Chapter 3

By Matthew Raspanti

LAWS OF MOTION

> *"If I have seen farther than others,*
> *it has been by standing on the shoulders*
> *of giants."*
>
> *Isaac Newton*

What Kepler lacked to explain the motions of the planets was a knowledge of "mechanics," the new science of motion that emerged from the research of Galileo and Newton.

GALILEO

Galileo Galilei (Italian, 1564-1642) was born twenty-one years after the death of Copernicus. In 1581, he entered the University of Pisa to study medicine, but soon became interested in mathematics. Four years later, lack of money forced him to stop his studies before receiving a degree, but he continued to devote himself to mathematics and natural science. In 1589, a paper he published earned him the title of "Archimedes of his time" and a post as mathematical lecturer at the University of Pisa. Here, he started to conduct experiments on the motion of falling bodies.

At the age of 28, he became professor of mathematics at the University of Padua, where he continued his research on the principles of motion. During the 18 years he spent there, he acquired an international reputation as a scientist and inventor. Although he did not invent the telescope, in 1609 he developed the first telescope suitable for astronomical observation, thus becoming the first person ever to gaze at the skies through a telescope.

With his new instrument, Galileo made a number of important discoveries, such as:

- The surface of the Moon is not smooth and polished, as a perfect celestial body was supposed to be, but rough and uneven.
- There are spots on the surface of the Sun, another supposedly perfect celestial body.
- Four satellites revolve around Jupiter, disproving Aristotle's theory that all celestial bodies revolve naturally around the center of the Universe.

Shortly after announcing his discoveries in 1610, at 46 Galileo was appointed "first philosopher and mathematician" to the Grand Duke of Tuscany, a post that assured him a large salary and more time for research.

For many years, Galileo had believed in Copernicus' theory, and had developed many arguments to support it. Fearing ridicule, however, he had kept his views to himself. The flattering reception he received at the papal court when he demonstrated his telescope encouraged him to defend openly the Copernican theory, and his views became very popular.

Galileo's astronomical discoveries cast serious doubts on Aristotle's System of the World, but they did not actually prove that the Earth revolved around the Sun. Galileo could prove, however, that the arguments raised against the Earth's motion were not valid. His opponents questioned, for instance, how the Earth could appear to be standing still if indeed it was rushing through space. Even a bird perched on a branch, they claimed, would be left behind in space the moment it let go of the branch! Galileo's answer was that the bird was moving with the Earth while it was on the branch, and it retained that motion even after it let go. For the same reason, if a stone is dropped from the mast of a ship, it will hit the deck at the foot of the mast, regardless of whether the ship is standing still or moving, because the stone retains whatever speed it had before it was dropped.

When some Aristotelian scholars started instigating Church authorities against Galileo, the Church did not support them at first. As soon as Galileo, however, tried to maintain that the Copernican theory could be reconciled with the Bible, he met opposition from the theologians. Concerned that the issue might undermine Catholicism in its fight against Protestantism, in 1616 (73 years after Copernicus' death) the Church declared the Copernican theory "false and erroneous". Galileo was admonished not to support the theory, although it could still be discussed as a mere "mathematical supposition."

For the next seven years, Galileo kept silent on the issue. In 1623, Maffeo Barberini - who as a cardinal had been a long-time friend and protector of Galileo - became Pope Urban VIII. Galileo asked his permission to discuss the arguments for the Sun-centered Copernican theory versus the Earth-centered Ptolemaic theory. His request was granted, provided he discussed Copernicus' theory only as a speculative hypothesis, and reached the conclusion that man could not presume to know how the world is really made, since God could have brought about the same effects in unimagined ways.

Nine years later, in 1632 Galileo published his "Dialogue Concerning The Two Chief World Systems - Ptolemaic and Copernican", which quickly won international fame. Galileo's enemies were quick to point out to the

Pope that, in spite of its final conclusion (the one the Pope had prescribed), the book was a compelling defense of the Copernican theory, and seemed to ridicule the Pope.

