Laurent Series and z-Transform
- Geometric Series
Double Pole Properties A

20181203 Mon

2 formulas of z

① $$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$$\Big\Updownarrow z^{-1}$$

② $$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

$$f(z) = \begin{cases} f_1(z) \\ f_2(z^{-1}) \end{cases} \qquad g(z) = \begin{cases} g_1(z) \\ g_2(z^{-1}) \end{cases}$$

$$X(z) = \begin{cases} X_1(z) \\ X_2(z^{-1}) \end{cases} \qquad Y(z) = \begin{cases} Y_1(z) \\ Y_2(z^{-1}) \end{cases}$$

① $\frac{3}{2} \frac{-1}{(z-0.5)(z-2)}$

$$\boxed{+\frac{1}{z-0.5} - \frac{1}{z-2}}$$

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \ \boxed{|z|<0.5}$$

causal $f_1(z)$

anti-causal $X_1(z)$

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \ \boxed{|z|>2}$$

anti-causal $f_2(z)$

causal $X_2(z)$

② $\frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)}$

$$\boxed{-\frac{2z}{(z-2)} + \frac{0.5z}{(z-0.5)}}$$

$$\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \ \boxed{|z|>2}$$

anti-causal $g_1(z)$

causal $Y_1(z)$

$$\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \ \boxed{|z|<0.5}$$

causal $g_2(z)$

anti-causal $Y_2(z)$

$f(z) \longleftrightarrow a_n$

$X(z) \longleftrightarrow x_n$

① – Ⓐ

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \ \boxed{|z|<0.5}$$

$$a_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$$x_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n < 1)$$

② – Ⓐ

$$\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \ \boxed{|z|>2}$$

$$a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n < 1)$$

$$x_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

① – Ⓑ

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \ \boxed{|z|>2}$$

$$a_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

$$x_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

② – Ⓑ

$$\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \ \boxed{|z|<0.5}$$

$$a_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

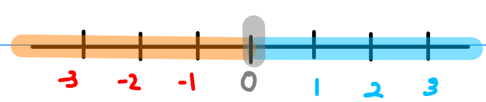
$$x_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

$$x_n = a_{-n}$$

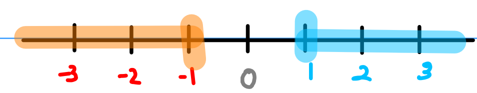
$$a_n = x_{-n}$$

$(n \geq 0) \longleftrightarrow (n < 1)$

$(n \geq 1) \longleftrightarrow (n < 0)$



$(n < 1)$ $(n \geq 0)$



$(n < 0)$ $(n \geq 1)$

$$(z^{-1}, R^{-1}) \Leftrightarrow (-n, -N)$$

① - Ⓐ

$$-\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \quad |z|<0.5$$

$$a_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$$x_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n < 1)$$

z^{-1}, R^{-1} ⟷

$-n, -N$ ⟷

② - Ⓐ

$$-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad |z|>2$$

$$a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n < 1)$$

$$x_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

① - Ⓑ

$$+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad |z|>2$$

$$a_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

$$x_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

z^{-1}, R^{-1} ⟷

$-n, -N$ ⟷

② - Ⓑ

$$+\frac{z}{1-0.5z} - \frac{z}{1-2z} \quad |z|<0.5$$

$$a_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

$$x_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

$$(z,\ R^{-1}) \Leftrightarrow (-1,\ N^{c})$$

① − Ⓐ

$$-\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \qquad |z|<0.5$$

$\Vert\ z$ $\qquad \Updownarrow\ R^{-1}$

① − Ⓑ

$$+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad |z|>2$$

② − Ⓐ

$$-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \qquad |z|>2$$

$\Vert\ z$ $\qquad \Updownarrow\ R^{-1}$

② − Ⓑ

$$+\frac{z}{1-0.5z} - \frac{z}{1-2z} \qquad |z|<0.5$$

① − Ⓐ

$$a_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \qquad (n \geq 0)$$

$\Updownarrow\ -1$ $\qquad \Updownarrow\ N^{c}$

① − Ⓑ

$$a_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \qquad (n < 0)$$

② − Ⓐ

$$a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \qquad (n < 1)$$

$\Updownarrow\ -1$ $\qquad \Updownarrow\ N^{c}$

② − Ⓑ

$$a_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \qquad (n \geq 1)$$

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$$x_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \qquad (n < 1)$$

$\Updownarrow\ -1$ $\qquad \Updownarrow\ N^{c}$

① − Ⓑ

$$x_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \qquad (n \geq 1)$$

② − Ⓐ

$$x_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \qquad (n \geq 0)$$

$\Updownarrow\ -1$ $\qquad \Updownarrow\ N^{c}$

② − Ⓑ

$$x_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \qquad (n < 0)$$

$$(a_n, N) \Leftrightarrow (x_{-n}, -N)$$

①-Ⓐ

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \ \boxed{|z|<0.5}$$

$$a_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$$\Updownarrow -n \qquad \Updownarrow -N$$

$$x_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n<1)$$

②-Ⓐ

$$\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \ \boxed{|z|>2}$$

$$a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n<1)$$

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①-Ⓑ

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \ \boxed{|z|>2}$$

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$$x_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

②-Ⓑ

$$\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \ \boxed{|z|<0.5}$$

$$a_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

$$\Updownarrow -n \qquad \Updownarrow -N$$

$$x_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0)$$

(z^{-1}, R) $\quad$ $(-n, (-N)^c)$

$(z^{-1}, R^{-1}) \rightarrow (z, R^{-1})$ $\quad$ $(-n, -N) \rightarrow (-1, N^c)$

