PUZZLER

For thousands of years the spinning Earth provided a natural standard for our measurements of time. However, since 1972 we have added more than 20 "leap seconds" to our clocks to keep them synchronized to the Earth. Why are such adjustments needed? What does it take to be a good standard? *(Don Mason/The Stock Market and NASA)*

chapter

1 Physics and Measurement

Chapter Outline

1.1 Standards of Length, Mass, and Time
1.2 The Building Blocks of Matter
1.3 Density
1.4 Dimensional Analysis
1.5 Conversion of Units
1.6 Estimates and Order-of-Magnitude Calculations
1.7 Significant Figures

Like all other sciences, physics is based on experimental observations and quantitative measurements. The main objective of physics is to find the limited number of fundamental laws that govern natural phenomena and to use them to develop theories that can predict the results of future experiments. The fundamental laws used in developing theories are expressed in the language of mathematics, the tool that provides a bridge between theory and experiment.

When a discrepancy between theory and experiment arises, new theories must be formulated to remove the discrepancy. Many times a theory is satisfactory only under limited conditions; a more general theory might be satisfactory without such limitations. For example, the laws of motion discovered by Isaac Newton (1642–1727) in the 17th century accurately describe the motion of bodies at normal speeds but do not apply to objects moving at speeds comparable with the speed of light. In contrast, the special theory of relativity developed by Albert Einstein (1879–1955) in the early 1900s gives the same results as Newton's laws at low speeds but also correctly describes motion at speeds approaching the speed of light. Hence, Einstein's is a more general theory of motion.

Classical physics, which means all of the physics developed before 1900, includes the theories, concepts, laws, and experiments in classical mechanics, thermodynamics, and electromagnetism.

Important contributions to classical physics were provided by Newton, who developed classical mechanics as a systematic theory and was one of the originators of calculus as a mathematical tool. Major developments in mechanics continued in the 18th century, but the fields of thermodynamics and electricity and magnetism were not developed until the latter part of the 19th century, principally because before that time the apparatus for controlled experiments was either too crude or unavailable.

A new era in physics, usually referred to as *modern physics,* began near the end of the 19th century. Modern physics developed mainly because of the discovery that many physical phenomena could not be explained by classical physics. The two most important developments in modern physics were the theories of relativity and quantum mechanics. Einstein's theory of relativity revolutionized the traditional concepts of space, time, and energy; quantum mechanics, which applies to both the microscopic and macroscopic worlds, was originally formulated by a number of distinguished scientists to provide descriptions of physical phenomena at the atomic level.

Scientists constantly work at improving our understanding of phenomena and fundamental laws, and new discoveries are made every day. In many research areas, a great deal of overlap exists between physics, chemistry, geology, and biology, as well as engineering. Some of the most notable developments are (1) numerous space missions and the landing of astronauts on the Moon, (2) microcircuitry and high-speed computers, and (3) sophisticated imaging techniques used in scientific research and medicine. The impact such developments and discoveries have had on our society has indeed been great, and it is very likely that future discoveries and developments will be just as exciting and challenging and of great benefit to humanity.

1.1 STANDARDS OF LENGTH, MASS, AND TIME

The laws of physics are expressed in terms of basic quantities that require a clear definition. In mechanics, the three basic quantities are length (L), mass (M), and time (T). All other quantities in mechanics can be expressed in terms of these three.

If we are to report the results of a measurement to someone who wishes to reproduce this measurement, a *standard* must be defined. It would be meaningless if a visitor from another planet were to talk to us about a length of 8 "glitches" if we do not know the meaning of the unit glitch. On the other hand, if someone familiar with our system of measurement reports that a wall is 2 meters high and our unit of length is defined to be 1 meter, we know that the height of the wall is twice our basic length unit. Likewise, if we are told that a person has a mass of 75 kilograms and our unit of mass is defined to be 1 kilogram, then that person is 75 times as massive as our basic unit.[1] Whatever is chosen as a standard must be readily accessible and possess some property that can be measured reliably—measurements taken by different people in different places must yield the same result.

In 1960, an international committee established a set of standards for length, mass, and other basic quantities. The system established is an adaptation of the metric system, and it is called the **SI system** of units. (The abbreviation SI comes from the system's French name "Système International.") In this system, the units of length, mass, and time are the meter, kilogram, and second, respectively. Other SI standards established by the committee are those for temperature (the *kelvin*), electric current (the *ampere*), luminous intensity (the *candela*), and the amount of substance (the *mole*). In our study of mechanics we shall be concerned only with the units of length, mass, and time.

Length

In A.D. 1120 the king of England decreed that the standard of length in his country would be named the *yard* and would be precisely equal to the distance from the tip of his nose to the end of his outstretched arm. Similarly, the original standard for the foot adopted by the French was the length of the royal foot of King Louis XIV. This standard prevailed until 1799, when the legal standard of length in France became the *meter,* defined as one ten-millionth the distance from the equator to the North Pole along one particular longitudinal line that passes through Paris.

Many other systems for measuring length have been developed over the years, but the advantages of the French system have caused it to prevail in almost all countries and in scientific circles everywhere. As recently as 1960, the length of the meter was defined as the distance between two lines on a specific platinum–iridium bar stored under controlled conditions in France. This standard was abandoned for several reasons, a principal one being that the limited accuracy with which the separation between the lines on the bar can be determined does not meet the current requirements of science and technology. In the 1960s and 1970s, the meter was defined as 1 650 763.73 wavelengths of orange-red light emitted from a krypton-86 lamp. However, in October 1983, the **meter (m) was redefined as the distance traveled by light in vacuum during a time of 1/299 792 458 second.** In effect, this latest definition establishes that the speed of light in vacuum is precisely 299 792 458 m per second.

Table 1.1 lists approximate values of some measured lengths.

[1] The need for assigning numerical values to various measured physical quantities was expressed by Lord Kelvin (William Thomson) as follows: "I often say that when you can measure what you are speaking about, and express it in numbers, you should know something about it, but when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind. It may be the beginning of knowledge but you have scarcely in your thoughts advanced to the state of science."

TABLE 1.1 Approximate Values of Some Measured Lengths

	Length (m)
Distance from the Earth to most remote known quasar	1.4×10^{26}
Distance from the Earth to most remote known normal galaxies	9×10^{25}
Distance from the Earth to nearest large galaxy (M 31, the Andromeda galaxy)	2×10^{22}
Distance from the Sun to nearest star (Proxima Centauri)	4×10^{16}
One lightyear	9.46×10^{15}
Mean orbit radius of the Earth about the Sun	1.50×10^{11}
Mean distance from the Earth to the Moon	3.84×10^{8}
Distance from the equator to the North Pole	1.00×10^{7}
Mean radius of the Earth	6.37×10^{6}
Typical altitude (above the surface) of a satellite orbiting the Earth	2×10^{5}
Length of a football field	9.1×10^{1}
Length of a housefly	5×10^{-3}
Size of smallest dust particles	$\sim 10^{-4}$
Size of cells of most living organisms	$\sim 10^{-5}$
Diameter of a hydrogen atom	$\sim 10^{-10}$
Diameter of an atomic nucleus	$\sim 10^{-14}$
Diameter of a proton	$\sim 10^{-15}$

Mass

The basic SI unit of mass, **the kilogram (kg), is defined as the mass of a specific platinum–iridium alloy cylinder kept at the International Bureau of Weights and Measures at Sèvres, France.** This mass standard was established in 1887 and has not been changed since that time because platinum–iridium is an unusually stable alloy (Fig. 1.1a). A duplicate of the Sèvres cylinder is kept at the National Institute of Standards and Technology (NIST) in Gaithersburg, Maryland.

