Functor (1A)

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This document was produced by using OpenOffice.

Based on

http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

Haskell in 5 steps
https://wiki.haskell.org/Haskell_in_5_steps

Typeclasses

Typeclasses are like interfaces

defines some behavior
  comparing for *equality*
  comparing for *ordering*
  enumeration

Instances of that typeclass
  types possessing such behavior

Such *behavior* is defined by
- **function definition**
- **type declaration**

```
(==) :: a -> a -> Bool        - a type declaration
x == y = not (x /= y)         - a function definition
```

a **type** is an **instance** of a **typeclass** implies
  the functions defined (implemented)
  by the typeclass with that type can be used

No relation with classes in Java or C++

http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

Car Type Example

the Eq typeclass

defines the functions **==** and ***/=***

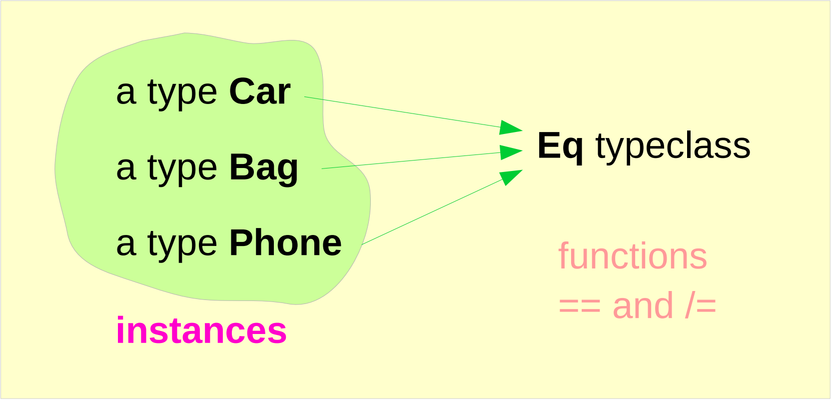
a type Car

comparing two cars c1 and c2 with the equality function ==

The Car type is an **instance** of Eq **typeclass**

Instances : various types

Typeclass : a group or a class of these similar types



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TrafficLight Type Example (1)

```
class Eq a where
    (==) :: a -> a -> Bool         - a type declaration
    (/=) :: a -> a -> Bool         - a type declaration
    x == y = not (x /= y)          - a function definition
    x /= y = not (x == y)          - a function definition
```

```
data TrafficLight = Red | Yellow | Green
```

```
instance Eq TrafficLight where
    Red    == Red     = True
    Green  == Green   = True
    Yellow == Yellow  = True
    _ == _            = False
```

```
ghci> Red == Red
True
ghci> Red == Yellow
False
ghci> Red `elem` [Red, Yellow, Green]
True
```

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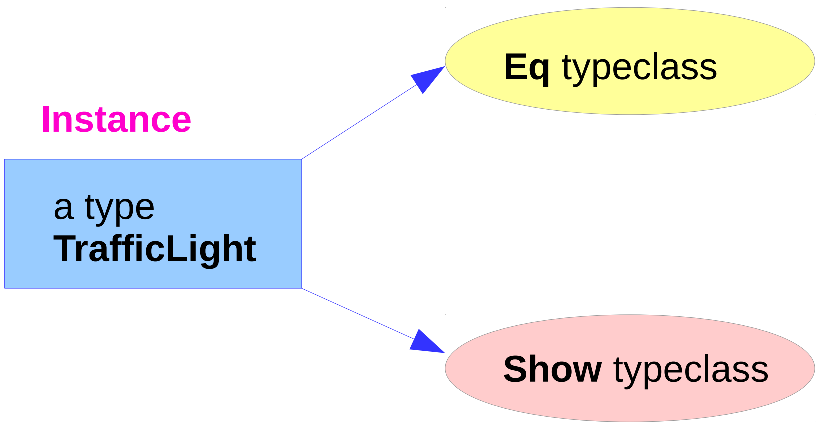
TrafficLight Type Example (2)

```
class Show  a where
    show :: a  -> String
    * * *
```

- a type declaration

```
data TrafficLight = Red | Yellow | Green
```

```
instance Show TrafficLight where
     show Red = "Red light"
     show Yellow = "Yellow light"
     show Green = "Green light"
```




```
ghci> [Red, Yellow, Green]
[Red light,Yellow light,Green light]
```

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Class Constraints

```
class (Eq a) => Num a where
  ...
```

```
class Num a where
  ...
```

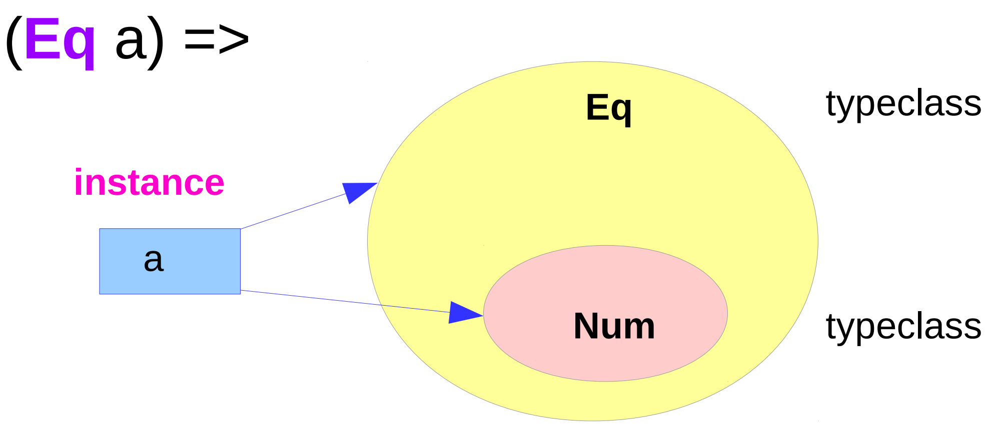
class constraint on a class declaration

an instance of **Eq**
before being an instance of **Num**

the required function bodies can be defined in

- the class declaration
- an instance declarations,

we can safely use == because a is a part of **Eq**



Num : a subclass of **Eq**

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Class Constraints : class & instance declarations

class constraints in **class declarations**

to make a typeclass a **subclass** of another typeclass

subclass

```
class (Eq a) => Num a where
   …
```

class constraints in **instance declarations**

to express **requirements** about the contents of some type.

