

Thevenin & Norton Equivalent Circuits (H.1)

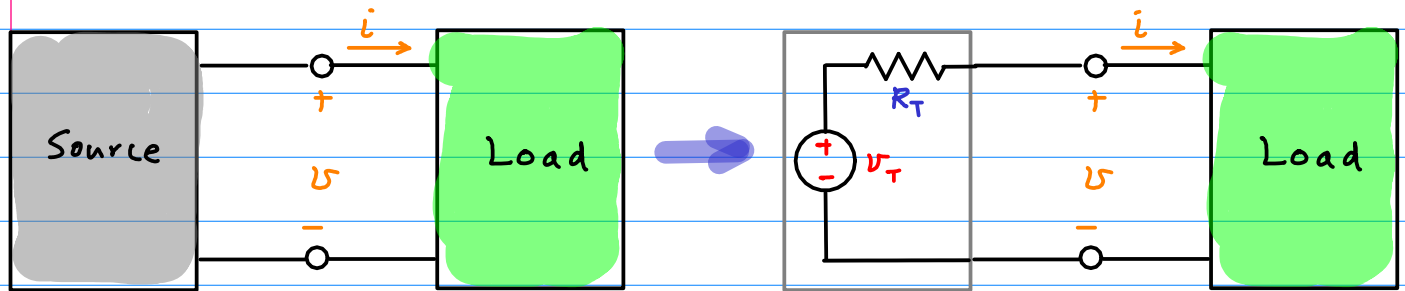
20170517

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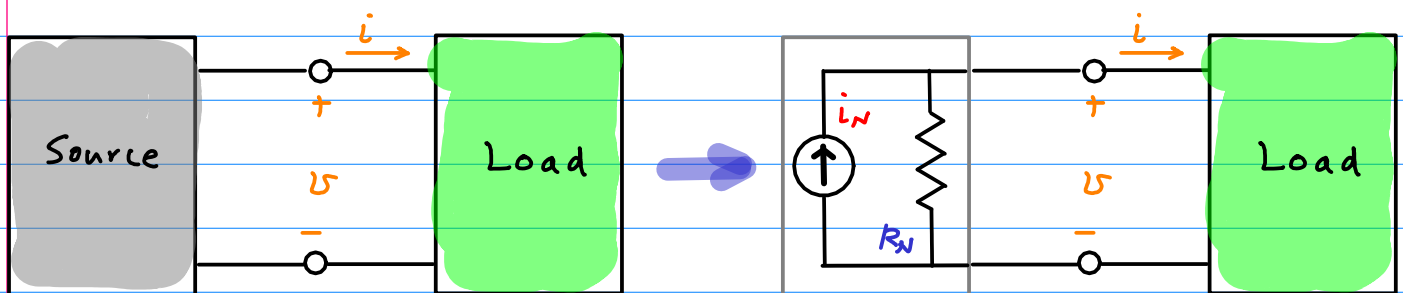
Thevenin & Norton Theorem

Thevenin's Theorem



$$U_T = U_{oc} \quad \text{when } i = 0 \quad \boxed{\text{max } U}$$
$$R_T = R_N$$

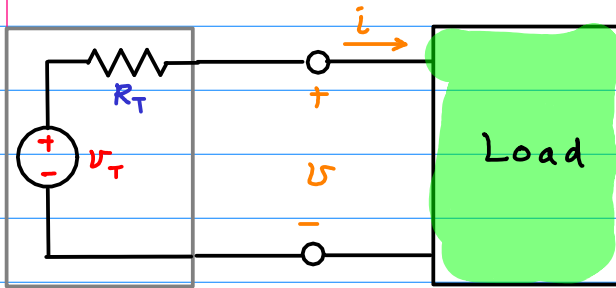
Norton's Theorem



$$i_N = i_{sc} \quad \text{when } U = 0 \quad \boxed{\text{max } i}$$
$$R_N = R_T$$

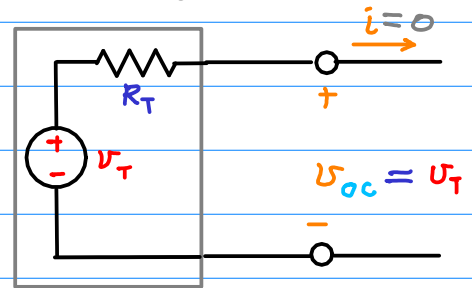
Max v and Max i conditions

Thevenin's Theorem



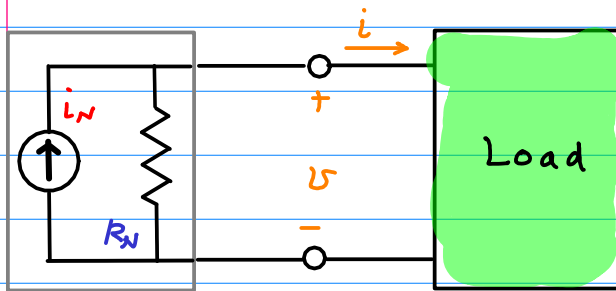
$V_T = V_{oc}$ when $i = 0$ **max v**
 $R_T = R_N$

no voltage drop

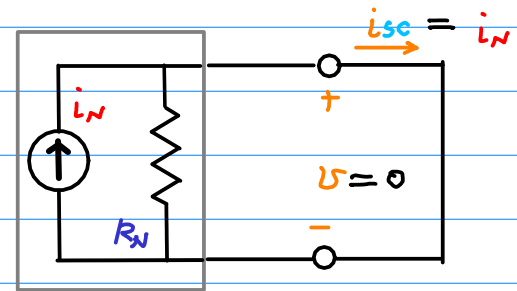


max v when O.C. ($R_L = \infty$)

Norton's Theorem

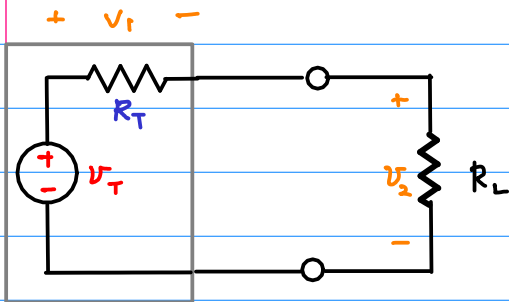


$i_N = i_{sc}$ when $v = 0$ **max i**
 $R_N = R_T$



max i when S.C. ($R_L = 0$)