The following year, even though many people in the Church supported Galileo and would continue to do so privately, Galileo was summoned to Rome, brought to trial and forced to recant. The sentence (which three cardinals refused to sign) would have called for imprisonment, but the Pope immediately commuted it into house arrest, which remained in effect throughout the last eight years of Galileo's life.

Even in the confinement of his small estate, Galileo continued his scientific work to the very end. In 1634, he completed "Dialogue Concerning Two New Sciences". (The new sciences were Strength of Materials and Mechanics). In the second half of the book, he summarized his experiments and thoughts on the principles of mechanics. His last telescopic discovery was made in 1637, only months before he became blind. He died in 1642 at the age of 78.

Almost three and a half centuries later, in October of 1992, Pope John Paul II made a speech vindicating Galileo, in which he said "In the 17th century, theologians failed to distinguish between belief in the Bible and interpretation of it. Galileo contended that the Scriptures cannot err but are often misunderstood. This insight made the scientist a wiser theologian than his Vatican accusers". [1]

Some Basic Definitions

Before summarizing the results of Galileo's research on motion, we need a few basic definitions and concepts.

Speed

Let us imagine ourselves driving along a straight road. If it takes us 2 hours to cover a distance of 100 miles, we say that we have traveled at the average speed of 50 miles per hour. Most likely, we have moved at times faster and at times slower than this average speed. Had we watched our odometer at half-hour intervals, we might have found that, in the first half-hour, we covered a distance of 15 miles, therefore moving at an average speed of 30 miles per hour. Again, within this half-hour interval, we may have moved sometimes faster and sometimes slower than the average speed for this interval. By considering average speeds for smaller and smaller intervals of time, we arrive at the concept of instantaneous (instant-by-instant) speed.

We say that a body (object) moves with straight uniform motion during some given interval of time, if both the speed and the direction of motion remain constant throughout this interval.

Acceleration

Let us assume now that a body moves, still along a straight line, at a speed that is not uniform. Let us assume, for instance, that the speed is 30 feet/second at some point in time, and it is 50 feet/second two seconds later. We say that the body has "accelerated" from 30 to 50 feet/second in 2 seconds, and we define the average acceleration during this time interval as the change in speed divided by the time interval, or (50-30)/2 = 10, that is, the speed has increased by an average of 10 feet/second each second. Conversely, if the speed of a body *decreases* from 50 to 30 feet/second in two seconds, we say that the body has "decelerated" at the average rate of 10 feet/second each second.

As we did for speed, we can consider smaller and smaller time intervals to arrive at the concept of instantaneous acceleration. In general, both the speed and the acceleration of a moving body may change from instant to instant.

We say that a motion along a straight line is uniformly accelerated during some interval of time, if the acceleration is constant during this interval; for instance, if its speed increases steadily at the rate of two feet per second every second.

Vectors

In general, an object will move along some curved path, rather than a straight line. Speed alone is thus not sufficient in general to describe the motion of a body, since we must also specify its direction, which may vary. For this reason, it is convenient to introduce a new quantity called "velocity", which specifies both the speed and the direction of a moving object.

Velocity is an example of a quantity that can be represented by an arrow that has a certain length, or "magnitude", and a certain direction. We call such an arrow a "vector". For a given moving object, at some instant, the direction of its velocity vector gives the direction of motion, while the length of the vector represents the speed of the object.

Another example of a quantity that can be represented by a vector is the *force* with which we might pull a boat at the end of a rope. The direction of the force vector is the direction of the taut rope; the length of the vector represents the strength with which we are pulling the boat. Another example of a force is what acts on a small piece of iron, when attracted by a nearby magnet.

Given two vectors, it is possible to replace them with a single equivalent vector, see **Figure 3.1**.

Galileo's results

Galileo proved wrong two major contentions of Aristotle's that had gone undisputed for many centuries:

- Heavy bodies by their nature fall faster than light bodies.
- The natural state of a body is to remain at rest. To put a body in motion and keep it moving requires the action of a force constantly in touch with the body.

To study motion, Galileo used small, smooth metal spheres of different weights rolling down a very smooth inclined plane. The use of small spheres and the smoothness of the plane greatly reduced the effects of air resistance and friction.

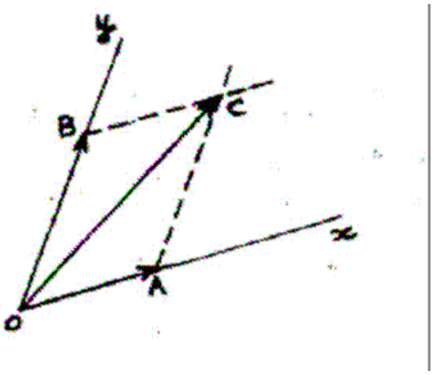

Figure 3.1 – Adding two vectors

Given two vectors OA and OB, we determine, as shown, their “sum” or “resultant,” the vector OC. (Lines OA and BC are parallel, and so are OB and AC.) A boat pulled by two forces represented by OA and OB will move as if it were pulled by a single force represented by OC. Conversely, given the vector OC and the directions x and y, we can determine the “components” of OC along the directions x and y.

To find the “difference” of two vectors, we simply add to the first the “opposite” of the other (same length but opposite direction).

A sphere rolling down an inclined plane is not affected by its full weight W (see **Figure 3.2**) but only by the component of W that is parallel to the inclined plane. The greater the slope of the plane, the greater is the component of W parallel to the plane. Thus, by changing the slope of the plane, Galileo could control how quickly, or how slowly, the spheres rolled down.

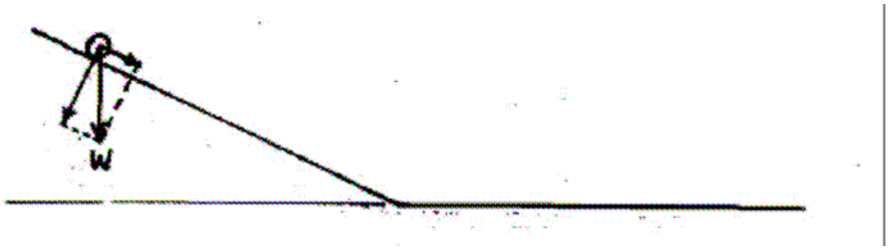

Figure 3.2 – Galileo’s inclined plane

Galileo's major findings and conclusions were:

- A sphere rolls down with constant acceleration. If, for instance, the sphere travels one inch during the 1st second, it will travel 3in. during the 2nd second, 5in. during the 3rd, 7in. during the 4th, and so on (thus, the acceleration is 2in. per second every second).
- For a given slope of the inclined plane, the acceleration does not depend on the weight of the rolling sphere.
- If we increase the slope (and thus the force acting on the sphere), the sphere will roll down with a higher constant acceleration. Galileo concluded from his experiments that, in a vacuum (i.e., in the absence of any resistance), regardless of weight, a body would fall with a constant acceleration "g" equal to about 32 feet/second every second.
- When the rolling sphere reaches the bottom of the incline, and moves on to a smooth horizontal plane, its speed remains constant, at least for a while. The sphere eventually stops because the horizontal surface is not perfectly smooth, and the resistance encountered by the sphere causes it to decelerate. If the smoothness is increased, the sphere will roll over a longer distance.

Galileo concluded that, if there were no decelerating forces, the sphere would continue to roll forever at constant speed. Contrary to what Aristotle had claimed, no force was needed to keep the sphere moving at constant speed.

Galileo studied also the motion of a cannon ball shot with some initial velocity V at an angle to the horizontal, see **Figure 3.3**. He assumed the motion to occur in a vacuum, with no forces present except for gravity. In the horizontal direction, the cannon ball travels at a constant speed, since there are no decelerating forces in this direction. In the vertical direction, instead, the cannon ball will be steadily decelerated by gravity until, having reached some height, it will come to a halt, and will then start falling with constant acceleration. Galileo could show mathematically that the trajectory of the cannon ball would be a "parabola", a "conic" curve well familiar to the Greeks.

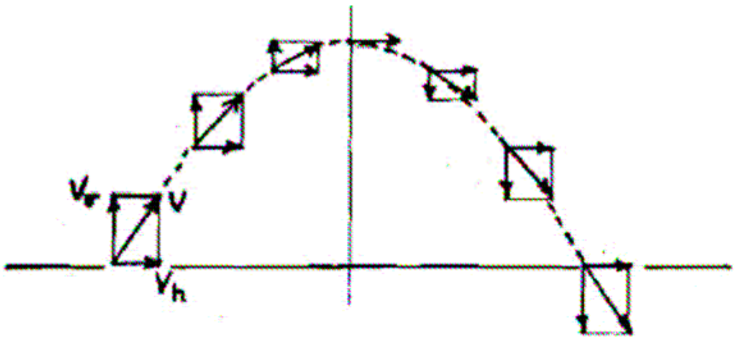

Figure 3.3 - Trajectory of a cannon ball

By concentrating on how motion would occur under idealized conditions, i.e., in the absence of any resistance, Galileo was able to understand the fundamental nature of motion better than his predecessors had been. The presence of a force that is in line with the motion of a body will cause the body to accelerate or decelerate, depending on the direction of the force relative to the motion. The absence of any forces will result in zero acceleration, that is, constant velocity. If the body is at rest, it will remain at rest; if the body is moving at some speed, it will continue to move at that speed in the same direction.

Galileo was the first to bring to the study of physical phenomena a method of investigation that combined reasoning, experimentation and mathematics. He created the modern idea of the experiment, in which special conditions are created to verify theoretical deductions.

About the role of mathematics in nature, Galileo wrote:

"Philosophy is written in this grand book, the universe, which stands ever

open to our gaze. But the book cannot be understood unless one learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles and other geometric figures without which it is humanly impossible to understand a single word of it; without these, one wanders about in a dark labyrinth."

NEWTON

The same year Galileo died, Isaac Newton (1642-1727) was born in a small village in central England. His father, a farmer, had died three months before. Two years later, his mother married a prosperous minister from a nearby village and moved there to raise three stepchildren, leaving her son in her mother's care. For nine years, until his stepfather died, young Isaac was separated from his mother. This early traumatic experience may have been the cause for the strong sense of insecurity that afflicted Newton throughout his life. Reluctant to face controversy and criticism, he would become much more interested in the private pursuit of science than in making his results known to others.

In 1661, he entered Trinity College in Cambridge. One of his professors, Isaac Barrow, encouraged him to study mathematics and optics (the study of phenomena pertaining to light). In less than a year, Newton mastered on his own the literature of mathematics, and started some original work. He kept to himself, however, the results of his private research.

In 1665, he received his bachelor's degree. That same year, when the Great Plague spread to Cambridge, the college was closed, and Newton returned to his native village. Here, during two extraordinary years, he laid the foundations of his monumental contributions to mathematics, optics and mechanics:

- He invented a new system of mathematical analysis called calculus.
- Experimenting with glass prisms, he discovered that a narrow beam of sun light, after passing through a prism, generates on a screen a series of colored bands in the familiar pattern of the rainbow: red, orange, yellow, green, blue, indigo and violet. He concluded that white light consists of rays of different colors, which are bent to different extents when passing through a prism, thus creating the rainbow spectrum on the screen.
- In mechanics, having analyzed uniform circular motion, he applied his analysis to the motion of the Moon and the planets, and derived the "inverse square law". This law states that, as a planet moves along its orbit, a force is acting on it that is inversely proportional to the square of

the planet's distance to the Sun[3], and is directed toward the Sun.

