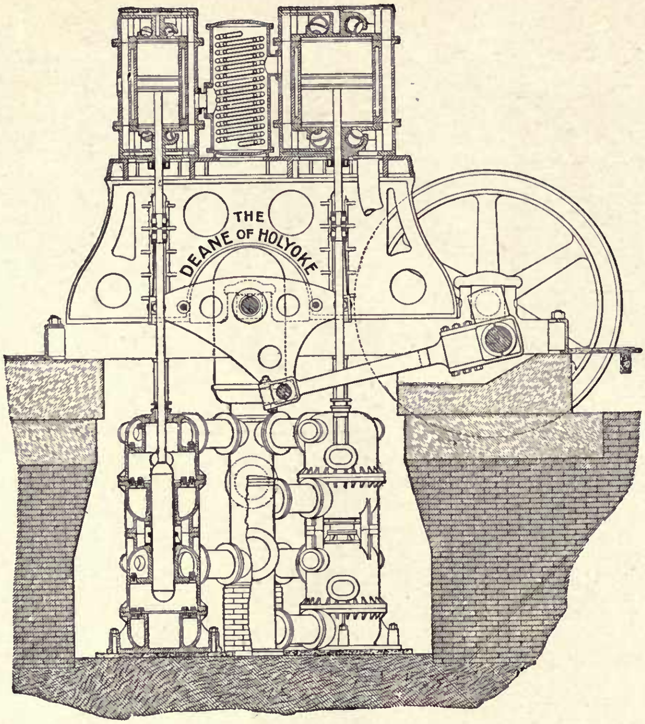
The Stirling Boiler consists of three upper steam-drums and a lower mud-drum. All the steam-drums are connected at the top, but the front and middle drums only are connected in the water space. Tubes, $3\frac{1}{4}''$ diameter; movement of flame shown by arrows around baffle plates.



The Abendroth and Root Boiler was the earliest of its type on the market, and has been modified in its details since its introduction to meet necessities which were developed by use. The section gives the latest form. The angles of the tubes, with the horizontal and the baffle plates beneath and around the tubes making a positive circulation of the flame, are of the original design, but longitudinal drums extend lengthwise over the tubes with a water circulation in the lower half toward the rear and downward by vertical pipes to the lower ends of the tubes. On these vertical pipes there are two cross-drums—the intermediate one to receive the feedwater, the lower for a mud-drum. The long drums connect with the steam-drum placed above and crosswise of them.

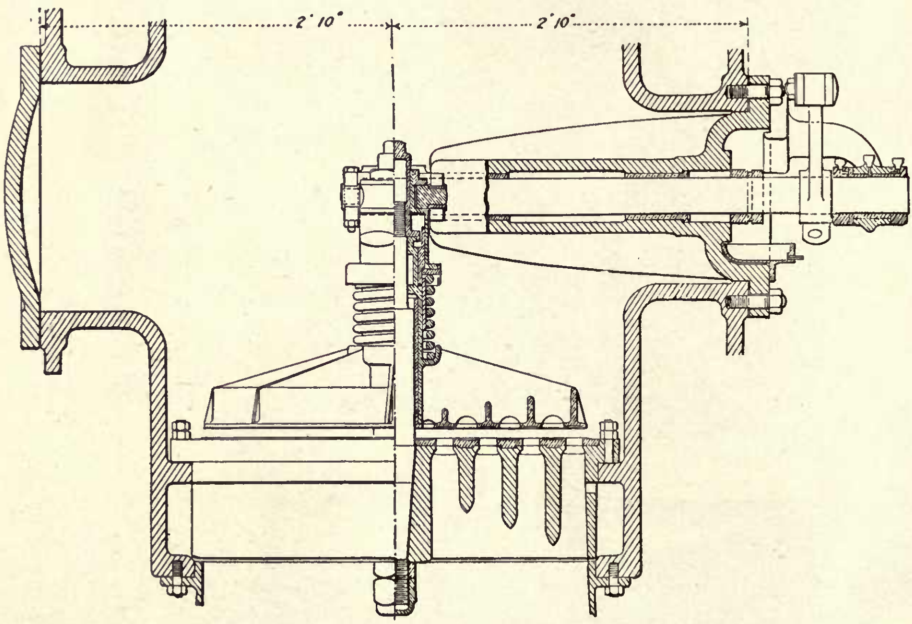


Corliss Steam Engine of the Fishkill Landing Machine Co.

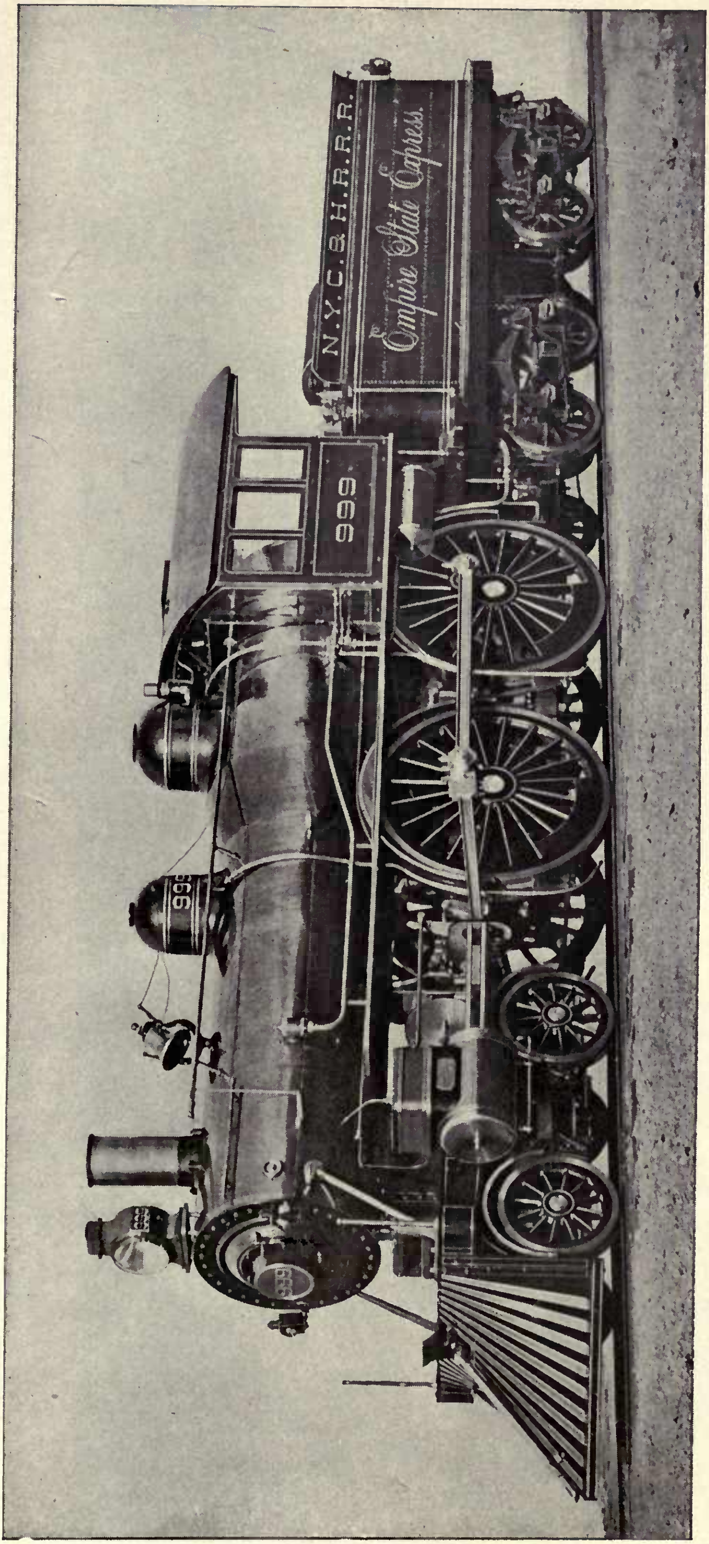


Andover, Mass., Steam Pumping Plant, by the Deane Steam Pump Co., Holyoke, Mass.

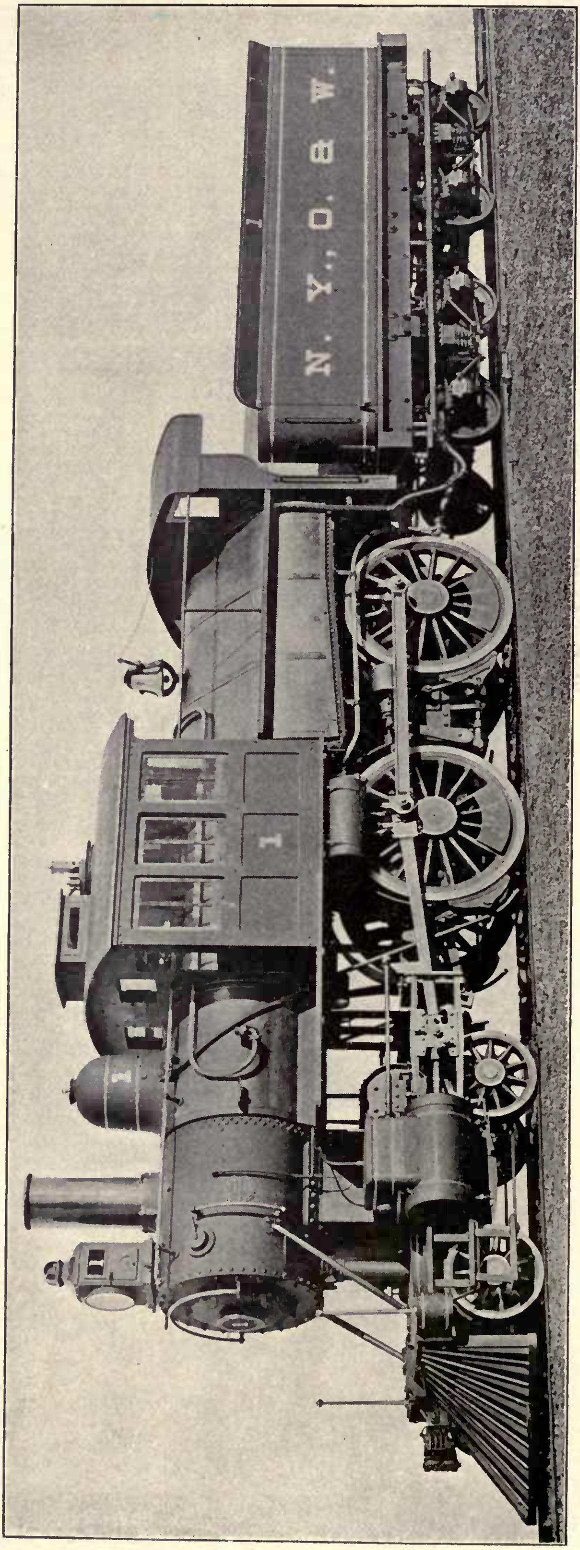
Diameter H. P. cylinder, 17″; L. P., 30″. Pump plunger, 8¾″. Stroke, 30″. Capacity at 205′ plunger speed, 1,205 gals. per minute. Average observed steam pressure, 90·3; water, 139·79; in test trial, pump exceeded contract duty of 125,000,000.



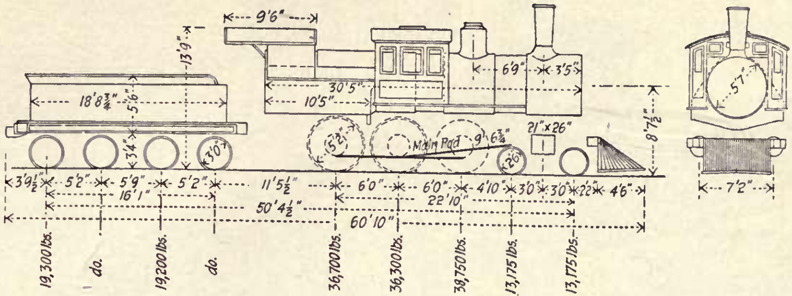
Reidler Valve, from Leavitt's Thames-Ditton Pump. (See page 365.)



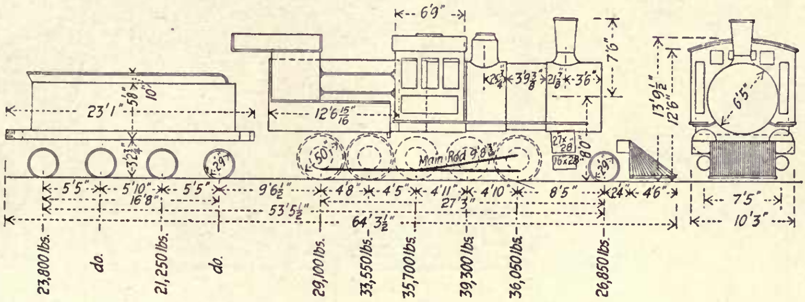
From the "Railroad Gazette."



From the "Engineering News."

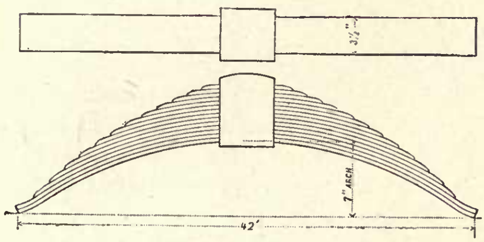


Class "R".

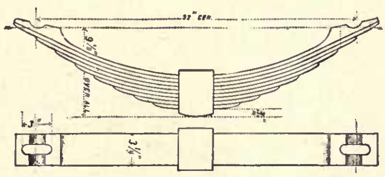


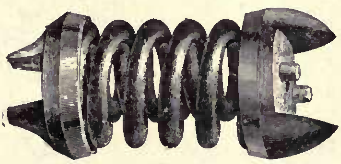
Class "S".

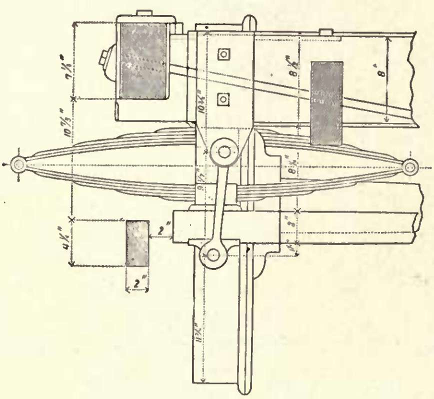
From the "Engineering News."

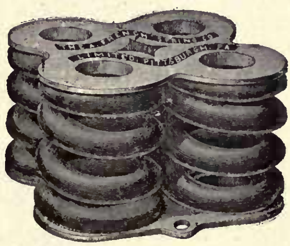


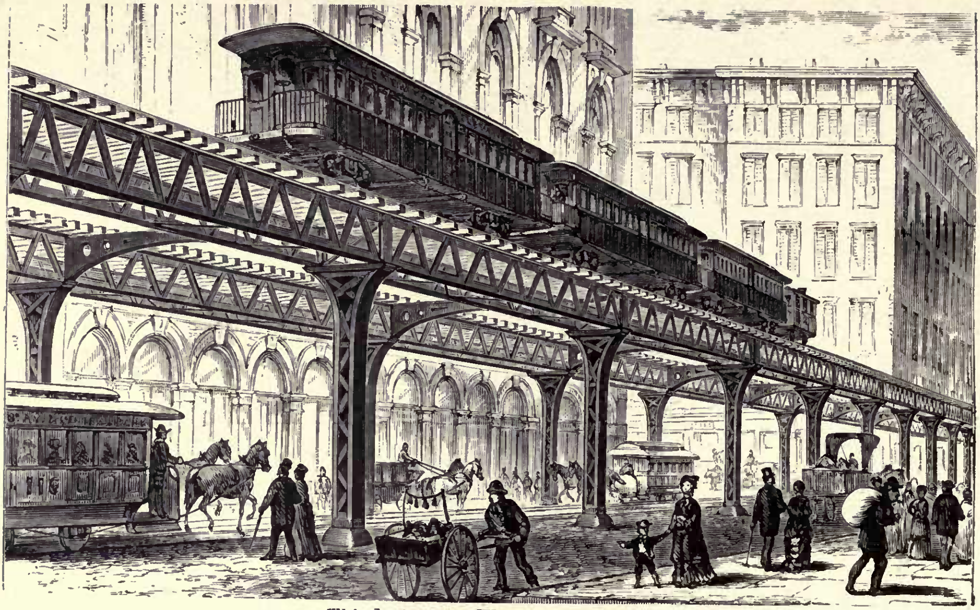
Tender Spring.


Driving Spring.

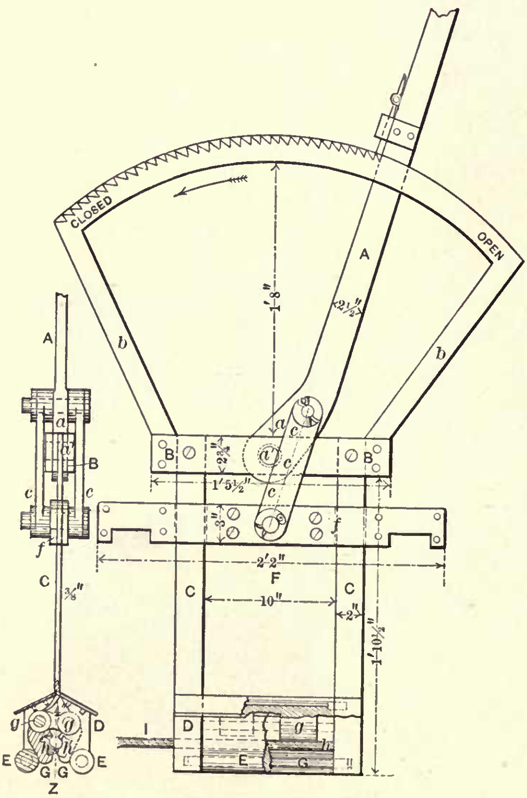

Equalizing Bar Spring.