Voltage divider

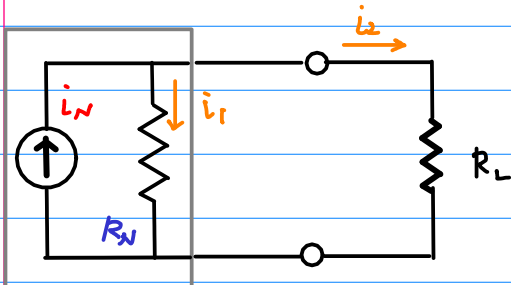


$$U_1 = \frac{R_T}{R_T + R_L} U_T$$

$$U_2 = \frac{R_L}{R_T + R_L} U_T$$

$$\lim_{R_L \rightarrow \infty} U_2 = U_T$$

Current divider



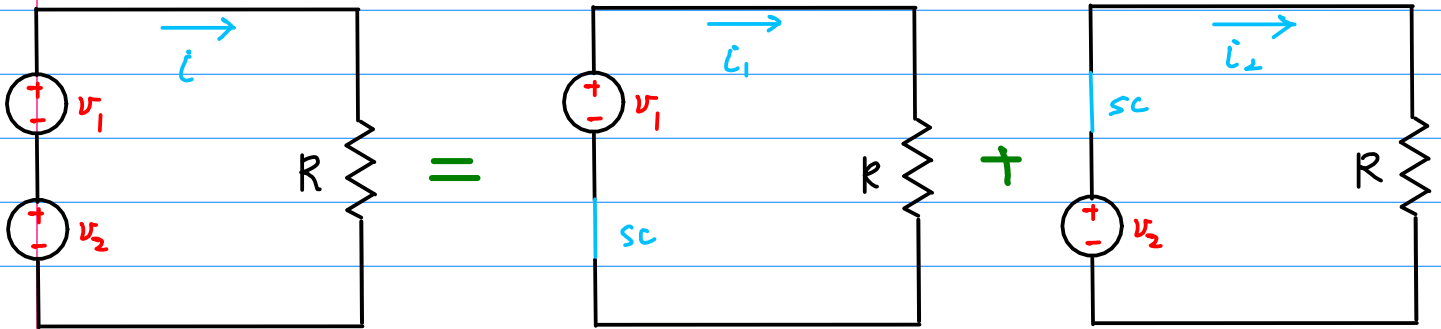
$$i_1 = \frac{R_L}{R_N + R_L} i_N$$

$$i_2 = \frac{R_N}{R_N + R_L} i_N$$

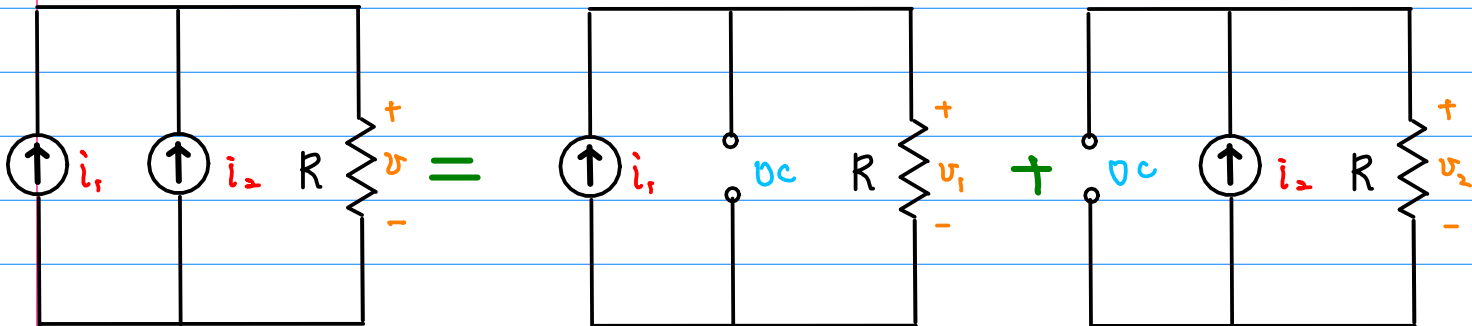
$$\lim_{R_L \rightarrow 0} i_2 = i_N$$

Super position

$$i = i_1 + i_2$$



$$v = v_1 + v_2$$

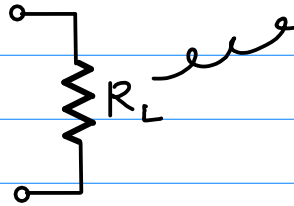


 \rightarrow sc (short circuit)

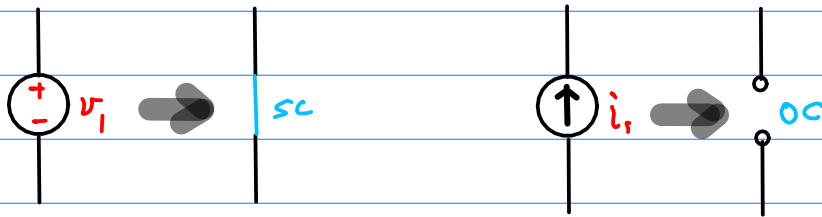
 \rightarrow dc (open circuit)

Equivalent Resistance $R_T = R_N$

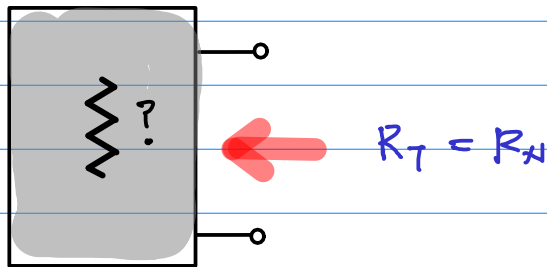
① remove R_L

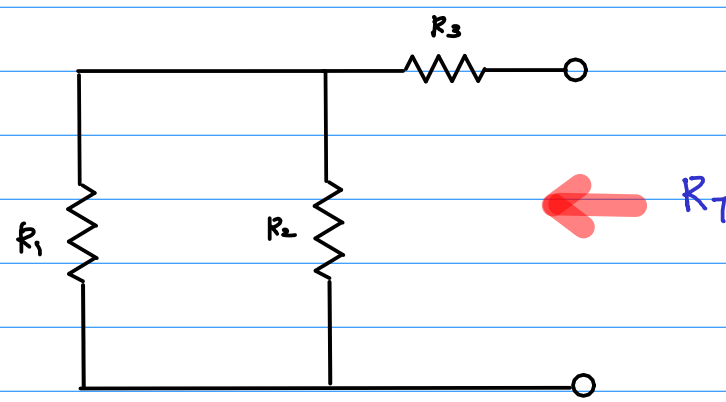
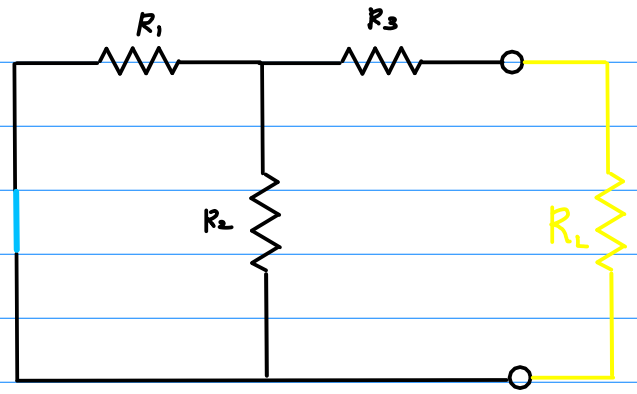
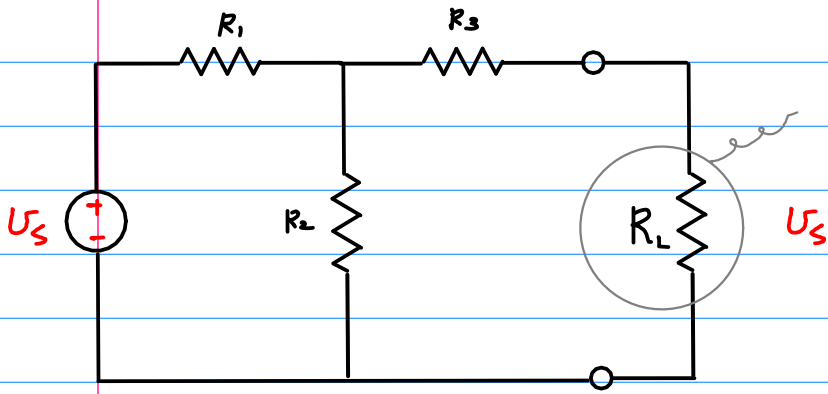