In 1667, after the university reopened, Newton returned to Trinity College. Two years later, at the age of 27, he was appointed professor of mathematics, succeeding Barrow, who recommended him as a man of "unparalleled genius".

In 1671, the Royal Society, England's prestigious academy of science, invited Newton to demonstrate a new type of telescope he had developed. Encouraged by the enthusiastic reception given to him, Newton volunteered to present a paper on his theory of colors early in 1672. The paper was well received but not without some dissent.

This led to the first of a number of bitter controversies that troubled Newton throughout his life, because he was incapable of dealing rationally with criticism. Less than a year later, he withdrew into virtual isolation, which lasted some twelve years. During these years, he immersed himself in alchemy.

By the mid-1680s, although he had already made his major discoveries in calculus, optics and mechanics, Newton had published very little. In the meantime, a number of scientists were trying to define a force originating from the Sun that could account for the planetary motions described by Kepler.

Among these scientists was Edmond Halley, the British astronomer after whom the famous comet was named. Like Newton, Halley and others had concluded that the force that keeps a planet in orbit must decrease as the square of the planet's distance from the Sun. They had not been able, however, to derive mathematically the orbit that would result from such a force.

In 1684, Halley visited Newton about this problem, and was astonished to find that Newton already had the solution. Newton's problem was that he had mislaid his calculations to prove it! Halley made him promise to send the proof, which arrived three months later.

In about 18 months between 1685 and 1686, Newton wrote in Latin his masterpiece "Philosophiae Naturalis Principia Mathematica" (Mathematical Principles of Natural Philosophy). The book, written in the style of Euclid's Elements, used intricate geometric arguments. It was published in 1687.

In the Principia, Newton was able to show how the behavior of objects falling near the surface of the Earth, the tides, and the motions of the celestial bodies could all be mathematically predicted from four laws he

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[3]. *When the distance to the Sun doubles, the force becomes 4 (2x2) times smaller; when the distance triples, the force becomes 9 (3x3) times smaller, and so on.*

postulated. Contrary to Aristotle's fundamental distinction between heavens and Earth, both followed the same mathematical laws! There were no precedents for such elegant simplicity and universality. His masterpiece immediately raised him to international fame, which was followed by many great honors.

In 1699, Newton became Master of the Mint (responsible for the coinage of money), a lucrative position that he held until his death. In 1703, he was elected president of the Royal Society. In 1705, he was knighted, becoming the first scientist ever to be so honored.

In 1704, Newton finally published his book "Optiks", which contained mostly work he had done some 30 years before on the study of light. His work on calculus, which was even older, was included merely as an appendix. In the meantime, calculus had been independently invented by the German mathematician and philosopher Leibniz, who published his work 20 years before Newton. A long and bitter dispute followed, with charges of plagiarism raised by supporters on both sides.

These accusations of dishonesty unleashed Newton's temper. He wrote papers in his defense and published them under the names of obliging supporters. As president of the Royal Society, he appointed an "impartial" committee to investigate the issue, and secretly wrote its report.

Yet, Newton was also capable of charming modesty, as suggested by the following remark, made shortly before his death:

> *"I do not know what I may appear to the world, but to myself*
> *I seem to have been only like a boy playing on the seashore,*
> *and diverting myself in now and then finding a smoother*
> *pebble or a prettier shell than ordinary, whilst the great*
> *ocean of truth lay all undiscovered before me."*

When Sir Isaac Newton died in 1727 at the age of 85, the greatest honors, including a royal funeral, were accorded to him.

The Laws of Motion

At the beginning of the Principia, in Euclidean fashion, Newton gives some definitions and then offers three laws of motion as unproven "axioms": the first two restate and generalize Galileo's findings, the third introduces a new and crucial concept.

Newton did not claim that his axioms or "laws" were self-evident. He presented them as working assumptions to be accepted only as long as they helped to explain in exact detail previously unexplained phenomena.

Before we can present these laws of motion, we need to broaden the concept of acceleration previously introduced for motion on a straight line. In the most general case, as a body moves along some curved path, its velocity vector may change from point to point in direction and/or speed.

The average acceleration from point X to point Y must now be defined as the *vector* difference of the two velocities at X and Y divided by the time it takes to go from X to Y. As we did before, by considering points X and Y that are closer and closer, we can arrive at the concept of an instantaneous acceleration vector. For our purposes, it will suffice to keep in mind that

- Like velocity, acceleration is a vector.
- Acceleration occurs whenever there is a change in speed and/or direction.

Another key concept is that of "mass". It was defined by Newton as the "quantity of matter" of an object, and was assumed to be constant. It is a purely numerical quantity, not a vector.

The mass of an object should not be confused with its weight: it is now well known, for instance, that the body of an astronaut weighs less on the Moon than on Earth, although it has the same mass in either place.

Law I (the law of inertia)

If a body is at rest or moving at constant speed in a straight line, it will remain at rest, or will continue to move at constant speed in a straight line, as long as no force is acting on it.

Passengers inside a car that is suddenly slowed down or stopped find themselves continuing in their forward motion, in accordance with the first law. They might even go through the windshield, if not restrained by a seat belt.

If the car turns suddenly and sharply, its passengers tend to continue in their original direction and slide across the seat. Thus, a body tends to maintain both its speed and its direction of motion.

Law II (the law of force)

If we wish a body of mass m to acquire an acceleration that has the value A in some direction, we must apply to it a force which has the same direction as the acceleration, and a value F equal to the mass m times the acceleration A

$$F = m \times A$$

The second law tells us that, the larger the mass of a body we want to accelerate, and the larger the acceleration we want it to attain, the larger must be the force we need to apply to the body.

For a given force, the larger the mass of the body to which the force is applied, the smaller the acceleration that will result; the smaller the mass, the larger the acceleration.

To maintain a constant speed (zero acceleration), no force is required, regardless of the speed. To maintain a steady acceleration, a constant force is required. To increase the acceleration, the force must be increased.

Law III (the law of action and reaction)

To every action, there is always an equal and opposite reaction, i.e., if P exerts a force on Q, then Q exerts an equal and opposite force on P.

What happens when we punch a brick wall provides a painful illustration of the third law! Another example is the recoil of a cannon as it propels an iron ball.

The Motion of a Body

Galileo's work showed that, if a body is already moving with some velocity, and a constant force is applied to it *in the direction of its motion*, the body will start accelerating at a constant rate in the same direction. If the force, instead, is in the *opposite* direction, the body will start decelerating at a constant rate.

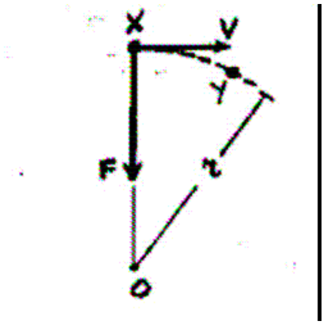

Figure 3.4 - Force and velocity vectors are perpendicular to one another

On the other hand, Newton analyzed what happens if a body is already moving with some velocity V, and a force F is applied to it *at 90 degrees* to the direction of motion as in **Figure 3.4**.

In the next instant, the speed of the body will not change, but its direction will be "bent" along a very short arc of a circle. The smaller is the radius of this circle, the larger is the bending. For a body with a given mass,

- the larger the velovity, the smaller the bending.
- the larger the force, the larger the bending.

Thus, we might say, there is a struggle going on between the velocity of a body and a perpendicular force applied to it: the velocity opposes bending, the force wants to create it.

In the most general case, the force vector applied to a body will have a component in the line of motion - which will either increase or decrease the speed - and a component perpendicular to the line of motion, which will cause a "bending" in direction.

If we know the position and velocity of a body at some particular point, and we know the "law" by which the force vector acting on the body changes in magnitude and direction from point to point, we can determine the subsequent path of the body.

The Law of Universal Gravitation

Newton made the very bold assumption that the attraction between the Sun and a planet was just a special case of a much more general situation: any two bodies in the universe with masses m1 and m2 exert on each other a force of attraction along the line connecting them. The magnitude of this force is

- directly proportional to the product of the two masses (i.e., if the product doubles, the force doubles; if the product triples, the force triples, and so on), and
- as previously mentioned, inversely proportional to the square of the distance.

Qualitatively, the law states that the larger the two masses, the larger the force of gravity between them; the larger the distance, the smaller the force.

Orbital paths of planets and comets

With his theory of gravity, Newton could account for Galileo's experimental results. He could also derive mathematically the three laws that Kepler, after years of painstaking labor, had obtained empirically from Tycho Brahe's data.

To understand in a general way the motion of a planet around the Sun along an elliptical orbit, we have to consider the interplay at any instant between the velocity of the planet and the gravitational pull of the Sun, which acts as a gigantic rubber band. In one half of the orbit, the "rubber band" *slows down* the planet while bending its path, until at some point, the planet starts coming around. In the other half of the orbit, the "rubber band" *speeds up* the planet while bending its path, until at some point the planet starts turning around, and the cycle repeats; see **Figure 3.5**.

The interplay between force and velocity explains also how orbits of different shapes and sizes are possible. If a comet, for instance, approaches the solar system with a sufficiently high velocity, its path will be deflected somewhat, but the comet will continue on its cosmic journey, never to return. (The "rubber band" never succeeds in turning the comet around.) On the other hand, if the approaching comet does not have sufficient velocity, it will be "captured" by the gravity pull of the Sun, and forced to orbit along an ellipse more or less elongated, depending on what its approach velocity was. The famous Halley's Comet, for instance, traveling at a speed of more than 80,000 miles per hour, passes by the Earth and the Sun at intervals of about 76 years.

In addition to the main interactions between the Sun and each planet, there are smaller gravitational interactions between planet and planet. Since the Sun accounts for more than 99% of the mass of the entire solar system, its effect on the planets is dominant, but not exclusive.

Perhaps the most dramatic confirmation of Newton's theory was provided by the discovery of the planet Neptune. In 1781, Sir William Hershel accidentally discovered a planet, which was later called Uranus. The orbit of the new planet, however, was not in agreement with Newton's theory, even taking into account the gravitational effects of the two large planets nearby, Jupiter and Saturn. If Newton's theory was valid, there had to be an undiscovered planet to account for the irregularities of Uranus.

The orbit of this hypothetical planet was computed by the French astronomer Le Verrier. On September 23, 1846, after only one hour of searching, the German astronomer Galle found the eighth planet, Neptune, at the precise spot in the sky that had been suggested to him by Le Verrier. (The ninth and last planet, Pluto, was accidentally discovered in 1930.)

The World as a Machine

The success of Newton's theory was so dramatic that he would dominate Western thinking almost as much as Aristotle had done for many centuries before. Eventually, Newtonian mechanics suggested to some that, if the positions and velocities of all particles of matter could be known at one particular instant, it would be possible - in principle at least - to determine the entire past and the entire future of every particle, and therefore the entire course of the universe.

In this view, the universe has the ordered structure, not of an Aristotelian organism, but of a machine, a gigantic clock that, once wound, proceeds on its own in predetermined, unalterable mechanical fashion.

This was not, however, the worldview held by Newton, who believed that God would have to intervene from time to time to maintain the stability of the solar system. A century later, the French mathematician, astronomer and physicist Pierre-Simon LaPlace proved that the solar system would remain stable on its own. Supposedly, when Napoleon asked him "What about God?", LaPlace replied "I did not need that hypothesis."

In his final remarks at the end of the Principia, Newton wrote: "it is not to be conceived that mere mechanical causes could give birth to so many regular motions. .... This most beautiful system of the sun, planets and comets could only proceed from the counsel and dominion of an intelligent and powerful Being. ..... All that diversity of natural things ... could arise from nothing but the ideas and will of a Being necessarily existing. .... Thus much concerning God; to discourse of whom from the appearances of things, does certainly belong to natural philosophy."

There was one aspect of the new theory that was disturbing even to Newton: *instantaneous action at a distance*. As he admitted in a letter, "that one body may act upon another at a distance through a vacuum without the mediation of anything else, ... is to me such an absurdity that I believe no man who has in philosophical matters a competent faculty of thinking can ever fall into it".

Newton did not claim to have an explanation for gravity. Toward the very end of the Principia, he remarked: "Hitherto I have not been able to discover the cause of those properties of gravity from phenomena, and I feign no hypothesis. .....To us it is enough that gravity does really exist, and act according to the laws which we have explained, and abundantly serves to account for all the motions of the celestial bodies, and of our sea."

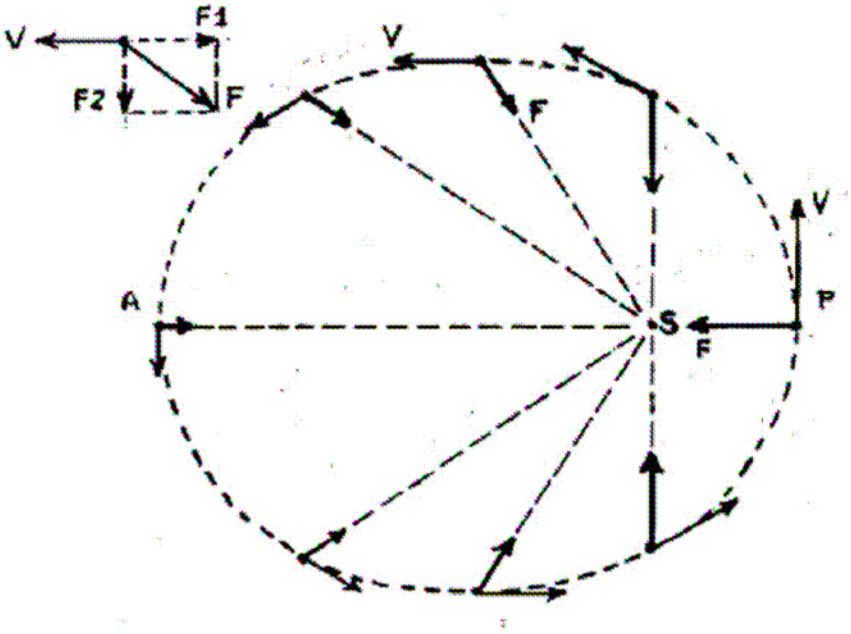

Figure 3.5 - A planet's elliptical orbit

Point S represents the Sun fixed at one focus of the ellipse. At any point on the orbit, the force vector F, which represents the pull of gravity, is always pointed toward S. Because of the inverse square law, its magnitude decreases with the planet's distance from point S. In the upper half of the orbit, from point P to point A, the angle between the force and velocity vectors is always more than 90 degrees, that is, the component of the force in the line of motion opposes the motion. As a result, the force causes the velocity to decrease in value, while at the same time forcing the path to bend.

In the lower half of the orbit, instead, the angle between the force and velocity vectors is always less than 90 degrees, that is, the component of the force in the line of motion aids the motion. As a result, the force causes the velocity vector to increase in value from A to P, while at the same time forcing the path to bend.

Starting from point P, we might say that the planet, under the impetus of its velocity, tries to escape the gravitational pull of the Sun, which keeps slowing down the planet and bending its path, until at point A, the planet starts coming around. The gravitational force now starts pulling the planet toward the Sun, increasing its speed and bending its path, until the planet is brought back to P, to start all over again.

Newton indeed had abundantly solved the riddle of the heavens, which had puzzled the human mind for thousands of years. Actually, we should say, he had found a solution. Less than two and a half centuries later, Albert Einstein would arrive at a totally different conception of gravity.