Example Random Processes

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September 6, 2019

Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles,Jr. and B. Shi

Outline

Gaussian Random Process

N Gaussian random variables

Definition

$$f_X(x_1, \cdots, x_N; t_1, \cdots, t_N) =$$

$$\frac{exp\left\{-(1/2)\left[x-\overline{X}\right]^t \left[C_X\right]^{-1}\left[x-\overline{X}\right]\right\}}{\sqrt{(2\pi)^N \left|[C_X]\right|}}$$

The Covariance Matrix
N Gaussian random variables

Definition

$$\overline{X}_i = E[X_i] = E[X(t_i)]$$

$$C_{ik} = C_{X_i X_k} = E\left[\left(X_i - \overline{X}_i\right)\left(X_k - \overline{X}_k\right)\right]$$

$$= E[(X_i - E[X(t_i)])(X_k - E[X(t_k)])]$$

$$C_{ik} = C_{X_i X_k} = C_{XX}(t_i t_k)$$

$$= R_{XX}(t_i, t_k) - E[X(t_i)]\,E[X(t_k)]$$

Stationary Gaussian Process

N Gaussian random variables

Definition

$$\overline{X}_i = E[X_i] = E[X(t_i)] = \overline{X} = const$$

$$C_{XX}(t_i, t_k) = C_{XX}(t_k - t_i)$$

$$R_{XX}(t_i, t_k) = R_{XX}(t_k - t_i)$$

Jointly Gaussian Process

N Gaussian random variables

Definition

the two random processes $X(t)$ and $Y(t)$ are jointly Gaussian
if the random variables $X(t_1), ..., X(t_N), Y(t'_1), ..., Y(t'_M)$
at times$t_1, ..., t_N$ for $X(t)$ and $t'_1, ..., t'_M$ for $Y(t)$
are jointly gaussian for any $N, t_1, ..., t_N, M, t'_1, ..., t'_M$

Poisson Random Process

N Gaussian random variables

Definition

$$f_X(x) = \sum_{k=0}^{\infty} \frac{(\lambda t)^k e^{-\lambda t}}{k!} \delta(x-k)$$