Table 1.2 lists approximate values of the masses of various objects.

web

Visit the Bureau at **www.bipm.fr** or the National Institute of Standards at **www.NIST.gov**

TABLE 1.2 Masses of Various Bodies (Approximate Values)

Body	Mass (kg)
Visible Universe	$\sim 10^{52}$
Milky Way galaxy	7×10^{41}
Sun	1.99×10^{30}
Earth	5.98×10^{24}
Moon	7.36×10^{22}
Horse	$\sim 10^{3}$
Human	$\sim 10^{2}$
Frog	$\sim 10^{-1}$
Mosquito	$\sim 10^{-5}$
Bacterium	$\sim 10^{-15}$
Hydrogen atom	1.67×10^{-27}
Electron	9.11×10^{-31}

Time

Before 1960, the standard of time was defined in terms of the *mean solar day* for the year 1900.[2] The *mean solar second* was originally defined as $(\frac{1}{60})(\frac{1}{60})(\frac{1}{24})$ of a mean solar day. The rotation of the Earth is now known to vary slightly with time, however, and therefore this motion is not a good one to use for defining a standard.

In 1967, consequently, the second was redefined to take advantage of the high precision obtainable in a device known as an *atomic clock* (Fig. 1.1b). In this device, the frequencies associated with certain atomic transitions can be measured to a precision of one part in 10^{12}. This is equivalent to an uncertainty of less than one second every 30 000 years. Thus, in 1967 the SI unit of time, the *second,* was redefined using the characteristic frequency of a particular kind of cesium atom as the "reference clock." The basic SI unit of time, **the second (s), is defined as 9 192 631 770 times the period of vibration of radiation from the cesium-133 atom.**[3] To keep these atomic clocks—and therefore all common clocks and

[2] One solar day is the time interval between successive appearances of the Sun at the highest point it reaches in the sky each day.

[3] *Period* is defined as the time interval needed for one complete vibration.

Figure 1.1 *(Top)* The National Standard Kilogram No. 20, an accurate copy of the International Standard Kilogram kept at Sèvres, France, is housed under a double bell jar in a vault at the National Institute of Standards and Technology (NIST). *(Bottom)* The primary frequency standard (an atomic clock) at the NIST. This device keeps time with an accuracy of about 3 millionths of a second per year. *(Courtesy of National Institute of Standards and Technology, U.S. Department of Commerce)*

watches that are set to them—synchronized, it has sometimes been necessary to add leap seconds to our clocks. This is not a new idea. In 46 B.C. Julius Caesar began the practice of adding extra days to the calendar during leap years so that the seasons occurred at about the same date each year.

Since Einstein's discovery of the linkage between space and time, precise measurement of time intervals requires that we know both the state of motion of the clock used to measure the interval and, in some cases, the location of the clock as well. Otherwise, for example, global positioning system satellites might be unable to pinpoint your location with sufficient accuracy, should you need rescuing.

Approximate values of time intervals are presented in Table 1.3.

In addition to SI, another system of units, the *British engineering system* (sometimes called the *conventional system*), is still used in the United States despite acceptance of SI by the rest of the world. In this system, the units of length, mass, and

TABLE 1.3 Approximate Values of Some Time Intervals

	Interval (s)
Age of the Universe	5×10^{17}
Age of the Earth	1.3×10^{17}
Average age of a college student	6.3×10^{8}
One year	3.16×10^{7}
One day (time for one rotation of the Earth about its axis)	8.64×10^{4}
Time between normal heartbeats	8×10^{-1}
Period of audible sound waves	$\sim 10^{-3}$
Period of typical radio waves	$\sim 10^{-6}$
Period of vibration of an atom in a solid	$\sim 10^{-13}$
Period of visible light waves	$\sim 10^{-15}$
Duration of a nuclear collision	$\sim 10^{-22}$
Time for light to cross a proton	$\sim 10^{-24}$

time are the foot (ft), slug, and second, respectively. In this text we shall use SI units because they are almost universally accepted in science and industry. We shall make some limited use of British engineering units in the study of classical mechanics.

In addition to the basic SI units of meter, kilogram, and second, we can also use other units, such as millimeters and nanoseconds, where the prefixes *milli-* and *nano-* denote various powers of ten. Some of the most frequently used prefixes for the various powers of ten and their abbreviations are listed in Table 1.4. For

TABLE 1.4 Prefixes for SI Units

Power	Prefix	Abbreviation
10^{-24}	yocto	y
10^{-21}	zepto	z
10^{-18}	atto	a
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^{1}	deka	da
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P
10^{18}	exa	E
10^{21}	zetta	Z
10^{24}	yotta	Y

example, 10^{-3} m is equivalent to 1 millimeter (mm), and 10^3 m corresponds to 1 kilometer (km). Likewise, 1 kg is 10^3 grams (g), and 1 megavolt (MV) is 10^6 volts (V).

1.2 THE BUILDING BLOCKS OF MATTER

A 1-kg cube of solid gold has a length of 3.73 cm on a side. Is this cube nothing but wall-to-wall gold, with no empty space? If the cube is cut in half, the two pieces still retain their chemical identity as solid gold. But what if the pieces are cut again and again, indefinitely? Will the smaller and smaller pieces always be gold? Questions such as these can be traced back to early Greek philosophers. Two of them—Leucippus and his student Democritus—could not accept the idea that such cuttings could go on forever. They speculated that the process ultimately must end when it produces a particle that can no longer be cut. In Greek, *atomos* means "not sliceable." From this comes our English word *atom.*

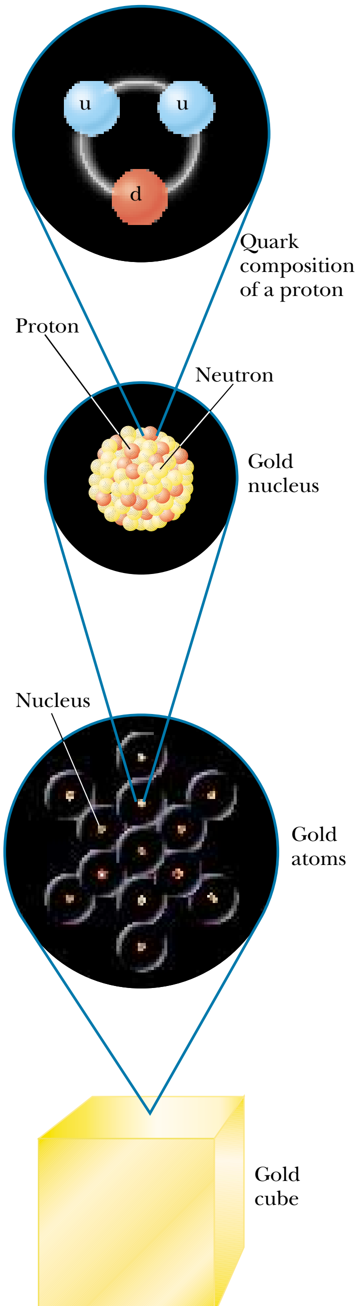


Figure 1.2 Levels of organization in matter. Ordinary matter consists of atoms, and at the center of each atom is a compact nucleus consisting of protons and neutrons. Protons and neutrons are composed of quarks. The quark composition of a proton is shown.

Let us review briefly what is known about the structure of matter. All ordinary matter consists of atoms, and each atom is made up of electrons surrounding a central nucleus. Following the discovery of the nucleus in 1911, the question arose: Does it have structure? That is, is the nucleus a single particle or a collection of particles? The exact composition of the nucleus is not known completely even today, but by the early 1930s a model evolved that helped us understand how the nucleus behaves. Specifically, scientists determined that occupying the nucleus are two basic entities, protons and neutrons. The *proton* carries a positive charge, and a specific element is identified by the number of protons in its nucleus. This number is called the **atomic number** of the element. For instance, the nucleus of a hydrogen atom contains one proton (and so the atomic number of hydrogen is 1), the nucleus of a helium atom contains two protons (atomic number 2), and the nucleus of a uranium atom contains 92 protons (atomic number 92). In addition to atomic number, there is a second number characterizing atoms—**mass number,** defined as the number of protons plus neutrons in a nucleus. As we shall see, the atomic number of an element never varies (i.e., the number of protons does not vary) but the mass number can vary (i.e., the number of neutrons varies). Two or more atoms of the same element having different mass numbers are **isotopes** of one another.

The existence of neutrons was verified conclusively in 1932. A *neutron* has no charge and a mass that is about equal to that of a proton. One of its primary purposes is to act as a "glue" that holds the nucleus together. If neutrons were not present in the nucleus, the repulsive force between the positively charged particles would cause the nucleus to come apart.

