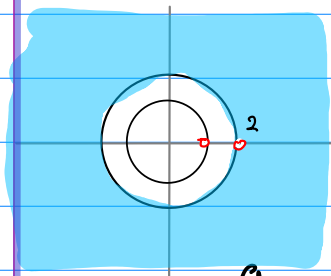


# Laurent Series and z-Transform Examples case 4.B

20171004

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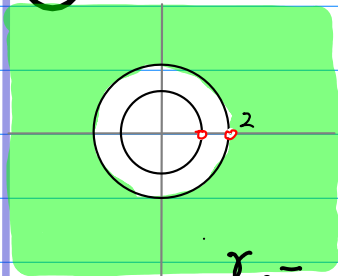
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$$a_n = \begin{cases} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

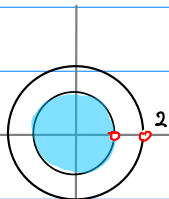
$$f(z) = \sum_{n=-1}^{-\infty} 1 \cdot z^n - \sum_{n=-1}^{-\infty} 2^{-n-1} z^n$$

Ⓘ



$$x_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

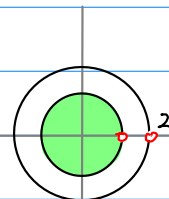
$$X(z) = \sum_{n=1}^{\infty} 1 \cdot z^{-n} - \sum_{n=1}^{\infty} 2^{n-1} z^{-n}$$



$$a_n = \begin{cases} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

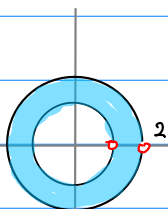
$$f(z) = \sum_{n=0}^{\infty} 2^{-n-1} \cdot z^n - \sum_{n=0}^{\infty} 1 \cdot z^n$$

Ⓛ



$$x_n = \begin{cases} 0 & (n > 0) \\ 2^{n+1} - 1 & (n \leq 0) \end{cases}$$

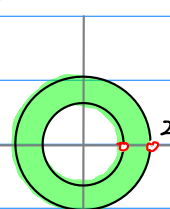
$$X(z) = \sum_{n=0}^{-\infty} 2^{n+1} \cdot z^{-n} - \sum_{n=0}^{-\infty} 1 \cdot z^{-n}$$



$$a_n = \begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} 1 \cdot z^n + \sum_{n=0}^{\infty} 2^{-n-1} \cdot z^n$$

Ⓜ



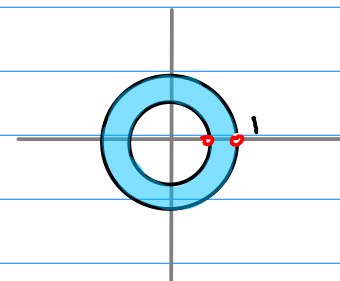
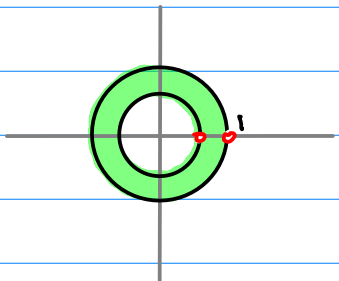
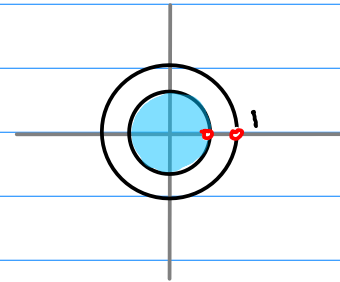
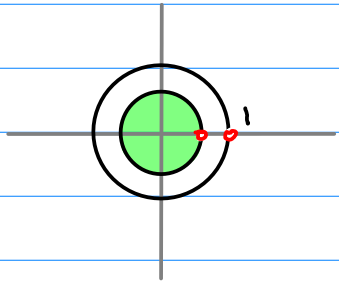
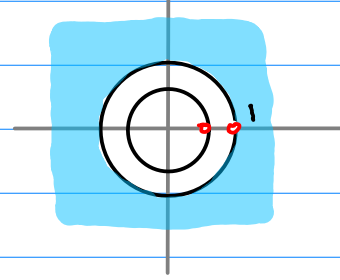
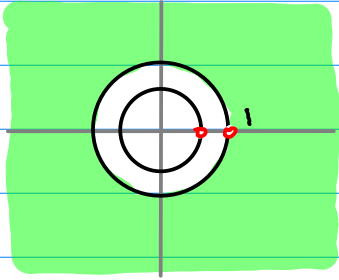
$$x_n = \begin{cases} 1 & (n > 0) \\ 2^{n+1} & (n \leq 0) \end{cases}$$

