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UNITED STATES DEPARTMENT OF AGRICULTURE • Economic Research Service

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(Continued on inside back cover.)

Agricultural Economics Research

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Charles E. Rogers, editor of this journal since April 1952, will relinquish his post with the publication of this issue. He retires from Government employment at the end of July. Elizabeth Lane has been appointed to succeed him and will enter upon her duties with the preparation of the October issue. Miss Lane has been a senior editor of economic, statistical, and marketing reports in the U.S. Department of Agriculture since 1957, having been recommended for the post by instructors of the editing course in the Graduate School of the Department as a student with exceptional qualifications. Before joining the Department she had done technical editing in the Brookings Institution and the National Bureau of Standards, and during World War II she was a WAC, working in Washington. She is a graduate of the George Washington University, a philosophy major with minors in economics and mathematics.

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Integration in Food and Agricultural Industries—A Perspective

By Allen B. Paul

Though integration has been well worked over by agricultural economists, it still deserves further thought, because of differences in the insight and outlook of people, and partly because integration deals with topics of enduring interest—shifting enterprise combinations. In this paper, the author discusses a few significant facts and ideas about integration that are not widely appreciated and, anyway, do merit restatement since they will help economists to focus on emerging problems. Some of the ideas in the paper were presented at the Western Regional Marketing Workshop for extension workers in agricultural marketing at Salt Lake City in 1961. The comments of Stephen Hiemstra, Martin Kriesberg, Ronald Mighell, William Waldorf, and William Wesson of ERS are gratefully acknowledged.

THE TENOR of much discussion about vertical integration in the food business is that it has been increasing relative to nonintegrated business. For important segments of the food business there is much substance to this view, but for the system as a whole there is considerable doubt.

First consider what census data show about the overlapping activities of food processors, wholesalers, retailers, and assemblers (table 1). Only in the case of wholesaling is there a strong overlap. Together, retailer warehouses and manufacturers sales branches did more than 40 percent of the wholesaling. But firms primarily in assembling, processing, or retailing did 96 percent or more of the assembling, processing, or retailing, respectively.

It may seem surprising that food chains are relatively unimportant processors. While they shipped $1.1 billion of food from their own plants to their stores in 1958, this amount was only 8.5 percent of their purchases, and only 3 percent of total purchases by all food retailers (*12*, Ch. 9).[1] Fully half of the dollar volume of these shipments consisted of coffee and bakery goods; the rest was thinly spread over many products (*12*, Ch. 10). Moreover, a point often missed, the relative importance of shipments from retailer processing plants to their own stores has not risen since 1930 (*6*, p. 77).

TABLE 1.—*Overlapping activity of food marketing firms, 1954*

Classified by primary activity	Employment in establishments of companies classified in		Total
	The primary activity	Other activities [4]	
	Percent	*Percent*	*Percent*
Assembling of farm products [1]	95. 9	4. 1	100
Food manufacturing [2]	97. 5	2. 5	100
Food wholesaling [3]	57. 7	42. 3	100
Food retailing	98. 1	1. 9	100

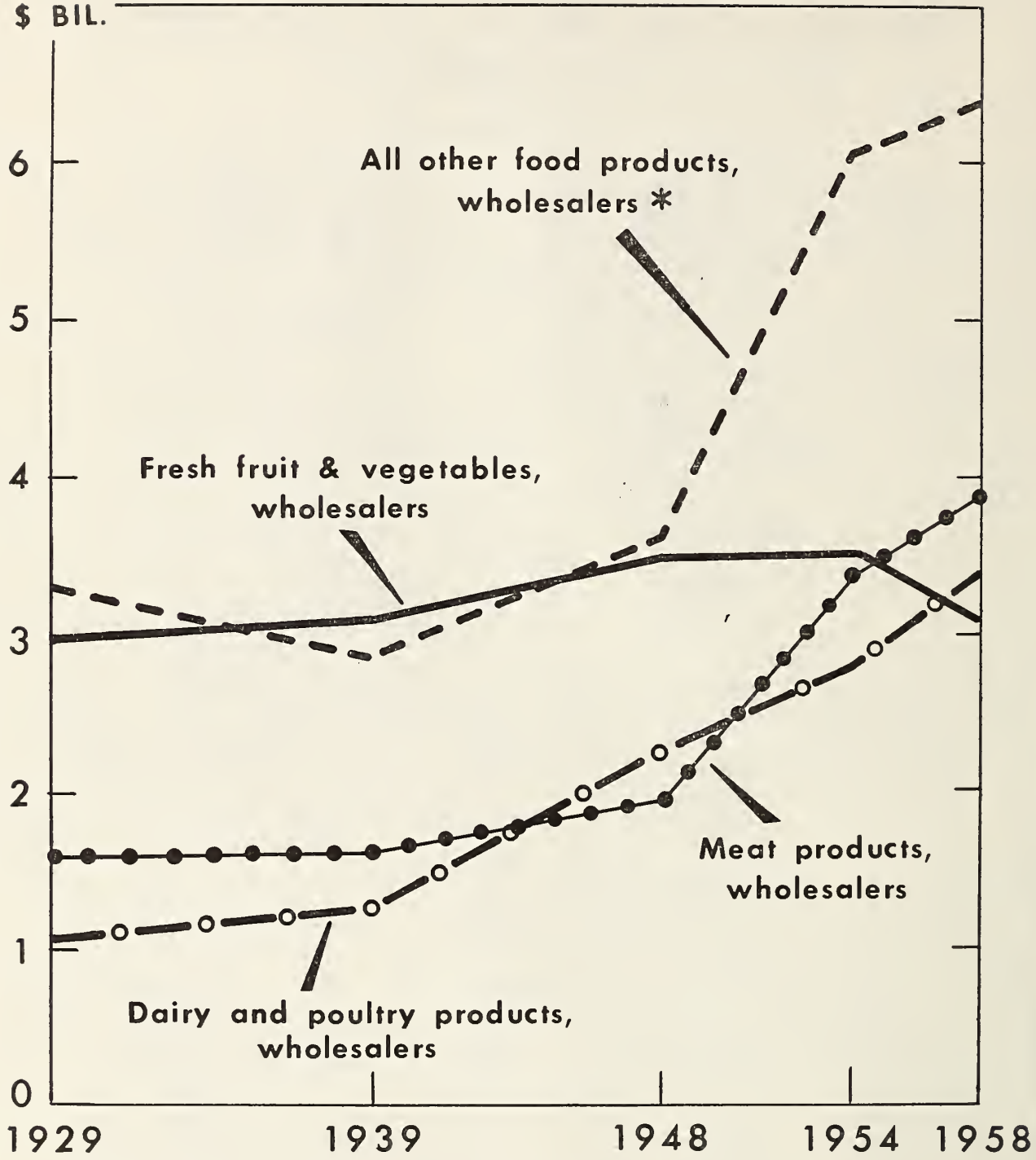
[1] Includes both food and nonfoods.
[2] Includes "vegetable and animal oils"; excludes beverages.
[3] Includes merchant wholesalers, manufacturers' sales branches and offices, and retailers' central offices and warehouses; excludes agents and brokers.
[4] Employment in these establishments amounted to 99.8 percent of the employment of all such establishments; the remaining 0.2 percent were employed in food establishments owned by companies primarily engaged in nonfood activities.

Source: Based on data from *Company Statistics*, Bulletin CS–1, U.S. Bureau of the Census. Partially estimated. While 1954 data are a little old they are not out of date; similar patterns will show in the 1958 data.

This view should be further tempered by changes in activities within establishments. Thus retail stores may process more chicken meat than formerly (although this is not certain), but they have given up credit and delivery service as well as packaging butter, cookies, sugar, and other items. The net effect might be to lessen vertical integration.

Sometimes the increase in retailers' private labels and their buying on specification are regarded as an increase in vertical integration in marketing. One must beware of drawing unwarranted implications. An essential feature of vertical integration is the fact that it broadens the area in which

[1] Italic numbers in parentheses refer to Literature Cited, p. 87.



* INCLUDES CANNED FOODS, FROZEN FOODS, CONFECTIONERY, FISH AND SEAFOOD, BAKERY PRODUCTS, FLOUR, RESTAURANT SUPPLY, SUGAR, AND OTHER SPECIALTY PRODUCT WHOLESALERS.

FIGURE 1.—Sales volume of U.S. specialty food wholesalers, 1929–58, expressed in 1958 prices.

a central administrative decision can determine how resources will be used. When a processor agrees to fabricate to a retailer's specifications, this fact itself is not evidence that the processor's decision on how to commit his resources is dictated by the retailer. Many suppliers who have a range of acceptable alternatives would be very much surprised to be told that they have lost jurisdiction over their affairs. Some of them partially finance the retailer. The greater truth is that, in the aggregate, the valuations of a larger number of market interests determine how the resources are used. It is only when the value of a supplier's assets is inimitably bound up with a specific buyer's operations, i.e., where there is a gap in the market valuation process, that the implication is warranted. Such cases are not obvious.

Most of the vertical integration in food marketing firms occurs in wholesaling. During 1958, chains bought $13 billion and affiliated wholesalers $5 billion of goods from manufacturers and assemblers (*12*). These were almost half the wholesale value of goods sold through all food stores. It is an impressive figure and warrants attention to buying practices of these organizations.

Yet such figures by themselves may obscure our perspective. About a quarter of the food purchased by consumers is provided by restaurants, schools, boardinghouses, hospitals, commissaries, or directly by growers or processors (*11*, p. 72). Thus, the volume moved by integrated wholesale-retail organizations may be more nearly a third than a half of the total civilian commercial food supply.

Then, too, the many independent, specialized food wholesalers still form the largest single factor in food wholesaling. This is not widely appreciated. Table 2 shows the relative volume of these wholesalers in 1939 and 1958 alongside the relative volume of general-line wholesalers, agents and brokers, manufacturers branches, and chain-store warehouses.

It is often overlooked that while transactions of mass buying organizations have grown greatly, so has the entire food market. Not only is there room for specialized wholesaling businesses; an expanding food market seems to depend on them. This may seem strange to those who have witnessed the decline of city terminal markets and merchant receivers and distributors.

TABLE 2.—*Percentage distribution of food sales through different wholesale outlets, United States, 1939 and 1958*

Outlet	1939	1958	Difference
	Percent	*Percent*	
Retail chain warehouses	15. 1	[1] 19. 4	+4. 3
Manufacturers sales branches and offices	21. 0	18. 2	−2. 8
Agents and brokers	22. 4	18. 8	−3. 6
General-line wholesalers	16. 2	14. 6	−1. 6
Cooperative and voluntary	6. 0	9. 1	+3. 1
Other	10. 2	5. 5	−4. 7
Specialty-line wholesalers	25. 2	29. 1	+3. 9
Total	100. 0	100. 0	

[1] Preliminary. Beyond this amount, chain organizations buy products from local suppliers that are not billed through their warehouses.

Source: Based on Bureau of Census data. Data adjusted to conform with reclassifications.

Who are the independent merchants? They handle almost every line of goods and they sell to almost everyone. Retailers, wholesalers, institutions, processors, and exporters depend on their services in varying degrees. They go about their work largely unrecognized and unsung.

The physical volume handled by such merchants by principal commodity lines from 1929 to 1958 is shown in figure 1.[2] With one noticeable exception, volume increased substantially in each major line. Willard Williams described the business of specialized wholesalers in the meat trade (*15*). Others handle frozen foods, eggs, canned goods, dairy products, confectionery, and a host of other lines.

The major exceptions noted are fresh fruit and vegetables. The volume moving through merchant specialists barely increased from 1929 to 1948, after which it leveled off and then declined. The 1958 volume was no higher than that of 1929.

This fits in with the procurement practices of large food retailing organizations described by William Folz (*2*). Evidently, the alternative marketing system for fresh produce may not have provided adequate methods of accomplishing the

[2] Census sales data (adjusted for changes in coverage between years) deflated by appropriate indexes of wholesale prices.

services expected by large retailers and they organized their enterprise on an integrated basis to provide them.

This raises some interesting questions about the future of these arrangements:

Under what conditions, if any, could independent enterprises arise to provide the services now provided by the chains for themselves?

Or will even closer ties between chains and growers occur?

Are the conditions of production and harvest unique?

Are there basic weakness in standards, grading, inspection, and informational services that might be remedied?

What influence might marketing order regulations exert on how marketing services are organized?

Questions like these merit attention as one tries to understand why existing arrangements have arisen and what one might expect of the future.

Something of the nature of the independent food merchant sector is shown in table 3. The 30,021 merchants in business in the United States during 1958 are classified by commodity line and employment size.

Horizontal integration in food marketing deserves some attention here. The term is used in different ways. Some limit its use to the acquisition of competitors. Others apply it to all means of horizontal expansion, including building on to an existing plant. In any case, the public questions about the subject are questions about efficiency and progress versus monopoly. I shall not deal with questions of operating efficiency except to note that the question of monopoly does not stand by itself. Would it be better, for example, to have one modern-type milk or bread plant serving an area, or several old-type plants?

The general question of whether monopoly in food industries has increased or decreased is unresolved. Our knowledge is not satisfactory. For example, two studies of food processing by well-known economists, presented in 1960, covered the same period. One concluded that characteristics of the food industry "have changed only slightly and certainly not in the direction of significantly increased concentration" (*1*). The other concluded that there is "fairly strong circumstantial evidence of high and growing market power in an economic sense" (*4*). Both cannot be right.

Part of the difficulty arises from differences in handling data. However, a fundamental diffi-

TABLE 3.—*Number of merchant food wholesalers in the United States, 1958**

Type of merchant	Number of establishments with paid employees numbering—					Total
	0 to 3	4 to 7	8 to 19	20 to 49	50 or more	
I. General-line:						
Voluntary and erop	39	50	144	230	210	673
Cash-earry	267	73	24	6	1	371
Other	52	121	538	364	134	1, 209
II. Speeialty-line:						
Dairy	1, 662	646	513	198	48	3, 067
Poultry	1, 048	461	400	147	26	2, 082
Confeetionery	1, 153	517	334	89	20	2, 113
Fish and seafood	771	350	314	131	25	1, 591
Meat	2, 138	974	903	368	76	4, 459
Fresh fruit, veg	3, 211	1, 300	1, 183	475	122	6, 291
Restaurant, hotel	380	232	221	89	26	948
Bakery produets	512	182	119	47	8	868
Canned foods	392	255	252	92	14	1, 005
Flour	82	48	52	11	1	194
Frozen foods	414	252	283	142	37	1, 128
Soft drinks	815	188	131	38	7	1, 179
Sugar	36	19	15	10	1	81
Other	1, 539	590	434	166	33	2, 762
Total	14, 511	6, 258	5, 860	2, 603	789	30, 021

*U.S. Census of Business, *Wholesale Trade*, Vol. III.

culty arises because industries generally are not classified on the basis of a meaningful measure of output. The output of the firm consists of services; to measure the market jurisdiction of a firm it is necessary to determine its relative importance in markets for such services. For analytical purposes, an industry should be constituted in terms of some economically homogeneous set.