But is this where the breaking down stops? Protons, neutrons, and a host of other exotic particles are now known to be composed of six different varieties of particles called **quarks,** which have been given the names of *up, down, strange, charm, bottom,* and *top.* The up, charm, and top quarks have charges of $+\frac{2}{3}$ that of the proton, whereas the down, strange, and bottom quarks have charges of $-\frac{1}{3}$ that of the proton. The proton consists of two up quarks and one down quark (Fig. 1.2), which you can easily show leads to the correct charge for the proton. Likewise, the neutron consists of two down quarks and one up quark, giving a net charge of zero.

1.3 DENSITY

A table of the letters in the Greek alphabet is provided on the back endsheet of this textbook.

A property of any substance is its **density** ρ (Greek letter rho), defined as the amount of mass contained in a unit volume, which we usually express as *mass per unit volume:*

$$\rho \equiv \frac{m}{V} \tag{1.1}$$

For example, aluminum has a density of 2.70 g/cm^3, and lead has a density of 11.3 g/cm^3. Therefore, a piece of aluminum of volume 10.0 cm^3 has a mass of 27.0 g, whereas an equivalent volume of lead has a mass of 113 g. A list of densities for various substances is given Table 1.5.

The difference in density between aluminum and lead is due, in part, to their different *atomic masses*. The **atomic mass** of an element is the average mass of one atom in a sample of the element that contains all the element's isotopes, where the relative amounts of isotopes are the same as the relative amounts found in nature. The unit for atomic mass is the *atomic mass unit* (u), where 1 u = $1.660\ 540\ 2 \times 10^{-27}$ kg. The atomic mass of lead is 207 u, and that of aluminum is 27.0 u. However, the ratio of atomic masses, 207 u/27.0 u = 7.67, does not correspond to the ratio of densities, (11.3 g/cm^3)/(2.70 g/cm^3) = 4.19. The discrepancy is due to the difference in atomic separations and atomic arrangements in the crystal structure of these two substances.

The mass of a nucleus is measured relative to the mass of the nucleus of the carbon-12 isotope, often written as ^{12}C. (This isotope of carbon has six protons and six neutrons. Other carbon isotopes have six protons but different numbers of neutrons.) Practically all of the mass of an atom is contained within the nucleus. Because the atomic mass of ^{12}C is defined to be exactly 12 u, the proton and neutron each have a mass of about 1 u.

One mole (mol) of a substance is that amount of the substance that contains as many particles (atoms, molecules, or other particles) as there are atoms in 12 g of the carbon-12 isotope. One mole of substance A contains the same number of particles as there are in 1 mol of any other substance B. For example, 1 mol of aluminum contains the same number of atoms as 1 mol of lead.

TABLE 1.5 Densities of Various Substances

Substance	Density ρ (10^3 kg/m^3)
Gold	19.3
Uranium	18.7
Lead	11.3
Copper	8.92
Iron	7.86
Aluminum	2.70
Magnesium	1.75
Water	1.00
Air	0.0012

Experiments have shown that this number, known as Avogadro's number, N_A, is

$$N_A = 6.022\ 137 \times 10^{23} \text{ particles/mol}$$

Avogadro's number is defined so that 1 mol of carbon-12 atoms has a mass of exactly 12 g. In general, the mass in 1 mol of any element is the element's atomic mass expressed in grams. For example, 1 mol of iron (atomic mass = 55.85 u) has a mass of 55.85 g (we say its *molar mass* is 55.85 g/mol), and 1 mol of lead (atomic mass = 207 u) has a mass of 207 g (its molar mass is 207 g/mol). Because there are 6.02×10^{23} particles in 1 mol of *any* element, the mass per atom for a given element is

$$m_{\text{atom}} = \frac{\text{molar mass}}{N_A} \tag{1.2}$$

For example, the mass of an iron atom is

$$m_{\text{Fe}} = \frac{55.85 \text{ g/mol}}{6.02 \times 10^{23} \text{ atoms/mol}} = 9.28 \times 10^{-23} \text{ g/atom}$$

EXAMPLE 1.1 How Many Atoms in the Cube?

A solid cube of aluminum (density 2.7 g/cm^3) has a volume of 0.20 cm^3. How many aluminum atoms are contained in the cube?

Solution Since density equals mass per unit volume, the mass m of the cube is

$$m = \rho V = (2.7 \text{ g/cm}^3)(0.20 \text{ cm}^3) = 0.54 \text{ g}$$

To find the number of atoms N in this mass of aluminum, we can set up a proportion using the fact that one mole of aluminum (27 g) contains 6.02×10^{23} atoms:

$$\frac{N_A}{27 \text{ g}} = \frac{N}{0.54 \text{ g}}$$

$$\frac{6.02 \times 10^{23} \text{ atoms}}{27 \text{ g}} = \frac{N}{0.54 \text{ g}}$$

$$N = \frac{(0.54 \text{ g})(6.02 \times 10^{23} \text{ atoms})}{27 \text{ g}} = \boxed{1.2 \times 10^{22} \text{ atoms}}$$

1.4 DIMENSIONAL ANALYSIS

The word *dimension* has a special meaning in physics. It usually denotes the physical nature of a quantity. Whether a distance is measured in the length unit feet or the length unit meters, it is still a distance. We say the dimension—the physical nature—of distance is *length.*

The symbols we use in this book to specify length, mass, and time are L, M, and T, respectively. We shall often use brackets [ ] to denote the dimensions of a physical quantity. For example, the symbol we use for speed in this book is v, and in our notation the dimensions of speed are written $[v] = \text{L/T}$. As another example, the dimensions of area, for which we use the symbol A, are $[A] = \text{L}^2$. The dimensions of area, volume, speed, and acceleration are listed in Table 1.6.

In solving problems in physics, there is a useful and powerful procedure called *dimensional analysis.* This procedure, which should always be used, will help minimize the need for rote memorization of equations. Dimensional analysis makes use of the fact that **dimensions can be treated as algebraic quantities.** That is, quantities can be added or subtracted only if they have the same dimensions. Furthermore, the terms on both sides of an equation must have the same dimensions.

TABLE 1.6 Dimensions and Common Units of Area, Volume, Speed, and Acceleration

System	Area (L^2)	Volume (L^3)	Speed (L/T)	Acceleration (L/T^2)
SI	m^2	m^3	m/s	m/s^2
British engineering	ft^2	ft^3	ft/s	ft/s^2

By following these simple rules, you can use dimensional analysis to help determine whether an expression has the correct form. The relationship can be correct only if the dimensions are the same on both sides of the equation.

To illustrate this procedure, suppose you wish to derive a formula for the distance x traveled by a car in a time t if the car starts from rest and moves with constant acceleration a. In Chapter 2, we shall find that the correct expression is $x = \frac{1}{2}at^2$. Let us use dimensional analysis to check the validity of this expression. The quantity x on the left side has the dimension of length. For the equation to be dimensionally correct, the quantity on the right side must also have the dimension of length. We can perform a dimensional check by substituting the dimensions for acceleration, L/T^2, and time, T, into the equation. That is, the dimensional form of the equation $x = \frac{1}{2}at^2$ is

$$L = \frac{L}{\cancel{T^2}} \cdot \cancel{T^2} = L$$

The units of time squared cancel as shown, leaving the unit of length.

A more general procedure using dimensional analysis is to set up an expression of the form

$$x \propto a^n t^m$$

where n and m are exponents that must be determined and the symbol $\propto$ indicates a proportionality. This relationship is correct only if the dimensions of both sides are the same. Because the dimension of the left side is length, the dimension of the right side must also be length. That is,

$$[a^n t^m] = L = LT^0$$

Because the dimensions of acceleration are L/T^2 and the dimension of time is T, we have

$$\left(\frac{L}{T^2}\right)^n T^m = L^1$$

$$L^n T^{m-2n} = L^1$$

Because the exponents of L and T must be the same on both sides, the dimensional equation is balanced under the conditions $m - 2n = 0$, $n = 1$, and $m = 2$. Returning to our original expression $x \propto a^n t^m$, we conclude that $x \propto at^2$. This result differs by a factor of 2 from the correct expression, which is $x = \frac{1}{2}at^2$. Because the factor $\frac{1}{2}$ is dimensionless, there is no way of determining it using dimensional analysis.

