Digital Signal Octave Codes (0A)

- Periodic Conditions

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using OpenOffice and Octave.

Based on
M.J. Roberts, Fundamentals of Signals and Systems
S.K. Mitra, Digital Signal Processing : a computer-based approach 2nd ed
S.D. Stearns, Digital Signal Processing with Examples in MATLAB

A Cosine Waveform

```
n= [0:29];
x= cos(2*pi*(n/10));
```

$$nT_s = n \cdot \frac{1}{10}$$

```
x= cos((2/10)*pi*n);
```

$$nT_s = n \cdot 1$$

$$\omega_0 n T_s = 2\pi f_0 n T_s = \frac{2\pi}{T_0} n T_s = 2\pi n \frac{T_s}{T_0}$$

$$\omega_0 t = 2\pi f t$$

$$\omega_0 n T_s = 2\pi f_0 n T_s = 2\pi \cdot 1 \cdot n \cdot \frac{1}{10}$$

$$\omega_0 n T_s = 2\pi f_0 n T_s = 2\pi \cdot \frac{1}{10} \cdot n \cdot 1$$

$f_0 = 1 \quad T_0 = 1 \quad T_s = 0.1$

$f_0 = 0.1 \quad T_0 = 10 \quad T_s = 1$

U of Rhode Island, ELE 436, FFT Tutorial

Many waveforms share the same sampled data

x

1.00000
0.80902
0.30902
-0.30902
-0.80902
-1.00000
-0.80902
-0.30902
0.30902
0.80902
1.00000
0.80902
0.30902
-0.30902
-0.80902
-1.00000
-0.80902
-0.30902
0.30902
0.80902
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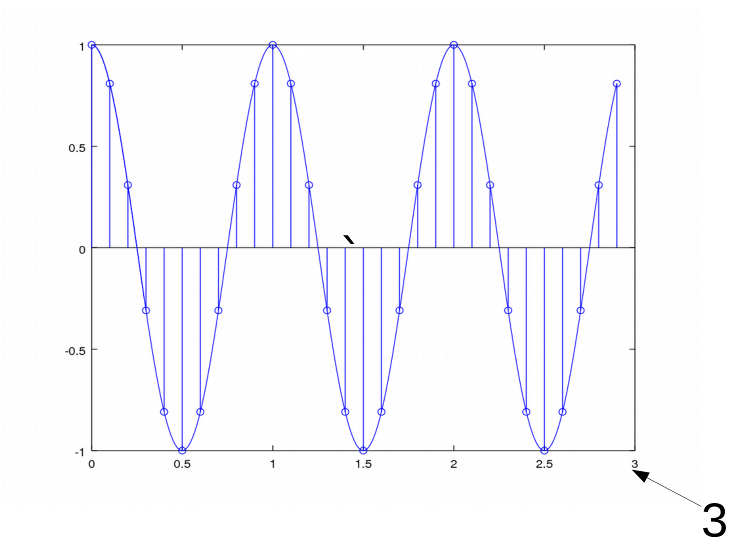
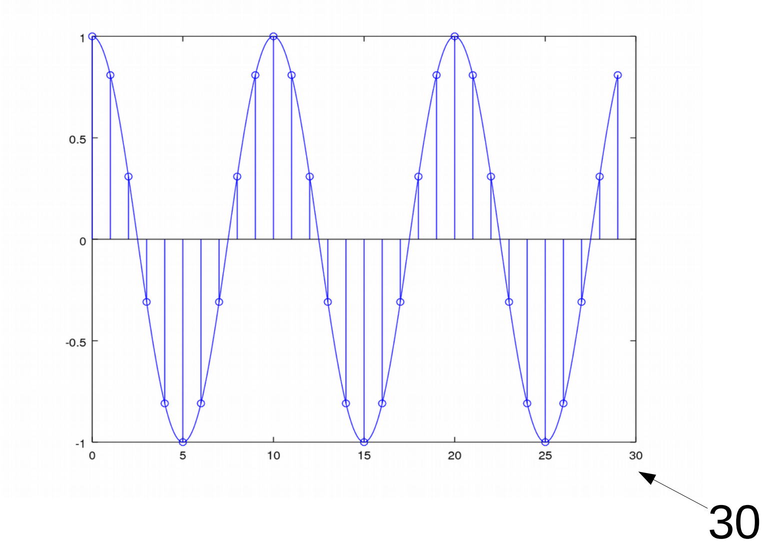
-0.30902
0.30902
0.80902





