**Prof. Dr. H. Brenner** **Osnabrück WS 2011/2012**

# Mathematics for natural sciences I

## Exercise sheet 25

### Warm-up-exercises

Exercise 25.1. Compute the definite integral

$$\int_0^{\sqrt{\pi}} x \sin x^2 \, dx \, .$$

In the following exercises, which involve the determination of antiderivative functions, consider an appropriate domain of definition.

Exercise 25.2. Determine an antiderivative of the function

$$\tan x \, .$$

Exercise 25.3. Determine an antiderivative of the function

$$x^n \cdot \ln x \, .$$

Exercise 25.4. Determine an antiderivative of the function

$$e^{\sqrt{x}} \, .$$

Exercise 25.5. Determine an antiderivative of the function

$$\frac{x^3}{\sqrt[5]{x^4+2}} \, .$$

Exercise 25.6. Determine an antiderivative of the function

$$\frac{\sin^2 x}{\cos^2 x} \, .$$

Exercise 25.7. Determine for which $a \in \mathbb{R}$ the function

$$a \longmapsto \int_{-1}^{2} at^2 - a^2 t \, dt$$

has a maximum or a minimum.

EXERCISE 25.8. According to recent studies the student's attention skills during the day are described by the following function

$$[8,18] \longrightarrow \mathbb{R},\ x \longmapsto f(x) = -x^2 + 25x - 100.$$

Here $x$ is the time in hours and $y = f(x)$ is the attention measured in micro-credit points per second. When should one start a one and a half hour lecture, such that the total attention skills are optimal? How many micro-credit points will be added during this lecture?

EXERCISE 25.9. Let $I$ be a real interval and let

$$f : I \longrightarrow \mathbb{R}$$

be a continuous function with antiderivative $F$. Let $G$ be an antiderivative of $F$ and let $b, c \in \mathbb{R}$. Determine an antiderivative of the function

$$(bt + c) \cdot f(t)\,.$$

EXERCISE 25.10. Let $n \in \mathbb{N}_+$. Determine an antiderivative of the function

$$\mathbb{R}_+ \longrightarrow \mathbb{R}_+,\ x \longmapsto x^{1/n},$$

using the antiderivative of $x^n$ and Theorem 25.4.

EXERCISE 25.11. Determine an antiderivative of the natural logarithm function using the antiderivative of its inverse function.

EXERCISE 25.12. Let

$$f : [a,b] \longrightarrow [c,d]$$

be a bijective, continuous differentiable function. Prove the formula for the antiderivative of the inverse function by the integral

$$\int_a^b f^{-1}(y)dy$$

using the substitution $y = f(x)$ and then integration by parts.

EXERCISE 25.13. Compute by an appropriate substitution an antiderivative of

$$\sqrt{3x^2 + 5x - 4}\,.$$

EXERCISE 25.14. Compute the definite integral of the function

$$f : \mathbb{R} \longrightarrow \mathbb{R},\ x \longmapsto f(x) = 2x^3 + 3e^x - \sin x\,,$$

on $[-1, 0]$.

EXERCISE 25.15. Compute the definite integral of the function

$$f : \mathbb{R}_+ \longrightarrow \mathbb{R},\ x \longmapsto f(x) = \sqrt{x} - \frac{1}{\sqrt{x}} + \frac{1}{2x+3} - e^{-x},$$

on $[1, 4]$.

**Hand-in-exercises**

EXERCISE 25.16. (4 points)

Compute the definite integral $\int_0^8 f(t)\,dt$, where the function $f$ is

$$f(t) = \begin{cases} t+1, \text{ if } 0 \leq t \leq 2\,, \\ t^2 - 6t + 11, \text{ if } 2 < t \leq 5\,, \\ 6, \text{ if } 5 < t \leq 6\,, \\ -2t + 18\,, \text{ if } 6 < t \leq 8\,. \end{cases}$$

EXERCISE 25.17. (3 points)

Determine an antiderivative of the function

$$x^3 \cdot \cos x - x^2 \cdot \sin x\ .$$

EXERCISE 25.18. (2 points)

Determine an antiderivative of the function

$$\arcsin x\ .$$

EXERCISE 25.19. (4 points)

Determine an antiderivative of the function

$$\sin(\ln x)\ .$$

EXERCISE 25.20. (5 points)

Determine an antiderivative of the function

$$e^x \cdot \frac{x^2+1}{(x+1)^2}\,.$$

EXERCISE 25.21. (5 points)

Let $I$ be a real interval and let

$$f : I \longrightarrow \mathbb{R}$$

be a continuous function with antiderivative $F$. Let $G$ be an antiderivative of $F$ and $H$ an antiderivative of $G$. Let $a, b, c \in \mathbb{R}$. Determine an antiderivative of the function

$$(at^2 + bt + c) \cdot f(t) \, .$$