

Laurent Series and z-Transform - Geometric Series Applications

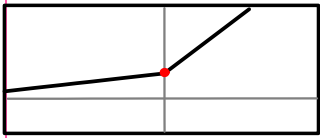


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a^n



*a
/a

Shifting a sequence

$$\frac{1}{1-az} \quad |z| < a^{-1}$$

$a^n u(n)$

$$\frac{a}{1-az} \quad |z| < a^{-1}$$

$a^{n+1} u(n)$

$$\frac{az}{1-az} \quad |z| < a^{-1}$$

$a^n u(n-1)$

$$\frac{z}{1-az} \quad |z| < a^{-1}$$

$a^{n-1} u(n-1)$

$$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \quad |z| > a^{-1}$$

$a^n u(-n-1)$

$$-\frac{z^{-1}}{1-a^{-1}z^{-1}} \quad |z| > a^{-1}$$

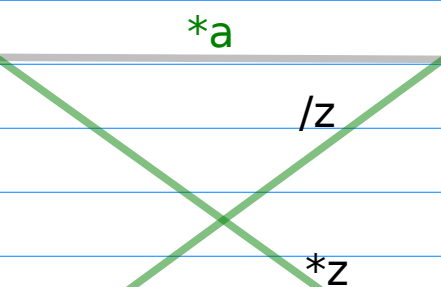
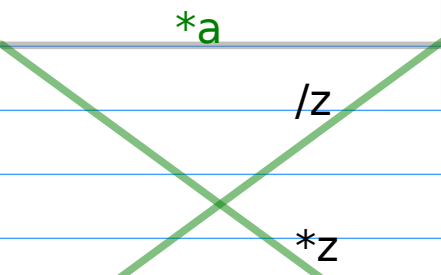
$a^{n+1} u(-n-1)$

$$-\frac{1}{1-a^{-1}z^{-1}} \quad |z| > a^{-1}$$

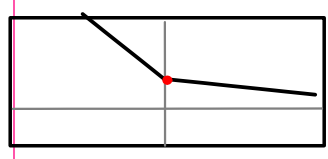
$a^n u(-n)$

$$-\frac{a^{-1}}{1-a^{-1}z^{-1}} \quad |z| > a^{-1}$$

$a^{n-1} u(-n)$



a^{-n}



/a
*a

Shifting a sequence

$\frac{1}{1-a^1z} \quad z < a$	$\xrightarrow{\quad /a \quad}$ $\xrightarrow{\quad /z \quad}$	$\frac{a^1}{1-a^1z} \quad z < a$
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$a^{-n} u(n)$

$a^{-n-1} u(n)$

$\frac{a^1z}{1-a^1z} \quad z < a$	$\xrightarrow{\quad *a \quad}$ $\xrightarrow{\quad *z \quad}$	$\frac{z}{1-a^1z} \quad z < a$
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$a^{-n} u(n-1)$

$a^{-n+1} u(n-1)$

$-\frac{a^1z^{-1}}{1-a^1z^{-1}} \quad z > a$	$\xrightarrow{\quad /a \quad}$ $\xrightarrow{\quad /z \quad}$	$-\frac{z^{-1}}{1-a^1z^{-1}} \quad z > a$
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$a^{-n} u(-n-1)$

$a^{-n-1} u(-n-1)$

$-\frac{1}{1-a^1z^{-1}} \quad z > a$	$\xrightarrow{\quad *a \quad}$ $\xrightarrow{\quad *z \quad}$	$-\frac{a}{1-a^1z^{-1}} \quad z > a$
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$a^{-n} u(-n)$

$a^{-n+1} u(-n)$

2 formulas

Simple Pole Form

$$\frac{1}{z - p}$$

$$\frac{1}{z^{-1} - p}$$

2 representations each

Geometric Series Form

$$\begin{array}{l} \frac{1}{z - p} \\ \swarrow \searrow \\ \frac{p^{-1}}{1 - p^{-1}z} \triangleq \overset{\text{causal}}{f(z)} = \overset{\text{anti-causal}}{\chi(z^{-1})} \\ \frac{z^{-1}}{1 - pz^{-1}} \triangleq \underset{\text{causal}}{\gamma(z)} = \underset{\text{anti-causal}}{g(z^{-1})} \end{array}$$

$$\begin{array}{l} \frac{1}{z^{-1} - p} \\ \swarrow \searrow \\ -\frac{p^{-1}}{1 - p^{-1}z^{-1}} \triangleq \overset{\text{causal}}{\chi(z)} = \overset{\text{anti-causal}}{f(z^{-1})} \\ \frac{z}{1 - pz} \triangleq \underset{\text{causal}}{g(z)} = \underset{\text{anti-causal}}{\gamma(z^{-1})} \end{array}$$

Simple Pole Form

Geometric Series Form

Geometric Series (1)

2 formulas

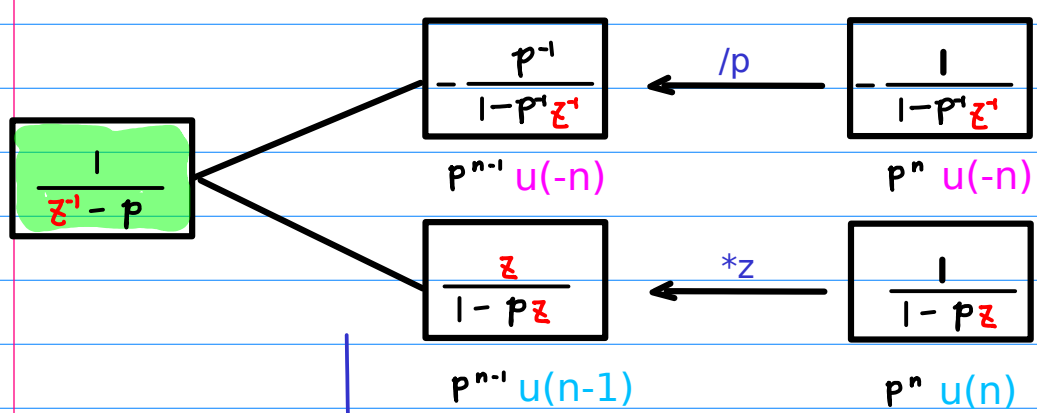
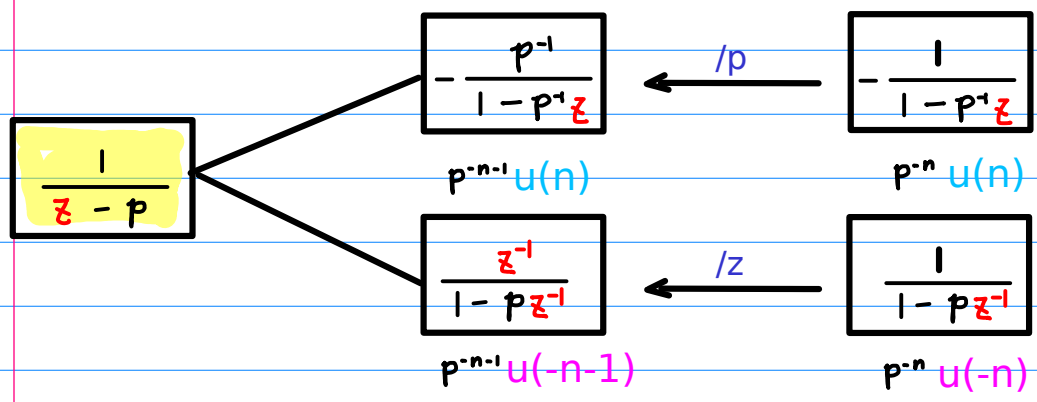
Simple Pole Form

$$\frac{1}{z - p}$$

$$\frac{1}{z^{-1} - p}$$

2 representations each

Geometric Series Form



Simple Pole Form

Geometric Series Form

Geometric Series (2)

2 formulas

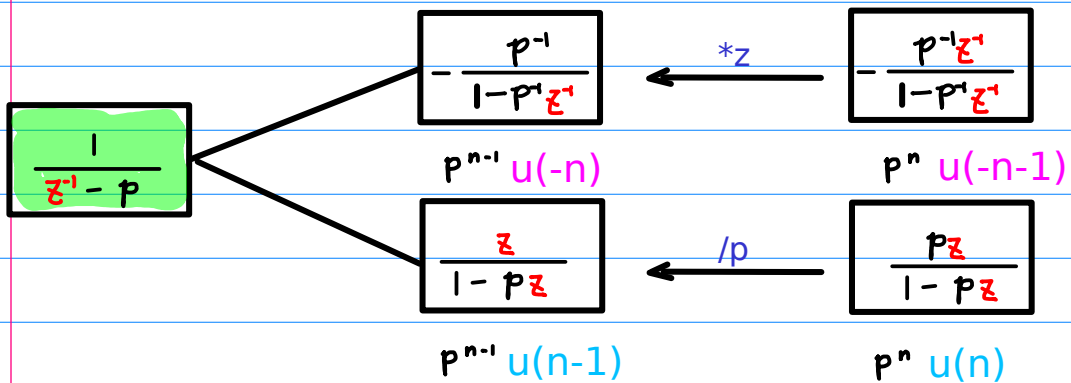
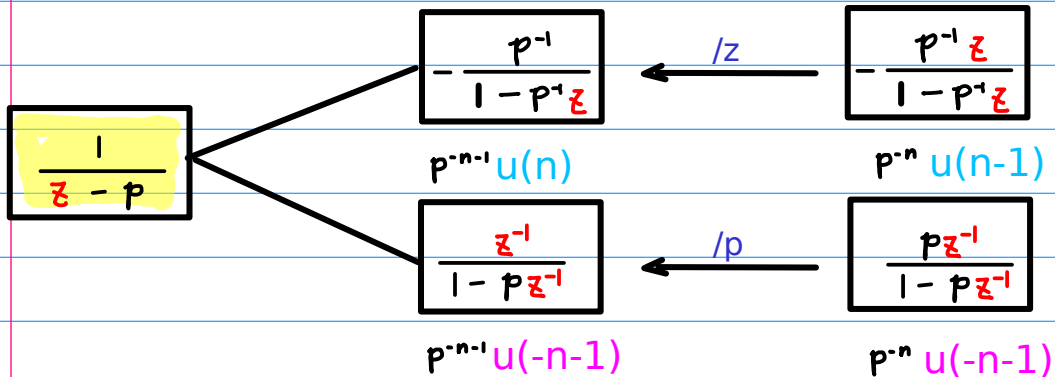
Simple Pole Form