Elliptic Spring applied to Car Truck.


Bolster Spring.



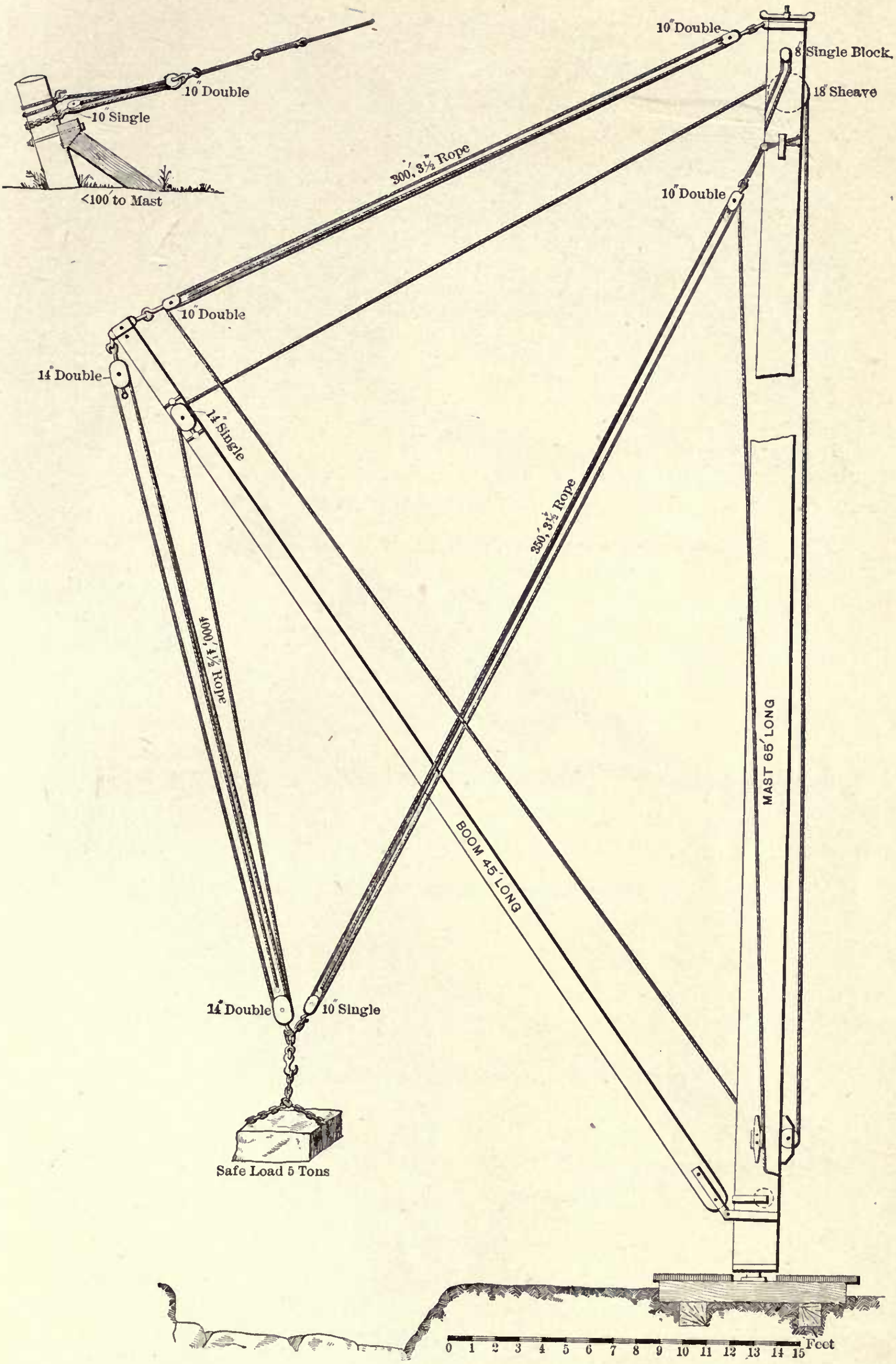
Third Avenue Elevated Railroad.



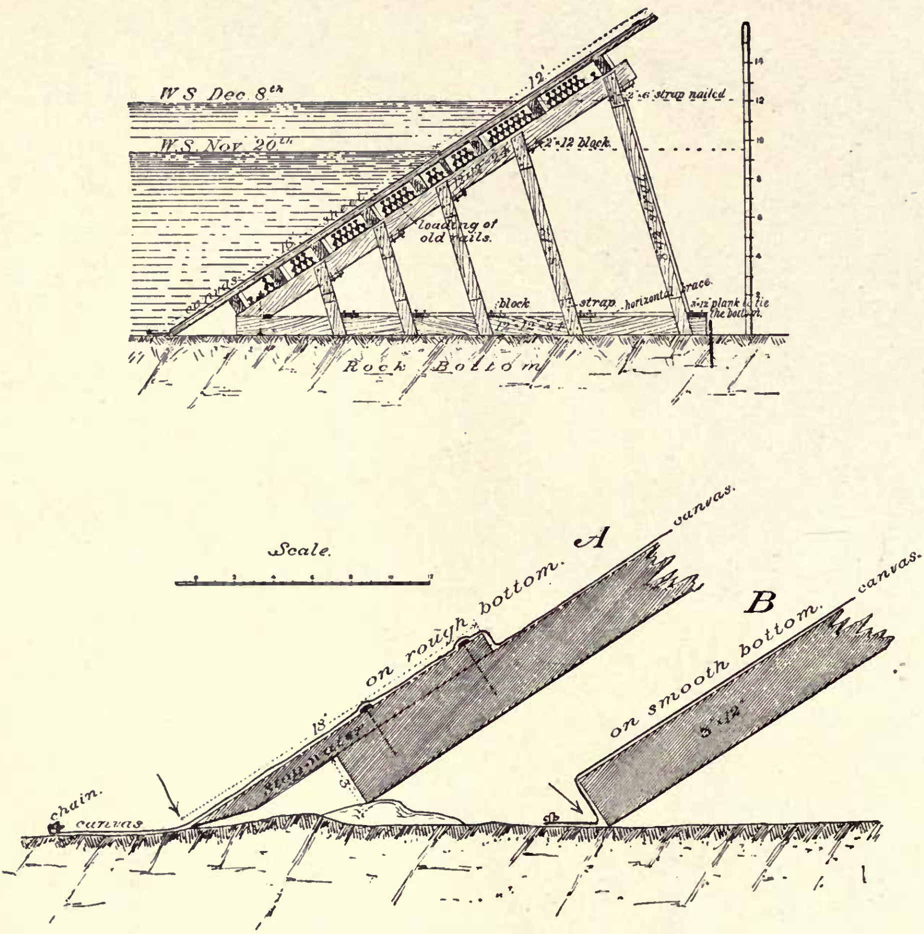
Cable-car Grip.

The accompanying figures represent a side elevation and an end view respectively of a *cable-car grip.*

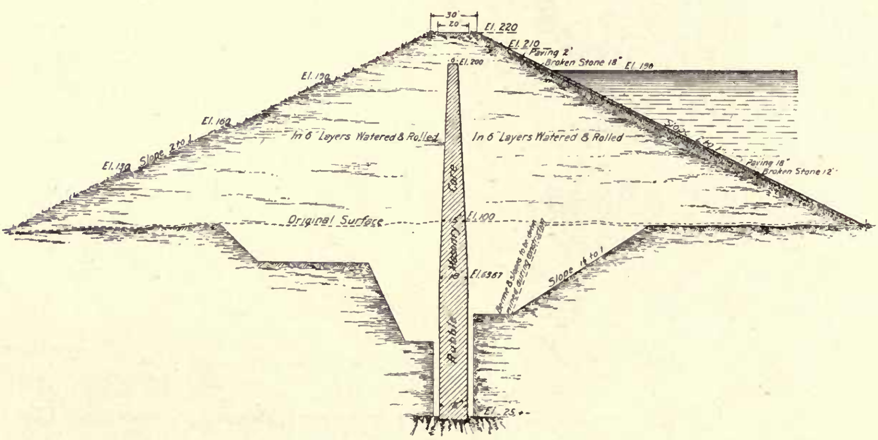
The gripping apparatus is shown in its open position, and the cable is therefore running inoperatively through it. When it is required to grip the cable it is merely necessary to pull over the lever A, whereupon the lever, its quadrant frame and shank attachments C, are raised up bodily about the fixed fulcrum of the link *c*, and the pair of rollers E, carried at the lower extremities of C, forces the jaws G close together, and tightly grasps the cable between the concave dies or packing pieces *h*.


10" Double
10" Single
<100' to Mast
10" Double
8" Single Block
18" Sheave
300', 3½" Rope
10" Double
10" Double
14" Double
14" Single
350', 3½" Rope
4000', 4½" Rope
MAST 65' LONG
BOOM 45' LONG
14" Double
10" Single
Safe Load 5 Tons
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 Feet

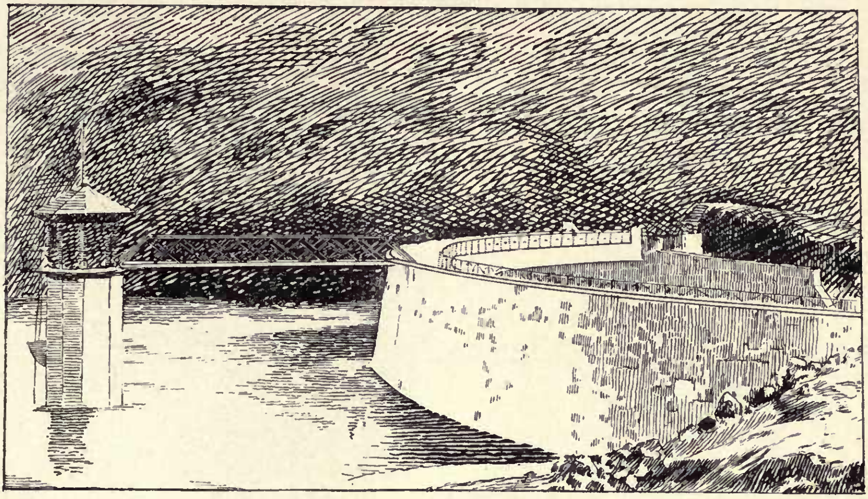
Derrick.



Canvas Dams. (From Trans. A. S. C. E.)

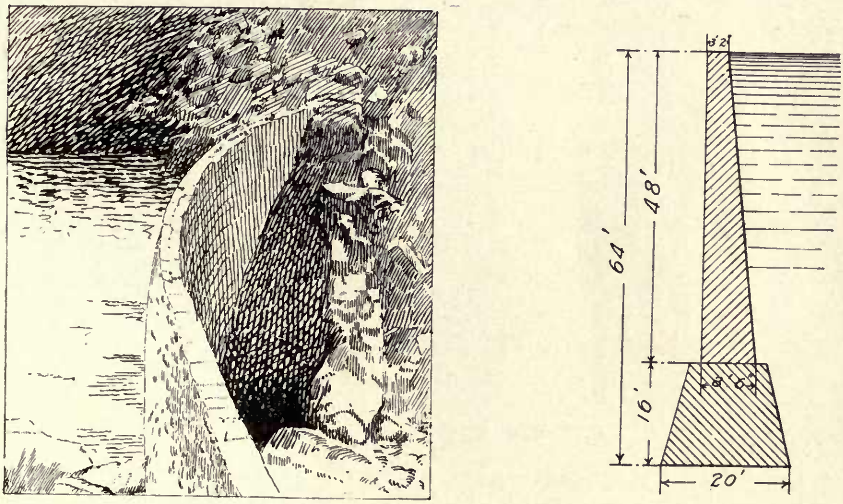


Earth Portion of the New Croton Dam, showing Rubble Masonry Core.



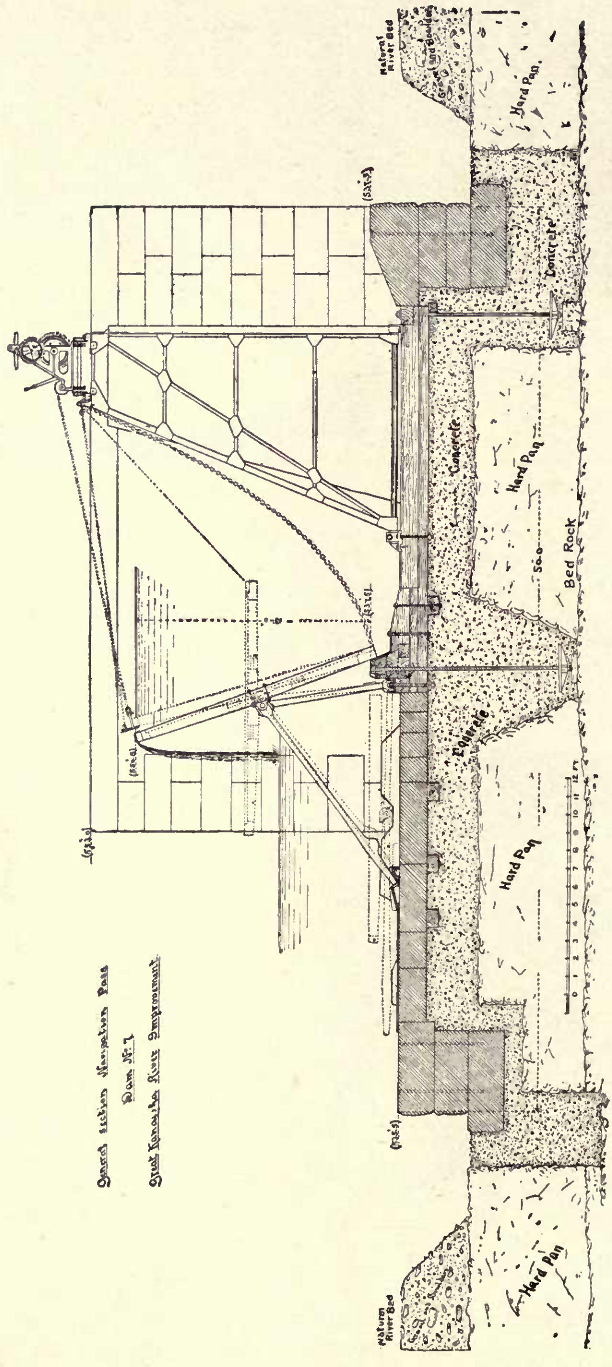
Sweetwater Arched Masonry Dam, Southern California.

The dimensions of this dam are : Thickness at base, 46 ft.; thickness at top, 12 ft.; height, 90 ft.; radius of arch at top, 222 ft. The upper face batter is 1 to 6 to within 6 ft. of the top, thence vertical; on the lower face it is 1 to 3 for 28 ft.; 1 to 4 for 32 ft.; thence 1 to 6 to the coping. In January, 1895, a freshet discharge flooded the waste weir and rose 22 inches over the parapet wall. The dam was subjected to this cataract action for a period of forty hours without injury.



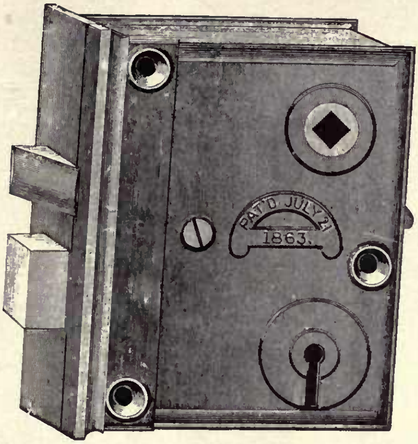
Bear Valley Arched Masonry Dam, California.

This receives a sufficient support from the arch-action, and has stood since 1884. Its length at top is 270 ft., and its radius of curvature is 355 ft. At the centre the deviation of the dam from a straight line is about 27 ft., this being the versed sine of the subtended angle. It is 38 inches in thickness at top and 102 inches at a point 48 ft. below.

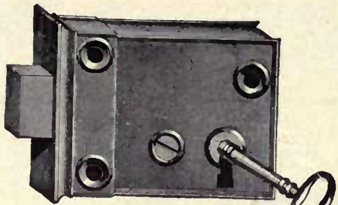


Movable Dam, Great Kanawha Improvement, Tripping Bars and Improved Hurters on Chauvine Wicket Dams. (Trans. A. S. C. E.)

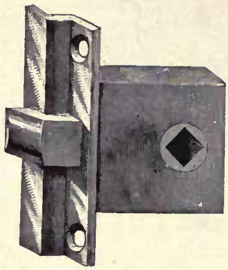
BUILDERS' HARDWARE.



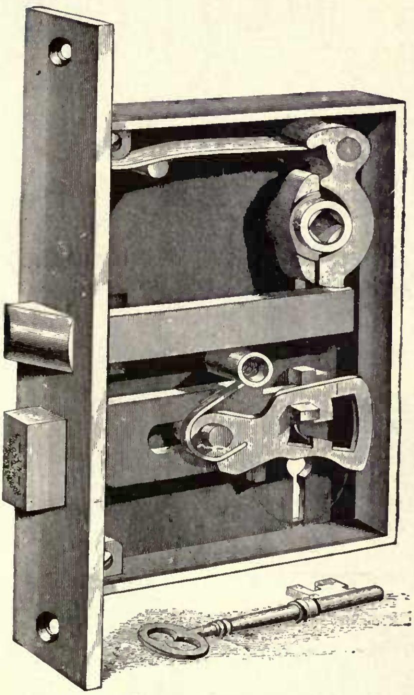
Rim Knob-Lock.