①–Ⓐ

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \boxed{|z|<0.5}$$

$$a_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

②–Ⓐ

$$\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \boxed{|z|>2}$$

$$a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n<1)$$

①–Ⓑ

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \boxed{|z|>2}$$

$$a_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0)$$

②–Ⓑ

$$\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \boxed{|z|<0.5}$$

$$a_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

①–Ⓐ

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \boxed{|z|<0.5}$$

$$x_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n<1)$$

②–Ⓐ

$$\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \boxed{|z|>2}$$

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①–Ⓑ

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \boxed{|z|>2}$$

$$x_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

②–Ⓑ

$$\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \boxed{|z|<0.5}$$

$$x_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0)$$

(z^{-1}, R) $\qquad$ $(-n, (-N)^c)$

$(z^{-1}, R^{-1}) \to (z, R^{-1})$ $\qquad$ $(-n, -N) \to (-1, N^c)$

① − Ⓐ

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \ \boxed{|z|<0.5}$$

② − Ⓑ

$$\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \ \boxed{|z|<0.5}$$

$a_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$

$a_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$

① − Ⓑ

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \ \boxed{|z|>2}$$

② − Ⓐ

$$\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \ \boxed{|z|>2}$$

$a_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$

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① − Ⓐ

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \ \boxed{|z|<0.5}$$

② − Ⓑ

$$\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \ \boxed{|z|<0.5}$$

$x_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n < 1)$

$x_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$

① − Ⓑ

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \ \boxed{|z|>2}$$

② − Ⓐ

$$\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \ \boxed{|z|>2}$$

$x_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$

$x_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$

ROC's of interests

$R1(z)$ $\quad$ $R1(z^{-1})$

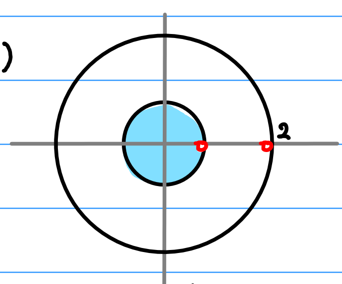
$R2(z)$ $\quad$ $R2(z^{-1})$

① $\frac{3}{2} \frac{-1}{(z-0.5)(z-2)} \xleftrightarrow{z^{-1}}$ ② $\frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)}$

$$\boxed{+\frac{1}{z-0.5} - \frac{1}{z-2}} \xleftrightarrow{z^{-1}} \boxed{-\frac{2z}{(z-2)} + \frac{0.5z}{(z-0.5)}}$$

$$p_1 = 0.5 \quad p_2 = 2 \xleftrightarrow{z^{-1}} p_1^{-1} = 2 \quad p_2^{-1} = 0.5$$

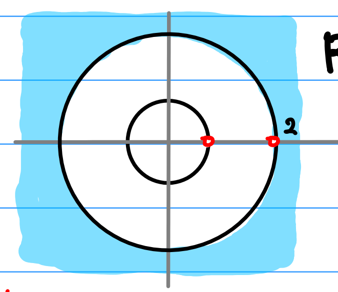
$R1(z)$



$|z| < 0.5$

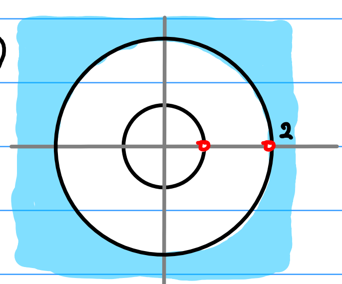
$\xleftrightarrow{z^{-1}}$

$R1(z^{-1})$



$|z^{-1}| < 0.5 \quad |z| > 2$

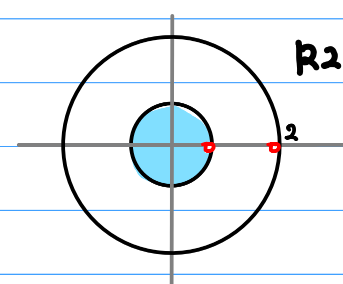
$R2(z)$



$|z| > 2$

$\xleftrightarrow{z^{-1}}$

$R2(z^{-1})$



$|z^{-1}| > 2 \quad |z| < 0.5$

$$R1(z^{-1}) = R2(z)$$
$$R2(z^{-1}) = R1(z)$$

2 Equations, each with 2 representations

$$① \ \frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} \quad \xleftrightarrow{z^{-1}} \quad ② \ \frac{3}{2}\,\frac{-z^{2}}{(z-2)(z-0.5)}$$

$$\left(\frac{1}{z-0.5}-\frac{1}{z-2}\right) \qquad\qquad \left(\frac{0.5z}{(z-0.5)}-\frac{2z}{(z-2)}\right)$$

$\boxed{z}$ $$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}$$

$\boxed{|z|<0.5}$ $\quad |2z|<1 \quad |0.5z|<1$

$\boxed{z^{-1}}$ $$\frac{0.5}{1-0.5z^{-1}}-\frac{2}{1-2z^{-1}}$$

$\boxed{|z|>2}$ $\quad |0.5z^{-1}|<1 \quad |2z^{-1}|<1$

$\boxed{z^{-1}}$ $$\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$$

$\boxed{|z|>2}$ $\quad |0.5z^{-1}|<1 \quad |2z^{-1}|<1$

$\boxed{z}$ $$-\frac{z}{1-2z}+\frac{z}{1-0.5z}$$

$\boxed{|z|<0.5}$ $\quad |2z|<1 \quad |0.5z|<1$

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}$$

$\cdot\frac{1}{2z}$ $\cdot 2z$ $\qquad \cdot\frac{2}{z}$ $\cdot\frac{z}{2}$

$$\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$$

$$\frac{0.5}{1-0.5z^{-1}}-\frac{2}{1-2z^{-1}}$$

$\cdot 2z$ $\cdot\frac{1}{2z}$ $\qquad \cdot\frac{z}{2}$ $\cdot\frac{2}{z}$

$$-\frac{z}{1-2z}+\frac{z}{1-0.5z}$$

Causal f(z) & X(z)

$$① \ \frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \xleftrightarrow{z^{-1}} ② \ \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$$

$$\left(\frac{1}{z-0.5} - \frac{1}{z-2}\right) \qquad \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

$\boxed{z}$ $-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$

$\boxed{|z| < 0.5}$ f(z) causal $(n \geq 0)$

$\boxed{z}$ $-\frac{z}{1-2z} + \frac{z}{1-0.5z}$

$\boxed{|z| < 0.5}$ f(z) causal $(n \geq 1)$

$\boxed{z^{-1}}$ $\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$

$\boxed{|z| > 2}$

X(z) causal $(n \geq 1)$

$\boxed{z^{-1}}$ $\frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}}$

$\boxed{|z| > 2}$

X(z) causal $(n \geq 0)$

Anti-causal $f(z)$ & $X(z)$

① $\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)}$ $\xleftrightarrow{z^{-1}}$ ② $\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)}$

