Resonance decay dynamics and their effects on p_T spectra of pions in heavy-ion collisions

Pok Man Lo
Institute of Theoretical Physics, University of Wroclaw, PL-50204 Wrocław, Poland
and Extreme Matter Institute (EMMI), GSI, Planckstrasse 1, D-64291 Darmstadt, Germany

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The influence of resonance decay dynamics on the momentum spectra of pions in heavy-ion collisions is examined. Taking the decay processes $\omega \to 3\pi$ and $\rho \to 2\pi$ as examples, I demonstrate how the resonance width and details of decay dynamics (via the decay matrix element) can modify the physical observables. The latter effect is commonly neglected in statistical models. To remedy the situation, a theoretical framework for incorporating hadron dynamics into the analysis is formulated, which can be straightforwardly extended to describe general N-body decays.

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I. INTRODUCTION

The problem posed by heavy-ion collisions is to deduce physical properties of the created hadronic matter based on information of the observed particles. Since most of the particles detected are connected with the system after the freeze-out stage, precise modeling at different levels is required to reconstruct the cooling history of the originally produced hot and dense medium.

Pion production is a dominant feature in heavy-ion collisions. The experimental data on momentum distributions of pions and other identified particles present a handle for discerning particles of different momenta. In many cases, the hadronic spectra are well reproduced by simple thermodynamical fits. However, the situation is more complicated for the soft pions. As demonstrated in a previous paper by Sollfrank *et al.* [1], pions originated from resonance decays have a different "shape" in their transverse momentum (p_T) spectrum compared to the purely thermal one. This tends to impact the low-momentum part of the spectrum and may help to explain why some of the conventional fluid-dynamical calculations [2,3] fail to describe the soft pions ($p_T \leqslant 0.3$ GeV) in recent measurements of the p_T spectra of identified particles produced in $\sqrt{s_{NN}} = 2.76$ TeV Pb + Pb collisions at the Large Hadron Collider (LHC) [4].

However, multiple mechanisms presumably contribute to the observed spectrum besides resonance decays. These include collective flow [5], influence of the medium [6,7], and nonequilibrium effects [8–12]. It is therefore no longer sufficient to perform data fitting within the framework of a single mechanism. Instead, a detailed examination of each effect is required, with its consistency with known hadron physics and symmetries of QCD inspected. Only then one can isolate the (possibly dominant) effect from the thermal medium and nonequilibrium dynamics, and eventually arrive at an internally consistent picture for heavy-ion collisions.

In this work, I focus on the p_T spectra of pions from resonance decays. It is known that a good description of resonances is essential for understanding the soft part of the spectrum. An extensive analysis based on statistical models was presented in Ref. [1]. However, in this and other studies [9,13–16], resonance decay dynamics has been neglected.

The purpose of this paper is to formulate a theoretical framework for incorporating hadron dynamics into the analysis, applicable to a general N-body decay. Details of this framework is discussed in Sec. II. In Sec. III, I apply the formalism to study the three-body decay of $\omega \to 3\pi$. In Sec. IV, I present the conclusion.

II. MOMENTUM DISTRIBUTIONS OF DECAY PARTICLES WITH DECAY DYNAMICS

A. Differential phase space

The first question to address is to determine the distribution $dn_1^{\mathrm{dec}}/d^3p_1$ of a particular decay particle 1 (in this case a pion) from a given distribution $dn_{\mathrm{res}}/d^3p_{\mathrm{res}}$ of the resonance. Note that the symbol n_X denotes the particle number density for the species X. A detailed account of this problem is given in textbooks [17,18]. The application in heavy-ion collisions is discussed in Ref. [1,19]. Here I briefly review the key steps of the calculation to establish the notations.

The pion momentum spectrum from resonance decay is given by

$$E_\pi \frac{dn_\pi^{\mathrm{dec}}}{d^3p_\pi} = \mathrm{br} \times \int d^3p_{\mathrm{res}} \frac{dn_{\mathrm{res}}}{d^3p_{\mathrm{res}}} \times E_\pi^\star \times \mathrm{dPS}(\vec{p}_\pi^{\,\star}). \quad (1)$$

Here br is the suitable branching ratio for the decay. The differential phase space function $\mathrm{dPS}(\vec{p}_\pi^{\,\star})$ is a key quantity of this study and will be addressed in detail. Throughout this work, variables in the resonance rest frame are denoted by $\star$, while those without are in the rest frame of the medium. The momentum variable $\vec{p}_\pi^\star$ should be understood as a function of $\vec{p}_{\rm res}$ and $\vec{p}_\pi$. The explicit expression is easily obtained by invoking Lorentz invariance of $p_\pi \cdot p_{\rm res}$, which dictates

$$E_\pi^\star = \frac{E_\pi E_{\rm res} - \vec{p}_\pi \cdot \vec{p}_{\rm res}}{m_{\rm res}}, \quad p_\pi^\star = \sqrt{E_\pi^{\star\,2} - m_\pi^2}. \tag{2}$$

In this study, only resonances from a static thermal source are considered. Hence one can put

$$\frac{dn_{\rm res}}{d^3p_{\rm res}} \to \frac{g_{\rm res}}{(2\pi)^3} \frac{1}{\xi_{\rm res}^{-1}\, e^{E_{\rm res}/T} - 1} \tag{3}$$

for a mesonic resonance of degeneracy $g_{\rm res}$ at finite temperature ($T = 120$ MeV is chosen) in both vanishing and finite chemical potential μ via a fugacity factor $\xi_{\rm res} = e^{\mu/T}$.

The function dPS involves an integral over the phase space of other decay particles and the decay matrix element in the resonance rest frame. For an N-body decay, the general definition reads [1,17,18,20,21]

$$\begin{aligned}\mathrm{dPS}(\vec{p}_1^\star) &\doteq \frac{1}{\gamma_{\rm res}} \frac{d\gamma_{\rm res}}{d^3p_1^\star} \\ &= \frac{1}{\gamma_{\rm res}} \frac{1}{2m_{\rm res}} \frac{1}{(2\pi)^3} \frac{1}{2E_1^\star} \\ &\quad\times \int \frac{d^3p_2^\star}{(2\pi)^3} \frac{1}{2E_2^\star} \frac{d^3p_3^\star}{(2\pi)^3} \frac{1}{2E_3^\star} \cdots \frac{d^3p_N^\star}{(2\pi)^3} \frac{1}{2E_N^\star} \\ &\quad\times (2\pi)^4 \delta^4(P - \sum_i p_i)\, |\Gamma_{{\rm res}\to 1+2+\cdots+N}|^2. \end{aligned}\tag{4}$$

Here $\gamma_{\rm res}$ is the width of the resonance, while $\Gamma_{{\rm res}\to 1+2+\cdots+N}$ denotes the decay matrix element for the relevant N-body decay process. The normalization of the dPS function is defined such that

$$\int d^3p_1^\star\, (\mathrm{dPS}) = 1. \tag{5}$$

A common approximation made by thermodynamical models when calculating this quantity is the assumption of isotropic (structureless) decay [1]. This amounts to replacing the decay matrix element Γ with the identity I and hence $\mathrm{dPS} \to \mathrm{dPS}^{(0)}$:

$$\begin{aligned}\mathrm{dPS}^{(0)} &= \frac{1}{\phi_N} \frac{d\phi_N}{d^3p_1^\star} \\ &= \frac{1}{\phi_N} \frac{1}{(2\pi)^3} \frac{1}{2E_1^\star} \\ &\quad\times \int \frac{d^3p_2^\star}{(2\pi)^3} \frac{1}{2E_2^\star} \frac{d^3p_3^\star}{(2\pi)^3} \frac{1}{2E_3^\star} \cdots \frac{d^3p_N^\star}{(2\pi)^3} \frac{1}{2E_N^\star} \\ &\quad\times (2\pi)^4 \delta^4\left(P - \sum_i p_i\right). \end{aligned}\tag{6}$$

Here I have introduced the N-body Lorentz invariant phase space ϕ_N:

$$\begin{aligned}\phi_N = \int d\phi_N &= \int \frac{d^3p_1^\star}{(2\pi)^3} \frac{1}{2E_1^\star} \frac{d^3p_2^\star}{(2\pi)^3} \frac{1}{2E_2^\star} \cdots \frac{d^3p_N^\star}{(2\pi)^3} \frac{1}{2E_N^\star} \\ &\quad\times (2\pi)^4 \delta^4\left(P - \sum_i p_i\right). \end{aligned}\tag{7}$$

To clarify the physical meaning of the differential phase space function dPS, the cases for isotropic two- and three-body decay are worked out explicitly, starting with the two-body case:

$$\begin{aligned}\mathrm{dPS}^{(0)} &= \frac{1}{\phi_2} \frac{1}{(2\pi)^3} \frac{1}{2E_1^\star} \times \int \frac{d^3p_2^\star}{(2\pi)^3} \frac{1}{2E_2^\star} \\ &\quad\times (2\pi)^4 \delta(m_{\rm res} - E_1^\star - E_2^\star)\delta^3(\vec{p}_1^\star + \vec{p}_2^\star). \end{aligned}\tag{8}$$