Quick Quiz 1.1

True or False: Dimensional analysis can give you the numerical value of constants of proportionality that may appear in an algebraic expression.

EXAMPLE 1.2 Analysis of an Equation

Show that the expression $v = at$ is dimensionally correct, where v represents speed, a acceleration, and t a time interval.

Solution For the speed term, we have from Table 1.6

$$[v] = \frac{\mathrm{L}}{\mathrm{T}}$$

The same table gives us L/T^2 for the dimensions of acceleration, and so the dimensions of at are

$$[at] = \left(\frac{\mathrm{L}}{\mathrm{T}^{\cancel{2}}}\right)(\cancel{\mathrm{T}}) = \frac{\mathrm{L}}{\mathrm{T}}$$

Therefore, the expression is dimensionally correct. (If the expression were given as $v = at^2$, it would be dimensionally *incorrect.* Try it and see!)

EXAMPLE 1.3 Analysis of a Power Law

Suppose we are told that the acceleration a of a particle moving with uniform speed v in a circle of radius r is proportional to some power of r, say r^n, and some power of v, say v^m. How can we determine the values of n and m?

Solution Let us take a to be

$$a = kr^nv^m$$

where k is a dimensionless constant of proportionality. Knowing the dimensions of a, r, and v, we see that the dimensional equation must be

$$\mathrm{L}/\mathrm{T}^2 = \mathrm{L}^n(\mathrm{L}/\mathrm{T})^m = \mathrm{L}^{n+m}/\mathrm{T}^m$$

This dimensional equation is balanced under the conditions

$$n + m = 1 \qquad \text{and} \qquad m = 2$$

Therefore $n = -1$, and we can write the acceleration expression as

$$a = kr^{-1}v^2 = k\frac{v^2}{r}$$

When we discuss uniform circular motion later, we shall see that $k = 1$ if a consistent set of units is used. The constant k would not equal 1 if, for example, v were in km/h and you wanted a in $\mathrm{m/s^2}$.

QuickLab

Estimate the weight (in pounds) of two large bottles of soda pop. Note that 1 L of water has a mass of about 1 kg. Use the fact that an object weighing 2.2 lb has a mass of 1 kg. Find some bathroom scales and check your estimate.

1.5 CONVERSION OF UNITS

Sometimes it is necessary to convert units from one system to another. Conversion factors between the SI units and conventional units of length are as follows:

$$1 \text{ mi} = 1\,609 \text{ m} = 1.609 \text{ km} \qquad 1 \text{ ft} = 0.304\,8 \text{ m} = 30.48 \text{ cm}$$

$$1 \text{ m} = 39.37 \text{ in.} = 3.281 \text{ ft} \qquad 1 \text{ in.} \equiv 0.025\,4 \text{ m} = 2.54 \text{ cm (exactly)}$$

A more complete list of conversion factors can be found in Appendix A.

Units can be treated as algebraic quantities that can cancel each other. For example, suppose we wish to convert 15.0 in. to centimeters. Because 1 in. is defined as exactly 2.54 cm, we find that

$$15.0 \text{ in.} = (15.0\ \cancel{\text{in.}})(2.54 \text{ cm}/\cancel{\text{in.}}) = 38.1 \text{ cm}$$

This works because multiplying by $\left(\frac{2.54 \text{ cm}}{1 \text{ in.}}\right)$ is the same as multiplying by 1, because the numerator and denominator describe identical things.

(Left) This road sign near Raleigh, North Carolina, shows distances in miles and kilometers. How accurate are the conversions? *(Billy E. Barnes/Stock Boston).*

(Right) This vehicle's speedometer gives speed readings in miles per hour and in kilometers per hour. Try confirming the conversion between the two sets of units for a few readings of the dial. *(Paul Silverman/Fundamental Photographs)*

EXAMPLE 1.4 The Density of a Cube

The mass of a solid cube is 856 g, and each edge has a length of 5.35 cm. Determine the density ρ of the cube in basic SI units.

Solution Because 1 g = 10^{-3} kg and 1 cm = 10^{-2} m, the mass m and volume V in basic SI units are

$$m = 856\ \cancel{\text{g}} \times 10^{-3}\ \text{kg}/\cancel{\text{g}} = 0.856\ \text{kg}$$

$$V = L^3 = (5.35\ \cancel{\text{cm}} \times 10^{-2}\ \text{m}/\cancel{\text{cm}})^3$$
$$= (5.35)^3 \times 10^{-6}\ \text{m}^3 = 1.53 \times 10^{-4}\ \text{m}^3$$

Therefore,

$$\rho = \frac{m}{V} = \frac{0.856\ \text{kg}}{1.53 \times 10^{-4}\ \text{m}^3} = \boxed{5.59 \times 10^3\ \text{kg/m}^3}$$

1.6 ESTIMATES AND ORDER-OF-MAGNITUDE CALCULATIONS

It is often useful to compute an approximate answer to a physical problem even where little information is available. Such an approximate answer can then be used to determine whether a more accurate calculation is necessary. Approximations are usually based on certain assumptions, which must be modified if greater accuracy is needed. Thus, we shall sometimes refer to the order of magnitude of a certain quantity as the power of ten of the number that describes that quantity. If, for example, we say that a quantity increases in value by three orders of magnitude, this means that its value is increased by a factor of $10^3 = 1000$. Also, if a quantity is given as 3×10^3, we say that the order of magnitude of that quantity is 10^3 (or in symbolic form, $3 \times 10^3 \sim 10^3$). Likewise, the quantity $8 \times 10^7 \sim 10^8$.

The spirit of order-of-magnitude calculations, sometimes referred to as "guesstimates" or "ball-park figures," is given in the following quotation: "Make an estimate before every calculation, try a simple physical argument . . . before every derivation, guess the answer to every puzzle. Courage: no one else needs to know what the guess is."[4] Inaccuracies caused by guessing too low for one number are often canceled out by other guesses that are too high. You will find that with practice your guesstimates get better and better. Estimation problems can be fun to work as you freely drop digits, venture reasonable approximations for unknown numbers, make simplifying assumptions, and turn the question around into something you can answer in your head.

EXAMPLE 1.5 Breaths in a Lifetime

Estimate the number of breaths taken during an average life span.

Solution We shall start by guessing that the typical life span is about 70 years. The only other estimate we must make in this example is the average number of breaths that a person takes in 1 min. This number varies, depending on whether the person is exercising, sleeping, angry, serene, and so forth. To the nearest order of magnitude, we shall choose 10 breaths per minute as our estimate of the average. (This is certainly closer to the true value than 1 breath per minute or 100 breaths per minute.) The number of minutes in a year is approximately

$$1\ \cancel{\text{yr}} \times 400 \frac{\cancel{\text{days}}}{\cancel{\text{yr}}} \times 25 \frac{\cancel{\text{h}}}{\cancel{\text{day}}} \times 60 \frac{\text{min}}{\cancel{\text{h}}} = 6 \times 10^5 \text{ min}$$

Notice how much simpler it is to multiply 400×25 than it is to work with the more accurate 365×24. These approximate values for the number of days in a year and the number of hours in a day are close enough for our purposes. Thus, in 70 years there will be $(70 \text{ yr})(6 \times 10^5 \text{ min/yr}) = 4 \times 10^7$ min. At a rate of 10 breaths/min, an individual would take $\boxed{4 \times 10^8 \text{ breaths in a lifetime.}}$

EXAMPLE 1.6 It's a Long Way to San Jose

Estimate the number of steps a person would take walking from New York to Los Angeles.

