Prof. Dr. H. Brenner **Osnabrück WS 2011/2012**

Mathematics for natural sciences I

Exercise sheet 28

Warm-up-exercises

EXERCISE 28.1. Let $x \in \mathbb{R}$ and consider the function

$$f : \mathbb{R}_+ \longrightarrow \mathbb{R},\, t \longmapsto f(t) = t^x e^{-t}.$$

Determine the extrema of this function.

EXERCISE 28.2. Prove that for the factorial function the relationship

$$\operatorname{Fak}\left(\frac{2k-1}{2}\right) = \frac{\prod_{i=1}^{k}(2i-1)}{2^k} \cdot \sqrt{\pi}$$

holds.

EXERCISE 28.3. a) Prove that for $x \geq 1$ the estimate

$$\int_1^\infty t^x e^{-t}\, dt \leq 1$$

holds.

b) Prove that the function $H(x)$ defined by

$$H(x) = \int_1^\infty t^x e^{-t}\, dt$$

for $x \geq 1$ is increasing.

c) Prove that $10! \geq e^{11} + 1$.

d) Prove that for the factorial function for $x \geq 10$ the estimate

$$\operatorname{Fak}(x) \geq e^x$$

holds.

EXERCISE 28.4. Solve the initial value problem

$$y' = \sin t \text{ with } y(\pi) = 7\,.$$

EXERCISE 28.5. Solve the initial value problem

$$y' = 3t^2 - 4t + 7 \text{ with } y(2) = 5\,.$$

Exercise 28.6. Find all the solutions for the ordinary differential equation

$$y' = y\,.$$

Exercise 28.7. Make clear and mathematically clear to yourself that in a location-independent differential equation (i.e. $f(t, y)$ does not depend on y) the difference between two solutions y_1 and y_2 does not depend on time, that is $y_1(t) - y_2(t)$ is constant. Show with an example that this may not happen in a time-independent differential equation.

Hand-in-exercises

Exercise 28.8. (2 points)

Prove that for the factorial function the relationship

$$\mathrm{Fak}\,(x) = \int_0^1 (-\ln t)^x\, dt$$

holds.

Exercise 28.9. (3 points)

Solve the initial value problem

$$y' = 3t^3 - 2t + 5 \text{ with } y(3) = 4\,.$$

Exercise 28.10. (3 points)

Find a solution for the ordinary differential equation

$$y' = t + y\,.$$

Exercise 28.11. (4 points)

Solve the initial value problem

$$y' = \frac{t^3}{t^2+1} \text{ with } y(1) = 2\,.$$