$$\frac{1}{z - p}$$

$$\frac{1}{z^{-1} - p}$$

2 representations each

Geometric Series Form



Simple Pole Form

Geometric Series Form

Geometric Series Form Combinations with a unit start term

$$(1) \quad \frac{1}{1 - az} \quad -a^n u(n)$$

$$(2) \quad \frac{1}{1 - az^{-1}} \quad -a^{-n} u(-n)$$

$$(3) \quad \frac{1}{1 - a^{-1}z^{-1}} \quad a^n u(-n)$$

$$(4) \quad \frac{1}{1 - a^{-1}z} \quad a^{-n} u(n)$$

$$(4)' \quad \frac{1}{1 - a^{-1}z} \quad -a^{-n} u(n)$$

$$(3)' \quad \frac{1}{1 - a^{-1}z^{-1}} \quad -a^{-n} u(-n)$$

$$(2)' \quad \frac{1}{1 - az^{-1}} \quad a^{-n} u(-n)$$

$$(1)' \quad \frac{1}{1 - az} \quad a^n u(n)$$

Geometric Series Form Combinations with a common-ratio start term

$$(5) \quad + \frac{a^{-1}z^{-1}}{1 - a^{-1}z^{-1}}$$

$a^n u(-n-1)$

$$(6) \quad + \frac{a^{-1}z}{1 - a^{-1}z}$$

$a^{-n} u(n-1)$

$$(7) \quad - \frac{az}{1 - az}$$

$a^n u(n-1)$

$$(8) \quad - \frac{az^{-1}}{1 - az^{-1}}$$

$a^{-n} u(-n-1)$

$$(8)' \quad + \frac{az^{-1}}{1 - az^{-1}}$$

$a^{-n} u(-n-1)$

$$(7)' \quad + \frac{az}{1 - az}$$

$a^n u(n-1)$

$$(6)' \quad - \frac{a^{-1}z}{1 - a^{-1}z}$$

$a^{-n} u(n-1)$

$$(5)' \quad - \frac{a^{-1}z^{-1}}{1 - a^{-1}z^{-1}}$$

$a^n u(-n-1)$

Geometric Series - a unit start term

Laurent Series

Geometric Series - a unit start term

z-Transform

Geometric Series - a unit start term

Laurent Series vs. z-Transform

Geometric Series - a unit start term

Laurent Series

(1)
$$-\frac{1}{1-az}$$

$(a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$

$-a^n u(n) \quad (n \geq 0)$

(2)
$$-\frac{1}{1-az^{-1}}$$

$(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$

$(\frac{1}{a} z^0 + (\frac{1}{a})^2 z^{-1} + (\frac{1}{a})^3 z^{-2} + \dots)$

$-(\frac{1}{a})^n u(-n) \quad (n < 1)$

(3)
$$+\frac{1}{1-a^{-1}z^{-1}}$$

$(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$

$a^n u(-n) \quad (n < 1)$

(4)
$$+\frac{1}{1-az}$$

$(a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$

$(\frac{1}{a} z^0 + (\frac{1}{a})^2 z^1 + (\frac{1}{a})^3 z^2 + \dots)$

$(\frac{1}{a})^n u(n) \quad (n \geq 0)$

(4)'
$$-\frac{1}{1-a^{-1}z}$$

$(a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$

$(\frac{1}{a} z^0 + (\frac{1}{a})^2 z^1 + (\frac{1}{a})^3 z^2 + \dots)$

$-(\frac{1}{a})^n u(n) \quad (n \geq 0)$

(3)'
$$-\frac{1}{1-az^{-1}}$$

$(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$

$-a^n u(-n) \quad (n < 1)$

(2)'
$$+\frac{1}{1-az^{-1}}$$

$(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$

$(\frac{1}{a} z^0 + (\frac{1}{a})^2 z^{-1} + (\frac{1}{a})^3 z^{-2} + \dots)$

$(\frac{1}{a})^n u(-n) \quad (n < 1)$

(1)'
$$+\frac{1}{1-az}$$

$(a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$

$a^n u(n) \quad (n \geq 0)$

Geometric Series - a unit start term

z-Transform ($n \rightarrow -n$)

(1)
$$\frac{1}{1 - az}$$
 $|z| < a^{-1}$

- $(a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$

- $((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \dots)$

$-a^n u(-n)$ ($n \geq 0$)

$-(\frac{1}{a})^n u(-n)$ ($n < 0$)

(2)
$$\frac{1}{1 - az^{-1}}$$
 $|z| > a$

- $(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$

- $((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \dots)$

$-(\frac{1}{a})^n u(-(-n))$ ($n < 0$)

$-a^n u(n)$ ($n \geq 0$)

(3)
$$+\frac{1}{1 - a^{-1}z}$$
 $|z| > a^{-1}$

$(a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \dots)$

$a^n u(-(-n))$ ($n < 0$)

$(\frac{1}{a})^n u(n)$ ($n \geq 0$)

(4)
$$+\frac{1}{1 - a^{-1}z^{-1}}$$
 $|z| < a$

$(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \dots)$

$(\frac{1}{a})^n u((-n))$ ($n \geq 0$)

$a^n u(-n)$ ($n < 0$)

(4)'
$$\frac{1}{1 - a^{-1}z}$$
 $|z| < a$

- $(a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$

- $((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \dots)$

$-(\frac{1}{a})^n u((-n))$ ($n \geq 0$)

$-a^n u(-n)$ ($n < 0$)

(3)'
$$\frac{1}{1 - a^{-1}z^{-1}}$$
 $|z| > a^{-1}$

- $(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$

- $((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \dots)$

$-a^n u(-(-n))$ ($n < 0$)

$-(\frac{1}{a})^n u(n)$ ($n \geq 0$)

(2)'
$$+\frac{1}{1 - az^{-1}}$$
 $|z| > a$

$(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \dots)$

$(\frac{1}{a})^n u(-(-n))$ ($n < 0$)

$a^n u(n)$ ($n \geq 0$)

(1)'
$$+\frac{1}{1 - az}$$
 $|z| < a^{-1}$

$(a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \dots)$

$a^n u((-n))$ ($n \geq 0$)

$(\frac{1}{a})^n u(-n)$ ($n < 0$)

Geometric Series - a unit start term

Laurent Series vs. z-Transform ($n \rightarrow -n$)

(1) $\frac{1}{1 - az}$ $|z| < a^{-1}$ $\frac{1}{1 - az^{-1}}$ $|z| > a$ (2)

- $(a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$

- $((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \dots)$

- $(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$

- $((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \dots)$

Laurent $-a^n u(n)$ $(n \geq 0)$

$-(\frac{1}{a})^n u(-n)$ $(n < 1)$

z-Trans $-(\frac{1}{a})^n u(-n)$ $(n < 1)$

$-a^n u(n)$ $(n \geq 0)$

(3) $\frac{1}{1 - a^{-1}z^{-1}}$ $|z| > a^{-1}$ $\frac{1}{1 - a^{-1}z}$ $|z| < a$ (4)

$(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \dots)$

$(a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \dots)$

Laurent $a^n u(-n)$ $(n < 1)$

$(\frac{1}{a})^n u(n)$ $(n \geq 0)$

z-Trans $(\frac{1}{a})^n u(n)$ $(n \geq 0)$

$a^n u(-n)$ $(n < 1)$

(4)' $\frac{1}{1 - a^{-1}z}$ $|z| < a$ $\frac{1}{1 - a^{-1}z^{-1}}$ $|z| > a^{-1}$ (3)'

- $(a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$

- $((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \dots)$

- $(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$

- $((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \dots)$

Laurent $-(\frac{1}{a})^n u(n)$ $(n \geq 0)$

$-a^n u(-n)$ $(n < 1)$

z-Trans $-a^n u(-n)$ $(n < 1)$

$-(\frac{1}{a})^n u(n)$ $(n \geq 0)$

(2)' $\frac{1}{1 - az^{-1}}$ $|z| > a$ $\frac{1}{1 - az}$ $|z| < a^{-1}$ (1)'

