Rob 501 - Mathematics for Robotics
Recitation #2

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1 Truth Tables

P	Q	$\sim$ P	$\sim$ Q	P $\wedge$ Q	P $\vee$ Q	$P \implies Q$	$\sim$ P $\vee$ Q	P $\wedge \sim$ Q

2 Negation of statements

1. Simple negation

   Ex:

   - $x > 2$
   - at least 3 elements
   - $p \wedge q$
   - $p \vee q$
   - $x \in \mathbb{R}, x \neq 0$
   - x satisfying $f(x) = 0$ is unique.

2. Statement with quantifiers Ex:

   - $\forall x \in X : P(x) < 0$
   - $\forall \, (x \in \mathbb{R}^n, \; x \neq 0) : x^T A x \geq 0$
   - $\forall \, \epsilon > 0, \; \exists \, N \in \mathbb{N} : \; \forall n \geq N, |x_n - x^*| < \epsilon$

3. Statement with implications

   - $p \implies q$
   - $\forall \, \epsilon > 0, \exists \, \delta > 0 : \forall \, x, \; |x - x_0| < \delta \Rightarrow |f(x) - f(x_0)| < \epsilon$

3 Proofs

For all integers $n \in \mathbb{N}$, Prove:

$$\sum_{i=1}^{N} i = \frac{n(n+1)}{2}$$

4 Subspace

Definition: Let $(\mathcal{X},\, \mathcal{F})$ be a vector space and let $\mathcal{Y} \subset \mathcal{X}$. Then $(\mathcal{Y},\, \mathcal{F})$ is a *subspace* of $(\mathcal{X},\, \mathcal{F})$ if $(\mathcal{Y},\, \mathcal{F})$ is a vector space when you use the rules of vector addition and scalar times vector multiplication defined on $(\mathcal{X},\, \mathcal{F})$.

Proposition: $(\mathcal{X},\, \mathcal{F})$ is a vector space and $\mathcal{Y} \subset \mathcal{X}$. The following are equivalent (TFAE):

- $(\mathcal{Y},\, \mathcal{F})$ is a subspace
- a) $\forall\, y_1,\, y_2 \in \mathcal{Y},\; y_1 + y_2 \in \mathcal{Y}$

  b) $\forall\, y \in \mathcal{Y},\; \forall\, \alpha \in \mathcal{F},\; \alpha y \in \mathcal{Y}$
- $\forall\, y_1,\, y_2 \in \mathcal{Y},\; \forall\, \alpha \in \mathcal{F},\; y_1 + \alpha\, y_2 \in \mathcal{Y}$
- $\forall\, y_1,\, y_2 \in \mathcal{Y},\; \forall\, \alpha_1,\; \alpha_2 \in \mathcal{F},\; \alpha_1 y_1 + \alpha_2\, y_2 \in \mathcal{Y}$

Which of the following are subspaces?:

1. $(\mathcal{X},\, \mathcal{F}) = (\mathbb{R}^3,\, \mathbb{R}),\; \mathcal{Y} = \{x \in \mathbb{R}^3 :\; Cx = b;\, C,\, b \text{ are given constants}\}$

2. $(\mathcal{X},\, \mathcal{F}) = (\mathbb{R}^{n \times n},\, \mathbb{R}),\; \mathcal{Y} = \{A \in \mathbb{R}^{n \times n} :\; A = A^\top\}$

3. $(\mathcal{X},\, \mathcal{F}) = (\mathbb{R}^{n \times n},\, \mathbb{R}),\; \mathcal{Y} = \{A \in \mathbb{R}^{n \times n} :\; A \text{ is not invertible}\}$

4. $(\mathcal{X},\, \mathcal{F}) = (\mathbb{R}^{n \times n},\, \mathbb{R}),\; \mathcal{Y} = \{P \in \mathbb{R}^{n \times n} :\; A^2 = A\}$