OpenMP Loop Parallelism (2A)

Loop

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The simplest way to create parallelism in OpenMP is to use the **parallel** pragma.

```
A block preceded by the omp parallel pragma is called a parallel region;
```

it is executed by a <u>newly created team</u> of **threads**.

This is an instance of the **SPMD** model: all **threads** execute the <u>same</u> segment of code.

```
#pragma omp parallel
{
  // this is executed by a team of threads
}
```

It would be pointless to have the block be executed identically by all threads.

One way to get a meaningful parallel code is to use the function omp_get_thread_num, to find out which thread you are, and execute work that is individual to that thread.

There is also a function **omp_get_num_threads** to find out the total number of threads.

Both these functions give a number relative to the current team; recall from figure 15.3 that new teams can be created recursively.

Immediately preceding the **parallel block**, one **thread** will be executing the code.

In the main program this is the initial thread.

At the <u>start</u> of the block, <u>a new team</u> of **threads** is <u>created</u>, and the **thread** that was <u>active</u> <u>before</u> the block becomes the **master thread** of that team.

After the block, <u>only</u> the **master thread** is <u>active</u>.

Inside the block there is **team** of **threads**:

each **thread** in the **team** <u>executes</u> the body of the block, and it will have <u>access</u> to all variables of the surrounding environment.

How many threads there are can be determined in a number of ways;

the **threads** that are <u>forked</u> are all **copies** of the **master thread**: they have <u>access</u> to all that was computed so far; this is their **shared data**

Of course, if the **threads** were completely <u>identical</u> the parallelism would be pointless, so they also have **private data**, and they can <u>identify</u> themselves: they know their **thread number**.

this allows you to do meaningful parallel computations with threads.

work sharing constructs
In a team of threads,
initially there will be replicated execution;
a work sharing construct divides
available parallelism over the threads.

OpenMP uses **teams** of **threads**, and inside a **parallel region** the **work** is <u>distributed</u> over the **threads** with a **work sharing construct**. **threads** can <u>access</u> **shared data**, and they have some **private data**.

An important difference between **OpenMP** and **MPI** is that parallelism in **OpenMP** is **dynamically activated** by a **thread spawning** a **team** of **threads**.

Furthermore, the **number** of **threads** used can <u>differ</u> between **parallel regions**, and **threads** can <u>create</u> **threads** recursively.

This is known as as dynamic mode.

By contrast, in an **MPI** program the **number** of running processes is (mostly) <u>constant</u> throughout the run, and determined by factors <u>external</u> to the program.

OpenMP parallel loops are an example of OpenMP `worksharing' constructs

take an amount of work and distribute it over the available threads in a parallel region.

The parallel execution of a loop can be handled a number of different ways.

For instance, you can create a parallel region around the loop, and adjust the **loop bounds**:

```
#pragma omp parallel
{
  int threadnum = omp_get_thread_num(),
     numthreads = omp_get_num_threads();

int low = N*threadnum/numthreads,
    high = N*(threadnum+1)/numthreads;

for (i=low; i<high; i++)
  // do something with i
}</pre>
```

use the **parallel for** pragma:

```
#pragma omp parallel
#pragma omp for
for (i=0; i<N; i++) {
    // do something with i
}</pre>
```

you don't have to calculate the loop bounds for the threads yourself,

but you can also tell OpenMP to assign the loop iterations according to different schedules

```
#pragma omp parallel
{
   code1();
   #pragma omp for
   for (i=1; i<=4*N; i++) {
      code2();
   }
   code3();
}</pre>
```

The code before and after the loop is executed identically in each thread; t

he loop iterations are spread over the four threads.

Without #pragma omp for

code1	code1	code1	code1
code2	code2	code2	code2
(1:N)	(1:N)	(1:N)	(1:N)
code2	code2	code2	code2
(N+1	(N+1	(N+1	(N+1
:2N)	:2N)	:2N)	:2N)
code2	code2	code2	code2
(2N+1	(2N+1	(2N+1	(2N+1
:3N)	:3N)	:3N)	:3N)
code2	code2	code2	code2
(3N+1	(3N+1	(3N+1	(3N+1
:4N)	:4N)	:4N)	:4N)
code3	code3	code3	code3

With #pragma omp for

code1	code1	code1	code1
code2 (1:N)	code2 (N+1 :2N)	code2 (2N+1 :3N)	code2 (3N+1 :4N)
code3	code3	code3	code3

Note that the **parallel do** and **parallel for** pragmas do not create a team of threads: they take the team of threads that is active, and divide the loop iterations over them.