U of Rhode Island, ELE 436, FFT Tutorial

Cosine Wave 1

```
n= [0:29];
x= cos(2*pi*n/10);
```

$$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0} \qquad f_0 = 1 \quad T_0 = 1 \quad T_s = 0.1$$

```
t = [0:29]/10;
y = cos(2*pi*t);
stem(t, y)
hold on
t2 = [0:290]/100;
y2 = cos(2*pi*t2);
plot(t2, y2)
```

$$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0} \qquad f_0 = 0.1 \quad T_0 = 10 \quad T_s = 1$$

```
t = [0:29];
y = cos(0.2*pi*t);
stem(t, y)
hold on
t2 = [0:290]/10;
y2 = cos(0.2*pi*t2);
plot(t2, y2)
```

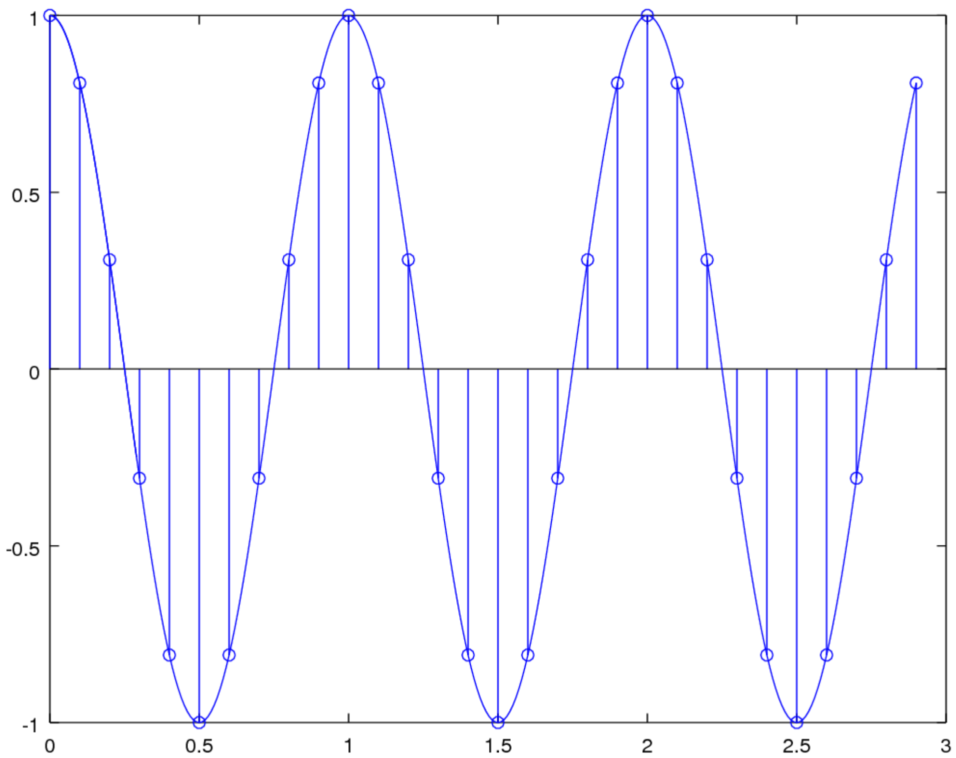
U of Rhode Island, ELE 436, FFT Tutorial

Cosine Wave 1

```
n= [0:29];
x= cos(2*pi*n/10);
```

$$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0} \qquad f_0 = 1 \quad T_0 = 1 \quad T_s = 0.1$$

```
t = [0:29]/10;
y = cos(2*pi*t);
stem(t, y)
hold on
t2 = [0:290]/100;
y2 = cos(2*pi*t2);
plot(t2, y2)
```