②

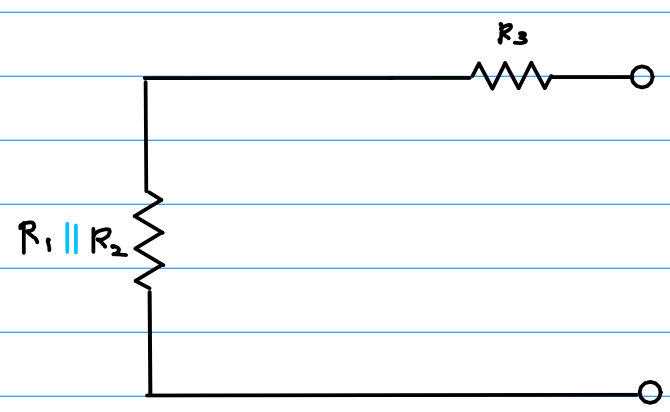


③ resistance seen from the R_L side



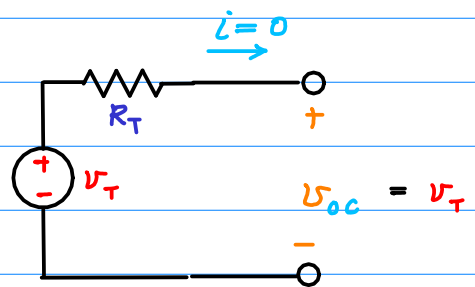
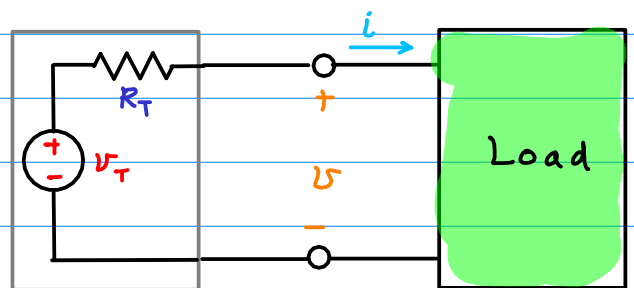
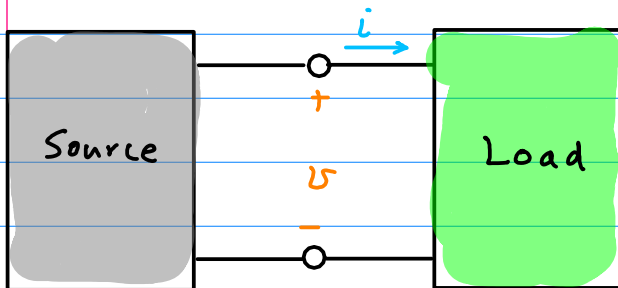


$$R_T = R_1 \parallel R_2 + R_3$$

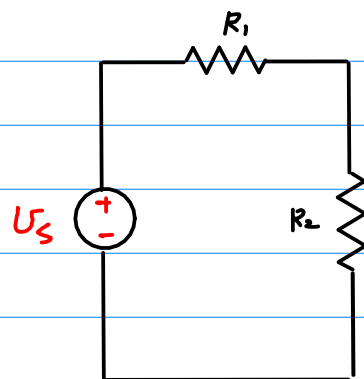
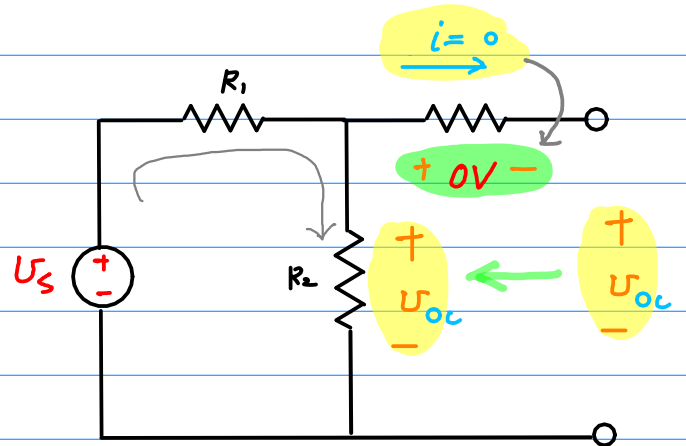
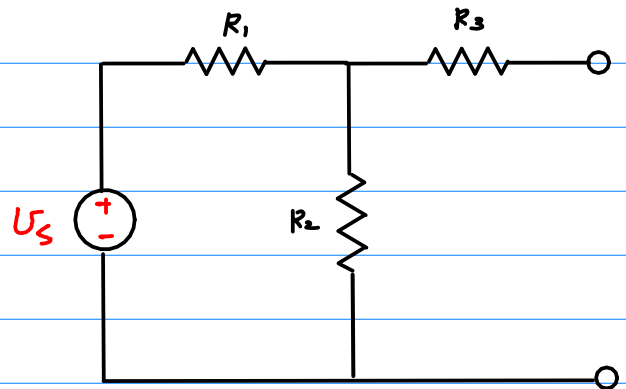
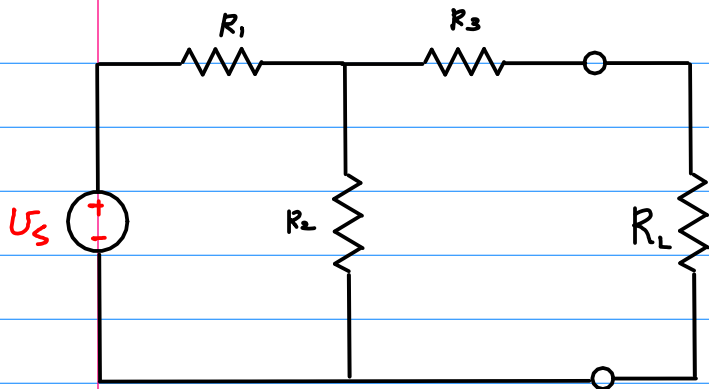