For example, if a firm is specialized in processing soybeans, its output of services is largely the milling plus ancillary services in milling beans into crude soybean oil and meal. The best available measure of this output is "value added," as defined by the Census (*14*). The firm's value added divided by the total value added by all oilseed processing services supplied *in the same market* would be its share of the total. Presumably the output contributed by most but not all soybean processing and some but not all cottonseed, linseed, and peanut crushing plants would be entered. The industry would be constituted on the basis of the elasticity of substitution of one processor's services for another's, as affected by difference in time and place as well as by the versatility of equipment. But if a firm processes soybeans, mixes feed, and refines oil, for example, its total value added must be apportioned among at least three different markets for its productive services, and its contribution to the total output in each must be reckoned separately. In case of feed-mixing services, many markets probably would be distinguished (because of the transport barrier) and the contribution of the firm to each should be determined separately (*7*).

Agriculture

To many people, integration means contract farming. Much of the excitement over such changes in agriculture is warranted. While the literature is already large, studies are currently underway to learn more about them (*5*). The following comments are confined to a few points that seem pertinent.

Contracts are means of dividing up enterprise and separately transferring the parts. Over the last four centuries, man has devised ingenious schemes for this. Agriculture, as we are now witnessing, is an active testing ground. Present arrangements to farm include leasing, partnership, and corporate organization, as well as special joint-account, forward-delivery, and service agreements. The possible ways of distributing enterprise responsibility in such a setting have no apparent limit (*9*, p. 316).

The importance of such arrangements is that when resources are collected into a larger bundle, from different owners, greater productivity is possible: Specialization, economies of scale, and application of new technology all become heightened.

Particular arrangements arise from historical circumstance. Great new possibilities in production—like the opening of a new territory, the discovery of a new process, or an unprecedented demand for output—attract capital through existing institutions. But innovations occur if the institutions cannot serve the purpose. The less dramatic but widespread possibilities of reducing costs by increasing scale of existing operations also draw capital.

The reservoirs of venture capital are broad; agriculture traditionally has been its own principal source but it has also drawn much outside capital. Conversely, much agricultural capital has entered various farm supply and marketing services through cooperative pooling and other means. Construction and machinery interests promoted every other cannery in Wisconsin in the pre-World War I era, taking first liens on the vegetable packs. Vegetable growers usually had residual claims on the annual proceeds after all expenses and tolls were deducted. It was the only feasible way a plant could be built in some communities. If the venture did not work, the loss hurt but did not destroy. The growers usually did not give up much to grow vegetables.

Today fresh sources of venture capital have entered farming. Broilers are the prime case but others are in evidence. Nonfarm capital investment in farming is not new, although the present applications are. The vast expansion of grain and cotton production during the last half of the 19th century, and the vast expansion of soybean and irrigated cotton in this century were financed in part by the unseen speculator in commodity markets.[3] Production of Maine potatoes and late onions for winter storage has been similarly financed for a number of years (*8*, *13*). Eastern

[3] For many years before development of full-fledged organized futures trading in soybeans, soybean oil, and soybean meal, a sizable "cash" forward market was the principal way that "outside" speculation occurred.

wholesale dealers financed, on a large scale, every truck crop region of importance which had to ship any considerable distance to market starting about the turn of the century (*10*, p. 60–74).

Excepting fresh milk and fresh fruit and vegetables for processing (wherein technical conditions warrant the assurance of market outlets through advance agreements), there are few permanent reasons for the present contractual arrangements. The production and financing advantages, however great they may be, can prove transitory. Technical knowledge is transferable; so are the alternative sources of capital. A particular organization for commodity production will survive as long as it satisfies the basic problems of production and investment as well or better than other arrangements.

For example, when the hazards of price change are large, individuals will seek some accommodation. William O. Jones, writing in 1951, showed that over the course of years California lettuce shipping gravitated to those dealers that put their venture on an actuarial basis; i.e., shipped lettuce 52 weeks of the year. They in turn contracted with growers to produce under large-scale, low-cost conditions at guaranteed minimums, plus a share of profits (*3*). Similar methods or organization in the cattle feeding industry and in other industries are evident. It enables low-cost methods of production to be followed. Profits are made on narrower "markups" but on larger volumes in each phase of the business—i.e., in having the commodity produced as opposed to producing the services entering into the commodity.

Finally, the business adaptations cited here occur in a market where there is a relatively free play of supply, demand, and price. Regulation of one or more of these would affect the economic bases for such business arrangements and perhaps other kinds would become more appropriate.

Some Implications

How does one identify a marketing problem? Among other things, one is influenced by an image of the marketing system and what the future holds for it. One cannot be sure his own image is correct and germane because agricultural production and marketing is one of the most complex schemes of man and our knowledge of it is incomplete. I have tried to convey, on the basis of available evidence, a reasonable image of the system. Assuming it is reasonable, what would it mean to anyone who is concerned with the functioning of the system?

It would mean that most problems of marketing are not very different today from those of the past—they just appear in modern dress. The enduring features of the market include specialization of production, proliferation of small business alongside large business, and their interaction in the struggle for survival and growth. There invariably are market imperfections and monopoly elements, differential impacts from technological change and trade dislocations, and insistent pressures to reduce costs by enlarging scale. Remaining invariably is the central role of market valuations and prices in coordinating economic activity and man-made institutions in which such activity occurs.

The new dress in which the enduring features appear captures our imagination. New techniques and products and new kinds of specialization in production have appeared, and both old and new operations are conducted with novel kinds of enterprise arrangements. This is what many people mean when they speak of the growing trend toward vertical integration. People should be concerned with them. One might reasonably expect further growth of the market economy, together with new technology, to cause such changes in production and business organization to continue.

But others see this matter differently. They fix attention almost exclusively on the expansion of integrated businesses. Such pictures, though incomplete, may not be inconsistent with the one sketched here: A growing market permits considerable expansion of integrated businesses before they increase relatively. Also, important parts of the agricultural marketing system have had more integration, both absolutely and relatively.

A conflict in views would arise only if one holds that, on balance, resources are now being directed more largely by administrative decision than by market valuation. The most extreme image holds that the entire production process, from farmer to retailer, will become coordinated exclusively by business administrators. Such conceptual systems would not escape the problem of competitive valuation because a scheme for simulating market valuations would have to be installed

in place of the real thing. It is difficult to visualize the conditions in the food industry in which this scheme could become successful.

Under the present image, the problems that may be identified include (1) updating institutions for the conduct of modern business—i.e., the commodity grades, inspections, information, price reporting, contract security, rules, regulations, and public laws respecting the conduct of trade, use of patents, acquisition of competitors, and so forth; and (2) giving assistance to smaller firms who might make successful adaptations to changing conditions.

These are continuing problems of an expanding market economy but they are no less important on this count. Many people are aware of various needs in farming and marketing. For example, frequent reference is made in the literature to needed farm adjustments to buyer requirements, improved methods of paying for quality, better production planning, more efficient scheduling of deliveries, needed developments in financing, and so forth. Also much attention is given to minimizing unfair competition, to helping small business adjust and, to a lesser extent, to serviceable patent laws and regulations. The critical matter is to think these problems through in terms that will be relevant to future conditions.

A major problem in the food economy today is that market adjustments depend as much as ever on prices but prices do not have as clear a meaning as they might. Geoffrey Shepherd noted that the decentralization of marketing generally reduced marketing costs but that it created problems of pricing and he called for "steps to bring market grades to the highest possible level of detail and accuracy, and disseminate market news on a decentralized basis comparable with decentralization of the marketing of physical product" (*11*, p. 59).

The problem of pricing is complicated further by tendencies for exchange to involve economic quantities that do not correspond well with physical products. The meaning of price becomes blurred when considerations of finance or promotion are joined with physical commodities, when selected services built into products (rather than products) are bought and sold, and when joint accounts substitute for open market transactions between parties. The endless change in types of economic packages entering into transactions makes it more difficult to establish a feasible reporting system.

These are recurring difficulties. Working closely with firms, sometimes one can recommend workable improvements. At other times one can only interpret the meaning of occurrences around him in terms of larger forces and thus raise the general level of understanding.

Finally, the importance of improved market arrangements can be restated in larger terms. National economic growth requires continued reduction in costs within the economic system. It is the only important means of growing available to us. The principal way costs can be reduced is by specialization, i.e., the further division of work-tasks, standardization of processes, and the further application of machine methods. The increase in specialization is governed by the extent of the market. There is no known limit to either.

An expanding market implies that it must have been tolerably well coordinated for growth to have occurred. Yet the process of growth is a source of shock that causes unusual gains and losses and creates waste which might be avoided. It puts pressure on people to reexamine the areas in which their comparative advantage would show, even though they already are committed to a definite course of action. It creates the need for each to become more aware of his latent capacities, developing opportunities, and the possible ways of filling them. One of the most useful services is to provide people with the information they need to make major decisions intelligently.

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Nonlinear Programming of Field and Plant Vegetable Processing Activities

By Robert H. Reed and James N. Boles

Empirical studies of plant costs and efficiency always have stressed the importance of both size of plant and length of operating season on the level and shape of the economies of scale curve as well as the technical organization of production implied in it. Because most processing firms are integrated systems of product assembly, processing, and distribution, the minimum-cost combination of plant size and hours operated must be considered in terms of total operation, not of any single "stage" or component. This paper presents a method for determining optimum combinations of hours of operation and size of plant (as measured by rates of output) for two components, field and plant operations, of an integrated system of preparing lima beans for freezing. Though the analysis is oriented to only two "stages" and a particular product, the authors are hopeful that it may point the way toward extension to additional "stages" and other products.

COST SYNTHESIS is an effective economic tool for reflecting in-plant cost functions, technical relationships, and operating characteristics of agricultural processing operations.[1] Through this procedure, data on elemental input-outpnt and plant records are used to develop cost-output relationships among individual operating stages with alternative production techniques. Comparison of such stage-cost functions provides the basis for selecting least-cost operating techniques and for the development of generalized plant-cost functions that are closely analogous to the long-run cost or planning functions of economic theory.

As long-run total and unit plant costs vary with both length of season and capacity output rates, optimum plant adjustment is determined by both variables simultaneously. This may be made clear by noting several general characteristics of long-run average cost behavior.

First, with any given pattern of daily operating hours and length of operating period, total hours per period are fixed and so differences in total planned volume per period requires changes in plant capacity (measured as an output rate). Consequently, each point on a particular long-run average cost curve represents unit cost with a different plant. With a given length of operating period, and over a wide range of output scale, unit costs as represented by such a curve decrease as plant capacity increases. The decrease results from more effective utilization of supervisory and other partially fixed labor inputs and the substitution of various cost-reducing techniques in the larger plants.

Second, plant capacity rates necessary to achieve any given season volume decrease as hours of operation per season increase. As capacity decreases, investment cost and the corresponding annual fixed costs are smaller, with the result that unit fixed costs decline. But variable costs tend to rise and some of the cost advantages of increased scale are lost. Thus, efficient plant organization calls for balancing the net cost effects of scale of plant and operating hours.[2]

Most processing firms are an integrated system of product procurement, processing, and distribution. With the technique of cost synthesis the analyst can develop a separate cost function for each "stage" or component operation of the integrated system. But he must consider minimum-cost combination of rates and hours in terms of the total operation, not of any single component.

[1] For detailed presentation of economic-engineering techniques in cost measurement, refer to B. C. French, L. L. Sammet, and R. G. Bressler, Jr., "Economic Efficiency in Plant Operations with Special Reference to the Marketing of California Pears." *Hilgardia*, Vol. 24, No. 19, July 1956, pp. 543–721. Also, Sammet, L. L., "Economic and Engineering Factors in Agricultural Processing Plant Design" (unpublished Ph. D. thesis, Department of Agricultural Economics, University of California, Berkeley, 1959), 434 pp.

[2] An example and discussion of the problem of finding optimum combinations of plant size and time of operation for any given season volume is given in: French, Sammet, and Bressler, *op. cit.*, pp. 684–704.

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If output rates for individual stages can be varied and part of the product stored between stages, the problem may usually be treated as a constrained minimum problem whereby a nonlinear objective function is minimized, subject to a series of linear restraints. A detailed development of the nonlinear programming problem in which the restraining functions are linear is given by Dorfman, Samuelson, and Solow.[3] These authors point out that, though "no sure fire practical method for solving nonlinear programming problems has yet been found, . . . special approximative methods can be tailor-made to solve individual problems as they arise."

In this paper we demonstrate a method for determining optimum combinations of hours of operation and rates of output for two components of an integrated system.[4] The example involves field and plant operations in preparing lima beans for freezing, using data developed in a 1958 study in California. The field operations consist of vining or shelling the beans and transporting them to the receiving station of the plant. At the freezing plant, the product is pumped, flumed, or otherwise conveyed from the receiving station through a series of in-plant operations. Substantial investment is required at both field and plant locations.[5]

The following assumptions and constraints reflect conditions consistent with actual experience: (1) A maximum of 8 hours of storage (with ice) is allowed between vining and in-plant processing, and is assumed to have no measurable effect on quality; (2) the daily operating hours and rates of output of vining and plant operations are such that the *total* volume vined or shelled per day equals the *total* daily volume processed; and (3) in recognition of time lost daily in lunch periods, changing shifts, cleanup, equipment servicing, rest periods, and other delays, a maximum of 16 operating hours per day is assumed.

Economic-engineering methods were used to synthesize cost functions for field and plant activities representing total and average planning costs for each activity. Specific equations are given by Reed.[6] These are expressed here solely in terms of rates of output (R) and hours operated per season (H) by substituting, in more general equations, particular values for the variables representing distance of haul, percentage manual grade-out, and proportions packed in retail, institutional, and bulk containers. That is:

$$(1)\quad TSC_1 = \$3{,}929 + \$2{,}633R_1 + \$0.3691H_1 + \$A_1R_1H_1$$

$$(2)\quad TSC_2 = \$15{,}353 + \$1{,}870R_2 + \$27.6177H_2 + \$A_2R_2H_2$$

where

TSC_1 is total annual costs of vining and hauling.