Solution Without looking up the distance between these two cities, you might remember from a geography class that they are about 3 000 mi apart. The next approximation we must make is the length of one step. Of course, this length depends on the person doing the walking, but we can estimate that each step covers about 2 ft. With our estimated step size, we can determine the number of steps in 1 mi. Because this is a rough calculation, we round 5 280 ft/mi to 5 000 ft/mi. (What percentage error does this introduce?) This conversion factor gives us

$$\frac{5\,000\ \cancel{\text{ft}}/\text{mi}}{2\ \cancel{\text{ft}}/\text{step}} = 2\,500 \text{ steps/mi}$$

Now we switch to scientific notation so that we can do the calculation mentally:

$$(3 \times 10^3\ \cancel{\text{mi}})(2.5 \times 10^3 \text{ steps}/\cancel{\text{mi}}) = 7.5 \times 10^6 \text{ steps}$$

$$\sim \boxed{10^7 \text{ steps}}$$

So if we intend to walk across the United States, it will take us on the order of ten million steps. This estimate is almost certainly too small because we have not accounted for curving roads and going up and down hills and mountains. Nonetheless, it is probably within an order of magnitude of the correct answer.

EXAMPLE 1.7 How Much Gas Do We Use?

Estimate the number of gallons of gasoline used each year by all the cars in the United States.

Solution There are about 270 million people in the United States, and so we estimate that the number of cars in the country is 100 million (guessing that there are between two and three people per car). We also estimate that the average distance each car travels per year is 10 000 mi. If we assume a gasoline consumption of 20 mi/gal or 0.05 gal/mi, then each car uses about 500 gal/yr. Multiplying this by the total number of cars in the United States gives an estimated total consumption of 5×10^{10} gal $\sim \boxed{10^{11} \text{ gal.}}$

[4] E. Taylor and J. A. Wheeler, *Spacetime Physics,* San Francisco, W. H. Freeman & Company, Publishers, 1966, p. 60.

1.7 SIGNIFICANT FIGURES

When physical quantities are measured, the measured values are known only to within the limits of the experimental uncertainty. The value of this uncertainty can depend on various factors, such as the quality of the apparatus, the skill of the experimenter, and the number of measurements performed.

Suppose that we are asked to measure the area of a computer disk label using a meter stick as a measuring instrument. Let us assume that the accuracy to which we can measure with this stick is ± 0.1 cm. If the length of the label is measured to be 5.5 cm, we can claim only that its length lies somewhere between 5.4 cm and 5.6 cm. In this case, we say that the measured value has two significant figures. Likewise, if the label's width is measured to be 6.4 cm, the actual value lies between 6.3 cm and 6.5 cm. Note that the significant figures include the first estimated digit. Thus we could write the measured values as (5.5 ± 0.1) cm and (6.4 ± 0.1) cm.

Now suppose we want to find the area of the label by multiplying the two measured values. If we were to claim the area is $(5.5 \text{ cm})(6.4 \text{ cm}) = 35.2 \text{ cm}^2$, our answer would be unjustifiable because it contains three significant figures, which is greater than the number of significant figures in either of the measured lengths. A good rule of thumb to use in determining the number of significant figures that can be claimed is as follows:

> When multiplying several quantities, the number of significant figures in the final answer is the same as the number of significant figures in the *least* accurate of the quantities being multiplied, where "least accurate" means "having the lowest number of significant figures." The same rule applies to division.

QuickLab

Determine the thickness of a page from this book. (Note that numbers that have no measurement errors—like the count of a number of pages—do not affect the significant figures in a calculation.) In terms of significant figures, why is it better to measure the thickness of as many pages as possible and then divide by the number of sheets?

Applying this rule to the multiplication example above, we see that the answer for the area can have only two significant figures because our measured lengths have only two significant figures. Thus, all we can claim is that the area is 35 cm^2, realizing that the value can range between $(5.4 \text{ cm})(6.3 \text{ cm}) = 34 \text{ cm}^2$ and $(5.6 \text{ cm})(6.5 \text{ cm}) = 36 \text{ cm}^2$.

Zeros may or may not be significant figures. Those used to position the decimal point in such numbers as 0.03 and 0.007 5 are not significant. Thus, there are one and two significant figures, respectively, in these two values. When the zeros come after other digits, however, there is the possibility of misinterpretation. For example, suppose the mass of an object is given as 1 500 g. This value is ambiguous because we do not know whether the last two zeros are being used to locate the decimal point or whether they represent significant figures in the measurement. To remove this ambiguity, it is common to use scientific notation to indicate the number of significant figures. In this case, we would express the mass as 1.5×10^3 g if there are two significant figures in the measured value, 1.50×10^3 g if there are three significant figures, and 1.500×10^3 g if there are four. The same rule holds when the number is less than 1, so that 2.3×10^{-4} has two significant figures (and so could be written 0.000 23) and 2.30×10^{-4} has three significant figures (also written 0.000 230). In general, **a significant figure is a reliably known digit** (other than a zero used to locate the decimal point).

For addition and subtraction, you must consider the number of decimal places when you are determining how many significant figures to report.

When numbers are added or subtracted, the number of decimal places in the result should equal the smallest number of decimal places of any term in the sum.

For example, if we wish to compute $123 + 5.35$, the answer given to the correct number of significant figures is 128 and not 128.35. If we compute the sum $1.000\,1 + 0.000\,3 = 1.000\,4$, the result has five significant figures, even though one of the terms in the sum, 0.000 3, has only one significant figure. Likewise, if we perform the subtraction $1.002 - 0.998 = 0.004$, the result has only one significant figure even though one term has four significant figures and the other has three. In this book, **most of the numerical examples and end-of-chapter problems will yield answers having three significant figures.** When carrying out estimates we shall typically work with a single significant figure.

Quick Quiz 1.2

Suppose you measure the position of a chair with a meter stick and record that the center of the seat is 1.043 860 564 2 m from a wall. What would a reader conclude from this recorded measurement?

EXAMPLE 1.8 The Area of a Rectangle

A rectangular plate has a length of (21.3 ± 0.2) cm and a width of (9.80 ± 0.1) cm. Find the area of the plate and the uncertainty in the calculated area.

Solution

$$\text{Area} = \ell w = (21.3 \pm 0.2 \text{ cm}) \times (9.80 \pm 0.1 \text{ cm})$$

$$\approx (21.3 \times 9.80 \pm 21.3 \times 0.1 \pm 0.2 \times 9.80) \text{ cm}^2$$

$$\approx \boxed{(209 \pm 4) \text{ cm}^2}$$

Because the input data were given to only three significant figures, we cannot claim any more in our result. Do you see why we did not need to multiply the uncertainties 0.2 cm and 0.1 cm?

EXAMPLE 1.9 Installing a Carpet

A carpet is to be installed in a room whose length is measured to be 12.71 m and whose width is measured to be 3.46 m. Find the area of the room.

Solution If you multiply 12.71 m by 3.46 m on your calculator, you will get an answer of 43.976 6 m^2. How many of these numbers should you claim? Our rule of thumb for multiplication tells us that you can claim only the number of significant figures in the least accurate of the quantities being measured. In this example, we have only three significant figures in our least accurate measurement, so we should express our final answer as $\boxed{44.0 \text{ m}^2}$.

Note that in reducing 43.976 6 to three significant figures for our answer, we used a general rule for rounding off numbers that states that the last digit retained (the 9 in this example) is increased by 1 if the first digit dropped (here, the 7) is 5 or greater. (A technique for avoiding error accumulation is to delay rounding of numbers in a long calculation until you have the final result. Wait until you are ready to copy the answer from your calculator before rounding to the correct number of significant figures.)

SUMMARY

The three fundamental physical quantities of mechanics are length, mass, and time, which in the SI system have the units meters (m), kilograms (kg), and seconds (s), respectively. Prefixes indicating various powers of ten are used with these three basic units. The **density** of a substance is defined as its *mass per unit volume.* Different substances have different densities mainly because of differences in their atomic masses and atomic arrangements.

The number of particles in one mole of any element or compound, called **Avogadro's number,** N_A, is 6.02×10^{23}.

The method of *dimensional analysis* is very powerful in solving physics problems. Dimensions can be treated as algebraic quantities. By making estimates and making order-of-magnitude calculations, you should be able to approximate the answer to a problem when there is not enough information available to completely specify an exact solution.

When you compute a result from several measured numbers, each of which has a certain accuracy, you should give the result with the correct number of significant figures.

QUESTIONS

1. In this chapter we described how the Earth's daily rotation on its axis was once used to define the standard unit of time. What other types of natural phenomena could serve as alternative time standards?