requirements

```
instance (Eq x, Eq y) => Eq (Pair x y) where
  Pair x0 y0 == Pair x1 y1 = x0 == x1 && y0 == y1
```

http://cmsc-16100.cs.uchicago.edu/2016/Lectures/07-type-classes.php

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Class constraints in instance declaration examples

```
instance (Eq m) => Eq (Maybe m) where
    Just x    ==  Just y        =    x == y   ⟸ Eq m
    Nothing  ==  Nothing        =    True
         _ == _                 =    False
```

```
instance (Eq x, Eq y) => Eq (Pair x y) where
    Pair x0 y0 == Pair x1 y1 = x0 == x1 && y0 == y1
               ⇑                   ⇑            ⇑
          Eq (Pair x y)          Eq x         Eq y
```

Derived instance

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A Concrete Type and a Type Constructor

a : a concrete type

Maybe : not a concrete type
: a type constructor that takes one parameter
produces a concrete type.

Maybe a : a concrete type

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Functor typeclass

the **Functor typeclass** is basically for things that can be *mapped over*

ex) mapping over lists

the list type is part of the Functor typeclass

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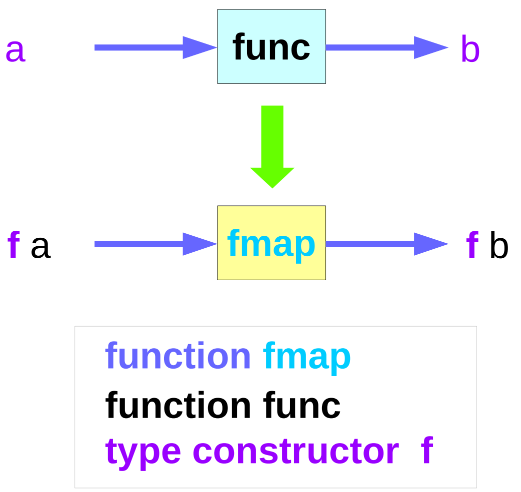
Functor typeclass

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```



The **Functor typeclass**

defines one function, **fmap**

no default implementation

the **type variable f**

not a concrete type (a concrete type can hold a value)

a **type constructor** taking one type parameter

Maybe Int : a concrete type

Maybe : a type constructor that takes one type as the parameter

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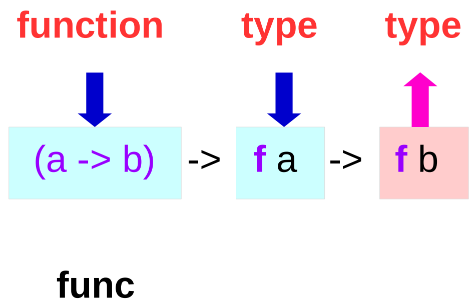
Function map & fmap

```
class Functor f where
    fmap :: (a -> b) -> f a -> f b
```



fmap takes

- a function from one type to another (a -> b)
- a Functor f applied with one type (f a)

fmap returns

- a Functor f applied with another type (f b)

```
map :: (a -> b) -> [a] -> [b]
```

map takes

- a function from one type to another (* 2)
- take a list of one type [1, 2, 3]
- returns a list of another type [2, 4, 6]

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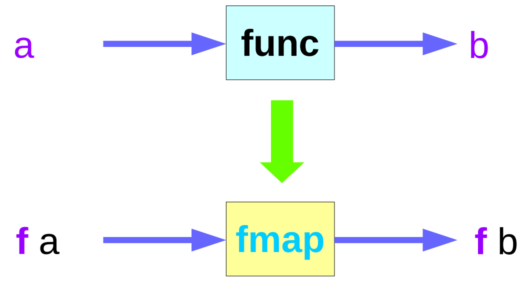
List : an instance of the Functor typeclass

```
class Functor f where
    fmap :: (a -> b) -> f a -> f b
```

```
map :: (a -> b) -> [a] -> [b]
```

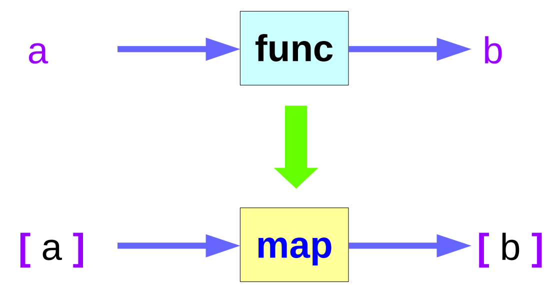


function fmap
function func
type constructor f

map is just a **fmap** that works only on **list**s

a **list** is an **instance** of the **Functor typeclass**.

```
instance Functor [ ] where
    fmap = map
```



f : a type constructor that takes one type

[ ] : a type constructor that takes one type

[a] : a concrete type ([Int], [String] or [[String]])

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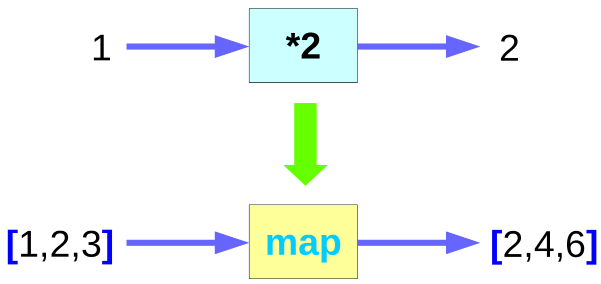
List Examples

```
class Functor f where
    fmap :: (a -> b) -> f a -> f b
```

```
map :: (a -> b) -> [a] -> [b]
```

```
instance Functor [ ] where
    fmap = map
```

```
map :: (a -> b) -> [a] -> [b]
ghci> fmap (*2) [1..3]
[2,4,6]
ghci> map (*2) [1..3]
[2,4,6]
```



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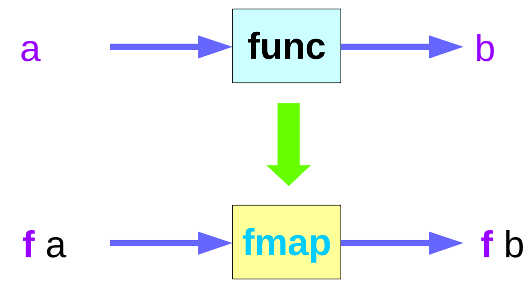
Maybe : an instance of the Functor typeclass