$\left(\frac{1}{z-0.5}-\frac{1}{z-2}\right)$

$\left(\frac{0.5z}{(z-0.5)}-\frac{2z}{(z-2)}\right)$

$\boxed{z}$ $-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}$

$\boxed{|z|<0.5}$ $f(z)$ causal $(n \geq 0)$

$X(z)$ anticausal $(n<1)$

$\boxed{z^{-1}}$ $\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$

$\boxed{|z|>2}$ $f(z)$ anticausal $(n<0)$

$X(z)$ causal $(n \geq 1)$

$\boxed{z^{-1}}$ $\frac{0.5}{1-0.5z^{-1}}-\frac{2}{1-2z^{-1}}$

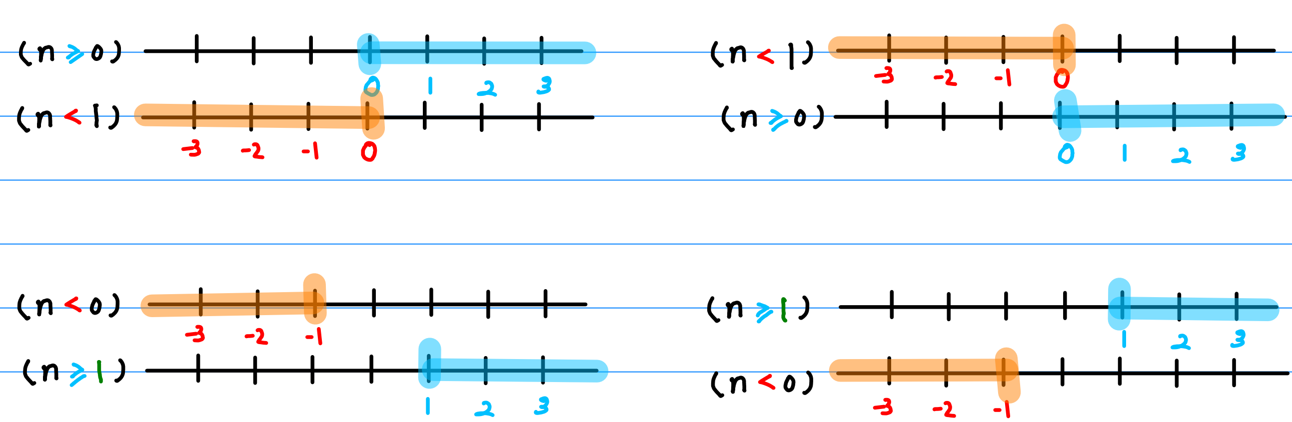
$\boxed{|z|>2}$ $f(z)$ anticausal $(n<1)$

$X(z)$ causal $(n \geq 0)$

$\boxed{z}$ $-\frac{z}{1-2z}+\frac{z}{1-0.5z}$

$\boxed{|z|<0.5}$ $f(z)$ causal $(n \geq 1)$

$X(z)$ anticausal $(n<0)$



Causal sequence a_n & x_n

$$① \ \frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \quad \xleftrightarrow{z^{-1}} \quad ② \ \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$$

$|z|<0.5$

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}$$

$$f(z) = -\left[2+2^2z^1+2^3z^2+\cdots\right] \quad -2^{n+1}$$
$$+\left[\left(\tfrac{1}{2}\right)+\left(\tfrac{1}{2}\right)^2z^1+\left(\tfrac{1}{2}\right)^3z^2+\cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n+1}$$

$|z|<0.5$

$$-\frac{z}{1-2z}+\frac{z}{1-0.5z}$$

$$f(z) = -\left[2^0z^1+2^1z^2+2^2z^3+\cdots\right] \quad -2^{n-1}$$
$$+\left[\left(\tfrac{1}{2}\right)^0z^1+\left(\tfrac{1}{2}\right)^1z^2+\left(\tfrac{1}{2}\right)^2z^3+\cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n-1}$$

$|z|>2$

$$\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$$

$$X(z) = +\left[\left(\tfrac{1}{2}\right)^0z^{-1}+\left(\tfrac{1}{2}\right)^1z^{-2}+\left(\tfrac{1}{2}\right)^2z^{-3}+\cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n-1}$$
$$-\left[2^0z^{-1}+2^1z^{-2}+2^2z^{-3}+\cdots\right] \quad -2^{n-1}$$

$*$ $n=$ 1 2 3

$|z|>2$

$$\frac{0.5}{1-0.5z^{-1}}-\frac{2}{1-2z^{-1}}$$

$$X(z) = +\left[\left(\tfrac{1}{2}\right)^1z^0+\left(\tfrac{1}{2}\right)^2z^{-1}+\left(\tfrac{1}{2}\right)^3z^{-2}+\cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n+1}$$
$$-\left[2^1z^0+2^2z^{-1}+2^3z^{-2}+\cdots\right] \quad -2^{n+1}$$

$*$ $n=$ 0 1 2

Anti-causal sequence a_n & x_n

$$① \quad \frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} \quad \xleftrightarrow{z^{-1}} \quad ② \quad \frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)}$$

$|z| < 0.5$

$$-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$$

$$f(z) = -\left[2 + 2^2 z^1 + 2^3 z^2 + \cdots\right] \quad -2^{n+1}$$
$$+\left[\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n+1}$$

$$\boxed{2 = \left(\tfrac{1}{2}\right)^{-1} \quad \left(\tfrac{1}{2}\right) = 2^{-1}}$$

$$X(z) = -\left[\left(\tfrac{1}{2}\right)^{-1} + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n-1}$$
$$+\left[2^{-1} + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right] \quad +2^{n-1}$$

$n = \quad 0 \quad -1 \quad -2$

$|z| < 0.5$

$$-\frac{z}{1-2z} + \frac{z}{1-0.5z}$$

$$f(z) = -\left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right] \quad -2^{n-1}$$
$$+\left[\left(\tfrac{1}{2}\right)^0 z^1 + \left(\tfrac{1}{2}\right)^1 z^2 + \left(\tfrac{1}{2}\right)^2 z^3 + \cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n-1}$$

$$X(z) = -\left[\left(\tfrac{1}{2}\right)^0 z^1 + \left(\tfrac{1}{2}\right)^{-1} z^2 + \left(\tfrac{1}{2}\right)^{-2} z^3 + \cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n+1}$$
$$+\left[2^0 z^1 + 2^{-1} z^2 + 2^{-2} z^3 + \cdots\right] \quad +2^{n+1}$$

$n = \quad -1 \quad -2 \quad -3$

$|z| > 2$

$$\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$

$$\boxed{2 = \left(\tfrac{1}{2}\right)^{-1} \quad \left(\tfrac{1}{2}\right) = 2^{-1}}$$

$$f(z) = +\left[2^0 z^{-1} + 2^{-1} z^{-2} + 2^{-2} z^{-3} + \cdots\right] \quad +2^{n+1}$$
$$-\left[\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^{-1} z^{-2} + \left(\tfrac{1}{2}\right)^{-2} z^{-3} + \cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n+1}$$

$$X(z) = +\left[\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^1 z^{-2} + \left(\tfrac{1}{2}\right)^2 z^{-3} + \cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n-1}$$
$$-\left[2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right] \quad -2^{n-1}$$