This means that the **omp for** or **omp do** directive needs to be <u>inside</u> a **parallel region**. It is also possible to have a <u>combined</u> **omp parallel for** or **omp parallel do** directive.

If your parallel region only contains a loop, you can combine the pragmas for the parallel region and distribution of the loop iterations:

```
#pragma omp parallel for
for (i=0; .....
```

Note that the **parallel do** and **parallel for** pragmas do not create a team of threads: they take the team of threads that is active, and divide the loop iterations over them.

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```
#pragma omp parallel for
for (i=0; .....
```

Loop Schedules (1)

more iterations in a loop than **threads** several ways to <u>assign</u> loop iterations to the **threads** OpenMP lets you specify this with the **schedule clause**.

#pragma omp for schedule(....)

Static schedules

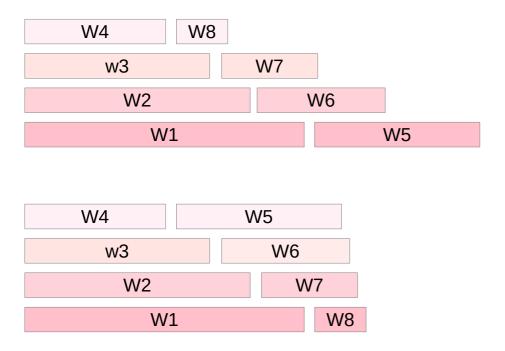
the iterations are assigned purely based on the <u>number of iterations</u> and the <u>number of threads</u> (and the **chunk parameter**; see later).

Dynamic schedules

iterations are assigned to threads that are <u>unoccupied</u>.

when iterations take an <u>unpredictable</u> amount of time, so **load balancing** is needed.

Loop Schedules (2)



Loop Schedules (3)

Static											
	thr0		th	r1		thr2		thr3			
Static,											
thr0	thr1	thr2	thr3	thr0	thr1	thr2	thr3	thr0	thr1	thr2	
Dynam	nic										
thr0	thr1	thr2	thr3	thr1	thr0	thr2	thr1	thr3	thr2	thr1	
Guideo	1										
thr0	thr	1	thr2	thr3	thr0	thr1	thr2	thr3	thr0	thr1	

Loop Schedules (4)

assume that each core gets assigned two (blocks of) iterations and these blocks take gradually less and less time.

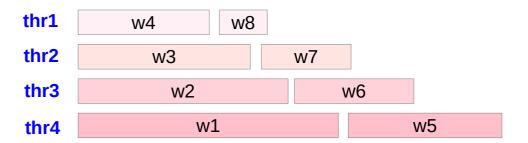
thread 1 gets two fairly long blocks, whereas thread 4 gets two short blocks,

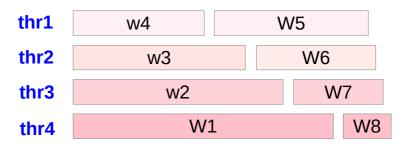
Thread 1 finishes much earlier.

Imbalance: unequal amounts of work

load balancing

thread 4 gets block 5, since it finishes the first set of blocks early. The effect is a perfect





Loop Schedule - Static (1)

The **default static schedule** is to assign one consecutive **block** of iterations to each **thread**.

#pragma omp for schedule(static)
#pragma omp for schedule(static, chunk)

With **static scheduling**, the compiler will <u>split up</u> the loop iterations <u>at compile time</u>,

When the iterations take <u>roughly</u> the <u>same</u> amount of time, this is the most <u>efficient</u> at runtime.

Static

thr0	thr1	thr2	thr3
uno.	U11 ±	UII Z	uno

Loop Schedule - Static (2)

#pragma omp for schedule(static,chunk)

If you want <u>different sized blocks</u> you can defined a **chunk size chunk**

The choice of a **chunk size** is often a balance between the **low overhead** of having only <u>a few chunks</u>, (big chunks) versus the **load balancing effect** of having <u>smaller chunks</u>. (many chunks)

Static, n



Loop Schedule - Static (3)

#pragma omp for schedule(static,chunk)

OpenMP <u>divides</u> the iterations into chunks of size <u>chunk-size</u>

distributes the chunks to **threads** in a circular order.

When <u>no</u> chunk-size is specified, OpenMP divides iterations into chunks that are approximately <u>equal in size</u> and distributes <u>at most</u> one **chunk** to each **thread**.

Static, n



Loop Schedule - Static (4)

Static scheduling is used when you know that each thread will do the approximately same amount of work at the **compile time**.

the following code can be parallelized using OMP. (assume only 4 threads)