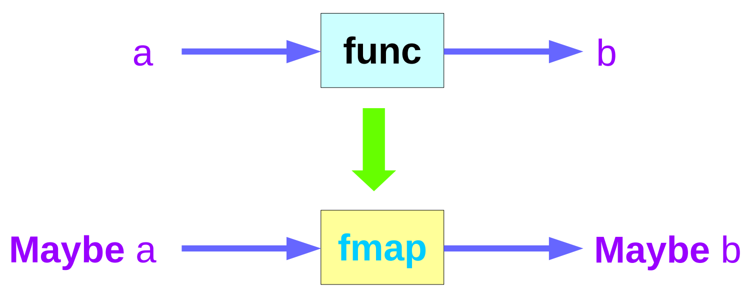
```
class Functor f where
    fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where
    fmap func (Just x) = Just (func  x)
    fmap func Nothing = Nothing
```

f	⟺	**Maybe**
f a	⟺	**Maybe** a
f b	⟺	**Maybe** b

(a -> b)	⟺	**func**





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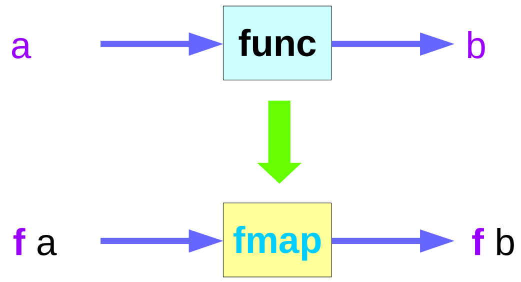
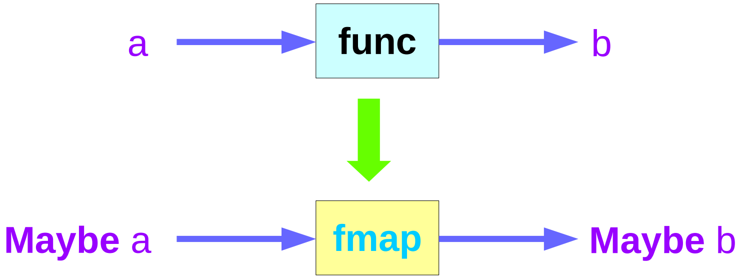
Maybe : a type constructor

```
class Functor f where
    fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where
    fmap func (Just x) = Just (func  x)
    fmap func Nothing = Nothing
```





f : a **type variable**

f : a **type constructor** taking one type parameter

Maybe : an **instance** of **Functor** typeclass

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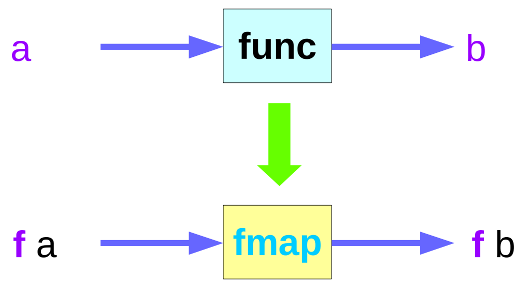
Maybe : an argument to fmap, together with a

```
class Functor f where
    fmap :: (a -> b) -> f a -> f b
```

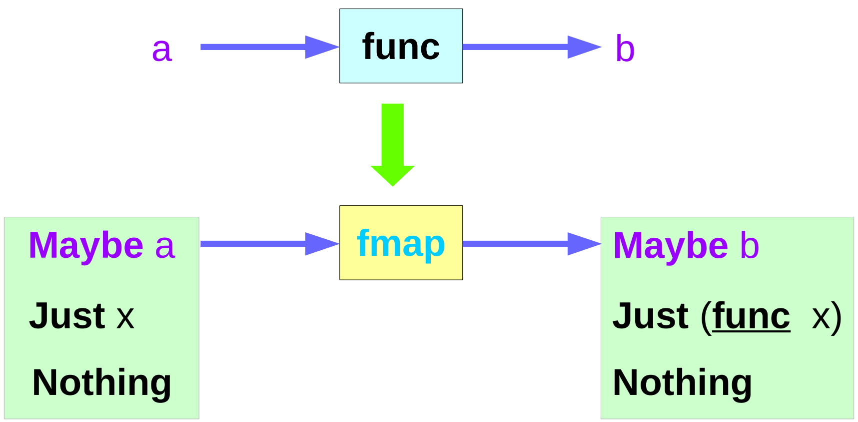
```
instance Functor Maybe where
    fmap func (Just x) = Just (func  x)
    fmap func Nothing = Nothing
```




```
fmap :: (a -> b) -> f a -> f b

fmap func (Just x) = Just (func  x)

fmap func Nothing = Nothing
```



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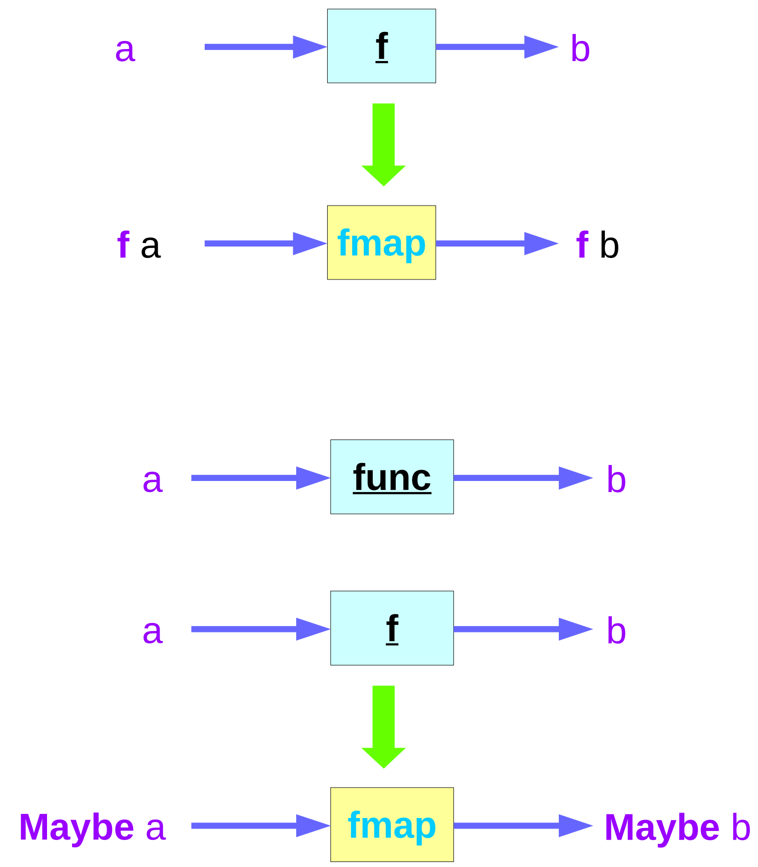
A function argument to fmap and a Functor f

```
class Functor f where
    fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where
    fmap func (Just x) = Just (func x)
    fmap func Nothing = Nothing
```

```
instance Functor Maybe where
    fmap f (Just x) = Just ( f x)
    fmap f Nothing = Nothing
```



