Laurent Series and z-Transform
- Geometric Series
Double Pole Properties A

20181204 Tue

2 formulas of z

① $$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$$\Big\updownarrow z^{-1}$$

② $$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

$$f(z) = \begin{cases} f_1(z) \\ f_2(z^{-1}) \end{cases} \qquad g(z) = \begin{cases} g_1(z) \\ g_2(z^{-1}) \end{cases}$$

$$X(z) = \begin{cases} X_1(z) \\ X_2(z^{-1}) \end{cases} \qquad Y(z) = \begin{cases} Y_1(z) \\ Y_2(z^{-1}) \end{cases}$$

① $\frac{3}{2} \frac{-1}{(z-0.5)(z-2)}$

$$\boxed{+\frac{1}{z-0.5} - \frac{1}{z-2}}$$

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \ \boxed{|z|<0.5}$$

causal $f_1(z)$

anti-causal $X_1(z)$

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \ \boxed{|z|>2}$$

anti-causal $f_2(z)$

causal $X_2(z)$

② $\frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)}$

$$\boxed{-\frac{2z}{(z-2)} + \frac{0.5z}{(z-0.5)}}$$

$$\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \ \boxed{|z|>2}$$

anti-causal $g_1(z)$

causal $Y_1(z)$

$$\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \ \boxed{|z|<0.5}$$

causal $g_2(z)$

anti-causal $Y_2(z)$

$f(z) \Longleftrightarrow a_n$

$X(z) \Longleftrightarrow x_n$

① – Ⓐ

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \quad \boxed{|z|<0.5}$$

$$a_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$$x_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n<1)$$

② – Ⓐ

$$\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \quad \boxed{|z|>2}$$

$$a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n<1)$$

$$x_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

① – Ⓑ

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \quad \boxed{|z|>2}$$

$$a_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0)$$

$$x_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

② – Ⓑ

$$\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \quad \boxed{|z|<0.5}$$

$$a_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

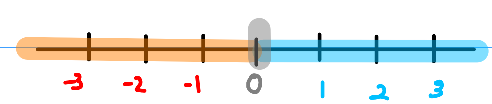
$$x_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0)$$

$$x_n = a_{-n}$$

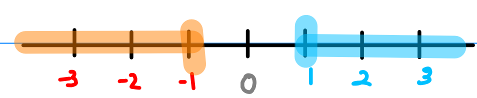
$$a_n = x_{-n}$$

$(n \geq 0) \Longleftrightarrow (n<1)$

$(n \geq 1) \Longleftrightarrow (n<0)$



$(n<1)$ $(n \geq 0)$



$(n<0)$ $(n \geq 1)$

$$(z^{-1},\ R^{-1}) \Leftrightarrow (a_{-n},\ -N)$$

①-Ⓐ		②-Ⓐ
$\boxed{-\dfrac{2}{1-2z} + \dfrac{0.5}{1-0.5z}}\ \boxed{\lvert z\rvert<0.5}$	$\xleftrightarrow{z^{-1},\ R^{-1}}$	$\boxed{-\dfrac{2}{1-2z^{-1}} + \dfrac{0.5}{1-0.5z^{-1}}}\ \boxed{\lvert z\rvert>2}$
$a_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}}\quad (n \geq 0)$	$\xleftrightarrow{-n,\ -N}$	$a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}}\quad (n<1)$
$x_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}}\quad (n<1)$	$\xleftrightarrow{-n,\ -N}$	$x_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}}\quad (n \geq 0)$

①-Ⓑ		②-Ⓑ
$\boxed{+\dfrac{z^{-1}}{1-0.5z^{-1}} - \dfrac{z^{-1}}{1-2z^{-1}}}\ \boxed{\lvert z\rvert>2}$	$\xleftrightarrow{z^{-1},\ R^{-1}}$	$\boxed{+\dfrac{z}{1-0.5z} - \dfrac{z}{1-2z}}\ \boxed{\lvert z\rvert<0.5}$
$a_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}}\quad (n<0)$	$\xleftrightarrow{-n,\ -N}$	$a_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}}\quad (n \geq 1)$
$x_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}}\quad (n \geq 1)$	$\xleftrightarrow{-n,\ -N}$	$x_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}}\quad (n<0)$

$$(z\ ,\ R^{-1}) \Leftrightarrow (-a_n,\ N^c)$$

① — Ⓐ

$$\boxed{-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}}\ \boxed{|z|<0.5}$$

① — Ⓑ $\quad \Big\| \ z \qquad \updownarrow R^{-1}$

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}}\ \boxed{|z|>2}$$

② — Ⓐ

$$\boxed{-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}}\ \boxed{|z|>2}$$

② — Ⓑ $\quad \Big\| \ z \qquad \updownarrow R^{-1}$

$$\boxed{+\frac{z}{1-0.5z}-\frac{z}{1-2z}}\ \boxed{|z|<0.5}$$

① — Ⓐ

$$a_n = \boxed{-2^{n+1}+\left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$\updownarrow -1 \qquad \updownarrow N^c$

① — Ⓑ

$$a_n = \boxed{+2^{n+1}-\left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

② — Ⓐ

$$a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1}+2^{n-1}} \quad (n < 1)$$

$\updownarrow -1 \qquad \updownarrow N^c$

② — Ⓑ

$$a_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1}-2^{n-1}} \quad (n \geq 1)$$

① — Ⓐ

$$x_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1}+2^{n-1}} \quad (n < 1)$$

$\updownarrow -1 \qquad \updownarrow N^c$

① — Ⓑ

$$x_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1}-2^{n-1}} \quad (n \geq 1)$$

② — Ⓐ

$$x_n = \boxed{-2^{n+1}+\left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$\updownarrow -1 \qquad \updownarrow N^c$

② — Ⓑ

$$x_n = \boxed{+2^{n+1}-\left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

$$(a_n, N) \Leftrightarrow (x_{-n}, -N)$$

① − Ⓐ

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \quad \boxed{|z|<0.5}$$

$$a_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$-n$ $\quad$ $-N$

$$x_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n < 1)$$

② − Ⓐ

$$\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \quad \boxed{|z|>2}$$

$$a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n < 1)$$

$-n$ $\quad$ $-N$

$$x_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

① − Ⓑ

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \quad \boxed{|z|>2}$$

$$a_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

$-n$ $\quad$ $-N$

$$x_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

② − Ⓑ

$$\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \quad \boxed{|z|<0.5}$$

$$a_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

$-n$ $\quad$ $-N$

$$x_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

$$(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c)$$

$$(z^{-1}, R^{-1}) \rightarrow (z, R^{-1}) \qquad (a_{-n}, -N) \rightarrow (-a_n, N^c)$$

