# Complex Random Processes

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles,Jr. and B. Shi

# Outline

## Definition

$$Z(t) = X(t) + jY(t)$$

$$E[Z(t)] = E[X(t)] + jE[Y(t)]$$

$$R_{ZZ}(t, t+\tau) = E[Z(t)Z^*(t+\tau)]$$

$$C_{ZZ}(t, t+\tau) = E[Z(t) - E[Z(t)]]\{E[Z(t+\tau) - E[Z(t+\tau)]]\}^*$$

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### Definition

$$R_{ZZ}(t,t+\tau) = E[Z(t)Z^*(t+\tau)]$$

$$C_{ZZ}(t,t+\tau) = E[Z(t) - E[Z(t)]]\{E[Z(t+\tau) - E[Z(t+\tau)]]\}^*$$

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$$\widetilde{R}_{ZZ}(t,t+\tau) = E[Z(t)Z(t+\tau)]$$

$$\widetilde{C}_{ZZ}(t,t+\tau) = E[Z(t) - E[Z(t)]]\{E[Z(t+\tau) - E[Z(t+\tau)]]\}$$

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**Definition**

A complex random process Z(t) is said to be proper if the pseudo-autocovariance function is identically zero.
If Z(t) is at least wide-sense stationary, the mean value becomes a constant

$$\overline{Z} = \overline{X} + j\overline{Y}$$

the correlation and pseudo-correlation functions are independent of absolute time

$$R_{ZZ}(t, t+\tau) = R_{ZZ}(\tau) \qquad \widetilde{R}_{ZZ}(t, t+\tau) = \widetilde{R}_{ZZ}(\tau)$$

$$C_{ZZ}(t, t+\tau) = C_{ZZ}(\tau) \qquad \widetilde{C}_{ZZ}(t, t+\tau) = \widetilde{C}_{ZZ}(\tau)$$

## Definition

$$R_{Z_i Z_j}(t, t+\tau) = E\left[Z_i(t) Z_j^*(t+\tau)\right]$$

$$C_{Z_i Z_j}(t, t+\tau) = E\left[\{Z_i(t) - E[Z_i(t)]\}\{Z_j(t+\tau) - E[Z_j(t+\tau)]\}^*\right]$$

$$R_{Z_i Z_j}(t, t+\tau) = E[Z_i(t) Z_j(t+\tau)]$$

$$C_{Z_i Z_j}(t, t+\tau) = E[\{Z_i(t) - E[Z_i(t)]\}\{Z_j(t+\tau) - E[Z_j(t+\tau)]\}]$$

## Definition

If the two processes are at least jointly wide-sense stationary

$$R_{Z_i Z_j}(t, t+\tau) = R_{Z_i Z_j}(\tau)$$

$$C_{Z_i Z_j}(t, t+\tau) = C_{Z_i Z_j}(\tau)$$

$$R_{Z_i Z_j}(t, t+\tau) = R_{Z_i Z_j}(\tau)$$

$$C_{Z_i Z_j}(t, t+\tau) = C_{Z_i Z_j}(\tau)$$

**Definition**

$Z_i(t)$and $Z_j(t)$ are uncorrelated processes if $C_{Z_iZ_j}(t, t+\tau) = 0$ and $\widetilde{C}_{Z_iZ_j}(t, t+\tau) = 0$

$Z_i(t)$and $Z_j(t)$ are orthogonal processes if $R_{Z_iZ_j}(t, t+\tau) = 0$ and $\widetilde{R}_{Z_iZ_j}(t, t+\tau) = 0$

$Z_i(t)$and $Z_j(t)$ are jointly proper processes if $\widetilde{C}_{Z_iZ_j}(t, t+\tau) = 0$