$n = \quad 1 \quad 2 \quad 3$

$|z| > 2$

$$\frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}}$$

$$f(z) = +\left[2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots\right] \quad +2^{n-1}$$
$$-\left[\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^{-1} + \left(\tfrac{1}{2}\right)^{-3} z^{-2} + \cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n-1}$$

$$X(z) = +\left[\left(\tfrac{1}{2}\right)^1 z^0 + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n+1}$$
$$-\left[2^1 z^0 + 2^2 z^{-1} + 2^3 z^{-2} + \cdots\right] \quad -2^{n+1}$$

$n = \quad 0 \quad 1 \quad 2$

Sequence a_n & x_n

VI

① $\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)}$ $\overset{z^{-1}}{\longleftrightarrow}$ ② $\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)}$

$|z|<0.5$

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}$$

$$f(z) = -\left[2 + 2^2 z + 2^3 z^2 + \cdots\right] + \left[\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right]$$

$$\boxed{a_n = -2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \qquad (n \geq 0)}$$

$$X(z) = -\left[\left(\tfrac{1}{2}\right)^{-1} + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right] + \left[2^{-1} + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right]$$

$$\boxed{a_n = -\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1} \qquad (n < 1)}$$

$|z|>2$

$$\frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}}$$

$$f(z) = +\left[2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots\right] - \left[\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^{-1} + \left(\tfrac{1}{2}\right)^{-3} z^{-2} + \cdots\right]$$

$$\boxed{a_n = +2^{n-1} - \left(\tfrac{1}{2}\right)^{n-1} \qquad (n < 1)}$$

$$X(z) = +\left[\left(\tfrac{1}{2}\right)^1 z^0 + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right] - \left[2^1 z^0 + 2^2 z^{-1} + 2^3 z^{-2} + \cdots\right]$$

$$\boxed{a_n = +\left(\tfrac{1}{2}\right)^{n+1} - 2^{n+1} \qquad (n \geq 0)}$$

$|z|>2$

$$\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$

$$f(z) = +\left[2^0 z^{-1} + 2^{-1} z^{-2} + 2^{-2} z^{-3} + \cdots\right] - \left[\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^{-1} z^{-2} + \left(\tfrac{1}{2}\right)^{-2} z^{-3} + \cdots\right]$$

$$\boxed{a_n = +2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \qquad (n < 0)}$$

$$X(z) = +\left[\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^1 z^{-2} + \left(\tfrac{1}{2}\right)^2 z^{-3} + \cdots\right] - \left[2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right]$$

$$\boxed{a_n = +\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1} \qquad (n \geq 1)}$$

$|z|<0.5$

$$-\frac{z}{1-2z} + \frac{z}{1-0.5z}$$

$$f(z) = -\left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right] + \left[\left(\tfrac{1}{2}\right)^0 z^1 + \left(\tfrac{1}{2}\right)^1 z^2 + \left(\tfrac{1}{2}\right)^2 z^3 + \cdots\right]$$

$$\boxed{a_n = -2^{n-1} + \left(\tfrac{1}{2}\right)^{n-1} \qquad (n \geq 1)}$$

$$X(z) = -\left[\left(\tfrac{1}{2}\right)^0 z^1 + \left(\tfrac{1}{2}\right)^{-1} z^2 + \left(\tfrac{1}{2}\right)^{-2} z^3 + \cdots\right] + \left[2^0 z^1 + 2^{-1} z^2 + 2^{-2} z^3 + \cdots\right]$$

$$\boxed{a_n = -\left(\tfrac{1}{2}\right)^{n+1} + 2^{n+1} \qquad (n < 0)}$$

$$(\mathbb{Z}^{-1}, \mathbb{R}^{-1}) \Leftrightarrow (-n, -N)$$

$$(\mathbb{Z}, \mathbb{R}^{-1}) \Leftrightarrow (-1, N^c)$$

$$(a_n, N) \Leftrightarrow (x_{-n}, -N)$$

$$(\mathbb{Z}^{-1}, \mathbb{R}) \qquad (-n, (-N)^c)$$

$$(\mathbb{Z}^{-1}, \mathbb{R}^{-1}) \to (\mathbb{Z}, \mathbb{R}^{-1}) \qquad (-n, -N) \to (-1, N^c)$$

$ROC(z) \quad f(z) \Longleftrightarrow a_n \quad RNG(n)$

$|z| < p \qquad n \geq 0$

Ⅰ $\boxed{ROC(z^{-1}) \quad f(z) \Longleftrightarrow -a_n \quad \overline{RNG}(n) \\ |z| > \frac{1}{p} \qquad n < 0}$

$ROC(z) \quad f(z) \Longleftrightarrow a_n \quad RNG(n)$

$|z| < p \qquad n \geq 0$

Ⅱ $\boxed{ROC(z^{-1}) \quad f(z^{-1}) \Longleftrightarrow a_{-n} \quad RNG(-n) \\ |z| > \frac{1}{p} \qquad n < 1}$

$ROC(z) \quad f(z) \Longleftrightarrow a_n \quad RNG(n)$

$|z| < p \qquad n \geq 0$

Ⅲ $\boxed{ROC(z) \quad f(z^{-1}) \Longleftrightarrow -a_{-n} \quad \ll RNG(n) \gg \\ |z| < p \qquad n \geq 1}$ Ⅰ + Ⅱ

$ROC(z) \quad f(z) \Longleftrightarrow a_n \quad RNG(n)$

$|z| < p \qquad n \geq 0$

Ⅳ $\boxed{ROC(z) \quad X(z) \Longleftrightarrow a_{-n} \quad RNG(-n) \\ |z| < p \qquad n < 1}$

$$\text{III} = \text{I} + \text{II}$$

III $\quad \mathrm{ROC}(z) \quad f(z^{-1}) \longleftrightarrow -a_{-n} \quad \ll \mathrm{RNG}(n) \gg \quad$ I + II

$|z| < p \qquad n \geq 1$

$\mathrm{ROC}(z) \quad f(z) \longleftrightarrow a_n \quad \mathrm{RNG}(n)$

$|z| < p \qquad n \geq 0$

I

$\mathrm{ROC}(z^{-1}) \quad f(z) \longleftrightarrow -a_n \quad \overline{\mathrm{RNG}}(n)$

