Mtg 12: Thu, 21 Jan 10

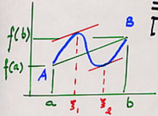
$G(t) = 0$, $G(x_i) = 0$, $i = 0, \ldots, n$ $\Rightarrow$ $G(\cdot)$ has $(n+2)$ zeros $(t, x_0, \ldots, x_n)$ — $(n+2)$ pts

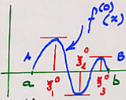
Rolle's thm: $G^{(1)}(\cdot)$ has at least $(n+1)$ zeros.

particular case of Deriv. MVT: $f: \mathbb{R} \to \mathbb{R}$ cont. diff. $\exists$ at least one pt $\xi \in [a,b]$ st

$$\frac{f(b) - f(a)}{b - a} = f^{(1)}(\xi)$$

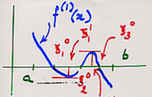
or: $f(b) - f(a) = f^{(1)}(\xi)(b-a)$





Rolle's thm: $f(a) = f(b) = 0$ (Const) $\Rightarrow \exists$ at least $\xi \in [a,b]$ st $f^{(1)}(\xi) = 0$

$f^{(1)}(\cdot)$ has 3 zeros: $\{\xi_1^0, \xi_2^0, \xi_3^0\}$



$f^{(2)}(\cdot)$ has 2 zeros: $\{\xi_1^1, \xi_2^1\}$

$f^{(3)}(\cdot)$ has 1 zero: $\{\xi_1^2\}$

Apply Rolle's thm to $G(\cdot)$:

$G^{(0)}(\cdot)$ has at least $(n+2)$ zeros

$G^{(1)}(\cdot)$ — $(n+1)$ —

$\vdots$

$G^{(n+1)}(\cdot)$ — 1 zero $\Rightarrow \exists\, \xi \in$ ~~$[a,b]$~~ I_t

st $G^{(n+1)}(\xi) = 0$

Note: $E^{(n+1)}(x) = f^{(n+1)}(x) - 0$ (HW) ($f_n^{(n+1)}(x)$)

Recall $f_n \in \mathcal{P}_n$

$q_{n+1}^{(n+1)}(x) = (n+1)!$ HW

$$G^{(n+1)}(x) = f^{(n+1)}(x) - \frac{(n+1)!}{q_{n+1}(t)} \underbrace{E(t)}_{\text{Const.}}$$

Let $x = \xi$:

$$G^{(n+1)}(\xi) = 0 = f^{(n+1)}(\xi) - \frac{(n+1)!}{q_{n+1}(t)} E(t)$$

$$E(t) = \frac{q_{n+1}(t)}{(n+1)!} f^{(n+1)}(\xi)$$

(end proof)

HW: p. 11-2

Let $f(x) = \log x$, $t = 2$, $x_0 = 3$, $x_1 = 4$, ..., $x_6 = 9$

Plot 3 figs similar to figs on p. 11-2 for $i = 3$ ($x_3 = 6$) and $x = 5.5$.