Logic Background (1A)

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Techniques in Artificial Intelligence - courses - MIT

https://courses.csail.mit.edu/6.825/

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Proposition : etymology

From Old French, from Latin prōpositiō
("a proposing, design, theme, case").

The content of an assertion
that may be taken as being **true** or **false**
and is considered abstractly without reference to
the linguistic sentence that constitutes the assertion.

Predicate : etymology

From Middle French predicate (French prédicat),

from post-classical Late Latin praedicatum (“thing said of a subject”),

a noun use of the neuter past participle of praedicare (“proclaim”)

From Latin predicātus, perfect passive participle of praedicō,

from prae + dicō (“declare, proclaim”), from dicō (“say, tell”).

Proposition

In Aristotelian logic a proposition is
a particular kind of sentence,
one which affirms or denies
a **predicate** of a subject.

In formal logic a proposition is considered as
objects of a formal language.
A formal language begins with different types of symbols.

Predicate

(grammar) The part of the sentence (or clause)

which states something about

the subject or the object of the sentence.

The dog barked very loudly

subject *predicate*

(logic) A term of a statement,

where the statement may be true or false

depending on whether the thing referred to

by the values of the statement's variables

has the property signified by that (predicative) term.

Propositional Logic

Propositional logic includes only

- operators and
- propositional constants

as symbols in its language.

The propositions in this language are

- propositional constants
  considered atomic propositions
- composite propositions
  recursive application of operators to propositions

Predicate Logic

Predicate logic include

- variables,
- operators,
- predicate and
- function symbols, and
- quantifiers

as symbols in their languages.

Logic

A Formal Language

Syntax – legal expressions

Semantics – the meaning of legal expressions

Proof System – a way of manipulating syntactic expressions
to get another syntactic expressions

Multiple Percepts → Conclusions

Current State, Operators → Next State Properties

Propositional Logic

Sentences (WFFs : Well Formed Formulas)

T and F are sentences

Propositional variables are sentences (A, B, C, …)

If A and B are sentences, the followings are also sentences

(A), $\neg A$, $A \wedge B$, $A \vee B$, $A \rightarrow B$, $A \leftrightarrow B$

Precedence of Connectives

$\neg$	highest
$\wedge$	
$\vee$	
$\Rightarrow$	
$\Leftrightarrow$	lowest

$A \vee B \wedge C \quad = \quad A \vee (B \wedge C)$

$A \wedge B \Rightarrow C \vee D \quad = \quad (A \wedge B) \Rightarrow (C \vee D)$

$A \Rightarrow B \vee C \Leftrightarrow D \quad = \quad (A \Rightarrow (B \vee C)) \Leftrightarrow D$

Semantics

Meaning of a sentence : **true** or **false**

Interpretation : an assignment of **true** or **false**
to the propositional variables

$\models_i \varphi$: Sentence φ is true in the interpretation **i**

$\not\models_i \varphi$: Sentence φ is true in the interpretation **i**

Semantic Rules

$\models_i \varphi$: Sentence φ is true in the interpretation **i**

$\models_i \varphi$: Sentence φ is true in the interpretation **i**

$\models_i \mathbf{T}$ for all **i**

$\not\models_i \mathbf{F}$ for all **i**

$\models_i \mathbf{T}$ for all **i**

$\models_i \neg\varphi$ iff $\not\models_i \varphi$

$\models_i \varphi \land \psi$ iff $\models_i \varphi$ **and** $\models_i \psi$

$\models_i \varphi \lor \psi$ iff $\models_i \varphi$ **or** $\models_i \psi$

$\models_i P$ iff $\mathbf{i}(P) = \mathbf{T}$

Since **i** is a mapping from variables to truth values,

Look P up in **i** and return the truth value assigned to P

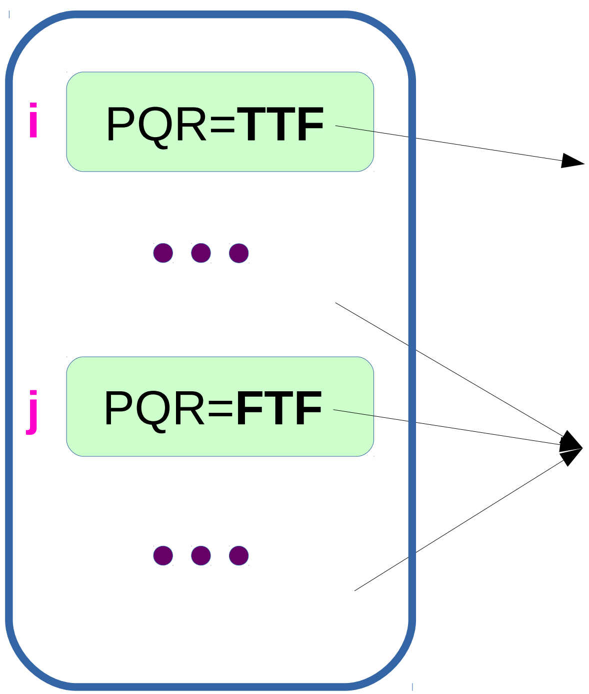
Semantics

Meaning of a sentence : **true** or **false**

Interpretation : an assignment of **true** or **false**

to the propositional variables (P, Q, R)

Interpretations



Interpretations

sentence

$P \land Q \land \neg R$ ➡ **true**

$\models_i P \land Q \land \neg R$

sentence

$P \land Q \land \neg R$ ➡ **false**

$\not\models_j P \land Q \land \neg R$

Valid, Satisfiable, Unsatisfiable

A sentence is **valid** iff

Its truth value is **T** in <u>all</u> interpretations

ex) **T**, ¬**F**, $P \vee \neg P$ (**tautology**: $\top$)

A sentence is **satisfiable** iff

Its truth value is **T** in <u>at least one</u> interpretation

ex) P, **T**, ¬P

A sentence is **unsatisfiable** valid iff

Its truth value is **F** in <u>all</u> interpretations

ex) **F**, ¬**T**, $P \wedge \neg P$ (**contradiction**: $\perp$)

Models

Semantics :

The relationship between sentences and interpretations

There are some set of interpretations that makes a sentence **true**

→ **models** of a sentence

An interpretation is a **model** of a sentence if the sentence is **true** in that interpretation

An interpretation ***i*** is a **model** of a sentence φ ***iff*** $\vDash_i \varphi$

a sentence

models

x = John
x = Socrates
x = Baker

F(x) ⇨ **true**

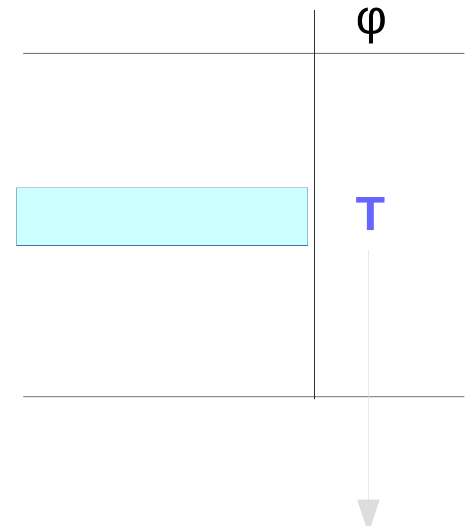
Models

An interpretation ***i*** is a **model** of a sentence φ ***iff*** $\models_i \varphi$

$$\models_i \varphi$$

an interpretation a sentence

interpretation ***i***



interpretation ***i***
is a model

Models and Interpretations

models

8 interpretations

P	Q	R	P∧(Q∨R)
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

T	T	T
T	T	F

T	T	T
T	F	T

T	F	T

Entailment

An interpretation i is a **model** of a sentence φ ***iff*** $\vDash_i \varphi$

A set of sentences KB **entails** a sentence φ

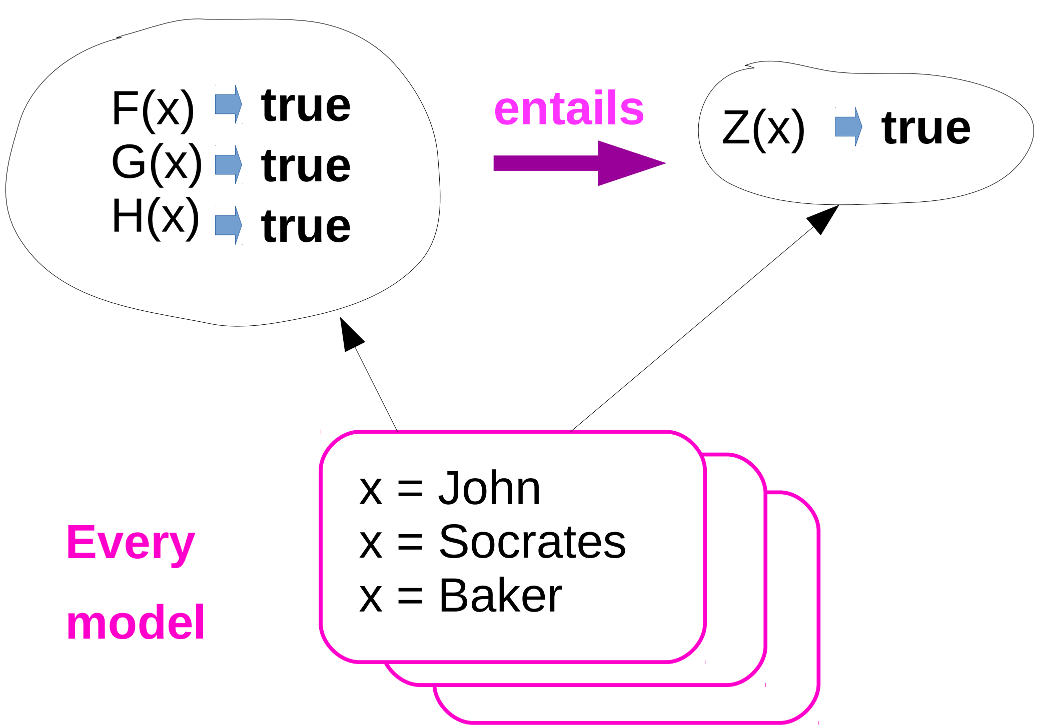
iff **<u>every</u>** **model** of KB is also a **model** of φ

KB: Knowledge Base
a set of sentences

every model of KB

every model of
each sentence in KB

the model of a sentence φ



Models and Entailment

A set of sentences KB **entails** a sentence φ

iff **<u>every</u>** **model** of KB is also a **model** of φ

entail

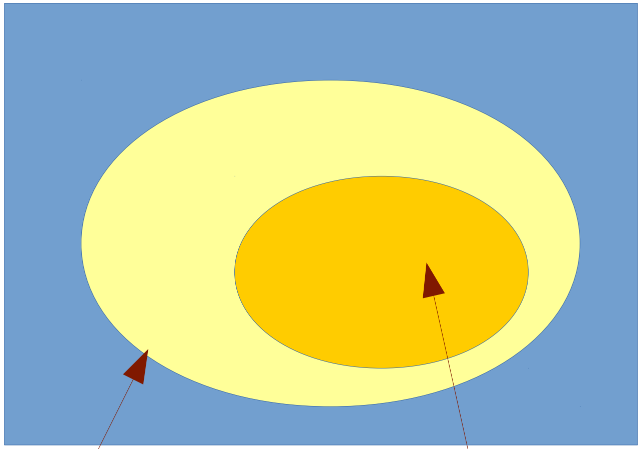
$$KB \models \varphi$$

a set of sentences

a sentence

A set of all models of KB $\subset$ A set of all models of φ

U: A set of all possible interpretation



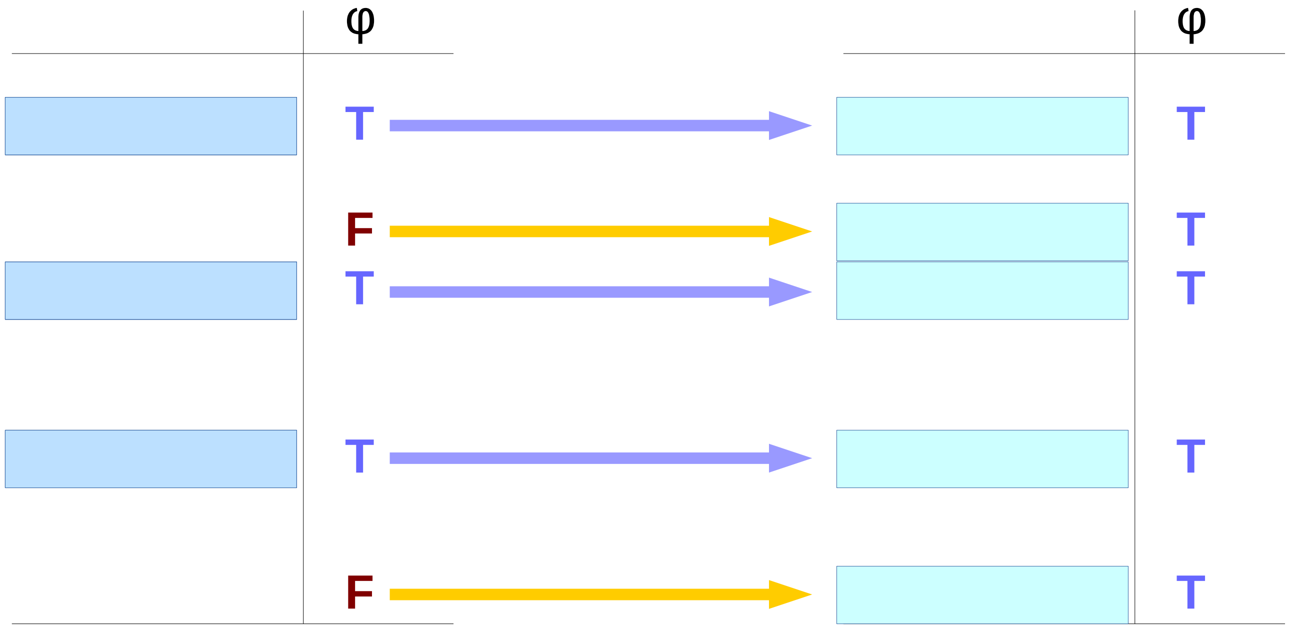
A set of all models of φ $\supset$ A set of all models of KB

Truth Tables and Entailment

A set of sentences KB **entails** a sentence φ
iff **<u>every</u>** **model** of KB is also a **model** of φ

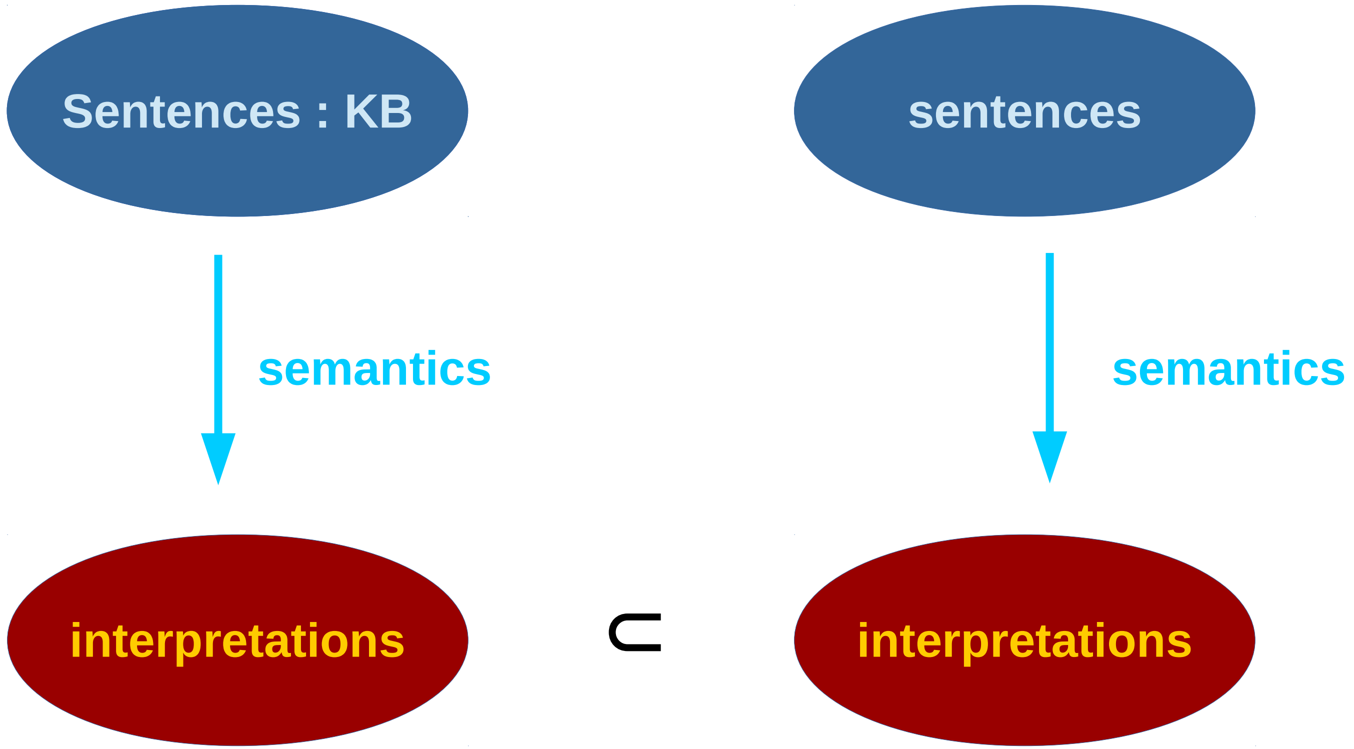
entail

$$KB \models \varphi$$



A set of all models of KB $\subset$ A set of all models of φ

Entailment


Sentences : KB
sentences
semantics
semantics
interpretations
⊂
interpretations

References

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