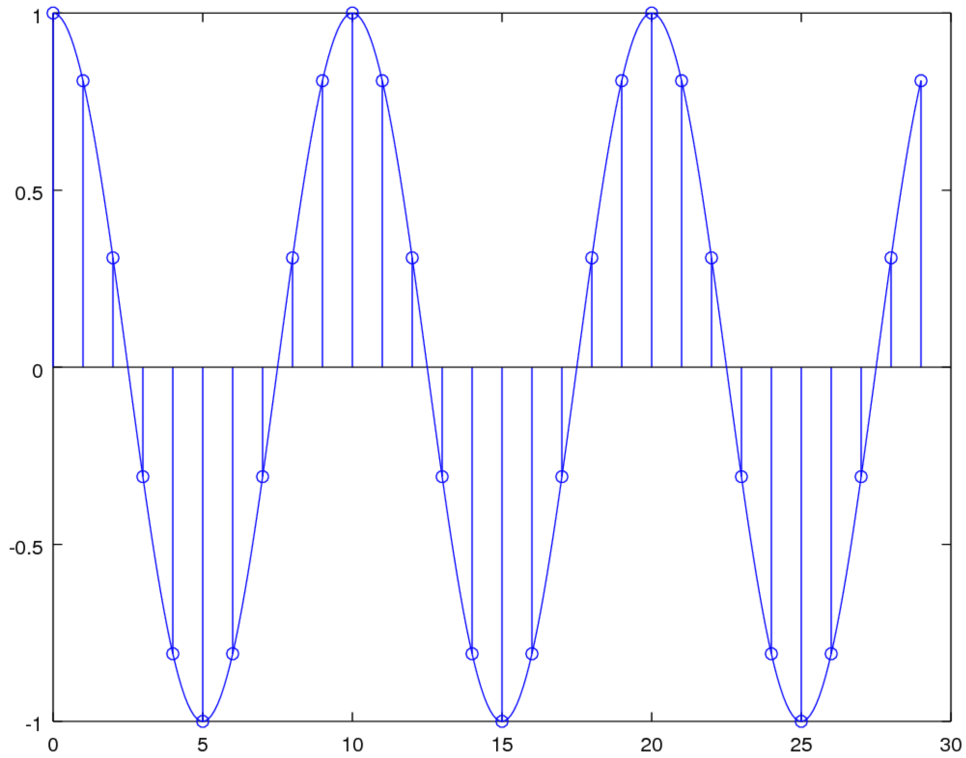
U of Rhode Island, ELE 436, FFT Tutorial

Cosine Wave 2

```
n= [0:29];
x= cos(2*pi*n/10);
```

$$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0} \qquad f_0 = 0.1 \quad T_0 = 10 \quad T_s = 1$$

```
t = [0:29];
y = cos(0.2*pi*t);
stem(t, y)
hold on
t2 = [0:290]/10;
y2 = cos(0.2*pi*t2);
plot(t2, y2)
```