2. Suppose that the three fundamental standards of the metric system were length, density, and time rather than length, mass, and time. The standard of density in this system is to be defined as that of water. What considerations about water would you need to address to make sure that the standard of density is as accurate as possible?

3. A hand is defined as 4 in.; a foot is defined as 12 in. Why should the hand be any less acceptable as a unit than the foot, which we use all the time?

4. Express the following quantities using the prefixes given in Table 1.4: (a) 3×10^{-4} m (b) 5×10^{-5} s (c) 72×10^{2} g.

5. Suppose that two quantities A and B have different dimensions. Determine which of the following arithmetic operations *could* be physically meaningful: (a) $A + B$ (b) A/B (c) $B - A$ (d) AB.

6. What level of accuracy is implied in an order-of-magnitude calculation?

7. Do an order-of-magnitude calculation for an everyday situation you might encounter. For example, how far do you walk or drive each day?

8. Estimate your age in seconds.

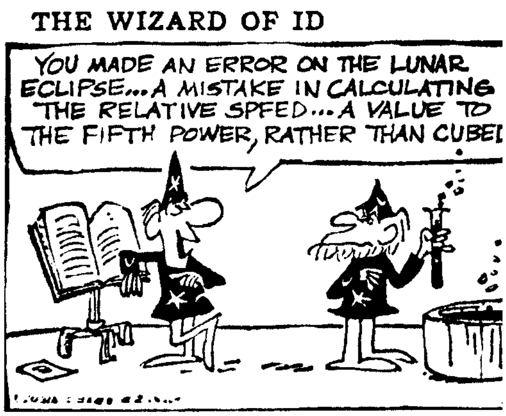
9. Estimate the mass of this textbook in kilograms. If a scale is available, check your estimate.

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging   ☐ = full solution available in the *Student Solutions Manual and Study Guide*
WEB = solution posted at **http://www.saunderscollege.com/physics/**   = Computer useful in solving problem   = Interactive Physics
☐ = paired numerical/symbolic problems

Section 1.3 Density

1. The standard kilogram is a platinum–iridium cylinder 39.0 mm in height and 39.0 mm in diameter. What is the density of the material?
2. The mass of the planet Saturn (Fig. P1.2) is 5.64×10^{26} kg, and its radius is 6.00×10^{7} m. Calculate its density.
3. How many grams of copper are required to make a hollow spherical shell having an inner radius of 5.70 cm and an outer radius of 5.75 cm? The density of copper is 8.92 g/cm^3.
4. What mass of a material with density ρ is required to make a hollow spherical shell having inner radius r_1 and outer radius r_2?
5. Iron has molar mass 55.8 g/mol. (a) Find the volume of 1 mol of iron. (b) Use the value found in (a) to determine the volume of one iron atom. (c) Calculate the cube root of the atomic volume, to have an estimate for the distance between atoms in the solid. (d) Repeat the calculations for uranium, finding its molar mass in the periodic table of the elements in Appendix C.



By permission of John Hart and Field Enterprises, Inc.

Figure P1.2 A view of Saturn from *Voyager 2*. *(Courtesy of NASA)*

6. Two spheres are cut from a certain uniform rock. One has radius 4.50 cm. The mass of the other is five times greater. Find its radius.

WEB **7.** Calculate the mass of an atom of (a) helium, (b) iron, and (c) lead. Give your answers in atomic mass units and in grams. The molar masses are 4.00, 55.9, and 207 g/mol, respectively, for the atoms given.

8. On your wedding day your lover gives you a gold ring of mass 3.80 g. Fifty years later its mass is 3.35 g. As an average, how many atoms were abraded from the ring during each second of your marriage? The molar mass of gold is 197 g/mol.

9. A small cube of iron is observed under a microscope. The edge of the cube is 5.00×10^{-6} cm long. Find (a) the mass of the cube and (b) the number of iron atoms in the cube. The molar mass of iron is 55.9 g/mol, and its density is 7.86 g/cm^3.

10. A structural I-beam is made of steel. A view of its cross-section and its dimensions are shown in Figure P1.10.

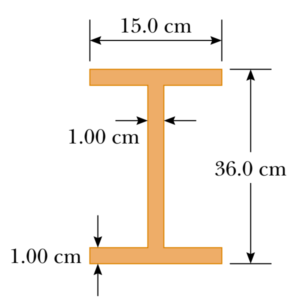


Figure P1.10

(a) What is the mass of a section 1.50 m long? (b) How many atoms are there in this section? The density of steel is 7.56×10^3 kg/m^3.

11. A child at the beach digs a hole in the sand and, using a pail, fills it with water having a mass of 1.20 kg. The molar mass of water is 18.0 g/mol. (a) Find the number of water molecules in this pail of water. (b) Suppose the quantity of water on the Earth is 1.32×10^{21} kg and remains constant. How many of the water molecules in this pail of water were likely to have been in an equal quantity of water that once filled a particular claw print left by a dinosaur?

Section 1.4 Dimensional Analysis

12. The radius r of a circle inscribed in any triangle whose sides are a, b, and c is given by

$$r = [(s - a)(s - b)(s - c)/s]^{1/2}$$

where s is an abbreviation for $(a + b + c)/2$. Check this formula for dimensional consistency.

13. The displacement of a particle moving under uniform acceleration is some function of the elapsed time and the acceleration. Suppose we write this displacement $s = ka^m t^n$, where k is a dimensionless constant. Show by dimensional analysis that this expression is satisfied if $m = 1$ and $n = 2$. Can this analysis give the value of k?

14. The period T of a simple pendulum is measured in time units and is described by

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

where ℓ is the length of the pendulum and g is the free-fall acceleration in units of length divided by the square of time. Show that this equation is dimensionally correct.

15. Which of the equations below are dimensionally correct?
(a) $v = v_0 + ax$
(b) $y = (2\text{ m})\cos(kx)$, where $k = 2\text{ m}^{-1}$

16. Newton's law of universal gravitation is represented by

$$F = \frac{GMm}{r^2}$$

Here F is the gravitational force, M and m are masses, and r is a length. Force has the SI units kg·m/s^2. What are the SI units of the proportionality constant G?

WEB **17.** The consumption of natural gas by a company satisfies the empirical equation $V = 1.50t + 0.008\,00t^2$, where V is the volume in millions of cubic feet and t the time in months. Express this equation in units of cubic feet and seconds. Put the proper units on the coefficients. Assume a month is 30.0 days.

Section 1.5 Conversion of Units

18. Suppose your hair grows at the rate 1/32 in. per day. Find the rate at which it grows in nanometers per second. Since the distance between atoms in a molecule is on the order of 0.1 nm, your answer suggests how rapidly layers of atoms are assembled in this protein synthesis.

19. A rectangular building lot is 100 ft by 150 ft. Determine the area of this lot in m^2.

20. An auditorium measures 40.0 m $\times$ 20.0 m $\times$ 12.0 m. The density of air is 1.20 kg/m^3. What are (a) the volume of the room in cubic feet and (b) the weight of air in the room in pounds?

21. Assume that it takes 7.00 min to fill a 30.0-gal gasoline tank. (a) Calculate the rate at which the tank is filled in gallons per second. (b) Calculate the rate at which the tank is filled in cubic meters per second. (c) Determine the time, in hours, required to fill a 1-cubic-meter volume at the same rate. (1 U.S. gal = 231 in.^{3})

22. A creature moves at a speed of 5.00 furlongs per fortnight (not a very common unit of speed). Given that 1 furlong = 220 yards and 1 fortnight = 14 days, determine the speed of the creature in meters per second. What kind of creature do you think it might be?

23. A section of land has an area of 1 mi^2 and contains 640 acres. Determine the number of square meters in 1 acre.

24. A quart container of ice cream is to be made in the form of a cube. What should be the length of each edge in centimeters? (Use the conversion 1 gal = 3.786 L.)

25. A solid piece of lead has a mass of 23.94 g and a volume of 2.10 cm^3. From these data, calculate the density of lead in SI units (kg/m^3).

26. An astronomical unit (AU) is defined as the average distance between the Earth and the Sun. (a) How many astronomical units are there in one lightyear? (b) Determine the distance from the Earth to the Andromeda galaxy in astronomical units.

