

Applicative Methods (3B)

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The definition of Applicative

```
class (Functor f) => Applicative f where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b
```

$f (a \rightarrow b) ::$ a **function** wrapped in f

$f a ::$ a **value** wrapped in f

The class has a two methods :

pure brings arbitrary values into the functor

(<*>) takes a **function** wrapped in a functor f
and a **value** wrapped in a functor f
and returns the result of the application
which is also wrapped in a functor f

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

The Maybe instance of Applicative

```
instance Applicative Maybe where
  pure          = Just
  (Just f) <*> (Just x) = Just (f x)
  _          <*> _      = Nothing
```

pure wraps the value with **Just**;

(<*>) applies

the function wrapped in **Just**

to the value wrapped in **Just** if both exist,

and results in **Nothing** otherwise.

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

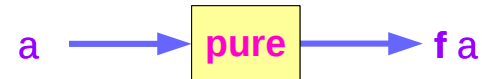
An Instance of the Applicative Typeclass

```
class (Functor f) => Applicative f where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b
```

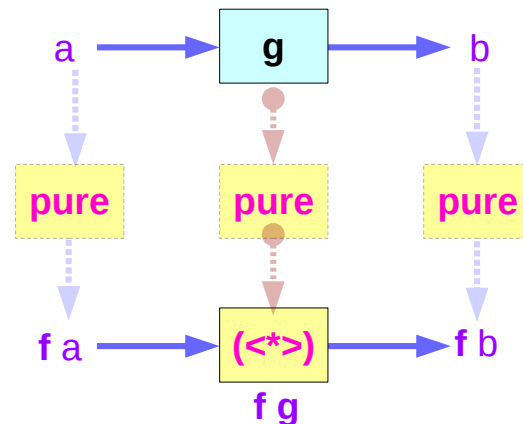
f : Functor, Applicative

```
instance Applicative Maybe where
  pure = Just
  Nothing <*> _ = Nothing
  (Just f) <*> something = fmap f something
```

f : function in a context



(Functor f) => Applicative f



(Functor f) => Applicative f

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

Left associative $\langle * \rangle$, `fmap`, and $\langle \$ \rangle$

```
pure g  $\langle * \rangle$  x  $\langle * \rangle$  y  $\langle * \rangle$  z
```

```
g :: f (a -> b -> c -> d)
x :: f a
y :: f b
z :: f c
```

```
fmap g x  $\langle * \rangle$  y  $\langle * \rangle$  z
```

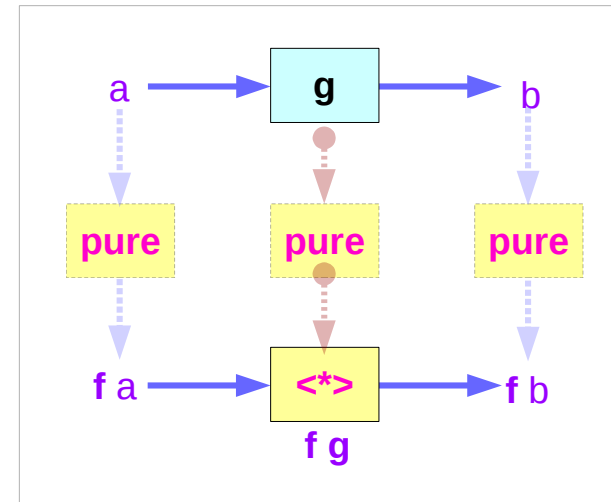
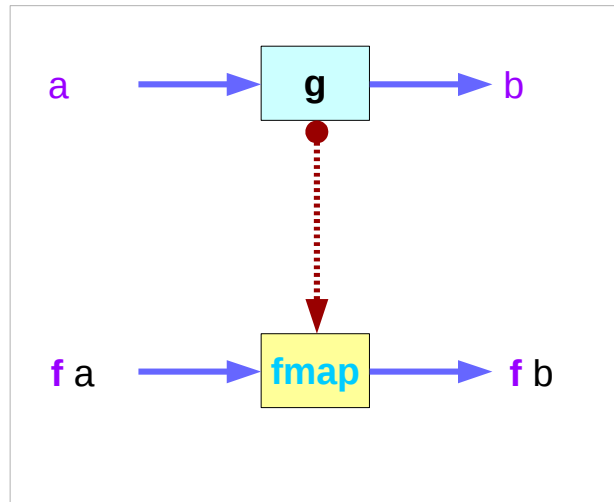


```
g  $\langle \$ \rangle$  x  $\langle * \rangle$  y  $\langle * \rangle$  z
```

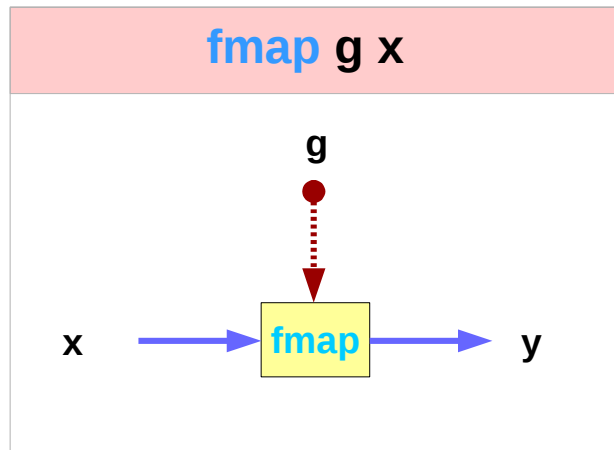
infix operator $\langle \$ \rangle$

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

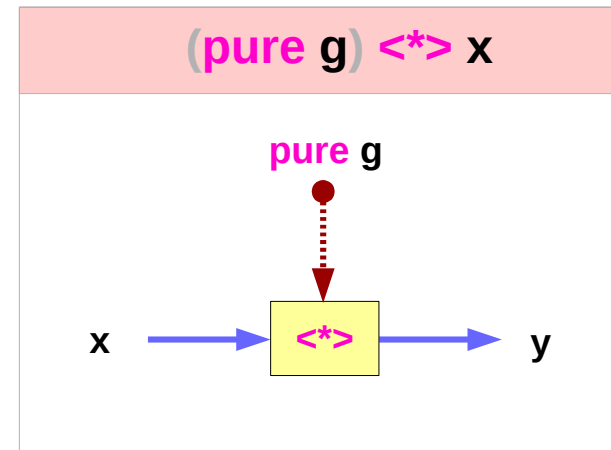
$$\text{fmap } g \ x = (\text{pure } g) \langle * \rangle x$$