$$\omega_0 n T_s = 2\pi f_0 n T_s = 2\pi \cdot \frac{1}{10} \cdot n \cdot 1$$

$$f_0 = 0.1 \quad T_0 = 10 \quad T_s = 1$$

U of Rhode Island, ELE 436, FFT Tutorial

Sampled Sinusoids

$$g[n] = A e^{\beta n}$$

$$g[n] = A z^{n} \qquad z = e^{\beta}$$

$$g[n] = A\cos(2\pi n/N_0 + \theta)$$

$$g[n] = A\cos(2\pi F_0 n + \theta)$$

$$g[n] = A\cos(\Omega_0 n + \theta)$$

$1/N_0$	$2\pi/N_0$
$= F_0$	$= 2\pi F_0$
$= \Omega_0/2\pi$	$= \Omega_0$

M.J. Roberts, Fundamentals of Signals and Systems

Sampling Period and Frequency

$$g(t) = A\cos(2\pi f_0 t + \theta)$$

$$g[n] = g(nT_s)$$

$$g[n] = A\cos(2\pi F_0 n + \theta)$$

$$g(nT_s) = A\cos(2\pi f_0 T_s n + \theta)$$

$$F_0 = f_0 T_s = f_0 / f_s$$

$$g[n] = A\cos(2\pi F_0 n + \theta)$$

$$T_s = \frac{1}{f_s}$$ sampling period

$$\frac{1}{T_s} = f_s$$ sampling frequency
sampling rate

M.J. Roberts, Fundamentals of Signals and Systems

Periodic Condition of a Sampled Signal

$$2\pi F_0 n = 2\pi m$$

$$F_0 n = m$$ Integers $n, \; m$

$$F_0 = \frac{m}{n}$$

$$F_0 = \frac{m}{n} = \frac{f_0}{f_s}$$

Fundamental Frequency

Sampling Frequency

Rational Number $F_0 = \frac{m}{n}$

$$g(nT_s) = A\cos(2\pi f_0 T_s n + \theta)$$

$$F_0 = f_0 T_s = f_0 / f_s$$

$$g[n] = A\cos(2\pi F_0 n + \theta)$$

M.J. Roberts, Fundamentals of Signals and Systems

Periodic Condition Examples

$$g(t) = A\cos(2\pi f_0 t + \theta)$$

$$g[n] = A\cos(2\pi F_0 n + \theta)$$

$$g[t] = 4\cos\left(\frac{72\pi t}{19}\right) = 4\cos\left(2\pi\cdot\frac{36}{19}\cdot t\right)$$

$$g[n] = 4\cos\left(\frac{72\pi n}{19}\right) = 4\cos\left(2\pi\cdot\frac{36}{19}\cdot n\right) \qquad T_s = 1$$

$$g(t) = 4\cos\left(2\pi\cdot\frac{36}{19}\cdot(t + T_0)\right) \qquad T_0 = \frac{19}{36}$$

Fundamental Period of $g(t)$

$$g[n] = 4\cos\left(2\pi\cdot\frac{36}{19}\cdot(n + N_0)\right) \qquad N_0 = 19$$

Fundamental Period of $g[n]$

$$N_0 \neq \frac{1}{F_0} \qquad \frac{N_0}{q} = \frac{1}{F_0} \qquad \frac{q}{N_0} = F_0$$

M.J. Roberts, Fundamentals of Signals and Systems

Periodic Condition Examples

$$g(t) = 4\cos\left(2\pi\cdot\frac{36}{19}\cdot(t + T_0)\right)$$

$T_0 = \frac{19}{36}$ Fundamental Period of $g(t)$

$$g[n] = 4\cos\left(2\pi\cdot\frac{36}{19}\cdot(n + N_0)\right)$$

$N_0 = 19$ Fundamental Period of $g[n]$

$$F_0 = \frac{36}{19} = \frac{q}{N_0}$$

⬅ the number of cycles in N_0 samples

⬅ the smallest integer : fundamental period

$N_0 \neq \frac{1}{F_0}$ $\qquad \frac{N_0}{q} = \frac{1}{F_0}$ $\qquad \frac{q}{N_0} = F_0$

M.J. Roberts, Fundamentals of Signals and Systems

Periodic Condition Examples

$$F_0 = \frac{36}{19} = \frac{q}{N_0}$$

- q ⟵ the number of cycles in N_0 samples
- N_0 ⟵ the smallest integer : fundamental period

"When F_0 is not the reciprocal of an integer (q=1), a discrete-time sinusoid may not be immediately recognizable from its graph as a sinusoid."