TSC_2 is total annual costs of in-plant processing.

R_1 and R_2 are hourly rates of vining and in-plant processing, respectively, in 1,000-pound units.

H_1 and H_2 are hours of vining and in-plant processing per season.

$Q = R_1H_1 = R_2H_2$ is total annual volume processed, in 1,000 pounds packed-weight equivalent.

A_1 and A_2 are constants whose values depend on those specified for the variables: distance of haul; percentage manual grade-out; and proportions packed in the various size containers.

Daily cost equations may be derived by dividing the coefficients by the number of days operated per season. As $R_1H_1 = R_2H_2 = Q$ by constraint (2) above—with Q expressed in packed-weight equivalent—the above equations can be written solely in terms of (H's) and (Q).

For any length of season (number of days operated), values of H_1 and H_2 and R_1 and R_2 can be found that minimize the combined costs of field and plant operations. The solution presented below assumes a 40-day operating season. (Solutions

[3] Robert Dorfman, Paul A. Samuelson, and Robert M. Solow, *Linear Programming and Economic Analysis* (New York: McGraw-Hill, Inc., 1958), 527 pp. Also, H. W. Kuhn and A. W. Tucker, "Nonlinear Programming," *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability*, ed. J. Neyman (Berkeley: University of California Press, 1951), pp. 481–492.

[4] Though oriented to two "stages" and a particular product, the analysis may be extended to additional stages, and to other products, involving similar considerations.

[5] A detailed analysis of lima bean assembly and processing operations and costs is given in: Robert H. Reed, *Economic Efficiency in Assembly and Processing Lima Beans for Freezing*, California AES Mimeographed Report No. 219 (Berkeley, 1959), 106 pp.

[6] Reed, *op. cit.*, pp. 78–80.

for seasons of different length are presented later in the paper.) Carrying through the substitutions noted, the *daily* cost equations for vining and in-plant processing are as follows:

$$(3)\quad TDC_1 = C_1 = \$98.225 + \frac{\$65.825Q}{H_1} + \$0.3691H_1 + a_1Q$$

$$(4)\quad TDC_2 = C_2 = \$383.825 + \frac{\$46.750Q}{H_2} + \$27.6177H_2 + a_2Q$$

The optimum combination of daily hours and output rates is obtained by minimizing $C = C_1 + C_2$, subject to the constraints of a maximum of 16 hours operation per day and a maximum temporary storage period of 8 hours per day.[7]

Thus:

$$H_1 - H_2 \leqq 8$$

$$H_2 - H_1 \leqq 8$$

$$H_1 \leqq 16$$

$$H_2 \leqq 16$$

These constraints are graphically depicted in figure 1. Every point on the graph corresponds to a pair of values for H_1 and H_2. Any point inside or on the boundary lines of the figure (OABCDE) corresponds to combinations of H_1 and H_2 that simultaneously satisfy all the constraints.

The solution to this problem is simplified by temporarily assuming there are no effective restrictions on daily hours of operation and that all variables except output and total cost are independent of hours operated per day. Then H_1 and H_2 are free to vary up to 24 hours per day with no increase in cost rates and equations C_1 and C_2 may be minimized separately, that is:

$$\frac{dC_1}{dH_1} = \frac{-65.825Q}{H_1^2} + 0.3691 = 0$$

$$\frac{dC_2}{dH_2} = \frac{-46.75Q}{H_2^2} + 27.6177 = 0$$

$$H_1 = 13.354(Q)^{1/2}$$

$$H_2 = 1.301(Q)^{1/2}$$

Thus, the locus of cost-minimizing combinations of H_1 and H_2 for different values of Q is given by the equation:

$$(5)\qquad H_1 = 10.264H_2$$

If total daily volume (Q) is allowed to increase from zero along the "expansion path" defined by equation (5), the extent of movement is constrained by the limitation $H_1 - H_2 = 8$. The values for H_1 and H_2—and consequently, for R_1, R_2, and Q—for which this constraint first becomes binding, are found by solving the pair of equations:

$$H_1 - H_2 = 8$$

$$H_1 = 10.624H_2$$

These equations imply that $H_1 = 8.86$, $H_2 = 0.86$, and $Q = 0.440$, which define point F in figure 1. Plants of such low capacity are below the range found in actual processing operations.

For larger volumes of daily output (Q), the constrained cost-minimizing expansion path follows the line $H_1 - H_2 = 8$. The next step then is to minimize the function $C = C_1 + C_2$ subject to the linear restraint $H_1 - H_2 = 8$. To do so, let $\theta = C - \lambda(H_1 - H_2 - 8)$, where λ is a Lagrange multiplier.

$$\frac{\delta\theta}{\delta H_1} = \frac{-65.825Q}{H_1^2} + 0.3601 - \lambda = 0$$

$$\frac{\delta\theta}{\delta H_2} = \frac{-46.75Q}{H_2^2} + 27.6177 + \lambda = 0$$

Adding these equations and clearing of fractions results in

$$(6)\quad -65.825QH_2^2 + 27.9868H_1^2H_2^2 - 46.75QH_1^2 = 0$$

This equation can be solved explicitly for H_1 and H_2 noting that $H_1 = H_2 + 8$. This gives a 4th degree equation, however, and it is easier to specify

[7] As the unit cost of temporary storage operations averages less than five-tenths of a mill per pound, the total daily cost function ($C = C_1 + C_2$) was not adjusted for these costs for each of the constraints on H_1 and H_2. Adjustment of the daily cost functions to account for variations in temporary storage costs as the constraints vary would have no significant effect on the solution obtained. Where temporary storage costs are important, however, costs should be adjusted to reflect such variation, or included in the analysis as an additional "stage" or component cost function.

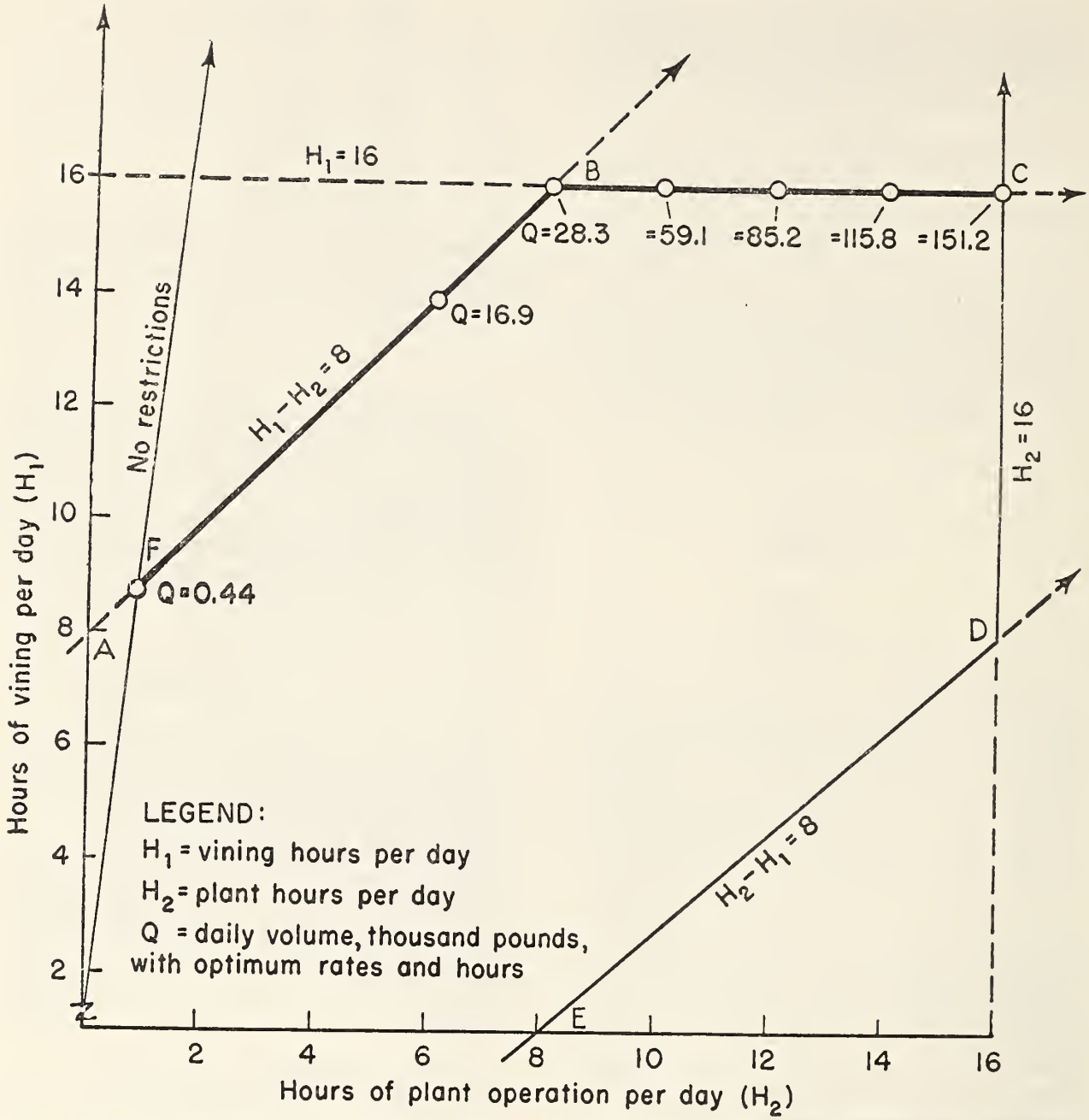

FIGURE 1.—Feasible and optimal combinations of daily hours of field and plant operations for frozen lima bean processing, California.

a point on the line ($H_1 - H_2 = 8$) and use the above equation to find the corresponding value of Q. For example, if H_1 is set at 14 hours and H_2 at 6 hours, the above equation implies that Q=16.9.

Movement along the line $H_1 - H_2 = 8$ can proceed with increasing daily volume, Q, until H_1 reaches the specified maximum of 16 hours. At this point (point B, figure 1), the value of H_1 is 16 and the value of H_2 is 8. With these values of H_1 and H_2, the daily volume Q implied by equation (6) is 28.3 and the expansion path becomes the horizontal line $H_1 = 16$. With H_1 fixed at 16 hours per day, C can be minimized for Q greater than 28.3 by minimizing C_2 independently, or as derived above, $H_2 = 1.301(Q)^{\frac{1}{2}}$. For example, if Q=85.2, the cost-minimizing value of H_2 is 12 hours.

As total daily volume increases further, H_2 can expand to its absolute limit of 16 hours (point C,

TABLE 1.—*Minimum-cost combinations of hours operated and rates of output for field and plant operations in processing lima beans for freezing, three lengths of operating season, California, 1958*

Hours operated per day		Output per hour		Daily volume	Season volume
Field (H_1)	Plant (H_2)	Field (R_1)	Plant (R_2)	(Q)	(q)
30-DAY SEASON					
Hours	*Hours*	*Pounds*	*Pounds*	*Thousands pounds*	*Million pounds*
16	8	1, 328	2, 657	21. 254	0. 637
16	10	2, 769	4, 430	44. 304	1. 329
16	12	3, 993	5, 324	63. 890	1. 917
16	14	5, 428	6, 203	86. 848	2. 605
16	16	7, 089	7, 089	113. 418	3. 403
16	16	9, 375	9, 375	150. 000	4. 500
16	16	12, 500	12, 500	200. 000	6. 000
16	16	18, 750	18, 750	300. 000	9. 000
16	16	25, 000	25, 000	400. 000	12. 000
40-DAY SEASON					
16	8	1, 771	3, 542	28. 338	1. 134
16	10	3, 692	5, 907	59. 072	2. 363
16	12	5, 324	7, 099	85. 186	3. 407
16	14	7, 237	8, 271	115. 797	4. 632
16	16	9, 452	9, 452	151. 224	6. 049
16	16	12, 500	12, 500	200. 000	8. 000
16	16	18, 750	18, 750	300. 000	12. 000
16	16	25, 000	25, 000	400. 000	16. 000
50-DAY SEASON					
16	8	2, 214	4, 428	35. 423	1. 771
16	10	4, 615	7, 384	73. 840	3. 692
16	12	6, 655	8, 874	106. 483	5. 324
16	14	9, 046	10, 339	144. 746	7. 237
16	16	11, 814	11, 184	189. 030	9. 452
16	16	12, 500	12, 500	200. 000	10. 000
16	16	18, 750	18, 750	300. 000	15. 000
16	16	25, 000	25, 000	400. 000	20. 000

figure 1), which corresponds to Q=151.2. No further adjustment of H_1 and H_2 is possible as Q expands and increasing daily volume beyond Q=151.2 can only be achieved with proportional increases in the hourly output rates (size of vining and plant facilities).

The number of days operated per season has an important effect on the least-cost combination of daily operating hours and size of facilities. The effect on rates and hours is directly proportional to the length of season. With a 30-day operating season, for example, the total daily volume (Q) corresponding to point C of figure 7 is 113.4, exactly three-fourths of the value for Q with a 40-day operating season. Table 1 gives selected values for combinations of hours and rates of field and plant operations, both daily and seasonal, for operating seasons of 30, 40, and 50 days.

The above example suggests that total combined costs of field and plant operations are lowest when storage time and field hours operated per day are maximized. The relatively low level of economies of scale found in the field operations suggests the same conclusion. In situations where assembly and storage costs are more important components of the total cost picture, a complete analysis would require their explicit consideration.

A Single Matrix Method for Several Problems

By Alvin C. Egbert

Matrix algebra has become a familiar research tool in recent years, but the teaching and learning problem is still formidable for many individuals. The purpose of this paper is to present a simple general-purpose method of handling matrices for solving simultaneous equations, including those involved in regression and linear programming problems. High-speed computers and different methods are now used in most practical analysis in this field, but teaching must rely on manual approaches to illustrate the mathematical principles. The method presented here is believed to shorten learning time and reduce the memory burden. In short, this is an introduction to matrix algebra in one easy lesson. The author wishes to thank Rex Daly and Martin Abel for suggestions that have helped to improve this article.

THIS PAPER SHOWS how a single method of handling matrices can be applied to problems involving simultaneous equations, regression analysis, and linear programming. All of this will be found in standard textbooks, but the conventional solutions for problems in each of these fields have been fragmented along lines that select a most efficient method for each purpose considered independently. The general-purpose approach presented here is a sort of least common denominator which has the pedagogical advantage of bringing out more clearly the interrelationships between the different types of problems.