pure = f

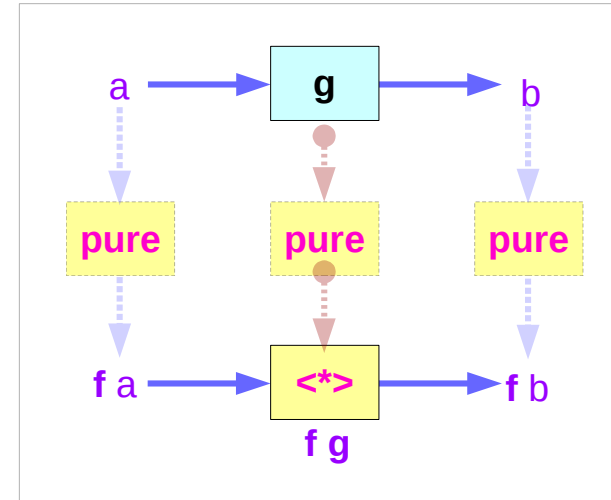
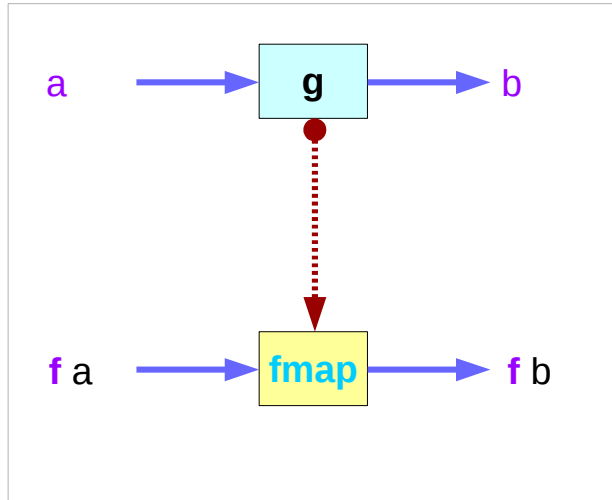


$x :: f \ a$
 $y :: f \ b$

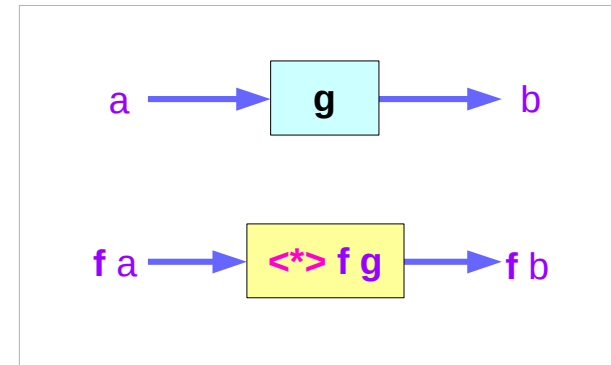


<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

$f a \langle * \rangle f g$



$pure = f$



<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

Left associative <*> examples

```
ghci> pure (+) <*> Just 3 <*> Just 5  
Just 8
```

```
pure (+) <*> Just 3 <*> Just 5
```

```
Just (+3) <*> Just 5
```

```
Just 8
```

```
ghci> pure (+) <*> Just 3 <*> Nothing  
Nothing
```

```
ghci> pure (+) <*> Nothing <*> Just 5  
Nothing
```

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

Infix Operators $\langle * \rangle$ vs $\langle \$ \rangle$ - a type view

h $\langle * \rangle$ **x** $\langle * \rangle$ **y**

$h :: f (a \rightarrow b \rightarrow c)$
 $x :: f a$
 $y :: f b$

$h :: f (a \rightarrow b \rightarrow c)$

$x :: f a$

$h \langle * \rangle x :: f (b \rightarrow c)$

$h :: f (a \rightarrow b \rightarrow c)$

$x :: f a$

$h \langle * \rangle x :: f (b \rightarrow c)$

$y :: f b$

$h \langle * \rangle x \langle * \rangle y :: f c$

g $\langle \$ \rangle$ **x** $\langle * \rangle$ **y**

$g :: (a \rightarrow b \rightarrow c)$
 $x :: f a$
 $y :: f b$

$g :: (a \rightarrow b \rightarrow c)$

$x :: f a$

$g \langle \$ \rangle x :: f (b \rightarrow c)$

$g :: (a \rightarrow b \rightarrow c)$

$x :: f a$

$g \langle \$ \rangle x :: f (b \rightarrow c)$

$y :: f b$

$g \langle \$ \rangle x \langle * \rangle y :: f c$

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

Infix Operators $\langle * \rangle$ vs $\langle \$ \rangle$ - a curried function view

$h \langle * \rangle x \langle * \rangle y$

$h :: f (a \rightarrow b \rightarrow c)$

$x :: f a$

$y :: f b$

$g \langle \$ \rangle x \langle * \rangle y$

$g :: (a \rightarrow b \rightarrow c)$

$x :: f a$

$y :: f b$

$h \langle * \rangle x$

$h :: f (a \rightarrow b \rightarrow c)$

$x :: f a \rightarrow \langle * \rangle \rightarrow h \langle * \rangle x$



$g \langle \$ \rangle x$

$g :: (a \rightarrow b \rightarrow c)$

$x :: f a \rightarrow \langle \$ \rangle \rightarrow g \langle \$ \rangle x$



<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

Infix Operators $\langle * \rangle$ vs $\langle \$ \rangle$ examples

h $\langle * \rangle$ **x** $\langle * \rangle$ **y**

Just (+) $\langle * \rangle$ Just 3 $\langle * \rangle$ Just 2
Just (+3) $\langle * \rangle$ Just 2
Just 5

g $\langle \$ \rangle$ **x** $\langle * \rangle$ **y**

(+) $\langle \$ \rangle$ Just 3 $\langle * \rangle$ Just 2
Just (+3) $\langle * \rangle$ Just 2
Just 5

h $\langle * \rangle$ **x**

Just (+)

Just 3 \rightarrow $\langle * \rangle$ \rightarrow Just (3+)

g $\langle \$ \rangle$ **x**

(+)

Just 3 \rightarrow $\langle \$ \rangle$ \rightarrow Just (3+)

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

the minimal complete definition

```
class (Functor f) => Applicative f where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b
```

the minimal complete definition

```
(<$>) :: (Functor f) => (a -> b) -> f a -> f b
g <$> x = fmap g x
```

Not in the minimal complete definition

`g :: a -> b, x :: f a`

```
instance Applicative Maybe where
  pure = Just
  Nothing <*> _ = Nothing
  (Just g) <*> something = fmap g something
```

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

The Applicative Typeclass

Applicative is a superclass of **Monad**.

every **Monad** is also a **Functor** and an **Applicative**

fmap, **pure**, **(<*>)** can all be used with **monads**.

a **Monad** instance

requires **Functor** and **Applicative** instances.

defines the types and roles of **return** and **(>>)**

fmap : defined in **Functors**

pure, **(<*>)** : defined in **Applicatives**

return, **(>>)** : defined in **Monads**

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

(<\$>) vs (\$)