f is different from the type constructor f

func : a -> b

f : a -> b

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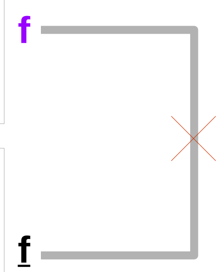
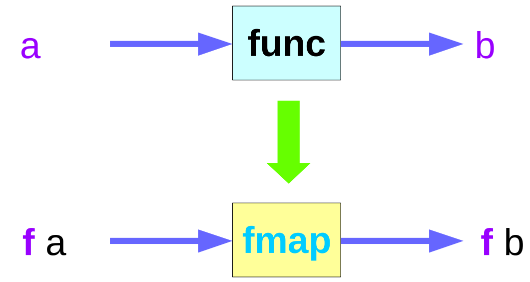
Maybe Examples (1)

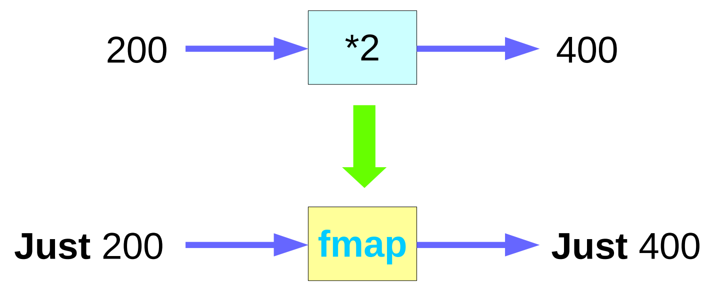
```
class Functor f where
    fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where
    fmap f (Just x) = Just (f x)
    fmap f Nothing = Nothing
```






```
ghci> fmap (*2) (Just 200)
Just 400
ghci> fmap (*2) Nothing
Nothing
```



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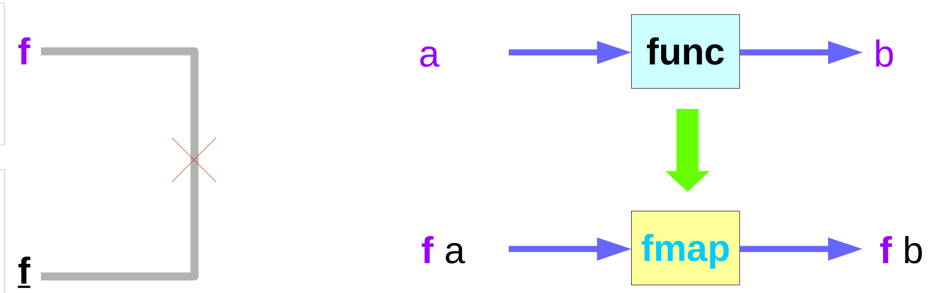
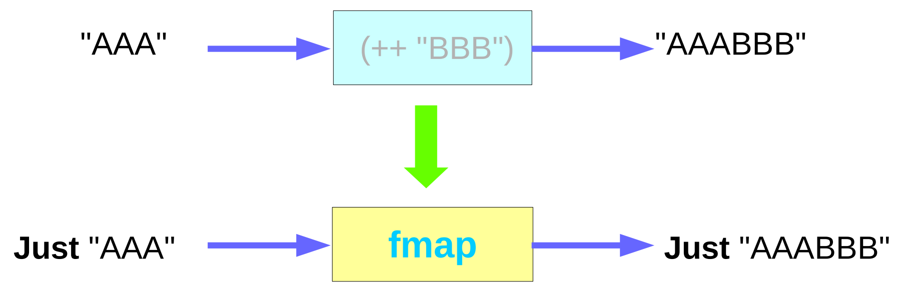
Maybe Examples (2)

```
class Functor f where
    fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where
    fmap f (Just x) = Just (f x)
    fmap f Nothing = Nothing
```




```
ghci> fmap (++ "BBB") (Just "AAA")
Just "AAABBB"
ghci> fmap (++ "BBB") Nothing
Nothing
```



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Maybe as a functor

Functor typeclass:

- transforming one type to another
- transforming operations of one type to those of another

Maybe a is an instance of a **functor** type class

Functor provides **fmap** method

maps functions of the base type (such as Integer)
to *functions* of the lifted type (such as Maybe Integer).

https://stackoverflow.com/questions/18808258/what-does-the-just-syntax-mean-in-haskell

