### Temporal Characteristics of Random Processes

Young W Lim

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Based on Probability, Random Variables and Random Signal Principles, P.Z. Peebles, Jr. and B. Shi



### Random Variable Definition

#### A random variable

a function over a sample space  $S = \{s_1, s_2, s_3, ..., s_n\}$ 

 $s \to X(s)$ x = X(s)

a function of the possible outcomes s of an experiment

### Random Variable Definition

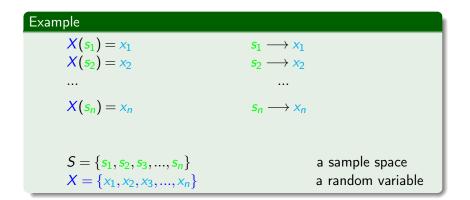
#### A random variable

- a random variable : a capital letter X
- a particular value : a lowercase letter x
- a sample space  $S = \{s_1, s_2, s_3, ..., s_n\}$
- an outcome (an element of S) : s

 $s \to X(s)$ x = X(s)

 $s \rightarrow x$ 

### Random Variable Example



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## Random Process (1)

#### A random process

a function of both outcome s and time t

X(t,s)

assigning a time function to every outcome  $s_i$ 

 $s_i \rightarrow x(t, s_i)$ 

(E)

### Random Process (2)

#### A random process

#### the family of such time functions is called a random process

 $x(t,s_i) = X(t,s_i)$ x(t,s) = X(t,s)

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# Random Process (3)

We have seen that a random variable X is a rule which assigns a number to every outcome e of an experiment. The random variable is a function X(e) that maps the set of experiment outcomes to the set of numbers.

A random process is a rule

that maps every outcome e of an experiment

to a function X(t, e).

A random process is usually conceived of

as a function of time,

but there is no reason to not consider random processes that are functions of other independent variables,

such as spatial coordinates.

The function X(u, v, e) would be a function

whose value depended on the location (u, v) and the outcome e,

# Ensemble of time functions

#### Time functions

A random process X(t,s) represents a family or ensemble of time functions

#### X(t, s) represents

- a single time function x(t,s)
- when *t* is a variable and *s* is fixed at an outcome

#### x(t, s) represents

- a sample function,
- an ensemble member,
- a realization of the process

## Short-form notation for time functions

#### The short-form notation x(t)

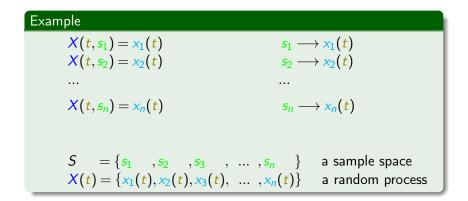
to represent a specific waveform of a random process X(t) for a given **outcome**  $s_i$ 

 $\mathbf{x}(t) = \mathbf{x}(t,s)$ 

X(t) = X(t,s)

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#### Random Process Example



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### Random variables with time

a random process X(t,s) represents a single time function when t is a variable and s is fixed at an outcome

a random process X(t,s) represents a single random variable when both t and s are fixed at a time and an outcome, respectively

$$X_i = X(t_i, s) = X(t_i)$$
 random variable

$$X(t,s) = X(t)$$
 random process

### An alphabet

#### the **alphabet** of X(t)

the set of its possible values

- the values of time t for which a random process is defined
- the **alphabet** of the random variable X = X(t) at time t

# Classification of Random Processes (1) Types of time and alphabet

- the values of time t for which a random process is defined
  - continuous time
  - discrete time
- the alphabet of the random variable X = X(t) at time t
  - continuous alphabet
  - discrete alphabet

# Classification of Random Processes (2) types of the random variable X(t) and the time t

- a continuous alphabet continuous time random process

  X(t) has continuous values and t has continuous values

  a discrete alphabet continuous time random process

  X(t) has discrete values and t has continuous values

  a continuous alphabet discrete time random process

  X(t) has continuous values and t has discrete values

  a continuous alphabet discrete time random process

  X(t) has continuous values and t has discrete values

  a discrete alphabet discrete time random process

  X(t) has continuous values and t has discrete values
  - X(t) has discrete values and t has discrete values

### Deterministic and Non-deterministic Random Processes

- A process is non-deterministic if future values of any sample function <u>cannot</u> be <u>predicted</u> exactly from observed past values
- A process is **deterministic** if future values of any sample function can be predicted from observed past values

#### Deterministic Random Process Example (1)

$$X(t) = A\cos(\omega_0 t + \Theta)$$

A,  $\Theta$ , or  $\omega_0$ (or all) can be random variables.

a <u>sample function</u> corresponds to the above equation with particular values of these random variables.

 $x_i(t) = A_i \cos(\omega_{0,i} t + \Theta_i)$ 

### Deterministic Random Process Example (2)

$$x_i(t) = A_i \cos(\omega_{0,i} t + \Theta_i)$$

the knowledge of the <u>sample function</u> prior to any time instance fully allows the prediction of the <u>sample function</u>'s future values because all the necessary information is known

$$\mathbf{x}_{i}(t)$$
  $t \leq 0$   $\implies$   $\mathbf{x}_{i}(t)$   $t > 0$ 

# Functions and variables of a random process $X(t, \theta)$ (1)

$X(t, \theta)$	a family of functions, an ensemble
$X(t, \theta_k)$	a single time function selected by the outcome $ heta_k$
$X(t_1, \theta)$	a random variable at the time $t = t_1$
$X(t_1, \theta_k)$	a number at the time $t = t_1$ , of the outcome $ heta_k$

https://www.cis.rit.edu/class/simg713/notes/chap7-random-process.pdf

# Functions and variables of a random process $X(t, \theta)$ (2)

- X(t, θ) is a family of functions. Imagine a giant strip chart recording in which each pen is identified with a different θ. This family of functions is traditionally called an ensemble.
- A single function X(t, θ<sub>k</sub>) is selected by the outcome θ<sub>k</sub>. This is just a time function that we could call X<sub>k</sub>(t). Different outcomes give us different time functions.
- If t is fixed, say  $t = t_1$ , then  $X(t_1, \theta)$  is a random variable. Its value depends on the outcome  $\theta$ .
- If both  $t_1$  and  $\theta_k$  are given then  $X(t_1, \theta_k)$  is just a number. https://www.cis.rit.edu/class/simg713/notes/chap7-random-process.pdf

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