(<\$>) infix operator

(<\$>) :: (Functor f) => (a -> b) -> f a -> f b

g <\$> x = fmap g x

The \$ operator is for avoiding parentheses

putStrLn (show (1 + 1))

putStrLn \$ show (1 + 1)

putStrLn \$ show \$ 1 + 1 – right associative

(\$) calls the function which is its left-hand argument of \$

on the value which is its right-hand argument of \$

The Applicative Laws

The identity law: $\text{pure id} \langle * \rangle v = v$ $\text{id} :: a \rightarrow a$ $v :: f\ a$

Homomorphism: $\text{pure g} \langle * \rangle \text{pure x} = \text{pure (g x)}$ $g :: a \rightarrow b$ $x :: a$

Interchange: $u \langle * \rangle \text{pure y} = \text{pure (\$ y)} \langle * \rangle u$ $u :: f\ (a \rightarrow b)$ $y :: a$

Composition: $u \langle * \rangle (v \langle * \rangle w) = \text{pure (.)} \langle * \rangle u \langle * \rangle v \langle * \rangle w$ $w :: f\ a$ $v :: f\ (a \rightarrow b)$ $u :: f\ (b \rightarrow c)$

Left associative $u \langle * \rangle v \langle * \rangle w = (u \langle * \rangle v) \langle * \rangle w$

$u :: f\ (c \rightarrow b \rightarrow a)$
 $v :: f\ c$
 $u \langle * \rangle v :: f\ (b \rightarrow a)$
 $w :: f\ b$
 $u \langle * \rangle v \langle * \rangle w = f\ a$

The Identity Law

The identity law

pure id <*> v = v

id :: a -> a

v :: f a

pure to inject values into the functor
in a *default, featureless* way,
so that the result is as close as possible
to the plain value.

applying the **pure id** morphism does nothing,
exactly like with the plain **id** function.

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

The Homomorphism Law

The homomorphism law

`pure g <*> pure x = pure (g x)`

`g :: a -> b`

`x :: a`

applying a "**pure**" function to a "**pure**" value is the same as applying the function to the value in the *ordinary way* and then using **pure** on the result.
means **pure** preserves function application.

applying a non-effectful function **g** to a non-effectful argument **x** in an effectful context **pure** is the same as just **applying** the function **g** to the argument **x** and then injecting the result (**f x**) into the effectful context with **pure**.

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

The Interchange Law

The interchange law

$$u \langle * \rangle \text{pure } y = \text{pure } (\$ y) \langle * \rangle u$$

$$u :: f (a \rightarrow b) \quad y :: a$$

$(\$ y)$ is the *function* that supplies y
as argument to another function
– a higher order function

Function $\$$ Argument

$\$ y$

(y) as a single argument

applying a morphism u to a "pure" value $\text{pure } y$
is the same as applying $\text{pure } (\$ y)$ to the morphism u

Just $(+3) \langle * \rangle$ Just 2

Just $(\$ 2) \langle * \rangle$ Just $(+3)$

when evaluating the application of
an effectful function (u) to a pure argument ($\text{pure } y$),
the order doesn't matter – commutative.

$u :: f (a \rightarrow b)$

$y :: a$

$u \langle * \rangle \text{pure } y :: f b$

$\text{pure } y :: f a$

$\text{pure } (\$ y) \langle * \rangle u :: f b$

$\text{pure } (\$ y) :: f (a)$

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

The Composition Law

The composition law $\text{pure } (.) \langle * \rangle u \langle * \rangle v \langle * \rangle w = u \langle * \rangle (v \langle * \rangle w)$ $w :: f a$ $v :: f (a \rightarrow b)$ $u :: f (b \rightarrow c)$

$\text{pure } (.)$ composes morphisms similarly
to how $(.)$ composes functions:

applying the composed morphism

$\text{pure } (.) \langle * \rangle u \langle * \rangle v$ to w

gives the same result $(u \langle * \rangle (v \langle * \rangle w))$

as applying u to the result $(v \langle * \rangle w)$

of applying v to w

it is expressing a sort of associativity property of $\langle * \rangle$.

```
w :: f a           -- value
v :: f (a -> b)    -- func1
u :: f (b -> c)    -- func2
```

```
v <*> w :: f b
u <*> (v <*> w) :: f c
```

```
pure (.) <*> u <*> v :: f (a -> c)
pure (.) <*> u <*> v <*> w :: f c
```

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

The Composition Law and Left Associativity

The composition law

$$\text{pure } (.) \langle * \rangle u \langle * \rangle v \langle * \rangle w = \boxed{u \langle * \rangle (v \langle * \rangle w)}$$

$$w :: f \ a \quad v :: f \ (a \rightarrow b) \quad u :: f \ (b \rightarrow c)$$

$$\text{pure } (.) \langle * \rangle \text{pure } g \langle * \rangle \text{pure } h \langle * \rangle \text{pure } x$$

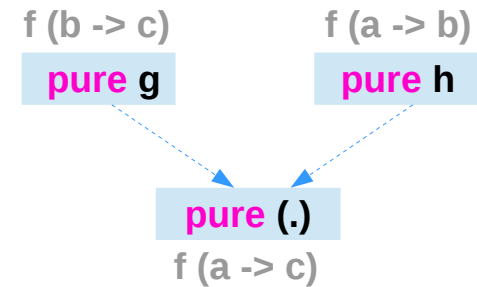
$$(g \cdot h) \ x$$

$$((\text{pure } (.) \langle * \rangle \text{pure } g) \langle * \rangle \text{pure } h) \langle * \rangle \text{pure } x$$



$$= \text{pure } g \langle * \rangle (\text{pure } h \langle * \rangle \text{pure } x)$$

$$g \ (h \ x)$$



$$u = \text{pure } g :: f \ (b \rightarrow c)$$

$$g :: (b \rightarrow c)$$

$$v = \text{pure } h :: f \ (a \rightarrow b)$$

$$h :: (a \rightarrow b)$$

$$w = \text{pure } x :: f \ a$$

$$x :: a$$

Left associative

$$u \langle * \rangle v \langle * \rangle w = \boxed{(u \langle * \rangle v) \langle * \rangle w}$$