Maybe as a functor

A *function* **f** transformed with **fmap**
can work on a **Maybe** value

```
case maybeVal of
  Nothing  -> Nothing         -- there is nothing, so just return Nothing
  Just val -> Just (f val)    -- there is a value, so apply the function to it
```

```
father :: Person -> Maybe Person
mother :: Person -> Maybe Person
```

```
       f :: Int           -> Int
  fmap f :: Maybe Integer -> Maybe Integer
```

a **Maybe** **Integer** value: **m_x**

fmap f **m_x**

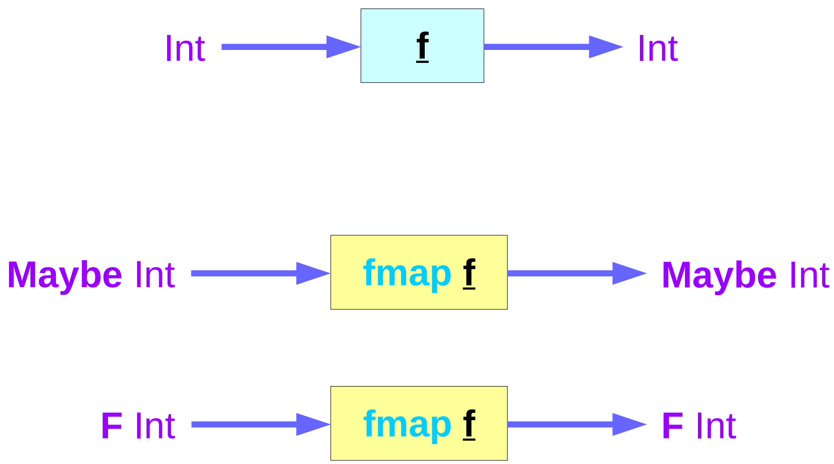
https://stackoverflow.com/questions/18808258/what-does-the-just-syntax-mean-in-haskell

Transforming operations

Functor provides **fmap** method

maps functions of the base type (such as Integer)

to *functions* of the lifted type (such as Maybe Integer).



https://stackoverflow.com/questions/18808258/what-does-the-just-syntax-mean-in-haskell

Maybe as a functor

m_x : a Maybe Integer value (Just 101, Nothing, …)
f :: Int -> Int

you can do **fmap** **f** m_x
to apply the function **f** directly to the Maybe Integer
without worrying whether it is Nothing or not

```
class Functor f where
    fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where
    fmap f (Just x) = Just (f x)
    fmap f Nothing = Nothing
```

Function of (a -> b)

fmap f m_x

lifted type

A Functor f applied with
one type **f** a or **f** b

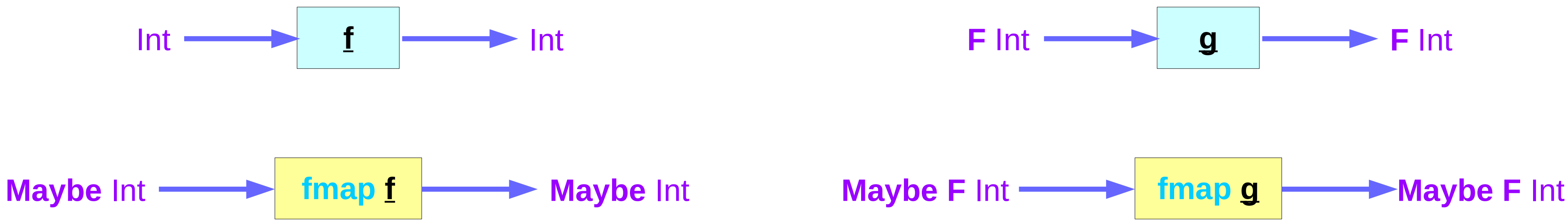
https://stackoverflow.com/questions/18808258/what-does-the-just-syntax-mean-in-haskell

Maybe as a functor

Can apply a whole chain of
lifted Integer -> Integer functions
to Maybe Integer values
and only have to worry about explicitly checking for Nothing
once when you're finished.

```
class Functor F where
    fmap :: (a -> b) -> F a -> F b
```

```
instance Functor Maybe where
    fmap f (Just x) = Just (f x)
    fmap f Nothing = Nothing
```



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Maybe class

The Maybe type definition

```
data Maybe a = Just a | Nothing
  deriving (Eq, Ord)
```

Maybe is
- an instance of **Eq** and **Ord** (as a base type)
- an instance of **Functor**
- an instance of **Monad**

For Functor, the fmap function f
- moves inside the Just constructor and
- is identity on the Nothing constructor.

For Monad,
- the bind operation passes through Just, while
- Nothing will force the result to always be Nothing.

https://wiki.haskell.org/Maybe

Maybe as a Monad

```
f::Int -> Maybe Int
f 0 = Nothing
f x = Just x
```

if x==0 then Nothing else Just x

```
g :: Int -> Maybe Int
g 100 = Nothing
g x     = Just x
```

if x==100 then Nothing else Just x

```
h ::Int -> Maybe Int
h x = case f x of
          Just n -> g n
          Nothing -> Nothing
```

if f x==Nothing then Nothing else g n

```
h' :: Int -> Maybe Int
h' x = do n <- f x
           g n
```

g (f x)

h & h' give the same results
h 0 = h' 0 = h 100 = h' 100 = Nothing;
h x = h' x = Just x

https://wiki.haskell.org/Maybe

Maybe as a Library Function

When the module is imported **import Data.Maybe**

maybe :: b->(a->b) -> Maybe a -> b

Applies the second argument (a->b) to the third Maybe a,
when it is Just x, otherwise returns the first argument (b).

isJust, **isNothing**

Test the argument, returing a Bool based on the constructor.

ListToMaybe , **maybeToList**

Convert to/from a one element or empty list.

mapMaybe

A different way to filter a list.

https://wiki.haskell.org/Maybe

Maybe as Monad

maybe :: b->(a->b) -> Maybe a -> b
The maybe function takes
a default value (b),
a function (a->b), and
a Maybe value (Maybe a).

If the Maybe value is Nothing,
the function returns the default value.
Otherwise, it applies the function to the value inside the Just and returns the result.

```
>>> maybe False odd (Just 3)
True

>>> maybe False odd Nothing
False
```

https://hackage.haskell.org/package/base-4.10.0.0/docs/Data-Maybe.html

Then Operator (>>) and do Statements

```
putStr "Hello"  >>
putStr " "      >>
putStr "world!" >>
putStr "\n"
```

```
do { putStr "Hello"
   ; putStr " "
   ; putStr "world!"
   ; putStr "\n" }
```

