

# Resolution (3A)

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# Argument

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$$\frac{(p \vee q) \quad (\neg p \vee r)}{q \vee r}$$

$$(p \vee q) \wedge (\neg p \vee r) \rightarrow q \vee r$$

# Truth Table









$p$	$q$	$r$	$p \vee q$
$T$	$T$	$T$	$T$
$T$	$T$	$F$	$T$
$T$	$F$	$T$	$T$
$T$	$F$	$F$	$T$
$F$	$T$	$T$	$T$
$F$	$T$	$F$	$T$
$F$	$F$	$T$	$F$
$F$	$F$	$F$	$F$

$p$	$q$	$r$	$\neg p$	$\neg p \vee q$
$T$	$T$	$T$	$F$	$T$
$T$	$T$	$F$	$F$	$F$
$T$	$F$	$T$	$F$	$T$
$T$	$F$	$F$	$F$	$F$
$F$	$T$	$T$	$T$	$T$
$F$	$T$	$F$	$T$	$T$
$F$	$F$	$T$	$T$	$T$
$F$	$F$	$F$	$T$	$T$

$p$	$q$	$r$	$(p \vee q) \wedge (\neg p \vee r)$	$q \vee r$
$T$	$T$	$T$	$T$	$T$
$T$	$T$	$F$	$F$	$T$
$T$	$F$	$T$	$T$	$T$
$T$	$F$	$F$	$F$	$F$
$F$	$T$	$T$	$T$	$T$
$F$	$T$	$F$	$T$	$T$
$F$	$F$	$T$	$F$	$T$
$F$	$F$	$F$	$F$	$F$

$$(p \vee q) \wedge (\neg p \vee r) \rightarrow q \vee r$$

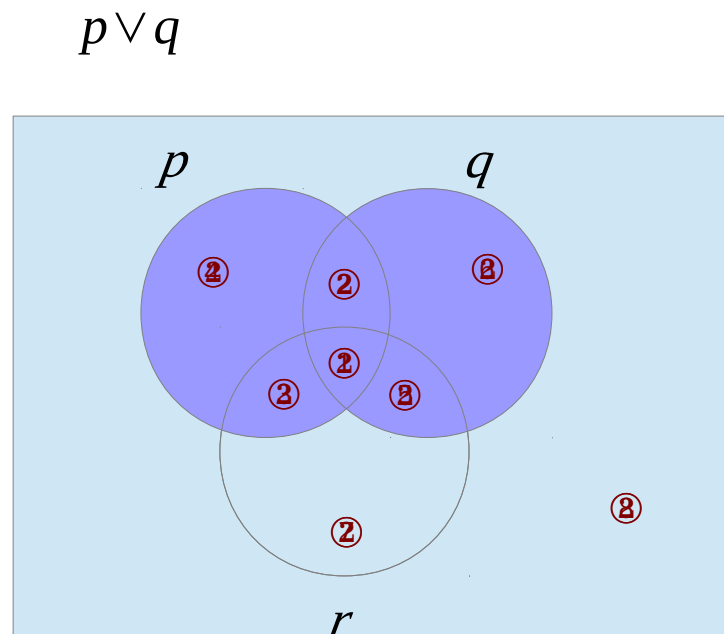
# Interpretation of this truth table

	$p$	$q$	$r$	$p \vee q$	$\neg p \vee r$	$(p \vee q) \wedge (\neg p \vee r)$	$q \vee r$
case ①	$T$	$T$	$T$	$T$	$T$	$T$ 	$T$
case ②	$T$	$T$	$F$	$T$	$F$	$F$ 	$T$
case ③	$T$	$F$	$T$	$T$	$T$	$T$ 	$T$
case ④	$T$	$F$	$F$	$T$	$F$	$F$ 	$F$
case ⑤	$F$	$T$	$T$	$T$	$T$	$T$ 	$T$
case ⑥	$F$	$T$	$F$	$T$	$T$	$T$ 	$T$
case ⑦	$F$	$F$	$T$	$F$	$T$	$F$ 	$T$
case ⑧	$F$	$F$	$F$	$F$	$T$	$F$ 	$F$

Whenever  $p \vee q$  and  $\neg p \vee r$  are **true**,  $q \vee r$  is **true**

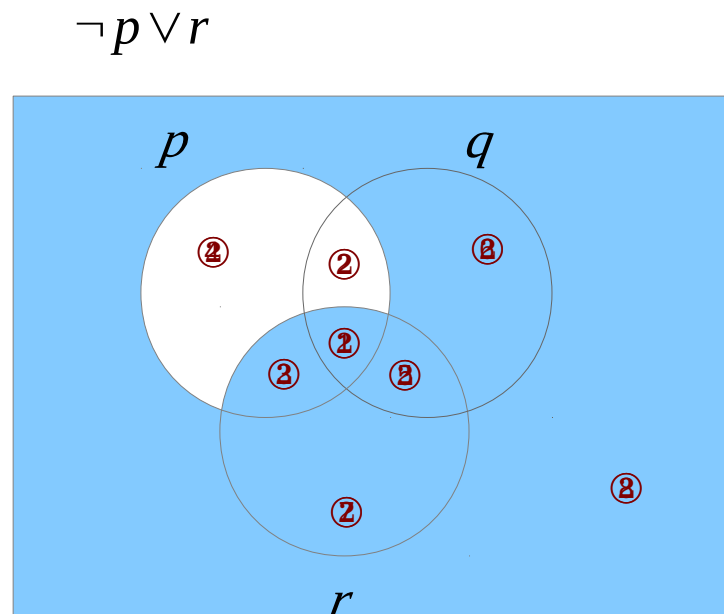
$$(p \vee q) \wedge (\neg p \vee r) \rightarrow q \vee r$$

# Venn diagram for $p \vee q$



	$p$	$q$	$r$	$p \vee q$
case ①	$T$	$T$	$T$	$T$
case ②	$T$	$T$	$F$	$T$
case ③	$T$	$F$	$T$	$T$
case ④	$T$	$F$	$F$	$T$
case ⑤	$F$	$T$	$T$	$T$
case ⑥	$F$	$T$	$F$	$T$
case ⑦	$F$	$F$	$T$	$F$
case ⑧	$F$	$F$	$F$	$F$

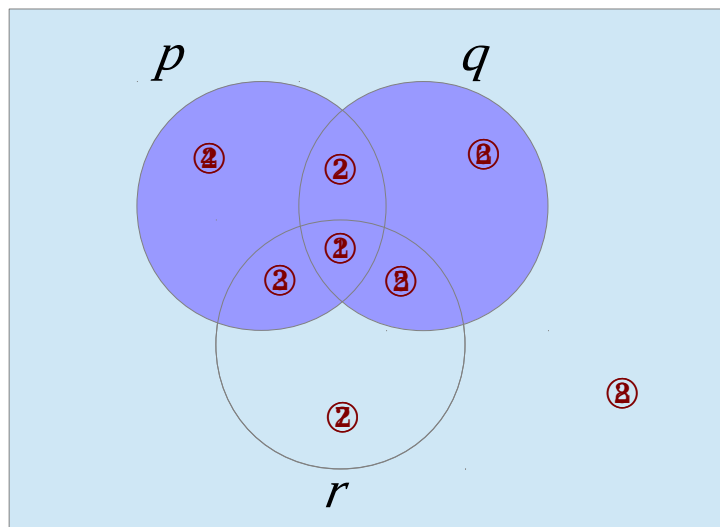
# Venn diagram for $\neg p \vee r$



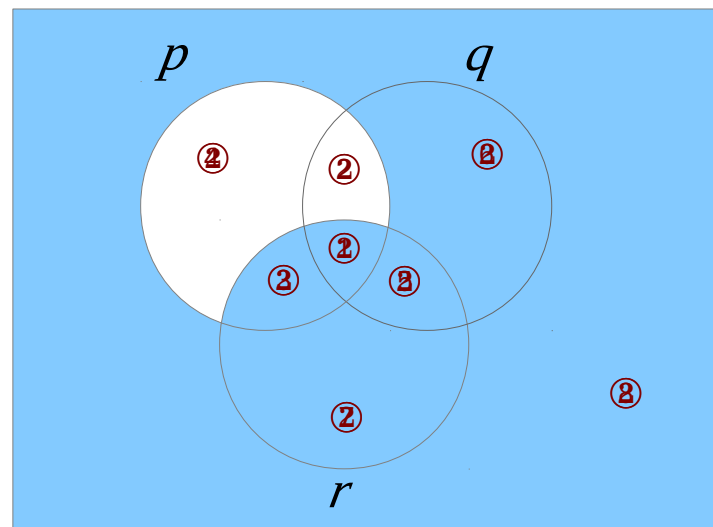
	$p$	$q$	$r$	$\neg p$	$\neg p \vee q$
case ②	$T$	$T$	$T$	$F$	$T$
case ②	$T$	$T$	$F$	$F$	$F$
case ②	$T$	$F$	$T$	$F$	$T$
case ②	$T$	$F$	$F$	$F$	$F$
case ⑧	$F$	$T$	$T$	$T$	$T$
case ⑧	$F$	$T$	$F$	$T$	$T$
case ②	$F$	$F$	$T$	$T$	$T$
case ⑧	$F$	$F$	$F$	$T$	$T$

# When $(p \vee q) \wedge (\neg p \vee r)$ is true

$p \vee q$



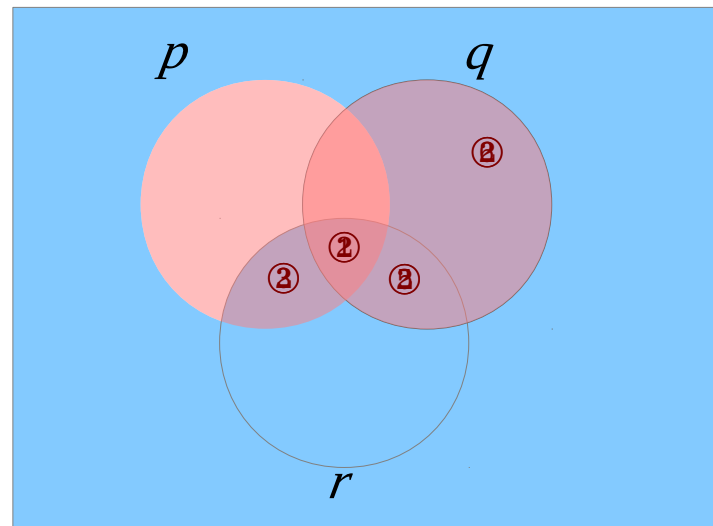
$\neg p \vee r$



When  $p \vee q$  is true  
and  $\neg p \vee r$  is true

$$(p \vee q) \wedge (\neg p \vee r)$$

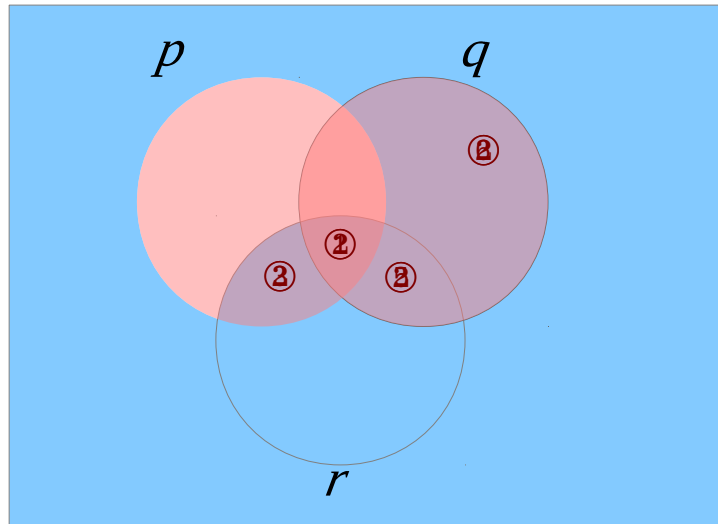
cases 2+3+5+6



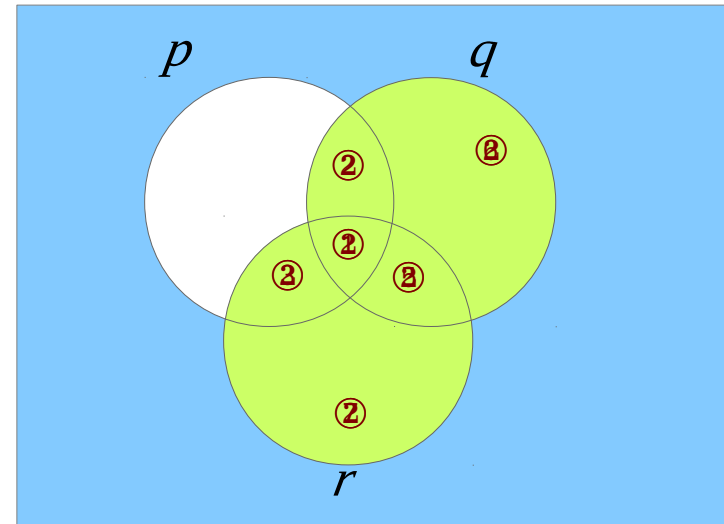


When  $(p \vee q) \wedge (\neg p \vee r)$  is true,  $q \vee r$  is also true

$p \vee q$   
 $\neg p \vee r$



$q \vee r$



cases ①+③+⑤+⑥

$\subset$

cases ①+③+⑤+⑥+②+⑦

$(p \vee q) \wedge (\neg p \vee r)$



$q \vee r$

# Argument

$$\begin{array}{c} p \vee q \\ \neg p \vee r \\ \hline q \vee r \end{array}$$

Case 1:  $p$  is false

$$\begin{array}{c} F \vee q \\ T \vee r \\ \hline q \end{array}$$

when  $p$  is false,  
 $q$  must be true.

Case 2:  $p$  is true

$$\begin{array}{c} T \vee q \\ F \vee r \\ \hline r \end{array}$$

when  $p$  is true,  
 $r$  must be true.

Therefore regardless of truth value of  $p$ ,  
If both premises hold,  
then the conclusion  $q \vee r$  is true

<http://en.wikipedia.org/wiki/Derivative>

# Resolution Examples

$$\frac{p \vee q \quad \neg p \vee r}{q \vee r}$$



$$\frac{\cancel{p \vee q} \quad \neg p \vee r}{q \vee r}$$



$$\frac{\cancel{p \vee q} \quad \cancel{\neg p \vee r}}{q \vee r}$$

$$\frac{p \vee q \quad \neg p}{q}$$



$$\frac{\cancel{p \vee q} \quad \cancel{\neg p}}{q}$$



$$\frac{\cancel{p \vee q} \quad \cancel{\neg p}}{q}$$

$$\frac{p \quad \neg p \vee r}{r}$$



$$\frac{\cancel{p} \quad \neg p \vee r}{r}$$



$$\frac{\cancel{p} \quad \cancel{\neg p \vee r}}{r}$$

# More Example (1)

$$\left( \begin{array}{l} p \vee q \\ p \vee \neg r \\ \neg p \vee q \\ \neg q \vee r \end{array} \right)$$

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$$\left( \begin{array}{l} q \\ p \vee \neg r \\ \neg q \vee r \end{array} \right)$$

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$$\left( \begin{array}{l} p \vee \neg r \\ r \end{array} \right)$$

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$$p$$

$$\begin{aligned} & (p \vee q) \wedge (p \vee \neg r) \wedge (\neg p \vee q) \wedge (\neg q \vee r) \\ \vdash & (p \vee q) \wedge (p \vee \neg r) \wedge (\neg p \vee q) \wedge (\neg q \vee r) \wedge (q) \\ \vdash & (p \vee q) \wedge (p \vee \neg r) \wedge (\neg p \vee q) \wedge (\neg q \vee r) \wedge (q) \wedge (r) \\ \vdash & (p \vee q) \wedge (p \vee \neg r) \wedge (\neg p \vee q) \wedge (\neg q \vee r) \wedge (q) \wedge (r) \wedge (p) \end{aligned}$$

## More Example (2)

$$p \rightarrow q \vee r$$

$$p \vee \neg q$$

$$r \vee q$$

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$$\neg p \vee q \vee r$$

$$p \vee \neg q$$

$$r \vee q$$

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$$q \vee \neg q \vee r$$

$$\neg q$$

$$r \vee q$$

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$$\neg q$$

$$r \vee q$$

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$$r$$

$$q \vee \neg q \vee r = T \vee r = T$$

## References

- [1] <http://en.wikipedia.org/>
- [2]