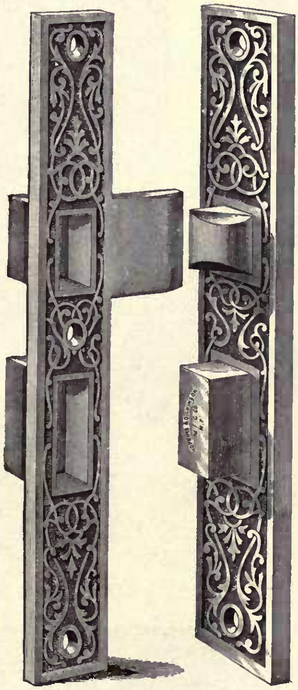
Rim Dead-Lock.



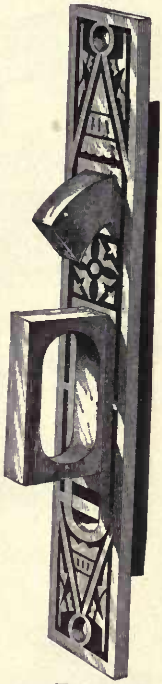
Rabbeted Knob-Catch.



Mortise-Lock, cover off.



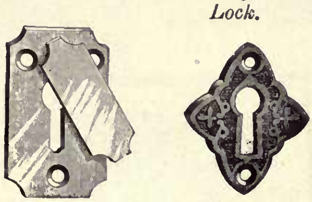
Boxed Strike, Front.



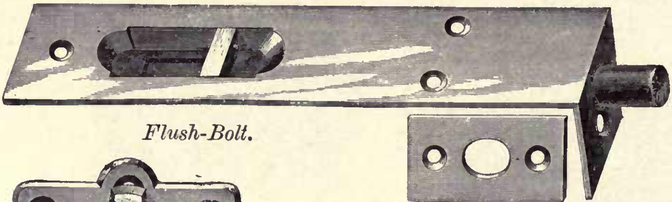
Front Sliding-door Lock.

Thumb-Piece.

Knob and Rose.

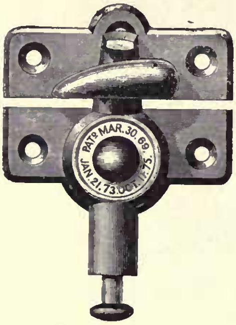


Escutcheons.

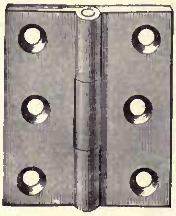


Flush-Bolt.

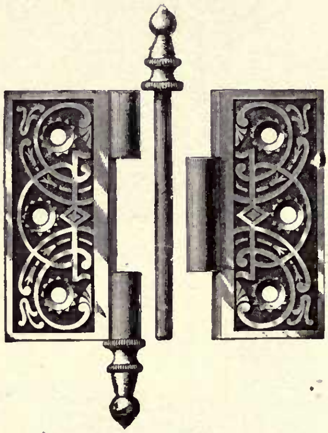
Flush Transom-Latch



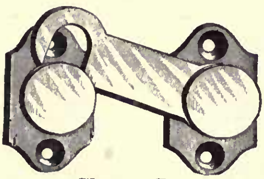
Sash-Lock.



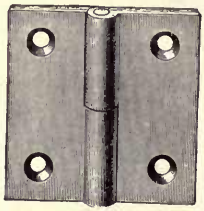
Fast-Joint Butt.



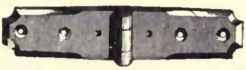
Loose-Pin Butt.



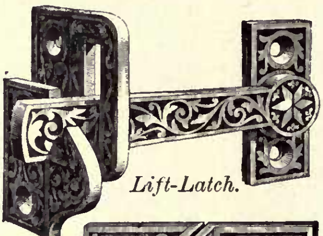
Shutter-Bar.



Loose-Joint Butt.



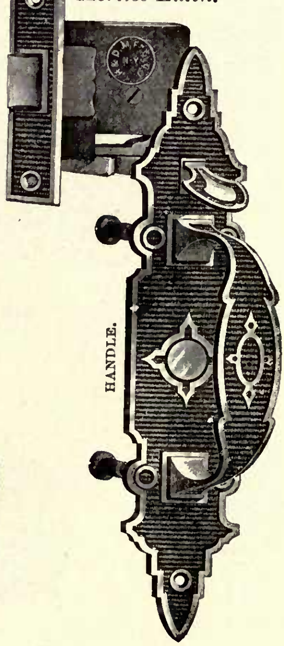
Strap-Hinge.



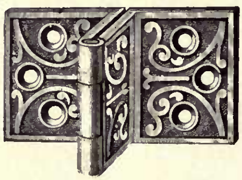
Lift-Latch.

Hasp.

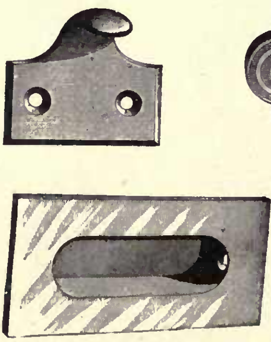
Mortise-Latch.



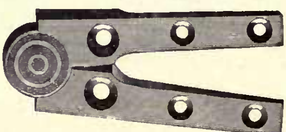
T-Hinge.



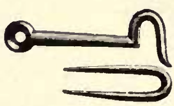
Three-Fold Shutter-Flap.



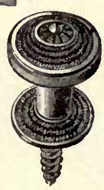
Sash-Lifts.



Pin-Hinge.

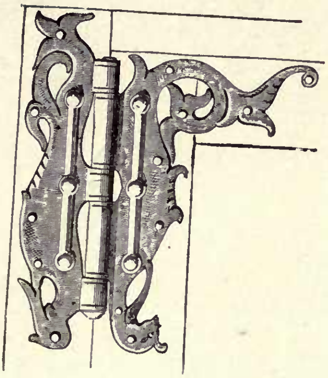


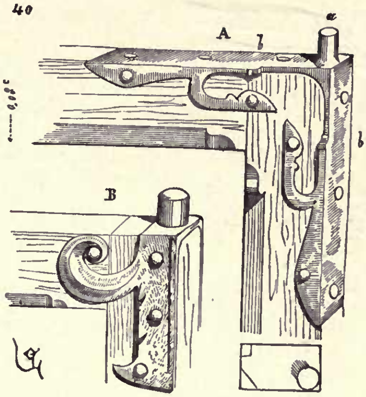
Hook and Eye.

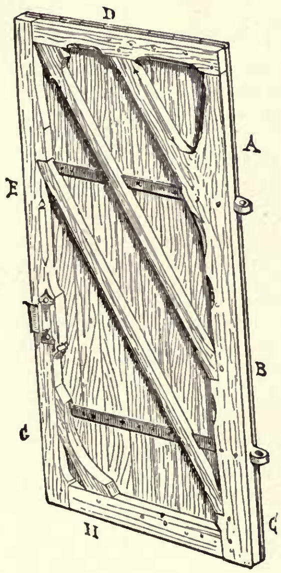


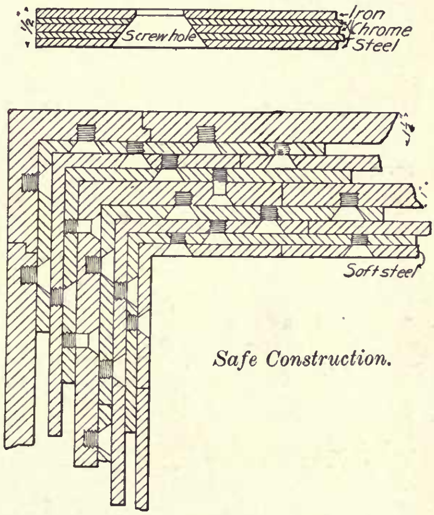
Shutter-Knob.

Examples of Ancient Hinges and Doors.

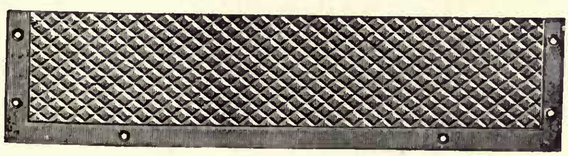




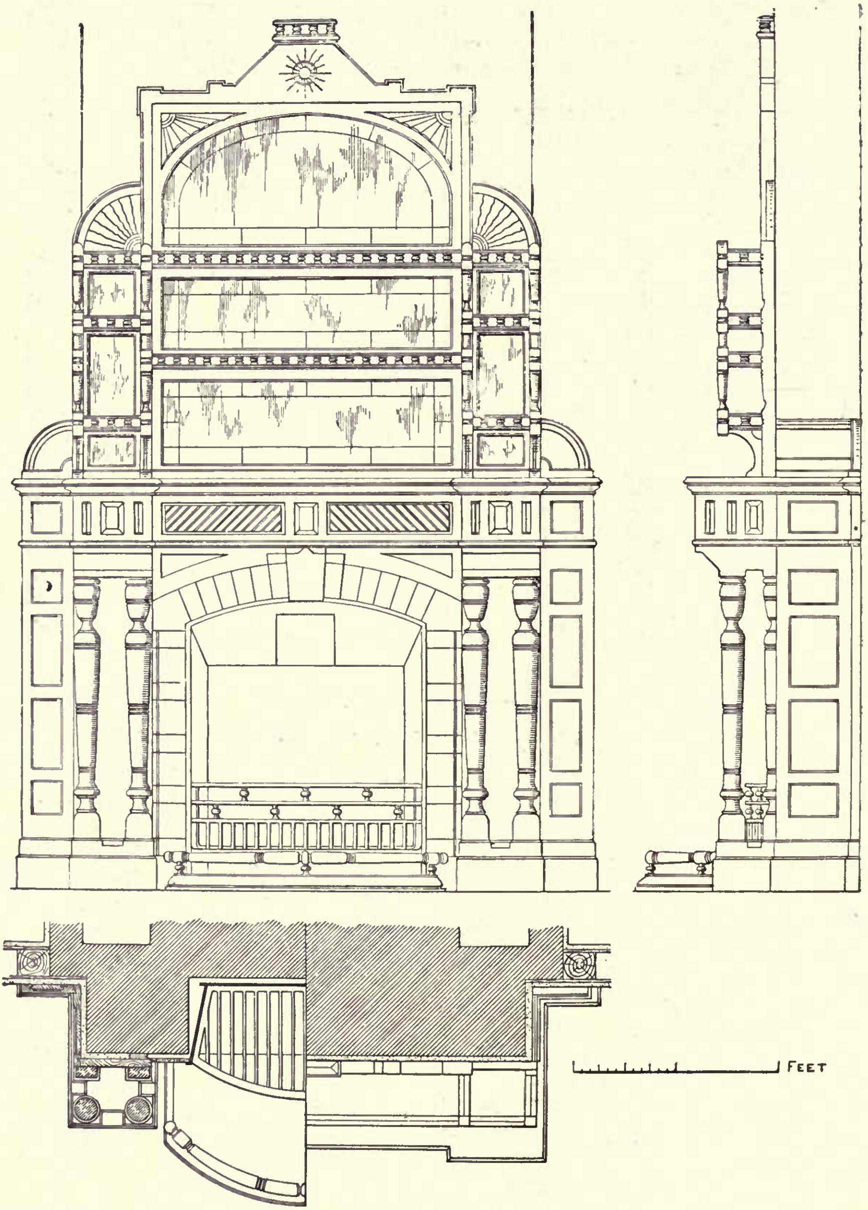




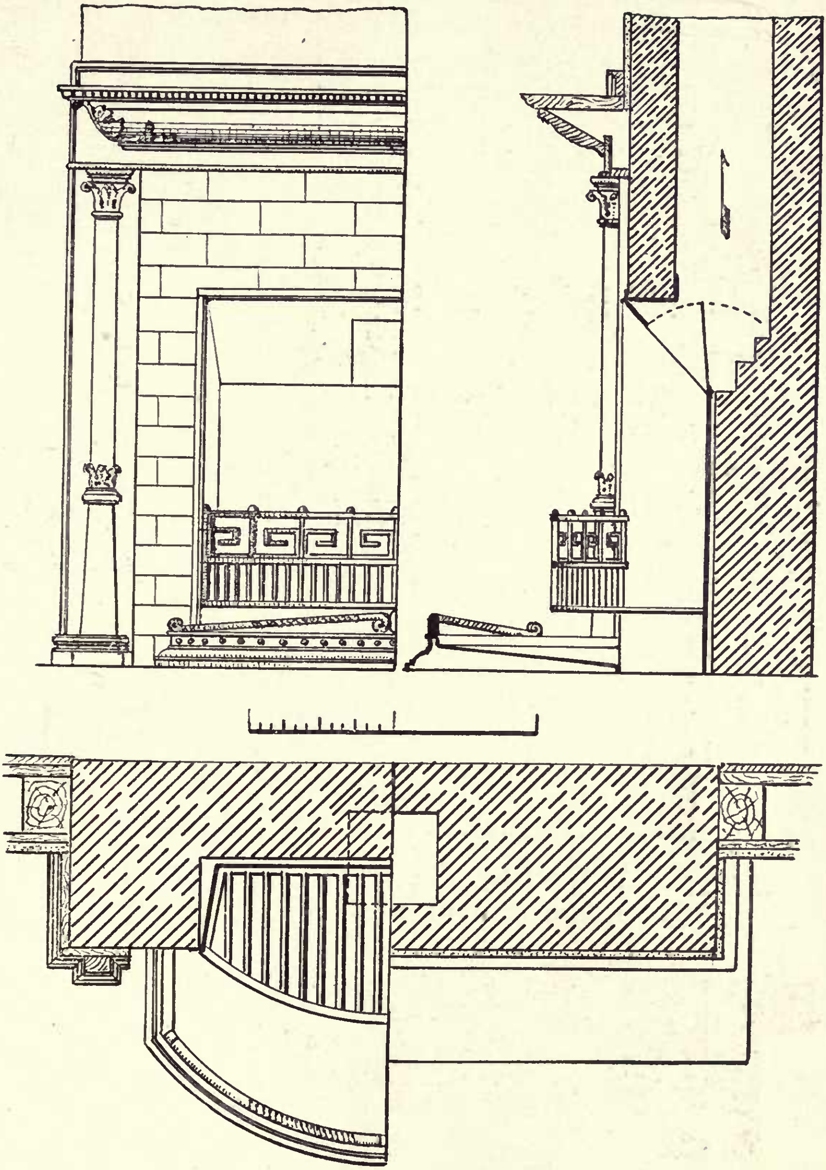
Safe Construction.



Cast-Iron Tread.

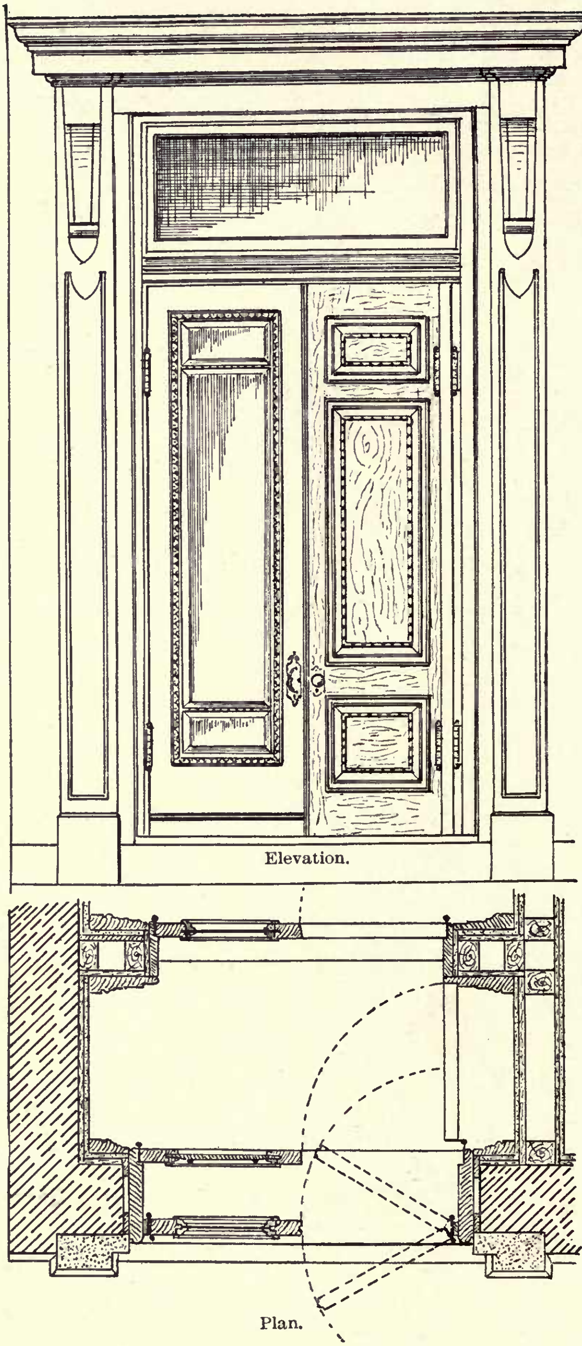


Plan, Section, and Elevation of a Wooden Mantel and Fire-Place.