27. The mass of the Sun is 1.99×10^{30} kg, and the mass of an atom of hydrogen, of which the Sun is mostly composed, is 1.67×10^{-27} kg. How many atoms are there in the Sun?

28. (a) Find a conversion factor to convert from miles per hour to kilometers per hour. (b) In the past, a federal law mandated that highway speed limits would be 55 mi/h. Use the conversion factor of part (a) to find this speed in kilometers per hour. (c) The maximum highway speed is now 65 mi/h in some places. In kilometers per hour, how much of an increase is this over the 55-mi/h limit?

29. At the time of this book's printing, the U. S. national debt is about \$6 trillion. (a) If payments were made at the rate of \$1 000/s, how many years would it take to pay off a \$6-trillion debt, assuming no interest were charged? (b) A dollar bill is about 15.5 cm long. If six trillion dollar bills were laid end to end around the Earth's equator, how many times would they encircle the Earth? Take the radius of the Earth at the equator to be 6 378 km. (*Note:* Before doing any of these calculations, try to guess at the answers. You may be very surprised.)

30. (a) How many seconds are there in a year? (b) If one micrometeorite (a sphere with a diameter of 1.00×10^{-6} m) strikes each square meter of the Moon each second, how many years will it take to cover the Moon to a depth of 1.00 m? (*Hint:* Consider a cubic box on the Moon 1.00 m on a side, and find how long it will take to fill the box.)

WEB **31.** One gallon of paint (volume = 3.78×10^{-3} m^3) covers an area of 25.0 m^2. What is the thickness of the paint on the wall?

32. A pyramid has a height of 481 ft, and its base covers an area of 13.0 acres (Fig. P1.32). If the volume of a pyramid is given by the expression $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height, find the volume of this pyramid in cubic meters. (1 acre = 43 560 ft^2)

Figure P1.32 Problems 32 and 33.

33. The pyramid described in Problem 32 contains approximately two million stone blocks that average 2.50 tons each. Find the weight of this pyramid in pounds.

34. Assuming that 70% of the Earth's surface is covered with water at an average depth of 2.3 mi, estimate the mass of the water on the Earth in kilograms.

35. The amount of water in reservoirs is often measured in acre-feet. One acre-foot is a volume that covers an area of 1 acre to a depth of 1 ft. An acre is an area of 43 560 ft^2. Find the volume in SI units of a reservoir containing 25.0 acre-ft of water.

36. A hydrogen atom has a diameter of approximately 1.06×10^{-10} m, as defined by the diameter of the spherical electron cloud around the nucleus. The hydrogen nucleus has a diameter of approximately 2.40×10^{-15} m. (a) For a scale model, represent the diameter of the hydrogen atom by the length of an American football field (100 yards = 300 ft), and determine the diameter of the nucleus in millimeters. (b) The atom is how many times larger in volume than its nucleus?

37. The diameter of our disk-shaped galaxy, the Milky Way, is about 1.0×10^5 lightyears. The distance to Messier 31—which is Andromeda, the spiral galaxy nearest to the Milky Way—is about 2.0 million lightyears. If a scale model represents the Milky Way and Andromeda galaxies as dinner plates 25 cm in diameter, determine the distance between the two plates.

38. The mean radius of the Earth is 6.37×10^6 m, and that of the Moon is 1.74×10^8 cm. From these data calculate (a) the ratio of the Earth's surface area to that of the Moon and (b) the ratio of the Earth's volume to that of the Moon. Recall that the surface area of a sphere is $4\pi r^2$ and that the volume of a sphere is $\frac{4}{3}\pi r^3$.

WEB **39.** One cubic meter ($1.00\ m^3$) of aluminum has a mass of 2.70×10^3 kg, and $1.00\ m^3$ of iron has a mass of 7.86×10^3 kg. Find the radius of a solid aluminum sphere that balances a solid iron sphere of radius 2.00 cm on an equal-arm balance.

40. Let ρ_{Al} represent the density of aluminum and ρ_{Fe} that of iron. Find the radius of a solid aluminum sphere that balances a solid iron sphere of radius r_{Fe} on an equal-arm balance.

Section 1.6 Estimates and Order-of-Magnitude Calculations

WEB **41.** Estimate the number of Ping-Pong balls that would fit into an average-size room (without being crushed). In your solution state the quantities you measure or estimate and the values you take for them.

42. McDonald's sells about 250 million packages of French fries per year. If these fries were placed end to end, estimate how far they would reach.

43. An automobile tire is rated to last for 50 000 miles. Estimate the number of revolutions the tire will make in its lifetime.

44. Approximately how many raindrops fall on a 1.0-acre lot during a 1.0-in. rainfall?

45. Grass grows densely everywhere on a quarter-acre plot of land. What is the order of magnitude of the number of blades of grass on this plot of land? Explain your reasoning. ($1\ \text{acre} = 43\ 560\ ft^2$.)

46. Suppose that someone offers to give you $1 billion if you can finish counting it out using only one-dollar bills. Should you accept this offer? Assume you can count one bill every second, and be sure to note that you need about 8 hours a day for sleeping and eating and that right now you are probably at least 18 years old.

47. Compute the order of magnitude of the mass of a bathtub half full of water and of the mass of a bathtub half full of pennies. In your solution, list the quantities you take as data and the value you measure or estimate for each.

48. Soft drinks are commonly sold in aluminum containers. Estimate the number of such containers thrown away or recycled each year by U.S. consumers. Approximately how many tons of aluminum does this represent?

49. To an order of magnitude, how many piano tuners are there in New York City? The physicist Enrico Fermi was famous for asking questions like this on oral Ph.D. qualifying examinations and for his own facility in making order-of-magnitude calculations.

Section 1.7 Significant Figures

50. Determine the number of significant figures in the following measured values: (a) 23 cm (b) 3.589 s (c) 4.67×10^3 m/s (d) 0.003 2 m.

51. The radius of a circle is measured to be 10.5 ± 0.2 m. Calculate the (a) area and (b) circumference of the circle and give the uncertainty in each value.

52. Carry out the following arithmetic operations: (a) the sum of the measured values 756, 37.2, 0.83, and 2.5; (b) the product $0.003\ 2 \times 356.3$; (c) the product $5.620 \times \pi$.

53. The radius of a solid sphere is measured to be (6.50 ± 0.20) cm, and its mass is measured to be (1.85 ± 0.02) kg. Determine the density of the sphere in kilograms per cubic meter and the uncertainty in the density.

54. How many significant figures are in the following numbers: (a) 78.9 ± 0.2, (b) 3.788×10^9, (c) 2.46×10^{-6}, and (d) 0.005 3?

55. A farmer measures the distance around a rectangular field. The length of the long sides of the rectangle is found to be 38.44 m, and the length of the short sides is found to be 19.5 m. What is the total distance around the field?

56. A sidewalk is to be constructed around a swimming pool that measures (10.0 ± 0.1) m by (17.0 ± 0.1) m. If the sidewalk is to measure (1.00 ± 0.01) m wide by (9.0 ± 0.1) cm thick, what volume of concrete is needed, and what is the approximate uncertainty of this volume?

ADDITIONAL PROBLEMS

57. In a situation where data are known to three significant digits, we write 6.379 m = 6.38 m and 6.374 m = 6.37 m. When a number ends in 5, we arbitrarily choose to write 6.375 m = 6.38 m. We could equally well write 6.375 m = 6.37 m, "rounding down" instead of "rounding up," since we would change the number 6.375 by equal increments in both cases. Now consider an order-of-magnitude estimate, in which we consider factors rather than increments. We write $500\ m \sim 10^3$ m because 500 differs from 100 by a factor of 5 whereas it differs from 1000 by only a factor of 2. We write $437\ m \sim 10^3$ m and $305\ m \sim 10^2$ m. What distance differs from 100 m and from 1000 m by equal factors, so that we could equally well choose to represent its order of magnitude either as $\sim 10^2$ m or as $\sim 10^3$ m?

58. When a droplet of oil spreads out on a smooth water surface, the resulting "oil slick" is approximately one molecule thick. An oil droplet of mass 9.00×10^{-7} kg and density $918\ kg/m^3$ spreads out into a circle of radius 41.8 cm on the water surface. What is the diameter of an oil molecule?

