

# Complex Random Processes

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Based on  
Probability, Random Variables and Random Signal Principles,  
P.Z. Peebles,Jr. and B. Shi



## Definition

$$Z(t) = X(t) + jY(t)$$

$$E[Z(t)] = E[X(t)] + jE[Y(t)]$$

$$R_{ZZ}(t, t + \tau) = E[Z(t)Z^*(t + \tau)]$$

$$C_{ZZ}(t, t + \tau) = E[Z(t) - E[Z(t)]] \{E[Z(t + \tau) - E[Z(t + \tau)]]\}^*$$

# Pseudo-correlation and covariance functions

$N$  Gaussian random variables

## Definition

$$R_{ZZ}(t, t + \tau) = E[Z(t)Z^*(t + \tau)]$$

$$C_{ZZ}(t, t + \tau) = E[Z(t) - E[Z(t)]] \{E[Z(t + \tau) - E[Z(t + \tau)]]\}^*$$

$$\tilde{R}_{ZZ}(t, t + \tau) = E[Z(t)Z(t + \tau)]$$

$$\tilde{C}_{ZZ}(t, t + \tau) = E[Z(t) - E[Z(t)]] \{E[Z(t + \tau) - E[Z(t + \tau)]]\}$$

# Proper Random Processes

$N$  Gaussian random variables

## Definition

A complex random process  $Z(t)$  is said to be proper if the pseudo-autocovariance function is identically zero.

If  $Z(t)$  is at least wide-sense stationary, the mean value becomes a constant

$$\bar{Z} = \bar{X} + j\bar{Y}$$

the correlation and pseudo-correlation functions are independent of absolute time

$$R_{ZZ}(t, t + \tau) = R_{ZZ}(\tau) \quad \tilde{R}_{ZZ}(t, t + \tau) = \tilde{R}_{ZZ}(\tau)$$

$$C_{ZZ}(t, t + \tau) = C_{ZZ}(\tau) \quad \tilde{C}_{ZZ}(t, t + \tau) = \tilde{C}_{ZZ}(\tau)$$

# Cross / Pseudo-cross, -corelation / -covariance

$N$  Gaussian random variables

## Definition

$$R_{Z_i Z_j}(t, t + \tau) = E [Z_i(t) Z_j^*(t + \tau)]$$

$$C_{Z_i Z_j}(t, t + \tau) = E [\{Z_i(t) - E[Z_i(t)]\} \{Z_j(t + \tau) - E[Z_j(t + \tau)]\}^*]$$

$$R_{Z_i Z_j}(t, t + \tau) = E [Z_i(t) Z_j(t + \tau)]$$

$$C_{Z_i Z_j}(t, t + \tau) = E [\{Z_i(t) - E[Z_i(t)]\} \{Z_j(t + \tau) - E[Z_j(t + \tau)]\}]$$

# Jointly Wide Sense Stationary Process

$N$  Gaussian random variables

## Definition

If the two processes are at least jointly wide-sense stationary

$$R_{Z_i Z_j}(t, t + \tau) = R_{Z_i Z_j}(\tau)$$

$$C_{Z_i Z_j}(t, t + \tau) = C_{Z_i Z_j}(\tau)$$

$$R_{Z_i Z_j}(t, t + \tau) = R_{Z_i Z_j}(\tau)$$

$$C_{Z_i Z_j}(t, t + \tau) = C_{Z_i Z_j}(\tau)$$



# Uncorrelated / Orthogonal / Jointly Proper Process

$N$  Gaussian random variables

## Definition

$Z_i(t)$  and  $Z_j(t)$  are uncorrelated processes if  $C_{Z_i Z_j}(t, t + \tau) = 0$  and

$$\tilde{C}_{Z_i Z_j}(t, t + \tau) = 0$$

$Z_i(t)$  and  $Z_j(t)$  are orthogonal processes if  $R_{Z_i Z_j}(t, t + \tau) = 0$  and

$$\tilde{R}_{Z_i Z_j}(t, t + \tau) = 0$$

$Z_i(t)$  and  $Z_j(t)$  are jointly proper processes if  $\tilde{C}_{Z_i Z_j}(t, t + \tau) = 0$