https://en.wikibooks.org/wiki/Haskell/do_notation

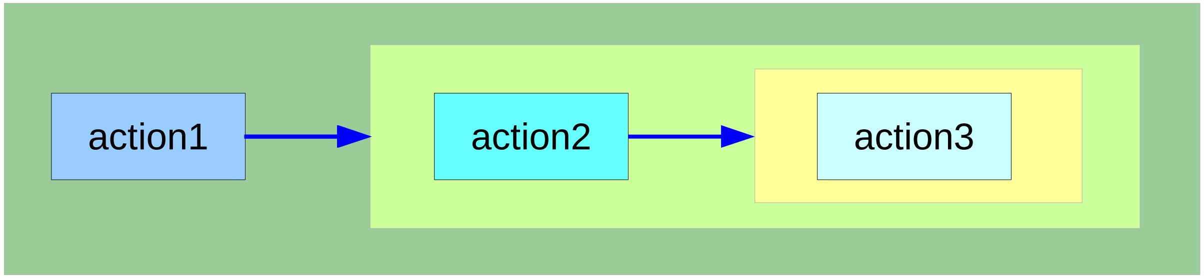
Translating in **do** notation

```
do { action1
   ; action2
   ; action3 }
```

```
action1 >>
do { action2
   ; action3 }
```

```
do { action1
   ; do { action2
        ; action3 } }
```

```
do { action1
   ; do { action2
        ; do { action3 } } }
```



can **chain** any actions
as long as all of them are
in **the same monad**

https://en.wikibooks.org/wiki/Haskell/do_notation

Bind Operator (>==) and **do** statements

The bind operator (**>>=**)

passes a value (the result of an action or function),
downstream in the binding sequence.

```
action1 >>= (\ x1 ->
  action2 >>= (\ x2 ->
    mk_action3 x1 x2 ))
```

anonymous function
(lambda expression)
is used

do notation assigns a variable name
to the passed value using the **<-**

```
do { x1 <- action1
   ; x2 <- action2
   ; mk_action3 x1 x2 }
```

https://en.wikibooks.org/wiki/Haskell/do_notation

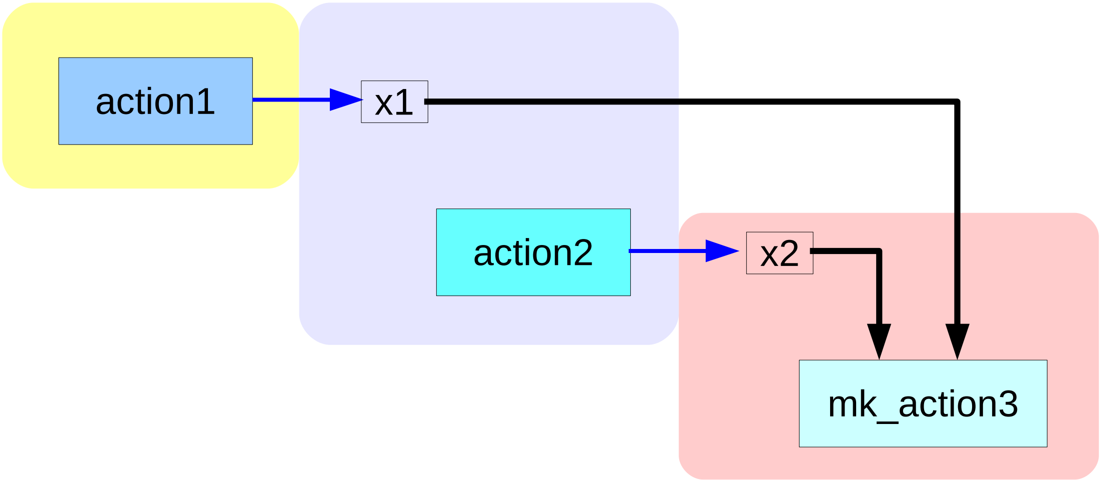
Translation using the bind operator (>>=)

```
do { x1 <- action1
     ; x2 <- action2
     ; mk_action3 x1 x2 }
```

```
action1 >>= (\ x1 -> action2 >>= (\ x2 -> mk_action3 x1 x2 ))
```

```
action1
  >>=
    (\ x1 -> action2
       >>=
         (\ x2 -> mk_action3 x1 x2 ))
```

```
action1 >>= (\ x1 ->
  action2 >>= (\ x2 ->
    mk_action3 x1 x2 ))
```



https://en.wikibooks.org/wiki/Haskell/do_notation

Anonymous Function

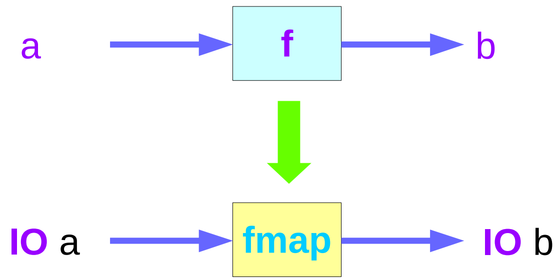
```
\x -> x + 1

(\x -> x + 1) 4
5 :: Integer

(\x y -> x + y) 3 5
8 :: Integer

addOne = \x -> x + 1
```

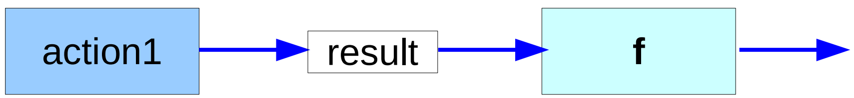
Lambda Expression

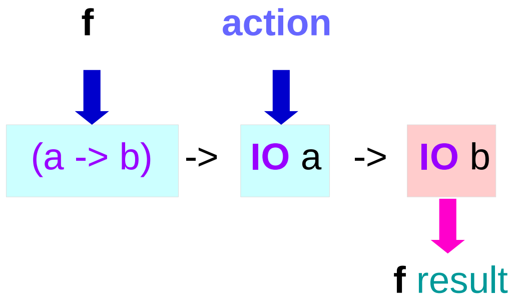
https://wiki.haskell.org/Anonymous_function

Functor Typeclass

```
instance Functor IO where
  fmap f action = do
    result <- action
    return (f result)
```








```
instance Functor Maybe where
  fmap func (Just x) = Just (func  x)
  fmap func Nothing = Nothing
```

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Functor Typeclass

```
main = do line <- getLine
          let line' = reverse line
          putStrLn $ "You said " ++ line' ++ " backwards!"
          putStrLn $ "Yes, you really said" ++ line' ++ " backwards!"

main = do line <- fmap reverse getLine
          putStrLn $ "You said " ++ line ++ " backwards!"
          putStrLn $ "Yes, you really said" ++ line ++ " backwards!"
```

```
instance Functor IO where
   fmap f action = do
      result <- action
      return (f result)
```

```
fmap reverse getLine = do
   result <- getLine
   return (reverse result)
```

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$ Operator

$ operator to avoid parentheses
Anything appearing after $
will take precedence over anything that comes before.

```
putStrLn (show (1 + 1))

putStrLn (show $ 1 + 1)
putStrLn $ show (1 + 1)
putStrLn $ show $ 1 + 1
```

https://stackoverflow.com/questions/940382/haskell-difference-between-dot-and-dollar-sign

. Operator

. operator to chain functions

putStrLn (**show** (1 + 1))

(1 + 1) is not a function, so the . operator cannot be applied
show can take an Int and return a String.
putStrLn can take a String and return an IO().