$$u :: f \ (c \rightarrow b \rightarrow a) \quad v :: f \ c \quad w :: f \ b$$

$$u :: f \ (c \rightarrow b \rightarrow a)$$

$$v :: f \ c$$

$$u \langle * \rangle v :: f \ (b \rightarrow a)$$

$$w :: f \ b$$

$$u \langle * \rangle v \langle * \rangle w = f \ a$$

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

liftA2

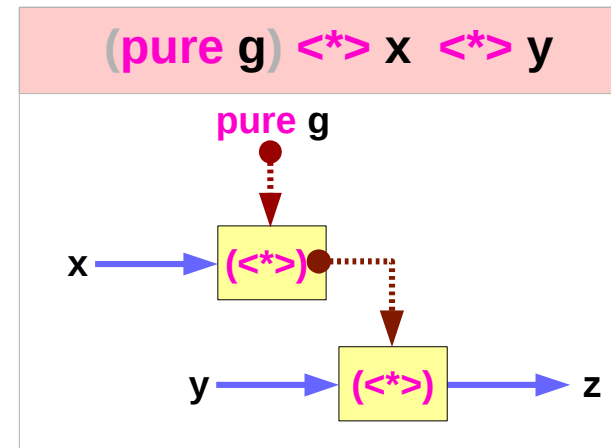
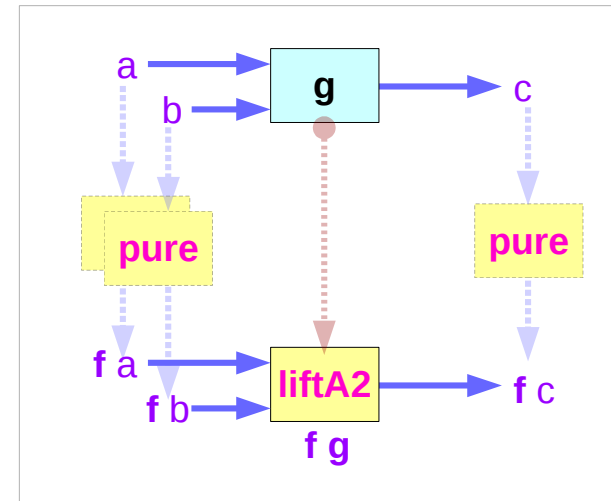
liftA2 :: (a -> b -> c) -> f a -> f b -> f c

lift a binary function (a->b->c) to actions.

Some functors support an implementation of **liftA2** that is more efficient than the default one.

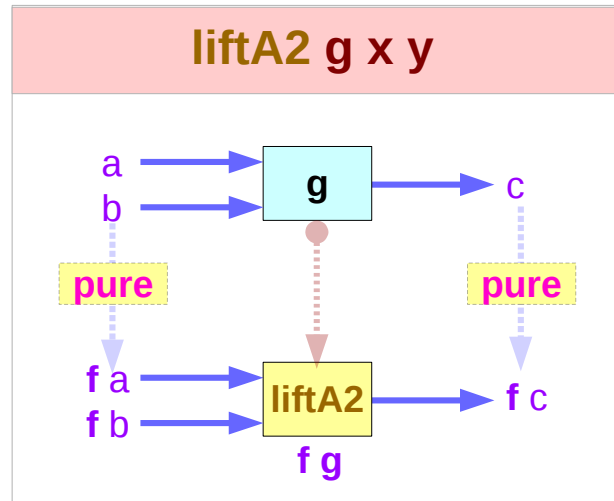
liftA2 may have an efficient implementation whereas **fmap** is an expensive operation,

sometimes better to use **liftA2** than to use **fmap** over the structure and then use <*>.

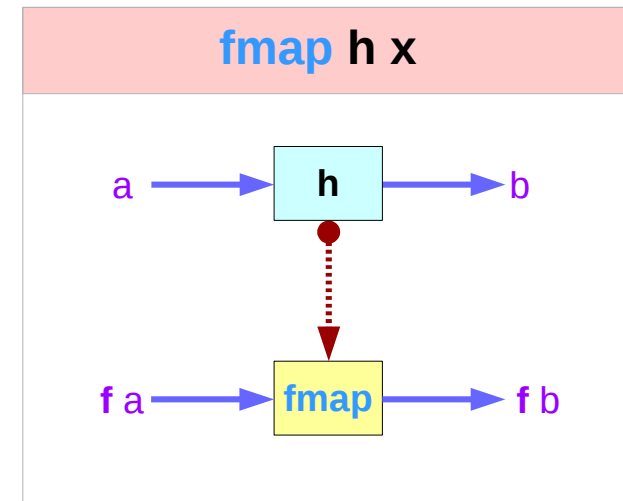
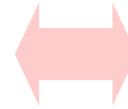
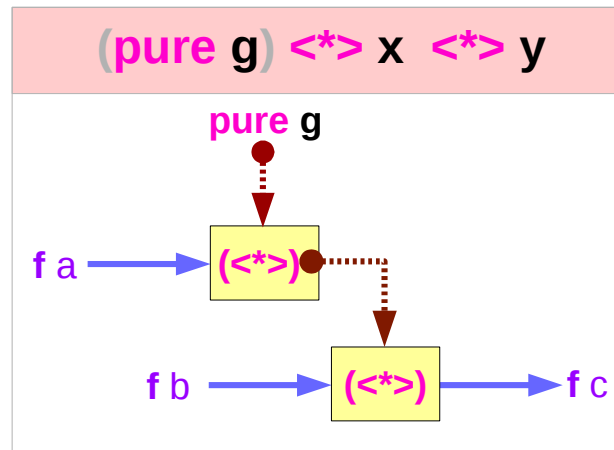


<http://hackage.haskell.org/package/base-4.10.1.0/docs/Control-Applicative.html#v:liftA2>