$(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \dots)$

$(a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \dots)$

Laurent $(\frac{1}{a})^n u(-n)$ $(n < 1)$

$a^n u(n)$ $(n \geq 0)$

z-Trans $a^n u(n)$ $(n \geq 0)$

$(\frac{1}{a})^n u(-n)$ $(n < 1)$

Geometric Series - a non-unit start term

Laurent Series

Geometric Series - a non-unit start term

z-Transform

Geometric Series - a non-unit start term

Laurent Series vs. z-Transform

Geometric Series - a non-unit start term

Laurent Series

(5)

$$+ \frac{a^{-1}z^{-1}}{1 - a^{-1}z^{-1}}$$

$$|z| > a^{-1}$$

$$(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \dots)$$

$$a^n u(-n-1) \quad (n < 0)$$

(6)

$$+ \frac{a^{-1}z}{1 - a^{-1}z}$$

$$|z| < a$$

$$(a^{-1}z^1 + a^{-2}z^2 + a^{-3}z^3 + \dots)$$

$$((\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + (\frac{1}{a})^3 z^3 + \dots)$$

$$(\frac{1}{a})^n u(n-1) \quad (n \geq 1)$$

(7)

$$\frac{az}{1 - az}$$

$$|z| < a^{-1}$$

$$-(a^1 z^1 + a^2 z^2 + a^3 z^3 + \dots)$$

$$-a^n u(n-1) \quad (n \geq 1)$$

(8)

$$\frac{az^{-1}}{1 - az^{-1}}$$

$$|z| > a$$

$$-(a^1 z^{-1} + a^2 z^{-2} + a^3 z^{-3} + \dots)$$

$$-((\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + (\frac{1}{a})^3 z^{-3} + \dots)$$

$$-(\frac{1}{a})^n u(-n-1) \quad (n < 0)$$

(8)'

$$+ \frac{az^{-1}}{1 - az^{-1}}$$

$$|z| > a$$

$$(a^1 z^{-1} + a^2 z^{-2} + a^3 z^{-3} + \dots)$$

$$((\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + (\frac{1}{a})^3 z^{-3} + \dots)$$

$$(\frac{1}{a})^n u(-n-1) \quad (n < 0)$$

(7)'

$$+ \frac{az}{1 - az}$$

$$|z| < a^{-1}$$

$$(a^1 z^1 + a^2 z^2 + a^3 z^3 + \dots)$$

$$a^n u(n-1) \quad (n \geq 1)$$

(6)'

$$\frac{a^{-1}z}{1 - a^{-1}z}$$

$$|z| < a$$

$$-(a^{-1}z^1 + a^{-2}z^2 + a^{-3}z^3 + \dots)$$

$$-((\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + (\frac{1}{a})^3 z^3 + \dots)$$

$$-(\frac{1}{a})^n u(n-1) \quad (n \geq 1)$$

(5)'

$$\frac{a^{-1}z^{-1}}{1 - a^{-1}z^{-1}}$$

$$|z| > a^{-1}$$

$$-(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \dots)$$

$$-a^n u(-n-1) \quad (n < 0)$$

Geometric Series - a non-unit start term

z-Transform ($n \rightarrow -n$)

(5)
$$+ \frac{a^{-1}z^{-1}}{1 - a^{-1}z^{-1}} \quad |z| > a^{-1}$$

$$(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \dots)$$

$$((\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + (\frac{1}{a})^3 z^{-3} + \dots)$$

$$a^{-n} u(-(-n)-1) \quad (n < 0)$$

$$(\frac{1}{a})^n u(n-1) \quad (n \geq 1)$$

(6)
$$+ \frac{a^{-1}z}{1 - a^{-1}z} \quad |z| < a$$

$$(a^{-1}z^1 + a^{-2}z^2 + a^{-3}z^3 + \dots)$$

$$((\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + (\frac{1}{a})^3 z^3 + \dots)$$

$$(\frac{1}{a})^{-n} u((-n)-1) \quad (n \geq 1)$$

$$a^n u(-n-1) \quad (n < 0)$$

(7)
$$- \frac{az}{1 - az} \quad |z| < a^{-1}$$

$$-(a^1 z^1 + a^2 z^2 + a^3 z^3 + \dots)$$

$$-((\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + (\frac{1}{a})^3 z^3 + \dots)$$

$$-a^{-n} u((-n)-1) \quad (n \geq 1)$$

$$-(\frac{1}{a})^n u(-n-1) \quad (n < 0)$$

(8)
$$- \frac{az^{-1}}{1 - az^{-1}} \quad |z| > a$$

$$-(a^1 z^{-1} + a^2 z^{-2} + a^3 z^{-3} + \dots)$$

$$-((\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + (\frac{1}{a})^3 z^{-3} + \dots)$$

$$-(\frac{1}{a})^{-n} u(-(-n)-1) \quad (n < 0)$$

$$-a^n u(n-1) \quad (n \geq 1)$$

(8)'
$$+ \frac{az^{-1}}{1 - az^{-1}} \quad |z| > a$$

$$(a^1 z^{-1} + a^2 z^{-2} + a^3 z^{-3} + \dots)$$

$$((\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + (\frac{1}{a})^3 z^{-3} + \dots)$$

$$(\frac{1}{a})^{-n} u(-(-n)-1) \quad (n < 0)$$

$$a^n u(n-1) \quad (n \geq 1)$$

(7)'
$$+ \frac{az}{1 - az} \quad |z| < a^{-1}$$

$$(a^1 z^1 + a^2 z^2 + a^3 z^3 + \dots)$$

$$((\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + (\frac{1}{a})^3 z^3 + \dots)$$

$$a^{-n} u((-n)-1) \quad (n \geq 1)$$

$$(\frac{1}{a})^n u(-n-1) \quad (n < 0)$$

(6)'
$$- \frac{a^{-1}z}{1 - a^{-1}z} \quad |z| < a$$

$$-(a^{-1}z^1 + a^{-2}z^2 + a^{-3}z^3 + \dots)$$

$$-((\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + (\frac{1}{a})^3 z^3 + \dots)$$

$$-(\frac{1}{a})^{-n} u((-n)-1) \quad (n \geq 1)$$

$$-a^n u(-n-1) \quad (n < 0)$$

(5)'
$$- \frac{a^{-1}z^{-1}}{1 - a^{-1}z^{-1}} \quad |z| > a^{-1}$$

$$-(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \dots)$$

$$-((\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + (\frac{1}{a})^3 z^{-3} + \dots)$$

$$-a^{-n} u(-(-n)-1) \quad (n < 0)$$

$$-(\frac{1}{a})^n u(n-1) \quad (n \geq 1)$$

Geometric Series - a non-unit start term

Laurent Series vs. z-Transform ($n \rightarrow -n$)

(5) $\boxed{+\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \quad |z| > a^{-1}}$ $\boxed{+\frac{a^{-1}z}{1-a^{-1}z} \quad |z| < a}$ (6)

$$(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \dots)$$

$$((\frac{1}{a})^{-1}z^{-1} + (\frac{1}{a})^{-2}z^{-2} + (\frac{1}{a})^{-3}z^{-3} + \dots)$$

$$(a^{-1}z^1 + a^{-2}z^2 + a^{-3}z^3 + \dots)$$

$$((\frac{1}{a})^1z^1 + (\frac{1}{a})^2z^2 + (\frac{1}{a})^3z^3 + \dots)$$

Laurent	a^n	$u(-n-1)$	$(n < 0)$
z-Trans	$(\frac{1}{a})^n$	$u(n-1)$	$(n \geq 1)$

$(\frac{1}{a})^n$	$u(n-1)$	$(n \geq 1)$
a^n	$u(-n-1)$	$(n < 0)$

(7) $\boxed{-\frac{az}{1-az} \quad |z| < a^{-1}}$ $\boxed{-\frac{az^{-1}}{1-az^{-1}} \quad |z| > a}$ (8)

$$-(a^1z^1 + a^2z^2 + a^3z^3 + \dots)$$

$$-((\frac{1}{a})^1z^1 + (\frac{1}{a})^2z^2 + (\frac{1}{a})^3z^3 + \dots)$$

$$-(a^1z^{-1} + a^2z^{-2} + a^3z^{-3} + \dots)$$

$$-((\frac{1}{a})^1z^{-1} + (\frac{1}{a})^2z^{-2} + (\frac{1}{a})^3z^{-3} + \dots)$$

Laurent	$-a^n$	$u(n-1)$	$(n \geq 1)$
z-Trans	$-(\frac{1}{a})^n$	$u(-n-1)$	$(n < 0)$

$-(\frac{1}{a})^n$	$u(-n-1)$	$(n < 0)$
$-a^n$	$u(n-1)$	$(n \geq 1)$

(8)' $\boxed{+\frac{az^{-1}}{1-az^{-1}} \quad |z| > a}$ $\boxed{+\frac{az}{1-az} \quad |z| < a^{-1}}$ (7)'

$$(a^1z^{-1} + a^2z^{-2} + a^3z^{-3} + \dots)$$

$$((\frac{1}{a})^1z^{-1} + (\frac{1}{a})^2z^{-2} + (\frac{1}{a})^3z^{-3} + \dots)$$

$$(a^1z^1 + a^2z^2 + a^3z^3 + \dots)$$

$$((\frac{1}{a})^1z^1 + (\frac{1}{a})^2z^2 + (\frac{1}{a})^3z^3 + \dots)$$

Laurent	$(\frac{1}{a})^n$	$u(-n-1)$	$(n < 0)$
z-Trans	a^n	$u(n-1)$	$(n \geq 1)$

a^n	$u(n-1)$	$(n \geq 1)$
$(\frac{1}{a})^n$	$u(-n-1)$	$(n < 0)$

(6)' $\boxed{-\frac{a^{-1}z}{1-a^{-1}z} \quad |z| < a}$ $\boxed{-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \quad |z| > a^{-1}}$ (5)'

$$-(a^{-1}z^1 + a^{-2}z^2 + a^{-3}z^3 + \dots)$$

$$-((\frac{1}{a})^1z^1 + (\frac{1}{a})^2z^2 + (\frac{1}{a})^3z^3 + \dots)$$

$$-(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \dots)$$

$$-((\frac{1}{a})^1z^{-1} + (\frac{1}{a})^2z^{-2} + (\frac{1}{a})^3z^{-3} + \dots)$$