M.J. Roberts, Fundamentals of Signals and Systems

Periodic Condition Examples

$$g[n] = 4\cos\left(2\pi\cdot\frac{36}{19}\cdot(n + N_0)\right)$$

$\frac{36}{19}\cdot(n + N_0)$ integer

$\frac{1}{19}\cdot N_0 = k$ integer

N_0 integer

$N_0 = 19$ Fundamental period of $g[n]$

$$g(t) = 4\cos\left(2\pi\cdot\frac{36}{19}\cdot(t + T_0)\right)$$

$\frac{36}{19}\cdot(t + T_0)$ integer

$\frac{36}{19}\cdot T_0 = k$ integer

T_0 ~~integer~~

$T_0 = \frac{19}{36}$ Fundamental period of $g(t)$

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Periodic Condition Examples

$$g[n] = 4\cos\left(2\pi\cdot\frac{1}{19}\cdot n\right)$$

$$g[n] = 4\cos\left(2\pi\cdot\frac{2}{19}\cdot n\right)$$

$$g[n] = 4\cos\left(2\pi\cdot\frac{3}{19}\cdot n\right)$$

$$g[n] = 4\cos\left(2\pi\cdot\frac{36}{19}\cdot n\right)$$

```
clf
n = [0:36]; t = [0:3600]/100;
y1 = 4*cos(2*pi*(1/19)*n);
y2 = 4*cos(2*pi*(2/19)*n);
y3 = 4*cos(2*pi*(3/19)*n);
y4 = 4*cos(2*pi*(36/19)*n);
yt1 = 4*cos(2*pi*(1/19)*t);
yt2 = 4*cos(2*pi*(2/19)*t);
yt3 = 4*cos(2*pi*(3/19)*t);
yt4 = 4*cos(2*pi*(36/19)*t);

subplot(4,1,1);
stem(n, y1); hold on;
plot(t, yt1);
subplot(4,1,2);
stem(n, y2); hold on;
plot(t, yt2);
subplot(4,1,3);
stem(n, y3); hold on;
plot(t, yt3);
subplot(4,1,4);
stem(n, y4); hold on;
plot(t, yt4);
```

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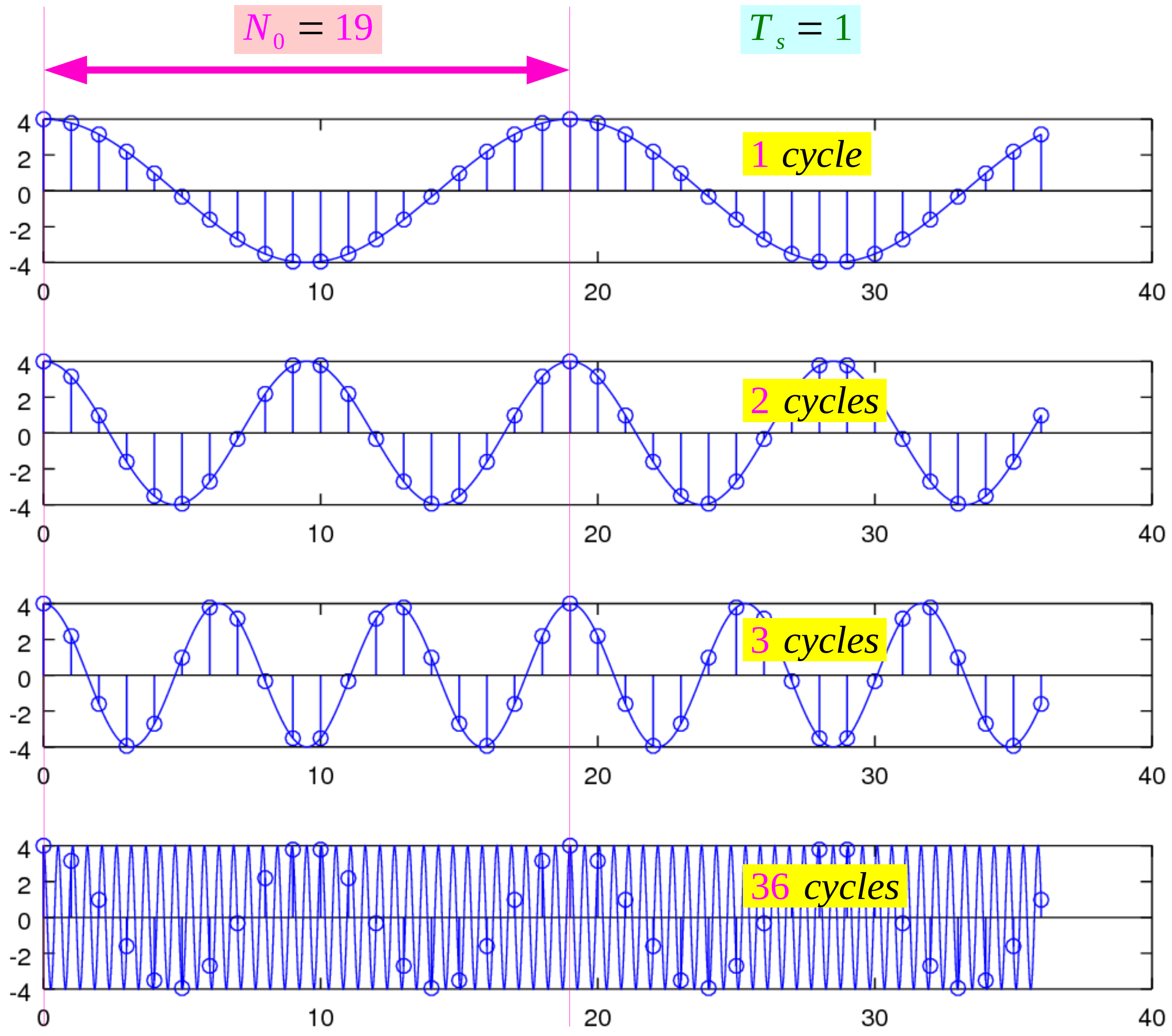
Periodic Condition Examples

$$g[n] = 4\cos\left(2\pi\cdot\frac{1}{19}\cdot n\right)$$

$$g[n] = 4\cos\left(2\pi\cdot\frac{2}{19}\cdot n\right)$$

$$g[n] = 4\cos\left(2\pi\cdot\frac{3}{19}\cdot n\right)$$

$$g[n] = 4\cos\left(2\pi\cdot\frac{36}{19}\cdot n\right)$$



Periodic Condition Examples

$$g(t) = A\cos(2\pi f_0 t + \theta)$$

$$g[n] = A\cos(2\pi F_0 n + \theta)$$

$$g_1(t) = 4\cos(2\pi\cdot 1\cdot t) \qquad t \leftarrow nT_1 \qquad g_1[n] = 4\cos(2\pi n T_{s1})$$

$$g_2(t) = 4\cos(2\pi\cdot 2\cdot t) \qquad t \leftarrow nT_2 \qquad g_2[n] = 4\cos(2\pi n T_{s2})$$

$$g_3(t) = 4\cos(2\pi\cdot 3\cdot t) \qquad t \leftarrow nT_3 \qquad g_3[n] = 4\cos(2\pi n T_{s3})$$

$$t \leftarrow nT_1 \qquad T_1 = \frac{1}{10} \qquad n = 0, 1, 2, 3, \cdots \Rightarrow 1\cdot t = 0, 0.1, 0.2, 0.3, \cdots$$

$$t \leftarrow nT_2 \qquad T_2 = \frac{1}{20} \qquad n = 0, 1, 2, 3, \cdots \Rightarrow 2\cdot t = 0, 0.1, 0.2, 0.3, \cdots$$

$$t \leftarrow nT_3 \qquad T_3 = \frac{1}{30} \qquad n = 0, 1, 2, 3, \cdots \Rightarrow 3\cdot t = 0, 0.1, 0.2, 0.3, \cdots$$

$$\{ g_1[n] \} \equiv \{ g_2[n] \} \equiv \{ g_2[n] \}$$

M.J. Roberts, Fundamentals of Signals and Systems

Periodic Condition Examples

$$g(t) = A\cos(2\pi f_0 t + \theta)$$

$$g[n] = A\cos(2\pi F_0 n + \theta)$$

$$g[n] = 4\cos\left(\frac{72\pi n}{19}\right)$$

$$= 4\cos\left(2\pi\left(\frac{36}{19}\right)n\right)$$

$$= 4\cos\left(2\pi\left(\frac{36}{19}\cdot(n + N_0)\right)\right)$$

smallest $N_0 = 19$

$$2\pi F_0 n = 2\pi m$$

$$\frac{36}{19} n = m$$

smallest n = 19

$$\frac{36}{19} = \frac{m}{n}$$

$$\frac{36}{19} = \frac{m}{n} = \frac{f_0}{f_s}$$

$$F_0 = \frac{q}{N_0}$$

$$1/N_0$$

$$= F_0$$

$$= \Omega_0/2\pi$$

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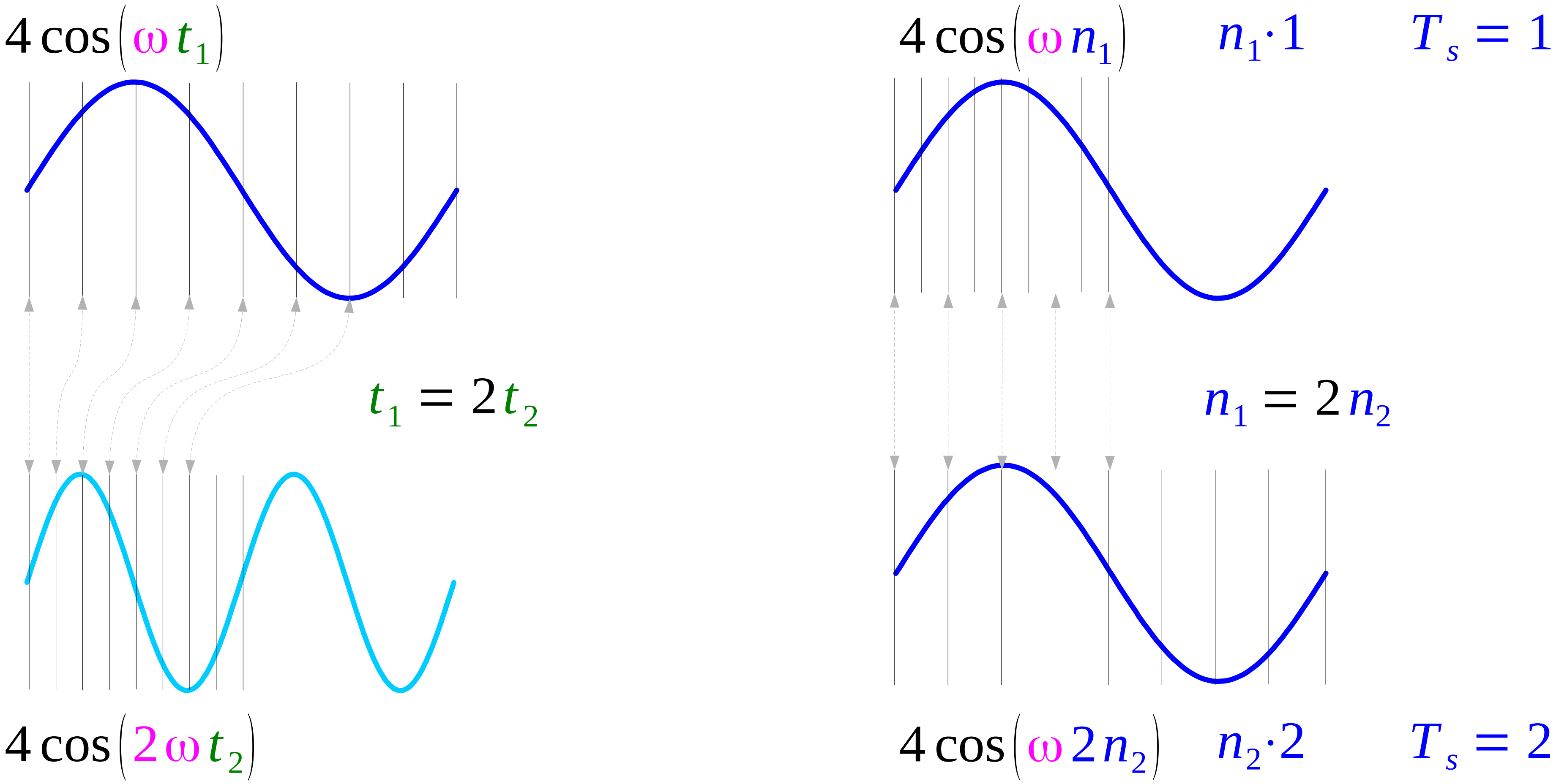
Periodic Condition Examples