① – Ⓐ

$$-\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \quad |z|<0.5$$

$$a_n = \boxed{-2^{n+1} + (\tfrac{1}{2})^{n+1}} \quad (n \geq 0)$$

② – Ⓐ

$$-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad |z|>2$$

$$a_n = \boxed{-(\tfrac{1}{2})^{n-1} + 2^{n-1}} \quad (n<1)$$

① – Ⓑ

$$+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad |z|>2$$

$$a_n = \boxed{+2^{n+1} - (\tfrac{1}{2})^{n+1}} \quad (n<0)$$

② – Ⓑ

$$+\frac{z}{1-0.5z} - \frac{z}{1-2z} \quad |z|<0.5$$

$$a_n = \boxed{+(\tfrac{1}{2})^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

① – Ⓐ

$$-\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \quad |z|<0.5$$

$$x_n = \boxed{-(\tfrac{1}{2})^{n-1} + 2^{n-1}} \quad (n<1)$$

② – Ⓐ

$$-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad |z|>2$$

$$x_n = \boxed{-2^{n+1} + (\tfrac{1}{2})^{n+1}} \quad (n \geq 0)$$

① – Ⓑ

$$+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad |z|>2$$

$$x_n = \boxed{+(\tfrac{1}{2})^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

② – Ⓑ

$$+\frac{z}{1-0.5z} - \frac{z}{1-2z} \quad |z|<0.5$$

$$x_n = \boxed{+2^{n+1} - (\tfrac{1}{2})^{n+1}} \quad (n<0)$$

$$(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c)$$

$$(z^{-1}, R^{-1}) \rightarrow (z, R^{-1}) \qquad (a_{-n}, -N) \rightarrow (-a_{n}, N^c)$$

① − Ⓐ

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \; \boxed{|z|<0.5}$$

$$a_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

② − Ⓑ

$$\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \; \boxed{|z|<0.5}$$

$$a_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

① − Ⓑ

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \; \boxed{|z|>2}$$

$$a_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

② − Ⓐ

$$\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \; \boxed{|z|>2}$$

$$a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n < 1)$$

① − Ⓐ

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \; \boxed{|z|<0.5}$$

$$x_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n < 1)$$

② − Ⓑ

$$\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \; \boxed{|z|<0.5}$$

$$x_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

① − Ⓑ

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \; \boxed{|z|>2}$$

$$x_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

② − Ⓐ

$$\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \; \boxed{|z|>2}$$

$$x_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

ROC's of interests

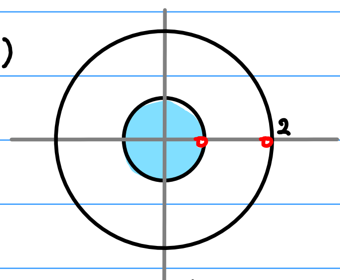
R1(z) R2(z)   R1(z^{-1}) R2(z^{-1})

$$① \ \frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} \ \xleftrightarrow{z^{-1}} \ ② \ \frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)}$$

$$\boxed{+\frac{1}{z-0.5}-\frac{1}{z-2}} \ \xleftrightarrow{z^{-1}} \ \boxed{-\frac{2z}{(z-2)}+\frac{0.5z}{(z-0.5)}}$$

$$p_1 = 0.5 \qquad p_1^{-1} = 2$$
$$p_2 = 2 \ \xleftrightarrow{z^{-1}} \ p_2^{-1} = 0.5$$

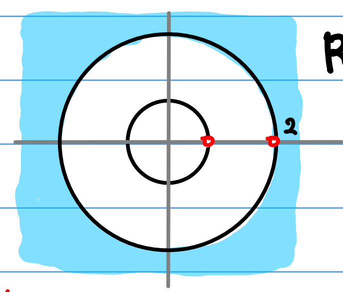
R1 (z)



$|z| < 0.5$

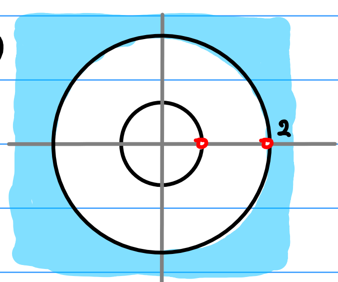
$\xleftrightarrow{z^{-1}}$

R1 (z^{-1})



$|z^{-1}| < 0.5 \qquad |z| > 2$

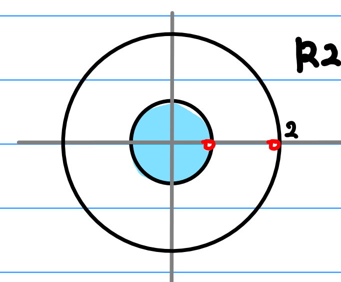
R2 (z)



$|z| > 2$

$\xleftrightarrow{z^{-1}}$

R2 (z^{-1})



$|z^{-1}| > 2 \qquad |z| < 0.5$

$$R1(z^{-1}) = R2(z)$$
$$R2(z^{-1}) = R1(z)$$

2 Equations, each with 2 representations

$$① \ \frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \xleftrightarrow{z^{-1}} ② \ \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$$

$$\left(\frac{1}{z-0.5}-\frac{1}{z-2}\right) \qquad \left(\frac{0.5z}{(z-0.5)}-\frac{2z}{(z-2)}\right)$$

$\boxed{z}$ $-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}$ $\qquad$ $\boxed{z^{-1}}$ $\frac{0.5}{1-0.5z^{-1}}-\frac{2}{1-2z^{-1}}$

$\boxed{|z|<0.5}$ $\ |2z|<1 \ \ |0.5z|<1$ $\qquad$ $\boxed{|z|>2}$ $\ |0.5z^{-1}|<1 \ \ |2z^{-1}|<1$

$\boxed{z^{-1}}$ $\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$ $\qquad$ $\boxed{z}$ $-\frac{z}{1-2z}+\frac{z}{1-0.5z}$

$\boxed{|z|>2}$ $\ |0.5z^{-1}|<1 \ \ |2z^{-1}|<1$ $\qquad$ $\boxed{|z|<0.5}$ $\ |2z|<1 \ \ |0.5z|<1$

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z} \qquad \frac{0.5}{1-0.5z^{-1}}-\frac{2}{1-2z^{-1}}$$

$$\cdot\frac{1}{2z}\Big\downarrow\Big\uparrow\cdot 2z \quad \cdot\frac{2}{z}\Big\downarrow\Big\uparrow\cdot\frac{z}{2} \qquad \cdot 2z\Big\downarrow\Big\uparrow\cdot\frac{1}{2z} \quad \cdot\frac{z}{2}\Big\downarrow\Big\uparrow\cdot\frac{2}{z}$$

$$\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \qquad -\frac{z}{1-2z}+\frac{z}{1-0.5z}$$

Causal $f(z)$ & $X(z)$

$$① \ \frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \quad \xleftrightarrow{z^{-1}} \quad ② \ \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$$

$$\left(\frac{1}{z-0.5}-\frac{1}{z-2}\right) \qquad \left(\frac{0.5z}{(z-0.5)}-\frac{2z}{(z-2)}\right)$$

$\boxed{z}$ $-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}$

$\boxed{|z|<0.5}$ $f(z)$ causal $(n \geq 0)$

$\boxed{z}$ $-\frac{z}{1-2z}+\frac{z}{1-0.5z}$

$\boxed{|z|<0.5}$ $f(z)$ causal $(n \geq 1)$

$\boxed{z^{-1}}$ $\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$

$\boxed{|z|>2}$

$X(z)$ causal $(n \geq 1)$

$\boxed{z^{-1}}$ $\frac{0.5}{1-0.5z^{-1}}-\frac{2}{1-2z^{-1}}$

$\boxed{|z|>2}$

$X(z)$ causal $(n \geq 0)$

Anti-causal f(z) & X(z)

III

$$① \ \frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} \quad \xleftrightarrow{z^{-1}} \quad ② \ \frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)}$$

$$\left(\frac{1}{z-0.5}-\frac{1}{z-2}\right)$$

$\boxed{z}$ $\quad -\dfrac{2}{1-2z}+\dfrac{0.5}{1-0.5z}$

$\boxed{|z|<0.5}$ $\quad f(z)$ causal $(n \geq 0)$

$X(z)$ anticausal $(n<1)$

$\boxed{z^{-1}}$ $\quad \dfrac{z^{-1}}{1-0.5z^{-1}}-\dfrac{z^{-1}}{1-2z^{-1}}$

$\boxed{|z|>2}$ $\quad f(z)$ anticausal $(n<0)$

$X(z)$ causal $(n \geq 1)$

$$\left(\frac{0.5z}{(z-0.5)}-\frac{2z}{(z-2)}\right)$$

$\boxed{z^{-1}}$ $\quad \dfrac{0.5}{1-0.5z^{-1}}-\dfrac{2}{1-2z^{-1}}$

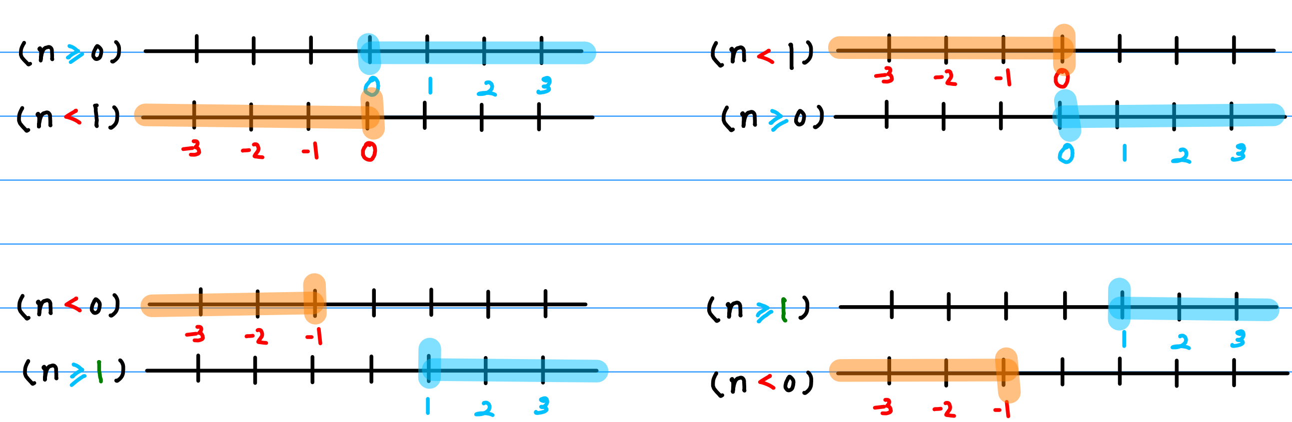
$\boxed{|z|>2}$ $\quad f(z)$ anticausal $(n<1)$

$X(z)$ causal $(n \geq 0)$

$\boxed{z}$ $\quad -\dfrac{z}{1-2z}+\dfrac{z}{1-0.5z}$

$\boxed{|z|<0.5}$ $\quad f(z)$ causal $(n \geq 1)$

$X(z)$ anticausal $(n<0)$



IV

Causal sequence a_n & x_n

$$① \ \frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \xleftrightarrow{z^{-1}} ② \ \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$$

$|z|<0.5$

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}$$

$$f(z) = -\left[2+2^2z^1+2^3z^2+\cdots\right] \quad -2^{n+1}$$
$$+\left[\left(\tfrac{1}{2}\right)+\left(\tfrac{1}{2}\right)^2z^1+\left(\tfrac{1}{2}\right)^3z^2+\cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n+1}$$

$|z|>2$

$$\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$$

$$X(z) = +\left[\left(\tfrac{1}{2}\right)^0z^{-1}+\left(\tfrac{1}{2}\right)^1z^{-2}+\left(\tfrac{1}{2}\right)^2z^{-3}+\cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n-1}$$
$$-\left[2^0z^{-1}+2^1z^{-2}+2^2z^{-3}+\cdots\right] \quad -2^{n-1}$$

$*\ n=$ 1 2 3

$|z|<0.5$

$$-\frac{z}{1-2z}+\frac{z}{1-0.5z}$$

$$f(z) = -\left[2^0z^1+2^1z^2+2^2z^3+\cdots\right] \quad -2^{n-1}$$
$$+\left[\left(\tfrac{1}{2}\right)^0z^1+\left(\tfrac{1}{2}\right)^1z^2+\left(\tfrac{1}{2}\right)^2z^3+\cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n-1}$$

$|z|>2$

$$\frac{0.5}{1-0.5z^{-1}}-\frac{2}{1-2z^{-1}}$$

$$X(z) = +\left[\left(\tfrac{1}{2}\right)^1z^0+\left(\tfrac{1}{2}\right)^2z^{-1}+\left(\tfrac{1}{2}\right)^3z^{-2}+\cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n+1}$$
$$-\left[2^1z^0+2^2z^{-1}+2^3z^{-2}+\cdots\right] \quad -2^{n+1}$$

$*\ n=$ 0 1 2

Anti-causal sequence a_n & x_n

$$① \ \frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \quad \overset{z^{-1}}{\longleftrightarrow} \quad ② \ \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$$

$2=\left(\frac{1}{2}\right)^{-1}$

$\left(\frac{1}{2}\right)=2^{-1}$

$|z|<0.5$

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}$$

$$f(z) = -\left[2+2^2z^1+2^3z^2+\cdots\right] \quad -2^{n+1}$$
$$+\left[\left(\tfrac{1}{2}\right)+\left(\tfrac{1}{2}\right)^2z^1+\left(\tfrac{1}{2}\right)^3z^2+\cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n+1}$$

$$X(z) = -\left[\left(\tfrac{1}{2}\right)^{-1}+\left(\tfrac{1}{2}\right)^{-2}z^1+\left(\tfrac{1}{2}\right)^{-3}z^2+\cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n-1}$$
$$+\left[2^{-1}+2^{-2}z^1+2^{-3}z^2+\cdots\right] \quad +2^{n-1}$$

$n=\quad 0 \quad -1 \quad -2$

$|z|<0.5$

$$-\frac{z}{1-2z}+\frac{z}{1-0.5z}$$

$$f(z) = -\left[2^0z^1+2^1z^2+2^2z^3+\cdots\right] \quad -2^{n-1}$$
$$+\left[\left(\tfrac{1}{2}\right)^0z^1+\left(\tfrac{1}{2}\right)^1z^2+\left(\tfrac{1}{2}\right)^2z^3+\cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n-1}$$

$$X(z) = -\left[\left(\tfrac{1}{2}\right)^0z^1+\left(\tfrac{1}{2}\right)^{-1}z^2+\left(\tfrac{1}{2}\right)^{-2}z^3+\cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n+1}$$
$$+\left[2^0z^1+2^{-1}z^2+2^{-2}z^3+\cdots\right] \quad +2^{n+1}$$

$n=\quad -1 \quad -2 \quad -3$

$2=\left(\frac{1}{2}\right)^{-1}$

$\left(\frac{1}{2}\right)=2^{-1}$

$|z|>2$

$$\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$$

$$f(z) = +\left[2^0z^{-1}+2^{-1}z^{-2}+2^{-2}z^{-3}+\cdots\right] \quad +2^{n+1}$$
$$-\left[\left(\tfrac{1}{2}\right)^0z^{-1}+\left(\tfrac{1}{2}\right)^{-1}z^{-2}+\left(\tfrac{1}{2}\right)^{-2}z^{-3}+\cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n+1}$$

$$X(z) = +\left[\left(\tfrac{1}{2}\right)^0z^{-1}+\left(\tfrac{1}{2}\right)^1z^{-2}+\left(\tfrac{1}{2}\right)^2z^{-3}+\cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n-1}$$
$$-\left[2^0z^{-1}+2^1z^{-2}+2^2z^{-3}+\cdots\right] \quad -2^{n-1}$$

$n=\quad 1 \quad 2 \quad 3$

$|z|>2$

$$\frac{0.5}{1-0.5z^{-1}}-\frac{2}{1-2z^{-1}}$$

$$f(z) = +\left[2^{-1}z^0+2^{-2}z^{-1}+2^{-3}z^{-2}+\cdots\right] \quad +2^{n-1}$$
$$-\left[\left(\tfrac{1}{2}\right)^{-1}z^0+\left(\tfrac{1}{2}\right)^{-2}z^{-1}+\left(\tfrac{1}{2}\right)^{-3}z^{-2}+\cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n-1}$$

$$X(z) = +\left[\left(\tfrac{1}{2}\right)^1z^0+\left(\tfrac{1}{2}\right)^2z^{-1}+\left(\tfrac{1}{2}\right)^3z^{-2}+\cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n+1}$$
$$-\left[2^1z^0+2^2z^{-1}+2^3z^{-2}+\cdots\right] \quad -2^{n+1}$$

$n=\quad 0 \quad 1 \quad 2$

Sequence a_n & x_n

VI

$$① \ \frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \quad \xleftrightarrow{z^{-1}} \quad ② \ \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$$

$|z|<0.5$

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}$$

$$f(z) = -\left[2+2^2 z+2^3 z^2+\cdots\right] + \left[\left(\tfrac{1}{2}\right)+\left(\tfrac{1}{2}\right)^2 z+\left(\tfrac{1}{2}\right)^3 z^2+\cdots\right]$$

$$\boxed{a_n = -2^{n+1}+\left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)}$$

$$X(z) = -\left[\left(\tfrac{1}{2}\right)^{-1}+\left(\tfrac{1}{2}\right)^{-2} z^1+\left(\tfrac{1}{2}\right)^{-3} z^2+\cdots\right] + \left[2^{-1}+2^{-2} z^1+2^{-3} z^2+\cdots\right]$$

$$\boxed{a_n = -\left(\tfrac{1}{2}\right)^{n-1}+2^{n-1} \quad (n < 1)}$$

$|z|>2$

$$\frac{0.5}{1-0.5z^{-1}}-\frac{2}{1-2z^{-1}}$$

$$f(z) = +\left[2^{-1}z^0+2^{-2}z^{-1}+2^{-3}z^{-2}+\cdots\right] - \left[\left(\tfrac{1}{2}\right)^{-1}z^0+\left(\tfrac{1}{2}\right)^{-2}z^{-1}+\left(\tfrac{1}{2}\right)^{-3}z^{-2}+\cdots\right]$$

$$\boxed{a_n = +2^{n-1}-\left(\tfrac{1}{2}\right)^{n-1} \quad (n < 1)}$$

$$X(z) = +\left[\left(\tfrac{1}{2}\right)^1 z^0+\left(\tfrac{1}{2}\right)^2 z^{-1}+\left(\tfrac{1}{2}\right)^3 z^{-2}+\cdots\right] - \left[2^1 z^0+2^2 z^{-1}+2^3 z^{-2}+\cdots\right]$$

$$\boxed{a_n = +\left(\tfrac{1}{2}\right)^{n+1}-2^{n+1} \quad (n \geq 0)}$$

$|z|>2$

$$\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$$

$$f(z) = +\left[2^0 z^{-1}+2^{-1}z^{-2}+2^{-2}z^{-3}+\cdots\right] - \left[\left(\tfrac{1}{2}\right)^0 z^{-1}+\left(\tfrac{1}{2}\right)^{-1}z^{-2}+\left(\tfrac{1}{2}\right)^{-2}z^{-3}+\cdots\right]$$

$$\boxed{a_n = +2^{n+1}-\left(\tfrac{1}{2}\right)^{n+1} \quad (n < 0)}$$

$$X(z) = +\left[\left(\tfrac{1}{2}\right)^0 z^{-1}+\left(\tfrac{1}{2}\right)^1 z^{-2}+\left(\tfrac{1}{2}\right)^2 z^{-3}+\cdots\right] - \left[2^0 z^{-1}+2^1 z^{-2}+2^2 z^{-3}+\cdots\right]$$

$$\boxed{a_n = +\left(\tfrac{1}{2}\right)^{n-1}-2^{n-1} \quad (n \geq 1)}$$

$|z|<0.5$

$$-\frac{z}{1-2z}+\frac{z}{1-0.5z}$$

$$f(z) = -\left[2^0 z^1+2^1 z^2+2^2 z^3+\cdots\right] + \left[\left(\tfrac{1}{2}\right)^0 z^1+\left(\tfrac{1}{2}\right)^1 z^2+\left(\tfrac{1}{2}\right)^2 z^3+\cdots\right]$$

$$\boxed{a_n = -2^{n-1}+\left(\tfrac{1}{2}\right)^{n-1} \quad (n \geq 1)}$$

$$X(z) = -\left[\left(\tfrac{1}{2}\right)^0 z^1+\left(\tfrac{1}{2}\right)^{-1}z^2+\left(\tfrac{1}{2}\right)^{-2}z^3+\cdots\right] + \left[2^0 z^1+2^{-1}z^2+2^{-2}z^3+\cdots\right]$$

$$\boxed{a_n = -\left(\tfrac{1}{2}\right)^{n+1}+2^{n+1} \quad (n < 0)}$$

$$(z^{-1},\ R^{-1}) \Leftrightarrow (a_{-n},\ -N)$$

$$(z,\ R^{-1}) \Leftrightarrow (-a_n,\ N^c)$$

$$(z^{-1},\ R) \Leftrightarrow (-a_{-n},\ (-N)^c)$$

$$(a_n,\ N) \Leftrightarrow (x_{-n},\ -N)$$

$(z^{-1}, R^{-1}) \rightarrow (z, R^{-1})$ $\qquad$ $(a_{-n}, -N) \rightarrow (-a_n, N^c)$

$ROC(z) \quad f(z) \Longleftrightarrow a_n \quad RNG(n)$

$|z| < p \qquad n \geq 0$

Ⅰ $ROC(z^{-1}) \quad f(z) \Longleftrightarrow -a_n \quad \overline{RNG}(n)$

$|z| > \frac{1}{p} \qquad n < 0$

$ROC(z) \quad f(z) \Longleftrightarrow a_n \quad RNG(n)$

$|z| < p \qquad n \geq 0$

Ⅱ $ROC(z^{-1}) \quad f(z^{-1}) \Longleftrightarrow a_{-n} \quad RNG(-n)$

$|z| > \frac{1}{p} \qquad n < 1$

$ROC(z) \quad f(z) \Longleftrightarrow a_n \quad RNG(n)$

$|z| < p \qquad n \geq 0$

Ⅲ $ROC(z) \quad f(z^{-1}) \Longleftrightarrow -a_{-n} \quad \ll RNG(n) \gg$ Ⅰ + Ⅱ

$|z| < p \qquad n \geq 1$

$ROC(z) \quad f(z) \Longleftrightarrow a_n \quad RNG(n)$

$|z| < p \qquad n \geq 0$

Ⅳ $ROC(z) \quad X(z) \Longleftrightarrow a_{-n} \quad RNG(-n)$

$|z| < p \qquad n < 1$

$$\text{III} = \text{I} + \text{II}$$

III

ROC(z)	$f(z^{-1})$	$\Longleftrightarrow$	$-a_{-n}$	« RNG(n) »
$\lvert z\rvert < p$				$n \geq 1$

I + II

ROC(z)	$f(z)$	$\Longleftrightarrow$	a_n	RNG(n)
$\lvert z\rvert < p$				$n \geq 0$

I

ROC(z^{-1})	$f(z)$	$\Longleftrightarrow$	$-a_n$	$\overline{\text{RNG}}$(n)
$\lvert z\rvert > \frac{1}{p}$				$n < 0$

$n \leq -1$

II

ROC(z)	$f(z^{-1})$	$\Longleftrightarrow$	$-a_{-n}$	$\overline{\text{RNG}}$(−n)
$\lvert z\rvert < p$				$n \geq 1$

Compare Ⅰ with Ⅳ

	$ROC(z)$	$f(z) \Longleftrightarrow$	a_n	$RNG(n)$
	$\|z\| < p$			$n \geq 0$
Ⅰ	$ROC(z^{-1})$	$f(z) \Longleftrightarrow$	$-a_n$	$\overline{RNG}(n)$
	$\|z\| > \frac{1}{p}$			$n < 0$
			-1	complement
Ⅳ	$ROC(z)$	$X(z) \Longleftrightarrow$	a_{-n}	$RNG(-n)$
	$\|z\| < p$			$n < 1$
			$-n$	$-n$
				symmetrical

$f(z)$ $\quad |z| < 0.5$ $\quad |z| > 2$

causal $\quad$ anticausal

① − Ⓐ $\frac{3}{2} \frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2} \right)$

$|z| < 0.5 \quad f(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \quad \boxed{-2^{n+1} + \left(\frac{1}{2}\right)^{n+1}} \quad (n \geq 0)$

$\frac{a}{1-az} \quad \frac{z^{-1}}{a^{-1}z^{-1}-1}$

$-\left(2 + 2^2 z + 2^3 z^2 + \cdots\right) + \left(\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 z + \left(\frac{1}{2}\right)^3 z^2 + \cdots\right)$

$n=0 \quad n=1 \quad n=2 \qquad n=0 \quad n=1 \quad n=2$

$|z| > 2 \quad f(z) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad \boxed{+2^{n+1} - \left(\frac{1}{2}\right)^{n+1}} \quad (n < 0)$

$\left(z^{-1} + \left(\frac{1}{2}\right)^1 z^{-2} + \left(\frac{1}{2}\right)^2 z^{-3} + \cdots\right) - \left(z^{-1} + 2z^{-2} + 2^2 z^{-3} + \cdots\right)$

$\left(2^0 z^{-1} + 2^{-1} z^{-2} + 2^{-2} z^{-3} + \cdots\right) - \left(\left(\frac{1}{2}\right)^0 z^{-1} + \left(\frac{1}{2}\right)^{-1} z^{-2} + \left(\frac{1}{2}\right)^{-2} z^{-3} + \cdots\right)$

$n=-1 \quad n=-2 \quad n=-3 \qquad n=-1 \quad n=-2 \quad n=-3$

② − Ⓐ $\frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)} \right)$

$|z| < 0.5 \quad f(z) = -\frac{z}{1-2z} + \frac{z}{1-0.5z} \quad \boxed{-2^{n-1} + \left(\frac{1}{2}\right)^{n-1}} \quad (n \geq 1)$

$-\left(z + 2z^2 + 2^2 z^3 + \cdots\right) + \left(z + \left(\frac{1}{2}\right) z^2 + \left(\frac{1}{2}\right)^2 z^3 + \cdots\right)$

$n=1 \quad n=2 \quad n=3 \qquad n=1 \quad n=2 \quad n=3$

$|z| > 2 \quad f(z) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}} \quad \boxed{+2^{n-1} - \left(\frac{1}{2}\right)^{n-1}} \quad (n < 1)$

$\left(\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 z^{-1} + \left(\frac{1}{2}\right)^3 z^{-2} + \cdots\right) + \left(2 + 2^2 z^{-1} + 2^3 z^{-2} + \cdots\right)$

$\left(2^{-1} + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots\right) + \left(\left(\frac{1}{2}\right)^{-1} + \left(\frac{1}{2}\right)^{-2} z^{-1} + \left(\frac{1}{2}\right)^{-3} z^{-2} + \cdots\right)$

$n=0 \quad n=-1 \quad n=-2 \qquad n=0 \quad n=-1 \quad n=-2$

$X(z)$ $\quad |z| < 0.5 \quad |z| > 2$

anticausal causal

① − Ⓑ $\frac{3}{2}\;\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$

$|z| < 0.5 \qquad X(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \qquad \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \qquad (n < 1)$

$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right) z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$

$-\left(\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$

n=0 n=-1 n=-2 n=0 n=-1 n=-2

$|z| > 2 \qquad X(z) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \qquad (n \geq 1)$

$\left(\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^1 z^{-2} + \left(\tfrac{1}{2}\right)^2 z^{-3} + \cdots\right) - \left(2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right)$

n=1 n=2 n=3 n=1 n=2 n=3

② − Ⓑ $\frac{3}{2}\;\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$

$|z| < 0.5 \qquad X(z) = -\frac{z}{1-2z} + \frac{z}{1-0.5z} \qquad \boxed{-\left(\tfrac{1}{2}\right)^{n+1} + 2^{n+1}} \qquad (n < 0)$

$-\left(z + 2z^2 + 2^2 z^3 + \cdots\right) + \left(z + \left(\tfrac{1}{2}\right) z^2 + \left(\tfrac{1}{2}\right)^2 z^3 + \cdots\right)$

$-\left(\left(\tfrac{1}{2}\right)^0 z + \left(\tfrac{1}{2}\right)^{-1} z^2 + \left(\tfrac{1}{2}\right)^{-2} z^3 + \cdots\right) + \left(2^0 z + 2^{-1} z^2 + 2^{-2} z^3 + \cdots\right)$

n=-1 n=-2 n=-3 n=-1 n=-2 n=-3

$|z| > 2 \qquad X(z) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}} \qquad \boxed{+\left(\tfrac{1}{2}\right)^{n+1} - 2^{n+1}} \qquad (n \geq 0)$

$\left(\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right) + \left(2 + 2^2 z^{-1} + 2^3 z^{-2} + \cdots\right)$

n=0 n=1 n=2 n=0 n=1 n=2

I

ROC(z^{-1})	$f(z)$	$\Longleftrightarrow$	$-a_n$	RNG(n)
$\lvert z\rvert > \frac{1}{p}$				$n<0$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$ $\quad X(z) \quad a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n<1)$

$|z| > 2$ $\quad X(z) \quad b_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n\geq 1)$

$\{|z|<0.5\} \cap \{|z|>2\} = \phi \Longrightarrow a_n + b_n = 0$

$a_n = -b_n$

ROC

$|z| < a$ $\quad X(z) = \dfrac{a}{1-az} = \displaystyle\sum_{n=0}^{\infty} a^{n+1} z^n \qquad a^{n+1} \qquad n\geq 0 \quad n\geq 1 \quad n<0 \quad n<1$

$-1\cdot$

ROC′

$|z| > a^{-1}$ $\quad X(z) = -\dfrac{z^{-1}}{1-a^{-1}z^{-1}} = -\displaystyle\sum_{n=0}^{\infty} a^{-n} z^{-n-1} \qquad -a^{n+1} \qquad n<0 \quad n<1 \quad n\geq 0 \quad n\geq 1$

$$= -\sum_{k=-1}^{-\infty} a^{k+1} z^k$$

$$\boxed{\frac{a}{1-az}} = \sum_{n=0}^{\infty} a^{n+1} z^n \qquad\qquad \boxed{\frac{z}{1-az}} = \sum_{n=1}^{\infty} a^{n-1} z^n$$

$$\boxed{-\frac{z^{-1}}{1-a^{-1}z^{-1}}} = -\sum_{n=0}^{\infty} a^{-n} z^{-n-1} \qquad\qquad \boxed{-\frac{a^{-1}}{1-a^{-1}z^{-1}}} = -\sum_{n=0}^{\infty} a^{-n-1} z^{-n}$$

$$= -\sum_{k=-1}^{-\infty} a^{k+1} z^k \qquad\qquad = -\sum_{k=0}^{-\infty} a^{k-1} z^k$$

$$\frac{a}{1-az} \Rightarrow \sum_{n=0}^{\infty} a^{n+1} z^n \qquad\qquad \frac{z}{1-az} \Rightarrow \sum_{n=1}^{\infty} a^{n-1} z^n$$

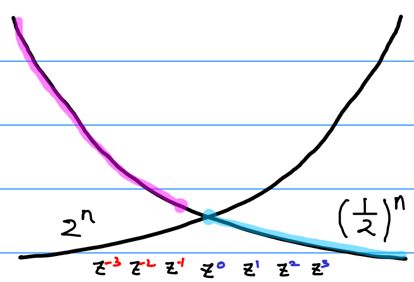
$$\| \qquad\qquad \| \qquad\qquad\qquad \| \qquad\qquad \|$$

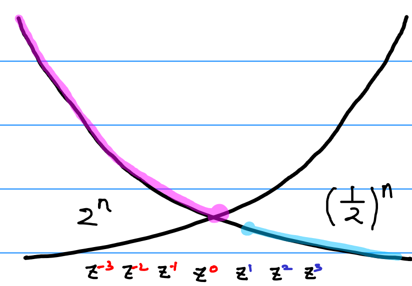
$$-\frac{z^{-1}}{1-a^{-1}z^{-1}} \Rightarrow -\sum_{n=0}^{\infty} a^{-n} z^{-n-1} \qquad\qquad -\frac{a^{-1}}{1-a^{-1}z^{-1}} \Rightarrow -\sum_{n=0}^{\infty} a^{-n-1} z^{-n}$$

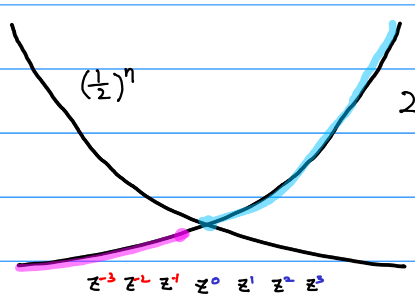
$$= -\sum_{k=-1}^{-\infty} a^{k+1} z^k \qquad\qquad = -\sum_{k=0}^{-\infty} a^{k-1} z^k$$

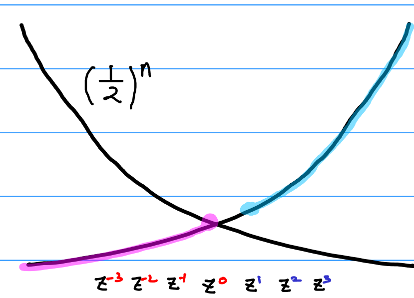
$$a + a^2 z^1 + a^3 z^2 + a^4 z^3 + \cdots \qquad\qquad z + a z^2 + a^2 z^3 + a^3 z^4 + \cdots$$

$$z^{-1} + a^{-1} z^{-2} + a^{-2} z^{-3} + a^{-3} z^{-4} + \cdots \qquad\qquad a^{-1} + a^{-2} z^{-1} + a^{-3} z^{-2} + a^{-4} z^{-3} + \cdots$$









IV	$ROC(z)$	$X(z)$	$\longleftrightarrow$	a_{-n}	$RNG(-n)$
	$\|z\| < p$				$n \leq 0$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$ $\qquad f(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$ $\qquad \boxed{-2^{n+1} + \left(\frac{1}{2}\right)^{n+1}}$ $\quad (n \geq 0)$

$$-\left(\underset{n=0}{2z^0} + \underset{n=1}{2^2 z^1} + \underset{n=2}{2^3 z^2} + \cdots\right) + \left(\underset{n=0}{\left(\tfrac{1}{2}\right) z^0} + \underset{n=1}{\left(\tfrac{1}{2}\right)^2 z^1} + \underset{n=2}{\left(\tfrac{1}{2}\right)^3 z^2} + \cdots\right)$$

$\|$

$|z| < 0.5$ $\qquad X(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$ $\qquad \boxed{-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}}$ $\quad (n \leq 0)$

$$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right) z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$$-\left(\underset{n=0}{\left(\tfrac{1}{2}\right)^{-1} z^0} + \underset{n=-1}{\left(\tfrac{1}{2}\right)^{-2} z^1} + \underset{n=-2}{\left(\tfrac{1}{2}\right)^{-3} z^2} + \cdots\right) + \left(\underset{n=0}{2^{-1} z^0} + \underset{n=-1}{2^{-2} z^1} + \underset{n=-2}{2^{-3} z^2} + \cdots\right)$$

ROC	$f(z) = \sum_{n=0}^{\infty} a^{n+1} z^n$	a^{n+1}	$n \geq 0$	$n \geq 1$	$n < 0$	$n < 1$
$\|$	$\|$ $\quad \sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n$	$\downarrow -n$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
ROC	$X(z) = \sum_{k=0}^{-\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k}$	$a^{-n+1} = \left(\frac{1}{a}\right)^{n-1}$	$n \leq 0$	$n \leq 1$	$n > 0$	$n > 1$

②	$ROC(z^{-1})$	$f(z^{-1})$	a_{-n}	$RNG(-n)$
	$\lvert z\rvert > \frac{1}{p}$			$n < 1$

$|z| < 1$

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z}$$

$$f(z) = -\left[1^1 + 1^2 z^1 + 1^3 z^2 + \cdots\right] + \left[\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right]$$

$$\boxed{a_n = -1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)}$$

$|z| > 1$

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$$

$$f(z) = -\left[\left(\tfrac{1}{1}\right)^{-1} z^0 + \left(\tfrac{1}{1}\right)^{-2} z^{-1} + \left(\tfrac{1}{1}\right)^{-3} z^{-2} + \cdots\right] + \left[2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots\right]$$

$$\boxed{a_n = -1^{n-1} + 2^{n-1} \quad (n < 1)}$$

ROC

$|z| < a$

$$f(z) = \frac{a}{1-az} = \sum_{n=0}^{\infty} a^{n+1} z^n \qquad a^{n+1} \qquad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$$

z^{-1} $\qquad\qquad -n$

ROC'

$|z| > a^{-1}$

$$f(z^{-1}) = \frac{a}{1-az^{-1}} = \sum_{n=0}^{\infty} a^{n+1} z^{-n} \qquad a^{-n+1} \qquad n < 1 \quad n < 0 \quad n \geq 0 \quad n \geq 0$$

$$= \sum_{k=0}^{-\infty} a^{-k+1} z^{k} \qquad = \left(\frac{1}{a}\right)^{n-1}$$

$z^{-1}X(z)$ Shifted Sequence

$$\overset{|z|>1}{} \qquad \overset{|z|>2}{}$$

$$X(z) = \frac{1}{1-z^{-1}} - \frac{1}{1-2z^{-1}} \quad (|z|>2)$$

$$a_n = 1^n + 2^n \quad (n \geq 0)$$

$$\left(1^0z^0 + 1^1z^{-1} + 1^2z^{-2} + \cdots\right) + \left(2^0z^0 + 2^1z^{-1} + 2^2z^{-2} + \cdots\right) \quad n \quad n = 0, 1, 2, \cdots$$

$\cdot z^{-1}$

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$\left(1^0z^{-1} + 1^1z^{-2} + 1^2z^{-3} + \cdots\right) + \left(2^0z^{-1} + 2^1z^{-2} + 2^2z^{-3} + \cdots\right) \quad n-1 \quad n = 1, 2, 3, \cdots$$

$$z^{-1}X(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad (|z|>2)$$

$$a_{n-1} = 1^{n-1} + 2^{n-1} \quad (n \geq 1)$$

$z f(z)$ Shifted Sequence

$$f(z) = (+1)\frac{1}{1-z} - \frac{1}{1-2z} \qquad (|z|<0.5)$$

$$a_n = 1^n - 2^n \qquad (n \geq 0)$$

$$(1^0 z^0 + 1^1 z^1 + 1^2 z^2 + \cdots) - (2^0 z^0 + 2^1 z^1 + 2^2 z^2 + \cdots) \quad n \qquad n = 0, 1, 2, \cdots$$

$\bullet z$

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \qquad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots) - (2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots) \quad n-1 \qquad n = 1, 2, 3, \cdots$$

$$z f(z) = (+1)\frac{z}{1-z} - \frac{z}{1-2z} \qquad (|z|<0.5)$$

$$a_{n-1} = 1^{n-1} - 2^{n-1} \qquad (n \geq 1)$$

$z^{-1} f(z^{-1})$ Shifted & Reflected Sequence

$$f(z) = \frac{1}{1-z} - \frac{1}{1-2z} \qquad (|z| < 0.5)$$

$$a_n = 1^n - 2^n \qquad (n \geq 0)$$

$$\left(1^0 z^0 + 1^1 z^1 + 1^2 z^2 + \cdots\right) - \left(2^0 z^0 + 2^1 z^1 + 2^2 z^2 + \cdots\right) \quad n \quad n = 0, 1, 2, \cdots$$

$\bullet z$ $\quad 1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$

$$\left(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots\right) - \left(2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right) \quad n-1 \quad n = 1, 2, 3, \cdots$$

$$z f(z) = \frac{z}{1-z} - \frac{z}{1-2z} \qquad (|z| < 0.5)$$

$$a_{n-1} = 1^{n-1} - 2^{n-1} \qquad (n \geq 1)$$

z $$\left(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots\right) + \left(2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right) \quad n \quad n = 1, 2, 3, \cdots$$

$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$

z^{-1} $$\left(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots\right) + \left(2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right) \quad -n \quad n = -1, -2, -3, \cdots$$

$$z^{-1} f(z^{-1}) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad (|z| > 2)$$

$$a_{-n-1} = 1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \qquad (n < 0)$$

$a_{-(n+1)}$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5 \quad X(z) \qquad a_n = \boxed{-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n<1)$

$|z| > 2 \quad X(z) \qquad b_n = \boxed{+\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$

$\{|z| < 0.5\} \cap \{|z| > 2\} = \phi \Rightarrow a_n + b_n = 0$

$a_n = -b_n$

$|z| < a \quad X(z) = \sum_{n=0}^{\infty} a^{n+1} z^n \qquad a^{n+1}$

$n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$

$$\left\| \quad \sum_{n=0}^{\infty}\left(\frac{1}{a}\right)^{-n-1} z^n \qquad \Big\downarrow -n$$

$|z| > a \quad X(z) = \sum_{k=0}^{-\infty}\left(\frac{1}{a}\right)^{k-1} z^{-k} \qquad a^{-n+1} = \left(\frac{1}{a}\right)^{n-1}$

$n < 0 \quad n < 1 \quad n \geq 0 \quad n \geq 1$

$a^{n+1} z^n$

$a(az)^n$

$a\left(\frac{1}{az}\right)^{-n}$

$\frac{a}{1-az} \quad \sum_{n=0}^{\infty} a^{n+1} z^n \qquad \frac{z}{1-az} \quad \sum_{n=1}^{\infty} a^{n-1} z^n$

$-\frac{z^{-1}}{1-a^{-1}z^{-1}} \quad -\sum_{n=0}^{\infty} a^{-n} z^{-n-1} \qquad -\frac{a^{-1}}{1-a^{-1}z^{-1}} \quad -\sum_{n=1}^{\infty} a^{-n-1} z^{-n}$

$\sum_{n=-1}^{-\infty} a^{n+1} z^n \qquad -\sum_{n=0}^{-\infty} a^{n-1} z^n$

$z X(z)$ Shifted & Reflected Sequence

$$X(z) = \frac{1}{1-z^{-1}} - \frac{1}{1-2z^{-1}} \quad (|z| > 2)$$

$$a_n = 1^n - 2^n \quad (n \geq 0)$$

$$\boxed{\begin{array}{lll} & (1^0 z^0 + 1^1 z^{-1} + 1^2 z^{-2} + \cdots) + (2^0 z^0 + 2^1 z^{-1} + 2^2 z^{-2} \quad) & n \quad n = 0, 1, 2, \cdots \\ \cdot z^{-1} \downarrow & 1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots & \downarrow \\ & (1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) + (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots) & n-1 \quad n = 1, 2, 3, \cdots \end{array}}$$

$$z^{-1} X(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad (|z| > 2)$$

$$a_{n-1} = 1^{n-1} - 2^{n-1} \quad (n \geq 1)$$

$$\boxed{\begin{array}{lll} z & (1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) + (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots) & n \quad n = 1, 2, 3, \cdots \\ \downarrow & 1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots & \downarrow \\ z^{-1} & (1^0 z^{1} + 1^1 z^{2} + 1^2 z^{3} + \cdots) + (2^0 z^{1} + 2^1 z^{2} + 2^2 z^{3} + \cdots) & -n \quad n = -1, -2, -3, \cdots \end{array}}$$

$$z X(z^{-1}) = \frac{z}{1-z} - \frac{z}{1-2z} \quad (|z| < 0.5)$$

$$a_{-n-1} = 1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$$

$$a_{-(n+1)}$$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$ $$f(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \qquad \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$$-\left(2z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$n=0$ $n=1$ $n=2$ $n=0$ $n=1$ $n=2$

$|z| < 0.5$ $$X(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \qquad \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n \leq 0)$$

$$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right) z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$$

$n=0$ $n=-1$ $n=-2$ $n=0$ $n=-1$ $n=-2$

ROC $\quad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^n \qquad a^{n+1} \qquad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$

$\Vert \quad \Vert \qquad \sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n \qquad \downarrow -n$

ROC $\quad X(z) = \sum_{k=0}^{-\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k} \qquad a^{-n+1} = \left(\frac{1}{a}\right)^{n-1} \qquad n < 0 \quad n < 1 \quad n \geq 0 \quad n \geq 1$

ROC $\quad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^{n} \quad a^{n+1} \quad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$

$z^{-1} \quad z^{-1} \quad -n$

$$\sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^{n}$$

ROC $\quad X(z) = \sum_{k=0}^{-\infty} (a)^{k-1} z^{-k} \quad \left(\frac{1}{a}\right)^{-n+1} \quad n < 0 \quad n < 1 \quad n \geq 0 \quad n \geq 1$

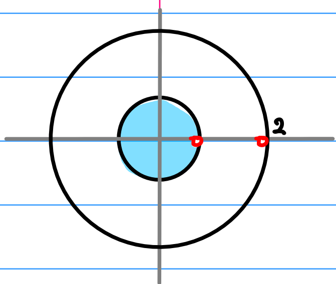
$= a^{n-1}$

Causal $\quad f(z) \quad X(z)$

$|z| < 0.5 \quad |z| > 2$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 2 \qquad |z| < 0.5$



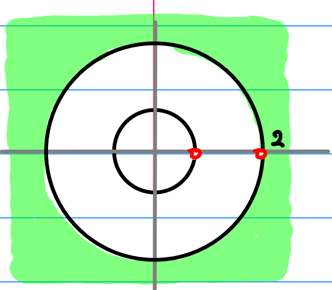
$$f(z) = (-2)\frac{0.5}{0.5-z} + (0.5)\frac{2}{2-z} \qquad \left(|z| < 0.5\right)$$

$$a_n = (-2)\;2^n + (0.5)\left(\tfrac{1}{2}\right)^n \qquad (n \geq 0)$$

$$-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}$$

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) \qquad |z| > 2$$

$|z| > 2 \qquad |z| > 0.5$



$$X(z) = 0.5\,\frac{z}{z-0.5} - 2\,\frac{z}{z-2} \qquad \left(|z| > 2\right)$$

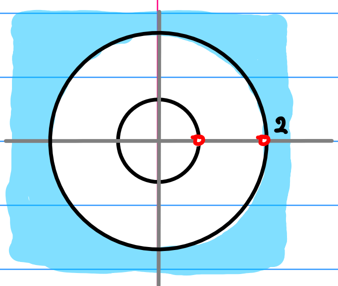
$$a_n = (0.5)\left(\tfrac{1}{2}\right)^n - 2 \cdot 2^n \qquad (n \geq 0)$$

$$\left(\tfrac{1}{2}\right)^{n+1} - 2^{n+1}$$

Anti-causal $\qquad f(z) \qquad X(z)$

$\qquad |z| > 2 \qquad |z| < 0.5$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$



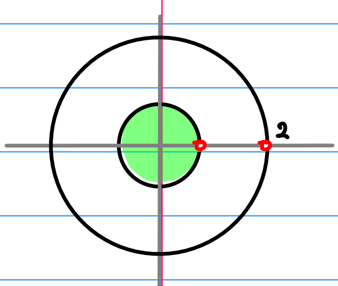
$|z|>2 \qquad |z|>0.5$

$$f(z) = (-2)\,\frac{-0.5}{0.5-z} + (0.5)\,\frac{-2}{2-z} \qquad (|z|>0.5)$$

$$a_n = (+2)\;2^n - (0.5)\left(\tfrac{1}{2}\right)^n \qquad (n<0)$$

$$+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}$$

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) \qquad |z|<2$$



$|z|<2 \qquad |z|<0.5$

$$X(z) = 0.5\,\frac{-z}{z-0.5} - 2\,\frac{-z}{z-2} \qquad (|z|<2)$$

$$a_n = -(0.5)\left(\tfrac{1}{2}\right)^n + 2 \cdot 2^n \qquad (n<0)$$

$$-\left(\tfrac{1}{2}\right)^{n+1} + 2^{n+1}$$