Laurent	$-(\frac{1}{a})^n$	$u(n-1)$	$(n \geq 1)$
z-Trans	$-a^n$	$u(-n-1)$	$(n < 0)$

$-a^n$	$u(-n-1)$	$(n < 0)$
$-(\frac{1}{a})^n$	$u(n-1)$	$(n \geq 1)$

Complement ROC Pairs - Original Geometric Series Form Combinations

unit	$-\frac{1}{1-az^{-1}} \quad z < a^{-1}$	$-a^n u(n)$	$-\frac{1}{1-az^{-1}} \quad z > a$	$-(\frac{1}{a})^n u(-n)$
non-unit	$\frac{a^1 z^{-1}}{1-a^1 z^{-1}} \quad z > a^{-1}$	$a^n u(-n-1)$	$\frac{a^1 z}{1-a^1 z} \quad z < a$	$(\frac{1}{a})^n u(n-1)$

unit	$\frac{1}{1-a^1 z^{-1}} \quad z > a^{-1}$	$a^n u(-n)$	$\frac{1}{1-a^1 z} \quad z < a$	$(\frac{1}{a})^n u(n)$
non-unit	$-\frac{az}{1-az} \quad z < a^{-1}$	$-a^n u(n-1)$	$\frac{az^{-1}}{1-az^{-1}} \quad z > a$	$-(\frac{1}{a})^n u(-n-1)$

unit	$-\frac{1}{1-a^1 z} \quad z < a$	$-(\frac{1}{a})^n u(n)$	$-\frac{1}{1-a^1 z^{-1}} \quad z > a^{-1}$	$-a^n u(-n)$
non-unit	$\frac{az^{-1}}{1-az^{-1}} \quad z > a$	$(\frac{1}{a})^n u(-n-1)$	$\frac{az}{1-az} \quad z < a^{-1}$	$a^n u(n-1)$

unit	$\frac{1}{1-a^1 z^{-1}} \quad z > a$	$(\frac{1}{a})^n u(-n)$	$\frac{1}{1-az} \quad z < a^{-1}$	$a^n u(n)$
non-unit	$-\frac{a^1 z}{1-a^1 z} \quad z < a$	$-(\frac{1}{a})^n u(n-1)$	$-\frac{a^1 z^{-1}}{1-a^1 z^{-1}} \quad z > a^{-1}$	$-a^n u(-n-1)$

start term

Complement ROC Pairs - Shifted Geometric Series Form Combinations

$-\frac{a}{1-az} \quad z < a^{-1}$	$-a^{n+1} u(n)$
$\frac{z^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$a^{n+1} u(-n-1)$

$-\frac{a}{1-a^{-1}z^{-1}} \quad z > a$	$-\left(\frac{1}{a}\right)^{n-1} u(-n)$
$\frac{z}{1-a^{-1}z} \quad z < a$	$\left(\frac{1}{a}\right)^{n-1} u(n-1)$

$\frac{z^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$a^{n+1} u(-n-1)$
$-\frac{a}{1-az} \quad z < a^{-1}$	$-a^{n+1} u(n)$

$\frac{z}{1-a^{-1}z} \quad z < a$	$\left(\frac{1}{a}\right)^{n-1} u(n-1)$
$\frac{a}{1-a^{-1}z^{-1}} \quad z > a$	$-\left(\frac{1}{a}\right)^{n-1} u(-n)$

$-\frac{a^{-1}}{1-a^{-1}z} \quad z < a$	$-\left(\frac{1}{a}\right)^{n+1} u(n)$
$\frac{z^{-1}}{1-az^{-1}} \quad z > a$	$\left(\frac{1}{a}\right)^{n+1} u(-n-1)$

$-\frac{a^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$-a^{n-1} u(-n)$
$\frac{z}{1-az} \quad z < a^{-1}$	$a^{n-1} u(n-1)$

$\frac{z^{-1}}{1-az^{-1}} \quad z > a$	$\left(\frac{1}{a}\right)^{n+1} u(-n-1)$
$-\frac{a^{-1}}{1-a^{-1}z} \quad z < a$	$-\left(\frac{1}{a}\right)^{n+1} u(n)$

$\frac{z}{1-az} \quad z < a^{-1}$	$a^{n-1} u(n-1)$
$-\frac{a^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$-a^{n-1} u(-n)$

Complement ROC Pairs - Reduced Shifted Geometric Series Form Combinations

$-\frac{a}{1-az}$ $ z < a^{-1}$	$-a^{n+1} u(n)$	$-\frac{a}{1-az^{-1}}$ $ z > a$	$-\left(\frac{1}{a}\right)^{n-1} u(-n)$
$\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $ z > a^{-1}$	$a^{n+1} u(-n-1)$	$\frac{z}{1-a^{-1}z}$ $ z < a$	$\left(\frac{1}{a}\right)^{n-1} u(n-1)$

$-\frac{a^{-1}}{1-a^{-1}z}$ $ z < a$	$-\left(\frac{1}{a}\right)^{n+1} u(n)$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $ z > a^{-1}$	$-a^{n-1} u(-n)$
$\frac{z^{-1}}{1-az^{-1}}$ $ z > a$	$\left(\frac{1}{a}\right)^{n+1} u(-n-1)$	$\frac{z}{1-az}$ $ z < a^{-1}$	$a^{n-1} u(n-1)$

$$2z$$

$$2z^{-1}$$

$$2^{-1}z^{-1}$$

$$2^{-1}z$$

$$|z| < 0.5$$

$$|z| > 2$$

$$|z| > 0.5$$

$$|z| < 2$$

$$- \frac{2}{1-2z} \xleftrightarrow{z^{-1}} - \frac{2}{1-2z^{-1}}$$

$$\cdot \frac{(2z)^{-1}}{(2z)^{-1}} \cdot \frac{(2z)}{(2z)}$$

$$\cdot \frac{(2z^{-1})^{-1}}{(2z^{-1})^{-1}} \cdot \frac{(2z^{-1})}{(2z^{-1})}$$

$$+ \frac{z^{-1}}{1-0.5z^{-1}} \xleftrightarrow{z^{-1}} + \frac{z}{1-0.5z}$$

scale(a)

scale(a)

Comp.ROC

$-\frac{1}{1-az} \quad z < a^{-1}$	$-\frac{a}{1-az} \quad z < a^{-1}$	$-\frac{1}{1-a^{-1}z^{-1}} \quad z > a$	$-\frac{a}{1-a^{-1}z^{-1}} \quad z > a$
$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$\frac{a^{-1}z}{1-a^{-1}z} \quad z < a$	$\frac{z}{1-a^{-1}z} \quad z < a$

scale(1/z)

scale(z)

Comp.ROC

$\frac{1}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$\frac{1}{1-a^{-1}z} \quad z < a$	$\frac{z}{1-a^{-1}z} \quad z < a$
$-\frac{az}{1-az} \quad z < a^{-1}$	$-\frac{a}{1-az} \quad z < a^{-1}$	$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \quad z > a$	$\frac{a}{1-a^{-1}z^{-1}} \quad z > a$

scale(1/a)

scale(1/a)

Comp.ROC

$-\frac{1}{1-a^{-1}z} \quad z < a$	$-\frac{a^{-1}}{1-a^{-1}z} \quad z < a$	$-\frac{1}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$
$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \quad z > a$	$\frac{z^{-1}}{1-a^{-1}z^{-1}} \quad z > a$	$\frac{az}{1-az} \quad z < a^{-1}$	$\frac{z}{1-az} \quad z < a^{-1}$

scale(1/z)

scale(z)

Comp.ROC

$\frac{1}{1-a^{-1}z^{-1}} \quad z > a$	$\frac{z^{-1}}{1-a^{-1}z^{-1}} \quad z > a$	$\frac{1}{1-az} \quad z < a^{-1}$	$\frac{z}{1-az} \quad z < a^{-1}$
$-\frac{a^{-1}z}{1-a^{-1}z} \quad z < a$	$-\frac{a^{-1}}{1-a^{-1}z} \quad z < a$	$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$

scale(a)

scale(a)

Comp.ROC	$-\frac{1}{1-az} \quad z < a^{-1}$	$-\frac{a}{1-az} \quad z < a^{-1}$	$-\frac{1}{1-az^{-1}} \quad z > a$	$-\frac{a}{1-az^{-1}} \quad z > a$
	$\frac{a^1 z^{-1}}{1-a^1 z^{-1}} \quad z > a^{-1}$	$\frac{z^{-1}}{1-a^1 z^{-1}} \quad z > a^{-1}$	$\frac{a^1 z}{1-a^1 z} \quad z < a$	$\frac{z}{1-a^1 z} \quad z < a$

scale(1/z)

scale(z)