(**putStrLn . show**) (1 + 1)

putStrLn . show $ 1 + 1

https://stackoverflow.com/questions/940382/haskell-difference-between-dot-and-dollar-sign

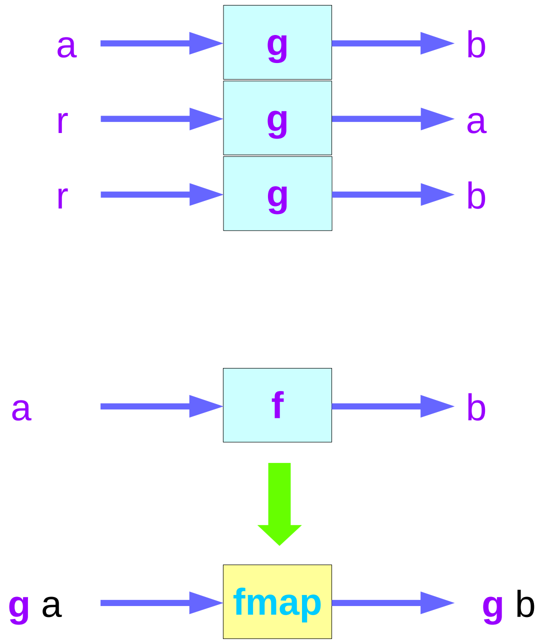
Functor Typeclass

```
instance Functor ((->) r) where
   fmap f g =  (\x -> f (g x))
```

```
instance Functor Maybe where
   fmap f (Just x) = Just (f x)
   fmap f Nothing = Nothing
```

A function takes any thing and returns any thing

```
g :: a -> b
g :: r -> a
```




```
fmap :: (a -> b) -> f a -> f b
fmap :: (a -> b) -> ((->) r a) -> ((->) r b)
fmap :: (a -> b) -> (r -> a) -> (r -> b)
```

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

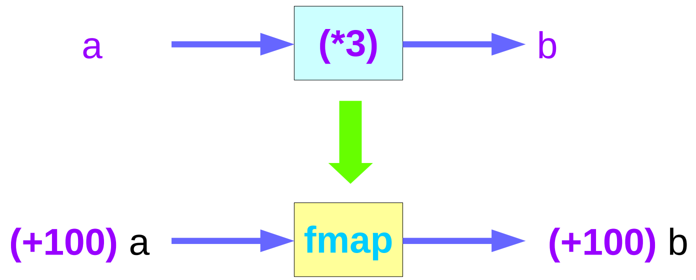
Functor Typeclass

```
instance Functor ((->) r) where
    fmap f g = (\x -> f (g x))

instance Functor ((->) r) where
    fmap = ( . )
```

```
instance Functor Maybe where
    fmap f (Just x) = Just (f x)
    fmap f Nothing = Nothing
```

```
ghci> :t fmap (*3) (+100)
fmap (*3) (+100) :: (Num a) => a -> a
ghci> fmap (*3) (+100) 1
303
ghci> (*3) `fmap` (+100) $ 1
303
ghci> (*3) . (+100) $ 1
303
ghci> fmap (show . (*3)) (*100) 1
"300"
```



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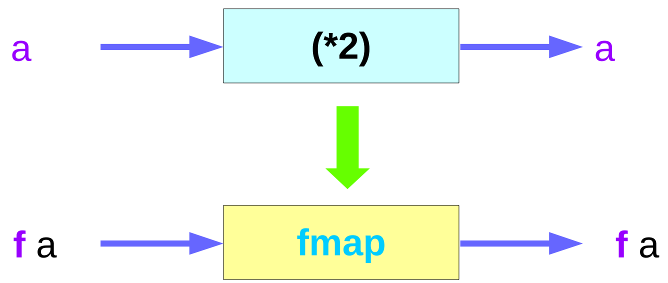
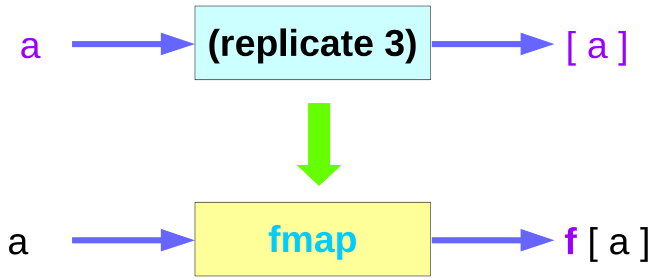
Functor Typeclass

ghci> :t **fmap** (*2)
fmap (*2) :: (Num a, Functor **f**) => **f** a -> **f** a



ghci> :t **fmap** (replicate 3)
fmap (replicate 3) :: (Functor **f**) => **f** a -> **f** [a]



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Functor Typeclass

```
ghci> fmap (replicate 3) [1,2,3,4]
[[1,1,1],[2,2,2],[3,3,3],[4,4,4]]

ghci> fmap (replicate 3) (Just 4)
Just [4,4,4]

ghci> fmap (replicate 3) (Right "blah")
Right ["blah","blah","blah"]

ghci> fmap (replicate 3) Nothing
Nothing

ghci> fmap (replicate 3) (Left "foo")
Left "foo"
```

