

Laurent Series and z-Transform

- Geometric Series

Permutations B

20240528 Tue

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A. Flipping

Base Inverting
Range Flipping

B. Range Shifting = Range Flipping + Range Complementing

Range Flipping
Range Complementing

C. Complementary Inverting

Base Inverting
Range Complementing

D. Flipping²

Base Inverting
Shifted Range Flipping = Exponent Shifting² + Range Flipping

E. Shifting² = Exponent Shifting² + Range Shifting

Shifted Range Flipping = Exponent Shifting² + Range Flipping
Range Complementing

F. Complementary Inverting

Base Inverting
Range Complementing

Base Inverting

$$2^{n+1} \longleftrightarrow 2^{-(n+1)}$$

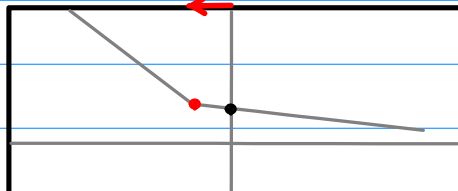
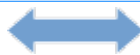
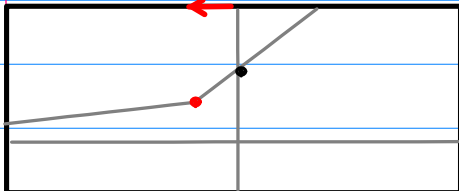
$$2^n \cdot 2^{+1} \longleftrightarrow 2^{-n} \cdot 2^{-1}$$

$$2^{n-1} \longleftrightarrow 2^{-(n-1)}$$

$$2^n \cdot 2^{-1} \longleftrightarrow 2^{-n} \cdot 2^{+1}$$

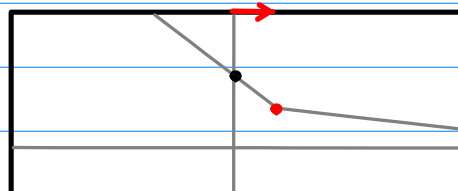
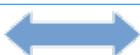
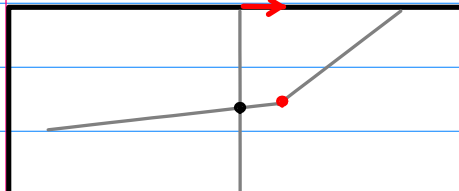
as a scaling

2^{n+1}



$2^{-(n+1)}$

2^{n-1}



$2^{-(n-1)}$

b^n	b^{-n}
a^n	a^{-n}
a^{-n}	a^n

b^n	$b^{-sh2(n)}$
$a^{(n+1)}$	$a^{-(n-1)}$
$a^{-(n+1)}$	$a^{(n-1)}$
$a^{(n-1)}$	$a^{-(n+1)}$
$a^{-(n-1)}$	$a^{(n+1)}$

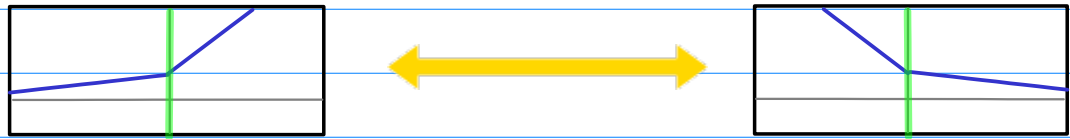
A.2a Flipping

- Base Inverting
- Range Flipping

$$a^n R(n) \longleftrightarrow a^{-n} R(-n)$$

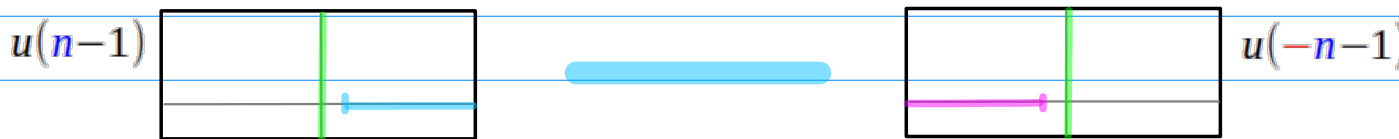
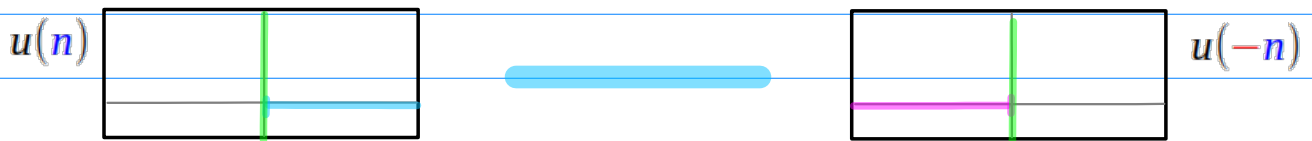
1) Base Inverting

$$a^n \longleftrightarrow a^{-n}$$



2) Range Flipping

$$R(n) \longleftrightarrow R(-n)$$

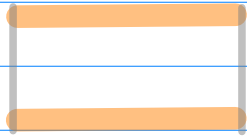


3) Flipping

$$a^n R(n) \longleftrightarrow a^{-n} R(-n)$$

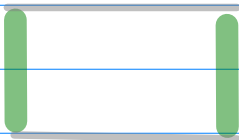


1) Base Inverting

 a^n a^{-n} 

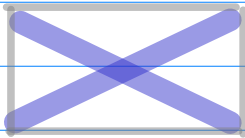
$$\boxed{0} \boxed{b_2} \boxed{b_1} \boxed{b_0} \longrightarrow \boxed{0} \boxed{b_2} \boxed{b_1} \boxed{\bar{b}_0}$$

2) Range Flipping



$$\boxed{0} \boxed{b_2} \boxed{b_1} \boxed{b_0} \longrightarrow \boxed{0} \boxed{b_2} \boxed{\bar{b}_1} \boxed{b_0}$$

3) Flipping



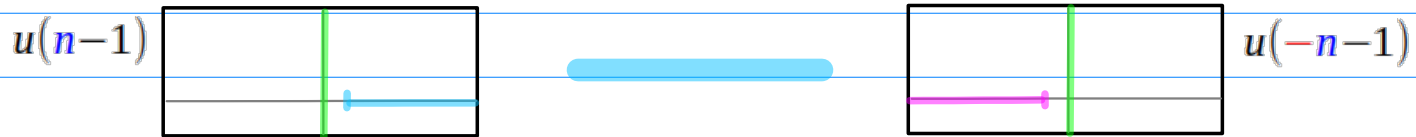
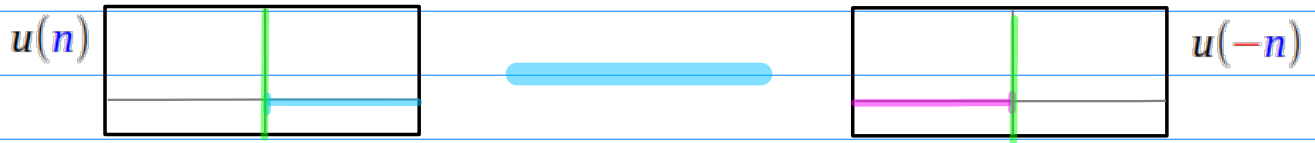
$$\boxed{0} \boxed{b_2} \boxed{b_1} \boxed{b_0} \longrightarrow \boxed{0} \boxed{b_2} \boxed{\bar{b}_1} \boxed{\bar{b}_0}$$

Range Shifting over (1) ~ (8)

$$R(n) \longleftrightarrow \overline{R(-n)}$$

(1) Range Flipping

$$R(n) \longleftrightarrow R(-n)$$



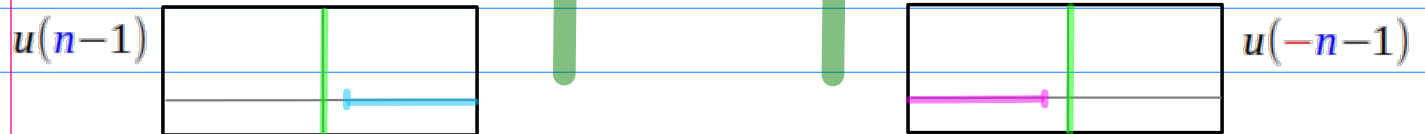
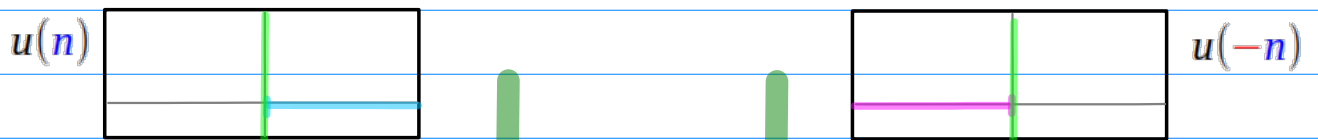
(2) Range Complementing

$$R(n) \longleftrightarrow \overline{R(n)}$$

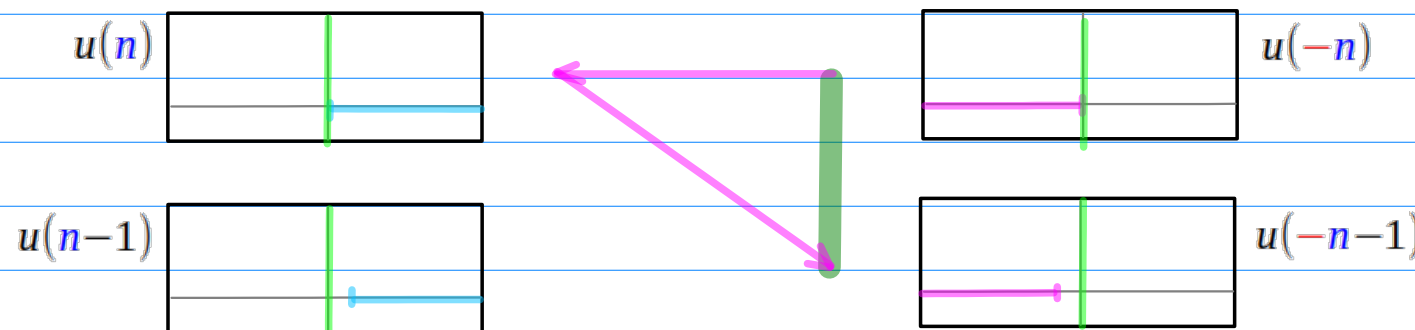
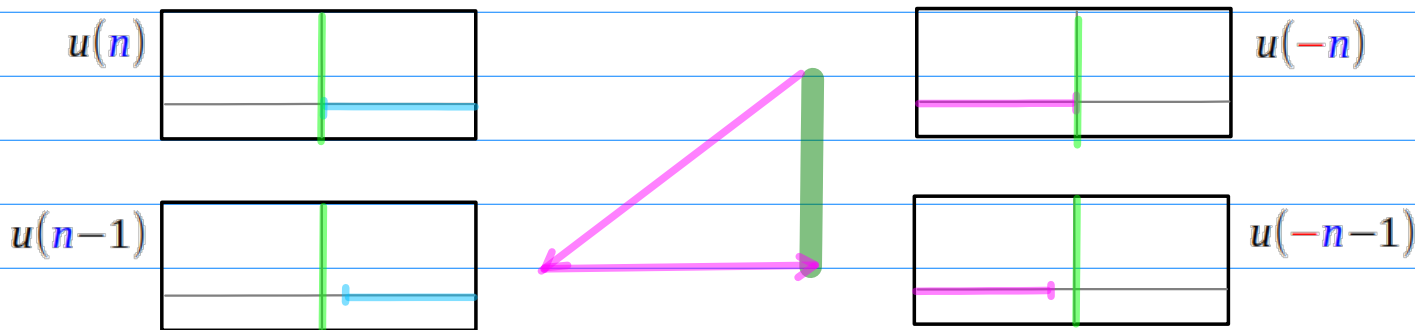
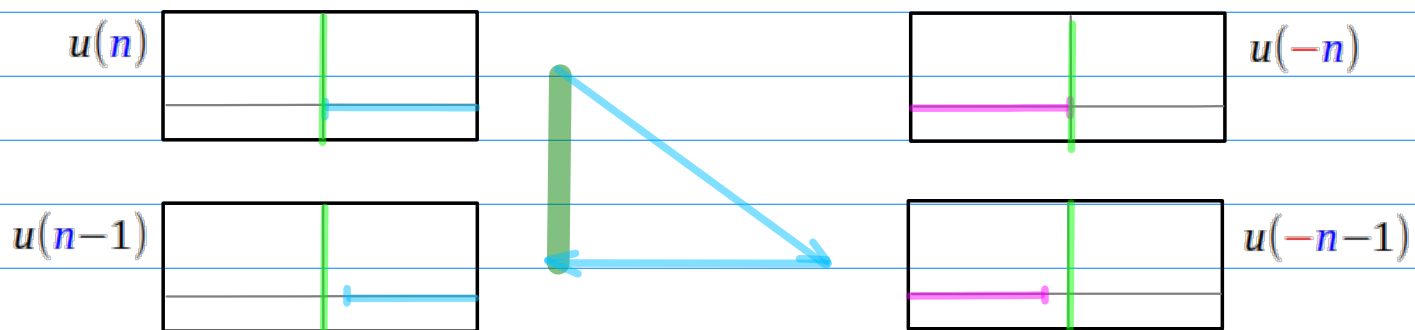
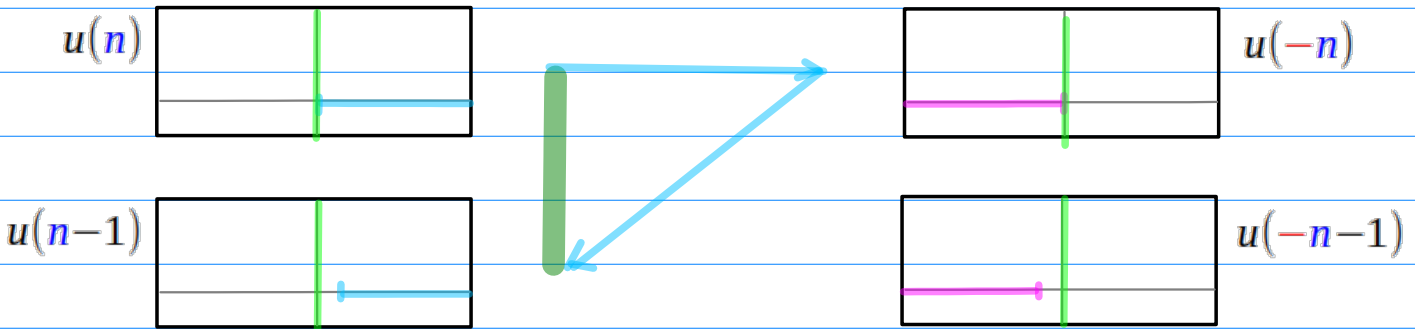