Comp.ROC	$\frac{1}{1-a^1 z^{-1}} \quad z > a^{-1}$	$\frac{z^{-1}}{1-a^1 z^{-1}} \quad z > a^{-1}$	$\frac{1}{1-a^1 z} \quad z < a$	$\frac{z}{1-a^1 z} \quad z < a$
	$-\frac{az}{1-az} \quad z < a^{-1}$	$-\frac{a}{1-az} \quad z < a^{-1}$	$\frac{a z^{-1}}{1-a z^{-1}} \quad z > a$	$\frac{a}{1-a z^{-1}} \quad z > a$

scale(1/a)

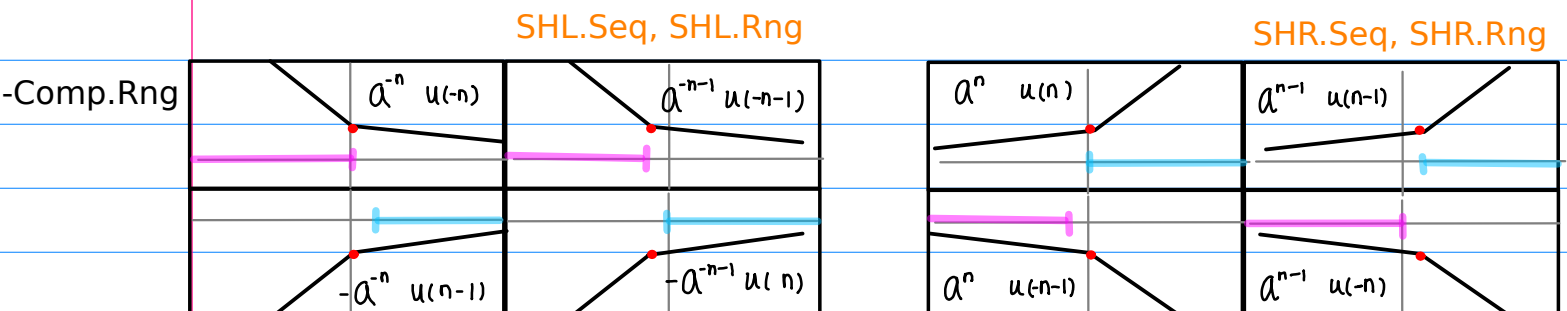
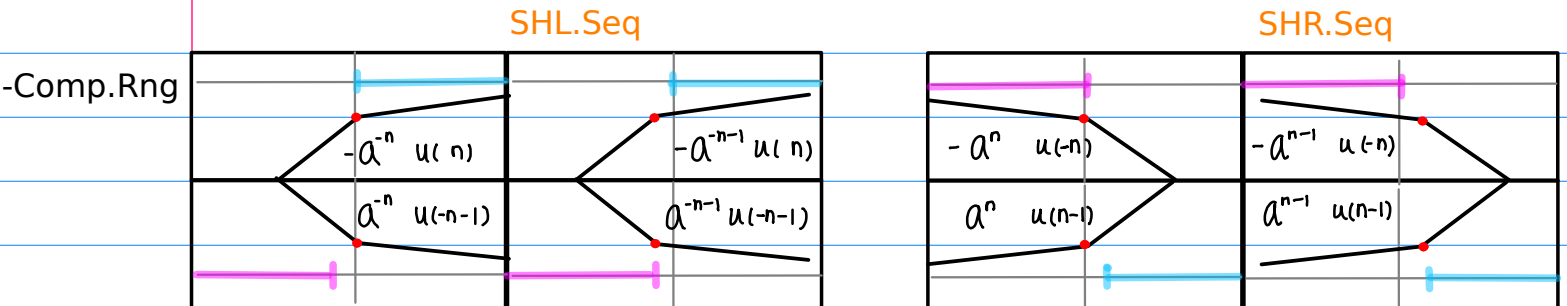
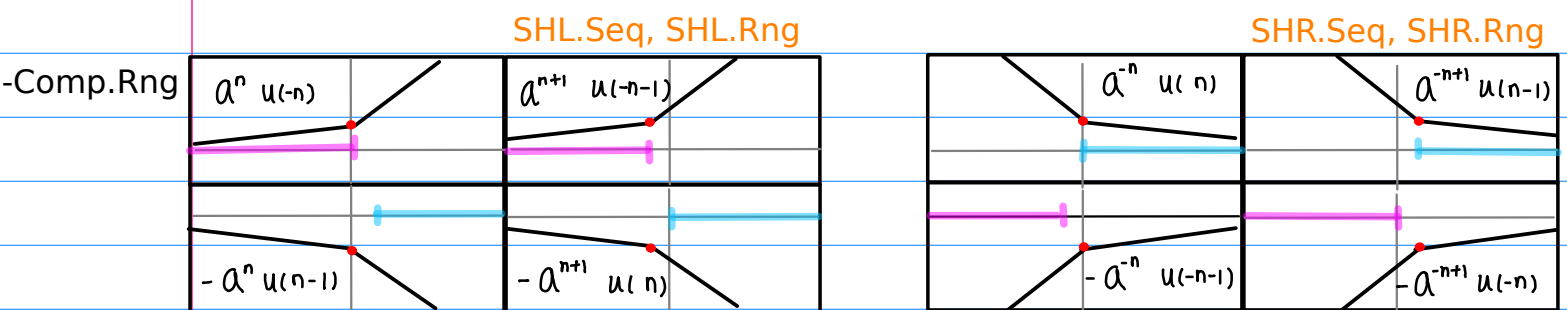
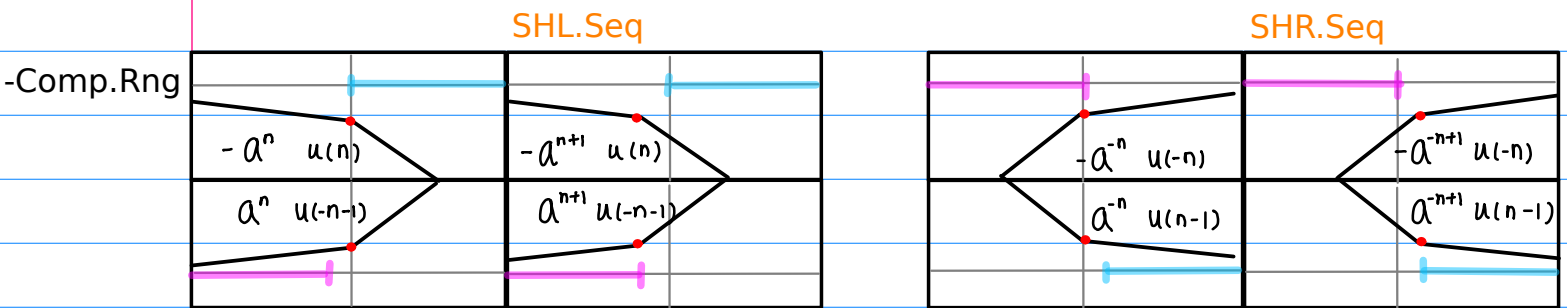
scale(1/a)

Comp.ROC	$-\frac{1}{1-a^1 z} \quad z < a$	$-\frac{a^1}{1-a^1 z} \quad z < a$	$-\frac{1}{1-a^1 z^{-1}} \quad z > a^{-1}$	$-\frac{a^1}{1-a^1 z^{-1}} \quad z > a^{-1}$
	$\frac{a z^{-1}}{1-a z^{-1}} \quad z > a$	$\frac{z^{-1}}{1-a z^{-1}} \quad z > a$	$\frac{a z}{1-az} \quad z < a^{-1}$	$\frac{z}{1-az} \quad z < a^{-1}$

scale(1/z)

scale(z)

Comp.ROC	$\frac{1}{1-a z^{-1}} \quad z > a$	$\frac{z^{-1}}{1-a z^{-1}} \quad z > a$	$\frac{1}{1-az} \quad z < a^{-1}$	$\frac{z}{1-az} \quad z < a^{-1}$
	$-\frac{a^1 z}{1-a^1 z} \quad z < a$	$-\frac{a^1}{1-a^1 z} \quad z < a$	$-\frac{a^1 z^{-1}}{1-a^1 z^{-1}} \quad z > a^{-1}$	$-\frac{a^1}{1-a^1 z^{-1}} \quad z > a^{-1}$



Left Shifted Sequence

Right Shifted Sequence

	scale(a)		scale(a)	
Comp.ROC	$-\frac{1}{1-az} \quad z < a^{-1}$	$-\frac{a}{1-az} \quad z < a^{-1}$	$-\frac{1}{1-az^{-1}} \quad z > a$	$-\frac{a}{1-az^{-1}} \quad z > a$
	$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$\frac{a^{-1}z}{1-a^{-1}z} \quad z < a$	$\frac{z}{1-a^{-1}z} \quad z < a$

	scale(1/z)		scale(z)	
Comp.ROC	$\frac{1}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$\frac{1}{1-a^{-1}z} \quad z < a$	$\frac{z}{1-a^{-1}z} \quad z < a$
	$-\frac{az}{1-az} \quad z < a^{-1}$	$-\frac{a}{1-az} \quad z < a^{-1}$	$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \quad z > a$	$\frac{a}{1-a^{-1}z^{-1}} \quad z > a$

	scale(1/a)		scale(1/a)	
Comp.ROC	$-\frac{1}{1-a^{-1}z} \quad z < a$	$-\frac{a^{-1}}{1-a^{-1}z} \quad z < a$	$-\frac{1}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$
	$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \quad z > a$	$\frac{z^{-1}}{1-a^{-1}z^{-1}} \quad z > a$	$\frac{az}{1-az} \quad z < a^{-1}$	$\frac{z}{1-az} \quad z < a^{-1}$

	scale(1/z)		scale(z)	
Comp.ROC	$\frac{1}{1-az^{-1}} \quad z > a$	$\frac{z^{-1}}{1-az^{-1}} \quad z > a$	$\frac{1}{1-az} \quad z < a^{-1}$	$\frac{z}{1-az} \quad z < a^{-1}$
	$-\frac{a^{-1}z}{1-a^{-1}z} \quad z < a$	$-\frac{a^{-1}}{1-a^{-1}z} \quad z < a$	$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$

SHL.Seq

SHR.Seq

-Comp.Rng

$-a^n u(n)$ $-(a^0, a^1, a^2, \dots)$	$-a^{n+1} u(n)$ $-(a^1, a^2, a^3, \dots)$
$a^n u(-n-1)$ $(\dots, \frac{1}{a^2}, \frac{1}{a^3}, \frac{1}{a^4})$	$a^{n+1} u(-n-1)$ $(\dots, \frac{1}{a^2}, \frac{1}{a^3}, \frac{1}{a^4})$

$-(\frac{1}{a})^n u(-n)$ $-(\dots, a^0, a^1, a^2)$	$-(\frac{1}{a})^{n-1} u(-n)$ $-(\dots, a^3, a^2, a^1)$
$(\frac{1}{a})^n u(n-1)$ $(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3}, \dots)$	$(\frac{1}{a})^{n-1} u(n-1)$ $(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2}, \dots)$

SHL.Seq, SHL.Rng

SHR.Seq, SHR.Rng

-Comp.Rng

$a^n u(-n)$ $(\dots, \frac{1}{a^2}, \frac{1}{a^3}, \frac{1}{a^4})$	$a^{n+1} u(-n-1)$ $(\dots, \frac{1}{a^2}, \frac{1}{a^3}, \frac{1}{a^4})$
$-a^n u(n-1)$ $-(a^1, a^2, a^3, \dots)$	$-a^{n+1} u(n)$ $-(a^1, a^2, a^3, \dots)$

$(\frac{1}{a})^n u(n)$ $(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2}, \dots)$	$(\frac{1}{a})^{n-1} u(n-1)$ $(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2}, \dots)$
$-(\frac{1}{a})^n u(-n-1)$ (\dots, a^3, a^2, a^1)	$-(\frac{1}{a})^{n-1} u(-n)$ (\dots, a^3, a^2, a^1)

SHL.Seq

SHR.Seq

-Comp.Rng

$-(\frac{1}{a})^n u(n)$ $-(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2}, \dots)$	$-(\frac{1}{a})^{n+1} u(n)$ $-(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3}, \dots)$
$(\frac{1}{a})^n u(-n-1)$ (\dots, a^3, a^2, a^1)	$(\frac{1}{a})^{n+1} u(-n-1)$ (\dots, a^2, a^1, a^0)

$-a^n u(-n)$ $-(\dots, \frac{1}{a^2}, \frac{1}{a^3}, \frac{1}{a^4})$	$-a^{n-1} u(-n)$ $-(\dots, \frac{1}{a^2}, \frac{1}{a^3}, \frac{1}{a^4})$
$a^n u(n-1)$ (a^1, a^2, a^3, \dots)	$a^{n-1} u(n-1)$ (a^0, a^1, a^2, \dots)

SHL.Seq, SHL.Rng

SHR.Seq, SHR.Rng

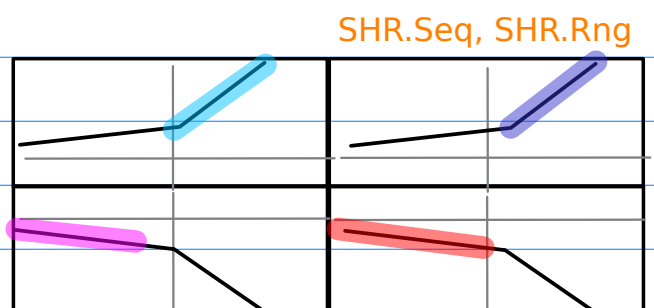
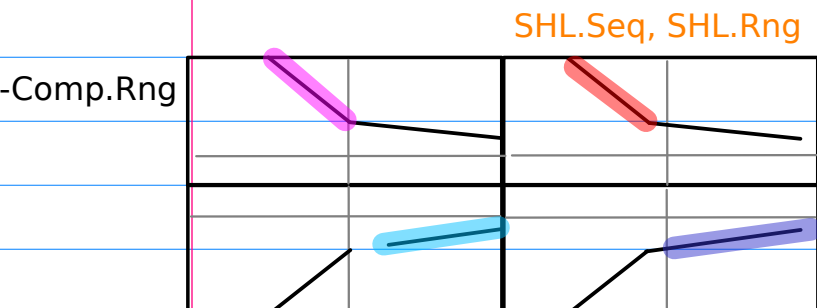
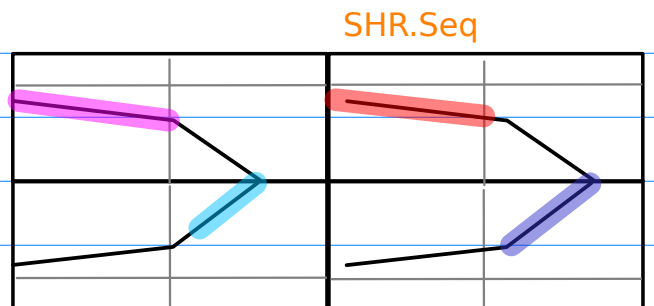
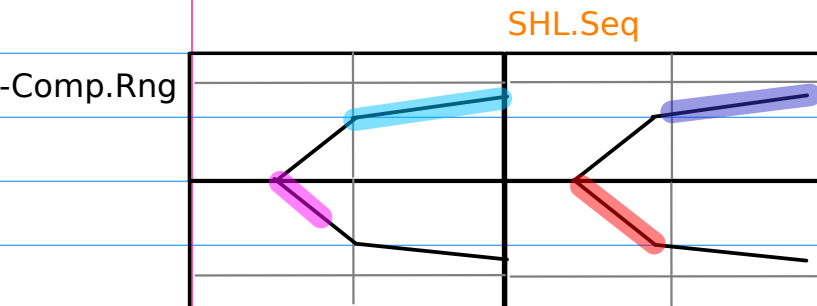
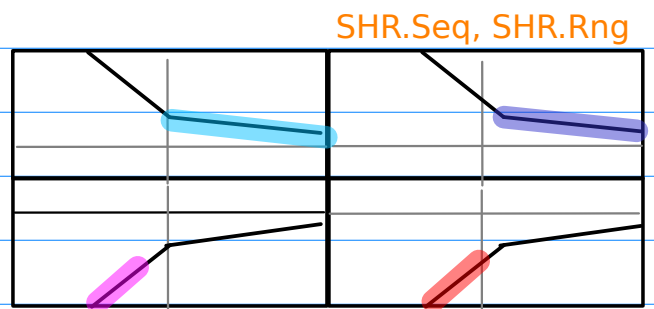
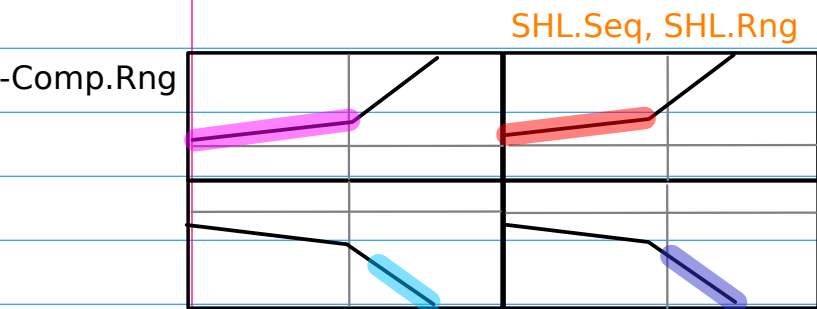
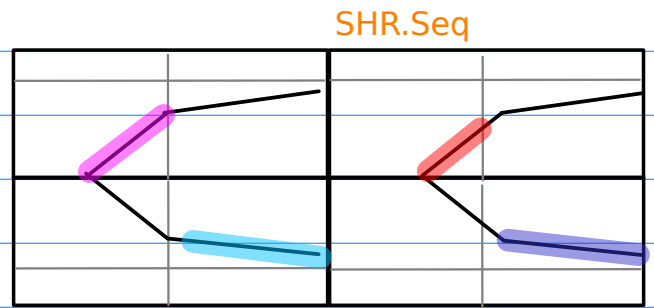
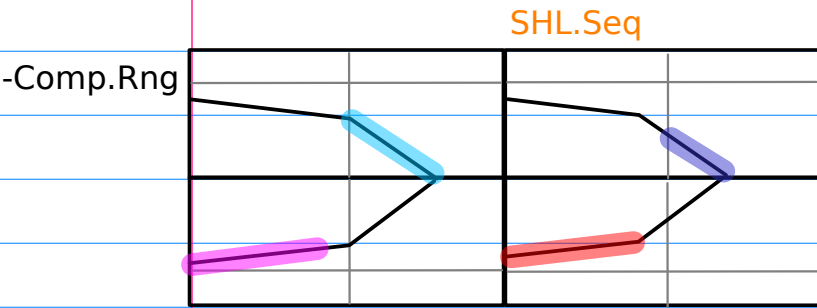
-Comp.Rng

$(\frac{1}{a})^n u(-n)$ (\dots, a^2, a^1, a^0)	$(\frac{1}{a})^{n+1} u(-n-1)$ (\dots, a^2, a^1, a^0)
$-(\frac{1}{a})^n u(n-1)$ $-(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3}, \dots)$	$-(\frac{1}{a})^{n+1} u(n)$ $-(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3}, \dots)$

$a^n u(n)$ (a^0, a^1, a^2, \dots)	$a^{n-1} u(n-1)$ (a^0, a^1, a^2, \dots)
$-a^n u(-n-1)$ $-(\dots, \frac{1}{a^2}, \frac{1}{a^3}, \frac{1}{a^4})$	$-a^{n-1} u(-n)$ $-(\dots, \frac{1}{a^2}, \frac{1}{a^3}, \frac{1}{a^4})$

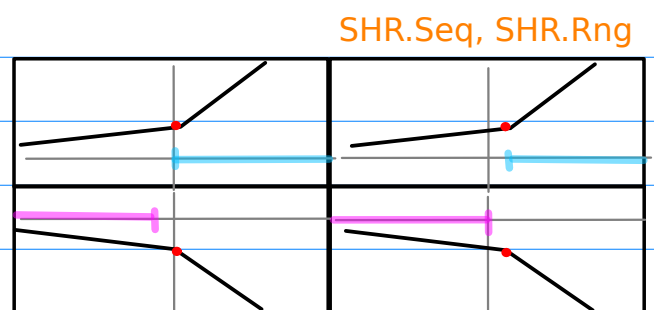
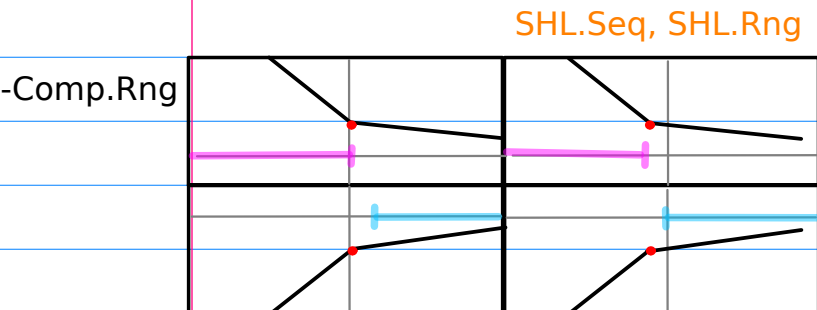
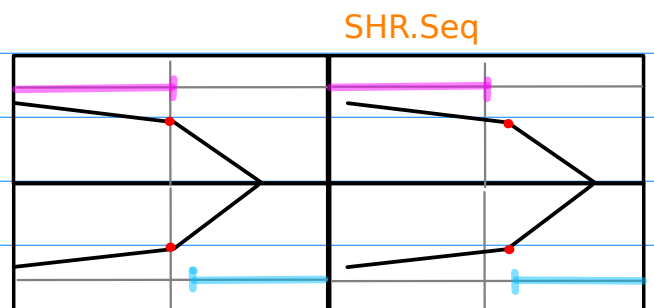
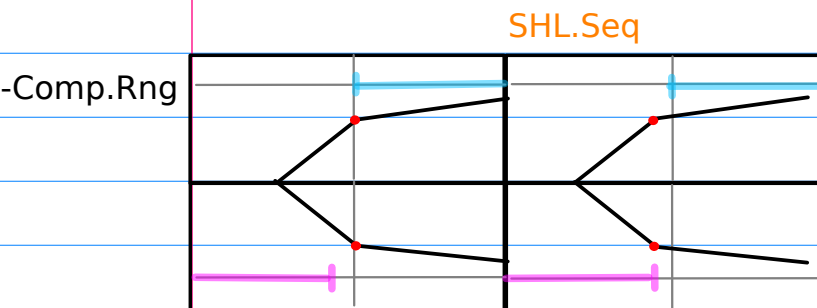
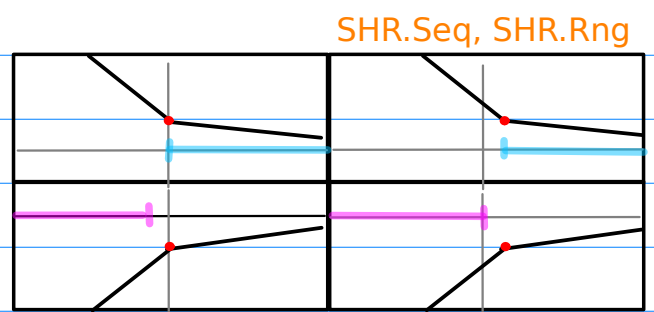
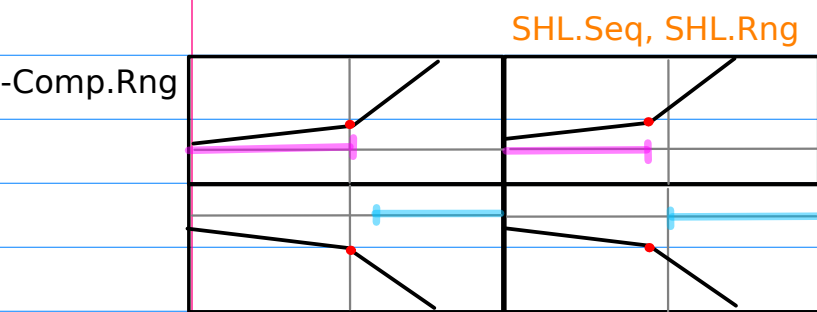
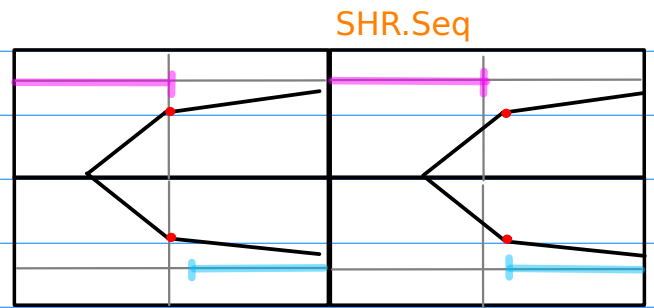
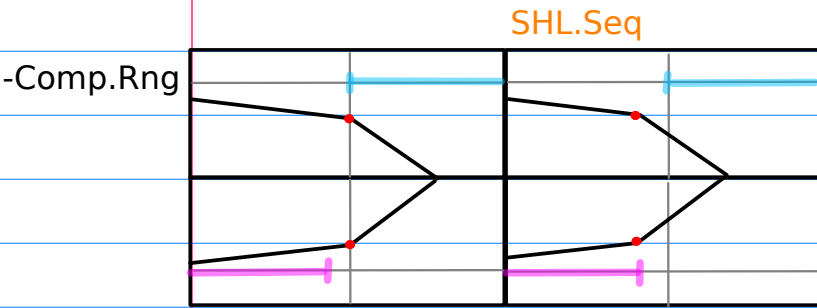
Left Shifted Sequence

Right Shifted Sequence



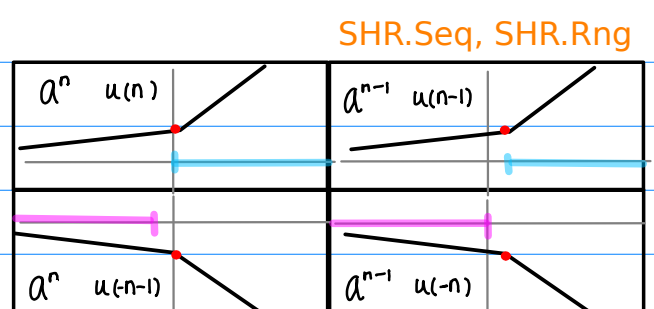
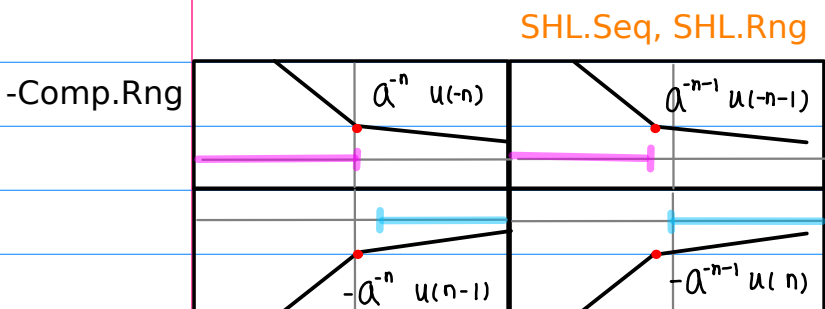
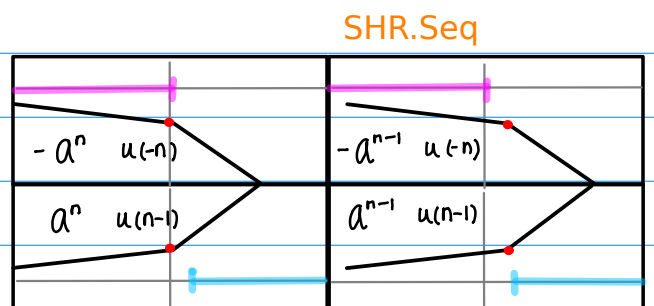
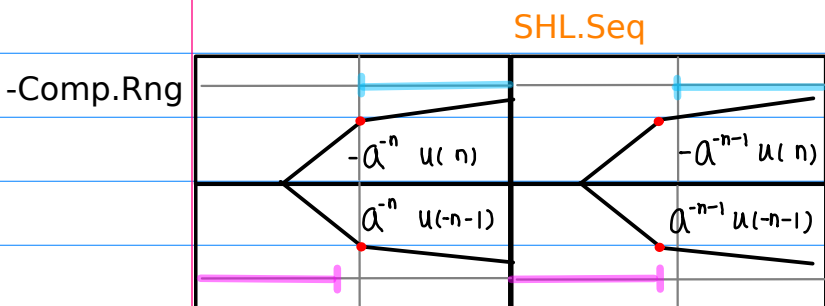
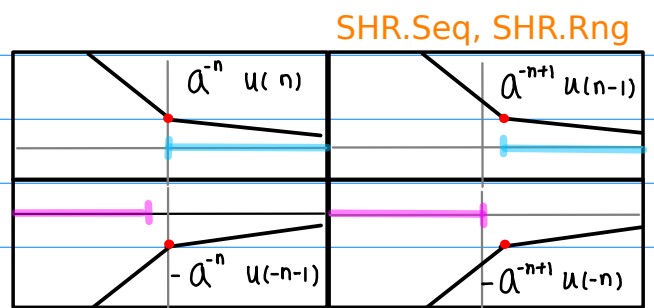
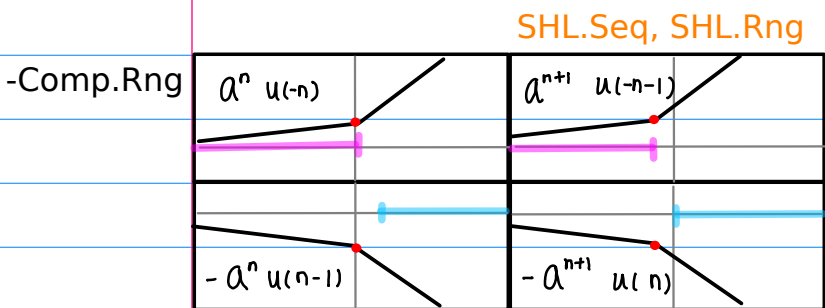
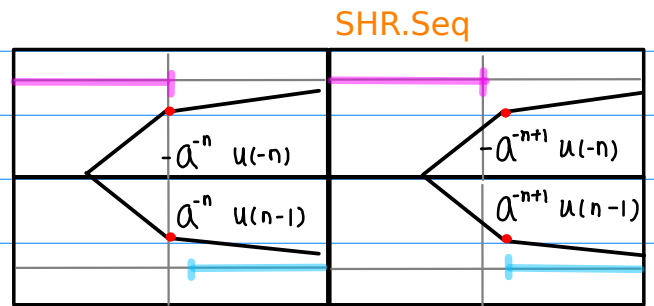
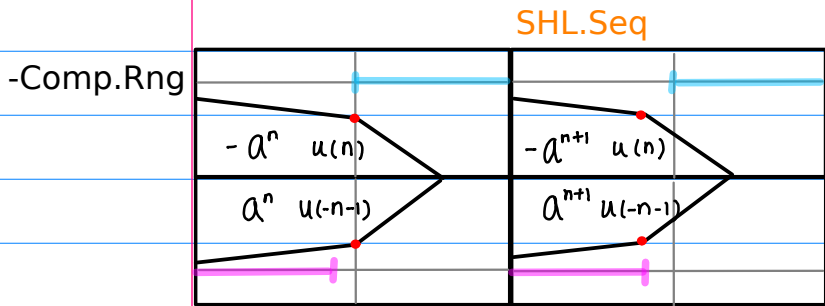
Left Shifted Sequence

Right Shifted Sequence



Left Shifted
Sequence

Right Shifted
Sequence

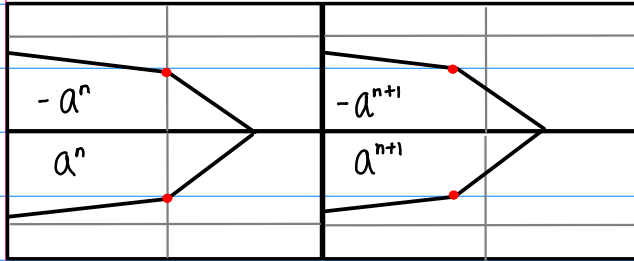


Left Shifted Sequence

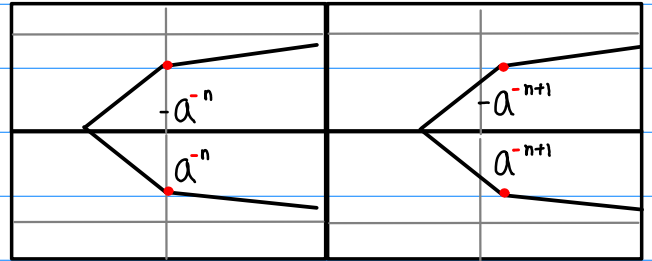
Right Shifted Sequence

a Sequence Function

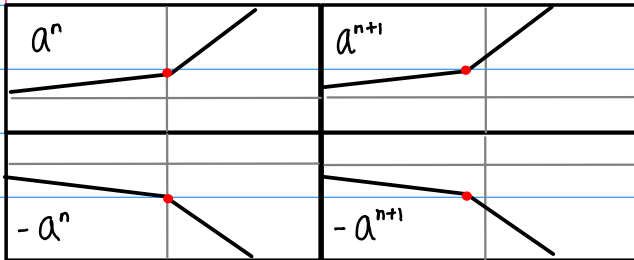
SHL.Seq



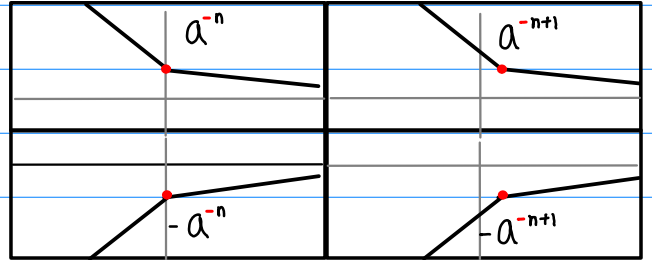
SHR.Seq



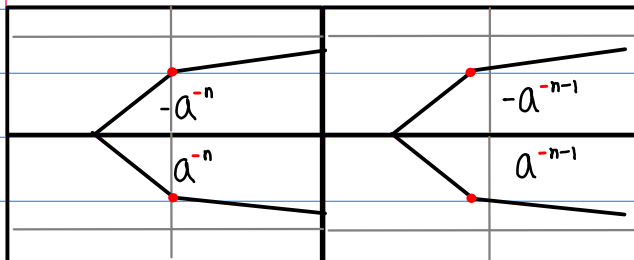
SHL.Seq, SHL.Rng



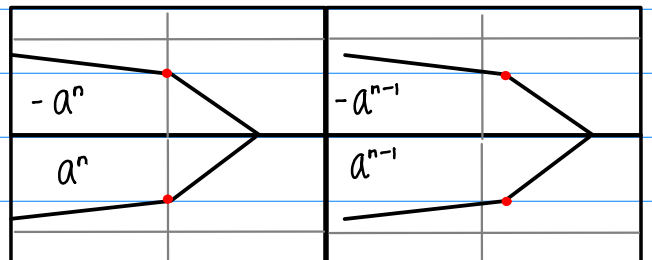
SHR.Seq, SHR.Rng



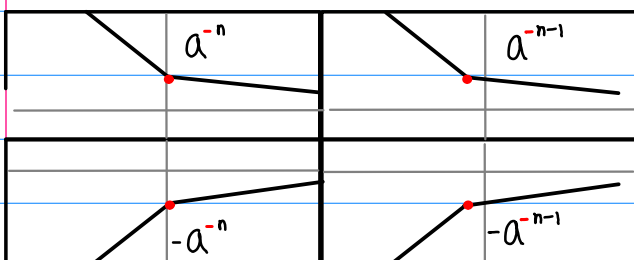
SHL.Seq



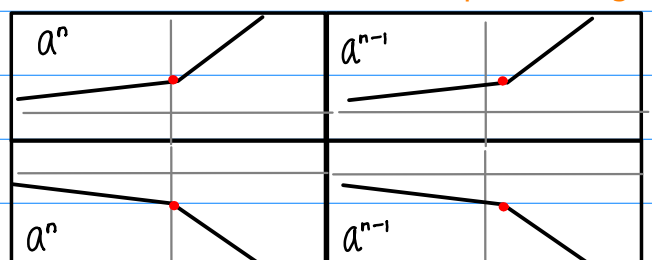
SHR.Seq



SHL.Seq, SHL.Rng



SHR.Seq, SHR.Rng



Left Shifted Sequence

Right Shifted Sequence

Range of a Sequence