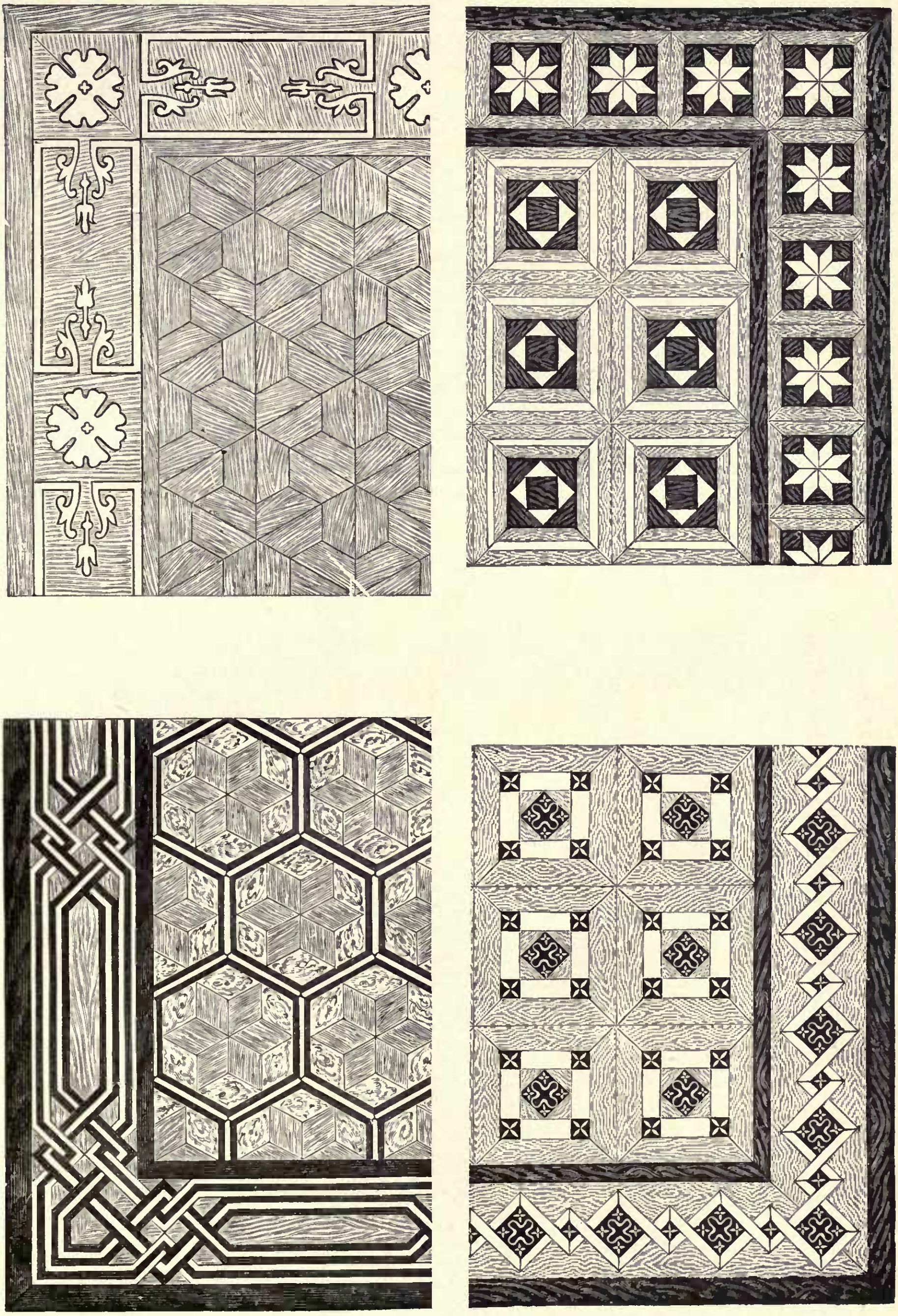


Sectional Plan of Grate and Flue.

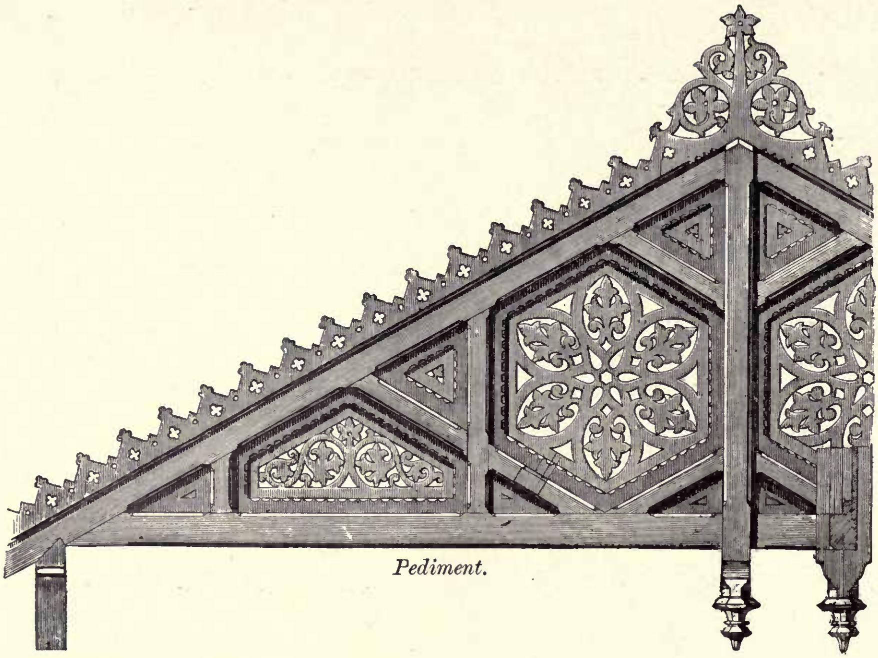
Details of Fireplace.



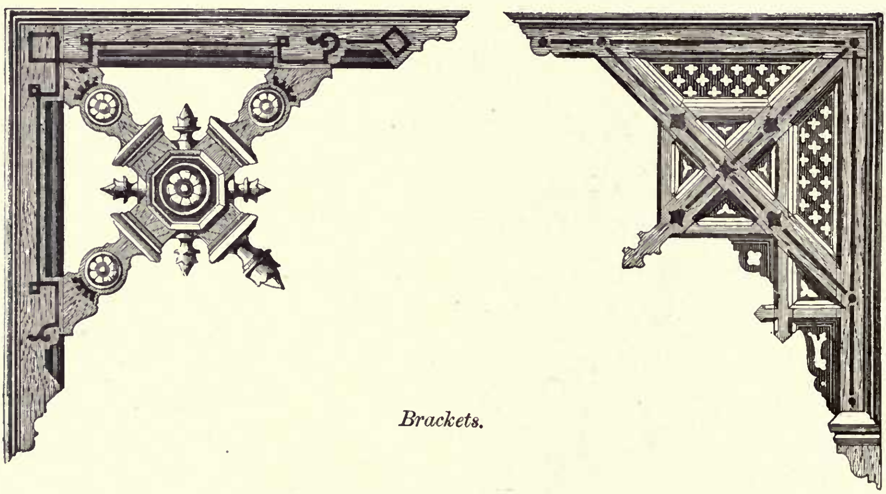
Vestibule Doors.



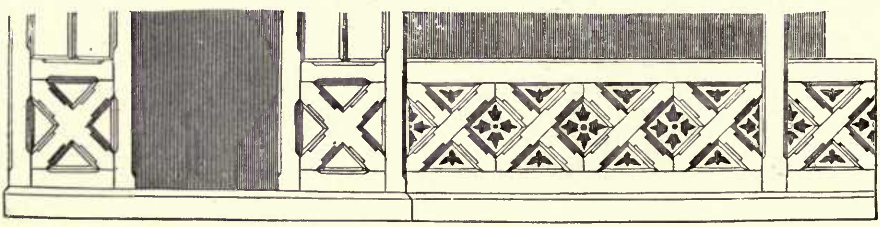
Examples of Inlaid Floors or Marquetry.



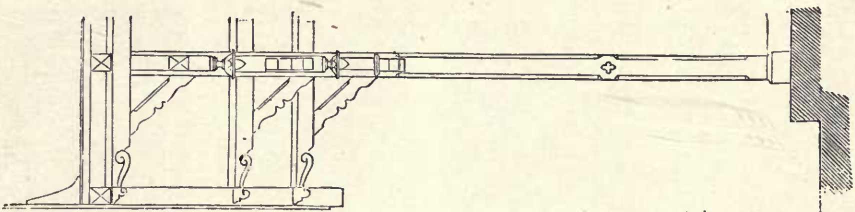
Pediment.

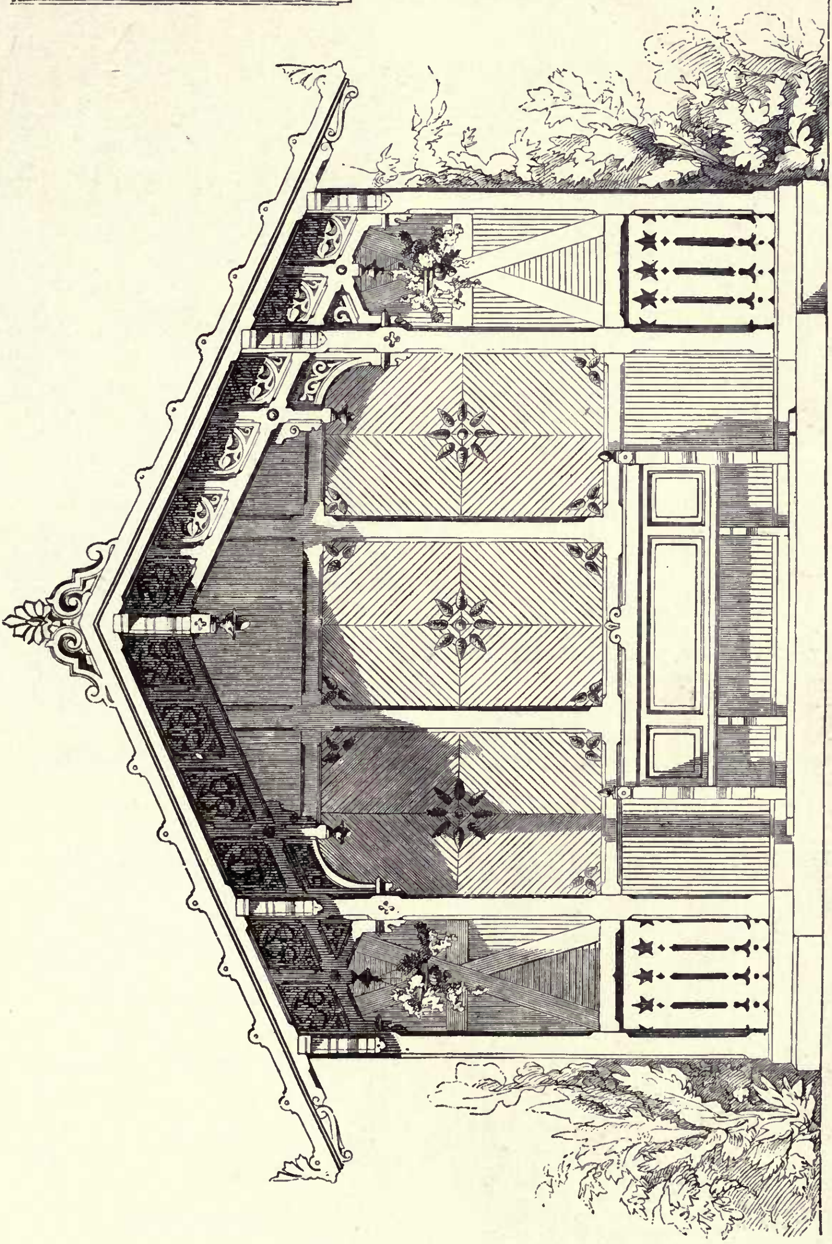


Brackets.



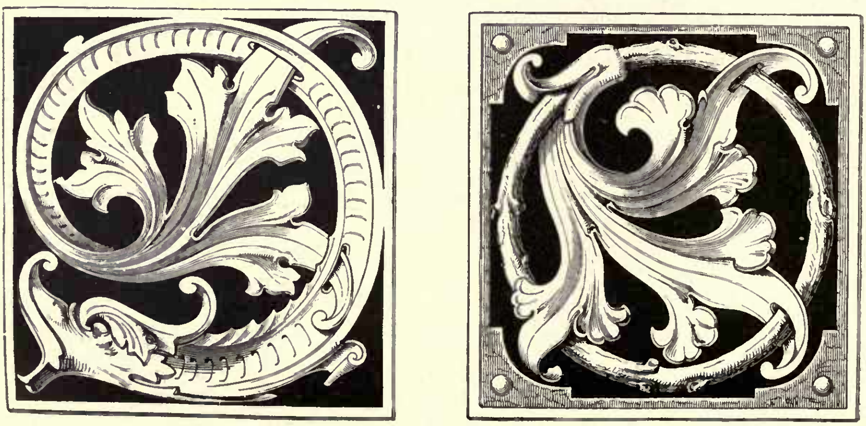
Railing.



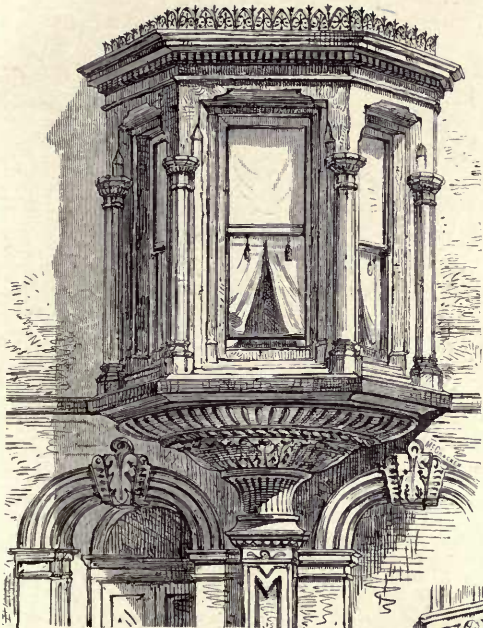


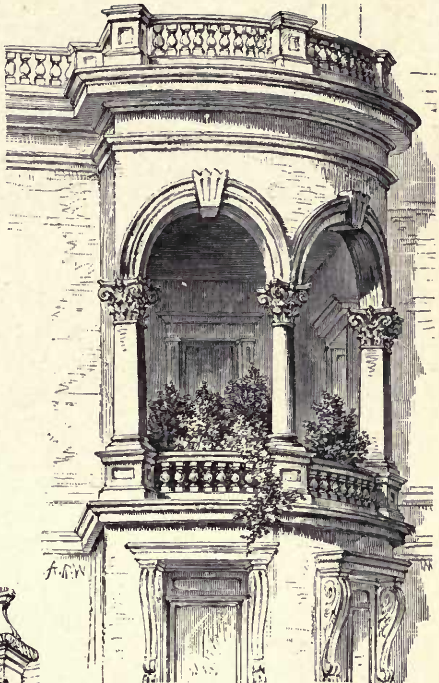
Elevation of Summer House.

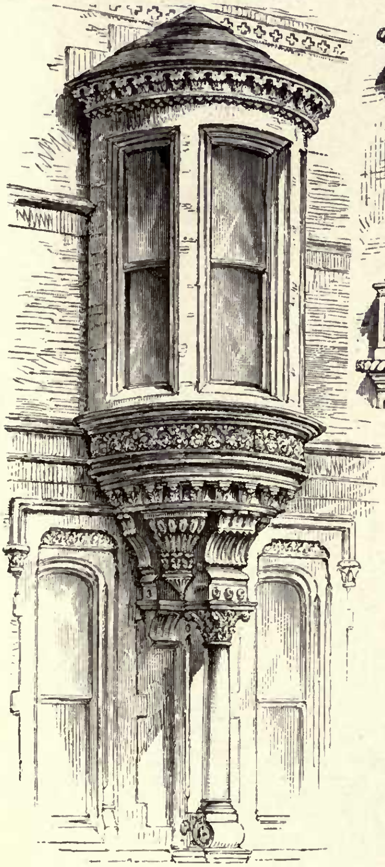
Enameled Tile.

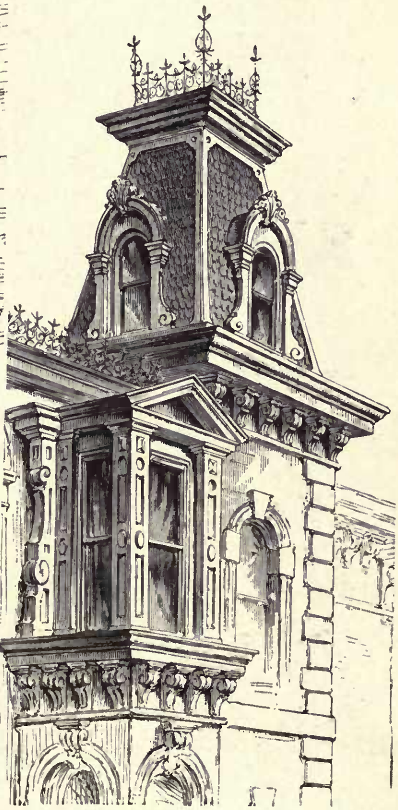


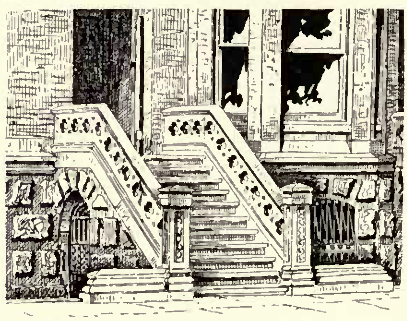
Terra Cotta.