$|z| > \frac{1}{p} \qquad n < 0$

$n \leq -1$

II

$\mathrm{ROC}(z) \quad f(z^{-1}) \longleftrightarrow -a_{-n} \quad \overline{\mathrm{RNG}}(-n)$

$|z| < p \qquad n \geq 1$

Compare I with IV

	ROC(z)	$f(z) \Longleftrightarrow a_n$	RNG(n)
	$\lvert z \rvert < p$		$n \geq 0$
I	ROC(z^{-1})	$f(z) \Longleftrightarrow -a_n$	$\overline{\text{RNG}}(n)$
	$\lvert z \rvert > \frac{1}{p}$		$n < 0$
		-1	complement
IV	ROC(z)	$X(z) \Longleftrightarrow a_{-n}$	RNG($-n$)
	$\lvert z \rvert < p$		$n < 1$
		$-n$	$-n$
			symmetrical

$f(z)$ $\quad |z| < 0.5$ $\quad |z| > 2$

causal $\quad$ anticausal

① − Ⓐ

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$

$$f(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \qquad \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$$\frac{a}{1-az} \qquad \frac{z^{-1}}{a^{-1}z^{-1}-1}$$

$$-\left(\underset{n=0}{2} + \underset{n=1}{2^2 z} + \underset{n=2}{2^3 z^2} + \cdots\right) + \left(\underset{n=0}{\left(\tfrac{1}{2}\right)} + \underset{n=1}{\left(\tfrac{1}{2}\right)^2 z} + \underset{n=2}{\left(\tfrac{1}{2}\right)^3 z^2} + \cdots\right)$$

$|z| > 2$

$$f(z) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

$$\left(z^{-1} + \left(\tfrac{1}{2}\right)^1 z^{-2} + \left(\tfrac{1}{2}\right)^2 z^{-3} + \cdots\right) - \left(z^{-1} + 2z^{-2} + 2^2 z^{-3} + \cdots\right)$$

$$\left(\underset{n=-1}{2^0 z^{-1}} + \underset{n=-2}{2^{-1} z^{-2}} + \underset{n=-3}{2^{-2} z^{-3}} + \cdots\right) - \left(\underset{n=-1}{\left(\tfrac{1}{2}\right)^0 z^{-1}} + \underset{n=-2}{\left(\tfrac{1}{2}\right)^{-1} z^{-2}} + \underset{n=-3}{\left(\tfrac{1}{2}\right)^{-2} z^{-3}} + \cdots\right)$$

② − Ⓐ

$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

$|z| < 0.5$

$$f(z) = -\frac{z}{1-2z} + \frac{z}{1-0.5z} \qquad \boxed{-2^{n-1} + \left(\tfrac{1}{2}\right)^{n-1}} \quad (n \geq 1)$$

$$-\left(\underset{n=1}{z} + \underset{n=2}{2z^2} + \underset{n=3}{2^2 z^3} + \cdots\right) + \left(\underset{n=1}{z} + \underset{n=2}{\left(\tfrac{1}{2}\right) z^2} + \underset{n=3}{\left(\tfrac{1}{2}\right)^2 z^3} + \cdots\right)$$

$|z| > 2$

$$f(z) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}} \qquad \boxed{+2^{n-1} - \left(\tfrac{1}{2}\right)^{n-1}} \quad (n < 1)$$

$$\left(\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right) + \left(2 + 2^2 z^{-1} + 2^3 z^{-2} + \cdots\right)$$

$$\left(\underset{n=0}{2^{-1}} + \underset{n=-1}{2^{-2} z^{-1}} + \underset{n=-2}{2^{-3} z^{-2}} + \cdots\right) + \left(\underset{n=0}{\left(\tfrac{1}{2}\right)^{-1}} + \underset{n=-1}{\left(\tfrac{1}{2}\right)^{-2} z^{-1}} + \underset{n=-2}{\left(\tfrac{1}{2}\right)^{-3} z^{-2}} + \cdots\right)$$

$X(z)$ $\quad |z| < 0.5 \quad |z| > 2$

anticausal $\quad$ causal

① − Ⓑ $\frac{3}{2}$ $\dfrac{-1}{(z-0.5)(z-2)} = \left(\dfrac{1}{z-0.5} - \dfrac{1}{z-2}\right)$

$|z| < 0.5$ $\quad X(z) = -\dfrac{2}{1-2z} + \dfrac{0.5}{1-0.5z}$ $\quad \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}}$ $\quad (n < 1)$

$$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right) z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$$

$n=0 \quad n=-1 \quad n=-2 \qquad n=0 \quad n=-1 \quad n=-2$

$|z| > 2$ $\quad X(z) = \dfrac{z^{-1}}{1-0.5z^{-1}} - \dfrac{z^{-1}}{1-2z^{-1}}$ $\quad \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}}$ $\quad (n \geq 1)$

$$\left(\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^1 z^{-2} + \left(\tfrac{1}{2}\right)^2 z^{-3} + \cdots\right) - \left(2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right)$$

$n=1 \quad n=2 \quad n=3 \qquad n=1 \quad n=2 \quad n=3$

② − Ⓑ $\frac{3}{2}$ $\dfrac{-z^2}{(z-2)(z-0.5)} = \left(\dfrac{0.5z}{(z-0.5)} - \dfrac{2z}{(z-2)}\right)$

$|z| < 0.5$ $\quad X(z) = -\dfrac{z}{1-2z} + \dfrac{z}{1-0.5z}$ $\quad \boxed{-\left(\tfrac{1}{2}\right)^{n+1} + 2^{n+1}}$ $\quad (n < 0)$

$$-\left(z + 2z^2 + 2^2 z^3 + \cdots\right) + \left(z + \left(\tfrac{1}{2}\right) z^2 + \left(\tfrac{1}{2}\right)^2 z^3 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^0 z + \left(\tfrac{1}{2}\right)^{-1} z^2 + \left(\tfrac{1}{2}\right)^{-2} z^3 + \cdots\right) + \left(2^0 z + 2^{-1} z^2 + 2^{-2} z^3 + \cdots\right)$$

$n=-1 \quad n=-2 \quad n=-3 \qquad n=-1 \quad n=-2 \quad n=-3$

$|z| > 2$ $\quad X(z) = \dfrac{0.5}{1-0.5z^{-1}} - \dfrac{2}{1-2z^{-1}}$ $\quad \boxed{+\left(\tfrac{1}{2}\right)^{n+1} - 2^{n+1}}$ $\quad (n \geq 0)$

$$\left(\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right) + \left(2 + 2^2 z^{-1} + 2^3 z^{-2} + \cdots\right)$$

