NAVAL POSTGRADUATE SCHOOL
INSTITUTE FOR JOINT WARFARE ANALYSIS

STUDIES IN ATTACK OPERATIONS IN THEATER MISSILE DEFENSE

Institute for Joint Warfare Analysis

NPS-IJWA-97-002

STUDIES IN ATTACK OPERATIONS
IN
THEATER MISSILE DEFENSE

Kneal T. Marshall

The Institute for Joint Warfare Analysis
Naval Postgraduate School
Monterey, California

Naval Postgraduate School
(NPS)

The Naval Postgraduate School mission is to increase the combat effectiveness of U.S. and Allied armed forces and enhance the security of the USA through advanced education and research programs focused on the technical, analytical, and managerial tools needed to confront defense-related challenges.

Institute for Joint Warfare Analysis
(IJWA)

The Institute for Joint Warfare Analysis was founded in 1994 with the mission of addressing the problems of the joint defense arena with the academic disciplines resident at NPS. It sponsors a wide ranging research program, curriculum development focused on joint warfare, and interaction with numerous services and DoD organizations.

Opinions, conclusions, and recommendations expressed or implied within are solely those of the authors and do not necessarily represent the views of the Naval Postgraduate School, the Department of Defense or any other U.S. Government agency.

DEDICATION

This document is dedicated to the memory of Lieutenant Commander Mark Alan Ehlers, USN who was lost at sea on 4 February, 1997 when his S3B Viking aircraft went down 80 miles off the coast of Israel while conducting operations from USS Roosevelt. Lieutenant Commander Ehlers graduated with an MS degree in Operations Research from the Naval Postgraduate School in September, 1992. His thesis documented the first research into Theater Missile Defense in the Operations Research Department. Among his important contributions to this field was the linkage he made between the concepts used in antisubmarine warfare and the problem of searching for and destroying mobile missile launchers. These ideas form the underpinnings of much of the material found in this document.

ACKNOWLEDGEMENTS

The research presented in this document has been supported in part by the Institute for Joint Warfare Analysis at the Naval Postgraduate School, Monterey, California, and by the Ballistic Missile Defense Office, Washington, D. C. Their support made these studies possible.

CONTENTS

1:Introduction

2:Linkages between Theater Missile Defense and ASW

3:A Joint Idea: An ASW Approach to TMD

4:Quantifying Attack Operations And Active Defense

5:Searching for Transient Objects along Roads

6:Supression Effects of Searching for Tels

7:A Pre-Hostilities Decision Model

8:Analyses of Unattended Ground Sensors in

1: Introduction

This document contains a number of chapters, all of which are related to understanding and analyzing various aspects of attack operations in Theater Ballistic Missile Defense (TMD). The material has been developed by NPS faculty members George W. Conner, CAPT., USN (Ret), Kneale T. Marshall and James J. Wirtz from our own research and that of our students in the Operations Analysis curriculum at the Naval Postgraduate School during the period July, 1992 through June, 1997: Mark A. Ehlers, Thomas W. Hair, Joseph P. Mattis, Paul A. Soutter, Vernon L. Junker, and Neil E. Fitzpatrick.

The three pillars of TMD are (a) Attack Operations - attacking the missile launch system prior to launch, (b) Active Defense - shooting down the missile at some point in its trajectory, and (c) Passive Defense - protecting the missiles' intended targets by hardening and other means. The models and analyses in this document are all concerned with various aspects of Attack Operations. Past history, from countering the V-2's in World War II through the attempts made to destroy Scud launchers in Iraq during Desert Storm, has shown how difficult it is to succeed when attacking launchers. It seems natural to assume that success in this area be measured simply by the fraction of launchers killed in a given campaign. The fact that the fraction of hard launcher kills has historically been very low, despite considerable attack operations' efforts, has led many to believe that efforts to achieve TMD should be concentrated in Active Defense. Indeed, considerable resources are being spent on difficult and highly technical methods of destroying the missile after it has been launched. The weapon used for Active Defense in Desert Storm was the Patriot missile. Far more sophisticated and expensive systems are currently under development to achieve higher missile kill fractions. These include the Air Force-sponsored laser beam weapon to destroy a missile in its boost phase, the Army-sponsored theater high altitude defense (THAAD) weapon and modifications to the Navy's Standard missile to destroy a missile in its reentry phase, and improvements to Patriot as well as new weapons to destroy a missile in its final phase. Compared to the resources being expended on these programs little is being spent on efforts to improve attack operations capability.

To be successful, attack operations against theater ballistic missiles (TBMs) require the following ordered set of information and procedures: (1) knowledge that mobile missile launchers are operating in a given area, (2) equipment and procedures that can find and identify the launchers from other moving vehicles of lesser interest, and (3) equipment and procedures with which to attack and destroy the launchers or their resupply points either before or after they have launched a missile. This is the sequence of events that has been in use by the Navy in the important and difficult task of countering enemy submarines since the U-boat war in the Atlantic in World War II. Chapter 2 discusses the similarities and differences between the TMD problem and the Anti-Submarine Warfare (ASW) problem. Although the physical and environmental differences between search, identifying and destroying a target on land are clearly different from those used against a target in water, the two missions have a great deal of *structural* similarity. Chapter 3 expands on the ideas in Chapter 2 and argues that, with the ASW model in mind, the attack operations in TMD are essentially a joint operations problem.

Chapter 4 contains an overall model of the TMD problem from loading a mobile launcher, setting up and firing its missile, missile trajectory, and launcher reload to repeat this cycle of operations. This model is based on a U-boat circulation model found in Reference [4] that was first adapted to the TMD problem by Mark A. Ehlers in his MS thesis. The object of the analysis shown there is to quantify the relative contributions of both attack operations and active defense to TMD. It is shown that a modest amount of success in attack operations significantly reduces the numbers of the (very expensive) weapons required for active defense. Not only would modest success in attack operations lead to considerable cost avoidance, it will be necessary to achieve feasible active defense with the number of defensive weapons the U.S. will be able to afford. This material has appeared in Reference [7] and is based in part on results presented in Reference [6].

Having demonstrated a need for successful attack operations the question arises as to how to achieve success in this area given the poor historical performance record. Chapter 5 presents results of models that describe aerial search along road systems. It is demonstrated how the problem is affected by factors such as the traffic density along the road and the mixture of launchers together with other traffic. As is the case in search at sea, the *coverage ratio*, that fraction of the road that can be overflown by a search vehicle while the missile launcher is visible, plays a major role in determining the effectiveness of the search effort.

Although hard-launcher-kills is clearly a major measure of effectiveness, it is not the only one. By this measure, historical records show that attack operations have been mostly unsuccessful. But a deeper study of the data available from Desert Storm indicates that significant launch suppression was achieved by having air search assets operating in the launch areas. Chapter 6 contains a simple model to measure this launch suppression in terms of equivalent missile kill rate, especially for campaigns of short duration.

Chapter 7 models the difficult decision problem of what type of operations should be undertaken in the period before hostilities break out. In particular, when would it pay to make a preemptive strike against launchers if mobilization of the opposing force has been observed? What data is required to analyze such problems and how should the decision maker weigh the intelligence at hand.

Chapter 8 presents models to optimally distribute ground sensors to monitor and detect time critical target traffic on road systems, and to determine the best operating policy to use with these sensors.

Much of the material in this report has appeared in various places, including the open literature, technical reports, MS student theses, and briefing slides. By gathering the various pieces together into a common source it is hoped that this document will be useful both in operations analysis courses and as a source of ideas for further studies to solve what is one the military's top priority problems, TMD.

This work represents an incomplete look at the entire TMD campaign. Additional studies should be undertaken to understand the entire TMD problem. Without this understanding, one cannot realistically and efficiently model, simulate, develop the necessary tactics, or develop an operational campaign to defeat, or at least suppress, this threat.

2: Linkages between Theater Missile Defense and ASW[1]

George W. Conner

Mark A. Ehlers

Kneale T. Marshall

2.1 Introduction

As evidenced by the Persian Gulf War, the short range ballistic missile (SRBM) is a highly effective political weapon even when its direct military effectiveness is, as in the case of the non-guided SCUD missile used by Iraq, relatively low (Reference [1]). Public fears and possible political repercussions created by SCUD missile launches forced the Allies to divert a significant percentage of air sorties, previously scheduled for other missions, to hunt for both fixed launch sites and mobile launchers. The degree to which the Iraqi government measured the success of their SRBM force is thought to be based largely on the capability of continuous SCUD missile launches throughout the war, independent of whether the intended target was destroyed or not. By this measure the mission was highly successful.

The scramble to destroy the mobile SRBM launchers became headline news as the war proceeded and a number of SCUD missiles penetrated the air defenses of the Coalition; a few reached their targets inside Israel and Saudi Arabia. The speculation that Iraq might use chemical warheads on its missiles increased the urgency of the launcher destroy mission. One can assume that other potential third world adversaries noted the success of Iraq's mobile missile force and might view them as an effective weapon system in which to invest. The threat appears to be increasing and will probably become more accurate and lethal with time.

2.2 The Mobile SRBM Counter Effort

The mission of defeating or significantly suppressing the mobile SRBM threat is difficult. Post Desert Storm analyses have revised downward the optimistic war time battle damage assessment (BDA) of a significant percentage of mobile launchers destroyed (Reference [2]). Some reports have indicated numbers close to zero for the estimated number destroyed during the conflict. The

---

[1] This material in this paper is taken from Ref. [6].

effectiveness of the PATRIOT system in defeating incoming missiles is also under debate. All reports agree that the inclusive counter effort failed to produce results that normally indicate mission success.

2.3 An Integrated Approach

Current research and development to counter the mobile SRBM threat is focused primarily on post-missile-launch hardware and tactics to counter both the missiles and the mobile launchers (Reference [3]). Air defense systems such as PATRIOT are being designed to kill incoming missiles. Weapon systems are being developed to allow for greater success in the prosecution of launchers after missile launch cuing data is received, referred to in this thesis as the flaming datum tactic. These approaches assume that the mobile SRBM problem begins *after* missile launch. This report focuses on the benefits and policy development of prosecuting the mobile launchers themselves prior to both missile launch and hostilities.

The analyst familiar with the general principles of anti-submarine warfare (ASW) find the poor results of the counter effort to suppress mobile SCUD launcher activity during the Persian Gulf War to be no surprise. An effective counter effort against a highly elusive target such as a mobile missile launcher or a submarine, should not begin after weapon release, as is the current focus with the SRBM, but well before the threat is in the position to do so. This section introduces the concept of using search tactics prior to missile launch as well as pre-hostility intelligence effort in countering the mobile launcher, and suggests using an existing structure to create an effective counter effort doctrine.

2.4 Anti-Submarine Warfare and SCUD Hunting

The capability to detect, track, classify, and if needed, destroy an enemy submarine has increased dramatically over the last half century. A Second World War air crewman, while visually (and later with the help of radar) searching the thousands of square miles of the Bay of Biscay for German U-boats, would have dismissed as impossible the idea of one day passively tracking a submarine while it is submerged. Today, this is commonplace. The ASW community has been effectively searching for increasingly invisible targets for many years; the lessons have already been learned and, in many cases, can be adapted to counter mobile missile launchers (Reference [8]).

The general principles that provide the structure for the current ASW doctrine have been developed through theory and tested by experience as the submarine gained in capability and stealth sophistication. Although the specific tactics and hardware will be different, many of these principles that have brought success to ASW apply directly to countering mobile missile launchers. Listed below is just a sampling of ASW principles that require consideration, each with a brief statement relating them to the mobile SRBM problem.

> a) **Strong community identification.** Like ASW, the mobile SRBM counter effort requires a dedicated community that is committed to defeating the threat. The predicted diversity of such an effort (possibly from special force units on the ground to satellites in space) will place a need for a strong community identification with a defined focal point for all aspects of the counter effort.
>
> b) **Intense scrutiny of enemy signatures.** Every possible signature, ranging from the obvious (infrared, electromagnetic, etc.) to the not so obvious (seismic, aural, tire patterns, etc.) needs careful examination for potential exploitation. Signatures play a crucial role in both detection and classification of targets.
>
> c) **Understanding enemy tactics.**The ability to predict or estimate the actions of the enemy mobile launcher force is invaluable in developing tactics for specific situations.
>
> d) **Environment considerations.** The environment of the counter effort will change from enemy to enemy, country to country. Future conflicts may not all be fought in a desert environment, as was the Persian Gulf War.
>
> e) **Heavy emphasis on intelligence.** Mobile launcher search without intelligence is much like a needle search in one of many haystacks. Intelligence (HUMINT, ELINT, etc.) can narrow the search to a single haystack, effectively giving the search effort a starting point.
>
> f) **Localization capabilities on many platforms.** The more platforms with the capability to localize a target the better. This increases the probability of a capable unit being in the vicinity of a reported datum and giving the potential target little or no time to evade.
>
> g) **Integrated weapon and sensor platforms.** This extends the last principle to target destruction. It is optimal for the same platform that localizes the threat to be ca-

pable of classification and destruction. This avoids potential time delays and communication failures associated with calling in an attack.

h) Large area search capabilities on a continuous basis. The capability to conduct continuous search of large areas is required to gain initial detection on possible targets. The system conducting the search must then be capable of providing a real time datum to a platform capable of target localization, classification, and destruction.

i) Base watch and choke point tactics. Intelligence effort focused on the locating and subsequent watching of launcher storage bases is vital to determine weapon mobilization and estimating enemy order of battle. A choke point can be thought of as an easily searched area where a target should pass through, usually due to geographic constraints, to get from point A to point B. For mobile launchers, this definition is simply extended to include paths of least resistance; highways and bridges, for example.

j) Tracking of all known threats at all times. Once a mobile launcher is detected and classified, there must be the capability to continue tracking until either hostilities erupt and it can be destroyed or it is no longer considered a threat.

k) Well exercised, coordinated prosecution. An optimal counter effort must combine the capabilities of all services as well as those of our Allies. Joint and NATO exercises are required to ensure all participants involved with the effort are in concert with each other.

l) Quick and successful response to reported datum. This encompasses many of the above principles. Once intelligence is received on a possible target, a capable platform must arrive expeditiously at datum and perform effective localization.

The mobile SRBM counter effort is still in its infancy. It should be thought of in broad terms, not simply as a science and/or engineering problem. The effort, like ASW, is multi-faceted and will need a broad array of disciplines including tactical modeling, risk analysis and decision modeling in addition to science and engineering. The general principles of ASW should be used as a basic structure, or guideline, to ensure the effort is focused in the direction to optimally counter the threat.

Many of the principles listed in this section involve or imply the prosecution of mobile launchers prior to receiving cuing data from a missile launch. Chapter 3 argues that attack operations in TMD will entail use of the various services in joint operations. Chapter 4 points out the benefits to be gained through the inclusion of both pre-launch search tactics and pre-hostility intelligence in the mobile launcher counter effort doctrine through the analysis of a circulation model.

3: A Joint Idea: An ASW Approach to TMD[1]

James J. Wirtz

3.1 Introduction

During the Gulf War, it became increasingly apparent that U.S. forces had failed to destroy Iraqi SCUDs on the ground before they could be launched against targets in Israel and Saudi Arabia. Despite the large number of air sorties devoted to eliminating the SCUD threat, the "flaming datum," used to target mobile missile launchers proved ineffective. Even though aircraft arrived in the general vicinity of a missile site only a few minutes after a missile launch, SCUD crews had plenty of time to "scoot" to predetermined hiding areas before U.S. warplanes arrived overhead.

Since the Gulf conflict, improving the ability of American units to defend themselves against ballistic missiles has remained a priority. The Clinton administration's counterproliferation policy emphasizes Theater Missile Defense (TMD), especially defense against missiles armed with Weapons of Mass Destruction (WMD).[1] The administration has concentrated on developing active defenses, for example upgrading the Army's Patriot missile system, and improving command, control, communications and intelligence (C3I) to counter the regional missile threat.[2] Still, improved active defenses and C3I are only two facets of effective TMD. To succeed, TMD requires both passive defenses and a counterforce capability.[3] Somehow, the services must improve the performance turned in against Iraqi SCUDs during the Gulf War by integrating the four major elements of TMD -- C3I, active defenses, passive defense and counterforce -- into an overall campaign strategy.

Many political issues complicate counterproliferation and TMD.[4] Devising a joint approach to C3I and multi-service air, ground and naval operations, however, poses its own unique set of military problems. In terms of organization and doctrine, TMD is difficult because it is "inherently a joint mission." As the authors of JP3-01.5 "Doctrine for Joint Theater Missile Defense" note, "joint force components, supporting CINCs, and multinational force TMD capabilities must be integrated

---

1. This paper is published in Ref. [8]. The sources referred to in this paper are included as Endnotes at the end of the paper.

toward the common objective of neutralizing or destroying the enemy's Theater Missile capability."[5] Accomplishing this integration, however, is no small task. New hardware, software or a single new weapon will not miraculously solve the TMD problem. What is needed is a "better idea" for organizing multiservice C3I, active defenses, passive defense and counterforce into an effective TMD strategy.

If one is willing to look for this organizing principle in unexpected places, then a tried and true method of destroying targets that rely on mobility and stealth to improve their survivability already exists: Anti-Submarine Warfare (ASW). As strange as it may sound, a TMD architecture based on an ASW philosophy offers a way to integrate the services' various capabilities into a coherent plan to stop an opponent's ballistic missiles from reaching their targets. Applying ASW principles to TMD also represents a novel development in joint warfare. Joint strategy can be achieved by using one service's approach to solving a specific problem as an integrating principle in a multiservice operation. In this case, an ASW approach allows each of the services to integrate what they do best into an overall joint campaign.

To support this argument, the article briefly sketches the fundamentals of ASW operations and applies them to the problem of locating and destroying mobile missiles before they can be launched. It then explains why each of the services should play a role in a TMD strategy inspired by ASW. It also suggests which CINC should take at least peacetime responsibility for promoting the TMD effort. The article concludes with some observations about the role of ideas in joint warfare.

3.2 Anti-Submarine Warfare

At first glance, it would seem easier to find a needle in a hay stack than to locate a submarine in the oceans' vast expanse. But, the U.S. Navy can detect, track, target and destroy submarines as they operate in the open oceans. In theory, the same ASW philosophy used to organize and prosecute attacks against submarines should prove to be effective against missile launchers that also rely on mobility and stealth to improve their pre-launch and post launch survivability.

ASW procedures are often divided into five categories: (1) continuous collection and analysis of intelligence; (2) continuous monitoring of probable launch areas; (3) generation of cueing (warning) when specific platforms move to a launch status; (4) the localization of specific systems; and (5) attack. Organized sequentially, each of these categories represents a stage in the ASW search and attack effort. As one moves from stage one to stage five not only does the area searched become increasingly restricted, but the time available to complete the task at hand becomes more limited. These five stages could form the core elements of a multi-service, multi-mission ASW approach to counterforce strikes against theater ballistic missiles.

Information, critical to the entire counterforce effort, can be gained through sustained collection and analysis of data about all known mobile missiles, the first stage of the ASW process. In tracking submarines, the opponent's inventory is followed by hull number; similar efforts would have to be made to track individual missile Transporter-Erector-Launchers (TELs). Missile production, storage and repair centers would have to be monitored to generate this order-of-battle intelligence; this fundamental intelligence work probably would provide the added benefit of uncovering clandestine installations in the opponents fixed missile infrastructure. This should produce information about the overall size, day-to-day readiness, and surge (alert-generation) capability of the opponent's systems. Training cycles, exercises, support vehicle activity, base egress and ingress and movement through "choke points" (well-maintained roads, heavy duty bridges, rail heads) would also be monitored. These efforts should yield a useful estimate of the general location of the opponent's mobile missiles, creating a baseline to assess deviation in the opponent's standard operating procedures. In effect, stage one creates an indications and warning baseline.

Surveillance of all probable launch areas, the second step in the ASW process, depends upon intelligence gathered about the opponent's overall missile capability: indications of when and where to look for mobile missiles are produced in stage one analyses. In stage two operations, visual signatures of areas of interest would be compared on a regular basis to look for changes (damage to plants, tire tracks or the presence of the weapons systems themselves). Similarly, acoustic, seismic, radar and communication signatures could be compared over time. Of special importance would be "life-support events," the logistical tail that could lead directly to a TEL in the field. Special attention would be paid to likely operating areas and negative search information (indications that terrain features make certain areas unsuitable for SCUD operations) would be used to develop an operating history of the opponent's TELs. Armed with this information, real-time "tracks" of fielded TELs could be monitored as long as possible; thus, a working knowledge of the location of all TELs in or near launch areas could be maintained.

Cueing, the third step in the ASW process, is characterized by intensive efforts to develop a more accurate and detailed track of a specific weapons system. It typically results when a TEL is detected in a launch area or when changes in activities or activity levels indicate that preparations are underway for an actual missile launch. This intelligence could come from a variety of sources. Stage one analyses might yield indications of changes in activity or the general location of a specific system. Stage two surveillance also might detect communication, acoustic or radiation signatures as TELs are made ready to fire. Cueing, however, is best viewed as a transitional step in counterforce efforts against mobile missiles: it is related to a decision by either U.S. authorities or the opponent to move to a war footing. Cueing is intended to establish a detailed track of a potential target, information that would allow for the quick prosecution of an attack.

The decision to engage in the localization (identification of the target's precise location) of cued TELs, the fourth stage of the counterforce operation, will likely be made by the National Command Authority. Although search activities related to cueing might require overflights of an opponent’s territory, localization will require armed aircraft or unmanned airborne vehicles to enter an opponent's airspace, an act of war. Piloted aircraft working to localize an opponent's TELs should possess a defense suppression capability. Localization begins from a starting point identified by intelligence collected and analyzed from the proceeding three stages of the ASW process; because of the short ranges involved, a wide variety of sensors can then be used to generate timely and detailed tracks of the target. Coordination of the platforms involved and fusion (receiving, analyzing and displaying) of the data produced by a variety of sensors plays a crucial role in localizing the target.

Over the years, the Navy also has discovered that practice facilitates localization efforts. The Navy was fortunate because the Soviets had for years provided opportunities to localize real targets on the open ocean. In other words, officers and policymakers cannot expect that the skills, experience, hardware and communication architectures (fusion) necessary to localize a target can be improvised at a moment's notice.[6]

The final step in the ASW process is to attack the target. Ideally, the attacking weapons system would have its own localization sensor. The Navy never carried out this final step during the Cold War, but exercises revealed that coordination and practice increased the likelihood of successful attacks. It would also be important following an attack to verify that the opponent's weapons system had been destroyed: crippled systems could be repaired and subsequently fired. This would be especially important if the mobile missiles under attack were armed with WMD. Ground forces would have to be inserted, deep behind enemy lines, to survey damaged sites or launch vehicles. These forces should be instructed to secure and remove intact warheads or to assess the extent of biological, chemical or nuclear hazards created by successful counterforce strikes. Even though damaged warheads and delivery systems are not militarily valuable, the hazardous materials they contain would still be valuable to terrorists or to enterprising criminals interested in making windfall profits on the black market. Indeed, given the extreme political sensitivity created by the threat of WMD attack, American political leaders will probably expect total certainty when it comes to damage assessments of WMD sites, the kind of certainty that has historically required the presence of ground forces.[7]

In sum, several aspects of an ASW approach to counterforce make it attractive as a framework for the destruction of TELs before missile launch. An ASW approach calls for continuous monitoring of the status and activities of an opponent's military forces. This would not only build order-of-battle and infrastructure intelligence, but it would also provide a basis for indications and warning estimates. An ASW approach also increases the defensive problem confronted by the opponent. Instead of counting on the ability to "shoot and scoot," opponents would have to assume that their forces are being hunted. In a situation when every stray electronic, seismic or acoustic emission might be used to attack a TEL, missile crews might become preoccupied with the defensive task of protecting their missiles. They might not be able to fire with the "hunters" on their trail. Moreover, because it does not rely on "flaming datum," an actual missile firing, to locate an opponent's weapon, an ASW inspired strategy probably is the most effective approach to counterforce. It is the only strategy that suggests that it is possible to locate and to destroy missiles after they have moved to the field but before they can be fired.[8]

3.3 TMD as Joint Warfare

It is unlikely that any one service could successfully undertake all four elements -- C3I, active defenses, passive defense and counterforce -- embodied in Theater Missile Defense. To succeed, an ASW approach to TMD would have to draw on the resources available within the entire U.S. defense and intelligence community. Indeed, the ASW approach to counterforce highlights the fact that TMD is primarily an exercise in peacetime intelligence gathering and analysis. Existing joint doctrine also acknowledges the important role played by national assets used by USSPACECOM, for example, in a joint TMD campaign.[9] An ASW approach, however, could help guide this peacetime collection and analysis by developing a highly specific set of intelligence requirements. New sensors also could be developed to facilitate day-to-day monitoring of potential opponents' mobile missile operations. Most importantly, work could begin to improve C3I between national intelligence resources and the service components that will need real-time intelligence to engage in the hunt for mobile missiles.

Each of the services also has a special role to play in an ASW approach to TMD. Air Force officers, given their expertise in the conduct of strategic bombardment, should be given responsibility for identifying and targeting the infrastructure that supports an opponent's mobile missile operations. To eliminate the possibility of sustained operations, the Air Force should work to destroy the logistical and industrial tail that supports an opponent's deployed missile force. Air Force experience in managing an overall air campaign also would suggest that they are the service of choice to tackle the C3I and resource allocation problems inherent in a massive TMD effort.

Naval officers have more than just expertise in ASW operations to contribute to TMD. Unlike their Air Force counterparts, naval aviators tend not to think in terms of strategic bombardment, but in terms of destroying specific military targets. The Navy should be given the mission of destroying missiles that have already been deployed. Because the Navy's Aegis system will soon possess limited capabilities against ballistic missiles, a Navy carrier battle-group also might serve as a sort of "emergency" TMD force. Naval aviation could conduct counterforce strikes against a few particularly threatening offensive systems while Aegis-equipped ships protect high-value coastal targets.

As the service operating the only demonstrated active defense -- the Patriot missile system -- against ballistic missiles, the Army has an obvious role to play in TMD. Others have been quick to identify the Army's Tactical Missile system, with a forty-kilometer range and antipersonnel/antimaterial submunitions, and the Apache attack helicopter, with a range in excess of 200 kilometers, as ideal counterforce weapons.[10] Less obvious, however, is the important role that ground forces play in an ASW approach to TMD. Ground forces, especially special forces, would prefer to exercise their ability to target and destroy installations and weapons deep behind enemy lines. But their greatest contribution to the TMD effort probably will take the less glamourous form of "policing the battlefield." In other words, ground forces will probably be required to conduct a whole host of operations after suspected missile sites have been subjected to attack. Small teams could guarantee that launchers and missiles damaged by air strikes were not just rendered temporarily inoperable by air attacks but were in fact destroyed. Primitive storage bunkers, difficult to identify from the air, might also be located by ground forces that quickly survey a damaged missile site. Most importantly, WMD warheads, already married to missiles or forward deployed near missile sites, will have to be secured. Even if launchers or missiles have been destroyed by air attack, operable warheads might still be used by an opponent or find their way onto the black market. U.S. forces would also benefit from a quick assessment of the chemical or radioactive hazard created by damaged warheads following a successful counterforce attack.

Who should be in charge of a TMD campaign influenced by an ASW philosophy? Several considerations shape the answer to this question. First, TMD is largely a peacetime intelligence activity. Second, TMD requires continuous coordination of offensive and defensive capabilities possessed by all the services. Third, the demand for TMD is not confined to a particular part of the globe. Regional CINCs must plan for TMD, but it might be more efficient if a separate command prepares TMD packages of multiservice C3I, active defense, passive defense and counterforce capabilities for insertion into a region.

Given these considerations, U.S. Strategic Command (STRATCOM) would be a good choice to head a TMD campaign. STRATCOM's Project "Silver Book," a peacetime effort to compile a TMD counterforce target list, could serve as an initial step in an ASW-inspired TMD strategy.[11] In its former incarnation as the Strategic Air Command, STRATCOM also has much experience in planning massive multiservice air campaigns which relied in part on real-time and national-level intelligence collection and analysis.[12] Alternately headed by Air Force and Naval officers, STRATCOM also brings together a unique combination of talents needed to make a TMD strategy based on ASW principles a reality: a history of planning joint counterforce attacks; an emphasis on large air operations; great familiarity with ASW; sustained intelligence gathering and real-time intelligence collection and assessment; a familiarity with special-forces operations against WMD targets; and a tradition as the primary command for U.S. nuclear operations.

3.4 Ideas and Joint Warfare

When applied to the problem of Theater Missile Defense, an ASW-philosophy provides a unifying idea that identifies goals and specifies tasks. It also supplies all concerned with an image of an entire process, based on extensive Navy experience, that can be used to evaluate how specific single-service initiatives might contribute to an overall TMD campaign. For those interested in fulfilling the scores of interrelated tasks identified in Doctrine for Joint Theater Missile Defense, the idea of ASW might supply a "point of departure": it specifies how one could begin to organize effective multiservice TMD with existing capabilities. In a sense, an ASW philosophy, borrowing a term from the philosophy of science, could serve as a "paradigm" for TMD: it identifies key problems that are in need of a solution; it specifies how one should proceed to overcome these key stumbling blocks; it allocates responsibility for solving specific parts of the problem; and it explains how the achievement of specific small tasks can produce a synergy that overcomes an extraordinarily complex problem.[13]

As a paradigm for TMD, however, anti-submarine warfare does suffer from a serious drawback: the term is forever linked to the Navy as one of its traditional, and quite important, mission areas. During the Cold War, a suggestion that one service possessed the key to American security was likely to provoke an outburst of interservice rivalry. Occasionally, a service endorsed an idea advanced by another to capitalize on political interest in a "war-winning" strategy or capability, but this tactic often backfired. The Navy's grudging recognition of the importance of "strategic bombardment" during the B-36 debate, for example, did not save their "super carrier."[14] Thus, an ASW approach to TMD might be misconstrued as an effort to develop a "single service strategy," a strategy that purportedly allows one service to single handedly win the next war.[15]

Unlike single service doctrines, however, an ASW philosophy is not an exclusionary paradigm. Much like the way the old Maritime Strategy organized all of the forces available to the Navy into a coherent campaign in the event of war along the Central Front, an ASW-philosophy also allows each of the services to contribute what they do best to solving the problem of Theater Missile Defense.[16] At its core, an ASW approach to TMD is a “joint strategy”: its central tenet is that only by working together can the services defend U.S. allies or U.S. forces stationed overseas from the mobile missile threat.

Still, it would be a mistake to underestimate the impact of interservice and intraservice rivalry, despite renewed congressional emphasis on fostering joint responses to security threats. STRATCOM’s Project Silverbook, for instance, has been superseded by a new initiative, The Theater Planning Support Document. Project Silverbook was abandoned apparently after other CINCs objected to what they perceived as STRATCOM’s effort to monopolize planning for counterforce strikes in support of TMD. At a time of shrinking or stable budgets, any effort to prompt a joint and, in this case, a potentially consolidated effort, is likely to meet with great resistance from some quarter of the defense establishment.

3.5 Conclusion

By adopting an ASW paradigm for TMD, the services would be embarking on a new form of joint warfare. Instead of reinventing the wheel, an idea used effectively by one service could be borrowed to address a complex multiservice problem. Indeed, breaking the taboo against borrowing ideas used by other services opens a whole range of possibilities. The danger always exists that some might choose to mimic blindly the capabilities possessed by other services, even though the size of post-Cold War defense budgets probably would greatly reduce the effectiveness of this budgetary tactic. But, just because an idea originates in one service does not mean that it forever must be banished from the effort to foster joint strategy.

3.6 Endnotes

1. Office of the Deputy Secretary of Defense, Report on Nonproliferation and Counterproliferation Activities and Programs May 1994.

2. David Mosher and Raymond Hall, "The Clinton Plan for Theater Missile Defenses: Costs and Alternatives," Arms Control Today Vol. 24, No. 7 (September 1994), pp. 15-20.

3. Kneale T. Marshall, "Quantifying Counterforce and Active Defense in Countering Theater Ballistic Missiles," Military Operations Research Vol. 1, No. 2 (Winter 1994), pp. 35-48; and Warner Schilling, "U.S. Strategic Concepts in the 1970s: The Search for Sufficiently Equivalent Countervailing Parity," International Security Vol. 6, No. 2 (Fall 1981), pp. 67-68.

4. The Clinton administration counterproliferation and TMD policies have raised much debate. Some analysts are concerned about the issues of preemption and preventive war raised by counterforce strategies see David C. Hendrickson, "The Recovery of Internationalism," Foreign Affairs Vol. 73, No. 5 (September/October 1994), p. 34-38. Others are concerned that TMD and the twenty-year old Anti-Ballistic Missile Treaty are on a collision course: active defense systems under development have capabilities that are apparently in violation of treaty provisions see Guy B. Roberts, "An Elegant Irrelevance: The Anti-Ballistic Missile Treaty in the New World Disorder," Strategic Review Vol. 23, No. 2 (Spring 1995), pp. 15-25. Those who champion more traditional nonproliferation strategies, for instance those associated with the Nonproliferation Treaty, believe that counterproliferation generally undermines their efforts to reduce states' incentives to acquire WMD and associated delivery systems see Leonard S. Sector, "No-Nonproliferation," Survival Vol. 37, No. 1 (Spring 1995), pp. 66-85. Others believe TMD will bolster traditional non-proliferation efforts by further reducing states' incentives to acquire and deploy WMD see Jonathan Sears, "The Northeast Asia Nuclear Threat," Proceedings Vol121/7/1,109 (July 1995), pp. 443-46.

5. Doctrine for Joint Theater Missile Defense, JP 3-01.5 (30 March 1994), p. I-2

6. Many successful wartime innovations, for instance, carrier aviation and the development of a U.S. Marine Corps amphibious capability, experienced a long peacetime gestation. Paradoxically, effective wartime innovation is facilitated by prior planning, see Stephen Rosen, Winning the Next War: Innovation and the Modern Military (Ithaca: Cornell University Press, 1991), pp. 76-85.

7. As part of the secret agreement ending the Cuban Missile Crisis, the Kennedy administration requested United Nations' inspections of Cuba to insure that the Soviets had eliminated their WMD capability from the Island. Castro, however, never granted permission for the inspections see Raymond L. Garthoff, Reflections on the Cuban Missile Crisis (Washington, D.C.: The Brookings Institution, 1989), p. 123.

8. For a discussion of the strategic and political benefits provided by this tactical advantage see James J. Wirtz, Counterforce & Theater Missile Defense: Can the Army benefit from an ASW Approach to the SCUD Hunt (Strategic Studies Institute, U.S. Army War College, March 1995).

9. "US forces that are part of multinational commands will normally be supported by national intelligence systems to augment their organic intelligence systems," Doctrine for Joint Theater Missile Defense, p. II-p. 10, and III-11.

10. John Gordon, "An Army Perspective of Theater Missile Defense," Proceedings Vol121/7/1,109 (July 1995), pp. 40-43.

11. Barbara Starr, "STRATCOM sees new role in WMD targeting," (14 January 1995), p. 3.

12. For a description of some of these activities see Ashton B. Carter, John D. Steinbruner and Charles Zraket, Managing Nuclear Operations (Washington, D.C. The Brookings Institution, 1987), pp. 217-352.

13. For a complete explanation of the term paradigm see Thomas Kuhn, The Structure of Scientific Revolutions (Chicago: University of Chicago Press, 1962). For a recent effort to apply the notion of paradigm to explain change in military organizations see Rosen, Winning the Next War.

14. Michael Palmer, Origins of the Maritime Strategy (Washington, D.C.: Naval Historical Center, 1988), pp. 44-52.

15. For a discussion of these "nonstrategies," See Edward Luttwak, Strategy: The Logic of War and Peace (Cambridge: Harvard University Press, 1987), pp. 156-174.

16. For a description of the Maritime Strategy see Linton Brooks, "Naval Power and National Security: The Case for the Maritime Strategy," International Security Vol. 11, No. 4 (Fall 1986), pp. 58-88.

4: Quantifying Attack Operations And Active Defense [1]

Kneale T. Marshall

4.1 Introduction

Chapter 2 (see also Reference [6]) contains a discussion of the similarities between theater ballistic missile (TBM) defense and anti-submarine warfare (ASW). Both missions require searching, detecting, localizing, classifying, and finally attacking the object of interest. A great deal has been learned over the past fifty years on how to accomplish a successful ASW mission, and many of lessons learned are pertinent to combatting TBMs. Notice that ASW was never referred as torpedo defense. Attempts were not made to kill the torpedo in the water; efforts were always concentrated on going after the launcher (the submarine) or the infrastructure necessary for it to operate. The purpose of this paper is to present and analyze a mathematical model of TBM launcher and missile flight operations so that comparisons can be made of the effectiveness of various strategies to counter the threat. The model presented here extends the earlier analysis and results found in Reference [6].

Figure 4.1 shows a schematic of the operations of a TBM launcher and the missile assumed in this report. Launchers are expected to be stored in some fixed storage area. When hostilities are about to commence the launchers will move to a forward area for assembly, fueling and mating with the missiles. From there a launcher will move to its launch area, and after launch will return to the forward area to prepare for the next launch. We assume that each launcher has the potential to launch m missiles, after which it must be taken out of service for an extended time. The reason could be that it must undergo extensive repair and refit, or it could run out of missiles. We also assume that each missile has n (≥ 1) warheads.

In this paper we assume that there are five phases in the TBM operation when the missile system could be attacked. These are

(a) Attack operations

1. Attack of the launcher with mated missile before launch between assembly area and launch site.[2]

---

[1] This paper is published in Ref. [7].

[2] It is understood that a launcher may employ a number of tactics on its way to or from the launch site, such as stopping in hide sites. The model summarizes the effects of these strategies in a single survival or kill probability.

2. Attack of the launcher after missile launch either at the launch site or on return to assembly area.

(b) Active Defense

3. Attack of the missile during the boost phase,
4. Attack the missile on reentry before multiple warheads separate,
5. Attack each warhead in the terminal phase.