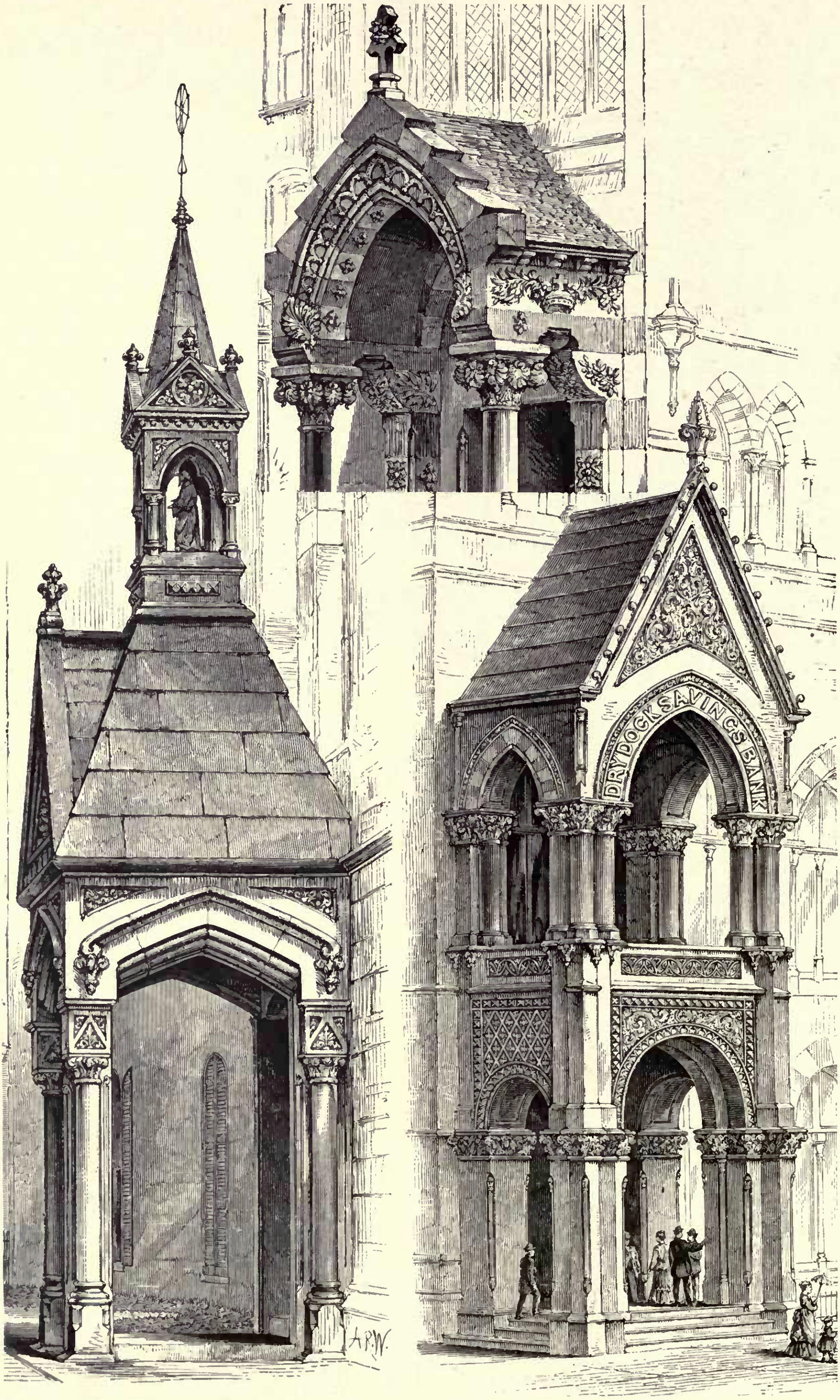


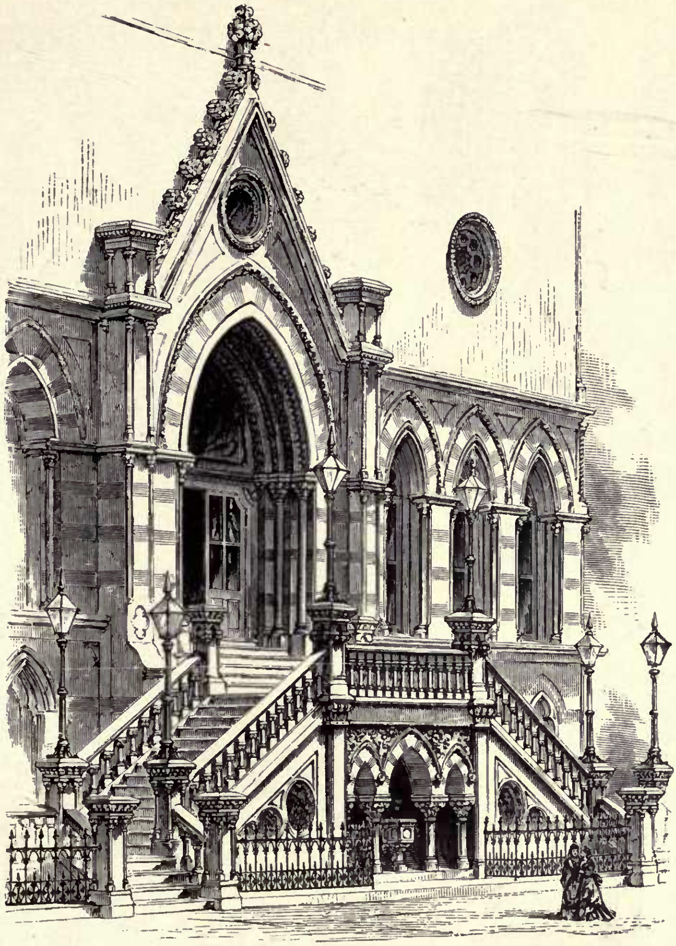


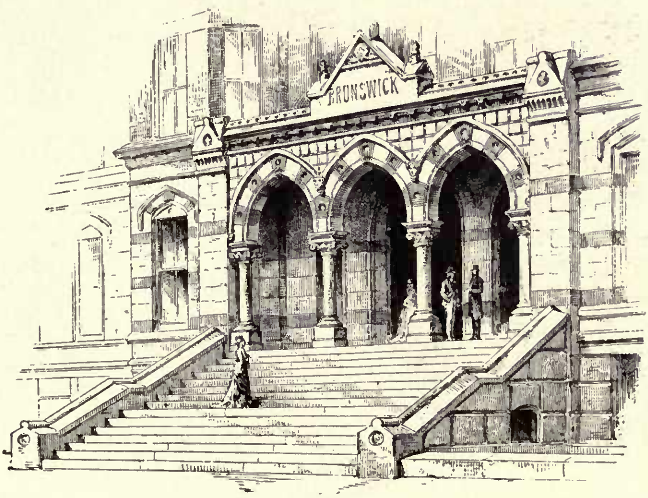


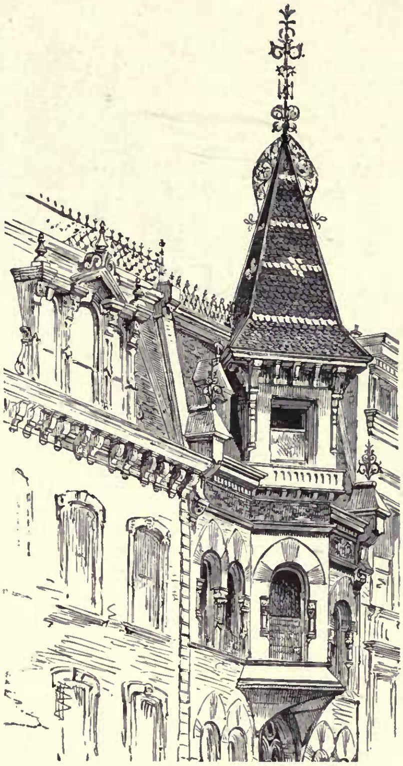


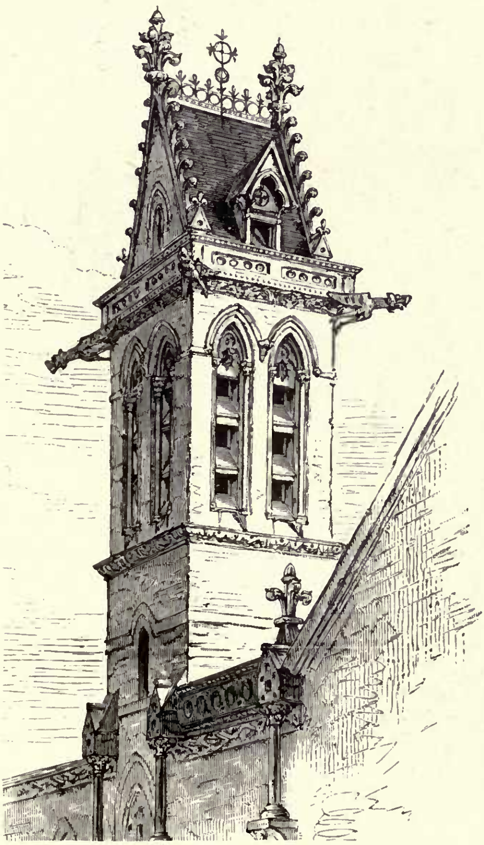


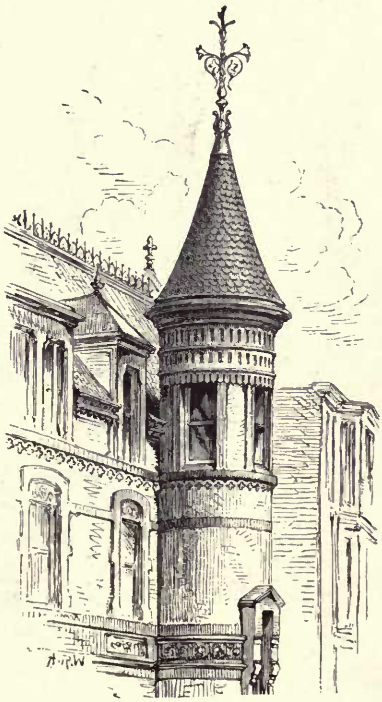

DRY DOCK SAVINGS BANK
A.P.W.