$n=0 \quad n=1 \quad n=2 \qquad n=0 \quad n=1 \quad n=2$

Ⓘ $\boxed{\begin{array}{llll} ROC(z^{-1}) & f(z) \Longleftrightarrow -a_n & & \overline{RNG}(n) \\ |z| > \frac{1}{p} & & & n < 0 \end{array}}$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$ $\quad X(z)$ $\quad a_n = \boxed{-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}}$ $\quad (n < 1)$

$|z| > 2$ $\quad X(z)$ $\quad b_n = \boxed{+\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}}$ $\quad (n \geq 1)$

$\{|z| < 0.5\} \cap \{|z| > 2\} = \phi \Longrightarrow a_n + b_n = 0$

$a_n = -b_n$

ROC

$|z| < a$ $\quad X(z) = \dfrac{a}{1 - az} = \displaystyle\sum_{n=0}^{\infty} a^{n+1} z^{n}$ $\quad a^{n+1}$ $\quad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$

$\Big\Vert$ $\quad \Big\downarrow {-1\cdot}$

ROC′

$|z| > a^{-1}$ $\quad X(z) = -\dfrac{z^{-1}}{1 - a^{-1}z^{-1}} = -\displaystyle\sum_{n=0}^{\infty} a^{-n} z^{-n-1}$ $\quad -a^{n+1}$ $\quad n < 0 \quad n < 1 \quad n \geq 0 \quad n \geq 1$

$$= -\sum_{k=-1}^{-\infty} a^{k+1} z^{k}$$

$$\boxed{\frac{a}{1-az}} = \sum_{n=0}^{\infty} a^{n+1} z^{n} \qquad \boxed{\frac{z}{1-az}} = \sum_{n=1}^{\infty} a^{n-1} z^{n}$$

$$\Big\Vert \qquad\qquad \Big\Vert \qquad\qquad \Big\Vert \qquad\qquad \Big\Vert$$

$$\boxed{-\frac{z^{-1}}{1-a^{-1}z^{-1}}} = -\sum_{n=0}^{\infty} a^{-n} z^{-n-1} \qquad \boxed{-\frac{a^{-1}}{1-a^{-1}z^{-1}}} = -\sum_{n=0}^{\infty} a^{-n-1} z^{-n}$$

$$= -\sum_{k=-1}^{-\infty} a^{k+1} z^{k} \qquad\qquad = -\sum_{k=0}^{-\infty} a^{k-1} z^{k}$$

$$\frac{a}{1-az} \Rightarrow \sum_{n=0}^{\infty} a^{n+1} z^{n}$$

$$\parallel \qquad \parallel$$

$$-\frac{z^{-1}}{1-a^{-1}z^{-1}} \Rightarrow -\sum_{n=0}^{\infty} a^{-n} z^{-n-1}$$

$$= -\sum_{k=-1}^{-\infty} a^{k+1} z^{k}$$

$$\frac{z}{1-az} \Rightarrow \sum_{n=1}^{\infty} a^{n-1} z^{n}$$

$$\parallel \qquad \parallel$$

$$-\frac{a^{-1}}{1-a^{-1}z^{-1}} \Rightarrow -\sum_{n=0}^{\infty} a^{-n-1} z^{-n}$$

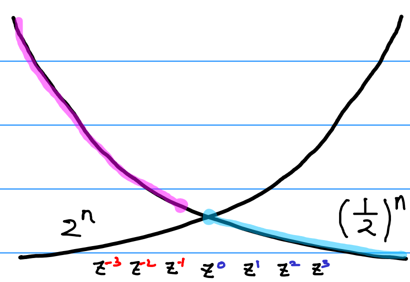
$$= -\sum_{k=0}^{-\infty} a^{k-1} z^{k}$$

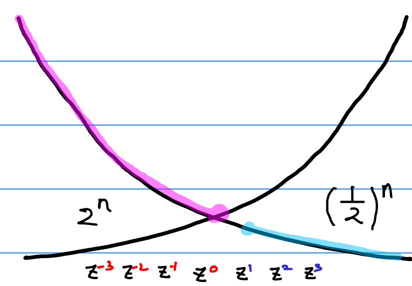
$$a + a^2 z^1 + a^3 z^2 + a^4 z^3 + \cdots$$

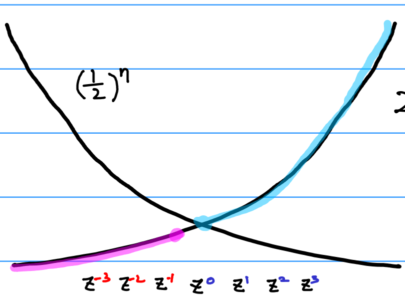
$$z^{-1} + a^{-1} z^{-2} + a^{-2} z^{-3} + a^{-3} z^{-4} + \cdots$$

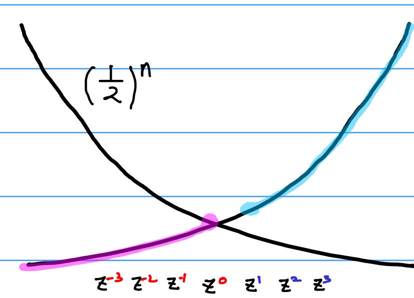
$$z + a z^2 + a^2 z^3 + a^3 z^4 + \cdots$$

$$a^{-1} + a^{-2} z^{-1} + a^{-3} z^{-2} + a^{-4} z^{-3} + \cdots$$









Ⅳ

ROC(z)	X(z) $\Longleftrightarrow$	a_{-n}	RNG(-n)
$\lvert z\rvert < p$			$n \le 0$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$ $\quad f(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$ $\quad \boxed{-2^{n+1} + (\tfrac{1}{2})^{n+1}}$ $(n \ge 0)$

$$-\left(2z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left((\tfrac{1}{2})z^0 + (\tfrac{1}{2})^2 z^1 + (\tfrac{1}{2})^3 z^2 + \cdots\right)$$

n=0 n=1 n=2 n=0 n=1 n=2

||

$|z| < 0.5$ $\quad X(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$ $\quad \boxed{-(\tfrac{1}{2})^{n-1} + 2^{n-1}}$ $(n \le 0)$

$$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left((\tfrac{1}{2})z^0 + (\tfrac{1}{2})^2 z^1 + (\tfrac{1}{2})^3 z^2 + \cdots\right)$$

$$-\left((\tfrac{1}{2})^{-1} z^0 + (\tfrac{1}{2})^{-2} z^1 + (\tfrac{1}{2})^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$$

n=0 n=-1 n=-2 n=0 n=-1 n=-2

ROC $\quad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^n$ $\quad a^{n+1}$ $\quad n \ge 0 \quad n \ge 1 \quad n < 0 \quad n < 1$

|| $\quad$ || $\quad \sum_{n=0}^{\infty} (\tfrac{1}{a})^{-n-1} z^n$ $\quad \downarrow -n$