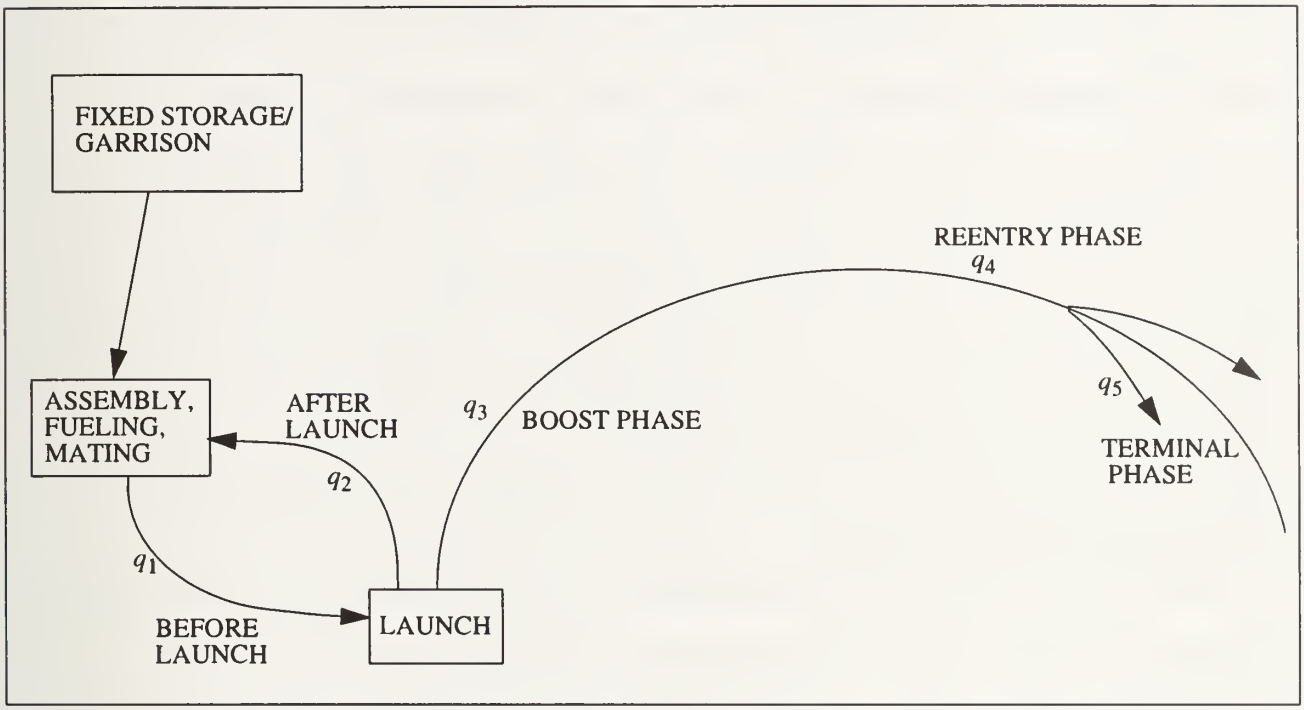


Figure 4.1: Schematic of Theater Ballistic Missile Operations

The effectiveness of attacking the system in each of these five phases is assume to be summarized by a kill probability p_i for the i-th phase, or equivalently by a survival probability q_i, where $q_i = 1 - p_i$. Although it is more usual to formulate a model in terms of kill probabilities, survival probabilities are used because of the simplification that results in model development and presentation of results. Our objectives are to find the probability distribution, mean, and variance of the number of warheads reaching the target area from each launcher, and the expected number of weapons required in each phase, in terms of the maximum number missiles per launcher (m), the number of warheads per missile (n) and the five survival probabilities for the five phases as shown in Figure 4.1. Using expressions for these quantities, we compare the effect of changing the model parameters to demonstrate that attack operations, with effectiveness measured by q_1 and q_2, will almost surely be a necessary part of a layered defense system; without at least a modest success rate in prosecuting the launchers effective active defense may not be feasible.

4.2 The Anti-TBM Model

We build the mathematical model in stages following the missile's path from being mounted on the launcher to its or its launcher's destruction, or the arrival of its warheads in the target area. First we develop the probability distribution, mean and variance of the number of missiles that are successfully launched from a given launcher. These clearly depend on the attack operations effort against the launcher. Next we derive the probability distribution, mean and variance of the number of missiles that survive the boost and reentry phases. Finally we find expressions for the probability distribution, mean and variance of the number of warheads that survive the final phase. The distribution of the warheads surviving to reach the target area is a complex mixture of binomial probabilities. The section ends with numerical examples to illustrate the results. A detailed analysis using the model is presented in the next section.

4.2.1 Launcher Movement Phases

Let X be the number of missiles launched from a given launcher before it is either destroyed or has launched m missiles. We assume independent attacks each time the launcher attempts an outward journey to the launch site, and similarly for each time it attempts an return journey to reload. Thus X is a random variable that can take on any integer value from 0 (the launcher is destroyed on the first outward journey) to m (all attempts to destroy the launcher fail). Note that $X > i$ if and only if the launcher survives the first outward journey, and then survives i succeeding cycles back to the reload point and out again to the launch site. Thus

$$Pr\{X>0\} = q_1$$

$$Pr\{X>1\} = q_1(q_1q_2)$$

$$Pr\{X>2\} = q_1(q_1q_2)^2$$

$$\ldots$$

$$Pr\{X>m\text{-}1\} = q_1(q_1q_2)^{m-1}$$

$$Pr\{X>m\} = 0.$$

The expected value of X is found by summing this cumulative tail distribution,

$$E[X] = \sum_{i=0}^{m-1} q_1 (q_1 q_2)^i = \frac{q_1(1-(q_1q_2)^m)}{(1-q_1q_2)} \quad . \tag{4.1}$$

This equation holds if both $0 \le q_1 < 1$ and $0 \le q_2 < 1$, and is equal to m when both q_1 and q_2 are equal to 1 (zero effect in killing the launcher before or after launch).

To find its variance we need to find its second moment. For a non-negative integer-valued random variable, say N, it is easy to show that

$$E[N^2] = 2\sum_{i=1}^{\infty} iPr\{N > i\} + E[N] \quad ,$$

so

$$E[X^2] = \frac{2q_1(q_1q_2)[1-m(q_1q_2)^{m-1}+(m-1)(q_1q_2)^m]}{(1-(q_1q_2))^2} + \frac{q_1(1-(q_1q_2)^m)}{1-(q_1q_2)} \quad . \tag{4.2}$$

This holds when both $0 \le q_1 < 1$ and $0 \le q_2 < 1$. When both q_1 and q_2 are equal to 1, $E[X^2] = m^2$. We find the variance of X in the usual way by subtracting the square of Equation (4.1) from Equation (4.2).

We now turn to finding the expected number of weapons required in the first two phases. Before attempting to do this it is necessary to make two important assumptions that are assumed to hold in all five phases. First, we assume that every time there is an opportunity to attack the launcher, the missile, or one of its warheads, this opportunity is taken and prosecuted with a single weapon. It may be that in practice more than one weapon is used, so that the numbers determined by the model in this report can be thought of as lower bounds. Second, the extreme case of some kill probability being zero in a given phase can be obtained in one of two ways, either (i) by not attempting an attack during that phase, or (ii) by attacking with a completely ineffective weapon system. In this paper we assume that the first of these is true; any time we use a p_i of zero (q_i of one) in phase i we assume no weapons are expended in phase i. The expected numbers of weapons required should not be interpreted as estimates of weapons requirements in actual operations. In this paper they are intended as an aid in gaining insight when comparing the effectiveness of changing kill probabilities in the various phases.

Let W_{BL} and W_{AL} be the numbers of weapons used in the "before launch" and "after launch" phases respectively against the launcher. Notice that if no missiles are launched, W_{AL} is zero (the launcher was destroyed on its first outward journey). It is easy to show that no matter how many missiles are launched from a given launcher, $W_{\mathrm{AL}} = X$ and its first two moments are given by Equations (4.1) and (4.2).

By following the cycle of the launcher one can see that the cumulative tail distribution of W_{BL} is given by

$$
\begin{aligned}
Pr\{W_{\mathrm{BL}} > i\} &= (q_1 q_2)^i \qquad \text{if } i = 0, 1, 2, \ldots, (m-1), \\
&= 0 \text{if } i \geq m.
\end{aligned}
$$

Thus,

$$E[W_{\mathrm{BL}}] = \frac{1-(q_1 q_2)^m}{1-q_1 q_2} \quad ,$$

and by comparing this with Equation (4.1) we see that

$$\mathrm{E}[W_{\mathrm{BL}}] = E[X]/q_1. \tag{4.3}$$

As our analysis progresses through the boost and reentry phases, expressions are found that require the probability mass function (pmf) of X. From the cumulative tail distribution above this is seen to be

$$
\begin{aligned}
&p_X(0) = 1 - q_1, \\
&p_X(i) = q_1(1 - q_1 q_2)(q_1 q_2)^{i-1} \quad ,i = 1, 2, \ldots, m-1, \\
&p_X(m) = q_1(q_1 q_2)^{m-1}.
\end{aligned} \tag{4.4}
$$

The Boost and Reentry Phases

The boost phase and reentry phase survival probabilities are q_3 and q_4 respectively (see Figure 4.1). Let the number of missiles surviving both of these phases (per launcher) be Y. Clearly this is also a random variable, and if we assume that the attempt to shoot down a given missile in either phase is independent of the outcomes of earlier or later attempts at other missiles, the conditional random variable $[Y|X]$ has a binomial distribution with parameters X and q_3q_4. Thus $E[Y|X] = Xq_3q_4$ and $Var[Y|X] = Xq_3q_4\,(1 - q_3q_4)$. By unconditioning on X, the expected number of warheads surviving the reentry phase is

$$E[Y] = q_3q_4\,E[X] \tag{4.5}$$

where $E[X]$ is given by Equation (4.1). The variance of Y is found using the standard conditional variance argument,

$$Var[Y] = E_X[Var[Y|X]] + Var_X[E[Y|X]],$$

so

$$Var[Y] = q_3q_4\,(1 - q_3q_4)E[X] + (q_3q_4)^2 Var[X], \tag{4.6}$$

where Equations (4.1) and (4.2) are used to find $Var[X]$.

To find the pmf of Y, note that

$$p_{Y|X}(j|i) = b(j;\, i,\, q_3q_4)$$

where $0 \le j \le i$, $0 \le q_3q_4 \le 1$, and $b(j;i,\,p) = \binom{i}{j} p^j (1-p)^{i-j}$ .Unconditioning on X we find

$$p_Y(j) = \sum_{i=j}^{m} b(i;j,\, q_3q_4)\,p_X(i) \quad , \quad j = 0, 1, 2, \ldots, m, \tag{4.7}$$

where the $p_X(i)$'s are given in Equation (4.4).

Let W_B and W_R be the number of weapons used in the boost and reentry phases respectively against the missiles from a given launcher, and assume that exactly one weapon is used against each in each phase. If X survive launch, $W_B = X$ and W_R is a binomial random variable with parameters X and q_3. Thus $E[W_B] = E[X]$, and $E[W_R] = q_3E[X]$.

The Final Phase

In the final phase the probability that a given warhead survives an attack is q_5. Again we assume independence among all attempts to destroy incoming warheads. Let the number of warheads surviving the final phase from the i-th incoming missile be Z_i, $i = 1, 2, \ldots, Y$. Each Z_i is a binomial random variable with parameters n and q_5, so $E[Z_i] = nq_5$ and $Var[Z_i] = nq_5(1-q_5)$. Let the number of warheads surviving the final phase (per launcher) be H, so

$$H = \sum_{i=1}^{Y} Z_i .$$

Conditioning on Y, $E[H|Y] = nYq_5$ and $Var[H|Y] = YVar[Z_i] = nYq_5(1-q_5)$. Unconditioning,

$$E[H] = nq_5E[Y] \tag{4.8}$$

and

$$Var[H] = nq_5(1-q_5)E[Y] + n^2q_5^2Var[Y], \tag{4.9}$$

where $E[Y]$ and $Var[Y]$ are given by Equations (4.5) and (4.6) respectively.

The pmf of H, $p_H(k)$, is found in a similar way by first conditioning on Y. If $Y = 0$ (no missiles survive through the reentry phase) no warheads can reach the target area, so $p_{H|Y}(0|0) = 1$. If $Y = j > 0$, H is the sum of j identically distributed binomials so that $p_{H|Y}(k\,|\,j) = b(k;\, nj,\, q_5)$.

Unconditioning,

$$p_H(k) = \sum_{j=k}^{m} b(k;nj,\, q_5)p_Y(j)\ ,\ k = 0, 1, 2, \ldots, mn, \tag{4.10}$$

where the $p_Y(j)$'s are given in Equation (4.7).

Let W_F be the number of weapons used in the final phase. If Y missiles survive the reentry phase and each carries n warheads, then $W_F = nY$. Thus the results on Y can be used to calculate the measures of interest on W_F.

Figure 4.2 demonstrates the model by showing the cumulative tail distribution of H for three different sets of survival probabilities. For all three cases the number of missiles per launcher (m) is 20, and the number of warheads per missile (n) is 10. The right-most curve is obtained using no (or completely ineffective) counter force ($q_1 = q_2 = 1$), boost and reentry survival probabilities (q_3 and q_4) of 0.7, and a final phase warhead survival probability (q_5) of 0.4. The center curve is obtained by decreasing q_3 and q_4 from 0.7 to 0.6, and q_5 from 0.4 to 0.3. The left-most curve is obtained using the original set of parameters but decreasing both q_1 and q_2 from 1 to 0.9. Clearly a modest increase in kill probability in attack operations from 0 to 0.1 has a dramatic effect on the number of warheads reaching the target area. An increase in kill probability from 0 to 0.1 in the two phases of the launcher shows a drop in the 10-th percentile from 52 warheads to 23, compared to a drop from 53 to 31 for a similar increase in kill probability in the boost, reentry and final phases. Another way to interpret the three curves is to note that the chance of *at most* 20 warheads (10% of a potential of 200) reaching the target area is 3% for the base case. With a given improvement in active defense this increases to 46%, but if that improvement were made in attack operations

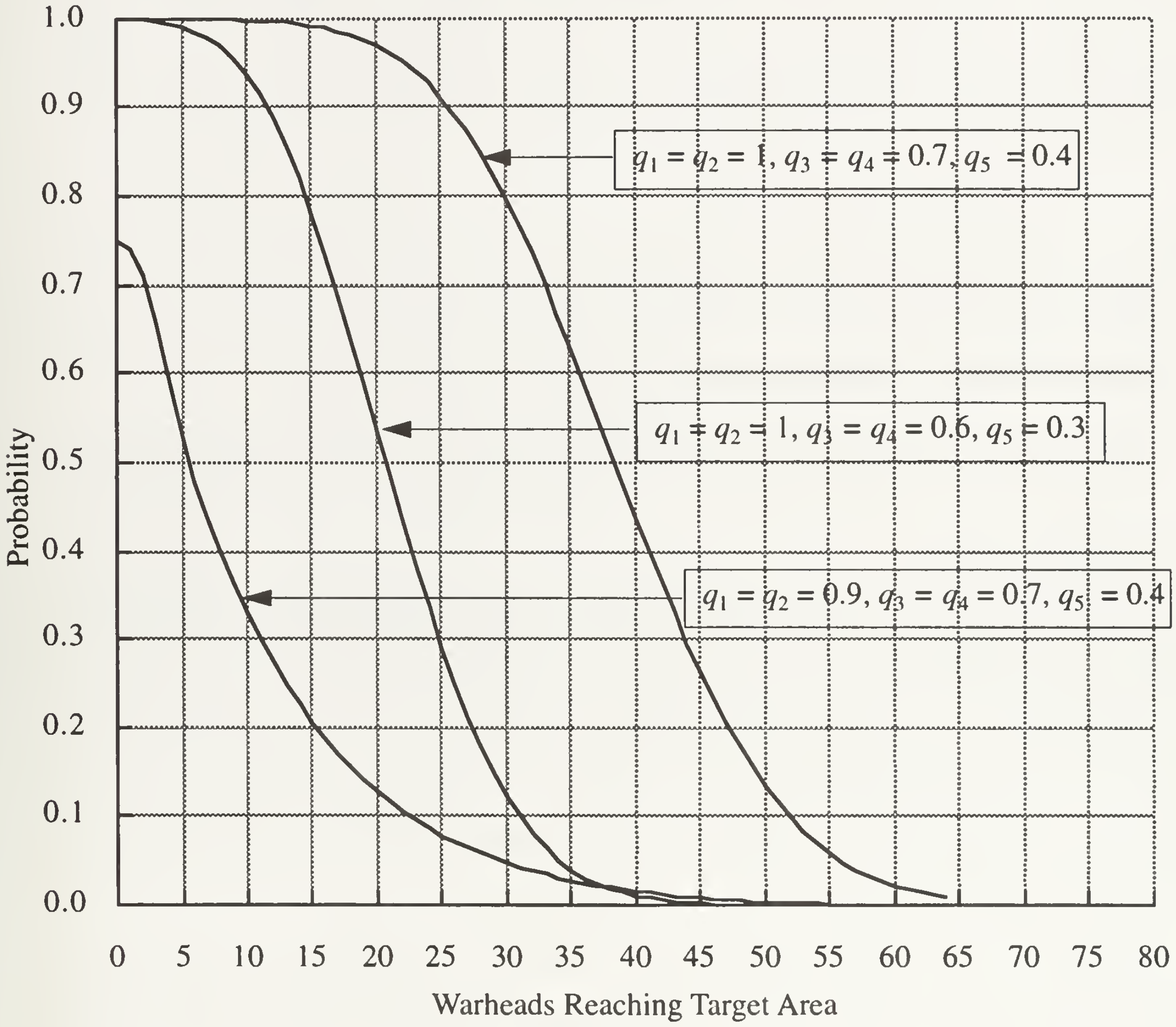


Figure 4.2: Cumulative Tail Distributions of Warheads Reaching Target Area

instead of active defense it would increase to 87%. These numbers are shown in Column 2 of Table 4.1. Columns 3 through 6 show the expected number of weapons used in each phase. A small improvement in attack operations effectiveness sharply decreases the expected number of weapons required for active defense. Note that the zero entries in columns 3 and 4 result from the assumption that when $q_1 = q_2 = 1$, it is assumed that no attack operations is attempted.

Case	$Pr\{H \le 20\}$	$E[W_{BL}]$	$E[W_{AL}]$	$E[W_B]$	$E[W_R]$	$E[W_F]$
$q_1 = q_2 = 1, q_3 = q_4 = 0.7, q_5 = 0.4$	0.03	0	0	20	14	98
$q_1 = q_2 = 1, q_3 = q_4 = 0.6, q_5 = 0.3$	0.46	0	0	20	12	72
$q_1 = q_2 = 0.9, q_3 = q_4 = 0.7, q_5 = 0.4$	0.87	5.2	4.7	4.7	3.3	23

Table 4.1: Sample Output for Numerical Example

The next section contains a more detailed analysis of the model as parameter values are varied.

4.3 Model Analysis

Throughout this section results are demonstrated using kill probabilities p_1 through p_5 rather than survival probabilities, where $p_i = 1 - q_i$. We refer to a kill probability vector which is defined to be $(p_1, p_2, p_3, p_4, p_5)$. For example (0, 0.2, 0.3, 0.5, 0.6) represents no chance of killing the launcher in its outward journey to the launch site, a 20% chance of kill on its return journey to reload, a 30% chance of killing the missile in its boost phase, a 50% chance in its reentry phase, and a 60% chance of killing each warhead in the final phase.

Theater anti-missile defense today consists primarily of the use of the PATRIOT system in the final phase. The navy Aegis ship anti-missile defense system is currently being considered for modification for the reentry phase of the anti-TBM mission, and the army is developing the THAAD (theater high altitude air defense) system for this same phase. The air force is currently developing boost phase systems. Although some work has been done on detecting and destroying launchers prior to or after a launch, operational experience in Desert Storm showed that current systems and operational doctrine are ineffective. This current state can be modeled by setting p_1, p_2, p_3 and p_4 all equal to 0. We can set p_5 at some value depending on how well one believes the

PATRIOT works. As a base case by which to measure possible system improvement we set p_5 to 0.7. Thus

$$\text{Base Case Kill Probability Vector} = (0, 0, 0, 0, 0.7). \tag{4.11}$$

Also as a base case we assume that a launcher can launch at most 20 missiles before requiring major overhaul, or before it runs out of missiles, so $m = 20$.

We look at three measures of effectiveness for the (random) number of warheads arriving in the target area, H. These are (i) the mean $E[H]$, (ii) the median, or that value h such that $Pr\{H \leq h\} = 0.5$, and (iii) the ninetieth percentile, or that value h such that $Pr\{H \leq h\} = 0.90$. We also look at the expected number of active defense weapons required ($E[W_B]$, $E[W_R]$, and $E[W_F]$), and the expected number of attack operations weapons ($E[W_C]$). We first look at today's case where there is only one warhead per missile ($n = 1$), and show how some performance measures are affected by improving kill probabilities in each of the first four phases. This is followed by a similar analysis when multiple warheads are considered.

4.3.1 Single Warhead Analysis

The mean numbers of warheads (and hence missiles since we are assuming one warhead per missile) that arrive in the target area shown plotted in Figure 4.3 as a function of the kill probability at a particular stage. The figure contains three curves. All three start at the same point (0,6) because the expected number of warheads reaching the target area, $E[H]$, is 6 when $m = 20$, $n = 1$, the base case probabilities are given in (4.11), and Equations (4.1), (4.5), and (4.8) are used. We investigate the effect on $E[H]$ of increasing each of the four zero kill probabilities in (4.11) one at a time.

The upper curve is found by increasing the kill probability of either the boost (p_3) or reentry (p_4) phase from its base value of 0 up to 0.8. In either case it decreases linearly with a slope of -6. The middle and lower curves are obtained by increasing p_2 and p_1 respectively over the same range. The difference in the effect of a small increase in kill probability in the attack operations phases when compared to the active defense stages is dramatic; an increase from 0 to 0.1 in either to boost or reentry phases reduces $E[H]$ from 6 to 5.4, whereas this same increase in the either of the attack operations stages reduces it from 6 to approximately 2.5. This significant improvement is caused by the fact that once a launcher (and its crew) is destroyed it can no longer fire missiles, causing a

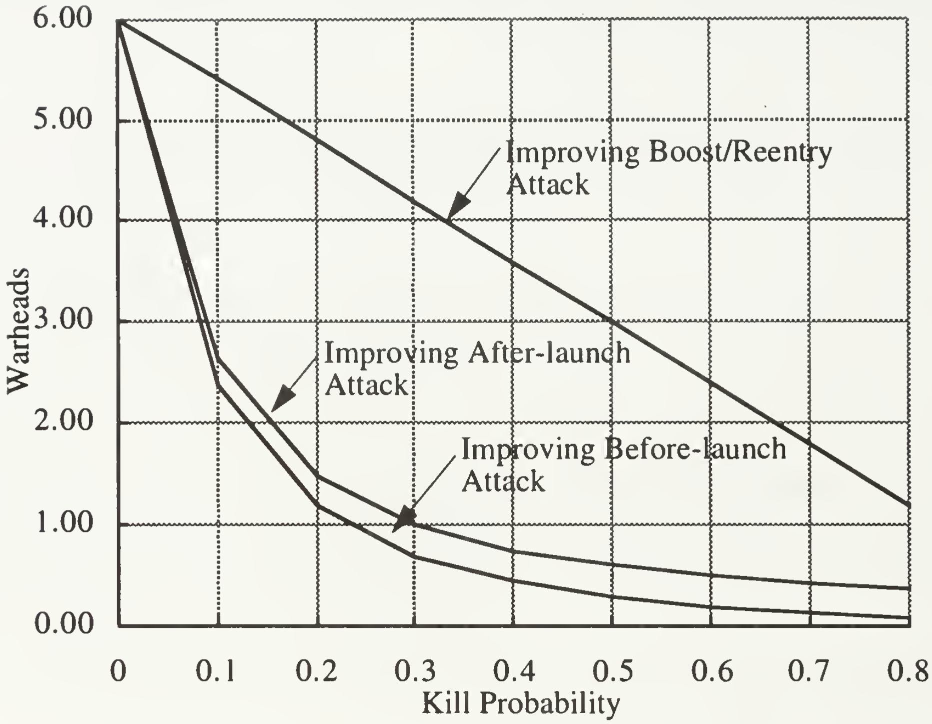


Figure 4.3: Mean Number of Warheads Reaching Target

geometric reduction in $E[H]$. In the active defense stages a kill results in the destruction of only one missile. The small improvement in increasing p_1 rather than p_2 is caused by the fact that keeping p_1 at zero means the first missile from a launcher will be launched for certain, whereas increasing p_1 gives a chance to destroy the launcher before its first missile flies.

Figure 4.4 contains a similar analysis using the median number of warheads reaching the target area rather than the mean. Similar results are found. For the base case the median of H is 5.4. Increasing the boost or reentry kill probabilities from 0 to 0.1 reduces this to 4.8, whereas this increase in p_1 or p_2 reduces it to 1.3 and 1.6 respectively. In other words, using a kill probability vector (0.1, 0, 0, 0, 0.7) there is a fifty percent chance that fewer than 1.3 warheads will reach the target area, whereas using (0, 0, 0, 0.1, 0.7) or (0, 0, 0.1, 0, 0.7) this number is 4.8.

Figure 4.5 contains a similar analysis using the ninetieth percentile of the number of warheads reaching the target. For the base case there is a ninety percent chance that the number of warheads reaching the target area from a given launcher is no more than 8.2. Increasing the boost or reentry kill probabilities from 0 to 0.1 reduces this to 7.6 whereas an increase from 0 to 0.1 in p_1 or p_2 re-

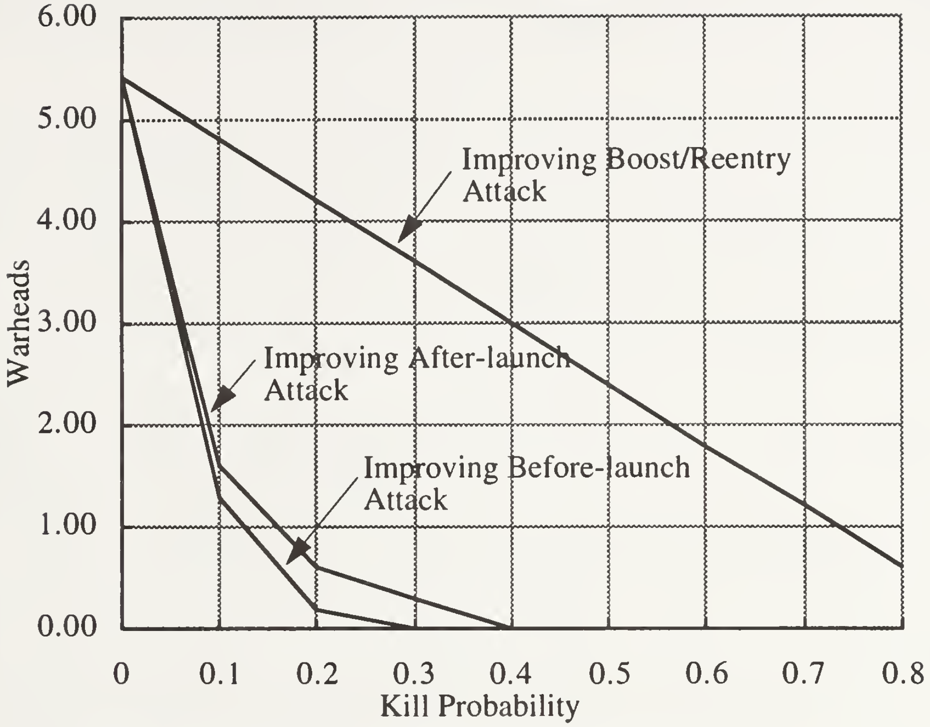


Figure 4.4: Median Number of Warheads Reaching Target

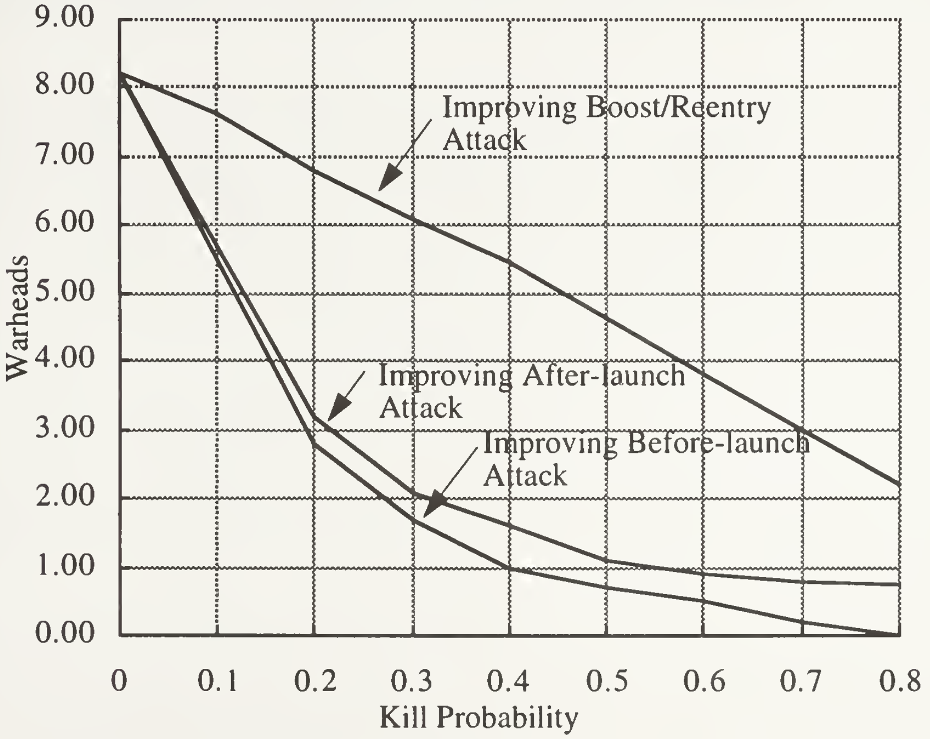


Figure 4.5: Ninetieth Percentile of the Number of Warheads Reaching Target

duces it to 5.5 or 5.7 respectively. Although by using this measure of effectiveness there is less of a difference between improving attack operations and active defense, the difference is still significant.

We now turn to measuring the effects of changing kill probabilities on the expected numbers of weapons used in each phase. Starting from the base case we assume that a zero kill probability in a given phase indicates that no attempt is being made to kill the launcher or missile in that phase. Table 4.2 demonstrates typical results that can be obtained from the model. For the base case the expected number of weapons used per launcher when no attempt is made to destroy the missile before the final phase, and assuming one weapon for each warhead, is equal to the number of missiles time warheads per missile that a launcher can launch. In this example that is 20. Also for the base case the expected number of warhead kills per weapon is equal to the final phase kill probability as should be expected. The remaining rows in Table 4.2 show the effect of increase the kill probability of each phase in turn from 0 to 0.2. Note the dramatic drop in the requirement for weapons in the final phase by having a modest effectiveness in attack operations versus the same effectiveness in the boost or reentry phases. In those phases a modest kill probability significantly *increases* the warhead kills/weapons used ratio.

Kill Probability Vector	$E[W_{BL}]$	$E[W_{AL}]$	$E[W_B]$	$E[W_R]$	$E[W_F]$	Expected Warheads Killed/Weapon
(0, 0, 0, 0, 0.7)-Base Case	0	0	0	0	20	0.70
(0, 0, 0, 0.2, 0.7)	0	0	0	20	16	0.42
(0, 0, 0.2, 0, 0.7)	0	0	20	0	16	0.42
(0, 0.2, 0, 0, 0.7)	0	4.94	0	0	4.94	1.88
(0.2, 0, 0, 0, 0.7)	4.94	0	0	0	3.95	2.11

Table 4.2: Effect of Increasing Kill Probabilities on Weapons Numbers and Effectiveness

Multiple Warhead Analysis

We repeat the analysis using the same base case kill probability vector shown in (4.11) and twenty missiles per launcher ($m = 20$), but in this section we assume each missile carries ten warheads ($n = 10$). The same types of results are illustrated in Figures 4.6, 4.7 and 4.8 as were seen in Figures 4.3, 4.4 and 4.5. In fact since the mean is linear in n the curves in Figure 4.5 are the same as those in Figure 4.3 except the vertical scale has changed by a factor of 10. There is no simple relationship between the median or the ninetieth percentile and n, although over some of the range of the kill probability the relationship appears to be approximately linear. For example, from Figure 4.4 with $n = 1$ we see that a median number 2 for H (90% kill of the twenty possible warheads) can be achieved if p_1 or p_2 are close to 0.08, whereas in the boost or reentry phases we would p_3 or p_4 to be 0.56 to achieve this success. From Figure 4.5 with $n = 10$ we see that a median number 20 for H (90% kill of the two hundred possible warheads) can be achieved if p_1 or p_2 are close to 0.1, whereas in the boost or reentry phases we would p_3 or p_4 to be 0.65. Similarly, from Figure 4.5 we see that to achieve a ninetieth percentile of 2 when $n = 1$ requires either a p_1 or p_2 of about 0.28 or a p_3 or p_4 of 0.81; from Figure 4.5 a ninetieth percentile of 20 when $n = 10$ requires either a p_1 or p_2 of about 0.30 or a p_3 or p_4 of 0.79.

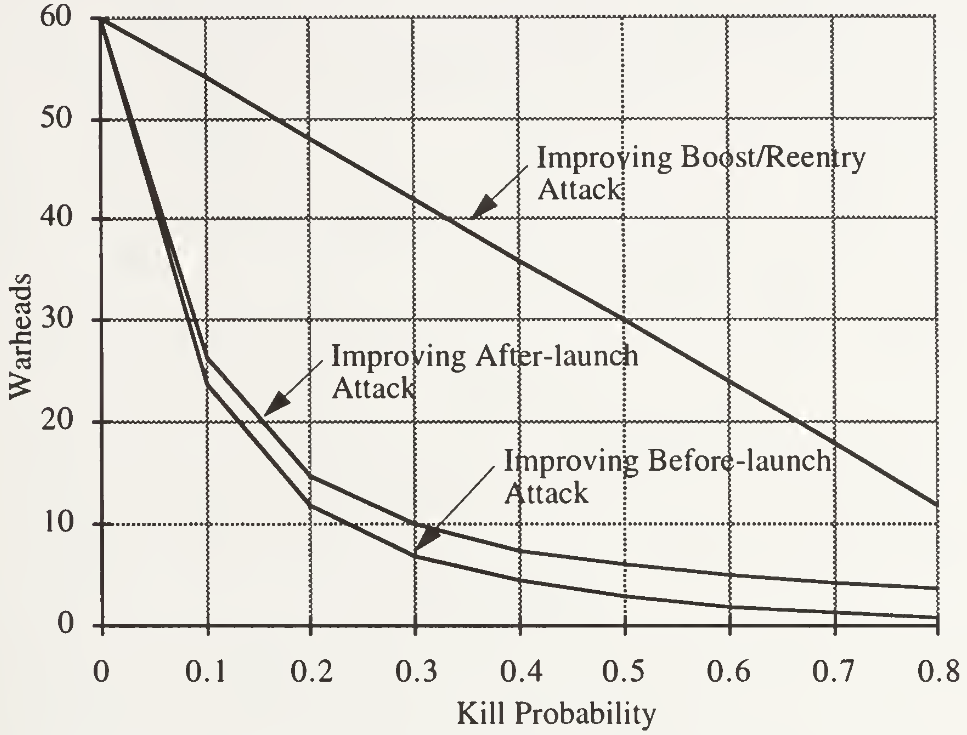


Figure 4.6: Mean Number of Warheads Reaching Target, Ten Warheads per Missile

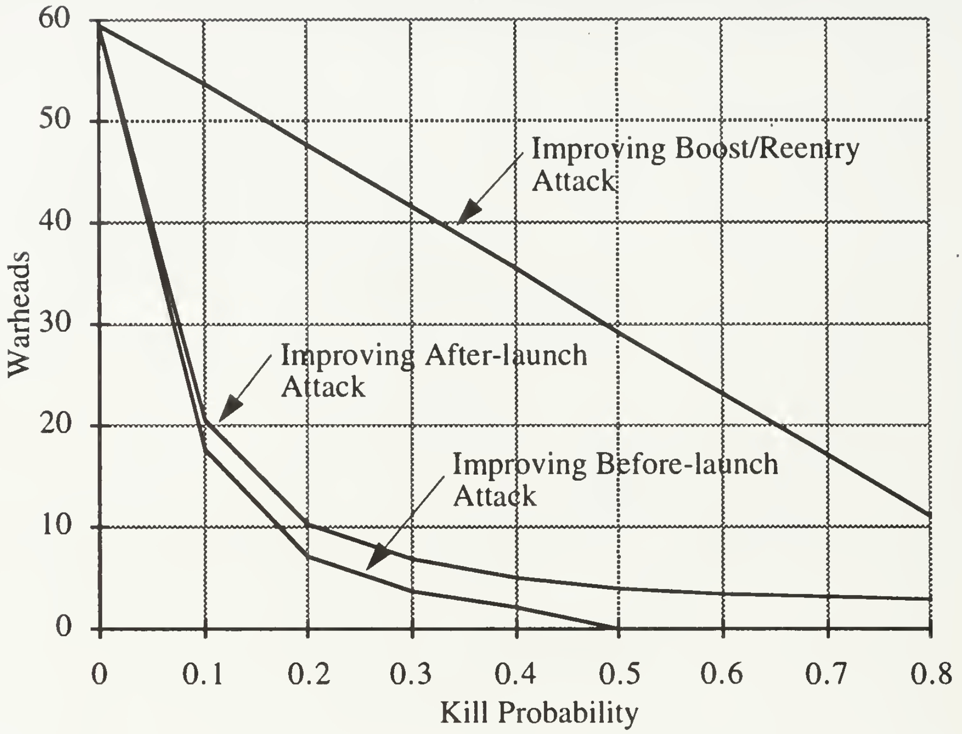


Figure 4.7: Median Number of Warheads Reaching Target, Ten Warheads per Missile

Table 4.3 shows the expected numbers of weapons required at each stage and the expected warhead kills per weapon when $n = 10$. By comparing the results with those in Table 4.2 it is clear that the required expected numbers of weapons at the attack operations, boost, or reentry phases does not change when warheads per missile increase from 1 to 10, but the number of weapons in the final stage increases by a factor of ten. These results should be expected since a successful kill at any phase before the warheads separate is assume to kill all n warheads. Note that the expected number of warheads killed per weapon increases significantly as n increases the earlier one can attack the TBM operation. In other words, attack operations is increasingly effective as the number of warheads carried by the missile increases.