```
float A[100][100];

for(int i = 0; i < 100; i++)
{
    for(int j = 0; j < 100; j++)
    {
        A[i][j] = 1.0f;
    }
}</pre>
```

100 * 100 iterations 10000 / 4 iterations / thread

https://stackoverflow.com/questions/15508128/using-omp-schedule-with-pragma-omp-for-parallel-scheduleruntime and the stackoverflow of the stackoverflow of

Loop Schedule - Static (5)

```
float A[100][100];
#pragma omp for schedule(static)
for(int i = 0; i < 100; i++)
  for(int j = 0; j < 100; j++)
    A[i][i] = 1.0f;
                                   (75:99)
 (0:24)
            (25:49)
                        (50:74)
                        (0.99)
                                    (0:99)
            (0.99)
 (0:99)
```

to use the default static scheduling, place pragma on the outer for loop,

then each thread will do 25% of the outer loop (i) work and eqaul amount of inner loop (j) work

Hence, the total amount of work done by each thread is <u>same</u>.

Hence, we could simply stick with the default static scheduling to give optimal load balancing.

https://stackoverflow.com/questions/15508128/using-omp-schedule-with-pragma-omp-for-parallel-scheduleruntime and the state of the sta

Loop Schedule - Static (6)

a for loop with 64 iterations 4 threads each row represents a thread.

schedule(static) has 16 iterations in the first row.

64 iterations and 4 threads \rightarrow 64 / 4 = 16

the first tread executes iterations 1, 2, 3, ..., 15 and 16.

the second thread executes iterations 17, 18, 19, ..., 31, 32.

Similar applies to the threads three and four.

For schedule(static),

OpenMP divides iterations into four chunks of size 16 and it distributes them to four threads.

For schedule(static, 4) and schedule(static, 8)

OpenMP divides iterations into chunks of size 4 and 8, respectively.

The static scheduling type is appropriate when all iterations have the same computational cost.

Loop Schedule - Static (6)

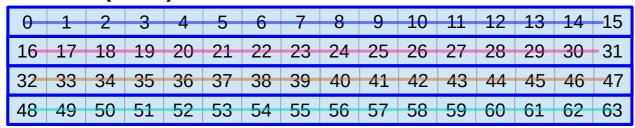
schedule(static, 4)

0	1	2	-3	16	17	18	19	32	33	34	35	48	49	50	51
4	5	6	7	20	21	22	23	36	37	38	39	52	53	54	55
8	9	10	11	24	25	26	27	40	41	42	43	56	57	58	59
12	13	14	1 5	28	29	30	31	44	45	46	47	60	61	62	-63

schedule(static, 8)

0-	1	2	3	4	-5	6	-7	32	33	34	35	36	37	38	3 9
8	9	10	11	12	13	14	1 5	40	41	42	43	44	45	46	-4 7
16	17	18	19	20	21	22	23	48	49	50	51	52	53	54	5 5
24	25	26	27	28	29	30	31	56	57	58	59	60	61	62	63

schedule(static)



Loop Schedule - Dynamic (1)

In **dynamic scheduling** OpenMP will put **blocks** of iterations in a **task queue**, (the **default chunk size** is **1**) and the **threads** take **one** of these tasks whenever they are **finished** with the previous.

#pragma omp for schedule(dynamic[,chunk])

While this schedule may give **good load balancing** if the iterations take very <u>differing</u> amounts of time to execute, it does carry **runtime overhead** for managing the **queue** of iteration tasks.

Dynamic

thr0	thr1	thr2	thr3	thr1	thr0	thr2	thr1	thr3	thr2	thr1
------	------	------	------	------	------	------	------	------	------	------

Loop Schedule - Dynamic (2)

<u>large</u> **chunks** carry the <u>least</u> **overhead**, but <u>smaller</u> chunks are <u>better</u> for **load balancing**.

If you don't want to decide on a schedule in your code, you can specify the schedule will then **at runtime** be read from the OMP SCHEDULE environment variable.

You can even just leave it to the runtime library by specifying **omp_set_schedule**.

Dynamic

thr0	thr1 th	hr2 thr3	thr1	thr0	thr2	thr1	thr3	thr2	thr1
------	---------	----------	------	------	------	------	------	------	------

Loop Schedule - Dynamic (3)

for schedule(dynamic, chunk-size)

the dynamic scheduling type

OpenMP divides the iterations into chunks of size chunk-size.

Each thread <u>executes</u>

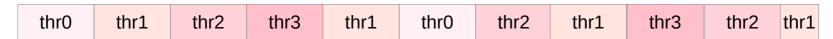
a chunk of iterations and
then <u>requests</u> another chunk
until there are <u>no</u> more chunks available.

There is <u>no particular order</u> in which the chunks are distributed to the threads.

The order changes each time when we execute the for loop.

If we do <u>not</u> specify **chunk-size**, it defaults to one.

Dynamic

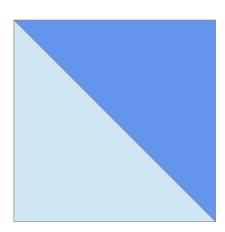


Loop Schedule - Dynamic (4)

Dynamic scheduling is used when you know that each thread will <u>not</u> do <u>same</u> amount of work by using static scheduling.