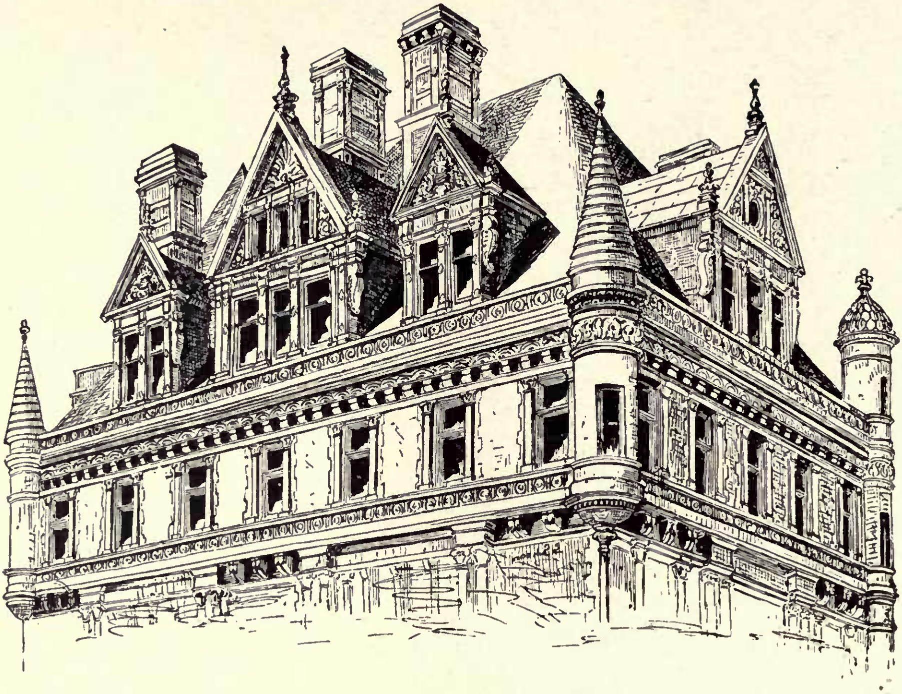


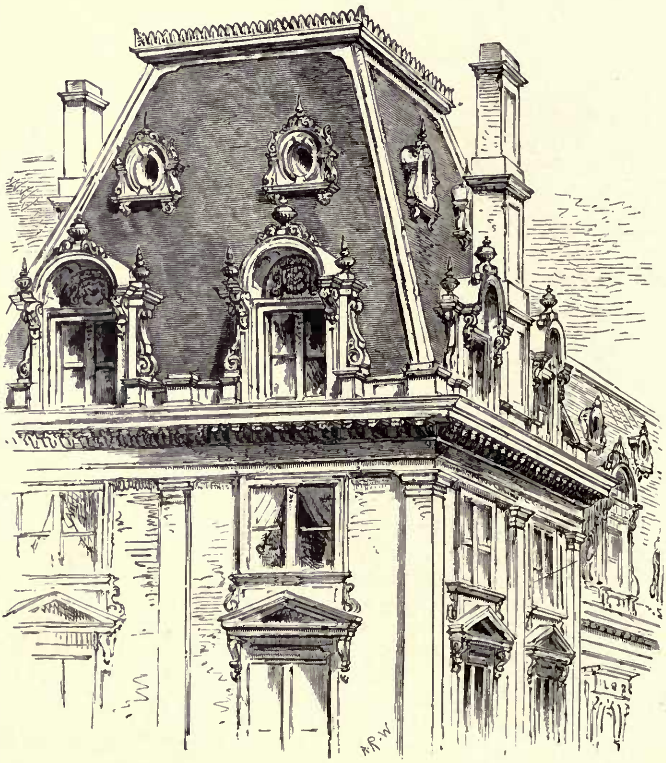

BRUNSWICK

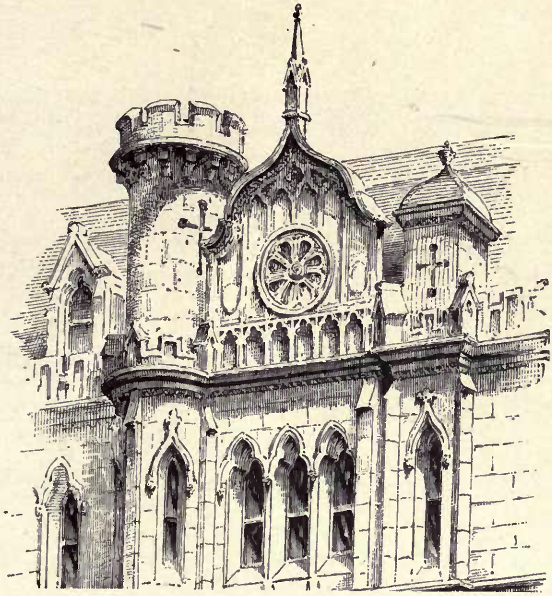


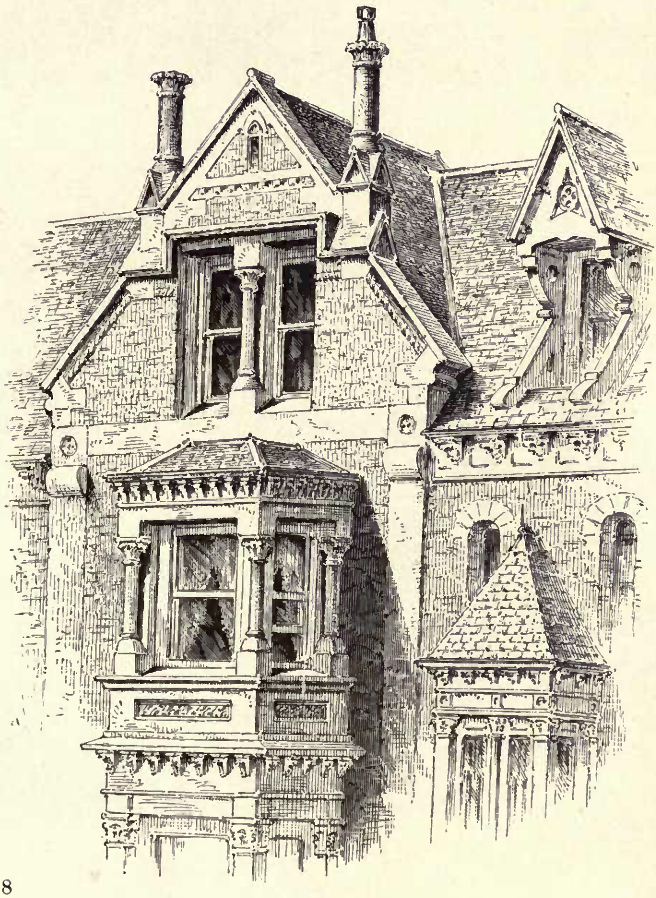







A.R.W





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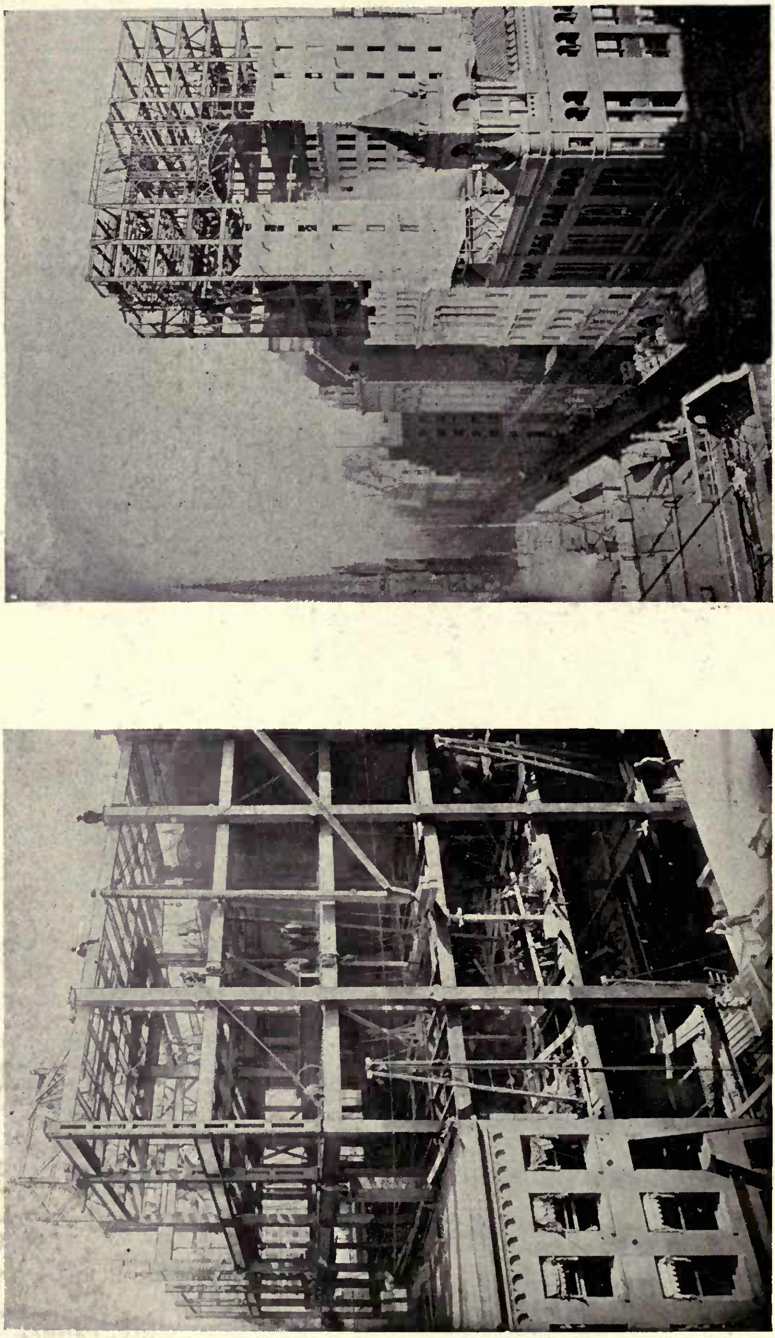
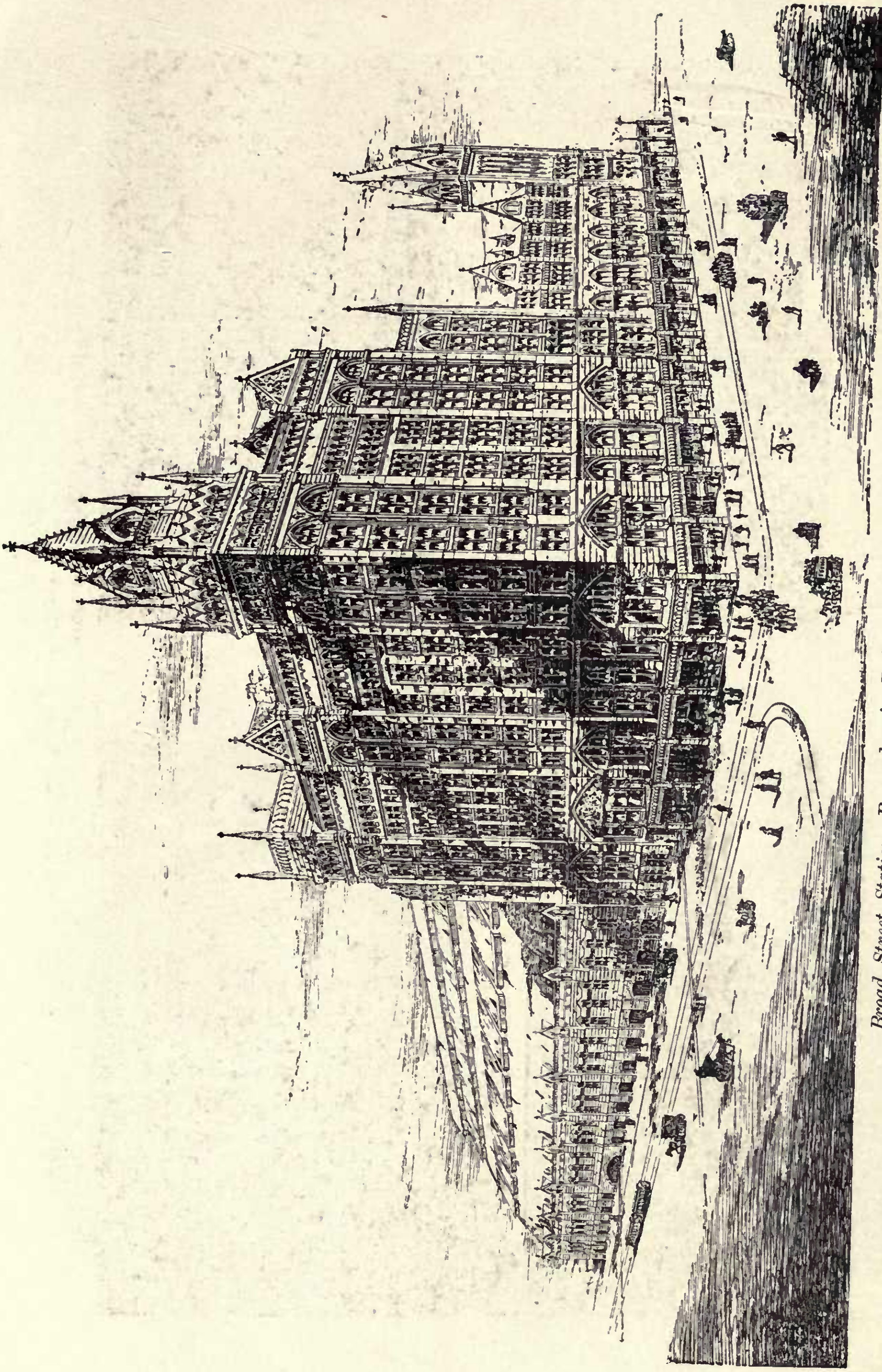
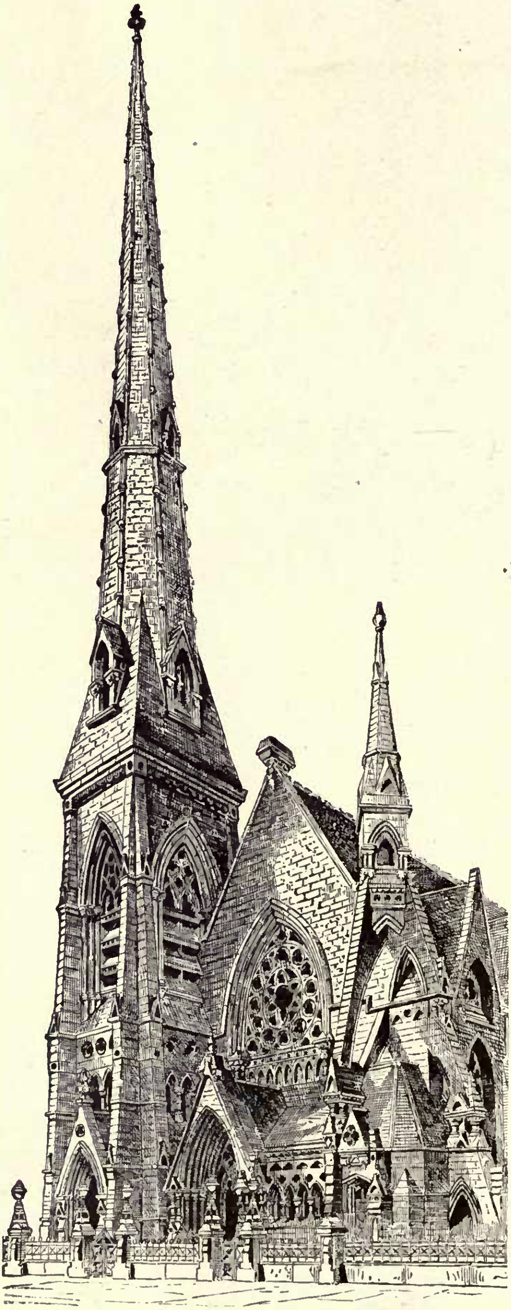
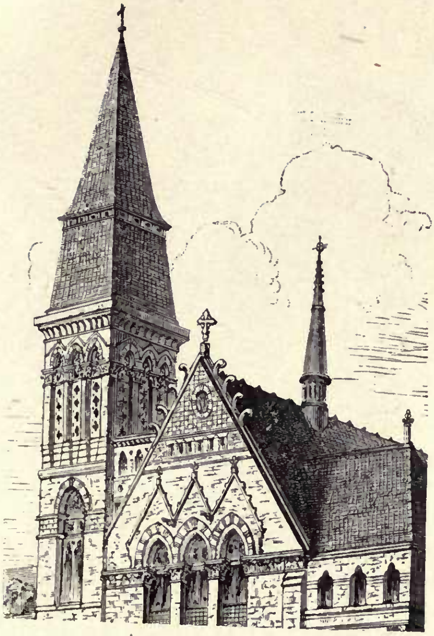


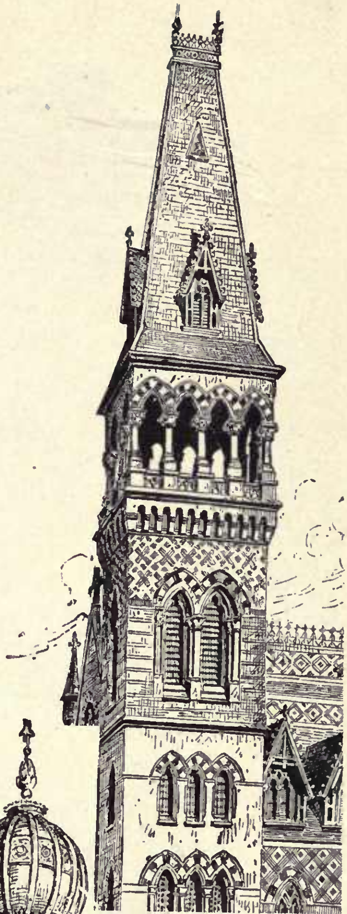
Skeleton Construction, Manhattan Life Insurance Building, New York City.
From "Engineering Magazine."

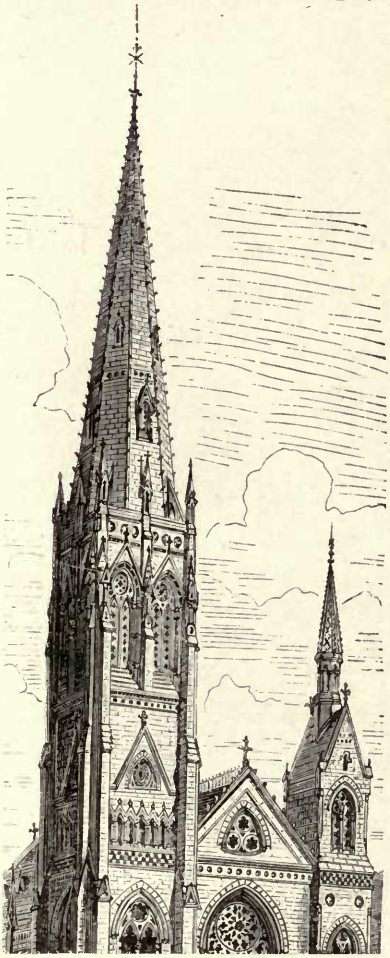


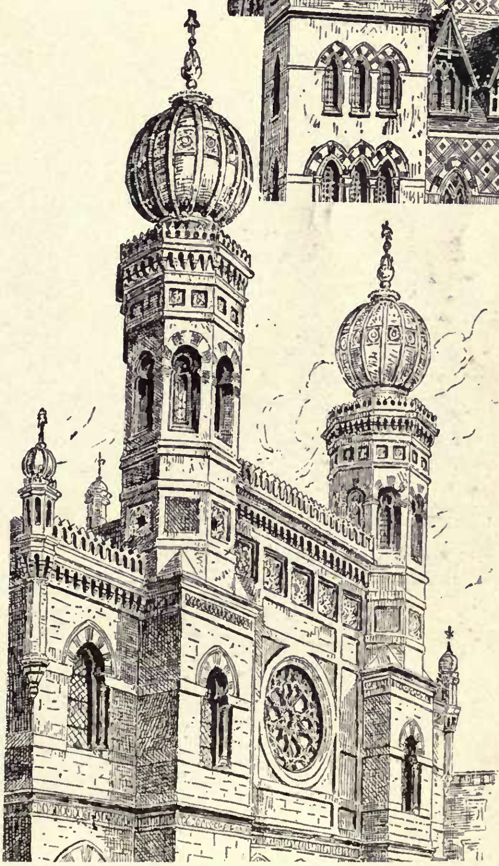
Broad Street Station, Pennsylvania R. R. Co., Philadelphia, Pa.

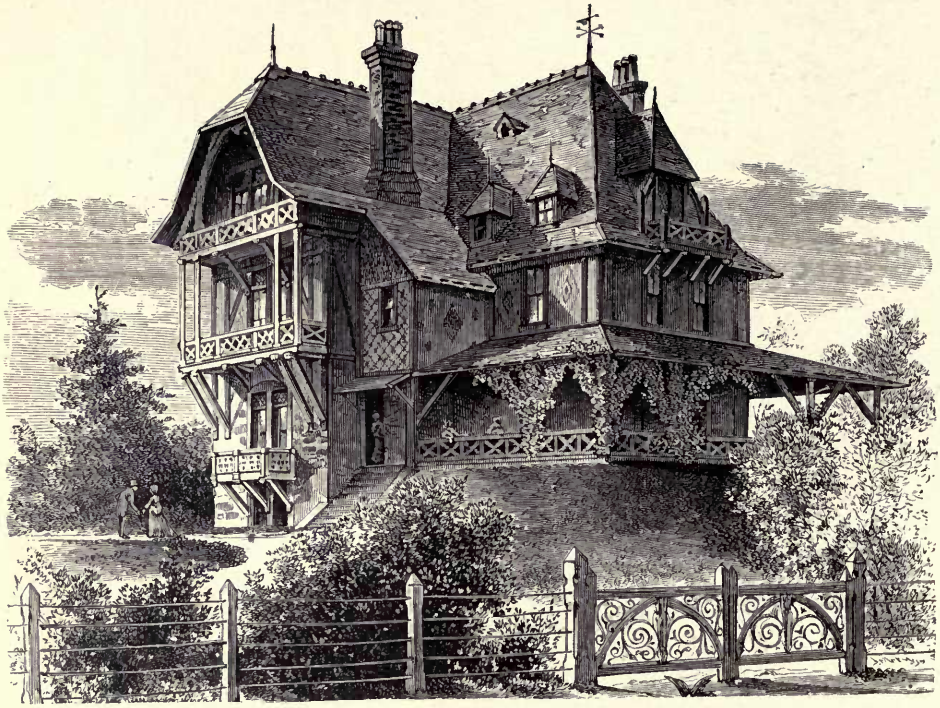


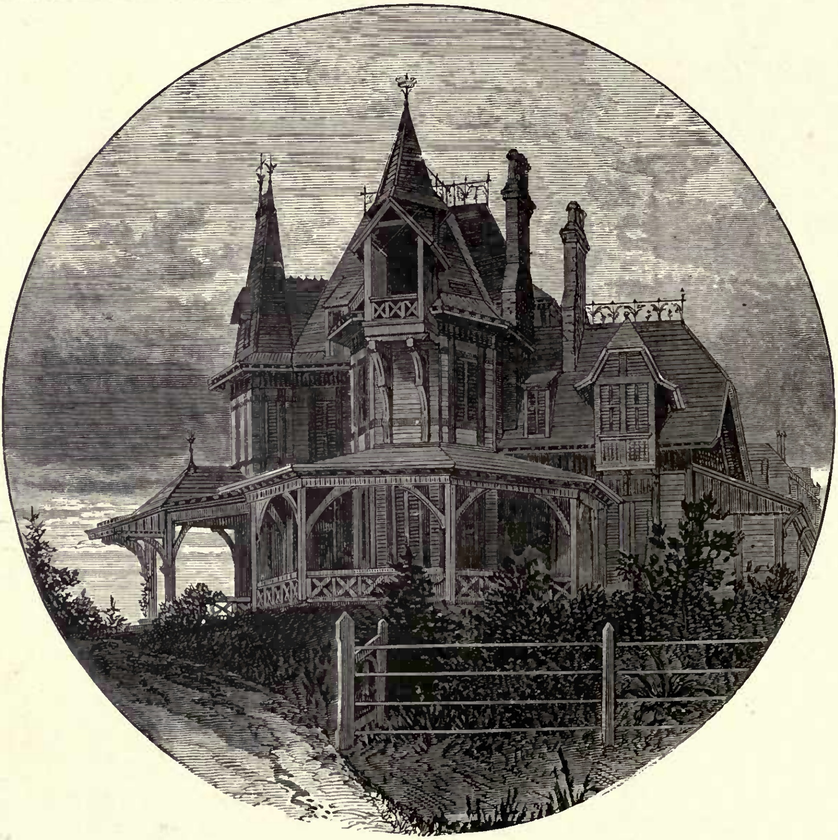


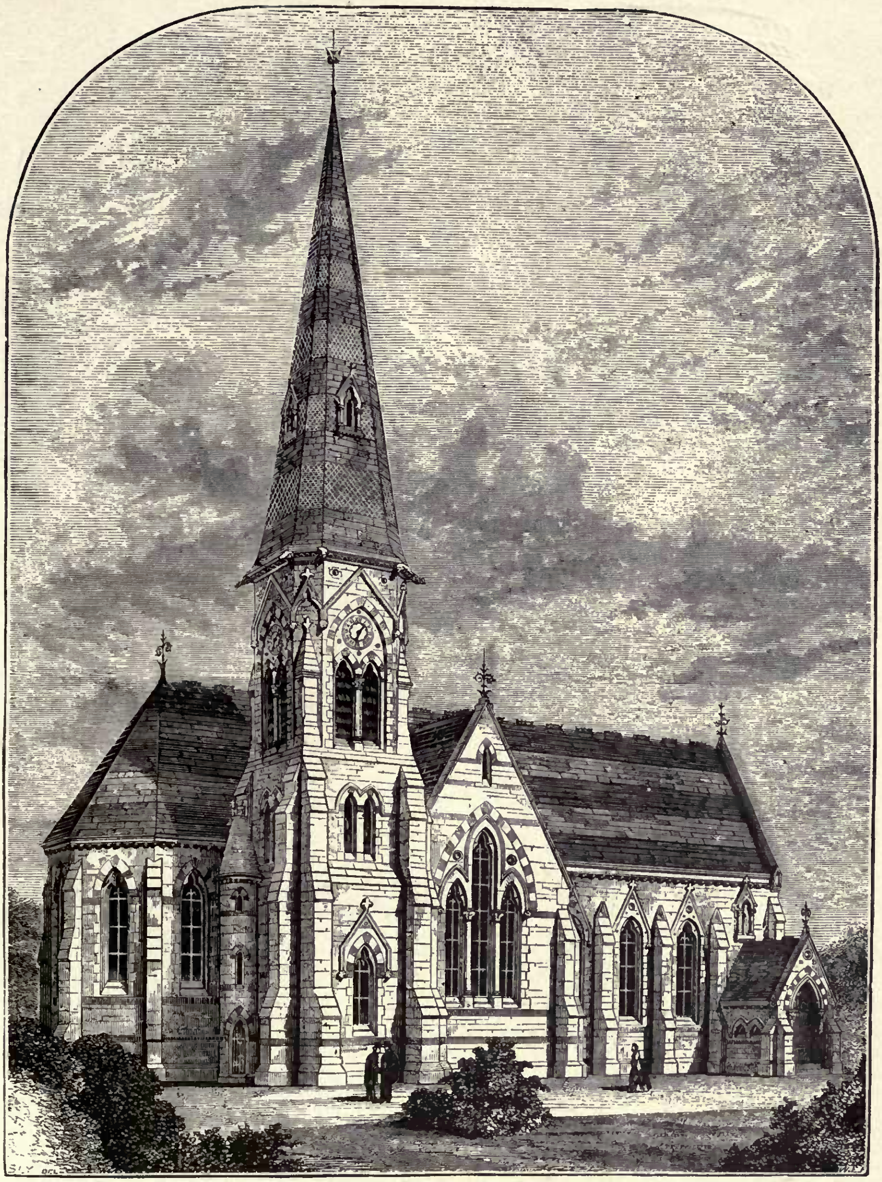


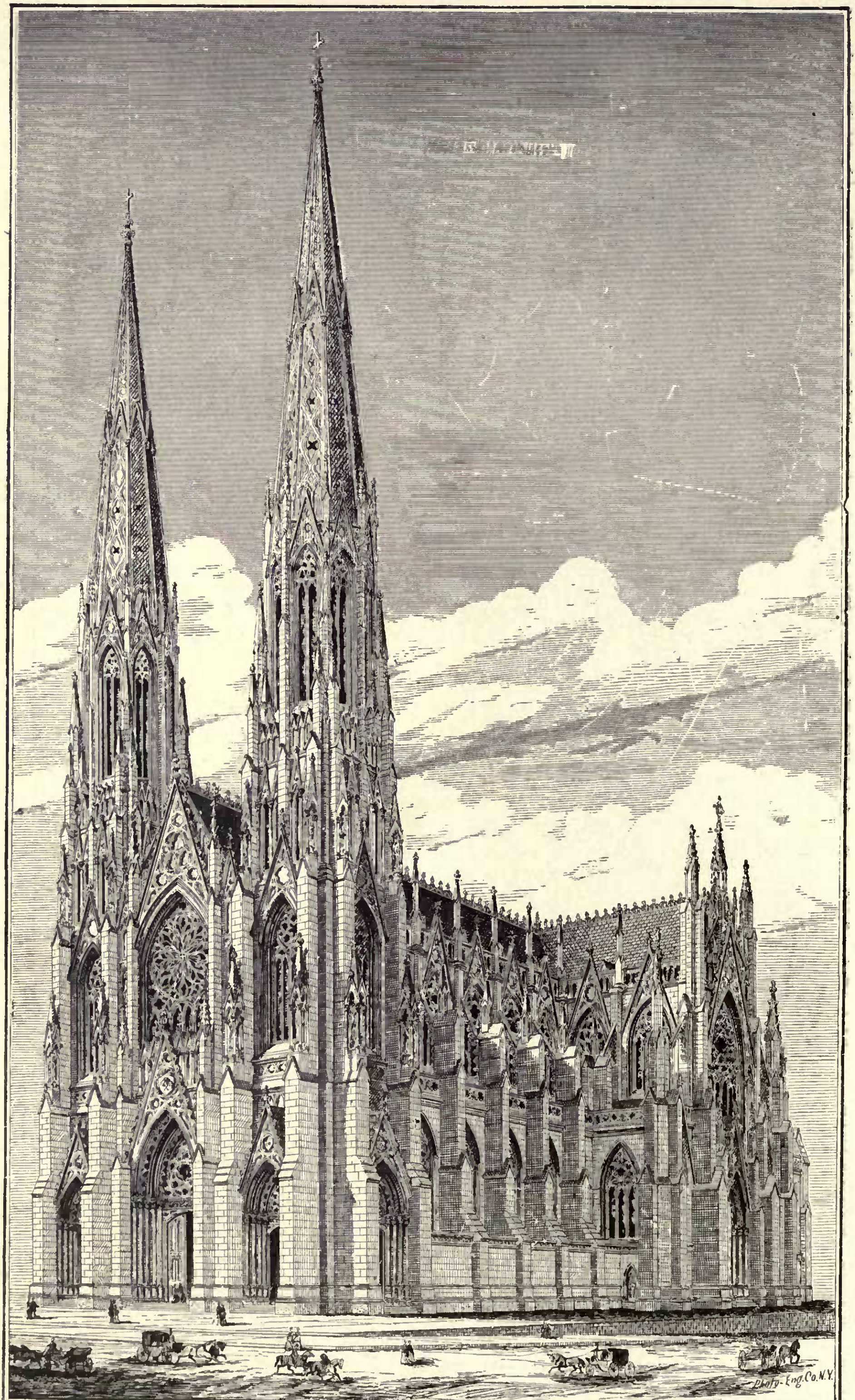












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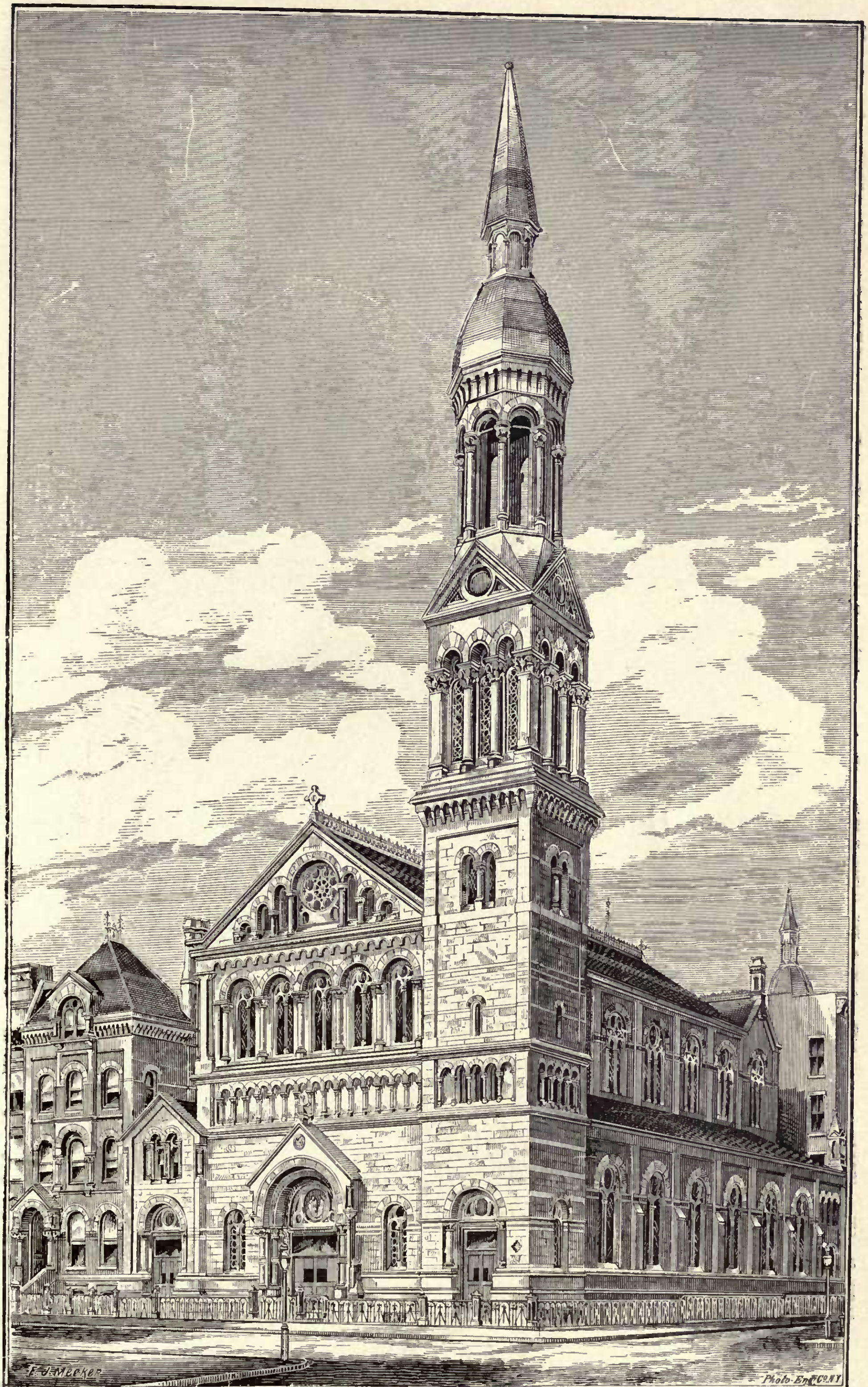
JAMES RENWICK ARCHITECT.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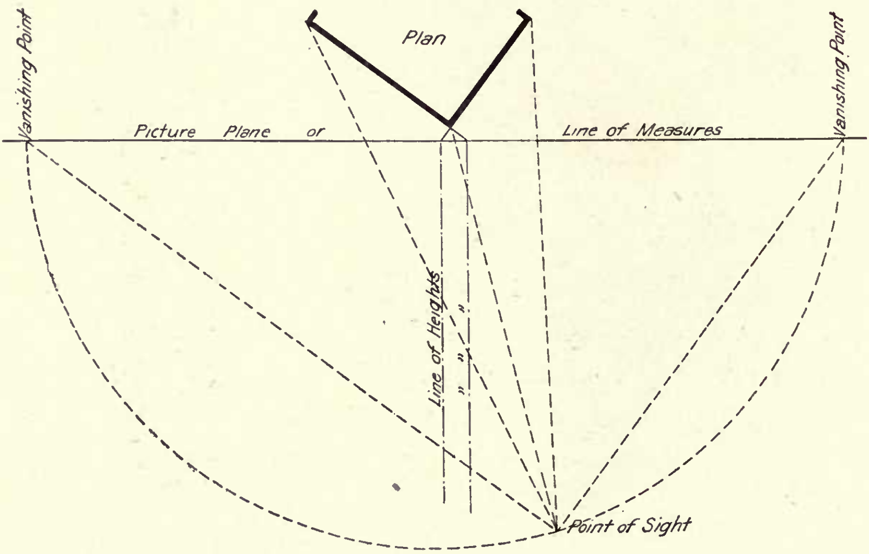
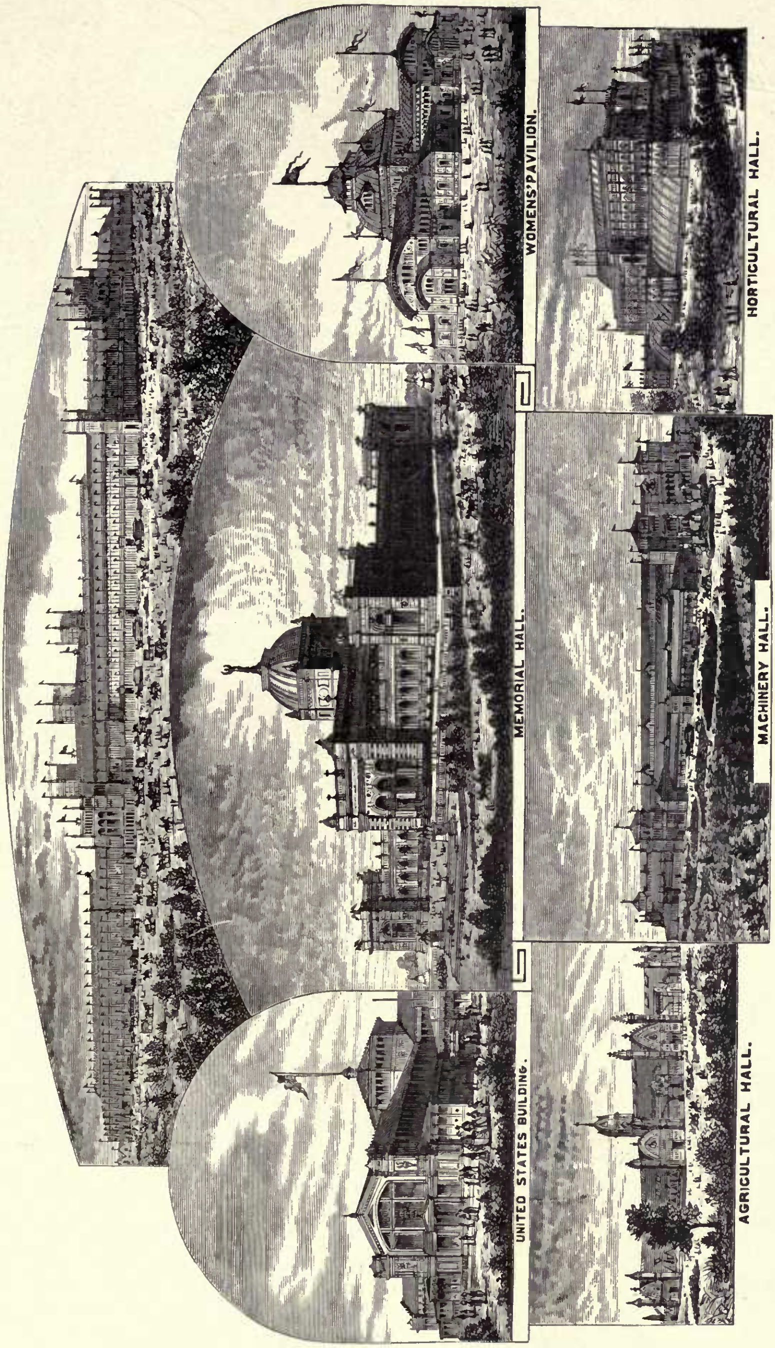
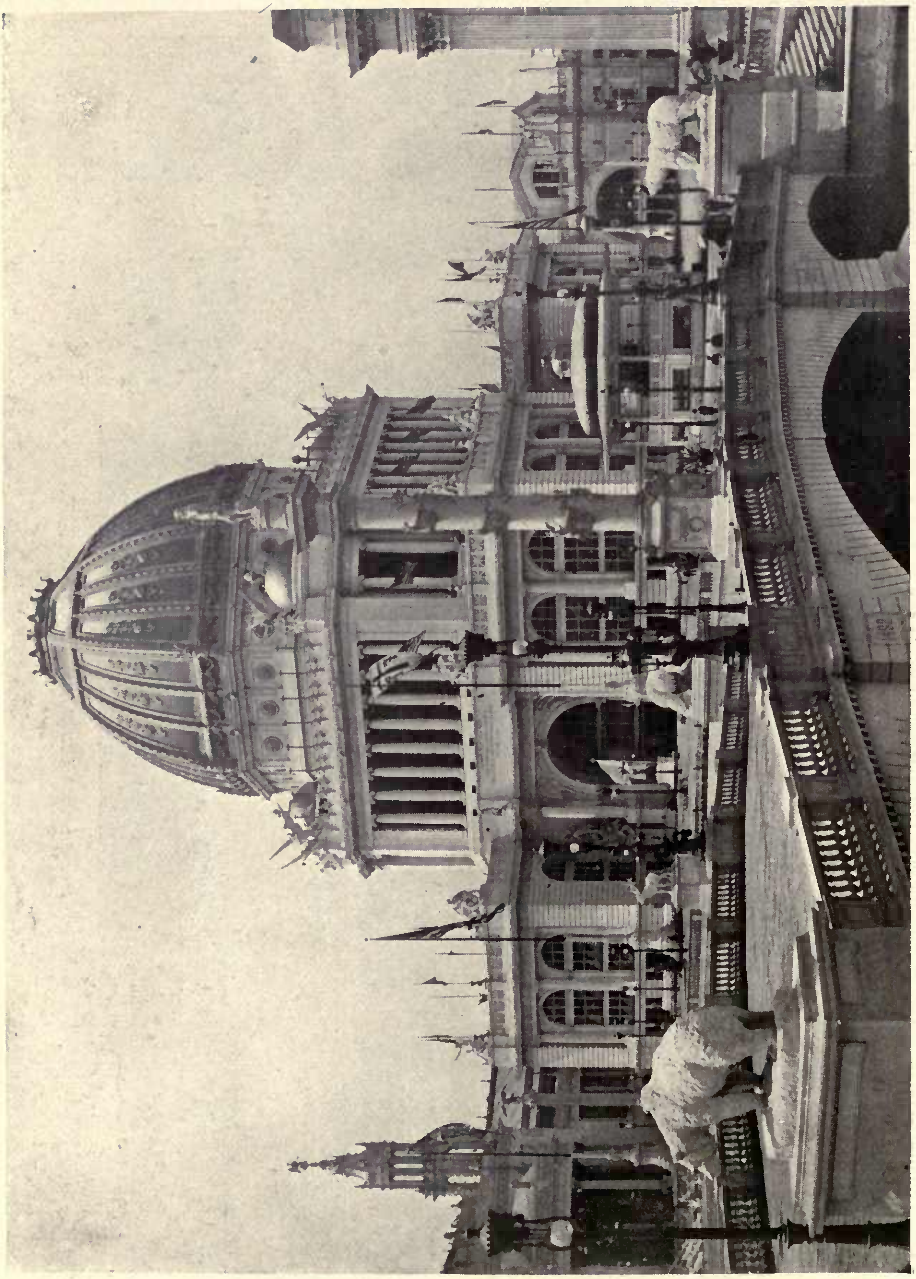


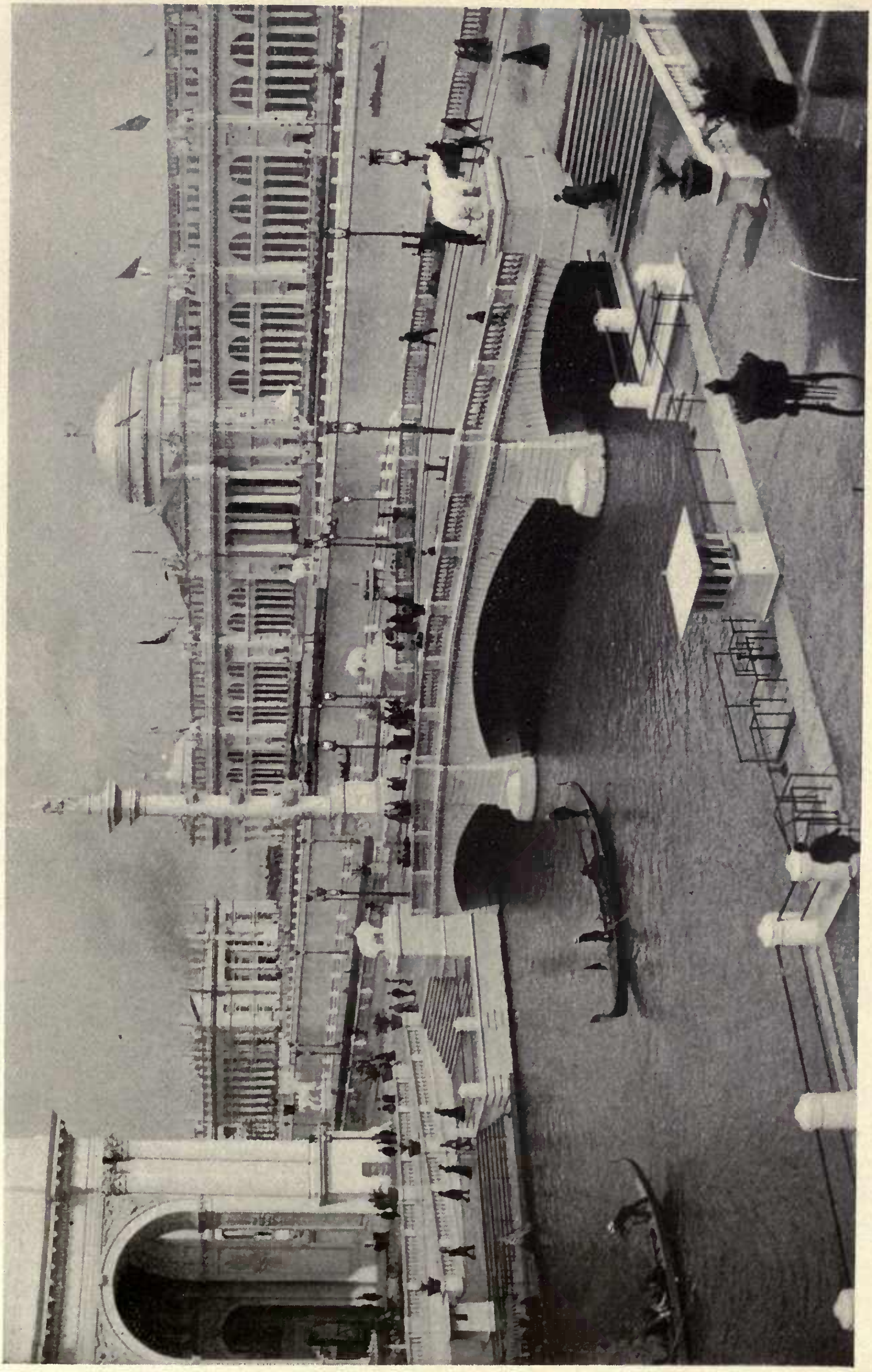
Perspective Diagram. (See page 714.)



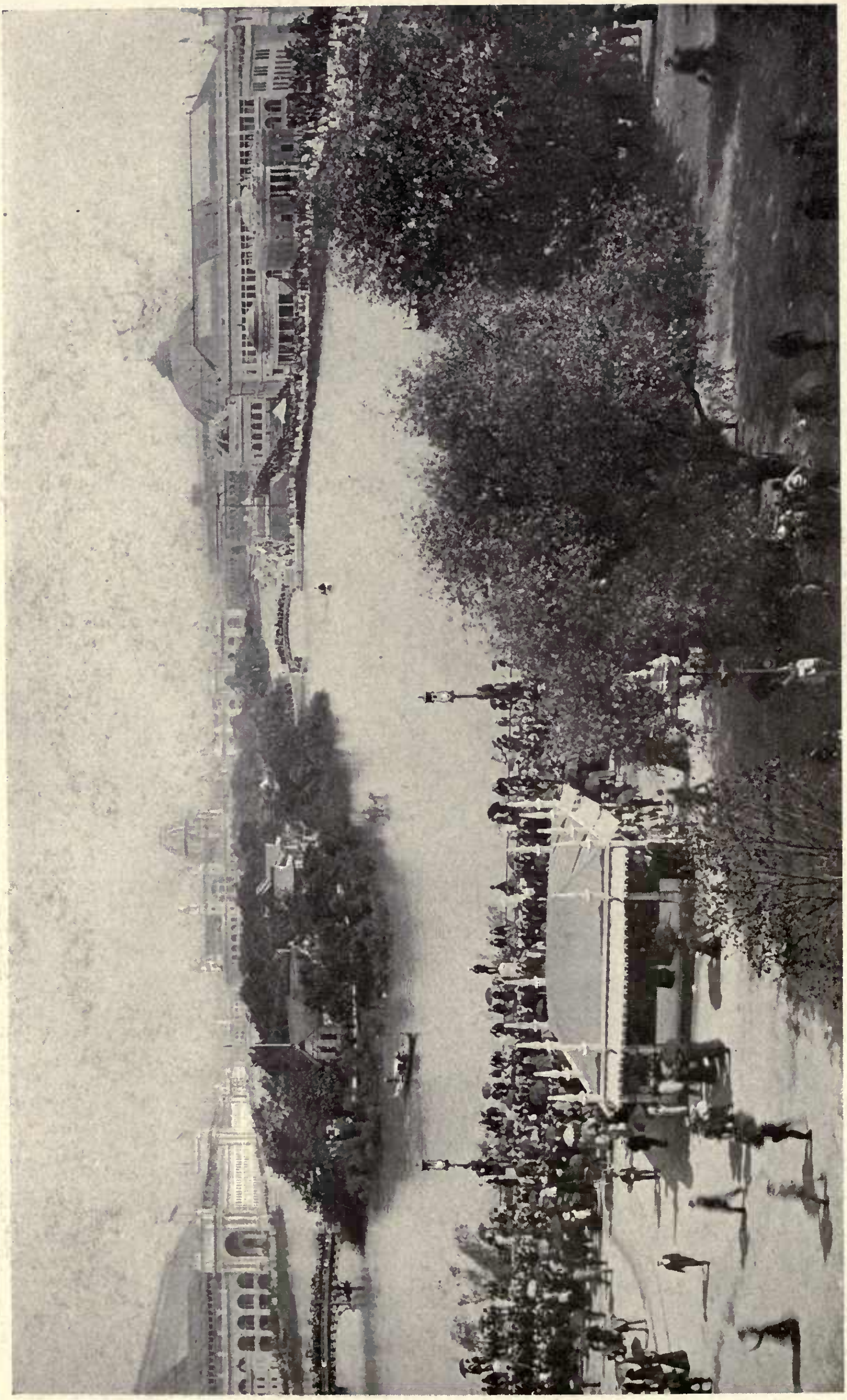
Buildings Centennial Exhibition, 1876.



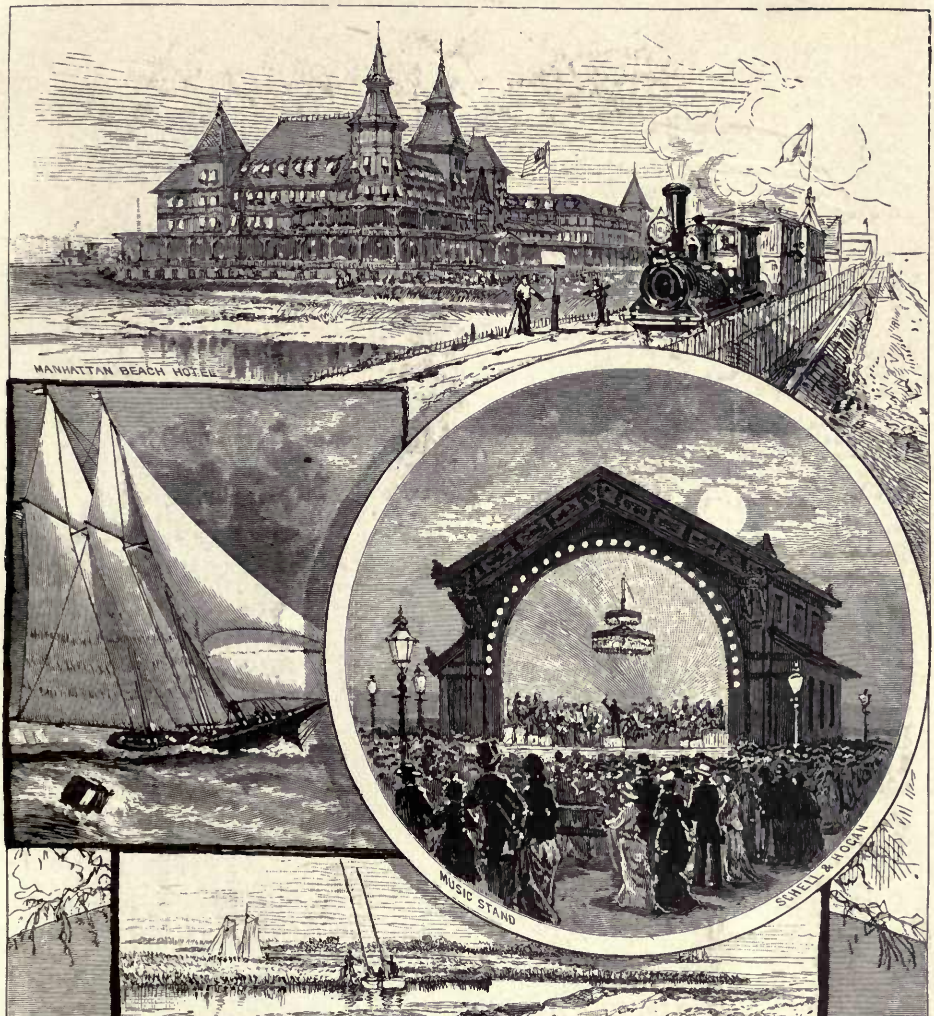
World's Fair, Administration Building.



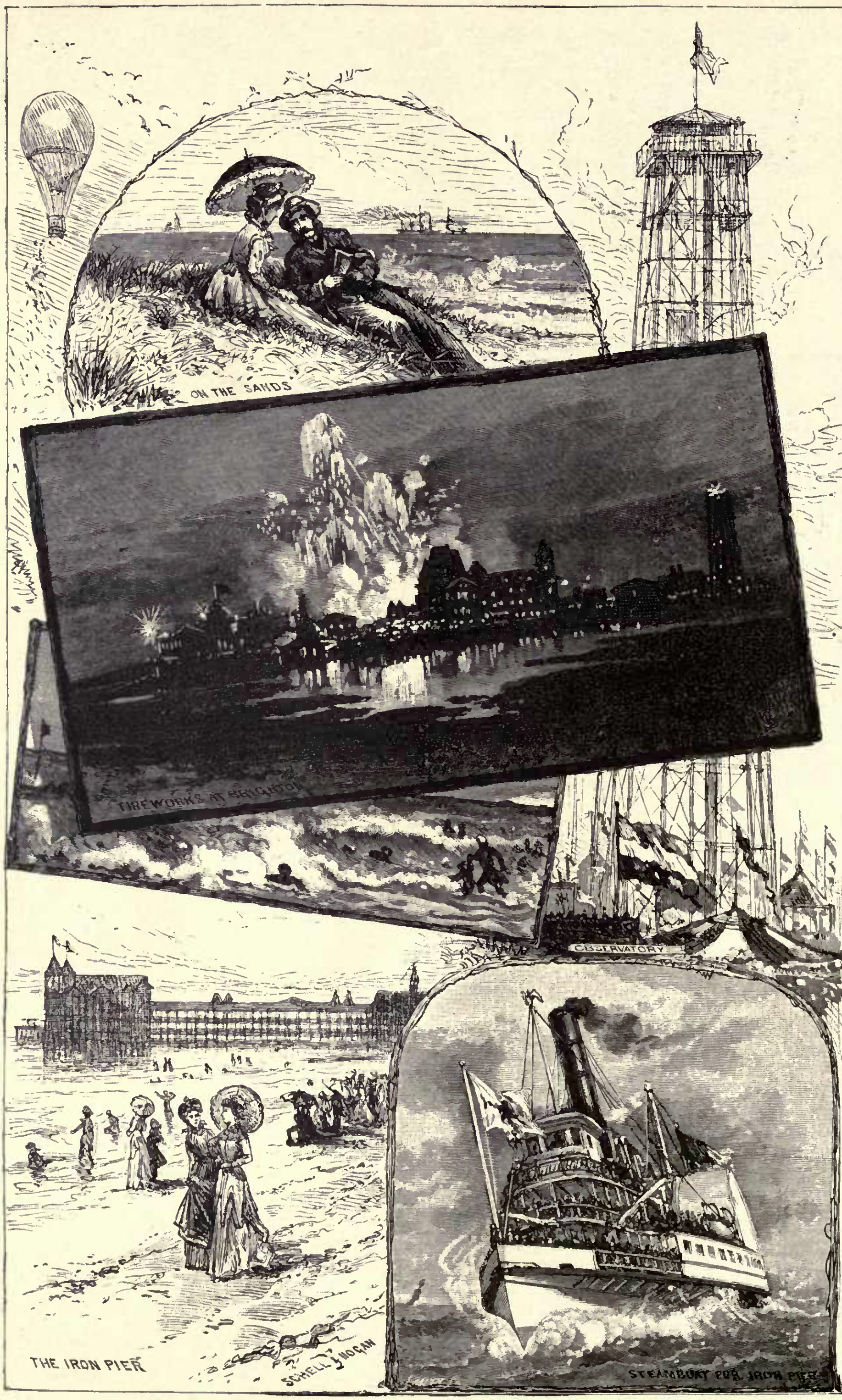
World's Fair, Agricultural Building.



World's Fair, General View.



Coney Island.



Coney Island.

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THE END.

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