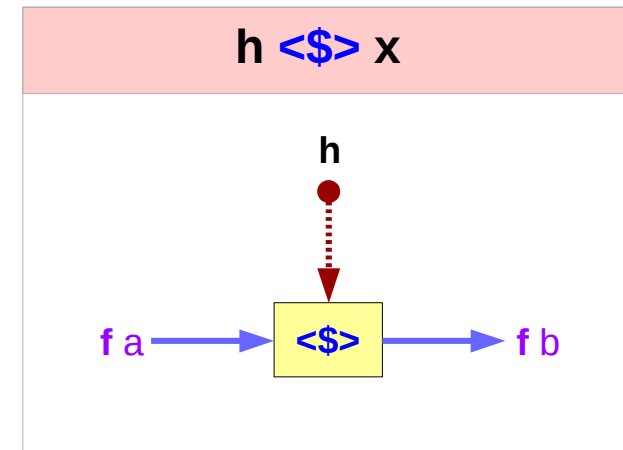
liftA2, <*>, fmap, <\$>



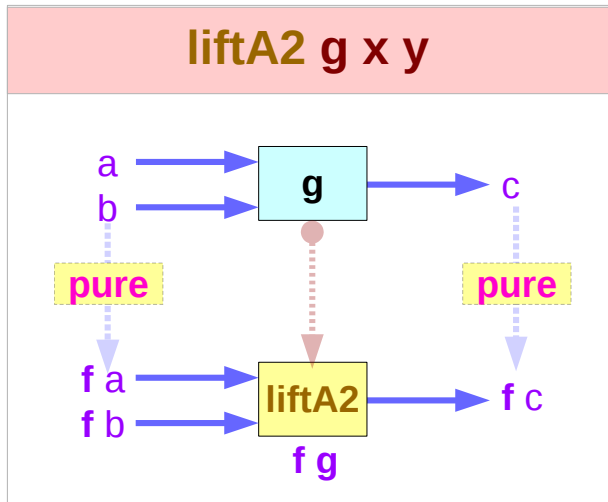
$g :: a \rightarrow b \rightarrow c$



$h :: a \rightarrow b$



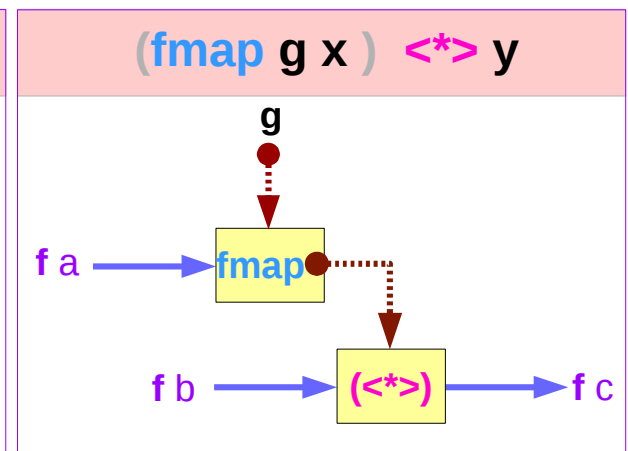
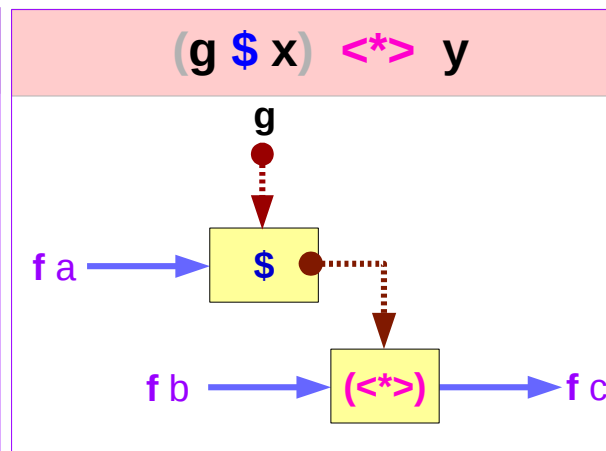
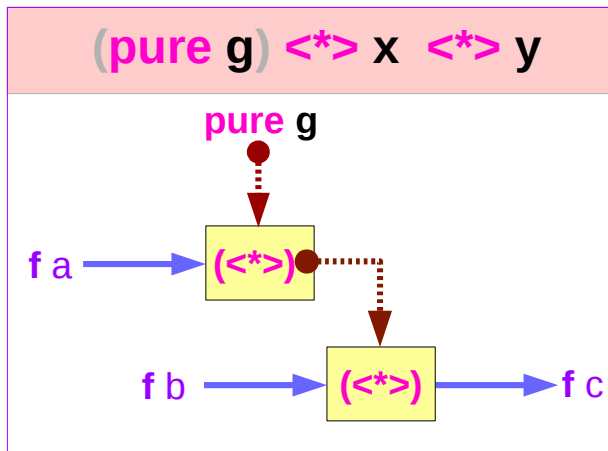
pure g <*> x <*> y equivalent



`g :: a -> b -> c`

`g :: a -> b -> c`

`g :: a -> b -> c`



liftA2

`liftA2 g x y`

`liftA2 :: (a -> b -> c) -> f a -> f b -> f c`

`g :: a -> b -> c`

`x :: f a`

`y :: f b`

`liftA2 g x y :: f c`

`pure g <*> x <*> y`

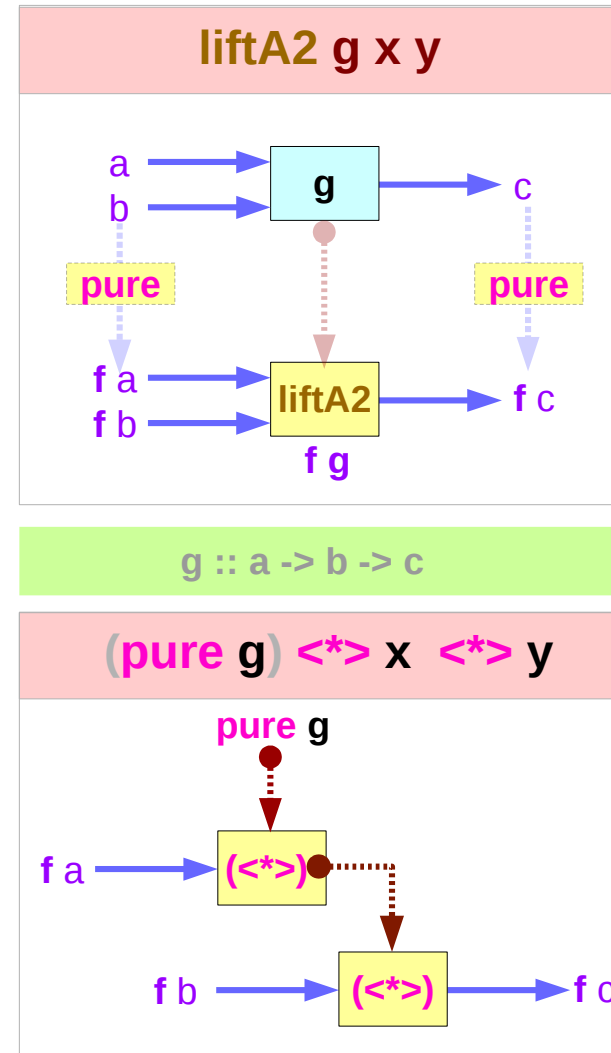
`g :: a -> b -> c`

`x :: f a`

`y :: f b`

`z :: f c`

`pure g <*> x <*> y :: f c`



https://wiki.haskell.org/Applicative_functor

Limitations of Functors

$(a \rightarrow b \rightarrow c) \rightarrow (f a \rightarrow f b \rightarrow f c)$ – let `fmap2` Functor as an extension of `fmap`

`fmap` :: $(a \rightarrow b) \rightarrow (f a \rightarrow f b)$

`fmap2` :: Functor `f` => $(a \rightarrow b \rightarrow c) \rightarrow (f a \rightarrow f b \rightarrow f c)$

`fmap2` `h` `fa` `fb` = undefined

`h` :: $a \rightarrow b \rightarrow c$

`fa` :: `f a`

`fb` :: `f b`

`h` :: $a \rightarrow (b \rightarrow c)$

`fmap` `h` :: $f a \rightarrow f (b \rightarrow c)$

`fmap` `h` `fa` :: $f (b \rightarrow c)$ – now `f (b -> c)` must be applied to `f b`

`fmap` gives us a way to apply functions $(a \rightarrow b)$ to values `(f a)` inside a Functor context, but `fmap` cannot be used to apply a functions $f (b \rightarrow c)$ which are themselves in a Functor context to values `f b` in a Functor context.