$$X(z) = \sum_{n=0}^{-\infty} 2^{n+1} \cdot z^{-n} + \sum_{n=1}^{\infty} 1 \cdot z^{-n}$$

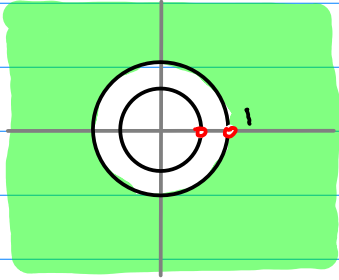


4.B

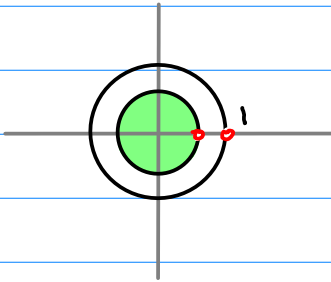
$$X(z) = \frac{-0.5z^2}{(z-1)(z-0.5)} \longrightarrow f(z) = \frac{-0.5z^2}{(z-1)(z-0.5)}$$



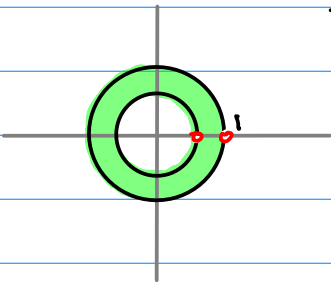
$$f(z) = \frac{-0.5z^2}{(z-1)(z-0.5)} = \frac{-z}{z-1} + \frac{0.5z}{z-0.5}$$



$$\begin{aligned} &= -\frac{(1)}{1-\left(\frac{1}{z}\right)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)} \\ &= -\sum_{n=0}^{\infty} (1)\left(\frac{1}{z}\right)^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)\left(\frac{1}{2z}\right)^n \\ &= -\sum_{n=0}^{\infty} z^{-n} + \sum_{n=0}^{\infty} 2^{-n-1} z^{-n} \\ &= \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1\right] z^{-n} \end{aligned}$$



$$\begin{aligned} &+ \frac{\left(\frac{z}{1}\right)}{1-\left(\frac{z}{1}\right)} - \frac{\left(\frac{z}{1}\right)}{1-\left(\frac{2z}{1}\right)} \\ &= + \sum_{n=0}^{\infty} (z)(z)^n - \sum_{n=0}^{\infty} (z)(2z)^n \\ &= \sum_{n=0}^{\infty} [1 - 2^n] z^{n+1} \\ &= \sum_{n=-1}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1\right] z^{-n} \end{aligned}$$



$$\begin{aligned} &\frac{-z}{z-1} + \frac{0.5z}{z-0.5} = -\frac{(1)}{1-\left(\frac{1}{z}\right)} - \frac{\left(\frac{z}{1}\right)}{1-\left(\frac{2z}{1}\right)} \\ &= -\sum_{n=0}^{\infty} (1)\left(\frac{1}{z}\right)^n - \sum_{n=0}^{\infty} \left(\frac{z}{1}\right)\left(\frac{2z}{1}\right)^n \\ &= -\sum_{n=0}^{\infty} z^{-n} - \sum_{n=0}^{\infty} 2^n z^{n+1} \\ &= -\sum_{n=0}^{\infty} z^{-n} - \sum_{n=-1}^{\infty} 2^{-n-1} z^{-n} \end{aligned}$$