4.4 Normal Approximations

For given values of m, n, and a kill probability vector, it is easy to calculate the expected value of H using Equations (4.1), (4.5), and (4.8); likewise one can easily find the variance using Equations (4.1), (4.2), (4.5), (4.6), and (4.9). But to find percentiles such as the median or the ninetieth percentile requires the distribution function of H, a much more complex calculation using Equa-

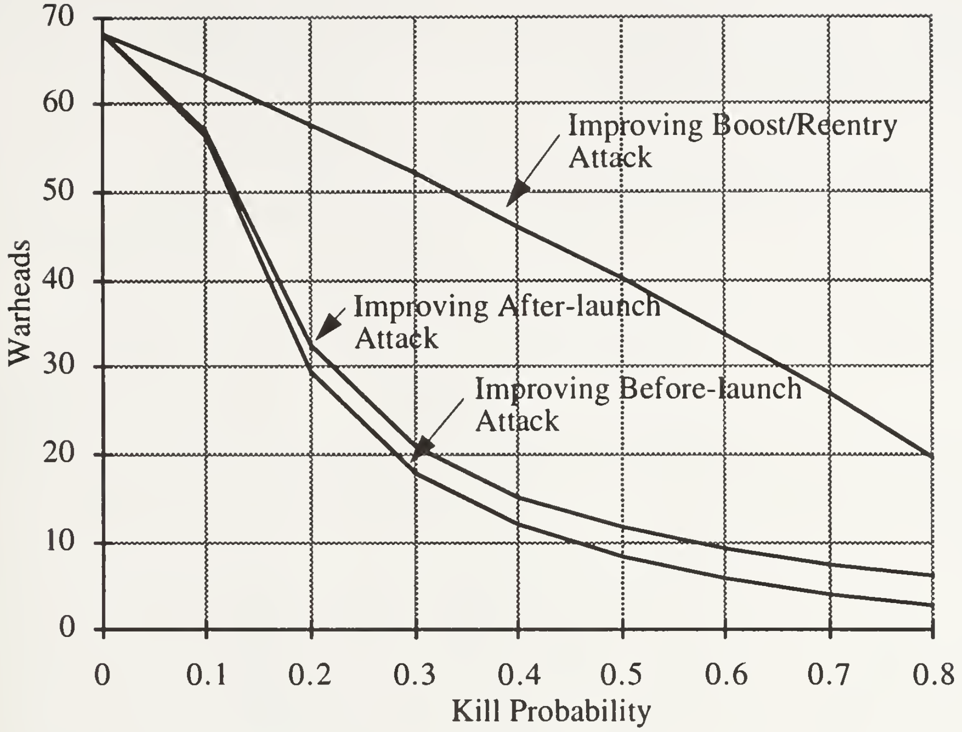


Figure 4.8: Ninetieth Percentile of Warheads Reaching Target, Ten Warheads per Missile

Kill Probability Vector	$E[W_{BL}]$	$E[W_{AL}]$	$E[W_B]$	$E[W_R]$	$E[W_F]$	Expected Warheads Killed/Weapon
(0, 0, 0, 0, 0.7)-Base Case	0	0	0	0	200	0.70
(0, 0, 0, 0.2, 0.7)	0	0	0	20	160	0.84
(0, 0, 0.2, 0, 0.7)	0	0	20	0	160	0.84
(0, 0.2, 0, 0, 0.7)	0	4.94	0	0	49.4	3.41
(0.2, 0, 0, 0, 0.7)	4.94	0	0	0	39.5	4.23

Table 4.3: Expected Weapons Numbers and Effectiveness with Ten Warheads per Missile

tions (4.4), (4.7), and (4.10). These equations were used to find the curves in Figures 4.2, 4.4, 4.5, 4.7, and 4.8. Recall that H is not a simple sum of independent random variables, but results from a complex set of five random events, the first two of which have a truncated geometric distribution, the next two a conditional binomial distribution, and the last is a random sum of these weighted binomials. Even so, one might suspect that its distribution is approximately normal for at least some range of the parameter values, in which case the percentiles can be estimated using only the mean and variance of H. We investigate the appropriateness of a normal approximation for the median and ninetieth percentiles of H in this section.

Since the normal is a symmetric distribution its mean and median are equal. Table 4.4 contains actual medians and normal approximations for the base case and kill probability vectors used in the previous sections, and an example that assumes positive kill probabilities in all five stages. The normal approximation seems to perform reasonably well for the ten warhead case when there are zero kill probabilities in the attack operations stages; it does less well in the single warhead case. When p_1 and/or p_2 are significantly larger than zero, the distribution of H is highly skewed and the normal approximation for the median is poor. The entries NA (not applicable) in the table indicate that the probability that H is zero is larger than 0.5 so that no median value exists.

Kill Probability Vector	Ten Warheads per Missile ($n = 10$)		One Warhead per Missile ($n = 1$)	
	Median	Normal Approximation	Median	Normal Approximation
(0, 0, 0, 0, 0.7)-Base Case	59.3	60.0	5.4	6.0
(0, 0, 0, 0.2, 0.7) or (0, 0, 0.2, 0, 0.7)	47.6	48.0	4.2	4.8
(0, 0.2, 0, 0, 0.7)	10.3	14.8	0.6	1.5
(0.2, 0, 0, 0, 0.7)	7.1	11.9	0.2	1.2
(0.2, 0.2, 0.3, 0.5, 0.7)	NA	2.3	NA	0.2

Table 4.4: Normal Approximation for the Median

Figure 4.9 contains cumulative tail distributions (solid lines) and normal approximations (dashed lines) for the kill probability vectors in Table 4.3 and one warhead per missile. For none of the examples is the normal approximation close to the actual distribution except in the extreme tails. It is particularly poor when there is a positive probability of kill by attack operations.

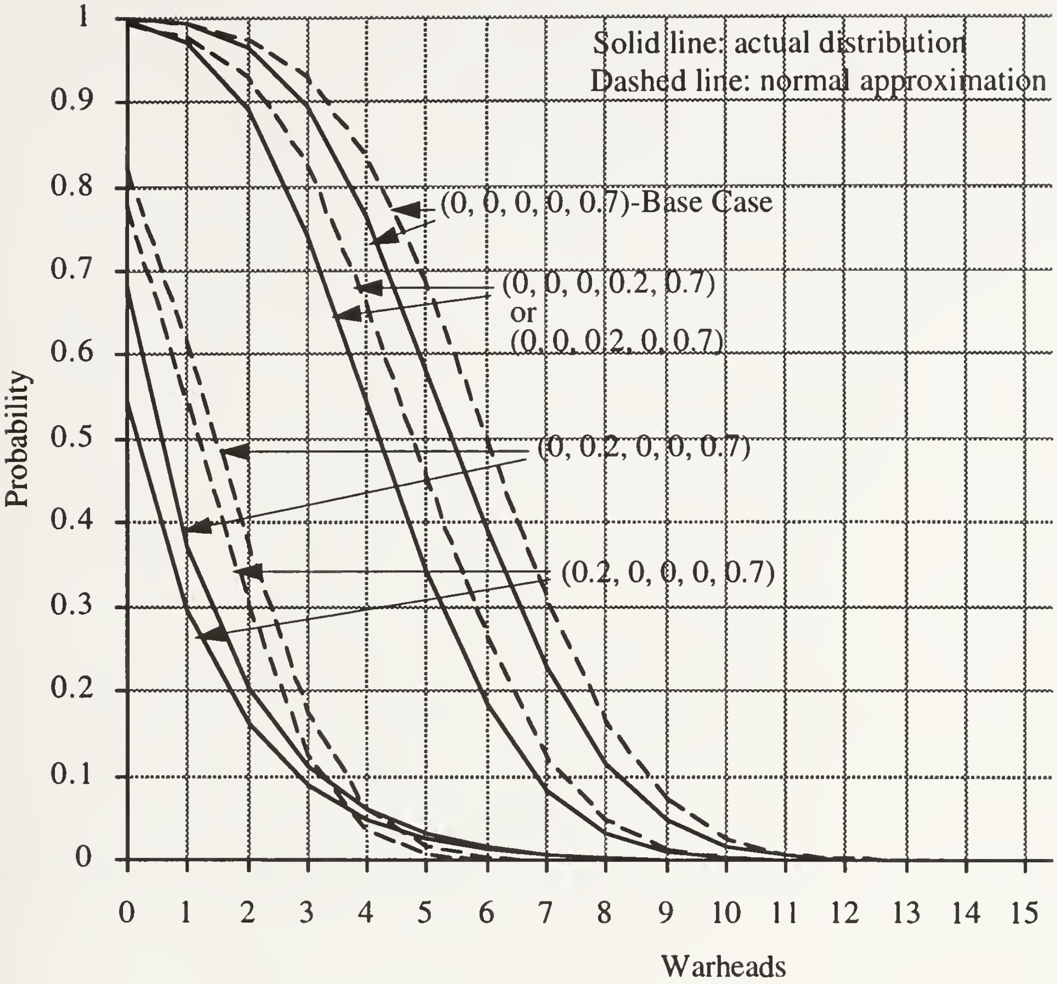


Figure 4.9: Cumulative Tail Distributions and Normal Approximations, $n = 1$

Figure 4.10 contains cumulative tail distributions (solid lines) and normal approximations (dashed lines) for the kill probability vectors in Table 4.3 and ten warheads per missile. When there is no attack operations the normal approximation is close to the actual distribution over the whole range, but again there are significant differences when there is a positive probability of kill by attack operations.

As one might expect the approximation does quite well when H is a fixed (non-random) sum of binomial random variables. Since this number is considerably larger when multiple warheads are present it does significantly better in this case. With positive attack operations probabilities the truncated geometric distribution of the number of missiles launch leads to skewing of the distribution of H. In this case the normal approximation shows significant error.

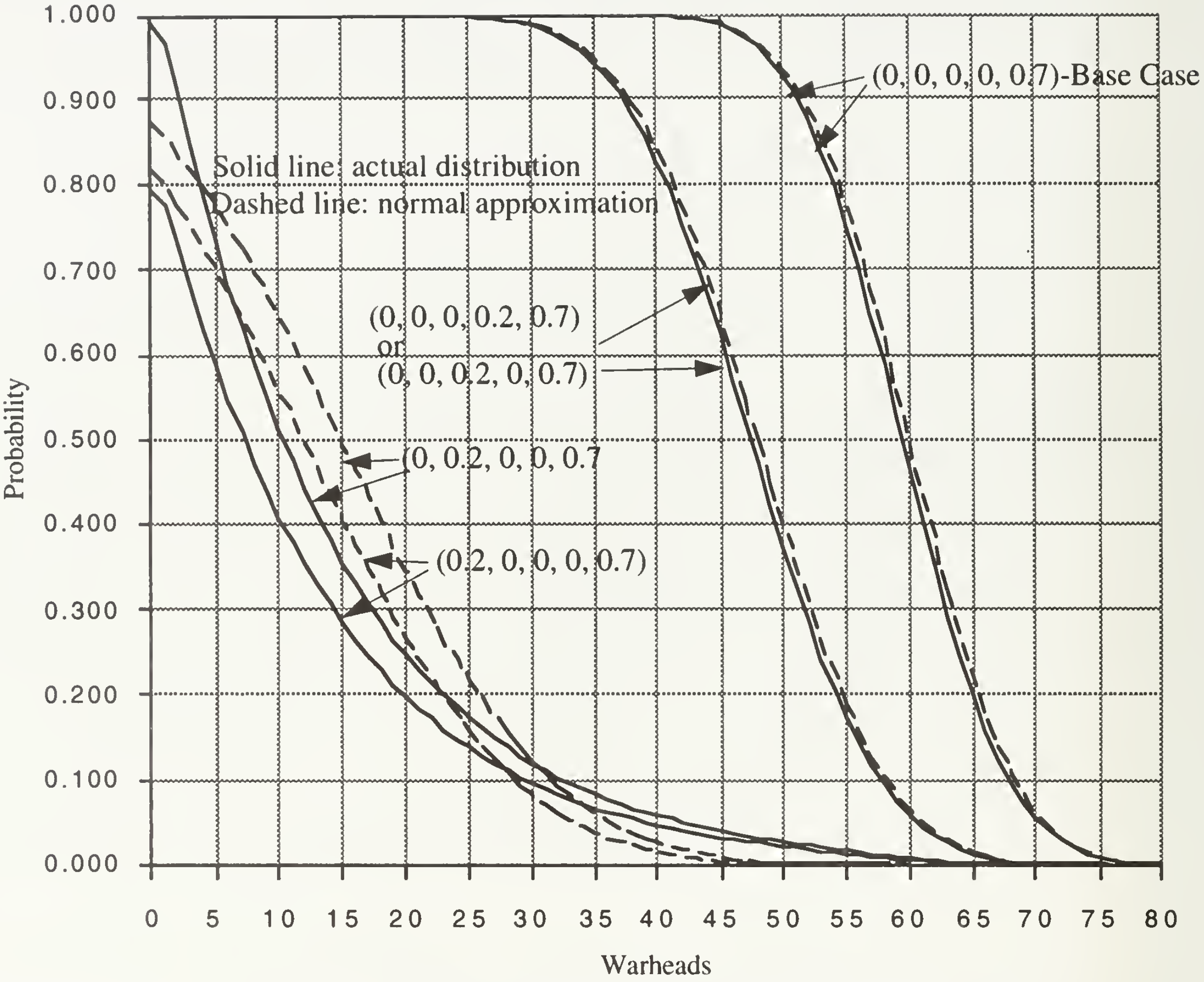


Figure 4.10: Cumulative Tail Distributions and Normal Approximations, *n* = 10

It is not recommended that the normal approximation be used for the median (or equivalently that the median and mean be assumed to take on the same value). Nor is it recommended that it be used as an approximation to the tail distribution unless multiple warheads are assumed to be present and the only significant source of uncertainty is in the final stages of the TBM operation.

4.5 Conclusions

The model in this report shows that both attack operations and active defense will form essential parts of any future successful system for theater ballistic missile defense. Without attack operations it will be relatively easy for the enemy to overwhelm a feasible active defense system. A system that can successfully destroy launchers and their crews will provide considerable leverage in reducing the numbers of active defense weapons required; this leverage increases dramatically as the number of warheads on each missile increases. The model allows the calculation of percentiles of the numbers of warheads destroyed rather than simple expected values.

Past experience in finding and destroying launchers has demonstrated little success in this areas. As was discussed in Chapter 2, success will most likely require a far more structured approach than has been used. A model for such a structure is that used in anti-submarine warfare where great experience has been gained in the past fifty years at finding and destroying torpedo underwater missile launchers. It is expected that successful attack operations against launchers on land will require efforts in cuing, search, detection, localization, classification and destruction. Current operations can be thought of as attempting to skip from cuing (for example, flaming datum information after launch) to attack.

5: Searching for Transient Objects along Roads[1]

Kneale T. Marshall

5.1 Introduction

The Transporter-Erector-Launcher (TEL) is the vehicle used to deliver theater ballistic missiles to their launch sites, set up and launch the missile, and return to a staging area for reloading. A TEL repeats this cycle for further missile launches until it runs out of missiles, is taken out of service for maintenance and repair, or is destroyed at some point in its cycle by attack operations. This paper presents a model for determining the probability of detecting a TEL during the limited time it is exposed during its reload/launch cycle, using an overhead aerial search platform that can monitor vehicular traffic on or close to a road.

It is assumed that a search vehicle (referred to as the searcher) patrols a road segment continuously; when it reaches one end of the road segment it reverses direction and repeats the search to the other end, after which it repeats the process. In order to detect a target it is assumed that the searcher must overfly the target during the period the target is exposed, and also must correctly identify it as a target while flying over it. Since TELs typically travel short distances relative to the length of road being patrolled we assume in this paper that the TEL remains essentially stationary during the time it is exposed.

5.2 Overfly Probabilities

Let the length of the road be l, the (constant) speed of the searcher v, and the length of time the TEL is exposed t. Let the fraction of the road that can be overflown during the TEL exposure time be x, so

$$x = \frac{vt}{l} \; . \tag{5.1}$$

---

[1] This paper is based on concepts found in theses written by Vernon L. Junker and Paul A. Soutter.

The quantity x plays a major role in the analysis. Note that it takes on the value 1 only if the searcher can overfly the entire road length once during the exposure time t. Accordingly x is referred to as the *coverage ratio*.

Note that if $x \geq 2$ the searcher will certainly overfly the exposed TEL no matter what the relative positions along the road the searcher and the TEL are when the TEL exposure time starts. For $x < 2$ it is possible that when the TEL exposure time starts, the TEL and searcher are located relative to each other in such a way that overflight by the searcher cannot occur. The purpose of this section is to determine the probability distribution of the number of times the searcher overflies the exposed TEL as a function of the coverage ratio x.

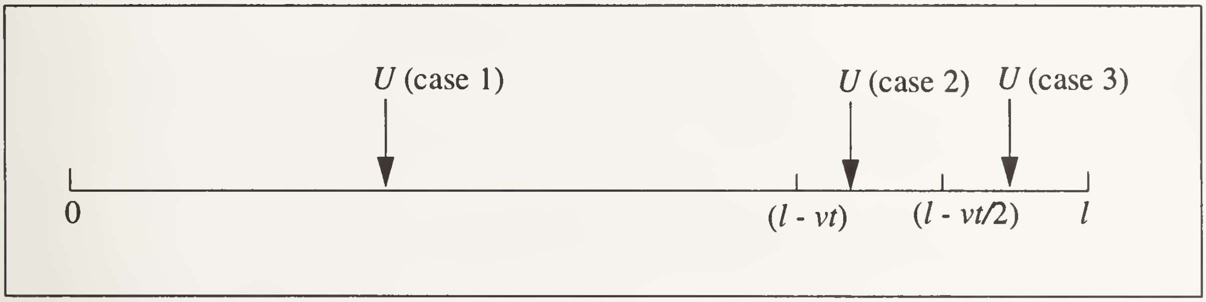


Figure 5.11: Schematic of Road and Searcher Position when TEL appears

First consider the case where $0 \leq x \leq 1$ ($vt \leq l$). Figure 5.11 shows a schematic of the road for this case. Assume that

A1: TEL exposure times occur as a stationary Poisson process,

A2: When TEL exposure occurs the TEL is located according to an independent stationary Poisson process along the road.

A3: The searcher patrols over the road at constant speed v.

These three assumptions imply that when a TEL exposes itself the position of the searcher and the position of the TEL are independent random variables, each uniformly distributed over $(0, l)$. A further assumption is made that when TEL exposure starts the searcher is traveling from left to right in Figure 5.11; since the problem is symmetric, this assumption causes no loss in generality.

Let U be the (uncertain) location of the searcher when TEL exposure occurs, measured from the left end of the road. If U is in the interval $(0, l - vt)$ it will overfly the TEL once only if the TEL's location is in the interval (U, $U + vt$). Note also that it can never overfly it twice when U is in this interval. These observations do not hold if U is in either of the intervals ($l + vt$, $l + vt/2$) or ($l + vt/2$, l). Each interval must be considered separately.

Let $p(i; x)$ be the probability the searcher overflies the TEL i times while it is exposed, given the coverage ration x, $i = 0, 1, 2$. To calculate this, first condition on the position of the searcher at exposure time by letting $U = u$, and let $p(i\ ; u, x)$ be this conditional probability.

Case 1: $0 \le u \le l - vt$. The searcher will overfly the TEL once if and only if the TEL is located in the interval (u, $u + vt$) when it becomes exposed, so

$$p(1; u, x) = \frac{vt}{l} = x. \tag{5.2}$$

The searcher cannot fly over the TEL twice in this case, so

$$p(2; u, x) = 0, \tag{5.3}$$

$$\text{and } p(0; u, x) = 1 - x. \tag{5.4}$$

Case 2: $l - vt \le u \le l - vt/2$. The searcher will overfly the TEL once if and only if the TEL is located in the interval (u, $2l - u - vt$) when it becomes exposed, so

$$p(1; u, x) = \frac{2(l-u)-vt}{l} = 2\left(1-\frac{u}{l}\right)-x. \tag{5.5}$$

The TEL is overflown twice in this case when it is in the interval ($2l$ - u - vt, l), so

$$p(2; u, x) = \frac{u-l+vt}{l} = x-\left(1-\frac{u}{l}\right), \tag{5.6}$$

$$\text{and } p(0; u, x) = \frac{u}{l}\ . \tag{5.7}$$

Case 3: $l - vt/2 \le u \le l$. The searcher will overfly the TEL once if and only if the TEL is located in the interval ($2l - u - vt$, u) at exposure time, so

$$p(1; u, x) = \frac{2u-2l+vt}{l} = x-2\left(1-\frac{u}{l}\right). \tag{5.8}$$

The TEL is overflown twice in this case when it is in the interval (u, l), so

$$p(2;\, u, x) = \frac{l-u}{l} = 1 - \frac{u}{l}\,, \tag{5.9}$$

$$\text{and}\ \ p(0;\, u, x) = 2 - \frac{u}{l} - x\,. \tag{5.10}$$

By unconditioning on u (recall it is uniformly distributed over (0, 1)), if $0 \le x \le 1$,

from equations (5.4), (5.7) and (5.10),

$$p(0;\, x) = 1 - x + \frac{x^2}{4}\,, \tag{5.11}$$

from (5.2), (5.5) and (5.8)

$$p(1;\, x) = x - \frac{x^2}{2}\,, \tag{5.12}$$

and from (5.3), (5.6) and (5.9)

$$p(2;\, x) = \frac{x^2}{4}\,. \tag{5.13}$$

Equations (5.11), (5.12) and (5.13) give the distribution of the number of overflights when the coverage ratio x is between 0 and 1. By again conditioning on u it is straightforward to show that the same expressions also hold for $1 \le x \le 2$. Again one needs to consider three cases, but now the intervals of interest are $(0, l - vt/2)$, $(l - vt/2, 2l - vt)$, and $(2l - vt, l)$.

By looking at each successive interval of length 2 one can show that:

For $2(n-1) \le x \le 2n$, $\quad n = 1, 2, 3, \ldots.,$

$$\begin{aligned} p(2n-2;\, x) &= p(0;\, x - 2(n-1)), \\ p(2n-1;\, x) &= p(1;\, x - 2(n-1)), \\ p(2n;\, x) &= p(2;\, x - 2(n-1)) \end{aligned} \tag{5.14}$$

$$p(j;\, x) = 0 \qquad \text{otherwise.}$$

Thus the expressions in (5.11), (5.12) and (5.13) are all that are required to find the distribution of the number of overflights for all feasible values of x. Figure 5.12 shows the distribution of overflights for all values of the x between 0 and 2. Note that at every point x the corresponding points on the three curves add to 1.

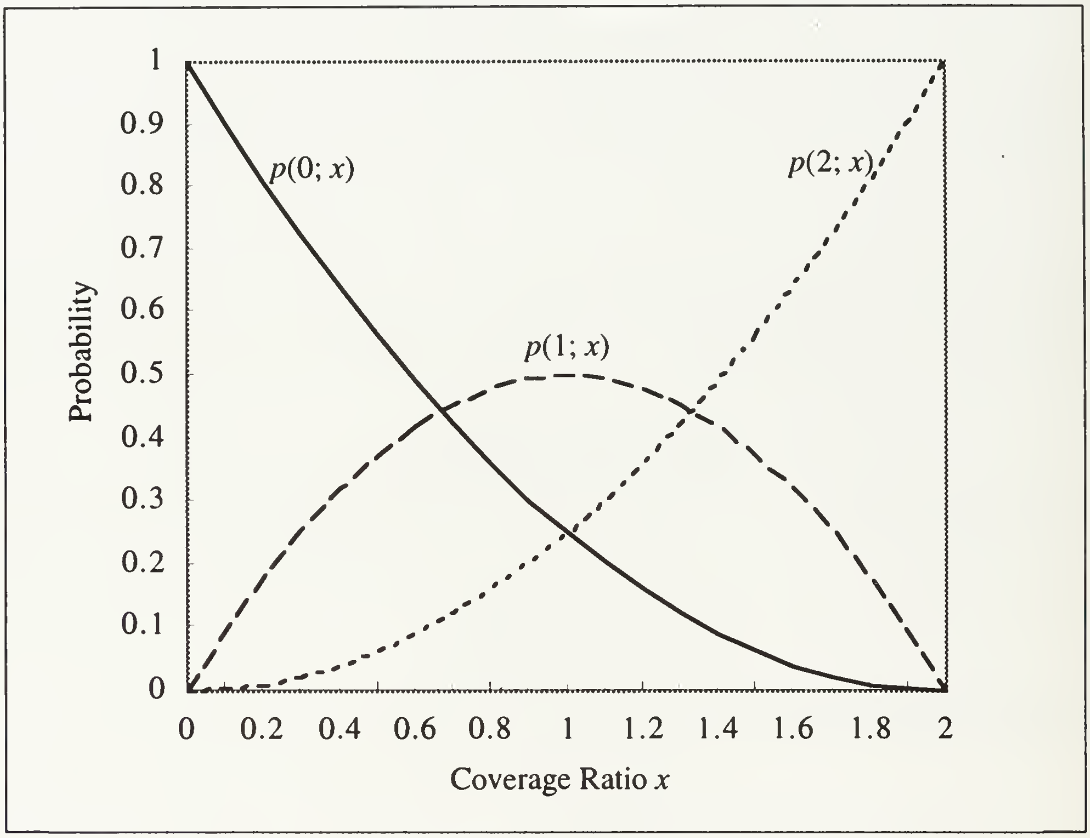


Figure 5.12: Overflight Probabilities when coverage ratio is less than 2

5.3 TEL Detection Probabilities, Single Overflight

Since sensors are not perfect, a vehicle that is being overflown by a sensor platform may or may not be correctly identified. The sensor may indicate that a certain vehicle type is present even though there is no vehicle beneath the sensor platform. We classify all vehicles on the road to be one of three types, a TEL, a decoy made specifically to appear to be a TEL, or some unrelated vehicle of no interest. In this section the objective is to find the probability that, given the sensor indicates a a certain type of vehicle is being overflown, that in fact what is beneath the search platform is indeed a TEL. This probability depends on both the performance capabilities of the sensor[2] aboard the search vehicle, and the traffic environment along the road being searched.

Let V be a random variable that denotes the actual vehicle type being overflown, where V can take on the outcomes t, d, o or ø for TELS, DECOYS, other vehicles and "no vehicle under the search platform", respectively. On any single overflight by the searcher the outcome of the sensor on board the searcher is a random variable S that has possible outcomes "t", "d", "o", and "ø". The quotes are used to help the reader distinguish the outcome of S from the outcome of V. For example, if $V = \text{t}$, a real TEL is being overflown, whereas if $S = $ "t", the sensor signals that a TEL is being overflown; with an imperfect sensor it may or may not be a real TEL. The outcome $S = $ "ø" indicates that the sensor does not identify any type of object as being overflown.

Consider a single overflight of a vehicle. Let $p_{S|V}(s \mid v)$ be the probability that the sensor indicates s, given that the vehicle type being overflown is v, for $s \in$ {"t", "d", "o", "ø"} and $v \in$ {t, d, o, ø}. These conditional probabilities would be determined by calibrating the sensor against known object types in controlled experiments. Table 5.5 shows an example of these conditional probabilities for a sensor that is reasonably discriminating. When used against real TELs it correctly identifies them 70% of the time; it indicates they are decoys 15% of the time, other vehicles 10% of the time, and gives no indication of a vehicle 5% of the time. It does not perform as well against decoys or other vehicles. For example, it correctly identifies decoys only 50% of the time and other vehicles 40% of the time. It indicates a vehicle is present 1% of the time when no vehicle is present.

Sensor outcome (S)	Vehicle type (V)			
	TEL (t)	Decoy (d)	Other (o)	None (ø)
TEL ("t")	0.70	0.30	0.20	0.003
DECOY ("d")	0.15	0.50	0.30	0.003
OTHER ("o")	0.10	0.15	0.40	0.004
NONE ("ø")	0.05	0.05	0.10	0.990

Table 5.5: Sensor performance against known vehicle types

---

[2] The term sensor can be interpreted to mean a suite of sensors working together to identify TELS.

Note that the columns each add to 1.0 since the elements in the table are probabilities conditioned on each vehicle type.

How well this sensor performs in a given situation depends on the mixture of vehicle types present and traffic density, that is the vehicular environment. Let $\mathbf{p}_V$ be the probability mass function in vector form of vehicle types in the order t, d, o. For example, $\mathbf{p}_V = (0.3, 0.3, 0.4)$ indicates that of all the vehicles on the road, 30% are TELs, 30% are DECOYS, and 40% other vehicles. Note the elements of this vector always add to 1.

Let f be the fraction of time there is a vehicle beneath the sensor platform so that f measures the traffic density. An f value of 0.1 indicates that on average only 10% of the road is covered by traffic at any time, whereas a value of 0.9 would indicate an extremely busy road.

Now we are in a position to find the conditional probability $p_{V|S}(v \mid s)$ that the vehicle type is v, given that the sensor indicates type s. These are found from $\mathbf{p}_V$, f and the probabilities in Table 5.5 using Bayes' rule,

$$\begin{aligned} p_{V|S}(v|s) &= \frac{p_{S|V}(s|v)p_V(v)f}{p_S(s)}, \text{ if } v = \text{t, d, o}, \\ &= \frac{p_{S|V}(s|\emptyset)(1-f)}{p_S(s)} \text{ if } v = \emptyset, \end{aligned} \tag{5.15}$$

where $p_S(s) = (p_{S|V}(s \mid \text{t})p_V(\text{t}) + p_{S|V}(s \mid \text{d})p_V(\text{d}) + p_{S|V}(s \mid \text{o})p_V(\text{o}))f + p_{S|V}(s \mid \emptyset)(1 - f)$.

The use of Equations (5.15) is illustrated with the following three examples that are used throughout the paper.

(i) TELS and Decoys Only

Let $\mathbf{p}_V = (0.8, 0.2, 0)$ and $f = 0.2$.This traffic vector indicates that the only vehicles on this road are TELs and decoys in a ratio of 4:1. the value of f indicates that at any point in time only 10% of the total road length is covered by traffic. For these conditions of light traffic that consists of only TELS and decoys the conditional probabilities of all possible sensor outputs and vehicle types calculated using equations (5.15) are shown in Table 5.6. First, note that there is no column for other vehicles; conditional probabilities are not well defined for events that occur with probability 0. Second, note that because the conditioning event is indicated by the row, the row sums are each equal to 1. Third, note that whenever the sensor indicates a TEL the vehicle is a TEL 88% of the

Sensor outcome (S)	Vehicle type (V)		
	TEL (t)	Decoy (d)	None (ø)
TEL (“t”)	0.88	0.090	0.02
DECOY (“d”)	0.52	0.43	0.05
OTHER (“o”)	0.63	0.24	0.13
NONE (“ø”)	0.01	0.00	0.99

Table 5.6: Sensor performance when $\mathbf{p}_X$ = (0.8, 0.2, 0)

time; 9% of the time it is a decoy and 2% of the time no vehicle is present. Fourth, note that a TEL could be present with probabilities 0.52, 0.63, and 0.01 if the sensor indicates a decoy, other vehicle, or fails to indicate any type of vehicle, respectively.

(ii) Approximately Equal Mixture of TELs, Decoys, and Other Traffic

For this example let $\mathbf{p}_V = (0.3, 0.3, 0.4)$ and $f = 0.2$; the traffic density on the road is the same as that used in example (i). The results for this case are shown in Table 5.7. These results show the effect of dilution, the reduction of the fraction of TELs in the traffic. When the sensor indicates a TEL, the vehicle being overflown is now a real TEL only 54% of the time, as compared to 88% in Example (i). But note that when the sensor indicates a decoy it is a TEL only 14% of the time compared to 52% of the time in Example (i).

Sensor outcome (S)	Vehicle type (V)			
	TEL (t)	Decoy (d)	Other (o)	None (ø)
TEL (“t”)	0.54	0.23	0.20	0.03
DECOY (“d”)	0.14	0.46	0.36	0.04
OTHER (“o”)	0.12	0.18	0.64	0.06
NONE (“ø”)	0.004	0.004	0.010	0.982

Table 5.7: Sensor performance when $\mathbf{p}_X$ = (0.33, 0.33 0.34)

(iii) Small Proportion of TELS

Let $\mathbf{p}_V = (0.05, 0.05, 0.9)$ and $f = 0.2$. Here 90% of the traffic consists of other traffic, so the problem is to correctly identify the small fraction of real TELs on the road. The results are shown in Table 5.8. When the sensor indicates a TEL there is only a 15% chance that it is correct. There is an 85% chance that it is a decoy, some other type of vehicle or nothing at all, a very high false alarm (or false positive) rate. This clearly indicates the difficulties of observing rarely occurring events among a large amount of clutter. In such cases the sensor needs to be extraordinarily discriminating to be useful.

Sensor outcome (*S*)	Vehicle type (*V*)			
	TEL (t)	Decoy (d)	Other (o)	None (ø)
TEL ("t")	0.15	0.06	0.74	0.05
DECOY ("d")	0.02	0.08	0.86	0.04
OTHER ("o")	0.01	0.02	0.93	0.04
NONE ("ø")	0.001	0.001	0.022	0.976

Table 5.8: Sensor performance when $\mathbf{p}_X$ = (0.05, 0.05 0.9)

5.4 TEL Detection Probabilities, Multiple Overflights

It is important to keep in mind that the probabilities shown in Tables 5.6, 5.7 and 5.8 apply to a single overflight of a vehicle. As shown above, the searcher may pass over the vehicle a number of times, depending on the coverage ratio. The goal of this section is to determine how well the sensor performs as a function of the coverage ratio *x* in addition to the road traffic environment.

Consider the case where the number of overflights is 2. There are sixteen (4^2) possible sensor outcomes, namely {("t", "t"), ("t", "d"), ("t", "o"), ("t", "ø"), ("d", "t"), ("d", "d"), ("d", "o"), ("d", "ø"), ("o", "t"), ("o", "d"), ("o", "o"), ("o", "ø"), ("ø", "t"), ("ø", "d"), ("ø", "o"), ("ø", "ø)}. Some assumption must be made as to how to resolve conflicting sensor outputs on the two or more overflights. Since time is among the most critical factors in successfully prosecuting TELs, in this paper it is assumed that *the first positive sensor output (any outcome other than "ø") is used to classify the object.*

For two overflights the vehicle would be classified as a TEL if and only if the output was a member of the set {(“t”, “t”), (“t”, “d”), (“t”, “o”), (“t”, “ø”), or (“ø”, “t”)}. In general the vehicle is classified to be the first non-empty observation.

Define T_n to be the event {sensor indicates a Tel, given n overflights of a vehicle}, so

T_n = {(“t”), or (“ø”, “t”), or . . . , or (“ø”, . . ,“ø”, “t”)}, where the last vector has $(n\text{-}1)$ “ø”’s. Since these events are mutually exclusive,

$$Pr\{\mathrm{T}_n|V=v\} = Pr\{\text{“t”}|V=v\} + Pr\{\text{“ø”, “t”}|V=v\} + \ldots + Pr\{\text{“ø”, . . , “ø”, “t”}|V=v\}.$$

Assuming conditional independence of each sensor output given the vehicle type, it is straight forward to show that for $n = 1, 2, \ldots$

$$\begin{aligned} Pr\{\mathrm{T}_n, V=v\} &= f p_V(v) p_{S|V}(\text{“t”}|v)(1-p_{S|V}(\text{“ø”}|v)^n)/(1-p_{S|V}(\text{“ø”}|v)),\ v = \text{t, d, o}, \\ &= (1-f) p_{S|V}(\text{“t”}|v)(1-p_{S|V}(\text{“ø”}|v)^n)/(1-p_{S|V}(\text{“ø”}|v)),\ v = \text{ø}. \end{aligned} \tag{5.16}$$

These are added over v to determine the unconditional probability

$$Pr\{\mathrm{T}_n\} = \textstyle\sum_v Pr\{\mathrm{T}_n, V=v\}. \tag{5.17}$$

Finally, using $\mathbf{p}_V$ and f together with Equations (5.16) and (5.17) the conditional probability of a vehicle being a particular type, given n overflights and it is identified as a TEL, is given by Bayes’ Rule to be

$$\begin{aligned} Pr\{V=v|\,\mathrm{T}_n\} &= r\{\mathrm{T}_n|V=v\}p_V(v)f/Pr\{\mathrm{T}_n\},\ v = \text{t, d, o}, \\ &= r\{\mathrm{T}_n|V=v\}(1-f)/Pr\{\mathrm{T}_n\},\ v = \text{ø}. \end{aligned} \tag{5.18}$$

Let D_n and O_n be the events that a vehicle is identified as a decoy and other vehicle, respectively, given n overflights. The analysis leading to Equation (5.18) can be repeated using either D_n and O_n in place of T_n to show that

$$\begin{aligned} Pr\{V=v|\,\mathrm{D}_n\} &= r\{\mathrm{D}_n|V=v\}p_V(v)/fPr\{\mathrm{D}_n\},\ v = \text{t, d, o}, \\ &= r\{\mathrm{D}_n|V=v\}(1-f)/Pr\{\mathrm{D}_n\},\ v = \text{ø}. \end{aligned} \tag{5.19}$$

and

$$\begin{aligned} Pr\{V=v|\,\mathrm{O}_n\} &= r\{\mathrm{O}_n|V=v\}p_V(v)f/Pr\{\mathrm{O}_n\},\ v = \text{t, d, o}, \\ &= r\{\mathrm{O}_n|V=v\}(1-f)/Pr\{\mathrm{O}_n\},\ v = \text{ø}. \end{aligned} \tag{5.20}$$

We are now in a position to calculate the probability that a sensor indicates a a given type of vehicle is present for a given coverage ratio x and a given sensor output. Let T(x) be the event {sensor indicates a Tel, given the coverage ratio is *x*). Then

$$Pr\{\mathrm{T}(x)|S=s\} = \sum_{n\geq 1} Pr\{T_n|S=s\}p(n;x), \tag{5.21}$$

where *p*(*n*; *x*) satisfies Equation (5.14). In the next section Equation (5.21) is used to show graphically the effects of changes in model parameters on the performance of the sensor under varying conditions.

5.5 Model Analysis

This section contains a number of graphs that have been calculated using Equation (5.21) in a Microsoft Excel spreadsheet. These graphs show the effectiveness of the sensor under various traffic conditions, as well as showing the effects of changing the sensor performance characteristics through the conditional probabilities in Table 5.5 on page 45.

5.5.1 Traffic Density Effects

Figure 5.13 shows a plot of the probability that a TEL is really present, given that the sensor indicates that a TEL is present, for a coverage ratio in the range (0,6). The plot was obtained using the probabilities in Table 5.5, the traffic vector in Example (ii), $\mathbf{p}_V = (0.3, 0.3, 0.4)$, and three values of the traffic density *f*. Note that changing the traffic density from 0.5 to 0.9 makes very little difference, but when the traffic density is low there is a significant degradation in sensor operational performance. Note the drop in performance as the coverage ratio exceeds 2. This is caused by the fact that the sensor can give positive readings when no traffic is present; when the traffic density is low there is a greater opportunity for the sensor to give false indications. Note also that a coverage ratio of approximately 2 is optimal for a very wide range of traffic densities.

5.5.2 Vehicle Mixture Effects

Figure 5.14 shows a plot of the probability that a TEL is really present, given that the sensor indicates that a TEL is present, for a coverage ratio in the range (0,6). The plot was obtained using the probabilities in Table 5.5, a traffic density of 0.2, and the traffic vectors in Examples (i), (ii) and (iii). We see again that a coverage factor of approximately 2 gives the best operational performance of a very wide mixture of traffic. The curves clearly demonstrate how the unreliable a sensor outcome can become as the fraction of traffic on the road that is of interest becomes small. With a coverage factor of 2 a sensor indication of a TEL is correct almost 90% of the time when 80% of the traffic is made up of TELS; it is correct only about 15% of the time when only 5% of the traffic is made up of TELS.

5.5.3 False Negatives

Figure 5.15 shows a plot of the probability that a particular vehicle type is really present, given that the sensor indicates that a TEL is present, for a coverage ratio in the range (0,6). The plot was obtained using the probabilities in Table 5.5 and the traffic vector and traffic density in Example (ii). For every value of x greater than 2 the points on the four curves add to 1.0 since they form a conditional probability mass function over possible vehicle types. For every value of x less than 2, the four curves add to the probability of one or more overflights. This graph clearly shows the monotone increasing (with x) probability of there being no vehicle present when the sensor indicates a TEL. If the entry in the lower right corner of Table 5.5, i.e. $\Pr\{S = \emptyset \mid V = \emptyset\}$, were 1, this line would be zero for all x. As a result the other three lines would level off at x=2 and would not decrease. It is the possibility that the sensor can indicate some type of vehicle is present when in fact no vehicle is present that leads to the presence of a maximum in the curves when the coverage factor is approximately 2.

5.5.4 False Positives

Figure 5.16 shows a plot of the probability that the vehicle present is a TEL, given that the sensor indicates each of the three vehicle types is present, for a coverage ratio in the range (0,6). The plot was obtained using the probabilities in Table 5.5 and the traffic vector and traffic density in Example (ii). Again the maximum probability that the vehicle is a TEL given that the sensor indicates a TEl is when x is approximately 2. Unfortunately this is also the value of coverage ratio that maximizes the probability it is a TEL given that it is a decoy or some other vehicle.

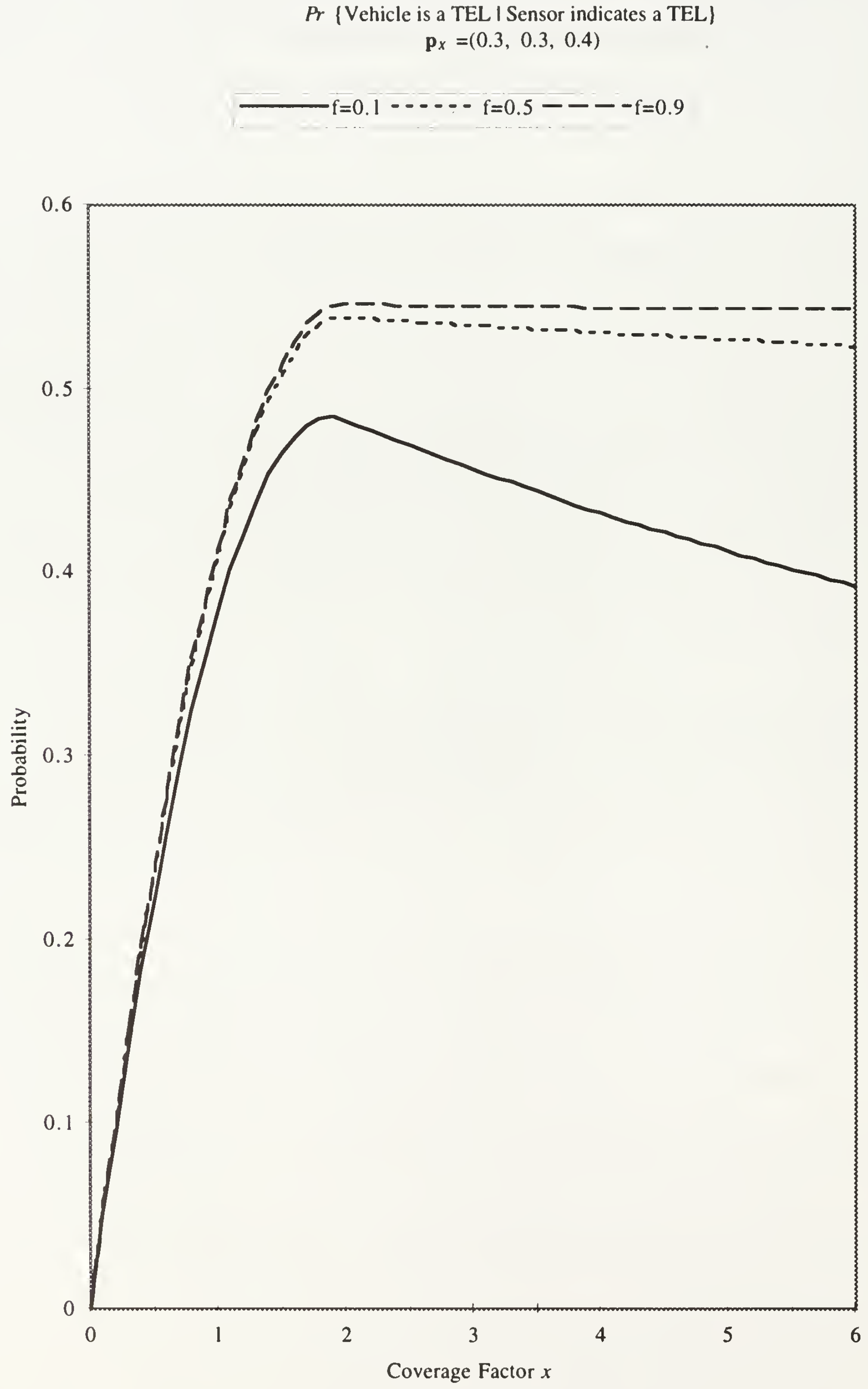


Figure 5.13: Effect of Traffic Density on Pr\{Vehicle is a TEL| Sensor indicates a TEL\}

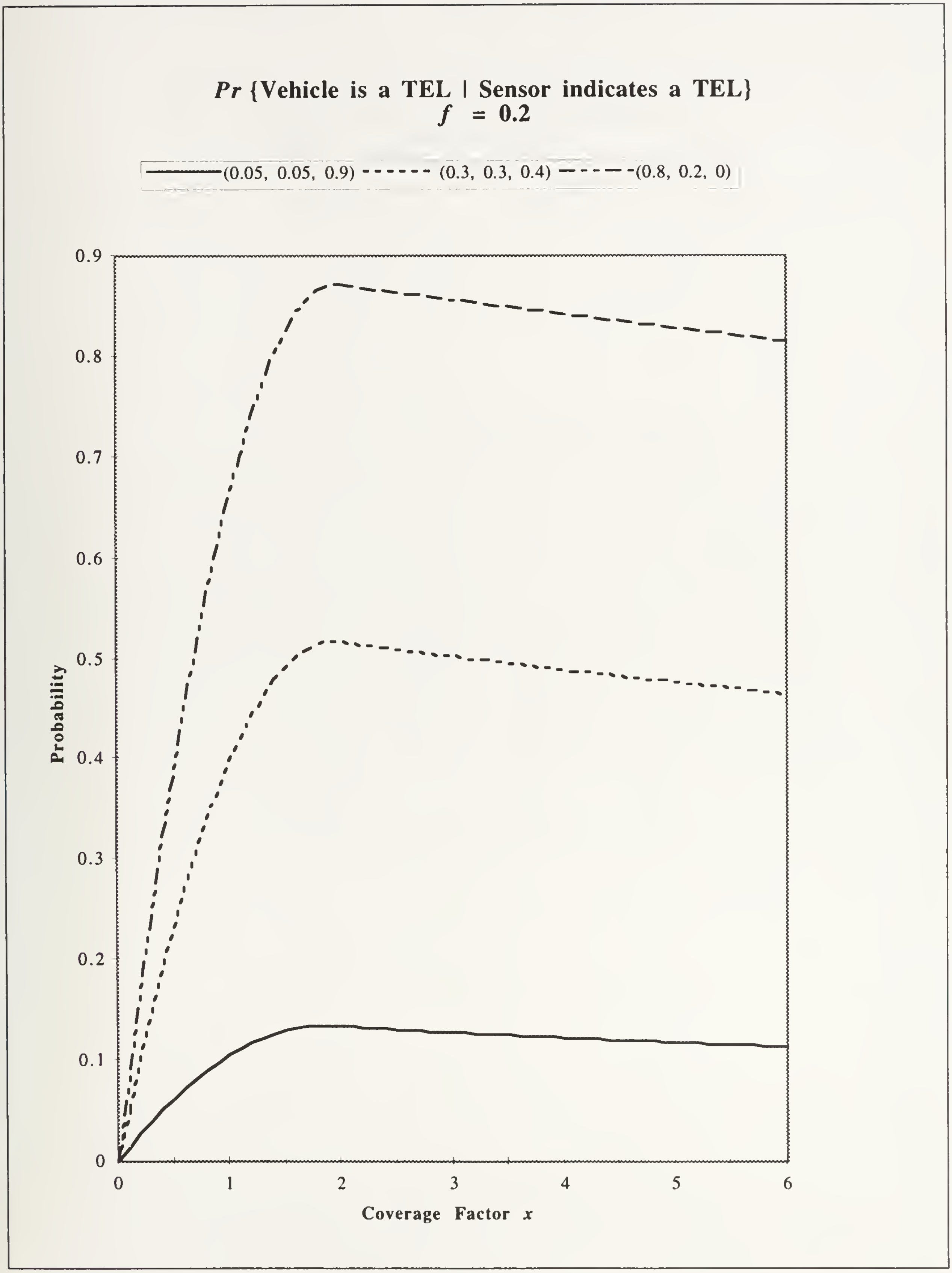


Figure 5.14: Effect of Traffic Mixture on *Pr*{Vehicle is a TEL| Sensor indicates a TEL}

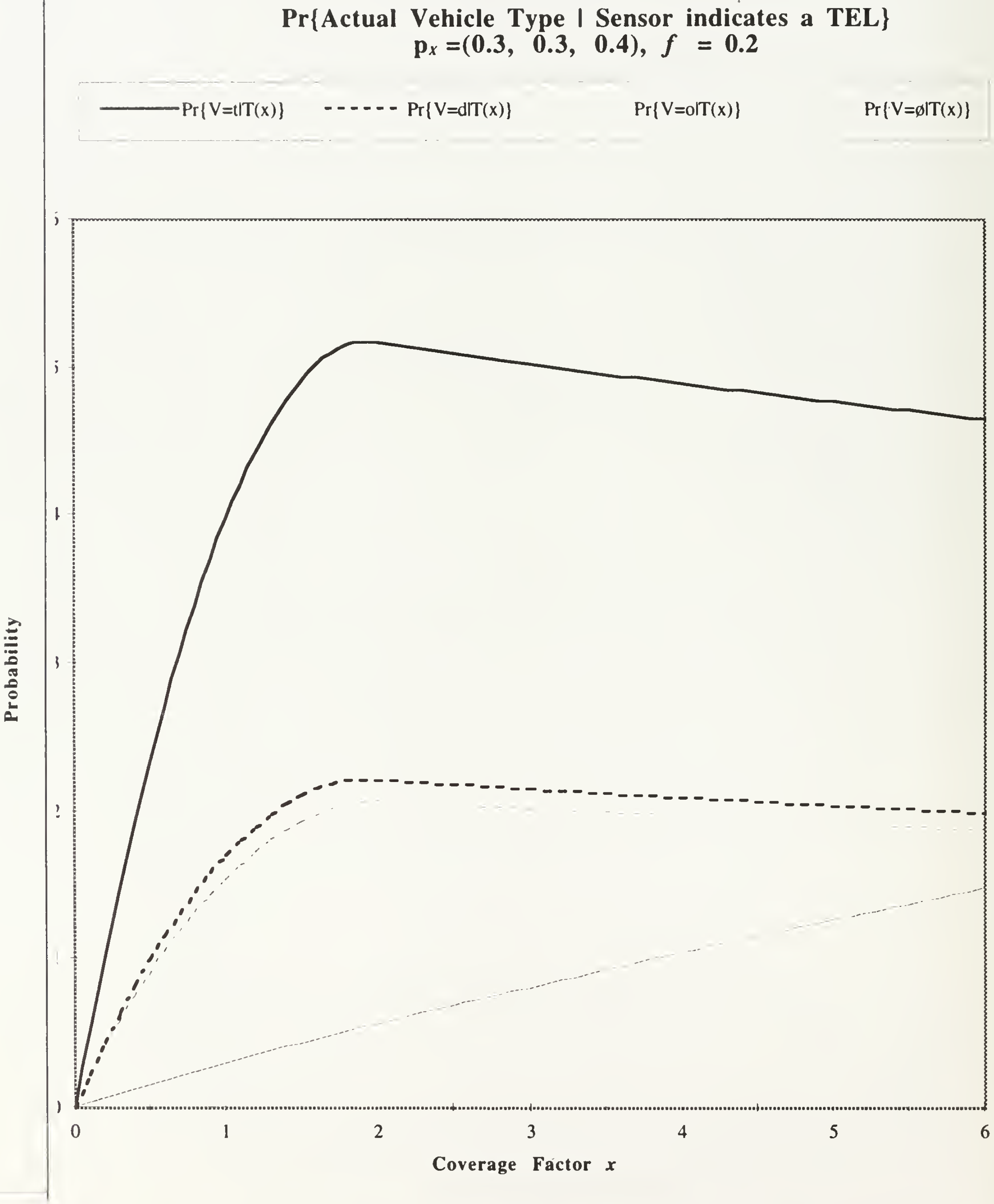


Figure 5.15: *Pr*{Actual Vehicle Type | Sensor indicates a TEL}

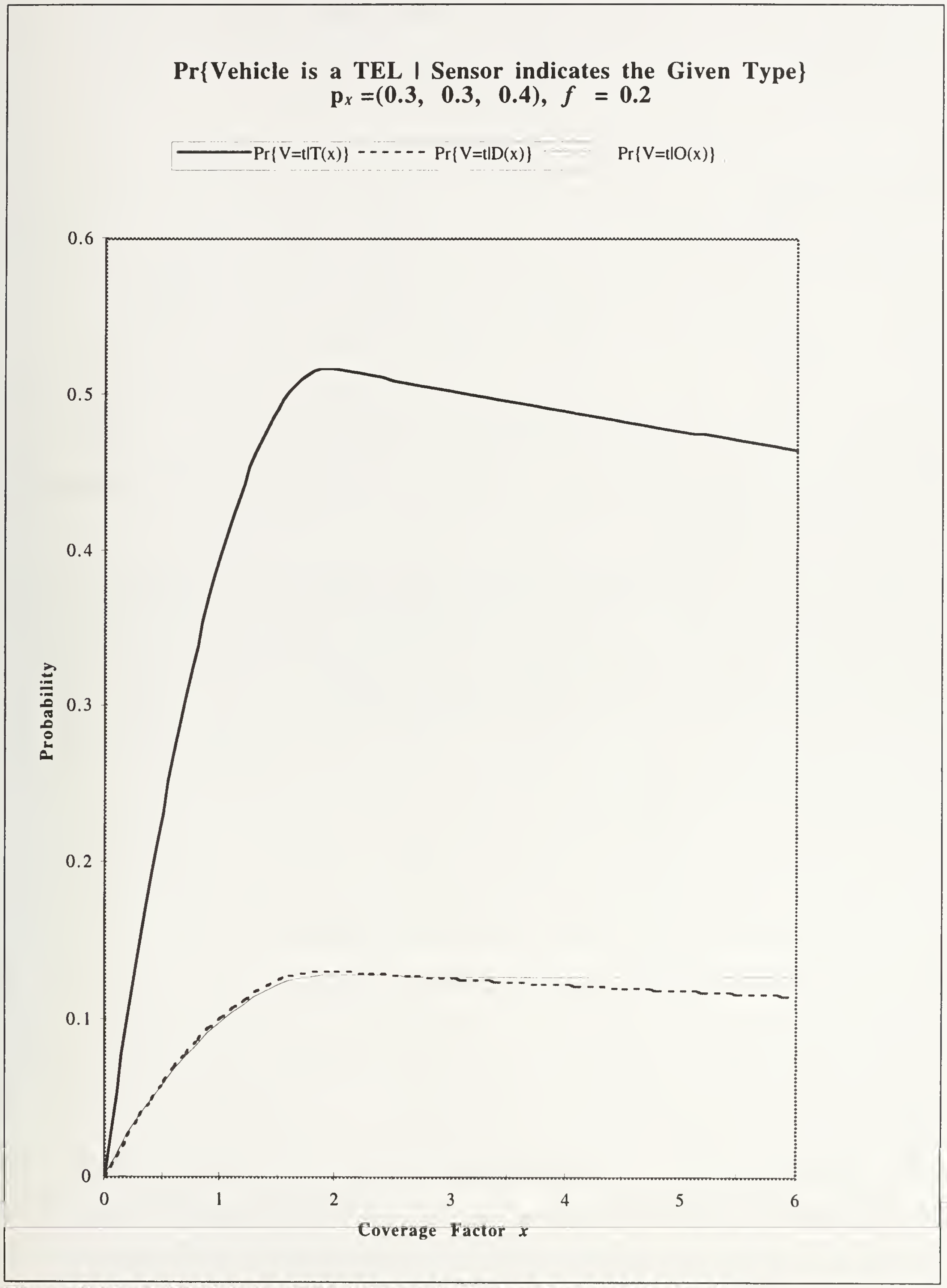


Figure 5.16: *Pr*{Vehicle is a TEL | Sensor indicates given vehicle type}

6: Supression Effects of Searching for Tels

Kneale T. Marshall

6.1 Introduction

In an earlier paper (Ref. [6]) Conner, Ehlers and Marshall showed how a simple circulation model based on submarine operations in world war two could be used to understand better the operations of theater tactical missile operations. This model was used to demonstrate that effective operations against the launchers could lead to significant payoff in solving the theater missile defense (TMD) problem. This circulation model was extended in Marshall (Ref. [7]), and used to quantify and demonstrate that a modest success against launchers, measured in terms of kill probability, can significantly reduce the numbers of boost-phase, upper-tier and lower-tier weapons required to counter a given threat.

Analysis of the data on anti-ballistic-missile-launcher operations from World War 2 through Desert Storm shows a discouraging lack of success in killing the launchers. This has resulted in the widely-held belief that the problem is too difficult so that concentration of effort is today focussed heavily on active defense. The author agrees that active defense will be an essential element to successful anti-TBM operations, but believes also that such operations could be easily overwhelmed without some success at disrupting launch operations. The paper takes a closer look at launcher operations with emphasis on quantifying and understanding the effects such operations might have on suppression of missile launches rather than hard kills. The ideas put forth below came in part from conversations with CAPT George Conner, USNR (Ret) whose navy career encompassed ASW operations. Other ideas came reading McCue (Ref. [9]) where it is shown how increasing transit times of U-boats across the Bay of Biscay in World War 2 was a significant factor in countering the U-boat campaign. In general, McCue compares improvements in technology to improvements in tactics and operations. In his overview he states:

> *"Much modern analysis aims to determine the "system requirements" for new or proposed weapon systems and to find ways to meet those requirements that cannot be defeated by enemy countermeasures. . . . , in the U-boat war, the introduction*

> *and assimilation of new hardware proceeded so slowly that an inherently defeatable device could have a useful career while the other side spent time realizing that the device had been deployed, arguing about what it was, and introducing a counterdevice. Moreover,. . . . . the "top-down" approach to weapon design, in which the design is derived from the requirements, is fundamentally inappropriate in cases involving possible enemy countermeasures. . . . . the finding of wartime operations researchers* [was] *that new tactics or operational policies can have as much impact as new equipment, and often more immediately.* " (Ref. [9], pp 3-4)

6.2 Measures of Effectiveness

The obvious measure of effectiveness to use in TMD is the number or fraction of hard kills of missiles and/or launchers during a campaign. Using these measures attack operations against launchers would be considered very ineffective. However, there is evidence in the data from Desert Storm that even though attack operations against missile launchers resulted in few, if any, verified launcher kills, the presence of search and destroy assets in the search area resulted in suppression of missile launch rates. Not only were the launch rates decreased but the ability of the enemy to coordinate launches into salvos was clearly reduced. Similar effects were observed by McCue in the anti U-boat campaign:

> “*From the "What if . . . " analyses emerges a detailed understanding of the relationships inherent in the U-boat campaign. As Donitz realized even before the war began,...the U-boats, despite their submersibility, moved and fought best on the surface and submerged only when threatened: they could be thought of as submersible torpedo boats* [Marshall: Mobile underwater missile launchers]. *Bay search turns out to have been effective largely because it encouraged U-boats to make lengthy submerged passages, cutting into the time they could spend at sea sinking merchant vessels.* (Ref. [9], p 17)

The effect in TMD similar to that found by McCue for U-boats is the tendency of launchers to go into hiding for substantial time periods when threatened with detection and destruction by search and destroy assets. These hide times can add significantly to the basic time it would take a launcher to return to its reload point, be prepared for another launch, refuel etc., and return to its firing location. The next section presents a model from which can be calculated the kill probability that is equivalent to increasing launcher cycle times, given a campaign of a certain length.

6.3 Basic Notation and Model

Consider a single launcher active in a campaign of length T. Assume that if this launcher can operate with impunity its cycle time from leaving the missile mating/refuelling area until its return after firing its missile, is T_C. Extending the notion used in Chapter 4, let

$\overline{m}$ = maximum number of missiles a launcher can launch if given unlimited launch time,
m = maximum number of missiles a launcher can launch given T and T_C,
q_1 = survival probability of launcher on outward leg to launch site,
q_2 = survival probability of launcher on inward leg from launch site to reload,
X = number of missiles launched per launcher (random variable).

From Equation (4.1) on page 23

$$E[X] = \frac{q_1(1-(q_1q_2)^m)}{(1-q_1q_2)}, \tag{6.1}$$

where

$$m = Min\left\{\overline{m}, \left\lfloor \frac{T}{T_C} \right\rfloor + 1\right\}. \tag{6.2}$$

Figure 6.17 shows the result using equations (6.1) and (6.2) with $\overline{m} = 16$ and $T = 144$ hours. The four curves are for the cases where the before-launch and after-launch kill probabilities are

S-1: $p_1 = p_2 = 0$,

S-2: $p_1 = 0$ and $p_2 = 0.05$,

S-3: $p_1 = 0.05$ and $p_2 = 0$,

S-4: $p_1 = p_2 = 0.05$.

The measure plotted on the horizontal axis is the launcher cycle time.

This simple model leads to some interesting observations. First, note that with a small success at killing launchers on either one of the legs of the launcher circulation (5% in this example) the

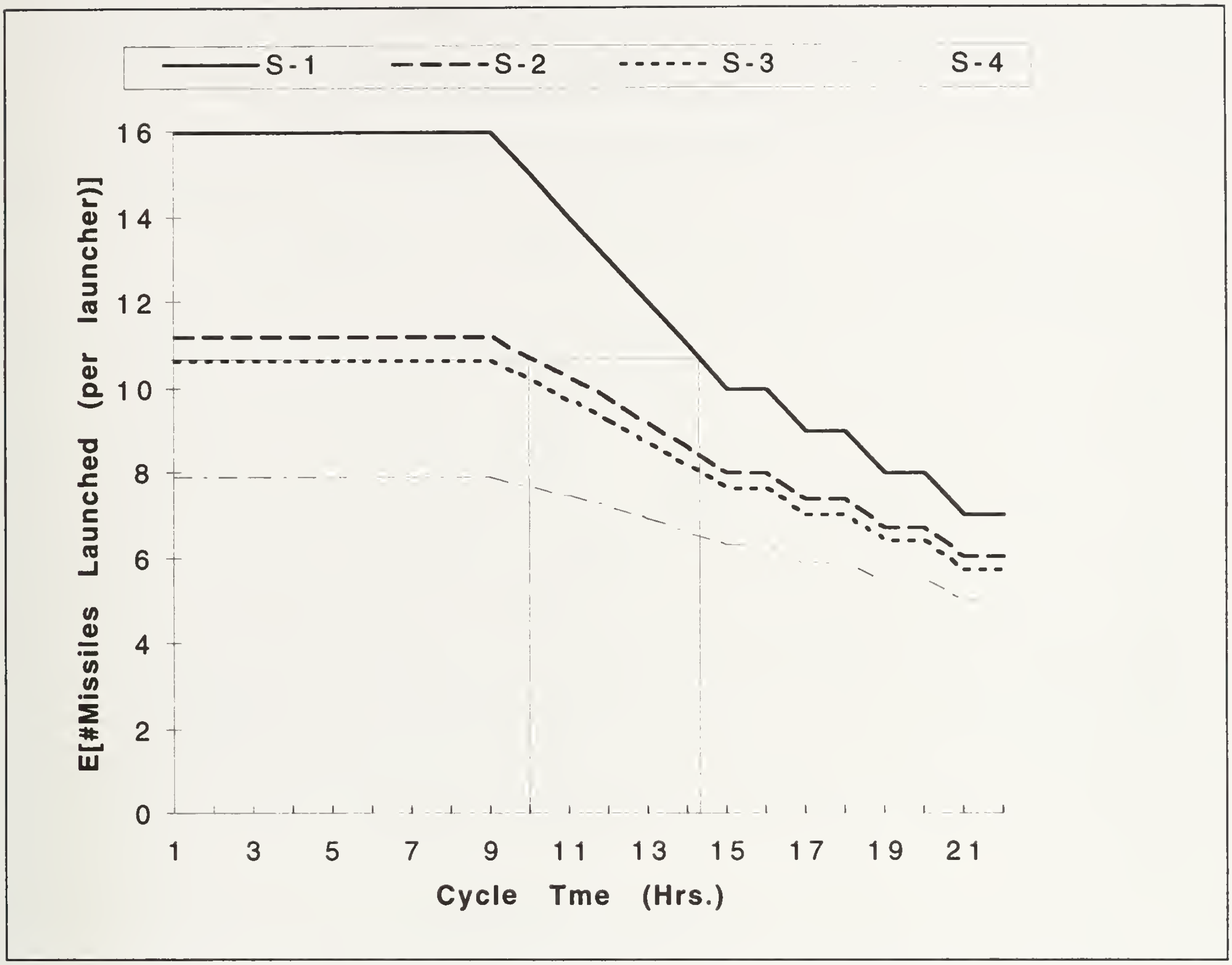


Figure 6.17: Expected Number of Missiles Launched vs. Cycle Time.

expected number of missiles launched per launcher, $E[X]$, is cut from 16 to about 11 when the launcher cycle time is less than 9 hours. Second, with 5% kill probabilities on both legs $E[X]$ is cut from 16 to about 8, cutting in half the missiles that would require the use of active defense weapons. Third, the S-1 curve shows that even when the launcher kill probabilities are both zero simply increasing cycle time above 9 hours leads to a significant reduction in $E[X]$. This effect is what we refer to as suppression. Fourth, note that S-4 is almost horizontal showing that the when both launcher kill probabilities are 0.05 the suppression effect is very small for all values of T_C up to about 24 hours.

From Equation (6.2) one can see that changing the campaign length relative to the cycle length will change the value of m and hence the value of $E[X]$. Rather than replotting Figure 6.17 whenever the campaign length changes, the same analysis can be carried out on a plot where the ratio T_C/T is plotted on the horizontal axis. Such a plot is shown in Figure 6.18.

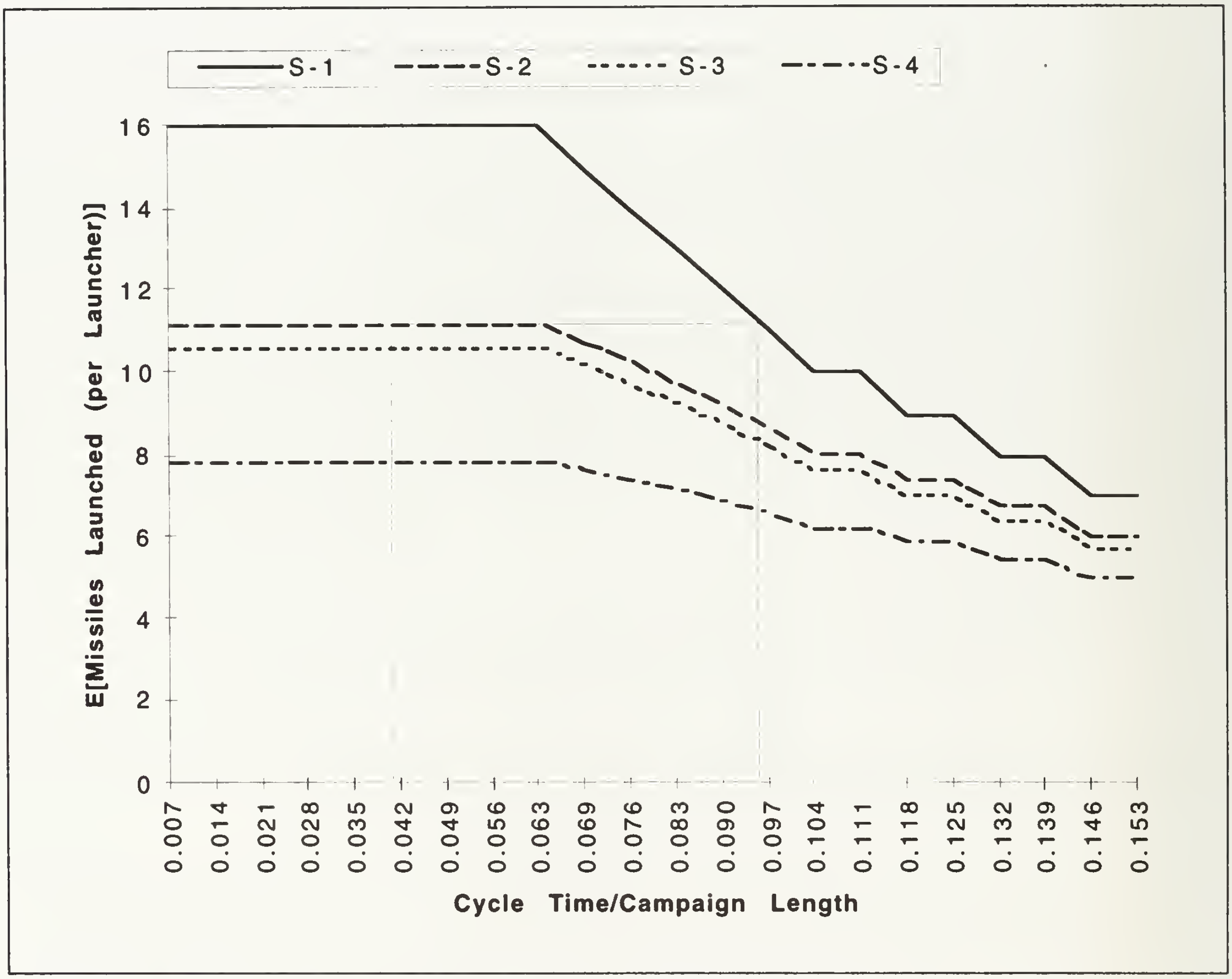


Figure 6.18: Expected Number of Missiles Launched vs. T_C/T.

6.4 Equivalent Kill Probabilities

The observations in the above paragraph lead to the definition of the concept of "equivalent kill probability" of suppression. Suppose that, when unhindered, launchers operate with a cycle time of 10 hours (see left vertical line in Figure 6.17). If the kill probabilities p_1 and p_2 are 0 and 0.05, respectively, each launcher would launch on average 10.7 missiles. But following the horizontal line in Figure 6.17 to where it meets S-1, we see that we could obtain this value of $E[X]$ by having p_1 and p_2 both 0 but extending the cycle time from 10 hours to 14.2 hours (see right vertical line in

Figure 6.17). Thus if by search and destroy tactics the launcher cycle time can be increased by about four hours, the effect would be the same as obtaining a 5% after-launch kill probability of the launcher. That is, the four hour cycle extension has an equivalent kill probability on the return leg of the cycle of 0.05.

An easy way to obtain the equivalent kill probability in a given situation is graphically as is demonstrated above. An example using Figure 6.18 are now given. Let the maximum number of launches from a launcher given unlimited launch time be sixteen as before. Suppose you expect a campaign to last ten days and that unimpeded launcher cycle times are expected to be twelve hours. These parameter values lead to $T_C/T = 0.05$. From Figure 6.18 we see that if $p_1 = 0$ and $p_2 = 0.05$, the expected number of launches per launcher would be about eleven. To achieve this value by suppression alone the launcher cycle time would need to be increased to $0.096 \text{x} 24 \text{x} 10 = 23$ hours. Thus the required increase in cycle time wound be nine hours. If the estimated campaign length were twelve days rather than ten days, the required increase in cycle time would be about sixteen hours. If the estimated campaign length were reduced to eight days, the required increase in cycle time would drop to about six hours.

We end this paper with quote from McCue concerning the usefulness to a decision maker of simple analytic models such as the one shown above, or those found in other papers in this document:

> *Many balk at the use of quantitative methods in the study of military matters, holding that judgment and experience - perhaps their own - have far more to offer than any calculation possibly could. Judgment and experience are valuable guides in any human endeavor. However, war - and especially the kind of search operation investigated here - entails a great deal of uncertainty. Those who deal with uncertainty in war by playing hunches seem likely to share the fate of those who use that approach in poker or backgammon.* (Ref. [9], p. 17).

7: A Pre-Hostilities Decision Model for TMD Attack Operations[1]

Kneale T. Marshall

7.1 Introduction

The United States does not have the resources to monitor all possible areas where mobile missile launchers may be deployed. The decision to expend resources in a given area before any hostilities break out, the level of resource allocation, and the methods of employing it, all must depend on the likelihood of hostilities in the area that would affect US national interests. The purpose of this paper is to present methods and models intended to aid a decision maker better understand the trade-offs and risks involved. The overall problem is complex, and the models presented here are intended to show the basic structure. Influence diagrams and decision trees are the tools used to help formulate and analyze the problem.

7.2 Problem Structure

The first assumption made is that the path to hostilities follows the three stages of peacetime, mobilization, and hostile action. The second assumption is that two distinct decisions must be made. The first is whether or not to gather intelligence in peacetime, and how to carry this out. The second is what action to take against the mobile missile launchers during mobilization. It is assumed that if hostilities occur the US would be involved[2] in reducing to a minimum the expected number of TBM's launched against friendly nations.

Let $\mathcal{D}_1$ be the set of alternatives available to the decision maker during peacetime with elements $d_1 \in \mathcal{D}_1$. We assume that it consists of three elements:

$d_1 = 0$ if no action is taken to gain intelligence,

$d_1 = 1$ if covert intelligence effort is undertaken,

$d_1 = 2$ if overt intelligence effort is undertaken.

---

[1] Based on material which first appeared in the thesis by Mark A. Ehlers.

[2] The political questions of possible coalition involvement are not relevant to this report. Our objective is to model the effects of pre-hostilities intelligence gathering on a possible future conflict.

Let $\mathcal{D}_2$ be the set of decisions available during mobilization with elements $d_2 \in \mathcal{D}_2$. We assume that it consists of three elements:

$d_2 = 0$ if no action is taken to gain intelligence

$d_2 = 1$ if overt intelligence effort is undertaken,

$d_2 = 2$ if interdiction of mobile launchers is undertaken.

It is a model assumption that interdiction at the mobilization decision stage is an option only if there has been intelligence performed during peacetime. Interdiction can take many forms. At this stage we do not specify how it is carried out, but simply assume that it will result in a lowering of the expected numbers of missiles launched.

Whichever decision is made during peacetime, the enemy decision to mobilize is uncertain. Let the random variable $M = 1$ if the enemy mobilizes and 0 if it does not. Inherent in this definition is the concept of a time period, say for example, a year. If mobilization does occur during the year, $M=1$. Otherwise it is zero and the decision problem can be repeated in the next period. The distribution of M is assumed to depend on the first decision with

$$Pr\{M=1|D_1=i\} = m_1 \qquad \text{if } i = 0,1,$$

$$Pr\{M=1|D_1=2\} = m_2.$$

Similarly, whichever decision is made during mobilization, the enemy decision to start hostilities is uncertain. Let H be a Bernoulli random variable that is 1 if hostilities break out and 0 if they do not. The distribution of H is assumed to depend on both decisions and on M with

$$Pr\{H=1|D_1=0, M=1, D_2=0\} = h_1,$$

$$Pr\{H=1|D_1=\mathrm{i}, M=1, D_2=2\} = h_2 \quad \text{if } i = 0,1,2.$$

We assume that if $D_2=3$ (interdiction after observing mobilization) hostilities are assumed to have broken out, or $H=1$ with probability 1.

Let $\mathcal{R}$ be the set of possible results of the decision process with elements $r \in \mathcal{R}$. An element r represents the expected number of TBM's launched. Clearly this will depend on the decisions made and on the behavior of the enemy. Let

$r = 0$ if hostilities do not break out,

$r = r_1$ if no intelligence effort is applied during peacetime or mobilization and hostilities break out,

$r = r_2$ if no intelligence effort is applied during peacetime but is carried out overtly during mobilization, and hostilities break out,

$r = r_3$ if covert intelligence effort is applied during peacetime, overt intelligence is applied in mobilization, and hostilities break out,

$r = r_4$ if overt intelligence effort is applied during both peacetime and mobilization, and hostilities break out,

$r = r_5$ if covert or overt intelligence effort is applied during peacetime and launcher interdiction is undertaken during mobilization.

Figure 7.19 shows the structure of the decision problem in an influence diagram. For a fuller treatment of influence diagrams and decision trees, with detailed demonstrations and examples of their use in building and solving decision models, see Ref. [5].

This figure shows the sequence of events in time from left to right, and the possible interactions among the events. The first event is the decision D_1 (decisions are depicted with square nodes) as to the type, if any, of intelligence gathering to undertake in peacetime. The next event is M, whether or not the enemy mobilizes (uncertain events are depicted by circles). This is followed by the mo-

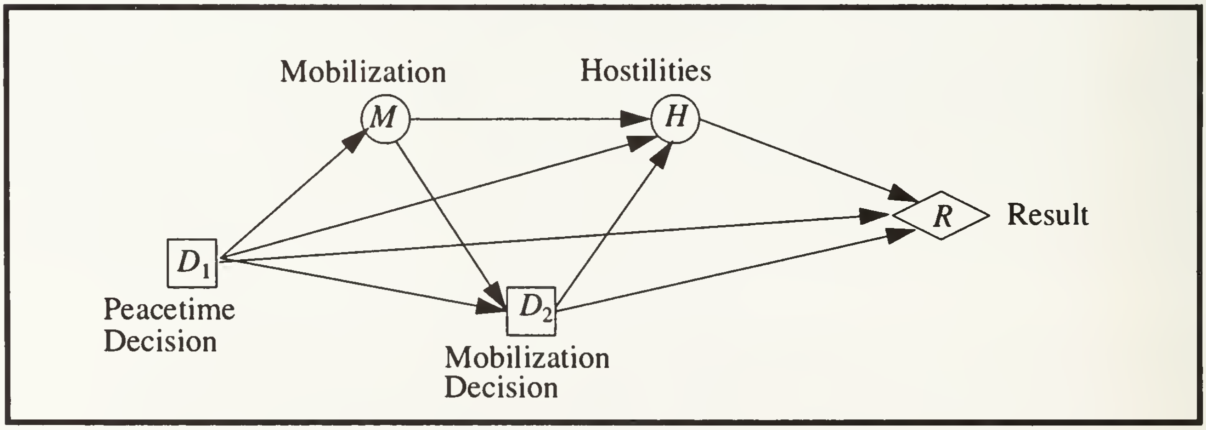


Figure 7.19: Influence Diagram for the Decision Problem

bilization decision D_2, then the observation of whether or not hostilities break out, H, and finally the result R is observed (shown as a diamond node).

Directed arcs between nodes imply possible dependence and are carefully defined. Consider the three arcs leaving D_1. The one that ends at D_2 indicates that D_1 is not forgotten when D_2 is made, and the mobilization decision can be affected by the peacetime decision. For example, if the peacetime decision is not to undertake intelligence gathering ($D_1 = 0$), the mobilization decision cannot be interdiction since we have no knowledge of the whereabouts of the missile launchers. If the peacetime decision is overt intelligence gathering ($D_1 = 2$) the overt character of these efforts may affect the enemy's decision to mobilize (change the distribution of H). If the peacetime decision is covert intelligence gathering ($D_1 = 1$) since these efforts would be unknown to the enemy they cannot affect H or M.[3] The arc from M to D_2 shows that the enemy mobilization decision is known before the US decision on intelligence gathering in this period is made. The arc from M to H shows that these random events are dependent. The arc from D_2 to H indicates the dependence of the distribution of H on D_2. Finally, once D_2 has been made and the outcome of H is known (H=1 if hostilities break out, 0 if not), the resulting expected number of SRBM's launched can be calculated.

The decision tree for this problem is shown in Figure 7.20. The sequence of the nodes follows that shown in the influence diagram. Extending from each decision node are branches that represent all possible decision options at that stage. Each of the decision branches terminates at a random event node representing possible mobilization.

The decision nodes are marked D_1 for the peacetime decision and D_2 for the mobilization decision. The random event nodes are labeled M for mobilization and H for hostilities. Extending from each decision node is a branch for each decision alternative in the sets $\mathcal{D}_1$ and $\mathcal{D}_2$, and from each random event node are two branches that represent the uncertain outcome, 1 or 0. The result values shown at the terminal diamond nodes, and the conditional probabilities on the branches leaving the M and H nodes, are those defined earlier in this section.

---

[3] Note that if overt operations were not considered from the outset we could remove the directed arcs between D_1 and both M and H, and from D_2 to H, thus simplifying the problem.

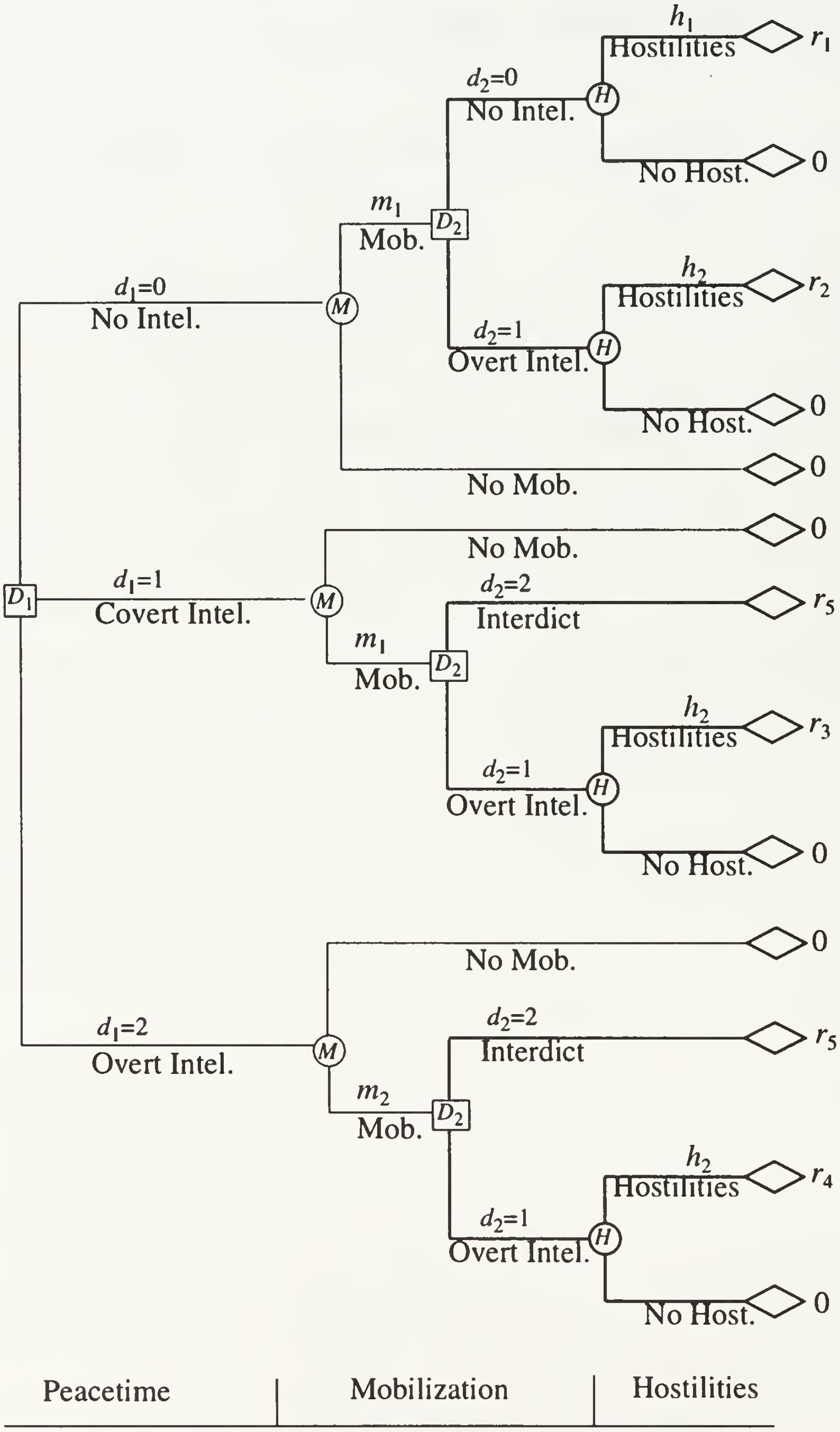


Figure 7.20: The Intelligence Effort Decision Tree

The notation m and h have been used to represent the probabilities of mobilization and hostilities occurring respectively. Subscripts are used to differentiate between estimated parameters under different policies. The following probabilities are required:

$m_1 = Pr\{$mobilization occurs, given no or covert intelligence effort in peacetime$\}$

$= Pr\{M=1|D_1=0\} = \Pr\{M=1|D_1=1\}$,

$m_2 = Pr\{M=1|D_1=2\}$.

$h_1 = Pr\{$hostilities occur, given no intel. effort in either peacetime or mobilization$\}$

$= Pr\{H=1|D_1=0, M=1, D_2=0\}$,

$h_2 = Pr\{$hostilities occur, given overt intelligence during mobilization$\}$

$= Pr\{H=1|M=1, D_2=1\}$,

These probabilities are attached to the appropriate branches of the decision tree.

Parameter Ordering Assumptions

From the structure of the problem thus far it is reasonable to make some assumptions on the relative values of the m_i's, h_i's and r_i's. First we assume that overt intelligence gathering in peacetime and/or mobilization acts as deterrence. It results in a probability of mobilization that is no larger than, and quite likely is smaller than, what it would be if no intelligence were gathered or it was gathered covertly. This implies that

$$m_1 \geq m_2. \tag{7.1}$$

Similarly, once mobilization has begun, it is assumed that overt intelligence will result in lowering (or at least not raising) the likelihood that hostilities break out. Thus

$$h_1 \geq h_2. \tag{7.2}$$

Second we assume that the more information we have the more successful we will be in reducing the expected number of missiles launched if hostilities break out. We also assume that the most effective way to reduce this number is to interdict the launchers during mobilization. Thus

$$r_1 \geq r_2 \geq r_3 \geq r_4 \geq r_5. \tag{7.3}$$

These assumptions play an important role in the next subsection.

Problem Solution

As in solving any decision tree we start at the terminal nodes in Figure 7.20. As we work back to the D_1 node we take expected values at the random nodes and minima at decision nodes. At the three D_2 nodes starting at the top of the diagram, the expected result for each is $Min\{h_1r_1, h_2r_2\}$, $Min\{r_5, h_2r_3\}$, and $Min\{r_5, h_2r_4\}$. From the inequalities in (7.2) and (7.3) the first of these minima can be resolved immediately. The optimal policy to pursue if no intelligence is gathered in peacetime and mobilization occurs, is to start overt intelligence gathering immediately with an expected result equal to h_2r_2. Resolution of the other two minima will depend on the relative value of h_2; we look at three cases.

Case I: $(r_5/r_4) \leq h_2 \leq 1$.

This implies that $r_5 \leq h_2r_4$, and with (7.3) that $r_5 \leq h_2r_3$. This shows us that if intelligence has been gathered in peacetime no matter whether it be covert or overt, if mobilization occurs the optimal policy for this case is to interdict the launchers.

Case II: $0 \leq h_2 \leq (r_5/r_3)$.

For this case $r_5 \geq h_2r_3$ and $r_5 \geq h_2r_4$. These show us that if intelligence has been gathered in peacetime no matter whether it be covert or overt, if mobilization occurs the optimal policy for this case is to continue gathering intelligence, but overtly.

Case III: $(r_5/r_3) \leq h_2 \leq (r_5/r_4)$.

For this case $r_5 \leq h_2r_3$ and $r_5 \geq h_2r_4$. These show us that if covert intelligence has been gathered in peacetime and mobilization occurs, the optimal policy for this case is to interdict the launchers. If overt intelligence has been gathered in peacetime and mobilization occurs, the optimal policy for this case is to continue overt intelligence.

With the minima resolved at the D_2 nodes we can take expectations at the M nodes and finally find the minimum at the D_1 node. At the top M node the expected return is $m_1h_2r_2$. At the other two M nodes it will depend which of the three cases pertains, so each must be analyzed separately.

Case I: $(r_5/r_4) \leq h_2 \leq 1$.

At the middle M node the expected payoff is m_1r_5 and at the lower one m_2r_5. At node D_1 we need to find

$$Min\{m_1h_2r_2, m_1r_5, m_2r_5\}.$$

Using the inequalities in (7.1), (7.2) and (7.3), if we denote by r^* the minimum expected result,

$$r^* = m_2r_5.$$

Case II: $0 \leq h_2 \leq (r_5/r_3)$.

At the middle and lower M nodes the expected payoffs are $m_1h_2r_3$ and $m_2h_2r_4$ respectively. We find the minimum of these and $m_1h_2r_2$, so that

$$r^* = m_2h_2r_4.$$

Case III: $(r_5/r_3) \leq h_2 \leq (r_5/r_4)$.

At the middle and lower M nodes the expected payoffs are m_1r_5 and $m_2h_2r_4$ respectively. We find the minimum of these and $m_1h_2r_2$, so that again

$$r^* = m_2h_2r_4.$$

To summarize the results,

1. In peacetime undertake overt intelligence gathering (D_1^*=2).
2. If mobilization occurs
    a. If $h_2 \leq (r_5/r_4)$, continue overt intelligence until hostilities occur (D_2^*=1),
    b. If $h_2 \geq (r_5/r_4)$, interdict launchers (d_2^*=2).

Figure 7.21 shows these results graphically, together with the expected result if no action is taken prior to hostilities.

7.3 Circulation and Decision Model Synthesis

In the decision model in the previous section assume the payoffs are measured in expected numbers of missiles launched. Equation (4.1) in Chapter 4 gives the expected number of missile launched by launcher i. The result measures from the decision model, (r_1 through r_5) can be expressed as

$$r_j = \frac{n^{(j)}q_1^{(j)}(1-(q_1^{(j)}q_2^{(j)})^m)}{1-q_1^{(j)}q_2^{(j)}}, \quad j=1,2,3,4,5. \tag{7.4}$$

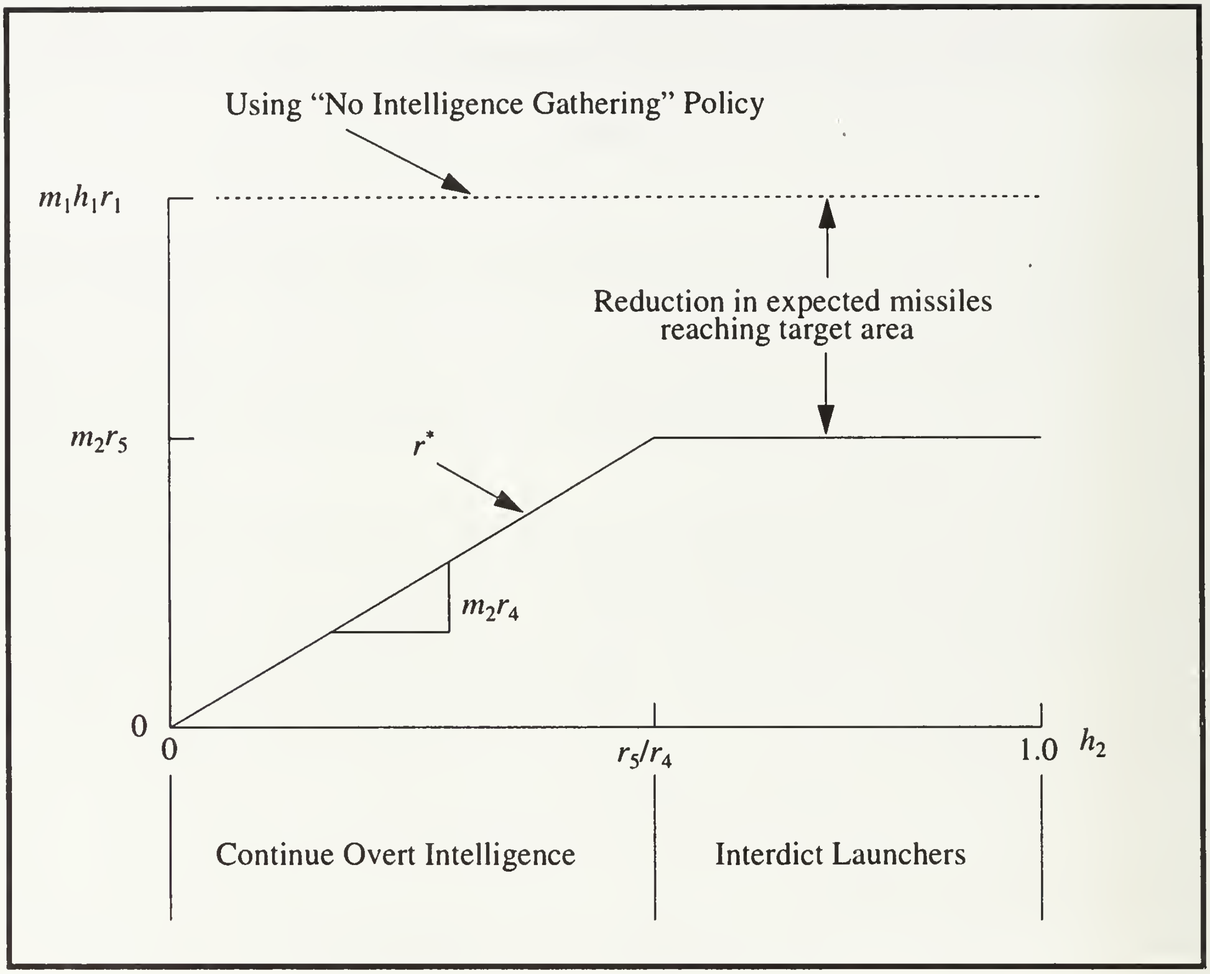


Figure 7.21: Optimal Policies and Expected Payoff Function

where the superscript (j) on the parameters refers to the subscript on r. For example, $n^{(1)}$ represents the estimated number of launchers remaining given the peacetime decision is no effort (D_1=0), mobilization occurs ($M = 1$), the mobilization decision is no effort (D_2=0), and hostilities occur ($H = 1$). The symbol m is the maximum number of missiles that can be launched from a single launcher (see Chapter 4).

Let n be the number of launchers before the start of any hostilities. It follows that

$$n^{(1)} = n^{(2)} = n^{(3)} = n^{(4)} = n, \tag{7.5}$$

as nothing is done prior to hostilities to reduce the number of launchers in these instances. If the interdiction decision is made we assume that action is taken against launchers on their way from storage to the forward staging area resulting in $n^{(5)} < n$.

It is assumed in this report that more intelligence leads to a reduced chance of launcher survival during the outbound leg of the cycle. This is expressed as

$$1 = q_1^{(1)} \geq q_1^{(2)} \geq q_1^{(3)} = q_1^{(4)} = q_1^{(5)}. \tag{7.6}$$

Once hostilities start, it is assumed the launcher survival probability q_1 on the outbound leg of the launcher cycle is approximately the same whether peacetime intelligence is followed by more intelligence, or by interdiction upon threat mobilization. Since post-missile-launch counter effort tactics are based on the flaming datum, they are assumed to be independent of pre-hostility intelligence effort. Therefore, it follows that all return transit launcher survival probabilities would be equal,

$$q_2^{(j)} = q_2, \tag{7.7}$$

for all j.

To determine the optimal policy, Figure 7.21 shows that the ratio r_5/r_4 is a critical quantity. Using Equation (4.1), the ratio can be expressed as

$$\frac{r_5}{r_4} = \frac{n^{(5)} q_1^{(5)} (1 - q_1^{(4)} q_2^{(4)})(1 - (q_1^{(5)} q_2^{(5)})^{m})}{n^{(4)} q_1^{(4)} (1 - q_1^{(5)} q_2^{(5)})(1 - (q_1^{(4)} q_2^{(4)})^{m})}.$$

From (7.5), (7.6), and (7.7) this equation reduces to

$$\frac{r_5}{r_4} = \frac{n^{(5)}}{n}$$

which is the estimated fraction of launchers remaining after interdiction. This result is shown in Figure 7.22, and can be stated as follows:

> *Given that mobilization has occurred and intelligence is being gathered overtly in this period, if the decision maker assesses the probability that hostilities will occur to be higher than the fraction of missile launchers that are expected to survive preemptive interdiction, then preemptive interdiction of the launchers is the optimal course of action.*

Intelligence Effort Level

Nothing has been said to this point about the type or level of intelligence effort. A detailed discussion of this topic is beyond the scope of this paper, but we include some remarks that show the relation between critical model parameters and resources spent on intelligence gathering.

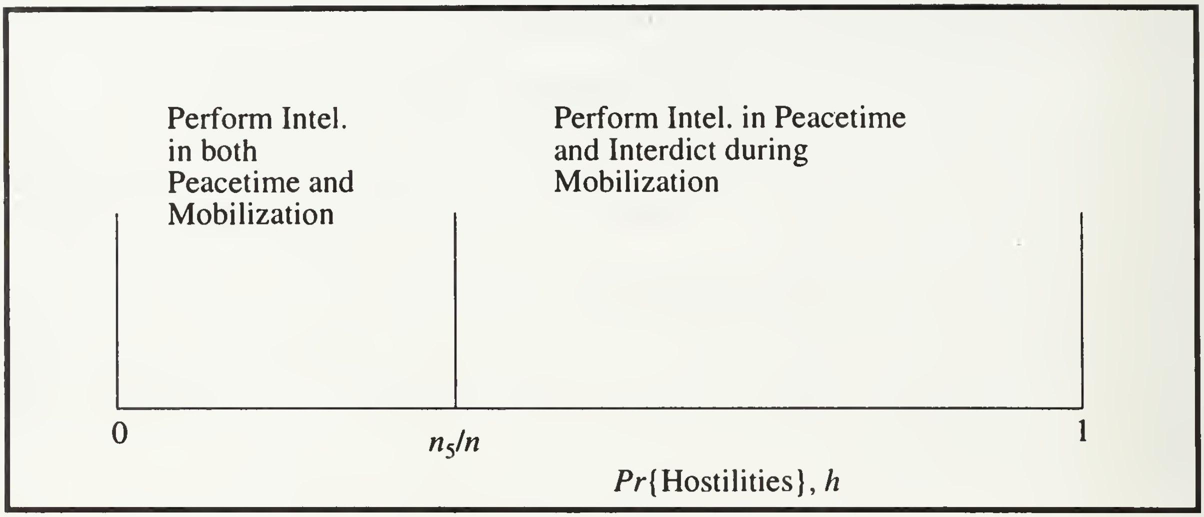


Figure 7.22: Optimal Policy in Terms of Fraction of Launchers Remaining

Figure 7.23 shows a typical curve of how the fraction of launchers that survive interdiction during mobilization is reduced by increasing intelligence gathering efforts.Also shown is a line depicting the effect of deterrence (and a dashed line showing no deterrence effect). For intelligence efforts at levels larger than this cross-over point the optimal strategy is to interdict launchers during mobilization.

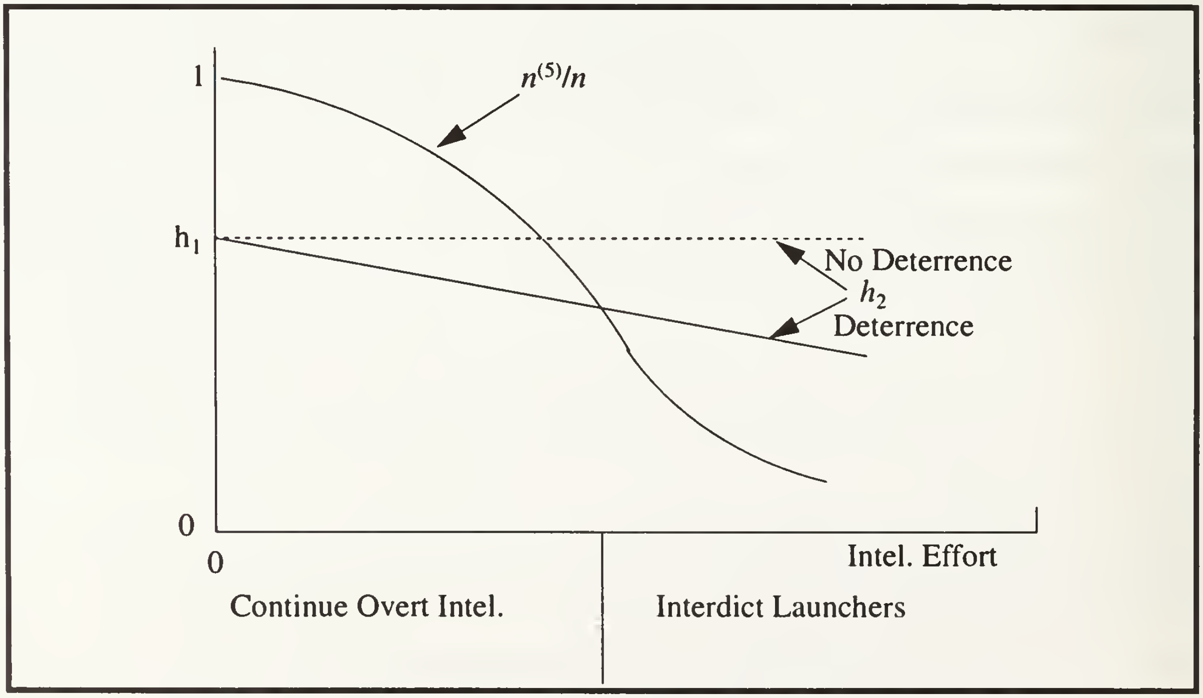


Figure 7.23: Optimal Policies in Terms of Intelligence Effort

The particular shape of the curve and the units used to measure the intelligence effort cannot be discussed in this report. An obvious unit is man-hours of effort; another might be surveillance flight hours. The S-shaped curve shown is typical of what one might expect, indicating little effect from small amounts in intelligence effort and little effect from amounts above a certain threshold. We can say little more at this preliminary stage of modeling.

8: Analyses of Unattended Ground Sensors in Theater Ballistic Missile Defense Attack Operations[1]

by

Richard J. Haberlin, Jr.

8.1 Introduction

Unattended ground sensors have a significant potential for improving Tactical Ballistic Missile Attack Operations. However, a lack of confidence in the systems and a lack of tactical doctrine have limited their employment to date. This paper provides analyses that demonstrate the effective use of sensor technology and provides recommendations as to how ground sensors may best be employed. The relatively disappointing effectiveness of active defense systems such as Patriot has illustrated the need for more reliable, yet cost-effective, means of defending theater forces.

Since the 1960's, ground sensors have been used to sharpen the theater tactical picture with limited effect. An early attempt, the McNamara Line of 1967, was aimed at slowing the flow of military goods along the Ho Chi Minh Trail by identifying targets for air strikes. These were arcane acoustic and seismic detectors with relaying aircraft, both manned and remotely piloted, used to monitor the output. The normal time between target acquisition and weapon delivery was approximately five minutes, yet few kills were confirmed (Ref. [10]). These poor results have been attributed to the unreliable and limited output of the sensors. Specifically, the sensors of the McNamara Line could determine if a target was personnel, wheeled vehicle, or tracked vehicle. For the United States, the losses fighting the sensor war were significant. Of the more than 600 planes and helicopters lost in Laos, one half to two-thirds were lost to defensive positions along the Ho Chi Minh Trail (Ref. [10]). Today's mission is similar, but the sensors have improved tenfold. The key, however, remains detecting and identifying a viable military target against which minimal assets are directed.

A Time Critical Target (TCT) is any military vehicle which, from its standard tactics, can be expected to remain on a road system for a period not to exceed 30 minutes. As a result, actions

---

[1] Edited version of LT Haberlin's MS paper in Operations Research

taken to prosecute this type of target must be made expeditiously. TCTs include Transporter-erector Launcher (TEL) units, command vehicles, missile fuel trucks, missile loading trailers, and mobile SAM units, among others. During the Gulf War American Pilots used random search tactics in an attempt to locate Iraqi TELs. Not only did this tactic prove largely unsuccessful, but it required the coalition to completely control the airspace above the battlefield. A more prudent tactic would be to send friendly aircraft over hostile territory only to prosecute a specific target already detected, located and identified by electronic means.

8.1.1 General Background

Theater ballistic missile (TBM) defense basically follows two discrete doctrines. Counterforce is aimed at the destruction of a TCT as it travels to and from its assembly area for reloading and maintenance. Active defense is aimed at the destruction of the individual tactical ballistic missile after launch during the boost, reentry or terminal phases. Only the counterforce strikes the target while it is slow-moving and, more importantly, only counter force prevents future use of the same TCT and its crew. It has been shown that counterforce, even in modest proportions, geometrically reduces the incoming tactical ballistic missile threat (Ref. [7]).

In the pre-hostility phase of a conflict, it will become necessary to locate and monitor TBM vehicles to prepare an adequate counter-attack should the conflict escalate. In addition to national collection assets, unattended ground sensors, placed strategically along known or suspected TCT travel routes, would aid in the development of a clear tactical picture to meet this end. Special Operations forces are also ideally suited for this type of work, but they constitute a limited and valuable commodity. Laying the sensor arrays, however, would definitely be a part of the pre-conflict Special Operations repertoire.

Should hostilities escalate to the point where armed response is required, a prime concern of the theater commander in the early phase of the conflict is the enemy TBM threat. It is in this early phase that the enemy has the greatest chance of surprise, and the full strength of its TBM force. TCT routes confirmed in the pre-hostility phase by ground sensors can now be covered by combat air patrol (CAP) or lethal unmanned aerial vehicles (UAV) awaiting a targeting order. Should a sensor array indicate the presence of a TCT, the theater commander can order the prosecution of that target by assets already on station. Turnaround times would be similar to those of the Mc-

Namara Line, but only against mission essential targets. Additionally, unlike the Gulf War, aircraft would be used for strike only, not for search. This would free additional sorties for the CAP missions.

There is also a political advantage to preparing a battlefield with unattended ground sensors. Through CNN America saw technology and training win a nearly bloodless battle in the Gulf War. Continuing this trend will require the reduction in the number of humans on the battlefield. Simply put,

> *There would be far less anger directed against an encounter in which the United States was putting hardware, not men, on the line and one which American casualty lists were dominated by decimated sensors, burnt-out computers, and downed RPVs* (Ref. [11]).

America's growing need for instant access to newsworthy information and demand for quickly resolved conflicts will require commanders to keep theater conflicts short and neat. It may be neither feasible nor desirable to await the final breakdown of negotiations before pinpointing enemy TBM forces. The use of unattended ground sensors could aid in defining hostile depots, staging areas, and hide sites prior to hostilities.

8.1.2 Analyses

The analyses provided in this document emphasize the use of unattended ground sensors in the locating and positive identification of time critical targets. Some degree of friendly intelligence capability is assumed in that candidate sensor array sites are chosen within the theater, and realistic enemy TCT populations are used. With the arrays in place, the decision maker must be able to evaluate the output of the array to determine, with some degree of confidence, if the target before the array is a TCT or a false indication.

The probabilistic decision model is developed and described in Section 8.2. It is based on there being a finite number of sensors available in a specific theater, with a fixed number of pre-determined candidate array sites. Further inputs include the approximate fraction of vehicle traffic assumed to be TCTs on each road to be seeded. This assumption is based on the belief that the intelligence analysts have pre-determined the most likely areas for TBM activity.

From the above input data, the model reports the optimal size array for each of the candidate locations. Additionally, the decision maker is provided with an optimal policy for determining the likelihood that the target is a TCT based on the number of sensors in agreement as to its identity. This policy may vary with each candidate array.

In Section 8.3, recommendations are provided on the placement of the arrays and their best configuration to maximize information gained while minimizing the likelihood of compromise. Specifics addressed include, inter-sensor spacing, placement patterns, array locations, and off-road distance.

Section 8.4 contains an analysis of the optimal location for the sensor array, along a given road. Considerations include locating at road intersections, along roads away from intersections, and at choke points through which roads pass.

Finally, Section 8.5 contains conclusions on all the analyses performed and provides a step-by-step procedure for defining, locating, and utilizing sensor arrays in a theater of operations.

8.2 Models and Assumptions

In the quest to determine the optimal sensor deployment scheme, it is most important to choose a model representative of actual Theater Ballistic Missile Defense (TBMD) operations. After some important clarifications, this section introduces the two separate models used in this paper.

First it is necessary to introduce the reader to the terminology common to TBM operations. Next, the reader l learns why the probability of detection can be assumed equal to one for an array of one or more sensors, and how this differs from the probability of correct identification. After introducing the equations which drive the numerical analysis of the probability of identification, the reader finds a description of the measures of effectiveness used to analyze the optimal policy. Finally, the assumptions common to both models are discussed to prepare the reader for the models, whose introductions end the paper.

8.2.1 Terminology

Before proceeding, we define the terminology used throughout this document.

1. Time Critical Target

A Time Critical Target (TCT) is any military vehicle which, from its standard tactics, can be expected to remain on the road system for a period not to exceed 30 minutes. As a result of this, actions taken to prosecute this type of target must be made expeditiously. TCTs include Transporter-erector Launcher (TEL) units, command vehicles, missile fuel trucks, missile loading trailers, and mobile Surface to Air Missile (SAM) units, among others.

2. "Steel Rattler" Unattended Ground Sensor

The "Steel Rattler" is a multi-component unattended ground sensor system with seismic, acoustic and infrared detection and identification capabilities produced by Sandia National Laboratory in Albuquerque, New Mexico. The seismic/acoustic system first detects a target of interest and attempts to match its signature with a pre-loaded signature database. The time of detection, system position, and identification are sent via satellite link to a decision maker in the fusion center. If a positive identification is not possible, the seismic and acoustic systems "wake up" the infrared sensor which is positioned further along the expected route of travel. The infrared sensor transmits to the fusion center a still-photographic thermal image of the target of interest at its closest point of approach. From this image, a system operator identifies the target. If the seismic and acoustic systems make an identification, the infrared sensor remains dormant. Current units are designed to be hand placed; air deployable units are undergoing testing. Air deployable units will not have infrared sensors. The analysis in this paper is based on the assumption that all sensors are identical

3. Sensor Array

An array consists of one or more unattended ground sensors operating as a single entity. That is not to say that their information is shared. Each sensor operates independently, but the information returned from the sensors in the array is treated as a single set of data.

4. Expected Loss

To measure properly the effects of varying array characteristics, it is necessary to derive a utility function defining the relative value of all possible outcomes. Typically, one of the payoff values is set equal to one, and the other outcomes are specified in relative terms. For the purpose of this analysis, a leaker is given a value of two and a false alarm a value of one. That is, it is considered twice as detrimental to miss a TCT than it is to prosecute an innocent vehicle. The appendices include analyses for other relative values of leakers and false alarms.

5. Probability of Detection

The probability of detection, P_d, is the single incident Bernoulli trial probability of success associated with the likelihood that the sensor identifies the presence of a target within its detection range. It is important to note that a single sensor may have more than one opportunity to detect the presence of a target, depending on the target's speed and the sensor sample rate. This phenomenon is covered in greater detail later in this section.

6. Sensor Forecasts

In reality, the output of any sensor is a forecast. With this in mind, it is easy to see that the probability of identification, f_1, is the probability that the sensor correctly identifies the target in front of it as a TCT given that it is a TCT. Similarly, the probability of false-identification, f_0, is the probability that the sensor incorrectly identifies the target before it as a TCT when, in fact, it is not. These are values typically specified by the manufacturer of the sensor after operational testing (Ref. [5]).

8.2.2 Probability of Detection

Modern acoustic, seismic and magnetic sensors sample at a fixed rate of t_s samples per second. The significance of this rate is that a TCT moving along a road may be investigated several times by the same sensor, thereby increasing the overall probability of detection. In fact, it can be shown that a “Steel Rattler” type of unattended ground sensor has a probability of detection approaching one against vehicles moving along roads at reasonable speeds.

To illustrate this effect, assume a single vehicle is traveling down a road with velocity v kph. Positioned along the road is an array of $n = 1$ sensor with maximum range $R_{\max}$ meters and sample rate t_s samples per second. The maximum *effective* range is determined by geometry to be r, and the sensor is positioned d_s meters perpendicular to the road, as shown in Figure 8.24, where $d_s < R_{\max}$.

