PAM (Pulse Amplitude Modulation)

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Generating the PAM Signal

- the message signal $m(t)$
- the PAM signal $s(t)$

Sample and Hold operation

- instantaneous sampling ($T_s = 1/f_s$)
- lengthening the duration of each sample (T)

Sample-and-Hold

Idealy Sampled Signal

$$m_\delta(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\delta(t - nT_s)$$

The rectangular Signal

$$h(t) = rect(\frac{t-T/2}{T}) \qquad \begin{cases} 1 & (0 < t < T) \\ 0.5 & (t = 0,\ T) \\ 0 & (otherwise) \end{cases}$$

Sample-and-Hold Filter

Sample and Hold operation

- Instantaneous Sampling ($T_s = 1/f_s$)
- Lengthening the duration of each sample (T)

Flat-top PAM Pulses

$$s(t) = \sum_{n=-\infty}^{\infty} m(nT_s)h(t - nT_s)$$ samples multiplied by pulses

$$s(t) = m_\delta(t) \star h(t)$$ convolution with a pulse

Flat-top PAM Pulses

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$$
\begin{aligned}
s(t) = m_\delta(t) \star h(t) &= \int_{-\infty}^{+\infty} m_\delta(\tau)h(t-\tau)d\tau \\
&= \int_{-\infty}^{+\infty} \left[\sum_{n=-\infty}^{\infty} m(nT_s)\delta(\tau - nT_s) \right] h(t-\tau)d\tau \\
&= \sum_{n=-\infty}^{\infty} m(nT_s) \left\{ \int_{-\infty}^{+\infty} [\delta(\tau - nT_s)]\, h(t-\tau)d\tau \right\} \\
&= \sum_{n=-\infty}^{\infty} m(nT_s)h(t - nT_s)
\end{aligned}
$$

Fourier Transform of PAM Pulses

Fourier Transform of a Sampled Signal

$$g_\delta(t) \leftrightarrows G_\delta(f) = f_s \sum_{m=-\infty}^{\infty} G(f - mf_s)$$ replicated spectrum

$$\sum_{n=-\infty}^{\infty} g(nT_s)\delta(t - nT_s) \leftrightarrows \sum_{n=-\infty}^{\infty} g(nT_s) exp(-j2\pi nT_s f)$$ DTFT

Fourier Transform of Flat-top PAM Pulses

$$s(t) = m_\delta(t) \star h(t)$$

$$S(f) = M_\delta(f)H(f) = \left[f_s \sum_{k=-\infty}^{\infty} M(f - kf_s) \right] H(f)$$

Recovering PAM Signal

Reconstruction

Given a PAM Signal $s(t) = m_\delta(t) \star h(t)$

$$S(f) = M_\delta(f)H(f) = \left[f_s \sum_{k=-\infty}^{\infty} M(f - kf_s) \right] H(f)$$

Low Pass (Reconstruction) Filter $H(f)$

the filtering output $M(f)H(f)$

But the filter $H(f)$ causes the message signal distorted

Amplitude Distortion $|H(f)|$
Phase Delay DIstortion $Arg(H(f))$ →aperture effect

Needs an equalizer

Equalization

Recovering the message signal from the PAM signal

- PAM signal $s(t)$
- Reconstruction filter
- Equalizer
- Message signal $m(t)$

Distortion of the rectangular pulse

Amplitude Distortion $|H(f)|$
Phase Delay DIstortion $Arg(H(f))$ →aperture effect
the amplitude Response of the equalizer: $\frac{1}{|H(f)|} = \frac{1}{Tsin(fT)} = \frac{\pi f}{sin(\pi f T)}$

Reference

[1] S. Haykin, M Moher, “Introduction to Analog and Digital Communications”, 2ed