The method presented employs what might be called a "desired goal approach." No proof of the method is given because this can be found elsewhere (*4*, ch. 1–4).[1] Nor is originality claimed. The procedure uses only a few principles of elementary matrix algebra and anyone who has used signed (positive and negative) numbers will have no difficulty in learning the method. The method is not the fastest one available for every situation. But it does get the job done with a minimum of mental effort.

[1] Italic numbers in parentheses refer to Selected References, p. 100.

The order of presentation is first to outline the procedure step by step using a simple example and then to show the several applications.

The Method

I. Some Definitions

EXHIBIT A

	P_1	P_2	P_3	P_4	P_5	P_6
R_1	6	4	2	1	0	0
R_2	4	9	3	0	1	0
R_3	2	3	5	0	0	1

A. Exhibit A is called a matrix. A matrix is simply a rectangular array of numbers.

B. P_1, P_2, etc., are labels or identifications for the columns.

C. A column, or column vector, is a vertical array of numbers, e.g., column P_1.

D. A row, or row vector, is a horizontal array of numbers, e.g., R_1.

E. A column is sometimes called a column matrix; a row is called a row matrix.

F. An element is any single number in a row, column or matrix. In a matrix, an element in row 2, col. 3 is identified as e_{23}—or 3 in Exhibit A.

G. An identity, or unit column is one containing the numbers, one (1-unity) in one position only, and zeros elsewhere, e.g., columns P_4, P_5, and P_6.

II. Objective

Vectors P_1, P_2, and P_3 are to be transformed into unit vectors like vectors P_4, P_5, and P_6 without disturbing the "relationship" between the rows and vectors (the reason why we want to do this will be clear later).

III. Procedure

A. First we need some information about what we can do without disturbing the relationship between the rows and columns. This information is stated without proof.

1. A row can be multiplied or divided by some number without disturbing the relationship.

2. A row or some multiple of a row (i.e. a row times 2 or row times ½ and so forth) can be added

to another row without disturbing the relationship.[2]

B. Armed with this information we can now proceed toward the objective.

1. We know that we want to get the number 1 or unity where element e_{11} or 6 now stands, hence, we divide row 1 by the number 6, element by element:

	P_1	P_2	P_3	P_4	P_5	P_6
R_1'----	1	.66667	.33333	.16667	0	0

2. We know also that we want to get zeros in the positions of elements e_{21} and e_{31}, i.e., rows 2 and 3 of column P_1. Accordingly, we subtract 4 times row 1′ from row 2, element by element.

	P_1	P_2	P_3	P_4	P_5	P_6
	4	9	3	0	1	0
	4	2.66668	1.33332	.66668	0	0
R_2'------	0	6.33332	1.66668	−.66668	1	0

3. And similarly we subtract 2 times row 1′ from row 3:

	P_1	P_2	P_3	P_4	P_5	P_6
	2	3	5	0	0	1
	2	1.33334	.66667	.33334	0	0
R_3'------	0	1.66666	4.33333	−.33334	0	1

Now we have a completely new matrix in which we have accomplished one-third of our task, i.e., column P_1 is in the desired form and the matrix at this point is as follows:

EXHIBIT B

	P_1	P_2	P_3	P_4	P_5	P_6
R_1'------	1	.66667	.33333	.16667	0	0
R_2'------	0	6.33332	1.66668	−.66668	1	0
R_3'------	0	1.66666	4.33333	−.33334	0	1

Beginning with Exhibit B, let us proceed with the next step of our objective, i.e., to change column P_2 into one like column P_5.

C. Since new column P_2 now has the number 6.33332 in second row, we must divide row 2′ by this number. We also operate on the other two rows in the same way as we did in the first step.

1. Row 2″ below is row 2′ in Exhibit B divided by 6.33332 in order to get 1 in element e_{22}.

	P_1	P_2	P_3	P_4	P_5	P_6
R_2''-------	0	1	.26316	−.10526	.15790	0

2. Row 1″ is obtained by subtracting .66667 times row 2″ from row 1′.

	P_1	P_2	P_3	P_4	P_5	P_6
	1	.66667	.33333	.16666	0	0
	0	.66667	.17544	−.07017	.10527	0
R_1''---	1	0	.15789	.23683	−.10527	0

3. Row 3″ is obtained by subtracting 1.66666 times row 2″ from row 3′.

	P_1	P_2	P_3	P_4	P_5	P_6
	0	1.66666	4.33334	−.33333	0	1
	0	1.66666	.43860	−.17543	.26317	0
R_3''--	0	0	3.89474	−.15790	−.26317	1

Collecting the transformed rows we have a new matrix:

EXHIBIT C

	P_1	P_2	P_3	P_4	P_5	P_6
R_1''-----	1	0	.15789	.23683	−.10527	0
R_2''-----	0	1	.26316	−.10526	.15790	0
R_3''-----	0	0	3.89474	−.15790	−.26317	1

D. With the results in Exhibit C we can proceed to the final step.

1. Divide row 3″ by 3.89474 to get the number 1 in row 3″ of column P_3 and thus obtain a new row 3‴.

	P_1	P_2	P_3	P_4	P_5	P_6
R_3'''-----	0	0	1	−.04054	−.06757	.25676

2. The final row, 1‴ is row 1″ minus .15789 times row 3‴:

	P_1	P_2	P_3	P_4	P_5	P_6
	1	0	.15789	.23683	−.10527	0
	0	0	.15789	−.00640	−.01067	.04054
R_1'''--	1	0	0	.24323	−.09460	−.04054

3. The final row, 2‴, is row 2″ minus .26316 times row 3‴.

	P_1	P_2	P_3	P_4	P_5	P_6
	0	1	.26316	−.10526	.15790	0
	0	0	.26316	−.01067	−.01778	.06757
R_2'''-	0	1	0	−.09459	.17658	−.06757

[2] The term column can be substituted for row and these statements are still true. But row and column operations cannot be intermingled. If we start with row operations, we must continue with them to the solution and vice versa.

The following exhibit presents the final rows:

EXHIBIT D—final matrix

	P_1	P_2	P_3	P_4	P_5	P_6
R_1'''	1	0	0	.24323	−.09460	−.04054
R_2'''	0	1	0	−.09459	.17568	−.06757
R_3'''	0	0	1	−.04054	−.06757	.25676

E. The original mission is now completed—vectors P_1, P_2, and P_3 are unit vectors in Exhibit D. In review the steps are:

1. Decide on objective (in the above example, this was the columns to become identity vectors and the elements in these vectors to become the number one or unity).

2. Select pivotal element. This is the element to be transformed to the number one, and is the element that designates the *operating column* and *critical row*. These were selected (so it appears) in arbitrary manner in the preceding example. In particular problems, the pivotal element will be selected by specific criteria.

3. Divide critical row by the pivotal element.

4. Multiply transformed *critical row* by the number located in *operating* column of row 1 and subtract products from row 1. Do this for all rows, except the critical row. This operation transforms all elements in the *operating* column (except the element of the critical row) to zeros.

5. Steps 2 through 4 are repeated for every column or vector that must be transformed into a unit vector.

Once these steps have been learned, it is usually more convenient when using a desk calculator to go directly from one intermediate matrix to another (A to B, B to C and so forth) without writing down the individual row multiplications and subtractions as we have done in this example. It is a good plan when using the direct method to have each successive matrix identified on a long sheet of paper so that the elements can be filled in as computations proceed.

Matrix Inversion Defined

The operations carried out in the preceding section have inverted a matrix. The final vectors P_4, P_5, and P_6 in Exhibit D form an inverse of original vectors P_1, P_2, and P_3 in Exhibit A.

If we let the symbol A stand for vectors P_1, P_2, and P_3 in Exhibit A and let B stand for vectors P_4, P_5, and P_6 in Exhibit D then:

$$AB=I$$

$$\text{where } I=\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}$$

and e_{11} or 1 is obtained by multiplying row 1 in A, by column 1 in B, element by element, and adding. For example, using row 1 of columns P_1, P_2, and P_3 in Exhibit A and column 4 in Exhibit D yields the following:

$$1=[6(0.24323)]+[4(0.09459)]+[2(-0.04054)]$$

In the same way, the zero element, e_{32}, of I is the sum of the inner products of row 3 of A and column 2 of B.

Using the Inverse To Solve Simultaneous Equations

If we let $\bar{y}$ stand for a column vector, $P_o=\begin{matrix}40\\50\\30\end{matrix}$, then we can write:

$$\bar{y}=A\bar{x}$$

where $\bar{x}$ is also a three-element column and A is the 3-column (P_1, P_2, P_3) matrix in Exhibit A. We can also write the above:

$$\begin{matrix}y\\\begin{bmatrix}40\\50\\30\end{bmatrix}\end{matrix}=\begin{matrix}A\\\begin{bmatrix}6&4&2\\4&9&3\\2&3&5\end{bmatrix}\end{matrix}\begin{matrix}\bar{x}\\\begin{bmatrix}X_1\\X_2\\X_3\end{bmatrix}\end{matrix}$$

Or it can be written as a set of simultaneous equations in conventional algebraic form:

$$\begin{aligned}40&=6X_1+4X_2+2X_3\\50&=4X_1+9X_2+3X_3\\30&=2X_1+3X_2+5X_3\end{aligned}\qquad(1)$$

This is an ordinary set of linear simultaneous equations for which unique numbers can be found for X_1, X_2, and X_3 if certain conditions hold.[3]

[3] We usually say that these equations have a unique solution if the matrix is nonsingular, which means that no row or column is some multiple of some other row(s) or column(s). Also, if a matrix has an inverse it is nonsingular. The A matrix has an inverse. Hence, it is nonsingular. But, if a matrix is singular and the method outlined here is used to solve a set of equations or invert a matrix, at some stage in the computations a row of zeros will appear.

The inverse can be used to get these X values since:

$$AB=BA=I$$

Where A is the original matrix and B the inverse (sometimes designated A^{-1}) of that matrix,

$$\text{then } By=BA\bar{x}=\bar{x}$$
$$BA=I \text{ and } I\bar{x}=\bar{x}$$

In terms of our example, the coefficients in the inverted matrix (B) (Exhibit D) times $\bar{y}$ are equal to $\bar{x}$, the values of X as follows:

EXHIBIT E

B			$\bar{y}$	$\bar{x}$
.24323	−.09460	−.04054	40	3.78300
−.09459	.17568	−.06757	50	=2.97330
−.04054	−.06757	.25676	30	2.70270

Accordingly, 40 (.24323)+50 (−.09460)+30 (−.04054)=3.78300, and so forth.

The Inverse in Input-Output Analysis

This example is also useful to show how the inverse is used in Leontief's input-output analysis. Without going into detail as to how a Leontief matrix is assembled, let us say only that it represents certain relationships within the economy that tie gross output ($\bar{y}$) to net output ($\bar{x}$). Now assume that our B matrix in Exhibit E is such an input-output matrix and the A matrix is its inverse. Accordingly, for any level of net output $\bar{x}$, we can find the corresponding level of gross output needed. If vector $\bar{x}$=3.78300, 2.97330, 2.70270, and represents the level of net output required of goods A, B, and C respectively, then the required gross outputs of A, B, and C would be 40, 50, and 30. For example, 40=6(3.78300)+4(2.97330)+2(2.70270) and so forth. In matrix notation, this operation is:

$$\bar{y}=A\bar{x}$$

which looks like the above simultaneous equation problem. But it differs in this respect. For the simultaneous equation problem, the $\bar{y}$ vector and the A matrix are known and we want to find a consistent $\bar{x}$ vector. For the input-output problem, the A matrix is known, the $\bar{x}$ vector is assumed and we want to find a consistent $\bar{y}$ vector.

Other Solutions to Simultaneous Equations

We do not need the inverse in order to solve simultaneous equations, as most readers know. The inverse was used above only to show how it can be used if it is available. Suppose we only want the solution to three equations such as:

EXHIBIT F

P_0	P_1	P_2	P_3
$40=6X_1+4X_2+2X_3$			
$50=4X_1+9X_2+3X_3$			
$30=2X_1+3X_2+5X_3$			

We can use the outlined procedure and work with the constants in columns P_0, P_1, P_2, and P_3 only. On carrying the computational procedure to completion, the solution is given by the final P_0 column. After performing the required steps the final matrix is:

EXHIBIT G

P_0	P_1	P_2	P_3
3.78300	1	0	0
2.97330	0	1	0
2.70270	0	0	1

Hence, X_1=3.78300, X_2=2.97330, and X_3=2.70270; which is the same answer obtained by using the inverse.

Regression Analysis

Let us assume that vectors P_0, P_1, P_2, and P_3 in Exhibit F represent the normal equations in a regression problem with three independent variables, X_1, X_2, and X_3 and the dependent variable Y (Exhibit H).

EXHIBIT H

$$(\Sigma X_1Y=40)=b_1(\Sigma X_1X_1=6)+b_2(\Sigma X_1X_2=4)+b_3(\Sigma X_1X_3=2)$$
$$(\Sigma X_2Y=50)=b_1(\Sigma X_1X_2=4)+b_2(\Sigma X_2X_2=9)+b_3(\Sigma X_2X_3=3)$$
$$(\Sigma X_3Y=30)=b_1(\Sigma X_1X_3=2)+b_2(\Sigma X_2X_3=3)+b_3(\Sigma X_3X_3=5)$$

Then again the values in the P_0 column of Exhibit G constitute the solution to these normal equations, i.e.:

$$3.78300=b_1,\ 2.97330=b_2,\text{ and } 2.70270=b_3$$

Also, the data required to obtain the normal equations or Exhibit H and the data in Exhibit D permit us to derive the standard regression statistics:

$$R^2 1.234=\frac{b_1\Sigma x_1y+b_2\Sigma x_2y+b_3\Sigma x_3y}{y^2}$$

$$S^2 1.234=\Sigma y^2-b_1\Sigma x_1y+b_2\Sigma x_2y+b_3\Sigma x_3y$$

$$S_{b_1}=\sqrt{\frac{S^2 1.234}{c_{11}}}$$

$$S_{b_2}=\sqrt{\frac{S^2 1.234}{c_{22}}}$$

etc.

where $c_{11}=.24323$

and $c_{22}=.17568$

Multiple regression problems can be solved then by these steps:

1. Use the formula

$$\Sigma i x_{1j}x_{jk}=\Sigma i X_{1j}X_{jk}-\frac{\Sigma i X_{1j}\Sigma i X_{jk}}{n}$$

to get the normal equations, where X_{1i} stands for Y.