<http://www.openhaskell.com/lectures/applicative.html>

pure, fmap, and liftA2

```
class Functor f => Applicative f where
  pure  :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b
```

```
pure    :: a -> f a           - fmap0 → pure
fmap    :: (a -> b) -> f a -> f b - fmap1 → fmap
fmap2   :: (a -> b -> c) -> f a -> f b -> f c - fmap2 → liftA2
```

```
liftA2 :: Applicative f => (a -> b -> c) -> f a -> f b -> f c
```

```
liftA2 h fa fb = (h `fmap` fa) <*> fb
```

```
liftA2 h fa fb = h <$> fa <*> fb
```

```
(<$>) :: Functor f => (a -> b) -> f a -> f b
```

```
(<$>) = fmap
```

```
liftA2 :: Applicative f => (a -> b -> c -> d) -> f a -> f b -> f c -> f d
```

```
liftA3 h fa fb fc = ((h <$> fa) <*> fb) <*> fc
```

<http://www.openhaskell.com/lectures/applicative.html>

liftA2 examples

liftA2 :: Applicative f => (a -> b -> c) -> f a -> f b -> f c

liftA2 (+) (Just 5) (Just 6) = Just 11

liftA2 h fa fb = (h `fmap` fa) <*> fb

liftA2 h fa fb = h <\$> fa <*> fb

fmap (+) (Just 5) = Just (+5)

(+) <\$> (Just 5) = Just (+5)

<*> :: Applicative f => f (a -> b) -> f a -> f b

(Just (+5)) <*> (Just 6) = Just 11

let v1 = IO (Just (+5))

let v2 = IO (Just 6)

liftA2 (<*>) v1 v2 = IO (Just 11)

<https://blog.ssanj.net/posts/2014-08-10-boosting-liftA2.html>

<*> or liftA2 implementations

liftA2 :: (a -> b -> c) -> f a -> f b -> f c

A minimal complete definition :

either one of the two

- 1) pure and <*>
- 2) pure and liftA2

- 1) pure g <*> x <*> y
- 2) liftA2 g x y

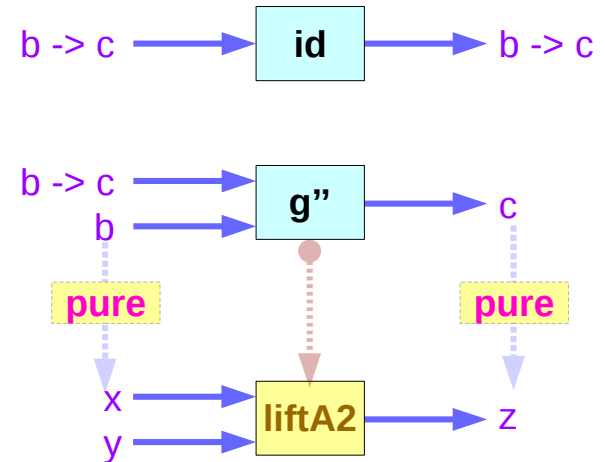
If it defines both, then they must behave the same as their default definitions:

<http://hackage.haskell.org/package/base-4.10.1.0/docs/Control-Applicative.html#v:liftA2>

$\langle * \rangle = \text{liftA2 id}$

$\text{liftA2 id } x \ y = \text{id } \langle \$ \rangle x \langle * \rangle y = x \langle * \rangle y$

$\text{liftA2 id } x \ y = x \langle * \rangle y$



$\text{liftA2 } g \ x \ y = g \langle \$ \rangle x \langle * \rangle y$

$g :: a \rightarrow b \rightarrow c$

$x :: f \ a$

$y :: f \ b$

$\text{liftA2 } g'' \ x \ y = g'' \langle \$ \rangle x \langle * \rangle y$

$g'' :: (b \rightarrow c) \rightarrow b \rightarrow c$

$x :: f \ (b \rightarrow c)$

$y :: f \ b$

$\text{liftA2 id } x \ y = \text{id } \langle \$ \rangle x \langle * \rangle y = x \langle * \rangle y$

$\text{id} :: (b \rightarrow c) \rightarrow (b \rightarrow c)$

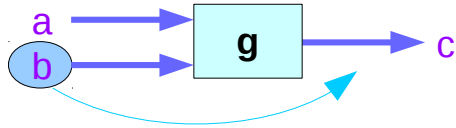
$x :: f \ (b \rightarrow c)$

$y :: f \ b$

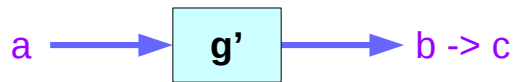
$\langle * \rangle = \text{liftA2 id}$

<http://hackage.haskell.org/package/base-4.10.1.0/docs/Control-Applicative.html#v:liftA2>

$g'' :: (b \rightarrow c) \rightarrow b \rightarrow c$



$g :: a \rightarrow b \rightarrow c$

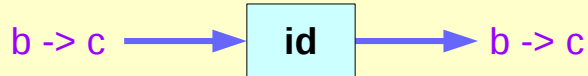


$g' :: a \rightarrow (b \rightarrow c)$

view the function
as having one input only

consider the case
when a is $(b \rightarrow c)$

Then g'' is the same as id



$id :: (b \rightarrow c) \rightarrow (b \rightarrow c)$



$g'' :: (b \rightarrow c) \rightarrow (b \rightarrow c)$

Results and effects in a scope

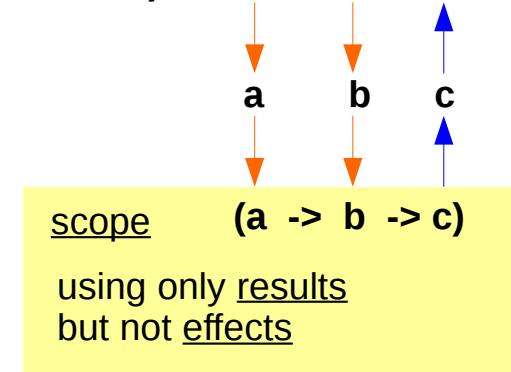
Actually, using the **liftA** commands
we can pull results of applicative functors
into a scope where we can talk

exclusively about functor results **c**
and not about effects. **f c**

Note that functor results can also be functions. **c**

This scope is simply a function,
which contains the code that we used in the non-functorial setting.

liftA2 :: (a -> b -> c) -> f a -> f b -> f c

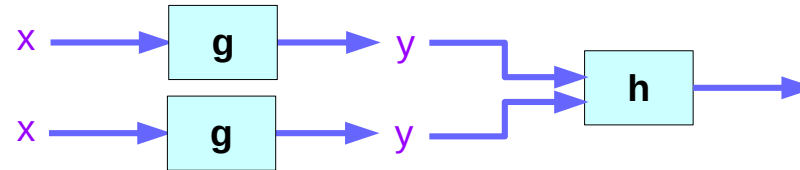


<http://hackage.haskell.org/package/base-4.10.1.0/docs/Control-Applicative.html#v:liftA2>

liftA3 – a non-functorial expression

Consider the non-functorial expression:

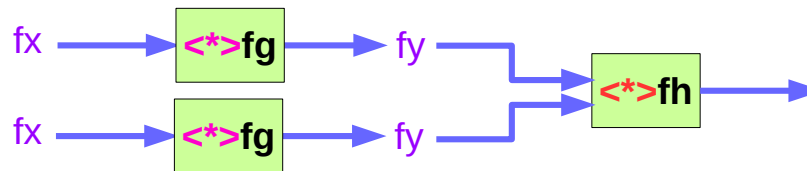
```
x :: x
g :: x -> y
h :: y -> y -> z
```



```
let y = g x
in h y y
```

generalization

```
fx :: f x
fg :: f (x -> y)
fh :: f (y -> y -> z)
```

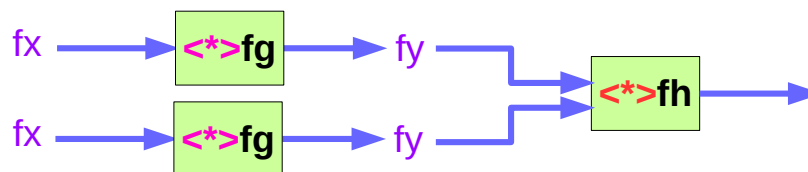


https://wiki.haskell.org/Applicative_functor

liftA3 – using `<*>` only

```
let fy = fg <*> fx
    in fh <*> fy <*> fy
```

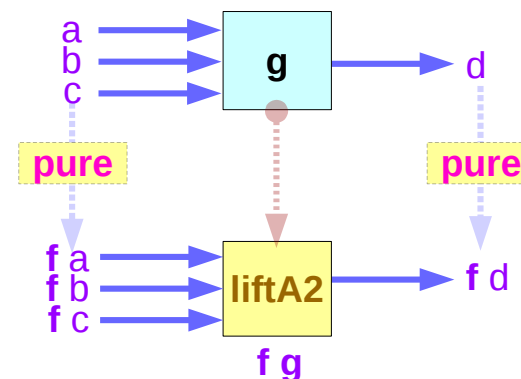
if **fy** writes something to the terminal
then **fh <*> fy <*> fy** writes twice.



this runs the effect of **fy** twice.

How the effect is run only once and the result is used twice?

→ utilize **liftA3**



https://wiki.haskell.org/Applicative_functor

liftA3 – using three input function

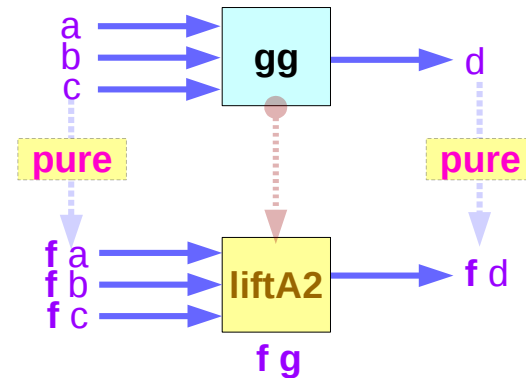
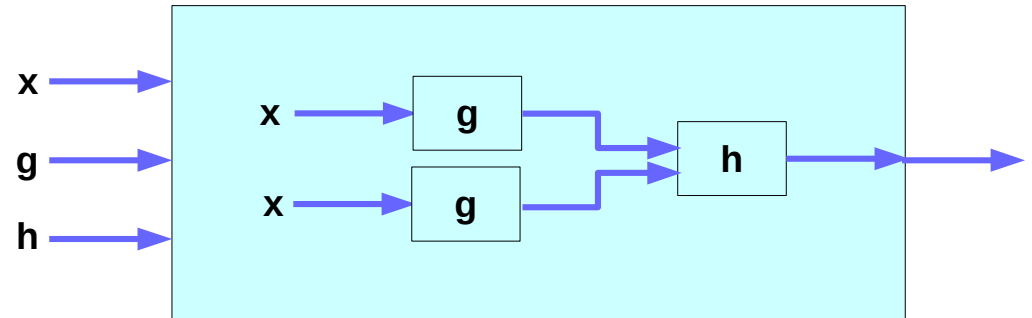
liftA3 :: Applicative f => (a -> b -> c -> d) -> f a -> f b -> f c -> f d

liftA3 h fa fb fc = ((h <\$> fa) <*> fb) <*> fc

liftA3

```
(\x g h -> let y = g x in h y y)
```

```
fx fg fh
```



<http://hackage.haskell.org/package/base-4.10.1.0/docs/Control-Applicative.html#v:liftA2>

liftA3 – effects, results and scopes

Actually, using the **liftA** commands

we can pull results of applicative functors

y from **fy**

into a scope where we can talk

y -> y -> z

exclusively about functor results

y

and not about effects.

fy

Note that functor results can also be functions.

y

This scope is simply a function,

y -> y -> z

which contains the code that we used in the non-functorial setting.

liftA3

```
(\x g h -> let y = g x in h y y)
```

```
fx fg fh
```

The order of effects is entirely determined by the order of arguments to liftA3

https://wiki.haskell.org/Applicative_functor

liftA2 (<*>)

10

down vote

accepted

The wiki article says that **liftA2 (<*>)** can be used to compose applicative functors.

It's easy to see how to use it from its type:

o :: (Applicative **f**, Applicative **f1**) =>

f (f1 (a -> b)) -> f (f1 a) -> f (f1 b)

o = liftA2 (<*>)

<https://stackoverflow.com/questions/12587195/examples-of-haskell-applicative-transformers>

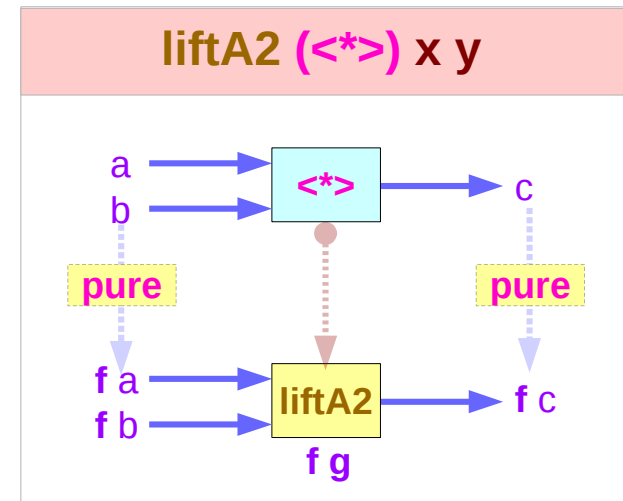
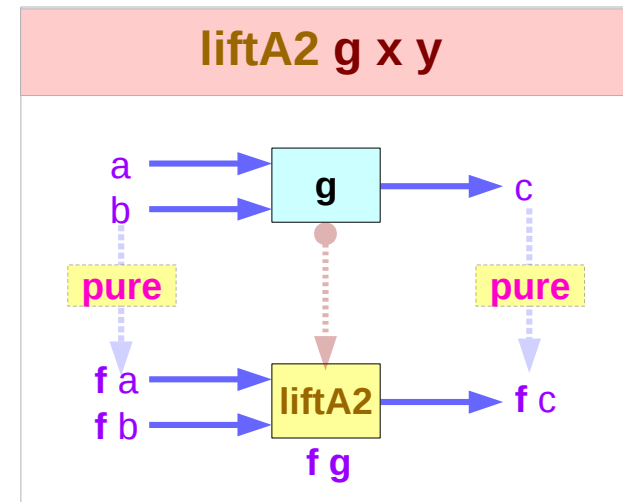
liftA2 (<*>) for composite applicative functors