59. The basic function of the carburetor of an automobile is to "atomize" the gasoline and mix it with air to promote rapid combustion. As an example, assume that 30.0 cm^3 of gasoline is atomized into N spherical droplets, each with a radius of 2.00×10^{-5} m. What is the total surface area of these N spherical droplets?

60. In physics it is important to use mathematical approximations. Demonstrate for yourself that for small angles ($< 20°$)

$$\tan \alpha \approx \sin \alpha \approx \alpha = \pi\alpha'/180°$$

where α is in radians and α' is in degrees. Use a calculator to find the largest angle for which $\tan \alpha$ may be approximated by $\sin \alpha$ if the error is to be less than 10.0%.

61. A high fountain of water is located at the center of a circular pool as in Figure P1.61. Not wishing to get his feet wet, a student walks around the pool and measures its circumference to be 15.0 m. Next, the student stands at the edge of the pool and uses a protractor to gauge the angle of elevation of the top of the fountain to be 55.0°. How high is the fountain?

Figure P1.61

62. Assume that an object covers an area A and has a uniform height h. If its cross-sectional area is uniform over its height, then its volume is given by $V = Ah$. (a) Show that $V = Ah$ is dimensionally correct. (b) Show that the volumes of a cylinder and of a rectangular box can be written in the form $V = Ah$, identifying A in each case. (Note that A, sometimes called the "footprint" of the object, can have any shape and that the height can be replaced by average thickness in general.)

63. A useful fact is that there are about $\pi \times 10^7$ s in one year. Find the percentage error in this approximation, where "percentage error" is defined as

$$\frac{|\text{Assumed value} - \text{true value}|}{\text{True value}} \times 100\%$$

64. A crystalline solid consists of atoms stacked up in a repeating lattice structure. Consider a crystal as shown in Figure P1.64a. The atoms reside at the corners of cubes of side $L = 0.200$ nm. One piece of evidence for the regular arrangement of atoms comes from the flat surfaces along which a crystal separates, or "cleaves," when it is broken. Suppose this crystal cleaves along a face diagonal, as shown in Figure P1.64b. Calculate the spacing d between two adjacent atomic planes that separate when the crystal cleaves.

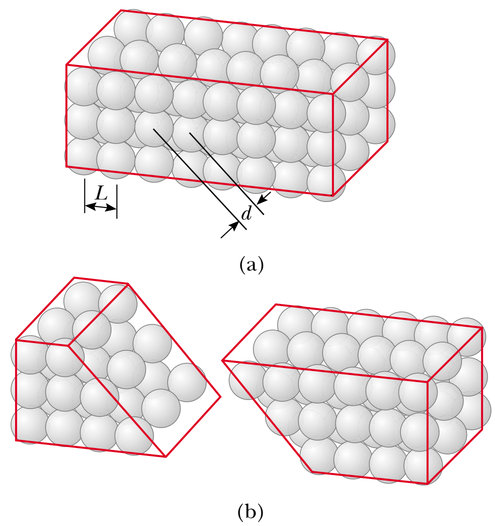


Figure P1.64

65. A child loves to watch as you fill a transparent plastic bottle with shampoo. Every horizontal cross-section of the bottle is a circle, but the diameters of the circles all have different values, so that the bottle is much wider in some places than in others. You pour in bright green shampoo with constant volume flow rate 16.5 cm^3/s. At what rate is its level in the bottle rising (a) at a point where the diameter of the bottle is 6.30 cm and (b) at a point where the diameter is 1.35 cm?

66. As a child, the educator and national leader Booker T. Washington was given a spoonful (about 12.0 cm^3) of molasses as a treat. He pretended that the quantity increased when he spread it out to cover uniformly all of a tin plate (with a diameter of about 23.0 cm). How thick a layer did it make?

67. Assume there are 100 million passenger cars in the United States and that the average fuel consumption is 20 mi/gal of gasoline. If the average distance traveled by each car is 10 000 mi/yr, how much gasoline would be saved per year if average fuel consumption could be increased to 25 mi/gal?

68. One cubic centimeter of water has a mass of 1.00×10^{-3} kg. (a) Determine the mass of 1.00 m^3 of water. (b) Assuming biological substances are 98% water, estimate the mass of a cell that has a diameter of 1.0 μm, a human kidney, and a fly. Assume that a kidney is roughly a sphere with a radius of 4.0 cm and that a fly is roughly a cylinder 4.0 mm long and 2.0 mm in diameter.

69. The distance from the Sun to the nearest star is 4×10^{16} m. The Milky Way galaxy is roughly a disk of diameter $\sim 10^{21}$ m and thickness $\sim 10^{19}$ m. Find the order of magnitude of the number of stars in the Milky Way. Assume the 4×10^{16}-m distance between the Sun and the nearest star is typical.

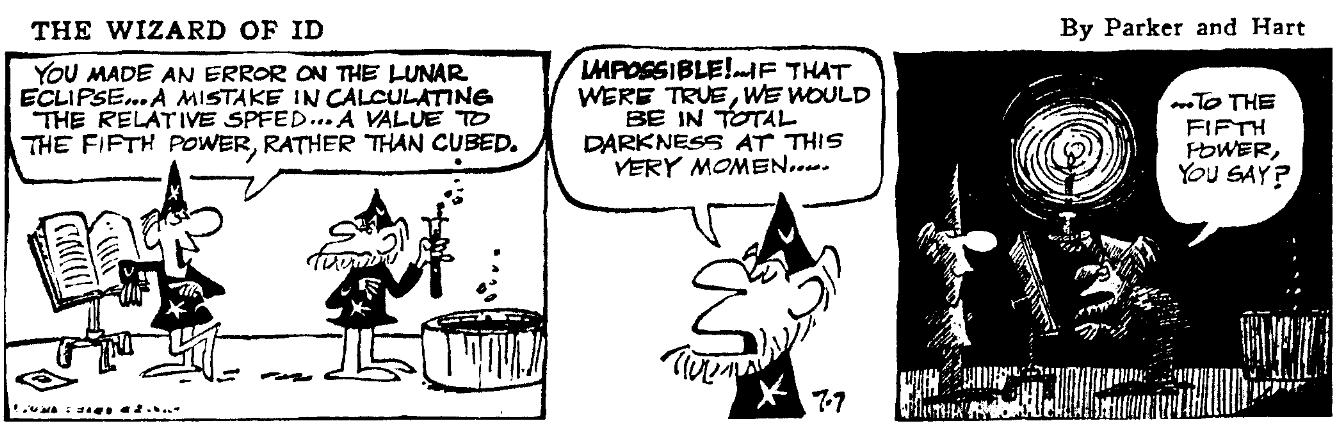
70. The data in the following table represent measurements of the masses and dimensions of solid cylinders of aluminum, copper, brass, tin, and iron. Use these data to calculate the densities of these substances. Compare your results for aluminum, copper, and iron with those given in Table 1.5.

Substance	Mass (g)	Diameter (cm)	Length (cm)
Aluminum	51.5	2.52	3.75
Copper	56.3	1.23	5.06
Brass	94.4	1.54	5.69
Tin	69.1	1.75	3.74
Iron	216.1	1.89	9.77

ANSWERS TO QUICK QUIZZES

1.1 False. Dimensional analysis gives the units of the proportionality constant but provides no information about its numerical value. For example, experiments show that doubling the radius of a solid sphere increases its mass 8-fold, and tripling the radius increases the mass 27-fold. Therefore, its mass is proportional to the cube of its radius. Because $m \propto r^3$ we can write $m = kr^3$. Dimensional analysis shows that the proportionality constant k must have units kg/m^3, but to determine its numerical value requires either experimental data or geometrical reasoning.

1.2 Reporting all these digits implies you have determined the location of the center of the chair's seat to the nearest $\pm$ 0.000 000 000 1 m. This roughly corresponds to being able to count the atoms in your meter stick because each of them is about that size! It would probably be better to record the measurement as 1.044 m: this indicates that you know the position to the nearest millimeter, assuming the meter stick has millimeter markings on its scale.



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