2. Write these down in matrix form with the identity matrix along side, just as was done in Exhibit A.

3. Perform standard steps to reduce X matrix to an identity.

4. Use this final P_0 column to specify regression equations, i.e.:

$$Y=a+3.78300X_1+2.97330X_2+2.70270X_3$$

(Note: We could have assumed some arbitrary numbers for the means and Σy^2 then computed the constant ($a=y-\Sigma ibx_i$) and standard error of the b's in the above equation. However, the purpose of this section is only to relate the computational method to regression analysis, not to give a complete explanation and interpretation.[4])

[4] See for example Anderson and Bancroft (*1*), chapters 13, 14, and 15.

In using this method to solve regression problems, as in other similar methods, it may be prudent to add a row sum or check column to the right of the identity matrix. If the same operations are performed on this column as are done on all the other columns, at any stage in the computations, the sum of all other elements in a row should equal the value of the element in the check column, of the same row. The completed computations can be checked by multiplying the original matrix by the inverse to check that AB=I:

$$a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31}+\ldots+a_{1n}b_{n1}=1$$

$$a_{11}b_{12}+a_{12}b_{22}+a_{13}b_{32}+\ldots+a_{1n}b_{n2}=0$$

Etc.

In making such checks, it may be found that the sums of the inner products do not equal 0 or 1, but are very near these values. Such discrepancies may be due to the number of decimal places carried in the computations. For most regression problems no more than eight decimal places need be carried. On small problems five or six places may be adequate. However, if there is a high degree of correlation between the independent variables more decimal places may be needed to prevent degeneracy (division by zero).

Linear Programming

With a few additional rules or steps, the procedure outlined in Section III can be used to solve linear programming problems. For programming problems the procedure is usually called the simplex method.

Looking at Exhibit H, let us assume that elements e_{01}, e_{02}, and e_{03} in the P_0 vector represent resources available to a particular firm, for example, 40=hours of labor, 50=hours of machine A time available, and 30=hours of machine B time available. Let the vectors P_1, P_2, and P_3 represent the quantities of each of these resources needed to produce one unit of products X, Y, and Z respectively. In linear programming each resource row must have an identity vector associated with it. Hence we need vectors P_4, P_5, and P_6 of Exhibit H. These vectors can appear in any position in the matrix; first, last or in the middle. And because in linear programming problems the numbers of columns does not need to equal the number of rows, let us add columns P_7, P_8, and P_9. Al-

EXHIBIT I

C_j →			9	4	7	0	0	0	3	8	5	
↓	Solution vector	P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	R
0	P_4	40	6	4	2	1	0	0	2	4	2	6.7
0	P_5	50	4	9	3	0	1	0	4	5	3	12.5
0	P_6	30	2	3	5	0	0	1	3	3	2	15.0
	Z_j	0	0	0	0	0	0	0	0	0	0	
	$Z_j - C_j$	0	−9	−4	−7	0	0	0	−3	−8	−5	

though we do not need to, we can assume that these vectors represent alternative ways of producing commodities X, Y, and Z.

Further, in order to have a linear programming problem, we need a profit row. This is usually written above the basic matrix and called the C_j row. We also need other rows, usually called the Z_j and the $Z_j - C_j$. The latter is, of course, the Z_j row minus the C_j row as in Exhibit I.

The Z_j row is computed by multiplying the C_j values of the basis (identity) vectors, i.e., P_4, P_5, and P_6, by each of the vectors P_0 through P_9. For example, Z_j for P_0 is $0(40)+0(50)+0(30)=0$ and Z_j for P_1 is $0(6)+0(4)+0(2)=0$. Because in this case all C_j of the basis vectors are zero, all Z_j values are zero. In numerous programming problems, however, especially minimizing problems, the C_j values of the initial basis are non-zeros. Once the Z_j row and then the $Z_j - C_j$ row have been computed, the Z_j can be omitted from subsequent computations.

Exhibit I is the standard format for linear programming problems. The computational procedure outlined in the first part of this paper can be used to obtain the solution. The objective here is different, however. In words, it is: To find some non-negative levels of the P_1 to P_9 that will maximize net returns, given the resources available, i.e., the P_0 column. Also, the method of selecting rows and columns for sequential operations is different. At the outset we do not know which columns we want to convert to unit vectors and which elements we want to be unity or the number 1. Finally, we need a criterion to tell us when the answer is found. But let's take one thing at a time:

1. The operating column is the column with largest *negative* $Z_j - C_j$ element. In Exhibit I this is P_1, since its $Z_j - C_j$ value is −9.

2. The critical row is the row with smallest positive ratio of P_0 element to operating column element.[5] For example, in Exhibit I, given P_1 as operating column, 40/6=6.7, 50/4=12.5, 30/2=15.0. Hence, row 1 is the critical row.

3. The optimal solution is obtained when all values in $Z_j - C_j$ row are non-negative (i.e., zero or positive). The optimal solution (maximum or minimum) is given by the values in the P_0 column.

The optimal profit solution to Exhibit I is derived by first converting column P_1 to a unit vector, with the number 1 in row one, the critical row. When this step is completed column P_9 or activity P_9 has the largest negative $Z_j - C_j$ value. It, therefore, is the operating column for the next step and row 3 is the critical row. After P_9 has been converted to a unit vector with the number 1 in row 3, all $Z_j - C_j$ values are non-negative, indicating the solution is optimal. These steps are not shown, to save space. The final matrix, after the described steps are completed, is given in Exhibit J.

The solution is as given by the P_0 column: 2.5 units of P_1 (product X), 12.5 units of P_9 (product Z), and 2.5 units of time on machine A unused or left idle. The profit is given in the pivotal element of the P_0 column and the $Z_j - C_j$ row.

Several checks are available to verify the optimal solution. One is the feasibility check which simply checks that resources are available to meet the specified levels of output. This can be checked by matrix multiplication, using the original vectors P_1, P_5, and P_9. We multiply

[5] In some problems this ratio may be zero. Computations can continue even though the ratio is zero. Also, the ratio for two rows may be the same. The selection of either row is permissible in this case.

EXHIBIT J

C_j ⟶			9	4	7	0	0	0	3	8	5
↓	Solution vector	P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9
9	P_1	2.5	1	.25	−.75	.25	0	−.25	−.25	.25	0
0	P_5	2.5	0	4.25	−3.75	−.25	1	−1.25	−.25	.25	0
5	P_9	12.5	0	1.25	3.25	−.25	0	.75	1.75	1.25	1
	$Z_j - C_j$	[1]85.0	0	4.50	2.50	1.00	0	1.50	3.50	.50	0

[1] Net profit.

these by the final P_0 vector and check to see that the product is equal to the original P_0 vector:

$$\begin{bmatrix} 6 & 0 & 2 \\ 4 & 1 & 3 \\ 2 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2.5 \\ 2.5 \\ 12.5 \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} 40 \\ 50 \\ 30 \end{bmatrix}$$

Multiplying each row of the left-hand matrix sequentially by the right-hand column or solution vector we see that the solution is feasible.

The net profit in the final matrix can be checked simply by multiplying the solution vector by the associated C_j value:

$$9(2.5)+0(2.5)+5(12.5) \stackrel{?}{=} 85$$

Another check is to multiply the original resource levels by the values in the $Z_j - C_j$ row of the final matrix columns P_4, P_5, and P_6, the original identity vectors.

$$1.0(40)+0(50)+1.5(30) \stackrel{?}{=} 85$$

Checking we see that the equality is satisfied.

The final $Z_j - C_j$ values associated with the original identity vectors are the shadow prices of the resources.

Summary

This paper has shown how a relatively simple computational technique can be used to solve several types of problems. Simple matrix algebra principles are stated. Then these principles are used to outline a uniform computational method that is easily memorized. With this method firmly in mind, the student can move easily from one type of problem to another without going to reference books for computational formulas that many times are difficult to follow.

Only hypothetical data are used in the examples presented. The basic theories of input-output analyses, multiple regression, and linear programming were not discussed beyond an attempt to show some of their similarities and dissimilarities. We did not discuss how data are collected and manipulated to build up the several matrices that are needed before computations can begin. Many references are available for those who need information on these subjects. For example, see (*1*), (*3*), (*5*), and (*6*).

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Book Reviews

Main Currents in Modern Economics; Economic Thought Since 1870

By Ben B. Seligman. The Free Press of Glencoe, New York. 887 pages. $11.75.

A BRIEF REVIEW cannot do justice to this scholarly, stimulating, and comprehensive volume. Essentially, it consists of some 70 chapters on most of the important or influential economists from Gustav Schmoller to Joseph Schumpeter and Paul Samuelson.

These chapters are grouped under three broad rubrics. Part I, "The Revolt Against Tradition," covers the Historical School, the Socialists and the Institutionalists. The latter group includes not only such stalwarts as Thorstein Veblen and John R. Commons, but also Gardner Means and J. K. Galbraith. Part II, "The Reaffirmation of Tradition," embraces various marginalists and "libertarians" including W. S. Jevons, J. B. Clark, and Friedrich Von Hayek. It also covers a number of equilibrium economists with mathematical propensities, such as Leon Walras, J. R. Hicks and Wassily Leontieff; plus a separate British group ranging from Phillip Wicksteed and Alfred Marshall to Sir Dennis Robertson and Lord Robbins. Part III, "The Thrust Toward Technique," devotes one section to the Swedish economists; a second to assorted Americans, including Irving Fisher, Frank Knight, and Milton Friedman; and a third to a loosely defined group that includes Joan Robinson, E. H. Chamberlin, J. A. Schumpeter, and Lord Keynes.

Seligman's volume is a valuable addition to the literature of economic thought. Among the particularly useful contributions are the chapters on the American Institutionalists and the Swedish School. The treatments of Commons and Wicksell seem especially good. Another feature is the inclusion of a fair number of contemporary economists such as Galbraith, Hansen, Samuelson, Leontieff, Friedman, Boulding, Myrdal, and Hicks. The fact that most of the chapters are more or less self-contained essays enables the reader to browse around among the economists in whom he is especially interested. On the debit side is a considerable amount of overlapping among some of the chapters.

The thing that provides unity and spice to Seligman's effort is the fact that he writes from a very definite point of view. Although he appreciates rigorous analysis, including the mathematical variety, he believes that the chief characteristic of modern economic thought "has been the thrust toward technique for its own sake," and that this characteristic has rendered economics progressively less useful in solving the social problems which are the roots of its existence. He believes that the great need is for economics to focus itself on significant problems, whether the method employed be "verbal, mathematical, statistical, historical, econometric."

Though I think that Seligman's point of view is basically valid, he does exhibit a tendency, like the supporter of any thesis, to overdo the matter. For example, in pointing out the shortcomings in the equilibrium system of Walras, he seems to forget that the demonstration of how the price-quantity variables in the economic system can be considered as an interrelated whole was by itself a monumental contribution to economics. Also, it seems to me that he has gone altogether too far in downgrading the contribution of Keynes by saying, for example, that "the human and the political did not penetrate the economic in Keynes' thinking." Joan Robinson in her recent book, *Economic Philosophy* (reviewed below), has a much better appraisal.

On balance, however, the merits of this volume overshadow its defects. Whether the 2,997 footnotes assembled in the last 79 pages fall in the first or second category will depend upon the tastes of the individual reader.

J. P. Cavin

Economic Philosophy

By Joan Robinson. Aldine Publishing Company, Chicago. 147 pages. $3.50.

MUCH OF THE BODY of doctrine and the ideas on which economics is based has always been ". . . partly a vehicle for the ruling ideology of each period as well as partly a method of scientific investigation." This quotation is from

Economic Philosophy, an iconoclastic appraisal of some basic economic principles and institutions which borders on economic heresy. Nevertheless, it is an excellent essay that economists, policy makers, and others will find interesting, provocative, and worthwhile.

The author lays waste many cherished ideas, principles, slogans, and beliefs about economics. Though the essay may be considered negative and destructive, its tone is probably an effort to emphasize the need ". . . to clear the decaying remnants of obsolete metaphysics out of the way before we can go forward." The critique devastates many fundamentals on which classical economics is based, yet one doubts that it will come as a great shock to many teachers and practicing economists.

A number of observations are made on present-day economic thinking and policy relating to unemployment, public vs. private investment, and the effectiveness of economic development under socialistic institutions. Many of these views are not shared by other economists and political philosophers, particularly in North America. However, Mrs. Robinson concludes that our patched-up economic system is probably the best in sight. There is no "better 'ole" in which to go. With respect to economics, she remarks that we must not ". . . abandon the hope that economics can make an advance towards a science . . . ." And the essay ends with an appeal: We must combat ". . . the ideology which pretends that values which can be measured in terms of money are the only ones that ought to count."

What prompts a distinguished economic theorist to say that economics ". . . limps along with one foot in untested hypotheses and the other in untestable slogans"? Possibly the thesis of the author's critique is a product of the times and merely a thread in the continuity of economic ideas. She seems to arrive at many of the conclusions reached by Veblen in his criticism of classical doctrine after the turn of the century.

The chapter on classical and neoclassical economic thought presents an interesting historical interpretation of important ideas of the time, including those of Adam Smith, Mandeville, Ricardo, Marx, Marshall, and Pigou. The author traces, for example, such concepts as the labor theory of value and the dilemma it presents for property ownership and profits, utility, laissez-faire, free trade, the attempts to develop a scientific underpinning for Marxism, and the Keynesian revolution.

The concept of utility and marginal utility in value theory swept away the ". . . labor theory, with its disagreeable smell . . . ." The whole point of the marginal analysis was to justify the doctrine of laissez-faire. Producers equate marginal costs to prices, and maximum possible satisfaction from available resources follows automatically. To the author, this is the ". . . ideology to end ideologies, for it has abolished the moral problem." The laissez-faire school strongly advocated free trade. In fact, this became the hallmark of an economist; protectionists were a lesser breed. The logical basis for free trade was the same as the case for unrestricted freedom and the pursuit of profit, and all countries would stand to gain. But the point was that ". . . Great Britain had everything to gain from other nations' adopting Free Trade . . . . Marshall, the old fox, had known perfectly well that it was all a case of national self-interest."