ROC $\quad X(z) = \sum_{k=0}^{-\infty} (\tfrac{1}{a})^{k-1} z^{-k}$ $\quad a^{-n+1}$ $\quad n \le 0 \quad n \le 1 \quad n > 0 \quad n > 1$

$$= (\tfrac{1}{a})^{n-1}$$

II

ROC(z^{-1})	$f(z^{-1})$	a_{-n}	RNG($-n$)
$\lvert z\rvert > \frac{1}{p}$			$n < 1$

$|z| < 1$

$$-\frac{1}{1 - z} + \frac{0.5}{1 - 0.5z}$$

$$f(z) = -\left[1^1 + 1^2 z^1 + 1^3 z^2 + \cdots\right] + \left[\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right]$$

$$\boxed{a_n = -1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)}$$

$|z| > 1$

$$-\frac{1}{1 - z^{-1}} + \frac{0.5}{1 - 0.5z^{-1}}$$

$$f(z) = -\left[\left(\tfrac{1}{1}\right)^{-1} z^0 + \left(\tfrac{1}{1}\right)^{-2} z^{-1} + \left(\tfrac{1}{1}\right)^{-3} z^{-2} + \cdots\right] + \left[2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots\right]$$

$$\boxed{a_n = -1^{n-1} + 2^{n-1} \quad (n < 1)}$$

ROC

$|z| < a$

$$f(z) = \frac{a}{1 - az} = \sum_{n=0}^{\infty} a^{n+1} z^n$$

a^{n+1}

$n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$

z^{-1} $\qquad$ $-n$

ROC'

$|z| > a^{-1}$

$$f(z^{-1}) = \frac{a}{1 - az^{-1}} = \sum_{n=0}^{\infty} a^{n+1} z^{-n} = \sum_{k=0}^{-\infty} a^{-k+1} z^{k}$$

$$a^{-n+1} = \left(\frac{1}{a}\right)^{n-1}$$

$n < 1 \quad n < 0 \quad n \geq 0 \quad n \geq 0$

$z^{-1}X(z)$ Shifted Sequence

$$\begin{array}{ccccc} & & |z|>1 & & |z|>2 & \\ X(z) & = & \dfrac{1}{1-z^{-1}} & - & \dfrac{1}{1-2z^{-1}} & (|z|>2) \\ & & \downarrow & & \downarrow & \\ a_n & = & 1^n & + & 2^n & (n \geq 0) \end{array}$$

$$\left(1^0 z^0 + 1^1 z^{-1} + 1^2 z^{-2} + \cdots\right) + \left(2^0 z^0 + 2^1 z^{-1} + 2^2 z^{-2} + \cdots\right) \quad n \quad n = 0, 1, 2, \cdots$$

$\cdot z^{-1}$

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$\left(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots\right) + \left(2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right) \quad n-1 \quad n = 1, 2, 3, \cdots$$

$$\begin{array}{cccccc} z^{-1}X(z) & = & \dfrac{z^{-1}}{1-z^{-1}} & - & \dfrac{z^{-1}}{1-2z^{-1}} & (|z|>2) \\ & & \downarrow & & \downarrow & \\ a_{n-1} & = & 1^{n-1} & + & 2^{n-1} & (n \geq 1) \end{array}$$

$z f(z)$ Shifted Sequence

$$f(z) = (+1)\frac{1}{1-z} - \frac{1}{1-2z} \quad (|z|<0.5)$$

$$a_n = 1^n - 2^n \quad (n \geq 0)$$

$\bullet z$

$$(1^0 z^0 + 1^1 z^1 + 1^2 z^2 + \cdots) - (2^0 z^0 + 2^1 z^1 + 2^2 z^2 + \cdots) \quad n \qquad n = 0, 1, 2, \cdots$$

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots) - (2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots) \quad n-1 \qquad n = 1, 2, 3, \cdots$$

$$z f(z) = (+1)\frac{z}{1-z} - \frac{z}{1-2z} \quad (|z|<0.5)$$

$$a_{n-1} = 1^{n-1} - 2^{n-1} \quad (n \geq 1)$$

$z^{-1} f(z^{-1})$ Shifted & Reflected Sequence

$$f(z) = \frac{1}{1-z} - \frac{1}{1-2z} \qquad (|z| < 0.5)$$

$$a_n = 1^n - 2^n \qquad (n \geq 0)$$

$$(1^0 z^0 + 1^1 z^1 + 1^2 z^2 + \cdots) - (2^0 z^0 + 2^1 z^1 + 2^2 z^2 + \cdots) \quad n \quad n = 0, 1, 2, \cdots$$

$\cdot z$

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \qquad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots) - (2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots) \quad n-1 \quad n = 1, 2, 3, \cdots$$

$$z f(z) = \frac{z}{1-z} - \frac{z}{1-2z} \qquad (|z| < 0.5)$$

$$a_{n-1} = 1^{n-1} - 2^{n-1} \qquad (n \geq 1)$$

z

$$(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) + (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots) \quad n \quad n = 1, 2, 3, \cdots$$

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \qquad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

z^{-1}

$$(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots) + (2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots) \quad -n \quad n = -1, -2, -3, \cdots$$

$$z^{-1} f(z^{-1}) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad (|z| > 2)$$

$$a_{-n-1} = 1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \qquad (n < 0)$$

$$a_{-(n+1)}$$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5 \quad X(z) \qquad a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n<1)$

$|z| > 2 \quad X(z) \qquad b_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$

$\{|z|<0.5\} \cap \{|z|>2\} = \phi \Rightarrow a_n + b_n = 0$

$a_n = -b_n$

$|z| < a \quad X(z) = \sum_{n=0}^{\infty} a^{n+1} z^n \qquad a^{n+1} \qquad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$

$\sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n \qquad \xrightarrow{-n}$

$|z| > a \quad X(z) = \sum_{k=0}^{-\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k} \qquad a^{-n+1} \qquad n < 0 \quad n < 1 \quad n \geq 0 \quad n \geq$

$= \left(\frac{1}{a}\right)^{n-1}$

$a^{n+1} z^n$

$a\,(az)^n$

$a\left(\frac{1}{az}\right)^{-n}$

$\frac{a}{1-az} \qquad \sum_{n=0}^{\infty} a^{n+1} z^n \qquad \frac{z}{1-az} \qquad \sum_{n=1}^{\infty} a^{n-1} z^n$

$-\frac{z^{-1}}{1-a^{-1}z^{-1}} \qquad -\sum_{n=0}^{\infty} a^{-n} z^{-n-1} \qquad -\frac{a^{-1}}{1-a^{-1}z^{-1}} \qquad -\sum_{n=1}^{\infty} a^{-n-1} z^{-n}$

$\sum_{n=-1}^{-\infty} a^{n+1} z^n \qquad - \sum_{n=0}^{-\infty} a^{n-1} z^n$

$zX(z)$ Shifted & Reflected Sequence

$$X(z) = \frac{1}{1-z^{-1}} - \frac{1}{1-2z^{-1}} \quad (|z| > 2)$$

$$a_n = 1^n - 2^n \quad (n \geq 0)$$

$\bullet z^{-1}$

$$(1^0 z^0 + 1^1 z^{-1} + 1^2 z^{-2} + \cdots) + (2^0 z^0 + 2^1 z^{-1} + 2^2 z^{-2} \quad) \quad n \quad n = 0, 1, 2, \cdots$$

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) + (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots) \quad n-1 \quad n = 1, 2, 3, \cdots$$

$$z^{-1}X(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad (|z| > 2)$$

$$a_{n-1} = 1^{n-1} - 2^{n-1} \quad (n \geq 1)$$

z

$$(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) + (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots) \quad n \quad n = 1, 2, 3, \cdots$$

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

z^{-1}

$$(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots) + (2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots) \quad -n \quad n = -1, -2, -3, \cdots$$

$$zX(z^{-1}) = \frac{z}{1-z} - \frac{z}{1-2z} \quad (|z| < 0.5)$$

$$a_{-n-1} = 1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$$

$$a_{-(n+1)}$$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$ $$f(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \qquad \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \qquad (n \geq 0)$$

$$-\left(2z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$n=0$ $n=1$ $n=2$ $n=0$ $n=1$ $n=2$

$|z| < 0.5$ $$X(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \qquad \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \qquad (n \leq 0)$$

$$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$$

$n=0$ $n=-1$ $n=-2$ $n=0$ $n=-1$ $n=-2$

ROC $$f(z) = \sum_{n=0}^{\infty} a^{n+1} z^n \qquad a^{n+1} \qquad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$$

$\|$ $\|$

$$\sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n \qquad -n$$

ROC $$X(z) = \sum_{k=0}^{-\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k} \qquad a^{-n+1} = \left(\frac{1}{a}\right)^{n-1} \qquad n < 0 \quad n < 1 \quad n \geq 0 \quad n \geq 1$$

ROC $\quad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^{n}$ $\quad a^{n+1}$ $\quad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$

$z^{-1} \qquad z^{-1} \qquad \sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^{n} \qquad -n$

ROC $\quad X(z) = \sum_{k=0}^{-\infty} (a)^{k-1} z^{-k}$ $\quad \left(\frac{1}{a}\right)^{-n+1}$ $\quad n < 0 \quad n < 1 \quad n \geq 0 \quad n \geq 1$

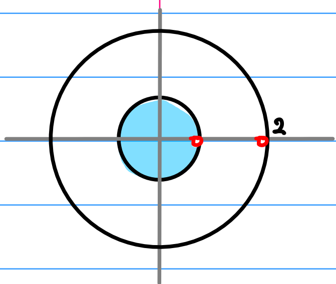
$= a^{n-1}$

Causal $\quad f(z)$ $\quad X(z)$

$|z| < 0.5 \quad |z| > 2$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 2 \qquad |z| < 0.5$



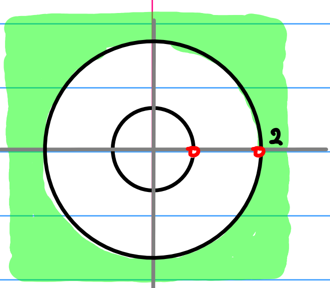
$$f(z) = (-2)\frac{0.5}{0.5-z} + (0.5)\frac{2}{2-z} \qquad (|z|<0.5)$$

$$a_n = (-2)\,2^n + (0.5)\left(\tfrac{1}{2}\right)^n \qquad (n \geq 0)$$

$$-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}$$

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) \qquad |z| > 2$$

$|z| > 2 \qquad |z| > 0.5$



$$X(z) = 0.5\,\frac{z}{z-0.5} - 2\,\frac{z}{z-2} \qquad (|z| > 2)$$

$$a_n = (0.5)\left(\tfrac{1}{2}\right)^n - 2 \cdot 2^n \qquad (n \geq 0)$$

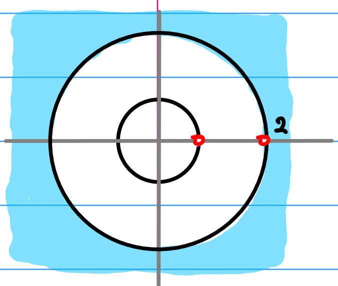
$$\left(\tfrac{1}{2}\right)^{n+1} - 2^{n+1}$$

Anti-causal $\quad f(z) \quad X(z)$

$\qquad |z| > 2 \qquad |z| < 0.5$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z|>2 \qquad |z|>0.5$



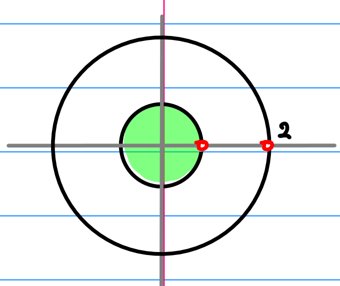
$$f(z) = (-2)\frac{-0.5}{0.5-z} + (0.5)\frac{-2}{2-z} \qquad (|z|>0.5)$$

$$a_n = (+2)\,2^n - (0.5)\left(\tfrac{1}{2}\right)^n \qquad (n<0)$$

$$+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}$$

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) \qquad |z|<2$$

$|z|<2 \qquad |z|<0.5$



$$X(z) = 0.5\,\frac{-z}{z-0.5} - 2\,\frac{-z}{z-2} \qquad (|z|<2)$$

$$a_n = -(0.5)\left(\tfrac{1}{2}\right)^n + 2\cdot 2^n \qquad (n<0)$$

$$-\left(\tfrac{1}{2}\right)^{n+1} + 2^{n+1}$$