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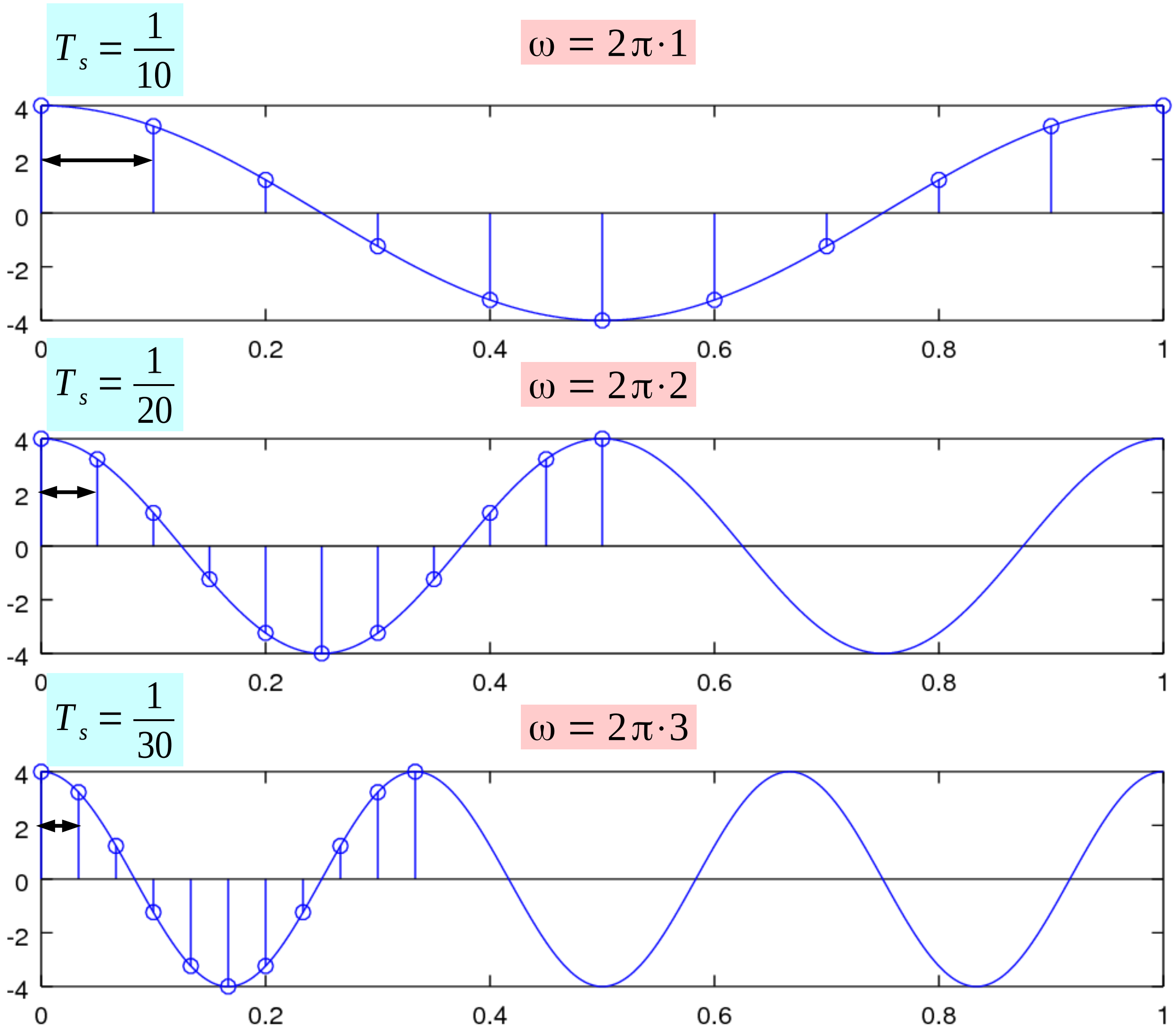
Periodic Condition Examples




```
clf
n = [0:10]; t = [0:1000]/1000;
y1 = 4*cos(2*pi*1*n/10);
y2 = 4*cos(2*pi*2*n/20);
y3 = 4*cos(2*pi*3*n/30);
yt1 = 4*cos(2*pi*t);
yt2 = 4*cos(2*pi*2*t);
yt3 = 4*cos(2*pi*3*t);

subplot(3,1,1);
stem(n, y1); hold on;
plot(t, yt1);
subplot(3,1,2);
stem(n/20, y2); hold on;
plot(t, yt2);
subplot(3,1,3);
stem(n/30, y3); hold on;
plot(t, yt3);
```

M.J. Roberts, Fundamentals of Signals and Systems

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] M.J. Roberts, Fundamentals of Signals and Systems
[4] S.J. Orfanidis, Introduction to Signal Processing
[5] K. Shin, et al., Fundamentals of Signal Processing for Sound and Vibration Engineerings

[6] A “graphical interpretation” of the DFT and FFT, by Steve Mann