Mrs. Robinson seems rather well pleased with Keynes' general theory and its attempts to patch up capitalistic doctrine and make it work. The general theory demolished some cherished doctrines which nurtured capitalism. But it sparked the development of national income accounting, encouraged attempts to fill those "empty economic boxes," and was concerned with the big picture. Accordingly, the analytical framework yielded recommendations on public policy in the area of taxation, fiscal and monetary policy, and other programs designed to maintain full employment. Although Keynes' general theory has become a classic, the author felt that the Keynesian revolution developed its own brand of metaphysics. Thus, we continue in the ". . . uncomfortable situation of having to think for ourselves."

Following World War II, a new question came to the fore—long-run economic development. Classical economists had been only incidentally interested in economic development. Keynes referred to the long period as ". . . a subject for the undergraduates." The author traces the fragmentary ideas of the classical economists on long-run economic growth as well as the more recent work by Harrod, Domar, and others. But she concludes that, in general, ". . . there is nothing very much for economic theory to say to the planner . . . ."

The final chapter of the book poses the question, "What are the rules of the game?" According to the author, the very nature of economics is rooted in nationalism. This is the ". . . one solid unchanging lump of ideology that we take so much for granted that it is rarely noticed." Behind this facade, according to the author, capitalistic nations have boosted trade and production, conquered territories, and adapted institutions to the advantage of their citizens. Nationalism continues a major world force influencing economic policy and economic institutions.

Rex F. Daly

Price Theory, A Provisional Text

By Milton Friedman. Aldine Publishing Company, Chicago. 285 pages. 1962.

VALUE AND DISTRIBUTION theories, treated separately by early economists, are merged into a unified theory in this provisional text. The theory of value concerned itself with prices of final products; that of distribution with prices of productive factors. General equilibrium theory merges these inquiries into a simultaneous determination of both sets of prices.

The view taken by Milton Friedman of equilibrium is not the one of microeconomics which seeks to explain behavior in individual consumers or firms. Nor is it the view of macroeconomics which seeks to explain general levels in an economy of business activity, employment, income, and money. Rather, Professor Friedman usually takes an intermediate view of an industry composed of many firms but operating in an economy of many industries. Most of the treatment concerns timeless equilibrium under perfect knowledge, but notable exceptions include discussions of the role of risk in utility theory and in size distribution of income and a discussion of time in relating income flows to resource stocks.

From his industry view, Professor Friedman develops the theory of consumer demand and of product supplies. His conception of demand and supply functions as boundaries of feasible areas is especially revealing in price theory. From the final product markets, factor demands are derived to be matched against factor supply curves leading to theories of distribution among resources as well as distribution among individuals. A final chapter relates flows of equilibrium analysis to the stocks of resources listed on the balance sheet of the industry and leads to theories of saving and investment and of equilibrium stocks of capital.

This price theory paperback will prove useful to those engaged in agricultural economics research. The treatment of the longrun supply of labor includes an interesting analysis of the net migration to the city from the country in terms of the comparative advantage of rural areas in the production of human capital. In the discussion of the factors of production are two schemes for classification of resources: One is according to the demand interests in the factor market and is in terms of substitutability of resources in production. The other is in terms of factor supply functions.

In the discussion of Euler's theorem, the author explains that it may be well to consider homogeneous production functions when taking the broad view. On the other hand, it is better to take the subproduction function approach, with diminishing returns to all factors under immediate control of the entrepreneur, when considering one firm at a time. The operational definition of length of run used by Friedman is practical in agricultural economics research, because it confines itself to deciding which factors are fixed and which are variable during the production period under consideration rather than attempting to span several time periods.

The chapters on industry supply functions for products and demand functions for factors demonstrate clearly the kinds of composition rules needed to aggregate firm responses into industry relationships in a timeless world of perfect knowledge. For example, the levels at which firms would regard average revenues and average factor costs as fixed would be set by the action of the industry.

In his discussion of distributive justice, the author shows that two ethical propositions postulated in Western societies are independent of marginal productivity analysis. The two propositions are: That each man should get what he produces, and that each man deserves what is produced by the resources that he owns. In separating the ethics from the economics, Friedman indicated that Karl Marx accepted capitalistic ethics but not marginal analysis in his theory of exploitation. On the other hand, in our own society, we sometimes use marginal productivity analysis of income distribution in our welfare economics while

changing the underlying ethic. Marginal productivity analysis of the determination of the rate of returns to resources does not, of itself, have ethical implications.

The easy-to-follow illustrations developed by Professor Friedman to clarify his text usually serve his purpose well, as when he uses a linear programming framework to develop the salient features of functional income distribution. Occasionally the illustrations are so limited as to be misleading, especially to graduate students treading the ground for the first time.

For example, in applying the law of variable proportions to the production function of a firm, the assertion is made that "the region of increasing average returns to one factor coincides with negative marginal returns to the other factor." (Page 130.) This may be valid for the simple illustration of two factors used in the production of one commodity with a homogeneous production function of degree one. But the example confines itself to but three of nine possible ways to combine three things two at a time. An example with three or more variables in a homogeneous production function would have led to a more complete explication of the law of variable proportions. For a pair of such variables, if one is in a stage of increasing average returns, decreasing average returns and positive marginal returns, or negative marginal returns, it is entirely possible that the other may be in any of the three stages of production.

In the preface, this publication is said to be more nearly a set of "scrappy class notes" than a bona fide textbook. Perhaps this explains why several errors creep into such an important contribution to price theory literature. Some may be there simply to see if the reader is awake, such as the pronouncement that society consists of four sectors, which is followed by a list of only three. Other minor difficulties include the assertion that determining the least-cost way of building a steam engine (or of reaching any single end with limited resources) is *not* an economic problem (page 6); that the decision to build a steam engine of maximum efficiency does not involve a value judgment (page 6); and elasticity is (loosely) slope (page 19).

This long-awaited, though provisional, price theory paperback is likely to prove durable. Another is not likely to come along unless Milton Friedman writes it. According to the publisher, Friedman's book has already been adopted as a text in several universities. Students across the country will benefit from the discussions, the problems, and the reading lists. But this book is also an opportunity for readers other than university students to benefit from Friedman's insights into price theory.

Clark Edwards

Agricultural Policy Under Economic Development

By Earl O. Heady. Iowa State University Press, Ames. 682 pages. 1962. $10.50.

EARL HEADY has made a major contribution by thoroughly reviewing U.S. agricultural policy against the long-run economic development trends of the agricultural sector. In pursuing a major thesis, he clearly demonstrates that some U.S. agricultural programs have resulted in less than a "Pareto-better" position because they have attempted short-run solutions to long-term problems.

The title is somewhat misleading as "the main focus of the analysis is on agriculture in the highly developed economy of the United States. However, the analysis of the basic interrelationships of agriculture and the total economy under economic growth is relevant regardless of the stage of development." A major theme of the work is the search for policy which will answer the question, "How can agriculture continue to contribute to national economic growth and consumer welfare without being penalized in income for doing so?"

After considering some of the underlying changes which have occurred, and are expected, in the demand and supply picture for agriculture, Heady turns to consider the trends in the performance of the agricultural sector compared with the performance of the rest of the economy. He examines the competitive structure of agriculture which, coupled with inelastic demand for agricultural products, results in great gains for the consumer but not for agriculture. The author deepens the analyses through consideration of the supply of resources used in agriculture and how the inflexibility of labor, land, and capital lead to inelastic supply responses. Study of expenditure patterns and demand potential demonstrates that in the United States there is little possibility in the next few decades that demand increases alone

will be sufficient to provide equitable returns to those in the agricultural sector.

Essential chapters follow which develop the criteria for policy and consider Games, Goals, and Political Processes before a synthesizing chapter presenting the Modern Need in Development and Policy. Heady concludes that "efficient policy would use such a time goal (a decade or two) hoping to bring agriculture into rough resource balance by this time but averting regional or area change at rates which are inconsistent with abilities of people and communities to adjust." The economic nature of different compensation and supply control policies is then examined. The following four chapters discuss ways policy may effect labor mobility, particularly through education, and ways it may effect the use of inputs, such as capital and land. Public policy on research and development is considered separately. The final chapter places the problems of U.S. agriculture in the perspective of world development. Particular emphasis is given to U.S. food and other aid programs in relation to the two economically separate questions of U.S. surplus disposal and development aid for low income countries.

A strength of the book is that the argument is pursued on two levels throughout, one by use of simple equations and the other through verbal means, thus permitting the reader to proceed at the level he prefers.

While feeling that the content of the work is of major value, this reviewer noted some shortcomings. From the point of view of economic development theory, it is disappointing to find that most of the mathematical formulations lack the specific time reference of dynamic systems. Turning to matters of style, the book could have been made less cumbersome by tighter discussions and a reduction in some of the illustrative material (e.g., chapter six on the structure of U.S. agriculture.) Unfortunately, also, the usefulness of the many excellent references is reduced because they are not cited in the customary way. Lack of references (p. 44, line 13, and Table 2.3, for example) occasionally mars the work. An augmented table of contents and a list of tables would have considerably increased the usefulness of the book, particularly for reference purposes.

This major review of United States agricultural policy is likely to be of greatest value for graduate students, members of the profession, and administrators who are concerned either with policy or with agricultural development.

Robert D. Stevens

The Public Lands: Studies in the History of the Public Domain

Edited by Vernon Carstensen. The University of Wisconsin Press, Madison. 522 pages. 1963. $6.75.

PRIVATE CITIZENS and public officials alike have a continuing interest in the history, disposal, extent, character, and use made of the publicly owned and administered lands. *The Public Lands: Studies in the History of the Public Domain* contains about 60 of the best historical articles on the public lands that have appeared in journal form since 1905. These articles provide a revealing review of significant plans and events in the disposal of public land to private citizens and corporations, in grants to railroads and States, and in the management of the remainder for public purposes.

May 20, 1962, marked the centennial of one of the most important land laws of the United States—The Homestead Act of 1862. The articles in *The Public Lands* impress the reader with the endless public actions since 1784, first, to encourage settlement and development of the public domain, and more recently, to provide for proper management of these lands. Not all the public plans to provide farms for the vast multitude of home seekers were successful. Yet, looking back today through the significant articles in *The Public Lands* there appear to have been enough honest successes over the two centuries of history to more than offset the failures and injustices. The public land disposal system illustrates the unchanging verities of good and bad in human behavior.

As expressed by Dr. Carstensen in the Introduction, "The land grabs . . . represent a . . . part of the story, but not the whole story. . . . The part . . . that involves the vast number of landseekers who got their land without violating either the spirit or the letter of the law . . . is the part that provided a lure so strong that it drew millions of people across the Atlantic."

In like manner, Dean Eugene Davenport of the College of Agriculture, University of Illinois, in 1915, discussed the distribution of the public domain, recognized the waste and abuse, and then added, "but we have these farms, these cities, these

railroads, and this civilization to show for it, and they are worth what they cost."

On the credit side of the public land ledger, some 30 States were carved out of the public domain, settled and developed by people from the older States and from other countries. The farm settlers brought with them generally wholesome ideas of citizenship and government, and practical and thrifty ways of farming and rural development. A fact often overlooked is that many settlers had good health, ability to work, and a little money or other property with which to start farming or stock raising. The predominantly family farm communities of the Piedmont and Mountain Southeast sent thousands of settlers to the Midwestern and Western Territories and new States. Many other thousands went from the more compact Northeastern towns and counties.

As to the debit side of the ledger, one is led by the articles in *The Public Lands* to ask why the many thousands of landless poor people of the South and East were not given a public helping hand to move west from 1850 to 1900. With this help they might have achieved home ownership in one of the new States less bound by ways of the past.

In 1862, well over a billion acres of public land, or over half the Mainland 48 States, was available for settlement. This was enough to share with the landless of the South and East, had they had the means to reach it. In retrospect, not only was there a big gap in actual efforts to settle our own landless people in the Civil War and post-Civil War years after 1962, there is a blank spot in the follow-up public actions to carry out the few proposals that were made at the time. The loss of several hundred thousand of the country's most active men and the other staggering costs of the Civil War and its aftermath evidently crowded out many things which might have been done for the general welfare.

In contrast, to the limited aid for landless poor, without property or a grubstake, was the opportunity for the man with a horse, a team and wagon, or a few dollars, who could go West by his own efforts and claim a piece of land. Public encouragement was given to advertisements to attract home seekers from other countries. Also, war veterans, beginning in Revolutionary War days, were rightly rewarded by land or redeemable claims for land and preferences in homesteading. The veterans and other settlers, who were able to travel and to live by their savings for a year or two, brought much to the new States by citizenship, thrift, and hard work.

While it may be fruitless to daydream of what might have been, it is worthwhile to appraise the past for what may be gleaned in solving present problems of equal home and job opportunities for all citizens.

Information apparently is not readily available on public efforts to aid the landless of the South and East to obtain free land in the West during the heyday of the Homestead period, 1865 to 1920. Limited searches indicate it would be useful today to review the original public land records, and the efforts to aid the landless people of the older States at a time when the Nation had ample land to give away.

The Public Lands should stimulate more original research to fill gaps in the history of public domain disposal and settlement. The articles show that too many publications have accepted and repeated generalized and often incomplete, inaccurate, or overdrawn versions of homesteading, land sales, and grants to railroads and States. *The Public Lands* will prove useful to the student and research worker, and to all who are concerned with land ownership, use, management, and land affairs generally, whether owners, farmers, professional people, business men, or public officials.

Hugh H. Wooten

Status and Methods of Research in Economic and Agronomic Aspects of Fertilizer Response and Use

By the Committee on Economics of Fertilizer Use of the Agricultural Board. Publication 918, National Academy of Sciences—National Research Council, 2101 Constitution Avenue, Washington, D.C. 89 pages. $2.00.

READERS INTERESTED in a summarization of the status of research on yield response to fertilizer will find this publication useful. Its scope includes discussion of the pertinent concepts and principles, the characteristics of different types of yield functions, and problems in obtaining suitable data. There is also a section on application of results. The importance of joint agronomic-economic effort is indicated.

The difference between basic laboratory research and research under field conditions is mentioned.

But emphasis is given to the use of yield functions for a refined analysis of field results. The inference is that precision in materials and tools of analysis may be comparable to that needed to quantify basic relationships studied under thoroughly controlled conditions. To many this will seem to give undue emphasis to the notion of finding a particular function that will reflect *the true* yield response relationship for field conditions.

Such a refinement may have little bearing on the role of fertilizer in the total context of striving for optimum use of resources on farms or in agriculture generally. The relevant range of response, when viewing either micro (firm) or macro problems, is that in which increments in yield decrease with additional applications, under stated conditions. Whether the rate of decreasing increments for a particular series of responses is constant or variable, or whether the function will reflect a "turn down" in absolute yield, somehow does not seem important for developing practical guides to profitable use of fertilizer. Neither do refined measurements contribute anything of substantial importance in the area of increasing increments where it is obvious that if fertilizer pays at all, increased applications will pay more.

Except as yields are limited by other factors, evidence of decreases in yield resulting from applying "too much" is hard to find. Where these other yield limiting factors cannot readily be removed, the important practical question still remains one of characterizing the range of decreasing increments under these restrictive conditions. Asymptotic functions derived from measuring the main effects of two or three independent variables have in some instances fitted the entire reported production surface as well as functions that allow for a turn down in response. This conclusion can be made more general with reference to the relevant area of the production surface. (Asymptotic functions are represented by curves that do not turn down, but approach a horizontal plane without reaching it.)

One important omission relates to the problem of obtaining a least squares fit when using asymptotic regression. H. D. Patterson writing in *Biometrics*, September 1956, describes a simple, direct method for obtaining almost fully efficient estimates of parameters through a system of weighting the incremental yields. This has been found superior to graphic methods, and less time consuming, when the latter are used with sufficient care to approximate a least squares fit. Patterson's methods represents a clear gain in methodology for those using this type of function.

One wonders whether more about the role of fertilizer in the whole picture of farm technology could not be learned by using simpler but larger scale field trials. In these, controllable factors other than the ones being tested could be set at levels that would not seriously limit response to the variable factors. This would focus attention on the main area of farmer operations and leave the more precise measurements (which are obscured in the field) to the basic researchers. Some information about the "fringes" of the production surface would be sacrificed in the interest of more knowledge of wider applicability in the area of profitable operations. The report might well have differentiated more clearly between the objectives and needs of basic research and those of supplying guides more readily applicable to farms and to major areas of production.

These comments are not intended to discount the value of the publication, which is a contribution as a documentation of the past, and a stimulant to future work. Although much of it is directed to mathematically minded readers, there are some sections that appeal to a wider audience. These sections include discussions on history of crop response research, application of findings, and some of the discussion of basic concepts in yield response to fertilizer. A list of 214 references forms a useful bibliography. The appendix lists land-grant colleges reporting agronomic-economic research in 1954 and in 1957, together with industry-sponsored research under way in 1957.

D. B. Ibach

Marketing Farm Products

By Geoffrey S. Shepherd. Iowa State University Press, Ames. 523 pages. 1962. 4th ed. $7.50.

PROFESSOR SHEPHERD directs his *Marketing Farm Products* toward undergraduate and beginning graduate students who are interested in economic analysis and agricultural marketing. As in previous editions, his fourth edition is functionally oriented around the economic elements of marketing. The development of the system, its changes, and its problems are shown as reactions to economic forces. Professor Shepherd

is quite successful in leading students into economic analyses of our marketing system, and doing so in a business-like, logical way with a minimum of economic jargon and a maximum of readable, down to earth, common sense.

The fourth edition follows closely the pattern of the third. Professor Shepherd begins by examining the general economic elements of marketing problems. He examines demand, then prices and price making, and, finally, costs of marketing. Using these tools, he analyzes techniques and problems of marketing livestock and meats, dairy products, poultry and eggs, grain, cotton, and fruits and vegetables.

During the 16 or 17 years since this book was first published, Dr. Shepherd has continued to gather evidence and present it in subsequent editions. In this edition he has substantially elaborated his presentation of the concept of income elasticity of demand, seasonal price movements, and futures trading. Recent changes in the structure of particular markets are evaluated. More attention is devoted to margins, and the significance of widening margins is examined in more detail. Professor Shepherd properly contends that changes in margins do not necessarily indicate changes in marketing efficiency, and he refers briefly to measures of efficiency. These formulations are not complete and Professor Shepherd indicates that he intends to explore these ideas further.

Selective references and charts are generally brought up to date. Liberal use of publications on marketing research published by the U.S. Department of Agriculture is evident in charts and references. Professor Shepherd's continuing work in bringing his books up to date is a valuable service to both teachers and students.

William C. Motes

Selected Recent Research Publications in Agricultural Economics Issued by the United States Department of Agriculture and Cooperatively by the State Universities and Colleges [1]

BALLINGER, ROY A., AND LARKIN, L. C. SWEETENERS USED BY THE DAIRY INDUSTRY—THEIR COMPETITIVE POSITION IN THE UNITED STATES. U.S. Dept. Agr., Econ. Res. Serv., Agr. Econ. Rpt. 30, 18 pp., illus. Apr. 1963.

Manufacturers of sweetened dairy products used about 4.7 percent of the total quantity of sugar, corn sirup, and dextrose delivered to consumers in the United States in 1961. This is the second of a group of reports to deal with the use of sweeteners and their competitive position in the various food industries.

BALLINGER, ROY A., AND LARKIN, L. C. SWEETENERS USED BY THE BEVERAGE INDUSTRY—THEIR COMPETITIVE POSITION IN THE UNITED STATES. U.S. Dept. Agr., Econ. Res. Serv., Agr. Econ. Rpt. 31, 15 pp., illus. May 1963.

The beverage industry in 1961 used 13.5 percent of all sugar consumed in the United States, and a fourth of that delivered to U.S. industrial users. This is the third of a group of reports to deal with the use of sweeteners by U.S. food processing industries.

BOHALL, ROBERT W. THE ORGANIZATION OF WHOLESALE FRUIT AND VEGETABLE MARKETS IN MIAMI AND TAMPA-ST. PETERSBURG. U.S. Dept. Agr., Mktg. Res. Rpt. 593, 41 pp., illus. Apr. 1963.

The Miami and Tampa-St. Petersburg produce markets are the main source of fresh fruits and vegetables for 3.5 million people from Key West 400 miles north to Orlando. Receipts of fresh fruits and vegetables were 30,000 carlots for Miami and 18,900 carlots for Tampa-St. Petersburg in 1958. This is the tenth in a group of reports under the general title, "The Organization of Wholesale Fruit and Vegetable Markets."

BROWN, LESTER R. AGRICULTURAL DIVERSIFICATION AND ECONOMIC DEVELOPMENT IN THAILAND: A CASE STUDY. U.S. Dept. Agr., Econ. Res. Serv., For. Agr. Econ. Rpt. 8, 34 pp., illus. March 1963.

Thailand formulated a 6-year plan in late 1961. But despite lack of previous planning, farm income, at constant prices, had climbed 60 percent during the past 6 years, and total exports, almost entirely agricultural, rose at a comparable rate. Thailand has traditionally been a rice monoculture, but during the last decade there has been a rapid growth in output of other crops, particularly corn, cassava, and kenaf.

COWHIG, JAMES D. EDUCATION, SKILL LEVEL, AND EARNINGS OF THE HIRED FARM WORKING FORCE OF 1961. U.S. Dept. Agr., Agr. Econ. Rpt. 26, 21 pp. March 1963.

Attention is focused on the relationships between kinds of farm and nonfarm jobs and educational attainment. In December 1961, 73 percent of all adult farm workers (those 25 years old and over) had no more than a grade school education (8 or fewer years of school completed). Only 1 out of 7 was a high school graduate.

COYNER, MARY S. AGRICULTURE AND TRADE OF HONDURAS. U.S. Dept. Agr., Econ. Res. Serv., ERS-Foreign 33, 23 pp., illus. May 1962.

[1] State publications may be obtained from the issuing agencies of the respective States.

Describes steps taken by the government of Honduras to expand the country's agricultural production within the next few years.

DWOSKIN, PHILIP B., HESTER, O. C., KERR, HOWARD W., JR., AND BAYTON, JAMES A. MARKET TEST OF INSTANT SWEETPOTATOES IN SELECTED INSTITUTIONAL OUTLETS. U.S. Dept. Agr., Agr. Res. Serv., Mktg. Res. Rpt. 580, 41 pp. Jan. 1963.

Study conducted in Cleveland and New Orleans indicated highly favorable reaction to instant sweetpotato flakes by chefs, kitchen help, and restaurant operators. When offered in the menu, 20 to 25 percent of the customers ordered them. Acceptance was favorable.

EDMAN, VICTOR G. PRICES AND MARKETING MARGINS FOR WASHINGTON DELICIOUS APPLES SOLD IN CHICAGO AND NEW YORK CITY, 1956–61. U.S. Dept. Agr., Mktg. Res. Rpt. 586, 10 pp., illus. Feb. 1963.

From 1956 to 1961 Delicious apples accounted for 22 percent of the total apple crop. The purposes of the study are to show the extent and nature of Washington Delicious apple price variations and to indicate the size of the total marketing margin for Washington Delicious apples. Retail prices averaged higher in Chicago than in New York City in 4 of the 5 seasons studied, but auction prices were higher in New York City during 4 of the 5 seasons.

FIENUP, DARRELL F., MOTES, WILLIAM C., HIEMSTRA, STEPHEN J., AND LAUBIS, ROBERT L. ECONOMIC EFFECTS OF U.S. GRADES FOR LAMB. U.S. Dept. Agr., Agr. Econ. Rpt. 25, 67 pp., illus. Feb. 1963.

Price analysis suggests that factors other than grading were responsible for the 1958–61 decline in lamb prices. Lamb prices are affected primarily by changes in the supply of lamb and beef. Federal grades promote competition and facilitate trade and they may lower total marketing costs.

FRIEND, REED E., AND BAUM, SAMUEL. ECONOMIC, SOCIAL, AND DEMOGRAPHIC CHARACTERISTICS OF SPANISH-AMERICAN WAGE WORKERS ON U.S. FARMS. U.S. Dept. Agr., Econ. Res. Serv., Agr. Econ. Rpt. 27, 21 pp. March 1963.

Approximately 40 percent of the 261,000 Spanish-American farm wage workers in 1960 were migratory. About 67 percent of the Spanish-American wage workers were born in the U.S., 23 percent in Mexico, 6 percent in Puerto Rico, 4 percent elsewhere. Median years of school of these wage workers were 7, compared with 8 for other workers.

HAVAS, NICK, AND FRYE, ROBERT E. PILOT FOOD STAMP PROGRAM—ITS EFFECT ON RETAIL FOOD STORE SALES IN FAYETTE COUNTY, PA., AND MC DOWELL COUNTY, W. VA. U.S. Dept. Agr., Econ. Res. Serv., Agr. Econ. Rpt. 29, 10 pp., illus. Apr. 1963.

Part of an overall research effort to evaluate effects of the Food Stamp Program on food consumption, nutritional intake of participants, food retailing, and farm income. Dollar sales in sample stores averaged 7 percent higher during a 4-week period in April–May 1962 than in the same period a year earlier, before the program was started.

HILL, ELTON B., AND HARRIS, MARSHALL. FAMILY FARM-OPERATING AGREEMENTS. Mich. Agr. Expt. Sta. Circular Bul. 234 (N. Cent. Reg. Res. Pub. 143), 42 pp., illus. (Econ. Res. Serv., Fedl. Ext. Serv., and Farm Foundation cooperating.)

Companion bulletin to "Family Farm Transfers and Some Tax Considerations," this publication is a revision of "Family Farm-Operating Agreements," published in 1951.

HILL, ELTON B., AND HARRIS, MARSHALL. FAMILY FARM TRANSFERS AND SOME TAX CONSIDERATIONS. Mich. Agr. Expt. Sta. Special Bul. 436 (N. Cent. Reg. Res. Pub. 127), 48 pp., illus. (Econ. Res. Serv., Fedl. Ext. Serv., and Farm Foundation cooperating.)

This publication is a complete revision and expansion of "Family Farm-Transfer Arrangements," published in 1951 as Illinois Circular 680. Included in the current publication is information on State and Federal taxes that may be involved in intra-family farm transfers and estate planning.

HODGES, EARL F. LIVESTOCK-PRODUCTION UNITS, 1910 TO 1961. U.S. Dept. Agr., Statis. Bul. 325, 20 pp. illus. Feb. 1963.

The balance between livestock production and the feed supply is of national concern during periods of both feed surpluses and feed shortages. Livestock-production units are an effective means of measuring the balance between livestock production and feed consumption. Statistics are given on cattle, hogs, milk and eggs, poultry, sheep and lambs, horses and mules, and goats.

HOLE, ERLING, AND VERMEER, JAMES. WHEAT GROWER'S MACHINERY COSTS, BY SIZE OF FARM, IN CENTRAL NORTH DAKOTA. U.S. Dept. Agr., Agr. Econ. Rpt. 24, 21 pp., illus. Feb. 1963.

Includes results of a study of costs of owning and operating machinery on three sizes of spring wheat farms in central North Dakota in 1960. Total cost per acre for major tractor-drawn and self-propelled machinery averaged about 15 percent higher on farms with 180 to 419 acres of cropland than on those with 660 to 899 acres. Costs on farms in the middle group—420 to 659 acres—averaged only slightly higher than those in the largest acreage group.

HOUSE, PETER. FARM TAXES ON THE RURAL URBAN FRINGE—A CASE STUDY OF FAIRFAX COUNTY, VIRGINIA. U.S. Dept. Agr., Agr. Econ. Res. Serv., ERS–102, 12 pp. March 1963.

Assessed 1961 value of farms averaged $266 per acre. Over a 5-year period assessed values of all farm properties had increased 163 percent. For the country as a whole, farms were assessed at about 35 percent of full value, compared with county target ratio of 40 percent.

HUNTER, ELMER C. CHANGES IN THE CATTLE FEEDING INDUSTRY ALONG THE NORTH AND SOUTH PLATTE RIVERS, 1953–1959. U.S. Dept. Agr.,

Econ. Res. Serv., ERS–98, 15 pp., illus. March 1963. (Colo. Agr. Expt. Sta. cooperating.)

Approximately 6 percent of the cattle fattened in the United States are fed in the irrigated valleys of the North and South Platte Rivers. The cattle-feeding industry directly or indirectly produces more than half of the area's agricultural income. During the period 1953–59, the number of cattle fed within the area increased by nearly half—from 536,500 to 797,055 head.

HUNTER, ELMER C. FORAGE PROGRAMS AND CATTLE SYSTEMS—COLORADO MOUNTAIN-MEADOW CATTLE RANCHES. U.S. Dept. Agr., Econ. Res. Serv., ERS–100, 18 pp., illus. Feb. 1963. (Colo. Agr. Expt. Sta. cooperating.)

Meadows produce practically all of the winter feed and some of the summer grazing for cattle in the mountainous portions of Colorado. The study examines five improved meadow-management programs to produce additional forage for a lower cost, and analyzes the relative profitableness of various livestock systems for a typical Colorado mountain-meadow ranch, when the meadow is managed under a rough-leveling, reseeding, and nitrogen-fertilization program.

HUNTER, JOHN H. JR. COSTS OF OPERATING EXEMPT FOR-HIRE MOTOR CARRIERS OF AGRICULTURAL COMMODITIES—A PILOT STUDY IN DELAWARE, MARYLAND, AND VIRGINIA. U.S. Dept. Agr., Econ. Res. Serv., ERS–109, 16 pp. Feb. 1963.

Cost information for 1960 was collected from 25 exempt for-hire trucking firms whose home offices are in Delaware, Maryland, and Virginia. Direct operating costs amounted to slightly more than 70 percent of total costs. Fixed costs were nearly 25 percent, and indirect costs about 5 percent of total costs.

JOHNSON, HUGH A., CARPENTER, J. RAYMOND, AND DILL, HENRY W., JR. EXURBAN DEVELOPMENT IN SELECTED AREAS OF THE APPALACHIAN MOUNTAINS. U.S. Dept. Agr., Econ. Res. Serv., ERS–111, 15 pp., illus. Apr. 1963.

In northwestern Virginia and adjacent areas of West Virginia, farm land is being taken over for recreational developments. Land use changes occurring on a sample area of approximately 15,400 acres from 1937 to 1962, involved 2,147 acres or about 14 percent of the entire area studied. In other parts of the country similar changes are occurring, where urban dwellers take advantage of opportunities for recreation.

LONG, MARY E. NEW ZEALAND INTRODUCES NEW TRADE CONTROLS FOR 1962–63. U.S. Dept. Agr., Econ. Res. Serv., ERS–Foreign–37, 8 pp. June 1962.

The report describes New Zealand's revised customs tariff and import licensing schedule for fiscal 1962–63, which may affect the country's agricultural trade with the United States.

LYNCH, JOHN V. URUGUAY'S AGRICULTURE—SITUATION AND OUTLOOK. U.S. Dept. Agr., Econ. Res. Serv., ERS–Foreign–18, 11 pp. Jan. 1962.

Uruguay's production of major crops and livestock in 1961 rose close to the levels attained prior to 1959 and 1960. Total exports, especially wool, were also larger during 1961 than a year earlier.

MCGRATH, EDWARD J., AND WEIDENHAMER, MARGARET. THE MARKET POTENTIAL FOR SUPERCONCENTRATED APPLE JUICE. U.S. Dept. Agr., Mktg. Res. Rpt. 582, 54 pp., illus. Jan. 1963.

The sales record of a new product—superconcentrated apple juice—compared with 80 other juice products indicates that it stands a reasonable chance of attaining commercial success. The market potential of this high-density (6 to 1) apple juice was studied in Fort Wayne, Ind., in a sample of 23 supermarkets in 1960. Most of the homemakers who were interviewed in the study said they served the new product betwen meals, using it in place of other fruit juices, soft drinks, or ades.

MIGHELL, RONALD L., AND JONES, LAWRENCE A. VERTICAL COORDINATION IN AGRICULTURE. U.S. Dept. Agr., Agr. Econ. Rpt. 19, 90 pp., illus. Feb. 1963.

This report is concerned in the main with vertical coordination in agriculture, especially between farms and the businesses that serve farms. It shows how patterns of coordination are constantly changing to meet new situations. New forms of vertical coordination, often adopted to gain market advantage, may be initiated by processors, suppliers, or farmers.

POATS, FREDERICK J. APPRAISAL OF TREATED BURLAP BAGS FOR SHIPPING WOOL. U.S. Dept. Agr., Econ. Res. Serv., ERS–110, 12 pp. Apr. 1963.

Presents the results of concerted efforts of the wool industry and USDA to find the answer to the problem of jute fiber contamination of wool. Regular burlap bags and latex rubber-treated bags were tested for suitability as shipping containers for wool. The results did not justify changing to the rubber-treated bags, because defects still appeared in cloth that was made from the wool shipped in the treated bags. These defects were identified as being from native grasses.

PODANY, JOSEPH C., AND FARRISH, RAYMOND O. P. TOMATO PRICES AND MARKET STRUCTURE IN THE LOWER RIO GRANDE VALLEY OF TEXAS. U.S. Dept. Agr., Econ. Res. Serv., Mktg. Res. Rpt. 588, 19 pp., illus. Feb. 1963.

Evaluates market performance, in terms of pricing efficiency, for the tomato market in the Lower Rio Grande Valley in 1961. Market performance is appraised with reference to the structure of the market. Important aspect of market structure is freedom of entry and exit of firms, and of market performance, the relation of f.o.b. and grower prices. When blend f.o.b. prices were compared with grower prices, the two series moved closely together over most of the season. In the latter part of the season, however, margins became wider and more variable.

ROSENBERRY, PAUL E. COSTS AND RETURNS, COMMERCIAL CORN BELT FARMS, 1962. U.S. Dept. Agr., Econ. Res. Serv., FCR–9, 5 pp., illus. Feb. 1963.

Part of a continuing nationwide study of costs and returns on farms and ranches by type and size in U.S. farming regions.

SANDERSON, AGNES G. BRITISH GUIANA—ITS AGRICULTURE AND TRADE. U.S. Dept. Agr., Econ. Res. Serv., ERS-Foreign-45, 42 pp., illus. March 1963.

British Guiana had a gross national product of $150 million in 1960, agriculture's share being 42 percent. Per capita income is $250, about average for Latin America. Over half the population depends on agriculture for a living. Sugar and rice are the dominant crops in the economy. Major problems for agriculture are the heavy expenditure needed for land reclamation and upkeep, and the difficulty and cost of transportation.

SANDERSON, AGNES G. THE AGRICULTURAL ECONOMY OF SURINAM (NETHERLANDS GUIANA). U.S. Dept. Agr., Econ. Res. Serv., ERS–Foreign–35, 30 pp., illus. June 1962.

Surinam's economy is dominated by bauxite, but agricultural production is increasing. Rice is the most important crop both domestically and in the export trade. Imports from the United States include many farm products, also machinery and equipment for use in capital investment programs.

STALLINGS, DALE G. MARKETING WESTERN FRUITS AND VEGETABLES—LONG-TERM OUTLOOK. U.S. Dept. Agr., Econ. Res. Serv., ERS–77, 14 pp., illus. March 1963.

The long-term outlook for marketing fruits and vegetables from the 11 western States is for continued growth. The Western Region now supplies about 65 percent of the noncitrus fruits, 45 percent of the vegetables, and 28 percent of the citrus fruit. Increases in the proportions of noncitrus fruits and vegetables and a small decrease in the proportion of citrus supplied by the West by 1975 are in prospect.

STODDARD, EVERETT O. COSTS AND RETURNS, COMMERCIAL EGG-PRODUCING FARMS, NEW JERSEY, 1962. U.S. Dept. Agr., Econ. Res. Serv., FCR–10, 9 pp., illus. March 1963.

Part of a continuing nationwide study of costs and returns on farms and ranches by type and size in U.S. farming regions.

VOLIN, LAZAR. THE AGRICULTURAL PICTURE IN U.S.S.R. AND U.S.A. U.S. Dept. Agr., Econ. Res. Serv., ERS–Foreign–27, 8 pp. Apr. 1963 (slightly revised).

Contrasts agricultural stagnation and food shortages in the Soviet Union with continuing high agricultural production in the United States. Higher farm productivity in the United States is attributed to wider use of improved technology, encouragement of innovation, and better incentives to managers and workers than the U.S.S.R. provides for its agricultural sector.

WARREN, CLINE. THE AGRICULTURAL ECONOMY OF THE SUDAN. U.S. Dept. Agr., Econ. Res. Serv., ERS–Foreign–26, 28 pp., illus. Apr. 1962. (Originally issued in Nov. 1958 as FAS–M–45.)

At least 90 percent of the inhabitants of the Sudan derive their livelihood from agriculture or animal husbandry. The report describes principal crops, irrigation methods, transportation, trade with the United States, and other aspects of the Sudan's agriculture.

WILLIAMS, F. W., AND ALLEN, M. B. THE SOUTHEASTERN VEGETABLE PROCESSING INDUSTRY: MARKETING PRACTICES AND MANAGEMENT PROBLEMS, 1960. U.S. Dept. Agr., Econ. Res. Serv., Mktg. Res. Rpt. 583, 20 pp., illus. Jan. 1963. (Ga. Agr. Expt. Sta. cooperating.)

This survey, made in 1961 in Alabama, Florida, Georgia, Mississippi, North Carolina, South Carolina, and Tennessee, showed fresh markets the dominant outlets for over 80 percent of southern vegetables, with less than 20 percent for processing. Total vegetable production in the Southeast declined 12 percent between 1955 and 1961.

YOUMANS, E. GRANT. AGING PATTERNS IN A RURAL AND URBAN AREA OF KENTUCKY. Ky. Agr. Expt. Sta. Bul. 681, 80 pp., illus. March 1963 (Econ. Res. Serv. cooperating.)

Report presents association of such factors as age, sex, color, marital status, and type of community with living conditions and behavior patterns of a sample of 1,236 persons aged 60 and over living in a rural and an urban area of Kentucky. Rural and urban data on economic status, health, leisure time activity, and mental outlook are given.

U.S. DEPARTMENT OF AGRICULTURE. HOW TO USE FARM INCOME STATISTICS. Misc. Pub. 920, 8 pp. Apr. 1963.

This leaflet is intended to help readers select and use reliable USDA statistical series. Descriptions of major USDA series of statistics on farm income are given. The leaflet tells how the series relate to each other and how each should be used. It is intended for general use. More detailed and technical information is contained in two other publications of the Department—Volume 3 of "Major Statistical Series of the U.S. Department of Agriculture" and "The Farm Income Situation" for July 1962.

U.S. DEPARTMENT OF AGRICULTURE. YOU AND THE USDA. U.S. Dept. Agr., PA–542, 48 pp., illus. 1963.

Gives a broad survey of the major responsibilities that the Congress has assigned to the Department in the 100 years since the Department was established. Describes research, marketing services, forest conservation, disaster relief, and other activities of the Department. Many of the programs are conducted in cooperation with State governments and land-grant colleges and universities.

U.S. ECONOMIC RESEARCH SERVICE. AGRICULTURE AND ECONOMIC GROWTH. U.S. Dept. Agr., Agr. Econ. Rpt. 28, 33 pp., illus. March 1963.

The United States provides an outstanding example of what an efficient agriculture can do for the economic growth of a country. Seven contributions are cited: (1) release of workers to industry; (2) lowering of food costs relative to income; (3) an expanding market for industrial goods; (4) large earnings from exports of farm products; (5) sustained output during economic depressions; (6) meeting wartime demands for food and fiber; and (7) assistance to economic development. The report was prepared by a study group established in 1961 by Willard W. Cochrane, Director of Economic Research.

U.S. Economic Research Service. LAND REDISTRIBUTION IN MEXICO. U.S. Dept. Agr., Econ. Res. Serv., ERS-Foreign-39, 11 pp. Aug. 1962. (Originally issued in March 1961 as FAS M–112.)

A history of the agrarian reform movement which began with the revolution of 1910 in Mexico, and description of some of the major changes in land distribution that have resulted.

U.S. Economic Research Service. 1960 GOVERNMENT PROGRAM SHIPMENTS TOTAL 30 PERCENT OF AGRICULTURAL EXPORTS. U.S. Dept. Agr., Econ. Res. Serv., ERS-Foreign-20, 12 pp., illus. Jan. 1962.

U.S. agricultural exports in calendar year 1960 totaled $4,824 million. Government program shipments under P.L. 480 and P.L. 666 were $1,469 million; more than half went to countries in Asia.

U.S. Economic Research Service. 1961 U.S. AGRICULTURAL EXPORTS SET RECORD; IMPORTS, AT 12-YEAR LOW. U.S. Dept. Agr., Econ. Res. Serv., ERS-Foreign-30, 10 pp. Apr. 1962.

Includes tabulation of quantities and dollar values of U.S. agricultural exports and imports during calendar 1960 and 1961, and value of exports to specific countries.

U.S. Economic Reseach Service. THE 1963 EASTERN EUROPE AGRICULTURAL SITUATION. U.S. Dept. Agr. Econ. Res. Serv., Suppl. 3 to The 1963 World Agricultural Situation, 53 pp. Apr. 1963

The Soviet Union and its East European satellites had another disappointing agricultural year in 1962. Only the Soviet Union and Bulgaria registered slight increases in net agricultural output; net agricultural output declined in six other countries. Agricultural imports in several Eastern European countries are expected to increase in 1963.

U.S. Economic Research Service. THE 1963 FAR EAST, COMMUNIST CHINA, OCEANIA AGRICULTURAL SITUATION. U.S. Dept. Agr., Econ. Res. Serv., Suppl. 4 to The 1963 World Agricultural Situation, 50 pp., illus. March 1963.

Agricultural production in the Far East in 1962 was 1.5 percent higher than in 1961. The rate of increase has declined progressively during the past several years. U.S. agricultural exports to the region in fiscal 1961–62 were about 14 percent below the level of a year earlier.

U.S. Economic Research Service. THE PHILIPPINES—LONG-TERM PROJECTION OF SUPPLY OF AND DEMAND FOR SELECTED AGRICULTURAL PRODUCTS. U.S. Dept. Agr., Econ. Res. Serv., ERS–Foreign–34, 256 pp.

Prepared under contract with Mercantile Incorporated. Manila, Philippines, the study sought to obtain a national projection to 1965 and 1975 of the import demands in the Philippines for rice, corn, wheat, meat, dairy products, tobacco, cotton, and tallow. Projections indicate that the Philippines should be self-sufficient in rice (milled), corn (shelled), and tobacco by 1965.

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Western Europe is the fastest growing advanced economic area in the Free World. This growth will probably continue in 1963, though perhaps at a lower rate than during the last 2 years. The report contains tabulations of agricultural production in 16 Western European countries for the past several years.

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U.S. agricultural exports totaled $4,946 million in fiscal 1960–61. The report shows dollar value of commodity groups exported to specific countries.

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(Continued from inside front cover.)

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Statistical Compilations

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