(1+2) Range Shifting

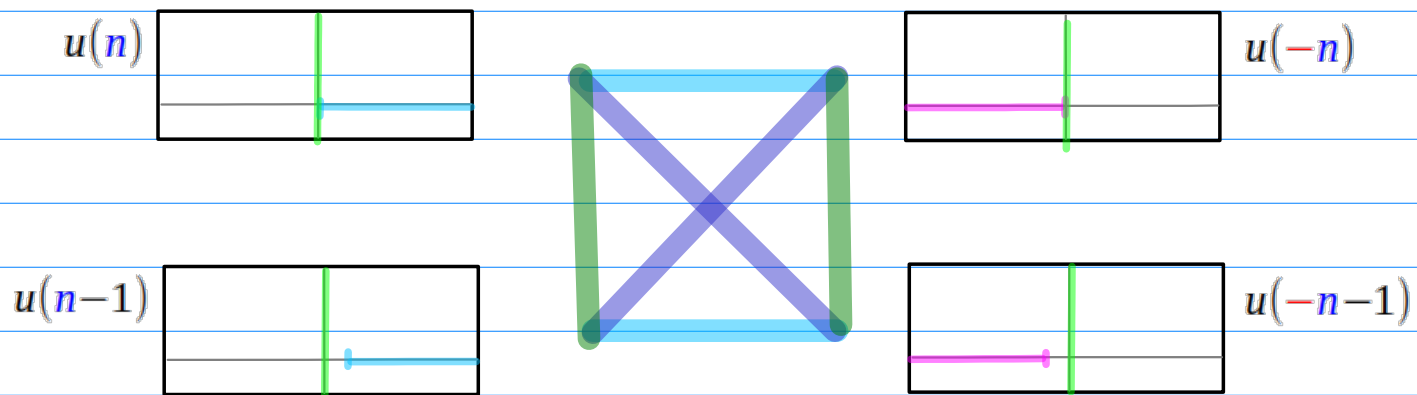
$$R(n) \longleftrightarrow \overline{R(-n)}$$



Range Shifting = Range Flipping + Range Complementing



Range Shifting, Range Flipping, Range Complementing



1) Range Flipping

$$R(n) \longleftrightarrow R(-n)$$

2) Range Complementing

$$R(n) \longleftrightarrow \overline{R(n)}$$

3) Range Shifting

$$R(n) \longleftrightarrow \overline{R(-n)}$$

$$R(n) \quad R(-n)$$

$$u(n) \quad u(-n)$$

$$u(n-1) \quad u(-n-1)$$

$$u(-n) \quad u(n)$$

$$u(-n-1) \quad u(n-1)$$

$$R(n) \quad \overline{R(n)}$$

$$u(n) \quad u(-n-1)$$

$$u(n-1) \quad u(-n)$$

$$u(-n) \quad u(n)$$

$$u(-n-1) \quad u(n-1)$$

$$R(n) \quad \overline{R(-n)}$$

$$u(n) \quad u(n-1)$$

$$u(n-1) \quad u(n)$$

$$u(-n) \quad u(-n-1)$$

$$u(-n-1) \quad u(-n)$$



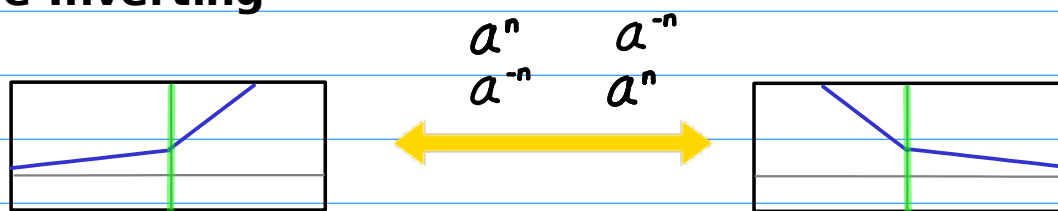
Complementary Inverting over (1) ~ (8)

$$a^n R(n)$$

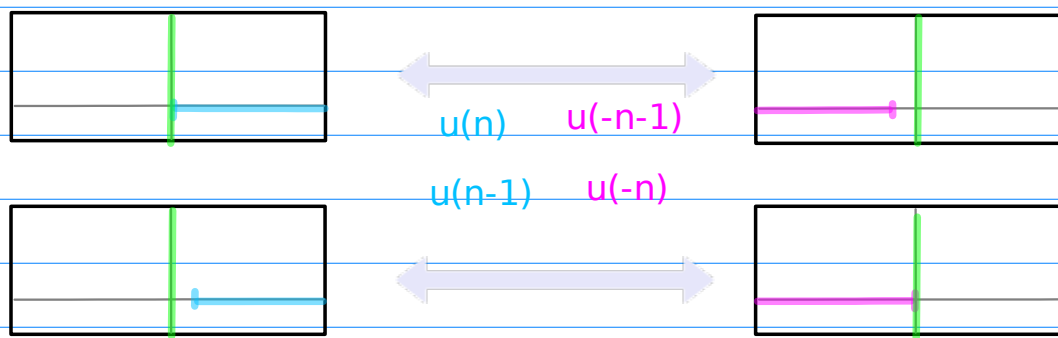


$$a^{-n} \overline{R(n)}$$

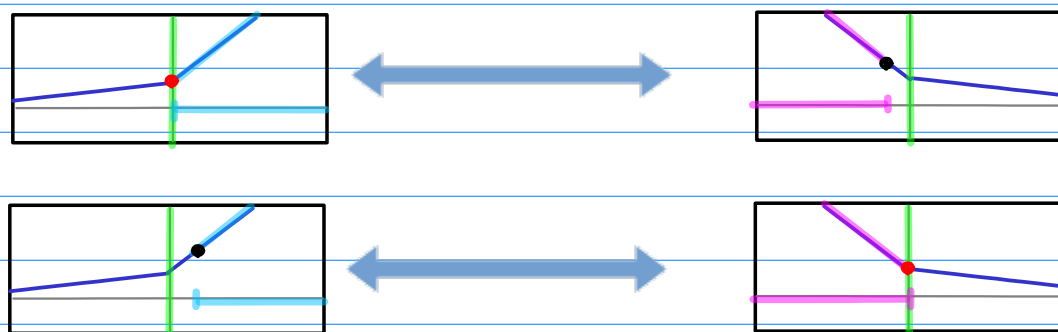
(1) Base Inverting



(2) Range Complementing



(1+2) Complementary Inverting

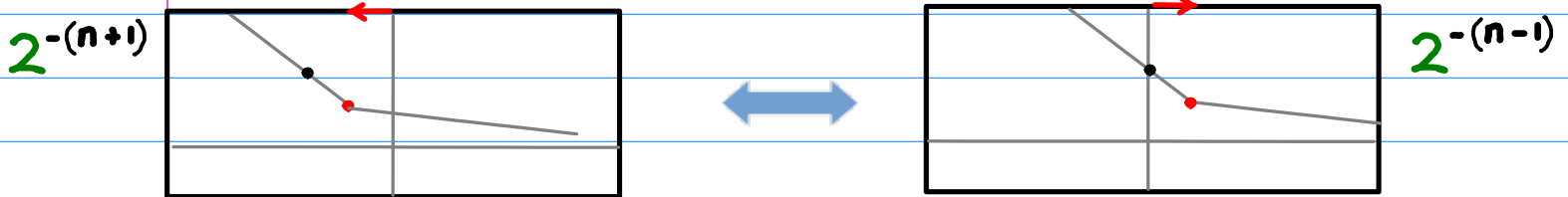
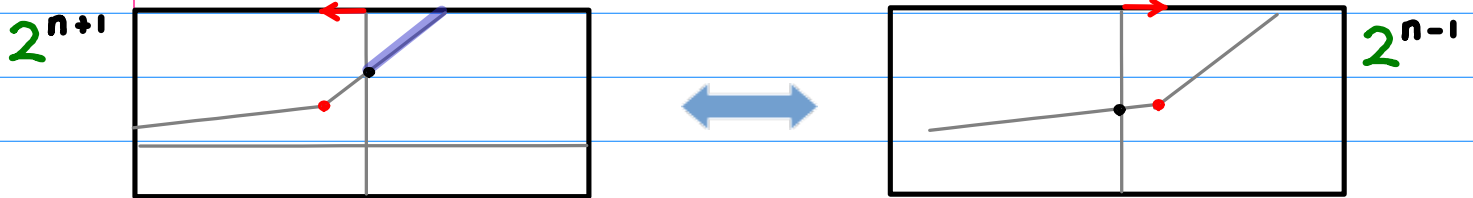




Exponent Shifting2 sh2(n)

$$2^{n+1} \longleftrightarrow 2^{n-1}$$

$$2^{-(n+1)} \longleftrightarrow 2^{-(n-1)}$$



Base Inverting

Shifted Range Flipping

$$a^n \longleftrightarrow a^{-n}$$

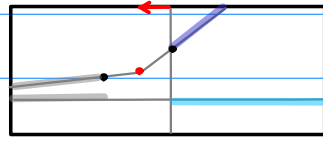
$$a^n R(n) \longleftrightarrow a^{sh2(n)} R(-n)$$

Exponent Shifting2	
b^n	$b^{sh2(n)}$
$a^{(n+1)}$	$a^{(n-1)}$
$a^{-(n+1)}$	$a^{-(n-1)}$
$a^{(n-1)}$	$a^{(n+1)}$
$a^{-(n-1)}$	$a^{-(n+1)}$

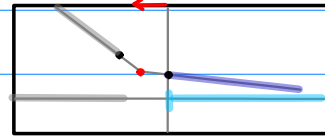
Exponent Shifting2 **Range Flipping**

$$a^n R(n) \longleftrightarrow a^{-sh2(n)} R(-n)$$

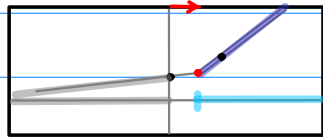




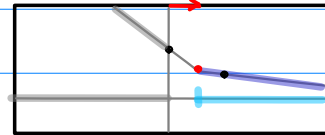
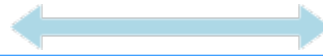
$$a^{n+1}$$



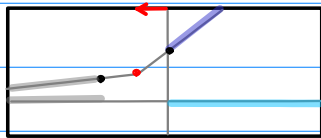
$$a^{-n-1}$$



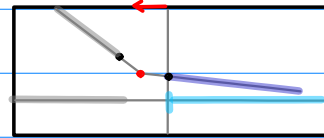
$$a^{n-1}$$



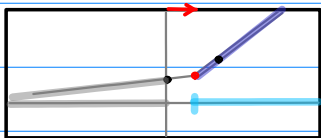
$$a^{-n+1}$$



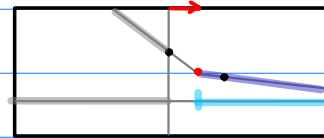
$$a^{n+1} u(n)$$



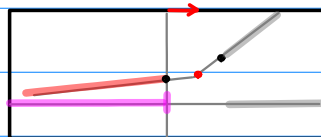
$$a^{-n-1} u(n)$$



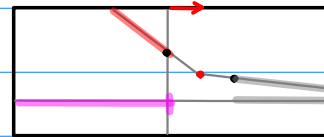
$$a^{n-1} u(n-1)$$



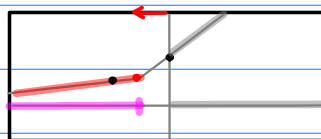
$$a^{-n+1} u(n-1)$$



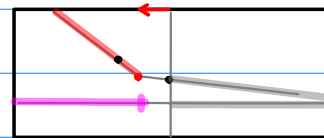
$$a^{n-1} u(-n)$$



$$a^{-n+1} u(-n)$$



$$a^{n+1} u(-n-1)$$



$$a^{-n-1} u(-n-1)$$

Shifted Range Flipping

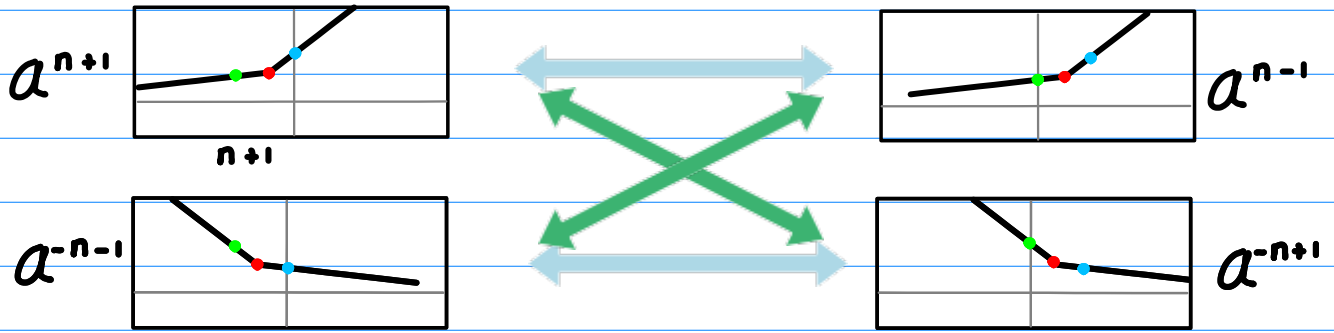
$$a^n R(n) \longleftrightarrow a^{sh2(n)} R(-n)$$

Exponent Shifting2

$$a^n \longleftrightarrow a^{sh2(n)}$$

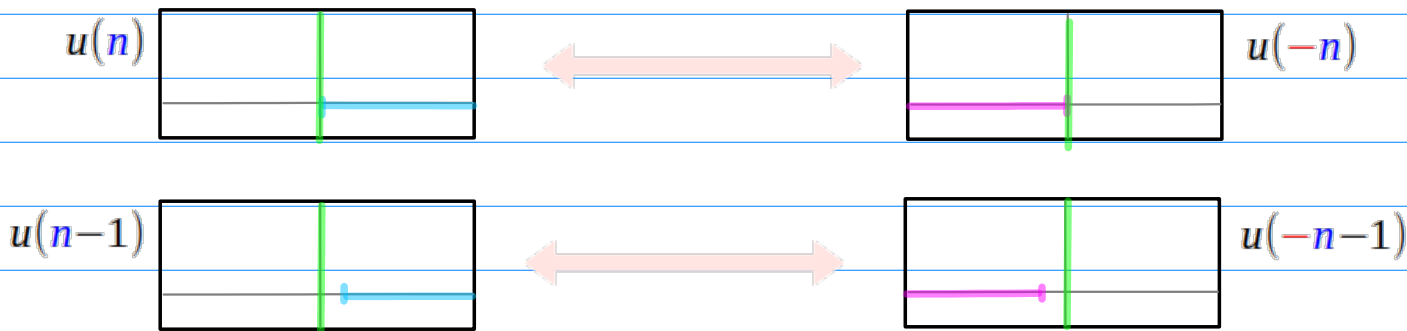
+ Base Inverting

$$a^n \longleftrightarrow a^{-sh2(n)}$$



Range Flipping

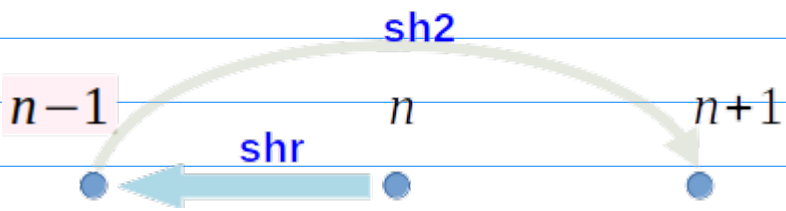
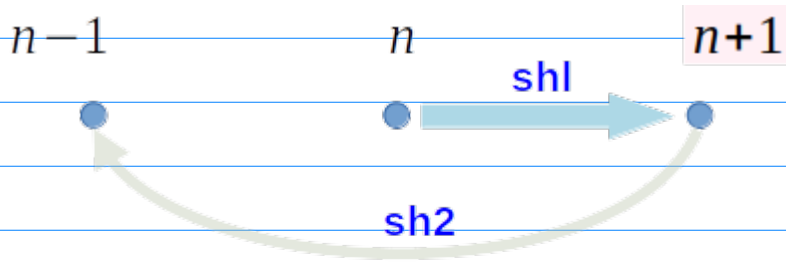
$$R(n) \longleftrightarrow R(-n)$$



b^n	$b^{-sh2(n)}$
$a^{(n+1)}$	$a^{-(n-1)}$
$a^{-(n+1)}$	$a^{(n-1)}$
$a^{(n-1)}$	$a^{-(n+1)}$
$a^{-(n-1)}$	$a^{(n+1)}$

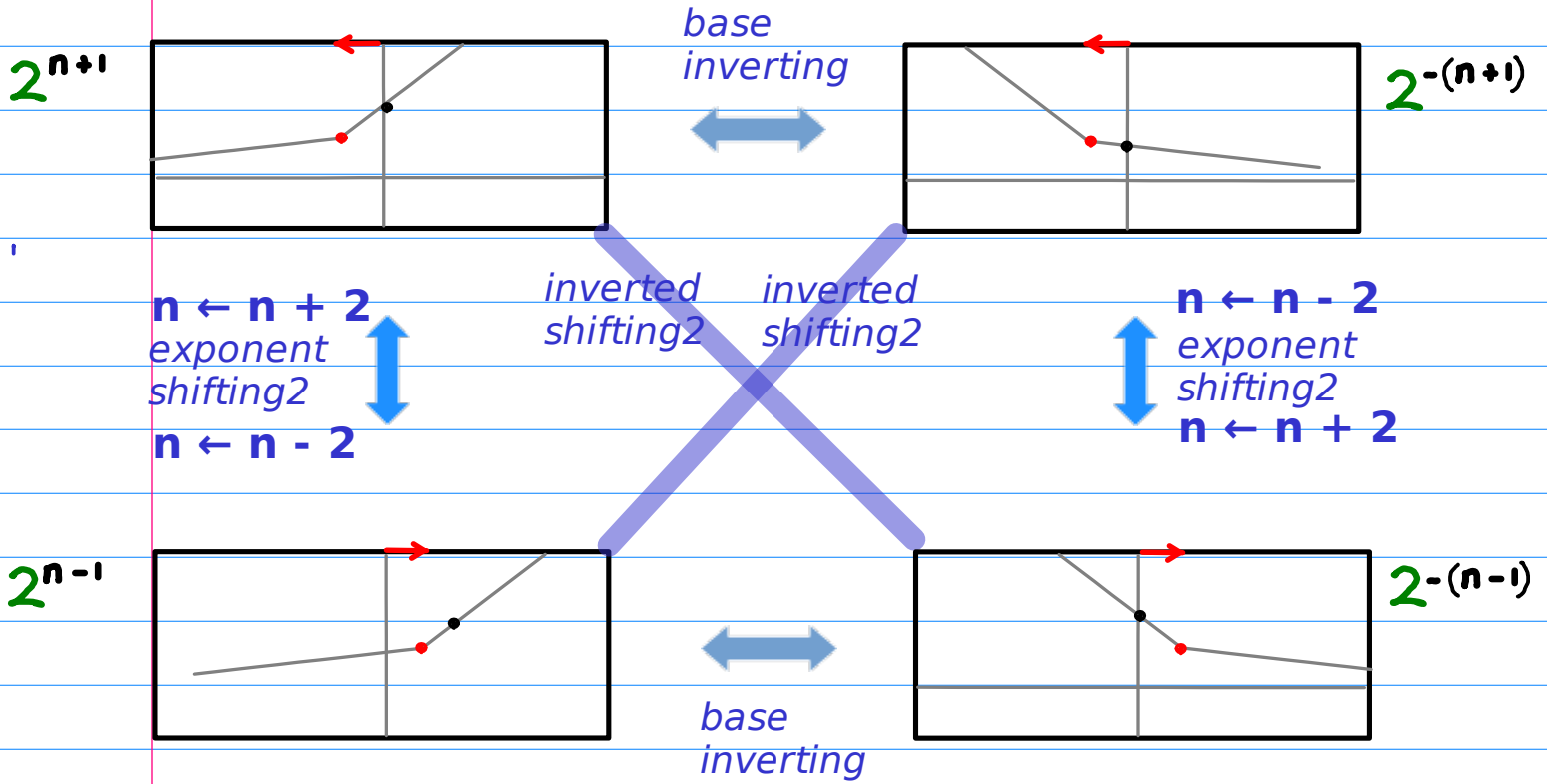
$R(n)$	$R(-n)$
$u(n)$	$u(-n)$
$u(n-1)$	$u(-n-1)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

Exponent Shifting2 $sh2(n)$



n		$sh2(n)$
$n+1$	$n \leftarrow n-2$	$n-1$
$n-1$	$n \leftarrow n+2$	$n+1$
$-(n+1)$	$n \leftarrow n-2$	$-(n-1)$
$-(n-1)$	$n \leftarrow n+2$	$-(n+1)$

Inverted Shifting2 -sh2(n)



Inverted Shifting2

b^n	$b^{-sh2(n)}$
$a^{(n+1)}$	$a^{-(n-1)}$
$a^{-(n+1)}$	$a^{(n-1)}$
$a^{(n-1)}$	$a^{-(n+1)}$
$a^{-(n-1)}$	$a^{(n+1)}$

$$a^n \longleftrightarrow a^{-n}$$

$$a^n R(n) \longleftrightarrow a^{sh2(n)} R(-n)$$

Exponent Shifting2 **Range Flipping**

$$a^n R(n) \longleftrightarrow a^{-sh2(n)} R(-n)$$

Inverted Shifting2

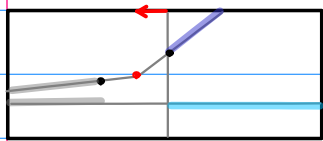
Shifted Range Flipping

Flipping2

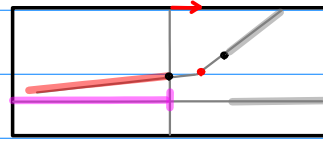
$$a^n R(n) \longleftrightarrow a^{sh2(n)} R(-n)$$

$$a^n R(n) \longleftrightarrow a^{-sh2(n)} R(-n)$$

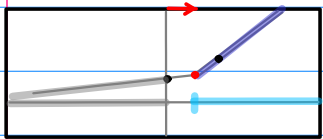
Flipping2 = Base Inverting + Shifted Range Flipping
 = Base Inverting + Exponent Shifting2 + Range Flipping



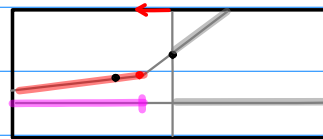
$$a^{n+1} u(n)$$



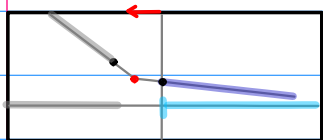
$$a^{n-1} u(-n)$$



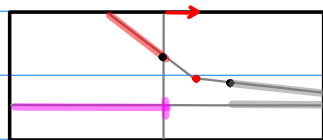
$$a^{n-1} u(n-1)$$



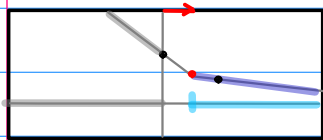
$$a^{n+1} u(-n-1)$$



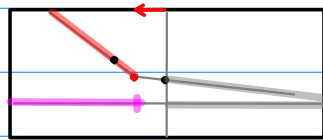
$$a^{-n-1} u(n)$$



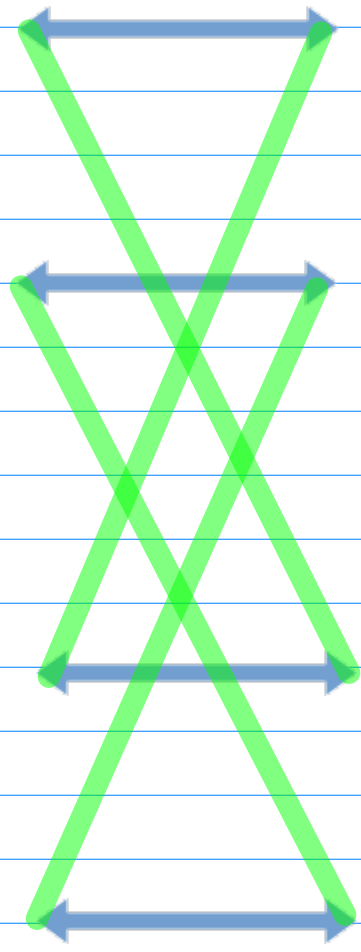
$$a^{-n+1} u(-n)$$



$$a^{-n+1} u(n-1)$$



$$a^{-n-1} u(-n-1)$$



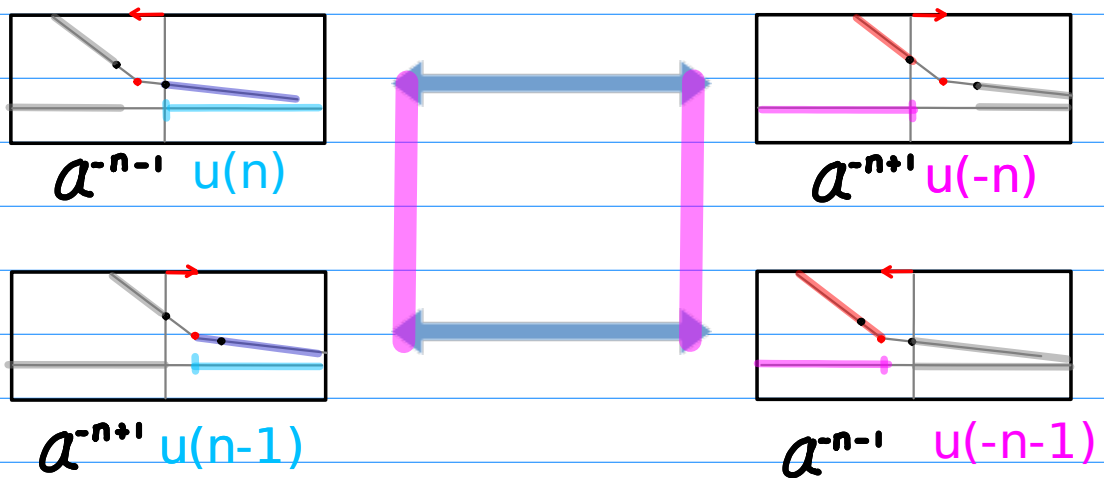
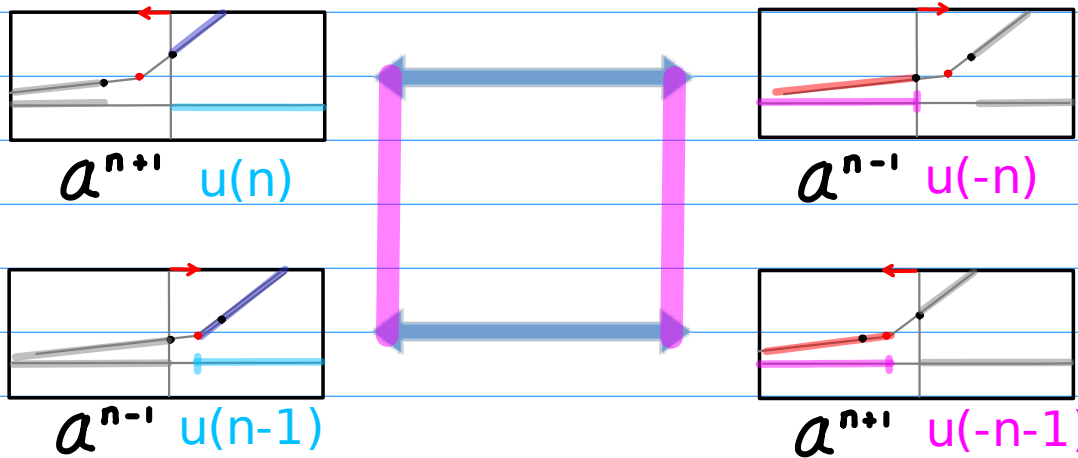
Shifted Range Flipping

$$a^n R(n) \longleftrightarrow a^{sh2(n)} R(-n)$$

Shifting2

$$a^n R(n) \longleftrightarrow a^{sh2(n)} \overline{R(-n)}$$

shifting2 = Shifted Range Flipping + Range Complementing
 = Exponent Shifting2 + Range (Flipping + Complementing)
 = Exponent Shifting2 + Range Shifting

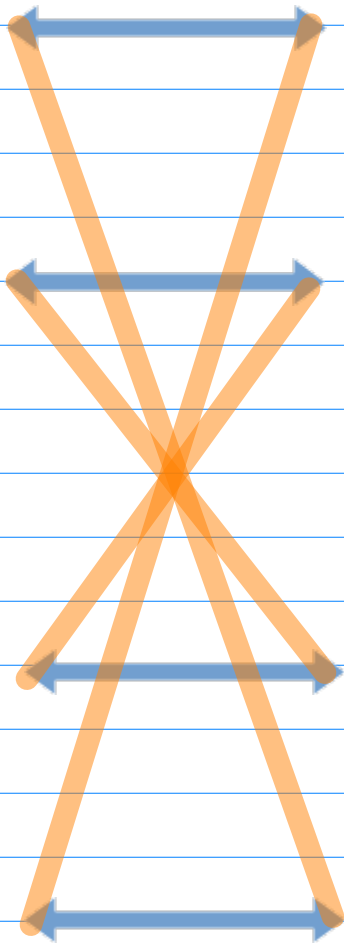
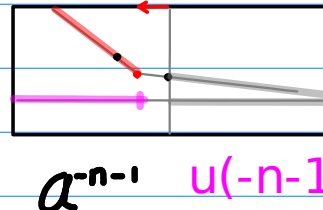
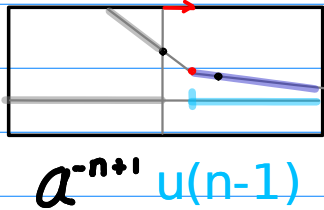
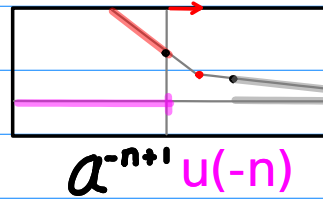
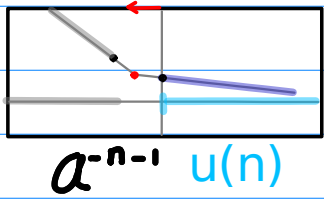
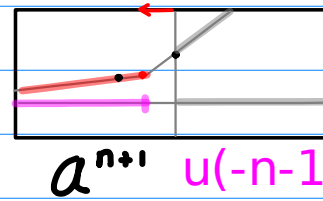
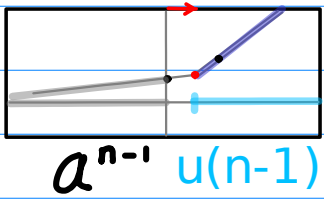
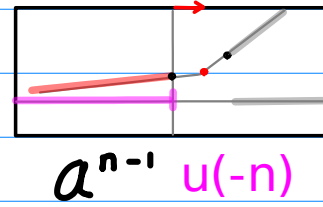
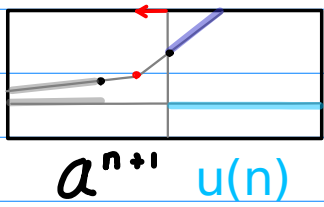


Shifted Range Flipping

Complementary Inverting

$$a^n R(n) \longleftrightarrow a^{sh2(n)} R(-n)$$

$$a^n R(n) \longleftrightarrow a^{-n} \overline{R(n)}$$



D. Flipping2

$$a^n R(n) \longleftrightarrow a^{-sh2(n)} R(-n)$$

$$a^n \longleftrightarrow a^{-n}$$

$$a^n R(n) \longleftrightarrow a^{sh2(n)} R(-n)$$

n		$sh2(n)$
$n+1$	$n \leftarrow n-2$	$n-1$
$n-1$	$n \leftarrow n+1$	$n+1$
$-(n+1)$	$n \leftarrow n+1$	$-(n-1)$
$-(n-1)$	$n \leftarrow n-1$	$-(n+1)$

$R(n)$	$R(-n)$
$u(n)$	$u(-n)$
$u(n-1)$	$u(-n-1)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

E. Shifting2

$$a^n R(n) \longleftrightarrow a^{\text{sh}2(n)} \overline{R(-n)}$$

$$a^n R(n) \longleftrightarrow a^{\text{sh}2(n)} R(-n)$$

$$R(n) \longleftrightarrow R(n)$$

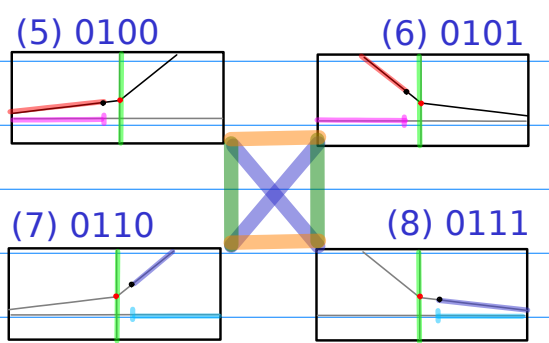
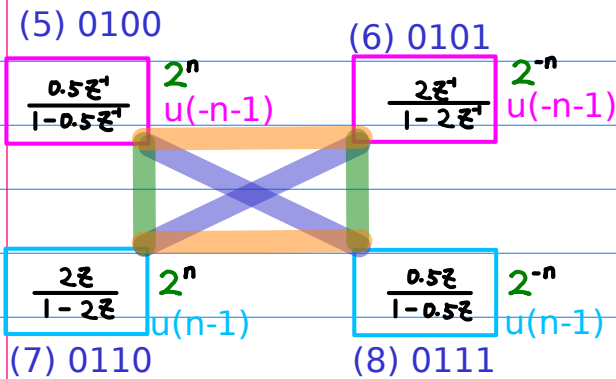
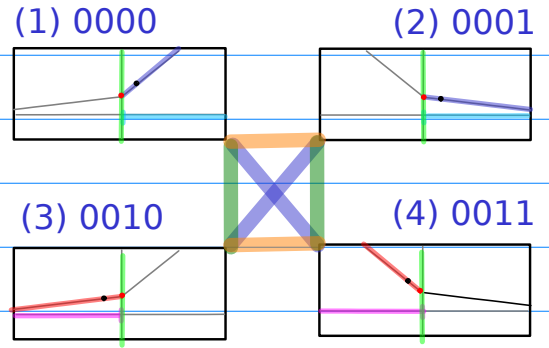
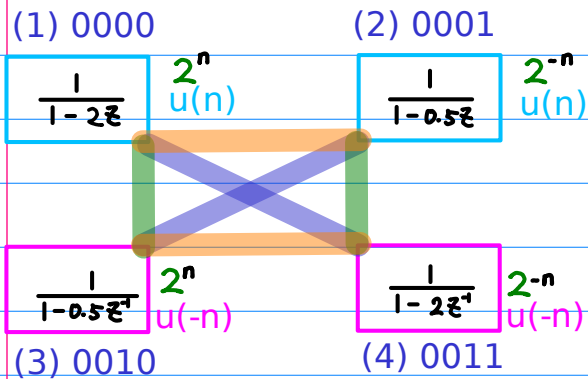
F. Complementary Inverting

$$a^n R(n) \longleftrightarrow a^{-n} \overline{R(n)}$$

$$\begin{array}{ccc} a^n & \longleftrightarrow & a^{-n} \\ R(n) & \longleftrightarrow & \overline{R(n)} \end{array}$$

<u>$R(n)$</u>	<u>$\overline{R(n)}$</u>
<u>$u(n)$</u>	<u>$u(-n-1)$</u>
$u(n-1)$	$u(-n)$
$u(-n)$	$u(n)$
<u>$u(-n-1)$</u>	<u>$u(n-1)$</u>

A.1 Flipping Base Inverting Range Flipping



Base Inverting

$$0 \ b_2 \ b_1 \ b_0 \longrightarrow 0 \ b_2 \ b_1 \ b_0$$

$$(1) \ 0 \ 0 \ 0 \ 0 \quad 0 \ 0 \ 0 \ 1 \quad (2)$$

Range Flipping

$$0 \ b_2 \ b_1 \ b_0 \longrightarrow 0 \ b_2 \ \bar{b}_1 \ b_0$$

$$(3) \ 0 \ 0 \ 1 \ 0 \quad 0 \ 0 \ 1 \ 1 \quad (4)$$

Flipping

$$0 \ b_2 \ b_1 \ b_0 \longrightarrow 0 \ b_2 \ \bar{b}_1 \ \bar{b}_0$$

$$(5) \ 0 \ 1 \ 0 \ 0 \quad 0 \ 1 \ 0 \ 1 \quad (6)$$

$$(7) \ 0 \ 1 \ 1 \ 0 \quad 0 \ 1 \ 1 \ 1 \quad (8)$$

$$a^n \longleftrightarrow a^{-n}$$

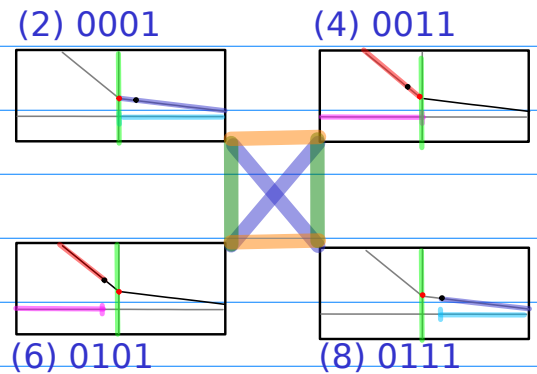
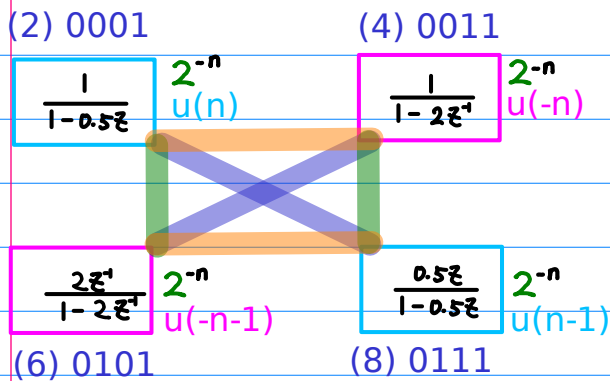
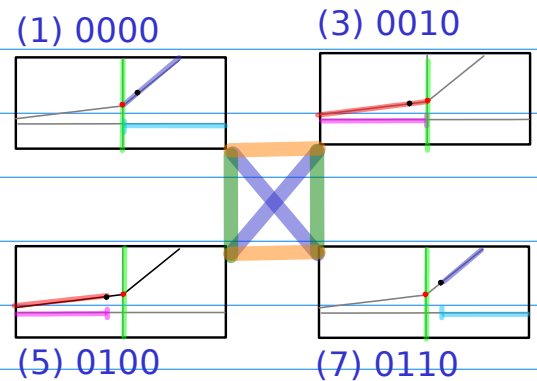
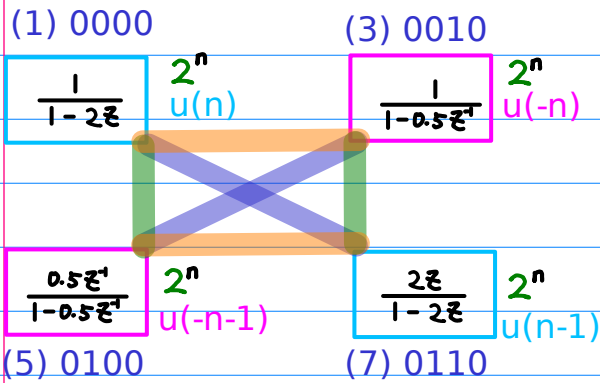
$$R(n) \longleftrightarrow R(-n)$$

$$a^n R(n) \longleftrightarrow a^{-n} R(-n)$$

B.1 Range Shifting = Range Flipping + Range Complementing

Range Flipping

Range Complementing



Range Flipping

$$0 \quad b_2 \quad b_1 \quad b_0 \quad \longrightarrow \quad 0 \quad b_2 \quad \bar{b}_1 \quad b_0$$

Range Complementing

$$0 \quad b_2 \quad b_1 \quad b_0 \quad \longrightarrow \quad 0 \quad \bar{b}_2 \quad b_1 \quad b_0$$

Range Shifting

$$0 \quad b_2 \quad b_1 \quad b_0 \quad \longrightarrow \quad 0 \quad \bar{b}_2 \quad \bar{b}_1 \quad b_0$$

$$R(n) \longleftrightarrow R(-n)$$

$$R(n) \longleftrightarrow \overline{R(n)}$$

$$R(n) \longleftrightarrow \overline{R(-n)}$$

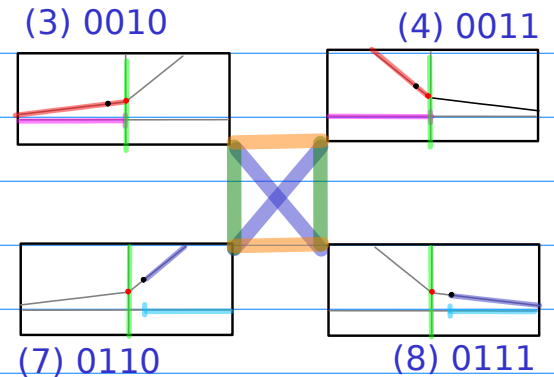
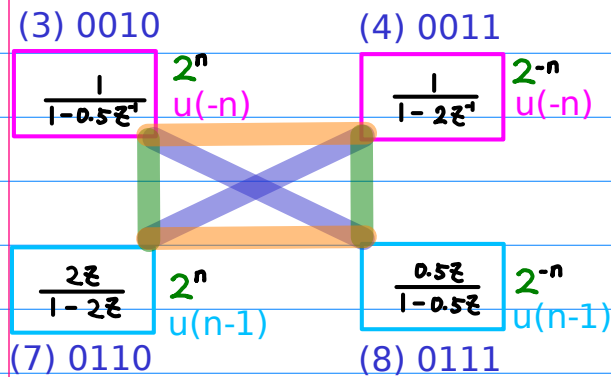
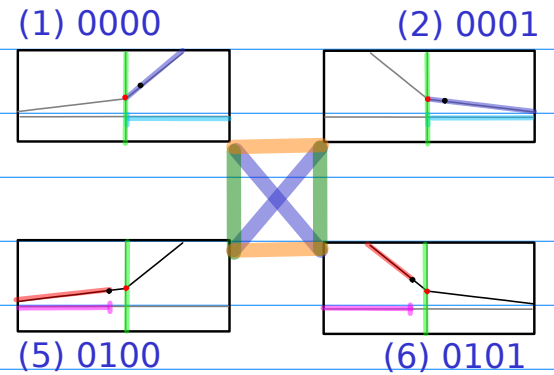
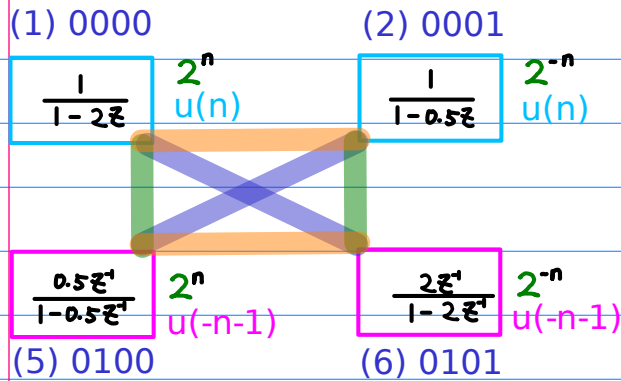
$$(1) \quad 0 \quad 0 \quad 0 \quad 0 \quad \quad \quad 0 \quad 0 \quad 1 \quad 0 \quad (3)$$

$$(5) \quad 0 \quad 1 \quad 0 \quad 0 \quad \quad \quad 0 \quad 1 \quad 1 \quad 0 \quad (7)$$

$$(2) \quad 0 \quad 0 \quad 0 \quad 1 \quad \quad \quad 0 \quad 0 \quad 1 \quad 1 \quad (4)$$

$$(6) \quad 0 \quad 1 \quad 0 \quad 1 \quad \quad \quad 0 \quad 1 \quad 1 \quad 1 \quad (8)$$

C.1 Complementary Inverting Base Inverting Range Complementing



Base Inverting

$$0 \ b_2 \ b_1 \ b_0 \longrightarrow 0 \ b_2 \ b_1 \ \bar{b}_0$$

$$(1) 0 \ 0 \ 0 \ 0 \qquad 0 \ 0 \ 0 \ 1 \ (2)$$

Range Complementing

$$0 \ b_2 \ b_1 \ b_0 \longrightarrow 0 \ \bar{b}_2 \ b_1 \ b_0$$

$$(5) 0 \ 1 \ 0 \ 0 \qquad 0 \ 1 \ 0 \ 1 \ (6)$$

Complementary Flipping

$$0 \ b_2 \ b_1 \ b_0 \longrightarrow 0 \ \bar{b}_2 \ b_1 \ \bar{b}_0$$

$$(3) 0 \ 0 \ 1 \ 0 \qquad 0 \ 0 \ 1 \ 1 \ (4)$$

$$(7) 0 \ 1 \ 1 \ 0 \qquad 0 \ 1 \ 1 \ 1 \ (8)$$

$$a^n \longleftrightarrow a^{-n}$$

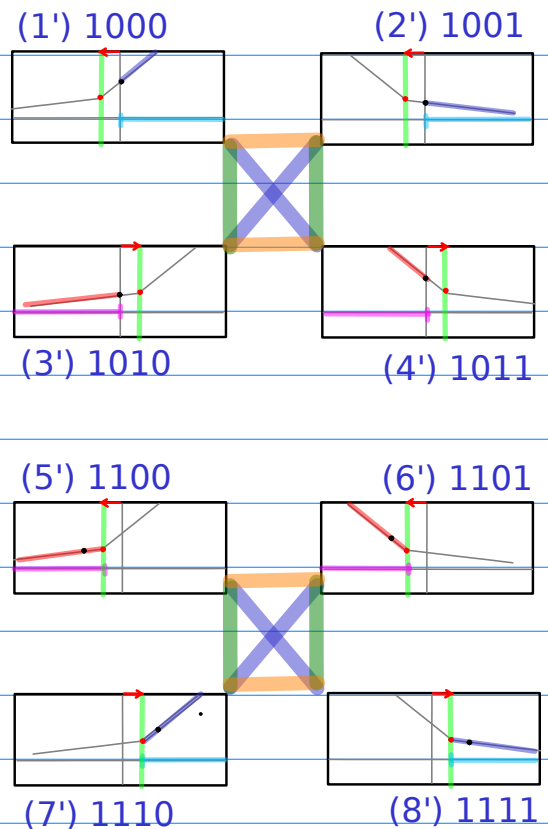
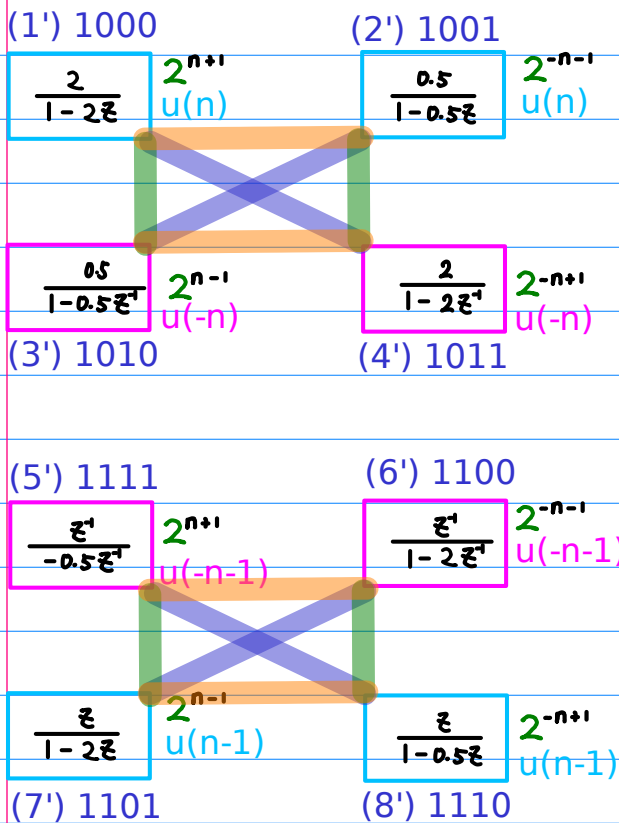
$$R(n) \longleftrightarrow \overline{R(n)}$$

$$a^n R(n) \longleftrightarrow a^{-n} \overline{R(n)}$$

D.1 Flipping2

Base Inverting

Shifted Range Flipping = Exponent Shifting2 + Range Flipping



Base Inverting



(1) 1 0 0 0 1 0 0 1 (2)

Shifted Range Flipping



(3) 1 0 1 0 1 0 1 1 (4)

Flipping2



(5) 1 1 0 0 1 1 0 1 (6)

(7) 1 1 1 0 1 1 1 1 (8)

$$a^n \longleftrightarrow a^{-n}$$

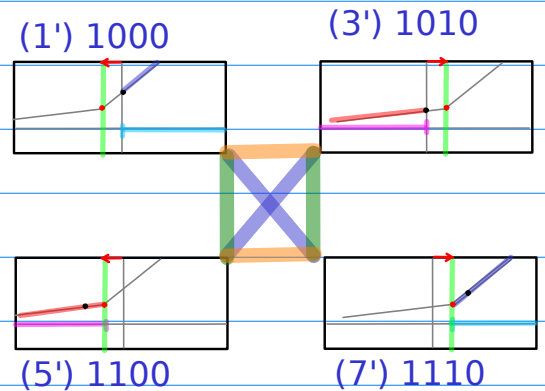
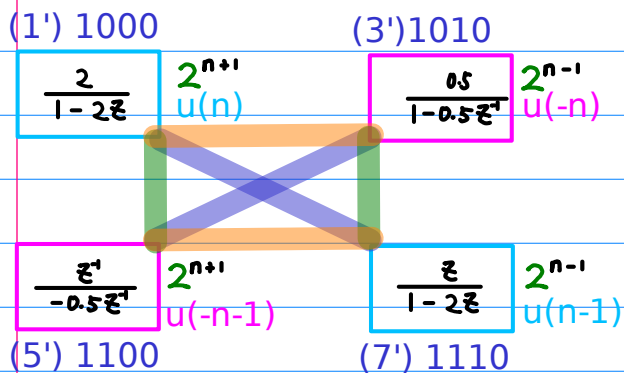
$$a^n R(n) \longleftrightarrow a^{sh2(n)} R(-n)$$

$$a^n R(n) \longleftrightarrow a^{-sh2(n)} R(-n)$$

E.1 Shifting2 = Exponent Shifting2 + Range Complementing

Shifted Range Flipping = Exponent Shifting2 + Range Flipping

Range Complementing



Shifted Range Flipping



Range Complementing



Shifting2



$$a^n R(n) \longleftrightarrow a^{sh2(n)} R(-n)$$

$$R(n) \longleftrightarrow R(n)$$

$$a^n R(n) \longleftrightarrow a^{sh2(n)} \overline{R(-n)}$$

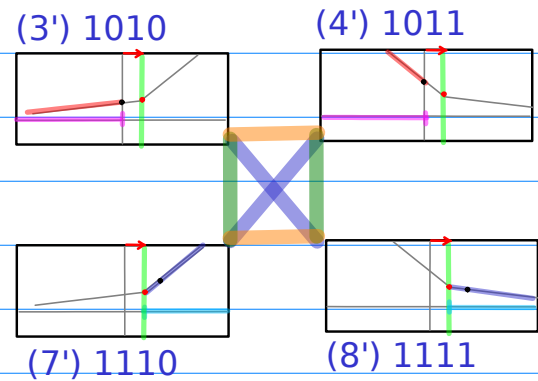
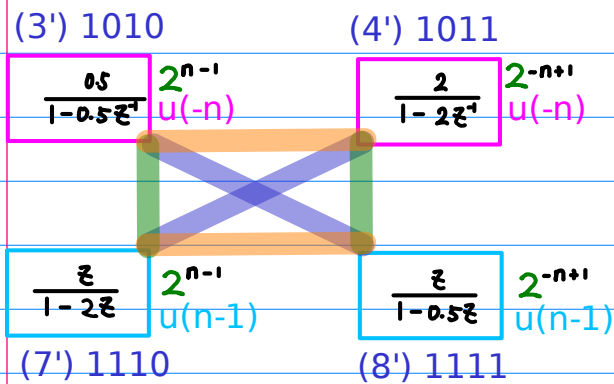
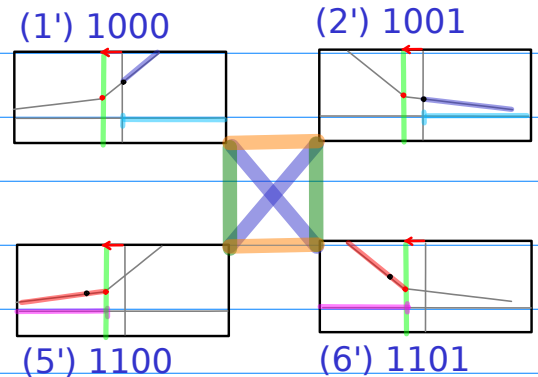
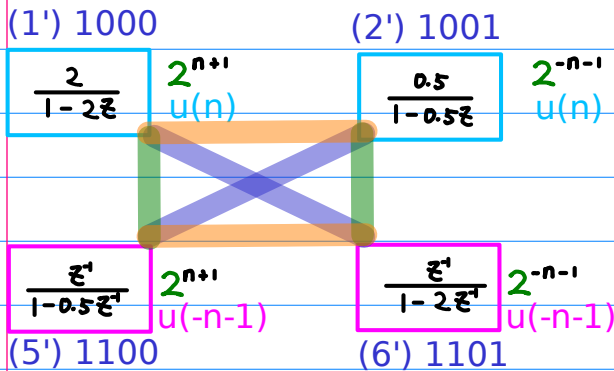
(1) 1 0 0 0 1 0 1 0 (3)

(5) 1 1 0 0 1 1 1 0 (7)

(2) 1 0 0 1 1 0 1 1 (4)

(6) 1 1 0 1 1 1 1 1 (8)

F.1 Complementary Inverting Base Inverting Range Complementing



Base Inverting



$$(1) 1 \bar{0} 0 0 \quad 1 \bar{0} 0 \bar{1} (2)$$

Range Complementing



$$(5) 1 \bar{1} 0 0 \quad 1 \bar{1} 0 \bar{1} (6)$$

Complementary Inverting



$$(3) 1 \bar{0} 1 0 \quad 1 \bar{0} 1 \bar{1} (4)$$

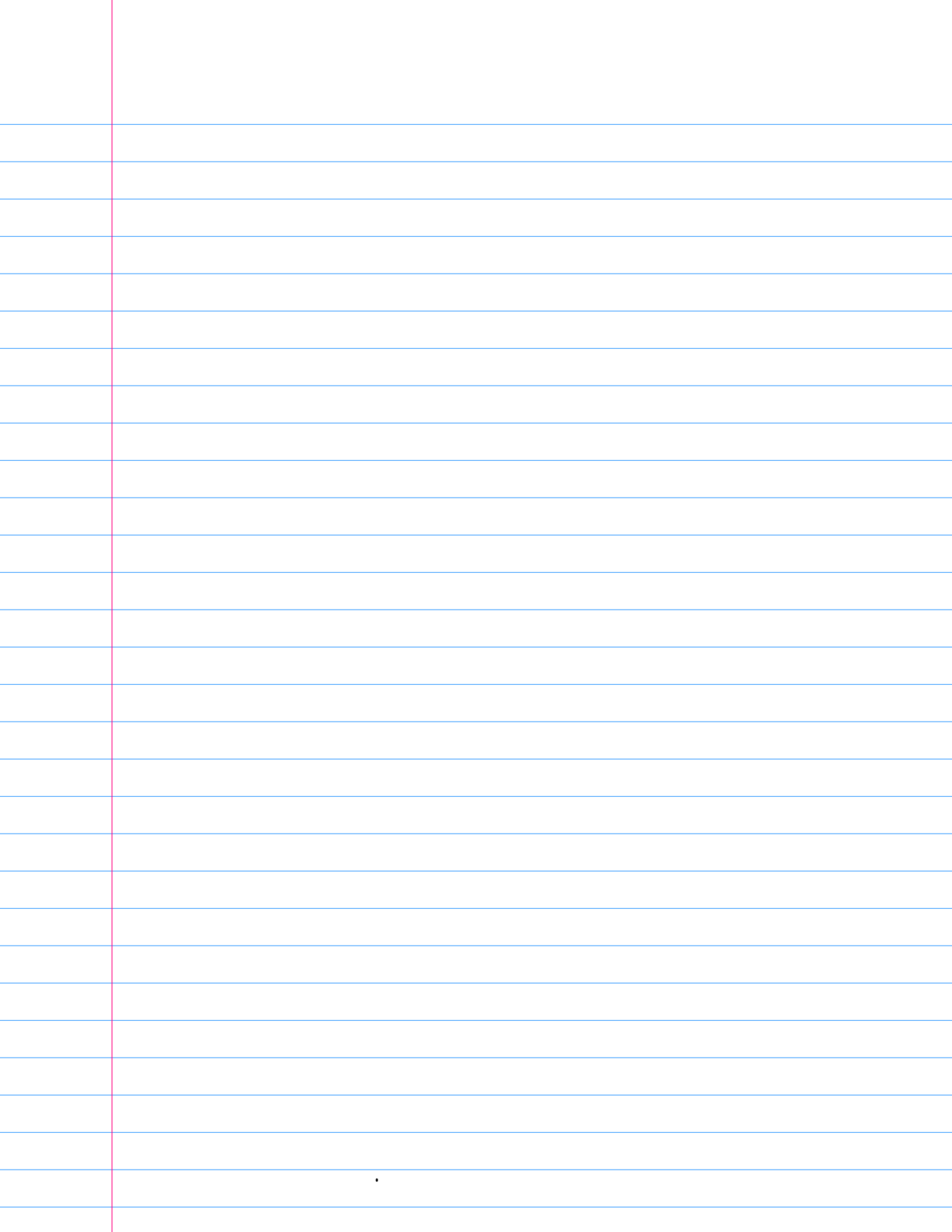
$$(7) 1 \bar{1} 1 0 \quad 1 \bar{1} 1 \bar{1} (8)$$

$$a^n \longleftrightarrow a^{-n}$$

$$R(n) \longleftrightarrow \overline{R(n)}$$

$$a^n R(n) \longleftrightarrow a^{-n} \overline{R(n)}$$





A.2b Flipping Base Inverting Range Flipping

$$a^n R(n) \longleftrightarrow a^{-n} R(-n)$$

1) Base Inverting

$$\boxed{0} \boxed{b_2} \boxed{b_1} \boxed{b_0} \longrightarrow \boxed{0} \boxed{b_2} \boxed{b_1} \boxed{\bar{b}_0}$$

2) Range Flipping

$$\boxed{0} \boxed{b_2} \boxed{b_1} \boxed{b_0} \longrightarrow \boxed{0} \boxed{b_2} \boxed{\bar{b}_1} \boxed{b_0}$$

3) Flipping

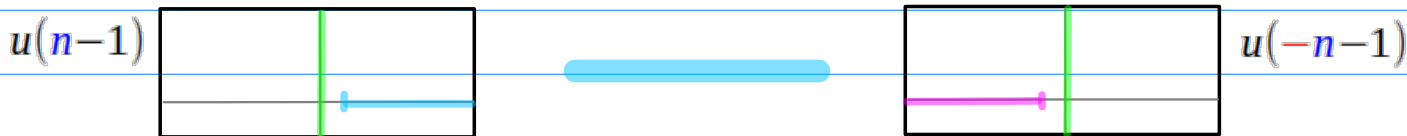
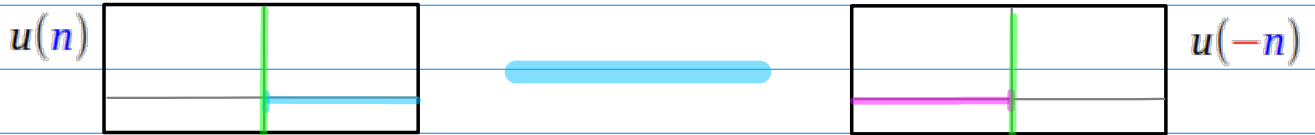
$$\boxed{0} \boxed{b_2} \boxed{b_1} \boxed{b_0} \longrightarrow \boxed{0} \boxed{b_2} \boxed{\bar{b}_1} \boxed{\bar{b}_0}$$

B.2a Range Shifting
Range Flipping
Range Complementing

$$R(n) \longleftrightarrow \overline{R(-n)}$$

1) Range Flipping

$$R(n) \longleftrightarrow R(-n)$$



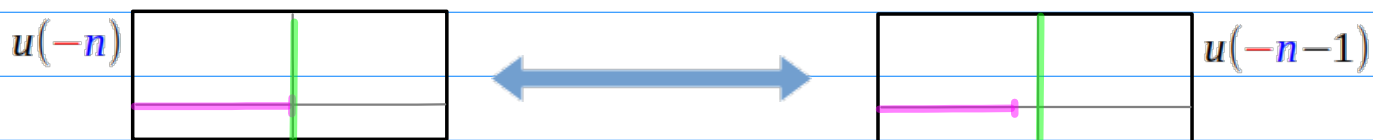
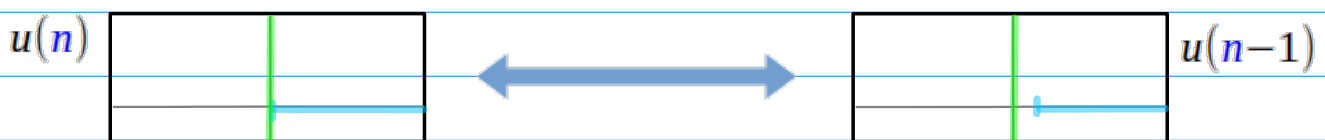
2) Range Complementing

$$R(n) \longleftrightarrow \overline{R(n)}$$



3) Range Shifting

$$R(n) \longleftrightarrow \overline{R(-n)}$$



B.2b Range Shifting Range Flipping Range Complementing

$$R(n) \longleftrightarrow \overline{R(-n)}$$

1) Range Flipping

$$\boxed{0} \boxed{b_2} \boxed{b_1} \boxed{b_0} \longrightarrow \boxed{0} \boxed{b_2} \boxed{\overline{b_1}} \boxed{b_0}$$

2) Range Complementing

$$\boxed{0} \boxed{b_2} \boxed{b_1} \boxed{b_0} \longrightarrow \boxed{0} \boxed{\overline{b_2}} \boxed{b_1} \boxed{b_0}$$

3) Range Shifting

$$\boxed{0} \boxed{b_2} \boxed{b_1} \boxed{b_0} \longrightarrow \boxed{0} \boxed{\overline{b_2}} \boxed{\overline{b_1}} \boxed{b_0}$$

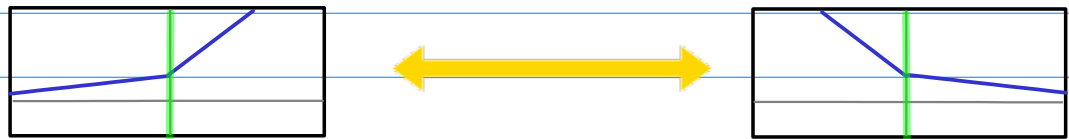


C.2a Complementary Inverting Base Inverting Range Complementing

$$a^n R(n) \longleftrightarrow a^{-n} \overline{R(n)}$$

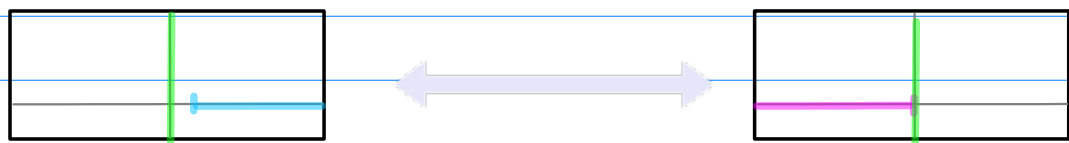
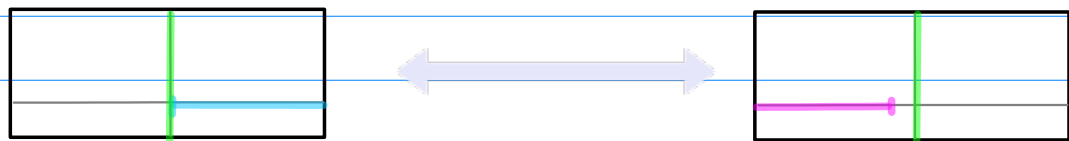
1) Base Inverting

$$0 \ b_2 \ b_1 \ b_0 \longrightarrow 0 \ b_2 \ b_1 \ \overline{b_0}$$



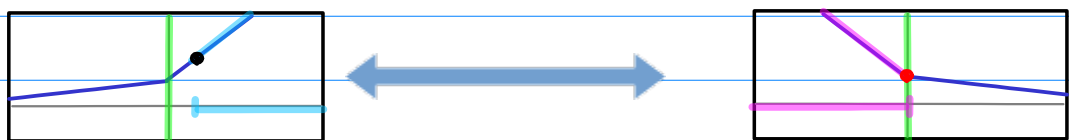
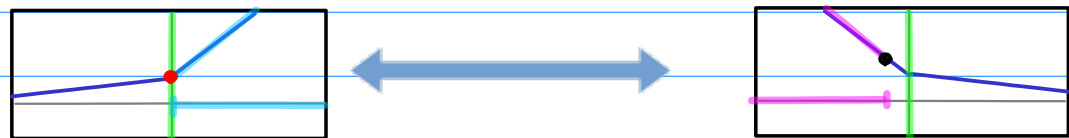
2) Range Complementing

$$0 \ b_2 \ b_1 \ b_0 \longrightarrow 0 \ \overline{b_2} \ b_1 \ b_0$$



3) Complementary Flipping

$$0 \ b_2 \ b_1 \ b_0 \longrightarrow 0 \ \overline{b_2} \ b_1 \ \overline{b_0}$$



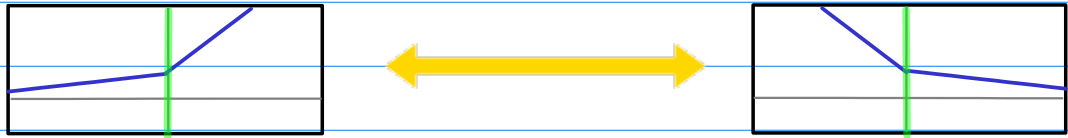
D.2a Flipping2
Base Inverting
Shifted Range Flipping

$$a^n \longleftrightarrow a^{-n}$$

$$a^n R(n) \longleftrightarrow a^{sh(n)} R(-n)$$

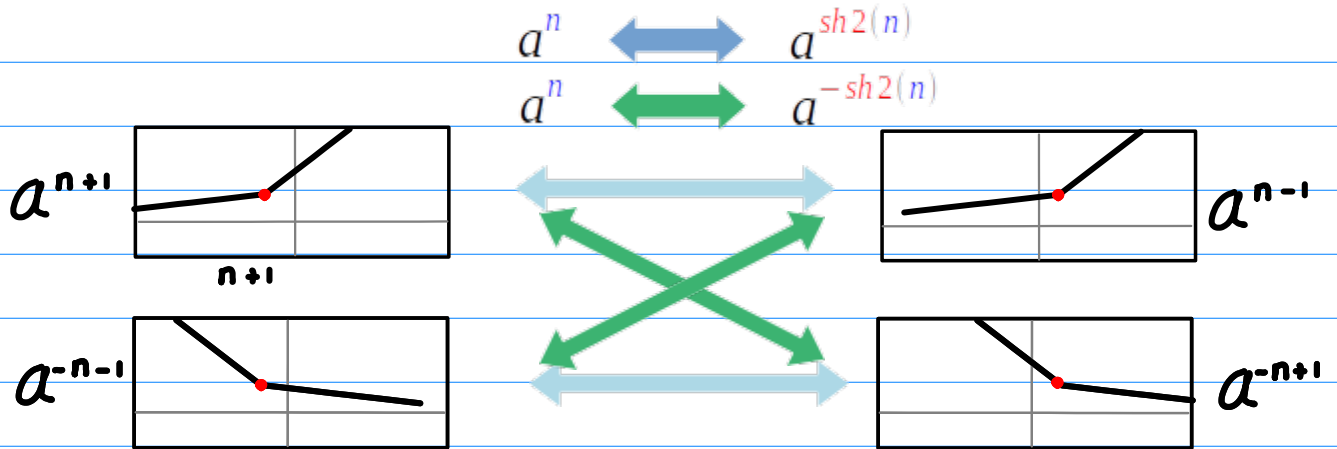
1) Base Inverting

$$a^n \longleftrightarrow a^{-n}$$

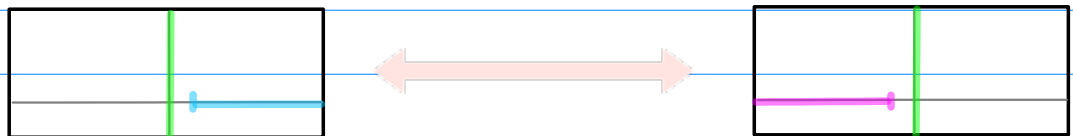
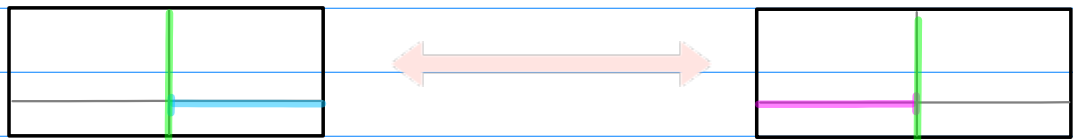


2) Shifted Range Flipping

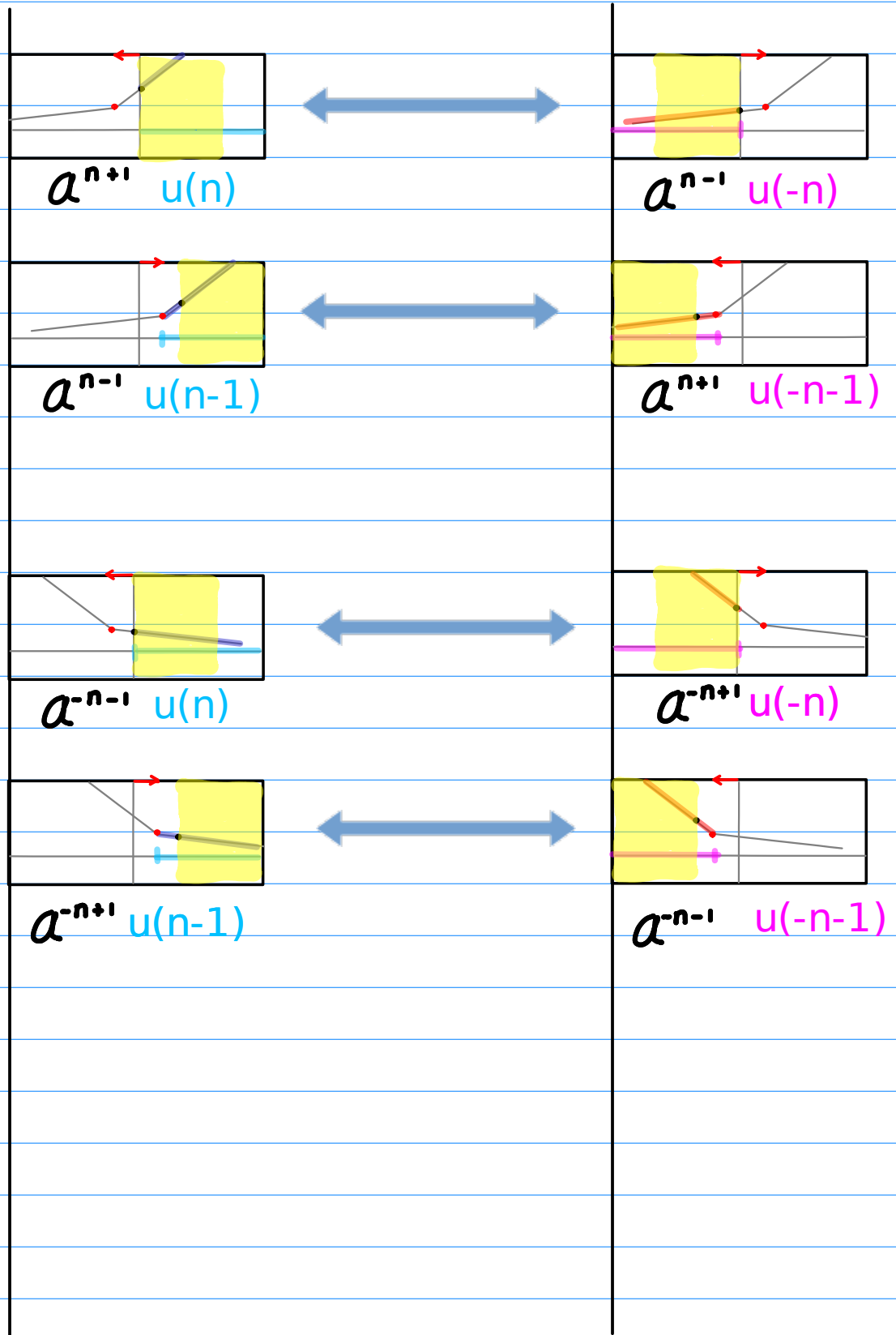
$$a^n R(n) \longleftrightarrow a^{-sh2(n)} R(-n)$$



$$R(n) \longleftrightarrow R(-n)$$

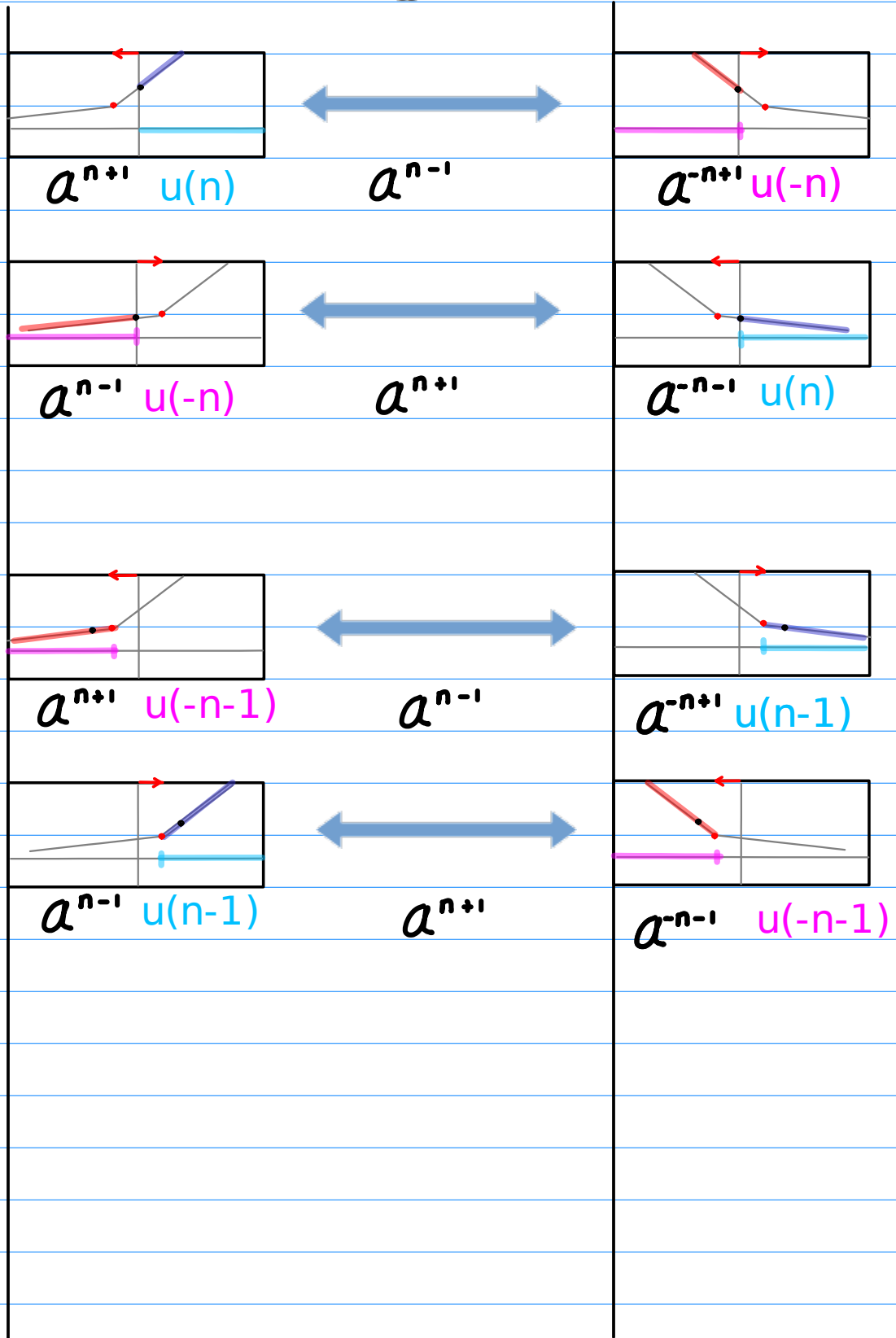


2) Shifted Range Flipping $a^n R(n) \leftrightarrow a^{sh2(n)} R(-n)$



3) Flipping2

$$a^n R(n) \xleftrightarrow{a^{sh(n)}} a^{-sh2(n)} R(-n)$$



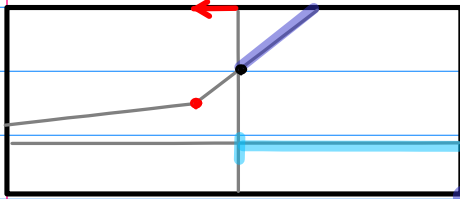


D. Flipping2

Base Inverting

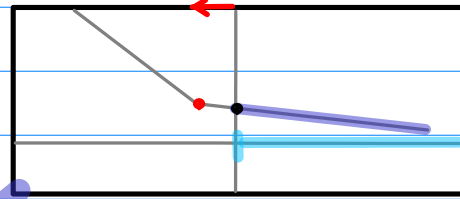
Shifted Range Flipping = Exponent Shifting2 + Range Flipping

(1') 1000



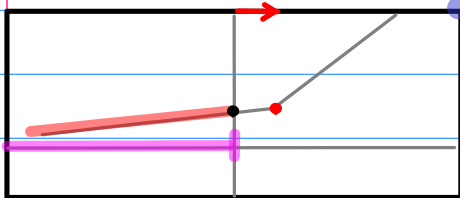
2^{n+1}
 $u(n)$

(2') 1001



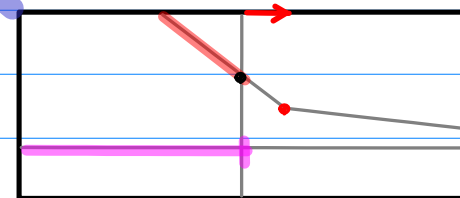
2^{-n-1}
 $u(n)$

(3') 1010



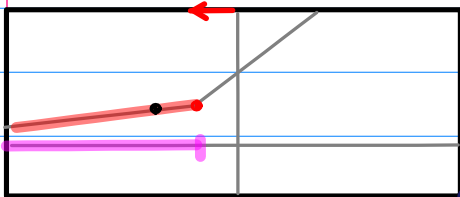
2^{n-1}
 $u(-n)$

(4') 1011



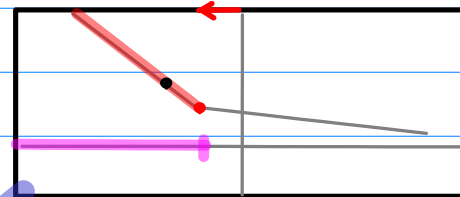
2^{-n+1}
 $u(-n)$

(5') 1111



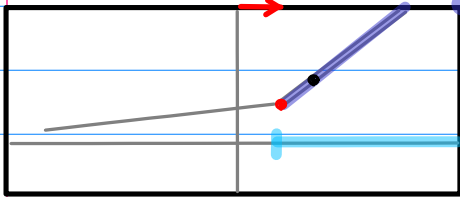
2^{n+1}
 $u(-n-1)$

(6') 1100



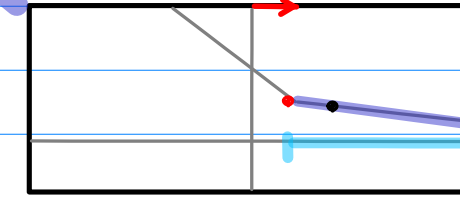
2^{-n-1}
 $u(-n-1)$

(7') 1101



2^{n-1}
 $u(n-1)$

(8') 1110



2^{-n+1}
 $u(n-1)$

D. Flipping2

Base Inverting

Shifted Range Flipping = Exponent Shifting2 + Range Flipping

1) Exponent Flipping = (Base Inverting, Exponent Shifting)

$$2^{n+1} \longleftrightarrow 2^{-(n-1)}$$

$$2^{n-1} \longleftrightarrow 2^{-(n+1)}$$

2) Range Flipping

$$u(n) \longleftrightarrow u(-n)$$

$$u(n-1) \longleftrightarrow u(-n-1)$$

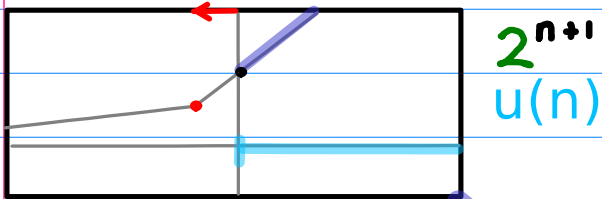
3) Exponent Flipping + Range Flipping

$$\begin{array}{|c|} \hline 2^{n+1} \\ \hline u(n) \\ \hline \end{array} \longleftrightarrow \begin{array}{|c|} \hline 2^{-(n-1)} \\ \hline u(-n) \\ \hline \end{array} \quad \begin{array}{|c|} \hline 2^{n+1} \\ \hline u(-n-1) \\ \hline \end{array} \longleftrightarrow \begin{array}{|c|} \hline 2^{-(n-1)} \\ \hline u(n-1) \\ \hline \end{array}$$

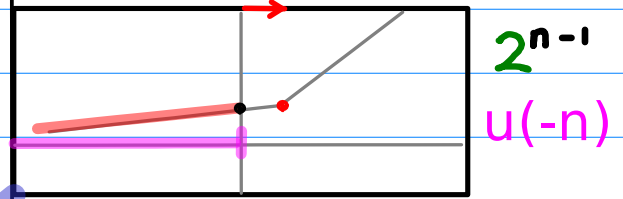
$$\begin{array}{|c|} \hline 2^{n-1} \\ \hline u(-n) \\ \hline \end{array} \longleftrightarrow \begin{array}{|c|} \hline 2^{-(n+1)} \\ \hline u(n) \\ \hline \end{array} \quad \begin{array}{|c|} \hline 2^{n-1} \\ \hline u(n-1) \\ \hline \end{array} \longleftrightarrow \begin{array}{|c|} \hline 2^{-(n+1)} \\ \hline u(-n-1) \\ \hline \end{array}$$

E. Shifting2 = Exponent Shifting2 + Range Shifting
Shifted Range Flipping = Exponent Shifting2 + Range Flipping
Range Complementing

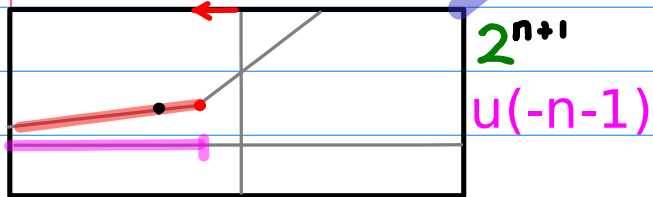
(1') 1000



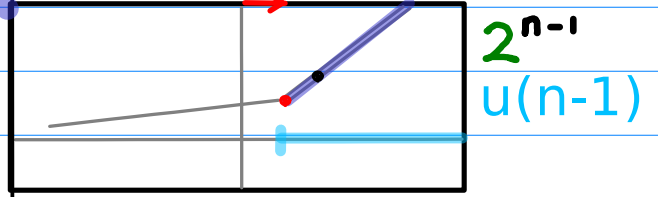
(3') 1010



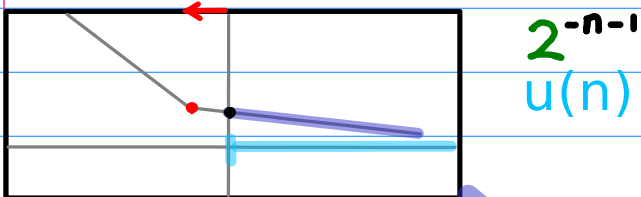
(5') 1111



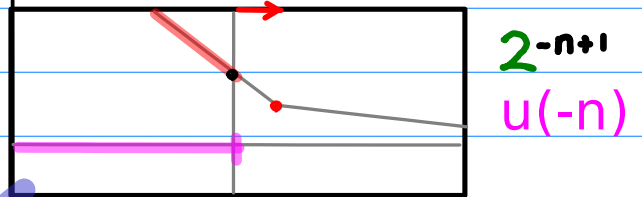
(7') 1101



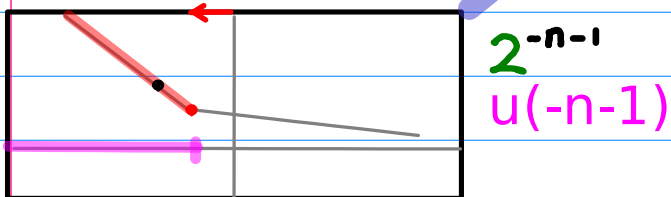
(2') 1001



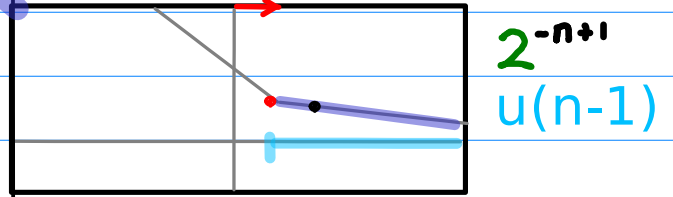
(4') 1011



(6') 1100



(8') 1110



E. Shifting2 = Exponent Shifting2 + Range Shifting
Shifted Range Flipping = Exponent Shifting2 + Range Flipping
Range Complementing

1) Exponent Shifting = (Exponent Shifting, ID)

$$2^{n+1} \longleftrightarrow 2^{n-1}$$

$$2^{-(n+1)} \longleftrightarrow 2^{-(n-1)}$$

2) Range Shifting

$$u(n) \longleftrightarrow u(n-1)$$

$$u(-n-1) \longleftrightarrow u(-n)$$

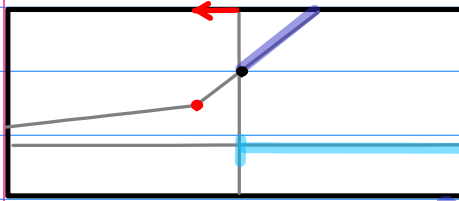
3) Shifting

$$\begin{array}{|c|} \hline 2^{n+1} \\ \hline u(n) \\ \hline \end{array} \longleftrightarrow \begin{array}{|c|} \hline 2^{n-1} \\ \hline u(n-1) \\ \hline \end{array} \quad \begin{array}{|c|} \hline 2^{-(n+1)} \\ \hline u(n) \\ \hline \end{array} \longleftrightarrow \begin{array}{|c|} \hline 2^{-(n-1)} \\ \hline u(n-1) \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline 2^{n-1} \\ \hline u(-n) \\ \hline \end{array} \longleftrightarrow \begin{array}{|c|} \hline 2^{n+1} \\ \hline u(-n-1) \\ \hline \end{array} \quad \begin{array}{|c|} \hline 2^{-(n-1)} \\ \hline u(-n) \\ \hline \end{array} \longleftrightarrow \begin{array}{|c|} \hline 2^{-(n+1)} \\ \hline u(-n-1) \\ \hline \end{array}$$

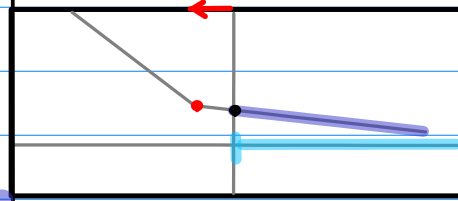
F. Complementary Inverting Base Inverting Range Complementing

(1') 1000



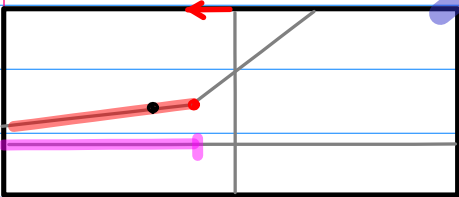
2^{n+1}
 $u(n)$

(2') 1001



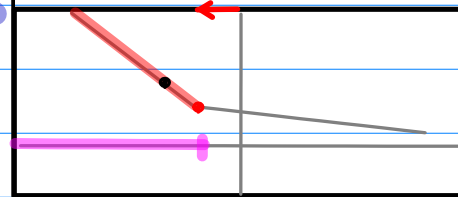
2^{-n-1}
 $u(n)$

(5') 1111



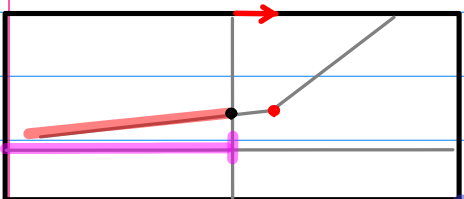
2^{n+1}
 $u(-n-1)$

(6') 1100



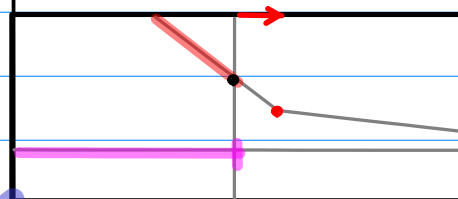
2^{-n-1}
 $u(-n-1)$

(3') 1010



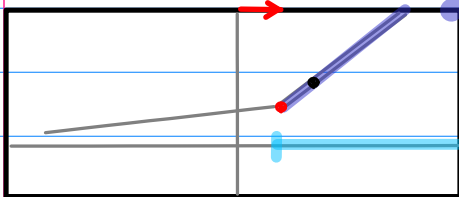
2^{n-1}
 $u(-n)$

(4') 1011



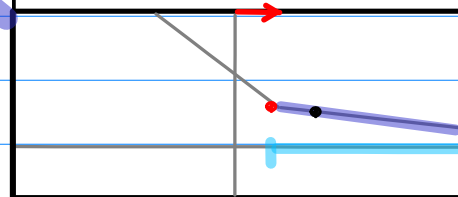
2^{-n+1}
 $u(-n)$

(7') 1101



2^{n-1}
 $u(n-1)$

(8') 1110



2^{-n+1}
 $u(n-1)$

F. Complementary Inverting Base Inverting Range Complementing

1) Base Inverting = (Base Inverting, ID)

$$2^{n+1} \longleftrightarrow 2^{-(n+1)}$$

$$2^{n-1} \longleftrightarrow 2^{-(n-1)}$$

2) Range Complementing

$$u(n) \longleftrightarrow u(-n-1)$$

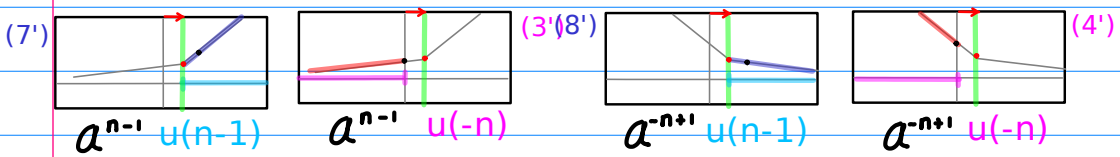
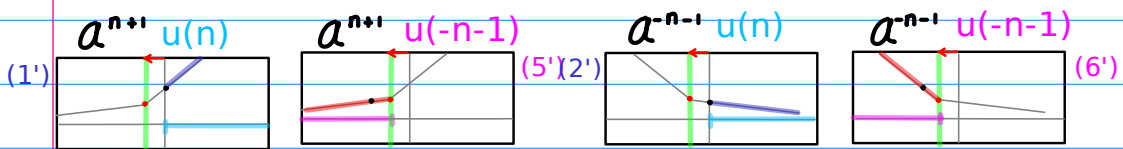
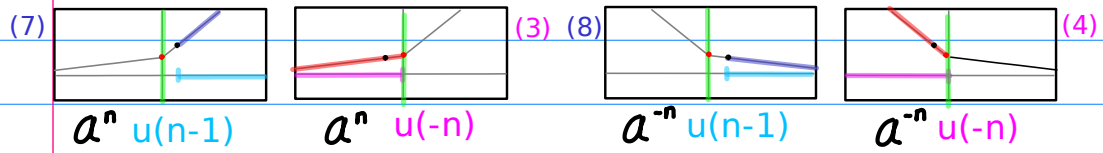
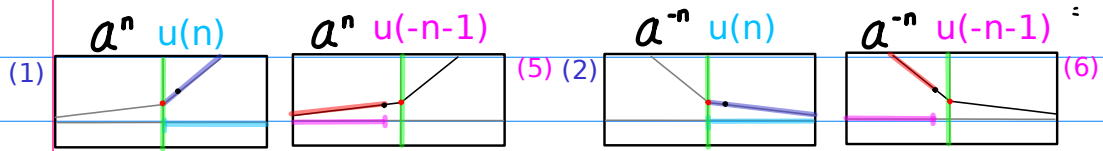
$$u(n-1) \longleftrightarrow u(-n)$$

3) Complementary Inverting

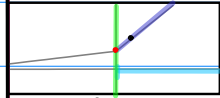
$$\begin{array}{|c|} \hline 2^{n+1} \\ \hline u(n) \\ \hline \end{array} \longleftrightarrow \begin{array}{|c|} \hline 2^{-(n+1)} \\ \hline u(-n-1) \\ \hline \end{array} \quad \begin{array}{|c|} \hline 2^{n-1} \\ \hline u(-n) \\ \hline \end{array} \longleftrightarrow \begin{array}{|c|} \hline 2^{-(n-1)} \\ \hline u(n-1) \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline 2^{-(n+1)} \\ \hline u(n) \\ \hline \end{array} \longleftrightarrow \begin{array}{|c|} \hline 2^{n+1} \\ \hline u(-n-1) \\ \hline \end{array} \quad \begin{array}{|c|} \hline 2^{-(n-1)} \\ \hline u(-n) \\ \hline \end{array} \longleftrightarrow \begin{array}{|c|} \hline 2^{n-1} \\ \hline u(n-1) \\ \hline \end{array}$$



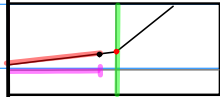


(1) 0000



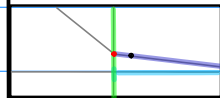
$$a^n u(n)$$

$$a^n u(-n-1)$$



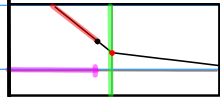
(5) 0100

(2) 0001



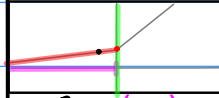
$$a^{-n} u(n)$$

$$a^{-n} u(-n-1)$$



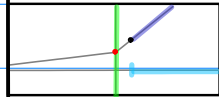
(6) 0101

(3) 0010



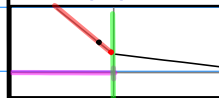
$$a^n u(-n)$$

$$a^n u(n-1)$$



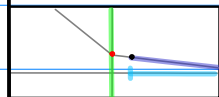
(7) 0110

(4) 0011



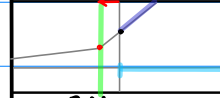
$$a^{-n} u(-n)$$

$$a^{-n} u(n-1)$$



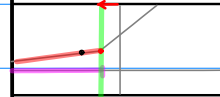
(8) 0111

(1') 1000



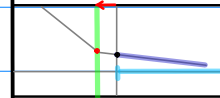
$$a^{n+1} u(n)$$

$$a^{n+1} u(-n-1)$$



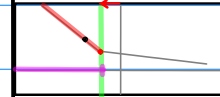
(5') 1100

(2') 1001



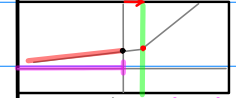
$$a^{-n-1} u(n)$$

$$a^{-n-1} u(-n-1)$$



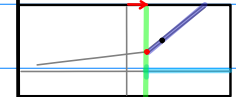
(6') 1101

(3') 1010



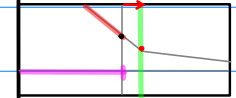
$$a^{n-1} u(-n)$$

$$a^{n-1} u(n-1)$$



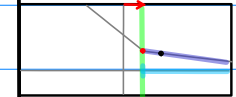
(7') 1110

(4') 1011



$$a^{-n+1} u(-n)$$

$$a^{-n+1} u(n-1)$$



(8') 1111