```
float A[100][100];

for(int i = 0; i < 100; i++)
{
    for(int j = 0; j < i; j++)
    {
        A[i][j] = 1.0f;
    }
}
```



https://stackoverflow.com/questions/15508128/using-omp-schedule-with-pragma-omp-for-parallel-scheduleruntime and the state of the sta

Loop Schedule - Dynamic (5)

```
float A[100][100];

for(int i = 0; i < 100; i++)
{
    for(int j = 0; j < i; j++)
    {
        A[i][j] = 1.0f;
    }
}</pre>
```



The inner loop variable **j** is <u>dependent</u> on the **i**.

the default static scheduling,

- the outer loop (i) work might be divided equally between the 4 threads,
- but the inner loop (j) work will be large for some threads.
- not equal amount of work
- not optimal load balancing

https://stackoverflow.com/questions/15508128/using-omp-schedule-with-pragma-omp-for-parallel-scheduleruntime and the state of the sta

Loop Schedule - Dynamic (6)

the dynamic scheduling

this scheduling is done at the run time can make sure optimal **load balance**.

Note:

you can also specify the chunk_size for scheduling. It depends on the **loop size**.

https://stackoverflow.com/questions/15508128/using-omp-schedule-with-pragma-omp-for-parallel-scheduleruntime

Loop Schedule - Dynamic (7)

for schedule(dynamic) and schedule(dynamic, 1)

OpenMP determines similar scheduling. the size of chunks is equal to **1** in both instances. the distribution of chunks between the threads is <u>arbitrary</u>.

For schedule(dynamic, 4) and schedule(dynamic, 8)

OpenMP divides iterations into chunks of size **4** and **8**, respectively. the distribution of chunks to the threads has <u>no pattern</u> (abitrary)

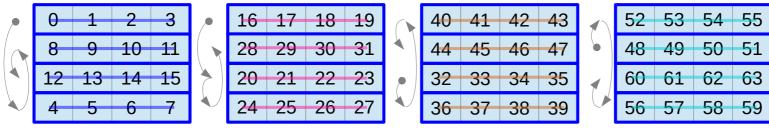
The dynamic scheduling type is appropriate when the iterations require different computational costs.

I.e, when the iterations are poorly balanced between each other.

the dynamic scheduling type has higher **overhead** than the static scheduling type because it dynamically distributes the iterations during the runtime.

Loop Schedule - Dynamic (6)

schedule(dynamic, 4)



schedule(dynamic, 8)

	16	17	18	19	20	21	22	23	1	40	41	42	43	44	45	46	4 7
	24	25	26	19 27	28	29	30	31	•	32	33	34	35	36	37	38	3 9
•	0-	1	2	3	4	5	6	-7	4	56	57	58	59	60	61	62	63 -55
•	8-	9	10	11	12	13	14	1 5		48	49	50	51	52	53	54	-5 5

schedule(dynamic)

16	1	50	35	36	5	22	Z	56	41	10	27	60	61	14	-1 5
48	33	34	51	52	37	38	5 5	40	57	42	59	44	29	46	-4 7
	49														
32	17	18	3	-4	53	54	39	24	25	2 6	43	12	45	62	31

Loop Schedule - Guided (1)

The **guided** scheduling type is similar to the **dynamic** scheduling type.

OpenMP again divides the iterations into chunks.

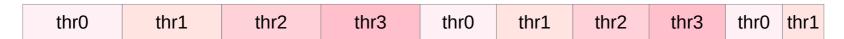
<u>Each thread</u> executes a chunk of iterations and then requests another chunk <u>until</u> there are no more chunks available.

The difference is in the size of chunks.

The size of a chunk is proportional to the number of unassigned iterations divided by the number of the threads.

Therefore the size of the chunks decreases as the execution goes on

Guided



Loop Schedule - Guided (2)

The minimum size of a chunk is set by 'chunk-size' in the scheduling clause:

for schedule(guided, chunk-size).

the chunk which contains the last iterations may have <u>smaller</u> size than chunk-size.

If we do not specify **chunk-size**, it defaults to **one**.

Guided

thr0	thr1	thr2	thr3	thr0	thr1	thr2	thr3	thr0	thr1
------	------	------	------	------	------	------	------	------	------

Loop Schedule - Guided (3)

the **size** of the **chunks** is <u>decreasing</u>. first chunk has always 16 iterations. 64 iterations and 4 threads \rightarrow 64 / 4 = 16

the **minimum chunk size** is determined in the schedule clause.

The only exception is the **last chunk**.

Its size might be lower then the prescribed minimum size.

The guided scheduling type is appropriate when the iterations are poorly balanced between each other.

The initial chunks are larger, because they reduce overhead.

The smaller chunks fills the schedule towards the end of the computation and improve load balancing.

This scheduling type is especially appropriate when poor load balancing occurs toward the end of the computation.

Loop Schedule - Guided (3)

schedule(guided)

64/4 = 16	64
(64-16) / 4 = 12	48
(48-12) / 4 = 9	36
(36-9) / 4 = 7	27
(27-7) / 4 = 5	20
(20-5) / 4 = 4	15
(15-4) / 4 = 3	11
(11-3) / 4 = 2	8
(8-2) / 4 = 2	6
(6-2) / 4 = 1	4
(4-1) / 4 = 1	3
(3-1) / 4 = 1	2
(2-1) / 4 = 1	1

schedule(guided, 4)

schedule(quided, 8)

64

64/4 = 16

Loop Schedule - Static (6)

schedule(static)

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47
48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63

schedule(guided)

64/4 = 16	64
(64-16) / 4 = 12	48
(48-12) / 4 = 9	36
(36-9) / 4 = 7	27
(27-7) / 4 = 5	20
(20-5)/A - A	15

45 46 47 48

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
16	17	18	19	20	21	22	23	24	25	26	27	49	50	51	52	(
28	29	30	31	32	33	34	35	36	53	54	55	61				
37	38	39	40	41	42	43	56	57	62				-			

59

58

63

$$(6-2) / 4 = 1$$

$$(4-1) / 4 = 1$$

$$(3-1) / 4 = 1$$

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
16	17	18	19	20	21	22	23	24	25	26	27	53	54	55	60
28	29	30	31	32	33	34	35	36	49	50	51	52	56	57	61
37	38	39	40	41	42	43	44	45	46	47	48	58	59	62	

Loop Schedule – Auto

The auto scheduling type <u>delegates</u> the decision of the scheduling to the **compiler** and/or **runtime** system.

Loop Schedule - Runtime

The schedule(runtime) clause tells it to set the schedule using the environment variable.

The environment variable can be set to any other scheduling type.

It can be set by

setenv OMP SCHEDULE "dynamic,5"

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Loop Schedules - Runtime

The **runtime** scheduling type <u>defers</u> the decision about the scheduling until the **runtime**.

different ways of specifying the scheduling type in this case.

One option is with the environment variable **OMP SCHEDULE**

and the other option is with the function omp_set_schedule.

Loop Schedules - Runtime

If the scheduling-type (in the schedule clause of the loop construct) is equal to runtime then OpenMP determines the scheduling by the internal control variable run-sched-var. We can set this variable by setting the environment variable OMP_SCHEDULE to the desired scheduling type. For example, in bash-like terminals, we can do

```
$ export OMP_SCHEDULE=sheduling-type
```

Another way to specify run-sched-var is to set it with omp set schedule function.

```
...
omp_set_schedule(sheduling-type);
...
```

Nested Parallelism (1)

```
void fun1()
  for (int i=0; i<80; i++)
main()
 #pragma omp parallel
    #pragma omp for
    for (int i=0; i<100; i++)
    #pragma omp for
    for (int i=0; i<10; i++)
       fun1();
```

```
the 2nd loop in main
can only be distributed to 10 threads
```

80 loop iterations in **fun1** which will be called **10** times in **main** loop.

total 800 iterations in fun1 and the main loop

This gives much more parallelism potential if parallelism can be added in both levels.

Nested Parallelism (2)

```
void fun1()
  #pragma omp parallel for
  for (int i=0; i<80; i++)
main
  #Pragma omp parallel
     #pragma omp for
     for (int i=0; i<100; i++)
    #pragma omp for
    for (int i=0; i<10; i++)
       fun1();
```

may either have <u>insufficient threads</u> for the 1st main loop as it has <u>larger loop count</u>, or

create exploded number of threads for the 2nd main loop when OMP_NESTED=TRUE.

The simple solution is to <u>split</u> the parallel region in main and create separate ones for each loop with a distinct thread number specified.

Nested Parallelism (3)

```
void fun1()
   #pragma omp taskloop
   for (int I = 0; i<80; i++)
main
  #pragma omp parallel
    #pragma omp for
    for (int i=0; i<100; i++)
    #pragma omp for
    for (int i=0; i<10; i++)
       fun1();
```

don't have to worry about the thread number changes in 1st and 2nd main loops.

Even though you still have a small amount of (10) threads allocated for 2nd main loop, the rest available threads will be able to be distributed through omp **taskloop** in fun1.

Nested Parallelism (4)

nested parallel regions is a way to <u>distribute</u> **tasks** by <u>creating</u> / <u>forking</u> more **threads**.

parallel region is the only construct determines execution thread number and controls thread affinity

Using **nested parallel regions** means each **thread** in **parent region** will yield multiple **threads** in enclosed regions, which in turn create a <u>product</u> of **thread number**.

Nested Parallelism (5)

omp tasking shows another way to explore parallelism by adding more **tasks**, instead of **threads**.

though the **thread number** is <u>unchanged</u> as specified <u>at the entry</u> of the **parallel region**, the <u>increased</u> **tasks** from the **nested tasking** constructs can be <u>distributed</u> and <u>executed</u> by any available/idle **threads** in the current team of the same parallel region.

This gives opportunities to fully use all threads' capability, and improve balance of workloads automatically.

Implicit task (1)

In addition to **explicit tasks** specified using the **task** directive, the OpenMP specification version 3.0 introduces the notion of **implicit tasks**.

An **implicit task** is a task generated

- by the implicit parallel region,
- when a parallel construct is encountered during execution.

The **code** for each **implicit task** is the code <u>inside</u> the **parallel construct**.

Each implicit task is

- assigned to a different thread in the team and is tied;
- <u>always executed</u> from <u>beginning</u> to <u>end</u>
 by the **thread** to which it is <u>initially assigned</u>.

https://docs.oracle.com/cd/E19205-01/820-7883/6nj43o69j/index.html

Implicit task (2)

All implicit tasks generated
when a parallel construct is encountered
are guaranteed to be complete
when the master thread exits the implicit barrier
at the end of the parallel region.

all **explicit tasks** generated <u>within</u> a **parallel region** are guaranteed to be <u>complete</u> on <u>exit</u> from the <u>next</u> <u>implicit</u> or <u>explicit barrier</u> <u>within</u> the **parallel region**.

https://docs.oracle.com/cd/E19205-01/820-7883/6nj43o69j/index.html

Implicit task (3)

When an **if clause** is present on a **task construct** and the value of the scalar-expression evaluates to **false**, the thread that encounters the task <u>must immediately</u> execute the task.

The **if clause** can be used to <u>avoid</u> the **overhead** of generating many **finely grained tasks** and placing them in the **conceptual pool**.

https://docs.oracle.com/cd/E19205-01/820-7883/6nj43o69j/index.html

Implicit barrier

Implicit BarriersSeveral OpenMP* constructs have implicit barriers

- parallel
- for
- single

Unnecessary barriers hurt performance

Waiting threads accomplish no work!

Waiting threads accomplish no work!
Suppress implicit barriers, when safe, with the nowait

https://www.intel.com/content/dam/www/public/apac/xa/en/pdfs/ssg/Programming_with_OpenMP-Linux.pdf

References

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