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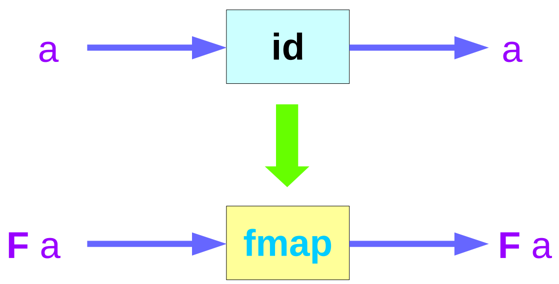
Functor Laws

fmap id = id

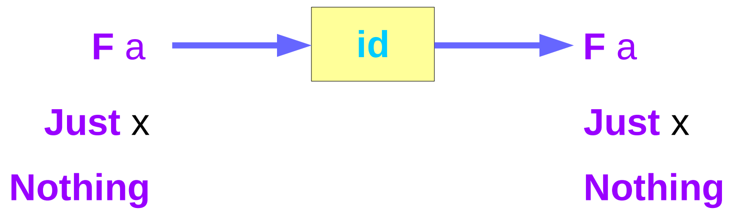
id :: a -> a
id x = x




```
instance Functor Maybe where
    fmap func (Just x) = Just (func x)
    fmap func Nothing = Nothing

instance Functor Maybe where
    fmap f (Just x) = Just (f x)
    fmap f Nothing = Nothing

instance Functor Maybe where
    fmap id (Just x) = Just (id x)
    fmap id Nothing = Nothing
```



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Functor Typeclass

ghci> **fmap** **id** (Just 3)
Just 3
ghci> **id** (Just 3)
Just 3
ghci> **fmap** **id** [1..5]
[1,2,3,4,5]
ghci> **id** [1..5]
[1,2,3,4,5]
ghci> **fmap** **id** []
[]
ghci> **fmap** **id** Nothing
Nothing

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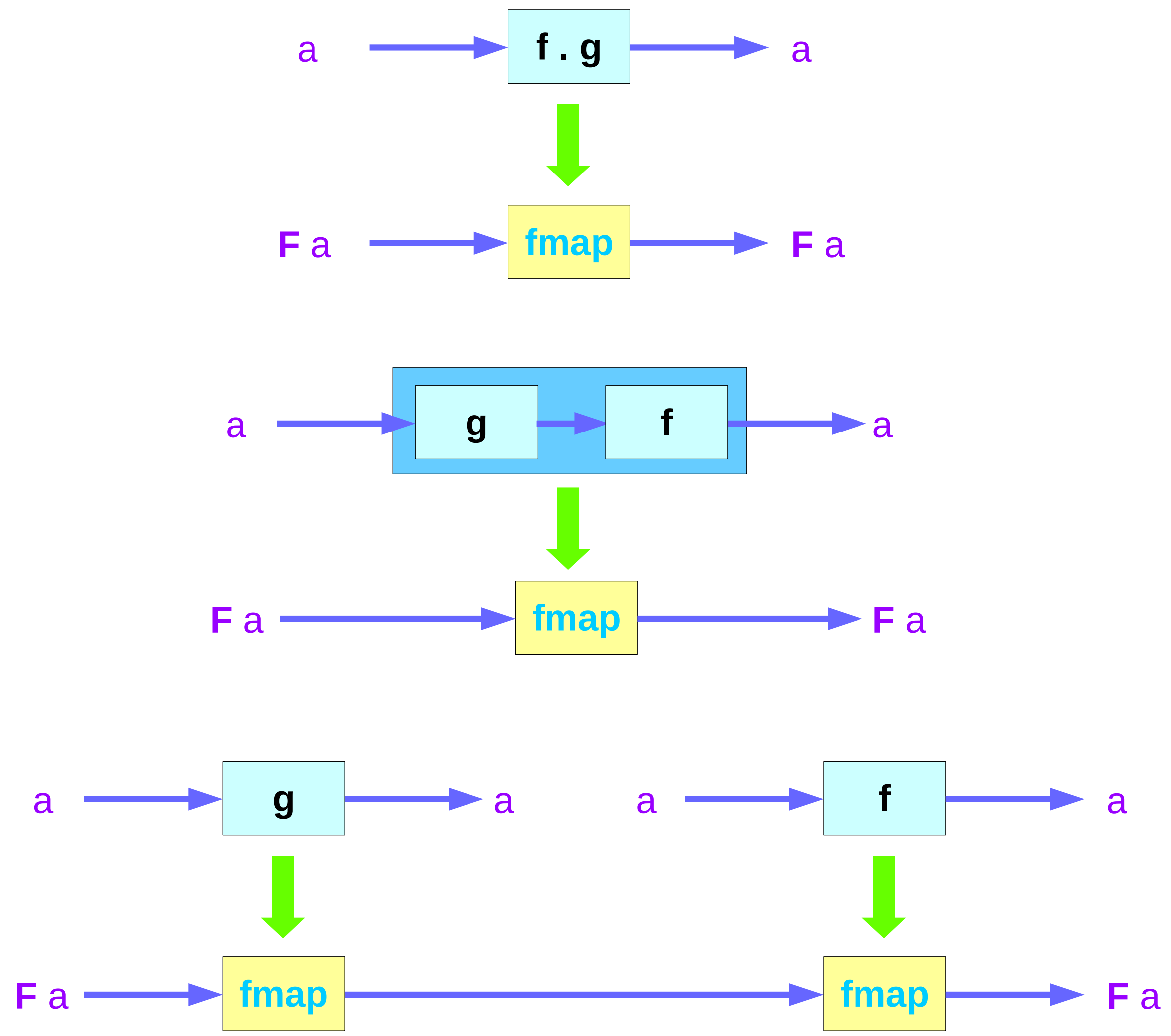
Functor Laws

fmap (**f . g**) = **fmap f** . **fmap g**

fmap (**f . g**) **F** = **fmap f** (**fmap g F**)



http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Functor Laws

```
fmap (f . g) = fmap f . fmap g
fmap (f . g) F = fmap f (fmap g F)

    instance Functor Maybe where
        fmap f (Just x) = Just (f x)
        fmap f Nothing = Nothing
```

```
fmap (f . g) Nothing = Nothing
fmap f (fmap g Nothing) = Nothing

fmap (f . g) (Just x) = Just ((f . g) x) = Just (f (g x))
fmap f (fmap g (Just x)) = fmap f (Just (g x)) = Just (f (g x))
```

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

References

[1] ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf
[2] https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf