

Differentiation

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Based on
Introduction to Matrix Algebra, Autar Kaw
<https://ma.mathforcollege.com>

Outline

- 1 Background on Differentiation
 - Tangent and Secant Lines

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 - Tangent and Secant Lines

Secant Lines

- Let P and Q be two points on the curve of $f(x)$
P $(a, f(a))$ and Q $(a + h, f(a + h))$
- the **secant line** is the straight line drawn through P and Q .
- the slope of the **secant line**

$$\begin{aligned}m_{\text{secant}} &= \frac{f(a + h) - f(a)}{(a + h) - a} \\ &= \frac{f(a + h) - f(a)}{h}\end{aligned}$$

Tangent Lines

- as $h \rightarrow 0$, $Q \rightarrow P$
and the **secant line** \rightarrow the **tangent line**
- the slope of the **tangent line**

$$\begin{aligned}m_{\text{tangent}} &= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{(a+h) - a} \\ &= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}\end{aligned}$$

Derivative of a function

the **derivative** of a function $f(x)$ at $x = a$

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

or

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

Finding equations of a tangent line

One of the numerical methods used to solve a **nonlinear equation** is called the **Newton- Raphson method**.

based on the knowledge of finding the **tangent line** to a curve at a point.

Theorems of differentiations (1)

- If $f(x) = k$, where k is a constant,
 $f'(x) = 0$
- The derivative of $f(x) = x^n$, where $n \neq 0$ is
 $f'(x) = nx^{n-1}$
- The derivative of $f(x) = kg(x)$, where k is a constant is
 $f'(x) = kg'(x)$
- The derivative of $f(x) = u(x) \pm v(x)$ is
 $f'(x) = u'(x) \pm v'(x)$

Theorems of differentiations (2)

- The derivative of $f(x) = u(x) \cdot v(x)$ is
$$f'(x) = \frac{d}{dx} u(x) \cdot v(x) + u(x) \cdot \frac{d}{dx} v(x)$$
- The derivative of $f(x) = \frac{u(x)}{v(x)}$ is
$$f'(x) = \frac{\frac{d}{dx} u(x) \cdot v(x) - u(x) \cdot \frac{d}{dx} v(x)}{(v(x))^2}$$
- The derivative of $f(x) = u(v(x))$ is
$$f'(x) = \frac{d}{dx} u(v(x)) \cdot \frac{d}{dx} v(x)$$

Implicit differentiation

- Sometimes, the function to be differentiated is not given explicitly as an expression of the independent variable.

- Find $\frac{dy}{dx}$ if $x^2 + y^2 = 2xy$

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(2xy)$$

$$2x + 2y \frac{dy}{dx} = 2y + 2x \frac{dy}{dx}$$

$$(2y - 2x) \frac{dy}{dx} = 2y - 2x$$

$$\frac{dy}{dx} = 1$$

Finding maximum and minimum of a function (1)

- The knowledge of the **first derivative** and the second derivative of a function is used to find the **minimum** and **maximum** of a function.
- Let $f(x)$ be a function in domain D , then
 - $f(a)$ is the **maximum** of the function if $f(a) \geq f(x)$ for all values of x in the domain D .
 - $f(a)$ is the **minimum** of the function if $f(a) \leq f(x)$ for all values of x in the domain D .
- The **minimum** and **maximum** of a function are also the **critical values** of a function.

Finding maximum and minimum of a function (2)

- An **extreme value** can occur in the interval $[c, d]$ at **end points** $x = c$, $x = d$.
- a point in $[c, d]$
 - where $f'(x) = 0$.
 - where $f'(x)$ does not exist.
- These critical points can be the **local maximas** and minimas of the function

Tables of derivatives (1)

$f(x)$	$f'(x)$
$x^n, n \neq 0$	nx^{n-1}
$kx^n, n \neq 0$	knx^{n-1}
a^x	$\ln(a)a^x$
$\ln(x)$	$\frac{1}{x}$
$\log_a^{-1}(x)$	$\frac{1}{x \ln(a)}$
e^x	e^x

Tables of derivatives (2)

$f(x)$	$f'(x)$
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\sec^2(x)$
$\sin^{-1}(x)$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1}(x)$	$\frac{-1}{\sqrt{1-x^2}}$
$\tan^{-1}(x)$	$\frac{1}{1+x^2}$

$f(x)$	$f'(x)$
$\sinh(x)$	$\cosh(x)$
$\cosh(x)$	$\sinh(x)$
$\tanh(x)$	$1 - \tanh^2(x)$
$\sinh^{-1}(x)$	$\frac{1}{\sqrt{1+x^2}}$
$\cosh^{-1}(x)$	$\frac{-1}{\sqrt{x^2-1}}$
$\tanh^{-1}(x)$	$\frac{1}{1-x^2}$

Tables of derivatives (3)

$f(x)$	$f'(x)$
$\csc(x)$	$-\csc x \cot x$
$\sec(x)$	$\sec x \tan x$
$\cot(x)$	$-1 - \cot^2 x$
$\csc^{-1}(x)$	$\frac{-1}{ x \sqrt{x^2-1}}$
$\sec^{-1}(x)$	$\frac{1}{ x \sqrt{x^2-1}}$
$\cot^{-1}(x)$	$\frac{-1}{1+x^2}$