The integrals can be explicitly worked out:

$$\begin{aligned} I_2 &= \frac{1}{(2\pi)^3} \frac{1}{2E_1^\star} \times \int \frac{d^3p_2^\star}{(2\pi)^3} \frac{1}{2E_2^\star} \\ &\quad\times (2\pi)^4 \delta(m_{\rm res} - E_1^\star - E_2^\star)\delta^3(\vec{p}_1^\star + \vec{p}_2^\star) \\ &= \frac{1}{4m_{\rm res} q} \frac{1}{(2\pi)^2} \delta(p_1^\star - q), \end{aligned}\tag{9}$$

$$\begin{aligned}\phi_2\left(m_{\rm res}^2, m_1^2, m_2^2\right) &= \frac{1}{8\pi m_{\rm res}^2} \sqrt{\lambda\left(m_{\rm res}^2, m_1^2, m_2^2\right)} \\ &= \frac{q}{4\pi m_{\rm res}}, \end{aligned}\tag{10}$$

where $\lambda(x,y,z)$ is the Källén triangle function [18],

$$\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz, \tag{11}$$

and q is the three-momentum of the decay particle in the resonance rest frame,

$$q = \frac{1}{2}\sqrt{m_{\rm res}^2} \sqrt{1 - \frac{(m_1+m_2)^2}{m_{\rm res}^2}} \sqrt{1 - \frac{(m_1-m_2)^2}{m_{\rm res}^2}}. \tag{12}$$

Finally, one arrives at the well-known result,

$$\mathrm{dPS}^{(0)} = \frac{1}{4\pi q^2} \delta(p_\pi^\star - q), \tag{13}$$

which may be alternatively obtained by inspection of the quantity $dN_1^{\rm dec}/d^3p_1^\star$ for a spherically symmetric two-body decay [1,19]. (Here $N_1^{\rm dec}$ is the number of particle 1 from the decay.)

The generalization to the case of three-body decay is straightforward:

$$\begin{aligned}\mathrm{dPS}^{(0)} &= \frac{1}{\phi_3} \frac{1}{(2\pi)^3} \frac{1}{2E_1^\star} \int \frac{d^3p_2^\star}{(2\pi)^3} \frac{1}{2E_2^\star} \frac{d^3p_3^\star}{(2\pi)^3} \frac{1}{2E_3^\star} \\ &\quad\times (2\pi)\delta(m_{\rm res} - E_1^\star - E_2^\star - E_3^\star) \\ &\quad\times (2\pi)^3 \delta^3(\vec{p}_1^\star + \vec{p}_2^\star + \vec{p}_3^\star), \end{aligned}\tag{14}$$

where the integrals can again be explicitly calculated:

$$\begin{aligned} I_3 &= \frac{1}{(2\pi)^3} \frac{1}{2E_1^\star} \int \frac{d^3p_2^\star}{(2\pi)^3} \frac{1}{2E_2^\star} \frac{d^3p_3^\star}{(2\pi)^3} \frac{1}{2E_3^\star} \\ &\quad\times (2\pi)\delta(m_{\rm res} - E_1^\star - E_2^\star - E_3^\star) \\ &\quad\times (2\pi)^3 \delta^3(\vec{p}_1^\star + \vec{p}_2^\star + \vec{p}_3^\star) \\ &= \frac{1}{(2\pi)^3} \frac{1}{2E_1^\star} \frac{1}{8\pi (P-p_1)^2} \sqrt{\lambda\left((P-p_1)^2, m_2^2, m_3^2\right)}, \end{aligned}\tag{15}$$

$$\phi_3(s) = \frac{1}{16\pi^2}\frac{1}{s}\int_{(m_2+m_3)^2}^{(\sqrt{s}-m_1)^2} ds'\sqrt{\lambda(s,s',m_1^2)} \times\phi_2(s',m_2^2,m_3^2). \quad (16)$$

Finally, the differential phase space for isotropic three-body decay reads

$$\mathrm{dPS}^{(0)} = \frac{I_3}{\phi_3(s=m_{\mathrm{res}}^2)}, \quad (17)$$

matching the result in Ref. [1].

The assumption of isotropic decay can be justified in some cases when the matrix element is a scalar (e.g., $\sigma \to \pi\pi$ decay via $\mathcal{L}_{\mathrm{int}} = -g\,\sigma\pi\pi$) or depends only on $s = P^2$. However, as I shall demonstrate in the example of the three-body decay $\omega \to 3\pi$, this approximation is problematic, especially for soft pions.

III. CASE STUDY: $\omega \to 3\pi$

A. dPS and p_T spectra

Multiple hadron models [22–25] are available to describe the decay of ω meson to three pions. The mechanisms involved are the Gell-Mann, Sharp, and Wagner (GSW) process [26,27] ($\omega \to \rho\pi \to \pi\pi\pi$) and possibly a direct process [22]. An effective Lagrangian describing these processes reads

$$\begin{aligned}\mathcal{L}_{\mathrm{int}} \sim\, & g_1\,\epsilon^{\mu\nu\alpha\beta}\partial_\mu\omega_\nu\left(\partial_\alpha\rho_\beta^-\pi^+ + \partial_\alpha\rho_\beta^0\pi^0 + \partial_\alpha\rho_\beta^+\pi^-\right)\\ & + i g_2\,\epsilon^{\mu\nu\alpha\beta}\omega_\mu\partial_\nu\pi^+\partial_\alpha\pi^0\partial_\beta\pi^-\\ & + i g_3\,\rho_\mu^0 \times (\pi^+\partial^\mu\pi^- - \pi^-\partial^\mu\pi^+) + \cdots . \end{aligned} \quad (18)$$

The first and third lines represent the processes $\omega \to \rho\pi$ and $\rho \to 2\pi$ respectively, while the second represents the direct process. Here the model of Ref. [25] is employed, which is a dispersive study based on isobar decomposition. Accordingly, the decay matrix element squared is given by

$$|\Gamma_{\omega\to3\pi}|^2 = \mathcal{P}|C_{V\to123}|^2, \quad (19)$$

where

$$\begin{aligned}\mathcal{P} &= -\frac{1}{3}\epsilon_{\mu\nu\alpha\beta}\epsilon_{abcd}P^\mu\, p_1^\nu\, p_2^\alpha\, P^a\, p_1^b\, p_2^c\, g^{\beta d}\\ &= \frac{1}{12}\left[s_{12}s_{23}s_{13} - m_\pi^2\left(m_{\mathrm{res}}^2 - m_\pi^2\right)^2\right]\end{aligned} \quad (20)$$

and

$$s_{ij} = (p_i + p_j)^2, \quad (21)$$

all subjected to the kinematic constraint

$$P^2 = m_{\mathrm{res}}^2 = s_{12} + s_{23} + s_{13} - m_1^2 - m_2^2 - m_3^2. \quad (22)$$

The factor $\mathcal{P}$ due to anomalous coupling, which is common to all models of the decay, dominates the properties of the matrix element. On the other hand, differences among models are limited to the different recipe for the amplitude function $C_{V\to123}$. It has been numerically checked that different choices of the latter only lead to minimal changes to the subsequent results.[1] The detail of the amplitude function employed in this work is given in Ref. [25] and the expression is reproduced here for convenience:

$$|C_{V\to123}|^2 = |\mathcal{N}|^2\,[1 + 2\alpha z + 2\beta z^{3/2}\sin(3\theta) + 2\gamma z^2 + 2\delta z^{5/2}\sin(3\theta)], \quad (23)$$

where

$$\begin{aligned}\sqrt{z}\,\cos(\theta) &= \frac{\sqrt{3}\,(s_{23}-s_{13})}{2\,m_{\mathrm{res}}(m_{\mathrm{res}}-3\,m_\pi)},\\ \sqrt{z}\,\sin(\theta) &= \frac{\sqrt{3}\,(s_c-s_{12})}{2\,m_{\mathrm{res}}(m_{\mathrm{res}}-3\,m_\pi)},\\ s_c &= \frac{1}{3}\left(m_{\mathrm{res}}^2 + 3m_\pi^2\right),\end{aligned} \quad (24)$$

with the model parameters

$$\alpha = 0.083,\quad \beta = 0.022,\quad \gamma = 0.001,\quad \delta = 0.014. \quad (25)$$

The normalization $\mathcal{N}$ is chosen such that the integrated width matches the experimental value.

The decay matrix element (19) is a function of phase space variables and thus cannot be pulled out of the integral in Eq. (4). In fact, the integration over the phase space of other decay particles is equivalent to the integration over the region of Dalitz decay:

$$\int d\phi_3 \cdots = \frac{1}{128\pi^3 M^2}\int_{\mathrm{Dalitz}} ds_{12}\, ds_{23} \cdots . \quad (26)$$

To examine the effects of decay dynamics on dPS, I numerically compute the integral in Eq. (4) together with the matrix element in Eq. (19). Explicitly, the integral reads

$$\begin{aligned}\mathrm{dPS} = & \frac{1}{2m_{\mathrm{res}}\gamma_{\omega\to3\pi}}\frac{1}{(2\pi)^3}\frac{1}{2E_1^\star}\\ & \times\int\frac{d^3p_2^\star}{(2\pi)^3}\frac{1}{2E_2^\star}\frac{d^3p_3^\star}{(2\pi)^3}\frac{1}{2E_3^\star}\\ & \times(2\pi)\delta(m_{\mathrm{res}} - E_1^\star - E_2^\star - E_3^\star)\\ & \times(2\pi)^3\delta^3(\vec{p}_1^\star + \vec{p}_2^\star + \vec{p}_3^\star)\times|\Gamma_{\omega\to3\pi}(s_{ij})|^2\end{aligned} \quad (27)$$

with

$$\gamma_{\omega\to3\pi} = \frac{1}{2m_{\mathrm{res}}}\int d\phi_3\,|\Gamma_{\omega\to3\pi}|^2. \quad (28)$$

In this implementation, the integral in Eq. (28) for $\gamma_{\omega\to3\pi}$ is by construction given by the experimental value $\gamma_{\mathrm{exp}} = (7.57 \pm 0.13)$ MeV [28]. The result for the dPS function is shown in Fig. 1. The key observation is that the full dPS function is substantially suppressed at low momenta. This is a direct consequence of the factor $\mathcal{P}$ in the decay matrix element. Note that both functions are normalized to unity when an integration over $\int d^3p_1^\star$ is performed. Furthermore, the cut at $p_1^\star \approx 0.33$ GeV simply reflects the kinematical situation where all three decay particles are collinear, with particles 1 going one way and the others going the opposite.

Next, the influence of dynamics on momentum distributions is studied. To construct the conventional p_T spectra studied in experiments, I perform an additional integration over the rapidity range on Eq. (1). At this stage, even the kinematic

[1] Deviation among models becomes appreciable when one dials up the meson width.

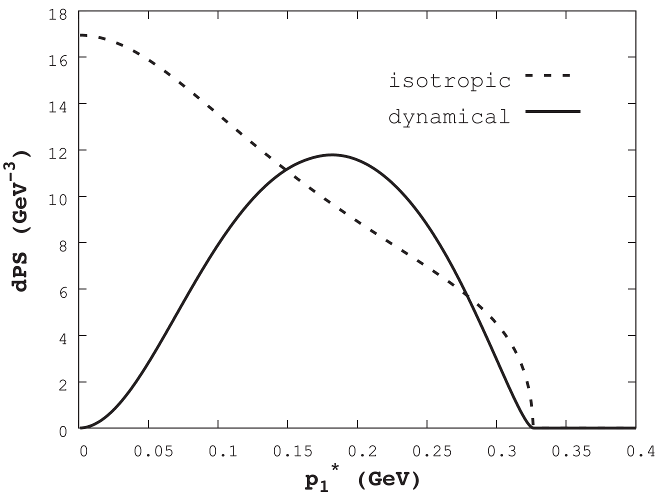

FIG. 1. Differential phase space function dPS calculated by Eq. (4) for the decay $\omega \to 3\pi$. Solid line corresponds to calculation with the full matrix element (19) while dashed line corresponds to result from the isotropic approximation.

cuts from a specific experimental analysis can be easily implemented. This may be essential for a realistic comparison with data [29]. For simplicity, I shall skip it here and consider only the midrapidity ($y = 0$) and rapidity-integrated p_T spectra of pions from the decay of ω meson. These are shown in Fig. 2.

It is somewhat surprising that despite the essential differences in the dPS functions (Fig. 1), the deviations in p_T spectra yielded by different treatments of decay dynamics are rather mild (Fig. 2). In both spectra, one sees that the correction from dynamics is limited to the low-p_T region. Moreover, deviation from the isotropic case is more visible in the midrapidity spectrum than in the integrated one. This is expected as features of dPS, and hence the influence of dynamics will be washed out when all the momentum variables are integrated over.

Previous study [1] suggests that the ω meson is one of the major sources of low-p_T pions. In this work, it is found that imposing the correct decay dynamics leads to a reduced contribution. More unexpected effects can come from other resonances and other types of decay. Further work is required to revise the resonance decay contribution to various physical observables based on the input of robust hadron physics.

B. Effects of finite width

Another consequence of hadron dynamics is the existence of resonance widths. These can be systematically included using the S-matrix formalism of Dashen *et al.* [30]. In this model, the Feynman amplitude for $3\pi \to 3\pi$ scattering can be constructed from the decay matrix element in Eq. (19) via an S-channel resonance exchange process

$$\begin{aligned} i\mathcal{M} &= \frac{-i\,|\Gamma|^2/\text{br}}{M^2 - \bar{m}_{\text{res}}^2 + iM\gamma_{\text{tot}}}, \\ \gamma_{\omega\to 3\pi} &= \frac{1}{2M}\int d\phi_3\,|\Gamma|^2, \\ \gamma_{\text{tot}} &\approx \gamma_{\omega\to 3\pi}/\text{br}, \end{aligned} \tag{29}$$

where M is the invariant mass, $\bar{m}_{\text{res}} = 0.783$ GeV is the pole mass of ω meson, and br $= 0.892$ is the branching ratio. In this model, other partial widths of ω meson are not explicitly calculated; instead, a factor of $1/\text{br}$ on $\gamma_{\omega\to 3\pi}$ is prescribed to obtain the total width γ_{tot}.

With the model scattering amplitude, the generalized phase shift function $\mathcal{Q}(M)$ and the effective spectral function $B(M)$ can be computed as follows [31–34]:

$$\begin{aligned} \mathcal{Q}(M) &= \frac{1}{2}\,\text{Im}\left[\ln\left(1 + \int d\phi_3\, i\mathcal{M}\right)\right], \\ B(M) &= 2\,\frac{d}{dM}\mathcal{Q}(M). \end{aligned} \tag{30}$$

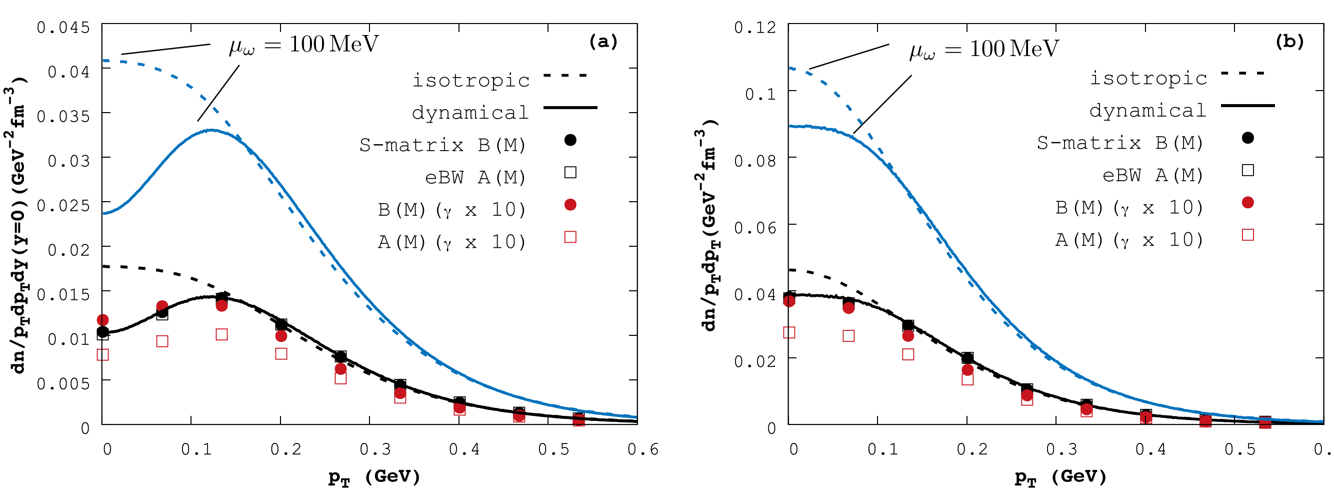

FIG. 2. The midrapidity (a) and rapidity-integrated (b) p_T spectra of decay pions from ω-meson decay [static source, $T = 120$ MeV at $\mu_\omega = 0$ MeV(black), 100 MeV(blue)], calculated for full decay matrix element in Eq. (19) and for the isotropic case. Also shown (as points) are the corresponding results (at $\mu_\omega = 0$) including the width of the ω meson via the S-matrix approach from Eq. (32), for the case of physical resonance width and the case in which the width is scaled up 10 times.

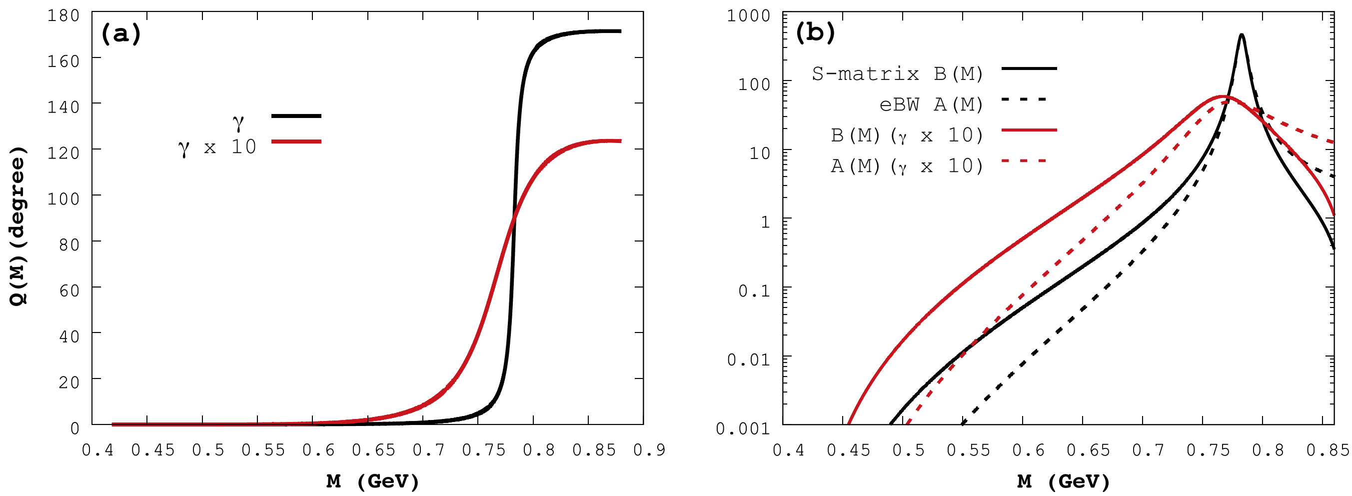

FIG. 3. Generalized phase shift function $\mathcal{Q}$ (a) and the effective spectral function B (b) computed from Eq. (30) for the model amplitude (29). The results are shown for the case of physical resonance width and the case in which the width is scaled up 10 times.

These functions are displayed in Fig. 3. Since the width of the ω meson is small, the phase shift function indeed behaves like a θ function $\pi \times \theta(M - \bar{m}_\omega)$, and the corresponding effective spectral function $B(M)$ is in practice well approximated by an energy-dependent Breit-Wigner function $A(M)$

$$A(M) = -2M \frac{\sin 2\mathcal{Q}(M)}{M^2 - \bar{m}_\omega^2}. \tag{31}$$

For the momentum spectrum, the influence of resonance width enters via [35,36]

$$E_\pi \frac{dn_\pi^{\text{dec}}}{d^3 p_\pi} = \text{br} \int^\Lambda \frac{dM}{2\pi} B(M) \times \int_{m_{\text{res}} \to M} d^3 p_{\text{res}} \frac{dn_{\text{res}}}{d^3 p_{\text{res}}} E_\pi^\star \text{dPS}(\vec{p}_\pi^{\,\star}). \tag{32}$$

One can perform an analogous numerical integration on Eq. (32) as in the zero-width case. Here $\Lambda = 0.88$ GeV is chosen, which is how far the model given in Eq. (29) is estimated to hold. The results are shown in Fig. 2. Almost no difference is found between this and the zero-width case. The vacuum width of the ω meson is so narrow that the zero-width approximation is justified.

On the other hand, substantial broadening of the ω width in the medium is suggested by model studies [37–42]. The actual extent of this effect on the measured pion spectrum depends on how rapid the freeze-out occurs. To investigate the dependence on resonance width of previous results, one simply scales the matrix element squared in Eq. (19) by a factor of 10. The resulting phase shift and the effective spectral functions are shown in Fig. 3.

In this particular model, an increase of width leads not only to the broadening of the effective spectral functions but also a reduction of their normalization although they are both normalized to unity in the limit of zero width. It also tends to reduce $\mathcal{Q}(M)$, and as $\gamma \to \infty$, the function will eventually approach the limit of $\pi/2$ at large invariant masses. As expected, this also leads to an overall drop in the magnitudes of the calculated p_T spectra (Fig. 2).

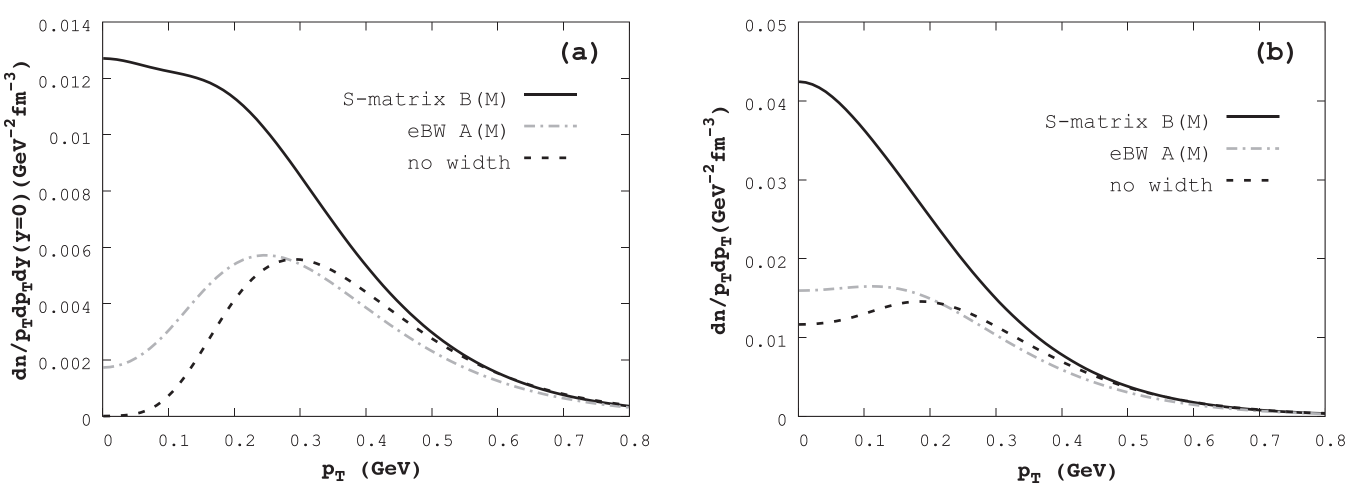

FIG. 4. The midrapidity (a) and rapidity-integrated (b) p_T spectra of decay pions from ρ-meson decay (static source, $T = 120$ MeV). The S-matrix treatment of ρ meson is discussed in Refs. [35,36]. The low-p_T enhancement from $B(M)$ is clearly visible.

Another important consequence of the broadening of resonance is the enhancement of low-p_T pions from the use of the effective spectral function $B(M)$ over that from the use of $A(M)$. This can be traced to the greater value of $B(M)$ at low invariant masses, which translates into a larger contribution to the soft part of p_T spectra due to the lesser Boltzmann suppression. A recent discussion of this effect is presented in Refs. [35,36] for the case of the ρ meson. The p_T spectra are shown in Fig. 4 for reference.[2] The enhancement in the low-p_T region from using $B(M)$ is clearly visible. In the case of the ω meson, similar effects become appreciable only when the resonance width is increased by a factor of 10. (See Fig. 2.)

IV. CONCLUSION

This study set out to investigate how details of hadron physics can modify heavy-ion collision observables. To this end, I formulate a theoretical framework for incorporating resonance decay dynamics into the analysis.

As an application, I consider the decay $\omega \to 3\pi$, and find that imposing the anomalous coupling feature of the decay matrix element leads to a reduction of low-p_T pions compared to the structureless decay treatment.

[2]For simplicity, I assume the decay vertex to be a function of $\sqrt{s}$ only, and thus the isotropic approximation is valid.

In many statistical models, the isotropic decay approximation is adopted instead of the full dynamics. The validity of this approximation has to be inspected case by case. Since multiple mechanisms are at work to produce the observed p_T spectra, it is necessary to perform a detailed examination of each effect based on existing knowledge of hadron physics. In the current study, the finding of reduced low-p_T pions from the ω meson makes room for other important effects such as higher order N-body decay and the influence of thermal medium and nonequilibrium effects to explain the unexpected enhancement of soft pions observed in the experiment.

It would be interesting to assess the effects of other important features of the strong interaction on these observables. In particular, coupled-channel dynamics [43–45] and the existence of complex objects like hadronic molecules and other exotics [46,47]. Such research is currently under way.

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