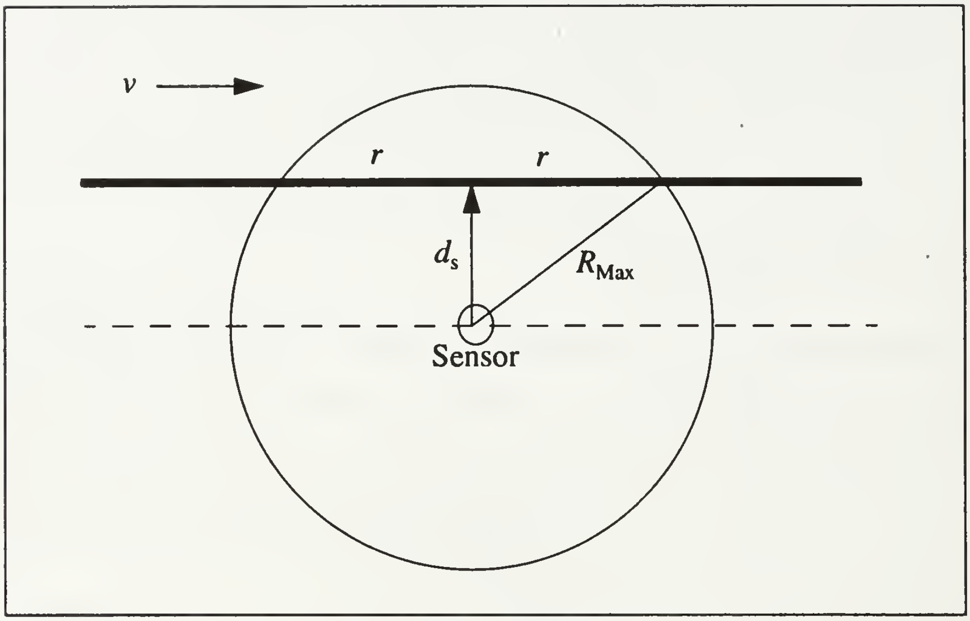


Figure 8.24: Single Sensor Probability of Detection

From Figure 8.24, it is evident that the vehicle will travel through 2*r* meters of sensor coverage for this particular sensor. Further, moving at speed *v*, the vehicle will travel $d_t = \frac{10v}{36t_s}$ meters between sensor samples. Rearranging the above to solve for the number of samples, *s*, available per sensor yields $s = \frac{2r}{d_t} = \frac{7.2rt_s}{v}$ sensor samples. Therefore, as a function of the off-road distance, d_s, the number of samples available per sensor is given by

$$s(d_s) = \left\lfloor \frac{7.2t_s}{v}\sqrt{(R_{max}{}^2 - d_s^2)} \right\rfloor. \tag{8.1}$$

Note the use of the floor function to allow only integer values of $s(d_s)$.

Figure 8.25 shows the relationship between number of samples and off-road distance for a sensor with sampling rate $t_s = 0.2$ samples per second, and maximum range $R_{max} = 500$ meters, against a target moving at $v = 15$ kph.

If the probability of detection for a single sensor sample is P_d, then the probability a vehicle is not detected on a single sample is $(1 - P_d)$. For a sensor sampling at rate t_s let $P\{\text{detect} \mid t_s\}$ be the probability that it detects the passing vehicle. If the sensor is shown as located in Figure 8.24, then assuming independence,

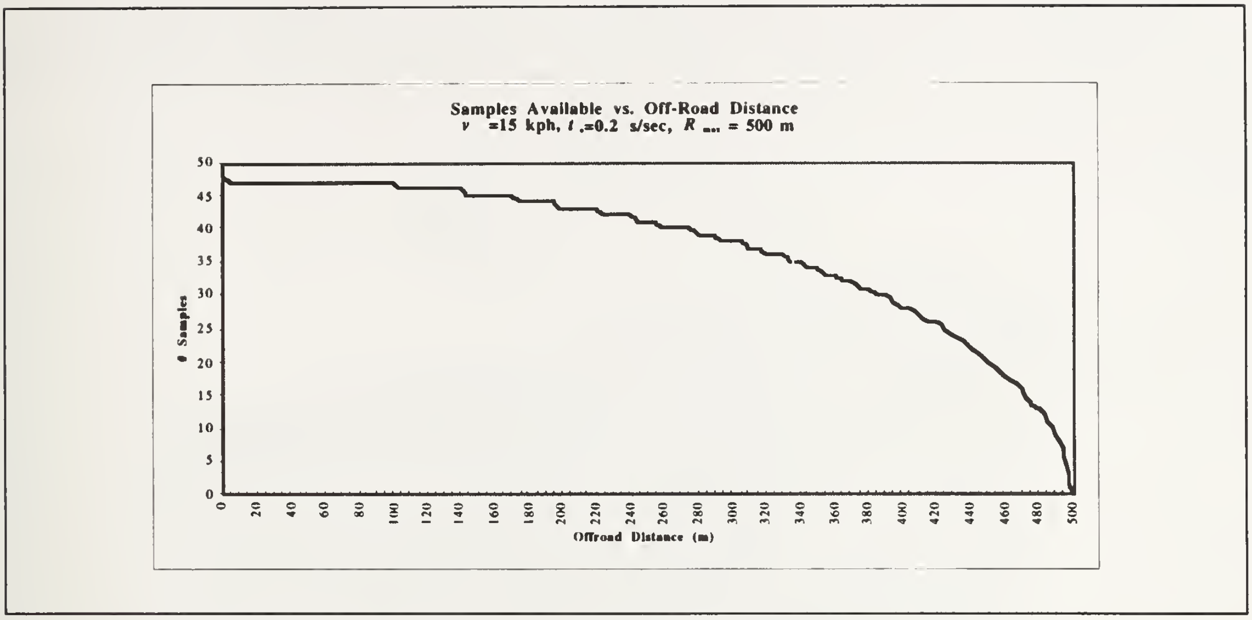


Figure 8.25: Samples as Function of Off-road Distance

$$P\{detect|t_s\} = 1-(1-P_d)^s, \tag{8.2}$$

where s is given by Equation (8.1). Values of $P\{\text{detect} \mid t_s\}$ are shown in Table 8.9 for a sample size $s = 10$ with probabilities of detection ranging from 0.1 to 0.9. It is clear that even at a distance near the maximum sensor range, the overall probability of detection is high, even for low values of P_d.

P_d	$P\{\text{detect} \mid t_s\}$
0.10	0.6513
0.20	0.8926
0.30	0.9718
0.40	0.9940
0.50	0.9990
0.60	0.9999
0.70	1.0000
0.80	1.0000
0.90	1.0000

Table 8.9: Sensor $P\{\text{detect} \mid t_s\}$ as a function of P_d

Typically, TCTs do not travel at speeds greater than $v = 60$ kph. A graph illustrating the number of samples per sensor as a function of off-road distance and target speed is shown for distances of zero to $R_{\max}$ and speeds from 5 to 60 kph in Figure 8.26.

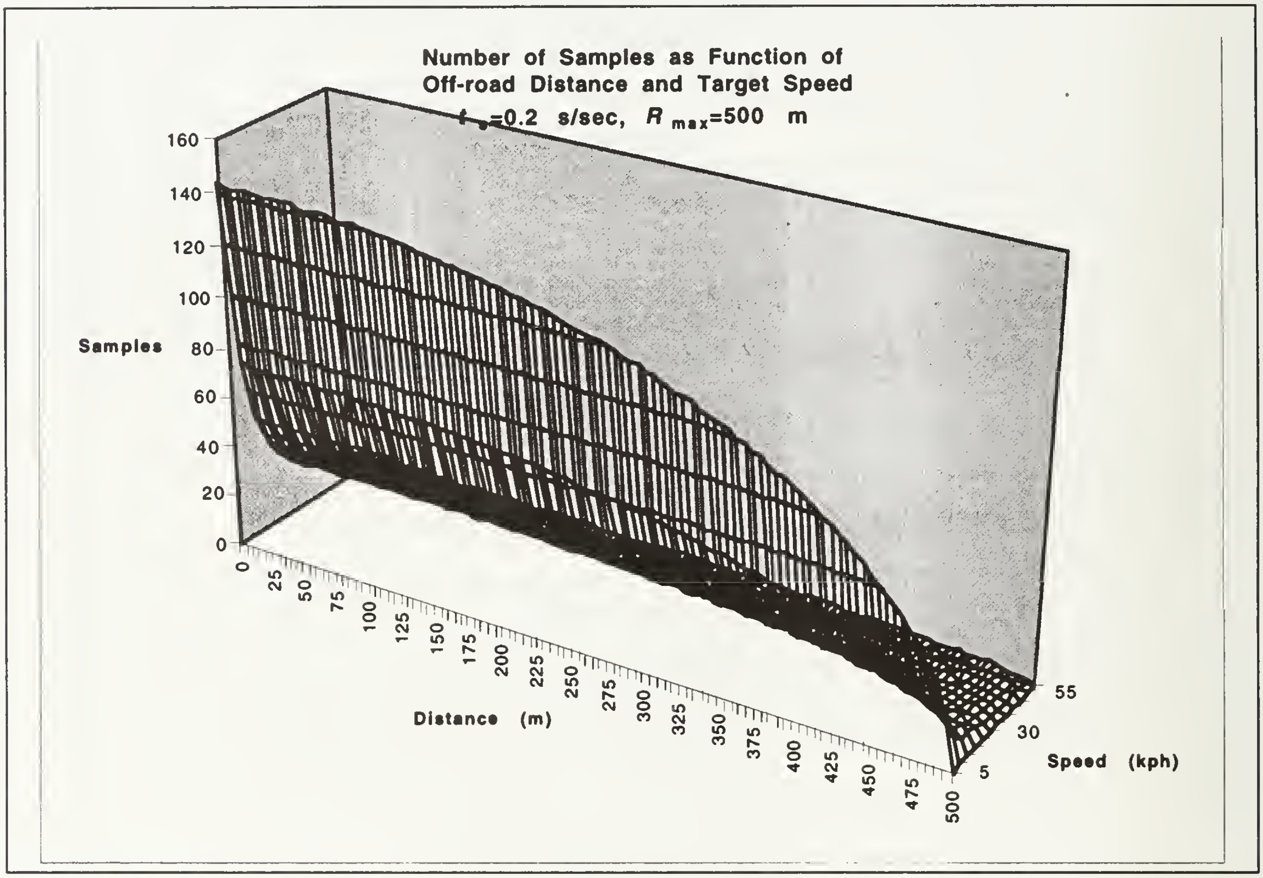


Figure 8.26: **Samples vs. Off-road Distance vs. Speed**

The curve drops to zero at an off-road distance equal to the sensor's maximum range for obvious reasons. Clearly, in most cases there are more than enough samples for a single sensor to warrant the assumption that any target present is always detected. The exception is for extremely poor sensors, placed near the maximum range from the road against high-speed targets. But, since this analysis concentrates on the "Steel Rattler" type of sensor, this case does not apply.

Sensor technology is represented by P_d, $R_{\max}$ and t_s. On the other hand, the operational characteristics of vehicle speed and off-road sensor distance are represented by v and d_s. Combining Equations (8.1) and (8.2) results in an expression for the probability of detection for a given sensor in terms of these variables,

$$P_{\text{detect}} = 1 - (1 - P_d)^{\left\lfloor \frac{7.2 t_s}{v} \sqrt{(R_{\max}^2 - d_s^2)} \right\rfloor}.$$

Detection probability versus off-road spacing is plotted for several values of P_d in Figure 8.27.

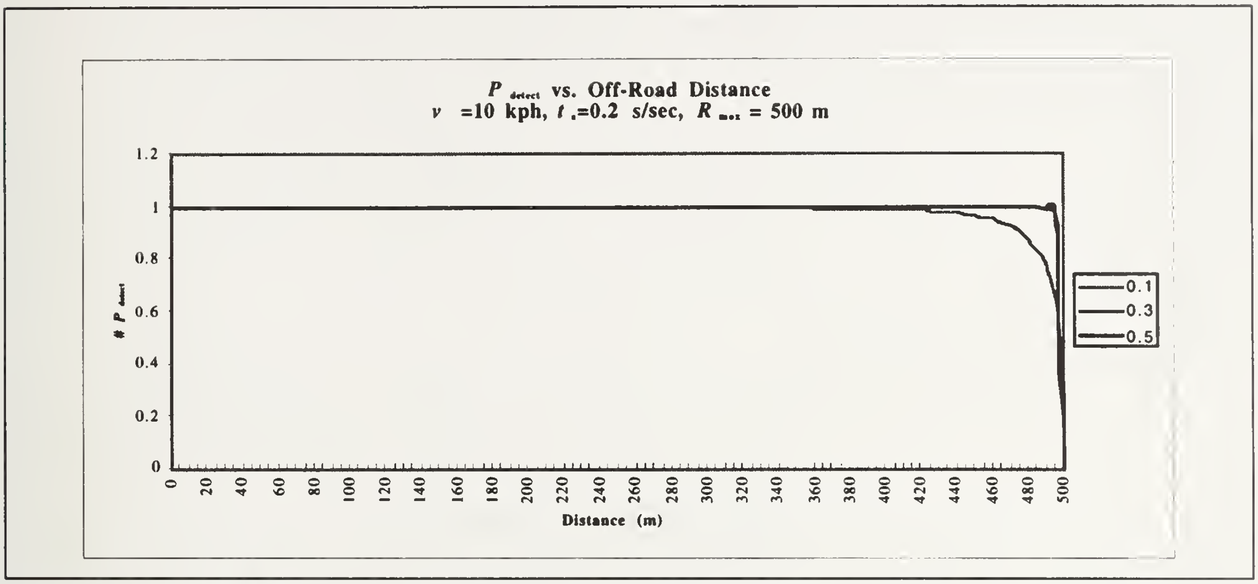


Figure 8.27: P_{detect} **as a Function of Off-road Distance**

In every case the P_{detect} drops to zero at $d_s = R_{max}$, as this reduces the effective sensor range to nil.

Figure 8.28 provides an alternate way of analyzing this relationship. It shows the maximum allowable off-road distance, d_s, such that the probability of detection is at least 0.95, as a function of the single sample probability of detection, P_d.

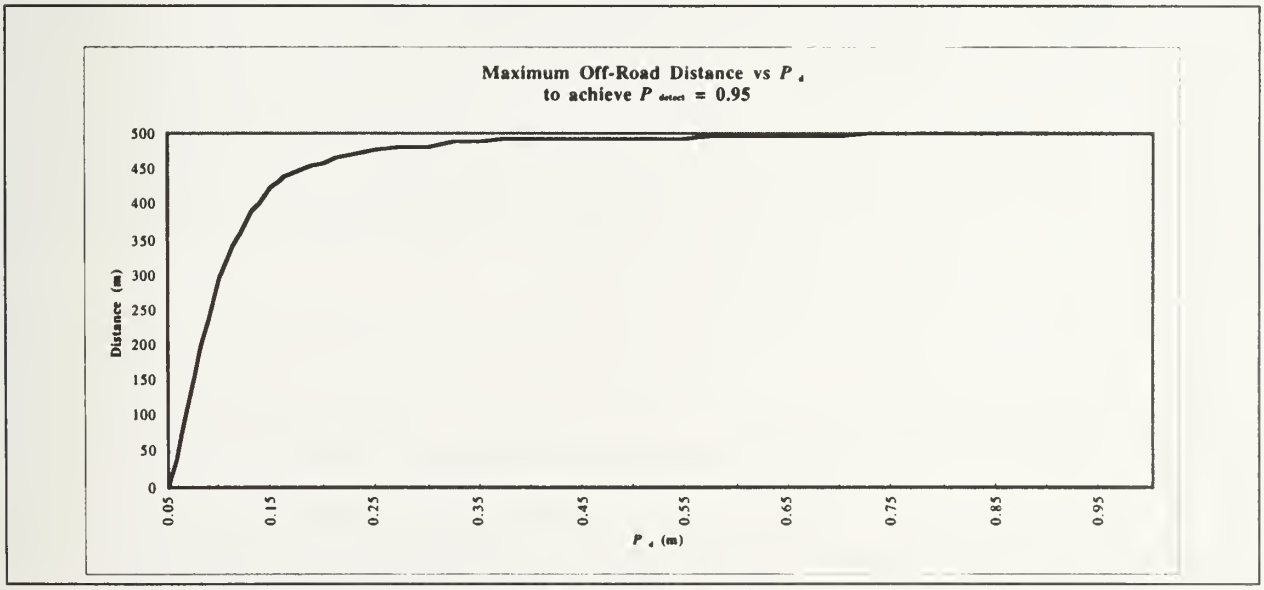


Figure 8.28: **Maximum Off-road Distance vs.** P_d

The sensors illustrated in these examples pale in comparison to the "Steel Rattler" type of sensor, yet still produce exceptional probabilities of detection. The "Steel Rattler" is advertised to have a sample rate, $t_s = 0.2$ samples per sec. and a probability of detection, $P_d = 0.95$ (Ref. [14]). These values make the probability of target detection virtually one through its entire range. Hereafter, because of the sensor's sample rate and maximum range, a vehicle is assumed to be detected with probability one.

8.2.3 Probability of Identification

In the previous section, the analysis of sensor detection indicates that a single sensor has nearly certain probability of detecting a vehicle moving along a road at a reasonable speed. The difficulty for the theater commander lies in determining when to commit scarce attack assets to prosecute a vehicle identified as a TCT based on data provided exclusively by ground sensors. The following discussion again assumes an array is comprised of *n* "Steel Rattler" type sensors, each of which relays a vehicle identification only once, at the closest point of approach, as it passes through the sensor's range. This identification is based on matching acoustic characteristics with an on-board signature file loaded prior to sensor deployment.

For each sensor in an array of size *n*, recall from Section 8.2.1, Definition 6 that

$f_1 = P\{\text{Sensor indicates a TCT, given a TCT is present}\}$, and

$f_0 = P\{\text{Sensor indicates a TCT, given no TCT is present}\}$.

Throughout this paper it is assumed that the sensor can discriminate between TCTs and other vehicles, so $f_1 > f_0$. Let P_x be the probability that an arbitrarily chosen vehicle traveling the road is a TCT (i.e. P_x is the fraction of vehicles that are TCTs).

The general formulation of probability of identification can be derived from Bayes' Rule and the Law of Total Probability. In the decision model described later in Section 8.3 the possible decision alternatives are to take action against a vehicle identified as a TCT, or take no action. The decision to take action is made when the theater commander has observed the number of sensors out of the entire array that indicate a TCT is present. Thus, the required identification probability to be used in the objective(Loss) function is $P\{\text{TCT} \mid k/n\}$, the probability that the detected vehicle is a TCT, given that exactly *k* out of *n* sensors in the array indicate it is. Assuming that the outputs of the array sensors are conditionally independent given the vehicle type, the decision probability is given by

$$P\{\text{TCT}|k/n\} = \frac{f_1^k(1-f_1)^{n-k}P_x}{f_1^k(1-f_1)^{n-k}P_x + f_0^k(1-f_0)^{n-k}(1-P_x)}. \tag{8.3}$$

To demonstrate the use of Equation (8.3), suppose there is an array of $n = 3$ sensors along a road with a TCT population of $P_x = 0.25$. The sensors are identical with $f_1 = 0.80$ and $f_0 = 0.15$. Table 8.10 shows the probability that a detected target is a TCT given k of n sensors indicate that it is for various values of k.

k	$P\{\text{TCT}|k/n\}$
0	0.0043
1	0.0896
2	0.6905
3	0.9806

Table 8.10: $P\{\text{TCT}\}$ Example Summary

The interested reader can find a more in-depth discussion of the derivation of Equation (8.3) in Appendix E.

8.2.4 Measures of Effectiveness

The theater commander must weigh all possible outcomes to arrive at his optimal policy. It is clear that this necessitates a common measure of effectiveness (MOE) used throughout the analyses. Possible candidate MOE's include the probability of an array correctly identifying a target and the loss incurred by acting incorrectly based on an array forecast. The former is maximized and the latter minimized.

Both of the above MOE's are used in this paper because they are intertwined. The best array size is achieved by optimizing the probability of correct identification subject to constraints on the number of sensors, available intelligence, and potential array sites. Given the optimal size, the incurred loss MOE is used to determine the theater commander's decision policy. This relationship is made clear in Section 8.3.

8.2.5 Common Model Assumptions

Before the models for optimal sensor deployment are introduced, it is necessary to make some assumptions about the environment in which the problem is to be solved. The following assumptions hold true for both models used in the analysis of sensor configuration. First, there must be a finite number of sensors available to a theater commander for his area of responsibility. The employment of these sensors, however, is left to his discretion. If reality indicates that sensors are abundant, then the above constraint can be thought of as a logistical limit to the maximum number in theater at a given time. Next, it is imperative that the intelligence community be able to evaluate or estimate the fraction of vehicles in the area of responsibility believed to be TCTs. This assumption varies slightly between the two models, as shown in the following subsection. It is not imperative that the intelligence specialists give an exact percentage. Because the optimization is performed through a sensitivity analysis, an approximate range is sufficient and can be calculated from the enemy order of battle (OOB). Finally, as Model 2 shows, the optimization is best performed with a specific theater in mind. In this case, the number of arrays desired is an input value, and would not make sense otherwise. These simple assumptions set the stage for the two decision models below.

Model 1 Introduction

The first model is representative of the current, automatic "decision making" in which one good decision is assumed to be the answer to all problems. It is included in this paper as a comparison to Model 2, which provides a response that changes with the specific theater, and is recommended. With the common assumptions above, the theater commander must choose n, the number of sensors in every array. Let N be the number of sensors available and let A be the number of candidate array sites. Then, $An \leq N$. The value of n may come from some tactical publication, a rule of thumb, or a hunch. In Model 1, n is chosen mathematically by dividing the number of sensors available by the number of array sites, and rounding down to the next lowest integer. Thus, $n = \left\lfloor \frac{N}{A} \right\rfloor$. Since all of the arrays are of equal size, the optimal solution may be found directly by arithmetic means. A significant drawback of this method is that sensors for additional arrays may not be available later in the conflict. Summarizing, in Model 1 the problem is to specify locations in the theater commander's AOR at which arrays should be placed. Array size is the direct result of distributing the available sensors equally among the locations.

Model 2 Introduction

The second model presents a more pragmatic, yet flexible approach to the sensor array problem. It uses the power of information gathered by intelligence analysts on the best candidate array sites for a given theater. In addition to a fixed N, it is assumed that a reasonable number of prospective sites has been chosen by professionals in the intelligence or special operations communities. In this case, the value $n = \left\lfloor \frac{N}{A} \right\rfloor$ may not represent the optimal array size for a given location. In fact, roads with differing fractions of TCT traffic are expected to be best covered by arrays of differing size. Since the array positions are chosen in advance, it is further assumed that some estimate of TCT traffic can be made for each road to be seeded. Let i be one of the roads to be seeded with an array. Further, let $P_x(i)$ be the fraction of traffic along road i comprised of TCTs. Then there is a specific n_i^* corresponding to each $P_x(i)$. For each of these A arrays there will be an optimal policy which dictates when the theater commander should commit an attack asset to a target.

Figure 8.29 illustrates the general problem flow and the solutions obtained. Section 8.2 has introduced material above the dashed line, including model inputs and primary outputs of n_i^* and k_i^*. Inputs include the maximum number of sensors available, N , the number of candidate array sites, A, and the estimated TCT fraction of the population, $P_x(i)$ on each road i. Section 8.3 demonstrates solutions to the problems displayed below the dashed line. A brief sojourn above the line gives definitive solutions for the n_i^* and k_i^* introduced in this section. Next, using the theater commander's maximum desired reporting time for the entire array, t_R, an inter-sensor spacing, d, is determined. As an aside, these values may be combined to evaluate P_c, the probability that the entire array is compromised given a single sensor is found. With the array size, policy and spacing found, the theater commander must now weight the array attributes of reporting, compromise, and countermeasures in accordance with his own view of the tactical situation. From these inputs the optimal array placement geometry may be found, concluding the problem.

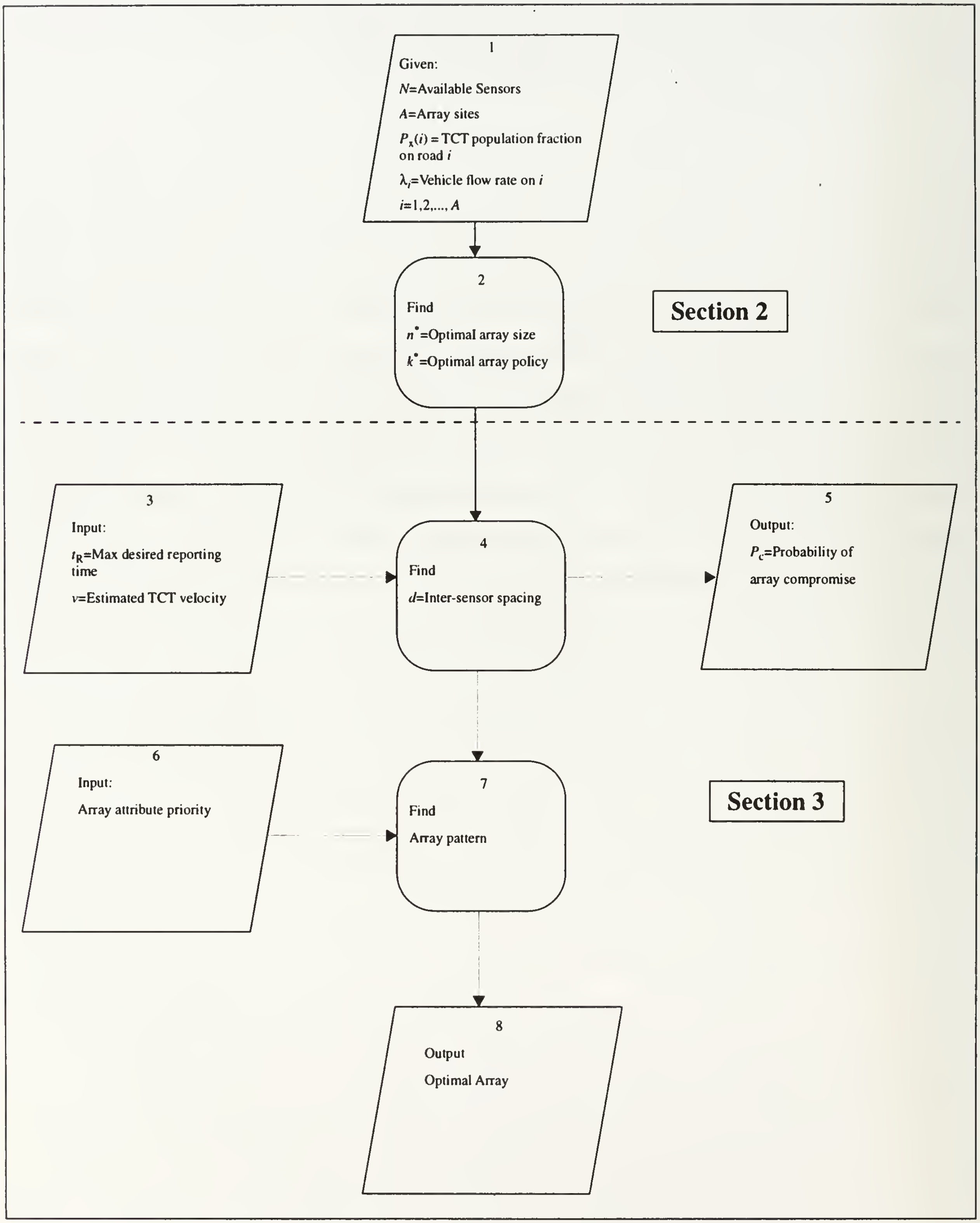


Figure 8.29: Problem Solution Flow

8.3 Array Configuration

8.3.1 Introduction

For reasons both tactical and financial, much emphasis is placed on the efficiency and reliability of the information reported by the ground sensors. With a finite number of sensors available, it is imperative that the arrays be configured so as to produce the most reliable forecast to the theater commander at the minimum unit cost.

Before continuing, it is necessary to introduce the problem to be solved by the models. In some specific theater, there is a network of roads along which time critical traffic, as well as normal civilian traffic, is known to travel. The fraction of vehicular traffic that is TCT is P_x, and the fraction along a specific road i of interest is $P_x(i)$, $i = 1,2,\ldots,A$. Further, the theater commander has been allocated N unattended ground sensors to locate and identify the TCT traffic, to be deployed at his discretion. Let these sensors have performance characteristics of $f_1 = 0.80$ and $f_0 = 0.15$, as described previously. Further, it is assumed that hostilities have erupted, so the "cost" of mis-identifying an actual TCT is $r_1 = 2$ and the "cost" of a false alarm is $r_2 = 1$. That is, a leaker is twice as costly as a false alarm.. Using this model, the following discussion details methods for determining the optimal array size, spacing and deployment pattern for the two models specified in Section 8.2.5 (p. 86).

8.3.2 Array Size

1. Model 1

The array size in Model 1 is the same for all locations and is predetermined to be n. This n is simply a function of the number of candidate array sites, A, and is given by $n = \left\lfloor \frac{N}{A} \right\rfloor$, where N is the number of sensors available. With the array size fixed, the sensitivity analysis consists of finding the optimal policy as a function of the overall TCT population in the theater. Let k_i be the minimum number of sensors indicating a vehicle is a TCT on road i required for the theater commander to commit an attack asset. The k_i or more of n policy defined in Section 8.2.3, may be found by solving the nonlinear program for each road, i:

$$\textit{Minimize } l_i(n, k_i)$$

subject to

$$1 \le k_i \le n,$$

$$k_i \quad \text{Integer}$$

$$i = 1, 2, \ldots, A,$$

where $l_i(n, k_i)$ is the expected loss function derived in Appendix F, and shown below as Equation (8.4).

$$l_i(n, k_i) = r_1 B(k_i - 1; n, f_1) P_x(i) + r_2 B(k_i - 1; n, f_0)(1 - P_x(i)). \tag{8.4}$$

In this eqation the binomial distribution function $\sum_{j=0}^{k} \binom{n}{j} f_0^j (1 - f_0^j)^{n-j}$ is abbreviated as $B(k; n, f_0)$.

Optimum k_i^* values for varying P_x and array sizes n are shown in Table 8.11 for arrays up to size $n = 10$.

	Array Size *n*									
P_x	1	2	3	4	5	6	7	8	9	10
0.10	1	2	2	3	3	4	4	5	5	6
0.20	1	2	2	3	3	4	4	5	5	6
0.30	1	1	2	2	3	3	4	4	5	5
0.40	1	1	2	2	3	3	4	4	5	5
0.50	1	1	2	2	3	3	4	4	5	5
0.60	1	1	2	2	2	3	3	4	4	5
0.70	1	1	1	2	2	3	3	4	4	5
0.80	1	1	1	1	2	2	2	3	3	4

Table 8.11: Model 1 Optimal Policies

For example, if the theater commander is allotted $N = 12$ UGS for his AOR consisting of $A = 3$ array sites, then he should deploy three arrays each of size $n = 4$. His intelligence team estimates the fraction of vehicles which are TCT's on the three roads are $P_x(1) = 0.30$, $P_x(2) = 0.10$ and $P_x(3) = 0.20$. Therefore, he knows from Table 8.11 that he should prosecute the target only if $k_1^* = 2$,

$k_2^* = 3$ and $k_3^* = 3$ or more of the four sensors in a given array identify the target as a TCT. These results are summarized in Table 8.12.

Array	$P_x(i)$	n	k_i^*
1	0.30	4	2
2	0.10	4	3
3	0.20	4	3

Table 8.12: Model 1 Example Results

The commander knows that this policy will minimize losses due to leakers and false alarms, thereby optimizing his asset allocation.

2. Model 2

Array size is the essence of the problem for Model 2. Remember that each array potentially has a different number of sensors based on the fraction of TCT's on the specified road, $P_x(i)$. Let n_i be the number of sensors used at location i, and let λ_i be the traffic flow rate along road i measured in vehicles per hour. In this case, the only restrictions on the n_i are that they be integers, that $\sum n_i \leq N$, and that each prospective location has a deployed array of at least one. This leads directly to an optimization of the form:

$$\begin{aligned} &\textit{Minimize} \sum_{i=1}^{A} \lambda_i l_i(k_i) \\ &\text{subject to} \\ &\sum_{i=1}^{A} n_i \leq N, \\ &n_j \geq 1, \qquad j = 1, 2, \ldots, A, \\ &1 \leq k_j \leq n_j, \qquad j = 1, 2, \ldots, A, \\ &k_j, n_j, \text{ integers,} \end{aligned}$$

where

$$l_i(n_i, k_i) = r_1 B(k_i - 1; n_i, f_1) P_x(i) + r_2 B(k_i - 1; n_i, f_0)(1 - P_x(i)) \tag{8.5}$$

is the optimal expected loss for array i, derived in Appendix F.

For example, assume the same theater commander is again allotted $N = 12$ UGS for his AOR, which has $A = 3$ candidate array sites. His intelligence team estimates the fraction of vehicles which are TCT's on these three roads are $P_x(1) = 0.30$, $P_x(2) = 0.10$ and $P_x(3) = 0.20$. Additionally, the flow rates for these roads are estimated at $\lambda_i = 1$ vehicle per hour, for all i. After solving the nonlinear integer program outlined above, the optimal array sizes are $n_1^* = 3$, $n_2^* = 4$ and $n_3^* = 5$. The optimal k_i^*, given these are $k_1^* = 2$, $k_2^* = 3$ and $k_3^* = 3$. These results are summarized in Table 8.13.

Array	$P_x(i)$	n_i	k_i^*
1	0.30	3	2
2	0.10	4	3
3	0.20	5	3

Table 8.13: Model 2 Example Results

Again, the commander knows that losses due to leakers and false alarms will be minimized if the k_i^* or more of n_i^* policy indicated is adhered to for a particular array. Given that k_i^* or more do indicate a TCT, he should prosecute the target, now identified as a TCT, with an available asset.

A comparison of the results reveals the advantages of Model 2 over Model 1. Continuing with the assumption that the same three roads were seeded in the given theater of operations gives the results summarized in Table 8.14. Model 1 determines a $k_1^* = 2$, $k_2^* = 3$ and $k_3^* = 3$ or more of $n = 4$ policy, and Model 2 determines the policies summarized in Table 8.13. In both cases the corresponding road populations, $P_x(i)$, are used for the loss function.

	Road			
	1	**2**	**3**	**Total**
Model 1	0.0930	0.0469	0.0819	0.2218
Model 2	0.1049	0.0469	0.0445	0.1963

Table 8.14: Minimum Expected Loss Comparison

Clearly, even for this small example, Model 2 shows superior performancewhen using twelve sensors for the theater.

8.3.3 Array Spacing

With the array size and identification policy determined, the next logical question to answer is how far apart should the sensors of a particular array be spaced, and how far from the road should they be. The second half of the question is more subjective, and therefore will be addressed first. In the case of air dropped sensors, it is likely that considerable error will be associated with the deployment and free-fall of the individual units. In this case, it seems smartest to aim at a position half of the maximum radius away from the road. Location error in either direction will then still allow the sensor to function with some or all of its capability. Sensors placed by special operations units are positioned with a greater degree of accuracy, and will always be within the range of the sensor's capabilities. Therefore, the off-road distance of the sensor should be left to the discretion of the insertion team leader, who should receive some training in placement. In general, the distance should be as close to the road as is operationally possible, without risking compromise of the sensor or the insertion team.

Inter-sensor spacing is a function of several different factors. Generally, the sensors need to be close enough to each other that the theater commander can consider the report cycle of all sensors in an array as a single event. On the other hand, they should be spaced far enough apart to minimize the likelihood that the entire array is compromised should a single sensor be discovered. Clearly, the probability that the entire array is compromised increases with decreasing inter-sensor distance. The goal is to find a middle ground acceptable to the theater commander.

To analyze this relationship, assume the hostile force has found a sensor and will conduct a random search of the surrounding area for some time, t, minutes. Then, assuming that m individuals each search randomly and uniformly at a rate of S m^2/second, these individuals cover a total area of $A_s = Smt$ m^2 during the search time. Using a standard area search model, the area to be searched is actually a circle with a radius equal to the distance between the farthest two sensors. This data is obtained by the enemy observing American standard operating procedures during the conflict, and correctly estimating n. An upper bound on this radius is found by assuming the sensors are arrayed in the line pattern configuration and that the sensor initially found is at the end of the line. This value is easily seen to be radius = $d(n\text{-}1)$. Let P_c be the probability an additional array sensor is compromised, given that a single sensor has been found. From Random Search Theory, the probability the hostile force finds one additional sensor in time t is given by see

$$P_c(d) = 1 - e^{\frac{60Smt}{\pi(d(n-1))^2}}$$ (Ref. [13]).

This probability of compromise is shown on the left hand curve in Figure 8.30 for varying inter-sensor range, d. Other parameters used in this illustration are:

- $n = 4$ sensors
- $S = 24\ \text{m}^2/$ second
- $m = 4$ searchers
  $t = 240$ minutes

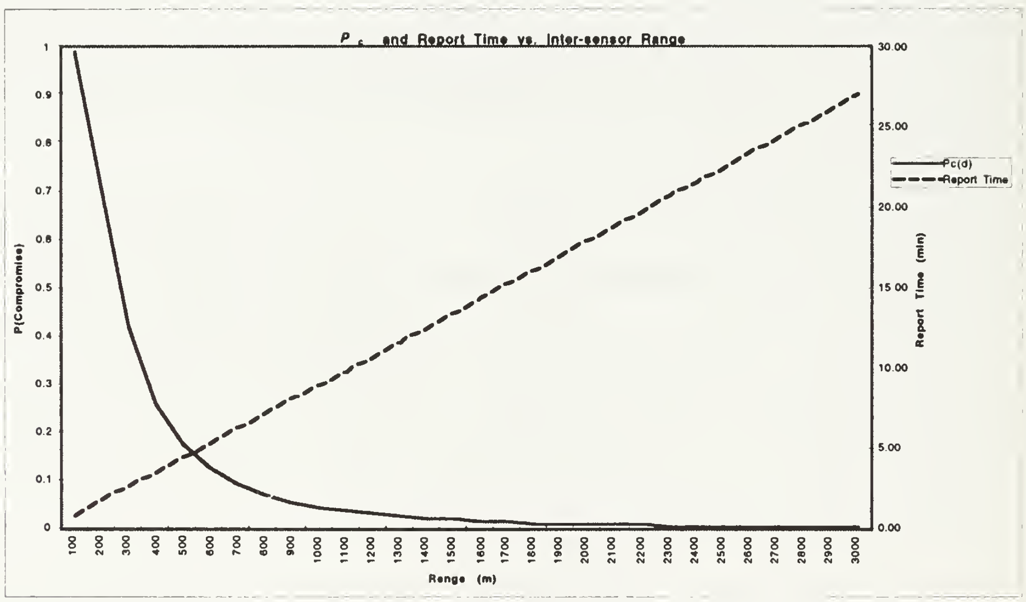


Figure 8.30: Effect of Inter-sensor Range on P_c and Report Time

For example, if four sensors are spaced at an inter-sensor distance of $d = 300$ meters, and the search effort is as indicated as above, then the probability that one additional sensor is discovered in the allotted search time is $P_c(300) = 0.419$. Therefore, there is a 41.9% chance that the searching party will find one additional sensor in $t = 240$ minutes. It is important to remember that the values shown in Figure 8.30 are obtained using the line pattern, and therefore represent an upper bound.

At the same time, the theater commander is awaiting the full report of his array. This time increases linearly with increasing inter-sensor distance d and is shown as the straight line in Figure 8.30. This figure assumes that the TCT is traveling at a constant speed, VTCT = 20 kph.

The graph illustrates two distinct features. The left hand curve depicts the decreasing probability that the search team compromises an additional sensor in the allotted time as the sensors become spaced further apart. The linear curve shows the increase in reporting time for the entire array as this inter-sensor distance increases. These two relations may be combined to obtain the probability of compromising one or more additional sensors as a function of array reporting time,

$$P_c(t_R) = 1 - e^{-\frac{60Smt}{\pi\left(\frac{25vt_R}{3}\right)^2}}, \tag{8.6}$$

where t_R is the reporting time for the entire array. A plot of P_c for varying t_R is shown in Figure 8.31 for the parameters specified above.

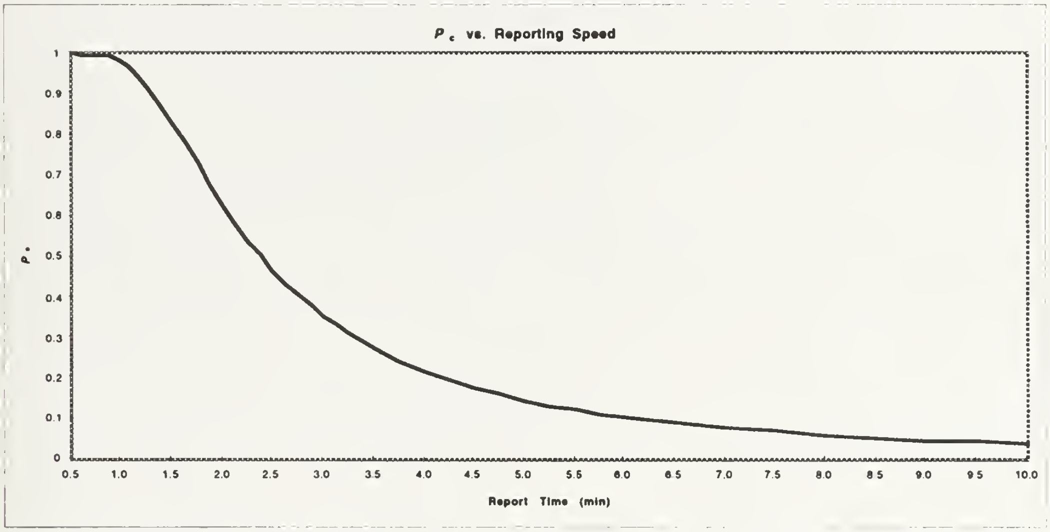


Figure 8.31: P_c as Function of Reporting Speed

It is important to remember that most of the factors involved in this curve are beyond the control of the theater commander. Placement errors from air drops or hand emplacement can influence the inter-sensor distance, and the opposing search team's capabilities, size, and available time are unknowns. The above formulas simply serve as a general reference as to the overall mission capability of the system using estimated parameters. Table 8.15 gives the inter-sensor range d for varying array sizes n and maximum reporting times, t_R, based on a TCT with estimated speed v = 20 kph. Tables for other common TCT speeds are compiled in Appendix D.

t_R (min)	Array Size n								
	2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	333	167	111	83	67	56	48	42	37
2	667	333	222	167	133	111	95	83	74
3	1000	500	333	250	200	167	143	125	111
4	1333	667	444	333	267	222	190	167	148
5	1667	833	556	417	333	278	238	208	185
6	2000	1000	667	500	400	333	286	250	222
7	2333	1167	778	583	467	389	333	292	259
8	2667	1333	889	667	533	444	381	333	296
9	3000	1500	1000	750	600	500	429	375	333
10	3333	1667	1111	833	667	556	476	417	370

Table 8.15: Inter-sensor Distance (meters) for TCT Speed v = 20 kph

Table 8.15 is based on Equation (8.7) which gives inter-sensor distance in meters as a function of array size, estimated TCT velocity in kph, and maximum array reporting time in minutes,

$$d = \frac{1000 v t_R}{60(n-1)}. \tag{8.7}$$

It is important to note that the number of sensors in the array for the look-up tables and for Equation (8.7) represent only the sensors not making simultaneous reports. In the case where *w* sensors report to the theater commander virtually simultaneously, only one of the *w* is used in computing the array size. This is described further in the following section.

8.3.4 Array Placement Patterns

This section covers the actual geometry of the sensor array as viewed from above. Each of the four proposed patterns has strengths and weaknesses when evaluated in the areas of reporting time, compromise and countermeasures as described below.

First, reporting time refers to the speed, in minutes, at which the entire array can deliver *n* reports to the theater commander. Next, compromise is the probability that additional array sensors are found given that one has been discovered. Finally, countermeasures refers to the likelihood that more than one sensor may be affected if some form of jamming is used in its proximity. Also included in this category are environmental effects which may hamper sensor performance, such as wind gusts or a falling branch.

When seeding a road, sensors spaced equidistant from each other fall into four proposed patterns: the line, the cross-hatch, triples, and the goal post. These are summarized and illustrated in Figures 8.32 through 8.35, below. In each figure *d* represents the inter-sensor spacing and d_s the off-road distance. Larger arrays than those shown can be built by adding two of the above simple pattern units, or by continuing the obvious pattern.

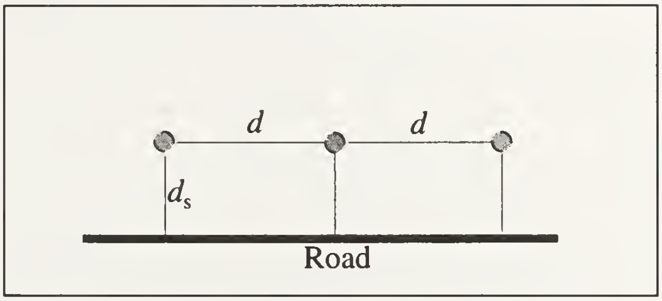


Figure 8.32: Line Pattern

The line pattern is the simplest of the building blocks used for array placement. Sensors are placed on one side of the road with a constant inter-sensor distance, *d* meters, and an off-road distance, d_s meters. This pattern is also the easiest to lay, either by hand or by air drop.

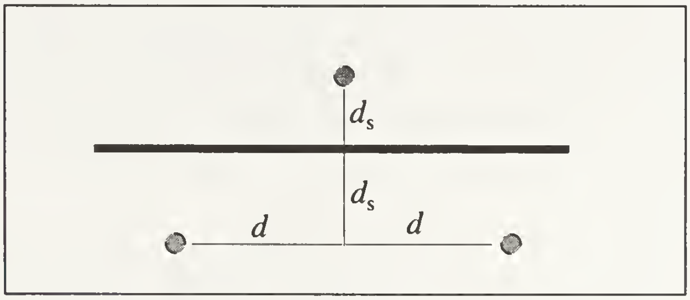


Figure 8.33: Cross-Hatch Pattern

The cross-hatch pattern alternates sensors on either side of the road, again with an effective along-road spacing of *d*. This pattern would be extremely difficult to deploy from the air. If it became necessary to do so, the aircraft would have to fly along either side of the road on separate runs, spacing the sensors a distance of 2*d* meters apart. Again, it would be unlikely that the configuration between the two rows would be properly aligned with this technique. With this in mind, it is recommended that this pattern be reserved for SOF deployment.

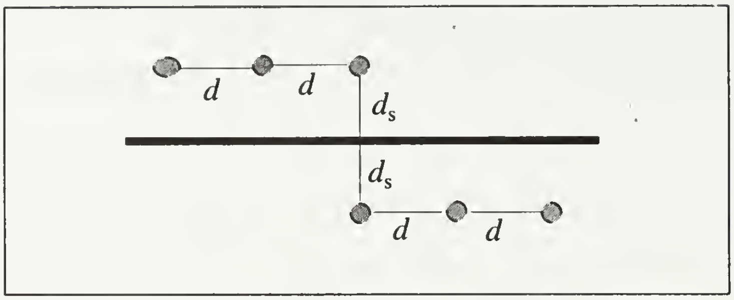


Figure 8.34: Triples Pattern

The triples pattern combines the line and the cross-hatch patterns. In fact, this pattern could also be produced with quadruples, or more. Similar deployment problems as with the cross-hatch are evident here, also. An advantage of the triples pattern is that every 2*d* meters of TCT transit there is a simultaneous report from two sensors.

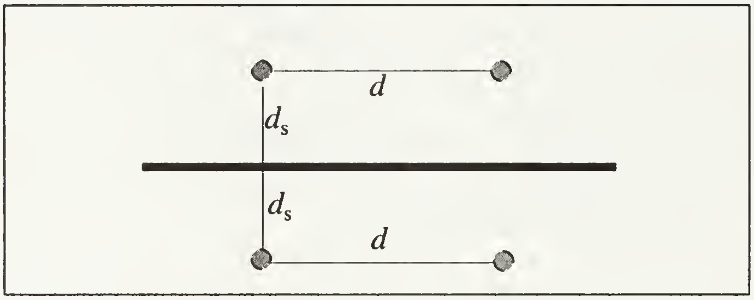


Figure 8.35: Goal Post Pattern

Finally, the goal post pattern is a combination of two line patterns, one on either side of the road. It can easily be deployed by both land and air forces. The greatest advantage of the goal post, is that the theater commander receives two sensor reports virtually simultaneously every *d* meters. The weakness is that any environmental effects or countermeasures affecting one sensor will probably also affect its counterpart across the road.

The performance of each of the patterns described above is scored on a scale of one to four and their relationships are illustrated on a policy diagram. In each case, a higher score is desired. Operationally, scores are obtained from the tactical experience of the theater commander. As an example, the author has provided scores for the overall performance of the sensor patterns when evaluated for reporting speed, likelihood of compromise, and susceptibility to countermeasures or environmental factors. These values are summarized in Table 8.16.

The triples and goal post patterns score high in the reporting category because two sensors relay information virtually simultaneously at most every 2*d* meters. This significantly speeds up the re-

Pattern	Reporting s_1	Compromise s_2	Counter Measures/ Environment s_3	$\sum_{i=1}^{3} w_i s_i$ $w_i = 1/3$
Line	1	1	3	1.67
Cross-hatch	1	4	4	3.00
Triples	3	3	2	2.67
Goal Post	4	2	1	2.33

Table 8.16: Placement Pattern Summary

porting time of the array. On the other hand, this arrangement could lead to greater susceptibility to countermeasures or compromise, and so they score lower in this attribute. The cross-hatch pattern scores high against both countermeasures and compromise because the inter-sensor distance on one side of the road is $2d$, or twice the actual inter-sensor distance. However, a report is only received every d/v time units rather than 2 reports every d/v time units, and therefore a lower reporting score.

The last column represents the relative values of the patterns if all three attributes are equally weighted, an unlikely occurrence for any decision maker. Therefore, a sensitivity analysis of the weights was performed using the ranks in Table 8.16. The policy space of the relative weights of the different categories is shown in Figure 8.36 where w_1 and w_2 represent the relative weights associated with reporting time and compromise. The sum of the weights on reporting time, compromise, and countermeasures is equal to one, or $\sum_{i=1}^{3} w_i = 1$.

The absence of the line pattern altogether is due to its domination by the cross-hatch pattern in the feasible region. For theater commanders who weigh reporting time heavily (greater than 0.5), goal post is the pattern of choice. Note that for a small range of the weights triples should be chosen, for example when there are approximately equal weights on reporting time and compromise and little weight on counter-measures. If all attributes are equally weighted, the diagram indicates cross-hatch should be chosen as was indicated in Table 8.16.

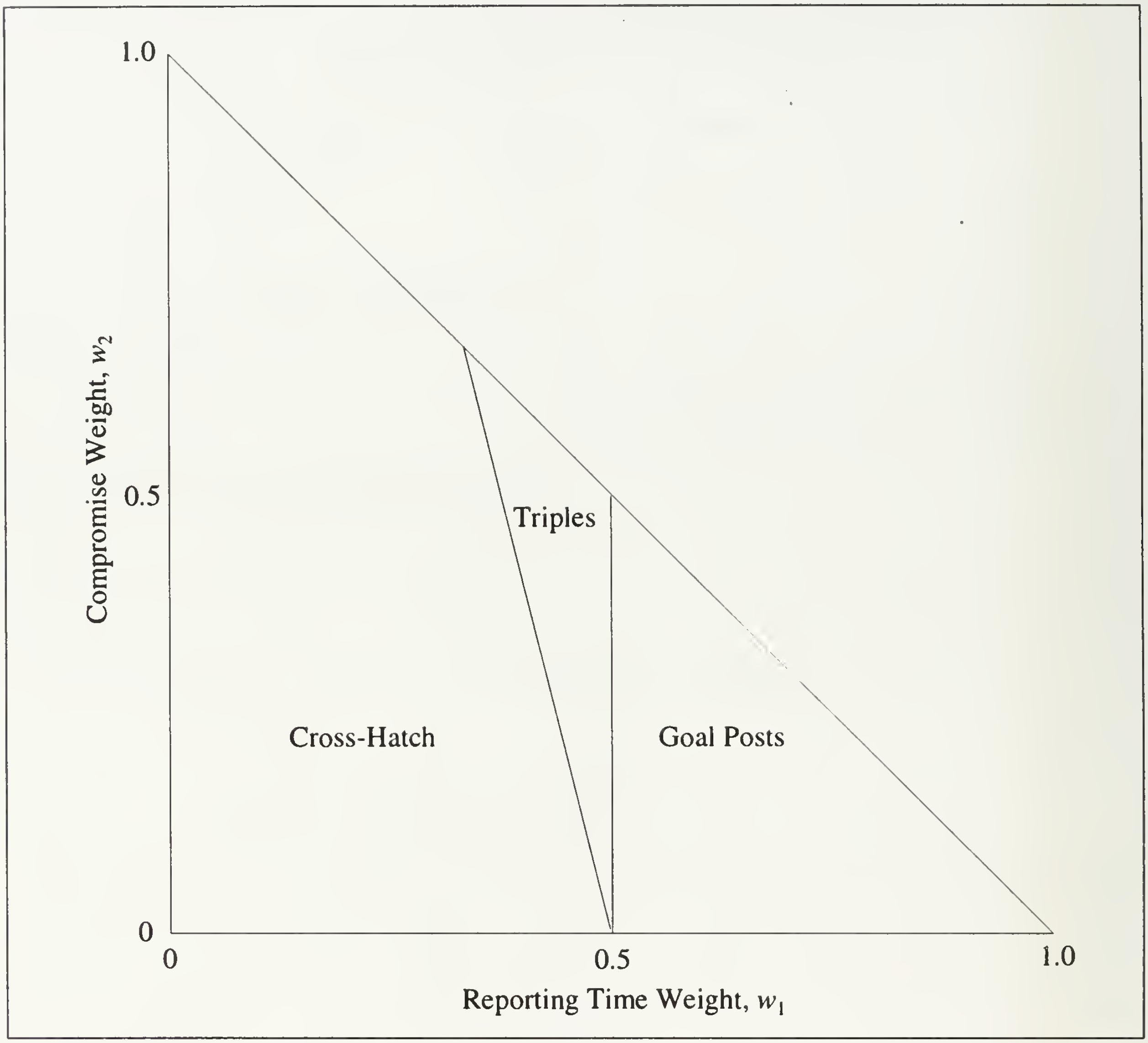


Figure 8.36: Placement Preference Regions

8.3.5 Conclusions

This section is best summarized by returning to Figure 8.29, reprinted below as Figure 8.37. This flow diagram may be used as a checklist for the theater commander when deploying his forces to minimize the TCT threat. The inputs include the array stockpile size, the candidate array sites, vehicle flow rates, and corresponding TCT population fractions for those sites. From the nonlinear program specified in Section 8.3.2 an optimal array size and k_i^* are obtained. Next, the theater commander must decide on his maximum array reporting time and estimated TCT velocity along the seeded road. These estimates, along with the array size above will yield the recommended inter-sensor distance when applied to Equation (8.7). With reasonable estimates of enemy capability, the probability of compromise may also be obtained at this point using Equation (8.6). Finally, the decision maker must prioritize the relative weights of reporting time, compromise, and countermeasures. These weights, when applied to Figure 8.36, return the optimal array pattern. Therefore, the complicated task of choosing the optimal array for a given set of roads in a theater of operations has been reduced to three simple decisions for the theater commander.

8.4 Array Location

8.4.1 Introduction

This section addresses the specific location of the array along a chosen road to maximize the array's strengths and minimize the chance of compromise in accordance with the theater commander's mission priorities. It is assumed that the intelligence community is responsible for choosing the particular roads to be seeded based on TCT traffic, mission criticality, and probability of mission success. The specific areas of placement considered are straight road segments, intersections, and geographic choke points. Each is described, then evaluated by sensitivity analysis to determine which best suits the theater commander's assessment of the tactical picture.

8.4.2 General Placement Concerns

Before the different possible locations are measured against one another, it is first necessary to determine the attributes most important to mission success. These attributes, placement and compromise, information, and tactical potential, are then used to evaluate the relative strength of each location. The performance of arrays in each of the possible locations described below is scored on a scale of one to four and their relationships are illustrated in a policy diagram. In each case, a higher score is desired.

Placement involves the ease with which the array may be deployed by SOF or by air. Placement not only includes the physical difficulty in laying the sensors, but also the likelihood that the insertion team is discovered before the array is completely deployed and camouflaged. Placement is grouped with compromise because they share the same strengths and weaknesses, thereby making their scores equal and uninteresting. That said, compromise, unlike the array spacing analysis, addresses the probability that the first sensor of an array is discovered by random sweeps conduct-

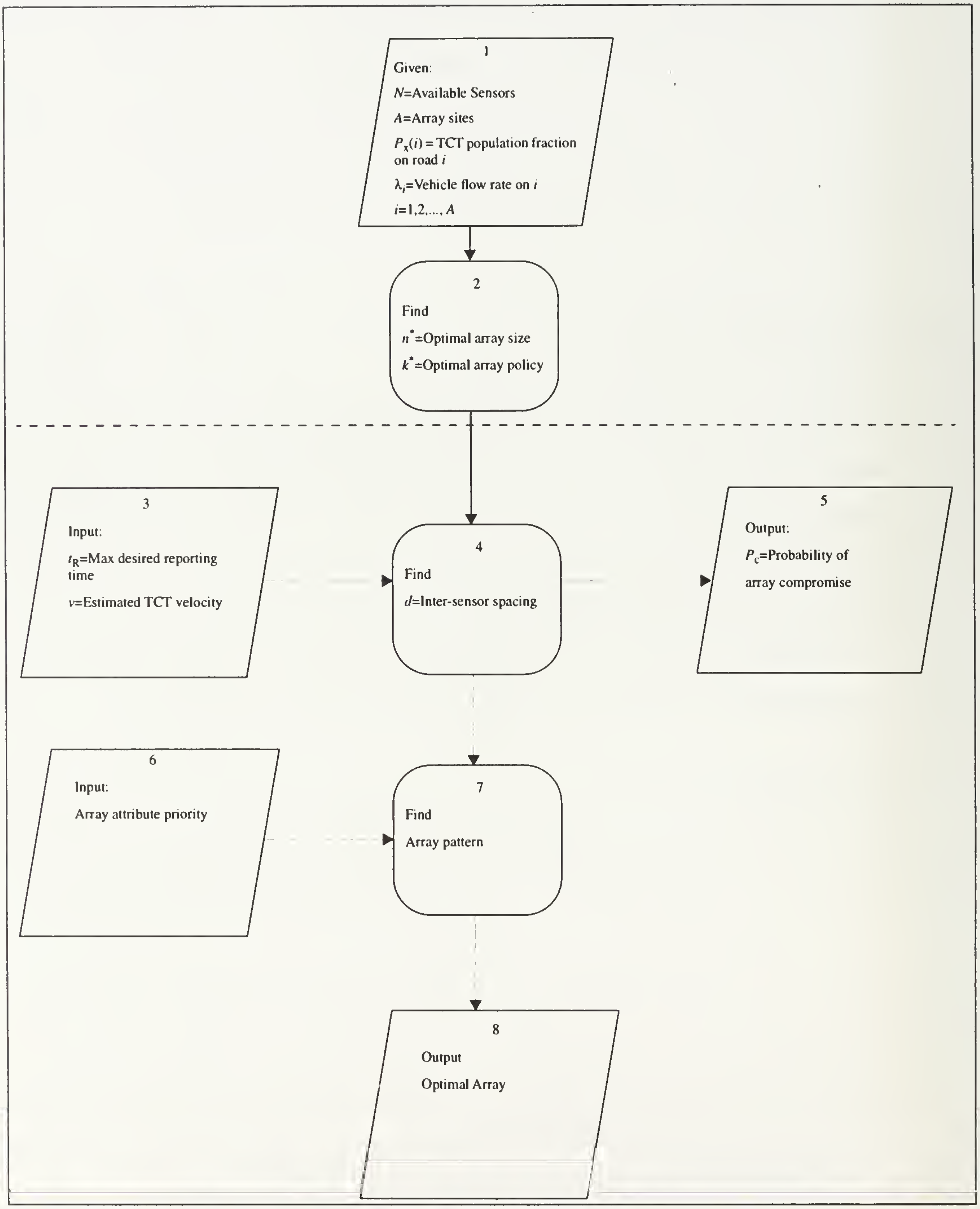


Figure 8.37: Array Deployment Checklist

ed by hostile forces. This is based on the assumption that the opponent knows the United States is using ground sensors for cueing strike assets, and is sweeping areas deemed most likely to be harboring arrays. Information is obtained from the raw data produced by the array. This includes both positive and negative information, which are defined as the presence of a TCT, and the lack of TCT contact, respectively. The tactical potential score is based on the clarity of the information received by the theater commander from the array. Specifically, the information attribute addresses the ability to transfer data to an airborne strike asset. Locations which allow an airborne platform to easily locate and positively correlate a vehicle with the array output are high on the scoring scale. Those that provide confusing or ambiguous information score low. The three attributes are weighted in relative importance by the theater commander and the optimal array location is read from the policy graph later in this section.

8.4.3 Straight Road Segment

Locating an array along a straight road segment is probably the simplest, and most practical insertion technique. Since straight roads are far more abundant than choke points or intersections, it is a simple matter for a trained unit to choose an easily accessible section of road, deploy the array and withdraw. Similarly, air-dropped arrays are equally effective along any section of a straight road, and the exact location may be chosen to minimize the possibility of action by hostile forces, both against the array and the deploying aircraft. Because road segments are so abundant, forces sweeping for arrays will have little success. It would be a difficult task, without some kind of cueing to isolate a particular section of straight road along which to conduct a search for an array. For this reason, it is hypothesized that enemy search forces will concentrate on sweeping choke points and intersections rather than on open roads. The geometry of road segments precludes excessive traffic, and the specific volume of traffic is a function of the road chosen, not the segment. Finally, road segments provide a clear tactical picture in that there is only one entry and one egress from a road segment. Therefore, a cued air asset should easily locate and visually identify a TCT traveling down a straight road. The scores for the straight road segment for the above attributes are shown in Table 8.17.

These scores are used in the sensitivity analysis against the other array locations.

Attribute	**Score** s_1
Placement & Compromise	3
Information	1
Tactical Potential	1

Table 8.17: Straight Road Segment

8.4.4 Intersection

Intersections naturally attract significant attention due to the seemingly endless possibilities they provide. A TCT is easily tracked until it reaches an intersection. Then, unless each of the exiting segments contains an array, it could simply vanish from the tactical picture of the theater commander. Similarly, if two arrays on opposite sides of an intersection gain contact in a reasonable time increment, who can positively state that there is only one TCT operating in the area? Perhaps the original TCT turned, and a second unit is passing the other sensor. These problems plague road intersections and may not be easily answered. The only definite solution is the use of a SOF team at the intersection to visually identify each TCT as it passes.

The general business of an intersection automatically makes array placement by SOF team more difficult. Although not all intersections are busy, they are by nature more traveled than straight road segments. Following the above hypopaper that enemy sweeping action will be concentrated at intersections and choke points, makes arrays placed at intersections more subject to compromise. Further, the likelihood that a given intersection is searched grows with the relative importance of that intersection as a military transit hub. Obviously, intersections near to forward assembly areas will be swept regularly. The real strength of intersections is the volume of information they produce. The sheer amount of traffic flowing through a busy intersection provides an excellent sample of vehicle population of all types, TCT and otherwise. A seeded intersection with no TCT contact provides as much, if not more, information as a positive contact along a straight road. The negative information associated with the intersection implies not only that the intersection sees no TCT traffic, but also that the road segments adjacent are not used by TCT's. This can significantly reduce the overall search area for other assets and can help determine array locations for future array deployment sites. Tactically an intersection provides little aid in the prosecution of TCT's by air assets. A TCT passing through an intersection is generally lost until it passes a more specific identification point, such as a choke point or an array along a straight road segment. Array output is generally too vague to determine which branch the TCT took when exiting the intersection. Table 8.18 the scores for an intersection as a placement location.

Attribute	**Score** s_2
Placement & Compromise	2
Information	3
Tactical Potential	1

Table 8.18: Intersection

8.4.5 Geographic choke point

Geographic choke points share the best and the worst characteristics of the above two locations. Placement is difficult due to the very nature of the choke point. Entry and egress to the area may be difficult, and it may be well patrolled because of its significance. Additionally, since the opposing forces must also realize this area is a choke point, it is a very likely candidate for sweeps, making the risk of compromise greater. With less area to search, arrays in these areas are at high risk. Depending on the particular choke point, information provided may be quite plentiful. If the area is one of few allowing passage between hostile depots and their forward staging areas, much information will be available. Similarly, a bridge or causeway frequently used to move military vehicles is a good target. Finally, the tactical use of a geographic choke point is incomparable. A targets moving into a choke point is restricted in movement and may be waited for as it egresses. This would allow an easy transition from ground information to air. The attribute scores for the geographic choke points are given in Table 8.19.

8.4.6 Sensitivity Analysis

A summary of the overall value of the different location areas when evaluated for placement and compromise, information, and tactical potential is given in Table 8.20. The last column indi-

Attribute	Score s_2
Placement & Compromise	3
Information	1
Tactical Potential	2

Table 8.19: Geographic Choke Point

cates the value of the location areas with all attributes weighted equally. Note that each area totals to a value of two. This implies that each of the areas has strengths and weaknesses in the attributes evaluated, and that the most suitable location depends greatly on the preferences of the theater commander.

Location Area	Information s_1	Tactical Potential s_2	Placement & Compromise s_3	$\sum_{i=1}^{3} w_i s_i$ $w_i = 1/3$
Straight Road	1	2	3	2
Intersection	3	1	2	2
Choke Point	2	3	1	2

Table 8.20: Location Area Summary

The policy space of the relative weights of the different categories is shown in Figure 4.1, where w_1 and w_2 represent the relative weights associated with information and tactical potential. The sum of the weights on information, tactical potential, and placement and compromise is equal to one, or $\sum_{i=1}^{3} w_i = 1$.

For example, the intelligence analyst, who is most concerned with information flow, may consider the information attribute paramount while having little concern for the tactical potential of the array. In his case, the array is best located at an intersection. On the other hand, the strike pilot is only interested in his ability to localize a target identified by the array. His choice would be for the choke point. The final extreme is represented by the SOF planner concerned with providing useful information without compromising the insertion team. His policy of choice would be the

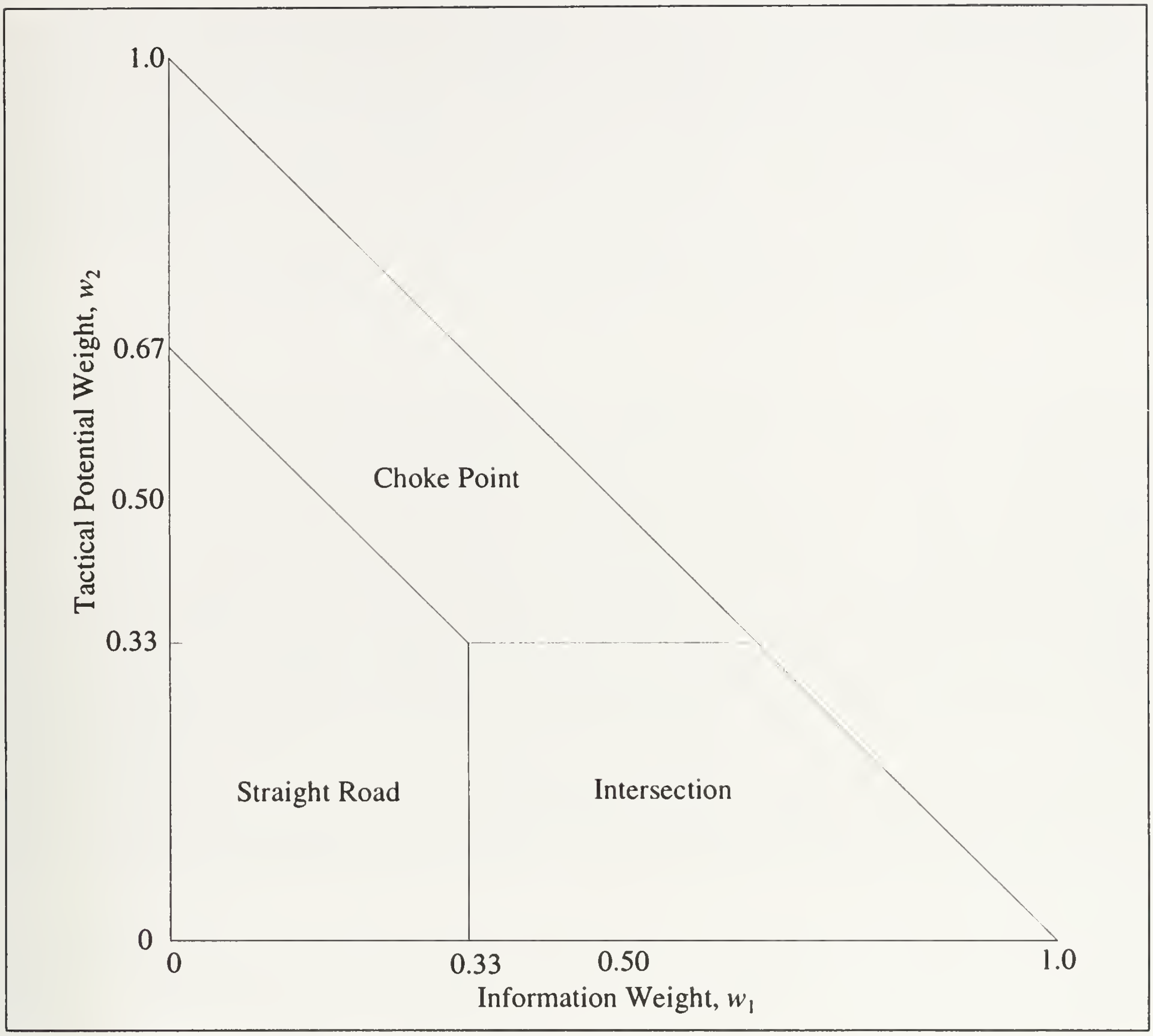


Figure 8.38: Placement Location Preference Regions

straight road. The diagram also provides a representation of the case illustrated in Table 8.20. This decision maker, who weighs all attributes equally, draws no information from the policy diagram

8.5 Conclusions

Used effectively, unattended ground sensors are a significant asset in the theater commander's TBMD toolbox. The key to accomplishing the optimal use of this resource, and those that rely upon it, is a step by step plan to follow for their employment. This paper provides the checklist to be used by theater commanders to maximize ballistic missile defense at minimal cost.

8.5.1 The Process

The process begins with a specific theater with an overlying road grid. The theater commander is allotted N unattended ground sensors to be deployed at his discretion. The intelligence shop, after careful analysis selects A candidate array sites based on time critical target traffic and mission criticality. Provided with the list of sites is a corresponding P_x, the fraction of vehicular traffic along that segment believed to be time critical and λ_i, estimated vehicle flow rate.

The theater commander then enlists a member of his staff to run the optimization given in Section 8.3 to minimize his expected losses based on the above parameters. The nonlinear program produces an optimal array size, n^*, and prosecution policy, k^*, for each of the A sites. If nonlinear programming software is unavailable, Appendix B may be used in which the maximum allowable probability of a leaker is compared to that of a false alarm to obtain an n^*. Appendix C then provides the optimal policy on prosecution using the array sizes specified in Appendix B. These look up tables are generated by enumerating the possible combinations of n and k, then choosing that which produces the minimal loss by Equation (8.5). It is important to note that the look up table procedure may not provide the actual optimal solution for the case where N is limited. The look up tables merely supply the optimal array size for discrete roads, without limiting the total count of sensors deployed.

With the array sizes and policies determined, the next step is to find the inter-sensor spacing to be used given that it is desirous to minimize the reporting time between adjacent sensors and also minimize the likelihood that the entire array is compromised by hostile forces. Appendix D provides look up tables for sensor distance based on array size, estimated TCT speed, and the maximum reporting time for the array. When entering the look up tables, it is important to subtract one for each pair of sensors that report virtually simultaneously because of their pattern orientation, as described in Section 8.3.4.

The geometry of the pattern is devised through the theater commander's relative weight of reporting time, compromise, and susceptibility to counter-measures. These weights are applied to Figure 8.36 and the optimal deployment pattern is read from the graph. To maximize the array effectiveness according to this theater commander's desires, sensors should be placed in the appropriate pattern with the spacing determined above.

After the specifics of the *A* arrays are complete, it is time to begin mission planning. The theater commander must meet with his intelligence analysts and operational planners to determine the relative weights of the factors affecting array locations, placement and compromise, information, and tactical potential. These weights are then applied to Figure 8.38 to ascertain the type of location most beneficial to the overall effort, yet in an area conducive to array deployment. These locations are broken into the categories of choke points, straight roads, and intersections.

8.5.2 Summary

As budgets continue to shrink and small theater actions become more common, optimal use of available assets exponentially increases in importance. The Vietnam era tradition of attrition warfare has given way to today's cost effective battlefield upon which fewer soldiers, and a large number of less expensive sensors are placed. This is the essence of Libicki's technological "Mesh" in which many small sensors perform all the data collection with the added advantage of being too numerous to kill, and thereby more robust. In fact, the role of unattended ground sensors can be summed up in that

> being there is necessarily a prerequisite to seeing there, and not necessarily a prerequisite to hitting there if the range set of one's own weapons is sufficiently dense.
> (Ref. [11])

Judiciously placed sensors, combined with lethal UAV's, artillery, or theater missiles would go a long way toward this vision. This document begins to satisfy the first portion of that equation.

Theater ballistic missiles pose an ominous threat to any theater commander in the battlefields of the future. It is only through the judicious use of all available assets that decisive action may be taken. Properly employed unattended ground sensors provide a cost effective and reliable option to assess the theater throughout the conflict.

APPENDIX A. THE "STEEL RATTLER" SENSOR

An excellent example of the advances in sensor technology is the "Steel Rattler" unattended ground sensor. The capabilities of both the sensor units and their deployment systems continue to evolve, but this appendix serves as a current-day ability profile. The sensors were designed and tested by Sandia National Laboratories in Albuquerque, New Mexico, from which most of this information was obtained. Further data was provided by Central MASINT Technology of Florida.

The "Steel Rattler" is a multi-component unattended ground sensor system with seismic, acoustic and infrared detection and identification capability. The seismic/ acoustic array first detects a target of interest and attempts to match its signature with a pre-loaded signature database. The time of detection, array position, and identification are sent via satellite link to a fusion center. If a positive identification is not possible, the seismic/ acoustic array "wakes up" the infrared sensor which is positioned further along the expected route of travel. The infrared sensor transmits a still photograph of the target of interest at its closest point of approach to the fusion center via a satellite link where a system operator must visually identify the contact. If the seismic/ acoustic array makes an identification, the infrared sensor will never be activated. There is no ability to turn on the infrared to confirm the sensor's identification. Similarly, if the seismic/ acoustic array fails to detect a target of interest, the infrared sensor has no means to detect on its own. It is possible to position a seismic/ acoustic array on either side of the infrared sensor to detect targets moving in either direction.

The seismic/ acoustic array field of regard is 360°. Therefore, the search area for the seismic/ acoustic array is circular with a radius equal to the maximum seismic/ acoustic range centered at the array position. This maximum range is approximately 500 meters, depending on the specific terrain in which the array is placed. For the purpose of this analysis, all sensors are assumed to be "cookie-cutter," implying that there is no chance of detecting a target outside the specified maximum range. In reality, some detections may occur in this region.

The "Steel Rattler" can be described as a system performing spot searches on a recurring basis. The actual sensor sample rate is once per second, but a maximum of five seconds are required to report a detection to the fusion center. For this reason, a five second sample rate is used in the model analysis of Section 8.3. The time required to check a target signature against the database is less than one second, and therefore considered negligible for this thesis (Ref[14]).

APPENDIX B. SIZE LOOK-UP TABLES

The tables in this appendix are based on the k^* or more of n^* policy given in Section 8.2 and the derivation in Appendix F. They are subdivided by the relative values of a leaker versus a false alarm, given as r_1 and r_2 respectively. Each table has a corresponding P_x value which is the fraction of traffic assumed to be TCTs. The sensors are assumed identical with $f_1 = 0.80$ and $f_0 = 0.15$, as previously described. The table is entered with a row value of the maximum allowable probability of prosecuting a non-TCT, and a column value of the maximum allowable probability of a leaker.

Table entries are obtained by enumerating possible values of n for the appropriate probabilities of a leaker and of a false alarm, and choosing the minimum array size. To summarize this procedure, let $k(n)$ be the optimal policy which minimizes $l(n,k)$. Further, let

$\alpha(n, k)$ be the probability of a leaker, $P(\text{Leaker})$, and

$\beta(n, k)$ be the probability of prosecuting a non-TCT, $P(\text{Hit F.T.})$.

The optimal array size may then be obtained from the math program given by

$$\begin{array}{l}
\textit{Minimize} \quad n \\
s.t. \\
\alpha(n,k(n)) \leq \text{Max. allowable } P(\text{Leaker}) \\
\beta(n,k(n)) \leq \text{Max. allowable } P(\text{Hit F.T.}) \\
n \geq 1 \\
1 \leq k \leq n \\
n,k \quad \textit{Integer}
\end{array}$$

For Example, assume leakers and false alarms have relative values of $r_1 = 1$ and $r_2 = 1$, respectively. Now assume that the theater commander wants to know the optimal array size given that he will allow a 5% leaker probability and a 20% chance of prosecuting a non-TCT. The intelligence shop estimates that 20% of vehicle traffic are TCT's. Then, from the $P_x = 0.4$ Table in Section B, the optimal array size is $n^* = 3$.

A. “Leaker,” $r_1 = 1$ and “False alarm,” $r_2 = 1$

$P_x = 0.1$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
Max P(Hit F.T.)	0.01	6	3	1	1	1	1
	0.05	6	2	1	1	1	1
	0.10	6	2	1	1	1	1
	0.15	6	2	1	1	1	1
	0.20	6	2	1	1	1	1
	0.25	6	2	1	1	1	1

$P_x = 0.2$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
Max P(Hit F.T.)	0.01	7	4	4	4	1	1
	0.05	7	3	2	2	1	1
	0.10	7	3	2	2	1	1
	0.15	7	3	2	2	1	1
	0.20	7	3	2	2	1	1
	0.25	7	3	2	2	1	1

$P_x = 0.3$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
Max P(Hit F.T.)	0.01	9	6	4	4	4	4
	0.05	8	3	3	2	2	2
	0.10	8	3	3	2	2	2
	0.15	8	3	1	1	1	1
	0.20	8	3	1	1	1	1
	0.25	8	3	1	1	1	1

$P_x = 0.4$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
Max P(Hit F.T.)	0.01	9	7	7	7	7	7
	0.05	6	3	2	2	2	2
	0.10	6	3	1	1	1	1
	0.15	6	3	1	1	1	1
	0.20	6	3	1	1	1	1
	0.25	6	3	1	1	1	1

$P_x = 0.5$

		Max *P* (Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
Max *P* (Hit F.T.)	0.01	9	7	7	7	7	7
	0.05	6	5	3	3	3	3
	0.10	6	4	1	1	1	1
	0.15	6	2	1	1	1	1
	0.20	6	2	1	1	1	1
	0.25	6	2	1	1	1	1

$P_x = 0.6$

		Max *P* (Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
Max *P* (Hit F.T.)	0.01	8	7	7	7	7	7
	0.05	8	4	3	3	3	3
	0.10	8	4	3	1	1	1
	0.15	8	2	2	1	1	1
	0.20	8	2	2	1	1	1
	0.25	8	2	2	1	1	1

$P_x = 0.7$

		Max *P* (Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
Max *P* (Hit F.T.)	0.01	8	5	5	5	5	5
	0.05	7	4	3	1	1	1
	0.10	7	2	2	1	1	1
	0.15	7	2	2	1	1	1
	0.20	7	2	2	1	1	1
	0.25	7	2	2	1	1	1

$P_x = 0.8$

		Max *P* (Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
Max *P* (Hit F.T.)	0.01	8	6	6	6	6	6
	0.05	5	4	4	4	1	1
	0.10	3	2	2	2	1	1
	0.15	3	2	2	2	1	1
	0.20	3	2	2	2	1	1
	0.25	3	2	2	2	1	1

$P_x = 0.9$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
Max P(Hit F.T.)	0.01	7	6	6	6	6	6
	0.05	3	2	2	2	2	2
	0.10	1	1	1	1	1	1
	0.15	1	1	1	1	1	1
	0.20	1	1	1	1	1	1
	0.25	1	1	1	1	1	1

B. "Leaker," $r_1 = 2$ and "False alarm," $r_2 = 1$

$P_x = 0.1$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
Max P(Hit F.T.)	0.01	6	6	6	6	6	6
	0.05	5	2	2	2	2	2
	0.10	5	2	2	2	2	2
	0.15	5	1	1	1	1	1
	0.20	5	1	1	1	1	1
	0.25	5	1	1	1	1	1

$P_x = 0.2$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
Max P(Hit F.T.)	0.01	7	4	4	4	4	4
	0.05	7	3	2	2	2	2
	0.10	7	3	2	2	2	2
	0.15	7	1	1	1	1	1
	0.20	7	1	1	1	1	1
	0.25	7	1	1	1	1	1

$P_x = 0.3$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
Max P(Hit F.T.)	0.01	9	7	7	7	7	7
	0.05	6	3	3	3	3	3
	0.10	4	3	3	3	3	3
	0.15	4	3	1	1	1	1
	0.20	4	2	1	1	1	1
	0.25	4	2	1	1	1	1

$P_x = 0.4$

		Max *P* (Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
Max *P* (Hit F.T.)	0.01	9	7	7	7	7	7
	0.05	6	3	3	3	3	3
	0.10	6	3	1	1	1	1
	0.15	6	3	1	1	1	1
	0.20	6	2	1	1	1	1
	0.25	6	2	1	1	1	1

$P_x = 0.5$

		Max *P* (Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
Max *P* (Hit F.T.)	0.01	10	7	7	7	7	7
	0.05	6	5	3	3	3	3
	0.10	6	4	1	1	1	1
	0.15	6	2	1	1	1	1
	0.20	6	2	1	1	1	1
	0.25	6	2	1	1	1	1

$P_x = 0.6$

		Max *P* (Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
Max *P* (Hit F.T.)	0.01	8	8	8	8	8	8
	0.05	7	6	3	3	3	3
	0.10	5	4	3	1	1	1
	0.15	5	2	2	1	1	1
	0.20	5	2	2	1	1	1
	0.25	5	2	2	1	1	1

$P_x = 0.7$

		Max *P* (Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
Max *P* (Hit F.T.)	0.01	8	8	8	8	8	8
	0.05	5	4	4	4	4	4
	0.10	5	2	2	2	2	2
	0.15	3	2	2	2	2	2
	0.20	3	2	2	2	2	2
	0.25	3	2	2	2	2	2

$P_x = 0.8$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
Max P(Hit F.T.)	0.01	8	6	6	6	6	6
	0.05	5	4	4	4	4	4
	0.10	3	2	2	2	2	2
	0.15	3	2	2	2	2	2
	0.20	1	1	1	1	1	1
	0.25	1	1	1	1	1	1

$P_x = 0.9$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
Max P(Hit F.T.)	0.01	7	7	7	7	7	7
	0.05	3	2	2	2	2	2
	0.10	1	1	1	1	1	1
	0.15	1	1	1	1	1	1
	0.20	1	1	1	1	1	1
	0.25	1	1	1	1	1	1

C. "Leaker," r_1 = 1 and "False alarm," r_2 = 2

P_x = 0.1

		Max P (Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
Max P (Hit F.T.)	0.01	6	3	1	1	1	1
	0.05	6	2	1	1	1	1
	0.10	6	2	1	1	1	1
	0.15	6	2	1	1	1	1
	0.20	6	2	1	1	1	1
	0.25	6	2	1	1	1	1

P_x = 0.2

		Max P (Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
Max P (Hit F.T.)	0.01	7	4	3	3	1	1
	0.05	7	4	2	2	1	1
	0.10	7	4	2	2	1	1
	0.15	7	4	2	2	1	1
	0.20	7	4	2	2	1	1
	0.25	7	4	2	2	1	1

P_x = 0.3

		Max P (Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
Max P (Hit F.T.)	0.01	9	6	4	4	4	4
	0.05	9	3	3	2	2	2
	0.10	9	3	3	2	2	2
	0.15	9	3	1	1	1	1
	0.20	9	3	1	1	1	1
	0.25	9	3	1	1	1	1

P_x = 0.4

		Max P (Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
Max P (Hit F.T.)	0.01	9	6	4	4	4	4
	0.05	9	3	3	2	2	2
	0.10	9	3	1	1	1	1
	0.15	9	3	1	1	1	1
	0.20	9	3	1	1	1	1
	0.25	9	3	1	1	1	1

P_x = 0.5

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
Max P(Hit F.T.)	0.01	9	6	4	4	4	4
	0.05	8	5	3	3	2	2
	0.10	8	5	1	1	1	1
	0.15	8	5	1	1	1	1
	0.20	8	5	1	1	1	1
	0.25	8	5	1	1	1	1

P_x = 0.6

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
Max P(Hit F.T.)	0.01	8	7	7	7	7	2
	0.05	8	4	3	3	3	2
	0.10	8	4	3	1	1	1
	0.15	8	4	3	1	1	1
	0.20	8	4	3	1	1	1
	0.25	8	4	3	1	1	1

P_x = 0.7

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
Max P(Hit F.T.)	0.01	8	5	5	5	5	5
	0.05	8	4	3	1	1	1
	0.10	8	2	2	1	1	1
	0.15	8	2	2	1	1	1
	0.20	8	2	2	1	1	1
	0.25	8	2	2	1	1	1

P_x = 0.8

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
Max P(Hit F.T.)	0.01	8	5	5	5	5	5
	0.05	8	4	3	3	1	1
	0.10	8	2	2	2	1	1
	0.15	8	2	2	2	1	1
	0.20	8	2	2	2	1	1
	0.25	8	2	2	2	1	1

P_x = 0.9

		Max *P* (Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
Max *P* (Hit F.T.)	0.01	6	6	6	6	6	6
	0.05	3	2	2	2	2	2
	0.10	1	1	1	1	1	1
	0.15	1	1	1	1	1	1
	0.20	1	1	1	1	1	1
	0.25	1	1	1	1	1	1

APPENDIX C. POLICY LOOK-UP TABLES

The tables in this appendix give the optimal k^* of the "k or more of n" policy described in Section 8.3. They are subdivided by the relative values of a leaker versus a false alarm, given as r_1 and r_2 respectively. Each table has a corresponding P_x value which is the local fraction of traffic assumed to be TCT. The sensors are assumed identical with $f_1 = 0.80$ and $f_0 = 0.15$. Table values are obtained by fixing P_x and n, then enumerating values of the loss function (Equation (8.5)) for varying k. The k corresponding to the minimum loss function value is k^*.

For Example, assume leakers and false alarms have relative values of $r_1 = 1$ and $r_2 = 2$, respectively. Now assume that the theater commander wants to know the optimal k or more of n^* policy given an array size of $n = 8$ sensors. The intelligence shop estimates that 40% of vehicle traffic are TCT's. Then, from the $P_x = 0.4$ column in Section B, the optimal value of k is $k^* = 4$. That is, the contact at the sensor should be prosecuted as a TCT if four or more of the eight sensors indicate that it is a TCT.

A. Leaker," r_1 = 1 and "False alarm," r_2 = 1

$P_x = 0.10$		$P_x = 0.20$		$P_x = 0.30$	
n	k^*	n	k^*	n	k^*
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	2	3	2
4	3	4	3	4	3
5	4	5	3	5	3
6	4	6	4	6	4
7	4	7	4	7	4
8	5	8	5	8	4
9	5	9	5	9	5
10	6	10	6	10	5

$P_x = 0.40$		$P_x = 0.50$		$P_x = 0.60$	
n	k^*	n	k^*	n	k^*
1	1	1	1	1	1
2	2	2	1	2	1
3	2	3	2	3	2
4	2	4	2	4	2
5	3	5	3	5	3
6	3	6	3	6	3
7	4	7	4	7	4
8	4	8	4	8	4
9	5	9	5	9	5
10	5	10	5	10	5

$P_x = 0.70$		$P_x = 0.80$		$P_x = 0.90$	
n	k^*	n	k^*	n	k^*
1	1	1	1	1	1
2	1	2	1	2	1
3	2	3	1	3	1
4	2	4	2	4	2
5	3	5	2	5	2
6	3	6	3	6	3
7	3	7	3	7	3
8	4	8	4	8	4
9	4	9	4	9	4
10	5	10	5	10	4

B. "Leaker," $r_1 = 2$ and "False alarm," $r_2 = 1$

$P_x = 0.10$		$P_x = 0.20$		$P_x = 0.30$	
n	k^*	n	k^*	n	k^*
1	1	1	1	1	1
2	2	2	2	2	1
3	2	3	2	3	2
4	3	4	3	4	2
5	3	5	3	5	3
6	4	6	4	6	3
7	4	7	4	7	4
8	5	8	4	8	4
9	5	9	5	9	5
10	6	10	5	10	5

$P_x = 0.40$		$P_x = 0.50$		$P_x = 0.60$	
n	k^*	n	k^*	n	k^*
1	1	1	1	1	1
2	1	2	1	2	1
3	2	3	2	3	2
4	2	4	2	4	2
5	3	5	3	5	2
6	3	6	3	6	3
7	4	7	4	7	3
8	4	8	4	8	4
9	5	9	4	9	4
10	5	10	5	10	5

$P_x = 0.70$		$P_x = 0.80$		$P_x = 0.90$	
n	k^*	n	k^*	n	k^*
1	1	1	1	1	1
2	1	2	1	2	1
3	1	3	1	3	1
4	2	4	2	4	1
5	2	5	2	5	2
6	3	6	3	6	2
7	3	7	3	7	2
8	4	8	4	8	3
9	4	9	4	9	3
10	5	10	4	10	4

C. "Leaker," $r_1 = 1$ and "False alarm," $r_2 = 2$

$P_x = 0.10$		$P_x = 0.20$		$P_x = 0.30$	
n	k^*	n	k^*	n	k^*
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	2
4	3	4	3	4	3
5	4	5	3	5	3
6	4	6	4	6	4
7	5	7	4	7	4
8	5	8	5	8	5
9	6	9	5	9	5
10	6	10	6	10	6

$P_x = 0.40$		$P_x = 0.50$		$P_x = 0.60$	
n	k^*	n	k^*	n	k^*
1	1	1	1	1	1
2	2	2	2	2	2
3	2	3	2	3	2
4	3	4	3	4	2
5	3	5	3	5	3
6	4	6	4	6	3
7	4	7	4	7	4
8	5	8	4	8	4
9	5	9	5	9	5
10	5	10	5	10	5

$P_x = 0.70$		$P_x = 0.80$		$P_x = 0.90$	
n	k^*	n	k^*	n	k^*
1	1	1	1	1	1
2	1	2	1	2	1
3	2	3	2	3	1
4	2	4	2	4	2
5	3	5	3	5	2
6	3	6	3	6	3
7	4	7	4	7	3
8	4	8	4	8	4
9	5	9	4	9	4
10	5	10	5	10	5

APPENDIX D. SPACING LOOK-UP TABLES

The tables in this appendix give the inter-sensor distance for array sensors as described in Section 8.3.3. Most of the factors affecting sensor spacing are beyond the control of the theater commander, so these values are based on some simple assumptions. It is important to note that the number of sensors in the array for the look-up represent only the sensors not making simultaneous reports. In the case where w sensors report to the theater commander virtually simultaneously, only one of the w is used in computing the array size. Table values are obtained from Equation (8.7) with the assumption that TCT velocity is constant through the array. A separate table is provided for speeds varying from 5 to 55 kph, and for one to ten sensor array sizes.

For Example, assume that the theater commander wants to know the optimal inter-sensor distance given his array size of $n = 8$ sensors. The intelligence shop estimates that TCT's along this stretch of road travel at approximately $v = 30$ kph and the theater commander wants his full array to report in no more than $t_R = 6$ minutes. Then, from the "Speed $v = 30$ kph" table, reading the $n = 8$ column and the $t_R = 6$ minutes yields a maximum sensor spacing of $d = 375$ meters.

Distance for TCT Speed $v = 5$ kph

t_R (min)	Array Size, n								
	2	3	4	5	6	7	8	9	10
1	83	42	28	21	17	14	12	10	9
2	167	83	56	42	33	28	24	21	19
3	250	125	83	63	50	42	36	31	28
4	333	167	111	83	67	56	48	42	37
5	417	208	139	104	83	69	60	52	46
6	500	250	167	125	100	83	71	63	56
7	583	292	194	146	117	97	83	73	65
8	667	333	222	167	133	111	95	83	74
9	750	375	250	188	150	125	107	94	83
10	833	417	278	208	167	139	119	104	93

Distance for TCT Speed $v = 10$ kph

t_R (min)	Array Size, n								
	2	3	4	5	6	7	8	9	10
1	167	83	56	42	33	28	24	21	19
2	333	167	111	83	67	56	48	42	37
3	500	250	167	125	100	83	71	63	56
4	667	333	222	167	133	111	95	83	74
5	833	417	278	208	167	139	119	104	93
6	1000	500	333	250	200	167	143	125	111
7	1167	583	389	292	233	194	167	146	130
8	1333	667	444	333	267	222	190	167	148
9	1500	750	500	375	300	250	214	188	167
10	1667	833	556	417	333	278	238	208	185

Distance for TCT Speed $v = 15$ kph

t_R (min)	Array Size, n								
	2	3	4	5	6	7	8	9	10
1	250	125	83	63	50	42	36	31	28
2	500	250	167	125	100	83	71	63	56
3	750	375	250	188	150	125	107	94	83
4	1000	500	333	250	200	167	143	125	111
5	1250	625	417	313	250	208	179	156	139
6	1500	750	500	375	300	250	214	188	167
7	1750	875	583	438	350	292	250	219	194
8	2000	1000	667	500	400	333	286	250	222
9	2250	1125	750	563	450	375	321	281	250
10	2500	1250	833	625	500	417	357	313	278

Distance for TCT Speed $v = 20$ kph

t_R (min)	Array Size, n								
	2	3	4	5	6	7	8	9	10
1	333	167	111	83	67	56	48	42	37
2	667	333	222	167	133	111	95	83	74
3	1000	500	333	250	200	167	143	125	111
4	1333	667	444	333	267	222	190	167	148
5	1667	833	556	417	333	278	238	208	185
6	2000	1000	667	500	400	333	286	250	222
7	2333	1167	778	583	467	389	333	292	259
8	2667	1333	889	667	533	444	381	333	296
9	3000	1500	1000	750	600	500	429	375	333
10	3333	1667	1111	833	667	556	476	417	370

Distance for TCT Speed $v = 25$ kph

t_R (min)	Array Size, n								
	2	3	4	5	6	7	8	9	10
1	417	208	139	104	83	69	60	52	46
2	833	417	278	208	167	139	119	104	93
3	1250	625	417	313	250	208	179	156	139
4	1667	833	556	417	333	278	238	208	185
5	2083	1042	694	521	417	347	298	260	231
6	2500	1250	833	625	500	417	357	313	278
7	2917	1458	972	729	583	486	417	365	324
8	3333	1667	1111	833	667	556	476	417	370
9	3750	1875	1250	938	750	625	536	469	417
10	4167	2083	1389	1042	833	694	595	521	463

Distance for TCT Speed $v = 30$ kph

t_R (min)	Array Size, n								
	2	3	4	5	6	7	8	9	10
1	500	250	167	125	100	83	71	63	56
2	1000	500	333	250	200	167	143	125	111
3	1500	750	500	375	300	250	214	188	167
4	2000	1000	667	500	400	333	286	250	222
5	2500	1250	833	625	500	417	357	313	278
6	3000	1500	1000	750	600	500	429	375	333
7	3500	1750	1167	875	700	583	500	438	389
8	4000	2000	1333	1000	800	667	571	500	444
9	4500	2250	1500	1125	900	750	643	563	500
10	5000	2500	1667	1250	1000	833	714	625	556

Distance for TCT Speed $v = 35$ kph

t_R (min)	Array Size, n								
	2	3	4	5	6	7	8	9	10
1	583	292	194	146	117	97	83	73	65
2	1167	583	389	292	233	194	167	146	130
3	1750	875	583	438	350	292	250	219	194
4	2333	1167	778	583	467	389	333	292	259
5	2917	1458	972	729	583	486	417	365	324
6	3500	1750	1167	875	700	583	500	438	389
7	4083	2042	1361	1021	817	681	583	510	454
8	4667	2333	1556	1167	933	778	667	583	519
9	5250	2625	1750	1313	1050	875	750	656	583
10	5833	2917	1944	1458	1167	972	833	729	648

Distance for TCT Speed $v = 45$ kph

t_R (min)	Array Size, n								
	2	3	4	5	6	7	8	9	10
1	750	375	250	188	150	125	107	94	83
2	1500	750	500	375	300	250	214	188	167
3	2250	1125	750	563	450	375	321	281	250
4	3000	1500	1000	750	600	500	429	375	333
5	3750	1875	1250	938	750	625	536	469	417
6	4500	2250	1500	1125	900	750	643	563	500
7	5250	2625	1750	1313	1050	875	750	656	583
8	6000	3000	2000	1500	1200	1000	857	750	667
9	6750	3375	2250	1688	1350	1125	964	844	750
10	7500	3750	2500	1875	1500	1250	1071	938	833

Distance for TCT Speed $v = 55$ kph

t_R (min)	Array Size, n								
	2	3	4	5	6	7	8	9	10
1	917	458	306	229	183	153	131	115	102
2	1833	917	611	458	367	306	262	229	204
3	2750	1375	917	688	550	458	393	344	306
4	3667	1833	1222	917	733	611	524	458	407
5	4583	2292	1528	1146	917	764	655	573	509
6	5500	2750	1833	1375	1100	917	786	688	611
7	6417	3208	2139	1604	1283	1069	917	802	713
8	7333	3667	2444	1833	1467	1222	1048	917	815
9	8250	4125	2750	2063	1650	1375	1179	1031	917
10	9167	4583	3056	2292	1833	1528	1310	1146	1019

APPENDIX E. PROBABILITY OF IDENTIFICATION

Section 8.2 introduces the probability of identification calculations used within the models. These probabilities are based on conditional probability and derived from both Bayes' Rule and the Law of Total Probability. This appendix shows the derivation of Equation (8.3) and Equation (8.4]).

A. SINGLE SENSOR ARRAY

The simplest case is an array consisting of a single sensor placed along a road. For the purposes of this analysis, it will still be considered an "array," and in fact will be the basic building block of all larger arrays. This array, as with all the arrays to be discussed, provides decision probabilities, $P(\text{TCT} \mid i/1)$, based on the sensor output and the fraction of vehicle traffic assumed to be TCT. $P(\text{TCT} \mid 1/1)$ is defined as the probability that the target is a TCT given the sensor reports it as TCT. Similarly, $P(\text{TCT} \mid 0/1)$ is the probability that the target is TCT given the sensor reports it as non-TCT . In all cases, P_x, the fraction of vehicles assumed to be TCT, must be provided by some intelligence estimate.(Ref[5])

Let

$$X = \begin{cases} 1 \text{ if the target moving past the sensor is a TCT,} \\ 0 \text{ otherwise.} \end{cases}$$

Each sensor outputs a forecast denoted by:

$$F_i = \begin{cases} 1 \text{ if the sensor identifies the target as TCT,} \\ 0 \text{ otherwise.} \end{cases}$$

Additional sensors are denoted using increasing subscripts, i.e. $i=1, 2, 3, \ldots.$

From sensor performance data provided by the manufacturer and field tests, the forecast likelihoods are given by

f_1 = P{Sensor indicates a TCT given a TCT present}, and

f_0 = P{Sensor indicates a TCT given no TCT present}.

That is,

$$f_1 = P\{F = 1 | X = 1\}, \text{ and}$$

$$f_0 = P\{F = 1 | X = 0\}.$$

For the simple one sensor case, using the forecast probabilities and Bayes' Rule, the decision probabilities are [Ref. 4]:

$$P(\text{TCT} | 1/1) = \frac{f_1 \cdot P_x}{f_1 \cdot P_x + f_0(1 - P_x)},$$

$$P(\text{TCT} | 0/1) = \frac{(1 - f_1) \cdot P_x}{(1 - f_1) \cdot P_x + (1 - f_0)(1 - P_x)}.$$

B. TWO SENSOR ARRAY

The two sensor array consists of two sensors spaced close enough to assume both identification calls reported to the fusion center are on the same target. The theater commander will be provided with the decision probabilities, P(TCT | k/n). As before, the estimate, P_x, must be provided by intelligence.

Each of the two sensors will output a forecast denoted by

$$F_1 = \begin{cases} 1 \text{ if sensor 1 identifies the target as TCT,} \\ 0 \text{ otherwise.} \end{cases}$$

$$F_2 = \begin{cases} 1 \text{ if sensor 2 identifies the target as TCT,} \\ 0 \text{ otherwise.} \end{cases}$$

If the sensors are assumed to be conditionally independent, then

$$P\{F_1 = i_1, F_2 = i_2 | X = x\} = P\{F_1 = i_1 | X = x\} P\{F_2 = i_2 | X = x\}, \text{ where } i, x \,\varepsilon\, \{0, 1\}.$$

Also, since the sensors are identical, $P(F_1 = f \mid X = x) = P(F_2 = f \mid X = x)$ for every f and x. Therefore $f_{1(1)} = f_{1(2)}$ and $f_{0(1)} = f_{0(2)}$. Henceforth, f_1 and f_0 will be used for the forecast likelihoods of all identical sensors.

The decision probabilities are given by [Ref. 4]

$$P(\mathrm{TCT} \mid 2/2) = \frac{f_1^2 P_x}{f_1^2 P_x + f_0^2 (1 - P_x)},$$

$$P(\mathrm{TCT} \mid 1/2) = \frac{(1 - f_1) f_1 P_x}{(1 - f_1) f_1 P_x + (1 - f_0) f_0 (1 - P_x)}, \text{ and}$$

$$P(\mathrm{TCT} \mid 0/2) = \frac{(1 - f_1)^2 P_x}{(1 - f_1)^2 P_x + (1 - f_0)^2 (1 - P_x)}.$$

C. GENERAL FORMALATION, n $\geq$ 2

Similar results hold for the general case involving $n \geq 2$ sensors. Again, assuming conditional independence allows

$$P\{F_1 = i_1, \ldots, F_n = i_n \mid X = x\} = P\{F_1 = i_1 \mid X = x\} \ldots P\{F_n = i_n \mid X = x\},$$

where i_n, $x \in \{0,1\}$. As above, the arrays are composed of identical sensors, and therefore

$$P\{F_1 = 1 \mid X = 1\} = P\{F_n = 1 \mid X = 1\} = f_1, \text{ and}$$

$$P\{F_1 = 1 \mid X = 0\} = P\{F_n = 1 \mid X = 0\} = f_0.$$

Now, let k equal the number of sensors in an array of size n to identify a passing target as a TCT. From Bayes' Rule and the Law of Total Probability the decision probabilities for exactly k of n sensors indicating a TCT are given by

$$P(\mathrm{TCT} \mid k/n) = \frac{f_1^k (1 - f_1)^{n-k} P_x}{f_1^k (1 - f_1)^{n-k} P_x + f_0^k (1 - f_0)^{n-k} (1 - P_x)}.$$

This equation can be used for any array size with a given intelligence estimate of the TCT population.

D. FINDING *k* OR MORE OF *n* POLICY

Again assume that an array of *n* identical sensors is in place with forecast likelihoods of f_1 and f_0. From Section C the exact *k* of *n* decision probability is given as

$$P(\mathrm{TCT} \mid k/n) = \frac{f_1^k (1-f_1)^{n-k} P_x}{f_1^k (1-f_1)^{n-k} P_x + f_0^k (1-f_0)^{n-k} (1-P_x)}.$$

APPENDIX F. *k* OR MORE OF *n* AND LOSS FUNCTION COMPUTATIONS

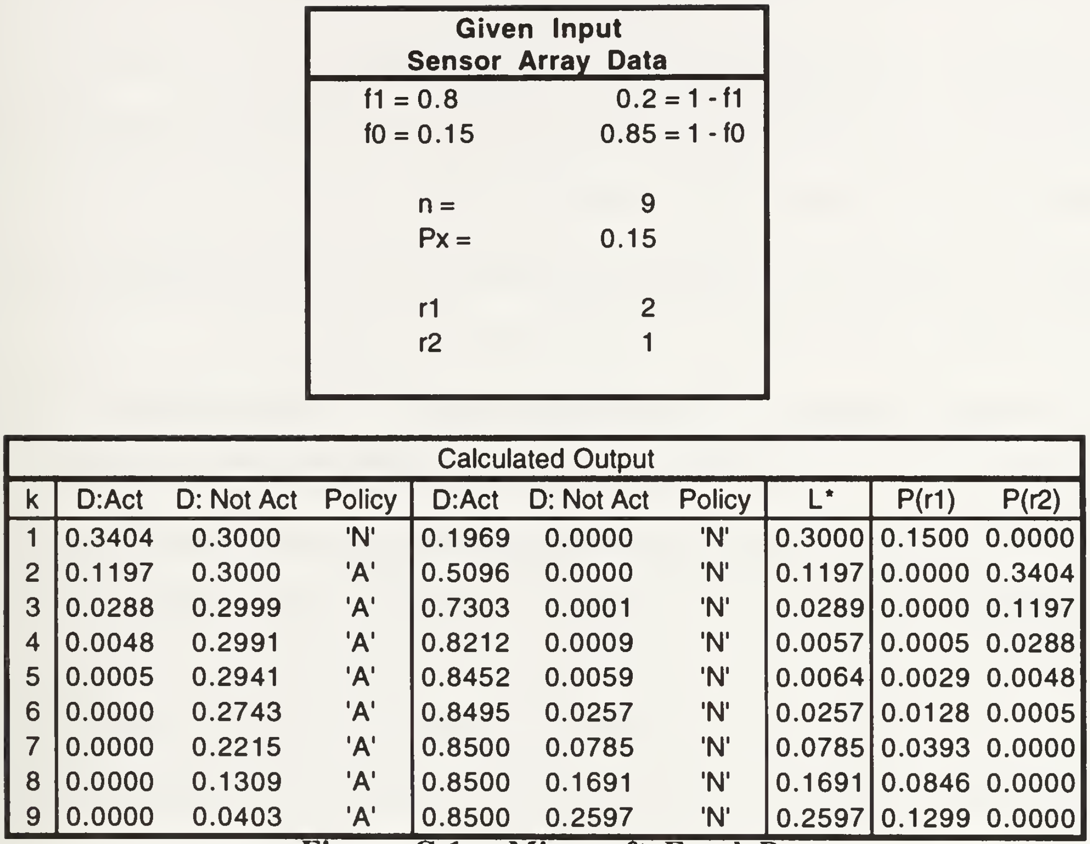
The "at least *k*-out-of-*n*" policy of Section 8.2 and the loss function introduced in Section 8.3 are the driving forces behind the two models used in this thesis. Their roots are based in decision theory and conditional probability. This appendix shows a complete derivation of the *k* or more of *n* policy, and how it is used to generate the loss function, $l(n,k)$, of the optimization.

The decision required in this thesis is to choose an integer k, where $0 \le k \le n$, such that the theater commander will take action if and only if at least k sensors indicate a vehicle is a TCT. Recall that r_1 is the loss obtained if no action is taken and the vehicle is a TCT, and r_2 is the loss obtained if a non-TCT is acted against. The expected loss is

$$\begin{aligned}
l(n,k) = {} & r_1\left(P\{X=1,S=0\}+P\{X=1,S=1\}+\ldots+P\{X=1,S=k-1\}\right)+ \\
& r_2\left(P\{X=0,S=k\}+P\{X=0,S=k+1\}+\ldots+P\{X=0,S=n\}\right) \\
= {} & r_1 P\{X=1,S\le k-1\}+r_2 P\{X=0,S\ge k\} \\
= {} & r_1 P\{S\le k-1,X=1\}P_x+r_2 P\{S\ge k,X=0\}(1-P_x) \\
= {} & r_1 B(k-1,n,f_1)P_x+r_2\left(1-B(k-1,n,f_0)(1-P_x)\right)
\end{aligned}$$

where $B(k_i,n,f_0)$ is the binomial distribution function given by $\sum_{j=0}^{k_i}\binom{n}{j} f_0^j (1-f_0)^{n-j}$.

Figure F.1 shows a Microsoft Excel v7.0 spreadsheet programmed to perform the above calculations in an interactive manner.



Given Input
Sensor Array Data

f1 = 0.8 0.2 = 1 - f1
f0 = 0.15 0.85 = 1 - f0

n = 9
Px = 0.15

r1 2
r2 1

Calculated Output									
k	D:Act	D: Not Act	Policy	D:Act	D: Not Act	Policy	L*	P(r1)	P(r2)
1	0.3404	0.3000	'N'	0.1969	0.0000	'N'	0.3000	0.1500	0.0000
2	0.1197	0.3000	'A'	0.5096	0.0000	'N'	0.1197	0.0000	0.3404
3	0.0288	0.2999	'A'	0.7303	0.0001	'N'	0.0289	0.0000	0.1197
4	0.0048	0.2991	'A'	0.8212	0.0009	'N'	0.0057	0.0005	0.0288
5	0.0005	0.2941	'A'	0.8452	0.0059	'N'	0.0064	0.0029	0.0048
6	0.0000	0.2743	'A'	0.8495	0.0257	'N'	0.0257	0.0128	0.0005
7	0.0000	0.2215	'A'	0.8500	0.0785	'N'	0.0785	0.0393	0.0000
8	0.0000	0.1309	'A'	0.8500	0.1691	'N'	0.1691	0.0846	0.0000
9	0.0000	0.0403	'A'	0.8500	0.2597	'N'	0.2597	0.1299	0.0000

Figure G.1 - Microsoft Excel Program

The numerical values on the table, with the exception of the probability columns represent the relative losses with the values of r_1, r_2, P_x, n and k given. Columns two through four represent the condition that k or more sensors indicate a TCT, while columns five through seven represent the condition that fewer than k do so. Finally, columns nine and ten are the probabilities that a "leaker" or a "false alarm" occur with the values given. It is these values, combined with the losses in column eight that provide the data for the look up tables in the earlier appendices.

Related MS Theses in Operations Research

at the Naval Postgraduate School

1. Ehlers, Mark A., Countering Short Range Ballistic Missiles, MS Thesis in Operations Research, Naval Postgraduate School, Monterey, CA, September, 1992.

2. Hair, Thomas W., The Application of Search Theory to the Timely Location of Tactical Ballistic Missiles, MS in Operations Research, Naval Postgraduate School, March 1993.

3. Mattis, Joseph P., The Application of Random Search Theory to the Detection of Tactical Ballistic Missile Launchers, MS in Operations Research, Naval Postgraduate School, September 1993.

4. Soutter, Paul A., On the Use of Unmanned Aerial Vehicles to Search for Tactical Ballistic Missile Transporter-Erector-Launchers, MS in Operations Research, Naval Postgraduate School, March 1994.

5. Junker, Vernon L., Tactical Unmanned Aerial Vehicles in a Proposed Joint Infrastructure to Counter Theater Ballistic Missiles, MS in Operations Research, Naval Postgraduate School, March 1995.

6. Fitzpatrick, Neil E., Tactical Model of Hyperspectral Imagery in Support of Theater Ballistic Missile Defense, MS in Operations Research, Naval Postgraduate School, June, 1997.

7. Haberlin, Richard J., Jr., Analyses of Unattended Ground Sensors in Theater Ballistic Missile Defense Attack Operations, MS in Operations Research, Naval Postgraduate School, September, 1997.

References

1. Schemmer,B. F., Special Ops Teams Found 29 SCUDS Ready to Barrage Israel 24 Hours Before Cease-Fire, *Armed Forces Journal International*, p.36, July 1991

2. Miller, M. C., Operation Desert Sham, *The New York Times*, p. A-21, 24 June 1992

3. Opall, B., SCUD Variants Pose New Threat to SDIO Plans, *Defense News*, v. 7, p. 40, 20-26 July 1992

4. Hall, J., Distribution of Losses in an Idealized Antishipping Campaign, Center for Naval Analyses Research Contribution No. 120, July 1969

5. Marshall, K. T. and Oliver, R. M., Decision Making and Forecasting, McGraw-Hill, 1995.

6. Conner, George W., Ehlers, Mark A., and Marshall, Kneale T., Countering Short Range Ballistic Missiles, Technical Report NPS-OR-93-009, Department of Operations Research, Naval Postgraduate School Monterey, CA 93940, January, 1993.

7. Marshall, Kneale T., Quantifying Counterforce and Active Defense in Countering Theater Ballistic Missiles, *Military Operations Research*, v.1, n.2 (1994).

8. Wirtz, James J., A Joint Idea: An Antisubmarine Warfare Approach to Theater Missile Defense, *Airpower Journal*, v. 11, n. 1 (1997).

9. McCue, Brian, U-Boats in the Bay of Biscay, National Defense University Press, Washington D.C. 20319 (1990).

10. Dickson, Paul, The Electronic Battlefield, Indiana University Press, 1976.

11. Libicki, Martin C., The Mesh and the Net, McNair Paper 28, Institute for Strategic Studies, National Defense University, March 1994.

12. Telephone conversation between Major Robert Rehoorn, USAF, of Central MASINT Technology Coordination Office and the author, 11 November, 1996.

13. Washburn, Alan R., Search and Detection, Institute for Operations Research and the Management Sciences, 1996.

14. Snyder, Derick, Sandia National Laboratory, Interview by author, November 1996.

☆U.S. GOVERNMENT PRINTING OFFICE: 1998-684-515/99081

Naval Postgraduate School
Monterey, California