Thevenin voltage U_T

max U

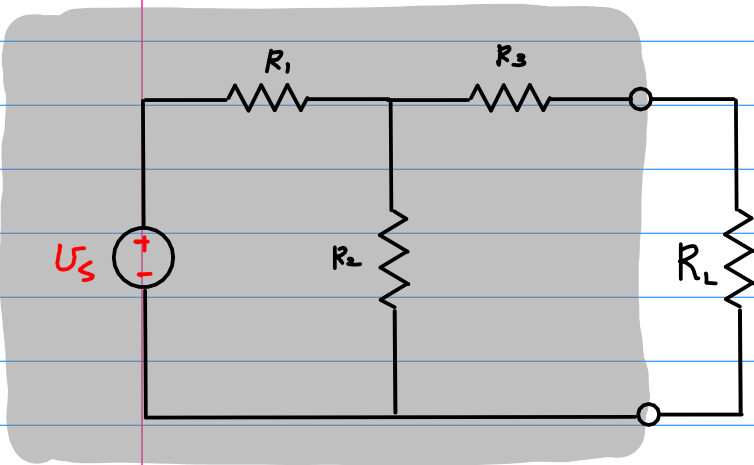


- ① remove $R_L \rightarrow OC$
- ② U_{oc}



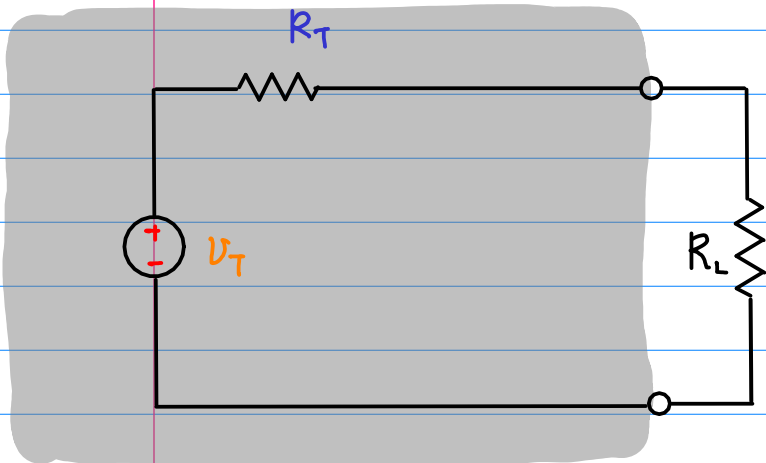
$$U_{oc} = \frac{R_2}{R_1 + R_2} U_s$$

$$\Rightarrow V_T$$



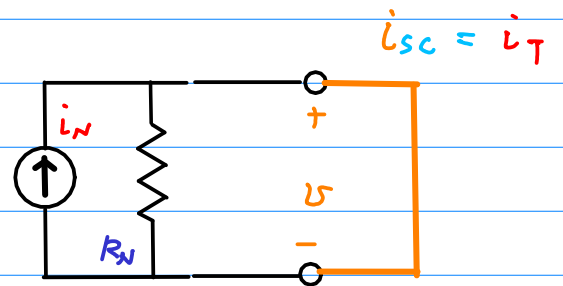
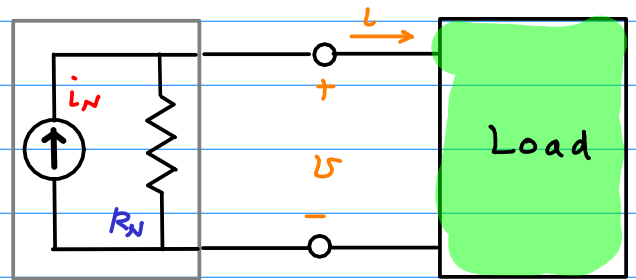
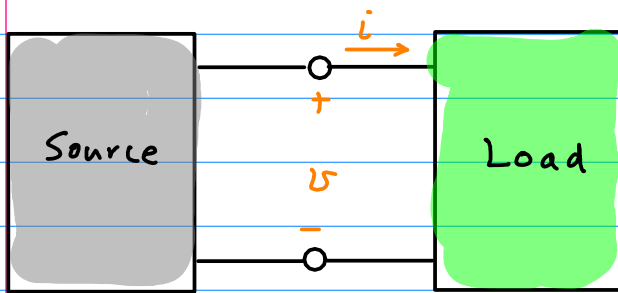
$$R_T = R_1 \parallel R_2 + R_3$$

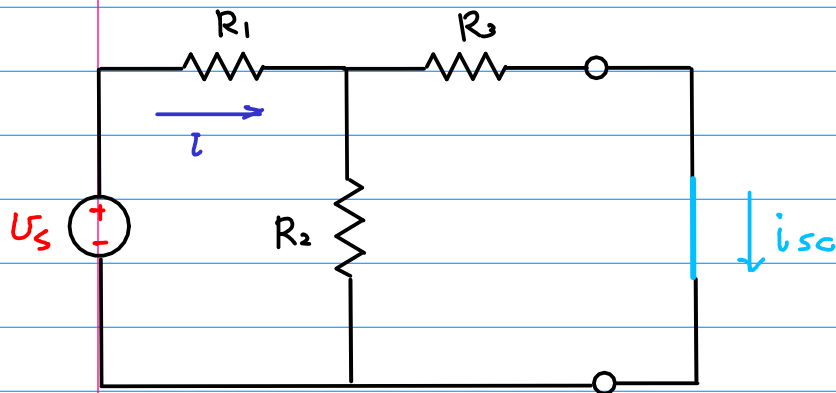
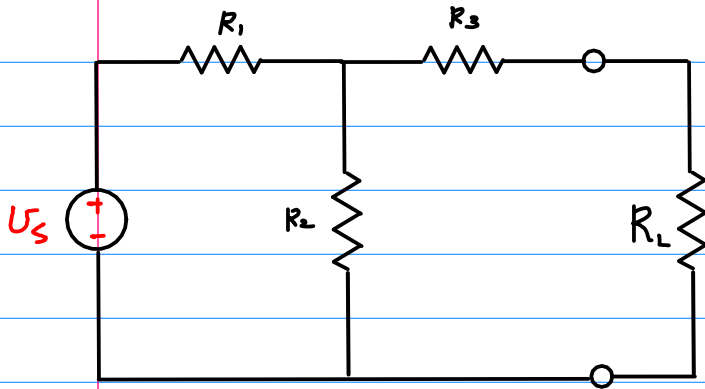
$$U_T = \frac{R_2}{R_1 + R_2} U_S$$



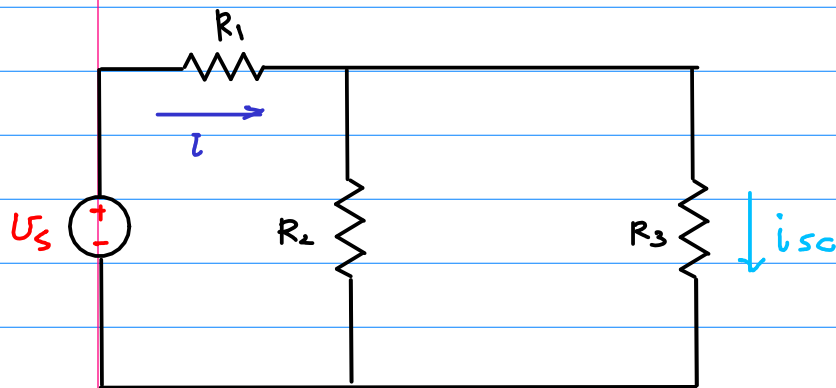
Norton Current i_T

max i





$$i = \frac{U_s}{R_1 + R_2 \parallel R_3}$$



$$i_{sc} = \frac{U_s}{R_1 + R_2 \parallel R_3} \frac{R_2}{R_2 + R_3}$$

$$\Rightarrow i_N$$

$$i = \frac{U_s}{R_1 + R_2 \parallel R_3}$$

$$i_{sc} = \frac{U_s}{R_1 + R_2 \parallel R_3} \frac{R_2}{R_2 + R_3}$$

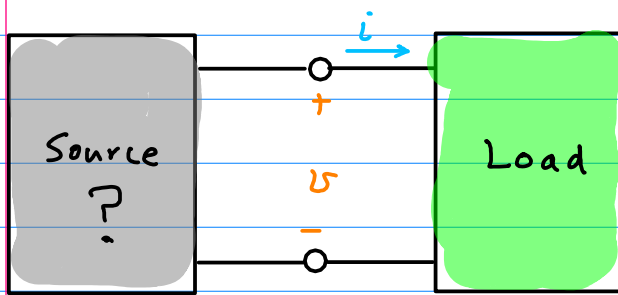
$$\Rightarrow i_N$$

$$R_2 \parallel R_3 = \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}} = \frac{R_2 R_3}{R_2 + R_3}$$

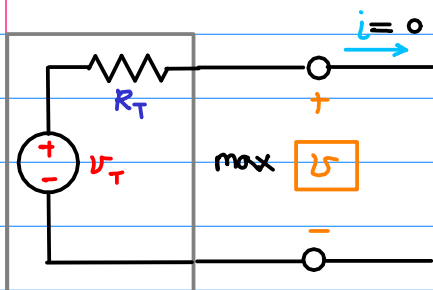
$$(R_1 + R_2 \parallel R_3)(R_2 + R_3) = R_1(R_2 + R_3) + R_2 R_3$$

$$\frac{R_2}{(R_1 + R_2 \parallel R_3)(R_2 + R_3)} = \frac{R_2}{(R_1 + R_3)R_2 + R_1 R_3}$$

$$i_N = \frac{R_2}{(R_1 + R_3)R_2 + R_1 R_3} U_s$$



Thévenin Voltage $\begin{matrix} + \\ \circlearrowleft \\ - \end{matrix} U_T$

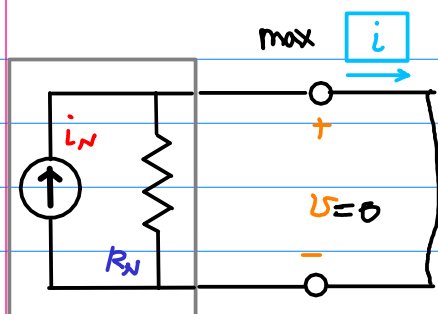


$$U_T = U_{oc} \text{ when } i = 0$$

max U

$$R_T = R_N$$

Norton Current $\begin{matrix} \uparrow \\ \circlearrowleft \end{matrix} i_N$



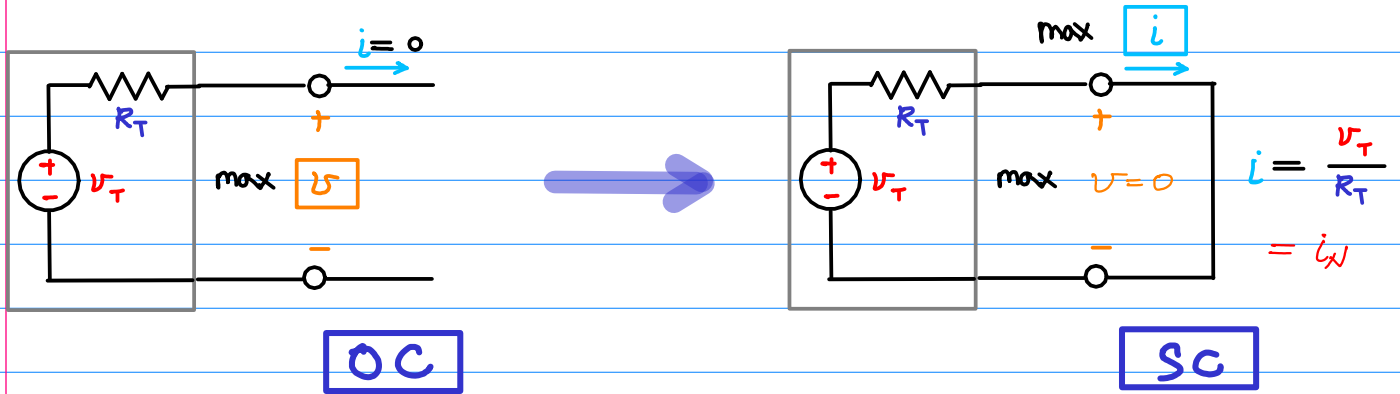
$$i_N = i_{sc} \text{ when } U = 0$$

max i

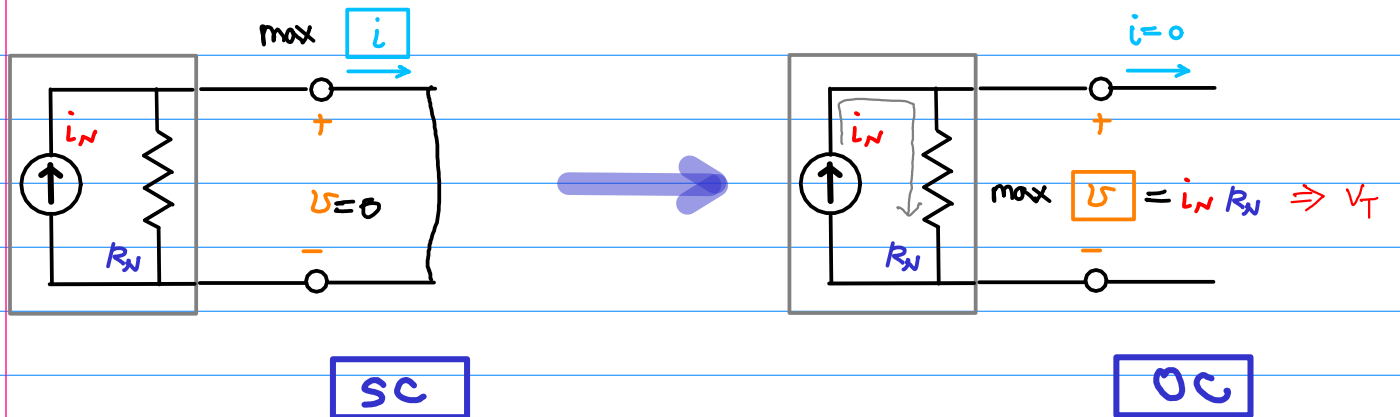
$$R_N = R_T$$

$$U_T = R_T i_N$$

Thévenin Equivalent Circuit

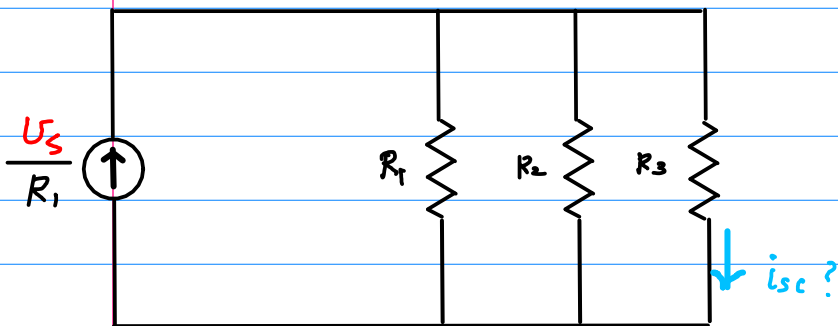
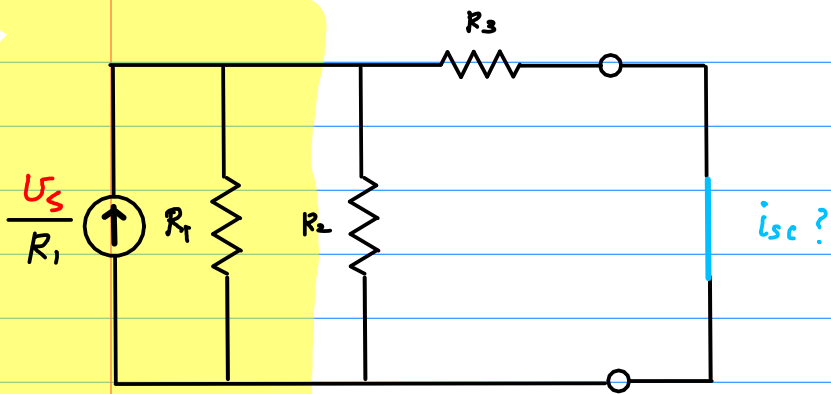
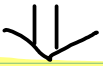
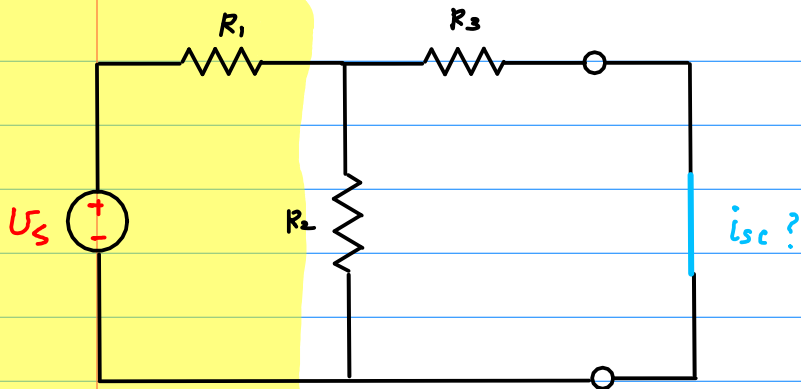


Norton Equivalent Circuit



$$V_T = R_T I_N$$

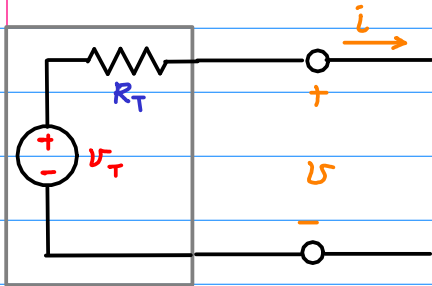
Source Transformation



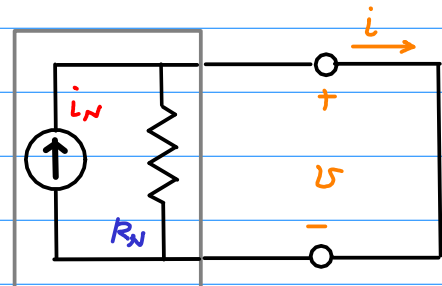
$$\begin{aligned}
 i_{sc} &= \frac{\frac{1}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \frac{U_s}{R_1} \\
 &= \frac{\cancel{R_1 R_2}}{R_2 R_3 + R_1 R_3 + R_1 R_2} \frac{1}{R_1} U_s \\
 &= \frac{R_2 U_s}{R_2 R_3 + R_1 R_3 + R_1 R_2}
 \end{aligned}$$

Source Side Equation R_T

max v $\leftarrow R_L = \infty$

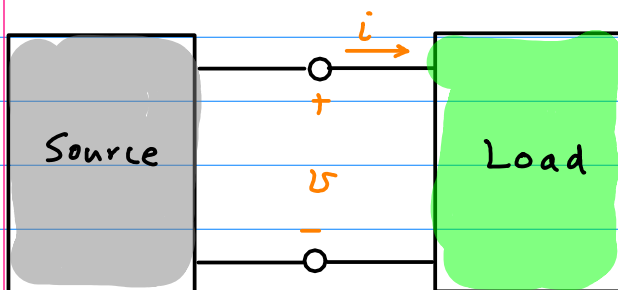


max i $\leftarrow R_L = 0$

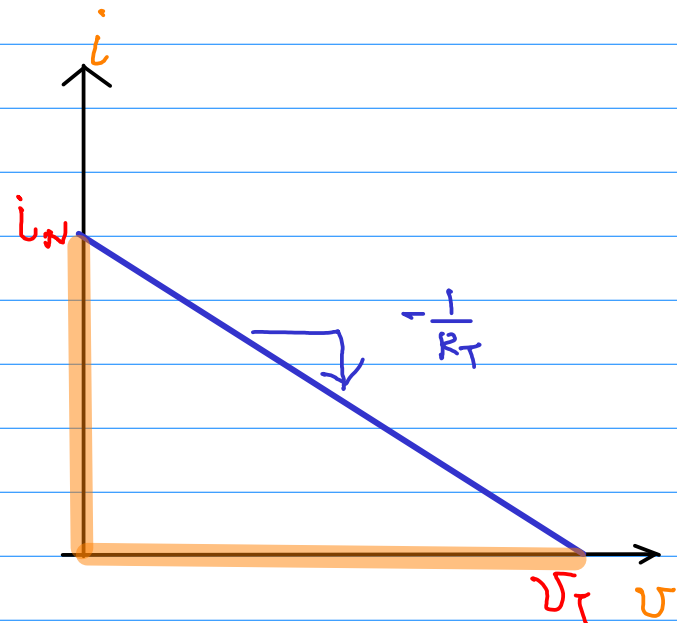


$i < i_N$
 $v < v_T$

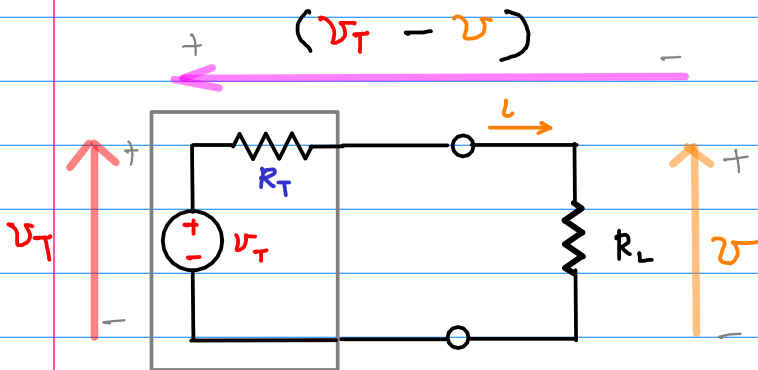
$0 < R_L < \infty$



$$i = \frac{1}{R_T} (v_T - v)$$



Load Line



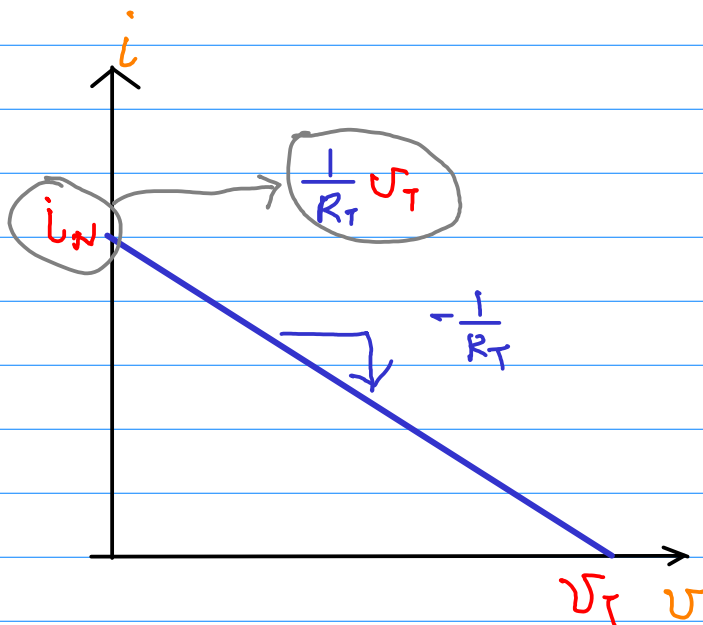
$$(V_T - v) = R_T \cdot i$$

$$i = \frac{1}{R_T} (V_T - v)$$

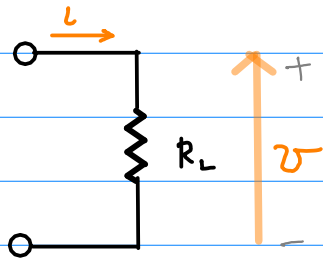
$$R_T = R_N$$

$$i = -\frac{1}{R_T} v + \frac{1}{R_T} V_T$$

$$y = -ax + b$$

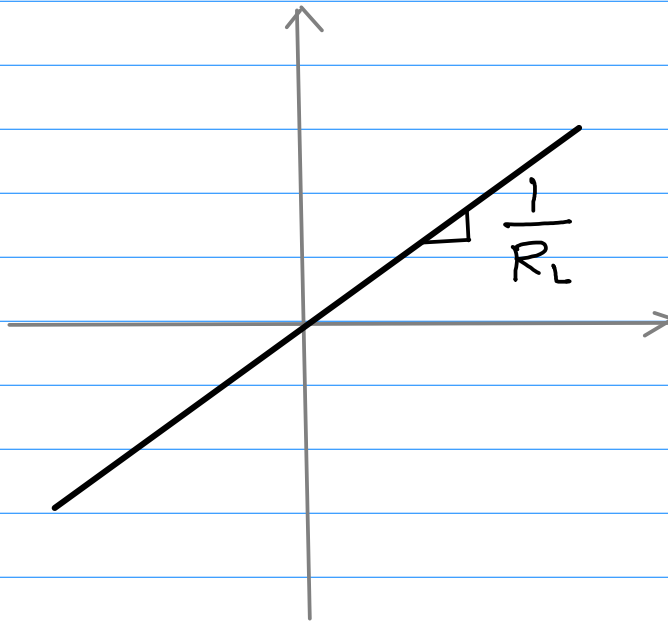


Load side Equation

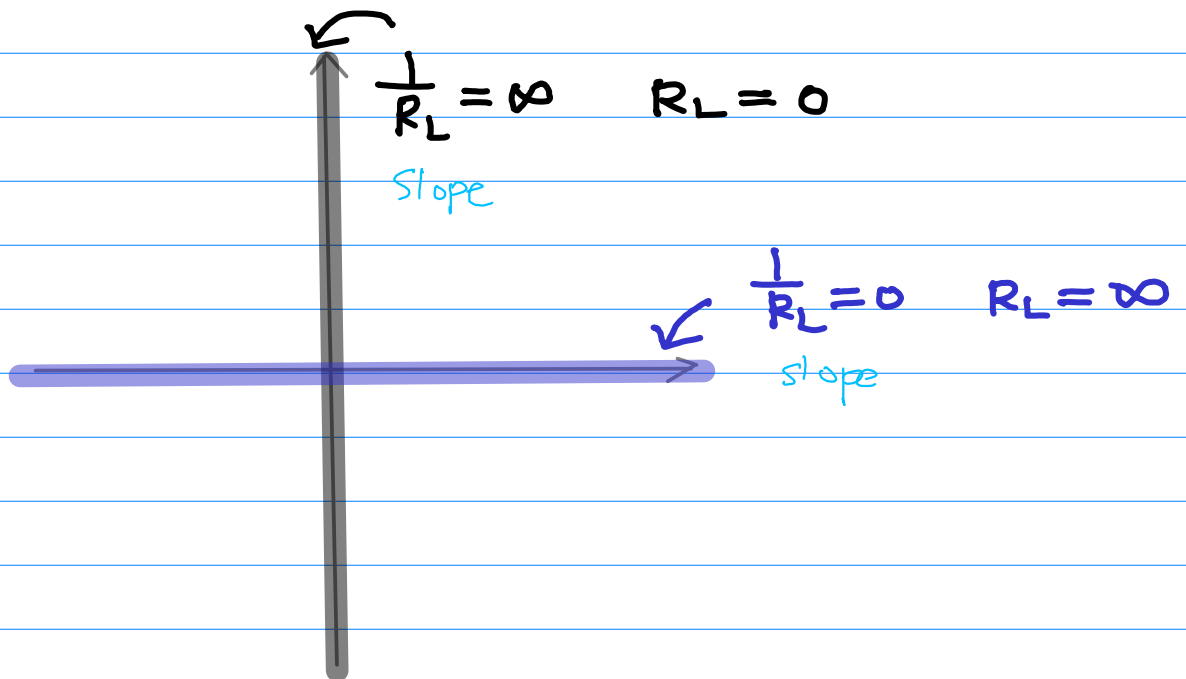


$$v = i R_L$$

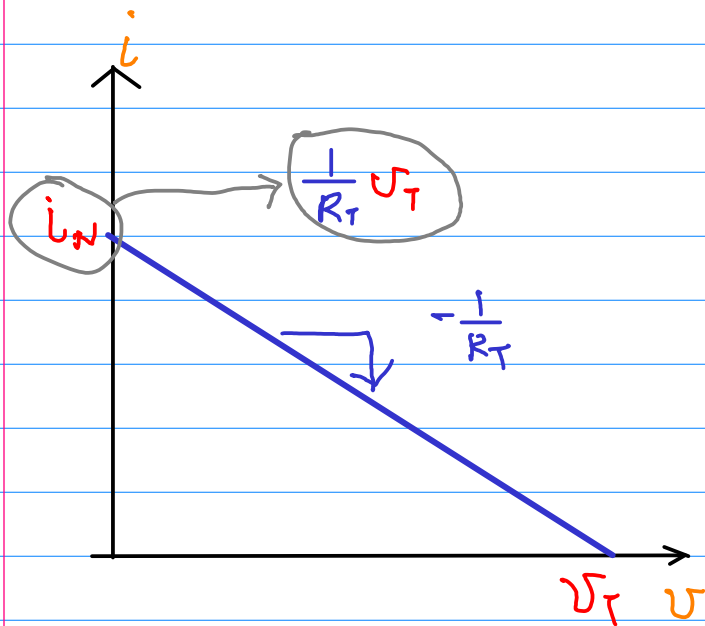
$$i = \frac{1}{R_L} v$$



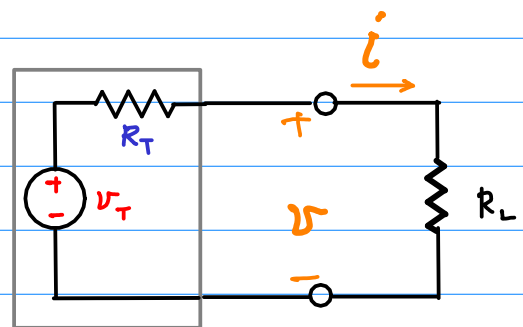
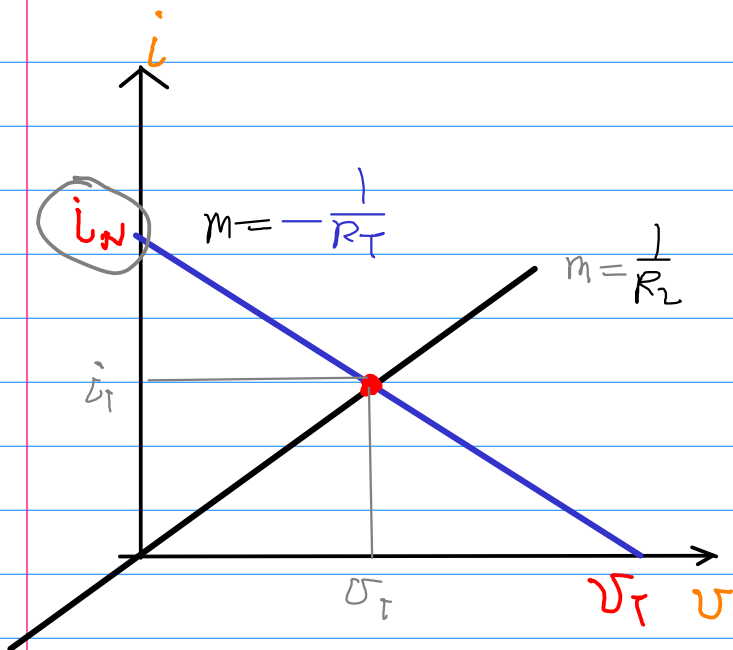
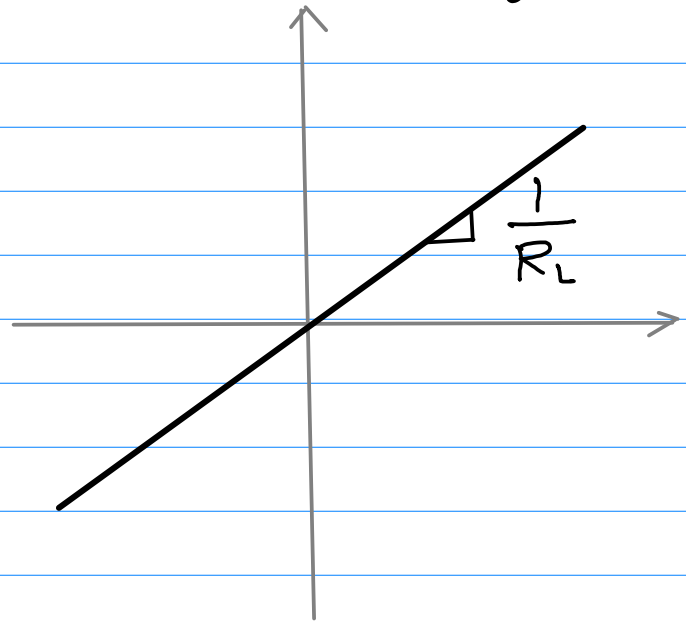
$$0 < R_L < \infty$$

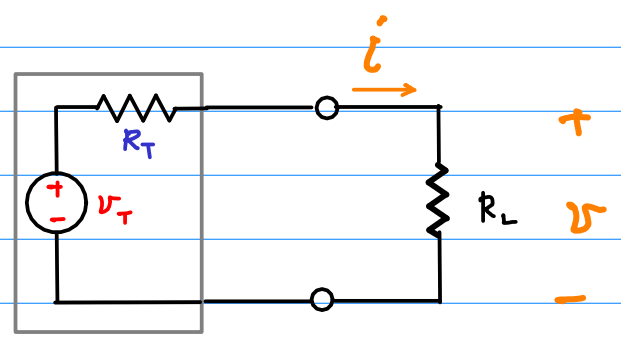
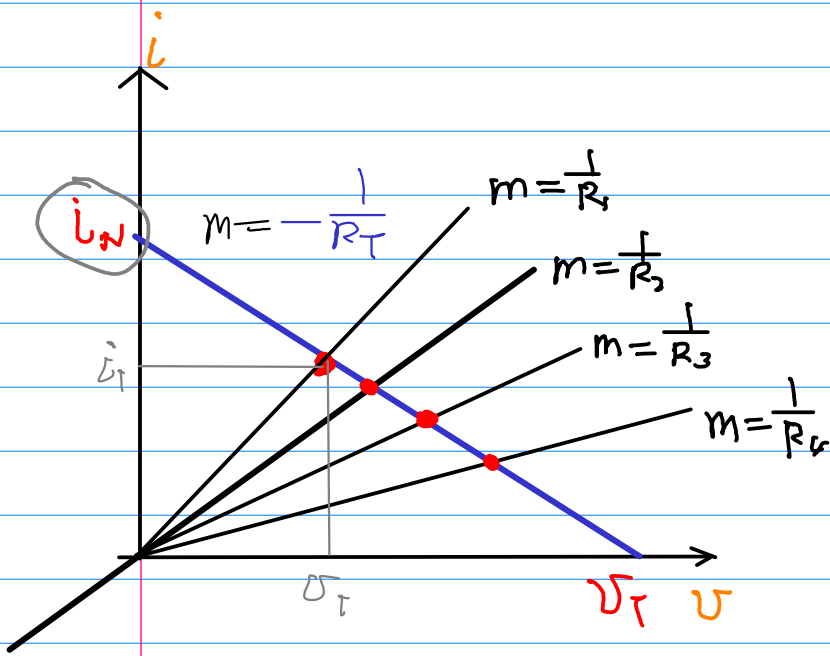


source side equation



load side equation

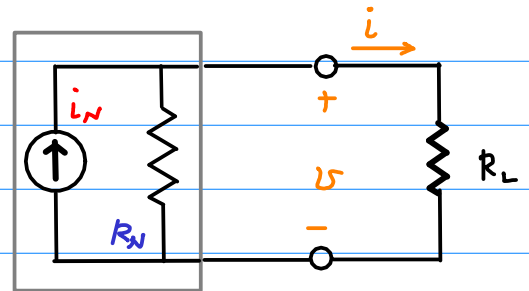
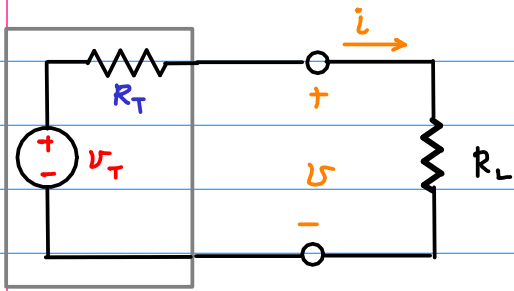




$R_L = R_1 < R_2 < R_3 \dots$

→

$i \downarrow \quad U \uparrow$



$$v = \frac{R_L}{R_T + R_L} v_T$$

$$i = \frac{R_N}{R_N + R_L} i_N$$

$$v = \frac{R_N}{R_N + R_L} i_N R_L$$

$$= \frac{R_L}{R_N + R_L} i_N R_N$$

$$= \frac{R_L}{R_T + R_L} v_T$$

$$v_T = i_N R_N$$

$$R_N = R_T$$

$$v = (R_N \parallel R_L) i_N$$

$$= \frac{R_N R_L}{R_N + R_L} i_N$$