`o :: (Applicative f, Applicative f1) =>`
`f (f1 (a -> b)) -> f (f1 a) -> f (f1 b)`

`o = liftA2 (<*>)`

`f1 (a -> b) <*> f1 a <*> f1 b`

liftA2 (<*>) can be used to compose applicative functors.



<https://stackoverflow.com/questions/12587195/examples-of-haskell-applicative-transformers>

liftA2 (<*>) Examples (1)

if `f` is `Maybe` and `f1` is `[]` we get:

`[(+1), (+6)]` `[1, 6]`

`Just [(+1),(+6)] `o` Just [1, 6]`

`(+1) [1, 6]`

`Just [2,7,7,12]`

`(+6) [1, 6]`

`liftA2 (<*>) Just [(+1),(+6)] Just [1, 6]`

`Just ([(+1),(+6)] <*> [1, 6])`

`Just [2, 7, 7, 12])`

<https://stackoverflow.com/questions/12587195/examples-of-haskell-applicative-transformers>

liftA2 (<*>) Examples (2)

if `f` is `Maybe` and `f1` is `[]` we get:

`[Just (+1),Just (+6)]`

`[Just 1, Just 6]`

`[Just (+1),Just (+6)] `o` [Just 1, Just 6]`

`Just (+1) [Just 1, Just 6]`

`[Just 2, Just 7, Just 7, Just 12]`

`Just (+6) [Just 1, Just 6]`

`liftA2 (<*>) [Just (+1),Just (+6)] [Just 1, Just 6]`

`[Just (+1) <*> Just 1, Just (+1) <*> Just 6, Just (+6) <*> Just 1, Just (+6) <*> Just 6]`

`[Just 2, Just 7, Just 7, Just 12]`

<https://stackoverflow.com/questions/12587195/examples-of-haskell-applicative-transformers>

liftA2 (:)

your ex function is equivalent to **liftA2 (:)**:

```
test1 = liftA2 (:) "abc" ["pqr", "xyz"]
```

To use **(:)** with deeper applicative stack
you need multiple applications of **liftA2**:

```
(liftA2 . liftA2) (:) (Just "abc") (Just ["pqr", "xyz"])  
Just ["apqr", "axyz", "bpqr", "bxyz", "cpqr", "cxyz"]
```

However it only works when both operands are equally deep.
So besides double liftA2 you should use pure to fix the level:

```
(liftA2 . liftA2) (:) (pure "abc") (Just ["pqr", "xyz"])  
Just ["apqr", "axyz", "bpqr", "bxyz", "cpqr", "cxyz"]
```

<https://stackoverflow.com/questions/12587195/examples-of-haskell-applicative-transformers>

<\$> related operators

Functor map <\$>

<\$> :: Functor f => (a -> b) -> f a -> f b

<\$:: Functor f => a -> f b -> f a

\$> :: Functor f => f a -> b -> f b

The <\$> operator is just a synonym
for the **fmap** function from the Functor typeclass.

This function generalizes the **map** function for lists
to many other data types, such as **Maybe**, **IO**, and **Map**.

<https://haskell-lang.org/tutorial/operators>

<\$> examples

```
#!/usr/bin/env stack
-- stack --resolver ghc-7.10.3 runghc
import Data.Monoid ((<>))

main :: IO ()
main = do
    putStrLn "Enter your year of birth"
    year <- read <$> getLine
    let age :: Int
        age = 2020 - year
    putStrLn $ "Age in 2020: " <> show age
```

<https://haskell-lang.org/tutorial/operators>

<\$, \$> operators

In addition, there are two additional operators provided which replace a value inside a Functor instead of applying a function.

This can be both more convenient in some cases, as well as for some Functors be more efficient.

value <\$ functor = const value <\$> functor

functor \$> value = const value <\$> functor

x <\$ y = y \$> x

x \$> y = y <\$ x

<https://haskell-lang.org/tutorial/operators>

<*> related operators

Applicative function application <*>

<*> :: Applicative f => f (a -> b) -> f a -> f b

(*>) :: Applicative f => f a -> f b -> f b

<*> :: Applicative f => f a -> f b -> f a

Commonly seen with <\$>, <*> is an operator that applies a wrapped function to a wrapped value.

It is part of the Applicative typeclass, and is very often seen in code like the following:

foo <\$> bar <*> baz

<https://haskell-lang.org/tutorial/operators>

<*> examples

For cases when you're dealing with a Monad, this is equivalent to:

```
do x <- bar
  y <- baz
  return (foo x y)
```

Other common examples including parsers and serialization libraries.

Here's an example you might see using the aeson package:

```
data Person = Person { name :: Text, age :: Int } deriving Show
```

```
-- We expect a JSON object, so we fail at any non-Object value.
```

```
instance FromJSON Person where
```

```
  parseJSON (Object v) = Person <$> v .: "name" <*> v .: "age"
  parseJSON _ = empty
```

<https://haskell-lang.org/tutorial/operators>

*> operator

To go along with this, we have two helper operators that are less frequently used:

`*>` ignores the value from the first argument. It can be defined as:

```
a1 *> a2 = (id <$ a1) <*> a2
```

Or in do-notation:

```
a1 *> a2 = do  
  _ <- a1  
  a2
```

For Monads, this is completely equivalent to `>>`.

<https://haskell-lang.org/tutorial/operators>

<* operator

<* is the same thing in reverse: perform the first action then the second, but only take the value from the first action.

Again, definitions in terms of <*> and do-notation:

(<*) = liftA2 const

a1 <* a2 = do

res <- a1

_ <- a2

return res

<https://haskell-lang.org/tutorial/operators>

(*> v.s. >>) and (pure v.s. return)

(*>) :: **Applicative** f => f a -> f b -> f b

(>>) :: **Monad** m => m a -> m b -> m b

pure :: **Applicative** f => a -> f a

return :: **Monad** m => a -> m a

the constraint changes from **Applicative** to **Monad**.

(*>) in **Applicative**

(>>) in **Monad**

pure in **Applicative**

return in **Monad**

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

References

- [1] <ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf>
- [2] <https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf>